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Topology optimization of acoustic systems via the TOBS-GT method

Otimização topológica de sistemas acústicos utilizando o método TOBS-GT

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RESUMO

CASTRO, T. A. R. **Otimização topológica de sistemas acústicos utilizando o método TOBS-GT.** 2022. 66 p. Dissertação (Mestrado em Engenharia Mecânica). Escola Politécnica da Universidade de São Paulo, São Paulo, Brasil.

A Otimização Topológica (OT) é um método computacional que vem ganhando cada vez mais notoriedade tanto no meio acadêmico, quanto no industrial. Esse crescimento se deve, em parte, aos avanços tecnológicos e ao consequente aumento da capacidade computacional, que permitem o uso de OT em problemas reais cada vez mais complexos. A ideia do método é criar um layout estrutural (desconhecido a priori) com ótimo desempenho independente da experiência do projetista, oferecendo assim grande liberdade de projeto na obtenção de estruturas ótimas. Embora os procedimentos de OT tenham alcançado um nível satisfatório de maturidade, um problema científico ainda desafiador nessa área é como adequar o método para conciliar diferentes físicas. Nesse contexto, a Otimização Topológica Acústica (OTA) é identificada como um tema aberto à pesquisa – com poucos estudos publicados até o momento, quando comparado a outras físicas. Este projeto propõe e investiga o uso do método de Otimização Topológica de Estruturas Binárias com Recorte de Geometria (TOBS-GT do inglês *Topology Optimization of Binary Structures with Geometry Trimming*) para sistemas acústicos. O método TOBS-GT combina variáveis binárias, linearização sequencial, filtro de sensibilidade e programação linear inteira. O Método dos Elementos Finitos (MEF) é utilizado para resolver a análise acústica, as sensibilidades são computadas pelo método adjunto via diferenciação automática e a malha de MEF é separada da malha de otimização. As regiões sólidas são removidas do domínio de análise a cada iteração para maior precisão do modelo físico. A metodologia proposta foi utilizada para otimizar uma sala e um dispositivo atenuador sonoro. Resultados promissores foram obtidos, onde topologias finais reduziram significativamente o nível de pressão sonora no domínio objetivo. Estudos mostraram um tempo computacional relativamente curto e que, conforme esperado, o MEF é o gargalo do processo de otimização. Para todos os exemplos propostos, o critério de convergência foi atingido em poucas iterações, sugerindo que o método TOBS-GT possa oferecer maior vantagem neste quesito em comparação a outros métodos de OT.

Palavras-Chave: Otimização Topológica. Acústica. Variáveis binárias. Método TOBS. Recorte de geometria.

ABSTRACT

CASTRO, T. A. R **Topology optimization of acoustic systems via the TOBS-GT method.** 2022. 66 p. Thesis (Masters). Escola Politécnica of the University of São Paulo, São Paulo, Brazil.

Topology optimization (TO) has been an active research area for over a century, and it is becoming more and more popular over the years in both academia and industry. One of the reasons of this growth is the rising computational power availability, that allows the usage of TO in increasingly complex real problems. The idea of the method is to find the entire structural layout (unknown a priori) with optimal performance regardless of the designer's experience, thus offering a great design freedom in obtaining optimal structures. Although the TO procedures have reached a satisfactory level of maturity, one challenging scientific problem is how to set its framework to account for different physics. In this context, acoustics is identified as a topic open to research with few studies published up to date, when comparing to other physics. This project proposes and investigates the use of Topology Optimization of Binary Structures with Geometry Trimming (TOBS-GT) for solving acoustic problems. The TOBS-GT method combines binary design variables, sequential problem linearization, sensitivity filtering and an integer programming solver. The forward problem is solved via finite element analysis (FEA) and the sensitivities are computed with the adjoint method via automatic differentiation. The FEA mesh is separated from the optimization mesh, and the solid regions are removed from the analysis at each iteration to accurately model physical phenomena. The proposed methodology was applied to room acoustics and to an acoustic attenuator. Promising results were obtained, where the final topologies significantly reduced the sound pressure level in the objective domain. Studies show a relatively small computational time and that, as expected, the FEA is the bottleneck of the optimization process. For all cases proposed, the convergence criteria was met in quite few iterations, showing that the TOBS-GT approach can offer an advantage in this regard when comparing with classical TO methods.

Keywords: Topology Optimization. Acoustics. Binary Design Variables. TOBS method. Geometry Trimming.

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LIST OF SYMBOLS AND ABBREVIATIONS

Abbreviations

ATO Acoustic Topology Optimization

BESO Bi-directional Evolutionary Structural Optimization

CAD Computer-aided design

CAE Computed Aided Engineering

ESO Evolutionary Structural Optimization

FEA Finite element Analysis

FEM Finite Element Method

ILP Integer Linear Programming

LP Linear Programming

LSM level-set method

MBB Messerschmidt-Bölkow-Blohm

PDE Partial Different Equation

RAMP Rational Approximation of Material Properties

SIMP Solid Isotropic Material with Penalization

SPL Sound pressure level

TO Topology Optimization

TOBS Topology Optimization of Binary Structures

TOBS-GT Topology Optimization of Binary Structures with Geometry Trimming

Greek letters

β Truncation error constraint parameter

δ Delta function

γ Penalty factor

μ Properties of a given isotropic material

Ω Domain

ω Angular frequency

Ω_a Air domain

Ω_d Design domain

Ω_s Solid domain

Ω_o Objective domain

$\Phi(\theta)$ Objective function

ρ Density

$\rho(x)$ Set of design variables as densities

$\rho(x_j)^\gamma$ Penalized density of the element j

ρ_{min} Lower bound density

τ Convergence criteria parameter

θ Set of design variables

θ_j Design variable for the element j

Symbols

$\mathbf{C}_{kj}$ Damping matrix

$\mathbf{f}_k$ Load vector

$\mathbf{K}_{kj}$ Stiffness matrix

$\mathbf{M}_{kj}$ Mass matrix

$\mathbf{r}$ Position

$\mathbf{S}_{kj}$ System matrix

$\mathbf{u}$	Nodal pressure vector
$\overline{g_i}$	Inequality constraint upper limit
C	Constant
c	Speed of sound
f	Frequency
$f_{n_x n_y}$	Eigenfrequency
g_i	Inequality constraint
H_{jm}	Filter weight factor
j	Finite element
K	Bulk modulus
k	Iteration number
$k_{n_x n_y}$	Eigenvalue
N_d	Number of design variables
N_g	Number of inequality constraints
N_h	Number of equality constraints
N_{jn}	Finite element shape function
p	Sound pressure
p_{ref}	Reference air pressure
p_{rms}	Root mean square pressure
Q_s	Volume velocity
r	Filter radius
t	Time
$V(\theta_j)$	Volume fraction of existing material
V_0	Design domain volume

V_f Prescribed final volume fraction

$\mathbf{n}$ Normal vector

CONTENTS

1 INTRODUCTION

Topology Optimization (TO) is a computational tool developed in the late eighties by Bendsøe and Kikuchi [1], which is used to provide an optimal geometry design for a given physical problem. The idea is to distribute material within a defined domain by fulfilling prescribed constraints in order to optimize an objective function. The final goal is to obtain a set of binary $\{0, 1\}$ pseudo-densities, also called design variables. Usually, for structural problems, 0 means the absence of material (void) and 1 represents the presence of solid material, whereas for acoustics the opposite is considered.

Despite the outstanding achievements of TO during the last couple decades, several problems are still open to research. Within these underexplored fields is acoustics, that only began to be investigated in the early 2000s. Sigmund and Jensen [2] present various results of applying topology optimization to structures and devices that are subjected to acoustic waves. Dühring et al. [3] developed a method for the optimization of both room acoustics and noise barriers, while Wadbro and Berggren [4] applied TO to an acoustic horn.

One challenge in TO is how to obtain the final structural layout represented by a binary design, which can be of great relevance for problems where explicit boundary description is important, as acoustics. One common approach is to relax the binary constraint by allowing intermediate design variables (between solid and acoustic fluid), as done so in the Solid Isotropic Material with Penalization (SIMP) method. In this case, some techniques were proposed to reduce or eliminate intermediate density (gray scale) elements in the final solutions, such as projection methods. Du and Olhoff [5] employed the SIMP model for minimizing the sound power radiated from a bi-material plate-like structure. Another approach consists of boundary-description based methods, e.g. the level-set method (LSM), in which an implicit function is used to describe the structure with clearly defined boundaries, with the disadvantage of the final solution being strongly affected by the initial design configuration. This method was recently used by Noguchi and Yamada [6] for two-layered acoustic metasurfaces, while Shu, Wang and Ma [7] employed the LSM to an acoustic-structure system.

A different way of approaching the problem is the use of discrete TO methods, with Bi-directional Evolutionary Structural Optimization (BESO), initially proposed by Xie and Steven [8], being the most established one. BESO consists in the update of the design variables as a fixed change in volume fraction every iteration, with the disadvantage of not being based on mathematical optimization. Some efforts were recently made to explore the field of acoustics with the BESO method. Azevedo et al. (2018) [9] apply the BESO method to acoustic muffler design and Pereira, Lopes and Pavanello (2022) [10] adapted the method to multiconstrained topology optimization of systems composed of rigid and porous materials. It is important to emphasize that it is imperative the existence of a volume constraint with the BESO method, and consequently it does not allow to solve unconstrained problems.

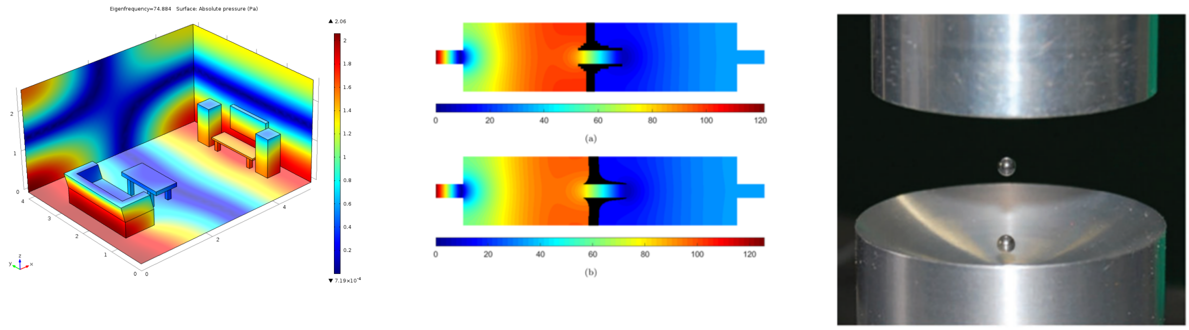
Within this context, Sivapuram and Picelli [11] proposed a binary method solved with formal mathematical programming, the so called Topology Optimization of Binary Structures (TOBS). This work investigates the use of TOBS with a geometry trimming technique (TOBS-GT) for acoustic problems, which is the first binary method to trim out the solid region of the acoustics topology optimization and to use only the acoustic domain in the analysis. Consequently, there is no vibration in the “rigid” solid. This method does not need continuation schemes as $\{0, 1\}$ solutions are always obtained due to the use of binary variables.

1.1 Justification

Acoustics has always been an important matter to humanity, but it has become even more relevant in modern life. Its presence is all over, from our daily lives – as when we listen to a speech in a room or when we are bothered by the sound of traffic –, to the most unthinkable applications – as in acoustic levitation purposes, where sound can actually levitate matter in air. Andrade, Marzo and Adamowski [12] present a review of technical challenges and future perspectives of acoustic levitation. Figure 1 exemplifies some acoustic problems, while Figure 2 presents the ”Wheel of Acoustics”, created by R. Bruce Lindsay in 1965 [13] and published in the Journal of the Acoustical Society of America. The Wheel categorizes the many sub-areas of acoustics, starting from four main disciplines of Earth Sciences, Engineering, Life Sciences and Arts, to more specific areas – that can even cover more than one of the main disciplines. The outer circle shows general fields, the inner circle shows typical subcategories, while the center of the wheel displays fundamental physical principles of acoustics. Acoustics is a broad, interdisciplinary field and therefore, there are many situations where it is desirable to improve some acoustic aspect.

Figure 1: Some examples of acoustic problems.

(a) Acoustic pressure distribution in a small room.
(b) Acoustic pressure distribution in an acoustic muffler.
(c) Acoustic levitation of steel spheres.



Source: (a) Jensen, 2015. [14] (b) Azevedo et. al., 2018. [9] (c) Andrade, Buiochi and Adamowski, 2010. [15]

Among different techniques, topology optimization represents a great tool for designing optimal structures. The objective of the TO can vary, but the most usual concerns compliance minimization – that is, making a structure as stiff as possible -, and most of the TO literature revolves around structural mechanics. As for Acoustic Topology Optimization (ATO) literature, it is very scarce when compared with other topics.

The final goal of TO is to obtain a set of binary $\{0, 1\}$ design variables, indicating the presence or absence of solid material. However, the trivial approach is to relax the design variables, which occasionally leads to grayscale regions, with intermediate properties between fluid and solid. One alternative to avoid this issue is the adoption of a binary TO method. While binary methods can be advantageous to acoustic problems, not many efforts were made yet to explore this subject, as the topic only began to be explored in the past few years.

The TOBS method is a relatively new binary method that was created in 2018 [16]. Since then, different problems were solved with the method, as fluid-structure interaction [17], microstructures [18], thermal expansion [11] and fluid pressure loads and buckling constraints [19]. Recently, the combination of the TOBS with a geometry trimming technique was suggested: the so-called TOBS-GT [20]. This work's most relevant contribution is the first usage of the TOBS-GT for acoustic problems.

1.2 Objectives and contributions

Within the present academic scenario, the **aim** of this work is to solve topology optimization of acoustic problems adopting the TOBS-GT method. The TOBS method was chosen due to its binary nature, that creates clear structural boundaries and interfaces, thus presenting a high potential for dealing with acoustics. In addition, a higher accuracy can be obtained with the geometry trimming technique.

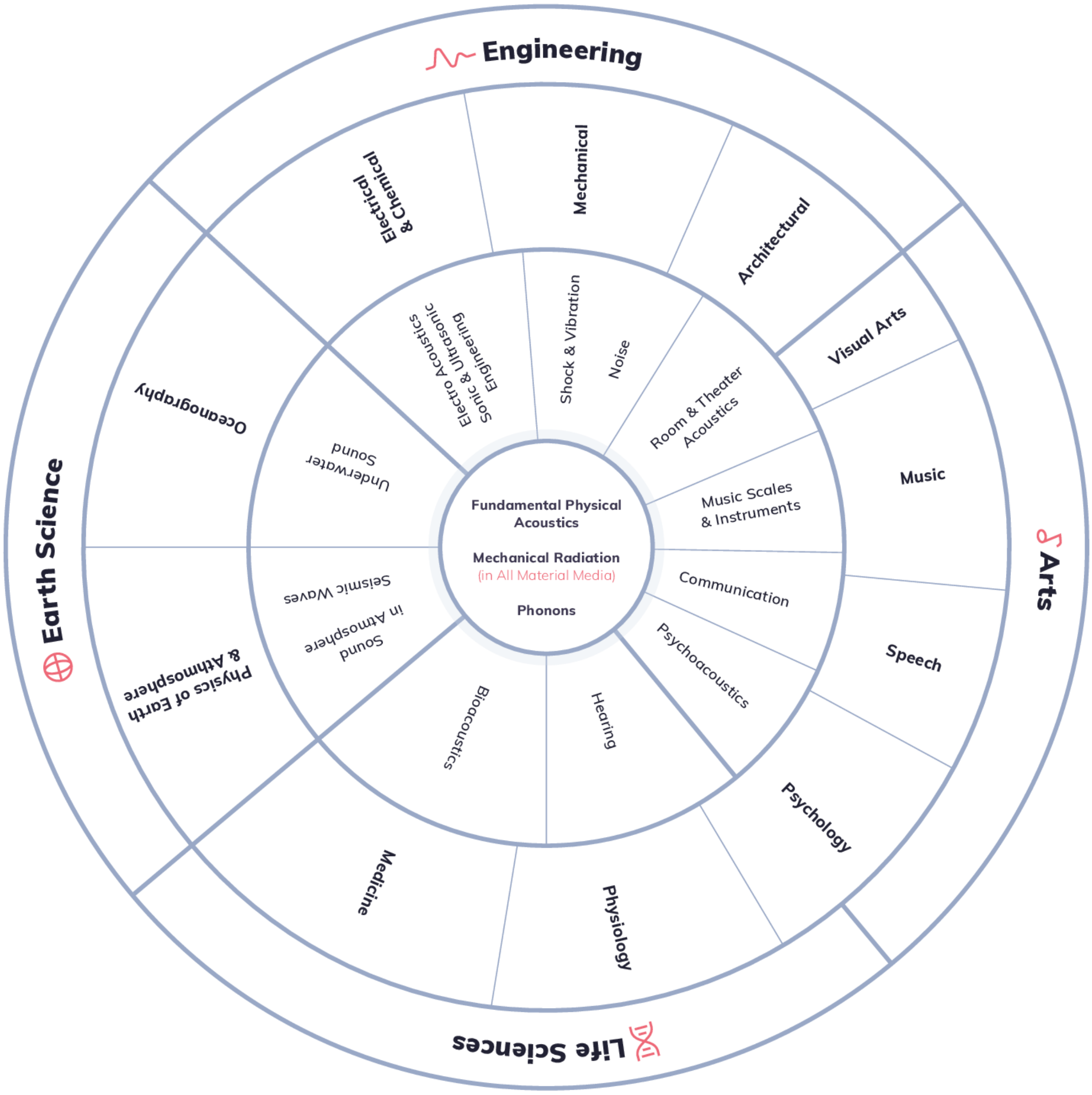
The **objectives** are:

1. To adapt and implement an acoustical topology optimization algorithm using the TOBS-GT method.
2. To investigate and discuss the characteristics of TOBS-GT method when employed on acoustic topology optimization problems.

1.3 Layout of the thesis

The remainder of this thesis is organized as follows. Chapter 2 presents a succinct literature review concerning topology optimization and optimization of wave-propagation problems. Chapter 3 contains fundamental concepts and theories regarding this research area, including essential definitions on structural optimization, available TO methods, sensitivity analysis, acoustic analysis and the finite element method. The proposed methodology for solving acoustic topology optimization problems is presented in chapter 4. Chapter 5 shows some results and discussions about the methodology. Finally, the key contributions of this research along with suggestions for future works are synthesized in chapter 6.

Figure 2: Lindsay's Wheel of Acoustics.



Source: R. Bruce Lindsay (1966) [13] in Acoucou, 2022. Wheel of Acoustics. Available in: [https://acoucou.org/about]. [21].

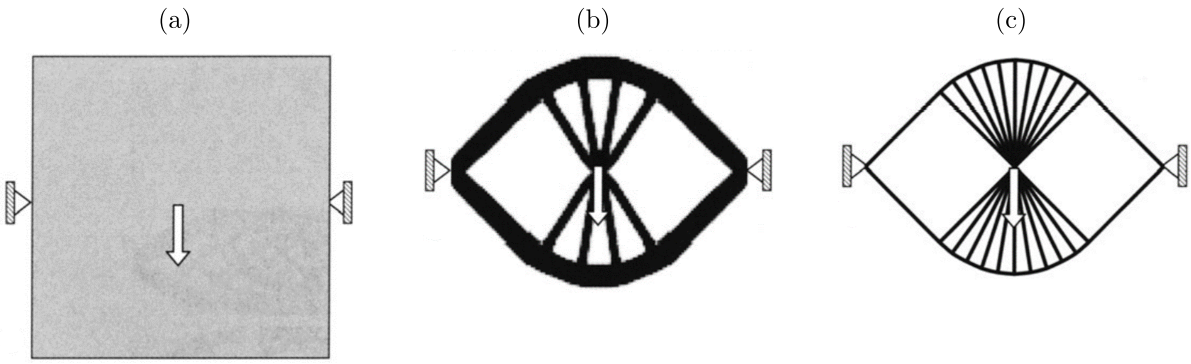
2 LITERATURE REVIEW

This chapter presents the literature review with analysis of the relevant literature of topology optimization and acoustics optimization. It serves as a basis for the investigation of the proposed work.

2.1 Topology Optimization

Topology optimization has been the most active research area in structural and multidisciplinary optimization in the past two decades [22] and has recently become the most popular engineering subfield [23]. This intensive research has pushed TO to reach a great level of maturity, with the consolidation of classical methods and the creation of promising new methods. Furthermore, the practical scope of TO has broadened beyond a few linear structural responses to include acoustics, fluid flow, heat transfer, materials design and other multiphysics disciplines.

Figure 3: A structure designed for minimum compliance, comparing TO with optimal Michell type structures [24]. (a) Design domain, (b) topology optimized solution and (c) Michell type optimal solution.



Source: Adapted from [25].

Although the starting date of TO cannot be precisely determined, its origins can be dated back to 19th century, with Maxwell's publication in 1870 [26]. Maxwell studied fundamental principles for the layout of truss structures of minimum weight and prescribed maximum stresses, concluding that the optimum truss design would be a set of unidimensional structures lying along the main stress lines. About 30 years later, Michell [24] applied Maxwell's theories to different cases and was able to validate and and deepen his research. Michell's optimization paper [24] from 1904 is considered by many as the first publication in the field, but his work is actually just an extension of Maxwell's achievement [23]. Fig. 3 depicts an example of a minimum compliance design with optimal Michell type structures and with topology optimization, where very similar results are obtained.

The problems studied in the early period of TO history were mainly academic, with no practical application. Therefore, no significant progress has been made at that time and there are not many publications available. Only in the 60's, with the advent of computer science and the Finite Element Analysis (FEA), practical problems were introduced – mainly in the aerospace industry –, and the topic began to rapidly expand. By the 80's, both optimization softwares and CAE (Computer Aided Engineering) softwares with optimization modules emerged. In the end of the decade, Bendsoe and Kikuchi [1] proposed a homogenization method to relate solid and void elements with their microstructure density. After that, several new computational concepts and procedures were formulated. Rozvany et al. [27] and Zhou and Rozvany [28] suggested a material interpolation technique for shape optimization, that was later applied in topology optimization [29] [30] and is known as the SIMP method. Ole Sigmund played an important role in the popularization of the SIMP scheme for topology optimization [31] [32]. Later, in 2001, he collaborated to further disseminate the technique with the publication of an outstanding article, consisting of a 99 line Matlab code for educational purposes [33]. Sigmund also substantially contributed with other aspects of the SIMP method, as checkerboard control [31] [34] and multiphysics applications [35], being an important leader up to date. Recently he published guidelines of good scientific practices in topology optimization [36], researches about TO considering local and global buckling response [37], among others.

Density-based methods underwent a rapid development and quickly became one of the most used TO methods, maintaining its popularity until nowadays. Over the years, the method was extended to different structural design problems, and some techniques were incremented to its formulation. As an alternative to the SIMP interpolation scheme, Stolpe and Svanberg (2001) [38] proposed the so-called RAMP interpolation (Rational Approximation of Material Properties). The versatile applications of SIMP, amongst many others, include vibration problems [39] [40] [41], stress constraints [42] [43], heat transfer and thermoelasticity [44] [45] [46], multiphysics [47] and wave-propagation [48] [49] [50]. Developments and applications of density-based methods can be found in the landmark structural topology optimization book by Bendsoe and Sigmund (2004) [25].

Along with SIMP, several new computational concepts and procedures were elaborated. The expansion of the field involves boundary-description, evolutionary and genetic methods, although the latter is less common. Boundary-variation methods, e.g. the Level-set Method, were developed in the early 2000s [51] [52] and, despite of its recent appearance, it is nowadays established as one of the main TO tools. A complete review of the LSM is presented by van Dijk et al. (2013) [53].

In the 90's the class of binary methods emerged, with the Evolutionary Structural Optimization (ESO) proposed by Yi Min Xie and Grant P. Steven [8] and its improved bi-directional version (BESO) [54]. The method became well known in the academia, despite of receiving severe criticism [55] due to its heuristic inspiration and lack of mathematical formulation. Xie and Huang (2010) [56] summarizes some of the latest developments in the BESO method to overcome these criticisms. Recently, Sivapuram and Picelli (2018) [16] combined features of BESO and the discrete topology optimization method of Svanberg and Werme [57] to improve the effectiveness of binary variable optimization, creating the so called TOBS method. The TOBS method adopts sequential problem linearization, constraints' relaxation, sensitivity filtering and an integer programming solver, and has been applied to different optimization problems as design-dependent fluid pressure and constant thermal expansion loads [11], microstructures settings [18], turbulent fluid flow [20] and pressure loads and buckling constraints [19].

In the last couple of decades, the research field of TO has undergone a rapid growth in academia, sponsored research and industrial application. Deaton and Grandhi [22] present a survey which highlights the advancements in topology optimization from 2000 to 2012. The survey contains the procedure and research stages of existent TO methods, along with recommendations and perspectives related to the field.

With this brief overview, the relevance and the potential of topology optimization in different areas become evident. There is a plurality of TO methods and tools available, each one with their weakness and strengths, which should be taken into account in order to choose the most suitable methodology. Yet, all methods have room for improvements and areas still to be explored.

2.2 Optimization of wave-propagation problems

When it comes to improving a general structure project, a simple approach is the *analysis*, where different possible configurations are evaluated for specific performance measures, and the best one is chosen. This formulation was adopted by Reich and Bradley [58] to improve the speech intelligibility in a classroom by comparing different locations of a fixed amount of sound absorbing material and similarly by Shen, Shen and Zhou [59] to improve the amplitude response from a loudspeaker in a room. However, this methodology does not guarantee for the structure to be the best possible, that is, to be optimal.

A different approach was employed by De Lima et al. [60], combining *parametric optimization* to evaluate the appropriate size of the inlet and outlet ducts of reactive silencers, and *shape optimization* to establish the proper profile of these ducts in order to improve the acoustic features. Shape optimization is a common and effective way to attack an acoustic problem. Habbal (1988) [61] applied shape optimization to noise reduction purposes. Bangtsson et al. (2003) [62] optimized the shape of an acoustic horn. Tran et al. [63] optimized the shape of two-dimensional lenses for acoustic cameras. In these cases, non-intuitive optimized shapes are obtained, but it is not possible to achieve holes in the structure and parts which are not attached to the boundary. This is made possible with *topology optimization.*

As mentioned in the previous section, many researches have explored the topology optimization field within different physics and applications, but especially within structural mechanics, around which most of the available literature revolves. However, acoustics is a much more recent topic within this research area, and much less literature exists about it.

The first publications regarding TO of wave-propagation problems emerged in the late 90's. Early studies included the design of structures with maximized band gaps [50,64,65], acoustic horns [4], room acoustics and sound barriers designs [3] and sound mufflers [66].

3 THEORETICAL BACKGROUND

This chapter introduces the theoretical foundation for this dissertation. It addresses the main ideas and important concepts concerning optimization of acoustic problems. Although this work addresses ATO, it is based in structural optimization, as it consists in defining the presence of solid structures to improve acoustical properties. Therefore, the fundamentals of structural optimization are also cover in this chapter.

3.1 Structural Optimization

Optimization can be defined as the process of finding the best possible outcome of a given task while satisfying specified restrictions. It is a fundamental principle which humankind has always seek – trying to save energy and resources in order to maximize output or profit. Amongst innumerable examples in the saga of human history, one can mention the invention of the lever and the pulley – a result of the pursuit to maximize mechanical efficiency. Regarding structural optimization, the objective is to find the best design according to a preselected quantitative measure of effectiveness [67].

There are two different manners to approach the development of a structural project: by analysis or by optimization. The first one consists of analysing different possible configurations of a structure by altering some parameters and evaluating them for specific performance measures and then choosing the best configuration. Therefore, in this approach the amount of required analysis increases as the number of parameters increase. For this reason, it can be efficient only when dealing with a small number of parameters. In contrast, the optimization – or synthesis – approach relies on computational procedures to find the best feasible configuration. The algorithm seeks – employing mathematical methods – the optimum design that provides best required performance within the possible solution space.

However, the term optimization is frequently misused. For instance, there are often works on engineering conferences that use this term when referring to a new configuration

– chosen based on trial and error – that enhanced the system's performance, but is not guaranteed to be the best. This misconception arose with the origination of CAE softwares based on FEA. Until the early sixties, the analysis was the main approach used to solve structural problems. Then, the origination of these softwares enabled and popularized the use of mathematical methods to solve complex engineering problems. The aerospace industry played an important role on the development of structural optimization, as it was the fist industry to recognize the importance of minimum weight design of structures.

3.1.1 Elements of an optimization problem

Structural optimization involves a broad field of study, and knowing its fundamental concepts is essential for a full understanding of the problem. Therefore, within this section, the three essential elements of an optimization problem formulation will be briefly introduced: design variables, objective function and constraints. Then, the standard formulation and different types of structural optimization will be discussed. These concepts will provide the basic knowledge for further understanding structural optimization problems.

- Design variables:

The notion of optimizing a structure performance involves the freedom to change certain structure's parameters within an allowed range. These parameters are denoted *design variables* and are expressed by a vector $\theta = (\theta_1, \theta_2, ..., \theta_n)$. A few examples of possible design variables are cross-section dimensions, member sizes and material properties.

Design variables can assume continuous or discrete values. Continuous design variables can take any value in a determined range of variation, allowing the reach of the very best value. Although this approach is easier to solve, in many situations the solution can be difficult to manufacture. For example, if the optimal thickness of a part was 2.5mm, but there are only 2mm or 3mm in the market. In such case, one possible manner to solve this issue is to treat the variables as discrete, that is, allow them to admit only isolated values. This is done with a branch of mathematical programming called *integer programming*.

- Objective function:

An *objective function* is a function that can be improved and is used as a measure of performance or effectiveness of the design. In structural optimization, weight, displacements, vibration frequencies and costs, for example, are frequently used as objective functions.

In an optimization problem, one can adopt a single function $\Phi(\theta)$ or multiple functions $\Phi(\theta) = [\Phi_1(\theta), \Phi_2(\theta), ..., \Phi_p(\theta)]$. The latter is referred to as *Multicriteria Optimization*, which is usually avoided due to its complexity [67]. There are two ways commonly used to handle this issue: generating a composite objective function that replaces all the objectives or selecting the most relevant as the objective function and setting limits to the other.

- Constraints

This idea of imposing a limit to a function introduces the notion of *constraints*. When the limits of the constraint are defined by an upper or a lower quantity, it is denominated *inequality constraint*; when a specific value must be achieved, it's denoted a *equality constraint*. It is preferable to work with inequality constraints, as equality constraints can be hard to handle.

3.1.2 Standard formulation

The standard optimization problem can be formulated, using the notation of this present work, as

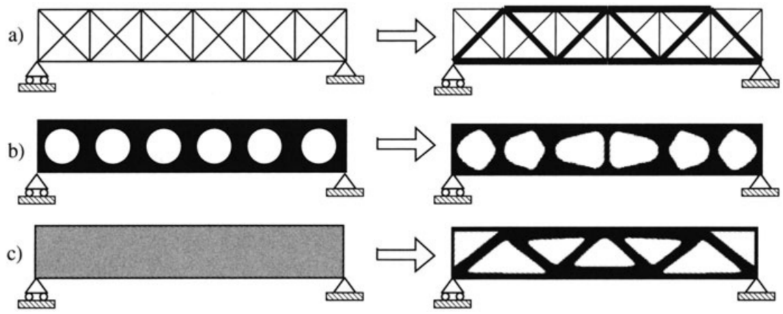
$$\begin{aligned} &\text{Minimize } \Phi(\theta) \\ &\text{Subject to } g_i(\theta) \leq 0,\ \ i \in [1, N_g] \\ &\qquad\qquad\quad h_b(\theta) = 0,\ \ b \in [1, N_h]. \end{aligned} \tag{3.1}$$

Here, θ represents the vector of design variables, Φ is the objective function and g_i and h_b denote the inequality and equality constraints, respectively. It is relevant to point out that although the problem is expressed as a minimization problem, it is possible to maximize a function simply by minimizing its negative.

3.1.3 Types of structural optimization

There are three main optimization methods available to solve a structural problem: parametric, shape and topology optimization. They cover different optimization purposes and imply distinct degrees of freedom to change the design. Figure 4 depicts the same problem addressed by the three optimization categories.

Figure 4: A truss problem solved by a) parametric optimization, b) shape optimization and c) topology optimization, with the initial designs on the left and the optimal designs on the right.



Source: Bendsoe and Sigmund (2004) [25].

- Parametric optimization:

Parametric optimization, also called *sizing optimization*, adopts one or more parameters as the design variables. A few examples of possible parameters are thickness, cross-section area, diameter of a hole, etc. The initial design of the structure is given based on the designer's experience, and is fixed during the optimization process. The parameters are changed in a way that minimizes (or maximizes) a physical quantity, such as the deflection, mean compliance, etc. In Figure 4.a an example can be observed where a truss is optimized regarding its bars' thickness.

- Shape Optimization:

In a shape optimization problem, the initial topology is given – that is, the location and number of holes -, and the goal is to optimize the objective function by modifying the shape of the holes and boundaries of the structure. An example of this application is seen in Figure 4.b. This method has a more complex formulation when compared to parametric optimization, but it also depends on the designer's ability to set the initial topology, which will dictate the quality of the solution.

- Topology optimization:

On the other hand, in a topology optimization, the solver is free to obtain not only the size or the shape of the structure, but the entire layout inside the specified design domain. In this case, the design variable is the presence or absence of material. This technique can achieve the best solution independent of the designer's previous experience, as it does not require the assignment of the initial setup. The computational development of the last decades resulted in the further expansion of TO methods, enabling its application to different physics and to complex problems.

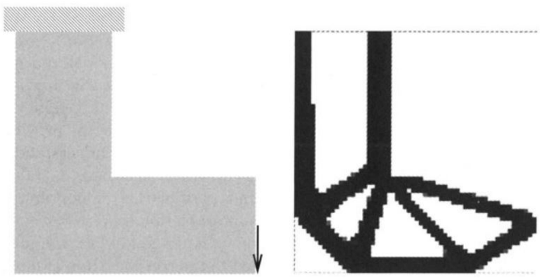
3.2 Topology Optimization

Topology optimization aims at finding an optimal material distribution over an specific design domain, that is, to determine which points should be *solid* or *void* – usually represented by 1 and 0, respectively. Therefore, the final objective is to find the final design variable vector containing {0,1} values, leading to an indicator function given as

$$\theta_j(\theta) = \begin{cases} 1 & \text{for } \Omega_s \\ 0 & \text{for } \Omega_a, \end{cases} \tag{3.2}$$

that designates whether the element j belongs to the solid Ω_s or air Ω_a domain. The solid domain is visually represented in black, and the air in white, similar to a black-and-white rendering of an image, as seen in Fig. 5. The function θ_j can indicate not only the presence or absence of the elements but also their material properties.

Figure 5: A black-and-white minimum compliance design for a loaded structure, with the design domain and boundary conditions on the left and the optimal design on the right.



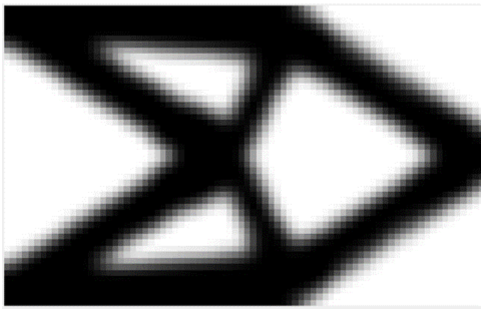
Source: Bendsoe and Sigmund (2004) [25].

The final design is expected to present only void or solid elements, that is, binary variables {0,1}. Managing integer variables can be challenging, and different topology optimization methods deal with this issue in distinct ways. Below follows a brief overview of the most notorious topology optimization methods.

- Density-based methods:

Instead of dealing with integer variables, density-based methods assume continuous design variables, expressed as a material density distribution $\rho_{min} \leq \rho(x) \leq 1$. Here $\rho(x)$ indicates the amount of solid material existent in each point x of the design domain, and a lower bound ρ_{min} is used to avoid singularity problems. This leads to results with intermediate densities and a gray-scale representation (see Fig. 6), requiring some form of penalty to lead the solution to discrete 0-1 values.

Figure 6: A SIMP topology optimization result for a cantilever beam, without a projection filter, presenting gray-scale elements.



The SIMP method [25] is the most notorious of this class, and features a very popular and efficient penalization scheme:

$$\mu(x_j) = \rho(x_j)^{\gamma}\mu_0. \tag{3.3}$$

Here, μ_0 represents the properties of a given isotropic material, $\rho(x_j)^{\gamma}$ is the penalized density of the element j, and γ is the penalty value. In SIMP one should use $\gamma > 1$, so that the stiffness obtained is small compared to the cost (volume) of material, thus intermediate densities are unfavourable in the optimal design.

This approach successfully circumvents numerical issues of strictly 0-1 problems and iteratively leads the solution towards a solid/void solution. The result is a robust method with a relatively simple implementation, justifying its application in a wide variety of problems and its solid development over the years.

- Boundary variation methods

Originated from shape optimization, boundary variation methods are another popular and consolidated class of TO. In this case, the design variables of the problem are the boundaries of the structure, which are controlled by implicit functions. The allowance for formation, disappearance and merger of void regions define a true topological design and distinguish boundary variation from shape optimization methods. [22]

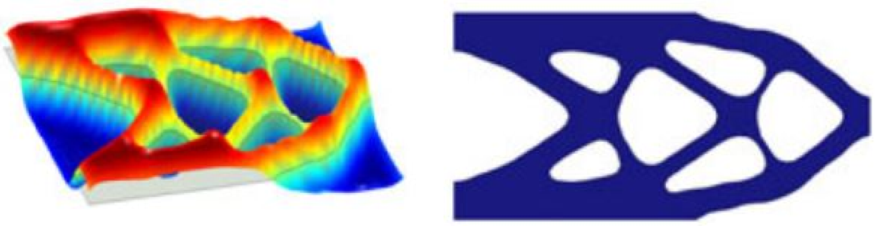
In the Level-set Method, a well-known technique with a boundary variation approach, a level-set function ϕ can be adopted to describe the material domain:

$$\theta_j(\theta) = \begin{cases} \phi(\theta) > C & \text{for } \theta \ \in \text{material domain} \\ \phi(\theta) = C & \text{for } \theta \ \in \text{interface} \\ \phi(\theta) = C & \text{for } \theta \ \in \text{void} \end{cases} \tag{3.4}$$

where C is a constant usually defined as 0. In other words, the boundaries are represented as the zero-level curve of the scalar function ϕ, as depicted in Fig. 7.

Figure 7: A level-set representation of a cantilever beam with (a) the level-set surface plot and (b) the structure topology at zero level set.

(a) (b)



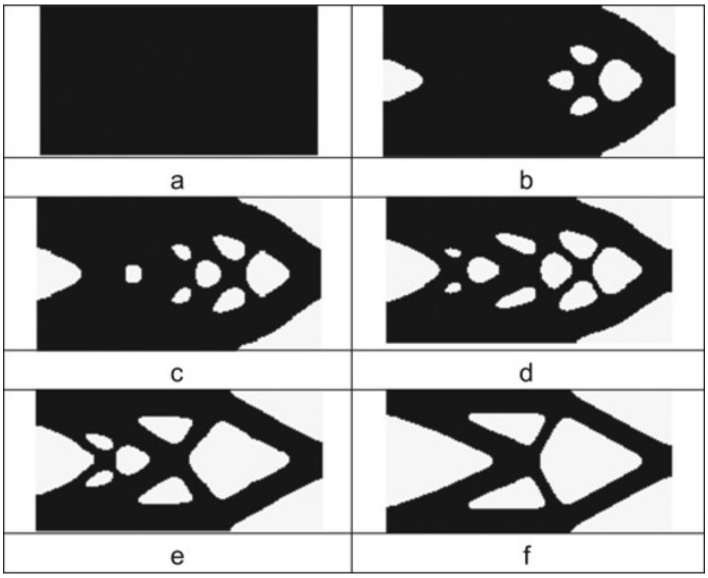
Source: Luo et. al, 2012. [68]

- Discrete methods

Discrete methods work by gradually removing or adding material from the design domain, iteratively steering the solution to the optimum. Just as density-based methods, it uses gradient information, and they both have similar procedures. However, for discrete methods, the design space is not relaxed. The design variables are taken as the existence ($\theta_j = 1$) or absence ($\theta_j = 0$) of material in each element j and a lower bound $\theta_j = \theta_{min}$ can also be utilized to avoid singular structural problems.

This approach results in crisply defined structural boundaries that are free of gray-scale intermediate design variables. The most notorious method of this category is the ESO method, originally proposed by Xie and Steven [8], and the later developed BESO, in which elements could be either added or removed. Figure 8 depicts part of the iteration process of a TO with the BESO method, where it's clearly seen that the boundaries are well defined throughout the process.

Figure 8: Iteration process of a topology optimization of a cantilever beam solved with BESO method.



Source: Zhao, 2014. [69]

The main criticism of these methods is the use of heuristic techniques and, therefore, the impossibility to handle constraints other than the volume. As the discrete update scheme is based on the amount of solid material in each iteration, a volume constraint is always present.

$$\frac{V(\theta_j)}{V_0} \leq V_f, \tag{3.5}$$

where $V(\theta_j)$ is the volume of the existent material, V_0 is the volume of the design domain, and V_f is the prescribed final volume fraction.

3.2.1 The TOBS method

TOBS is within the class of discrete TO methods. It was first proposed by Sivapuram and Picelli [16] in 2018 and was formulated in order to align the use of binary design variables with formal mathematical programming. This is an unusual combination in the TO literature that carries potential advantages in several applications.

A generic binary optimization problem with inequality constraints can be formulated as

$$
\begin{aligned}
\underset{\theta}{\text{Minimize}} \quad & \Phi(\theta) \\
\text{Subject to} \quad & g_i(\theta) \leq \overline{g_i}, \quad i \in [1, N_g] \\
& \theta_j \in \{0, 1\}, \quad j \in [1, N_d],
\end{aligned}
\tag{3.6}
$$

where Φ is the objective function – which depends on the vector of design variables θ of size N_d –, g_i is the i^{th} inequality constraint, $\overline{g_i}$ is the associated upper bound and N_g is the number of inequality constraints in the optimization problem.

Since general topology optimization problems are highly nonlinear and nonconvex, the method proposes the use of sequential approximation of the optimization problem. Considering a Taylor's series expansion, the objective and constraints functions can be expressed as follows, for the optimization iteration k:

$$
\begin{aligned}
\Phi(\theta) &= \Phi(\theta^k) + \frac{\partial \Phi(\theta^k)}{\partial \theta} \cdot \Delta\theta^k + O(\left|\left|\Delta\theta^k\right|\right|_2^2), \\
g_i(\theta) &= g_i(\theta^k) + \frac{\partial g_i(\theta^k)}{\partial \theta} \cdot \Delta\theta^k + O(\left|\left|\Delta\theta^k\right|\right|_2^2).
\end{aligned}
\tag{3.7}
$$

TOBS employs truncated linear approximations of the objective and constraint functions, so the truncation error is given as $O(\left|\left|\Delta\theta^k\right|\right|_2^2)$, and $\Delta\theta^k$, the vector that represents the changes in the design variables, is used to solve the optimization problem 3.6. These changes $\Delta\theta^k$ should be restricted in order to keep the design variable with integer (i.e., binary) values. For instance, considering an element j as solid ($\theta_j = 1$), the change in the design variable must be either $\Delta\theta_j = \{0\}$ to remain solid, or $\Delta\theta_j = \{-1\}$ to become void in the optimization iteration. The same procedure is analogous for a void element ($\theta_j = 0$). Therefore, the bound constraints for $\Delta\theta_j$ can then be expressed as,

$$
\begin{cases}
0 \leq \Delta\theta_j^k \leq 1 & \text{if } \theta_j^k = 0, \\
-1 \leq \Delta\theta_j^k \leq 0 & \text{if } \theta_j^k = 1,
\end{cases}
\tag{3.8}
$$

or, in a unified form,

$$\Delta\theta_j^k \in \{-\theta_j^k, 1 - \theta_j^k\}, \tag{3.9}$$

where $\Delta\theta_j^k \in \{-1, 0, 1\}$.

In order to maintain the linear approximation from Eq. 3.7 valid, the truncation error $O(\left|\left|\Delta\theta^k\right|\right|_2^2)$ should be reasonably small. One can control the truncation error by adding the constraint

$$\left|\left|\Delta\theta^k\right|\right|_1 \leq \beta N_d, \tag{3.10}$$

where β is an additional parameter added to the subproblem to restrict the the number of elements that may turn from solid to void and vice-versa to a fraction of the total number of elements (N_d).

By considering the sequential linear approximations from Eq. 3.7, and the extra constraints from Eq. 3.9 and Eq. 3.10, one can write the approximate integer linear subproblem as

$$
\begin{aligned}
\underset{\Delta\theta^k}{\text{Minimize}} \quad & \frac{\partial \Phi(\theta^k)}{\partial \theta} \cdot \Delta\theta^k, \\
\text{Subject to} \quad & \frac{\partial g_i(\theta^k)}{\partial \theta} \cdot \Delta\theta^k \leq \overline{g_i} - g_i(\theta^k) := \Delta g_i^k, \ i \in [1, N_g], \\
& \left|\left|\Delta x^k\right|\right|_1 \leq \beta N_d, \\
& \Delta\theta^k \in \{-\theta_j^k, 1-\theta_j^k\}, \ j \in [1, N_d].
\end{aligned}
\tag{3.11}
$$

This subproblem (Eq. 3.11) can be solved through Integer Linear Programming (ILP), which is essentially the same as a Linear Programming (LP) problem, but imposing additional constraints to ensure that the design variables can only achieve integer values. It is also a naturally understandable choice, since we aim to achieve a binary $\{0, 1\}$ solution. It should be noted that the output of the ILP solver is the optimum change vector $\Delta\theta$, thereby the design variable θ is updated as

$$\theta^{k+1} = \theta^k + \Delta\theta^k. \tag{3.12}$$

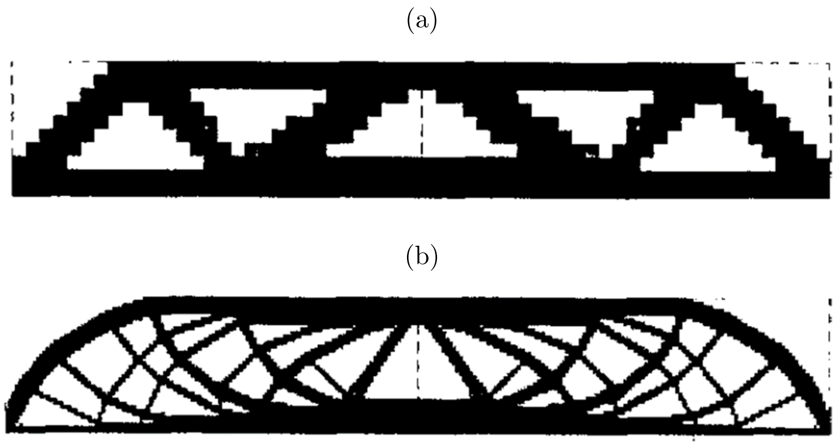
In this work, the ILP problem is solved by using the branch-and-bound algorithm from the CPLEX® optimization library, which is developed by IBM® . The branch-and-bound method consists of an algorithm based on a tree data structure, in which the ILP problem is first solved without any integer constraints (by using a linear optimization technique such as the Simplex method); then, branches of LPs are created with additional inequality constraints being imposed on the design variables in order for the solution to be yielded as integer [70, 71]. More details on the TOBS method can be found in [16, 72].

3.2.2 Sensitivity filtering

It is well-known that density-based TO methods are prone to solutions with checkerboard patterns and to mesh dependency issues. The checkerboard pattern consists of the formation of regions with alternating solid and void elements ordered in a checkerboard like fashion (see Fig. 9), i.e. bars or regions connected only by the elements' nodes. They are formed due to bad numerical modelling and result in a structure with an artificially high stiffness [34]. To overcome this issue, Sigmund (1994) [31] proposed a filtering technique based on image processing. The filter is used in each iteration of the algorithm and makes the sensitivity of an element depend on a weighted average over the element itself and its eight direct neighbours. However, this filter was still subject to mesh dependence, which, in turn, refers to the problem of not obtaining the same solution for different mesh-sizes or discretizations. An example of mesh dependence occurrence is illustrated in Fig. 10), where a TO of a well-known Messerschmidt-Bölkow-Blohm (MBB) beam problem results in different topologies for different discretizations.

Figure 9: (a) The checkerboard pattern and (b) a typical topology with checkerboard pattern.

Figure 10: An example of the mesh dependency problem, with (a) a solution for 600 elements discretization and (b) a solution with 5400 elements discretization.



Source: Adapted from Sigmund, 1998 [34].

Later, other filters were developed to solve checkerboard and mesh-dependency matters, as well as other numerical problems. For the TOBS method, Picelli (2020) [72] suggests the use of a filter similar to the standard sensitivity filter used in SIMP-based topology optimization, with the difference that element densities do not take part in the TOBS filtering procedure, as it uses only binary variables. In this case, the filtered sensitivities field $\widetilde{\frac{\partial \Phi}{\partial \theta_j}}$ is given as

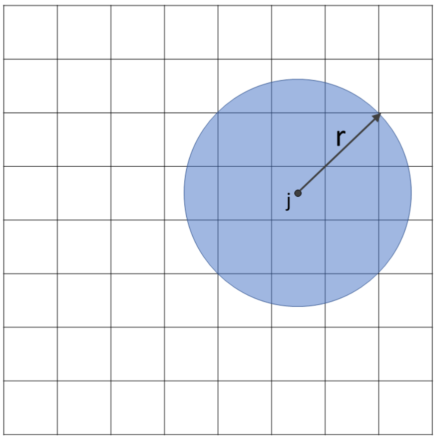
$$\widetilde{\frac{\partial \Phi}{\partial \theta_j}} = \frac{1}{\sum_{m \in N_m} H_{jm}} \sum_{m \in N_m} H_{jm} \frac{\partial \Phi}{\partial \theta_m}, \tag{3.13}$$

where N_m is the set of elements m for which the center-to-center distance $dist(x_j, x_m)$ from element j is smaller than the filter radius r and the weight factor H_{jm} is given by

$$H_{jm} = max(0, r - dist(x_e, x_m)). \tag{3.14}$$

This means that the elements closer to the element j have a higher contribution to the filtered sensitivities of element j than the elements located farther away from it.

Figure 11: Sensitivity filter scheme for an element j with a radius r.



Still according to Picelli (2020) [72], other types of filters could also be used with the TOBS method. One example is the BESO filter, which employs nodal averaging of the sensitivity field and then applies a spatial filter to recover element sensitivity field from the nodal sensitivity field (Huang and Xie, 2007) [73].

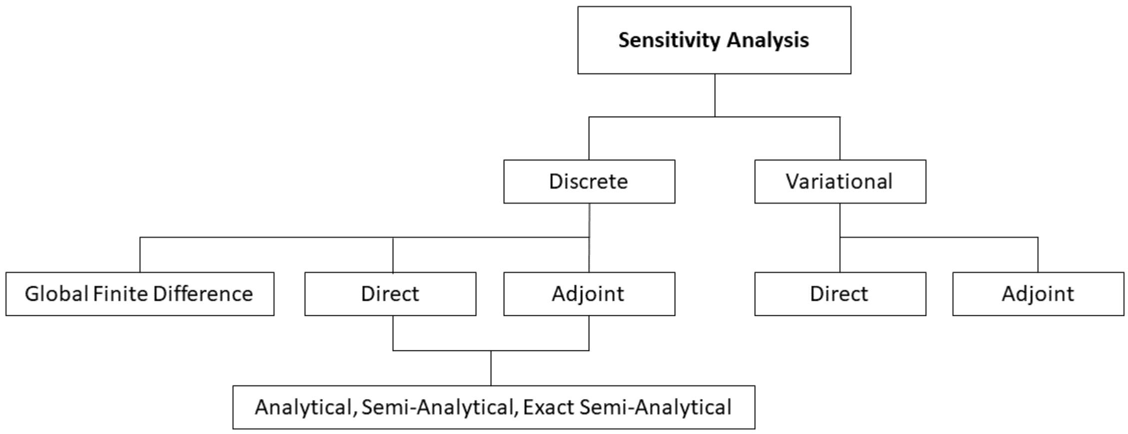
The sensitivity filter scheme is an effective strategy to overcome many numerical problems in topology optimization, such as checkerboard and mesh-dependency, as it is very easy to implement and requires little extra computational time. This approach is specially important for binary topology optimization as it smoothens out the original sensitivity field and extrapolates it to void regions, increasing the chances of void elements returning to solid.

3.3 Sensitivity analysis: available methods

Sensitivity analysis is a numerical evaluation of how a change in an input affects the output. As TOBS is a gradient-based method, it requires sensitivity analysis of the objective and restrictions functions in respect to the design variables. This is a crucial step in TO and several methods are available. Those can be classified in classes as: overall finite differences; discrete derivatives; continuum derivatives; and computational or automatic differentiation [74]. All these categories come in direct and adjoint methods, except for the finite differences class. A scheme of these classes is depicted in Fig. 12.

Given that the calculation of the sensitivities is often the major computational cost of the optimization process, it is essential to use efficient algorithms for this purpose. [67] The choice of which method to use should be mainly based according to the computational capability available and the accuracy required. [74]

Figure 12: Sensitivity Analysis Methods



3.3.1 Global Finite Difference

The overall or global finite difference method approximates the derivative of a function by a finite difference formulation. Given a function $\Phi(\theta)$, with a design variable θ, the first-order forward-difference approximation $\Delta\Phi/\Delta\theta$ to the derivative $d\Phi/d\theta$ is given as

$$\frac{\Delta\Phi}{\Delta\theta} = \frac{\Phi(\theta + \Delta\theta) - \Phi(\theta)}{\Delta\theta}. \tag{3.15}$$

In most cases a simple first-order (forward or backwards) approximation is used, a second-order central approach is not uncommon, but higher-order applications are rarely used in optimization due to its high computational cost. The reason is that if we need to find the derivatives of a function with respect to n design variables, the forward-difference approximation would require n additional analysis, the central-difference $2n$ additional analysis and higher order would be even more costly.

When employing a finite-difference approximation, two sources of errors occur: truncation and condition errors. The first one arises when a large step size Δx is used, and is related to the omitted terms in the Taylor series expansion. The latter can be caused by round-off errors when the step size adopted is too small. These issues lead us to the so called "step-size dilemma", and in some cases finding an adequate step size, which results in an acceptable error, may be unreachable [67].

Given these points, it can be noted that the global finite-difference is very easy to implement, making it a simple tool for calculating derivatives. Although, it presents accuracy problems and it is often computationally expensive, therefore it is only applicable when having a few number of design variables. An useful application of this method is to certify the efficiency of analytical approaches.

3.3.2 Variational derivatives

Variational and further discrete approaches distinguish from the global finite difference approach due to the utilization of analytical derivatives. The variational sensitivity analysis differentiates the continuum governing equations before they are discretized. It makes a convenient formulation in combination with existing FEA codes, as it can be performed without modifying these codes [75]. Another advantage is the possibility of using adjoint sensitivity analysis, which is beneficial for problems with a large number of design variables and a small number of response functions, as TO.

3.3.3 Discrete derivatives

As previously mentioned, the discrete formulation utilizes analytical derivatives, just as the variational method. The difference between these methods is the sequence of discretization and differentiation. For the discrete approach the differentiation takes place after the discretization, that is, the discretized governing equation is differentiated. In this case, more information from the FEA code is required.

3.3.4 Direct sensitivity analysis

Both variational and discrete sensitivity analysis can be derived in the direct and adjoint formulation. The sensitivity calculation in the direct method is obtained by an explicit evaluation of a state derivative for each design variable. This means that for a FEA based TO problem, where the number of design variables is extremely large and the number of response functions is small, this method would require both tremendous numerical effort and memory space. Therefore, this method is inadequate for TO problems.

3.3.5 Adjoint sensitivity analysis

In contrast to the direct method, the adjoint formulation is suitable for FEA based TO problems, as it is very efficient for conditions with many design variables and few response functions. It requires less computational memory and processing time than the direct method for this class of problems. For this reason, this was the chosen method for solving the sensitivity analysis in the present work.

3.3.6 Semi-analytical method

Features from both discrete and finite difference methods are combined in this formulation to overcome the drawbacks of analytical sensitivity analysis. The advantages here are the small programming effort, predictable numerical effort and the prevention of the need to modify existing element routines, and the drawbacks are related to the approximation errors of the finite difference. [75]

3.4 Fundamentals of acoustics

Acoustics can be defined as the generation, transmission, and reception of energy as vibrational waves in matter [76]. The most familiar acoustic occurrences are the ones associated with the sensation of sound, that usually relies between 20 Hz and 20.000 Hz frequencies. Yet, acoustics also includes infrasonic (below 20 Hz) and ultrasonic frequencies (above 20.000 Hz). When elastic waves propagate in a material medium, the molecules of this material (fluid or solid) are displaced from their equilibrium position, inducing the emergency of an internal elastic restoring force, that causes elements to move back and forth, producing adjacent regions of compression and rarefaction. This force is what enables matter to take part in oscillatory vibrations and, therefore, to generate and transmit acoustic waves.

One can describe an acoustic wave by indicating the displacements of the particles in the medium, that vibrates about their mean positions. Nevertheless, the velocity of particle displacement is usually considered rather than the displacement itself [77]. As this vibration yields some particles to be compressed and others rarefied, it causes variations of gas density and pressure, both of which are functions of time and position. Therefore, the sound pressure is defined as the difference between the instantaneous pressure and the static pressure.

The general differential equation governing sound propagation in any lossless fluid is the so called wave equation [77]

$$\nabla^2 p = \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2}. \tag{3.16}$$

Here p is the sound pressure, t is the time and c is the speed of sound defined by $c^2 = K/\rho$, where K and ρ are the bulk modulus and the density of the medium, respectively. To reduce the complexity of the analysis, we assume the time-independent form of the wave equation, known as Helmholtz equation

$$\nabla \cdot (\rho^{-1}\nabla p) + \omega^2 K^{-1} p = 0, \tag{3.17}$$

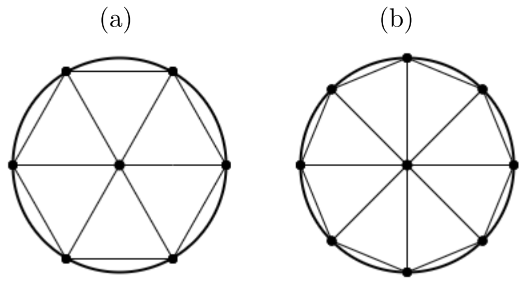
where ω is the angular frequency and p, ρ and K depend on the position vector $\mathbf{r}$.

3.5 Finite element analysis

Classical analytical methods in engineering provide exact solutions for simple structures. However, more complex geometries and material distributions require numerical solutions. Numerical methods are suitable for structures of any shape and general boundary condition and, while they provide approximate results, it can have a reasonable accuracy. In this work the Finite Element Method (FEM) is employed. Also known as Finite Element Analysis, the method obtains approximate solutions of partial differential equations (PDEs) by dividing a continuous domain into discrete subregions, called finite elements, which are represented and interconnected by nodes. The solution of the PDEs is computed at the nodes and interpolated throughout the elements.

The precision attained by this approximation is proportional to the number of nodes and elements used to discretize the domain. A simple example to illustrate this concept is found in fig. 13, where a circle is divided into triangular elements. The approximate area of the circle can be computed by summing the areas of the triangles. It is clear to note that the error gets smaller when the mesh is more refined (i.e. more elements are used).

Figure 13: Circular shape geometry discretized into (a) 6 and (b) 8 triangular elements.



The acoustic nodal pressures are collected in a field vector defined as $\mathbf{p} = [p_1, p_2, ..., p_J]$ and are discretized using sets of finite element basis functions $\{N_{jn}(\mathbf{r})\}$

$$p_j(\mathbf{r}) = \sum_{n=1}^{N} p_{jn} N_{jn}(\mathbf{r}), \tag{3.18}$$

where $\mathbf{r}$ is the position vector and the degrees of freedom corresponding to each field are assembled in the vectors $\mathbf{p}_j = \{p_{j1}, p_{j2}, ..., p_{jN}\}^T$. The governing equation (Eq. 3.17) is discretized as

$$\sum_{j=1}^{J} \mathbf{S}_{kj}\mathbf{p}_j = \sum_{j=1}^{J} (\mathbf{K}_{kj} + i\omega\mathbf{C}_{kj} - \omega^2\mathbf{M}_{kj})\mathbf{p}_j = \mathbf{f}_k. \tag{3.19}$$

Here, the stiffness matrix $\mathbf{K}_{kj}$, the damping matrix $\mathbf{C}_{kj}$ and the mass matrix $\mathbf{M}_{kj}$ contribute to the system matrix $\mathbf{S}_{kj}$, and $\mathbf{f}_k$ is the nodal load vector. The problem gets complex valued when the damping matrix is different from zero.

4 METHODOLOGY

Considering the theoretical concepts regarding acoustics and topology optimization previously presented, the use of the TOBS approach with geometry trimming (henceforth called TOBS-GT) for solving acoustic optimization problems is proposed in this work. The code implementation was developed using the integrated softwares COMSOL Multiphysics® and MATLAB® . In this chapter some important aspects of the methodology are initially described and the full computational procedure scheme is summarized afterwards.

4.1 Proposed formulation

Considering a domain Ω exited by a source that is emitting sound waves, the aim of the topology optimization is to find the best material distribution in the design domain Ω_d that optimizes an objective function Φ. The objective function adopted is the average of the Sound Pressure Level (SPL) in the objective domain Ω_o. The sound pressure level is given by

$$SPL = 20\log(\frac{p_{rms}}{p_{ref}}), \tag{4.1}$$

where $p_{ref} = 20\mu$ Pa is the pressure reference for air and p_{rms} is the root mean square pressure.

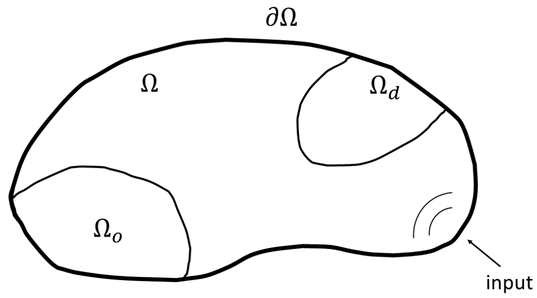
Under those circumstances the formulation of the optimization problem takes the form

$$\begin{aligned} &\underset{\theta}{\text{Minimize}}\ \Phi(r,\theta(r)) = \frac{1}{\int_{\Omega_o} dr}\int_{\Omega_o} SPL\ dr, \\ &\text{Subject to}\ \theta_j \in \{0,1\},\ j \in [1, N_d], \end{aligned} \tag{4.2}$$

where θ is the vector of design variables of size N_d. Note that, even though a volume restriction is imposed to most TO problems, we do not include it here, as wave-propagation problems are usually not optimal for extreme volume fractions (Sigmund and Jensen,

2003) [50]. A sketch of the problem is found in Fig. 14.

Figure 14: Illustration of an Acoustic Topology Optimization problem. The aim is to optimize an objective function in the objective domain Ω_o by distributing air and solid material in the design domain Ω_d.



4.2 Design variables and material interpolation

The design domain is discretized into small elements, represented by a vector of design variables θ, such that $\theta = 0$ corresponds to solid and $\theta = 1$ corresponds to air. Therefore, the material properties μ can be defined as

$$\mu(\theta) = \begin{cases} \mu_s, & \text{if } \theta = 0 \\ \mu_a, & \text{if } \theta = 1 \end{cases}, \tag{4.3}$$

where μ_s and μ_a are respectively the material properties of the solid and acoustic (air) regions. In this case, the properties used are the density ρ and the bulk modulus K and, motivated by their appearance in an inverse form in Eq. 3.17, their inverse can interpolated linearly as

$$\begin{aligned} \rho^{-1} &= \rho_s^{-1} + \theta(\rho_a^{-1} - \rho_s^{-1}), \\ K^{-1} &= K_s^{-1} + \theta(K_a^{-1} - K_s^{-1}). \end{aligned} \tag{4.4}$$

Thus, satisfying Eq. 4.3. The properties adopted for the air here are $\rho_a = 1.204$ kg/m^3 and $K_a = 141.921$ kPa and ρ_s and K_s are the properties of a solid material. However, there is no need to consider the solid material properties, as in TOBS-GT the solid regions are trimmed out of the domain, as previously explained in Sec. 4.3. Consequently, we can assume

$$\begin{aligned} \rho^{-1} &= \theta \cdot \rho_a^{-1}, \\ K^{-1} &= \theta \cdot K_a^{-1}. \end{aligned} \tag{4.5}$$

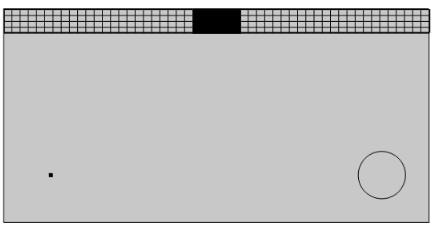
It is relevant to notice that a penalization scheme as SIMP is not necessary with the TOBS-GT method, so it eliminates the dependency on such parameters.

4.3 Geometry trimming and meshing

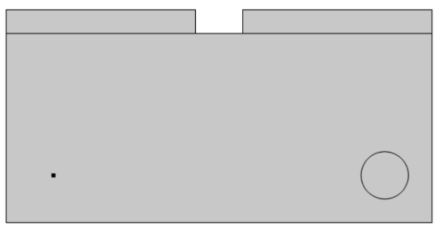
One of the key concepts of the present methodology is the so called geometry trimming technique. The TOBS-GT method was first proposed by Picelli et al. (2022) [20] for fluid-structure interaction problems. The method is based on the separation of the *optimization mesh* and the *FEA mesh*, and consists of removing the solid regions from the analysis domain to accurately model physical phenomena. The optimization mesh is regular structured, defines the design variables, and is immutable during the optimization process; whereas the FEA mesh needs to be generated again at each iteration and is used to compute acoustic pressures and sensitivities. This allows to obtain crispier topologies, as the optimization mesh can be finely refined and the FEA – which is the bottleneck of computational time within the optimization – can maintain the mesh in a certain size with a reasonable computational cost.

Figure 15: An example illustrating some steps of the TOBS-GT method.

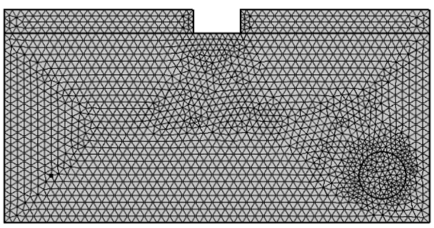
(a) A {0,1} model, with the design domain defined in a regular structured mesh (optimization mesh). The the black region represents solid, and the grey areas represent air.



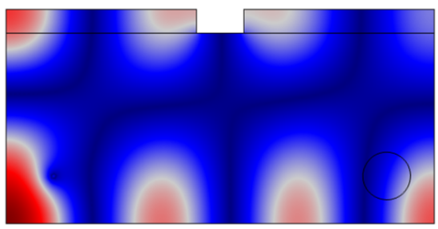
(b) CAD model, with the solid region extracted via geometry trimming.



(c) Freely generated mesh by the FEA software (FEA mesh).



(d) Acoustic pressure distribution computed by the FEA software.



Some illustration of the process is found in Fig. 15, and it goes as follows. First, the optimization mesh and an initial guess {0,1} for the design variables are defined (Fig. 15a) and the contours of the holes (solid regions) are saved as `.dxf`. In the FEA software, a new computer-aided design (CAD) geometry is created by trimming out the holes from the analysis domain (Fig. 15b). This is carried in the geometry building section from COMSOL Multiphysics® with the command `difference`. Next, the geometry is freely meshed by using the option `physics controlled` in COMSOL Multiphysics® (Fig. 15c). The option takes into account some built-in physics requirements when meshing. The forward (Fig. 15d) and adjoint problems are solved, computing acoustic pressures and sensitivities.

In this way, the modeled solid regions are not considered in the simulation. This procedure eliminates the influence of the emulated solid, which is not actually truly solid in the standard density-based approaches. The potential benefits here are higher accuracy and the possibility to obtain sharper topologies without additional computational cost.

4.4 Sensitivity computation

As previously mentioned, the TOBS method is a gradient-based algorithm, thus it requires the computation of the derivatives (sensitivities) of the objective function. The adjoint method is here employed via automatic differentiation at COMSOL Multiphysics® .

The generic adjoint equation is expressed as

$$\left(\frac{\partial \boldsymbol{R}}{\partial \boldsymbol{p}}\right)^T \boldsymbol{\lambda} = -\left(\frac{\partial \Phi}{\partial \boldsymbol{p}}\right)^T, \tag{4.6}$$

where Φ is the objective function, $\boldsymbol{p}$ and $\boldsymbol{\lambda}$ are the vectors of state and adjoint variables, respectively. Considering $\boldsymbol{R}$ as the residual, the sensitivities can be computed as

$$\left(\frac{dL}{d\theta}\right) = \left(\frac{\partial \Phi}{\partial \theta}\right)^T + \boldsymbol{\lambda}^T \left(\frac{\partial \boldsymbol{R}}{\partial \theta}\right). \tag{4.7}$$

This is a general formulation to compute the sensitivities of any function. The sensitivities with respect to the design variables can be exported via `fsens(dtopo1.theta_c)/dvol`. In COMSOL Multiphysics, the variable `theta_c` is used to represent the vector of design variables here expressed by θ_j and `dvol` is the mesh volume scale factor.

The sensitivities are computed using nodal variables at the finite element software and are interpolated to a set of grid points coincident with the optimization mesh. The finite element shape functions can be used to interpolate the sensitivities at such points. The sensitivities for the solid regions are set as zero. Back to the optimization module, the sensitivities are smoothed with a standard spatial filtering, as explained in subsection 3.2.2. The filtered sensitivities are obtained with Eq. 3.13. The filter adopted smoothens out the original sensitivity field and avoids the well known checkerboard problem. Moreover, it populates the void (solid) regions with sensitivities, thus increasing the chances of void elements to return to air.

Nevertheless, the sensitivity filtering might lead to inaccurate assessment of the sensitivities for adding elements (originally void) in regions that are not involved in the finite element analysis. Therefore, Picelli (2020) [72] suggests that, in the TOBS method, in addition to sensitivity filtering, one could use stabilization techniques, as the one proposed by Huang and Xie (2007) [73]. It consists of averaging the sensitivities over two consecutive iterations. However, as ATO problems are highly unstable, here the average of sensitivities over three consecutive iterations was adopted, as

$$\frac{\widetilde{\partial\Phi}^{k}}{\partial\theta_j} \leftarrow \frac{\frac{\widetilde{\partial\Phi}^{k}}{\partial\theta_j} + \frac{\widetilde{\partial\Phi}^{k-1}}{\partial\theta_j} + \frac{\widetilde{\partial\Phi}^{k-2}}{\partial\theta_j}}{3}. \tag{4.8}$$

This procedure favors the generation of more stable results.

4.5 Convergence criteria

The optimization process is concluded when a certain criteria is reached. In the present work, the convergence criteria is based in the method adopted in Huang and Xie (2007). It defines that the convergence is achieved when the objective function is stable for at least $2N$ iterations, that is, the computed error

$$error = \frac{|\sum_{i=1}^{N}(\Phi_{k-i+1} - \Phi_{k-N-i+1})|}{\sum_{i=1}^{N}\Phi_{k-i+1}} \leq \tau \tag{4.9}$$

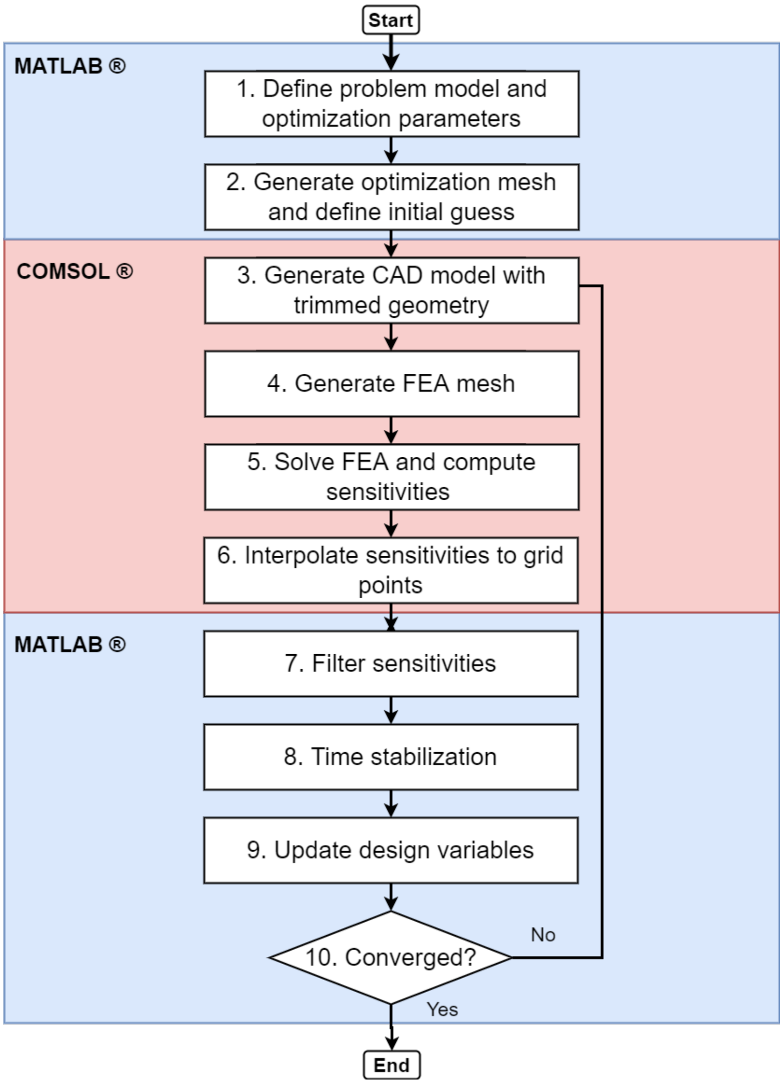
must be lower than the convergence error tolerance τ. Here k is the current iteration number, and N is an integral number, which dictates how many successive iterations are being evaluated. For instance, if $N = 5$, it means that 10 consecutive iterations are being considered. Once this requirement is fulfilled, the final topology is assumed as the optimum solution.

4.6 Computational procedure

The proposed optimization algorithm is illustrated in the flowchart in Fig. 16 and is composed by 10 main steps, as follows:

1. Define the design domain, boundary conditions, optimization parameters (r, τ, β, f) and material properties (ρ and K).
2. Generate optimization mesh and define initial design variables (initial guess).
3. Generate a CAD model with trimmed geometry.
4. Discretize the structure with a finite element mesh, using the option `physics controlled` in COMSOL Multiphysics® .
5. Solve FEA and compute sensitivities.
6. Interpolate sensitivities from FE mesh to the optimization mesh.
7. Filter sensitivities using Eq. 3.13.
8. Employ time stabilization technique (Eq. 4.8).
9. Solve the linearized optimization subproblem from Eq. 3.11 using the ILP solve and update design variables (Eq. 3.12).
10. Evaluate the convergence of the objective function (Eq. 4.9). If converged, stop. If not converged, repeat from step 3.

Figure 16: Flowchart of the proposed methodology algorithm.



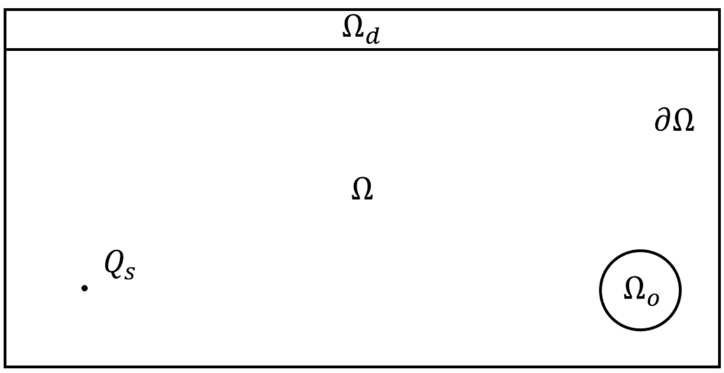
5 RESULTS AND DISCUSSIONS

Two acoustic models were chosen to investigate the effectiveness of the proposed methodology. The first one consists of the minimization of sound in a room, that is a known model in the literature and is here used to validate the methodology. The second model, consists of a device to attenuate the sound emitted from a source. The latter was chosen to test the method for a model with increased complexity.

5.1 Room acoustics

The first application investigated in this work concerns room acoustics, which is, problems related to wave propagation in closed spaces. This sort of problem is relevant for many applications where the construction of acoustically better environments is desirable, as industrial machinery noise control in closed spaces, reducing engine noise in car cabins and so on.

Figure 17: Rectangular room model in 2D, with design domain Ω_d, output domain Ω_o and a point source with volume velocity Q_s.



The problem chosen is a rectangular room in 2D, described by a domain Ω initially filled with air, as illustrated in Fig. 17. The dimensions of the model are indicated in Table 1. The model adopted was inspired by the work of Dühring et al. [3], with the purpose of verifying the results obtained against a reference case. The boundary conditions are a perfectly reflecting surface in the rigid walls of the room

$$\mathbf{n} \cdot (\rho^{-1}\nabla\hat{p}) = 0 \tag{5.1}$$

and a point source

$$\nabla \cdot (\rho^{-1}\nabla\hat{p}) + \omega^2 K^{-1}\hat{p} = -i\omega Q_s \delta(r - r_0). \tag{5.2}$$

A point source is considered a point that radiates sound equally in all directions. The point is located at $r_0 = (2, 2)$, it is vibrating with volume velocity $Q_s = 0.02$ m^2/s and emitting sinusoidal sound waves of angular frequency ω. The normal unit vector pointing out of the domain is represented by $\mathbf{n}$.

Table 1: Room model dimensions in meters.

Description	Value (m)
Room width	18.00
Room height	9.00
Design domain height	1.00
Objective domain center position	(16.00,2.00)
Objective domain radius	1.00
Point source position	(2.00,2.00)

5.1.1 Eigenfrequency and Frequency Response analysis

The first step was to run a eigenfrequency analysis to find out the patterns of vibration of the room. The eigenvalues for rectangular rooms with rigid boundaries in two dimensions can be obtained by [77]

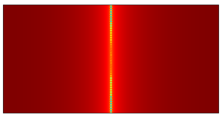
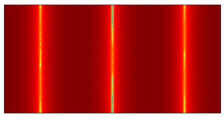
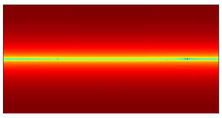
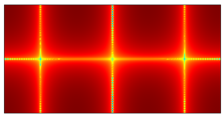
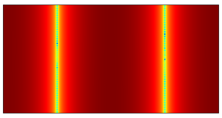
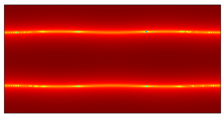
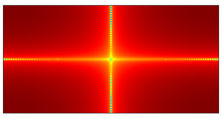
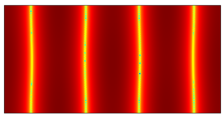
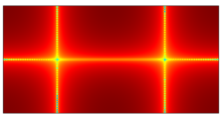
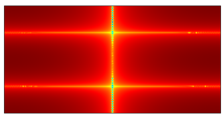
$$k_{n_x n_y} = \pi \left[\left(\frac{n_x}{L_x}\right)^2 + \left(\frac{n_y}{L_y}\right)^2 \right]^{1/2}, \tag{5.3}$$

where n_x and n_y are non-negative integers and $L_x = 18$ and $L_y = 9$ are the dimensions of the room. The eigenfrequencies corresponding to the eigenvalues of Eq. 5.3, which are real because of the boundary condition 5.1, are given by

$$f_{n_x n_y} = \frac{c}{2\pi} k_{n_x n_y}, \tag{5.4}$$

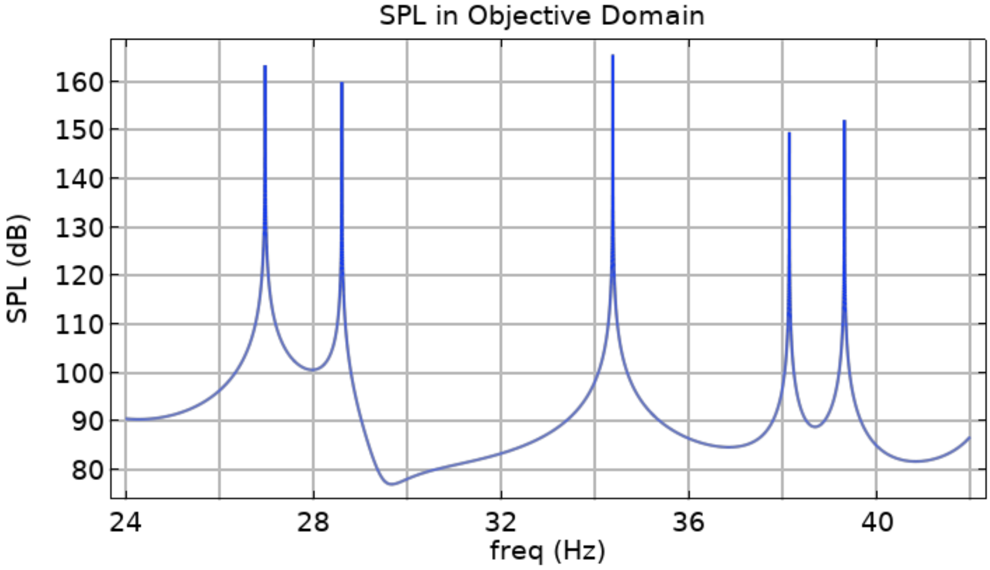
where c is the speed of sound and is given by $c = \sqrt{K/\rho}$. The result of the ten first eigenfrequencies are condensed in Table 2. The analysis was also carried out in the software COMSOL Multiphisics® and the sound pressure level distribution for each eigenfrequency is also presented in Table 2.

Table 2: Eigenfrequencies of the 2D rectangular room model.

f_n (Hz)	n_x	n_y	SPL plot	f_n (Hz)	n_x	n_y	SPL plot
9.537	1	0		28.611	3	0	
19.074	0	1		34.386	3	1	
19.074	2	0		38.148	0	2	
21.325	1	1		38.148	4	0	
26.974	2	1		39.322	1	2	

To assess how the frequencies influence the objective function in the oputput domain, a FRF (Frequency Response Comparison) was performed. The result is presented in Fig. 18. It can be observed, as expected, that the average of the SPL in the objective domain tends to get higher when the frequency is close to the eigenfrequencies of the room.

Figure 18: Frequency response analysis for the initial room design.



5.1.2 Target frequency analysis

The optimization is performed for a single frequency, so a study was made to analyse how the method performs for different target frequencies. The frequency range around the seventh eigenfrequency (third peak in Fig. 18) was chosen for this study. Two TO were performed for target frequencies of $f = 34.35$ Hz and $f = 34.4$ Hz, which are respectively slightly lower and higher than the seventh natural frequency (34.386 Hz) for the initial design of the room, and the results were compared. The initial design is a domain filled with air ($\theta = 1$) and the parameters used for both cases were $r = 0.3$ m, $\beta = 0.03$ and $\tau = 0.001$. The optimization mesh was set to 720×40 (28.800 elements).

The optimization history, with the objective function and air volume fraction, for the case with target frequency $f = 34.35$ Hz is presented in Fig. 19. The solver gradually decreases the objective function by adding solid material in the design domain (reducing the air volume fraction) until about iteration 15. From that point, the objective function slightly increases, and the algorithm circumvents this by removing solid material until it reaches convergence, in iteration 23. The SPL was reduced from 118.64 dB to 76.29 dB in the output domain and the final volume fraction was 53.1% of air in the design domain. Figure 20 shows SPL plots for the initial and final designs. It can be observed that the material was distributed between the nodal lines, at the high sound pressure of the initial design, and well defined structures were obtained.

For the second case studied, with a target frequency of $f = 34.4$ Hz, the SPL decreased

Figure 19: Optimization history for the case of the room with f = 34.35 Hz, with the objective function in the left axis and the air volume fraction in the right axis.

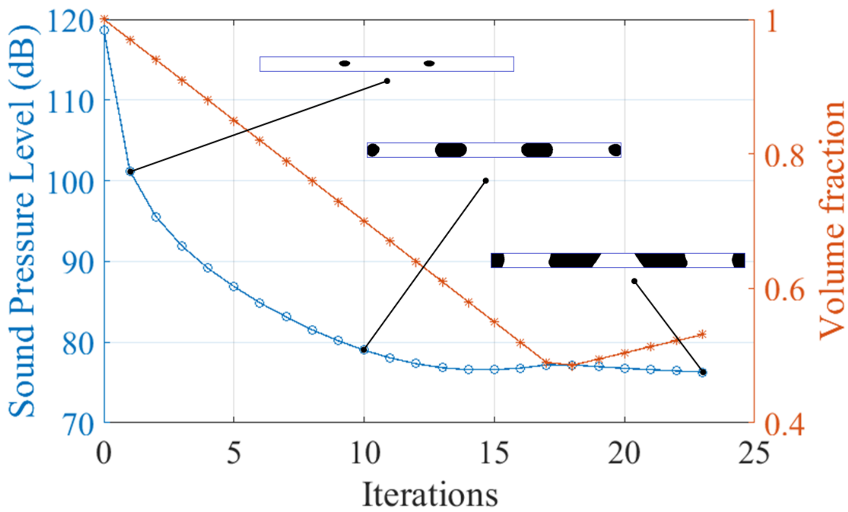
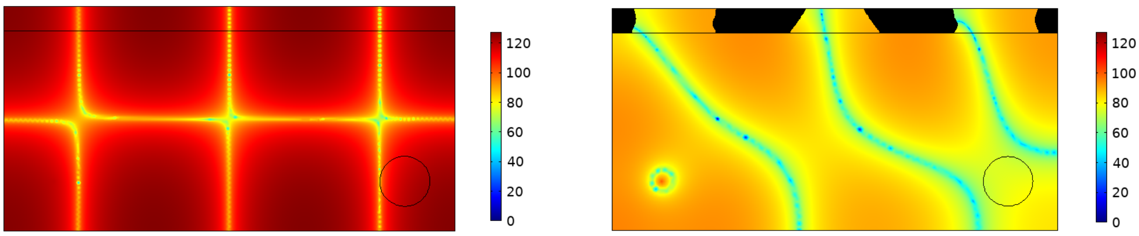


Figure 20: Sound Pressure Level plots for the case of the room with f = 34.35 Hz, where the black regions represent solid material and the colorbar defines the SPL scale in dB.

(a) Initial design (b) Final design



from 126.7 dB in the initial design to 87.1 dB with the optimized design and the final volume fraction was 59% of air. Regarding the location of solid material distribution, it can be noted that the opposite happened in comparison to the previous case, as here the material was deposited in the nodal lines of the initial domain (see Fig. 22). The optimization met the convergence criteria in 18 iterations and the optimization history for this problem is depicted in Fig. 21. The optimization process was very stable, as the objective function gradually decreased until convergence. The air volume fraction declined linearly until the fourteenth iteration, and then increased a little until the final iteration.

When comparing both cases, the first case presented here achieved a lower objective function value. Figure 23 also suggests that the objective function is minimized most for both target frequencies in the case where the target frequency is smaller than the resonance frequency. Table 3 shows how the different designs obtained behave for the different target frequencies used.

Figure 21: Optimization history for the case of the room with f = 34.4 Hz, with the objective function in the left axis and the air volume fraction in the right axis.

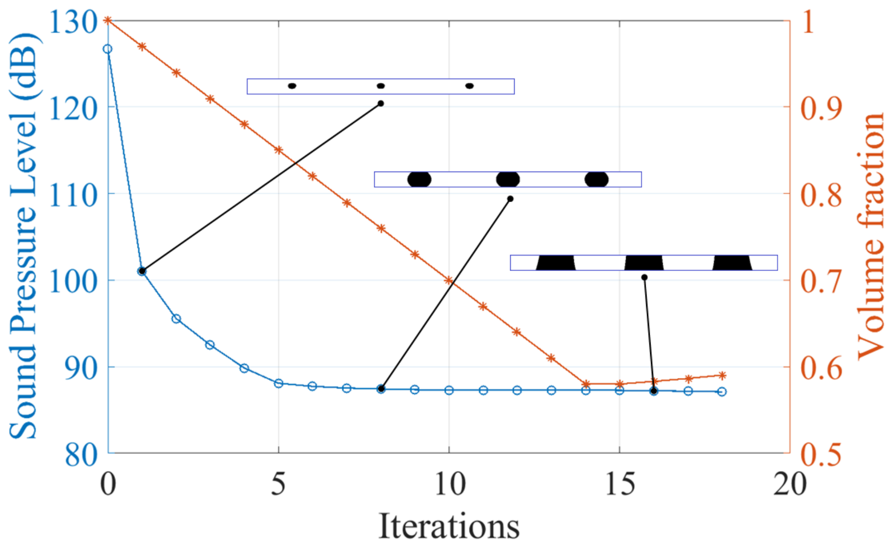
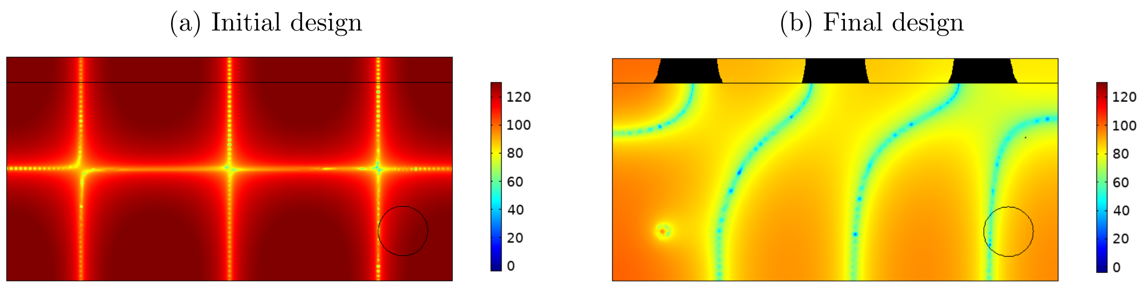


Figure 22: Sound Pressure Level plots for the case of the room with f = 34.4 Hz, where the black regions represent solid material and the colorbar defines the SPL scale in dB.

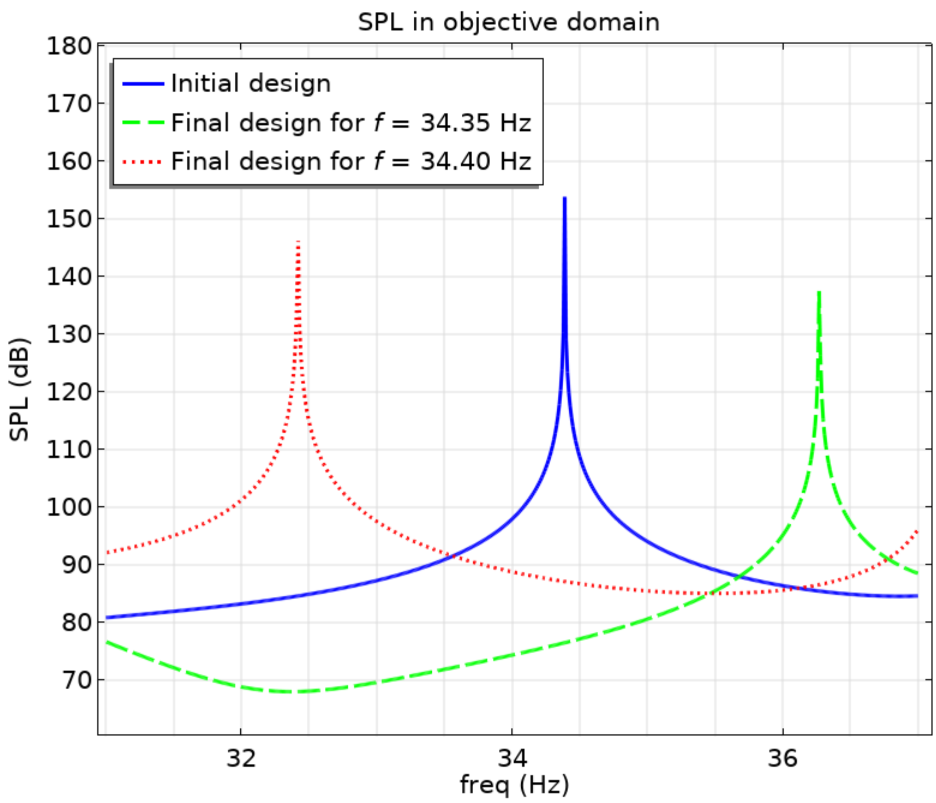


It is noted that for the optimization performed with the target frequency smaller than the natural frequency, there was a tendency for material to be added in the higher pressure regions. This caused the natural frequency to move to a higher value. In contrast, when the target frequency is higher than the natural frequency, the material is placed on the nodal lines and causes the natural frequency to shift to a lower value. This aspects can be clearly observed in Fig. 23. As this phenomena occurs, it seem like the final objective function value is related to the “valley” between the resonance frequencies. For instance, the lowest value of the objective function between the sixth and seventh eigenfrequencies (second and third peaks in Fig. 18) is in the 70-80 dB range, and the final Φ value for the problem with the target frequency smaller than the seventh eigenfrequency was also in that range (76.29 dB). The same happened for the other case, as the “valley” was in the 80-90 dB range and the final Φ value obtained was 87.102 dB.

Table 3: Objective function value for the frequencies 34.35 and 34.40 Hz for the two designs from Fig. 20b and 22b.

Frequency f (Hz)	Φ for optimized design for $f = 34.35$ Hz	Φ for optimized design for $f = 34.40$ Hz
34.35	76.290	87.287
34.40	76.580	87.102

Figure 23: Frequency response comparison for the initial and optimized room designs.



5.1.3 Optimization parameters analysis

In this section it is intended to investigate how the optimization parameters influence the solver and the final solution. In this case, the parameters studied are the size of the radius of the sensitivity filter r, and the truncation error constraint parameter β.

The case analysed is the one presented in the previous section, with the target frequency $\omega = 34.4$ Hz.

- **Truncation error constraint parameter:**

As previously explained, the TOBS method requires small changes between iterations to control the truncation error and maintain the linear approximation valid. For this purpose, the β parameter is introduced to control the number of elements that change from solid to void and vise versa at each iteration. Figure 24 displays the air volume fraction history during the optimization for three different β values and Fig. 25 shows the final topologies obtained.

Figure 24: Optimization histories of the air volume fraction using different β's.

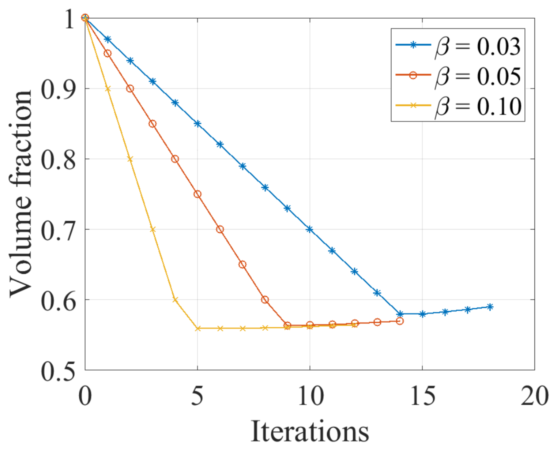
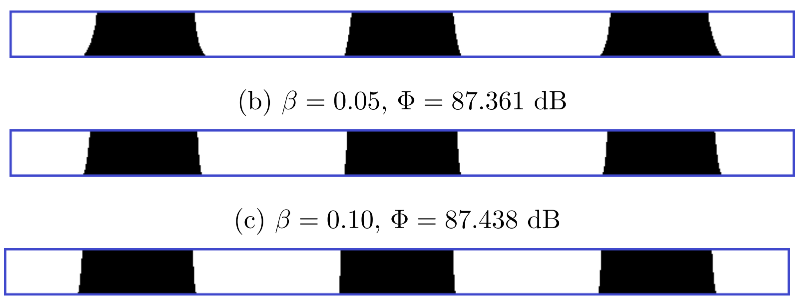


Figure 25: Final topologies obtained using different β's.

(a) $\beta = 0.03$, $\Phi = 87.102$ dB

(b) $\beta = 0.05$, $\Phi = 87.361$ dB

(c) $\beta = 0.10$, $\Phi = 87.438$ dB



It can be conclude from Fig. 24 that this parameter dictates the pace on which the solver removes the solid elements from the design domain, and consequently it also influences on how fast it converges to the optimum solution. The lowest value $\beta = 0.03$ required 18 iterations for convergence, while $\beta = 0.05$ and $\beta = 0.10$ demanded 14 and

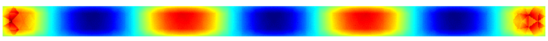
12 iterations, respectively. Although higher β values lead to faster convergence, it results in a more straight-like shape (see Fig. 25) and a slightly higher final objective function value. One can consider the case with $\beta = 0.03$ as the best result, as it yields better local minima and a structure with a more refined outline. The optimizer gets unstable when β is too small and fails to find a feasible solution.

- **Filter radius:**

The sensitivity filter concept was introduced in section 3.2.2 and herein it is intended to investigate how different radius size affect the results. The sizes used in the analysis were $r = 0.3$ m, $r = 0.6$ m and $r = 1.2$ m. The sensitivities field for the initial guess, unfiltered and filtered with the mentioned radius sizes is presented in 26. The filter can smooth the sensitivities field, but when the size is too large it deforms and mischaracterizes the field. Usually, the TOBS method requires a filter large enough to encompasses just a few elements. But from this analysis we can conclude that, for acoustic problems, a larger filter is necessary. When the radius is too small, the solver gets to a point where it starts to add sparsely small amounts of solid material, thus resulting in irregular geometries, and the solver gets unstable and difficult to reach the convergence criteria.

Figure 26: Sensitivity filter effect.

(a) Unfiltered sensitivities



(b) Sensitivities filtered with $r = 0.3$ m

(c) Sensitivities filtered with $r = 0.6$ m

(d) Sensitivities filtered with $r = 1.2$ m

Figure 27: Final topologies obtained using different r's.

(a) $r = 0.3$ m, $\Phi = 87.102$ dB

(b) $r = 0.6$ m, $\Phi = 87.315$ dB

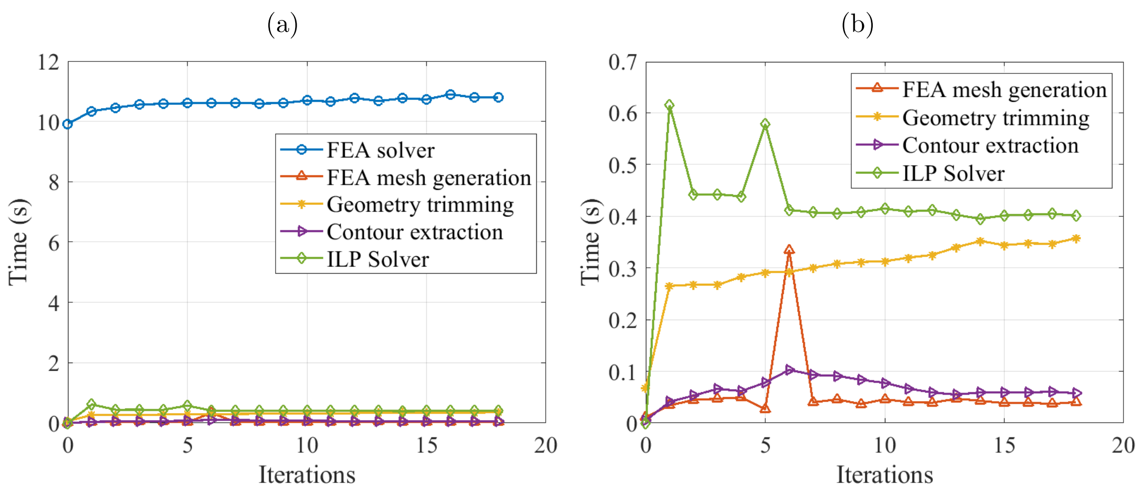
(c) $r = 1.2$ m, $\Phi = 87.403$ dB

Figure 27 shows the final topologies obtained for these cases. The effect observed here is very similar to the previous analysis, of the β parameter, where higher values generate more straight-like shapes with a worse objective function result. Consequently, there seems to be an inverse relation between the β and the r parameters. As the usage of a small value for one of these parameters, allows the increase of the other guaranteeing the same quality of result.

5.1.4 Computational time study

The separation of the FEA and optimization meshes – one of the main features of the TOBS-GT method – can increase the computational efficiency. This is attainable due to the possibility to use a very refined optimization mesh – to obtain crispier topologies - and a less refined FEA mesh – to reduce the computational time. In this section the computational time of the main steps of the optimization is analysed.

Figure 28: Computational time breakdown for the case with $f = 34.4$ Hz, $\beta = 0.03$, $r = 0.3$ m and a 720×40 optimization mesh (a) for all main steps and (b) omitting the FEA solver times.



The case analysed settings was a target frequency of $f = 34.4$ Hz, a truncation error constraint parameter of $\beta = 0.03$, a filter radius of $r = 0.3$ m and an optimization mesh set to 720×40 (28,800 elements). The FEA mesh is generated again at each iteration, using the option `physics controlled` in COMSOL Multiphysics® with the `element size` option set to `extra fine`. The initial and final FEA meshes were composed by $3,566$ and $14,283$ elements, respectively. It is important to highlight that the optimization mesh comprises only the design domain, while the FEA mesh comprises the entire domain. Considering only the design domain, the initial and final FEA meshes were discretized respectively into 402 and 4,188 elements.

The total running time was 20.0499 minutes, using an Intel Xeon Silver 4114 - 2x CPU 2.20 GHz - 128 GB RAM. Figure 28 presents the the breakdown computation times for the main steps. It can be seen that the FEA required a significant computational time when compared to the remaining steps, as expected for TO methods. The FEA took an average of 10.614 seconds per iteration, including both forward and adjoint problems. This relatively small value is due to the simplicity of the proposed problem, but it still keeps the FEA as the bottleneck of the optimization. All the other steps required less than a second to compute. The second most time-consuming step is the ILP solver. Despite the uncommon usage of integer linear programming in topology optimization, this step is fairly cheap, taking an average of 0.433 seconds per iteration. The contour extraction usually takes up to 0.1 seconds while the geometry trimming takes an average of 0.3 seconds. The FEA mesh has to be generated again each iteration, and the time it takes increase with the complexity of the structure during optimization. This illustrates that the proposed TOBS-GT method can be a relatively cheap design method for acoustic problems.

5.2 Acoustic attenuator

This example shows a different acoustic problem consisting of a device that could be used to attenuate the sound emitted by a source. The model is axisymmetric and, in contrast with the previous example, the domain is not a completely closed space. An infinite domain in granted by the Perfectly Matched Layer (PML) feature and the sound waves are uniform and equally strong in all directions. Figure 29 illustrates the axisymmetric CAD model and table 4 compiles its dimensions. In Fig. 29, the blue dashed line depicts the axial symmetry and the red circle located in the origin represents the monopole point source. The design domain is delimited by the red rectangle and the blue region represents the waveguide - a region that must remain free for the sound waves to flow through it, i.e. a non-design domain.

A frequency response analysis is presented in Fig. 30. Unlike the previous room example, for this case the peaks of the FRF did not match the eigenfrequencies of the model. Either way, two frequencies, before and after one of the peaks were chosen for this study. Each optimization is performed for a single frequency and the target frequencies chosen for this analysis were 380 and 400 Hz. For both analysis the optimization mesh was set to 30 x 200 elements, and the results are presented next.

Figure 29: Computational model for the attenuator case.

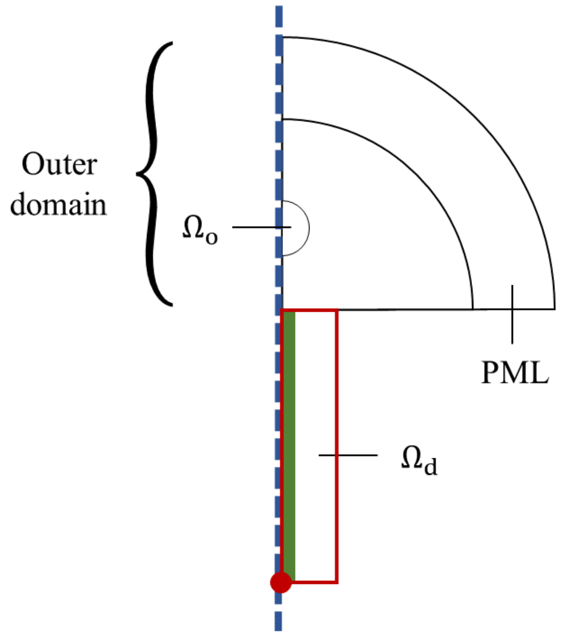


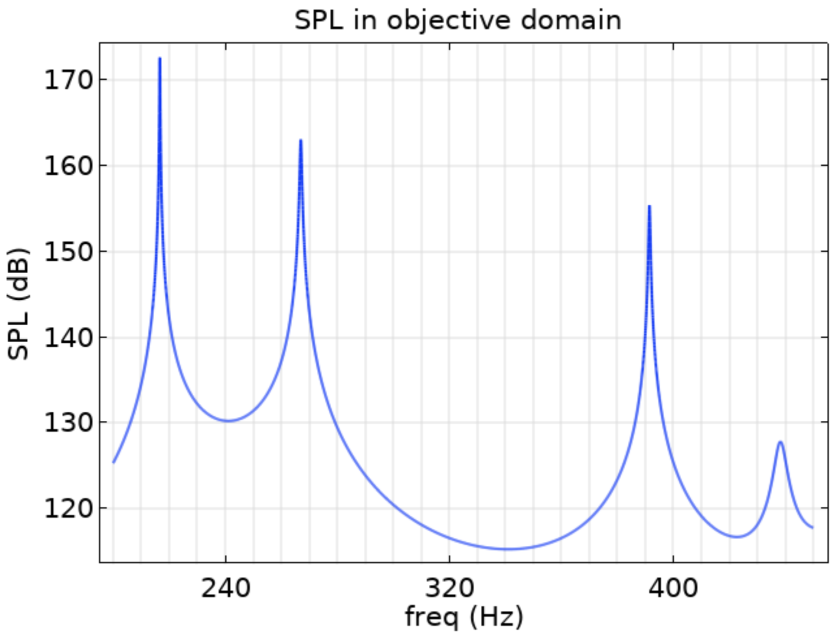
Table 4: Attenuator model dimensions in meters.

Description	Value (m)
Design domain height	1.00
Design domain width	0.15
Non-design domain width	0.05
Objective domain center position	(0.00,1.30)
Objective domain radius	0.10
Outer domain	1.00
PML thickness	0.30

5.2.1 Target frequency of 380 Hz

For this case, the parameters used were a filter radius $r = 1.2$ m and a truncation error constraint parameter $\beta = 0.01$. The optimization process run very smoothly, as it can be observed in Fig. 31. In the first iterations two structures were created and, from iteration 28 a third structure appears in the bottom, but almost completely vanishes until

Figure 30: Frequency response analysis for the attenuator model.



the last iteration. The process took 36 iterations and the objective function reduced from 123.41 dB to 76.12 dB and the final volume was composed by 71.2% of air in the design domain, including the waveguide region. The SPL plots for the initial and final designs can be seen in Fig. 32.

5.2.2 Target frequency of 400 Hz

The parameters used in this example were the same as the previous one: a filter radius $r = 1.2$ m and a truncation error constraint parameter $\beta = 0.01$. The optimization took 34 iterations to reach the convergence criteria, and the SPL and volume fraction histories are illustrated in Fig. 33. It can be observed that the solver gradually decreases the objective function by adding two structures in the design domain. After iteration 23 until the end, there is an oscillating pattern, that is caused by the repetitive attachment and detachment of the structures to the right wall. If you look closely to Fig. 33 you can see the top and middle structures detached at iteration 26, and the bottom structure detached at iteration 29. It is important to notice that even with this oscillation, the convergence criteria was met and in the final topology all structures are connected to the right wall. A similar oscillating issue, for ATO with a binary method is reported in the literature [78]. The objective function reduced from 125.45 dB to 48.18 dB and the final volume fraction is composed by 74.8% of air in the design domain, including the waveguide region. The SPL plots for the initial and final designs can be seen in Fig. 34.

Figure 31: Optimization history for the attenuator model with a target frequency of 380 Hz, with the objective function in the left axis and the air volume fraction in the right axis.

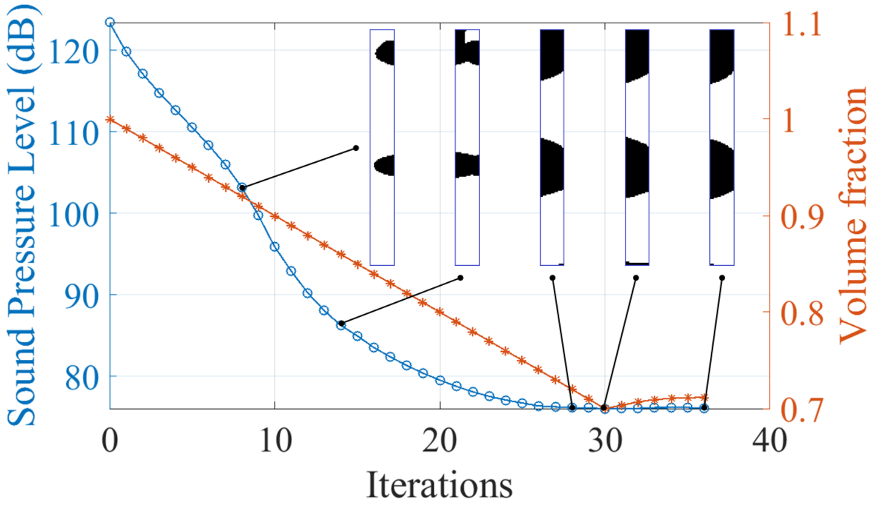
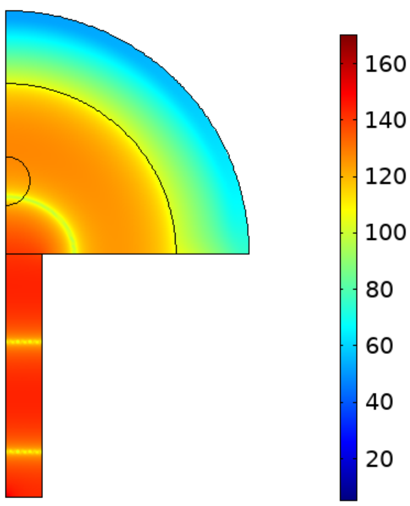


Figure 32: Sound Pressure Level plots for attenuator model with f = 380 Hz, where the colorbar defines the SPL scale in dB.

(a) Initial design



(b) Final design

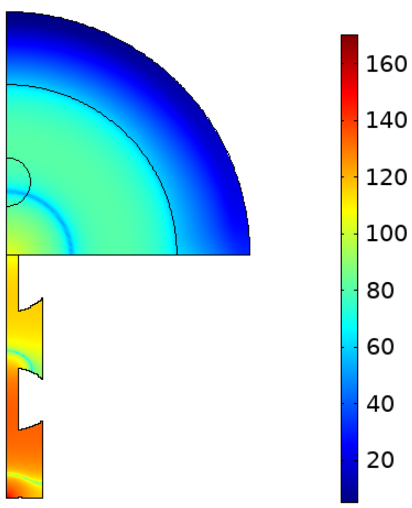


Figure 33: Optimization history for the attenuator model with a target frequency of 400 Hz, with the objective function in the left axis and the air volume fraction in the right axis.

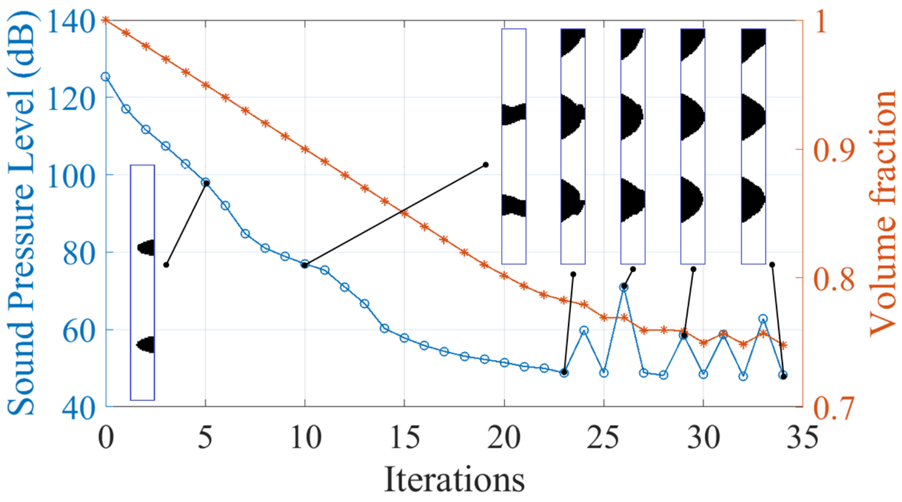
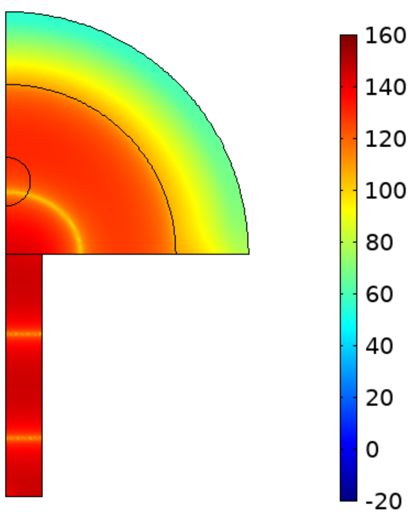
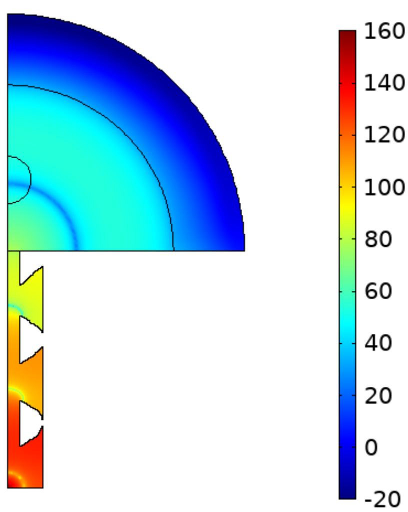


Figure 34: Sound Pressure Level plots for attenuator model with f = 400 Hz, where the colorbar defines the SPL scale in dB.

(a) Initial design



(b) Final design



6 CONCLUSIONS

The most relevant contribution of this work is the extension of the TOBS-GT method to acoustic optimization problems. In this scheme, the separation of the FEA and optimization meshes guarantees accurate modelling of the physical phenomena. The proposed formulation effectively solved the numerical examples and avoided common issues related to classical TO approaches, such as intermediate densities and mesh dependency, and dependency on material interpolation, as in this methodology it is not necessary to penalize intermediate densities.

The binary characteristic of the method favored the computational cost and took a small number of iterations until meeting convergence criteria. For the numerical examples presented in this dissertation, the highest number of iterations presented was 23 for the 2D model and 36 for the axysimmetric model, which is quite small when compared to other methods. In comparison with the FEA, the proposed methodology does not represent a substantial increase in the total computation time for the problems studied. The resulting topologies not only show a significant decrease in the objective function, but also were feasible and could be manufactured for an industry application.

6.1 Closing remarks and future work

In short, the potential of this methodology for solving acoustic problems is substantial. There are many different paths to continue and deepen this research field, either to increase its complexity or to improve the effectiveness of the method. Among the available research topics, it is worth suggesting:

- to increase problem complexity by performing TO of more realistic cases, implementing three-dimensional models, and including other physics in addition to acoustics (e.g. acoustic-structure);
- to perform the optimization with distinctions in the objective function, such as

minimization of the SPL for a range of frequencies instead of one single frequency, maximization problems, multi-objective function, different acoustic properties as the objective function (e.g. transmission loss, reflections back into the waveguide caused by a horn, etc.)

- to develop new techniques to further reduce the instability during the optimization process.

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