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Física além do Modelo Padrão: Desafios Teóricos e Fenomenológicos

Testes de Precisão Eletrofracos

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Electroweak Precision Measurements: Present and Future

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Resumo

Graças ao trabalho de milhares de físicos nos últimos 80 anos, atualmente nós temos uma compreensão extraordinária de como a natureza se comporta em altas energias. Tudo no Universo, do quasar mais distante ao menor átomo, é constituído do que chamamos de partículas fundamentais. Essas partículas são governadas por quatro diferentes forças: forte, fraca, eletromagnética e gravitacional. Desenvolvido no início da década de 1970, o chamado Modelo Padrão da física de partículas é capaz de descrever como as partículas estão relacionadas a três dessas forças.

Desde que foi proposto, o Modelo Padrão conseguiu explicar diversos resultados experimentais com uma precisão excepcional. Além disso, foi capaz de prever corretamente a existência de muitas novas partículas, incluindo o famoso bóson de Higgs. No entanto, embora seja nossa melhor descrição do mundo subatômico até agora, o Modelo Padrão não explica o quadro completo. Um dos maiores problemas é que a teoria falha em descrever a gravidade e explicar o porquê dessa força fundamental ser muito mais fraca do que as demais. Outra grande dificuldade é o que os físicos chamam de problema da hierarquia, que se refere à sensibilidade da massa do Higgs à novas escalas. Há ainda muitas outras questões deixadas sem resposta pelo Modelo Padrão, como a assimetria entre matéria e antimatéria, a natureza da matéria escura e da energia escura, o problema da violação CP forte e a massa dos neutrinos. Assim, ficou claro que deve haver nova física escondida no Universo.

Nos últimos anos, os físicos têm se dedicado ao desenvolvimento de diferentes extensões do Modelo Padrão. Como os testes de precisão foram cruciais para estabelecer a validade da nossa atual descrição, eles também serão importantes para determinar se nova física já está se manifestando nos dados experimentais.

Focando nas restrições fenomenológicas que os testes de precisão podem impor sobre o setor eletrofraco, pretendemos buscar um novo programa de precisão no qual serão consideradas as modificações mais genéricas devido à nova física. Nós aplicaremos este formalismo à teorias que estendem a simetria de *gauge* do Modelo Padrão com um novo grupo Abeliano chamado $U(1)_X$. O bóson de *gauge* associado a $U(1)_X$ pode se misturar com o bóson Z e o fóton do Modelo Padrão através de um termo cinético. Além disso, dependendo de como escolhemos quebrar essa simetria extra, o novo bóson de *gauge* X pode também se misturar com ambos a partir de um termo de massa. Como mostraremos, tais misturas implicam em três novos auto-estados. O fóton e o bóson Z que observamos agora serão uma mistura dos campos do modelo padrão e do bóson X. O mesmo é verdade para o terceiro auto-estado observável, conhecido como bóson Z'. Neste trabalho, propomos que o Z' tenha uma massa na região do MeV-GeV. Essa faixa de massa tem sido de grande interesse para os físicos, já que novas partículas podem ser muito leves e, ainda assim, não terem sido descobertas nos aceleradores de partículas.

Acreditamos que as modificações dos observáveis eletrofracos devido à presença de um bóson Z' leve não foram estudadas sistematicamente na literatura. Portanto, nossa análise consiste em realizar um *fit* global para a massa do bóson W e outros onze observáveis medidos na ressonância do Z. Isso nos permite determinar uma região de exclusão no espaço de parâmetros do nosso modelo, estabelecendo a faixa de massa do bóson Z' que é consistente com os atuais dados experimentais. Finalmente, podemos verificar se nosso modelo é capaz de explicar a tensão entre os valores teórico e experimental do momento magnético do múon. Essa discrepância tem sido considerada uma das restrições mais rigorosas sobre os potenciais efeitos de nova física.

Palavras-chave: Modelo Padrão, Portais Vetoriais, Boson Z', Testes de Precisão Eletrofracos, Momento Magnético Anômalo do Múon.

Abstract

Thanks to the work of thousands of physicists over the past 80 years, we now have a remarkable understanding of how nature behaves at high energies. Everything in the Universe, from the most distant quasar to the smallest atom, is made of what we call fundamental particles. These particles are governed by four different forces: strong, weak, electromagnetic and gravitational. Developed in the early 1970s, the so-called Standard Model of particle physics is capable of describing how particles are related to three of these forces.

Since it was first proposed, the Standard Model has successfully explained several experimental results to an outstanding precision, and correctly predicted the existence of many new particles, including the famous Higgs boson. However, even though it is currently our best description of the subatomic world, the Standard Model does not explain the complete picture. One major problem is that the theory fails to describe gravity and to answer why this fundamental force is much weaker than the others. Another big difficulty is what physicists call the hierarchy problem, which refers to the sensitivity of the Higgs mass to new scales. Besides these two, there are many other questions left unanswered by the Standard Model, such as the matter-antimatter asymmetry, the nature of dark matter and dark energy, the strong CP problem, and the mass of neutrinos. Thus, it became clear there must be new physics hidden deep in the Universe.

In the past few years, physicists have dedicated themselves to the development of different extensions of the Standard Model. Since precision experiments have been crucial in establishing the validity of our current description, they will also be instrumental to assess whether new physics is already manifesting itself in experimental data.

Focusing on the phenomenological constraints that precision measurements can provide on the gauge sector of the electroweak group, we aim to pursue a new precision program in which the most generic modifications due to new physics will be considered. We intend to apply this formalism to theories that extend the Standard Model gauge symmetry by a new Abelian group called $U(1)_X$. The gauge boson associated with $U(1)_X$ can mix with both the Standard Model Z boson and photon through the kinetic term. Furthermore, depending on how we choose to break this extra symmetry, the new gauge boson X can also have a mass mixing with them. As we will show, such mixings imply in three new eigenstates. The photon and Z boson we observe are now a mixture of the Standard Model fields and the X boson field. The same is true for the third observable eigenstate, which is known as Z' boson. In this work, we propose the Z' to be in the MeV-GeV mass range. Such mass range has been of great interest to physicists since they realized new particles can be quite light and still have evaded discovery in particle accelerators.

Modifications of electroweak observables due to the presence of a light Z' boson have not been studied systematically in the literature to date. Thus, our analysis consists in performing a global fit to the W boson mass and other eleven observables measured at the Z resonance. This allows us to determine an exclusion region in the parameter space of our model, and establish the mass range of the Z' boson consistent with current experimental data. Finally, we can check whether our model is able to explain the tension between the theoretical and experimental values of the muon magnetic moment. Such discrepancy is now considered one of the most stringent constraints on potential new physics effects.

Keywords: Standard Model, Vector Portals, Z' Boson, Electroweak Precision Measurements, Muon Anomalous Magnetic Moment.

Contents

Chapter 1

Introduction

The gauge principle has proven to be a tremendously successful method in elementary particle physics to generate interactions between matter fields through the exchange of (massless and massive) gauge bosons. The best known-example is, of course, Quantum Electrodynamics (QED). Since the photon is the only particle that was recognized as a field before it was detected as a particle, it is natural that the formalism of Quantum Field Theory (QFT) was developed as an effort to better understand electromagnetic phenomena. But the attempts of applying this same formalism to the other fundamental forces of nature has led to one of the most consistent theories ever created, the Standard Model of particle physics.

The Standard Model is a theory based on the gauge symmetry group $SU(3)_C \times SU(2) \times U(1)_Y$. The $SU(3)$ part describes the so-called Quantum Chromodynamics (QCD) [1], a non-Abelian gauge theory in which the matter fields are spin 1/2 fermions (quarks) carrying a color charge. There are six different flavors of quarks with three possible colors for each one (red, green and blue). The gauge bosons of QCD are known as gluons and there are eight of them, one for each generator of $SU(3)$. The strong interactions do not mix flavor, implying that the gluons must be flavor neutral. However, due to the non-Abelian character of $SU(3)$, they also carry color charge, which allows gluons to interact with themselves.

The non-Abelian nature of QCD has another very important physical effect. Because of its negative β-function, the coupling constant decreases for increasing energies. This behavior is known as asymptotic freedom and is a consequence of an anti-screening due to gluonic interactions that prevails over the screening due to quark/anti-quark pairs. On the other hand, at low energies (in the infrared) a non-perturbative behavior is observed: quarks and gluons confine to form color neutral states called hadrons. Since quarks have never been observed as asymptotically free states, the concept of mass is tightly connected with the method by which they are extracted from hadronic properties.

The $SU(2) \times U(1)_Y$ symmetry was introduced by Glashow, Weinberg and Salam in the 1960s. The GWS model, as it became known, gives a unified description of weak and electromagnetic interactions [2, 3, 4, 5, 6]. The electroweak unification is based on the spontaneous symmetry breaking pattern $SU(2) \times U(1)_Y \to U(1)_{EM}$. The high-energy $U(1)_Y$ symmetry is called hypercharge, while the low-energy $U(1)_{EM}$ is associated with electromagnetism. The mentioned symmetry breaking happens due to a complex doublet, known as the Higgs multiplet, which acquires a vacuum expectation value (vev) at 246 GeV.

The GWS model also predicts the existence of four gauge bosons, three of them being massive and only one massless (the photon). These gauge bosons can couple to all fermionic matter fields through neutral and charged currents. In fact, the discovery of neutral currents in

[1] A great pedagogical introduction to the theoretical foundations of QCD and Chiral Perturbation Theory (CPT) can be found at [1].

1973 gave the first experimental verification of the Standard Model. Later, at the beginning of the 1980s, the discovery of the massive W and Z bosons, and the validation of their properties, finally established the electroweak unification. Since then, the electroweak gauge sector has been tested over an enormous range of probes and scales.

Although the Standard Model has a great number of parameters, it is an over-constrained theory. Assuming Dirac neutrino masses, the Standard Model will have 27 parameters. However, there are enough independent measurements that can be done in order to constrain the theory. Through these tests, we are able to connect our quantum field theory at 1-loop (or more) to measured quantities. Radiative corrections will play an important role in the gauge sector of the electroweak theory and in the arena of flavor physics. But since in the latter the non-perturbative effects of QCD become very significant, we will focus mainly on electroweak precision measurements. They have been crucial to prove the validity of our current description of nature, and will continue to be essential in the search of new physics.

The Standard Model, whilst being very successful until now, is still incapable of describing the complete picture of what happens in our Universe. Physicists have tried to describe gravity using the QFT formalism, but this only led to a non-renormalizable theory. We also expect that, at the scale where quantum fluctuations of the gravitational field become important, there will be profound changes in physics as we know it. Thus, if we consider the Standard Model as an effective field theory up to some cutoff Λ, we find that the Higss mass parameter is sensitive to this new scale or, in other words, it will be quadratically divergent. This sensitivity is known as the hierarchy problem [2].

There are many other questions left unanswered by the Standard Model, such as the matter-antimatter asymmetry, the nature of dark matter and dark energy, the strong CP problem, and the mass of neutrinos [3]. This is the reason why physicists have been working on new theoretical developments that aim to extend the Standard Model. Several models were proposed, including string theory, supersymmetry, extra dimensions, multiverse, and composite Higgs.

In this work, we aim to develop a new phenomenological analysis of one of the simplest extensions of the Standard Model. In such theories, the electroweak gauge structure is extended by an additional $U(1)$ symmetry. It was usually believed that the gauge boson associated with this symmetry should be very heavy (i.e., heavier than several hundred GeV's at least). However, since physicists realized such gauge boson could be light, provided they are also very weakly coupled, the eV-GeV mass range became of great interest. Thus, we will explore ways to extend the Standard Model by a new Abelian group called $U(1)_X$ in which the associated gauge boson X could be light and weakly coupled.

In order to give a mass to the gauge boson X, we must also spontaneously break the extra $U(1)$ symmetry. In fact, the spontaneous symmetry breaking pattern is now given by $SU(2) \times U(1)_Y \times U(1)_X \to U(1)_{EM}$. Since we want to guarantee the breaking of both $U(1)$ groups, our theory must contain at least two scalar fields: the Higgs doublet and a complex singlet. As we are going to show, depending on how these scalar fields are charged under the whole gauge group, the X boson can mix with the Standard Model photon and Z boson through a mass term. Furthermore, a kinetic mixing with the Standard Model B field is not forbidden by gauge invariance. As a consequence, the photon and Z boson we observe are now a mixture of the Standard Model fields and the X boson field. The same is true for the third observable eigenstate, which is known as Z' boson.

Our phenomenological analysis consists in determining how the electroweak observables

[2]For a review of the physical meaning of the hierarchy problem see Chapter 1 of [7].

[3]Symmetry Magazine (a joint Fermilab/SLAC online publication) published a nice story about some of the mysteries that the Standard Model cannot explain: https://www.symmetrymagazine.org/article/five-mysteries-the-standard-model-cant-explain.

change in the presence of the so-called Z' boson. Although we are interested mainly in corrections up to tree-level, if this new gauge boson is very light, its contribution to the electron magnetic moment cannot be neglected a priori, given that the leading order correction to $(g-2)_e$ is at 1-loop. Thus, at first we aim to perform a global fit to the W boson mass and other eleven observables measured at the Z resonance. This allows us to determine exclusion regions in the parameter space of our model, and establish a mass range of the Z' boson consistent with current experimental data. Finally, we can use our model to try to explain the tension between the theoretical and experimental values of the muon magnetic moment. Since this discrepancy can be evidence of new physics lying beyond the Standard Model, we can look for regions in the parameter space in which the Z' contribution can account for such anomaly.

Chapter 2

Weak Interactions

Any quantum field theory textbook [2, 3, 4, 5, 6] provides indispensable theoretical tools for the understanding of the properties of elementary particles, forming the backbone of one of the most remarkable achievements of humankind: the Standard Model of particle physics. In these textbooks, the GWS model for electroweak unification is always presented as the principal application of spontaneously broken symmetries. Authors often like to discuss how precise and successful are the predictions of the GWS model. Since the main focus of this thesis is the analysis of the precision tests of the Standard Model, the best way to start it is by briefly describing the GWS model. This will also allow us to set our notation for the following chapters.

2.1 The GWS Model

Symmetry concepts dominate our current understanding of modern fundamental physics, both in quantum theory and relativity. A symmetry can be exact, approximate or broken. In particle physics, broken symmetries have a crucial role. The study of symmetry breaking goes back to Pierre Curie [1], who determined that, for the occurrence of certain phenomena, the original underlying symmetry group must be lowered (broken) to some of its subgroups. Thus, a symmetry can be explicitly broken, i.e., when one or more terms in the Lagrangian are not invariant under the symmetry group considered, or it can be spontaneously broken, such that the full Lagrangian is invariant, but the ground state of the theory is not.

The GWS model is based on the spontaneous symmetry breaking of $SU(2) \times U(1)_Y \to U(1)_{EM}$, which occurs through the Higgs mechanism. Since gauge invariance does not allow mass terms in the Lagrangian for both gauge bosons and chiral fermions, this mechanism is extremely useful to generate particle masses. The symmetry will be broken by the vacuum expectation value of some complex field introduced to our theory. If we consider a gauge Lagrangian for the $SU(2) \times U(1)_Y$ symmetry and a scalar Lagrangian for a complex doublet with hypercharge 1/2, i.e., the multiplet included is charged under both groups, one can generically write

$$\mathscr{L} = -\frac{1}{4} W^a_{\mu\nu} W^{\mu\nu a} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + (D_\mu H)^\dagger (D^\mu H) + m^2 H^\dagger H - \lambda (H^\dagger H)^2, \tag{2.1.1}$$

where $B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$ and $W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g\epsilon^{abc} W^b_\mu W^c_\nu$. Due to the non-Abelian nature of $SU(2)$, we can see that the W fields will have three and four-point self-interactions. The covariant derivative acting on H is defined as

[1] For a brief description of the historical roots and emergence of the concept of symmetry in modern physics see https://plato.stanford.edu/entries/symmetry-breaking/.

$$D_\mu H = \partial_\mu H - \frac{1}{2} i g W_\mu^a \tau^a H - \frac{1}{2} i g' B_\mu H. \tag{2.1.2}$$

Here, g and g' are the couplings of $SU(2)$ and $U(1)_Y$ groups, respectively. The complex doublet H is the so-called Higgs multiplet. Minimizing its potential, we can easily see that the induced vev is given by

$$\left\langle H^\dagger H \right\rangle = \frac{v^2}{2}, \tag{2.1.3}$$

where we have used $v^2 = m^2/\lambda$. Thus, the Higgs multiplet can be expanded around the ground state as

$$H(x) = U(x) \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}, \tag{2.1.4}$$

such that $U(x)$ is a $SU(2)$ gauge transformation leaving H a complex-valued field, and $h(x)$ is a real-valued fluctuation around the vev. Since the field $h(x)$ annihilates the vacuum, i.e., $\langle h \rangle = \left\langle h^2 \right\rangle = 0$, Eq. (2.1.4) will satisfy Eq. (2.1.3).

Without any loss of generality, we can use a representation in which $U(x)$ is given in terms of the broken generators, i.e., those generators that will not annihilate the vacuum anymore. According to Goldstone's theorem, to each broken generator corresponds a massless Nambu-Goldstone boson, whose symmetry properties are tightly connected to the generator in question. For now, we will work in the unitary gauge and set $U(x) = 1$ or, equivalently, set the Goldstone fields to zero. Thus, plugging only the vev in the covariant derivative, one finds that

$$(D_\mu H)^\dagger (D^\mu H)\Big|_{vev} = \frac{g^2 v^2}{8} \left[2 W_\mu^- W^{\mu +} + (\tan\theta_w B_\mu - W_\mu^3)^2 \right], \tag{2.1.5}$$

where we have defined $\tan\theta_w = g'/g$ and the combinations $W_\mu^\pm = \frac{1}{\sqrt{2}}(W_\mu^1 \mp i W_\mu^2)$. Since the kinetic terms are already canonically normalized, we can rotate B_μ and W_μ^3 to find the following diagonalized combinations:

$$\left.\begin{array}{l} Z_\mu = \cos\theta_w W_\mu^3 - \sin\theta_w B_\mu \\ A_\mu = \sin\theta_w W_\mu^3 + \cos\theta_w B_\mu \end{array}\right\} \Leftrightarrow \left\{ \begin{array}{l} B_\mu = \cos\theta_w A_\mu - \sin\theta_w Z_\mu \\ W_\mu^3 = \sin\theta_w A_\mu + \cos\theta_w Z_\mu \end{array}\right. . \tag{2.1.6}$$

Note if we had chosen a different direction for the Higgs vev, these definitions would have changed. However, the final (and physical) outcome would still be the same. Thus, after spontaneous symmetry breaking, the covariant derivative generates mass terms for the W and Z bosons:

$$(D_\mu H)^\dagger (D^\mu H)\Big|_{vev} = M_W^2 W_\mu^- W^{\mu +} + \frac{M_Z^2}{2} Z_\mu Z^\mu, \tag{2.1.7}$$

with $M_W = \frac{1}{2}(vg)$ and $M_Z = \frac{1}{2}(v\sqrt{g^2 + g'^2})$. The fact that A_μ is massless already gives an indication that this could be the photon field. Another unambiguous prediction comes from these results: the W boson should be lighter than the Z boson. If we use the definition of the mixing angle θ_w into the expressions of the gauge boson masses, it is easy to show that

$$M_Z = \frac{M_W}{\cos\theta_w}. \tag{2.1.8}$$

The gauge kinetic terms in the Lagrangian of Eq. (2.1.1) will provide interaction terms among $W_\mu^\pm$, A_μ and Z_μ. Since gauge bosons transform in the adjoint representation of a group, their interactions are determined by commutators. Defining the generators $\tau^\pm = \frac{1}{\sqrt{2}}(\tau^1 \pm i\tau^2)$, we obtain

$$\begin{aligned} gW_\mu^1 W_\nu^3 \left[\frac{\tau^1}{2}, \frac{\tau^3}{2}\right] + gW_\mu^2 W_\nu^3 \left[\frac{\tau^2}{2}, \frac{\tau^3}{2}\right] = -\frac{1}{2}\big[g \sin\theta_w W_\mu^+ A_\nu \tau^+ - \\ - g\sin\theta_w W_\mu^- A_\nu \tau^- + g\cos\theta_w W_\mu^+ Z_\nu \tau^+ - g\cos\theta_w W_\mu^- Z_\nu \tau^-\big]. \end{aligned} \tag{2.1.9}$$

Thus, up to an overall factor, the previous result indicates that if one identifies A_μ as truly being the photon field, the electromagnetic coupling strength can be set by $e = g\sin\theta_w = g'\cos\theta_w$. This leads to

$$\begin{aligned} gW_\mu^1 W_\nu^3 \left[\frac{\tau^1}{2}, \frac{\tau^3}{2}\right] + gW_\mu^2 W_\nu^3 \left[\frac{\tau^2}{2}, \frac{\tau^3}{2}\right] \approx eW_\mu^+ A_\nu \tau^+ - eW_\mu^- A_\nu \tau^- + \\ + \frac{e}{\tan\theta_w} W_\mu^+ Z_\nu \tau^+ \frac{e}{\tan\theta_w} W_\mu^- Z_\nu \tau^-, \end{aligned} \tag{2.1.10}$$

implying that the W bosons have electric charges ± 1 in units of e, and couplings to the Z boson given by $\pm\frac{1}{\tan\theta_w}e$. The only interaction that vanishes is between the photon and the Z boson. They are both linear combinations of B_μ and W_μ^3 and, as a consequence, the commutator appearing in their interaction is zero ($\left[\tau^3, \tau^3\right] = 0$).

To continue our analysis, it will be interesting to incorporate the Nambu-Goldstone bosons back to the complex doublet H. We already know that the spontaneous symmetry breaking pattern is $SU(2)\times U(1)_Y \to U(1)_{EM}$. The $SU(2)$ group has three generators (the Pauli matrices), while each $U(1)$ group possesses only one. Hence, our theory starts with a total of four generators and, after spontaneous symmetry breaking, just one of them still annihilates the vacuum. Since for every broken generator there is an associated Goldstone boson, we conclude that H must also describe three extra scalar fields. Choosing a convenient representation, we can immediately write

$$H(x) = \begin{pmatrix} w^+(x) \\ \frac{1}{\sqrt{2}}\left(v + h(x) + iz(x)\right) \end{pmatrix} \text{ and } H^\dagger(x) = \begin{pmatrix} w^-(x) \\ \frac{1}{\sqrt{2}}\left(v + h(x) - iz(x)\right) \end{pmatrix}. \tag{2.1.11}$$

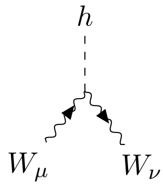
If now we substitute $H(x)$ in Eq. (2.1.1) and expand out the Lagrangian, we find several terms with significant physical meanings:

$$\begin{aligned} \mathscr{L}_{Higgs} = -\frac{1}{2}h(\Box + m_h^2)h - g\frac{m_h^2}{4M_W}h^3 - \frac{g^2 m_h^2}{32M_W^2}h^4 + \\ + \left(\frac{h^2}{v^2} + 2\frac{h}{v}\right)\left(M_W^2 W_\mu^- W^{+\mu} + \frac{M_Z^2}{2} Z_\mu Z^\mu\right), \end{aligned} \tag{2.1.12}$$

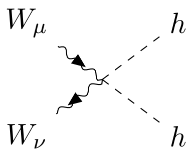
$$\begin{aligned} \mathscr{L}_{\text{Int with NGBs}} = -i\frac{e}{2\cos\theta_w}\left[w^+(\partial_\mu h) - h(\partial_\mu w^+)\right] W^{-\mu} + \\ + i\frac{e}{2\cos\theta_w}\left[w^-(\partial_\mu h) - h(\partial_\mu w^-)\right] W^{+\mu} - \\ - \frac{e}{2\sin\theta_w\cos\theta_w}\left[z(\partial_\mu h) - h(\partial_\mu z)\right] Z^\mu, \end{aligned} \tag{2.1.13}$$

$$\mathscr{L}_{\text{NGBs eaten}} = iM_W(\partial_\mu w^+)W^{-\mu} - iM_W(\partial_\mu w^-)W^{+\mu} + M_Z(\partial_\mu z)Z^\mu. \tag{2.1.14}$$

The first Lagrangian describes the Higgs boson field h. Note we have defined the H doublet as being charged under both the weak and hypercharge gauge groups. However, the Higgs boson is a fluctuation around the vev and, as a consequence, it can have no charge under these symmetries. The particle mass was identified as $m_h = \sqrt{2}m$, where m is one of the parameters of the original Lagrangian. The interaction terms show that the Higgs boson couples to the W and Z bosons through their masses. Thus, the Feynman rules derived from this Lagrangian are

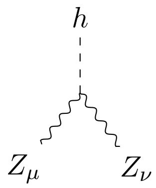


$$= i\frac{e}{\sin\theta_w}M_W g_{\mu\nu}, \tag{2.1.15}$$

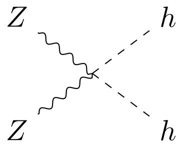


$$= i\frac{e^2}{2\sin^2\theta_w}g_{\mu\nu}, \tag{2.1.16}$$

for a Higgs boson interacting with W bosons, and



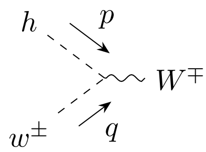
$$= i\frac{e}{\sin\theta_w\cos^2\theta_w}M_W g_{\mu\nu}, \tag{2.1.17}$$



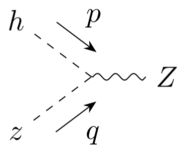
$$= i\frac{e^2}{2\sin^2\theta_w\cos^2\theta_w}g_{\mu\nu}, \tag{2.1.18}$$

for a Higgs coupling to Z bosons. These interactions come to rescue when we try, for example, evaluate the cross section of scattering longitudinal modes of the W and Z bosons. If we compute the total matrix element of such processes considering only contributions from purely W-Z interactions at high energies, it will seem that our result grows indefinitely with the energy [6]. However, the unitarity bound, a consequence of the optical theorem, says that amplitudes cannot be arbitrarily large. This means that we must be missing some other contribution. One can immediately think on loop contributions. They should be important since theories with massive gauge bosons are non-renormalizable. On the other hand, if m_h is not too large, adding the Higgs can be sufficient to cancel the high-energy behavior.

From the second Lagrangian we are able to derive the Feynman rules for interactions among the Higgs, the gauge bosons and the Goldstones. Although we can always go in the unitary gauge, some physical quantities are easier to compute if we consider the contributions coming from the Goldstone bosons. Thus, the vertices are given by



$$= \pm i \frac{e}{2\sin\theta_w}(p-q)_\mu, \tag{2.1.19}$$



$$= \frac{e}{2\sin\theta_w \cos\theta_w}(p-q)_\mu. \tag{2.1.20}$$

Finally, the last Lagrangian confirms the intricate role of the Goldstone bosons in the spontaneous symmetry breaking of $SU(2) \times U(1)_Y$. Earlier in our discussion, we have eliminated them using a convenient gauge choice. However, one can argue that the Goldstone bosons have not completely disappeared. In fact, the contributions of Eq. (2.1.14) show that the Goldstone bosons are "eaten" by the gauge bosons. This means they can be identified as the longitudinal degrees of freedom of the massive W and Z bosons. Therefore, if we try to compute again the cross section of scattering longitudinal modes of the W and Z bosons, but this time replacing the gauge fields by Goldstone bosons, the fact that we get exactly the same result is not a coincidence.

So far we have focused only on the gauge and Higgs sectors of the $SU(2) \times U(1)_Y$ group. But now we must also discuss how to couple these particles to all the known fermions in the Standard Model. It turns out that, well before the discovery of the W and Z bosons, experiments indicated that the theory of weak interactions is chiral and maximally parity-violating, i.e., the $SU(2)$ gauge bosons only couple to left-handed fermions. Thus, it is intuitive to pair them up as doublets that transform under the fundamental representation of this group. Actually, there are three generations of doublet pairs of leptons and quarks:

$$L^i = \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}, \begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix}, \begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix}, \quad Q^i = \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \begin{pmatrix} c_L \\ s_L \end{pmatrix}, \begin{pmatrix} t_L \\ b_L \end{pmatrix}. \tag{2.1.21}$$

Each left-handed spinor transforms under the $\left(\frac{1}{2}, 0\right)$ representation of the Lorentz group. Although the three generations have the same quantum numbers (including hypercharge), their masses are very different. The third generation is always the heaviest one.

On the other hand, right-handed fermions are singlets under $SU(2)$, i.e, they are uncharged under the weak interactions:

$$e_R^i = \{e_R, \mu_R, \tau_R\}, \quad u_R^i = \{u_R, c_R, t_R\}, \quad d_R^i = \{d_R, s_R, b_R\}. \tag{2.1.22}$$

Each right-handed spinor transforms under the $\left(0, \frac{1}{2}\right)$ representation of the Lorentz group. Note that, originally, the GWS model did not include right-handed neutrinos [4]. Since there was no experimental evidence that neutrinos were in fact massive, right-handed components were not needed. However, even though we still cannot observe such degress of freedom, we now know that neutrinos do have a mass [6]. Therefore, it is useful to already add right-handed neutrinos to the Standard Model: $\nu_R^i = \{\nu_{eR}, \nu_{\mu R}, \nu_{\tau R}\}$.

Writing Y_Q and Y_L as the hypercharges of left-handed fields, and Y_e, Y_ν, Y_u and Y_d for the right-handed fields, the gauge couplings to fermions will come from

$$
\begin{aligned}
\mathscr{L} =& i\bar{L}_i \left(\not{\partial} - \frac{1}{2} i g \not{W}^a \tau^a - i g' Y_L \not{B} \right) L_i + i \bar{Q}_i \left(\not{\partial} - \frac{1}{2} i g \not{W}^a \tau^a - i g' Y_Q \not{B} \right) Q_i + \\
&+ i \bar{e}_R^i \left(\not{\partial} - i g' Y_e \not{B} \right) e_R^i + i \bar{\nu}_R^i \left(\not{\partial} - i g' Y_\nu \not{B} \right) \nu_R^i + \\
&+ i \bar{u}_R^i \left(\not{\partial} - i g' Y_u \not{B} \right) u_R^i + i \bar{d}_R^i \left(\not{\partial} - i g' Y_d \not{B} \right) d_R^i .
\end{aligned}
\tag{2.1.23}
$$

If we isolate the neutral gauge bosons W_μ^3 and B_μ, and then change to the A_μ-Z_μ basis, keeping only terms proportional to A_μ, we get

$$
\begin{aligned}
\mathscr{L} = e & \left\{ \bar{L}_i \left(\frac{\tau^3}{2} + Y_L \right) \not{A} L_i + \bar{Q}_i \left(\frac{\tau^3}{2} + Y_Q \right) \not{A} Q_i \right\} + \\
&+ e \left\{ \bar{e}_R^i Y_e \not{A} e_R^i + \bar{\nu}_R^i Y_\nu \not{A} \nu_R^i + \bar{u}_R^i Y_u \not{A} u_R^i + \bar{d}_R^i Y_d \not{A} d_R^i \right\} .
\end{aligned}
\tag{2.1.24}
$$

Since the electric charges are the coefficients of the coupling to a massless gauge boson, namely the photon, one can relate the hypercharges and the electric charges through the above expression. For example, using that the electron is conventionally defined to have charge -1 in units of e, we see that $Y_L = -\frac{1}{2}$ and $Y_e = -1$.

Through this same analysis, but also keeping terms proportional to Z_μ, we are able to define the so-called neutral currents of $SU(2) \times U(1)_Y$:

$$
\mathscr{L} \supset \frac{e}{\sin\theta_w} Z_\mu J_Z^\mu + e A_\mu J_{EM}^\mu ,
\tag{2.1.25}
$$

where

$$
J_Z^\mu = \frac{1}{\cos\theta_w} \left(J_3^\mu - \sin^2\theta_w J_{EM}^\mu \right) ,
\tag{2.1.26}
$$

$$
J_3^\mu = \sum_i \bar{\psi_i^L} \gamma^\mu \frac{\tau^3}{2} \psi_i^L ,
\tag{2.1.27}
$$

$$
J_{EM}^\mu = \sum_i \left(\frac{\tau^3}{2} + Y_i \right) \left(\bar{\psi_i^L} \gamma^\mu \psi_i^L + \bar{\psi_i^R} \gamma^\mu \psi_i^R \right) .
\tag{2.1.28}
$$

We can see that, in the electromagnetic current, when τ^3 acts on right-handed spinors, namely the singlets of $SU(2)$, the result must be zero. Since they were predicted only by the GWS model, the weak neutral currents are historically very important. Their discovery in 1973 gave the first experimental verification of what later became the Standard Model of particle physics.

Doing the same procedure for the charged vector bosons, we find

$$
\mathscr{L} \supset \frac{e}{\sqrt{2}\sin\theta_w} \left(W_\mu^+ J_+^\mu + W_\mu^- J_-^\mu \right) ,
\tag{2.1.29}
$$

with

$$
J_+^\mu = \frac{1}{\sqrt{2}} \sum_i \left(\bar{L}_i \, \gamma^\mu \, \tau^+ \, L_i + \bar{Q}_i \, \gamma^\mu \, \tau^+ \, Q_i \right) ,
\tag{2.1.30}
$$

$$J^{\mu}_{-} = \frac{1}{\sqrt{2}} \sum_i \left(\bar{L}_i \, \gamma^{\mu} \, \tau^{-} \, L_i + \bar{Q}_i \, \gamma^{\mu} \, \tau^{-} \, Q_i \right) . \tag{2.1.31}$$

Since τ^+ and τ^- are non-diagonal matrices, we can see that charged currents will mix flavor. Thus, due to the nature of these currents, we are able to describe processes such as nuclear β decay and μ decay. In fact, even the low energy limit of the charged interactions gives a very accurate description of these phenomena. The so-called 4-Fermi theory can be understood as an effective description of the weak interactions in which the massive gauge bosons were integrated out:

$$\mathscr{L}_{Fermi} = -\frac{4G_F}{\sqrt{2}} \left(J_{+\mu} \, J^{\mu}_{-} \right) , \tag{2.1.32}$$

where we have defined the Fermi constant as $\frac{G_F}{\sqrt{2}} = \frac{e^2}{8 \, M_W^2 \sin^2 \theta_w}$. As one can see, the above Lagrangian describes a local four-fermion interaction, which was introduced by Fermi around 30 years before the W and Z bosons were first proposed.

Now that we have derived all the formalism about the spontaneous symmetry breaking of $SU(2) \times U(1)_Y$, and discussed how particles couple in the Standard Model, we are ready to analyze more subtle consequences of this theory.

2.2 Fermion Masses and Mixing Angles

In theories like QED, left- and right-handed fermions are connected by a Dirac mass term. However, we can see that adding to our Lagrangian $\bar{\psi}_L \psi_R$ explicitly breaks $SU(2)$ invariance, thus this term is forbidden. We can use the Higgs multiplet to establish that fermion masses only appear after electroweak symmetry breaking. Yukawa terms like

$$\mathscr{L}_{Yukawa} = -y_i \, \bar{L}^i H e^i_R + h.c. \tag{2.2.1}$$

will correctly generate a Dirac mass term for the charged leptons after H gets a vev. Hence, their masses must be related to both Higgs vev and Yukawa couplings: $m_i = \frac{1}{\sqrt{2}} y_i v$. Through this same construction, we can also give mass to the down-type quarks (d, s, b).

To give mass to the remaining fermions (neutrinos and up-type quarks), we have to introduce the field $\tilde{H} = i\tau^2 H^*$. This field transforms in the fundamental representation of $SU(2)$ and has hypercharge $-\frac{1}{2}$. Then we can write, equivalently to Eq. (2.2.1),

$$\mathscr{L}_{Yukawa} = -y'_i \, \bar{L}^i \tilde{H} \nu^i_R + h.c. \tag{2.2.2}$$

It is interesting to notice that using this formalism, one can "naturally" introduce a Dirac mass to neutrinos.

We will focus first on the quark sector, such that, if one includes all generations, the quark masses can be given by

$$\mathscr{L}_{\text{Quark-mass}} = -Y^d_{ij} \, \bar{Q}^i H d^j_R - Y^u_{ij} \, \bar{Q}^i \tilde{H} u^j_R, \tag{2.2.3}$$

where $Y_{d,u}$ are now 3×3 complex Yukawa matrices. Each term above is invariant under the whole Standard Model group $SU(3)_C \times SU(2) \times U(1)_Y$. After spontaneous symmetry breaking, they become

$$\mathscr{L}_{\text{Quark-mass}} = -\frac{v}{\sqrt{2}}\left(\bar{d}_L\, Y_d\, d_R + \bar{u}_L\, Y_u\, u_R\right), \tag{2.2.4}$$

which are not diagonal in the so-called flavor basis. Thus, to diagonalize the masses, we can use that a matrix can be generically written as $Y = UMK^\dagger$, where U and K are unitary matrices, and M is a diagonal matrix. Substituting on the result above, we get

$$\mathscr{L}_{\text{Quark-mass}} = -\frac{v}{\sqrt{2}}\left[\bar{d}_L\left(U_d M_d K_d^\dagger\right) d_R + \bar{u}_L\left(U_u M_u K_u^\dagger\right) u_R\right]. \tag{2.2.5}$$

Now we can freely change to the mass basis by rotating left-handed fields as $d_L \to U_d\, d_L$ and $u_L \to U_u\, u_L$, and right-handed fields as $d_R \to K_d\, d_R$ and $u_R \to K_u\, u_R$. Then, the mass Lagrangian will be given by

$$\mathscr{L}_{\text{Quark-mass}} = -\bar{d}^i_L\, m^i_d\, d^i_R - \bar{u}^i_L\, m^i_u\, u^i_R, \tag{2.2.6}$$

where we have defined the quark masses as $m^i_{d,u} = \frac{1}{\sqrt{2}} v (M_{d,u})^{ii}$. Note that there is still a residual $U(1)^6$ global symmetry under which the down-type quarks transform as $d^j_{L,R} \to e^{i\alpha j} d^j_{L,R}$, and the up-type quarks as $u^j_{L,R} \to e^{i\beta j} u^j_{L,R}$.

However, not only the mass terms will be modified by those flavor rotations. The gauge interactions will be also different in the mass basis. In fact, if we carefully analyze the structure of the neutral currents, we can see that they remain invariant under these changes. The neutral currents do not mix left- and right-handed quarks or different flavors, thus they cannot be sensitive to such rotations. The same will be true for the kinetic terms. On the other hand, the charged currents that couple to the W bosons do mix flavor and, as a consequence, will be the only different terms in the mass basis:

$$W^+_\mu\, \bar{u}^i_L\, \gamma^\mu\, d^i_L + h.c. \;\to\; W^+_\mu\, \bar{u}^i_L\, \gamma^\mu\, (V)_{ij}\, d^j_L + h.c. \tag{2.2.7}$$

The matrix $V = U^\dagger_u\, U_d$ is called CKM matrix, and it gives all the interesting mixing effects in the quark sector. The CKM matrix is a complex unitary matrix with nine real degrees of freedom. Knowing that if V were completely real, it would be an $O(3)$ matrix with only three rotation angles, one can identify the nine degrees of freedom of the CKM matrix as being three angles and six phases.

Under that $U(1)^6$ residual symmetry the CKM matrix transforms as $(V)_{ij} \to e^{-\beta_i}\,(V)_{ij}\, e^{i\alpha_j}$. We can write a total of nine relative phases, but only five of them will be independent. Thus, they can be chosen to cancel exactly five out of the six phases from the CKM matrix, leaving an overall of four degrees of freedom: three angles and just one phase.

It is important to emphasize that this whole time we were calling the mass basis in the same way we have labeled the flavor basis. This is not a problem since they are fairly close to each other and, as consequence, the CKM matrix is nearly diagonal. The measurements of the CKM elements have confirmed this approximation [6].

Now one may ask what happens to the W boson interactions if we apply a CP transformation. The result is easily found when we use how fermions and gauge bosons transform under parity and charge conjugation:

$$W^+_\mu\, \bar{u}^i_L\, \gamma^\mu\, (V)_{ij}\, d^j_L + h.c. \;\to\; W^+_\mu\, \bar{u}^j_L\, \gamma^\mu\, (V^*)_{ji}\, d^i_L + h.c. \tag{2.2.8}$$

The Standard Model Lagrangian will be invariant under CP only if the CKM matrix satisfies $V = V^*$, i.e., the CKM matrix has to be real. But we already know that one of its independent parameters is a phase, which leaves V strictly complex. Therefore, the CKM matrix implies that CP is violated in the Standard Model. If there were only two generations of quarks, V would be a 2×2 complex matrix with four real degrees of freedom: one angle and three phases. Using a residual $U(1)^4$ chiral symmetry, one can remove these three phases and take the CKM matrix to be real. The remaining degree of freedom is known as the Cabibbo angle. Then, considering only two generations of quarks CP is not violated.

2.3 Neutrinos

Experiments have shown that, although very light, neutrinos are in fact massive. We have seen that, if we assume the existence of both left- and right-handed neutrinos, their masses can be generated after electroweak spontaneous breaking by terms like the one in Eq. (2.2.2). Since L^i and $\tilde{H}$ are charged in the same way under $SU(2) \times U(1)_Y$, ν_R cannot have any weak or hypercharge quantum numbers. This implies that the most general way to write mass terms for leptons is including Majorana masses to right-handed neutrinos. Note that Majorana terms are allowed by the electroweak symmetry. However, if neutrinos have any other quantum number (e.g. lepton number), these terms are forbidden and neutrino masses have to be strictly Dirac masses. Thus, the most general renormalizable mass Lagrangian for neutrinos is given by

$$\mathscr{L}_{\nu\text{-mass}} = -Y_\nu^{ij}\,\bar{L}^i\tilde{H}\nu_R^j - iM_{ij}(\nu_R^i)^c\nu_R^j + h.c. \tag{2.3.1}$$

Assuming that left- and right-handed neutrinos are different particles due to the hierarchy between their masses, one can construct Dirac spinors out of single Weyl spinors,

$$\psi_L = \begin{pmatrix} \nu_L \\ i\tau^2\nu_L^* \end{pmatrix}, \qquad \psi_R = \begin{pmatrix} i\tau^2\nu_R^* \\ \nu_R \end{pmatrix}, \tag{2.3.2}$$

to find that, after H acquires a vev, the above Lagrangian can be rewritten as

$$\begin{aligned} \mathscr{L}_{\nu\text{-mass}} &= -m\bar{\psi}_L\psi_R - \frac{M}{2}\bar{\psi}_R\psi_R + h.c. \\ &= -\left(\bar{\psi}_L\ \bar{\psi}_R\right)\begin{pmatrix} 0 & m \\ m & M \end{pmatrix}\begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}. \end{aligned} \tag{2.3.3}$$

Diagonalizing the above mass matrix and taking the limit of $M \gg m$, we find two mass eigenvalues, one heavy and one much lighter: $m_{heavy} \approx M$ and $m_{light} \approx \frac{m^2}{M}$. But why should we take the limit of Majorana masses being so large? The Wilsonian renormalization group picture tells us that we can interpret quantum field theory as a manifestation of high-energy phenomena below some cutoff Λ. If our theory includes scalars and right-handed neutrinos, the most generic Lagrangian will be written as

$$\mathscr{L}_{QFT} = \Lambda^4\, f\left(\frac{\phi}{\Lambda}, \frac{\nu_R}{\Lambda^{3/2}}\right). \tag{2.3.4}$$

The arbitrary function f can be expanded to generate mass terms for both ϕ and right-handed neutrinos:

$$\mathscr{L}_{QFT} \approx c_1\,\Lambda^2\,\phi^2 + c_2\,\Lambda\,\nu_R\,\nu_R. \tag{2.3.5}$$

If the coefficients $c_{1,2}$ were almost 1, the degrees of freedom would be pushed back above the cutoff. Thus, since we want to describe the low-energy regime, $c_{1,2}$ must be chosen to be much smaller than 1. This choice is what physicists call fine tuning. The corrections to each one of the masses reflect very physical concepts. The scalar mass will be quadratically sensitive to the cutoff Λ, such as in the case of the Higgs boson. But the spinor masses are protected by an uderlying symmetry and, as a consequence, the radiative corrections to their masses are proportional to the masses themselves. Therefore, the Majorana masses will roughly remain at the same scale in which they have been generated. This justifies why one can take them to be very large, so that at low energies we do not have to deal with these unobservable particles.

Taking the Majorana masses to be very large, and the Dirac masses to be extremely light, of the order of the electroweak scale, we obtain the so-called see-saw mechanism. However, one does not need to include right-handed neutrinos to give neutrinos mass. If we allow non-renormalizable terms in the Lagrangian, the neutrino masses could be generate from a 5-dimensional operator:

$$\mathscr{L}_{\nu,\, 5-dim} = -M_{ij} \left(\bar{L}_i \, \tilde{H}\right) \left(\tilde{H}^T L^c_j\right). \tag{2.3.6}$$

After spontaneous symmetry breaking, we get

$$\mathscr{L}_{\nu,\, 5-dim} = -\frac{v^2}{2} \, M_{ij} \, (\bar{\nu}^c_L)^i \, \nu^j_L. \tag{2.3.7}$$

Thus, as in the quark sector, the mass terms are not diagonal. Note that M is a symmetric matrix, so it can be diagonalized using $M = U \, m \, U^T$. If we rotate the neutrino fields as $\nu_L \to U_\nu \, \nu_L$ and $\bar{\nu}^c_L \to \bar{\nu}^c_L \, U^T_\nu$, the Lagrangian is given in the mass basis by

$$\mathscr{L}_{\nu,\, 5-dim} = -\frac{1}{2} \nu^i_L \, m^i_\nu \, \nu^i_L, \tag{2.3.8}$$

where $m^i_\nu = v^2 (m)^{ii}$. This is exactly a Majorana Lagrangian for left-handed neutrinos. Hence, we can easily see that it cannot have any residual $U(1)$ symmetry.

The procedure to diagonalize the mass Lagrangian of the charged leptons is the same as we did for the quarks. However, since there are only three different flavors in the charged lepton sector, and if we ignore the existence of right-handed neutrinos, the final Lagrangian will actually have a residual $U(1)^3$ symmetry.

Applying these changes of basis to the W boson interactions, we find that

$$W^+_\mu \, \bar{\nu}^i_L \, \gamma^\mu \, e^i_L + h.c. \;\to\; W^+_\mu \, \bar{\nu}^i_L \, \gamma^\mu \, (U^\dagger)_{ij} \, e^j_L + h.c. \tag{2.3.9}$$

The matrix $U = U^\dagger_e \, U_\nu$ is called the PMNS matrix, and it is the lepton analog of the CKM matrix. Under the residual $U(1)^3$, the PMNS matrix transforms as $U_{ij} \to e^{-i\beta_i} \, U_{ij}$. Thus, we can use these three phases to reduce the degrees of freedom of U to three rotation angles and three phases. Note that if neutrino masses were purely Dirac, these two extra phases could have been absorbed. But no matter what is the origin of neutrino masses, CP will always be violated in the lepton sector of the Standard Model.

In the case of neutrinos, the mass and flavor basis are not fairly close to each other. Experiments have shown that neutrino flavor eigenstates oscillate as they propagate through space. Only in the mass basis their propagators are diagonal. This means we need to relate both situations through an unitary matrix, namely the PMNS matrix: $\nu^f_L = U^{f1} \, \nu^1 + U^{f2} \, \nu^2 + U^{f3} \, \nu^3$. Charged leptons can only couple to their respective neutrino partner, but they will couple in different ways to each neutrino mass component.

2.4 CP Violation

2.4.1 Weak CP Violation

We have already shown that both CKM and PMNS matrices violate CP invariance. This is known as the weak CP violation and is observed in rare processes involving hadrons. What we did earlier was to derive a condition for which the mixing terms of the electroweak Lagrangian were invariant under CP. We found that the Standard Model would not violate CP only if the CKM and PMNS matrices were real. However, it is not truly correct to take any conclusions based on the fact that a matrix is real or complex. This property of matrices is not a basis-invariant statement. For example, in the flavor basis, the W interactions are flavor diagonal and the mass matrix is complex, such that even with $V = \mathbb{1}$ we would still observe CP violation. On the ohter hand, in the mass basis, for which the mass matrix is diagonal and V is complex, there could be some residual chiral rotation able to remove the phase that makes V complex. Thus, in this scenario, we would not see CP violation anymore. In order to measure the CP violation in the Standard Model, it would be useful to find a condition that is actually basis-independent.

Remember that we have related the Yukawa couplings to diagonal mass matrices through $Y = U\, M\, K^\dagger$. Then, we rotated the left- and right-handed fields in order to get rid of U and K. But we could equally well have rotated only the right-handed fields as $d_R \to K_d\, U_d^\dagger\, d_R$ and $u_R \to K_u\, U_u^\dagger\, u_R$, such that the new Yukawa couplings would be hermitian:

$$\mathscr{L}_{\text{Quark-mass}} = -\frac{v}{\sqrt{2}}\left[\bar{d}_L \left(U_d\, M_d\, U_d^\dagger\right) d_R + \bar{u}_L \left(U_u\, M_u\, U_u^\dagger\right) u_R\right] + h.c. \tag{2.4.1}$$

If these couplings were simultaneously diagonalizable, there would be no mixing between generations in the W interactions, which is equivalent to say that $V = \mathbb{1}$. In this case, there is no CP violation since the CKM matrix is essentially real.

There is a theorem stating that if two hermitian matrices commute, then they are simultaneously unitarily diagonalizable, i.e., there exists an unitary matrix that diagonalizes both hermitian matrices at the same time. Thus, one can realize that CP violation is encoded in the commutator

$$-\,iC = \left[U_u\, M_u\, U_u^\dagger, U_d\, M_d\, U_d^\dagger\right] = U_u\left[M_u, V\, M_d\, V^\dagger\right]U_u^\dagger. \tag{2.4.2}$$

The matrix C is traceless and also hermitian. Using these properties, we can look to its determinant as the obvious basis-invariant quantity:

$$det(C) = -\frac{16}{v^2}(m_t - m_c)(m_t - m_u)(m_c - m_u)(m_b - m_s)(m_b - m_d)(m_s - m_d)J, \tag{2.4.3}$$

where $Im(V_{ij}\, V_{kl}\, V_{il}^*\, V_{kj}^*) = J \sum_{m,n} \epsilon_{ikm}\epsilon^{jln}$ is the Jarlskog invariant [6]. If there is no CP violation, i.e., if V is real, then the Jarlskog invariant vanishes, implying $det(C) = 0$. But if either two up-type or two down-type quarks are degenerate, we also get $det(C) = 0$. This is consistent with the possibility of choosing a basis with some residual chiral rotation that could remove the phase from the CKM matrix. Since $det(C)$ has many factors of the type $(m_{d,u}^i - m_{d,u}^j) \ll v$, it will in general be quite small, such that the physical manifestations of CP violation are bound to be very modest. In fact, current measurements from kaon systems show that effects due to CP violation are always of order 10^{-3} [6].

2.4.2 Strong CP Violation

There is another source of CP violation arising from anomalous symmetries, i.e., when the classical action is invariant under a symmetry transformation but the path integral measure is not. Usually, global chiral symmetries are anomalous, leading to extra terms in the Standard Model Lagrangian. The so-called θ-terms not only are allowed, but they must be included to renormalize some divergences of the theory [6]. There are three of these terms in the Standard Model, each one corresponding to the gauge groups $SU(3)_C$, $SU(2)$, and $U(1)_Y$. However, since the $SU(2)$ and $U(1)_Y$ terms can be completely removed by chiral rotations, they are unphysical. On the other hand, the θ-term corresponding to $SU(3)_C$ will receive corrections from the Yukawa couplings. Since the corrected term is left unchanged by chiral rotations, it will lead to observable consequences:

$$\mathscr{L}_{\text{CP-viol.}} \supset \theta_{QCD} \frac{g_s^2}{32\pi^2} \epsilon^{\mu\nu\rho\lambda} G^a_{\mu\nu} G^a_{\rho\lambda}. \tag{2.4.4}$$

Here $G^a_{\mu\nu}$ is the $SU(3)_C$ field strength tensor. It is straightforward to see that this term violates CP.

To see why θ_{QCD} produces physical effects, we must revisit the Yukawa couplings one more time. We can generically write that the Yukawa couplings are also related to diagonal matrices by $Y = UMU^\dagger K^\dagger$. The freedom of choosing what kind of rotations we want to perform to eliminate K and U allow us to induce a chiral transformation like

$$\begin{aligned} \begin{pmatrix} u_R \\ d_R \end{pmatrix} &\to \begin{pmatrix} K_u U_u & 0 \\ 0 & K_d U_d \end{pmatrix} \begin{pmatrix} u_R \\ d_R \end{pmatrix} \equiv R \begin{pmatrix} u_R \\ d_R \end{pmatrix}, \\ \begin{pmatrix} u_L \\ d_L \end{pmatrix} &\to \begin{pmatrix} U_u & 0 \\ 0 & U_d \end{pmatrix} \begin{pmatrix} u_L \\ d_L \end{pmatrix} \equiv L \begin{pmatrix} u_L \\ d_L \end{pmatrix}. \end{aligned} \tag{2.4.5}$$

For multiple generations, the rotation angle related to this chiral transformation is given by the relation $det(R^\dagger L) = r\, e^{i\theta}$, with $r \in \mathbb{R}$. This means that θ_{QCD} will be corrected by

$$\begin{aligned} \theta &= arg[det(R^\dagger L)] \\ &= arg[det(K_u^\dagger\, K_d^\dagger)] \\ &= -arg[det(K_u\, K_d)] \\ &= -arg[det(Y_u\, Y_d)]. \end{aligned} \tag{2.4.6}$$

Thus, any other chiral rotation will move both phases back and forth, but the combination $\bar{\theta} = \theta_{QCD} + \theta$ is always left unchanged. This combination is called strong CP phase. It is a basis-independent measure of CP violation, and can have physical consequences. But since one can write $\epsilon^{\mu\nu\rho\lambda} G^a_{\mu\nu} G^a_{\rho\lambda}$ as a total derivative of a quantity known as Chern-Simons current, $\bar{\theta}$ can only appear through non-perturbative effects [6].

Despite its physical implications, the strong CP phase has never been obeserved. What physicists know is an experimental bound due to the neutron electric dipole moment, indicating that this phase is extremely small when compared to the weak CP phase [6]. The smallness of CP violation by the θ-term despite the large amount coming from the weak sector is known as the strong CP problem.

Chapter 3

Electroweak Precision Measurements

Precision electroweak physics is interested mainly on observables constructed out of fermions and electroweak gauge bosons. The measurement of such observables have been crucial in both validating the Standard Model and in providing directions for the search of new physics. The so-called precision program started in 1973 with the discovery of weak neutral currents. Since then it confirmed the gauge principle in the Standard Model, established its gauge group and representations, and tested its one-loop structure, which validated the basic principles of renormalization [8].

There are many well measured quantities sensitive only to electroweak physics [9, 10]. Usually, they are associated with processes at the Z pole resonance, such as Z partial widths, left-right asymmetries, and forward-backward asymmetries; and at low energies, such as the nuclear weak charge from atomic parity violation, effective electron couplings from neutrino-electron scattering, and coupling-related quantities from inelastic neutrino scattering. Furthermore, we can also include among these well measured observables the W boson mass.

Since this chapter is mainly based on [6], we are going to follow its precision program and, just for simplicity, consider only five observables that have been measured extremely well:

1. Electron magnetic dipole moment $\frac{1}{2}g_e = 1.001\,159\,652\,180\,73 \pm 2.8 \times 10^{-13}$;
2. Muon lifetime $\tau_\mu^{-1} = 2.995\,98 \times 10^{-19}\,GeV$;
3. Z boson pole mass $M_{Z,pole} = 91.1876 \pm 0.0021\,GeV$;
4. W boson pole mass $M_{W,pole} = 80.385 \pm 0.015\,GeV$;
5. Polarization asymmetry in Z boson production $A_e = 0.1515 \pm 0.0019$.

At leading order in perturbation theory, each one of these observables can be described in terms of three electroweak parameters: the couplings of $SU(2)$ and $U(1)_Y$, respectively g and g', and the Higgs vev v. These parameters can be translated into the QED coupling e (or the fine-structure constant α_e), the Fermi constant G_F, and the weak mixing angle θ_w (if one is considering lepton universality). Thus, if we want to know whether all the experimental quantities given above are consistent with the Standard Model, we need to define renormalization conditions to the electroweak parameters involved.

We proceed with defining what will be our inputs, i.e., the measured quantities we need to use in the renormalized expressions in order to verify their validity. The obvious choice is to extract α_e, G_F and $\sin\theta_w$ from the best well measured observables. From now on, in order to distinguish observable values from Standard Model quantities, we are going to denote the latter with a circumflex.

Since g_e is known extremely well, we can use it to determine an experimental value for $\hat{\alpha}_e$. At 1-loop they are related by $g_e - 2 = \frac{\hat{\alpha}_e}{\pi}$. In more recent works, this relation is already known to even higher orders in $\hat{\alpha}_e$ [11]. Thus, we can use the full Standard Model expression to fix the value of the fine-structure constant at low energies: $\alpha_e(0) = (137.035\,999\,074 \pm 0.000\,000\,044)^{-1}$. Knowing that the coupling runs with the energy, we need to choose another energy scale in which $\hat{\alpha}_e$ is going to be given. For precision electroweak physics, since most of the observables are measured at the Z resonance, the more convenient choice is to work at the scale of the Z mass [1]: $\alpha_e(M_Z) = (127.944 \pm 0.014)^{-1}$. Furthermore, using that $\hat{\alpha}_e = \frac{\hat{e}^2}{4\pi}$, we can also extract $e(M_Z)$.

The second best measured quantity is the muon lifetime τ_μ. We can use the 4-Fermi theory, Eq. (2.1.32), to determine a decay rate formula for the muon and, consequently, find a relation between τ_μ and $\hat{G}_F$:

$$\begin{aligned}\tau_\mu^{-1} &= \Gamma\left(\mu^- \to \nu_\mu e^- \bar{\nu}_e\right) \\ &= \hat{G_F}^2 \frac{\hat{m}_\mu^5}{192\pi^3}\left[1 - 8\frac{\hat{m}_e^2}{\hat{m}_\mu^2} + 8\left(\frac{\hat{m}_e^2}{\hat{m}_\mu^2}\right)^3 - \left(\frac{\hat{m}_e^2}{\hat{m}_\mu^2}\right)^4 - 12\left(\frac{\hat{m}_e^2}{\hat{m}_\mu^2}\right)^2 \log\left(\frac{\hat{m}_e^2}{\hat{m}_\mu^2}\right)\right].\end{aligned} \tag{3.0.1}$$

Substituting the experimental values $m_\mu = 105.658\,371\,5\,MeV$ and $m_e = 0.510\,998\,910\,MeV$ [6], one can easily extract $G_F(m_\mu) = 1.163\,93 \times 10^{-5}\,GeV^{-2}$. Differently from what is done for the fine-structure constant, there is no need to evolve the Fermi constant to the energy scale of the Z boson mass. Since loops from QED give very small contributions to the running of G_F, the value at M_Z is almost the same as the one obtained at m_μ [12].

We know that the mixing angle $\hat{\theta}_w$ is determined through its sine. One could define $\sin\hat{\theta}_w$ in many different ways but, for precision tests, it is convenient to use the third best measured observable, the Z boson mass $M_Z = M_{Z,pole}$. For simplicity, we are going to use $\sin\theta_w = s$ and $\cos\theta_w = c$, such that

$$s^2(1-s^2) = \frac{\pi\,\alpha_e(M_Z)}{\sqrt{2}\,G_F\,M_Z^2}. \tag{3.0.2}$$

From the relation $s^2 \approx 1 - \frac{\hat{M}_W^2}{\hat{M}_Z^2}$ we can determine the correct root in order to find $s^2 = 0.234\,289$.

If we plug all these numbers into the tree-level Standard Model expressions for $\hat{M}_W$ and A_e, we find as results

$$M_{W,pole} = \hat{M}_W = \sqrt{1-s^2}M_Z = 79.794\,GeV, \tag{3.0.3}$$

$$A_e \equiv \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} = \frac{(\frac{1}{2} - s^2)^2 - s^4}{(\frac{1}{2} - s^2)^2 + s^4} = 0.1252, \tag{3.0.4}$$

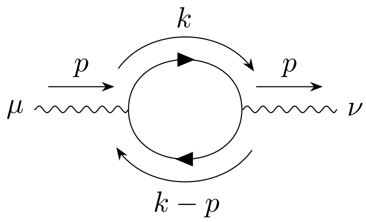
where $\sigma_{L,R}$ is the cross section for a left- or right-handed incident electron [6]. Both values are well outside the experimental bounds – nearly 40 standard deviations for $\hat{M}_W$, and 14 for A_e. However, this does not necessarily indicate a contradiction within the Standard Model. Instead, we still need to include loop corrections and carefully renormalize our theory to make any conclusion.

[1] Both experimental values $\alpha_e(0)$ and $\alpha_e(M_Z)$ given in [6] are from the 2002 version of the Particle Data Group (PDG) book. Updated values can be found in [10].

3.1 Oblique Corrections

All observables mentioned above receive radiative corrections from many loops. But since they are given at tree-level by gauge boson exchange, the most important contributions will come from vacuum polarization diagrams, i.e., diagrams in which loops affect the gauge boson propagator. These are the so-called oblique corrections.

Let us begin with the photon self-energy. There are radiative corrections that modify the photon structure due to virtual electron-positron pairs. At order-$\hat{\alpha}_e$, there is only one diagram contributing, which is known as the vacuum polarization diagram:

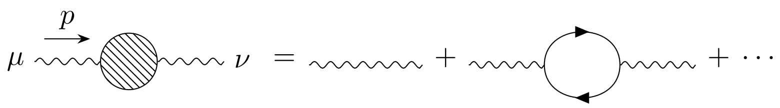


$$= i\Pi^{\mu\nu}(p). \tag{3.1.1}$$

We know that the tensor structure of $\Pi^{\mu\nu}(p)$ must be of the form $\left(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}\right)$ because it guarantees the photon to be massless. Or, if we consider Ward identities, $p_\mu \Pi^{\mu\nu}(p) = 0$, one can deduce that $\Pi^{\mu\nu}(p)$ must be proportional to this projector. Therefore, it is convenient to write

$$i\Pi^{\mu\nu}(p) = i\left(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}\right)\Pi(p^2). \tag{3.1.2}$$

Thus, the exact two-point function is the sum of all one-particle irreducible (1PI) diagrams, i.e., the sum of all diagrams that cannot be split in two by removing a single line [4]:

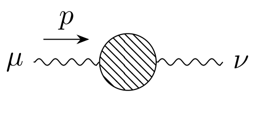


$$= -\frac{i}{p^2\left[1-\Pi(p^2)\right]}\left(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}\right) - \frac{i}{p^2}\left(\frac{p^\mu p^\nu}{p^2}\right). \tag{3.1.3}$$

Since at least one end of this propagator will be connected to on-shell spinors, which we will consider to have momentum q_1 and q_2, we can use the equations of motion $\bar{\psi}(q_1)\not{q}_1 = -m\bar{\psi}(q_1)$ and $\not{q}_2\psi(q_2) = m\psi(q_2)$ to show that all terms proportional to $p^\mu p^\nu$ vanish, giving no physical effects:

$$\begin{aligned}
p_\mu p_\nu\left[\bar{\psi}(q_1)\gamma^\nu\psi(q_2)\right] = p_\mu\left[\bar{\psi}(q_1)\not{p}\,\psi(q_2)\right] &= p_\mu\left[\bar{\psi}(q_1)\left(\not{q}_1 + \not{q}_2\right)\psi(q_2)\right] \\
&= p_\mu\left[\bar{\psi}(q_1)\left(-m+m\right)\psi(q_2)\right] \\
&= 0.
\end{aligned} \tag{3.1.4}$$

Then, the photon propagator can be rewritten as



$$= -\frac{ig^{\mu\nu}}{p^2 - \Pi_{\gamma\gamma}(p^2)}, \tag{3.1.5}$$

where we have defined $\Pi_{\gamma\gamma}(p^2) = p^2\Pi(p^2)$.

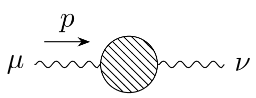
The same analysis can be made for massive gauge bosons, but with a slight difference. The photon has its mass forbidden at all orders in perturbation theory due to gauge invariance. However, W and Z bosons acquire a mass through the Higgs mechanism, such that the Lorentz structure of their 1-loop diagrams do not need to be the same as the one from the photon. Imposing only Lorentz invariance, we get

$$i\Pi^{\mu\nu}_{WW} = i\Pi_{WW}g^{\mu\nu} + i\Pi^{pp}_{WW}p^\mu p^\nu \text{ and } i\Pi^{\mu\nu}_{ZZ} = i\Pi_{ZZ}g^{\mu\nu} + i\Pi^{pp}_{ZZ}p^\mu p^\nu. \tag{3.1.6}$$

The massive gauge bosons couple to chiral currents, which can be easily decomposed into vector and axial components. The latter is not conserved, indicating that terms proportional to $p^\mu p^\nu$ will now contribute:

$$\begin{aligned}
\frac{p_\mu p_\nu}{M^2}\left[\bar\psi(q_1)\gamma^\nu\gamma^5\psi(q_2)\right] &= \frac{p_\mu}{M^2}\left[\bar\psi(q_1)\not{p}\gamma^5\psi(q_2)\right] \\
&= \frac{p_\mu}{M^2}\left[\bar\psi(q_1)(\not{q}_1\gamma^5 - \gamma^5\not{q}_2)\psi(q_2)\right] \\
&= \frac{p_\mu}{M^2}\left[\bar\psi(q_1)(-m\gamma^5 - \gamma^5 m)\psi(q_2)\right] \\
&= -2p_\mu\frac{m}{M^2}\left[\bar\psi(q_1)\gamma^5\psi(q_2)\right],
\end{aligned} \tag{3.1.7}$$

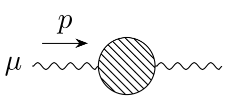
where we have used $\gamma^\nu\gamma^5 = -\gamma^5\gamma^\nu$. However, we can see that these contributions will be proportional to m/M^2, where m is the fermion mass and M is the mass of the gauge boson. Since all the observables in which we are interested have the massive gauge bosons coupled essentially to electrons, the $p^\mu p^\nu$ terms will be highly suppressed by the factor m/M^2. Thus, we can neglect such terms and approximate the corrected propagator to



$$= -\frac{ig^{\mu\nu}}{p^2 - M_I^2 - \Pi_{II}(p^2)}, \tag{3.1.8}$$

with $I = W, Z$. So we can see that, for massive gauge bosons, the pole mass is the quantity related to the renormalized Lagrangian mass at 1-loop.

The optical theorem says that the imaginary part of the scattering amplitude is proportional to the total scattering cross section when the initial state is a two-particle state. However, if the initial state is a one-particle state, then the decay rate is the quantity proportional to the imaginary part of the process' amplitude: $Im[\mathscr{M}] = M\,\Gamma_{tot}$. When $\Gamma_{tot} \ll M_{pole}$, all the contributions from non-1PI diagrams are suppressed [6], such that $Im[\Pi(M^2_{pole})] = M_{pole}\,\Gamma_{tot}$. This quantity is non-zero for unstable particles, as the W and Z bosons. Hence, to keep the pole mass real, one can define



$$= -\frac{ig^{\mu\nu}}{p^2 - M_I^2 - Re\left[\Pi_{II}(M^2_{I,pole})\right] - iM_{I,pole}\,\Gamma_{tot}}, \tag{3.1.9}$$

where $M^2_{I,pole} = M_I^2 + Re\left[\Pi_{II}(M^2_{I,pole})\right]$ is the actual pole mass (or the Breit-Wigner mass). This gives us the first relation between measured and renormalized parameters:

$$\hat M_Z^2 = \hat M^2_{Z,pole} - Re[\Pi_{ZZ}(M_Z^2)] = M_Z^2 - Re[\Pi_{ZZ}(M_Z^2)]. \tag{3.1.10}$$

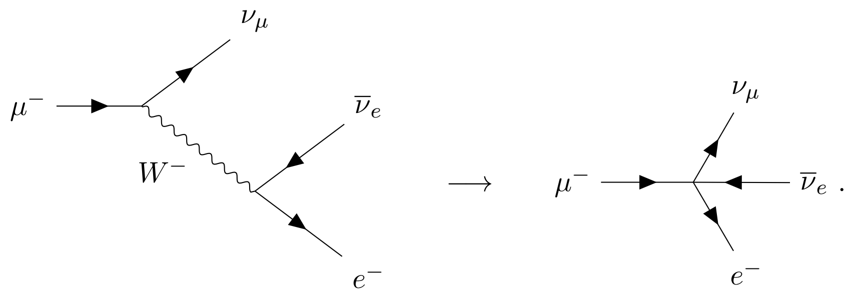
If we use Π_{ZZ} only to 1-loop order, it does not matter which Z boson mass is used in the argument since the difference is higher order in perturbation theory.

Now we must relate the renormalized electric charge to the measured quantity $\alpha_e(M_Z)$. The effects of replacing the tree-level photon propagator with the exact propagator from Eq. (3.1.5) were already known in QED, and they showed how the electric charge had a momentum dependence [4]. Using the full vacuum polarization contributions, we find that

$$\hat{e}^2 = e^2(M_Z)\left[1 - \frac{\Pi_{\gamma\gamma}(M_Z^2)}{M_Z^2}\right]. \tag{3.1.11}$$

This equation relates the renormalized parameter $\hat{e}$, which appears in the Standard Model Lagrangian, to the value of the coupling extracted from the measurement of $(g-2)_e$ and evolved to M_Z.

In order to relate the muon lifetime, i.e., the quantity G_F, to the renormalized parameters, we need to take into account the low energy limit of the process $\mu^- \to e^- \nu_\mu \bar{\nu}_e$:



(3.1.12)

Thus, using the corrected propagator of the W boson in the 4-Fermi contribution, Eq. (2.1.32), yields

$$\frac{G_F}{\sqrt{2}} = \frac{\hat{e}^2}{8\hat{M}_Z^2 \cos^2\hat{\theta}_w \sin^2\hat{\theta}_w}\left[1 - \frac{\Pi_{WW}(0)}{\hat{M}_W^2}\right]. \tag{3.1.13}$$

If we invert the above expression and plug in Eq. (3.1.10) and Eq. (3.1.11), we find a relation between the mixing angle $\hat{\theta}_w$ and the renormalized parameters:

$$\hat{s}^2\hat{c}^2 = \frac{\sqrt{2}\, e^2(M_Z)}{8\, G_F\, M_Z^2}\,(1 + \Pi_R)\,, \tag{3.1.14}$$

with

$$\Pi_R = \frac{Re[\Pi_{ZZ}(M_Z^2)]}{M_Z^2} - \frac{Re[\Pi_{\gamma\gamma}(M_Z^2)]}{M_Z^2} - \frac{Re[\Pi_{WW}(0)]}{\hat{M}_W^2}. \tag{3.1.15}$$

Considering perturbations as $\hat{s}^2 = s^2 + A\,\Pi_R$, we can separate $\sin\hat{\theta}_w$ and $\cos\hat{\theta}_w$ in two different expressions:

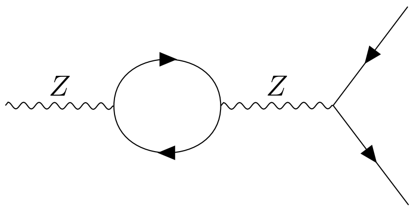
$$\hat{s}^2 = s^2\left(1 + \frac{c^2}{c^2 - s^2}\Pi_R\right), \tag{3.1.16}$$

$$\hat{c}^2 = c^2\left(1 - \frac{s^2}{c^2 - s^2}\Pi_R\right). \tag{3.1.17}$$

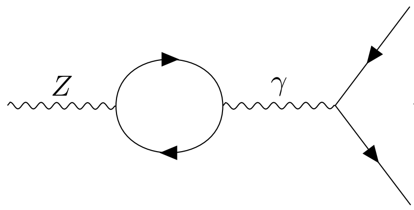
Finally, we can write the Z boson production asymmetry in terms of the renormalized parameters. This quantity is determined by how the Z boson couples to fermions, more specifically to electrons. As follows from Eqs. (2.1.25) – (2.1.28), the Z and the photon couplings are given by

$$\mathscr{L}_{\gamma Z} = -\frac{\hat{e}}{\hat{s}\hat{c}} Z_\mu \left[\left(\frac{1}{2} - \hat{s}^2 \right) \bar{e}_L \gamma^\mu e_L - \hat{s}^2 \bar{e}_R \gamma^\mu e_R \right] - \hat{e} A_\mu \left[\bar{e}_L \gamma^\mu e_L + \bar{e}_R \gamma^\mu e_R \right]. \tag{3.1.18}$$

This Lagrangian implies that, at 1-loop, there are two contributions to the Z boson propagator, one of them due to the electron-photon interaction:

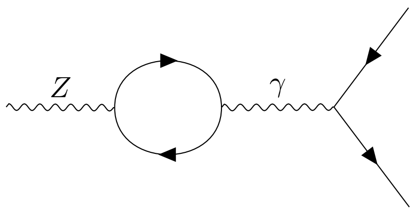


and



. (3.1.19)

We know that Π_{ZZ} corrects the Z boson pole mass. But since the polarization asymmetry $\hat{A}_e$ must not depend on $\hat{M}_Z$ at tree-level, the effects of Π_{ZZ} are of higher orders in perturbation theory. Thus, the first diagram can be neglected. On the other hand, the second diagram gives a correction to the Z boson propagator by a factor proportional to the electric charge and, therefore, must be considered. This contribution is given by



$$= i Z_\mu \left[\hat{e} \frac{\Pi_{\gamma Z}(p^2)}{p^2} \right] J^\mu. \tag{3.1.20}$$

Evaluating at the resonance, i.e., $p^2 = \hat{M}_Z^2$, the above diagram corrects the Z coupling as

$$\mathscr{L}_Z = -\frac{\hat{e}}{\hat{s}\hat{c}} Z_\mu \left[\left(\frac{1}{2} - s_{eff}^2 \right) \bar{e}_L \gamma^\mu e_L - s_{eff}^2\, \bar{e}_R \gamma^\mu e_R \right], \tag{3.1.21}$$

where

$$s_{eff}^2 = \hat{s}^2 - \hat{s}\hat{c} \frac{\Pi_{\gamma Z}(M_Z^2)}{\hat{M}_Z^2} \tag{3.1.22}$$

is the effective mixing angle. It can be written in terms of the observables M_Z, s and c as

$$s_{eff}^2 = s^2 \left(1 + \frac{c^2}{c^2 - s^2} \Pi_R \right) - sc \frac{\Pi_{\gamma Z}(M_Z^2)}{M_Z^2}. \tag{3.1.23}$$

The remaining quantity that needs to be expressed in terms of the renormalized parameters is the W boson mass. Using all the results we have previously derived, we can find that

$$M_{W,pole}^2 = \hat{M}_W^2 \left\{ 1 - \frac{s^2}{c^2 - s^2} \Pi_R - \frac{Re[\Pi_{ZZ}(M_Z^2)]}{M_Z^2} + \frac{Re[\Pi_{WW}(M_W^2)]}{\hat{M}_W^2} \right\}. \tag{3.1.24}$$

Next, we need to compute these vacuum polarization amplitudes in order to compare our predictions to the experimental values.

3.2 Electroweak Vacuum Polarization Loops

Since we considered the gauge boson propagators being corrected only by fermion loops, the largest contributions will come from the top and bottom quarks (such corrections will be prportional to the ratio m_f^2/M_Z^2, with m_f being the fermion mass). Although the top quark is much heavier than the bottom, the latter is required by $SU(2)$ invariance. The loops must be performed considering both left- and right-handed quarks. Their tensor structure implies that $\Pi_{LL} = \Pi_{RR}$ and $\Pi_{LR} = \Pi_{RL}$. Thus, using Feynman parametrization and dimensional regularization, these amplitudes can be computed for a generically pair of fermions with masses m_1 and m_2, such that [6]

$$\Pi_{LL} = \frac{e^2}{8\pi^2}\left\{\frac{1}{\epsilon}\left(m_1^2 + m_2^2 - \frac{2}{3}p^2\right) + \int_0^1 \mathrm{d}x\, ln\left(\frac{\mu^2}{a^2}\right)\left[a^2 - (1-x)xp^2\right]\right\}, \tag{3.2.1}$$

$$\Pi_{LR} = -\frac{e^2}{4\pi^2}m_1 m_2\left\{\frac{1}{\epsilon} + \frac{1}{2}\int_0^1 \mathrm{d}x\, ln\left(\frac{\mu^2}{a^2}\right)\right\}, \tag{3.2.2}$$

where $a^2 = xm_1^2 + (1-x)(m_2^2 - xp^2)$. One can also define the vector-vector vacuum polarization amplitude as $\Pi_{VV} = \Pi_{LL} + \Pi_{RR} + \Pi_{LR} + \Pi_{RL}$, resulting in [6]

$$\begin{aligned}\Pi_{VV} = \frac{e^2}{4\pi^2}&\left\{\frac{1}{\epsilon}\left[(m_1 - m_2)^2 - \frac{2}{3}p^2\right]\right\} + \\ &+ \frac{e^2}{4\pi^2}\left\{\int_0^1 \mathrm{d}x\, ln\left(\frac{\mu^2}{a^2}\right)\left[a^2 - (1-x)xp^2 - m_1 m_2\right]\right\}.\end{aligned} \tag{3.2.3}$$

As one can easily confirm, all these amplitudes are proportional to the fermion masses, which is why the top quark gives the largest effect.

Knowing how the gauge bosons couple to the left- and right-handed currents, and to the vector current, one can immediately derive that

$$\Pi_{\gamma\gamma}(p^2) = N\sum_i Q_i^2 \Pi_{VV}(a_{ii}^2), \tag{3.2.4}$$

$$\Pi_{\gamma Z}(p^2) = \frac{N}{sc}\sum_i Q_i\left(\frac{\tau_i^3}{4} - s^2\, Q_i\right)\Pi_{VV}(a_{ii}^2), \tag{3.2.5}$$

$$\Pi_{ZZ}(p^2) = \frac{N}{s^2c^2}\sum_i\left\{\left(\frac{\tau_i^3}{2}\right)^2\Pi_{LL}(a_{ii}^2) + s^2 Q_i\left(s^2 Q_i - \frac{\tau_i^3}{2}\right)\Pi_{VV}(a_{ii}^2)\right\}, \tag{3.2.6}$$

$$\Pi_{WW}(p^2) = \frac{N}{2s^2}\left|V_{tb}\right|^2\Pi_{LL}(a_{tb}^2), \tag{3.2.7}$$

where N is the number of colors, $i = t, b$ and $a_{ij}^2 = xm_i^2 + (1-x)(m_j^2 - xp^2)$. For the third generation of quarks, we have $Q_t = \frac{2}{3}$, $Q_b = -\frac{1}{3}$, $\tau_t^3 = 1$ and $\tau_b^3 = -1$.

Substituting these expressions into Eq. (3.1.23) and Eq. (3.1.24), we can check that the divergent parts of s_{eff}^2 and $M_{W,pole}^2$ are, respectively, given by

$$s_{eff}^2 \supset \frac{c^2 s^2}{(c^2 - s^2)}\frac{1}{\epsilon}\left[\frac{3e^2}{16\pi^2 M_Z^2\, c^2 s^2}\left(1 - |V_{tb}|^2\right)\left(m_b^2 + m_t^2\right)\right], \tag{3.2.8}$$

$$M^2_{W,pole} \supset \frac{c^2 s^2}{(s^2-c^2)}\frac{1}{\epsilon}\left\{\frac{3e^2}{16\pi^2 M_Z^2\, c^2 s^2}\left(1+\frac{(c^2-s^2)}{s^2}\right)+\frac{e^2}{8\pi^2 s^4}(1-2c^2)\right\}\left(1-|V_{tb}|^2\right). \quad (3.2.9)$$

Thus, we can conclude they are finite only if $|V_{tb}|^2 = 1$. For $|V_{tb}|^2 \neq 1$, it would be necessary to include all other quark loops to see the finiteness. Again the corrections due to the light quarks can be neglected, and the only contributions will be from top-bottom, top-strange and top-down graphs. Taking into account the unitarity of the CKM matrix, this is equivalent to consider only the top-bottom correction with $|V_{tb}|^2 = 1$. Therefore, all the divergences will cancel.

As we have already mentioned in Chapter 1, the concept of mass for quarks is tightly connected with the method by which they are extracted from hadronic properties. If we correctly choose what values of top and bottom masses to use, $m_t(m_t) = 163.0\,GeV$ and $m_b = 4.18\,GeV$ [6], we can predict that

$$M_{W,pole} = 80.368\,GeV, \quad (3.2.10)$$

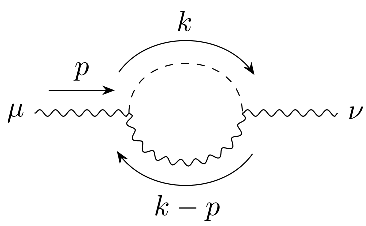
$$s^2_{eff} = 0.2313 \implies A_e = 0.1491, \quad (3.2.11)$$

which are now in a good agreement with the experimental values – almost 1 standard deviation for both of them. This is a remarkable result that validates our currently description of nature.

3.3 Oblique Corrections from the Higgs Boson

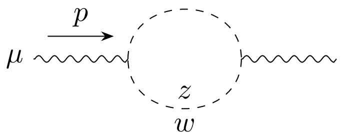
In addition to the top contribution, the Higgs boson will also give a large correction to the gauge boson propagators. There are three different diagrams that will be important to our analysis, and one of them includes the Nambu-Goldstone bosons. If we work in the unitary gauge, the final answer will not count all the possible physical contributions. In fact, the divergences are only cancelled if we also consider loops involving just Nambu-Goldstone bosons.

Since the vertices of W and Z bosons always differ by an extra factor of $\sin\theta_w$ or $\cos\theta_w$, we can compute that



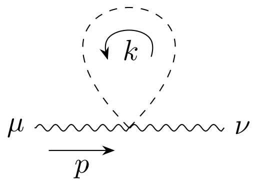
$$= -ig^{\mu\nu}\frac{e^2 \hat{M}_W^2}{(4\pi)^2 A^2}\Gamma\left(\frac{\epsilon}{2}\right)\int_0^1 \mathrm{dx}\left(\frac{\mu^2}{a^2}\right)^{\frac{\epsilon}{2}}, \quad (3.3.1)$$

where $A = \sin\theta_w$ for the W boson, and $A = \sin\theta_w \cos^2\theta_w$ for the Z boson. Furthermore, $a^2 = xm_h^2 + (1-x)(\hat{M}_I^2 - xp^2)$, with $I = W, Z$. The next contribution comes from a loop with the Higgs boson and the Nambu-Goldstone bosons z and $w^{\pm}$:



$$= ig^{\mu\nu}\frac{A^2 e^2}{(4\pi)^2 s^2}\frac{\Gamma\left(\frac{\epsilon}{2}\right)}{(2-\epsilon)}\int_0^1 \mathrm{dx}\left(\frac{\mu^2}{a^2}\right)^{\frac{\epsilon}{2}} a^2. \quad (3.3.2)$$

Finally, the last correction will be given by



$$= -\, i g^{\mu\nu} \frac{e^2}{4(4\pi)^2 A} \Gamma\left(\frac{\epsilon}{2}\right) \times \tag{3.3.3}$$

$$\times \int_0^1 \mathrm{dx} \left\{ \left(\frac{4-\epsilon}{2-\epsilon}\right) a^2 + \left[(1-x)^2 p^2 - \hat{M}_I^2 \right] \right\} \left(\frac{\mu^2}{a^2}\right)^{\frac{\epsilon}{2}} .$$

To compute the shift these graphs will give to the W mass and to the effective mixing angle, we have to sum them, take the limit of $\epsilon \to 0$ and m_h large, and fix the renormalization scale at $\mu^2 = \hat{M}_W^2$. After this procedure, we get

$$\Pi_{ZZ}(p^2) = -\frac{e^2}{64\pi^2 s^2 c^2} \left[\frac{2}{\epsilon} - ln \left(\frac{m_h^2}{\hat{M}_W^2} \right) \right] \left(3M_Z^2 + \frac{p^2}{3} \right), \tag{3.3.4}$$

$$\Pi_{WW}(p^2) = -\frac{e^2}{64\pi^2 s^2} \left[\frac{2}{\epsilon} - ln \left(\frac{m_h^2}{\hat{M}_W^2} \right) \right] \left(3\hat{M}_W^2 + \frac{p^2}{3} \right). \tag{3.3.5}$$

Since we know the divergences will be cancelled by other diagrams containing only Goldstone bosons [3], we can already neglect them. Thus, plugging these results in Eqs. (3.1.23) and (3.1.24), we see that the Higgs boson will change our predictions by

$$M_{W,pole}^2 \to M_{W,pole}^2 - \frac{11\alpha_e}{48\pi} \frac{c^2 M_Z^2}{c^2 - s^2} ln \left(\frac{m_h^2}{\hat{M}_W^2} \right), \tag{3.3.6}$$

$$s_{eff}^2 \to s_{eff}^2 + \frac{\alpha_e (1 + 9s^2)}{48\pi (c^2 - s^2)} ln \left(\frac{m_h^2}{\hat{M}_W^2} \right). \tag{3.3.7}$$

Using the value $m_h = 125\, GeV$ [6], we get $M_{W,pole} = 80.333$ and $A_e = 0.1470$, which yields 3 standard deviations for the W mass, and nearly 2 for the polarization asymmetry. In order to see how the Higgs correction truly improves both predictions, we must also include the two-loop electroweak diagrams, as well as the hadronic and perturbative QCD contributions [13].

It is also important to highlight that, although we have directly used the correct values of the top and Higgs masses to compute predictions for both observables in the last two sections, historically it was the other way around. The measurement of such observables at the Large Electron-Positron (LEP) Collider and the SLAC Linear Collider (SLC) led to predictions for the masses of the top quark and Higgs boson [13], which were only confirmed later on.

Chapter 4

Extending the Standard Model: a New $U(1)$ Symmetry

Since physicists realized new physics was necessary to explain the Universe we live in (see Chapter 1), extensions of the Standard Model by an extra $U(1)$ symmetry have been broadly studied. In fact, these extensions are predicted in well-motivated ultraviolet completions, such as Grand Unified Theories (GUTs), extra-dimensional models, and string theory (for a more complete discussion on the major classes of these models and their issues see [14]). Although such extensions do not solve directly all the problems we have previously mentioned, they represent a rather minimal and in some sense natural alternative for new physics. Traditionally, extra $U(1)$ symmetries arised from the decomposition of $SO(10)$, E_6 or even larger GUT groups. For example, E_6 contains the subgroup $SO(10) \times U(1)_\psi$, and $SO(10)$ can be further decomposed into the subgroup $SU(5) \times U(1)_\chi$. Thus, many models considered new gauge bosons arising from a linear combination of $U(1)_\psi$ and $U(1)_\chi$ [15]. Recently, it was discovered that another class of gauge bosons is also natural in string compactifications, and they can suppress proton decay in theories with a quantum gravity scale much smaller than the Planck scale [16].

Our aim in this thesis is to develop a phenomenological analysis of the gauge boson associated with a generic extra $U(1)$ symmetry. As we will see, this new gauge boson must be massive, neutral, colorless and self-adjoint, i.e., it is its own antiparticle [17]. Since physicists believed for many years that new physics could only be heavy, many of them worked in TeV scale extensions of the Standard Model. These included supersymmetry, various forms of dynamical symmetry breaking and little Higgs models, which often predicted the existence of a new gauge boson also at the TeV scale [14]. Thus, modifications of Standard Model parameters due to the presence of a heavy new gauge boson were usually computed through an Effective Field Theory (EFT) approach [18]. However, today it is widely accepted that, if such gauge bosons are sufficiently weakly coupled to Standard Model fields, they can be light and still have evaded discovery in particle accelerators [16, 19, 20]. Therefore, in this chapter, we present a new minimal model for a massive and weakly coupled gauge boson.

4.1 Theoretical Motivation

The purpose of this section is to elaborate why a weakly coupled, very light gauge boson is physically allowed [16]. We begin from the simplest relativistic Lagrangian for a complex scalar field, which is given by

$$\mathscr{L} = (\partial_\mu \phi)^\dagger (\partial^\mu \phi) + m^2 \phi^\dagger \phi - \frac{\lambda}{4} \left(\phi^\dagger \phi\right)^2 . \tag{4.1.1}$$

Such theory has an exact global $U(1)$ symmetry, i.e., the Lagrangian is invariant under the transformation $\phi(x) \to e^{i\theta}\phi(x)$. If we want to gauge this symmetry, we must add a vector field that transforms as $X_\mu(x) \to X_\mu(x) + \frac{1}{g}(\partial_\mu \theta)$, where now the parameter θ is also x-dependent. Thus, the Lagrangian can be written in terms of ϕ and X_μ as

$$\mathscr{L} = -\frac{1}{4} X_{\mu\nu} X^{\mu\nu} + \left[(\partial_\mu - igX_\mu)\phi\right]^\dagger \left[(\partial^\mu - igX^\mu)\phi\right] + m^2 \phi^\dagger \phi - \frac{\lambda}{4}\left(\phi^\dagger \phi\right)^2. \tag{4.1.2}$$

As one can easily check, the first term above is a kinetic term for the four-vector field X_μ, while g corresponds to the coupling between X_μ and the complex scalar field ϕ. If we consider that $m^2 < 0$, the theory is said to be stable, i.e., it has only a single ground state. In this case, the symmetry gauged by X_μ is already *linearly realized*. However, if $m^2 > 0$, the theory will have an infinite number of equivalent vacua. As it happens for the Higgs multiplet, choosing a particular ground state to expand $\phi(x)$ around it will break the symmetry. The expansion of ϕ can be parameterized as

$$\phi(x) = \left[\frac{v}{\sqrt{2}} + \frac{\sigma(x)}{\sqrt{2}}\right] e^{\frac{i}{f}\pi(x)}, \tag{4.1.3}$$

with $\sigma(x)$ and $\pi(x)$ being real fields, v is the vev, and f is just a mass scale. This is exactly the same procedure as the one we used to describe the spontaneous symmetry breaking in the Standard Model. It was first introduced by Coleman, Wess and Zumino [21] in 1969, where the parameterization given by Eqs. (2.1.4) and (4.1.3) is said to be a *nonlinear realization* of the considered symmetry.

Substituting Eq. (4.1.3) into the Lagrangian, one can verify that $\sigma(x)$ has a mass given by $m_\sigma = \sqrt{2}\,m$, while $\pi(x)$ remains massless. The field σ can be decoupled by taking $m, \lambda \to \infty$ with a fixed v. The only terms left will relate the four-vector gauge field X_μ to the Nambu-Goldstone boson π

$$\mathscr{L} = -\frac{1}{4} X_{\mu\nu} X^{\mu\nu} + \left(\frac{vg}{\sqrt{2}}\right)^2 \left[X_\mu - \frac{1}{fg}(\partial_\mu \pi)\right]\left[X^\mu - \frac{1}{fg}(\partial^\mu \pi)\right]. \tag{4.1.4}$$

Note that we could identify π as being the Nambu-Goldstone boson because the above Lagrangian is only invariant under $\pi(x) \to \pi(x) + f\theta(x)$. On the other hand, X_μ will acquire a mass given by $M = vg$.

In order to keep $\pi(x)$ canonically normalized, we can set $f = v/\sqrt{2}$. Defining a new gauge boson $U_\mu = X_\mu - \frac{\sqrt{2}}{vg}(\partial_\mu \pi)$, we reduce our Lagrangian to the so-called Proca Lagrangian, which describes a massive spin-1 gauge boson

$$\mathscr{L} = -\frac{1}{4} U_{\mu\nu} U^{\mu\nu} + \frac{M^2}{2} U_\mu U^\mu. \tag{4.1.5}$$

Thus, the gauge field X_μ has blended with the Nambu-Goldstone mode to form the gauge invariant massive field U_μ. In this case, we can say that π has been “eaten” by X_μ to give it mass.

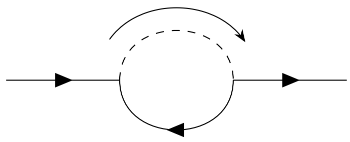
Now suppose our theory also includes fermions charged under the symmetry gauged by X_μ. Then the most generic gauge invariant Lagrangian can be given by

$$\mathscr{L} = -\frac{1}{4} U_{\mu\nu} U^{\mu\nu} + \frac{M^2}{2} U_\mu U^\mu + \bar{\psi}\left[i\gamma^\mu(\partial_\mu - igQU_\mu) - m_\psi\right]\psi, \tag{4.1.6}$$

where Q is the fermionic charge. If we rewrite the interaction term as a function of X_μ and π, this will lead to a five-dimensional operator

$$\mathscr{L}_{5-dim} = -\frac{\sqrt{2}}{v} \left(\bar{\psi}\gamma^{\mu} Q \psi\right) (\partial_{\mu}\pi) \,. \tag{4.1.7}$$

Since the coupling $1/v$ has negative mass dimension, such operator is non-renormalizable. In fact, it will contribute to a quantum loop as



$$\sim \frac{p^2}{(4\pi)^2\, v^2} \sim \frac{(g\Lambda)^2}{(4\pi)^2\, M^2} \tag{4.1.8}$$

for some ultraviolet scale Λ. Thus, the Lagrangian that contains this non-renormalizable operator must be interpreted as an EFT whose predictions will depend on a low-energy expansion in powers of $1/\Lambda$. As a consequence, the low-energy behavior of the theory we have described so far is strongly connected to how high Λ is relative to $(4\pi)v = (4\pi)M/g$.

If the gauge boson is very light compared with the ultraviolet scale ($M \ll g\Lambda/4\pi$), then the theory will be renormalizable only when the five-dimensional operator is redundant, i.e., when we are able to remove it by a field redefinition. A sufficient condition for an interaction of the form $J^{\mu}\,(\partial_{\mu}\pi)$ to be redundant is if the equations of motion for ψ imply that the current is conserved: $\partial_{\mu}J^{\mu} = 0$. This would be equivalent to replacing the five-dimensional operator by a total divergence, which can be easily eliminated by requiring the fields to be well behaved at infinity. Such procedure is only consistent with X_{μ} gauging an *exact linearly realized* symmetry, but spontaneously broken.

However, if the gauge boson initially gauges a *nonlinear realized* symmetry, the fermions are not required to provide a linear representation of the correspondent group. In fact, it can be an anoumalous representation in which currents are not conserved. Therefore, one can expect $\left(\bar{\psi}\gamma^{\mu}Q\psi\right)(\partial_{\mu}\pi)$ to not be redundant. In this situation, the naturalness condition is to have an upper bound on the ultraviolet scale: $\Lambda < 4\pi M/g$.

We can finally conclude that a new gauge boson can only be light if it gauges a spontaneously broken, *exact linearly realized* symmetry. Furthermore, even if the ultraviolet scale lies above the TeV, the relation $M \ll g\Lambda/4\pi$ shows that the gauge boson can still be in the eV-GeV mass range for small enough couplings.

4.2 Anomaly Cancellation

Symmetries are extremely important in physics and they are used to determine the structure of a theory. Therefore, anomalies also play a crucial role in the development of new models. They were first understood through Feynman diagrams, although this is not the easiest way to understand them [6]. An anomalous symmetry can be defined as a symmetry of a classical theory that is not maintained at a quantum level. Noether's theorem states that there is a conserved current associated to each continuous global symmetry of a theory. However, if the symmetry is anomalous, then it is not actually a symmetry and the associated current is no longer conserved. Recall that we have used this fact to explain one of the sources of CP violation in the Standard Model in Section 2.4. Since the classical action is invariant under a *global* chiral symmetry transformation, but the path integral is not, it was necessary to include extra terms in the Standard Model Lagrangian. The so-called θ-terms are CP violating and have physical consequences. Note that global anomalies are not only used to understand the strong CP problem, but also to explain baryon number violation and why the η' meson is so heavy.

However, if the anomalous symmetry is associated with a gauge boson, this can lead to a disaster. Gauge symmetries are crucial in demonstrating the unitarity and renormalizability of a theory. Furthermore, in order to correctly describe a massless vector boson, we must make sure that its longitudinal components do not couple to matter, which is only true when the current is conserved. Thus, all gauged symmetries of a consistent quantum theory must be *anomaly free* [6].

If one assumes that the new gauge symmetry $U(1)_X$ acts on ordinary fermions, it is expected that they will contribute to gauge anomalies. Since the Standard Model must be anomaly free, i.e., all gauge anomalies must be absent, an important issue is how they are ultimately cancelled. There are two main anomaly-cancellation scenarios: all anomalies cancel among the Standard Model fields themselves, or new particles are required.

For example, we could imagine the new gauge symmetry to be a linear combination of one or more of the four classical global symmetries: baryon number (B), electron number (L_e), muon number (L_μ), and tau number (L_τ) [16]. There are three independent combinations of these symmetries that are truly anomaly free, but they are all inconsistent with our requirements for a new light gauge boson. In GUTs, it is common to have a gauge boson associated with $B-L$, where $L = L_e + L_\mu + L_\tau$. Since this would allow the proton to decay, the bounds on the proton lifetime imply that such gauge boson should be very heavy ($\gtrsim 10^{16}$ GeV) [6]. One could also gauge $L_e - L_\mu$ or $L_e - L_\tau$, but neutrino oscillations make it unlikely that these symmetries are exact. Such vector forces are constrained by atmospheric neutrino data. Defining α_V as the fine-structure constant of a vector force, when the range of these forces is the Earth-Sun distance, one finds the following bounds [22]:

$$\alpha_V(e\,\mu) < 5.5 \cdot 10^{-52} \quad \text{and} \quad \alpha_V(e\,\tau) < 6.4 \cdot 10^{-52}. \tag{4.2.1}$$

These bounds imply a very small coupling constant for a force mediated by a light gauge boson. However, $L_e - L_\mu$ and $L_e - L_\tau$ forces can significantly influence neutrino oscillations. This is a consequence of the long range nature and the flavor dependence of the potential generated by the correspondent gauge bosons. In fact, to generate correct values for the mixing angle and the neutrino mass splitting is necessary to break $L_e - L_\mu$ and $L_e - L_\tau$ [23]. Therefore, neutrino oscillations are only consistent with *approximate* ΔL symmetries.

Another way to cancel anomalies is through the Green-Schwarz mechanism, which can arise from many plausible ultraviolet physics, such as low energy string models [16]. There are six types of anomalies that appear when one extends the Standard Model with a new gauge symmetry. These anomalies can all be related to the generators of the Standard Model and the $U(1)_X$ gauge groups, as well as the generators of the Lorentz group, by

$$\begin{aligned}
A\left[U(1)_X, U(1)_X, U(1)_X\right] &= \mathrm{Tr}\left[\chi\left\{\chi,\chi\right\}\right] = 2\,\mathrm{Tr}\left[\chi^3\right], \\
A\left[U(1)_X, U(1)_X, U(1)_Y\right] &= \mathrm{Tr}\left[\chi\left\{\chi,Y\right\}\right] = 2\,\mathrm{Tr}\left[\chi^2 Y\right], \\
A\left[U(1)_X, U(1)_Y, U(1)_Y\right] &= \mathrm{Tr}\left[\chi\left\{Y,Y\right\}\right] = 2\,\mathrm{Tr}\left[\chi Y^2\right], \\
A\left[U(1)_X, SU(2), SU(2)\right] &= \mathrm{Tr}\left[\chi\left\{\frac{\tau^a}{2},\frac{\tau^b}{2}\right\}\right], \\
A\left[U(1)_X, SU(3), SU(3)\right] &= \mathrm{Tr}\left[\chi\left\{\frac{\lambda^a}{2},\frac{\lambda^b}{2}\right\}\right], \\
A\left[U(1)_X, grav, grav\right] &= \mathrm{Tr}\left[\chi\left\{\frac{\tau^a}{2},\frac{\tau^b}{2}\right\}\right].
\end{aligned} \tag{4.2.2}$$

We use χ to denote the generator of the new symmetry, Y is the Standard Model hypercharge, τ^a and λ^a represent the Pauli and Gell-Mann matrices, respectively. Since the action of the Lorentz group on fermions is essentially equivalent to $SU(2)$ transformations, its generators are also given by Pauli matrices. Just for simplicity, we can define a new symmetry generator $V = \chi + \xi Y$, such that the anomalies will only contribute with $\mathrm{Tr}\,[VVV]$, $\mathrm{Tr}\,[VYY]$, $\mathrm{Tr}\,[V\tau^a\tau^b]$ and $\mathrm{Tr}\,[V\lambda^a\lambda^b]$ (the Standard Model ensures the absence of anomalies like $\mathrm{Tr}\,[Y^3]$).

We know that the anomalies of the gauge group $SU(3)_C \times SU(2) \times U(1)_Y \times U(1)_X$ can be written in an invariant form when we perform symmetry transformations on the action,

$$\delta S = \int d^4x\, \theta(x) \left(\partial^\mu J^5_\mu\right), \tag{4.2.3}$$

where $\theta(x)$ is the symmetry parameter. Using the Adler-Bell-Jackiw anomalies, we can rewrite the derivative of the axial current for abelian and non-abelian symmetries as

$$\begin{aligned} \partial^\mu J^5_\mu &= -c\, \epsilon^{\alpha\beta\gamma\delta}\, F_{\alpha\beta} F_{\gamma\delta}, \\ \partial^\mu J^5_\mu &= -d\, \epsilon^{\alpha\beta\gamma\delta}\, F^a_{\alpha\beta} F^a_{\gamma\delta}, \end{aligned} \tag{4.2.4}$$

respectively. The coefficients c and d are calculable, and F is the correspondent field strength tensor. Finally, this leads to

$$\begin{aligned} \delta S = &- \int d^4x\, \theta(x) \times \\ &\times \epsilon^{\alpha\beta\mu\nu} \left[c_V\, V_{\alpha\beta} V_{\mu\nu} + c_Y\, B_{\alpha\beta} B_{\mu\nu} + d^1_a\, W^a_{\alpha\beta} W^a_{\mu\nu} + d^2_a\, G^a_{\alpha\beta} G^a_{\mu\nu} - c_L\, R_{\alpha\beta}{}^{\gamma\delta} R_{\mu\nu\gamma\delta}\right], \end{aligned} \tag{4.2.5}$$

where $V_{\alpha\beta} = X_{\alpha\beta} + \xi B_{\alpha\beta}$. Note that $X_{\alpha\beta}$, $B_{\alpha\beta}$, $W^a_{\mu\nu}$ and $G^a_{\mu\nu}$ are the gauge boson field strength tensors related to $U(1)_X$, $U(1)_Y$, $SU(2)$ and $SU(3)_C$, respectively. Furthermore, $R_{\alpha\beta\gamma\delta}$ is the Riemann tensor. The above expression can be coupled to the Goldstone field in the kinetic Lagrangian of the theory as

$$\begin{aligned} \mathscr{L}_{kin} \supset &-\frac{1}{4} X_{\mu\nu} X^{\mu\nu} + \frac{M^2}{2} \left[X_\mu - \frac{\sqrt{2}}{M} (\partial_\mu \pi)\right] \left[X^\mu - \frac{\sqrt{2}}{M} (\partial^\mu \pi)\right] + \\ &+ \frac{\sqrt{2}}{v} \pi \epsilon^{\alpha\beta\mu\nu} \left[c_V\, V_{\alpha\beta} V_{\mu\nu} + c_Y\, B_{\alpha\beta} B_{\mu\nu} + d^1_a\, W^a_{\alpha\beta} W^a_{\mu\nu} + d^2_a\, G^a_{\alpha\beta} G^a_{\mu\nu} - c_L\, R_{\alpha\beta}{}^{\gamma\delta} R_{\mu\nu\gamma\delta}\right]. \end{aligned} \tag{4.2.6}$$

Since the Goldstone boson transforms as $\pi(x) \to \pi(x) + \frac{v}{\sqrt{2}}\theta(x)$, this term is not invariant. Thus, such variation precisely cancels the fermion anomaly. Note, however, that the cancelling term is a dimension-five operator, which makes it non-renormalizable. As before, for $M > g\Lambda/4\pi$ this operator is suppressed. On the other hand, if $M < g\Lambda/4\pi$, we also need to extend the Goldstone boson to a linear representation of the symmetry.

Finally, one could also introduce a new sector of fermions charged under the $U(1)_X$ symmetry. Depending on the model, they can be singlets under the Standard Model, or they can carry nontrivial Standard Model quantum numbers. Since we are going to work mainly with Z pole observables, we can assume that these fermions are heavy degrees of freedom and, therefore, they will not contribute to our computations. In fact, at low energies a theory can appear to be anomalous just because the anomaly-cancellation degrees of freedom lie above the cutoff.

4.3 The Lagrangian

The next step is to write a Lagrangian invariant under a $SU(2)\times U(1)_Y\times U(1)_X$ symmetry group. As we mentioned before, in order to guarantee the breaking of both $U(1)$ groups, there must be *at least* two scalar fields in our theory, which we will take to be the Higgs doublet and a complex singlet. They can have arbitrary charges under the whole gauge group. Such charges will determine the presence of a mixing between the gauge boson X and the Standard Model fields. Many well-established models consider $U(1)_X$ to be broken by a singlet under the Standard Model and the Higgs doublet to be uncharged under the extra gauge group, such that there is no need to consider a mass mixing. However, for a general analysis, we will consider the effects of this possible mixing.

Since the most generic Lagrangian must include all gauge invariant terms, a kinetic mixing is also allowed. Such term couples the field strength tensors of the Standard Model gauge field B and the new gauge field X. Even if one considers the kinetic mixing parameter to be zero at tree-level, this term could still be generated by loop effects or via renormalization group flow [14].

With these considerations in mind, the simplest – yet most general – Lagrangian of interest for us is given by

$$\begin{aligned}\mathscr{L} = &-\frac{1}{4}W^a_{\mu\nu}W^{a\,\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}X_{\mu\nu}X^{\mu\nu} - \frac{\sin(\beta)}{2}B_{\mu\nu}X^{\mu\nu}+\\ &+(D_\mu H)^\dagger(D^\mu H) + m_H^2(H^\dagger H) - \lambda_H(H^\dagger H)^2+\\ &+(D_\mu S)^*(D^\mu S) + m_S^2(S^* S) - \lambda_S(S^* S)^2 - \lambda_{HS}(H^\dagger H)(S^* S),\end{aligned} \tag{4.3.1}$$

where m_I and λ_I are the bare mass and coupling, respectively, for $I = H, S$, and β is the kinetic mixing parameter. The term that couples the scalar fields H and S is also allowed by symmetry. We denote its coupling by λ_{HS}. Finally, the covariant derivatives can be written as

$$D_\mu H = \left(\partial_\mu - ig\frac{\tau^a}{2}W^a_\mu - ig'\frac{1}{2}B_\mu - ig_X\,\chi_H X_\mu\right)H, \tag{4.3.2}$$

with χ_H being the charge of the Higgs doublet under the new symmetry group $U(1)_X$, and

$$D_\mu S = \left(\partial_\mu - ig' Y_S B_\mu - ig_X\,\chi_S X_\mu\right)S, \tag{4.3.3}$$

where Y_S and χ_S are the charges of the singlet under $U(1)_Y$ and $U(1)_X$, respectively. The $U(1)_X$ coupling is represented by g_X. It is interesting to highlight that both χ_H and Y_S guarantee a mass mixing.

As in the Standard Model, the gauge group $SU(2)\times U(1)_Y\times U(1)_X$ is spontaneously broken into $U(1)_{EM}$ when the scalar fields H and S acquire a vacuum expectation value. Let $v = m_H/\sqrt{\lambda_H}$ be the vev acquired by the Higgs doublet, and $w = m_S/\sqrt{\lambda_S}$ the one acquired by the singlet. In the unitary gauge, the scalar fields can be expanded around their vevs as

$$H(x) = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v + h(x)\end{pmatrix}, \quad S(x) = \frac{1}{\sqrt{2}}\left[w + s(x)\right]. \tag{4.3.4}$$

Note that the real scalar fields h and s are not the mass eigenstates. As we have seen in Eq. (4.3.1), a term that mixes the Standard Model doublet H and the new complex singlet S is allowed by symmetry. Consequently, we must diagonalize the scalar mass matrix in order to find the true physical states. One of these mass eigenstates can be identified as the observable Higgs boson, while the other scalar is expected to be as light as the new gauge boson. And since they will both couple to the massive gauge bosons, a light scalar can significantly contribute to some physical processes. Furthermore, it can also contribute to the invisible decay of the Higgs boson. However, the analysis of such contributions is not in the scope of this thesis.

Since the W boson mass is not modified, and the $W^{\pm}$ fields do not mix kinetically with X, we will focus on the kinetic and mass Lagrangians for the neutral vectors Z, A, and X. In order to distinguish before and after the diagonalization of both kinetic and mass terms, we will denote the still-mixed fields by hat or tilde, reserving the variables Z, A, and Z' only for the final, diagonalized fields. After spontaneous symmetry breaking and weak rotation, the Lagrangian can be written explicitly as

$$\mathscr{L} \supset -\frac{1}{4}\hat{\mathbf{V}}^T_{\mu\nu} K \hat{\mathbf{V}}^{\mu\nu} + \frac{1}{2}\hat{\mathbf{V}}^T_{\mu} M^2 \hat{\mathbf{V}}^{\mu}, \tag{4.3.5}$$

where $\hat{\mathbf{V}}$ is the gauge-field-valued vector

$$\hat{\mathbf{V}} = \begin{pmatrix} \hat{Z} \\ \hat{A} \\ \hat{X} \end{pmatrix}, \tag{4.3.6}$$

K is the non-diagonal kinetic matrix

$$K = \begin{pmatrix} 1 & 0 & -\frac{g'}{\sqrt{g^2+g'^2}}\sin(\beta) \\ 0 & 1 & \frac{g}{\sqrt{g^2+g'^2}}\sin(\beta) \\ -\frac{g'}{\sqrt{g^2+g'^2}}\sin(\beta) & \frac{g}{\sqrt{g^2+g'^2}}\sin(\beta) & 1 \end{pmatrix}, \tag{4.3.7}$$

and M^2 is the most general mass matrix

$$M^2 = \begin{pmatrix} \hat{M}_Z^2 + \frac{g'^2}{g^2}\hat{M}_A^2 & -\frac{g'^2}{g^2}\hat{M}_A^2 & -\hat{M}_X\hat{M}_Z - \frac{g'}{g}\hat{m}_X\hat{M}_A \\ -\frac{g'^2}{g^2}\hat{M}_A^2 & \hat{M}_A^2 & \hat{m}_X\hat{M}_A \\ -\hat{M}_X\hat{M}_Z - \frac{g'}{g}\hat{m}_X\hat{M}_A & \hat{m}_X\hat{M}_A & \hat{m}_X^2 + \hat{M}_X^2 \end{pmatrix}. \tag{4.3.8}$$

The fields $\hat{A}$ and $\hat{Z}$ are the Standard Model gauge bosons given by Eq. (2.1.6). The mass parameters $\hat{M}_Z$, $\hat{M}_A$, $\hat{M}_X$, and $\hat{m}_X$ are conveniently defined in terms of gauge couplings, vevs and charges as

$$\begin{aligned} \hat{M}_Z &= \tfrac{v}{2}\sqrt{g^2+g'^2}, \\ \hat{M}_A &= \tfrac{g\,g'}{\sqrt{g^2+g'^2}}\, w\, Y_S, \\ \hat{M}_X &= g_X\, v\, \chi_H, \\ \hat{m}_X &= g_X\, w\, \chi_S. \end{aligned} \tag{4.3.9}$$

One can worry about the fact that the Standard Model photon $\hat{A}$ also picks up a mass. However, what is physically relevant is that one of the mass eigenstates is massless. Such eigenstate will be identified as the observable photon. Furthermore, one can notice we have defined $\hat{M}_Z$ as the

Standard Model mass for the $\hat{Z}$ boson. This will allow us to determine the physical Z mass as the Standard Model prediction plus a correction due to mixings.

In order to study the implications of the new gauge boson to any electroweak observable, we need to work in the physical (or mass) eigenbasis. Going to such basis requires diagonalizing both kinetic and mass Lagrangians. This is a two-step process in which the starting point is to diagonalize the kinetic term. The matrix in Eq. (4.3.7) is easily put into canonical form by letting

$$\hat{\mathbf{V}} \equiv L\tilde{\mathbf{V}} \equiv \begin{pmatrix} 1 & 0 & \frac{\sin(\beta)}{\sqrt{1-\sin^2(\beta)}}\frac{g'}{\sqrt{g^2+g'^2}} \\ 0 & 1 & -\frac{\sin(\beta)}{\sqrt{1-\sin^2(\beta)}}\frac{g}{\sqrt{g^2+g'^2}} \\ 0 & 0 & \frac{1}{\sqrt{1-\sin^2(\beta)}} \end{pmatrix} \begin{pmatrix} \tilde{Z} \\ \tilde{A} \\ \tilde{X} \end{pmatrix}. \tag{4.3.10}$$

The transformation L will also have effects on the mass matrix. Thus, the final matrix we must diagonalize is the result of how Eq. (4.3.8) changes under the diagonalization of the kinetic terms: $L^T\, M^2\, L$. Unfortunately, it is not easy to find a simple analytical expression for all the eigenvalues and eigenvectors of such matrix. Instead, we need to consider some restrictions from electroweak precision measurements in order to determine which parameters of our model must be small. Then, we can use perturbation theory to find the observable masses and states. This will allow us to better interpret the physical consequences of an extra $U(1)$ symmetry.

The characteristic polynomial of $L^T\, M^2\, L$ is of the form $\lambda\,(-\lambda^2 + F_1\,\lambda + F_0)$, which means that one of the eigenvalues is $\lambda_1 = 0$. This indicates that our model provides a massless eigenstate, i.e., the photon. Furthermore, knowing that

$$\begin{aligned} F_1 = 1+&\frac{g'^2}{g^2+g'^2}\frac{\sin^2(\beta)}{1-\sin^2(\beta)} + \frac{1}{1-\sin^2(\beta)}\frac{g^2+g'^2}{g^2}\frac{\hat{M}_A^2}{\hat{M}_Z^2} + \frac{1}{1-\sin^2(\beta)}\frac{\hat{m}_X^2+\hat{M}_X^2}{\hat{M}_Z^2} - \\ &- 2\frac{\sin(\beta)}{1-\sin^2(\beta)}\left[\frac{\sqrt{g^2+g'^2}}{g}\frac{\hat{m}_X\hat{M}_A}{\hat{M}_Z^2} + \frac{g'}{\sqrt{g^2+g'^2}}\frac{\hat{M}_X}{\hat{M}_Z}\right], \end{aligned} \tag{4.3.11}$$

$$\begin{aligned} F_0 = -\frac{1}{1-\sin^2(\beta)}&\frac{g^2+g'^2}{g^2}\frac{\hat{M}_A^2\hat{M}_X^2}{\hat{M}_Z^4} + 2\frac{1}{1-\sin^2(\beta)}\frac{g'}{g}\frac{\hat{m}_X\hat{M}_A\hat{M}_X}{\hat{M}_Z^3} - \\ &- \frac{1}{1-\sin^2(\beta)}\frac{\hat{m}_X^2+\hat{M}_A^2}{\hat{M}_Z^2} + 2\frac{\sin(\beta)}{1-\sin^2(\beta)}\frac{g}{\sqrt{g^2+g'^2}}\frac{\hat{m}_X\hat{M}_A}{\hat{M}_Z^2}, \end{aligned} \tag{4.3.12}$$

we can determine the other two eigenvalues by $\lambda_{2,3} = \frac{\hat{M}_Z^2}{2}\left(F_1 \pm \sqrt{F_1^2 + 4F_0}\right)$. One of them can be identified as the mass of the physical Z boson, while the other corresponds to the mass of the new gauge boson. Since the Standard Model prediction of the Z mass is consistent with experiments, any correction coming from the mixings must be very small.

In order to determine the correct small combinations from Eqs. (4.3.11) and (4.3.12), we need to study separately the three possible scenarions for a non-vanishing mass mixing:

1. $\chi_H = 0$, $Y_S \neq 0$, $\chi_S \neq 0$,
2. $\chi_H \neq 0$, $Y_S = 0$, $\chi_S \neq 0$,
3. $\chi_H \neq 0$, $Y_S \neq 0$, $\chi_S \neq 0$.

	$SU(3)_C$	$SU(2)$	$U(1)_Y$	$U(1)_X$
Q^i	**3**	**2**	$1/6$	0
u^i_R	**3**	**1**	$2/3$	0
d^i_R	**3**	**1**	$-1/3$	0
L^i	**1**	**2**	$-1/2$	0
e^i_R	**1**	**1**	-1	0
ν^i_R	**1**	**1**	0	0
H	**1**	**2**	$1/2$	0
S	**1**	**1**	Y_S	χ_S

Table 4.1: Quantum numbers of the fields contained in our model. The index i runs over the three Standard Model generations.

In this work, we will focus on the first situation in which the singlet alone is responsible for the breaking of $U(1)_X$. As one can see, taking $\chi_H = 0$ means that all terms proportional to $\hat{M}_X$ will vanish. Therefore, this scenario is already less complicated than the third one. It is important to highlight that the third case has never been considered in the literature to date. On the other hand, the second case, which has been extensively studied by physicists [14, 15, 17, 24, 25, 26], only takes into account a mass mixing between $\hat{Z}$ and $\hat{X}$. However, when $\chi_H = 0$ and $Y_S \neq 0$, a mass mixing with the Standard Model photon is also generated.

Throughout our analysis, we are going to consider the Standard Model fermions to be uncharged under the extra gauge group $U(1)_X$. We are also going to neglect the existence of new fermions, such that the only additional degrees of freedom are the complex singlet S and the gauge boson X. Thus, Table (4.1) provides the charges assigned to each field in our model.

4.4 The Minimal Model

As we have already mentioned, this thesis focus on the scenario where the $\hat{X}$ boson has non-vanishing mass and kinetic mixings with both Standard Model fields $\hat{Z}$ and $\hat{A}$. In order to diagonalize such mixings and correctly determine the physical eigenstates, we must establish which parameters of our model are small. Just for simplicity, we start by analyzing the non-zero eigenvalues when both $\hat{M}_X$ and β are zero:

$$\begin{aligned}\lambda_{2,3}\Big|_{\hat{M}_X=0,\beta=0} &= \frac{\hat{M}_Z^2}{2}\times \\ &\times\left[1+\frac{\hat{m}_X^2}{\hat{M}_Z^2}+\frac{g^2+g'^2}{g^2}\frac{\hat{M}_A^2}{\hat{M}_Z^2}\pm\sqrt{\left(1+\frac{\hat{m}_X^2}{\hat{M}_Z^2}+\frac{g^2+g'^2}{g^2}\frac{\hat{M}_A^2}{\hat{M}_Z^2}\right)^2-4\left(\frac{m_X^2}{M_Z^2}+\frac{M_A^2}{M_Z^2}\right)}\,\right].\end{aligned} \tag{4.4.1}$$

Given these expressions, we can argue that the term proportional to $\hat{M}_A$ must be a small correction to the Standard Model Z mass. This allow us to define

$$\frac{\hat{M}_A^2}{\hat{M}_Z^2} = \frac{g^2}{(g^2+g'^2)}\frac{g'^2}{(g^2+g'^2)}4Y_S^2\frac{w^2}{v^2} \equiv \frac{g^2}{(g^2+g'^2)}\frac{g'^2}{(g^2+g'^2)}a_2, \tag{4.4.2}$$

$$\frac{\hat{m}_X^2}{\hat{M}_Z^2} = \frac{g_X^2}{(g^2+g'^2)}4\chi_S^2\frac{w^2}{v^2} \equiv a_4, \tag{4.4.3}$$

It is important to highlight that, at this point, a_4 does not have to be small. Since it is the leading order term of one of the eigenvalues, a_4 can assume any value. Furthermore, to recover the Standard Model, we must decouple the new $U(1)_X$ symmetry by taking the limit of a_2 going to zero. Such limit can be compatible with a small singlet hypercharge or a small ratio between the vevs. However, as we are going to show, the only possible scenario is Y_S small. Note that, if one wants to obtain the Standard Model from the above equations, the limit of a_4 going to infinity is also allowed. This scenario is consistent with previous works in which the extra gauge boson was very heavy, and corrections to Standard Model observables were given by an EFT approach.

Turning the kinetic mixing back on (i.e., $\beta \neq 0$), we also expect the second term of

$$F_1\Big|_{\hat{M}_X=0} = 1+\frac{g'^2}{g^2+g'^2}\frac{\sin^2(\beta)}{1-\sin^2(\beta)} + \frac{1}{1-\sin^2(\beta)}\frac{g^2+g'^2}{g^2}\frac{\hat{M}_A^2}{\hat{M}_Z^2} + \frac{1}{1-\sin^2(\beta)}\frac{\hat{m}_X^2}{\hat{M}_Z^2} - \\ - 2\frac{\sin(\beta)}{1-\sin^2(\beta)}\frac{\sqrt{g^2+g'^2}}{g}\frac{\hat{m}_X\hat{M}_A}{\hat{M}_Z^2} \tag{4.4.4}$$

to be small. If we rewrite it as

$$\frac{\sin^2(\beta)}{1-\sin^2(\beta)} \equiv a_1, \tag{4.4.5}$$

then the third term will be proportional to $(1+a_1)\,a_2$.

Finally, the matrix we want to diagonalize can be written as

$$\frac{(1+r^2)^2}{\hat{M}_Z^2}\left(L^T M^2 L\right) = \begin{pmatrix} (1+r^2)^2 + r^4 a_2 & -r^3 a_2 & f_1 \\ -r^3 a_2 & r^2 a_2 & f_2 \\ f_1 & f_2 & f_3 \end{pmatrix}, \tag{4.4.6}$$

where, just for simplicity, we have defined $r \equiv g'/g$ and

$$\begin{aligned} f_1 &\equiv r\sqrt{1+r^2}\left\{-r\sqrt{(1+r^2)a_2a_4(1+a_1)} + \sqrt{a_1}\left[1+r^2(1+a_2)\right]\right\}, \\ f_2 &\equiv r\sqrt{1+r^2}\left(-r a_2\sqrt{a_1} + \sqrt{(1+r^2)a_2a_4(1+a_1)}\right), \\ \tfrac{f_3}{(1+r^2)^2} &\equiv \left\{a_4(1+a_1) + r\left[-2\sqrt{a_1a_2a_4\tfrac{(1+a_1)}{(1+r^2)}} + \tfrac{r}{(1+r^2)}\,a_1(1+a_2)\right]\right\}. \end{aligned} \tag{4.4.7}$$

In order to determine the eigenvalues and eigenvectors of this matrix, it is easier to use perturbation theory. We can apply the perturbation method to this scenario because Eq. (4.4.6) can be decomposed as $M_0^2 + \epsilon\, M_1^2 + \epsilon^2\, M_2^2$, where ϵ is a small parameter. However, to use perturbation theory, we must make a very important assumption: a_4 is never equal to one. Throughout this work, we have used non-degenerate perturbation theory, which means the fields $\hat{Z}$, $\hat{A}$ and $\hat{X}$ are non-degenerate mass states. On the other hand, if $a_4 = 1$, then the non-degerate perturbation method breaks down since $\hat{Z}$ and $\hat{X}$ now have the same mass.

However, before applying perturbation theory to our matrix, it is worth studying closely the analytical expression of the massless eigenstate,

$$
\begin{aligned}
A = & \frac{1}{\sqrt{\left[(1+r^2)^2(1+a_1) + r\left(-2\sqrt{(1+r^2)a_1\frac{a_2}{a_4}(1+a_1)} + r\frac{a_2}{a_4}(1+a_1)\right)\right]}} \times \\
& \times \left[\frac{r^2}{\sqrt{1+r^2}}\sqrt{a_1\frac{a_2}{a_4}}\tilde{Z} + \left(-\frac{r}{\sqrt{1+r^2}}\sqrt{a_1\frac{a_2}{a_4}} + (1+r^2)\sqrt{1+a_1}\right)\tilde{A} - r\sqrt{\frac{a_2}{a_4}}\tilde{X}\right].
\end{aligned}
\tag{4.4.8}
$$

Although we already know this is an *exact* massless eigenstate, we must make sure it does behave like the observable photon in all other aspects. Furthermore, it can help us to determine which combinations of parameters are truly small. As one can easily see, Eq. (4.4.8) depends on the ratio a_2/a_4, which means it is actually independent of the ratio between the vevs. Thus, besides a_1, the only possible small parameter is the singlet hypercharge Y_S. At this point, one could argue whether there is one more viable scenario: g_X small. The problem with a small coupling is that it takes our model too far away from the Standard Model. In this case, the mixing angle between the gauge fields B_μ and X_μ is almost $\pi/2$, and even though we could still find a massless eigenstate, it would not couple to the electromagnetic current as the observable photon.

In order to verify whether Eq. (4.4.8) is indeed the observable photon, and a small Y_S is physically allowed, we can compute the electric charge operator. The interaction Lagrangian is initially given by

$$
\mathscr{L}_{int} = \hat{\mathbf{V}}_\mu^T \mathbf{J}^\mu \equiv g\,\hat{Z}_\mu J_Z^\mu + \frac{g\,g'}{\sqrt{g^2+g'^2}}\hat{A}_\mu J_A^\mu + g_X\,\hat{X}_\mu J_X^\mu, \tag{4.4.9}
$$

where J_Z and J_A are the Standard Model currents, and J_X is a current that couples the X boson to the real scalar fields. After diagonalizing the kinetic term, the interaction Lagrangian takes the form

$$
\begin{aligned}
\mathscr{L}_{int} =& g\,\tilde{Z}_\mu J_Z^\mu + \frac{g\,g'}{\sqrt{g^2+g'^2}}\tilde{A}_\mu J_A^\mu + \\
& + \tilde{X}_\mu\left[g_X\sqrt{1+a_1}\,J_X^\mu + \frac{g\,g'}{\sqrt{g^2+g'^2}}\sqrt{a_1}\left(-\frac{g'}{\sqrt{g^2+g'^2}}\,J_A^\mu + \frac{g'}{g}\,J_Z^\mu\right)\right].
\end{aligned}
\tag{4.4.10}
$$

Therefore, using Eq. (4.4.8), we can obtain the complete interaction Lagrangian for the massless gauge boson of our model:

$$
\begin{aligned}
\mathscr{L}_{int}^A =& \frac{g'\sqrt{(g^2+g'^2)(1+a_1)}}{g\sqrt{\left[\left(\frac{g^2+g'^2}{g^2}\right)^2(1+a_1) + \frac{g'}{g}\left(-2\frac{\sqrt{(g^2+g'^2)}}{g}\sqrt{a_1\frac{a_2}{a_4}(1+a_1)} + \frac{g'}{g}\frac{a_2}{a_4}(1+a_1)\right)\right]}} \times \\
& \times A_\mu\left(J_A^\mu - \frac{Y_S}{\chi_S}J_X^\mu\right).
\end{aligned}
\tag{4.4.11}
$$

From this equation, we can see that A does not couple to J_Z, i.e., it does not couple to the Standard Model axial current. And since preserving gauge invariance means that a massless photon cannot couple to any axial current, we can conclude that a coupling to J_X is also allowed. Furthermore, it is straightforward to identify the new electric charge as

$$e = \frac{g' \sqrt{(g^2+g'^2)(1+a_1)}}{g\sqrt{\left[\left(\frac{g^2+g'^2}{g^2}\right)^2(1+a_1) + \frac{g'}{g}\left(-2\frac{\sqrt{(g^2+g'^2)}}{g}\sqrt{a_1\frac{a_2}{a_4}(1+a_1)} + \frac{g'}{g}\frac{a_2}{a_4}(1+a_1)\right)\right]}}, \tag{4.4.12}$$

and the modified electromagnetic current as

$$J^\mu_{EM} = J^\mu_A - \frac{Y_S}{\chi_S} J^\mu_X. \tag{4.4.13}$$

Knowing the generators and quantum numbers associated with the currents J_A and J_X, we can define the electric charge operator as

$$Q = \frac{\tau_3}{2} + Y - \frac{Y_S}{\chi_S}\chi. \tag{4.4.14}$$

Even for a small Y_S, both scalars in our model are electrically neutral, as they should be. Thus, the massless eigenstate from Eq. (4.4.8) can certainly be the observable photon.

For all the eigenstates, we can use perturbation theory to determine corrections to their masses. Therefore, the physical masses predicted by our model are given by

$$M_Z^2 = \hat{M}_Z^2 \left[1 + \frac{1}{(1-a_4)}\frac{g'^2}{(g^2+g'^2)}\left(a_1 + \frac{g'^2}{(g^2+g'^2)}a_2 - 2\frac{g'}{\sqrt{g^2+g'^2}}\sqrt{a_1 a_2 a_4}\right)\right], \tag{4.4.15}$$

$$M_A^2 = 0, \tag{4.4.16}$$

$$\begin{aligned} M_{Z'}^2 = \hat{m_X}^2 + \frac{\hat{M}_Z^2}{(-1+a_4)}\frac{g^4}{(g^2+g'^2)^2}\times \\ \times\left[-1 + \frac{(g^2+g'^2)}{g^2}a_4\right]\left[\frac{(g^2+g'^2)}{g^2}a_1\,a_4 + \frac{g'}{g}\left(\frac{g'}{g}a_2 - 2\frac{\sqrt{g^2+g'^2}}{g}\sqrt{a_1\,a_2\,a_4}\right)\right]. \end{aligned} \tag{4.4.17}$$

As expected, the observable photon is massless at all orders in perturbation theory. On the other hand, both Z and Z' bosons receive small corrections to their masses. Additionally, still using perturbation theory, we can determine the eigenvectors and, most importantly, how they couple to the SM currents and to J_X:

$$\begin{aligned} \mathscr{L}^Z_{int} = Z_\mu\, g\left\{1 + \frac{g'^2}{(g^2+g'^2)}\frac{\left[a_1(1-2a_4) + \frac{g'^2}{(g^2+g'^2)}a_4\left(-a_2 + 2\frac{\sqrt{g^2+g'^2}}{g'}\sqrt{a_1\,a_2\,a_4}\right)\right]}{2(-1+a_4)^2}\right\} J^\mu_Z + \\ + Z_\mu \frac{g\,g'}{\sqrt{g^2+g'^2}}\left\{\frac{g\,g'}{(g^2+g'^2)}\frac{\left[a_1 + \frac{g'^2}{(g^2+g'^2)}\left(a_2 - 2\frac{\sqrt{g^2+g'^2}}{g'}\sqrt{a_1\,a_2\,a_4}\right)\right]}{(-1+a_4)}\right\} J^\mu_A + \\ + Z_\mu\, g_X\left[\frac{g'^2}{(g^2+g'^2)}\frac{\sqrt{1+a_1}}{(-1+a_4)}\left(\sqrt{a_2\,a_4} - \frac{\sqrt{g^2+g'^2}}{g'}\sqrt{a_1}\right)\right] J^\mu_X, \end{aligned} \tag{4.4.18}$$

$$
\begin{aligned}
\mathscr{L}^{A}_{int} = A_\mu \frac{g\,g'}{\sqrt{g^2+g'^2}} &\left(1 + \frac{g^2 g'}{(g^2+g'^2)^{\frac{3}{2}}}\sqrt{\frac{a_1\,a_2}{a_4}} - \frac{g^2 g'^2}{(g^2+g'^2)^2}\frac{a_2}{2a_4}\right) J^\mu_A + \\
&+ A_\mu\, g_X \left(-\frac{g\,g'}{(g^2+g'^2)}\sqrt{\frac{(1+a_1)a_2}{a_4}}\right) J^\mu_X,
\end{aligned}
\tag{4.4.19}
$$

$$
\begin{aligned}
\mathscr{L}^{Z'}_{int} = Z'_\mu\, g &\left[\frac{g'^2}{(g^2+g'^2)}\frac{\left(-\sqrt{a_2 a_4} + \frac{\sqrt{g^2+g'^2}}{g'}\sqrt{a_1}\right)}{(-1+a4)} + \frac{g'}{\sqrt{g^2+g'^2}}\sqrt{a_1}\right] J^\mu_Z + \\
&+ Z'_\mu \frac{g\,g'}{\sqrt{g^2+g'^2}}\left(\frac{g\,g'}{(g^2+g'^2)}\sqrt{\frac{a_2}{a_4}} - \frac{g}{\sqrt{g^2+g'^2}}\sqrt{a_1}\right) J^\mu_A + \\
&+ Z'_\mu\, g_X \sqrt{1+a_1}\left[1 - \frac{g^2 g'^2}{(g^2+g'^2)^2} C_{Z'}\right] J^\mu_X,
\end{aligned}
\tag{4.4.20}
$$

where, just for simplicity, we have defined

$$
C_{Z'} \equiv \frac{\left[a_2\left(1 + a_4(-2 + \frac{(g^2+g'^2)}{g^2}a_4)\right) + a_4\frac{(g^2+g'^2)}{g^2}\left(a_1 - 2\frac{g'}{\sqrt{g^2+g'^2}}\sqrt{a_1\,a_2\,a_4}\right)\right]}{2(-1+a_4)^2}. \tag{4.4.21}
$$

We can summarize the physical effects of the presence of an additional $U(1)_X$ as follows:

1. The Z boson mass is modified;
2. The coupling of the physical Z boson to the Standard Model J_Z current is modified, and Z now also couples to the Standard Model electromagnetic current;
3. The coupling of the physical photon to the Standard Model electromagnetic current is modified;
4. The new massive Z' boson couples to both Standard Model currents, which opens a new channel for weak neutral current processes;
5. The presence of a non-vanishing J_X adds new Z and photon interactions that can be potentially relevant for their phenomenology;
6. If $\lambda_{HS} \neq 0$ in Eq. (4.3.1), then an h-s mixing is generated, which can have important consequences on the Higgs phenomenology.

Chapter 5

Z' Phenomenology

After constructing our minimal model, we now make contact with experiments. As we know from Eq. (4.4.20), the Z' boson couples to both Standard Model J_A and J_Z currents. Moreover, it also modifies the couplings of the physical Z boson and photon – Eqs. (4.4.18) and (4.4.19). Thus, we expect the presence of this new particle to have a great impact on electroweak physics.

As we have seen in Chapter 3, high-precision measurements of quantities fundamentally sensitive only to electroweak physics were instrumental in establishing the validity of the Standard Model. In the 1990's, particle accelerators like LEP and SLC performed important measurements of various observables at the Z resonance. For example, in the seven years that LEP operated at an energy close to $100\,GeV$, it produced around 17 million Z particles [1]. Their precise observations of how Z bosons were created and then, shortly after, decayed into other particles were a critical test of the Standard Model. Nowadays, these measurements can be used to constrain new physics contributions to electroweak observables, which is precisely our goal in this chapter [2].

5.1 Re-Expressing the Model in Terms of 'Standard' Parameters

The EW Standard Model Lagrangian is expressed in terms of three theoretical parameters: the electric charge $\hat{e}$, the Z boson mass $\hat{M}_Z$, and the weak mixing angle $\hat{\theta}_w$. For electroweak physics, they are sufficient to describe all the observables. However, as we did in Chapter 3, we must eliminate $\hat{e}$, $\hat{M}_Z$ and $\hat{\theta}_w$ in terms of reference measured quantities. For this purpose, it is standard to choose the best-measured observables as input: the electromagnetic fine-structure constant, the physical Z mass, and the Fermi constant. But we first need to ask ourselves how the existence of a Z' boson can affect these quantities. As we will show, in the presence of new physics the relation between these theoretical parameters and the reference observables changes [9, 28]. Thus, the values inferred from experiments for a given parameter will differ from what would be found within the Standard Model. Furthermore, since new physics parameters are already small, it is sufficient to compute most of these changes at tree-level.

5.1.1 Corrections to $(g-2)_e$ and the Fine-Structure Constant

The fine-structure constant is determined through the measurement of the electron anomalous magnetic moment. Hence, in order to compute the correct contribution of Z' to $\hat{\alpha}_e$, we must

[1]More informations about LEP can be found in the website https://home.cern/science/accelerators/large-electron-positron-collider.

[2]Rare decays of K and B mesons can also be used as sensitive probes of new physics [27]. Although such processes can constrain many models with light new states, in this work we are interested only in limits coming from electroweak precision measurements.

calculate its effects to $(g-2)_e$. The first step is to evaluate how the new coupling of the photon can modify this quantity. Second, we need to compute the correction to the magnetic moment coming from the interection between the electron and the Z' boson. Although we are interested only in corrections up to tree-level, the leading order contribution to the electron magnetic moment is already at 1-loop. If the Z' was very heavy, such correction would be highly suppressed and, therefore, negligible. But if the new gauge boson is light, it is not clear we can ignore this effect.

In order to determine how the observable photon contributes to $(g-2)_e$, we start by neglecting the scalar current J_X in Eq. (4.4.19):

$$\mathscr{L}_{A-e} = A_\mu \frac{g\, g'}{\sqrt{g^2+g'^2}} (1+\eta) \left(-\bar{e}\,\gamma^\mu\, e\right), \tag{5.1.1}$$

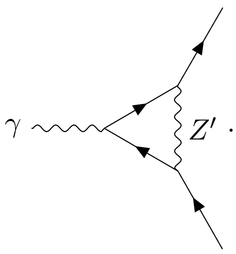
where, just for simplicity, we have defined

$$\eta \equiv \frac{g^2 g'}{(g^2+g'^2)^{\frac{3}{2}}} \sqrt{\frac{a_1 a_2}{a_4}} - \frac{g^2 g'^2}{(g^2+g'^2)^2} \frac{a_2}{2a_4}. \tag{5.1.2}$$

We can easily compute that the new photon coupling yields an extra term to the electron magnetic moment, which is now given by

$$(g-2)_e^{photon} = \frac{\hat{\alpha}_e}{\pi} (1+2\eta)\,, \tag{5.1.3}$$

where we have already used the Standard Model relation $4\pi\hat{\alpha}_e = \frac{(g\,g')^2}{(g^2+g'^2)}$. On the other hand, neglecting once more the physics of the extended Higgs sector, we know from Eq. (4.4.20) that the only new Feynman diagram contributing to the electron magnetic moment is



(5.1.4)

As we mentioned before, if the Z' boson was heavy, such contribution would be highly supressed. For example, this is what happens with the electroweak contribution of the Z boson, which can be neglected at leading order. However, since the Z' can be light, it is not clear we can do the same in our model. Furthermore, also from Eq. (4.4.20), we know exactly how the Z' boson couples to the electron:

$$\mathscr{L}_{Z'-e} = Z'_\mu \frac{g\, g'}{\sqrt{g^2+g'^2}}\, \bar{e}\,\gamma^\mu \left(\delta + \sigma\gamma^5\right) e, \tag{5.1.5}$$

where we have defined the couplings to the vector and axial currents as

$$\begin{aligned} \delta \equiv & \frac{3}{4}\frac{\sqrt{g^2+g'^2}}{g}\sqrt{a_1} - \frac{g\,g'}{(g^2+g'^2)}\sqrt{\frac{a_2}{a_4}} + \\ & + \frac{1}{4}\frac{(-g^2+3g'^2)}{g\sqrt{g^2+g'^2}}\frac{1}{(-1+a_4)}\left(-\frac{g'}{\sqrt{g^2+g'^2}}\sqrt{a_2 a_4} + \sqrt{a_1}\right), \end{aligned} \tag{5.1.6}$$

$$\sigma \equiv \frac{1}{4}\left[\frac{1}{(-1+a_4)}\left(-\frac{g'}{g}\sqrt{a_2 a_4}+\frac{\sqrt{g^2+g'^2}}{g}\sqrt{a_1}\right)+\frac{\sqrt{g^2+g'^2}}{g}\sqrt{a_1}\right]. \tag{5.1.7}$$

Employing the Feynman rules, and after some fair amount of algebra, we were able to extract the form factor that gives the electron its anomalous magnetic moment [3]. Thus, we can determine that the Z' boson changes the $(g-2)_e$ Standard Model value by

$$\begin{aligned}(g-2)_e^{Z'} =& \frac{2\hat{\alpha}_e}{\pi}\delta^2 \int_0^1 dx \frac{x(1-x)^2}{(1-x)^2 + x\left(\frac{M_{Z'}}{m_e}\right)^2} + \\ &+ \frac{2\hat{\alpha}_e}{\pi}\sigma^2 \left\{\left(\frac{m_e}{M_{Z'}}\right)^2 \int_0^1 dx \frac{(1-x)^3(1+x)}{(1-x)^2 + x\left(\frac{M_{Z'}}{m_e}\right)^2} + 2\log\left[\left(\frac{m_e}{M_{Z'}}\right)^2 + 1\right]\right\}.\end{aligned} \tag{5.1.8}$$

This result, together with Eq. (5.1.3), can be interpreted as an effective shift to the Standard Model fine-structure constant.

Physicists have already been able to perform QED calculations of the electron magnetic moment up to 4-loop level [11]. However, they failed to compare theory to experiment since QED results are expressed as a function of $\hat{\alpha}_e$, which could not be measured precisely any other way except through the measurement of $(g-2)_e$ [4]. As it is usually done in the literature, we are going to use the high-precision measurement of the electron magnetic moment, which yields $\alpha_e(0) = (137.035\,999\,084 \pm 0.000\,000\,051)^{-1}$ [32], to define the renormalized value of the fine-structure constant:

$$\begin{aligned}\alpha_e(0) = \hat{\alpha}_e + 2\hat{\alpha}_e &\left[\eta + \delta^2 \int_0^1 dx \frac{x(1-x)^2}{(1-x)^2 + x\left(\frac{M_{Z'}}{m_e}\right)^2}\right] + \\ &+ 2\hat{\alpha}_e\sigma^2 \left\{\left(\frac{m_e}{M_{Z'}}\right)^2 \int_0^1 dx \frac{(1-x)^3(1+x)}{(1-x)^2 + x\left(\frac{M_{Z'}}{m_e}\right)^2} + 2\log\left[\left(\frac{m_e}{M_{Z'}}\right)^2 + 1\right]\right\}.\end{aligned} \tag{5.1.9}$$

Recall from Section 3.1 that, in order to relate the renormalized parameter to the value of the coupling evolved to an energy scale equals to the Z boson mass, we had to use the full vacuum polarization contributions to the photon propagator – Eq. (3.1.11). Such relation indicates that if new physics contributes to the running of the fine-structure constant, it must be only through $\Pi_{\gamma\gamma}(M_Z^2)$. Taking into account how the Z' boson changes the photon coupling, we can easily compute that $\Pi_{\gamma\gamma}$ is now given by

$$\Pi_{\gamma\gamma}(p^2) = N\frac{\hat{\alpha}_e}{\pi}(1+\eta)^2 \sum_i Q_i^2 \left\{-\frac{2p^2}{3\epsilon} + \int_0^1 dx \log\left(\frac{4\pi\mu^2}{a^2}\right)\left[a^2 - (1-x)xp^2 - m_i^2\right]\right\}, \tag{5.1.10}$$

[3] The computation is very similar to the one performed in [29], where they determined the Z contribution to the anomalous magnetic moment of the muon.

[4] This has been changing for the past few years. Using methods from atomic physics, $\hat{\alpha}_e$ has already been measured directly. Although physicists found an agreement between both experimental values at first [30], recent measurements are now pointing to a 2.5σ tension with the Standard Model theoretical prediction [31]. However, since we do not understand well enough how atomic measurements are performed in order to account possible corrections coming from our model, we decided to keep using the measurement of $(g-2)_e$ to extract $\hat{\alpha}_e$.

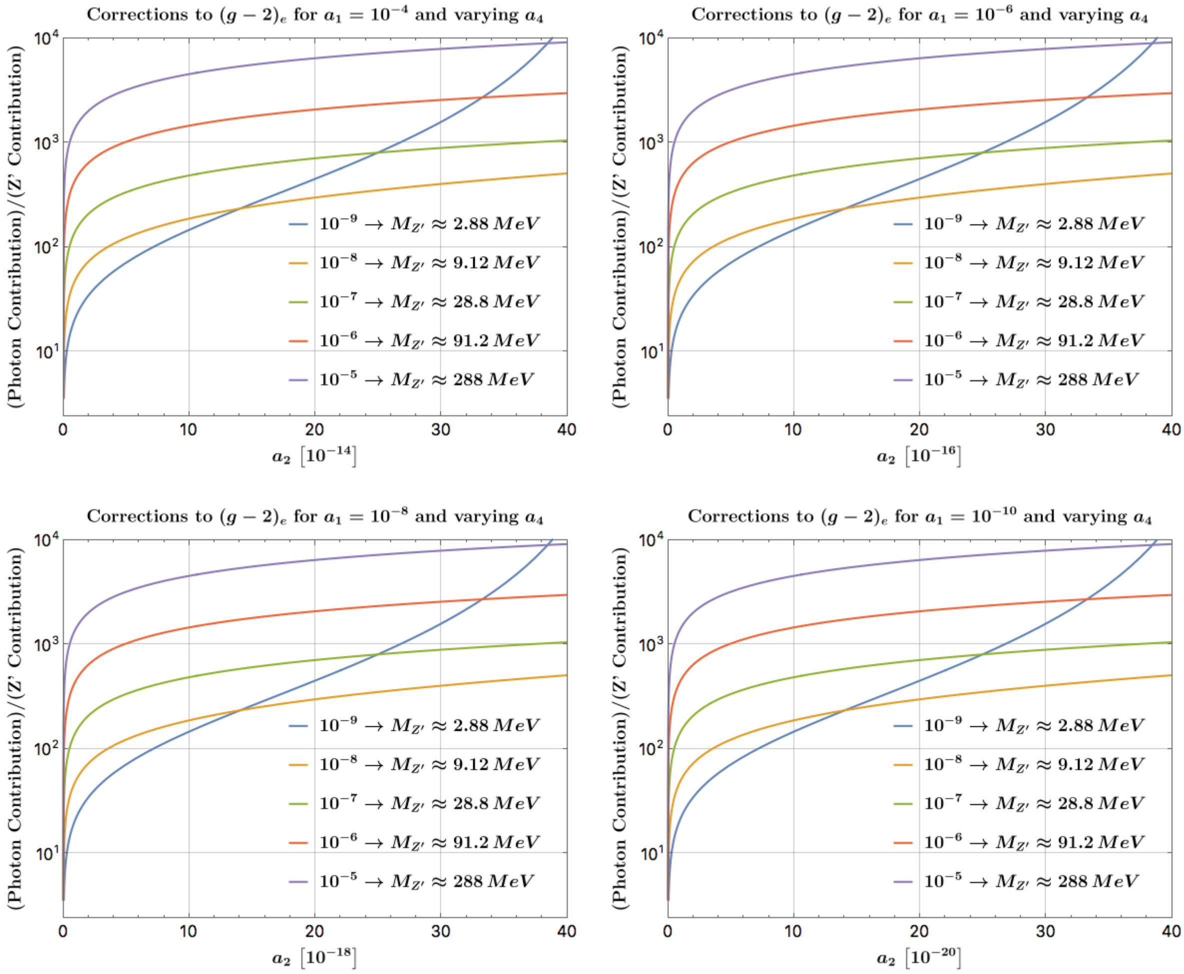


Figure 5.1: Ratio between the photon and Z' contributions to $(g-2)_e$ for fixed values of a_1 and varying values of a_4 (diffetent color lines). For a Z' boson within a mass range around the MeV, the photon contribution is the dominant one by at least two orders of magnitude.

with $a^2 = xm_i^2 + (1-x)(m_i^2 - xp^2)$. Knowing that η must be small, we can use $(1+\eta)^2 \approx 1+2\eta$. Thus, the leading term is exactly the Standard Model one, as expected. The second term yields a contribution to $\hat{\alpha}_e$ of order $\hat{\alpha}_e^2\,\eta$, which is negligible. This means the presence of a Z' boson does not give a significant contribution to $\Pi_{\gamma\gamma}$, such that the running of the fine-structure constant in our model can be considered the same as in the Standard Model. Therefore, we can relate $\alpha_e(M_Z) = (127.955 \pm 0.0010)^{-1}$ [10] to the shifted fine-structure constant in our model by

$$
\begin{aligned}
\alpha_e(M_Z) = \hat{\alpha}_e + 2\hat{\alpha}_e \left[\eta + \delta^2 \int_0^1 dx \frac{x(1-x)^2}{(1-x)^2 + x\left(\frac{M_{Z'}}{m_e}\right)^2} \right] + \\
+ 2\hat{\alpha}_e \sigma^2 \left\{ \left(\frac{m_e}{M_{Z'}}\right)^2 \int_0^1 dx \frac{(1-x)^3(1+x)}{(1-x)^2 + x\left(\frac{M_{Z'}}{m_e}\right)^2} + 2\log\left[\left(\frac{m_e}{M_{Z'}}\right)^2 + 1\right] \right\},
\end{aligned}
\tag{5.1.11}
$$

where $\hat{\alpha}_e$ is still the Standard Model parameter, but now containing contributions from $\Pi_{\gamma\gamma}$. Inverting this equation and linearizing the result in terms of η, δ^2, and σ^2, we get

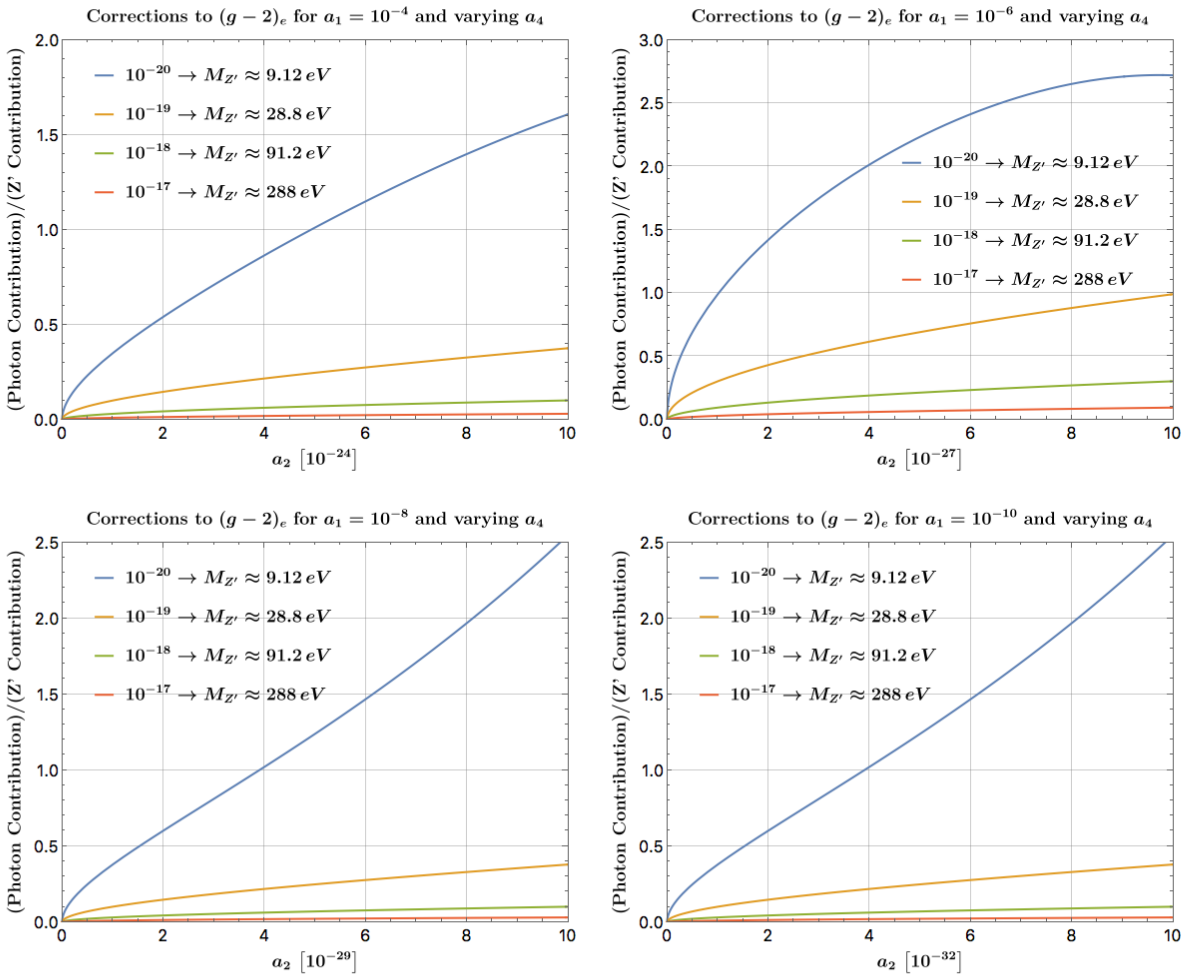


Figure 5.2: Ratio between the photon and Z' contributions to $(g-2)_e$ for fixed values of a_1 and varying values of a_4 (diffsame color lines). For a Z' boson within a mass range around the eV, the Z' contribution can be the dominant one, or nearly the same as the photon contribution.

$$
\begin{aligned}
\hat{\alpha}_e \approx \alpha_e(M_Z) - 2\alpha_e(M_Z)\left[\eta + \delta^2 \int_0^1 dx \frac{x(1-x)^2}{(1-x)^2 + x\left(\frac{M_{Z'}}{m_e}\right)^2}\right] - \\
- 2\alpha_e(M_Z)\,\sigma^2 \left\{\left(\frac{m_e}{M_{Z'}}\right)^2 \int_0^1 dx \frac{(1-x)^3(1+x)}{(1-x)^2 + x\left(\frac{M_{Z'}}{m_e}\right)^2} + 2\log\left[\left(\frac{m_e}{M_{Z'}}\right)^2 + 1\right]\right\}.
\end{aligned}
\tag{5.1.12}
$$

We can see that Eq. (5.1.12) is tightly related to the mass of the Z' boson. If our Z' is heavy (i.e., $M_{Z'} \gg m_e$), then the corrections proportional to δ^2 and σ^2 can be neglected. However, this is not the case for a Z' lighter or as light as the electron. For this reason, we analyzed the behavior of both photon and Z' contributions as a function of a_1, a_2, and a_4. From Eq. (4.4.17) we know the parameter a_4 determines how heavy the Z' boson is going to be. Furthermore, recall that a_4 cannot be equal to 1, otherwise the non-degenerate pertubation theory is no longer valid and our equations must be modified. Thus, since we are interested in a Z' lighter than the Z boson, we have restricted our analysis to $a_4 < 1$.

Fig. (5.1) shows the ratio between the photon and Z' corrections to $(g-2)_e$. We varied the value of a_4 such that the Z' is within a mass range of nearly 3 MeV to almost 300 MeV. In this scenario, we have found that the photon contribution can be at least two orders of magnitude greater than the Z' one. On the other hand, we can see that for decreasing values of a_2 with a_1 and a_4 fixed, i.e., taking the limit of a_2 going to zero, the Z' contribution will dominate:

$$\eta \to 0, \quad \sigma \to 0, \quad \delta \to \frac{g}{\sqrt{g^2 + g'^2}} \sqrt{a_1}. \tag{5.1.13}$$

As showed in Fig. (5.2), for even smaller values of a_4, which corresponds to a Z' within the eV mass range, we have two possibilities: both particles give nearly the same contribution, or the Z' contribution will also dominate. Thus, for a very light Z' boson, all terms must be taken into account.

In this thesis, just for simplicity, we chose to work in the regime where the photon contribution to $\hat{\alpha}_e$ is the relevant one. Therefore, we consider a Z' in the MeV-GeV mass range, such that the fine-structure constant is shifted by

$$\hat{\alpha}_e \approx \alpha_e(M_Z)\,(1 - 2\eta)\,. \tag{5.1.14}$$

This approximation can be understood as taking into account only tree-level contributions from the Z' boson. In fact, many recent works have been proposing extra gauge bosons with a mass around a few MeVs (e.g. [16]), but they usually neglect any contribution from these new particles to the fine-structure constant.

5.1.2 Z Boson Mass

In order to eliminate the Standard Model parameter $\hat{M}_Z$ in favor of the physical mass $M_Z = 91.1876 \pm 0.0021\, GeV$ [10], we only need to invert Eq. (4.4.15):

$$\hat{M}_Z^2 \approx M_Z^2 \left[1 - \frac{1}{(1-a_4)} \frac{g'^2}{(g^2+g'^2)} \left(a_1 + \frac{g'^2}{(g^2+g'^2)} a_2 - 2 \frac{g'}{\sqrt{g^2+g'^2}} \sqrt{a_1 a_2 a_4} \right) \right]. \tag{5.1.15}$$

5.1.3 Fermi Constant

As we have already mentioned in Section 4.3, the Standard Model Lagrangian for the W boson does not change in the presence of a Z'. Thus, for the weak mixing angle $\hat{\theta}_w$ it is convenient to define its sine and cosine so that the Fermi constant measured in muon decay is given exactly by the Standard Model formula:

$$G_F = \frac{\pi\,\hat{\alpha}_e}{\sqrt{2}\,\hat{s}^2\,\hat{c}^2\,\hat{M}_Z^2} \equiv \frac{\pi\,\alpha_e(M_Z)}{\sqrt{2}\,s^2\,c^2\,M_Z^2}, \tag{5.1.16}$$

where we have used $\hat{s} = \frac{g'}{\sqrt{g^2+g'^2}}$ and $\hat{c} = \frac{g}{\sqrt{g^2+g'^2}}$. Substituting Eqs. (5.1.14) and (5.1.15) into the left-hand side of the above expression, we can infer that

$$\hat{s}^2\,\hat{c}^2 = s^2\,c^2 \left[1 - 2\left(\hat{s}\,\hat{c}^2 \sqrt{\frac{a_1 a_2}{a_4}} - \hat{s}^2\,\hat{c}^2 \frac{a_2}{2a_4}\right) + \frac{\hat{s}^2}{(1-a_4)} \left(a_1 + \hat{s}^2\,a_2 - 2\,\hat{s}\,\sqrt{a_1 a_2 a_4}\right)\right]. \tag{5.1.17}$$

Since all these corrections are small, we can directly replace the Standard Model parameters $\hat{s}$ and $\hat{c}$ with the measured ones on the right-hand side of this relation. Therefore, we find that the weak mixing angle is modified by

$$\hat{s}^2 = s^2 \left\{ 1 + \frac{c^2}{c^2 - s^2} \left[s^2 c^2 \frac{a_2}{a_4} - 2sc^2 \sqrt{\frac{a_1 a_2}{a_4}} + \frac{s^2}{(1-a_4)} \left(a_1 + s^2 a_2 - 2s\sqrt{a_1 a_2 a_4} \right) \right] \right\} , \tag{5.1.18}$$

$$\hat{c}^2 = c^2 \left\{ 1 - \frac{s^2}{c^2 - s^2} \left[s^2 c^2 \frac{a_2}{a_4} - 2sc^2 \sqrt{\frac{a_1 a_2}{a_4}} + \frac{s^2}{(1-a_4)} \left(a_1 + s^2 a_2 - 2s\sqrt{a_1 a_2 a_4} \right) \right] \right\} . \tag{5.1.19}$$

5.1.4 Rewriting the Interactions

We can finally re-express the interaction Lagrangians of Eqs. (4.4.18), (4.4.19), and (4.4.20) in terms of the standard parameters $\alpha_e(M_Z)$, s, and c. As before, we will neglect the coupling to the scalar current J_X. Throughout the process we have assumed that the corrections are small, such that all expressions can be linearized in terms of the parameters of our model. The Z boson Lagrangian is then given by

$$\mathscr{L}^Z_{int} = \frac{\sqrt{4\pi\, \alpha_e(M_Z)}}{s\, c} Z_\mu \bar{f} \gamma^\mu \left[\left(g_L^{f,SM} + \delta g_L^{ff} \right) P_L + \left(g_R^{f,SM} + \delta g_R^{ff} \right) P_R \right] f, \tag{5.1.20}$$

where $P_{L,R}$ are the left- and right-handed projectors, $g_L^{f,SM} = T_f^3 - Q_f s^2$ and $g_R^{f,SM} = -Q_f s^2$ are the Standard Model couplings, and

$$\delta g_L^{ff} = \frac{\left(a_1 a_4 + s^2 a_2 - 2s\sqrt{a_1 a_2 a_4}\right)}{(-1+a_4)} s^2 \left[-\frac{1}{2(-1+a_4)} \left(T_f^3 - Q_f s^2 \right) + \frac{c^4}{(c^2 - s^2)} \frac{Q_f}{a_4} \right] , \tag{5.1.21}$$

$$\delta g_R^{ff} = \frac{\left(a_1 a_4 + s^2 a_2 - 2s\sqrt{a_1 a_2 a_4}\right)}{(-1+a_4)} Q_f s^2 \left[\frac{s^2}{2(-1+a_4)} + \frac{c^4}{(c^2 - s^2)} \frac{1}{a_4} \right] , \tag{5.1.22}$$

are the corrections to both $g_L^{f,SM}$ and $g_R^{f,SM}$, respectively. On the other hand, the observable photon Lagrangian takes the same form as in the Standard Model:

$$\mathscr{L}^A_{int} = \sqrt{4\pi\, \alpha_e(M_Z)} A_\mu \left(Q_f \bar{f} \gamma^\mu f \right) . \tag{5.1.23}$$

At last, the Z' Lagrangian can be rewritten as

$$\mathscr{L}^{Z'}_{int} = \frac{\sqrt{4\pi\, \alpha_e(M_Z)}}{c} Z'_\mu \bar{f} \gamma^\mu \left(\delta\tilde{g}_L^{ff} P_L + \delta\tilde{g}_R^{ff} P_R \right) f, \tag{5.1.24}$$

where its left- and right-handed couplings are given by

$$\begin{aligned} \delta\tilde{g}_L^{ff} = & T_f^3 \left[\frac{s}{(-1+a_4)} \left(\frac{\sqrt{a_1}}{s} - \sqrt{a_2 a_4} \right) + \sqrt{a_1} \right] + \\ & + Q_f \left\{ s \left[c^2 \sqrt{\frac{a_2}{a_4}} - \frac{s^2}{(-1+a_4)} \left(\frac{\sqrt{a_1}}{s} - \sqrt{a_2 a_4} \right) \right] - \sqrt{a_1} \right\} , \end{aligned} \tag{5.1.25}$$

$$\delta\tilde{g}_R^{ff} = Q_f \left\{ s \left[c^2 \sqrt{\frac{a_2}{a_4}} - \frac{s^2}{(-1+a_4)} \left(\frac{\sqrt{a_1}}{s} - \sqrt{a_2 a_4} \right) \right] - \sqrt{a_1} \right\} . \tag{5.1.26}$$

Thus, Eqs. (5.1.20), (5.1.23), and (5.1.24) can now be used to predict the implications of our Z' boson to any desired observable.

5.2 Calculation of Observables

In this section our goal is to compute the expressions for sixteen observables in terms of the parameters of our model. Just as we did before, we limit ourselves to work only to linear order in the corrections given by the presence of a Z' boson.

We start with seven observables measured in e^+e^- collisions at the Z resonance which are sensitive only to changes to the Standard Model couplings. These include the total Z width Γ_Z, and all partial widths Γ_f for $Z \to \bar{f}\, f$, where f can be the charged leptons ($l = e,\, \mu,\, \tau$), the neutrinos, or the quarks lighter than the Z boson ($q = u,\, c,\, d,\, s,\, b$). It is convenient to use the variables Γ_Z,

$$R_l \equiv \frac{\Gamma_{had}}{\Gamma_l}, \quad R_q \equiv \frac{\Gamma_q}{\Gamma_{had}}, \quad \sigma_{had} \equiv \frac{12\pi\, \Gamma_e\, \Gamma_{had}}{M_Z^2\, \Gamma_Z^2}, \tag{5.2.1}$$

where $\Gamma_{had} = \Gamma_u + \Gamma_c + \Gamma_d + \Gamma_s + \Gamma_b$ is the partial width into hadrons [8]. There are three measured values for R_l, but only two for R_q ($q = c,\, b$).

At tree-level and neglecting the fermion masses, the Z boson partial widths are given by

$$\Gamma_f = \Gamma_f^{SM} \left[1 + \frac{2 g_L^{f,SM}}{\left(g_L^{f,SM}\right)^2 + \left(g_R^{f,SM}\right)^2}\, \delta g_L^{ff} + \frac{2 g_R^{f,SM}}{\left(g_L^{f,SM}\right)^2 + \left(g_R^{f,SM}\right)^2}\, \delta g_R^{ff} \right], \tag{5.2.2}$$

in which

$$\Gamma_f^{SM} = \frac{\alpha_e(M_Z)\, M_Z}{6\, s^2\, c^2} \left[\left(g_L^{f,SM}\right)^2 + \left(g_R^{f,SM}\right)^2 \right] \tag{5.2.3}$$

are the predictions from the Standard Model. Since these expressions will only depend on s^2, we can use that $s^2 = 0.23$ [9] to find the following partial widths

$$\Gamma_l = \Gamma_l^{SM} \left[1 - (0.23 a_2 + a_1 a_4 - 0.96\sqrt{a_1 a_2 a_4})\, \frac{(0.16 + 0.07 a_4)}{(-1 + a_4)^2 a_4} \right], \tag{5.2.4}$$

$$\Gamma_\nu = \Gamma_\nu^{SM} \left[1 - (0.23 a_2 + a_1 a_4 - 0.96\sqrt{a_1 a_2 a_4})\, \frac{0.23}{(-1 + a_4)^2} \right], \tag{5.2.5}$$

$$\Gamma_{u,c} = \Gamma_{u,c}^{SM} \left[1 - (0.23 a_2 + a_1 a_4 - 0.96\sqrt{a_1 a_2 a_4})\, \frac{(0.45 - 0.22 a_4)}{(-1 + a_4)^2 a_4} \right], \tag{5.2.6}$$

$$\Gamma_{d,s,b} = \Gamma_{d,s,b}^{SM} \left[1 - (0.23 a_2 + a_1 a_4 - 0.96\sqrt{a_1 a_2 a_4})\, \frac{(0.32 - 0.09 a_4)}{(-1 + a_4)^2 a_4} \right], \tag{5.2.7}$$

$$\Gamma_{had} = \Gamma_{had}^{SM} \left[1 - 0.13\, (0.23 a_2 + a_1 a_4 - 0.96\sqrt{a_1 a_2 a_4})\, \frac{(2.7 - a_4)}{(-1 + a_4)^2 a_4} \right]. \tag{5.2.8}$$

Therefore, the predictions for the shifts in the electroweak precision observables Γ_Z, R_l, R_q, and σ_{had} are given by

$$\Gamma_Z = \Gamma_Z^{SM} \left[1 - 0.04\, (0.23 a_2 + a_1 a_4 - 0.96\sqrt{a_1 a_2 a_4})\, \frac{(4.2 + a_4)}{(-1 + a_4)^2 a_4} \right], \tag{5.2.9}$$

$$R_l = R_l^{SM}\left[1 + 0.20\frac{(0.23a_2 + a_1a_4 - 0.96\sqrt{a_1a_2a_4})}{(-1+a_4)a_4}\right], \tag{5.2.10}$$

$$R_b = R_b^{SM}\left[1 - 0.05\frac{(0.23a_2 + a_1a_4 - 0.96\sqrt{a_1a_2a_4})}{(-1+a_4)a_4}\right], \tag{5.2.11}$$

$$R_c = R_c^{SM}\left[1 + 0.09\frac{(0.23a_2 + a_1a_4 - 0.96\sqrt{a_1a_2a_4})}{(-1+a_4)a_4}\right], \tag{5.2.12}$$

$$\sigma_{had} = \sigma_{had}^{SM}\left[1 + 0.15\frac{(0.23a_2 + a_1a_4 - 0.96\sqrt{a_1a_2a_4})}{(-1+a_4)a_4}\right]. \tag{5.2.13}$$

Other very important constraints follow from the measurements of eight Z pole asymmetries. Since they are defined through the cross section of $e^+e^- \to \bar{f}\,f$, besides the contributions sensitive only to changes to the Z boson couplings, we must also include those coming from Z' exchange. After removing QED corrections and contributions from photon exchange, $\gamma - Z$ interference, and electroweak boxes [10], the left-right asymmetries A_{LR}^f are given by

$$\begin{aligned} A_{LR}^f = (A_{LR}^f)_{SM} + \\ + 4\frac{g_L^{f,SM}\, g_R^{f,SM}}{\left[\left(g_L^{f,SM}\right)^2 + \left(g_R^{f,SM}\right)^2\right]^2}\left(g_R^{f,SM}\,\delta g_L^{ff} + g_L^{f,SM}\,\delta g_R^{ff}\right) + \left[\frac{\left(\delta\tilde{g}_L^{ff}\right)^2 - \left(\delta\tilde{g}_R^{ff}\right)^2}{\left(\delta\tilde{g}_L^{ff}\right)^2 + \left(\delta\tilde{g}_R^{ff}\right)^2}\right], \end{aligned} \tag{5.2.14}$$

in which

$$(A_{LR}^f)_{SM} = \frac{\left(g_L^{f,SM}\right)^2 - \left(g_R^{f,SM}\right)^2}{\left(g_L^{f,SM}\right)^2 + \left(g_R^{f,SM}\right)^2} \tag{5.2.15}$$

are the Standard Model predictions, just as the polarization asymmetry of Eq. (3.0.4). Substituting $s^2 = 0.23$, we obtain

$$\begin{aligned} A_{LR}^l =& (A_{LR}^l)_{SM} + 2.0\frac{(0.23a_2 + a_1a_4 - 0.96\sqrt{a_1a_2a_4})}{(-1+a_4)a_4} - \\ &- \frac{a_4(-1.0+a_4)(0.14a_2 + 0.60a_2a_4 - 0.58\sqrt{a_1a_2a_4})}{D_1}, \end{aligned} \tag{5.2.16}$$

$$\begin{aligned} A_{LR}^b =& (A_{LR}^b)_{SM} + 0.16\frac{(0.23a_2 + a_1a_4 - 0.96\sqrt{a_1a_2a_4})}{(-1+a_4)a_4} - \\ &- \frac{a_4(-3.1+a_4)(0.14a_2 + 0.60a_2a_4 - 0.58\sqrt{a_1a_2a_4})}{D_2}, \end{aligned} \tag{5.2.17}$$

$$\begin{aligned} A_{LR}^c =& (A_{LR}^c)_{SM} + 0.87\frac{(0.23a_2 + a_1a_4 - 0.96\sqrt{a_1a_2a_4})}{(-1+a_4)a_4} - \\ &- \frac{a_4(-1.2+a_4)(0.20a_2 + 0.88a_2a_4 - 0.85\sqrt{a_1a_2a_4})}{D_3}, \end{aligned} \tag{5.2.18}$$

where, for simplicity, we have defined

$$\begin{aligned} D_1 \equiv & a_1 a_4 \left[0.95 + (-1.8 + a_4) a_4\right] + a_2 \left[0.22 + (-0.43 + 0.23 a_4) a_4\right] + \\ & + \sqrt{a_1 a_2 a_4} \left[-0.91 + (1.8 - 0.96 a_4) a_4\right], \end{aligned} \tag{5.2.19}$$

$$\begin{aligned} D_2 \equiv & a_1 a_4 \left[0.95 + (-0.62 + a_4) a_4\right] + a_2 \left[0.22 + (-0.14 + 0.23 a_4) a_4\right] + \\ & + \sqrt{a_1 a_2 a_4} \left[-0.91 + (0.59 - 0.96 a_4) a_4\right], \end{aligned} \tag{5.2.20}$$

$$\begin{aligned} D_3 \equiv & a_1 a_4 \left[1.1 + (-1.8 + a_4) a_4\right] + a_2 \left[0.26 + (-0.42 + 0.23 a_4) a_4\right] + \\ & + \sqrt{a_1 a_2 a_4} \left[-1.1 + (1.7 - 0.96 a_4) a_4\right]. \end{aligned} \tag{5.2.21}$$

The forward-backward asymmetries A^f_{FB} are defined as [8]

$$A^f_{FB} \equiv \frac{\sigma^f_F - \sigma^f_B}{\sigma^f_F + \sigma^f_B}, \tag{5.2.22}$$

in which $\sigma^f_{F,B}$ is the total cross section for a forward or backward scattering of the fermion f with respect to the direction of the incident electron. Since LEP performed the measurements for an initial e^- polarization equals to zero [10], we can use the effective tree-level expressions

$$A^l_{FB} = \frac{3}{4} A^e_{LR} A^l_{LR}, \quad A^q_{FB} = \frac{3}{4}\left(1 - k_A \frac{\alpha_s}{\pi}\right) A^e_{LR} A^q_{LR}, \tag{5.2.23}$$

where the factor $\left(1 - k_A \frac{\alpha_s}{\pi}\right)$ comes from QCD radiative corrections – we can set the numerical value 0.93 [9]. Using the above relations for the left-right asymmetries, we find that

$$\begin{aligned} A^l_{FB} =& (A^l_{FB})_{SM} + 0.48 \frac{(0.23 a_2 + a_1 a_4 - 0.96\sqrt{a_1 a_2 a_4})}{(-1 + a_4) a_4} - \\ & - \frac{a_4(-1.0 + a_4)(0.03 a_2 + 0.14 a_2 a_4 - 0.14\sqrt{a_1 a_2 a_4})}{D_1}, \end{aligned} \tag{5.2.24}$$

$$\begin{aligned} A^b_{FB} =& (A^b_{FB})_{SM} + 1.3 \frac{(0.23 a_2 + a_1 a_4 - 0.96\sqrt{a_1 a_2 a_4})}{(-1 + a_4) a_4} - \\ & - 0.70 \frac{a_4(-1.0 + a_4)(0.13 a_2 + 0.56 a_2 a_4 - 0.54\sqrt{a_1 a_2 a_4})}{D_1} - \\ & - 0.70 \frac{a_4(-3.1 + a_4)(0.04 a_2 + 0.15 a_2 a_4 - 0.15\sqrt{a_1 a_2 a_4})}{D_2}, \end{aligned} \tag{5.2.25}$$

$$\begin{aligned} A^c_{FB} =& (A^c_{FB})_{SM} + 1.0 \frac{(0.23 a_2 + a_1 a_4 - 0.96\sqrt{a_1 a_2 a_4})}{(-1 + a_4) a_4} - \\ & - 0.70 \frac{a_4(-1.0 + a_4)(0.09 a_2 + 0.40 a_2 a_4 - 0.39\sqrt{a_1 a_2 a_4})}{D_1} - \\ & - 0.70 \frac{a_4(-1.2 + a_4)(0.04 a_2 + 0.18 a_2 a_4 - 0.18\sqrt{a_1 a_2 a_4})}{D_3}. \end{aligned} \tag{5.2.26}$$

The remaining asymmetries are the polarization asymmetry $A_\tau(P_\tau)$, which is defined through the cross section for the reaction $e^+e^- \to \tau^+\tau^-$, and the joint forward-backward/left-right asymmetry $A_e(P_\tau)$, which is defined through the cross section for a left- or right-handed incident electron to produce a τ traveling in the forward or backward hemisphere. Due to lepton universality, $A_\tau(P_\tau)$ and $A_e(P_\tau)$ are given by the same expression as A^l_{LR}.

Therefore, the asymmetries we need to include in our analysis are A^l_{FB}, $A_\tau(P_\tau)$, $A_e(P_\tau)$, A^b_{FB}, A^c_{FB}, and A^e_{LR}.

The last observable is the only one that comes from charged-current data. From Eqs. (2.1.8), (5.1.15), and (5.1.19), we know that the observable W mass must be given by

$$M_W = M_W^{SM} \times \times \left\{ 1 - \frac{s^2c^2(a_1 + s^2a_2 - 2s\sqrt{a_1a_2a_4})}{2(c^2-s^2)(-1+a_4)} - \frac{s^2}{2(c^2-s^2)}\left(s^2c^2\frac{a_2}{a_4} - 2sc^2\sqrt{\frac{a_1a_2}{a_4}} \right) \right\}. \tag{5.2.27}$$

Using the numerical value for s^2, this expression becomes

$$M_W = M_W^{SM}\left\{ 1 + 0.16\frac{\left[-a_1a_4 + a_2(0.23 - 0.46a_4) + \sqrt{a_1a_2a_4}(-0.96 + 1.9a_4)\right]}{a_4(-1+a_4)} \right\}. \tag{5.2.28}$$

It is important to highlight that, since the variables which parameterize our model are small, we can work to any loop order in the Standard Model predictions we have used above.

5.3 Constraints on the Z' Mass

We already know that electroweak precision measurements can be used to search for and set limits on deviations from the Standard Model. At this point, we are not interested in the agreement between the data and the model as a whole. Instead, we aim to determine which realisations of our model, specified by distinct sets of (a_1, a_2, a_4), are in best agreement with the given data.

Our analysis involves a set of sixteen measurements $(x_{exp})_{i=1,\dots,16}$, given in Table (5.1), described by a corresponding set of theoretical expressions $(x_{theo})_{i=1,\dots,16}$, which we derived in the previous section. The theoretical expressions, as we have seen, are functions of the Standard Model predictions, also given in Table (5.1), and the parameters of our model (a_1, a_2, a_4). Assuming that the measurements are normally distributed, we can define the chi-square as [33]

$$\chi^2(a_1,a_2,a_4) = \left[x_{exp} - x_{theo}(a_1,a_2,a_4)\right]_i \left(cov^{-1}\right)_{ij} \left[x_{exp} - x_{theo}(a_1,a_2,a_4)\right]_j . \tag{5.3.1}$$

The covariance matrix is given in terms of the correlation among the observables by $(cov)_{ij} = \sigma_i\,(cor)_{ij}\,\sigma_j$, where σ is the standard deviation of each measurement. There are two correlation matrices we need to take into account [34]: an 8×8 matrix relating the Z-lineshape observables Γ_Z, σ_{had}, R_l and A^l_{FB}, and a 4×4 matrix relating the heavy-flavor observables R_b, R_c, A^b_{FB} and A^c_{FB}. However, comparing the leptonic quantities R_e, R_μ and R_τ, and A^e_{FB}, A^μ_{FB} and A^τ_{FB}, we can see they agree within the experimental errors – Table (5.1). Thus, it is possible to impose the additional requirement of lepton universality and proceed our analysis using the combined results $R_l = 20.767 \pm 0.025$ and $A^l_{FB} = 0.0171 \pm 0.0010$, with Standard Model predictions given by $R_l^{SM} = 20.739$ and $A^l_{FB} = 0.01642$ [34]. In this scenario, we must use two 4×4 correlation matrices, which are given in Table (5.2).

Quantity	Experiment	Standard Model
Γ_Z	2.4952 ± 0.0023	2.4942
σ_{had}	41.540 ± 0.037	41.481
R_e	20.804 ± 0.050	20.737
R_μ	20.785 ± 0.033	20.737
R_τ	20.764 ± 0.045	20.782
R_b	0.21629 ± 0.00066	0.21582
R_c	0.1721 ± 0.0030	0.17221
A^e_{FB}	0.0145 ± 0.0025	0.01618
A^μ_{FB}	0.0169 ± 0.0013	0.01618
A^τ_{FB}	0.0188 ± 0.0017	0.01618
A^b_{FB}	0.0992 ± 0.0016	0.01030
A^c_{FB}	0.0707 ± 0.0035	0.0735
$A_\tau(P_\tau)$	0.1439 ± 0.0043	0.1469
$A_e(P_\tau)$	0.1498 ± 0.0049	0.1469
A^e_{LR}	0.15138 ± 0.00216	0.1469
M_W	80.370 ± 0.019	80.358

Table 5.1: Principal electroweak observables [34] and their Standard Model predictions [10]. We are using experimental values for the Z pole observables measured by LEP and SLC. The value for the W mass, which is the only observable measured from charged-current data, is the one obtained by the ATLAS collaboration.

	Γ_Z	σ_{had}	R_l	A^l_{FB}
Γ_Z	1.000			
σ_{had}	-0.297	1.000		
R_l	0.004	0.183	1.000	
A^l_{FB}	0.003	0.006	-0.056	1.000

	R_b	R_c	A^b_{FB}	A^c_{FB}
R_b	1.00			
R_c	-0.18	1.00		
A^b_{FB}	-0.10	0.04	1.00	
A^c_{FB}	0.07	-0.06	0.15	1.00

Table 5.2: Correlation matrices among the Z-lineshape and the heavy-flavor observables [34].

The χ^2 can be used to determine the quality of the agreement between the data and the various realisations of our model. However, if one wants to estimate confidence intervals for the complete set of parameters, it is easier to use the offset-corrected test statistics [35]

$$\Delta\chi^2(a_1, a_2, a_4) = \chi^2(a_1, a_2, a_4) - \chi^2_{min}. \tag{5.3.2}$$

χ^2_{min} denotes the absolute minimum value of the χ^2 function, obtained when letting (a_1, a_2, a_4) free to vary within their respective bounds. We assumed the same bound for all three parameters: $0 < a_1, a_2, a_4 < 10^{-3}$. The lower bound was chosen based on Section 4.4, in which they are all defined to be positive. On the other hand, we already knew both a_1 and a_2 should be very small since they are related to the kinetic and mass mixings, respectively. Therefore, we chose as an upper bound 10^{-3}. As for a_4, we have mentioned before that we are interested in a region where the Z' boson is lighter than the Z, which corresponds to $a_4 < 1$. However, if we let a_4 run freely to values closer to 1, the corrections to the observables become very large. This is a consequence of using non-degenerate perturbation theory, which makes $a_4 = 1$ a singular point in our model. To avoid extending our formulas to the degenerate and almost-degenerate cases, we chose the upper bound of a_4 to also be 10^{-3} since it is a value sufficiently distant from 1 that guarantees the validity of the non-degenerate perturbation theory, and it is also compatible with our proposal of a Z' within the MeV-GeV mass range. That said, we find $\chi^2_{min} = 15.841$ for $a_1 = 4.5085 \times 10^{-4}$, $a_2 = 1.3851 \times 10^{-8}$, and $a_4 = 7.0037 \times 10^{-6}$. This result is slightly better

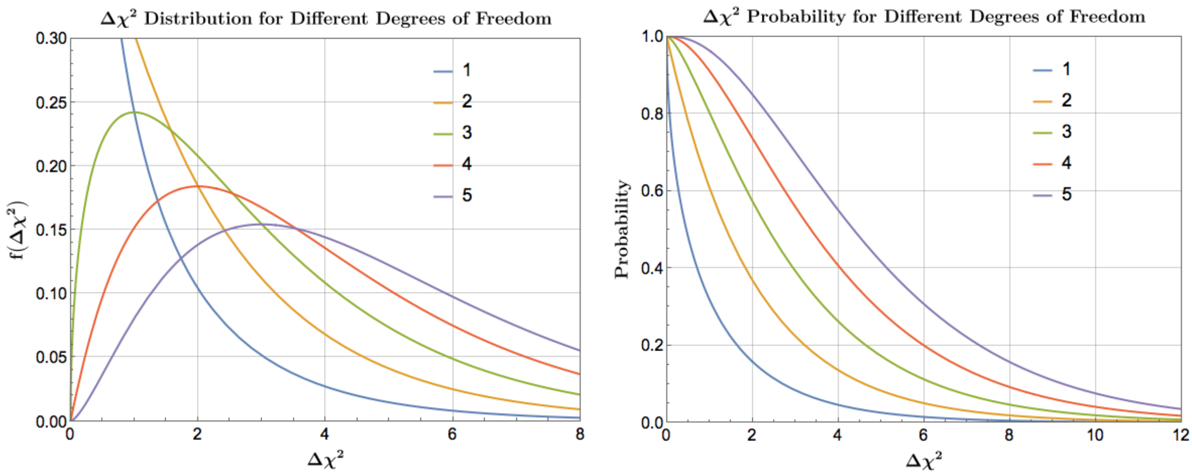


Figure 5.3: $\Delta\chi^2$ distribution and probability for a varying number of degrees of freedom.

than the fit to the Standard Model, which yields $(\chi^2_{min})_{SM} = 16.234$.

For a Gaussian problem like ours, the test statistics follows a χ^2 distribution, which is given by

$$f(\Delta\chi^2) = \frac{1}{2^{\frac{n}{2}}\Gamma\left(\frac{n}{2}\right)} e^{-\frac{\Delta\chi^2}{2}} (\Delta\chi^2)^{\frac{n}{2}-1}, \tag{5.3.3}$$

where n is the number of degrees of freedom. This means that the quantity $f(\Delta\chi^2)d(\Delta\chi^2)$ gives the probability that a particular value of $\Delta\chi^2$ falls between $\Delta\chi^2$ and $\Delta\chi^2 + d(\Delta\chi^2)$. Therefore, the exclusion confidence level (CL) can be defined as [35]

$$1 - CL(a_1, a_2, a_4) = Prob\left(\Delta\chi^2, n\right) = \int_{\Delta\chi^2_{n,1-CL}}^{\infty} f(\Delta\chi^2)d(\Delta\chi^2). \tag{5.3.4}$$

If for a probability $p = 1 - CL$ we find that $\Delta\chi^2(\hat{a}_1, \hat{a}_2, \hat{a}_4) > \Delta\chi^2_{n,p}$, then our model predicts the observation at a probability of less then $(100 \cdot p)\%$. In other words, we are $100(1-p)\%$ confident in rejecting our model at the set $(\hat{a}_1,\ \hat{a}_2,\ \hat{a}_4)$.

Fig. (5.3) shows both $\Delta\chi^2$ distribution and probability for different numbers of degrees of freedom. Since our model has three parameters, we must follow the curve for $n = 3$. Usually, physicists are interested in CLs of 68%, 95%, and 99%. Thus, from Fig. (5.3), we are able to determine that these CLs correspond to a $\Delta\chi^2$ of about 3.51, 7.81, and 11.3, respectively.

In order to better understand our results, we made the substitutions

$$a_1 \to \frac{\sin^2(\beta)}{\sqrt{1-\sin^2(\beta)}}, \quad a_2 \to \frac{M^2_{Z'}}{M^2_Z}\frac{(g^2+g'^2)Y_S^2}{(\chi_S\, g_X)^2}, \quad a_4 \to \frac{M^2_{Z'}}{M^2_Z}. \tag{5.3.5}$$

From Eq. (4.4.17), we know that the parameter a_4 is the leading contribution to one of the mass eigenvalues. And since we have already defined that a_1, a_2 and a_4 are all small, we can certainly approximate a_4 by the physical Z' mass. Thus, for fixed values of β and $\chi_S\, g_X$, we have derived constraints in the two-dimensional model parameter space $(M_{Z'},\ Y_S)$. In this analysis, we have used the experimental values and the Standard Model predictions presented before, as well as the theoretical expressions we have previously computed.

Fig. (5.4) displays the resulting 68% (red), 95% (blue) and 99% (green) CL excluded regions for $\beta = 0.01$ and $\chi_S\, g_X = 10^{-3}, 10^{-2}, 10^{-1}, 1$. Since we do not want our theory to be strongly

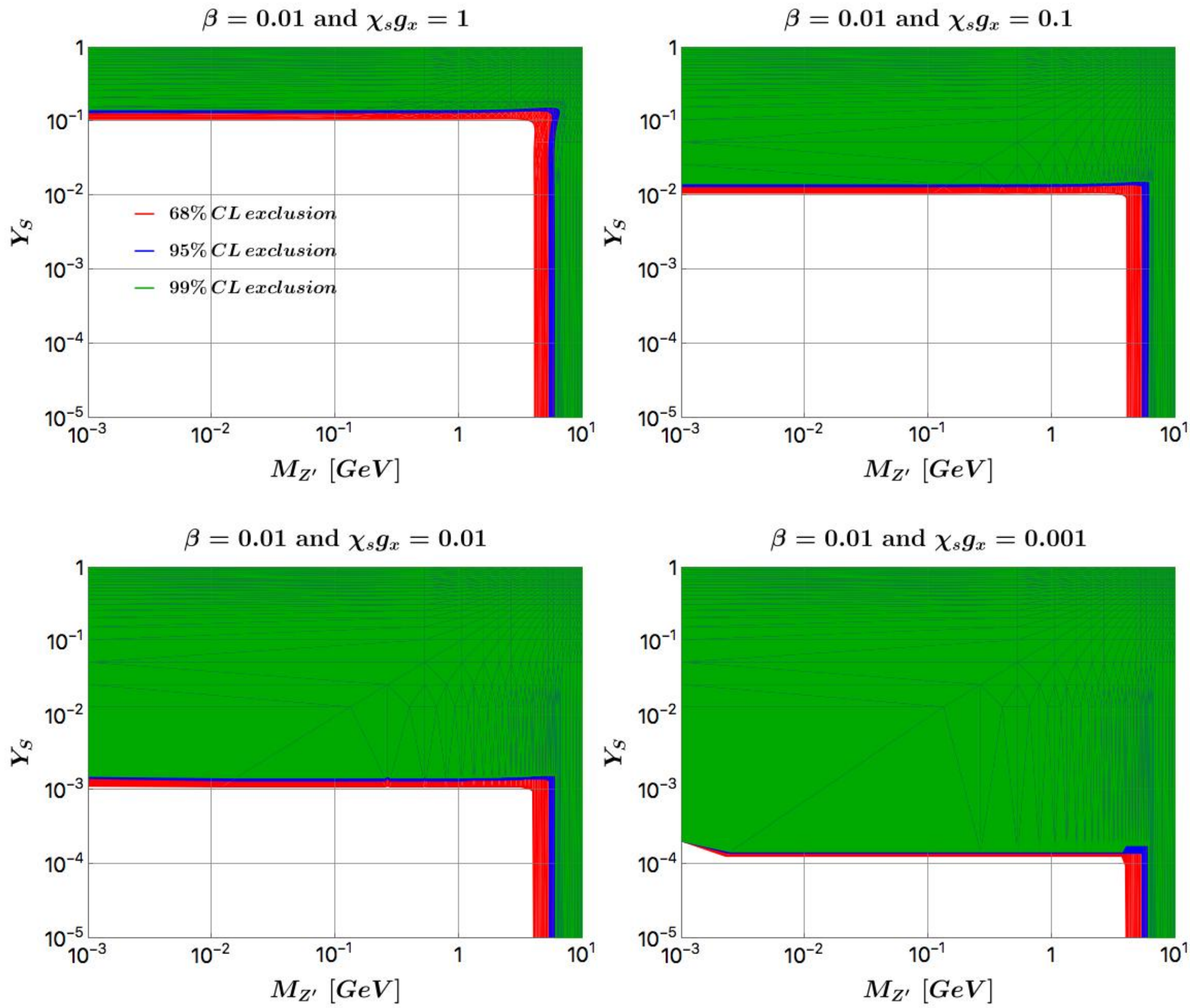


Figure 5.4: Parameter space for fixed values of β and $\chi_S\, g_X$ with 68% (red), 95% (blue) and 99% (green) CL exclusion regions. This analysis takes into account all eleven Z-pole observables, and the mass of the W boson.

coupled, we have set $\chi_S\, g_X = 1$ as an upper limit. One can see that, for increasing values of $\chi_S\, g_X$, the allowed region of Y_S is larger. However, as expected, the singlet hypercharge must always be less than 1. Furthermore, a Z' mass greater than a few GeVs is not in agreement with the experimental data.

It is important to highlight that the results showed in Fig. (5.4) are the same for negative values of the singlet hypercharge. This is a consequence of the fact that a_2 depends only on Y_S^2.

Chapter 6

Muon Anomalous Magnetic Moment

Over the years, the tension between the theoretical and experimental values of the muon magnetic moment, $a_\mu \equiv (g-2)_\mu/2$, has given rise to many theoretical speculations on new physics solutions to this discrepancy. Physicists believe this disagreement may arise from effects of undiscovered particles, such as the vector bosons predicted in extensions of the Standard Model by an extra $U(1)$ gauge group [20, 36].

The final result of the experimental value, $a_\mu^{exp} = 116\,592\,089(63)\times 10^{-11}$, was first published in 2004 by the Brookhaven National Laboratory (BNL) [37]. At that time, the Standard Model theoretical value a_μ^{SM} was calculated employing two different methods, which led to a discrapancy of 2.7σ and 1.4σ for each one of them [38]. However, physicists knew such difference could be a result of uncertainties coming from hadronic vacuum polarization. The enthusiasm about possible new physics only appeared later on, when a comprehensive experimental and theoretical effort led to an improvement of the Standard Model prediction. The difference now was around 3-4σ [38], and gained the attention of the particle physics community.

Theoretical physicists are still trying to develop new methods to improve the precision of the Standard Model evaluation since the uncertainty of a_μ^{SM} remains dominated by strong interaction contributions. To this end, the *Muon g-2 Theory Initiative* was created to better evaluate all aspects of the Standard Model and determine a single value against which new experimental results should be compared (the Fermilab *Muon g-2* collaboration is currently taking and analysing data). The first published value of this initiative was $a_\mu^{SM} = 116\,591\,810(43)\times 10^{-11}$ [38]. It differs from the Brookhaven measurement by

$$\Delta a_\mu \equiv a_\mu^{exp} - a_\mu^{SM} = 279(76)\times 10^{-11}. \tag{6.0.1}$$

As we have already discussed in Section 5.1, the presence of our Z' gives two distinct contributions to the magnetic moment of leptons:

$$\begin{aligned}(g-2)_l =& \frac{\hat{\alpha}_e}{\pi}(1+2\eta) + \frac{2\hat{\alpha}_e}{\pi}\delta^2 \int_0^1 dx \frac{x(1-x)^2}{(1-x)^2 + x\left(\frac{M_{Z'}}{m_l}\right)^2} + \\ &+ \frac{2\hat{\alpha}_e}{\pi}\sigma^2 \left\{ \left(\frac{m_l}{M_{Z'}}\right)^2 \int_0^1 dx \frac{(1-x)^3(1+x)}{(1-x)^2 + x\left(\frac{M_{Z'}}{m_l}\right)^2} + 2\log\left[\left(\frac{m_l}{M_{Z'}}\right)^2 + 1\right] \right\},\end{aligned} \tag{6.0.2}$$

where the couplings η, δ and σ are given by Eqs. (5.1.2), (5.1.6) and (5.1.7), respectively. Thus, using $m_\mu = 105.658\,371\,5\,MeV$ [6], we can try to find a region in the parameter space of our theory where Δa_μ is explained by these extra contributions to $g-2$.

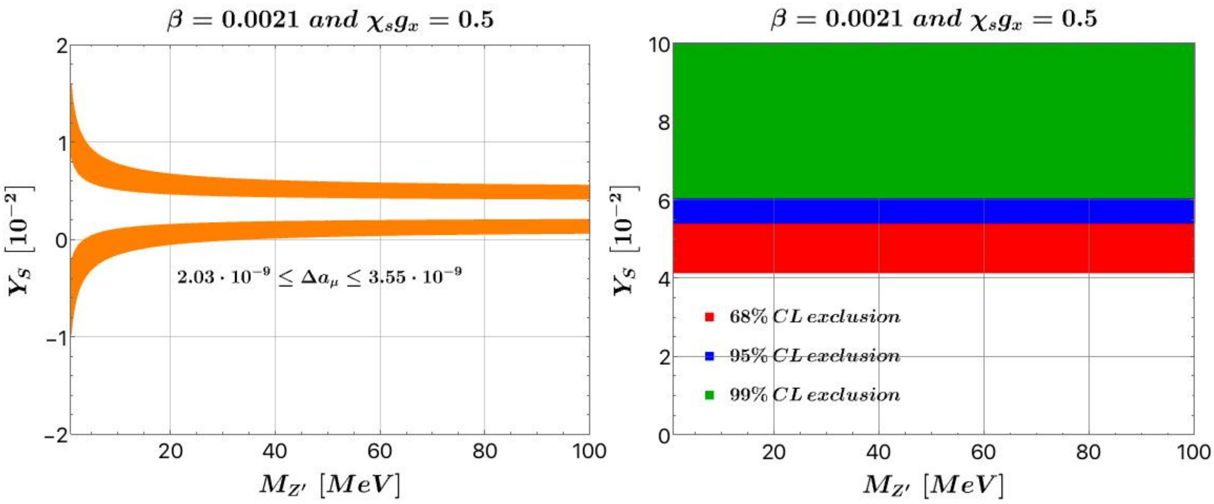


Figure 6.1: Both figures show the parameter space for fixed values of β and $\chi_S\, g_X$. However, the figure on the left shows a region in which our model can account for the discrepancy Δa_μ (orange), while the figure on the right shows the 68% (red), 95% (blue) and 99% (green) CL exclusion regions.

Fig. (6.1) shows the parameter space for $\beta = 0.0021$ and $\chi_S\, g_X = 0.5$. In both orange regions, our model is capable of explaining the muon anomaly within one standard deviation. In fact, there are two allowed regions since all expressions depend on Y_S^2 and, therefore, we will always find two possible roots. The CL exclusion regions lie way above (or below) the regions in which the presence of a light Z' boson can account for such discrepancy.

Chapter 7

Conclusions

We extended the Standard Model of particle physics by a new Abelian gauge group $U(1)_X$, and allowed the associated gauge boson X to mix with the Standard Model photon and Z boson through both kinetic and mass terms. Besides the new field X, in order to spontaneously break $U(1)_X \times U(1)_Y$ and give mass to the gauge bosons, we had to add a second field to our model. To guarantee the breaking of both $U(1)$ groups, this new field needed to be a complex singlet. Since the most general Lagrangian must contain an invariant term that couples the Higgs doublet to this new singlet, our model can have important consequences on the Higgs phenomenology. However, this was not the main puorpose of the thesis.

As a consequence of the kinetic and mass mixings, the photon and Z boson we observe are now a mixture of the Standard Model fields and the X boson field. The same is true for the third observable eigenstate, which we have called Z'. If we do not want to be taken too far away from the Standard Model, the only possible small parameters in our model must be the kinetic mixing β and the singlet hypercharge Y_S. This allowed us to use perturbation theory to determine expressions for the observed fields and their masses. Although our photon is different from the Standard Model one, all the important features of a true photon remained: it is massless at all orders in perturbation theory, it does not couple to axial currents, and it keeps both scalars of our theory electrically neutral. Additionally, a non-vanishing J_X would add new Z and photon interactions that could be potentially relevant for their phenomenology. However, this was also not one of the main purposes of the thesis.

At this point, we already know that the presence of an additional $U(1)_X$ modifies the Z boson mass and its couplings to the Standard Model neutral currents, and it also modifies the coupling of the photon to the electromagnetic current. Furthermore, the new Z' boson couples to both Standard Model neutral currents, having a greater impact on electroweak physics. Thus, we have shown how the relation between the theoretical Standard Model parameters and the reference measured quantities changes in the presence of this Z'. For the fine-structure constant, we assumed that the photon contribution was the dominant one. As we have already discussed, this restricted ourselves to a Z' within the MeV-GeV mass range. If we wanted to make the Z' boson even lighter, we would have to take into account the full expression for the elecron magnetic moment.

Our analysis involved a set of eleven Z pole observables, and the W boson mass. We found 68%, 95% and 99% CL excluded regions for fixed values of β and $\chi_S\, g_X$. We have concluded that a Z' within the whole MeV mass range is in agreement with the electroweak precision data. We are not aware that any other systematic analysis like ours has been done in the literature to date. However, there is always room for improvements. We can still add to our analysis the low energy observables, such as those measured in neutrino-electron and inelastic neutrino scatterings. We can also consider the full correction to the electron magnetic moment in order to extend the Z' mass to the eV-GeV range.

Finally, we tried (successfully) to explain the discrepancy between the experimental and theoretical values of the muon magnetic moment. We have shown that our model can account for such disagreement within one standard deviation when the Z' has a mass around the MeV.

As we said, although there is still room for improvements, our model has already proved to be very promising. It shows that many possibilities of extra vector bosons were left unexplored by the particle physics community in the last few years.

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