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Integration of Real Time Optimization with Model Predictive Control applied to a Gas-lift System: A comparative study.

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Integration of Real Time Optimization with Model Predictive Control applied to a Gas-lifted System: A comparative study.

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And we know that God causes everything to work together for the good of those who love God and are called according to his purpose for them. (Romans 8:28)

For I can do everything through Christ, who gives me strength. (Phil 4:13)

ABSTRACT

There is a rising need for more practical, efficient and sustainable techniques for improving industrial system operation in the face of a highly competitive market. The integration of real time optimization (RTO) and model predictive control (MPC) is a classical approach applied in the industry for improving processes. In this work, we analyze the application of the two most common RTO and MPC integration strategies – one-layer and two-layer – to a gas-lifted system. We analyzed the performance of the economic cost function and efficiency in handling disturbance for each strategy, as well as consider practical industrial application.

In the two-layer strategy, an upper economic optimization layer uses a rigorous nonlinear steady state model to compute the optimal process decision variables and send to the controller as an optimizing target which then computes the optimal control actions to achieve these targets. For this strategy, the hybrid RTO (HRTO) technique is implemented in the upper layer and an established controller - infinite horizon MPC with zone control- in the lower dynamic control layer. The hybrid RTO stems from the modification of the traditional static RTO found in the industry, to deal with the steady state wait time challenge. This is achieved by updating the optimizer with dynamic information rather than the static used in the traditional RTO. The zone control strategy allows the controller to focus on reaching a desired input target supplied by the optimization layer if the outputs are kept within their specified zones and constraints are respected. In the one-layer strategy, the gradient of the economic cost function is included in the controller cost function to be considered when computing the manipulated variable used to achieve optimal process operation. It was proposed with the main aim of practical industrial application.

The two strategies were applied to a gas-lifted system and their results are compared and discussed considering economic objective. The results show that the IHMPC can reach the desired input targets despite abrupt disturbances of the uncertain parameter while keeping the outputs within the desired zone. Therefore, the HRTO can efficiently work with the IHMPC implemented in achieving optimal operation under uncertainties interfering as disturbance. It also shows that the one-layer strategy gives similar results to the two-layer strategy, implying that it can also achieve similar economic objective. However, the two-layer strategy using HRTO technique is more efficient in handling the disturbances.

Keywords: Real-time Optimization (RTO), Model predictive control (MPC), Zone control, Extended Kalman filter (EKF), Online estimation.

RESUMO

Otimização em tempo real (no inglês, RTO) e controle preditivo baseado em modelo (no inglês, MPC) é uma abordagem clássica na indústria para melhorar processos industriais. Nesta dissertação, duas técnicas mais comuns de integração de RTO e MPC – uma e duas camadas – são aplicadas em um sistema de *gas-lift*. Compara-se a performance da função objetivo econômico bem como a rejeição à distúrbios das duas configurações, considerando aplicação industrial prática.

Na configuração de duas camadas, utiliza-se um modelo não-linear rigoroso de estado estacionário em uma camada de otimização econômica, que tem por função calcular variáveis de decisão ótimas para o processo. Estas variáveis são então enviadas para a camada inferior, responsável pelo controle dinâmico do sistema e por atingir os valores ótimos computados na camada superior. Nesta configuração, utiliza-se a técnica de RTO híbrida (do inglês, HRTO) na camada superior e uma estratégia de controle estabelecida na camada inferior – controle por zona de horizonte infinito. HRTO vem de modificações feitas na RTO estática com o propósito de lidar com a necessidade de aguardar o estado estacionário. Isso é feito por atualizações do otimizador com informações dinâmicas ao invés de estáticas, como é feito na RTO tradicional. A estratégia de controle por zona permite ao controlador focar em alcançar um alvo para as variáveis manipuladas uma vez que as variáveis controladas se encontrem dentro de suas zonas operacionais e as restrições do processo sejam satisfeitas. Para a configuração de uma camada, o gradiente da função objetivo econômico é incluso na função objetivo do controlador, para que o objetivo econômico seja considerado ao se computar valores para as variáveis manipuladas que acarretem operação ótima.

As duas configurações foram aplicadas a um sistema de *gas-lift* e seus resultados são comparados com respeito ao objetivo econômico. Os resultados mostram que o MPC alcança os objetivos econômicos mesmo na presença de distúrbios no parâmetro estimado em tempo real concomitantemente mantendo as saídas dentro das zonas operacionais desejadas. Sendo assim, a HRTO opera juntamente com o MPC afim de levar a operação ótima, mesmo com incertezas perturbadas. Fica evidente que a estratégia de uma camada fornece resultados simulares à estratégia de duas camadas, implicando que também maximiza o objetivo econômico. Porém, a estratégia de duas camadas que utiliza a técnica de HRTO é mais eficiente em detectar e responder a distúrbios para este sistema.

Palavras-chave: Otimização em tempo real, controle preditivo baseado em modelo, controle por zonas, filtro de Kalman estendido, estimação em tempo real.

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LIST OF ABBREVIATIONS AND ACRONYMS

CEKF	Constrained Extended Kalman filter
DAE	Differential algebraic equation
DMC	Dynamic matrix control
DRTO	Dynamic real time optimization
EKF	Extended Kalman filter
EMPC	Economic model predictive control
ESP	Electrical submersible pumping
EPSAC	Extended Prediction Self Adaptive control
FCC	Fluid catalytic cracking
GLS	Gas lift system
GOR	Gas to oil ratio
GPC	Generalized predictive control
HRTO	Hybrid real time optimization
IHMPC	Infinite horizon model predictive control
ISOPE	Integrated System Optimization
IPR	Inflow Performance Relationship
IVP	Initial value problem
LDMC	Linear Dynamic Matrix Control
LMI	Linear Matrix Inequalities
MHE	Moving horizon estimation

MPA	Model parameter adaptation
MPC	Model predictive control
MPHC	Model predictive heuristic control
NLP	Non-linear programming
NMPC	Non-linear Model predictive control
ODE	Ordinary differential equation
OPOM	Output prediction oriented model
QDMC	Quadratic Dynamic Matrix control
QP	Quadratic Programming
rEKF	Reduced rank Extended Kalman Filter
ROPA	Real-time optimization with persistent parameter adaptation
RTO	Real- time optimization
SRTO	Static/Steady state real- time optimization
SQP	Sequential Quadratic Programming
SS	Steady state
TPR	Well tubing performance relation
UKF	Unscented Kalman filter
UPC	Unified Predictive control

LIST OF SYMBOLS

A	MPC state transition Matrix
B	MPC matrix relating system inputs and states
C	MPC state space model matrix of outputs
Civ	Valve flow coefficient for the downhole injection valve
Cpc	Valve flow coefficient for the production choke
C_{rh}	Valve flow coefficient for the riser head valve
C_t	Auxiliary vector of the objective function for quadratic programming problems
D^0	Matrix of the coefficients obtained by partial fraction expansion of transfer function
$\widetilde{D}^0$	Matrix of D^0 along the control horizon
D^d	Matrix of residual of the transfer function
$\widetilde{D}^u$	Matrix of I_{nu} along the control horizon
D	Diameter
d_0	Transfer function gain coefficients
e	Output prediction error
F	Matrix with the dynamic stable nodes
$f(x, u, \varsigma)$	Dynamic function
f_{ss}	Steady state function
F_c	System differential function as constraints

F_{eco}	Economic objective function
F_k	EKF Model Linearization matrix
G	Algebraic function constraints
$G(s)$	System transfer function
g_m	Inequality optimization constraints
$h(x, u, \varsigma)$	Output function
H_k	EKF output linearization Matrix at each sampling time
H_a	Height of the annulus
H_{bh}	Height of the bottom-hole
H_w	Height of the well
H_r	Height of the riser
i	Time instants
I_n	Identity matrix in the n dimension
I_{nu}	Auxiliary Identity matrix for formulation of input constraints
I_{ny}	Auxiliary Identity matrix for computation of cost function
I_O	Auxiliary matrix for $I_{nu.m} - M$
J	Economic objective function
J_k	Controller objective function
k	Time steps
K_f	Kalman filter gain

K_p	Process gain
L_a	Length of the annulus
L_{bh}	Length of the bottom-hole
L_w	Length of the well
L_r	Length of the riser
M_w	Molecular weight
M	Auxiliary MPC matrix
m	Control horizon
N	Number of time instants
na	Order of system transfer function
nb	Number of past instants considered for the input
nd	Component dimension ($nd = nu * ny * na$)
nu	Number of inputs
n_w	Number of wells
ny	Number of outputs
P	Pressure
P_r	Reservoir pressure
P_s	Separator pressure
PI	Reservoir production index
p	Prediction horizon

P_k	Estimated covariance matrix
Q_ς	Covariance matrix of uncertain parameters
Q'_k	Covariance matrix for Gaussian noises
R_k	Covariance matrix for Gaussian noises
Q_y	Controller output weighting Matrix
Q_u	Controller input weighting matrix
$\bar{Q}$	Controller states weighting matrix along the control horizon
$\bar{R}$	Controller input weighting matrix
R	Gas constant
R_k	EKF covariance matrix for output random Gaussian noises
S	Slack variables weighting matrix
S_u	Controller input slack variables weighting matrix
S_y	Controller output slack variables weighting matrix
T	Sampling time
T_a	Temperature in the annulus
T_w	Temperature in the well
T_r	Temperature in the riser
u	Vector of system inputs
u_k	System inputs at time k

U	Vector of Δu
u_{des}	Optimizing input targets computed by the upper optimization layer
u_{max}	Upper inputs constraint
u_{min}	Lower inputs constraint
v_k	Normally distributed measurement noise with zero mean and covariance R
W_{eco}	Weight on the gradient of the economic cost in the controller cost function
wgl_i	Gas lift injection rate for each well (inputs)
w_k	Normally distributed process noise with zero mean and covariance Q
w'_k	Augmented normally distributed process noise with zero mean and covariance Q
x	Process states
x_k	Process states at time k
$\hat{x}_k$	Predicted states at time k
$\hat{x}'_k$	EKF augmented predicted states at time k
x'	EKF augmented states at time
x^s	Output prediction at steady state
x^d	Evolution of process stable nodes
y	Vector of process outputs
y_k	Measured output at time k
$\hat{y}$	Predicted output
y_{min}	Lower limit of the output control zone

y_{max}	Upper limit of the output control zone
y^{sp}	System output set point
$y_{sp,k}$	Output set point computed by the controller at time k
Δu	Control actions (manipulated input moves)
$\Delta\bar{u}$	Vector of total manipulated input moves
Δu_{max}	Maximum admissible control action
ΔU_{min}	Vector containing input action along the control horizon
ΔP_{fric}	Frictional pressure drop
ρ_o	Density of oil
Ψ	Controller state matrix
Φ	Auxiliary matrix for Ψ formation
Θ	Controller input matrix
ς	Uncertain parameters
$\hat{\varsigma}_k$	Estimated parameters
δ	Slack variable
$\delta_{y,k}$	Output slack variable for deviation between outputs and output set-points
$\delta_{u,k}$	Input slack variable for deviation between inputs and optimizing input targets
μ	Viscosity
$\forall_i$	Mapping

γ	Gradient of the economic cost function

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1 INTRODUCTION

1.1 Process industry optimization

The primary objective of industrial processes is to efficiently and safely produce valuable products at the least cost possible. The interconnection of different components with different objectives make these processes inherently complicated. Moreover, there is an immense pressure on the industries - from a highly competitive market, to produce these quality products under strict environments (exogenous and endogenous constraints), at the least cost possible. Therefore, there is an increasing interest in research areas that facilitate better techniques for improving these industrial processes. With the advancement of numerical methods, modeling tools and techniques for handling industrial process complexities; process modeling, simulation, optimization and control are the leading research areas towards handling these large scale process complexities.

Process optimization can basically be described as finding the maximum or minimum point (known as optimum points) of a process with constraints given an objective function. Typically, the optimal point of an operation would likely be found at the overlap of the different system constraints, therefore to safely and optimally operate these complex and mostly large processes, goals need to be met at different time scales (DARBY et al., 2011). The hierarchical approach of process control is widely accepted for managing the production chain in the process industry, it has been successful in facilitating optimal operations through distinction of roles for each layer, ease of tracking faults and communication.

The hierarchical structure of the process control framework can be identified by a functional or temporal decomposition. The control objectives in an order of decreasing consequence are handled by the functional decomposition (i.e. ensuring safe operation, meeting product quality and environmental constraints and ensuring maximum profit is achieved). Temporal decomposition determines the formulation of the control framework with respect to the dynamics of the state variables (DARBY et al., 2011; MENDOZA et al., 2016). Figure 1.1 presents a schematic of the process control hierarchy.

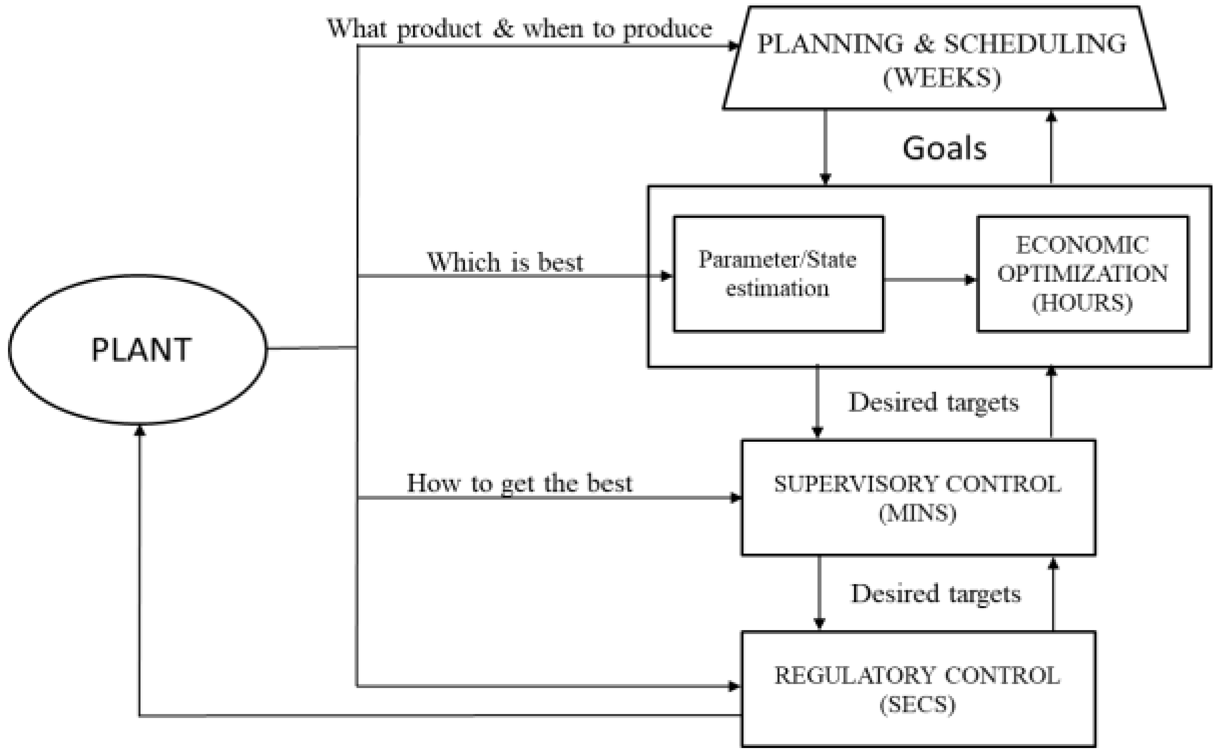

Figure 1.1 - Process optimization decision hierarchy

Source: (own elaboration)

As seen in Figure 1.1, the planning and scheduling layer of the decision hierarchy determines the organizations objectives such as product, quality, production schedules, coverage and expenses and making available the resources required. The optimization and control layer work towards implementing and achieving the given objectives considering the system dynamics, constraints and disturbances. The optimization and control layer can be said to be the core of the process, here the set-points for optimal operations are computed by the optimization layer (using a rigorous nonlinear stationary plant model) and sent to the control layers, where the input trajectories (control actions) to achieve and maintain the trajectories of the given optimum set-points, are computed (supervisory control) and implemented (regulatory control), with consideration to the system constraints (ENGELL, 2007; DARBY et al., 2011; GAO; WENZEL; ENGELL, 2016; DAOUTIDIS et al., 2018; AHMAD; GAO; ENGELL, 2019).

1.2 Real-time optimization and Model predictive control (RTO&MPC)

The optimization and control theory presents fundamental mathematical basis, whereas automation and control systems enable implementation, monitoring and verification of processes (DAOUTIDIS et al., 2018). Real-time optimization and the model predictive control are both online optimization techniques that can handle large scale processes with constraints. They bring about efficiency in production while considering process disturbances. Since the introduction of the RTO in the 1980's, it has improved the economic efficiency of industrial production as compared to when the controller is burdened with optimizing and controlling the processes. The real-time optimizer - finds specifically and reliably the optimal operating point, within or at boundary intersections in the presence of disturbances and process nonlinearities. Although this has brought about some challenges (structural and hierarchical), it has been a welcomed improvement for some complex processes. And ongoing research continues to reinvent the online optimization techniques for increased efficiency.

The scope of this study covers the optimization and supervisory control layer; it presents the application of an improved process optimization and control techniques. Existing RTO technique is modified to handle some of its challenges with respect to the system under consideration, it is then implemented with stable linear model predictive controller. The control strategy has a guaranteed stability and feasibility for a nominal case. The integration of this modified RTO method with MPC will be studied and the results evaluated for optimal operations considering economic profits in the presence of uncertainties.

1.3 Motivation and Objective

1.3.1 Motivation

Understanding the needs of large scale industrial processes and testing new ideas to improve these processes, so as to achieve optimal operations, can be very risky, time consuming and expensive. Likewise, there is progression in the development of tools and techniques for improving processes to meet the increasing production demands. Industries continue to invest huge amounts into researching and designing these tools/techniques.

The traditional RTO and MPC are established technologies that have been successfully used in industries, however, application comes with challenges. Many of the new methods presented to handle these challenges are mostly theoretically and have not been successfully implemented. As most researchers thrive towards developing new methods for improving dynamic processes, it will also be beneficial to understand the limits of the existing established methods and find ways to improve these limitations to fit peculiar problems, rather than using the new complicated methods for systems that might have been easier solved with the existing methods. Researchers have been working on ways to improve the RTO and manage the challenges which (DARBY et al., 2011; GAO; WENZEL; ENGELL, 2016; MENDOZA et al., 2016) described in details.

A technique was proposed by KRISHNAMOORTHY; FOSS; SKOGESTAD, (2018) and MATIAS; LE ROUX, (2018), called ‘Hybrid Real Time Optimization (HRTO)’ and ‘Real-time optimization with persistent parameter adaptation (ROPA)’ respectively to curb the most limiting challenge of the traditional RTO which is the steady state wait time. Using online parameter estimation to update the optimizer, more reliable information is used for the steady state optimization. This technique eliminates the need for steady state detection and allows for a more frequent optimization. Therefore, the process is not operated sub optimally when there are changes in parameters. The RTO uses a rigorous nonlinear model to find the optimal decision variable for the system and sends it down to the MPC as a desired target or set-point, the MPC then finds and maintains the trajectory to achieve this optimum. The interaction between the RTO and the MPC continues to pose questions, due to the possible conflict in their models, objectives and implementation time scale.

The main contribution of this work is, the application of the two commonly used RTO/MPC integration strategies to a gas-lifted system. Considering application to real systems, we use practically implemented optimization tools, and we compare the results obtained with respect to achieving optimal operations and maximizing revenue. This can help in understanding the extent of RTO benefits to processes, especially slow processes with parameters that are dynamic and can disturb the optimal operating point. Furthermore, the implementation of the RTO with a practical MPC that have been applied to a complex process as the Fluid catalytic cracking (FCC), on another complicated process as gas-lifted system demonstrates the efficiency and robustness of these existing optimization tools. It shows that while research in this area is ongoing and new ideas are proposed to solve existing gaps in the industry - especially as it relates to models capturing data accurately enough to be feasible practically. Some modifications can be made on existing tools and techniques to address some of the challenges, as they are more realistic for large-scale processes.

In this study, the novel hybrid RTO is implemented with a zone control infinite horizon MPC and applied to a gas-lifted oil production system network. For the production of oil and gas at the upstream, daily production optimization is required to find the optimal decision variables that can efficiently operate the production process and maximize revenue. The utilization of mathematical models to optimize the process performance is typical, moreover these models predict the outcomes of production decision variables such as choke and valve openings or gas-lift rates. Oil and gas production system constitute multiphase flow and pipelines pressure drops, therefore modelling such system can be quite complex, with significant uncertainties (KRISHNAMOORTHY; FOSS; SKOGESTAD, 2016). In trying to avoid too much complications, the process models could be oversimplified and some parameters ignored or their impact minimized. This can affect the quality of optimization performed significantly, as the optimal solutions computed could be infeasible in real systems or the system would be operated sub-optimally. Therefore, there is a need to perform online optimization with models that are not too complex but accurate and efficient enough to be applied in real processes.

The gas-lifted system is one of such complex systems that requires less complicated but efficient online optimization tools and techniques. This system is suitable because its optimal decision variable changes as the system encounters disturbance. This brings about uncertainty especially when the disturbance becomes quite frequent, hence there is need to consider these disturbances in the optimization models for the system. Furthermore, it has so many outputs to be manipulated with fewer inputs, which fits into the kind of problem the zone control MPC is used to solve. With the main objective of maximizing revenue, the RTO/MPC techniques applied in this work can aid the process achieve optimal operations, very close enough to what is obtainable with the dynamic optimizations. Furthermore, in the study, the application of the one-layer RTO/MPC strategy (a strategy developed for the purpose of practical large-scale process application) to the gas-lifted system is done. Overall, we compare and discuss the two-layer

RTO/MPC integration strategy and the one-layer RTO/MPC integration as applied to the gas-lifted system.

1.3.2 Objective

The main objective of this study is to apply the two RTO/MPC integration strategies commonly found in the industries to a gas-lifted system and compare their optimal operational objectives.

1.4 Dissertation Overview

This dissertation containing six (6) chapters is structured as follows:

Chapter 1 introduces the scope of the research work, lays out the motivation, objectives and structure of this research.

Chapter 2 gives a review of the literature, presenting an overview of traditional RTO and its shortcomings, steps taken to overcome these shortcomings; an overview of MPC structure and algorithms. Also, here we present the different RTO/MPC integration strategies commonly applied in the industry.

Chapter 3 describes methodology in details, showing the underlying concept and the mathematical preliminaries under consideration; it describes the implementation of novel hybrid optimization as well as the integration of RTO with zone control infinite horizon MPC (IHMPC) in two major strategies considered.

Chapter 4 describes the process under consideration in detail, this involve the process description, and process modeling and assumptions made, process optimization and then discusses the process simulation and results obtained.

Chapter 5 gives a comparison between the two-layer strategy and the one-layer strategy applied to the system under consideration

Chapter 6, conclusions are drawn from the result analysis, and insights with suggestion on future works is provided.

2 LITERATURE REVIEW

The quest for the improving industrial processes continues to evolve through the years, from the coming of optimal control in the 1960s to the advanced control and the introduction of technologies such as model predictive control (MPC). These developments were to tackle specific optimization and control needs, such as maintaining process trajectories in the presence of disturbances, considering their multi-objectives and operating constraints. The advanced controllers also enabled the economical optimization of these processes online (LI; QI, 2010; TRAN; LINGA; MACIEJOWSKI, 2014; MATIAS; LE ROUX, 2018).

The development of the real-time optimization (RTO) in the mid-1980s, where rigorous steady state process models are utilized to achieve economic optimization provided a clear separation, between the optimization and the control of processes (ENGELL, 2007; DARBY et al., 2011). Researchers continue to develop new techniques, and also improve existing techniques to meet the increasing optimization demands of the industry.

2.1 Real-time optimization (RTO)

RTO (Real-time Optimization) is a successful process automation technology for optimal operations, it is concerned with implementing economic decisions in real time based on attuned non-linear models.

Traditional RTO uses a rigorous nonlinear steady-state models from the fundamental first principles equations such as hydraulic effects or reaction kinetics, multi-component mass and energy balances, vapor–liquid equilibrium expressions etc. It is used only where it is economically viable and justified as it may not be suitable for all continuous processes (ADETOLA; GUAY, 2010; DE SOUZA; ODLOAK; ZANIN, 2010; DARBY et al., 2011). Engell, (2007) suggested that the RTO is a “well-established” optimization approach which can be used to create a connection between economic optimization and regulatory control. The author further describes RTO as “a model based upper-level control system that is operated in closed loop and provides set-points to the lower-level control systems in order to maintain the process operation as close as possible to the economic optimum”. Although there are other online optimization approaches in the literature, the traditional RTO also known as Model Parameter

Adaptation (MPA) or Static Real-time optimization (SRTO), is commonly found in the process industry (ENGELL, 2007; DARBY et al., 2011).

The traditional RTO is implemented based on the assumption that the disturbances and dynamics of transient measurements can be neglected if the time at which the optimization is executed is long enough; the process can be assumed to be at steady-state (ENGELL, 2007; ADETOLA; GUAY, 2010). Basically, the SRTO can be described by three main steps briefly summarized below:

- Steady-state detection and data pre-processing – the first step is to detect the steady state. The steady-state detection is based on the analysis of the data obtained from the process. It identifies if the system is operating at or close enough to steady-state. This is a very important step which sets the RTO cycle, since the RTO uses a rigorous steady state model.
- Static Parameter estimation: if steady state is confirmed, measurement data are reconciled mostly based on material and energy balances to sort out unreasonable erroneous data and compensate for systematic errors. The reconciled model parameters are then updated to match current data at operating point, using regression techniques. Significant knowledge of the process is required to decide on the crucial parameters to be updated.
- Static Optimization: Given an economic objective function, system constraints and an updated model with the reconciled values, the optimum set-points are computed using mathematical optimization methods (ENGELL, 2007; KRISHNAMOORTHY; FOSS; SKOGESTAD, 2018).

Th summary of the steps described is given by Figure 2.1:

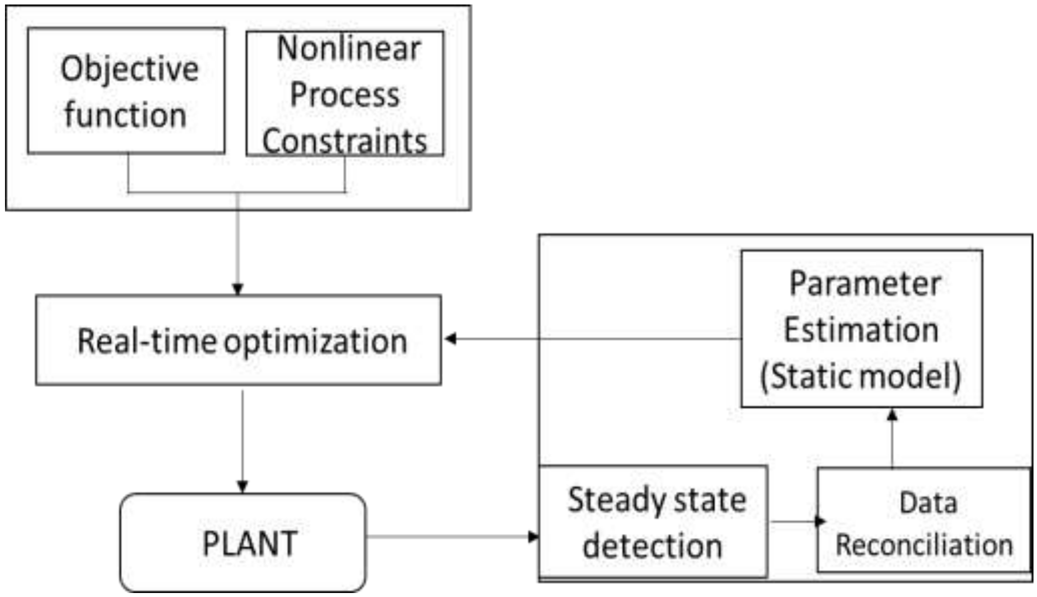

Figure 2.1 - Traditional Real time optimization

Source: (own elaboration)

The mathematical formulation based on KRISHNAMOORTHY; FOSS; SKOGESTAD, (2018) are presented in equations (2.1) - (2.5):

Consider a discrete time system:

$$x_{k+1} = f(x_k, u_k, \varsigma_k) \tag{2.1}$$

$$y_k = h(x_k, u_k) \tag{2.2}$$

Where $x_k \in \mathbb{R}^{n_x}$ are the states, $u_k \in \mathbb{R}^{n_u}$ are the process inputs and $y_k \in \mathbb{R}^{n_y}$ are the process measurements at time step k. The model contains a set of time varying parameters and disturbances represented as $\varsigma_k = [p_k^T, d_k^T] \in \mathbb{R}^{n_\varsigma}$. The static part of the model is given by:

$$y = f_{ss}(x, u, \varsigma) \tag{2.3}$$

Where f_{ss}: $\mathbb{R}^{n_x} \times \mathbb{R}^{n_u} \times \mathbb{R}^{n_\varsigma} \rightarrow \mathbb{R}^{n_y}$ describing the static input-output mapping

Step 1: Parameter Estimation

$$\hat{\varsigma}_k = arg\min_{\emptyset} \|y_{meas} - f_{ss}(x_k, u_k, \varsigma)\|_2^2 \tag{2.4}$$

Step 2: Static Optimization

$$u^* = arg\min_u J(y,u) \quad (2.5)$$

$$\text{s.}t.\ x_k = f_{ss}(u, \hat{\varsigma}_k)$$

$$g(y,u) \leq 0$$

Where $y_{meas} \epsilon \mathbb{R}^{n_y}$ denotes the measurements from the plant $J: \mathbb{R}^{n_u} \times \mathbb{R}^{n_y} \rightarrow \mathbb{R}$ describes the objective function, $g: \mathbb{R}^{n_u} \times \mathbb{R}^{n_y} \rightarrow \mathbb{R}^{n_c}$ describes vector of nonlinear constraints that may be imposed and ς are the system parameters.

In process optimization the RTO requires an accurately performing MPC to be successfully implemented.

RTO Challenges

As stated earlier in the introduction, SRTO is a technology that have been successfully implemented in process industry. However, it comes with limitations and challenges which reduces its applicability. These challenges include:

- Model uncertainties /mismatch such as lack of process measurements, measurement noises and structural uncertainties.
- System identification and parameter update challenges due to frequent data changes, this reduces the relevance of steady-state optimization as the system optimum point would have moved from the steady-state being computed as at the time it is being sent as set points.
- Dynamic limitations such as infeasibility due to constraint violation.
- Low frequency of set-points update and miscommunication with the control layer due to different time scales and models used (ENGELL, 2007; DARBY et al., 2011; MENDOZA et al., 2016; KRISHNAMOORTHY; FOSS; SKOGESTAD, 2018; AHMAD; GAO; ENGELL, 2019).

Tackling Challenges

In a bid to tackle these challenges, different techniques have been proposed to improve the efficiency of the RTO. More of the current research tend towards Dynamic Real time optimization and Economic MPC. Since most systems are dynamic and disturbed, and they also do not stay at steady state for such a time long enough for the steady state optimization as assumed, the Dynamic RTO (using dynamic models) and the economic MPC (where the RTO is completely removed and the economic optimization is done at the MPC layer) are gaining more attention for research. Although the DRTO may seem logical given the challenges of the SRTO, because it gives a closer representation of real systems, it is more expensive, computationally exhaustive. Moreover, there are very few successes recorded in the literature and stability is still an open discussion. Furthermore, some processes will achieve an economic optimum or close using SRTO at a cheaper computational cost as compared to using DRTO (KRISHNAMOORTHY; FOSS; SKOGESTAD, 2018; MATIAS; LE ROUX, 2018).

In an RTO overview review DARBY et al., (2011) pointed out that the fundamental limiting factor of the SRTO is the steady-state detection and wait time that is required for online update of model (parameter adaptation). To this effect (KRISHNAMOORTHY; FOSS; SKOGESTAD, 2018; MATIAS; LE ROUX, 2018) proposed a method of steady-state optimization using transient data, where the parameter estimation step is done using online estimators with dynamic models. They demonstrated that, using online parameter estimation will eliminate the difficult steady-state detection step, and also the steady-state wait-time since the updated measurement used is transient. Therefore, the technique will provide the optimizer with more reliable data as compared to the outdated information obtained using a static model parameter estimator. There are different methods of online parameter estimation approaches suggested in the literature such as Modifier adaptation, integrated system optimization (ISOPE) (GRACIANO, 2016).

For this study the method proposed by KRISHNAMOORTHY; FOSS; SKOGESTAD, (2018) is applied and will be discussed in details in section 2.3.

2.1.1 Dynamic real time optimization (DRTO)

Real industrial processes are dynamic and continuous. The static RTO is a simplified representation of the real process based on some assumptions. This poses a challenge because it does not necessarily capture some intrinsic details, leading to suboptimal operations when applied to real systems.

The dynamic real-time optimization is a technique developed to overcome the challenges of the traditional RTO. It basically involves the use of dynamic models rather than static models for economic optimization. Since the real processes are dynamic, process details will be better captured using the dynamic models, also there is no need to wait for the process to get steady-state before optimization takes place, this eliminates the wait time especially for highly transient and slow processes. While this method has been an interesting and more logical research area in recent times, current tools available are less practical especially for large scale systems. It is more expensive and computationally exhaustive to implement. Although more research in this area are explored, few successes have been recorded in the literature. The dynamic RTO technique can be mathematically represented in (2.6) and ((2.7):

Step 1: Dynamic Parameter Estimation

$$\hat{\varsigma}_k = arg\min_{\theta} \left\| y_{mea,k} - h(x_k, u_k) \right\| \tag{2.6}$$

$$s.t.\ x_k = f(x_{k-1}, u_{k-1}, \varsigma)$$

Step 2: Dynamic real time optimization (DRTO)

$$u_k^* = arg\min_{u_k} \sum_{k=1}^{N} J(y_k, u_k) \tag{2.7}$$

$$s.t.\ x_k = f(x_{k-1}, u_{k-1}, \hat{\varsigma}_k)$$

$$y_k = h(x_k, u_k)$$

$$g(y_k, u_k) \leq 0$$

$$x_k = \hat{x}_k \qquad \forall_k \in \{1,2 \ldots, N\}$$

Where the subscript $*_k$ represents optimal point at each sampling time in the optimization horizon of length N. Figure 2.2 shows the dynamic RTO

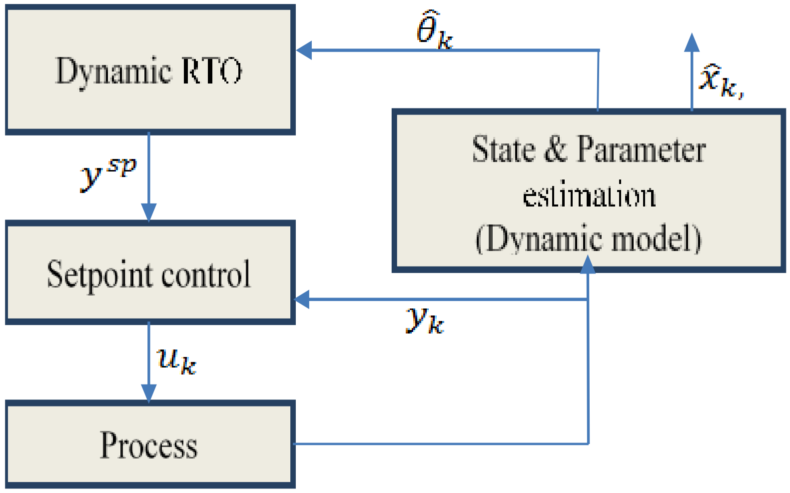

Figure 2.2 - Dynamic RTO

Source: (KRISHNAMOORTHY; FOSS; SKOGESTAD, 2018)

2.1.2 Hybrid Real-Time Optimization

The Hybrid Real-Time Optimization (HRTO) is a modification of traditional RTO, it was proposed by Krishnamoorthy; Foss; Skogestad, (2018) to handle some of the limitations of the traditional RTO found in the industry. This technique tackles the challenge of steady-state wait time and eliminates the steady-state detection step of the SRTO mentioned in section 2.1 above. The idea here is to use transient data for steady-state optimization rather than the conventional static data, which is usually obsolete as at the time of optimization.

To achieve this, the parameter estimation step which is a very important step in real-time optimization is done using online parameter estimators (dynamic models) rather than static estimators. Moreover, in reality these parameters that are used for the optimization are not always static but can be uncertain and change with time. After the parameters are estimated and the model data is updated to current data, the parameters are sent to the static optimizer which optimizes at steady state. Since the primary objective is to optimize the system at steady-state, the introduction of the dynamic terms of the system is only required at the model adaptation step.

The HRTO tends to serve as an intermediary between the more successfully established SRTO and the computationally expensive but efficient theoretical DRTO. HRTO enjoys the best of both worlds such that, it constantly approximates the optimal operation point, since the parameters are constantly updated with current data. It also enjoys the less computational cost of the steady-state model by relying on its already established literature and software for implementation in large processes. Note that HRTO does not seek continuous optimization as in dynamic optimization rather it pursues continuous improvement of optimal economic decisions by computing steady-state optimum persistently until the process attains steady-state optimum (MATIAS; LE ROUX, 2018). It is also important to understand that it cannot be as exact as the steady-state optimization or dynamic optimization, because it carries both properties it might take more time than the SRTO and more number of parameter or states to be estimated but it will achieve an efficiency that is close to the dynamic RTO at a relatively reduced cost and time. (KRISHNAMOORTHY; FOSS; SKOGESTAD, 2018; MATIAS; LE ROUX, 2018).

This method is suitable for most real processes that require frequent parameter update but are not very complex dynamic models that require intrinsic optimization, this should be noted during implementation and the process for which it will be used must be critically analyzed to validate its efficiency on the process. Figure 2.3 demonstrates the Hybrid RTO.

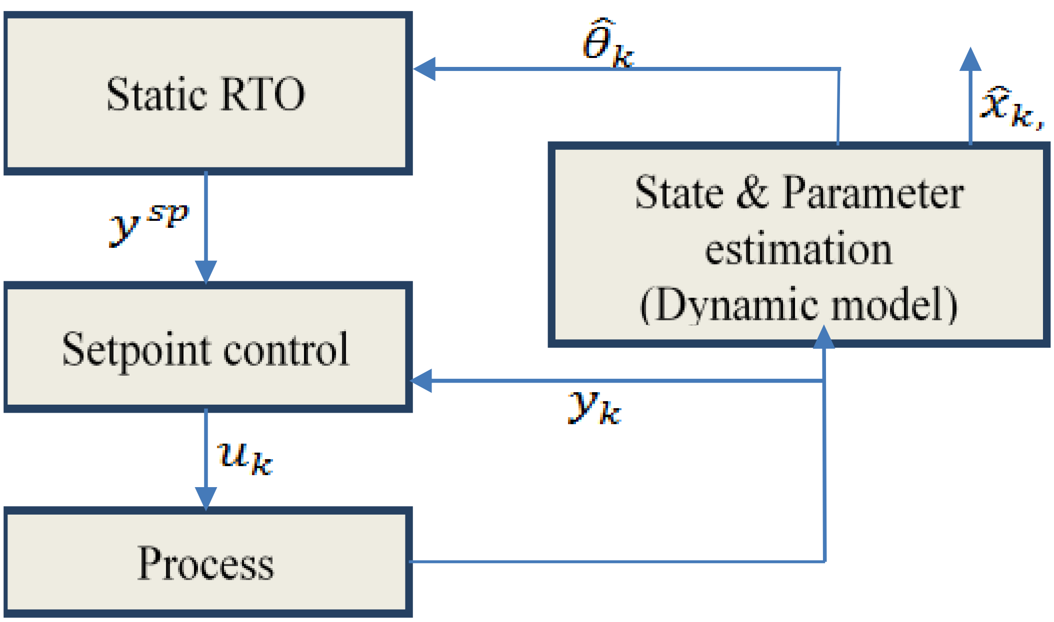

Figure 2.3 - 'Hybrid Real-Time Optimization'(HRTO)

Source: (KRISHNAMOORTHY; FOSS; SKOGESTAD, 2018)

The mathematical representation for the HRTO is based on KRISHNAMOORTHY; FOSS; SKOGESTAD, (2018) and it is presented in the equations (2.8) - (2.10) :

Step 1: Dynamic Parameter Estimation

$$\hat{\varsigma}_k = arg\min_{\theta}\left\| y_{mea,k} - h(x_k, u_k) \right\|$$
$$s.t.\; x_k = f(x_{k-1}, u_{k-1}, \varsigma) \tag{2.8}$$

Step 2: Static Optimization

$$u^* = arg\min_{u} J(y,u) \tag{2.9}$$

$$s.t.\; x_k = f_{ss}(x, u, \hat{\varsigma}_k)$$
$$g(y,u) \leq 0 \tag{2.10}$$

2.2 Model predictive control (MPC)

Model Predictive control (MPC), also referred to as Receding horizon control, is an online optimization based supervisory control technique that optimizes a performance index over a control horizon by taking advantage of a dynamic nominal process, while accounting for process constraints (ELLIS; DURAND; CHRISTOFIDES, 2014).

Since its development in the 70s, it has gained popularity for its ability to handle multivariable complex systems with strict constraints. The formulation of MPC takes into consideration the product specification, equipment limitation, safety and environmental constraints. MPC is also a successful technology widely applied in the continuous process industry. It is required to drive the process to and maintain the operations at optimum trajectories. In the two-layer strategy, it is common practice to implement RTO in cascade to MPC. When the optimal value for the system is found at the RTO layer, this value is sent down to the MPC controller as set-point. MPC then finds the optimal trajectory that drives the process to the desired set-point. It is important to note that, RTO requires a properly working controller to be successfully implemented (DARBY et al., 2011).

The optimization in MPC is implemented at each sampling time using the actual process output measurements or estimated state variables and parameters when the process measurements are not available or measuring tools are not reliable. MPC computes the series of control actions for the manipulated input variables that would drive the system along its optimal trajectory, from the series of control actions computed, only the first one is applied to the system and these steps are repeated over a given time horizon (MACIEJOWSKI, 2002).

Different types of predictive controllers have been introduced for the optimization and control of industrial processes, as well as different approaches in their implementation. MPC have distinguished itself with the following advantages:

- It can be easily implemented by operators with limited knowledge of control as the underlying concepts are quite intuitive and it is also relatively easy to tune.
- It has the ability to handle a great variety of processes, from relatively simple dynamics to quite complex multivariate processes, including processes with long time delays, dead times, non-minimum phase or unstable.
- It introduces a feed forward control in an organic way to compensate for disturbances thereby resulting in an easy to implement control law.
- It has the ability to handle constraints in a conceptually simple way, hence can be systematically included in the controller design.
- It is an open and flexible methodology with fundamental principles that allow for future extensions or modifications.

The conventional model predictive control requires the tracking of the trajectory of an optimum set point within a given prediction horizon. The formulation of the cost function is such that, the quadratic deviation between the process controlled/manipulated variables and the set-point trajectory is penalized with respect to the degrees of freedom of control and also the control effort (CAMACHO; BORDONS, 2007).

As the control technique advanced, different approaches towards achieving efficiency and recursive stability of processes emerged. (ALLGOWER; ZHENG, 1991; SANTOS; AFONSO; BIEGLER, 2001; HOLKAR; WAGHMARE, 2010; DARBY et al., 2011; ORUKPE, 2012; AL-NAUMANI; ROSSITER, 2015; ALANQAR; ELLIS; CHRISTOFIDES, 2015; CHU; YOU, 2015; GUIDO et al., 2017; MULLER; ALLGOWER, 2017) are just a few of the numerous papers that have given an overview of the concept, theory and application of MPC in the industry.

Although the early articles with relative interest in MPC emerged in the 1970s with the model predictive heuristic control (MPHC) as the first reported in 1978 by Richalet et al. Constituting an impulse response model, a quadratic objective function, the input and output constraints, the optimal input values computed by heuristic iterations over a finite prediction horizon.

The dynamic matrix control (DMC), followed the MPHC. In the dynamic matrix control (DMC), an unconstrained multivariable control was reported by a team of Shell engineers Cutler and Ramaker in 1979. The DMC formulation involved a linear step response system model, a quadratic objective function, optimal inputs computed by solving a least square problem over a finite prediction horizon. These two developments are regarded as the first MPC technology, they were closely related to the minimum time optimal control problem.

The structure of MPHC and the DMC posed the challenge of handling process constraints. To solve this challenge in 1986 Garcia and Morshedi developed the QDMC (Quadratic Dynamic Matrix Control) and in 1985 (Morshedi, Cutler, Skrovanek) developed the LDMC (Linear Dynamic Matrix Control), in which the DMC is modified to fit into quadratic programming and linear programing respectively so that it can handle explicit constraints in the optimization problem, solving MPC problems in the standard QP solvers. Subsequently, the formulation of MPC in the state space context allowed for the use of well-known state space theorems and facilitate the generalization of MPC to more complex cases such as multivariable processes, nonlinear processes and systems with disturbances and noise in measured variables.

Algorithms like LDMC, Linear Matrix Inequality (LMI), EMPC, infinite horizon MPC, NMPC to mention a few have been proposed, developed or modified to consider robustness and stability and achieve optimal process operations. Several successful practical application of MPC have been documented in the literature since the first algorithm of MPC was proposed.

2.2.1 Mechanism for MPC algorithm

Generally, all MPC algorithms constitute the same fundamental components which can be developed using different strategies. The following discussion involves these components and some of the methodologies by which they are developed:

- Prediction model representation
- Objective function and
- Attaining the control law

2.2.1.1 MPC Model representation

The process model is an intrinsic component in implementing the MPC. The model representation defines the algorithm of the technique to be used for solving the control problem. Therefore, it is very important that the characteristic of the system considered is critically analysed. The process model should fully capture the process dynamics as best as possible, yet maintain simplicity for ease of computation (CAMACHO; BORDONS, 2007). The process model used for prediction in MPC is usually a dynamic linear model. The models can be obtained through two classes of modelling namely:

- Non-parametric models: these are essentially represented by impulse or step response models.
- Parametric models represented by transfer function models or state space models.

The different model representation that demonstrate the relationships between the outputs and inputs, some of the commonly used models are described as follows:

- **Impulse response model:** the impulse response model the also known as weighting or convolution models are characterised by a sequence of values that corresponds to the response of the system when a unit impulse is introduced in the system. See equation ((2.11)

$$y(k) = \sum_{j=1}^{N} h_j u(k-j) \tag{2.11}$$

Where h_j are the j^{th}sampled output values when the process is disturbed by a unitary pulse and u is the input at N values. This limits the method to only stable processes without integrators, unstable systems cannot benefit from it. With no prior information about the process needed besides the stability of N, the method gives a significant advantage, in that process identification is simplified.

- **Step response:** with the exception of the input signal being a step, that is the model corresponds to the coefficients of a unit step, the step response is similar to the impulse response model, shown in equation ((2.12):

$$y(k) = y_0 + \sum_{i=1}^{N} g_i \Delta u(k-i) \tag{2.12}$$

Where g_i are the i^{th}sampled output values when the process is disturbed by a unitary step and Δu is the incremental input. The advantage and disadvantage of this method is the same as in the impulse response method. It can easily be shown that the two models are similar in that, an impulse can be considered as the difference between two steps with a lag of one sampling period as can be seen in equation ((2.13).

$$h_j = g_i - g_{i-1} \qquad g_i = \sum_{j=1}^{i} h_j \tag{2.13}$$

The application of this method can be found in DMC and its variants.

- **Transfer function model:** This can usually be obtained as a transitional step to the construction of a step or impulse model. In practice, transfer function is usually identified from experimental data and then a step unit or impulse is applied to produce the desired model required for MPC implementation. In the discrete form, the transfer function model of the order na uses the concept of $G = B/A$ so that the relationship between the output to the input in discrete form is given by equations ((2.14) and ((2.15):

$$A(z^{-1})y(t) = B(z^{-1})u(t) \tag{2.14}$$
$$A(z^{-1}) = 1 + a_1 z^{-1} + a_2 z^{-2} + \cdots + a_{na} z^{-na}$$
$$B(z^{-1}) = 1 + b_1 z^{-1} + b_2 z^{-2} + \cdots + b_{nb} z^{-nb}$$

Given a system

$$y(k) = G(z^{-1})u(k) \tag{2.15}$$

Such that,

$$G(z^{-1}) = \frac{y(k)}{u(k)} = \frac{B(z^{-1})}{A(z^{-1})} = \frac{b_0 z^n + b_1 z^{n-1} + \cdots + b_{nb}}{z^n + a_1 z^{n-1} + \cdots + a_{na}}$$

Where, $y(k)$ corresponds to the system output and $u(k)$ correspond to the input. The coefficients $b_0, \ldots, b_{nb}$ and $a_1, \ldots, a_{nb}$ are the model parameters. This method gives a significant advantage of applicability to any kind of system, stable or unstable and it only needs fewer parameters to represent the system compared to the parametric models, although priori knowledge of the process is essential, especially the order of the polynomials A and B (na and nb) for process identification. This method can be found in GPC, UPC, EPSAC. Furthermore, for implementation, the transfer function or step response can be easily converted to a state space model.

- **Linear State space model:** considering the discrete form of a system, the state space model can be presented in equations ((2.16)and ((2.17):

$$x(k+1) = Ax(k) + Bu(k) \tag{2.16}$$

$$y(k) = Cx(k) \tag{2.17}$$

Where, x corresponds to the states, y corresponds to the system output, $u(k)$ correspond to the input and A, B, C, D are matrices of the system. The prediction for this model can be represented by equation (2.18):

$$\hat{y}(t+k|t) = C\hat{x}(t+k|t) = C[A^k x(t) + \sum_{i=1}^{k} A^{k-i} Bu(t+i-i)] \tag{2.18}$$

The ability to be easily and conveniently used for multivariate processes gives this method an advantage. The control law is simply the feedback of a linear combination of the state vector. Although if the states are not available or accessible the calculations may become complicated requiring an observer/estimator.

- **Linear State space model in incremental form:** in this case, the input of the state space model is represented in incremental form such that:

$$x(k+1) = Ax(k) + B\Delta u(k) \tag{2.19}$$

$$y(k) = Cx(k)$$

Where $\Delta u(k) = u(k) - u(k-1)$ and other components of the equation is as already defined in the equation ((2.16).

- **Output Prediction Oriented Model (OPOM)**

With respect to the IHMPC by RAWLINGS and MUSKE (1993), (ODLOAK, 2004) proposed the OPOM with objective of practical implementation in the industries in mind, the technique:

- Reproduces exactly the step response model with fewer parameters including the cases where time delays are present.
- Some of the states are related with the output steady state.
- Makes easier the development of infinite horizon controllers.

Consider a SISO system with a transfer function as follows:

$$\frac{y(s)}{u(s)} = \frac{b_0 + b_1 s + b_2 s^2 + \cdots + b_{nb} s^{nb}}{1 + a_1 s + a_2 s^2 + \cdots + a_{na} s^{na}} \tag{2.20}$$

Where $\{na, nb \in \mathrm{N} | nb < na\}$. Assuming that the system has only stable poles with single multiplicity, the system step response at time t can be written as:

$$S(t) = d_0 + \sum_{j=1}^{na} [d_d(j)] e^{r_j t},$$

$$S(t) = y(t) = d_0 + d_1 e^{r_1 t}, d_2\, e^{r_2 t}. \ ;$$

For example, assuming that the system have distinct poles $r_1\ and\ r_2$ and the coefficients d_0, d_1, d_2 can be obtained by partial fraction expansion of the system transfer function, we can write the equation ((2.20) as:

$$\frac{y(s)}{u(s)} = \frac{b_0 + b_1 s}{a_2 (s - r_1)(s - r_2)} \tag{2.21}$$

Then, if $u(s) = 1/s$

$$y(s) = \{\frac{d_0}{s} + \frac{d_1}{s - r_1} + \frac{d_2}{s - r_2}\}$$

The state space model:

$$\begin{aligned} x(k+1) &= Ax(k) + B\Delta u(k) \\ y(k) &= Cx(k) \end{aligned} \tag{2.22}$$

can then be written as:

$$x(k+1) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & e^{r_1 t} & 0 \\ 0 & 0 & e^{r_2 t} \end{bmatrix} \begin{bmatrix} p_0 \\ p_1 \\ p_2 \end{bmatrix}_k + \begin{bmatrix} d_0 \\ d_1 e^{r_1 t} \\ d_2\, e^{r_2 t} \end{bmatrix} \Delta u(k)$$

$$y(k) = [1 \quad 1 \quad 1] \begin{bmatrix} p_0 \\ p_1 \\ p_2 \end{bmatrix}_k$$

Now considering a MIMO system with nu inputs and ny outputs, analogously the transfer function relating inputs u_j to output y_j is represented by:

$$G_{i,j}(s) = \frac{b_{i,j,0} + b_{i,j,1}s + b_{i,j,2}s^2 + \cdots + b_{i,j,nb}s^{nb}}{1 + a_{i,j,1}s + a_{i,j,2}s^2 + \cdots + a_{i,j,na}s^{na}} \tag{2.23}$$

and

$$S(t) = d_0 + \sum_{j=1}^{na} [d_d(j)]e^{r_j t}$$

The following coefficient matrices can be conveniently defined as:

$$D^0 = \begin{bmatrix} d_{1,1}^0 & \dots & d_{1,nu}^0 \\ \vdots & \ddots & \vdots \\ d_{ny,1}^0 & \dots & d_{ny,nu}^0 \end{bmatrix}, D^0 \in \Re^{ny \times ny}$$

$D^d = diag(d_{1,1,1}^d \cdots d_{1,1,na}^d \cdots d_{1,nu,1}^d \cdots d_{1,nu,na}^d \cdots d_{ny,1,1}^d \cdots d_{ny,nu,1}^d \cdots d_{ny,nu,na}^d$,

$D^d \in \Re^{nd \times nd}$

For such a system, a discrete time space model representing the process is as in equations ((2.24 -(2.25).

$$\begin{bmatrix} x^s(k+1) \\ x^d(k+1) \end{bmatrix} = \begin{bmatrix} I_{ny} & 0 \\ 0 & F \end{bmatrix} \begin{bmatrix} x^s(k) \\ x^d(k) \end{bmatrix} + \begin{bmatrix} D^0 \\ D^d FN \end{bmatrix} \Delta u(k) \tag{2.24}$$

$$y(k) = [I_{ny} \quad \Psi] \begin{bmatrix} x^s(k) \\ x^d(k) \end{bmatrix} \tag{2.25}$$

Where

$x^s = [x_1 \cdots x_{ny}]^T$, $x^s \in \Re^{ny}$, $x^d = [x_{ny+1} \; x_{ny+2} \; \cdots \; x_{ny(nu\,na+1\,)}]^T$,

$x^d \in C^{nd}$, $nd = nu\;na\;ny$

$F = diag(r_{1,1,1} \cdots r_{1,1,na} \cdots r_{1,nu,1} \cdots r_{1,nu,na} \cdots r_{ny,1,1} \cdots r_{ny,nu,1} \cdots r_{ny,nu,na})$,

$$F \in C^{nd\times nd}, N = \begin{bmatrix} J_1 \\ \vdots \\ J_{ny} \end{bmatrix}, N \in \Re^{nd\times nu}, \; J_1 = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 0 & \cdots & 0 \\ & & \ddots & \\ 0 & 0 & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix}, N \in \Re^{nu\, na\times nu}$$

$$\Psi = \begin{bmatrix} \Phi & & 0 \\ & \ddots & \\ 0 & & \Phi \end{bmatrix}, \Psi \in \Re^{ny\times nd} \; \Phi = [1 \quad \cdots \quad 1], \Phi \in \Re^{nu\times na}$$

- **Nonlinear Models**: Most real industrial system models are nonlinear represented as:

$$x_{k+1} = f(x_k, u_k, \varsigma_k) \tag{2.26}$$

$$y_k = h(x_k, u_k) \tag{2.27}$$

Where, x_k corresponds to the states, y_k corresponds to the outputs measurement, u_k correspond to the inputs, all at time step k, f represents the system nonlinear equations and h the output measurement equations.

2.2.2 MPC Algorithms

The main objective of the supervisory controller is to compute the optimal input trajectory that drives the system to the optimal operating point. There are various algorithms of control formulated to achieve this objective. Depending on the process model, objective and constraints, the MPC algorithm are generally the same; developed to achieve feasibility, stability and optimality (CAMACHO; BORDONS, 2007).

The objective function of the controller is defined as a quadratic function with respect to the inputs, output and constraints of the process under consideration. The model developed is mostly based on past process data obtained and can be linear or non-linear, the traditional MPC is linear, although the non-linear MPC is under intensive research, and linear MPC is the practically attainable one currently. The algorithm is implemented in a strategy known as the ‘receding horizon’. The receding horizon idea is such that, given the previous history of the output trajectory, a set point which the output should follow and input injected into the process at the current time, an output is obtained which is used as the previous point for the next time step. Using the information the controller computes the best input that will lead to and maintain the given set point trajectory and continues to adapt process iteratively along this trajectory within the control horizon until the input becomes constant (MACIEJOWSKI, 2002).

2.2.2.1 Objective function:

Depending on the MPC algorithm considered, different cost function for achieving a control law can be presented. The main aim is that the prediction of the future output (y) within a given prediction horizon should follow a given reference trajectory, while at the same time penalizing the input actions required for achieving the aim.

The objective function considers:

- **Parameters**: such as the prediction horizon within which the output trajectory follows the reference trajectory, the control horizon- usually a smaller range compared to the prediction horizon, which is the interval by which necessary control actions are implemented, the output and input weights used to penalize deviation from the objective. These parameters can be used to tune the controller.
- **Reference trajectory (set-point)**: knowing the future evolution of the reference a priori can avoid the effects of delay in process response since the system can react before the change is effectively made. This is one of the advantages of MPC, as knowing ahead what to expect facilitates getting ahead of any disturbance (CAMACHO; BORDONS, 2007).
- **Constraints**: all industrial processes are subject to certain constraints. These constraints come from safety measures, environmental measures or production deliverables. Therefore, it is necessary to factor in these constraints in the objective function to achieve optimal condition necessary. This consideration is also an advantage of MPC. Inclusion of process constraints can complicate the minimization solution, but it is necessary to achieve optimal operations.

2.2.2.2 Obtaining the control Law:

In order to obtain the sought values of input actions, the objective function have to be minimized. This is achieved by computing the values of predicted output as a function of the past inputs, outputs and future targets, using the model and substituting in the objective function. This obtains an expression that leads to the values sought (CAMACHO; BORDONS, 2007).

The MPC can be implemented within a prediction horizon or in an infinite horizon. Process operation is not always smooth and steady, the conditions of operation changes when disturbed by changes in some parameters or introduction of impurities, therefore the robust MPC's are developed to handle processes uncertainties and disturbances. Within the scope of this work, the following section describes different MPC algorithms categorised with respect to the type of process model, time horizon and kind of control.

2.2.3 Quadratic Model Predictive controller

The Quadratic DMC is an extension of the DMC to accommodate constraints and fit into a quadratic programming problem. DMC is based on a linear state space model in incremental form as in equation ((2.19), where the state is the following:

$$x(k) = [y(k+1|k)^T \; y(k+2|k)^T \ldots \; y(k+p|k)^T]^T$$

Where $y(k+1|k)^T$ is the output prediction computed within the prediction horizon at a time k.

The QDMC leads to an optimization problem that has the structure:

$$\min_{u} J; \;\; J = \frac{1}{2} x^T H x + c^T x \quad (2.28)$$

$$s.t. Ax \leq b$$

Where H is a symmetric matrix $nx \times nx$, c is a vector of nx dimension, A is a matrix of $m \times n \; n$ dimension and b is a vector of m dimension. The basic concept of the QMDC is to fit the DMC into the different parameters defined in equation ((2.29) while considering constraints.

2.2.3.1 Conventional MPC (SET-POINT TRACKING)

The conventional set point tracking MPC involves the prediction of the trajectory for a given set point provided either by the optimization layer or the operator in a given prediction horizon. The equations ((2.29) – ((2.40) show the mathematical representation of the conventional MPC.

Consider a discrete - time system with state variables x and control inputs u:

$$x_{k+1} = Ax_k + Bu_k + w_k \tag{2.29}$$

$$y_k = Cx_k + v_k \tag{2.30}$$

Where $u \in \mathbb{R}^{nu}$, $y \in \mathbb{R}^{ny}$, nu is the number of manipulated variables(input control action) and the ny is the number of controlled variables (output), x_k is the state variables at time k; w_k is the disturbance, v_k is the measurement noise of the system; A is the matrix of state, B is the input matrix and C is the output matrix

Given a controller cost:

$$J_k = \sum_{J=1}^{p} \|y(k+j|k) - y^{sp}\|_Q^2 + \sum_{J=0}^{m-1} \|\Delta u(k+j|k)\|_Q^2 \tag{2.31}$$

Where p is the prediction horizon; m is the input horizon, y^{sp} is the desired set-point (output), $\Delta u(k+j|k) = u(k+j|k) - u(k+j-1|k)$, Q and R are the matrices of appropriate dimensions.

The equation ((2.31) is developed based on the equations ((2.29) and ((2.30) as shown below:

$$y(k+1|k) = Cx(k+1|k) = CAx(k) + CBu(k|k)$$

$$y(k+2|k) = CAx(k+1|k) + CBu(k+1|k)$$

$$= CA^2x(k) + CABu(k|k) + CBu(k+1|k)$$

$$y(k+3|k) = CA^3x(k) + CA^2Bu(k|k) + CABu(k+1|k) + CBu(k+2|k)$$

$$\vdots \qquad \vdots$$

$$y(k+j|k) = CA^jx(k) + CA^{j-1}Bu(k|k) + CA^{j-2}Bu(k+1|k) + \cdots + CBu(k+2|k)$$

If we suppose that

$$u(k+m|k) = u(k+m+1|k) = \cdots u(k+m-1|k) \tag{2.32}$$

Meaning that the control input remains constant after time instant $k+m$. With this assumption in equation ((2.35), the output predictions can be written as:

$$y(k+m+1|k) = CA^{m+1}x(k) + CA^{m}Bu(k|k) + CA^{m-1}Bu(k+1|k) + \cdots +$$

$$+[CAB + CB]u(k+m-1|k)$$

$$y(k+m+2|k) = CA^{m+2}x(k) + CA^{m+1}Bu(k|k) + CA^{m}Bu(k|k) + \cdots$$

$$+[CA^{2}B + CAB + CB]u(k+m-1|k)$$

$$\vdots \qquad\qquad \vdots$$

$$y(k+p|k) = CA^{p}x(k) + CA^{p-1}Bu(k|k) + CA^{p-2}Bu(k+1|k) + \cdots +$$
$$CA^{p-m+1}Bu(k+m-2|k) + [CA^{p-m}B + CA^{p-m-1}B + \cdots + CB]u(k+m-1|k)$$

Therefore, the vector of output predictions can be written as such:

$$\begin{bmatrix} y(k+1|k) \\ y(k+2|k) \\ \vdots \\ y(k+m|k) \\ y(k+m+1|k) \\ \vdots \\ y(k+p|k) \end{bmatrix} = \begin{bmatrix} CA \\ CA^{2} \\ \vdots \\ CA^{m} \\ CA^{m+1} \\ \vdots \\ CA^{p} \end{bmatrix} x(k) + \begin{bmatrix} CB & CB & 0 & 0 \\ CAB & CAB & \vdots & \vdots \\ \vdots & \vdots & 0 & 0 \\ CA^{m-1}B & CA^{m-2}B & \cdots & CB \\ CA^{m}B & CA^{m-1}B & \cdots & C\tilde{A}_{1}B \\ \vdots & & \cdots & \\ CA^{p-1}B & CA^{p-2}B & 0 & C\tilde{A}_{p-m}B \end{bmatrix} \begin{bmatrix} u(k|k) \\ u(k+1) \\ \vdots \\ u(k+m-1|k) \end{bmatrix} \qquad (2.33)$$

Where $\tilde{A}_{1} = A + I$; $\tilde{A}_{2} = A^{2} + A + I$; $\cdots$ $\tilde{A}_{p-m} = A^{p-m} + A^{p-m-1} + \cdots + I$

Then the output prediction given in equation ((2.30) can be represented in the form

$$\bar{y}(k) = \Psi x(k) + \Theta u_{k} \qquad (2.34)$$

And defining the set point vector $\bar{y}^{sp} = \Big[\underbrace{y^{sp} \quad \cdots \quad y^{sp}}_{p}\Big]^{T}$ and the weight matrix

$\bar{Q} = \Big[\underbrace{Q \quad \cdots \quad Q}_{p}\Big]^{T}$, then the first term of the right hand side of equation ((2.31) becomes:

$$J_k = \sum_{J=1}^{P} (y(k+j|k) - y^{sp})^T Q(y(k+j|k) - y^{sp})$$

$$= (\Psi x(k) + \Theta u_k - \bar{y}^{sp})^T Q(\Psi x(k) + \Theta u_k - \bar{y}^{sp})$$

Now, to develop the second term of equation we can write:

$$\begin{bmatrix} \Delta u(k|k) \\ \Delta u(k+1) \\ \vdots \\ \Delta u(k+m-1|k) \end{bmatrix} = \begin{bmatrix} u(k|k) - u(k-1) \\ u(k+1|k) - u(k|k) \\ \vdots \\ u(k+m-1|k) - u(k+m-2|k) \end{bmatrix} = u_k - Mu_k - \bar{I}u(k-1)$$

Such that:

$$M = \begin{bmatrix} 0_{nu} & 0_{nu} & \cdots & 0_{nu} & 0_{nu} \\ I_{nu} & 0_{nu} & \cdots & 0_{nu} & 0_{nu} \\ 0_{nu} & I_{nu} & \cdots & 0_{nu} & 0_{nu} \\ \vdots & \vdots & \cdots & 0_{nu} & \vdots \\ 0_{nu} & 0_{nu} & \cdots & I_{nu} & 0_{nu} \end{bmatrix}, M \in \Re^{(m.nu)\times(m.nu)}, \bar{I} = \begin{bmatrix} I_{nu} \\ 0_{nu} \\ \vdots \\ 0_{nu} \end{bmatrix}, \bar{I} \in \Re^{(m.nu)\times(m.nu)}$$

Therefore, the second term of equation ((2.30) can be written as:

$$\sum_{J=0}^{m-1} \Delta u(k+j|k)^T R \Delta u(k+j|k) = [(I_{nu.m} - M)u_k - \bar{I}u(k-1)]^T R[(I_{nu.m} - M) - \bar{I}u(k-1)]$$

The two parts of the equation ((2.34) developed can be expressed as:

$$J_k = (\Psi x(k) + \Theta u_k - \bar{y}^{sp})^T \bar{Q}(\Psi x(k) + \Theta u_k - \bar{y}^{sp}) + [I_O u_k - \bar{I}u(k-1)]^T \bar{R}[I_O u_k - \bar{I}u(k-1)] \quad (2.35)$$

Where: $I_O = I_{nu.m} - M$, $\bar{R} = diag[\underbrace{R \cdots R}_{m}]$

The Objective function can thus be reduced to the quadratic form:

$$J_k = u_k^T H u_k + 2C_f^T u_k \quad (2.36)$$

Where:

$H = \Theta^T \bar{Q} \Theta + {I_O}^T \bar{R} I_O$

$C_f = (\Psi x(k) - \bar{y}^{sp})^T \bar{Q} \Theta - u(k-1)^T \, I^T \bar{R} I_O$

$c = (\Psi x(k) - \bar{y}^{sp})^T \bar{Q} (\Psi x(k) - \bar{y}^{sp}) + u(k-1)^T \, I^T \bar{R} \bar{I} u(k-1)$

The control law for the MPC can finally be written as:

$$\min_{u_k} u_k^T H u_k + 2 C_f^T u_k \tag{2.37}$$

$s.t.$

$$u_{min} \leq u(k+j|k) \leq u_{max}, \;\; j = 0,1,\ldots,m-1 \tag{2.38}$$

$$-\Delta u_{max} \leq u(k+j|k) - u(k+j-1) \leq u_{max}, \;\; j = 0,1,\ldots,m-1 \tag{2.39}$$

2.2.3.2 Infinite Horizon MPC

The infinite horizon controller implements a control law which minimizes the difference between the predicted outputs and the future outputs trajectory of a system along an infinite prediction horizon. Proposed by RAWLINGS; MUSKE, (1993), the authors showed that stability can be guaranteed for the undisturbed regulator as IHMPC would stabilize an ideal system regardless of the controller tuning parameter. It is characterized by the following objective:

Consider the equation ((2.31) previously mentioned in the conventional finite horizon MPC from the state model given in equation ((2.29), but this time we replace the prediction horizon p with infinity. Adapting the proposed IHMPC by RAWLINGS; MUSKE, (1993) to the state space model in incremental form, the following cost function is formed (RODRIGUES; ODLOAK, 2003):

$$J_k = \sum_{J=1}^{\infty} \| e(k+j|k) \|_Q^2 + \sum_{J=0}^{m-1} \| \Delta u(k+j|k) \|_R^2 \tag{2.40}$$

And simplifying equation (2.40) the control problem is reduced to the optimization problem as follows:

$$\min_{\Delta u_k}[\Delta u_k^T H \Delta u_k + 2c_t^T \Delta u_k] \quad (2.41)$$

$s.t.$

$$u_{min} \leq u(k+j|k) \leq u_{max}, \;\; j = 0,1,\dots,m-1 \quad (2.42)$$

$$-\Delta u_{max} \leq \Delta u(k+j|k) \leq \Delta u_{max}, \;\; j = 0,1,\dots,m-1 \quad (2.43)$$

$$e^s + D_m^0 \Delta u_k = 0 \quad (2.44)$$

Where $D_m^0 = \left[\overbrace{D^0 \quad D^0 \quad \cdots \quad D^0}^{m} \; 0 \; \cdots 0\right] \in \mathbb{R}^{ny \times m.nu}$

$e^s = y^{sp} - x^s(k)$

The terminal constraint in equation (2.44) keeps the control objective bounded

2.2.3.3 Zone control MPC

As mentioned, the conventional MPC receives set points, and tracks the trajectory that leads to achieving the given set-points. For processes in the places as pharmaceutical industries where exact product quality is a major priority, the conventional setpoint tracking MPC is necessary. However, in some chemical processes where the desired output can be within a range of given values with an upper and lower boundary beyond which is unacceptable, the ‘zone control’ MPC can be implemented in the supervisory control layer. The zone control strategy is mostly applied to processes where the exact values of controlled outputs are not required as long as they are within a specified acceptable range.(MACIEJOWSKI, 2002; GONZÁLEZ; ODLOAK, 2009). For this type of control strategy, efficient application stems from defining the implementation objectives of the control problem in a hierarchy such as:

- first objective is to find a feasible solution based on the given constraints and boundaries,

- secondly, reach and maintain the outputs within the given zones,

- and to steer the inputs as close as possible to the desired target.

As long as the higher priority objective is obtained, the lower objective can be satisfied with the remaining degrees of freedom. This strategy can be seen as moving the solution to within a range rather than the general exact ouput (FERRAMOSCA et al., 2012).

The zone control strategy is suitable for most real dynamic systems with high economic objective, process systems with highly correlated outputs to be controlled with fewer inputs to control all the outputs, biological systems etc. Figure 1.1 and equation (2.45) - (2.50) represents the MPC strategy described:

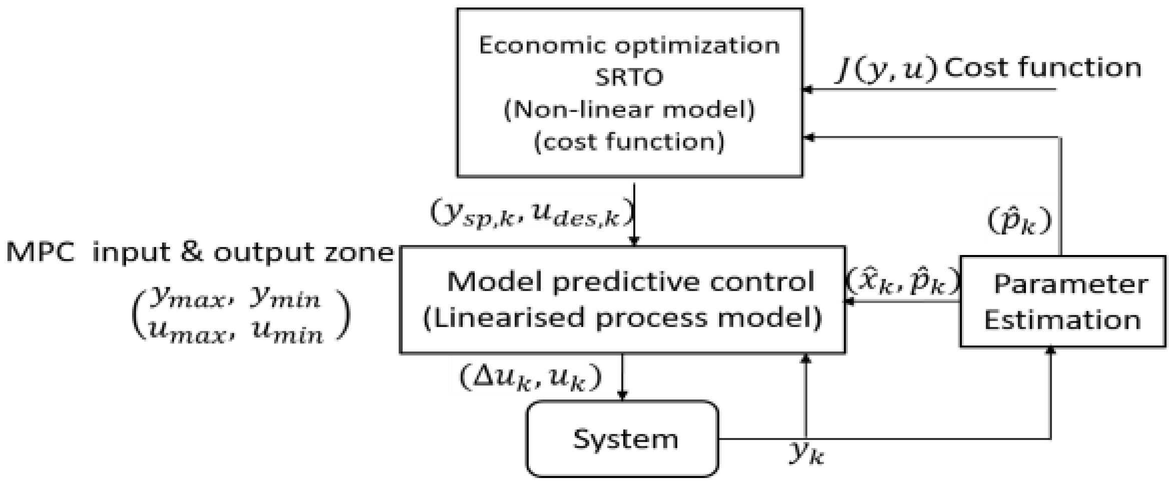

Figure 2.4 – Structure of Model predictive control (MPC) with Zone control

Source:(GONZÁLEZ; ODLOAK, 2009)

The cost function can then be described mathematically as:

$$\min_{\Delta U_k, y_{sp,k}} J_k(y,u) \tag{2.45}$$

$$J_k = \sum_{j=1}^{\infty} \left\| y(k+j|k) - y_{sp,k} \right\|_{Q_y}^2 + \sum_{j=1}^{m} \left\| \Delta u(k+j|k) \right\|_R^2 + \sum_{j=1}^{m} \left\| \left(u(k+j|k) - u_{des,k} \right) \right\|_{Q_u}^2 \tag{2.46}$$

$s.t$:

$$x_{k+1} = Ax_k + Bu_k \tag{2.47}$$

$$y_{k+1} = Cx_k$$

$$u_{min} \leq u(k-1) + \sum_{i=0}^{j} \left(\Delta u(k+i|k) \right) \leq u_{max} \;\; j = 0, \dots, m-1 \tag{2.48}$$

$$\Delta u_{min} \leq \Delta u(k+i|k) \leq \Delta u_{max} \tag{2.49}$$

$$\Delta u(k+j|k) \epsilon\, U$$

$$y_{min} \leq y_{sp,k} \leq y_{max} \tag{2.50}$$

Where m is the control horizon; $y_{sp,k}$ is the output set-point, $u_{des,k}$ is the input desired target, $\Delta u(k+j/k) = u(k+j/k) - u(k+j-1/k)$ is the input move computed at time k to be applied at time $k+j$; Q_y, Q_u and R are the positive weighting matrices of appropriate dimensions. Note that the $y_{sp,k}$ can assume any value within the output zone, and $u_{des,k}$ varies whenever the operating objective changes.

2.3 Parameter Estimation

Measurement data coming from the plant/process through measuring tools such as sensors and transmitters are combined with internal and external disturbances or noises and are not very reliable. These data are filtered for the relevant and actionable information to validate and align these measurements to the actual process under consideration through a process called parameter adaptation (estimation). (SOROUSH, 1998; RAWLINGS; BAKSHI, 2006; HEDENGREN, 2016; HEDENGREN; EATON, 2017).

In the traditional RTO model encountered in commercial RTOs, unknown or uncertain parameters for the system are estimated based on measurement data from the system. These data are filtered and the validated data updated to the RTO layer to compute optimal values that is assumed to drive the process to optimum operation iteratively (CHACHUAT; SRINIVASAN; BONVIN, 2009; KRISHNAMOORTHY; FOSS; SKOGESTAD, 2018).

Parameter estimation is very essential for RTO implementation and in the HRTO method above, online estimation is used for parameter estimation; this allows transient data to be used for the optimization. The online estimator sends updates to the model at current time regardless of if it is in steady state. It needs to infer the most likely state/ parameter estimate based on a dynamic model and the available measurements as they come from the system, hence the choice of estimator is very important. It has to be robust(RAWLINGS; BAKSHI, 2006).

For this study, the online estimator that will be used here is the Extended Kalman Filter (EKF) since the system is a constrained nonlinear system, other types of estimators that can be used are Kalman filter (for linear systems), rEKF, Unscented Kalman Filter (UKF), Moving

Horizon Estimation (MHE), etc found in the literature(RAWLINGS; BAKSHI, 2006; HEDENGREN; EATON, 2017).

EKF is the most commonly used parameter estimator for nonlinear systems. It involves the linearization of nonlinear model of the system considered, it takes advantage of the recursive strategy and computational efficiency of the Kalman filter. However, linearization of highly non-linear systems could not be as efficient considering it is an approximation, and most researchers argue its implementation on large systems due to the stress of computing the Jacobian (GOLDENSTEIN, 2004; UMAMAGESWARI; IGNATIOUS; VINODHA, 2012; SHI; O'BRIEN, 2019). EKF is applied based on (SIMON, 2006; KRISHNAMOORTHY; FOSS; SKOGESTAD, 2018).

2.4 Integration of RTO with MPC

The introduction of the RTO in the 80s gave an apparent separation of concerns and timescales between the RTO system and the process control system, the use of RTO in cascade to MPC has become a norm in the industries as the RTO requires a properly working MPC to be successfully implemented. This is not the case with the MPC which can be implemented successfully without the RTO. The RTO performs economic optimization on a time scale of hours to days whereas the control system functions on a shorter time scale of second to hours, this could lead to an erroneous conclusion that dynamics do not matter and that the introduction of the RTO layer could significantly increase the complexity of the control system, bringing about extra costs in design implementation and maintenance. Although operators could know which variables should be kept within their bounds, optimization of set-points with respect to their constraints and disturbance encountered is beyond their ability. The purpose of economic optimization is to find the point in which processes can be operated at an economic optimum. And as mentioned, this optimal point can be found at the intersection of constraints, therefore the multilayer structure of RTO – MPC integration is a classical approach.

The integration of RTO with MPC is mostly implemented in three ways namely:

2.4.1 Two-layer strategy

The two-layer structure of integrating RTO with MPC is a common approach. As mentioned earlier the RTO uses rigorous steady state non-linear model to compute the optimizing targets/set points of decision variables and sent to the advanced controller. The controller then uses a linear dynamic model to compute the best trajectory, to drive and maintain the system to the optimal operating point provided by the upper layer. This approach is suitable for some processes such as the ethylene plants, this is because units always operate at maximum production and there are certain tradeoffs that MPC alone cannot address, the pricing and the cost here are well defined and there is a lack of significant inventories. Also RTO has better response to disturbances and feasibility is maintained between RTO implementations, but for refining applications this method could create complications (DARBY et al., 2011). In a case where RTO is integrated with MPC in a two-layer structure, there is a clear separation of tasks performed by the optimization and the control layer (MILETIC; MARLIN, 1998; ENGELL, 2007).

The challenge with the two-layer technique is that: RTO and MPC layers of the optimization hierarchy are implemented with different objective function, different models and at different time-scale, this could bring about conflict arising from model mismatch (set-point computed by RTO nonlinear model inconsistent with linear model used by the MPC) or competing objectives (differences in the degrees of freedom). Consequently, leading to steady state set-point implementation offset and in some cases infeasibility and poor performances of the MPC. The time interval between consecutive RTO implementations must be large enough for the plant to reach steady state. It is a difficulty achieving this, moreover steady-state detection itself is not an easy task, especially since most large processes considered are dynamic and frequently disturbed. Therefore, making the RTO infrequently implemented. Furthermore, the MPC is conservatively designed to accommodate constraints hence the economic optimal points computed by RTO that likely changes in the presence of disturbances may not be considered by the MPC or result in instability (YING; JOSEPH, 1999; DE SOUZA; ODLOAK; ZANIN, 2010; DARBY et al., 2011; HINOJOSA, 2015; MENDOZA et al., 2016). Figure 2.5 shows the scheme of the two-layer approach.

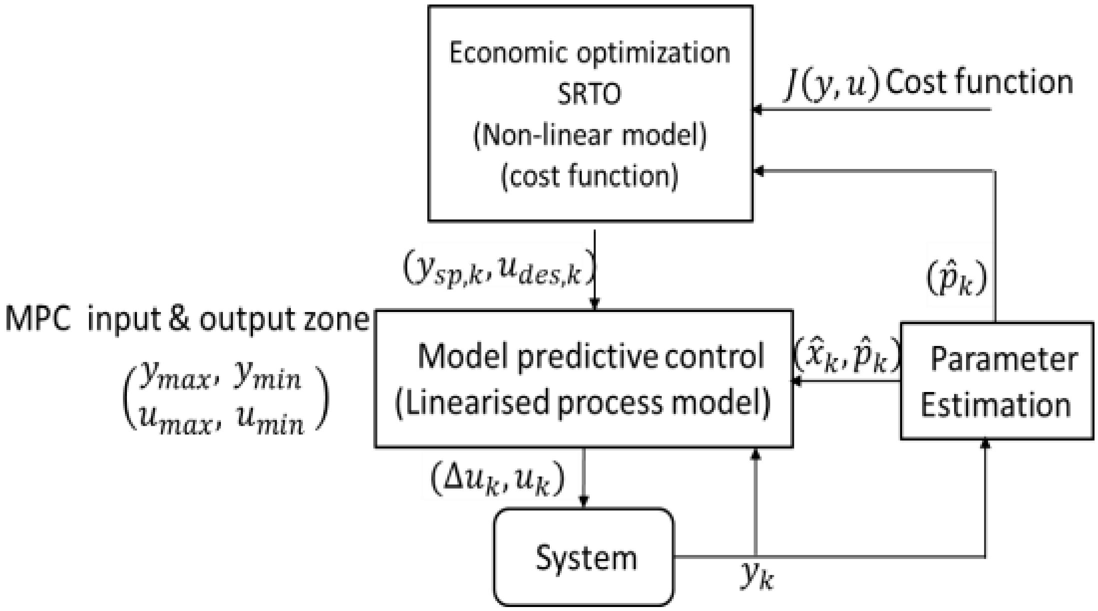

Figure 2.5 - Simplified representation of a two-layer RTO/MPC approach

(Own elaboration)

2.4.2 Three-layer strategy

With the challenges of the two-layer approach mentioned in the preceding section, the need for optimal set-points computed to be consistent with the principal model used in the MPC algorithm led to another approach for integrating RTO with MPC, the three-layer structure. In this approach, a linear optimization layer is introduced between the non-linear optimization (RTO) layer and the linear dynamic advanced controller (MPC). The objective of the second optimization layer is to enable compatibility of the nonlinear model in the first layer with the dynamic linear model of the third layer, it is formulated such that the difference between the optimizing targets it computes with the one computed by the upper RTO layer is minimized. This steady state target optimizer (SSTO) to choose the best admissible input and output targets for the MPC to work with. (YING; JOSEPH, 1999) gave a detailed comparison between the two layer RTO/MPC and the three layer strategy.(DANG; BANJERDPONGCHAI, 2013; ALAMO et al., 2014; WANG et al., 2017; PAN; ZHONG; WANG, 2018) all proposed and implemented different algorithms concerning this strategy. Figure 2.6 describes the scheme of the three layer strategy.

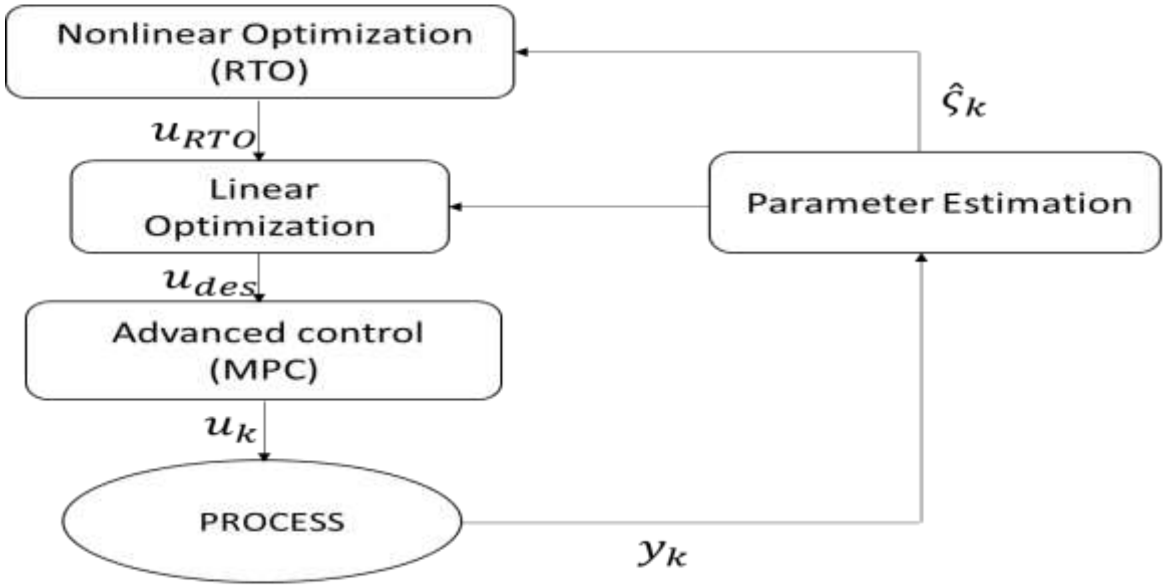

Figure 2.6 - The three-layer approach

(Own elaboration)

2.4.3 One-layer strategy

The one-layer approach is yet another solution proposed to solve the RTO/MPC integration. The idea presented here is to include the economic cost function in the RTO directly in the MPC controller cost function, the controller then does the job of optimizing the process, choosing the set-points and computing the manipulated variables trajectory. This will eliminate the need for designing RTO models and also tackle the time scale conflict challenge of the two-layer structure (ENGELL, 2007; ADETOLA, 2008; DE SOUZA; ODLOAK; ZANIN, 2010; ALAMO et al., 2014; FERRAMOSCA et al., 2014; TRAN; LINGA; MACIEJOWSKI, 2014; HINOJOSA, 2015).

The limitations of this proposed method is that, there are some processes that require the RTO because the MPC alone cannot handle the dynamics of these processes (ENGELL, 2007; DARBY et al., 2011). In situations where the optimum always lies at the constraints, there is a possibility that the constraint can change and in such case the RTO can more efficiently find the optimal point. This implies that in some processes, there are tradeoffs that RTO can account for and MPC cannot, due to nonlinearities or by a robust consideration. Moreover, RTO has a better response to frequent disturbance and feasibility is maintained between executions. An example of such process is the ethylene plant, with reasons being that units always run at maximum production, price and cost are defined and there is lack of major liquid inventories, such complicated tradeoffs cannot be addressed by MPC (DARBY et al., 2011). Figure 2.7 describes the steps for the one-layer approach

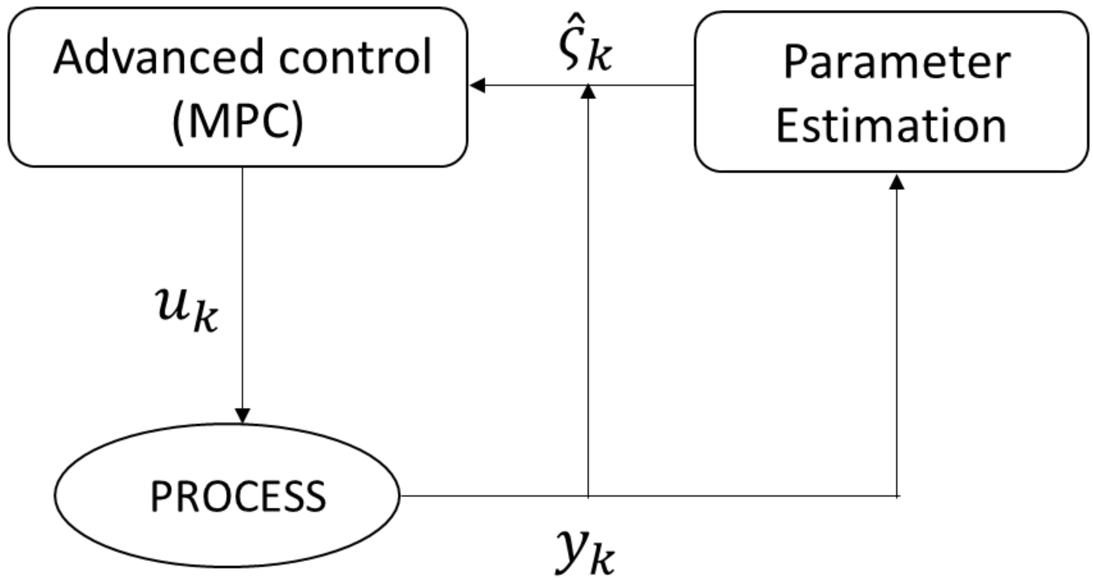

Figure 2.7 - Simplified representation of a one layer RTO/MPC

(Own elaboration)

3 ALGORITHMS FOR THE INTEGRATION OF RTO WITH MPC

In the preceding section 2.4, we described the three commonly used strategies for the integration of the RTO with MPC for the optimization of large-scale processes. There are several techniques and algorithms presented in the literature for implementing these strategies. While new process optimization strategies are continually proposed with the advancement of numerical tools, in the same manner, modifications to some of the existing and successfully applied techniques are proposed to improve their performance and efficiencies.

In this research, these modified practical techniques are applied. The two-layer and the one-layer, strategies which are the commonly applied approaches in the industry, are considered and the modified techniques to improve these strategies as proposed recently in the literature is applied as well.

For the optimization layer of the process control hierarchy as in Figure 1.1, we implement the steady-state real-time optimization modified by KRISHNAMOORTHY; FOSS; SKOGESTAD, (2018) to address the challenges of steady state wait time and parameter update mismatch. While this technique addresses the structural challenge of the RTO layer, it does not consider the hierarchical challenge with the supervisory control layer; the set-point tracking nonlinear MPC which is mostly theoretical, was used for this layer. The linear MPC is the controller mostly implemented in the industry for large-scale processes, therefore its compatibility with the modified RTO technique needs to be considered for practical implementation. In this work, for the supervisory control layer, we implement the infinite horizon MPC with zone control represented with OPOM (output prediction oriented model) model as proposed by GONZÁLEZ; ODLOAK, (2009); and DE SOUZA; ODLOAK; ZANIN, (2010) in the two strategies of RTO/MPC integration considered for this work.

The first controller we use for the two-layer approach as proposed by GONZÁLEZ; ODLOAK, (2009) was designed for nominal stable systems and have been successfully implemented on an FCC system. Likewise the controller we implement for the one-layer approach where we include the gradient of the RTO in the controller objective function as modified by DE SOUZA; ODLOAK; ZANIN, (2010) to computationally simplify the initial technique proposed by ZANIN; TVRZSKÁ DE GOUVÊA; ODLOAK, (2002). The following section describes the mathematical preliminaries for the algorithms considered and applied in this work.

3.1 The Two-layers algorithm

The two layer strategy includes:

- Layer 1: data filtering, reconciliation, updating step and the real time optimization step
- Layer 2: Supervisory control step (model predictive control).

3.1.1 The Traditional RTO (Static RTO/MPA)

Steady state detection and data processing is the initial step in implementing the classic RTO commonly found in the commercial RTO used in the industries. Using statistical or heuristic tools, a condition for which a steady state can be said to be attained is set and the plant measurements are tested to detect if the steady state condition has been closely fulfilled. Once the process is believed to be operating close enough to this steady state condition the parameter estimation step is initiated, this include data reconciliation and model adaptation of the model parameters. It is essentially to understand the process so that the choice of important parameters that significantly affect the process, hence the need for frequent update is made correctly. With an updated model, a given economic objective and process constraint, the optimal decision variables are computed by the optimization layer using a numerically optimization tool. The steps involved is represented in equations ((3.1) – ((3.2):

- Steady state detection:
- **Step 1a: Model adaptation/Parameter Estimation**

$$\hat{\varsigma}_k = arg \min_{\hat{\varsigma}} \|y_{meas} - f_{ss}(u_k, \varsigma)\|_P^2 \quad (3.1)$$

- **Step 1b: Steady state Optimization**

$$u^* = arg \min_{u} J(y, u) \quad (3.2)$$

$$s.t. \; x_k = f_{ss}(u, \hat{\varsigma}_k)$$

$$g(y, u) \le 0$$

3.1.2 Steady state RTO using transient measurement (Hybrid RTO)

In the implementation of the 'hybrid real time optimization' proposed by KRISHNAMOORTHY; FOSS; SKOGESTAD, (2018), the parameter estimation (information updating) step is the novelty of the technique. In this approach the information update is carried out using online parameter estimation tools. This implies that the information from the dynamic process, measured with tools such as sensors, which come with some noises or measurement errors, is sent directly to be filtered by a dynamic parameter estimator without having to wait for the process steady state to be detected, therefore the economic optimization step is updated with a more timely and adequate information for the computation of optimal decision variables. The following describes the computation steps for the hybrid RTO technique.

- **Step 1a: Dynamic Parameter Estimation**

$$\hat{\varsigma}_k = arg\min_{\varsigma}\left\|y_{mea,k} - h(x_k, u_k)\right\| \tag{3.3}$$

$$s.t.\ x_k = f(x_{k-1}, u_{k-1}, \varsigma)$$

As earlier mentioned, the parameter estimation step is the distinction (modification made) between the traditional RTO with this new RTO technique, therefore the choice of a good information filter is necessary. There are so many online estimators in the literature, popular amongst them is the group of Kalman filters such as the Kalman filter, extended Kalman filter, reduced extended Kalman filter, unscented Kalman filter, other filters that are optimization based such as the moving horizon estimation, constrained extended Kalman filter are also gaining more research attention.

For this research, the EKF is used. The Kalman filter though simpler to implement is only suitable for linear unconstrained process, that is not the case in most of the real industrial processes, the EKF an extension of the linear Kalman filter, and it is used for nonlinear processes as other members of the Kalman filter family (UKF, CEKF, rEKF), but the EKF is popularly applied due to its simplicity of implementation to nonlinear processes and computational speed. It achieves the desired result through approximation of the process about a point and it does not require solving the nonlinear optimization problems online. Since it is an approximation of the process, the limitations of the EKF comes when it has to deal with highly nonlinear processes. To implement the EKF we use augmented state vector constructed using the state and parameters (uncertain parameter to be estimated) as shown:

$$x' = [x^T, \varsigma^T]^T \epsilon\ \mathbb{R}^{n_x + n_\varsigma}$$

The uncertain parameters augmented system is given by:

$$x'_{k+1} = \begin{bmatrix} x_{k+1} \\ \emptyset_{k+1} \end{bmatrix} = f'(x', u_k) + w'_k \tag{3.4}$$

$$y_k = [h'(x', u_k) \quad 0] \begin{bmatrix} x_k \\ \varsigma_k \end{bmatrix} + v_k$$

Where $w_k \sim N(0, Q)$ is the normally distributed measurement noise with zero mean and covariance Q, and the augmented system $f'(x_k, u_k, \varsigma_k)$ is constructed as:

$$f'(x_k, u_k, \varsigma_k) = \begin{bmatrix} f(x_k, u_k, \varsigma_k) \\ \varsigma_k \end{bmatrix} \tag{3.5}$$

Where $v_k \sim N(0, R)$is the normally distributed process noise with zero mean and covariance R.

The discrete –time EKF for the augmented system is:

$$\hat{x}'_{k|k-1} = f'(\hat{x}'_{k-1|k-1}, u_k, \hat{\varsigma}'_{k-1|k-1}) \tag{3.6}$$

$$P_{k|k-1} = F_{k-1} P_{k-1|k-1} F_{k-1}^T + Q'_k \tag{3.7}$$

$$K_{f,k} = P_{k|k-1} H_k^T (H_k P_{k|k-1} H_k + R_k)^{-1} \tag{3.8}$$

$$\hat{x}'_{k|k} = \hat{x}'_{k|k-1} + K_k(y_{meas,k}, -h(\hat{x}'_{k|k-1}, u_k)) \tag{3.9}$$

$$P_{k|k} = (I - K_k H_k) P_{k|k-1} \tag{3.10}$$

$$F = \left. \frac{\partial f'(x,u,\emptyset)}{\partial x'} \right|_{x' = \hat{x}'} \qquad H = \left. \frac{\partial h(x,u)}{\partial x'} \right|_{x' = \hat{x}'}$$

(3.11)

(3.12)

The augmented covariance Q' is given by

$$Q' = \begin{bmatrix} Q & 0 \\ 0 & Q_\varsigma \end{bmatrix}$$

The estimated parameter $\hat{\varsigma}$ is then used in the static optimizer as shown in (3.13)

Step 1b: Static Optimization

$$u^* = arg \min_u J(y, u) \tag{3.13}$$

$$s.t.\ 0 = f_{ss}(u, \hat{\varsigma}_k)$$

$g(y, u) \leq 0$

Step 2: Zone control IHMPC with optimizing targets

The controller model is presented based on the output prediction oriented model (OPOM) developed by ODLOAK, (2004). The controller applied here is as developed by GONZÁLEZ; ODLOAK, (2009) with the goal of practical applications.

Consider a stable system with nu inputs and ny outputs and assume the poles related to u_i and output y_j are non-repeated. Suppose that the state space model:

$$x(k+1) = Ax(k) + B\Delta u(k) \tag{3.14}$$

$$y(k) = Cx(k)$$

Is represented as the equations (3.15) and (3.16)

$$\underbrace{\begin{bmatrix} x^s(k+1) \\ x^d(k+1) \end{bmatrix}}_{x(k+1)} = \underbrace{\begin{bmatrix} I_{ny} & 0 \\ 0 & F \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x^s(k) \\ x^d(k) \end{bmatrix}}_{x(k)} + \underbrace{\begin{bmatrix} D^0 \\ D^d F N \end{bmatrix}}_{B} \Delta u(k) \tag{3.15}$$

$$y(k) = \underbrace{\begin{bmatrix} I_{ny} & \Psi \end{bmatrix}}_{C} \underbrace{\begin{bmatrix} x^s(k) \\ x^d(k) \end{bmatrix}}_{x(k)} \tag{3.16}$$

Where

$x^s = [x_1 \cdots x_{ny}]^T$, $x^s \in \Re^{ny}$, $x^d = [x_{ny+1} \; x_{ny+2} \cdots x_{ny(nu\,na+1\,)}]^T$,

$x^d \in C^{nd}$, $nd = nu\,na\,ny$, $F \in C^{nd \times nd}$, $N \in \Re^{nu\,na \times nu}$, $\Delta u(k) = u(k) - u(k-1)$

$\Psi = \begin{bmatrix} \Phi & & 0 \\ & \ddots & \\ 0 & & \Phi \end{bmatrix}$, $\Psi \in \Re^{ny \times nd}$ $\Phi = [1 \quad \cdots \quad 1]$, $\Phi \in \Re^{nu \times na}$

For this control structure, at each time step k, an upper economic optimization layer computes the optimal target $u_{des,k}$, for the manipulated inputs. In this case it is assumed that the MPC is dedicated to ensuring the outputs are kept within the provided output range and at the same time guiding the manipulated inputs to the desired target provided by the upper economic optimization layer. This target provided changes with changes in operating objectives, entrance of disturbances or parameter uncertainties. The controller is able to follow the trajectory of this optimal inputs sent down to it from the upper layer. Therefor the control problem is given as:

$$\min_{\Delta u_k, y_{sp,k}, \delta_{y,k}, \delta_{u,k}} J_k \tag{3.17}$$

$$J_k = \sum_{j=0}^{\infty} \left\| y(k+j|k) - y_{sp,k} - \delta_{y,k} \right\|_{Q_y}^2 + \sum_{j=0}^{m-1} \|\Delta u(k+j|k)\|_R^2 \dots \tag{3.18}$$

$$+ \sum_{j=0}^{m-1} \left\| \left(u(k+j|k) - u_{des,k} - \delta_{u,k} \right) \right\|_{Q_u}^2 + \left\| \delta_{y,k} \right\|_{S_y}^2 + \left\| \delta_{u,k} \right\|_{S_u}^2$$

$s.t.$

$$\Delta u(k+j|k) \in U; \quad j = 0,1,\cdots,m-1 \tag{3.19}$$

$$\Delta u(k+j|k) = 0 \qquad j \geq m \tag{3.20}$$

$$y_{min} \leq y_{sp,k} \leq y_{max} \tag{3.21}$$

$$x^s(k) + \widetilde{D}^0 \Delta u_k - y^{sp,k} - \delta_{y,k} = 0 \tag{3.22}$$

$$u(k-1) + \widetilde{D}^u \Delta u_k - u_{des,k} - \delta_{u,k} = 0 \tag{3.23}$$

Where S_y, S_u are positive matrices of the appropriate dimensions and $\delta_{y,k} \in \mathfrak{R}^{ny}, \delta_{u,k} \in \mathfrak{R}^{nu}$ are the corresponding slack variables that eliminate a possible infeasibility of the terminal constraints and equations (3.22) and (3.23) of the constraints ensure that the control objective is bounded.

$$\widetilde{D}^0 = [\underbrace{D^0 \ \cdots \ D^0}_{m}] \ , \quad \widetilde{D}^u = [\underbrace{I_{nu} \ \cdots \ I_{nu}}_{m}]$$

Now to develop the control optimization problem defined above to implement the zone control and enforce economic target, equation (3.18) can be written as:

$$J_{k,u} = \sum_{j=0}^{m} \left\| y(k+j|k) - y_{sp,k} - \delta_{y,k} \right\|_{Q_y}^2 + \sum_{j=0}^{m} \left\| \left(u(k+j|k) - u_{des,k} - \delta_{u,k} \right) \right\|_{Q_u}^2 \tag{3.24}$$

$$+ \sum_{j=0}^{m-1} \|\Delta u(k+j|k)\|_R^2 + \|x^d(k+m|k)\|_{\bar{Q}}^2 + \left\| \delta_{y,k} \right\|_{S_y}^2 + \left\| \delta_{u,k} \right\|_{S_u}^2$$

Where $\bar{Q}$ is calculated from the Lyapunov equation:

$$\bar{Q} = \Psi^T Q_y \Psi + F^T \bar{Q} F \tag{3.25}$$

Simplifying further we have:

$$J_{k,u} = \sum_{j=0}^{m} \left\| y(k+j|k) - y_{sp,k} - \delta_{y,k} \right\|_{Q_y}^{2} + \sum_{j=0}^{m} \left\| \left(u(k+j|k) - \ u_{des,k} - \delta_{u,k} \right) \right\|_{Q_u}^{2} + \sum_{j=0}^{m-1} \| \Delta u(k+j|k) \|_{R}^{2} + \| F_x x^d(k) + F_u \Delta u_k \|_{\bar{Q}}^{2} + \left\| \delta_{y,k} \right\|_{S_y}^{2} + \left\| \delta_{u,k} \right\|_{S_u}^{2} \tag{3.26}$$

A point of note for this controller is that, even if the desired steady state is not admissible, the controller will still be stable. Resulting from the decreasing property of the cost function and the slack variables included in the optimization, since the open loop system is assumed stable. Therefore, the system will evolve to a point in which the slack variables are as small as possible, but not equal to zero. This feature is very important in this controller because in a practical scenario, disturbance may move the system to a point in which it is impossible to reach the targeted steady state. This controller will be able to compensate the disturbance while keeping the system under control when such scenario presents itself (GONZÁLEZ; ODLOAK, 2009).

When the state estimator is fast enough such that the state estimation converges to the true system state in negligible time, the controller will stabilize the closed loop system. The stability and feasibility of this controller have been tested on an FCC system and it will be used in this work for a gas-lift system.

3.2 The One-layer algorithm

In the previous chapter, in section (2.4.3), we discussed the one layer RTO/MPC, where the economic cost function used in the optimization layer is included in the controller cost function instead, this way there is no desired target required to be computed by an upper layer optimizer.

For this research work, we implement the one layer RTO/MPC approach proposed by DE SOUZA; ODLOAK; ZANIN, (2010). The technique modified the one layer approach initially proposed by ZANIN; TVRZSKÁ DE GOUVÊA; ODLOAK, (2002), the paper proposed the inclusion of the economic cost function into the controller, this led to a complex nonlinear problem. Considering the implication of the complexity and the computational burdens of solving a large scale NLP, the computational delay, and also to avoid closed loop system stability jeopardy.

DE SOUZA; ODLOAK; ZANIN, (2010) proposed a simplified one layer RTO/MPC integration with the aim of maintaining the advantages of the proposed one layer – a technique that is promising practical industrial application- while reducing the complexity of solving the resulting NLP problem for large scale systems. The simplified one layer RTO/MPC integration includes the gradient of the economic cost in the controller instead of the direct nonlinear economic cost. This will enable low cost computation by allowing the resulting optimization/control problem to be solved as a QP problem.

In this work, the controller proposed by DE SOUZA; ODLOAK; ZANIN, (2010) is extended to the infinite horizon MPC with zone control and the system model for the controller is represented using the OPOM model described in section 2.2.1.1. Extending to the infinite horizon controller is to ensure recursive stability, considering also the presence of uncertain parameters which can disturb the system. The controller should be able to keep all the outputs within their corresponding zones while computing the optimal operating point. The following describes the technique we implement in this research for the one layer approach:

Consider a multivariable system with ny controlled output variables and nu manipulated input variables. At any instant k we can represent the system outputs and inputs as:

$$y(k) = \left[y_1(k),\ y_2(k), \cdots, y_{ny}(k)\right]^T \qquad u(k) = [u_1(k),\ u_2(k), \cdots, u_{nu}(k)]^T$$

With the predicted steady state controlled variable corresponding to u represented as y, the economic objective function associated to the steady state will be represented by:

$$J = f_{eco}(\hat{y}, u) \tag{3.27}$$

Modifying the control vector to $u + \Delta\bar{u}$, the first order approximation of the gradient of the objective function at this point is:

$$\gamma_{u+\Delta u} = \left.\frac{dJ}{du}\right|_{u+\Delta\bar{u}} = \left.\frac{dJ}{du}\right|_{u} + \frac{d^2J}{du^2}\Delta\bar{u} \tag{3.28}$$

In a case of an unconstrained optimization, a point where $\gamma = 0$ corresponding to the extreme point of the economic cost function, then we can expand equation ((3.28) as:

$$\frac{dJ}{du} = \frac{\partial J}{\partial \hat{y}}.\frac{\partial \hat{y}}{\partial u} + \frac{\partial J}{\partial u} \tag{3.29}$$

and

$$\frac{d^2J}{du^2} = \left(\frac{\partial \hat{y}}{\partial u}\right)^T \frac{\partial^2 J}{(\partial \hat{y})^2}\frac{\partial \hat{y}}{\partial u} + \frac{\partial^2 J}{\partial \hat{y}\partial u}\frac{\partial \hat{y}}{\partial u} + \frac{\partial J}{\partial \hat{y}}.\frac{\partial^2 \hat{y}}{\partial u^2} + \left(\frac{\partial \hat{y}}{\partial u}\right)^T .\frac{\partial^2 J}{\partial u\partial \hat{y}} + \frac{\partial^2 J}{(\partial u)^2} \tag{3.30}$$

Where $\frac{\partial \hat{y}}{\partial u}$ is the process gain known as K_p and can be computed using the nonlinear steady state model of the process considered so also can $\frac{\partial^2 \hat{y}}{\partial u^2}$. Substituting therefore ((3.29) and (3.31) into equation ((3.28), we have:

$$\gamma_{u+\Delta u} = \underbrace{\left[\frac{\partial J}{\partial \hat{y}}.K_p + \frac{\partial J}{\partial u}\right]}_{v} + \underbrace{\left[K_p^T\frac{\partial^2 J}{(\partial \hat{y})^2}K_p + \frac{\partial^2 J}{\partial \hat{y}\partial u}K_p + \frac{\partial J}{\partial \hat{y}}\left(\frac{\partial^2 \hat{y}}{\partial u^2}\right) + K_p^T\frac{\partial^2 J}{\partial u\partial \hat{y}} + \frac{\partial^2 J}{\partial u^2}\right]}_{Z}\Delta\bar{u} \tag{3.31}$$

Represented as:

$$\gamma_{u+\Delta u} = v + Z\Delta\bar{u}$$

Where:

$\Delta\bar{u} = u(k+m-1) - u(k-1)$ is the vector of total input moves

The controller cost function is given by equation (3.32):

$$J_k = \sum_{j=0}^{\infty}\left\|y(k+j|k) - y_{sp,k} - \delta_k\right\|_{Q_y}^2 + \sum_{j=0}^{m-1}\|\Delta u(k+j|k)\|_R^2 + \|\gamma_{u+\Delta u}\|_{W_{eco}}^2 + \|\delta_k\|_{S_y}^2 \tag{3.32}$$

Where W_{eco} is the matrix of appropriate weight assigned to the economic cost included in the controller cost function. To develop the control problem from the cost function in equation (3.32), we simplify each term on the objective function. The equation can be re-written as:

$$J_k = \sum_{j=0}^{m} \left\| y(k+j|k) - y_{sp,k} - \delta_k \right\|_{Q_y}^2 + \sum_{j=1}^{\infty} \left\| y(k+j|k) - y_{sp,k} - \delta_k \right\|_{Q_y}^2 + \sum_{j=0}^{m-1} \|\Delta u(k+j|k)\|_R^2 + \|\gamma_{u+\Delta u}\|_{W_{eco}}^2 + \|\delta_k\|_{S_y}^2 \quad (3.33)$$

Equation can be further simplified as:

$$J_k = \underbrace{\sum_{j=0}^{m} \left\| y(k+j|k) - y_{sp,k} - \delta_k \right\|_{Q_y}^2}_{J_{k,1}} + \underbrace{\|x^d(k+m|k)\|_{\bar{Q}}^2}_{J_{k,2}} + \|\delta_k\|_{S_y}^2 + \underbrace{\sum_{j=0}^{m-1} \|\Delta u(k+j|k)\|_R^2}_{J_{k,3}} + \underbrace{\|\gamma_{u+\Delta u}\|_{W_{eco}}^2}_{J_{k,4}} \quad (3.34)$$

Considering each term of the equation (3.34) and simplifying we have:

$$J_{k,1} = \left[\left\| \bar{I}_{ny} x^s(k) + \widetilde{D}^0 \Delta u_k + \Psi_1 F_x x^d(k) + \Psi_1 F_u \Delta u_k - \bar{I}_{ny} y_{sp,k} - \bar{I}_{ny} \delta_{y,k} \right\|_{Q_{y1}}^2 \right]$$

$$J_{k,2} = (F_x x^d(k) + F_u \Delta u_k)^T \bar{Q} (F_x x^d(k) + F_u \Delta u_k)$$

$$J_{k,3} = [\Delta u(k|k)^T \;\; \Delta u(k+1|k)^T \; \cdots \; \Delta u(k+m-1|k)^T] \begin{bmatrix} R & 0 & \cdots & 0 \\ 0 & R & \cdots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & \cdots & R \end{bmatrix} \begin{bmatrix} \Delta u(k|k) \\ \Delta u(k+1|k) \\ \vdots \\ \Delta u(k+m-1|k) \end{bmatrix}$$

$$J_{k,4} = [v + Z_k \Delta u_k]^T W_{eco} [v + Z_k \Delta u_k]$$

$$Z_k = \underbrace{[Z \;\; Z \; \cdots Z]}_{m}, \; \Delta u_k = \begin{bmatrix} \Delta u(k|k) \\ \Delta u(k+1|k) \\ \vdots \\ \Delta u(k+m-1|k) \end{bmatrix}$$

Substituting these equations into equation (3.34) we have:

$$J_k = \Delta u_k^T\big(\Xi^T Q_{y1}\Xi + F_u^T\bar{\bar{Q}}F_u + R + Z_k^T W_{eco} Z_k\big)\Delta u_k + y_{sp,k}^T \bar{I}_{ny}^T Q_{y1}\bar{I}_{ny} y_{sp,k} +$$
$$\delta_k^T \bar{I}_{ny}^T Q_{y1}\bar{I}_{ny}\delta_k - \Delta u_k^T \Xi^T Q_{y1}\bar{I}_{ny} y_{sp,k} - y_{sp,k}^T \bar{I}_{ny}^T Q_{y1}\Xi\Delta u_k + y_{sp,k}^T \bar{I}_{ny}^T Q_{y1}\bar{I}_{ny}\delta_k +$$
$$\delta_k^T \bar{I}_{ny}^T Q_{y1}\bar{I}_{ny} y_{sp,k} + \delta_k^T S_y \delta_k - \Delta u_k^T \Xi^T Q_{y1}\bar{I}_{ny}\delta_k - \delta_k^T \bar{I}_{ny}^T Q_{y1}\Xi\Delta u_k +$$
$$2\Big[\Big(\Delta u_k^T\Big(\Xi^T Q_{y1}\Big(\bar{I}_{ny} x^s(k) + \Psi_1 F_x x^d(k)\Big) + F_u^T\bar{\bar{Q}}(F_x x^d(k) + Z_k^T W_{eco} v\Big) +$$
$$y_{sp,k}^T(\bar{I}_{ny}^T Q_{y1}\Big(\bar{I}_{ny} x^s(k) + \Psi_1 F_x x^d(k)\Big) + \delta_k^T(\bar{I}_{ny}^T Q_{y1}\Big(\bar{I}_{ny} x^s(k) + \Psi_1 F_x x^d(k)\Big))\Big)\Big] +$$
$$\Big[\Big(\bar{I}_{ny} x^s(k)\Big)^T Q_{y1}\Big(\bar{I}_{ny} x^s(k)\Big) + \Big(\bar{I}_{ny} x^s(k) + \Psi_1 F_x x^d(k)\Big)^T Q_{y1}\Big(\bar{I}_{ny} x^s(k) +$$
$$\Psi_1 F_x x^d(k)\Big) + (F_x x^d(k))^T\bar{\bar{Q}}(F_x x^d(k) + v^T W_{eco} v\Big]$$

Arranging the terms together we have:

$$J_k = \begin{bmatrix}\Delta u_k{}^T & y_{sp,k}^T & \delta_{y,k}^T\end{bmatrix}\Bigg[\underbrace{\begin{matrix} h_{11} & h_{12} & h_{13}\\ h_{21} & h_{22} & h_{23}\\ h_{31} & h_{32} & h_{33}\end{matrix}}_{H}\Bigg]\begin{bmatrix}\Delta u_k\\ y_{sp,k}\\ \delta_{y,k}\end{bmatrix} + 2C_t{}^T\begin{bmatrix}\Delta u_k\\ y_{sp,k}\\ \delta_{y,k}\end{bmatrix} + c$$

$$h_{11} = \underbrace{(\widetilde{D}^0 + \Psi_1 F_u)}_{\Xi}{}^T\bar{Q}_y\underbrace{(\widetilde{D}^0 + \Psi_1 F_u)}_{\Xi} + F_u^T\bar{\bar{Q}}F_u + R + Z_k^T W_{eco} Z_k$$

$$h_{12} = h_{21}^T = h_{13} = h_{31}^T = -\underbrace{(\widetilde{D}^0 + \Psi_1 F_u)}_{\Xi}{}^T\bar{Q}_y\bar{I}_{ny}$$

$$h_{22} = h_{23} = h_{32} = \bar{I}_{ny}{}^T\bar{Q}_y\bar{I}_{ny}$$

$$h_{33} = \bar{I}_{ny}{}^T\bar{Q}_y\bar{I}_{ny} + S_y \; h_{44} = \bar{I}_{nu}{}^T\bar{Q}_u\bar{I}_{nu} + S_u$$

$$C_{t,1} = \underbrace{(\widetilde{D}^0 + \Psi_1 F_u)}_{\Xi}{}^T\bar{Q}_y(\bar{I}_{ny} x^s(k) + \Psi_1 F_x x^d(k)) + \widetilde{D}^{u\,T}\bar{Q}_u(u(k-1) - u_{des,k} +$$
$$F_u^T\bar{Q}F_x x^d(k)$$

$$C_{t,2} = -\bar{I}_{ny}{}^T\bar{Q}_y(\bar{I}_{ny} x^s(k) + \Psi_1 F_x x^d(k))$$

$$C_{t,3} = -\bar{I}_{ny}{}^T\bar{Q}_y(\bar{I}_{ny} x^s(k) + \Psi_1 F_x x^d(k))$$

$$C_{t,4} = \bar{I}_{ny}{}^T\bar{Q}_y(u(k-1) - u_{des,k})$$

$$c = \Big(\bar{I}_{ny} x^s(k)\Big)^T\bar{Q}_y\Big(\bar{I}_{ny} x^s(k)\Big) + \Big(\bar{I}_{ny} x^s(k) + \Psi_1 F_x x^d(k)\Big)^T\bar{Q}_y\Big(\bar{I}_{ny} x^s(k) +$$
$$\Psi_1 F_x x^d(k)\Big)$$

$$+(F_x x^d(k))^T \bar{\bar{Q}}(F_x x^d(k) + v^T W_{eco} v$$

$$J_k = [\Delta u_k \quad y_{sp,k} \quad \delta_{y,k}]^T H [\Delta u_k \quad y_{sp,k} \quad \delta_{y,k}] + 2C_t [\Delta u_k \quad y_{sp,k} \quad \delta_{y,k}] + c$$

Where:

$$H = \Xi^T \bar{Q}_y \, \Xi + \widetilde{D}^u Q_u \widetilde{D}^u + F_u^T \bar{\bar{Q}} F_u + R + Z_k^T W_{eco} Z_k$$

Therefore the control problem is posed as ((3.35) – ((3.38):

$$\min_{\Delta u_{k,} y_{sp,k}, \delta_{k,}} J_k \qquad (3.35)$$

$s.t.$

$$\Delta u(k+j|k) \in U; \quad j = 0,1,\cdots,m-1 \qquad (3.36)$$

$$\Delta u(k+j|k) = 0 \qquad j \geq m$$

$$y_{min} \leq y_{sp,k} \leq y_{max} \qquad (3.37)$$

$$x^s(k) + \widetilde{D}_m^0 \Delta u_k - y^{sp,k} - \delta_{y,k} = 0 \qquad (3.38)$$

Equation (3.38) ensures the control cost remains bounded. The stability of this control law for a nominal case is guaranteed.

4 APPLICATION OF THE RTO/MPC ALGORITHMS TO A GAS-LIFTED SYSTEM

4.1 Process Description

In the production of oil and gas from an oil reservoir to the surface (well head), the reservoir pressure should be enough to lift the oil from the reservoir to the surface using the natural drive mechanism. Figure 4.1 shows a well deliverability curve.

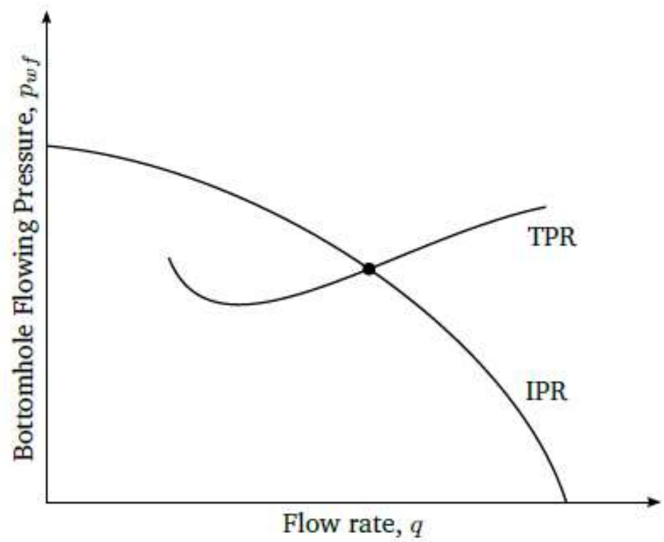

Figure 4.1 - Well deliverability curve

Source: (HÜLSE, 2015)

Where IPR is the inflow performance relationship and TPR is the well tubing performance relation. As production continues over a period of time the bottom-hole pressure builds up and the reservoir energy declines. Therefore, it is no longer sufficient to drive the oil to the surface naturally at an economic rate or even at all (i.e. in the situation where the oil can move from the reservoir into the well tubing but cannot be produced at the wellhead). In this case, an artificial lift mechanism is applied to assist in moving the oil to the surface and at a desired rate. There are different types of artificial lift mechanism that can be applied depending on numerous factors such as implementation cost, oil viscosity, type of well, reservoir properties, gas to liquid ratio, safety and environmental factor and the list goes on. The common artificial lift mechanisms used include:

i) Gas lift (continuous and intermittent)

ii) Electrical submersible pumping (ESP)

iii) Sucker rod pump

iv) Plunger lift

v) Progressive Cavity Pumping (PCP)

Gas lift is the most common artificial lift method used in the production of oil to increase, maintain or revive production. This is the process of injecting compressed gas into the well tubing through the annulus of the well to lift the oil from the reservoir to the surface. This injected compressed gas goes into the well and reduces the fluid density due to the continuous aeration process in the well thereby making the oil lighter and also reducing the hydrostatic pressure in the well tubing. This will allow more oil to flow in from the reservoir into the well tubing as the flowing bottom hole pressure is lowered (EIKREM; AAMO; FOSS, 2008; JAHANSHAHI; SKOGESTAD; HANSEN, 2012; MUKHTYAR; SHASTRI; GUDI, 2013). Consequently assisting the lifting of oil to the surface to be produced.

In using gas lift method, as gas is injected into the well, it gets to a point where the benefit of increased production is no longer enjoyed due to increase in frictional pressure loss. In the case where there is much gas present in the well tubing, this effect increases the pressure in the bottom-hole of the well and lowers fluid production. Therefore, to obtain an efficient operation, the rate at which the compressed gas is injected into the well has to be optimal. An appropriate design of the gas lift system is required for this efficiency and different components play important roles in ensuring the effectiveness of the lifting mechanism implemented. These components place constraints such as availability of compressed gas (cost), the capacity of the surface facility to handle and treat the quantity of oil, water and gas to be produced. Every well has an individual optimal gas lift injection rate but when more than one well is considered in a gathering network, the gas lift rate differs compared to when the well is being optimized independently(MUKHTYAR; SHASTRI; GUDI, 2013). Figure 4.2 shows the gas-lift well oil production curve and Figure 4.3 shows the workings of a gas lift well.

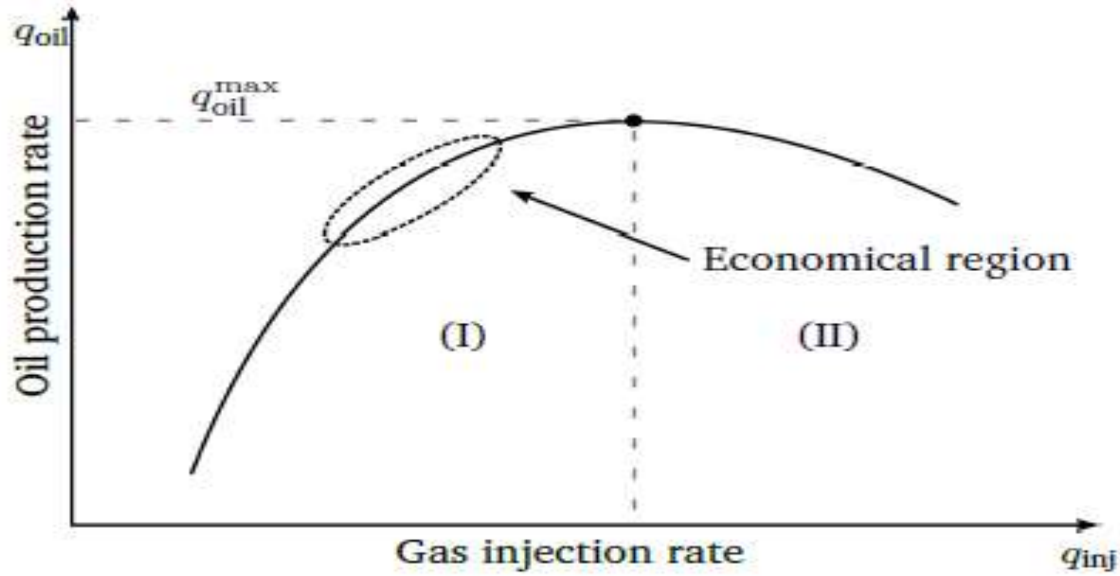

Figure 4.2 - A gas-lift well oil production curve

Source: (HÜLSE, 2015)

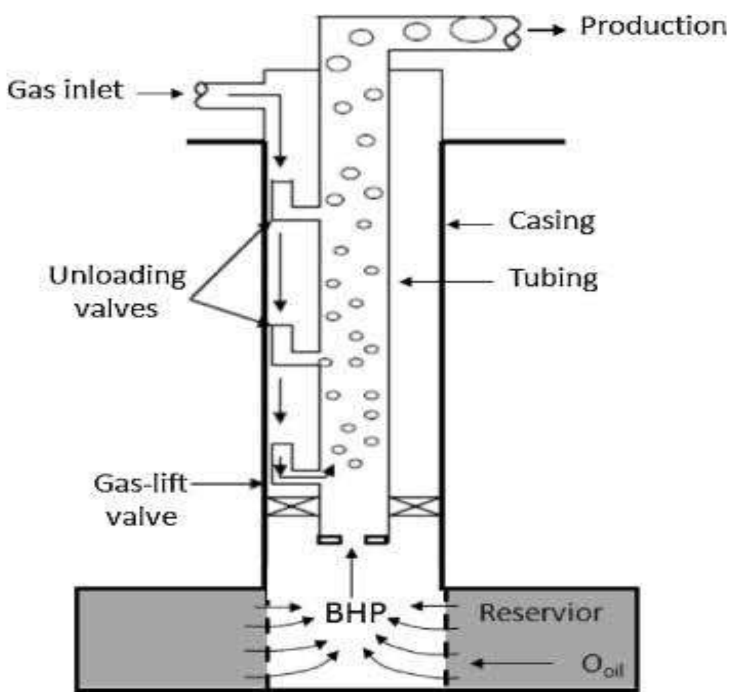

Figure 4.3 - Typical gas lift process

Source: (SHI et al., 2016)

4.2 System modelling

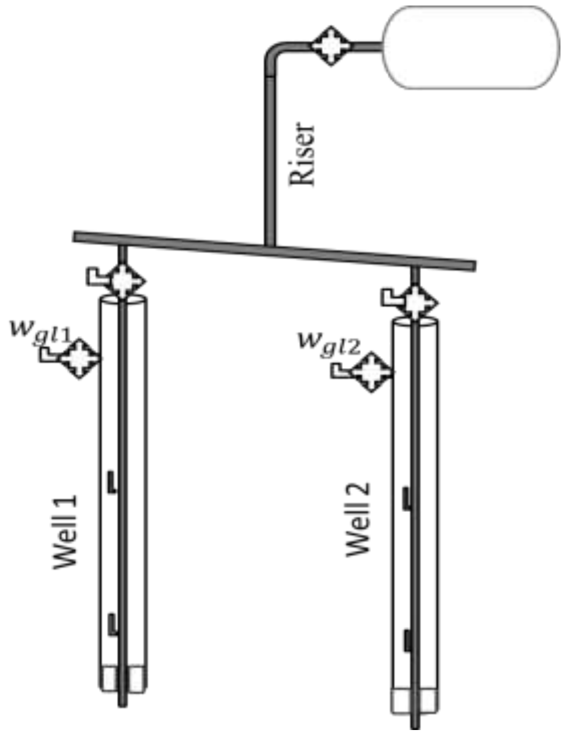

Figure 4.4 - Gas-lift Network with two wells connected by a common manifold and riser.
(Own elaboration)

For this research we consider two producing wells connected to a common riser by a manifold as shown in Figure 4.4.

The oil production process is dynamic therefore, the gas lift model has to be a dynamic model capturing essential details as it changes with time. The instabilities and uncertainties in flow of oil that results from interplay between the gas in the casing and the fluid in the tubing is reflected in the rigorous oscillations in the oil within a short period of time. Over the years models for gas-lift well have been modified to capture the dynamics of the system (EIKREM; AAMO; FOSS, 2008; KRISHNAMOORTHY; FOSS; SKOGESTAD, 2016, 2018). The modeling and assumptions made for the gas-lift process in this study is based on the Eikrem model (EIKREM; AAMO; FOSS, 2008) which include:

1. Temperature is almost constant
2. All liquid produced from the reservoir is oil

The model that describes the dynamics involved in gas lift network system of a multiphase fluid (oil and gas) produced from two different wells interconnected at the same production manifold; the model description is represented in four parts:

(i) Multiphase mass balance

(ii) Density equations

(iii) Pressure equations

(iv) Flowrate equation (EIKREM; IMSLAND; FOSS, 2003; KRISHNAMOORTHY; FOSS; SKOGESTAD, 2018).

4.2.1 Mathematical representation of the model

The system under consideration described in the preceding section, leads to a system of semi-explicit index-1 differential algebraic equations (DAE) of the form:

$$\dot{x} = F_c(x, z, u, \varsigma) \quad (4.1)$$

$$0 = G(x, z, u, \varsigma) \quad (4.2)$$

Where $F_c(x, z, u, \varsigma)$ is a set of differential equations and $G(x, z, u, \varsigma)$ is a set of algebraic equations, $x \epsilon \mathbb{R}^{nx}$ are the set of differential variables, $z \epsilon \mathbb{R}^{nz}$ are the set of algebraic variables, $u \epsilon \mathbb{R}^{nu}$ are the set of control inputs and $\varsigma \epsilon \mathbb{R}^{n\varsigma}$ are the set of uncertain parameters.

The summary of the model parameters and variables as used for the simulation is given below as:

$$x_i = [m_{ga_i}\, m_{gt_i}\ m_{ot_i}\, m_{gr}\ m_{or}]^T$$

$$z_i = [P_{a_i}\, P_{w_i}\ P_{wh_i}\, P_{bh_i}\, P_{rh}\ P_m\ \ \rho_{a_i}\ \rho_{m_i}\, \rho_r\, w_{iv_i}\, w_{pc_i}\, w_{pg_i}\, w_{po_i}\, w_{ro_i}\, w_{rg_i}\, w_{rh}\, w_{to}\, w_{tg}\ ]^T$$

$$u = [w_{gl_i}]^T$$

$$p = [GOR_i]^T$$

This gives 8 differential state variables for each well and the riser, 30 algebraic state variables for each well and the riser, 2 input variables for each well and 2 uncertain parameters for each well. The appendix A shows full details of model description.

For this process, the economic optimization objective is to maximize profit, given by:

$$\max_{w_{gl_i}} J = \left(\$_o \sum_{i \in \mathrm{N}} w_{po_i} - \$_{gl} \sum_{i \in \mathrm{N}} w_{gl_i} \right) \tag{4.3}$$

s.t.

$$F_c(x, z, u, \varsigma) = 0 \tag{4.4}$$

$$0 = G(x, z, u, \varsigma)$$

$$\textstyle\sum_{i \in \mathrm{N}} w_{pg_i} \leq w_{g_{max}} \tag{4.5}$$

Where the $\$_o$ and $\$_{gl}$ are the unit price of oil and the cost of gas compression for the gas-lift process respectively,w_{po_i}, w_{pg_i} and w_{gl_i} are the flow rates of the produced oil, produced gas and gas lift for each well respectively, $w_{pg_{max}}$ is the total capacity of the surface gas treatment facility constraint. The process model (4.1) and (4.2) are enforced as equality constraint which makes it equivalent to RTO.

Here, the flow rate of gas injected for the gas lift process from the surface is the input, and the best choice of this input is required to obtain maximum oil production rate. The gas used for the gas-lift process is compressed at a cost leading to the consideration of compressed gas that can be available for gas-lift distribution between the wells and then the cost of the oil produced. Considering the model constraint, the optimization is performed. For the two layer implementation for both the SRTO and the HRTO, the upper optimization layer provides the optimizing targets (desired decision variables) for the IHMPC to control the trajectory in achieving these targets.

4.3 SIMULATION

All the simulations in this work are carried out using MATLAB R2015a programming software, we simulate the dynamic gas-lifted system using the Euler method at a small step size, and we also carried out the system linearization for the linear MPC model and the EKF using numerical linearization methods. In the oil and gas production process, the annulus pressure, bottom-hole pressure, wellhead pressure, manifold pressure, riser head pressure, total oil and gas flowrates produced and sent to the separator are commonly available measurements (KRISHNAMOORTHY; FOSS; SKOGESTAD, 2018). In the case considered for this work, the annulus pressure for each well, the manifold pressure, the riser-head pressure and total gas produced at the riser are the output measurements used in the EKF for parameter and state estimation.

The NLP problem of the upper optimization layer as well as the control problem in equations (3.13) and (3.17) respectively are solved using the interior point algorithm of '*fmincon*' in MATLAB. The uncertain parameter GOR varies in the plant simulator, interfering as disturbance. We perform the simulation for a period of 10h. For the MPC layer, we use the infinite horizon MPC with zone control, this ensures the stability of the controller and furthermore the controller sampling time is set to 5min.

4.4 Two-layer strategy implementation

4.4.1 Standard Steady-state RTO (Model Parameter Adaptation)

We implement the standard RTO for comparison to the hybrid RTO and the one layer RTO/MPC approach. Here for the steady state detection, an algorithm is set to compare the variance of the current and previous output measurement within a 98% difference tolerance, when the algorithm detects the system steady state, the chosen uncertain parameter is estimated from the output measurement at that time. The parameter estimation is carried out as in equation (3.1) by minimizing the sum of least squared errors between the system process and predicted outputs at steady state. The steady state economic optimization in equation (3.2) is then solved and the optimal decision variables obtained are sent to the controller for implementation. The simulation results will be discussed in section 4.6.

4.4.2 Steady-State RTO using transient measurements (Hybrid RTO)

We consider the implementation of the steady state RTO using transient measurement (HRTO) already discussed. Here, the online parameter estimation is used to estimate the uncertain parameter, which is used to update the state optimizer for economic optimization. The uncertain parameter considered in this case is the gas-to oil ratio (GOR). This is a reservoir parameter hence cannot be measured, this parameter can affect the optimal input required to achieve optimal operations. The discrete extended Kalman filter (EKF) is implemented for the dynamic parameter estimation, the EKF estimates the parameter at the same sampling time as the optimizer, and the optimizer is updated using the estimated GOR parameter. The optimal gas-lift rate is then computed by the optimizer at steady state as in the traditional RTO and send to the controller for desired target tracking.

The estimator tuning parameters play very significant roles in the efficiency of the EKF, since it is just an approximation. Here we assume the model is perfect, therefore a high weight is placed on R demonstrating the trust level of the model predictions and a smaller weight on the Q. The results obtained from the simulation will also be discussed in section 4.6.

4.4.3 MPC implementation in the two layer

As mentioned, after the RTO computes the optimal decision variables, the values are sent to the controller which computes the trajectory for achieving these values. The process considered for this work is a nonlinear, therefore a nonlinear model in equation (4.1) and (4.2) is used at the at the upper optimization layer, but for the MPC layer a linear process model is used, therefore the nonlinear process linearized about the steady state point. We implement the infinite horizon model predictive controller with input targets and control zones in equations (3.17) - (3.23), the output prediction oriented model (OPOM) described in section 2.2.1.1, is used for the controller model. The transfer function model obtained from the linearized system model is shown in appendix A. MPC computes the sequence of optimal input trajectory for the next prediction time step based on the process model prediction and implements just the first value of the computed input trajectory to create a feedback control law (CAMACHO; BORDONS, 2007).

For the system considered in this work, the controller has 2 manipulated input variables, the gas-lift injection flowrates for each of the two wells, 5 controlled output variables. For this work, the annulus pressure of each well, manifold pressure, riser-head pressure and the flowrate of total gas produced at the riser flowing into the separator. The output control zones chosen are with respect to the optimal outputs computed from the RTO layer. The MPC tuning parameters are also shown in Appendix A. EKF is used for the state feedback. We show and discuss the controller results in the result discussion section.

4.5 One-layer Implementation

We consider the implementation of the one layer RTO/MPC based on the simplified one layer RTO/MPC proposed by DE SOUZA; ODLOAK; ZANIN, (2010) discussed earlier. As mentioned, the gradient of RTO economic objective is included in the controller cost function as in equation (3.32). Therefore, we do not have a different layer for the RTO, rather the controller computes the optimal manipulated input variable that it implements in the system. We calculate the gradient of the economic objective equation (4.3) with respect to the inputs at predicted steady state numerically using the forward finite differences sensitivity. The sensitivity computes the changes in the objective function and output with change small changes in the input as shown in the equations (4.6) - (4.7) :

$$J' = \lim_{\Delta t} \frac{h(x,u,\varsigma + \Delta t) - h(x,u,\varsigma)}{\Delta t} \tag{4.6}$$

$$J'' = \lim_{\Delta t} \frac{\frac{h(x,u,\varsigma + 2\Delta t) - h(x,u,\varsigma + \Delta t)}{\Delta t} - \frac{h(x,u,\varsigma + \Delta t) - h(x,u,\varsigma)}{\Delta t}}{\Delta t}$$

$$J'' = \lim_{\Delta t} \frac{h(x,u,\varsigma + \Delta t) - 2h(x,u,\varsigma) - h(x,u,\varsigma - \Delta t)}{(\Delta t)^2} \tag{4.7}$$

Where $J'$is the gradient of the economic cost, J'' is the hessian, Δt is the small change in the input and h is the output function. An infinite horizon zone control MPC is also applied rather than the finite MPC used in the original paper proposing the technique. The results obtained will be discussed in section 4.7.

Table 4.1 and Table 4.2 show the parameters used in the simulation of the gas lift network system and Table 4.3 shows the values of the GOR used for the plant simulation. The full model description is shown in the appendix A.

Parameter	Units	Well 1	Well 2
L_w	[m]	1500	1500
H_w	[m]	1000	1000
D_w	[m]	0.121	0.121
L_{bh}	[m]	500	500
H_{bh}	[m]	500	500
D_{bh}	[m]	0.121	0.121
L_a	[m]	1500	1500
D_a	[m]	1000	1000
H_a	[m]	0.189	0.189
ρ_o	[kg/m^3]	800	800
Civ	[m^2]	0.1E-3	0.1E-3
Cpc	[m^2]	2E-3	2E-3
P_r	[bar]	150	155
PI	[kg s^{-1} bar^{-1}]	0.7	0.7
T_a	[^{o}C]	28	28
T_w	[^{o}C]	32	32
GOR	[kg/kg]	0.1 ± 0.05	0.1 ± 0.05

Table 4.1 – List of well parameters and their respective values used in the results

Parameter	Units	Riser
L_r	[m]	500
H_r	[m]	500
D_r	[m]	0.121
C_{rh}	[m^2]	10E-3
P_s	[bar]	20
T_r	[^{o}C]	30
M_w	[g/mol]	20
R	[J mol^{-1} K^{-1}]	8.314

Table 4.2 – List of well parameters and their respective values used in the results

GOR well 1	0.1	0.1033	0.0817	0.1198	0.0757	0.0864	1104	0.0854
GOR well 2	0.12	0.115	0.1127	0.1278	0.1197	0.1295	1176	0.1176

Table 4.3 – GOR values used in the well simulator

4.6 Simulation results discussion

- **Parameter and state estimation**

First, we discuss the parameter estimation step. Figure 4.5 and Figure 4.6 show the process measurement used for the parameter estimation; the annulus pressure for each well, manifold pressure, riser-head pressure and total gas produced.

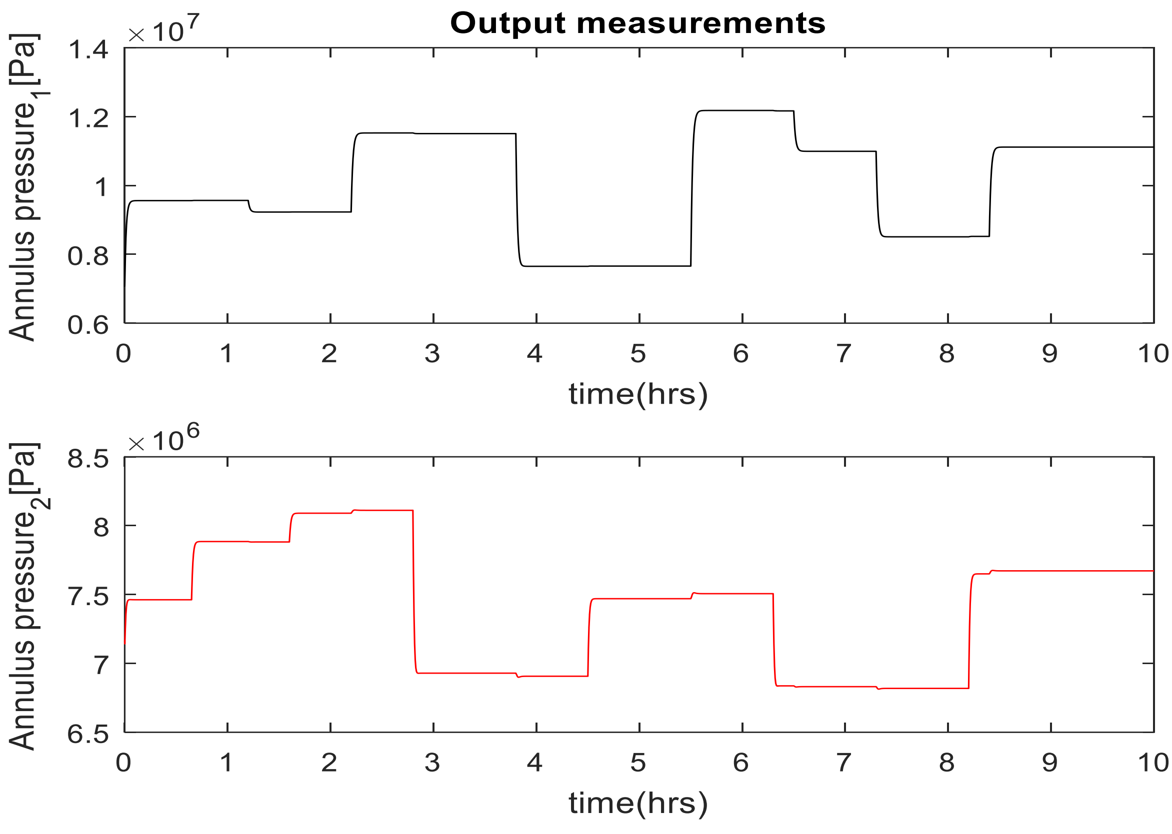

Figure 4.5 – Output measurements used for the EKF parameter estimation

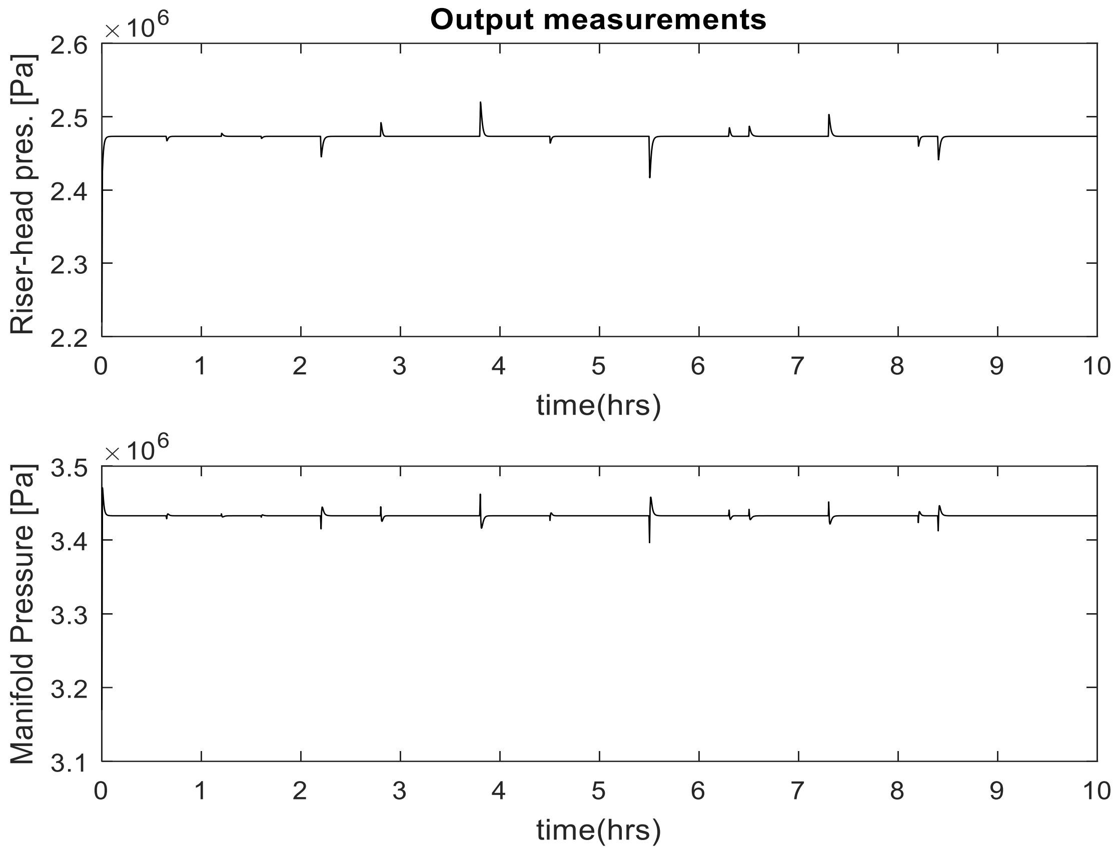

Figure 4.6 – Output measurements used for the EKF parameter estimation

The parameter estimation is a fundamental step in the optimization process, moreover it is the differentiator between the traditional RTO found in the industries and the hybrid RTO technique proposed by KRISHNAMOORTHY; FOSS; SKOGESTAD, (2018) , which we use for this work. We update the optimization layer with the information coming from the filter. Figure 4.7 - Figure 4.10 show the static and dynamic parameter estimations respectively. We can see that Figure 4.7 (static estimator) shows a delay from the parameter values sent to the optimizer by the estimator. While the parameter in the plant has already changed, the parameter estimation algorithm did not pick that change up immediately until a steady state was detected. Furthermore, the change in parameter causes the system to go back to the transient state, and the parameter estimation step can only be carried out when the system steady state is detected. Therefore, during the period where the process is in transient state due to the change in parameter, the process is operating sub-optimally since the parameters used for optimization are not up to date as in the system.

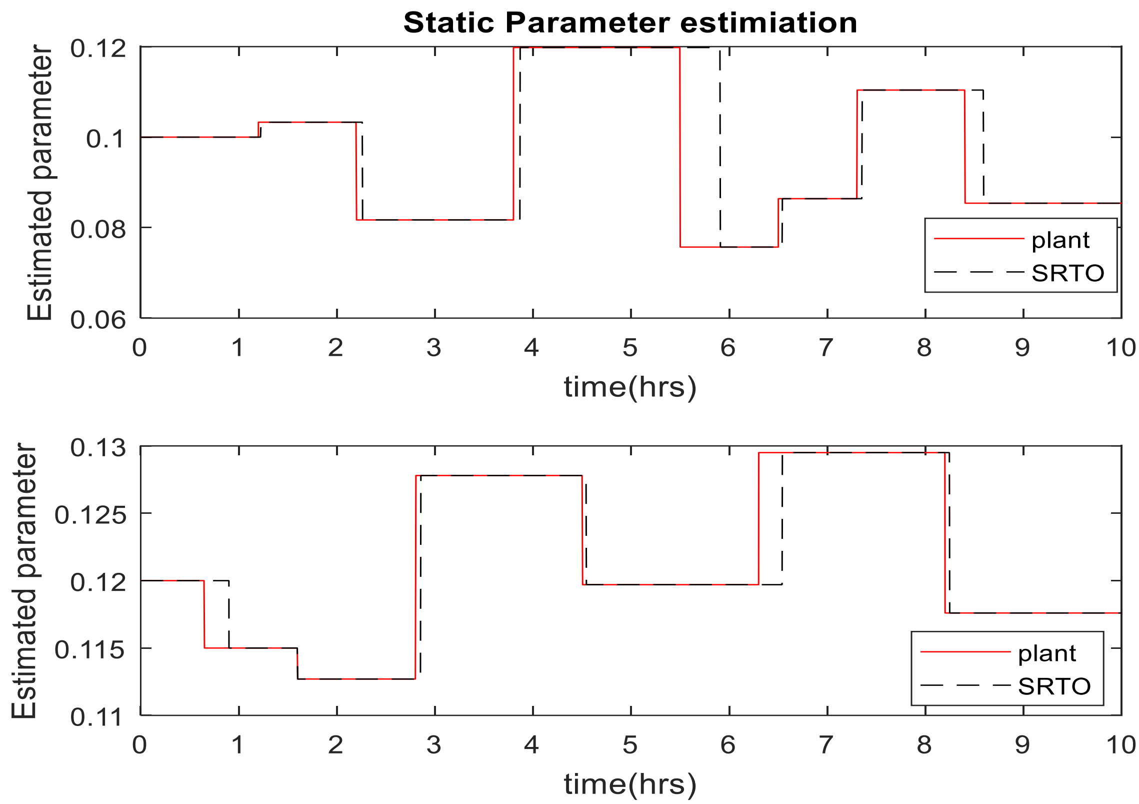

Figure 4.7 – SRTO Estimated parameters (GOR)

However in the hybrid RTO technique, the dynamic estimator performs the parameter estimation online, thereby updating the economic optimizer with a more accurate information as the parameters change in the plant. From Figure 4.8, we can see that the dynamic parameter estimation performed with EKF is consistent with the plant, likewise the state estimations shown in Figure 4.9. Therefore, EKF is working properly and will efficiently transmit a more accurate data to the RTO and MPC.

The results show that the use of an online estimator eliminates the need for steady state detection and consequently the steady-state wait time challenge of the traditional RTO. Figure 4.10 shows the estimation error for the state and parameter estimation. We can see that the EKF (even though with an approximation of the nonlinear model), if efficiently tuned can estimate the parameters and the state with as minimum error as possible. It is important to note that in this case a perfect nominal model is assumed (no model mismatch), so the tuning parameters selected assumes the model used can be trusted.

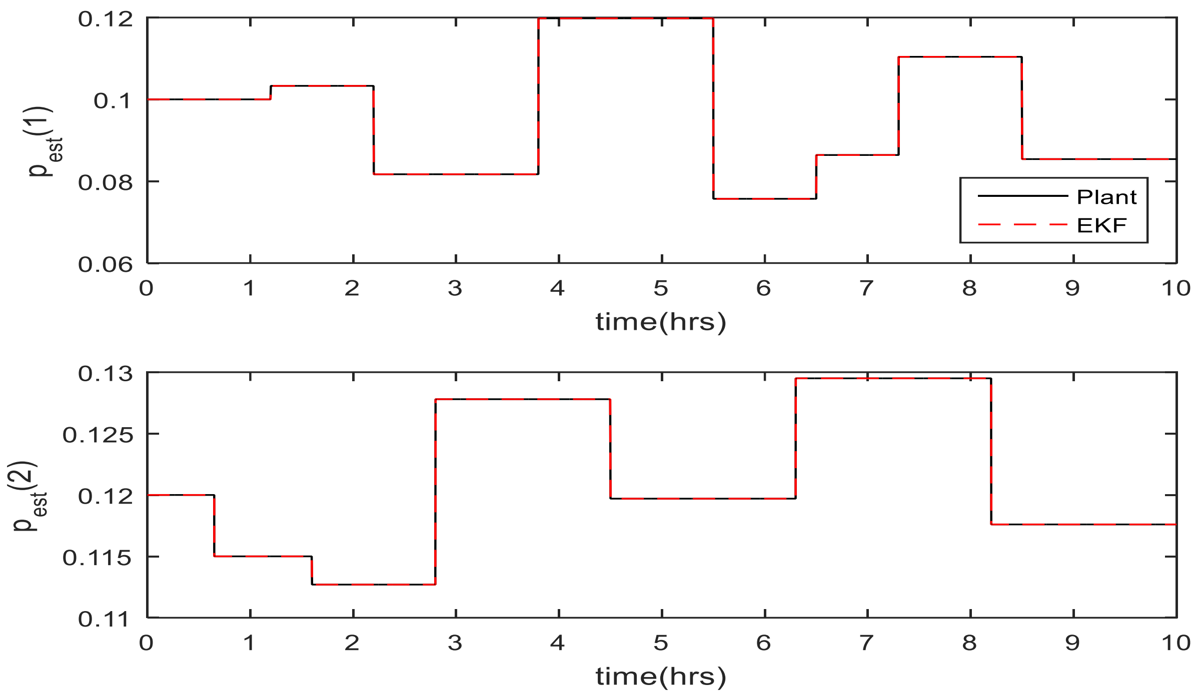

Figure 4.8 – EKF estimated parameters

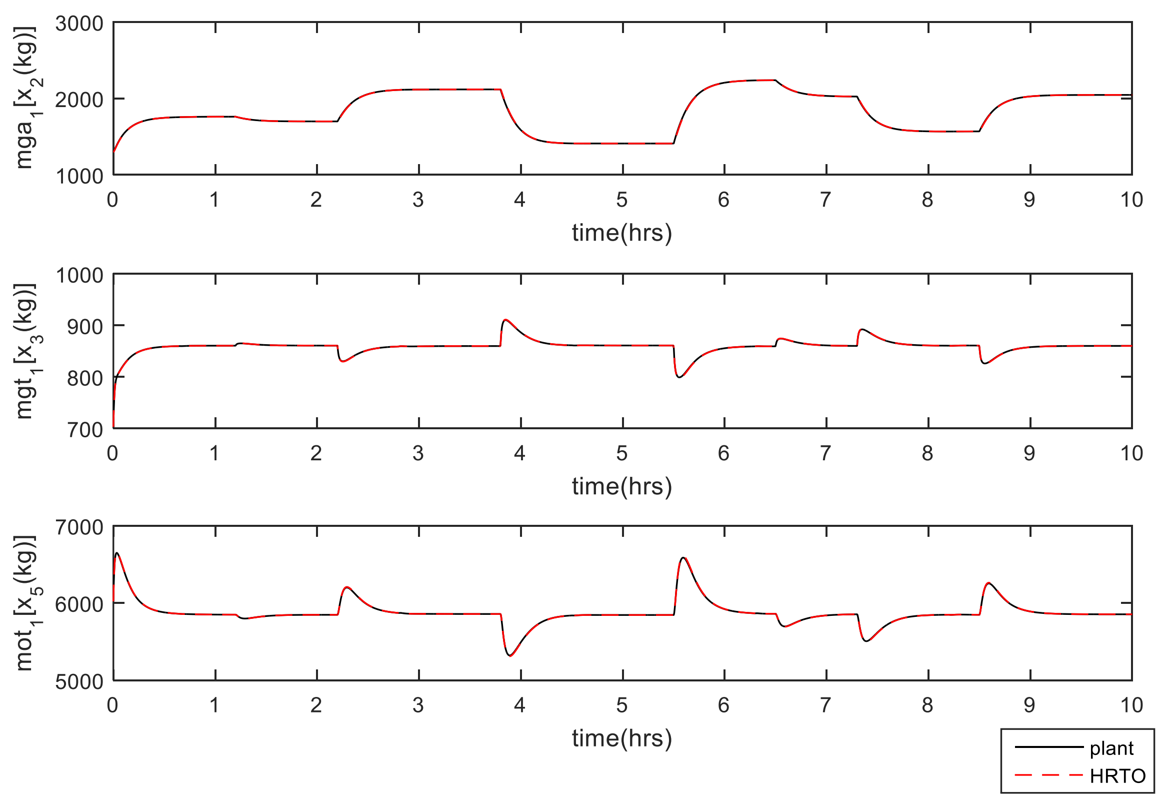

Figure 4.9 – Some of the state trajectories showing the parameter estimation with EKF

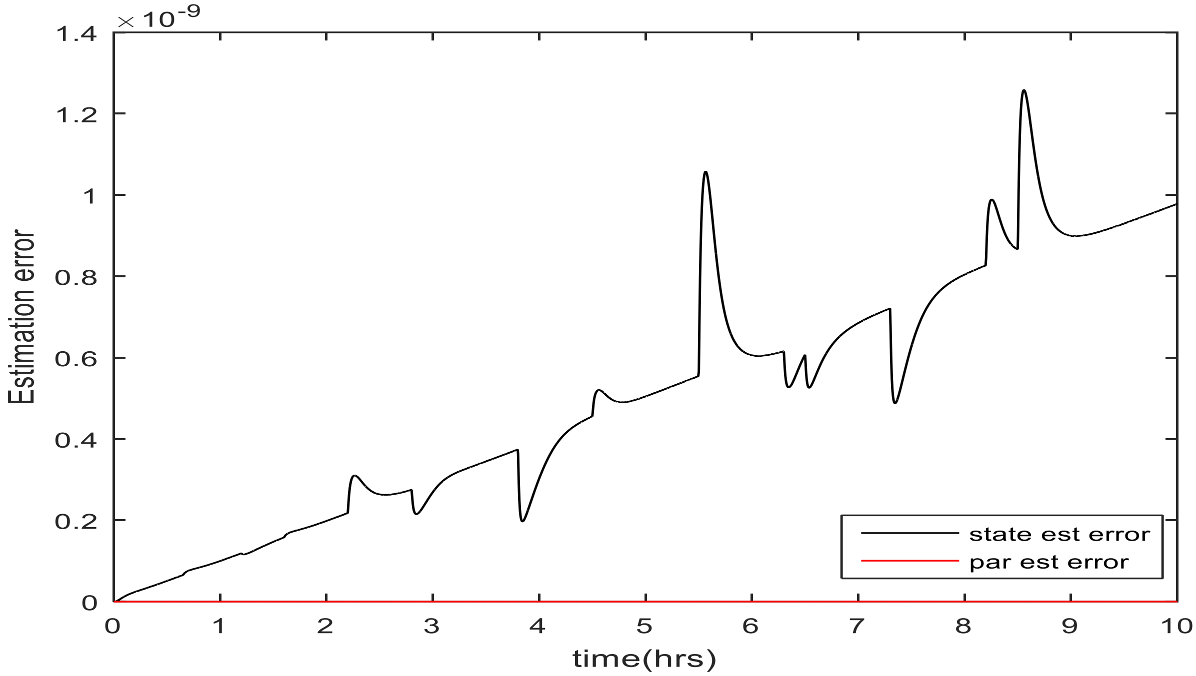

Figure 4.10 – Plant Vs EKF Estimation error

- **Steady state optimization**

Here we discuss the steady state economic optimization. As mentioned, the EKF updates the optimizer at every optimization sampling time, thus the information used for performing the steady state optimization is more accurate and consistent with the system under consideration. Figure 4.11 - Figure 4.12 show the optimal decision variable (input variable) and the economic objective respectively. From Figure 4.11 we can see that the optimal input changes with change in the uncertain parameter (GOR), so the use of the online estimator to update this parameter is justified. Neglecting this parameter could lead to sub optimal operation of the process. It also shows that, a change in the uncertain parameter (GOR) in one well leads to a change in the optimal input for both wells, justifying the point that: for a gas-lift system with more than one well it is essential to update this parameter for each well, as it influences the choice of the optimal input required to achieve the desired economic optimal outcome for the overall process.

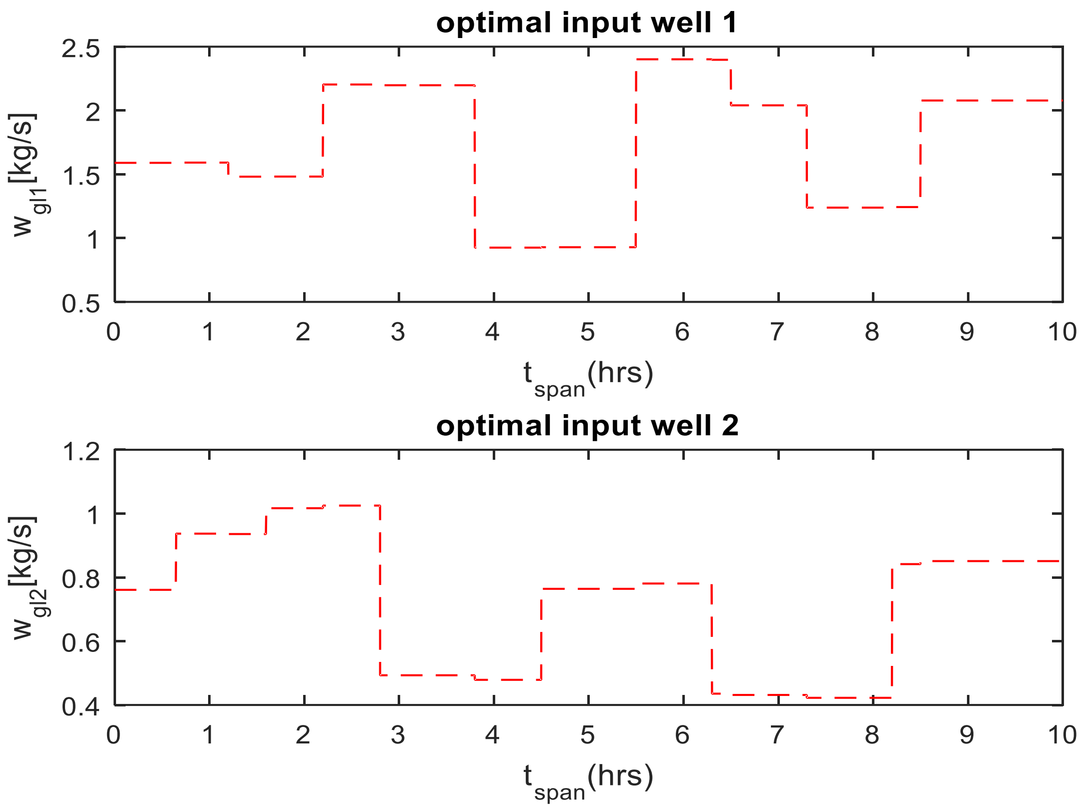

Figure 4.11 – Optimal inputs computed using HRTO

From Figure 4.12 showing the economic cost, we can see how the changes in the GOR causes the system to be in transient state for a significant duration. Therefore, the hybrid RTO enables optimal process operation by computing new steady state gas lift rates (input) as the GOR changes in the system. Table 4.4 shows the optimal gas lift rates as the GOR changes.

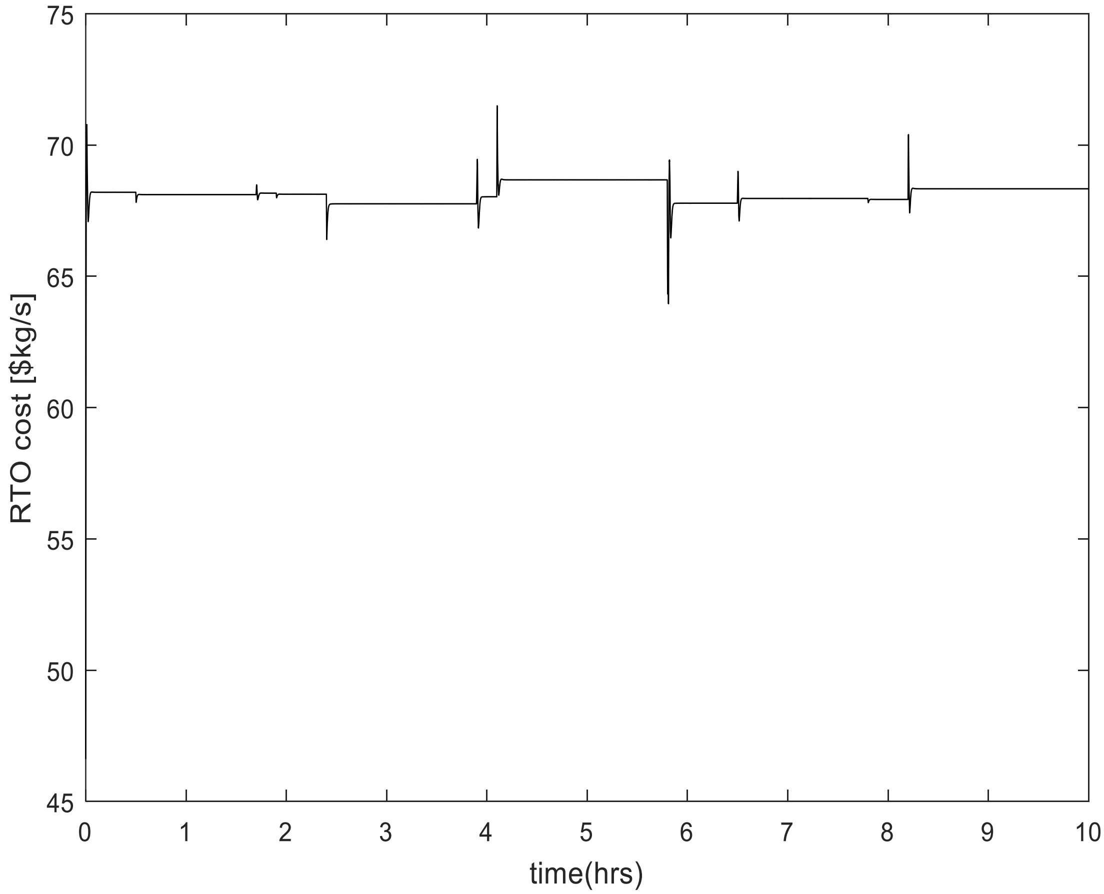

Figure 4.12 - Optimal RTO objective function using HRTO

$w\_gl_1^*$ [kg/s]	1.5898	1.5916	1.4823	2.2037	2.1979	0.9255	0.9285	2.4013	2.3975	1.2388
$w\_gl_2^*$ [kg/s]	0.7609	0.9370	1.0168	1.0250	0.4931	0.4788	0.7640	0.7807	0.4355	0.4224

Table 4.4 – Optimal gas-lift injection rates with changes in the GOR

- **MPC implementation of the two-layer approach**

Here we discuss the implementation of the hybrid RTO with the MPC. This is one of the key contributions of this work, we study the relationship between the two online optimization layers, considering the decision hierarchy challenge which is mostly encountered during implementation in real industrial processes. Figure 4.13 - Figure 4.16 show the simulation results of the RTO with the zone control IHMPC. As mentioned in section 3.1, we implement the hybrid RTO with a controller that has been applied practically. For the zone control IHMPC, the controller objective is also in a hierarchy.

First, the controller ensures that the outputs are within a given optimal interval/region, once this objective is achieved then the attention of the controller moves to manipulating the input to achieve the desired input target provided by the RTO layer. In the event where the outputs go out of the zone, the controller refocuses on forcing it back into the zone. We can see from Figure 4.13 and Figure 4.14 showing the output trajectory, that the controller is working properly and meeting the desired objectives. The controller can efficiently keep the outputs within their given zones.

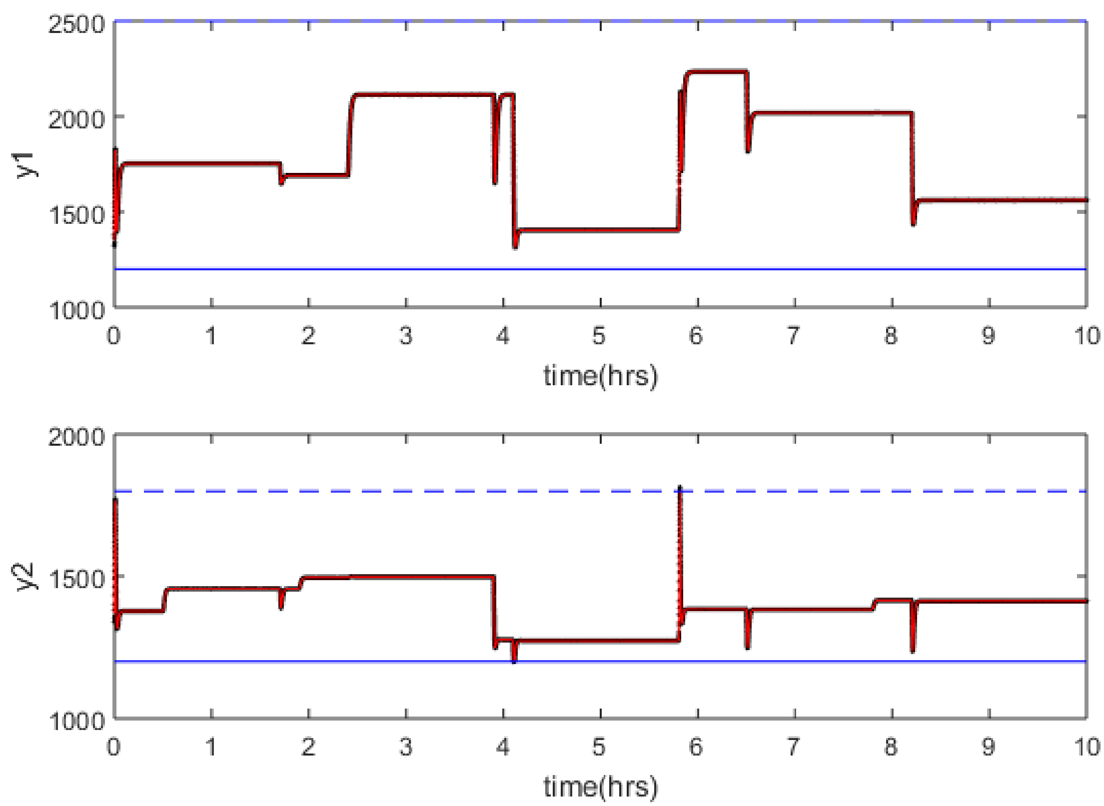

Figure 4.13- HRTO with Zone control MPC Output trajectory

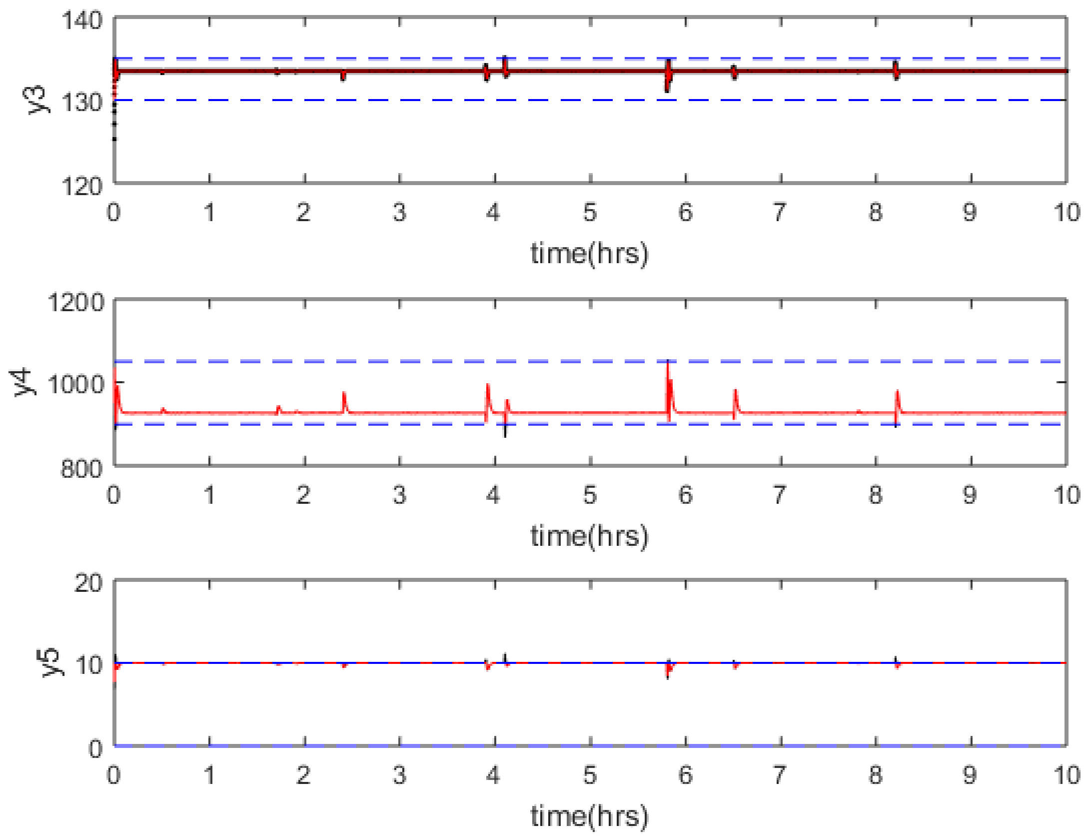

Figure 4.14- HRTO with Zone control IHMPC Output trajectory

Figure 4.15 shows the inputs computed by the controller to reach the desired target. Figure 4.16 shows the input computed by the RTO with the input computed by the controller, we can see that despite the intermittent changes of the GOR leading to consequent abrupt changes of optimal decision variable (input), the controller is still able to reach the desired target sent by the RTO layer.

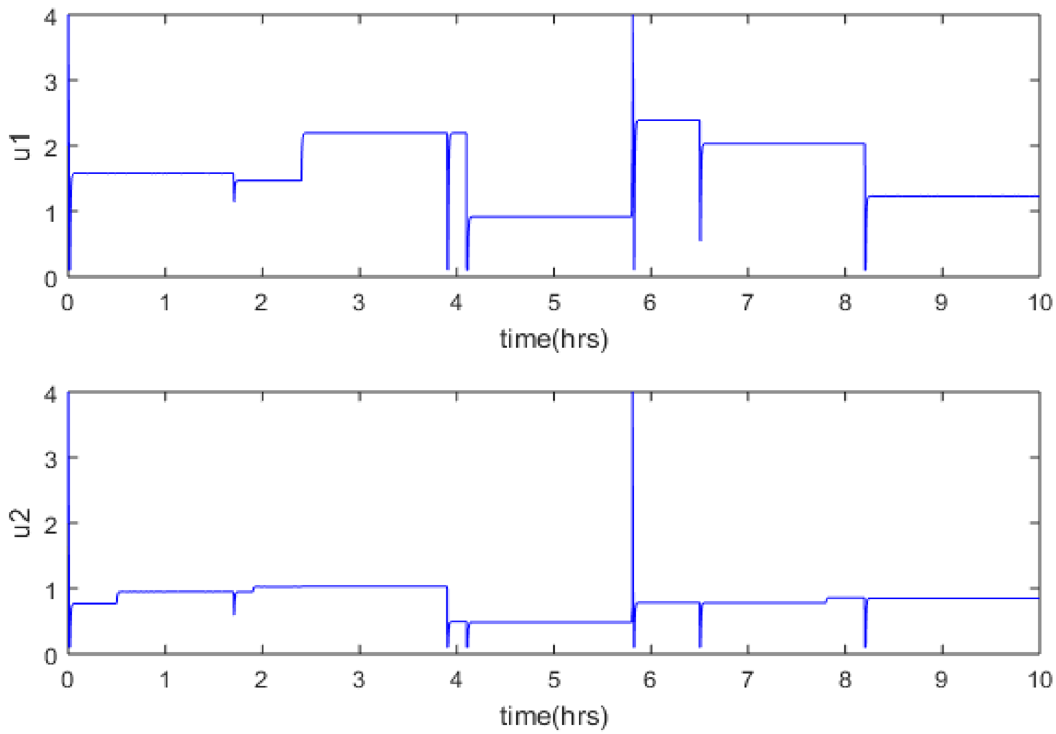

Figure 4.15- Manipulated input targets trajectory HRTO with IHMPC

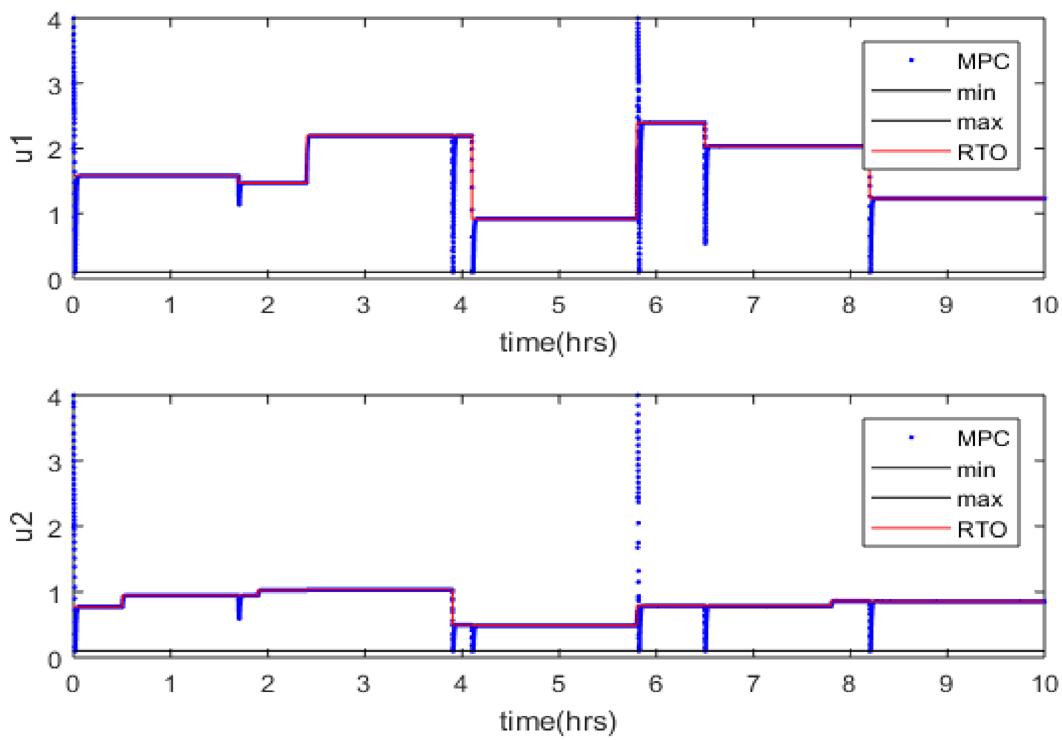

Figure 4.16- Manipulated input targets trajectory HRTO with IHMPC

4.7 One-layer implementation Results

In the second part of this research, we consider the implementation of the one-layer RTO/MPC on a gas-lifted system and examine the difference with the two-layer implemented in the preceding section.

As described earlier, the gradient of the economic objective is included in the controller cost, this allows the controller to consider this when computing the manipulated variable rather than receive the targets and try to reach it. We implement the zone control IHMPC which has also been practically applied. Figure 4.17 - Figure 4.20 show the results obtained. The simulation results show that this controller can optimize the gas lift system and achieve the economic objective for the process very similarly to the two-layer strategy but faster. For this MPC implementation, we also use the EKF for updating the states in the controller. We see from Figure 4.17 and Figure 4.18 showing the output trajectory, that the outputs are maintained within the optimal range provided. The effect of the changes in the GOR is also evident in the outputs.

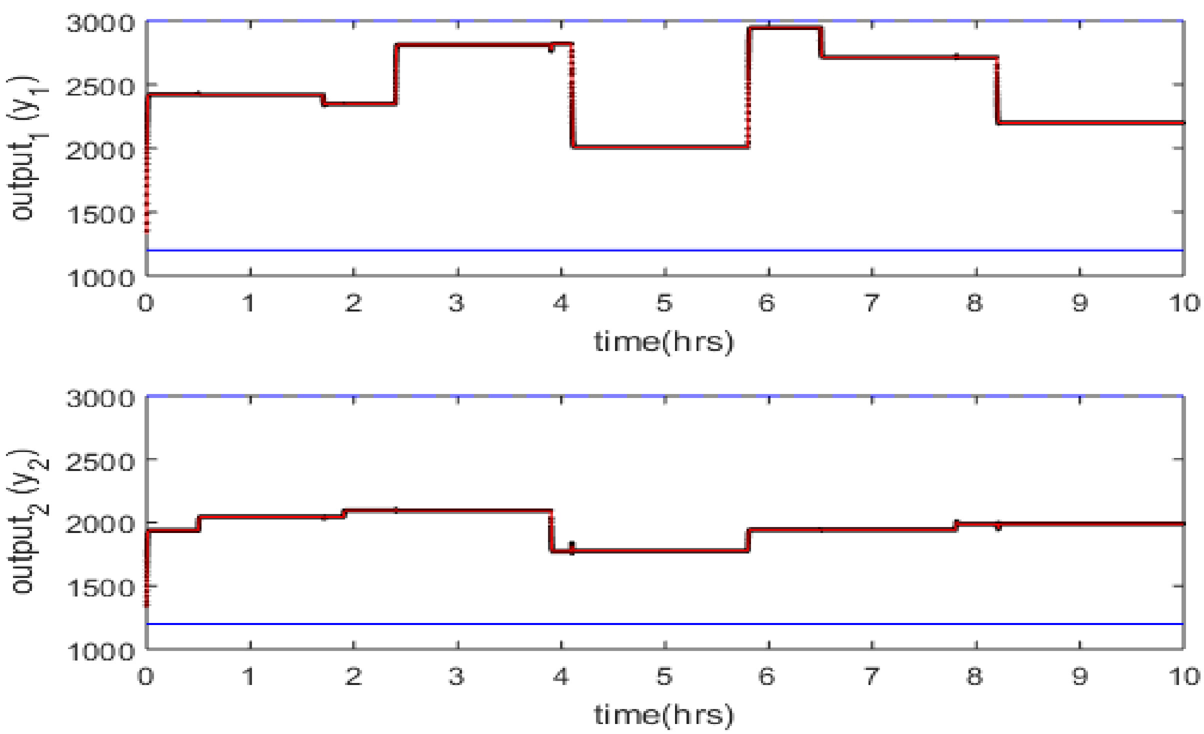

Figure 4.17 – One-layer RTO with Zone control IHMPC Output trajectory

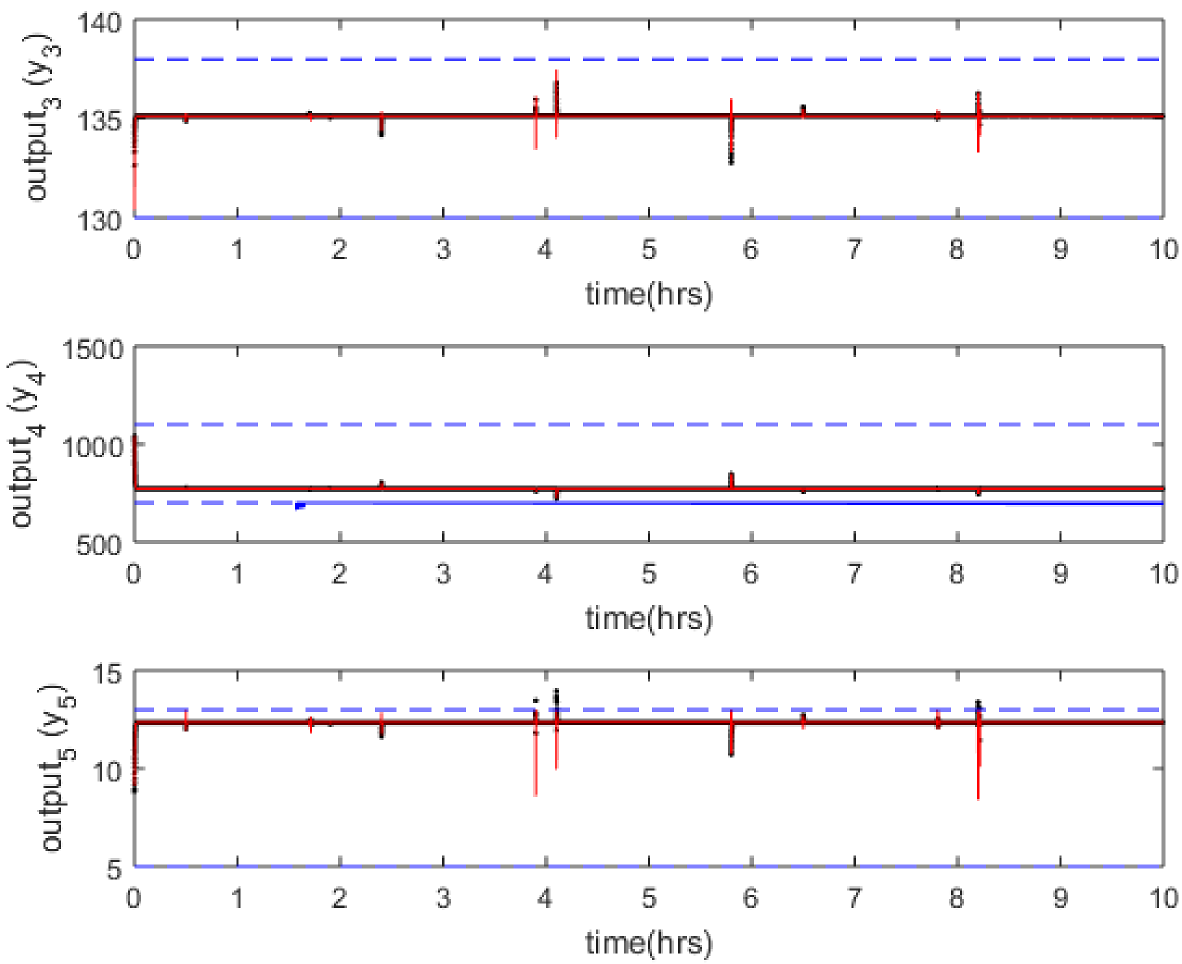

Figure 4.18 – One-layer RTO with Zone control IHMPC Output trajectory

Figure 4.19 shows the manipulated input variable. Here we can see that, as expected the control signal for the one-layer is not exact as the two-layer but gives a similar trajectory. Figure 4.20 shows a similar economic objective as the two layer strategy is achieved. In the next chapter, we will be discussing the differences in the simulation results obtained in the two optimization strategies applied for this work.

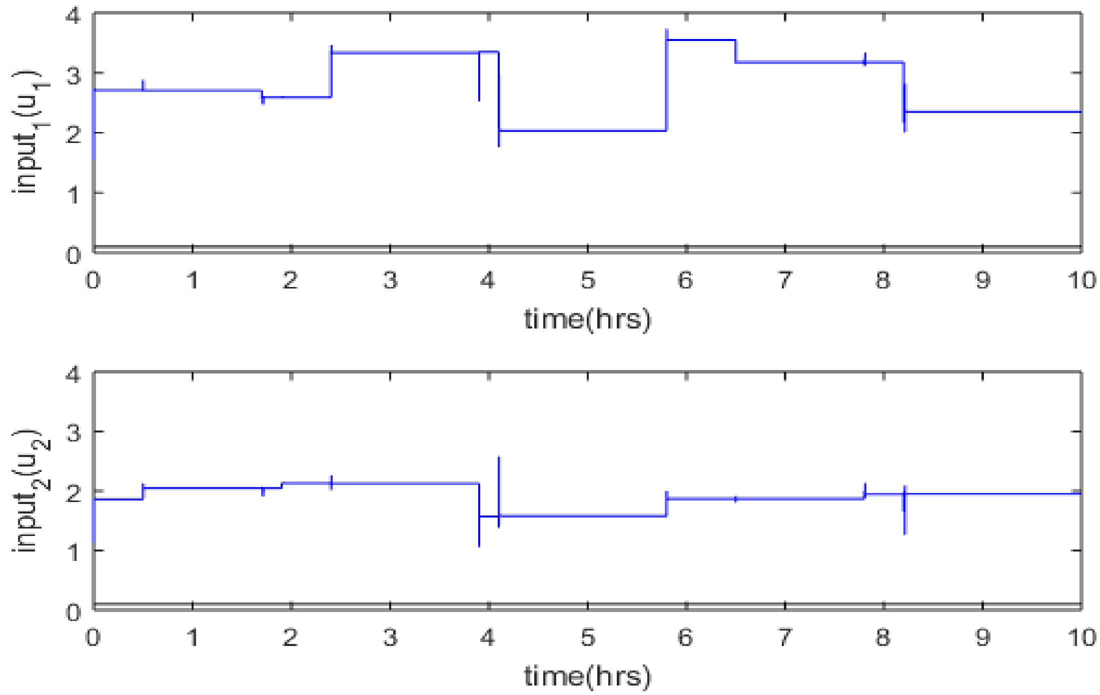

Figure 4.19 – One-layer RTO with Zone control IHMPC manipulated input trajectory

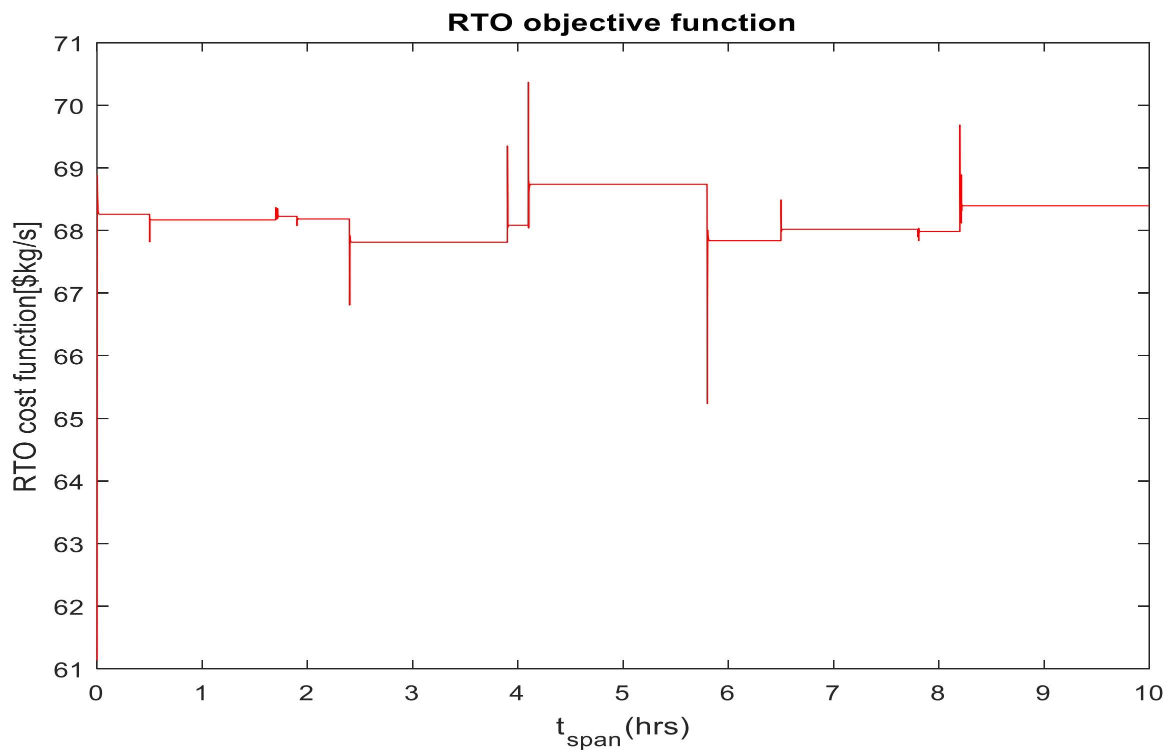

Figure 4.20 – One-layer RTO Economic Profit

5 COMPARISON OF THE TWO INTEGRATION STRATEGIES

In this chapter, we discuss the two optimization strategies considered in this research work, and compare the results we obtained from the simulation. The performance metrics considered for this work, are mainly the economic cost, the optimal decision variable (the gas lift rate) computed to meet these economic objectives considering the GOR interfering as an uncertainty and the time of implementation.

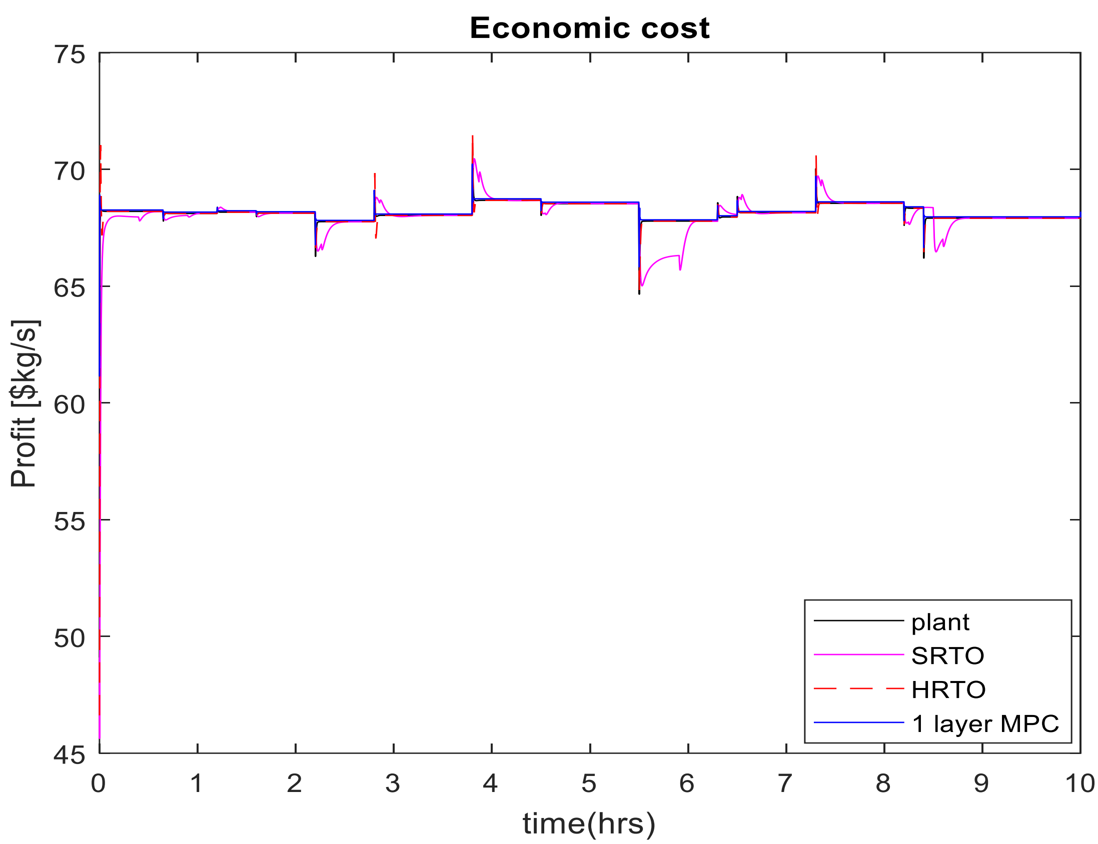

Figure 5.1 - Comparison of the economic performance

First, we compare the economic objective for the system as given in equation (4.3). Figure 5.1 shows the profit computed using the two layers – SRTO and HRTO - and the one layer strategies. We can see from Figure 5.1 that the three optimization techniques are able to give a similar economic cost function, except for the delay in the SRTO due to the steady state wait time (hence the need for HRTO). The HRTO and the one-layer IHMPC are overlapping each other, this shows that, a similar economic objective can be achieved using both HRTO and the one-layer

IHMPC. It also shows the dynamics of how the system steady state is affected by the disturbance. We can see that the system quickly compensates the disturbance, and maintain optimal operation. The one layer also gives an optimal operation, however, for the system under consideration, it does not handle disturbance as well as the two-layer technique. It requires a longer time to return to steady state after the system is disturbed, therefore the choice of sampling time and weighting matrices play a significant role in the simulation. As expected, the SRTO delays, thus making the system operate sub optimally.

In Figure 5.2 and Figure 5.3 – **Comparison of the manipulated input 2 (gas-lift rate)**, we show the optimal inputs computed in the two-layer and one-layer techniques. We can see that the inputs follow the same trajectory finding new optimal input (decision variable) as the uncertain parameter (GOR) changes. Furthermore, the two-layer (HRTO) and the one-layer optimization strategy gives similar response, although the one-layer is more sensitive to the changes in the GOR, it still tries to find and stabilize at the optimal value. We can also see the delay in the response of the traditional two-layer (SRTO) approach found in the industry. The simulation result demonstrates the need for the HRTO proposed, where this uncertain parameter that can affect the optimal operation of the system should be consistently updated.

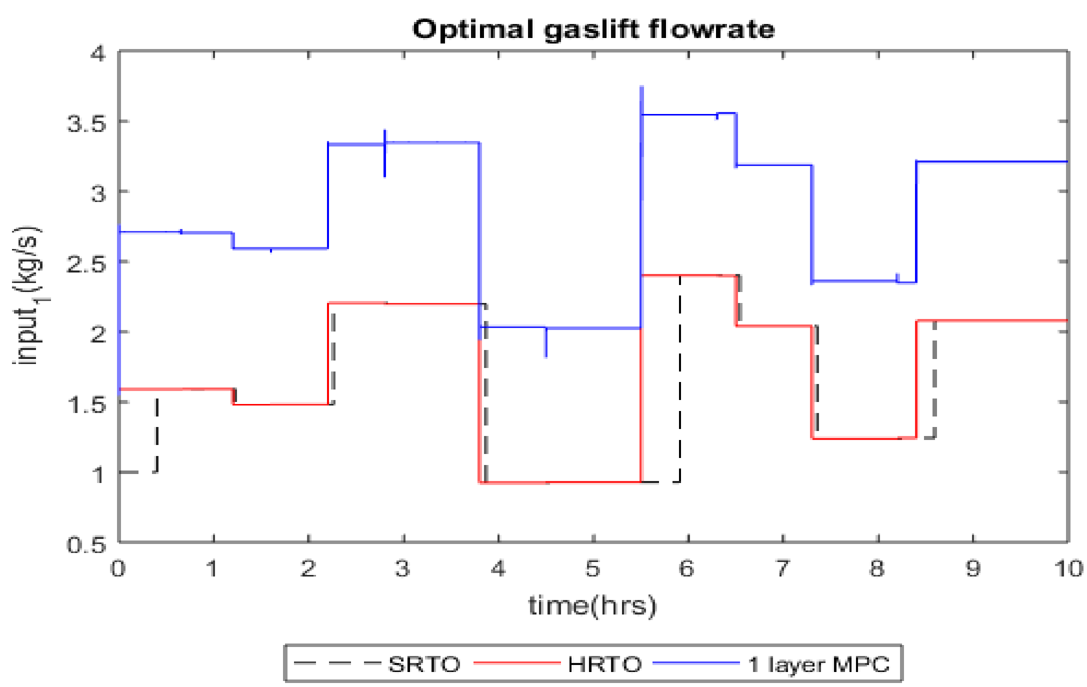

Figure 5.2 – Comparison of the manipulated input 1 (gas-lift rate)

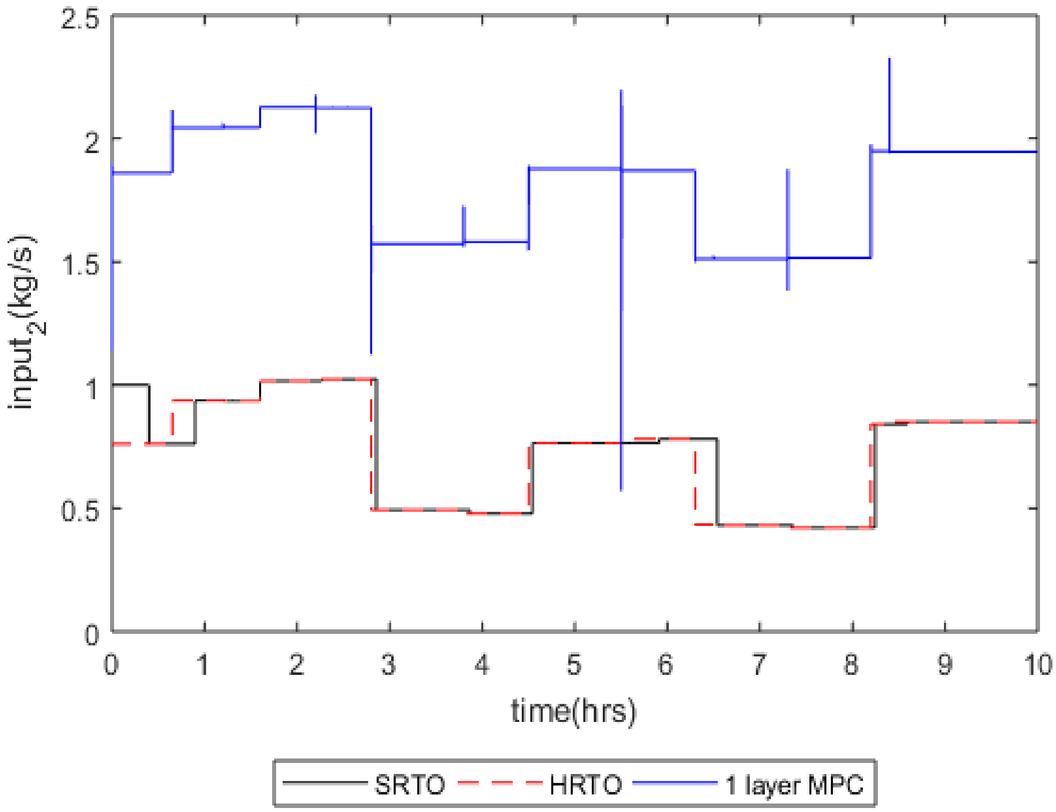

Figure 5.3 – Comparison of the manipulated input 2 (gas-lift rate)

Table 5.1 shows the performance index of the two strategies, it can be seen that while the HRTO gives a little more value of total profit than the one-layer, the one-layer gives a very similar economic cost at a shorter computation time. Moreover, the idea behind the simplified one layer RTO/MPC integration used in this work is to reduce computation effort and time for similar or close economic cost, this could be an advantage over the two-layer strategy depending on the trade-off considered. The table also shows a higher percentage increase in production in the two-layer strategy compared to the one-layer. This is because of the initial point of optimization in both strategies. While the steady state RTO (two-layer) has particular optimal points for each GOR value at steady state, therefore obtains this value from the first optimization point until the GOR changes. The one-layer RTO/MPC continues to find the optimal operating point at each point of the optimization until the system gets to steady state.

Strategy	Increase in production [%]	Avg time [s]	Total profit [x $10^6$$]
Two layer (SRTO)	38.3	0.3037	2.449
Two layer (HRTO)	45.7	0.3288	2.454
One layer	11.5	0.0876	2.453

Table 5.1 – Performance comparison

5.1 DISCUSSION

The production of petroleum is chiefly an economic objective process, therefore achieving an optimal operation and maximizing revenue depends on a tradeoff. In the current clime, where the constraints is getting more strict profit still have to be made. Of course representing the real system for optimization is necessary, this have shifted the research discussions towards the nonlinear online optimization techniques (dynamic RTO, EMPC and NMPC), these are instinctively the points of reference when discussing optimal operations since they best represent the process. However, for large-scale and quite complex systems such as the gas-lifted system, these techniques could further magnify the problems they set out to solve. This is as a result of limited computing resources currently available for large-scale application. With large-scale system operations such as a gas-lifted system, which constitute a large number of variables and constraints, the use of dynamic online optimization techniques usually lead to large nonlinear programming problems. Therefore the addition of the time dimension can result in computational exhaustion as the tools are limited, consequently problems of closed loop instabilities and sullied performance arises (ALLGÖWER; FINDEISEN; NAGY, 2004).

With the limitations of these sophisticated 'ideal' techniques it is not clear when the available existing techniques are enough to achieve reasonable optimal operations, for the petroleum production process, the question continues to arise as asked by FOSS; KNUDSEN; GRIMSTAD, (2018) when is the static RTO technique enough to give the desired optimal operation? Could the profit obtained using a sophisticated, expensive but comprehensive technique be justified compared to the profit obtained using a modified simple, cheaper and available technique. For processes where the changes are frequent and mostly in transient states, the dynamic online optimization would be more adequate and efficient, but for the gas-lifted system where the changes in system parameters are not as frequent, and steady state could be achieved for a significant amount of time before another change, the static optimization could be enough.

As pointed out by KRISHNAMOORTHY; FOSS; SKOGESTAD, (2018) the computing power challenge is more evident when the optimization has a discrete integer decision variables such as cyclic operational processes, start up and shutdown etc. The paper compares the hybrid RTO to the dynamic RTO, demonstrating that the static RTO used in the hybrid RTO gives a preferred formulation present in real industrial applications. Moreover, the modification of the parameter estimation aspect of the traditional RTO gives it more benefit to be applied, as it addresses the steady state wait time challenge which is one of the structural issues plaguing the static RTO.

As mentioned, the one-layer RTO/MPC strategy was proposed to handle the decision hierarchy challenge, that is, avoiding conflict of interest between the two optimization layers (RTO and Supervisory control), as well as to enable practical application in real processes. And as shown in the comparative simulation results already discussed, it is also able to achieve quite similar economic objective as the two-layer. The one-layer RTO/MPC could be preferred in the case where the traditional RTO is implemented with the MPC due to the steady state wait time challenge, as the one-layer technique would consider the change in the parameter faster than when the traditional RTO is used as demonstrated by ZANIN; TVRZSKÁ DE GOUVÊA; ODLOAK, (2002). The main challenge with the one-layer is the inclusion of the nonlinear economic cost, which leads to the NLP, making it demand higher computing power and time. Therefore, the simplified RTO/MPC of the one-layer technique proposed by DE SOUZA; ODLOAK; ZANIN,

(2010) to reduce the computational burden makes it even more attractive for some large-scale industrial processes, especially when practical applications are considered.

The argument made that the introduction of the RTO further complicates the control system, and incur additional cost since the implementation can be expensive; given that from extended knowledge and experience, it is likely that the operator could easily know which variables are to be kept within their bonds. However, knowing that optimal operating points are likely found in the intersection of constraints and the influence of disturbance over the optimal operating points, especially if the system constraints can change, it is safe to point out that operators would not efficiently detect, identify or optimize these changes especially if frequent and abrupt. As explained in a detailed RTO/MPC application review by ENGELL, (2007) and DARBY et al., (2011), the introduction of the RTO layer in the optimization hierarchy gives a clear separation of objective and implementation time scale between the RTO and process control, this can be a benefit or a disadvantage depending on the system under consideration.

For processes where optimal operation point can be consistently determined by the MPC, even if it lies at the constraint, then RTO would not be necessary, whereas in processes where the optimal operating points varies due to changes in process constraints, process nonlinearities and significant economic variations, then RTO will be required to find this optimal point consistently with the operating conditions. Ultimately, RTO could account for tradeoffs in some cases and MPC cannot (DARBY et al., 2011).

The need for the use of RTO layer and by extension the two-layer strategy will always attract scrutiny because of the ability of the MPC to perform the functions of the RTO, therefore the use of the RTO for the optimization of a process must be justified. Moreover, if the variability of process variables is not so frequent, the benefit is minimal because the process steady state will not be highly disturbed and consequently the optimal operating point will necessarily be affected thus application of the RTO not be cost effective. As pointed out by DARBY et al., (2011), while a process like the ethylene plants enjoy the benefits of the two layer strategy, refining processes like the FCC and crude unit face issues with the strategy. However, the hybrid RTO addresses the issue of the steady-state wait time, which is the main challenge on the delay in optimization cycles, making it possible for a more frequent optimization cycle than in the traditional RTO.

As mentioned, the simplified one-layer RTO/MPC approach applied to the gas-lifted system in this work, was proposed for the purpose of practical application on large-scale industrial process. Furthermore, in this work the controller decision variables include the output set-point which enables the controller to compute the reference value for optimal operation and a slack variable to ensure feasibility. Therefore, in this case the controller does not require the reference point to be provided by the RTO layer or the operator. The controller, considers the gradient of the economic cost when computing the optimal input and the reference value, thereby relegating the need for the RTO layer. This approach is more practical to implement in the industry especially for large-scale processes. Moreover, the simplified one-layer RTO/MPC is faster and simpler to implement, than the two-layer and the one-layer RTO/MPC (where the nonlinear economic cost is directly included in the controller cost function), and it also achieves similar results. However, it is prone to more numerical problems since it mostly involves a lot of approximations of the nonlinear process under consideration, the choice of tuning parameters used play a significant role.

Ultimately, there is no one size fits all approaches to optimizing large-scale systems, it always comes down to understanding the nature of the system and the trade-off to be made with respect to the performance in either time, profit or intrinsic details. Therefore, for a process like the gas-lifted system which we are considering for this research, where the uncertain parameter change can be considered as intermediately frequent and have a significant effect on the optimal decision variables for optimal operations, the two-layer approach using the hybrid RTO strategy shows better performance in handling the disturbance (GOR changes). However, the one-layer strategy is a more practical approach that is simpler and faster to implement, giving a similar result (economic cost and optimal input trajectory) as the two-layer. Unlike the two-layer strategy that has a particular optimal operating point at steady state, and depends on the MPC to find the trajectory that drives the system to this point, the one-layer is more dynamic, it finds the optimal operating point of the system at each sampling time until it achieves steady state.

6 CONCLUSION AND RECOMMENDATION

6.1 CONCLUSION

This research work presents a comparative study between the two main RTO/MPC integration techniques found in the industry, applied to a gas-lifted system. The contribution of this work is in two ways.

First, the two layer RTO/MPC integration strategy is applied to a gas-lift system by implementing a modified RTO technique called 'hybrid RTO' with a nominal zone control IHMPC, with the objective of studying the practical application of this modified technique. The hybrid RTO technique proposed by KRISHNAMOORTHY; FOSS; SKOGESTAD, (2018) addresses the issue of the standard RTO steady state wait time which is a major challenge in the practical application bringing to question the need for RTO. The technique enables the RTO to be updated with more reliable information and a more frequent successive RTO cycle. The simulation results show that the nominal zone control IHMPC applied can efficiently reach the desired targets provided by the so called hybrid RTO, while keeping the outputs in their given zones. The technique was able to achieve 45% increase in the given economic cost from the initial point of optimization.

Secondly, the one layer RTO/MPC integration strategy proposed by DE SOUZA; ODLOAK; ZANIN, (2010) is applied to the gas-lifted system for comparison with the hybrid RTO technique and the existing traditional RTO found in the industries. In this work, the MPC is extended to an infinite prediction horizon and the decision variables include the output reference value and a slack variable to ensure feasibility. The one-layer RTO was implemented in a shorter time and achieved similar economic objective as the two-layer HRTO, it is also takes into account the variation of the uncertain parameter which comes in as a disturbance, however it does not handle this disturbance as well as the two-layer. Moreover, the two-layer returns to steady-state faster after encountering disturbance.

Overall, the study shows that, the modification made in the traditional RTO leading to the hybrid RTO is a useful development that should attract better reception of RTO in the industry. Moreover it solves a major challenge of the controversial steady state wait time, bridges the gap between the traditional RTO and the dynamic optimization techniques, and can serve as an intermediary techniques for some systems with enough disturbance that the traditional RTO will be too limited and the dynamic optimizations would be overreaching. The one-layer on the other hand, with a simpler and faster implementation achieves a similar result as the modified two-layer (HRTO), which gives it an advantage for practical application.

6.2 RECOMMENDATION

In furtherance of this work:

- More large scale processes should be considered to test the new RTO technique with MPC.
- The robustness of the controller applied in this work for both the two-layer and one-layer technique could be tested using output zone tracking.
- It would also be interesting to consider a robust MPC based on a multi-model representation to account for the nonlinearities that was not considered in the nominal MPC. Moreover, it will also be interesting to see if a robust MPC could handle the disturbances better than in the case of the one-layer.
- It will be interesting to consider more implementation of practical controllers on oil and gas production systems. Consider more variables within the system that can be controlled to maximize revenue.
- It will also be interesting to consider more strict constraints such as environmental restrictions to allow for a more robust optimization.

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APPENDIX A

The gas-lift system under consideration consist of two wells connected to a riser through which the total produced oil is transported to a separator. The system can be described using first principle differential and algebraic equations. The mass balances in each well and the riser models the dynamics in the systems represented in equations A1-A5

$$\mathbb{N} = \{1,2 \ldots . n_w\}$$

$$\frac{dx_{1i}}{dt} = wgl_i - wiv_i \tag{A.0.1}$$

$$\frac{dx_{2i}}{dt} = wiv_i - wpg_i + wrg_i \tag{A.0.2}$$

$$\frac{dx_{3i}}{dt} = wro_i - wpo_i \tag{A.0.3}$$

$$\frac{dx_4}{dt} = \sum_{i=1}^{n_w} wpg_i - wtg \tag{A.0.4}$$

$$\frac{dx_5}{dt} = \sum_{i=1}^{n_w} wpo_i - wto \tag{A.0.5}$$

where, $x_{1i}$is the mass of gas in the annulus, $x_{2i}$is the mass of gas in the well tubing, $x_{3i}$is the mass of oil in the well tubing $x_4$is the mass of gas in the riser and $x_5$is the mass of oil in the riser, $wgl_i$is the gas lift injection rate, $wiv_i$is the gas flow from the annulus into the tubing, $wpg_i$and $wpo_i$are the produced gas and oil flow rates respectively and, $wrg_i$and $wro_i$are the gas and oil flow rates from the reservoir for each well i,wtg and wto are the total gas and oil flow rates respectively.

The densities in the model are represented by equations (A.0.6) - (A.0.11) for the two wells and the riser head:

$$\rho a_i = \frac{M_w Pa_i}{Ta_i R} \tag{A.0.6}$$

$$\rho m_i = \frac{x_{2i} + x_{3i} - \rho_o L_{bhi} A_{bhi}}{L_{wi} A_{wi}} \tag{A.0.7}$$

$$\rho_r = \frac{x_4 + x_5}{L_r A_r} \tag{A.0.8}$$

$$\rho a_i = \frac{M_w Pa_i}{Ta_i R} \tag{A.0.9}$$

$$\rho m_i = \frac{x_{2i} + x_{3i} - \rho_o L_{bhi} A_{bhi}}{L_{wi} A_{wi}} \tag{A.0.10}$$

$$\rho_r = \frac{x_4 + x_5}{L_r A_r} \tag{A.0.11}$$

Where ρa_i is the density of gas in the annulus of each well and ρm_i is the fluid mixture density in the tubing for each well and $\rho_r$fluid mixture density in the riser, $M_w$is the molecular weight of the gas, R is the gas constant, Ta_i is the temperature in the annulus in each well, ρo is the density of oil in the reservoir,L_{wi} and $L_{bhi}$are the lengths of each well above and below the injection point respectively and $A_{wi}$and $A_{bhi}$are the cross-sectional area of each well above and below the injection point respectively , $L_r and\ A_r$are the length and the cross sectional area of the riser manifold.

Equations (A.0.12) - (A.0.18) show the pressure models for the two wells and the riser head:

$$Pa_i = \left(\frac{T_{ai} R}{V_{ai} M_w} + \frac{g L_{ai}}{L_{ai} A_{ai}}\right) x_{1i} \tag{A.0.12}$$

$$Pwh_i = \frac{T_{wi} R}{M_w}\left(\frac{x_{2i}}{L_{wi} A_{wi} + L_{bhi} A_{bhi} - \left(\frac{x_{3i}}{\rho_o}\right)}\right) - 0.5\left(\frac{x_{2i} + x_{3i}}{L_{wi} A_{wi}} g H_{wi}\right) \tag{A.0.13}$$

$$Pw_i = Pwh_i + \frac{g(x_{2i} + x_{3i} - \rho_o L_{bhi} A_{bhi}) H_{wi}}{L_{wi} A_{wi}} + \Delta P_{fric}^{t} \tag{A.0.14}$$

$$Pbh_i = Pwh_i + \rho m_i g H_{bhi} + \Delta P_{fric}^{bh} \forall_i \in \mathbb{N} \tag{A.0.15}$$

$$Prh = \frac{T_r R}{M_w}\left(\frac{x_4}{L_r A_r}\right) \tag{A.0.16}$$

$$Pm = Prh + \rho r g H_r + \Delta P_{fric}^{r} \forall_i \in \mathbb{N} \tag{A.0.17}$$

$$\Delta P_{fric} = \frac{128 L \mu Q}{\pi D_c^4} \qquad \text{(as given by Darcy − Weisbach equation)} \tag{A.0.18}$$

The annulus pressure Pa_iwell- head pressure Pwh_i, well injection point pressure $Pw_i$and the bottom-hole pressure Pbh_i, manifold pressure $Pm$and the riser head pressure $Prh.L_{ai}$and $A_{ai}$are the length and cross sectional area of each annulus, $T_{wi}$is the temperature in each well tubing, $H_{bhi}$and $H_{wi}$are the vertical height of each well tubing below and above the injection point respectively and g is the acceleration of gravity constant. $\Delta P_{fric}^{t}$and ΔP_{fric}^{bh} represents the frictional pressure drop in the well tubing above and below the gas injection point respectively.

The flowrates in the model are represented by equations (A.0.19) - (A.0.27) for the two wells and the riser head:

$$wiv_i = Civ_i\sqrt{max\big(0, \rho a_i(Pa_i - Pw_i)\big)} \tag{A.0.19}$$

$$wpc_i = Cpc_i\sqrt{max\big(0, \rho w_i(Pwh_i - Pm)\big)} \tag{A.0.20}$$

$$wpg_i = \frac{x_{2i}}{x_{2i} + x_{3i}} wpc_i \tag{A.0.21}$$

$$wpo_i = \frac{x_{3i}}{x_{2i} + x_{3i}} wpc_i \tag{A.0.22}$$

$$wro_i = PI_i(Pr_i - Pbh_i) \tag{A.0.23}$$

$$wrg_i = GOR_i.wro_i \forall_i \in \mathbb{N} \tag{A.0.24}$$

$$wrh = Crh\sqrt{\rho r(Prh - Ps)} \tag{A.0.25}$$

$$wtg = \frac{x_4}{x_4 + x_5} wrh \tag{A.0.26}$$

$$wto = \frac{x_5}{x_4 + x_5} wrh \tag{A.0.27}$$

Where wiv_i is the flow through the down-hole gas lift injection valve, $wpc_i$is the total flow through the production choke, wpg_i and wpo_i are, the produced gas and oil flow rates respectively, $Civ_i$and $Cpc_i$are the valve flow coefficients for the down-hole injection valve and the production choke for each well respectively, $PI_i$is the reservoir productivity index, $Pr_i$is the reservoir pressure and $GOR_i$is the gas-oil ratio for each well. The two wells produce to a common manifold, where the manifold pressure is denoted by Pm and the flow rates from the two well mixes together. The total flow through the riser head choke wrh,$wtg$and wto are the total produced oil and gas rates respectively, Crh is the valve flow coefficient for the riser head valve and Ps is the separator pressure, which is assumed to be held at a constant value.

The transfer function model obtained from the system under consideration is:

$$G(s)=\begin{bmatrix} \frac{-0.1476s^2+0.002668s+6.018e-06}{s^2+0.004914s+6.018e-06} & \frac{-0.1847s^2+0.0002337s-1.013e-21}{s^2+0.004914s+6.018e-06} \\ \frac{0.06846s^2-0.0002876s+1.491e-20}{s^2+0.004914s+6.018e-06} & \frac{0.05803s^2+0.002015s+6.018e-06}{s^2+0.004914s+6.018e-06} \\ \frac{-0.2769s^2+0.002245s+6.802e-06}{s^2+0.004914s+6.018e-06} & \frac{-0.3845s^2+0.001879s+6.606e-06}{s^2+0.004914s+6.018e-06} \\ \frac{0.1226s^2+0.003901s+7.744e-06}{s^2+0.004914s+6.018e-06} & \frac{0.3217s^2+0.003355s+5.029e-06}{s^2+0.004914s+6.018e-06} \end{bmatrix} \quad \text{(A.0.28)}$$

The decision variables for the controller to achieve this objective are: $[\Delta u_k, y_{sp,k}, \delta_{y,k}, \delta_{u,k}]$.

Steady state values: $x_{ss} = [1456.1; 1478; 0822.8; 0878.5; 6324.4; 5935.8; 0133.2; 0956.0]\ kg$

$u_{ss} = [1; 1]$

Manipulated input: $[w_{gl1};\ w_{gl2}]$

Controlled outputs: $[p_{a1};\ p_{a2}; p_m;\ p_{rh};\ w_{tg}]$

Controller tuning parameters for the two-layer strategy:

$m = 3;\ ny = 5; nu = 2; R = diag[1000\ 1000]; Q = diag[1\ 1\ 1\ 1\ 1];$

$y_{max} = [2000\ 1800\ 140\ 1100\ 10];\ \ y_{max} = [1200\ 1000\ 120\ 900\ 0]$

$u_{max} = [4\ 4];\ u_{min} = [0.01\ 0.01];\ \ \Delta u_{max} = [1\ 1]$.

Controller tuning parameters for the one layer strategy:

$m = 3;\ ny = 5; nu = 2; R = diag[5e4\ 5e4]; Q = diag[5e2\ 5e2\ 5e2\ 5e2\ 5e5]; W3 = 2e1;$

$y_{max} = [3000\ 3000\ 140\ 1100\ 13];\ \ y_{min} = [1200\ 1000\ 120\ 700\ 0]$

$u_{max} = [4\ 4];\ u_{min} = [0.01\ 0.01];\ \ \Delta u_{max} = [1\ 1]$.