EDUARDO STEVE RODRIGUEZ CANALES

Formation Static Output Control of Linear Multi-Agent Systems With Hidden Markov Switching Network Topologies

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RESUMO

Esta dissertação aborda o controle estático de saída de formação $\mathcal{H}_2$, $\mathcal{H}_\infty$ e $\mathcal{H}_2/\mathcal{H}_\infty$ misto para sistemas lineares multiagentes a tempo contínuo com topologias de rede de comutação Markoviana. Supõe-se que o modo de operação da topologia da rede não pode ser medido diretamente, mas em vez disso, pode ser estimado por um detector imperfeito. Para modelar este problema consideramos um modelo de Markov oculto em tempo contínuo, no qual o componente oculto representa o modo real de operação da topologia da rede enquanto o componente observado representa a informação emitida pelo detector e disponível para o controlador. Também é assumido que apenas uma informação parcial das variáveis de estado dos sistemas multiagente está disponível. Usando uma formulação LMI (linear matrix inequality), um controlador de saída estático distribuído que muda de acordo com as informações do detector é projetado para garantir a estabilidade no sentido da média quadrática do sistema de malha fechada, bem como um limite superior para um índice de desempenho. Três situações são consideradas para os critérios de desempenho: a norma $\mathcal{H}_2$, a norma $\mathcal{H}_\infty$ e a norma mista $\mathcal{H}_2/\mathcal{H}_\infty$. O trabalho é concluído com exemplos numéricos para ilustrar a eficácia dos resultados teóricos.

Palavras-Chave – Controle de formação $\mathcal{H}_2$, $\mathcal{H}_\infty$ e $\mathcal{H}_2/\mathcal{H}_\infty$ misto, topologias de comutação Markoviana, desigualdades matriciais lineares, sistema linear multiagente, controle estático de saída.

ABSTRACT

This dissertation addresses the $\mathcal{H}_2$, $\mathcal{H}_\infty$ and mixed $\mathcal{H}_2/\mathcal{H}_\infty$ formation static output control of continuous-time linear multi-agent systems with Markovian switching network topologies. It is assumed that the mode of operation of the network topology cannot be directly measured but, instead, can be estimated by an imperfect detector. To model this problem we consider a continuous-time hidden Markov model, in which the hidden component represents the real mode of operation of the network topology while the observed component represents the information emitted from the detector and available for the controller. It is also assumed that only a partial information from the state variables of the multi-agent systems is available. By using an LMI (linear matrix inequality) formulation, a distributed static output controller which switches according to the detector information is designed to guarantee the stability in the mean square sense of the closed-loop system as well as an upper bound for an index performance. Three situations are considered for the performance criteria: the $\mathcal{H}_2$ norm, the $\mathcal{H}_\infty$ norm, and the mixed $\mathcal{H}_2/\mathcal{H}_\infty$ norms. This work is concluded with numerical examples to illustrate the effectiveness of the theoretical results.

Keywords –$\mathcal{H}_2$, $\mathcal{H}_\infty$ and mixed $\mathcal{H}_2/\mathcal{H}_\infty$ formation control, hidden Markov switching topologies, linear matrix inequalities, linear multi-agent system, static output control.

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LIST OF ABBREVIATIONS

MAS	Multi-agent system
MJLS	Markov jump linear system
TVF	Time-varying formation
LMI	Linear matrix inequality
MSS	Mean square stability
SS	Stochastic stability

LIST OF SYMBOLS

x,x_i	State, i-th agent state
u,u_i	Control input, i-th control input
w,w_i	Exogenous disturbance, i-th exogenous disturbance
z,z_i	Controlled output, i-th controlled output
y,y_i	Measured output, i-th measured output
A, B, C, D, E, F, L	System matrices
v	Number of agents
$\mathbb{B}(\mathbb{X}, \mathbb{Y})$	Banach space $\mathbb{X}$ into $\mathbb{Y}$
$\|\|.\|\|$	Uniform induced norm
$\mathscr{T}$	Linear Operator
$\mathbb{R}^n$	n-dimensional real euclidian space
A'	Transpose of matrix A
$tr(A)$	Trace of matrix A
$Her(A)$	$A + A'$
$A \otimes B$	Kronecker product of matrix A and matrix B
Ω	Sample space
$\mathscr{F}$	Filtration
Pr	Probability function
$\mathbf{E}$	Mathematical expectation
$\mathcal{G}$	Graph
$\mathbb{V}$	Vertice
$\mathcal{E}$	Edge
d_i	i-th agent degree
$\mathcal{N}_i$	Neighbors of i-th agent
$\mathcal{A}$	Adjacency matrix
$\mathcal{D}$	Degree matrix

$\mathcal{L}$	Laplacian matrix
I_n	n-dimensional indentity matrix
$\mathbf{1}_n$	n-dimensional column vector with all ones
$\theta(t)$	Markov chain
$\widehat{\theta}(t)$	Hidden Markov chain
$\nu_{(pk)(rl)}$	Transition rate
$h(t)$	Formation vector
$\delta(t)$	Average error
γ	$\mathcal{H}_\infty$ performance level
φ	$\mathcal{H}_2$ performance level

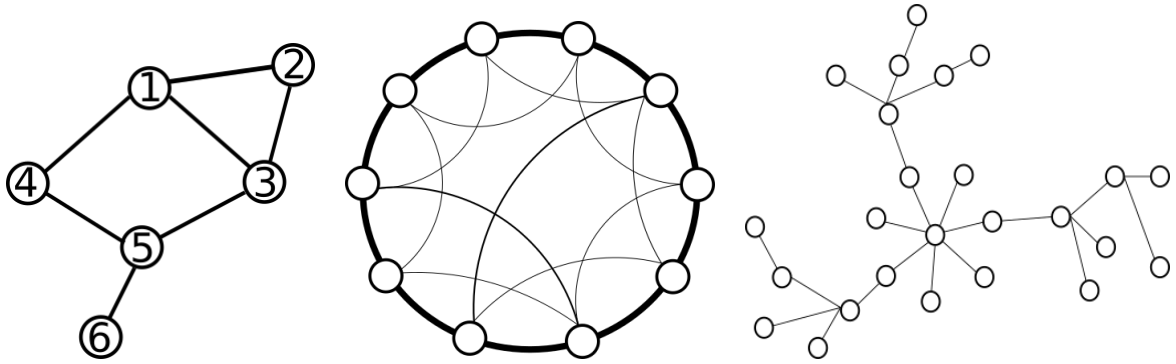
CONTENTS

1 INTRODUCTION

The synchronization of collective systems is a behavior inherent to nature. Because of that, the study of networks of coupled dynamical systems has long aroused the curiosity and interest of the academic community. It is possible to imagine these phenomena as a large set of individuals or agents with their own dynamics that, when interacting with each other, generate a network with new dynamic properties; this approach is known as complex system dynamics (BARRAT; BARTHéLEMY; VESPIGNANI, 2008). The dynamics of complex systems are currently studied in different fields of human knowledge such as sociology, biology, physics, chemistry, and engineering. One that interests us particularly is biology, where the organization of life occurs in all its scales, from cells and microorganisms to entire ecosystems. These relationships have been favored by natural selection since their origins. It is interesting how animals instinctively tend to generate synchronized networks; we can imagine, for example, a flock of birds migrating due to seasonal changes or a fish school swimming in groups to confuse predators (see Figure 1). Related to the collective motion of animal groups (REYNOLDS, 1987), three parameters that allow its characterization are proposed: collision avoidance, velocity matching, and flock centering.

(a) Flock birds in migration. (b) Fish school.

Figure 1: Collective motion of animal groups.



(a) Regular network. (b) Small-world network. (c) Scale-free network.

Figure 2: Network architectures.

The study and characterization of cooperative networks are based on graph theory as shown in Figure 2a and, together with other areas of mathematics such as statistics and dynamic systems, allow modeling the topology of different types of complex networks such as neurons, oscillators, internet, etc. (STROGATZ, 2001). Depending on the process being investigated, these networks can have purely deterministic, chaotic, or stochastic dynamics. A well-known case is that of small-world networks (WATTS; STROGATZ, 1998) presented in Figure 2b, a simple arrangement in which the nodes have a probability between 0 and 1 of being connected, this causes network configurations of short paths between them and a large clustering coefficient. In real networks, some nodes are highly connected than others; this approach shown in Figure 2c is known as scale-free networks (BARABÁSI; BONABEAU, 2003), in this type of network, by using a statistical distribution, it is possible to model their growth in more realistic conditions, the internet, power systems, and social networks are examples of this type of systems.

In the last decades, the study and application of automatic control systems have allowed the development of new, more efficient and safest technologies in various fields such as aeronautics, energy distribution, medicine, industry, etc. Currently, advances in hardware with more compact and powerful computers have allowed the implementation of new control strategies that address systems with non-linear behavior and multiple variables. Robust control strategies have also been developed to ensure the stability and performance of systems under uncertainties, unmodeled dynamics, disturbances and measurement noises (SKOGESTAD; POSTLETHWAITE, 2007), a general control structure is shown in Figure 3.

Recently, the need to optimize the operation of network systems such as power sys-

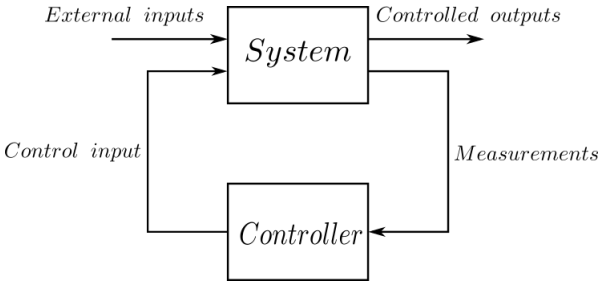


Figure 3: General control configuration.

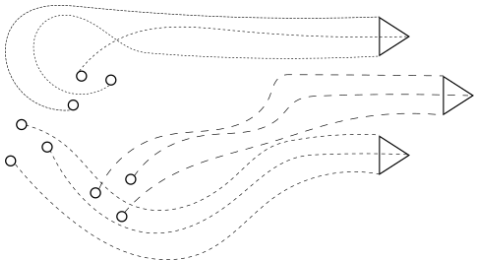


Figure 4: Multi-agent formation.

tems, unmanned vehicles or sensor networks (BUTENKO; MURPHEY; PARDALOS, 2013) have inspired research that addresses the problem of cooperative control, achieving interesting results from different approaches such as multi-agent system (MAS) control, distributed systems, control of networks, swarm systems, etc. (SHAMMA, 2007). As part of the large and diverse field of study that cooperative systems represent is the formation control, which can be described by three factors: the agents, the communication between them and their geographical position; taking this into account, it is possible to develop formation strategies that allow agents to follow trajectories while maintaining predefined geometric patterns, also known as topologies (AHN, 2020), in the Figure 4 the formation convergence of twelve agents in a topology formed by three triangular groups is shown.

Due to changes in the environmental conditions or transmission failures, it is reason-

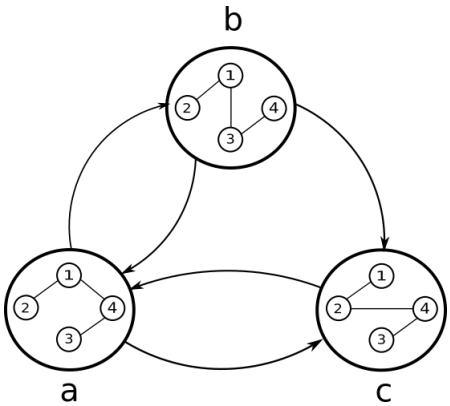


Figure 5: Formation switching topologies modeled as a Markov chain.

able to consider scenarios in which the formations need to vary without losing stability, this problem known as time-varying formation (TVF) control is receiving particular attention from the academic community (DONG et al., 2016; RAHIMI; ABDOLLAHI; NAQSHI, 2014; ANTONELLI et al., 2014). In practice, the events mentioned can happen randomly; it is possible to model these dynamic processes as a Markov jump linear system (MJLS) (COSTA; FRAGOSO; TODOROV, 2012) to deal with this. A MJLS can be described as a linear system with a certain number of states, these states can pass from one to another randomly, and the probability of changing to the next state depends only on the current state. In the case of MAS formations, it is possible to imagine the states of the Markov chain as possible configurations in the formation network topology as seen in Figure 5.

Inspired by the above discussions and bearing in mind the current studies of MAS and MJLS; this work addresses the time-varying formation control of MAS, where the topology switching is modeled in the framework of MJLS with partial observations focused on the *detector approach* formulation presented in (STADTMANN; COSTA, 2017); the so-called hidden Markov model allows to represent the case of "asynchronous behavior" as in (NGUYEN; KIM, 2020). We try to formulate sufficient conditions based on LMI to synthesize controllers in the framework of $\mathcal{H}_\infty$ and $\mathcal{H}_2$ control and investigate the mixed $\mathcal{H}_2/\mathcal{H}_\infty$ control problem based on these results.

2 LITERATURE REVIEW

This chapter presents a literature review to provide a perspective about the cooperative control of multi-agent systems under Markovian switching topologies. First, we address the literature concerning the Markov Jump Linear Systems, considering the cases of systems with complete observations and systems with partial observations modeled as hidden MJLS. Next, a review of cooperative control on multi-agent systems is presented, taking into account different approaches such as consensus control, leader-following control and formation control. Finally, recent studies about the control of multi-agent systems with Markovian switching network topologies are considered.

2.1 Markov Jump Linear Systems

Research on Markov Jump Linear Systems began in the early 1960s, a time in which researchers were interested in studying the stability of systems with random parameters as a function of time. The well-known Lyapunov stability theory was used to set stability conditions for this type of stochastic systems (KATS; KRASOVSKII, 1960), taking advantage of developments in computing, optimal control theory and dynamic programming (BELLMAN, 1954; KALMAN et al., 1960). Using Bellman's dynamic programming concepts, a solution to the optimal control problem in partial differential equations is presented for dynamical systems with Markov processes. For the special case of linear systems with Gaussian random components, a computable solution is defined by (FLORENTIN, 1961) starting the formal study of Markovian jump linear systems.

2.1.1 The complete observation case

The case of complete observation was the first one to be studied; in this approach, the system states and mode of operation are considered known (SWORDER, 1969). The optimal control of discrete-time MJLS was studied for the case of complete observations, a jump linear quadratic (JLQ) control law was proposed by using coupled Riccati

equations to stabilize the system in the infinite time horizon (CHIZECK; WILLSKY; CASTANON, 1986). Similarly, for continuous-time MJLS, the necessary and sufficient conditions to computing a JQL control law were established in the Lyapunov stochastic stability framework (JI; CHIZECK, 1990). Recently, modern control approaches have been applied in the study of robust stability of MJLS, such is the case of (FARIAS et al., 2000), in this work, the stability conditions in terms of LMI for the output feedback control of continuous-time MJLS are studied, and expressions are presented for achieving stability in the mean square sense by solving both the $\mathcal{H}_\infty$ problem and the $\mathcal{H}_2$ problem. In (CHENG; ZHANG, 2006) is presented a stabilization approach of MJLS based on adaptive control, with a switching law designed from quadratic Lyapunov functions. The switching transition rates case is tackled in (BOLZERN; COLANERI; NICOLAO, 2010), characterizing the transition rates as piecewise-constants, sufficient LMI conditions are proposed to guarantee the mean square stability in the dwell-time between switching instants. In (BOLZERN; COLANERI; NICOLAO, 2014) the stochastic stability of positive Markov jump linear systems (PMJLS) is addressed, by using a novel notion of stability (Exponential mean stability), sufficient conditions with different conservatism levels are studied. The problem of synthesizing controllers that fulfill multiple performance criteria has drawn a great deal of attention in the literature. A useful framework in this direction is the mixed $\mathcal{H}_2/\mathcal{H}_\infty$ control problem, which combines the minimization of a quadratic functional, related to the $\mathcal{H}_2$ control while ensuring some degree of robustness to the closed-loop system, the $\mathcal{H}_\infty$ control problem (CHEN; ZHANG, 2004; MA; ZHANG; HOU, 2012; HUANG; ZHANG; FENG, 2008). Another interesting formulation for the mixed $\mathcal{H}_2/\mathcal{H}_\infty$ problem is based on game theory associated to Nash equilibrium between two performance indexes, as presented, for instance in (ZHU; ZHANG; BIN, 2014), in which it is desired to minimize the output energy for the given control law whenever the worst-case disturbance is applied to a MJLS governed by Ito-type equation.

Due to the possibility of representing systems with abrupt changes, MJLS has many modeling and control applications. Using MJLS, it is possible to model economic systems in continuous-time and to calculate the optimal regulation policy (BLAIR; SWORDER, 1975). There are also applications related to the modeling and control of aerial vehicles, such as the modeling of flight systems susceptible to electromagnetic disturbances and the control of flight dynamics related to wing deployment (GRAY; GONZALEZ, 1998; STOICA; YAESH, 2002). Aerospace applications, such as the formation of satellites with communications noise, were also studied (MESKIN; KHORASANI, 2009).

2.1.2 The partial observation case

Because the controller may not always be able to access the states' or modes of operation, the case of partial observation is studied in the literature. Such is the case of continuous-time MJLS with parametric uncertainties, which is expressed in the framework of $\mathcal{H}_\infty$ control by using matrix inequalities (FARIAS; GEROMEL; VAL, 2002). The principle of separation is studied in (COSTA; TUESTA, 2003) by using two coupled Riccati equations, one for controlling the state variable once it is available and the other for filtering; the optimal control problem is solved. Robust control strategies were also proposed for the case of partially known transition rates through characterization by LMI to solve the general problem and the problem of uncertainties in the system and transition matrices (XIONG et al., 2005; ZHANG; BOUKAS, 2009). The mode of operation may be unknown; this would imply that the controller assigned for a certain mode of operation would not be appropriate to ensure the system's stability. This problem is addressed by assuming that the controller can only access random samples characterized by a hidden Markov process, by using LMI, the $\mathcal{H}_\infty$ and $\mathcal{H}_2$ control problems are solved (OGURA et al., 2018). Another approach is the use of an output feedback controller to solve the infinite horizon problem, the necessary conditions for the MJLS stability are set up transforming it, from an optimal problem to a $\mathcal{H}_2$ control problem, and by an iteration algorithm, the controller can be estimated (DOLGOV; HANEBECK, 2017).

From the practical point of view, the controller may not always have access to the mode of operation of the system (the Markov parameter $\theta(t)$), so that it is important to consider the case of partial observations. For continuous-time MJLS this has been analyzed by considering an exponential hidden Markov approach in (STADTMANN; COSTA, 2017, 2018; OLIVEIRA et al., 2020; OLIVEIRA; COSTA, 2021b) for the $\mathcal{H}_2$ state-feedback, $\mathcal{H}_\infty$ static output feedback control problems, and for the mixed $\mathcal{H}_2/\mathcal{H}_\infty$ dynamic control problem. In these cases the controller relies only on the information coming from a detector device (represented by $\widehat{\theta}(t)$), and that the joint process $Z(t) = (\theta(t), \widehat{\theta}(t))$ is an exponential hidden Markov chain, with $\widehat{\theta}(t)$ being the observable part.

As far as the author is aware, there are not many experimental results in the literature concerning the control of hidden MJLS, for instead in (VARGAS; COSTA; VAL, 2013) the authors address the problem of controlling a DC Motor subject to abrupt power failures considering it a discrete MJLS with no mode observation, using a variational method to obtain a gain matrix that satisfies the optimal condition.

2.2 Multi-Agent Systems Distributed Control

In the last two decades, the study of multi-agent systems (MAS) has become relevant. A MAS can be defined as a group of coupled autonomous entities performing complex cooperative tasks based on interaction with their neighbors. These tasks can be executed in parallel and asynchronously between agents, with advantages such as robustness, scalability, efficiency, and planning (DURFEE; LESSER; CORKILL, 1989; DORRI; KANHERE; JURDAK, 2018). Currently, the control of MAS can be divided into two groups: centralized control and distributed control. In reduced MAS, the centralized control approach is easier to design and implement, but as the system scales, the central station's processing and communications load increases rapidly (LI; TAN, 2019). For controlling complex MAS, the distributed control approach shows improved efficiency, reliability, and performance when compared to the centralized control approach (TSITSIKLIS; ATHANS, 1984).

2.2.1 MAS consensus control

The consensus control is the essence of distributed control of MAS; since the introduction of the Laplacian matrix by the authors of (MURRAY, 2002), they have introduced an approach to transform the consensus problem from qualitative description to the theoretical analysis based on graph theory. Under this framework, it is possible to design control protocols that guarantee the equilibrium and convergence of MAS (FAX; MURRAY, 2002; OLFATI-SABER; MURRAY, 2004). The consensus problem has been tackled from different directions. For instance, one important issue is the communication constraints since the distributed control approach is based on the connection between agents. In (YANG; BERTOZZI; WANG, 2008) the consensus problem for second-order systems with time-delay is addressed by frequency analysis; a condition of convergence is proposed for second-order MAS and generalized for high order MAS. The uncertainties and time-delay are tackled by (LIU; JIA, 2011), by using a LMI formulation, a $\mathcal{H}_\infty$ controller for MAS is designed for a given disturbance attenuation level. In (WANG et al., 2014) the optimal consensus control problem is solved for MAS with time-delay and disturbances; by using a Kalman filter to obtain the states and solving a set of Riccati equations, a feedforward-feedback optimal control protocol is presented. The stochastic control framework is introduced in (ZONG; LI; ZHANG, 2019) for consensus control of MAS with measurement noise and time-delay; sufficient conditions for stochastic consensus are provided by using a Lyapunov functional. In (WANG et al., 2020), the consensus

for MAS with additive noise is addressed, the Lyapunov equation solution allows to derive a control protocol for the bounded consensus. The event-triggered mean square consensus control with measurement multiplicative noise is studied in (CONG; MU; HU, 2021).

Some improvements to the consensus problem have been developed to replicate practical cases. The leader-following consensus control addresses the MAS consensus between one leader and multiple followers; the objective is to achieve the leader dynamics via feedback control (LIU; CHEN, 2011). In (MA; LI; ZHANG, 2010) the leader-following control problem is solved for MAS systems with measurement noises; by using stochastic analysis and graph theory, a controller is designed for the cases of fixed and switching topologies. In (XIAO; SHI; LI, 2015) the leader-following consensus for second-order MAS with switching topologies is considered via the average dwell time approach, and the necessary condition for the agents reaching leader-following consensus is obtained. The leader-following consensus for MAS under measurement noises and communication time-delay is addressed in (ZHANG et al., 2018b) by using a Lyapunov functional, sufficient conditions for mean square stability with additive and multiplicative noise are provided. An interesting approach is the leader-following consensus control of heterogeneous MAS, in (XIAO; CHEN, 2017) this problem is tackled for general linear systems.

In practice, the agents must be able to realize multiple parallel tasks, the group consensus control addresses the design of control protocols to achieve different consensus values in a MAS divided into groups or clusters, in (FENG; XU; ZHANG, 2014) the group consensus problem is investigated for double-integrator MAS with fixed communication topology. In (HUANG et al., 2015) the group consensus control for heterogeneous systems is addressed, by using Lyapunov functions, sufficient conditions to achieve the group consensus are obtained. In (REN; LIU; SUN, 2020), the authors deal with the $\mathcal{H}_\infty$ group consensus control for linear MAS with external disturbance under directed switching topologies; the problem can be solved if a correct L_2 gain and an appropriate intra-cluster coupling are chosen. For systems with highly complex clustering behavior, the concepts of multi-consensus and multi-tracking are introduced, in (LI; GUAN; CHEN, 2015), a feedback controller for nonlinear MAS is designed to drive the system to achieve multi-consensus. In (ZHANG et al., 2018a), the robust multi-tracking consensus problem for MAS with uncertainties and disturbances is solved by providing a distributed impulsive control protocol. The multi-tracking consensus in MAS under intermittent communication is studied in (HUANG et al., 2021), the authors tackled the problem with a novel pinning control protocol and a Lyapunov functional to provide sufficient conditions to consensus under a dual subsystem framework.

There exist several and diverse applications for MAS consensus control, for instead in (WANG; DING; SHENG, 2014) the consensus control of multiple networked robots with input delays are implemented and validated by experimental results, in (ZHONGHE et al., 2013) a feedback controller is designed to reduce the traffic congestion in urban road traffic networks, the control of energy storage in microgrids is addressed in (KHAZAEI; MIAO, 2016) to synchronize the state-of-charge (SoC) and power levels of batteries with limited information exchange, in (GÜZEY; DUMLU, 2018) the nonlinear consensus synchronizing control for networked DC motors to tracking a desired position or velocity is developed, also, the consensus control of MAS is continuously applied in the distributed control of unmanned vehicle groups (SHI et al., 2017; ZHANG et al., 2019; KADA; KHALID; SHAIKH, 2020).

2.2.2 MAS formation control

In the MAS distributed cooperative control framework, the formation control is one of the most actively studied topics, the main objective is to drive the agents to the desired behavior, based on the sensing ability and interaction topologies (OH; PARK; AHN, 2015). The most basic formation control scheme is position-based formation control. In this approach, the sensing capability of the MAS and a global coordinate system is essential because each agent needs to know their absolute position to keep the formation, in (REN; ATKINS, 2007) a distributed position-based protocol control with double-integrator agents is developed. A similar concept is treated in (DONG; FARRELL, 2008), with the design of a position-based control protocol for nonholonomic mobile agents under communication delay.

The global coordinate system is not always accessible for the multi-agent system. In this case, the displacement-based formation control shows some improvements, and the MAS only needs to have access to the local coordinate system; in contrast, the interaction with neighbors is necessary to know their relative positions (REN; BEARD; MCLAIN, 2005; LIN; FRANCIS; MAGGIORE, 2007). In (WU; SUN; WANG, 2015) the displacement-based formation control is addressed for multi-agent systems with size scaling by assuming the formation description and the relativity velocities are known to all the agents. In (YAO; LIU; HUANG, 2018) the authors deal with the displacement-based control problem to maintain a circular formation of unicycle-type agents; by using a Lyapunov design, a control protocol is presented for time-invariant formation. For MAS systems with energy consumption constraints, in (BABAZADEH; SELMIC, 2018) the displacement-based formation optimal control problem is solved by using a state-

dependent Riccati equation (SDRE) method and weighting cost matrices.

If the agents cannot access their local coordinate system, the distance-based control becomes a good alternative for MAS distributed formation. In this approach, it is necessary to set the distances between between the neighbors and consider the entire formation as a rigid body. In (DIMAROGONAS; JOHANSSON, 2008) the stability of a distance-based formation control is tackled, by using a tree graph structure, a negative gradient control law is provided. The case of nonholonomic agents is addressed in (BAROGH; WERNER, 2017); in this work, the asymptotically stability is achieved for MAS distance-based formation with collision avoidance. In (HOU; YU, 2018) the authors deal with the distance-based formation control with hybrid communication topology, which means that directed and undirected edges form the formation graph. The distance-based formation control with exogenous disturbance is addressed in (BAE; LIM; AHN, 2020), by using an adaptive gradient controller, the local stability is guaranteed. Other approaches include, for instead, the flocking control based on Reynolds rules for collective behavior (LEI; LI, 2008; CAO et al., 2010), the angle-based distributed formation control based on bearing measurements (JING; CAIXIA; MEIJIN, 2015; JING et al., 2019; CHEN et al., 2020) and the containment control, in which the follower agents are driven to formation by autonomous leaders (DONG et al., 2014, 2015; HU; BHOWMICK; LANZON, 2020).

The distributed formation control has several applications in real engineering problems, specially, in unmanned vehicle control. Such is the case of the formation control of mobile robots (YANGYANG; YUPING, 2007; BAZOULA; MAAREF, 2007; DU; YANG; JIA, 2016), cooperative missions of unmanned aerial vehicles (DIERKS; JAGANNATHAN, 2009; ZHU et al., 2017; ALI; SHAFIQ; FARHI, 2018) or marine autonomous vehicles (HUANG et al., 2016; YU; FU, 2018; LIU; HU; WANG, 2021), also, can be founded in aerospace applications such as satellites (WOOLFSON, 2004; GAUTAM; SOH; CHU, 2008) and spacecrafts (GAO; LV; WANG, 2011; WU; CAO, 2018).

2.3 Markovian Switching Networked Systems

Several authors have been using a Markov chain formulation for the MAS topology. As a sample of these works, we can mention (PARK et al., 2014), which considers the problem of leader-following consensus stability and stabilization for multi-agent systems with interval time-varying delays and Markovian switching interconnection information among agents, by using a Lyapunov Krasovskii functional, a set of LMI based consensus stability conditions are provided. In (LI; MU, 2020), the authors analyze the leader-

following consensus of MAS with random switching topologies, where the dwell time in each topology consists of a fixed part and a random part, and a semi-Markov process models the topology switching signal in the random part. In (ZHOU et al., 2019), a distributed formation control based on a modified integral sliding mode (ISM) controller is designed to deal with bounded accelerations and disturbances for a group of quadrotor unmanned aerial vehicles (UAV), which a finite-time Markov chain models the switching topologies with partially unknown transition rates. (DING; GUO, 2015) deals with the sampled-data leader-following consensus with Markovian switching network topologies and delay communication by employing a Lyapunov-Krasovskii functional, and the weak infinitesimal operation a stability criterion is derived, which ensures that the consensus of nonlinear multi-agent systems can be globally exponentially achieved in mean-square sense.

In (HE; MU; MU, 2020) the authors tackle the $\mathcal{H}_\infty$ leader-following consensus problem for nonlinear MAS under semi-Markovian switching topologies, by using stochastic techniques, sufficient conditions are derived to achieving a performance $\mathcal{H}_\infty$ index despite external perturbations and partially unknown transition rates. In (SHANG, 2016) the authors deal with the stochastic consensus problem for MAS over Markovian switching networks with a fixed maximal allowable upper bound of time-varying delays and topology uncertainties which are not caused by the Markov process. (GE; HAN, 2017) addresses the consensus problem for a MAS with Markovian network topologies, external disturbance and partial observation of system states by using an overlapping set of shared modes approach, a distributed control protocol that relies only on a specific group and partial modes is designed to achieve the consensus under a prescribed $\mathcal{H}_\infty$ performance level. (CONG; MU, 2019) investigates the $\mathcal{H}_\infty$ consensus of MAS where the switching network topologies are modeled by a semi-Markov model and there exist multiplicative measurement noises in the information that each agent receives of its neighbors.

In (XUE et al., 2013), a formation controller is designed in terms of LMI and a Lyapunov functional considering the nonlinear dynamics of each agent and potential functions to stabilize a time-delay system, in which a finite-time Markov process governs the switch between communication network topologies. (LI et al., 2020) analyzes the TVF control of MAS with communication noises described as independent white noises, where the communication topology switches from several different topologies following a Markov chain, a stochastic control protocol is given to achieve stability in the mean square sense. (NGUYEN; KIM, 2020) deals with the problem of leader-following consensus for MAS with an asynchronous control mode of the Markov parameter by using a

mode-dependent Lyapunov function and a relaxation process, sufficient conditions for the stochastic leader-following consensus are given. In (DONG et al., 2020) it is investigated the $\mathcal{H}_\infty$ output consensus problem for MAS with Markov jumps and external disturbance in both continuous-time and discrete-time domains by considering an output feedback controller based on a hidden Markov model. Within the discrete-time set up, (ZHANG et al., 2016) studies the $\mathcal{H}_\infty$ consensus control problem for MAS with switching network topologies subject to a heterogeneous Markov chain, under the hypothesis of partial information exchange among neighbor agents and (MO; GUO; YU, 2018) addresses the velocity-constrained mean-square consensus problem of heterogeneous MAS with Markovian switching topologies and time-delay, which consist of first-order and second-order agents.

2.4 Contribution and Structure

The contributions of this dissertation are mainly concerned with the design of formation controllers for MAS under Markovian switching topologies with partial observations and are summarized as follows:

- We propose design conditions in terms of LMI for the synthesis of an static output TVF controller for MAS *that depends only on* $\widehat{\theta}(t)$ and such that the closed-loop MAS is mean square stable with $\mathcal{H}_\infty$ norm less than a given $\gamma > 0$.
- Similarly as above, we also treat the $\mathcal{H}_2$ case, and propose design conditions such that the $\mathcal{H}_2$ norm of the closed-loop MAS is less than a given $\varphi > 0$.
- By combining the previous results we tackle the mixed $\mathcal{H}_2/\mathcal{H}_\infty$ case.
- We illustrate our results by means of numerical examples of a TVF control of a MAS consisting of six agents.

The structure of this work is briefly described below:

- In Chapter 3, we introduce basic concepts about the stability and calculation of the $\mathcal{H}_\infty$ and $\mathcal{H}_2$ norms of MJLS, as well as some topics in MAS, including graph theory and protocol formulation for the consensus and formation problem.
- In Chapter 4, the concept of hidden MJLS is addressed, we describe some possible cases for control systems, depending only on the observed state $\widehat{\theta}(t)$, that can be modeled with this approach.

- In Chapter 5, we design a formation control protocol for MAS with partial observations on the network. Then, we present sufficient conditions in LMI for the design of $\mathcal{H}_2$ and $\mathcal{H}_\infty$ static output feedback controllers for MAS formation with switching network topologies depending only on the observed variable $\widehat{\theta}(t)$, and if jointly solved, allows a mixed $\mathcal{H}_2/\mathcal{H}_\infty$ controller formulation.
- Finally, in Chapter 6, we conclude with some numerical examples to study the effectiveness of the proposed method. $\mathcal{H}_2$, $\mathcal{H}_\infty$ and mixed $\mathcal{H}_2/\mathcal{H}_\infty$ controllers are studied for the formation control of a linear MAS formed by six agents with possible mismatching between the detector and network operation modes.

The results presented in Chapters 5 and 6 were published in the IEEE Access Journal (RODRIGUEZ-CANALES; COSTA, 2021).

3 PRELIMINARIES

This chapter presents some necessary concepts for the derivation of the results. First, the notation used in this work is described. Then, we introduce some definitions related to graph theory and Markov processes. Finally, the control problem is formulated in the stochastic control framework, and some results in optimal control are presented.

3.1 Notation

Spaces

For the Banach spaces $\mathbb{X}$ and $\mathbb{Y}$, we define $\mathbb{B}(\mathbb{X}, \mathbb{Y})$ as the space of all bounded linear operators with uniform induced norm denoted by $||.||$. $\mathbb{R}^n$ represents the the n-dimensional real Euclidian space and $\mathbb{R}^+$ the interval $[0, \infty)$. The bounded linear space of all $m \times n$ real matrices is denoted by $\mathbb{B}(\mathbb{R}^n, \mathbb{R}^m)$, with $\mathbb{B}(\mathbb{R}^n) \triangleq \mathbb{B}(\mathbb{R}^n, \mathbb{R}^n)$. For N and M positive integers we set $\mathscr{N} \triangleq \{1, \ldots, N\}$, $\mathscr{M} \triangleq \{1, \ldots, M\}$ and $\mathscr{V} \subseteq \mathscr{N} \times \mathscr{M}$.

Matrices

Consider the matrix A. The transpose of the matrix A is represented by A', if $A = A'$ thus A is a symmetric matrix. The eigenvalues of a matrix A are denoted by $\lambda_i(A)$. $A < 0$ and $A \leq 0$ denote a negative definite and semi-definite matrix, respectively, which means that the eigenvalues are strictly negative. The trace is denoted by $tr(A) = \sum_{i=1}^{n} a_{ii}$ where a_{ii} denotes the entry on the ith row and ith column of A and $Her(A) = A + A'$. The Schur Complement says that

$$M = \begin{bmatrix} Q & S \\ S' & R \end{bmatrix} < 0,$$

if and only if $Q < 0$ *and* $R - S'Q^{-1}S < 0$,

if and only if $R < 0$ *and* $Q - SR^{-1}S' < 0$.

Blocks induced by symmetry in a square matrix is represent by $\bullet$

$$\begin{bmatrix} A & B \\ B' & C \end{bmatrix} = \begin{bmatrix} A & \bullet \\ B' & C \end{bmatrix}.$$

For $A = [a_{ij}] \in \mathbb{R}^{mxn}$ and $B = [b_{ij}] \in \mathbb{R}^{pxq}$ the Kronecker product of A and B is

$$A \otimes B = \begin{bmatrix} a_{11}B & a_{12}B & \dots & a_{1n}B \\ a_{21}B & a_{22}B & \dots & a_{2n}B \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1}B & a_{m2}B & \dots & a_{mn}B \end{bmatrix}.$$

The identity matrix of size $n \times n$ is given by I_n and $\mathbf{1}_n$ denotes the n-dimensional column vector with all ones. In what follows let $\widetilde{B}$ be the orthogonal complement matrix of the row space of a matrix B, so that $B\widetilde{B} = 0$. We introduce the following version of the Finsler lemma (see (FINSLER, 1937)) that will be needed later in the results demonstration.

Lemma 3.1.1. *The following statements are equivalent:*

a) $\widetilde{B}'A\widetilde{B} > 0$.

b) $A + XB + B'X' > 0$ *for some matrix* X.

Stochastic processes: The probability space is defined by $(\Omega, \mathscr{F}, \Pr)$, with a right-continuous filtration $\mathscr{F}_t$. $\mathbf{E}(\cdot)$ denotes the mathematical expectation with respect to Pr and $L_2^r(\Omega, \mathscr{F}, \Pr)$ (or just L_2^r for simplicity) the set of square integrable stochastic processes $z = \{z(t) \in \mathbb{R}^r, t \in \mathbb{R}^+\}$ with $z(t)$ being $\mathscr{F}_t$-measurable for each $t \in \mathbb{R}^+$. In this case we set $\|z\|_2^2 = \int_0^\infty \mathbf{E}(\|z(t)\|^2)dt$. Finally, the Dirac measure over the set $A \in \mathscr{F}$ is denoted by $\mathbb{1}_A(\cdot)$ such as

$$\mathbb{1}_A(\omega) = \begin{cases} 1 & \text{if } \omega \in A \\ 0 & \text{otherwise.} \end{cases} \tag{3.1}$$

3.2 Multi-Agent Systems

3.2.1 Graph theory

A graph represented by $\mathcal{G}(\mathbb{V}, \mathcal{E})$ (REN; CAO, 2010) is a set of objects where the vertices (also known as nodes or agents) $\mathbb{V}$ are linked by the edges $\mathcal{E} \subseteq \mathbb{V} \times \mathbb{V}$. The objects

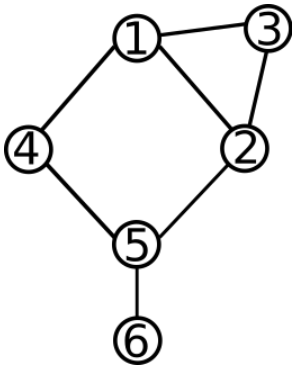


Figure 6: Undirected graph with 6 vertices and 7 edges.

in the graph coincides with the number of vertices and are represented by $\mathbb{V} = \{1, \ldots, v\}$. The graphs where the edges are unordered pair of vertices $(i, j) \in \mathcal{E} = (j, i) \in \mathcal{E}$ are known as undirected graphs as shown in the Figure 6.

Two nodes are adjacent if a node have a edge with another node. The neighbors of a node i are all those that have an edge coming from node i denoted by $\mathcal{N}_i$ and the degree of this node is represented by $d_i = |\mathcal{N}_i|$, where

$$\mathcal{N}_i = \{j \in \mathbb{V}, (i, j) \in \mathcal{E}\}.$$

Let $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{vxv}$ be the adjacency matrix with

$$a_{ij} = \begin{cases} 1 & \text{if } i \neq j \text{ and } (i, j) \in \mathcal{E} \\ 0 & \text{otherwise.} \end{cases} \tag{3.2}$$

Let $\mathcal{D} = diag(d_i, \ldots, d_v) \in \mathbb{R}^{v \times v}$ be the degree matrix with $d_i = \sum_{j=1}^{v} a_{ij}$. The Laplacian matrix of the graph $\mathcal{G}$ is defined as $\mathcal{L} = \mathcal{D} - \mathcal{A}$ where

$$l_{ij} = \begin{cases} d_i & \text{if } i = j \\ -1 & \text{if } i \neq j \text{ and } (i, j) \in \mathcal{E} \\ 0 & \text{otherwise.} \end{cases} \tag{3.3}$$

Example 1. *Taking the graph in Figure 6 as an example, we have that the degree, adjacency and Laplacian matrix are*

$$\mathcal{D} = \begin{bmatrix} 3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad \mathcal{A} = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}, \quad \mathcal{L} = \begin{bmatrix} 3 & -1 & -1 & -1 & 0 & 0 \\ -1 & 3 & -1 & 0 & -1 & 0 \\ -1 & -1 & 2 & 0 & 0 & 0 \\ -1 & 0 & 0 & 2 & -1 & 0 \\ 0 & -1 & 0 & -1 & 3 & -1 \\ 0 & 0 & 0 & 0 & -1 & 1 \end{bmatrix}.$$

3.2.2 Consensus

An important result in MAS control is the consensus algorithm. The objective of this algorithm is that all agents achieve the same state as their neighbors (REN; CAO, 2010). This simple paradigm can be extended to more complex problems; to illustrate this, we consider a group of v agents with a single-integrator dynamic given by

$$\dot{x}_i(t) = u_i(t), \quad i = 1, \ldots, v, \tag{3.4}$$

where $\dot{x}_i(t)$ and $u_i(t)$ are the state and control input of the i-th agent, respectively. Assuming all agents are one dimension for simplicity, a basic consensus algorithm has the form

$$u_i(t) = \sum_{j=1}^{v} a_{ij}[x_i(t) - x_j(t)], \tag{3.5}$$

where a_{ij} is the (i, j)-th term of the adjacency matrix $\mathcal{A}$. The consensus is achieved if for all $i, j = 1, \ldots, v$, the system satisfies $|x_i(t) - x_j(t)| \to 0$ as $t \to \infty$.

3.2.3 Formation

Considering the multi-agent system described in (3.4), we introduce a predefined vector $h(t) = [h_1^T(t), \ldots, h_v^T(t)] \in \mathbb{R}^v$ representing the desired time-varying formation (TVF) (DONG; HU, 2016) and the following TVF algorithm

$$u_i(t) = \sum_{j=1}^{v} a_{ij}[(x_j(t) - h_j(t)) - (x_i(t) - h_i(t))]. \tag{3.6}$$

The multi-agent system (3.4) is said to achieve time-varying formation if $|x_i(t) - h_i(t) - x_j(t) - h_j(t)| \to 0$ as $t \to 0$ for all $i, j = 1, \ldots, v$. Note that when $h(t) \equiv 0$, the multi-agent system (3.4) achieves consensus if it achieves formation. Therefore, for the system described in (3.4) the consensus problem is a special case of the formation problem.

3.3 Markov Jump Linear Systems

Previously, we mentioned that systems subject to abrupt changes could be modeled as Markov jump linear systems. This case can be considered a set of modes subjected to an exponential distribution for the jump times between events, also called continuous-time Markov chains. Combining linear systems with continuous-time Markov chains is a continuous-time Markov jump linear system (CT-MJLS). This section presents the

mathematical formulation of the Markov process and a static output control formulation for MJLS.

3.3.1 Markov process

Let $(\Omega, \mathscr{F}, \Pr)$ be a complete probability space, for the Markov chain, we also define the set $\mathscr{N} = \{1, \ldots, N\}$ where N is a positive integer. The set $\mathscr{N}$ defines the operations modes of the linear system denoted by $\theta(t)$, where $\theta(t) \in \mathscr{N}, t \in \mathbb{R}^+$. The transition probability of the continuous-time Markov chain is defined as follows (COSTA; FRAGOSO; TODOROV, 2012):

$$\Pr\{\theta(t+h) = r | \theta(t) = p\} = \begin{cases} \lambda_{pr}h + o(h), & p \neq r \\ 1 + \lambda_{pp}h + o(h), & p = r, \end{cases} \tag{3.7}$$

where $\Pr[\cdot]$ is the probability measure, $p, r \in \mathscr{N}$ such that $\theta(t) = p$ if the p-th topology is chosen at time instant t, $o(h)$ denotes a function such that $\lim_{h\to 0} o(h)/h = 0$, λ_{pr} is the transition rate from p to r with $\lambda_{pr} \geq 0$ if $(p \neq r)$ and $\lambda_{pp} = -\sum_{p\neq r} \lambda_{pr}$. The transition matrix $\mathit{\Pi} = [\lambda_{pr}]_{1\leq r,p\leq N} \in \mathbb{R}^{NxN}$ of $\{\theta(t), t \geq 0\}$ is given by

$$\mathit{\Pi} = \begin{bmatrix} \lambda_{11} & \lambda_{12} & \ldots & \lambda_{1N} \\ \lambda_{21} & \lambda_{22} & \ldots & \lambda_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \lambda_{N1} & \lambda_{N2} & \ldots & \lambda_{NN} \end{bmatrix}. \tag{3.8}$$

3.3.2 Static output feedback control of MJLS

On the probability space $(\Omega, \mathscr{F}, \Pr)$, with $\mathscr{F}_t$ a right-continuous filtration, we consider a continuous-time Markov jump linear system (MJLS) given by:

$$\mathcal{G} : \begin{cases} \dot{x}(t) & = A_{\theta(t)}x(t) + B_{\theta(t)}u(t) + E_{\theta(t)}w(t) \\ y(t) & = F_{\theta(t)}x(t) \end{cases} \tag{3.9}$$

where $x(t) \in \mathbb{R}^n$ denotes the vector of states, $u(t) \in \mathbb{R}^{n_u}$ denotes the vector of control, $w(t) \in \mathbb{R}^{n_w}$ an external disturbance, $y(t) \in \mathbb{R}^{n_y}$ the measured output, and $\{\theta(t)\}$ is a Markov chain taking values in the set $\mathscr{N}$ and with transition rates λ_{pr}, with $\lambda_{pr} \geq 0$ for all $p \neq r$. All matrices are considered to be of compatible dimensions.

The feedback control law depends only on the variables $y(t)$ and $\theta(t)$, so that it takes

the form

$$u(t) = K_{\theta(t)} y(t). \tag{3.10}$$

By applying (3.10) into (3.9), we get that the closed loop system is given by

$$\mathcal{G}_{cl} : \begin{cases} \dot{x}(t) &= \widetilde{A}_{\theta(t)} x(t) + E_{\theta(t)} w(t) \\ y(t) &= F_{\theta(t)} x(t) \end{cases}, \tag{3.11}$$

where

$$\widetilde{A}_{\theta(t)} = A_{\theta(t)} + B_{\theta(t)} K_{\theta(t)} F_{\theta(t)}. \tag{3.12}$$

3.4 Results

In this section, some results on the stability and optimal control of MJLS are presented.

3.4.1 Stability

Let us consider the reduced linear system:

$$\dot{x}(t) = A_{\theta(t)} x(t) + B_{\theta(t)} u(t). \tag{3.13}$$

Definition 3.4.1 (Stochastic Stability (SS) (COSTA; FRAGOSO; TODOROV, 2012))**.** *The system 3.13 is considered mean square stabilizable if there exist a set of controllers* $\mathscr{K} = \{K_1, \ldots, K_n\}$ *that for any initial conditions* x_0, θ_0 *it holds that*

$$\int_0^\infty E(||x(t)||^2) dt < \infty. \tag{3.14}$$

Definition 3.4.2 (Mean Square Stability (MSS) (COSTA; FRAGOSO; TODOROV, 2012))**.** *System* (3.13) *is said to be mean square stable if for arbitrary initial conditions* $\theta_0 \in \mathscr{N}$*, and second order initial state vector* x_0*, we have that*

$$\lim_{t \to \infty} \mathbf{E}(\|x(t)\|^2) = 0.$$

Remark 1. *The notions of stochastic stability and mean square stability are equivalent (the proof can be found in (COSTA; FRAGOSO; TODOROV, 2012)).*

3.4.2 Optimal Control

For the cases of $\mathcal{H}_2$ and $\mathcal{H}_\infty$ norms, the following continuous-time Markov jump linear system (MJLS) is considered:

$$\mathcal{G}: \begin{cases} \dot{x}(t) &= A_{\theta(t)}x(t) + B_{\theta(t)}u(t) + E_{\theta(t)}w(t) \\ z(t) &= C_{\theta(t)}x(t) + D_{\theta(t)}u(t) + L_{\theta(t)}w(t)\ , \\ y(t) &= F_{\theta(t)}x(t) \end{cases} \tag{3.15}$$

where $z(t) \in \mathbb{R}^{n_z}$ is the vector of output variables. The feedback control system for a control law $u(t) = K_{\theta(t)}y(t)$ is given by:

$$\mathcal{G}_{cl}: \begin{cases} \dot{x}(t) &= \widetilde{A}_{\theta(t)}x(t) + E_{\theta(t)}w(t) \\ z(t) &= \widetilde{C}_{\theta(t)}x(t) + L_{\theta(t)}w(t) \end{cases}, \tag{3.16}$$

where

$$\widetilde{C}_{\theta(t)} = C_{\theta(t)} + D_{\theta(t)}K_{\theta(t)}F_{\theta(t)}. \tag{3.17}$$

3.4.2.1 $\mathcal{H}_2$ control

For the definition of the $\mathcal{H}_2$ norm we consider in system (3.16) that $L_p = 0$. In what follows we set $\Pr(\theta(0) = p) = \mu_p \geq 0$, $p \in \mathscr{N}$.

Definition 3.4.3 ($\mathcal{H}_2$ Norm (COSTA; FRAGOSO; TODOROV, 2012))**.** *Suppose that system* (3.16) *is MSS. The $\mathcal{H}_2$-norm for system (3.16) is defined as follows: for $x_0 = 0$,*

$$\|\mathcal{G}_{cl}\|_2^2 = \sum_{s=1}^{n_w} \sum_{(p,k)\in\mathscr{V}} \mu_p \|z_{s,(p)}\|_2^2,$$

where $z_{s,(p)}(t)$ is the controlled output of (3.16) for $w(t) = v_s\delta(t)$, $\delta(t)$ is the unitary impulse, v_s is the s^{th} element of the canonical basis of $\mathbb{R}^{n_w}$ and $\theta(0) = p$.

3.4.2.2 $\mathcal{H}_\infty$ control

Definition 3.4.4 ($\mathcal{H}_\infty$ Norm (COSTA; FRAGOSO; TODOROV, 2012))**.** *Suppose that system* (3.16) *is MSS. The $\mathcal{H}_\infty$-norm for system (3.16) is defined as:*

$$\|\mathcal{G}_{cl}\|_\infty = \sup\left\{\frac{\|z\|_2}{\|w\|_2};\ w \in L_2^{n_w},\ w \neq 0\right\}.$$

We will need the following result along this dissertation (the proof can be found in (COSTA; FRAGOSO; TODOROV, 2012)). The bounded-real lemma establishes an LMI condition for the stability and optimally of a dynamic system:

Lemma 3.4.1 (Bounded Real Lemma)**.** *System* (3.16) *is MSS with an* $\mathcal{H}_\infty$ *cost smaller than* γ *if, for all* $p \in \mathscr{N}$*, there exist* $R_p > 0$ *such that*

$$\begin{bmatrix} R_p\widetilde{A}_p + \widetilde{A}_p'R_p + \sum_{r\in\mathscr{N}} \lambda_{pr}R_r & \bullet & \bullet \\ E_p'R_p & -\gamma^2 I_{n_w} & \bullet \\ \widetilde{C}_p & L_p & -I_{n_z} \end{bmatrix} < 0. \tag{3.18}$$

4 HIDDEN MARKOV JUMP LINEAR SYSTEMS

Due to the possibility of imperfect detection of the operation mode, we introduce the concept of hidden Markov model. The detector operation mode is considered as the information emitted by a detector that observes the operation mode, this estimation of the operation mode is represented by $\widehat{\theta}(t)$. By combining the mode of operation $\theta(t)$ and the detector mode $\widehat{\theta}(t)$ in a joint process, it is possible to represent MJLS with uncertainties in the observation of the operation mode. In this chapter, the framework of hidden Markov process is presented, as well as the extended results of stability and control of MJLS in Subsection 3.3 to the hidden MJLS approach.

4.1 Hidden Markov process

It is assumed that $\theta(t)$ is not known but, instead, there is an estimation $\widehat{\theta}(t)$ for this variable, and that $Z(t) = (\theta(t), \widehat{\theta}(t))$, $t \in \mathbb{R}^+$, is a continuous-time hidden Markov model, with the hidden state $\theta(t)$ taking values in $\mathscr{N}$, and the observation state $\widehat{\theta}(t)$ taking values in $\mathscr{M}$. It is assumed that $Z(t)$ is a homogeneous Markov process having transition rates $\nu_{(pk)(r\ell)}$, with $\nu_{(pk)(r\ell)} \geq 0$ for $(r,\ell) \neq (p,k)$ and $-\nu_{(pk)(pk)} = \sum_{(r\ell)\neq(pk)} \nu_{(pk)(r\ell)}$. The transition rates $\nu_{(pk)(r\ell)}$ of $Z(t) = (\theta(t), \widehat{\theta}(t))$, are given by

$$\begin{aligned} &\Pr(Z(t+h) = (r,\ell) \mid Z(t) = (p,k)) = \\ &\begin{cases} \nu_{(pk)(r\ell)}h + o(h), & (r,\ell) \neq (p,k) \\ 1 + \nu_{(pk)(pk)}h + o(h), & (r,\ell) = (p,k), \end{cases} \end{aligned} \tag{4.1}$$

where

$$\nu_{(pk)(r\ell)} = \begin{cases} \alpha^k_{r\ell}\lambda_{pr}, & p \neq r, \ell \in \mathscr{M}, \\ q^p_{k\ell}, & r = p, \ell \neq k, p \in \mathscr{N}, \\ \lambda_{pp} + q^p_{kk}, & r = p, \ell = k, \\ 0, & \text{otherwise} \end{cases}$$

and $\alpha_{r\ell} \geq 0$, $\sum_{\ell \in \mathscr{M}} \alpha_{r\ell}^{k} = 1$, $q_{k\ell}^{p} \geq 0$, $\ell \neq k$, $\lambda_{pp} = -\sum_{r \in \mathscr{N}} \lambda_{pr}$, $q_{kk}^{p} = -\sum_{\ell \in \mathscr{M}} q_{k\ell}^{p}$. We denote by $\mathscr{V} \subseteq \mathscr{N} \times \mathscr{M}$ an invariant set of $Z(t)$, that is, $P(Z(t) \in \mathscr{V}) = 1$ whenever $Z(0) \in \mathscr{V}$.

Remark 2. *Notice that, for the observed state $\widehat{\theta}(t)$, simultaneous or spontaneous jumps with respect to $\theta(t)$ are modeled by the parameters $\alpha_{r\ell}^{k}$ and $q_{k\ell}^{p}$ respectively. Indeed, recalling that λ_{pr} represents the transition rate of $\theta(t)$, we get that α_{rl}^{k} and q_{kl}^{p} models simultaneous and spontaneous jumps of $\widehat{\theta}(t)$, that is, for small $h > 0$, $\Pr(\widehat{\theta}(t+h) = \ell \mid \theta(t+h) = r, Z(t) = (p,k)) = \alpha_{r\ell}^{k} + r(h)$ for some function such that $\lim_{h \to 0} r(h) = 0$, and $\Pr(\widehat{\theta}(t+h) = \ell \mid \theta(t+h) = p, Z(t) = (r,k)) = q_{k\ell}^{r} h + o(h)$. See (OLIVEIRA et al., 2020) for more details.*

Remark 3. *The above approach allows modeling the following cases (see (STADTMANN; COSTA, 2018)):*

- *Mode-dependent case: $\mathscr{M} = \mathscr{N}$, $q_{k\ell}^{p} = 0$, $\alpha_{rr}^{k} = 1$, and $\alpha_{r\ell}^{k} = 0$ for $r \neq \ell$, with invariant set $\mathscr{V} = \{(p,p) \in \mathscr{N} \times \mathscr{N}\}$. Note that in this case $\theta(t)$ and $\widehat{\theta}(t)$ will be equal.*
- *Mode-independent case: $\mathscr{M} = \{1\}$, $q_{kl}^{p} = 0$, and $\alpha_{r1}^{1} = 1$. In this setting, the detector would be always equal to 1.*
- *No Mutual Jumps: $\alpha_{rk}^{k} = 1$ and $\alpha_{rl}^{k} = 0$ for $k \neq l$.*
- *The Cluster Case: In this case the Markov chain states can be written as the union of $M \leq N$ disjoint sets (clusters) $\mathscr{N}_\ell$ so that $\mathscr{N} = \cup_{\ell \in \mathscr{M}} \mathscr{N}_\ell$. By defining $g : \mathscr{N} \to \mathscr{M}$ such that $g(p) = \ell$ we have that this function represents the cluster where the Markov state belongs to, and thus the controller would have access to $g(p)$. This would be equivalent to take $q_{k\ell}^{p} = 0$ and $\alpha_{pg(p)}^{k} = 1$, so that whenever $\theta(t)$ jumps to p, $\widehat{\theta}(t)$ would jump simultaneously to $g(p)$.*

Example 2. *We consider that the MJLS, adopted from (OLIVEIRA; COSTA, 2021a), is modeled by a Markov chain $\theta(t)$ with three operation modes, that is $\mathscr{N} = \{1,2,3\}$. The transition probability matrix is given by*

$$[\lambda_{pr}] = \begin{bmatrix} 0.5 & -0.2 & -0.3 \\ -0.7 & 0.3 & 0.4 \\ 0.8 & -1.0 & 0.2 \end{bmatrix}, \tag{4.2}$$

Now, we consider the partial observation case with $\mathscr{N} = \mathscr{M}$, where the mode $\theta(t) = 1$ is perfectly detected ($\Pr(\widehat{\theta}(t) = 1 | \theta(t) = 1) = 1$), and the detector have a probability of

estimating a wrong mode in $\theta(t) = 2$ *and* $\theta(t) = 3$ *given by the following matrix*

$$[\alpha_{r\ell}^{k}] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.2 & 0.8 \\ 0 & 0.8 & 0.2 \end{bmatrix}, \forall k \in \mathscr{M}, \tag{4.3}$$

we consider the case of only simultaneous jumps by setting $[q_{k\ell}^{p}] = 0$*. Then, the invariant set is given by*

$$\mathscr{V} = \{(11), (22), (23), (32), (33)\}, \tag{4.4}$$

and the transition rate matrix is given by

$$[\nu_{(ik),(j\ell)}] = \begin{array}{c} (ik)\,|\,(j\ell) \\ (11) \\ (22) \\ (23) \\ (32) \\ (33) \end{array} \begin{array}{c} \begin{array}{ccccc} (11) & (22) & (23) & (32) & (33) \end{array} \\ \begin{bmatrix} 0.5 & -0.04 & -0.16 & -0.24 & -0.06 \\ -0.7 & 0.3 & 0 & 0.32 & 0.08 \\ -0.7 & 0 & 0.3 & 0.32 & 0.08 \\ 0.8 & -0.2 & -0.8 & 0.2 & 0 \\ 0.8 & -0.2 & -0.8 & 0 & 0.2 \end{bmatrix} \end{array}. \tag{4.5}$$

4.2 Static output feedback control of hidden MJLS

Now, the feedback control law depends on the observable variables $y(t)$ and $\widehat{\theta}(t)$, so that it takes the form

$$u(t) = K_{\widehat{\theta}(t)} y(t). \tag{4.6}$$

The feedback control system for a control law $u(t) = K_{\widehat{\theta}(t)} y(t)$ is given by:

$$\mathcal{G}_{cl} : \begin{cases} \dot{x}(t) & = \widetilde{A}_{\theta(t)\widehat{\theta}(t)} x(t) + E_{\theta(t)} w(t) \\ z(t) & = \widetilde{C}_{\theta(t)\widehat{\theta}(t)} x(t) + L_{\theta(t)} w(t) \end{cases}, \tag{4.7}$$

where, for $(r, \ell) \in \mathscr{N} \times \mathscr{M}$,

$$\widetilde{A}_{r\ell} = A_r + B_r K_\ell F_r, \;\; \widetilde{C}_{r\ell} = C_r + D_r K_\ell F_r. \tag{4.8}$$

Remark 4. *In practice, it is very difficult to find an ideal controller* $K_{\widehat{\theta}(t)}$ *for the system* (4.7)*; in this work, by using LMI, sufficient conditions for stabilization are given to synthesize these controllers considering a suboptimal performance cost.*

4.3 Results

In this section, the results that were presented in Subsection 3.4 are generalized for systems with imperfect observations from the detector.

4.3.1 Stability

We present now the condition of mean square stability (MSS) and operators to obtain differential equations for the second moment of $x(t)$ in (4.7).

Definition 4.3.1 (Linear Operators (OLIVEIRA; COSTA, 2021a)). *We now introduce conditions for verifying the MSS of* (4.7)*. For that we define the linear operator* $\mathscr{T}$ *from* $\mathbb{H}^n$ *to* $\mathbb{H}^n$ *such that*

$$\mathscr{T}_{pk}(\mathbf{R}) \triangleq \widetilde{A}_{pk}R_{pk} + R_{pk}\widetilde{A}'_{pk} + \sum_{(j\ell)\in\mathscr{V}} \nu_{(pk)(r\ell)}R_{r\ell}, \tag{4.9}$$

for $\mathbf{R} \in \mathbb{H}^n$.

We have the following lemma (see (COSTA; FRAGOSO; TODOROV, 2012)).

Lemma 4.3.1. *The system* $\dot{x}(t) = \widetilde{A}_{\theta(t)\hat{\theta}(t)}x(t), x(0) = x_0 \in \mathbb{R}^n$*, is MSS if and only if there exists* $\mathbf{R} \in \mathbb{H}^{n+}$ *such that*

$$\mathbf{R} > 0, \mathscr{T}(\mathbf{R}) < 0. \tag{4.10}$$

The set of admissible controllers (4.6) is given by

$$\mathscr{K} \triangleq \{(K_1, \ldots, K_M) \text{ such that (4.10) holds for (4.7)}\}.$$

4.3.2 Optimal control

4.3.2.1 $\mathcal{H}_\infty$ control

Definition 4.3.2 ($\mathcal{H}_\infty$ Norm (COSTA; FRAGOSO; TODOROV, 2012)). *Suppose that system* (4.7) *is MSS. The* $\mathcal{H}_\infty$*-norm for system* (4.7) *is defined as:*

$$\|\mathcal{G}_{cl}\|_\infty = \sup\left\{\frac{\|z\|_2}{\|w\|_2};\, w \in L_2^{n_w},\, w \neq 0\right\}.$$

Notice that the norm defined above represents a measure for the worst-case effect of finite-energy disturbances on the output.

The following lemma provides conditions for hidden MJLS stability (the proof can be found in (COSTA; FRAGOSO; TODOROV, 2012).

Lemma 4.3.2 (Bounded Real Lemma)**.** *System* (4.7) *is MSS with an* $\mathcal{H}_\infty$ *cost smaller than* γ *if, for all* $(p,k) \in \mathscr{V}$*, there exist* $R_{pk} > 0$ *such that*

$$\begin{bmatrix} R_{pk}\widetilde{A}_{pk} + \widetilde{A}'_{pk}R_{pk} + \mathcal{R}_{pk}(R) & \bullet & \bullet \\ E'_pR_{pk} & -\gamma^2 I_{n_w} & \bullet \\ \widetilde{C}_{pk} & L_p & -I_{n_z} \end{bmatrix} < 0, \tag{4.11}$$

where $\mathcal{R}_{pk}(R) = \sum_{(r,\ell)\in\mathscr{V}} \nu_{(pk)(r\ell)}R_{r\ell}$.

4.3.2.2 $\mathcal{H}_2$ control

For the definition of the $\mathcal{H}_2$ norm we consider in system (4.7) that $L_p = 0$, since the output $z(t)$ in this case is related to the quadratic cost of the state and control variables, and not to the external input. Now, we set $\Pr(Z(0) = (p,k)) = \mu_{pk} \geq 0$, $(p,k) \in \mathscr{V}$.

Definition 4.3.3 ($\mathcal{H}_2$ Norm (STADTMANN; COSTA, 2017))**.** *Suppose that system* (4.7) *is MSS. The* $\mathcal{H}_2$*-norm for system* (4.7) *is defined as follows: for* $x_0 = 0$*,*

$$\|\mathcal{G}_{cl}\|_2^2 = \sum_{s=1}^{n_w} \sum_{(p,k)\in\mathscr{V}} \mu_{pk}\|z_{s,(p,k)}\|_2^2,$$

where $z_{s,(p,k)}(t)$ *is the controlled output of* (4.7) *for* $w(t) = v_s\delta(t)$*,* v_s *is the* s^{th} *element of the canonical basis of* $\mathbb{R}^{n_w}$ *and* $\theta(0) = p$*,* $\widehat{\theta}(0) = k$*.*

For obtaining conditions for an upper-bound for the $\mathcal{H}_2$ norm of (4.7), we can resort to the following lemma (see (STADTMANN; COSTA, 2017)):

Lemma 4.3.3. *System* (4.7) *is MSS with an* $\mathcal{H}_2$ *cost smaller than* φ *if, for all* $(p,k) \in \mathscr{V}$*, there exist* $R_{pk} > 0$ *such that*

$$\sum_{(r,\ell)\in\mathscr{V}} \mu_{r\ell}Tr(E'_rR_{r\ell}E_r) < \varphi^2, \tag{4.12}$$

$$Her(R_{pk}\widetilde{A}_{pk}) + \sum_{(r,\ell)\in\mathscr{V}} \nu_{(pk)(r\ell)}R_{r\ell} + \widetilde{C}'_{pk}\widetilde{C}_{pk} < 0. \tag{4.13}$$

5 FORMATION STATIC OUTPUT CONTROL OF MAS WITH HIDDEN MARKOV SWITCHING NETWORK TOPOLOGIES

5.1 Hidden Markov Switching Topologies

In a complete probability space $(\Omega, \mathscr{F}, \Pr)$ with a right-continuous filtration $\mathscr{F}_t$, consider a multi-agent time-varying topology represented by the undirected graph $\mathcal{G}_{\theta(t)} = \mathcal{G}(\mathbb{V}, \mathcal{E}_{\theta(t)}, \mathcal{A}_{\theta(t)})$, where $\theta(t) \in \mathscr{N}$ denotes the network topology mode, $\mathbb{V} = \{1, \ldots, v\}$ and $\mathcal{E}_{\theta(t)} \subseteq \{(i,j)|i,j \in \mathbb{V}, i \neq j\}$ are the set of nodes and edges respectively. An edge $(i,j) \in \mathcal{E}_{\theta(t)}$ represents a connection of node i and j. $\mathcal{A}_{\theta(t)} = [a_{ij,\theta(t)}] \in \mathbb{R}^{v \times v}$ is the adjacency matrix, with $a_{ij,\theta(t)} = 1$ if $(i,j) \in \mathcal{E}_{\theta(t)}$ and $a_{ij,\theta(t)} = 0$ otherwise. Let $\mathcal{D}_{\theta(t)} = diag(d_{1,\theta(t)}, \ldots, d_{v,\theta(t)}) \in \mathbb{R}^{v \times v}$ be the degree matrix with $d_{i,\theta(t)} = \sum_{j=1}^{v} a_{ij,\theta(t)}$. The Laplacian matrix of graph $\mathcal{G}_{\theta(t)}$ is defined as $\mathcal{L}_{\theta(t)} = \mathcal{D}_{\theta(t)} - \mathcal{A}_{\theta(t)}$.

We consider that the topology switching process is governed by a continuous-time Markov process $\theta(t)$ taking values in $\mathscr{N}$ as described in Section 4.1, which is not observable for the controller. Instead, it can be estimated by an imperfect detector $\widehat{\theta}(t)$ taking values in $\mathscr{M}$, with $Z(t) = (\theta(t), \widehat{\theta}(t))$, $t \in \mathbb{R}^+$ being a homogeneous hidden Markov model with transition rate matrix given by (4.1).

5.2 Problem formulation

Consider the linear multi-agent system

$$\mathscr{G} : \begin{cases} \dot{x}_i(t) = Ax_i(t) + Bu_i(t) + Ew_i(t) \\ y_i(t) = Fx_i(t) \\ x_i(0) = x_i^0, \quad i = 1, 2, \ldots, v. \end{cases} \tag{5.1}$$

Let $x(t) = [x_1'(t), \ldots, x_v'(t)]'$, $u(t) = [u_1'(t), \ldots, u_v'(t)]'$ and $y(t) = [y_1'(t), \ldots, y_v'(t)]'$ be the aggregate vectors of the states $x_i(t) \in \mathbb{R}^n$, control inputs $u_i(t) \in \mathbb{R}^{n_u}$ and measured

outputs $y_i(t) \in \mathbb{R}^{n_y}$ respectively. We define $\{w_i(t)\} \in L_2^{n_w}$ as the i-th external disturbance with aggregate vector $w(t) = [w_1'(t), \ldots, w_v'(t)]'$. Moreover it is considered that only the signal $y_i(t)$ and the observed mode of operation $\widehat{\theta}(t)$ are available for the control strategy. For the i-th agent, we consider the following distributed control protocol

$$u_i(t) = K_h F h_i(t) + K_{\widehat{\theta}(t)} \sum_{j \in \mathbb{V}} a_{ij,\theta(t)} \Big((y_j(t) - F h_j(t)) - (y_i(t) - F h_i(t)) \Big), \tag{5.2}$$

where K_h is set to manage the formation vector $h_i(t)$ and $K_{\widehat{\theta}(t)}$ will be designed to drive the states of the MAS (5.1) to achieve the desired time-varying formation under switching topologies. Notice that the feedback gain matrix $K_{\widehat{\theta}(t)}$ depends only on the observed mode of operation $\widehat{\theta}(t)$ while the adjacency matrix $a_{ij,\theta(t)}$ depends on the Markov process $\theta(t)$, with the joint process $Z(t) = (\theta(t), \widehat{\theta}(t))$ defined as in (4.1). The formation vectors $h_i(t)$ satisfy the following dynamic equations:

$$\dot{h}_i(t) = (A + B K_h F) h_i(t). \tag{5.3}$$

Substituting the control protocol (5.2) into the multi-agent system (5.1) we get that

$$\begin{aligned} \dot{x}_i(t) =& A x_i(t) + B K_h F h_i(t) + B K_{\widehat{\theta}(t)} F \Big(\sum_{j \in \mathbb{V}} a_{ij,\theta(t)} ((x_j(t) \\ & - h_j(t)) - (x_i(t) - h_i(t))) \Big) + E w_i(t). \end{aligned} \tag{5.4}$$

The output controlled variable $z_i(t) \in \mathbb{R}^{n_z}$, $i = 1, \ldots, v$, is defined as

$$\begin{aligned} z_i(t) &= C(x_i(t) - h_i(t)) + D(u_i(t) - u_{h_i}(t)) + L w_i(t), \\ u_{h_i}(t) &= K_h F h_i(t), \end{aligned} \tag{5.5}$$

where C, D and L are weighting matrices related to the state error, control effort, and external disturbance. Let $\varepsilon_i(t) = x_i(t) - h_i(t)$. From (5.3) and (5.4) we get that

$$\dot{\varepsilon}_i(t) = A \varepsilon_i(t) + B K_{\widehat{\theta}(t)} F \Big(\sum_{j \in \mathbb{V}} a_{ij,\theta(t)} (\varepsilon_j(t) - \varepsilon_i(t)) \Big) + E w_i(t), \tag{5.6}$$

which, in a compact way, can be re-written as

$$\dot{\varepsilon}(t) = (I_v \otimes A) \varepsilon(t) - (\mathcal{L}_{\theta(t)} \otimes B K_{\widehat{\theta}(t)} F) \varepsilon(t) + (I_v \otimes E) w(t). \tag{5.7}$$

We define the average error $\delta_i(t)$ as

$$\delta_i(t) = (x_i(t) - h_i(t)) - \frac{1}{v} \sum_{j=1}^{v} (x_j(t) - h_j(t)), \tag{5.8}$$

which can be re-written in a compact form as

$$\delta(t) = [(I_v - J_v) \otimes I_n](x(t) - h(t)) = [(I_v - J_v) \otimes I_n]\varepsilon(t), \tag{5.9}$$

where $J_v = \frac{1}{v}\mathbf{1}_v\mathbf{1}_v'$. Notice that for any symmetric $v \times v$ matrix S and any $n \times n$ matrix Z, we have that $(J_v \otimes I_n)(S \otimes Z) = (S \otimes Z)(J_v \otimes I_n)$. Thus we get that

$$\begin{aligned}[(I_v - J_v) \otimes I_n](I_v \otimes A) =&(I_v \otimes A)[(I_v - J_v) \otimes I_n],\\ [(I_v - J_v) \otimes I_n](\mathcal{L}_p \otimes BK_kF) =&(\mathcal{L}_p \otimes BK_kF)[(I_v - J_v) \otimes I_n].\end{aligned}$$

The output averaged controlled variable $z_i^a(t)$ is defined as

$$z_i^a(t) = z_i(t) - \frac{1}{v}\sum_{j=1}^{v} z_j(t) \tag{5.10}$$

and $z(t) = [z_1^{a\prime}(t), \ldots, z_v^{a\prime}(t)]'$ is the aggregate vector of the averaged controlled outputs, with $z(t) \in \mathbb{R}^{n_z v}$. Considering the average error $\delta(t)$ in (5.9) and system (5.7), and the controlled output variables (5.5), (5.10), we get the dynamical system for the error $\delta(t)$ and $z(t)$ as

$$\mathscr{G}_{cl} : \begin{cases} \dot{\delta}(t) = \widetilde{A}_{\theta(t)\widehat{\theta}(t)}\delta(t) + \widetilde{E}w(t) \\ z(t) = \widetilde{C}_{\theta(t)\widehat{\theta}(t)}\delta(t) + \widetilde{L}w(t), \end{cases} \tag{5.11}$$

where

$$\begin{aligned}\widetilde{A}_{pk} &= I_v \otimes A - \mathcal{L}_p \otimes BK_kF, \;\; \widetilde{E} = (I_v - J_v) \otimes E,\\ \widetilde{C}_{pk} &= I_v \otimes C - \mathcal{L}_p \otimes DK_kF, \;\; \widetilde{L} = (I_v - J_v) \otimes L.\end{aligned}$$

The goal is to obtain K_k in (5.2), $k \in \mathscr{M}$, such that we have TVF mean square stability and either an $\mathcal{H}_\infty$ or $\mathcal{H}_2$ performance (or both), as described next:

1. TVF mean square stability: system $\mathscr{G}_{cl}$ in (5.11) is MSS, that is, with $w(t) = 0$,

$$\lim_{t\to\infty} \mathbf{E}(\|\delta(t)\|^2) = 0, \tag{5.12}$$

   for any initial conditions $\delta(0)$ and $(\theta(0), \widehat{\theta}(0)) \in \mathscr{V}$.

2. $\mathcal{H}_\infty$ performance: for some performance level γ, we have that $\|\mathscr{G}_{cl}\|_\infty < \gamma$, that is,

$$\mathbf{E}\int_0^\infty ||z(t)||^2 dt < \gamma^2 \mathbf{E}\int_0^\infty ||w(t)||^2 dt, \tag{5.13}$$

   for any $w \in L_2^{vn_w}$, $w \neq 0$.

3. $\mathcal{H}_2$ performance: for some performance level φ, we have that

$$\|\mathscr{G}_{cl}\|_2 < \varphi. \tag{5.14}$$

5.3 Main Results

In this section we present LMI conditions to obtain K_k in (5.2), $k \in \mathscr{M}$, such that (5.12) and either (5.13) or (5.14) (or both) are satisfied. For this we need to make the following assumption.

Assumption 1: F has full row rank matrix.

Note that Assumption 1 is a standard assumption to avoid redundant measurements. From Assumption 1 we have that there exists a non-singular matrix T such that

$$FT = \begin{bmatrix} I_{n_y} & 0 \end{bmatrix}. \tag{5.15}$$

In what follows we define, for $(p,k) \in \mathscr{V}$,

$$\begin{aligned} \mathscr{V}_{(p,k)} &= \{(r,\ell) \in \mathscr{V}; (r,\ell) \neq (p,k) \text{ and } \nu_{(p,k)(r,l)} \neq 0\} \\ &= \{r_{(p,k)}(1), \ldots, r_{(p,k)}(\tau_{(p,k)}); r_{(p,k)}(\iota) \in \mathscr{V}, \iota = 1, \ldots, \tau_{(p,k)}\}. \end{aligned}$$

Consider $n \times n$ matrices $X_{pk} > 0$, $(p,k) \in \mathscr{V}$, and set

$$\begin{aligned} \Pi_{pk} &= \left[\sqrt{\nu_{(p,k)r_{(p,k)}(1)}}(I_v \otimes I_n) \ldots \sqrt{\nu_{(p,k)r_{p,k}(\tau_{(p,k)})}}(I_v \otimes I_n)\right], \\ \mathscr{D}_{pk} &= diag(I_v \otimes X_{r_{(p,k)}(1)}, \ldots, I_v \otimes X_{r_{(p,k)}(\tau_{(p,k)})}). \end{aligned}$$

Notice that

$$\begin{aligned} \sum_{(r,\ell)\in\mathscr{V}} \nu_{(p,k)(r,\ell)}(I_v \otimes X_{rl}^{-1}) &= \sum_{(r,\ell)\in\mathscr{V}_{(p,k)}} \nu_{(p,k)(r,\ell)}(I_v \otimes X_{r\ell}^{-1}) + \nu_{(p,k)(p,k)}(I_v \otimes X_{pk}^{-1}) \\ &= \Pi_{pk}\mathscr{D}_{pk}^{-1}\Pi_{pk}' + \nu_{(p,k)(p,k)}(I_v \otimes X_{pk}^{-1}). \end{aligned} \tag{5.16}$$

5.3.1 $\mathcal{H}_\infty$ Control

The following theorem, based on the results in (STADTMANN; COSTA, 2018), presents a solution for the $\mathcal{H}_\infty$ problem for the cooperative control of multi-agent system (5.1) under hidden Markov switching topologies, based on the solution of a set of LMI.

Theorem 5.3.1. *Consider a fixed upper-bound $\gamma > 0$ and suppose that for all $(p,k) \in \mathscr{V}$, there exist matrices $X_{pk} > 0$, G_k and V_k and a scalar $\epsilon_\infty > 0$ such that the following set of LMI is satisfied:*

$$\begin{bmatrix} \nu_{(p,k)(p,k)} I_v \otimes X_{pk} & \bullet & \bullet & \bullet & \bullet \\ (I_v - J_v) \otimes E' & -\gamma^2 (I_v \otimes I_{n_w}) & \bullet & \bullet & \bullet \\ 0 & (I_v - J_v) \otimes L & -I_v \otimes I_{n_z} & \bullet & \bullet \\ I_v \otimes X_{pk} & 0 & 0 & 0 & \bullet \\ \Pi'_{pk}(I_v \otimes X_{pk}) & 0 & 0 & 0 & -\mathscr{D}_{pk} \end{bmatrix} +$$

$$Her\left(\begin{bmatrix} I_v \otimes (ATG_k) - \mathcal{L}_p \otimes B \begin{bmatrix} V_k & 0 \end{bmatrix} \\ 0 \\ I_v \otimes (CTG_k) - \mathcal{L}_p \otimes D \begin{bmatrix} V_k & 0 \end{bmatrix} \\ -I_v \otimes (TG_k) \\ 0 \end{bmatrix} \begin{bmatrix} \epsilon_\infty (I_v \otimes I_n) \\ 0 \\ 0 \\ I_v \otimes I_n \\ 0 \end{bmatrix}' \right) < 0, \tag{5.17}$$

with G_k in the following form:

$$G_k = \begin{bmatrix} G_{k1} & 0 \\ G_{k2} & G_{k3} \end{bmatrix}. \tag{5.18}$$

Then the multi-agent system (5.1) is mean square stable with a closed-loop norm $\|\mathscr{G}_{cl}\|_\infty < \gamma$ whenever the distributed control protocol (5.2) is applied, with the feedback controller matrices K_ℓ given by:

$$K_k = V_k G_{k1}^{-1}, \quad k \in \mathscr{M}. \tag{5.19}$$

Proof. From (5.17) we have that $I_v \otimes (TG_k) + I_v \otimes (G'_k T') > 0 = I_v \otimes (TG_k + G'_k T') > 0$ so that it follows that $TG_k + G'_k T' > 0$, which implies that G_k is non-singular, so that the inverse in (5.19) is well defined. From (5.18) and (5.19) we have that $V_k = K_k G_{k1}$ and

$$K_k \begin{bmatrix} I_{n_y} & 0 \end{bmatrix} G_k = \begin{bmatrix} K_k & 0 \end{bmatrix} G_k = \begin{bmatrix} V_k & 0 \end{bmatrix}, \tag{5.20}$$

so that from (5.15) and (5.20) we have that

$$\begin{aligned} & I_v \otimes (ATG_k) - \mathcal{L}_p \otimes B \begin{bmatrix} V_k & 0 \end{bmatrix} \\ & = I_v \otimes (ATG_k) - \mathcal{L}_p \otimes BK_k \begin{bmatrix} I_{n_y} & 0 \end{bmatrix} G_k \\ & = I_v \otimes (ATG_k) - \mathcal{L}_p \otimes BK_k FTG_k \\ & = (I_v \otimes A - \mathcal{L}_p \otimes BK_k F)(I_v \otimes TG_k). \end{aligned}$$

Similarly, we have that

$$I_v \otimes (CTG_k) - \mathcal{L}_p \otimes D \begin{bmatrix} V_k & 0 \end{bmatrix} = (I_v \otimes C - \mathcal{L}_p \otimes DK_kF)(I_v \otimes TG_k).$$

Set

$$\widetilde{A}_{pk} = I_v \otimes A - \mathcal{L}_p \otimes BK_kF, \ \widetilde{E} = (I_v - J_v) \otimes E, \tag{5.21}$$

$$\widetilde{C}_{pk} = I_v \otimes C - \mathcal{L}_p \otimes DK_kF, \ \widetilde{L} = (I_v - J_v) \otimes L, \tag{5.22}$$

so that (5.17) can be re-written as

$$\Phi_{pk} + Her \left(\begin{bmatrix} \widetilde{A}_{pk} \\ 0 \\ \widetilde{C}_{pk} \\ -I_v \otimes I_n \\ 0 \end{bmatrix} I_v \otimes (TG_k) \begin{bmatrix} \epsilon_\infty (I_v \otimes I_n) \\ 0 \\ 0 \\ I_v \otimes I_n \\ 0 \end{bmatrix}' \right) < 0, \tag{5.23}$$

where

$$\Phi_{pk} = \begin{bmatrix} \nu_{(p,k)(p,k)} I_v \otimes X_{pk} & \bullet & \bullet & \bullet & \bullet \\ \widetilde{E}' & -\gamma^2 (I_v \otimes I_{n_w}) & \bullet & \bullet & \bullet \\ 0 & \widetilde{L} & -I_v \otimes I_{n_z} & \bullet & \bullet \\ I_v \otimes X_{pk} & 0 & 0 & 0 & \bullet \\ \Pi'_{pk}(I_v \otimes X_{pk}) & 0 & 0 & 0 & -\mathscr{D}_{pk} \end{bmatrix}.$$

Defining

$$\widetilde{W}_{pk} = \begin{bmatrix} I_v \otimes I_n & 0 & 0 & 0 \\ 0 & I_v \otimes I_{n_w} & 0 & 0 \\ 0 & 0 & I_v \otimes I_{n_z} & 0 \\ \widetilde{A}'_{pk} & 0 & \widetilde{C}'_{pk} & 0 \\ 0 & 0 & 0 & I_v \otimes I_n \end{bmatrix},$$

$$W_{pk} = \begin{bmatrix} \widetilde{A}'_{pk} & 0 & \widetilde{C}'_{pk} & -I_v \otimes I_n & 0 \end{bmatrix} \tag{5.24}$$

it follows that $\widetilde{W}_{pk}$ has full rank and that $W_{pk}\widetilde{W}_{pk} = 0$, so that from Finsler's lemma (see Lemma 3.1.1) and (5.23) we have that

$$\widetilde{W}'_{pk} \Phi_{pk} \widetilde{W}_{pk} < 0. \tag{5.25}$$

From (5.25) we conclude that

$$\begin{bmatrix} \mathscr{Z}_{pk} & \bullet & \bullet & \bullet \\ \widetilde{E}' & -\gamma^2(I_v \otimes I_{n_w}) & \bullet & \bullet \\ \widetilde{C}_{pk}(I_v \otimes X_{pk}) & \widetilde{L} & -I_v \otimes I_{n_z} & \bullet \\ \Pi'_{pk}(I_v \otimes X_{pk}) & 0 & 0 & -\mathscr{D}_{pk} \end{bmatrix} < 0,$$

where

$$\mathscr{Z}_{pk} = \nu_{(p,k)(p,k)} I_v \otimes X_{ik} + \widetilde{A}_{pk}(I_v \otimes X_{pk}) + (I_v \otimes X_{pk})\widetilde{A}'_{pk}.$$

From Schur's complement, we get that

$$\begin{bmatrix} \bar{\mathscr{Z}}_{pk} & \bullet & \bullet \\ \widetilde{E}' & -\gamma^2(I_v \otimes I_{n_w}) & \bullet \\ \widetilde{C}_{pk}(I_v \otimes X_{pk}) & \widetilde{L} & -I_v \otimes I_{n_z} \end{bmatrix} < 0,$$

where

$$\bar{\mathscr{Z}}_{pk} = \mathscr{Z}_{pk} + (I_v \otimes X_{pk})\Pi_{pk}\mathscr{D}_{pk}^{-1}\Pi'_{pk}(I_v \otimes X_{pk}).$$

Multiplying on the left hand side and right hand side by $diag((I_v \otimes X_{pk}^{-1}), (I_v \otimes I_{n_w}), I_v \otimes I_{n_z})$ we get that

$$\begin{bmatrix} \widetilde{\mathscr{Z}}_{pk} & \bullet & \bullet \\ \widetilde{E}'(I_v \otimes X_{pk}^{-1}) & -\gamma^2(I_v \otimes I_{n_w}) & \bullet \\ \widetilde{C}_{pk} & \widetilde{L} & -I_v \otimes I_{n_z} \end{bmatrix} < 0, \tag{5.26}$$

where, from (5.16),

$$\begin{aligned} \widetilde{\mathscr{Z}}_{pk} =& (I_v \otimes X_{pk}^{-1})\widetilde{A}_{pk} + \widetilde{A}'_{pk}(I_v \otimes X_{pk}^{-1}) + \nu_{(p,k)(p,k)}(I_v \otimes X_{ik}^{-1}) + \Pi_{pk}\mathscr{D}_{pk}^{-1}\Pi'_{pk} \\ =& (I_v \otimes X_{pk}^{-1})\widetilde{A}_{pk} + \widetilde{A}'_{pk}(I_v \otimes X_{pk}^{-1}) + \sum_{(r,\ell)\in\mathscr{V}} \nu_{(p,k)(r,\ell)}(I_v \otimes X_{rl}^{-1}). \end{aligned} \tag{5.27}$$

By combining (5.26) and (5.27) we have that (4.11) is satisfied by taking $R_{pk} = I_v \otimes X_{pk}^{-1}$. From Lemma 4.3.2 and considering the representation in (5.11) for $\mathscr{G}_{cl}$ we get the desired result. □

The next algorithm provides the way of computing a $\mathcal{H}_\infty$ controller such as is presented in Theorem 5.3.1.

Algorithm 1 $\mathcal{H}_\infty$ controller design procedure

1: Set a gain K_h such that (5.3) is satisfied.
2: Calculate matrices $X_{pk} > 0$, G_k and V_k such that the LMI (5.17) is satisfied.
3: With G_k and V_k obtained from the previous step, design a stochastic stabilizing state-feedback gain K_k^∞ by means of relation (5.19).

5.3.2 $\mathcal{H}_2$ Control

We present next a solution for the $\mathcal{H}_2$ problem for the cooperative control of multi-agent system (5.1) under hidden Markov switching topologies, based on the solution of a set of LMI. We recall that in this case we consider $L = 0$ since the output $z(t)$ is only related to the quadratic cost of the state and control variables.

Theorem 5.3.2. *Consider a fixed upper-bound $\varphi > 0$ and suppose that for all $(p,k) \in \mathscr{V}$, there exist matrices $W_{pk} > 0$, $X_{pk} > 0$, G_k and V_k and a scalar $\epsilon_2 > 0$, such that the following set of LMI is satisfied:*

$$\sum_{(p,k)\in\mathscr{V}} \mu_{pk} Tr(W_{pk}) < \varphi^2, \tag{5.28}$$

$$\begin{bmatrix} W_{pk} & \bullet \\ (I_v - J_v) \otimes E & I_v \otimes X_{pk} \end{bmatrix} > 0, \tag{5.29}$$

and

$$\begin{bmatrix} \nu_{(p,k)(p,k)} I_v \otimes X_{pk} & \bullet & \bullet & \bullet \\ 0 & -I_v \otimes I_{n_z} & \bullet & \bullet \\ I_v \otimes X_{pk} & 0 & 0 & \bullet \\ \Pi'_{pk}(I_v \otimes X_{pk}) & 0 & 0 & -\mathscr{D}_{pk} \end{bmatrix} + Her\left(\begin{bmatrix} I_v \otimes (ATG_k) - \mathcal{L}_p \otimes B \begin{bmatrix} V_k & 0 \end{bmatrix} \\ I_v \otimes (CTG_k) - \mathcal{L}_p \otimes D \begin{bmatrix} V_k & 0 \end{bmatrix} \\ -I_v \otimes (TG_k) \\ 0 \end{bmatrix} \begin{bmatrix} \epsilon_2 (I_v \otimes I_n) \\ 0 \\ I_v \otimes I_n \\ 0 \end{bmatrix}' \right) < 0, \tag{5.30}$$

with G_k as in (5.18). Then the multi-agent system (5.1) is mean square stable with a closed-loop norm $\|\mathscr{G}_{cl}\|_2 < \varphi$ whenever the distributed control protocol (5.2) is applied, with the feedback controller matrices K_ℓ given by (5.19).

Proof. As before, from (5.23) we have that G_k is non-singular, so that the inverse in (5.19) is well defined. As in the proof of Theorem 5.3.1 and using the same notation as in (5.21), (5.22), we have that (5.30) can be re-written as

$$\Phi_{pk} + Her\left(\begin{bmatrix} \widetilde{A}_{pk} \\ \widetilde{C}_{pk} \\ -I_v \otimes I_n \\ 0 \end{bmatrix} I_v \otimes (TG_k) \begin{bmatrix} \epsilon_2 (I_v \otimes I_n) \\ 0 \\ I_v \otimes I_n \\ 0 \end{bmatrix}' \right) < 0, \tag{5.31}$$

where

$$\Phi_{pk} = \begin{bmatrix} \nu_{(p,k)(p,k)} I_v \otimes X_{pk} & \bullet & \bullet & \bullet \\ 0 & -I_v \otimes I_{n_z} & \bullet & \bullet \\ I_v \otimes X_{pk} & 0 & 0 & \bullet \\ \Pi'_{pk}(I_v \otimes X_{pk}) & 0 & 0 & -\mathscr{D}_{pk} \end{bmatrix}.$$

Defining

$$\widetilde{W}_{pk} = \begin{bmatrix} I_v \otimes I_n & 0 & 0 \\ 0 & I_v \otimes I_{n_z} & 0 \\ \widetilde{A}'_{pk} & \widetilde{C}'_{pk} & 0 \\ 0 & 0 & I_v \otimes I_n \end{bmatrix}, \; W'_{pk} = \begin{bmatrix} \widetilde{A}_{pk} \\ \widetilde{C}_{pk} \\ -I_v \otimes I_n \\ 0 \end{bmatrix}, \tag{5.32}$$

it is easy to see that $\widetilde{W}_{pk}$ has full rank and that $W_{pk}\widetilde{W}_{pk} = 0$, so that from Finsler's lemma (see Lemma 3.1.1) and (5.32) we have that

$$\widetilde{W}'_{pk}\Phi_{pk}\widetilde{W}_{pk} < 0. \tag{5.33}$$

From (5.33) we conclude that

$$\begin{bmatrix} \mathcal{Z}_{pk} & \bullet & \bullet \\ \widetilde{C}_{pk}(I_v \otimes X_{pk}) & -I_v \otimes I_{n_z} & \bullet \\ \Pi'_{pk}(I_v \otimes X_{pk}) & 0 & -\mathscr{D}_{pk} \end{bmatrix} < 0, \tag{5.34}$$

where

$$\mathcal{Z}_{pk} = \nu_{(p,k)(p,k)} I_v \otimes X_{ik} + \widetilde{A}_{pk}(I_v \otimes X_{pk}) + (I_v \otimes X_{pk})\widetilde{A}'_{pk}.$$

By applying the Schur's complement in (5.34) we get that

$$\mathcal{Z}_{pk} + (I_v \otimes X_{pk})\Pi_{pk}\mathscr{D}_{pk}^{-1}\Pi'_{pk}(I_v \otimes X_{pk}) + (I_v \otimes X_{pk})\widetilde{C}'_{pk}\widetilde{C}_{pk}(I_v \otimes X_{pk}) < 0.$$

Multiplying on the left and right by $I_v \otimes X_{pk}^{-1}$ we get that

$$\widetilde{\mathcal{Z}}_{pk} + \widetilde{C}'_{pk}\widetilde{C}_{pk} < 0, \tag{5.35}$$

where $\widetilde{\mathcal{Z}}_{pk}$ is as in (5.27). By combining (5.35) and (5.27) we have that (4.13) is satisfied by taking $R_{pk} = I_v \otimes X_{pk}^{-1}$. Moreover from Schur's complement in (5.29) we get that $W_{pk} > (I_v - J_v) \otimes E'(I_v \otimes X_{pk})(I_v - J_v) \otimes E$ so that from (5.28) we get that $\sum_{(p,k)\in\mathscr{V}} Tr((I_v - J_v) \otimes E'(I_v \otimes X_{pk})(I_v - J_v) \otimes E) < \varphi^2$ showing that (4.12) is also satisfied. From Lemma 4.3.3 we get the desired result. □

The next algorithm provides the way of computing a $\mathcal{H}_2$ controller such as is presented in Theorem 5.3.2.

Algorithm 2 $\mathcal{H}_2$ controller design procedure

1: Set a gain K_h such that (5.3) is satisfied.
2: Calculate matrices $W_{pk} > 0$, $X_{pk} > 0$, G_k and V_k such that the LMI (5.28), (5.29), (5.30) is satisfied.
3: With G_k and V_k obtained from the previous step, design a stochastic stabilizing state-feedback gain K_k^2 by means of relation (5.19).

5.3.3 Mixed $\mathcal{H}_2/\mathcal{H}_\infty$ Control

The following corollary is straightforward after combining the results from Theorems 5.3.1 and 5.3.2.

Corollary 5.3.2.1. *Consider fixed upper-bounds $\gamma > 0$ and $\varphi > 0$. If for all $(p,k) \in \mathscr{V}$, there exist matrices $W_{pk} > 0$, $X_{pk} > 0$, G_k and V_k and scalars $\epsilon_\infty > 0$, $\epsilon_2 > 0$, such that the LMI (5.17), (5.28), (5.29), (5.30) are satisfied, where G_k is as in (5.18) then the multi-agent system (5.1) is mean square stable with a closed-loop norm $\|\mathscr{G}_{cl}\|_2 < \varphi$ and $\|\mathscr{G}_{cl}\|_\infty < \gamma$ whenever the distributed control protocol (5.2) is applied, with the feedback controller matrices K_ℓ given as in (5.19).*

From the previous results the following LMI optimization problems could be defined:

1) $\mathcal{H}_\infty$ control problem: min γ^2 such that the LMI (5.17) is satisfied.

2) $\mathcal{H}_2$ control problem: min φ^2 such that the LMI (5.28), (5.29), (5.30) are satisfied.

3) Mixed $\mathcal{H}_2/\mathcal{H}_\infty$ control problems:

   3.a) for $\beta_2 \geq 0$, $\beta_\infty \geq 0$, min $\beta_2\varphi^2 + \beta_\infty\gamma^2$ such that the LMI (5.17), (5.28), (5.29), (5.30) are satisfied.

   3.b) for fixed $\varphi > 0$, min γ^2 such that the LMI (5.17), (5.28), (5.29), (5.30) are satisfied.

   3.c) for fixed $\gamma > 0$, min φ^2 such that the LMI (5.17), (5.28), (5.29), (5.30) are satisfied.

6 CASE STUDY

In this section, numerical examples are presented to illustrate the effectiveness of the proposed method. The first one deals with a comparison between the $\mathcal{H}_2$ and $\mathcal{H}_\infty$ costs for the synchronous mode and asynchronous mode cases, by varying the parameters $\alpha_{r\ell}^k$ and $q_{k\ell}^p$ in (4.1). Next, the average error responses $\delta_i(t)$ are studied for the controllers $\mathcal{H}_2$, $\mathcal{H}_\infty$ and $\mathcal{H}_2/\mathcal{H}_\infty$. Finally, the ability of the control protocol $\mathcal{H}_2/\mathcal{H}_\infty$ to achieve the TVF in the MAS is verified.

Consider the multi-agent system (5.1), consisting of six agents with $x_i(t) = [x_{i1}(t)\ x_{i2}(t)\ x_{i3}(t)\ x_{i4}(t)\ x_{i5}(t)\ x_{i6}(t)]'$ $(i = 1, 2, \ldots, 6)$ and state matrices adopted from (LI et al., 2020), defined as

$$A = \begin{bmatrix} 0_{3\times3} & I_3 \\ -I_3 & 0_{3\times3} \end{bmatrix}, \quad B = \begin{bmatrix} 0_{3\times3} \\ I_3 \end{bmatrix}, \quad E = \begin{bmatrix} 0.8 \\ 0.5 \\ 1 \\ 0_{3\times1} \end{bmatrix},$$

$$C^{(\infty)} = C^{(2)} = I_6, \qquad F = \begin{bmatrix} I_3 & 0_{3x3} \end{bmatrix}.$$

The Markovian mode-dependent network topologies, represented by the undirected graphs $\mathcal{G}_1$ and $\mathcal{G}_2$ in Fig. 7, are described by the following Laplacian matrices $\mathcal{L}_p$ with $p \in \mathscr{N} \triangleq \{1, 2\}$

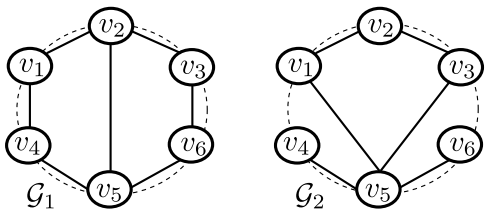


Figure 7: Network topologies.

$$L_1 = \begin{bmatrix} 2 & -1 & 0 & -1 & 0 & 0 \\ -1 & 3 & -1 & 0 & -1 & 0 \\ 0 & -1 & 2 & 0 & 0 & -1 \\ -1 & 0 & 0 & 2 & -1 & 0 \\ 0 & -1 & 0 & -1 & 3 & -1 \\ 0 & 0 & -1 & 0 & -1 & 2 \end{bmatrix},$$

$$L_2 = \begin{bmatrix} 2 & -1 & 0 & 0 & -1 & 0 \\ -1 & 2 & -1 & 0 & 0 & 0 \\ 0 & -1 & 2 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \\ -1 & 0 & -1 & -1 & 4 & -1 \\ 0 & 0 & 0 & 0 & -1 & -1 \end{bmatrix},$$

and the transition rate matrix is given by

$$\Lambda = \begin{bmatrix} -0.3 & 0.3 \\ 0.5 & -0.5 \end{bmatrix}.$$

The observed mode $\widehat{\theta}(t)$ is set with $M = 2$ along with

$$[\alpha_{r\ell}^k] = \begin{bmatrix} \varsigma_1 & 1-\varsigma_1 \\ 1-\varsigma_2 & \varsigma_2 \end{bmatrix}, \forall k \in \mathscr{M},$$

$$[q_{k\ell}^p] = \begin{bmatrix} -\varrho_1 & \varrho_1 \\ \varrho_2 & -\varrho_2 \end{bmatrix}, \forall p \in \mathscr{N},$$

for $\varsigma_1, \varsigma_2 \in [0, 1]$ and $\varrho_1, \varrho_2 \in [0, 0.5]$. The desired time-varying formation for the six agents is a periodic rotation parallel hexagon, where the formation vector $h_i(t)$ $(i = 1, 2, \ldots, 6)$ is specified by

$$h_i(t) = \begin{bmatrix} 2sin(2t + \frac{(i-1)\pi}{3}) - 2cos(2t + \frac{(i-1)\pi}{3}) \\ sin(2t + \frac{(i-1)\pi}{3}) + cos(2t + \frac{(i-1)\pi}{3}) \\ 4cos(2t + \frac{(i-1)\pi}{3}) \\ 4sin(2t + \frac{(i-1)\pi}{3}) + 4cos(2t + \frac{(i-1)\pi}{3}) \\ 2cos(2t + \frac{(i-1)\pi}{3}) - 2sin(2t + \frac{(i-1)\pi}{3}) \\ -8sin(2t + \frac{(i-1)\pi}{3}) \end{bmatrix}.$$

With the purpose of determining the performance of the proposed solutions, we set the

gain matrix $K_h = -3I_3$ that satisfies (5.3) as follows

$$\dot{h}_i(t) = \begin{bmatrix} 4cos(2t + \frac{(i-1)\pi}{3}) + 4sin(2t + \frac{(i-1)\pi}{3}) \\ 2cos(2t + \frac{(i-1)\pi}{3}) - 2sin(2t + \frac{(i-1)\pi}{3}) \\ -8sin(2t + \frac{(i-1)\pi}{3}) \\ 8cos(2t + \frac{(i-1)\pi}{3}) - 8sin(2t + \frac{(i-1)\pi}{3}) \\ -4sin(2t + \frac{(i-1)\pi}{3}) - 4cos(2t + \frac{(i-1)\pi}{3}) \\ -16cos(2t + \frac{(i-1)\pi}{3}) \end{bmatrix} = \begin{bmatrix} 0_{3\times3} & I_3 \\ -4I_3 & 0_{3\times3} \end{bmatrix} h_i(t).$$

Let the initial states $x_i(0) = [x_{i1}(0)\ x_{i2}(0)\ x_{i3}(0)\ x_{i4}(0)\ x_{i5}(0)\ x_{i6}(0)]'$ $(i = 1, 2, \ldots, 6)$ be random values uniformly chosen between -10 and 10 and the external disturbance input $w_i(t)$ as follows

$$w_{i1}(t) = \begin{cases} 2, & \text{for } t \in [0, 10) \cup [20, 30) \\ -2, & \text{for } t \in [10, 20) \cup [30, 40) \\ 0, & \text{otherwise} \end{cases},$$

$$w_{i2}(t) = \begin{cases} 1, & \text{for } t \in [0, 10) \cup [20, 30) \\ -1, & \text{for } t \in [10, 20) \cup [30, 40) \\ 0, & \text{otherwise} \end{cases},$$

$$w_{i3}(t) = \begin{cases} 2, & \text{for } t \in [0, 10) \cup [20, 30) \\ -2, & \text{for } t \in [10, 20) \cup [30, 40) \\ 0, & \text{otherwise} \end{cases}.$$

In order to compare the $\mathcal{H}_2$ and $\mathcal{H}_\infty$ costs for the synchronous and asynchronous modes, we set $\varsigma_1 = \varsigma_2 = 1$ and $\varrho_1 = \varrho_2 = 0$ for the synchronous case $(\theta(t) = \widehat{\theta}(t))$ and $\varsigma_1 = \varsigma_2 = 0.6$, and $\varrho_1 = \varrho_2 = 0.3$ for the asynchronous case, which indicates the imperfect information case (we could have $\theta(t) \neq \widehat{\theta}(t)$). Taking this into account, the invariant set is given by

$$\mathscr{V} = \{(11), (12), (21), (22)\}, \tag{6.1}$$

and the transition rate matrix is given by

$$[\nu_{(ik),(j\ell)}] = \begin{array}{c} (ik)\backslash(j\ell) \\ (11) \\ (12) \\ (21) \\ (22) \end{array} \begin{array}{c} \begin{array}{cccc} (11) & (12) & (21) & (22) \end{array} \\ \begin{bmatrix} \lambda_{11} + q_{11}^1 & q_{12}^1 & \alpha_{21}^1\lambda_{12} & \alpha_{22}^1\lambda_{12} \\ q_{21}^1 & \lambda_{11} + q_{22}^1 & \alpha_{21}^2\lambda_{12} & \alpha_{22}^2\lambda_{12} \\ \alpha_{11}^1\lambda_{21} & \alpha_{12}^1\lambda_{21} & \lambda_{22} + q_{11}^2 & q_{12}^2 \\ \alpha_{11}^2\lambda_{21} & \alpha_{12}^2\lambda_{21} & q_{21}^2 & \lambda_{22} + q_{22}^2 \end{bmatrix} \end{array}. \tag{6.2}$$

Table 1: $\mathcal{H}_2$ and $\mathcal{H}_\infty$ costs.

Operation mode	φ	γ
Synchronous	0.1189	0.0249
Asynchronous	0.1368	0.0295

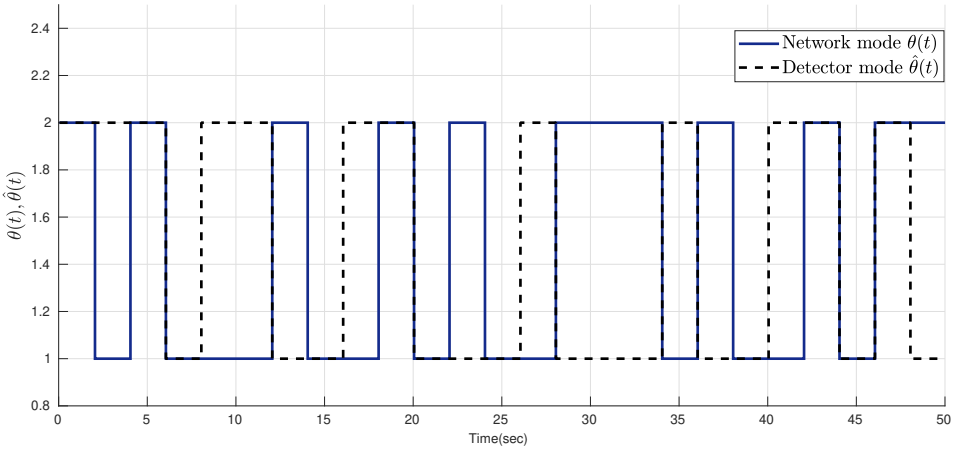


Figure 8: Asynchronous switching signal.

The $\mathcal{H}_2$ controller is obtained by solving the LMI (5.28), (5.29), (5.30) in Theorem 5.3.2 with $\epsilon_2 = 20$, which yields the performance values shown in Table 1. Notice that the values $\varphi = 0.1189$ for the synchronous mode and $\varphi = 0.1368$ for the asynchronous mode differ by only 13.1%. Similarly, for the $\mathcal{H}_\infty$ controller obtained from Theorem 5.3.1 by fixing $\epsilon_\infty = 10$ and solving the LMI (5.17), the values between the synchronous mode ($\gamma = 0.0249$) and the asynchronous mode ($\gamma = 0.0295$) differ only by 15.6%. These results indicate that the performance and robustness are maintained even if the detector emits mismatching signals concerning the network mode of operation.

Based on the asynchronous mode control gains, Fig. 8 shows the evolution of the network and detector modes performed in the simulation. We notice that there are mismatches between the modes of the network $\theta(t)$ and the detector $\widehat{\theta}(t)$ at some times during the simulation. Fig. 9 and Fig. 10 show the time-varying formation average error of each agent, denoted by $\delta_i(t) = [\delta_{i1}(t)\ \delta_{i2}(t)\ \delta_{i3}(t)\ \delta_{i4}(t)\ \delta_{i5}(t)\ \delta_{i6}(t)]'$ $(i = 1, 2, \ldots, 6)$. With the purpose to study the average error responses of the $\mathcal{H}_2$ and $\mathcal{H}_\infty$ solutions, we consider two parameters: the velocity of the response, characterized by the time at which the signal reaches a value very close to zero, denoted by $\zeta_i = [\zeta_{i1}\ \zeta_{i2}\ \zeta_{i3}\ \zeta_{i4}\ \zeta_{i5}\ \zeta_{i6}]'$ $(i = 1, 2, \ldots, 6)$ and the maximum overshoot magnitude, denoted by $\vartheta_i = [\vartheta_{i1}\ \vartheta_{i2}\ \vartheta_{i3}$Fig. 9 and Fig. 10 show the time-varying formation average error of each agent, denoted by $\delta_i(t) = [\delta_{i1}(t)$ $\delta_{i2}(t)\ \delta_{i3}(t)\ \delta_{i4}(t)\ \delta_{i5}(t)\ \delta_{i6}(t)]'$ $(i = 1, 2, \ldots, 6)$. With the purpose to study the average error responses of the $\mathcal{H}_2$ and $\mathcal{H}_\infty$ solutions, we consider two parameters: the velocity of

Table 2: Selected ζ_i and ϑ_i values from Fig. 9, Fig. 10 and Fig. 11.

	ζ_i(sec)			ϑ_i		
$\delta_{iv}(t)$	$\mathcal{H}_2$	$\mathcal{H}_\infty$	$\mathcal{H}_2/\mathcal{H}_\infty$	$\mathcal{H}_2$	$\mathcal{H}_\infty$	$\mathcal{H}_2/\mathcal{H}_\infty$
$\delta_{11}(t)$	1.8	2.4	2	-4.1	-3	-3.4
$\delta_{41}(t)$	1.8	2.8	2.1	4.5	3.4	3.5
$\delta_{12}(t)$	0.6	0.7	0.7	1.2	1.1	0.8
$\delta_{62}(t)$	2.5	3.2	2.9	0.4	0.8	0.1
$\delta_{43}(t)$	1.3	2	1.8	-6	-4.5	-5
$\delta_{63}(t)$	2.2	2.5	2.4	3.4	3.2	2.5

the response, characterized by the time at which the signal reaches a value very close to zero, denoted by $\zeta_i = [\zeta_{i1}\ \zeta_{i2}\ \zeta_{i3}\ \zeta_{i4}\ \zeta_{i5}\ \zeta_{i6}]'$ $(i = 1, 2, \ldots, 6)$ and the maximum overshoot magnitude, denoted by $\vartheta_i = [\vartheta_{i1}\ \vartheta_{i2}\ \vartheta_{i3}\ \vartheta_{i4}\ \vartheta_{i5}\ \vartheta_{i6}]'$ $(i = 1, 2, \ldots, 6)$. The results for some selected responses are summarized in Table $\vartheta_{i4}\ \vartheta_{i5}\ \vartheta_{i6}]'$ $(i = 1, 2, \ldots, 6)$. The results for some selected responses are summarized in Table 2. In general, the $\mathcal{H}_2$ control shows faster times in the parameter ζ_i than the $\mathcal{H}_\infty$ control as, for instance, the value $\zeta_{41}^2 = 1.8$ s (Fig. 9a) which is 35.7% lower than the value $\zeta_{41}^\infty = 2.8$ s (Fig. 10a). In contrast, the $\mathcal{H}_\infty$ control shows lower values ϑ_i than the $\mathcal{H}_2$ control. This effect becomes evident in states with high initial values such as $\vartheta_{43}^2 = -6$ (Fig. 9c) in comparison with $\vartheta_{43}^\infty = -4.5$ (Fig. 10c), with a 25% difference between them. These results suggest a robust response to a worst-case situation. In addition to these improvements, in both solutions $\mathcal{H}_2$ and $\mathcal{H}_\infty$ the TVF average error $\delta_i(t)$ converges to zero, showing that the presented method is capable to stabilize the multi-agent system even in the presence of uncertainties concerning to the mode of operation $\theta(t)$.

We return to Corollary 5.3.2.1 in order to investigate the $\mathcal{H}_2/\mathcal{H}_\infty$ control problem considering case 3.a (min $\beta_2\varphi^2+\beta_\infty\gamma^2$). By setting $\beta_2 = \frac{1}{3}$, $\beta_\infty = \frac{2}{3}$ and $\epsilon_\infty = \epsilon_2 = 15$, the LMI (5.17), (5.28), (5.29), (5.30) are solved for the case of asynchronous mode operation in Fig. 8. This method achieves values $\gamma = 0.1068$ and $\varphi = 0.2188$. It is worth pointing out that the optimal values for the cost are influenced by the scalars ϵ_2 and ϵ_∞. Figure 11 shows that the TVF average error $\delta_i(t)$ converges to zero despite the topology network changes and divergences between the mode of operation $\theta(t)$ and the detector $\widehat{\theta}(t)$. The $\mathcal{H}_2/\mathcal{H}_\infty$ control also combines the fast response and overshoot attenuation of the $\mathcal{H}_2$ and $\mathcal{H}_\infty$ control respectively (Table 2), for instance, the value $\zeta_{11}^{2/\infty} = 2$ s (Fig. 11a) differs only in 10% with respect to $\zeta_{11}^2 = 1.8$ s (Fig. 9a), and the value $\vartheta_{63}^{2/\infty} = 2.5$ (Fig. 11c) is even 28.12% lower than $\vartheta_{63}^\infty = 3.2$ (Fig. 10c).

Figure 12 displays snapshots of the six agents at $t = 0$ s, $t = 15$ s, $t = 20$ s and $t = 30$ s for the $\mathcal{H}_2/\mathcal{H}_\infty$ asynchronous control, where the states of the agents are denoted by the

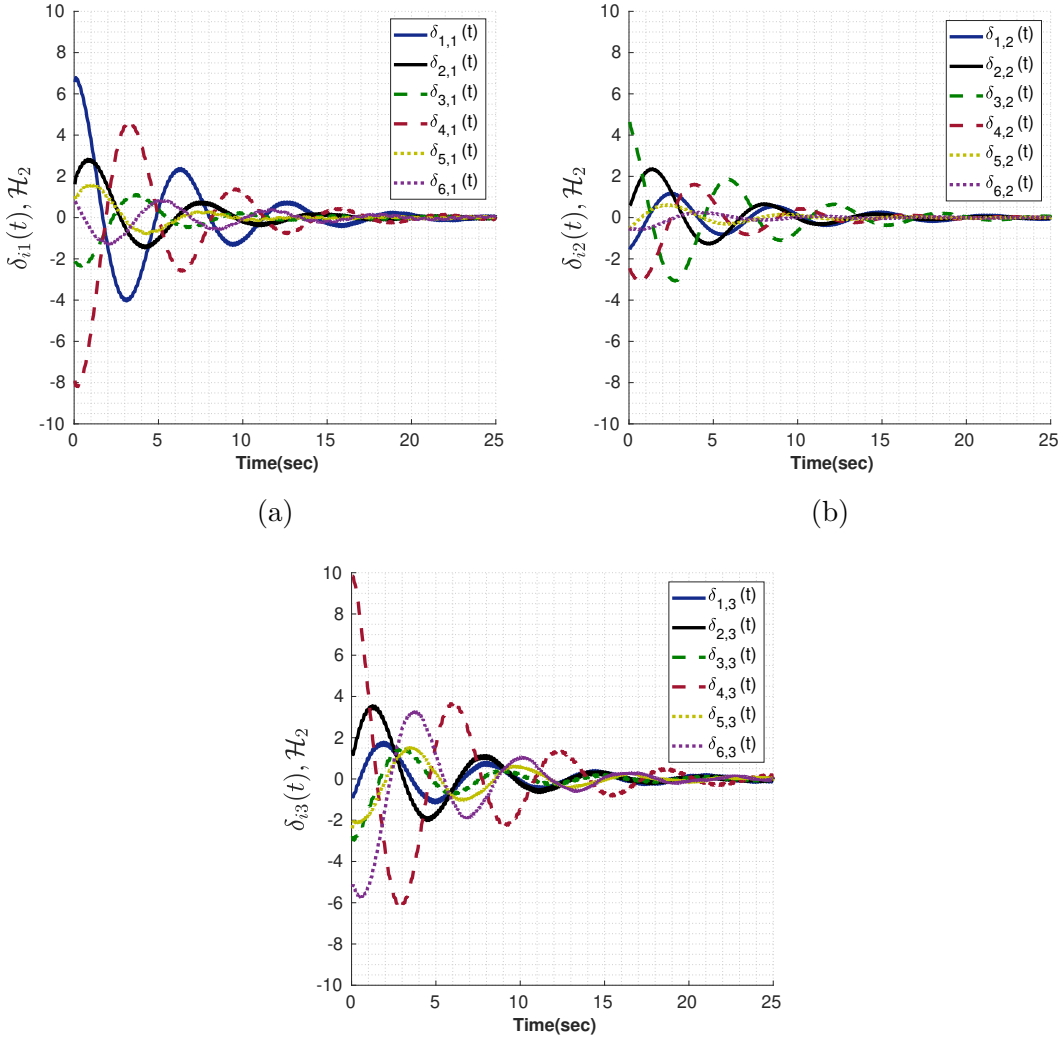


Figure 9: $\mathcal{H}_2$ average TVF error response $\delta_i(t)$.

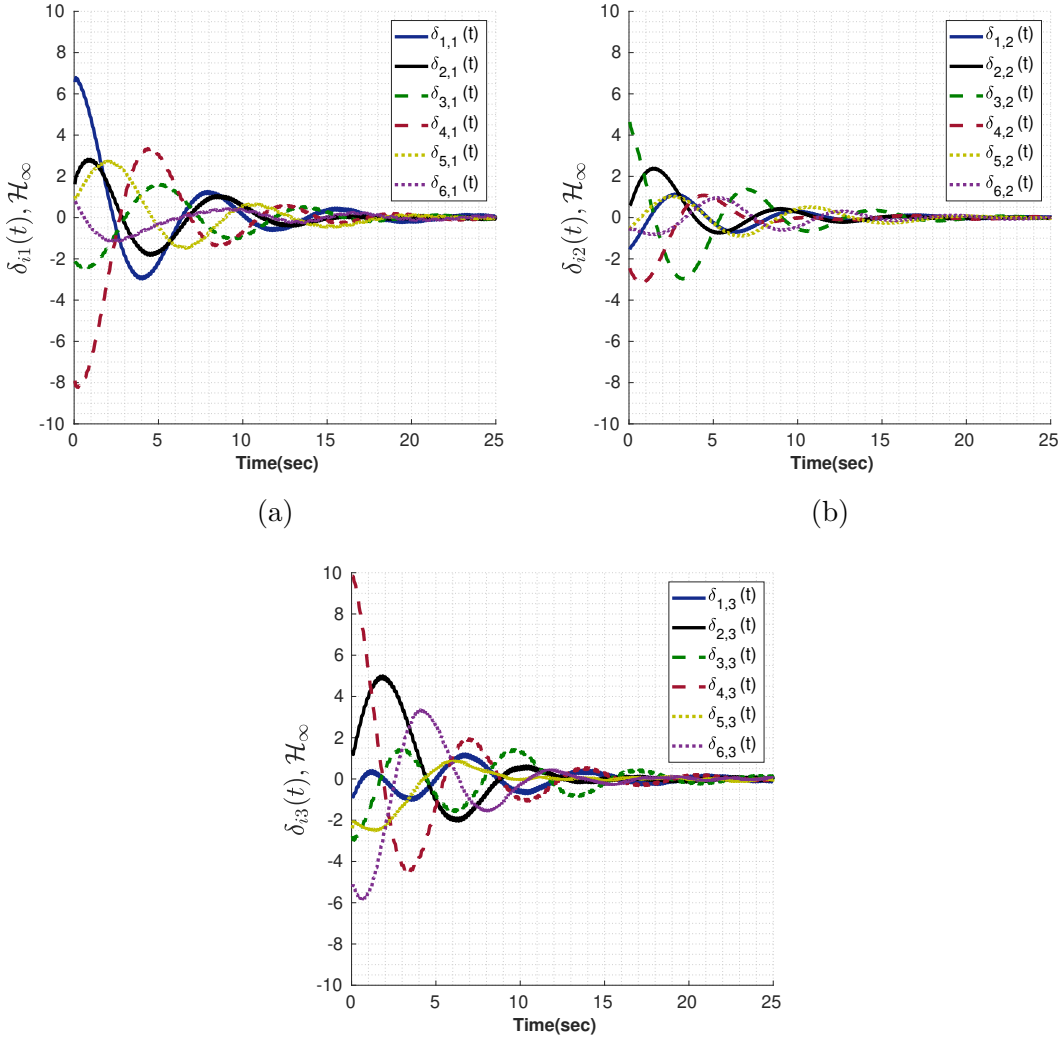


Figure 10: $\mathcal{H}_\infty$ average TVF error response $\delta_i(t)$.

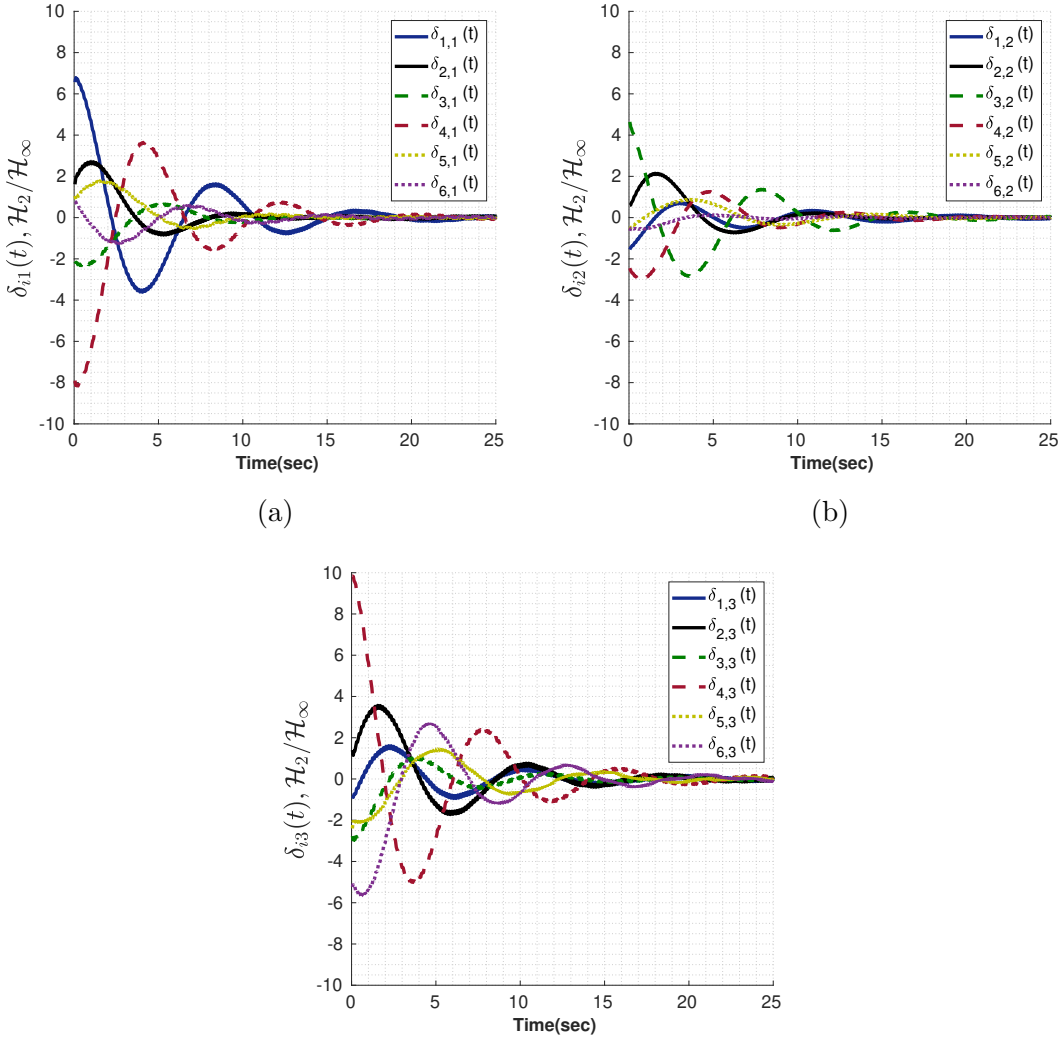
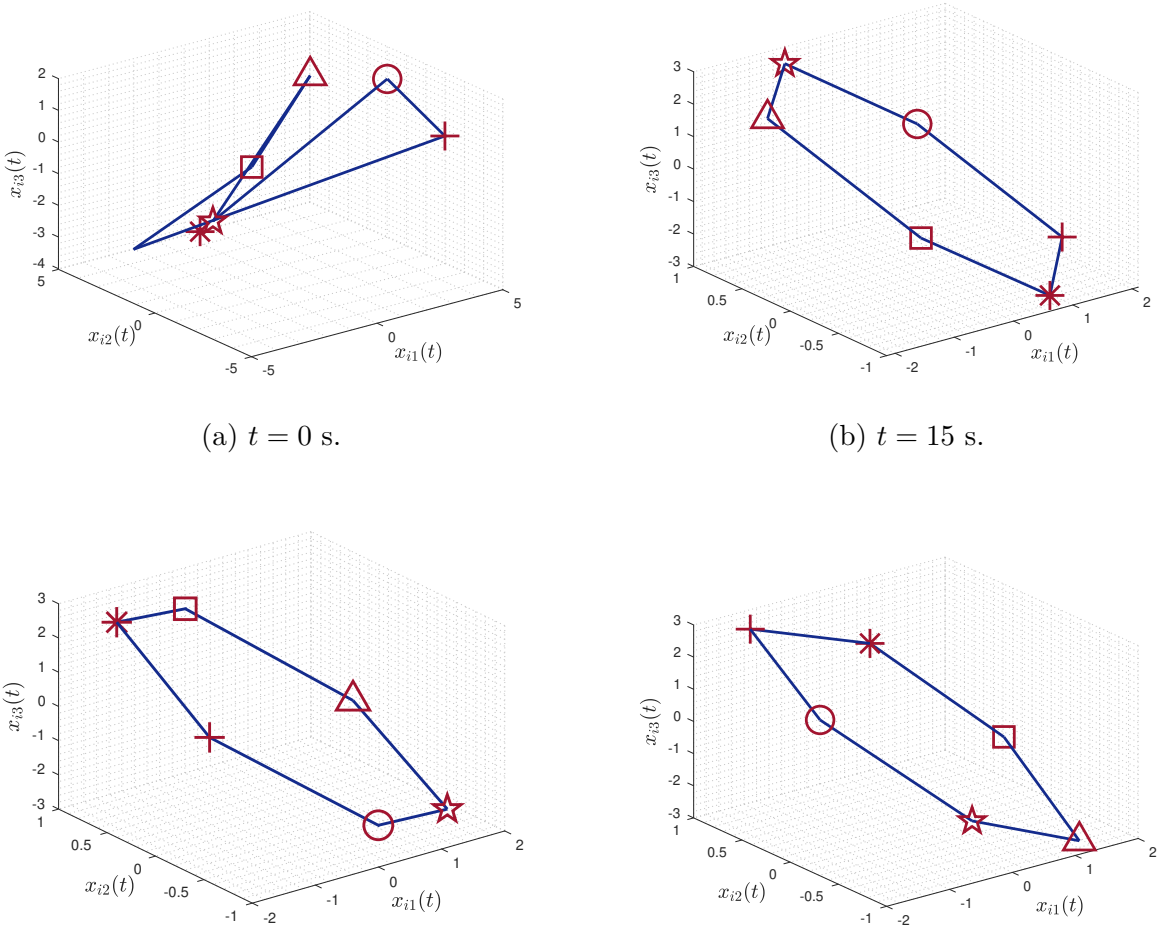


Figure 11: $\mathcal{H}_2/\mathcal{H}_\infty$ TVF average error response $\delta_i(t)$.



(a) $t = 0$ s.

(b) $t = 15$ s.

(c) $t = 20$ s.

(d) $t = 30$ s.

Figure 12: States of the six agents at different time instants t.

pentagram, triangle, square, asterisk, cross and circle respectively. From Figure 12 it can be observed that the states of the six agents keep a parallel hexagon formation and the edge of the parallel hexagon keeps rotating in a time-varying formation. We believe that these results verify the effectiveness of the proposed method.

7 CONCLUSIONS AND PERSPECTIVES

7.1 Summary

In this work, we have studied the control of continuous-time multi-agent systems under Markovian switching network topologies under partial information on the Markov parameter. To represent the possible mismatching between the detector and network modes, a hidden Markov model is considered. Notice that in comparison with other existing works, the controller relies only on the information coming from a detector device (represented by $\widehat{\theta}(t)$), and that the joint process $Z(t) = (\theta(t), \widehat{\theta}(t))$ is an exponential hidden Markov chain, with $\widehat{\theta}(t)$ being the observable part. It is important to point out that a key difference with respect to (DONG et al., 2020; NGUYEN; KIM, 2020), which deals only with the $\mathcal{H}_\infty$ case, lies in the model representing the detector. The formulation considered in these papers is based on a conditional probability condition that must hold for each time t (DONG et al., 2020) which can be hard to be checked, while in our formulatio, the model $Z(t)$ is assumed to be an exponential hidden Markov process so that the time evolution of the process $Z(t)$ is well defined and can be easily simulated. Notice also that this formulation encompasses the so-called mode-dependent case, mode-independent case, and cluster case.

The design technique is based on LMI optimization problems so that the powerful toolboxes available for this class of problems can be used. A set of LMI conditions are provided to design a distributed static output controller that guarantees the closed-loop stability of MAS with the following performance criteria:

- $\mathcal{H}_2$ **control.** For the $\mathcal{H}_2$ control, a static output TVF controller is provided in terms of LMI, for this case, we consider the quadratic cost only in the states and control variables. The theorem 5.3.2 shows that, if the LMI (5.28), (5.29), (5.30) are satisfied, there exists a controller that stabilizes the MAS formation in the MSS with a $\mathcal{H}_2$ norm less than a given $\varphi > 0$.

- $\mathcal{H}_\infty$ **control.** Similarly, in the $\mathcal{H}_\infty$ control case, we propose a solution for the static output TVF control. The theorem 5.3.1 shows that if the LMI (5.17) is satisfied, then the MAS is MSS with a closed-loop $\mathcal{H}_\infty$ norm less than a given $\gamma > 0$ whenever the distributed control protocol is applied, as pointed out, this norm represents the worst-case effect of finite-energy disturbances on the output.

- $\mathcal{H}_2/\mathcal{H}_\infty$ **control.** By combining the above results, we propose in the corollary 5.3.2.1 that if the LMI (5.17), (5.28), (5.29), (5.30) are satisfied, there exist a $\mathcal{H}_2/\mathcal{H}_\infty$ controller that minimizes the quadratic functional while providing robustness to the closed-loop MAS.

To verify the effectiveness of this method, numerical examples were performed, showing that it is possible to design controllers able to stabilize a time-varying formation of MAS while achieving a minimal performance cost. Furthermore, it is shown that the mixed control $\mathcal{H}_2/\mathcal{H}_\infty$ successfully combines the control properties of the pure $\mathcal{H}_2$ and $\mathcal{H}_\infty$ strategies.

7.2 Future works

There are several open problems in the formation control of MAS with partial observations on the network topology. Some are mentioned below:

- In this work, only the static output control has been considered; for this reason, the dynamic output control problem for MAS formation under partial observations can be tackled.

- The numerical examples that were used to verify the theoretical results are based on simple multi-agent systems. In the future, more complex experiments, considering realistic MAS models such as unmanned vehicles or mobile robots on large-scale networks, could be considered.

- This work considers uncertainties in the operation mode detection. In addition, communication signals in TVF control of MAS may be affected by transmission noise and/or communication delays. Thus, it would be interesting to deal with the TVF control of MAS under the hidden Markov switching topology as proposed in this dissertation, but also incorporating communication noises and time-delay.

APPENDIX A – NUMERICAL RESULTS OF CHAPTER 6

The controllers calculated in Chapter 6 are shown in this appendix.

A.1 Synchronous mode

A.1.1 $\mathcal{H}_2$ controller

$$\left[\begin{array}{c} K_1 \\ \hline K_2 \end{array}\right] = \left[\begin{array}{ccc} -0.0400 & -0.0200 & -0.0320 \\ -0.0200 & -0.0100 & -0.0160 \\ -0.0320 & -0.0160 & -0.0256 \\ \hline -0.0323 & -0.0162 & -0.0259 \\ -0.0162 & -0.0081 & -0.0129 \\ -0.0259 & -0.0129 & -0.0207 \end{array}\right] \tag{A.1}$$

A.1.2 $\mathcal{H}_\infty$ controller

$$\left[\begin{array}{c} K_1 \\ \hline K_2 \end{array}\right] = \left[\begin{array}{ccc} -0.0058 & -0.0015 & -0.0024 \\ -0.0015 & -0.0036 & -0.0012 \\ -0.0024 & -0.0012 & -0.0048 \\ \hline -0.0041 & -0.0009 & -0.0015 \\ -0.0009 & -0.0027 & -0.0007 \\ -0.0015 & -0.0007 & -0.0034 \end{array}\right] \tag{A.2}$$

A.2 Asynchronous mode

A.2.1 $\mathcal{H}_2$ controller

$$\left[\begin{array}{c} K_1 \\ \hline K_2 \end{array}\right] = \left[\begin{array}{ccc} -0.0401 & -0.0195 & -0.0312 \\ -0.0195 & -0.0109 & -0.0156 \\ -0.0312 & -0.0156 & -0.0261 \\ \hline -0.0319 & -0.0155 & -0.0248 \\ -0.0155 & -0.0086 & -0.0124 \\ -0.0248 & -0.0124 & -0.0207 \end{array}\right] \tag{A.3}$$

A.2.2 $\mathcal{H}_\infty$ controller

$$\left[\begin{array}{c} K_1 \\ \hline K_2 \end{array}\right] = \left[\begin{array}{ccc} -0.0101 & -0.0020 & -0.0032 \\ -0.0020 & -0.0071 & -0.0016 \\ -0.0032 & -0.0016 & -0.0086 \\ \hline -0.0072 & -0.0012 & -0.0020 \\ -0.0012 & -0.0054 & -0.0010 \\ -0.0020 & -0.0010 & -0.0063 \end{array}\right] \tag{A.4}$$

A.2.3 Mixed $\mathcal{H}_2/\mathcal{H}_\infty$ controller

$$\left[\begin{array}{c} K_1 \\ \hline K_2 \end{array}\right] = \left[\begin{array}{ccc} -0.0526 & -0.0251 & -0.0401 \\ -0.0251 & -0.0150 & -0.0201 \\ -0.0401 & -0.00201 & -0.0345 \\ \hline -0.0427 & -0.0204 & -0.0326 \\ -0.0204 & -0.0121 & -0.0163 \\ -0.0326 & -0.0163 & -0.0280 \end{array}\right] \tag{A.5}$$

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