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Periodic supply vessels planning problem integrating berth allocation decisions and schedule robustness

São Paulo

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To my kids (Bela, Cacá and Tix), my mum, my brother & sister...
...and to you.

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Thanks also to my family who supported me on carrying out this study.

"Não te deixes destruir…
Ajuntando novas pedras
e construindo novos poemas.
Recria tua vida, sempre, sempre.
Remove pedras e planta roseiras e faz doces. Recomeça.
Faz de tua vida mesquinha
um poema.
E viverás no coração dos jovens
e na memória das gerações que hão de vir.
Esta fonte é para uso de todos os sedentos.
Toma a tua parte.
Vem a estas páginas
e não entraves seu uso
aos que têm sede.."

"Mesmo quando tudo parece desabar, cabe a mim decidir entre rir ou chorar, ir ou ficar, desistir ou lutar; porque descobri, no caminho incerto da vida, que o mais importante é o decidir."

– Cora Coralina

ABSTRACT

Cruz, Roberto E. **Periodic supply vessels planning problem integrating berth allocation decisions and schedule robustness**. 2023. Thesis (PhD) – Polytechnic School, University of São Paulo, São Paulo, 2023.

The periodic supply vessel planning problem (PSVPP) consists in determining a periodic schedule and the respective fleet composition for servicing offshore units on a regular basis. In this Thesis, two extensions of the PSVPP have been studied. The first includes berth allocation decisions to the problem, which links vessel´s and harbour´s planning, allowing the evaluation of the impact of the number of berths on the fleet composition. The second is to extend the problem to include stochastic demand and travel time and evaluate the schedule robustness. The integration with the berth allocation decisions makes the size of problem increases significantly. In order to reduce the search space and to speed up convergence, the proposed solution strategy consists of sequentially solving models, that capture different aspects of the problem, starting with models that are simpler to solve. The solution found in one step provides a lower bound to the next step. The proposed solution strategy was applied to real instances resulting in good quality solutions and improvements to the computational time. Regarding the schedule robustness, one of the challenges for the periodic supply vessel planning problem is to determine reliable schedules with a good compromise between reliability and cost. A novel methodology based on a voyage-based model to deal with the schedule robustness is introduced. Basically, key statistical parameters related to the routes demand and execution time are generated and they are used together with a set of probability combinations in order to incorporate the schedule reliability into the optimization model. Therefore, the schedule reliability is an input parameter in the optimization model. A comparison of the new methodology to conventional approaches is presented and a Monte Carlo simulation is used to evaluate the quality of the solutions. The proposed methodology can generate robust schedules at lower cost, compared to the conventional approaches. The proposed methodology might be applied to other stochastic problems, where the schedule reliability is a key parameter for the problem.

Keywords: Periodic Supply Vessel Planning Problem. Offshore Supply Vessel Operations. Berth Allocation. Schedule Reliability. Periodic Routing Problem under uncertainty.

ABSTRACT

Cruz, Roberto E. **Periodic supply vessels planning problem integrating berth allocation decisions and schedule robustness**. 2023. Tese (Doutorado) – Escola Politécnica, Universidade de São Paulo, São Paulo, 2023.

O problema de programação periódica de navios de suprimentos (PSVPP) consiste em determinar um cronograma periódico e a respectiva composição da frota para atendimento das unidades offshore de forma regular. Nesta Tese, duas extensões do PSVPP foram estudadas. A primeira é incluir as decisões de alocação de berços ao problema, que vincula o planejamento de navios e portos, permitindo a avaliação do impacto do número de berços na composição da frota. A segunda é estender o problema para incluir demanda estocástica e tempo de viagem e avaliar a robustez do cronograma. A integração com as decisões de alocação de berços faz com que o tamanho do problema aumente significativamente. Para reduzir o espaço de busca e acelerar a convergência, a estratégia de solução adotada consiste em resolver sequencialmente modelos que capturam diferentes aspectos do problema, partindo de modelos mais simples de resolver. A solução encontrada em uma etapa fornece um limite inferior para a próxima etapa. A estratégia de solução proposta foi aplicada a instâncias reais resultando em soluções de boa qualidade e melhorias no tempo computacional. Com relação à robustez do cronograma, um dos desafios para o problema de planejamento periódico de navios de suprimento é determinar cronogramas confiáveis com um bom compromisso entre confiabilidade e custo. Uma nova metodologia baseada em um modelo baseado em rotas para lidar com a robustez do cronograma é apresentada. Basicamente, são gerados os principais parâmetros estatísticos relacionados à demanda das rotas e o tempo de execução e estes são utilizados junto com um conjunto de combinações de probabilidades para incorporar a confiabilidade do cronograma ao modelo de otimização. Portanto, a confiabilidade do cronograma é um parâmetro de entrada no modelo de otimização. Uma comparação da nova metodologia com abordagens convencionais é apresentada e uma simulação de Monte Carlo é usada para avaliar a qualidade das soluções. A metodologia proposta é capaz de gerar cronogramas robustos a um custo menor, em comparação com as abordagens convencionais. A metodologia proposta pode ser aplicada a outros problemas estocásticos, onde a confiabilidade do cronograma é um parâmetro chave para o problema.

Palavras-chave: Problema de Programação Periódica de Navios de Suprimentos. Operação de navios de suprimento. Alocação de Berços. Confiabilidade do Cronograma. Problema de Roteirização Periódica sob Incertezas.

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LIST OF ABBREVIATIONS AND ACRONYMS

ALNS	*Adaptive large neighbourhood search*
BT	*Berth set up time*
CI	*Confidence level*
CM	*Complete mathematical model*
DT	*Distance*
EC	*Number of occurrences of cargo exceedance*
ED	*Undelivered demand*
EV	*Relative exceedance of the number of vessels*
E&P	*Exploration and Production*
HT	*Cargo handling time at the harbor*
KPI	*Key performance indicator*
LT	*Leg travel time*
MIP	*Mixed integer programming*
OT	*Offshore cargo handling time*
PSS	*Proposed solution strategy*
PSV	*Platform supply vessel*
PSVPP	*Periodic supply vessel planning problem*
PVRP	*Periodic vehicle routing problem*
RO	*Robust optimization*
RT	*Overall route duration*
S-PSVPP	*Stochastic periodic supply vessel planning Problem*
SVRP	*Stochastic vehicle routing problem*
TSP	*Traveling salesman problem*
TT	*Travel time*
VRP	*Vehicle routing problem*

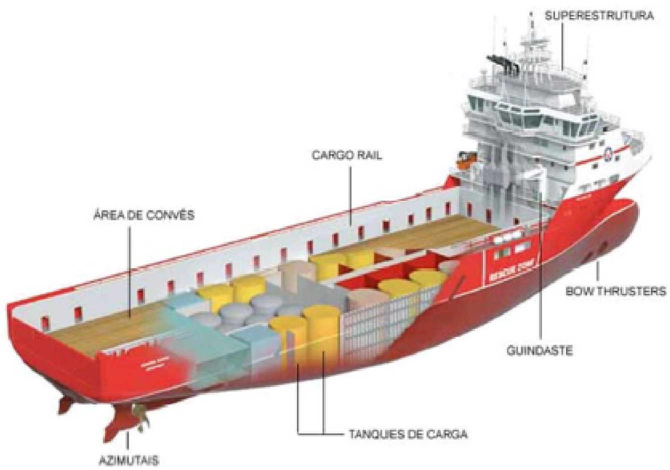
ALGORITHMS

CONTENTS

1 INTRODUCTION

Supply vessels play an important role in an offshore oilfield development, as they are present throughout the whole of the offshore exploration and production (E&P) life cycle (exploration, production and demobilization). The supply vessels' costs are one of the most significant cost elements in the upstream oil and gas segment (Aas, Halskau-Sr e Wallace (2009)). These authors described the supply vessels' role as delivering goods from an onshore supply base to one or more offshore units and returning items from these units to the onshore base. The service provided by those vessels has to be reliable because late or missing deliveries may cause considerable losses in case offshore production and operations are affected. An increase in service reliability usually implies in higher costs, and a balance between reliability and costs is very challenging. The impact of reducing costs seems to be quite relevant, mainly for oilfields with far distance from the shore and relatively high density in terms of number of offshore units to be serviced.

A supply vessel (or platform supply vessel - PSV) usually has the capacity to carry both bulk cargo and deck cargo. While the former - consisting of water, diesel and special fluids - is stored in tanks under the main deck, the latter consists of general cargo that is stored in offshore containers which are lifted by cranes from the vessels' main decks. In most cases the E&P operators charter the fleet of PSVs rather than owning them. The hiring rates are highly dependent on the contract duration - long term (time charter) or short term (spot) - and on the deck area capacity, as described in Døsen e Langeland (2015).



(http://www.oceanica.ufrj.br/deno/prod_academic/relatorios/2008/LuizFelipePimentel/relat1/index.htm)

Figure 1 – Platform Supply Vessel (PSV) - general arrangement

The supply vessel requires an onshore supply base for loading and unloading cargo to and from the offshore units. The supply base has a given capacity to operate with a number of vessels simultaneously and the capacity is usually driven by the number of berths available.

Figure 2 – Onshore supply base

The Periodic Supply Vessel Planning Problem (PSVPP) consists of determining a schedule and the respective fleet composition for servicing offshore units regularly. The schedule is made up of routes assigned to departure days from the supply base periodically. For the periodic schedule, a fleet composition should be defined in order to ensure both vessel capacity and availability for executing the planned schedule.

In this Thesis, two extensions of the PSVPP have been studied. The first includes the berth allocation decisions to the problem, which links vessel´s and harbor´s planning and the impact of the number of berths on the fleet composition might be evaluated. The second extends the problem to include stochastic demand and travel time and evaluate the schedule robustness.

For the PSVPP integrated with berth allocation decisions the fleet planning must also take into consideration the berth capacity constraints in a 24x7 continuous operation, being the berth time dependent on the amount of cargo loaded rather than a fixed time. A mathematical model is proposed and a solution strategy to solve the mathematical model in steps is devised, in order to reduce the processing time. The solution strategy adopted consists of sequentially solving models that capture different aspects of the problem, by starting with models that are simpler to solve. The solution found in one step provides a lower bound to the next step.

For the PSVPP with stochastic demand and stochastic travel time we introduce a novel methodology based on a voyage-based model to deal with the servicing schedule's reliability. The schedule disruptions due to bad weather conditions are not addressed, as other publications propose several methodologies and strategies. The treatment to cope with such conditions may vary, e.g., hiring extra vessels in the spot market during harsh weather conditions. This study focuses on determining reliable schedules at a lower cost facing the challenge of coping with stochastic demand and stochastic travel time under regular operational conditions. Therefore, the stochastic travel time takes into account the variation of the travel time under regular operations and not under bad weather conditions, in which the service of the offshore units is interrupted.

1.1 Problem description

The PSVPP is to define a periodic schedule to serve the offshore units. The schedule is defined in a planning period, and it should be repeated over the subsequent planning periods. Basically, regular weekly schedules must be established to serve the offshore units. This is the current praxis and brings the required discipline for all agents involved in the process - suppliers, supply base operators, cargo planners and clients. The weekly regular schedules might be updated from time to time, mostly due to location changes of mobile units or due to special operations (e.g., installation campaigns, decommissioning campaigns, or planned maintenance). The regular weekly schedule consists of routes and corresponding departure days over the planning period. Each route is also associated with a vessel or a vessel type according to the vessel capacity. Therefore, assigning the vessel type to the routes is part of the problem. The vessels are assigned to routes that depart from and return to a given onshore supply base.

The planning period is one week, but the schedule is considered within a two-week planning horizon to deal with the "end of the week" effect. Extending the planning horizon to two planning periods makes it possible to take into account routes that might start in a period and be completed in the subsequent period. Additionally, it is possible to check if the routes do not overlap, and it opens the possibility that one vessel is not bounded to repeat the same route in subsequent planning periods, thus giving more flexibility to the vessel assignment. In Fig. 3 there is an example of a PSVPP schedule. The routes are indicated by a rectangle. Inside the rectangle, the route number is indicated, and the departure day is the first day when the rectangle starts. In this example, the vessel of type 1 indicated in the first row starts route 1362 on the first day of week 1 (day 1). However, it is not the same vessel that will repeat the same route on the first day in week 2 (day 8).

Another characteristic of the PSVPP is that the frequency of visits requested by an offshore unit within a week is known and depends on several factors. Usually the production units have a more stable demand with lower fluctuations and, as a consequence,

<table>
<tr><th rowspan="2">Type of vessel #</th><th colspan="7">Week1</th><th colspan="7">Week 2</th></tr>
<tr><th>1</th><th>2</th><th>3</th><th>4</th><th>5</th><th>6</th><th>7</th><th>8</th><th>9</th><th>10</th><th>11</th><th>12</th><th>13</th><th>14</th></tr>
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</table>

Figure 3 – Example of departures routes per day.

the demand is more predictable. In this case, a lower frequency is required. On the other hand, demand is more unpredictable for drilling rigs due to unforeseen problems that often occur. Therefore, for drilling rigs and for some special units that work in campaigns, e.g., maintenance units, higher frequencies of visit are imposed. In order to ensure the required frequency of visits, the departure days of the routes that will serve an offshore unit should be evenly spread for a continuous supply. The combination of departure days for a given frequency of visits is previously defined. An example of departure patterns from the supply base for frequencies of one-, two- and three-weekly visits can be seen in Fig 4. There is an example of a PSVPP schedule for two vessels; the respective routes and offshore units are indicated therein, in Fig. 5. In this schedule, offshore unit C7 has a frequency of visits equal to 2 and is served by routes that depart on day four and day seven in the planning period.

Freq	Mon	Tue	Wed	Thu	Fri	Sat	Sun
	x						
		x					
			x				
1				x			
					x		
						x	
							x
	x			x			
	x				x		
		x			x		
2		x				x	
			x			x	
			x				x
				x			x
	x		x		x		
	x		x			x	
	x			x		x	
3		x		x		x	
		x		x			x
		x			x		x
			x		x		x

Figure 4 – Departure patterns for frequencies of one-, two- and three-weekly visits.

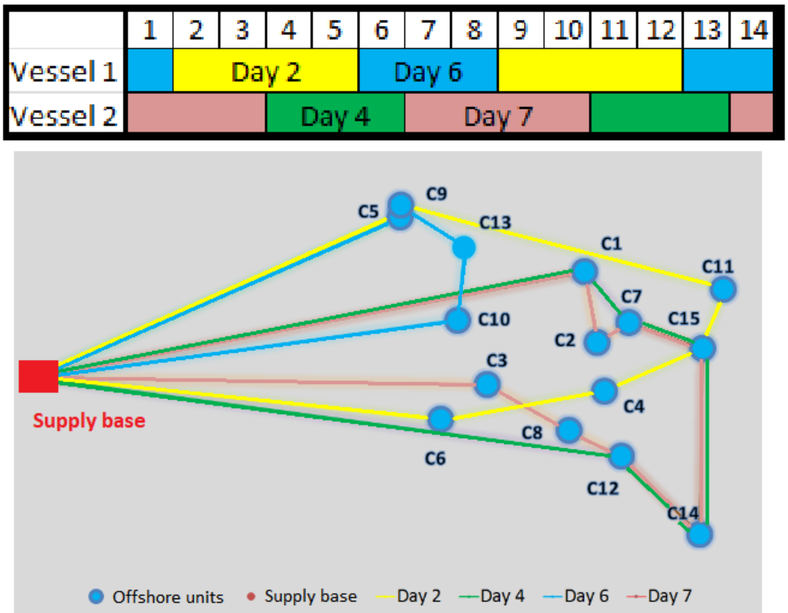

Figure 5 – Example of vessels' allocation per day for 15 offshore units.

A specific characteristic for the Brazilian offshore operation is that the offshore units might be grouped into clusters. A cluster consists of a group of nearby platforms that are served together, as a separate entity from the others. They belong to the same oilfield and are operated by the same onshore base. This segregation is observed in the Brazilian offshore E&P, and allows a better management of the logistics process, including cargo prioritization and solving disputes more easily. Figure 6 presents a set of 79 offshore units grouped into nine clusters. Each cluster is indicated by a different color and each dot represents an offshore unit. Even though platforms are grouped into clusters, the frequency of visits is defined individually for each platform.

The PSV fleet is categorized by the vessels' deck capacity. The main focus is on the delivery of general cargo. The pickup cargo, also known as backload, is assumed not to be critical for the problem, as it could be placed on the vessel's deck without violating its capacity. Usually, a free area is left on the deck in order to allow for any cargo handling that may be necessary. Detailed information on this subject can be found in Seixas et al. (2016). Three classes of PSV have been considered in this study. The first one is PSV4500 with deck capacity of up to $900m^2$, the second one is PSV3000 with deck capacity of up to $600m^2$, and the third is PSV1500 with deck capacity of up to $300m^2$.

No opening hours are considered for either the onshore base or the offshore unit. The offshore units are open 24 hours a day, except when performing operations that restrict cargo transfer (e.g., diving or helicopter operations) and special cases may be treated by reversing the route-visiting sequence or by changing the sequence of a few visits, resulting in a minor impact on the overall performance. Moreover, both the onshore base and the offshore units operate seven days a week, 24 hours per day.

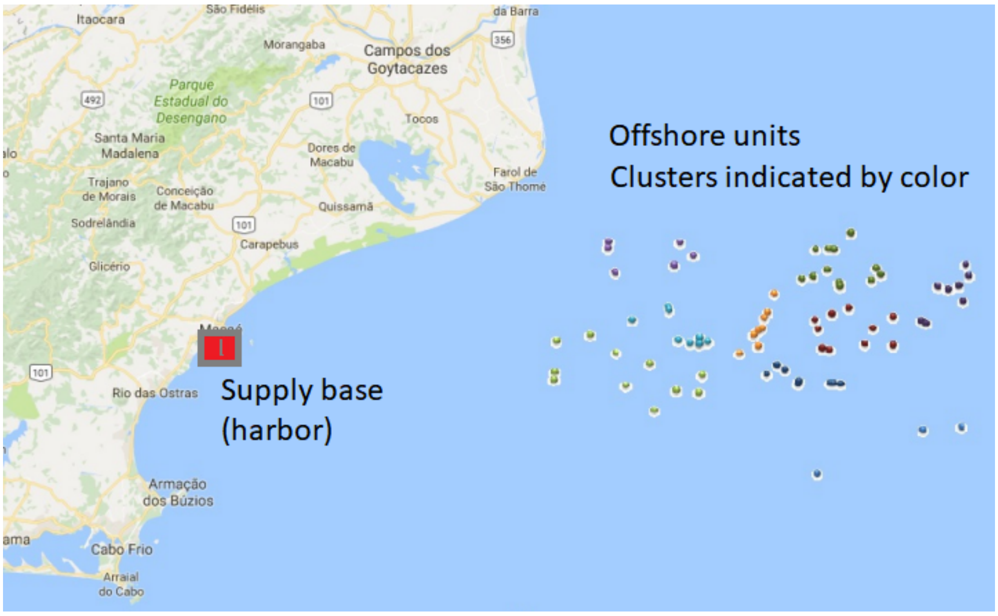

Figure 6 – Supply base and offshore installations.

The maximum number of offshore units that can be visited in the same route is limited by each vessel's capacity, and it is common for the PSVPP to define a limit for the maximum number of offshore units that can be visited in a route and the maximum route duration, as longer voyages are more sensitive to delays and to adverse weather conditions. The main reason is to avoid routes of long duration that are not realistic in practice. In this study, the maximum number of units in a route is limited to eight, and the route execution is limited to one week.

The vessels are not linked to routes and thus may not repeat the past voyages on subsequent weeks. Rather, vessels are considered as a common resource to be used whenever requested, and this is expected to produce good overall fleet allocation.

Regarding the onshore supply base operation, the port time of a vessel at the supply base is calculated by considering a fixed setup time (vessel berthing time) added to the loading time, which is given by the amount of cargo to be loaded divided by an average loading rate. Given that routes cover different numbers of offshore units, the loading time is not considered fixed but, rather, dependent on the total amount of cargo to be transported. Based on the actual loading rates and on the historical average amount of cargo transported, the maximum number of departures from each berth on each day is limited to two. Each departure from each berth position is nominated as a departing position. The vessel may begin loading at any time during the day. Consequently, there is a risk of not attending the imposed departing day if the loading process begins late in a day. However, in order to give flexibility to the berth planning process, a tolerance of up to 12 hours is considered. For example, the departure day of a given vessel may be considered as belonging to day l even if the vessel leaves the port by 12 pm of the following day l+1. All parameters are known in advance and no stochastic data are used for the harbor operation.

Regarding the stochastic demand, the demand variation is very challenging to the PSVPP, and two possible violations may occur. The first one is related to the demand that may exceed the vessel capacity. In such a case, some cargo may be left behind, resulting in huge losses if the production is affected or the unit stops its operation due to the lack of supply. The second possible violation is related to the vessel schedule, which happens when a vessel does not complete its current route on time, thus delaying its next voyage if no other vessel is available. The schedule violation is affected not only by high demands - as longer times are spent both at the harbor and the units for the cargo handling and transshipment, but also by the actual travel time.

Regarding the stochastic travel time, it may deviate from the expected value due to several factors such as, the vessel's design characteristics, the coordination of the expected arrival time for the vessel in its next destination, approximation and maneuvering issues, the proximity between two consecutive units, and the weather conditions even under the operation limits for regular operations (in this study the disruption due to bad weather is disregarded). Consequently, planning a route using a constant travel time along all the transportation legs may result in a non-realistic duration, likely affecting the overall schedule.

The stochastic PSVPP (S-PSVPP) is thus a relevant research topic involving the generation of low-cost, reliable schedules in the presence of stochastic demand and travel time. Specifically, for the S-PSVPP, the following aspects are considered: i) the probability distribution of each offshore unit's demand per visit, assumed to be independent among the units; ii) the probability distribution of the travel time. The travel time is calculated by combining the duration of each route leg (or between units), and the probability distribution of each leg's duration is known and independent of each other; iii) the joint probability of the offshore units' demands and travel time is based on independent events.

1.2 Motivation and Contributions

In this Thesis, two extensions of the PSVPP have been studied. The first includes the berth allocation decisions to the problem, which links vessel´s and harbor´s planning. The second is to extend the problem to include stochastic demand and travel time and evaluate the schedule robustness. The studies carried out for both extensions of the PSVPP have been published in Cruz et al. (2019) and Cruz, Mendes e Bahiense (2023).

For the PSVPP integrated with berth allocation decisions the fleet planning must take into consideration the berth capacity constraints and berth assignment decisions. This makes the problem size to explode resulting in a interesting problem to solve. A mathematical model is proposed and a solution strategy to solve the mathematical model in steps is devised, in order to reduce the processing time. The berth allocation decisions should be done together with the PSVPP in order to ensure solutions that are feasible and the supply base capacity constraint is not violated. This is relevant for problems that the supply base operates close to its maximum capacity.

The demand and travel time variation is very challenging to the PSVPP and huge losses might occur if the production is affected or the offshore unit stops its operation due to the lack of supply. It is thus a relevant research topic involving the generation of low-cost, reliable schedules in the presence of stochastic demand and travel time. For the PSVPP with stochastic demand and stochastic travel time, a novel methodology based on a voyage-based model to deal with the servicing schedule's reliability is introduced.

1.3 Objectives

The objective of this Thesis is to solve two extensions of the PSVPP. The first includes the berth allocation decisions to the problem while the second extends the problem to include stochastic demand and travel time and evaluate the schedule robustness.

The specific objectives are:

a) Introduce a novel Mathematical Model for the PSVPP including the berth allocation decisions to the voyage based model of the PSVPP;

b) For the berth allocation decisions added to the problem, develop and test a solution strategy to solve it and speed up the convergence using a mixed integer programming (MIP) optimization software;

c) Introduce a novel Mathematical Model for the PSVPP including the schedule robustness; and

d) Compare the performance and the quality of the solution obtained for several different approaches and the novel methodology;

1.4 Methodology

The following methodology was adopted in the development of this Thesis:

a) the literature review to understand the PSVPP modeling, its variants and the solutions strategies;

b) the input data analysis and the generation of relevant instances based on real cases data; and

c) the development and testing of the mathematical model (model development and computational studies.)

1.5 Thesis Structure

This Thesis has 6 chapters, including the introduction (Chapter 1). The chapters are organized according to the following: Chapter 2 presents the literature review; in Chapter 3 one might find the deterministic mathemathical model for the PSVPP and the mathematical models for the extensions that are object of this Thesis; in Chapter 4 the solution method for the extensions are presented; in Chapter 5 the computational results obtained are presented and the main conclusions are indicated in Chapter 6.

2 LITERATURE REVIEW

The planning of supply operations has been addressed by several authors. In Aas, Halskau-Sr e Wallace (2009) a general overview of the supply vessel role in the oil industry is presented. Aas et al. (2007) studied the pickup and delivery problem in the offshore oil industry involving one single vessel, and considered limited storage capacity at the platforms and on the vessel. A mathematical model was proposed taking into consideration that any platform may be visited twice in order to have its demand fulfilled, given the limited storage capacity. Gribkovskaia, Laporte e Shlopak (2008) tackled the same problem, and proposed a tabu search procedure to solve larger instances. The route-planning problem with the purpose of assessing the cost impact of not being able to serve some offshore units during the night is addressed by Fagerholt e Lindstad (2000). Candidate routes were generated by considering the night closures and a set partition model was proposed including the fixed cost of using a vessel in the planning horizon.

The PSVPP was better described in Halvorsen-Weare et al. (2012), with periodicity being incorporated in the mathematical model. The latest deterministic formulation used in the PSVPP that considers flexible departures, i.e., vessels not being bounded to a departure day and route over the periods, is presented in Kisialiou, Gribkovskaia e Laporte (2018a), Cruz et al. (2019), and Vieira et al. (2021).

In Halvorsen-Weare et al. (2012), different from the previous contribution, the authors included the departing day in the route selection decision variable, which allowed the interval between consecutive voyages to be controlled, in accordance with a list of candidate departure patterns. A more sophisticated set partition model was proposed, based on a voyage generator, and instances with up to 14 offshore units were reported. This problem was also solved by a large neighbourhood search in Shyshou et al. (2012), and reports on solving larger instances were given. The same problem was later studied by Halvorsen-Weare e Fagerholt (2017) who proposed a new formulation based on arc-flow variables. Kisialiou, Gribkovskaia e Laporte (2018a) extended the work of Halvorsen-Weare et al. (2012) by allowing flexible departure times from the onshore base, instead of considering a fixed departure time for all voyages. The authors also dealt with the 'end of week effect', by which a vessel that started a trip by the end of a week may not be able to repeat the routes performed in the beginning of the previous week. The authors proposed an ALNS heuristic and the results were compared to a voyage-based model solved by CPLEX.

A single vessel pickup and delivery problem in the offshore industry is addressed by Cuesta et al. (2017). The problem considers that it is not mandatory to attend to the whole demand; however, in such cases, a cost penalty is introduced to consider the losses due to unattended demand. A mathematical model was proposed to select the sets of cargo to be transported, followed by the vessel routing. Another situation was considered, where transportation of all cargo was compulsory, even if additional vessels were incorporated into the fleet. The problem was solved by an adaptive large neighborhood search.

In the oil industry, besides the aforementioned references such as Halvorsen-Weare et al. (2012) and Halvorsen-Weare e Fagerholt (2017), other contributions related to fleet sizing include Shyshou, Gribkovskaia e Barceló (2010) who proposed a discrete event simulation model in an anchor-handling operational context. Maisiuk e Gribkovskaia (2014) studied a platform supply vessels' fleet-sizing problem under uncertainty by combining optimization and discrete event simulation. Eskandari, Gribkovskaia e BarcelÓ (2016) proposed a multi-objective discrete event simulation model for a supply vessel fleet-sizing problem, and Stålhane et al. (2016) developed a two-stage stochastic optimization model in a fleet-sizing problem related to maintenance activities.

The period routing problem is an important variation of the classical vehicle routing problem as many practical applications impose multiple visits to the customers during the planning period, as in Christofides e Beasley (1984) and Baptista, Oliveira e Zúquete (2002). A general overview of existing models, solution approaches and applications is given in Francis, Smilowitz e Tzur (2008). The integration of periodic routing with fleet sizing is also found in another class of problems known as the periodic location routing problems, which extends the period routing and fleet-sizing problem by considering location decisions. Prodon (2011), Hemmelmayr (2015) and Koç (2016) offer a general overview of existing models and solution approaches.

Berth allocation problems have been mostly investigated in the container industry, under different configurations. For example, Lim (1998) considered the problem with the continuous quayside, while Imai, Nagaiwa e Tat (1997) studied the discrete quayside. Imai, Nishimura e Papadimitriou (2001) considered that the ships arrive dynamically and in Cordeau et al. (2005) time windows for berthing the ships are imposed. In Agra e Oliveira (2018) a more complex version of the problem was considered by integrating berth planning with the planning of cranes. For a comprehensive overview, one can refer to Bierwirth e Meisel (2015). Applications in other types of terminals can be found, for example, in Ribeiro et al. (2016) and in Pratap et al. (2017). The extension of the PSVPP including the berth allocation decisions have been studied by Cruz et al. (2019), and Vieira et al. (2021). In Vieira et al. (2021) a powerful adaptive large neighborhood search (ALNS) heuristic has been developed and good quality solutions were obtained in a low computational time.

Some extensions of the PSVPP consider the uncertainty in the planning. Halvorsen-Weare e Fagerholt (2011) and Halvorsen-Weare e Fagerholt (2017) considered weather uncertainty and developed a three-phase optimization-simulation heuristic approach for the robust supply vessel planning. Feasible candidate voyages are generated in the first phase. A simulation of each voyage is performed in the second stage when a robustness measure is assigned to the voyages. In the last phase, a set covering model is solved. In Maisiuk e Gribkovskaia (2014), the problem of determining the fleet size of offshore supply vessels under stochastic sailing and service times is studied. A discrete-event simulation model is used for evaluating the optimal solution produced by the deterministic MIP model presented in Halvorsen-Weare et al. (2012). The simulation considers uncertainty in weather conditions and future spot vessel rates and adds one vessel from the spot market each time a vessel is lacking for attaining the prescribed schedule. In Norlund, Gribkovskaia e Laporte (2015a), a simulation-optimization tool was presented for evaluating the environmental performance of speed optimization strategies under weather uncertainty. The objective is to minimize the expected schedule cost computed over the execution horizon. The schedule cost is calculated taking into account the implementation of operational modifications in case of infeasibility during the schedule execution.

Kisialiou, Gribkovskaia e Laporte (2018b) considered weather uncertainty and developed a methodology for generating robust supply vessel schedules for realistic large-size instances, taking into account the trade-off between service levels and vessel costs. The methodology involves generating multiple vessel schedules with different robustness levels using an ALNS heuristic and a subsequent discrete event simulation procedure to assess the service levels. Following a similar approach, Kisialiou, Gribkovskaia e Laporte (2019) studied the PSVPP with stochastic demand. The methodology is based on ALNS heuristics combined with a discrete event simulation procedure with recourse actions. A reliability level related to not violating the vessel capacity by the route aggregated demand is used in the ALNS for defining the feasible solutions. The solutions obtained in the ALNS algorithm are evaluated in the simulation model that uses recourse actions to mitigate infeasible voyages. For each reliability value, the ALNS solution's cost is compared to the cost obtained by the simulation with recourse actions. Recently, Yauheni, Gribkovskaia e Laporte (2021) solved a periodic supply vessel planning problem under demand and weather uncertainty in the offshore oil and gas industry. The ALNS algorithm in Kisialiou, Gribkovskaia e Laporte (2019) is extended to incorporate the voyage duration infeasibility under uncertain weather conditions. A simulation model is used to assess the schedule performance over its execution horizon, and visit relocation options are applied to ensure the schedule feasibility. The ALNS metaheuristic and the simulation model are integrated into a single optimization-simulation decision support tool enabling the construction of schedules of least expected cost for large-size instances under demand and weather uncertainty.

The PSVPP with stochastic parameters is, to some extent, correlated to the stochastic vehicle routing problem (SVRP). One of the first references to the SVRP with stochastic demand is due to Bertsimas (1992). A summary of the scientific literature on stochastic vehicle routing problems may be found in Gendreau, Laporte e Séguin

(1996), where different problems are described within a broad classification scheme. In Dror, Laporte e Trudeau (1989), one may find a description of potential methodologies to solve the SVPR with stochastic demand. More recently, Gendreau, Jabali e Rei (2016) presented an updated survey on the main classes of the SVRP, the modeling paradigms, and the existing exact and approximate solution methods. The SVRP may be solved using stochastic programming based on chance constraint and the two-stage program with recourse. For the chance constraint approach, a confidence level is defined as a protection level against constraints violations. In the two stages with recourse, the problem is solved in two stages; firstly, by deciding without knowing the outcome of the stochastic parameters. Then, in the second stage, recourse actions are taken with the realization of the stochastic parameters.

When the probability distributions for the stochastic parameters are not known, one can resort to an uncertainty set, wherein the uncertain data resides. The robust optimization paradigm assumes that constraint violation cannot occur for any realization of the data in the uncertainty set (Ben-Tal, Ghaoui e Nemirovski (2009), Bertsimas e Sim (2003), Bertsimas e Sim (2004)). Gorissen, Yanıkoğlu e den Hertog (2015) present a tutorial on robust optimization, while Ordóñez (2014) gives a tutorial specifically on robust vehicle routing problems. In Bertsimas, Brown e Caramanis (2011), there is a broad survey on the known landscape of RO's theory and applications. Munari et al. (2019) presents a compact formulation and branch and cut method for the robust VRP with time-windows. The application of robust optimization also is found to other variants of the VRP. In Vega, Munari e Morabito (2020), the classical VRP is extended to consider time-windows and multiple deliverymen. Robustness is incorporated for uncertainties in the demands and in the travel times. In Agra et al. (2013), a robust optimization approach is proposed for a maritime transportation problem with time-windows, admitting routes that are feasible for all values of the travel times in a predetermined uncertainty polytope. Applications of robust optimization in other contexts can be found, for instance, in Raad, Sinske e Vuuren (2009), Kalaï e Vanderpooten (2011), Paiva, Rocco e Morabito (2020) and González, Bert e PodestÁ (2022).

A map indicating the main subject for each reference listed in the bibliography is presented in Fig. 7.

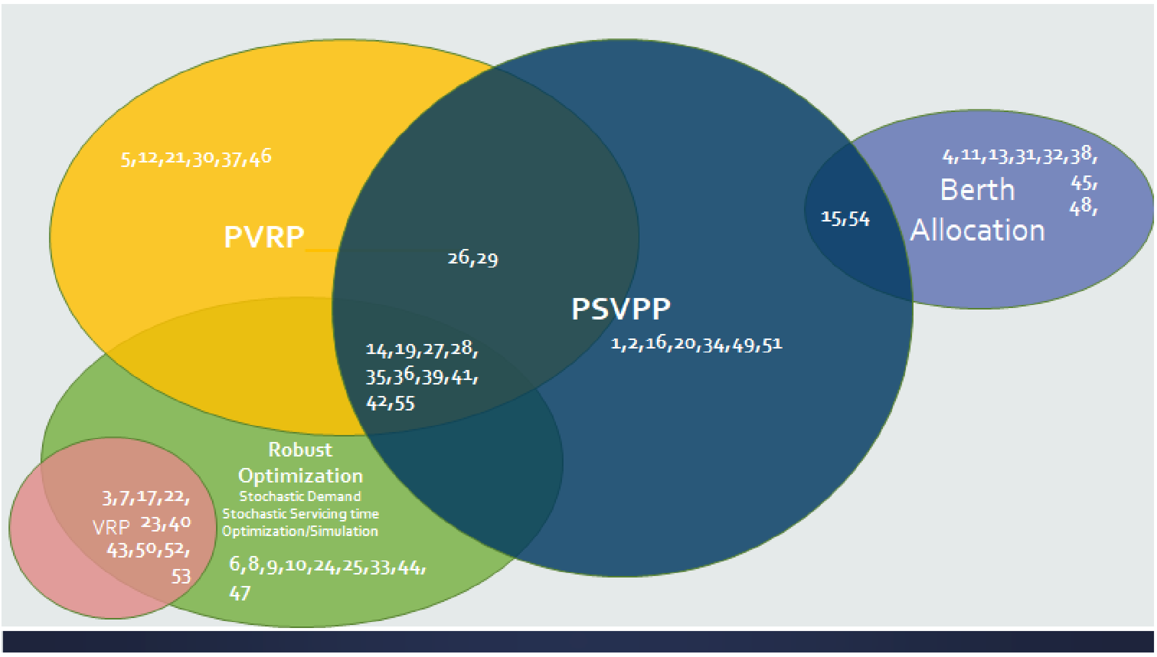

Figure 7 – Map of the main subject for each reference from the bibliography.

3 MATHEMATICAL MODEL

In this section, we first present a consolidated deterministic mathematical model for the PSVPP based on the existing literature (Halvorsen-Weare et al. (2012),Kisialiou, Gribkovskaia e Laporte (2018a) and Cruz et al. (2019)). Afterwards, the mathematical models for the PSVPP with berth allocation and the PSVPP with stochastic demand and travel time are introduced.

3.1 PSVPP Deterministic Mathematical Model

This section presents the PSVPP deterministic mathematical model, which is described after Tables 1, 2 and 3 containing the definitions of the sets, indices, parameters and decision variables used in it.

Table 1 – PSVPP Deterministic Model - Sets and indices

Sets and Index	Description
V	Set of vessel classes (index v).
J	Set of offshore units (index j).
R	Set of routes (index r).
R_v	Subset of routes that vessels of class v can sail ($R_v \subset R$).
S	Set of departure patterns (i.e. departure days) from the supply base (index s).
S_j	Subset of departure patterns for the frequency of visits imposed by the offshore unit j ($S_j \subset S$).
T	Planning horizon of the vessels (indices l and t, t:1..14).
L	Planning horizon of the offshore units ($L \subset T$, index l, l:1..7).

Table 2 – PSVPP Deterministic Model - Parameters

Parameters	Description
F_v	Fixed cost of vessels belonging to class v.
C_{vr}	Cost of route r for a vessel belonging to class v.
H	Maximum number of departures from the supply base in a day.
A^0_{rj}	Binary parameter that is 1 if route r visits offshore unit j, and 0 otherwise.
A^1_{sl}	Binary parameter that is 1 if the departure pattern s has a departure on day l, and 0 otherwise.
B_{vrlt}	Binary parameter that is set to 1 if a vessel of class v departs from the supply base on day l to perform route r has not returned to the supply base by day t, and 0 otherwise.

Table 3 – PSVPP Deterministic Model - Decision Variables

Decision Variables	Description
n_v	Number of utilized vessels of class v.
b_{sj}	Binary variable that is 1 if the departure pattern s is chosen for offshore unit j, and 0 otherwise.
x_{vrl}	Binary variable that is 1 if a vessel of class v departs on day l to execute route r, and 0 otherwise.

The mathematical model is defined as:

$$minZ = \sum_{v \in V} F_v n_v + \sum_{v \in V} \sum_{r \in R_v} \sum_{l \in L} C_{vr} x_{vrl} \tag{3.1}$$

subject to:

$$\sum_{s \in S_j} b_{sj} = 1 \qquad j \in J \tag{3.2}$$

$$\sum_{v \in V} \sum_{r \in R_v} x_{vrl} A^0_{rj} = \sum_{s \in S_j} b_{sj} A^1_{sl} \qquad l \in L,\, j \in J \tag{3.3}$$

$$\sum_{v \in V} \sum_{r \in R_v} x_{vrl} \leq H \qquad l \in L \tag{3.4}$$

$$n_v \geq \sum_{r \in R} \sum_{l \in L} x_{vrl} B_{vrlt} \qquad v \in V,\, t \in T \tag{3.5}$$

$$n_v \in \mathbb{Z}^+,\, b_{sj} \in \{0, 1\}\,,\, x_{vrl} \in \{0, 1\}\,. \tag{3.6}$$

The objective function (3.1) minimizes the vessels' fixed costs and the routing costs. Constraints (3.2) ensure that one departure pattern is assigned to each offshore unit.

Constraints (3.3) ensure that there is one vessel departing from the supply base each day belonging to each offshore unit's selected departure pattern. Constraints (3.4) limit the number of routes departing from the supply base per day. Constraints (3.5) calculate the number of vessels per class as the maximum number of vessels in use at the same time on any day. Lastly, Constraints (3.6) define the variables' domain.

The deterministic mathematical model to define the PSVPP frequently used in the literature is a set partition model (SPM) that uses a voyage-based structure as in Halvorsen-Weare e Fagerholt (2011), Halvorsen-Weare et al. (2012), Norlund, Gribkovskaia e Laporte (2015b), Kisialiou, Gribkovskaia e Laporte (2018a) and Cruz et al. (2019). The problem can be solved using a MIP software, for instances with a limited number of units (around 15 to 20) and routes. In case the number of units and the associated set of feasible routes are too large, heuristics are the most suitable way to obtain good-quality solutions within a reasonable processing time (Kisialiou, Gribkovskaia e Laporte (2018a)).

In order to solve the PSVPP using a MIP software, the following steps are observed: i) generate all feasible routes observing the imposed limits (route duration, the maximum number of visits) and the other constraints; ii) with the defined set of routes, generate all the parameters of the mathematical model; iii) solve the MIP model. This procedure is presented in Fig. 8.

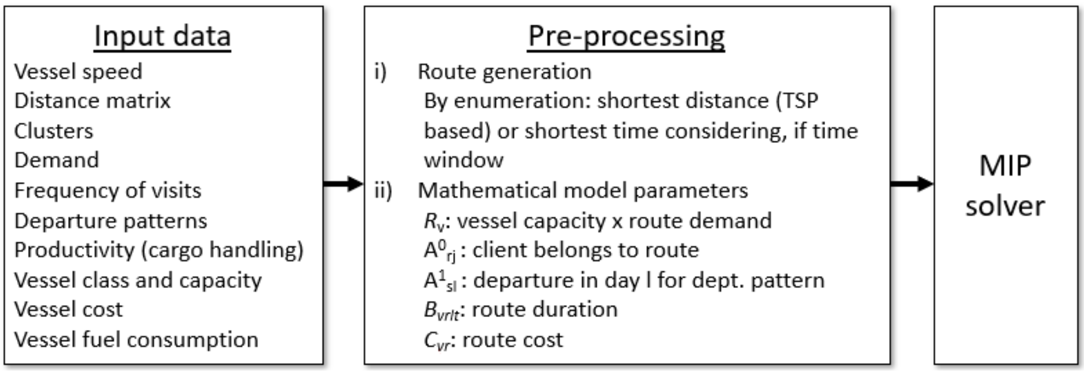

Figure 8 – PSVPP procedure for the deterministic problem using a MIP solver.

3.1.1 Route generation

The set of routes is built by first generating all possible combinations of up to eight offshore units belonging to the same cluster. Only the combinations whose aggregated demand does not violate the largest vessel capacity are maintained as routes. The subset R_v and the parameter A^0_{rj} are generated. The sequence of visits for each route is obtained by solving the associated TSP. The final step is to calculate the route execution duration that depends on the route aggregated demand and the travel time. The route execution time is used to create the binary parameter B_{vrlt}, with fractional values being rounded to the least integer greater than or equal to it. For example, a vessel with a route execution time of 2.5 days is allocated for three days, starting on the departure day.

The overall route duration (RT) is calculated according to the sum of the following: i) a fixed setup time for the vessel in the harbor (BT), ii) the cargo handling time in the harbor that is dependent on the route aggregated demand (HT), iii) the travel time (TT) that is the sum of the travel time of each route leg (a leg is defined as two waypoints in the route) and iv) the sum of the cargo handling time on each offshore unit served by the route (OT). The procedure for the route generation is presented in Algorithm 1, where D_j is offshore unit j's demand and LT is a leg's travel time.

Algorithm 1: Route Generation

Input: j; D_j; Distance Matrix; Vessel Capacity and Route Cost per Vessel Type v
Output: R; R_v; A^0_{rj}; B_{vrlt}; C_{vr}

for every combination of up to 8 customers **do**
  Calculate the route demand by summing D_j of the units in the combination
  if the route demand is lower or equal to the maximum vessel capacity **then**
    Solve the TSP and define the route sequence
    Define A^0_{rj}
    for each class v compatible with the route demand **do**
      Calculate TT based on each LT
      Calculate the route duration RT
      Create parameter B_{vrlt} based on RT for every day in the planning horizon
      Calculate the route cost C_{vr}
      Update subset R_v
    end for
  end if
end for
return R; R_v; A^0_{rj}; B_{vrlt}; C_{vr}

3.2 PSVPP with berth allocation

This section presents a mathematical model for the PSVPP with berth allocation decisions. In order to include the berth allocation decisions in the model, new sets regarding the berth positions at the supply base, new parameters related to the harbor operation and respective productivity factors and new decisions variables to cope with the sequence of the operations at the harbor are introduced. The sets, parameters and decision variables are presented in Tables 4, 5 and 6 respectively.

Table 4 – PSVPP with berth allocation - Sets and indices

Sets and Index	Description
V	set of vessel classes (index v).
J	set of offshore units (index j).
R	set of routes (index r).
R_v	subset of routes that vessels of class v can sail ($R_v \subset R$).
S	set of departure patterns (i.e. departure days) from the supply base (index s).
S_j	subset of departure patterns compatible with the frequency of visits imposed by offshore unit j ($S_j \subset S$).
T	planning horizon of the vessels (indices l and t, t:1..14).
L	planning horizon of the offshore units ($L \subset T$, index l, l:1..7).
B	set of berths (index b).
P	set of departures per day for each berth (index p). For the sake of simplicity, as each vessel is always assigned to a pair (berth, departing position) at the onshore base, a berth position (b,p) notation is used.

Table 5 – PSVPP with berth allocation - Parameters

Parameters	Description
F_v	fixed cost of vessels belonging to class v
C_{vr}	route r cost of vessels belonging to class v
D_j	offshore unit j demand (m^2) given by the mean plus two times the standard deviation of the weekly demand divided by the imposed frequency of visits (in order to cover possible demand variations)
A^0_{rj}	binary parameter that is 1 if route r visits offshore unit j, and 0 otherwise
A^1_{sl}	binary parameter that is 1 if the departure pattern s has a departure on day l, and 0 otherwise
ST_{rv}	route r sailing time of vessels belonging to class v
PB	productivity factor (days/m^2) for cargo handling at the onshore base
PO_j	productivity factor (days/m^2) for cargo handling at offshore unit j
SB	berth set up time (average time for approaching and mooring)
LL_{lb}	lower time limit for starting loading of a vessel on berth b and day l
LU_{lb}	upper time limit for finishing the loading of a vessel on berth b and day l.

The specific parameters introduced due to the berth allocation are indicated with double letters. The parameters LL_{lb} and LU_{lb} enables the tolerance in the departure time of a vessel being loaded at berth position (b,p).

Table 6 – PSVPP with berth allocation - Decision Variables

Decision Variables	Description
n_v	number of utilized vessels of class v
b_{sj}	binary variable that is 1 if departure pattern s is chosen for offshore unit j, and 0 otherwise (refer to Figure 4 for the list of departure patterns)
x_{vbprl}	binary variable that is 1 if a vessel of class v departs from the berth position (b,p) on day l to execute route r, and 0 otherwise
$c_{vbpt_1t_2}$	binary variable that is 1 if a vessel of class v departing from the berth position (b,p) on day t_1 is still operating its route by day t_2, and 0 otherwise
sl_{bpl}	loading time spent on berth position (b,p) on day l
s_{bpl}	instant of time that the loading operation of the berth position (b,p) on day l is concluded and the vessel starts the voyage
t_{bpl}	round trip voyage duration of the vessel that occupies the berth position (b,p) on day l.

The mathematical model is defined as:

$$minZ = \sum_{v \in V} F_v n_v + \sum_{v \in V} \sum_{b \in B} \sum_{p \in P} \sum_{r \in R_v} \sum_{l \in L} C_{vr} x_{vbprl} \tag{3.7}$$

subject to:

$$\sum_{s \in S_j} b_{sj} = 1 \qquad j \in J \tag{3.8}$$

$$\sum_{v \in V} \sum_{b \in B} \sum_{p \in P} \sum_{r \in R_v} x_{vbprl} A^0_{rj} \geq \sum_{s \in S_j} b_{sj} A^1_{sl} \qquad l \in L,\, j \in J \tag{3.9}$$

$$\sum_{v \in V} \sum_{r \in R_v} x_{vbprl} \leq 1 \qquad l \in L,\, b \in B,\, p \in P \tag{3.10}$$

$$\sum_{v \in V} \sum_{r \in R_v} x_{vb2rl} \leq \sum_{v \in V} \sum_{r \in R_v} x_{vb1rl} \qquad l \in L,\, b \in B \tag{3.11}$$

$$s_{b11} \geq sl_{b11} \qquad b \in B \tag{3.12}$$

$$s_{b2l} \geq s_{b1l} + sl_{b2l} \qquad b \in B,\, l \in L \tag{3.13}$$

$$s_{b1(l+1)} \geq s_{b2l} + sl_{b1(l+1)} \qquad b \in B,\, l \in L,\, l \geq 2 \tag{3.14}$$

$$LL_{lb} + sl_{bpl} \leq s_{bpl} \leq LU_{lb} \qquad l \in L,\, b \in B,\, p \in P \tag{3.15}$$

$$sl_{bpl} \geq \sum_{v \in V} x_{vbprl}(SB + \sum_{j \in J:\, A^0_{rj}=1} D_j PB) \qquad l \in L,\, r \in R,\, b \in B,\, p \in P \tag{3.16}$$

$$t_{bpl} \geq \sum_{v \in V} x_{vbprl}(ST_{rv} + \sum_{j \in J:\, A^0_{rj}=1} D_j PO_j) \qquad l \in L,\, r \in R,\, b \in B,\, p \in P \tag{3.17}$$

$$\sum_{t_2=l}^{|T|} c_{vbplt_2} \geq s_{bpl} + t_{bpl} - l - (1 - c_{vbpll})|T| \qquad v \in V,\, b \in B,\, p \in P,\, l \in L \tag{3.18}$$

$$c_{vbpll} \geq x_{vbprl} \qquad l \in L,\, v \in V,\, r \in R,\, b \in B,\, p \in P \tag{3.19}$$

$$c_{vbpt_1t_2} = 0 \qquad v \in V,\, b \in B,\, p \in P,\, t_1 \in L,\, t_2 \in L,\, t_2 < t_1 \tag{3.20}$$

$$c_{vbpt_1(t_2+1)} - c_{vbpt_1t_2} \leq 0 \qquad v \in V,\, b \in B,\, p \in P,\, t_1 \in L,\, t_2 \in T, t_2 \geq t_1 \tag{3.21}$$

$$c_{vbp(t_1+|L|)(t_2+|L|)} = c_{vbpt_1t_2} \qquad v \in V,\, b \in B,\, p \in P,\, t_1 \in L,\, t_2 \in L \tag{3.22}$$

$$n_v \geq \sum_{t_1 \in T} \sum_{b \in B} \sum_{p \in P} c_{vbpt_1t_2} \qquad v \in V,\, t_2 \in T \tag{3.23}$$

$$n_v \in \mathbb{Z}^+,\, b_{sj} \in \{0,1\},\, x_{vbprl} \in \{0,1\},\, c_{vbpt_1t_2} \in \{0,1\},\, s_{bpl} \in \mathbb{R}^+,\, sl_{bpl} \in \mathbb{R}^+,\, t_{bpl} \in \mathbb{R}^+. \tag{3.24}$$

The objective function (3.7) minimizes the vessels' fixed costs and the routing costs. Constraint (3.8) ensures that one departure pattern is assigned to each offshore unit. Constraint (3.9) ensures that there is at least one vessel departing on each day belonging to the selected departure days of each offshore unit. Constraint (3.10) limits to at most one the number of routes departing per day for each berth position. Constraint (3.11) forces that berth position two is only assigned if position one was used. Constraint (3.12) initializes the berth's departure time based on the berth's loading time of the first day. Constraint (3.13) ensures that the departure time for position two of any berth depends on the position one departure time added to the position two loading time. Constraint (3.14) ensures that the departure time for berth position 1 is greater than the instant in which the loading operation of the previous day was concluded, for $l \geq 2$. Constraint (3.15) indicates that the departure time for any berth position is within the time window defined by LL_{lb} and LU_{lb}. Constraint (3.16) ensures that the operating time of any berth position must be greater than the sum of the vessel's set up time and the loading time. Constraint (3.17) ensures that the vessel's round trip duration must be greater than the sum of the vessel's sailing time and the cargo handling time at each visited offshore unit. Constraint (3.18) ensures that a binary counter is activated (i.e. equal to 1), for the time period that a vessel is in use. Constraint (3.19) associates the binary counter with the x_{vbprl} variable. Constraint (3.20) ensures that the binary counter is not activated in any period t_2 prior to a candidate's departing day t_1. Constraint (3.21) links the counter variable of one day to its previous day. Constraint (3.22) replicates a vessel class assignment for the subsequent period. Constraint (3.23) calculates the number of vessels per class as the maximum number of vessels in use at the same time. Constraint (3.24) define the variables' domain. Constraints (3.18) to (3.23) ensure that voyages of the same vessel should not overlap in time.

An example of the calculation of variable $c_{vbpt_1t_2}$ in terms of the departure day t_1 (rows) and the respective days that the vessel is in use t_2 (columns) is given in Table 7. The example is based in a case where a vessel of class 1 is assigned to perform a route that departs on day 5 from berth 1 and position 1. Besides, it is assumed that the vessel should be in use for 3.6 days, according to (3.18). Therefore, the minimum number of vessels for each class n_v must be equal to the maximum number of vessels for the respective class in use at any time t_2 (the columns in Table 7), as indicated in (3.23).

In order to illustrate the use of parameters LL_{lb} and LU_{lb} consider, for instance, a loading operation that takes place on day 5, where $LL_{lb} = 4.0$ and $LU_{lb} = 5.5$. If two vessels are assigned to berth 2 with loading times of 0.5 and 0.6 (for positions 1 and 2 respectively), and assuming that there is no loading on berth 2 on the previous day then, according to (3.16): $sl_{215} \geq 0.5$ and $sl_{225} \geq 0.6$. Constraint (3.15) defines the interval for departure of each position as $4.0 + 0.5 \leq s_{215} \leq 5.5$ and $4.0 + 0.6 \leq s_{225} \leq 5.5$. Constraint (3.13) ensures that position 2 only starts after position 1, $s_{225} \geq s_{215} + sl_{215}$ and, if we use the minimum values, the constrain is $s_{225} \geq 4.5 + 0.6$. This means that any vessel assigned for berth position 2 starts loading on day 4.5 (or after) and departs from the harbor on day 5.1 (or after). According to constraint (3.14) the departure for the next day (day 6) for berth 2 should consider that the loading in position 1 starts after vessel from position 2 on the previous day has departed: $s_{216} \geq 5.1 + sl_{216}$. By the use of the upper limit extended to

Table 7 – Example of the calculation of variable $c_{vbpt_1t_2}$.

$v = 1$		$c_{vbpt_1t_2}$													
$b = 1$		t_2													
$p = 1$		1	2	3	4	5	6	7	8	9	10	11	12	13	14
t_1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	5	0	0	0	0	1	1	1	1	0	0	0	0	0	0
	6	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	7	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	8	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	9	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	10	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	11	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	12	0	0	0	0	0	0	0	0	0	0	0	1	1	1
	13	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	14	0	0	0	0	0	0	0	0	0	0	0	0	0	0

the next day it is possible to accommodate better berth allocation, without compromising the evenly spread departures from the harbor. This is a very important relaxation in the case of a harbor that operates close to the maximum capacity.

3.3 PSVPP with stochastic demand and travel time

As one may notice, the demand and the travel time are not explicit parameters in the set partition model (3.1) - (3.6). Instead, those parameters are used as input data in the route generation algorithm that generates the candidate routes and the related parameters. Likewise, stochastic demand and stochastic travel time are considered in the route generation algorithm. Therefore, three approaches to deal with the stochastic parameters in the PSVPP to calculate the route demand and the route duration are presented, the robust approach, confidence level applied to each individual probabilistic parameter and confidence level applied to the routes. After, a new mathematical model that includes the schedule reliability is presented.

In order to clarify how this process takes place, a numerical example is first presented. Consider two routes that serve four customers. The first route visits customers {1,2}, while the second route visits customers {3,4}. The customers' demands expressed by their mean values and standard deviations (in m^2) are respectively: {42±38,108±63,103±94,164±57}. A normal distribution is assumed to represent the demand. In this study, the maximum value of the stochastic parameters is set to comply with a confidence level (CI) of 99.865%, which is sufficiently large for the intended application. In such a case, the corresponding demands are {156, 297, 385, 335} in m^2. The route leg's travel time depends on the distance (DT), and it is assumed to be represented by two normal distributions. The first applies when the distances are inferior to ten nautical miles, typically within a cluster. For the first case, the travel time (mean, standard deviation) is given by $0.006124DT \pm 0.002764DT$ (days per km), and for the latter case, $0.004904DT \pm 0.001143DT$ (days per km). In the given example, the routes are small, having only three navigation legs. Although the assumption of independence of the travel times of the segments is valid, one can decide to calculate the overall duration of the route using other statistical assumptions and apply the proposed methodology to attest the schedule reliability.

3.3.1 PSVPP: route generation with stochastic parameters

Three approaches to deal with the stochastic parameters in the PSVPP to calculate the route demand and the route duration are presented, the robust approach, confidence level applied to each individual probabilistic parameter and confidence level applied to the routes.

3.3.1.1 Robust approach

The first approach for generating the route parameters is based on robust optimization. Whenever the probability distributions for D_j and LT cannot be defined, or its cumulative distribution function can only be numerically calculated, the use of the robust approach is straightforward if the mean and the maximum values are known. The routes' parameters are calculated according to a predefined uncertainty budget (Γ), which can be translated as a protection level against a given number of variations to extreme values. The uncertainty budget (Γ) defines how many variations from the mean to the maximum value can be accepted within each route. This procedure is similar to the robust discrete optimization paradigm presented in (BERTSIMAS; BROWN; CARAMANIS, 2011). Route duration must be protected against a given number of delays per voyage defined by Γ, regardless of whether it is due to the variation of the demand D_j or the variation of the leg travel time LT. All combinations are tested during the route generation procedure, and the worst realization is selected. A pseudo-code is presented in Algorithm 2. The route duration is defined as $RT = RT_m + RT(\Gamma)$, where RT_m is the route duration considering the parameters' mean values and $RT(\Gamma)$ is the duration increase when Γ parameters are set to their maximum values.

In the given example, for route #1, considering the mean demand of each customer and the mean travel time of each route's leg, $RT_m = 1.288$ days and the $RT(\Gamma) = (0.000,$ 0.519, 0.831, 1.107, 1.383, 1.389) in days, for $\Gamma = (0,1,2,3,4,5)$. If $\Gamma = 0$, the mean values of the parameters are used, and no robustness is added to the problem. Then, the Γ value ranges from 1 to 5, representing the route added time due to variation both in the demands and in the route legs' travel time. For example, if $\Gamma = 2$, then RT = 2.119 days, being this variation due to the increase in cargo handling time both at the harbor and the offshore units. In this example, the route demand is set to 453 m^2.

Algorithm 2: Route generation - Robust approach

Input: j; D_j; LT; Vessel Capacity and Route Cost per Vessel Type v

Output: R; R_v; A^0_{rj}; B_{vrlt}; C_{vr}

for every combination of up to 8 customers **do**
- Calculate the route demand by summing the mean value of D_j of the units in the combination
- **if** the route demand is lower than or equal to the maximum vessel capacity **then**
  - Solve the TSP and define the route sequence
  - **for** each class v compatible with the route demand **do**
    - Calculate the mean TT as the sum of the mean value of each LT in the route
    - Calculate the route duration RT_m using the mean demand and the mean TT
    - **for** every offshore unit in the route **do**
      - Define the maximum demand of D_j to cover for a confidence level of 99.865%
    - **end for**
    - **for** every route leg in the route **do**
      - Define the maximum LT to cover for a confidence level of 99.865%
    - **end for**
    - Define the combination of size Γ of elements that most affect the route duration (demand or travel time)
    - **if** any D_j belongs to 'worst' combination **then**
      - Set D_j to its maximum value
    - **end if**
    - **if** the vessel capacity is not violated for the worst case demand **then**
      - Define A^0_{rj}
      - Calculate $RT(\Gamma)$ and RT
      - Create parameter B_{vrlt} based on RT for every day in the planning horizon
      - Calculate the route cost C_{vr}
      - Update subset R_v
    - **end if**
  - **end for**
- **end if**

end for

return R; R_v; A^0_{rj}; B_{vrlt}; C_{vr}

3.3.1.2 Confidence level applied to each individual probabilistic parameter

As the probability functions for the offshore units' demands and the legs' travel time are known, one may also consider each parameter to cover for given confidence level (CI) and then apply the route generation procedure defined in 3.1.1. A pseudo-code is indicated in Algorithm 3.

In the given example and for a CI of 99.865%, for route #1: i) customer#1 demand

Algorithm 3: Route generation - Confidence level applied to each individual probabilistic parameter

Input: j; D_j; LT; Vessel Capacity and Route Cost per Vessel Type v
Output: R; R_v; A^0_{rj}; B_{vrlt}; C_{vr}

for every combination of up to 8 customers **do**
  Set each demand D_j to the value corresponding to the CI
  Calculate the route demand by summing D_j of the units in the combination
  if the route demand is lower than or equal to the maximum vessel capacity **then**
    Solve the TSP and define the route sequence
    Define A^0_{rj}
    for each class v compatible with the route demand **do**
      for every route leg in the route **do**
        Set each LT to the value corresponding to the CI
      end for
      Calculate TT based on each LT
      Calculate the route duration RT
      Create parameter B_{vrlt} based on RT for every day in the planning horizon
      Calculate the route cost C_{vr}
      Update subset R_v
    end for
  end if
end for
return R; R_v; A^0_{rj}; B_{vrlt}; C_{vr}

is set to 156 m^2, and customer#2 demand is set to 297 m^2, causing the overall route demand to be 453 m^2; ii) each leg travel time is computed to meet the confidence level; and iii) the overall route duration is 2.674 days.

3.3.1.3 Confidence level applied to the routes

Another approach is to calculate the route's statistical parameters based on the joint probability functions derived from the D_j and LT. The route's statistical parameters are the route demand and the route duration. This is done by adding independent random variables. With the routes' statistical parameters calculated, it is possible to set the routes parameters to cover for a given CI. A pseudo-code is indicated in Algorithm 4.

Algorithm 4: Route generation - Confidence level applied to the routes

Input: j; D_j; LT; Vessel Capacity and Route Cost per Vessel Type v
Output: R; R_v; A^0_{rj}; B_{vrlt}; C_{vr}

for every combination of up to 8 customers **do**
  Calculate the route demand based on the joint probability of D_j to cover for a given CI
  if the route demand is lower than or equal to the maximum vessel capacity **then**
    Solve the TSP and define the route sequence
    for each class v compatible with the route demand **do**
      Define A^0_{rj}
      Calculate the probability function of the route duration from D_j and TT
      Set RT to cover for a given CI
      Create parameter B_{vrlt} based on RT for every day in the planning horizon
      Calculate the route cost C_{vr}
      Update subset R_v
    end for
  end if
end for
return R; R_v; A^0_{rj}; B_{vrlt}; C_{vr}

In the proposed example, route #1 demand is normally distributed $\sim N(42 + 108; 38^2 + 63^2)$, or 150 ± 73.6, resulting in an overall route demand of 371 m^2 for a 99.865% confidence level. The same applies to the route duration, which has an aggregate duration of 1.288 ± 0.239 days and an overall duration of 2.006 days at the desired confidence level. One may notice that the confidence level applied to the route duration leads to a lower overall route duration than the route duration calculated using the same confidence level applied to each individual parameters.

3.3.2 Schedule reliability

In contrast to the proposed approaches for dealing with the stochastic parameters in the route generation process, a new mathematical model that includes the schedule reliability is presented. A deterministic schedule is composed of routes with fixed departing days and known duration, as shown in Fig. 9. Each rectangle corresponds to a route assigned to a vessel of different sizes, represented by different colors. The numbers one to fourteen are references to days. The schedule presented covers two planning periods of one week each. As an example, route 1094 has a duration of 3 days and two departure days on this horizon: day 2 and day 9. The PSVPP is concerned with defining the optimal schedule and determining the fleet composition needed for servicing the offshore units on a regular basis. In such cases, the fleet is given by the maximum number of vessels simultaneously in use on any given day.

<table>
<tr><th></th><th></th><th colspan="14">4B</th></tr>
<tr><th>Class</th><th>#</th><th>1</th><th>2</th><th>3</th><th>4</th><th>5</th><th>6</th><th>7</th><th>8</th><th>9</th><th>10</th><th>11</th><th>12</th><th>13</th><th>14</th></tr>
<tr><td>PSV 4500</td><td>#1</td><td colspan="3">1131</td><td colspan="3">1013</td><td colspan="3">1362</td><td colspan="3">1331</td><td></td><td></td></tr>
<tr><td>PSV 4500</td><td>#2</td><td colspan="4">39</td><td colspan="3">1336</td><td colspan="4">39</td><td colspan="3">1336</td></tr>
<tr><td>PSV 4500</td><td>#3</td><td></td><td colspan="3">1094</td><td colspan="3">1238</td><td colspan="3">1131</td><td colspan="3">1013</td><td></td></tr>
<tr><td>PSV 4500</td><td>#4</td><td></td><td></td><td colspan="3">1331</td><td colspan="3">686</td><td colspan="3">1094</td><td colspan="3">1238</td></tr>
<tr><td>PSV 3000</td><td>#1</td><td colspan="2">972</td><td colspan="3">22</td><td colspan="2">167</td><td colspan="2">972</td><td colspan="3">22</td><td colspan="2">167</td></tr>
<tr><td>PSV 3000</td><td>#2</td><td></td><td colspan="2">113</td><td colspan="2">267</td><td colspan="3">31</td><td colspan="2">113</td><td colspan="2">267</td><td></td><td></td></tr>
<tr><td>PSV 1500</td><td>#1</td><td colspan="2">265</td><td colspan="2">933</td><td>49</td><td colspan="2">937</td><td colspan="2">265</td><td colspan="2">933</td><td>49</td><td colspan="2">937</td></tr>
</table>

Figure 9 – Schedule representation

In the stochastic version of the PSVPP, a major concern is how to achieve a reliable schedule, as the stochastic demand and the stochastic travel times may cause the following violations: i) vessel capacity being inferior to the actual route's demand; ii) a vessel not being available for the next assignment (trip) in due time.

As an example, consider two given routes, named routed #1 and route #2, whose departure day is the same, day 1, and both routes are assigned to the same vessel type. The mean route duration and the corresponding standard deviation are 1.28795 ± 0.23943 ($days$) for route #1 and 1.44292 ± 0.31637 for route #2. The statistical values for route #1 and route #2 are calculated by the route generation procedure presented in Algorithm 4. The duration distribution of route #1 shows that the probabilities for a vessel finishing the route in up to one, two, and three days are 11.46%, 99.85%, 100.00%, respectively (it has been assumed 100% of probability if the route duration is above the mean plus three times the standard deviation). Analogously, for route #2, the probabilities of finishing the route in up to one, two, and three days are 8.08%, 96.09%, and 100.00%. For a confidence level of 99.865% and assuming that the routes depart on the same day, two vessels are required from day one to day three. It may also be noticed that, on day three, there is only a 0.15% of probability that a vessel will be executing route #1 and 3.91% of probability that a vessel will be executing route #2. By combining these probabilities, it can be realized that the probability that no vessel will be in use on day three is 95.95%, the probability of one or less vessel will be in use on that day is 99.99%, and the probability of two vessels being in use is only 0.01%. This means that one of the vessels may be assigned to its next trip on day three, incurring a very low risk of not being able to execute the proposed schedule. The detailed calculation for combining the statistics of routes #1 and #2 on day 3 is presented in Fig. 10.

The main challenge for incorporating the schedule reliability in the problem is to find out an efficient way to address the statistics of the violations for the schedule, given a large number of possible routes and departing days combinations in the problem. This research uses a new methodology based on the routes' duration probability function and the joint probability function for the routes' combinations. It comprises a set of ranges of

(a) Probability of finishing the route for each day

	Day 1	Day 2	Day 3
Route #1	11.46%	99.85%	100.00%
Route #2	8.08%	96.09%	100.00%

(b) Probability of operating at day 3

Route #1	0.15%
Route #2	3.91%

(c) Probability combinations of operating at day 3

# of Vessels	Route #1	Route #2	Prob Route #1	Prob Route #2	Prob Routes #1 and #2
0	0	0	99.85%	96.09%	95.95%
1	1	0	0.15%	96.09%	0.14%
1	0	1	99.85%	3.91%	3.90%
2	1	1	0.15%	3.91%	0.01%

# of Vessels	Prob. Execution	Prob. Accumulated
0	95.95%	95.95%
1	4.04%	99.99%
2	0.01%	100.00%

Figure 10 – Numerical example of how to combine statistics of two routes departing on the same day.

probabilities for a given vessel type to be executing a route in a given day and a set for the combinations of the probability ranges. The range of probability is a predefined interval of probability; for example, a range of probability could be set from 0% to 10%, which indicates the chance that a vessel is in use lies within this interval. The combination of the ranges is the Cartesian product of the probability ranges of the routes being simultaneously executed. For example, if six ranges of probabilities are set and the maximum number of routes in execution is set to six, there are 923 possible combinations of ranges of probability (combination with repetition). The required number of vessels for a specific vessel type is calculated for each combination of probability ranges and the desired confidence level.

In the given example for day 3, two ranges of probabilities for a vessel being in use in a given day are arbitrarily defined as C1, with a range equal to [0%;0.3%], and C2 with a range equal to]0.3%;5%]. Then, all combinations for those probability ranges are generated, and the number of required vessels are defined, considering the desired confidence level and using the upper value of each range. Combining C1 and C1 results in a 99.401% probability that no vessel will be in use on day three, a 99.999% probability that one or fewer vessels will be in use, and 100% probability that two or fewer vessels will be in use on day three. Combining C1 and C2 results in a 94.715% probability that no vessel will be in use on day three, 99.985% probability that one or fewer vessels to be in use, and 100% probability that two or fewer vessels will be in use. For a 99.865% confidence level, the combination of C1 and C2 only requires one vessel on day three. Fig. 11 illustrates the combination of the ranges of probabilities C1 and C2 and the respective calculation for the number of vessels required.

(a) Probabily range of a vessel operating at day 3

C1	[0.0%;0.3%]
C2	]0.3%;5.0%]

(b) Combinations of probability ranges

# of Vessels	**Combination**		**# of Vessels**	**Combination**	
0	-	-	**2**	C1	C1
1	C1	-	**2**	C1	C2
1	-	C2	**2**	C2	C2

(c) Probability combinations of operating at day 3

			Combinations C1 and C1			**Combinations C1 and C2**			**Combinations C2 and C2**		
# of Vessels	**Route #1**	**Route #2**	**Prob Route #1**	**Prob Route #2**	**Prob Routes #1 and #2**	**Prob Route #1**	**Prob Route #2**	**Prob Routes #1 and #2**	**Prob Route #1**	**Prob Route #2**	**Prob Routes #1 and #2**
0	0	0	99.70%	99.70%	99.401%	99.70%	95.00%	94.715%	95.00%	95.00%	90.250%
1	1	0	0.30%	99.70%	0.299%	0.30%	95.00%	0.285%	5.00%	95.00%	4.750%
1	0	1	99.70%	0.30%	0.299%	99.70%	5.00%	4.985%	95.00%	5.00%	4.750%
2	1	1	0.30%	0.30%	0.001%	0.30%	5.00%	0.015%	5.00%	5.00%	0.250%

(d) Probability combinations per number of vessels of operating at day 3

			Combinations C1 and C1		**Combinations C1 and C2**		**Combinations C2 and C2**	
# of Vessels	**Route #1**	**Route #2**	**Prob. Execution**	**Accumulated Prob.**	**Prob. Execution**	**Accumulated Prob.**	**Prob. Execution**	**Accumulated Prob.**
0	0	0	99.401%	99.401%	94.715%	94.715%	90.250%	90.250%
1	0	1	0.599%	99.999%	5.270%	99.985%	9.500%	99.750%
2	1	1	0.001%	100.00%	0.015%	100.00%	0.250%	100.00%

Figure 11 – Combinations of C1 and C2 for different number of vessels.

3.3.2.1 A novel mathematical model for the S-PSVPP

This section presents the mathematical model considering the schedule robustness, which is based on the definitions of the sets, indices, parameters and decision variables described in Tables 8, 9 and 10, and on that previously detailed in Tables 1, 2 and 3 from Section 3.3.2.1.

Table 8 – S-PSVPP - Sets and indices

Sets and Index	Description
P	Set of ranges of probabilities for a vessel to be in use in a given day (index p).
A	Combinations of the ranges of probabilities (index a).

Table 9 – S-PSVPP - Parameters

Parameters	Description
W_a	Required number of vessels to ensure a given confidence level for (non-exceedance limit) for the combination in a.
Z_{ap}	Number of vessels in the combination of probabilities a that are in the range of probability p.
K_{vrltp}	Binary parameter that is 1 if a vessel v that is assigned to a route r that departs from supply base in day l is in use on day t with the probability of be in use in day t within the range of p, and 0 otherwise.
$\tilde{B}_{vrlt}$	Binary parameter that is 1 if a vessel v that is assigned to a route r that departs from supply base in day l is in use on day t and the probability of the vessel to be executing the route in day t is above the maximum range of probabilities defined in p, and 0 otherwise.

Table 10 – S-PSVPP - Decision Variables

Decision Variables	Description
m_{vta}	Binary variable equal to 1 if the probability combination of vessels in use a is selected (equals zero otherwise) for vessel type v on day t.

The mathematical model taking into account the schedule robustness is defined as:

$$minZ = \sum_{v \in V} F_v n_v + \sum_{v \in V} \sum_{r \in R_v} \sum_{l \in L} C_{vr} x_{vrl} \tag{3.25}$$

subject to:

$$\sum_{s \in S_j} b_{sj} = 1 \qquad j \in J \tag{3.26}$$

$$\sum_{v \in V} \sum_{r \in R_v} x_{vrl} A^0_{rj} = \sum_{s \in S_j} b_{sj} A^1_{sl} \qquad l \in L,\, j \in J \tag{3.27}$$

$$\sum_{v \in V} \sum_{r \in R_v} x_{vrl} \leq H \qquad l \in L \tag{3.28}$$

$$n_v \geq \sum_{r \in R} \sum_{l \in L} x_{vrl} \tilde{B}_{vrlt} + \sum_{a \in A} m_{vta} W_a \qquad v \in V,\, t \in T \tag{3.29}$$

$$\sum_{a \in A} m_{vta} Z_{ap} \geq \sum_{r \in R} \sum_{l \in L} x_{vrl} K_{vrltp} \qquad v \in V,\, t \in T,\, p \in P \tag{3.30}$$

$$\sum_{a \in A} m_{vta} = 1 \qquad v \in V,\, t \in T \tag{3.31}$$

$$n_v \in \mathbb{Z}^+,\, b_{sj} \in \{0,1\},\, x_{vrl} \in \{0,1\},\, m_{vta} \in \{0,1\}. \tag{3.32}$$

The objective function (3.25) and constraints (3.26) to (3.28) are the same of those one presented in the SPM model, in Section 3.3.2.1. Constraint (3.29) calculates the number of vessels per class as the maximum number of vessels in use at the same day for a given confidence level. Constraint (3.30) ensures that the number of vessels in use with the probability range p is greater or equal to the number of vessel in the range of probability p for the combination a selected at any day in t and vessel type v. Constraint (3.31) ensures that only one probability combination a is selected at any day in t and vessel type v. Lastly, Constraint (3.32) defines the variables' domain.

4 SOLUTION METHOD

The solution methods applied to solve the 2 extensions of the PSVPP are presented in this section.

4.1 PSVPP with berth allocation

The model proposed for the PSVPP with berth allocation decisions is difficult to solve, due to its combinatorial nature. In order to achieve a good-quality solution or even the optimal solution, the problem is solved in four steps. The initial steps simplify many important constraints, which are progressively incorporated. The idea is that the solution of any given step provides a lower bound to the following step and, in the case of the fourth step, a bound is provided for the complete model. Although the complexity increases with each step, the informed bounds are meant to reduce the processing time. In the following, each step is detailed.

Step 1 - The berth allocation constraints are relaxed and a limit on the maximum number of departures (i.e. routes) per day is imposed as two times the number of berths, as no more than two departures per berth are expected to occur in each day. The routing costs are also eliminated from the objective function. Lastly, instead of calculating the number of vessels as proposed by constraint (3.23), the vessels' operating time (in days) are summed, for each class, and divided by seven (one week). This is an approximation that is refined in Step 2, taking into account the vessels' temporal distribution.

Step 2 - The difference from Step 1 is the refinement in the fleet-sizing process. The fleet is now defined by the maximum number of vessels in use in each day, for each class, as in constraint (3.23). The fleet cost of Step 2 cannot be inferior to the Step 1 fleet cost, and this is represented by a constraint.

Step 3 - The routes selected in Step 2 are not necessarily optimal if the routing costs are considered, despite being able to yield the lowest fleet cost, which was accurately calculated in Step 2. In Step 3, the same model proposed for Step 2 is considered except that the routing costs are incorporated in the objective function. As in the previous case, the fleet cost of Step 3 cannot be inferior to the fleet cost of Step 2, and a constraint is added to the model in this respect.

Step 4 - The problem solved in Step 3 is complete except for the berth allocation decisions, which are considered in this step. The routes from the Step 3 solution are used as input to Step 4.

4.1.1 Step 1 Mathematical Model

In order to solve Step 1, new decision variables are introduced to allow working with the simplified model, w_{vrl} - binary variable that is 1 if a vessel of class v executes route r on day l, and 0 otherwise; and tc_{vrl} - integer variable that registers the cycle time for a vessel of class v when executing route r on day l. The cycle time is the time span between the beginning of the vessel loading at the harbour until its return to the onshore base, after performing a route. Z_1 is the value of the objective function, and $\underline{Z_1}$ is the lower-bound value obtained with the model processing. The model can be stated as follows:

$$min\ Z_1 = \sum_{v \in V} F_v n_v \tag{4.1}$$

subject to:

$$\sum_{s \in S_j} b_{sj} = 1 \qquad j \in J \tag{4.2}$$

$$\sum_{v \in V} \sum_{r \in R_v} w_{vrl} A^0_{rj} \geq \sum_{s \in S_j} b_{sj} A^1_{sl} \qquad l \in L,\ j \in J \tag{4.3}$$

$$\sum_{v \in V} \sum_{r \in R_v} w_{vrl} \leq |B|\ |P| \qquad l \in L \tag{4.4}$$

$$tc_{vrl} \geq w_{vrl} \left[SB + ST_{rv} + \sum_{j \in J:\ A^0_{rj}=1} (PB + PO_j) D_j \right] \qquad l \in L,\ v \in V,\ r \in R_v \tag{4.5}$$

$$n_v \geq \sum_{l \in L} \sum_{r \in R_v} tc_{vrl}\ /\ |L| \qquad v \in V \tag{4.6}$$

$$n_v \in \mathbb{Z}^+, \quad tc_{vrl} \in \mathbb{Z}^+, \quad b_{sj} \in \{0,1\}, \quad w_{vrl} \in \{0,1\}. \tag{4.7}$$

The objective function (4.1) minimizes vessels' fixed costs based on their average use during the planning period. Constraint (4.2) ensures that one departure pattern is assigned for each offshore unit. Constraint (4.3) ensures that there is at least one vessel departing on each day belonging to the selected departure pattern of each offshore unit. Constraint (4.4) limits the number of departures per day based on the number of available berth positions. Constraint (4.5) determines the cycle time for a vessel of class v when assigned to route r on day l. Constraint (4.6) calculates the number of vessels per class, based on the average utilization in the considered period (one week). Constraint (4.7) defines the variables' domain.

4.1.2 Step 2 Mathematical Model

In order to solve Step 2, the following variable is needed: $y_{vrt_1t_2}$ - binary variable that is 1 if a vessel of class v departing on day t_1 is still operating on route r by day t_2, and 0 otherwise. This variable plays a similar role as $c_{vbpt_1t_2}$ in the complete model, and allows for computing the maximum number of class v vessels in use. Z_2 is the value of the objective function, and $\underline{Z_2}$ is the lower-bound value obtained with the model processing. The model can be stated as:

$$min\ Z_2 = \sum_{v \in V} F_v n_v \tag{4.8}$$

subject to:

$$\sum_{s \in S_j} b_{sj} = 1 \qquad j \in J \tag{4.9}$$

$$\sum_{v \in V} \sum_{r \in R_v} w_{vrl} A^0_{rj} \geq \sum_{s \in S_j} b_{sj} A^1_{sl} \qquad l \in L, \quad j \in J \tag{4.10}$$

$$\sum_{v \in V} \sum_{r \in R_v} w_{vrl} \leq |B|\,|P| \qquad l \in L \tag{4.11}$$

$$\sum_{t_2=t_1}^{|T|} y_{vrt_1t_2} \geq w_{vrl} \left[SB + ST_{rv} + \sum_{j \in J:\ A^0_{rj}=1} (PB + PO_j) D_j \right] \qquad l \in L,\ t_1 \in L,\ t_2 \in T,\ v \in V,\ r \in R \tag{4.12}$$

$$y_{vrll} \geq w_{vrl} \qquad l \in L,\ v \in V,\ r \in R_v \tag{4.13}$$

$$y_{vrt_1t_2} = 0 \qquad t_1 \in L,\ t_2 \in L,\ v \in V,\ r \in R_v,\ t_2 < t_1 \tag{4.14}$$

$$y_{vrt_1(t_2+1)} - y_{vrt_1t_2} \leq 0 \qquad t_1 \in L,\ t_2 \in T,\ v \in V,\ r \in R,\ t_2 \geq t_1 \tag{4.15}$$

$$y_{vr(t_1+|L|)(t_2+|L|)} = y_{vrt_1t_2} \qquad t_1 \in L,\ t_2 \in L,\ v \in V,\ r \in R \tag{4.16}$$

$$n_v \geq \sum_{t_1 \in T} \sum_{r \in R} y_{vrt_1t_2} \qquad v \in V,\ t_2 \in T \tag{4.17}$$

$$\sum_{v \in V} F_v n_v \geq \underline{Z_1} \tag{4.18}$$

$$n_v \in \mathbb{Z}^+,\, b_{sj} \in \{0,1\}\,,\, w_{vrl} \in \{0,1\}\,,\, y_{vrt_1t_2} \in \{0,1\}\,. \tag{4.19}$$

The objective function (4.8) minimizes the vessels' fixed costs based on the maximum number of vessels that is required for each vessel class in the planning period. Constraints (4.9) to (4.11) have the same purposes as stated in the previous model. Constraint (4.12) ensures that a binary counter is activated (i.e. equal to 1), for the time period that a vessel of class v is executing route r. Constraint (4.13) forces variable $y_{vrt_1t_2}$ to be 1 if a vessel of class v departs from the port on day t_1 to execute route r. Constraint (4.14) ensures that the binary counter is not activated in any period t_2 prior to a candidate departing day t_1. Constraint (4.15) links the counter variable of one day to its previous day. Constraint (4.16) replicates a vessel class assignment for the subsequent period. Constraint (4.17) calculates the number of vessels per class, as the maximum number of vessels in use at the same time. Constraint (4.18) imposes a lower-bound value on the fleet cost, based on the Step 1 lower bound. Constraint (4.19) defines the variables' domain.

4.1.3 Step 3 Mathematical Model

In Step 3, the same model from Step 2 is solved except that the objective function incorporates the routing costs. The solution found in Step 2 is used as an initial solution, and also provides a lower bound to the objective function. Z_3 is the value of the objective function, and $\underline{Z_3}$ is the lower-bound value obtained with the model processing. The model is stated as:

$$min\ Z_3 = \sum_{v \in V} F_v n_v + \sum_{v \in V} \sum_{r \in R_v} \sum_{l \in L} C_{vr} w_{vrl} \tag{4.20}$$

subject to:

(4.9) to (4.17), (4.19)

$$\sum_{v \in V} F_v n_v \geq \underline{Z_2}. \tag{4.21}$$

The objective function (4.20) minimizes the vessels' fixed costs and the routing costs, and constraint (4.21) is the lower bound.

4.1.4 Step 4 Mathematical Model

In Step 4, the berth allocation decisions are made based on input from the set of routes that were selected in Step 3. These routes are represented by the subset $R_3 \subset R$. In Step 4, the complete model is complemented by the indicated route selection constraints and the lower-bound constraints. Z_4 is the value of the objective function, and $\underline{Z_4}$ is the lower-bound value obtained with the model processing. The model can be stated as:

$$min\ Z_4 = \sum_{v \in V} F_v n_v + \sum_{v \in V} \sum_{b \in B} \sum_{p \in P} \sum_{r \in R_v} \sum_{l \in L} C_{vr} x_{vbprl} \tag{4.22}$$

subject to:

(3.8) to (3.24)

$$\sum_{v \in V} \sum_{b \in B} \sum_{p \in P} \sum_{l \in L} x_{vbprl} = 1 \qquad r \in R_3 \tag{4.23}$$

$$\sum_{v \in V} \sum_{b \in B} \sum_{p \in P} \sum_{l \in L} x_{vbprl} = 0 \qquad r \in R \setminus R_3 \tag{4.24}$$

$$\sum_{v \in V} F_v n_v \geq \underline{Z_2} \tag{4.25}$$

$$\sum_{v \in V} F_v n_v + \sum_{v \in V} \sum_{b \in B} \sum_{p \in P} \sum_{r \in R_v} \sum_{l \in L} CR_{vr} x_{vbprl} \geq \underline{Z_3} \tag{4.26}$$

The objective function (4.22) minimizes the fleet's fixed costs and the routing costs. Constraints (4.23) and (4.24) assign the routes obtained in Step 3 and discard all the others. Constraint (4.25) defines that the fleet cost must be greater than or equal to the Step 2 lower bound, and constraint (4.26) defines that the objective function must be greater than or equal to the Step 3 lower bound.

In the problem description, it was considered that a vessel meant to depart on day l could be delayed up to 12 hours, thus leaving the port any time before 12 pm of day l+1. This could happen in situations where a vessel was not able to start loading earlier on day l, due to limited berth capacity. The berth scheduling process therefore focuses on how to accommodate all the vessels in such a way that all departures fit in a one-week period. The berth scheduling may have a direct impact on the fleet size as well. The $c_{vbpt_1t_2}$ binary decision variable is equal to 1 each day that a vessel of class v is in use, after leaving the port. However, according to constraint (3.18), the sum of the $c_{vbpt_1t_2}$ variables is influenced by the departure time, which is given by s_{bpl}. For example, if the route duration is 2.3 days and the departure time is 0.5, then the assigned vessel is in use until instant 2.8, and three $c_{vbpt_1t_2}$ variables are set to 1; but, if the departure takes place in instant 0.9, the assigned vessel is in use until instant 3.2, and therefore four $c_{vbpt_1t_2}$ variables are set to 1. In this case, the maximum number of vessels in use, which is assessed by constraint (3.23), may indicate a different fleet.

If the solution found in Step 3 indicates a number of departures per day that are inferior or equal to the number of available berths, then the Step 4 solution is the same as for Step 3. However, as this may not be the case, in Step 4, the complete model is processed with the following simplifications in order to make the model processing more tractable: the routes that were generated in Step 3 are retained, and the decisions regarding the departure pattern (i.e. the departing days for each offshore unit) and the vessel class that is assigned to each route are released. This means that the routes selected in Step 3 can be sailed by a vessel from a different class and on a different day other than the ones established in that previous step, thus giving more flexibility to the solution procedure. Although this procedure may not lead to the optimal solution, good-quality solutions are expected to be obtained which would not be otherwise possible if one was to resort to solving the complete model. Finally, the objective function of Step 3 is a natural lower bound on the objective function of Step 4, and the Step 4 solution provides an upper bound to the complete model.

4.2 S-PSVPP: Probabilities combinations and relevant parameters

For the stochastic case, the introduction of new sets, parameters, and decision variables demands an extra step in generating the input parameters for the mathematical model. The following procedure is added to that stage: i) input parameters: a set of ranges of probabilities for vessels in use (set P), the maximum number of simultaneous routes in execution to define the combinations of the range of probabilities (set A), and the desired confidence level (non-exceedance limit; CI); ii) generate all combinations of ranges of probabilities in groups of one up to the maximum number of routes in execution simultaneously; iii) define the set A and the parameter Z_{ap} according to (ii); iv) calculate the joint probability of the combination using the upper value in the range of probability; v) define W_a using the joint probability of the combination and the confidence level. A pseudo-code is presented in Algorithm 5.

Algorithm 5: Schedule Reliability Parameters

Input: P; maximum number of simultaneous routes in execution; CI;
Output: A; Z_{ap}; W_a
for Every combination of ranges of probabilities (P) from 1 to the max. number of routes **do**
    Update A
    Define Z_{ap}
    Calculate the joint probability of the combination using the upper value in the range
    Define W_a to cover for a given CI
end for
return A; Z_{ap}; W_a

Besides this added step, the route generation has to be updated regarding the generation of the parameters $\tilde{B}_{vrlt}$ and K_{vrltp}. The route statistics are calculated according to 3.3.1.3. An upper limit for the route duration is set to a confidence level of 99.865%.

If the probability of the vessel being used in a given route on day t is above the upper probability range defined in p, only B_{vrlt} is set to one. In case this probability lies in one of the ranges defined in p, only the corresponding K_{vrltp} is set to 1. If the day t is beyond the upper limit for route duration with a departure on day l, both $\tilde{B}_{vrlt}$ and K_{vrltp} are set to zero. A pseudo-code is presented in Algorithm 6. A diagram to illustrate the key input data and the generation of the mathematical model parameters for the proposed method is presented in Fig. 12.

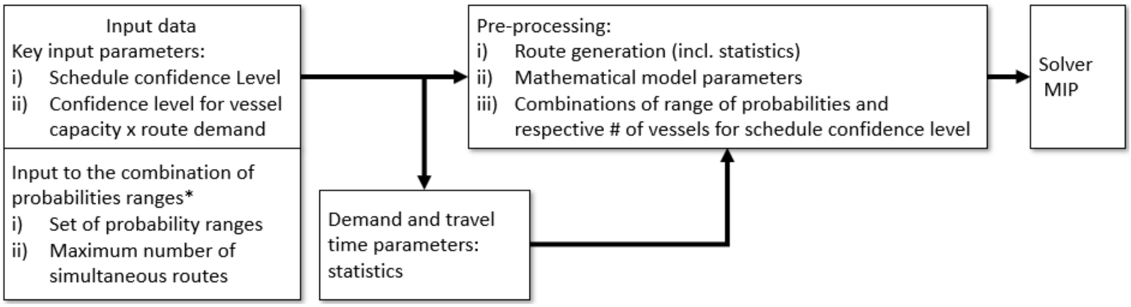

Figure 12 – S-PSVPP procedure using a MIP solver.

Algorithm 6: Route Generation - Schedule Reliability

Input: j; D_j; LT; Vessel Capacity and Route Cost per Vessel Type v; P
Output: R; R_v; A^0_{rj}; $\tilde{B}_{vrlt}$; K_{vrltp}; C_{vr}

```
for every combination of up to 8 customers do
    Solve the TSP and define the route sequence
    Calculate the route demand based on the joint probability of D_j to cover for a
      given CI
    if The route demand is lower than or equal to the maximum vessel capacity
      then
        Define A^0_rj
        for each class v compatible with the route demand do
            Calculate the probability function of the route duration from D_j and TT
            Set RT to cover for a given CI
            Calculate the route cost C_vr
            Update subset R_v
            Define B~_vrlt and K_vrltp based on RT for every day in the planning
              horizon, observing each day's route duration statistics
        end for
    end if
end for
return R; R_v; A^0_rj; B~_vrlt; K_vrltp; C_vr
```

As an example, the following has been used in the study: i) 6 ranges of probabilities: [0%; 0.3%],]0.3%; 0.5%],]0.5%; 0.7%],]0.7%; 1%],]1%; 5%] and]5%; 10%]; and ii) maximum of 6 routes in execution simultaneously. Therefore, 923 combinations are generated. The combination #642 has 6 vessels (or routes) with the probably of each vessels to be in use executing the routes of 0.3%, 0.5%, 0.7%, 10%, 10% and 10%. For this combination there is 71.81%, 96.83%, 99.86%, 99.998%, 99.9999%, 99.99999999% and 100% of probability that 0, 1, 2, 3, 4, 5 and 6 or less vessels are in use on that day, respectively. If the confidence level is set to 99%, W_{642} should be set to 2.

5 COMPUTATIONAL RESULTS

The models applicability are demonstrated by real-based cases. Four cases are presented: these are cases C10, C15, C41 and C79, having 10, 15, 41 and 79 offshore units, respectively. In cases C10 and C15, routes are generated considering all possible combinations for all units. In cases C41 and C79, routes are generated considering all possible combinations of the units belonging to their clusters (i.e. groups of offshore units).

In the offshore E&P there are permanent units such as the production platforms that remain fixed in their positions for as long as 25 years, and mobile units, related to drilling rigs and maintenance platforms, that are constantly moved from one oilfield to another. Those mobile units usually require a different service level in terms of frequency of visits and different types of cargo when compared to the permanent units. Therefore, an oil company may consider grouping near-by permanent units and near-by mobile units in order to form clusters. Another policy is to group near-by units irrespective of their type (permanent or mobile). Instances were generated to compare these two policies. Those with the suffix S indicate that the mobile units are segregated from the permanent units.

The study is divided in two main sections, one dedicated to the berth allocation and the other dedicated to the stochastic demand and travel time.

5.1 PSVPP with berth allocation

Table 17 presents some key features regarding each instance. The columns indicate the following data: OU - number of offshore units to be served; Routes - number of generated routes to be used as input data in the mathematical model; Clusters - number of clusters (groups of platforms to be serviced together and apart from the others) for each instance; OUmax - maximum number of offshore units served in a cluster; OUmin - minimum number of offshore units served in a cluster; Rmax - maximum number of generated routes in a cluster; and Rmin - minimum number of generated routes in a cluster.

Table 11 – Berth Allocation - Instances

Case	OU	Routes	Clusters	OUmax	OUmin	Rmax	Rmin
C10	10	912	1	10	10	912	912
C15	15	10,021	1	15	15	10,021	10,021
C41	41	1,372	5	11	6	822	47
C41S	41	438	7	8	4	199	14
C79	79	2,168	9	12	6	882	47
C79S	79	1,107	12	10	4	315	14

Case C10 is the easiest to solve and is used to verify the effectiveness of the step solution strategy performance, when compared to the use of the complete model. Case C15 allows us to demonstrate the difficulty of solving an instance with a high overall number of routes. Cases C41 and C79 are intermediate and hard-to-solve instances, respectively, and allow for assessing the proposed solution strategy. Not only the overall number of routes defines hard-to-solve instances. A large number of offshore units divided into clusters might produce hard-to-solve instances, even with a limited number of routes in each cluster. This is clearly demonstrated in the results obtained for cases C79.

For all cases, 4 berths were considered available with a maximum of two departures per berth each day. A tolerance of half a day was allowed for the vessel departure time, to allow better berth utilization.

All models were implemented in C++ and solved by Gurobi. The results were obtained using a 2.27 GHz Intel(R) Xeon(R) E5520, with 16 cores, 48 GB RAM, and solved by GUROBI 7.0.2. The routes were previously generated in a Excel spreadsheet using VBA. The processing time for route generation varies from 30 to 2,100 seconds depending on the instance. The code for the route generation has not been developed for optimal performance and its processing time has been disregarded from the computational study. The maximum time limit of 99,000 seconds has been set for each run, in order to verify how far it was possible to go with the proposed solution strategy.

In the tables used to present the main results, the lines represent the following: OF - objective function value; VC - vessels' costs; RC - routing costs; V0 - number of PSV4500 vessels; V1 - number of PSV3000 vessels; V2 - number of PSV1500 vessels; NR - total number of sailed routes; ND - maximum number of departures at any day; B - number of berths; GAP - percentage difference from the upper and lower bounds given by Gurobi; Time - processing time in seconds; and CTime - cumulative processing time in seconds for the steps in the proposed solution strategy, and the processing time in seconds in case of the complete model. As for the columns, they are organized according to each step proposed in the solution strategy: S1 refers to Step 1, S2 refers to Step 2, and so on. In some cases, Step 4 is tested for a number of berths other than four, indicated in line B. Column CM refers to the complete model, defined in 3.2.

5.1.1 Results for Cases C10 and C15

The results for cases C10 and C15 are provided in Table 12. It can be noticed that case C10 presented no variation in the number of vessels, with each solution step. The maximum number of departures on the same day decreased significantly from Step 1 to Step 2 because of the vessel usage consideration. However, a considerable difference can be noticed for the processing time. The complete model took 84,340 seconds to process, while the total processing time for the solution strategy was 115 seconds.

In case C15 the fleet varied with each step, except for the last one. In Step 1,

Table 12 – Berth Allocation - Results for cases C10 and C15

C10	S1	S2	S3	S4	CM
OF	50	50	53.77	53.77	53.77
VC	50	50	50	50	50
RC	-	-	3.77	3.77	3.77
V0	0	0	0	0	0
V1	0	0	0	0	0
V2	2	2	2	2	2
NR	6	7	6	6	6
ND	3	1	1	2	1
B	4	4	4	4	4
Gap	0	0	0	0	0
Time	4	11	16	84	84,340
CTime	4	15	31	115	84,340
C15	**S1**	**S2**	**S3**	**S4**	**CM**
OF	71	75	80.37	80.37	89.43†
VC	71	75	76	76	83
RC	-	-	4.37	4.37	6.43
V0	1	0	2	2	0
V1	1	0	0	0	1
V2	0	3	0	0	2
NR	5	11	4	4	9
ND	1	2	1	1	2
B	4	4	4	4	4
Gap	0	0	0	0	57.90%
Time	1,862	4,448	496	6	21,289
CTime	1,862	6,310	6,806	6,812	21,289

† Objective function value not proven to be optimal.

two vessels were needed to perform five routes. However, when the fleet was calculated more accurately, three small vessels were chosen to perform 11 routes. In Step 3, as the routing costs were computed, two large vessels were selected to perform four routes. The fleet cost increased from 75 to 76, but this was compensated for by a reduction in the routing costs. In Step 2, the routing costs, which are not shown in Table 12, were 8.04, and in Step 3, the routing costs were 4.37, yielding an overall cost reduction of 2.67. As for the complete model, an out-of-memory error interrupted the processing at instant 21,289 seconds, with a 57.90% gap. The proposed solution strategy thus proved to be efficient and an optimal solution could be found in 6,812 seconds. For both cases C10 and C15, given that optimal solutions were attained at Steps 1 to 3, and that there were enough berths to accommodate all the departures generated in Step 3, the Step 4 solutions are also optimal to the complete model.

Table 13 – Berth Allocation - Results for cases C41 and C41S

C41	S1	S2	S3	S4	S4(3B)	S4(2B)	CM
OF	243	243	256.59	256.59	256.59	270.20†	289.23†
VC	243	243	243	243	243	256	276
RC	-	-	13.59	13.59	13.59	14.20	13.23
V0	4	4	4	4	4	5	4
V1	2	2	2	2	2	2	3
V2	1	1	1	1	1	0	1
NR	19	19	19	19	19	19	18
ND	4	4	4	4	4	3	4
B	4	4	4	4	3	2	4
Gap	0	0	0	0	0	3.1%	62.6%
Time	21	41	77	348	173	99,000	99,000
CTime	21	62	139	487	312	99,139	99,000
C41S	**S1**	**S2**	**S3**	**S4**	**S4(3B)**	**S4(2B)**	**CM**
OF	248	248	261.99	261.99	-	261.99	305.00†
VC	248	248	248	248	-	248	291
RC	-	-	13.99	13.99	-	13.99	14.00
V0	5	5	5	5	-	5	7
V1	1	1	1	1	-	1	0
V2	1	1	1	1	-	1	1
NR	18	18	17	17	-	17	17
ND	4	4	3	3	3	3	4
B	4	4	4	4	-	2	4
Gap	0	0	0	0	-	0	60.46%
Time	10	170	81	6	-	61	99,000
CTime	10	180	261	267	-	322	99,000

† Objective function value not proven to be optimal.

5.1.2 Results for Cases C41 and C41S

Table 13 presents the results for cases C41 and C41S, without and with segregation regarding servicing mobile units apart from the permanent units, respectively. Different from cases C10 and C15, these cases were also tested for two and three berths. It can be noticed that in case C41, the number of vessels remained the same, except for the case with two available berths. The maximum number of departures were four and three, for cases C41 and C41S, respectively, and Steps 1 to 3 were optimally solved. Therefore, as with cases C10 and C15, the Step 4 solutions are also optimal. In order to test if the solution would change in the case that fewer berths were available, the Step 4 model was run considering three and two berths, and the results are presented in Table 13 in columns S4(3B) and S4(2B), respectively.

In case C41S, the results for three berths were omitted, as the maximum number of departures in Step 3, with four available berths, was three and, therefore, the solution would not change. Also, when only two berths were available, the solution remained the same. As for the complete model processed with four berths, solutions with poor lower bounds were obtained, despite their long processing times, and the computational efficiency of the proposed solution strategy could thus be verified once again. While the total processing times for considering all steps were 1,234 seconds and 267 seconds, for cases C41 and C41S, respectively, the complete model was run for 99,000 seconds for both cases, and gaps exceeding 60% were obtained. The servicing policies can be compared. If four berths are available, the option to service the mobile units and permanent units together is better than segregating them. The costs would be 2.1% lower in this case. This analysis is valid for the instances presented and might diverge in other cases.

Berth allocation for case C41 can be seen in Figure 13. On the left, the berth number (B) and the departing position number (P) are indicated. On the top, each column represents one day, and the solutions with four, three and two berths are compared. The numbers indicated in the colored spaces are the route number for each berth-position and day. The color indicates the type of vessel that was used: red for large size, yellow for mid-size and green for small size.

		4B							3B							2B						
B	P	1	2	3	4	5	6	7	1	2	3	4	5	6	7	1	2	3	4	5	6	7
1	1		1094			1336	686		933	267	1238	167		1131	113	686	1131	267	113	265	1013	1131
1	2								22			937						39	937			
2	1	972		933	267	49	167			1013	1336	686	1331	265	1094	167	972	1238	1336	31	933	49
2	2													39		22					1362	1094
3	1	265	113	22			937		1362		49	31		972								
3	2																					
4	1	1131		1331	1013	1238	31	1362														
4	2	39																				

Figure 13 – Berth allocation for case C41.

In case C41, the results for three berths were the same as for four berths both in terms of fleet composition and in terms of routing costs. Given that the maximum number of departures is four and the number of available berths is three, the routes could be rearranged on different departing days, as indicated in the middle section of Figure 13. When instance C41 was processed with two berths, a different fleet composition was obtained. The positive gap of 3.1% indicates a lower bound equal to 261.82, which is greater than the objective function value of S4 with three or four berths. Thus, a greater fleet is necessary to allow more departures throughout the planning horizon.

The departure times for case C41 can be seen in Figure 14. The importance of allowing flexibility regarding the departure times (i.e. to allow a departure to take place by 12 pm of the following day) was noticed in the case C41. This actually happened on 4B day 1, 3B day 6, and 2B day 3. In case 3B, for example, there is a departure scheduled to take place on day 6 which actually happens at instant 6.2 (day 7). If no tolerance in the departure time is allowed, one extra large vessel would be needed in place of a mid-size vessel, incurring a cost increase of 2% (around 5 MM USD). This reinforces that finding a feasible berth allocation with the minimum fleet configuration was made possible due to such a tolerance in the departure time.

		4B							3B							2B						
B	P	1	2	3	4	5	6	7	1	2	3	4	5	6	7	1	2	3	4	5	6	7
1	1		1.68			4.69	5.73		0.33	1.41	2.74	3.48		5.75	6.47	0.73	1.75	2.41	3.57	4.33	5.69	6.75
1	2								0.79			3.79						3.10	3.88			
2	1	0.45		2.33	3.41	4.21	5.48			1.69	2.69	3.73	4.68	5.33	6.70	0.48	1.45	2.74	3.69	4.56	5.33	6.21
2	2													6.2		0.95					5.94	6.89
3	1	0.33	1.47	2.47			5.31		0.62		2.21	3.56		5.45								
3	2																					
4	1	0.75		2.68	3.69	4.74	5.56	6.62														
4	2	1.43																				

Figure 14 – Departure instants for each berth-position for case C41.

When a vessel is assigned to a route, it can begin loading at the very beginning of the scheduled departure day; or it may have to wait for a vessel from a previous day to release the berth; or it may happen that a vessel is the second to occupy a berth on a given day. The waiting time before starting to load a vessel, added to the loading time and to the time at sea, indicates the vessel cycle time. The rounded value of the cycle time to its upper integer value indicates the number of $c_{vbpl_1l_2}$ binary variables set to 1, which allows for computing the maximum number of vessels in use - refer to constraints (3.18) to (3.23). Figure 15 indicates the cycle time for all departures for case C41. For example, for 3B day 6, route 39 starts loading at instant 5.33, which is the time the previous vessel left the berth (see Figure 14). The loading time is 0.87, and the sailing time added to the trans-shipment time at sea is 2.29. The cycle time is given by $0.33 + 0.87 + 2.29 = 3.49$, thus requiring the vessel to be in use for four days.

		4B							3B							2B						
B	P	1	2	3	4	5	6	7	1	2	3	4	5	6	7	1	2	3	4	5	6	7
1	1		2.76			2.91	2.94		1.53	1.85	2.98	1.10		2.97	1.89	2.94	2.97	1.85	1.98	1.56	2.82	2.97
1	2								2.71			1.89						3.56	1.98			
2	1	1.89		1.53	1.85	0.97	1.10			2.82	2.91	2.94	2.87	1.56	2.78	1.10	1.89	2.98	2.91	2.71	1.53	0.97
2	2													3.49		2.86					2.99	2.97
3	1	1.56	1.89	2.38			1.41		2.67		0.97	2.71		1.89								
3	2																					
4	1	2.97		2.87	2.82	2.98	2.71	2.67														
4	2	3.90																				

Figure 15 – Cycle times for each berth-position for case C41.

According to Table 13, in case C41 the fleet is composed of seven vessels. Figures 16 to 18 present possible arrangements for the vessels' assignments depending on the number of berths. In the figures, the number of berths is indicated in the first line, and the days are indicated in the second line. Then, each additional line corresponds to a vessel, indicated by its class and its number. For each vessel, the rectangles are as large as the cycle time of each route, whose number is indicated inside the rectangle. The colors indicate the vessel classes: red for large size, yellow for mid-size and green for small size. The light and dark colors are meant to highlight the routes' assignments over each week. As one may notice, the routes selected for the large-size vessels cannot be repeated for the same vessel in two consecutive weeks. If the vessels were required to perform the same routes from one period to the other the proposed solution would not be feasible. However, by considering the vessels as belonging to a pool of available resources, more sophisticated and cost-effective solutions can be built.

	4B													
Class #	1	2	3	4	5	6	7	8	9	10	11	12	13	14
PSV 4500 #1	1131			1013			1362			1331				
PSV 4500 #2	39				1336			39				1336		
PSV 4500 #3		1094			1238			1131			1013			
PSV 4500 #4			1331			686			1094			1238		
PSV 3000 #1	972		22			167		972		22			167	
PSV 3000 #2		113		267		31			113		267			
PSV 1500 #1	265		933		49	937		265		933		49	937	

Figure 16 – Vessels' allocation for case C41 (four berths).

	3B													
Class #	1	2	3	4	5	6	7	8	9	10	11	12	13	14
PSV 4500 #1	1362			686			1094			1238				
PSV 4500 #2		1013			1331			1362			686			
PSV 4500 #3			1238			1131			1013			1331		
PSV 4500 #4			1336			39				1336				
PSV 3000 #1	22			167		972		22			167			
PSV 3000 #2		267		31			113		267		31			
PSV 1500 #1	933		49	937		265		933		49	937		265	

Figure 17 – Vessels' allocation for case C41 (three berths).

	2B													
Class #	1	2	3	4	5	6	7	8	9	10	11	12	13	14
PSV 4500 #1	686			113		1013			1331			31		
PSV 4500 #2		1331			31			686			113			
PSV 4500 #3		972		1336			1131			1238				
PSV 4500 #4			39				1094			39				
PSV 4500 #5			1238			1362			972		1336			
PSV 3000 #1	22			937		933		22			937		933	
PSV 3000 #2	167		267		265		49	167		267		265		49

Figure 18 – Vessels' allocation for case C41 (two berths).

5.1.3 Results for Cases C79 and C79S

Cases C79 and C79S were also tested and the results are presented in Table 14. In case C79 with four berths, Steps 1 to 3 did not present any variation related to the fleet composition. However, in Step 4, the solution from Step 3 could not be confirmed, and a mid-sized vessel was added to the fleet. The gap was 5.58% with no progress of the lower bound throughout the whole processing time. Step 4 was also tested for five and six berths (5B and 6B) and only with six berths the Step 3 solution could be scheduled with the same fleet. In case C79S with four berths, the model processing in Step 1 and in Step 2 were not able to close the gap, which was 0.56% after 99,000 seconds. The Step 1 solution was obtained after 1,549 seconds, and the gap was 2.04%. Then it took almost 97,500 seconds to reduce the gap to 0.56%. In Step 2, the final solution was generated at instant 5,160 with a gap equal to 0.56%. The Step 3 model was interrupted after 61,180 seconds due to an out-of-memory error without having generated any primal solution.

Table 14 – Berth Allocation - Results for cases C79 and C79S

C79	S1	S2	S3	S4	S4(5B)	S4(6B)	CM
OF	527	527	558.55	591.55†	591.55†	558.55	-
VC	527	527	527	560	560	527	-
RC	-	-	31.55	31.55	31.55	31.55	-
V0	8	8	8	8	8	8	-
V1	6	6	6	7	7	6	-
V2	1	1	1	1	1	1	-
NR	38	38	38	38	38	38	-
ND	8	8	8	7	6	7	-
B	4	4	4	4	5	6	4
Gap	0	0	0	5.58%	5.58%	0	-
Time	3,274	5,755	36,270	99,000	99,000	64,803	99,000
CTime	3,274	9,029	45,299	144,299	144,299	110,102	99,000

C79S	S1	S2	S3	S2*	S3*	S4(4B)*	CM
OF	540†	540†	570.46†	540	570.37	570.37	-
VC	540	540	-	540	540	540	-
RC	-	-	-	-	30.37	30.37	-
V0	9	9	-	9	9	9	-
V1	6	6	-	6	6	6	-
V2	0	0	-	0	0	0	-
NR	36	35	-	35	34	34	-
ND	8	6	-	6	6	6	-
B	4	4	4	4	4	4	4
Gap	0.56%	0.56%	0.49%	0	0	0	-
Time	99,000	99,000	61,180	161	1,771	8,653	99,000
CTime	99,000	198,000	259,180	99,161	100,932	109,585	99,000

† Objective function value not proven to be optimal.
* Upper bound of a previous solution step informed instead of the lower bound.

We also tested how Steps 2, 3 and 4 would perform if the upper bound of a previous solution step was informed in constraints (4.18), (4.21), (4.25) and (4.26) instead of the lower bound. This is different from the previous cases, where the initial steps always produced optimal solutions, and thus the lower bounds and the upper bounds were the same. These new tests are registered in the columns marked with an asterisk. It was observed that this procedure produced rapid convergence in the model and very good-quality solutions could be achieved, although we cannot claim to have obtained the optimal solution. It was possible to find a solution serving 79 offshore units using only four berths and the solution obtained in Step 4 has a gap of only 0.56% (the same as in the Step 1 solution). Therefore, we consider this alternative as a valid and useful solution strategy that could be used in practice. The complete model was tested for both cases to assess the effectiveness of the proposed methodology. In both cases, no feasible solution was obtained after 99,000 seconds.

5.1.4 Proposed Solution Strategy Performance

The comparison between complete mathematical model (CM) with the proposed solution strategy (PSS) considering the overall results from Step 1 to Step 4 is presented in Table 15. Based on the presented results, one may conclude that the proposed methodology is capable of significantly reducing the computational time and to provide an optimal solution for most of the cases considered. For case C10 both PSS and CM achieved the optimal solution and the PSS processing time is significantly lower than the CM processing time. For cases C15, C41 and C41S the PSS was able to achieve the optimal solution and the CM could not. As one might notice, the PPS spent a significant reduced amount of time to generate optimal solution. For the cases with 79 offshore units, in C79 a good solution has been found by the PPS but no solution has been found by the CM. For case C79S both PPS and the CM struggle to find a feasible solution. A large processing time is obtained for a few cases, particularity for cases C79 and C79S. For this kind of problem, a large processing time of thousands of seconds is not an issue, once it has to be solved around every quarter of a year or more.

An assessment to further investigate the effectiveness of breaking down the complete model into steps has been performed for cases C15, C41 and C79. The results are presented in Table 16 and the following denomination has been used in the columns: i) S1-3 is the consolidated result for steps 1 to 3; ii) S3’ is the result for the Step 3 model without the lower-bound constraint (4.21) and without using the Step 2 solution as an initial solution; iii) S3” is the result for Step 3 without the use of the Step 2 solution as an initial solution; iv) S3 is the result for Step 3 (previously calculated); v) S4” is the result of Step 4 without the use of the Step 3 solution as a constraint, i.e. without considering constraints (4.23) and (4.24); vi) S4 is the result for Step 4 (previously calculated).

If one compares S1-3 to S3’ it is possible to verify the effectiveness of breaking

Table 15 – Berth Allocation - Comparison between the proposed solution strategy and the complete model

C10	PSS	CM	C15	PSS	CM
OF	53.77	53.77	OF	80.37	89.43†
Gap	0	0	Gap	0	57.90%
Time	115	84,340	Time	6,812	21,289
C41	PSS	CM	C41S	PSS	CM
OF	256.59	289.23†	OF	261.99	305.00†
Gap	0	62.60%	Gap	0	60.46%
Time	487	99,000	Time	267	99,000
C79	PPS	CM	C79S	PPS	CM
OF	591.55†	-	OF	-	-
Gap	5.58%	-	Gap	-	-
Time	144,299	99,000	Time	-	-

† Objective function value not proven to be optimal.

Table 16 – Berth Allocation - Performance assessment of Step 3 and Step 4

C15	S1-3	S3'	S3"	S3	S4"	S4
Time	6,806	3,251	438	496	13,006	6
OF	80.37	80.37	80.37	80.37	80.37	80.37
GAP	0	0	0	0	0	0
C41	S1-3	S3'	S3"	S3	S4"	S4
Time	140	394	122	77	99,000	348
OF	256.59	256.59	256.59	256.59	294.95†	256.59
GAP	0	0	0	0	13.00%	0
C79	S1-3	S3'	S3"	S3	S4"	S4
Time	42,685	95,192	99,000	36,270	99,000	99,000
OF	558.55	558.55	558.55†	558.55	705.24†	591.55†
GAP	0	0	0.02%	0	20.80%	5.58%

† Objective function value not proven to be optimal.

down the Step 3 model in Step 1 and Step 2. For cases C41 and C79, the reduction on the processing time is clearly demonstrated. However, as one might notice in case C15 it is more efficient to run Step 3 without Step 1 and 2, and the use of the lower bound and of the initial solution has not been demonstrated as advantageous for Step 3 model. If one compares S3" to S3 it is possible to verify the effectiveness of using the Step 2 solution in Step 3, as for cases C41 and C79 the processing time has been significantly reduced. For case C15, the processing time remains almost the same. If one compares S4" to S4 it is possible to verify the effectiveness of using the routes obtained in Step 3 as constraints (4.23) and (4.24). For cases C15 and C41, there was a significant reduction in the processing time (by a thousand times less). For case C79, neither S4 nor S4" has closed the gap to zero; however a better solution could be provided by using the Step 3 solution as an input to Step 4.

This assessment demonstrates that the effectiveness of breaking down the model into steps (1 to 3) is not always the most efficient manner to solve the problem, although it was efficient for some cases. This assessment also demonstrates the effectiveness of using Step 3 routes as the initial solution to Step 4 for all evaluated instances. The overall evaluation of the method presented in Table 15 demonstrates the overall effectiveness of breaking down the model into steps for all instances.

5.2 S-PSVPP Computational results

All models were implemented in Python and solved by Gurobi. The results were obtained using a 2.2 GHz Intel(R) Core(TM) i5-5200U, with two processors and 8 GB RAM, and solved by Gurobi 8.1.1. The routes and the mathematical model input parameters were previously generated in a Python script (Spyder/Python 3.6). The processing time for the route generation varies from 60 to 3.600 seconds, depending on the instance. The route generation code has not been developed for optimal performance, and its processing time has been disregarded in the computational study. The maximum time limit of 1.200 seconds has been set for each run. For some specific cases, the time limit has been extended, aiming to achieve smaller gaps. Such cases are identified with a star (*).

Table 17 presents some key features regarding each instance, and the columns indicate the following data: OU - number of offshore units; CL - number of clusters; MAX - the maximum number of OU in a cluster and MIN - the minimum number of OU in a cluster; FV1, FV2, FV3: number of units with a frequency of visit 1, 2 and 3 days in a week respectively, and #R is the number of routes generated for each case.

Table 17 – S-PSVPP - Instances.

Case	OU	CL	MAX	MIN	FV1	FV2	FV3	#R
C10	10	1	10	10	5	5	0	1012
C15	15	1	15	15	7	7	1	22818
C41	41	5	11	6	28	8	5	2935
C41S	41	7	8	4	28	8	5	585
C79	79	9	12	6	54	17	8	6398
C79S	79	12	10	4	54	17	8	2298

5.2.1 Stochastic demand

The average standard deviation of the demand lies between 50% and 62% of the average demand per visit. The units' demands per visit have a large dispersion to the mean value, as the standard deviation is larger than the mean value in a few specific cases. The offshore units' demands probability distributions are assumed to follow normal distributions, with no negative demand being allowed for any customer; in such cases, the demand is set to zero.

5.2.2 Stochastic travel time

A route's leg travel time is assumed to follow a normal distribution and depends on the distance (DT). Based on real data of vessel's departure time and arrival time for voyages under regular operations, two probability distributions for the routes' leg travel times are derived: i) for the cases that the distance between two waypoints is lower than ten nautical miles (18 km) and ii) for the cases that the distance is greater than ten nautical miles. For distances inferior to ten nautical miles, the distribution (mean, standard deviation) is given by $0.006124DT \pm 0.002764DT$ (days per km), and $0.004904DT \pm 0.001143DT$ (days per km), otherwise. No negative travel time is allowed; in such cases, it is set to zero.

5.2.3 Robustness and confidence levels

Robustness levels and confidence levels must be defined when testing different route generation schemes. The values should be chosen with two primary purposes: i) to allow verifying the solution behavior according to the risk-averse level; ii) to allow comparing the solutions generated from different approaches for the same level of constraint violation. The robust approach is tested for three robustness levels, B1, B2, and B3, corresponding to an uncertainty budget (Γ) of 1, 2, and 3, respectively. There is no need to extend the uncertainty budget beyond 3. It may be noticed in the computational results that an uncertainty budget of 3 already provides very conservative results for the instances in this study.

Applying a confidence level to each individual stochastic parameter is tested for four confidence levels (CI): 84.134%, 93.319%, 97.725%, and 99.865%, and they are labeled as I1, I2, I3, and I4, respectively. The CI values correspond to the mean plus one, one and a half, two, and three standard deviations for a normal distribution. One may notice that I4 also corresponds to the robust approach with the uncertainty budget set to the maximum value possible. Applying a confidence level to the routes' statistics is tested for three confidence levels (RC) of 97.725%, 99.379%, and 99.865%, namely R1, R2, and R3. The RC values correspond to the mean plus two, two and a half, and three standard deviations for a normal distribution.

The new proposed methodology is tested for confidence levels of 97.725%, 99.379%, and 99.865% for the vessel capacity and the schedule probability, namely N1, N2, and N3. It has been defined six ranges of probabilities for the combination of vessels in use [0%; 0.3%],]0.3%; 0.5%],]0.5%; 0.7%],]0.7%; 1%],]1%; 5%], and]5%; 10%], namely F3, F5, F7, F10, F50 and F100 respectively. The combinations are limited to a maximum of six routes that can be simultaneously executed.

5.2.4 Monte Carlo simulation

A Monte Carlo simulation has been used in order to compare the solutions obtained for each case and approach. The simulations run in a Python script and comprise the following: 1. Sample generation of customers' demands per visit using a normal distribution with the same mean and standard deviation used in the mathematical model. 2. Sample generation of each leg's travel time for distances below and above ten nautical miles (18 km). 3. The simulation starts by reading the schedule (routes, vessels, and departure days) and uses the data samples to calculate how many days a vessel will be in use while executing a route and check if any route demand violates the vessel capacity. 4. The number of vessels simultaneously in use per day is recorded. 5. If there is a vessel capacity violation, the exceedance is recorded, and the vessel executes the route assuming the maximum vessel capacity. 6. No negative demand for a customer is allowed, and the demand is set to zero in such a case. Even if the demand is zero, the vessel has to visit the offshore unit.

The samples for the demands and the leg's travel time are the same within a case, so it is possible to compare the different approaches for each case studied. The simulation runs for 1000 evaluation periods or 7000 days.

5.2.5 Key performance indicators (KPI)

Assessing the performance of the different approaches is not a straightforward task. The main challenge is to compare solutions that use robustness or confidence level targets in different problem parameters. In order to structure the comparison among the different approaches, the following key performance indicators are considered: 1. Solution cost (objective function value). 2. Relative exceedance of the number of vessels (EV), which is the number of days that the required number of vessels has exceeded the maximum number of vessels in the fleet, divided by the number of days in the simulation. 3. Relative exceedance of vessel capacity (EC), defined as the number of occurrences of cargo exceedance in each route divided by the number of routes executed in the simulation. 4. Total undelivered demand due to the lack of vessel capacity (ED). 5. Fleet utilization rate (UR), defined as the average number of vessels in use divided by the total number of vessels in the fleet.

5.2.6 Results

The main results are presented in Tables 18 to 21. The first column and first row indicate the instance, and the subsequent lines in the first column indicate the approach used according to the definitions presented in 5.2.3. The approaches based on IC, RC, and N are grouped to the same level of KPIs to facilitate comparing the different approaches. The deterministic case using the mean value of each stochastic parameter is labeled as DT and included in the table for comparison. The robust approach is grouped apart, as the KPI correlation to the other approaches is not possible. In those tables, the columns represent the following: PR - robustness factor or confidence level applied according to the definitions given in 5.2.3; SOL - solution value; GAP - integrality gap of the solution; LB - lower bound; RT - runtime in seconds; NV - number of vessels for each class [n_1, n_2 and n_3] for PSV1500, PSV3000 and PSV4500 respectively; EV - relative exceedance of the number of vessels; NR - number of departures (routes) in one evaluation period; EC - relative exceedance of vessel capacity; ED - total undelivered demand due to the lack of vessel capacity (in m^2); UR - fleet utilization rate.

Table 18 – S-PSVPP - Results for instance C79.

C79	PR	SOL	GAP	LB	RT	NV	EV	NR	EC	ED	UR
N3*	99.865%	574.67	8.3%	526.90	3600	[1, 2, 12]	0.000%	32	0.041%	627	75.3%
R3	99.865%	607.96	0.7%	603.51	1200	[1, 3, 12]	0.057%	32	0.056%	627	71.2%
I3	97.725%	708.10	1.0%	701.17	1200	[0, 9, 10]	0.000%	37	0.054%	884	64.9%
N2	99.379%	530.09	6.9%	493.35	1200	[1, 3, 10]	0.029%	30	0.230%	3235	79.3%
R2	99.379%	533.36	0.0%	533.29	1200	[0, 5, 9]	0.029%	30	0.260%	3608	78.5%
I2	93.319%	592.63	0.0%	592.63	537	[1, 6, 9]	0.071%	33	0.324%	5459	71.6%
N1	97.725%	490.42	5.8%	461.86	1200	[1, 3, 9]	0.214%	28	0.711%	10400	82.1%
R1	97.725%	491.50	2.9%	477.36	1200	[1, 3, 9]	0.971%	29	0.838%	11985	83.7%
DT	50.000%	321.49	7.0%	299.99	1200	[5, 3, 2]	55.19%	28	29.59%	832320	95.6%
I1	84.134%	488.93	0.5%	486.66	1200	[3, 6, 5]	0.243%	33	2.924%	47605	82.6%
B1	1	482.10	1.0%	477.28	1200	[2, 2, 9]	0.586%	28	1.771%	28366	81.6%
B2	2	696.87	2.1%	681.90	1200	[0, 4, 14]	0.014%	37	0.005%	78	79.3%
B3	3	825.89	1.9%	809.83	1200	[2, 4, 16]	0.000%	42	0.021%	189	61.1%
I4	99.865%	944.91	0.0%	944.75	1200	[1, 6, 18]	0.000%	45	0.004%	31	55.8%

Table 19 – S-PSVPP - Results for instance C79S.

C79S	PR	SOL	GAP	LB	RT	NV	EV	NR	EC	ED	UR
N3	99.865%	584.16	5.0%	554.91	1200	[0, 3, 12]	0.000%	32	0.031%	388	76.8%
R3	99.865%	627.55	0.0%	627.55	1194	[0, 2, 14]	0.000%	32	0.025%	347	72.1%
I3	97.725%	733.40	1.4%	722.83	1200	[2, 7, 11]	0.029%	41	0.220%	3169	66.3%
N2	99.379%	534.71	6.9%	497.83	1200	[0, 5, 9]	0.086%	31	0.203%	3170	80.8%
R2	99.379%	565.20	3.9%	543.05	1200	[1, 4, 10]	0.214%	32	0.228%	3058	76.9%
I2	93.319%	614.20	1.9%	602.37	1200	[2, 7, 8]	0.100%	35	0.600%	9527	70.8%
N1	97.725%	509.19	6.2%	477.84	1200	[3, 2, 9]	0.486%	30	0.820%	12950	79.7%
R1	97.725%	498.11	1.3%	491.78	1200	[4, 2, 8]	0.657%	33	0.912%	16441	83.0%
DT	50.000%	307.81	1.6%	302.89	1200	[3, 3, 3]	60.37%	23	26.61%	693406	98.3%
I1	84.134%	494.33	0.1%	493.86	1200	[2, 8, 4]	1.486%	34	2.256%	38710	86.2%
B1	1	483.27	2.7%	470.03	1200	[2, 2, 9]	0.657%	28	1.779%	42001	82.1%
B2	2	677.65	0.0%	677.65	619	[0, 0, 17]	0.000%	33	0.027%	478	69.5%
B3	3	831.74	0.0%	831.66	579	[2, 3, 17]	0.000%	42	0.029%	391	62.2%
I4	99.865%	992.02	0.0%	991.73	1200	[2, 10, 15]	0.000%	52	0.008%	70	57.0%

Table 20 – S-PSVPP - Results for instance C41.

C41	PR	SOL	GAP	LB	RT	NV	EV	NR	EC	ED	UR
N3*	99.865%	274.15	8.5%	250.83	7200	[0, 1, 6]	0.000%	16	0.013%	132	76.5%
R3	99.865%	268.13	0.0%	268.13	585	[0, 2, 5]	0.029%	15	0.020%	54	73.1%
I3	97.725%	311.47	0.0%	311.47	265	[2, 4, 3]	0.014%	23	0.170%	1152	72.5%
N2	99.379%	249.31	10.0%	224.38	1200	[1, 3, 3]	0.100%	15	0.300%	1827	72.0%
R2	99.379%	248.04	0.2%	247.66	1200	[2, 1, 4]	0.286%	18	0.183%	1298	82.5%
I2	93.319%	271.67	0.0%	271.66	352	[2, 4, 2]	0.000%	19	0.368%	1913	71.5%
N1	97.725%	219.01	3.7%	210.94	1200	[0, 4, 2]	0.414%	14	0.929%	6671	82.5%
R1	97.725%	220.87	0.0%	220.87	366	[1, 1, 4]	0.971%	14	0.500%	4830	83.1%
DT	50.000%	143.69	0.2%	143.40	00	[4, 1, 0]	29.77%	17	31.07%	455283	98.9%
I1	84.134%	229.75	0.2%	229.20	1200	[3, 2, 2]	0.429%	19	2.968%	23510	84.1%
B1	1	213.93	0.0%	213.93	51	[2, 0, 4]	2.314%	16	0.475%	2350	89.7%
B2	2	294.57	0.0%	294.57	294	[1, 2, 5]	0.000%	18	0.022%	79	71.4%
B3	3	362.80	0.1%	362.39	1200	[1, 4, 50]	0.000%	22	0.009%	29	63.7%
I4	99.865%	418.35	0.0%	418.35	32	[0, 3, 8]	0.000%	20	0.000%	0	54.2%

Table 21 – S-PSVPP - Results for instance C41S.

C41S	PR	SOL	GAP	LB	RT	NV	EV	NR	EC	ED	UR
N3	99.865%	283.45	6.5%	264.95	1200	[2,2,4]	0.014%	20	0.040%	225	80.5%
R3	99.865%	295.46	0.9%	292.92	1200	[1,2,5]	0.000%	18	0.050%	250	72.1%
I3	97.725%	336.65	0.0%	336.65	45	[0,4,5]	0.000%	19	0.011%	59	64.8%
N2	99.379%	255.75	5.1%	242.70	1200	[1,2,4]	0.186%	16	0.200%	995	76.6%
R2	99.379%	255.94	0.0%	255.91	256	[1,2,4]	0.214%	16	0.119%	470	76.2%
I2	93.319%	288.22	1.2%	284.80	1200	[0,6,2]	0.014%	19	0.247%	1642	73.4%
N1	97.725%	239.91	2.2%	234.56	1200	[3,0,4]	0.186%	17	0.706%	4266	79.7%
R1	97.725%	235.14	0.0%	235.14	360	[3,1,2]	1.129%	18	1.056%	8834	81.0%
DT	50.000%	152.04	0.0%	152.04	915	[3,2,0]	26.87%	16	25.71%	422064	96.3%
I1	84.134%	237.73	0.0%	237.71	258	[2,3,2]	0.300%	17	1.412%	12667	78.9%
B1	1	221.30	0.0%	221.30	18	[1,1,4]	1.643%	13	1.400%	13090	80.3%
B2	2	319.35	0.0%	319.32	368	[0,0,8]	0.000%	17	0.006%	9	70.1%
B3	3	386.89	0.0%	386.89	51	[0,2,8]	0.000%	21	0.000%	0	62.6%
I4	99.865%	443.08	0.0%	443.06	3	[0,6,6]	0.000%	23	0.000%	0	54.9%

Based on the results of the KPIs for the evaluated instances, it is possible to notice that the robust approach (B) and the confidence level applied to each stochastic parameter (I) produced higher-cost solutions when compared to the confidence level of the routes (R) or the schedule reliability (N). This is more clear if one observes the cost of the solutions that ensure low relative exceedance for EV (number of vessels) and EC (vessel capacity), and also for solutions with low values of ED (total undelivered demand due to the lack of vessel capacity). This is an indication that applying a confidence level to individual parameters instead of observing the grouped statistics may not be the best approach as it leads to costly solutions. On the other hand, the solutions obtained by applying a confidence level to the route's statistics (R) and the schedule reliability (N) usually have lower costs and good coherence between the confidence level and the exceedance values of the KPIs. Regarding the comparison of the R and N approaches, one may notice that in most cases, the N approach produces solutions with lower or equivalent costs than the solutions obtained by the R approach.

In Fig. 19, it is possible to realize the benefits of the proposed methodology and the importance of considering the schedule reliability in the problem. The resulting schedule of the vessels of class PSV4500 of instance C79S is displayed, with routes as lines and periods as columns. Value one indicates that a vessel executes the route with a probability that it exceeds the probability ranges' upper limit. If the probability lies within the range of probabilities, it is indicated which range it fits into, e.g., F10 means the range between 0.7% and 1%. The last two lines indicate the number of needed vessels according to the R3 and the N3 approaches, respectively. In this solution, route #5 departs on day one, and the vessel has a chance of up to 0.7% of executing this route on day 4. The combined statistics for the schedule on day four for routes 5, 11, 12, 13, and 20 indicate that there is 76.624%, 97.776%, 99.902%, 99.998%, 99.999..% and 100% of chance that zero, one, two, three, four, five or fewer vessels will be in use. Therefore, only two vessels are required to meet the schedule reliability at a 99.865% of the acceptance limit instead of five vessels. In this case, there is a 99.902% of chance that two or fewer vessels will be required, which surpasses the imposed limit. If one compares the solution with R3, five vessels are required for these routes. In Table 22, this example is presented. The first row indicates the route number. PR is the probability range of the vessel executing the route on day four, and NVS is the number of vessels executing routes simultaneously on day four. NVSP is the probability that NVS or fewer vessels execute a route on day four, given the routes' probability range and using the upper value in the range to compute the joint probability.

Table 22 – S-PSVPP - Example for case C79s in day 4 using approach N3.

Route	5	11	12	13	20
PR	]0.5%; 0.7%]	]5%; 10%]	]1%; 5%]	]1%; 5%]	]1%; 5%]
NVS	0	1	2	3	4
NVSP	76.624%	97.776%	99.902%	99.998%	99.999...%

T	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	0	0	0	1	1	1	F50	0	0	0	1	1	1	F50
2	0	0	0	0	0	1	1	1	0	0	0	0	1	1
3	0	0	0	0	0	0	1	1	F3	0	0	0	0	1
4	0	0	1	1	1	F3	0	0	0	1	1	1	F3	0
5	1	1	1	F7	0	0	0	1	1	1	F7	0	0	0
6	0	0	1	1	1	F5	0	0	0	1	1	1	F5	0
7	0	1	1	1	F5	0	0	0	1	1	1	F5	0	0
8	0	0	0	0	0	1	1	1	F50	0	0	0	1	1
9	0	0	0	0	1	1	1	0	0	0	0	1	1	1
10	0	0	0	0	0	0	1	1	1	1	0	0	0	1
11	1	1	1	F100	0	0	0	1	1	1	F100	0	0	0
12	1	1	1	F50	0	0	0	1	1	1	F50	0	0	0
13	1	1	1	F50	0	0	0	1	1	1	F50	0	0	0
14	0	0	0	1	1	1	F50	0	0	0	1	1	1	F50
15	0	0	0	1	1	1	F100	0	0	0	1	1	1	F100
16	0	1	1	1	0	0	0	0	1	1	1	0	0	0
17	0	0	0	0	1	1	1	F5	0	0	0	1	1	1
18	0	0	0	0	1	1	1	F7	0	0	0	1	1	1
19	0	1	1	1	F50	0	0	0	1	1	1	F50	0	0
20	1	1	1	F50	0	0	0	1	1	1	F50	0	0	0
21	0	0	0	0	1	1	1	F50	0	0	0	1	1	1
22	0	0	0	1	1	1	F50	0	0	0	1	1	1	F50
23	0	0	0	0	0	0	1	1	1	F50	0	0	0	1
24	0	0	0	1	1	1	F10	0	0	0	1	1	1	F10
25	0	0	0	0	0	0	1	1	1	F10	0	0	0	1
R3	5	8	10	15	13	13	15	14	13	13	15	13	13	15
N3	5	8	10	12	12	12	12	12	12	12	12	12	12	12

Figure 19 – S-PSVPP - C79S, N3 solution for vessel class PSV4500.

One might argue that the improvement obtained by using the N approach is marginal or small if compared to the R approach for some of the cases. This could happen if the solution results in a schedule whose joint probability of vessels simultaneously in use does not reduce the number of vessels for a given confidence level. On top of that, special attention should be given to the probability combination discretization in the N approach. A higher discretization for the probability combination might lead to better solutions, while a poor discretization may limit the results' quality. The number of probability ranges and the maximum number of vessels simultaneously in use define the number of probability combinations present in the model. Ideally, the ranges of probabilities should be set to cover small intervals from 0 to 100%, and, at the same time, the maximum number of simultaneous routes should not be limited. However, the number of probability combinations would be very high, and the model would not be tractable. On the other hand, if only a limited number of broad ranges of probabilities is used, it may not be possible to capture the benefits of applying the joint probability of vessels simultaneously in use. For example, if the ranges of probabilities started in F50* ([0%;5%]), three vessels would be required instead of two on day four, for the case presented in Fig. 19. One possibility is to use the R approach results as a reference to devise the number of simultaneous routes to be considered in the combinations of probability ranges. On top of that, it is possible to try out variations in the probabilities range that would lead to better results using the N method given a desired confidence level.

The drawback of the N approach compared to the R approach is the computational time. As the N approach introduces new decision variables, combinations, and assignments, it also requires more computational time to close the gap. A good example is case C41, where N3 spent 7200s and still had a gap of 8.51%. To improve the convergence of the N method, one possibility is to use the solution obtained in the R approach as an initial solution to N. This procedure is valid provided that the route duration upper limit used to generate the routes in the N approach is equal or lower than the route duration confidence level used in R. For instance, case R3 has a confidence level of 99.865% for the route duration, and the same value is used in N1, N2, and N3 as the route duration upper limit to generate the routes. Therefore, the solution obtained in R3 is always a valid solution for the cases N1, N2, and N3. The improvement in the convergence for instances with a larger number of routes by applying this procedure may be seen in Table 23 for case C15. The brackets' values are obtained by introducing the R3 solution as an initial solution for N1, N2, and N3. A final remark is that all the methods can generate good solutions for small instances with a limited number of routes, as case C10, shown in Table 24.

Table 23 – S-PSVPP - Results for instance C15.

C15	SOL	GAP	LB	RT
N1	93.36 (70.09)	29.3% (3.9%)	66.04 (67.33)	1200
N2	93.91 (75.22)	25.4% (0.7%)	70.07 (74.69)	1200
N3	80.56 (80.34)	7.2% (6.9%)	74.78 (74.78)	1200
R1	69.94	0.2%	69.78	1200
R2	75.24	6.8%	70.14	1200
R3	80.34	6.8%	74.88	1200
I1	80.33	7.1%	74.65	1200
I2	93.76	0.2%	93.54	1200
I3	104.19	0.0%	104.19	122
B1	66.83	7.2%	62.02	1200
B2	75.25	4.2%	72.09	1200
B3	96.61	0.0%	96.61	337
I4	138.28	2.0%	135.50	1200

Table 24 – S-PSVPP - Results for instance C10.

C10	SOL	GAP	LB	RT
N1	40.05	0.0%	40.05	24
N2	40.05	0.0%	40.05	13
N3	40.05	0.0%	40.05	26
R1	40.05	0.0%	40.05	7
R2	40.05	0.0%	40.05	7
R3	40.05	0.0%	40.05	4
I1	40.05	0.0%	40.05	6
I2	53.93	0.0%	53.93	11
I3	61.20	0.0%	61.20	7
B1	34.82	0.0%	34.82	3
B2	40.05	0.0%	40.05	3
B3	60.82	0.0%	60.82	8
I4	74.24	0.0%	74.24	7

6 CONCLUDING REMARKS

The integration of berth allocation decisions to the PSVPP is a real complex and hard-to-solve problem found in some oil and gas industries, such as in the Brazilian case, where the supply operation is uninterrupted, both at the onshore base and the offshore units. In this Thesis the berth scheduling problem is considered together with the periodic routing problem. Also, in order to model the fleet size, a pool of vessels was considered, one for each class, and this modeling strategy seemed to be effective. A solution strategy scheme to solve real instances has been presented, as the complete mathematical model is hard to solve due to its combinatorial nature and many constraints. This strategy consists of sequentially solving relaxed versions of the problem, adding more complexity with each step. The proposed strategy consisted of forcing the routes obtained in Step 3 to be scheduled in the fourth step, which could not be feasible from the berth-scheduling perspective. To deal with this issue, the Step 4 model allowed the modification of the departing days, by selecting any valid departure pattern, by considering the use of vessels from different classes, and by allowing late departures up to given tolerance. Although this solution approach may hinder the achieving of the optimal solution, from the practical point of view, good-quality solutions were obtained. Instances were solved based on a real case from a Brazilian oil and gas company, including a case with up to 79 offshore units grouped into clusters.

The occurrence of non-deterministic demand and variations in the travel time affect both the fleet composition and the route schedule. As this operation is recurrent, the unfulfillment of the schedule, both in terms of left-behind cargo due to the lack of deck capacity and late departures, is highly undesirable and should be avoided to not cause a major disruption in the supply chain. On the other hand, vessels are very costly, and the smallest fleet is thus desired. In this research, elaborating a reliable schedule in the presence of stochastic parameters was addressed. Several approaches to deal with stochastic demand and stochastic travel time in the S-PSVPP were compared using KPIs derived from simulating the solutions obtained in each method: the robust approach (B), the approach of applying a confidence level to each stochastic parameter (I), the approach of applying a confidence level to the routes' statistics (R), and the new method devised for applying a confidence level to the schedule (N).

In general, the proposed approach that takes into account the statistics for the routes (R) and the statistics for the schedule (N) are the ones that produce more coherent and lower-cost solutions for the desired confidence level and the level of exceedance values in the KPIs. A new methodology to evaluate the reliability of the schedule (N) is introduced. It has generated better solutions, especially for instances with a large number of units and routes. The drawback of the new method is the increased computing time. One possibility for accelerating the convergence is to use the solution obtained in the R approach as an initial solution input to the N approach. Future work could investigate the impact of the probability's discretization on the overall efficiency. Another potential future work is to devise a methodology to deal with non-independent events and correlated probabilities among routes, e.g., bad weather or adverse environmental conditions.

Other future research topic might be the development of heuristic methods, and the integration of the berth allocation with stochastic demand and stochastic travel times.

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