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DEVELOPMENT OF A MULTI-OBJECTIVE TUNING TECHNIQUE FOR MODEL PREDICTIVE CONTROLLERS

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ABSTRACT

Two multi-objective optimization based tuning techniques for Model Predictive Control (MPC) were developed. Both take into account the sum of the squared errors between closed-loop trajectories and reference responses based on pre-defined goals as tuning objectives; one solves a lexicographic optimization to obtain an optimum set of tuning parameters (LTT), whereas the other solves a compromise optimization problem (CTT). The main advantages are an automated framework, and straightforward goal definition, which are capable of taking into account a specification on the process dynamics, a time-domain metrics, and of embedding the control engineer’s knowledge into a reliable approach. A fluid catalytic cracking tuning case study unveiled the goal definition flexibility of the LTT, with respect to output tracking and variable coupling. A heavy oil fractionator in closed-loop with a MPC case study compared both tuning techniques developed here, and it was observed that the LTT in fact prioritizes the main objectives, whereas the CTT yields an average solution, in terms of the tuning objectives. The CTT was compared to another multi-objective tuning technique from the literature, in the tuning of a MPC with input targets and output zone control in closed-loop with a crude distillation unit model. The simulation results showed that the CTT allows for faster results, regarding the computational time to compute the tuning parameters and there is no need of a posteriori decisions to select the best non-dominated solution. Real MPC applications are strongly hindered by model uncertainty. This limitation was addressed by the extension of the tuning techniques to account for multi-plant model uncertainty, thus obtaining optimum robustly tuned parameters for nominal controllers, addressing the trade-off between robustness and performance. A robustly tuned Infinite Horizon MPC (IHMPC) was compared to a Robust IHMPC, in closed-loop with a C3/C4 splitter system model. It was observed in a simulation that even though the latter yields better output responses, it is two orders of magnitude slower than the former in online operation.

Keywords: Model Predictive Control tuning, multi-objective optimization tuning, robust tuning

RESUMO

Neste trabalho foram desenvolvidas duas técnicas de sintonia para controladores preditivos por modelo. Ambas visam minimizar a soma do erro quadrático entre respostas do sistema em malha fechada e trajetórias de referência pré-definidas; a primeira resolve um problema de otimização lexicográfica enquanto a segunda resolve um problema de otimização de compromisso. As vantagens dos métodos apresentados são: maior automatização, definição de objetivos de sintonia intuitiva que considera especificações na dinâmica do processo, uma métrica no domínio do tempo e é capaz de incluir o conhecimento do engenheiro de controle em uma técnica de sintonia confiável. Um estudo de caso no sistema de craqueamento catalítico ilustrou a flexibilidade de definição dos objetivos da técnica lexicográfica. Um estudo de caso sobre uma coluna de fracionadora de óleo pesado em malha fechada com um controlador preditivo por modelo comparou ambas as estratégias de sintonia desenvolvidas aqui e pode-se concluir que a técnica lexicográfica dá prioridade aos objetivos importantes enquanto a técnica de compromisso calcula uma solução média, com respeito aos objetivos. A técnica de compromisso foi comparada a um método de sintonia da literatura quanto a aplicação em um controlador preditivo de horizonte infinito com targets para as entradas e controle por faixas das saídas com uma coluna de destilação. Observou-se que a técnica desenvolvida aqui é computacionalmente mais rápida e não requer a escolha de uma solução não-dominada dentre um conjunto de soluções de Pareto. Aplicações reais de controle preditivo são severamente afetadas por incerteza de modelo. Estendeu-se as técnicas desenvolvidas aqui para considerar o caso de incerteza multi-planta, calculando parâmetros de sintonia robustos para controladores nominais, visando tratar o compromisso entre performance e estabilidade e robustez da malha fechada. Um controlador preditivo de horizonte infinito foi sintonizado de forma robusta e comparado com um controlador preditivo robusto em malha fechada com um modelo de separadora C3/C4. Observou-se que este consegue controlar melhor o processo, entretanto, tem um tempo de computação duas ordens de grandeza maior que o controlador nominal, em operação on-line.

Palavras-chave: sintonia de controladores baseado em modelo, sintonia robusta, sintonia por otimização multi-objetivo.

LIST OF ILLUSTRATIONS

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NOMENCLATURE

A	State transition matrix
A_{eq}	Equality constraint coefficient matrix
A_{in}	Inequality constraint coefficient matrix
B	States to system inputs relationship matrix
b_{eq}	Equality constraint independent vector
$b_{i,j,k}$	k-th coefficient of the numerator of transfer function $G_{i,j}(s)$
b_{in}	Inequality constraint independent vector
C	States to system outputs relation matrix
c	QP constant term
c_f^T	QP linear term vector
D_i	Robust IHMPC auxiliary matrix
D_m	Step-response model representation dynamic matrix
$d_{i,j}^0$, $d_{i,j,k}^d$, $d_{i,j}^i$	Step-response coefficients
e	Number of equality constraints in a multi-objective optimization problem
$F(x)$	Vector of objectives
$F^{\circ}(x)$	Utopia point
$F_i(x)$	i-th objective of the vector $F(x)$
F_j^{N*}	j-th optimum objective cost obtained in a previous lexicographic step
f	Vector of stacked free responses over p
f_{ref}	Vector of percentages multiplying FO reference transfer functions time constants
$f(k+i)$	System free response in step response model representation at time instant $k+i$
$G_{des}(s)$	Reference trajectory transfer function matrix
$G_{des,i}(s)$	i-th diagonal entry of matrix $G_{des}(s)$

$G_{des,i}^{FO}(s)$	Desired response first order transfer function approximation
$G_{i,j}(s)$	Transfer function between output y_i and input u_j
$G(z)$	Transfer function in the *z*-domain
g_i	Step response coefficient at time step *i*; SISO systems
$g_{i,k}$	Step response coefficient at time step *i* for system output *k*; MIMO systems
$g_j(x)$	Generic inequality constraint
$\overline{\overline{g}}_{i,j}$	Matrix containing the $g_{i,k}$ step response coefficients at time step *i* for an unitary step in input *j*
H	QP hessian matrix
h_i	Impulse response coefficient at time step *i*; SISO systems
h_i^{kj}	Impulse response coefficient of output *j* at time step *i* for input *k*; MIMO systems
$h_i(x)$	Generic equality constraint
I_η	Identity matrix of dimension η x η
$\overline{I}_{nu}$	Auxiliary matrix used in the MPC input constraint
$J^{w'}$	(Chapter 3) Sum of all the objectives up to the lexicographic optimization step *w'*
K	Hessian matrix of the unconstrained MPC problem
$K_{i,j}$	Open-loop gain of transfer function $G_{i,j}(s)$
$K_{i,j}^{N}$	Normalized open-loop gain
k	(Appendix B) Input number (Remaining Chapters) Discretized time instant
L	Number of system models in Ω, which describes the multi-plant uncertainty
LB	Lower bounds on the decision variables of a optimization problem matrix
$\overline{M}$	Auxiliary matrix used in the MPC input constraint matricial form
m	MPC control horizon

N	(Appendix B) DMC model horizon (Remaining Chapters) Nominal plant model
n_{dec}	Number of decision variables in a multi-objective optimization
$\hat{n}(k+i\,\vert\,k)$	Step/impulse response model disturbance at time instant $k+i$
nu	Number of system inputs
ny	Number of system outputs
p	MPC output prediction horizon
$\bar{Q}$	Terminal cost matrix of the infinite horizon MPC
Q_u	Weighting matrix on the differences between system outputs and input targets
$\bar{Q}_u$	Extended weighting matrix of the inputs
Q_y	Weighting matrix on the differences between system outputs and output setpoints
$\bar{Q}_y$	Extended weighting matrix of the outputs
$q_{u,j}$	j-*th* diagonal entry of matrix Q_u
$q_{y,i}$	i-th diagonal entry of matrix Q_y
R	Weighting matrix on input increments
$\bar{R}$	Extended weighting matrix of the input increments
$r_{i,j,k}$	k-th pole of transfer function $G_{i,j}(s)$
r_j	j-th diagonal entry of matrix R
S_y, S_i	IHMPC weighting matrices on slack variables δ_y, δ_i
S_r, S_t	LTT slack variables weighting matrices
s	(Section 7.1.3) Parameterization vector of the NBI method (Remaining Sections) Laplace variable
T_s	Sampling time
U	Decision maker's 'real' utility function
UB	Upper bounds on the decision variables of an optimization problem
u^c	Vector of stacked inputs

$u(k)$	System input at instant *k*
$u(k)_j^N$	*j-th* normalized input at time instant *k*
$u_{\min}, u_{\max}$	System input minimum and maximum bounds
$V_{1,a},\ V_{1,b}(\omega_l)$	(R)LTT cost function value calculated for plant model ω_l
$V_{2,b},\ V_{2,c}(\omega_l)$	(R)CTT cost function value calculated for plant model ω_l
z	Number of inequality constraints in a multi-objective optimization problem
w	Number of objectives in a multi-objective optimization problem
w^1	Output-related tuning objectives
w^2	Input-related tuning objectives
w'	(Chapter 3) Number of output objectives taken into account in the LTT
$y(k)$	System output at time instant *k* (SISO)
y^c	Vector of stacked output predictions
$y_i^{ref}(k)$	Discretized reference trajectory of the *i*-th output at time instant *k*
$y(k)_i^N$	*i-th* normalized output
$y_j(k)$	System output *j* at time instant *k* (MIMO)
y_0	System output reference values vector
$y_{sp}(k)$	Output set point vector at time step *k*
$\hat{y}(k+j\,\vert\,k)$	Output prediction vector at time instant *k*+*j* computed at time instant *k*
$\boldsymbol{X}$	Feasible design space in multi-objective optimization problems
x	Multi-objective optimization decision vector
$x(k)$	System states vector at time instant *k*
x_c^*	Compromise tuning optimum solution
x_{utop}	Utopia solution decision variables

$x^d(k)$	State-space model stable states
$x^i(k)$	State-space model integrating states
$x_j^i(k)$	Stacked integrating states used in the RIHMPC state-space representation
$x^s(k)$	State-space model integrating states introduced by the incremental form
$\boldsymbol{Z}$	Feasible criterion space in multi-objective optimization problems
z	Variable used in z-transform transfer functions
$z_j(k)$	State-space model auxiliary state variable that accommodates the past input increments

Greek letters

γ	RCTT supremum
$\Delta u(k)$	System input increment vector at time instant k
Δu_k	Vector of stacked input increments
$\Delta u_{\max}$	Maximum input increment bound
λ	Decision variable of the NBI method
$\delta_{i,k}$	IHMPC slack variable for the integrating states
δ_r	RLTT robustness constraint slack variable
δ_t	LTT performance constraint slack variable
$\delta_{y,k}$	IHMPC slack variable, system output and set point value deviation
$\theta_{i,j}$	Dead time of transfer function $G_{i,j}(s)$
$\theta_{\max}$	Maximum dead time of a system
θ_{set}	Input reference trajectory horizon
θ_t	Tuning horizon
Φ	Pay-off matrix of the NBI method
Ω	Set of system models that define the multi-plant uncertainty
ω_i, ω_N, ω_T	*i*-th system model in Ω; most probable, or nominal model in Ω; ‘real system’ model in Ω

LIST OF ACRONYMS

AB	Air Blower
ASTM	American Society for Testing Materials
CARIMA	Controlled Auto-regressive and Integrated Moving Average
CDU	Crude Distillation Unit
CETAI	Acronym in Portuguese for Technology and Industrial Automation Excellence Center
(R)CTT	(Robust) Compromise Tuning Technique
DMC	Dynamic Matrix Control
FCC	Fluid Catalytic Cracking
FO	First-order transfer function
FOPDT	First-order-plus-dead-time transfer function
HCGO	Heavy Cycle Gas Oil
HOF	Heavy Oil Fractionator
IAE	Integral of Absolute Error
ICGO	Intermediate Cycle Gas Oil
IDCOM-M	Identification and Command – Multivariable
IHMPC	Infinite Horizon Model Predictive Control
ISE	Integral of Square Error
ITAE	Integral of Time Multiplied by the Absolute Error
ITSE	Integral of Time Multiplied by the Square Error
LQR	Linear Quadratic Regulator
(R)LTT	(Robust) Lexicographic Tuning Technique
MF	Main Fractionator
MIMO	Multiple-Input Multiple-Output
MINLP	Mixed-integer Nonlinear Programming
MOO	Multi-objective Optimization
MPC	Model Predictive Control
NBI	Normal Boundary Intersection
PID	Proportional-integral-derivative
PSO	Particle Swarm Optimization
QP	Quadratic Programming

RG	Regenerator Vessel
RGA	Relative Gain Array
RIHMPC	Robust Infinite Horizon Model Predictive Control
RX	Reactor
SISO	Single-Input Single-Output
SS	Stripping Section
SSE	Sum of Square Errors
ST	Steam Turbine
VRU	Vapor Recovery Unit
WGC	Wet Gas Compressor

CONTENTS

1. INTRODUCTION

Model Predictive Control (MPC) has been widely used in industry, especially in oil refining and petrochemical plants. It is a successful control strategy because it accounts for process constraints and can be easily extended to Multiple-Input Multiple-Output (MIMO) systems. The earliest reported MPC application in industry dates back to the 1970’s. Motivated by industrial needs, the academic contributions started to improve the early MPC formulations, increasing robustness, enhancing performance and stability and reducing the computational cost. For detailed information about the evolution of the MPC technology, the reader is referred to Qin & Badgwell, (2003). The usual control structure in most industrial plants is as follows: the lower automation level is based on Proportional-Integral-Derivative (PID) controllers arranged in a distributed control system (DCS) framework. One level above, a MPC controller calculates optimum input increments, based on the process model and input and output target values, which are calculated by an upper Real Time Optimization (RTO) layer.

While earlier MPC controllers were based on a step or an impulse response model, which need large amounts of data storage, the recent realizations of MPC are based on state-space models, which need significantly less data to accurately describe the system behavior (Lee, Morari & Garcia, 1994). Unfortunately, the state-space models are not as intuitive as the step (impulse) response models, and the state variables, which are introduced to establish a bridge between the system inputs and outputs, may not have physical meaning and therefore, it is usually impossible to measure them. Along with the advantages brought by the improvements in MPC formulations, also came a plethora of parameters that interfere in its stability, robustness, and overall performance. The parameters that directly affect the controller behavior, or tuning parameters, vary according to the controller formulation.

1.1. Motivation

A joint action with PETROBRAS’ research center on industrial automation (CETAI, in Portuguese) identified the most prominent concerns of the process control engineers in the MPC commissioning scenarios:

- Time spent during the tuning procedure;
- Prioritization of output process constraints;
- Appropriate definition of tuning goals

Concomitantly, the most pressing concern observed in the literature is the definition of compatible tuning objectives, representing the desired control performance. Moreover, one can identify several shortcomings of the current MPC tuning methods, such as: excessively complex based on heuristics on one hand and unrealistic assumptions on the other, make room for novel breakthroughs in the MPC tuning research field.

In industry the MPCs are usually tuned by trial and error based on the experience of process and control engineers (Liu & Wang, 2000; Al-Ghazzawi, Ali, Nouh, & Zafiriou, 2001). In fact, the trial and error fine tuning step may be indispensable because the tuning results obtained in simulations may not be feasible for the real application. However, in Qin & Badgwell (2003), it is recommended to tune the MPCs using an automated tuning framework in a simulated environment nonetheless.

The trial and error technique should be ruled out as an early tuning strategy because it is cumbersome, time consuming, and does not allow for a proper tuning goal definition. In this way, it is impossible to set up an automated trial and error method, which is its main limitation (Garcia, Prett & Morari, 1989). A survey showed that over 70% of the plant automation strategy providers and 60% of their clients consider that the human cost is the most relevant economic factor in the commissioning step of the control system (Bauer & Craig, 2008). Therefore, since the tuning parameters affect the closed-loop performance of the system, there is a potential economic gain in tuning the controllers properly.

Some tuning parameters are tightly connected to the system operation and the available computational facilities and therefore, they cannot be freely manipulated. Furthermore, the tuning literature provides reliable techniques for some parameters. This section will discuss some tuning guidelines for N, p, m, T_s in DMC, and m, T_s, S_y, S_i in IHMPC and RIHMPC. The tuning techniques developed here address the weighting matrices Q_y, Q_u, and R.

1.2. Selecting the un-addressed tuning parameters

1.2.1. Sampling time

The sampling time, sampling rate, or sampling frequency, indicates the time interval in which subsequent data samples are collected to convert a continuous signal into a discrete signal. Usually, the sampling time is selected considering the available computing power and the system dynamics. Faster dynamics usually calls for shorter sampling times. Slow dynamic processes are often found in the petrochemical and oil processing industries.

1.2.2. Model horizon

The DMC process model is based on a step (or impulse) response representation, which stores all the coefficients of the open-loop step (or impulse) response up to the model horizon N. Therefore, N should be large enough to accommodate the dynamics of all the input $\times$ output pairs of the system.
Georgiou, Georgakis & Luyben(1988) propose to choose N as at least 95% of the slowest step response settling time, which yields a good compromise between the amount of stored data and an accurate representation of the system. This tuning guideline is used throughout this thesis.

1.2.3. Prediction horizon

The prediction horizon p defines the time window in which the difference between predicted system outputs and the output set points are considered in the control cost function. The tuning guidelines for the prediction horizon are usually based on the system dynamics. Some authors suggest a percentage of the largest time constant, while others recommend a minimum value based on the relationship between the model horizon and the control horizon (Garriga & Soroush, 2010). For open loop stable systems, a larger prediction horizon usually leads to more stable and more computationally demanding controllers.

1.2.4. Control horizon

In the usual MPC strategies, it is assumed that the increments of the system inputs, or the control actions, will only vary along a short time interval, known as the control horizon (*m*). This means that the input values will remain constant beyond the control horizon, and, therefore the input increments will be zero. Large values of *m* are likely to result in aggressive control actions, whereas small values of *m* tend to increase robustness and to reduce computational expense, but decrease the aggressiveness.

In the literature one can find different suggestions for the value of the control horizon. Set *m* equal to 1 in order to minimize the computational demand. Set *m* equal to the number of unstable poles to guarantee that there will be enough degrees of freedom to cancel these poles and the end of the control horizon. Finally, it also recommended to set *m* as a percentage of the prediction horizon (Garriga & Soroush, 2010). In this work, the control horizon is chosen within the range 3-6, which represents an adequate tradeoff between control performance and computational expense.

1.2.5. Slack variables weighting matrices

Even though in Santoro & Odloak (2012) and in Martins et al. (2013) it is suggested a two-step solution algorithm in order to guarantee the control problem numerical convergence, an equivalent result is obtained through sufficiently high penalization on the integrating slack variables. Actually, according to Alvarez et al. (2009), any positive definite S_i will yield a converging solution over time in the two step IHMPC.

The one-step IHMPC and RIHMPC algorithms will be used here, and S_y and S_i are set to at least four orders of magnitude larger than R, to guarantee numerical convergence(Alvarez et al., 2009).

1.3. Tuned parameters

1.3.1. The output error weight Q_y

The entries of matrix Q_y contemplate the relative importance among the system outputs, which is usually obtained from process knowledge or economic goals. Nonetheless, Q_y provides additional degrees of freedom to solve the tuning problem. The literature recommends, on one hand, to include it in the tuning problem, and disregard the original priority relationship between outputs (Liu & Wang, 2000; Cairano & Bemporad, 2009; Shah & Engell, 2010); while on the other hand, it is recommended to tune R using a pre-defined Q_y (Shridhar & Cooper, 1997, 1998; Huusom et al., 2012).

1.3.2. The input weight Q_u

As far as the author's knowledge goes, the tuning literature does not provide tuning guidelines for Q_u, probably because few realistic MPC algorithms are considered in the literature on the MPC tuning methods.

1.3.3. The input move weight R

Small input variations yield smooth output and input profiles, and large variations yield faster output tracking performance. In MPC literature, R *is* considered the most important tuning parameter to be defined in the commissioning stage of the controller.

1.4. Objectives

The main objective of this thesis is to develop a new MPC tuning framework that addresses the shortcomings of the current methods from the literature and that suits the needs of industrial applications.

Another issue in the controller commissioning scenarios is the way to address model uncertainty. In real applications, the model identification step is extensively time consuming; therefore, it is highly unlikely for the control engineer to have sufficient information about model uncertainty at his/her disposal. A common approach in the literature for plant-model mismatch is adopting robust cost-contracting controllers, in which non-linear constraints force the value of the cost function evaluated for each possible model inside an uncertainty polytope to be lower or equal to the value of the same cost at the previous time step. This non-linear constraint takes its toll on quadratic programming solvers, generally used in nominal MPC strategies, making the robust controller problem solvable only by non-linear optimization algorithms, which might prove prohibitively time consuming, depending on the process size or the sampling time. This problem is addressed in the development of a robust tuning strategy for nominal predictive controllers. Such approach allows for offline calculation of robustly optimum tuning parameters, redirecting the online computational burden of solving a non-linear optimization problem to an offline step.

The global objectives above lead to punctual ones, such as: choosing an appropriate tuning framework where the tuning goals can be defined straightforwardly; accessing multi-objective optimization strategies to solve the tuning problem; testing the tuning technique proposed here in typical MPC applications on significant benchmark processes from the literature and relevant processes from the petrochemical and oil refining industries.

1.5. Contributions

Two MPC tuning techniques were developed in this work to address the shortcomings of the current algorithms. Both techniques consider as tuning goals input or output reference trajectories, which can be defined, for example, in terms of open-loop input-output transfer functions. The two tuning techniques differ in the multi-objective optimization approach used to solve the tuning problem. The first one was based on a lexicographic optimization approach, resulting in better performances for high priority goals, while the other is based on the compromise optimization approach, resulting in satisfactory results for all goals. The methodologies were extended to the tuning of controllers that include input targets.

The tuning techniques were also extended to deal with multi-plant model uncertainty and the optimum parameters result from a trade-off between robustness and performance.

1.6. List of publications

1.6.1. Journal publications

Yamashita, A.S., Zanin, A.C., Odloak, D. Tuning Of Model Predictive Control With Multi-Objective Optimization. *Brazilian Journal of Chemical Engineering.* Under review, submitted in December the 4th, 2014.

1.6.2. Congress proceedings

Yamashita, A.S. & Odloak, D. Sintonia automática de controladores MPC. *In 7th Congresso Brasileiro em P&D em Petróleo e Gás - PDPETRO*, Aracaju, Brazil, 2013.

Yamashita, A.S. & Odloak, D. Reference Trajectory Based Tuning Strategy for Model Predictive Controllers. *In 5th International Symposium on Advanced Control of Industrial Process*, Hiroshima, Japan, 2014.

Yamashita, A.S., Odloak, D. Compromise Optimization Tuning Strategy for Model Predictive Controllers. In *14th AIChE Annual Meeting*, Atlanta, GA, 2014.

1.7. Thesis structure

This thesis is structured as follows: Chapter 1 states the main objectives pursued here; the literature is reviewed in Chapter 2 and begins with two review papers about MPC tuning strategies. It follows with three MPC review papers that provide some heuristic guidelines. Then, industrial and academic surveys on MPC tuning are assessed. These works, spanning from the 1980's to 2014, were the underlying basis that guided the subsequent research work on the thesis main topics, and yielded some relevant contributions to the field. Chapter 3 introduces the Lexicographic Tuning Technique (LTT), and Chapter 4, the Compromise Tuning Technique (CTT). In Chapter 5, we apply the tuning techniques to four system models to evaluate their efficiency. The nominal and robust scenarios are considered. The first case study proposes a tuning methodology for a Fluid Catalytic Cracking unit in closed-loop with a DMC. The second case study considers the Heavy Oil Fractionator benchmark system in closed loop with a MPC. The third one addresses the tuning of a Crude Distillation Unit in closed-loop with an MPC with input targets and output zone control, and finally, the robust tuning techniques are assessed on a C3/C4 splitter system. The thesis closes with conclusions and directions for further work in Chapter 6.

Appendix A summarizes the transfer functions of the systems considered in this thesis to illustrate the application of the tuning techniques. Appendix B contemplates the MPC formulations considered in this work and Appendix C presents a brief review of techniques usually applied to solve multi-objective optimization problems.

2. LITERATURE REVIEW

MPC formulations take into account a model to predict the behavior of the system and a rolling horizon strategy, in which optimum control moves are calculated as the solution of a constrained optimization problem at each sampling time. The first control action is injected into the system and the procedure is repeated at the next sampling instant. The control cost function incorporates at least two weighted sum terms; the first one considers the deviations between the outputs and the output set points along a prediction horizon, weighted by a positive definite matrix, and the second one considers the control moves along a control horizon, weighted by a positive semi-definite matrix. The closed-loop performance is affected by several parameters, including the input and output horizons and the weighing matrices of the control cost function. Equation (2-1) describes a generic finite horizon control cost function.

$$J = \sum_{i=1}^{p} \left\| y\left(k+i \mid k\right) - y^{sp} \right\|_{Q_y}^{2} + \sum_{i=0}^{m-1} \left\| \Delta u\left(k+i \mid k\right) \right\|_{R}^{2} \tag{2-1}$$

where p is the prediction horizon, m is the control horizon, Q_y, Q_y>0 and R, R≥0 are weighting matrices, y^{sp} is a output reference value, $y\left(k+i \mid k\right)$ is the output prediction calculated for time instant $k+i$ using information available at time instant k and a state-space or equivalent model representation, $\Delta u\left(k+i \mid k\right)$ is an input increment, or control action, that affects the system at time instant $k+i$. The parameters p, m, Q_y, R, directly affect the controller performance. Observe that more parameters might be considered according to the complexity of the control cost formulation and the model used to calculate $y\left(k+i \mid k\right)$.

Depending on the approach that is followed to obtain the optimum tuning parameters, existing MPC tuning methods are usually divided into two major groups. The first one encompasses the methods based on analytical expressions obtained through some level of simplification, either in the process description or process model, or in the arbitrary selection of some of the parameters. The second group concerns the techniques based on multi-objective optimization. In the latter approach, the techniques differ according to the goal definition and to which multi-objective optimization algorithm is used to solve the tuning problem. The methods show different tuning goal definitions that may take into consideration time domain characteristics (e.g. settling time, rise time, overshoot); time domain mathematical metrics (e.g. Integral of Square Error (ISE), Integral of Absolute Error (IAE)); frequency domain sensitivity function norms; or a combination of the previously mentioned possibilities. The time domain metrics can measure the controller closed-loop performance directly, as seen in Figure 2-1, which usually requires closed-loop simulations. Figure 2-2 shows a classification chart of the tuning methods.

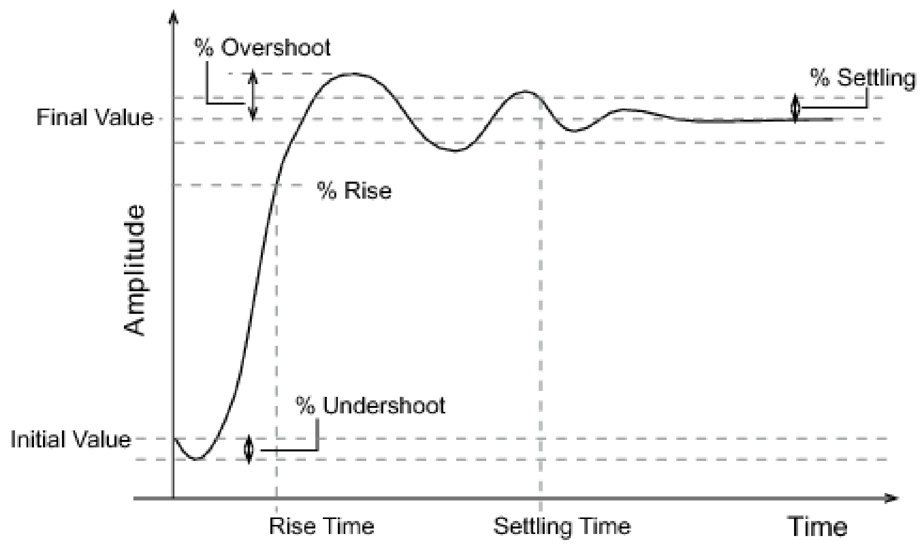


Figure 2-1: Time domain performance metrics (MATLAB, 2013).

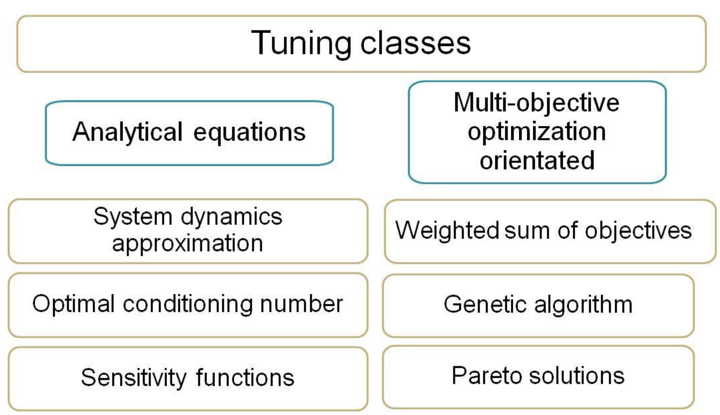


Figure 2-2: Tuning techniques classification chart.

Rani & Unbehauen (1997) compared the tuning approaches for Generalized Predictive Control (GPC) and DMC developed from 1985 to 1994 and proposed a new procedure based on a compilation of the previously observed results to tune the prediction horizon, and the move suppression coefficient in SISO systems, which corresponds to R in (2-1). The authors developed an analytical expression to select p depending on the system sampling time, T_s, and dead-time, θ. Furthermore, they observed that R and p are correlated and a linear relationship between these variables, in which the linear and angular coefficients were adjusted based on a set of practical results, was proposed and it yielded satisfactory results. The authors compared their method to other strategies from the literature considering as a metric the Integral of the Square Error (ISE) index between the system outputs and set points along an arbitrary simulation horizon. However, they remarked that the ISE is not reliable enough to compare and rate the performances of the tuning methods, because from the results, they observed that low values of p lead to overshoot and oscillatory behavior, although the observed ISE values were low. In many cases, the ISE does not represent the system performance straightforwardly, and they suggested that a reliable tuning strategy should also take into account the presence of output inverse responses and oscillatory behavior.

Garriga & Soroush (2010) extensively reviewed the available tuning methods. Tuning strategies for p, m, Q_y, R, as well as the parameters related to a state observer (the covariance matrix and the Kalman filter gain) were compared. Not only GPC and DMC controllers were considered, but also more recent control frameworks such as the max-plus-linear approach and state-space based MPC. The case study considered in their work was a non-linear continuous stirred tank reactor model as the plant model, and local linearizations of the rigorous model at different steady states as the controller models. Although the robust control was not directly addressed, the authors observed that all the tuning strategies that were considered yielded a trade-off between computational cost and robustness on one hand and ease of tuning and narrow application ranges on the other. The auto-tuning strategies did not require a large amount of system knowledge, and the tuning parameters were supposedly optimum throughout the whole operation range. The computational cost was a major drawback that makes such algorithms unfeasible for large systems. The authors also emphasized two points: (i) model identification is a decisive step to yield satisfactory tuning results; (ii) the engineers who intend to use a tuning strategy to obtain better control effectiveness will have to deal with the trade-off between robustness and performance.

Garcia & Morari (1982) developed general tuning guidelines based on the observation of industrial IMC applications. T_s, Q_y, R, p, and m were analyzed and three SISO case studies illustrated the methodology, which yielded satisfactory results for DMC. However, the obtained guidelines are specific for a limited array of controller formulations, and do not yield a systematic methodology. Until the late 80's, the tuning strategies were based on sufficient stability conditions derived from the Linear Quadratic Regulator (LQR) optimal control literature. Therefore, the tuning strategies from the literature were developed aiming at stability, while the performance improvements were sought through tedious and time-consuming trial and error approaches.

As stated by Morari & Lee (1999), m and p do not affect the closed-loop performance as much as Q_y and R in LQR controllers. A pressing issue to be addressed by future research is the fact that the MPC performance deteriorates throughout its operational cycle, and a controller that allows for easy upkeep will perform better in the long run. Even though the main deterioration factor is a faulty model identification, a strategy that can tune the MPC sporadically would improve its performance (Morari & Lee, 1999).

Along the operation cycle of the process, some of the system inputs or outputs may become unavailable due to hardware failure or valve saturation; therefore, the original system size may change dynamically. Qin & Badgwell (2003) stated three important observations: (i) thin systems have more outputs than inputs ($ny>nu$) and yield an over-determined control problem, whereas fat systems have less controlled variables than manipulated variables ($ny<nu$) and yield under-determined problems. The control engineer selects appropriate, usually square ($ny=nu$), sub-systems, based on the input-output relationships to determine a control structure for larger systems. (ii) The initial controller tuning attempts are carried out offline, using closed-loop simulations; the tuning parameters are assessed regarding their sensitivity to the plant-model mismatch. Once a control framework is commissioned, a second tuning attempt is performed, in which the fine tuning takes place. In some cases, optimizing the whole plant parameters simultaneously may prove infeasible due to its size or the lack of degrees of freedom. Industrial controllers might rank the outputs by priority, and enforce that the performance obtained by the most important variables do not decrease to improve the performance of lower-ranked ones. (iii) In a similar fashion, it is possible to prioritize the system inputs, allowing the important ones to be

driven towards their targets, assuming that there are degrees of freedom available. The tuning guidelines provided by Qin & Badgwell (2003) are quite simple. *p* was selected long enough as to contemplate the steady-state output responses.

In accordance with Kulhavy, Lu & Samad (2001), from the industrial point of view, the MPC commissioning is not ready for global business optimization strategies because the MPC does not perform well enough along broad operational ranges, and during fast and slow transition between different operating points. The business optimization might be the driving force that will take MPC to the next level, improving its overall stability, robustness and performance.

Froisy (2006) reported the development of an industrial Infinite Horizon MPC (IHMPC), which focused on improving the user interface so that the process engineers would not need specific theoretical knowledge to operate it. Offline tests showed that the methodology was straightforward. However, performance tuning was carried on in a simplistic fashion and the options were limited to either faster or slower responses, by switching between two predefined values of *R*.

Bauer & Craig (2008) reported the results of a survey regarding MPC commissioning, and maintenance prices. Their most important considerations were: (i) it is difficult to measure the operation improvement subsequent to the MPC implementation, and the MPC economic assessment has been a research topic since 1987; (ii) 70% of the survey respondents believe that the most attractive feature of MPCs is to yield higher throughputs and better product quality; (iii) 70% of the MPC companies and 60% of the users consider that the most important cost factor in MPC commissioning is the manpower (engineers, technicians, operators) and 30% of the respondents mentioned maintenance as one of the three most important cost factors. The MPC tuning affects the commissioning and upkeep phases and, from the previous statistics, we infer that, in practice, it is remarkably important to develop a tuning tool to accelerate the MPC commissioning.

In the following paragraphs, specific tuning techniques from the literature, separated in the two main groups presented in Figure 2-2 are reviewed chronologically. Most of the authors justify their tuning methods providing simulation examples or pilot-plant scale applications, while just a few include real applications. The MPC tuning problem is essentially a constrained optimization problem, in which the tuning goals may be defined either in the time domain or in the frequency domain. A particularly prominent trend in the literature is to incorporate multiple objectives (robustness, stability and performance) into a single objective function, and solve the problem using either a heuristic or a goal attainment algorithm. The tuning constraints might be related to the enforcement of specification requirements on the system outputs, for example.

2.1. Tuning techniques based on analytic equations

Marchetti, Mellichamp, & Seborg (1983) developed general tuning guidelines based on the performance assessment of unconstrained convoluted MPC such as the DMC, and IMC and PID controllers in simulated scenarios. The DMC model horizon should match the settling time of the process, and it should be large enough to avoid truncation problems; the selection of p and m is subject to the process dynamics, sampling time, and the available computational facilities. The authors demonstrated that an appropriate choice of R allows for a higher system dynamic matrix conditioning number, thus allowing a better conditioning of the control problem; however, the authors did not provided any analytical expression or heuristic guidelines for the selection of the tuning parameters.

On the same direction, Maurath et al.,(1988) developed tuning guidelines to improve the numerical conditioning of DMC algorithms. The control horizon should be large enough to encompass all the control actions needed to track any programmed set point change, considering the possible active constraints. The prediction horizon should be large enough to contemplate the significant dynamics of the process; a value between 80 and 90% of the slower input-output pair of the system model open-loop settling time is a reasonable value. The dynamic matrix of the DMC was studied from the principal component analysis viewpoint, allowing the authors to calculate the number of useful components, and to select an adequate R to obtain a balanced compromise between robustness and performance.

Banerjee & Shah (1992) considered the small gain theorem to draw general tuning guidelines regarding the effect of the tuning parameters on the closed-loop performance of the GPC in model mismatch scenarios. According to their results, higher values of R and p increase the controller robustness, at the cost of decreasing the performance.

Lee & Yu (1994) stated that even though the MPC technology facilitates process control, allowing for the straightforward incorporation of process constraints and extension to MIMO systems; it lacks easy tuning guidelines that take into account performance and robustness goals. In the light of this observation, a tuning strategy for state-space MPC was developed, using the frequency-domain robust control theory. The results demonstrated that in output uncertainty scenarios, it is recommended to tune the state observer instead of the MPC, in order to achieve robust performance. However, in the presence of input uncertainty, it is recommended to tune the MPC, especially by selecting an appropriate value for R. Moreover, the tuning parameters that are indirectly related to the closed-loop response of the system should be set to pre-defined values.

Shridhar & Cooper (1997, 1998) developed a tuning technique to select an optimum R for the unconstrained DMC in closed-loop with stable SISO and MIMO systems, respectively. The core idea of their strategy was to approximate the system transfer functions by first-order-plus-dead-time (FOPDT) functions in the tuning step. The conditioning number of the Hessian of the DMC control problem was set equal to 500, which represents a reasonable trade-off between closed-loop robustness and performance. These assumptions allowed the development of an analytical expression for R. The strategy resulted in satisfactory results for both output tracking and disturbance rejection goals. The tuning approach focused on performance, while stability and robustness were not investigated. The authors concluded that: (i) large p and small m are recommended for stability purposes; (ii) Q_y should not be used as a tuning parameter because it usually contemplates either the scaling factors, predefined economic goals, or the priority of particular outputs.

Through further exploration of the frequency domain controller properties, Campi, Lecchini & Savaresi (2002) developed a tuning strategy in which the cost function is the L^2-norm of the complementary closed-loop sensitivity function. A similar class of the transfer-function-based controllers in the z-domain studied by Ali & Zafiriou (1993a) was tuned. In this strategy, the transfer functions were obtained through model identification with operational data and the tuning goals were defined as output reference trajectories. The optimum parameters were obtained for a pre-defined controller order, differently from (Ali & Zafiriou, 1993a) that also tuned the polynomial order.

Wojsznis, et al. (2003) developed an experimental formula to tune R, based on the observations of a DMC in closed-loop with a first-order process. They show that this parameter has a strong role in the control robustness, assuming that the model identification step was done correctly, and that the plant behaves similarly to the controller model. They preferred to use time-invariant Q_y and R; and they show that processes with large dead times may require more robust controllers. Their tuning technique yielded satisfactory results in applications with up to 50% additive uncertainty.

Another tuning technique in the frequency domain was developed by Trierweiler & Farina (2003). The tuning objectives were defined in terms of the complementary closed-loop sensitivity functions, taking into account desired output characteristics in the time-domain, such as offset, settling time, rise time and overshoot. The cost function includes a novel performance metric named Robust Performance Number, which gauges the controllability of a system. The case studies proposed by the authors demonstrated that the technique can be used to correct the control structure of a system, and also take into account nonlinearities and uncertainties.

Following the ideas of Campi, Lecchini & Savaresi (2002); Adam & Marchetti (2004) proposed a tuning technique in the frequency domain for structured SISO controllers. The model uncertainty was defined by the lower and upper bounds of the system transfer function parameters (gain, time constant, dead time). Their strategy is suitable for feedforward and feedback-feedforward controllers. In the former, the tuning cost function is the H_∞-norm of the closed-loop sensitivity function, and in the latter, a mixed sensitivity function, including the inputs sensitivity is used instead.

Abu-Ayyad, Dubay, & Kember (2006) proposed a new DMC structure, in which R is a hollow matrix (the main diagonal entries are zero). In their approach, the dimension of R is independent of both m and the number of inputs, and its optimum value is selected from a set of possible solutions given by the overlapping points of the approximated values of the hollow matrix and the real values of the condition number of the DMC control problem. The optimum R is the one that yields simultaneously a small dynamic matrix condition number and a large determinant value. The advantage of the novel control formulation is that it allows for longer control horizons without increasing the number of tuning parameters.

Lee, Huang, & Tamayo (2008) developed a tuning strategy to reduce the process variance in online tuning applications. First, the method examines the process data to assess the sensitivity of the MPC cost function with respect to the process variables and coupled variables. Once the variables are identified, constraint relaxation or variability attenuation procedures are suggested to achieve the tuning goal, which is defined in terms of a desired benefit potential that is a percentage of the ideal potential benefit calculated assuming zero process variance due to constraint violation.

Garriga & Soroush (2008) proposed a pole-placement tuning strategy for unconstrained MPC. It is based on an analytical analysis of how m, p, Q_y, and R affect the location of the eigenvalues of the closed-loop transfer function. Tuning objectives are given in terms of eigenvalue placement, and it also accounts for the stability goals.

Huusom et al. (2012) developed tuning guidelines for a SISO Autoregressive Exogenous based MPC in a disturbance rejection scenario. The process noise was modeled as a combination of a white noise and integrated white noise, both incorporated into the system model. R was tuned considering the trade-off between the variance of system states and outputs, calculated analytically. The authors proposed two approaches: the first one aims to optimize the state and output variances, which are a function of R, minimizing a specific performance criterion; in the second one, the authors chose R based on the trade-off between input and output variance, and select its value based on the inflection point of a log-log input variance/output variance plot.

Sarhadi, Salahshoor, & Khaki-Sedigh (2012) formulated analytic expressions for Q_y and R, taking into account a MPC with control horizon equals to one, in closed-loop with systems represented by first order plus dead time transfer functions. Assuming that the control constraints are inactive, the authors calculated the MPC gain matrix and matched it to a desired gain matrix, selected to meet performance criteria, thus calculating analytic equations for the tuning variables.

2.2. Tuning techniques based on the multi-objective optimization framework

Ali & Zafiriou (1993a) tuned nonlinear controllers in closed-loop with unstable and slow dynamics systems. The authors worked with a standard Non-Linear MPC and a modified one, in which the prediction horizon is also a decision variable of the control problem. Q_y and R were tuned using an offline optimization approach that takes into account the performance specifications modeled as output envelope constraints. It allows for setting the speed of the output response and overshoot bounds, under model uncertainty and disturbances. The control horizon was optimized through grid search. The drawbacks of the strategy are (i) the lack of stability guarantees and (ii) the multiplicity of local optima, which was addressed by repeated optimization runs, starting from different initial guesses.

Ali & Zafiriou (1993b) developed a tuning technique for variable order controllers in the z-domain, using extensive search to obtain the minimum order that satisfies the tuning objectives. The numerator and the denominator coefficients of the controller transfer function are the decision variables of the tuning problem, cast as a constrained optimization problem by selecting an ISE-based cost function and the performance objectives as soft constraints, which are similar to the constraint envelopes defined in (Ali & Zafiriou, 1993a). Its strongest point is to allow for the inclusion of as many time-domain performance objectives as necessary, as long as they are consistent among themselves and are within the system physical limitations. Also, the technique is suitable for different MPC formulations and is extendable to the plant-model mismatch case, assuming polytopic uncertainty.

Ali & Zafiriou (1993b) drew two important conclusions: (i) conventional tuning objective functions (ISE, IAE and ITAE) do not contemplate important closed-loop performance characteristics, reinforcing the observation made in (Rani & Unbehauen, 1997). Moreover, the latter aspects are only evident after closed-loop simulations; (ii) meeting the performance criteria is generally more important than obtaining the minimum value of an objective function in optimization based tuning strategies. Therefore, in order to save computational time, the tuning procedure can be interrupted once all the performance goals are attained.

Transfer function based controllers were tuned by Abbas & Sawyer (1995) considering the underlying ideas of the multiple criteria decision analysis theory. The tuning goals were indicators such as the overshoot, rise time, settling time, steady-state error and the maximum output values. The performance indices were evaluated for different values of tuning parameters, chosen according to the process dynamics, yielding a Pareto front from which the decision-maker chose a feasible and optimum solution, unaware of its achievable performance. The weakness of the method, according to the authors, lies in the fact that it requires the solution of several optimization problems, which might prove time consuming.

The input-output variable coupling, observed in MIMO systems, makes the relationships between inputs and outputs even more complex and, in practice, tuning is generally carried on according to the experience of the process and control engineers (Liu & Wang, 2000). Liu & Wang (2000) developed a tuning strategy for unconstrained DMC, built upon a mixed-integer non-linear optimization problem. The tuning goals were defined as the sensitivity of the control cost function to p, m, Q_y and R. The cost function Hessian and Jacobian, with respect to the tuning parameters, were calculated numerically and the optimum ones were obtained through a multi-objective optimization framework, solved by a goal programming algorithm. The strategy may be implemented online, yielding good results in systems in which the process model assumes time-varying coefficients.

Al-Ghazzawi et al.(2001) developed an online tuning strategy for constrained MPC that uses linear approximations of the process dynamics to develop analytical expressions for the sensitivity functions between Q_y, and R and a tuning cost function including the output constraint envelopes, in the same fashion as in (Ali & Zafiriou, 1993b). The sensitivity functions were calculated using Lagrange multipliers when the controller constraints were active, allowing for online applications. Moreover, the authors pointed out that the tuning parameters might not affect the closed-loop performance if the input constraints are active.

Li & Du (2002) proposed a tuning strategy for GPC, in which the tuning objectives are represented by fuzzy membership functions, including operational goals and constraints. Optimum R values were obtained online, using a gradient search method.

Han, Zhao, & Qian (2006) developed a tuning strategy for unconstrained DMC and additive model uncertainty. They proposed an objective function that takes into account the output performance goals in the time-domain to soften the conservativeness of min-max optimizations. The tuning problem was solved using the Particle Swarm Optimization method, which is a metaheuristic optimization algorithm that works well for complex non-linear optimization problems. The advantage of the technique lies in the fact that it can be used to tune any parametric controller, and it also works well for robust controllers.

Building upon the frequency domain tuning strategy by Adam & Marchetti (2004), Vega, Francisco, & Sanz (2007) proposed a tuning method for the unconstrained DMC where the objective function is a mixed sensitivity function comprised of two terms: a sensitivity function between the process disturbances and the output signal, and another one that considers the process disturbances and the input signal. Disturbance rejection goals and bounds on the inputs and outputs were included in the tuning problem as constraints. The method allows for an automated tuning framework for SISO systems. Vega, Francisco, & Tadeo, 2008, extended the method to account for polytopic uncertainty, by enforcing the disturbance rejection constraint for every plant model inside the polytope. In (Francisco, et al., 2010), the algorithm was modified again to take into account asymmetric input and output bounds, which made it more applicable to real cases. Also, the technique was applied to a constrained DMC by (Francisco & Vega, 2010), assuming that a static set of active constraints is known throughout its operation. Such consideration shows to be rather strong because different constraints might become active at any time; however, it suffices for research purposes, but the DMC constraints might conflict with the input and output tuning constraints, hindering the tuning method. Finally, it was proposed in (Francisco, Vega, & Revollar, 2011), an extension to MIMO systems, through the straightforward redefinition of the mixed sensitivity functions for matricial transfer functions. The proposed method might work for several unconstrained controller formulations, and the time domain objectives can be straightforwardly included as constraints. Its main drawbacks are: it is restricted to disturbance rejection cases, and the definition of the tuning constraints requires frequency-domain knowledge.

van der Lee, Svrcek & Young (2008) proposed a tuning strategy based on the solution of a multi-objective optimization problem. A fuzzy cost function, including the ISE between the process outputs and reference values, and time-domain performance goals were included. Differently from (Li & Du, 2002), who used a deterministic algorithm to solve the fuzzy optimization problem, the authors used a genetic algorithm. Their technique is highly flexible, and relies on a closed-loop simulation to calculate the system output values simultaneously to the tuning problem solution. The latter can make it computationally demanding for large systems.

Susuki *et al.* (2008) developed a tuning technique for unconstrained state feedback controllers, aiming to improve transient operational characteristics during the plant startup and stationary operation, especially in large dead time or model uncertainty scenarios. The technique takes both the system model and the controller weighting matrices as tuning parameters, and the tuning objectives are time-domain output performance goals. The tuning objective function is evaluated numerically using Particle Swarm Optimization (PSO).

Cairano & Bemporad (2009) used a linear controller gain, chosen by pole-placement as the tuning goal in a strategy developed for constrained state-feedback MPC. Q_y and R were optimized so that the MPC behaves as the target linear controller, in the absence of active constraints. The tuning problem was solved using Linear Matrix Inequalities; its cost function is the squared norm of the difference between the favorite controller gain and the unconstrained MPC gain. Lastly, the authors proposed that the MPC behaves as a LQR when the constraints are inactive, making the controller matching optimization problem independent of the MPC control horizon. In (Cairano & Bemporad, 2010), the strategy was extended to output-feedback MPC formulations.

Exadaktylos & Taylor (2010) developed a tuning strategy for state-feedback MPC based on non-minimum state space models using an integral-of-error state to replace the state observer. The tuning objectives were given in terms of the Integral of Absolute Error (IAE) between the closed-loop output responses and reference levels for dynamic decoupling, and the IAE between the closed-loop outputs and the designed responses, (e.g. first-order transfer function with a specified time constant). The optimization problem was solved using the goal attainment algorithm, which allows for the definition of optimistic goals, without rendering the problem infeasible, to obtain optimum values for Q_y and R. Similarly to (Susuki *et al.*, 2008; van der Lee, Svrck & Young, 2008), the tuning cost function was evaluated numerically.

Shah & Engell (2010) developed a tuning strategy for the SISO GPC, using the Controlled Auto-Regressive and Integrated Moving Average system models. The tuning goals were defined in the same fashion as in (Exadaktylos & Taylor, 2010), using output reference transfer functions. The tuning problem was posed as an optimization problem in which the decision variables are Q_y and R. Horizon p is chosen according to a guideline that provides enough degrees of freedom to allow for good tuning results, and m is fixed at 1. The tuning problem was solved by equaling the closed-loop characteristic polynomial to the desired closed-loop response characteristic polynomial, obtained from the tuning objectives. In (Shah & Engell, 2011), the authors extended the technique to MIMO systems using a two-step tuning algorithm. The objectives were defined in the same way as before, but the decoupling effects must be taken into account. In the first step, the closed-loop controller gain was calculated based on the tuning objectives, using a frequency domain approximation. In the second step, the GPC matrices Q_y and R were calculated to match the previously calculated gain. The method was still limited to m=1.

Reynoso-Meza et al. (2013) proposed a flexible multi-objective tuning strategy for several MPC formulations. The tuning objectives were the IAEs between the system outputs and their set point values; the integral of absolute values of the input increments, coupling effect measurements, robust stability requirements, and other financial control commissioning objectives. The authors suggested using as few objectives as possible, because the more the objectives, the harder it is to calculate a Pareto front, and to choose the best non-dominated solution within it. An evolutionary algorithm was used to solve the tuning problem and the procedure was tested on two controllers, a PID and a state-space model based MPC in closed-loop with a 2x2 non-linear system. The authors also developed a graphical tool to analyze Pareto fronts and to guide a decision among non-dominated optimum solutions.

Reinforcing the multi-objective optimization role in the controller tuning literature, Vallerio, van Impe & Logist, (2014) developed a strategy for non-linear MPC for both online and offline applications. The tuning objective was defined as the usual control cost function. However, the authors showed that it does not yield a uniform Pareto frontier distribution and proposed two interchangeable parameterizations, namely the Normal Boundary Intersection (NBI) and the Extended Normalized Normal Constraint. The tuning technique was developed for output tracking scenarios. However, two application examples showed that it performs well considering disturbance rejection goals and with mild plant-model mismatch scenarios. In online applications, the preferences of the decision-maker were defined *a priori*, and the algorithm chooses the most appropriate non-dominated solution automatically.

Oliveira, et al. (2014) proposed an automatic tuning strategy for sliding mode generalized predictive controllers, in which the control actions are divided in two terms: the first one is the classic control actions in the MPC literature, and the second one drives the system through a sliding region to address other economic goals. Simulation results illustrated the technique, and according to the authors, their strategy successfully improved robustness in uncertainty scenarios, compared to a manually tuned GPC. The tuning problem is solved using PSO, and considers both the minimization of tracking error and control effort as goals.

Júnior, Martins, & Kalid (2014) developed a tuning technique based on the solution of a mixed-integer nonlinear optimization problem. Both the control and prediction horizons and the weighting matrices of the MPC were considered as tuning parameters, and the tuning cost function included both the tracking error and control effort. A PSO algorithm solves the tuning problem for the worst-case model mismatch scenario, which is obtained by taking into account both the condition number od the dynamic matrix and the another metric that measures the resiliency and controllability of a system.

2.3. Miscellaneous tuning strategies based on multi-objective optimization

Messac & Wilsont (1998) proposed a computational control approach instead of the MPC approach because it takes into account more than one control framework and it optimizes the variables with physical meaning directly. From the practical point of view, its main drawback is the computationally demanding non-linear programming formulation. The authors used the Physical Programming approach (Messac, 1996) to tackle the computational control problem. The decision-maker specifies acceptable ranges iteratively, which differs greatly from the MPC tuning procedure. The MPC problem provides little insight regarding acceptable tuning values, whereas in the computational control approach, the parameters are chosen intuitively. The Physical Programming based computational control algorithm was compared to available control approaches from the literature in a case study with model uncertainty, yielding satisfactory results.

Bemporad & Morari (1999) integrated the process characteristics, (e.g. phenomenological equations, logical rules, and operating constraints) into a single control framework. The logical statements were accounted for as inequalities through the inclusion of auxiliary variables, using the digital network design, computational inference, and the gain scheduling techniques. The nonlinear equations were regarded as mixed-integer linear inequalities. The control cost function was posed as a Lyapunov function and therefore, the controller is asymptotically stable. Moreover, since the MILP problems are intrinsically more computationally demanding than the nominal finite horizon MPC formulations, the authors stated that intermediate feasible solutions might be used, but hindering the tracking performance.

In (Kerrigan et al., 2000), the authors reshaped the logical dynamic control framework introduced in (Bemporad & Morari, 1999) as a lexicographic optimization problem, using slack variables to substitute logic related inequalities by numeric ones. They included the slack variables in the control cost function, weighted by appropriate penalization matrices, which yielded a simpler QP problem. The effect of the slack weighting matrices on the control problem was not investigated. Kerrigan & Maciejowski (2002) presented rigorous mathematical definitions of conventional objective functions to develop a lexicographic control framework. A sufficient condition to guarantee the lexicographic minimum uniqueness was provided.

Gambier (2008) formulated both a PID tuning problem and a MPC control problem as multi-objective optimization problems. In agreement with the author, the method works well in the former case, since most of the computational effort is done offline. However, in the latter case the method is not as promising, especially for fast-dynamic processes.

Bemporad & Muñoz de la Peña (2009) proposed a multi-objective optimization approach to solve a state-space based MPC problem. The Pareto optimal control action was calculated at each time instant, using a set of dynamic objectives. A weighted-sum multi-objective optimization method with time-varying and state-dependent weighting vectors were included to solve the control problem and asymptotic stability was enforced by a cost contraction constraint. The approach can take into account multiple control goals simultaneously. Once the objectives are established, the Pareto optimal solutions are calculated offline as a piecewise affine function of the process states, which allows for a small online computational burden.

Zavala & Flores-Tlacuahuac (2012) developed a multi-objective MPC based on the solution of a compromise optimization problem. The approach does not require the direct computation of the Pareto curve and asymptotic convergence is ensured by a state terminal cost constraint. The authors provided the guidelines for the MPC implementation and explained how the definition of the distance between the compromise solution and the Utopia point affects the control performance.

Subbu et al. (2006) developed a multi-objective optimization control framework based on a neural network model representation. The optimization problem was solved using an evolutionary algorithm and they tested the control algorithm in a complex power plant, obtaining satisfactory results.

Geyer, Papafotiou & Morari (2009) tackled the direct torque control of a mechanical system using a MPC. The state-space system model was obtained from phenomenological equations, the control framework takes into account multiple objectives and it is solved using an exhaustive search along the control horizon and a point-to-point evaluation of the optimum control profile. They concluded that the MPC-based control strategy offers better performance than the usual direct torque control strategies.

Villarroel et al. (2010) proposed a finite-state MPC in the control of a direct matrix converter system. They proposed a formulation in which the Pareto optima were calculated for each state and ranked following subjective criteria. The decision-maker selects the top ranked solution, as in *a posteriori* multi-objective optimization methods. The authors highlighted the absence of tuning parameters as a positive factor and pointed out that further works will improve the efficiency of the optimization algorithm. *Rojas* et al. (2013) applied the finite control set approach to a torque and flux system following an approach similar to the method studied by Geyer, Papafotiou & Morari (2009). The authors stated that since the set of decision variables is finite, it is possible to evaluate the cost function for each feasible solution and select the optimum control move based on the exhaustive search method. Comparing the results obtained by these authors and by Geyer, Papafotiou & Morari (2009), one concludes that the first strategy avoids the controller tuning step but is more time consuming in online applications.

In the torque and flux systems field (Geyer, Papafotiou & Morari, 2009; Rojas et al., 2013), there have been attempts to skip the controller tuning step altogether, abandoning the classic weighted-sum based MPC formulation. The approaches exploit the fact that due to the system characteristics, there are finite feasible control actions at each sampling time; which allowed for the development of the finite state MPC. These controllers perform an exhaustive search over a finite number of possible solutions to select an optimum control action (Geyer, Papafotiou & Morari., 2009, Villarroel et al., 2010). These approaches work well for torque control systems, but are infeasible strategies in the petrochemical and oil processing industries, because it is difficult to obtain a finite set of control actions, unless major simplifications are included.

3. LEXICOGRAPHIC TUNING TECHNIQUE (LTT)

3.1. Background

The lexicographic tuning strategy was developed aiming at the industrial process control needs, namely: reliable ways to decrease the controller commissioning time.

The frequency domain tuning techniques often disregard the control performance, and prioritize the robustness and stability goals. Some authors, however, tried to include time-domain performance objectives as frequency-domain tuning constraints (Campi, Lecchini & Savaresi, 2002; Han, Zhao & Qian, 2006; Reynoso-Meza et al., 2013; Vega, Francisco & Sanz, 2007) or to include robustness and stability goals in the time-domain tuning strategies (Banerjee & Shah, 1992; Han, Zhao & Qian, 2006). In Garriga & Soroush (2010), it is shown that the tuning approaches need to deal with the trade-off between robustness and performance. Also, it is clear that a single tuning cost metric usually fails to define the tuning goals adequately (Rani & Unbehauen, 1997; Giovanini & Marchetti, 1999).

In general, it is difficult to obtain the closed-form of the tuning cost function (Susuki et al., 2012) and its differentiability conditions, as well as the expressions for the Hessian and Jacobian. Then, several tuning methods use a closed-loop simulation to calculate the output trajectories and evaluate the tuning objectives. Examples of these objectives are the overshoot, the rise time, and settling time of the closed-loop response (Abbas & Sawyer, 1995; Han, Zhao & Qian,, 2006; Reynoso-Meza et al., 2013; Suzuki et al., 2012; van der Lee, Svrcek & Young, 2008). The tuning objective can also be the IAE or the ISE between the output trajectory and an output constraint envelope with variable height (Ali & Zafiriou, 1993b; Al-Ghazzawi et al., 2001). The limit in which the height goes to zero characterizes the reference trajectory tuning goals (Exadaktylos & Taylor, 2010). However, other tuning methods (Shridhar & Cooper, 1998; Trierweiler & Farina, 2003) derived analytical equations for the tuning parameters, using simplified system models. The simplifying assumptions might make their tuning strategy specific for a narrow range of situations. In this fashion, either heuristic or rigorous optimization-based techniques are preferred. The former does not provide formal convergence guarantees and the latter does not guarantee that a global optimum will be obtained, since the tuning problems are usually non-convex.

The tuning goals in the lexicographic method were chosen according to output (Al-Ghazzawi et al., 2001; Ali & Zafiriou, 1993b; Campi, Lecchini & Sacaresi, 2002; Exadaktylos & Taylor, 2010; Shah & Engell, 2011) or input reference trajectories. However, depending on the number of goals and constraints, which are directly related to the number of system outputs and inputs, the tuning problem can become cumbersome. The tuning technique described here uses the sum of squared error (SSE) between the reference trajectories and the closed-loop simulated trajectories along a tuning horizon as objectives. In order to minimize the computation time to simulate the closed-loop system, and because the technique should obtain the very best tuning parameters, the MPC physical constraints are disregarded, allowing for an analytical solution of the control problem. As reported by Wojsznis et al. (2003), this assumption is not a critical issue because only the first control action is actually fed to the plant.

A brief review on lexicographic optimization is available in Appendix C.

3.2. Nominal LTT

The approach developed here is based on the minimization of the sum of squared errors between the closed-loop trajectories and the user-defined reference trajectories that take into account desired process characteristics (e.g. overshoot, rise time and settling time). To each of the system outputs, is assigned a priority coefficient, and input-output pairs are defined to characterize the desired reference trajectories. In this way, a lexicographic tuning method is developed, in which the most important outputs are driven closer to their reference trajectories, compared to the less important ones. The performance improvement of low priority outputs is only taken into account if there is no significant sacrifice in the performance of the high priority outputs.

In SISO systems, it suffices to set Q_y to a fixed value and tune R to obtain an optimum control performance, because the ratio of Q_y to R unequivocally determines a unique control profile, even though the total control cost might vary. In MIMO systems, it suffices to make one entry of Q_y constant and tune the remaining entries of Q_y as well as all the entries of R (Ali & Zafiriou, 1993b; Al-Ghazzawi et al., 2001).

This approach is highly recommended in tuning procedures because otherwise, we will attempt to solve an ill-posed problem.

The method, summarized in Table 3-1, is detailed in the following paragraphs and application examples are provided in Chapter 5. For the remainder of this work, $q_{y,i}$, $i=1,\ldots,ny$, $q_{u,j}$, $j=1,\ldots,nu$ and r_j, $j=1,\ldots,nu$ are the main diagonal entries of Q_y, Q_u and R.

Table 3-1: Lexicographic technique steps summary

Step	Procedure
1	Define the outputs importance
2	Normalize inputs, outputs and model gains
3	Specify input-output pairs
4	Specify the tuning objectives
5	Perform the Lexicographic optimization

3.2.1. Output priority assignment

Selecting the most important inputs and outputs is a common scenario for process engineers (Lee, Huang & Tamayo, 2008; Qin & Badgwell, 2003; Wojsznis et al., 2003). Important outputs are usually chosen based on economic, environmental, or safety factors, whereas input importance is mostly defined by economic factors.

3.2.2. Normalization

According to Al-Ghazzawi et al. (2001), it is important to work with normalized process variables (inputs, outputs, set points and targets, transfer function gains). Francisco et al. (2010) adopted normalized disturbances to avoid numerical problems. The algorithm presented here uses normalized transfer functions gains and variables to reduce the numerical problems, and to attenuate output scaling problems. A suggested normalization algorithm, used in an advanced control framework of PETROBRAS, Brazil is shown in Section 5.1.1. Other strategies, such as the straightforward normalization using the upper and lower bounds of the variables are also applicable.

3.2.3. Specify input-output pairs

In the conventional regulatory control, the PID control loops are usually designed following the importance ordering of the controlled variables. Therefore the idea of ranking the variables is well established in industry. In the previous section, the outputs priority were defined and here, suitable inputs are assigned to the outputs according to their priority, in order to establish input-output pairs, which will define the output reference trajectories.

There are cases in which process knowledge alone can dictate the selection of the input-output pairs. However, when such information is either unavailable or insufficient, pairing methods like the Relative Gain Array (Bristol, 1966) (RGA) or the Singular Value Analysis are used. The RGA does not take into account the process dynamics and the reader is referred to van de Wal & de Jager (2001) for detailed information about this method. A simple open-loop gain analysis consists in plotting the unitary step responses of all the system transfer functions, and choosing, by visual inspection the input which has the bigger impact on y_1. Once the first pair is selected, repeat the process for i=2,…,ny, discarding the previously selected inputs at each step.

Observe that there may be processes in which $ny \neq nu$, and the reader is oriented to either: (i) assign more than one input to high priority outputs, if nu>ny; or (ii) clump low priority outputs to a single input otherwise.

3.2.4. Specifying tuning objectives

The output reference trajectories allow the user to define time-domain performance goals such as overshoot, rise time, and settling time. The SSE between the reference trajectories and the closed-loop output trajectories is evaluated and minimized during the tuning procedure. The reference trajectories can also be chosen according to different criteria. One example is to use open-loop transfer functions related to each input-output pair defined in Section 3.2.3 to generate approximated first-order (FO) transfer functions and set the time constant of the reference transfer function as a fraction of the time constant of the approximated process. This strategy has its merits because control engineers are fond of stable, swift, and non-oscillatory responses. Nonetheless, higher order transfer functions or tailored time-domain responses can also be adopted. Ali & Zafiriou (1993) used constraint envelopes as tuning targets, however, they observed that in some instances, output goals defined as wide envelopes lead to oscillatory responses.

Observe that reference trajectory goals are applicable to both disturbance rejection, and output tracking tuning scenarios. In this thesis, the output reference trajectories are given by a diagonal transfer function matrix, $G_{des}(s)$, comprised of $G_{des,i}(s)$, $i=1,\ldots,w^1$, w^1 is the number of output objectives.

The reference trajectories that characterize the inputs dynamics are defined in terms of the $\Delta u_{\max}$, even though the constraint on the control moves is ignored in the tuning problem. A total of w^2 input objectives can be defined and included in the optimization problem.

3.2.5. Lexicographic optimization

In this approach, the tuning problem is solved by a lexicographic optimization problem, which sequentially obtains optimum tuning parameters for subsystems of the original system, until all the relevant components of the weighting matrices $Q_y,\ Q_u,\ and\ R$ are tuned.

Equation (3-1) defines a function corresponding to the SSE between the output reference trajectories and the closed-loop responses.

$$F_i\ (x)=\sum_{k=1}^{\theta_t}\left(y_i^{ref}(k)-y_i(k)\right)^2,\ i=1,\ldots w^1 \tag{3-1}$$

θ_t is the tuning horizon, $y_i^{ref}(k)$ is the discretized reference trajectory of output *i*, $y_i(k)$ is the closed-loop trajectory of output *i*, $k=1,\ldots,\theta_t$, x is the vector of decision variables containing the appropriate diagonal entries of matrices $Q_y,\ Q_u,\ and\ R$ and w^1 is the number of output reference goals. Observe that $y_i(k)$ is evaluated considering the control moves calculated by the controller in closed-loop. Therefore, it is a function of the tuning parameters. Analogously, one can define the input goals as the distance between the input closed loop trajectory and the reference trajectory, as follows:

$$F_i\ (x) = \sum_{k=1}^{\theta_t} \left(u_i^{ref}(k) - u_i(k) \right)^2,\ i = 1, \ldots w^2 \qquad (3\text{-}2)$$

where $u_i^{ref}(k)$ is the discretized input reference trajectory at time step *k*, $u_i(k)$ is the closed-loop input value at time instant *k*. Observe that even though the unconstrained MPC is considered in the tuning scenarios, it is not possible to obtain a single analytical solution to the tuning problem because (3-1) and (3-2) are discretized cost functions, and the non-linearity of the MPC cost function with respect to the tuning parameters would lead to very cumbersome expressions for the partial derivatives of the objectives defined in (3-1) and (3-2).

The importance of the process outputs defined previously also characterizes the lexicographic optimization steps. In this approach, the input goals are only addressed after all the output goals have been addressed.

The lexicographic optimization tuning approach solves the following problem:

Problem 1a

$$\min_{x,\delta_t} V_{1,a} = \sum_{i=1}^{w'} F_i(x) + \delta_t^T S_t \delta_t \qquad (3\text{-}3)$$

subject to

$$F_j(x) - F_j^{N*} - \delta_t(j) \leq 0,\ \ j = 1, \ldots, w' - 1 \qquad (3\text{-}4)$$

$$\delta_t(j) \geq 0,\ \ j = 1, \ldots, w' - 1 \qquad (3\text{-}5)$$

$$LB \leq x \leq UB \qquad (3\text{-}6)$$

where $w = w^1 + w^2$, $w' \leq w$, *w’* is defined as an intermediate lexicographic step, $\delta_t \in \Re^{w'}$ is a vector of slack variables, $S_t \in \Re^{w' \times w'}$ is a diagonal weighting matrix, *LB* and *UB* are lower and upper bounds of the decision variables, and F_i^* is the optimum value of the objective goal, defined in (3-1) and (3-2), for the variable y_i, obtained at the *i*-th lexicographic tuning step. Observe that once a F_i^* is obtained, it remains constant throughout the lexicographic method. The constraints defined in (3-4) enforce that the optimum performance obtained for higher priority outputs will not deteriorate when lower priority output goals are addressed. The slack variable is included to ensure that Problem 2a is always feasible.

3.3. Robust LTT

The robustness goals can be included in tuning problems through constrictive frequency-domain constraints in terms of the H_∞-norm (Lee & Yu, 1994; Vega, Francisco & Tadeo, 2008). Han, Zhao & Qian,(2006) tackled robustness and performance goals in a two-step optimization problem to address additive model uncertainty.

Here, the lexicographic tuning method is extended to account for multi-plant uncertainty, in which *L* plant models are included in the set Ω. The 'real plant' model is ω_T and the nominal, most likely model is ω_N. The steps 1 to 4 of Table 3-1 remain unchanged. The robust tuning problem must assure that the decision variable optimum values will be chosen such that the instability effects caused by plant-model mismatch are minimized. It is expected that in most cases, the weights on the control moves will be driven towards their upper bounds to limit the control actions and attenuate the oscillatory behavior of the system. Nonetheless, there may be sufficiently low penalization that allows for both acceptable oscillations and satisfactory control performance, which is our main goal.

The sum of the elements in vector $F(x)$ is defined according to (3-7), and the robust tuning problem is defined as follows.

$$J^{w'} = \sum_{i=1}^{w'} F_i(x) \tag{3-7}$$

Problem 1b

$$\min_{x,\delta_t,\delta_r} V_{1,b} = J^{w'} + \delta_t^T S_t \delta_t + \delta_r{}^T S_r \delta_r \tag{3-8}$$

subject to

$$F_j^N(x) - F_j^{N*} - \delta_t(j) \le 0,\ \ j = 1,\ldots,w^{'} - 1 \tag{3-9}$$

$$J^{w^{'}}(\omega_l) - J^{w^{'}}(\omega_N) - \delta_r(\omega_l) \le 0,\ \ l = 1,\ldots,L,\ \ l \ne N \tag{3-10}$$

$$\delta_t(j) \ge 0,\ \ j = 1,\ldots,w^{'} - 1 \tag{3-11}$$

$$\delta_r(\omega_l) \ge 0 \ \ \ l = 1,\ldots,L,\ \ l \ne N \tag{3-12}$$

$$LB \le x \le UB \tag{3-13}$$

where $J^{w^{'}}(\omega_l)$ is calculated using (3-7) for model ω_l, $l = 1,\ldots,L$, $\delta_r \in \Re^{L^{'}}$ is a vector of slack variables comprised of entries $\delta_r(\omega_l)$, $l = 1,\ldots,L-1$, $S_r \in \Re^{L^{'} \times L^{'}}$ is a diagonal positive definite weighting matrix, x is the decision vector. Constraint (3-10) of Problem 1b tries to enforce that the total cost for each model in Ω is smaller or equal to the total cost of the nominal model. Observe that this approach is similar to a min-max optimization in which the worst mismatch scenario cost function is minimized. However, in this framework, it is assumed that the nominal plant model will yield the best closed-loop responses. The slack variables $\delta_r(\omega_l)$, $l = 1,\ldots,L-1$ are included to ensure that Problem 1b is always feasible.

It is important to emphasize that, since no formal demonstration of the closed loop stability is provided, the technique might not be able to yield robust controllers in plant-model mismatch scenarios. The proposed approach only tries to guarantee that the tuning goals will be bounded for the possible process models.

4. COMPROMISE TUNING TECHNIQUE (CTT)

Even though the Lexicographic method can deal with different goal definitions, in practical sense it is too heuristic. The Compromise method was developed to address this shortcoming by solving a compromise optimization problem, which calculates the closest feasible solution to the Utopia solution.

4.1. Background

The Compromise Tuning Technique was based on the tuning technique developed in (Vallerio, van Impe & Logist, 2014) for non-linear controllers, where the Normal Boundary Intersection (NBI) and the Extended Normalized Normal Constraints multi-objective optimization techniques are applied to obtain a finite set of Pareto solutions. Then, a subjective criterion is used to choose a solution from the available set of solutions. As already mentioned in this work, several works have developed tuning methods that are based on multi-objective (Exadaktylos & Taylor, 2010; Han, Zhao & Qian, 2006; van der Lee, Svrcek & Young,, 2008). In other works the technique is used to solve the control problem directly. Messac & Wilsont (1998) used Physical Programming to develop a computational control platform, which does not require any tuning weights but relies on the solution of a non-linear optimization at each sampling time. Bemporad & Morari (1999) developed a logical optimization framework, which uses logical statements from the computational theory to include conflicting time-domain goals into the control cost function, using a solver of mixed-integer optimization. A similar approach was seen in (Kerrigan et al., 2000), where the mixed-integer problem was posed as a QP problem by the inclusion of slack variables in the logical constraints. The advantage is that there are plenty of algorithms to solve the latter problem quickly and efficiently. Considering the current state of the art of computers and non-linear solvers, the computational predictive controller is currently limited to slow-dynamic processes, PID algorithms (Gambier, 2008) and some torque control applications in which there is a finite number of control actions and the optimal one can be found by exhaustive search (Geyer, Papafotiou & Morari, 2009; Rojas et al., 2013). The compromise approach can be carried out offline and therefore, the computational burden of solving a potentially

complex non-linear multi-objective optimization problem in a short time is not a pressing issue.
A brief review on compromise optimization is available in Appendix C.

4.2. Nominal CTT

In the nominal Compromise Technique, one assumes that the tuning goals are defined as reference trajectories, as seen in Chapter 3. The tuning goals, $F_i(x)$, are defined according to (3-1) and (3-2).
Once all the goals are defined, the Utopia solution is calculated, by solving the optimization problem defined in (4-1) and (4-2) for each goal. The Utopia point vector, $F^{\circ}(x) \in \Re^{w}$, is defined according to (4-3).

Problem 2a

$$F_i^{\circ}(x) = \min_x F_i(x) \tag{4-1}$$

subject to

$$LB \leq x \leq UB \tag{4-2}$$

LB and UB denotes the lower and upper bounds on the decision variables.
The utopia vector is defined as follows:

$$F^{\circ} = \left[F_1^{\circ}(x) \quad \cdots \quad F_i^{\circ}(x) \right],\ i = 1,\ldots,w \tag{4-3}$$

Observe that it is not possible to obtain a single x that corresponds to the Utopia decision vector, since each $F_i^{\circ}(x)$ is obtained for a particular set of decision variables, unless all the optimization problems defined by (4-1) and (4-2) share the same solution. The compromise solution is defined as the feasible point that is the closest to the Utopia solution, in terms of the Euclidian distance. The Compromise problem is defined by Equations by (4-4) and (4-5).

Problem 2b

$$\min_{x}\left\| F^{\circ} - F\left(x\right)\right\|^{2} \tag{4-4}$$

subject to

$$LB \le x \le UB \tag{4-5}$$

4.3. Robust CTT

The nominal formulation was extended to account for some model mismatch. As usual in this thesis, the model uncertainty is described as the multi-plant uncertainty. The robust compromise problem is solved in terms of a min-max optimization, obtaining the optimum tuning parameters for the worst-case scenario.

First, we write (4-4) in terms of a generic model in Ω, denoted by ω_l, according to (4-6). Observe that the Utopia solution vector is calculated for each model in Ω, assuming that the plant and the controller share the same model.

$$V_{2,c}\left(\omega_l\right) = \left\| F_{\omega_l}^{\circ} - F_{\omega_l}\left(x\right)\right\|^{2},\ l = 1,\ldots,L \tag{4-6}$$

The CTT solution is obtained by solving the min-max optimization problem described by (4-7) and (4-8).

$$\min_{x}\max_{\omega_l} V_{2,c}\left(\omega_l\right),\ l = 1,\ldots,L \tag{4-7}$$

subject to

$$LB \le x \le UB \tag{4-8}$$

This problem can be recast as a minimization problem by introducing the auxiliary variable γ, which acts as a supremum value for all admissible values of $V_{2,c}\left(\omega_l\right)$.

Problem 2c

$$\min_x \gamma \tag{4-9}$$

subject to

$$V_{2,c}\left(\omega_l\right) \le \gamma,\ l = 1,\ldots,L \tag{4-10}$$

$$LB \le x \le UB \tag{4-11}$$

5. CASE STUDIES

This chapter puts into perspective the applicability of the tuning techniques in simulated industrial systems. In Section 5.1, the lexicographic approach is applied to tune a DMC in closed-loop with a 4x4 subsystem of the Fluid Catalytic Cracking (FCC) reactor-regenerator system presented in (Grosdidier et al., 1993). Also, the Lexicographic and Compromise techniques are applied to tune a MPC in closed-loop with the Shell Heavy Oil Fractionator benchmark system (Maciejowski, 2002). Next, the compromise technique is compared to another multi-objective optimization tuning approach from the literature and the resulting controllers are compared through simulation to the existing MPC of the Crude Distillation Unit (CDU) of the refinery of Capuava, Brazil. Finally, Section 5.2 addresses the robust tuning of the C3/C4 splitter (Porfírio, Neto & Odloak, 2003). It is compared the performances of a robustly tuned IHMPC and a RIHMPC.

5.1. Nominal applications

5.1.1. Fluid catalytic cracking case study

Figure 5-1 shows a schematic representation of the complete combustion FCC unit studied in (Grosdidier et al. 1993), which is an important process unit in oil refining plants. Its internal catalytic recycling loop poses an interesting challenge from the process control point of view. The names, tags, ranges and engineering units of the inputs and outputs are given in Table 5-1. This case study takes into account a 4x4 adapted subsystem of the original FCC system. The original transfer functions were adapted to disregard the dead-time, and are presented in Appendix A.
The importance order of the outputs was defined in the following sequence: 1) The O_2 concentration in the flue gas (y_1) should be kept at low values, otherwise, energy is wasted as a result of the excess of O_2 being blown into the regenerator. Therefore, its set point is set to 0.12%. 2) The regenerator bed temperature (y_2) is usually kept within a zone because high values causes undesired effects in the product yield and low values can limit the processing capability of the plant. Its set point was chosen slightly below the mean value between the zone upper and lower limits, at 725°C. 3)

The output of the controller of the pressure at the wet gas compressor (output y_3) should not stay above a maximum bound to make sure that the gas compressor subsystem is able to maintain the compressor suction pressure at its desired value. Its set point was set to 57.5%. 4) The least important output is the fuel gas flow (y_4), which should not surpass a maximum bound, otherwise it can no longer be handled by the sponge oil absorber. Its set point was set equal to 12 t/h.

Table 5-2 contains the normalized transfer function open-loop gains, which were considered in the selection the input-output pairs. The process variables were normalized according to (5-1) and (5-2) and the output $\times$ input matching step resulting in the following pairs: y_1-u_1, y_2-u_2, y_3-u_3, y_4-u_4.

Table 5-1: FCC unit input and output list.

Variable name	Tag	Range	Unit
Inputs			
Hot gasoil flow	u_1	90-110	m^3/h
Combined cold gasoil and recycle oil flow	u_2	90-110	m^3/h
Riser outlet temperature	u_3	515-535	°C
Recycle oil flow rate controller output	u_4	20-80	%
Outputs			
Flue gas O_2 concentration	y_1	0.1-5	%
Regenerator bed temperature	y_2	705-735	°C
Output of the controller of the wet gas pressure	y_3	20-70	%
Fuel gas flow rate	y_4	5-15	t/h

$$range_{u,j} = u_{\max}^{j} - u_{\min}^{j}, \quad j = 1,\ldots,nu \tag{5-1}$$

$$factor_{y,i} = \max\left(\left|K_{i,j}\right| range_{u,j}\right), \; i = 1,\ldots ny; \; j = 1,\ldots,nu \tag{5-2}$$

where $K_{i,j}$ is the unitary step response gain of $G_{i,j}(s)$, $u_{\min}^{j}$ and $u_{\max}^{j}$ are the *j-th* entries of vectors $u_{\min}$ and u_{max}. The system inputs, outputs (as well as upper, lower bounds, and set points) and transfer function gains were normalized according to (5-3), (5-4), and (5-5).

$$u(k)_j^N = \frac{u_j(k) - u_{\min}^j}{range_{u,j}}, \ j = 1, \ldots, nu \tag{5-3}$$

$$y(k)_i^N = \frac{y_i(k)}{factor_{y,i}}, \ i = 1, \ldots, ny \tag{5-4}$$

$$K_{i,j}^N = K_{i,j} \frac{range_{u,j}}{factor_{y,i}}, \ i = 1, \ldots, ny; \ j = 1, \ldots, nu \tag{5-5}$$

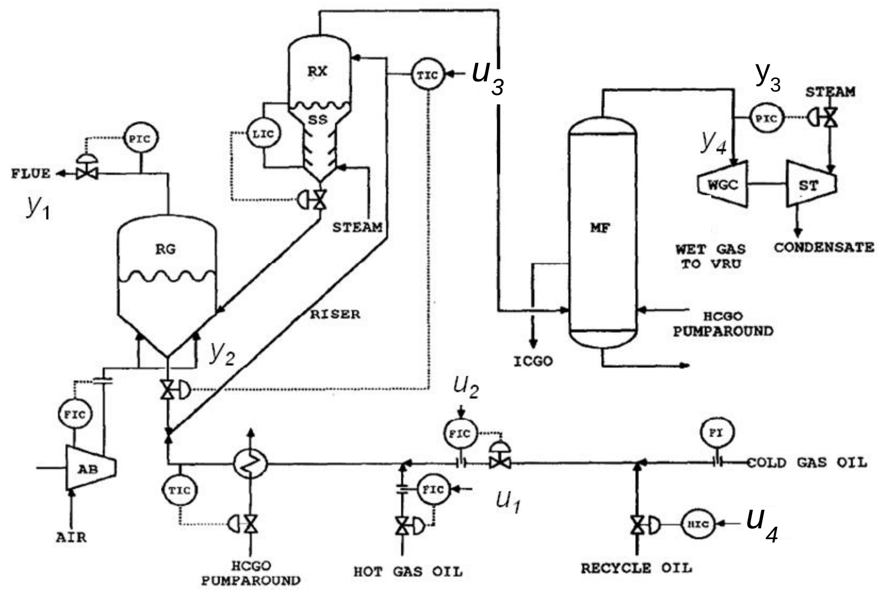


Figure 5-1: FCC schematic representation (Adapted from Grosdidier et al., 1993).

Table 5-2: FCC unit gain matrix considering normalized transfer functions.

Normalized gain matrix				
$K_{i,j}$	u_1	u_2	u_3	u_4
y_1	-1.00	-0.11	-0.09	0
y_2	0.51	0.51	0.69	1
y_3	0.36	0.36	1.00	0.34
y_4	0.05	0.52	1.00	0.17

In the tuning step, the system starts from $y_0 = [0.3 \quad 720 \quad 45 \quad 10]^T$, $u_0 = [147.5 \quad 100 \quad 525 \quad 50]^T$, the output set points are $y^{sp} = [.012 \quad 725 \quad 57.5 \quad 12]^T$

, considering the engineering units in Table 5-1. Grosdidier et al. (1993) report that an IDCOM-M controller was successfully implemented in this FCC unit. This commercial controller is based on an impulse response model, and allows for output zone control and input targets (Qin & Badgwell, 2003).

In this case study, we address the tuning of a DMC, which is depicted Appendix B. The following un-addressed parameters were chosen as: N=120, p=60, m=3, T_s=1 min. The initial guess for the decision variables of the tuning problem were $Q_{y,0} = diag\begin{bmatrix}10 & 6 & 4 & 2\end{bmatrix}$, $R_0 = diag\begin{bmatrix}0.1 & 0.1 & 0.1 & 0.1\end{bmatrix}$. The DMC formulation used here is detailed in Appendix B.

5.1.1.1. *Definition of the tuning goals*

The reference trajectories were selected based on the open-loop system dynamics. The original transfer functions, shown in Appendix A, were approximated by first order transfer functions, and the final reference trajectories were obtained multiplying the time constant of the first order transfer function by response factors that define the output priority, as illustrated in Figure 5-2. The variable responses are normalized.

The time constants of the trajectories that define the performance goals for the outputs were defined based on arbitrary fractions of the time constants of the approximated transfer functions. The following values were adopted for outputs y_1 to y_4 respectively: 50%, 70%, 80% and 100%. The reference trajectories are defined in Equation (5-6). The goal definition considered here assumes that faster responses are better than the slower ones.

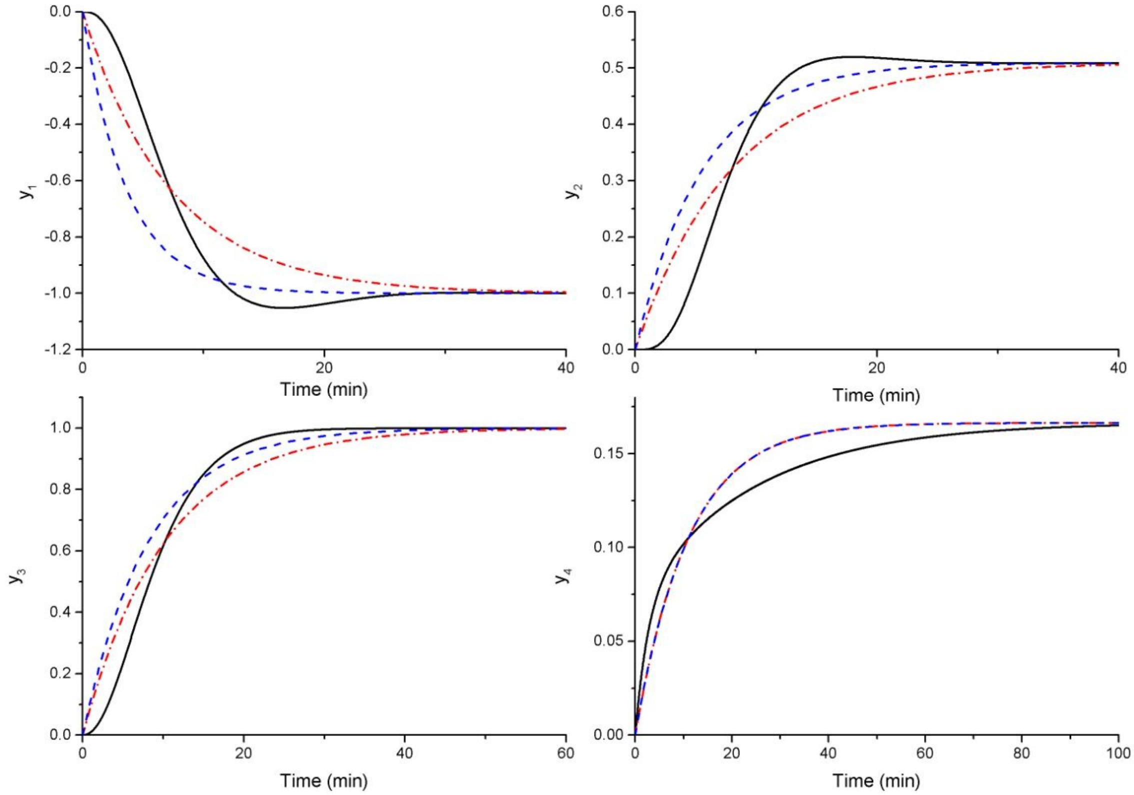


Figure 5-2: FCC reference trajectories, open-loop response (——), first-order approximation (–·–) and final reference trajectory (– –).

$$G_{des}(s) = \begin{bmatrix} \frac{1}{7.3s+1} & 0 & 0 & 0 \\ 0 & \frac{1}{8.05s+1} & 0 & 0 \\ 0 & 0 & \frac{1}{10.25s+1} & 0 \\ 0 & 0 & 0 & \frac{1}{11.05s+1} \end{bmatrix} \tag{5-6}$$

In order to illustrate the flexibility of the Lexicographic technique, two tuning scenarios were proposed here. In both scenarios, w' represents the current lexicographic step.

Scenario I: all system outputs are subject to set point changes at every lexicographic tuning step. The values of $F_j^{N*},\ j=1,\ldots,w'-1$ are defined according to the optimum cost function values obtained in the previously tuned subsystems.

Scenario II: only the new output included in the present tuning step is subject to a set point change. $F_j^{N*},\ j=1,\ldots,w'-1$ are calculated considering the same input profile obtained in the previous tuning step. The SSE index between output responses and their initial values characterizes the coupling effects between the most recent input and more important outputs.

The Lexicographic approach parameters were defined as follows: The simulation time period was $\theta_t = 60$. The tuning parameters were constrained to $10^{-3} < r_j < 10^2,\ j=1,\ldots,w';10^{-3} < q_{y,i} < 10^2,\ i=1,\ldots,w',\ w' \le w$ where w indicates the total number of objectives whereas w' indicates an intermediate lexicographic step.

Matrix S_t was chosen large enough to guarantee that the slacks related to the more important goals are heavily weighted in the tuning cost function. The DMC control problem was solved analytically disregarding all constraints and the lexicographic tuning problem was solved using *fmincon* (trust-region reflective algorithm, function tolerance = 10^{-12}, decision variable tolerance = 10^{-8}, max function evaluations = 4×10^3 and constraint tolerance = 10^{-10}) in MATLAB® 2013.

5.1.1.2. *Tuning of the DMC*

Scenario I

Table 5-3 summarizes the tuning parameters resulting from the Lexicographic method. As the size of the tuned controller increases, more inputs are available to achieve the reference tracking goal. However, the coupling effects decrease their efficiency; in fact, the best tracking performance of y_1 is achieved at step 1, since its corresponding input pair was optimally selected based on the open-loop gains defined in Table 5-2. In the subsequent steps, the method tries to preserve the initial performance, while optimally driving the additional outputs to their set points.

Figure 5-4 shows the responses of the outputs along the tuning procedure. Observe that at step 4, the responses of all the other outputs are sacrificed to make the response of y_1 closer to its desired trajectory. The slack variables weighting matrix was defined as $S_t = diag\begin{bmatrix}10^3 & 10^2 & 10\end{bmatrix}$. Figure 5-3 compares the set point tracking responses of y_1 considering the optimum tuning parameters for step 1 and step 4. The first response is the SISO subsystem response; whereas in the second response, the 4x4 system is considered but only the set point of y_1 is tracked, while the output weights on the other outputs are set to zero. Both responses are faster than the reference trajectory and the performance obtained by step 1 is 2.6% better, in terms of the SSE.

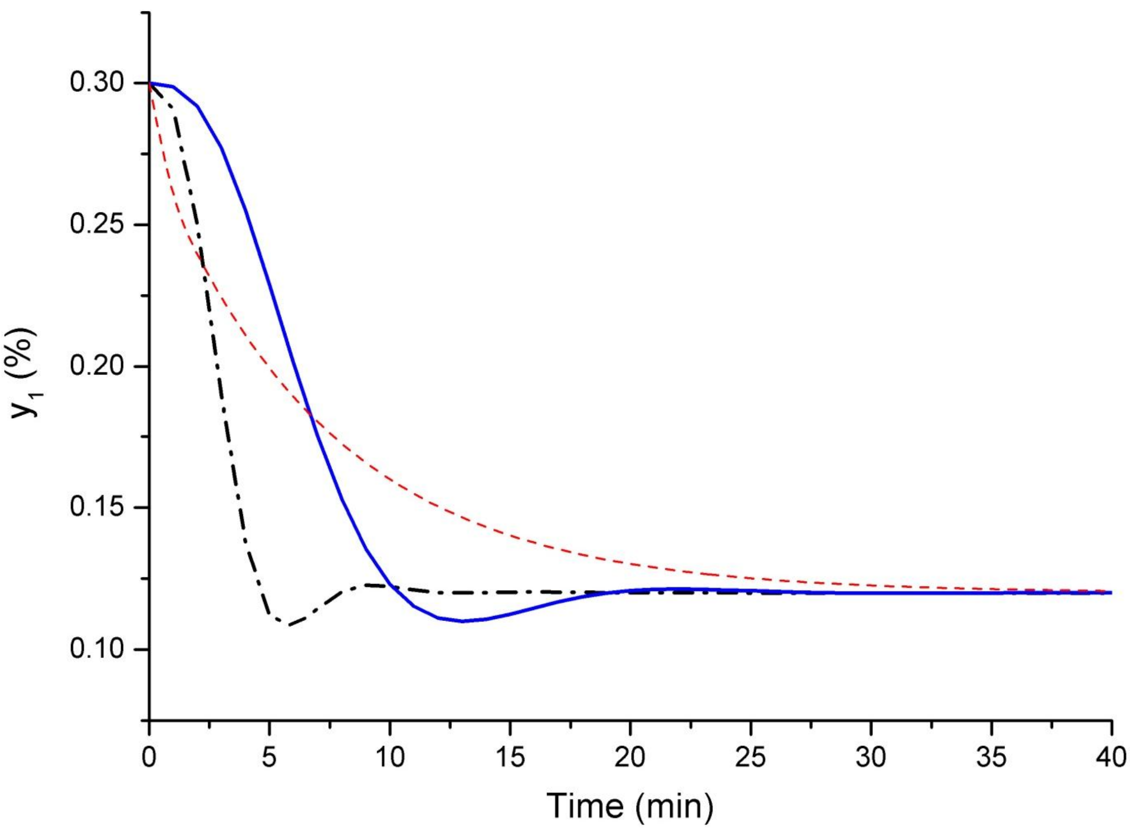


Figure 5-3: Closed-loop output responses obtained in the first tuning step (——), last tuning step (–·–) and reference trajectory (– –).

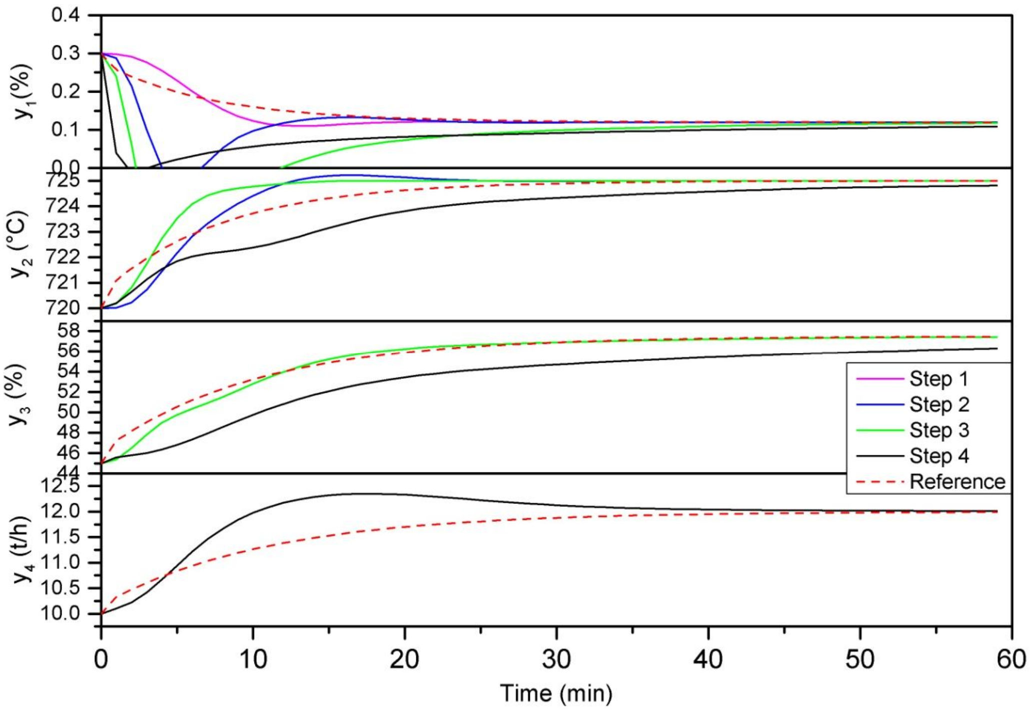


Figure 5-4a: Evolution of output responses in the tuning of the FCC unit using the goals defined in Scenario I, outputs.

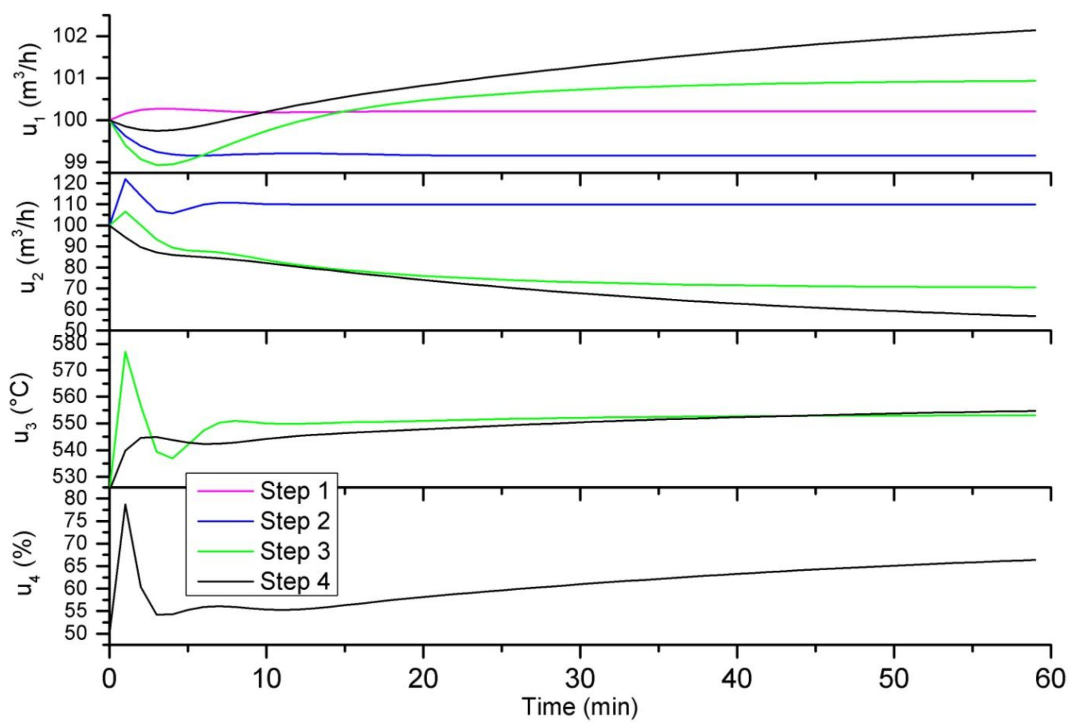


Figure 5-3b: Evolution of output responses in the tuning of the FCC unit using the goals defined in Scenario I, inputs.

Table 5-3: FCC lexicographic tuning results, Scenario I.

	Q_y*				R*			
Step	y_1	y_2	y_3	y_4	r_1	r_2	r_3	r_4
1	10				10			
2	10	0.1685			8.5	0.001		
3	10	0.9208	0.006		8.3	0.01	0.001	
4	10	0.03	0.03	0.01	8	0.001	0.002	0.001

Table 5-4: Lexicographic tuning slack variables and cost function values, Scenario I.

Slack variables				Cost function			
Step	δ^*_1	δ^*_2	δ^*_3	V^*_1	V^*_2	V^*_3	V^*_4
1				6.89×10^{-5}			
2	6×10^{-3}				0.0293		
3	0.005	0.0028				0.1286	
4	0.0012	0.037	0.253				2.85

Scenario II

Scenario II takes into account the input profile obtained in a previous tuning step to evaluate the closed-loop coupling effects and hence define the tuning problem constraints. The procedure is as follows: suppose that optimum q_y and r are calculated at the first tuning step, for the input-output pair u_1-y_1. The lexicographic goal of the second tuning step is defined to minimize the coupling effects between the input u_2 and the output y_1. Therefore, the current tuning cost function contribution of output y_1 is calculated with respect to a fixed set point, instead of a reference trajectory. This sum of square error must be equal or minor to the sum of square errors between the closed-loop response of variable y_1 and its reference trajectory obtained in the tuning step 1. A similar procedure is applied in the following tuning steps and the constraint values vector is $F^{N*}=\begin{bmatrix}0.222 & 0.126 & 0.66\end{bmatrix}$.

The resulting tuning parameters, slack variables and values of the tuning objective function are shown in Table 5-5 and Table 5-6. Figure 5-4, which depicts the responses obtained in tuning step 4 of Scenario I and the reference trajectory responses obtained at each step of Scenario II, shows that Scenario II yields worse performances than Scenario I, especially regarding y_1 and y_2. This result was expected because the constraint defined in Scenario II is trying to minimize the coupling effects whereas the constraints defined in Scenario I aim to maintain the tracking performance of the output variables.

Table 5-5: Lexicographic tuning results, Scenario II.

	Q_y				R			
Step	y_1	y_2	y_3	y_4	r_1	r_2	r_3	r_4
1	10				10			
2	10	0.43			100	0.001		
3	10	10.49	1.44		0.01	55.15	0.001	
4	10	10.51	24.51	8.59	2.19	0.001	0.092	0.03

Table 5-6: Lexicographic slack variables and cost function values, Scenario II.

Step	δ_1	δ_2	δ_3	V^*_1	V^*_2	V^*_3	V^*_4
1				6.23×10^{-5}			
2	*0*				0.022		
3	*0*	*0*				0.2412	
4	0	0	0				0.3085

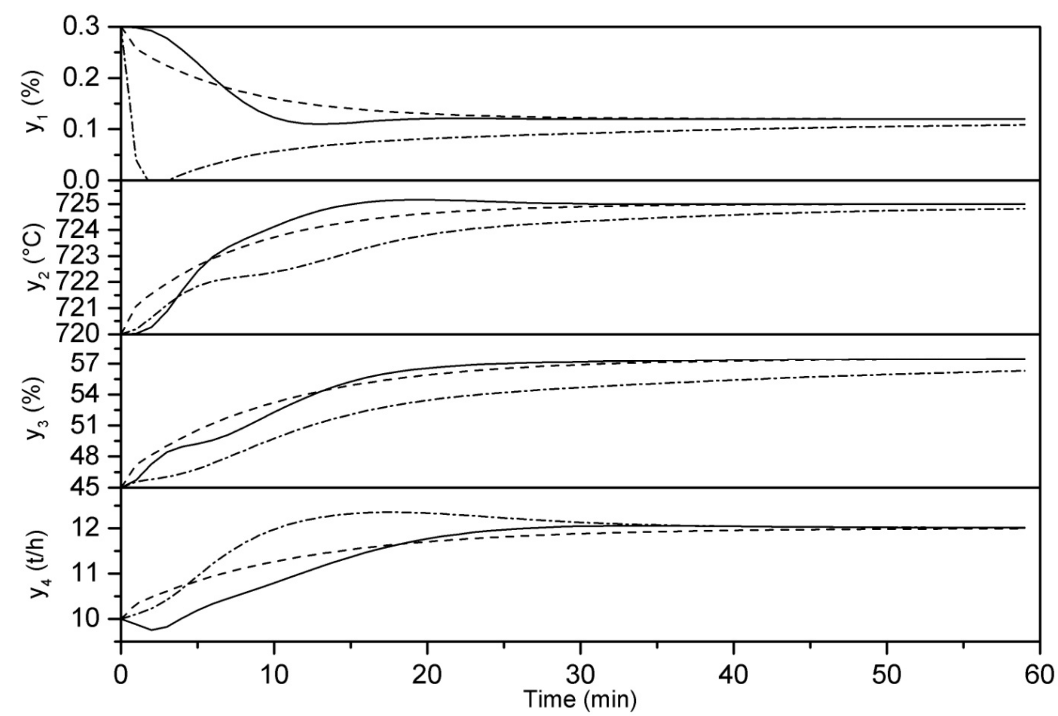


Figure 5-4: FCC closed-loop responses resulting from tuning scenarios I (——) and II (–·–) and reference trajectories (– –).

5.1.1.3. *Simulation results*

A simulation was carried out to compare the output set point tracking performance of a DMC tuned according to Scenarios I and II. The 4x4 FCC subsystem was driven towards different operating points following the set point changes defined in Table 5-7. The remaining variables and DMC parameters were chosen as: N=120, p=60, m=3, T_s=1 min, the simulation time was 4500 min, and the system initial point was $y_0 = \begin{bmatrix} 0.3 & 720 & 45 & 10 \end{bmatrix}^T$ and $u_0 = \begin{bmatrix} 147.5 & 100 & 525 & 50 \end{bmatrix}^T$. The maximum input increments were $\Delta u_{\max} = \begin{bmatrix} 0.9 & 0.3 & 1.3 & 0.05 \end{bmatrix}^T$, considering the engineering units presented in Table 5-1, which also includes the input and output ranges. In this simulation, the constrained version of the DMC was simulated, according to Appendix B, which was solved through *quadprog* (default settings) of MATLAB® 2013.

Table 5-7: FCC simulation study, set point changes.

Time (min)	$y_{1,sp}$ (%)	$y_{2,sp}$ (°C)	$y_{3,sp}$ (%)	$y_{4,sp}$ (t/h)
10	0.2	720	45	10
100	0.2	722	45	10
1500	0.2	722	47	10
3500	0.2	722	47	10.3

Figure 5-5 and Figure 5-6 show the output and input responses respectively throughout the simulation. We can conclude that Scenario I yields better results for y_1 in set point tracking scenarios and poor responses for the remaining outputs, while Scenario II gives a poor response to output y_1 showing offset after the set point in y_3, until the set point change in y_4 and a large excursion at 1500 minutes. Observing variables y_2 and y_3 and y_4 from 0 to 300 minutes, it is noted that both were successfully decoupled from the set point changes in y_1; on the other hand, the controller tuned using the goals defined by Scenario I yielded more sluggish performance, with larger excursions for the set point changes in y_3 and from 200 min to 1000 min in y_4, respectively.

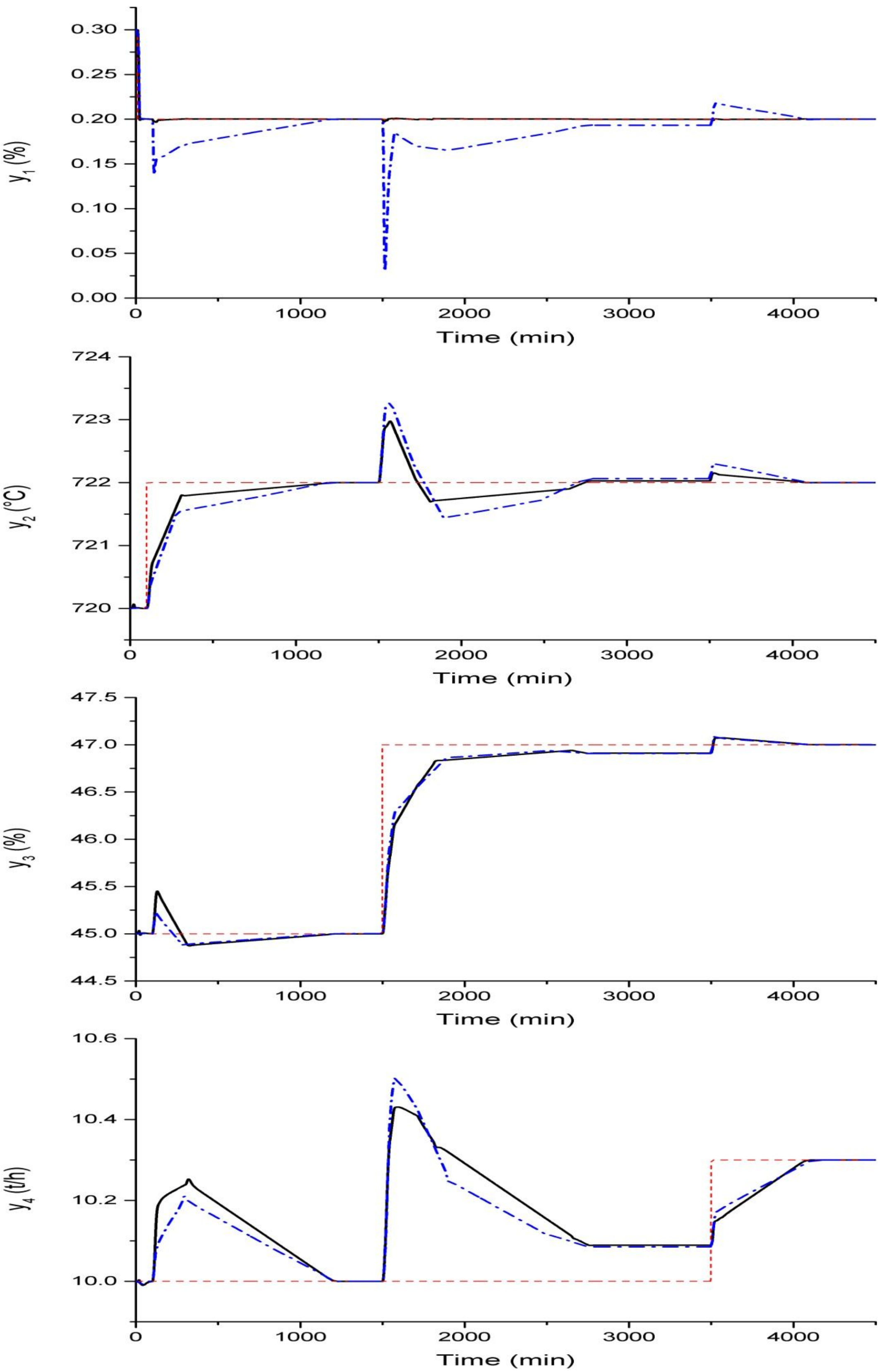


Figure 5-5: Output tracking of the FCC unit simulation, Scenario I (——), Scenario II (–·–). Set points (– –).

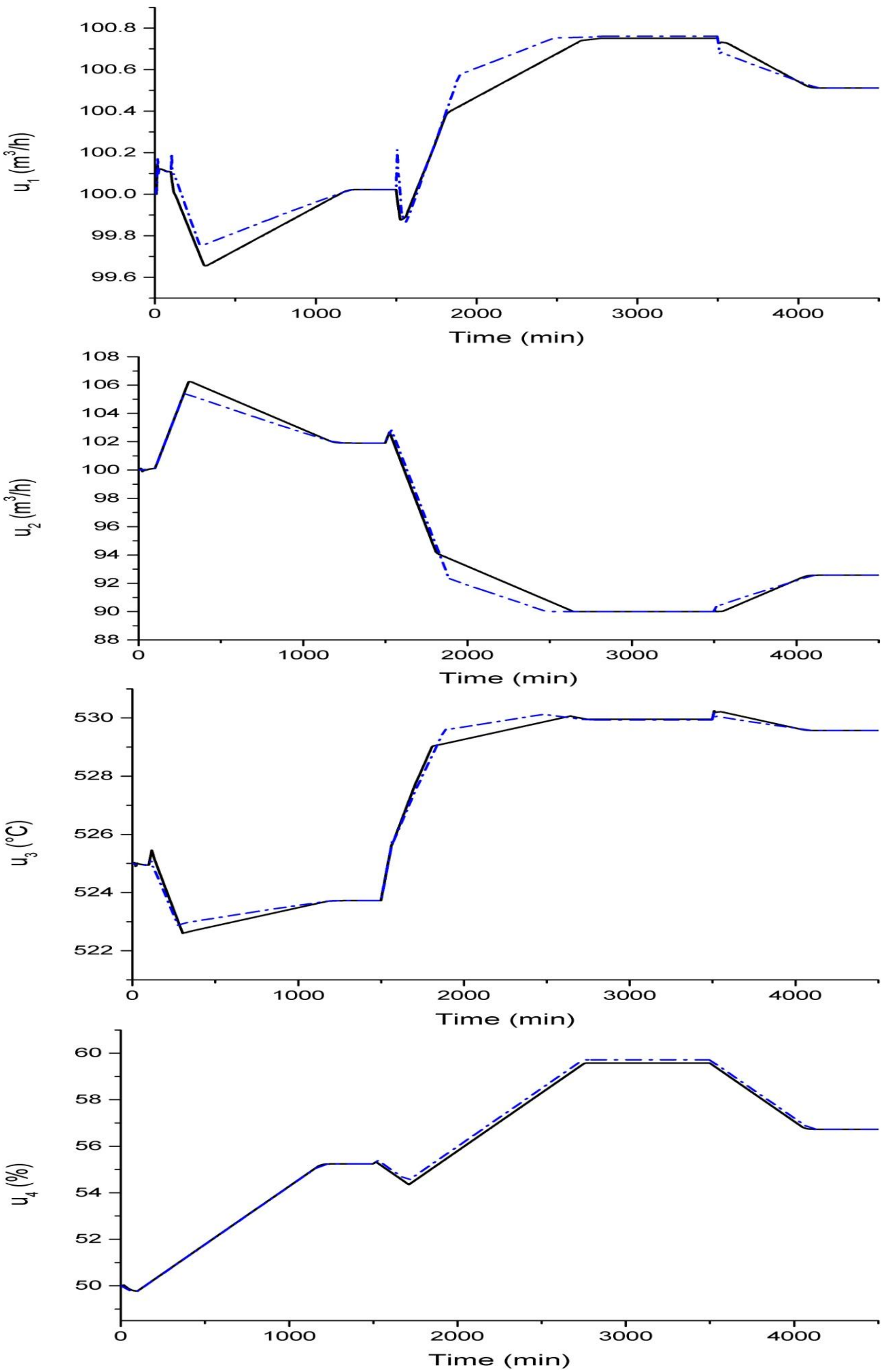


Figure 5-6: Inputs of the FCC unit simulation, Scenario I (——), Scenario II (– · –).

It is observed in Figure 5-6 that the calculated input increments are close to their maximum values, which is expected due to the low values of *R* obtained in Scenario II. Moreover, the inputs u1, u2 and u4 saturate from 2500 to 3500 minutes, when the DMC is trying to track the set point change of output y_2. It is concluded that the goal definitions defined in Scenarios I and II are potentially capable of defining a multi-objective optimization tuning problem that represents the desired behavior of the control system.

5.1.2. Heavy oil fractionator case-study

To compare the application of the tuning techniques proposed here, a MPC controller implemented in a subsystem of the Shell Heavy Oil Fractionator (HOF) benchmark system (Maciejowski, 2002) was tuned.

5.1.2.1. *Process description*

The inputs of this subsystem are the top drawn flow rate (u_1), the side drawn flow rate (u_2) and the bottoms reflux heat duty (u_3). The controlled outputs are the top end point composition (y_1), the side end point composition (y_2) and the bottoms reflux temperature (y_3). Figure 5-7 shows a schematic representation of the process and equation (5-7) defines the transfer functions that represent the HOF, with normalized gains. The tuning strategies proposed here are applied to the MPC of the HOF. The simulated scenarios involve the output tracking and the disturbance rejection.

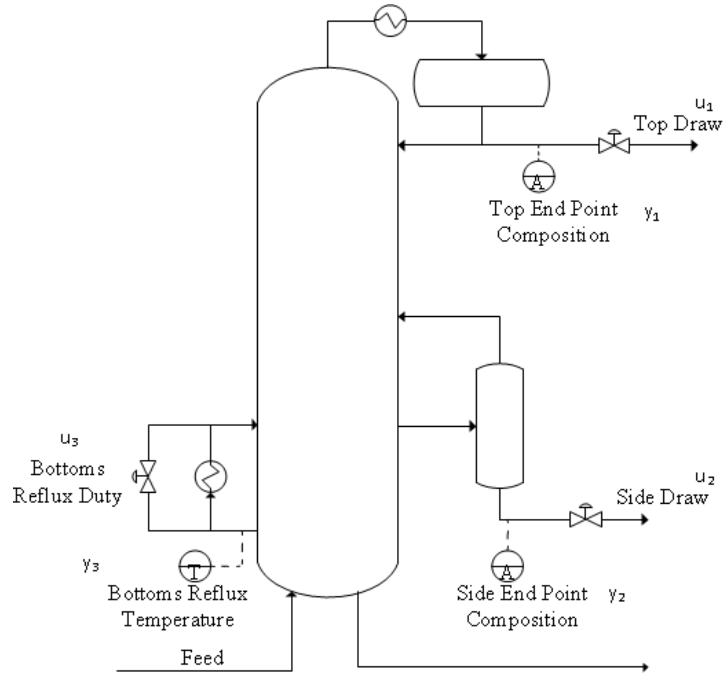


Figure 5-7: Shell Heavy Oil Fractionator 3x3 subsystem schematic representation.

$$G(s)=\begin{bmatrix} \dfrac{4.05e^{-27s}}{50s+1} & \dfrac{1.77e^{-28s}}{60s+1} & \dfrac{5.88e^{-27s}}{50s+1} \\ \dfrac{5.39e^{-18s}}{50s+1} & \dfrac{5.72e^{-14s}}{60s+1} & \dfrac{6.90e^{-15s}}{40s+1} \\ \dfrac{4.38e^{-20s}}{33s+1} & \dfrac{4.42e^{-22s}}{44s+1} & \dfrac{7.20}{19s+1} \end{bmatrix} \tag{5-7}$$

5.1.2.2. *Tuning goals*

The reference trajectories corresponding to the tuning goals were defined according to the input-output pairing presented in (Li, Zhang & Zhu, 2005). The open-loop transfer functions, $G_{i,j}(s)$ had their time constants multiplied by a response factor, $f_{ref}(i)$, i=1,…,ny, to obtain the reference transfer functions and reference trajectories for each output, $G_{des,i}(s)$. The input-output pairs were selected as y_1-u_1, y_2-u_2 and y_3-u_3. Based on the process information available in (Li, Zhang & Zhu, 2005), the vector of selected response factors was the following f_{ref}=[0.10 0.15 0.30]^T. Response factors smaller than one indicate that the reference trajectory is faster than the corresponding open-loop response. Considering that the reference trajectory corresponds to a first order plus dead time transfer function, $G_{des}(s)=\frac{K_i e^{(-\theta_i s)}}{\tau_i s+1}$, the resulting model parameters that define the reference trajectories for the tuning methods proposed here are given in Table 5-8.

Table 5-8: Parameters of the reference transfer function

Output	K_i	τ_i	θ_i
y_1	1	5	27
y_2	1	9	14
y_3	1	5.7	0

For all the outputs, the tuning horizon, θ_t, was assumed to be equal to 450 min, which is large enough to encompass the responses of set point moves that drive the closed loop system to different operating points. For the tuning procedure, the input and output initial values are $y_0=[0\quad 0\quad 0]^T$ and $u_0=[0\quad 0\quad 0]^T$. The output set points are changed to $y_{sp}=[0.2\quad 0.2\quad 0.2]^T$ at the initial time instant, to $y_{sp}=[0.0\quad 0.4\quad 0.1]^T$ at time instant 150 min and to $y_{sp}=[0.1\quad 0.3\quad 0.0]^T$ at 300 min. Observe that this tuning scenario might be overly demanding in real applications, because only in rare occasions more than one output set point is driven towards new values at the same time. The control horizon is set equal to 5 and the prediction horizon is set equal to 70. All the problems pictured here were solved using an Intel® Core i5 320 GHz, 4 GB RAM computer. We assume that the process model considered in the controller is ideal and that the system states are fully measured.

5.1.2.3. *MPC implemented in this case study*

The controller assumes that the system is represented by the state space model in the incremental form (5-8).

$$\begin{aligned} x(k+1) &= Ax(k)+B\Delta u(k) \\ y(k) &= Cx(k) \end{aligned}, \tag{5-8}$$

where $x \in \Re^{nx}, u \in \Re^{nu}, y \in \Re^{ny}$. Matrices A, B and C are the model matrices that carry all the system information required for future output predictions.

The MPC cost function includes the weighed sum of the squared deviation of the predicted outputs and the set point values over the prediction horizon, and the weighted sum of squared input increments over the control horizon. The control problem can be summarized as follows.

$$\min_{\Delta u_k} \sum_{j=0}^{p} \left\| y(k+j\,|\,k) - y_{sp} \right\|_{Q_y}^{2} + \sum_{j=0}^{m-1} \left\| \Delta u(k+j\,|\,k) \right\|_{R}^{2} \tag{5-9}$$

subject to (5-8) and the bounds on the inputs and input increments. The tuning algorithms consider the unconstrained version of the MPC, disregarding the latter bounds, so that their effects on the system inputs and input increments do not affect the tuning results.

5.1.2.4. *Tuning the MPC of the HOF system with LTT*

The Lexicographic tuning problem (Problem 1) was solved for the HOF system with the Matlab® routine *fmincon*, which solves a NLP. At any step *w'*=1,…,*nu* of the tuning method, the vector of decision variables is $x = \left(q_{y,1},...,q_{y,w'},r_1,...,r_{w'}\right)$, and the initial guess is $x_0 = \left[5 \quad 1_{1,w'-1} \quad 1_{1,w'} \times 10^{-1}\right]$, with the lower and upper bounds of the decision variables equal to $LB = \left[5 \quad 1_{1,w'-1} \times 10^{-2} \quad 1_{1,w'} \times 10^{-3}\right]$ and $UB = \left[5 \quad 1_{1,w'-1} \times 10^{2} \quad 1_{1,w'} \times 10^{2}\right]$ respectively. Observe that the weight corresponding to output y_1 is kept at a fixed value ($q_{y,1}$=5). Otherwise, the tuning problem would show multiple equivalent solutions.

The weighting matrix S_t of the slack variables was chosen considering the expression $s_{t,i} = 10^{(w'-i)\times 2}$, i=1, …, w'-1, where $s_{t,i}$ denotes the i-th diagonal element of matrix S_t. This approach guarantees that the slacks related to the more important goals will be more heavily weighted in the tuning cost function. Observe that in the Lexicographic approach, at any step the value of *w*' corresponds to the size of the subsystem that is considered.

Table 5-9 and Table 5-10 show the values of Q_y, R and δ_t resulting from the application of the method to the controller of the HOF considering the goal definition used in Scenario 1 of Section 5.1.1. The required computational time for this method was 4.27 hours. Observe that in Table 5-10 the values of δ_t at the third step are small, indicating that the Lexicographic approach was able to properly adjust the closed-loop responses of outputs y_2 and y_3 to their reference trajectories without significantly degrading the response of y_1. This result is also observed in Figure 5-8, which shows the evolution of system outputs in closed-loop throughout the Lexicographic method. The response related to output y_1 improves from step 1 to step 2, and degrades only a little from step 2 to step 3, while the response related to y_2 remains nearly the same from step 2 to step 3. The parameters shown in the last row of Table 5-9 are the optimum tuning parameters obtained by the lexicographic method.

Table 5-9: Lexicographic optimum tuning parameters.

	Q_y			R		
Step	1	2	3	1	2	3
1	5			8.63		
2	5	1.32		2.62	6.49	
3	5	1.54	1.57	1.49	7.46	0.50

Table 5-10: Lexicographic slack variables, δ_t.

Step	1	2
1		
2	0	
3	0.013	0.010

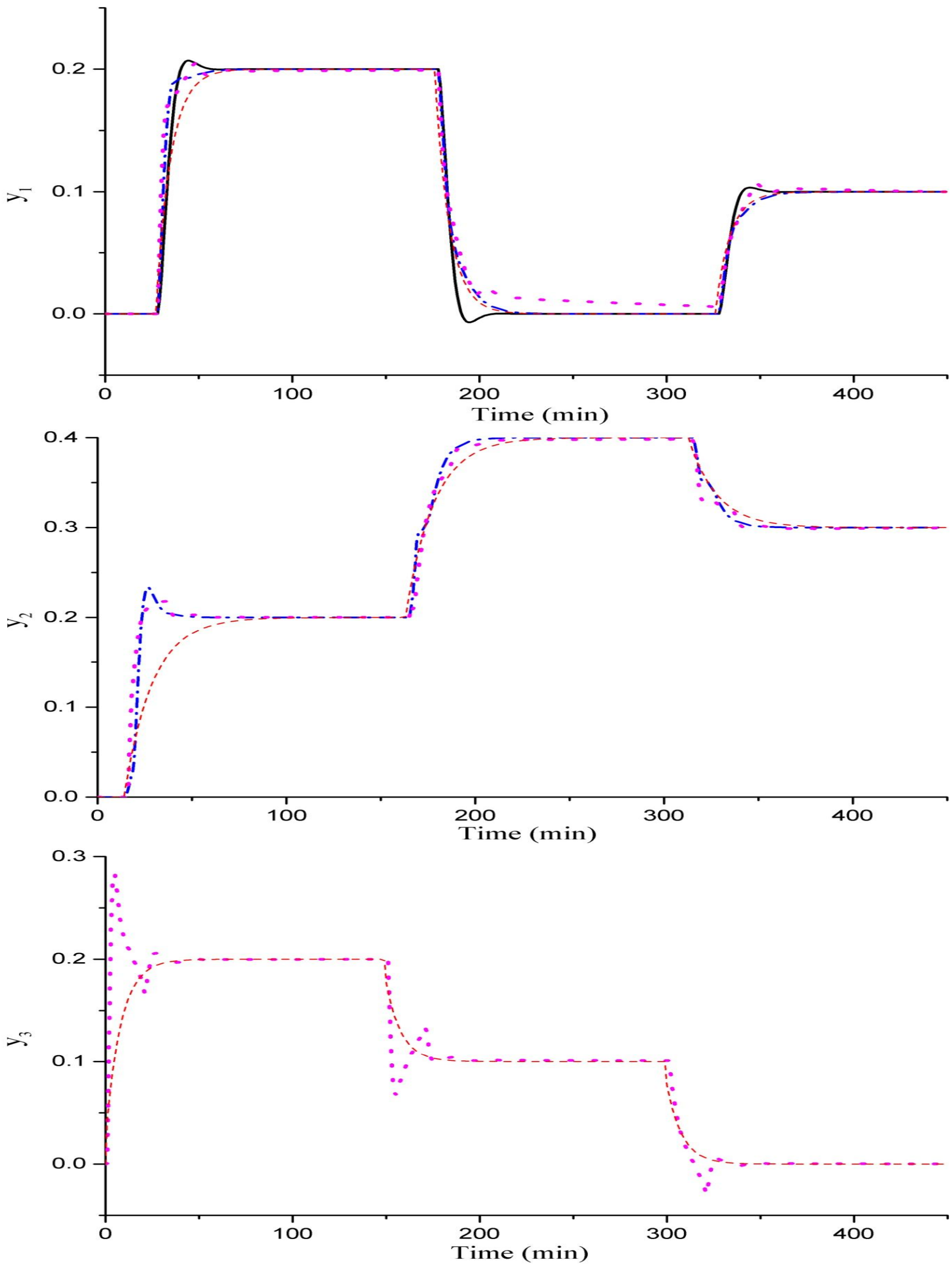


Figure 5-8a: Responses of *the HOF* system outputs with LTT calculated in Step 1(——), Step 2 (−·−) and Step 3 (···) and reference trajectories (−−).

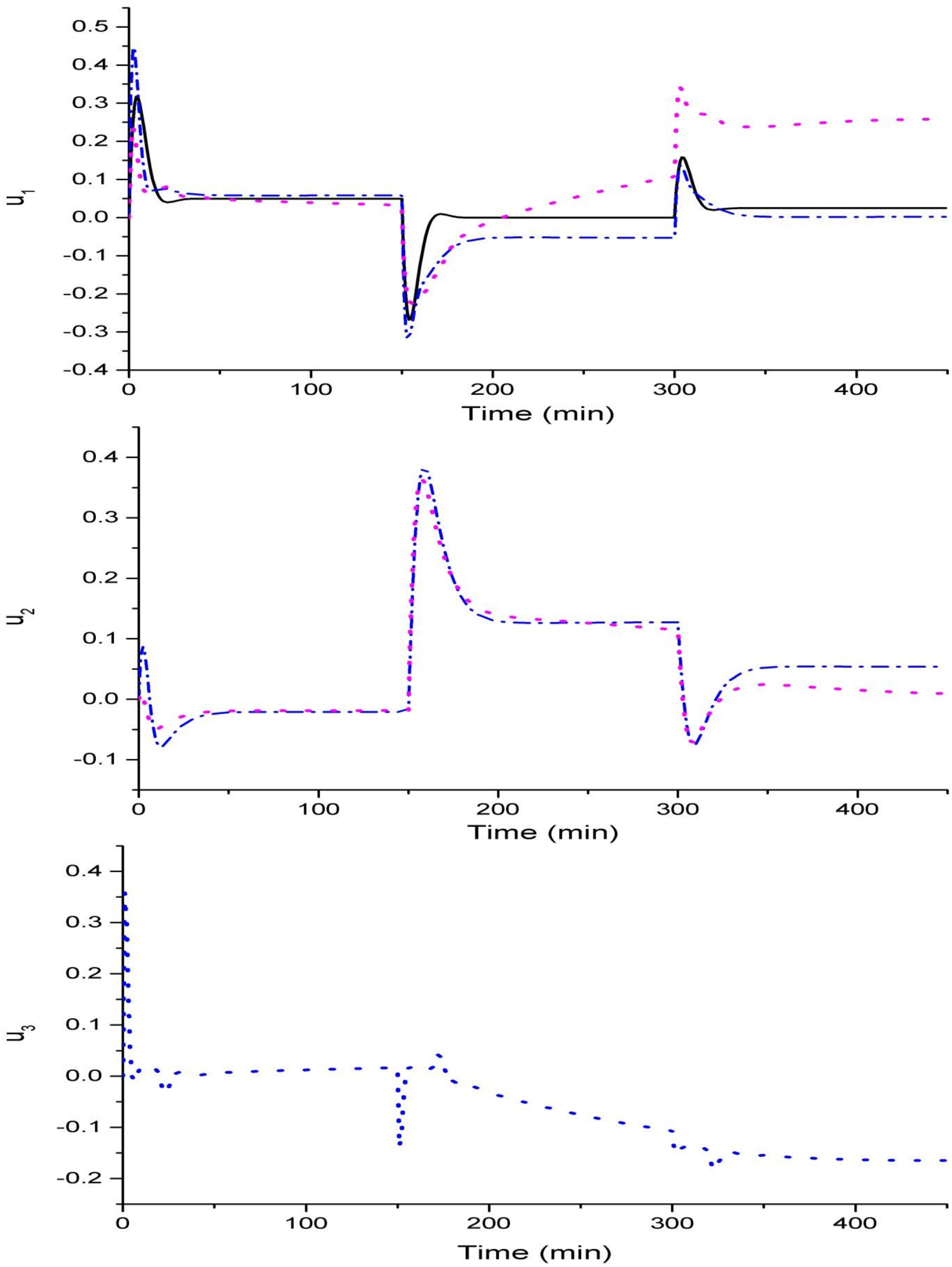


Figure 5-9b: Responses of *the HOF* system inputs with LTT calculated in Step 1(——), Step 2 (–·–) and Step 3 (···).

5.1.2.5. *Tuning the MPC of the HOF system with CTT*

Here, it is assumed the same tuning goals as in the Lexicographic method. In the application of the Compromise method to the Heavy Oil Fractionator, the individual objectives of the multi-objective optimization problem are built for each controlled output y_i, according to (3-1) and, this function is minimized with respect to all the tuning parameters to calculate the Utopia solutions, which means that each Utopia solution defines a solution $x_{utop} = \left(q_1,...,q_{ny},r_1,...,r_{nu}\right)$.

The Compromise method (Problem 2) is solved for the tuning of the MPC of the HOF system using *fmincon* in MATLAB 2013®, with the following initial guess for the tuning parameters $x_0 = \begin{bmatrix}5 & 1_{1,2} & 1_{1,3}\times10^{-1}\end{bmatrix}$. The lower and upper bounds of the decision variables are the same as in the Lexicographic solution of the tuning problem. The resulting Utopia vector is $F^0 = \begin{bmatrix}1.893 & 0.465 & 0.004\end{bmatrix}^T$ and the optimum solution, with an execution time of 53 min, including the time required to obtain the Utopia solution, is $Q_y^* = diag\left(\begin{bmatrix}5 & 4.96 & 2.91\end{bmatrix}\right)$ and $R^* = diag\left(\begin{bmatrix}10^{-3} & 2.39\times10^{-2} & 0.982\end{bmatrix}\right)$.

Differently from the lexicographic approach, the number of objective defined for the compromise tuning method is not limited to the number of inputs of the tuned system and neither is its application to square (*ny*=*nu*) systems. It is possible to define as many objectives as the number of system outputs, and all of them are treated simultaneously, independently of the number of inputs. It is also observed that the values of the parameters obtained with the compromise method are quite different from the values of the same parameters obtained with the lexicographic method, as seen in Table 5-11. Then, the performances of the controllers with these two sets of tuning parameters need to be compared through closed-loop simulations.

5.1.2.6. *Simulation results*

Here, we analyze two different operating scenarios to compare the MPC controllers with the parameters obtained with the tuning techniques presented earlier, addressed in the figure captions and tables as LTT (Lexicographic technique) and CTT (Compromise technique). The first scenario corresponds to nearly the same conditions in which the MPC controlling the HOF system was tuned. Outputs are subject to set point changes one at a time, and finally driven back to the initial steady-state point, following the sequence of changes defined in Table 5-12. The second simulation considers set point changes that are given in Table 5-13, as well as unmeasured input disturbances. The different scenarios are used to validate the tuning results. In both simulations, the constrained version of the MPC problem defined in Section 5.1.2.3 is considered. A perfect process model and fully measured states are assumed, and the initial operating point of the system is $y_0 = [0 \quad 0 \quad 0]^T$ and $u_0 = [0 \quad 0 \quad 0]^T$. The input lower and upper bounds and maximum input increments are $u_{\min} = [-0.5 \quad -0.5 \quad -0.5]^T$, $u_{\max} = [0.5 \quad 0.5 \quad 0.5]^T$, $\Delta u_{\max} = [0.05 \quad 0.05 \quad 0.05]^T$.

Table 5-11: Optimum tuning parameters summary.

Method	Q_y^*	R^*	Computational Time
LTT	$[5 \quad 1.54 \quad 1.57]$	$[1.49 \quad 7.46 \quad 0.50]$	4.27h
CTT	$[5 \quad 4.96 \quad 2.91]$	$[10^{-3} \quad 2.39\times10^{-2} \quad 0.98]$	53 min

Figure 5-10 and Figure 5-11 show the output and input responses respectively, corresponding to the scenario defined as Simulation I with the set-points changes represented in Table 5-12. These figures compare the behavior of the MPC tuned according to the methods proposed here. We observe that, although the numerical values of the tuning parameters provided by the methods are quite different from each other, the responses of the closed-loop system are not too different. The responses corresponding to method LTT tend to be not the same as the CTT but they are still close from a practical viewpoint. The same considerations can be given to the input responses corresponding to the two methods. However the input responses corresponding to the Lexicographic approach seems to be slightly smoother.

Figure 5-12 and Figure 5-13 show the responses for Simulation II with the set point changes defined in Table 5-13. The scenario is affected by an unmeasured step disturbance of intensity 0.05 on input u_1 from 40 min to 100 min. In this simulation, for the set point changes, the responses of the two controllers seem to follow the same patterns as in the previous case. However, we can observe some differences on the responses for the disturbance rejection. The overshoots are different for the two methods, but the largest overshoot depends on the output, and consequently, there is not a clear superiority of any of the methods. Concerning the inputs, the two methods seem to perform similarly, but the Lexicographic approach gives smoother responses again. Table 5-14 shows the ISE index calculated for the output responses and their deviation from the set points. It is observed that in both simulations, the ISE index of the compromise strategy is lower than the ISE index of the lexicographic strategy, and the results also support the claim that the choice of exceedingly fast reference trajectory of output y_1 lead to poor performance.

Table 5-12: Simulation I set points.

Time (min)	y_1^{sp}	y_2^{sp}	y_3^{sp}
1 to 80	0.2	0.2	0.2
80 to 200	0.0	0.4	0.1
200 to 300	0.1	0.3	0.0
300 to 400	0	0	0

Table 5-13: Simulation II set points.

Time (min)	y_1^{sp}	y_2^{sp}	y_3^{sp}
1 to 120	-0.2	0	0
120 to 200	-0.2	0	0.3
200 to 300	0	-0.2	0.3
300 to 400	0.3	0	0.2

Table 5-14: ISE index calculated for the simulation responses.

	Variable Technique	y_1	y_2	y_3	Total
Simulation I	LTT	3.23	3.64	0.43	7.3
	CTT	3.3	3.49	1.04	7.82
Simulation II	LTT	6.22	1.7	1.34	9.26
	CTT	5.74	1.55	1.69	8.98

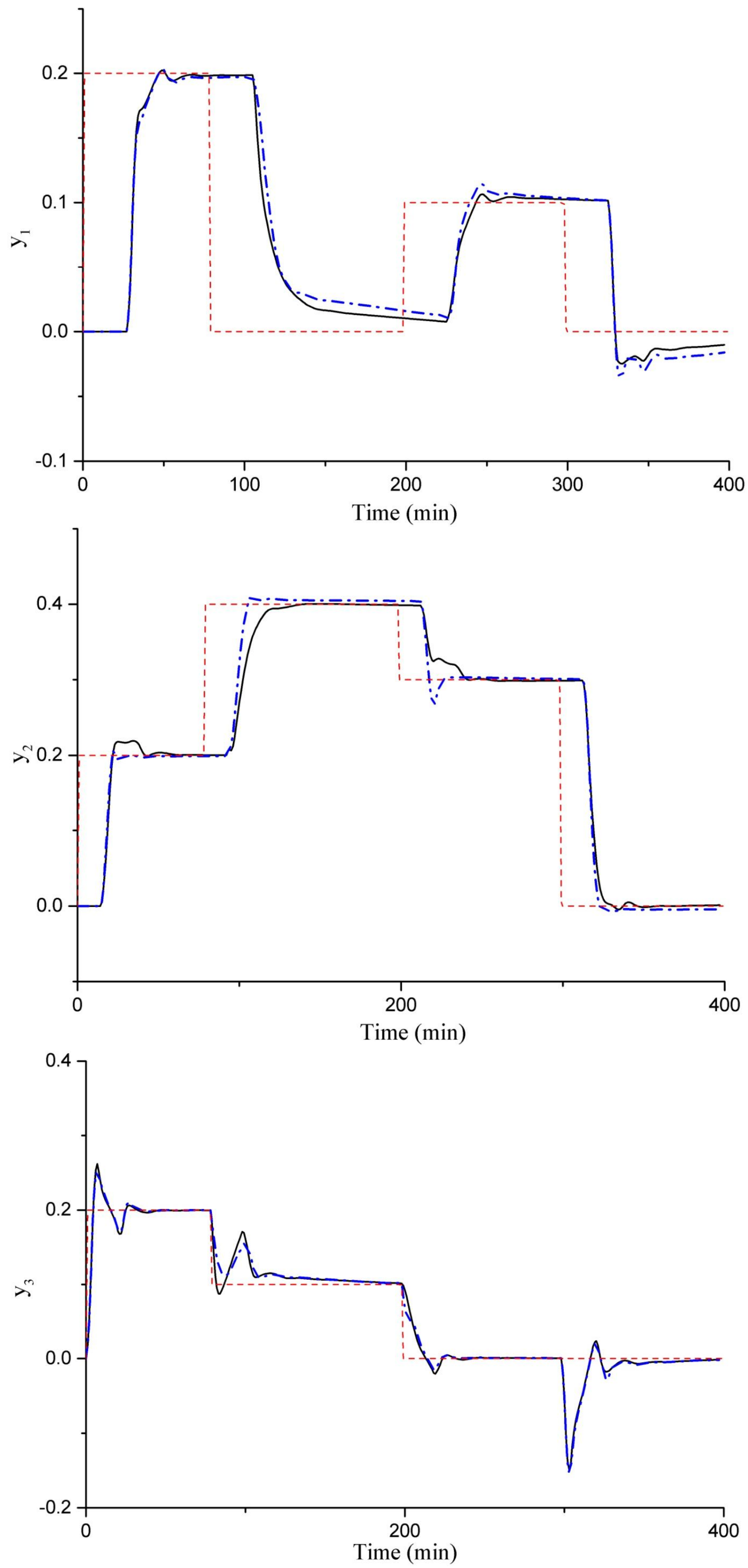


Figure 5-10: Simulation I. HOF outputs to set point changes (– –), LTT (——) and CTT (–·–).

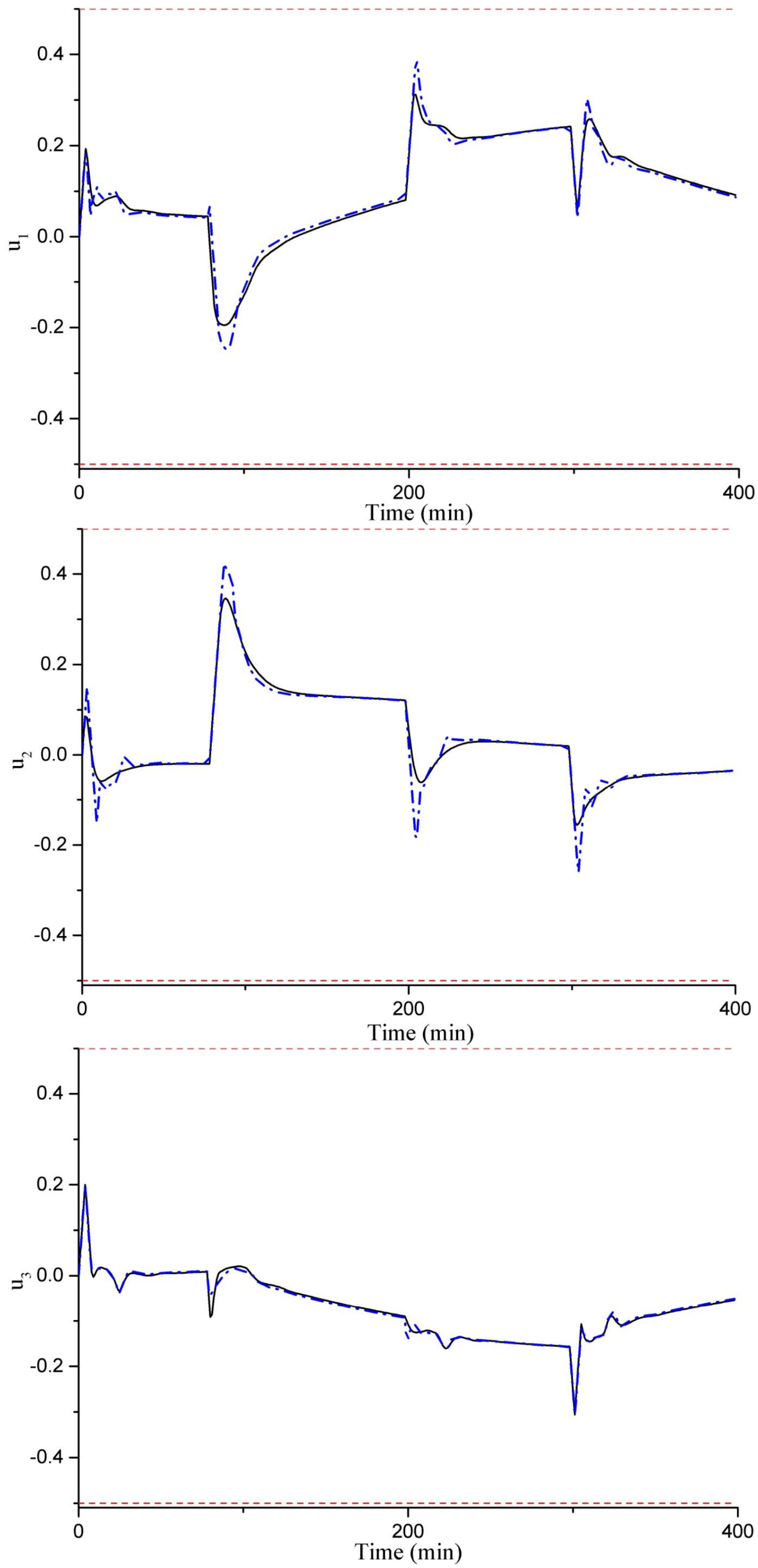


Figure 5-11: Simulation I. HOF inputs, LTT (——) and CTT (−·−) and the upper and lower bounds (−−).

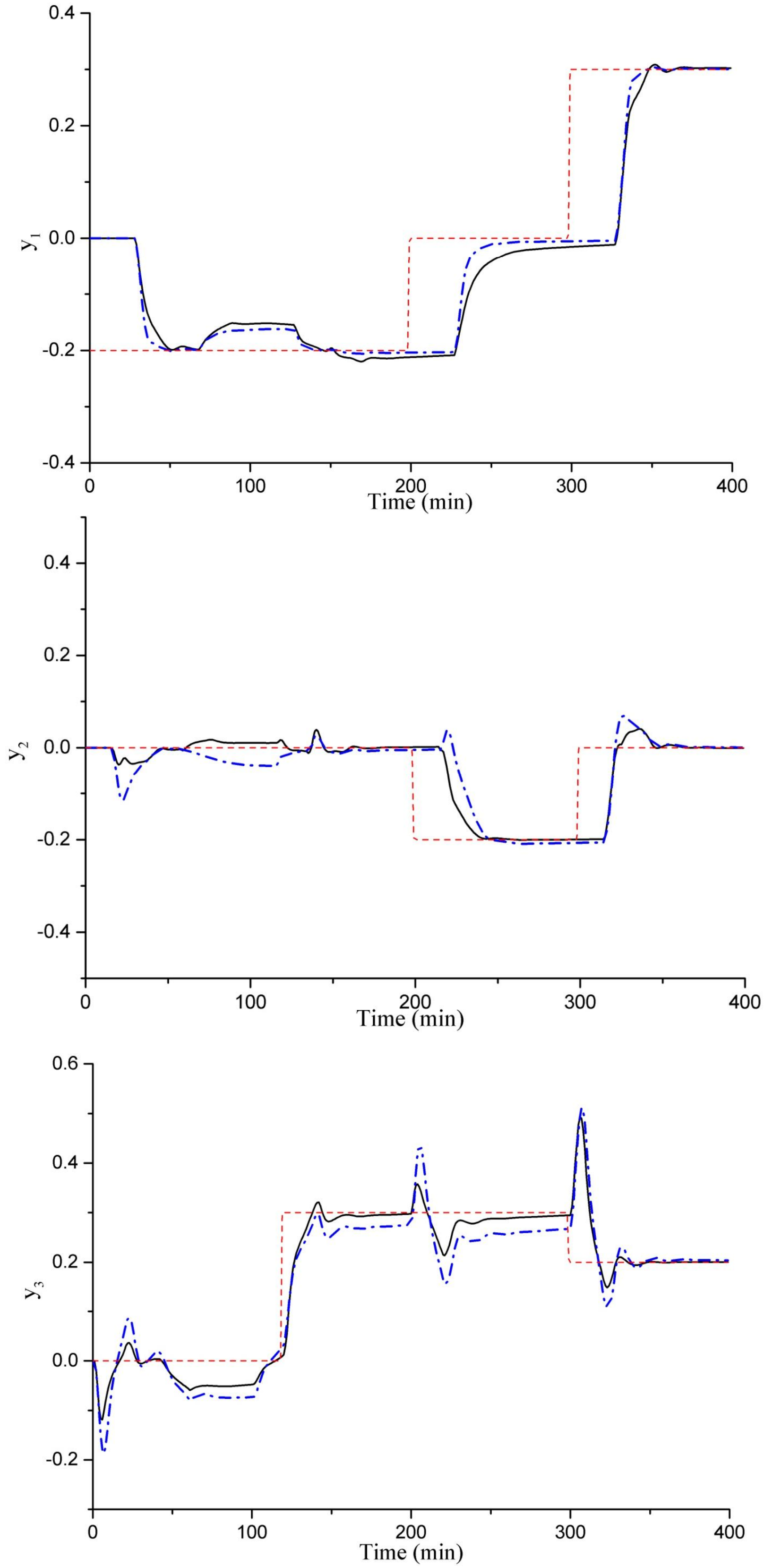


Figure 5-12: Simulation II. HOF output to set point changes (– –) and unmeasured disturbances, LTT (——) and CTT (–·–).

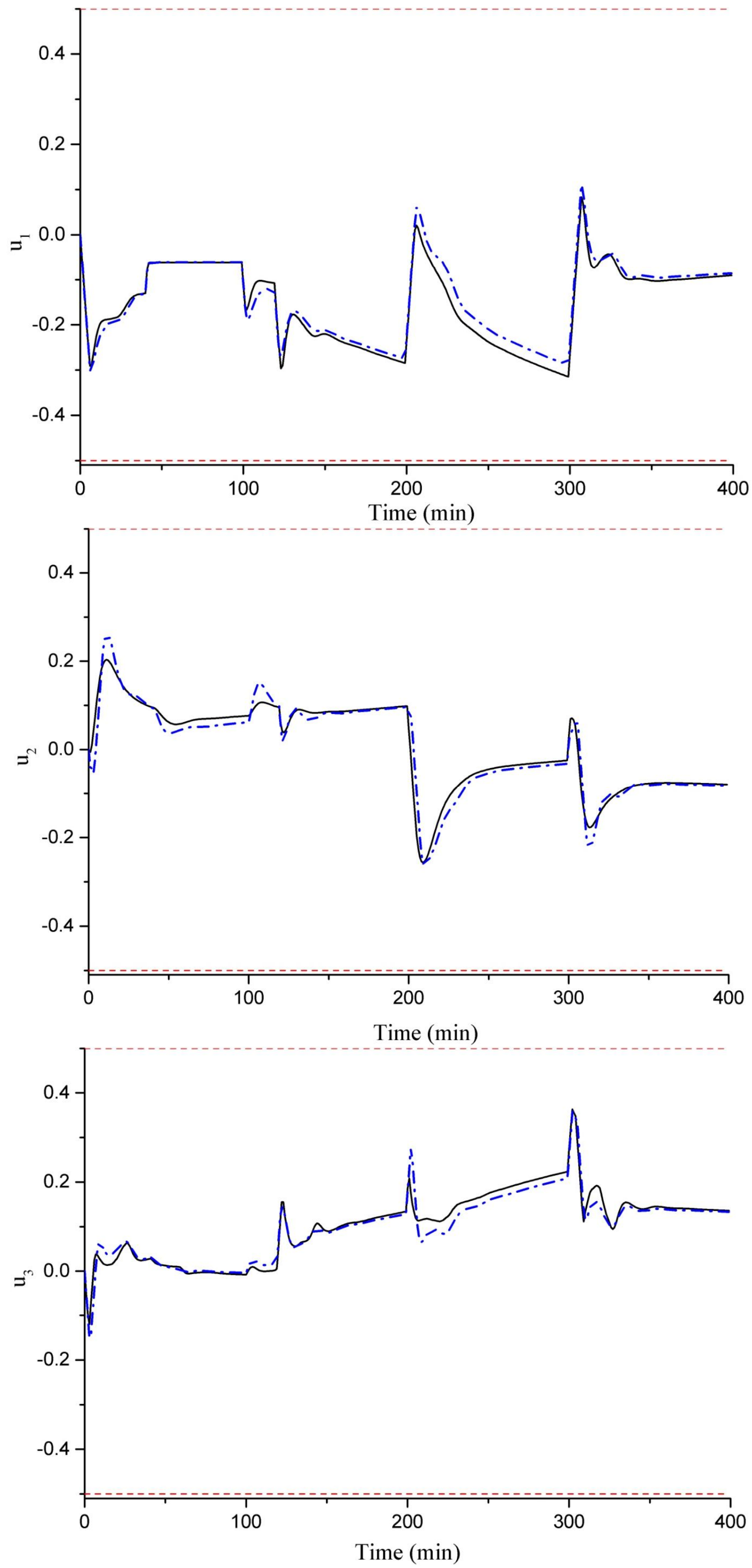


Figure 5-13: Simulation II. HOF inputs, LTT (——) and CTT (–·–) and the upper and lower bounds (– –).

5.1.3. Crude distillation unit case study

In this case study, the compromise technique is applied to a Crude Distillation Unit (CDU) in closed-loop with an IHMPC control framework. First, we provide a brief description of CDU, along with specific information about the particular unit from the Capuava Refinery, Brazil. Then, we define the goals and show the optimum tuning parameters. Finally, we compare tuning results to another multi-objective tuning technique from the literature and to the pre-existing parameters in a simulation including set point changes and disturbance rejection scenarios.

The CDUs are certainly one of the most important, complex and energy-intensive processes in oil refineries (Liau, Yang & Tsai, 2004; Luo, Wang & Yuan, 2013). It is also one of the most common separation process units in the chemical industry (Seo, Oh, & Lee, 2000). Moreover, all the crude oil must initially go through a CDU before being further processed in subsequent refining units. Even though chemical reactions do not take place along the distillation process, the amount of mass and energy involved, the number of equipment, the environmental and operational constraints, and the conflicting financial goals are sufficient to make the CDU system a challenging control problem.

The crude oil stream contains an array of different kinds of hydrocarbons along with other organic and inorganic compounds (Seo, Oh & Lee, 2000). It is not practical to precisely determine the feedstock composition and to set the operating conditions of the distillation unit in order to deal with that specific composition, therefore, a reliable automated control framework is imperative to enforce safe and stable operation, and to achieve high product quality while meeting production rate goals (Mizoguchi, Marlin & Hrymak, 1995).

Another complicating issue arises from the fact that it is not straightforward to assess product quality. Diesel, kerosene, gas oil and naphtha are the main products of a CDU and there is few expensive and hard to maintain hardware available to measure density, viscosity, flash points (Chatterjee & Saraf, 2004), gravity, sulfur content and composition of such product streams (Liau, Yang & Tsai, 2004). In practice, it is common to correlate the temperature and flow rates measures of the outlet streams with physical properties resulting from offline laboratorial assessments of ASTM (America Society for Testing Materials) standardized tests, such as the D86 curves of distillated oil products. For example, Chatterjee & Saraf (2004) established a method to infer product properties such as densities, flash points, pour points and recoveries for gas oils, and the freezing point of the kerosene from the crude oil true boiling point and other routinely measured variables during the CDU operation. Trial inference applications showed satisfactory results; however, the authors recommend extensive tests prior to real applications.

CDUs are usually comprised of a pre-flash column, an atmospheric distillation column and a vacuum distillation column, along with side stripping sections, heat exchangers, and pump around reflux streams. The heat recovery problem, i.e. determine an optimum arrangement of pump around streams to minimize the heat required to raise the crude oil temperature up to its required value at the inlet of the pre-flash or atmospheric distillation column, is just one of the complications that arise in CDU optimization. The oil industry is also concerned with maximizing profit and enforcing product standards.

The control and optimization of the CDU unit has been extensively studied in the literature. There are different approaches to address CDU optimization. Liau, Yang & Tsai (2004) developed a neural network model to correlate quality parameters and measurable process variables to optimize the CDU operation. The neural network training data was collected over a period of six months and the results showed that the method can provide useful insight for inexperienced operators. Luo, Wang & Yuan (2013) used a shortcut method (SCFrac) in ASPEN Plus® to simulate distillation columns using real input parameters such as column pressure, estimated product flow, number of theoretical stages, steam flow and product specifications. A multi-objective optimization was assembled to find an optimum operating point considering energy cost and profitability goals. The authors solved the problem using PSO. Seo, Oh & Lee (2000) developed an algorithm to determinate the optimum feed tray, in terms of maximizing heat recovery along the process, and to determine optimum operational conditions to yield the minimum annual operational cost. The optimization problem was posed as a MINLP, solved using GAMS®. Mizoguchi, Marlin & Hrymak (1995) proposed a first principle description of a CDU from Petro-Canada. Process optimization was done in terms of the advanced control framework, in which an upper RTO layer provides optimum controlled and manipulated variables targets for a lower QDMC control algorithm. In the simulated studies, the QDMC uses a linearized process model, whereas the real plant is represented through to the first-principle model. The results showed that control strategies should be carefully studied since some variables might drive the process to better operational points. Moreover, the authors concluded that it is imperative to ensure that the control system can take into account the process disturbances.

5.1.3.1. *Process description*

The system studied here is a process unit of the RECAP refinery of Petrobras, located in São Paulo, Brazil that processes about 50,000 bbl/day of light crude oil. Figure 5-14 shows a schematic representation of the unit, including the relevant process equipment, streams and regulatory control loops. The crude preheating system consists of one preheating train with two desalters operating in series. The pre-flash column N-507 separates light naphtha and light diesel from the crude. Its bottom product is preheated in two parallel trains of heat exchangers and is partially vaporized in the furnace L-506, before being fed to the atmospheric fractionator N-506, which produces: heavy naphtha, kerosene, heavy diesel and the atmospheric residue. Kerosene and heavy diesel are blended to produce the commercial diesel product. The bottom product is sent directly to a Residue Catalytic Cracking Unit.

5.1.3.2. *Control strategy, variables and economic goals*

The economic objectives of the CDU, in order of priority, are: (i) maximize the diesel production by minimizing the flow rates of heavy naphtha and atmospheric residue, and enforce that its specifications are satisfied (minimum bound on the flash point and a maximum bound on the ASTM D-86 distillation curve 95%); (ii) supply the required amount of light naphtha that feeds the solvent unit; (iii) minimize the flow rate of stripping steam injected into the system; (iv) minimize the consumption of fuel oil at the preheating furnace.

Based on the objectives defined above that are related to the profit optimization, the unit operating goals are listed as follows: 1) The crude oil flow rate (u_1) should be maximized in all the scenarios. 2) The flow rate of the stripping steam to the pre-flash column (u_3) should be driven to its target that is calculated by the RTO layer. 3) The light naphtha outlet stream flow rate (y_9) and the ASTM D-86 end point (y_7) are important controlled variables to ensure the appropriate quantity and quality of the light naphtha sent to the solvent production unit. 4) The pre-flash overhead temperature set-point (u_2) is a manipulated variable with direct influence on the light naphtha quality, but it is constrained by the top reflux flow rate (y_8). 5) Two properties of the diesel must be enforced as constraints, its flash point (y_6) and its ASTM D86 95% (y_5). The former is important in the minimization of the fractionator top temperature (u_4). This minimization is constrained by the minimum bound on the heavy naphtha flow rate (y_2) and by the maximum bound on the reflux flow rate (y_3). The ASTM D86 95% constraint tends to become active when the outlet temperature of the crude furnace (u_8) is maximized, which is also constrained by the maximum bound on the furnace heat duty (y_{10}). 6) The pumparound flow rate (u_6) should be maximized in order to save fuel in the furnace. It is constrained by the ratio between kerosene and heavy diesel that can be represented by the kerosene withdraw temperature (y_4). 7) The diesel pumpdown reflux flow rate (u_5) should be manipulated to improve the fractionation between the heavy diesel and the atmospheric residue.

The controlled and manipulated variables are illustrated in Figure 5-14, and are described in Table 5-15 and Table 5-16, respectively, along with their operational bounds and usual values. The output set points, or control zones are enforced directly by the MPC, while the input targets are calculated at the RTO layer. The reader can find the system transfer functions, obtained from operational data of the CDU industrial system, in Appendix A.

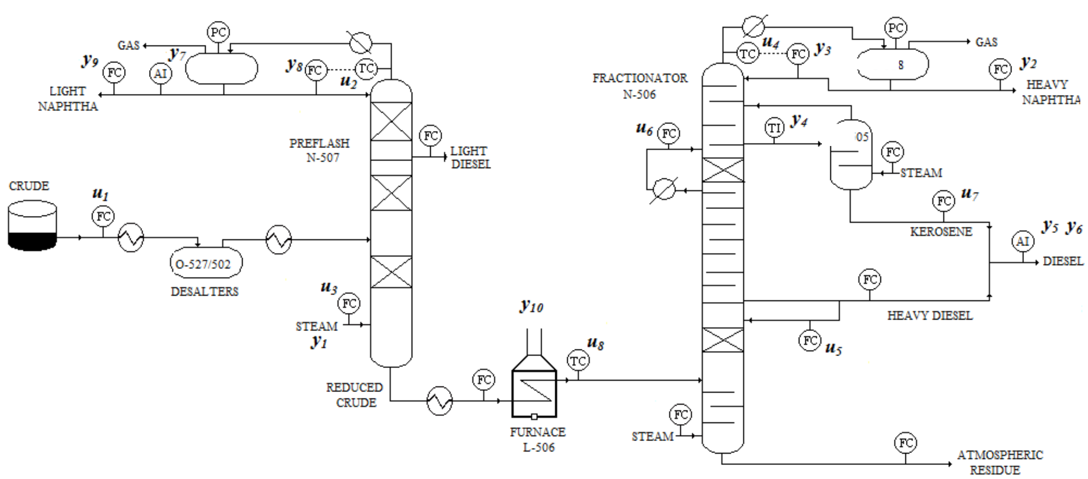


Figure 5-14: Schematic Representation of Crude Distillation Unit.

Table 5-15: CDU controlled outputs.

Tag	Variable name	Unit	Bounds		y_0
y_1	N-507 bottom stripping steam to reduced crude ratio	kg/m^3	1	4	1.03
y_2	Heavy naphtha flow rate	m^3/d	40	380	220
y_3	N-506 top reflux flow rate	m^3/d	800	1850	1670.41
y_4	Kerosene withdraw temperature	°C	180	190	184.38
y_5	Diesel ASTM D-86 95%	°C	368	370	368.6
y_6	Diesel flash point	°C	35	65	39.7
y_7	Light naphtha ASTM D-86 end point	°C	160	180	173.68
y_8	N-507 top reflux flow rate	m^3/d	600	1290	1050
y_9	Light naphtha flow rate	m^3/d	600	1342	911.62
y_{10}	L-506 heat duty	Gcal/h	15	21	17.7

Table 5-16: CDU manipulated inputs.

Tag	Variable name	Unit	Bounds		u_0	$\Delta u_{\max}$	u_{des}
u_1	Crude feed flow rate	m^3/d	8000	8500	8000	20	8100
u_2	N-507 top temperature set-point	°C	128	131	129.95	0.1	129.9
u_3	N-507 bottom stripping steam flow rate	t/h	0.3	1	0.3	0.03	0.5
u_4	N-506 top temperature set-point	°C	105	110	108.54	0.1	109
u_5	Heavy diesel pumpdown reflux flow rate	m^3/d	1000	1670	1380	10	1365
u_6	Diesel pumparound reflux flow rate	m^3/d	5800	6400	6400	8	6400
u_7	Kerosene outlet flow rate	m^3/d	900	1050	1050	6	1049.5
u_8	L-506 outlet temperature	°C	365	372	368.98	0.07	368.5

5.1.3.3. *Definition of the tuning goals*

In the previous section, it was listed all the important inputs that should be driven to their targets and the outputs that should be kept inside their control zones. The MPC tuning analysis presented in this work will consider two different scenarios. The first one will focus only at the output tracking performance of the controller. This means that the weighting matrices Q_y, and R are selected solely based on the performance of the output responses. The second scenario will consider output and input tracking simultaneously, and tuning will focus on Q_y, Q_u, and R.

The compromise method proposed here is compared with a MPC tuning technique from the literature that is also based on multi-objective optimization. Hereafter this technique is addressed as the Normal Boundary Intersection (NBI) approach. It is based on a *posteriori* choice of the optimum solution from a set of non-dominated solutions. The set of solutions is obtained through a grid search over an evenly spaced parameterized unitary segment (s_i) for each objective. The searching points lie on the (quasi-) normal direction $(-\lambda\Phi e)$ to a plane defined by the individual optimum solutions, indicated by Φs. The following optimization problem is solved:

Problem 3

$$\min_{x\in X,\lambda} -\lambda \tag{5-10}$$

Subject to

$$\Phi s - \lambda\Phi 1_{ny,1} = F(x) - F^{\circ} \tag{5-11}$$

$s_i \geq 0$, $\sum_{i=1}^{\sigma} s_i = 1$. The *i-th* column of the pay-off matrix Φ is defined as $F(x_i^{\circ}) - F^{\circ}$

The reader is referred to (Das & Dennis, 1998; Vallerio, van Impe & Logist, 2014) and the references therein for more information. The NBI method was implemented using an increment of 0.5 to define the vector s. The decision regarding the best Pareto solution was done following the weighted sum method (Pohekar & Ramachandran, 2004), in which a performance index named WSM is calculated according to (5-12). The non-dominated solution that yields the largest index value is selected as the best solution.

$$WSM = \sum_{j=1}^{w} a_{ij} f_j,\ i = 1,\ldots,M \tag{5-12}$$

where w is the number of objectives, M is the number of non-dominated solutions, f_j is the *j-th* objective weight and a_{ij} is the value of the cost function of the *j-th* objective calculated using the *i-th* Pareto solution.

The objective weights of the output tuning approach are the inverse of the response factors, i.e. $f_{output} = \frac{1}{f_{res}}$, and the input tuning approach weighting vector was chosen arbitrarily as $f_{input} = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{bmatrix}$.

The two tuning approaches considered here consider the following lower and upper bounds for the tuning parameters, which correspond to Q_y, Q_u, and R: $LB = \begin{bmatrix} 1_{ny\times1} & 1_{nu\times1}\times10^{-3} & 1_{nu\times1}\times10^{-2} \end{bmatrix}$, $UB = \begin{bmatrix} 1_{ny\times1}\times20 & 1_{nu\times1}\times0.1 & 1_{nu\times1}\times10^{2} \end{bmatrix}$; and the tuning horizon is $\theta_t = 300$ min.

In all the cases considered here, the transfer function gains, inputs and outputs are normalized considering the input and output operating ranges. The control horizon is chosen as m=5. These parameters were selected following the literature guidelines and also based on the analysis of the simulation results. The MPC control problem was solved analytically, disregarding the constraints. The CTT multi-objective optimization problem (Problem 2) and the NBI problems (Problem 3) ware solved using *fmincon* (default settings), MATLAB® 2013, on an Intel ® Core i5, 3.20Ghz, 4Gb RAM computer. The MPC with input targets and output zone control and its analytical solution are introduced in Appendix B.

5.1.3.4. *Tuning the MPC for the output set point tracking scenario*

The selection of the input-output pairs for the definition of the reference trajectories is performed based on the input-output relationship matrix represented in Table 5-17 and the RGA approach (Bristol, 1966). Since the closed loop transfer functions relate the outputs with their set points, all the gains are unitary, and the time constants are obtained by multiplying the time constants of the approximate transfer functions by the response factors defined below:

$f_{res} = \left[0.25 \quad 0.30 \quad 0.45 \quad 0.50 \quad 0.55 \quad 0.70 \quad 0.85 \quad 0.90 \quad 1.0 \quad 1.0\right]$

Table 5-18 summarizes the transfer functions that define the reference trajectory for each output. In the tuning scenario corresponding to the output tracking, matrix Q_u is not included as a decision variable of the tuning problem. Therefore, the objective function of the MPC considered here does not include the term corresponding to the input target. Also, y_{sp} is considered as a pre-defined parameter, instead of a decision variable of the MPC problem, because the main focus of this tuning step is to obtain a set of parameters such that the MPC will be capable of adequately driving the outputs back to their zones when a disturbance affects the CDU system. To compute the values of function (3-1) that characterizes the performance of the controller, the system starts from a steady state that corresponds to 50% of the nominal output values and corresponding input values. The outputs are all inside of their control zones and the output set points are moved to a value equal to 75% of their nominal value at time instant 1 min. At time instant 100 min, they are moved to values equal to 50% of the nominal values and finally, at time 200 min, the set points are fixed at 25 % of the nominal values.

The tuning parameters resulting from the application of the proposed Compromise method to this first scenario are the following:

$Q_y = diag\left[1 \quad 2.07 \quad 1 \quad 1 \quad 1 \quad 2.93 \quad 8.3 \quad 20 \quad 10.2 \quad 1.34\right]$,

$R = diag\left[0.65 \quad 0.011 \quad 0.01 \quad 0.01 \quad 0.01 \quad 0.01 \quad 0.01 \quad 0.01\right]$

and the tuning parameters obtained with the application of the NBI method are the following:

$Q_y = diag\left[1 \quad 20 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 20 \quad 1.54 \quad 4.41\right]$,

$R = diag\left[0.01 \quad 3.49 \quad 0.11 \quad 0.01 \quad 0.012 \quad 0.01 \quad 0.015 \quad 0.235\right]$. The parameters originally implemented in the MPC of the real CDU system are $Q_y = diag\left[5 \quad 2 \quad 1 \quad 3 \quad 10 \quad 5 \quad 5 \quad 1 \quad 10 \quad 20\right]$, $Q_u = diag\left[1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1\right]$, $R = diag\left[0.1 \quad 6 \quad 2.93 \quad 5 \quad 0.15 \quad 4 \quad 1 \quad 20\right]$.

The performance of the controller with this tuning set, addressed here as ‘Existing’, is also compared to the other tuning sets obtained here.

Table 5-17: CDU input-output relationship matrix.

	u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8
y_1			x					
y_2				x			x	
y_3		x		x		x	x	x
y_4				x		x	x	x
y_5				x	x		x	x
y_6				x		x	x	
y_7		x	x					
y_8	x	x	x					
y_9	x	x	x					
y_{10}	x	x	x					x

Table 5-18: Reference trajectories transfer function time constants.

Output	Input	Time constant
y_1	u_3	1.1080
y_2	u_4	12.6710
y_3	u_6	1.0684
y_4	u_7	10.6011
y_5	u_5	8.3741
y_6	u_4	2.7598
y_7	u_2	8.1778
y_8	u_3	7.3582
y_9	u_1	32.0422
y_{10}	u_8	3.8556

It is observed that there are significant differences between some of the tuning parameters produced by the two methods. We also observe that the existing tuning parameters of the CDU controller are quite different from the parameters obtained here.

The performances of these three sets of parameters are compared in a simulated scenario in which five outputs are driven to different set points, following the pattern proposed in the tuning section, while the remaining outputs are assumed to remain within their control zones. Figure 5-15 shows the closed-loop responses of the selected outputs. It is also shown the responses of the controller with the parameters corresponding to the utopia solution.

It is observed from Figure 5-15 that the responses of the MPC with the tuning parameters defined through the compromise method are closer to the Utopia responses than the MPC tuned through the NBI method. We also observe that the existing tuning parameters are quite conservative and lead to more sluggish responses than the tuning parameters obtained here. The controllers resulting from the two tuning techniques analyzed here successfully tracked all the output set points, but the existing controller failed to track the set point moves of y_3 within the simulation horizon considered here.

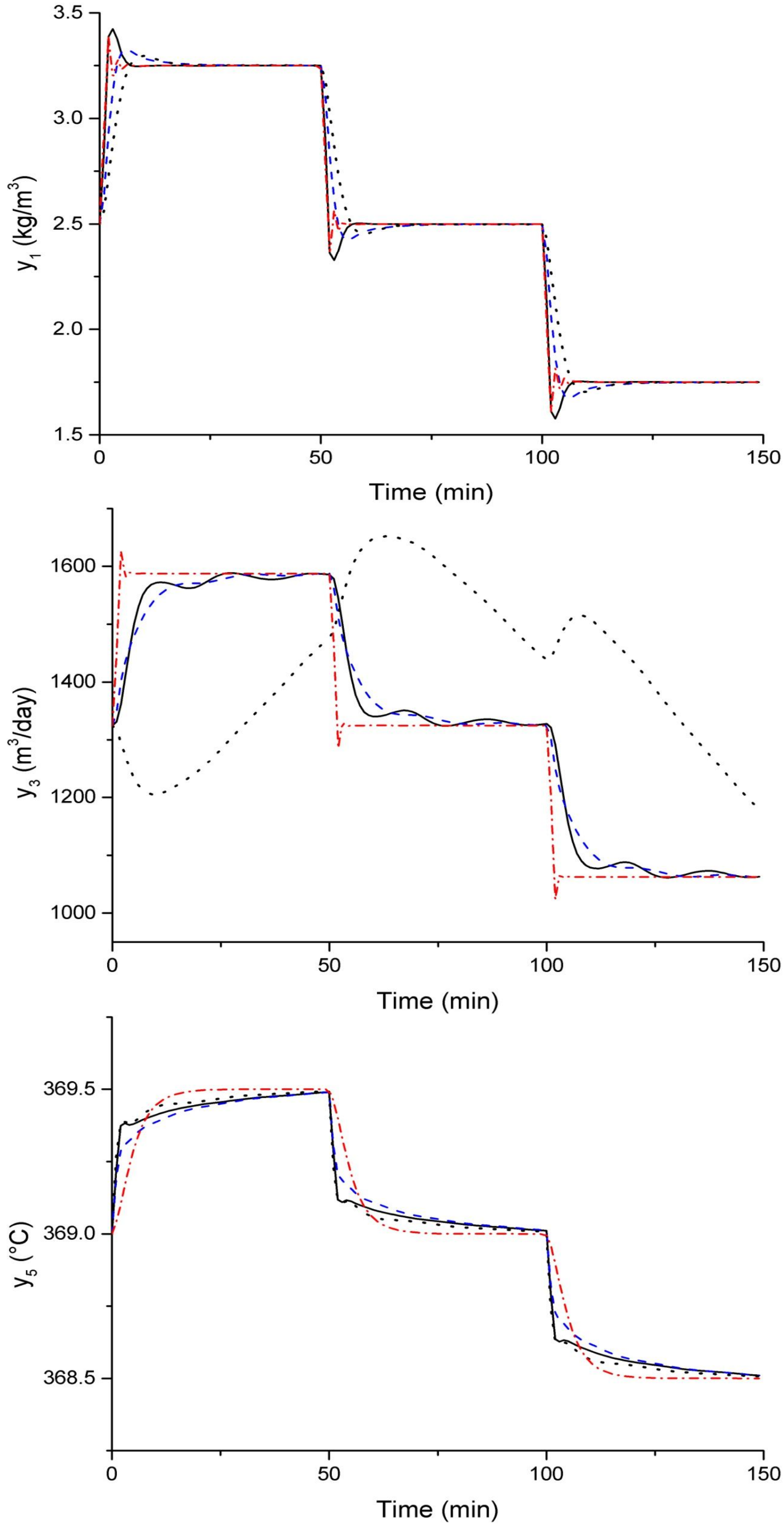


Figure 5-15a: Output tracking tuning analysis, CTT (——), NBI (– –), existing controller (···), Utopia (–·–)

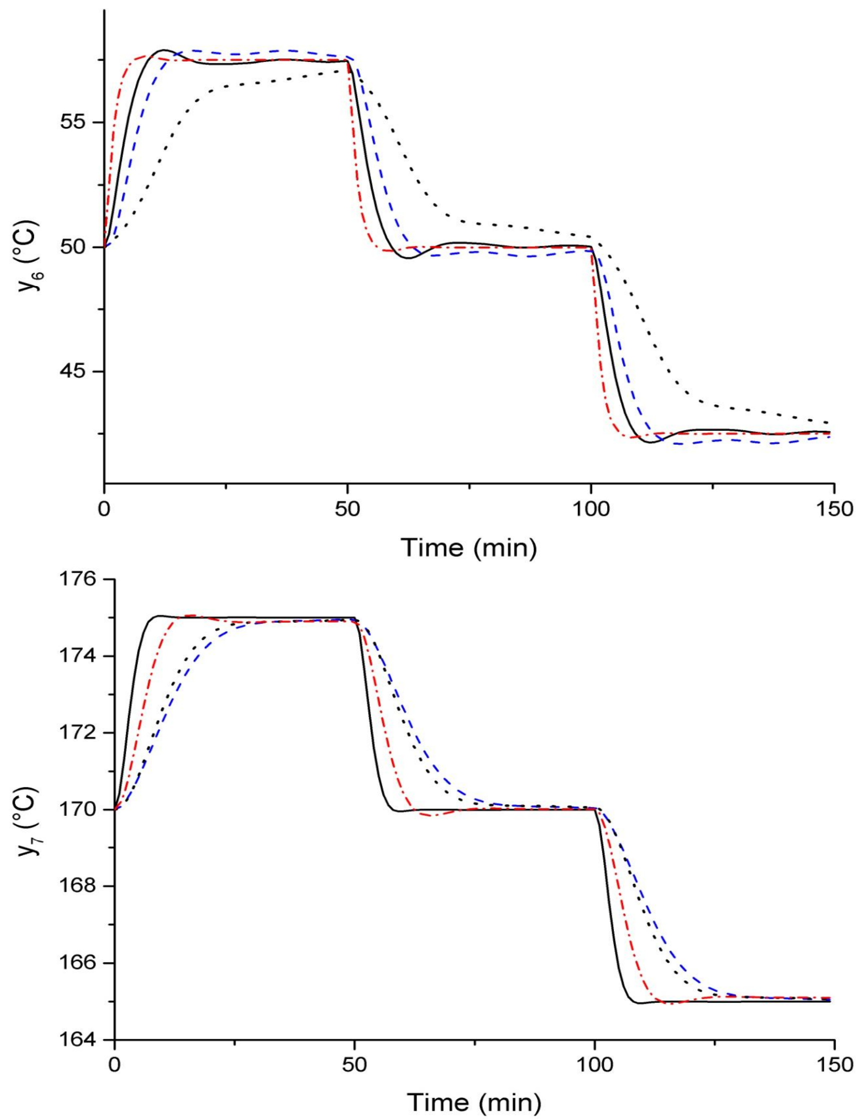


Figure 5-15b: Output tracking tuning analysis, CTT (——), NBI (– –), existing controller (···), Utopia (– · –)

5.1.3.5. *Tuning for the input target tracking scenario*

As seen in Table 5-18, each input of the CDU system is related to several outputs through non-zero transfer functions. This multivariable character may result in a conflict between the input targets and the output zones. This means that some of the outputs may tend to be driven to outside their zones when the MPC forces the inputs toward their targets. The tuning parameters of the MPC should be selected such that this conflict is minimized in the practical case. This means that if the input targets are unreachable, the priority should be to keep the outputs inside the control zones and allow for offset in the inputs. The following paragraphs present some specific scenarios where an input is driven to a target while one or more associated outputs tend to be driven to the border of their control zones, becoming active constraints. The reference trajectories for the inputs are defined as linear functions with angular coefficient corresponding to $\Delta u_{\max}$. The output constraints over the input optimization are included for the outputs strongly correlated with the input. Observe that, here the definition of the input tuning objective is more complex, and it is heavily dependent on a practical knowledge of the process system and its operating peculiarities. It is assumed that the output initial value lies in the middle of its operating range and at time instant 1 min, their set points are changed to 75% of their nominal values. The inputs initial conditions and targets, in the units shown in Table 5-15 and Table 5-16 are $u_0 = \begin{bmatrix} 8000 & 128 & 1 & 110 & 1271.5 & 6400 & 1050 & 372 \end{bmatrix}^T$

and $u_{des} = \begin{bmatrix} 8125 & 128.75 & 0.825 & 108.75 & 1439.1 & 6250 & 1012.5 & 370.25 \end{bmatrix}^T$.

Each of the cases described below considers one input with its target. The outputs more heavily connected to this input and that can reach a constraint are also considered one at a time, even though, in practice, the input may be constrained by more than one output simultaneously. In the definition of the tuning goal related with each case, it is assumed that the output should be driven to a set point, while the input should follow a trajectory towards its target.

Objective I – Flow rate of crude to the CDU

The target to the flow rate of crude oil (u_1) is defined by the RTO layer that tries to force the maximization of the flow rate of diesel that is produced by the CDU. The outputs that may constrain the input optimization are the light naphtha flow rate (y_9)

and the furnace heat duty (y_{10}). Only the latter is considered in the definition of this tuning objective.

Objective II – Flow rate of diesel pumparound

The diesel pumparound flow rate (u_6) is set by the RTO to maximize the heat that is recovered in the crude oil preheating trains and to save fuel oil/gas in the heating furnace. This input is mainly constrained by the top reflux flow rate of the main fractionator (y_3).

Objective III – Flow rate of stripping steam

The RTO layer computes an optimum target to the flow rate of the stripping to the bottom of the pre-flash column (u_3) in order to maximize the flow rate of light naphtha that is produced by the CDU. The related output constraints are the pre-flash reflux flow rate (y_8) and the ratio between the flow rate of the stripping steam and the flow rate of reduced crude (y_1). The former is chosen in the definition of this objective.

Objective IV – Temperature at the top of the pre-flash column

The set point to the controller of the temperature at the top of the pre-flash column (u_2) is set by the RTO to maximize the production of light naphtha. The constraints associated with this target are the bounds of the light naphtha flow rate (y_9) and the maximum D-86 end point of the light naphtha (y_7). Here, the objective is formulated in terms of y_9 only.

Objective V – Temperature at the top of the atmospheric column

The RTO sets a target to the set point of the temperature controller at the top of the fractionator (u_4) in order to maximize the production of heavy naphtha. The constraints associated with this input are the bounds on the heavy naphtha flow rate (y_2) and the bounds on the diesel flash point (y_6). The objective pair is selected in terms of y_2

Objective VI – Flow rate of heavy diesel pumpdown reflux

The target to the flow rate of the heavy diesel pumpdown reflux in the fractionator (u_5) is set by the RTO to maximize the production of diesel. This target may be constrained by the diesel ASTM D-86 95% (y_5).

Objective VII – Kerosene outlet flow rate

The kerosene outlet flow rate (u_7) is also set by the RTO when the diesel production is maximized. This target may be constrained by the temperature of the tray where the kerosene is drawn from the atmospheric column (y_4).

Objective VIII – Furnace temperature

The temperature at the outlet of the crude heating furnace (u_8) is also set by the RTO. This target may be constrained by the diesel ASTM D-86 95% specification (y_5).

Two different approaches are proposed: in Scenario II-A, the optimum values of Q_y and R obtained in Section 5.1.3.4 are inherited and used as fixed parameters instead of decision variables. In Scenario II-B, the elements of matrices Q_y, Q_u and R are considered as decision variables of the tuning problem.

The input tracking analysis compares the closed-loop responses with the controller tuned through the Compromise method using approaches II-A and II-B to the controller with the existing parameters. The following pairs were chosen to illustrate the closed-loop performance of the MPC for the input tracking scenario: y_3-u_6, y_8-u_3, and y_{10}-u_1, which represent the objectives II, III, and I, respectively. The remaining outputs are assumed to be kept within their control zones and other inputs do not have active targets, where the input references are also included. Table 5-19 shows the optimum tuning parameters From Figure 5-16, we observe that the output responses of the controllers tuned with the two scenarios are fast and not too different from each other, but the CTT II-A tends to produce an oscillatory behavior, which is not desirable in practice. The performance of the MPC based on the CTT II-B method is slightly more sluggish but not oscillatory. Regarding the input reference trajectories, none of the proposed tuning approaches yielded an outstanding tracking. This shows that the compromise approaches prioritize the control of the system outputs while the optimizing input targets will be tracked only after the outputs have been driven to their set points.

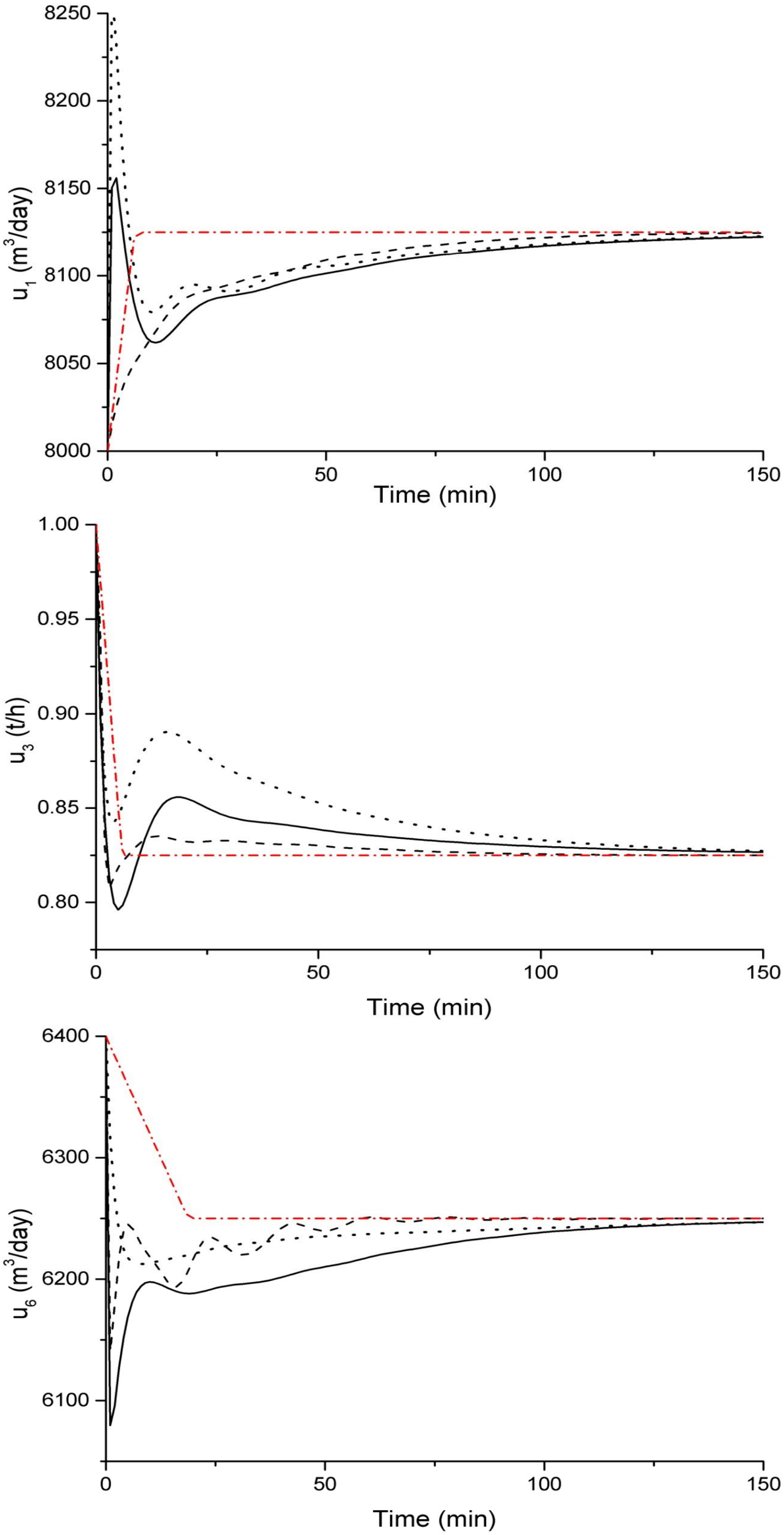


Figure 5-16a: Input tracking tuning analysis, CTT II-A (– –), CTT II-B (——), existing controller (···), reference trajectory (–·–).

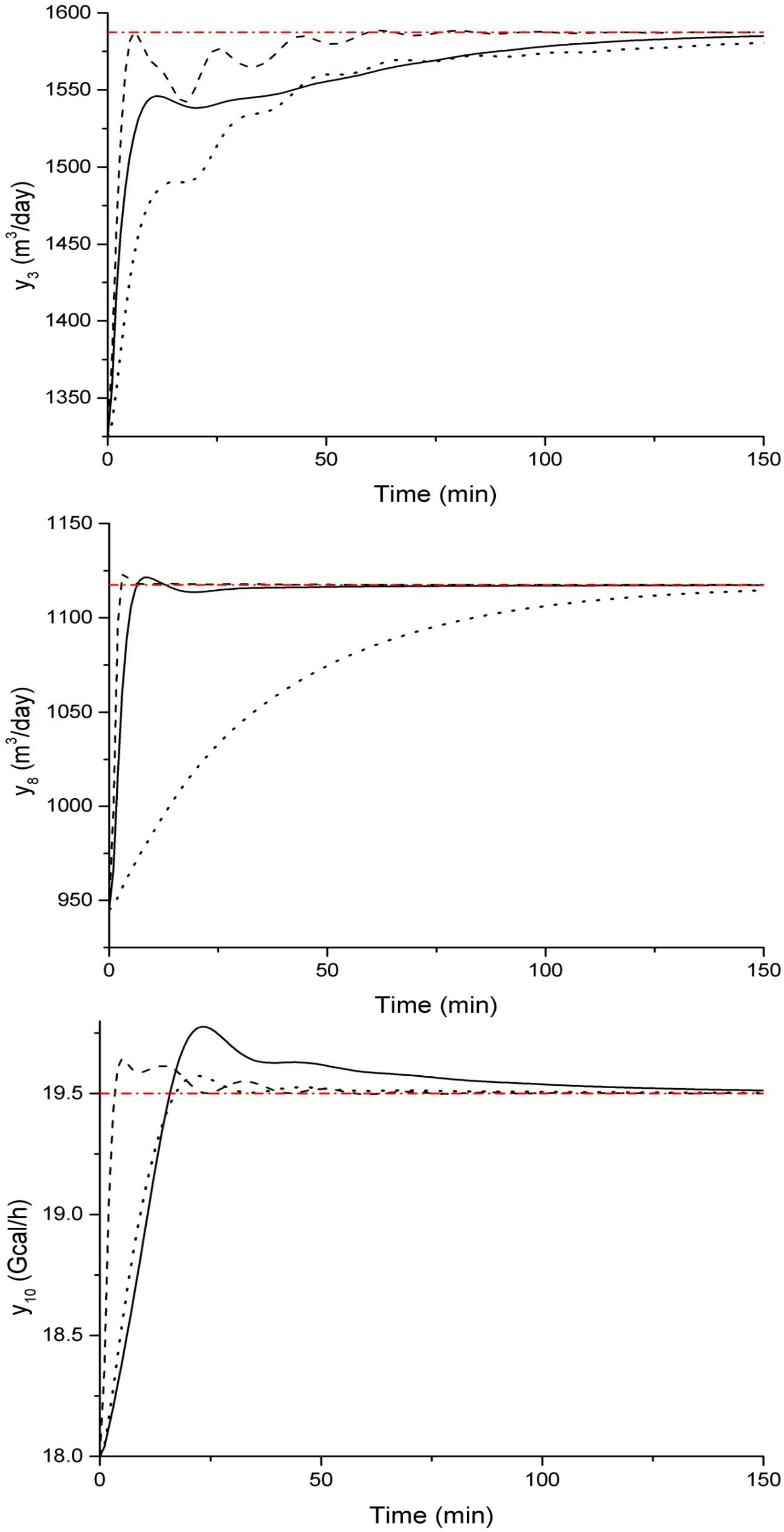


Figure 5-16b: Input tracking tuning analysis, CTT II-A (– –), CTT II-B (——), existing controller (···), reference trajectory (–·–).

Comparing the values of the tuning parameters obtained with the compromise technique and the existing tuning parameters that have been used in the real plant, in the existing controller all the values of Q_u are set equal to 1, which is quite large compared to the values of Q_u obtained here. The effect that is observed in Figure 5-16 is that with the existing controller the process response is slower than in the controller tuned with the compromise method. The result will be a slower output response if the input target is changed because the RTO has computed a new optimum operating point. The consequence is that we may lose the product specification for a significant period of time and the expected economic benefit will be jeopardized.

5.1.3.6. *Additional remarks and simulations*

Regarding the NBI method, when it is implemented in the output tuning case, there are 10 objectives (one objective per output), which require a total of 55 Pareto solutions. In the input tuning case, the NBI method requires 8 additional objectives (one per input), which lead to 36 Pareto solutions. Therefore, the number of solutions to be considered in the NBI method is much larger than the number of solutions that need to apply the compromise method. In the NBI method, the increment interval of parameter s affects the distribution and spread of the Pareto solutions over the Pareto curve. The adopted value of s was large to implement the method; a smaller value would require too many solutions.

Table 5-19 shows the optimum tuning parameters obtained with the CTT and NBI methods, whereas Table 5-20 summarizes the computational times and the resulting minimum values of the tuning cost function for each method. It is clear that the proposed method (CTT) is much faster than the existing method of the MPC tuning literature (NBI). In the input tracking scenario, the adopted upper bound on Q_u is 10 times lower than the lower bound of Q_y. With these constraints, we expect that the input targets will be tracked only if the related outputs are lying in their zones.

To wrap up the performance assessment of the tuning technique developed here, we include a simulation, considering the constrained MPC with input targets and zone control, tuned according to the techniques CTT II-B and NBI II-B, which yielded the best results in terms of the tuning cost function values shown in Table 5-20 and the existing tuning parameters. The system initial condition, output zone values and input targets are given Table 5-15 and Table 5-16 and correspond to a typical operating condition. It is assumed that the system starts from a condition where the input target and all the computed output set points lie in their operating ranges and, consequently, the inputs are not constrained by the output zones. In this case, the MPC can drive the system inputs to their targets without offset. This scenario can be observed in Figure 5-17 and Figure 5-18, from 0 min until 1000 min. In this case, all the inputs reach their targets and the outputs stabilize in the control zones.

Table 5-19: Tuning results for Scenario II-A and II-B.

CTT				NBI			
II-A		II-B		II-A		II-B	
Q_u	Q_y	Q_u	R	Q_u	Q_y	Q_u	R
0.1	1	0.1	0.035	0.001	1	0.099	0.082
0.1	2.359	0.1	0.03	0.1	2.067	0.001	0.01
0.1	1	0.1	0.142	0.001	1.746	0.001	0.01
0.1	1	0.1	0.044	0.1	1.6	0.1	1.497
0.001	1	0.001	0.01	0.1	1.006	0.1	0.202
0.1	1	0.1	0.01	0.1	3.857	0.001	0.075
0.1	1.924	0.1	9.435	0.1	1	0.018	0.028
0.061	3.012	0.018	5.048	0.1	1	0.076	2.499
	1				1		
	1				4.616		

Table 5-20: Tuning strategies comparison.

Method	Elapsed time (h)	Total cost fuction value
Scenario I		
CTT	2.54	9.34
NBI	68.37	12.52
Utopia		1.03
Scenario II-A		
CTT	0.69	237
NBI	24.5	385.7
Utopia		1.4
Scenario II-B		
CTT	7.34	180.5
NBI	34.5	241.76
Utopia		2.91

At time instant 1000 min, the target of input u_5 is changed from $u_{des,5} = 1365\,m^3/d$ to $u_{des,5} = 1250\,m^3/d$, which is unreachable because it corresponds to a steady state where y_5 would lie above its maximum bound. From Figure 5-17, we observe that the output responses of the closed loop system with MPC tuned with the proposed method are similar to the responses of MPC with the existing parameters, however the responses obtained by the set of parameters obtained by the NBI were strange, with large excursions. The three controllers try to drive u_5 to its new target, which cannot be reached because y_5 reaches its maximum bound. We observe that all the controllers with the different sets of tuning parameters behave adequately in the sense that the maximum bound on y_5 is not surpassed but inputs u_4, u_5, u_7 and u_8 that have non-zero transfer functions relating them to y_5, tend to show offsets with respect to their targets. Figure 5-18 shows that the three controllers stabilize the system at a new steady state where the offsets in the inputs are distributed differently by each controller. We also observe that the responses of inputs u_6 and u_7 show some large excursions for the controller tuned with the NBI method.

At time instant 2000 min, the target of input u_1 is changed from $u_{des,1} = 8100\,m^3/d$ to $u_{des,1} = 8470\,m^3/d$. With this new target, the controller tends to be constrained by the upper bound on output y_{10}. Again the three controllers try to drive the CDU system to an optimum point that is unreachable, and the offset is distributed between inputs u_1, u_2, u_3 and u_8, which are the inputs that are related to y_{10}. The MPC tuned with the method proposed here behaves similarly to the existing MPC tuned with ad-hoc procedures based on trial and error. However, the former shows large excursions upon the second input target change for u_6 and u_8, which as attributed to the low value of the upper bounds on Q_u. Again, the MPC tuned with the NBI method shows unjustified large excursions of inputs u_2 and u_3.

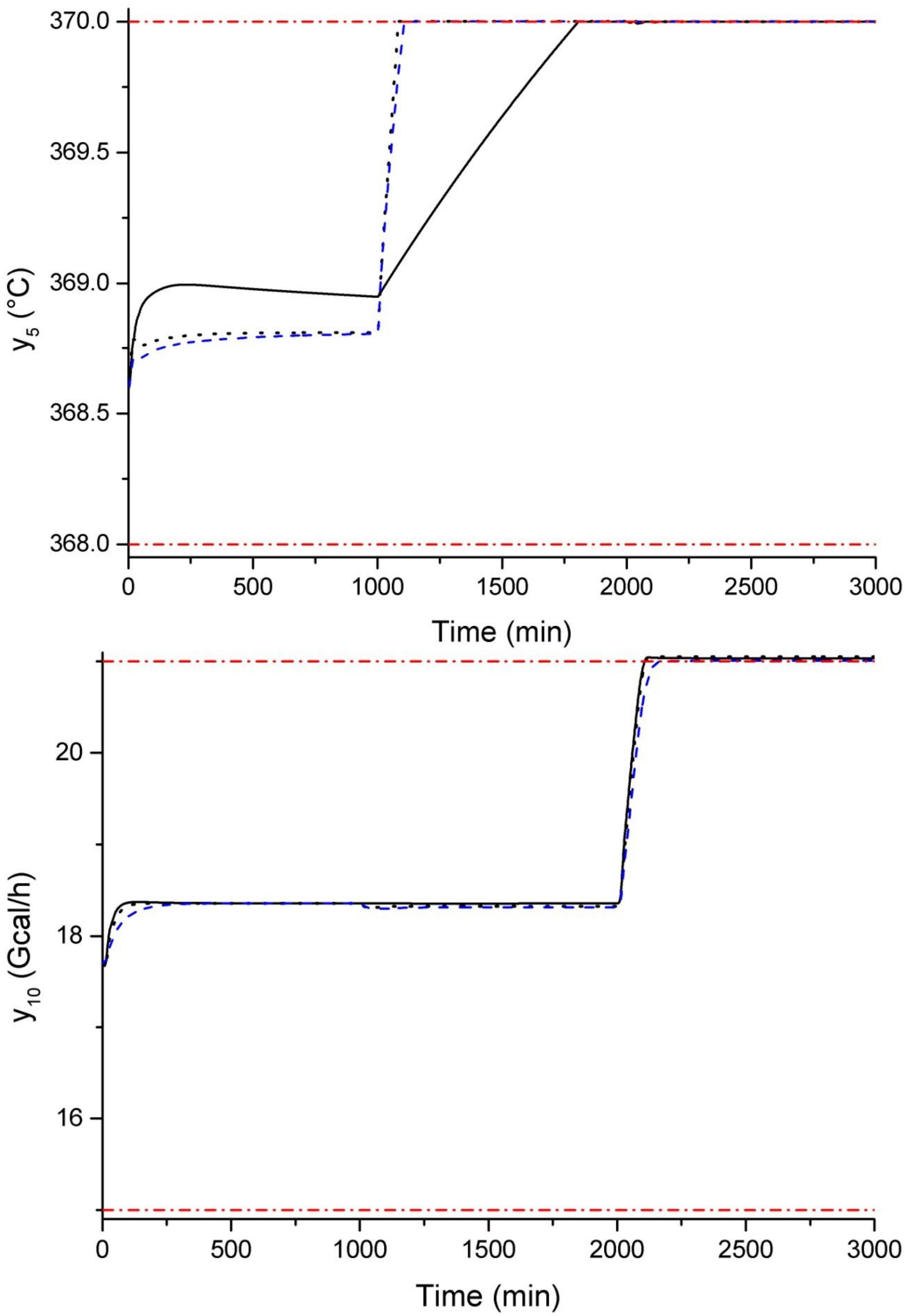


Figure 5-17: Outputs of the CDU in closed loop simulation with MPC tuned with CTT II-B (——), NBI II-B (– –), existing controller (···) and bounds (–·–).

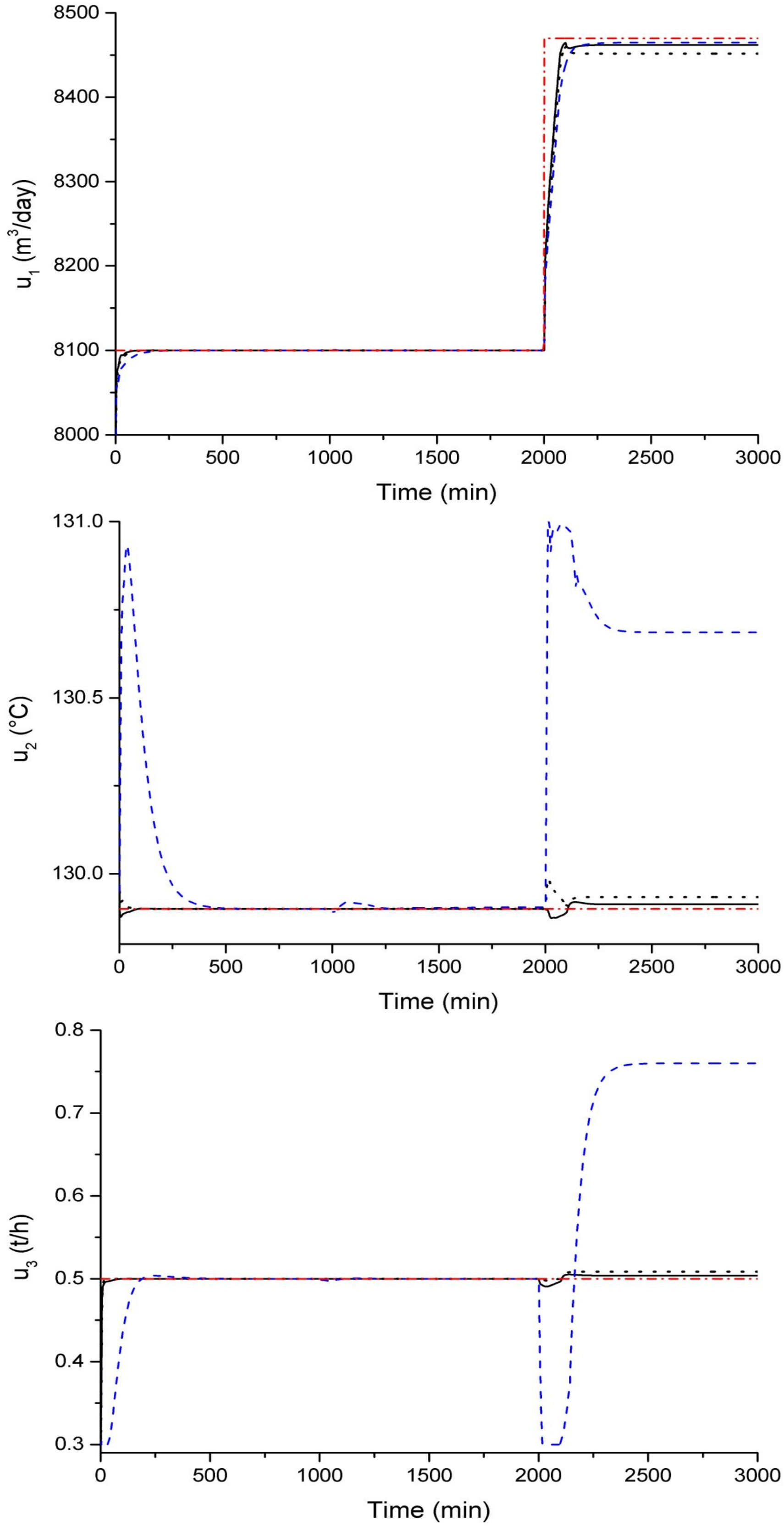


Figure 5-18a: Inputs of the CDU in closed loop simulation with MPC tuned with CTT II-B (———), NBI II-B (– –), existing controller (···) and targets (–·–).

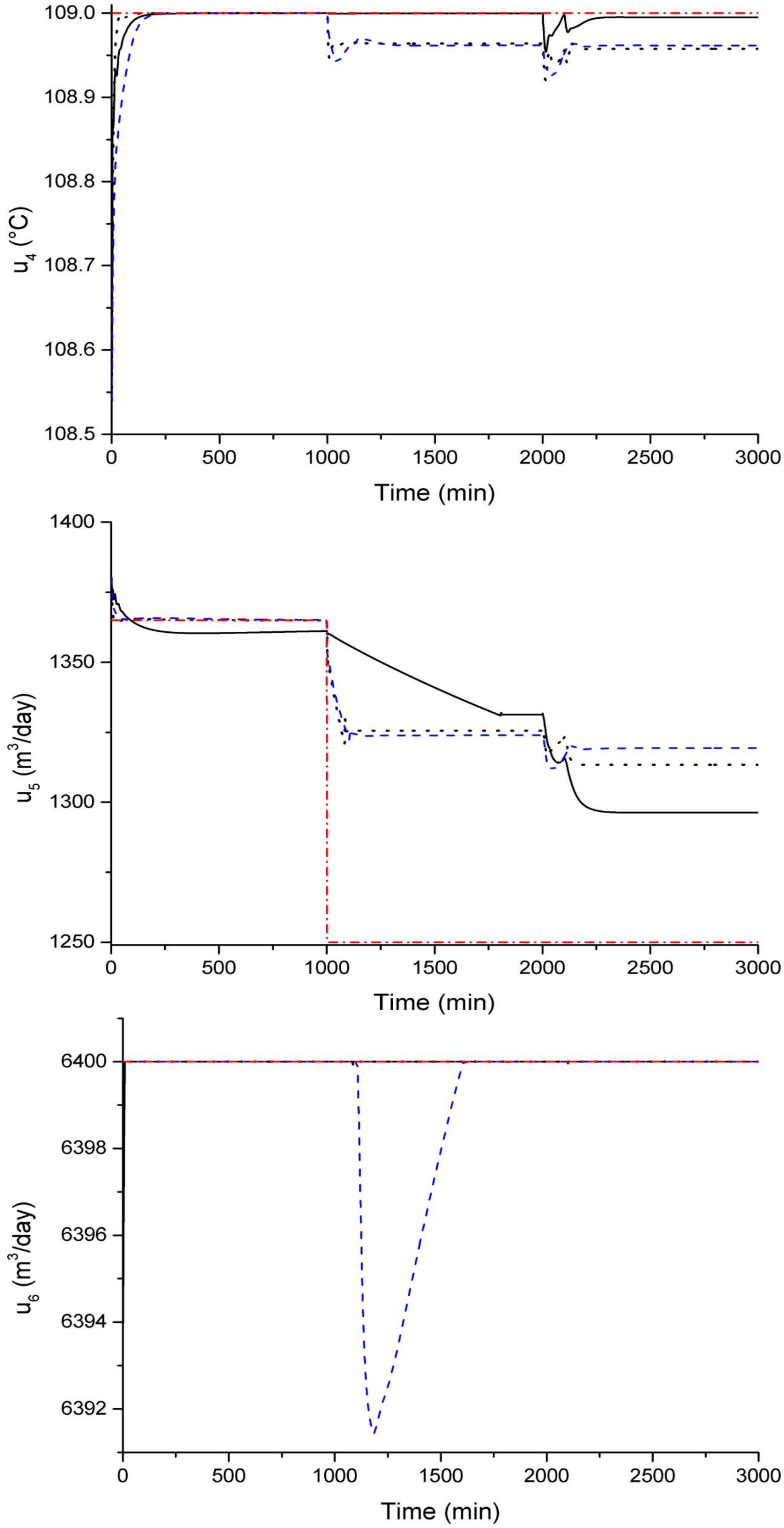


Figure 5-18b: Inputs of the CDU in closed loop simulation with MPC tuned with CTT II-B (——), NBI II-B (– –), existing controller (···) and targets (–·–).

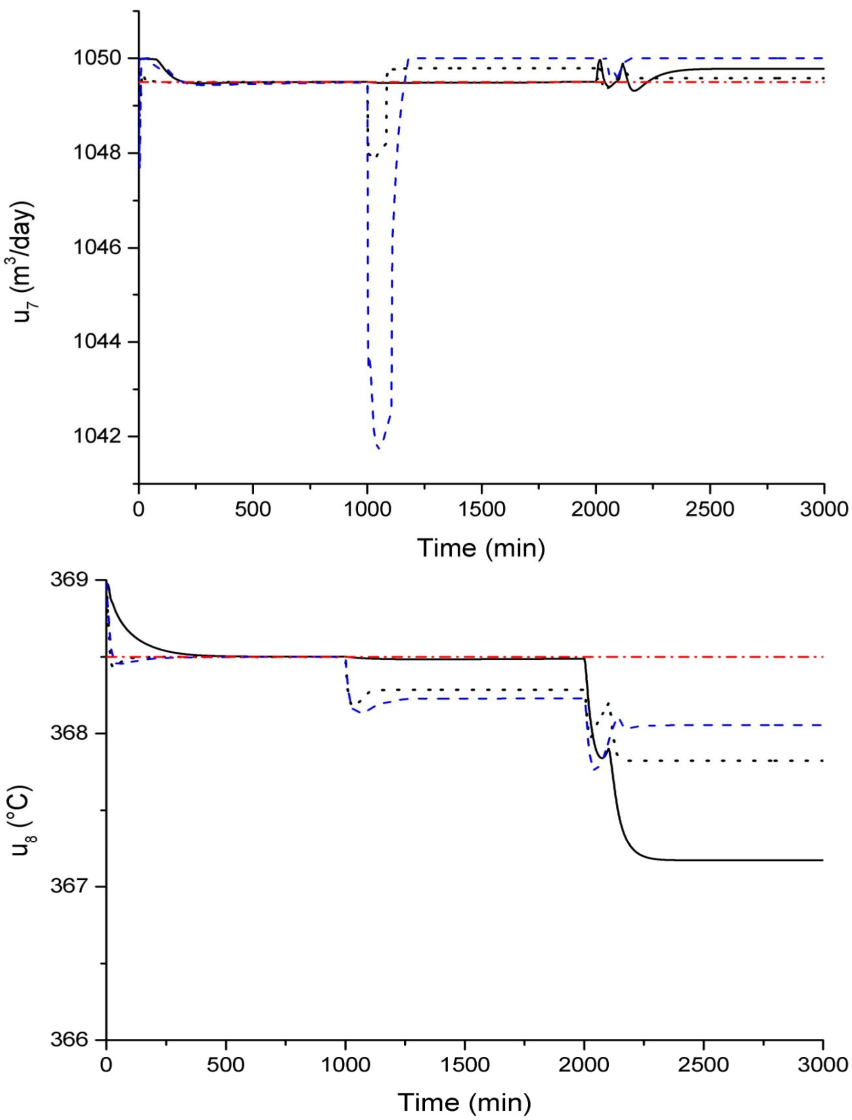


Figure 5-18c: Inputs of the CDU in closed loop simulation with MPC tuned with CTT II-B (———), NBI II-B (– –), existing controller (···) and targets (–·–).

5.2. Tackling the model uncertainty

The case study presented here is an application of the extended robust tuning techniques developed here to an infinite horizon MPC in closed-loop with a C3/C4 splitter system. The formulation of the IHMPC and RIHMPC used in this case study are presented in Appendix B.

5.2.1. Nominal IHMPC performance under plant-model mismatch

Porfírio, Neto & Odloak, (2003) identified 6 different transfer function models from the operational data of a C3/C4 splitter, as seen in Appendix A. Figure 5-19 shows the system schematic representation and Table 5-21 lists the tags, names, ranges and engineering units of the inputs and outputs.

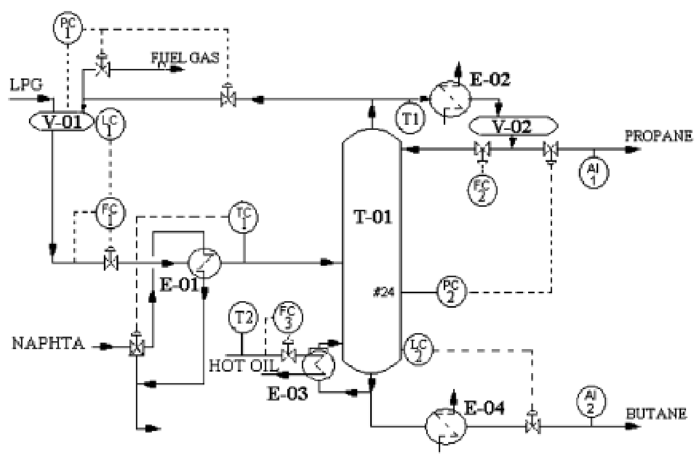


Figure 5-19: Schematic view of the C3/C4 splitter system, (Porfírio, Neto & Odloak, 2003).

Table 5-21: List of variables of the C3/C4 splitter system.

Tag	Variable	Range	Unit
y_1	C3 % in the butane stream	0.80 - 1.20	%
y_2	Top temperature	43 – 54	°C
u_1	Reflux flowrate	2000 - 4100	m^3/d
u_2	Hot fluid flowrate	1200 - 2200	m^3/d

Table 5-22: Set point changes of the C3/C4 Simulation I.

Time instant (min)	$y_{1,sp}$ (%)	$y_{2,sp}$ (°C)
0	0.99	48
100	0.85	49
300	0.9	51
500	0.95	50

The simulation shown in Figure 5-20, addressed as Simulation I, illustrates the plant-model mismatch effects. The initial state of inputs and outputs is $y_0 = \begin{bmatrix} 1.1 & 50 \end{bmatrix}^T$ and $u_0 = \begin{bmatrix} 2200 & 1300 \end{bmatrix}^T$. The upper and lower bounds on the inputs and outputs were taken from Table 5-21, and $\Delta u_{\max} = \begin{bmatrix} 50 & 25 \end{bmatrix}^T$. The simulation ran for 550 minutes and the set point changes are listed in Table 5-22, the system was in closed-loop with an IHMPC, with the following tuning parameters: T_S=1 min, m=5, $S_y = diag\left(\begin{bmatrix} 10^5 & 10^5 \end{bmatrix}\right)$, $S_i = diag\left(\begin{bmatrix} 10^5 & 10^5 \end{bmatrix}\right)$, $Q_y = diag\left(\begin{bmatrix} 10 & 0.5 \end{bmatrix}\right)$ and $R = diag\left(\begin{bmatrix} 0.71 & 0.06 \end{bmatrix}\right)$. Matrices Q_y and R were obtained using the nominal lexicographic approach, whose procedure was omitted for brevity. Considering that the IHMPC is based on the real plant model that is assumed to be model 6. A Kalman filter was used to estimate the system states, based on the difference between current and predicted output values; the process and measurement noise covariance matrices were set equal to unit matrices of appropriate dimensions. The 'real' plant changes from the nominal model (model 6) to model 1 at 100 minutes. Observe that the nominally tuned IHMPC is unable to track the set point changes and yields oscillatory responses.

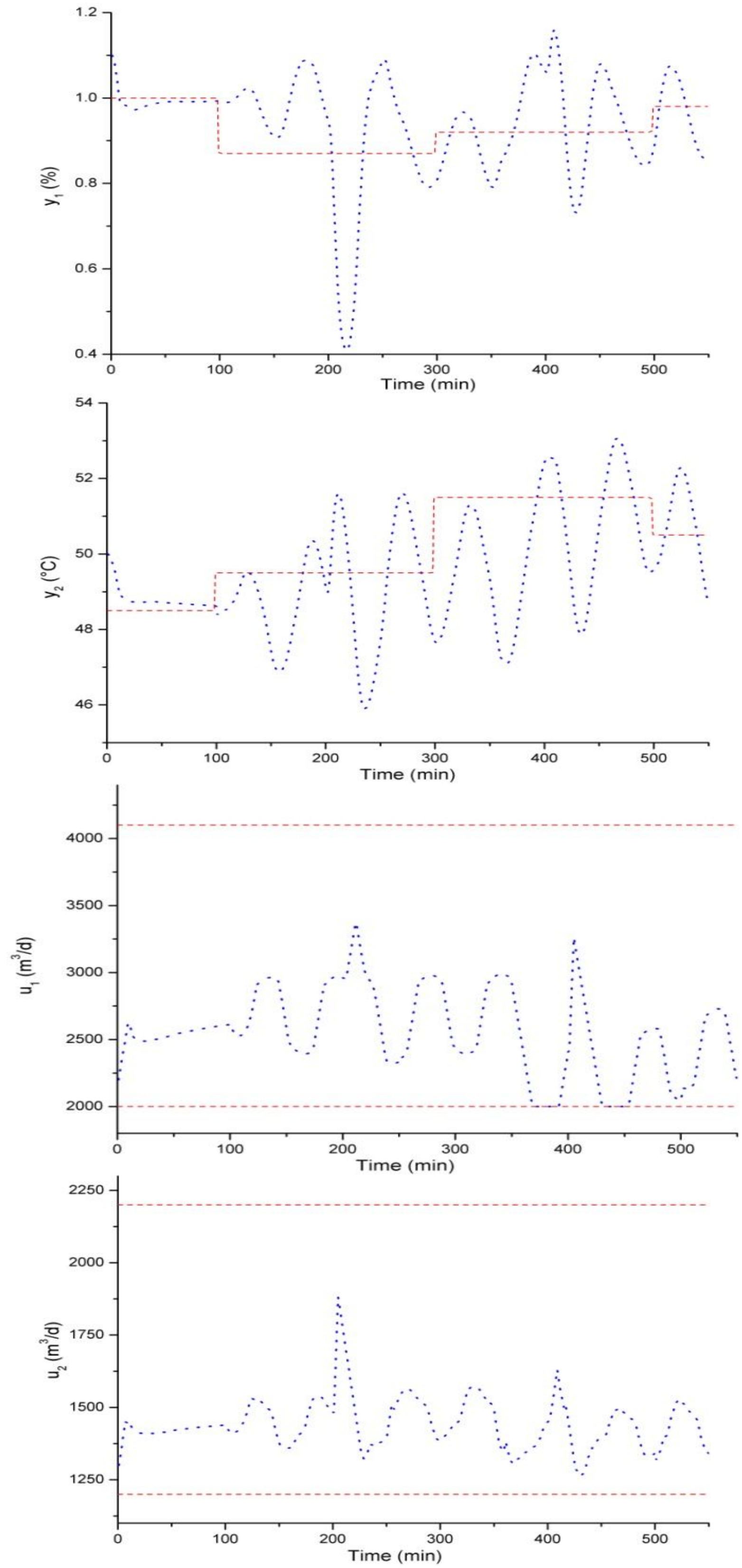


Figure 5-20: C3/C4 splitter model-plant mismatch Simulation I, LTT IHMPC responses $(\cdots)$ and set points or bounds $(--)$.

5.2.2. Robust LTT tuning

The robust Lexicographic approach takes into account all the 6 models simultaneously, as described in Section 3.3, to find a set of tuning parameters that yields a robust operation in plant-model mismatch scenarios.

The tuning horizon was chosen as $\theta_t = 60$, and the pair y_1-u_2 was deemed more important than y_2-u_1. $G_{des}(s)$ was chosen based on first order approximations of the open-loop step responses of transfer functions $G_{2,1}(s)$ and $G_{1,2}(s)$ of the nominal model. The time constants of $G_{des,1,1}(s)$ and $G_{des,2,2}(s)$ were set to 10% and to 30% of the first-order approximations as shown in (5-13). The system initial conditions and output set points were: $y_0 = \begin{bmatrix} 0.277 & 1.630 \end{bmatrix}^T$, $u_0 = \begin{bmatrix} 0.6 & 0.7 \end{bmatrix}^T$, $y_{sp} = \begin{bmatrix} 0.272 & 1.581 \end{bmatrix}^T$, considering the normalized values defined in (5-3) and (5-4); the robust Lexicographic problem, defined by equations(3-9) - (3-13), was solved using *fmincon* (*tolfun* = 10^{-10}, *tolx* = 10^{-10}, *maxfunevals* = 4×10^3, *tolcons* = 10^{-6}, *maxiter* = 4×10^2) and the IHMPC control problem was solved analytically in MATLAB® 2013. The decision variable vector was $x = \begin{bmatrix} q_{y,1} & \dots & q_{y,w'} & r_1 & \dots & r_{w'} \end{bmatrix}$; its initial guess, lower, and upper bounds for the tuning parameters were: $x_0 = \begin{bmatrix} 10 & \mathbf{1}_{w'-1} & \mathbf{1}_{w'} \times 10^{-2} \end{bmatrix}$, $LB = \begin{bmatrix} 10 & \mathbf{1}_{w'-1} \times 0.5 & \mathbf{1}_{w'} \end{bmatrix}$ and $UB = \begin{bmatrix} 10 & \mathbf{1}_{w'-1} \times 10^2 & \mathbf{1}_{w'} \times 10^3 \end{bmatrix}$; the initial values, lower and upper bounds of slack variables were vectors of ones, zeros and 10^6 with appropriate dimensions, respectively; $S_r = diag\left(\begin{bmatrix} 10^8 & 10^8 & 10^6 & 10^8 & 10^6 & 10^3 \end{bmatrix}\right)$ and $S_p = I_6 \times 10^6$. The optimum tuning parameters obtained by the robust Lexicographic technique were Q_y=diag([10 3.87]) and R=diag([129.91 100]). The total time elapsed was 247 minutes on an Intel® Core™ i5 3.20 GHz, 4Gb RAM computer. Figure 5-21 and Figure 5-22 show the set point tracking output responses for all six models, and its reference trajectory.

$$G_{des}(s) = \begin{bmatrix} \dfrac{1}{2s+1} & 0 \\ 0 & \dfrac{1}{7.8s+1} \end{bmatrix} \tag{5-13}$$

Figure 5-22 shows that the optimum solution yields poor performances for models 4 and 3, i.e. excessive sluggishness, and large overshoot, respectively. The right hand graph of Figure 5-22 shows that the y_2 set point tracking is only achieved for model 6, within the tuning horizon. However, all the trajectories are stable. The sluggish responses observed in the left hand graph, for variable y_1 are necessary in order to stabilize all the responses obtained in the model mismatch scenario, taking into account all the models in Ω.

Penalizing the robustness slack variable of model 2 improved the reference trajectory tracking performance in the second tuning step. On the other hand, model 4 did not improve.

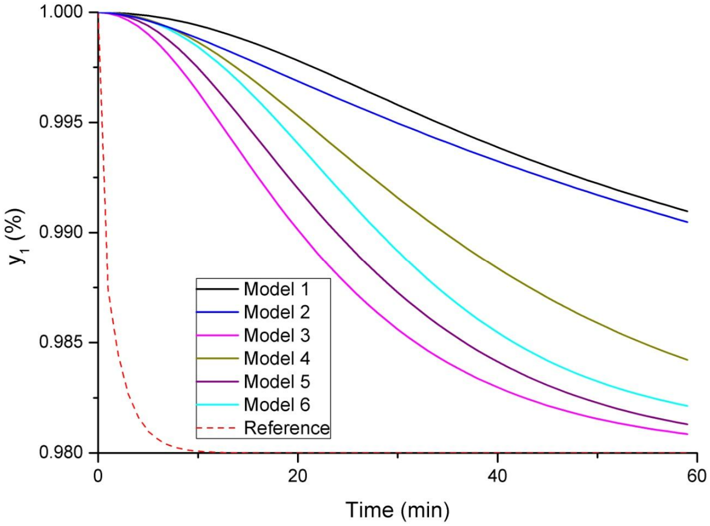


Figure 5-21: C3/C4 splitter RLTT, output responses of the first step.

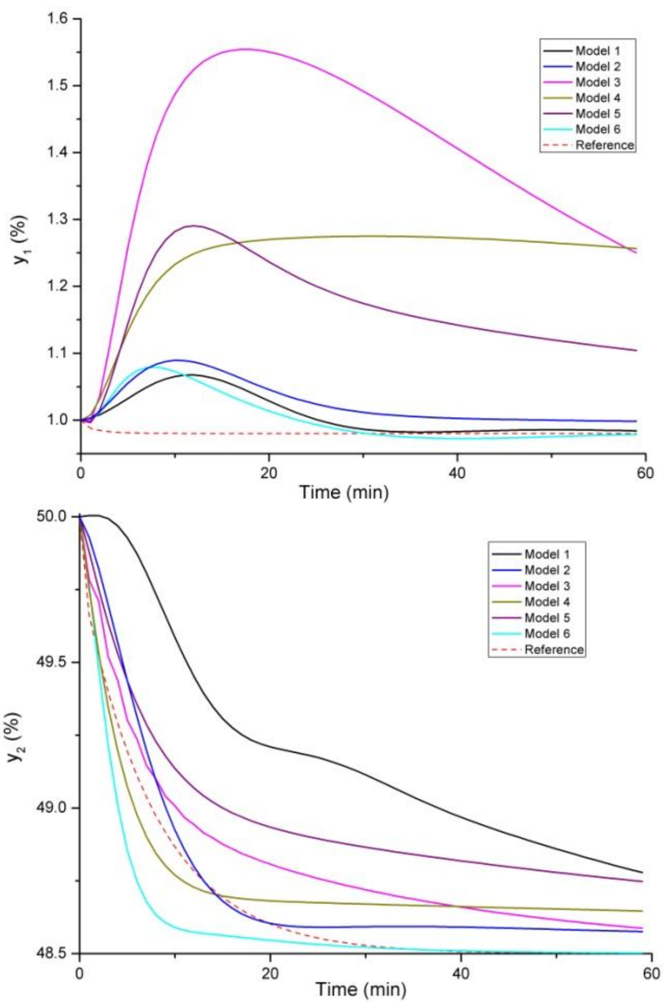


Figure 5-22: C3/C4 splitter RLTT, output responses of the second step.

5.2.3. Robust CTT application

In this section, we use the robust compromise approach to tune an IHMPC in closed-loop with the C3/C4 splitter model. The tuning goals were defined in the same fashion as in the Lexicographic approach. Problem 2, defined by equations (4-9) - (4-11) was solved using fmincon (tolfun = 10^{-10}, tolx = 10^{-10}, maxfunevals = 4×10^{3}, tolcon = 10^{-6}, and maxiter = 400) in MATLAB 2013®. The decision variable vector was $x=\begin{bmatrix} q_{y,1} & \dots & q_{y,w} & r_1 & \dots & r_w & \lambda \end{bmatrix}$ and the initial guess was $x0=\begin{bmatrix} 10 & 1 & 100 & 100 & 2000 \end{bmatrix}$, The lower and upper bounds were $LB=\begin{bmatrix} 10 & 0.5 & 10^{-2} & 10^{-2} & -100 \end{bmatrix}$ and $UB=\begin{bmatrix} 10 & 100 & 10^{3} & 10^{3} & 10^{6} \end{bmatrix}$. Table 5-23 shows the values of $F°$ obtained for each model. Observe that, as expected, the lowest objective function values was obtained with model 6.

Table 5-23: C3/C4 RCTT results, $F°$ for each model.

Model	1	2	3	4	5	6
F°_{1}	0.0484	0.1060	0.0091	0.0131	0.0076	0.0015
F°_{2}	0.3591	0.1339	0.0605	0.0401	0.0729	0.0041

The optimum tuning parameters calculated by the Compromise technique are Q_y=diag[(10 8.96]) and R=diag([217.95 50]). Comparing these results with the ones obtained by the robust Lexicographic approach, it is observed that both techniques yielded a large r_1, which means that aggressive control actions of u_1 lead to instability in the plant-model mismatch scenario.

Comparing Figure 5-22 and Figure 5-23, we observe that in the former, more sluggish output responses are obtained; and model 2 yields the best trajectories, while in the latter, there are some oscillatory responses, especially for models 1 and 2, and the nominal model has good performance.

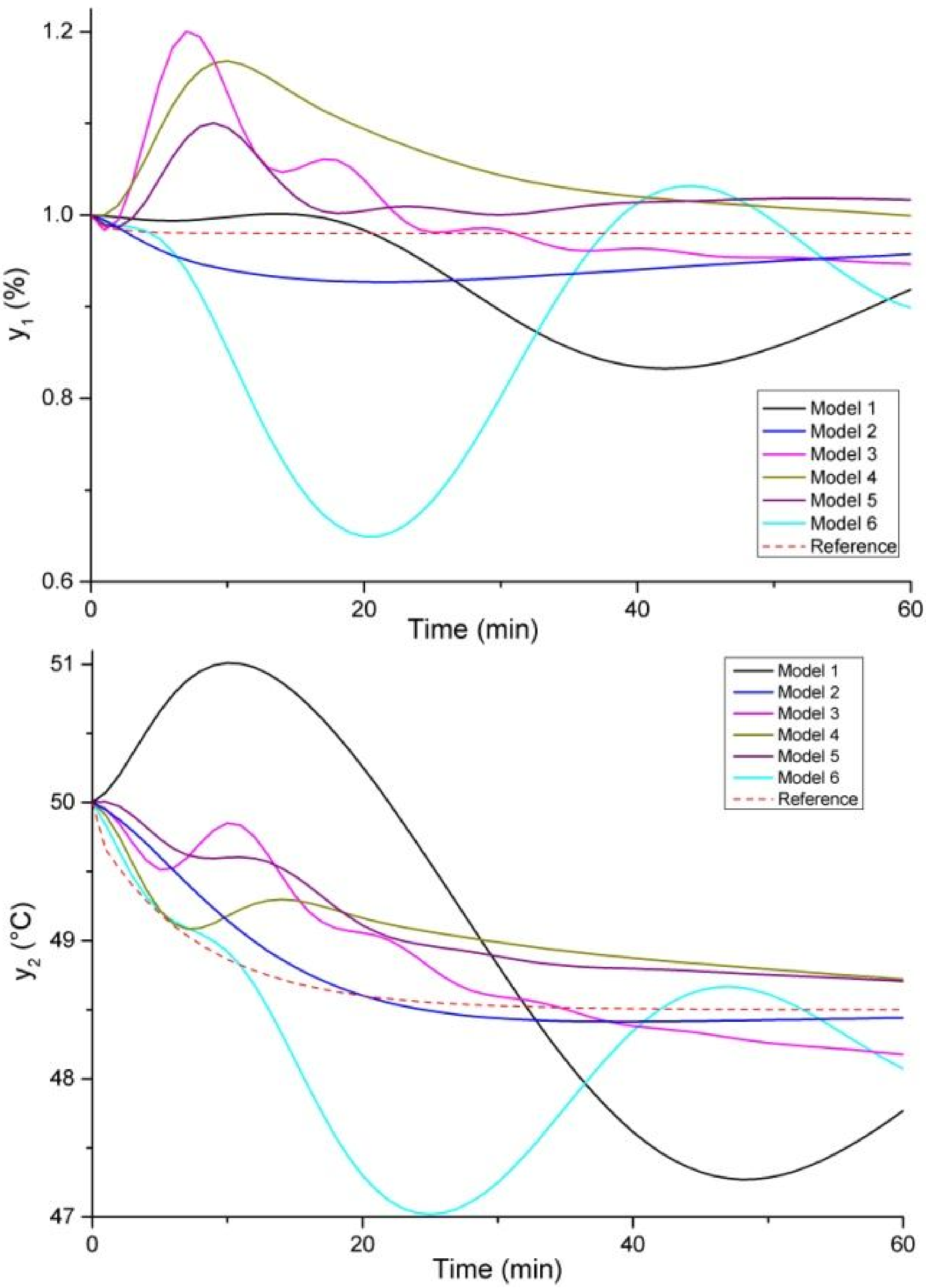


Figure 5-23: C3/C4 RCTT tuning results, outputs.

5.2.4. Comparing the RLTT and RCTT in a simulation example

The settings from Simulation I were implemented to compare two different scenarios: (i) the closed-loop responses of the nominally tuned IHMPC and the robustly tuned IHMPCs; and (ii) the performance of the robustly tuned IHMPC and a Robust IHMPC, tuned by trial and error.

The RIHMPC control problem, defined in Appendix B, was solved using the *CONOPT* nonlinear solver in GAMS® 23.6. The RIHMPC tuning parameters were $Qy = diag\left(\left[9\times10^5 \quad 8\times10^4\right]\right)$, $Q_u = diag\left(\left[0 \quad 0\right]\right)$, $R = diag\left(\left[5\times10^2 \quad 5\times10^3\right]\right)$, $S_y = diag\left(\left[8\times10^6 \quad 10^6\right]\right)$, $S_u = diag\left(\left[0 \quad 0\right]\right)$, and m=6. The IHMPC control problem was solved using *quadprog* (default settings) in MATLAB® 2013.

Figure 5-24 compares the set point tracking and disturbance rejection capabilities of an IHMPC tuned using the robust tuning techniques develop here and the nominal lexicographic approach. We observe that the undesired oscillatory behavior was eliminated by the robust tuning techniques; larger values of R resulted in more conservative controllers. We observe that both robust tuning techniques attenuated the closed-loop oscillatory behavior, and improved both set point tracking and disturbance rejection capabilities, however, they yielded sluggish responses.

Figure 5-25 compares the responses of an IHMPC tuned by the robust tuning techniques proposed here and a RIHMPC tuned by trial and error in the scenario defined in Simulation I. The RIHMPC simulation took 269 seconds while the IHMPC took only 6 seconds, both running on an Intel® Core™ i5 3.20 GHz, 4Gb RAM computer. Considering the plant was simulated over 550 time instants, the IHMPC (the values of tuning parameters do not interfere in the computing time) has a real time/time instant ratio of 0.019, while the ratio of the RIHMPC is 0.489. We observe that the RIHMPC yields sluggish performance for y_1, remarkable tracking and disturbance rejection capabilities for y_2 and non-oscillatory responses.

The value of r_1 obtained by both tuning techniques shows that the input u_1 is on one hand responsible for the oscillatory behavior in model-plant mismatch scenarios, but on the other, is the most important input used to drive y_1 to its set points.

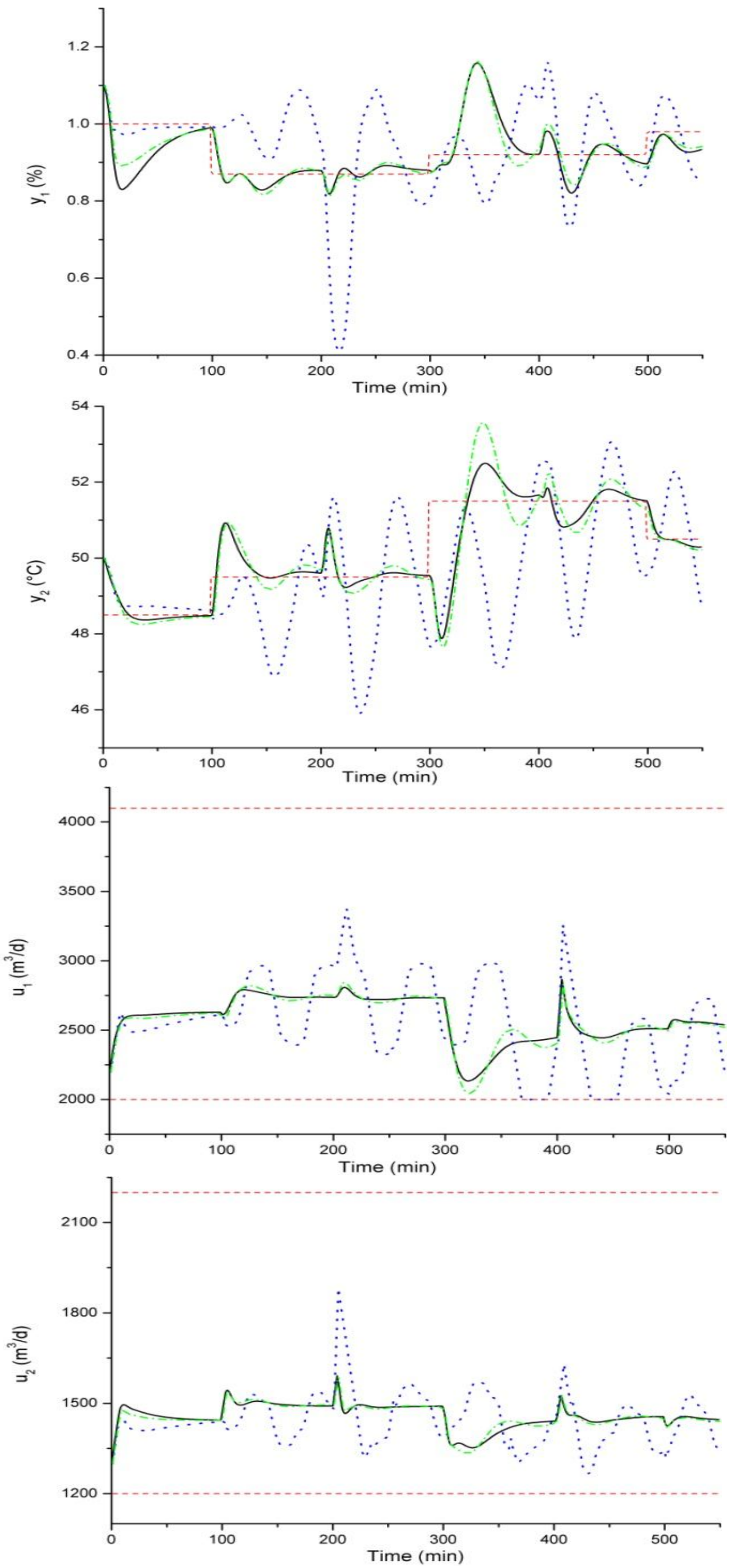


Figure 5-24: Responses of the IHMPC tuned with RLTT (——), RCTT (–·–) and LTT (···). Set points and input bounds (– –).

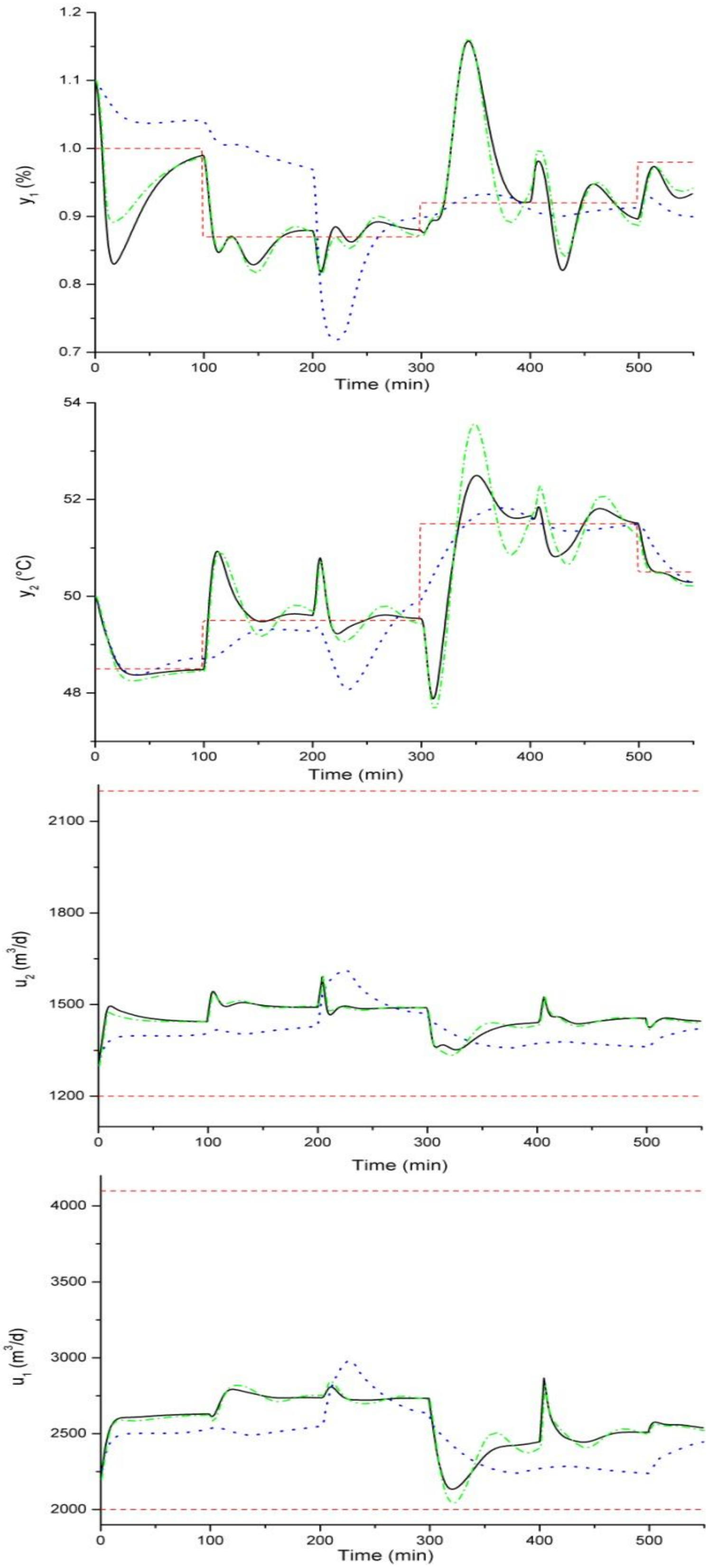


Figure 5-25: C3/C4 splitter Simulation I. IHMPC with RLTT (——), RCTT (−·−) and a RIHMPC (···). Set points and input bounds (−−).

In order to further investigate how the robust tuning techniques perform compared to the robust controller, we propose another scenario, named Simulation II. The initial conditions are $u_0 = [3000 \quad 1800]^T$, $y_0 = [0.9 \quad 50]^T$, the time instants at which the set points are changed and its values are given in Table 5-24. The nominal model is model 6, the plant model starts with model 6, changes to model 1 at 100 min, to model 3 at 1200 min and to model 4 at 3100 min. The simulation runs for 7000 minutes.

Table 5-24: C3/C4 Simulation II set points.

Time (min)	y^{sp}_1 (%)	y^{sp}_2 (°C)
0	1	50
1000	0.95	49
3000	0.95	48.5
5000	0.9	48.5

Figure 5-26 show the results of Simulation II. We observe that the RIHMPC has an overall better performance for y_2 than the robustly tuned controller, although it yields poor tracking capabilities for the set point of y_1, it is more sluggish than the RLTT IHMPC. The RCTT yields oscillatory behavior for both outputs, from 3000 to 4000 min, when the plant is represented by plant model 4. Figure 5-27 shows the output responses for the set point change from time 1000 min until 1500 min. It is observed that the IHMPCs tuned with the robust techniques yield oscillatory for output y_1, however, the response of the RIHMPC is sluggish and it takes almost 1000 minutes to drive y_1 to its set point. However, the RIHMPC response for y2 is faster and smoother than the responses of the robustly tuned IHMPCs. The IHMPC tuned with the robust compromise technique yields large amplitude oscillations whereas the lexicographic technique yields a controller with low amplitude oscillations.

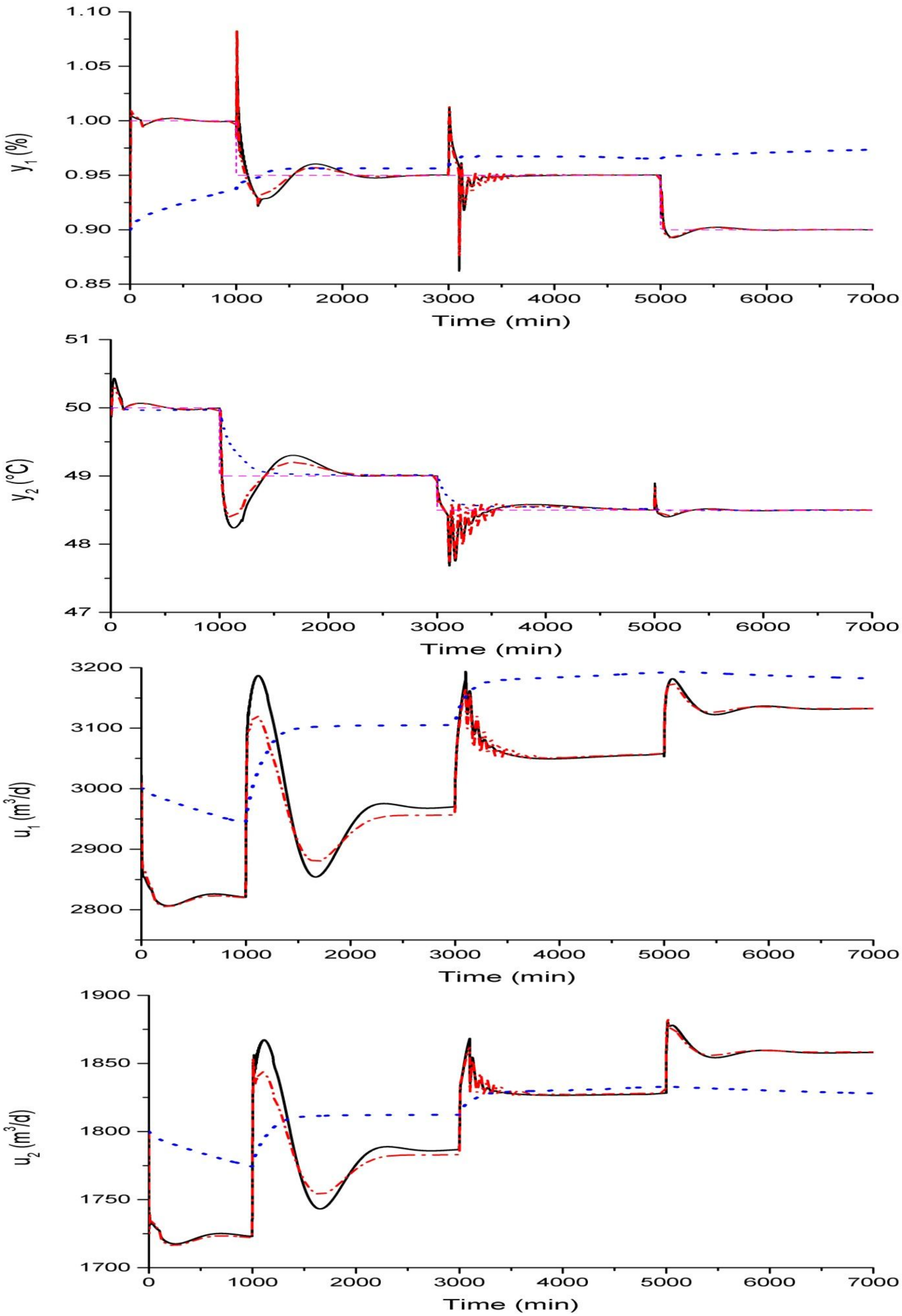
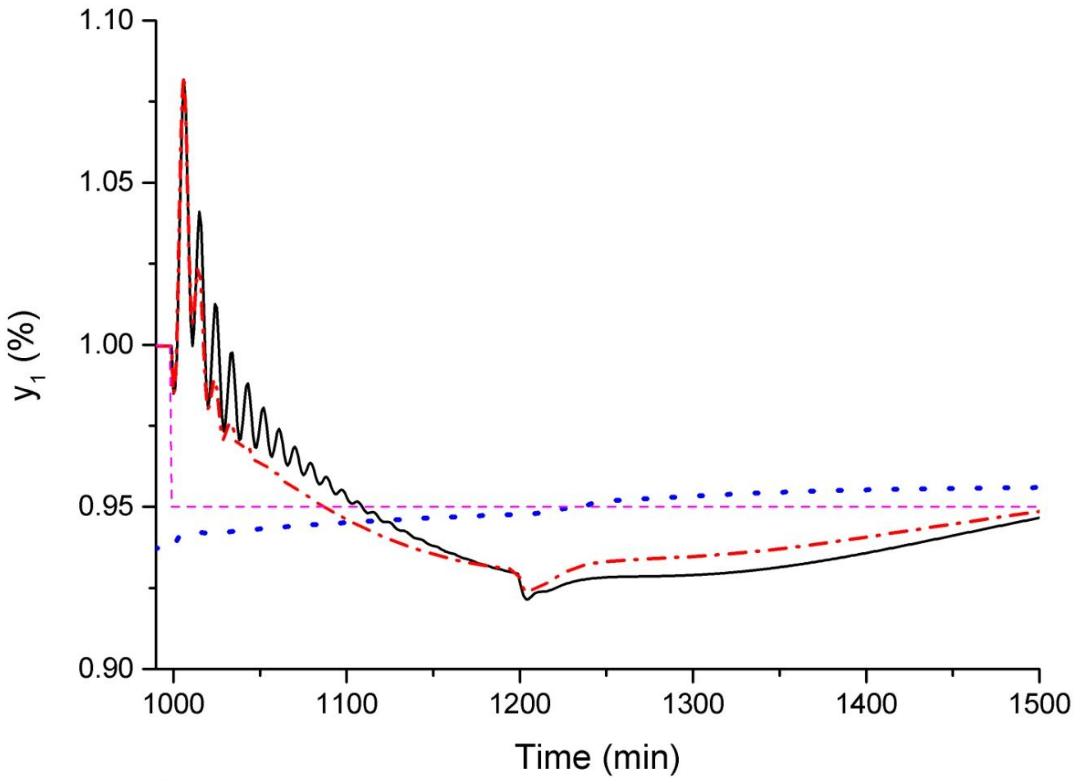


Figure 5-26: C3/C4 splitter Simulation II. IHMPC with RLTT (——), IHMPC with RCTT (–·–), RIHMPC (···). Set points (– –).



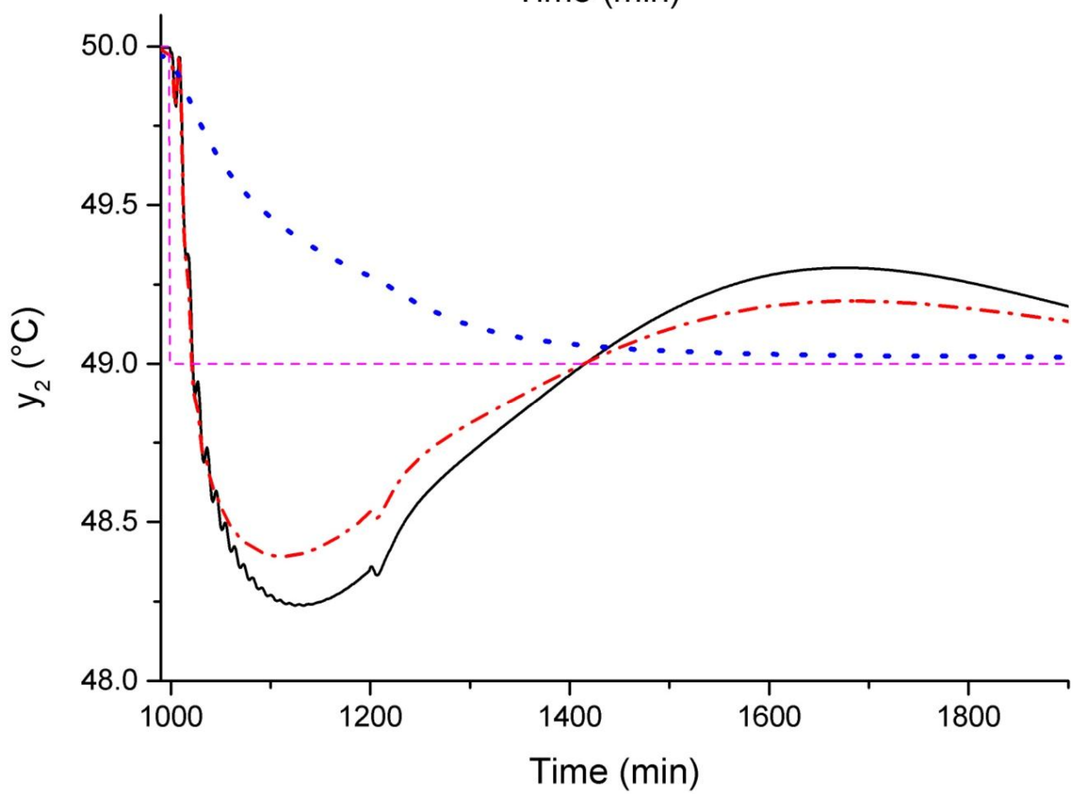


Figure 5-27: Simulation II from 1000 min to 1500 min. IHMPC with RLTT (——), IHMPC with RCTT (−·−), RIHMPC (···). Set points (−−).

The input profiles calculated by the nominal controllers were similar, except that the compromise technique yields larger and more aggressive control moves. The RIHMPC profiles, on the other hand, are smoother but tend to take much time to reach the steady state. The trial and error tuning of the RIHMPC yielded a set of tuning parameters that allows a better tracking performance of y_2. When the value of $q_{y,1}$ is increased, the system response becomes oscillatory, especially when the plant model is model 1.

The results shown here indicate that because of the sluggish responses of the RIHMPC, it tends to oscillate less than the robustly tuned IHMPCs. However, the computational time of the robust IHMPC is at least one order of magnitude larger than the computational time of the robustly tuned controllers, for this relatively small system. Regarding the nominal MPC implemented in the mismatch scenario, the robustly tuned controller reduces oscillation and improves set point tracking and disturbance rejection capabilities.

6. FINAL CONSIDERATIONS

6.1. Conclusions

The conclusions drawn from the case studies in this thesis are summarized here.

The FCC case study showed that the Lexicographic approach is more suitable for systems in which *ny=nu.* It allows the calculation of appropriate tuning parameters to achieve the best performance of the prioritized outputs in reasonable time, while a trial and error tuning approach would take a lot of computer time. Also, this case study unveils the flexibility of the technique to account for different goal definitions. Even though the Lexicographic technique is able to respect output priority, and in real application, there are as many degrees of freedom as the system inputs to define the subsystem pairs, the method may still require much heuristics or fail to address systems with *ny≠nu* straightforwardly.

The HOF case study showed that the proposed techniques lead to different values of the corresponding optimum tuning parameters. The Lexicographic technique follows the usual tuning guidelines of the industry, in which the goals are defined according to the number of available inputs as degrees of freedom. The Lexicographic method showed to be more suitable for systems in which the number of outputs is equal to the number of inputs, whereas the compromise approach can take into account as many objectives as necessary and is independent of the size of the system. The lexicographic method successfully prioritizes the more important objectives, whereas the Compromise method obtains the best attainable performance considering all objectives simultaneously.

A MPC with output zone control and input targets was tuned using the compromise approach in the CDU case study. The method was compared to a similar multi-objective tuning approach, based on an *a posteriori* solution. The latter was, in average, 22 times more expensive in terms of computational time than the proposed method, which may result in an even more remarkable difference for large scale systems, although both methods are carried out off-line. Two different strategies were assessed: in the first strategy (Scenario II-A), matrices Q_y and R are tuned in a first step, considering the output tracking goals and in the second step, matrix Q_u is tuned, assuming that the optimum values of Q_y and R are inherited from the previous step and the input tracking goals are addressed. The second tuning strategy (Scenario II-B) considers the case in which Q_y, Q_u and R are tuned considering the input and output tracking goals simultaneously. In both strategies, typical operating scenarios of the CDU are defined, in which an output is assumed to act as an active constraint to an input reference trajectory tracking. The results showed that the methodology defined in Scenario II-A might lead to fast but oscillatory responses. A simulation study was performed, considering the output zones and input targets defined in a real operating scenario of the CDU. The results showed that the compromise method yielded similar responses to the existing set of tuning parameters that were obtained by trial and error. The input tracking capability of the latter was more efficient, but the former yielded better results in the output tracking scenario analysis. Nonetheless, the trial and error approach, used to obtain the existing tuning parameters, is cumbersome and time consuming and therefore, should be used as a supplementary tuning method, instead of in the early stages of the MPC tuning procedure.

In the last case study, we considered the C3/C4 splitter model, but with multi-plant model uncertainty and assessed the robust formulations of the tuning techniques. The analysis of the robust tuning results indicates which models are most different from the nominal model, as well as what outputs are most likely to render the system unstable. A robustly tuned IHMPC was compared to a RIHMPC in two scenarios. In the first one, setpoint changes, plant model changes and unmeasured disturbances take place in short time intervals. The simulation showed that the RIHMPC, although much smoother than the robustly tuned IHMPC, was considerably more sluggish in some conditions while the robust tuning techniques were able to find a compromise between robustness and performance, through the inclusion of either a cost contracting constraints (RLTT) or min-max optimization (RCTT). In the second scenario, in which the time intervals between setpoint changes and plant model changes are larger, in general the RIHMPC performed better than the robustly tuned controllers. Oscillatory response for some plant models and large overshoot in disturbance rejection were the main drawbacks from the latter. However, we kept track of the simulation time and compared how long the RIHMPC and IHMPC take to complete the 550 time instants simulation. The robust controller about 0.489 sec/iteration while the robust tuned controller took about 0.019 sec/iteration. The difference, which is over one order of magnitude, might not be significant for a small system as the C3/C4 splitter, however, time is crucial in industrial applications of MPCs, and RIHMPCs have been questioned due to their computational expense. Robust tuning techniques for nominal controllers might address this problem by shifting the heavy computational burden to offline applications. The simulation results indicate that tuning an existing controller for a different scenario might be a cheap solution to improve its performance. In certain circumstances, the financial cost involved in the project and commissioning of a new control platform is prohibiting, but a simple change in the tuning parameters of the existing controller might yield a profit increase due to less oscillatory responses.

6.2. Directions for further work

A major limitation for industrial application of the tuning techniques developed here is the lack of knowledge about model uncertainty, which highly affect the system operation over long time intervals, due to equipment deterioration; or over short intervals, due to changes in operational points, and system disturbances. Properly identifying a set of models, over a significant range of operating conditions is almost impossible or, at least not practical and cumbersome from a practical point of view.

From the findings in this thesis, robust tuning techniques seem a valid alternative for robust predictive controllers, under the assumption that the user is able to provide a reliable uncertainty representation. Therefore, two suggestions are due: first, alternatives for model identification in different conditions and operational points should be developed. Second, online tuning techniques, on one hand capable of dealing with model uncertainty based on a database of previous control actions and system behavior, and on the other, fast enough to run and optimize tuning parameters once model mismatch is detected. These are interesting alternatives for the current industrial needs.

Moreover, a straightforward extension of the tuning methods proposed here for robust MPC is also a possibility.

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APPENDIX A – Transfer function for the case studies

Fluid Catalytic Cracking

Table A1: FCC Reactor Regenerator.

	u_1	u_2
y_1	$\frac{-0.87}{13s^3+17.9s^2+5.9s+1}$	$\frac{-0.023s-0.092}{6.29s^4+15.76s^3+15.27s^2+6.8s+1}$
y_2	$\frac{0.55}{27s^3+35.7s^2+9.7s+1}$	$\frac{0.55}{17s^4+29.33s^3+21.99s^2+7s+1}$
y_3	$\frac{0.25}{17s^3+24s^2+8s+1}$	$\frac{0.25}{5.1s^3+8s^2+5.1s+1}$
y_4	$\frac{0.014}{46s^3+54.5s^2+9.5s+1}$	$\frac{0.14}{78.2s^4+111.1s^3+65.55s^2+10.6s+1}$

	u_3	u_4
y_1	$\frac{-0.2273s^2-0.4026s-0.074}{120.9s^4+109s^3+46.1s^2+10s+1}$	$\frac{-0.48s}{40s^2+14s+1}$
y_2	$\frac{0.8051s^2+1.732s+0.74}{143s^4+171.9s^3+75.1s^2+14.3s+1}$	$\frac{0.36}{33s^2+6.5s+1}$
y_3	$\frac{0.448s+0.7}{39s^3+34s^2+10s+1}$	$\frac{0.4977s+0.079}{24s^2+12.01s+1}$
y_4	$\frac{2.765s^2+4.493s+0.27}{689s^4+670s^3+227s^2+30s+1}$	$\frac{0.18s+0.015}{66s^2+27s+1}$

C3/C4 Splitter

Transfer function model

$$Gi,j(s)=\frac{\left(b_{i,j,0}+b_{i,j,1}s\right)e^{-\theta_{i,j}s}}{1+a_{i,j,1}s+a_{i,j,2}s^2} \qquad \text{A-1}$$

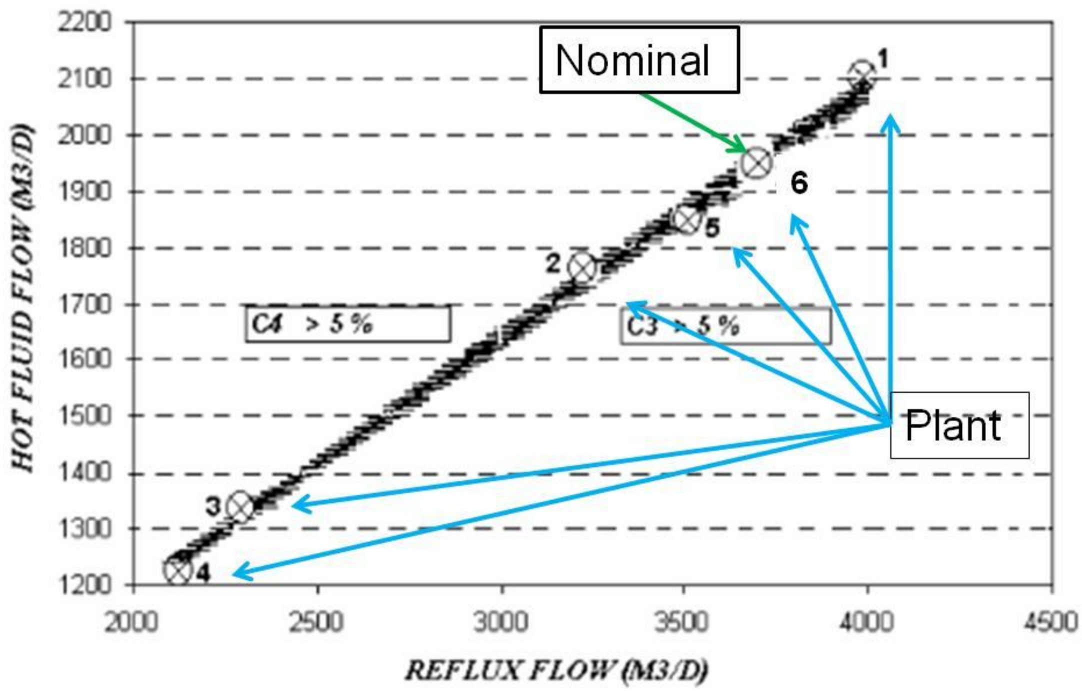

Figure A1: Operating range and operational points of the C3/C4 splitter (Adapted from Porfítio, Neto and Odloak, 2003).

Table A2: C3/C4 transfer function coefficients.

(i,j)	b_0	b_1	a_1	a_2	θ
Operating point 1					
(1,1)	1.09E-05	4.23E-05	1.09E-01	2.43E-02	0
(1,2)	-3.82E-05	-1.21E-04	1.34E-01	2.43E-02	0
(2,1)	-1.12E-04	-8.73E-05	1.32E-01	7.30E-03	0
(2,2)	7.00E-03	1.30E-03	2.26E+00	1.37E-01	0
Operating point 2					
(1,1)	4.22E-04	-2.72E-04	1.66E+00	2.53E-01	0

(1,2)	-1.41E-04	-2.18E-04	1.32E-01	1.17E-02	0
(2,1)	-6.30E-03	-3.40E-03	2.07E+00	2.43E-01	0
(2,2)	4.50E-03	2.00E-04	8.35E-01	8.12E-02	0
Operating point 3					
(1,1)	1.53E-03	-8.60E-04	1.19E+00	9.12E-02	0
(1,2)	-7.81E-04	-3.77E-04	3.40E-01	1.81E-02	0
(2,1)	-8.00E-04	-3.40E-03	4.02E-01	3.65E-02	0
(2,2)	8.90E-03	6.40E-03	1.90E+00	1.95E-01	0
Operating point 4					
(1,1)	4.88E-04	-1.11E-04	9.88E-01	6.46E-02	0
(1,2)	-1.86E-04	-1.76E-04	2.61E-01	9.10E-03	0
(2,1)	-2.50E-03	-3.90E-03	8.87E-01	8.40E-02	0
(2,2)	2.90E-03	5.50E-03	8.60E-01	3.92E-02	0
Operating point 5					
(1,1)	5.65E-04	-3.54E-04	8.17E-01	-8.09E-02	0
(1,2)	-4.78E-04	-1.43E-04	3.42E-01	2.59E-02	0
(2,1)	-2.10E-03	-1.90E-03	1.17E+00	1.07E-01	0
(2,2)	8.10E-03	5.30E-03	2.42E+00	1.76E-01	0
Operating point 6					
(1,1)	5.66E-04	-2.22E-04	3.49E+00	5.90E-01	0
(1,2)	-1.45E-03	7.41E-05	2.70E+00	4.02E-01	0
(2,1)	-1.24E-03	-1.14E-03	1.64E+00	9.85E-02	0
(2,2)	2.00E-03	-3.00E-04	2.43E+00	6.51E-02	0

Crude Distillation Unit

The omitted entries in Table A4 represent null transfer functions.

Table A4: CDU.

	u_1
y_8	$\dfrac{7.31\times10^{-4}s^5-1.82\times10^{-3}s^4+1.49\times10^{-2}s^3+2.67\times10^{-3}s^2+1.32\times10^{-3}s+1.71\times10^{-4}}{s^6+2.18s^5+5.83s^4+2.14s^3+0.56s^2+0.03s+9.07\times10^{-4}}$
y_9	$\dfrac{-2.63\times10^{-2}s^6-0.17s^5+0.36s^4+0.15s^3+0.12s^2+5.55\times10^{-3}s+8.86\times10^{-4}}{s^7+8.16s^6+29.45s^5+19.87s^4+7.98s^3+1.11s^2+0.10s+3.20\times10^{-3}}$
y_{10}	$\dfrac{-1.88\times10^{-4}s^5+1.11\times10^{-4}s^4+1.53\times10^{-4}s^3+8.67\times10^{-5}s^2-1.76\times10^{-5}s+3.90\times10^{-5}}{s^6+3.83s^5+6.67s^4+2.84s^3+0.68s^2+0.12s+4.88\times10^{-3}}$

	u_2
y_3	$\dfrac{-1.29s^2-1.37s-2.13}{s^3+1.99s^2+0.99s+0.19}$
y_7	$\dfrac{4.60\times10^{-3}s^2+0.06s+4.11\times10^{-3}}{s^3+0.39s^2+0.07s+2.93\times10^{-3}}$
y_8	$\dfrac{11.31s^6+102.90s^5-58.29s^4-171.20s^3-362.10s^2-159.70s-56.27}{s^7+8.84s^6+38.89s^5+71.07s^4+83.25s^3+53.24s^2+19.11s+3.16}$
y_9	$\dfrac{-8.80s^5+13.69s^4+8.60s^3+5.95s^2+0.89s+0.18}{s^6+4.13s^5+4.94s^4+2.19s^3+0.65s^2+0.11s+7.57\times10^{-3}}$
y_{10}	$\dfrac{-4.67\times10^{-3}s^2-5.59\times10^{-4}s-6.76\times10^{-4}}{s^3+0.32s^2+0.06s+5.19\times10^{-3}}$

	u_3
y_1	$\dfrac{0.46s^3-1.15s^2+7.40s+0.91}{s^4+2.57s^3+11.90s^2+3.96s+0.32}$
y_7	$\dfrac{-0.20s^3-1.08s^2+3.67s+3.13}{s^4+9.85s^3+35.88s^2+12.34s+1.62}$
y_8	$\dfrac{-0.83s^2+15.19s+4.86}{s^3+1.85s^2+0.46s+0.06}$
y_9	$\dfrac{0.56s^2+14.74s+5.08}{s^3+1.96s^2+0.52s+0.05}$

y_{10}	$\dfrac{1.82\times10^{-3}s^3+0.20s^2-0.41s-0.78}{s^4+3.55s^3+14.10s^2+9.24s+2.22}$

	u_4
y_2	$\dfrac{0.24s^6-0.14s^5+1.71s^4+3.63s^3+3.72s^2+0.66s+0.05}{s^7+5.58s^6+19.65s^5+39.65s^4+11.93s^3+3.85s^2+0.20s+4.13\times10^{-3}}$
y_3	$\dfrac{0.14s^5+3.82s^4-7.56s^3-12.02s^2-0.30s-0.39}{s^6+5.22s^5+18.97s^4+17.63s^3+3.14s^2+0.32s+0.01}$
y_4	$\dfrac{-0.13s^2+0.26s+0.29}{s^3+4.16s^2+1.90s+0.32}$
y_5	$\dfrac{-6.42\times10^{-4}s^6-0.02s^5+0.04s^4+0.05s^3+0.04s^2+0.03s+6.327\times10^{-3}}{s^7+4.27s^6+16.04s^5+10.95s^4+6.98s^3+1.72s^2+0.51s+0.02}$
y_6	$\dfrac{0.03s^3-0.12s^2+0.60s+0.42}{s^4+3.26s^3+13.41s^2+7.94s+1.64}$

	u_5
y_5	$\dfrac{-6.499s^2\times10^{-4}-6.382s\times10^{-4}-1.923\times10^{-4}}{s^3+0.58s^2+0.09s+5.371\times10^{-3}}$

	u_6
y_3	$\dfrac{-0.05s^3+0.52s^2-1.92s-2.32}{s^4+4.56s^3+17.19s^2+17.01s+5.58}$
y_4	$\dfrac{0.01s^3+0.03s^2+0.05s-0.44}{s^4+7.28s^3+29.63s^2+53.21s+33.54}$
y_6	$\dfrac{9.46\times10^{-4}s^2-1.39\times10^{-5}s+1.72\times10^{-4}}{s^3+0.46s^2+0.12s+0.01}$

	u_7
y_2	$\dfrac{0.01s^5-0.02s^4-0.02s^3-0.01s^2-1.84\times10^{-4}s-1.89\times10^{-4}}{s^6+3.39s^5+4.97s^4+2.45s^3+0.47s^2+0.03s+9.58\times10^{-4}}$
y_3	$\dfrac{-1.84\times10^{-3}s^2-6.35\times10^{-4}s-1.07\times10^{-4}}{s^3+0.26s^2+0.02s+3.83\times10^{-4}}$
y_4	$\dfrac{-6.67s^4\times10^{-4}+1.26\times10^{-3}s^3+1.71\times10^{-3}s^2+1.68\times10^{-4}s+6.58\times10^{-5}}{s^5+2.48s^4+3.34s^3+0.66s^2+0.08s+4.09\times10^{-3}}$

y_5	$\dfrac{-8.51\times10^{-4}s^5-1.52\times10^{-4}s^4-1.48\times10^{-4}s^3-1.01\times10^{-5}s^2-1.32\times10^{-6}s-6.\times10^{-8}}{s^6+0.31s^5+0.21s^4+0.04s^3+6.14s^2\times10^{-3}+2.93\times10^{-4}s+5.26\times10^{-6}}$
y_6	$\dfrac{-8.431\times10^{-4}s^3+5.02\times10^{-5}s^2-0.02s-3.67\times10^{-3}}{s^4+3.34s^3+13.05s^2+3.09s+0.25}$

	u_8
y_3	$\dfrac{0.5407s^6+1.312s^5+0.5546s^4+0.3327s^3+0.06388s^2+0.005414s+0.000}{s^7+0.4485s^6+0.599s^5+0.1516s^4+0.06437s^3+0.009542s^2+0.0005524s+1.2}$
y_4	$\dfrac{0.008947s^3-0.3574s^2+1.39s+0.9825}{s^4+4.132s^3+15.16s^2+7.629s+1.396}$
y_5	$\dfrac{-0.02s^5-0.26s^4+0.74s^3+0.21s^2+0.11s+0.04}{s^6+5.85s^5+20.46s^4+13.84s^3+4.03s^2+0.67s+0.04}$
y_{10}	$\dfrac{0.03s^3-0.10s^2+0.51s+0.11}{s^4+2.96s^3+12.66s^2+5.60s+0.70}$

APPENDIX B – MPC formulations

DMC Controller

Industry has accepted Dynamic Matrix Control (DMC), formalized in (Cutler & Ramaker, 1980), as a standard advanced control strategy because it straightforwardly includes process constraints in the control problem. It is applicable to both MIMO and SISO systems. The DMC is based on a step-response or an impulse-response model, which are more intuitive than the state-space models.

Model Representation

In a SISO system, an output impulse response is represented by (B-1).

$$y(k) = \sum_{i=1}^{\infty} h_i u(k-i), \qquad \text{(B-1)}$$

h_i is a sampled output when the system is excited by a unitary impulse, $u(k)$ and $y(k)$ are the input and output values at time instant *k*.

It is assumed that the impulse response is asymptotically stable and the model horizon, *N*, is defined as the upper bound on the impulse response horizon. The output prediction, $\hat{y}(k+j \mid k)$, at time instant *k*+*j*, calculated using system information available at time instant *k*, is defined in (B-2).

$$\hat{y}(k+j \mid k) = \sum_{i=1}^{N} h_i u(k+j-i \mid k) \qquad \text{(B-2)}$$

It is easy to write (B-2) using a step response model representation. The difference between the two representations is the characteristic of the input signal; in the step response model it is a step perturbation. The truncated step response at time step *k* is shown in (B-3).

$$y(k) = y_0 + \sum_{i=1}^{N} g_i \Delta u(k-i), \tag{B-3}$$

where y_0 is the output reference value that can be set to 0 without loss of generality, g_i is the sampled output value when the system is excited by an unitary step and $\Delta u(k) = u(k) - u(k-1)$. The output prediction at time instant *k*+*j*, for SISO systems, is calculated as follows:

$$\hat{y}(k+j \mid k) = \sum_{i=1}^{N} g_i \Delta u(k+j-i \mid k) \tag{B-4}$$

and the following relationships hold:

$$g_i = \sum_{j=1}^{i} h_j, \quad i = 1, \ldots, N \tag{B-5}$$

$$h_i = g_i - g_{i-1}, \quad i = 1, \ldots, N \tag{B-6}$$

Equations (B-5) and (B-6) show the relationships between the impulse and step responses coefficients.

Assuming that the system disturbances are constant until time step *k*+*N*, the prediction error is represented in (B-7) and the predicted output value is calculated as the sum of the system free response and the input contribution, according to (B-8).

$$\hat{n}(k+i \mid k) = \hat{n}(k \mid k) = y(k) - \hat{y}(k \mid k), \quad i = 1, \ldots, N \tag{B-7}$$

$$\begin{aligned} \hat{y}(k+i \mid k) &= \sum_{j=1}^{i} g_j \Delta u(k+i-j) + \sum_{j=i+1}^{N} g_j \Delta u(k+i-j) + \hat{n}(k+i \mid k) \\ &= \sum_{j=1}^{i} g_j \Delta u(k+i-j) + \sum_{j=i+1}^{N} g_j \Delta u(k+i-j) + y(k) - \sum_{j=1}^{N} g_j \Delta u(k-j) \\ &= \sum_{j=1}^{i} g_j \Delta u(k+i-j) + f(k+i), \end{aligned} \tag{B-8}$$

$f(k+i)$ is the system free response at time instant *k+i*; the part of the system response that does not depend on the future control actions. Since the system is assumed to be asymptotically stable, $g_{j+1} - g_j = 0,\ j > N$.

In the right hand side of (B-8), the first term contains the effects of future control actions, the second term contains the effects of the past control actions (which are known and already implemented into the system) and the last term represents the constant disturbances, according to (B-9).

$$f(k+i) = y(k) + \sum_{j=1}^{N} \left(g_{j+i} - g_j\right)\Delta u(k-j) \tag{B-9}$$

Then, the output predictions along the prediction horizon, considering *m* control actions in the future, are calculated as follows:

$$\begin{aligned}
\hat{y}(k+1|k) &= g_1\Delta u(k) + f(k+1) \\
\hat{y}(k+2|k) &= g_2\Delta u(k) + g_1\Delta u(k+1) + f(k+2) \\
&\vdots \\
\hat{y}(k+p|k) &= \sum_{i=p-m+1}^{p} g_i\Delta u(k+p-i) + f(k+p)
\end{aligned} \tag{B-10}$$

It is convenient to define the system dynamic matrix, D_m, arranging the step response coefficients as in (B-11) to write (B-10) in the compact form, seen in (B-12).

$$D_m = \begin{bmatrix} g_1 & 0 & \cdots & 0 \\ g_2 & g_1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ g_m & g_{m-1} & \cdots & g_1 \\ \vdots & \vdots & \ddots & \vdots \\ g_p & g_{p-1} & \cdots & g_{p-m+1} \end{bmatrix} \tag{B-11}$$

$$y^c = D_m\Delta u_k + f \tag{B-12}$$

Equation (B-12) relates the predicted outputs to the control moves, and it will be used to calculate the optimum input moves to track the output set-points.

The discussion above holds for MIMO systems. The model equations are extended to accommodate the vectors and matrices of large dimensions. In a system with *nu* inputs and *ny* outputs, the notation in (B-13) is used.

$$y_j(k) = \sum_{l=1}^{nu}\sum_{i=1}^{N} h_i^{lj} u^l(k-i), \tag{B-13}$$

h_i^{lj} is the sampled value of output *j* at time instant *i* for an unitary impulse in input *l*, *j*=1,…,*ny*; *l*=1,…,*nu*; *i*=1,…,*N*. Assuming model linearity, and using the superposition principle, y^c, Δu_k and *f* are defined for MIMO systems according to (B-14), (B-15) and (B-16)

$$y^c = \left[y_1(k+1|k) \ \cdots \ y_{ny}(k+1|k) \ \cdots \ y_1(k+p|k) \ \cdots \ y_{ny}(k+p|k) \right]^T \tag{B-14}$$

$$\Delta u_k = \left[\Delta u_1(k) \ \cdots \ \Delta u_{nu}(k) \ \cdots \ \Delta u_1(k+m-1) \ \cdots \ \Delta u_{nu}(k+m-1) \right]^T \tag{B-15}$$

$$f = \left[f_1(k+1|k) \ \cdots \ f_{ny}(k+1|k) \ \cdots \ f_1(k+p|k) \ \cdots \ f_{ny}(k+p|k) \right]^T \tag{B-16}$$

$y^c \in \Re^{ny\cdot p}$, $\Delta u_k \in \Re^{nu\cdot m}$, $f \in \Re^{ny\cdot p}$, $D_m \in \Re^{ny\cdot p \times nu\cdot m}$ and the MIMO dynamic matrix is defined in (B-17)**.**

$$D_m = \begin{bmatrix} \overline{\overline{g}}_{1,1} & 0 & \cdots & 0 \\ \overline{\overline{g}}_{2,1} & \overline{\overline{g}}_{12} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \overline{\overline{g}}_{p,1} & \overline{\overline{g}}_{p-1,2} & \cdots & \overline{\overline{g}}_{p-m+1,nu} \end{bmatrix}, \tag{B-17}$$

$\overline{\overline{g}}_{i,j}$, $i = 1,\ldots,p$; $j = 1,\ldots,nu$ are submatrices containing the coefficients $g_{i,k}$ of the *i-th* step response for the *k-th* output value corresponding to a step perturbation in the *j-th* input.

Control Algorithm

The DMC cost function is calculated according to (B-18), and its control problem is solved by minimizing (B-18) subject to the constraints defined in (B-19) and (B-20).

$$\min_{\Delta u_k} V_k = \sum_{i=0}^{p} \left\| \hat{y}(k+i|k) - y_{sp}(k+i) \right\|_{Q_y}^2 + \sum_{i=0}^{m-1} \left\| \Delta u(k+i|k) \right\|_R^2 \tag{B-18}$$

subject to

$$u_{\min} \le u(k+j) \le u_{\max},\ j = 0,\ldots,m-1 \tag{B-19}$$

$$-\Delta u_{\max} \le \Delta u(k+j) \le \Delta u_{\max},\ j = 0,\ldots,m-1 \tag{B-20}$$

$y_{sp}(k+i) \in \Re^{ny}$ is the output set point, $u_{\min} \in \Re^{nu}$, $u_{\max} \in \Re^{nu}$ are the lower and upper bound of the inputs and $\Delta u_{\max} \in \Re^{nu}$ is the maximum allowed control move. $Q_y \in \Re^{ny \times ny}$, $Q_y > 0$ and $R \in \Re^{nu \times nu}$, $R \ge 0$ are diagonal matrices. The problem is cast as a QP, using (B-12). Assuming a constant output setpoint, $y_{sp}(k)$, the set point vector is defined as $y^{sp} = \left[\underbrace{y_{sp}(k) \ \cdots \ y_{sp}(k)}_{p} \right]$, $y^{sp} \in \Re^{ny \cdot p}$. Equation (B-12) can be used to correlate the output prediction to the control actions, which are the decision variables of the DMC control problem. Equations (B-19) and (B-20) can also be written in terms of Δu_k as follows:

$$\begin{bmatrix} u(k|k) \\ u(k+1|k) \\ \vdots \\ u(k+m-1|k) \end{bmatrix} = \underbrace{\begin{bmatrix} I_{nu} & 0 & \cdots & 0 \\ I_{nu} & I_{nu} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ I_{nu} & I_{nu} & \cdots & 0 \end{bmatrix}}_{\bar{M}} \begin{bmatrix} \Delta u(k|k) \\ \Delta u(k+1|k) \\ \vdots \\ \Delta u(k+m-1|k) \end{bmatrix} + \underbrace{\begin{bmatrix} I_{nu} \\ I_{nu} \\ \vdots \\ I_{nu} \end{bmatrix}}_{\bar{I}_{nu}} u(k-1) \tag{B-21}$$

$$or,\ \begin{bmatrix} u(k|k) \\ u(k+1|k) \\ \vdots \\ u(k+m-1|k) \end{bmatrix} = \bar{M}\Delta u_k + \bar{I}_{nu} u(k-1)$$

$$\underbrace{\begin{bmatrix} -\Delta u_{\max} \\ \vdots \\ -\Delta u_{\max} \end{bmatrix}}_{-\Delta U_{\max}} \le \Delta u_k \le \underbrace{\begin{bmatrix} \Delta u_{\max} \\ \vdots \\ \Delta u_{\max} \end{bmatrix}}_{\Delta U_{\max}} \tag{B-22}$$

$I_{nu} \in \Re^{nu \times nu}$ is the identity matrix of dimension $nu \times nu$ and the 0's are null matrices of appropriate dimensions. A generic QP is defined as follows:

QP Problem

$$\min_x x^T H x + 2c_f^T x + c \tag{B-23}$$

subject to

$$A_{in} x \le b_{in} \tag{B-24}$$

$$A_{eq} x = b_{eq} \tag{B-25}$$

$$LB \le x \le UB \tag{B-26}$$

The DMC control problem can be converted to the QP Problem form using $x = \Delta u_k$, and

$$\begin{aligned} H &= D_m^T \bar{Q}_y D_m + \bar{R} \\ c_f &= D_m^T \bar{Q}_y \left(f - y^{sp} \right) \\ c &= f^T \bar{Q}_y f + {y^{sp}}^T \bar{Q}_y y^{sp} \end{aligned} \tag{B-27}$$

$$\begin{aligned} A_{in} &= \begin{bmatrix} \bar{M} \\ -\bar{M} \end{bmatrix} \\ b_{in} &= \begin{bmatrix} \bar{I}_{nu} \left(u_{\max} - u(k-1) \right) \\ \bar{I}_{nu} \left(u(k-1) - u_{\min} \right) \end{bmatrix} \end{aligned} \tag{B-28}$$

$$\begin{aligned} A_{eq}, b_{eq} &= 0 \\ LB &= -\Delta U_{\max} \\ UB &= \Delta U_{\max} \end{aligned} \tag{B-29}$$

where $\bar{Q}_y = blockdiag\left(\left[\underbrace{q_y \quad \cdots \quad q_y}_{p}\right]\right)$, $\bar{R} = blockdiag\left(\left[\underbrace{r \quad \cdots \quad r}_{m}\right]\right)$.

The potential inconveniences of the DMC algorithm are the large dimension of the step response model and the inability to cope with integrating systems.

The parameters that affect stability and performance in the DMC are the prediction horizon (p), the control horizon (m), the model horizon (N), the sampling time (T_s), and the cost function weighting matrices on the differences between predicted outputs and their set points (Q_y) and on the control moves (R).

MPC with input targets and zone control

The MPC studied here is based on the state-space model presented in (González & Odloak, 2009). The MPC cost function takes into account the deviation between the predicted system outputs, calculated using a state space system model over the prediction horizon, and the output set points; the penalization of the input increments; and the deviation between the system inputs and the input targets, both over the control horizon. The MPC considered here is defined through the solution to the following problem:

$$\min_{\Delta u_k, y_{sp}} \sum_{i=1}^{p} \left\| y(k+i|k) - y_{sp,k} \right\|_{Qy}^2 + \sum_{i=0}^{m-1} \left\| \Delta u(k+i|k) \right\|_R^2 + \sum_{i=0}^{m-1} \left\| u(k+i|k) - u_{des,k} \right\|_{Q_u}^2 \quad \text{(B-30)}$$

subject to the state space model of the system, the bounds on the inputs and input increments ((B-19) and (B-20)) and

$$y_{\min} \le y_{sp,k} \le y_{\max} \quad \text{(B-31)}$$

The parameter $u_{des,k} \in \Re^{nu}$ is an input target, assumed to be defined by an upper layer of the control structure and $y_{sp,k} \in \Re^{ny}$ is the output set point that is an additional decision variable of the control problem; assumed to be restricted within a control zone, $y_{\min} \in \Re^{ny}$ and $y_{\max} \in \Re^{ny}$ are the output bounds. $Q_y \in \Re^{ny \times ny}$, $Q_u \in \Re^{nu \times nu}$ and $R \in \Re^{nu \times nu}$ are positive definite diagonal weighting matrices.

The MPC approach assumes the rolling horizon strategy (Maciejowski, 2002), in which it is solved at a time instant *k*, and the input increment corresponding to the first time step of the control horizon value is fed to the real system represented by the state-space model. The new system inputs and outputs are obtained and the procedure is repeated at time instant *k*+1. The MPC formulation used here is a simplified, finite horizon version of the one proposed in (González & Odloak, 2009), for open-loop stable systems.

In order to reduce the computational time required to tune the MPC, an analytical solution to the MPC problem was developed, disregarding constraints (B-19), (B-20), and (B-31), which represent the bounds on the inputs, input increments and outputs respectively, and using y_{sp} as a fixed parameter instead of a decision variable. Basically, we need to write the output prediction and the input values as a function of Δu_k.

Using the incremental state-space model defined in (B-32), the output prediction vector from time instants *k*+1 to *k*+*p* can be written as follows:

$$\begin{aligned} x(k+1) &= Ax(k) + B\Delta u(k) \\ y(k) &= Cx(k) \end{aligned}, \qquad \text{(B-32)}$$

$$\underbrace{\begin{bmatrix} y(k+1) \\ y(k+2) \\ \vdots \\ y(k+m) \\ y(k+m+1) \\ \vdots \\ y(k+p) \end{bmatrix}}_{y^c} = \underbrace{\begin{bmatrix} CA \\ CA^2 \\ \vdots \\ CA^m \\ CA^{m+1} \\ \vdots \\ CA^p \end{bmatrix}}_{\Psi} x(k) + \underbrace{\begin{bmatrix} CB & 0 & \cdots & 0 \\ CAB & CB & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ CA^{m-1}B & CA^{m-2}B & \cdots & CB \\ CA^m B & CA^{m-1}B & \cdots & CAB \\ \vdots & \vdots & \ddots & \vdots \\ CA^{p-1}B & CA^{p-2}B & \cdots & CA^{p-m}B \end{bmatrix}}_{\Theta} \underbrace{\begin{bmatrix} \Delta u(k) \\ \Delta u(k+1) \\ \vdots \\ \Delta u(k+m-1) \end{bmatrix}}_{\Delta u_k}$$

or, $y^c = \Psi x(k) + \Theta \Delta u_k$ (B-33)

where $y^c \in \Re^{ny \cdot p}$, $\Psi \in \Re^{ny \cdot p \times nx}$, $\Theta \in \Re^{ny \cdot p \times nu \cdot m}$; $\Delta u_k \in \Re^{nu \cdot m}$, is defined as a vector containing the stacked input increments up to the control horizon. The input values can be written in terms of their increments as follow:

$$\underbrace{\begin{bmatrix} u(k|k) \\ u(k+1|k) \\ \vdots \\ u(k+m-1|k) \end{bmatrix}}_{u^c} = \underbrace{\begin{bmatrix} I_{nu} & 0 & \cdots & 0 \\ I_{nu} & I_{nu} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ I_{nu} & I_{nu} & I_{nu} & I_{nu} \end{bmatrix}}_{\bar{M}} \underbrace{\begin{bmatrix} \Delta u(k|k) \\ \Delta u(k+1|k) \\ \vdots \\ \Delta u(k+m-1|k) \end{bmatrix}}_{\Delta u_k} + \underbrace{\begin{bmatrix} I_{nu} \\ I_{nu} \\ \vdots \\ I_{nu} \end{bmatrix}}_{\bar{I}_{nu}} u(k-1) \qquad \text{(B-34)}$$

or, $u^c = \bar{M}\Delta u_k + \bar{I}_{nu}u(k-1)$ (B-35)

where $u^c \in \Re^{nu\cdot m}, \bar{M} \in \Re^{nu\cdot m\times nu\cdot m}, \bar{I}_{nu} \in \Re^{nu\cdot m\times nu}$, I_{nu} is a $nu \times nu$ identity matrix. Finally, the modified MPC control problem can be written in terms of Δu_k using (B-34) and (B-35).

$$\min_{\Delta u_k} \left\| \Psi x(k) + \Theta\Delta u_k - \bar{I}_{ny} y_{sp} \right\|_{\bar{Q}_y}^2 + \left\| \Delta u_k \right\|_{\bar{R}}^2 + \left\| \bar{M}\Delta u_k + \bar{I}_{nu}u(k-1) - \bar{I}_{nu}u_{des} \right\|_{\bar{Q}_u}^2 \qquad \text{(B-36)}$$

$\bar{I}_{ny} = \underbrace{\begin{bmatrix} I_{ny} \\ \vdots \\ I_{ny} \end{bmatrix}}_{p}$, $\bar{I}_{ny} \in \Re^{p\cdot ny\times ny}$, I_{ny} is a $ny \times ny$ identity matrix. It has the following analytic solution:

$$\Delta u_k^* = -K\left\{\Theta^T\bar{Q}_y\left(\Psi x(k) - \bar{I}_{ny}y_{sp}\right) + \bar{M}^T\bar{Q}_u\bar{I}_{nu}\left(u(k-1) - u_{des}\right)\right\} \qquad \text{(B-37)}$$

$K = \left(\Theta^T\bar{Q}_y\Theta + \bar{R} + \bar{M}^T\bar{Q}_u\bar{M}\right)^{-1}$, $\bar{Q}_y = diag\left[\underbrace{Q_y \ \cdots \ Q_y}_{p}\right]$, $\bar{Q}_y \in \Re^{p\cdot ny\times p\cdot ny}$,

$\bar{Q}_u = diag\left[\underbrace{Q_u \ \cdots \ Q_u}_{m}\right]$, $\bar{Q}_u \in \Re^{m\cdot nu\times m\cdot nu}$, $\bar{R} = diag\left[\underbrace{R \ \cdots \ R}_{r}\right]$, $\bar{R} \in \Re^{m\cdot nu\times m\cdot nu}$.

Infinite Horizon MPC

The IHMPC controller used in this thesis was built upon the work of Santoro & Odloak (2012). Differently from the DMC algorithm, the output predictions are calculated using a state-space model, which is not as intuitive as the step response model, but it makes the control algorithm computationally faster. Moreover, the IHMPC is closed-loop stable in the absence of plant-model mismatch.

Model representation

The controller assumes that the system is represented by a state-space model. $G_{i,j}(s)$ is a transfer function between the input-output pair (y_i,u_j) in the Laplace domain.

$$G_{i,j}(s) = \frac{b_{i,j,0} + b_{i,j,1}s + \ldots + b_{i,j,nb}s^{nb}}{s(s - r_{i,j,1})(s - r_{i,j,2})\ldots(s - r_{i,j,na})} e^{-\theta_{i,j}s}, \tag{B-38}$$

nb is the numerator order, na is the denominator order, and $r_{i,j,l}$, $l = 1,\ldots,na$; $i = 1,\ldots,ny$; $j = 1,\ldots,nu$, are the unrepeated poles of $G_{i,j}(s)$. Then, the step response corresponding to the transfer function defined in (B-38) is defined in (B-39). In the DMC algorithm, all the coefficients of the step (impulse) response were stored over the model horizon to predict the outputs; here only the coefficients $d^0_{i,j}$, $d^d_{i,j,k}$, $d^i_{i,j}$, and $r_{i,j,k}$ $i = 1,\ldots,ny$; $j = 1,\ldots,nu$; $k = 1,\ldots,na$ are necessary.

$$\frac{G_{i,j}(s)}{s} = \frac{d^0_{i,j}}{s} e^{-\theta_{i,j}s} + \frac{d^d_{i,j,1}}{s - r_{i,j,1}} e^{-\theta_{i,j}s} + \ldots + \frac{d^d_{i,j,na}}{s - r_{i,j,na}} e^{-\theta_{i,j}s} + \frac{d^i_{i,j}}{s^2} e^{-\theta_{i,j}s} \tag{B-39}$$

The $x(k)$ vector is comprised of four groups: the states introduced by the incremental form ($x^s(k)$); the integrating ($x^i(k)$) and stable states ($x^d(k)$) of the original system; and the states representing the past input increment values ($z_i(k)$, $i = 1,\ldots,\theta_{\max}$). $\theta_{\max}$ is the maximum dead-time of the system.

$$x(k) = \begin{bmatrix} x^s(k)^T & x^d(k)^T & x^i(k)^T & z_1(k)^T & z_2(k)^T & \cdots & z_{\theta_{\max}}(k)^T \end{bmatrix}^T \tag{B-40}$$

$$z_j(k) = \Delta u(k - j) \tag{B-41}$$

The system then is written as a state-space model, as in (B-32). The matrices *A*, *B* and *C* are arranged accordingly, so that the analytical step response coefficients are stored correctly with respect to each state group. The reader is referred to (Santoro & Odloak, 2012) for detailed information.

Control algorithm

The IHMPC cost function is more complex because, besides the classic control goals (minimize the tracking error and control effort) other ones such as input targets and output zone control are considered. Slack variables are included in Equations (B-43) and (B-44) to ensure that the additional stability constraints do not render the control problem infeasible. Simultaneously, the slack variables are heavily weighted in the control cost function, so that they are not used unless it is necessary. The IHMPC control problem, hereafter addressed to as Problem 6, is described by Equations (B-42), (B-43), (B-44), the state-space model and the bounds on the input (B-19) and input increments (B-20).

$$\min_{\Delta u_k,\delta_{y,k},\delta_{i,k}} V_{6,k} = \sum_{j=0}^{\infty}\left\| y(k+j\,|\,k) - y_{sp} - \delta_{y,k} - jT_s\delta_{i,k}\right\|_{Q_y}^2 + \sum_{j=0}^{m-1}\left\|\Delta u(k+j\,|\,k)\right\|_R^2 + \sum_{j=0}^{m-1}\left\|u(k+j\,|\,k) - u_{des}\right\|_{Q_u}^2 + \left\|\delta_{y,k}\right\|_{S_y}^2 + \left\|\delta_{i,k}\right\|_{S_i}^2 \tag{B-42}$$

subject to

$$x^s(k+m+\theta_{\max}\,|\,k) - y_{sp,k} - \delta_{y,k} = 0 \tag{B-43}$$

$$x^i(k+m+\theta_{\max}\,|\,k) - \delta_{i,k} = 0 \tag{B-44}$$

and (B-19), (B-20), (B-32).

$\delta_{y,k} \in \Re^{ny}$ *and* $\delta_{i,k} \in \Re^{ny}$ are the slack variables. $S_y \in \Re^{ny\times ny}$, $S_i \in \Re^{ny\times ny}$ are weighting matrices, $u_{des} \in \Re^{nu}$ is the input target vector. The infinite summation in (B-42) is replaced by the finite summation up to the control horizon, the constraint (B-43) on the integral states and constraint (B-44) on the incremental states and the terminal cost related to the stable states (Maciejowski, 2002). Santoro & Odloak (2012) provided a formal demonstration of the IHMPC closed-loop stability, resulting from the solution of Problem 6 in two steps. In the first step, the optimum control moves are calculated in order to zero the integrating states $x_i(k)$ and in the second one, the output and input targets are addressed and the value of the sum of control moves obtained in the first step is passed to the second one as an additional constraint. For practical purposes, it suffices to set weight S_i to a very high value (e.g. 10^6) in order to force the zeroing of $x_i(k)$ in the one-step solution of Problem 6 (Santoro & Odloak, 2012).

The IHMPC control problem is convex and, therefore, is solved efficiently using QP algorithms. Observe that if constraints (B-19), (B-20) are disregarded, and if the output zone control reduced to a fixed y_{sp}, it is possible to solve the control problem analytically. The parameters that affect the robustness and performance of IHMPC controllers are m, T_s, Q_y, R, Q_u, S_y and S_i.

Robust IHMPC

The Robust IHMPC developed in (González, Marchetti & Odloak,. 2007) was extended by Martins et al. (2013), using the state-space model developed by Santoro & Odloak (2012), to account for time delayed systems. The time delay states, $z_i(k)$, defined in (Santoro & Odloak, 2012) were merged with the integrating ones to create new states $x_j^i(k)$, $j = 1,\ldots,\theta_{\max}$ for a clearer representation. The model matrices A, B and C were modified to accommodate the state redefinition.

It was assumed the multi-plant model uncertainty, in which L different models are included in a set of models Ω. Each model in Ω is characterized by a unique values of A, B, and C. Model ω_N is the most likely, or nominal model, and ω_T is the 'real' model. Nominal IHMPC applications in the literature consider that $\omega_N = \omega_T$, whereas in real applications it is extremely difficult to obtain ω_T. Therefore, robust MPC applications assume that $\omega_T \in \Omega$. The states $x_j^i(k)$, $j = 1,\ldots,\theta_{\max}$ are unaffected by the model uncertainty because they are defined in terms of the past input increments.

RIHMPC algorithm

$$\min_{\Delta u_k, \delta_{y,k}(\omega_{n=1,\ldots,L}), \delta_{i,k}} V_k(\omega_N) \tag{B-45}$$

$$\begin{aligned} V_k(\omega_N) = \sum_{j=1}^{\infty} & \left\| \begin{matrix} \hat{y}_N(k+j\,|\,k) - y_{sp,k}(\omega_N) - \delta_{y,k}(\omega_N) \\ -(j-m-\theta_{\max}) T_s D^i(\omega_N)\delta_{i,k} \end{matrix} \right\|_{Q_y}^2 \\ & + \sum_{j=0}^{m-1} \left\| \Delta u(k+j\,|\,k) \right\|_R^2 + \sum_{j=0}^{m-1} \left\| u(k+j\,|\,k) - u_{des} \right\|_{Q_u}^2 \\ & + \left\| \delta_{y,k}(\omega_N) \right\|_{S_y}^2 + \left\| \delta_{i,k} \right\|_{S_i}^2 \end{aligned} \tag{B-46}$$

subject to

$$x_n^s(k+m+\theta_{\max}\,|\,k) - y_{sp,k}(\omega_n) - \delta_{y,k}(\omega_n) = 0 \quad n = 1,\ldots,L \tag{B-47}$$

$$x_0^i(k+m+\theta_{\max}\,|\,k) - \delta_{i,k} = 0 \tag{B-48}$$

$$V_k(\omega_n) \le \tilde{V}_k(\omega_n) \quad n = 1,\ldots,L \tag{B-49}$$

and (B-19), (B-20), (B-32).

Equations (B-43) and (B-47) are equivalent and guarantee that the states representing the integrating modes of the original system are zeroed at time instant $\theta_{\max} + m$. Model ω_n, $n = 1,\ldots,L$ is a particular model in Ω. The constraints (B-47), (B-48), and (B-49) are enforced for all the models in Ω, although V_k is minimized considering the nominal model. D^i is an auxiliary matrix containing coefficients of the integrating poles arranged appropriately. The integrating states are defined with respect to the system inputs, and therefore, $S_i \in \Re^{nu \times nu}$. This assumption makes constraint (B-48) possible for all the possible models. However, it restricts the RIHMPC application to a limited group of systems (Martins et al., 2013). Constraint (B-49) enforces that for each model the value of V_k is lower or equal to $\tilde{V}_k$, which is calculated using a feasible, but not necessarily optimum, solution of the control problem at time instant *k*, obtained through a slight modification of the optimum solution obtained at time instant *k*-1. Since constraint (B-49) is enforced for all ω, $\omega \in \Omega$, it leads to a robust closed-loop system. The reader is referred to (Martins et al., 2013) for detailed explanation. The authors also demonstrate the two-step solution stability in the same fashion as in (Santoro & Odloak, 2012) and again, for practical purposes, setting S_i to high values (e.g. 10^6) is sufficient to accomplish asymptotic stability.

Since the constraint (B-49) is nonlinear, the RIHMPC control problem cannot be solved as a QP. Its solution calls for a nonlinear programming solver, usually available in commercial control and optimization software. The control performance of the RIHMPC is affected by m, T_s, Q_y, R, Q_u, S_y and S_i..

APPENDIX C - Multi-objective optimization

The brief multi-objective optimization review presented here is based on the work by Marler & Arora (2004). The objective is to provide the reader sufficient background in order to understand how the optimization methods can solve the MPC tuning problems proposed in this thesis.

Multi-objective optimization methods solve problems with competing goals. There are two main alternatives to deal with the trade-off between diverging objectives: properly weighting of the objectives prior to the problem solution or choosing an optimum solution based on subjective criteria, from a set of Pareto, or non-dominated optimum solutions. A general multi-objective problem is posed as follows:

$$\min_{x} F(x) = \left[F_1(x) \quad F_2(x) \quad \cdots \quad F_w(x) \right]^T \tag{C-1}$$

subject to

$$g_j(x) \leq 0, \; j = 1, \ldots, z \tag{C-2}$$

$$h_l(x) = 0, \; l = 1, \ldots, e \tag{C-3}$$

where $F(x)$ is a vector comprised of w objectives $F_i(x)$. Functions $g_j(x)$ and $h_l(x)$ are related with the inequality and equality constraints, respectively, $x \in \Re^{n_{dec}}$ is the vector of decision variables and n_{dec} is the number of decision variables. The feasible design space is defined as $\mathbf{X} = \left\{ x \in \Re^{n_{dec}} \mid g_j(x) \leq 0, j = 1, \ldots, z \text{ and } h_l(x) = 0, l = 1, \ldots, e \right\}$, and the feasible criterion space is defined as $\mathbf{Z} = \left\{ z \in \Re^{w} \mid z = F(x), x \in \mathbf{X} \right\}$. The objectives $F_i(x)$ are defined in terms of preferences, imposed by the decision-maker.

Pareto optimality

The following statements characterize the optimum solutions in the multi-objective optimization problem.

Definition 1: A point $x^* \in \mathbf{X}$ is a Pareto optimum *if and only if* there does not exist another point $x \in \mathbf{X}$, such that $F(x) \le F(x^*)$, and $F_i(x) < F_i(x^*)$ for at least one *i*.

Definition 2: A vector of objective functions, $F(x^*) \in \mathbf{Z}$, is non-dominated *if and only if* there does not exist another vector, $F(x) \in \mathbf{Z}$, such that $F(x) \le F(x^*)$ with at least one $F_i(x) < F_i(x^*)$. Otherwise, $F(x^*)$ is dominated.

Definition 3: A point $F^\circ(x) \in \mathbf{Z}$ is an Utopia point *if and only if* for each $i = 1,\ldots,w$ then $F_i^\circ(x) = \min_x \{F_i(x) \mid x \in \boldsymbol{X}\}$.

Definition 4: A lexicographic minimum is defined as a solution in which an objective F_i can be reduced only at the expense of increasing one of the higher-prioritized objectives $\{F_1,\ldots,F_{i-1}\}$ and the lexicographic minimizer is the corresponding set of decision variables (Kerrigan & Maciejowski, 2002).

Every multi-objective optimization optimum solution lies on a frontier of ***Z***, defined as the Pareto curve or the Pareto frontier. However, one might be interested in finding the compromise solution, which is defined as the solution $F(x_c^*)$, $x_c^* \in \mathbf{X}$, which is the closest to $F^\circ(x)$.

The multi-objective optimization techniques are divided into two main groups, according to how the objectives are considered and how the final solution is chosen: *a priori* or *a posteriori*.

A priori articulation of preferences

In this approach, appropriate weights are used to combine the multiple objectives into a single cost function; or the multi-objective problem is solved addressing one objective at a time. The most popular variant in the first group is the weighted sum method, although the weighted product and exponential methods are also common. The second group includes the lexicographic optimization method, in which goals are arranged in order of importance, and the problem is solved following a sequence of steps.

All the methods mentioned above produce a single optimum solution, which may or may not be a Pareto optimum. The Pareto optimality conditions vary according to the problem formulation and weight values.

A posteriori articulation of preferences

Here, a decision-maker selects the most attractive solution from a set of Pareto solutions. Usually. varying the weights of *a priori* weighed sum solution strategy and solving the optimization problem repeatedly to calculate the Pareto set is not efficient, because the relationship between the weighting parameters and the optimum solutions in ***Z*** is not straightforward (Das & Dennis, 1997). Therefore, different methods, such as the Normal Boundary Intersection (NBI), the Normal Constraint (NC) (Das & Dennis, 1998), and the physical programming (Messac & Wilsont, 1998; Messac, 1996) were developed. These approaches were shown to yield well distributed, accurate and evenly ranged solutions along the Pareto frontier.

It is still an arduous task for the decision-maker to pick a good solution among the Pareto solutions in problems with more than three objectives, because it is not possible to represent it graphically; also, even in two-dimension problems, one must carefully examine all the possibilities. Therefore, *a posteriori* methods are only recommended for small multi-objective problems (Marler & Arora, 2004).

No articulation of preferences

The evolutionary algorithms belong to a different class of solvers that do not require an articulation of preferences. These methods were developed based on the biological evolution of a population where a group of randomly generated solutions is driven closer and closer to an optimum solution throughout the solver iterations. The genetic algorithms are attractive because they do not require mathematical information of the objective function (e.g. continuity and differentiability). However, depending of the problem size, it might become computationally expensive.

Although the genetic algorithms and the Pareto optimization theory do not share any theoretical background, the solution of a genetic algorithm is likely to converge to a Pareto optimum.

Further discussion on specific multi-objective optimization techniques

Das & Dennis (1997) reported several weaknesses of the weighted sum technique. The interior points of a non-convex Pareto curve cannot be easily obtained by weighting the combinations of the vectors corresponding to the individual objectives. On the other hand, if the Pareto curve is convex, evenly spaced sets of weighting parameters fail to produce evenly spread solutions. Moreover, the authors stated that it is impossible to accurately define weighting sets that actually lead to an evenly spread Pareto curve, unless its shape is known *a priori*. The weight-based multi-objective optimization methods suffer from the inner loop limitation, in which the weights are selected iteratively. Depending of the problem complexity, its solution might be impractical in real applications (Messac, 1996).

In (Das & Dennis, 1998), the authors developed a novel method to select the multi-objective optimization weights so that the solutions are evenly spread along the Pareto curve. The method, named Normal-Boundary Intersection (NBI), minimizes the distance between a frontier of attainable solutions and a normal vector originating from the points located in the set of convex combinations of the differences between the non-dominated solutions and the Utopia solution.

The goal programming strategy is a particular application of the NBI method, in which the search region is limited to the Pareto optimum, which is the closest to a targeted solution. The technique is applicable to large problems as well, however, selecting a final optimal solution from a Pareto set with more than three dimensions (objectives) is a challenging task by itself. According to Messac & Mattson (2002), the NBI method might obtain non-dominated solutions along its search process, which leads to higher computational cost.

Messac (1996) developed *a priori* multi-objective optimization approach named Physical Programming. The objective functions are defined in terms of three classes, according to whether the objective values should be minimized, maximized, or kept at a constant value. The classes are further segmented to precisely define tolerable zones for each objective. Finally, once all the objective zones and constraints are properly defined, a cost function comprised of the weighted sum of the objectives is minimized.

Tind & Wiecek (1999) solved multi-objective optimization problems calculating the dual augmented Lagrangian of the Tchebychev-norm between the objective functions and a pre-defined ideal point. Non-concave Pareto curves were addressed by the lexicographic Tchebychev approach to ensure optima efficiency. The authors claim that the weighted sum multi-objective optimization approach is cumbersome for large problems, whereas their approach is straightforward. Nonetheless, the latter do not provide a realistic utility function to distinguish between weakly non-dominated and non-dominated solutions. Therefore, the authors suggest using the Tchebychev approach to obtain the optimum points and to use the quadratic weighted norm to evaluate their utility function values.

Messac & Mattson (2002) applied the Physical Programming strategy (Messac, 1996) as an *a posteriori* multi-objective optimization method. The Physical Programming weights were initially selected as a pseudo-preferences set, leading to an initial optimal solution. In the next iteration, the pseudo-references values are shifted slightly and the optimization problem is solved again, until sufficient Pareto solutions have been obtained. The authors compared the novel approach to the weighted-sum and the exponentially-weighted sum methods with respect to accuracy, spread and distribution of the optima. The results showed that their method yields evenly spread, well-distributed, and accurately placed points throughout the whole Pareto curve. The authors concluded that their approach is as efficient as the NBI method (Das & Dennis, 1998).

Gatti & Amigoni (2005) developed *a priori* multi-objective optimization approach based on cooperative negotiation and bargaining models, which solves a cooperative problem using a competitive algorithm to improve its efficiency. The sequential bargaining model developed by the authors was shown to converge to a Pareto optimal. The mathematical proof was supported by experimental results, which yielded solutions close to the real Pareto curve. The technique is presented as the first step towards an integrated technique, capable of solving more complex, dynamic multi-objective problems.

Okabe, Jin & Sendhoff, (2003) surveyed performance metrics to evaluate *a posteriori* multi-objective optimization methods. From their survey, the Euclidian distance between solutions or the volume occupied by an *n*-dimensional polytope defined by the solutions are commonly used to assess the multi-objective algorithm efficiency. The authors tried to identify a performance index capable of evaluating the performance of a set of Pareto solutions individually or to allow for a comparison between two or more sets. The authors concluded that there is no single performance index capable of accomplishing such task and suggested a combination of two or more metrics to address the problem.

Lexicographic or hierarchical optimization

The lexicographic optimization approach works well when the optimization structure is better represented by an objective function ranking instead of a scalar-value objective function (Luptáčik & Turnovec, 1991). Also, Tind & Wiecek (1999), stated that it is difficult to accurately represent the goals of the decision maker, which is fundamental to select a single solution from a set of non-dominated solutions (Messac & Mattson, 2002). However, for Gambier (2008), the lexicographic methods are computationally straightforward, but the low priority objectives may not be satisfied.

Waltz (1967) pointed out that translating a real world problem and its numerous cost and constraint factors into a mathematical problem is a pressing issue for multi-objective optimization solvers. Considering the dynamic nature of the weighting factors in the MPC literature, the author recommended *a posteriori* solution based on the weighted-sum approach. A hierarchical multi-objective optimization technique was used to solve a control problem with two objectives. Such approach is recommended for the case where one of the objectives is very important, but the remaining ones are also significant. The lexicographic solution yielded a better overall control performance through the relaxation of the optimum control profile obtained with a single-objective approach. The author stated that even though the hierarchical optimization is based on the solution of a sequence of optimization problems, it might be computationally faster than an *a priori* approach.

Luptáčik & Turnovec (1991) transformed the lexicographic optimization framework into a convex geometric programming problem, through the definition of a Lagrange function that includes the problem constraints and objectives, as well as the appropriate weighting parameters. Its solution leads to a proper lexicographical optimum.

The lexicographic optimization approach requires extensive process knowledge to define the tuning goals. Rentmeesters et al. (1996) stated that the Kuhn-Tucker conditions for optimality do not define a lexicographic optimum unless the Utopia solution is feasible. By relaxing the Kuhn-Tucker conditions using weighting vectors in the constraint functions, the authors formally described the characteristics of a lexicographic optimum.

The lexicographic optimization was used by Kerrigan et al. (2000) and Kerrigan & Maciejowski (2002) to solve a multi-objective control problem. The authors identified some weakness of the lexicographic method concerning the fulfillment of lowly prioritized goals, and they showed how important it is to choose the goals correctly.

The lexicographic optimization framework implemented by the authors assumes that the goals and their respective ranking are defined by the user. At each step of the tuning process, a single objective optimization problem is solved, where a single goal is addressed following the ordering of importance. In the subsequent steps, the previously obtained optimum cost function value is included as a constraint in the new optimization problem. The latter addresses a new, less important goal while preserving the performance of the previous, more important goals.

Compromise optimization

The compromise optimization approach also requires weights to address both scaling and preference issues (Messac, 1996), and from Ballestero & Romero (1991), the compromise optimum is also a non-dominated solution, but the compromise method is attractive because in real cases it is reasonable to assume that the decision-maker seeks a solution close to the Utopia solution (Zelany, 1974). Gambier (2008) proposed different definitions for the compromise optimum: the Nash, the Kalai-Smorodinsky, and the egalitarian definitions.

Ballestero & Romero (1991) interpreted the compromise solution framework from the economic point of view. The authors considered two-objective problems to develop necessary and sufficient conditions to characterize a compromise optimum as a Lagrangian optimum. They concluded that in several practical applications, the compromise optimization could successfully replace the classic optimization approaches.

Zelany (1974) studied different approaches to reduce the set of Pareto solutions. The author listed drawbacks of the weighted-sum multi-objective optimization method, which drove other authors to reinforce their criticism to this method (Das & Dennis, 1997; Marler & Arora, 2009). Regarding the compromise solution strategy, the author combined mathematical functions and fuzzy criteria to define a distance evaluation metric, which was used either in the cardinal sense (e.g. solving a min-max optimization problem), or in the geometrical sense (e.g. minimizing quadratic or cubic norms). Another comparison metric, similar to the entropy of a system, was recommended when substantial information about the problem is available. After using any of the aforementioned techniques to reduce the number of non-dominated solutions, the author observed that the remaining points were displaced towards the Utopia solution. The process is iterative and it converges to a single solution at the cost of considerable knowledge about the problem. However, it suffices to reduce the original set to a new one containing ‘few enough’ alternatives, from which a final solution is chosen according to subjective criteria.

The compromise optimization approach used here solves a multi-objective optimization problem finding the closest feasible solution, in terms of the Euclidian distance, to the Utopia point. Figure C1 shows the geometric representation of the compromise solution, considering a 2-objective problem.

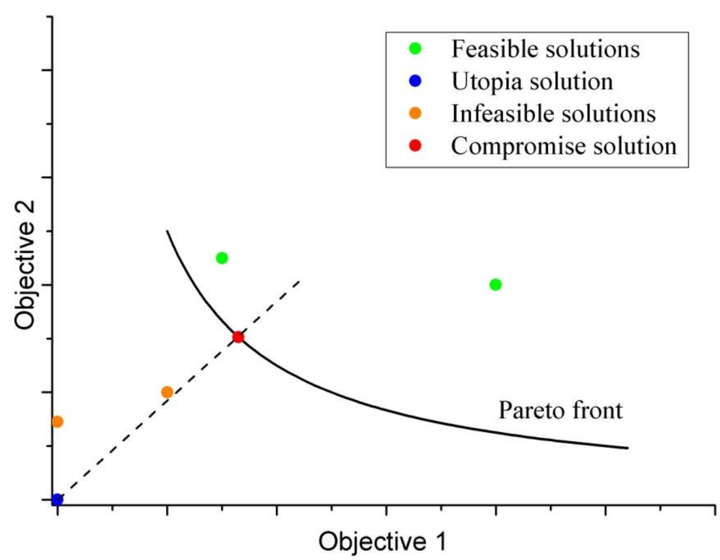

Figure C1: Geometric representation of the compromise optimization method considering two objectives.