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An Immediately Post-Walras Generation of Mathematical Economists

A Geração do Imediato Pós-Walras de Economistas Matemáticos

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Agradecimentos

De certa maneira, eu sempre achei peculiar o conceito de agradecimentos em um trabalho acadêmico. O leitor pega em mãos uma tese e encontra na capa um título que na maioria das vezes soa algo bastante "científico", abrindo então o trabalho e, encontrando, repentinamente, uma seção inicial no qual o autor lista as pessoas importantes de sua vida particular que foram importantes em sua trajetória. Talvez por esta estranheza eu nunca tenha escrito um agradecimento até o momento. Sim, você leitor acostumado com a cultura acadêmica, pode me julgar: eu não agradeci formalmente um ser vivo na terra em nenhum dos meus trabalhos até agora. Definitivamente não foi por acreditar que o trabalho de conclusão de curso de graduação e a dissertação de mestrado, por exemplo, haviam surgido mágica e unicamente pelo meu esforço próprio, sem ajuda de ninguém – como um Robinson Crusoé da vida acadêmica. Mas a estranheza com essa instituição social que são os agradecimentos e, obviamente, um certo desconcerto em encontrar as palavras certas, fizeram com que eu não registrasse em papel créditos devidos. Entretanto, a percepção insuperável de que uma tese de doutorado é produto de uma comunidade – não só acadêmica, mas com o suporte essencial de pessoas fora dela – fizeram desta seção algo bastante indispensável. E, muito provavelmente, minha última oportunidade de reconhecer formalmente as pessoas que, sem dúvidas, merecem um "muito obrigada" – assim mesmo, oficialmente em português.

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Resumo

Essa tese estuda a história da matematização da economia, em particular, a disseminação da teoria do equilíbrio geral walrasiana no início do século XX. Duas principais perspectivas são usadas. Analisamos primeiramente a disseminação por meio das obras de três autores que adotavam as ideias de Walras: Henry Ludwell Moore nos Estados Unidos, e Albert Aupetit e Étienne Antonelli na França. A segunda perspectiva é a da disseminação na sala de aula. Para isto, investigamos os livros utilizados para o ensino à época, que apresentavam o modelo walrasiano. Acreditando que a falta de interesse dos jovens pela teoria de Walras era devido à falta de conteúdo empírico, Henry Moore, professor da *Columbia University* desenvolve uma versão do modelo walrasiano que poderia ser testada empiricamente. Albert Aupetit, por sua vez, acreditava também no papel importante que a estatística poderia desempenhar na teoria; entretanto, sua principal preocupação fora com o aspecto monetário do modelo de Walras. Étienne Antonelli interessava-se pelas ideias de Walras em utilizar a matemática na economia, bem como suas ideias acerca do movimento cooperativo. Tanto Aupetit quanto Antonelli foram reprovados duas vezes no processo seletivo para tornarem-se professores na França, e dedicaram-se a outras atividades fora da academia: Aupetit se torna secretário geral do Banco da França e Antonelli um político francês, responsável pela lei que estabeleceu o sistema de seguridade social no país em 1928. O trabalho mostra, portanto, como a instituição do *concours d'agrégation* pode ter impossibilitado o ensino da teoria do equilíbrio geral walrasiano na França. O trabalho também mostra que provavelmente alguns dos principais seguidores de Walras no país no início do século XX estavam fora da academia, envolvidos na formulação de políticas econômicas. Nos três capítulos iniciais um tema é central: o *Abrégé* de Walras, a versão de sua teoria desenvolvida para ser utilizada em sala de aula, escrita para que Aupetit utilizasse-o na França – o que não ocorre com a reprovação de Aupetit no processo seletivo para se tornar professor, como demonstramos na tese. O trabalho também aponta como Walras tentou obter uma tradução do *Abrégé* por Moore. É Antonelli, entretanto, após a morte de Walras em 1910, quem primeiro publica o trabalho, com algumas adaptações, em 1914. Entretanto, a teoria do equilíbrio geral torna-se mais conhecida com a publicação dos livros de Cassel (1924) e Bowley (1924), que apresentavam o modelo sem atribuir autoria a Walras. Analisamos os livros de Antonelli (1914), Cassel (1924) e Bowley (1924) no quarto capítulo. Concluímos que, no capítulo quatro, acerca da disseminação da teoria do equilíbrio geral walrasiana, apesar dos três livros apresentarem o modelo de Walras, eles apresentavam simultaneamente perspectivas antagônicas sobre a definição da ciência econômica e seu escopo. Logo, a teoria foi ensinada em quadros teóricos bastante diferentes, e nenhum grupo único apropriou-se do modelo à época.

Palavras-chave: matematização da economia, teoria do equilíbrio geral, Léon Walras.

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Abstract

This thesis studies the history of mathematization of economics, in particular, the dissemination of the Walrasian general equilibrium theory at the beginning of the 20th century. Two main perspectives were used. I first analyzed this dissemination through the works of three authors who adopted Walras's ideas: Henry Ludwell Moore in the United States, and Albert Aupetit and Étienne Antonelli in France. The second perspective is the dissemination of Walrasian ideas in the classroom. For this, I investigated general treatises used for teaching at that time, which presented the Walrasian model. Believing that the lack of interest in Walrasian theory by young people was due to its lack of empirical content, Henry Moore, a professor at *Columbia University*, developed a version of the Walrasian model that could be empirically tested. Albert Aupetit, for his part, also believed in the important role that statistics could play in the theory; however, his main concern was with the monetary aspect of Walras's model. Étienne Antonelli was interested in Walras's ideas about using mathematics in economics and about the cooperative movement. Both Aupetit and Antonelli failed twice in the selection process to become professors in France, and devoted themselves to other activities outside the academy: Aupetit became secretary general of the Bank of France and Antonelli became a French politician, responsible for the law that established the social security system in France in 1928. The work shows, therefore, how the institution of the *concours d'agrégation* may have made it difficult for the Walrasian general equilibrium theory to be taught in France. The thesis also shows that probably some of Walras's main followers in France at the beginning of the 20th century were in policy-making, and outside the academy. In the three first chapters one theme is central: Walras's *Abrégé*, the version of his theory developed for the classroom, written for Aupetit to use in France – with no success, since Aupetit failed the *concours d'agrégation*. The thesis also shows how Walras tried to get a translation of the work by Moore, without success too. Finally, after Walras's death in 1910, Antonelli published the *Abrégé* in 1914, with some modifications. However, the general equilibrium theory became famous with the publication of two books: Cassel (1924) and Bowley (1924), which presented the model without attributing authorship to Walras. In the fourth chapter I analyzed three books – Cassel (1924), Bowley (1924) and Antonelli (1914). The chapter concluded that, regarding the dissemination of the Walrasian general equilibrium theory, although the three books presented Walras's framework, they also presented at the same time antagonistic fundamental beliefs about what was economic science and its scope. Therefore, Walras's model was taught in very different theoretical frameworks: no one dominant group appropriated it.

Keywords: mathematization of economics, general equilibrium theory, Léon Walras.

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Introduction

We know that the history of the mathematization of economics is certainly a complex chronicle. However, one of its undeniable features is that the Walrasian general equilibrium theory, at beginning of the twentieth century, was a very important development. The general equilibrium theory, as constructed by Léon Walras (1834-1910), was a resound "yes" to this question: is it possible to apply the same methodology used by mathematicians and physicists to the study of man as a social being and to his economic behavior? One of Walras's main concerns was to transform economics through the use of mathematics, accusing those who opposed this transformation of making the science looks less credible (Walker, 1970, p. 687). Besides being related to the transformation of economics using the natural sciences, Düppe and Weintraub (2014, p. xiii) also argued that the general equilibrium theory influenced the very process of demarcation of economics as an autonomous science, and Weintraub and Gayer (2001, p. 421) argued the theory was developed in more than one discipline and in at least more than two continents. This knotty chronicle of the history of the general equilibrium theory, more specifically, the dissemination of Walras's ideas regarding the mathematization of economics, is the main subject of this dissertation.

In his mission to transform economics using mathematics, Walras, as his letters reveal, sought the support from different groups in different countries, but he was ignored and rejected most of the time. Breton (1992) mentions that Walras's first relations with the French Liberal School were actually positive: in 1859 he started writing for the *Journal des Économistes* and become, in the following year, a member of the *Société d'Économie Politique de Paris*. However, a break happened the moment Walras expressed his desire to use mathematics to reconstruct economic theory (Breton, 1992, p. 34). Regarding the hostility of French economists to Walras's work, Gallois (2011) argued that three were the group's biggest objections to the work of the general equilibrium theorist. The first was related to economic training, given that a significant part of those who analyzed economic problems were trained on literary or legal studies. The second was that the theorists who had at least some knowledge of mathematics probably only knew arithmetic or elementary algebraic calculations – and were unaware of new techniques, and could not understand how mathematics could be used in economics. Therefore, by proposing the reconstruction of the field based on techniques such as the infinitesimal calculus, Walras made it very difficult his integration into the network of authors of that time (Gallois, 2011, p. 14). The third of the problems is also well known. French economists considered the variables that influenced man's behavior as innumerable, making the role of mathematics in economics very limited (Gallois, 2011, p. 14). Cot and Lallement (2006) reiterate that Walras's lack of recognition by his contemporaries was due much more to the resistance to his mathematical formulations than to his allegedly socialist ideas: *"La liberté humaine ne*

se laisse pas mettre en équations!" – used to say Walras's critics (Cot & Lallement, 2006, 379).

Even French mathematicians did not support his ideas. Turk (2012) and Ingrao and Israel (2015, 141-152) mention how Henri Poincaré and Émile Picard had strong reservations to the Walrasian project of using mathematics in economics. Weintraub (2002) cites the correspondence exchanged between Walras and Hermann Laurent, which ended with Walras suggesting that "Laurent was part of a plot against him" (Weintraub, 2002, 156). One of the many setbacks in his home country was the appointment of a successor to the chair of political economy of the *Collège de France*, in 1878, chair that Walras clearly desired. Michel Chevalier[1], endorsed the candidacy of his son-in-law, Paul Leroy-Beaulieu[2], leading an embittered Walras to assert, about Beaulieu, *"Ce n'est qu'un journaliste"*[3], also arguing that [he] *"has none of the qualities of a professor or man of science"* (Gallois, 2011).

Like the experience of Jesus in Nazareth, the relationship between Walras and France could exemplify the age-old verse that "there is no prophet without honor, except in his own country". However, Walras's attempts to raise disciples in other countries were also unsuccessful. Regarding England, Walras stated that "the English economists have taken a determined position not to make my theory known in England" (Walker, 1970, 699), arguing that their mentality was "an obstinate mixture of stupidity and ill will" (Walker, 1970, p. 699). Furthermore, since, according to Walras, Jevons had not given credit to him properly, he even accused the English author – and, by the way, also Wicksteed – of plagiarizing his works (Walker, 1970, p. 700). Schumpeter (1954, p. 796) reports that in England, Marshall's teaching excluded any influence of Walrasian ideas until Bowley introduced the Walras-Pareto system in a textbook, in 1924. Alcouffe (2013), studying the first reception of Walras's ideas at German universities, concludes that some of his reformist ideas were disseminated, but that it was illusory to expect more adherence in such a troubled period, given the methodological debates in the country. Regarding Walras's situation in Lausanne, Schumpeter stated:

> At the present time, when it would be hard to find a theorist who does not acknowledge Walras's influence, the statement will read strange that he formed no personal school. But the students of law who had the opportunity of listening to him at Lausanne were hardly accessible to his scientific message: his professorship brought him peace and security but very little influence. And his professional contemporaries were mostly indifferent or hostile (Schumpeter, 1954, p. 796)

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[1]A Saint-Simonian French economist, who would become an early member of the *Société d'Économie politique* and was elected *député*.

[2]French economist opposed to protectionist and collectivist ideas.

[3]"He's just a journalist".

Schumpeter also discusses the developments in Italy of Walrasian theory, stating that Barone and Pantaleoni were more receptive to Walras's ideas, and that it was probably through Pantaleoni that Walras met his successor, Pareto. In Italy then appeared a "School of Lausanne", but, according to Schumpeter, as a coherent school, this would be restricted only to that country, being, in reality, more "Paretian" than properly "Walrasian" (Schumpeter, 1954, 796).[4]

Despite the cold reception in academia at the time, mentioned briefly in this introduction and well explored by the literature, Walras managed to gain support from some theorists, such as, in addition to those already mentioned, Gustav Cassel, Irving Fisher, Henry Schultz, Albert Aupetit, Étienne Antonelli and Henry Ludwell Moore (Busino, 2010, p. 120). This dissertation investigates the relationship between Walras's ideas and these last three authors. The three theorists were almost the same age: Moore was born in 1869, followed by Aupetit in 1876 and Antonelli in 1879. Thus, I study a generation following Walras's. Studying each of the three authors allows us, in a big picture, to analyze the dissemination of some of Walrasian ideas – especially those related to the theory of general equilibrium, particularly in two countries: the United States and France.

Given the resistance hitherto experienced by Walras in Europe, his hope was renewed with the prospect of a market for his ideas in the New World with Henry Moore (1869–1958). The latter believed that the lack of interest in the general equilibrium theory came from the absence of empirical work to support it. The American then departed from the original Walrasian model and developed a version he believed to be subject to empirical analysis. Moore also tried to demonstrate that the formulation of the general moving equilibrium allows for the empirical test of the productivity theory of distribution. This attempt to make the Walrasian model statistically operative is explored in Chapter 1. From the analysis of some reviews of Moore's works, we also tried to evaluate how the academic community received his developments on Walrasian theory. The chapter has a twofold perspective: 1) the analysis of the personal relationship between Walras and Moore, mostly through their exchanged letters, in which we highlight Walras's attempts to spread his theory in the United States and; 2) theoretical inquiry, exploring the work developed by the American. We also provided some space to discuss the general climate at Columbia University – where Moore was a professor – regarding the application of statistics in the social sciences.

Walras considered Aupetit (1876–1943) his first disciple in France. In a letter to Gustave Maugin Walras wrote that "he is the best and most brilliant disciple and successor I may wish to have" (Jaffé, 1965, 352). Aupetit sought to reformulate the Walrasian general equilibrium model integrating the monetary feature, which Walras recognized as the most problematic aspect of his theory (Ostroy, 1987). But the young Frenchman,

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[4]Among the most recent literature, Pomini and Tusset (2009), for example, address in more detail the Paretian School in Italy and the attempts to make the system dynamic.

labelled as a Walras's disciple, as we will address, faced obstacles to be admitted as a professor at French universities. He then pursued a career at the Bank of France, where he held the position of secretary-general for over half a decade. The relationship between Aupetit and Walras's ideas is the subject of Chapter 2.

Declaring open affiliation with Walrasian theory, in a time of reticent support for the French master, Antonelli (1879–1971) is the first to implement a course in France on general equilibrium theory and mathematical economics (Diemer, 2006, p. 5). However, despite the interest in the mathematization of economics, it is Walras the "socialist economist" who first attracted Antonelli's attention. Influenced by Proudhon's theory, Antonelli interpreted the Walrasian theory as a research program suitable for analyzing the evolution of the economic and social systems. For him, therefore, it was possible to analyze the transformations of capitalism employing the theory of general equilibrium. Besides his academic activities, Antonelli began early on a political career, contributing to the consolidation of modern social legislation in France by promoting the French law that established a compulsory social insurance. Moreover, he also dedicated himself to the popularization of economic knowledge. We shall study in chapter 3 how Walrasian ideas influenced him in these two activities: as a policy-maker and as a knowledge-broker.

Besides these activities, Antonelli also wrote some textbooks, which also allow us to investigate, along with the study of some other selected manuals, how Walrasian theory was taught in the early years after Walras's death (1910). The textbook that Antonelli wrote, presenting the general equilibrium theory is studied in Chapter 4, along with the analysis of two other important books that presented Walras's system: the *Mathematical Groundwork of Economics: An Introductory Treatise* (1924), by Arthur Bowley; and *The Theory of Social Economy* (1924), by Gustav Cassel. I used in the chapter content analysis techniques and quantitative text summarization, and I commented on the rhetorical elements employed by the authors. Therefore, in Chapter 4, I studied not only the ideas of the generation immediately following Walras's, but also how this generation taught general equilibrium theory to students.

1 General Equilibrium Theory and the search of its empirical endorsement: Henry Ludwell Moore

"I hope that you flourish in Probabilities"

– Letter from Francis Ysidro Edgeworth to Karl Pearson, 11 September 1893.

"The economist is like a scion of a noble race who is proud of his honorable descent and not a little ashamed of his own scant achievements"

– Henry Ludwell Moore, *Synthetic Economics*, 1929.

Abstract

The article analyzes an attempt to make the Walrasian model statistically operative: the approach developed by the American Henry Ludwell Moore in the early twentieth century. Thus, the paper analyzes the dissemination of Walrasian theory in the immediately post-Walras generation. From the analysis of some reviews, this study also aims to evaluate how the academic community received Moore's work on walrasian theory. The paper has a twofold goal: 1) the analysis of the personal relationship between Walras and Moore, mostly through their exchanged letters, in which we highlight Walras's attempts to spread his theory in the New World and; 2) theoretical inquiry, exploring properly the work developed by the American. We conclude that the main analytical tool used by Moore was the treatment of data using a secular trend. As this trend is empirically derived, according to Moore, there is a transition from a purely rational construction to a real and dynamic situation. Hence, in the author's works, this transition from a static analysis to a dynamic analysis was intrinsically related to the empirical basis of the theory. Moore also tried to demonstrate that the formulation of the general moving equilibrium allow him to test the productivity theory of distribution. We will also provide some space to discuss the general climate at Columbia University – where Moore was a professor – regarding the application of statistics in the social sciences. Additionally we will make particular considerations about the parallel development of pure statistical theory. Finally, we will draft some related comments about the original Walrasian theory.

1.1 Introduction

With the process of axiomatization of the general equilibrium theory, that culminated in the publication of Arrow, Debreu and McKenzie's seminal articles in 1954, researchers distanced themselves from the attempts of empirical verification of the system. This article aims to analyze a work in which the Walrasian equilibrium model was adapted in a form that it could be empirically tested. This work is the theoretical construction of the American Henry Moore, developed in the early twentieth century. Thus, the paper analyzes the dissemination of Walrasian theory in the immediately post-Walras generation. From the analysis of some reviews, this study also aims to evaluate how the academic community received Moore's work on walrasian theory. Looking at the reception of the

endeavor, from reading the reviews, it is possible to analyze the limitations of the work not based on the degree of development of contemporary economic and statistical theory, but from the state-of-the art of the respective theories at the time. Further, we also aim to delineate the relationship between this specific theoretical development with the rest of Moore's work. The paper has a twofold goal: 1) The analysis of the personal relationship between Walras and Moore, mostly through their exchanged letters, in which we highlight Walras's attempts to spread his theory in the New World and; 2) theoretical inquiry, exploring properly the work developed by the American author.

Regardless of controversies about the paternity of disciplines, there are numerous references in the literature addressing the importance of Moore at the beginning of statistical economics.[5] G. J. Stigler (1962, p. 1) argues that no other author was so influential in developing the subject.[6] Epstein (2014, p. 13), writing a history of econometrics, chose Moore as the first author to be approached, and pointed out that "modern econometrics really began with an analysis of the labor market by the American Henry L. Moore". Le Gall (1999) asserted that Moore pioneered at least two questions: the application of spectral methods in economics and introducing celestial bodies as a source of economic fluctuations.[7] Boumans and Dupont-Kieffer (2011, p. 26) also indicated that in the literature Moore appears as the "quintessential pioneer" of early-twentieth-century econometrics. One last example is G. J. Stigler (1954). Discussing the history of empirical studies of consumer behavior, the article described that "Moore was the single most influential economist in the popularization of statistical demand analysis" (G. J. Stigler, 1954, p. 112).

Despite of references in the literature regarding his importance, not all mentions about Moore were positive. Spanos (2006, p. 16), for example, asserted that "some of the crucial weaknesses of the current textbook approach can be traced back to Moore". Wulwick (1992, p. 182) also points out that he is often portrayed as a "bungler", while Le Gall (1999), that he is pictured as a "raving madman". Epstein (2014) mentioned that, after some criticism, Moore started to avoid as much as possible professional meetings. Also, in an interview, Samuelson, speaking about Moore, told that "H.L. Moore was a strange man, who had some psychiatric problems later in his life" (Freedman, 2010, p. 166). Ambiguously, when referring to Moore, Douglas (1939, p. 104) pointed he out as "that lovable and nervous genius".

Apart from these selected examples from the literature, the importance of Moore's work might still be evidenced by using some citation data. Quandt (1976), making a

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[5]Mirowski (1990) discusses the question of the genealogy of econometrics and Henry Moore, pointing out the artificiality of the title of founder of a given branch of knowledge.

[6]However, Stigler points out this influence is not for priority or excellence, since in both cases Moore was not superior to the French Marcel Lenoir (G. J. Stigler, 1954, p. 112)

[7]Some works that address the theory of economic oscillations developed by Moore are Morgan et al. (1990), Klein and Klein (1997) e Raybaut (1991).

quantitative study of the academic literature on economics, showed that during the 1930s, Moore is the third most cited economist in a sample of eight selected journals.[8] Moore, in number of citations received, is only behind Marshall and Keynes, and ahead of authors such as Pigou, Hansen, Robbins, and Schumpeter (Quandt, 1976, p. 754).[9]

In addition to the references that the literature makes about Moore, there are also the references associating the American to the Walrasian theory – either evidencing the intellectual approach or the distance between the two. Schumpeter is one author relating the American professor to the Walrasian theory. In his 1954 book, he told that "In the United States, Walras acquired two-first rank followers, Fisher and Moore, but was practically ignored by the rest of the profession" (Schumpeter, 1954, p. 796). Another work to point out the relationship between Moore and Walrasian theory is Jolink and Knot (1993). The authors asserted that despite the hostility suffered by Walras by economists, engineers and mathematicians, "the Walrasian dream of a mathematical economics was revived at the turn of the century by Henry L. Moore" (Jolink & Knot, 1993, p. 166). Ingrao and Israel (2015) also claim that Moore was Walras's first and most convinced follower among American economists, however "Moore's respectful but decided *breakaway* from the original Walrasian programme in favour of an inductive approach is a further sign of the isolation imposed by a quirk of fate on the author of the *Éléments*" (Ingrao & Israel, 2015, p. 107). Another author who suggests a distance of Moore from the theory of general equilibrium is Epstein (2014, p. 14), who stated that "Moore increasingly came to view himself as a kind of rebel in economic theory. His admiration for the neoclassical analysis of the labor market did not extend to the full model of general equilibrium". Therefore, the present work will also allow us to analyze with more detail the relationship between one of the first researchers in statistical economics and Walras and walrasian theory.

This paper is divided into three sections, apart from this introduction. The following part will explore the personal relationship between Moore and Walras, highlighting attempts to disseminate Walrasian theory in the United States and in the English language. The second section will deal with Moore's Walrasian theory, which, according to the American, could be empirically verified, as already indicated. This section will also provide space to discuss the general climate at Columbia University – where Moore was a professor – regarding the application of statistics in the social sciences. We will also make particular considerations about the parallel development of pure statistical theory.

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[8]American Economic Review, Journal of Political Economy, Quarterly Journal of Economics, Economic Journal, Economica, Econometrica, Southern Economic Journal and the Review of Economics and Statistics.

[9]We also did a brief quantitative analysis using data from Google Scholar, the most complete bibliometric basis for the period. From a search for "statistical economics", until 1930, ranking the data by the index provided by Google, the first two most important works given by the search are 1) Moore's 1911 book, Law of Wages and; 2) Moore's 1908 paper, Statistical Complement of Pure Economics. More information about the Google's rank calculation is given in Harzing (2010, p. 22).

Finally, as pointed out, Moore's work will be further presented and evaluated taking into account the reviews available at the time. The last section presents the concluding remarks of the paper.

1.2 The personal relationship between Henry Moore and Léon Walras

It dates from mid-1898 the first mention made to Moore in Walras correspondences. An unidentified employee of Briquet et Fils, a Geneva-based bookstore, informed the Frenchman that there was a young foreigner, probably one of his disciples, who was strongly insisting on obtaining a photograph of the Professor. Unsure if Walras would allow it, the representative then wrote to him, asking if they could provide the photo and, if so, which one could be given. Without getting an answer from Walras – he never answered the bookstore – the employee repeated the question again a few days later in another correspondence, adding that he knew that the young man was an American and his name was H.L. Moore. Moore also left his address with the seller, in case the bookstore could mail the request to the United States (Jaffé, 1965, p. 24-5).

Five years later, in 1903, discovering through a publication of the American Economic Association that Alvin Saunders Johnson, an economist and later professor at Chicago and Stanford, wrote a paper on income distribution, Walras sent to him the *Eléments d'économie politique pure*, with some additional comments on Saunders' publication. Johnson replied that he would admit advantages in the mathematical approach to economic theory. The American also wrote that Walras would likely be interested to know that a Columbia member, Professor H.L. Moore "a devoted disciple of Cournot" gave a course of mathematical economics in the previous semester, and that this course aroused much interest among PhD students (Jaffé, 1965, p. 229).

Coincidentally, in the same month of this letter from Johnson, Moore, who was in Geneva, wrote to Walras and asked if he could meet him personally. The American at the time was writing in the history of economic thought and then went to Europe to retrieve a Cournot's manuscript.[10] His interest in Walras was twofold: he was the first to pay due respect to the French economist and one of the few who developed his method. He concluded his letter, however, pointing out that "aside from the Cournot interest, I should steem it a privilege to know you" (Jaffé, 1965, p. 230).

The two professors then met a few days later at Clarens, Walras's residence. Walras recommended that, apropos of the publication of Cournot's manuscript, Moore could talk to Charles Gide or with Gabriel Tarde (a task that Moore later reported that he had no opportunity to do).[11] Moore also insisted that Walras would consider elaborating an

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[10]Moore, in his doctoral dissertation, for example, analyzed von Thünen's ideas about natural wages.

[11]In a correspondence with Gide, Walras reported that he met Moore and described him as a *"homme*

autobiography (Jaffé, 1965, p. 233).

Few months later, Walras mentioned to Moore that a recent event had made him actually consider writing some autobiographical notes, and that he would let him know when he was done.[12] The Frenchman also reported that he read the Papers and Proceedings series of the 16th annual meeting of the American Economic Association and that Fetter's work, "The Relations between Rent and Interest", interested him, but it was quite painful to follow an article in "ordinaire" language, especially when the problem addressed would be much simpler, clearer and easier to solve through a mathematical approach. For him, the reading experience could be compared to that of a hypothetical mathematical researcher forced to listen a discussion among *littérateurs* about the motion of the planets. At the time, he alluded to the fact that it took fifty years before Laplace could be understood by his community. Therefore Walras wrote that he resigned himself to belong to the group that sowed without being certain that he would reap the rewards of the work of the harvest (Jaffé, 1965, p. 247).

Moore replied, days later, that he did not agreed that Walras was one of those who "sowed without seeing the harvest": "I wish you could have seen the enthusiasm with which a small class of my students at Columbia studied your "theory of exchange", during the past winter" (Jaffé, 1965, p. 249). Moore reported that there were not really many willing to undergo such hard work as the quantitative approach, but that in the next ten years, those students would be the leaders of American research in Economics. The American also said that he would start a series of courses in Mathematical Statistics and Mathematical Economics, and that he also intended to extend this last course to a duration of three semesters, dealing, in the first semester, with elementary concepts of statistics and *"pure economics"*, in the second one, with applications of probability theory in statistics, and in the third one, covering topics in advanced Mathematical Economics (Jaffé, 1965, p. 249 and 250).

In December 1904 Walras informed Moore that he had completed the autobiographical work, but that the American should publish it only after his death. They also exchanged correspondences about an article that Moore would write, by Walras's suggestion, about Cournot, in a special edition of *the Revue de métaphysique et de morale* (Jaffé, 1965, p. 256, 260, and 275).[13]

Knowing that Walras had completed the autobiographical draft, Moore stressed that one of the strongest means to serve the scientific cause would be a series of biographical studies, showing that the best economists were those who believed, "heart and soul", in

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parfait, aimable et don j'ai été enchanté" (Jaffé, 1965, p. 232).

[12]Walras's French disciples, Albert Aupetit – our subject of study in the next chapter – failed the test to be a Professor at the Law School, allegedly for his defense of the mathematization of Economics.

[13]When Moore mentioned this work in a later correspondence, he pointed out that "I am glad if you find any good in the article on Cournot. The editor Mr. X. Leon published the article without waiting for the corrected proof to arrive from New York" (Jaffé, 1965, p. 276).

the supreme value of abstract theory. The American professor further marked out that he was truly grateful that Walras entrusted him with publishing his work after his death, and that he would honor this desire (Jaffé, 1965, p. 278).

In a later correspondence, replying to the question of the biographical works, Walras pointed out to Moore that five economists were the founders of pure economics – for which, therefore, there was a justification for such a work as a biographical one – his father, A. Walras, Cournot, Gossen, Jevons and himself. The works about his father, Gossen and Walras, then, were missing. Making a first comment about the translation of his own work, Walras stated that such translation would be a task too big to be accomplished at that moment, but a good start would be an English version of the teaching material he had prepared for his French disciple, Albert Aupetit.[14] According to Walras, he prepared this teaching material for high schools, so translating it would be a very productive way to begin the propagation of his work in the United States (Jaffé, 1965, p. 279). Walras would send the work to Moore when the American signalled that he could begin the translation. It is worth noting an additional episode. Writing about such correspondences to Gustave Maugin, Walras pointed out that he agreed with Moore on the publication of his work in English since Columbia University paid the translation of his *Abrégé* and published the work in the United States (Jaffé, 1965, p. 282).

Moore, in a letter of November 1905, pointed out that he would like to see the outline of the *cours élémentaire d'économie politique pure*, since he likewise adopted in his courses in Columbia the original version of the *Éléments* (along with additional materials on economic principles), and a work done by Walras especially devised for teaching would thus be very helpful. However, according to him, about the translation, the French underestimated the difficulty inherent in the content of his work:

> As to whether it would be wise to translate and publish it, I could, of course offer an opinion only after going through the work. You, I am sure, underestimate the difficulty of your method and theories. On the other hand several attempts have recently been made to present the elements of the mathematical method. In English there are Wicksteed's "Alphabet of Economics", and Cunynghame's "The Geometrical Political Economy", 1904. In French, Laurent's *"Petit Trait d'Economie politique mathématique"*, 1902, which opens with the following: *"Dans ce petit traité, l'économie politique sera exposée of a make-up to the nouvelle and accordingly to the preconceptions to Lausanne by M. Walras, puis by M. Pareto"*. In Italian, there is Vergilii [*sic*!] Garibaldi's "Introduzione alla economia matematica", 1899 (Jaffé, 1965, p. 281).

On the difficulty of the course, Walras responded to Moore that he had given the program to law students and had used only the knowledge of algebra and two-dimensional

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[14]It is important to emphasize this point: so far, there is no written record of any conversation between Moore and Walras about the possibility of the first translating Walras's books into English. They may have talked in person, but this is the first time the subject appeared in the correspondences.

analytic geometry. Walras concluded that in special schools of trade and industry the enterprise would be consequently even less problematic (Jaffé, 1965, p. 284).

Despite the content of the previous letter, Walras anyway sent his *Abrégé* to Moore. The American replied the professor's letter suggested a hope that he would translate the work, and continued: "but if you recall my letter, you will notice that I was quite careful to say that I was sure you underestimated the difficulties of your work, and that I doubted the wisdom of a translation" (Jaffé, 1965, p. 295). Moore concluded that by reading the material, he confirmed his doubts. Apart from the difficulty, he explained that for reasons of health and the workload of the University, he could not make the translation (Jaffé, 1965, p. 295).

Walras replied in early 1906 he had expressed himself badly about hoping for a translation from Moore: what he intended to say was that without the translation, the publication of his biography made no sense.[15] Walras surprisingly ended the letter positively, citing that a magazine, the *Revue du Mois*, would soon publish an article called *La méthode mathématique et les sciences sociales*. He concluded that was approaching the time when the public would be further attentive in the ideas endorsed by them (Jaffé, 1965, p. 298).

Apart from the correspondence exchanged between the two professors, there are also letters in which Walras cited Moore with other correspondents. For example, in a letter addressed to Poincare, Walras reported that Moore told him he met Poincare, and the latter agreed, at first, to the use of the mathematical method in political economy. This understanding had satisfied Walras (Jaffé, 1965, p. 315). Further, there are several correspondences in which Walras cited Moore as his American disciple. In a particular letter addressed to Gustave Maugin, he pointed out that Moore was very diligent and kind, but complain about the slowness with which the American was performing one assigned task – the translation of his work (Jaffé, 1965, p. 325).

After more than a year since Moore's refusal to translate Walras's work, the latter sent comments on the young man's recent book, "The Differential Law of Wages". Walras began by pointing out that he was truly interested in any empirical work on supply and demand curves, as they were the "*base of notre économique*". Walras also said that probably a Sorbonne professor, Emile Borel, would contact Moore about his father's biography, which the *Revue du Mois* would publish in an article (Jaffé, 1965, p. 351). In addition, he wrote to Moore about a work that two of his disciples, Aupetit and Barriol, would publish, in an encyclopedia. They – all the defenders of mathematical economics — were, according to Walras, officially researchers of *Mathématiques Appliquées*. Walras highlighted that the academic community would probably discriminate Moore, as one defender of the *Mathématiques Appliquées*. Nevertheless, the young man's adherence to

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[15]In the original: *"Si j'ai exprimé l'espoir* (hope) *d'une traduction de mes Éléments ou de mon Abrégé, soit par vous, soit par quelque autre, l'expression a dépassé mes intentions. J'ai voulu seulement exprimer ma conviction que, sans une telle traduction, la publication de ma biographie n'a pas de raison d'être"*(Jaffé, 1965, p. 298).

his work was a delight to Walras (Jaffé, 1965, p. 351).

Given the failed attempt to get a translation from Moore, in March 1908, Walras wrote to Henry Walcott Farnam of Yale University, reporting that his American disciple, for a variety of reasons, could not do the task, and Walras recognized someone else would have to make the translation. According to the Frenchman, he expected at least Moore's help to find someone who could do the job, but since the latter also could not help him in this task, he thought it best to take it for himself, the reason he was writing to Farnam (Jaffé, 1965, p. 354). The American answered him just two days later (Jaffé, 1965, p. 355), saying that in response to Walras's request, he wrote to Fisher to find out if the latter had any information about the whereabouts of the manuscript that the Frenchman gave to Moore – which in reality was not exactly what Walras asked him.

Walras wrote to Moore about the episode, as he "did not know what Fisher could do other than write to the youngman", and add that he also did not want any misunderstanding between them – everything he wished was to add some contributors to the difficult task of getting his *Abrégé* translated (Jaffé, 1965, p. 355). Moore said that no disciple would like to see more the professor's work in English than him, but that, given his present conditions, he would hand over the copy to whom Walras trusted. The American also commented that he was eager to read the book *Économique et Mécanique* – which Walras alluded to in the previous letter. According to Moore, Jevons was the first one to observe the parallelism between the two sciences, economics and mechanics, but no one, to the extent he knew it, apart from Walras, had in fact showed such similarity (Jaffé, 1965, p. 356 e 357).

Still in the same letter, Moore asked Walras's opinion on the attempts to empirically test some conclusions of pure economic theory. He said that one reason the science did not attract young students, was the absence of inductive demonstrations, adding that "I have, therefore, assumed that the present generation of scholars could render most effective service by attacking inductively the problems which you and others have treated so brilliantly on a deductive manner" (Jaffé, 1965, p. 357). That was the reason, according to him, why he wrote the "Differential Law of Wages" (Jaffé, 1965, p. 357). There is no record of Walras's answer to Moore's question.[16] A few days later Moore then sent the *Abregé* manuscript back to Walras (Jaffé, 1965, p. 361). Still in the same month of this letter, July 1908, Moore paid a visit to Walras, and in his message of acknowledge for the professor's attention at the time, pointed out that "I also hope that you now understand very clearly why I returned your volume" (Jaffé, 1965, p. 365).

On Walras's seventy-year anniversary, the University of Lausanne organized a tribute ceremony for the professor, who invited Moore to the event. The American, then, in

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[16]Mirowski (1990, p. 595) points out that Moore had "never received an answer, perhaps because Walras was never that enamored of statistics" or "perhaps because the self-pity of those convinced of their unjustly neglected genius rarely has room for sympathy for another, different species of neglected genius".

the reply, recalled that the first time he visited Walras, they talked about how maybe it would take approximately 50 years for a revolution happen in Economics, but "it is scarcely fifty months since that first visit, and now you writings are about to receive a unique recognition of merit on the part of you Lausanne colleagues!" (Jaffé, 1965, p. 369).

Less than a year before his death in March 1909, Walras sent his father's biography, published in the *Revue du Mois*, to the American, and pointed out that he could publish the translation if it interested him. Walras further stated that it surprised him when he discovered that someone published his autobiography in Italian, albeit briefly, in the announcement of his jubilee. However, when he "thought better", he recalled that a few years earlier he had given Pantaleoni a previous version of the story of his life[17].

Finally, in May 1909, Walras received a letter from Farnam talking about the translation of his work into English. The American had met with Seligman and discussed the possibility of the publication of the work in the Columbia University Studies, but he said it would be impossible to publish in the series, given the high cost and the expected low return. He then sent again the material in hand to Walras'a residence, with an additional apology for not being able to collaborate with the task (Jaffé, 1965, p. 416). The English translation, in fact, would only be done in 1954 by William Jaffé. However, the dissemination of Walrasian theory in the United States would begin earlier: Moore would develop his own approach to the theory, from the main equations of the Walrasian model, as we will see in the next section.

1.3 Moore and the development of his empirical approach to the general equilibrium theory

The first child of fifteen brothers, Moore was born in Maryland, United States, in 1869 and received his PhD from Johns Hopkins University at the age of 27 in 1896. However, even though he obtained his degree in the United States, as pointed out by Mirowski (1990, p. 589), "Moore was a member of that generation of fledging American scholars who travelled to Europe to round out their education, aspiring to a level of sophistication which was absent in the American academic scene of that period".[18] Before obtaining

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[17]Shortly before his death, Walras faced some noticeable health difficulties. Schumpeter, for example, told Jaffe that on his visit to the Frenchman, the professor had praised the book that the young man sent him, but had so far believed it was in fact the work of Schumpeter's father. The latter corrects the misunderstanding, but on saying goodbye, Walras congratulates the author's father again for the "excellent book". (Jaffé, 1965, p. 385).

[18]Still on Moore's education, as Christ (1985, p.42) pointed out, at the time of Moore as a student at John Hopkins University, there were merely two courses listed as "Economics" that had any mathematical content: "10 lectures on Economics as an Exact Science" by Simon Newcomb and "25 lectures on Statistics" by Elgin Gould. The course records of Moore are filed in the University of Chicago Archives. This document of Johns Hopkins shows that his formal education was fairly substantial in the historical approach: the American attended, for example, courses like Historical Seminary, Germanic History, Church History, English Constitutional Law and History, Ethnological History of the Indo-European Peoples, Methods of Historical Research, Prussian History, Elements and History Political Economy, Economic

his degree, Moore attended lectures at the University of Vienna, and participated in Karl Pearson's courses on mathematical statistics at the University of London (Mirowski, 1990, p. 589). As noted before, in his early career, Moore devoted himself to the history of economic theory, although he taught, for example, in 1896, a course named "mathematical economics" at Johns Hopkins.

At the same time, Columbia University's engagement with the statistical approach had been noticeable since about the 1880s, when Richmond Mayo-Smith, the first economist of Columbia's School of Political Science, incorporated Statistics in the field of Political Economy (Camic & Xie, 1994, p. 794). When Mayo-Smith died in 1901, Columbia University, unable to find a suitable senior replacement, hired two assistant professors to replace Mayo-Smith: Henry Seager and Henry Moore.[19]

One point, therefore, is worth underlining: until entering Columbia, Moore had not yet produced any work involving the statistical method. Bernert (1983, p. 238) specified that "Columbia University served as a portal for the English statistics". Apart from Moore in Economics, three were other researchers at the University in different fields using statistics, with a dual purpose. These were Franklin H. Giddings in sociology, James McKeen Cattell in psychology and Franz Boas in anthropology (Camic & Xie, 1994, 773). The dual purpose was 1) to present their conformance with acceptable scientific model data and; 2) to establish a special form of analysis to differentiate their disciplines from separate areas (Camic & Xie, 1994, p. 773).[20] Camic and Xie (1994), therefore, pointed out that Columbia, determined to preserve a given institutional advantage, provided a supportive environment for the multidisciplinary process of incorporating the statistical methods into the social sciences.

After some activities on the history of ideas – analyzing, for example, von Thünen's and Cournot's ideas – Moore published his first work directed at statistically testing a statement of pure economic theory in 1907. His first goal was analyzing the differential law of wages. The differential law of wages determines the distribution among different workers of the products of the labor. According to Moore statistics and pure economic theory, were so distant at the time that researchers had to establish a series of hypotheses so they could use statistical data and statistical methods to make pure economics theories effective. Given the failure, to empirically analyze the theory of differential wages, accordingly to

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and Social History of Europe, History of the Nineteenth Century and History of Economic Theories.

[19]Ginzberg (1990) marked out that in the late 1920s, there was another big change in the Economics Department of Columbia. In 1928 Moore retired and disappeared from the academic scene, R.A. Seligman, the department chairman, had a mild heart attack and Henry Seager passed away, the same summer, on a visit to the Eastern Europe. The three were replaced by Harold Hotelling of Stanford, Carter Goodrich of the University of Michigan, and Leon Wolman of the National Bureau of Economic Research (Ginzberg, 1990, p. 14).

[20]The authors pointed out that this dual task, showing conformity with established scientific methods and, at the same time, differentiating their emerging disciplines from others, can be characterized as a "dilemma" whose one solution found was the use of statistics. At least for some contemporaries, statistical tools were both demonstrably scientific and capable of diversification.

Moore, this later should be "regarded as without significance so far as scientific uses are concerned" (Moore, 1907a, p. 639).

This issue of hypotheses is a query that Moore considered notably relevant: in the absence of accurate data, researchers should take special care to preserve "a befitting sobriety in the use of hypotheses". The assumptions he employed for the test of the theory were 1) premises about the distribution of workers' efficiency and; 2) allowance for particular time and place. Moore assumed that the distribution of workers' sagacity and energy followed a Gaussian law (Moore, 1907a, 642). In his work in 1908, he re-asserts a related point: it is necessary to give priority to formulations with higher affinity with the normal curve: "in cases where the normal law is evidenced both by a priori reasoning and observed fitness, there can be no hesitation about preferring that law" (Moore, 1908, p. 6)[21]. He quotes Pearson and Lee (1903) to support his idea that the normal can describe, within the limits of random sampling, the distribution of men's main physical characteristics. This assumption that general sagacity follows a normal law appears in several of Moore's works on wages, such as one of his major books, the Law of Wages, in 1911 – although he also analyzed cases whose distribution was asymmetric. We might still stress one point about hypothesis: at the beginning of his attempts at empirical verification of theories, Moore believed it was very important to consider differences in space and time analyzed – thus being maybe problematic to consider a theory like the general equilibrium.

However, the query of the universality of theoretical laws presents itself intricately in Moore's approach. In his 1911 work, he reasoned that there were two kinds of laws: those determining mass phenomena and those that relating particular cases, depending on the time and place. Global laws did not require explaining all particular phenomena. However, how much, for example, the price oscillates with the supply depends on the location: "The statistical law of the variations of price with the supply of a commodity has one form in a highly competitive center and quite a different form in an agricultural community" (Moore, 1911, p. 21). In his 1914 book, addressing some difficulties of establishing a law of demand, Moore suggested that agricultural goods were a special case in relation to other economic goods. One of the main difficulties was because of changes that occurred in the market, at the time researchers got the data. But at least for staple commodities, they could overcome this difficulty using the method that Moore presented:

> This usually means that, during the interval surveyed in the statistical series, important changes occur in the condition of the market. **But in case of staple**

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[21]Moore, later in his work, made no assumption about the distribution of variables. One possible explanation is the criticism received. Edgeworth (1912), for example, condemned the use of the hypothesis in its review of the Law of Wages: An Essay in Statistical Economics.

commodities, such as the agricultural products with which we shall have to deal, the effects of those changes in the condition of the market that obscure the relation between prices and amounts of commodity may be largely eliminated (Moore, 1914, p. 68).[22]

Before addressing the method that Moore presented, some other points are worth examining. Much of Moore's work, in addition to addressing wages and the labor market, aimed at analyzing crop yields for specific commodities such as corn, hay, oats, potatoes (Moore, 1914), cotton (Moore, 1917), and specific markets such as the US, (Moore, 1919), France, the United Kingdom (Moore, 1920a), and the Dakotas (Moore, 1920b). In reporting the history of probability and statistics, Tabak (2014) pointed out that many researchers, since late nineteenth century, also dedicated their works to measure crop yield - chemists, botanists, and other. These scientists generated a multitude of informations, but frequently, they did not place such data in a specific theoretical framework (Tabak, 2014, p. 142). In its turn, Moore analyzed crop yields while addressing economic cycles, which was related in some degree with the empirical verification of the general equilibrium theory.

The first work fully concerned with statistical analysis of economic theory is the 1908 paper, *The Statistical Complement of Pure Economics*, an article published in The Quarterly Journal. Previously, empirical verification appeared in his works, but secondarily. In this 33-page essay, Moore pointed out that his three goals were three: 1) To show that the major contributors to economic development – Cournot, Jevons, Edgeworth, and Pareto for example – had in mind an inductive statistical complementary science; 2) Describe the fundamental statistical processes that should be employed in inductive analysis; 3) Show how economic theory and the statistical field were being used together in the development of Statistical Complement of Pure Economics (Moore, 1908, p. 2).

Moore quoted Jevons to argue that the only insurmountable obstacle in the way of economics being an exact science was that they had not yet developed a "perfect system of statistics" (Moore, 1908, p. 5). Deductive Economics, according to Jevons and Moore, should be verified and made useful by the purely empirical science: "Theory must be invested with the reality and life of fact" (Moore, 1908, p. 5).

Moore argued that during the nineteenth century there was a detached development in Statistics and Pure Economics. However, there was also attempts to use both together, and he aimed to present some of these works in Scientific Realism (Moore, 1908, p. 23). The most promising field of exact research seemed to be the investigation of demand and supply curves (Moore, 1908, p.23). Jevons's work on the corn demand curve promised

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[22]Walras also, in his Studies in Applied Economics, pointed to the peculiarity of some agricultural goods: "Among ordinary commodities, there are those, like certain agrarian products, which tend to increase regularly in rareté and value, independently of weekly, monthly or annual fluctuations" (Walras, 2008, p. 95). Therefore, for Walras, there were some commodities that showed no trend. Moore, in his model, as we shall see ahead, later adopted the idea that all types of goods had a trend in their prices.

an immediate connection between economic theory and statistics. However, according to Moore, the best known empirical law of economics was still Pareto's law of income distribution, although this law was a purely empirical one, whose origin Pareto had not yet offered an explanation. Pareto's method was similar to that of physicists such as Boyle, Gay-Lussac, and Avograd, who first established their cases from purely empirical results (Moore, 1908, p. 28).

Discussing these evolutions in neoclassical theory and Probability and Statistics, Mirowski (1989, p. 223) argues that one of the most curious aspects of the development of the neo-classical theory is that many of the marginalists were also instrumental in the development of probability theory and statistics, such as Jevons, Edgeworth, Bowley, Keynes, Slutsky and Wald. However, between the 1870-1925 period none of these theorists actually made explicit links between stochastic theory and the neoclassical approach.

This first 1908 work on the development of economic statistics ended with Moore marking that "it is not unreasonable to say that at the point which economics has now reached further fecunt scientific ideas and abiding practical results are to be found in the development of the *Statistical Complement of Pure Economics*" (Moore, 1908, p. 33). Given the numerous references to other authors who devoted themselves, albeit remotely, to statistics and empirical laws, we may conjecture that in this first paper, Moore used the authority argument to legitimize all the remaining work in statistical verification that he will develop in the following years.

We mentioned earlier that Moore, in many of his works, developed an analysis of the law of demand in specific markets such as corn, hay, oats and potatoes. It is therefore surprising that the first mathematical model incorporating the idea of general equilibrium is found in his 1917 book, "Forecasting the yield and the price of cotton". This is the first time the price of a commodity formally appears to be dependent not only on its own demand, but also on other factors:

$$x_0 = \phi(x_1, x_2, x_3, ..., x_n) \tag{1}$$

Where x_0 is the percentage change in the price of the commodity 0, x_1 is the percentage change in the quantity demanded of this product and $x_2, x_3, ..., x_n$ are "percentage change in other factors" (Moore, 1917, p. 152). According to Moore three were the problems innate to formulations of the law of demand in general: 1) The form of ϕ is unknown; 2) the influences of the factors $x_2, x_3, ..., x_n$ are usually ignored and; 3) the interactions between $x_2, x_3, ...x_n$ had not yet been determined. Moore assumed that the ϕ function was linear, so that $\phi(x_1, x_2, x_3, ..., x_n) = \mu = a_0 + a_1 x_1 + a_2 x_2 + ... + a_n x_n$ and that the relationship between $x_1, x_2, x_3, ...x_n$ was also linear; for example, $x_1 = b_1 + b_2 x_2$ (Moore, 1917, p. 153). The value of $a_0, a_1, a_2, ..., a_n$ should be chosen such that the correlation, R, between x_0 and μ was maximum. Meanwhile $S = \sigma_0 \sqrt{1 - R^2}$ which measures the root-mean-square value

of forecasts should be minimal. It is pertinent to point out that, at the time, Pearson had already developed a version of the χ^2 test to obtain the probabilistic measure of the accuracy of the estimated curves (Tabak, 2014). The development of significance tests was also of concern, for example, Fisher (Tabak, 2014). Another concern to some extent missing from Moore's discussions was the stability of statistical ratios, one of the main themes of continental econometrician's staticians (Aldrich, 2010).

This method of calculating the multiple coefficients can, according to Moore, solve the three limitations simultaneously (Moore, 1917, p. 152). The approach, according to the American, was in direct contrast to Marshall's *coeteris paribus* method of assuming all other factors constant. In his 1914 book, "Economic Cycles: Their Law and Cause", Moore also condemned Marshall's approach[23]: "the "other things" that are supposed to remain equal are seldom mentioned and are never completely enumerated" (Moore, 1914, p. 66). Later, he continued:

> The fruitfulness of the statistical theory of correlation stands in significant contrast to the vast barrenness of the method that has just been described, and the two methods follow opposed courses in dealing with a problem of multiple effects. Take, for example, the question of the effects of weather upon crops. What a useless bit of speculation it would be to try to solve, in a hypothetical way, the question as to the effect of rainfall upon the crops, other enumerated elements of weather remaining remaining constant? The question as to the effect of temperature, *coeteris paribus*? How finally, would a synthesis be made of the several individual effects? The statistical method of multiple correlation formulates no such vain questions. It inquires, directly, what is the relation between crop and rainfall, not *coeteris paribus*, but other things changing according to their natural order; what is the relation between crop and temperature, other things conforming to the observed changes in temperature (Moore, 1914, p. 67).

On this point of Moore's theory, G. J. Stigler (1954, p. 110) argued that while criticizing theorists for assuming that other things remain equal, the very method that Moore adopted, the link relative method – to be presented ahead in this paper – does the exact same thing. The approach, according to G. J. Stigler (1954), eliminates the factors that did not remain constant in the data. Moreover, although he criticized the *coeteris paribus* hypothesis in his 1914 and 1917 books, Moore ended the latter, surprisingly, commenting that despite the high correlation between x_0 and x_2, there was little advantage in forecasting accuracy when considering x_0 as a function of the two variables x_1 and x_2 instead of the simple linear relationship between x_0 and x_1. Moore concluded the book by pointing out that "The seal of the true science is the confirmation of the forecasts. Economists

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[23]Interestingly, Moore taught a course in Columbia, Quantitative Economics II: Mathematical Economics, whose content included Marshall's mathematical methods. The program also included the approaches of Walras and Pareto.

theoretical and practical have grown impatient with any form of speculation that is not of immediate use" (Moore, 1917, p. 163).

In many parts of his work Moore pointed the need for a theory to be useful. For example, in Moore (1911, p. 1) he argued that the usefulness of the theory must be measured by its ability to work on three different issues: 1) definition and analysis of concepts; 2) the discovery of appropriate methods for dealing with mutually dependent social phenomena and; 3) the ability to provide a general representation of the economy. However, while defending the importance of the theory's usefulness to the business man, Moore does not write to these businessmen: his works were clearly meant to be read by his academic peers. In their turn, we can see some incredulity of these peers regarding the predictability of the theory. Burns (1931, p. 95), for example, reviewing Moore's book, Synthetic Economics, pointed out that "one reason why economic forecasting can never have the quality of perfection of the Nautical Almanac, is that the number of variables in the economic system is itself a variable".

Later in his works, the importance of forecasting appeared even more prominently. For example, in Moore (1920b, p. 205) the author asserted that the role of economic science was to be able to evidence, by looking at data, the elements that were routine, to determine their interrelationships, and to use this knowledge to forecast. It is worth recalling that the issue of forecast had been present since the very development of Statistics itself: one of the earliest record and process of data, for example, was meant to *predict* Halley's comet reappearance (Tabak, 2014, p. 39). The discussion about the usefulness of the theory has also been present since the original Walrasian program. Walras in his book "Elements of Pure Economics or the Theory of Social Wealth" pointed out that the theorist has the right to develop science for its own sake. However, the truths of pure economics generate solutions to important problems of applied economics (Walras, 2013, p. 71).

Discussing not the difference between Moore's work and the original Walrasian theory, but the former in relation to subsequent generations, there may yet be a point to make regarding the role of theoretical constructions. The post-Moore generation of macroeconomists were much more concerned with introducing structural changes than merely producing forecasts (Epstein, 2014, p. 7). In this next generation there was a central belief that economic policy should somehow change the fundamental economic structure of society (Epstein, 2014, p.7). Alternately, for Walras, the role of science was to formulate the ideas of justice and advantageousness, and to indicate the means for this realization – the rest was specifically political work (Walras, 2010, p. 245).

Regarding the theory of general equilibrium in Moore's works, we addressed until now mainly his works of 1908 and 1917. Another important paper of Moore in the area is a 1925 article, *A Moving Equilibrium of Demand and Supply*, published in the Quartely Journal of Economics. One of his main points is that "the concrete determination of the

laws of supply and demand leads to the conception of a moving equilibrium of demand and supply" (Moore, 1925, p. 358). Here is one of the central features of the development of Moore's empirical basis of the general equilibrium theory: the explicit association between empirically analyzing the theory and making it dynamic. In fact, even in his 1917 paper, this association between dynamics and empirical verification was already evident: "our law of demand is a dynamic law, it is a summary description of routine in concrete affairs" (Moore, 1917, p. 147). Moore then uses, shortly, "dynamic" and "empirical" as essentially synonymous. This becomes even more explicit when Moore describes the development of the history of general equilibrium theory:

> There are three stages in the development of the theory of general equilibrium. In the first stage, the whole economic system is seen as a complex of interdependent parts, the interrelations of which must be apprehended before the working of any single part can be adequately understood. Cournot was the first to see clearly this characteristic of social science and to suggest the method appropriate to its treatment. In the second stage, the device of the static state is introduced, and the interrelations of the parts of the economic system are enumerated and expressed symbolically in the form of general equations. Walras and Pareto worked out this part of the general problem. In the third stage, the transition is made from statics to dynamics, and the equations expressing the relations between the parts of the economic system receive the definite, numerical form in which theory admits of empirical testing. The object of the present paper is to treat this phase of the subject (Moore, 1926b, p. 28).

Moore presented the complete Walrasian model in his 1925 and 1926 articles. But the American author gave its definite form in the last book – and work in general – that he wrote, the Synthetic Economics, in 1929. Moore began the work by pointing out the advantages he had over, for example, Cournot, since his generation had new tools and materials to deal with theoretical issues (Moore, 1929, p. 4). Moore reasons that the title of the book, *Synthetic Economics*, "is intended to indicate a concrete, positive description of moving equilibria, oscillations, and secular change, by a method which presents all of the interrelated economic quantities in a synthesis of simultaneous real equations" (Moore, 1929, p. 5). According to him, as far as he was aware, neither Walras nor Pareto used the term before.[24] For Moore, to find the solution of the general equilibrium, it is not enough just to prove that there were many equations as the number of variables, but also to show that the equations could be empirically derived: the problem should admit a *real* solution (Moore, 1929, p. 6).

There are three advantages, according to the author, of the Synthetic Economics approach. First, it deals with the issue of remuneration of the production factors (Moore,

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[24]Walras employed the term "synthetic socialism" or "synthesism'' to refer to his theory. For the French, synthetic socialism opposed both individualism and communism, and proposed a synthesis of the rights and duties of the state and the individual (Walras, 2013, p. 158).

1929, p. 6). Second, it allow us to identify when a solution to an economic problem has actually been reached (Moore, 1929, p. 7). The third is that "it gives ground for the hope of introducing into economic life rational forecasting and enlightened control" (Moore, 1929, p. 8).

Moore, in this book, returned to the question of assumptions adopted by models. One hypothesis that he claimed to abandoned is that of absolute competition. He called the free competition premise a "spurious superfoetation". For him the fundamental hypothesis should be "competition" only in the real sense that each economic factor seeks maximum net gain. Thus, his theory, accordingly to him, was not based on any "unrealistic premise" (Moore, 1929, p. 107). However, according to Burns (1931, p. 95) the premise of perfect competition is implicitly introduced when Moore assumed that in equilibrium costs are equal to prices and that an industry's total product is the sum of the marginal product of the factors of production multiplied by the units of each factor.

The theoretical model developed by Moore is quite extensive: it has 164 equations. Importantly, the theory he had as his starting point was not only the general equilibrium theory, but the Walrasian general equilibrium theory: "it is desirable, in the interest both of science and of personal loyalty, to adhere as far as possible not only to Walras' terms but also to his symbols" (Moore, 1929, p. 17).

One of the most important concepts Moore employed developing his equations is the elasticity of demand and supply. Already in his 1914 book, "Economic Cycles: Their Law and Cause", Moore pointed out that by using relative change in demand, $\Delta D \backslash D$, rather than the absolute change in demand, eliminated the effect of the rising population on the variable, while using the relative price $\Delta p \backslash p$ partially eliminated errors due to general price fluctuation (Moore, 1914, p. 69). The elasticity of demand, in infinitesimal terms as usual, is given by $dD \backslash dp \cdot p \backslash D$ and is denoted by η. In turn, the so-called coefficient of flexibility of prices is given by $dp \backslash dD \cdot D \backslash p$ and is denoted by ϕ.

Moore started with the simple demand of just one product depending on its own price. If the quantity placed on the market is the independent variable, the demand functions can be built from the elasticity of demand:

$$\eta = \begin{cases} \beta, \text{ or} \\ \beta + \beta'p, \text{ or} \\ \beta + \beta'p + \beta''p^2 \end{cases} \tag{2}$$

That is, the elasticity of demand can be a constant, or to depend linearly on the price, or be a quadratic function of this price. Each of these assumptions about the elasticity

generated a different typical demand function:

$$
D = \begin{cases} Bp^\beta, \text{ or} \\ Bp^\beta e^{\beta' p}, \text{ or} \\ Bp^\beta e^{\beta' p + \frac{1}{2}\beta'' p^2} \end{cases} \tag{3}
$$

Where B is the integration constant to be determined by the observations and e is the neperian number. The issue is to derive from the data the parameter values of these functions. The problem, however, is that both price and quantity are constantly changing secularly. Although, according to Moore, there are many methods to deal with the question, he chose the trend-ratios method.[25] By the trend ratios method prices are expressed as a ratio to their trends (Moore, 1929, p. 42). The law of demand for the method are then:

$$
\frac{D}{\overline{\overline{D}}} = F\left(\frac{p}{\overline{p}}\right) \tag{4}
$$

Moore gave a numerical example of the estimate for the potato market (Moore, 1929, p. 43). He determined the trend for the period from 1881 to 1913 for both production and price. The correlation coefficient gives the quality of the fit of the estimate between production-ratios and price-ratios. Finding, $r = -0.84$, Moore pointed out that it was sufficient evidence of a high relationship between the two series. Next he started from the function:

$$
\frac{p}{\overline{p}} = A\left(\frac{D}{\overline{\overline{D}}}\right)^\alpha e^{\alpha'(D\setminus\overline{D})} \tag{5}
$$

He applied the logarithm to both sides of the equation and estimated the following expression, by the least squares method: (Moore, 1929, p. 46)

$$
log\left(\frac{p}{\overline{p}}\right) = \alpha log\left(\frac{D}{\overline{\overline{D}}}\right) + \alpha'\left(\frac{D}{\overline{\overline{D}}} - 1\right) log(e) \tag{6}
$$

The author estimated the demand equation taking its price as an independent variable, instead of the quantity, using the same method.[26] Thereafter, he dealt with the more

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[25]Moore references Schultz (1925) for an inquiry into other methods. Akhabbar (2010, p. 51) indeed included Schultz as one of the three leading proponents of the neoclassical research program on empirical studies of the law of supply and demand. The two other were Moore and Harold Hotelling. However, Akhabbar (2010) pointed out that the three programs were abandoned roughly because of a "curse": Moore suffered from psychological problems after his 1929 book, Schultz died in a car accident in 1938 and Hotelling abandoned the project after Schultz's death. The author also added two other important research programs on the empirical studies of supply and demand, then, with a total of five programs: 1) Leontief's, which brought him a controversy with Frisch, and that Leontief later abandoned; 2) Milton Friedman's, who when confronted with the analytical difficulties abandoned not the approach, but the positivist epistemology that accompanied it. Friedman opted for an instrumentalist epistemology that freed the theory from its constraints on realism.

[26]Judge (1968, p. 1707) pointed out that this practice of estimating two different equations, one with quantity and the other with price as the dependent variable, was common. Schultz summed up the situation humorously: "If, however, we are asked to determine the effect of a change in conditions of

difficult problem of estimating the demand equation of a commodity as a function of the price of all other commodities. Since the price of $(m-1)$ commodities is expressed in terms of the monetary standard, commodity A, the Walrasian demand functions are:

$$
\begin{aligned}
D_b &= F_b(p_t, p_p, p_k, ..., p_b, p_c, p_d, ...), \\
D_c &= F_c(p_t, p_p, p_k, ..., p_b, p_c, p_d, ...), \\
D_d &= F_d(p_t, p_p, p_k, ..., p_b, p_c, p_d, ...), ...
\end{aligned}
\tag{7}
$$

Where p_t, p_p and p_k are respectively the price of land, people and capital services and p_b, p_c and p_d are the prices of commodities B, C, D and all others (Moore, 1929, p. 52 and 53). Regarding the fact all commodities are subject to forces that give each a secular trend, the new demand equation for commodity C, for example, is given by:

$$
\frac{D_c}{\overline{D_c}} = F_c\left(\frac{p_t}{\overline{p_t}}, \frac{p_p}{\overline{p_p}}, \frac{p_k}{\overline{p_k}}, ..., \frac{p_b}{\overline{p_b}}, \frac{p_c}{\overline{p_c}}, \frac{p_d}{\overline{p_d}}, ...\right)
\tag{8}
$$

Presenting this formula, Moore argued that "While this hypothesis is simple it is the means of making the transition from a purely rational construction to a real situation" (Moore, 1929, p. 54). He also assumed a new elasticity of demand for one variable relative to all others – the partial elasticity. For example:

$$
\eta_{cp_t \cdot p_p p_k ... p_b p_c p_d} = \frac{p_t}{D_c} \cdot \frac{\partial D_c}{\partial p_t}
\tag{9}
$$

Using the idea that demand elasticity could be equal to a constant, a linear or a quadratic function, Moore developed the new equations. For example, assuming that partial elasticity-price was equal to $\beta_{ct} + \beta'_{ct}p_t$ the demand function for commodity C was:

$$
\frac{D_c}{\overline{D_c}} = Constant \left(\frac{p_t}{\overline{p_t}}\right)^{\beta_{ct}} \left(\frac{p_p}{\overline{p_p}}\right)^{\beta_{cp}} (...) \times \left(\frac{p_b}{\overline{p_b}}\right)^{\beta_{cb}} \left(\frac{p_c}{\overline{p_c}}\right)^{\beta_{cp}} e^{\beta'_{ct}\left(\frac{p_t}{\overline{p_t}}\right) + \beta'_{cp}\left(\frac{p_p}{\overline{p_p}}\right) + ...}
\tag{10}
$$

The author argued that for most practical problems, to calculate the two demand functions derived from the two simplest price elasticities was already sufficient. Moore also addresses in his works the issue of "simplicity" of theories. In Moore (1908, p. 16) he pointed out that one criterion for choosing fitting curves is was simplicity, priori validity, and fecundity. However, "the impossibility of rigidly defining what is simple and what is complex has not escaped statisticians". A formula may be simple because it has few constants, but involves the use of highly tortuous processes to be determined (Moore, 1911, p. 16). Moore also argued in his 1929 book that starting from the simplest solutions

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supply-say the imposition of a tariff-on prices, imports and consumption, we need to know among other things the elasticity of demand of the commodity in question, and we cannot conveniently say to the legislator "your tariff will have one effect if the elasticity of demand is computed from the regression of price on quantity and quite a different effect if it is derived from the regression of quantity on prices".

is a valid aproach to solve particularly the problem of the general equilibrium theory:

> The chief difficulty in the way of extending the realistic treatment and of making the transition from particular equilibria to general equilibria is the necessity in case of the more complex inquiry, of working with functions of many variables. Is it not probable that help toward overcoming this difficulty may be obtained by extending the theories which facilitated the solutions of the simpler problem of particular equilibria? (Moore, 1926a, p. 393)

Moore's attention to the simplicity of theories might be understood from the criticism received by him in his past works. As we have seen, one of the most important critics of his works was Edgeworth. Edgeworth (1912, p. 70) addressing his statistical approach to the law of wages pointed out that "not only has he employed a steam-engine to crack a nut; but the nut is blind". Edgeworth (1912, p. 70) still added that "we cannot acquit our author of the charge – often brought too justly against mathematical economists and statisticians – of having overlaid a simple matter with useless and cumbrous technicalities". Moore replied that Professor Edgeworth was the first economist to regard the question as "simple" (Moore & Edgeworth, 1912, p. 315). Edgeworth, in his rejoinder, pointed out that he did not affirm that the relationship between workers' habilities and wages was a simple matter. What he meant was that the method of dealing with the relationship was considered simple when "divested of disguise" (Moore & Edgeworth, 1912, p. 318).

Treating the trend series was a central point of Moore's approach, as can be seen. This discussion about the trend data had also been present since his first works. For example, in his 1907 paper, Moore (1907b, p. 63) criticized official government statistics for not strictly trending the data. In Moore (1917, p. 121), the author marked out that data variations had three distinct sources: 1) secular change; 2) cyclical changes and; 3) random changes. Therefore, by Moore's own standards there was still two problems left that the trend could not solve – cyclic changes and random changes.[27] The idea of the multiple causes of price changes likewise appeared in his 1921 work: "No one familiar with the theory of prices and with their *multitudinous* causes of changes would expect the record of general wholesale prices to show an exact mathematical precision in the working of any one cause" (Moore, 1921, p. 515).

Still discussing trends, in the original Walrasian theory, concerns with price comparison at different time periods were already present. Walras pointed out that researchers should exercise caution when comparing separate prices over a longer period of time, given that "one would run the risk of comparing non-comparable things. Ideally, high-tide prices

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[27]In his book "Economic Cycles: Their Law and Cause", Moore used Fourier's theorem to analyze periodic phenomena and the periodogram to separate natural-cause fluctuations from spurious-cause fluctuations (Moore, 1914, p. 10-4).

should be compared with high-tide prices, or low-tide prices" (Walras, 2008, p. 25). Indeed, economic series were compiled and presented with their trends for a long time, but until the turn of the twentieth century they did not use them to indicate any kind of causal relationship (Epstein, 2014, p. 12). However, a discussion of the original Walrasian theory was absent from Moore's theory about trends: While admitting it, Moore did not discuss whether the trend would be negative or positive.

The criticism Moore's work received also addressed the method of the trend ratios. Allen (1930, p. 112) for example, pointed out that "the method of trend ratios is made to bear a greater burden than it can really support". Burns (1931, p. 93) also pointed out that Moore used the concept of "secular change" just to support the adoption of trend-ratios, but that the professor did not consider that the very existence of secular trends was an object of curiosity by itself. Ezekiel (1930, p. 678) also pointed out that the trend-ratios method was perhaps not the most appropriate to deal with data problems: "Simply fitting empirical trends to series of data, and expressing them in percentages of trend, is not an adequate treatment of the dynamic element in economic activity. In such a statistical process, the influences of *all* factors which change progressively in time are eliminated, and this may include economic factors just as well as others". Ezekiel (1930, p. 678) pointed out that new methods were being developed to deal with price fluctuations. Such approaches attempted to understand the changes that occurred over time, rather than placing them all under the same category of "secular change".

Some remarks about the estimation method, the least squares, are also worth noticing. Moore did not specifically justify the choice of method in his 1929 work. In Moore (1907a), he used, for example, alternatively, the method of moments. In Moore (1908) he compared Cauchy's method, used by Pareto, against the least squares method. His conclusion was that the least squares, although more difficult to calculate, has a higher degree of accuracy. In Moore (1911), the Professor further used the method developed by Pearson for the derivation of the coefficient of mean square contingency (Moore, 1911, p. 112). Moore (1917) used two different approaches for estimation: the method of progressive averages and the method of percentage changes. The latter used the hypothesis that there was a close relationship between the percentage change in price from one year to another and the percentage change in production from one year to the next. Alternatively, in Moore (1925) the author employed the trend ratios method, and the method of link ratios, to estimate the supply curves.

A peculiar point about Moore's method is the query of outliers. The author seemed to consider removing the discrepant observations as scientifically disloyal. For example, in trying to predict cotton production from climate reports, the author pointed out that "it would have been possible, on several occasions, to increase the coefficients by omitting one or two rainfall-ratios which, in consequence of torrential storms, presented unduly large values; but no such liberty has been taken with the crude material" (Moore, 1917,

p. 117). In analyzing the history of statistics before 1900, S. M. Stigler (1986) marked out that this practice of informally discarding extremely discordant observations was actually common among astronomers, for example.

So far we have presented the demand functions developed by Moore. In chapter V of *Synthetic Economics*, the author reasoned that "the fundamental symmetry with which demand and supply co-operate in the determination of price suggests the possibility, and indicates the desirability that the typical functions descriptive of supply may be of the same general forms as those which have been found useful when dealing with demand" (Moore, 1929, p. 65). Like the elasticity of demand, the elasticity of supply could have different patterns:

$$S\eta_{tp_cp_tp_pp_kp_bp_d} = \frac{p_c}{S_t} \cdot \frac{\partial S_t}{\partial p_c} = \begin{cases} \gamma_c & , or \\ \gamma_c + \gamma_c'p & , or \\ \gamma_c + \gamma_c'p + \gamma''p_c{}^2 \end{cases} \quad (11)$$

Therefore, supply equations could be obtained analogously to demand equations. However, the data that Moore employed to obtain the law of supply was the price of the previous year. That is, the output was built as a function of the price of the preceding period. Commenting on this approach, (Wright, 1930, p. 332) pointed out that except in agricultural goods, there was no apparent reason why the lag should be one year. As Wright reasoned, "it is fairly obvious" that in drafting the law of demand, if the author finds a high negative correlation between price and quantity, using the value with lag one will find a high but positive correlation. Wright (1930, p. 333) argued also that "it raises the question whether the curves so derived can with propriety be called demand and supply curves". Demand may be inelastic, while supply need not necessarily be inelastic as well (Wright, 1930, p. 333). Schultz, commenting Wright's review, pointed out that the lag method was not essential to Moore's main thesis. Wright (1930, p. 333) responded to Schultz that "I did not mean to imply that Moore might not accept other methods of deriving supply and demand functions. What methods he might accept but did not use I do not know". One final criticism by Wright of the supply curve is that it looked nothing like the neoclassical supply curve, since it had no connection to marginal cost. In fact, (Wright, 1930, p. 338) prefers to refer to both curves as "Moore curves" rather than supply and demand.

There are four original equations of the Walrasian model. The first two, demand and supply, have already been presented. The third equation expresses the equality between

the quantity of services demanded and the quantity of services offered:

$$a_t D_a + b_t D_b + c_t D_c + d_t D_d + ... = S_t$$
$$a_p D_a + b_p D_b + c_p D_c + d_p D_d + ... = S_p \quad (12)$$
$$a_k D_a + b_k D_b + c_k D_c + d_k D_d + ... = S_k$$

Where a_t, for example, is the production coefficient that expresses the amount of factor T (land) used in the production of the commodity (A). The fourth equation indicates that the cost of production of a commodity must be equal to its price. Taking commodity (A) as *numéraire*:

$$a_t p_t + a_p p_p + a_k p_k + ... = 1$$
$$b_t p_t + b_p p_p + b_k p_k + ... = p_b \quad (13)$$
$$c_t p_t + c_p p_p + c_k p_k + ... = p_c$$

To develop the algebraic formations of demand and supply functions, Moore used the theory of partial elasticity of demand and the theory of partial elasticity of supply. To find the two last equations Moore will use the theory of partial relative efficiency of organization. ω was defined as the ratio of the relative change in total production to the relative change in total cost. If we assume that Q_c represents the quantity of the commodity (C) that is produced, and that land, persons, and services of capital services are represented by T_c, P_c, K_c, the function of production is given by: $Q_c = \Psi(T_c, P_c, K_c, ...)$. Therefore, the relative efficiency of organization is:

$$\omega_{ct \cdot pk \cdot \cdot} = \frac{T_c}{Q_c} \cdot \frac{\partial Q_c}{\partial T_c}$$
$$\omega_{cp \cdot tk \cdot \cdot} = \frac{P_c}{Q_c} \cdot \frac{\partial Q_c}{\partial P_c} \quad (14)$$
$$\omega_{ck \cdot tp \cdot \cdot} = \frac{K_c}{Q_c} \cdot \frac{\partial Q_c}{\partial K_c}$$

Such coefficients, in turn, may be constant, or depend on the cost linearly or quadratically. For example:

$$\omega_{ct \cdot pk \cdot \cdot} = \begin{cases} \epsilon_{ct} & , or \\ \epsilon_{ct} + \epsilon'_{ct} T_c + & , or \\ \epsilon_{ct} + \epsilon'_{ct} T_c + \epsilon''_{ct} T_c^2 \end{cases} \quad (15)$$

In Walrasian notation $T_c \backslash Q_c = c_t$, $P_c \backslash Q_c = c_p$ and $K_c \backslash Q_c = c_k$. Moore assumed that the coefficient of relative efficiency of organization was a linear function. Another assumption was that the use of each factor in production was carried to the point where the value of the product imputed to the final increment of the factor was just equal to the price of the increment of the factor (Moore, 1929, p. 120). The author further assumed that the most likely value of p is its trend value, $\bar{p}$. Therefore, the constant production coefficients

by which Walras describes equation (14) can be replaced and this equation became:

$$\left[\epsilon_{at} + \epsilon'_{at}\left(\frac{T_a}{\overline{\overline{T_a}}}\right)\right]\frac{\overline{p_a}}{p_t}D_a + \left[\epsilon_{bt} + \epsilon'_{bt}\left(\frac{T_b}{\overline{\overline{T_b}}}\right)\right]\frac{\overline{p_b}}{p_t}D_b + \epsilon'_{bt}\left(\frac{T_c}{\overline{\overline{T_c}}}\right)\frac{\overline{p_c}}{p_c}D_c + ... = S_t \qquad (16)$$

In place of Walras cost and price equations that depended on the assumption of fixed coefficients of production, Moore also replaced this fourth equation. Still assuming that the coefficient of relative efficiency of the organization is a linear function:

$$\left[\epsilon_{ct} + \epsilon'_{ct}\left(\frac{T_c}{\overline{\overline{T_c}}}\right)\right]\frac{\overline{p_c}}{p_t}p_t \qquad (17)$$

Moore concluded that "these four groups of equations (...) like Walras' equations (...) determine a general equilibrium, but the equilibrium with which they are concerned is *real* and not *hypothetic*, is *moving* and not *static*. It is a moving equilibrium about the lines of general trend" (Moore, 1929, p. 126).

The model presented so far is the simplest version of the book: later Moore will distinguish the price of capital goods from the price of services of these goods. The model will also take into account the supply of credit, the interest rate and the creation of capital goods. Although more mathematically complex, Moore justified that such changes supported for greater similarity with economic facts (Moore, 1929, p. 128). However, apart from this more complete model, Moore also addressed two other important points. The first one is that American marked out his approach allowed to statistically test the productivity theory of distribution. Moore reasoned that:

> There are three cardinal features of that doctrine which seems to present insuperable obstacles in the way of statistical verification: (a) The productivity theory obtains only when consumption and production have reached a state of equilibrium. But how may one know when the state of equilibrium is reached? (b) The productivity applies to marginal increments. But how may we isolate the marginal increments? (c) The productivity theory asserts that each factor in production receives an income equal to the number of units of the factor multiplied by its marginal product. But can one prove that the sum of all the several incomes determined by this formula is equal to the product of industry? All three of these difficulties may be removed by the preceding analysis (Moore, 1929, p. 143).

As for the first problem, Moore argued that the theory of a moving general equilibrium solves the issue: trend prices and trend products are equilibrium prices and equilibrium products (Moore, 1929, p. 143). The second query, according to the author, is also solved. After we determine the statistical values of the model's constants, the marginal product of any factor can be immediately calculated. To demonstrate the third point, Moore made some algebraic manipulations from the equation that expresses that the price of

commodities is equal to the cost of production. The author arrived at the expression:

$$\overline{T}_c\frac{\partial \overline{Q}_c}{\partial \overline{T}_c} + \overline{P}_c\frac{\partial \overline{Q}_c}{\partial \overline{P}_c} + \overline{K}_c\frac{\partial \overline{Q}_c}{\partial \overline{K}_c} + ... = \overline{Q}_c \qquad (18)$$

Which, according to the author is proof, in a form that can be statistically tested, that in an equilibrium state the industrial product is partitioned according to the productivity formula (Moore, 1929, p. 145).

The second important point of Moore's theory is that his last chapter of the 1929 book dealt with economic oscillations. For him, the theory of economic equilibrium is a prerequisite for understanding the fluctuations of the economic activity. The oscillations appeared in the model as a result of perturbations of a system under the influence of forces that lead it to a moving general equilibrium. As in the rest of the book, he first analyzed a particular equilibrium and then the general case, building *the synthetic theory of economic oscillations*. On the occasion of Moore's death, Columbia University, in its Memorial Minute, pointed out that "Moore's final work, "Synthetic Economics" provided a significant bridge between the now classic work of Walras and Pareto in the field of mathematical economics and the more recent formulations of Hotelling and Samuelson" (Columbia, 1959).

As pointed out at the beginning of this paper, Moore's main objective was to adapt the Walrasian model so that it could be statistically tested. According to the American author, it was precisely this lack of empirical verification that caused the absence of interest in Walrasian theory at the time (Moore, 1914, p. 82). Persons (1925, p. 194) argues that Mitchell, in his Presidential address before the American Economic Association, raised the question of whether, in fact, the economic theory developed so far could be used as a first approximation to reality, to be tested empirically. In this sense, according to (Persons, 1925, p. 195): "The fact remains, however, that there are two views in regard to this question. Moore's view is that it is possible, while Mitchell's view is that it is impossible, starting with economic theory as it exists today, to develop a statistical complement of pure economics". Persons (1925, p. 195), then continues: "These are the views of two leading investigators in the field of economic statistics, both optimists as to the future contributions of statistics to the science of economics".[28] However, Persons (1925, p. 195) argues that overall, studies in economic statistics have supported Moore's defense that it is possible to develop the statistical complement to the body of economic theory. Burns (1931, p. 88), in turn, points out that the general opinion at the time, even among the exponents of the mathematical school, was that the empirical derivation of the Walrasian system was practically an unreachable goal. According to the author, the required statistical work "would be extraordinarily, almost superhumanly, laborious"

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[28]Comparing Mitchell and Moore's work on Economic Cycles Magee (1915) pointed out that Moore's approach was poorer in terms of sense of reality than Mitchell's.

(Burns, 1931, p. 92).

The difficulty of obtaining the data was also a point that appeared in the reviews. Flux (1931, p. 272) argued that Moore did not seem to regard obtaining the information as a very important obstacle to determine the parameters of his model.[29] Still on the amount of data required, Ezekiel (1930, p. 677) made an estimate: if demand for cotton depended on 100 prices, it would be necessary, for example, 150 independent observations to establish the estimate, and with annual data "the determination of the elasticity of demand for cotton would thus have to be based on the entire history of cotton production in this country, even back to well before the cotton gin was invented!". Ezekiel (1930, p. 677) therefore criticized the fact that existed two extreme models, one in which demand for a good depended only on its own price, and the other in which demand depended on *all* other prices.

However, despite this criticism, Ezekiel (1930, p. 679) ended his review pointing out that "It cannot be said that Synthetic Economics will always remain a dream". Curiously, Moore's last book also ended with a section called "economic dreams". According to it, the society needed both forecasting and control. For Moore, Economics should go long beyond a "philosophy of the closet". Marxism, for example, would not yield to refutation, ridicule, and persecution: the only countermeasure of society was to mitigate the grievances of the working class.

1.4   Concluding remarks

The main purpose of this paper was to present Henry Moore's attempt to make the Walrasian model of general equilibrium statistically operative. The perspective of analysis was twofold, considering both the personal relationship between Moore and Walras, and the theoretical development, exploring the American's work. The letters exchanged between the two professors demonstrated Walras's academic isolation and his attempt to disseminate his theory in the United States. Further, we have seen that Walras's foreground was the translation of a teaching material: thus the propagation of his theory in the New World would begin in the classroom. We also conclude that the main analytical tool used by the author was the treatment of data using a secular trend. As this trend is empirically derived, according to Moore, there was a transition from a purely rational construction to a real and dynamic situation. Therefore, in Moore's work, the transition from a static analysis to a dynamic analysis was intrinsically related to the empirical basis of the theory. The author also claimed that the formulation of the general moving equilibrium allows the empirical test of the productivity theory of distribution.

We also observed that the question of the empirical grounding of the general equi-

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[29]A point highlighted by Flux (1931, p. 272) was also that: "the dangers of assuming that a formula that fits the relations of the past will be reliable as applied to the relations of the future hardly need to be emphasized here".

librium theory was something more prominent in Moore's later work. The main effort on Walrasian theory, his 1929 book, was also the last work the American author ever wrote. He retired shortly thereafter and never again engaged in any academic activity. Curiously, in one of his books, Moore mentioned Darwinism as a specific case of theory development and dissemination (Moore, 1905, p. 370). Darwin, according to Moore, succeeded in propagate his theory as a result of several favorable conditions: independent income, leisure time, great ability to work, a subject close to the public's interest, and *"courageous, able, aggressive disciples"*. As – in the words of Walras – his chief American disciple, it is undoubtedly that Moore is important in explaining the manner in which the Walrasian theory disseminated in the United States – or, more precisely, how it did not.

Also doubtless, one of the most important works in the history of general equilibrium theory is Ingrao and Israel (2015). The book's title, *The Invisible Hand: Economic Equilibrium in the History of Science*, is fairly suggestive: such theory, in its essence is inseparable from Smith's poetic notion of the forces of society moving without much help toward a state of, in a broad sense, optimal balance. We have seen, however, that for Moore, the empirical grounding of the theory was primarily aimed at forecasting, and forecasting has a very explicit purpose: it is possible to improve the outcome of the economy if we can know what the future holds.

2 Albert Aupetit and the tough assignment: monetary issues and the Walrasian General Equilibrium Theory

"Money is a State responsibility and, moreover, an international concern. The States [..] would do as well to agree in order to avert monetary crises as they would to protect themselves against cholera"

– Léon Walras, 1898.

"Nearly incomprehensible mass of formulae"

– van Daal and Jolink, about Walras's monetary model, 1994.

"Money has always been an awkard puzzle for neoclassical general equilibrium theory"

– Tobin, 1985.

Abstract

One of the most problematic issues of the Walrasian general equilibrium theory was integrating money into the theory's framework. Walras used to argue it was his monetary ideas that changed the most over the years. The aim of this paper is to study the work of Albert Aupetit, the first French disciple who tried dealing with the complex problem of money and Walras's model. Given how much Walras's ideas changed – and, consequently, how many ideas could be attributed to him – analyzing Aupetit's work allow us to examine how the Walrasian theory about money was assimilated by the French generation after Walras. According to Aupetit, his main contribution was related to the problem of measuring value. I first empashized this aspect of his ideas, as well as Walras's comments about it, registered in their exchanged letters. Further, I commented on two aspects of what could be Aupetit's actual contributions, although not acknowledged by him: 1) to study Walras's ideas about money using statistical data and 2) his arguments in the methodological defense of using mathematics in political economy. Furthermore, Aupetit tried twice the aggregation exam to be a professor in France, failing in both occasions. Since Walras's asked his son to watch the exam and keep him posted, his letters are a valuable source of information about these attempts, and these registers can show how institutions, in this case the *concours* in France, might have delayed the dissemination of mathematical economics in the French classroom. Moreover, these letters may be a colorful register in the history of science in general, presenting a debate about Aupetit's true reason for failing the exam: his ideas or his personality. Finally, failing the aggregation exam, Aupetit started working at the French Central Bank and developed ideas about applied monetary issues, such as the role of the Central Bank. We commented on these theories in the final section. We used as sources for this paper Aupetit's published books, Walras's letters, newspaper clippings and Aupetit's written material for the aggregation exam, available in the French National Library archive, and documents of the French Central Bank archive.

Introduction

The use of analogies and metaphors is a very good resource when authors try to make difficult concepts easier to be understood. Adam Smith, for example, developed a famous metaphor when trying to explain the role of bank money: according to Smith, the gold and silver money that circulated in any country could be compared to a highway – which do not produce a single thing by itself, but it is responsible for the important role of circulating the country's goods – while the use of banking money could be compared to a "sort of wagon-way through the air", allowing the space previously used as highways to become pastures and increasing a country's production of wealth (Smith, 1979, 321). But the fact that, in this case, the use of metaphors generated even more debates – for example, Rist, about Smith's analogy, stated that "it is difficult to imagine a more complete confusion of ideas" (Rist, 1966, 83) – is a good illustration of how troublesome were monetary issues at that time. The history of economic ideas about money is distinguished, for example, among innumerous others, by debates on the very definition of bank money – and its difference from gold coin or wooden token – the effect of the production of precious metals on the price level and on the interest rate, the difference between money and credit and the definition of the velocity of money. At the period that we studied – the beginning of the 20th century – other more contemporary issues were also added on the discussion, for example, the role of Central Banks.

However, even more problematic than monetary issues in general was the specific problem of introducing money into the general equilibrium theory. Walras himself is the one who admitted that was his monetary theory the one that changed the most: "Chiefly [...] it was my theory of money that made the most important changes"(Walras, 1954, 38); "My research has gradually led me to a total change of opinion about money and the issue of banknotes" (Walras, 2008, 54). Walras's recurrent reflections on the subject were due to the high importance given by him to money problems. For Walras, monetary issues were an important way in which his ideas could prove applicable: "I am doubly interested in the question of money, because it is important and topical, and even more perhaps because it is one of the first and most decisive applications of my system of economics" (Walras, 2008). In second, it could also be a way to get more followers (Bridel, 1997, 100).

Despite the difficulties involved in the relation of money and general equilibrium theory, at least in France aroused a Walras's disciple willing to deal with the question: Albert Aupetit. Aupetit wrote in 1901 a thesis entitled *Essai sur la théorie genérale de la monnaie*, in which he made some reflections about monetary phenomena using the Walrasian theoretical framework. Walras argued that Aupetit was his most brilliant disciple: "He is in agreement with my social economics as well as with my pure and applied economics. He is the best and most brilliant disciple and successor I may wish to have" (Jaffé, 1965,

352).[30] Jaffé (1983, 123) stated that "France has rendered justice to Léon Walras, although tardily, especially in the teaching of writers and professeurs such as Etienne Antonelli, Aupetit, Bompaire, and Gaëtan Pirou (Jaffé, 1983, 123). Walker (2006, 278-79) affirmed "In France, Walras did not have many followers during his lifetime, with the notable exceptions of Albert Aupetit, who attempted to elaborate the theory of money in a general equilibrium setting (1901) [...]". Dupriez (1958, 631) also pointed out Walras as Aupetit's "master"[*maître*], while Jacoud (1994, 258) claimed that Aupetit was Walras's first true disciple. Aline, Walras's daughter, in a correspondence with Aupetit, also stated that her father considered him as his best disciple (Bousquet, 1951, 152).

In addition to being mentioned as a Walras's follower, the literature also pictures Aupetit as an important reference at that time. He was read, for example, by Fisher – "Fisher, when he formulated this proposition, made reference to Aupetit" (A. Béraud, 2013, 349) – and by Patinkin – "Patinkin read some old-fashioned post-Walrasian French authors like Antonelli (1914), Aupetit (1901) and Divisia (1927) (Bridel, 2002, 289). Rist (1966, 270), addressing some authors that investigated the relationship between price movement and gold production, stated that Aupetit's work was known to everyone, comparing it with Fisher's and Cassel's books: "In this connexion special mention should be made of the work of a young statistician, Marcel Lenoir, too early lost to science; the work of Aupetit, Fisher and Cassel is already well known to all". Bridel (1997, 139) even stated that Aupetit was "responsible for introducing single-handed into Continental economic theory the Lagrangian multipliers method". Semedo also emphasizes that the concepts of forecast and anticipation errors and Central Bank independence were already present in the works of Aupetit (Semedo, 2000).

Besides being mentioned as a Walras's follower and an important author at that time, the literature pointed out that Aupetit had an even more relevant role in the French Central Bank. After failing the aggregation *concours* – as we will address later – Aupetit began a career at the French Bank. Yves Barroux, past general secretary of the institution, stated that an important section of economic studies was created in the bank at the end of the 19th century, but it was only with Aupetit that this section started developing formalized economic analysis (Le Maux, 2007, 9). More than that, Aupetit also appears as an economist interested in statistics (Le Maux, 2007, 64). He was not only one of the first members of the Econometrics Society (Fisher, 1933), but also became president of the *Société de Statistique de Paris*, and was a member of the *Institut International de Statistique* (French National Library, 1933). Dupriez (1958, 630), in a review published in the *Econometrica* pointed out that Aupetit is one of the first researchers of econometrics: "*Il apparait, par la, comme l'un des premiers theoriciens et praticiens de 1 'econometrie*".

This last point, that Aupetit is an author dedicated to statistical analyzes, is quite

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[30]In the original: "*Il adhère à mon économie sociale comme à mon économique pure et appliquée. C'est le meilleur et le plus brilliant disciple et successeur que je puisse souhaiter*".

relevant, since, according to Walras, the last step in developing a rational theory of money was related to statistics – a fact that is not mentioned very often in the literature: "The last problems to be solved with a view to completion of a rational theory of money are of a statistical nature. After some reflection, I believe it would be better to present them to the statisticians than attempt to solve them myself (Walras, 2008, 27).[31] This article proposes, therefore, to emphasize this aspect of Aupetit's contribution: to adopt the Walrasian monetary model and simultaneously work to analyse empirical data.

Another aspect of Aupetit's contribution that I intend to highlight is his methodological reflections on the use of mathematics in political economy. Aupetit's main book, *Essai sur la théorie générale de la monnaie*, presents an entire chapter devoted to a rare methodological defense of the use of mathematics in economic science in 20th century France, with arguments beyond those used by Walras. Aupetit tried to demonstrate that more than one path could be used in the study of monetary issues, and in economics science in general.

This article also has two additional main contributions. First, by studying the letters exchanged between Walras and Aupetit, and Aupetit's failures in the exam to become a professor in France, we can show how institutions – in this case the *concours d 'aggregation* in France – may have been a reason for delaying the teach of mathematical economics in the country. Further, it is a material for the debate in the history of science of how authors' individual personalities might influence the development of a particular field: one reason for Aupetit being rejected as a professor was his personality. If Aupetit had a different personality, would he been approved as an aggregate professor and the teaching of mathematical economics in France have been introduced earlier? Such questioning is even more relevant when we note Walras wrote a version of his theory to be used by Aupetit to teach, in the classroom, his ideas in France – which, with Aupetit's failure, was left aside until Étienne Antonelli published it after Walras's death.

Second, we saw that for Walras, monetary analyzes were an important way for applying his theory in the real world. We analyzed materials written by Aupetit as head of the *secretariat* of the Bank of France, which address various issues, for example, the role of central banks. We also consulted the written material of Aupetit's aggregation exam, available in the French National Library archive, as well as newspaper clippings. To the best of my knowledge, these documents have not yet been used in any previous research.

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[31]Walras's only empirical work was an attempt to measure price changes in France and Switzerland (Walras, 2008, xi).

2.1 Enthusiasm and disappointment: The relationship between Walras and Aupetit

Aupetit's first correspondence to Walras dates from May 1901, in which the first sent his PhD dissertation in Economic Sciences, *Essai sur la théorie générale de la monnaie*, stating to Walras that the dissertation was a homage made by a *disciple*, and he knew that until then Walras probably have never heard of him before. Still according to the Frenchman, although perhaps Walras would not find the work much original, the essay showed, at least, that among the young people of France there was recognition of the importance of the work developed in Lausanne, even though these young students were surrounded by an audience that did not recognize the value of Lausanne's work. Three days later, Walras replied that he had already started reading the *Essai* and that it seemed to him, so far, quite "remarkable concise and precise" (Jaffé, 1965, 152).[32] Walras continued the message reporting that Pareto, his successor in Lausanne, was going to teach a mathematical economics course at the *École de Hautes Études Sociales* in Paris, that November. Of these two good news – receiving Aupetit's work and Pareto's report – according to Walras, the first was undoubtedly the most satisfactory, given that Pareto, even though a supporter of pure political economy, was still quite attached to *École Économiste Individualiste*, while Aupetit seemed more willing to adopt his ideas regarding the production and distribution of wealth (151). The author of Lausanne stated that, in addition to suffering from the loss of his wife, he was also afflicted because of the slowness that the "force of truth" – referring to his works – was spreading in academia. The letter ended, despite the praise for the dissertation, with a warning from Walras because of the excessive complication that Aupetit presented his concepts in the essay, recommending that he should make easier in an eventual exhibition for students. In addition, Walras also mentioned that a young German, from the *Université de Berlin*, Alfred Dambitsch, was developing a thesis similar to Aupetit's, but was happy that the latter had finished it first.

In October 1901, five months after the exchange of these messages, Walras finally sent his opinion after reading the whole dissertation, and stated that he had found it *"excellent except for a few unimportant details"* (155).[33] However, he also had founded some problems in the issue of measuring value as developed by the student, and indicating, among other comments, an alternative equation of the total utility of the money. We will discuss with more detail this evaluation made by Walras of Aupetit's work in the next sections. Despite these notes, Walras stated that he was satisfied because at least the disciple had approached the problems, and he trusted, since he also believed that Aupetit was on the right path, that the student would sooner or later solve them. His advice was

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[32]In the original: *"remarquable de concision et d'exactitude"*. Among Walras' findings in Lausanne, there is the copy of Aupetit's dissertation, with the notes written by Walras (152).

[33]In the original: *"excellent sauf quelques détails sans importance"*.

that the Frenchman should wait a while until the ideas could mature in his mind – *"je crois que vous ferez bien de le laisser mûrir un peu dans votre esprit"*, and he also advised that Aupetit should acquire a solid base of economic statistics – which he knew that the student was already, in any case, working to get (156).

Figure 1: Albert Aupetit.
French National Library

Still in this letter, Walras also reported that Melle Dick May[34] had sent him the prospectus of the course that would be lecture by Pareto at the *École des Hautes Études Sociales*, and Walras mentioned Aupetit's name to Dick May – who was organizing the course – suggesting to May that, after an introduction given by Pareto, Aupetit could finish lecturing the course. Walras then warned Aupetit that likely Melle Dick May would contact him in the future about the issue (156).

After exchanging letter with Léon Winiarski[35], in which he reported that he had received from Aupetit a "brilliant" thesis of mathematical economics, Walras later reported to Georges Renard that Pareto had sent him the 16-page document named *L'économie pure*, which he was going to use in Paris's course (170, 175, 176). According to Walras, Pareto quoted him in the document, and compared his works about the system of

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[34]Pseudonym of Melle L. Weill, a French sociologist.

[35]Polish sociologist and Professor at the University of Geneva. Until 1900, a year before the date of this letter, Winiarski used to teach the discipline "social mechanics", which was the application, for example, of chemistry and physics, to sociological analysis.

free competition equilibrium with Newton's works, in a flattering manner. Meanwhile Pareto, to Walras's dissatisfaction, also mentioned several errors of pure economic theory in his system, and the French, responding Pareto, argued that such supposed errors were, in fact, only divergences of opinions about economic phenomena. According to Walras, Pareto also exposed a miscalculation made by him, miscalculation that Walras insisted that was not true, despite – Walras's words – the superiority in terms of mathematical knowledge of his successor in Lausanne (175, 176).

Despite mentioning these differences with Pareto, Walras also reported to Renard that he had no intention of getting involved in controversies that, besides being useless, would also be tiring, concluding with a complain for not having a disciple in France who would have asked him for an opinion before writing the course's material. Walras further regretted that Aupetit could have been this man, but that he had not heard from him anymore, despite having welcomed him very well on the occasion that Aupetit had made contacted (176).

Meanwhile, Aupetit was trying for the first time to become a professor in France, through the *concours d'aggrégation*. Aupetit wrote an essay for the exam discussing the protection of French workers from the competition of immigrants. Aupetit argued that international migration was a normal process, and that there was a general tendency towards equilibrium of labour offer, defending that immigration was also necessary for France (Aupetit, 1901).

One of the last letters sent by Walras in 1901 was addressed to Adrien Louis Lachenal, politician, lawyer and President of *Swiss Confederation*, with the intention to advertise his work. Walras reported, in an optimistic tone, that in France, albeit slowly, his work was being introduced: the mathematicians of the *Institut des Actuaires* had agreed to include his ideas in the agenda of discussion, Poincaré, then Professor of mathematics in Sorbonne, had also accepted some of his concepts and also Charles Ange Laisant[36], in the December edition of the *Revue Génerale des Sciences* had claimed that economists would have a lot to gain using Walras's ideas as inspiration. Even lawyers, "a field of research that was only apparently scientific", were also on their way to adhere to Walrasian work: Emile Bouvier, Dean of the *Faculté de Droit de Lyon*, had published in the *Revue d'économie politique* a series of articles defending the mathematical method in political economy, adding that one of the first thesis on the subject had been defended – Aupetit's work. Given these facts, Walras hoped to see very soon his political economy being taught, at the French Faculties of Science and, at least in an elementary manner, also at the Faculties of Law (184, 185).

After a few days, in a correspondence with Emile Bouvier, Walras described how only a year before his work was still being attacked in France, mainly by Laurent – vice president

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[36]French mathematician and politician, professor at the *Collège Sainte Barbe* and at the *École Polytechnique*.

of the *Institut des Actuaires* – who questioned "how to accept that satisfaction can be measured?", also stating that "a mathematician would never agreed to this"[37] (188). However, according to Walras, since then, a change had happened in the *Institut des Actuaires.* The December edition of the Institute's bulletin contained a note presenting the equation of the net profit rate as developed in the *Éléments d'Économie Politique Pure* by Walras, as well as a note from Laurent on the ideas of the Lausanne School, agreing with the application of mathematics in economics. In addition, Walras also reported the invitation made by Dick May to teach lessons in mathematical political economy in France, a task given to Pareto, as already mentioned, because of Walras's health conditions (188, 189).

Still in this correspondence to Emile Bouvier, Walras stated that Poincaré had agreed with the measurement of the *rareté* of the goods, as long as the author could found a way to do it. According to Walras, in the same way that was possible to measure the heat by using the expansion of the mercury, the Lausanne professor believed that measuring the *rareté* was also feasible, even tought not simple. Walras then mentioned that his young disciple, Aupetit, did not accept this idea of measurement, but this was not something that concerned Walras. Actually, what really was bothering him was the three lessons given by Pareto in the *École des Hautes Études Sociales.* Walras had read the summary of the classes, and according to him, they were "*non-mathématiques*" (189). Walras concluded that nothing had yet been done in concrete to introduce mathematical economics in the Faculties of Science, and besides this unfortunate fact, Aupetit still had not responded him. Walras ended this section of the letter stating that, many times, in the course of his career, he saw that a door opened when someone gave up knocking, and perhaps that was the case of Paris (189). In the meantime, Géorges Pallain, then *gouverneur* of the Bank of France, read Aupetit's work on monetary issues and invited him to join the bank's service, initially as *commis* (French National Library, 1933).

More or less a month after the letter to Bouvier, Walras finally received an answer from Aupetit, in February 1902. The young Frenchman apologized for the time that he took to write it, and explained that he had made the *concours d'agrégation* of the *Facultés de Droit* and this task had demanded much of his attention. Aupetit reported that he had failed the exam, because he was still too young and lacked training, but that he could still go through the process again in two years. In the meantime, he had decided to join the Bank of France, where, if he failured in academic life, he would find, perhaps, a promising future. All these activities, according to Aupetit, had prevented him from dedicating time to studies of pure economics, but maybe this delay would have positive effects, because his ideas could then mature – as suggested by Walras. The young man also stated that, regarding Dick May's request to give some classes, he decided to postpone the project,

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[37]In the original: "*Comment accepter qu'une satisfaction puisse être mesurée? Jamais un mathématicien n'y consentira.*" (188)

given his current occupations, but that perhaps he could do it the following year (193, 194).

Aupetit also reported in the letter that he had met, in Paris, Pareto and Gide, and that he had been delighted to contact theorists with a similar view. Pareto, in turn, describing this meeting with Aupetit, in a letter to Pantaleoni, stated that he was modest, friendly, full of goodwill, and that he had advised him to compete in Geneva. Pareto also mentioned that the theme of his aggregation exam – the first that Aupetit failed – was *"la culture maraîchère*[38]*: Quel est le meilleur usage à faire du boni des sociétés coopératives"*, and stated to Pantaleoni: "here they call such things political economy!"[39] (194).

In a quick response to Aupetit's letter, Walras reported that, although he hoped to raise more disciples in France, he was happy that he had not lost the first one, and also to know that the young man's omission was for a good reason. Walras limited himself to give some advice on preparing for the next aggregation exam: to delve into issues of *économie politique pure* and *solutions empiriques* (194, 195).

Walras obtained another letter from Aupetit only fifteen months after sending this response, in May 1903. In the meantime, Walras wrote to Laurent, arguing that topics on economics and mathematical statistics should be included in the aggregation exams, mentioning not only the case of the young Frenchman but also of other young examples: there was, according to Walras, a group of candidates for aggregation who were also very interested in pure political economics. In this sense, it was not justified, in such exams, that mathematicians were at a disadvantage in relation to lawyers (205).

Hermann Laurent, responding, explained that he hoped that the small treatise he had written, *Traité d'économie politique mathématique*, could inspire young people to study Walras's work, and also reported that he had met Aupetit, and was surprised with how young he was: "he is 26 years old!". Laurent reported that he had read the young man's Essay on monetary matters, and contacted him immediately. According to the mathematician, there was in Aupetit what was needed to be a scientist *"illustre"*. Laurent also stated that Walras's student was not yet "very strong in mathematics"[40], but that he studied with passion and added that Aupetit was "very intelligent, he will be our future president [of the *Institut des Actuaires*], I hope" (207)[41]. However, regarding teaching in France, Laurent stated that the time was not yet right for the modification of the programs – Rome had not been built in a day – but the moment would come when

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[38]Cultivation of vegetables outdoors or under shelter. The theme of the Aupetit exam document, therefore, was related to the cooperatives of such farmers.

[39]In the original: "qui, questa roba, la chiamano economica poplitica!"(194)

[40]In the original: *"très fort en mathématiques"*.

[41]"In the original: *remarquablement intelligent, ce sera notre futur président,je l'espère"*. Laurent also signed the letter to the Lausanne professor as *"votre bien dévoué disciple"* (207). It seems, therefore, that even with a strong initial aversion, Walras had not exaggerated about Laurent having changed his mind. Aupetit is also mentioned in the archives of the French National Library as a member of the *Institut des Actuaires*.

they would be the majority in the *Institut des Actuaires* (207).

Walras reported, in a subsequent correspondence with Renard, Laurent's encouragement for Aupetit to go deeper into mathematics, in which he also happily mentioned that Emile Bouvier, a professor of political economy in Lyon, had published the work *La Method Mathématique en Économie Politique*. After addressing other issues, Walras returned, in the same correspondence, to talking about his French disciple, stating that, given his many occupations – mainly with the preparation for the next aggregation exam – it would be better not to demand him to teach the course in the *École des Hautes Études Sociales*. Furthermore, Walras stated that he agreed with Melle Dick May that the French were anyway too little mathematicians and too ignorant to be interested in pure political economy (209).

Although Walras stated that it might be better for Aupetit to spend more time preparing for the exam, the young man sent a letter in May 1903, mentioning that, attending the request of *Monsieur Gide*, he accepted lecturing a course on *Principe et les Applications de la Méthode mathématique en Économie Politique*, at the *École des Hautes Études Sociales*. Aupetit took the opportunity to ask Walras for advice on what to do with an insufficiently prepared audience, as he believed that Walras had managed to solve the problem in his Lausanne courses. Aupetit also mentioned that, since the last letter, he had translated Jevons's Theory of Political Economy, and that the Englishman, despite developing some elements of the theory of mathematical economy, had not succeeded, as Walras had done, in postulating the idea of general economic equilibrium. Therefore, he concluded that it was really the French, Cournot and Walras – and the latter being also the "true creator of the pure or rational economics" – to whom we should give credit for the advances in mathematical economics. The young man ended the letter by reaffirming that he remained, therefore, faithful to the one who called him his "first French disciple" (228).

Regarding the advice on the best way to captivate the students' interest, Walras argued in his courses in Lausanne he had managed to present pure mathematical economy in a way that seemed, at the same time, accessible and scientific, replacing algebra for geometry. The Professor mentioned he had used, as material, pieces from the second edition of his *Éléments*, and that Aupetit could also use Appendix I of the fourth edition. The letter ended with Walras affirming his belief that Aupetit would soon make a remarkable contribution to science (229, 230).

After this last letter, Walras received a message from Charles Gide – to whom Aupetit had asked to write an introduction to the translation of Jevons's work – reporting to Walras that he had recommended his disciple to the judges of the aggregation exam, even though he had declined the task of being one of the judges himself – not informing in the letter, until then, the reason for the refusal (232). A few days later, Walras received a message from Carlos Cagé, also reporting details of Aupetit's test. According to Cagé,

the members of the jury were Leveillé[42], Levasseur, Cauwès, Perrot[43] and Villet (234, 235).

A mission similar to Cagé's – informing news about Aupetit's exam – was also given to Georges, Walras's son. In October 1903, he wrote to Aline, his sister, reporting, first, that the President of the Jury would be Leveillé and that there were twelve other candidates: Garice, Fochiez, Trinquel, Martin[44], Alglave, Raynaud, Lebeau, Oubert, Dolleans[45], Bigollet, Schatz[46] and Dubois[47]. Georges also told Aline that the questions would be chosen twenty-four hours in advance and Aupetit was the first to present his answers, and the question for him was: "what is the difference between the banks of Algeria, Guadeloupe and Indochina?".[48] Georges then described Aupetit's physique: his appearance was *agréable*, of medium height, brown hair and dark blond beard. According to Georges, Aupetit did not have the appearance of an ascetic, nor that of a "scientist with glasses", and he expressed himself clearly, precisely and without difficulty. Regarding the presentation, specifically, he pointed out that it had been methodical and pertinent – although he was a layman and, therefore, someone whose assessment could not be totally trusted.[49] A historical exposition on the topic was necessary and the presentation had lasted forty-five minutes (236). His written work on the *concours d'aggrégation*, on the other hand, addressed the issue "*Les Plus-Values foncières dans les villes*", in which Aupetit discussed surplus value in cities, mentioning authors as de Thünen, Henri George and David Ricardo – explicitly disagreeing with the last two on some points. The aim of the work was to investigate the causes and characteristics of the surplus value generated in the price of land in the cities and the means of mitigating it. The cause, for him, of the *plus-value* was relatively simple: the excess demand for land. However, the surplus value generated by the price of land in the cities could not grow indefinitely, and therefore it could not be considered social injustice. If we wanted to punish people for profiting from real estate speculation, then we should reimburse people when they lose money too. He therefore saw such plus-value as a reward for risk. For him, the attempt to maintain equality of land values in the name of justice would be an "unstable and chimerical equality" [*égalité toujours instable et chimérique*] (Aupetit, 1903).

The result was released, and Georges informed Aline and her father in the following

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[42]Professor at the Faculty of Law in Paris and French politician (235).

[43]Also Professor at the Faculty of Law of Paris (235).

[44]Germain Martin, economic historian, French politician and Professor at Dijon, Montepellier and Paris (236).

[45]Edouard Dólleans, French economist and Professor at the Faculty of Law (236).

[46]Albert Lucien Schatz, French economist (236).

[47]Auguste Dubois, historian of economic doctrines and founder of the *Revue de l'histoire des doctrines économiques et sociales* (237).

[48]The other two questions reported by Georges refer to Raynaud exam – "should we create cotton colonies and, if so, where should we cultivate them?" – and Lebeau – "by what types of contracts can we guarantee land investments in the new countries?" (235).

[49]Georges Walras was captain of the infantry.

correspondence. Three candidates were considered qualified, in the order of classification: Dubois, Martin and Schatz. According to Georges, there was a discussion about approving Schatz or Aupetit, but, finally, the second was disqualified. Aupetit was the youngest of the candidates, being 25 years old[50], while the oldest was Dubois, 38 years old. According to Georges, his father's disciple was considered too young; however he was ranked among the "strongest". Another exam would be done in two years, and Aupetit could try again, for the third time (238).

Two weeks later, Georges reported the message he had received from Ferdinand Giraud about the exam, in which the latter stated that he did not know the specific reasons for Aupetit's failure, but believed that age had been a major part of it. Giraud had been surprised, too, that the young man seemed willing to give up his high position at the Bank of France to pursue a career in a university, and not only presented himself as a disciple of the ideas of Walras but, further, his apostle [in the original: *apôtre*] (238).

After the competition, Walras mentioned to Renard that Aupetit had asked for advice on teaching mathematical economics and in response, he was drafting a basic course for the classes, based on his fourth edition of the *Éléments*, also hoping that the course would eventually be given not only at the *École des Hautes Études Sociales*, but at technical, commercial and law schools. This effort, in a period in which, according to Walras, he should be resting, had devastated him. However, whether he was successful or not, he stated that it would be the last undertaking trying to disseminate his work, pointing out that he had spent forty-five years in political economy and, meanwhile, regarding France, the only result had been a significant waste of his wealth and a *névrose cérébrale*. Walras regretted that posthumous successes were considered the most beautiful ones, but he used to consol himself because he was, first, a "man of science" and he had not only dedicated his life to looking for the "true", but he actually had founded it (239).

Responding to Walras, Renard mentioned that he did not know whether Aupetit would lecture the course or not, and that he had been told that the young man was not approved mainly because of the aversion of Leveillé and Pérot, professors at the Faculty of Law. Gide regretted not being on the jury. However, Aupetit, at least held an important position at the Bank of France – the young man was already *chef du secrétariat*. However, in spite of the favourable situation, possibly these occupations would keep him away from pure science (240).

After reading Renard's letter, knowing the probable reason for Aupetit's failure, Walras wrote to Bouvier and justified that writing to him, at least in part, would relieve his "obsession" with the question. He then complained to Bouvier that one of his best disciples, after presenting his work brilliantly, had failed, mainly, according to his source, due to the opposition of Léveillé and Pérot – to whom Cauwès probably also had joined. Walras criticized the fact that the jury had five members, three of them professors at

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[50]In the correspondence dated sixteen months earlier, Laurent stated that Aupetit was 26 years old.

the Faculty of Law, including two non-economists. They, according to Walras, in a very arrogant way, gave themselves the right to prohibit the study of a method – mathematical economics – that had already managed to recruit followers even among the most qualified candidates for aggregation (241). Bouvier, for his part, did not deny Walras's information, but asked how he had obtained it. Regarding the Professor's complaints, he pointed out: "what can be done?" – in any case, for him, *concours* were still the best way to recruit professors and, personally, he wished that the way of selection could remain for a long time with no change (241).

Uneasy by the silence of Walras, who had not responded to his letters for weeks, and after hearing from Renard that the Frenchman was ill and had been greatly affected by Aupetit's failure, Gide decided to write a letter addressed to Switzerland in February 1904. Gide tried to reassure Walras by stating that, if the professor's illness was due to the disciple's failure, he did not need to suffer anymore: the failure had not been caused by a dislike of mathematical economists by the jury, given that the test had not even addressed the question. First, Aupetit was not chosen because of an older candidate, and furthermore, it appeared that the jury was not very satisfied with the young man's presumption and arrogance. Gide himself reported having regretted paying so much praise to Aupetit directly, after seeing his behavior after the competition, stating that "such behavior" was not at all appropriate. Gide further complained that in the past eight months he had sent Aupetit several books for reviews, and had not obtained either the reviews or the devolution of his books.[51] An additional complaint is made because the young man had never confirmed whether he would lecture the course at the *École des Études Sociales* (245). Walras, on the other hand, replied to Gide that he had not really gotten any more answers from Aupetit, but he believed that, like him, the disciple had been strongly affected by the result of the aggregation exam. The professor also explained that he did not know if he had the courage to stimulate the young man to engage further in science – even though he affirmed that he had done this in the past in a more difficult situation than his disciple's (247).

A different opinion from Renard's about the reason for Aupetit's failure is given by Charles Rist[52], in October 1906. He stated that he considered himself lucky that Walras considered him a disciple, even though he did not have the mathematical skills of his friend Aupetit. He regretted how someone so talented could not access the Faculty of Law, and attributed the failure to the hostility and ignorance of certain members of the jury. Regarding the teaching of mathematical economics in France, Rist argued that the renovation that Walras was hoping for would sooner or later happen, and that he had knew young people with independent spirit and intellectual value willing to assist in the

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[51]In a later letter, Renard also reported to Walras that, apparently, Aupetit usually did not respond to his correspondence and missed appointments that he himself had proposed (246).

[52]French economist, professor of political economy at *Faculté de Droit de Montpellier* and later professor of political economy, social economy and history of economics in the *Faculté de Droit de Paris*.

transformation (308, 309).

Finally, in October 1907, Aupetit informed Walras that he was planing to visit him, since he would be in Lausanne because of a issue of the Bank of France. In a letter to Gustave Maugin, Walras pointed out that this visit would be one of the most pleasant events and probably one of the most remarkable of his scientific career, stressing that he still had faith the young man would continue his work. The French professor also had in his hands the material of the elementary course, which he had waited for four years to give to Aupetit (335).

Aupetit visited Walras in Switzerland in 1907. After the visit, the latter wrote to Walras thanking him for his friendly welcome and stating that he would not only fulfill his promise to lecture the course, but would also write a book they talked about, *Économique rationnelle*, for a series by a publisher named Doin, *L'encyclopédie scientifique*, edited by Maurice d'Ocagne. Walras still expected a second visit from Aupetit, to be made in the following months, and had entrusted the disciple with copies of the *Économie pure et Économie appliquée*, so that he could do the *Abrégé*, a version to be used in the classroom (333).

Before his second visit, Aupetit sent Walras two letters he received, one from Laurent and the other from an unidentified correspondent.[53] In the letter sent by Laurent, the latter pointed out that he was disappointed because Aupetit was going to publish the treatise on political economy in Doin's collection – the series mentioned in the previous paragraph – stating that in the same edition he also would write a treatise on statistics. His disappointment – *"Je suis navré!"* – was due to the fact that he had only recently heard about Aupetit's publication, because otherwise they could have done a much better job together. Anyway, consoling himself with the fact that *"ce qui est fait est fait"*, Laurent asked if the the author could at least send him a copy of the work, when ready, for him to include in the library do *Institut des Actuaires*, thus helping to spread the good doctrine. In addition to these considerations, Laurent pointed out that he would very much like to see the young man as a member of the Institute one day, even if it meant resigning himself of the position of vice president for Aupetit to take his place: *"Je suis tout ready to donner a vice-presidential release for your succession"* (337). According to Laurent, Aupetit could not only develop the "true" economic theory but also some projects in the field of statistics. The letter ended with a request: if Aupetit could not mention to anyone the confidences he had made in the message (337).

In the second letter that Aupetit attached to the correspondence, the unknown writer made some considerations about the young man's work. According to him, Aupetit really knew a lot calculus and mechanics and the author was pleased with the way that he introduced mathematics into his work. Furthermore, Aupetit's definition of total utility

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[53]Apparently, the letter's handwriting does not allow the reader the identification of the author. Aupetit sent it to Walras given the "authority" of the writer's opinion (336).

reminded him of the way that he presented the concept of light intensity in his course on optics. The author also pointed out that intellectual loyalty at that time was something quite rare. The unknown professor then raised two questions about the use of mathematics in economics: is the method fertile? it helps to discover economic laws? The writer also stated that he was not exactly an economist, and therefore had no authority answering them, but an important reflection was that, even if the theory was not yet very fruitful, this could be explained because of its immaturity: there had not yet been enough time for the theory to yield results (337, 228).

In December 1907, Walras sent Aupetit a draft of his latest book, *Économique et Mécanique*, stating that the work was extremely important for the dissemination of the method among mathematicians and proposed that they could discuss it on the next visit of the youngman to Clarens. Meanwhile, Walras received a letter from Alfred Barriol, director of the *Institut des Finances et des Assurances*, who also introduced himself as Walras's disciple, and dedicating to him his work *Théorie et Pratique des Opérations Financières*. As well as Aupetit, Barriol had also received an invitation to collaborate with the *Mathématiques appliquées* section of Ocagne's encyclopedia (345).

In February 1908, Walras wrote to Aupetit commenting on some changes he had made to his *Économique et Mécanique*, and arguing that it would be interesting to share the material with Poincaré, given his adoption of the method they used. If, however, Poincaré was not interested, other candidates would be Picard and Émile Borel. Walras added that these three professors would participate in the *Congrès international de Mathématiques* in Rome, and argued that perhaps this was a unique opportunity to publicize his work (347).

Months later, in July, in a correspondence with Renard, Walras reported, however, that he had heard no more news from Aupetit – neither about the course, nor about the *Économique et Mécanique*. In this absence of news from France, he stated that he had taken the time to contact Henri Fehr[54], who, after the congress held in Rome, organized a commission on the teaching of mathematics. Walras reported that he sent a report on teaching political economy using mathematics and, according to him, had been well received (362). A few days later, he mentioned this contact with Fehr again in a letter to Ernest Roguin, in which he also recounted that he had received Aupetit's visit the previous year and that the young man had taken his *Abrégé des éléments d'Économie politique pure* and was about to inaugurate the teaching of mathematical economics in Paris (365).

In October 1908, Walras received a message from Schumpeter: he sent his work *Das Wesen und der Hauptinhalt der theoretischen Nationalökonomie*, expressing that although the Frenchman did not know him, the book was the work of a disciple. Schumpeter also mentioned that Walras probably did not read German, but that he should at least

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[54]Swiss mathematician and professor of geometry and advanced algebra at the *University of Geneva*.

accepted the work as a kind of homage (378). The French professor, in turn, replied that he appreciated the message very much and that he would make the necessary effort to obtain a translation or perhaps he would be able to do the reading on his own. In the same correspondence, Walras pointed out that the success of mathematical economics would be slow, but it was sure: Italy was already an important center and, in France, his two disciples, Aupetit and Barriol, would probably teach courses according to his *Éléments* (379).

In January 1909, a year before his death, Walras wrote his last letter to Aupetit, from which he would never get an answer. Renard was looking for a professor for the mathematical political economy course that would be inaugurated at the *Collège libre des Sciences Sociales*, and he would like to know if the young man would be interested, since otherwise he would have to ask Renard some extra time to indicate the candidate to an opportunity that he believed to be both excellent and unexpected (386).

Ten days after this letter, with no reply, Walras wrote to Renard, arguing that if the young man no longer wished to be his disciple in France, as it seemed to be the case, he would then have to find another follower in Paris who could take his place. The Frenchman described to Renard what a potential candidate should look like: he should have a bachelor's degree in mathematics or be a former student of the *École Normale* or the *École Polytechnique*. With the help of Darboux, Tannery and Borel he could also read Walras's *Mémoire: Économique et Mécanique*, in the *Académie des Sciences*, and he could also inaugurate the course on pure political economy, divided into 38 lessons, in the *Collège libre des sciences sociales* (388).

In the same letter to Renard, Walras also made other comments about teaching mathematical economics. The French stated that, as in the physical-mathematical sciences, political economy – also a science *physico-mathématique* – was based entirely on a differential equation, the equation of maximum utility, and this justified its analysis using infinitesimal calculus and the method of limits, the latter through the calculation of nontranscendental functions. The professor also mentioned that he had been engaged for a long time constructing the pure mathematical economics and making a basic course about it, a course that would not require any knowledge other than arithmetic, geometry, algebra and basic notions of analytical geometry and, with this simplicity, it could be incorporated even in secondary schools (388).

Walras continued the letter stating that his goals of reformulating the educational system was the reason he wrote the introduction of the third edition of *Éléments d'économie politique pure*, and this was also why he only used the geometric approach in the *Théorie géométrique de la détermination des prix*. These two, the introduction and the *Théorie géométrique*, were part of his *Abrégé*, which he had used in the course of Lausanne. Walras also pointed out that he had carefully revised his fourth edition of the *Éléments* so that Aupetit would eventually use it in a course of mathematical economics, but that he

had not done so until now, as mentioned before. Finally, according to Walras, the popularization of the education of political economy in secondary education was only part of the process of change, which should be completed by justifying the method before the capable men, a task that he had aimed when writing his work *Économique et Mécanique* (388).

Apparently, the last news received by Walras about Aupetit was in two correspondences, one from Barriol and the other from Renard, in November 1909. The French professor passed away less than two months later. Barriol sent him the program of the course he taught in Paris, at the *Institut des Finances et des Assurances*, and also informed that he had drafted his book *Économique rationnelle* and hoped that Aupetit – his friend – could help him, even though he knew that the young man was quite overwhelmed at work. Renard pointed out that, at least, he tried to welcomed Aupetit: he had offered him several times an opportunity to teach in his program, without getting any answer (436, 440).

2.2 Aupetit and his methodological defense of the use of mathematics in economics

In 1901 Emile Bouvier wrote, as mentioned in Walras's letters, a series of articles for the *Revue d'Économie Politique* in which he made a methodological defense of the use of mathematics in economics (Bouvier, 1901a) (Bouvier, 1901b). These works would become an important reference in France in subsequent years (Moret, 1915). Bouvier openly disagreed with authors such as Cauwès and Leroy-Beaulieu, arguing that even the exact sciences did not have many relations between variables that could truly be studied using mathematics. Bouvier criticized those who, according to him, without knowing mathematics, defended that it could not be applied to economics (Bouvier, 1901a, 834). Further, he said that mathematics should be learned as any other language, like Latin and Greek, also stating that those who "could not read Greek probably found Athenian law rather obscure" (Bouvier, 1901b, 1034) ["*Ceux qui ne savent pas le grec trouvent le droit des Athéniens bien obscur*"].

In a slightly more conciliatory tone, in the same year Aupetit wrote his main book, *Essai sur la Théorie Générale de la Monnaie*, discussed earlier. Despite being a treatise on monetary issues, surprisingly, the first part of the book is a long analysis about scientific methodology. One of the first defenses of the book is "the true scientific method does not exclude any form of research, not even, for example, the analytical formulas of M. Walras and the descriptive observations of Jevons or the statistics of M. Foville" (Aupetit, 1901, 4).[55]

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[55]In the original: "*La véritable méthode scientifique n'exclut aucune forme de recherche, pas plus, par exemple, les formulas analytiques de M. Walras que les descriptives observations de Jevons ou les*

The beginning of all science is marked by an act of faith – the belief that there is a permanent order in the Universe. Aupetit remembered Pascal's statement: "We know the truth not only by reason, but also by heart" (6) ["*Nous connaissons la vérité non seulement par la raison , mais encore par le cœur*"]. No matter how chaotic the world appears to the researcher at first sight, it is possible to find a pattern that describe the world. And how to find such a pattern – how to transcribe the chaotic world into general laws that explain phenomena? There are two methods available to the researcher: analysis and synthesis (11). The aim of his work is to show how these two methods are indispensable for the development of any science and can be used togheter, which he will try to do when studying monetary phenomena.

Analysis – or as he uses as synonymous, induction or experimental method – starts from the particular to the general. The analytical method has four steps: 1) to observe and to classify the phenomena; 2) subject the observed phenomena to rigorous numerical determinations; 3) establish the relationships between the variables that were quantified in step 2. It is at this point that the general and permanent laws are obtained, and the phase that demands greater analytical capacity from the scientist. In the last and 4) step, a broader generalization of such laws is made (10–11).

Some comments are pertinent at this point in Aupetit's presentation. First he emphasized the issue of measuring variables, and one of his main disagreements with Walras was over the problem of measuring value, as we will discuss. However, in addressing monetary issues later in the book, Aupetit mentioned the difficulty in measuring many of the basic variables of monetary theory, for example, the stock of money. According to him, measuring any value requires knowledge of three essential elements: the quantity to be measured, a quantity of the same kind to be chosen as the unit of account and a quantity that can express the final result (25). To measure the money stock of a country, we know the quantity to be measured and the unit of measurement, but the dispersion of phenomena in time and space makes the comparison of measurements quite complex (25). According to him, it is necessary to be satisfied with often inexact approximations. The problem of measuring the money stock is still affected by personal interests, preconceived ideas and partisan passions (26). Still according to him, in statistics – the instrument for measuring social phenomena – there are no fixed causes of errors in measurement. Statistics is different from physics, for example, which wrong measurements can be easily corrected with a more suitable instrument, while statistics involves much more complex alterations (26). He still mentioned one other example of difficult in mensuarements in social sciences. If we consider the value of a merchandise any given market, the price table for that market provides a precise numerical expression of this value, but there is a lot of uncertainty about the elements of this measurement (26). This happens because even if we consider a commodity in a particular country, within that country, such commodity

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statistiques by M. Foville".

has numerous prices. Is it legitimate to use the average? How to calculate it, in a manner that this measurement provides an unequivocal characteristic of a given economic state? (26). He still did not have an answer.

However, would these statistical measurement difficulties be an impediment for monetary issues to be properly developed in a scientific manner? The answer seems to be no, and we find it in the very title of his book: it is an essay on the *general theory* of money, which implies that monetary analyzes were already at the stage of broader generalization – the fourth stage mentioned earlier. A second point worth mentioning, at least briefly, in addition to the measurement problem, is the recurrent use, as Walras did, of astronomy as an example: Aupetit mentions the discoveries of Tycho-Brahé, Kepler and Newton as an illustration of the development of science (8). A third point is about Aupetit comments on the specific characteristics of scientists: as we have seen, according to him, the scientist must have an especially greater analytical capacity in step three of the synthesis method. On more than one occasion, in fact, Aupetit commented on the skills necessary for someone to be a scientist, despite being somewhat obscure passages: for example, in one of them, he compares the scientist's ability to analyze the unexplained residues in his research with a sculptor "using the chisel to give life to a lifeless material" (16).[56] Although the passages are not very clear about the qualifications necessary for a scientist, one conclusion can be drawn, at least: being a scientist requires specific skills not likely to be available to all individuals.

After describing the analysis – or induction, or experimental economics – Aupetit describes the synthesis – pure economics, or rational economics. The aim of the synthesis is the same as that of analysis: to determine the general and permanent relationships existing between certain phenomena (11). The main instrument of synthesis is the abstraction – more or less intuitive – which replaces concrete objects for ideal ones and defines the essential properties of these ideal types (11). Once these properties are established, we need to separate and compare them to deduce the implicit connections between these properties. This logical operation, in turn, is done using signals that translate such properties and represent them. If the algorithm chosen for this is the ordinary language, most likely the study will be limited (12). An example of this is Archimedes's theorems: when you reach the end, you have already forgotten the beginning. In the case of economics, he also argued that Ricardo expressed himself poorly and often incorrectly, as did J. Stuart Mill, especially in his "Principles of Political Economy" (31).

The ideas are simplified if we use algebraic language for this translation: proofs that would require a page in ordinary language can be expressed in algebra in a few lines. However, he distinguished analysis with elementary algebra and analysis with Lagrange

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[56]In the original: *"Pour réaliser cette adaptation finale des signes intelligibles aux phénomè nes sensibles, il faut plus que l'habileté du praticien qui inscrit me silhouette humaine dans la géométrie rigoureuse du marbre, il faut tout le génie du maître dont le ciseau donne la vie à l'insensible matière"*.

functions (13). Elementary algebra can only express exact relationships, for example:

$$pv = \frac{p_o v_o (a + t)}{a + t_o} \tag{19}$$

Lagrange relations such as $u = f(x, y, z, ...)$, however, can reach a higher degree of indeterminacy and, therefore, can be used to greater generalizations, being adapted to many situations and analyses. This generalization can reach the point where the study of any phenomena can be reduced only to the study of a certain function (13). Again, mentioning economics, he argued that common logic, with its verbal expression, is not a sufficiently flexible instrument to study the full complexity of the problems of the science. Mathematics, far from "reducing everything to numbers", could be able to leave several of the quantities undetermined and could make the theory much more adaptable (31). However, regardless of the language used in the translation, according to Aupetit, logical deduction will never depart too far from reality, as long as we use adequate premises. For the author, there is no deduction made with true premises that has not been proved by experience (14).

Even using true premises, however, the confrontation of deductions with reality often leaves an inexplicable residue, and it is necessary to introduce some complications. This is the phase in which, as discussed above, the researcher often needs to have a more specific skill – the ability to mould theory to data (18). This operation of adapting the theory is compared, also somewhat obscurely, to that of a merchant who provisionally stops using the valuable metal in his transactions to use bank note, until the day he exchanges his earnings for the metal again. He ended this part by summarizing that the deductive method is formed by three defined parts: induction, reasoning and adaptation (16).

After defining the two methods, the deductive method and the inductive method, Aupetit defends a specific idea: the specialization of work in science. For him, both synthesis and analysis are necessary for the advancement of a field, but he argued that the two cannot be confused: for getting out of a dangerous forest, it is better to follow a unique path than trying to take several paths altogether. It is good science to follow experimental analysis and rational synthesis separately. There is less risk of erecting empirical laws into general theories or imposing arbitrary constructions of the mind on true realities (17). In the immense path to be travelled by science, analysis goes to the right and synthesis goes to the left: one day we will be able to see the progress achieved simultaneously by both paths (23). This is how more advanced fields develop, for example, mathematical physics and experimental physics (18). Despite the distinction, it is eventually necessary to build a bridge connecting the two ways. But there is a condition: building a bridge involves the willingness on both sides.

For Aupetit, despite many developments, political economy had not yet defined its object of research and its methods (18). The main cause of this delay was the sense of

immediacy. Again, a division of tasks was necessary: the researcher had to leave to others the task of enriching States and individuals. To achieve an evolution in science, we could not confuse the study of what was fair, with the study of what was necessary and useful. Addressing applied economics, he presented yet another justification for the division of labour: human nature itself, which has two separate and well-defined functions: thinking and acting.

At the end of his methodological section, Aupetit tried to answer the main objections to using mathematics in economics. Many authors said that, for example, to study problems such as supply and demand, it was totally unnecessary to use complicated calculations, because they were easily observable in the real world. However, Aupetit argued that their criticism would apply to astronomy as well, given that the stars were very easily observable phenomena too – we only needed to look at the sky (33). He also listed authors who used the mathematical method in economics, in addition to Cournot and Walras in France: Marshall, Edgeworth and Wicksteed in England, Laungardt and Lehr in Germany, Auspitz and Lieben in Austria, Wicksell in Sweden, Pantaleoni and Barone in Italy, Pareto in Switzerland and Clark, Gidding and Irving Fisher in the United States (34). Further, he responded the objection that economic mathematics used certain quantities – rarity, utility, ophelimity – that did not seem to be rigorously defined (31). This problem, for Aupetit, was not so relevant. First, he thought it possible that in the future, they would be able to measure such quantities. In any case, he mentions Pareto and argued that the author was able to reconstruct the entire system of economic equilibrium equations without having to use these difficult-to-measure quantities (32).

Some critics also argued that the method was only capable of dealing with static problems. However, according to Aupetit, it was already possible to transform some dynamics problems into statics problems (31). Finally, he defended scientific advancement for the sake of scientific advancement, and that even what may seem useless today could inspire concrete action in the future. To those who doubt the method, he recalled d'Alembert and his response when asked about the fundamentals of infinitesimal calculus: "Go ahead, faith will come to you". ["*Allez de l'avant, la foi vous viendra*"] (23).

After defending using the two main research methods separately, analysis and synthesis, Aupetit will make an explicit demarcation in his book, also dividing it into two parts. In the first of these, *Économique Rationnelle*, Aupetit dealt with the monetary problem using analysis, while in the second, *Économique Expérimentale*, he studied money from the point of view of the synthesis method.

2.3   *Économique Rationnelle* and Aupetit's contributions to the Walrasian theoretical model, according to Aupetit

Aupetit divided his *Economique Rationnele* section into three major parts, each devoted to a different function of money: unit of accounting, medium of exchange and store of value. In this division, Aupetit already differs from the French tradition of the mid-19th and early 20th centuries, which did not usually mention the last of the functions; for example, Michael Chevalier, Colson and Leroy-Beaulieu, considered that money served only as a medium of exchange and the unit of accounting – or valorimenter, in Leroy-Beaulieu's terminology (Rist, 1966, 324).[57] For Bridel (1997, p. 139) Aupetit's model is the last step in the evolution of Walras's monetary theory, although his contribution to monetary theory *stricto sensu* was modest. Aupetit himself made a disclaimer at the beginning of his work, arguing that on most points the monetary theory was already done and he could not add much. His intention was to contribute with a specific point of the Walrasian model: the difficulty of finding a mathematical interpretation for the human sensations or desires. The difficulty, according to him, was serious – the best Pareto could do, for example, was to dodge it. However, on the contrary, his intention was to face the problem once and for all [*tenté de l'abord de front*] (Aupetit, 1901, p. 35).

The problem of measuring value is then dealt in Chapter II, *Fonction de numéraire*, in the section "La mesure de la valeur". He enquired: is it possible that the *numéraire* could be a measure of value? (83). Aupetit argued, first, that the very notion of "value" had no scientific meaning – it was just a remnant from the time when economists, before developing a technical language, needed to borrow terms from the quotidian vocabulary. According to Aupetit, "value" could be replaced by the dual notion of production cost and utility [*l'intensité du besoin*]. We could, then, develop the monetary study without using this "parasite" term of "value" (85). Aupetit's next step is to prove that the intensity of desire is a measurable quantity. This goal is important for the monetary model since, according to him, before investigating the conditions under which money provides a unit of accounting, we need to determine the measurable quantity itself and how we could measure it (85).

First, Aupetit showed that in the equilibrium state, the price of two commodities is a relation between the intensity of the desire of each one. At this point, he further specified the problem of the measure of value: under what conditions can the *numéraire* provide a common measure of comparison between the desires of the same individual in two different states, or of two individuals in the same state?

Defining the *numéraire* as a commodity, the intensity of the individual 1 desire for it

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[57]The list of money functions is quite old, backing thousands of years. However, during the 19th century, particularly after Ricardo, many authors started ignoring the third of the functions (Rist, 1966, 324). Walras, as part of the many of the changes in thinking about money, started stressing the role of money as a means of providing for the future.

is expressed by:

$$\beta_{1,a} = \frac{k_1}{q_{1,a}} \tag{20}$$

In which k is an unknown constant and, as usual, q is the quantity. According to Aupetit, this particular form of the desire intensity function for the *numéraire* commodity is the analogue of a form developed by Bernoulli in his probability calculations (92). For the same individual in two different states, the relationship is:

$$\frac{\varphi_{1,a}(q) = q'}{\varphi_{1,a} = q} \tag{21}$$

We still need to develop an expression for comparing two different individuals in the same state. The only solution is to assume all members of an economic system to be identical – a certain average type. If we use this assumption in the equilibrium system, we have:

$$\begin{aligned}
p_b &= \frac{\varphi_b(q_{1,b})}{\varphi_a(q_{1,a})} = \frac{\varphi_b(q_{2,b})}{\varphi_a(q_{2,a})} = \frac{\varphi_b(q_{3,b})}{\varphi_a(q_{3,a})} = \cdots = \frac{\varphi_b(q_{\theta,b})}{\varphi_a(q_{\theta,a})} \\
p_c &= \frac{\varphi_c(q_{1,c})}{\varphi_a(q_{1,a})} = \frac{\varphi_c(q_{2,c})}{\varphi_a(q_{2,a})} = \frac{\varphi_c(q_{3,c})}{\varphi_a(q_{3,a})} = \cdots = \frac{\varphi_c(q_{\theta,c})}{\varphi_a(q_{\theta,a})} \\
&\qquad\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
p_m &= \frac{\varphi_m(q_{1,m})}{\varphi_a(q_{1,a})} = \frac{\varphi_m(q_{2,m})}{\varphi_a(q_{2,a})} = \frac{\varphi_m(q_{3,m})}{\varphi_m(q_{3,m})} = \cdots = \frac{\varphi_m(q_{\theta,m})}{\varphi_a(q_{\theta,a})}
\end{aligned} \tag{22}$$

If we understand by V_a the value of the absolute unit previously defined for any individual, we have the relation:

$$\varphi_a(q_{1,a}) = m_1 V_a \quad \varphi_a(q_{2,a}) \quad \cdots \quad \varphi_a(q_{0,a}) = m_0 V_a \tag{23}$$

According to Aupetit, measuring the intensity of desire is thus feasible given the double hypothesis: 1) the function of intensity of desire for the *numéraire* commodity has the particular form k/q; 2) the functions of intensity of desire for all the other commodities and expressed in *numéraire* have for all individuals the same form (95). Aupetit then concluded that "The problem of the measurement of value, in the new form that we have given it, is therefore completely solved and we have, at the same time, justified the hypothesis that the intensity of desire is a measurable quantity"[58](95).

To these ideas, Walras made two objections, recorded in one of his letters to Aupetit. First, for Walras, the individual utility equation of the commodity *numéraire*, as a

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[58]In the original: *"Le problème de la mesure de la valeur, sous la forme nouvelle que nous lui avons donnée, se trouve par là complètement résolu et nous avons, du même coup, justifié l'hypothèse primordiale que l'intensité du besoin is une grandeur mesurable"*.

commodity and as money, is given by:

$$q_{1,a} = \psi_{1,a}(\beta_1, a) + \frac{h_1}{\beta 1, a} \qquad (24)$$

Which can be approximated to $\beta_{1,a} = \frac{k_1}{q_{1,a}}$ given the insignificance of the function $\psi_{1,a}(\beta_1, a)$ with respect to $\frac{h_1}{\beta_{1,a}}$. However, for Walras, it is not possible to easily admit that the parameter k is a constant. The second objection is related to the replacement of a *besoin moyen* – the same for all men – by the desire for a *homme moyen* (Jaffé, 1965, p. 1900). However, despite the objections, it is important to remember the enthusiasm that Walras showed when receiving Aupetit's work, already highlighted in the second section.

A particularity of the pure economics section of the young Frenchman's book is also that, despite having defended that common language is normally not a good expression for economic relations, he translated many mathematical expressions into literary statements: "These formulas are translated by the following statement" [*Ces formules sont exactement traduites par l'énoncé suivant*] is an expression that appeared frequently in his book. Another particular point is that Aupetit was quite aware of the static feature of the presented monetary model: "the true theory of circulation would be a dynamic theory (...). This theory is still to be done"[59] (128). This conclusion shows us that the absence of attempts to dynamize the Walrasian system by his immediate disciples was not due to such disciples thinking that the problem was already solved: it was very clear, for example, for Aupetit, the shortcomes of the model.

After much emphasis in monetary theory on aspects of the money *supply* as a determinant of the value of money, at the beginning of the 20th century there is a growing tendency to emphasize *demand* playing a role in this value – Walras himself being one of the relevant authors in this process. However, we see that Aupetit chose, when approaching monetary theory from the perspective of Rational Economics, to emphasize something different: the measurement of the intensity of desire – given that, as highlighted, he thought that before investigating any monetary problem it was necessary to determine the quantities to be measured and how to measure they. However, it is in the empirical section that he emphasizes the idea – adopted by Walras – that demand is important in determining the value of money.

2.4 Statistical work

The method of synthesis section begins with Aupetit arguing that all experimental study starts with observation and classification (Aupetit, 1901, 169). Therefore, before analyzing data related to the monetary phenomenon, in the first part of this section, he described what money is and its different classifications, what he called the "anatomy

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[59]In the original: "*la véritable théorie de la circulation serait une théorie dynamique (...). Cette théorie est encore à faire*".

of money" [*anatomie de la monnaie*] and the "physiology of money" [*physiologie de la monnaie*]. Once again, the justification is found in other sciences: physics and chemistry also began first with descriptions and classifications (169).

Describing the anatomy of money, Aupetit stated that the first type of money was the "true money" [*vraie monnaie*]. True money was made up of gold, silver or two precious metals, and its main feature was that the value of the coin was not different from the value of the metal (170). The second type of money, the *billon* could be made of bronze, nickel or silver, and the coin value was different from the value of the metal used to produce the coin (170). This division differs from the usual classifications of early 20th century French authors, who used the term money to metallic currency and called fiduciary money simply "notes" (Rist, 1966, 321). By studying the so-called true money, Aupetit analyzes the functioning of a monometallic system in comparison with a bimetallic system. His text is quite descriptive, but many criticisms are explicit: for example, he criticized the currency reform of 1793 – true to the Walrasian tradition of remembering the *assignats* with horror – and called "monetary nihilists" those who believe that a bimetallic system could exist without fixing a legal value for the two metals (173). Addressing the *billon*, he also argued that the countries that had better organized their monetary system used the trust of the population, and not force, for the circulation of it. However, regardless of the system – using force or trust – the issuance of money was a state monopoly: "In both cases the monopoly is essential, and it is moreover universally consecrated in practice" [*Dans les deux cas le monopole s'impose, il est d'ailleurs universellement consacré das la pratique*]. Despite the state monopoly, he emphasized that seigniorage was an immorality and a danger (184). His full analysis, in general, according to him, was limited in the sense that, given the increasing complexity of the instruments used as money, his work on physiology would not include many of the new instruments, because of the limits of the scope of the book.

In a review for the *Journal of Political Economy* in 1901, the Chicago professor Herbert J. Davenport openly criticized Aupetit's book: "the quantitative or quantity theory – which is it? – is accepted practically without discussion and in naïve disregard of the fact, that in other parts of the world this question is seriously, perhaps even bitterly, in controversy" (Davenport, 1901, 149). However, this criticism is unjustified. In fact, Aupetit made it clear that he believed that quantity theory was an oversimplification. For him, first of all, the money stock, as we discussed earlier, is difficult to measure. In this part of the book, he added: the money stock is not just the metallic money in circulation, because there is an important part that is immobilized in the houses, in "wool socks", or in large public or private deposits and "trunks" (192). One of the reasons, therefore, that quantity theory is not actually a exact relation was, for him, the French proclivity of hoarding – proclivity mentioned extensively in the literature of the time. Second, in addition to hoarding, other elements such as banknotes and paper money were added to

the metallic stock (192). Third, it was still necessary to consider the issue of the velocity of money (193). This velocity of circulation of money depended on a lot of factors, such as commercial customs and the and the velocity of circulation of goods (193). The velocity of circulation of money, therefore was "an extremely difficult element to clearly define and determine" ["*un élément extrement difficile à bien définir et à déterminer*"] (193).

Reservations with the quantity theory of money went even further: "The proportionality of prices to the quantity of money appears even more precarious if we no longer consider only the static determination of prices but a dynamic variation in the stock of money, inflation or a contraction crisis".[60] Aupetit also mentioned price rigidity arising, for example, from the existence of contracts, and from the habits of traders, who are often resistant to changing prices (193). There is also a desynchronization of price adjustment depending on the sector of the economy (194).

Still on the issue of the quantity theory of money, Aupetit echoed much of Walras's 1880 work *Théorie mathématique du billet de banque*, in which Walras developed his theory on forced savings and the non-neutrality of money. According to Aupetit, monetary crises always entailed a passive or active crisis of production – although Aupetit did not explicit the difference between passive and active crises (194). The conclusion of the entire section is: "Be that as it may, we are forced to conclude that the law of quantity, assuming it to be theoretically exact, must be combined in practice with a thousand factors capable of more or less completely concealing its rigorous effects"[61] (195).

More than the theory of forced savings, Aupetit also believed the price of money was determined by the demand size, as mentioned. According to the French author, three were the determinants of prices in general, emphasizing that the first two were not independent of each other: 1) the natural relations of exchange between commodities; 2) *l'encaisse desirée*; 3) the monetary stock (196). The *encaisse désirée* – the amount of money people wish to hold – has other synonyms: "*la circulation à desservir*" , or le "*tantième monétaire*". This last term was used first on a book by H. Cernuschi, a French-Italian banker, who wrote a piece called *Mécanique de l'Echange* (191). According to Aupetit, the demand for money was determined by factors such as industrial development and the general wealth of a country (272).

As pointed out earlier, the first part of the empirical section of the work was dedicated to definitions and classifications. After these definitions, Aupetit tried to develop measurements of monetary aggregates – always stressing that only approximations were feasible. In fact, he argued that studies that seemed too accurate, such as those made by

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[60]In the original: "*la proportionnalité des prix à la quantité de monnaie apparaît encore plus précaire si l'on consider non plus l'établissement statique de l'ensemble des prix mais une variation dynamique du stock monétaire, the development of an inflation or contraction crisis*".

[61]In the original: "*Quoi qu'il en soit, nous sommes obligés de conclure que la loi de quantité, à la supposer theoriquement exacte, doit se combinar en pratique avec mille facteurs susceptibles d'en dissimuler plus ou moins complètement les effets rigoreux*".

McLean Hardy in 1895, published in the Journal of Political Economy, were absolutely illusory – Hardy's study presented the variation of the monetary stock of the United States year by year in a very precise way.[62] According to Aupetit, to present statistics as a science so exact, under a scientific label, was to expose it unnecessarily to criticisms (221). According to him, both extremes were bad: condemning statistics for its errors of uncertainty or ignoring those errors (221). He further stressed that the measurement of phenomena is not a goal in itself, but a mean to develop further analysis. Also, more important than the absolute values, were the relationships between the variables (223).

Given that monetary aggregates were difficult to measure, Aupetit's choice is quite clear: try to compile as much statistical data as possible related do money. Aupetit offers then a summary of numerous monetary aggregates, for example, the production of gold and silver, the quantity of these metals used in the industry, coinage rates [*tarif des frais de frappe*], the composition of the medium of exchange (*billets*, gold, *ecus*), quantities of different monetary instruments (gold, silver, paper money, commercial bonds), import and export of gold, the discount rate and numerous price indices.

The empirical section ended with a conclusion about the possibility of having experimental laws in economics. According to Aupetit, the impossibility of experimenting did not prevent astronomy, for example, from fully developing these laws. One of the available manoeuvres in economics was to use periods of crisis: for example, John Law's attempt – who believed that to create money was to create wealth – to establish his bank of issue in France and the *assignats*' "adventures" in the French revolution were a richer source of information than centuries of normal and regulated monetary practices (256). During crises, the abrupt change in the money supply did not allow time for other factors to change, and it was therefore, for example, easier to identify the effect of money stock changes on real variables (272). Despite using the other sciences as an example, the conclusion of Aupetit's book is: "Finishing the book, should we try to make some single and general conclusion? We will not. Scientific work's do not have the same traditional harmony as purely literary compositions. We are not looking for the conclusion of a treatise on physics (294)".[63]

In general, we can observe that it is in the statistical part that Aupetit presented more definitions related to monetary theory: it is when he defined, for example, what is the stock of money and the velocity of circulation. If his aim was to show that the two

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[62]Professor Sarah Hardy was a Chicago fellow at the time. Broadly, in this article published in the Journal of Political Economy, she examined the quantitative theory of money using statistical data. A noteworthy comment is that, writing this present article, it was frustrating to search information about Professor Hardy's and to find academic literature commenting on how beautiful she was and her habit of taking walks with someone else's husband.

[63]In the original: *Parvenus au terme que nous avions assigné à cette étude , devons - nous tenter d'en ramener tous les éléments à quelque conclusion unique et générale? Nous ne pensons pas . Les travaux d' ordre scientifique ne comportent pas la même harmonie traditionnelle que les compositions purement littéraires . On ne cherche pas la conclusion d' un traité de physique (294).*

methods were complementary – analysis and synthesis – he somehow fails to address these basic questions using the mathematical framework. It is also in this empirical section that he draws conclusions about the dynamics of the system, for example, when discussing the validity of the quantity theory of money. As we can see, it is also in this section that he places more emphasis on demand as a determinant of the price of money. This is in agreement, to some extent, with, for example, Bridel, who argued "Walras warns the reader that the agents' demand function for money is simply empirically given"(91).

2.5 Walras's monetary theory applied?

Walras advocated a bimetallic gold system with a regulatory *billon* to stabilize the price level. According to the author, regarding the implementation of his system, "I should leave that to my descendants. Fortunately, the system exists and is functioning. We don't need to get it accepted. It suffices to make sure that it will not be renounced. In these conditions success might be easier; anyway, the attempt would be more attractive" (Walras, 2008, 9). In addition to his bimetallic gold system with regulatory *billon*, Walras also had some ideas of his own about how central banks should work.

As we have seen, in 1902 Aupetit joined the Bank of France as *commis*, was promoted to *chef de service* and reached the position of head of the general secretary. The general secretary was responsible for the administration of the bank, mainly the relations between the institution and public authorities and other French and foreign banks (French Central Bank, 2000). Moreover, it is in the general secretary that economic studies were carried out, making the section the *centre nerveux de la Banque de France* (French Central Bank, 2002).

Regarding Walras's bimetallic system, in theory, Aupetit defended it, at least in his original thesis. First, Aupetit presented the system (Aupetit, 1901, 217). According to him, the objections raised by critics did not seem well founded (218). The only two most serious problems had been exposed by Walras himself. The first was the difficulty in knowing when the *billon* should be introduced. Variations in the production of precious metals seemed an insufficient criterion, for example (218). The second is that the introduction of *billon* raised prices, including the metal that is used for making the *vraie monnaie*, creating a demonetization. The rise in prices would also lead to an increase in imports and an export of money. This, however, could be alleviated with international cooperation (218). As for the implementation of the model, he asked: is the system possible and even desirable?, while the answer is "It is up to art and economic policy to answer these questions" [*Il n'appartient qu'à l'arte et à la politique économique de répondre à ces questions*].

Analyzing to what extent Aupetit, in fact, tried to implement Walras's ideas is a problematic issue, given that most of the documents of the bank's general secretary, at the

time, did not inform the author. However, the bank keeps, for example, a course given by Aupetit in 1909, for the *Société Internationale pour le Développement de l'Enseignnement Commercial*, covering the functioning of central banks, in particular, the French central bank, and applied monetary policy issues (Aupetit, Albert, 1909).

First of all, far from condemning the existence of paper money, according to Aupetit, banking paper was a sign of a more advanced civilization (5) – while Walras, for example, even defended the abolition of paper money (Álvarez, 2020, 5). According to Aupetit, all western civilizations, given the volume of commercial transactions, needed to issue paper money, that were more easily manipulated. Despite criticizing again the *assignats* and the creation of Law's bank, and highlighting the dangers of issuing too much, he argued that there was no problem in issuing more than the convertible in gold. It was the duty of the State to set the minimum amount that must be kept in reserve.

How to determine how much should be kept in reserve? Aupetit mentioned England, in which the limit was fixed according to a reserve held in state funds, and also Germany, in which the limit was three times the reserve. France, in turn, did not have this "artificial" rigor (7). The rule adopted by the country was to use the signature of three people to create credit. Emission had a maximum nominal value that was modified each time "they felt the need" [*"le besoin s'en est fait felt"*]. The legislator was much more concerned with the quality than the quantity of money issued (7). Aupetit further highlighted what he called the superiority of the French system, which was capable of helping other countries in crisis and preventing these crises from reaching France.

The general organization of the Bank of France could be divided into three main points: 1) it was an institution founded by private capital; 2) a strictly national organ; and 3) it was the institution that the State entrusted the privilege of issuance (2). Furthermore, Aupetit also stressed that the bank should accumulate reserves – something already defended, for example, long before, by Thorton, and which forms the so-called "classical theory of banks of issue". Rist (1966, 415) stressed that "between 1870 and 1914, there gradually arose the idea that central banks of issue were and should be first and foremost "central reserve banks"".

Regarding the Bank of France as a private bank, the author stressed that this feature ensured a truly commercial management, and could protect, in times of war, the bank from being plundered by belligerent leaders (4). The Bank of France was a private institution whose goal was to help the commerce and the industry, and never the army: it was an organization independent of the State (4). However, given the danger of the excess of issuance, the state had the right to supervise the bank, and the right to appoint the governor and deputy governors of the institution. The State also had the right to collect taxes from the bank like any other private establishment (8). Further on, the State also carried out operations with the Bank, since the bank kept the Treasury funds: the Bank of France was "*le grand Caissier de l'État* (16)".

Despite being a private bank, Aupetit emphasized that it was also a national bank. As a national bank, it should be concerned, for example, with credit for the local agriculture, helping, for example, the *Caisses de crédit agricole*. The bank must be concerned not only with its own matters, but the interests of the whole nation. In this sense, the author did not criticize the suspension of conversion in moments of crisis (11). In addition to the functions already highlighted, the Central Bank should also be the *Clearing House* of France, the common bank of the French financial world (10): *la banque des banques*. It is much more a rediscount bank than a lending bank [*une banque de réescompte plutôt qu'une banque d'escompte proprement dite*].

Aupetit stated that the defense of competition in issuance was a buried idea, given that monopoly was the universally accepted concept in modern states (6).[64] A controversy in France, but in Aupetit's time, was whether the issuing bank was able to control the discount rate or not (Rist, 1966, 408). For Aupetit, the Bank was not able to fix the rate, but only "recordes" it. The rate is freely set in the financial markets. Given the mobility of capital between countries, the Bank had a limited role in maintaining a stable rate (15).

Although he did not criticize the suspension of conversion in moments of crisis after the First World War, together with Georges Robineau, as the bank's governor, Aupetit was one of the economists who believed that it was possible to restore the franc's pre-war value. However, the fiscal sacrifices imposed for this goal made the measure impractical and Georges Robineau was fired. According to the archives of the National Library of France, with Robineau's resignation, Aupetit realizing that the Bank's independence was compromised, resigned. The few records about Aupetit show that he went to work in private banks in the following years, and that in 1936 he applied for a post as professor of Political Economy, given the decease of Auguste Deschamps, but that he later withdrew his candidacy (Académie des Sciences Morales and Politiques, n.d.) .

2.6   Concluding remarks

Studying the Walrasian monetary theory is not a simple task, as it involves his many publications, related not only to his theoretical model of general equilibrium but also to his ideas on applied monetary policy. These ideas not only changed a lot over the course of his life, but they were often conflicting. This article focused, first, on showing Aupetit's contributions to the Walrasian model from the point of view of the author himself, who considered that his main contribution to the issue was the measurement of value. We also analyzed his dialogue with Walras about these theoretical developments, recorded

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[64]The French Central Bank acquired the monopoly right in 1848, although under criticism of authors such as Michel Chevalier and Courcelle-Seneuil. Walras, in turn, after supporting the free enterprise of bankers, in his *Théorie Mathématique du Billet de Banque*, formally pronounces himself against the freedom of issuance (Jacoud, 1994, 278).

in his exchanged letters. The article also highlighted that Walras saw that the next step in his monetary theory would be in the empirical field, and we highlight Aupetit's exhaustive work of compiling statistics related to the monetary phenomenon. Given how much Walras's ideas have changed over the years – and the multitude of monetary ideas that can be attributed to him – studying Aupetit, the first and for a long time only French disciple interested in Walras's monetary issues, may elucidate how his ideas were initially assimilated by the subsequent generation after Walras. We saw, for example, that Aupetit was quite aware of the shortcomes of the model and the criticisms made at Walras's ideas. Aupetit pointed out these problems and tried to respond to some criticisms, such as those directed at the Walras's bimetallic model with regulating *billon*.

Aupetit's main book is also surprising because of its opening chapter, which, far from being related to the study of money, is an elaborate defense of the use of mathematics in economics. There were few authors at the time who defended the use of mathematics in economics in France. At the time of Aupetit, however, the articles published by Bouvier in the *Revue d'Economie Politique* became famous – and Walras himself mentioned them in his correspondence. However, we have seen that Aupetit appears as a much more conciliatory author in his defense, speaking, for example, of the importance of building bridges between the different methods – and how the construction involves the goodwill of both sides – and, unlike Bouvier, without directly mentioning the names of authors with whom he disagreed with.

Despite his tone seeming more conciliatory than that of contemporary authors, it is Aupetit's personality that appeared as one of those responsible for his failure in teaching competitions. Given that Walras had prepared his teaching material for Aupetit to use in French universities, this failure is, in fact, a relevant factor in the history of the dissemination of Walrasian theory in France. However, Walras's letters do not really allow us to conclude the ultimate cause of Aupetit's failure, given that, as we have seen, his theses on the aggregation contests covered quite different subjects, such as the culture *maraîchère* and the plus value of city lands. One point in Aupetit's professional career, however, is quite interesting: the governor of the Bank of France read his thesis and invited him to work at the bank. The banking sector apparently did not care about theoretical divergences from French academia, or were not averse to the use of mathematics in economic analysis.

Walras, as highlighted at the beginning of the chapter, saw his monetary theory as an important way of applying his ideas. However, despite defending Walrasian ideas in theory, the documents available from Aupetit as an employee of the French bank show that he differed from the master on several points. For example, he was in favor of issuing more than the value in gold, although his career at the bank ended when he resigned in favor of the restoration of the franc. There is little information about his life after this episode. Newspaper records show that he worked in private banks and considered

applying for academic positions. Being in favor of the specialization of labor in science – dividing the world between theorists and practitioners – Aupetit, who started his career as an academic and was barred from being a professor, is forced to stay in the world of applied theory.

3 An economic advisor *manqué* finds a follower: Léon Walras and Étienne Antonelli

"Le titre de M. Antonelli à l'actualité, c'est donc d'être descendu un jour de la région sereine des idées et des théories, d'avoir répudier aussi cette littérature ennuyeuse qu'est l'économie politique, et de nous avoir donné – et sans le vouloir peut-être – une précieuse leçon non de droit, mais de réalité parlementaire"

– *La Revue Universelle*, 19-.

"Lorsque la tempête bat le navire, il importe moins d'expliquer et de comprendre que d'agir et de prévoir. Le savant ne peut pas, ne doit pas être un simple contemplatif, mais un homme d'action, se mêlant à la vie"

– Étienne Antonelli, *Autour d'une Renaissance Proudhonienne*, 1910.

"E. Antonelli est en dialogue permanent avec Léon Walras. Dans la mesure où l'on choisit ses maîtres, ce choix était courageux"

– François Perroux, *Étienne Antonelli et l'analyse scientifique des phénomènes économiques*, 1953.

Abstract

Léon Walras's letters show he spent much of his life trying to arouse interest in his ideas and theories. Attempts in his home country, France, though, were even less fruitful than in other places. Nevertheless, there were exceptions and this paper aims to present an outstanding case in the French community: Étienne Antonelli. Still young, Antonelli proclaimed open affiliation to Walrasian theory. This affiliation, however, made his academic life more difficult – he failed twice in the examination to be a Professor *agrégé*. I conjecture that he reacted to these failures by looking for alternatives, and he dedicated himself to two other activities: policy-maker and the popularization of economic knowledge. The purpose of the paper is to analyze how Walrasian ideas influenced these last two activities. I went through some newspaper material, available documents published by Antonelli, some selected documents written by Walras and Walras's exchanged letters. Throughout his life, Walras made occasional excursions outside the Ivory Tower and defended the formation of mutual assistance cooperatives and increasing workers' share in capital formation. Antonelli became a politician in 1924, and he was one of the chief authors of the law that established modern social security in France. He also wrote several best-sellers translated into many languages, and he tried to promote a law that would facilitate workers' participation in industrial capital. The literature about Walras points out that there was, after his death, a period of "quiescence" in the development of Walrasian agenda. Nonetheless, this paper shows that, at least in the case of France, perhaps the main followers just were not in the academy, and Walras's ideas were being applied in other spheres.

3.1 Introduction

In a letter of January 1901, Walras wrote, about his attempts to find a follower in France, that he felt like a man in love, who promised to himself not to look for the beloved one anymore, but at the first sight of hope, he could not help himself.[65] Walras was engaged in trying to find support and recognition for his work, and he spent almost three quarters of his inheritance in publishing and disseminating his books. Although at the end of his life Walras found some recognition in other countries – for instance, the American Economic Association elected him an honorary member in 1892, and the International Statistical Institution in Rome nominated him member in 1887[66] – he could find no one who would carry his legacy in his home country, France, despite his many efforts.

In his 1975 article, "Léon Walras, an Economic Advisor *Manqué*", Jaffe (1975) outlined that the French author, although many times without being invited by the public authorities, also attempted to contribute to the policy-making process. However, all the times Walras engaged himself in such activities he encountered, at the best, a polite refusal. It is well known that the French, as his father, defended policies as the nationalization of all lands and the abolishment of taxes (Walker, 2006), but it is less mentioned that furthermore Walras attempted to elaborate some law projects: for instance, he proposed a reform of the French Trade Code, to include legislations about mutual societies of credit, production and consumption (Walras, 1865).

According to Jaffe (1975, 822), given the failure of his endeavors to influence public policies, he resigned himself to defend that, after all, the division of labour between theoreticians and practitioners was useful. Later in his life, Walras proclaimed that "it was always with the greatest reluctance that he ever entertained the idea of taking part in public affairs, and then only under special circumstances" (822).[67]

Even though Walras died in 1910 thinking he had no dedicated follower in France, this paper aims to present one outstanding counter example: Étienne Antonelli. Still in the beginning of his carrier, he proclaimed open affiliation with the Walrasian theory. Yet in 1910, he wrote a paper mourning the fact that Walras was recognized in several countries, but in France his ideas were not respected. According to Antonelli, Walras was criticized because his opponents did not comprehend his work (Antonelli, 1910b, 175). For this reason, he decided to change this scenario and explain the ideas of Walras.

Before his death, Walras had prepared his *Abrégé*, a material aimed to be used in mathematical economics courses. Commenting on what happened to this manual after

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[65]Letter to Georges Renard (Antonelli, 1914).

[66]This recognition came about a decade after the publication of his main work, in 1874, the *Éléments d'économie politique pure, ou théorie de la richesse sociale*.

[67]Jaffe's thesis in this article is that, although Walras and his father endorsed an individualistic view, the fact that Auguste Walras called it "socialism" and the son called "scientific socialism" was enough for Walras being disqualified in the public administration arena.

Walras's decease, Walker (2006, 248) mentions that "Walras's daughter, Aline, gave the abridgement to Étienne Antonelli, who, in 1911, became the first person to use it in a classroom in France". Actually, more than the *Abrégé*, Antonelli was responsible for the preservation of a great part of Walras's correspondence upon Aline's donation (Jaffé, 1983, 17). He was also the first to publish some letters from Walras. Antonelli justified the effort by sustaining that Walras's correspondences were the richest source of documents about modern doctrines of pure economics (Walras, Cournot, Jevons, & Antonelli, 1935, 119).

The avowal to the ideas of Walrasian theory, though, made Antonelli's academic life troublesome. In 1910, in his lecture of the *concours d'agrégation*, the examination to be a Professor at the *Faculté de Droit de Paris*, he defended the development of the pure abstract economics, using mathematics as a language. In his turn, Cauwès, president of the jury, told him that while he stayed as president of the jury, Antonelli would never become a Professor *agrégé*[68]. Antonelli asked the reason of this objection, and Cauwès replied that Antonelli was trying to develop the science in a manner that he did not appreciate (Morini-Comby, 1953, 358). After this episode, Antonelli failed one more time in the aggregation examination.

Antonelli eventually became a Professor – Cauwès himself would later appoint him to a place – and wrote several *textbooks*, gave many courses and taught students as François Perroux. Nonetheless, this initial difficulty possibly made him search for alternative activities, and Antonelli also made notable contributions to the policy-making. He was one of the main founders of the *Comité de Démocratie Sociale*[69] and was elected congressman in 1924. As a politician, he wrote the law that established the social security system in France. Moreover, he also worked to explain this new system to the population, writing a guide in 1928, the *Guide pratique des assurances sociales*, that was reprinted three more times. Antonelli also worked in the Ministry of the Colonies, where he helped preparing the report that was used as base for the French negotiations in the Treaty of Versailles. Again, after his experience as policy-maker, he wrote a volume meant to be read by a wide audience, *L'Afrique et la Paix de Versailles*. Another publication that he wrote, whose public was not just his fellow academicians, was *La Russie Bolchéviste*, a book published after he returned from the war, in 1919. The work had ten editions and was translated into English, Portuguese, Italian and Spanish.

We can see, then, that Antonelli was a man involved in public affairs. Nonetheless, we also can see that he was involved in another activity: turning knowledge accessible to a wide audience. Therefore, this paper aims to analyze how the ideas of Walras influenced

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[68]This selection of full professors in France, called *agrégation* provides a lifetime position in the university.

[69]The Committee aimed to 1) study social reforms that could be carried out in the short run, 2) create ways to advertise these reforms and 3) bring these reforms to the Parliament for approval (Milhau, 1953).

Étienne Antonelli as a policy-maker and as a knowledge broker[70]. Antonelli was also a socialist influenced by Proudhon – whose ideas Walras criticized in one of his first works in economics (Walras, 1860). This paper, therefore, has a secondary goal to analyze how apparently these two conflicting influences – Walras and Proudhon – conjoined in the thought of Étienne Antonelli[71].

Among the historiography of Walrasian theory, Walker (2006, 14) wrote about Walras's followers in the period shortly after his death, and mentioned Wicksell, Barone, Moore, and Schumpeter as his continuators. Walker (2006, 14) also explains that, after the development of this Walrasian strand, a period of "quiescence" followed the Walrasian research during the 1930s. Therefore, this work aims to contribute to the broader literature on the dissemination of Walrasian theory after Walras's death, and including this period of "quiescence". This study also aims to extend Perroux's 1953 paper, which underlined the dialogue between the work of Antonelli and Walras (Perroux, 1953)[72]. Finally, this article aims at addressing Forget (2020) invitation to give space in the history of economics to understand the complex task of knowledge brokers in our society.

3.2   Étienne Antonelli: Early life

Étienne Antonelli was born in 1879, in Spain, into a family of wine merchants. The family's activities explain the theme of his first doctoral thesis, *La protection de la Viticulture par l'État*. The thesis was presented in 1905, and it studied the French legislation regarding wine's production protection. Antonelli emphasized that, when protecting a branch of activities, the government should not harm other interests. However, in France, such protection was undermining the wine traders (Antonelli, 1905, 3). To understand this first criticism of policy-makers made by Antonelli, it is necessary to understand the issue. In 1870, there was an agricultural plague, the phyllorex, that destroyed many wineries in Europe, mainly French. France then needed to import many wines from places not affected by the plague, giving opportunity to a new group of wine-importing merchants to profit – of which we assume that the Antonelli family was a part. With the end of the plague, French producers had to face this new competition in the domestic market. The French government then acted to protect local producers by imposing a series of restrictions on wine imports. However, according to Antonelli, not only were merchants

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[70]About knowledge brokers, Forget (2020, 1) asserts that "they popularize economic insights in best-selling books, and work for think tanks or with various organizations, but they are also textbook writers, journal editors, and conference organizers, all of whom jointly contribute to the edifice of economic knowledge and facilitate communication". Hayek discussed a similar concept, but calling them "second hand dealers in ideas". According to him, some intellectuals have a decisive weight in shaping the ideas that become public policies.

[71]It is worth remembering that for a period, Proudhon was also a member of the French Parliament.

[72]Perroux aimed to analyze the dialog between Walras and Antonelli focusing on three specific points: 1) the relationship between the social milieu and the economic system; 2) the pure economics of the static and dynamic systems and; 3) the division of economics in pure economics, applied economics and social economics.

being harmed. Consumers were also suffering – deprived of a larger offer at a lower price – and despite having more disperse interests than producers, they should be taken into consideration by policy makers (4). Antonelli also defended that such protectionist policy was the outcome of a "prestigious" economic theory, but he didn't specify what was this theory (16). According to the French author, this legislation was creating a monopoly with the sacrifice of the rest of the population (116). However, Antonelli did not make any argument about monopoly based on Walras's ideas – for whom the existence of monopolies was justified only in the case of natural ones.[73] Therefore, it is not possible to say whether Antonelli had already had contact with Walrasian theory in his student days. Despite the statements about the legislation of French viticulture, Antonelli concluded the book underlining that the issue was challenging, and there was no unique formula for solving the question – only a few ideas that could eventually guide the legislator (Antonelli, 1905, 223). The ideas about the economic and legal problems of wine production's legislation reappeared in a book published in 1908, *La législation sur le mouillage et le sucrage des vins.* At the time, Antonelli was a lecturer at the *Faculté de Montpellier*, and he wrote the book together with E. La Clavière, an official at the French Ministry of Finance (Antonelli & La Clavière, 1908).

Explicit references to Walrasian theory appeared for the first time in Antonelli's 1910 work, the same year of Walras's death. Antonelli wrote an article about Walras's life and scientific production, published in the *Revue d'histoire des doctrines économiques et sociales.* This 1910's paper is important for us because it stresses which parts of Walras's ideas, in Antonelli's view, deserved to be highlighted. He started the work comparing Walras to Cournot and criticizing the French community – who should be proud of Walras's work – for not recognizing his value: "A great spirit, aware of his value, misunderstood like his master Cournot, by the very people who should have been most proud of his glory. He peregrinate through a long life; a life made of obstinate and serene scientific work, always carrying a bleeding wound of a strong soul, stiffened against unjust fate" (Antonelli, 1910b, 169).[74] Given Walras's important contribution to economic theory, Antonelli underlined it was shameful that, until then, there was not a single French professor teaching his ideas and lecturing political economy using mathematics (174).

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[73]Walras worked as a young man at a railway company and used the example of railways to approach monopoly police. Despite contributing by definiting terms, the transition from Walras's pure economic theory to applied economics, in this case, was problematic. For Walras's discussions about monopoly theory applied to railways, see Ekelund and Hébert (2003). Walras also accepted the existence of monopolies in cases in which the usefulness of the goods could not be appreciated by individuals alone, but only by the collectivity as a whole – the moral monopolies (N. G. Béraud Alain, 2019). Later, Antonelli also mentioned that in every organized state there were activities that, by their very nature, were of public responsibility, criticizing Smith and Leroy-Beaulieu for refusing to admit these special cases (Antonelli, 1966a).

[74]In the original: "*Grand esprit, conscient de sa valeur, incompris, comme son maître Cournot, de ceux-là mêmes qui auraient dû se montrer le plus fiers de sa gloire, il promena à travers une longue vie, faite toute de labeur scientifique opiniâtre et serein, la toujours saignante blessure d'une âme forte, raidie contre le destin injuste*".

Antonelli highlighted Walras's defense of the cooperative movement, and cited the three lessons he gave in 1865 to the *Cercle des Sociétés savantes.* On this occasion, Walras presented his law project on popular associations and stated the purpose of them: the possibility for workers to take part in capital formation and to gain the revenue provided by this capital. According to Walras, this was the very definition of progress: the participation of all citizens in all sources of revenues (Walras, 1865, 14). Walras indicated that there were three main types of cooperatives: consumption, production and credit.

Associations of consumption could buy essential items in large volume, which would then be resold to the families of the cooperative at a lower price. Production associations could use the capital, as the name suggests, for the production of a given industry, and credit associations could lend money for different purposes, including the personal needs of members. There were two ways to form the capital of the associations, according to Walras: the workers could pay a monthly contribution, or they could obtain external funds – funds of non-members. The legal challenge, however, was to guarantee the legal right of these third parties in relation to the association, and the legal right of the members in relation to each other. For Walras, the existing legislation was failing to adequately contemplate the case in which capital came from third parties. The purpose of 1865 Walras's work was, then, to analyze the different types of associations covered by French law. The two types existing at the time were the association with limited liability and the association with joint liability. Walras proposed the adoption of a third type, the association with proportional responsibility. For him, there should be a proportional distribution of profits and a proportional distribution of all responsibilities to all partners, according to how much they had contributed to the total capital (Hébert, 1988, 259).

After affirming that a reform of the legislation was essential, Walras also pointed out that it was an obligation of political economy to indicate these important changes to legislators. Walras concluded the work stressing that a reform was needed to prevent a revolution:

> It is indeed, gentlemen, a consideration of which I am deeply determined, and which, no doubt, you will share with me that any reform of the Code must be made, as much as possible, in accordance with the spirit of the Code, and in accordance with the requirements of political economy; because otherwise it would not be a reform, but a revolution, and if nothing is more illusory than a reform when a revolution is needed, nothing is more foolish than to make a revolution when a reform is enough (Walras, 1865, 103).[75]

[75] In the original: "*C'est en effet, Messieurs, une considération dont je suis, pour ma part, très-pénétré, et que, sans doute, vous partagerez avec moi, que toute réforme du Code doit être faite, autant que possible, conformément à l'esprit du Code, en même temps que conformément aux exigences de l'économie politique; car autrement elle serait non une réforme, mais une révolution, et si rien n'est plus illusoire que d'accorder une réforme quand il faut une révolution, rien n'est plus sot que de faire une révolution*

According to Walras, through the two ways to improve the living conditions of the working class, mutual insurance and cooperative associations, the poor could improve their lives without the help from others – unlike philanthropy (Walras, 2010, 43). This quality of the human being in control of his own destiny will appear, as we will see, also in Antonelli's thought.[76] Additionally, according to Walras, men only need an initial incentive, because when faced with the choice between two sides they tend "more and more to choose the better part" (Walras, 1954, 55). Still according to him, mutual assistance associations could protect them from diseases and disasters. Consumer cooperatives could reduce spending; production and credit associations could increase income; and the three – consumer, production and credit cooperatives – could make savings easier. If savings started, it could be quickly multiplied by capitalization and then, according to Walras, "poverty is already conquered" (Walras, 2010, 43).

Before we continue the analysis of the 1910 paper written by Antonelli, which highlighted Walras's main works according to the young French, and mainly this important Walras'1865 work – some other points about Walras's involvement with the cooperative movement deserve to be mentioned. Walras became interested in the middle of 1863 on the cooperative issue, from reading the works of Anselme Batbie[77], and also inspired by the German model of Hermann Schulze-Delitzsch of popular banks, based on the principle of mutual assistance (Hébert, 1988)[78]. Despite the fact that cooperatives were a very popular subject in 19th century France – and also in other countries – Walras was a singular author, with a particular economist view of the issue (257). Walras disagreed even with the Schulze-Delitzsch and Lassalle movement, mainly because they argued that the State could help cooperatives financially: for Walras, they should be autonomous (262). He disagreed with Proudhon and his disciples, the "empirical socialism", because of the defense that cooperatives should lend money without charging interest for it (262). For Walras, the cooperatives had as main objective to increase savings, and not be a new organization of work, or a new form of social revolution (263). He also strongly advocated that market mechanisms should be respected, for example, encouraging competition (261). His economic view, therefore, isolated him from other cooperative movements: no cooperative school claimed his ideas, not even the *École de Nîmes*, founded by Charles Gide, close friend of Walras (262).

Now we go back to Antonelli's 1910 paper. Antonelli also noted that Walras wrote an article in 1859 refuting Proudhon's theories, but the youngman made no further comments specifically on the ideas discussed in this 1859 article (Antonelli, 1910b, 170). He

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quand il suffit d'une réforme".

[76]But Antonelli did not highlighted, as Walras did, that this control of destiny was derived from man's ability to the division of work.

[77]Batbie was a French economist, associated with the University of Toulouse and also elected deputy and senator. The author wrote a three-volume treatise on political economy.

[78]Schulze-Delitzsch was a German politician and economist influenced by Ferdinand Lassalle, and studied law at Leipzig University.

highlighted the role played by Walras as administrator of the *Caisse d'Escompte des Associations Populaires*; and he also commented Walras's role as editor, with Léon Say, of the cooperative movement's newspaper *Le Travail* (171).

Addressing more specifically the Walrasian theory, Antonelli remembered that Walras's work was formed by three distinctive parts – pure economics, applied economics and social economics (174). The problem of his critics was not understanding this separation and condemning Walras's economic theory as a whole (175). While pure theory did not depend on time and space, recommendations of applied economics did. But, in some moments, Antonelli condemned Walras himself for generalizing considerations on applied economics (182). However, this assertion made by Antonelli, that pure economics did not depend on time and space, also contradicts his claim that general equilibrium theory – which belongs to pure economics – applied only to the prevailing economic system, the capitalist.

Antonelli remarked that French policy-makers could use Walras's ideas about applied economics, as well as his considerations about cooperativism and its monetary ideas. Antonelli even highlighted that six years before being implemented, Walras was already defending the decision of suspending the free coinage in India (181). Antonelli concluded his 1910 article celebrating Walrasian theory, and arguing that some of Walras's followers were unable to perceive the sensitive humanity of the author's work, and that the Walrasian theory was an eloquent protest against the selfish ideas of all schools of *laissez-faire* (190).

In 1910, Antonelli, as previously mentioned, competed in the *concours d'agrégation* of the *Faculté de Droit de Paris* and ended his presentation by making a fierce defense of the study of abstract economics and of Walras's ideas:

> But, gentlemen, in this economic science each day closer to the concrete social reality, each day closer to being realistic, in the broad sense of the word, there is room, there must be room, for an abstract science of pure economics, which will serve as a framework for applied economic science, that is, practical science [...] As applied economics becomes more realistic, pure economics must become more mathematical.
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> These ideas have entered France. A committee has recently been formed – which includes members of the Institute, professors at the Collège de France, and it is chaired by a professor of the *Faculté de Droit*, to perpetuate the memory of the lessons of the great unknown economist that was Léon Walras [...] The time has come for you to decide if these theories should be represented in French Higher Education, as they are in all other countries. For my part, I trust your scientific judgment.[79]

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[79]In the original: *Mais, Messieurs, à côte de cette science économique chaque jour plus rapprochée de la réalité sociale concrète, chaque jour plus proche du fait, soit réaliste au sens large du mot, il y a place, il doit y avoir place pour une science abstraite d'économie pure qui servira, en quelque sorte, de squelette à*

As already mentioned, Cauwès, head of the jury, strongly denied the chair to Antonelli. Before going to fight the World War I – when he was severely injured – the French author also wrote a new work in 1912, *Les actions de travail dans les sociétés anonymes à participation ouvrière*. In this new book, Antonelli argued, as did Walras, in favor of a substancial reform in order to avoid a rupture of the capitalist system. The reform, in this case, would involve increasing the labor share in the formation of industrial capital. This system in which workers have a bigger share of capital would be a "new equilibrium of forces", and this new equilibrium would be achieved through the gradual and reciprocal action of the elements of the system, through the *tâtonnement* method (Antonelli, 1912b, 88). Then, through this reciprocal action, it would be possible to transform the spirit of the authoritarian Napoleonic social organization into a new democratic union organization (16). We see, therefore, that Antonelli directly applied notions developed by Walras in the field of pure economics – the mention of an equilibrium and the *tâtonnement* process – into an applied problem – to increase the participation of workers in the formation of capital.

One characteristic of the capitalist system was the increasing social distance between workers and owners, and workers' participation in the formation of capital was a way to approximate the two groups. Capitalists and proletariats were like combatants, who would like to fight directly in a decisive conflict, but thought that probably they would break their backs if they did so. Then they approach each other slowly, for the last battle, but when they are finally close enough, they realize that the very combat's reason – the distance between them – had disappeared. The new legislation that facilitates worker participation in industrial capital was one of those small steps of approximation between them (Antonelli, 1912b, 17).[80]

An important point is also to understand with whom Antonelli dialogues in the book. The preface was written by Aristide Briand, who would be six times the French Prime Minister[81], four times the French Minister of Foreign Relations[82], three times the Minister of Justice[83], and would win the Nobel Peace Prize in 1926. The book's ideas – reforming

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la science économique appliquée, c'est-à-dire à la science vivante [...] A mesure que l'économie appliquée devient plus réaliste, l'économie pure doit devenir plus mathématique. Ces idées ont pénétré en France. Un comité s'est formé récemment – qui comprendra des membres de l'Institut, des professeurs au Collège de France et qui est présidé par un professeur de la Faculté de Droit – pour perpétuer la mémoire de l'enseignement du grand économiste méconnu que fut Léon Walras [...] Vous penserez peut-être que le moment est venu pour vous de décider que ces théories doivent être représentées dans l'Enseignement supérieur français des Facultés, comme elles le sont dans tous les autres pays. Pour ma part, j'ai confiance dans votre scientifique perception.

[80]This defense of increasing workers' share, the *actions de travail*, would become the main characteristic of the republican-socialist program of France from 1911 to 1934, the element that used to identify the party, even with its successive changes (Billard, 1996).

[81]He was prime minister in the 1909-1911 period, two months in 1913, 1915-1917, 1921-1922, 1925-1926, and five months in 1929.

[82]During 1915-1917, 1921-1922, 1925–1926, and 1926–1932.

[83]1908–1909, 1912–1913 and 1914–1915.

the legislation in order to help increase workers' capital share – were developed in 1909 within the *Démocratie Sociale Comité* – the committee created to study social reforms – presented to the Parliament by Briand and transformed into a book by Antonelli. Briand argued that in democracies, new laws did not emerge from a magic wand: it was necessary a gestation period, in which the laws were submitted to public opinion. That was, then, according to Briand, the important role that Antonelli's book was playing.[84] The law, however, was criticized not only by the conservative part of the Parliament but also by socialists, such as Raoul Briquet (a congressman and member of the *Section française de l'Internationale ouvrière*); and by unions who accused the proposal of jeopardizing their power.

Two points are still relevant to be addressed about this 1912 book. The first is the technical question of the project. The central idea was that any anonymous association could become an association with worker participation. The companies would benefit from this because they would have preference in contracts with the French government. In return, a quarter of the company should be owned by the workers, as well as a quarter of the company's management should be run by a council elected by the workmen. Each collective of workers could choose what to do with their share of the dividends, for example, they could distribute it or use it for a sickness insurance fund. We see that, unlike Walras, the purchase of shares would not be deducted from the workers' wages: these would earn their share by right. The second point is that, in addition to discussing the technical issue, an important part of the work was devoted to defending the project from an economic point of view. Many socialist critics argued that these associations were aiming at extraordinary profits – which was to some degree immoral, according to them – and not just the return on capital, but Antonelli engaged in a defense of profit as a mean of encouraging investment, and battled against the idea of socialization of profits. But also unlike Walras, who saw cooperatives as only a means of increasing savings, and not a new way of organizing work, Antonelli suggested that cooperatives would be a mean of changing the way in which society was organized, given that the then prevailing regime in which there were wage earners would not last forever.

A brief comment on the reception of this 1912 book is that, still in the same year, we can already find an English review of the work, which may be an indicator of the extent of its circulation. Fay (1912) stated that, given the proposed incentives, and the preference in contracts with the Government, the companies imagined by Antonelli, "resemble those herring boats of which Adam Smith said that they were fitted out to catch not the fish, but the bounty" (Fay, 1912), raising questions, therefore, about the economic efficiency of these companies owned partially by workers.

In 1914, Antonelli wrote the book *Principes d'économie pure - La théorie de l'échange*

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[84]Antonelli also used to write Briand's speeches, who was known as a politician who did not liked improvisation and preferred to use ready-made texts.

sous le régime de la livre concurrence, which reproduced a course given two years earlier by him at the *Collège libre des sciences sociales*. According to Antonelli, the book was a faithful exposition of Walras's doctrines of pure economy. A few years before his death, Walras had prepared a version of his work to be used in classes, the *Abrégé*. Initially, the material was made by Walras for Albert Aupetit, but Aupetit never actually used the material, as we saw. Walras also tried to translate the document through one of his American followers, Henry Moore, but with no success, as we also addressed. It is Antonelli, then, who not only used the *Abrégé* in the classroom for the first time, but also transformed it into a published book. The reception of the work, however, was not much better than Walras had encountered years before. One review stated, also written in English, that "the neatness and conciseness of the exposition here set forth on the lines laid down by Walras do not convince us that recent developments in this direction, in other countries more noticeably indeed than in France, are devoid of serious danger" (P, 1914, 884). Georges Renard, who wrote the book's preface, remarked that although it was hard to admit it, in the beginning of the 20th century in France, Antonelli's publishing the book was something quite courageous (Antonelli, 1914, vii).

Despite the reception of this 1914's work, when he returned from the war in 1919, Antonelli wrote another book, *La Russie Bolchéviste*, and it was a best-seller translated into several languages. An interesting point is that the first sentence is a claim that the work made an unbiased presentation: "Reader, this book is honest; there is in it neither interest nor prejudice. It says what is. It lets men and facts speak. It wants to be about history, it is not a pamphlet" (Antonelli, 1919, 5).[85] Although in fact the work is quite descriptive – addressing the political, economic and legal characteristics of Bolshevik Russia – inevitably, we can find some value judgements in the work. For example, in concluding, Antonelli pointed out that in Russia people thought that they had overthrow slavery, but in fact, it was still present, in different ways. He further pointed out that "I do not believe that Bolshevism is a viable social regime. We cannot establish a real society if this society is against culture and intelligence" (Antonelli, 1919, 272).[86] Further, Antonelli argued that the country symbolized the opposite of the socialism he believed in, still being, as it was at the time of the czars, a dictatorship ruled by a small group.

This idea of presenting questions impartially also appeared two years later in his work *L'Afrique et la Paix de Versailles*, in which he argued that the book only presented the information necessary to illuminate the question, and was not intended to be used to indoctrinate, but only to make readers think about (Antonelli, 1921, ii). The book also aimed to be read by a larger audience than the academy. Its goal was to show the role

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[85]In the original: "*Lecteur, ce livre est honnête; Il n'est à la solde ni d'un intérêt ni d'un préjugé. Il dit ce qui est. Il laisse parler les hommes et les faits. Il veut être une histoire, il n'est pas un pamphlet*".

[86]In the original: "*Je ne crois pas que le bolchévisme soit un régime social viable. On ne fonde pas une société contre la culture et l'intelligence*".

that colonies, especially in Africa, could play in France, and in the civilized world. One problem, according to Antonelli, was that the question did not arise interest in the society in general, as it should have. Until then, the colonization problem was something that occupied only theorists, publicists, administrators, capitalists of the African market, and all other "parasites" (Antonelli, 1921, i). However, again, despite claiming that the book was presented without bias, Antonelli, for example, argued that the Treaty of Versailles was full of contradictions, uncertain and imprecise. Antonelli also criticized the feeling that Africa was a cake that could be cut into pieces and distributed. There were social, geographical and historical questions that should be respected. However at the same time he also defended that the "African French" should be organized, with urgency. Therefore, it is difficult for the reader to conclude whether Antonelli was in fact against or in favor of French colonization policies in Africa.

This idea of trying to write books without bias, however, ended up being criticized by the community. Regarding the manual about colonial legislation, Antonelli complained about one criticism made by a "professor that he respected very much". This professor condemned him precisely for leaving aside "questions of principles". Antonelli argued that for many, it seemed disappointing to find a book that only presented ordered and classified facts, but he would not give up continuing producing books with the same intention of being impartial (Antonelli, 1927, xv).

Antonelli wrote many pieces throughout his life. But in this first period, before becoming a politician, he wrote two papers that deserve to be mentioned. The first is a criticism of recent books that addressed socialist theory. In it, Antonelli commented Aftalion's work, *Les fondements du socialism*. Aftalion criticized socialist theory and proposed a mixed regime, a "socialisme rectifié". Aftalion's intention in proposing this ideal socialism was, in a way, to reconcile marginalism with the French socialism of Proudhon and Saint-Simon (Frobert, 1999). Antonelli then criticized Aftalion's essay by accusing him of using terms from one economic system, such as productivity and profit, to discuss another economic system, the socialist system. However, ultimately, what Aftalion proposed – to improve a given economic structure – is not exactly different from what Antonelli defended in general, that is, a reform of the capitalist system in order to adjust some issues. Furthermore, as we have seen, Antonelli defended the idea that economics was divided into pure economics, social economics and applied economics, as did Walras. For him, the conclusions of pure economics did not depend on time and space. Therefore, if we start from the idea that concepts such as productivity and profit belong to the sphere of pure economics, there is nothing wrong – even by Antonelli's conceptions – in using them in different economic systems, because they are conceptions that did not depend on time and space.

Furthermore, Antonelli ended the article, commenting that the most recent theories were introducing more and more a "spiritual element" into social life, and making theories

more "vivid" (*vivante*) (Antonelli, 1923a, 690). This new trend in theories was counterbalancing the "mechanistic spirit" that existed until then. Antonelli argued that this new trend was a desirable development, because it should exist an equilibrium between these different forces – this new trend with theories more vivid, and the old spirit mechanistic. One remark that we can make is that the last book that Walras wrote, a year before his death, was the *Économique et Mécanique*, in which Walras tried to show precisely the parallelism between economics and mechanics. Walras, therefore, spent his last effort, even when severely sick, to show that economics was permeated by this "mechanistic spirit".

Although the meaning of "vivid" (*vivante*) for Antonelli, is somewhat obscure in this article, the rest of his work illuminates the question. For example, in his 1910's article, Antonelli condemned authors who restricted themselves only to the domain of pure economics, "anatomists working with death", who forgot the social part of analysis. The main mistake of Walras's followers was exactly that they did not perceive Walras's sensible and **vivid** humanity (Antonelli, 1910b, 187). Therefore, a more vivid theory was a theory that also addresses social issues.

The second piece that we highlight is an article in which Antonelli is a knowledge broker. In 1923, he wrote that very rarely theorists had time to select foreign works that would be worth reading. So, he decided to write an article underlining what works and trends in the French academy he, as French researcher, thought were worthwhile to be read, to serve as a guide for Anglo-speakers (Antonelli, 1923b). Antonelli then chose two works that presented, in his view, "the highest interest to a Frenchman". Both were *general treatises* of political economy used by students in law schools. They were the *Cours d'Economie Politique*, by Camille Perreau et the *Cours d'Economie Politique*, by Henri Truchy.[87] He chose those two because their authors held the most important chairs in Economics in the University of Paris for a long time, and, therefore, represented the teaching of economics in France.

One additional note about this period is that Antonelli recognized the value of Walras's ideas regarding monetary policies, however he refrained himself on commenting on this type of policy, despite France's difficult high inflation period between 1924 and 1926. One of the only references made to monetary guidelines, is in his book on French colonies, in which he suggested that Algeria could benefit from broader credit policies (Antonelli, 1925, 336).[88]

Finally, additionally to writing articles and books, in these early years, Antonelli also

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[87]Despite highlighting Truchy's work in his own 1927 textbook, Antonelli criticized some of the author's ideas, accusing him of confusing economic notions with legal notions (Antonelli, 1927, 24).

[88]Later, he would have the opportunity, however, to put his defenses into practice. In the service of Parliament, he was the *rapporteur* of a project to reform the monetary statute of the Bank of Algeria (Milhau, 1953). Taking advantage of his experiences with the colonization of Africa, he still presented a project, in 1931, on the collection of Algerian's product and income duties, and is also the *rapporteur* of a law authorizing Algeria to contract a large loan for the execution of some programmes.

worked as journalist. He was the chief editor of the *Démocratie sociale*, the journal of the *Comité de la Démocratie Sociale*, and presented the manifest of the group (Frobert, 1997). He collaborated weekly with the *Lyon Républicain*, a newspaper of the radical wing of Lyon, and also wrote a weekly article for the *Le Peuple*, for the *Annales Coloniales* and for the *Dépêche Dauphinoise*. These activities as journalist did not end when he became a parliamentarian, but he even started writing monthly from 1927, an article in the Argentine newspaper *La Nación* (Milhau, 1953). As noted earlier, Walras also worked in a newspaper, *Le Travail*, dedicated to the cooperative movement.

Also in 1927, Antonelli wrote another political economy treatise, the *Traité d'Economie Politique*, and made a severe criticism of the French economic-advisers, claiming that, even among the strictest ones, there were those devoted only to political propaganda and false nationalism. They were making up data, distorting observations and ridiculing science. Some of these policy-makers were selling their economic advice the same way as drug dealers were selling their products (Antonelli, 1927, x).

Summarizing, the list of activities of the young Antonelli is quite impressive. Until the mid-1920s, Antonelli wrote two doctoral theses, numerous articles, eight books – and among them, three textbooks and two best-sellers – worked as a professor and journalist, worked in the Ministry of the Colonies and fought in the first World War. However, above all, as we mentioned in the introduction, the left party also elected him member of the Parliament. His first term began in 1924, and he had a very prominent role in implementing the compulsory social security's French law in 1928.

3.3   Étienne Antonelli: Maturity in the Parliament

It is in social security's issue that the influence of Proudhon's ideas is most explicit. Antonelli defended an independent social security system, controlled mainly by its members, instead of the State, the same as the mutualist spirit defended by Proudhon. Antonelli also advocated for integral syndicalism, the idea that all groups in society should be assembled in unions, not just workers. Although Proudhon was influenced by Smith and the labor theory of value, Proudhon and Walras equally advocated cooperativism as a way of improving workers' living conditions, and both also distrusted the ability of charity to ameliorate these circumstances. However, Antonelli supported Walras's ideas instead of Proudhon's in two occasions: first, Antonelli also criticized the theory of labor-value and second, unlike Proudhon, he believed in the absence of conflict between justice and material well-being.

Despite the differences, one important matter united Walras and Proudhon's ideas and was also present in Antonelli's thought. Both authors believed that man was not just a "superior" animal, but a living being with reasoning and freedom, capable of promoting progress. Both scholars criticized authors like Say for – allegedly – defending fatalism;

and they also criticized the idea that society's course followed a natural order – without the possibility of being significantly modified by man and his free will.

This defense of the humanism – the central role developed by man in the course of society's history – accompanied Antonelli until the end of his life. In his essay entitled "My last lesson", the French author summarized what he had taught until then, and concluded that: "I think that you can recognize that humanism, in this framework of the present economy, occupies the whole foundation, either in the evolution of ideologies, or in the structural evolution of institutions. It regulates the deep trends of economic and social life of our time" (Antonelli, 1967, 219).[89] It is, therefore, this recognition of men's active role in the development of society that united, in Antonelli's thought, such contrasting influences as Proudhon and Walras, as well as the defense of the autonomy of workers.

As already mentioned, Antonelli did not only defend implementing a unified social security system, but also wanted this system to be mutualist, with more independence and less State intervention. Before being voted on the Parliament, Antonelli argued in his final speech that the law would give workers the opportunity to be emancipated, materially and morally:

> Gentlemen, I have the right to say that the law you are going to vote [...] is the broad and bright beginning of a new chapter in our social history, in which we will see the democratic working class of this country, rejecting all arrangement made for a classist and governmental social protection, ensuring its security, its well-being, and its social dignity by its own resources and by its own agents (Antonelli, 1966b, 1064)[90]

A first version of the new system's law was voted in 1924, and Antonelli was responsible for being the *rapporteur*, which allowed him to make some alterations. However, there were several interests that needed to be reconciled. On the one hand, the unions themselves were concerned that the obligatory contribution would reduce the already low wages of workers, on the other hand some companies disliked losing control of the funds previously managed by them. However, Antonelli did not only work on the law's creation, but gave numerous lectures across the country, trying to explain the new institutional scheme to workers and entrepreneurs (Le Van Lemesle, 1994). The law was then voted on March 5,

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[89]In the original: "*Je crois qu'à la réflexion vous reconnaîtrez que l'humanisme, dans cette peinture de l'économie présente, occupe tout le fond du tableau, soit dans l'évolution des idéologies, soit dans l'évolution des institutions structurelles, c'est lui qui marque les tendances profondes de la vie économique et sociale de notre temps*".

[90]In the original: "*Messieurs, j'ai le droit de dire que la loi que vous allez voter – car vous allez la voter, n'est-ce pas ? – c'est la vaste et lumineuse préface d'un chapitre nouveau de notre histoire sociale, où l'on verra la démocratie travailleuse de ce pays, écartant toutes méthodes usées d'une protection sociale de classe ou d'Etat, assurer elle-même par ses propres ressources et par ses propres mandataires, sa sécurité, son mieux-être, et sa dignité sociale*".

1928 and supplemented by the law of April 30, 1930.[91]

Besides writing the *Guide pratique des assurances sociales* – a book that had three editions and aimed to explain the new system's rules to the general public – Antonelli was also responsible, for decades, of publishing annual reports in the journal *Revue d'économie politique*.[92] The annual reports summarized the important events concerning the social security system that year. The first publication was in 1935, requested by Marcel Porte, president of the *Union nationale des caisses autonomes mutualistes*. Antonelli wrote this report annually until the mid-1960s. Over the years it is possible to see his disappointment with the growing participation of the State in the administration of security. In his 1956 report he stated that it was necessary a new reform of the system, mainly to simplify it after the numerous subsequent changes (Antonelli, 1959, 768). Further, another recurring theme in the reports was the measurement of the institute's deficit. In 1966, Antonelli asked "how long will we discuss this imaginary Social Security's deficit?" (Antonelli, 1966b, 1064).[93] Some years before he also tried to show some data to discuss the "controversial" deficit of 1951 (Antonelli, 1953b). We see, therefore, again, that even in the role of knowledge broker – presenting an annual report on the system – Antonelli continued to give his opinions openly.

Another matter highlighted in the annual reports, was the social security's role in the distribution of income, comparing it to using taxes for the same purpose. Antonelli advised that tax's income was a flawed instrument of distribution. Nothing could guarantee that the income would be effectively used for the social good. Distribution was also done very slowly. There was a wide time's gap between the moment that the State collected the money and the moment that this money satisfied the citizens' need. The social security system quickly distributed resources to those who needed it the most: the elderly, the unemployed, the sick, and the injured. Society's disbursements would not be maximum, but *optimal* (890). For that reason, everyone should be in the social security system – associated with the idea of integral syndicalism – since it made little sense to think about redistributing the *national* income if all citizens were not associated, even employers (891).

This, however, is only one of the few times that Antonelli commented on tax issues. Despite not believing tax's income were a good distribution tool, Antonelli is far from the Walrasian perception that all taxes were unfair – Walras defended that people's income belonged only to them and the State should finance public services renting land (Walras, 2010, 153). This is perhaps also more surprising, from the point of view of its theoretical

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[91]Other important activity of Antonelli as a Parliamentarian was a project that aimed to establish a broad French program for the construction of popular housing. The author also defended in Parliament an electoral reform. Finally, he was also a proeminent member of the Finance Comission of the French Parliament (Milhau, 1953).

[92]Antonelli also wrote an article in 1953, commenting on the journal's creation analyzed from the point of view of the correspondences between Walras and Charles Gide (Antonelli, 1953a).

[93]In the original: *Quand cessera-t-on de nous battre les oreilles avec le déficit inventé et entretenu de la "Securité sociale?"*.

influences, given that Proudhon also advocated, at some point, for the abolition of taxes. Antonelli, therefore, seems to be a much more pragmatic author, in the sense of exploring only policies that were more feasible and easier to be implemented in the short term. In addition, despite having written several papers presenting Walras's ideas, Antonelli only mentioned in one occasion Walras's defense of the nationalization of land, even though this point had been repeated several times by Walras.

The reports also presented another interesting subject. In the 1956's edition, Antonelli pointed out that three were the destinations of a country's income: consumption, investment and social security. The division between these three should not be maximum, but *optimal*, targeting a point of economic equilibrium. If necessary, the State could intervene to help reaching this point (Antonelli, 1959, 890). However, Antonelli never came to formalize a model in which national income was divided between these three destinations, despite defending the mathematization of economics.

Although not defending that taxes were unfair, Antonelli argued in favor of a very Darwinian thought: there were the most capable and the least capable individuals in society, and the community would be better allowing the most accomplished have all their income, without subtracting a part to the government. If we consider the two groups as coexisting species, we would see that the least able would gain nothing with the extinction of the better adapted (Antonelli, 1927, 232). By allowing the most apt to have incentives, for example, a bigger income without taking away a tax share, the production of wealth as a whole would be bigger, and the less adapted would also benefit – despite not making it very clear how they would benefit. One point is still worth highlighting: he differentiated the most able from the least able considering the difference that each provided of *utilité* to society, a central notion in Walrasian theory (231). Commenting on the fact that this ideas may be seen as too unrightful, Antonelli argued that the inequalities of individuals' capacities were often more moral than intellectual or physical (232). Inequalities came mainly from addiction, laziness, lethargy or desire for luxury (232). Walras, for his part, also believed that the State should not intervene to alleviate the consequences of human incompetence, because it would be a contravention of natural law (Jaffé, 1983, 49).

Besides writing the guide explaining the social security system, and writing the annual reports, Antonelli also taught a course at the *Conservatoire des Arts et Métiers*, as professor of the Social Security chair. In an interview with a newspaper[94], Antonelli explained that he taught the management of the security funds, for example, legal issues of their administrative and financial control. However, the most important thing is that Antonelli declared that his course was a course of pure science: "My classes are classes of pure science; I try to make them as simple as possible and more easily assimilated for all who listen to me"[95]. We can see, therefore, that Antonelli used to apply the vocabulary

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[94]It was not possible to identify the name of the newspaper.

[95]In the original: *Mes cours sont des cours de science pure ; je m'efforce de les rendre de plus simple*

of Walrasian theory – for example, using the term "pure science" – to distinct areas, such as the administration of insurance funds, and he also again related the notion of pure science to a applied problem – the management of the funds. Secondary literature on Antonelli, for example, Frobert (1997, 1534), mentions that Antonelli read Walras using a Proudhian and solidarist filter, and that the theory of general equilibrium was seen by the young Frenchman as a research program that allowed the analysis of the evolution of the economic and social systems. We can see that there is, in fact, an attempt to related pure theory to the study of issues of the economic system, however, there is still some confusion in Antonelli's statements about the specific divisions of economic science in its broad areas: pure, applied and social. The concept of pure science, for example, appears here associated with something that is easily assimilated by the public.

In one of his works, Jaffe accused Walras of being a terrible historian of ideas: "Léon Walras's occasional excursions into the history of economics show him up as an execrable historian" (116). According to Jaffe, if Walras, for example, had read Smith properly, he would have understood that there were much more similarities in the ideas of the two than he believed. By his turn, Antonelli wrote articles about Walras's work, but the latter was not the only one about whom Antonelli wrote. The French author many times also synthesized the knowledge of the past by writing on the history of economic ideas – another one of his facets of knowledge broker. In addition to address Walras's intellectual production – including publishing some of his letters, as already mentioned – Antonelli also wrote pieces on Auguste Walras (Antonelli, 1923c), on the revival of Proudhon's thought in France (Antonelli, 1910a), on the French philosopher Henri Bergson (Antonelli, 1912a) and on Constantin Pecqueur, a 19th century economist (Antonelli, 1930). Antonelli was also professor of the history of economic doctrines of the *Faculté de Droit de Lyon*. Finally, Schultz even thanked Antonelli, as well as Boninsegni, for answering his questions about the history of the marginal productivity theory. He especially thanked Antonelli for authorizing the publication of a letter exchanged between Walras and Barone, since Antonelli was responsible for its preservation (Schultz, 1929).

Still on the issue of history of ideas, Antonelli pointed out that until then, the most useful general treatises of political economy had been exactly those of the "history of doctrines". These guides were able to order and classify the massive amount of scientific production written in that last decades (Antonelli, 1927, xii). However, despite valuing the history of doctrines, Antonelli also argued that, particularly in that troubled period of transformation, the scientist would hardly have time to do long exegesis, because they had to spent increasingly more time in public positions (Antonelli, 1910a, 562). This assessment of scientific exegesis also appeared in 1927, when he reasoned that it was time to elaborate a synthesis, instead of working on several exegesis. Quoting Proudhon, he remarked that when you spend a lot of time studying leaves, you may end up forgetting the

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possible et assimilables pour tous ceux qui m'écoutent.

forest (Antonelli, 1927, xii). This parallelism with nature in the history of ideas appeared again in 1929, when he suggested that analyzing ideas in the same way of dissecting corpses was a tedious activity. We should show the living men behind the formulas and theories – and again we can perceive the underlying notion of making theories seem more vivid (Antonelli, 1929, 1493).

Antonelli was a socialist much more influenced by Proudhon and Kropotkin than by Marx. However, by the end of his carrier, he commented on how he had taught the history of ideas and emphasized that he had not made the mistake of presenting theories that classified and studied some time periods in isolation. The ideas of the present were the dialectic antithesis of previous ideas, and we could not look at the present in isolation, without considering the past or the future. He further stated that every human phenomenon engendered its own negation. The economy of that time – the 1960s – was the negation of the previous capitalist economy, which in turn was the negation of Middle Ages' system (Antonelli, 1967, 216). Notetheless, despite referring to Marx, he further taught the neo-liberalism of Rueff, Hayek, von Mises, Röke and Allais – calling the latter "school of equilibrium" (219).

Economic systems, according to Antonelli, should not be seen as isolated from others. The system was a continuous flow, with no well-defined divisions. Yet he did not see the progress of civilization as something linear: previous civilizations were not necessarily inferior, or "primitive". Notwithstanding the fact that the economic system behaved like a flow, it was impossible to specify what the future system would look like. In admitting this – that we could not know the future, even with scientific studies – we could loose in the sense of not having a correct and reliable systematic analysis. But there was a gain, though, from the point of view of life. Life, in its wholeness, was something full of apprehension, hope, discouragement and uncertainty[96] (Antonelli, 1967, 218).

Although the future economic system was unknown, it was possible to study the complex current economic system by looking at its "ideal" structure. It is in this Antonelli's defense that we see most clearly the use of the theory of general equilibrium for the analysis of the economic system. In terms of war organization, for example, even if a soldiers' alignment in a battle was not actually formed by perfect lines and columns, it was important for Antonelli to study the ideal model. He disagreed with Ricardo, who, according to him, sustained the uselessness to analyze the ideal structure because it did not correspond to reality. Antonelli argued that society was studied assuming respect for private property, but there were thieves in the real world. For every real social organization, there must be an ideal type of system. For every real economic organization, there

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[96]In his article, Perroux (1953) asserted that he would like Antonelli to also offer some thoughts on the pure economy of non-capitalist systems. According to him, this was not too much to ask, because he knew Antonelli's capability and how much he could offer to the field. Perroux argued that when this wish could be fulfill, we would be able to appreciate more the content of the pure economy of the current system.

was – as there was for Walras – a *pure economics* (Antonelli, 1927, 227).

3.4 Concluding Remarks

This article's goal was to analyze how the Walrasian thought influenced Étienne Antonelli in his roles as a policy-maker and knowledge broker. First, we can conjecture that to adhere to Walras's ideas, made him to seek these options outside the academy in the first place. Therefore, more than just influencing these activities, being a Walras disciple forced him to *choose* them. The transcript of his speech in the competition – in which he explicitly quoted Walras – and the jury's answer, undoubtedly showed that presenting himself as a follower of Walras, and of the mathematization of economics, made more difficult the access to stable positions as professor. We could also see that Antonelli made a conscious effort to present, according to him, only "facts", setting aside personal opinions, most likely trying to safeguard himself from further criticism, and we observed this mainly in his work as knowledge broker. However, we could see that even this strategy was also condemned – although Antonelli argued he would not change his approach because of critics, and would continue to write texts without bias. As a knowledge broker, relating to main points defended by Walras, we also saw Antonelli's important role in trying to convince the public opinion, with his 1912 book, about the new legislation that would facilitate the participation of workers in companies, and after that, with his travels in France trying to explain the new institutional scheme of the law on the mutual social security system.

We also saw that Walras's and Proudhon's ideas were conjoined in Antonelli's thought given the central role of man, with their free will, in the process of development of society. By the end of his life, Antonelli commented that humanism and the idea that all individuals were endowed with some sort of potential pervaded his entire research agenda. Workers, as human beings capable of taking care of their own interests, should be increasingly emancipated – hence Antonelli's defense, as a politician, of a mutual security system, integral syndicalism and increasing workers' share in the capital of companies. Antonelli disagreed with Walras on some points about the formation of societies with worker participation, for example, Walras argued that workers should pay monthly participation while Antonelli believed that the shares should be given to workers. However, Antonelli maintained a very economical concern with respect, for example, to productive efficiency. He also defended that it should exist competition in the markets even with the associations. Finally, the human ability to alter any historical course makes the future very uncertain. However, despite this chaotic uncertainty, we could – and should – use models to analyze economic system at an ideal point of equilibrium, according to Antonelli.

We also started this work highlighting Walras's effort at finding a French disciple, sending his works assiduously throughout his correspondence. However, there is no evi-

dence that Walras knew the student's existence – Antonelli was not mentioned in any of the correspondences of Walras. Therefore, we cannot state how the young man discovered Walras's work. However, it is possible that the defense of a "scientific socialism" attracted the young socialist. The same way as Walras abandoned the socialism of Saint-Simon for believing it lacked a logical basis and was too vague, perhaps Antonelli also found in Walras an alternative to the socialist theories of his time. By the end of his life, Walras used to say that he had resigned himself to knowing that he would not see the fruits of his work. Antonelli was, undoubtedly, one of those first fruits in France that, unfortunately, Walras would never get to know.

4 Teaching General Equilibrium Theory in the Early 20th Century: An Analysis of Treatises

Abstract

This paper aims to analyze the teaching of general equilibrium theory at the beginning of the 20th century. I studied three general treatises: The Mathematical Groundwork of Economics: An Introductory Treatise (1924), by Arthur Bowley; The Theory of Social Economy (1924), by Gustav Cassel; and *Principes d'Économie Pure: La Théorie de l'Échange sous le Régime de la Libre Concurrence* (1914), written by Étienne Antonelli. Despite having some original contributions, the three works were undoubtedly written for students, and analysing them allow us 1) to contribute to the literature on economics education in the pre-war, and 2) to analyze from a singular point of view the emergence of mathematical economics as a field within economic science. I used content analysis techniques and quantitative text summarization, and I commented on the rhetorical elements employed by the authors. With respect to the dissemination of the Walrasian general equilibrium theory, I conclude that, although the three books presented Walras's framework, they also presented at the same time antagonistic fundamental beliefs about what was economic science and its scope; therefore, Walras's model was taught in very different theoretical frameworks: no one dominant group appropriated it. In spite of that, the content analysis technique shows that the three treatises share similarities in their structure, which already indicates some degree of standardization in the first decades of the 20th century.

4.1 Introduction

If there were no books to summarize and to pass on accumulated information from one generation to the next, a critical part of the knowledge of the humanity would be lost in generational changes. Therefore, long before the modern textbook as we know it today, *general treatises* played a very important role in science. Part of the literature explores the importance of such books as depositories of knowledge – like Kuhn's traditional view – while other authors emphasize the active role of these books, not as just passive depositories, given their fundamental function in training subsequent generations. Regardless of how we choose to study textbooks, though, their importance is seldom questioned, and the history of economics is increasingly giving due importance to its analysis in the development of economics.

However, despite this recently more intense interest in economics's textbooks, most of these new articles analyzes modern publications, such as Forder (2015), studying the presentation of the Phillips curve in different works, Giraud (2018), examining Samuelson's canonical publication, and Teixeira (2014), on the mass production of textbooks associated to MIT. Notwithstanding the importance of these investigations, less attention has been given to the production of economics teaching books in the pre-war period. Therefore, this paper aims to analyze general treatises of mathematical economics, and more specifically, of general equilibrium theory, in the first decades of the 20th century. This

analysis will allow us to study from a singular point of view the emergence of mathematical economics as a delimited and separate field of political economy.

Furthermore, this paper aims to investigate general treatises that presented the *Walrasian* general equilibrium model. Three books were chosen: The Mathematical Groundwork of Economics: An Introductory Treatise (1924), by Arthur Bowley; The Theory of Social Economy (1924), by Gustav Cassel; and *Principes d'Économie Pure: La Théorie de l'Échange sous le Régime de la Libre Concurrence* (1914), written by Étienne Antonelli. Schumpeter (1954, 796) argued that Marshall's teaching in England excluded any other contender until Bowley presented Walras-Pareto model in his 1924 textbook. Boianovsky (2016, 310) stated that it was Cassel's version of the Walrasian system of simultaneous equations that made general equilibrium system known to German economists and English speakers. Wicksell (1934) also discussed Cassel's *popularization* of the Walrasian system (Boianovsky, 2016, 310). Therefore, Cassel and Bowley are doubtless important to our inquiry. But why to choose Antonelli's book – a much less known author – as the third work to be analyzed?

Economic education highly mattered to Walras. The author argued education in France, "so bourgeois in its narrowness", was divided into two separate compartments: on one side, forming calculators with no knowledge of sociology, philosophy, history or economics, and in the other extreme, "cultivating men of letters devoid of any notion of mathematics". That is why he thought it was critical to change the system and to form students who were able to think both inductively and deductively (Walras, 1954, 48). Walras also wrote several papers – later published by the Center Auguste et Léon Walras[97] – on the teaching of Economics, in which he exposed his dissatisfaction with the educational system (Walker, 1989). In the last decade of his life, even though he was sick, he also spent substantial time preparing his *Abrégé*, a version of his ideas to be used in the classroom, hoping to be implemented by a French disciple, Aupetit, who never did it. Walras's letters also show he tried many times to get an English translation of the work, but without success (Jaffé, 1965). It was Antonelli, however, who first published Walras's *Abrégé*, in 1914, in France. Therefore, given the paper's aim – to analyze the teaching of Walrasian theory – to study the first published version of a work that he spent so much effort to write, and even more, also published in his native country, is valuable to understand Walras's original legacy[98].

I will use in this paper content analysis techniques, an established methodology in

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[97]The Center Auguste and Léon Walras was founded by a group of economists from the University Lumière Lion 2 aiming at safeguarding and enhancing the documents left by Aline Walras to the University. The group was formed by Pierre Dockès, Pierre-Henri Goutte, Claude Hébert, Claude Mouchot, Jean-Pierre Potier and Jean-Michel Servet.

[98]Antonelli wrote an updated version of the textbook, also presenting the model of general Walrasian equilibrium in 1939, *L'économie pure du capitalisme*. However, we will focus the analysis on the 1914 book, which was published in time closer to the other two, although we also make some comments about the 1939 book.

the education literature, also used particularly for the analysis of textbooks in economics. King and Millmow (2003), for example, used the method to compare between the textbook written by Robinson and Eatwell, and Samuelson's 1973 edition. In his "The Content Analysis Guidebook", Neuendorf (2002, 1) specified that "*Content analysis* may be briefly defined as the systematic, objective, quantitative analysis of message characteristics". The author also stated that frequently the term is used incorrectly, and must be differentiated from other types of message analysis, such as, among others, Rhetorical Analysis, Normative Analysis, Discourse Analysis and Critical Analysis (Neuendorf, 2002, 4). Besides the systematic analysis of content, I will also employed quantitative text summarization, using tag clouds – or, as is most often used with word analysis – word clouds. The quantitative analysis of the most used words in the text, and its final presentation, correlating the font size with the frequency of the term, is a simple, but useful tool to summarize a text and, in our case, to visually compare each work. Finally, despite basing our study on the systematic content analysis, I believe that all authors are engaged in some degree in a rhetorical discourse, trying, in our case, to assure to each student that their perspective is the most appropriate, even they did not always do this explicitly. Therefore, in a complementary way, I will try to explore, to a small extent, this angle, the rhetorical aspect, of the analysis of textbooks.

4.2   Our authors and a first text overview

Arthur Bowley (1869-1957) was an English economist and statistician educated at Trinity School during Marshall's tenure, and became a professor at the London School of Economics in 1895. In spite of being a notable statistician, as we will see later, his book has no statistical content – and it is indeed Cassel's book that used data series to support the theory of cyclical movements. Bowley actually wrote a textbook on statistics, which was quite well received, the *Elements of Statistics*, in 1901, and he was one of the first to estimate the British national income; but his *Mathematical Groundwork* presented all equations with indeterminate coefficients, and no type of data analysis. Bowley also played an important role, together with Lionel Robbins, in shaping the structure of the London School. It is valid to remember that was Robbins who defined, in his canonical 1932 text, the scope of economic science as all textbooks use today: "Economics is the science which studies human behaviour as a relationship between ends and scarce means which have alternative uses" (Robbins, 2007, 15). Other features of Robbins's conception, that are also disseminated in modern textbooks, are, first, that our science must explain economic phenomena from individual choices, and second, that economics and ethics are two separate fields. We will also see later how each author defined our field and its scope.

Gustav Cassel (1866-1945) was a Swedish mathematician who presented his thesis in 1894 at the University of Uppsala, and taught Economics at the University of Stock-

holm from 1903 until 1936, lecturing "National Economy and Finance". His 1918 book, *Theoretische Sozialökonomie*, was translated into English in 1924, and had five German editions, two different English translations and French, Japanese and Swedish versions. The book was undoubtedly written for students, and its aim was to make knowledge as accessible as possible:

> I hope that English students will find the present treatise, although its reading without doubt requires a certain amount of thinking, to be the easiest and most direct way to get access to those great economic and social problems of the day for the understanding of which they undertake the trouble of economic training (Cassel, 1924, vii).

Regardless of being a book for students, Cassel's work portrayed a typical feature of past general treatises, offering original contributions to economic science. His microeconomic growth theory was important to von Neumann's work, his new macroeconomic growth theory directly inspired Harrod, and his theory of optimal depletion of mines drew attention to a forgotten problem (Brems, 1986). Besides the original contributions, Cassel used much of the work done in Lausanne, but: "Cassel followed Walras and Pareto, mentioned neither, and never paid tribute to anybody" (Brems, 1986, 23).

Étienne Antonelli (1879-1971) was a notable French politician and one of the main responsible for creating the law establishing the modern French social security system. He was also one founder of the *Committee de Démocratie sociale*, working alongside Aristide Briand, former Prime Minister of France. Despite his remarkable career outside the academy, we will concentrate on his role as professor. He studied Law – contrasting with Bowley's training as a statistician and Cassel's training as a mathematician – and he was probably the first to implement a course in France on general equilibrium theory and mathematical economics (Diemer, 2006). He started teaching in 1910 at the Faculty of Law in Paris, although he was not an associate professor. Therefore, Antonelli had some – but not huge – teaching experience at the time he published the book, in 1914. In 1919 he became a professor of history of economic thought at the Lyon Faculty of Law, and after retiring as a politician in 1934, he devoted the rest of his life to teaching at Montpellier University.

This last point, classroom experience, is worth some discussion: does experience as professor greatly influence the quality of a textbook? King and Millmow (2003), for example, conjectured that the fact that neither Robinson nor Eatwell had much experience teaching first-year students, in some way prejudiced their textbook. Of course we cannot objectively answer the question, even more because all of our three authors were professors. However, we will see that, at least in Antonelli's case, his interest in the history of economic thought, the subject matter of his class, was reflected in his work.

We saw earlier that Cassel did not mention Walras in the treatise, not even a single time. At this point, a first visual analysis of Antonelli's text, with the most used words in his book[99], is a welcomed tool for comparison:

Figure 2: The 250 most frequent words in Antonelli (1914)

A first noticeable feature is that the term "Walras" is one of the book's most frequent words. The name of the French author appears repeatedly already in the book's preface, written by Georges Renard. Renard begins the text by stating that the work is an effort to *popularize* Walras's name and doctrine in France. The author's name appears even in the titles and subtitles of the chapters; for example, the first part of the book is named "Walras's process of pure economics" (*Le Procès de l'Économie pure de Walras*). The entire first chapter of this part is also dedicated to addressing "The work of Léon Walras" (*L'oeuvre de Léon Walras*), and is divided into: 1. The teaching of pure economics in France. *"L'abrégé"* de Walras (*"L'enseignement de l'économie pure en France. "L'abrégé" de Walras*); 2. Walras and the problem of general equilibrium (*Walras et le problème de l'équilibre économique général*); 3. Walras and the two schools of pure economics: the psychological school and realistic school (*Walras et les deux écoles d'économie pure: l'école psychologique et l'école réaliste*), and, finally; 4. Léon Walras's

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[99]The list was made removing the stopwords. Stopwords are the most used words in a language, but usually irrelevant to natural language processing. There is no consensus on which groups of words should be removed, but in our analysis, we used the R package "stopwords", and we also removed the word "être" (the verb "be"), the white spaces, the punctuation and the numbers. Since there is no official translation of the work, we decided to study Antonelli's book in the original French.

life (*La vie de Léon Walras*).

The title of the chapters and the fact that "school" (*école*) is one of the words that appears most frequently suggests regardless of being a book for students, it does not present economic knowledge as something definitive and consensual, but that there were different beliefs within the science. Such feature is quite understandable in the French academic scene at the time: when you want to present something new, and challenge the *status quo*, it is important to show that such *status* is not the only existing path. We could suggest that the mention of the term "school" refers only to the history of economic thought – not necessarily schools of the present – but, when you introduce to a student the idea that history is marked by divergences of thought, who can guarantees that the present also does not offer other alternatives?

Bowley, for his part, despite of not discussing controversies of the theory, highlighted the fact that this theory was not yet finished, with several paths not explored. He presented the general equilibrium equations, and argued that in analyzing the *static* solution it was possible to determine in which direction the system would move with some change, for example, with increasing land or capital. But with the *dynamic* solution, when changes occur continuously, this would involve more complicated analysis, and little progress had been made in such an investigation (Bowley, 1924, 53). Antonelli presented the Walrasian system, but also reminded that the theory was not finished, and that several problems should be solved by next generations (Antonelli, 1914, 201).

An advantage of using word clouds is their ability to show the key themes of a work. Undoubtedly Antonelli's book deals with the general equilibrium theory: the words price (*prix*), equilibrium (*équilibre*), demand (*demande*), offer (*offre*), exchange *(échange)*, commodity *merchandise)*, quantity (*quantité*) are some of the most frequently used words in the book. Also analyzing the figure, we see that the word "two" (*deux*) is one of the most frequent term in the book[100]. It would be curious for anyone who was not educated with textbooks in economics and already familiarized with the traditional exchange model with only two goods, employed in the book. The work, therefore, presented such a feature of the original Walrasian model, first studying a simpler economy with two goods and two individuals. In fact, Walras was also one of the first economists to use, for pedagogical purposes, the method of increasing complexity. One last point of this visual analysis is that we can see some themes that the book also addresses, but with less emphasis: production, money (*monnaie*) and value (*valeur*).

Besides the visual analysis, Table 1 also shows the ten most employed terms in the work, in decreasing order:

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[100]Is it the 13º most frequent word in the text.

Table 1: The then most frequent words in Antonelli (1914)

	Word	Frequency
1	Price (*prix*)	369
2	Exchange (*échange*)	183
3	Walras	171
4	Theory (*théorie*)	170
5	Equilibrium (*équilibre*)	148
6	Problem (*problème*)	148
7	Market (*marché*)	147
8	Economy (*économie*)	146
9	Plus (*plus*)	145
10	Demand (*demande*)	141

In comparison to Antonelli, Cassel's work seems to place a much greater emphasis on the issue of production[101]:

Figure 3: The 250 most frequent words in Cassel (1924)

It is relevant to highlight a point that had not been mentioned until now: the size of each book. While Antonelli's work has 206 pages and Bowley's has only 98 – with a fifth of

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[101]We also removed the stopwords from the R package, and the words "the", and "means", the white spaces, the punctuation and the numbers. The same applied to Bowley's book.

the book devoted to the appendix – Cassel's work is almost 700 pages long. Cassel divides his work into four main books: 1) General Survey of the Social Economy; 2) The Pricing of the Factors of Production; 3) Money and; 4) The theory of conjuncture-movements. In this last book, Cassel devotes two chapters to production: Chapter XIV, "The influence of conjunctures on production" , and Chapter XVII, "The influence of conjunctures on the permanent material means of production". We see, therefore, despite being a much more extensive book than the others, *production* appears relatively more frequently given that, first, one of the four main books is dedicated entirely to the price of the factors of production, and, second, the production processes is also discussed in the analysis of economic cycles, one of the other four main books.

An identifiable feature is that regardless of being the book that appears in the literature as the one that popularized the theory of general equilibrium, "equilibrium" is not a term discussed relatively very often. In fact, the ten most frequent terms in the work are:

Table 2: The then most frequent words in Cassel (1924)

	Word	Frequency
1	Production	1426
2	Capital	1241
3	Prices	993
4	Must	979
5	Demand	737
6	Economy	685
7	Interest	669
8	Money	668
9	Price	641
10	General	640

The first thing to notice is that "price" and "prices" are two terms that together appear most in the book: 1634 times. In fact, the cornerstone of Cassel's book is to build a theory of prices. The first page of the work states that:

> From the first beginnings of my studies of this science I have felt that it ought to be possible to do away with the whole of the old theory of value as an independent chapter of economics and build up the science from the beginning on the theory of prices, and that we in this matter would be able to rid ourselves a lot of unnecessary discussions, mostly of a rather scholastic nature, which had burdened earlier treatises on economics (Cassel, 1924, v).

Cassel argued we should ignore any concept of "value" that is not synonymous with "prices". When not a quantitative measure, value only means a subjective object, that cannot be scientifically analyzed. His criticism covered not only the idea of marginal utility as a source of value, but also the labour theory of value, which he considered a

hindrance to the development of a scientific theory, because it transferred the controversy to the ethical political field (Cassel, 1924, 193). But his approach also made him liable to a straightforward criticism: it is not possible to eliminate the concept of value just by excluding the word (Moll, 1947). Cassel criticized books published until his work, that all began the analyses with a hypothetical economy with pure exchange, without money. Such an economy, for him, was only imaginary and therefore of no interest for real analysis. We see in this matter the contrast with Antonelli's approach of increasing difficulty for pedagogical reasons. Furthermore, Cassel also criticized the hypothesis of perfect competition: "It should be clear that to take free competition as the starting-point for a general theory of prices is of very little use" (Cassel, 1924, 126).

An intriguing feature of the book's most frequent word list is the use of the term "must". It can certainly mean only a linguistic choice, but it is possible to conjecture two other explanations: 1) the book contains a higher normative tone than other books or; 2) the book repeatedly uses incisive expressions when explaining some ideas and the consequent conclusions of these ideas; in simpler terms, something such as: A causes B, and B causes C, then A *must* causes C. As for the first possible explanation, before all else, it is important to state that I believe, along with Walker (1984), that the original Walrasian theory has as its central point the idea that markets are interrelated, which in itself is not a normative concept. Established this point, an inspection of Cassel's book shows us, maybe, some examples of the use of the term "must" with a higher normative tone. For example, one of the first uses of the word is about how to do economic science: "Economic science *must* aim at thoroughly and exhaustively elucidating the entire network of causes and effects, and it *must* therefore take a complete economy as the subject of its investigations. This economy *must* be "self-contained": it *must* have no relations with anything outside itself" (Cassel, 1924, 4). An example of the second case, causal conclusions, can also be found at the beginning of the book: "[...] a less important need must not be met in preference to a more important, and a need that is in itself important must not be so exclusively regarded as to neglect others. A sound economy [then] *must* make a certain very general, possibly unconscious, classification of needs according to their importance" (Cassel, 1924, 8).

Finally, we have the first overview of Bowley's text in an image:

Figure 4: The 250 most frequent words in Bowley (1924)

And we also have, in table 3, the 10 most frequent words.

We can see an indication of a first main difference between Bowley's text and the others: Bowley is the author who presented the book with the highest proportion of equations. In fact, looking at the list of the most cited words, we see that "equation" and "equations" together appear 266 times in the book, more than twice the second term "curve", which appears 131 times. Actually, the book's title "Mathematical groundwork" already seems to suggest that it intended to construct a mathematical basis for economic science. One of the book's reviews defined it as a basic structure: "Like a skeleton it is closely articulated, bare of softer parts and white, with the white light of reason" (Tappan, 1925, 334). Indicating the book's intended public Bowley stated that:

> Though the simpler applications of mathematics made by competent writers and lecturers can be appreciated by any intelligent readers and students, the more complicated analyses are only within the power of those who have mathematical aptitude, and it is for them that this book is arranged
> (Bowley, 1924, v).

Therefore, despite of being a book for students, the work was not promptly available to

Table 3: The then most frequent words in Bowley (1924)

	Word	Frequency
1	Equations	192
2	Curve	131
3	Supply	126
4	Demand	115
5	Production	90
6	One	87
7	Price	86
8	Utility	79
9	May	77
10	Equation	74

all of them, given the difficulty of the mathematical language. But for these students not so familiarized with algebra, he wrote the appendix: "The actual number of mathematical theorems used is quite small, but among them are some uses of the calculus which do not form part of the usual elementary curriculum, and these are brought together in an appendix" (Bowley, 1924, v). This appendix, as we pointed out earlier, occupies almost a fifth of the book, and also appeared as one of the most frequently cited terms in the work, indicating that Bowley probably had to resort several times to appendix's information in the course of the book.

Finally, some last comments on the visual analysis of Bowley's work. First, "two" is a term that also appeared as one of the most frequent, as well as "equilibrium", "supply" and "demand". Second, money is a term that appears, but relatively less than in the other works. Some terms also indicate other points addressed by the book: "monopoly" – Bowley presented equilibrium equations for the case of perfect competition and monopoly; "indifference", "utility" and "surface" – the book presented Edgeworth's and Fisher's indifference curves and utility surface. Although it is also a book whose purpose is to provide the basic mathematical structure of pure economics, " taxation" is a theme that appears, albeit modestly. Curiously, in Bowley's book, instead of the term " must", "may" is one of the words that is on the most frequent list.

4.3 Three approaches to the presentation of the general equilibrium theory: content analysis

Table 4 presents the content comparison of each of the chapters of the three books. We highlighted with similar colors chapters with similar content. The ones without color are those that do not have correspondents in the other books. We see, first, that, as expected, the chapter "Léon Walras's work" of Antonelli's book does not have a comparable in the other two works. Cassel's book is the only one to present a chapter on international payments, and a significant part of the book is dedicated to analysis of economic cycles.

It is also in this analysis that Cassel used statistical data to build his theory. Bowley's book, on the other hand, analyzed monopolies and taxation, both in the chapter on applications of general equilibrium theory.

The parts on general equilibrium theory were highlighted in red. While the subject is the last one to be presented in Antonelli's, and one of the last discussed in Bowley's book, Cassel presented the general equilibrium equations before most of his other chapters. Cassel, even more singularly, though, stated the chapter could be skipped. The equations were presented in the section named *Arithmetical treatment of the problem of equilibrium*, and the author argued that the part was written in mathematical language, but could be understood by any educated person with a knowledge of systems of equations with several unknown quantities. But he added: "The work is, however, so arranged that the section may be passed over without breaking the general connectedness. In that case the reader would merely have to be content with a less profound insight into the aforesaid questions" (Cassel, 1924, 134).

Antonelli, following Walras, on the problem of existence, highlighted the possibility of finding equilibrium if the number of equations were equal to the number of unknowns (Antonelli, 1914, 215). About the question of unicity, Bowley, for his part, is the only one to point out that it was possible that there were multiple solutions, but, if we had any numerical values, *likely* it would not be difficult to know which set was the most appropriate (Bowley, 1924, 53). He also discussed the stability of equilibrium: in the case in which no producer was dominant, *likely* would be oscillation in the neighborhood of the equilibrium's price (Bowley, 1924, 37). In the same way, Cassel also stated only in words, that the economy would oscillate around the equilibrium, because "any material deviation from the principle of cost provokes, as a rule, counteracting forces" (Cassel, 1924, 127).

Cassel also perceived economic science as a discipline with linear development, in which it was possible to resolve and set aside some questions: "A progressive science will always find it necessary, in order to make room for new investigations, to leave out such old matters and old discussions as are no longer of essential importance" (Cassel, 1924, v). In this sense, Cassel wanted to distance himself from almost all the theories developed until that moment, stating that his ideas were different from all that had already been developed. Its reviewers, however, criticized the allegedly originality of the book: for example, Phillips (1924, 237) argued that "the novelty of his discoveries is to that extent exaggerated", while Jones and Hart (1934, 356) stated " his advance beyond Marshall may not be so great as he believes". Edgeworth (1920, 535), in his comments, argued "Professor Cassel is the last man to appreciate this reverence for the past", and "we could wish that Professor Cassel had employed his sagacity in discovering the wisdom which is latent in the practice of the classical economists" – Cassel not only criticized the doyen of English economists, Marshall, but also much of classical theory.

Table 4: Comparison of Antonelli (1914), Bowley (1924) and Cassel (1924)

Antonelli (1914)	Cassel (1924)	Bowley (1924)
Chap.1: Léon Walras's work	Chap.1: Economy in General	Chap.1: Simple Exchange of two commodities
Chap. 2: Pure Economics. General notions	Chap. 2: Exchange Economy	Chap. 2: Multiple Exchange
Chap. 3: The problem of exchange of two commodities	Chap. 3: The Economic Principle in the Exchange Economy	Chap. 3: Production
Chap. 4: The maximum satisfaction equation	Chap.4: The Mechanism of Pricing	Chap.4: Supply and Demand of the factors of production
Chap. 5: The exchange of several goods	Chap.5: The Pricing of the Factors of Production: Introduction	Chap. 5: General Equations of Supply and demand in a stationary population
Chap. 6: The theory of production	Chap. 6: The Interest on Capital	Chap. 6: Applications of the general equations
Chap. 7: The problem of capitalization and credit	Chap. 7: Ground Rents and the Prices of Natural Materials	Chap. 7: Surplus value, rent and taxation
Chap. 8: The problem of circulation	Chap. 8: Wages	
Chap. 9: The problem of general equilibrium	Chapter 9: Analysis of the monetary system on the basis of its evolution	
	Chap. 10: Bank currency	
	Chap. 11: The value of money	
	Chap. 12: International Payments	
	Chap. 13: The theory of conjuncture-movements: Introduction	
	Chap. 14: The influence of conjunctures on production	
	Chap. 15: The influence of conjunctures on labour	
	Chap. 16: The influence of conjunctures on the permanent material means of production	
	Chap. 17: The influence of conjunctures upon the fixing of prices, income, and capital	
	Chap. 18: The influence of conjunctures on the capital market	
	Chap. 19: The determining factors of conjuncture-movements	

Almost antagonistically, Bowley, at the book's preface, stated that his presentation was based on the work already done by the community in the past: "I have attempted to reduce to a uniform notation, and to present as a properly related whole, the main part of the mathematical methods used by Cournot, Jevons, Pareto, Edgeworth, Marshall, Pigou, and Johnson" (Bowley, 1924, v). He made explicit that he thought he was not doing anything new: "I have not intended to advance any new theorems in economics, nor do I claim any originality in mathematical results" (Bowley, 1924, vi). But his critics disagreed: Tappan (1925, 337) argued that he developed a whole that was more than the sum of its parts, and Young (1925, 134) stated that to relate two previously unrelated theorems was, actually, to advance a new theorem. While Dalton (1924, 233) defined Cassel as an "orthodox iconoclast" for his criticism of the mainstream economics, Bowley's work was seen as a synthesis of the work done until then. Furthermore, the synthesis was qualified as neutral: "Professor Bowley presents the most complete and uniform and – if one may so – neutral treatment, of I am aware, of the mathematical problem of statical equilibrium in exchange" (Tappan, 1925, 337). On other occasions, Bowley's book is pointed out as a *survey*. Allyn Young, then president of the American Economic Association, asserted that until Bowley's book was released, the best survey of economic mathematics was the *Les mathématiques appliquées à l'économie politique*, written by Zadwadski, but Bowley's book had surpassed Zadwadski's because its scope was broader and more systematic (Young, 1925, 133). But what is the scope of Bowley's book – what is the definition of economics and which part of it did he intend to study?

We saw that he intended to study the *mathematical methods* presented by, among other authors, Cournot, Jevons and Pareto. After the preface, the first sentence of the book is the definition of economics: "Economics deals with the production, exchange, possession, consumption, and use of material goods and immaterial services" (Bowley, 1924, 1). Moreover, he also pointed out that the subject of wealth and welfare has two aspects, one subjective – moral or psychological – and the other objective or material. The first could not be measured arithmetically – for example, the satisfaction of obtaining a good; the latter could be measured by quantity or money value. But, although it is not possible to make operations such as addition or multiplication with the subjective aspect, it is still possible to identify equality, continuity, relationship and other properties. From this, the objective aspect could be compared with a "measurable shadow" of an undefined object (Bowley, 1924, 1).

Despite being a lengthier book, the scope of study in economics was actually narrower for Cassel. He stated that "the ultimate aim of economic science must be to discover those necessities which are of a purely economic nature and which cannot be arbitrarily mastered by the will of men" (Cassel, 1924, vi). The demanded quantity of a product, at a given price, has a quantitative nature. However, the psychological process behind this fact does not belong to the domain of economic theory (Cassel, 1924, 81). Antonelli also believed

that exchanges were social facts, as opposed to individual psychological facts. However, he is the only one to make explicit that there were two competing theories in this matter, the Mathematical-Psychological School, or Anglo-Germanic, and the French school – whose main exponent was Cournot. Further, he traced the development of the psychological school since its advent within psychology, presenting the work of the German author Herbart. In the psychological school, Antonelli also included Gossen, Winiarsky, Pareto and Edgeworth.[102] However, still regarding the scope of Antonelli's work, he further specified that, from Walras' works, he presented only the science of general economic equilibrium. He is also the only one to point out the existing division in economics between pure economics, social economics and applied economics, and we should not use "pure economics" as a synonym for "mathematical economics". The confusion of terms used to be a source of many of the misunderstandings between "orthodox" economists and mathematicians. The mathematical method could be applied to many economic problems, for example, the problem of bimetallism, and not only to analyze exchanges (Antonelli, 1914, 41).

There are some further comments on categorizations. In addition to dividing economic science into three broad categories, Antonelli also presented divisions between different types of money and thirteen different types of capital. Cassel, in turn, divided natural factors of production in two types: land and natural material. But he emphatically criticized the classic differentiation between productive and unproductive work. For Cassel – and also later Robbins – this differentiation should be completely abolished, as well as the idea that the general aim of economic activity was solely to produce *material* goods (Cassel, 1924, 22). Bowley, as we saw, despite not directly attacking this classical differentiation, also included the "immaterial services" as an object of study of economics.

In addition to avoiding classifications between productive and unproductive work, Cassel also criticized the attempt to differentiate rational from irrational economic behaviour. Trying to limit the scope of economics only to address rational conduct would make the theory far from the real world (Cassel, 1924, 11). But he chose to consider only human desires that could be expressed in terms of money and that were relatively scarce. This last point is very relevant: the entire price determination process for Cassel was based on the scarcity principle. Cassel did not see cost as determining prices and for him it was the price that determined the marginal value, and not the other way around.

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[102]Although, evidently, he is not the first to do this, Antonelli's division of authors according to countries is worth a remark. Jaffé (1982) argued that Walras refused to see the resemblances between his work and Adam Smith's because of his anglophobia. Jaffé (1982) further argued that such anglophobia had been inherited from his father, a child at the time of the Napoleonic wars. Therefore, it is possible that Antonelli also retained this feature of much considere the countries in which the theories were developed. An Italian author, commenting on Antonelli's work, pointed out that he exaggerated the distinction between the two groups of general equilibrium theorists – in addition to state that the presentation was boring – but that many points that could be considered superfluous in Italy, might not be superfluous in France, where new economic trends were generaly less accepted (V., 1914).

We then have two relevant points to discuss: the scarcity issue and the marginal calculation issue. The title of the first paragraph of Bowley's book is "Marginal utility, indifference curves, offer curves", of the chapter "simple exchange of two commodities", in which he presented the simple exchange model based on the marginal utility of each good exchanged. The marginal principle is further discussed in the section on money prices, in which he used marginal utility of money in his analysis. In his theory of production, the marginal increment is proportional to the price of each factor (Bowley, 1924, 30), and he pointed out that there was an oscillation in the price level given by the equation "marginal price for each = selling price"

Antonelli, as expected, also resorted to marginal principles. He argued that maximum satisfaction was achieved when the *rareté* of the last satisfied desire was equal to prices (Antonelli, 1914, 103). In contrast, when addressing the theory of marginal utility, Cassel argued that such theory was superfluous in economic science, and was only an attempt to input the psychology of demand in an abstract mathematical form: "we must further observe that this deductive inference of the nature of the demand from a single principle, in which so much childish pleasure has been taken, was impossible without artificial constructions and a considerable distortion of the reality" (Cassel, 1924, 82). For him, the idea that the price was equal to the marginal cost was not reasonable. Even if a good could be satisfied in successive doses, it would not mean that the last dose was equal to the price (Cassel, 1924, 82). Prices were equal to the cost of production, what he referred by the "principle of cost". By its nature, this principle of cost was the result of the price system, that regulated the entire economy (Cassel, 1924, 91). This economy could eventually deviate from the principle of cost, though. But he intended to study economics as astronomers first study the motion of the planets, as if they were unaffected by other bodies. Only after this study he would consider these other disturbances (Cassel, 1924, 82). Edgeworth, commenting on Cassel's work, criticized this approach of discarding marginal values. "Our author has not much use for margins. What competent economist of any school can deny it? The fear of making the weak brother to offend should not deter us from using the principle" (Edgeworth, 1920, 533).

So far, we have analyzed the definition and scope of economics for each of our authors, and we have seen that Cassel's demarcation was strict, studying only the phenomena related to human desires that are scarce and could be expressed in terms of money. Antonelli also pointed out that economics studies the phenomena of exchange as social factors, as opposed to individual psychological facts – although he pointed out that there were different views about the issue. And finally, Bowley was seen as middle way between those who saw economic phenomena as related to hedonistic psychological factors and those who studied economics without referring to underlying psychological elements.

But in which economic structures we could apply the theory developed by the authors? According to Cassel, his ideas referred not only to the capitalist system, but the conclu-

sions were also valid in socialism: the results applied to any exchange economy, whatever its particular organization (Cassel, 1924, 113). In fact, studying the pricing system in a socialist economy would be advantageous from a theoretical point of view, according to Cassel. Such a study could be able to show that the free competition hypothesis was unnecessary for the validity of the principle of cost, and show how important this cost principle was for all types of economies (Cassel, 1924, 129). For Cassel the study of pricing in a socialist economy is interesting because it could be considered a simpler economy, and some more complicated features of our system could be elucidated. Besides, the comparison between capitalism and socialism could indicate which institutions were in fact necessary. Moreover, studying socialism would enable us to correct some unrealistic and romanced ideas formed about it (Cassel, 1924, 130). Finally, one of Cassel's appendices was still made to prove that his ideas applied even in war economies, since one of the possible objections to the book was that the world war had changed things so deeply that the economic theory presented in the book was no longer valid. He decided to write the appendix to analyze the phenomenon of war economies, from the point of view of the price theory developed in the book, and concluded again that this theory was broad enough to include not only the traditional economic order but also a socialist system, and both even in cases of war structures (Cassel, 1924, 629).

Bowley, for his part, did not comment economic systems, but discussed, within his system, as previously mentioned, cases of free competition and monopoly. In his chapter on applications of general equilibrium theory, Bowley analyzed the case of only one producer, and concluded, as usual, that the price was higher in equilibrium in comparison with free competition, despite pointing out that the results depend on the elasticity of consumption (Bowley, 1924, 60). But further, he commented on the issue of monopoly in several cases: bilateral monopoly, three monopolists producers, the scenario where all commodities were monopolized, and also discussed the difference in taxation when existed a monopoly. It would not be unusual, perhaps, to have such a detailed discussion, but in a book with only 98 pages, of which 16 was the appendix, this attention to the case of monopolies may be intriguing. We can speculate that Bowley was particularly uncomfortable with the Fabian approach – the London School of Economics was founded by four members of the Fabian Society – who regarded monopolistic companies as acceptable, given the possibility of workers having better living conditions in them. Antonelli, for his part, did not discuss monopoly – which is to be expected when the book's name is *Principles of pure economics: The theory of exchange under the regime of free competition* – but it is curious that he used the example of monopolies to justify the employment of mathematics in economics. For the author, the goal of a method was not always to bring something theoretically new. Just only repeating what authors had already previously said, but in an algebraical form, mathematical economists could be useful in perfecting existing theories, and this was the case with monopolies, in which the mathematical school had had

considerably improved classical theories (Antonelli, 1914, 52).

Although there was some allegation that Cassel did not drastically innovate in relation to classical economics, some novelty is credited to the book, such as the issue of depletion of mines and its microeconomic growth theory – as highlighted in the beginning of this paper. However, still other original contributions are important. First, he developed the concept of purchasing power parity, discussing International Payments, and he highlighted the role of interest in the economic process, which would be a theoretical source later for Keynes. Third, about land rent, he showed that land supply was not fixed. If the price went up, the supply could be increased artificially. Land that had hitherto not been cultivated because of the distance – overseas, for example – could be cultivated, and the higher price would offset transport costs. Unlike Ricardo, Cassel perceived land as a factor of the pricing process, and whose value was also fixed by scarcity. He criticized authors who had developed a hostile tendency to ground rents, citing John Stuart Mill in England and Henry George in the United States. It is in this discussion, too, that it seems that Cassel was referring directly to Walras for the first time. He criticized the "radical" group that "went so far as to want to confiscate existing ground rents, [and] believed that this would suffice to raise the entire revenue of the State, and they accordingly put forward the "single tax" program (Cassel, 1924, 254).

Other innovations are still relevant. Cassel is perhaps the first economist to dynamize the general equilibrium within the so-called "uniformly progressive state", and advancing in relation to Walras, introduced the concept of stationary equilibrium with reproduction of capital. His model of business cycles was also important, but he was unable to identify the ultimate cause of fluctuations, probably because of his limited scope of work – since the main cause of the cycles was outside his strict scope of economics (Phillips, 1924, 239).

We mentioned that Cassel criticized "radicals" who wanted to confiscate the rent of land and to implement a single tax program. Not only regarding ground rent, Cassel was in general an attacker of governmental intervention, and he condemned socialist economics, although his main criticism was in his 1928 book "Socialism or Progress" (Cassel, 1928). The only exception regarding intervention was related to Central Banks, and for that he was accused of being an uncritical believer in an "invisible hand" (Jones & Hart, 1934). The tax system should be constructed in a way that collective wants were compared to those of individual households (Cassel, 1924, 71), and the system should be built aiming to promote growth. The greater weight of taxation on the wealthier prejudices the formation of capital, given that this group is also the one who most promote saving (Cassel, 1924, 229). Still about taxation, Antonelli did not discuss the matter: he only mentioned that it was something that Walras had discussed in his works (Antonelli, 1914, 198).

In addition to taxation, some other controversial questions were discussed, for example, the wage gap between men and women, and the race issue. In the chapter on wages,

Cassel tried to answer the question: why female labor generally receives a wage lower than the value that the equilibrium between supply and demand would fix in the pricing process? Cassel replied that it was possible, in fact, that female labor did not occasionally receive the real market price, because of lack of resistance power or because of the lower geographic mobility of woman. But he argued that if, in fact, women were paid less than they deserved, employers would increasingly replace male work with female work, and "there you have the heart of the question" (Cassel, 1924, 315). Therefore, for Cassel, seeing the real world, he could not recognize the allegedly equality of quality of work, as the theorists used to claim. Men and women do different types of work, so they get different wages. Cassel concluded "the claim that female labour shall be paid equally with male without regard to the state of the market, on some theoretical or emotional "grounds of justice", is on the same level as the ordinary Socialist program of wages" (Cassel, 1924, 316). It would be easy to anachronistically label Cassel as sexist, but it is important to remember that Cassel had engaged in a long battle against socialist ideas. To admit that women systematically received less than men was to admit a flaw in the pricing system, and in the automatism of the market. It is also important to highlight that Cassel spent about a quarter of the book to study economic fluctuations, and his whole motivation was to respond to critics who questioned why, in the capitalist system, there was not a perfect adjustment of the productive forces to demand (Cassel, 1924, 569).

Regarding the race issue, Cassel criticized the "iron law of wages", the tendency for wages to fall to the minimum required for the workers' subsistence. This law could not be observed in modern democratic societies. Wages had risen systematically, and this had not been translated into faster population growth. On the contrary, it was precisely the upper strata of the working class that adhered to birth control, and used the higher wages to improve living standards. But it was necessary to remember that these conclusions applied only to the most advanced countries, "the people of our own *race*" (Cassel, 1924, 296). But although Cassel argued that his ideas could be applied to any exchange economy, whether capitalist or even socialist, we see in this discussion of wages that he clearly pointed out some structures – the world of the people of other races – in which his conclusions could not be applied. Later, Cassel also argued that the characteristics of workers, their health, physical and mental capacity, differed in terms of countries, and were still more conspicuous when comparing one race with another (Cassel, 1924, 322).

Antonelli did not once discuss the issue of races in his 1914 work, but in 1939, in his updated textbook, included an entire chapter about races. In this book, instead of discussing the origin of races by the Darwinian theory, Antonelli preferred to present Moritz Wagner's theory of migration. From the point of view of economic sociology, the human race could be divided into four large groups: the inferior races, the Mongolian race, the Semitic race and the Indo-European race (*les races inférieures, la race mongole, la race*

sémite, la race indo-européene). Men of inferior races, for example Indians and Eskimos, had an intelligence limited only to the field of immediate observation (Antonelli, 1939, 97). However, Antonelli concluded the chapter stating that we should not exaggerate the importance of race from an economic and social point of view (Antonelli, 1939, 101).

We argued that Bowley's work came from authors such as Cournot, Jevons, and Pareto; Cassel did not mention anyone as a precursor, and Antonelli openly declared that he was following Walras's footsteps.[103]. But, furthermore, what types of sources were used in the textbooks? Antonelli singularly used not only Walras's published works but also resorted to his letters and speeches. Antonelli actually used letters as a source of information not only in this work, but in his 1939's textbook. Discussing the definitions of pure economics, he published several pages of exchanged letters between Walras and Mesnil-Marigny. The organization of the presentation was still peculiar: instead of putting it inside chapters or appendices, he published the letters between some chapters and called it a "note" (Antonelli, 1939). In passing, still about the book's structures, Antonelli was also the only one to present, in the introduction to each chapter, a brief summary outlining the key points discussed in it for pedagogical reasons. Regarding again the sources, and the use of speeches, he used, for example, Walras's jubilee's speech at the University of Lausanne, and concluded Walras always saw, behind the problem of price, the problem of general equilibrium (Antonelli, 1914, 11). Still on the same section of this speech, he also resorted to an article published by Walras in 1874 in the *Journal des Economistes*, that addressed the relationship between the price problem and general equilibrium theory.

Cassel, besides not mentioning anyone as precursor, used to reefer to some of his own other books. One of his most referenced works is *The Nature and Necessity of Interest*, published in 1903. He did cite some other works, but mainly to criticize them: for example he cited Marshall's *Principes of Economics*, to criticize the idea of the consumer surplus (Cassel, 1924, 84), and the idea that cost represents a sacrifice made to obtain a good (Cassel, 1924, 90). In addition to Marshall, Cassel mentioned Mill's *Principles*, criticizing the law of decreasing returns on land (Cassel, 1924, 266). Other sources are also founded, for example, in discussing interest rates and life expectancy, Cassel used a survey by the Swedish Ministry of Finance, dealing with hereditary wealth (Cassel, 1924, 236), and discussing money, he cited the Annual Report of the Director of the Mint, published in 1911 (Cassel, 1924, 381). Moreover, as stated before, Cassel also used series of statistical data, in his analysis of business cycles, for example, the British labour statistics, the Journal of the Royal Statistical Society, and the Statistical Abstract of the United States. Marshall's *Principles* is also referenced by Bowley, in his discussion of applications of general equilibrium theory (Bowley, 1924, 65), but he usually made no

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[103]Despite defending Walras's ideas, Antonelli did not present the Walrasian theory uncritically. For example, presenting Cournot's demand equation as a function of prices, Antonelli argued that Walras followed some of Cournot's mistakes, and that reservations should be made about the hypothesis of continuity of the functions (Antonelli, 1914, 18)

reference to other specific works, except when mentioning an article published by W. E. Johson, also addressing applications of general equilibrium theory (Bowley, 1924, 57).

4.4   They are all trying to convince someone: Rhetoric

We began this article with an analysis of the most used words in each book, employing word clouds as a means of summarizing and comparing the three works. We turn again to *words* in this section, with a rhetorical analysis – or as McCloskey defines it, "wordcraft" (McCloskey, 1998, 5). McCloskey (1998) begins her book by defining rhetoric as "a theory of how words persuade even scientists" (McCloskey, 1998, xi). So if rhetoric starts from the idea that words are capable of convincing even scientists, we can start from the idea that students are probably even more vulnerable to the art of persuasion – and what better way to analyze the rhetorical arguments used with students than by studying textbooks? Therefore, we will try to briefly analyze the rhetorical and literary resources employed by the authors in presenting their theories.

The most evident of the rhetorical and literary arguments is the appeal to authority. Bowley just stated that his work was based on the ideas of established authors in economic science: Cournot, Jevons, Pareto, Edgeworth, Marshall, Pigou, and Johnson. Antonelli and Bowley are two peculiar cases. First, Antonelli relied on the authority of Walras, little recognized in France at the beginning of the 20th century. His choice, then, is to show that there were disagreements on how to do economic science, between different schools. In this sense, presenting different views, he *undermines* the appeal to authority of the mainstream schools: their authority was not incontestable. Another perspective is also possible: by demonstrating that Walras was a follower of Cournot, Antonelli claimed to belong to a tradition, and even more, a French tradition, isolating the other contenders; as Keynes did, claiming to be a follower of Malthus, and not Ricardo (Arida, 1983). Cassel, in turn, as we have seen repeatedly, decided to break with the past, "to do away with the whole of the old theory of value, [...] mostly of a rather scholastic nature, which had burdened earlier treatises on economics" (Cassel, 1924, v). In this case, his bet is clear: he preferred to demonstrate the originality and uniqueness of his work, something that students could not find anywhere else. Cassel's main rhetorical feature, however, is the aesthetic criteria of simplicity.

Although simplicity is not an easily defined quality, in Cassel's case, simplicity is evidenced with fewer variables involved in his explanations. First, he excluded the abstract concept of "value" by the tangible concept of "price". But further, he defined that the entire price-determination process was based mainly on only one element: the scarcity principle. He accused his opponents of being much more complicated – the pedantic scholastics – and of ignoring points that he considered to be fundamental. On the Marshallian theory, he argued, for example that the whole of Marshall's system was an attempt

to dispense with the principle of scarcity, or to reject scarcity as a determining factor of prices (Cassel, 1924, 162).

Although Cassel based the simplicity of his theory on a single variable, the scarcity principle, Antonelli and Bowley tried to present a simpler theory abstracting as many complications as possible, starting only with exchanges between two individuals. But Antonelli and Bowley also tried to make the theory more accessible – even though we know that this is not synonymous with simpler – with graphics to illustrate their ideas, something that Cassel did not employ. Bowley also tried to make his theory more accessible by adding the mathematical appendix to help those students with more difficulty in keeping track with the notations. Unfortunately, this effort failed to convince readers about the book's accessibility. A review of the *Journal of the Royal Statistical Society* stated that few people would have the patience to follow economic theories written in mathematical form, and that the notation was valuable only to the author. Even those who had more mathematical skills and were tempted to read the work, would probably enjoy less than expected (F., 1924, 620). Even sympathetic critics pointed to the difficulty of following the author's work: "Professor Bowley's excellently proportioned book is not for us, if we have not reached the alphabet of mathematics" (Tappan, 1925, 334). We see, therefore, that formalization as a way of convincing, in fact, requires an audience capable of understanding, which was not the case of economic science in the early 20th century.

A similar criticism was also made in the *Journal of the Royal Statistical Society* to Antonelli's work. The author argued that the popular interest in economics could be threatened by excessive technicality, which would be a great loss for both professional economists and the lay public. And "the loss is inadequately compensated by the wholesome abstention of the uninstructed from rough awkward meddling with delicate refinements" (P, 1914, 884). Despite the fact that economic science is an autonomous science for our authors – we pointed out previously how each one discussed the object of economics as a singular field – we see that the community perceived a book that was inaccessible to people outside the profession as something undesirable – that is, addressing only fellow academicians it does not necessarily impose greater convincing power.

Still related to the idea that economics is a delimited and separate field from others, we saw that Cassel, criticizing the theory of labour value, argued that it was a hindrance to the development of a scientific theory of labour, mainly because it transferred the controversy to the ethical political field. We see, therefore, a veiled manner of disqualifying the opponent's argument: it simply belongs to a field other than economic science. Antonelli also resorted to this argument: he stressed that the school that he criticized, despite including Pareto and Edgeworth, was the "psychological" school. About Pareto, for example, Antonelli pointed out that he based his entire system on the notion of *ophélimite* which was an essentially psychological notion (Antonelli, 1914, 15). About the school as a whole, he stated that their mathematical economics also was based solely

in a psychological principle (Antonelli, 1914, 16).

Discussing the mathematization of economics, Debreu (1984), stated that simplicity is an important quality of a theory, but besides simplicity, another important quality was generality (Debreu, 1984, 406). Generality makes it possible to apply a theory to a wide range of situations. In this case, Cassel is undoubtedly the one who allegedly presented the most general theory: his ideas applied to all types of exchange economy, whether capitalist, socialist, or even in exceptional cases of war economy. Bowley in turn presented the case of free competition, but he also addressed monopolies, and Antonelli is the author who discussed only the case of free competition. We could say that Antonelli only approached the economy of free competition to make the book as short as possible, however, his work is almost three times longer than Bowley's, so not presenting the monopoly theory is a price to be paid when you spend a large part of your book discussing different existing theories.

In addition to simplicity and generality, another regularly mentioned quality of a good theory – and of a theory that imposes greater convincing power – is the descriptive acuity, the ability it has to describe the world as it really is, or the closest possible. Cassel's book is the one most concerned with highlighting this quality: he argued that it was not necessary to use the hypothesis of perfect competition and rational behaviour, nor to begin the analysis with a theory that had no money. He also did not like to categorize elements – to categorize is to order a world that in reality is chaotic. Furthermore, he is also the only author to use real statistical data, presenting yet another nuance of descriptive acuity. Finally, we also saw that he is the author who used the language in a more incisive way – with the widespread use of "must", instead of Bowley's "may" – perhaps showing greater certainty in his statements – even though we know that it is not possible, of course, to prove that this approach imposes greater convincing power.

4.5   Concluding remarks

Textbooks are capable of showing the maturity of a science, and its increasing degree of homogenization. This paper had as main goal to analyze the teaching of the Walrasian general equilibrium theory, in the first decades of the 20th century, and we chose the works of Bowley, Cassel and Antonelli for this purpose. We could expect that, given the relatively new field, that there would be a need to defend the usefulness of mathematics in economics. But with the exception of mentions made by Antonelli, the authors did not spend much effort in this justification, and Cassel simply pointed out that it was possible to read the book and skip the mathematical part. But an expected feature of a nascent field, which was indeed observed, was the heterogeneity of the books. There was no consensus on fundamental elements, such as the very definition of economics and its scope, the use of assumptions about the market structure and the rationality of individuals, the

adoption of marginal calculations, and the issue of value. Despite disagreements over fundamental matters, however, the content analysis showed that they shared a similar structure. The three essentially discussed exchange, production and factors of production, and the equations of general equilibrium, although they differed in terms of the additional topics discussed in the works, such as Cassel and his analysis of economic cycles and Bowley discussing monopolies and taxation. In the bigger picture of the dissemination of the walrasian general equilibrium theory, we can conclude that Walras's model was presented to students within quite different frameworks, with underlying antagonistic fundamental beliefs about economic science, in the first decades of the 20th century.

This text also started and ended the analysis with *words* as its central element. In the first part, we saw how essentially the quantitative analysis can provide us important information for summarization, and how they can be useful for visually compare different works. However, we ended the discussion by showing how their choice, even in a science whose main language is algebraic, can mean more than just literary style and permeate the authors' argumentative power.

5 Final remarks

This work addressed the dissemination of Walras's theory in two specific ways: studying authors who were interested in his ideas, and tried to advance his model, and the dissemination in the classroom. What we saw first was that defending the mathematization of economics at the beginning of the 20th century was a difficult job. Moore suffered several attacks from the academy, the main example being Edgeworth, who believed that he used an unnecessarily complicated apparatus to study economic science – a criticism commonly raised to Walras as well. Moore published his major book in 1929, *Synthetic Economics*, and retired shortly thereafter, still quite young. Despite the criticism, however, Moore did not suffer any major setbacks in his career for defending Walras in the United States. In contrast, Aupetit and Antonelli had a much worse fate: they tried the *concours d'agregation* in France twice and failed both attempts, not succeeding, thus, in pursuing an academic career. This thesis demonstrated how the institution of the *concours d'agregation* may have been an important factor explaining why mathematical economics was not taught in the country, at least the Walrasian model.

This thesis also shows that the history of the mathematization of economics is also the history of economic statistics. Moore, one of the main authors of early econometrics, had as one of his starting points the empirical verification of the theory of general equilibrium. Albert Aupetit, in turn, also appears as a practitioner of econometrics, and devoted half of his main work to the compilation and analysis of data. The literature also mentions he not only developed these analyzes in the theoretical field, but was one of the main authors responsible for the development of formalized analyzes at the French Central Bank. However, despite the fact that both were Walrasian disciples interested in statistics, there is no record that the two knew each other: there is no cross-citation nor records in Moore's personal archives at Columbia University. Would they have been more successful if they had joined forces? Curiously, it is by studying them that we also observe the question of the authors' personalities influencing, in a way, their personal career: Moore is remembered as an eccentric genius, while Aupetit is pointed out as an arrogant academician – arrogance that is credited to his failure at the *concours*.

Certainly, the main character of this thesis is Walras's *Abrégé*. We saw in Chapter 2 that it was written especially for Aupetit to start teaching Walrasian theory in France. Later, Walras unsuccessfully tries to get Moore to translate it into English and publish it in the United States. However, it is Antonelli, the disciple that Walras was unaware of, who first published it in France in 1914, even reprinting it in 1939. But Walras's ideas became famous in the classroom with no attribution to him, with Cassel's books – which mentions no one – and Bowley's – who did not specifically mention Walras.

As historians of economic thought, we may be instigated by debates happening in neighboring disciplines, for example, the history of philosophy. A development worth

mentioning in the latter are the attempts to build a history of philosophy with no gaps, in which attention is given to "minor figures" in the history of the field. One of the reasons given to study these figures is that they usually were outside thinkers who used to critique the mainstream strands – the reason sometimes they were neglected – and also because they are valuable to understand the context in which more "famous" authors developed their ideas. Is it relevant to talk about building a "history of economic thought without any gaps"? Certainly, this question could not be answered without a collective discussion. But if the answer is "yes", for sure it will be important for this project to study figures outside academia and the history of economic ideas in the classroom.

References

Académie des Sciences Morales and Politiques. (n.d.). *Revue des travaux de l'académie des sciences morales et politiques et comptes rendus de ses séances.* French National Library.

Akhabbar, A. (2010). L'étrange Victoire. Leontief et la Transformation de la Science économique. de la "planification sans Théorie" à la "Mesure sans Théorie", 1920-1949. *Revue Européenne des Sciences Sociales. European Journal of Social Sciences*(145), 33–62.

Alcouffe, A. (2013). La première réception de Léon Walras chez les économistes allemands. In *Léon Walras un siècle après (1910-2010).* Lyon: 7 Colloque de l'Association Internationale Walras.

Aldrich, J. (2010). The Econometricians' Statisticians, 1895–1945. *History of Political Economy*, *42*(1), 111–154.

Allen, W. M. (1930). Review: Synthetic Economics by H. L. Moore. *Economica*.

Álvarez, A. (2020). Léon Walras and Augustin Cournot on the regulation of paper money: Rules vs. discretion at the end of the 19th century. *Iberian Journal of the History of Economic Thought*, *7*(1), 1–13.

Antonelli, E. (1905). *La Protection de la Viticulture par l'état.* Chevalier et Rivière.

Antonelli, E. (1910a). Autour d'une Renaissance Proudhonienne. *Wissen und Leben*, *6*, 562–568.

Antonelli, E. (1910b). Léon Walras. *Revue d'Histoire des Doctrines Économiques et Sociales*, *3*, 169–190.

Antonelli, E. (1912a). Bergson et le Mouvement Social Contemporain. *Wissen und Leben*, *10*, 627–635.

Antonelli, E. (1912b). *Les Actions de Travail dans les Sociétés Anonymes à Participation Ouvrière* (Vol. 12). F. Alcan.

Antonelli, E. (1914). *Principes d'économie Pure: la Théorie de l'échange sous le Régime de la Libre Concurrence.* Librairie des Sciences Politiques et Sociales.

Antonelli, E. (1919). *La Russie Bolchéviste.* B. Grasset.

Antonelli, E. (1921). *L'Afrique et la Paix de Versailles.* B. Grasset.

Antonelli, E. (1923a). Les Fondements du Socialisme: A propos de quelques livres récents. *Revue d'Économie Politique*, *37*(5), 664–691.

Antonelli, E. (1923b). Recent Tendencies in French Theoretical Economics. *Journal of Political Economy*, *31*(4), 561–564.

Antonelli, E. (1923c). Un économiste de 1830: Auguste Walras. *Revue d'histoire Économique et Sociale*, 516–538.

Antonelli, E. (1925). *Manuel de Législation Coloniale.* Presses Universitaires de France.

Antonelli, E. (1927). *Traité d'économie Politique. I. Introduction. Sociologie économique.*

Economie Pure. Impr. des Presses Universitaires de France.

Antonelli, E. (1929). La Doctrine Socialiste. *Revue d'Économie Politique*, *43*(6), 1492–1514.

Antonelli, E. (1930). Constantin Pecqueur. *Revue d'Histoire Économique et Sociale*, 482–504.

Antonelli, E. (1939). *L'économie Pure du Capitalisme.* Librairie M. Rivière.

Antonelli, E. (1953a). La Naissance de la Revue d'économie Politique vue a Travers la Correspondance de Charles Gide et Léon Walras. *Revue d'Économie Politique*, *63*(6), 826–856.

Antonelli, E. (1953b). La Sécurité Sociale. *Revue d'Économie Politique*, *63*(4/5), 808–813.

Antonelli, E. (1959). La Sécurité Sociale. *Revue d'Économie Politique*, *67*(4/5), 754–769.

Antonelli, E. (1966a). Du Dirigisme à la Planification. *Revue d'économie Politique*, *76*(6), 1183–1195.

Antonelli, E. (1966b). La Sécurité Sociale. *Revue d'Économie Politique*, *76*(4/5), 1064–1070.

Antonelli, E. (1967). Ma Dernière Leçon. *Revue d'Économie Politique*, *77*(2), 215–219.

Antonelli, E., & La Clavière, E. (1908). *La Législation sur le Mouillage et le Sucrage des Vins.* Librairie des Sciences Politiques et Sociales.

Arida, P. (1983). A história do pensamento econômico como teoria e retórica. *Texto para discussão*.

Aupetit, A. (1901). *Essai sur la théorie générale de la monnaie.* Paris: Guillaumin.

Aupetit, A. (1903). *Composition d'économie Politique.*

Aupetit, Albert. (1909). *La banque de France. Cours d'expansion commerciale du havre.* Société Internationale pour le Développement de l'Enseignement Commercial.

Béraud, A. (2013). French Economists and the Purchasing Power of Money. *The European Journal of the History of Economic Thought*, *20*(2), 349–371.

Béraud, N. G., Alain. (2019). Léon Walras's Theory of Public Interest Goods: Toward an Organic View of the State. *Journal of the History of Economic Thought*, *41*(4), 553–572.

Bernert, C. (1983). The Career of Causal Analysis in American Sociology. *British Journal of Sociology*, 230–254.

Billard, Y. (1996). Un Parti Républicain-Socialiste a Vraiment Existé. *Vingtieme siecle. Revue d'Histoire*, 43–55.

Boianovsky, M. (2016). Knut Wicksell on Utility and Market Aggregation. *History of Political Economy*, *48*(2), 307–340.

Boumans, M., & Dupont-Kieffer, A. (2011). A History of the Histories of Econometrics. *History of Political Economy*, *43*(1), 5–31.

Bousquet, G. H. (1951). Lettres de Léon et Aline Walras a Albert Aupetit. *Revue d'Histoire Économique et Sociale*, 146-155.

Bouvier, E. (1901a). La méthode mathématique en économie politique: II: La méthode mathématique est-elle nécessaire? *Revue d'économie politique*, 1029–1086.

Bouvier, E. (1901b). La méthode mathématique en économie politique: Ii: La méthode mathématique est-elle nécessaire?(suite). *Revue d'économie politique*, 1029–1086.

Bowley, A. L. (1924). *The Mathematical Groundwork of Economics: An Introductory Treatise*. Oxford: Clarendon Press.

Brems, H. (1986). Gustav Cassel's Contributions to Economic Theory. *BEBR Faculty Working Paper*(1282).

Breton, Y. (1992). L'économie politique et les mathématiques en France 1800-1940. *Histoire & Mesure*, 25–52.

Bridel, P. (1997). Money and General Equilibrium Theory.

Bridel, P. (2002). Patinkin, walras and the'money-in-the-utility-function'tradition. *European Journal of the History of Economic Thought*, *9*(2), 268–292.

Burns, A. (1931). Review: Synthetic Economics. *Journal of the American Statistical Association*.

Busino, G. (2010). Note di storia economica e storia delle dottrine economiche. *Revue Européenne des Sciences Sociales*, *48*(145), 101–163.

Camic, C., & Xie, Y. (1994). The Statistical Turn in American Social Science: Columbia University, 1890 to 1915. *American Sociological Review*, 773–805.

Cassel, G. (1924). *The Theory of Social Economy*. Harcourt.

Cassel, G. (1928). *Socialism eller framåtskridande*. Norstedt.

Christ, C. F. (1985). Early Progress in Estimating Quantitative Economic Relationships in America. *The American Economic Review*, *75*(6), 39–52.

Columbia, U. (1959). *Henry Ludwell Moore*. Memorial Minute.

Cot, A.-L., & Lallement, J. (2006). 1859-1959: de Walras à Debreu, un siècle d'équilibre général. *Revue économique*, *57*(3), 377–388.

Dalton, H. (1924). Review: The Theory of Social Economy, by Gustav Cassel. *Economica*(11), 223-226.

Davenport, H. J. (1901). Review: Essai sur la théorie genérale de la monnaie: économique rationnelle by Albert Aupetit. *Journal of Political Economy*.

Debreu, G. (1984). Economic theory in the mathematical mode. *The Scandinavian Journal of Economics*, *86*(4), 393–410.

Diemer, A. (2006). Existe-t-il une Tradition Française de la Théorie de l'équilibre Général? *Document de travail. Project "Histoire des Savoirs", Centre National de la Recherche Scientifique*.

Douglas, P. H. (1939). Henry Schultz as Colleague. *Econometrica: Journal of the Econometric Society*, 104–106.

Düppe, T., & Weintraub, E. R. (2014). *Finding equilibrium: Arrow, Debreu, McKenzie and the problem of scientific credit*. Princeton: Princeton University Press.

Dupriez, L. (1958). Review: Essai sur la théorie genérale de la monnaie: économique rationnelle by Albert Aupetit. *Econometrica*.

Edgeworth, F. Y. (1912). Review: Laws of Wages: An Essay in Statistical Economics. *The Economic Journal*.

Edgeworth, F. Y. (1920). Professor Cassel's Treatise. *The Economic Journal*(120), 530-536.

Ekelund, R. B., & Hébert, R. F. (2003). Ethics, Engineering, and Natural Monopoly: the "Modern Debate" between Léon Walras and Jules Dupuit. *History of Political Economy*, *35*(4), 655–678.

Epstein, R. J. (2014). *A History of Econometrics* (Vol. 165). Amsterdã: Elsevier.

Ezekiel, M. (1930). Moore's Synthetic Economics. *The Quartely Journal of Economics*.

F., A. W. (1924). Review: The Mathematical Groundwork of Economics, by A. L. Bowley. *Journal of the Royal Statistical Society*, *87*(4), 619-621.

Fay, C. (1912). Review: Les Actions de Travail dans les Sociétés Anonymes à Participation Ouvrière. *The Economic Journal*.

Fisher, I. (1933). List of fellows of the econometric society. *Econometrica*.

Flux, A. W. (1931). Review: Synthetic economics by henry ludwell moore. *The Economic Journal*.

Forder, J. (2015). Textbooks on the Phillips curve. *History of Political Economy*, *47*(2), 207–240.

Forget, E. L. (2020). 2018 HES Presidential Address: Folk Wisdom in Economics. *Journal of the History of Economic Thought*, *42*(1), 1–18.

Freedman, C. (2010). The Chicago School. A conversation with Paul Samuelson. *History of Economic Ideas*, *18*(3), 161–185.

French Central Bank. (2000). *Secretariat general 1812-1969 – versement 1060200001*.

French Central Bank. (2002). *Inventaire du fonds du secretariat general – service administratif – versements 1060200103 à 1060200110*.

French National Library. (1933). *Clipping of a newspaper from an unidentified day. Copy in possession of the author*.

Frobert, L. (1997). L'économie politique d'Etienne Antonelli (1879-1971). *Revue économique*, 1529–1559.

Frobert, L. (1999). Albert Aftalion on Socialism. *Journal of the History of Economic Thought*, *21*(2), 145–161.

Gallois, N. (2011). The French Liberal School against the heterodox economist Léon Walras. *L'Économie politique*(3), 7–32.

Ginzberg, E. (1990). Economics at Columbia: Recollections of the Early 1930s. *The American Economist*, *34*(2), 14–19.

Giraud, Y. (2018). Textbooks in the History of Recent Economics: The Case of Samuelson's Economics. *History of Political Economy*, *50*(3), 579–585.

Harzing, A.-W. (2010). *The Publish or Perish Book*. Tarma Software Research Pty Limited.

Hébert, C. (1988). Léon Walras et les associations populaires coopératives. *Revue d'économie politique*, 252–272.

Ingrao, B., & Israel, G. (2015). *The invisible hand: economic equilibrium in the history of science*. Cambridge: MIT Press, MA.

Jacoud, G. (1994). Stabilité monétaire et régulation étatique dans l'analyse de Léon Walras. *Revue économique*, 257–288.

Jaffé, W. (1965). *Correspondence of Léon Walras and Related Papers*. Amsterdam: North-Holland Publishing Company.

Jaffe, W. (1975). Léon Walras, an Economic Adviser Manqué. *The Economic Journal*, 810–823.

Jaffé, W. (1982). William Jaffés Essays on Walras. In D. Walker (Ed.), (chap. A Centenarian on a Bicentenarian: Leon Walras's Elements on Adam Smith's Wealth of Nations). Cambridge University Press.

Jaffé, W. (1983). *William Jaffé's Essays on Walras*. Cambridge University Press.

Jolink, A., & Knot, G. (1993). Procrustean beds and all that: the irrelevance of Walras for a Mirowski thesis. *De Marchi (1993)*, 159–174.

Jones, H., & Hart, A. (1934). Professor Cassel's Treatise. *Journal of Political Economy*, *42*(4), 555-561.

Judge, G. G. (1968). The Search for Quantitative Economic Knowledge. *American Journal of Agricultural Economics*, *50*(5), 1703–1717.

King, J. E., & Millmow, A. (2003). Death of a Revolutionary Textbook. *History of Political Economy*, *35*(1), 105–134.

Klein, J. L., & Klein, D. (1997). *Statistical Visions in Time: A history of time series analysis, 1662-1938*. Cambridge University Press.

Le Gall, P. (1999). A world ruled by Venus: On Henry L. Moore's transfer of periodogram analysis from physics to economics. *History of Political Economy*, *31*(4), 723.

Le Maux, L. (2007). Mesurer la monnaie. Banques centrales et construction de l'autorité monétaire (xixe-xxe siècle). In *Annales. histoire, sciences sociales* (Vol. 62, pp. 1472–1474).

Le Van Lemesle, L. (1994). Antonelli, etienne (1879-1971). Professeur d'Assurances Sociales (jurisprudence et contentieux)(1932-1935). *Publications de l'Institut National de Recherche Pédagogique*, *19*(1), 108–116.

Magee, J. (1915). Review: Economic Cycles: Their Law and Cause by Henry Ludwell Moore. *Journal of Political Economy*.

McCloskey, D. N. (1998). *The Rhetoric of Economics*. Univ of Wisconsin Press.

Milhau, J. (1953). L'Action Politique d'Etienne Antonelli. *Revue d'Histoire Économique et Sociale*, 364–382.

Mirowski, P. (1989). The Probabilistic Counterrevolution, or how Stochasticconcepts Came to Neoclassical Economic Theory. *Oxford Economic Papers*, *41*(1), 217–235.

Mirowski, P. (1990). Problems in the paternity of econometrics: Henry Ludwell Moore. *History of Political Economy*, *22*(4), 587–609.

Moll, B. (1947). Gustavo Cassel (1866-1945). *El Trimestre Económico*.

Moore, H. L. (1905). The personality of Antoine Augustin Cournot. *The Quarterly Journal of Economics (1886-1906)*, 370.

Moore, H. L. (1907a). The Differential Law of Wages. *Journal of the Royal Statistical Society*, *70*(4), 638–651.

Moore, H. L. (1907b). The Variability of Wages. *Political Science Quarterly*, *22*(1), 61–73.

Moore, H. L. (1908). The statistical complement of pure economics. *The Quarterly Journal of Economics*, *23*(1), 1–33.

Moore, H. L. (1911). *Laws of Wages: An Essay in Statistical Economics*. Macmillan.

Moore, H. L. (1914). *Economic Cycles: Their law and cause*. Macmillan.

Moore, H. L. (1917). *Forecasting the Yield and the Price of Cotton*. Macmillan.

Moore, H. L. (1919). Crop-Cycles in the United Kingdom and in the United States. *Journal of the Royal Statistical Society*, *82*(3), 373–389.

Moore, H. L. (1920a). Crop-Cycles in the United Kingdom and in France. *Journal of the Royal Statistical Society*, *83*(3), 445–454.

Moore, H. L. (1920b). Forecasting the Crops of the Dakotas. *Political Science Quarterly*, *35*(2), 204–235.

Moore, H. L. (1921). Generating Cycles Reflected in a Century of Prices. *The Quarterly Journal of Economics*, *35*(4), 503–526.

Moore, H. L. (1925). A Moving Equilibrium of Demand and Supply. *The Quarterly Journal of Economics*, *39*(3), 357–371.

Moore, H. L. (1926a). Partial elasticity of demand. *The Quarterly Journal of Economics*, *40*(3), 393–401.

Moore, H. L. (1926b). A Theory of Economic Oscillations. *The Quarterly Journal of Economics*, 1–29.

Moore, H. L. (1929). Synthetic Economics.

Moore, H. L., & Edgeworth, F. Y. (1912). A Reply to Professor Edgeworth's Review of Professor H.L. Moore's "Laws of Wages". *The Economic Journal*.

Moret, J. (1915). *L'emploi des mathématiques en économie politique*. M. Giard & É. Brière.

Morgan, M. S., et al. (1990). *The History of Econometric Ideas*. Cambridge University Press.

Morini-Comby, J. (1953). La Carrière Enseignante d'Etienne Antonelli. *Revue d'Histoire économique et Sociale*, 355–363.

Neuendorf, K. A. (2002). *The Content Analysis Guidebook*. Sage.

Ostroy, J. M. (1987). Money and general equilibriun theory. In *The New Palvgrave Dictionary of Economics* (pp. 6736–6742). Basingstoke: Palgrave Macmillan.

P, L. L. (1914). Review: Principes d'économie Pure by Etienne Antonelli. *Journal of the Royal Statistical Society*.

Pearson, K., & Lee, A. (1903). On the Laws of Inheritance in Man: I. Inheritance of Physical Characters. *Biometrika*, *2*(4), 357–462.

Perroux, F. (1953). Étienne Antonelli et l'Analyse Scientifique des Phénomènes économiques: Le Dialogue Léon Walras-Etienne Antonelli. *Revue d'Histoire Économique et Sociale*, 339–354.

Persons, W. M. (1925). Statistics and economic theory. *The Review of Economics and Statistics*, *7*(3), 179–197.

Phillips, H. (1924). Review: The Theory of Social Economy, by Gustav Cassel and Joseph Mccabe. *The Economic Journal*(134), 234-241.

Pomini, M., & Tusset, G. (2009). Habits and expectations: dynamic general equilibrium in the Italian Paretian School. *History of Political Economy*, *41*(2), 311–342.

Quandt, R. E. (1976). Some Quantitative Aspects of the Economics Journal Literature. *Journal of Political Economy*, *84*(4, Part 1), 741–755.

Raybaut, A. (1991). L'économie synthétique d'H.L. Moore: Un compromis original entre la théorie walrasienne de l'équilibre et l'analyse des cycles d'affaires. *Revue d'Économie Politique*, *101*(6), 907-923.

Rist, C. (1966). *History of Monetary and Credit Theory from John Law to the Present Day, trans.*

Robbins, L. (2007). *An Essay on the Nature and Significance of Economic Science*. Macmillan.

Schultz, H. (1925). The Statistical Law of Demand as Illustrated by the Demand for Sugar. *Journal of Political Economy*, *33*(6), 577–631.

Schultz, H. (1929). Marginal Productivity and the General Pricing Process. *Journal of Political Economy*, *37*(5), 505–551.

Schumpeter, J. A. (1954). *History of Economic Analysis*. Routledge.

Semedo, G. (2000). Réhabiliter Albert Aupetit, disciple et continuateur de L. Walras, par sa mise en évidence du lien entre la monnaie, les échanges extérieurs et la grande dépression du XIXe siècle dans un contexte de change or. *Brussels Economic Review*, *167*, 345–380.

Smith, A. (1979). *An Inquiry into the Nature and Causes of the Wealth of Nations. Volume I of the Glasgow Edition of the Works and Correspondence of Adam Smith*. Libert Classics.

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Spanos, A. (2006). Econometrics in Retrospect and Prospect. *New Palgrave Handbook of Econometrics*, *1*, 3–58.

Stigler, G. J. (1954). The Early History of Empirical Studies of Consumer Behavior. *Journal of Political Economy*, *62*(2), 95–113.

Stigler, G. J. (1962). Henry L. Moore and statistical economics. *Econometrica*, 1–21.

Stigler, S. M. (1986). *The history of statistics: The measurement of uncertainty before 1900*. Harvard University Press.

Tabak, J. (2014). *Probability and statistics: The science of uncertainty*. Infobase Publishing.

Tappan, M. (1925). The Mathematical Groundwork of Economics by A.L. Bowley. *Economica*(15), 334-338.

Teixeira, P. (2014). Serving the Institute and the Discipline: The Changing Profile of Economics at MIT as Viewed from Textbooks. *History of Political Economy*, *46*(suppl_1), 153–174.

Turk, M. H. (2012). The mathematical turn in economics: Walras, the French mathematicians, and the road not taken. *Journal of the History of Economic Thought*, *34*(2), 149–167.

V., G. D. (1914). Review: Principes d'économie politique Pure. *Giornale degli Economisti e Rivista di Statistica*, *20*(4), 265.

Walker, D. A. (1970). Léon Walras in the light of his correspondence and related papers. *Journal of Political Economy*, *78*(4, Part 1), 685–701.

Walker, D. A. (1984). Is Walras's Theory of General Equilibrium a Normative Scheme? *History of Political Economy*, *16*(3), 445–469.

Walker, D. A. (1989). Book Review: Mélanges d'économie Politique et Sociale. *History of Political Economy*, *21*(3), 556–559.

Walker, D. A. (2006). *Walrasian Economics*. Cambridge University Press.

Walras, L. (1860). *L'économie Politique et la Justice; Examen Critique et Réfutation des Doctrines économiques de MPJ Proudhon: Précédés d'une introduction à l'étude de la question sociale*. Librairie de Guillaumin.

Walras, L. (1865). *De l'Organisation Financière et de la Constitution Légale des Associations Populaires*. Impr. A. Parent.

Walras, L. (1954). *Elements of Pure Economics; or, The Theory of Social wealth /Translated by William Jaffé*. Homewood, III, Published for the American Economic Association by Richard D. Irwin.

Walras, L. (2008). *Studies in Applied Economics*. Routledge.

Walras, L. (2010). *Studies in Social Economics* (Vol. 111). Routledge.

Walras, L. (2013). *Elements of Pure Economics*. Routledge.

Walras, L., Cournot, A., Jevons, S., & Antonelli, E. (1935). Léon Walras et sa Corréspondance avec Augustin Cournot et Stanley Jevons. *Econometrica*, 119–127.

Weintraub, E. R. (2002). *How economics became a mathematical science.* Durham: Duke University Press.

Weintraub, E. R., & Gayer, T. (2001). Equilibrium proofmaking. *Journal of the History of Economic Thought*, *23*(4), 421-442.

Wicksell, K. (1934). Professor cassel's system of economics. *Trans. by S. Adler. In Lectures on Political Economy*, *1*, 219–273.

Wright, P. G. (1930). Moore's "synthetic economics". *Journal of Political Economy*.

Wulwick, N. J. (1992). The Folklore of HL Moore on the Demand for Pig Iron. *Journal of the History of Economic Thought*, *14*(2), 168–188.

Young, A. (1925). Review: The Mathematical Groundwork of Economics, by A. L. Bowley. *Journal of the American Statistical Association*, *20*(149), 133-135.