Universidade de São Paulo
Instituto de Física

Massa de Neutrinos e Matéria Escura em Modelos de Twin Higgs

Davi Bastos Costa

Davi Bastos Costa

Orientador: Prof. Dr. Gustavo Alberto Burdman

Dissertação de mestrado apresentada ao Instituto de Física da Universidade de São Paulo, como requisito parcial para a obtenção do título de Mestre em Ciências.

Banca Examinadora:
Prof. Dr. Gustavo Alberto Burdman - Orientador - IFUSP
Prof. Dr. Chee Sheng Fong - UFABC
Prof. Dr. Andre Paniago Lessa - UFABC

São Paulo
2020

FICHA CATALOGRÁFICA
Preparada pelo Serviço de Biblioteca e Informação do Instituto de Física da Universidade de São Paulo

Costa, Davi Bastos

Massa de neutrinos e matéria escura em modelos de Twin Higgs. São Paulo, 2020.

Dissertação (Mestrado) – Universidade de São Paulo. Instituto de Física, Depto. de Física Matemática.

Orientador: Prof. Dr. Gustavo Burdman

Área de Concentração: Física

Unitermos: 1. Bóson de Higgs; 2. Matéria escura; 3. Neutrinos.

USP/IF/SBI-077/2020

University of São Paulo
Physics Institute

Neutrino Masses and Dark Matter in a Twin Higgs Model

Davi Bastos Costa

Supervisor: Prof. Dr. Gustavo Alberto Burdman

Dissertation submitted to the Physics Institute of the University of São Paulo in partial fulfillment of the requirements for the degree of Master of Science.

Examining Committee:
Prof. Dr. Gustavo Alberto Burdman - Supervisor - IFUSP
Prof. Dr. Chee Sheng Fong - UFABC
Prof. Dr. Andre Paniago Lessa - UFABC

São Paulo
2020

Prologue

What is the purpose of this work? This is a dissertation, so it is a requirement of my funding agency FAPESP and also of my home institution USP. Apart from these bureaucratic requirements, is there any other purpose? The physics community is driven by peer-review papers that are available online in repositories such as the Arxiv[1]. Because of that, for me it does not make much sense to write in this dissertation what I would otherwise write in a paper as I already did a couple of times by now [1, 2, 3]. Therefore, I used the bureaucratic requirement of doing this dissertation as an opportunity to expose more broadly my thoughts and current understanding of physics, something that is not a good practice in specialized papers. I tried to start each subsection with an informal discussion that a wit reader should be able to follow. These informal discussions are then followed by a technical presentation where I tried to condense and summarize what I take as the main physical ideas involved in each subject. I hope you enjoy reading as I enjoyed learning all these fascinating themes.

[1] `https://arxiv.org/`

Acknowledgements

What seems to be the common practice is to acknowledge people who were in a sense directly related to the work being presented. However, being completely honest, I have difficulty thinking of a precise meaning for this. For example, should I include São Paulo citizens who paid taxes and financed USP and FAPESP? Should I thank the workers who produced the food that fed me during my life? Should I thank the previous generations that created the technological and intellectual tools that allowed my way of life as well as my research? If so, I must thank the parents of modern science like Newton and Galileo, or I must go back in time, and thank the Greeks like Euclid? But it doesn't seem right to thank both of them without referring to the Arabs who preserved and developed the knowledge between the two. Nor does it seem right to me not to thank the Chinese and Indians who contributed to the development of mathematics without which no science would be possible. But these are facts that I am aware of, certainly there are many more events in the human history to which, if I had a more extensive historical knowledge, I would think necessary to acknowledge. All these events concerns human history, however, and maybe going a little too far, I could thank other kinds of things that also contributed to this work. For example, should I thank the air molecules that allowed me to breath? Or going back in time should I thank the Nilo and Yellow river that allowed the born of civilization? Or more broadly to nature who allowed our existence, maybe thanking each existent elementary particles, without which, nothing we know would exist? These general facts of which we are all beneficiaries and that connect humanity and the universe as a whole do not usually appear in acknowledgements. So, although I want to leave my recognition to all humanity and the universe here, I will reserve this space to thank some human beings that I met personally.

Much of my joy comes from my beloved son Aruê. It seems to me that none of the ideas directly related to this work came from him, but I would say that without joy, I would not have been able to complete this work. Some people where particularly important for my training as a physicist and to deepen my understanding of particle physics. I would like to thank my supervisor Gustavo Burdman, my collaborators Bogdan Dobrescu, Patrick Fox, Roni Harnik and Pedro Machado as well as my professors at USP, Renata Funchal and Enrico Bertuzzo.

Abstract

In this dissertation, I will present a twin Higgs model relating the spontaneous breaking of $\mathbb{Z}_2$ symmetry, the cosmological constraints of twin Higgs models, and neutrino masses. The $\mathbb{Z}_2$ symmetry is exact in the model and it contains an $SU(2)$ triplet with hypercharge 1 and its twin copy in addition to the degrees of freedom present in the original twin Higgs model. The vacuum expectation value of the scalar sector breaks spontaneously $\mathbb{Z}_2$ and electroweak symmetry. Then, by type-II seesaw mechanism, the Standard Model neutrinos get small masses and their twin gets large masses mitigating the dark radiation problem.

Keywords: Particle Physics; Hierarchy problem; Neutrino Masses; Dark Matter; Twin Higgs Model.

Contents

1 Introduction

What are things made of?

Your body is made of about 10^{13} cells [4]; each cell is made of about 10^7 proteins; a protein of 10^5 molecules; a molecule of atoms; each atom of electrons, protons and neutrons; and nucleons of up and down quarks. It is not known if the electron, the up and down quarks have an internal structure and are therefore made of other things. However, if they are not, there seems to be no way to know that for sure. A reason that can be grasped intuitively, is because the way to tell if something is composite is by breaking it. But to break something requires enough energy. Therefore, the fact that we see no structure in the electron, the up and down quarks, only tells us that the energies involved in their binding must be larger than the available in the experiments designed to break them.

When we consider a familiar object and start to break it into its constituents as we did in the human body example, in most cases we will end up in electrons, up and down quarks. That is because atoms seem to be the only objects, as far as we know, that can form the complex structures as the ones that made familiar objects such as animals, rocks, books, etc. However, by colliding electrons and protons, at very high energies in kilometers long particle accelerators, we find that new objects are produced that are not made either of electrons or protons. The objects which we can't break using current experimental techniques are the known elementary particles. The list of these particles with its corresponding mass is [5]:

Particles	Log_{10}(Mass/MeV)
photon	$\emptyset$
gluons	$\emptyset$
electron neutrino	?
muon neutrino	?
tau neutrino	?
electron	-1
up	0
down	0
strange	1
muon	2
charm	3
tau	3
bottom	3
W boson	4
Z boson	4
Higgs	5
top	5

Table 1: List of elementary particles.

From the discussion presented above, it should be clear that these elementary particles are not necessarily elementary in the sense that they are not made of other things [6]. However, these are the only things we know today that have the chance of being elementary. Therefore we should take elementary as a notion about the possibility of being truly elementary. And this possibility depends on the state of development of technology and on the energies we have available to break and produce other things. With this technology-dependent notion of elementary, it is then correct to say that particle physics is the research field that studies elementary particles, and we can then declare that this dissertation is on particle physics.

For those with a mathematical bend, I want to briefly explore the realm of beings by considering the structure that emerges when we consider the formal properties of the constituent relation [7]. If a thing a is made of another thing b we can conversely say that b is a constituent of a. We can understand this relation formally as a relation $\geq$ defined in the set of things Ω with the following properties

$$\text{Reflexive: } a \geq a \tag{1.1}$$

$$\text{Antisymmetric: if } a \geq b \text{ and } b \geq a \text{ then } a = b \tag{1.2}$$

$$\text{Transitive: if } a \geq b \text{ and } b \geq c \text{ then } a \geq c \tag{1.3}$$

for all $a, b, c \in \Omega$. In words, this formal properties means that a given thing is one of its constituents, that if two things are the constituents one of the other they are the same thing, and lastly, if a thing b is the constituent of a and if c is the constituent of b then c is the constituent of a. The set of all things Ω with the constituent relation $\geq$ is a partially ordered set and it may have minimal elements. An element $m \in \Omega$ is minimal if there is no $a \neq m$ in Ω such that $m \geq a$. An open question in our endeavor to understand nature is if $(\Omega, \geq)$ have minimal elements, and what they are. The elementary, or fundamental constituents of all things, which are equal to the minimal elements of Ω, are the referents of parts of high energy physics such as string theory. A possibility thought in antiquity, and not invalidated by theoretical physics, is that everything has constituents and therefore that the set of minimal elements of Ω is the empty set $\emptyset$. Particle physics should be contrasted with these other parts of high energy physics because its referents are defined by the technological degree to which it is possible to investigate the constituents of things. To formalize this we can define a new set Ω_E that is the set of things that have binding energies smaller then E. Then, $(\Omega_E, \geq)$ have minimal elements and they are the referents of particle physics. Now is ended the mathematical interlude.

I've been talking rather generically on breaking things apart. However, in particle physics, to break something does not always coincide with our intuitive notion of splitting a thing into its different parts. An important example to consider is the proton. We know that it is made of quarks and gluons, but we've never detected a free quark or gluon after colliding two protons. In a proton-proton collision, a jet of particles is produced that include many elementary particles and also other composite particles made of quarks and gluons such as the pion and the kaon. From the study of these jets of particles, physicists were able to discover the existence and the properties of the quarks. The conclusion is that if a given thing is made of something this does not mean that you can find its parts after breaking it. There are cases, such as protons and neutrons, in which the constituent parts are not found directly in experiments but rather indirectly after theoretical analysis.

The theory that describes these elementary particles and allows such an analysis is the Standard Model (SM). Before giving a more technical overview of it let's expand on the notion of binding energy presented before. The binding energy of an object is associated with some force that keeps its constituents together. As an example, the atoms are bind together in molecules due to electromagnetic forces; protons and neutrons are bind together in the nucleus due to the strong force; and the nuclei of an atom and its electrons are bind together again due to electromagnetic forces. This way of describing composite objects as some parts held together by some force seems to vindicate two different ontological notions: matter and force. However, in the SM these two notions are much alike because the forces are also described by elementary particles.

The electromagnetic force is associated with the photon, strong force to gluons, the weak force to the Z and W boson, and gravity to gravitons. All these force particles, and all the other elementary particles of the SM listed in (1) are in fact excitations of different quantum fields. Also, radiation, which could be thought of as a different type of substance, is a particular state of the photon fields. We see, therefore, that particle physics has achieved an ontological unification of great significance. Matter, radiation, and force are all the same thing: quantum fields.

In a sense, the concept of a quantum field naturally emerges when ones tries to put together quantum mechanics an special relativity. From quantum mechanics comes the notion of superposition and of unitary evolution, and from special relativity the notion of locality. However, the dynamics and properties of quantum fields is a topic of advanced courses in the physics curriculum and are part of a large framework called quantum field theory (QFT) [8, 9, 10, 11, 12]. Most parts of this dissertation will assume that the reader is familiar with the subject except the introductory parts of each section, where I will try to give an intuitive account of what will be presented. For completeness, however, I will give a very brief and schematic summary of QFTs that can be formulated with a Lagrangian focusing on the SM.

The Standard Model

Lagrangian QFTs, including the SM, are defined by just two ingredients:

Gauge group of symmetries
Quantum fields and their representations

Table 2: Scheme for what defines a QFT

With these two ingredients, one can write all renormalizable terms that respect the Lorentz and gauge group to determine the Lagrangian of the theory. With the Lagrangian, in many cases, one can make predictions to determine the values of the free parameters of the theory. An interesting feature of QFTs, and a subject of present research, is that not all choices of gauge groups and quantum fields yields a valid theory [13, 14, 15, 16]. For the choices which yields an inconsistent theory, it is commonly said that the theory have an anomaly. My previous works [1, 2, 3] where related with this topic, in particular with local anomalies, however, the topic is much broader and includes global anomalies for example [17, 18].

The SM gauge group of symmetries is

$$G_{\mathrm{SM}} = SU(3) \times SU(2) \times U(1) \tag{1.4}$$

and the quantum fields are particular representations of G_{SM}. The quantum fields are

Scalar field	SU(3)	SU(2)	U(1)
H	1	**2**	3

Table 3: Scalar field of the SM and its representation under G_{SM}

plus 3 copies of

Fermion fields	SU(3)	SU(2)	U(1)
Q	$\mathbf{3}$	$\mathbf{2}$	1
D	$\overline{\mathbf{3}}$	$\mathbf{1}$	2
L	$\mathbf{1}$	$\mathbf{2}$	-3
U	$\overline{\mathbf{3}}$	$\mathbf{1}$	-4
E	$\mathbf{1}$	$\mathbf{1}$	6

Table 4: Fermion fields of the SM and their representations under G_{SM}

where **3** and **2** are the fundamental representation of $SU(3)$ and $SU(2)$ respectively, $\overline{\mathbf{3}}$ is the conjugate of the **3** representation, and **1** is the singlet. Following the conventions of previous work [1, 2], I assumed that fermions are taken as left-handed fields which can be done via charge conjugation. I also assumed that hypercharge is given for a normalization of the gauge couplings where all charges are integers with no common divisor and the biggest charge, in absolute value, is positive. The SM is an anomaly-free theory and it is rather astonishing to see how each piece that makes the SM combine in a precise way to yield an anomaly-free theory.

The electroweak subgroup $SU(2) \times U(1)$ of G_{SM} makes the SM a chiral theory. This means that no mass term can be formed for the SM fermions using the SM fermion fields without violating electroweak symmetry. Fermion masses are generated because the electroweak subgroup $SU(2)\times U(1)$ is spontaneously broken to the electromagnetism $U(1)$ [19, 20, 21]. These spontaneous symmetry breaking also gives mass to the weak gauge bosons by what is called the Anderson-Higgs mechanism [22, 23, 24, 25]. I should add that this feature is not an additional input to the theory, this breakdown is a dynamical process that takes place because the Higgs field gets a vacuum expectation value which does not respect the electroweak subgroup. To be more precise, the Higgs potential is

$$V(H) = m^2 H^\dagger H + \frac{1}{2}\lambda(H^\dagger H)^2 \tag{1.5}$$

which respects the electroweak subgroup $SU(2) \times U(1)$. If $m^2 > 0$ the minimum of the potential is at 0 and the vacuum expectation value (VEV) of the Higgs field will respect the symmetry G_{SM}. If $m^2 < 0$ the VEV will be

$$\langle H^\dagger H \rangle = -\frac{m^2}{\lambda}. \tag{1.6}$$

and will spontaneously break $SU(2) \times U(1)$. The fact that $m^2 < 0$ and that electroweak symmetry is spontaneously broken to electromagnetism is not an additional input of the SM, but a empirical finding. It is somewhat in the same level as the empirical discovery that the top Yukawa coupling is approximately equal to 1 at electroweak scale and not other value.

Because $SU(3) \subseteq G_{\mathrm{SM}}$ is not broken, the three colors of quarks are equal in all respects and in this sense indistinguishable. That is the reason why we talk generically about the up quark and not the "blue" or "red" up quark. No process in nature is able to distinguish any of them. In contrast, $SU(2) \times U(1)$ is spontaneously broken, giving different identities to the $SU(2) \times U(1)$ components of the SM fermion fields. For example, the left-handed electron and neutrino fields are components of L, the first forms a mass term together with E which is the conjugate of the right-handed electron field. Both make an massive Dirac particle. The neutrino, on the other hand, remains massless. This sort of statement have real physical meaning and are independent of the way we align the VEV of the Higgs. However, to understand the details of the SM we need to choose a VEV alignment and work with it. The dominant convention for the VEV alignment of the Higgs is

$$\langle H \rangle = \begin{pmatrix} 0 \\ v \end{pmatrix}. \tag{1.7}$$

With this choice we can identify the $SU(2)$ components of the fermion fields as

$$L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}, \quad Q = \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \quad D = (d_R)^c, \quad U = (u_R)^c, \quad E = (e_R)^c. \tag{1.8}$$

This identification of the Weyl fermion inside the fermion fields gives more substance to the previous statement that the SM is a chiral theory. Simply note that left-handed and right-handed fields are in different representations of $SU(2)$. Furthermore, we should recall that u_L, d_L, u_R and d_R are in the **3** representation of $SU(3)$.

The most general renormalizable Lagrangian written with the quantum fields of the SM in a way consistent with Lorentz and G_{SM} symmetry have many free parameters. However, not all of them have physical significance. After eliminating the meaningless ones, we end up with 19 free parameters [5]

Free parameters	Physical meaning
α	strength of electromagnetism
θ_W	relates electromagnetism and weak interaction couplings
g_3	strong interactions coupling
v	Higgs vacuum expectation value
λ	Higgs quartic coupling
$\theta_{12}, \theta_{23}, \theta_{13}, \delta_{13}$	determine mixings of quarks of different flavor
9 Yukawa coupling	determine the masses of the charged fermions
θ	strong CP violation

Table 5: Free parameters of the SM

The SM has "standard" in its name because it is essentially universally accepted. After tens of thousands of measurements, the model continues to agree with experimental results. As examples, the SM predicted the existence of the W, Z, top quark and many of their properties before these particles were observed [26, 27, 28, 29, 30]. In addition to this, the SM provides what is perhaps the most accurate prediction of all physics. The fine-structure constant α can be theoretically calculated in terms of the anomalous magnetic dipole moment of the electron which can be measured with high precision [31]. The fine-structure constant α can also be determined from atom-recoil measurements [32]. Checking the consistency of such measurements tests the theory. The agreement found this way is to within ten parts in a billion (10^{-8}) which makes the SM one of the most accurate physical theories constructed thus far. The observation of the Higgs boson in 2012 marks the experimental confirmation of the last ingredient of the SM [33, 34].

Some problems of the Standard Model

Although the SM has demonstrated huge successes in providing experimental predictions it has problems that demonstrate that there needs to be physics beyond it. The problems of the SM are diverse and very different. There are observational problems related to the existence of facts that are not accounted for by it such as:

- Neutrino masses,
- Dark matter,
- Baryon asymmetry,

there are severe theoretical problems related to the extrapolation of the theory to higher energies such as:

- Gravity,
- Landau pole,

and there are features of the SM that seems to be unnatural and to require an explanation

- Hierarchy problem,
- Cosmological constant,
- Strong CP problem,
- Fermion masses,
- Quantization of electric charge.

There are many different theoretical proposals to account for these different problems. In a sense, the work of a theoretical particle physicist is to design extensions of the SM that address some of these problems and that have testable consequences. By doing so, they guide the design of experiments and enhance the chances of the community to discover new physics. In this endeavor, it is most attractive to build extensions that relate the solution of different problems in a single framework. That is our aim in this dissertation. The structure will be the following. Section (3) is the most important part and is the only part that contains new and original research. In it, we will present a model that relates the solution of the hierarchy problem, with neutrino masses and dark matter. Before presenting this model we will describe in (2) the framework which we will use, the twin Higgs models. And in the next subsections, we will describe in more detail the problems that we are willing to solve. The level of difficulty of each section increases from the beginning to the end. As an exercise to myself, I tried to make the start of each section understandable to a non-physicisit.

1.1 Neutrino masses

Where are the other particles?

We started the previous section with the reducionist ontological question, “what are things made of?”. Starting from your body, we dive into your constituents until we get to the quarks, gluons, and electrons that bound together to make your atoms. Just after, I presented the list of elementary particles (1) know to this day which includes many other particles along with the quarks, gluons, and electrons from which you are made. So one may wonder, where are the other particles?

With a few exceptions, the answer now is nowhere. The reason is that almost all particles of the SM are unstable and decay promptly to stable particles. We are aware of them because they are produced in high energy collisions events that happen in particle physics experiments, such as the Large Hadron Collider. However, the list of stable elementary particles of the SM is very short

Stable elementary particles
photon
neutrinos
electron

Table 6: List of stable elementary particles of the SM.

Each of these particles is forbidden to decay because nature has symmetries and for each symmetry there is an associated conservation law. For example, conservation of energy forbids a particle to decay to a heavier one, therefore massless photons are stable. Conservation of charge forbids a charged particle to decay to a set of particles such that the sum of charges is different. These two conservation laws then imply that the electron is stable. The only particles that are lighter than the electron are neutrinos, photons, and gluons. However, they are all electrically neutral, so any combination of them have zero total charge. Therefore, there is nothing that the electron can decay to. For short, because the electron is the lightest particle that has an electric charge, it is stable.

The proton is also stable in the SM. However, it is not in many of its extensions as in Grand Unified Theories [35, 36]. Therefore, probably the proton is not stable but long-lived. They are so long-lived that at most a minuscule fraction of them have decayed since the Big Bang. The other rather long-lived particle is the neutron, which when on its own, outside an atomic nucleus, lives just 15 minutes or so. But neutrons inside many atomic nuclei can live far longer than the age of the universe. The other neutrinos are also long-lived and hence stable for practical purposes.

Now we see that it is not a coincidence that we end up in electrons, and quarks bound in protons and neutrons when we dive in the constituents of your body. But the list of stable elementary particles (6) also includes the photon. We are used to deal with the photon field in our everyday lives, and we encounter it as a coherent state in the form of light. Light does not form complex structures as the atoms. However, without photons, we would not be able to see anything. It is the fact that the light propagates almost freely thru space that allows us to get so much information about objects by looking at them. We see therefore that the photon and the long-lived stable atoms seem to correspond to the old-fashioned answer that things are made either of matter or radiation. But what about neutrinos?

Neutrinos are more like photons in the sense that they do not form complex structures as atoms do. But more fundamentally he is more like the electron and the quarks because all of them are fermions. The neutrino interacts only via weak and gravitational force, which are the weakest forces in the SM. The weak force has a very short range, the gravitational interaction is extremely weak, and neutrinos do not participate in the strong or electromagnetic interaction. Thus, neutrinos typically pass through normal matter unimpeded and undetected. That is why there is no old fashioned word (such as matter and radiation) that corresponds to them: they where discovered recently in human history. Neutrinos are created by various radioactive decays. The majority of neutrinos that are detected about the Earth are from nuclear reactions inside the Sun. At the surface of the Earth, the flux is about 10^{10} solar neutrinos, per second per square centimeter. And interestingly, the radioactive decay that happens in a banana already produces around 10^6 per day.

From an earthly perspective, neutrinos may seem to be unimportant. We are made of atoms and we see because of light, so matter and radiation are certainly very important for our lives and our existence. Neutrinos, on the other hand, seem to not affect us as they typically pass through everything unimpeded. However, without neutrinos stars would not burn and would not form the heavy atoms that formed the earth, give emergence to life which evolved to us. Therefore, taking a cosmological perspective we see that they are indeed very important.

Masses from oscillations

The problem with neutrino masses is that in the SM they are massless but we know they are not. So let's expand a bit on the fact that in the SM they are massless. With the identification of the fermions inside the fermion fields (1.8) it is clear that there can be no neutrino masses because there is no ν_R to form a Dirac mass term and because it is not possible to make a Majorana mass term with ν_L. This argument however relies on the fact that we have already identified the components inside the fermion fields. A somewhat more generic argument can be made. By counting the number of degrees of freedom associated with Weyl fermions inside the fermion fields (4) of the SM, we find that there are $|Q|+|D|+|L|+|U|+|E| = 2\times 3+3+2+3+1 = 15$ per family. A Dirac mass term is made using two Weyl fermions therefore, it is already clear that we can have at most $\lfloor 15/2 \rfloor = 7$ Dirac mass terms per family in the SM. For the first family, these are the electron, 3 colors of up and down quark. There is left one Weyl fermion which in principle could be used to make a Majorana mass term. However, there is no way to make Majorana mass terms from the fermion fields of the SM. Therefore, there can be no mass term for neutrinos. This Weyl fermion counting argument shows already that the solution to the neutrino mass problem will need to introduce new Weyl fermions to generate neutrino masses, or other fields in a precise representation of G_{SM} in such a way that a Majorana mass term is allowed.

Although theoretically, neutrinos are massless, we know empirically that they are not. We know that because neutrino oscillations have been firmly established [37, 38, 39] which implies that neutrinos are massive particles.

First of all, what are neutrinos oscillations? Roughly is the following. Neutrinos have three flavors: there are the electron, the muon, and the tau neutrino. Each of these neutrinos is produced by a different process that we see in nature. The discovery of neutrino oscillations was the discovery that a fraction of neutrinos produced by a process that produces just a particular flavor of neutrinos, after traveling a long distance, where detected as another type of neutrino. This change from one flavor to another as neutrinos travel thru space is what is know as neutrino oscillations. What oscillates is the flavor of neutrinos as they fly.

A second question is why neutrino oscillations imply that neutrinos are massive. Oscillations come about because the flavor eigenstates are not the same as the mass eigenstates. When a neutrino of definite flavor is produced it is a superposition of different mass eigenstates and each of which evolves differently as it propagates thru space depending on the mass. This different evolution of the different mass eigenstates appears as an oscillation in the flavor basis. Due to the implication of the existence of neutrinos masses, the experimental discovery of neutrino oscillation, and thus neutrino mass, by the Super-Kamiokande Observatory and the Sudbury Neutrino Observatories was recognized with the 2015 Nobel Prize for Physics [40].

To better understand how neutrino having masses explains neutrino oscillations consider two families of neutrinos which are related to neutrino mass eigenstates by a generic orthogonal matrix, that is

$$\begin{pmatrix} \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_2 \\ \nu_3 \end{pmatrix}. \tag{1.9}$$

The mass eigenstates are the eigenstates of the free Hamiltonian and not the flavor eigenstates. In the ultrarelativistic limit, one can show that after propagating a distance L, the initial mass eigenstates $|\nu_i\rangle$ become

$$|\nu_i(L)\rangle = \exp\left(-i\frac{m_i^2 L}{2E}\right)|\nu_i(0)\rangle\,. \tag{1.10}$$

Then it is straightforward to show that the probability of oscillation from one flavor to the other is given by

$$P(\nu_\mu \to \nu_\tau) = |\,\langle\nu_\mu(L)|\nu_\tau\rangle\,|^2 = \sin^2 2\theta \sin^2\left(\frac{\Delta m^2 L}{4E}\right). \tag{1.11}$$

Although dealing with only two flavors, these formulas are good approximations for different situations. This formula is often appropriate for discussing the transition $\nu_\mu \leftrightarrow \nu_\tau$ in atmospheric mixing since the electron neutrino plays almost no role in this case. It is also appropriate for the solar case of $\nu_e \leftrightarrow \nu_x$, where ν_x is a superposition of ν_μ and ν_τ. Also, this simple model shows the parameters that can be measured from neutrino oscillation. We see that the probabilities of oscillations depend on $\Delta m_{ij}^2 = m_i^2 - m_j^2$. Therefore, the measurement of neutrino oscillations puts bounds on the masses of neutrinos. Cosmology is also an avenue to put such bounds.

Besides giving masses to neutrinos, another and related problem is to understand why neutrino masses are so small compared with the other SM particles. From the observations of neutrino oscillations, we can find the difference between their squares and we can put bounds on their masses from this. Presently we know that $m_\nu < eV$. Comparing with the electron which is the lightest charged lepton of the SM with a mass of $0.5MeV$, we see that neutrinos are at least 10^6 times lighter. If neutrino mass is proportional to the Higgs boson vacuum expectation value

$$m_\nu = y_\nu v \tag{1.12}$$

as the other SM particles, since $m_\nu < 0.1eV$ and $v \approx 100GeV$, this would imply that $y_\nu < 10^{-12}$. The fact that this Yukawa coupling would be so small is interpreted as unnatural and motivates the introduction of a mechanism that circumvents the attribution of such a small number. We should add that fermion masses although unnatural are technically natural because there is an enhanced symmetry when their masses are zero: chiral symmetry. This is not generically the case for scalar masses and gives rise to the hierarchy problem which we will see in section (1.3).

From what was presented above we see that neutrino masses are a central theme of present particle physics. It is arguably, an incontestable evidence of the existence of physics beyond the SM. There are different proposals that address this problem. Now we will briefly present a particular one, the seesaw mechanism. Our interest is because it will be incorporated in our model and it is probably the most well-known mechanism that gives neutrino masses.

Seesaw mechanism

The seesaw mechanism in its three incarnations, the type-I [41, 42, 43, 44, 45, 46], type-II [46, 47, 48, 49], and type-III [50], gives an explanation for why neutrino masses are nonzero and so small compared with the other fermions of the SM. In this section, we will present the seesaw mechanism type-I, and then the type-II which is the one we will incorporate in our model to address the neutrino mass problem.

When we presented the problem of neutrino masses, we anticipated that a solution would need to introduce the right-hand component of the neutrino field or a new fermion field in a particular representation of $G_{\rm SM}$ such that a Majorana mass term is allowed. The seesaw type-I goes in the first direction by introducing new fermion singlets which becomes the right-handed component of the neutrino field. The type-II goes in the other direction introducing a scalar triplet that allows a Majorana mass term for ν_L. For simplicity, we will describe these two mechanisms for a single family of the SM (see [51] or [52] for more details).

The seesaw mechanism type-I [41, 42, 43, 44, 45, 46] works by extending the SM with a new fermion field ν_R which is a singlet under $G_{\rm SM}$. This extension is appealing because it enhances the structural symmetry between quarks and leptons. This additional fermion field allows us to write a Yukawa term that generates mass to the neutrinos with the spontaneous breaking of electroweak symmetry in complete analogy to what happens with the up quark. However, a singlet under $G_{\rm SM}$ may be equal to its anti-particle and therefore can have a Majorana mass. More precisely, with the additional degree of freedom

Scalar field	SU(3)	SU(2)	U(1)
$(\nu_R)^c$	1	**1**	0

Table 7: Right-handed neutrino and its representation under $G_{\rm SM}$. This SM extension generates neutrinos masses by the seesaw mechanism.

it is possible to write the following Yukawa and mass term

$$\mathcal{L}_{\nu \text{ mass}} = Y_\nu \overline{L}^T \sigma_2 H^* \nu_R + \frac{1}{2} M \nu_R^T C \nu_R + h.c. \tag{1.13}$$

Introducing

$$\nu = \nu_L + C\overline{\nu_L}^T, \tag{1.14}$$

$$N = \nu_R + C\overline{\nu_R}^T, \tag{1.15}$$

with electroweak symmetry breaking we get

$$\mathcal{L}_{\nu \text{ mass}} = \frac{1}{2} m(\overline{\nu}N + \overline{N}\nu) + \frac{M}{2}\overline{N}N \tag{1.16}$$

where $m = Y_\nu v$. The mass matrix for ν and N is

$$\begin{pmatrix} 0 & m \\ m & M_R \end{pmatrix}. \tag{1.17}$$

The eigenvalues for this matrix are

$$m_1 = \frac{1}{2}(M + \sqrt{M^2 + 4m^2}), \tag{1.18}$$

$$m_2 = \frac{1}{2}(M - \sqrt{M^2 + 4m^2}). \tag{1.19}$$

If $M \gg m$, the eigenvectors are approximately equal to N and ν with eigenvalues

$$m_1 \approx M, \tag{1.20}$$

$$m_2 \approx -\frac{m^2}{M}, \tag{1.21}$$

respectively. The field N is then identified with a heavy fermion and ν to the neutrino. Formula (1.21) then motivates the name of the mechanism since if m_1 gets up then m_2 gets down. I should add that it is necessary at least two singlets to account for both solar and atmospheric neutrino mass differences.

The seesaw mechanism type-II [46, 47, 49, 48] extends the SM with a scalar triplet Δ of $SU(2)$ with hypercharge $Y = 2$. With this additional scalar field, the following term is allowed by the symmetry group of the SM

$$\mathcal{L}_{\nu \text{ mass}} = Y_\Delta L^T C \sigma_2 \Delta L + h.c. \tag{1.22}$$

Neutrinos get a mass when the triplet gets a VEV

$$m_\nu = Y_\Delta \langle \Delta \rangle. \tag{1.23}$$

The VEV results from the scalar interaction with the Higgs

$$\Delta V = b H^T \sigma_2 \Delta^* H + M^2 \mathrm{Tr} \Delta^\dagger \Delta + \lambda (\mathrm{Tr} \Delta^\dagger \Delta)^2. \tag{1.24}$$

Before starting the next subsection, note that I answered “where are the other particles?” saying that now they are nowhere. The “now” in the answer is not an unimportant detail because depending on the time you take into consideration, the answer could be “everywhere”. The reason is that in the past the universe was much hotter than it is today. At high temperatures, there is a higher probability for non-stable particles to be created by energy collisions as happens today in particle accelerators. Depending on the temperature, the time it takes for a non-stable particle to be created could be smaller than the time it take to decay. Therefore, the early and hot universe was filled with SM particles, even the non-stable ones. As we will see, this fact will put constraints in the model we will consider in the next section. However, to better understand these issues we need to introduce some concepts to talk about the cosmological history of the universe, which we will do next.

1.2 Dark Matter and Cosmology

What is there?

What is there? The Milkway, the Andromeda galaxy, the Solar System, the Sun, the Moon, Earth’s atmosphere, Himalaia, human societies, you, me, and many more things. All these things are so diverse that it may seem impossible to compare all of them reasonably. However, because things are made of more elementary constituents, a simple way to compare two things is by counting their parts and comparing these numbers. For example, you are made of about 10^{27} atoms and the sun of about 10^{57}, therefore we could say that you are a part in 10^{30} of the sun. Although this procedure works for things made approximately of the same constituents, it does not allow us to compare things of different kinds, such as you and the radiation emitted by the sun. Therefore we may wonder if there is a more general way to compare things.

The answer is yes, because any two things can be compared with respect to the amount of energy they contain [53]. For example, the energy of a photon is equal to its wavelength, and your energy is equal to mc^2 with m your mass. Using this two facts we can compare the total energy radiated by the sun in one second which is approximate 10^{26} Joules, with your energy which is about 10^{17} Joules. In this sense you are a part in 10^9 of the radiation emitted in a second by the sun. More generally, the fact that everything can be compared in terms of energy allow us to look to the universe as a certain amount of energy and to pose questions as, how much of the universe is a particular kind of thing and how much is another.

Rather generically, the energy of something is equal to the energy of its parts plus the binding energy that keeps the parts together. An interesting fact is that for most things, the binding energy is much smaller than the energy of its constituents. The sole exception we know is the proton and neutron. The energy of the proton and the neutron is about $1GeV$ and the sum of the energy of their constituents is about 10^3 smaller, around $1MeV$. For these baryons, the binding energy is many orders of magnitude larger than the energies of their constituent quarks, and this has to do with confinement and the strength of the strong force. This should be contrasted with complex objects built with these baryons. The binding energy of the hydrogen atom is around $13eV$ which is 10^6 smaller than the energy of its constituent proton. The binding energy of molecules is also much smaller than the energy of the protons and neutrons that makes them. The consequence of this fact is that the total energy of a particular thing made of baryons, such as you, your book, planet Earth, etc. is approximately equal to $1GeV$ times the number of baryons that make it. For example:

$$E(\text{you}) \approx \#(\text{barions } \in \text{ you}) \times GeV \tag{1.25}$$

As showed in the beginning of the previous subsection, apart from ordinary objects which are made of baryons and electrons, the other stable particles of the SM are the photon and neutrinos. Photons and neutrinos do not bound in complex objects as baryons and electrons do, therefore the total energy of a collection of photons and neutrinos is simply the sum of their energies.

From these arguments, if the universe is made just of SM particles then a rough prediction is that

$$E(\text{universe}) \approx \#(\text{barions } \in \text{ universe}) \times GeV + \sum_{\text{photons}} E(\text{photons}) + \sum_{\text{neutrinos}} E(\text{neutrino}) \tag{1.26}$$

However this is not true. In fact, at present, the contributions of photons and neutrinos to the total energy density of the universe is negligible, around 5% comes from baryons, 25% is made of Dark Matter, and the other 70% is Dark Energy. Now we will make a summary of how we come to this knowledge.

How to know the existence of something we don't know

For short, we know Dark Matter exists because of gravity. In the theory of General Relativity the

Einstein field equations

$$R_{\mu\nu} + Rg_{\mu\nu} = 8\pi G T_{\mu\nu} \tag{1.27}$$

relates the geometry of spacetime $R_{\mu\nu} + Rg_{\mu\nu}$ with the distribution of energy within it $T_{\mu\nu}$. This statement condenses the reason why physicists were able to discover that the universe is not made just of SM particles. Because energy is a universal property, by measuring parameters associated with the geometry of spacetime we can discover what the universe is made of.

The early indications of the existence of DM where based on the comparison of our predictions for the movement of bodies based on what we could see, and their actual movement. For example, astronomers can infer the location of celestial bodies by looking at them and their masses from luminosity. Using these two inputs in a theory of gravity one can predict the subsequent movement of these celestial bodies including their velocities. However, using the Doppler shift, astronomers can also measure the velocity of these objects and compare them to the velocities predicted from their positions and masses. Roughly this procedure allowed physicists to discover indications of DM in different scales from galaxies [54] to clusters of galaxies [55, 56] (see [57] for more detail).

Another way to find dark matter comes from a deeper understanding of gravity. In the theory of general relativity massive objects curve spacetime and by doing so they bend the trajectory of light. They work this way as a gravitational lens. The signature of this phenomena are ellipses of light in the sky. By measuring this ellipses, astronomers can infer the amount of mass in a given region of space and this had provided yet another observation of DM [58].

This indications of DM should be contrasted with the evidences that come from what is sometimes called the era of high precision cosmology [59]. In part, this era started when physicists started to understand some history. Not human history, but the history of the universe. Two events in the cosmological history of the universe provide a huge part of the information we have about the early universe, and also about the different energy densities that fill our universe. These are Big Bang Nucleosynthesis (BBN) and the releasing of the Cosmic Microwave Background (CMB). These two events also provide most of the constraints on the model we will soon present, therefore we will take the chance to explain in some detail what they are and to define the parameters that will appear after. However, to understand the CMB and BBN we need to have some understanding of the cosmological history of the universe which is attained by the ΛCDM model.

ΛCDM model

Our understanding of the cosmological history of the universe is built from postulating two properties of space in the large-scale universe:

Homogeneity
Isotropy

Table 8: Postulates of modern cosmology concerning space.

These postulated properties of space imply that the large-scale universe is described by the Robertson-Walker metric [60, 61, 62, 63]

$$ds^2 = -dt^2 + a(t)^2 \left(\frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right), \tag{1.28}$$

with $a(t)$ the scale factor and $k = 1$, 0, or -1 depending on whether the universe is a closed 3-sphere, flat, or an open 3-hyperboloid, respectively.

The Robertson-Walker metric is a kinematical consequence of homogeneity and isotropy. To understand the dynamics of spacetime we need Einstein field equations (1.27) and the form of the energy-momentum tensor. Assuming the perfect fluid form for the energy-momentum tensor and imposing that it is homogeneous and isotropic leads to

$$T_{\mu\nu} = \rho \delta^0_\mu \delta^0_\nu + p g_{ij} \delta^i_\mu \delta^j_\nu \tag{1.29}$$

with ρ and p respectively the energy density and the pressure in the rest frame of the fluid, g_{ij} the spacial part of the Robertson-Walker metric (1.28) and δ the ordinary Kronecker deltas. With this form for the energy-momentum tensor it is possible to derive the Friedmann equation and the evolution equation [62, 63], respectively:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\sum_i \rho_i - \frac{k}{a^2}, \tag{1.30}$$

$$\frac{\ddot{a}}{a} + \frac{1}{2}\left(\frac{\dot{a}}{a}\right)^2 = -4\pi G\sum_i p_i - \frac{k}{2a^2}. \tag{1.31}$$

where ρ_i is the energy density of the i type of thing that makes the universe. The Friedmann equation relates the rate of increase of the scale factor, as encoded by the Hubble parameter

$$H \equiv \frac{\dot{a}}{a}. \tag{1.32}$$

to the total energy density of the universe. The Hubble parameter relates how fast the most distant galaxies are receding from us to their distance from us via Hubble's law $v \approx Hd$.

The energy density is in general expressed in terms of the density parameter

$$\Omega_i = \frac{\rho_i}{\rho_c}, \tag{1.33}$$

where

$$\rho_c = \frac{3H^2}{8\pi G}, \tag{1.34}$$

which would be the total energy density if spacial sections where flat ($k = 0$), as one can check from the Friedmann equation. As we will see, precision measurements of the cosmic microwave background, show the universe today to be extremely spatially flat. Therefore

$$\sum_i \Omega_i \approx 1 \tag{1.35}$$

and Ω_i ends up being almost equal to the fraction of each component of the energy of the universe.

An additional important feature of the ΛCDM model is cosmological inflation [64, 65, 66, 67]. Inflation is a period shortly after the Big Bang of exponential expansion of space driven by the inflaton field. It is somewhat controversial for its ad hoc introduction, but currently it is quite established for its importance in explaining the large-scale structure of the universe, the horizon, flatness and monopole problems.

In a universe with flat spacial section ($k = 0$) we can use the Friedmann equation to calculate the functional form of $\rho(a)$ and $a(t)$ for the most relevant types of things that exist in our universe. The table below shows these results:

Type of energy	$\rho(a)$	$a(t)$
Matter	a^{-3}	$t^{2/3}$
Radiation	a^{-4}	$t^{1/2}$
Dark energy	constant	e^{Ht}

Table 9: Energy density as a function of a and a as a function of time for different types of energy.

For our universe which is almost flat, these expression allow us to rewrite the Friedmann equation as

$$H(a) \equiv \frac{\dot{a}}{a} = H_0\sqrt{(\Omega_b + \Omega_c)a^{-3} + \Omega_r a^{-4} + \Omega_\Lambda}, \tag{1.36}$$

which can be integrated to give the expansion history $a(t)$ for any chosen values of the cosmological parameters. These expression shows very explicitly that the different components of energy of the universe have observational impacts in the geometry of spacetime. The fit of the values for the density parameters in the ΛCDM model are [5, 68, 69]

$$\Omega_\Lambda = 0.685(7), \tag{1.37}$$

$$\Omega_c = 0.265(7), \tag{1.38}$$

$$\Omega_b = 0.0493(6), \tag{1.39}$$

$$\Omega_\gamma = 5.38(15) \times 10^{-5} \tag{1.40}$$

The relative amount of each energy component justifies the name of the model: the universe in the ΛCDM model contains three major components: first Ω_Λ, a cosmological constant associated with dark energy; second Ω_{DM}, the postulated cold dark matter (CDM); and third Ω_B, ordinary matter. This is the formal counterparts of what I told before: at present the contributions of photons and neutrinos to the total energy density of the universe is negligible, around 5% comes from baryons, 26.5% from Dark Matter, and the other 68.5% from dark energy.

In addition to showing in greater depth how dark matter fits into our understanding of the universe, we introduced these concepts because the energy density of non-visible radiation will be one of the main constrains to the model we will present in the next section. So we also want to show where ρ fits in the current cosmological model for our universe. As anticipated, the constraints to dark radiation comes mainly from BBN and CMB, which are two of the most important sources of information of the early universe. For that reason in the next two paragraphs we will give a briefly summary of them.

BBN is a period from a few seconds to a few minutes after inflation in the early universe when neutrons and protons bound together in deuterium, helium, lithium and other light elements. This event in the early universe relates present-day deuterium to hydrogen ratio D/H abundance and the overall density of baryons in the universe. Therefore, by measuring the D/H abundance physicists can estimate the energy density of baryons in the universe Ω_b. The formation of these elements happened in the photon-baryons plasma, therefore the amount of photon impact directly these ratios. In particular, if the energy density in radiation has a fraction of dark radiation, that is, some form of relativistic energy such as neutrinos, that behaves differently from photons, this has effects in these ratios.

CMB is electromagnetic radiation released in an early stage of the universe. There was a time after the Big Bang when the universe was an extremely dense plasma of charged particles and photons. At this time photons have a very short mean free path because they interact with the charged particles. However, the universe expanded and cooled until it reached what is known as the epoch of recombination. At the epoch of recombination the charged particles that made the plasma bound together in neutral atoms and the universe became transparent to electromagnetic radiation. The photons released from this last scattering exist today as the CMB. The CMB is a nearly perfect blackbody with a temperature of about $2.7255(6)K$ [5, 70]. Although the CMB is extraordinarily uniform but it have some anisotropies (fluctuations). The fluctuations in the CMB are indications of initial density perturbations that allowed for the formation of early gravitational wells as well as the dynamics of the photon-baryon fluid. In this manner, the temperature fluctuations of the CMB are dependent on the baryons and radiation energy density in the universe at the time of recombination. In fact, different features of the CMB power spectrum constrains essentially all of the cosmological parameter [71, 72].

For all that was said, to understand the nature of dark matter is one of the central problems of particle physics. Next, we will present a particular framework that tries to address this problem. Later we will see how this framework is incorporated into our model.

Asymmetric Dark Matter

We will present the framework of Asymmetric Dark Matter (ADM) [73, 74, 75, 76, 77, 78] which are naturally accomodate in the twin Higgs framework that we will present in the next section. For more detailed reviews see [79, 80, 81].

The motivation for ADM comes from the observation that DM and baryon abundances are very close to each other observationally [5, 68, 69], that is

$$\rho_{DM} \approx 5\rho_B. \tag{1.41}$$

In many models describing the early universe, this coincidence is just an accident and these two quantities are unrelated. For example, in baryogenesis [82, 83, 84], the baryon density is set by CP-violating parameters and out-of-equilibrium dynamics associated with baryon number violating processes. On the other hand, in the WIMP freeze-out DM paradigm [85, 86] this density is fixed by the annihilation cross-section of DM. While it is possible that this coincidence is an accident, or that this ratio is set anthropically, dynamics may also play a role. The ADM hypothesis simply states that the present-day DM density is similarly due to a DM particle-antiparticle asymmetry. The DM density is then set by its asymmetry, which can be directly connected to the baryon asymmetry

$$n_X - n_{\overline{X}} \approx n_b - n_{\overline{b}}. \tag{1.42}$$

rather than by its annihilation cross-section. Because (1.41) and (1.42) a prediction of the asymmetric dark matter framework is that $m_{DM} \approx 5m_b \approx 5GeV$.

Asymmetric dark matter is a particular framework to pursue models that address the dark matter problem which is motivated by the comparable densities of dark matter and ordinary matter. For all that was said, to understand the nature of dark matter is one of the most salient problems of particle physics. In section (2) we will explain how dark matter can be understood in a twin Higgs model, a framework that addresses the hierarchy problem which we will discuss next.

1.3 The Hierarchy Problem

Nature is like an onion

At the beginning of this section, we introduced the elementary particles of the SM with the warning that they are not necessarily elementary in the sense of not being made of other objects. I adverted that is possible that we see no structure in these elementary particles because the energies involved in their binding are larger than the available in the experiments designed to break them. The meaning of these sentences can be grasped intuitively, however, deep theoretical arguments are underlying it and constitute one of the most significant recent developments of theoretical physics [87, 88, 89].

Before a more formal presentation, let's expand a little on the intuitive notions involved by giving an example. If you let the apparatus you are using to read this dissertation to fall on the ground from rest, its perpendicular motion will be approximately described by

$$h(t) = h_0 - \frac{1}{2}gt^2 \tag{1.43}$$

with h_0 the height your apparatus is from the ground, $g \approx 10m/s^2$ the acceleration of gravity. One may wonder, why g is constant? Newton's law of gravitation teaches us that the acceleration is proportional to the distance square, and not constant. Furthermore, we know Newton was overcome by Einstein, so we should use Einstein's field equations (1.27) instead. But as mentioned in the introduction, Einstein's theory of gravity breaks when we extrapolate it to arbitrarily high energies, so by this reasoning we should use the correct theory of gravity, which may be string theory.

The fact that we don't need to use the final theory of gravity to find out the approximate trajectory of your apparatus if you let it fall, highlights a profound fact about nature which in turn had allowed the development of science. Nature is like an onion made of different layers, and each layer gives rise to patterns that are approximately insensitive to the details of the more interior layers. This aspect of nature allows us to built our understanding of nature by dissecting these different layers and exploring their independent features as we move in. After understanding the dynamics of a layer we move to the next and then we can understand how the specificity of the inner layer produces small corrections to the other one. This intuitive idea of nature as an onion can be made precise with the renormalization group formalism. Instead of layers, we have energy scales, with higher energies corresponding to more internal layers.

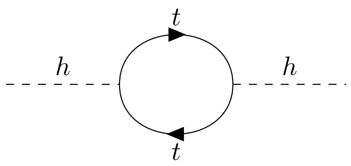
Following these developments of theoretical physics, the modern view on the SM is that it is an Effective Field Theory (EFT) [90, 91, 92, 93]. In addition to the two ingredients that define a QFT presented before (2), in an EFT we also have a certain energy interval bounded above by the theory cutoff Λ in which the EFT is valid. In an EFT the Lagrangian includes high-order operators and not just renormalizable, the cutoff divide appropriately these non-renormalizable terms whose effects are suppressed when considering process below the cutoff. Energies above this domain are generically called ultraviolet (UV), and many times one imagines that there is a UV theory for which the EFT is an effective description. This UV theory may be another EFT but it is not necessarily a QFT. For example, Fermi theory is an EFT and the SM is its UV completion which is also an EFT. The UV completion of the SM can be string theory which is not a QFT.

Why the Higgs is light?

Having introduced the idea of EFT we are in a position to explain the electroweak hierarchy problem. It has to do with the sensitivity of the mass of a scalar to UV physics combined with the discovery that the Higgs mass is much smaller than the apparent cutoff Λ of the SM. An indication of the problem can be seen by calculating the quantum corrections δm^2 to the Higgs mass parameter.

$$m_h^2 = m_0^2 + \delta m^2 \tag{1.44}$$

Considering the SM as an effective field theory up to some scale Λ and using cutoff regularization to calculate the quantum correction one can find that the biggest contribution comes from the top loop which gives



$$= -\frac{3y_t^2}{8\pi^2}\Lambda^2. \tag{1.45}$$

Therefore

$$\delta m^2 \supset \frac{3y_t^2}{4\pi^2}\Lambda^2 \tag{1.46}$$

which depends quadratically on the cutoff. We see that if the cutoff Λ is greater than a few TeV we have δm^2 much bigger than the value found experimentally for the Higgs mass parameter $m^2 = (90GeV)^2$. Then the bare mass parameter must be finely tunned against the quantum corrections to reproduce the observed value of the Higgs mass.

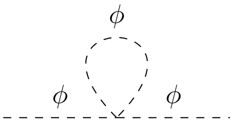
As mentioned before, the fact that quantum corrections are quadratic with the cutoff should be regarded as an indication of the sensitivity of the Higgs mass parameter to UV physics. That is because, in a sensible renormalization scheme that factorizes scales properly, such as dimensional regularization, there are no Λ^2 loop contributions. However, imposing a hard cutoff in momentum space can be thought of as a modification of the UV physics that signals this sensitivity. In a sense, this is the bottom-up argument for the hierarchy problem.

We can better appreciate the hierarchy problem from a top-down perspective by considering the following toy model (see [93] for more detail). Consider the theory that describes a light scalar ϕ with mass m coupled to a heavy-fermion ψ with mass $M \gg m$:

$$S = \int d^4x \left[\frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{1}{4!}\lambda\phi^4 + \overline{\psi}i\partial\!\!\!/\psi - M\overline{\psi}\psi + y\phi\overline{\psi}\gamma^5\psi\right]. \tag{1.47}$$

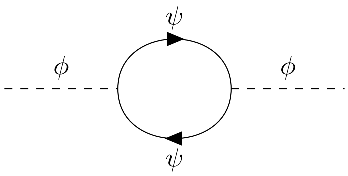
Because $M \gg m$, we can integrate out the heavy-fermion and obtain an effective field theory for the scalar which is valid for energies $E \ll M$. The tree-level effective action S^0_{eff} for the scalar is equal to the action (1.47) with $\psi = 0$. To calculate the quadratic part of the 1-loop effective action $S^{(1)}_{\text{eff}}$, we need to compare the 1-loop correction to the scalar propagator obtained using the tree-level EFT and the full theory.

In dimensional regularization, the 1-loop correction to the scalar propagator obtained using the tree level EFT yields



$$= -\frac{i\lambda\widetilde{\mu}^{\varepsilon}}{2}\int\frac{d^{4-\varepsilon}k}{(2\pi)^{4-\varepsilon}}\frac{-i}{k^2+m^2-i\epsilon} = \frac{i\lambda}{32\pi^2}m^2\left[\frac{2}{\varepsilon}+1+\ln\left(\frac{\mu^2}{m^2}\right)+\mathcal{O}(\varepsilon)\right] \quad (1.48)$$

In the full theory, this same diagram appears and therefore it drops out when comparing the 1-loop corrections of the two theories. Therefore, the 1-loop corrections to the effective action must give the same as the diagram with a fermion running in the loop

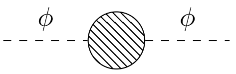


$$= -(-y)^2\int\frac{d^4k}{(2\pi)^4}\mathrm{Tr}\left[\frac{-i(-\not{k}-\not{p}+M)}{(k+p)^2+M^2-i\epsilon}\gamma^5\frac{-i(-\not{k}+M)}{k^2+m^2-i\epsilon}\gamma^5\right] \quad (1.49)$$

Recalling that γ^5 is not well defined for $d \neq 4$ we need to eliminate it using $(\gamma^5)^2 = 1$ and $\gamma^5\not{p}\gamma^5 = -\not{p}$ before starting dimensional regularization. Then this diagram gives

$$-\frac{iy^2}{4\pi^2}\left\{\frac{1}{\varepsilon}(p^2+2M^2)+\frac{p^2}{2}\ln\left(\frac{\mu^2}{M^2}\right)+m^2\left[1+\ln\left(\frac{\mu^2}{M^2}\right)\right]+\mathcal{O}(\varepsilon, M^{-2})\right\} \quad (1.50)$$

and using $\overline{\mathrm{MS}}$ scheme we get



$$= -\frac{iy^2}{4\pi^2}\left\{\frac{p^2}{2}\ln\left(\frac{\mu^2}{M^2}\right)+m^2\left[1+\ln\left(\frac{\mu^2}{M^2}\right)\right]+\mathcal{O}(\varepsilon, M^{-2})\right\}. \quad (1.51)$$

From this result, we see that the quadratic part of the effective action at 1-loop is given by

$$S_{\text{eff}}^{(0)} + S_{\text{eff}}^{(1)} = \int d^4x\left[-\frac{1}{2}\mathcal{Z}(\mu)\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}\mathcal{M}^2(\mu)\phi^2 + ...\right] \quad (1.52)$$

with Wilson coefficients

$$\mathcal{Z}(\mu) = 1 + \frac{y^2}{8\pi^2} \ln\left(\frac{\mu^2}{M^2}\right), \tag{1.53}$$

$$\mathcal{M}^2(\mu) = m^2 + \frac{y^2}{4\pi^2} M^2 \left[1 + \ln\left(\frac{\mu^2}{M^2}\right)\right]. \tag{1.54}$$

These results show that the scalar mass is proportional to M^2 and therefore quadratically sensitive to the UV scale, which in this case is equal to the fermion mass M. By fine-tuning the three-level and loop contributions at a particular scale μ one can circumvent this problem. However, such fine-tuning would not survive if we changed the renormalization scale by a relative factor of $O(1)$, and is thus regarded as "unnatural". This fine-tunning would be worst if we add other heavy fermions for example. In any case, this simple toy model shows that, in a sense, a natural expectation is that the Higgs mass parameter is proportional to the square of the cutoff Λ of the SM. In this example, the cutoff is given by the mass of the heavy-fermion, but rather generic new physics would give similar contributions. Even more generally, if there is new physics that renders the Higgs mass calculable, then the SM fields will also give contributions of this sort.

This toy model shows very explicitly that the scalar mass parameter has a quadratic dependence on the fermion mass which defines the cutoff scale Λ for this low energy EFT. As mentioned before, the quadratic divergence (1.46) can be thought of as a stand-in for the finite threshold corrections. Or conversely, we can argue that from the bottom-up perspective, the quadratic divergences are a handy way to estimate the effects of new physics.

At this point, we presented the fact that a scalar mass is quadratically sensitive to UV physics. Follows from this fact that a natural expectation is that the mass of any scalar is of the order of the cutoff of its EFT. In the SM the Higgs is a scalar and its mass is $125GeV$. However, we know that the SM is valid up to energies of the order of TeV, therefore its cutoff seems to be at least this high. The failure of this prediction for the Higgs mass is the hierarchy problem.

The electroweak hierarchy problem sometimes called the little hierarchy problem, associated with the stabilization of the Higgs mass to TeV scales, should be contrasted with the big hierarchy problem, which requires the stabilization of the Higgs mass beyond. Solutions to the big hierarchy in general involve supersymmetry or compositeness. However, these solutions to the big hierarchy problem, seems to require low energy model building to properly address the little hierarchy problem [94]. As we will see, the Twin Higgs model is an attempt to solve the little hierarchy problem that can be incorporated in both supersymmetric and composite Higgs models to address the big hierarchy problem.

The electroweak hierarchy problem can be converted into a strategy for new physics. We imagine that the Higgs mass is natural because the theory changes not far from the weak scale. This motivates the search of models in which the quadratic divergences are canceled by some mechanism. We are going to do this next.

2 Twin Higgs

In the previous section, we saw some of the problems of the SM, now we will focus on solutions starting with the hierarchy problem. We saw that the hierarchy problem arises from the combination of the fact that the Higgs mass parameter is sensitive to UV physics and the discovery that the Higgs mass is $125 GeV$. A solution to this problem would need to explain why this mass is apparent so much smaller than the cutoff of the SM which seems to be at least of order TeV.

Theories beyond the SM that address the hierarchy problem generally involve new particles that cancel the top quark quadratic divergences (1.46) which are called top partners. If top partners are charged under the SM color group they are subjected to stringent limits from searches at the LHC [95, 96, 97, 98, 99, 100, 101]. Therefore, besides having a mechanism that explains why the Higgs mass is smaller than 1 TeV, a valid solution to the hierarchy problem should account for the fact that top partners were not found by LHC.

Twin Higgs models [102, 103, 104, 105] combine two ideas to address the hierarchy problem and the associated problem related with top partners:

$$\mathbb{Z}_2 \qquad \left\{ \boxed{\begin{array}{c}\text{The Higgs is a pseudo-Nambu-Goldstone boson}\\[1em] \text{Neutral naturalness}\end{array}} \right\}$$

Table 10: Main idea of twin Higgs models. The fact that the Higgs is a pseudo-Nambu-Goldstone boson is combined with neutral naturalness using a $\mathbb{Z}_2$ symmetry.

Neutral naturalness [106, 107] is the framework where top partners carry no charge under the strong interactions and therefore avoid stringent limits from LHC. In the next subsection, we will explain what it means to have the Higgs as a pseudo-Nambu-Goldstone boson (pNGB).

2.1 Nambu-Goldstone bosons

Nambu-Goldstone bosons (NGBs) arise whenever a continuous global symmetry is spontaneously broken. If the symmetry is exact, the NGBs are exactly massless and have only derivative couplings. The simplest example is provided by the theory of a complex scalar. The Lagrangian is given by

$$\mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi + m^2 \phi^* \phi + \frac{1}{2}\lambda^2 (\phi^* \phi)^2 \tag{2.1}$$

which is invariant under the $U(1)$ symmetry

$$\phi \to e^{i\alpha}\phi, \qquad \phi^* \to e^{-i\alpha}\phi^* \tag{2.2}$$

for $\alpha \in \mathbb{R}$.

If m^2 is positive, the minimum of the potential is at $\phi = 0$. The vacuum is unique, the theory is in the unbroken phase and there are two elementary excitations with mass m corresponding to the real and imaginary components of ϕ. If m^2 is negative, the minimum of the potential is at

$$|\phi|^2 = v \equiv -\frac{m^2}{\lambda^2}. \tag{2.3}$$

The vacuum is not unique, because there is a continuous set of vacuum states related by each other by a phase rotation. All these vacua are physically equivalent.

Near the vacuum, the field ϕ can be represented as

$$\phi(x) = v + \frac{1}{\sqrt{2}}\varphi(x) + \frac{i}{\sqrt{2}}\chi(x) \tag{2.4}$$

where $\varphi(x)$ and $\chi(x)$ are real fields. Then in terms of this fields

$$\mathcal{L} = \frac{1}{2}\left(\partial_\mu \varphi \partial^\mu \varphi + \partial_\mu \chi \partial^\mu \chi\right) - \left[\lambda^2 v^2 \varphi^2 + \frac{\lambda^2 v}{\sqrt{2}}\varphi(\varphi^2 + \chi^2) + \frac{\lambda^2}{8}(\varphi^2 + \chi^2)^2\right]. \tag{2.5}$$

We can see that the mass of the excitation φ is $\sqrt{2}\lambda v$ and that excitation χ is massless.

The broken phase of the complex scalar field theory is arguably the simplest realization of Goldstone's theorem, which states that there is a massless mode for each broken symmetry generator of a spontaneous broken continuous symmetry. The massless modes are known as NGBs. In the example above the spontaneously broken symmetry is $U(1)$ which has a single broken generator, hence the NGBs χ is unique.

In the SM, as mentioned in the introduction, we have the spontaneous breaking of electroweak to electromagnetism symmetry, $SU(2)\times U(1) \to U(1)$. This pattern of symmetry breaking produce $|SU(2)\times U(1)| - |U(1)| = |SU(2)| + |U(1)| - |U(1)| = |SU(2)| = 2^2 - 1 = 3$ NGBs. However, because these are gauge symmetries the NGBs are absorbed by the gauge bosons giving their masses. This mechanism for giving masses to the gauge bosons are known as the Anderson-Higgs mechanism.

Now we will present another example that will be useful in the model we will present in the next section. We will look in more detail to a model that is often considered in the literature in which the electroweak symmetry breaking is accomplished by the Higgs doublet plus a complex scalar triplet with hypercharge $Y = 1$. First lets analyse the relation between the generators of $SU(2)\times U(1)$ and the vacuum expectation value of the theory. Fist note that the three fields of an $SU(2)$ triplet can be combined in a 2×2 traceless matrix $\Delta = \Delta_a \sigma_a$, where σ_a with $a = 1, 2, 3$ are the Pauli matrices. It can be shown that Δ transforms as $\Delta \to U\Delta U^\dagger$ under $SU(2)$ with $U \in SU(2)$. Taking the standard convention for the VEV of the Higgs in the SM we know that $Q = T_3 + Y$ and with this fact we can check that

$$Q\langle\Delta\rangle Q = 0 \tag{2.6}$$

just for $\langle\Delta\rangle = 0$ or $\langle\Delta\rangle \neq 0$ but $Y = 1$ or -1. We see therefore, that adding an $SU(2)$ triplet to the SM with a non-zero vacuum expectation value, will break $SU(2) \times U(1)$ to $U(1)_{\text{em}}$ when $Y = 1, -1$, and will break it to nothing otherwise. We see therefore, that depending on the assignment of the hypercharge of the triplet, the symmetry breaking pattern will be very different. In particular the counting of NGBs is different in each case. For the case in which $Y = 1, -1$ we have $SU(2) \times U(1) \to U(1)_{\text{em}}$ which gives $(3 + 1) - 1 = 3$ NGBs. When $Y \neq 1, -1$ we have $SU(2) \times U(1)$ breaking to nothing and therefore we have $3 + 1 = 4$ NGBs.

From a Lagrangian point of view these statements can be checked rather explicitly. The general potential for an $SU(2)$ doublet with hypercharge $1/2$ and an $SU(2)$ triplet with hypercharge 1 is

$$
\begin{aligned}
V =& m^2 H^\dagger H + \lambda (H^\dagger H)^2 + M^2 \mathrm{Tr}\Delta^\dagger\Delta + \lambda_1 (\mathrm{Tr}\Delta^\dagger\Delta)^2 + +\lambda_3 \mathrm{Tr}\Delta^\dagger\Delta\Delta^\dagger\Delta \\
&+ \lambda_4 H^\dagger H \mathrm{Tr}\Delta^\dagger\Delta + \lambda_5 H^\dagger \Delta\Delta^\dagger H + \left(b^* H^\dagger \Delta H^* + \text{h.c.}\right).
\end{aligned}
\tag{2.7}
$$

Notice that when the hypercharge of the triplet is different from 1 or -1, from a Lagragian perspective means that $b = 0$ because the $H^\dagger \Delta H^*$ is not gauge-invariant anymore. In the case $b = 0$, this potential have a minimum for

$$
\langle H^\dagger H \rangle = v^2 = \frac{M^2(\lambda_4 + \lambda_5) - 2m^2(\lambda_1 + \lambda_3)}{4\lambda(\lambda_1 + \lambda_3) - (\lambda_4 + \lambda_5)^2}, \tag{2.8}
$$

$$
\langle \Delta^\dagger \Delta \rangle = V^2 = \frac{m^2(\lambda_4 + \lambda_5) - 2\lambda M^2}{4\lambda(\lambda_1 + \lambda_3) - (\lambda_4 + \lambda_5)^2}. \tag{2.9}
$$

Instead of working with m^2 and M^2 we can invert the relations above to get

$$
M^2 = -(\lambda_4 + \lambda_5) v^2 - 2V^2(\lambda_1 + \lambda_3), \tag{2.10}
$$

$$
m^2 = -2\lambda v^2 - V^2(\lambda_4 + \lambda_5). \tag{2.11}
$$

Expanding the theory with

$$
H = \begin{pmatrix} H^+ \\ h + v + ia \end{pmatrix}, \qquad \Delta = \begin{pmatrix} \Delta^{++} & \frac{\Delta^+}{\sqrt{2}} \\ \frac{\Delta^+}{\sqrt{2}} & \Delta^0_s + V + i\Delta^0_a \end{pmatrix}, \tag{2.12}
$$

and using the expressions for m^2 and M^2 in terms of v^2 and V^2 we find that the fields H^+ and Δ^+ mix and their mixing is encoded by the mass matrix

$$
\frac{\lambda_5}{2} \begin{pmatrix} (H^+)^* & (\Delta^+)^* \end{pmatrix} \begin{pmatrix} -V^2 & \sqrt{2} v V \\ \sqrt{2} v V & -v^2 \end{pmatrix} \begin{pmatrix} H^+ \\ \Delta^+ \end{pmatrix}, \tag{2.13}
$$

which have mass eigenstates

$$S_+ = \sin\theta\Delta^+ + \cos\theta H^+, \tag{2.14}$$

$$S_- = \cos\theta\Delta^+ - \sin\theta H^+, \tag{2.15}$$

with $\theta = \arctan(\sqrt{2}V/v)$ and masses respectively equal to

$$m_+ = 0, \tag{2.16}$$

$$m_- = -\frac{\lambda_5}{2}(v^2 + V^2). \tag{2.17}$$

The scalar fields h_A and Δ_s^0 also mix with an mass matrix

$$\begin{pmatrix} h_A & \Delta_s^0 \end{pmatrix} \begin{pmatrix} 4\lambda v^2 & 2vV(\lambda_4+\lambda_5) \\ 2vV(\lambda_4+\lambda_5) & 4V^2(\lambda_1+\lambda_3) \end{pmatrix} \begin{pmatrix} h_A \\ \Delta_s^0 \end{pmatrix}, \tag{2.18}$$

which have two non-zero eigenvalues, corresponding to two massive scalar particles. Finally, a, Δ_a^0 and Δ^+ are mass eigenstates with masses respectively equal to

$$m_a = 0, \tag{2.19}$$

$$m_{\Delta_a^0} = 0, \tag{2.20}$$

$$m_{\Delta^+} = -\lambda_5 v^2 - 2\lambda_3 V^2. \tag{2.21}$$

We see that for this symmetry breaking pattern we end up with 4 NGBs: S_+, a and Δ_a^0. Recall that S_+ is a complex scalar field, and therefore contains two degrees of freedom. This number of NGBs is in agreement with the counting of broken generators because setting $b = 0$ is equivalent, from a Lagrangian perspective, to have the hypercharge of Δ different from $1, -1$. However, when $b \neq 0$ the symmetry breaking pattern is the usual $SU(2)_L \times U(1)_Y \to U(1)_{\rm em}$ and we will have just 3 NGBs. This can be checked explicitly by including the term with b coupling in the Lagrangian. However, the formulas are excessively complicated to include here. In any case, an interesting conclusion is that one of the NGBs will acquire a mass which will be proportional to b.

We should add that if the symmetry which is spontaneously broken is not exact, then the would-be NGBs get a mass proportional to the parameters that break the symmetry explicitly. In these cases, they are known as pseudo-Nambu-Goldstone bosons (pNGBs). An important example to consider is the pion of QCD. The pion is a pNGBs associated with the spontaneous breaking of chiral symmetry, and their masses are proportional to the quark masses, which are the parameters that explicitly break chiral symmetry.

The generic idea behind twin Higgs models is to have the Higgs as a pNGB of an spontaneously broken global symmetry [108, 109, 110]. The actual implementation of this idea, however, is not so simple because generically the electroweak gauge and Yukawa interactions explicitly break the global symmetry to which the pNGB Higgs is associated and reintroduce the quadratic divergences to the Higgs mass parameter. This problem can be addressed by controlling radiative corrections from gauge and Yukawa interactions. This can be done by breaking symmetry collectively [111, 112, 113], by making the size of the gauge group generating the pNGB Higgs large and thus separating the momentum cutoff scale from the cutoff of the theory [114, 115], or by using a discrete symmetry as in Twin Higgs models [102, 105]. By having a discrete symmetry controlling the radiative correction from gauge and Yukawa interactons, Twin Higgs models end up with colorless top parners. Next, we will see in detail how this happens.

2.2 $\mathbb{Z}_2$ symmetry

In Twin Higgs models the SM is extended with a twin sector and the Higgs boson is a pNGB of broken global symmetry. There is an $\mathbb{Z}_2$ exchange symmetry between the SM and the twin sector that ensures that the radiative corrections to the Higgs potential respects the global symmetry and therefore does not reintroduce the quadratic divergences to the Higgs boson mass.

We will illustrate the basic idea by way of a simple example where the global symmetry is $SU(4)\times U(1)$ and is realized linearly. We assume that the SM Higgs doublet is part of a complex scalar field H which transforms in the fundamental representation of $SU(4)$. The potential for H is

$$V(H) = m^2 H^\dagger H + \lambda (H^\dagger H)^2 \tag{2.22}$$

and respects $SU(4)$. Gauge interactions are introduced by gauging the $SU(2)_A \times U(1)_A \times SU(2)_B \times U(1)_B$ subgroup of $SU(4) \times U(1)$. The field H is taken to be

$$H = \begin{pmatrix} H_A \\ H_B \end{pmatrix} \tag{2.23}$$

with H_A having the same quantum numbers as the Higgs field (1) under $SU(2)_A \times U(1)_A$ and being a singlet of $SU(2)_B \times U(1)_B$, and conversely for H_B. The subgroup $SU(2)_A \times U(1)_A$ is identified with the electroweak gauge group of the SM and H_A with the Higgs field. If m^2 is negative, H will acquire a VEV

$$\langle H \rangle = \frac{m}{\sqrt{2\lambda}} = f \tag{2.24}$$

that spontaneously breaks $SU(4) \times U(1)$ to $SU(3) \times U(1)$. This symmetry breaking pattern will produce $|SU(4)| - |SU(3)| = (4^2 - 1) - (3^2 - 1) = 7$ massless NGBs. Depending on the alignment of the VEV, different NGBs will be eaten. If it is aligned along H_B, the SM Higgs doublet H_A will remain massless.

However, the $SU(4) \times U(1)$ global symmetry is broken explicitly by Yukawa and gauge interactions generating a mass to the NGBs that are proportional to the parameters that explicit break the symmetry. The top Yukawa interactions take the form

$$\lambda_A H_A q_A t_A + \lambda_B H_B q_B t_B \tag{2.25}$$

These interactions generate quadratically divergent corrections to the Higgs potential at one-loop order that take the form

$$\Delta V = \frac{3\lambda_A^2\Lambda^2}{8\pi^2}H_A^\dagger H_A + \frac{3\lambda_B^2\Lambda^2}{9\pi^2}H_B^\dagger H_B, \tag{2.26}$$

below the cutoff Λ of the theory. Adding an $\mathbb{Z}_2$ symmetry that exchanges $A \leftrightarrow B$ enforces that $\lambda_A = \lambda_B \equiv \lambda$ which makes

$$\Delta V = \frac{3\lambda^2\Lambda^2}{8\pi^2}(H_A^\dagger H_A + H_B^\dagger H_B) = \frac{3\lambda^2\Lambda^2}{8\pi^2}H^\dagger H, \tag{2.27}$$

respecting the $SU(4)$ symmetry. Therefore, it does not contribute any more to the mass of the pNGBs. The other gauge and the Yukawa interactions will also generate corrections to the Higgs potential that respects the $U(4)$ due to the $\mathbb{Z}_2$ symmetry that exchanges the two sectors. In conclusion, there are no quadratically divergent contributions to the Higgs mass at one-loop order. However, gauge and Yukawa interactions still breaks the $U(4)$ symmetry and these introduces logarithmically divergent corrections. The leading contributions are

$$\Delta V \supset \frac{3y^4}{16\pi^2}\left(|H_A|^4 \log\frac{\Lambda^2}{y^2|H_A|^2} + |H_B|^4 \log\frac{\Lambda^2}{y^2|H_B|^2}\right). \tag{2.28}$$

To sum up, the $\mathbb{Z}_2$ ensures that the corrections to the Higgs potential respects $SU(4)$, and therefore, $\mathbb{Z}_2$ protects the pNGB from quadratic divergences. The leading contributions to the SM Higgs potential are logarithmic, this other divergence restricts the mechanism of twin Higgs models to protect the weak scale from radiative corrections up to scales of order 5-10 TeV.

The previous argument was based on the simple example where the global symmetry is $SU(4) \times U(1)$ and is realized linearly. However, the cancellation is independent of the details of the UV theory. To see this we will consider the low-energy EFT for the light pNBG in which the symmetry is realized nonlinearly.

We parametrize the pNGB degrees of freedom in terms of fields $\Pi^a(x)$ that transform nonlinearly under the broken symmetry. Then

$$H = \begin{pmatrix} H_A \\ H_B \end{pmatrix} = \exp\left(\frac{i}{f}\Pi\right) \begin{pmatrix} 0 \\ 0 \\ 0 \\ f \end{pmatrix}, \tag{2.29}$$

where f is the vacuum expectation value of H and Π is given by

$$\Pi = \begin{pmatrix} 0 & 0 & 0 & h_1 \\ 0 & 0 & 0 & h_2 \\ 0 & 0 & 0 & 0 \\ h_1^* & h_2^* & 0 & 0 \end{pmatrix}, \tag{2.30}$$

where we assumed unitary gauge where all the B sector NGBs have been eaten by the corresponding vector bosons. Expanding the exponential gives

$$H = \begin{pmatrix} \mathbf{h}\frac{if}{\sqrt{\mathbf{h}^\dagger \mathbf{h}}} \sin\left(\frac{\sqrt{\mathbf{h}^\dagger \mathbf{h}}}{f}\right) \\ 0 \\ f\cos\left(\frac{\sqrt{\mathbf{h}^\dagger \mathbf{h}}}{f}\right) \end{pmatrix}, \tag{2.31}$$

showing that

$$H_A = \mathbf{h}\frac{if}{\sqrt{\mathbf{h}^\dagger \mathbf{h}}} \sin\left(\frac{\sqrt{\mathbf{h}^\dagger \mathbf{h}}}{f}\right) = i\mathbf{h} + ..., \tag{2.32}$$

$$H_B = \begin{pmatrix} 0 \\ f\cos\left(\frac{\sqrt{\mathbf{h}^\dagger \mathbf{h}}}{f}\right) \end{pmatrix} = \begin{pmatrix} 0 \\ f - \frac{1}{2f}\mathbf{h}^\dagger \mathbf{h} + ... \end{pmatrix}. \tag{2.33}$$

where $\mathbf{h} = (h_1, h_2)^T$ is the Higgs doublet of the SM. Using this expressions in (2.25) we get

$$i\lambda_t \mathbf{h} q_A t_A + \lambda_t \left(f - \frac{1}{2f}\mathbf{h}^\dagger \mathbf{h} \right) q_B t_B. \tag{2.34}$$

We can evaluate the radiative contributions to the Higgs mass parameter using this Lagrangian from the diagrams

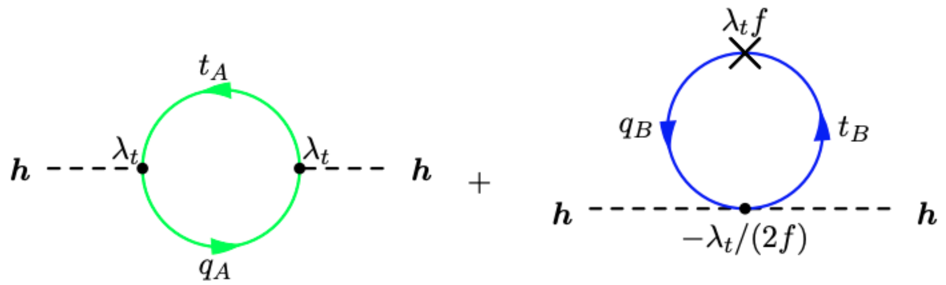


Figure 1: Cancellation mechanism of twin Higgs model. The different colors of the green top loop and the blue twin-top loop are meant to emphasize that they are charged under different color groups. The first in the SM QCD, the second in twin-QCD.

Evaluating these diagrams we find out that the quadratic divergences coming from the first one exactly cancel the ones coming from the second one. We see that the twin-top particles are responsible for the stabilization of the Higgs potential. As mentioned at the beginning of this section, twin Higgs models combine the idea that the Higgs is a pNGB of a spontaneous broken global symmetry with the idea of neutral naturalness using the $\mathbb{Z}_2$ symmetry. The diagram above shows very explicitly that in the twin Higgs model the top-partners, the twin-top, are not charged under the SM color group (corrigir). These theories demonstrate that, contrary to the conventional wisdom, stabilizing the weak scale does not require new light particles transforming under the SM gauge groups.

The Goldstone Higgs is protected from radiative corrections from $\mathbb{Z}_2$-symmetric physics above the scale f. Therefore, to address the hierarchy problem it is necessary to generate a mild hierarchy $v < f$ so that in the strong coupling limit the cutoff $\Lambda \approx 4\pi f$ is of order 5 TeV. Therefore, the $\mathbb{Z}_2$ symmetry can't be exact. In original twin Higgs proposal [102], vacuum misalignment between electroweak and new physics scales is realized by adding explicit $\mathbb{Z}_2$ breaking terms. Alternatively, it is possible to have $\mathbb{Z}_2$ broken spontaneously [116, 117, 118, 119, 120] or it may remain exact as long as the contributions are correctly suppressed [121]. In the model we will present in the next section, we have $\mathbb{Z}_2$ broken spontaneously.

While the mirror Twin Higgs addresses the little hierarchy problem, it does not address the big hierarchy problem, as nothing stabilizes the scale f against radiative corrections. However, the Twin Higgs model can be incorporated in composite Higgs [122, 123, 124] or supersymmetric [125, 126, 127, 128] models to address the big hierarchy problem. Alternativelly, one can postpone the solution of the big hierarchy problem with additional copies of the Twin Higgs mechanism [129].

2.3 Twin Dark matter and Cosmology

We saw in the introduction that the ideas of matter and radiation where unified by modern physics with the concept of the quantum field. Nevertheless, matter and radiation are indeed very different things. One of the most striking differences between the two is the diversity we see in the matter and that we cannot see in radiation. The fact that simple atoms can combine in molecules, and they combine in proteins and so on, gives rise to all the richness we see in the world, including you. Interestingly, we are only able to see the richness of forms of matter because radiation is quite homogeneous and does not interact with itself. If the photons that are now reaching you from these letters decided to interact with themselves forming interesting bound states as the proton and electron do, you would not be able to see anything. Therefore, in a very concrete sense, the fact that photons are boring is what allows us to see.

In any case, in many models, Dark Matter is more like radiation than matter. Not in the sense of being massless, which we know can't be the case due to the gravitational effects of Dark Matter, but in the sense that they do not form complex structures as matter. On the other hand, in twin Higgs models, Dark Matter can have components very similar to ordinary matter. If the $\mathbb{Z}_2$ symmetry where exact, dark-quarks would form dark-protons and dark-neutrons which would bound with dark-electrons in dark-atoms and then make dark-molecules. It would be possible to have a dark-planets orbiting around dark-starts. In a particular dark-planet, due to particular dark-environmental conditions, some dark-molecules may have form dark-proteins, and from that, dark-life may have emerged. The process of evolution would also be at play in this dark-planet, and it is conceivable that life had evolved to more complex forms, maybe a dark-human. All this hypothetical but possible scenario could have happened here. The only interaction between the two sectors is from the Higgs boson and gravitational. Therefore apart from the huge perturbation on the orbits of the solar system due to a dark-Sun, it would be possible that a dark-person, just like you, exists where you are. You can't see him, and you can occupy the same space as him because no interaction forbids this to happen. And maybe, this dark-person is a physicist trying to understand why the Higgs boson mass parameter is small, and why there is a small fraction of matter that is not "ordinary-matter" (which for us is dark matter). And he could be considering a twin-Higgs model in which his dark-matter (our matter) is a twin-sector.

This somewhat wild speculation shows that twin Higgs models are a natural environment in which to implement the ADM idea. More formally, because the SM and the twin sector are related by the $\mathbb{Z}_2$ symmetry, it is reasonable to suppose that the mechanism responsible for baryogenesis will produce a similar asymmetry in the twin sector. If there is a small $\mathbb{Z}_2$ breaking and twin baryons are a bit heavier or their numbers slightly larger than ordinary baryons, they would have an abundance that can match with the observed abundance of dark matter. This provides an elegant explanation for dark matter, the asymmetric twin dark matter [130, 131, 132, 133, 134]. There are other ways to accommodate dark matter in twin Higgs models, as WIMPs [18, 135, 136], SIMPs [137] and Freeze-in [138]. More generally, the twin sector contains suitable multiple candidates for dark matter: twin-neutrino [139], the neutralino of supersymmetric twin Higgs [140], some exotic twin baryons and atoms [141]. Therefore, twin Higgs models can address the DM and the hierarchy problem in a single framework.

Although twin Higgs models have suitable candidates for dark matter, they are constrained by cosmology [103, 142, 143, 144, 145, 146]. Because the Higgs portal coupling that makes the Higgs natural also keeps the two sectors in thermal equilibrium down to $\mathcal{O}(GeV)$ temperatures. The presence of twin relativistic degrees of freedom at the time of decoupling of the two sectors adds to dark radiation at late times and conflicts with the tight constraints on dark radiation from BBN [147, 69] and from the CMB [68, 69]. This limit on dark radiation is often quoted as a limit on the effective additional number of neutrinos ΔN_{eff}. Therefore, before explaining in more detail the cosmological problem of twin Higgs let's define the meaning of ΔN_{eff}.

The energy density ρ_i for a relativistic particle i in thermal equilibrium can be calculated as a function of its temperature T_i and is given by

$$\rho_i = g_i \int \frac{d^3p}{(2\pi)^3} f_i(p, T_i) E(p) = \frac{a_B}{2} g_i T_i^4 \times \begin{cases} 1 & \text{boson} \\ \frac{7}{8} & \text{fermion} \end{cases} . \tag{2.35}$$

As showed in the previous sections photons and neutrinos are relativistic today and contribute to the present energy density in radiation

$$\rho = \rho_\gamma + \rho_\nu = \rho_\gamma \left[1 + 3 \times \frac{7}{8} \times \left(\frac{T_\nu}{T_\gamma} \right)^4 \right]. \tag{2.36}$$

Their temperature today is different because they are not in thermal equilibrium anymore. After most of the neutrinos had decoupled from the thermal bath, which contained photons, electrons, and positrons, the electron-positron annihilation occurred increasing the photon bath temperature. Using conservation of comoving entropy it is possible to show that

$$\frac{T_\nu}{T_\gamma} \approx \left(\frac{4}{11} \right)^{1/3}. \tag{2.37}$$

This motivates us to define the effective *number of neutrino species* N_{eff}, by the equation

$$\rho = \rho_\gamma \left[1 + N_{\text{eff}} \times \frac{7}{8} \left(\frac{4}{11} \right)^{4/3} \right], \tag{2.38}$$

or conversely

$$N_{\text{eff}} = \frac{8}{7} \left(\frac{11}{4} \right)^{4/3} \left(\frac{\rho}{\rho_\gamma} - 1 \right). \tag{2.39}$$

This definition is used because there are several ways that an additional number of neutrinos impact the anisotropy of the CMB. Current CMB constraints on N_{eff} are dominated by the impact of the neutrino energy density on the expansion rate, affecting the CMB damping tail [148]. It also has effects on BBN. Present bounds of CMB and BBN on ΔN_{eff} are

$$\Delta N_{\text{eff}} \equiv N_{\text{eff}} - N_{\text{eff}}^{\text{SM}} = 0.11 \pm 0.23 \tag{2.40}$$

at 2σ confidence level [5, 68, 69]. I should add that the SM prediction is not $N_{\text{eff}}^{\text{SM}} = 3$ because neutrinos had decoupled from the photons before complete electron-positron annihilation. The best SM prediction is $N_{\text{eff}}^{\text{SM}} = 3.046$.

Now that we know the bounds on ΔN_{eff} 2.40 and its meaning (2.39) we can understand the cosmological problem of twin Higgs. In the context of twin Higgs models, we have $\rho = \rho_A + \rho_B$ and therefore

$$\Delta N_{\text{eff}} = \frac{8}{7}\left(\frac{11}{4}\right)^{4/3}\frac{\rho_B}{\rho_\gamma}. \tag{2.41}$$

We see that to find out the contribution to ΔN_{eff} from twin Higgs models we need to estimate the ratio between ρ_B and ρ_γ. Conversely, we can find the ratio between ρ_B and ρ_A and use the fact that $\rho_A \approx 0.6\rho_\gamma$ which can be seen from (2.38) and $N_{\text{eff}} \approx 3$.

In the twin Higgs framework, the cancellation of quadratic divergences arises from a Higgs portal interaction between the SM Higgs doublet and its twin partner. This interaction keeps the two sectors in thermal equilibrium down to $\mathcal{O}(GeV)$ temperatures [144, 143]. Then the universe cools down and the energy density in the two sectors changes independently affecting CMB and BBN.

To estimate the ratio of energy densities at CMB and BBN times, first, note that when all i species are in thermal equilibrium the energy density (2.35) is given simply by $\rho = g_* T^4$ with

$$g_* = N_{\text{bosons}} + \frac{7}{8}N_{\text{fermions}}. \tag{2.42}$$

the *effective number of relativistic species*. As the universe cools, degrees of freedom become nonrelativistic and annihilate changing the energy density of the bath. This change can be found from thermodynamics using the conservation of comoving entropy. For species in thermal equilibrium, the comoving entropy is given by $s \propto g_* T^3$. By conservation of comoving entropy, when a sector transitions from an initial effective number of degrees of freedom g_{*i} at temperature to a lower one g_{*f} with all species in thermal equilibrium at both stages, we have

$$\frac{T_f}{T_i} = \left(\frac{g_{*i}}{g_{*f}}\right)^{1/3} \tag{2.43}$$

and therefore the ratio of energy densities increases

$$\frac{\rho_f}{\rho_i} = \frac{g_{*f}}{g_{*i}} \left(\frac{T_f}{T_i}\right)^4 = \frac{g_{*f}}{g_{*i}} \left(\frac{g_{*i}}{g_{*f}}\right)^{4/3} = \left(\frac{g_{*i}}{g_{*f}}\right)^{1/3} . \tag{2.44}$$

Because the bounds on energy density in hidden radiation come from BBN and CMB, we are interested in evaluating (2.41) at these times which happened after the decoupling of the two sectors at $O(GeV)$ temperatures. Using (2.44) with i =decoupling and f =BBN we have

$$\left.\frac{\rho_B}{\rho_A}\right|_{\mathrm{BBN}} \approx \left(\left.\frac{g_{*B}}{g_{*A}}\right|_{\mathrm{D}}\right)^{1/3} \left(\left.\frac{g_{*A}}{g_{*B}}\right|_{\mathrm{BBN}}\right)^{1/3} \left.\frac{\rho_B}{\rho_A}\right|_{\mathrm{D}} = \left(\left.\frac{g_{*A}}{g_{*B}}\right|_{\mathrm{BBN}}\right)^{1/3} \left(\left.\frac{g_{*B}}{g_{*A}}\right|_{\mathrm{D}}\right)^{4/3} \tag{2.45}$$

where we use that at decoupling the temperature of both sectors, are equal and therefore

$$\left.\frac{\rho_B}{\rho_A}\right|_{\mathrm{D}} = \left.\frac{g_{*B}}{g_{*A}}\right|_{\mathrm{D}} . \tag{2.46}$$

Using this expression in (2.41) and the fact that today $\rho_\gamma \approx 0.6\rho_A$ gives

$$\Delta N_{\mathrm{eff}} = \frac{8}{7}\left(\frac{11}{4}\right)^{4/3} \left.\frac{\rho_B}{\rho_\gamma}\right|_{BBN} \approx \frac{1}{0.6}\frac{8}{7}\left(\frac{11}{4}\right)^{4/3} \left.\frac{\rho_B}{\rho_A}\right|_{BBN} \approx 7.3 \left(\left.\frac{g_{*A}}{g_{*B}}\right|_{\mathrm{BBN}}\right)^{1/3} \left(\left.\frac{g_{*B}}{g_{*A}}\right|_{\mathrm{D}}\right)^{4/3} . \tag{2.47}$$

Now we see that if $\mathbb{Z}_2$ is exact than $g_{*B} = g_{*A}$ both at decoupling and at BBN which imply that $\Delta N_{\mathrm{eff}} \approx 7.3 > 0.1$ is at least 3 sigma out of the measured value and is therefore excluded by current bounds. This argument shows that a viable twin Higgs can't have an exact $\mathbb{Z}_2$.

This bound may be somewhat relaxed if the dark radiation scatters with a short mean free path, as opposed to free streaming like neutrinos [149]. Another possibility is to remove all of the "unnecessary" light degrees of freedom from the twin sector. This is accomplished in Fraternal Twin Higgs models [150] in which the twin sector is taken to contain only the third generation of fermions, as well as the twin EW and QCD gauge bosons. One can further remove light degrees of freedom by assuming that the twin sector is vector-like [151].

Even another possibility is highlighted by this rough calculation. By equation (2.47) we see that having more degrees of freedom in the A sector than in the B sector at decoupling would decrease ΔN_{eff}. This was considered in [132], which explored a scenario in which decoupling occurred between the two QCD phase transitions. However, this is not enough. Let's review the argument: suppose we have neutrinos, electrons, positrons, and photons in the twin-sector at decoupling

$$g_{*B}\big|_D = 2 + 2 \times 2 \times \frac{7}{8} + 3 \times 2 \times \frac{7}{8} = 10.75 \tag{2.48}$$

and in the SM sector we have the same plus the muon, the 3 light quarks, and gluons which have not confined yet

$$g_{*A}\big|_D = 10.75 + 8 \times 2 + 3 \times 3 \times 2 \times 2 \times \frac{7}{8} + 2 \times 2 \times \frac{7}{8} = 61.75. \tag{2.49}$$

At BBN both sectors have the same g_*, then

$$\Delta N_{\text{eff}} \approx 7.3 \left(\frac{10.75}{61.75}\right)^{4/3} \approx 0.70, \tag{2.50}$$

which ameliorates the tension but does not resolve it. Even if we also include the charm quark and the tau lepton in the SM sector at decoupling, we would have $g_{*A} = 75.75$ which would give $\Delta N_{\text{eff}} = 0.5$ which is still in tension with current bounds.

However if twin-neutrinos are heavy at decoupling, having masses larger than few GeV, then

$$g_{*B}\big|_D = 5.5 \tag{2.51}$$

and at BBN the twin-sector will have just 2 relativistic degrees of freedom. In the SM, on the other hand, g_* will be 3.4 at BBN because neutrinos are not in thermal equilibrium with photons anymore and their lower temperature suppress their contribution to the energy density. Using this we end up with

$$\Delta N_{\text{eff}} \approx 7.3 \left(\frac{3.4}{2}\right) \left(\frac{5.5}{61.75}\right)^{4/3} \approx 0.5 \tag{2.52}$$

or including the other degrees of freedom in the SM at decoupling

$$\Delta N_{\text{eff}} \approx 7.3 \left(\frac{3.4}{2}\right) \left(\frac{5.5}{75.75}\right)^{4/3} \approx 0.37. \tag{2.53}$$

All these results are still in tension with the ΔN_{eff} measurement but giving neutrino masses and having the decoupling temperature between the two QCD phase transitions the model is not excluded. In the next section, we will show how our model accomplishes this by giving heavy masses to twin neutrinos at the same time that gives small masses to SM neutrinos and spontaneously breaks the $\mathbb{Z}_2$ symmetry.

3 Neutrino Masses in Twin Higgs with Spontaneous $\mathbb{Z}_2$ Breaking

So far we had shown that the twin Higgs model was constructed as a solution to the electroweak hierarchy problem and that they can also accommodate dark matter. Then we showed that it has problems related to the presence of dark radiation at the time when the two sectors decoupled. We showed how this problem could be mitigated if twin-neutrinos have masses above a few GeV. Now we will show how spontaneously breaking the $\mathbb{Z}_2$ symmetry in twin Higgs models can be used to give SM neutrino masses at the same time that the cosmological problem that affects twin Higgs models is ameliorated.

In original twin Higgs model, vacuum misalignment between electroweak and new physics scales is realized by adding explicit $\mathbb{Z}_2$ breaking terms. Alternatively, it is possible to have $\mathbb{Z}_2$ broken spontaneously [116, 117, 118, 119, 120] or it may remain exact as long as the contributions are correctly suppressed [121]. In the model we will present next, we have $\mathbb{Z}_2$ broken spontaneously.

To sum up, the $\mathbb{Z}_2$ symmetry is exact in the model and it contains an $SU(2)$ triplet with hypercharge 1 and its twin copy in addition to the degrees of freedom present in the original twin Higgs model. The vacuum expectation value of the scalar sector breaks spontaneously $\mathbb{Z}_2$ and electroweak symmetry. Then, by type-II seesaw mechanism, the SM neutrinos get small masses in comparison to other SM fermions and their twin gets large masses mitigating the dark radiation problem. In section (3.1) we will present the model and show the pattern of spontaneous $\mathbb{Z}_2$ symmetry breaking, in section (3.2) we will investigate the vacuum expectation value of the theory and find relations between the physical quantities of the model and in section (3.3) we will explain neutrino masses.

3.1 Spontaneous Z_2 breaking

Following the example presented in the original proposal [102], we assume the Higgs field H_A to be part of a complex scalar

$$H = \begin{pmatrix} H_A \\ H_B \end{pmatrix} \tag{3.1}$$

which transform in the fundamental representation of $SU(4)$. This global symmetry is broken explicitly by gauge and Yukawa interactions, but it is assumed to hold approximatelly. The novelty of our model is to assume that the $\mathbb{Z}_2$ symmetry that exchange the SM with the twin sector is exact and the introduction of an $SU(2)$ triplet with hypercharge 1:

$$\Delta_A = \begin{pmatrix} \Delta_A^{++} & \Delta_A^{+}/\sqrt{2} \\ \Delta_A^{+}/\sqrt{2} & \Delta_A^{0} \end{pmatrix}, \tag{3.2}$$

and its B copy. The introduction of this triplet will allow $\mathbb{Z}_2$ and electroweak spontaneous symmetry breaking.

Ignoring quarks, the most general Lagrangian respecting the $\mathbb{Z}_2$ symmetry is

$$\begin{aligned}
-\mathcal{L} = &-(DH_A)^\dagger DH_A - (DH_B)^\dagger DH_B + m^2 H^\dagger H + \lambda (H^\dagger H)^2 + \tilde{\lambda} H_A^\dagger H_A H_B^\dagger H_B \\
&- (D\Delta_A)^\dagger D\Delta_A - (D\Delta_B)^\dagger D\Delta_B + M^2(\mathrm{Tr}\Delta_A^\dagger \Delta_A + \mathrm{Tr}\Delta_B^\dagger \Delta_B) \\
&+ \lambda_1 \left(\mathrm{Tr}\Delta_A^\dagger \Delta_A + \mathrm{Tr}\Delta_B^\dagger \Delta_B\right)^2 + \lambda_2 \left(\mathrm{Tr}\Delta_A^\dagger \Delta_A\right)\left(\mathrm{Tr}\Delta_B^\dagger \Delta_B\right) + \lambda_3 \left(\mathrm{Tr}\Delta_A^\dagger \Delta_A \Delta_A^\dagger \Delta_A + \mathrm{Tr}\Delta_B^\dagger \Delta_B \Delta_B^\dagger \Delta_B\right) \\
&+ \lambda_4 H^\dagger H \left(\mathrm{Tr}\Delta_A^\dagger \Delta_A + \mathrm{Tr}\Delta_B^\dagger \Delta_B\right) + \lambda_5 \left(H_A^\dagger \Delta_A \Delta_A^\dagger H_A + H_B^\dagger \Delta_B \Delta_B^\dagger H_B\right) \\
&+ \left\{ b^* \left(H_A^\dagger \Delta_A H_A^* + H_B^\dagger \Delta_B H_B^*\right) + \kappa \left(L_A^T \epsilon \Delta_A \epsilon L_A + L_B^T \epsilon \Delta_B \epsilon L_B\right) + \text{h.c.} \right\}.
\end{aligned} \tag{3.3}$$

where $\epsilon_{12} = -\epsilon_{21} = 1$. We want to investigate the minimum of the potential associated with this model and show that this minimum can produce spontaneous $\mathbb{Z}_2$ and electroweak symmetry breaking. However, it is hard to find analytical formulas and meaningful results approaching this problem in full generality. But we can gain understanding investigating the case in which b is small compared with the other-dimensional parameters of the model because in this case the potential greatly simplifies and we can find analytical formulas that hold to first order in b. Furthermore, when $b = 0$, lepton number is conserved.

To proceed we define

$$\begin{aligned}
\langle H_A \rangle &\equiv v + \delta v b + \mathcal{O}(b^2), \qquad \langle H_B \rangle \equiv f + \delta f b + \mathcal{O}(b^2), \\
\langle \Delta_A \rangle &\equiv V + \delta V b + \mathcal{O}(b^2), \qquad \langle \Delta_B \rangle \equiv F + \delta F b + \mathcal{O}(b^2).
\end{aligned} \tag{3.4}$$

The potential associated with the Lagrangian (3.3) in the ground state, neglecting terms of order b is

$$\begin{aligned} & m^2(v^2+f^2)+\lambda(v^2+f^2)^2+\tilde{\lambda}(v^2f^2)+M^2(V^2+F^2) \\ & +\lambda_1(V^2+F^2)^2+\lambda_2V^2F^2+\lambda_3(V^4+F^4)+\lambda_4(v^2+f^2)(V^2+F^2)+\lambda_5(v^2V^2+f^2F^2) \end{aligned} \tag{3.5}$$

This potential has a critical point for

$$v^2=\frac{M^2(2\lambda\lambda_5+\tilde{\lambda}(\lambda_4+\lambda_5))-m^2(2\tilde{\lambda}(\lambda_1+\lambda_3)+\lambda_5(\lambda_4+\lambda_5))}{2\lambda\left(4\tilde{\lambda}(\lambda_1+\lambda_3)+\lambda_5^2\right)+\tilde{\lambda}(\tilde{\lambda}(\lambda_1+\lambda_3)-\lambda_4(\lambda_4+\lambda_5))}, \tag{3.6}$$

$$f^2=\frac{m^2(\lambda_4\lambda_5-2\tilde{\lambda}(\lambda_1+\lambda_3))+M^2(\tilde{\lambda}\lambda_4-2\lambda\lambda_5)}{2\lambda\left(4\tilde{\lambda}(\lambda_1+\lambda_3)+\lambda_5^2\right)+\tilde{\lambda}(\tilde{\lambda}(\lambda_1+\lambda_3)-\lambda_4(\lambda_4+\lambda_5))}, \tag{3.7}$$

$$F^2=\frac{\tilde{\lambda}\left(m^2(2\lambda_4+\lambda_5)-M^2(4\lambda+\tilde{\lambda})\right)}{2\lambda\left(4\tilde{\lambda}(\lambda_1+\lambda_3)+\lambda_5^2\right)+2\tilde{\lambda}(\tilde{\lambda}(\lambda_1+\lambda_3)-\lambda_4(\lambda_4+\lambda_5))}, \tag{3.8}$$

and $V=0$.

The Hessian matrix associated with this critical point is

$$\begin{pmatrix} 2\lambda_1+2\lambda_3 & 2\lambda_1+\lambda_2 & \lambda_4+\lambda_5 & \lambda_4 \\ 2\lambda_1+\lambda_2 & 2\lambda_1+2\lambda_3 & \lambda_4 & \lambda_4+\lambda_5 \\ \lambda_4+\lambda_5 & \lambda_4 & 2\lambda & 2\lambda+\tilde{\lambda} \\ \lambda_4 & \lambda_4+\lambda_5 & 2\lambda+\tilde{\lambda} & 2\lambda \end{pmatrix} \tag{3.9}$$

where the order of the lines/columns is related to v,f,V and F respectively. This matrix is definite-positive for different regions of the parameter space. Therefore, the critical point (3.8) can be indeed a minimum of the potential. It is easy to see that there are regions of the parameter space in which

$$r=\frac{f}{v}>3 \tag{3.10}$$

as phenomenologically needed [152]. And it is also possible to show that these regions have overlap with the regions in which the Hessian matrix (3.9) is definite-positive, that is, the region in which the critical point is a minimum. We see, therefore, that the introduction of the triplet has lead to a phenomenologically viable pattern of $\mathbb{Z}_2$ and electroweak spontaneous breaking.

As a consistent check one can compute the corrections of order b to the values (3.8) and verify that they are indeed of order b. The results of the next sections will be presented using the zero-order approximation (3.8), therefore, all results should be taken as tree-level exact up to order b corrections. An important feature of this model is that $\langle \Delta_A \rangle$ is not zero for non-zero b. To be precise, we have

$$\langle \Delta_A \rangle = \frac{\lambda_5}{2(r^2-1)\left(\lambda_5^2 - \tilde{\lambda}\lambda_2 + 2\tilde{\lambda}\lambda_3\right)} b + \mathcal{O}(b^2). \tag{3.11}$$

This no null vacuum expectation value for the triplet contributes to the T parameter and puts a bound on b. However, the bounds that comes from neutrinos will be more stringent, so we will not worry with this by now. Similarly the corrections to $\langle H_A \rangle$, $\langle H_B \rangle$ and $\langle \Delta_B \rangle$ can be calculated and are

$$\delta v = \frac{v\left(2\tilde{\lambda}\left(r^2-1\right)\left(\lambda_4^2 - 2(2\lambda+\tilde{\lambda})(\lambda_1+\lambda_3)\right) + \lambda_5^2 r^2(2\lambda+\tilde{\lambda}) + \tilde{\lambda}\lambda_4\lambda_5\left(3r^2-2\right)\right)}{4F\lambda_5\left(\lambda\left(4\tilde{\lambda}(\lambda_1+\lambda_3)+\lambda_5^2\right) + \tilde{\lambda}(\tilde{\lambda}(\lambda_1+\lambda_3) - \lambda_4(\lambda_4+\lambda_5))\right)} \tag{3.12}$$

$$\delta f = \frac{v\left(2\tilde{\lambda}\left(r^2-1\right)\left(4\lambda(\lambda_1+\lambda_3)-\lambda_4^2\right) - 2\lambda\lambda_5^2 r^2 + \tilde{\lambda}\lambda_4\lambda_5 r^2\right)}{4F\lambda_5 r\left(\lambda\left(4\tilde{\lambda}(\lambda_1+\lambda_3)+\lambda_5^2\right) + \tilde{\lambda}(\tilde{\lambda}(\lambda_1+\lambda_3) - \lambda_4(\lambda_4+\lambda_5))\right)} \tag{3.13}$$

$$\delta F = \frac{4\lambda\lambda_5 + \lambda_5 r^2(-(8\lambda+\tilde{\lambda})) + 2\tilde{\lambda}\lambda_4\left(r^2-1\right)}{4\left(r^2-1\right)\left(\lambda\left(4\tilde{\lambda}(\lambda_1+\lambda_3)+\lambda_5^2\right) + \tilde{\lambda}(\tilde{\lambda}(\lambda_1+\lambda_3) - \lambda_4(\lambda_4+\lambda_5))\right)} \tag{3.14}$$

The dimensionful parameters M^2 and m^2 are not directly observable in this model and have no simple physical meaning, therefore it is useful to rewrite them in terms of v^2 and r^2 giving:

$$M^2 = -\frac{\left(\lambda_4\lambda_5 + 2\tilde{\lambda}\left(r^2-1\right)(\lambda_1+\lambda_3) + \lambda_5 r^2(\lambda_4+\lambda_5)\right)}{\lambda_5} v^2, \tag{3.15a}$$

$$m^2 = -\frac{\left(2\lambda\lambda_5 + \lambda_5 r^2(2\lambda+\tilde{\lambda}) + \tilde{\lambda}\lambda_4\left(r^2-1\right)\right)}{\lambda_5} v^2. \tag{3.15b}$$

We will use these expressions in the next section to analyze the spectrum of the scalars of the theory.

They will allow us to express the masses in terms of the physical parameters v and r instead of m and M. We can also use these expressions to rewrite

$$F^2 = \frac{\tilde{\lambda}(r^2-1)}{\lambda_5} v^2. \tag{3.16}$$

This expression shows that, although we have an additional triplet in our model, we have only two scales which are set by the vacuum expectation value of H_A and H_B. Since we are most interested in the case in which these two values are comparable, in the end, all masses are determined by the scale of v. Note that when $r = 1$, $F = V = 0$ and $\mathbb{Z}_2$ is not spontaneously broken.

3.2 Scalar spectrum

To investigate the spectrum of the scalar sector of the theory we need to expand the scalar field around the minimum (3.8) for a particular choice of the alignment of the vacuum expectation values. We choose

$$H_A = \begin{pmatrix} H_A^+ \\ v + h_A + ia_A \end{pmatrix}, \qquad H_B = \begin{pmatrix} H_B^+ \\ f + h_B + ia_B \end{pmatrix},$$

for the scalar doublet to agree with the conventions of the SM, and

$$\Delta_A = \begin{pmatrix} \Delta_A^{++} & \frac{\Delta_A^+}{\sqrt{2}} \\ \frac{\Delta_A^+}{\sqrt{2}} & \Delta_{A_s}^0 + i\Delta_{A_a}^0 \end{pmatrix} \quad \text{and} \quad \Delta_B = \begin{pmatrix} \Delta_B^{++} & \frac{\Delta_B^+}{\sqrt{2}} \\ \frac{\Delta_B^+}{\sqrt{2}} & F + \Delta_{B_s}^0 + i\Delta_{B_a}^0 \end{pmatrix}. \tag{3.17}$$

for the scalar triplets. Writting the potential of (3.3) for this choice of vacuum alignment we can find the spectrum of masses by analysing the terms quadratic with the fields.

For the doublets we have 3 Goldstone bosons in the A sector

$$m_{H_A^+}^2 = 0, \tag{3.18a}$$

$$m_{a_A}^2 = 0, \tag{3.18b}$$

and one in the B

$$m^2_{a_B} = 0. \tag{3.19a}$$

For the triplets of the A sector we have

$$m^2_{\Delta_A^{++}} = \frac{v^2\left(\tilde{\lambda}\left(r^2-1\right)\left(\lambda_2-2\lambda_3\right)-\lambda_5^2 r^2\right)}{\lambda_5}, \tag{3.20a}$$

$$m^2_{\Delta_A^{+}} = \frac{\tilde{\lambda}\left(r^2-1\right)v^2(\lambda_2-2\lambda_3)}{\lambda_5} + \frac{1}{2}\lambda_5\left(1-2r^2\right)v^2, \tag{3.20b}$$

$$m^2_{\Delta^0_{A_a}} = -\frac{\left(r^2-1\right)v^2\left(-\tilde{\lambda}\lambda_2+2\tilde{\lambda}\lambda_3+\lambda_5^2\right)}{\lambda_5}, \tag{3.20c}$$

$$m^2_{\Delta^0_{A_s}} = m^2_{\Delta^0_{A_a}}, \tag{3.20d}$$

The spectrum of Δ_A is not algebraically independent because the 3 different masses are defined in terms of just 2 adimensional parameters λ_5 and $\tilde{\lambda}(\lambda_2 - 2\lambda_3)$. By inverting these expressions we can express both adimensional couplings in terms of the physical quantities and we are left with $3 - 2 = 1$ constraint equation relating the masses. The constraint equation is

$$m^2_{\Delta_A^{++}} - 2m^2_{\Delta_A^{+}} + m^2_{\Delta^0_{A_a}} = 0 \tag{3.21}$$

and the couplings can be given by

$$\lambda_5 = \frac{1}{v^2}\left(m^2_{\Delta_{A^0_a}} - m^2_{\Delta_A^{++}}\right), \tag{3.22}$$

$$\tilde{\lambda}(\lambda_2 - 2\lambda_3) = \frac{\lambda_5}{r^2-1}\frac{1}{v^2}\left(r^2 m^2_{\Delta_{A^0_a}} - (r^2-1)m^2_{\Delta_A^{++}}\right). \tag{3.23}$$

For the B sector we have

$$
m^2_{\Delta_B^{++}} = -\frac{v^2\left(2\tilde{\lambda}\lambda_3\left(r^2-1\right)+\lambda_5^2 r^2\right)}{\lambda_5}, \tag{3.24a}
$$

$$
m^2_{\Delta^0_{B_a}} = 0. \tag{3.24b}
$$

using the previous equations allow us to write

$$
\tilde{\lambda}\lambda_3 = -\frac{1}{2(r^2-1)}\frac{1}{v^2}\left(m^2_{\Delta_{A_a^0}} - m^2_{\Delta_A^{++}}\right)\left(\frac{m^2_{\Delta_B^{++}}}{v^2} + \frac{r^2}{v^2}\left(m^2_{\Delta_{A_a^0}} - m^2_{\Delta_A^{++}}\right)\right) \tag{3.25}
$$

The complex scalar fields H_B^+ and Δ_B^+ mix and their terms in the potential are

$$
\begin{pmatrix} (H_B^+)^* & (\Delta_B^+)^* \end{pmatrix} \begin{pmatrix} \tilde{\lambda} - \left(r^2-1\right)v^2 & \frac{1}{2}F\lambda_5 r v \\ \frac{1}{2}F\lambda_5 r v & -\frac{1}{4}\lambda_5 r^2 v^2 \end{pmatrix} \begin{pmatrix} H_B^+ \\ \Delta_B^+ \end{pmatrix} \tag{3.26}
$$

which have mass eigenstates

$$
S_{B+} = \cos\theta\Delta_B^+ + \sin\theta H_B^+, \tag{3.27}
$$

$$
S_{B-} = \cos\theta\Delta_B^+ - \sin\theta H_B^+, \tag{3.28}
$$

with $\theta = \arctan(rv/2F)$ and masses respectively equal to

$$
m^2_{S_{B+}} = 0 \tag{3.29}
$$

$$
m^2_{S_{B-}} = -\frac{1}{4}v^2\left(4\tilde{\lambda}\left(r^2-1\right)+\lambda_5 r^2\right). \tag{3.30}
$$

This leads to

$$\tilde{\lambda} = -\frac{1}{4(r^2-1)}\left(4\frac{m^2_{S_{B-}}}{v^2} + \frac{r^2}{v^2}\left(m^2_{\Delta_{A^0_a}} - m^2_{\Delta_A^{++}}\right)\right) \tag{3.31}$$

From, (3.31), (3.23),(3.25),(3.22), we see that we can express $\tilde{\lambda}, \lambda_2, \lambda_3$ and λ_5 using the physical tree level masses of the scalar particles. In particular, we can use this expression in (3.16) to write another equation relating physical parameters of the model, namely

$$\left(m^2_{\Delta_{A^0_a}} - m^2_{\Delta_A^{++}}\right) F^2 + \left[m^2_{S_{B-}} + \frac{r^2}{4}\left(m^2_{\Delta_{A^0_a}} - m^2_{\Delta_A^{++}}\right)\right] v^2 = 0. \tag{3.32}$$

An interesting feature of this formula is that it relates A sector masses with B sector masses. It could be used to infer the mass of S_{B-} by measuring at LHC the masses of $\Delta_{A^0_a}$ and Δ_A^{++} once we know F.

We also have the mixing of h_A, h_B and $\Delta^0_{B_s}$ encoded by the mass matrix

$$\begin{pmatrix} h_A & h_B & \Delta^0_{B_s} \end{pmatrix} \begin{pmatrix} 4\lambda v^2 & 2rv^2(2\lambda+\tilde{\lambda}) & 2F\lambda_4 v \\ 2rv^2(2\lambda+\tilde{\lambda}) & 4\lambda r^2 v^2 & 2Frv(\lambda_4+\lambda_5) \\ 2F\lambda_4 v & 2Frv(\lambda_4+\lambda_5) & \frac{4\tilde{\lambda}(r^2-1)v^2(\lambda_1+\lambda_3)}{\lambda_5} \end{pmatrix} \begin{pmatrix} h_A \\ h_B \\ \Delta^0_{B_s} \end{pmatrix} \tag{3.33}$$

When, $r = 1$ the field $\Delta^0_{B_s}$ is massless and

$$2v^2 \begin{pmatrix} h_A & h_B \end{pmatrix} \begin{pmatrix} 2 & 2\lambda+\tilde{\lambda} \\ 2\lambda+\tilde{\lambda} & 2 \end{pmatrix} \begin{pmatrix} h_A \\ h_B \end{pmatrix}, \tag{3.34}$$

which have eigenvectors

$$h_+ = h_B + h_A, \tag{3.35}$$

$$h_- = h_B - h_A, \tag{3.36}$$

with eigenvalues

$$m_{h+}^2 = 2v^2(4\lambda + \tilde{\lambda}), \tag{3.37}$$

$$m_{h-}^2 = -2v^2\tilde{\lambda}. \tag{3.38}$$

Because the mass terms are continuous functions of r and the eigenvalues are also a continuous function of the entries of a matrix, for r not equal to 1 but close to it the eigenvalues and eigenvectors will be similar to the ones above.

We see therefore that in the A side the Goldstone bosons are H_A^+ and a_A, and that in the B sector they are a_B, $\Delta_{B_a}^0$ and S_{B+}. The 3 Goldstone bosons of the A sector will be eaten by the $SU(2)_L \times U(1)_Y$ gauge bosons giving their masses as is the case in the SM. In the B sector, notice, that when $b = 0$ this is effectively the same as having the hypercharge of Δ_B different from -1. But when the hypercharge of Δ_B is different from $-1, 0, 1$ this means that the spontaneous breaking pattern is from $SU(2) \times U(1)$ to nothing, and therefore there are 4 Goldstones associated with this spontaneous symmetry breaking pattern. Indeed, in the B sector we have a_B, $\Delta_{B_A}^0$ and S_{B+} as Goldstones, a total of 4 degrees of freedom, recall that S_{B+} is a complex scalar field and therefore have 2 degrees of freedom. Notice that, when $b \neq 0$ one of this Goldstones will acquire a mass proportional to b and the three other will be eaten by the $(SU(2) \times U(1))_B$ gauge bosons. The conclusions is that we will have a scalar particle with Lagrangian mass parameter proportional to b in the B sector.

At this point, a possible concern is that the region of the parameter space which gives a spectrum of masses consistent with phenomenological constraints have no intersection with the region of the parameter space such that the critical point is a minimum and that the adimensional parameters of the model are in the perturbative region. To demonstrate that this is not the case, notice that for $r = 3$, $m_{\Delta_{A_a^0}}, m_{\Delta_{A_a^{++}}}$ both are of the order TeV to avoid phenomenological constraints from searches at the LHC, and such that the difference between their square divided by v is order one, gives perturbative adimensional parameters and one can have a definite positive Hessian matrix in this case. For concreteness, consider $m_{\Delta_{A_a^0}} = 5v$ and $m_{\Delta_{A_a^{++}}} = \sqrt{25 - 0.1}v$. With this choice $\lambda_5 = -0.1$, and the Hessian matrix (3.9) can be definite positive with the other dimensional parameters in the perturbative region.

To sum up, the scale of the scalar spectrum of this model is given by the VEV of the Higgs v. We see that we can naturally avoid phenomenological constraints assuming order one couplings with no fine tunning. In addition, the expression for the tree level masses are not algebraically independent. Due to this fact we where able to find algebraic relations between them which translates in the physical relations (3.21) and (3.32). This physical relations should be taken as finger prints of our model.

3.3 Neutrino Masses and Dark Matter

With Δ_B getting a large vacuum expectation value (3.16) and Δ_A only a tiny one (3.11) from a small b term we have large neutrino masses in the B sector and tiny ones in the A sector from the type-II seesaw mechanism [46, 48, 49]. To be more precise, with electroweak and $\mathbb{Z}_2$ spontaneous symmetry breaking we will have

$$\mathcal{L} = \kappa \left(L_A^T \epsilon \Delta_A \epsilon L_A + L_B^T \epsilon \Delta_B \epsilon L_B \right) + \text{h.c.} \supseteq \kappa (\langle \Delta_A \rangle \nu_A^T C \nu_A + \langle \Delta_B \rangle \nu_B^T C \nu_B) + \text{h.c} \tag{3.39}$$

and therefore

$$m_{\nu_B} = \kappa \langle \Delta_B \rangle = \kappa \sqrt{\frac{\tilde{\lambda}(r^2-1)}{\lambda_5}} v, \tag{3.40}$$

$$m_{\nu_A} = \kappa \langle \Delta_A \rangle = \frac{\lambda_5 \kappa}{2(r^2-1)\left(\lambda_5^2 - \tilde{\lambda}\lambda_2 + 2\tilde{\lambda}\lambda_3\right)} b. \tag{3.41}$$

For three families κ is a matrix. From current bounds on neutrino masses we find that b is of order eV [5]. Note that the large hierarchy between v and b have no explanation in this model. However, as is the case for Yukawa couplings, it is technically natural to have b small seems lepton number is attained when $b = 0$. In addition, it is arguably more elegant to have different dimensional parameters giving masses to neutrinos and to the other fermions of the SM.

As anticipated in (2.53), having large B side neutrino masses will mitigate the cosmological problem that affects twin Higgs models related to dark radiation. As presented before, twin-neutrinos will not be relativistic when the two sectors decouple and therefore we will have

$$\Delta N_{\text{eff}} \approx 7.3 \left(\frac{3.4}{2}\right) \left(\frac{5.5}{75.75}\right)^{4/3} \approx 0.37. \tag{3.42}$$

which is in tension with current bounds but is not excluded yet. Recall that the symmetry breaking pattern that we have, will produce a scalar in the B sector with mass proportional to b which we know is order an eV. However, recall that the mass of scalar fields are quadratically sensitive to UV physics and that just the Higgs is protected by the approximate $SU(4)$ symmetry, therefore we expect that the radiative corrections will increase its mass to electroweak scales and hence it will not contribute significantly to N_{eff}.

Finally, these model have different candidates for dark matter as is the case in other twin Higgs models [130, 131, 132, 133, 134, 18, 135, 136, 137, 138, 139, 140, 141]. We should mention that if we tried to mitigate even more the cosmological problem by breaking B electromagnetism, another problem would be created. The B electrons and positrons would not annihilate sufficiently and would become overabundant and overclose the universe.

4 Conclusions

The hierarchy problem, dark matter, and neutrino masses are some of the problems of current particle physics. In this work, we presented Twin Higgs models which are a solution to the hierarchy problem. We saw that in these models the $\mathbb{Z}_2$ symmetry needs to be broken. Then, we constructed a Twin Higgs model with an additional triplet that leads to the spontaneous breaking of the $\mathbb{Z}_2$ and electroweak symmetry. This symmetry breaking pattern gives small masses to SM neutrinos and big masses to twin-neutrino solving the neutrino mass problem and mitigating the radiation problem associated with twin Higgs. In addition, our model has many dark degrees of freedom. These new particles could be the actual things causing the phenomena associated with dark matter and therefore could solve the dark matter problem.

There are many proposals to solve the problems of particle physics. It is arguably more aesthetical to have different problems solved in a coordinated way by a single framework. In addition, our model has phenomenological implications and signatures that could be searched for by present experiments.

References

[1] Davi B. Costa, Bogdan A. Dobrescu, and Patrick J. Fox. General Solution to the U(1) Anomaly Equations. *Phys. Rev. Lett.*, 123(15):151601, 2019.

[2] Davi B. Costa, Bogdan A. Dobrescu, and Patrick J. Fox. Chiral Abelian gauge theories with few fermions. *Phys. Rev. D*, 101(9):095032, 2020.

[3] Davi B. Costa. Anomaly-free $U(1)^m$ extensions of the Standard Model. 7 2020.

[4] Eva Bianconi, Allison Piovesan, Federica Facchin, Alina Beraudi, Raffaella Casadei, Flavia Frabetti, Lorenza Vitale, Maria Chiara Pelleri, Simone Tassani, Francesco Piva, Soledad Perez-Amodio, Pierluigi Strippoli, and Silvia Canaider. An estimation of the number of cells in the human body. *Annals of Human Biology*, 40(6):463–471, 2013. PMID: 23829164.

[5] P.A. Zyla et al. Review of Particle Physics. *PTEP*, 2020(8):083C01, 2020.

[6] Steven Weinberg. What is an elementary particle? *SLAC Beam Line*, 27N1:17–21, 1999.

[7] Mario Bunge. Treatise on Basic Philosophy: Ontology.

[8] Michael E. Peskin and Daniel V. Schroeder. *An Introduction to quantum field theory.* Addison-Wesley, Reading, USA, 1995.

[9] Matthew D. Schwartz. *Quantum Field Theory and the Standard Model.* Cambridge University Press, 3 2014.

[10] Steven Weinberg. *The Quantum theory of fields. Vol. 1: Foundations.* Cambridge University Press, 6 2005.

[11] Steven Weinberg. *The quantum theory of fields. Vol. 2: Modern applications.* Cambridge University Press, 8 2013.

[12] Steven Weinberg. *The quantum theory of fields. Vol. 3: Supersymmetry.* Cambridge University Press, 6 2013.

[13] Stephen L. Adler. Axial vector vertex in spinor electrodynamics. *Phys. Rev.*, 177:2426–2438, 1969.

[14] William A. Bardeen. Anomalous Ward identities in spinor field theories. *Phys. Rev.*, 184:1848–1857, 1969.

[15] Howard Georgi and Sheldon L. Glashow. Gauge theories without anomalies. *Phys. Rev. D*, 6:429, 1972.

[16] David J. Gross and R. Jackiw. Effect of anomalies on quasirenormalizable theories. *Phys. Rev. D*, 6:477–493, 1972.

[17] Edward Witten. An SU(2) Anomaly. *Phys. Lett. B*, 117:324–328, 1982.

[18] Lei Wang and Jin Min Yang. Dark matter and Higgs phenomenology predicted by left-right twin Higgs model in light of CDMS II results. *JHEP*, 05:024, 2010.

[19] Steven Weinberg. A Model of Leptons. *Phys. Rev. Lett.*, 19:1264–1266, 1967.

[20] Abdus Salam. Weak and Electromagnetic Interactions. *Conf. Proc. C*, 680519:367–377, 1968.

[21] Sheldon L. Glashow. The renormalizability of vector meson interactions. *Nucl. Phys.*, 10:107–117, 1959.

[22] Philip W. Anderson. Plasmons, Gauge Invariance, and Mass. *Phys. Rev.*, 130:439–442, 1963.

[23] Peter W. Higgs. Broken Symmetries and the Masses of Gauge Bosons. *Phys. Rev. Lett.*, 13:508–509, 1964.

[24] F. Englert and R. Brout. Broken Symmetry and the Mass of Gauge Vector Mesons. *Phys. Rev. Lett.*, 13:321–323, 1964.

[25] G.S. Guralnik, C.R. Hagen, and T.W.B. Kibble. Global Conservation Laws and Massless Particles. *Phys. Rev. Lett.*, 13:585–587, 1964.

[26] G. Arnison et al. Experimental Observation of Isolated Large Transverse Energy Electrons with Associated Missing Energy at s**(1/2) = 540-GeV. *Phys. Lett. B*, 122:103–116, 1983.

[27] M. Banner et al. Observation of Single Isolated Electrons of High Transverse Momentum in Events with Missing Transverse Energy at the CERN anti-p p Collider. *Phys. Lett. B*, 122:476–485, 1983.

[28] G. Arnison et al. Experimental Observation of Lepton Pairs of Invariant Mass Around 95-GeV/c**2 at the CERN SPS Collider. *Phys. Lett. B*, 126:398–410, 1983.

[29] P. Bagnaia et al. Evidence for $Z^0 \to e^+e^-$ at the CERN $\bar{p}p$ Collider. *Phys. Lett. B*, 129:130–140, 1983.

[30] F. Abe et al. Observation of top quark production in $\bar{p}p$ collisions. *Phys. Rev. Lett.*, 74:2626–2631, 1995.

[31] G. Gabrielse, D. Hanneke, T. Kinoshita, M. Nio, and Brian C. Odom. New Determination of the Fine Structure Constant from the Electron g Value and QED. *Phys. Rev. Lett.*, 97:030802, 2006. [Erratum: Phys.Rev.Lett. 99, 039902 (2007)].

[32] Pierre Clade, Estefania de Mirandes, Malo Cadoret, Saida Guellati-Khelifa, Catherine Schwob, Francois Nez, Lucile Julien, and Francois Biraben. Determination of the Fine Structure Constant Based on Bloch Oscillations of Ultracold Atoms in a Vertical Optical Lattice. *Phys. Rev. Lett.*, 96:033001, 2006.

[33] Serguei Chatrchyan et al. Observation of a New Boson at a Mass of 125 GeV with the CMS Experiment at the LHC. *Phys. Lett. B*, 716:30–61, 2012.

[34] Georges Aad et al. Observation of a new particle in the search for the Standard Model Higgs boson with the ATLAS detector at the LHC. *Phys. Lett. B*, 716:1–29, 2012.

[35] Jogesh C. Pati and Abdus Salam. Unified Lepton-Hadron Symmetry and a Gauge Theory of the Basic Interactions. *Phys. Rev. D*, 8:1240–1251, 1973.

[36] H. Georgi, Helen R. Quinn, and Steven Weinberg. Hierarchy of Interactions in Unified Gauge Theories. *Phys. Rev. Lett.*, 33:451–454, 1974.

[37] Q.R. Ahmad et al. Direct evidence for neutrino flavor transformation from neutral current interactions in the Sudbury Neutrino Observatory. *Phys. Rev. Lett.*, 89:011301, 2002.

[38] Y. Fukuda et al. Evidence for oscillation of atmospheric neutrinos. *Phys. Rev. Lett.*, 81:1562–1567, 1998.

[39] K. Eguchi et al. First results from KamLAND: Evidence for reactor anti-neutrino disappearance. *Phys. Rev. Lett.*, 90:021802, 2003.

[40] The nobel prize in physics 2015.

[41] Peter Minkowski. $\mu \to e\gamma$ at a Rate of One Out of 10^9 Muon Decays? *Phys. Lett. B*, 67:421–428, 1977.

[42] Rabindra N. Mohapatra and Goran Senjanovic. Neutrino Mass and Spontaneous Parity Nonconservation. *Phys. Rev. Lett.*, 44:912, 1980.

[43] Tsutomu Yanagida. Horizontal gauge symmetry and masses of neutrinos. *Conf. Proc. C*, 7902131:95–99, 1979.

[44] Murray Gell-Mann, Pierre Ramond, and Richard Slansky. Complex Spinors and Unified Theories. *Conf. Proc. C*, 790927:315–321, 1979.

[45] S.L. Glashow. The Future of Elementary Particle Physics. *NATO Sci. Ser. B*, 61:687, 1980.

[46] J. Schechter and J.W.F. Valle. Neutrino Masses in SU(2) x U(1) Theories. *Phys. Rev. D*, 22:2227, 1980.

[47] M. Magg and C. Wetterich. Neutrino Mass Problem and Gauge Hierarchy. *Phys. Lett. B*, 94:61–64, 1980.

[48] Rabindra N. Mohapatra and Goran Senjanovic. Neutrino Masses and Mixings in Gauge Models with Spontaneous Parity Violation. *Phys. Rev. D*, 23:165, 1981.

[49] George Lazarides, Q. Shafi, and C. Wetterich. Proton Lifetime and Fermion Masses in an SO(10) Model. *Nucl. Phys. B*, 181:287–300, 1981.

[50] Robert Foot, H. Lew, X.G. He, and Girish C. Joshi. Seesaw Neutrino Masses Induced by a Triplet of Leptons. *Z. Phys. C*, 44:441, 1989.

[51] Carlo Giunti and Chung W. Kim. *Fundamentals of Neutrino Physics and Astrophysics.* 4 2007.

[52] G. Senjanovic. Neutrino mass: From LHC to grand unification. *Riv. Nuovo Cim.*, 34(1):1–68, 2011.

[53] M. Bunge. Energy: Between Physics and Metaphysics. *Science & Education*, pages 459–463, 2000.

[54] Vera C. Rubin and Jr. Ford, W.Kent. Rotation of the Andromeda Nebula from a Spectroscopic Survey of Emission Regions. *Astrophys. J.*, 159:379–403, 1970.

[55] F. Zwicky. Die Rotverschiebung von extragalaktischen Nebeln. *Helv. Phys. Acta*, 6:110–127, 1933.

[56] F. Zwicky. On the Masses of Nebulae and of Clusters of Nebulae. *Astrophys. J.*, 86:217–246, 1937.

[57] Katherine Garrett and Gintaras Duda. Dark Matter: A Primer. *Adv. Astron.*, 2011:968283, 2011.

[58] D. Walsh, R.F. Carswell, and R.J. Weymann. 0957 + 561 A, B - Twin quasistellar objects or gravitational lens. *Nature*, 279:381–384, 1979.

[59] Alan H. Guth. Inflation and the New Era of High-Precision Cosmology. *mit physics annual*, 2002:12, 2002.

[60] G. Lemaitre. The expanding universe. *Gen. Rel. Grav.*, 29:641–680, 1997.

[61] H.P. Robertson. Kinematics and World-Structure. *Astrophys. J.*, 82:284–301, 1935.

[62] A. Friedmann. On the Possibility of a world with constant negative curvature of space. *Z. Phys.*, 21:326–332, 1924.

[63] A. Friedman. On the Curvature of space. *Z. Phys.*, 10:377–386, 1922.

[64] Alan H. Guth. The Inflationary Universe: A Possible Solution to the Horizon and Flatness Problems. *Adv. Ser. Astrophys. Cosmol.*, 3:139–148, 1987.

[65] K. Sato. First Order Phase Transition of a Vacuum and Expansion of the Universe. *Mon. Not. Roy. Astron. Soc.*, 195:467–479, 1981.

[66] Andrei D. Linde. A New Inflationary Universe Scenario: A Possible Solution of the Horizon, Flatness, Homogeneity, Isotropy and Primordial Monopole Problems. *Adv. Ser. Astrophys. Cosmol.*, 3:149–153, 1987.

[67] Andreas Albrecht and Paul J. Steinhardt. Cosmology for Grand Unified Theories with Radiatively Induced Symmetry Breaking. *Adv. Ser. Astrophys. Cosmol.*, 3:158–161, 1987.

[68] N. Aghanim et al. Planck 2018 results. VI. Cosmological parameters. *Astron. Astrophys.*, 641:A6, 2020.

[69] Julien Lesgourgues and Sergio Pastor. Massive neutrinos and cosmology. *Phys. Rept.*, 429:307–379, 2006.

[70] D.J. Fixsen. The Temperature of the Cosmic Microwave Background. *Astrophys. J.*, 707:916–920, 2009.

[71] M Trodden and S.M. Carroll. Tasi lectures: Introduction to cosmology (`https://ned.ipac.caltech.edu/level5/Sept03/Trodden/Trodden_contents.html`).

[72] Daniel Baumann. Primordial cosmology. *PoS*, TASI2017:009, 2018.

[73] S. Nussinov. TECHNOCOSMOLOGY: COULD A TECHNIBARYON EXCESS PROVIDE A 'NATURAL' MISSING MASS CANDIDATE? *Phys. Lett. B*, 165:55–58, 1985.

[74] R.Sekhar Chivukula and Terry P. Walker. TECHNICOLOR COSMOLOGY. *Nucl. Phys. B*, 329:445–463, 1990.

[75] Stephen M. Barr, R.Sekhar Chivukula, and Edward Farhi. Electroweak Fermion Number Violation and the Production of Stable Particles in the Early Universe. *Phys. Lett. B*, 241:387–391, 1990.

[76] David B. Kaplan. A Single explanation for both the baryon and dark matter densities. *Phys. Rev. Lett.*, 68:741–743, 1992.

[77] Dan Hooper, John March-Russell, and Stephen M. West. Asymmetric sneutrino dark matter and the Omega(b) / Omega(DM) puzzle. *Phys. Lett. B*, 605:228–236, 2005.

[78] David E. Kaplan, Markus A. Luty, and Kathryn M. Zurek. Asymmetric Dark Matter. *Phys. Rev. D*, 79:115016, 2009.

[79] Kathryn M. Zurek. Asymmetric Dark Matter: Theories, Signatures, and Constraints. *Phys. Rept.*, 537:91–121, 2014.

[80] Kalliopi Petraki and Raymond R. Volkas. Review of asymmetric dark matter. *Int. J. Mod. Phys. A*, 28:1330028, 2013.

[81] Hooman Davoudiasl and Rabindra N. Mohapatra. On Relating the Genesis of Cosmic Baryons and Dark Matter. *New J. Phys.*, 14:095011, 2012.

[82] Andrew G. Cohen, D.B. Kaplan, and A.E. Nelson. Progress in electroweak baryogenesis. *Ann. Rev. Nucl. Part. Sci.*, 43:27–70, 1993.

[83] David E. Morrissey and Michael J. Ramsey-Musolf. Electroweak baryogenesis. *New J. Phys.*, 14:125003, 2012.

[84] Antonio Riotto and Mark Trodden. Recent progress in baryogenesis. *Ann. Rev. Nucl. Part. Sci.*, 49:35–75, 1999.

[85] Torsten Bringmann and Christoph Weniger. Gamma Ray Signals from Dark Matter: Concepts, Status and Prospects. *Phys. Dark Univ.*, 1:194–217, 2012.

[86] Lars Bergstrom. Dark Matter Evidence, Particle Physics Candidates and Detection Methods. *Annalen Phys.*, 524:479–496, 2012.

[87] Kenneth G. Wilson. The Renormalization Group and Strong Interactions. *Phys. Rev. D*, 3:1818, 1971.

[88] Kenneth G. Wilson. Renormalization group and critical phenomena. 1. Renormalization group and the Kadanoff scaling picture. *Phys. Rev. B*, 4:3174–3183, 1971.

[89] Kenneth G. Wilson. Renormalization group and critical phenomena. 2. Phase space cell analysis of critical behavior. *Phys. Rev. B*, 4:3184–3205, 1971.

[90] H. Georgi. Effective field theory. *Ann. Rev. Nucl. Part. Sci.*, 43:209–252, 1993.

[91] Ben Gripaios. Lectures on Effective Field Theory. 6 2015.

[92] Aneesh V. Manohar. Introduction to Effective Field Theories. *Les Houches Lect. Notes*, 108, 2020.

[93] Riccardo Penco. An Introduction to Effective Field Theories. 6 2020.

[94] Riccardo Barbieri and Alessandro Strumia. The 'LEP paradox'. In *4th Rencontres du Vietnam: Physics at Extreme Energies (Particle Physics and Astrophysics)*, 7 2000.

[95] Morad Aaboud et al. Search for a scalar partner of the top quark in the jets plus missing transverse momentum final state at $\sqrt{s}$=13 TeV with the ATLAS detector. *JHEP*, 12:085, 2017.

[96] M. Aaboud et al. Search for direct top squark pair production in final states with two leptons in $\sqrt{s} = 13$ TeV pp collisions with the ATLAS detector. *Eur. Phys. J. C*, 77(12):898, 2017.

[97] Morad Aaboud et al. Search for direct top squark pair production in events with a Higgs or Z boson, and missing transverse momentum in $\sqrt{s} = 13$ TeV pp collisions with the ATLAS detector. *JHEP*, 08:006, 2017.

[98] Morad Aaboud et al. Search for new phenomena in final states with an energetic jet and large missing transverse momentum in pp collisions at $\sqrt{s} = 13$ TeV using the ATLAS detector. *Phys. Rev. D*, 94(3):032005, 2016.

[99] Albert M Sirunyan et al. Search for supersymmetry in proton-proton collisions at 13 TeV using identified top quarks. *Phys. Rev. D*, 97(1):012007, 2018.

[100] Albert M Sirunyan et al. Search for supersymmetry in multijet events with missing transverse momentum in proton-proton collisions at 13 TeV. *Phys. Rev. D*, 96(3):032003, 2017.

[101] Vardan Khachatryan et al. Search for supersymmetry in the all-hadronic final state using top quark tagging in pp collisions at $\sqrt{s} = 13$ TeV. *Phys. Rev. D*, 96(1):012004, 2017.

[102] Z. Chacko, Hock-Seng Goh, and Roni Harnik. The Twin Higgs: Natural electroweak breaking from mirror symmetry. *Phys. Rev. Lett.*, 96:231802, 2006.

[103] Riccardo Barbieri, Thomas Gregoire, and Lawrence J. Hall. Mirror world at the large hadron collider. 9 2005.

[104] Z. Chacko, Yasunori Nomura, Michele Papucci, and Gilad Perez. Natural little hierarchy from a partially goldstone twin Higgs. *JHEP*, 01:126, 2006.

[105] Z. Chacko, Hock-Seng Goh, and Roni Harnik. A Twin Higgs model from left-right symmetry. *JHEP*, 01:108, 2006.

[106] Gustavo Burdman, Z. Chacko, Hock-Seng Goh, and Roni Harnik. Folded supersymmetry and the LEP paradox. *JHEP*, 02:009, 2007.

[107] Gustavo Burdman, Zackaria Chacko, Roni Harnik, Leonardo de Lima, and Christopher B. Verhaaren. Colorless Top Partners, a 125 GeV Higgs, and the Limits on Naturalness. *Phys. Rev. D*, 91(5):055007, 2015.

[108] David B. Kaplan and Howard Georgi. SU(2) x U(1) Breaking by Vacuum Misalignment. *Phys. Lett. B*, 136:183–186, 1984.

[109] David B. Kaplan, Howard Georgi, and Savas Dimopoulos. Composite Higgs Scalars. *Phys. Lett. B*, 136:187–190, 1984.

[110] Howard Georgi and David B. Kaplan. Composite Higgs and Custodial SU(2). *Phys. Lett. B*, 145:216–220, 1984.

[111] Nima Arkani-Hamed, Andrew G. Cohen, and Howard Georgi. Electroweak symmetry breaking from dimensional deconstruction. *Phys. Lett. B*, 513:232–240, 2001.

[112] N. Arkani-Hamed, A.G. Cohen, E. Katz, A.E. Nelson, T. Gregoire, and Jay G. Wacker. The Minimal moose for a little Higgs. *JHEP*, 08:021, 2002.

[113] N. Arkani-Hamed, A.G. Cohen, E. Katz, and A.E. Nelson. The Littlest Higgs. *JHEP*, 07:034, 2002.

[114] Roberto Contino, Yasunori Nomura, and Alex Pomarol. Higgs as a holographic pseudoGoldstone boson. *Nucl. Phys. B*, 671:148–174, 2003.

[115] Kaustubh Agashe, Roberto Contino, and Alex Pomarol. The Minimal composite Higgs model. *Nucl. Phys. B*, 719:165–187, 2005.

[116] Brian Batell and Christopher B. Verhaaren. Breaking Mirror Twin Hypercharge. *JHEP*, 12:010, 2019.

[117] Hugues Beauchesne, Kevin Earl, and Thomas Grégoire. The spontaneous $\mathbb{Z}_2$ breaking Twin Higgs. *JHEP*, 01:130, 2016.

[118] Tae Hyun Jung. Spontaneous Twin Symmetry Breaking. *Phys. Rev. D*, 100(11):115012, 2019.

[119] Jiang-Hao Yu. Radiative-$\mathbb{Z}_2$-breaking twin Higgs model. *Phys. Rev. D*, 94(11):111704, 2016.

[120] Jiang-Hao Yu. A tale of twin Higgs: natural twin two Higgs doublet models. *JHEP*, 12:143, 2016.

[121] Csaba Csáki, Cong-Sen Guan, Teng Ma, and Jing Shu. Twin Higgs with Exact Z_2. 10 2019.

[122] Riccardo Barbieri, Davide Greco, Riccardo Rattazzi, and Andrea Wulzer. The Composite Twin Higgs scenario. *JHEP*, 08:161, 2015.

[123] Puneet Batra and Z. Chacko. A Composite Twin Higgs Model. *Phys. Rev. D*, 79:095012, 2009.

[124] Matthew Low, Andrea Tesi, and Lian-Tao Wang. Twin Higgs mechanism and a composite Higgs boson. *Phys. Rev. D*, 91:095012, 2015.

[125] Adam Falkowski, Stefan Pokorski, and Martin Schmaltz. Twin SUSY. *Phys. Rev. D*, 74:035003, 2006.

[126] Spencer Chang, Lawrence J. Hall, and Neal Weiner. A Supersymmetric twin Higgs. *Phys. Rev. D*, 75:035009, 2007.

[127] Nathaniel Craig and Kiel Howe. Doubling down on naturalness with a supersymmetric twin Higgs. *JHEP*, 03:140, 2014.

[128] Andrey Katz, Alberto Mariotti, Stefan Pokorski, Diego Redigolo, and Robert Ziegler. SUSY Meets Her Twin. *JHEP*, 01:142, 2017.

[129] Pouya Asadi, Nathaniel Craig, and Ying-Ying Li. Twin Turtles. *JHEP*, 02:138, 2019.

[130] Isabel Garcia Garcia, Robert Lasenby, and John March-Russell. Twin Higgs Asymmetric Dark Matter. *Phys. Rev. Lett.*, 115(12):121801, 2015.

[131] Hugues Beauchesne. Mirror neutrons as dark matter in the Mirror Twin Two Higgs Doublet Model. 7 2020.

[132] Marco Farina. Asymmetric Twin Dark Matter. *JCAP*, 11:017, 2015.

[133] Marco Farina, Angelo Monteux, and Chang Sub Shin. Twin mechanism for baryon and dark matter asymmetries. *Phys. Rev. D*, 94(3):035017, 2016.

[134] Wan-Zhe Feng and Jiang-Hao Yu. Twin Cogenesis. 5 2020.

[135] Ethan M. Dolle and Shufang Su. Dark Matter in the Left Right Twin Higgs Model. *Phys. Rev. D*, 77:075013, 2008.

[136] Isabel Garcia Garcia, Robert Lasenby, and John March-Russell. Twin Higgs WIMP Dark Matter. *Phys. Rev. D*, 92(5):055034, 2015.

[137] Yonit Hochberg, Eric Kuflik, and Hitoshi Murayama. Twin Higgs model with strongly interacting massive particle dark matter. *Phys. Rev. D*, 99(1):015005, 2019.

[138] Seth Koren and Robert McGehee. Freezing-in twin dark matter. *Phys. Rev. D*, 101(5):055024, 2020.

[139] Hsin-Chia Cheng, Lingfeng Li, and Rui Zheng. Coscattering/Coannihilation Dark Matter in a Fraternal Twin Higgs Model. *JHEP*, 09:098, 2018.

[140] Marcin Badziak, Giovanni Grilli Di Cortona, and Keisuke Harigaya. Natural Twin Neutralino Dark Matter. *Phys. Rev. Lett.*, 124(12):121803, 2020.

[141] John Terning, Christopher B. Verhaaren, and Kyle Zora. Composite Twin Dark Matter. *Phys. Rev. D*, 99(9):095020, 2019.

[142] Zackaria Chacko, David Curtin, Michael Geller, and Yuhsin Tsai. Cosmological Signatures of a Mirror Twin Higgs. *JHEP*, 09:163, 2018.

[143] Zackaria Chacko, Nathaniel Craig, Patrick J. Fox, and Roni Harnik. Cosmology in Mirror Twin Higgs and Neutrino Masses. *JHEP*, 07:023, 2017.

[144] Nathaniel Craig, Seth Koren, and Timothy Trott. Cosmological Signals of a Mirror Twin Higgs. *JHEP*, 05:038, 2017.

[145] Csaba Csaki, Eric Kuflik, and Salvator Lombardo. Viable Twin Cosmology from Neutrino Mixing. *Phys. Rev. D*, 96(5):055013, 2017.

[146] Riccardo Barbieri, Lawrence J. Hall, and Keisuke Harigaya. Minimal Mirror Twin Higgs. *JHEP*, 11:172, 2016.

[147] Richard H. Cyburt, Brian D. Fields, Keith A. Olive, and Tsung-Han Yeh. Big Bang Nucleosynthesis: 2015. *Rev. Mod. Phys.*, 88:015004, 2016.

[148] Zhen Hou, Ryan Keisler, Lloyd Knox, Marius Millea, and Christian Reichardt. How Massless Neutrinos Affect the Cosmic Microwave Background Damping Tail. *Phys. Rev. D*, 87:083008, 2013.

[149] Daniel Baumann, Daniel Green, Joel Meyers, and Benjamin Wallisch. Phases of New Physics in the CMB. *JCAP*, 01:007, 2016.

[150] Nathaniel Craig, Andrey Katz, Matt Strassler, and Raman Sundrum. Naturalness in the Dark at the LHC. *JHEP*, 07:105, 2015.

[151] Nathaniel Craig, Simon Knapen, Pietro Longhi, and Matthew Strassler. The Vector-like Twin Higgs. *JHEP*, 07:002, 2016.

[152] Georges Aad et al. Measurements of the Higgs boson production and decay rates and constraints on its couplings from a combined ATLAS and CMS analysis of the LHC pp collision data at $\sqrt{s} = 7$ and 8 TeV. *JHEP*, 08:045, 2016.