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Beyond GDP: essay of welfare convergence in Brazilian States

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Supervisor Nakabashi, Luciano

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Statement of Authorship

I hereby declare that the thesis submitted is my own work. All direct or indirect sources used are acknowledged as references. I further declare that I have not submitted this thesis at any other institution in order to obtain a degree.

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"One Ring to rule them all, One Ring to find them, One Ring to bring them all and in the darkness bind them.."

J. R. R. Tolkien

Além do PIB: ensaio sobre convergência de bem-estar dos Estados Brasileiros.

Resumo

O trabalho de Jones and Klenow (2016) cria uma medida para o bem-estar econômico usando alternativo ao PIB per capita, utilizando-se dados de consumo, lazer e expectativa de vida afim de suprir as fraquezas reconhecidas da medida do PIB per capita. Este trabalho estende esta metodologia para verificar à convergência do bem-estar dos estados brasileiros entre 2002 e 2017. Os resultados indicam um aumento real de bem-estar para quase todos os estados, sendo a expectativa de vida o fator que mais contribuiu para este crescimento. Ademais, houve convergência tanto em pib per capita quanto em bem-estar, sendo que este último teve uma taxa de crescimento de 2.98%, duas vezes maior que a do pib per capita. Estimamos através da medida de meia-vida, que seria necessário 29 anos para que os estados alcancem metade da convergência de bem-estar, enquanto que para o pib per capita, seria necessário 55 anos.

Palavra-chave: Bem-estar, Consumo, Convergência, PIB.

Beyond GDP: essay of welfare convergence in Brazilian States

Abstract

The work of Jones and Klenow (2016) creates a measure for economic well-being using an alternative to GDP per capita, using consumption, leisure and life expectancy data in order to address the recognized weaknesses of the measure of GDP per capita. This work extended this methodology to verify the convergence of the well-being of Brazilian states between 2002 and 2017. The results indicate a real increase in well-being for almost all states, with life expectancy being the factor that most contributed to this growth. Furthermore, there was convergence both in GDP per capita and in well-being, however, the latter had a growth rate of 2.98%, twice as high as that of GDP per capita. We estimate, using the half-life measure, that it would take 29 years for states to reach half of the welfare convergence, while for GDP per capita, it would take 55 years.

Keywords: Welfare, Consumption, Convergence, GDP.

List of Figures

List of Tables

Contents

1 Introduction

A person's income allows him to acquire goods and services, ultimately providing well-being. For example, Easterlin (2001) finds a positive relationship between income level and subjective well-being measures. However, despite the close relationship between living standards and income measures, as GDP per capita, the latter does not reflect essential elements affecting society's welfare [1].

Although GDP per capita has been the most used metric in the literature for measuring living standards, there is growing criticism about this statistic since it does not consider some important factors, such as population growth and the use of natural resources. Nordhaus and Tobin (1972)'s study represents a significant step since it challenges the use of GDP per capita by considering elements other than just production capacity.

To implement other elements influencing well-being besides income, Jones and Klenow (2016) combine measures of national consumption (private and public), leisure, life expectancy, and inequality to construct a theoretically appealing welfare index. They found that the proposed indicator is highly correlated with income level, but rich countries do better in terms of well-being than income would suggest.

Another relevant issue considering welfare is income convergence across regions or nations. For example, if the poorer regions of a country tend to have a higher income growth, those in more need will experience higher benefits from the country's economic growth. However, if income is not an adequate welfare measure, it would be more relevant to analyze convergence using other measures that better capture the well-being of nations or regions.

Many studies tested the convergence hypothesis across countries, but they usually find only conditional income convergence as Barro (2016),

[1] In the present study, welfare and well-being are used as synonyms

Miller and Upadhyay (2002), and Mankiw, Romer and Weil (1992). The convergence hypothesis is valid when comparing groups of countries that are more similar (Johnson; Papageorgiou, 2020).

Despite the absence of absolute income convergence across countries, some studies find converge in fundamental aspects of living standards. Soares (2007), Kenny (2005), and Neumayer (2003) find converge on life expectancy, educational measures, infrastructure measures, among other variables related to well-being across countries. Royuela and García (2015) find similar results using a regional-level dataset for Colombia.

The present study follows those mentioned above, i.e., we analyze the income and well-being convergence process, but for the Brazilian states and using an aggregate welfare function developed by Jones and Klenow (2016). We estimated the welfare of the Brazilian states for 2002, 2008, and 2017 using the consumption data from POF [2], leisure from PNAD, and life expectancy from IBGE.

The methodology of Jones and Klenow (2016) is adequate to study the convergence process since it is more related to welfare than income. Also, it is possible to verify the role of consumption, leisure, and life expectancy in the convergence process. A comprehensive understanding of how each of these factors contributes both to growth and to the convergence of well-being is essential to determining which public policies will increase the quality of life. Therefore, we can better understand the channels that explain the convergence or divergence process across the Brazilian States.

The Brazilian regions are a good test bed for the convergence process since it is a country with great disparities in income and welfare. Our database shows that in 2017, the highest income per capita region in Brazil - the Federal District - had almost 7.0 times the income of the poorest state - Roraima. Considering the well-being, the distance between the highest (the Federal District) and lowest (Maranhão) regions was 6.3 times.

The contribution of the present paper to the literature is to measure

[2] A sample survey carried out in all Brazilian states that identifies household members' consumption over time

the welfare of the Brazilian states for 2002, 2008, and 2017, analyze the evolution of welfare in this period, to check for welfare convergence, and the differences between the income and welfare convergence in the Brazilian states.

The main results indicate an increase in well-being in most Brazilian states between 2002 and 2017, except for Rio de Janeiro (RJ), Distrito Federal (DF), Amazonas(AM), and Roraima (RR). Increases in life expectancy were the most critical factor contributing to an increase in the state's well-being. Furthermore, the results indicate a convergence of states regarding GDP per capita and welfare. Compared to GDP per capita, well-being has a higher convergence rate. Half-life measures indicate that, on average, the Brazilian states reach half convergence in welfare in 29 years, while GDP per capita takes 55 years.

This study is divided into four sections in addition to this introduction. The following section brings the literature on different metrics and convergence. The third section offers an in-depth presentation of the model, economic parameters, and databases. The results for relative welfare and convergence are in the fourth and fifth sections, respectively. The concluding section discusses the paper's main contributions and limitations.

2 Literature Review

2.1 A brief review of GDP as a welfare measure

Economic and social indicators are constructed and computed to measure social phenomena. According to Herculano (2000) indicators help in the comparison, understanding, and study of phenomena of a qualitative or quantitative nature. According to SANDRONI (1999) economic indicators are obtained to provide an idea of the progress of a given field of analysis. However, due to the complexity of these phenomena, one of the challenges for economists is to have reasonable measures for them.

Following Arndt (1981), economists such as Adam Smith focused on increasing nations' wealth. Material progress was the term commonly used by economists up to World War II when referring to economic development. Thus, the term development was linked to the efficient exploitation of resources to promote economic growth.

The difficulty was and still is in measuring extra-economic factors proposed as an alternative to better capture the complexity of development. Herculano (2000) states that because these indicators are not expressed in monetary terms, they are not universally accepted. However, this problem was not able to prevent the attempts to expand the concept of development beyond the economic aspects.

Since the mid-twentieth century, researchers, governments, and multilateral agencies have invested in alternatives ways of measuring development. For example, there was a recovery of interest in the discussion of collective happiness indicators, as proposed by the indicator Gross National Happiness (GNH) in the 1970s (Dowbor (2009); Stiglitz et al. (2009)).

Rising concerns about GDP as a measure of welfare have increased, and the need to consider other measures such as well-being, environmental and social sustainability, and happiness. In general, there is a need to identify the limitations of GDP as an indicator of social progress and welfare. The following subsection discusses criticisms regarding the use of GDP as a measure of social well-being in more depth.

2.2 Problems of using GDP as measures of welfare

One of the first criticisms of using GDP as a measure of social welfare is that of Nordhaus and Tobin (1972). They stated that well-being is not just related to production but also to what people do with it. In other words, the aim of production is consumption. In addition to this criticism, the authors proposed using aggregate consumption from the National Accounts to measure well-being.

Fleurbaey (2009) carries out an extensive review on this subject, attributing the name of "GDP Correction Project". In this project, several studies aim to create indicators considering aspects such as sustainability, consumption-based measures, and equivalent income, especially the willingness to pay or receive for specific changes in the economic environment, measures of subjective well-being, in which issues such as personal satisfaction and happiness, among other factors, are evaluated, and synthetic indicators that are groupings of different dimensions, each with a specific weight.

Sustainability-based indicators gained prominence. Given their nature of including the implicit "costs"of production not accounted for in aggregate income measures. The main contribution of this research area is to provide visibility to the limits imposed by income growth, looking at the environmental impact generated by increased production over time. Some of the leading indicators are the Genuine Progress Indicator (GPI), the Genuine Savings Index (GSI), and the Ecological Footprint (EF) (Lawn, 2007).

Although the above-cited research area includes sustainability and other aspects with a significant effect on well-being, it needs an understanding of how individual choices can influence people's standards and quality of life. The economic theory provides a framework to deal with this problem considering agents' preference via utility functions. In this context, equivalence measures are used to assess in which situations an agent is indifferent to an eventual change of state. However, according to Fleurbaey (2009), this approach is open to criticism, as it leads to consider only objective economic aspects.

A complicating factor of externalities is that they arise in practically all actions of people or companies. Holcombe and Sobel (2000) examine this issue and make an analogy between the externalities of production and consumption, comparing the differences between the profit functions of companies and the utility of households. In addition, the authors assess situations in which externalities justify some public policies to mitigate their effects.

The first studies, such as Nordhaus and Tobin (1972), have tried to incorporate externality into the well-being discussion by estimating monetarily the value of the "disadvantages of living in large urban centers."In particular, the authors included negative externalities associated with urbanization in their calculation. Urbanization generates two types of externalities: i) one related to the environment, such as pollution; and ii) those related to "disamenities"resulting from population density. Nordhaus and Tobin (1972) estimated an "attractiveness premium"for living in large centers based on the difference between the salary paid between small and large cities. This premium aimed to adjust the agents' aggregate income before transforming into consumption.

However, Sen (1976) argues that income inequality plays a central role in this debate. Other studies deepen the discussion on inequality by assessing the distribution of "extended income", incorporating domestic activities not offered in the market into monetary income. This type of income presents a more accurate picture of income inequality, particularly between poorer and richer families who spend different amounts of time on domestic production. It makes the income between poorer families, who spend more time on domestic production, comparable to richer families' income.

(Krueger; Perri, 2006) and (Attanasio; Pistaferri, 2016) distinguish between income inequality and consumption inequality, stating that there is not necessarily a direct relationship between them. In the first case, income inequality refers to the distribution of current income, while consumption inequality refers to the distribution of current consumption. The effects of income inequality on social welfare depend on how households are able to smooth consumption over time. Therefore, the importance of income inequality depends partially on how households smooth income over their lifetime. Attanasio and Pistaferri (2016) also find that the effect of income inequality on social welfare depends on how people smooth consumption over time.

Jones and Klenow (2016) bring all these aspects together in an equivalent consumption measure using the representative agent approach. The starting point is an economy that is already in a steady state. Additionally, the microdata represents the entire population, serving to calculate the demographic adjustment. Distributional aspects of consumption and leisure affect utility as individuals prefer smoothing their consumption and leisure over time."

In Brazil, research on well-being is usually related to the level and distribution of income. Méndez and Waltenberg (2016) show evidence of inequality aversion in all social strata, indicating a preference to smooth consumption and leisure over time. The model also assumes that each agent chooses his or her optimal combination of consumption and leisure based on a utility function that embodies utility maximization, risk aversion, and intertemporal substitution of consumption.

Lôbo and Nakabashi (2020) follow the approach proposed by Jones and Klenow (2016) for the Brazilian states. They found a Spearman correlation of approximately 95% between GDP per capita and the measure of well-being. Therefore, in the Brazilian states, GDP per capita can serve as a useful initial indicator of welfare at the state level.

2.3 Convergence literature

Several concepts of income convergence are used, most notably sigma-convergence and beta-convergence (Holzinger; Knill, 2005). The most common version is sigma convergence, which denotes a reduction in income differences across units over time. On the other hand, beta-convergence refers to catching up or converging towards a certain level.

In welfare state research, sigma convergence is typically used to measure changes in income inequality over time via the Gini coefficient, for example. Beta-convergence is used to measure changes in economic development over time, such as changes in GDP per capita or other indicators of economic growth.

In addition to these two approaches, scholars have developed other methods for measuring welfare performance, such as the Human Development Index (HDI), which assesses economic, health, and educational outcomes across countries (Helliwell, Layard and Sachs (2017), Chaaban, Irani and Khoury (2016)). This index combines different well-being indicators and provides a holistic view of welfare. Other metrics, such as the Social Progress Index, examine a variety of social indicators to assess the overall quality of life in a given country or region (Imperative, 2015).

As neoclassical growth theory describes, beta-convergence is the phenomenon where developing countries grow faster than rich ones. In other words, the growth rate is inversely related to the initial income level (Barro; Mankiw; Martin, 1992). In the long run, this catch-up process means that countries reach a steady-state equilibrium rate. It follows that sigma-convergence entails beta-convergence, whereas beta-convergence does not necessarily leads to sigma-convergence - for example, when laggards overtake pioneers.

Starting in the late 1990s, there have been quite a few studies testing for beta-convergence in variables related to welfare (Alonso, Galindo and

Sosvilla-Rivero (1998); Alsasua, Bilbao-Ubillos and Olaskoaga (2007); Attia and Berenger (2007)). In general, there is considerable evidence for beta convergence in spending ratios, particularly regarding public social, health, and unemployment spending (Starke; Obinger; Castles, 2008). Castles (2004) uses the term 'steady-state welfare state' to describe the catch-up movement in social spending dynamics across OECD countries. However, the evidence of converging levels of social rights, namely replacement rates, is much weaker (Caminada; Goudswaard; Vliet, 2010).

3 Methodology

3.1 The main setup

Jones and Klenow (2016)'s methodology aims to measure welfare using standard economics tools. They make two crucial hypotheses. First, a single representative agent is assumed. Second, his preferences are represented by a utility function. These hypotheses are fundamental for evaluating and comparing the level of welfare across regions and time. In their model, individuals' utility comes from consumption and leisure and varies over time.

In Jones and Klenow (2016)'s model, utility assigns weights to the determinants of welfare. The utility level establishes how much agents value a given situation (levels of utility can be listed in order of preference), and the marginal utility establishes the preference between two or more goods.

There are two types of utility: intertemporal (U) and instantaneous (u). The intertemporal utility function is the sum of instantaneous utility weighted by a personal discount factor ($\beta \in (0, 1)$). The instantaneous utility is a function of the consumption (C) and leisure (ℓ) assigned to each individual in the household. To introduce uncertainty in the model, the an individual has a survival probability $S(a)$, called a lottery. The term a denotes the age reached. Hence, $S(a)$ is an additional intertemporal discount factor to β. Therefore, in addition to impatience, the representative agent assesses his chances of being alive in the future to establish his consumption trajectory.

The lottery in this model reflects the economic reality faced by the representative agent. As described by (Jones; Klenow, 2016), the agent is uncertain about their future wealth, work ethic, leisure opportunities, or even whether they will fall victim to a fatal disease before enjoying much of their life. The agent's birthplace determines their life conditions. The optimal choice to maximize utility is made by the representative agent, who subsequently experiences the conditions of each location in which they reside. The intertemporal utility function for the representative agent in locality i is represented by:

$$U_i = E \sum_{a=1}^{100} \beta^a u(C_{ai}, \ell_{ai}) S_i(a) \tag{3.1}$$

For each location (state), a representative agent is assumed to exist. As part of the model's assumptions, all agents share the same preferences. Consequently, agents in different locations possess identical intertemporal (U) and instantaneous (u) utility functions, discount factor β, and all other components of the utility functions.

Jones and Klenow (2016) employed two distinct types of instantaneous utility functions. The first, referred to as the base case, assumes additive separability between consumption and leisure. In contrast, the second, general case, does not make this assumption. The primary implication is the requirement for consumption (basket of goods) and leisure data to come from the same survey, ensuring consistency in the agent's choices regarding work and leisure time allocation.

The Appendix of this study demonstrates how, under specific conditions, the base case is a special case of the general case. Equations 3.2 and 3.3 below represent the base case and general case, respectively.

$$u(C, \ell) = \bar{u} + \log(C) - \frac{\theta\epsilon}{1+\epsilon}(1-\ell)^{\frac{1+\epsilon}{\epsilon}} \tag{3.2}$$

$$u(C, \ell) = \bar{u} + \frac{(C + \underline{c})^{1-\gamma}}{1-\gamma}\left(1 + (\gamma - 1)\frac{\theta\epsilon}{1+\epsilon}(1-\ell)^{\frac{1+\epsilon}{\epsilon}}\right)^{\gamma} - \frac{1}{1-\gamma} \tag{3.3}$$

The term $\bar{u}$ represents the intercept of the agent's lifetime utility flow. This value should be assigned as "the monetary value of the remaining life"of an average individual from the reference location (Jones; Klenow, 2016). In the (Jones; Klenow, 2016) paper, the reference was the monetary value estimate, under uncertainty of consumption and leisure, of a 40-year-old individual with residence in the USA, in 2006. The Value of a Statistical Life

(VSL) is the methodology to estimate the monetary value of the remaining life. The VSL represents people's willingness to pay for a reduction in the risk of death, according to the definitions presented by Ortiz, Markandya and Hunt (2009).

Also, in equation 3.2, the individual's choice of leisure (ℓ) establishes the labor supply that is represented by $(1-\ell)$. The parameter θ represents the weight given to labor disutility. The term ϵ represents the elasticity of labor supply, known as the Frisch elasticity, the elasticity of hours worked to salary. In the equation 3.3, γ represents the elasticity of substitution between consumption and leisure, the case where $\gamma \to 1$ represents exactly the base case.

However, for the Brazilian states, it is not possible to estimate welfare using equation 3.3, since the leisure and consumption data come from different sources and households. Therefore, we use the base case, equation 3.2, in this study.

Using this methodology, a summary statistic is created for the welfare of each Brazilian state as a consumption equivalent scale concerning the reference state. This focus on consumption-equivalent welfare follows Lucas and Lucas (1987) who calculate the welfare benefits of eliminating the business cycle versus raising the growth rate.

In equation 3.2, the index of consumption-equivalent welfare consists of introducing the term λ into the consumption of each period. This parameter is adjusted until the expected utility is the same for the representative individual of the reference state, i, and that of state j. Equation 3.4 represents the functional form of the intertemporal utility function with the introduction of λ in consumption. Equation 3.5 shows the condition under the agent is indifferent between where to live.

$$U_i(\lambda_j) = E \sum_{a=1}^{100} \beta^a u(\lambda_j C, \ell) S_j(a) \tag{3.4}$$

$$U_i(\lambda_j) = U_j(1) \tag{3.5}$$

Equation 3.5 provides an insightful way to understand the consumption-equivalent welfare measure used in the present study. It represents the utility of living in the reference state, i, adjusted by the consumption-equivalent factor λ_j, to the point where the individual is indifferent about living in the reference state or state j. This parameter captures the relative welfare difference between two states in terms of their consumption levels.

To illustrate, let us consider a scenario where the consumption and leisure in states i and j are the same, but the life expectancy of state i is higher than that of state j. In this case, the utility of living in state i is higher than living in state j, as shown in equation 3.2. However, by reducing the consumption of the representative individual living in state i through the factor λ_j, we can make him indifferent between living in state i or j. This consumption-equivalent factor λ_j represents the reduction of consumption in state i that would result in the same utility as living in state j. Thus, the condition $U_i(\lambda_j) = U_j(1)$ represents the consumption-equivalent condition in this example.

By computing the difference between the agent's utilities in state i before and after the adjustment from the factor λ_j, we obtain the utility difference that can be transformed into a consumption-equivalent measure. This specific case exemplifies how the consumption-equivalent welfare measure can capture the value that an individual places on life expectancy in consumption terms.

Given the hypothesis of additive separability and equation 3.5, we can obtain a closed-form solution for the consumption-equivalent measure. Let us consider the tuple j, a, i, where j is the individual in the sample, a is their age, and i is the location where they live. We denote the reference location as "ref".

Let ω^i_{ja} be the sample weight of individual j living in state i at age a, and N^i_a be the number of individuals aged a residing in state i. Assuming that population consumption and leisure coincide with what is observed in the sample for each age group and locality, we do not need to make any assumptions about the population distribution over these variables. The weight term ω^i_{ja} is normalized to be in the interval $[0,1]$. Therefore, we have:

$$\bar{\omega}^i_{ja} = \frac{\omega^i_{ja}}{\sum_{j=1}^{N^i_a} \omega^i_{ja}} \tag{3.6}$$

Following the reinterpretation of the "veil of ignorance" proposed by Jones and Klenow (2016), where the representative agent does not know the distributions of probabilities under the locality he is, his expected utility will be:

$$U_i = \sum_{a=1}^{100} \beta^a S^i_a \sum_{j=1}^{N^i_a} \bar{\omega}^i_{ja} u(c^i_{ja} e^{ga}, \ell^i_{ja}) \tag{3.7}$$

where S^i_a represents the survival rate of the agent until the age a in the locality i. Note that the survival rate depends only on the agent's age because his locality is already known.

Given the equation 3.5, that is $U_{ref}(\lambda_i) = U_i(1)$, we can rewrite the equation 3.2 as:

$$u_i(\lambda C, \ell) = \bar{u} + \log(\lambda C) + v(\ell) = \bar{u} + \log(\lambda) + \log(C) + v(\ell) = u(C, \ell) + \log(\lambda) \tag{3.8}$$

where $v(\ell) = -\frac{\theta}{1+\epsilon}(1-\ell)^{\frac{1+\epsilon}{\epsilon}}$

Thus, the instantaneous utility for the reference state, which is on the left side of the equivalence condition described by equation 3.5, can be obtained through the following steps:

$$\begin{aligned} u^{ref}_a(C, \ell) &= \bar{u} + \sum_{j=1}^{N^{ref}_a} \bar{\omega}^{ref}_{ja} \left[\log(c^{ref}_{ja} e^{ga} + v(\ell^{ref}_{ja}) \right] \\ &= \bar{u} + \sum_{j=1}^{N^{ref}_a} \bar{\omega}^{ref}_{ja} \left[\log(e^{ag}) + \log(c^{ref}_{ja}) + v(\ell^{ref}_{ja}) \right] \\ &= \bar{u} + ag + \sum_{j=1}^{N^{ref}_a} \bar{\omega}^{ref}_{ja} \left[\log(c^{ref}_{ja}) + v(\ell^{ref}_{ja}) \right] \end{aligned}$$

Therefore, the intertemporal utility function is given by:

$$U_{ref}(\lambda_i) = \sum_{a=1}^{100} \beta^a S_a^{ref} \big[u_a^{ref} + \log(\lambda_i)\big] \tag{3.9}$$

where

$$u_a^{ref} \equiv \bar{u} + ga + \sum_{j=1}^{N_a^{ref}} \bar{\omega}_a^{ref} \big[\log(c_{ja}^{ref}) + v(\ell_{ja}^{ref})\big] \tag{3.10}$$

In order to obtain an equivalent equation as 3.9 to another state, we take the same steps, only changing the superscript "ref"to i, disregarding the term $\log(\lambda)$. The equivalence given by equation 3.5 makes it possible to isolate the term $\log(\lambda_i)$.

$$\begin{aligned}
\sum_a \beta^a S_a^{ref}\big[u_a^{ref} + \log(\lambda_i)\big] &= \sum_a \beta^a S_a^i u_a^i && \text{(rewriting eq 3.5))}\\
\sum_a \beta^a S_a^{ref} u_a^{ref} + \sum_a \beta^a S_a^{ref} \log\lambda_i &= \sum_a \beta^a S_a^i u_a^i && \text{(rearranging } \textstyle\sum_a \beta^a S_a^{ref})\\
\sum_a \beta^a S_a^{ref} \lambda_i &= \sum_a \beta^a S_a^i u_a^i - \sum_a \beta^a S_a^{ref} u_a^i && \text{(isolating the term } \log\lambda_i)\\
\log\lambda_i &= \frac{1}{\sum_a \beta^a S_a^{ref}}\left[\sum_a \beta^a S_a^i u_a^i - \sum_a \beta^a S_a^{ref} u_a^{ref}\right] && \text{(dividing by } \textstyle\sum_a \beta^a S_a^{ref})\\
\log\lambda_i &= \frac{1}{\sum_a \beta^a S_a^{ref}}\left[\sum_a \beta^a \left(S_a^i u_a^i - S_a^{ref} u_a^{ref}\right)\right] && (\beta \text{ in evidence})\\
\log\lambda_i &= \frac{1}{\sum_a \beta^a S_a^{ref}}\left[\sum_a \beta^a \left(S_a^i u_a^i - S_a^{ref} u_a^{ref} \pm S_a^{ref} u_a^i\right)\right] && (\pm S_a^{ref} u_a^i)
\end{aligned}$$

$$\log\lambda_i = \frac{1}{\sum_a \beta^a S_a^{ref}}\left[\sum_a \beta^a \left(S_a^i u_a^i - S_a^{ref} u_a^i\right) + \left(S_a^{ref} u_a^{ref} - S_a^{ref} u_a^i\right)\right] \quad \text{(rearranging)}$$

Therefore, the log of consumption that equals the expected utility at both locations is:

$$\log\lambda_i = \frac{1}{\sum_a \beta^a S_a^{ref}} \sum_a \beta^a [(S_a^i - S_a^{ref}) u_a^i + S_a^{ref}(u_a^i - u_a^{ref})] \tag{3.11}$$

To simplify, define:

$$s_a^{ref} \equiv \frac{\beta^a S_a^{ref}}{\sum_a \beta^a S_a^{ref}} \tag{3.12}$$

$$\Delta s_a^i \equiv \frac{\beta^a (S_a^i - S_a^{ref})}{\sum_a \beta^a S_a^{ref}} \tag{3.13}$$

Following Jones and Klenow (2016), define the demographically-adjusted variables:

- demographically-adjusted average consumption

$$\bar{c}_i \equiv \sum_a s_a^{ref} \sum_{j=1}^{N_a^i} \bar{\omega}_{ja}^i c_{ja}^i e^{ga} \tag{3.14}$$

- demographically-adjusted average leisure

$$\bar{\ell}_i \equiv \sum_a s_a^{ref} \sum_{j=1}^{N_a^i} \bar{\omega}_{ja}^i \ell_{ja}^i \tag{3.15}$$

- demographically-adjusted average utility from consumption

$$E \log c_i \equiv \sum_a s_a^{ref} \sum_{j=1}^{N_a^i} \bar{\omega}_{ja}^i \log(c_{ja}^i e^{ga}) \tag{3.16}$$

- demographically-adjusted average utility from leisure

$$Ev(\ell_i) \equiv \sum_a s_a^{ref} \sum_{j=1}^{N_a^i} \bar{\omega}_{ja}^i v(\ell_{ja}^i) \tag{3.17}$$

- the ratio between income of locality i and location ref

$$\tilde{y}_i \equiv \frac{y_i}{y_{ref}}$$

From equation 3.11, because of the additive in log consumption, we can achieve an additive decomposition of welfare differences in terms of consumption equivalents (see Appendix):

$$\begin{aligned}
\log \frac{\lambda_i}{\tilde{y}_i} = {} & \sum_a \Delta s_a^i u_a^i && \textit{Life Expectancy} \\
& + \log \frac{\bar{c}_i}{c_{ref}} - \log \frac{y_i}{y_{ref}} && \textit{Consumption Share} \\
& + v(\bar{\ell}_i) - v(\bar{\ell}_{ref}) && \textit{Leisure} \\
& + E \log c_i - \log \bar{c}_i - (E \log c_{ref} - \log \bar{c}_{ref}) && \textit{Consumption Inequality} \\
& + Ev(\ell_i) - v(\bar{\ell}_i) - (Ev(\ell_{ref}) - v(\bar{\ell}_{ref})) && \textit{Leisure Inequality}
\end{aligned}$$

When considering the welfare relative to income levels, the interpretation of consumption in the decomposition changes. In this case, there are five components that contribute to welfare: i) life expectancy, ii) consumption share, iii) leisure, iv) consumption inequality, and v) leisure inequality. The welfare measure derived from life expectancy reflects the fact that increasing years of life generates a greater flow of utility and leisure over time. The consumption share indicates how much income is spent on consumption, which can raise welfare. The welfare from leisure comes from the disutility of work and is indirectly computed by the function $v(\ell)$. All terms are relative to a reference location, in this case, the state of São Paulo.

The inequalities in consumption and leisure measure the dispersion of these variables throughout the representative agent's lifetime. Unlike traditional measures of inequality that assess the distribution across individuals in all age groups, the inequality in the present study is an intertemporal measure. That is, if the representative agent has large fluctuations in the level of consumption (or leisure) throughout his life, then the intertemporal inequality of these variables will be high, with a negative effect on well-being since the representative agent has a preference for smoothing consumption and leisure over time. In the appendix of the present study, there is a simple example to demonstrate the welfare determinants under some hypotheses to simplify the algebra.

Jones and Klenow (2016) measured welfare using country-level data and measured the relative welfare of countries relative to the United States. In the present study, we analyze the relative welfare of Brazilian States and the *Distrito Federal* (DF) in relation to São Paulo (SP) state.

The methodology of Jones and Klenow (2016) allows us to estimate the average welfare growth over the years. We denote the growth of State i as g_i. Since there is no available data year-to-year, we compute the average growth between two available periods. For example, the welfare growth in State i from 2002 to 2008 was 24%, representing a yearly average growth of 4%. We can represent this using the following equation:

$$U_{i_{1+T}}(\lambda_i) = U_{i_t}(1); \tag{3.18}$$

$$g_i \equiv -\frac{1}{T}\log \lambda_i \tag{3.19}$$

The available data allows for a welfare convergence analysis. If convergence occurs, it is possible to map which components were most important for this.

The following section discusses the parameters used to compute the Brazilian states' welfare. The main variables and sources of information are presented. Finally, it discusses the period of analysis and the computational strategy adopted.

3.2 Structural parameters

The structural parameters are those determined exogenously. They define how the representative agent behaves in certain situations. For example, the personal discount factor determines how impatient the agent representative is and how much he values future consumption in present values. The parameters used for the base case are the same used in the paper of Jones and Klenow (2016).

Although it may not be plausible to use the same structural parameter values at the state level, some parameters of Jones and Klenow (2016) are in the interval estimated by the Brazilian literature as the value of life for a 40-year-old computed in Ortiz, Markandya and Hunt (2009) for the São Paulo city. The authors concluded that it is between US$ 0,77 mi and US$ 6,1 mi. The value used in the base case belongs to this interval.

Furthermore, Ortiz, Markandya and Hunt (2009) suggest a range between US$0.77 million and US$1.31 when evaluating public policies. We use the exchange rate given by the Brazilian Central Bank to compute the value of life for a 40-year-old in Brazilian currency. The table 1 is from Lôbo and Nakabashi (2020).

Term	Description	Base case	Reference
β	Personal discount rate	0.96	Junior and Bugarin (2003)
ϵ	Frisch elasticit	1.00	Moura (2015)
θ	Disutility of work	14.17	Jones and Klenow (2016)
g	Consumption growth	2.00	Table1193 IBGE (SIDRA)
VSL	Value of Life	6.00	Ortiz, Markandya and Hunt (2009)

Table 1 – Structural parameters values and reference works.

We adopted the discount factor (β) of 0.96 as in Junior and Bugarin (2003). They also suggest using a value of 0.9. (Moura, 2015) use the value of $\epsilon \approx 0.25$ to estimate Brazil's fiscal and investment multipliers from 1999 to 2003. We assume it grows at the same rate as GDP for 2003. Although evidence shows that, in general, consumption grows at a slower rate than income, this scenario was adopted as a baseline case.

Finally, consistent estimations were not found concerning the parameter θ (disutility of work). Therefore, this aspect is left to be developed in future research.

3.3 Convergence

The convergence analysis is one of the main ways of assessing the evolution of the well-being of locations over time. In this context, absolute convergence is one of the most used approaches, where the welfare growth rate depends on the initial welfare. The following equation is used to test for convergence:

$$\frac{1}{T}\ln\left[\frac{\lambda_{i,t}}{\lambda_{i,t-1}}\right] = \alpha + \beta\ln(\lambda_{i,t-1}) + \varepsilon_{i,t} \tag{3.20}$$

In the equation (3.20), the current period is represented by t. The term $\lambda_{i,t}$ denotes the welfare of state i in year t. The difference between the period t and $t-1$ is represented by T. $\ln\left[\frac{\lambda_{i,t}}{\lambda_{i,t-1}}\right]$ means the welfare growth rate; the parameters α and β denote the intercept and the speed of convergence respectively. The term ε represents the random error term.

The equation 3.20 was estimated using the ordinary least squares method. From the estimation of β, it is possible to calculate the speed of convergence, indicated by the equation 3.21, as well as the time required for a given state to travel halfway to reach its steady state, which is called the half-life. The former is found by the equation 3.22.

$$\theta = -\frac{\ln(1 + T\beta)}{T} \tag{3.21}$$

$$\eta = \frac{-\ln(2)}{\ln(1 + \beta)} \tag{3.22}$$

It is a measure of the rate at which a system changes over time. The higher the β value, the faster the system will converge to its steady state. A lower β value indicates that the system will take longer to reach its steady state. This speed of convergence is important to analyze how long, if there is convergence, it would take States to reach the same level of well-being. It is also possible to compare the convergence speed of GDP per capita in each State, then verify their relationship.

3.4 Data

We have two main categories of data. In the first one, we have individual and household data. In this category are income, leisure hours, age, and consumption. The second one includes the macroeconomic and demographic data at the state level: mortality rate; government transfers; GDP; and population. In both categories, we use the data from 2002, 2008, and 2017.

We summarized the period and source of all variables from PNAD and POF in Table 2. Work hours and consumption are from PNAD and POF, respectively. The survival rates were computed using the mortality rate from IBGE; GDP per capita is available from IBGE; government transfers and current expenditures of the states are provided by the Ministry of Finance.

The leisure hours are calculated as the difference between the total annual and total hours worked that year. The worked hours are self-declared by the individuals at PNAD and correspond to those at the primary,

Variable	Dataset	Source
Work hours	PNAD	IBGE
Consumption	POF	IBGE
Income	POF	IBGE
Age	POF	IBGE
Mortality rate	Tábua de Mortalidade	IBGE
Government transfers	Government transfers	Ministry of Finance
Government expenses	Government expenses	Ministry of Finance
GDP	Gross Domestic Product	IBGE
Population	Population	IBGE

Table 2 – Source and number of observations of used variable

secondary, and others jobs. The dataset provides weekly worked hours. Then, we compute the total hours worked that year, given 52 weeks of the year. The worked hours are normalized to a range between 0 and 1.

The income data is from POF. This data consists of all monetary income, government transfers, other resources, asset variation, and non-monetary income. Since Brazil is a developing country, non-monetary income is essential part of households' income and consumption.

The survival rate, extracted from the IBGE Mortality Table, is calculated for five-year groups: "1 to 4 years", "5 to 9 years", and so on. Furthermore, before 2010 the survey was taken every ten years. Therefore, we interpolated the data to find the survival rates for 2002 and 2008. We used the survival rate from 1980, 1990, 2000, and 2010 to estimate the survival rates for 2002 and 2008.

3.5 Consumption basket and hypothesis

The POF survey monitors approximately 59 thousand families (approximately 190 thousand individuals) representing the Brazilian population. Families participating in the POF survey report items consumed in forms categorized into different periodicities (daily, weekly, monthly, and annually).

The consumption data for households is annualized and weighted, according to the household sample, so that the total consumption of individuals accurately represents the entire population of Brazil. While the POF dataset provides detailed information about household consumption patterns, it is comparatively less detailed at the individual level. Although this fact is a limitation, only some places worldwide conduct a national survey on individual consumption. This limitation is pointed out by Jones and Klenow (2016) and used as a justification for estimating individual consumption by using two allocation methods. We will use only one of these in this study, the simple average household consumption per individual. Verifying the robustness of the results by using the second approach is an important step for future works.

The Jones and Klenow (2016) approach emphasizes consumption variables. The consumption basket is the main component of utility and, ultimately, of well-being. It is a complex variable because of the amount of information in the POF survey. Its construction depends on some assumptions that can bring limitations to the calculation of well-being. For example, how to account for expenses on high-value-added items, such as vehicles and real estate, that generate a utility flow over time? In such cases, the immediate expense does not necessarily represent the utility of the agent's consumption. In addition, it is necessary to consider the need for minimum consumption.

In addition to individual consumption, another sensitive aspect of POF is categorizing types of expenses. The categories used in the surveys of different years are distinct, so a careful analysis was necessary to maintain a comparable consumption basket across periods. However, slight differences in the consumption basket across periods cannot be ruled out, given the large number of consumption items - approximately 13 thousand. The main concern regarding possible inconsistencies in the consumption baskets is the possible reallocation of the same product in different categories or products that are no longer registered in distinct surveys. The following table presents the classification of consumption categories:

(a) Food: consumed inside and outside the home.

(b) Housing: expenses with rent and taxes, among other expenses related to housing.

(c) Clothing: acquisition of men's, women's, and children's clothing.

(d) Transport: urban and rural transport and fuel costs.

(e) Hygiene and personal care: personal beauty articles.

(f) Health care: expenses with health insurance, medication, among others.

(g) Education: expenses with books, school uniforms, and monthly fees.

(h) Recreation and leisure: subscriptions to newspapers, travel, toys, etc.

(i) Tobacco: tobacco for pipe, tobacco for cigarettes, and other articles for smokers.

(j) Other expenses: family parties, condominium, and specialized services.

(k) Other current expenses: pensions, labor expenses, social security, and taxes.

(l) Increase in assets: acquisition of real estate, construction, renovations, among others.

(m) Decrease in liabilities: payments of debts with personal and judicial loans.

(n) Sporadic trips: sporadic trips to other municipalities.

According to Jones and Klenow (2016), expenses related to vehicle acquisition (for both transport and leisure purposes) and expenses related to the acquisition, renovation, and improvement of properties were excluded from the composition of the consumption basket. Maintenance expenses were also discarded. This exclusion was justified on the basis that the utility of these goods is spread over time.

Furthermore, since many of these assets were acquired in previous periods, it is often impossible to obtain data on their previous purchases. Therefore, a methodology is needed to estimate the average acquisition value and the flow of utility of the assets, as proposed by Krueger and Perri (2006). While we have not yet followed this approach, it is a promising avenue for future research.

Fuel expenses were included in the consumption basket because they reflect the utility of transportation at the time of expenditure. Similarly, expenses for licensing, insurance, and other accessories related to the use of the vehicle were also considered.

Expenses such as pension payments, legal salary deductions, and debts determined in court were excluded from the consumption basket, as they do not generate utility. Likewise, payments made to service debts and credits contracted by the household were not considered as part of the consumption basket. Finally, financial transactions related to savings, investments in assets, and donations were also excluded.

The consumption basket includes expenses for food, clothing, housing (in the case of own property, the estimated rent was used), public utilities (such as electricity, water, and sewage), communication services, medical services, transport, education, and cultural expenses. However, expenses related to the acquisition of durable goods, renovation, repairs in general, maintenance, and transfers to family members and philanthropic entities were excluded. Our approach closely follows the consumption basket proposed by Jones and Klenow (2016).

4 Results

4.1 The components of welfare measure

The first section of the results presents a visual analysis to explore the relationship between components of social well-being and GDP per capita. In this initial analysis, the Federal District (DF) was excluded as an outlier, and GDP per capita was measured relative to São Paulo State (SP) as the reference state. The graphs will be displayed at the end of this section.

Life expectancy at birth is a vital indicator of social well-being, and in Brazil, it increased notably from 71.0 years in 2002 to 74.6 years in 2017. Figure 1 presents two graphs illustrating the relationship between life expectancy and GDP per capita for each state in 2002 (left) and 2017 (right). The 2002 graph shows a positive but weak correlation, with some exceptions like Santa Catarina (SC), which had a higher life expectancy than Rio de Janeiro (RJ), even though RJ had a higher GDP per capita. The 2017 graph reveals a noticeable increase in life expectancy across all states, with Maranhão (MA), the state with the lowest life expectancy in both periods, experiencing an increase of about six years.

Another essential aspect of welfare is consumption as a share of income (CSI), which reflects the average proportion of income households consume. A higher CSI value implies that families spend a larger percentage of their income. However, the left graph of Figure 2, showing data for CSI and GDP per capita in 2002, does not indicate a robust correlation between these variables. The right graph of Figure 2 shows a slightly negative relationship between CSI and GDP per capita in 2017, suggesting that states with lower GDP per capita spend a higher proportion of their income on consumption. However, the relationship is not strong enough to confirm that it is negative. Notably, the GDP per capita is presented in absolute values, not relative to any reference state. Examining the data from an intertemporal perspective, most states had a CSI of less than 50% in 2017, as opposed to 2002. This trend could be due to the average growth of GDP per capita, enabling households to maintain their consumption levels while utilizing a smaller portion of their disposable income.

According to the theoretical model by Jones and Klenow (2016), leisure is a crucial factor affecting well-being. Leisure is calculated as the number of non-working hours in a day, allowing us to examine the relationship between GDP per capita and total working hours. Figure 3 displays graphs showing the average weekly hours worked per capita for each state, along with GDP per capita in 2002 and 2017, for individuals over 18 years old.

According to the theoretical model by Jones and Klenow (2016), leisure is a crucial factor affecting well-being. Leisure is calculated as the number of non-working hours in a day, allowing us to examine the relationship between GDP per capita and total working hours. Figure 3 displays graphs showing the average weekly hours worked per capita for each state, along with GDP per capita in 2002 and 2017, for individuals over 18 years old.

Figure 3 shows the average weekly hours worked per capita in each state for 2002 and 2017, plotted against GDP per capita. The vertical axis ranges from 15 to 44 hours per week, in accordance with the limit defined by the Brazilian constitution for working hours.

In 2002, the relationship between GDP per capita and the average number of total hours worked was weakly positive, with some deviations, such as Rio de Janeiro (RJ), which had a lower average weekly workload of 24 hours, despite having a GDP per capita that was 94 percent of the reference state. In contrast, states like Santa Catarina (SC), Rio Grande do Sul (RS), and Paraná (PR) had a significantly higher average workload than RJ, even with a lower relative GDP per capita.

In 2017, the positive relationship between working hours and GDP per capita becomes stronger. However, Rio de Janeiro (RJ) remains an outlier because the number of hours worked for the states has decreased, and their relative GDP per capita has increased.

Overall, we can observe a certain degree of correlation between the social well-being components and GDP per capita across Brazilian states, as suggested by the analysis of the previous section. However, it is important to note that a comprehensive assessment of welfare requires a more integrated approach that accounts for the interdependence and trade-offs among different dimensions of well-being. Therefore, in the following section, we present a more holistic measure of well-being that incorporates multiple factors and their interactions over time. Specifically, we examine how the well-being metric proposed by Jones and Klenow (2016) has evolved across states and years, and how each of the contributing factors has influenced this trend. By doing so, we aim to provide a more nuanced and accurate picture of the welfare dynamics in Brazil and to inform policy decisions that promote sustainable and inclusive development.

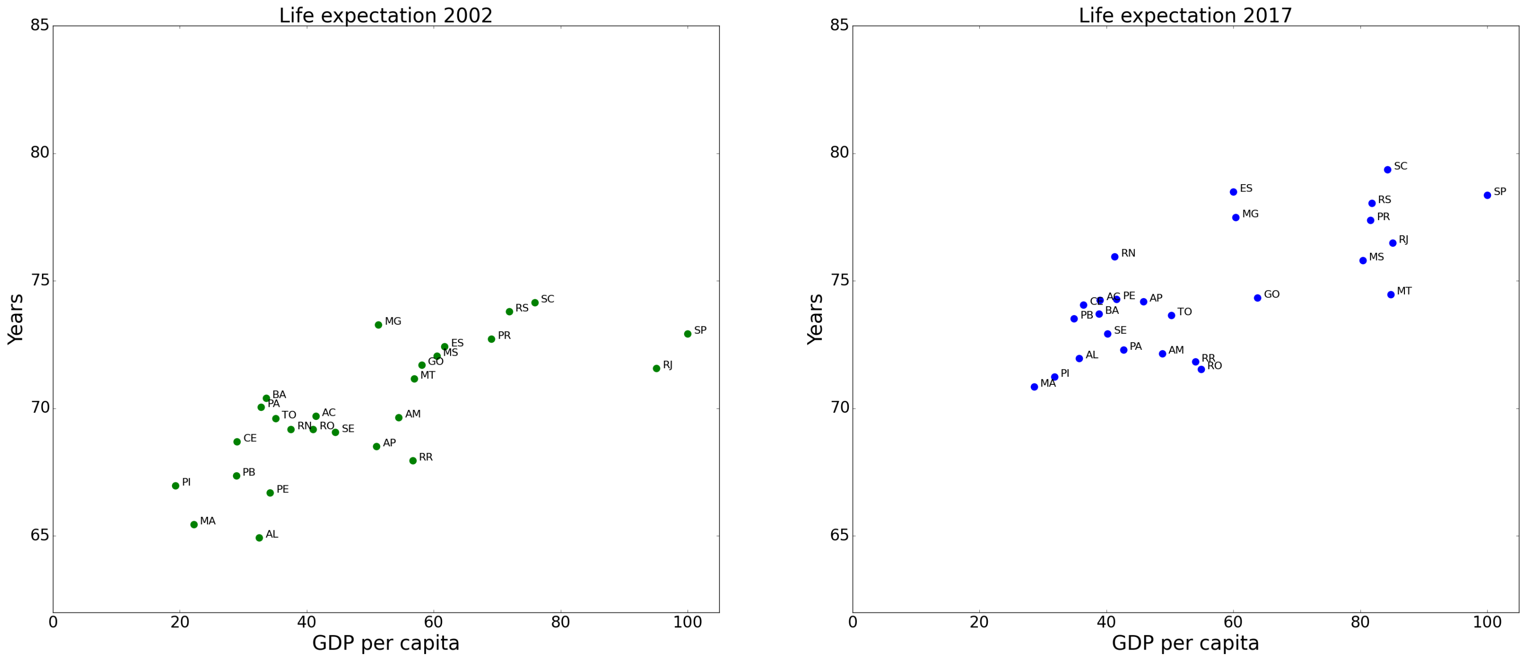

Figure 1 – life expectancy and their relationship with GDP per capita

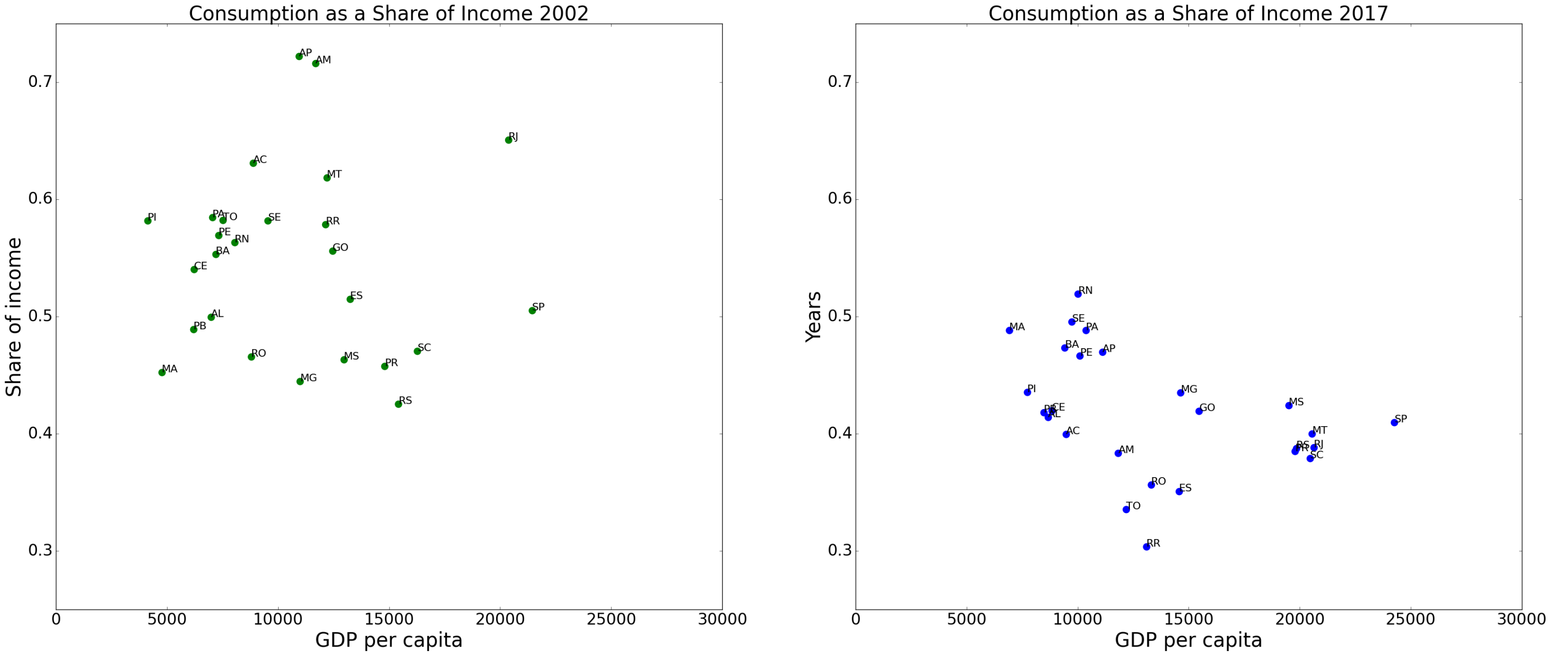

Figure 2 – Consumption as Share of income and their relationship with GDP per capita

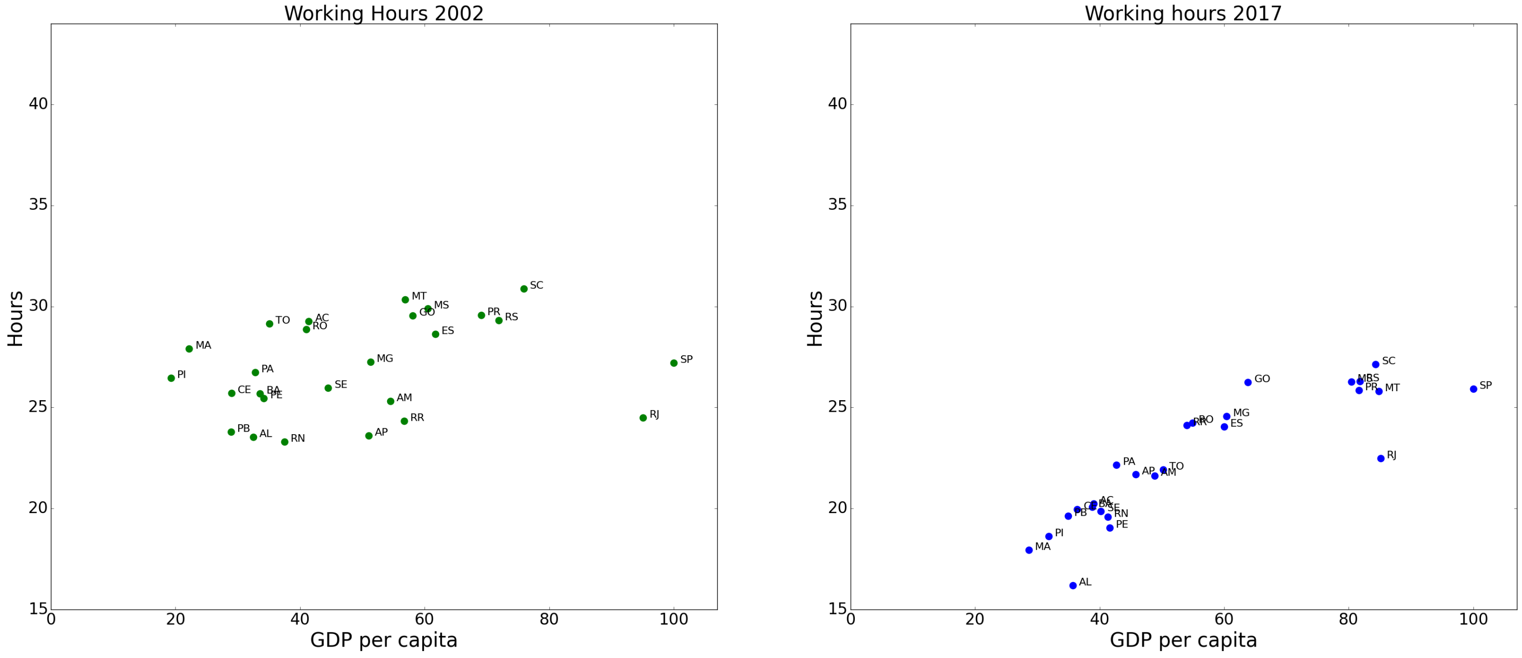

Figure 3 – Working hours and their relationship with GDP per capita

4.2 Brazilians states welfare

The previous section analyzed three components of well-being: life expectancy, consumption share of income, and leisure hours (work disutility). While these components provide valuable insights into a state's welfare, a more comprehensive measurement of well-being requires understanding how each factor influences individuals' overall well-being. In this section, we will delve deeper into these factors and assess their contributions to the overall well-being measure over time.

This section aims to offer a more accurate measurement of Brazilian states' well-being by estimating their relative welfare. By doing so, we can better understand the factors that most significantly impacted each state's welfare. It is also crucial to analyze the relationship between this well-being measure and GDP per capita to determine whether the latter is an appropriate metric for evaluating states' welfare.

Figure 4 displays the relationship between average well-being and GDP per capita for 2002. The dashed line represents the 45°line, which indicates perfect equality between GDP per capita and well-being. States falling below this line imply that GDP per capita overestimates social welfare, while those above suggest that income underestimates well-being. In some cases, this discrepancy is more pronounced. For example, Santa Catarina (SC) has a relative welfare level of 90 points, while its relative GDP per capita is below 80%. Conversely, Alagoas (AL) has a well-being score of approximately 15 welfare relative points, while its relative GDP per capita is around 32%.

Figure 5 presents the same relationship for 2017. Several states are above the 45°line, meaning that the relative GDP per capita measure overestimates the well-being of these states compared to São Paulo state. For instance, Roraima (RR) has a relative well-being level of approximately 15 points, while its relative GDP per capita is close to 55%. Brazilian states have made more progress in catching up with the reference state regarding income than welfare. To better comprehend this dynamic, it is necessary to

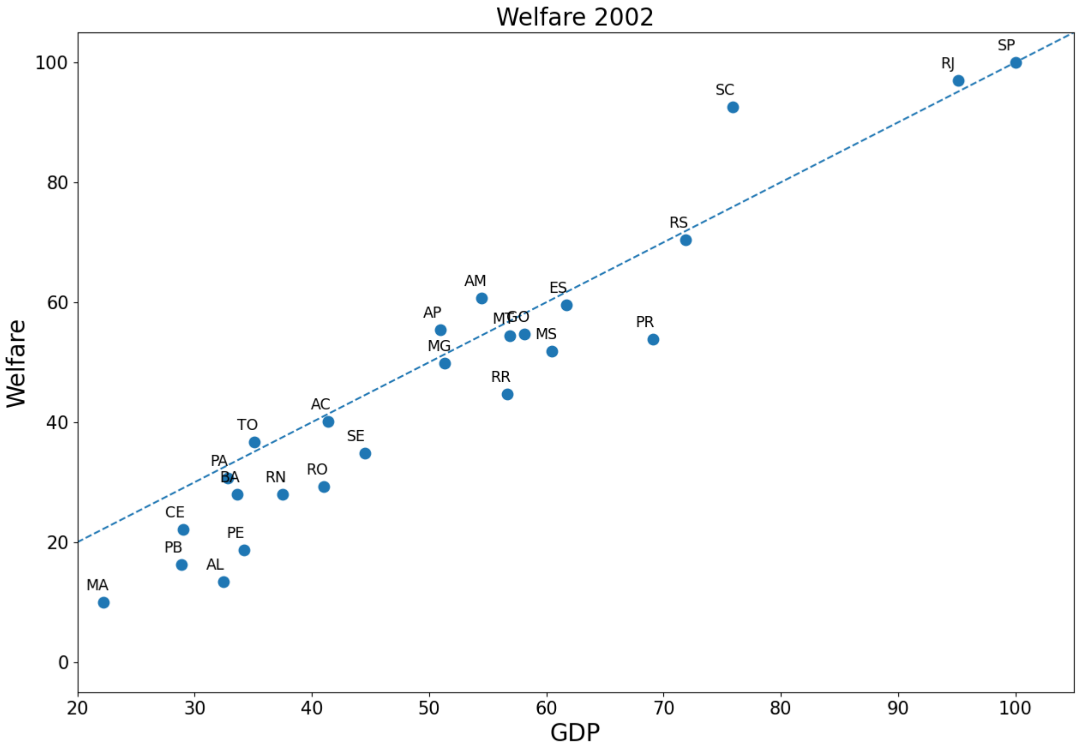

Figure 4 – Relation between Welfare and GDP per capita - 2002

examine which factors most impact their relative well-being.

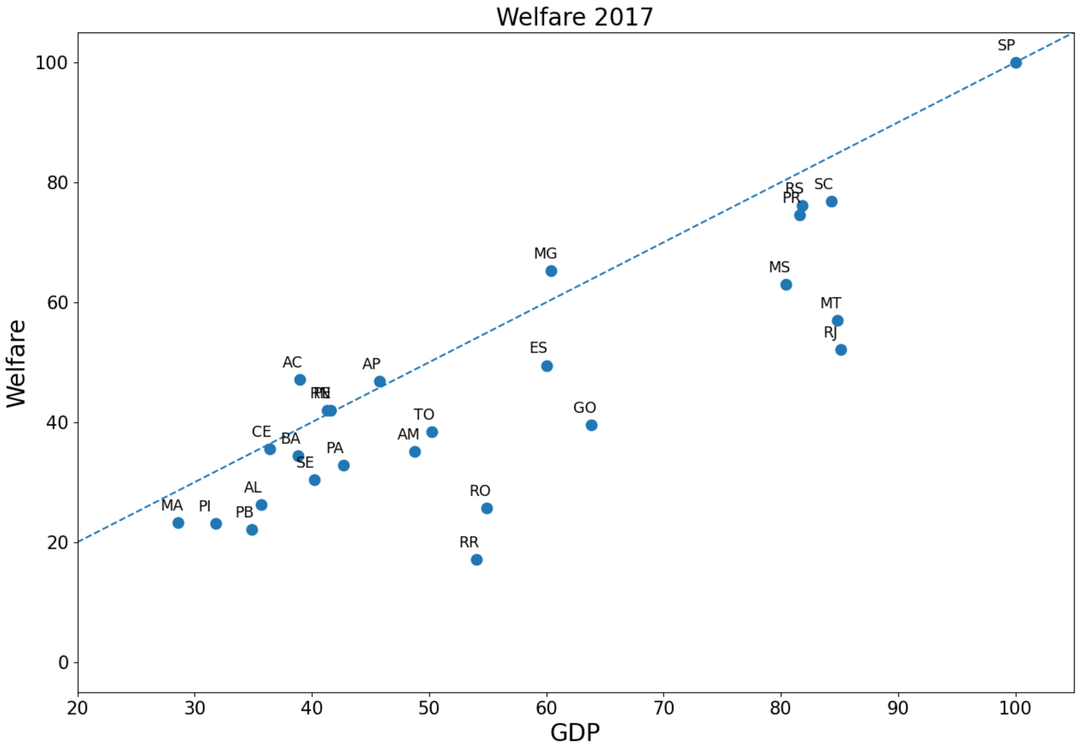

Figure 5 – Relation between Welfare and GDP per capita - 2017

The table below compiles the results obtained for the λ_{level} alongside each of the well-being determinants in 2017, with the tables referring to 2002 and 2008 in the appendix section. The states (including the Federal District) are ordered from the highest to the lowest welfare. The sum of the determinants equals the change expressed by $\log(\frac{\lambda}{\bar{y}})$.

The Federal District (DF) experienced a significant decline of 44% in relative well-being compared to GDP per capita, primarily due to consumption inequality, which affected the indicator by approximately 31% - the highest among all states. This finding contrasts with the fact that the DF's economy depends predominantly on the public sector. The functional stability of this sector guarantees an almost certain income flow over time, but this guarantee has not been reflected in the temporal flow of consumption.

The other states in the Midwest region, including Mato Grosso (MT), Mato Grosso do Sul (MS), and Goiás (GO), exhibit lower welfare levels compared to the Federal District. Specifically, Goiás (GO) has a relative

	λ	$\tilde{y}$	$\log \frac{\lambda}{\tilde{y}}$	Life Exp.	$\frac{C}{Y}$	ℓ	C ineq.	ℓ ineq.
DF	115.8	179.0	-0.44	-0.00	-0.18	0.02	-0.31	0.02
SP	100.0	100.0	0.00	0.00	0.00	0.00	0.00	0.00
SC	76.8	84.3	-0.09	0.09	-0.16	-0.02	-0.01	-0.00
RS	76.1	81.8	-0.07	-0.03	-0.07	-0.02	0.05	-0.01
PR	74.6	81.6	-0.09	-0.09	-0.08	-0.00	0.08	0.00
MG	65.3	60.4	0.08	-0.09	0.07	0.01	0.08	0.00
MS	63.0	80.4	-0.24	-0.25	-0.02	-0.00	0.03	0.00
MT	57.0	84.8	-0.40	-0.38	-0.04	0.01	-0.00	0.01
RJ	52.1	85.1	-0.49	-0.18	-0.10	0.03	-0.27	0.02
ES	49.5	60.0	-0.19	-0.00	-0.18	0.02	-0.06	0.02
AC	47.2	39.0	0.19	-0.39	0.24	0.06	0.22	0.05
AP	46.8	45.8	0.02	-0.41	0.19	0.05	0.15	0.04
RN	42.0	41.3	0.02	-0.22	0.20	0.07	-0.07	0.04
PE	42.0	41.6	0.01	-0.37	0.19	0.07	0.07	0.05
GO	39.6	63.8	-0.48	-0.37	-0.09	-0.00	-0.02	-0.00
TO	38.4	50.2	-0.27	-0.44	-0.08	0.05	0.16	0.04
CE	35.6	36.4	-0.02	-0.39	0.15	0.06	0.11	0.04
AM	35.1	48.8	-0.33	-0.58	0.11	0.05	0.04	0.05
BA	34.5	38.8	-0.12	-0.44	0.20	0.06	0.01	0.04
PA	32.9	42.7	-0.26	-0.56	0.22	0.04	0.00	0.03
SE	30.4	40.2	-0.28	-0.49	0.16	0.07	-0.06	0.04
AL	26.3	35.7	-0.30	-0.59	0.09	0.10	0.01	0.08
RO	25.7	54.9	-0.76	-0.61	-0.19	0.03	-0.01	0.02
MA	23.3	28.6	-0.20	-0.69	0.26	0.08	0.07	0.07
PI	23.1	31.8	-0.32	-0.63	0.09	0.08	0.08	0.06
PB	22.2	34.9	-0.45	-0.42	-0.01	0.07	-0.13	0.04
RR	17.1	54.0	-1.15	-0.55	-0.36	0.03	-0.29	0.02
Weighted Aver.	61.08	69.91	-0.17	-0.23	0.019	0.02	-0.001	0.02

Table 3 – Welfare measure decomposition of 2017

welfare of 39.6 points, while MS and MT have welfare levels around 60 points. The primary contributing factor to their low well-being is their relatively low life expectancy.

Most of the northeastern states, such as Maranhão (MA), Piauí (PI), Ceará (CE), Rio Grande do Norte (RN), Paraíba (PB), Pernambuco (PE), Alagoas (AL), and Sergipe (SE), display relative welfare levels below 50 points. The most significant factor accounting for this observation is life expectancy, with AL's life expectancy representing a loss of approximately 59% of well-being. However, these states benefit from a considerable share of consumption in income and experience minor variations in consumption, which can be attributed to government income transfer programs. These programs include monetary initiatives like Auxílio Brasil and non-monetary forms such as gas vouchers and distribution of essential food baskets. Studies conducted by (Sperandio et al., 2017) and (Lignani et al., 2011) investigated the impact of these programs on food consumption for families in the northeast region, revealing a significant and positive effect on increasing consumption.

To enhance the welfare of the North states of Brazil, it is essential to implement effective public policies focused on improving basic health infrastructure and sanitation. Like the Northeast states, the North states primarily have low welfare due to low life expectancy. Measures such as increased investment in healthcare facilities, trained medical personnel, and access to clean water and sanitation can significantly improve the well-being of the population in these regions.

The Southern states of Brazil, including Rio Grande do Sul (RS), Santa Catarina (SC), and Paraná (PR), have welfare levels closest to São Paulo (SP). This is primarily due to their lower inequality in intertemporal consumption compared to São Paulo, with the exception of SC, which has high welfare because of its high life expectancy. The consumption share in these states is lower than the reference state, which accounts for their lower welfare.

Rio de Janeiro (RJ) is a unique case in Southeast Brazil. Although it has the second-highest relative GDP per capita in Brazil, behind only the Federal District (DF), its welfare is close to 50 points. Consumption inequality is the main factor explaining this outcome. In Minas Gerais (MG), the consumption share and variation make its welfare closer to the reference state than GDP per capita. In Espirito Santo (ES), the lower relative well-being can be attributed to the lower share of consumption compared to the reference state.

4.3 Welfare Growth

In previous sections, we presented a welfare measure in relation to a reference state. Consequently, a state with decreasing welfare from one period to another is not necessarily worse off.

Figure 6 shows the relationship between relative growth of welfare—the difference in λ between 2002 and 2017—and GDP per capita for each state, excluding the DF. We observe that higher growth in GDP per capita generally corresponds to higher well-being growth, but the relationship is not perfect. States above the 45°line, such as PR and MG, experienced higher well-being growth than GDP per capita growth relative to the reference state. Conversely, most states, like RJ, experienced lower welfare growth than anticipated by income expansion. While RJ had a 10% relative decrease in income, its reduction in relative welfare was 45 points.

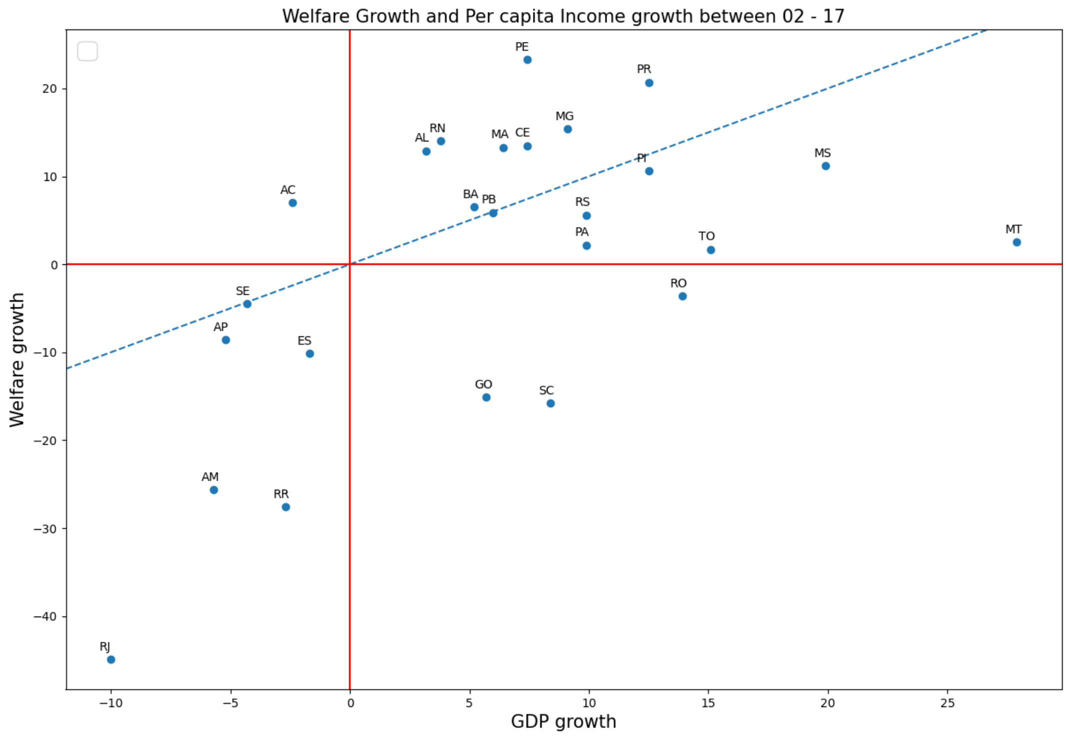

Figure 6 – Relation between 2002 - 2017 of Welfare growth and GDP per capita growth

Table 4 displays the relative welfare levels of each state over the

	λ_{2002}	λ_{2008}	λ_{2017}
DF	234.2	146.9	115.8
SP	100.0	100.0	100.0
RJ	97.0	80.5	52.1
SC	92.6	79.4	76.8
RS	70.5	74.2	76.1
AM	60.7	53.1	35.1
ES	59.6	78.0	49.5
AP	55.4	51.1	46.8
GO	54.7	45.8	39.6
MT	54.5	55.0	57.0
PR	53.9	66.5	74.6
MS	51.8	53.9	63.0
MG	49.9	55.5	65.3
RR	44.7	45.5	17.1
AC	40.2	48.2	47.2
TO	36.7	42.6	38.4
SE	34.9	40.8	30.4
PA	30.7	32.8	32.9
RO	29.3	36.9	25.7
RN	28.0	40.2	42.0
BA	28.0	31.4	34.5
CE	22.1	28.4	35.6
PE	18.7	23.7	42.0
PB	16.3	25.0	22.2
AL	13.4	20.3	26.3
PI	12.5	17.7	23.1
MA	10.0	18.2	23.3

Table 4 – Welfare measure over years

years, ordered in descending order based on the values of λ_{2002}. The Federal District (DF) exemplifies how relative welfare has changed over time. In 2002, DF's welfare was more than double that of the reference state, but it decreased to 115.8 in 2017. Similarly, RJ's welfare was nearly the same as the reference state in 2002, with $\lambda_{2002} = 97$, but dropped to 52.1 in 2017.

Northeast states experienced the highest growth in relative welfare. From 2002 to 2017, the region's well-being grew by an average of 11.81 points. PE stands out with a welfare growth of 23.3 points. Except for Sergipe (SE), which experienced a decrease of 4.5 points, other states showed improvements in relative welfare. The states with the worst rankings in 2002 had the fastest growth.

However, these variations reflect relative growth to the reference

	λ	$\tilde{y}$	$\log\frac{\lambda}{\tilde{y}}$	Life Exp.	$\frac{C}{Y}$	ℓ	C ineq.	ℓ ineq.
DF	195.3	202.6	-0.04	0.68	-0.43	0.03	-0.38	0.06
SP_{2017}	157.0	113.1	0.33	0.68	-0.31	0.01	-0.08	0.04
SC	128.9	95.4	0.30	0.78	-0.46	-0.01	-0.05	0.04
RS	118.6	92.6	0.25	0.62	-0.36	-0.01	-0.04	0.03
PR	112.5	92.3	0.20	0.54	-0.38	0.01	-0.01	0.04
MG	98.8	68.3	0.37	0.55	-0.25	0.02	0.01	0.04
MS	95.5	91.0	0.05	0.33	-0.29	0.01	-0.04	0.04
MT	81.6	95.9	-0.16	0.18	-0.33	0.02	-0.08	0.05
RJ	80.6	96.2	-0.18	0.43	-0.40	0.04	-0.31	0.06
ES	75.4	67.9	0.10	0.63	-0.48	0.04	-0.14	0.06
AP	65.0	51.8	0.23	0.10	-0.10	0.06	0.08	0.08
RN	63.0	46.7	0.30	0.35	-0.09	0.08	-0.12	0.08
AC	62.7	44.2	0.35	0.15	-0.11	0.08	0.15	0.09
GO	57.0	72.2	-0.24	0.16	-0.37	0.01	-0.07	0.04
PE	56.7	47.1	0.19	0.16	-0.13	0.09	-0.02	0.08
TO	48.9	56.8	-0.15	0.08	-0.44	0.06	0.07	0.08
CE	46.8	41.2	0.13	0.13	-0.20	0.08	0.04	0.08
BA	46.5	43.9	0.06	0.07	-0.12	0.08	-0.05	0.08
AM	44.7	55.2	-0.21	-0.10	-0.22	0.06	-0.04	0.09
PA	43.2	48.3	-0.11	-0.09	-0.07	0.06	-0.07	0.06
SE	41.7	45.4	-0.09	-0.00	-0.12	0.08	-0.12	0.08
AL	32.9	40.4	-0.21	-0.13	-0.23	0.11	-0.07	0.12
RO	32.7	62.1	-0.64	-0.16	-0.50	0.04	-0.08	0.06
PB	29.6	39.5	-0.29	0.06	-0.30	0.08	-0.21	0.08
PI	29.4	36.0	-0.20	-0.20	-0.22	0.09	0.03	0.10
MA	28.5	32.3	-0.12	-0.26	-0.06	0.09	0.00	0.10
RR	23.8	61.1	-0.94	-0.13	-0.61	0.04	-0.31	0.06
Weighted Aver.	92.2	77.9	0.11	0.368	-0.28	0.038	-0.07	0.06

Table 5 – Welfare measure decomposition of 2017 using São Paulo at 2002 as reference

state over time. To analyze absolute welfare changes, we employ a different approach. We set São Paulo as the reference point for calculating relative well-being in 2002. Therefore, shifts in welfare over time represent absolute changes in states' well-being.

Table 11 presents welfare and its decomposition in 2017, using São Paulo's 2002 welfare as the reference. As a result, when comparing the welfare of DF to SP in 2017, the relative well-being was 115.8. However, when comparing the DF in 2017 to SP in 2002, the former reached a more favorable position with $\lambda = 195.3$. Analyzing SP_{2017}, we notice a 57-point increase in its welfare compared to 2002.

Figure 7 illustrates the relationship between welfare growth and GDP

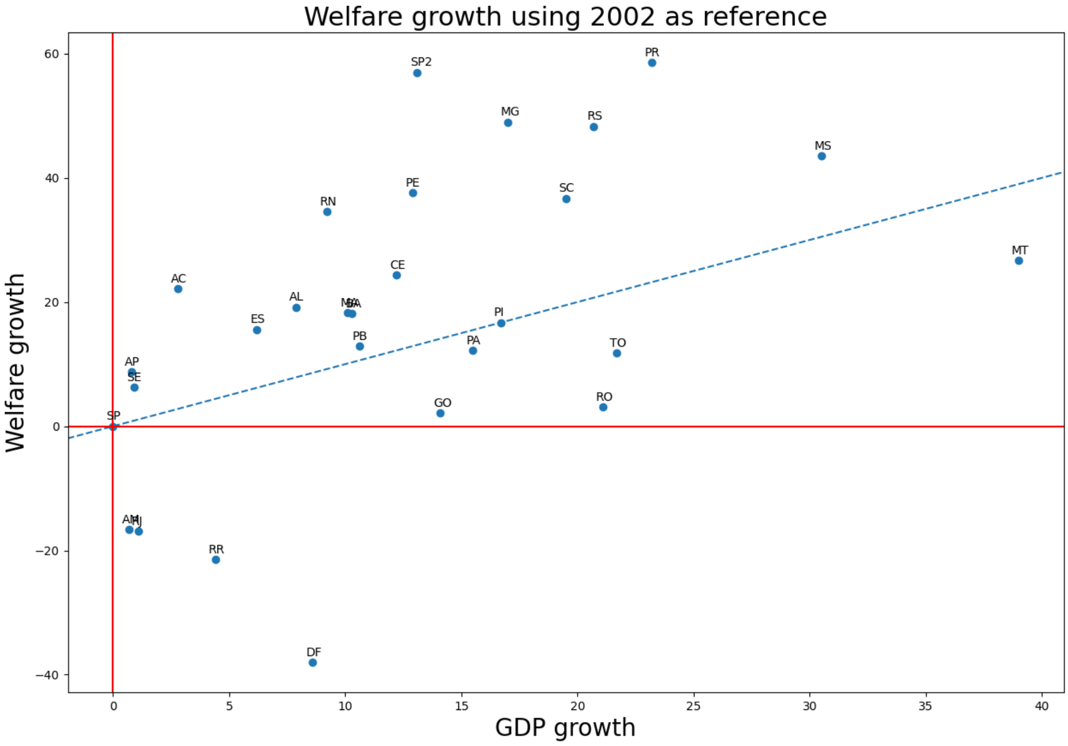

Figure 7 – Relation welfare growth and gdp per capita growth between 2002-2017 using São Paulo 2002 as reference state

per capita, using São Paulo state in 2002 as the reference. The 45-degree line represents a hypothetical scenario where GDP per capita increase perfectly reflects changes in states' well-being. With the new reference, most states lie above the dashed line, indicating their well-being has increased more than predicted by income growth. These states demonstrated an average welfare growth of more than $\lambda = 50$, while their GDP per capita growth did not surpass 25. Notably, the states with the highest welfare growth during this period were SP, PR, MG, and RS.

The following section investigates the welfare growth of Brazilian states and their correlation with initial welfare to determine if convergence has occurred.

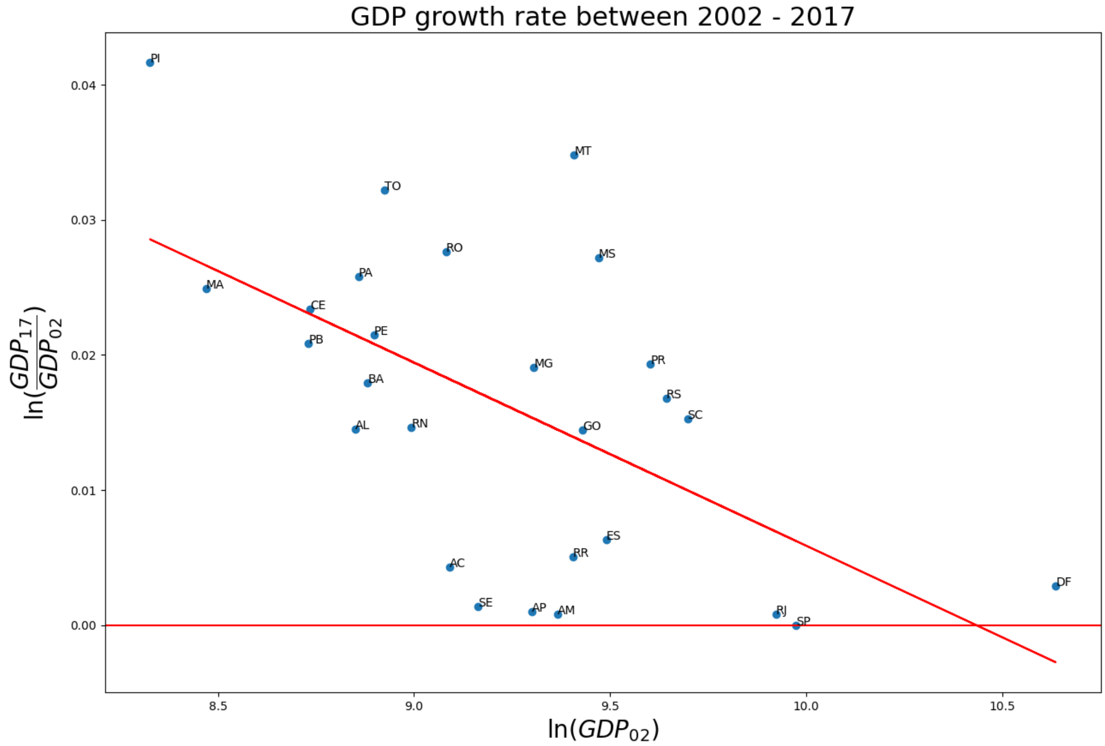

Figure 8 – GDP per capita Convergence between 2002 and 2017

4.4 Welfare Convergence

In this section, we aim to evaluate whether a convergence process exists by examining the relationship between the initial level of well-being and its growth over time. As described in the methodology section, we use the β-absolute convergence analysis. We also conduct a convergence analysis considering GDP per capita, an extensively researched indicator of convergence.

Convergence occurs when the income growth rate is negatively related to its initial value. In other words, the growth rate of a state should be lower the higher its initial income value. Figure 8 presents this relationship. We observe a negative relationship between the income growth rate and its initial value, which suggests that convergence is occurring, albeit at a slow pace.

Table 6 shows the initial income's estimated coefficient (β). The estimated coefficient is negative and statistically significant, around -0.13,

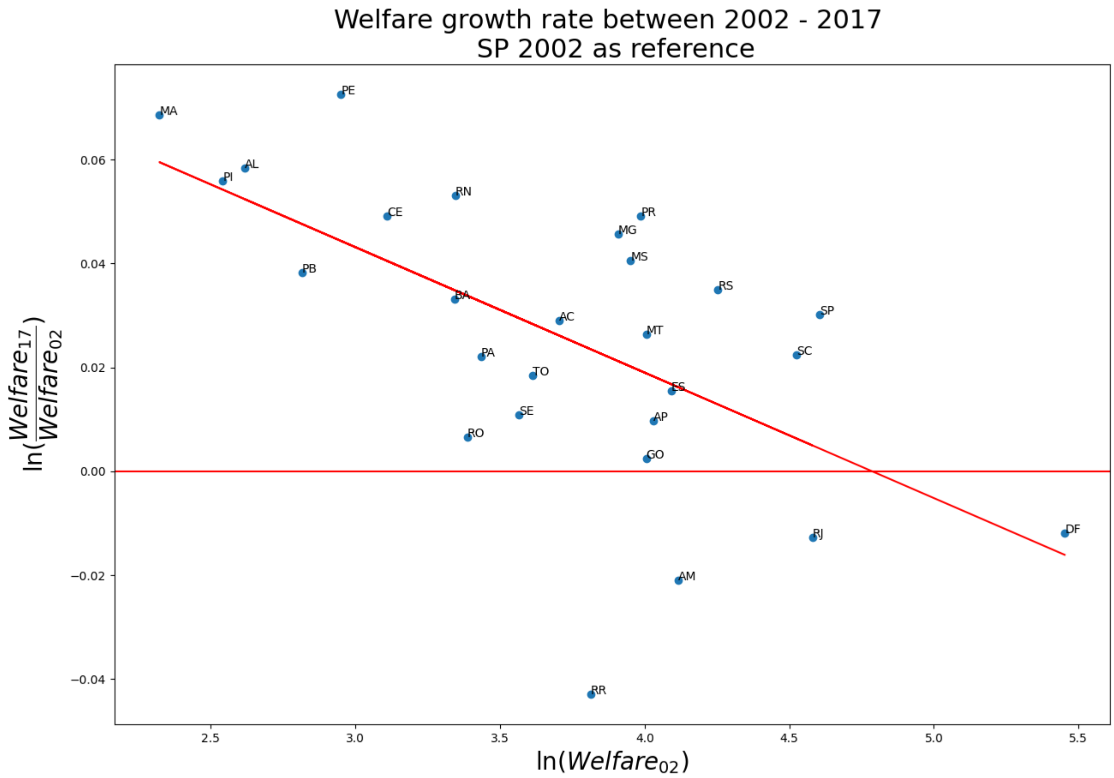

Figure 9 – Welfare Convergence between 2002 and 2017

indicating absolute income convergence in the Brazilian states.

$\mathbf{\Delta}(\ln(\cdot))$	**Coef**	**Std err**	**P>|t|**
$\ln(\text{GDP}_{2002})$	-0.0126	0.004	0.003
$\ln(\lambda_{\mathbf{2002}})$	-0.0241	0.006	0.001

Table 6 – Welfare and GDP β-absolute convergence between 2002-2017

Figure 9 brings the relationship between the well-being growth rate - approximated by the welfare log difference - and its initial value. We can see a negative relationship, indicating that the states with lower initial well-being experienced higher welfare growth rates.

Table 6 shows the results of the regression of $\frac{1}{T}\ln(\frac{\lambda_{2017}}{\lambda_{2002}})$ against $\ln(\lambda_{2002})$. The result obtained is -0.024, a negative and statistically significant estimated coefficient. Therefore, the states with lower initial well-being had a greater improvement in welfare between 2002 and 2017, meaning absolute convergence in well-being.

When examining the results in Table 7, the difference between the

	Speed (%)	**Half-life**
GDP	1.40%	55 years
Welfare	2.98%	29 years

Table 7 – Speed and Half-life of convergence for GDP per capita and Welfare between 2002-2017

	$\ln(\frac{\lambda_{17}}{\lambda_{02}})$	$\ln(\frac{y_{17}}{y_{02}})$	$\ln(\frac{\lambda_{17}}{\lambda_{02}}) - \ln(\frac{y_{17}}{y_{02}})$	Life Exp.	$\frac{C}{Y}$	ℓ	C ineq.	ℓ ineq.
DF	-0.18	0.04	-0.22	0.50	-0.64	0.01	-0.13	0.03
SP_{2017}	0.45	0.12	0.33	0.68	-0.31	0.01	-0.08	0.04
SC	0.34	0.23	0.10	0.62	-0.43	0.05	-0.19	0.05
RS	0.52	0.25	0.27	0.50	-0.21	0.04	-0.11	0.04
PR	0.74	0.29	0.44	0.57	-0.25	0.05	0.02	0.06
MG	0.69	0.29	0.40	0.52	-0.15	0.04	-0.06	0.04
MS	0.61	0.41	0.20	0.46	-0.25	0.05	-0.12	0.06
MT	0.40	0.52	-0.13	0.42	-0.57	0.08	-0.12	0.07
RJ	-0.19	0.01	-0.20	0.63	-0.70	0.02	-0.19	0.04
ES	0.23	0.10	0.14	0.71	-0.56	0.06	-0.16	0.08
AP	0.15	0.02	0.13	0.70	-0.57	0.03	-0.06	0.03
RN	0.80	0.22	0.58	0.85	-0.34	0.05	-0.03	0.05
AC	0.43	0.07	0.37	0.60	-0.56	0.12	0.08	0.13
GO	0.04	0.22	-0.18	0.33	-0.52	0.05	-0.12	0.08
PE	1.09	0.32	0.77	0.97	-0.28	0.08	-0.10	0.09
TO	0.28	0.48	-0.21	0.52	-0.78	0.12	-0.14	0.08
CE	0.74	0.35	0.39	0.66	-0.36	0.08	-0.06	0.07
BA	0.50	0.27	0.23	0.43	-0.25	0.08	-0.08	0.05
AM	-0.32	0.01	-0.33	0.35	-0.69	0.04	-0.07	0.05
PA	0.33	0.39	-0.05	0.28	-0.30	0.06	-0.16	0.07
SE	0.16	0.02	0.14	0.52	-0.36	0.08	-0.16	0.05
AL	0.88	0.22	0.66	0.92	-0.38	0.09	-0.05	0.09
RO	0.10	0.42	-0.32	0.31	-0.53	0.05	-0.22	0.06
PB	0.57	0.31	0.26	0.77	-0.39	0.06	-0.23	0.05
PI	0.84	0.62	0.22	0.49	-0.45	0.11	-0.02	0.08
MA	1.03	0.37	0.66	0.66	-0.15	0.14	-0.12	0.12
RR	-0.64	0.07	-0.72	0.52	-0.89	0.00	-0.35	0.00
Weighted Aver.	0.47	0.23	0.25	0.59	-0.35	0.05	-0.10	0.06

Table 8 – Decomposition of Welfare growth rate between 2002 and 2017

speed of convergence in income and welfare is clear. The well-being speed of convergence is approximately 2.98 percent, while the GDP per capita speed of convergence is 1.40 percent. It takes 55 years for GDP per capita to reach half-life convergence. Considering welfare, the half-life convergence is 29 years. Therefore, the speed of convergence is considerably higher in terms of well-being than income. In order to understand the convergence process in well-being, we must look at the evolution of the welfare components.

As illustrated Table 8, the growth rate of welfare, approximated by $\ln \lambda_{17} - \ln \lambda_{02}$, minus the growth rate of income, approximated by $\ln(y_{17}) - \ln(y_{02})$, is decomposed by the factors affecting welfare. Between 2002 and 2017, well-being grew by 47 percent weighted by the average population of the period, with the results in the first column of Table 8. Only DF, RR, AM, and RJ experienced negative growth. Consumption factors were the most important influencing the welfare decrease in these states

Overall, in all states, life expectancy and leisure favor welfare variation in relation to income growth. However, the reduction in the consumption share of income in all states leads to a reduction in welfare concerning income growth rate. Consumption inequality also favors income growth rates in relation to welfare.

5 Conclusion

Measuring well-being is a multifaceted endeavor. The innovative approach proposed by Jones and Klenow (2016) takes life expectancy, leisure, and consumption into account when evaluating well-being. When applied to Brazilian states, this model reveals that nearly all states exhibit lower well-being relative to income when compared to the state of São Paulo.

Between 2002 and 2017, the well-being of Brazilian states increased, on average, by 47%. This growth rate was 25 percentage points higher than income growth, signifying a substantial difference. The primary driver of this disparity between well-being and GDP per capita growth is the improvement in life expectancy.

During this period, there was a convergence in GDP per capita and real welfare among the states. However, the rate of well-being convergence was notably higher than GDP per capita convergence. Utilizing the half-life measure, it would take 29 years for welfare to reach half of the convergence, whereas per capita GDP would require 55 years.

Potential future research on this topic could include examining varying preferences across Brazilian states or investigating the impact of different consumption categories on well-being.

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APPENDIX A – Proofs

In this section some mathematical derivations of the main results are demonstrated from the work of Jones and Klenow (2016) that were omitted in the original article. The demonstration of the particular case where non-separable utility and separable utility are equivalent will be presented in the first part. Next, we present the decomposition of well-being in the illustrative example used by the authors.

A.1 Non-separable utility to the base case

Non-separable preferences in consumption and leisure can be represented by the following utility function:

$$u(C,\ell) = \bar{u} + \frac{(C+\underline{c})^{1-\gamma}}{1-\gamma}\left(1+\gamma-1)\frac{\theta\epsilon}{1+\epsilon}(1-\ell)^{\frac{1+\epsilon}{\epsilon}}\right)^{\gamma} - \frac{1}{1-\gamma} \quad \text{(A.1)}$$

The definition of all parameters are the same from the section Methodology. Take $v(\ell) = -\frac{\theta}{1+\epsilon}(1-\ell)^{\frac{1+\epsilon}{\epsilon}}$. Assume that $\gamma \to 1$ and $\underline{c}$=0. Then:

$$u(C,\ell) = \bar{u} + \frac{(C+\underline{c})^{1-\gamma}}{1-\gamma}\left(1-(\gamma-1)v(\ell)\right)^{\gamma} - \frac{1}{1-\gamma} \quad \text{(using definition of } v(\ell))$$

$$u(C,\ell) = \bar{u} + \underbrace{\frac{(C)^{1-\gamma}}{1-\gamma}}_{\text{a}}\underbrace{\left(1-(\gamma-1)v(\ell)\right)^{\gamma}}_{\text{b}} - \frac{1}{1-\gamma} \quad \text{(using } \underline{c}=0)$$

Note that the previous equation is not defined for the point $\gamma = 1$, so the notation $\gamma \to 1$ was used. In this way, it is possible to use the Taylor approximation of first order at this point, for the terms highlighted by the letters a and b, as placed at follow:

$$(a)\frac{(C+\underline{c}^{1-\gamma}}{1-\gamma} \approx \frac{1}{1-\gamma} + \log C - \frac{1}{2}(\gamma-1)\log C + \mathcal{O}((\gamma-1)^2)$$

$$(b)(1-(\gamma-1)v(\ell)^{\gamma} \approx 1 - v(\ell) + \mathcal{O}((\gamma-1)^2)$$

The term $\mathcal{O}(\cdot)$ represents the superior order term in the fast convergence approximation to zero. We can rewrite the utility $u(C,\ell)$ as:

$$u(C,\ell) \approx \bar{u}+\left(\frac{1}{1-\gamma}+\log C+D(\gamma)\right)\left(1-v(\ell)(\gamma-1)+\mathcal{O}((\gamma-1)^2)\right)-\frac{1}{1-\gamma} \tag{A.2}$$

distributing the terms in parentheses, we have:

$$\begin{aligned} u(C,\ell) &\approx \bar{u} \\ &+ \underbrace{\frac{1}{1-\gamma}}_{\text{cancel}} - \frac{1}{\cancel{1-\gamma}} v(\ell)\cancel{(1-\gamma)} + \frac{1}{1-\gamma}\mathcal{O}((\gamma-1)^2) && \text{(Line I)} \\ &+ \log C - \log C \cdot v(\ell)(\gamma-1) + \log C \cdot \mathcal{O}((\gamma-1)^2) && \text{(Line II)} \\ &+ D(\gamma) + D(\gamma)\cdot v(\ell)(\gamma-1) + D(\gamma)\mathcal{O}((\gamma-1)^2) && \text{(Line III)} \\ &- \underbrace{\frac{1}{1-\gamma}}_{cancel} \end{aligned}$$

Taking the limit of each of the lines highlighted earlier when $\gamma \to 1$, we have:

$$\text{Line I:} \lim_{\gamma\to 1} v(\ell) + \underbrace{\lim_{\gamma\to 1} \frac{1}{1-\gamma}\mathcal{O}((\gamma-1)^2)}_{=\frac{0}{0}\textbf{not determined}} = v(\ell) + \underbrace{\lim_{\gamma\to 1} \frac{\left[\mathcal{O}((\gamma-1)^2)\right]'}{(-1)}}_{\textbf{L' Hopital}} = v(\ell)$$

$$\begin{aligned} \text{Line II:} &\lim_{\gamma\to 1} \log C - \log C \cdot v(\ell) \lim_{\gamma\to 1}(\gamma-1) + \log C \lim_{\gamma\to 1} \mathcal{O}((\gamma-1)^2) \\ &= \log C - \log C \cdot v(\ell)\cdot 0 \\ &= \log C \end{aligned}$$

$$\text{Line III:} \lim_{\gamma\to 1}\left[D(\gamma) + D(\gamma)\cdot v(\ell)(\gamma-1) + D(\gamma)\cdot\mathcal{O}((\gamma-1)^2)\right] = 0$$

Using the same steps presented above and taking advantage of the result that the error is zero at the limit. Finally, merging lines I, II and III, and not forgetting the term $\bar{u}$, we have that:

$$u(C,\ell) = \bar{u} + \log C + v(\ell) \ \square$$

A.2 Example Case: Simple Case

We use this illustrative example to clarify the model, the representative agent does not discount future consumption and there is no growth in consumption over time, that is, we consider $\beta = 1$ and $g = 0$. In this example, leisure is known and constant over time, that is, there is no distribution for leisure. Furthermore, the economy is in steady state. Consider also the following functional form of the separable utility:

$$u(C, \ell) = \bar{u} + \log C + v(\ell) \tag{A.3}$$

where C denotes the consumption and $v(\ell)$ the disutility from work.

It follows that the intertemporal utility of the representative agent is given by:

$$U_i = E \sum_{a}^{100} S_i(a) \Big(\bar{u} + \log C_i + v(\ell_i) \Big) \tag{A.4}$$

where $S_i(a)$ denotes the survival rate of the representative agent in locality i for each age a. Suppose that consumption is zero, then the term $\sum_a^{100}$ depict the born' life expectancy of the representative agent, denote as e_i where i denotes the agent.

Jones and Klenow (2016) assume that consumption in each location has a distribution log normal between individuals at a given point in time (regardless of age and death rate) with arithmetic mean $\bar{c}_i$ and variance σ_i^2. In this way, we have that $E(\log C) = \log c_i - \frac{\sigma_i^2}{2}$. Then, equation A.4 can be rewritten as:

$$U_i^{simple} = e_i \left(\bar{u} + \log c_i + v(\ell) - \frac{1}{2}\sigma_i^2 \right) \tag{A.5}$$

Starting from the equivalence condition, represented by equation 3.5, and the functional form of the utility given by equation A.3, we arrive at the following equality:

$$e_j \left(\bar{u} + \log(\lambda^{simple} c_j) + v(\ell_j) - \frac{1}{2}\sigma_j^2 \right) = e_i \left(\bar{u} + \log c_i + v(\ell_i) - \frac{1}{2}\sigma_i^2 \right) \tag{A.6}$$

where j denotes the reference location, and i the compared one, with $i \neq j$.

Rewriting:

$$\log \lambda^{simple} = \frac{e_j - e_i}{e_j}\bar{u} + \overbrace{\underbrace{\frac{e_j}{e_j}}_{=1} \log c_i - -\frac{e_j}{e_i} \log c_j}^{A} + \overbrace{\frac{e_i}{e_j} \log v(\ell_i) - \underbrace{\frac{e_j}{e_j}}_{=1} v(\ell_j)}^{B}$$

$$- \overbrace{\frac{1}{2}\frac{e_i}{e_j}\sigma_i^2 + \frac{1}{2}\underbrace{\frac{e_j}{e_j}}_{=1}\sigma_j^2}^{C}$$

Solving separately the terms A, B and C from the previous equation, we have:

$$A = \frac{e_i}{e_j} \log c_i - \log c_j \pm \frac{e_j}{e_j} \log c_i = \left(\frac{e_i - e_j}{e_j}\right) \log c_i - \log c_j + \frac{e_j}{e_j} \log c_i$$

$$A = \left(\frac{e_i - e_j}{e_j}\right) \log c_i + \log c_i - \log c_j$$

$$B = \frac{e_i}{e_j} v(\ell_i) - v(\ell_j) \underbrace{\frac{e_j}{e_j}}_{=0} = \left(\frac{e_i - e_j}{e_j}\right) v(\ell_i) - v(\ell_j) + \frac{e_j}{e_j} v(\ell_i)$$

$$B = \left(\frac{e_i - e_j}{e_j}\right) v(\ell_i) + v(\ell_i) - v(\ell_j)$$

$$C = -\left(\frac{1}{2}\frac{e_i}{e_j}\sigma_i^2 - \frac{1}{2}\sigma_j^2 \pm \frac{1}{2}\frac{e_j}{e_j}\sigma_i^2\right) = -\left[\left(\frac{e_i - e_j}{e_j}\right)\frac{1}{2}\sigma_i^2 - \sigma_i^2 + \frac{e_j}{e_j}\sigma_j^2\right]$$

$$C = \left(\frac{e_i - e_j}{e_j}\right)\sigma_i^2 - \sigma_i^2 + \sigma_j^2$$

We can rewrite equation as:

$$\log \lambda_i^{simple} = \overbrace{\frac{e_i - e_j}{e_j}\left(\bar{u} + \log c_i + v(\ell_i) - \frac{1}{2}\sigma_i^2\right)}^{\text{Life expectancy}} + \underbrace{+ \log c_i - \log c_j}_{\text{Consumption}}$$

$$+ \underbrace{v(\ell_i) - v(\ell_j)}_{\text{Leisure}} - \underbrace{\frac{1}{2}(\sigma_i^2 - \sigma_j^2)}_{\text{Consumption inequality}} \quad \square$$

APPENDIX B – Tables

In this section, the remaining tables and graphs for the years 2002 and 2008 will be presented.

	λ	$\tilde{y}$	$\log\frac{\lambda}{\tilde{y}}$	Life Exp.	$\frac{C}{Y}$	ℓ	C ineq.	ℓ ineq.
DF	234.2	194.0	0.19	0.18	0.21	0.02	-0.25	0.03
SP	100.0	100.0	0.00	0.00	0.00	0.00	0.00	0.00
RJ	97.0	95.1	0.02	-0.20	0.30	0.03	-0.13	0.02
SC	92.6	75.9	0.20	0.16	-0.03	-0.06	0.14	-0.01
RS	70.5	71.9	-0.02	0.12	-0.15	-0.05	0.07	-0.01
AM	60.7	54.5	0.11	-0.46	0.48	0.02	0.03	0.04
ES	59.6	61.7	-0.03	-0.08	0.08	-0.03	0.01	-0.02
AP	55.4	51.0	0.08	-0.61	0.46	0.03	0.14	0.05
GO	54.7	58.1	-0.06	-0.17	0.15	-0.04	0.05	-0.04
MT	54.5	56.9	-0.04	-0.25	0.24	-0.06	0.04	-0.02
PR	53.9	69.1	-0.25	-0.03	-0.14	-0.04	-0.02	-0.02
MS	51.8	60.5	-0.16	-0.13	-0.04	-0.05	0.08	-0.03
MG	49.9	51.3	-0.03	0.02	-0.10	-0.02	0.07	-0.00
RR	44.7	56.7	-0.24	-0.66	0.28	0.04	0.04	0.06
AC	40.2	41.4	-0.03	-0.46	0.45	-0.04	0.06	-0.04
TO	36.7	35.1	0.05	-0.45	0.34	-0.06	0.21	-0.00
SE	34.9	44.5	-0.24	-0.54	0.23	0.00	0.04	0.03
PA	30.7	32.8	-0.07	-0.38	0.23	-0.00	0.09	-0.00
RO	29.3	41.0	-0.34	-0.49	0.03	-0.01	0.13	-0.00
RN	28.0	37.5	-0.29	-0.52	0.25	0.04	-0.09	0.03
BA	28.0	33.6	-0.18	-0.37	0.13	-0.00	0.03	0.03
CE	22.1	29.0	-0.27	-0.55	0.16	-0.00	0.11	0.01
PE	18.7	34.2	-0.60	-0.83	0.16	0.00	0.08	-0.01
PB	16.3	28.9	-0.57	-0.73	0.09	0.02	0.02	0.03
AL	13.4	32.5	-0.89	-1.07	0.15	0.02	-0.02	0.03
PI	12.5	19.3	-0.44	-0.71	0.23	-0.02	0.05	0.02
MA	10.0	22.2	-0.80	-0.94	0.08	-0.04	0.12	-0.02

Table 9 – Welfare measure decomposition of 2002

	λ	$\tilde{y}$	$\log \frac{\lambda}{\tilde{y}}$	Life Exp.	$\frac{C}{Y}$	ℓ	C ineq.	ℓ ineq.
DF	146.9	177.4	-0.19	0.04	0.01	0.03	-0.29	0.02
SP	100.0	100.0	0.00	0.00	0.00	0.00	0.00	0.00
RJ	80.5	96.8	-0.18	-0.15	0.06	0.03	-0.15	0.01
SC	79.4	82.6	-0.04	0.07	-0.06	-0.04	0.00	-0.02
ES	78.0	79.3	-0.02	-0.04	0.08	-0.00	-0.05	0.00
RS	74.2	71.3	0.04	0.01	0.01	-0.02	0.04	-0.00
PR	66.5	72.3	-0.08	-0.01	-0.02	-0.02	-0.02	-0.01
MG	55.5	57.9	-0.04	-0.04	-0.01	-0.00	-0.00	0.00
MT	55.0	70.1	-0.24	-0.24	0.00	-0.02	0.04	-0.02
MS	53.9	63.3	-0.16	-0.15	-0.06	-0.02	0.08	-0.01
AM	53.1	55.6	-0.05	-0.41	0.24	0.02	0.11	-0.00
AP	51.1	50.8	0.01	-0.33	0.18	0.05	0.05	0.06
AC	48.2	41.3	0.15	-0.33	0.26	0.01	0.19	0.02
GO	45.8	59.5	-0.26	-0.19	0.00	-0.02	-0.04	-0.02
RR	45.5	51.6	-0.13	-0.48	0.28	-0.01	0.08	0.00
TO	42.6	40.8	0.04	-0.32	0.26	-0.02	0.12	0.01
SE	40.8	46.5	-0.13	-0.36	0.11	0.05	0.03	0.04
RN	40.2	39.7	0.01	-0.19	0.12	0.03	0.05	0.00
RO	36.9	49.0	-0.28	-0.41	0.00	0.01	0.08	0.02
PA	32.8	36.1	-0.09	-0.34	0.17	0.01	0.06	0.01
BA	31.4	34.6	-0.10	-0.28	0.11	0.03	0.01	0.03
CE	28.4	29.9	-0.05	-0.27	0.11	0.01	0.09	0.01
PB	25.0	31.0	-0.22	-0.37	0.14	0.05	-0.06	0.03
PE	23.7	32.7	-0.32	-0.37	0.03	0.04	-0.03	0.00
AL	20.3	30.9	-0.42	-0.52	0.06	0.06	-0.06	0.04
MA	18.2	25.8	-0.35	-0.52	0.21	0.02	-0.08	0.02
PI	17.7	22.1	-0.22	-0.42	0.16	0.01	0.01	0.02

Table 10 – Welfare measure decomposition of 2008

	λ	$\tilde{y}$	$\log \frac{\lambda}{\tilde{y}}$	Life Exp.	$\frac{C}{Y}$	ℓ	C ineq.	ℓ ineq.
DF	264.3	200.5	0.277	0.382	0.107	0.011	-0.256	0.032
SP2	174.8	113.0	0.436	0.309	0.085	-0.020	0.052	0.010
SC	144.3	93.3	0.436	0.415	0.049	-0.058	0.038	-0.008
ES	131.2	89.7	0.381	0.222	0.185	-0.025	-0.013	0.011
RS	130.1	80.6	0.479	0.323	0.103	-0.039	0.084	0.008
RJ	128.2	109.4	0.159	0.067	0.168	0.014	-0.111	0.020
PR	114.2	81.7	0.335	0.274	0.082	-0.039	0.013	0.004
SP	100.0	100.0	0.000	0.000	0.000	0.000	0.000	0.000
MG	93.0	65.5	0.352	0.228	0.092	-0.025	0.041	0.015
MS	84.1	71.5	0.163	0.041	0.041	-0.038	0.117	0.002
MT	80.8	79.2	0.020	-0.117	0.119	-0.044	0.070	-0.009
AP	71.0	57.5	0.211	-0.250	0.266	0.032	0.098	0.066
AM	70.7	62.9	0.117	-0.375	0.340	-0.004	0.150	0.005
GO	69.3	67.3	0.029	-0.027	0.090	-0.043	0.019	-0.011
AC	66.6	46.7	0.355	-0.257	0.359	-0.010	0.236	0.026
RN	60.1	44.9	0.292	-0.042	0.224	0.007	0.090	0.014
TO	58.9	46.1	0.246	-0.249	0.319	-0.040	0.193	0.022
RR	58.0	58.3	-0.006	-0.480	0.350	-0.034	0.146	0.012
SE	54.8	52.5	0.043	-0.320	0.214	0.029	0.068	0.053
RO	48.1	55.4	-0.141	-0.393	0.095	-0.008	0.130	0.035
PA	44.2	40.8	0.081	-0.298	0.269	-0.007	0.098	0.020
BA	43.7	39.2	0.110	-0.204	0.212	0.008	0.049	0.044
CE	39.6	33.8	0.157	-0.196	0.211	-0.009	0.134	0.017
PB	32.5	35.0	-0.075	-0.357	0.238	0.029	-0.024	0.039
PE	30.7	37.0	-0.187	-0.363	0.133	0.022	0.006	0.016
AL	23.5	34.9	-0.398	-0.622	0.150	0.038	-0.010	0.046
PI	21.9	25.0	-0.133	-0.461	0.254	-0.012	0.054	0.032
MA	20.8	29.1	-0.335	-0.644	0.321	0.005	-0.052	0.034

Table 11 – Welfare measure decomposition of 2008 using São Paulo at 2002 as reference