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DOES THE CENTRAL BANK OF BRAZIL CARES ABOUT EXPECTED INFLATION?

O Banco Central do Brasil se Preocupa com a Inflação Esperada?

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ABSTRACT

The purpose of this dissertation is to characterize Brazil's monetary policy conduction under the inflation targeting regime that started in 1999 for different Central Bank of Brazil (BCB) chairman tenures. The methodology adopted consists in estimating a forward-looking Taylor rule with parameters that can vary through time using Kalman filter. Specifically, to handle with the endogeneity problem of forward-looking rules estimated with ex-post data under a time-varying parameters framework, we adopt a similar approach to the two-stage method originally suggested by Kim and Nelson (2006). Regarding results, using data from 2000 to 2020, estimates suggest that BCB chairman's were – for the great majority of time for the analyzed period – seeking a stabilizing monetary policy and following the guidelines of the inflation target framework. In this sense, given the results and what they suggest, we attribute the failure to meet the target in certain years much more to the fact that Brazil is a newly democratic emerging market economy subject to many domestic and external shocks than with a disregard of the BCB with monetary policy stabilization.

Key words: Taylor rule; inflation targeting; time-varying parameters; Kalman filter.

JEL Classification: C5, C32, E52, E58.

RESUMO

O objetivo desta dissertação é caracterizar a condução da política monetária do Brasil durante o regime de metas para inflação iniciado em 1999 para diferentes mandatos presidenciais do Banco Central do Brasil (BCB). A metodologia adotada consiste na estimação de uma Regra de Taylor *forward-looking* com parâmetros que podem variar ao longo do tempo utilizando Filtro de Kalman. Especificamente, para lidar com problemas de endogeneidade nos regressores quando dados *ex-post* são utilizados em um arcabouço de parâmetros variantes no tempo, adotamos um procedimento semelhante ao de dois estágios originalmente proposto por Kim e Nelson (2006). Em relação aos resultados, usando dados de 2000 a 2020, as estimativas sugerem que os presidentes do BCB estavam - para a grande maioria do período analisado - buscando tanto uma política monetária estabilizadora quanto seguindo o arcabouço do regime de metas para inflação. Nesse contexto, considerando os resultados e o que eles sugerem, nós atribuímos o não cumprimento com a meta em determinados anos muito mais ao fato de que o Brasil é uma economia emergente recém democratizada sujeita a muitos choques internos e externos do que ao descaso do BCB com a estabilidade monetária.

Palavras-chave: Regra de Taylor; metas para inflação; parâmetros variantes no tempo; Filtro de Kalman.

Código JEL: C5, C32, E52, E58.

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1 INTRODUCTION AND OBJECTIVES

Taylor's (1993) noteworthy paper was the starting point for many research studies focused on monetary policy evaluation in a variety of countries. The idea proposed by the author, although simple, was groundbreaking: a good policy rule should empirically relate the policy rate (i.e., the interest rate used as monetary policy instrument) to the economic objectives of the Central Bank (e.g., inflation and output). It's clear that this type of understanding was mostly based on Lucas (1976) *critique*¹, which shed light to flaws in traditional econometric policy evaluation, and in Kydland and Prescott (1977) work, which demonstrates the superiority of rules over time inconsistent discretion policies².

For a brief period after Taylor's (1993) paper, the vast majority of works was focused in analysing United States monetary policy³, however this quickly changed as researchers started to pay attention to other "laboratories" for their experiments. Due to it's idiosyncrasies, such as the megainflationary period⁴ that prevailed between 1980 and 1994, the subsequent new republic economic stabilization plans with the success of the Real Plan in 1994 and the high levels of government indebtedness⁵, Brazil is object of many macroeconomic studies. Moreover, since the adoption of the Inflation Targeting (IT) in 1999, many works have focused on Brazil's monetary policy conduction under the IT regime.

As exposed by Fraga et al. (2003), after the adoption of the target system, inflation expectations and Central Bank credibility became the core of Brazil's monetary policy. Indeed, BCB long-run objective with the IT framework is to achieve a bigger control over the price level, without the costs associated with traditional monetary policy arrangements, such as the unemployment-inflation trade-off (Mendonça, 2009).

With all that said, in order to develop a better comprehension of Brazil's monetary policy conduction after 1999, the purpose of this dissertation is to estimate a policy reaction function for the Central Bank of Brazil (BCB) during the inflation targeting era. Specifically, we want to evaluate changes in monetary policy stance for different BCB

¹ In general terms, Lucas emphasizes the impact of private agents expectations on economic results, especially with regard to the evaluation of public policies. The so-called Lucas Critique originates from the argument that traditional methods of evaluation do not adequately encompass the impact that policy has on agents' expectations. Therefore, such public policies end up not being evaluated and implemented optimally (Lucas, 1976).

² See Kydland and Prescott (1977).

³ See the literature review.

⁴ This period was not characterized by a typical hyperinflation, given that it was mostly marked by a big inflationary inertia. Inflation became inertial because contracts had an indexation clause that re-established their real value at a fixed frequency. In this way, a process of automatic readjustment of the price level was formed, based on past inflation (Simonsen, 1991).

⁵ According to Fincke and Greiner (2015), from 1980 to 2012 the average government debt ratio for Brazil was 61% against 46% for a sample of peer emerging market economies (Indonesia, India, Mexico, Malasya, Thailand, Turkey and South Africa).

chairmans that had their tenures during the IT regime⁶, therefore we use a time-varying parameters framework.

Moreover, as we are focusing on the IT era, we estimate a forward-looking Taylor rule. Regarding that, one needs to deal with the endogeneity problem, given that regressors could be possibly correlated to monetary policy shocks. Indeed, as exposed by Clarida, Gali and Gertler (2000), when using ex-post data in this type of rule, the regressors are going to be correlated with the error term⁷. Therefore, to handle with this problem under a time-varying parameters with Kalman filter framework, we adopt a similar approach to the two-stage Heckman (1974) type procedure originally proposed by Kim and Nelson (2006).

In this context, the objectives of this research are twofold. First, in order to gauge the evolution of monetary policy conduction in Brazil under inflation targeting, we estimate a forward-looking Taylor rule in which the coefficients associated with expected inflation deviation from target and output gap are allowed to vary over time. Not only this procedure allows one to classify a certain BCB governor monetary policy as stabilizing or not (using the benchmarks proposed by Clarida, Gali and Gertler (2000) and exposed in subsection 3.2), but also it enables assessing whether adopting a stabilizing policy leads to actual monetary stability. It's particular interesting analysing this question, considering that Brazil is a newly democratic emerging market economy with lots of idiosyncrasies.

Second, we follow Carvalho (2021) and estimate a time-varying implicit inflation target for the BCB policy rule. This procedure allows one to observe BCB inflation preferences over time and compare them to the target imposed by the National Monetary Council (CMN), providing another tool to assess monetary policy stance. Indeed, we can verify if the BCB governors have adopted a more dovish or hawkish approach vis-à-vis the official target. Note that both objectives can contribute to a better characterization of the Brazilian monetary policy under IT for different chairman tenures⁸.

In terms of results, they suggest that in general the Central Bank of Brazil (BCB) governors have adopted a stabilizing monetary policy for the sample considered. Specifically, regarding the Taylor principle, with exception of a brief period during Tombini's tenure, all BCB chairman's have respect it, and the coefficient associated with the output gap has stayed mostly above the zero line. Furthermore, the pattern found for the implicit target

⁶ Arminio Fraga Neto (3/4/1999 to 1/1/2003), Henrique de Campos Meirelles (1/1/2003 to 12/31/2010), Alexandre Antonio Tombini (1/1/2011 to 6/9/2016), Ilan Goldfajn (6/9/2016 to 2/27/2019) and Roberto Campos Neto (2/28/2019 to 12/31/2025).

⁷ As argued by Baxa, Horváth and Vasicek (2014), endogeneity should be viewed as a problem in foward-looking rules regardless of the use of real-time data. This because real-time forecasts are only going to be truly exogenous if there's an belief that the policy rate will remain unchanged throughout the forecast horizon, which is a quite strong hypothesis (Boivin, 2006).

⁸ Its worth mentioning that this dissertation is mostly based on Medeiros and Aragón (2015) and Carvalho (2021) works. Nevertheless, we use a larger sample, different set of variables and also try to provide a different interpretation for the results.

deviation from the official one suggests that BCB governors were monitoring inflation expectations and trying to anchor them. Moreover, the BCB has operated with a implicit target predominantly within the upper and lower bound imposed by the CMN, although the estimated target has exceeded the upper limit in some situations after 2011. All in all, results suggest that governors were acting under the IT framework.

In this sense, our contribution is to support the idea that BCB chairman's were - in the great majority of time for the analyzed period - seeking a stabilizing monetary policy and following the guidelines of the IT framework. Therefore, given the results, we attribute the failure to meet the target in certain years much more to the fact that Brazil is a newly democratic emerging market economy subject to many domestic and external shocks than with a disregard of the BCB with monetary policy stabilization.

Indeed, as argued by Moura and Carvalho (2010), monetary policy results for developing economies cannot be isolated from a wide range of external factors. Consequently, they defend that instead of focusing on the results of the policy, attention should be paid to discriminating the different stances based on the characterization of a certain monetary policy. We agree with that and try to go a little further, by analysing both monetary policy posture and outcome for the Brazilian case.

The remaining of this dissertation will be organized as follows. In chapter 2, we expose a brief literature review. In chapter 3 and 4 we present the methodology and data, respectively. Finally, in chapter 5 one can find the results and in chapter 6 the conclusion.

2 LITERATURE REVIEW

Probably, one of the first works to point out that Federal Reserve's (Fed) reaction function changed over time was the one proposed by Rudebusch and Judd (1998). Basically, the authors used Ordinary Least Squares (OLS) to estimate Fed's policy function for three different subsamples distinguished by chairman⁹ and concluded that the Taylor rule parameters varied notably in each estimation. Of course, we also need to cite Clarida, Gali and Gertler (2000) research, that verified by GMM estimation of a forward-looking policy reaction function a considerable shift in Fed's sensibility to expected inflation during Paul Volcker's tenure¹⁰. It's clear that these two papers underscore that policymakers stance affects monetary policy condunction.

In terms of estimation of time-varying parameters Taylor rules for the United States, we can cite the work of Shen, Hakes and Brown (1999). Using a nonlinear Kalman filter approach, the authors concluded that a reaction function for the Federal Reserve with fixed parameters was inappropriate to describe monetary policy conduction over the period of 1953-1994. Furthermore, the research of Kim and Nelson (2006) innovates as it deals with the endogeneity problem of forward-looking Taylor rules under a time-varying parameters framework. Specifically, the authors propose a Heckman (1974) two-step estimation procedure together with a Kalman filter maximum likelihood method. One can say that the greater contribution of the work is to propose a novel division for monetary policy conduction in United States between 1960Q1 and 2001Q2, with three sub-periods: 1970's, 1980's and 1990's.

Concerning the literature regarding the estimation of Taylor rules with time-varying implicit inflation targets, the research of Leigh (2008) innovates by relaxing the common assumption of a constant inflation target in United States used at the time. By observing Federal Reserve (Fed) policymakers public statements over the period of 1979Q3 to 2004Q1, the author had a suggestion that the target for the inflation rate was varying over time. Indeed, using the Kalman filter, a significant reduction in the Fed's inflation target is found over the course of Greenspan's tenure, for example. Moreover, Klein (2012) compares South Africa Reserve Bank (SARB) implicit inflation target with the explicit one, again using a Kalman filter framework, for a sample that ranges from 2001 to 2011. The main conclusion of the paper is that the estimated inflation target presented a upward movement toward the upper band of the interval of tolerance after the 2008 financial crisis, which may suggest a higher tolerance for inflation in order to support economic activity.

Regarding Brazil, many works have studied reaction functions for the Brazilian

⁹ Specifically, Burns (1970Q1-1978Q2), Volcker (1979Q3-1987Q2) and Greenspan (1987Q3-1997Q4).

¹⁰ From the baseline estimation, the authors find out that pre-Volcker period the coefficient associated with inflation expectations was less than one ($\beta = 0.83$), and considerably above the unit ($\beta = 2.15$) under Volcker-Greenspan period.

Central Bank (BCB). Probably one of the first to investigate Brazil's monetary policy approach after the adoption of inflation targeting in 1999 was the one elaborated by Minella et al. (2002). The main conclusion for a sample that span the period of 2000M1 to 2002M6 was that BCB has acted under the Taylor's principle, reacting strongly to inflation expectations. Furthermore, in order to characterize different stances in monetary policy before and after the Real plan in Brazil, De Losso (2008) estimates with GMM a forward-looking Taylor rule. Somewhat unexpected, results suggest that Taylor's principle was respected before Real plan and not after it.

Again for the Brazilian case, but considering Taylor rule estimation with timevarying parameters, we cite the work of Medeiros and Aragón (2015), which addresses the endogeneity problem of forward-looking policy rules under a Kalman filter framework using the methodology developed by Kim and Nelson (2006). Using data from 2001M1 to 2011M12, the authors estimate two different rules: one with actual inflation and other with market's expected inflation. Regarding the major results, they find that the coefficient associated with expected inflation deviation from the target was negative after 2011 and that policy rule regressors indeed present endogeneity.

Furthermore, again addressing the endogeneity problem of forward-looking rules with Kim and Nelson (2006) methodology, Carvalho (2021) also estimate a time-varying monetary policy rule for the Central Bank of Brazil; specifically being the first one - as far as we know - to estimate a time-varying implicit inflation target for the BCB. Using three different specifications for the rule, the results suggest that the Taylor principle was respected for the great majority of the sample (2004Q1 to 2020Q3). Moreover, a relation between the implicit inflation target and expected inflation is found, and the estimated target was until 2011 predominantly below the explicit one and has oscillated above it afterwards.

Lima et. al. (2015) estimate a policy rule for Brazil using a Markov-Switching (MS) model for the period after the adoption of the Real plan in 1994, finding evidence that monetary policy can be characterized by four different regimes during this period. It's also important to cite the work of Policano and De Losso (2006), in which the Brazilian Central Bank time-varying monetary policy rule is estimated for two different periods: 1995-2005 (complete sample) and 2000-2005. For the complete sample - in which the authors add as additional regressors to the Taylor rule international reserves and exchange rate variation - it's found that until January 1999 (managed exchange rate regime) the policy rate has negatively responded to reserves and positively reacted to the output gap. Nevertheless, for the same sample, from January 1999 to June 1999 - transition period from the managed exchange rate regime to the inflation target one - there is a sudden increase in the Selic's response to the exchange rate devaluation. Furthermore, for the period of 2000-2005, there is evidence that Brazil's monetary policy was predominantly guided by expected inflation

deviation from the target.

Finally, to conclude our literature review, we cite the work of Moura and Carvalho (2010), in which the authors seek to characterize monetary policy conduction for seven Latin America countries: five with inflation targeting (i.e., Brazil, Chile, Colombia, Mexico and Peru) and two without it (i.e., Argentina and Venezuela). Using a sample ranging from 1999-2008 and selecting the most adequate Taylor rule specification from sixteen different functional forms - based on out-of-sample measures -, they find evidence that monetary policy rules are quite good in representing monetary policy conduction in the selected emerging market economies. Regarding Brazil, results suggest an asymmetric monetary policy rule: policymakers have been adjusting the policy rate more heavily when inflation is below the target, when compared to when it's above it.

3 EMPIRICAL MODEL

3.1 Forward-Looking Taylor Rule

With the interest of characterizing different monetary policy stances for the Central Bank of Brazil chairman's after the adoption of the IT regime in 1999, we begin with the simple forward-looking Taylor rule as proposed by Clarida, Gali and Gertler (2000):

$$i_t^* = i^* + \beta_\pi \left(\mathbb{E}_{t-1} \left[\pi_{t,k} \right] - \pi^* \right) + \beta_y \mathbb{E}_{t-1} \left(\tilde{y}_t \right)$$
(1)

where i_t^* is the target interest rate followed by the Central Bank at time t; i^* is the nominal neutral interest rate (i.e., the nominal rate when inflation and output are at their target levels), $\pi_{t,k}$ is the inflation rate between periods t and t + k; π^* is the target for inflation rate; \tilde{y}_t is the output gap, the percentage difference between output and potential output (i.e., what can be produced if the economy were operating with employment at its natural level, without generating inflation pressure); and \mathbb{E}_{t-1} is the expectation formed with information conditional at time t - 1.

3.2 Implied Real Rate Rule

As a way to better understand the Taylor rule intuition, we can define the *ex-ante* optimal real interest rate (r_t^*) as:

$$r_t^* \equiv i_t - \mathbb{E}_{t-1}\left[\pi_{t,k}\right] \tag{2}$$

Therefore, after substituting (2) in (1) and doing some algebra, one can find a forward-looking Taylor rule for the *ex-ante* real interest rate:

$$r_t^* = r^* + (\beta_{\pi} - 1) \left(\mathbb{E}_{t-1} \left[\pi_{t,k} \right] - \pi^* \right) + \beta_y \mathbb{E}_{t-1} \left(\tilde{y}_t \right)$$
(3)

where $r^* \equiv i^* - \pi^*$ is the long-run equilibrium real interest rate.

It's clear from this specification that the response of the *ex-ante* real interest rate after deviations of inflation and output from their respective targets will depend on the magnitude of the coefficients β_{π} and β_{y} . Indeed, a rule with $\beta_{\pi} > 1$ (i.e., Taylor principle) will tend to stabilize economic activity and inflation, while $\beta_{\pi} \leq 1$ can be seen as destabilizing. The same logic applies to the sign of β_{y} , with $\beta_{y} > 0$ stabilizing the economy and $\beta_{y} \leq 0$ destabilizing. In this sense, we have two benchmarks ($\beta_{\pi} = 1$ and $\beta_{y} = 0$) to evaluate changes in monetary policy conduction (Clarida, Gali and Gertler, 2000).

3.3 Taylor Rule with Interest Rate Smoothing and Exogenous Shocks

One can argue that the rule proposed in equation (1) does not represent the way a central bank adjust its policy rate in reality. Two problems arise when considering this

specification: (1) it disregards general monetary authority desire to smooth the interest rate adjustment¹¹ and (2) it ignores shocks that can effect the control of interest rates¹². In this sense, following again Clarida, Gali and Gertler (2000), we take into account these two problems by postulating a partial adjusment term for the *actual* nominal interest rate (i_t) :

$$i_t = \rho i_{t-1} + (1 - \rho) \, i_t^* + v_t \tag{4}$$

where v_t is a *iid* zero mean, homokedastic, exogenous interest rate shock and $\rho \in [0, 1]$ can be interpreted as the degree of smoothing of interest rate changes (Clarida, Gali and Gertler, 2000). Basically, equation (4) is defining the nominal interest rate as a linear combination between the past interest rate and the optimal interest rate as defined by the Taylor rule. Specifically, the monetary authority adjusts its policy rate to remove a fraction $(1 - \rho)$ of the difference between its current target and the previous rate (Kim and Nelson, 2006).

Finally, by combining equations (1) and (4), and doing some algebra, one can find the policy reaction function:

$$i_{t} = (1 - \rho) \left[i^{*} + \beta_{\pi} \left(\pi_{t,k} - \pi^{*} \right) + \beta_{y} \tilde{y}_{t} \right] + \rho i_{t-1} + e_{t}$$
(5)

where $e_t = v_t - (1 - \rho) \left[\beta_{\pi} \left(\pi_{t,k} - \mathbb{E}_{t-1} \left(\pi_{t,k} \right) \right) + \beta_y \left(\tilde{y}_t - \mathbb{E}_{t-1} \left(\tilde{y}_t \right) \right) \right]$. Due to the specification of e_t (i.e., forecast errors for inflation and output gap), it's possible to observe a natural source of endogeneity, given that π_t and \tilde{y}_t will be correlated with the error term.

This research is focused on Brazil's inflation target era, so we slightly change the rule, following De Losso (2008) and Minella et al. (2002). Indeed, we can observe expected inflation¹³ and the inflation target is non-constant¹⁴, therefore Brazil Central Bank (BCB) monetary rule can be described by:

$$i_t = (1 - \rho) \left[i^* + \beta_\pi \left(\pi_t^e - \pi_t^* \right) + \beta_y \tilde{y}_t \right] + \rho i_{t-1} + e_t \tag{6}$$

where π_t^e is the market's expected inflation and we can define $x_t \equiv \pi_t^e - \pi_t^*$ as the market's expected inflation deviation from the target. Here, we observe endogeneity again, because π_t^e and \tilde{y}_t can be correlated to policy shocks¹⁵.

¹¹ Here we cite Goodfriend (1991), which recognizes that central banks have a preference for interest rate smoothing, in order to avoid financial market turbulence; and Sack (1998), which underscores that smoothing is important when dealing with monetary policy uncertainty.

¹² Indeed, a central bank wont always be able to put the interest rate at the desired level indicated by the Taylor rule (Clarida, Gali and Gertler, 2000).

¹³ In Brazil, the BCB collects market participants expectation for different macroeconomic variables and public disclose the information.

¹⁴ In Brazil, the CMN informs to BCB the numerical target for the inflation rate usually three-years in advance, in order to anchor inflation expectations. We treat the inflation target as exogenous (Favero and Giavazzi, 2002).

¹⁵ Indeed, \tilde{y}_t is ex-post data, because it's subject to revised data. Furthermore, as argued in the introduction, forecasts would only be exogenous if they weren't conditioned to moves of the policy rate within the period for the projection, which doesn't apply for market expectations. For the determinants and properties of inflation forecasts, see Carvalho and Minella (2012).

3.4 Monetary Rule with Time-varying Parameters

As previously mentioned, we want to evaluate shifts in BCB monetary policy conduction for different governors after the adoption of the IT in 1999. For the sake of this purpose, we postulate a policy reaction function with time-varying parameters, specifically assuming a random walk process without drift for these parameters¹⁶.

As argued in the introduction, two models will be estimated. To characterize a certain BCB governor monetary policy as stabilizing or not, we estimate a model in which β_{π} and β_{y} can vary through time:

$$i_{t} = (1 - \rho) \left[i^{*} + \beta_{\pi,t} \left(\pi_{t}^{e} - \pi_{t}^{*} \right) + \beta_{y,t} \tilde{y}_{t} \right] + \rho i_{t-1} + e_{t}, \quad e_{t} \stackrel{iid}{\sim} \mathcal{N}(0, \sigma_{e}^{2})$$
(7)

$$\beta_{\pi,t} = \beta_{\pi,t-1} + \varepsilon_{1,t}, \quad \varepsilon_{1,t} \stackrel{iid}{\sim} \mathcal{N}(0, \sigma_{\varepsilon,1}^2) \tag{8}$$

$$\beta_{y,t} = \beta_{y,t-1} + \varepsilon_{2,t}, \quad \varepsilon_{2,t} \stackrel{ind}{\sim} \mathcal{N}(0, \sigma_{\varepsilon,2}^2) \tag{9}$$

Furthermore, as a way to estimate the implicit inflation target for the BCB, and compare it to the explicit one for the different chairman tenures, we consider a rule in which only the inflation target follows a random walk¹⁷:

$$i_{t} = (1 - \rho) \left[i^{*} + \beta_{\pi} \left(\pi_{t}^{e} - \pi_{t}^{**} \right) + \beta_{y} \tilde{y}_{t} \right] + \rho i_{t-1} + e_{t}, \quad e_{t} \stackrel{iid}{\sim} \mathcal{N}(0, \sigma_{e}^{2})$$
(10)

$$\pi_t^{**} = \pi_{t-1}^{**} + \varepsilon_{3,t}, \quad \varepsilon_{3,t} \stackrel{iid}{\sim} \mathcal{N}(0, \sigma_{\varepsilon,3}^2) \tag{11}$$

3.5 Dealing with Endogeneity: A Two-step Procedure

As underscored in subsection 3.3, π_t^e and \tilde{y}_t could be correlated with the error term in the policy reaction function (6). Nevertheless, Kalman filter derivation requires orthogonality between regressors and disturbances, therefore we cannot trivially estimate (7) to (9), or (10) to (11), with the Kalman filter via Maximum Likelihood (Medeiros and Aragón, 2015).

To solve this endogeneity problem, we follow a similar approach to the Kim and Nelson (2006) two-step procedure. Specifically, bias corrections terms obtained from a first step regression are added to the equation with endogeneity, which in turn is estimated by Kalman filter.

First, we postulate the relationship between the endogenous regressors $(\pi_t^e \text{ and } \tilde{y}_t)$

¹⁶ We cite here Kim and Nelson (2006) and Boivin (2006), whose works have assumed random walk processes without drift for time varying parameters. Furthermore, we also present in the Appendix a method proposed by Kim and Nelson (1988) to test the constancy of regression coefficients against the alternative of a random walk.

¹⁷ Leigh (2008), Klein (2012) and Carvalho (2021) also assume a random walk for the estimated target. Here we do not estimate the standard deviation for the random walk process, but rather fix it at the sample standard deviation for the explicit target series, without loss of generality.

and a vector of instrumental variables z_t :

$$\pi_t^e = z_t' \delta_1 + u_{1,t}, \quad u_{t,1} \stackrel{iid}{\sim} \mathcal{N}(0, \sigma_{u,1}^2)$$
(12)

$$\tilde{y}_t = z'_t \delta_2 + u_{2,t}, \quad u_{2,t} \stackrel{iid}{\sim} \mathcal{N}(0, \sigma^2_{u,2})$$

$$\tag{13}$$

To start the two-step procedure, π_t^e and \tilde{y}_t are decomposed into predicted and prediction error components. In matrix notation, we have:

$$\begin{bmatrix} \pi_t^e \\ \tilde{y}_t \end{bmatrix} = \mathbb{E} \begin{bmatrix} \pi_t^e \\ \tilde{y}_t \end{bmatrix} + \begin{bmatrix} u_{1,t} \\ u_{2,t} \end{bmatrix}$$
(14)

$$\begin{bmatrix} u_{1,t} \\ u_{2,t} \end{bmatrix} = \Omega^{1/2} \begin{bmatrix} u_{1,t}^* \\ u_{2,t}^* \end{bmatrix}, \quad \begin{bmatrix} u_{1,t}^* \\ u_{2,t}^* \end{bmatrix} \stackrel{iid}{\sim} \mathcal{N} \left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right)$$
(15)

where Ω is the covariance matrix for the vector of predicted errors $u_t = \begin{bmatrix} u_{1,t} & u_{2,t} \end{bmatrix}'$, $u_t^* = \begin{bmatrix} u_{1,t}^* & u_{2,t}^* \end{bmatrix}'$ is a vector of standardized prediction errors and Ψ_{t-1} is the information available at time t-1.

The source of endogeneity in equation (6) comes from the non-zero correlation between u_t and e_t . Therefore, without loss of generality, assume the following covariance structure between u_t^* and e_t :

$$\begin{bmatrix} u_t^* \\ e_t \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} 0_2 \\ 0 \end{bmatrix}, \begin{bmatrix} I_2 & \theta \sigma_e \\ \theta' \sigma_e & \sigma_e^2 \end{bmatrix} \right)$$
(16)

where I_2 is a (2×2) identity matrix, 0_2 is a (2×1) vector of zeros and $\theta = \begin{bmatrix} \theta_1 & \theta_2 \end{bmatrix}'$ is a constant correlation vector¹⁸.

Following Kim and Nelson (2006), it's possible to perform the Cholesky decomposition of the covariance matrix in $(16)^{19}$, in order to apply a linear transformation to u_t^* and e_t such that the transformed variables are uncorrelated:

$$\begin{bmatrix} u_t^* \\ e_t \end{bmatrix} = \begin{bmatrix} I_2 & 0_2 \\ \theta' \sigma_e & \sqrt{(1 - \theta' \theta)} \sigma_e \end{bmatrix} \begin{bmatrix} \epsilon_t \\ \omega_t \end{bmatrix}, \quad \begin{bmatrix} \epsilon_t \\ \omega_t \end{bmatrix} \stackrel{iid}{\sim} \mathcal{N} \left(\begin{bmatrix} 0_2 \\ 0 \end{bmatrix}, \begin{bmatrix} I_2 & 0_2 \\ 0'_2 & 1 \end{bmatrix} \right)$$
(17)

 $\overline{18}$ We have

$$\begin{bmatrix} u_{1,t}^* \\ u_{2,t}^* \\ e_t \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 & \theta_1 \sigma_e \\ 0 & 1 & \theta_2 \sigma_e \\ \theta_1 \sigma_e & \theta_2 \sigma_e & \sigma_e^2 \end{bmatrix}\right)$$

where $\theta_1 \sigma_e$ is the covariance between $u_{1,t}^*$ and e_t , and $\theta_2 \sigma_e$ is the covariance between $u_{2,t}^*$ and e_t . ¹⁹ Indeed,

$$L = \begin{bmatrix} I_2 & 0_2 \\ \theta' \sigma_e & \sqrt{(1 - \theta' \theta)} \sigma_e \end{bmatrix}$$

is a lower triangular matrix which satisfies

$$LL' = \begin{bmatrix} I_2 & \theta \sigma_e \\ \theta' \sigma_e & \sigma_e^2 \end{bmatrix}$$

where $u_t^* = \epsilon_t$, $e_t = \theta_1 \sigma_e u_{1,t}^* + \theta_2 \sigma_e u_{2,t}^* + \sqrt{(1 - \theta_1^2 - \theta_2^2)} \sigma_e \omega_t$, and ϵ_t and ω_t are uncorrelated.

Let us define ω_t^* as:

$$\omega_t^* = \sqrt{(1 - \theta_1^2 - \theta_2^2)} \sigma_e \omega_t, \quad \omega_t^* \sim \mathcal{N}\left(0, (1 - \theta_1^2 - \theta_2^2)\sigma_e^2\right)$$
(18)

which is also uncorrelated with $\epsilon_t = u_t^*$.

Therefore, e_t can be expressed as:

$$e_t = \theta_1 \sigma_e u_{1,t}^* + \theta_2 \sigma_e u_{2,t}^* + \omega_t^* \tag{19}$$

which is basically showing us that the error term e_t in equation (6) can be decomposed into two parts: (1) $u_{1,t}^*$ and $u_{2,t}^*$, which are correlated with π_t^e and \tilde{y}_t , and (2) ω_t^* , which is uncorrelated with π_t^e and \tilde{y}_t .

Finally, it's possible to find our policy reaction function with bias correction term by substituting (19) into (6):

$$i_{t} = (1 - \rho) \left[i^{*} + \beta_{\pi} \left(\pi_{t}^{e} - \pi_{t}^{*} \right) + \beta_{y} \tilde{y}_{t} \right] + \rho i_{t-1} + \theta_{1} \sigma_{e} u_{1,t}^{*} + \theta_{2} \sigma_{e} u_{2,t}^{*} + \omega_{t}^{*}$$
(20)

where the new error term is uncorrelated with $u_{1,t}^*$ and $u_{2,t}^*$, and consequently with π_t^e and \tilde{y}_t .

Therefore, considering our two proposed models with time varying parameters, (7) to (9) and (10) to (11), the two-step estimation procedure is:

- 1. Estimate equations (12) and (13) with Ordinary Least Squares (OLS) and collect the standardized prediction errors $\hat{u}_{1,t}^*$ and $\hat{u}_{2,t}^*$.
- 2. Estimate with Maximum Likelihood using Kalman filter equations:

$$i_{t} = (1 - \rho) \left[i^{*} + \beta_{\pi,t} \left(\pi_{t}^{e} - \pi_{t}^{*} \right) + \beta_{y,t} \tilde{y}_{t} \right] + \rho i_{t-1} + \theta_{1} \sigma_{e} \hat{u}_{1,t}^{*} + \theta_{2} \sigma_{e} \hat{u}_{2,t}^{*} + \omega_{t}^{*}$$
(21)

and

$$i_{t} = (1 - \rho) \left[i^{*} + \beta_{\pi} \left(\pi_{t}^{e} - \pi_{t}^{**} \right) + \beta_{y} \tilde{y}_{t} \right] + \rho i_{t-1} + \theta_{1} \sigma_{e} \hat{u}_{1,t}^{*} + \theta_{2} \sigma_{e} \hat{u}_{2,t}^{*} + \omega_{t}^{*}$$
(22)

together with equations (8) to (9) and (11), respectively.

3.6 Augmented Kalman Filter

Following Medeiros and Aragón (2015), consider the general state-space form for the monetary authority policy reaction function with bias correction terms:

$$i_t = X'_t \xi_t + A' x_t + \omega_t^*, \quad \omega_t^* \sim \mathcal{N}\left(0, \sigma_{w^*}^2\right)$$
(23)

$$\xi_{t+1} = F\xi_t + \varepsilon_{t+1}, \quad \varepsilon_t \stackrel{iid}{\sim} \mathcal{N}(0, Q) \tag{24}$$

where i_t is the policy rate; ξ_t is the state vector; X_t , A and F are possibly vector-value functions; x_t is a vector of exogenous variables; and ω_t^* and ε_t are disturbances with variance $\sigma_{w^*}^2$ and covariance matrix Q, respectively. Equation (23) is called observation equation and (24) is referred as state equation, as it postulates the dynamics of the state vector.

Considering our model, the Kalman Filter algorithm is given by iterating the equations below for t = 1, ..., T:

$$\xi_{t|t-1} = F\xi_{t-1|t-1} \tag{25}$$

$$P_{t|t-1} = FP_{t-1|t-1}F' + Q \tag{26}$$

$$\eta_{t|t-1} = i_t - X'_t \xi_{t-1|t-1} - A' x_t \tag{27}$$

$$H_{t|t-1} = X_t' P_{t|t-1} X_t + \sigma_{\omega^*}^2$$
(28)

$$\xi_{t|t} = \xi_{t|t-1} + P_{t|t-1} X_t H_{t|t-1}^{-1} \eta_{t|t-1}$$
(29)

$$P_{t|t} = P_{t|t-1} - P_{t|t-1} X_t H_{t|t-1}^{-1} X_t' P_{t|t-1}$$
(30)

Conditioned on the Maximum Likelihood estimates of the parameters, the Kalman Filter above provide us with correct inferences on ξ_t , given that these inferences are properly conditioned on the bias correction terms. Nevertheless, the variances given by $P_{t|t}$ and $P_{t|t-1}$ are not correct, because they shouldn't depend on these correction terms. Consequently, we follow Kim (2008) and calculate the correct variances by iterating the equations below for $t = 1, \ldots, T$:

$$H_{t|t-1}^* = X_t' P_{t|t-1}^* X_t + \sigma_e^2$$
(31)

$$P_{t|t}^{*} = P_{t|t-1}^{*} - P_{t|t-1}^{*} X_{t} H_{t|t-1}^{*-1} X_{t}^{\prime} P_{t|t-1}^{*}$$

$$(32)$$

$$P_{t+1|t}^* = F P_{t|t}^* F' + Q \tag{33}$$

Kim (2004) argues in favor of the smoothed values for the state vector²⁰, which are given by iterating for t = T - 1, T - 2, ..., 1 the equations below. Nonetheless, smoothing uses information that is not available for the policy maker at the time the policy is implemented, therefore we present in this dissertation the filtered estimates for the states.

$$\xi_{t|T} = \xi_{t|t} + P_{t|t} P_{t+1|t}^{-1} \left(\xi_{t+1|T} - \xi_{t+1|t} \right)$$
(34)

$$P_{t|T}^{*} = P_{t|t}^{*} + P_{t|t}^{*} P_{t+1|t}^{*-1} \left(P_{t+1|T}^{*} - P_{t+1|t}^{*} \right)^{*} P_{t+1|t}^{*-1} P_{t|t}^{*'}$$
(35)

²⁰ Smoothing uses all information available in the sample. For a complete derivation of the filtering and smoothing algorithms, see Hamilton (1994).

4 DATA

For the estimation of the brazilian monetary policy reaction functions previously proposed, all data is quarterly²¹ and span the period from 2000Q1 to 2020Q1, allowing one to analyze 20 years of monetary policy conduction under inflation targeting. Furthermore, all the series here described are public available and were obtained either from the Institute of Applied Economic Research (IPEA), Brazil Central Bank Time Series Management System (BCB SGS) or Independent Fiscal Institution (IFI) site.

The nominal policy rate variable is the annualized (252) monthly Selic rate²², inflation is 12-month accumulated percentage variation of the Broad Consumer Price Index $(IPCA)^{23}$ and the output gap (the percentage difference between output and potential output) series is the one calculated and disclosed by IFI^{24} . Furthermore, we also use the percentage quarter-over-quarter variation of the Brazil international reserves (liquidity concept) and of the nominal exchange rate USDBRL (period average).

For the construction of the series for expected inflation, the methodology proposed by De Losso (2008) was used. Assuming that inflation is distributed evenly throughout the year, it is possible to weight the median of inflation expectations, creating a series that considers at instant t the expected inflation for the next 12 months:

$$\pi_t^e = \ln\left(\frac{1 + \pi_{t,m}^e}{1 + \pi_{t,m}}\right) + \frac{m}{12}\ln\left(1 + \pi_{t,m}^{e+1}\right)$$
(36)

where $\pi_{t,m} = \prod_{j=0}^{m-1} (1 + \pi_{t-j}) - 1$ is the realized inflation between month 1 and m $(m = 1, 2, ..., 12), \pi_{t,m}^e$ is the expected inflation for the current year in period t, to which corresponds a month m, and $\pi_{t,m}^{e+1}$ is the inflation expectation for next year collected at period t^{25} .

Of course, in order to correctly measure the difference between inflation expectations and the target, it's necessary to create a series for the center of the inflation target²⁶. Again, the methodology proposed by De Losso (2008) was used, which enable us to find in t the center for the inflation target for the next 12 months:

²¹ To transform monthly data into quarterly one, we take the mean of the respective quarters. All data is log-linearized before estimation procedures.

 $^{^{22}}$ Here we are referring to the *actual* Selic rate, not the Selic rate *target*.

²³ By Resolution 2,615, of June 30, 1999, it was determined by the National Monetary Council (CMN) that the index to be used as a reference for the IT regime would be the IPCA. The IPCA series was deseasonalized.

²⁴ One can find a detailed overview of the methodology in <http://www2.senado.leg.br/bdsf/bitstream/ handle/id/536764/EE_04_2018.pdf>. The IFI is a prestigious institution linked to Brazil's Federal Senate that spends significant amount of resources in macro- and microeconomic analysis.

²⁵ The inflation expectations used were those generated by the market and disclosed in the Market Focus Report.

²⁶ In the IT framework adopted by Brazil, the National Monetary Council (CMN) determines a center for the annual inflation target. In addition, a variation band of usually 1.5 p.p. is defined. We apply the same procedure to construct the upper and lower bound.

$$\pi_t^* = \left(\frac{12-m}{12}\right) \ln\left(1+\pi_{t,m}^T\right) + \frac{m}{12} ln\left(1+\pi_{t,m}^{T+1}\right)$$
(37)

where $\pi_{t,m}^T$ is the center for inflation target for the current year at period t and in each month m = 1, 2, ..., 12; and $\pi_{t,m}^{T+1}$ is the center for inflation target for the next year at period t. In the Appendix one can find descriptive statistics for all series used as well as stationarity tests.

5 RESULTS

5.1 Standard Taylor Rule

The idea of this subsection is to estimate the standard Taylor Rule given by equation (6). Not only this procedure allows one to have a first glance on monetary policy conduction in Brazil during inflation targeting (IT), but it also provides us with suitable initial values for initializing the Kalman Filter algorithm (Hamilton, 1994). The Table 1 below presents the results for the Generalized Method of Moments (GMM) estimation of the Taylor Rule²⁷, considering a set of instruments that follows closely De Losso (2008)²⁸.

Coefficient	Estimate	Std. Error	t-Statistic	
ho	0.90	0.01	49.1*	
i^*	6.47	1.35	4.76^{*}	
eta_π	8.03	1.20	6.65^{*}	
$eta_{m{y}}$	0.61	0.19	3.21^{*}	
Observations	77	Adjusted \mathbb{R}^2	0.95	
Instrument Rank	25	Prob(J-statistic)	0.91	
$H_0: \beta_\pi = 1?$	Reject*, $\beta_{\pi} > 1$			

Table 1 – Taylor Rule - GMM

Equation: $i_t = (1 - \rho) [i^* + \beta_\pi (\pi_t^e - \pi_t^*) + \beta_y \tilde{y}_t] + \rho i_{t-1} + e_t.$

Instruments: constant, 1-4 lags of interest rate, inflation, exchange rate variation, reserves variation, output gap and expected inflation deviation from target.

* significant at 1%, ** significant at 5%, *** significant at 10%.

Focusing on parameters analysis, the estimates show a high degree of smoothing for interest rate changes in Brazil during IT and a neutral nominal interest rate of 6.5%. For the coefficient associated with the output gap, a positive and statistically significant value suggests a stabilizing behavior. Finally, the parameter related to expected inflation deviations from the target is quite high and above the unit, indicating that the Central Bank of Brazil (BCB) has been reacting strongly to these deviations²⁹, in line with the IT

²⁷ In light of the recent discussion brought by Carvalho, Nechio and Tristão (2021), one could argue in favor of OLS estimation of the Taylor Rule. Nevertheless, given the use of a set of instruments in the first stage of the estimation procedure, we opt for using the same instruments in this estimation to maintain coherence.

²⁸ Augmenting the rule with the exchange rate, given that Brazil is a small open economy, is a possilibity. Nevertheless, Taylor (2001) disagrees with this idea, because interest rate movements already account indirectly for the effects of the exchange rate variation, considering it's impact on inflation. We take into consideration this fact by incorporating reserves and exchange variation on the set of instruments, as done by De Losso (2008), because the information set of the policy maker includes them when choosing the policy rate.

²⁹ Silva and Lopes (2017) have found similar high values for the long-run β_{π} parameter the Brazilian

framework. Indeed, the Wald test suggests that the Taylor Principle has been followed by the BCB for the sample considered.

5.2 Time-varying Taylor Rule

Now we use the framework developed in Section 3 to estimate time-varying measures for the coefficients associated with expected inflation deviation from the target and output gap, in order to characterize Brazil's monetary policy under Inflation Targeting (IT) for different BCB chairman's. The first stage estimation is reported in the Appendix and the Table 2 below provides the Maximum Likelihood estimates of the parameters for the reaction function (21).

Coefficient	Estimate	Std. Error	z-Statistic
ho	0.87	0.04	23.0^{*}
i^*	7.79	1.97	3.95^{*}
$ heta_1$	-0.20	0.21	-0.95
$ heta_2$	-0.23	0.16	-1.47
σ_e	0.56	0.07	7.61^{*}
$\sigma_{arepsilon,1}$	1.48	0.68	2.18^{**}
$\sigma_{arepsilon,2}$	0.83	0.41	2.02**
Observations	77	Log Likelihood	-92.3
$\hat{\sigma}_{w^*} = \sqrt{(1 - \hat{\theta}_1^2 - \hat{\theta}_2^2)\hat{\sigma}_e^2}$	0.53		

Table 2 – Taylor Rule - Time Varying β_{π} and β_{y} with correction

Equation: $i_t = (1 - \rho) \left(i^* + \beta_{\pi,t} \left(\pi_t^e - \pi_t^* \right) + \beta_{y,t} \tilde{y}_t \right) + \rho_{t-1} + \theta_1 \sigma_e \hat{u}_{1,t}^* + \theta_2 \sigma_e \hat{u}_{2,t}^* + w_t^*$ with $\beta_{\pi,t} = \beta_{\pi,t-1} + \varepsilon_{1,t}, \ \beta_{y,t} = \beta_{y,t-1} + \varepsilon_{2,t}, \ \varepsilon_{1,t} \stackrel{iid}{\sim} \mathcal{N}(0, \sigma_{\varepsilon,1}^2), \ \varepsilon_{2,t} \stackrel{iid}{\sim} \mathcal{N}(0, \sigma_{\varepsilon,2}^2)$ and $w_t^* \stackrel{iid}{\sim} \mathcal{N}(0, \sigma_{w^*}^2).$

* significant at 1%, ** significant at 5%, *** significant at 10%.

Results suggest a high degree of interest rate smoothing and a nominal neutral interest rate of 7.8%, pretty much in line with what have been found with the GMM estimation. Furthermore, the estimates for the bias correction terms have the same negative sign found by Medeiros and Aragón (2015), although none one them turns out to be statistically significant. In terms of the standard errors for the random walk processes, we found that they are statistically significant at 5%. Finally, the figures below presents the behavior of the time-varying parameters estimated.

By analysing Figure 1, one can see that the Central Bank of Brazil (BCB) has reacted strongly to expected inflation deviations from the target, respecting the Taylor

forward-looking Taylor rule with data from 2003M1 to 2016M12, with estimates around 10 for different proxies for the output gap.

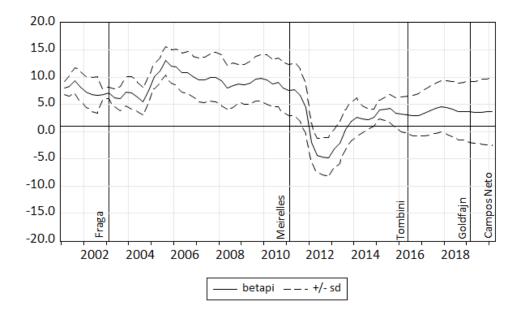


Figure 1 – Filtered Estimates for $\beta_{\pi,t}$.

principle for most of the chairman tenures (i.e., in line with the Inflation Target framework). Specifically, during Fraga and Meirelles mandates it's possible to observe that the coefficient has the greater magnitude for the entire sample. Considering the newly implemented inflation targeting system in 1999, it makes sense this more conservative approach from the BCB, probably to build credibility.

Nevertheless, once we turn to the beginning Tombini's era, the coefficient suddenly presents a sharp decrease until becoming negative. A similar result was also found by Medeiros and Aragón (2015) and Carvalho (2021) and suggests a non-stabilizing monetary policy from the beginning of 2012 until the end of 2013. Indeed, it is not for lack of coincidence that during this period the BCB was criticized by many brazilian economists, who argued that the authority was yielding to political pressures, by cutting interest rates even with inflation expectations rising since 2010. As argued by Martins (2022), such a reorientation of the monetary policy - with a Central Bank more lenient with expected inflation - could be related to the so-called New Macroeconomic Matrix implemented during the first Dilma Rousseff administration³⁰.

Even so, from the middle of Tombini's tenure onwards, the coefficient quickly recovers and starts to respect the Taylor principle, in line with the continued increase of the expected inflation in relation to the target. For Goldfajn and Campos Neto tenures,

³⁰ The New Macroeconomic Matrix - as it is called -, was a series of developmental economic measures taken between 2011 and 2014. For the implementation of this new economic model, the Macroeconomic Tripod of 1999 (i.e., floating exchange rate, inflation targeting and target for government primary result) was supposedly abandoned. The New Macroeconomic Matrix was based on: more devalued exchange rate, tax exemptions, credit expansion and administered prices control.

one can see that the coefficient has staid relatively stable in a level that also respects the Taylor principle. This period in specific was marked by a big fall in inflation expectations.

The Table 3 below presents an overview of the behavior of the coefficient in question for the different governors and reinforces the perception that the BCB has acted under the Taylor principle throughout the great majority of the analysed period, in line with the Inflation Target framework.

Table 3 – Monetary policy stability benchmark for $\beta_{\pi,t}$ for different BCB Chairman's

Statistic \Chairman	Fraga	Meirelles	Tombini	Goldfajn	Campos Neto
# of quarters	8	32	22	11	4
$eta_{\pi,t} > 1$	8 (100%)	32 (100%)	15 (68%)	11 (100%)	4 (100%)
$\boldsymbol{eta}_{\pi,t} \leq 1$	$0 \\ (0\%)$	$0 \\ (0\%)$	7 (32%)	$0 \\ (0\%)$	$\begin{array}{c} 0 \ (0\%) \end{array}$

Note: $\beta_{\pi,t}$ is the coefficient associated with market's expected inflation deviation.

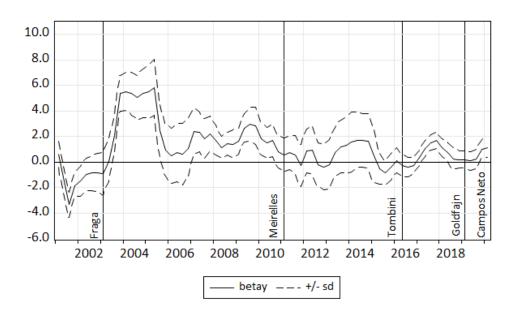


Figure 2 – Filtered Estimates for $\beta_{u,t}$.

Turning now to the behavior of the parameter associated with the output gap, one can see that the coefficient has stayed predominately below the zero line during Fraga's tenure, suggesting a non-stabilising monetary policy towards the output gap. Nevertheless, once we enter in Meirelles's era, it's possible to observe a sharp increase in the parameter. Indeed, from 2004 until 2008 Brazil has experienced a period of continued large increases in GDP³¹, which may have led the central bank to adopt a more aggressive stance with the output gap

For Tombini's tenure, the parameter has oscillated, becoming negative in some periods. Once we enter in the Goldfajn era, the coefficient becomes positive again after some quarters and presents a slightly increase in the middle of the mandate, indicating a more active response towards the output gap³². For the mandate of Campos Neto, the parameter stays above the zero line and presents what it seems to be a upward movement.

The Table 4 below presents an overview of the behavior of the coefficient in question for different governors and suggests that the BCB has positively reacted to the output gap during the majority of tenures for the analyzed period.

Statistic \Chairman	Fraga	Meirelles	Tombini	Goldfajn	Campos Neto
# of quarters	8	32	22	11	4
$eta_{y,t} > 0$	$\frac{1}{(13\%)}$	$31 \\ (97\%)$	14 (64%)	9 (82%)	4 (100%)
$\boldsymbol{\beta}_{\boldsymbol{y},t} \leq 0$	7 (87%)	$\frac{1}{(3\%)}$	$\frac{8}{(36\%)}$	2 (18%)	$\begin{matrix} 0\\ (0\%) \end{matrix}$

Table 4 – Monetary policy stability benchmark for $\beta_{y,t}$ for different BCB Chairman's

Note: $\beta_{y,t}$ is the coefficient associated with the output gap.

5.3 Implicit Inflation Target

As underscored in the introduction, we also use the framework developed in Section 3 to estimate a measure for the Central Bank of Brazil (BCB) implicit inflation target. This procedure allows one to characterize monetary policy for different chairman's by compering BCB implicit inflation preferences over time with the explicit target set by the National Monetary Council (CMN). The Table 5 below provides the Maximum Likelihood estimates of the parameters for the reaction function (22).

Results suggest a degree of interest rate smoothing of 0.65, a elevated coefficient for expected inflation deviations from target, a nominal neutral interest rate of 12% and a small negative value for the parameter associated to the output gap (although only the first three are statistically significant). Furthermore, the Wald test also suggest that the Taylor principle was respected during the analyzed period. In terms of estimates for the bias correction terms, they have again the same negative sign previously found and the first one is statistically significant at 1%. The figure 3 below shows the trajectory of the estimated implicit target together with the explicit one.

³¹ This period of prosperity in Brazil is mainly attributed to the 2000s commodities boom, mostly driven by chinese demand.

³² It's worthy mentioning that Brazil has experienced a severe economic crisis during 2015 and 2016.

Coefficient	Estimate	Std. Error	z-Statistic		
ρ	0.65	0.07	9.03^{*}		
i^*	12.1	1.31	9.18^{*}		
eta_π	3.78	0.68	5.52^{*}		
$eta_{m{y}}$	-0.01	0.18	-0.08		
$ heta_1$	-0.54	0.11	-4.57^{*}		
$ heta_2$	-0.20	0.12	-1.62		
σ_e	0.40	0.04	9.28^{*}		
Observations	77	Log Likelihood	-83.2		
$\hat{\sigma}_{w^*} = \sqrt{(1-\hat{\theta}_1^2-\hat{\theta}_2^2)\hat{\sigma}_e^2}$	0.33	$H_0: \beta_{\pi} = 1?$	Reject*, $\beta_{\pi} > 1$		
Equation: $i_t = (1 - \rho) [i^* + \beta_\pi (\pi_t^e - \pi_t^{**}) + \beta_y \tilde{y}_t] + \rho i_{t-1} + \theta_1 \sigma_e \hat{u}_{1,t}^* + \theta_2 \sigma_e \hat{u}_{2,t}^* + w_t^*$					

Table 5 – Taylor Rule - Time Varying inflation target with correction

Equation: $i_t = (1 - \rho) [i^* + \beta_\pi (\pi_t^e - \pi_t^{**}) + \beta_y \tilde{y}_t] + \rho i_{t-1} + \theta_1 \sigma_e \hat{u}_{1,t}^* + \theta_2 \sigma_e \hat{u}_{2,t}^* + w_t^*$ with $\pi_t^{**} = \pi_{t-1}^{**} + \varepsilon_{3,t}, \ \varepsilon_{3,t} \stackrel{iid}{\sim} \mathcal{N}(0, \sigma_{\varepsilon,3}^2)$ and $w_t^* \stackrel{iid}{\sim} \mathcal{N}(0, \sigma_{w^*}^2)$.

* significant at 1%, ** significant at 5%, * ** significant at 10%.

By analysing the chart, one can see two distinct patterns. First, during Fraga and Meirelles tenures the BCB has followed more closely the explicit target, even adopting in some times a more hawkish approach vis-à-vis the target, with a implicit target below the explicit. Second, from Tombini's term onwards the BCB has assumed a more dovish approach vis-à-vis the target, with a implicit target strictly above the explicit, exceeding the upper bound for some periods. In this sense, results suggest that BCB was more committed to the boundaries imposed by the CMN prior to 2011³³. The Table 6 below presents a summary for the behavior reported in the chart, and reinforces the aforementioned interpretation³⁴. Note that in terms of mean squared error, Fraga and Meirelles can be considered the BCB governors who have followed more closely the explicit target.

We adopt a slightly different approach from the one proposed by Carvalho (2021) and - in order to better analyse the implicit target trajectory vis-à-vis the explicit one -, we plot in Figure 4 the deviation of this two measures (i.e., implicit minus official) together with our measure for market's expected inflation deviation. One can see that, following the spike in inflation expectations in 2002-2003, the BCB has adopted with some lag a more hawkish approach vis-à-vis the explicit target, which suggests an active behavior to anchor expected inflation. After that, expected inflation fall and start to oscillate around the target; and again we observe an opposite movement from the BCB after some lag, with the estimated target rising towards the explicit and oscillating around it. Furthermore,

 $^{^{33}}$ Even so, if we consider the entire sample, the BCB has stayed within the explicit upper/lower bounds for 80% of the time.

 $^{^{34}}$ The idea for this table comes from Carvalho (2021).

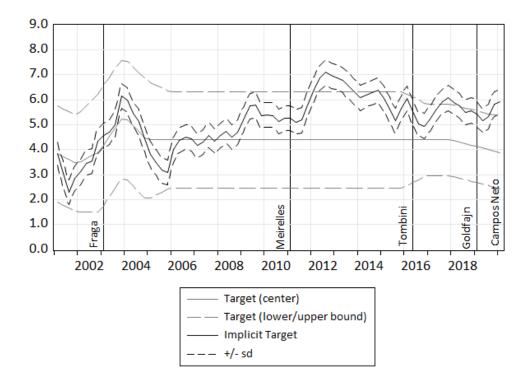


Figure 3 – Filtered Estimates for π_t^{**} .

we observe a Central Bank more dovish during 2009, after the 2007-2008 subprime crisis, which has led to a fall in expected inflation in Brazil.

Statistic \Chairman	Fraga	Meirelles	Tombini	Goldfajn	Campos Neto
# of quarters	8	32	22	11	4
$\pi^{**}_t > \pi^u_t$	$0 \\ (0\%)$	$0 \\ (0\%)$	$9 \\ (41\%)$	4 (36%)	$2 \\ (50\%)$
$\pi^*_t \leq \pi^{**}_t \leq \pi^u_t$	$\frac{1}{(12\%)}$	$22 \\ (69\%)$	13 (59%)	7 (64%)	$2 \\ (50\%)$
$\pi_t^l \leq \pi_t^{**} \leq \pi_t^*$	7 (88%)	$ \begin{array}{c} 10 \\ (31\%) \end{array} $	$0 \\ (0\%)$	$0 \\ (0\%)$	$\begin{array}{c} 0 \ (0\%) \end{array}$
$\pi^{**}_t < \pi^l_t$	$0 \\ (0\%)$	$0 \\ (0\%)$	$0 \\ (0\%)$	$0 \\ (0\%)$	$\begin{matrix} 0\\ (0\%) \end{matrix}$
MSE	0.37	0.48	3.24	1.58	2.63

Table 6 – Descriptive Statistics for Implicit Inflation Target for different BCB Chairman's

Note: The mean squared error (MSE) is the mean squared deviation of the implicit inflation target from the official one; π_t^{**} : estimated target; π_t^* : explicit target; π_t^u : explicit target upper bound, π_t^l : explicit target lower bound

Nevertheless, from the mid 2011 until the end of 2012, it seems that this pattern is lost: although inflation expectations are rising since 2010, the BCB continues to adopt a more dovish approach vis-à-vis the official target. This outlook is related to the abrupt surge in the implicit target in Tombini's mandate, a result also found by Carvalho (2021), with estimates exceeding the upper bound. Indeed, this outcome is in line with what have been found in Figure 1, suggesting a non-stabilising monetary policy in the middle of Tombini's tenure.

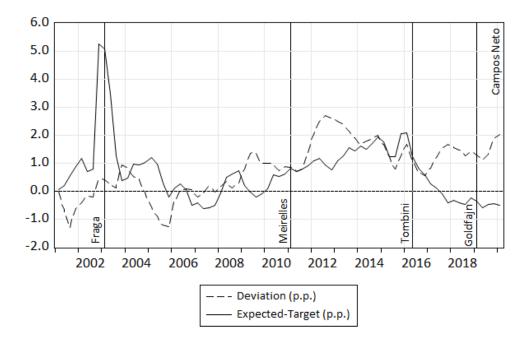


Figure 4 – Discrepancy between estimated and explicit target together with market's expected inflation deviation.

From 2013 onwards, we begin to observe the aforementioned pattern again: inflation expectations are high and the BCB brings the estimated target closer to the explicit; and them expected inflation starts to fall sharply and with some lag the BCB adopts a more dovish approach vis-à-vis the target.

All in all, with exception of a part of Tombini's tenure, Figure 4 suggests that the BCB was not only monitoring expected inflation deviation from target, but also adopting a behavior to anchor these expectations. In a simple Granger Causality test with 5 lags, the null that market's expected inflation deviation from the official target does not Granger cause the implicit target deviation from official is rejected at $10\%^{35}$, suggesting that expected inflation can be a important variable to gauge BCB behavior, which is line with the IT framework.

Finally, to conclude we plot in the Figure 5 below again the implicit target deviation from the official but this time together with the policy rate. As one can see, the two series

³⁵ The p-value for the test is 0.066. Carvalho (2021) plots the implicit target deviation from official together with the expected inflation and find evidence that the deviation granger-cause inflation expectations. We do not find Granger causality in this direction considering our slightly different approach.

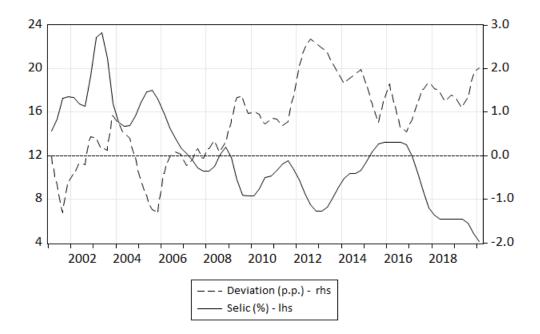


Figure 5 – Discrepancy between estimated and explicit target together with policy rate.

present a negative correlation³⁶ such that: (1) cycles of expansionary monetary policy (e.g., 2006-2013 and 2017-2020) coincide with periods in which BCB was increasing the implicit target relative to the explicit; and (2) cycles of monetary tightening (e.g., 2004-2005 and 2013-2017) coincide with periods in which the BCB was decreasing the implicit target in relation to the official. Therefore, results suggest a relation between Brazil's monetary policy cycles and BCB implicit inflation preferences.

 $[\]overline{^{36}}$ The coefficient of correlation is -0.72.

6 CONCLUSION

Considering the benchmarks proposed by Clarida, Gali and Gertler (2000) to evaluate changes in monetary policy conduction, the results suggest that - for the great majority of the analyzed period - the Central Bank of Brazil (BCB) governors have adopted a stabilizing monetary policy. Furthermore, the pattern found for the implicit target deviation from the explicit also corroborates this understanding, by suggesting that BCB was monitoring expected inflation and trying to anchor it. These results and the fact that BCB has operated with a estimated target predominantly within the upper and lower bound imposed by the National Monetary Council (CMN) indicate that the BCB was acting under the IT framework.

Table 7 – Summary of Monetary Policy Stability Statistics Under IT

Statistic	$\beta_{\pi,t} > 1$	$\beta_{y,t} > 0$	$\pi_t^l \le \pi_t^{**} \le \pi_t^u$
# of Quarters	$70 \\ (91\%)$	$59 \\ (77\%)$	$62 \\ (80\%)$

Note: $\beta_{\pi,t}$: coefficient associated with market's expected inflation deviation; $\beta_{y,t}$: coefficient associated with the output gap; π_t^{**} : estimated target, π_t^u : explicit target upper bound, π_t^l : explicit target lower bound.

The Table 7 above - which summarizes the results found for the monetary policy stability benchmarks studied in the dissertation -, supports the above-stated. As one can see, considering the entire sample and what the results suggest, the Taylor principle was respected for 91% of the time. Furthermore, a stabilizing behavior towards the output gap was found in 77% of the sample and the BCB has stayed within the explicit target upper and lower bounds in 80% of the time.

Nevertheless, although the aforementioned interpretation suggests a Central Bank of Brazil committed to monetary policy stability and inflation expectations, we had five moments (i.e., 2001, 2002, 2003, 2015, 2017) for the sample considered in which BCB has failed to fulfill its mandate under the IT framework³⁷. In this sense, considering our results, the most plausible interpretation for this fact is that: albeit BCB was predominantly seeking for a stabilizing monetary policy, it cannot fully control uncertainty and shocks.

For example, it's known that Brazil has suffered several adverse domestic and external shocks during 2001-2002 (e.g., Brazilian energetic crisis, 2002 presidential elections, Argentina crisis and the September 11 attacks in United States), which had contributed to a significant rise in inflation and inflation expectations (Arestis, de Paula and Ferrari,

³⁷ Failure to meet the pre-established target for a given year is associated with extrapolation of the upper or lower band.

2009). Hence, considering our results, the extrapolation of the inflation target upper bound in 2001, 2002 and 2003 could be much more related to these shocks.

For 2015, one could argue that the non-stabilizing behavior from BCB suggested by the results in the middle of Tombini's tenure could have been the main driver for a rise in inflation, which had culminated in the extrapolation of the upper bound in this year. Nevertheless, as argued by Barbosa Filho (2017), the period of 2011-2014 was marked by a combination of poorly formulated economic policies which had in the end led to the 2015-2016 Brazilian economic crisis. Therefore, we cannot ignore that this context could also have had a major contribution to the failure to meet the target in 2015. For 2017, surprisingly, the BCB has failed to fulfill its mandate because inflation ended the year slightly below the lower bound³⁸.

All in all, considering the aforementioned analysis and the results, the contribution of this dissertation is to provide novel evidence that the Central Bank of Brazil governors were seeking in general a stabilizing monetary policy and also acting under the Inflation Target framework for the period of 2001-2020. We truly think that meeting the target for 74% of times for the sample considered is a great achievement for a emerging market economy with such idiosyncrasies as Brazil.

 $^{^{38}\,}$ The official inflation number for 2017 is 2.95%, with a lower bound of 3.00%.

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APPENDIX A -

A.1 Statistics and Stationarity Tests

The Table A.1 below shows some summary statistics for all the series used in the research.

Statistic	i_t	π_t	π^e_t	π_t^*	$ ilde{y}_t$	d_t	h_t
Mean	12.10	6.10	5.02	4.35	-1.23	1.02	2.75
Median	11.64	5.90	4.96	4.40	-0.70	-0.13	1.01
Max	23.29	15.50	9.23	5.50	3.97	31.58	29.53
Min	4.09	2.50	3.38	3.48	-7.33	-15.95	-29.68
sd	4.21	2.47	1.11	0.33	3.23	7.58	7.01
Obs.	81	81	81	81	81	81	81

Table A.1 – Summary Statistics

 i_t : policy rate, π_t : inflation, π_t^e : expected inflation, π_t^* : inflation target, \tilde{y}_t : output gap, d_t : USDBRL, h_t : international reserves.

In Table A.2, one can find Ng and Perron (2001) stationarity test for all series. We opt for this test instead of ADF or Phillips-Perron because they have very low power against highly persistent stationarity processes. Moreover, when deterministic terms are included, the power of these tests becomes even lower.

Variable	Deterministic	$\mathrm{MZ}_{lpha}(\mathrm{k})$	$MZ_t(k)$
i_t	$^{ m c,t}$	-11.35(2)	-2.34(2)
Δi_t	$^{\rm c,t}$	-77.92^{*} (1)	-6.23^{*} (1)
π_t	$^{ m c,t}$	$-66.43^{*}(5)$	-5.76^{*} (5)
π^e_t	$^{\rm c,t}$	-12.29(3)	-2.45(3)
$\Delta \pi^e_t$	$^{\rm c,t}$	-81.78^{*} (2)	-6.39^{*} (2)
$ ilde{y}_t$	$^{\rm c,t}$	-7.74(1)	-1.91(1)
$\Delta \tilde{y}_t$	$^{\rm c,t}$	-35.97^{*} (0)	$-4.23^{*}(0)$
π_t^*	С	$-11.52^{*}(1)$	-2.39^{**} (1)
d_t	$^{\rm c,t}$	$-51.11^{*}(1)$	-5.05^{*} (1)
h_t	$^{\rm c,t}$	-30.09^{*} (0)	-3.87^{*} (0)

Table A.2 – Unit Root Tests

 i_t : policy rate, π_t : inflation, π_t^e : expected inflation, π_t^* : inflation target, \tilde{y}_t : output gap, d_t : USDBRL, h_t : reserves.

k: number of lags, c: constant, t: trend, Δ : first difference.

* significant at 1%, ** significant at 5%, *** significant at 10%.

The test statistics provides us with evidence that inflation, inflation target, exchange rate variation and international reserves variation are stationary. For the output gap, expected inflation and policy rate, the null of unit root is only reject in the first difference, suggesting that these series are non-stationary in level. Nevertheless, it's know that interest rates are highly persistent stationary processes³⁹ and even Ng and Perron test suffers from low power⁴⁰, therefore we treat the policy rate as stationary. In terms of inflation expectations, previous works have shown that Brazilian expected inflation can be taken as stationary (Gomes and Leme, 2011) and we do not find reasonable to have a stationary inflation with a non-stationary inflation expectation.

Table A.3 – Chow Breakpoint Test

Equation	regression of \tilde{y}_t in a constant
F-statistic	155.9
p-value	0.00
null hypothesis:	no breaks at 2015Q3.
~ , ,	

 \tilde{y}_t : output gap.

For the output gap series, visual inspection suggests a structural break in the 2015 crisis, which is confirmed by the Chow Test above. In this sense, we perform a Unit Root test with structural break, which provides us with evidence that, contrary to the Ng and Perron test, the output gap series is stationary in level. Considering all of the above mentioned, we treat all series as stationary in level in the dissertation, a standard approach in the literature.

Table A.4 – Unit Root with Break Test

Variable	exogenous	lags	t-stat
Output gap (\tilde{y}_t)	$^{\rm c,t}$	1	-5.13^{**}

c: constant, t: trend. break specification: intercept. ** significant at 5%.

³⁹ See for example Cerrato et. al. (2010).

⁴⁰ In general, unit root tests are all quite misleading, so caution is always necessary when analysing results.

A.2 First Stage

The Tables A.5 and A.6 below provides the results for the first stage estimation. Note that the F-statistics are above the threshold value of 10 suggested by Stock et al. (2002) to rule out weak instruments problem.

Variable	Estimate	Std. Error	t-Statistic
Constant	4.09	0.34	11.7^{*}
i_{t-1}	-0.02	0.13	-0.16
i_{t-2}	-0.06	0.26	-0.21
i_{t-3}	0.04	0.25	0.17
i_{t-4}	-0.005	0.13	-0.03
π_{t-1}	0.17	0.14	1.25
π_{t-2}	-0.03	0.18	-0.20
π_{t-3}	-0.10	0.17	-0.62
π_{t-4}	0.10	0.11	0.95
d_{t-1}	0.02	0.01	2.12^{**}
d_{t-2}	0.004	0.01	0.35
d_{t-3}	-0.01	0.01	-0.74
d_{t-4}	-0.005	0.01	-0.42
h_{t-1}	0.03	0.01	1.92^{***}
h_{t-2}	-0.05	0.02	-2.70^{*}
h_{t-3}	0.01	0.01	1.36
h_{t-4}	0.007	0.01	0.61
\widetilde{y}_{t-1}	0.11	0.09	1.25
\widetilde{y}_{t-2}	-0.08	0.12	-0.69
\widetilde{y}_{t-3}	0.03	0.12	0.31
\widetilde{y}_{t-4}	-0.03	0.09	-0.30
x_{t-1}	0.92	0.18	5.05^{*}
x_{t-2}	-0.48	0.23	-2.10^{**}
x_{t-3}	0.32	0.22	1.44
x_{t-4}	0.09	0.19	0.49
Observations	77	R^2	0.82
F-Statistic	10.21	Adjusted \mathbb{R}^2	0.74

Table A.5 – First Stage - Expected Inflation

 i_t : policy rate, π_t : inflation, π_t^e : expected inflation, π_t^* : inflation target, \tilde{y}_t : output gap, d_t : exchange rate, h_t : reserves, $x_t \equiv \pi_t^e - \pi_t^*$: expected inflation deviation from target.

 ${\it Heteroskedastic\ robust\ standard\ errors\ reported}.$

 \ast significant at 1%, $\ast\ast$ significant at 5%, $\ast\ast\ast$ significant at 10%.

Variable	Estimate	Std. Error	t-Statistic
Constant	-0.12	0.56	-0.22
i_{t-1}	-0.44	0.22	-1.95^{***}
i_{t-2}	0.54	0.42	1.25
i_{t-3}	-0.21	0.41	-0.50
i_{t-4}	0.14	0.21	0.65
π_{t-1}	-0.29	0.22	-1.28
π_{t-2}	0.14	0.29	0.48
π_{t-3}	0.13	0.28	0.47
π_{t-4}	-0.11	0.18	-0.61
d_{t-1}	-0.04	0.02	-2.30^{**}
d_{t-2}	-0.008	0.02	-0.37
d_{t-3}	0.01	0.02	0.58
d_{t-4}	0.03	0.02	1.65
h_{t-1}	0.02	0.03	0.73
h_{t-2}	-0.01	0.03	-0.57
h_{t-3}	-0.01	0.02	-0.55
h_{t-4}	0.03	0.02	1.89^{***}
\tilde{y}_{t-1}	0.95	0.15	6.23^{*}
\tilde{y}_{t-2}	-0.13	0.19	-0.67
\tilde{y}_{t-3}	0.39	0.19	2.05^{**}
\widetilde{y}_{t-4}	-0.27	0.15	-1.87^{***}
x_{t-1}	0.56	0.29	1.87^{***}
x_{t-2}	-0.31	0.37	-0.84
x_{t-3}	-0.01	0.36	-0.30
x_{t-4}	0.08	0.31	0.27
Observations	77	R^2	0.94
F-Statistic	39.4	Adjusted \mathbb{R}^2	0.92

Table A.6 – First Stage - Output Gap

 i_t : policy rate, π_t : inflation, π_t^e : expected inflation, π_t^* : inflation target, \tilde{y}_t : output gap, d_t : exchange rate, h_t : reserves, $x_t \equiv \pi_t^e - \pi_t^*$: expected inflation deviation from target.

Heteroskedastic robust standard errors reported.

* significant at 1%, ** significant at 5%, *** significant at 10%.

Kim and Nelson Constant Parameters test A.3

As a way to corroborate the decision to model the time-varying parameters as random walks, we perform a test proposed by Kim and Nelson (1988) that allows one to test the null of coefficient constancy against the alternative of a random walk process. The test is basically another version of the Breusch and Pagan (1979) heteroskedasticity test.

Under the alternative hypothesis, Ordinary Least Square (OLS) residuals present a particular form of heteroskedasticity that depends on $x_t^2 \times t$, where x_t is a vector of regressors. As demonstrated by Breusch and Pagan (1979), half of the Explained Sum of Squares $(ESS = \sum_{t=1}^{T} (\hat{y}_t - \overline{y})^2/2)$ of a regression of $\hat{u}_t^2/\hat{\sigma_u}^2$ on $x_t^2 \times t$ follows a Chi-Square distribution with k degrees of freedom under the null of coefficient stability. Here, \hat{u}_t is the residual from the short-run version of equation (6), and $\hat{\sigma}_u$ its standard error. In this sense, we calculate the joint test statistic from the following regression:

$$\frac{\hat{u}_t^2}{\hat{\sigma}_u^2} = c_0 + c_1 t + c_2 (\pi_t^e - \pi_t^*)^2 t + c_3 \tilde{y}_t^2 t + c_4 i_{t-1}^2 t + e_t, \quad t = 1, \dots, \text{T}$$
(38)

	Coe	fficient					
c_1	c_2	c_3	c_4	k	$\chi^2(k)$	p-value	R^2
-0.02^{*} (0.009)	-	-	-	1	11.58	0.00	0.08
-	-0.003 (0.003)	-	_	1	3.37	0.06	0.01
-	-	-0.0003^{***} (0.0002)	_	1	5.16	0.02	0.03
-	-	-	-6.5e-5 (8.5e-5)	1	1.03	0.31	0.007
-0.03^{***} (0.02)	-0.002 (0.003)	8.4e-5 (0.0003)	2.9e-5 (9.8e-5)	4	12.40	0.01	0.09

Note: Null hypothesis of coefficient constancy; standard errors reported in parenthesis. * significant at 1%, ** significant at 5%, *** significant at 10%.

Table A.7 – Kim and Nelson (1988) Test