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Essays on International Spillovers of Monetary Policy

Ensaios sobre efeitos de transbordamento de política monetária

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RESUMO

Esta tese de doutorado é composta por três artigos voltados à análise dos efeitos transbordamento de política monetária internacional. No primeiro artigo, investiga-se a presença desses efeitos entre os países da OCDE. Primeiro, constata-se que, mais recentemente, as decisões de política monetária divergem das indicadas pela Regra de Taylor. Além disso, esses desvios são correlacionados entre diferentes países. Diante desses fatos, o artigo visa verificar se existe efeito transbordamento nas decisões de taxa de juros dos bancos centrais. Para contornar o problema da endogeneidade e o fato de que as relações entre os países são distintas, utiliza-se o arcabouço metodológico da econometria espacial. Considerando o modelo SAR e uma matriz de pesos W que mostra a relação comercial entre os países, encontra-se que além da taxa inflação e do hiato do produto, os países levam em conta as taxas de juros internacionais nas decisões de política monetária.

No segundo artigo, tendo como base os modelos clássicos de DSGE, analisa-se a presença de efeitos transbordamento de política monetária, através da inclusão da taxa de juros internacional no processo de decisão do agente monetário. O modelo mostra que a relação entre a taxa de juros doméstica e a internacional depende dos parâmetros utilizados, em especial, da elasticidade intertemporal de consumo e do grau de abertura do país. Considerando valores razoáveis para estes parâmetros, encontra-se uma relação positiva entre a taxa de juros externa e a doméstica. Além disso, no modelo estimado para o Brasil considerando a Regra de Taylor para o índice de preços, verificou-se que um choque negativo na taxa de juros americana gera uma queda na taxa de juros brasileira, estimulando a atividade e o consumo. Como consequência, tem-se uma alta dos preços domésticos. No entanto, o índice de preços recua com a apreciação da moeda, levando a uma lenta retomada da taxa de juros ao *steady-state*.

Por fim, o terceiro paper busca mensurar os custos de bem-estar gerado pelos efeitos transbordamento de política monetária. Partindo de um modelo não linear, aplica-se a metodologia proposta por Schmitt-Grohe e Uribe (2004b) e encontra-se que independente da importância da taxa de juros externa nas decisões domésticas, a política monetária ótima é condizente com uma resposta agressiva dos juros à inflação e uma reação mais contida com relação ao hiato do produto. Já comparando as diferentes regras, obtém-se algumas evidências de que ao considerar a taxa de juros internacional, o banco central incorre em perda de bem-estar social. Vale destacar que o custo deste bem-estar depende dos coeficientes da Regra de Taylor e, em maior medida, do grau de abertura da economia.

Palavras-chave: Banco Central, política monetária internacional, efeito transbordamento.

ABSTRACT

This doctoral dissertation is composed of three articles focused on analyzing international spillovers of monetary policy. The first article investigates the spillover effects of monetary policy among OECD countries. First, it appears that, more recently, monetary policy decisions diverge from those indicated by the Taylor Rule. Thus, the article aims to verify if there is a spillover effect in the interest rate decisions of central banks. We apply the methodological framework of spatial econometrics to circumvent the endogeneity problem. Considering the SAR model and a W weight matrix that shows the trade relationship between countries, it is found that countries take into account the international interest rate, besides considering the differential between inflation and its target and the output gap.

In the second article, based on classical DSGE models, we analyze the spillover effects of monetary policy by including the foreign interest rate in the domestic monetary agent's decision process. The model shows that the relationship between domestic and foreign interest rates depends on the parameters used, particularly the intertemporal elasticity of consumption and the country's degree of openness. Considering reasonable values for these parameters, we found a positive relationship between the foreign and domestic rates. Furthermore, in the model estimated for Brazil considering the Taylor Rule with Consumer Price Index (CPI), it was found that a negative shock in the US interest rate generates a drop in the Brazilian rate, stimulating activity and consumption. Therefore, domestic prices rise. However, the CPI declines with the appreciation of the exchange rate, leading to a slow return of the interest rate to the steady-state.

The third paper seeks to measure the welfare costs generated by the spillover effects of monetary policy. Starting from a non-linear model, we apply the methodology proposed by Schmitt-Grohe and Uribe (2004b) and found that regardless of the importance of the foreign interest rate in domestic decisions, the optimal monetary policy is consistent with an aggressive response of interest rates to inflation and a more mild reaction to the output gap. Comparing different rules, we found some evidence that the central bank incurs social welfare loss when considering the international interest rate. Finally, it is worth noting that the cost depends on the

Taylor Rule's coefficients and, to a greater extent, on the degree of openness of the economy.

Keywords: Central Bank, international monetary policy, spillover effect.

LIST OF FIGURES

LIST OF TABLES

TABLE OF CONTENTS

1 International Monetary Policy Spillovers: Spatial Econometric Approach

With André Chagas [1.0.1]

1.1 Introduction

In terms of trade as well as financial transactions, global integration has been intensified strongly in recent years. One consequence of this greater integration is increased spillover effects between countries. Shocks from one economy, for example, via monetary policy, can influence others in different ways.

The effects of international spillover from monetary policies have been subject to intense economic debate since the inter-war period. However, after the 2008 global financial crisis, these spillover effects gained even more attention due to the intense use of monetary measures to stimulate the economy.

The central banks take the monetary policy decisions worldwide, considering the current environment and the domestic economy's prospects. In a seminal paper, (Taylor 1993) suggested a reaction function to represent the Federal Reserve's monetary policy decisions, which assumes that the Central Bank adjusts its nominal interest rate to deviations of the inflation and output from some target.

In general, before the 2008 financial crisis, the Taylor Rule was a fair representation of the Central Banks' monetary policy decisions. However, in recent years, the nominal interest rate has increasingly diverted from what is suggested by the rule. This behavior can be evidence that there is some relevant variable in the Central Bank reaction function that the Taylor Rule has neglected.

Considering the significant increase in global integration and, consequently, a possible rise in the spillover effects of monetary policies, the central assumption of the present study is that, currently, the Central Bank of one country also

[1.0.1] Natalia received financial support from *Coordenação de Aperfeiçoamento de Pessoal de Nível Superior - Brasil* (CAPES - Finance Code 001).

considers the actions of other Central Banks to make its decision. In this context, spillover effects are included in the original Taylor Rule. As traditional econometric methodology would generate biased and inefficient estimators, this study applies econometric spatial methods to test if the rule with spatial dependence fits the data better than the traditional interest rate rules.

Besides this introduction, this article is divided into five more sections. There is a brief bibliography revision about monetary policy and financial integration in the next one. The third contains the main details of the methodology applied to estimate the Taylor Rule with spatial effects. Section four gives some information about the database and presents the main results. Section five shows other interesting results, and Section 6 concludes this study.

1.2 Bibliography Revision

Central Banks make their monetary policy decision considering the current environment and the prospects for the domestic economy. In a seminal paper, (Taylor 1993) examined how research on policy rules might apply in a practical policy-making environment. Taylor compared the performance of three different interest-rate rules and found that policies that focus on the deviations of the exchange rate from some target or deviations of the money supply from some target do not deliver good performance as policies that focus on the price level and output gap directly. In other words, monetary rules in which the interest rate is raised when inflation and activity are above the target and is lowered if both are below the target seem to work better.

This simple rule fitted the policy performance of the Federal Reserve (Fed) remarkably well during the 1987-1992 period. Therefore, many studies examined the Taylor Rule from different perspectives during the last decades.

According to (Svensson 2003), the rule proposed by (Taylor 1993) can be examined from a descriptive and prescriptive perspective. From the first one, it has been evaluated to what extent this rule is a good empirical description of Central Bank behavior. From the other one, it has been studied how the Taylor

Rule performs in different macroeconomic models.

(Clarida et al. 2001) estimate a forward-looking monetary policy reaction function for the United States between 1960 and 1996. From the empirical side, the forward-looking rule also provides a good description of the central bank's response as it allows for the monetary authority to consider a broad array of information. According to these authors, this framework can be viewed as a particular case of the rule proposed by (Taylor 1993): when a combination of the lagged inflation and output gap is a sufficient statistic for forecasting future changes in the price level. (Clarida et al. 2001) concluded that the Federal Reserve response to a deviation of inflation is less aggressive for the pre-Volcker period than for the Volcker-Greenspan period. Moreover, although significant in both times, the sensitivity to the cyclical variable is only marginal for the more recent period.

The extensive research about the Taylor Rule provides many important insights about monetary policy from the perspective approach. One of these insights is the so-called Taylor principle, which established that the equilibrium with price stability is achieved only if the short interest rate's response to inflation is larger than one.

(Woodford 2001) showed that the Taylor Rule incorporates several features of an optimal monetary policy. First, he argued that the Taylor principle is necessary and sufficient to guarantee an equilibrium price level with rational expectations. After that, (Woodford 2001) demonstrated that, because of the Taylor Principle, the problem of unstable inflation dynamics is not relevant. The sort of feedback from the inflation and output gap present in the Taylor Rule dampens such an inflationary spiral.

The author also supported that the Taylor Rule is consistent with an optimal equilibrium if it includes a time-varying intercept, which represents the Wicksellian natural rate of interest (real equilibrium rate under flexible prices), and if it shows a commitment to historical behavior, in particular to more gradual adjustments of the level of the interest rate.

Evaluating the performance of different interest rate rules in a business cycle model for the United States (US), (Schmitt-Grohe e Uribe 2004) found that an optimal monetary policy has an inflation coefficient higher but close to one, in line with the Taylor Principle. At the same time, they identified a mute response to output and no interest rate smoothing.

Although the Taylor Rule's initial proposal was to give a simple illustration of the United States rate policy, it has become a popular gauge for assessments of the monetary policy stance in both emerging markets and advanced economies. The Taylor Rule provided helpful guidance for monetary policy in different countries for many years; nevertheless, since the mid-2000s, and mainly after the 2008 global financial crisis, policy rates in all these countries have been below the levels recommended by the Taylor Rule.

This deviation also has been the subject of a large number of studies. (Hoffman e Bogdanova 2012) computed Taylor Rule for an aggregate of 11 advanced economies and a group composed of 17 emerging market economies over 1995 to 2012. Since the early 2000s, the authors found that global policy rates have usually been below the levels indicated by the Taylor Rule. According to the authors, this gap is mainly driven by the emerging market economies, which have an average of deviations of about 4.5 percentage points since 2003, while interest rates in advanced economies have been below Taylor Rule since 2001, but with a minor deviation (average less than 2.0 percentage points).

In their research, (Hoffman e Bogdanova 2012) highlighted some factors that could explain these global deviations of rates. First, concerns about the macroeconomics tail risks associated with financial instability could have driven down real interest rates, thereby opening a wedge between policy rates and Taylor rule implied rates. Other explanations can be derived from the worries about the effects of unwelcome capital flow and exchange rate movements that may, in turn, have transmitted low-interest rates in advanced economies to emerging markets. In this sense, Central Banks may aim to avoid large interest rate differences, so their policy rates become implicitly tied to the interest rate of other Central Banks. Other factors that can explain adopting a more accommodative monetary policy than implied in the Taylor Rule are the fall in the natural equilibrium rate, and many Central Banks have approached the zero lower bound.

The fact that there is a gap between the actual policy rates and the levels implied by the Taylor Rule indicates that there could be some relevant variable (or variables) in the Central Bank reaction function that the Taylor Rule has neglected. Considering the high degree of global integration and the growth of spillover effects of monetary policy, the hypothesis that one Central Bank decision influences the others arises.

The topic of spillover effects of monetary policy is not new and has been the object of intense debate. However, the first formal models of monetary policy in an open economy were formulated by (Mundell 1963) and (Fleming 1962). Both authors independently extended the Keynesian model to incorporate the possibility of capital flow amid economies. The model, subsequently named Mundell-Fleming, proposes two main transmission channels for monetary policy: demand effect and portfolio effect. In the first case, a fall in the interest rate in a major economy boosts domestic demand, which stimulates other countries' exports. On the other hand, with a lower interest rate, investments go from the central economy to the foreign ones, leading to an appreciation of the foreign currencies that is harmful to the foreign economy due to the fall of exports.

In a more empirical approach, many studies have shown that leading Central Banks, such as the Federal Reserve (Fed) and European Central Bank (ECB), affect monetary conditions and interest rates worldwide. (Beckworth e Crowe 2012) developed the monetary superpower hypothesis, which advocates that the Federal Reserve is a monetary superpower capable of shaping global liquidity conditions and impacting other economies' monetary policy decisions. The authors' central argument is that the United States has a superpower status as the world's main currency reserve manager. Thus, when the Fed lowers its interest rate, the dollar depreciates relative to other currencies. The other economies cannot afford an over-valuated currency, and then they also have to ease their monetary policy.

Using data between 1992 and 2010, (Beckworth e Crowe 2012) found strong evidence that the Fed policy influences the Eurozone's monetary environment, with no indication of any influence in the opposite direction. According to the research, US monetary policy in Eurozone can explain up to 40 percent of the policy forecast error. More than heavily affect ECBs decisions, the authors showed that the Fed's influence could be felt across the global economy. The authors concluded their study by arguing that the Fed, as a monetary superpower, needs to be more cognizant of its global economic influence, in particular, when making monetary policy decisions.

(Taylor 2013) also highlighted the importance of the international coordination of monetary policy nowadays. In the 1980s and 1990s, researchers found that the gains from coordination were small. According to the author, this view needs to be reexamined, not because the model or the theory used was wrong, but rather because the assumptions have changed. In particular, the author underlined that there were similar deviations from the Taylor Rule at many central banks, an apparent spillover that helped break down the international monetary balance.

According to (Taylor 2013), there are at least four reasons for spillover in monetary policy. First, when a central bank lowers its interest rate, other central banks also cut their interest rate to prevent a significant exchange rate appreciation in their currencies. Second, lower international interest rates and the exchange rate appreciation cause an increasing risk-taking. Again, one way to offset the appreciation is to cut the interest rate. A third reason is the existence of a common global shock, and a fourth one defends only that central banks may simply follow each other.

Perhaps more interesting in Taylor's paper is the description of a policy spillover amplification mechanism caused by central banks' response to each other. For example, if Fed cuts its interest rate, considering the spillover effect, the rest of the world's rate will also be reduced, which causes the Fed to cut it again, leading to another fall in the rest of the world rate, and so on. To sum up, the new equilibrium is an interest rate much lower than necessary to stimulate the domestic economy without coordination. This is a fundamental conclusion, as we live in a world with interest rates in their historical lows and central banks have less and less space to use the interest rates to stimulate their economies in a new adverse scenario.

In a more recent study, (Beckmann et al. 2017) examined the causes for the deviations from the standard Taylor rule in the central industrial countries. According to the authors, incorporating international spillovers and allowing for nonlinear dynamics improve the Taylor reaction function.

Lately, there has been considerable empirical evidence of the foreign interest rate's impact on a domestic central bank policy decision. Perhaps the best evidence comes from Central Banks themselves. Some central banks are more explicit as the Norges Bank (NB). In a Monetary Policy Report of 2010[1.2.1], the NB highlighted that the policy rule that includes the external interest rate fitted the policy performance of the central bank better than the policy rule without considering the foreign monetary policy. Since then, Norges Bank has been dividing the factors behind the changes in its key policy rate, not only by domestic demand, prices, and wages but also by exchange rate and foreign interest rates.

Similarly, the Central Bank of Mexico, Banxico, in its monetary policy statement in October 2018[1.2.2], justified the decision to maintain its policy rate by citing the domestic prices, the Mexican economy, and the US monetary policy. Commenting on the country's monetary policy's conduction, Banxico highlighted three factors that could affect the next decisions: changes in prices because of variations in the exchange rate, the evolution of the Mexican economy, and the difference in monetary policy between the US and Mexico.

Besides that, the vast majority of the central banks do not explicitly show the importance of foreign policy rates for their decisions. The Fed frequently emphasizes the importance of domestic factors and minimizes external factors, such as the monetary policy decisions of other Central Banks. However, many times, Fed members recognize the importance of the global interest rate. Ben Bernanke, former Chairman of the Federal Reserve, highlighted the implication of the financial integration for the US monetary policy in a Stanford speech. According to Bernanke[1.2.3]: *"At the broadest level, globalization influences the conduct of monetary policy through its powerful effects on the economic and financial environment in which monetary policy must operate. As you know, several decades of global economic integration have left a large imprint on the structure of the US economy, including changes in patterns of production, employment, trade, and financial flows.*

[1.2.1]https://static.norges-bank.no/contentassets/5baf110bd2a94e109939e11e5ba37194/en/mpr$_{21}$0.$pdf?v =$ 03/09/2017123118$ft =$ .pdf.

[1.2.2]https://www.banxico.org.mx/publicaciones-y-prensa/minutas-de-las-decisiones-de-politica-monetaria/%7B50ED7644-47A9-934D-4C49-3CDF170A0FFC%7D.pdf.

[1.2.3]https://www.federalreserve.gov/newsevents/speech/bernanke20070302a.htm.

Besides contributing to general economic and financial stability, monetary policy can do little to affect these structural changes or the powerful economic forces that drive them. However, to make effective policy, the Federal Reserve must have as full an understanding as possible of the factors determining economic growth, employment, and inflation in the US economy, whether those influences originate at home or abroad. Consequently, one direct effect of globalization on Federal Reserve operations has been to increase the time and attention that policymakers and staff must devote to following and understanding developments in other economies, in the world trading system, and in world capital markets.".

In the same way, Mark Carney, former president of the Bank of England (BoE), in a speech for the IMF in 2017, pointed out that all Central Banks must consider how global factors influence the stance of domestic monetary policy. Moreover, Carney highlighted that *"global integration affects the transmission mechanism of domestic monetary policy, the degree of spillovers from foreign monetary policies, and the equilibrium rate of interest itself."*.

To sum up, even when a Central Bank does not show explicitly that external variables are relevant in its monetary policy, in a more subjective way, it has been emphasizing the relevance of the international environment for its policy rate decision. In this context, this paper aims to show the impact of one Central Bank's decision on the others, based on the argument that the rise in the financial integration intensifies the spillover effects.

1.3 Methodology

1.3.1 Taylor Rule

(Taylor 1993) found that, in most countries, it is preferable to place a positive weight on both, i.e., the lagged inflation and output gap in the interest-rate rule. More than that, the economist suggested that one policy rule that captures the spirit of the research and is straightforward is:

$$i_t = i^* + 0.5(\pi_{t-1} - \bar{\pi}) + 0.5\hat{y}_t \tag{1.3.1}$$

Where i_t is the nominal interest rate, i^* is the nominal equilibrium rate, π_t is the rate of inflation over the previous four quarters, $\bar{\pi}_t$ is the inflation target, and $\hat{y}_t$ is the output gap.

(Taylor 1993) used Equation (1.3.1) to describe the US' monetary policy, using the same coefficient for output gap and deviation of the inflation rate from the target. Specifically, to represent the Federal Reserve's policy rule, the author considered the inflation target of 2% and an equilibrium real interest rate of 2%. Thus, if the price level and real GDP are on target, the federal funds rate would be equal to 4%. For a first analysis, we estimate Taylor Rule considering the following Equation:

$$i_t = i^* + \beta_1(\pi_{t-1} - \bar{\pi}) + \beta_2\hat{y}_t + \epsilon_t \tag{1.3.2}$$

Where β_1 and β_2 are estimated using Ordinary Least Squares (OLS).

As highlighted by (Clarida et al. 2001), however, Equation (1.3.2) can be too restrictive to describe actual changes in the nominal interest rate. Therefore, we also take into account the following central bank reaction function:

$$i_t = (1 - \rho)(i^* + \beta_1(\pi_{t+1} - \bar{\pi}) + \beta_2\hat{y}_t) + \rho i_{t-1} + \epsilon_t \tag{1.3.3}$$

Different from the rule proposed by (Taylor 1993), in Equation (1.3.3), the monetary authority responds to the expected inflation rate (π_{t+1}), which allows the central bank to consider a more extensive range of information. Moreover, aligned with some empirical studies, this last specification assumes that the monetary authority tends to smooth changes in the nominal interest rate.

We will describe the observable and unobservable variables before estimating Equations (1.3.2) and (1.3.3). The data covers 36 countries quarterly, from the first quarter of 2000 until the fourth quarter of 2019. The policy rate is the annual rate defined by each central bank. The lagged inflation rate is defined as the year on year variation of the consumer price index (CPI) over the previous four quarters, while the expected inflation is the annual change of the CPI in the next four quarters[1.3.1] The inflation target is explicitly defined by each central bank and can vary over the period. Finally, using the quarterly real GDP, we estimate the output gap.

1.3.2 Output Gap

The interest in estimating the output gap has been rising, particularly in an environment of near-zero interest rates, benign economic growth, and low inflation. Because of this, we had a proliferation of techniques for estimating the potential output. (Alvarez e Gomez-Loscos 2017) divided the menu of available estimation methods into two categories: univariate approach, in which trend output is measured only based on actual output, and multivariate approach, which incorporates valuable information from some other variables.

As this study aims to find an interest-rate rule that fits better with the data, we opt to use a simple and widely applied technique to estimate the potential output, known as the Hodrick-Prescott (HP) filter. This univariate method estimates a trend that simultaneously minimizes a weighted average of the output gap and the rate of change in trend output, as in the Equation below:

$$\min_{\tau}\left(\sum_{t=1}^{T}(y_t-\tau_t)^2+\lambda\sum_{t=2}^{T-1}[(\tau_{t+1}-\tau_t)-(\tau_t-\tau_{t-1})]^2\right) \tag{1.3.4}$$

Where τ is a trend component.

Despite its intense use, the HP filter has some drawbacks. First, the result depends on the weight factor's arbitrary choice that controls the smoothness of the trend line (λ). High values of λ reduce the variation of the trend. In the limit, when the weight factor tends to infinite, the trend converges to the mean growth rate for the entire estimated period. In this paper, λ equals 1600 (value typically chosen for applications with quarterly data). Another weakness of the HP filter

[1.3.1] Align with the conventional wisdom regarding the lag with which monetary policy affects inflation and activity.

is the endpoint problem, in which the last points have an outsized impact on the trend. To minimize this problem, the HP filter will be applied to a series extended with ARIMA[1.3.2]. Thus, for each country, we extended the GDP series through the fourth quarter of 2022 using an ARIMA model[1.3.3].

1.3.3 Empirical Evidence

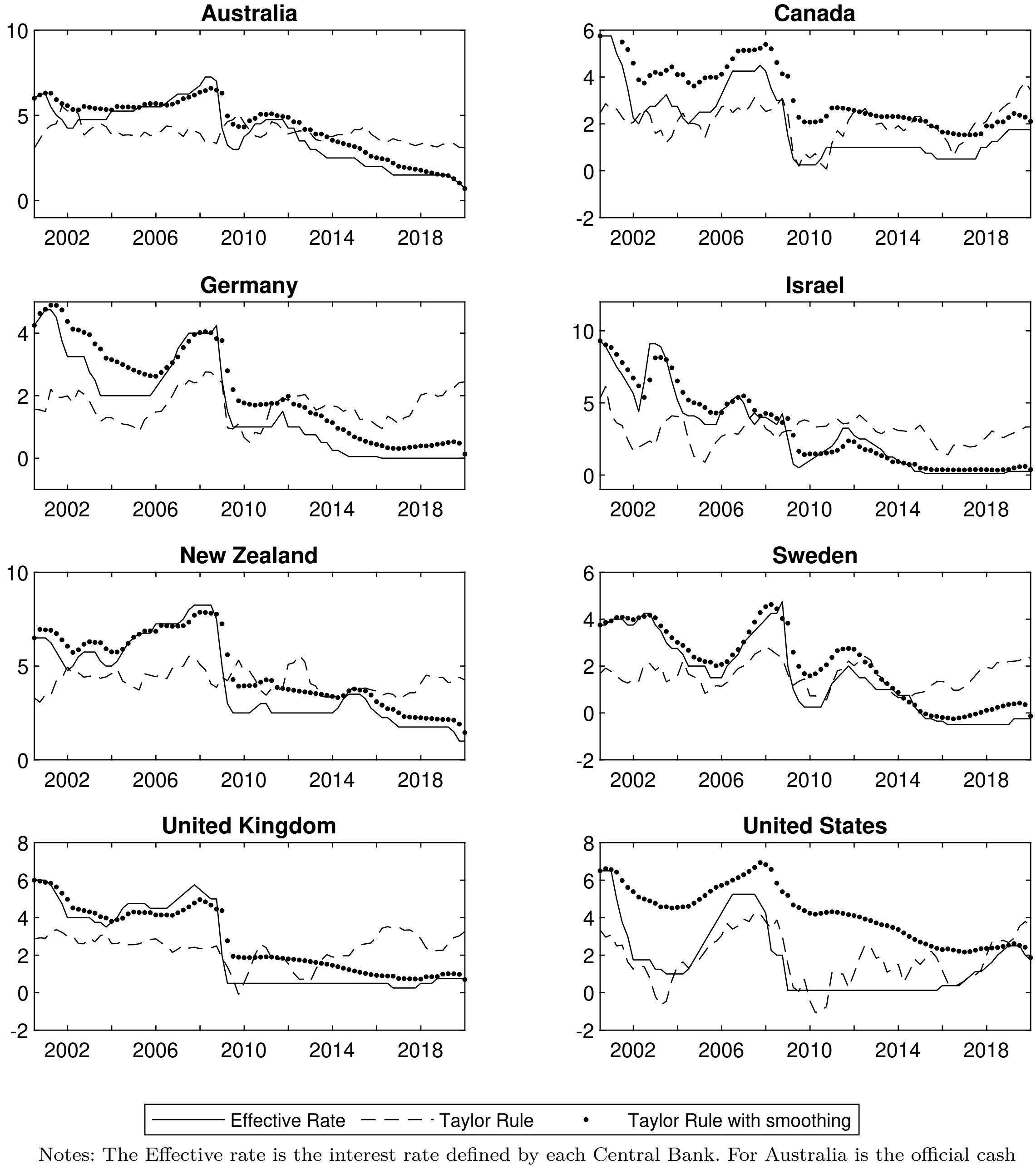
Figure 1.3.1 displays for some countries the actual path of nominal interest rate and the implied by Equations (1.3.2) and (1.3.3). The chart shows that recently, especially after 2008, the actual interest rate has been lower than the level suggested by the Taylor Rule described by Equation (1.3.2). Besides, considering Equation (1.3.3), although we have improved data adherence, we continue to verify that many central banks recently deviated from the interest rate suggested by the reaction function.

Moreover, Table 1.3.1 shows the main statistics of each country's deviation presented in Figure 1.3.1 for two data sets: the complete one, from first quarter 2000 to fourth-quarter 2019, and from the first quarter of 2008 until the fourth-quarter 2019. The results show that the deviations were more pronounced after 2008, with all the eight countries registering a median deviation close to one percentage point for the Taylor Rule without smoothing and nearby 0.5 for the Taylor Rule with smoothing.

[1.3.2](Kaiser e Maravall 2002) show through simulation exercises that applying the HP filter to a series extended with ARIMA forecasts generally provides a cycle estimator for recent periods that requires smaller revisions.

[1.3.3]To define the ARIMA model, we pursue the following steps: First, we applied the Augmented Dickey-Fuller test (ADF) to verify the presence of a unit root. Second, we used the Autocorrelation (ACF) and the Partial Autocorrelation (PACF) to determine the AR and MA terms. Finally, we used these models to forecast the GDP for the months ahead

Figure 1.3.1 – Effective interest rate versus interest rate suggested by the Taylor Rule



Notes: The Effective rate is the interest rate defined by each Central Bank. For Australia is the official cash rate, Canada the policy interest rate, Germany the ECB's main refinancing operations, Israel the bank of Israel interest rate, New Zealand the official interest rate, Sweden the deposit rate, United Kingdom the base rate and the United States the Fed Funds rate. Besides, the Taylor Rule was estimated for each country using the Ordinary Least Square (OLS) methodology. Finally, the Taylor Rule with smoothing was estimated for each country using 2SLS.

Table 1.3.1 – Descriptive Statistics

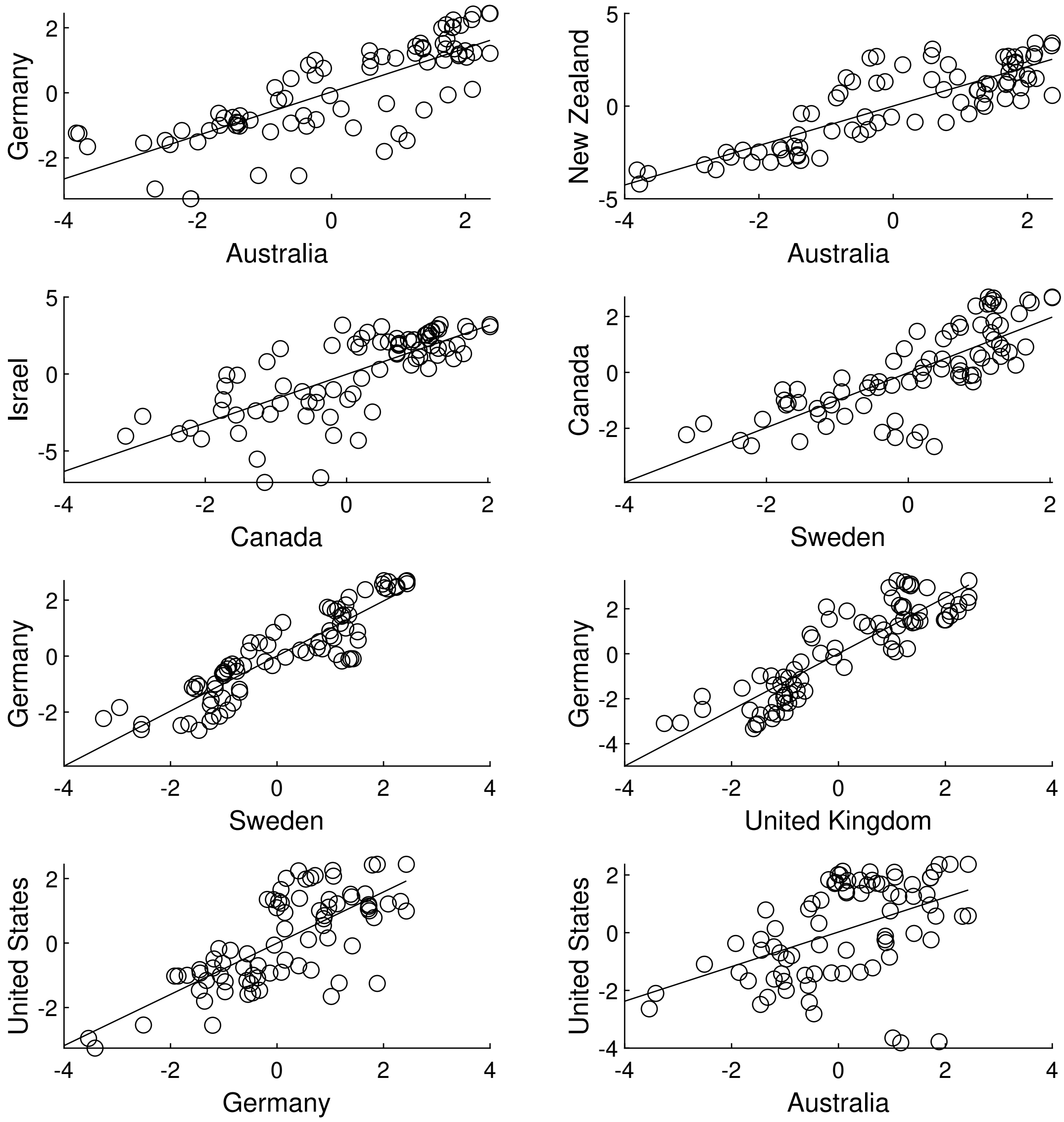
Panel A: Data from 2000-Q1 to 2019 - Q4								
	Taylor Rule without smoothing				Taylor Rule with smoothing			
	Maximum	Minimum	Median	St. Dev	Maximum	Minimum	Median	St. Dev
Australia	2.4	-3.8	0.1	1.7	2.1	-0.8	0.4	0.5
Canada	2.0	-3.3	0.3	1.3	2.5	0.0	1.2	0.5
Germany	2.4	-3.3	-0.1	1.4	1.5	-0.4	0.6	0.4
Israel	3.2	-7.0	1.1	2.6	2.0	-2.5	0.3	0.7
New Zealand	3.4	-4.2	0.4	2.1	2.6	-0.7	0.5	0.6
Sweden	2.7	-2.7	-0.1	1.5	1.8	-0.7	0.4	0.5
United Kingdom	3.2	-3.3	0.2	2.0	2.4	-1.0	0.4	0.7
United States	2.4	-3.5	0.1	1.3	5.1	0.0	2.7	1.4
Panel B: Data from 2008-Q1 to 2019 - Q4								
	Taylor Rule without smoothing				Taylor Rule with smoothing			
	Maximum	Minimum	Median	St.Dev	Maximum	Minimum	Median	St.Dev
Australia	2.4	-3.8	1.4	1.5	2.1	-0.8	0.5	0.6
Canada	2.0	-0.9	1.0	0.6	2.5	0.4	1.3	0.5
Germany	2.4	-1.7	1.1	1.0	1.3	-0.4	0.5	0.3
Israel	3.2	-1.3	2.0	1.0	2.0	-1.3	0.2	0.6
New Zealand	3.4	-4.2	1.4	1.6	2.6	-0.4	0.7	0.6
Sweden	2.7	-2.4	0.9	1.2	1.8	-0.7	0.6	0.5
United Kingdom	3.2	-2.9	1.5	1.4	2.4	-0.5	0.7	0.6
United States	2.4	-1.2	0.9	0.9	5.1	0.0	3.2	1.5

Notes: The deviation is calculated as the difference between the interest rate defined by each Central Bank and the one proposed by the Taylor Rule.

Interestingly, different countries' deviations seem to be correlated. (Gray 2013) has already reported that the correlation between US deviation from Taylor Rule and deviation from the Taylor Rule in several other countries is quite strong. Nevertheless, Figure 1.3.2 shows that there is also a robust correlation between other countries, even when the Federal Reserve decision is not considered. Differently from many papers that suggest that only changes in US monetary policy could affect other economies, this study aims to show that the impact of monetary policy decisions occurs in two ways. In other words, the Federal Reserve's actions impact other countries, but other central banks' decisions also affect US monetary policy.

In order to capture the correlation between monetary policies from different countries, this paper suggests an alternative rule, which adds in the policy rule of one central bank the decisions made by other central banks. For this reason, this study applies the spatial econometric methodology, detailed in the following subsection.

Figure 1.3.2 – Deviations from the Taylor Rule of two large countries



Notes: The deviation is calculated as the difference between the interest rate defined by each Central Bank and the one proposed by the Taylor Rule.

1.3.4 Taylor Rule with spatial effects

This paper adopts the spatial econometric methodology to contemplate the correlation between monetary policy decisions from different countries. Although empirical works in regional science have been applying this methodology recurrently, in the field of macroeconomics and monetary policy, studies that use spatial econometrics are scarcer.

This section provides more details about the necessity of using spatial econometrics in the present study and describes the different spatial models.

It should be noted that the present data exclude the possibility of adopting traditional econometrics: first, the spatial dependence between the observations; second, the presence of heterogeneity in the relationship estimated. These two issues violate Gauss-Markov assumptions, generating estimators biased and inefficient when the traditional methods are applied.

The spatial dependence refers to the fact that observation i depends on other observation j, formally:

$$y_i = f(y_j), i = 1, ..., n, j \neq i \tag{1.3.5}$$

This dependence occurs if the spatial dimension of economic activity is an important aspect of the modeling problem. In this study, spatial dependence results from our main assumptions: countries with significant economic integration tend to have a higher cross-country dependency on the interest rate decision.

The other condition leading spatial econometrics, the spatial heterogeneity, refers to a variation in the relationship over different observations. A linear relationship that describes this can be written as the equation below:

$$y_i = X_i \beta_i + \epsilon_i, i = 1, ..., n \tag{1.3.6}$$

Here we assume that depending on the economic integration level, the dependence between two countries' interest rate decisions could be different.

Given a sample N, as it is not possible to estimate N beta parameters for each relation because there is a degree of freedom's problem, it is necessary to provide a specification for variation over the observations to proceed with the analysis, that it is possible through the spatial weight matrix described below.

1.3.5 Spatial Weight Matrix (W)

A spatial weight matrix (W) is a N x N matrix in which each element w_{ij} represents the prior strength of the interaction between location i and location j. By convention, the diagonal elements are all equal to zero ($w_{ii} = 0$). Moreover, for computational simplicity, the weights are often standardized so that the elements in each row sum to 1 ($\sum_i w_{ij} = 1$). The specification of the spatial weight matrix is an important issue in applied spatial econometrics. As explained by (L. Julie L G 2008), in practice, it is more common to choose w based on geographic criteria; however, specifications that incorporate notions of "economic"distances are increasingly used as well.

This paper uses an "economic"distance to construct its spatial weight matrix. Based on the assumption that monetary policy spillovers are correlated with the level of global economic integration, the "distances"between the two countries will be measured by their trade flow.

The international trade data was extracted from the OECD Inter-Country Input-Output (ICIO) tables. In these Tables, the diagonal blocks represent domestic transaction flows of intermediate goods and services across industries, while the off-diagonal blocks represent the inter-country flows of intermediates via exports and imports.

OECD ICIO Tables have data from 45 different sectors, such as Agriculture, Mining, Construction, Wholesale... Considering the importance of global economic integration to this paper's assumption, most estimations are based on the sum of flows of all 45 sectors.

We made some fundamental transformations in the OECD ICIO Tables to be compatible with a spatial weight matrix (W). First, OECD ICIO Tables were separated into 45 matrices (w_0^k) with k=1,2,...,45 and the dimension NxN, each representing one sector. Then, the elements $w_{k,ij}^0$ of each matrix were added (w_1).

After that, as a spatial weight matrix must be symmetrical, w_1 was multiplied by its transposed ($w_2 = w_1' w_1$). This step emphasizes that it is not important what a particular country exports or imports; the relevant information is the flow between two countries. That represents their level of integration.

The next step is to set all the diagonal's elements of w_2 equal to zero, as long as flows within the country, from different industries, are not relevant for the present study. Figures 1.3.3 and 1.3.4 show w_2 (Total) for two different years, 1999 and 2011.

From both figures, it is possible to verify that the intensity of the commercial flows between countries (darker equals to more commercial integration) is different across sectors. The machinery and equipment sector, for example, has a higher international integration. On the other hand, education is a sector with less global dependency.

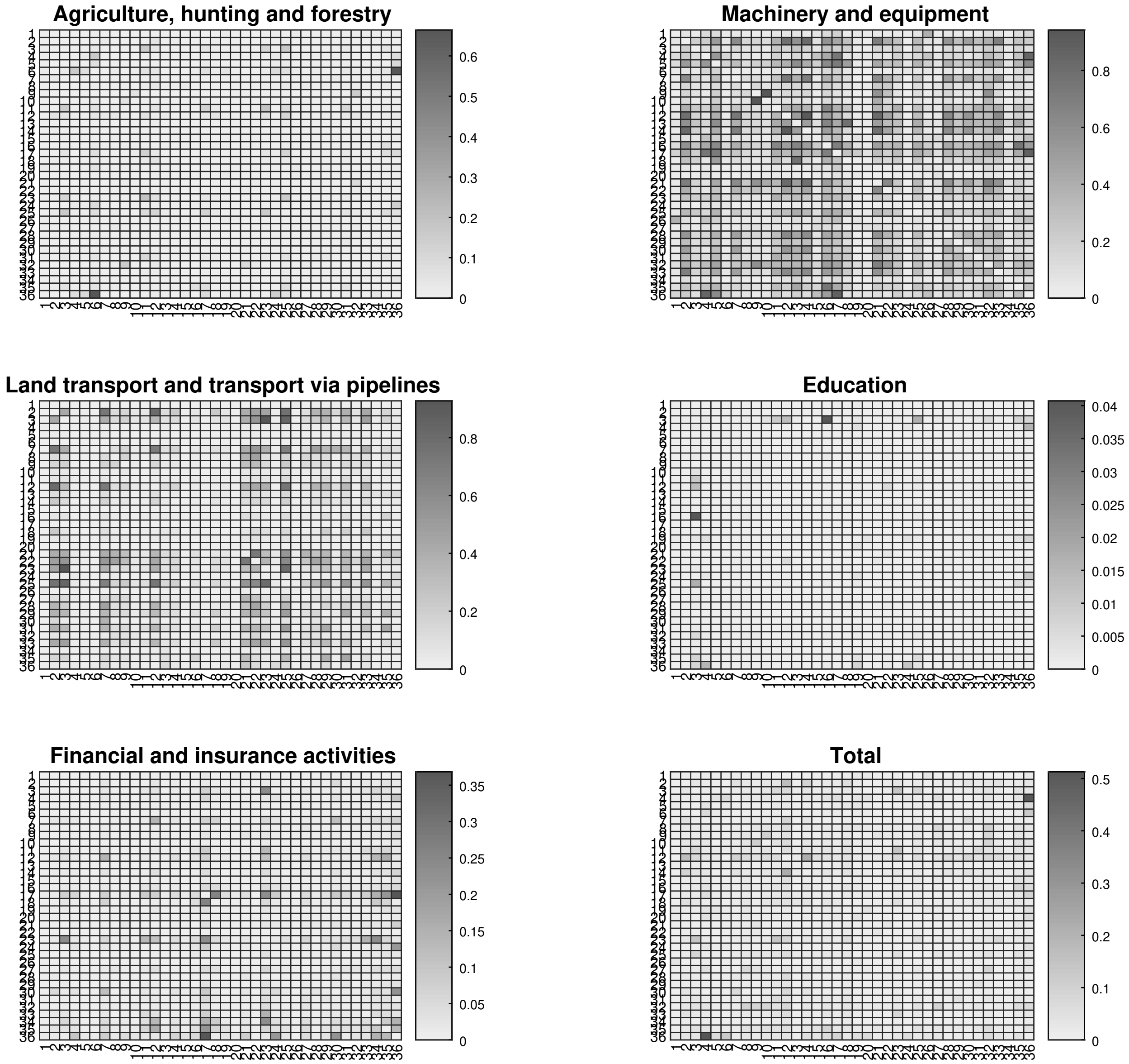
To conclude the transformations, it should be noted that the spatial weight matrix commonly used was purely cross-sectional. To extend its use to panel models, first it is necessary to assume that w remains constant over time, then apply the following transformation:

$$W = I_T \otimes W_N \tag{1.3.7}$$

Where W is the weight matrix for the cross-sectional dimension and I_T, the identity matrix of dimension T.

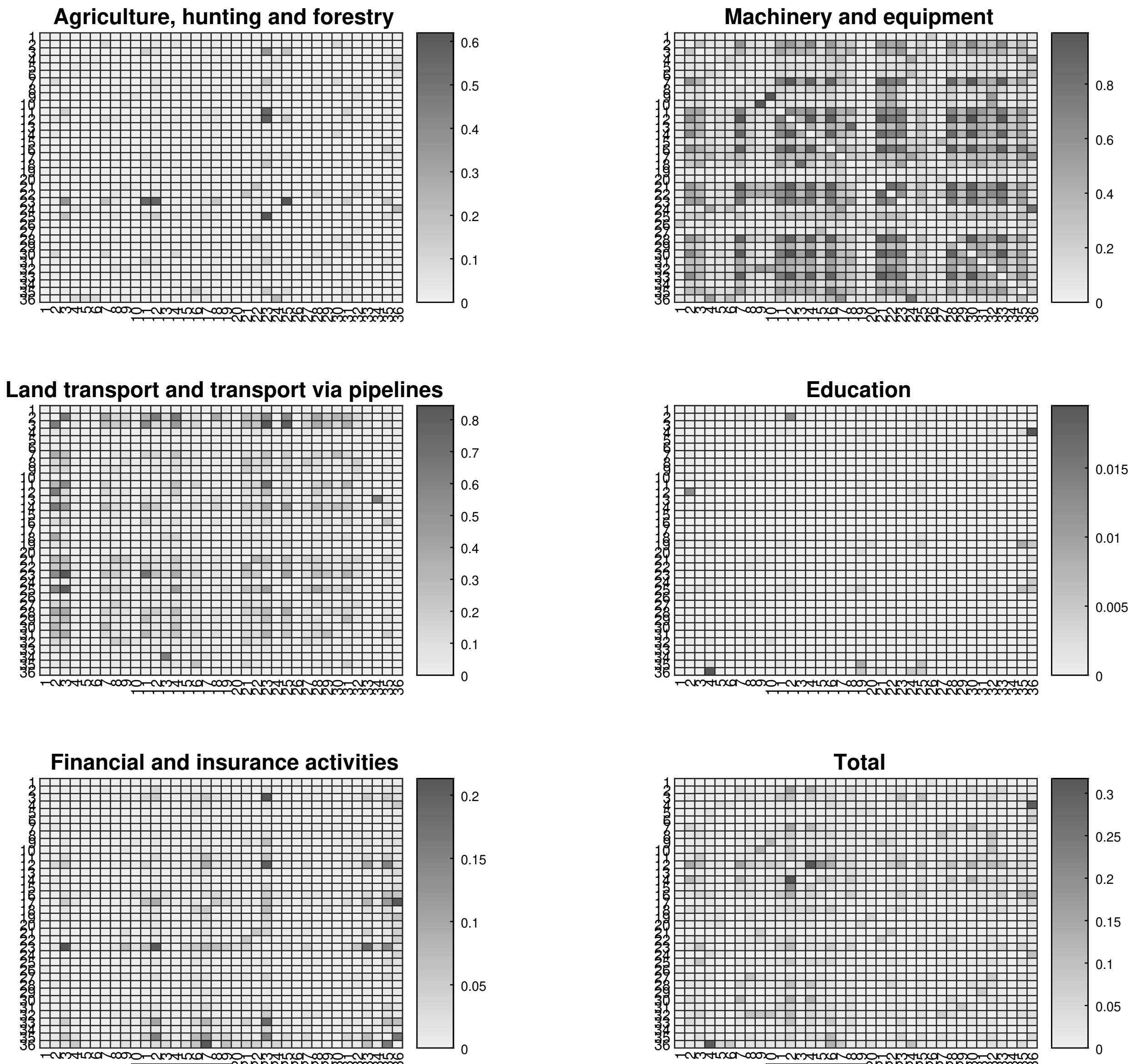
After choosing the adequate spatial weight matrix (W), it is essential to decide which model fits the data better. Various specifications may be considered to take spatial autocorrelation into account. The spatial dependence could be associated with the dependent variable, as in the Simultaneous Auto-Regressive Model (SAR), interacting with the error term, as in Spatial Error Model (SEM), combined with dependent and independent variables, as in the Spatial Durbin Model (SDM), or yet, combine with the independent variables and error term as

Figure 1.3.3 – Neighbor weights matrix in 1999 for different sectors



Notes: Each column/row represents a country: 1.Australia, 2.Austria, 3.Belgium, 4.Canada, 5.Chile, 6.Colombia, 7.Czech Republic, 8.Denmark, 9.Estonia, 10.Finland, 11.France, 12.Germany, 13.Greece, 14.Hungary, 15.Iceland, 16.Ireland, 17.Israel, 18.Italy, 19.Japan, 20.Korea, 21.Latvia, 22.Lithuania, 23.Luxembourg, 24.Mexico, 25.Netherlands, 26.New Zealand, 27.Norway, 28.Poland, 29.Portugal, 30.Slovak Republic, 31.Spain, 32.Sweden, 33.Switzerland, 34.Turkey, 35.United Kingdom, and 36.United States. First, we normalize the commercial flow by line and then multiply by its transpose to obtain a symmetric matrix.

Figure 1.3.4 – Neighbor weights matrix in 2011for different sectors



Notes: Each column/row represents a country: 1.Australia, 2.Austria, 3.Belgium, 4.Canada, 5.Chile, 6.Colombia, 7.Czech Republic, 8.Denmark, 9.Estonia, 10.Finland, 11.France, 12.Germany, 13.Greece, 14.Hungary, 15.Iceland, 16.Ireland, 17.Israel, 18.Italy, 19.Japan, 20.Korea, 21.Latvia, 22.Lithuania, 23.Luxembourg, 24.Mexico, 25.Netherlands, 26.New Zealand, 27.Norway, 28.Poland, 29.Portugal, 30.Slovak Republic, 31.Spain, 32.Sweden, 33.Switzerland, 34.Turkey, 35.United Kingdom, and 36.United States. First, we normalize the commercial flow by line and then multiply by its transpose to obtain a symmetric matrix.

in Spatial Durbin Error Model (SDEM). The details of each model are presented below.

1.3.6 Simultaneous Auto-Regressive Model (SAR)

In the Simultaneous Auto-Regressive Model (SAR), spatial dependence is represented by the dependent variable and can be described as follows:

$$Y = \beta X + \rho WY + u \tag{1.3.8}$$

where $u \sim N(0, \sigma^2)$. According to (LeSage 1998), the model is termed SAR because it represents a spatial analogy to the lagged dependent model from the time series analysis.

On average, the value of Y (policy rate) is explained by the values of the explanatory variables X and those associated with all the neighbors' observations (policy rate of other countries).

In terms of estimation, a simple Ordinary Least Square (OLS) would generate biased coefficients, as shown by (LeSage 1998). The result's intuition is that as the spatial dependence WY is not fixed in repeated sampling, only the assumption that E(u) = 0 does not eliminate the bias. Moreover, the fact that WY changes rule out the consistency of the OLS estimate.

Given that OLS will produce biased and inconsistent estimates, maximum likelihood estimation for the SAR will be applied[1.3.4]. The steps are described below:

1. Estimate an OLS for the model $Y = X\beta + \epsilon_0$
2. Estimate an OLS for the model $WY = X\beta_L + \epsilon_L$

[1.3.4] All spatial models in this study are estimated using Maximum Likelihood (ML). In the empirical literature, the Instrumental Variable (IV) and the Generalized Method of Moments (GMM) were also applied for spatial panel models. (Franzese Jr e Hays 2007) compared the performance of IV and ML estimators of panel data models with the spatially dependent variable and found that ML offers weakly dominant and generally unbiased estimators. Unfortunately, the authors did not consider spatial fixed or random effects.

3. Compute the residuals $e_0 = Y - X\widehat{\beta}$ e $e_L = WY \breve{} X\widehat{\beta}_L$

4. Given e_0 e e_L find ρ that maximizes the concentrate likelihood function:

$$L = -(NT2)ln(2\pi\sigma^2) - (12\sigma^2)(e_0 - \rho e_L)'(e_0 - \rho e_L) + Tln|I - \rho W|$$

5. Given $\widehat{\rho}$ compute $\widehat{\beta} = (\widehat{\beta_0} - \rho\widehat{\beta_L})$ and $\widehat{\sigma_\epsilon^2} = (1n)(\epsilon_0 - \rho\epsilon_L)'(\epsilon_0 - \rho\epsilon_L)$

1.3.7 Spatial Error Model (SEM)

In contrast to the Spatial Auto-Regressive Model, a Spatial Error Model (SEM) does not require a theoretical model for spatial interaction. Instead, it is a particular case of a non-spherical error covariance matrix. SEM can be described as follow:

$$\begin{aligned} Y &= \beta X + u \\ u &= \lambda W u + \epsilon \end{aligned} \tag{1.3.9}$$

with $\epsilon \sim$ N(0,σ^2).

In the Spatial Error Model, it is well-known from traditional econometrics that autocorrelated errors do not affect the property of unbiasedness of the OLS estimator. The same applies to Equation (1.3.8), where OLS still provides unbiased coefficients. However, they are inefficiently estimated as their standard errors are biased. Thus, the procedure to estimate this model is based on the maximum likelihood method and pursue the following iterative approach:

1. Estimate an OLS for the model $Y = \beta X + u$

2. Estimate an OLS for the model $u = \lambda W u + e$

3. Compute the residuals $u_0 = Y - X\widehat{\beta}$ and $e_0 = u - \widehat{\lambda} W u$

4. Finds a value of λ that maximizes the log likelihood conditional on the OLS β values

$$L = -(\frac{NT}{2})ln(2\pi\sigma^2) - (\frac{1}{2\sigma^2})(u_0 - e_0\lambda W)'(u_0 - e_0\lambda W) - Tln|I - \lambda W|$$

5. Updates the OLS values of β using the value of λ found in step 4.

6. This process continued until the convergence in the residuals.

1.3.8 Spatial Durbin Model (SDM)

The Spatial Durbin Model (SDM) contains a spatially lagged dependent variable (WY) and spatially lagged explanatory variables (WX). The procedure to estimate the model is:

$$Y = \beta_1 X + \rho WY + \beta_2 WX + u \tag{1.3.10}$$

where u $\sim$ N(0,σ^2).

The procedure to estimate the model is:

1. Considering $\tilde{x}$ = [XWX]

2. Estimate a OLS for the model: $Y = \tilde{x}\beta_1 + e_1$

3. Estimate a OLS for the model: $WY = \tilde{x}\beta_2 + e_2$

4. Compute the residuals: $e_1 = Y - \tilde{x}\hat{\beta}_1$ and $e_2 = WY - \tilde{x}\hat{\beta}_2$

5. Given e_1 and e_2 find ρ that maximizes:
$$L = Tln|I - \rho W| - (\frac{1}{2\sigma^2})(e_1'e_1 - 2\rho e_2'e_1 + \rho^2 e_2''e_2)$$

6. Compute $\hat{\beta} = (\beta_1 - \hat{\rho}\beta_2)$ and $\sigma^2 = \frac{(y - \hat{\rho}Wy - \tilde{x}\hat{\beta})'(y - \hat{\rho}Wy - \tilde{x}\hat{\beta})'}{TN}$

1.3.9 Spatial Durbin Error Model (SDEM)

The last model is the Spatial Durbin Error Model (SDEM) which contains a spatial error dependence and spatially lagged explanatory variables (WX):

$$Y = \beta_1 X + \beta_2 \sum WX + u$$
$$u = \lambda W u + \epsilon \tag{1.3.11}$$

To estimate the model we follow the steps below:

1. Estimate a OLS for the model: $Y = X\beta_1 + XW\beta_2 + e_1$
2. Estimate a OLS for the model: $u = \lambda W u + e_2$
3. Compute the residuals: $e_1 = Y - X\hat{\beta}_1 - XW\hat{\beta}_2$ and $e_2 = u - \hat{\lambda} W u$
4. Finds a value of λ that maximizes the log likelihood conditional on the OLS β values
$$L = -(\frac{NT}{2})ln(2\pi\sigma^2) - (\frac{1}{2\sigma^2})(u_0 - e_0\lambda W)'(u_0 - e_0\lambda W) - Tln|I - \lambda W|$$
5. Updates the OLS values of β using the value of λ found in step 4.
6. This process continued until the convergence in the residuals.

1.4 Results

1.4.1 Data Analysis

(Elhorst 2014) suggested a mixed procedure, in which first the non-spatial model is estimated to test it against the spatial lag and spatial error model. If the non-spatial model was rejected, the SDEM and SDM are estimated to test if it can be simplified to either SAR or SEM.

Then, before verifying the presence of spatial autocorrelation, tests for traditional panel data were applied. Table 1.4.1 shows the main results considering three types of estimations: Pooled Ordinary Least Squares (OLS) (column 1), Panel data with fixed effects (column 2), and panel data with random effects (column 3).

Comparing the models, we can see that the results of simple panel data estimations align with the empirical literature. In all three models, the coefficient

Table 1.4.1 – Panel data Regression

Variable	OLS	Fixed Effects	Random Effects
Constant	2.7778***		2.82941***
	(0.07)		(0.42)
Inflation gap	1.266***	1.2507***	0.9118***
	(0.02)	(0.02)	(0.02)
Output gap	0.1601***	0.1589***	0.1396***
	(0.03)	(0.03)	(0.03)
R^2	0.509	0.514	0.679
R^2 adj.	0.508	0.514	0.679

Notes: $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$

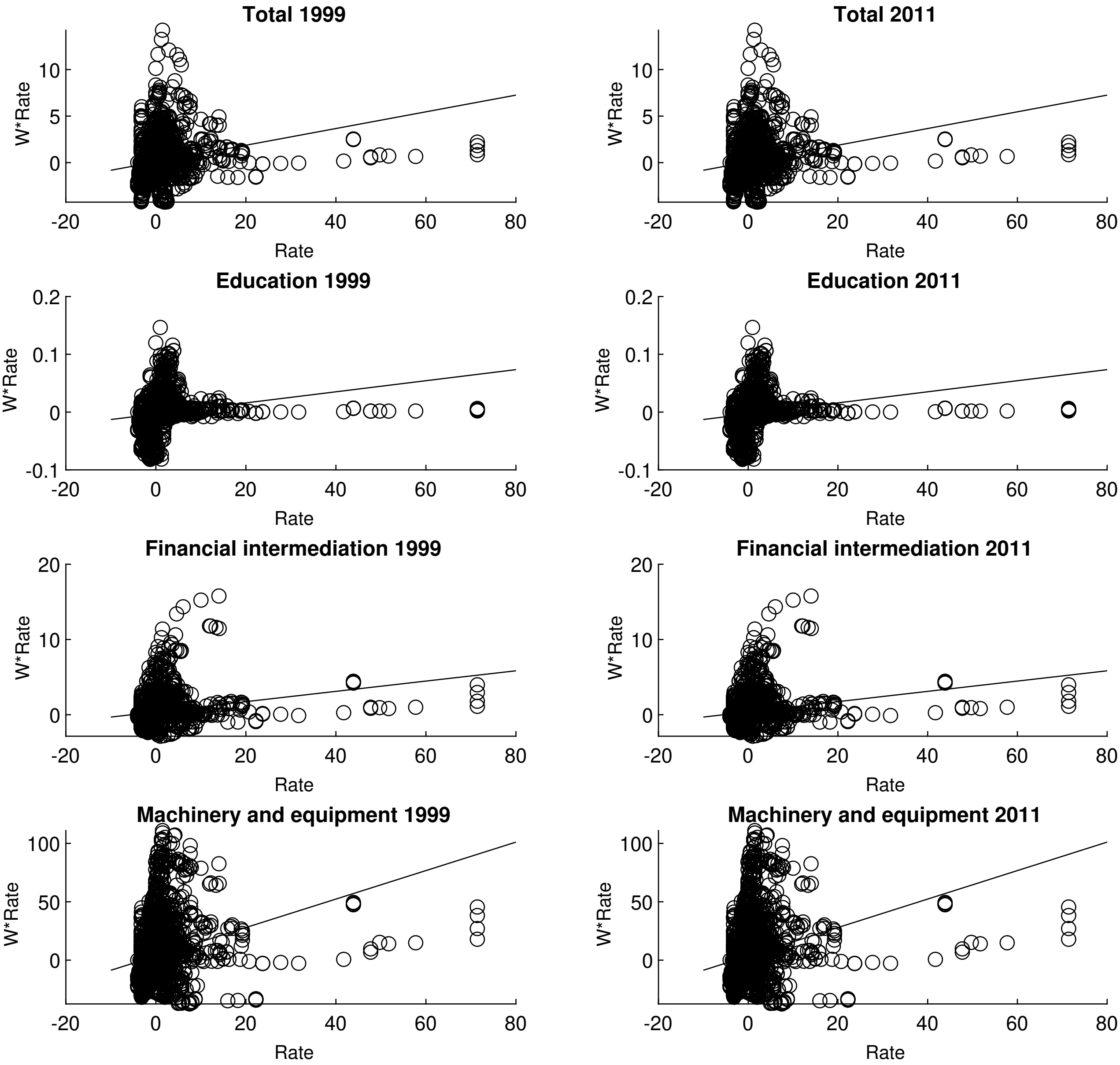
for inflation gap is significant and greater than one, respecting the Taylor principle. Moreover, following many studies, the output gap coefficient is significant but smaller than described by (Taylor 1993), suggesting some muted response to activity.

Additionally, the presence of unobserved specific effects is tested. Applying the test F, the p-value close to zero indicates that the fixed effects model is preferred to a pooled OLS model. Besides, the Breusch-Pagan LM test can reject the null hypothesis, indicating that the random effect model is preferred over pooled OLS. Thus, to choose between fixed or random effects, we applied the Hausman Test, where the null hypothesis indicates that the random effect is the preferred model compared to fixed effects. With the statistic χ^2 close to 35, we can reject H_0 with 5% of significance, indicating that the most appropriate model is the fixed effects.

Thus, using panel data with fixed effects, the next step is to check the presence of spatial dependence in error. A simple way to examine the existence of spatial dependence is the Moran scatter plot, shown in Figure 1.4.1.

The Moran scatter plot shows the relation between the interest rate (dependent variable y in deviation means form) and its spatial lag vector Wy, which shows the interaction between the interest rate and economic integration level. Figure 1.4.1 displays a positive association between y and Wy, suggesting a positive spatial dependence in countries' monetary policy decisions. The magnitude of the slope of the line fitted through the points in the Moran scatters plot is equal to

Figure 1.4.1 – Moran scatter plot



Notes: On the left side, we use the spatial weight matrix of 1999, and on the right side, it was applied the w for 2011.

Moran's I statistic. The slopes vary from 0.002 for the education sector to 0.8 in the machinery and equipment sector. These different behaviors across sectors were somewhat expected. Education could be considered a non-tradable sector, thus, less affected by economic integration. On the other hand, as financial and machinery sectors have more inter-country flows, demonstrated in the W matrix, they illustrate better the level of economic integration.

It is worth noting that Moran's I statistic stays close to 0.1 for the W matrix that considers all sectors. Moreover, even with some registering small magnitudes, all Moran's I statistics are different from zero with 95% confidence, corroborating the presence of spatial dependence.

Another commonly used test to verify spatial correlation is based on the Lagrange Multiplier (LM). One of the main advantages of this test is that it only requires estimating the model under the null hypothesis (no spatial dependence).

LM statistics are based on a pooled regression model (or a panel data with spatial and time specific fixed effects) and the spatial weight matrix (W). (4) specified the LM test for spatial error correlation and spatial lagged dependent variable as follows:

$$LM_E = \frac{[e'(I_T \otimes W_N)e\hat{\sigma}^2)]^2}{TTw} \tag{1.4.1}$$

$$LM_L = \frac{[e'(I_T \otimes W_N)y\hat{\sigma}^2)]^2}{J} \tag{1.4.2}$$

Given these formulas, the robust counterparts will take the form[1.4.1]:

[1.4.1] (Elhorst 2010)

$$RLM_E = \frac{[e'(I_T \otimes W)e\hat{\sigma}^2 - [TTw/J]e^T(I_T \otimes W)y\hat{\sigma}^2]^2}{TTw[1 - TTw/J]^{-1}} \tag{1.4.3}$$

$$RLM_L = \frac{[e'(I_T \otimes W)y\hat{\sigma}^2 - e'(I_T \otimes W)e\hat{\sigma}^2]^2}{J - TTw} \tag{1.4.4}$$

Where I_T denotes the identity matrix, $TTw = trace(WW + W'W)$, $J = \frac{1}{\hat{\sigma}^2}[(I_T \otimes W)X\hat{\beta})'(I_{NT} - X(X'X)^{-1}X')(I_T \otimes W)X\hat{\beta} + TTw\sigma^2]$ and e is a residual vector of a pooled regression or panel data with spatial and/or time period fixed effects.

Since the LM robust tests' outcome depend on which effects are included in the model, it is highly recommended to carry out these LM tests for different data specification. The results of the LM tests for a polled OLS and considering spatial and time fixed effects are presented in Table 1.4.2.

Table 1.4.2 – Panel data Regression

	Polled OLS	Spatial FE	Time FE	Spatial and Time FE
constant	2.8105***			
	(0.06)			
Inflation gap	1.2385***	0.8733***	1.2367***	0.7867***
	(0.03)	(0.03)	(0.03)	(0.03)
Output gap	0.1568***	0.1367***	-0.1402	-0.1294
	(0.03)	(0.03)	(0.04)	(0.03)
R^2	0.496	0.348	0.515	0.343
LL	-7853	-7226	-7543	-6673
LM_L	532.3	1162	1.6	0.7
LM_E	630.5	1727	0.4	0.2
Robust LM_L	163.5	97.6	2.5	2.1
Robust LM_E	114.5	162.2	1.3	1.6

Notes: $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$

According to the results, when the model is estimated using Pooled OLS or spatial fixed effects, the LM tests and its robust counterparts suggest the presence of spatial lag dependent variable. The LM_L and Robust LM_L statistics displayed in columns 1 and 2 indicate that the null hypothesis of no spatially lag dependent variable can be rejected at 10% of significance.

However, when we include the time-fixed effect, the results in Table 1.4.2 are less conclusive. (Elhorst 2010) highlighted that applied researchers often find weak evidence of spatial correlation when the model also accounts for time fixed effect. Intuitively, the explanation relies on the fact that many variables tend to increase and decrease together in different spatial units over time.

Besides, some empirical studies also found some significant differences among coefficients estimates for models with or without spatial fixed effects. This happens because these models use distinct parts of the variation between observations. In other words, spatial effects models utilize the time series component of the data, while the pooled OLS uses the cross-sectional components of the data. Moreover, the inclusion of spatial fixed effects could omit the estimation of variables that do not change over time or only vary because demeaning transformation wipes out these variables. For this reason, some researchers suggest not controlling for spatial fixed effects.

However, if a critical variable such as spatial or time-fixed effects is omitted from the regression equation, the estimators of the coefficients are biased and inconsistent. Consequently, in addition to these tests, we also applied the Lagrange Multiplier (LR) test to verify the presence of spatial and time fixed effects, as suggested by (Elhorst 2010). In this case, the LR test for time and spatial fixed effects has a statistic $\chi^2(77) = 620$ and $\chi^2(34) = 1254$, respectively, suggesting the inclusion of both effects in the model.

1.4.2 Main Results

We estimate the four main spatial models following the previous section's results, including spatial and time fixed effects. It should be noted that the spatial (time) fixed effects can only be consistently estimated when N(T) is sufficiently larger. Increasing the number of observations in the cross-section domain (time-series domain) worsens the problem as more unknown coefficients have to be estimated.

Fortunately, the inconsistency of the fixed effects is not transmitted to estimating the slope coefficients in a demeaned equation since the coefficient estimator is not a function of the estimated fixed effects. We are not interested in the fixed effects parameters, so we will apply the demeaning procedure to estimate consistent slope coefficients.

From the first column of Table 1.4.3, we see that considering the SAR model, the inflation gap coefficient is positive and statistically different from zero, while the coefficient for the output gap is significant but negative. The larger coefficient to the deviation of inflation from its target was expected. Although (Taylor 1993) gave the same weight for both variables when analyzing the Federal Reserve decisions, the GDP's lower importance is a common finding in the recent related literature. Some studies pointed out the acyclicality of the monetary policy. (Schmitt-Grohe e Uribe 2004) found that the coefficient of the output gap is close to zero in an optimal operational interest rate rule. Moreover, column 1 also indicates that the interest rate exhibits a positive spatial dependence ($\rho = 0.26$) even after accounting for the effects of the deviations of inflation and GDP.

Table 1.4.3 – Panel data Regression

Variable	SAR	SEM	SDM	SDEM
Gap inflation	0.7844***	0.7864***	0.7832***	0.7844***
	(0.02)	(0.02)	(0.03)	(0.03)
Output gap	-0.1278***	-0.1293***	-0.1276***	-0.1277***
	(0.03)	(0.03)	(0.03)	(0.03)
W*(interest rate) (ρ)	0.2676***		0.2611***	
	(0.04)		(0.04)	
Spatial error (λ)		0.0289		0.0499
		(0.05)		(0.05)
W*Gap inflation (θ_1)			0.0665	0.3000*
			(0.16)	(0.16)
W*Output gap (θ_2)			0.0071	-0.0351
			(0.17)	(0.18)
R^2	0.6778	0.7779	0.7783	0.7783
LL	-6672	-6673	-6671	-6070

The SEM model in column 2 shows that the inflation gap is significant to explain the interest rate's behavior. In contrast, the output gap has a negative effect on the interest rate. Further, the SEM results suggest that after considering the influence of explanatory variables, we do not have a spatial correlation in the residuals, as parameter λ is negative and no significant different from zero.

The SDM model results in the third column of Table 1.4.3 show that the inflation coefficient remains lower than one, but positive and statistically different from zero, while the output gap coefficient is negative. The spatially lagged for the output and inflation is positive, but its coefficient is not significant. However, even including the spatial lagged explanatory variables, the interest rate maintains its spatial dependence, verified in the SAR model ($\rho = 0.26$).

Finally, considering the SDEM model, the results are similar for the explanatory variables: inflation and the output gap. Besides, the model shows no significant spatial correlation in the residuals, in line with the SEM results. The spatially lagged inflation gap is positive and statistically significant. Again, the coefficient for the spatially lagged output gap is not statistically different from zero.

Considering the estimations above, it is essential to decide which model specification is the most appropriate for the present study. As the alternative models can be considered as nesting each other, this task is not so complicated.

According to (LeSage 1998), the presence of a negative spatial correlation, such as we found in the SEM and SDEM models, suggests that these are not the most appropriate specifications for this data as a negative relationship is a result counter to intuition. Thus, we focus our analysis on the spatial lag models.

It should be noted that whereas the parameters estimated in non-spatial models represent the marginal effect of a change in the independent variable on the dependent one, this is not the case for some spatial models. For models with spatial lag, such as SAR and SDM, to fully interpret changes, direct and indirect effects have to be calculated because of the feedback effects that arise due to impacts passing through neighboring countries back to the countries themselves.

Following (Elhorst 2010), the direct effect is viewed as an impact of a change of explanatory variable in location i on the dependent variable in the same area. The indirect effect is an impact of a change of explanatory variable in location i on a change of the dependent variable in location j, with $j \neq i$.

In SAR and SDM models, the direct effect is the sum of diagonal elements of the matrix that contained the partial derivatives of the dependent variable for the k^{th} explanatory variable divided by N. The total effect is the sum of all elements of this matrix divided by N. Therefore, the indirect effect is based on off-diagonal elements and is calculated as the difference between the total and direct effects[1.4.2].

Table 1.4.4 displays the direct and indirect effects of our SAR models[1.4.3]. Regarding the SAR model, the inflation gap has positive and significant direct and indirect effects. The first effect highlighted the Central Bank's response to a deviation from domestic inflation from its target. The indirect effect (0.29) suggests the presence of spillover effects in monetary policy. Inflation rate above the target on foreign countries leads to a higher interest rate in the domestic country. The total effect of the inflation gap is larger than one[1.4.4], in line with the Taylor principle. In other words, considering the feedback effects, Central Banks have a more than a one-to-one response of interest rate to inflation below or above its target.

The SAR model's direct and indirect effects of the output gap are negative, corroborating some studies that found a low impact of the activity on the interest rate.

The results above are in line with the argument of (Taylor 2013). For the author, the spillover effects could cause more significant deviations from the policy rules. This can happen because as a Central Bank rises (cuts) its interest rate to counter (stimulate) its economy; other central banks will consider this decision. They will also increase (cut) their rate, leading to another rise (fall) in the interest

[1.4.2] Actually both the direct and indirect effects are different across units. However, to improve the "surveyability"of these estimations, (LeSage e Pace 2009) proposed to report one direct measured by the average of the diagonal elements of the partial derivative matrix and one indirect effect estimated by the average sum of the non-diagonal elements.

[1.4.3] According to (Elhorst 2010), one important limitation of the SAR model is that the ratio between direct and indirect effects of a specific explanatory variable is independent of β; thus, this ratio is the same for every explanatory variable, with its magnitude depending on the spatial autoregressive parameter ρ and the spatial weight matrix w. On the other hand, as the direct and indirect effects of SDM model depend on the θ estimation, their ratio may be different for distinct explanatory variables.

[1.4.4] Although not statistically significantly higher than one.

Table 1.4.4 – SAR Model - Direct and Indirect Effects

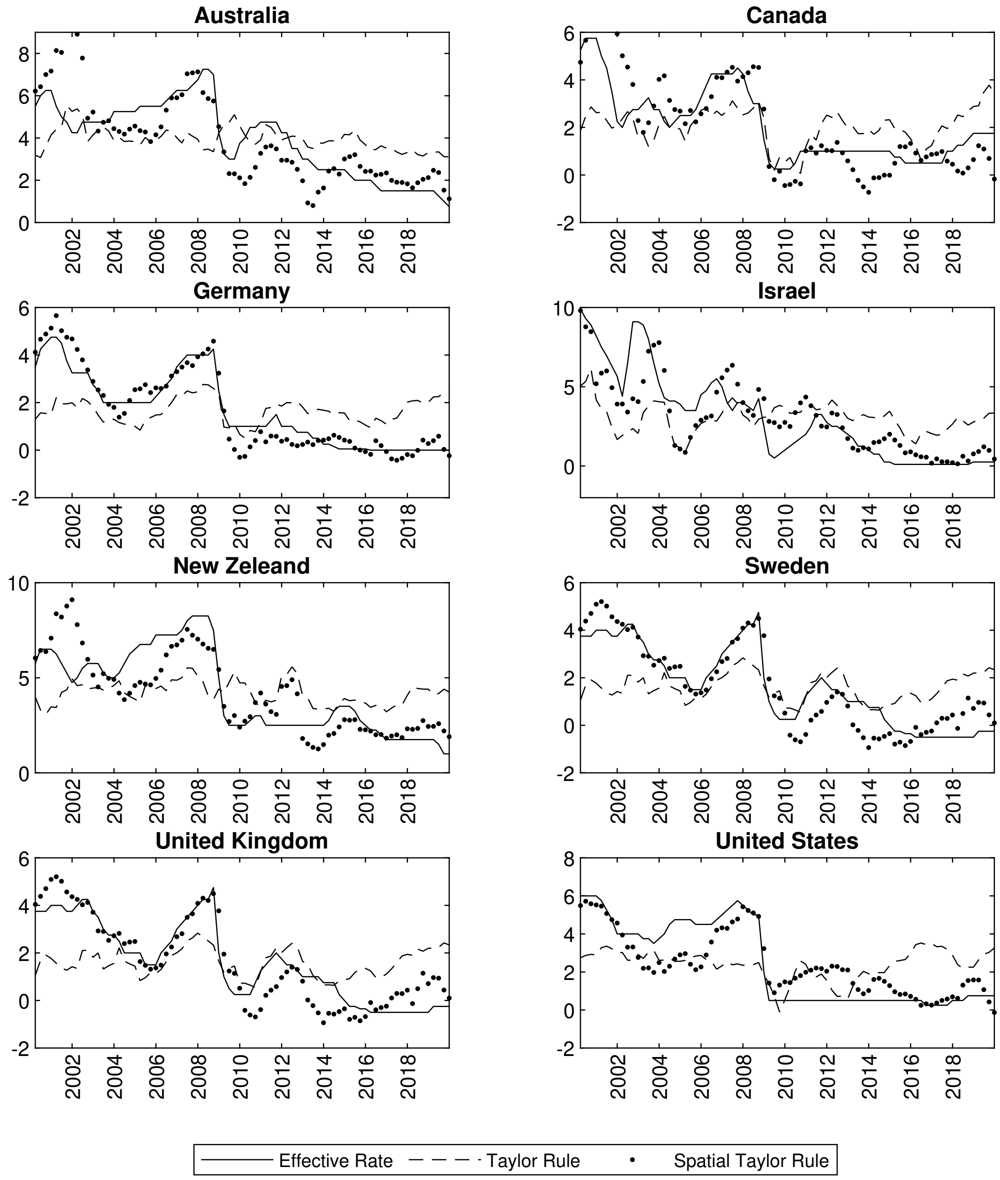
Direct Effects			
	Coefficient	t-estat	t-prob
Inflation gap	0.7882	36.93	0.00
Output gap	-0.1294	-4.05	0.00
Indirect Effects			
	Coefficient	t-estat	t-prob
Inflation gap	0.2913	4.39	0.00
Output gap	-0.0479	-2.88	0.00
Total Effects			
	Coefficient	t-estat	t-prob
Inflation gap	1.0795	14.75	0.00
Output gap	-0.1777	-3.88	0.00

rate of the first central bank, and so on. In this scenario, the new equilibrium is an interest rate higher (lower) than necessary to counter (stimulate) the economy.

These findings could be helpful in the present situation since many Central Banks have their interest rates at historically low levels. Coordination between Central Banks could lead to a higher global interest rate, giving monetary policy agents more space to act when the next economic crisis happens.

The last step shows the Taylor Rule's performance with spatial effects, vis-a-vis the traditional one. As shown in Figure 1.4.2, for most countries, considering other Central banks' monetary policy decisions could improve the performance of the Taylor Rule. In other words, we can reduce the deviations between the effective rate and the one suggested by the Taylor Rule by including some spatial dependence.

Figure 1.4.2 – Effective interest rate versus interest rate suggested by the Taylor Rule and other alternative rules



Note: Spatial Taylor Rule is the interest rate estimated using the SAR model.

1.5 Other results

1.5.1 Evaluating the spatial weight matrix

One of our main assumptions is that higher economic integration leads to a more intense monetary policy spillover. It was essential to choose a matrix that showed the commercial flows between countries in this context.

Thus, this section will evaluate this spatial matrix's choice, estimating the model with a spatial weights matrix of distances. Table 1.5.1 shows that, in this case, the results are considerably altered and, in most cases, lose their economic sense.

Table 1.5.1 – Spatial Models with Fixed Effects - (w distance)

Variable	SAR	SEM	SDM	SDEM
Gap inflation	0.8999***	0.9009***	0.9021***	0.9004***
	(0.02)	(0.02)	(0.02)	(0.02)
Output gap	0.1564***	0.1611***	0.1819***	0.1825***
	(0.03)	(0.03)	(0.03)	(0.03)
W*interest rate (ρ)	-0.3583***		-0.3735***	
	(0.05)		(0.05)	
Spatial error (λ)		-0.3949***		-0.3631***
		(0.05)		(0.05)
W*Gap inflation (θ_1)			0.1931	0.0025
			(0.22)	(2.5)
W*Output gap (θ_2)			0.6431***	0.6154**
			(0.18)	(0.19)
R^2	0.6988	0.6873	0.7006	0.6888
LL	-7136	-7132	-7129	-7129

When we consider the distances between the countries and not the economic flow between them, some results are not valid anymore. Specifically, this is the case for the spatial lag dependence in the SAR and SDM models. The coefficient ρ although significant, suggests some negative spatial correlation.

These results align with our hypothesis that the spillover effects in monetary policy could result from the higher economic integration between countries.

Considering only the distances, we do not verify some positive spatial correlation in the interest rate decisions; however, when the w from the OCDE matrix is applied, we could verify the presence of spillover effects, in which the higher inflation rate in one country also causes the increase in the interest rate of others.

1.5.2 Evaluating the spatial domain

One can assume that the presence of many Eurozone countries (16 of the 36 countries of the data belong to the Eurozone) could lead to higher cross-country monetary policy dependence, as they are all submitted to the decision of the same Central Bank. With that in mind, we estimate the same spatial panel models with spatial and time fixed effects for data considering just the 19 OECD countries plus Eurozone. The main results are present in Tables 1.5.4 and 1.5.5.

Table 1.5.2 – Spatial Models with Fixed Effects - (Eurozone plus 19 countries)

Variable	SAR	SEM	SDM	SDEM
Gap inflation	0.9403***	0.9383***	0.9354***	0.9350***
	(0.03)	(0.03)	(0.03)	(0.03)
Output gap	-0.3050***	-0.3069***	-0.3193***	-0.3158***
	(0.06)	(0.06)	(0.06)	(0.06)
W*(interest rate) (ρ)	0.1707***		0.1802***	
	(0.04)		(0.04)	
Spatial error (λ)		0.0249		0.0099
		(0.04)		(0.04)
W*Gap inflation (θ_1)			-0.3498	-0.1869
			(0.26)	(0.26)
W*Output gap (θ_2)			0.5952**	0.5554**
			(0.24)	(0.24)
R^2	0.786	0.786	0.787	0.787
LL	-4212	-4211	-4209	-4209

In general, the conclusions are remarkably similar to the ones obtained from the original data. However, some points should be noted. Again, the coefficient for the inflation gap is positive and different from zero, considering 1% of significance, while the coefficient of the output gap is negative.

Table 1.5.3 – SAR Model - Direct and Indirect Effects - (Eurozone plus 19 countries)

Direct Effects			
	Coefficient	t-estat	t-prob
Inflation gap	0.9433	30.72	0.00
Output gap	-0.3059	-4.71	0.00
Indirect Effects			
	Coefficient	t-estat	t-prob
Inflation gap	0.1928	3.66	0.00
Output gap	-0.0626	-2.78	0.01
Total Effects			
	Coefficient	t-estat	t-prob
Inflation gap	1.1361	17.32	0.00
Output gap	-0.3686	-4.54	0.00

As in the primary estimations, SAR and SDM models show that the interest rate has a spatial lag dependence, even when we control the explanatory variables, while SEM and SDEM suggest no spatial correlation in the residuals.

The direct and indirect effects of the SAR model also suggest the presence of spillover effects on monetary policy. Moreover, both the direct and indirect effects are bigger, considering the Eurozone than the desegregate data.

1.5.3 Evaluating the time-series domain

Figure 1 shows that the deviation of the actual rate from the rate suggested by the Taylor Rule became more pronounced after the 2008 financial crisis. With this in mind, we divided the database into two periods: between 2000 and 2008 and after 2008. The main results are presented in Table 1.5.4.

It is interesting to note the spatial coefficients' magnitude change between the two periods. Before 2010, the spatial coefficients of the SAR and SDM models are close to zero. However, after 2010, there was a significant increase in rho's value, from 0.09 to 0.16 (SAR) and from 0.07 to 0.14 (SDM). More than that, both spatial

Table 1.5.4 – Spatial Models with Fixed Effects - Different period

	1Q00 - 4Q08		1Q09 - 4Q19	
Variable	SAR	SDM	SAR	SDM
Gap inflation	0.7289***	0.7228***	0.3952***	0.3826***
	(0.03)	(0.03)	(0.02)	(0.02)
Output gap	-0.3614***	-0.3758***	-0.0356*	-0.0290
	(0.05)	(0.05)	(0.02)	(0.02)
W*interest rate (ρ)	0.2201***	0.2201***	0.4268***	0.3793***
	(0.06)	(0.06)	(0.04)	(0.05)
W*Gap inflation (θ_1)		-0.0338		0.3644*
		(0.42)	¨	(0.15)
W*Output gap (θ_2)		0.6382*		-0.1981
		(0.31)		(0.15)
R^2	0.831	0.831	0.804	0.806
LL	-3140	-3138	-2515	-2507

coefficients are statistically different from zero, with a significance of 1%. These results reveal that the global deviations of rates documented by several authors, such as (Svensson 2003) and (Hoffman e Bogdanova 2012) have intensified over the last few years.

Contrary to our view, Table 1.5.7 also showed a significant variation in the magnitude of the inflation gap coefficient. Although in all cases, the inflation gap's coefficient is statistically significant at the 1% level, before 2010, the weight of the difference between the current inflation and its target was more significant than one, in line with the result established by the Taylor principle. However, the post-2010 result goes against what Taylor's principle suggests. In this case, the beta of the inflation gap stays close to 0.18.

To assess the magnitude and signs of the two explanatory variables, the summary measures of direct, indirect, and total impacts are presented in Table 1.5.5.

Table 1.5.5 – SAR Model - Direct and Indirect Effects - (Different period)

Direct Effects	1Q00 - 4Q08			1Q09 - 4Q19		
	Coefficient	t-estat	t-prob	Coefficient	t-estat	t-prob
Inflation gap	0.7301	23.95	0.00	0.4002	19.90	0.00
Output gap	-0.3628	-6.65	0.00	-0.0359	-1.80	0.08
Indirect Effects	1Q00 - 4Q08			1Q09 - 4Q19		
	Coefficient	t-estat	t-prob	Coefficient	t-estat	t-prob
Inflation gap	0.2222	2.82	0.00	0.2954	4.66	0.00
Output gap	-0.1107	-2.55	0.01	-0.0260	-1.68	0.10
Total Effects	1Q00 - 4Q08			1Q09 - 4Q19		
	Coefficient	t-estat	t-prob	Coefficient	t-estat	t-prob
Inflation gap	0.9523	10.74	0.00	0.6957	9.73	0.00
Output gap	-0.4736	-5.68	0.08	-0.0623	-1.78	0.08

Although the indirect effect of the inflation gap is lower after 2010 than in the first ten years of the data, proportionally, they still suggest a higher spillover effect in recent years. The indirect effect represents 9.9% of the total effect between 1999 and 2009, but this rate raises to 14% for the period after the first quarter of 2010.

1.6 Conclusion

The increase in global integration has increased the spillover effects between the economies. These spillover effects could also be found in the monetary policy decisions and could be one of the causes for the recent deviations of the actual interest rate from the one suggested by the Taylor Rule.

The methodology of spatial econometric was adopted to include these spillover effects in the Taylor Rule. With a spatial weight matrix that considers the economic flows between countries, the first assumption is that economies with high integration (considering 34 sectors) have bigger spillover effects of monetary policy.

Testing for spatial dependence, Morans' I statistics, and LM test indicate the existence of spatial dependence in the data. Moreover, comparing the different specifications of spatial dependence, we choose models that include a spatially lagged dependent variable.

The results show that the spatial coefficient ρ is positive and significant, indicating the presence of spatial dependence in the data. Moreover, the total effect of the inflation gap is statistically different from zero and bigger than one, corroborating the previous research. The coefficient of the output gap became statistically insignificant.

To conclude, a comparison was made between the traditional Taylor Rule and the one with spatial dependence, and the last one showed a modest better performance.

2 International spillovers of monetary policy

With Marcio Nakane. [2.0.1]

2.1 Introduction

In light of the higher global financial integration, the role of external factors on domestic activity has been a subject of intense debate. Particularly, some studies have found that central banks react to foreign interest rate movements, besides reacting to inflation deviations and the domestic output gap.

Based on (Clarida et al. 2005), and (Schmitt-Grohe e Uribe 2007), this paper aims to extend the New-Keynesian model for a small open economy to take into account the spillover effect of monetary policy. The main idea is to link the domestic natural interest rate to the foreign interest rate. Thus, external shocks in the foreign monetary policy will directly impact the domestic nominal interest rate.

The remainder of the paper is organized into six sections. Section 2 presents an overview of the existing literature. Section 3 provides an empirical analysis with an estimation of a Taylor Rule for a panel of 28 countries that adopted the inflation target regime. Section 4 presents the linear model. Section 5 provides the parameter estimations and the model solution, and Section 6 provides concluding remarks.

2.2 Bibliography Revision

In a world with a growing financial integration, the importance of external factors on domestic activity has also been increasing. Remarkably, after the 2008 financial crisis and the unprecedented movements in policy rates, a wide range

[2.0.1] Natalia received financial support from *Coordenação de Aperfeiçoamento de Pessoal de Nível Superior - Brasil* (CAPES - Finance Code 001).

of studies have highlighted the role that foreign interest rates could have in the domestic interest rate decision.

The New Keynesian models have become the workhorse for analyzing monetary policy. This framework is described by optimizing private sector behavior in the presence of nominal rigidities, as the staggered price setting proposed by (Calvo 1983) and (Yun 1996). With sticky prices, monetary policy is non-neutral in the short term. That is to say, the equilibrium path of real variables cannot be determined independently of how the nominal interest rate evolves. Moreover, nominal rigidities make room for welfare improvement interventions by the monetary authority.

In a seminal paper, (Taylor 1993) found that policies that focus on exchange rate or money supply do not deliver good performance as policies that directly focus on the domestic price level and the output gap. However, more recently, many papers have analyzed the impact of external variables in the central bank's decisions.

(Clarida et al. 2001) highlighted that the external factors are relevant in the monetary analysis to the degree they affect domestic inflation or the real equilibrium rate. Moreover, how aggressively the central bank should adjust its nominal rate in response to a deviation of inflation depends on the degree of openness.

(Corsetti e Pesenti 2001) emphasized the lack of consensus in the literature about external factors' role in the domestic monetary policy. In this spirit, they accounted for the possibility of deviations from the perfect exchange rate pass-through to export prices. According to the paper, the degree of pass-through and exchange rate exposure are crucial elements determining the impact of external conditions in the optimal monetary policy. Along the same line, (Clarida et al. 2005) showed that incomplete pass-through alters the canonical New Keynesian optimal model.

Some studies searched for empirical evidence that central banks in small open economies react to external factors. (Adolfson 2001), analyzing developed countries with inflation target regimes, found that central banks from Canada and England respond to exchange rate movements. Nevertheless, (Mohanty e Klau 2004) and (Aizenman et al. 2011) found that the real exchange rate and inflation are important determinants of policy interest rates for most emerging economies.

More recently, some studies have considered the rate of foreign monetary policy as a determinant of the domestic interest rate. (Hofmann e Takats 2015) highlighted that interest rates had moved nearly together internationally despite the business cycle, often at different stages across countries. Analyzing the post-2000 period, the authors found that US interest rates affect emerging markets and developed economies' rates. In the same vein, (Caputo e Herrera 2017) showed that inflation-targeting central banks, besides reacting to inflation and output gap, also respond to moves in the US interest rate.

2.3 Empirical evidence

2.3.1 Model

To assess the importance of foreign interest rate for the conduct of monetary policy in small open economies, this study follows (Caputo e Herrera 2017) and estimates the interest rate rule below:

$$i_{i,t} = \mu_i + \rho i_{i,t-1} + \phi_\pi E_t(\pi_{i,t-j} - \bar{\pi}_{i,t-j}) + \phi_y(y_{i.t} - \bar{y}_{i,t}) + \phi_i i_t^* + \delta X_{i,t} + v_{i,t} \quad (2.3.1)$$

where $i_{i,t}$ is the nominal interest rate of country i at time t, $(\pi_{t+1} - \bar{\pi}_{i,t+1})$ denotes the inflation deviation from the target, $(y_{i.t} - \bar{y}_{i,t})$ output gap, i_t^* is the foreign interest rate and $X_{i,t}$ is a vector that contains additional endogenous or exogenous variables.

This empirical part differs from the (Caputo e Herrera 2017) study in two main points. First, it will view the spillover effect of two foreign interest rates; second, it will contemplate an extended period and a broader set of countries.

2.3.2 Data

The empirical analysis considers 28 countries (12 advanced economies and 16 emerging markets and developing economies)[2.3.1]. The data starts in the first quarter of 1999[2.3.2] or in the quarter in which the respective central bank explicit adopted the inflation target regime[2.3.3] and ended in the last quarter of 2018[2.3.4].

In Equation (2.3.1), the interest rate ($i_{i,t}$) is defined by each central bank, the inflation rate consists in the year-over-year variation of the price index two quarters ahead, the inflation target is the one explicit announced by each central bank[2.3.5], the output gap is the deviation of output from the potential[2.3.6], and the two foreign interest rates are nominal Fed Funds rate (Fed_t) and the ECB interest rate on the main refinancing operations (ECB_t). Also, some other variables are considered: exchange rate (e_t), US output gap (y_t^{US}), change in the WTI oil price (WTI_t) and the G20 inflation rate (π_t^{G20}).

Before estimating the Taylor Rule is essential to verify whether the variables are stationary. Applying the ADF and Phillips-Perron unit root tests[2.3.7] in all variables, the null hypothesis that all panels contain a unit root can not be rejected for the Fed and ECB interest rate. Because of that, both variables are used in the first difference.

Equation (2.3.1) includes a lagged dependent variable as one of the regressors. In this case, the most common approach is to apply the Least Square Dummy Variable (LSDV) estimator. However, it is well known that this technique could

[2.3.1] Following the International Monetary Fund's country group division.

[2.3.2] This paper uses data after 1999 because the European Central Bank (ECB) interest rate will be used as one of the external interest rates.

[2.3.3] Countries that adopted the inflation target after the first quarter of 2009 were not included in the data to no generate bias estimation problems.

[2.3.4] (Caputo e Herrera 2017) studied the data until 2010 arguing that the monetary policy was very accommodative after this period. Thus, we will use a dummy variable for the period between 2010 and 2016. Moreover, after 2015, several central banks started raising the policy rate as the economy improved. Thus, movements in the policy rate could be again tracked by the Taylor Rule.

[2.3.5] Could be time-varying.

[2.3.6] Estimated by the Hodrick Prescott (HP) filter.

[2.3.7] The Fisher-type tests allow for an unbalanced panel, and it is the most appropriate when data has a finite number of panels and a large number of time-period.

generate a biased estimator when the panel's time dimension is small. Only countries that adopted the inflation target regime before the first quarter of 2009 will be considered to deal with this problem. (Judson e Owen 1999) showed that the LSDV estimator has a smaller bias than the Anderson-Hsiao and Arellano-Bond estimators when T>30.

Table (2.3.1) presents the main results for a forward-looking interest rate rule. The use of expected inflation as a regressor causes another problem; there may be a correlation between the error term and the expected inflation deviation, generating a bias and inconsistent estimators. Because of that, this paper follows (Clarida et al. 1999) and (Clarida et al. 2000) and estimates the LSDV applying a GMM-IV approach. As instruments for the inflation expected deviation, the contemporaneous and lagged values of the output gap, foreign interest rate, G20 inflation, WTI oil price, exchange rate, and the lagged value of the policy rate were considered.[2.3.8]

Considering all 28 countries, Columns (1) and (2) show a positive and significant coefficient for the Fed Funds rate and ECB interest rate, even after controlling for other external variables, such as exchange rate, US output gap, G20 inflation, and WTI oil price. These results align with (Clarida et al. 1999) and (Caputo e Herrera 2017). The first authors concluded that external rates were significant in determining the policy rate of some central banks in Europe. Meanwhile, (Caputo e Herrera 2017) showed that central banks react to Fed funds rate movements. To our knowledge, this is one of the first studies that found that the ECB rate also plays an important role in the policy rate of other countries.

Considering the lagged interest rate coefficient, Table (2.3.1) shows a high degree o policy inertia (approximately 0.89 for all countries). This coefficient is higher than the previous found in the empirical literature. Since this study also uses the most recent data, this result confirms the conventional wisdom that central banks became more cautious in moving the interest rate after the 2008 financial crisis, maintaining the rates more accommodating.

[2.3.8] The instruments were chosen based on three tests: under-identification test, weak identification test, and Hansen J statistics.

Table 2.3.1 – GMM estimation

	All countries		Developed		EM	
	(1)	(2)	(3)	(4)	(5)	(6)
i_{t-1}	0.891***	0.892***	0.901***	0.918***	0.891***	0.890***
	(0.013)	(0.013)	(0.030)	(0.030)	(0.014)	(0.014)
$E_t(\pi_{i,t-j} - \bar{\pi}_{i,t-j})$	0.203***	0.196***	0.100***	0.084**	0.237***	0.234***
	(0.028)	(0.028)	(0.033)	(0.034)	(0.036)	(0.035)
$(y_{i.t} - \bar{y}_{i,t})$	0.109	0.079	0.104	0.074	-0.045	-0.107
	(0.092)	(0.092)	(0.072)	(0.072)	(0.460)	(0.460)
Fed_t	0.194***		0.192***		0.150***	
	(0.055)		(0.068)		(0.011)	
ECB_t		0.296***		0.532***		0.110
		(0.090)		(0.068)		(0.108)
e_t	-0.001	0.001	-0.009**	-0.005	0.006	0.007
	(0.005)	(0.005)	(0.005)	(0.004)	(0.008)	(0.008)
y_t^{US}	0.065***	0.053***	0.067***	0.034***	0.063***	0.063***
	(0.013)	(0.013)	(0.018)	(0.010)	(0.020)	(0.021)
π_t^{G20}	0.125***	0.096***	0.097**	0.062	0.156***	0.136***
	(0.029)	(0.029)	(0.049)	(0.050)	(0.043)	(0.044)
WTI_t	-0.003**	-0.003***	0.001	0.001	-0.006***	-0.005***
	(0.001)	(0.001)	(0.001)	(0.001)	(0.002)	(0.002)
n^o obs	1914	1914	910	910	1004	1004
R^2	0.9385	0.9391	0.9472	0.9502	0.9360	0.9361
Root MSE	0.8018	0.7978	0.5126	0.4977	0.994	0.9932
Countries	28	28	12	12	16	16

Notes: HAC standard errors in brackets.
Notes:*Significant at 10%. **Significant at 5%. ***Significant at 1%

Considering the domestic variables, while the response to inflation deviations is positive and statistically significant, the output gap's coefficient is not statistically different from zero. Some empirical studies also found that central banks pay less attention to the output gap.

It is also relevant to analyze how these results differ across the two groups of countries. Both emerging and developed economies showed a significant and positive policy response to the Fed Funds rate. However, only for the last group, the coefficient for the ECB interest rate is also statistically different from zero. The policy response to the ECB interest rate is considerably more prominent for the developed countries than the Fed funds rate.

The behavior of the other foreign variables is also different across the two groups. Beyond the Fed funds rate, the US output gap, the G20 inflation rate, and the changes in the WTI oil price are also relevant to determine the policy rate in emerging and developing countries. The variation in the WTI oil price found a negative response for emerging countries, suggesting that a rise in the oil price could be viewed as a negative supply shock.

The US output gap has a positive and significant effect on both groups. This result suggests synchronicity between the business cycle in the US and all other countries. Finally, unlike the results for emerging markets, the policy response to exchange rate movements is negative and statistically significant for the developing countries.

Table 2.3.2 calculates the long-run coefficients[2.3.9]. Even controlling for external variables, the long-run response to inflation deviations from the target is above 1.0, in line with the Taylor Principle. Interestingly, for developed countries, this coefficient is only slightly higher than 1.0, while for emerging economies, the response to inflation deviations seems to be more intense.

Table 2.3.2 – Long Run Coefficients

	All countries		Developed		EM	
	(1)	(2)	(3)	(4)	(5)	(6)
$(\pi_{i,t-j} - \bar{\pi}_{i,t-j})$	1.862	1.814	1.010	1.012	2.174	2.127
$y_{i,t} - \bar{y}_{i,t}$	1.000	0.731	1.050	0.891	-0.412	-0.973
Fed_t	1.780		1.940		1.376	
ECB_t		2.740		6.409		1.000

To conclude, the long-run policy response to the foreign interest rate is also larger than one for all three groups, suggesting some spillover effects in the monetary policy decisions of different countries.

We will develop a model that considers the foreign interest rate in policy decisions, considering this empirical evidence. As important as verifying the presence

[2.3.9] The long-run coefficients are computed by dividing the estimates by $\frac{1}{1-\rho}$

of international monetary policy spillover is understanding how this effect influences the domestic economy.

2.4 Model

This paper considers a small open economy model with nominal rigidity, based on (Clarida et al. 2005) and (Schmitt-Grohe e Uribe 2007). The domestic economy is small in the sense that it does not influence foreign output, the foreign price level, or the foreign interest rate. The nominal rigidity is in the form of a staggered price. Following (Calvo 1983), firms producing intermediate goods maintain their price constant unless they receive a signal to revise them, which arrives at the beginning of each period with a constant probability.

We extended the previous models to include the foreign interest rate in the central banks' reaction function, through the real natural interest rate, to evaluate the welfare consequence of the spillover effects of monetary policy verified in the empirical part of this study.

2.4.1 Households

The preferences are defined over streams of consumption and labor. Thus, the lifetime utility function of the representative household can be defined as:

$$E_0 \sum_{t=0}^{\infty} \beta^t [U(c_t) - V(h_t)] \tag{2.4.1}$$

where β represents the subjective discount factor, the utility function U is assumed to be strictly increasing and strictly concave. The function V is strictly increasing and strictly convex, h_t is the hours worked, and c_t is a composite consumption index defined by:

$$c_t = (1-\alpha)c_{H,t} + \alpha c_{F,t}$$

Where $\alpha \ \epsilon \ [0,1]$ can be defined as the measure of openness, and $c_{H,t}$ and $c_{F,t}$ are the consumption of domestic and foreign goods, respectively.

Domestic and foreign consumption goods are produced with a continuum of differentiated goods $j \in [0,1]$:

$$c_{H,t} = \left(\int_0^1 c_{H,t}^{\frac{\epsilon-1}{\epsilon}}(j)dj\right)^{\frac{\epsilon}{\epsilon-1}}$$

$$c_{F,t} = \left(\int_0^1 c_{F,t}^{\frac{\epsilon-1}{\epsilon}}(j)dj\right)^{\frac{\epsilon}{\epsilon-1}}$$

Where $\epsilon > 1$ represents the elasticity of substitution between varieties of products within any country.

Thus for a given expenditure level, the optimal allocation for each category is given by:

$$c_{H,t}(j) = (\frac{P_{H,t}(j)}{P_{H,t}})^{-\epsilon} c_{H,t} \tag{2.4.2}$$

$$c_{F,t}(j) = (\frac{P_{F,t}(j)}{P_{F,t}})^{-\epsilon} c_{F,t} \tag{2.4.3}$$

Finally, the optimal allocation between foreign and domestic goods:

$$c_{H,t} = (1-\alpha)(\frac{P_{H,t}}{P_t})c_t \tag{2.4.4}$$

$$c_{F,t} = \alpha(\frac{P_{F,t}}{P_t})c_t \tag{2.4.5}$$

Where the Consumption Price Index (CPI) is defined by: $P_t = P_{H,t}^{1-\alpha} P_{F,t}^{\alpha}$. Considering the effective terms of trade $s_t = \frac{P_{F,t}}{P_{H,t}}$, the CPI inflation can be described as:

$$\pi_t = \pi_{H,t}(\frac{s_t}{s_{t-1}})^{\alpha} \tag{2.4.6}$$

Households choose contingent plans c_t, h_t, m_t (real demand for money) d_{t+1}^* (foreign debt) and d_{t+1} (domestic debt) to maximize the lifetime utility function (2.4.1) subject to the sequential budget constrains (2.4.7) and the cash in advance constraint (2.4.8), taken as given P_t, the lump sum taxes τ_t^L, the income tax rate τ_t^D, the real wage w_t, the real profits ϕ_t, the exchange rate ε_t and the stochastic discount factors $q_{t,t+1}$ and $q_{t,t+1}^*$:

$$\begin{aligned} c_t + m_t + \tau_t^L + E_t q_{t,t+1}\frac{d_{t+1}}{P_t} + E_t q_{t,t+1}^* \frac{\varepsilon_t d_{t+1}^*}{P_t} = \\ \frac{d_t}{P_t} + (1-\tau_t^D) w_t h_t + \frac{P_{t-1}}{P_t} m_t + \frac{\varepsilon_t d_t^*}{P_t} + \phi_t \end{aligned} \tag{2.4.7}$$

$$m_t \geq \nu^h c_t \tag{2.4.8}$$

Considering $\beta^t \lambda_t$ and $\beta^t \lambda_t \xi_t$ the Lagrange multiplier of equations (2.4.7) and (2.4.8), respectively, the first order conditions associated with the households problem is given by:

$$U'_{ct} = \lambda_t(1 + \xi_t \nu) \tag{2.4.9}$$

$$U'_{ht} = -\lambda_t(1 - \tau_t^D)w_t \tag{2.4.10}$$

$$\lambda_t(1 - \xi_t) = \beta E_t \lambda_{t+1} \frac{P_t}{P_{t+1}} \tag{2.4.11}$$

$$\lambda_t E_t q_{t,t+1} = \beta E_t \lambda_{t+1} \frac{P_t}{P_{t+1}} \tag{2.4.12}$$

$$\lambda_t E_t q^*_{t,t+1} = \beta E_t \lambda_{t+1} \frac{P_t}{P_{t+1}} \frac{\epsilon_{t+1}}{\epsilon_t} \tag{2.4.13}$$

By combining the first-order conditions for consumption and hours of work, respectively, we obtain:

$$\frac{U'_{ht}}{U'_{ct}} = \frac{(1 - \tau_t^D)w_t}{1 + \xi_t \nu} \tag{2.4.14}$$

After log-linearization, we obtain the following condition:

$$\begin{aligned} U'_{ht} - U'_{ct} = w_t - \frac{\xi\nu}{1 + \xi\nu}\xi_t - \frac{\tau^D}{1 - \tau^D}\tau_t^D = \\ w_t - \phi_\xi \xi_t - \phi_\tau \tau_t^D \end{aligned} \tag{2.4.15}$$

From equations (2.4.11) and (2.4.12), we have that $E_t q_{t,t+1} = (1 - \xi_t)$. Finally, combining the first order conditions (2.4.9) and (2.4.11), we have:

$$U'_{ct} E_t q_{t,t+1} = \beta E_t U'_{ct+1} \frac{1}{\pi_{t+1}} \tag{2.4.16}$$

From the first order conditions, it is possible to verify that the income tax rate distorts the leisure-labor choice, while the money constraint distorts both the leisure-labor choice and the inter-temporal allocation of consumption.

2.4.2 Government

In the domestic economy the government prints money M_t, issues risk free bonds D_t, collects taxes $P_{H,t}\tau_t$ and faces exogenous expenditure g_t. Thus, the government's period-by-period budget constraint is given by:

$$M_t + D_t = i_{t-1}D_{t-1} + M_{t-1} + P_{H,t}g_t - P_{H,t}\tau_{H,t} \tag{2.4.17}$$

where i_t denotes the nominal interest rate. By a non-arbitrage condition, i_t is equal to the inverse of to the price at time t of a portfolio that pays one dollar in t+1 ($i_t = \frac{1}{E_t q_{t,t+1}}$). Combining this expression with the optimal condition, we have that the opportunity cost of holding money equals the gross nominal rate $i_t = \frac{1}{1-\xi_t}$.

We made two important assumptions for the government expenditure g_t: first, g_t denotes per capita government spending only on domestic goods; second, the government minimizes the cost of producing g_t. Thus, the public demand is given by $g_t(j) = (\frac{P_{H,t}(j)}{P_{H,t}})^{-\epsilon} g_t$.

As we define the real liabilities as $l_t = \frac{M_t + i_t D_t}{P_{H,t}}$ and $m_t = \frac{M_t}{P_{H,t}}$, equation (17) can be written as:

$$l_t = \frac{i_t}{\pi_{H,t}} l_{t-1} + i_t(g_t - \tau_t) - m_t(i_t - 1) \tag{2.4.18}$$

To analyze different fiscal policy specifications, we have both lump sum τ_t^L and distortionary income taxation τ_t^D. Thus, total taxes revenues is equal to:

$$\tau_t = \tau_t^L + \tau_t^D yt \tag{2.4.19}$$

Finally, the fiscal regime is defined by the following rule:

$$\tau_t - \tau^* = \gamma_1(l_{t-1} - l^*) \quad (2.4.20)$$

Where γ_1 is a fiscal parameter, and τ^* and l^* are the steady-state values of total taxes revenues and government liabilities, respectively. Equation (2.4.20) displays a simple fiscal rule, where the government chooses its taxes as a linear function of the real value of its liabilities.

For future reference, the log linearized form of equations (2.4.18), (2.4.19), and (2.4.20) around the zero-inflation steady state is given by:

$$l_t = I(i_t + l_{t-1} - \pi_{H,t} + + \frac{(g - \tau)}{l} i_t + \frac{g}{l} g_t + \frac{\tau}{l} \tau_t - \frac{m}{l}(m_t + i_t)) \quad (2.4.21)$$

$$\tau_t = \phi_{\tau^L} \tau_t^L + \phi_{\tau^D}(\tau_t^D + y_t) \quad (2.4.22)$$

$$\tau_t = \gamma_1(l_t) \quad (2.4.23)$$

2.4.3 Firms

A typical firm in the domestic economy produces a differentiated good with a linear technology represented by the production function:

$$Y_t(j) = Z_t H_t(j) \quad (2.4.24)$$

Where $j \in [0, 1]$ is a firm-specific index. Z_t represents the level of technology and $z_t = log(Z_t)$ follows the AR(1) process:

$$log(z_t) = \rho_z log(z_{t-1}) + \varepsilon_t^z \quad (2.4.25)$$

It is assumed that firms set prices in a staggered fashion, following (Calvo 1983).

Thus, with an exogenous probability $\theta \in (0,1)$, a firm must keep its price unchanged, and with probability $(1-\theta)$ firm resets its prices independent of the time elapsed since the last adjustment.

As described by (Yun 1996), the firm picks $P^*_{H,t}(j)$ that maximize the following present value of profits:

$$\sum_{k=0}^{\infty} \theta^k E_t[Q^f_{t,t+k}(P^*_{H,t}(j)Y_{t+k/t}(j) - \Psi_{t+k}(Y_{t+k/t}(j))] \tag{2.4.26}$$

Where $Q^f_{t,t+k} = \beta^k \frac{\lambda_{t+k}}{\lambda_t}\frac{P_{H,t}}{P_{H,t+k}}$ is the discount stochastic factor for nominal payoffs[2.4.1], $\Psi(Y_{t+k/t})$ is the cost function and $Y_{t+k/t}$ the output in t+k for a firm that last rest its price in t.

Subject to the sequences of demand constraint:

$$Y_{t+k/t}(j) = [\frac{P^*_{H,t}(j)}{P_{H,t+k}}]^{-\varepsilon} Y_{t+k} \tag{2.4.27}$$

Thus, the first-order condition associated with this problem is described by:

$$\sum_{k=0}^{\infty} \theta^k E_t[Q^f_{t,t+k}(Y_{t+k/t}(j)\frac{P_{H,t+k}}{P^*_{H,t}(j)}(\frac{P^*_{H,t}(j)}{P_{H,t+k}} - \frac{\varepsilon}{\varepsilon-1}\Psi'_{t+k/t}))] \tag{2.4.28}$$

Note that when $\theta = 0$, the price that optimizes equation (2.4.28) is the same as under flexible prices ($P^*_{H,t} = \frac{\varepsilon}{1-\varepsilon}\Psi'_{t+k/t}$). Thus, $\frac{\varepsilon}{1-\varepsilon}$ can be interpret as the desired mark up in the absence of constraints on the frequency of price adjustments.

Defining the real marginal cost as $MC_{t+k/t} = \frac{\Psi'_{t+k/t}}{P_{H,t+k}}$, equation (2.4.28) can be described as:

$$\sum_{k=0}^{\infty} \theta^k E_t[Q^f_{t,t+k}(Y_{t+k/t}(\frac{P^*_{H,t}(j)}{P_{H,t+k}})^{-\varepsilon-1}(mc_{t+k/t} - \frac{\varepsilon-1}{\varepsilon}\frac{P^*_{H,t}}{P_{H,t+k}}))] \tag{2.4.29}$$

[2.4.1] $Q^f_{t,t+1} \neq Q_{t,t+1}$, as the first depends on the domestic price $P_{H,t}$ and the latter the overall prices P_t

For simplicity, this result can be divided as:

$$x_{1,t} = E_t \sum_{k=0}^{\infty} Q^f_{t,t+k} \theta^k (\frac{P^*_{H,t}}{P_{H,t+k}})^{-\varepsilon-1} Y_{t+k} mc_{t+k}$$
$$= (\frac{P^*_{H,t}}{P_{H,t}})^{-1-\varepsilon} Y_t mc_t + \theta\beta E_t \frac{\lambda_{t+1}}{\lambda_t} \frac{P_{H,t}}{P_{H,t+1}} (\frac{P^*_{H,t}}{P^*_{H,t+1}})^{-1-\varepsilon} x_{1,t+1} \tag{2.4.30}$$
$$= \tilde{P_{H,t}}^{-\varepsilon-1} Y_t mc_t + \theta\beta E_t \frac{\lambda_{t+1}}{\lambda_t} (\frac{\tilde{P_{H,t}}}{\tilde{P_{H,t+1}}})^{-1-\varepsilon} \pi^{\varepsilon}_{H,t+1} x_{1,t+1}$$

$$x_{2,t} = E_t \sum_{k=0}^{\infty} Q^f_{t,t+k} \theta^k (\frac{P^*_{H,t}}{P_{H,t+k}})^{-\varepsilon} Y_{t+k}$$
$$= (\frac{P^*_{H,t}}{P_{H,t}})^{-\varepsilon} Y_t + \theta\beta E_t \frac{\lambda_{t+1}}{\lambda_t} \frac{P_{H,t}}{P_{H,t+1}} (\frac{P^*_{H,t}}{P^*_{H,t+1}})^{-\varepsilon} x_{2,t+1} \tag{2.4.31}$$
$$= \tilde{P_{H,t}}^{-\varepsilon} Y_t + \theta\beta E_t \frac{\lambda_{t+1}}{\lambda_t} (\frac{\tilde{P_{H,t}}}{\tilde{P_{H,t+1}}})^{\varepsilon} (\pi_{H,t+1})^{1-\varepsilon} x_{2,t+1}$$

Where $\tilde{P}_{H,t} = \frac{P^*_{H,t}}{P_{H,t}}$. Log-linearizing aroud the zero-inflation steady-state yields the following expressions:

$$x_{1,t} = (1-\theta\beta)[(-1-\varepsilon)\tilde{p_{H,t}} + y_t + mc_t]$$
$$+\theta\beta[\Delta\lambda_{t+1} - (-1-\varepsilon)\Delta\tilde{p}_{H,t+1} + \varepsilon\pi_{H,t+1} + x_{1,t+1}] \tag{2.4.32}$$

$$x_{2,t} = (1-\theta\beta)[-\varepsilon\tilde{p_{H,t}} + y_t] + \theta\beta[\Delta\lambda_{t+1} + \varepsilon\Delta\tilde{p_{H,t+1}} + (1-\varepsilon)\hat{\pi}_{H,t+1} + x_{2,t+1}] \tag{2.4.33}$$

Defining $S_t \subset [0,1]$ as the set of firms that is not re-optimizing in period t and using the fact that all firms resetting prices will choose identical prices $P^*_{H,t}$, we have:

$$P_t = [\int_{s(t)} P_{H,t-1}(j)^{1-\varepsilon} dj + (1-\theta) {P^*_{H,t}}^{1-\varepsilon}]^{\frac{1}{1-\varepsilon}} =$$
$$[\theta P^{1-\varepsilon}_{H,t-1} + (1-\theta) {P^*_{H,t}}^{1-\varepsilon}]^{\frac{1}{1-\varepsilon}} \tag{2.4.34}$$

That also can be described as:

$$1 = \theta\pi_{H,t}^{\varepsilon-1} + (1-\theta)[\frac{P_{H,t}^{*}}{P_{H,t}}]^{1-\varepsilon} \tag{2.4.35}$$

Combining the log-linearization of equation (2.4.35) around the zero-inflation steady-state $(\frac{\theta}{1-\theta})\pi_{H,t} = p_{H,t}^{*} - p_{H,t}$, with $x_1 = x_2$, yields the popular New Keynesian Phillips Curve (NKPC):

$$\pi_{H,t} = \beta\pi_{H,t+1} + \frac{(1-\theta)(1-\theta\beta)}{\theta}]\hat{mc}_t \tag{2.4.36}$$

The relation (2.4.36) determining the domestic inflation as a function of the deviations of marginal cost from its steady-state value does not depend on any of the parameters that characterize the open economy.

2.4.4 Equilibrium

In equilibrium, the domestic output is equal to the domestic and foreign consumption of domestic goods and the government expenditure. Thus, we can define the domestic resource constraint as:

$$\begin{gathered} Y_t(j) = c_{H,t}(j) + c_{H,t}^{*}(j) + g_t(j) = \\ (\frac{P_{H,t(j)}}{P_{H,t}})^{-\varepsilon}[(1-\alpha)(\frac{P_{H,t}}{P_t})^{-1}c_t + \alpha(\frac{P_{H,t}}{\epsilon_{*,t}P_{F,t}^{*}})^{-1}(\frac{P_{F,t}^{*}}{P_t})^{-1}c_t^{*} + g_t] \end{gathered} \tag{2.4.37}$$

For all $j \in (0,1)$. Combining equation (2.4.37) with the definition of aggregate domestic output $Y_t = [\int_0^1]Y_t(j)^{1-\frac{1}{\epsilon}}dj]^{\frac{\epsilon}{\epsilon-1}}$ yields:

$$Y_t = (1-\alpha)(\frac{P_{H,t}}{P_t})^{-1}c_t + \alpha(\frac{P_{H,t}}{\epsilon_{*,t}P_{F,t}^{*}})^{-1}(\frac{P_{F,t}^{*}}{P_t})^{-1}c_t^{*} + g_t \tag{2.4.38}$$

Defining the bilateral real exchange rate $Q_t = \frac{\varepsilon_{*,t}P^*_{F,t}}{P_t}$ and $P_{F,t} = \varepsilon_{*,t}P^*_{F.t}$, equation (39) can be written as:

$$Y_t = (\frac{P_{H,t}}{P_t})^{-1}[c_t(1-\alpha) + \alpha Q_t c^*_t] + g_t \tag{2.4.39}$$

Finally, using the fact that $S_t^{-\alpha} = \frac{P_{H,t}}{P_t}$[2.4.2] and $c^*_t = y^*_t$, we have the following relation between domestic and foreign output:

$$Y_t - g_t = S_t^{\alpha}[c_t(1-\alpha) + \alpha Q_t y^*_t] \tag{2.4.40}$$

Considering that under specif parameters defined in this model, the relation between the bilateral exchange rate and the bilateral terms of trade can be determined by $Q_t = S_t^{1-\alpha}$, log-linearizing equation (2.4.40) yields:

$$yg_t = c_t + \alpha y^*_t + \alpha(1-\alpha)s_t \tag{2.4.41}$$

Where $yg_t = log(\frac{y_t - g_t}{y-g})$.

A similar condition will hold for the foreign country. Thus, under the assumption of complete markets, for a household in a foreign country, we have:

$$\frac{V_{t,t+1}}{\epsilon^*_t P^*_t}(c^*_t)^{-\sigma} = \frac{1}{Q_{t,t+1}}\beta(c^*_{t+1})^{-\sigma}\frac{1}{\epsilon^*_{t+1}P^*_{t+1}}\frac{1+\xi_t\nu}{1+\xi_{t+1}\nu} \tag{2.4.42}$$

Thus, the relation between the consumption of a household in the domestic economy and the consumption of a household in any country is given by:

$$c_t = c^*_t Q_t^{\frac{1}{\sigma}} \tag{2.4.43}$$

[2.4.2] We can use this equality because the elasticity between domestic and foreign goods is equal to one. See the definition of the composite consumption index c_t.

That also can be written as $c_t = c_t^* + (\frac{1-\alpha}{\sigma})s_t$. Log linearizing the equation above and combing with (41), yields:

$$yg_t = y_t^* + \frac{1+\alpha(\omega-1)}{\sigma}s_t \tag{2.4.44}$$

Where $\omega = \sigma + (1-\alpha)(\sigma-1)$.

2.4.5 Functional Form

We assume that the period utility function is given by:

$$U(c_t, h_t) = \frac{c_t^{1-\sigma}-1}{1-\sigma} + \varphi log(\bar{h} - h_t)$$

Thus, using equation (2.4.16), we can derive the usual Euler equation:

$$\frac{c_t^{-\sigma}}{1+\xi_t \nu^h} = \beta E_t[i_t \frac{c_{t+1}^{-\sigma}}{1+\xi_{t+1}\nu^h}\frac{1}{\pi_{t+1}}] \tag{2.4.45}$$

The log-linearized form is given by:

$$c_t = E_t c_{t+1} - \frac{1}{\sigma}(i_t - E_t\pi_{t+1} + log(\beta) - \phi_\xi \Delta\xi_{t+1}) \tag{2.4.46}$$

Combining the Euler equation with equation (2.4.41) yields:

$$yg_t = yg_{t+1} - \frac{1}{\sigma}[i_t - E_t\pi_{t+1} - \rho - \phi_{xi}E_t\Delta\xi_{t+1}] - \frac{\alpha\omega}{\sigma}E_t\Delta s_{t+1} \tag{2.4.47}$$

Considering the relation between π_t and $\pi_{H,t}$:

$$yg_t = E_t yg_{t+1} - \frac{1+\alpha(\omega-1)}{\sigma}[i_t - E_t\pi_{t+1} - \rho - \phi_\xi E_t\Delta\xi_{t+1}] + \alpha(\omega-1)E_t\Delta y_t^* \quad (2.4.48)$$

Equation (2.4.48) shows that the degree of openness influences the sensitivity of output to any given change in the domestic rate discount by the cost of holding money $(i_t - E_t\pi_{t+1} - \phi_\xi E_t\Delta\xi_{t+1})$. If $\omega > 1$, higher values of α increase that sensitivity, as the negative effect of an increase in the real rate on aggregate demand and output is amplified by the induced real appreciation.

In an open economy, due to the existence of a wedge between output and consumption and between domestic and consumer prices, the real marginal cost can be defined as a function of the domestic and foreign output:

$$\begin{aligned} mc_t = -v + w_t - p_{H,t} - z_t = \\ -v + \varphi y_t + \sigma y_t^* + s_t - (1+\varphi)z_t + \phi_\xi\xi_t + \phi_\tau\tau_t^D = \\ -v + \varphi y_t + [\sigma - \sigma_a]y_t^* + \sigma_a yg_t - (1+\varphi)z_t + \phi_\xi\xi_t + \phi_\tau\tau_t \end{aligned} \quad (2.4.49)$$

Where $\sigma_a = \frac{\sigma}{1+\alpha(\omega-1)}$. Defining $yg_t = \phi_y y_t + \phi_g g_t$, where $\phi_y = \frac{y}{y-g}$ and $\phi_g = \frac{g}{y-g}$, we have:

$$mc_t = -v + (\varphi + \phi_y\sigma_a)y_t + [\sigma - \sigma_a]y_t^* - (1+\varphi)z_t + \phi_\xi\xi_t + \phi_\tau\tau_t - \sigma_a\phi_g g_t \quad (2.4.50)$$

By equation (2.4.50), it can be seen that the change in domestic output affects the real marginal cost through its impact on employment (captured by φ) and the terms of trade (captured by σ_a). At the same time, the foreign affects the marginal cost through its effect on consumption (σ) and the terms of trade. However, the sign of the impact is ambiguous and depends on the assumption of ω. Suppose $\omega > 1$ the term $\sigma - \sigma_a > 0$ implies that an increase in the foreign output leads to a rise in the marginal cost. Intuitively, this happens because, in this case, the real appreciation needed to absorb the change in relative supplies is small, with its adverse effects on marginal cost more than offset by the positive effect from a higher real wage. Moreover, it should be noted that if $\alpha = 0$ only domestic variables impact the domestic real marginal cost.

Defining the natural level of output equal to the output level under flexible price, and considering that under flexible price $mc_t = -\mu$. Then, the natural level of output y_t^n can be described as:

$$y_t^n = \frac{-\mu + v}{\varphi + \sigma_b} - \frac{\sigma - \sigma_a}{\varphi + \sigma_b} y_t^* + \frac{1 + \varphi}{\varphi + \sigma_b} z_t - \frac{\phi_\xi}{\varphi + \sigma_b} \xi_t - \frac{\phi_\tau}{\varphi + \sigma_b} \tau_t^d + \frac{\phi_g \sigma_a}{\varphi + \sigma_b} g_t \quad (2.4.51)$$

$$y_t^n = \Gamma_c + \Gamma_* y_t^* + \Gamma_z z_t + \Gamma_\xi \xi_t + \Gamma_\tau \tau_t^d + \Gamma_g g_t \quad (2.4.52)$$

Note that the sign of the effect of foreign output on y_t^n depends on the sign of the effect of the former on the marginal cost, which in turn depends on the relative importance of the terms of trade.

Let the output gap be denoted by $\tilde{y}_t = y_t - y_t^n$. Then, the real marginal cost as a deviation of its natural level can be defined as:

$$\hat{mc}_t = [\varphi + \sigma_b]\tilde{y}_t \quad (2.4.53)$$

Combining the previous expression with equation (2.4.36), we have a new version of the NKPC:

$$\pi_{H,t} = \beta\pi_{H,t+1} + \frac{(1-\theta)(1-\beta\theta)(\varphi+\sigma_b)}{\theta}\tilde{y_t} \tag{2.4.54}$$

More generally, the degree of openness affects the dynamics of inflation only through its influence on the slope of the NKPC.

Using (2.4.48) we derive the dynamic IS equation in terms of the output gap:

$$\tilde{y_t} = E_t\tilde{y_{t+1}} - \frac{1}{\sigma_b}[i_t - \pi_{H,t+1} - r_{H,t}] \tag{2.4.55}$$

Where the natural domestic rate $r_{H,t}$ is equal to:

$$\begin{aligned} r_{H,t} = \rho - \sigma_b\Gamma(z)(1-\rho_z)z_t + \frac{\sigma_b}{\sigma}(\Gamma_* + \alpha(\omega-1))r_t^* + (\sigma_b\Gamma_\xi + \phi_\xi)\Delta\xi_{t+1} + \\ \sigma_b\Gamma_\tau\Delta\tau_{t+1}^d + (\sigma_b\Gamma_g - \sigma_a\phi_g)\Delta g_{t+1} \end{aligned} \tag{2.4.56}$$

Where we use the fact that $y_t^* = \frac{1}{\sigma^*}r_t^*$. From Equation (2.4.56), it is possible to note that the domestic interest rate will depend on domestic factors and the foreign interest rate. Again, the sign of the impact of changes in the foreign interest rate on the natural domestic one depends on ω, which depends on σ and the degree of openness α. As expected, a higher α increases the sensitivity of the natural domestic rate to the foreign one.

Combining the household's first-order conditions (2.4.12) and (2.4.13), we can derive the Uncovered Interest rate Parity (UIP) ($i_t = i_t^* + E_t \Delta e_{t+1}$). Thus, we can describe the total natural interest rate as:

$$\bar{r}_t = (1-\alpha)\bar{r}_{H,t} + \alpha r_t^* =$$
$$(1-\alpha)[\rho - \sigma_a \Gamma_z (1-\rho_z)\hat{z}_t + (\sigma_a \Gamma_\xi + \phi_\xi)\Delta\hat{\xi}_{t+1} + (\sigma_a \Gamma_g - \phi_g)\Delta \hat{g}_{t+1} + \sigma_a \Gamma_\tau \Delta \hat{\tau}_{t+1})] +$$
$$(1-\alpha)\frac{\sigma_a}{\sigma}(\Gamma_* + \alpha(\omega - 1))r_t^* + \alpha r_t^* \tag{2.4.57}$$

For simplicity, we define $\Lambda = \phi_h - \phi_h \alpha + \phi_h \alpha\omega + \sigma$ and $\Omega = 1 - \alpha + \alpha\omega$:

$$\bar{r}_t = \frac{(1-\alpha)}{\Lambda}[\rho - \sigma(1+\phi_h)(1-\rho_z)\hat{z}_t + \phi_\xi(1+\Omega)\Delta\hat{\xi_{t+1}} + \phi_\tau \sigma \Delta\hat{\tau}_{t+1} - \frac{\sigma\phi_g}{\Omega}((1+\phi_h)\Omega + \sigma)\Delta\hat{g}_{t+1}] + \frac{\alpha}{\Lambda}(\frac{(1-\alpha)\alpha(\omega-1)}{\Omega}(\Lambda - \sigma) + \Omega)r_t^* \tag{2.4.58}$$

$$\bar{r}_t = \frac{(1-\alpha)}{\Lambda} r_t^{close} + \frac{\alpha}{\Lambda} r_t^{**} \tag{2.4.59}$$

With:

$$r_t^{close} = \rho - \sigma(1+\phi_h)(1-\rho_z)\hat{z}_t + \phi_\xi(\Lambda + \Omega)\Delta\hat{\xi_{t+1}} + \phi_\tau\sigma\Delta\hat{\tau}_{t+1} - \frac{\sigma\phi_g}{\Omega}((1+\varphi)\Omega + \sigma)\Delta\hat{g}_{t+1} \tag{2.4.60}$$

$$r_t^{**} = (\frac{(1-\alpha)\alpha(\omega-1)}{\Omega}(\Lambda - \Omega) + \Omega)r_t^* \tag{2.4.61}$$

Equation (2.4.59) is central for our model as it shows that the total natural rate can be represented as a linear combination between the closed economy natural rate and the real foreign one. The latter term describes the monetary spillover effect in our model.

To conclude, two important observations should be made. First, it is essential to highlight that with $\omega > 1$ we guarantee that the impact of the foreign rate on the domestic one is positive, as $\Lambda > 0$[2.4.3]. Second, considering $0 < \phi_h < 1$, we have that a more integrated domestic economy (higher α) has a more intense spillover effect of monetary policy. Thus, besides reacting to domestic variables, the monetary authority responds to movements in the foreign rate.

2.4.6 Central Bank

The monetary authority chooses short term nominal rate according to a simple rule belonging to the following class of (Taylor 1993) types rule. (Clarida et al. 2005) proposed a domestic inflation based rule of the form:

$$i_t = \bar{r}_{H,t} + \alpha_\pi \pi_{H,t+1} + \alpha_y \tilde{y}_t \tag{2.4.62}$$

Where $\bar{r}_{H,t}$ is the domestic real natural rate of interest under the assumption of flexible prices and $\tilde{y}_t$ demotes the output gap, with the potential output equals to the output under flexible prices.

Considering the Phillips curve $\pi_{H,t} = \beta \pi_{H,t+1} + \kappa_a \tilde{y}_t$ and the Euler equation $\tilde{y}_t = \tilde{y}_{t+1} - \frac{1}{\sigma_b}(i_t - \pi_{H,t+1} - r_{H,t}$, the equilibrium dynamics for $\tilde{y}_t$ and $\pi_{H,t+1}$ can be represented by means of the system of difference equations:

$$\begin{bmatrix} \tilde{y}_t \\ \pi_{H,t+1} \end{bmatrix} = A \begin{bmatrix} E_t \tilde{y}_{t+1} \\ E_t \pi_{H,t+1} \end{bmatrix} \tag{2.4.63}$$

where

$$A = \begin{bmatrix} \frac{\sigma_b}{\sigma_b + \alpha_y} & \frac{(1-\alpha_\pi}{\sigma_b + \alpha_y} \\ \frac{\kappa_a \sigma_b}{\sigma_b + \alpha_y} & \frac{\kappa a(1-\alpha_{pi})}{\sigma_b + \alpha_y} + \beta \end{bmatrix}$$

The characteristic polynomial of A is given by $p(\lambda) = \lambda^2 + a_1 \lambda + a_0$ where $a_0 = \frac{\beta \sigma_b}{\sigma_b + \alpha_y}$ and $a_1 = -(\frac{\sigma_b}{\sigma_b + \alpha_y} + \frac{\kappa_a(1-\alpha_\pi}{\sigma_b + \alpha_y} + \beta)$. For uniqueness we need both eigenvalues

[2.4.3] This result is also found in (Hofmann e Takats 2015) and (Caputo e Herrera 2017) of A to be inside the unit circle, in other words, $| a_0 |< 1$ and $| a_1 |< 1 + a_0$. In this case, these conditions are satisfied if:

$$\alpha_y \geq 0 \tag{2.4.64}$$

$$-\kappa_a(1 - \alpha_\pi) - \alpha_y(\beta + 1) < 2(1 + \beta)\sigma_b \tag{2.4.65}$$

$$\kappa_a(1 - \alpha_\pi) - \alpha_y(\beta + 1) > 0 \tag{2.4.66}$$

Thus, if conditions (2.4.64) to (2.4.66) are satisfied, there is a locally unique stationary equilibrium under the domestic inflation-based Taylor rule.

Condition (2.4.66) is also usually referred to as the Taylor principle, as it ensures that the Central Bank responds with sufficient strength to deviations of inflation from the target level. As extensively highlighted by the literature, this condition is viewed as a desirable feature of any interest rate rule.

Despite the importance of the above interest rate rule, in practical terms, the domestic inflation-based Taylor rule has some important limitations. First and foremost, empirical evidence suggests that most central banks explicit target the CPI inflation instead the domestic price index. This, defining equation (2.4.62) in terms of CPI inflation, we have:

$$i_t = \bar{r}_t + \alpha\Delta s_{t+1} + \alpha_\pi(\pi_{t+1} - \alpha\Delta\bar{s}_{t+1}) + \alpha_y\tilde{y}_t \tag{2.4.67}$$

From equation (2.4.67) we can derive 3 interest rate rules, depend on the assumptions. The more unrestricted one is:

$$i_t = \bar{r}_t + \alpha_\pi\pi_{t+1} + \alpha_y\tilde{y}_t - \alpha E_t(\phi_\pi\Delta s_{t+1} - \Delta\bar{s}_{t+1}) \tag{2.4.68}$$

However, if we consider that $s_t = \bar{s}_t$, then we have:

$$i_t = \bar{r}_t + \alpha_\pi \pi_{t+1} + \alpha_y \tilde{y}_t + \alpha(1-\alpha_\pi)\Delta s_{t+1} \tag{2.4.69}$$

Finally, if we consider $\alpha_\pi = 1$ equation could be simplified by:

$$i_t = \bar{r}_t + \pi_{t+1} + \alpha_y \tilde{y}_t \tag{2.4.70}$$

Considering the two main types of interest rate rule, domestic and CPI, we will evaluate how the parameters of our model influence the dynamics of the main variables, considering a foreign interest rate shock. Subsequently, we estimate the main parameters for the Brazilian economy to assess the dynamic response of the Brazilian Central Bank to a foreign interest rate shock.

2.5 Results

2.5.1 Domestic Taylor Rule

As extensively highlighted in the previous sections, the impact of the foreign interest rate on the domestic variables depends mainly on the parameter ω, which is a function of the inter-temporal elasticity of consumption σ and the degree of openness α. For that reason, using a calibrated model, we will evaluate how different values of these parameters affect the relationship of the main variables.

We calibrate the model to the Brazilian economy, choosing the time unity to be one quarter. In line with the literature, we assign a value of 0.99 to the subjective discount factor β, implying a real annual interest rate of 4%. The parameter θ is set equal to 0.75, consistent with an average period of one year between price adjustments. It is assumed that $\bar{h}$ equals 3.0 and the steady-state value of hours of work of unity. These parameter values ensure a Frisch elasticity of labor supply of 2. Following (Clarida et al. 2005), the elasticity of substitution between differentiated goods ϵ is set equal to 6, resulting in a steady-state mark-up of 20%.

Considering the money constraint, we define $\nu^h = 0.35$, which means that households hold money balances equivalent to 35% of their quarterly consumption. From the fiscal side, considering the Brazilian data for 2018, we set the government expenditure as a percentage of the GDP of 0.40, the share of total tax revenues as 0.27, and the domestic debt as a percentage of GDP equals 0.77. We define the fiscal parameter γ_1 as 0.35. Moreover, for simplicity, we consider in this part of the paper that the government only relies on lump-sum taxes ($\tau_t = \tau_t^L$ and $\tau_t^D = 0$)[2.5.1]. For the interest rate rule's parameters, we follow the empirical part of the paper. Thus, α_π is equal to 1.8, and α_y close to 1[2.5.2].

In order to calibrate the stochastic properties of the exogenous drive forces, we fit AR(1) process to (log) total GDP for Brazil (a proxy for domestic productivity), (log) government final consumption expenditure for Brazil, and (log) US GDP, using quarterly data over the period 1996:1 - 2020:4[2.5.3]. Finally, following (Caporale e Gil-Alana 2017), we set the serial correlation of the Fed Funds rate equals 0.70. Table 3 presents the parameter values implied by our calibration strategy.

Figure 1 displays the relative impulse response to a negative shock on the foreign interest rate (r_t^*), considering $\sigma = 2$, four different values for the degree of openness: $\alpha = [0.1, 0.3, 0.6, 0.9]$, and the domestic inflation Taylor Rule (equation (2.4.62)).

As expected, the effect on the total natural interest rate $\bar{r}_t$ and the domestic natural interest rate $\bar{r}_{H,t}$ is more prominent when the degree of openness is higher. Consequently, the central bank's response to a fall in the foreign interest rate is more significant as the value of α is bigger.

In terms of economic activity, the lower nominal interest rate stimulates consumption and output. Moreover, with higher consumption and a fall in the cost of holding money (positively correlated with the interest rate), the demand for money also rises.

It should be emphasized that with reasonable values of α, the domestic

[2.5.1] Considering only a foreign interest rate shock, the results for distortionary taxes are the same.
[2.5.2] For more details see columns (1) and (2) from Table 2.
[2.5.3] For all variables we use a quadratically detrended data.

Table 2.5.1 – Structural parameters

Parameter	Value	Description
β	0.99	Subjective discount rate
θ	0.75	Calvo parameter
$\bar{h}$	3	Labor parameter
ϵ	6	Elasticity of substitution between differentiated goods
ν^h	0.35	Money constraint parameter
α_π	1.8	Taylor rule coefficient for CPI
α_{π_H}	1.8	Taylor rule coefficient for domestic inflation
α_y	1	Taylor rule coefficient for output gap
γ_1	0.35	Fiscal parameter
G	0.40	Steady-state level of government purchases
τ	0.27	Steady-state level of taxes revenue
D	0.77	Steady-state level of government liabilities
ρ_g	0.73	Serial correlation of government spending
ϵ_g	0.02	Standard Deviation of innovation to government purchases
ρ_{y^*}	0.80	Serial correlation of US output
ϵy^*	0.25	Standard Deviation of innovation to US output
ρ_z	0.70	Serial correlation of productivity shock
ϵ_z	0.25	Standard Deviation of innovation to productivity shock
ρ_{r^*}	0.70	Serial correlation of Fed Funds rate
ϵr^*	0.25	Standard Deviation of innovation to Fed funds rate

interest rate rule delivers flexible price allocation[2.5.4]. Given the more pronounced fall of the foreign interest rate, the uncovered interest parity implies a nominal exchange rate appreciation. As the domestic prices are constant, we have a reduction in the CPI.

Figure 2 shows the impulse response to a negative shock on the foreign interest rate, considering $\alpha = 0.6$ and four different values for the inter-temporal elasticity of consumption: $\sigma = [0.5, 1, 1.5, 2]$.

Since the impact of a foreign interest rate shock on $\bar{r}_t$ depends on the degree of openness directly, the fall in the former leads to a drop in the second. However, the effects on $r_{H,t}$ and i_t are very different depending on the value of ω and the inter-temporal elasticity of consumption. If $\sigma < 1$ ($\omega < 1$), we negatively correlate the foreign interest rate and the domestic natural and nominal interest

[2.5.4] Even when we consider a degree of openness equal or higher than 0.9, domestic inflation's movement is marginal.

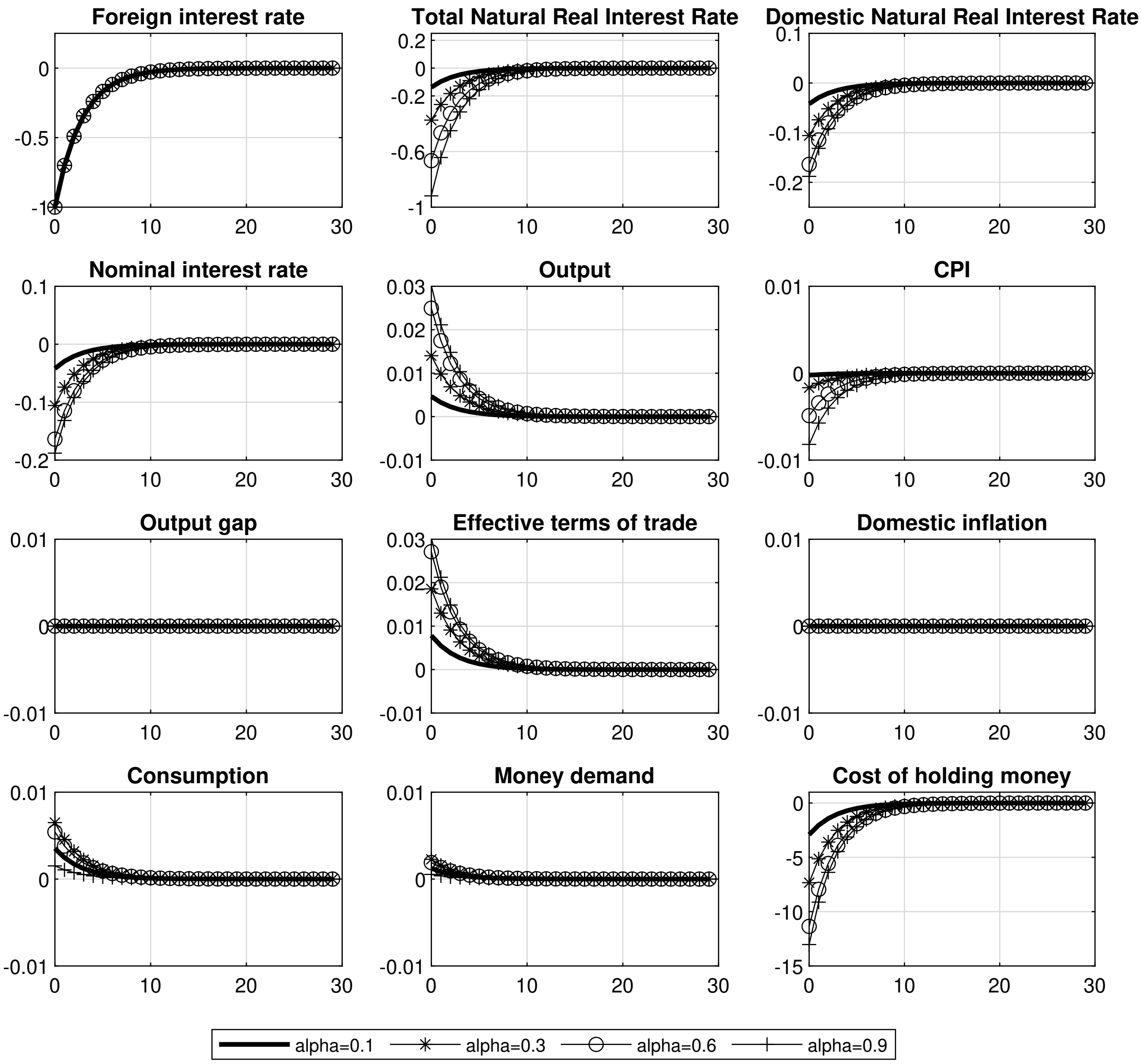


Figure 2.5.1 – Domestic interest rate rule - Foreign interest rate shock

rate. Contrarily, with σ equals to one, the relation between the foreign interest and the domestic variables disappears[2.5.5]. Finally, if the inter-temporal elasticity of consumption is higher than one, as a response to a fall in the foreign interest rate, the domestic central bank cuts its nominal rate, boosting output and consumption.

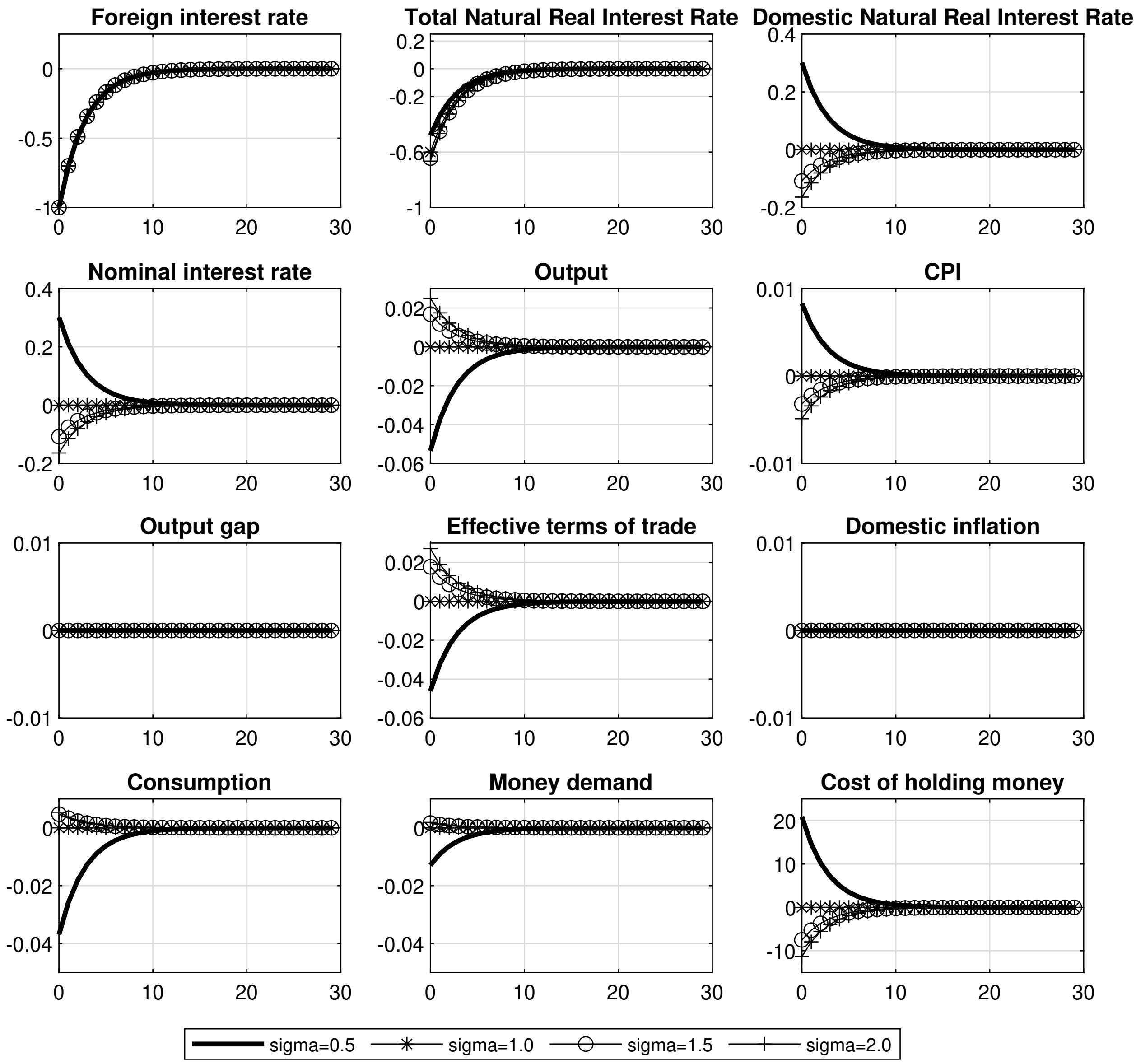


Figure 2.5.2 – Domestic interest rate rule - Foreign interest rate shock

It should be noted that independent of the value of σ, Figure 2.5.2 shows

[2.5.5] As we also consider a unitary elasticity of substitution between domestic and foreign goods.

that Equation (2.4.62) delivers the flexible domestic price allocation.

2.5.2 CPI Taylor Rule

This subsection considers the CPI Taylor Rule described in Equation (2.4.69). For the first analysis, we also assume the intra-temporal elasticity of consumption equals 2.0 and four different values for the degree of openness: $\alpha = [0.1, 0.3, 0.6, 0.9]$. The Brazilian economic response is displayed in Figure 2.5.3.

As expected, the fall in the domestic interest rate is more pronounced when the degree of openness is more significant, indicating that when the domestic economy is more integrated, the international spillover of monetary policy is also higher.

Contrary to the domestic interest rate rule, in this case, the domestic price and output gap are affected by the foreign interest rate shock. With an expansionary monetary policy, consumption and output grow. The fall in the nominal interest rate combined with the rise in activity generates pressure on domestic prices. The same behavior is verified for the Consumer Price index, which has a smaller increase due to the exchange rate appreciation. Given the higher fall of the foreign interest rate, the uncovered interest parity implies a nominal appreciation and an improvement in terms of trade. As was foreseeable, more significant values of α result in higher exchange rate appreciation.

In this case, the potential output remains constant, leading to an increase in the output gap. With higher inflation and activity, the domestic interest rate returns to its steady-state value in the following quarters.

Finally, considering $\alpha = 0.6$ and four different values for the inter-temporal elasticity of consumption: $\sigma = [0.5, 1, 1.5, 2]$, Figure 2.5.4 shows that the response of a negative shock on r_t^* on the domestic natural real rate and on the nominal interest rate depends on the value of σ. Only for values of the inter-temporal elasticity of consumption higher than 1.0, we have a positive correlation between foreign and domestic interest rates, meaning that the domestic central bank responds to a fall in its interest rate with a drop in the foreign one.

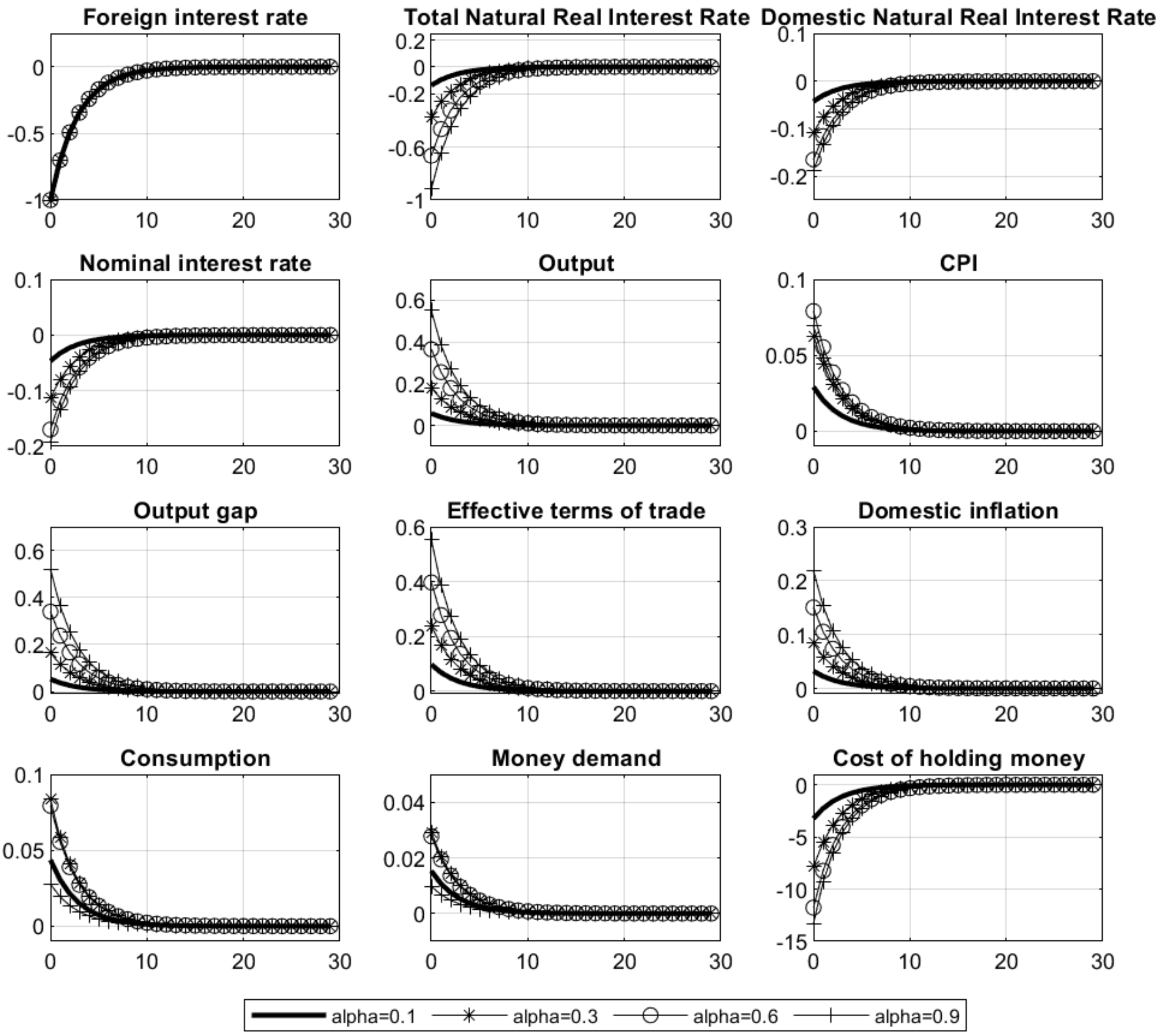


Figure 2.5.3 – CPI interest rate rule - Foreign interest rate shock

As highlighted in the previous section, to guarantee that the correlation between foreign and domestic interest rates is positive, we need $\omega > 1$, which is not valid when σ is smaller than one an α equals 0.6.

To conclude, it is fundamental to compare these results with an environment with no international monetary policy spillover. In the latter case, a fall in the foreign interest rate will not directly affect the domestic central bank response (there is no direct effect on the domestic natural real rate or the total real rate). However, indirectly we could consider that a fall in r_t^* that would lead to a rise in the foreign output could boost domestic GDP through exports. As output grows, inflation rises, and the monetary policy should adopt a contractionary monetary policy. Thus, without considering the effect of the foreign interest rate on the natural rate, these two rates could be negatively correlated, contradicting the most recent empirical results[2.5.6].

In contrast, when we consider the total real natural interest rate in the Taylor Rule, we add a new and direct effect of the foreign rate on the nominal domestic one. Moreover, assuming some values for specific parameters, we could find a positive relationship between distinct countries' monetary policy decisions, aligning with our first empirical results.

2.5.3 Bayesian estimation - Domestic inflation rule

In this section, we estimate the main parameters of the model. For this, we have five shocks: foreign interest rate shock, foreign output shock, government expenditure shock, productivity shock, and a domestic interest rate shock. The five observable variables are Brazilian GDP, US GDP, the Government Brazilian expenditure[2.5.7], the quarterly Brazilian nominal interest rate (Selic Rate) and Fed Funds rate[2.5.8].

The parameters that assume a value between 0 and 1 we consider beta

[2.5.6](Caputo e Herrera 2017) and (Hofmann e Takats 2015).

[2.5.7]For the three variables we use the seasonally adjusted data, we estimate the per capita value (divided by the labor force) and detrended the log of the data using a quadratic detrend approach.

[2.5.8]For the interest rates, we use the quarterly log data subtracted by its mean value.

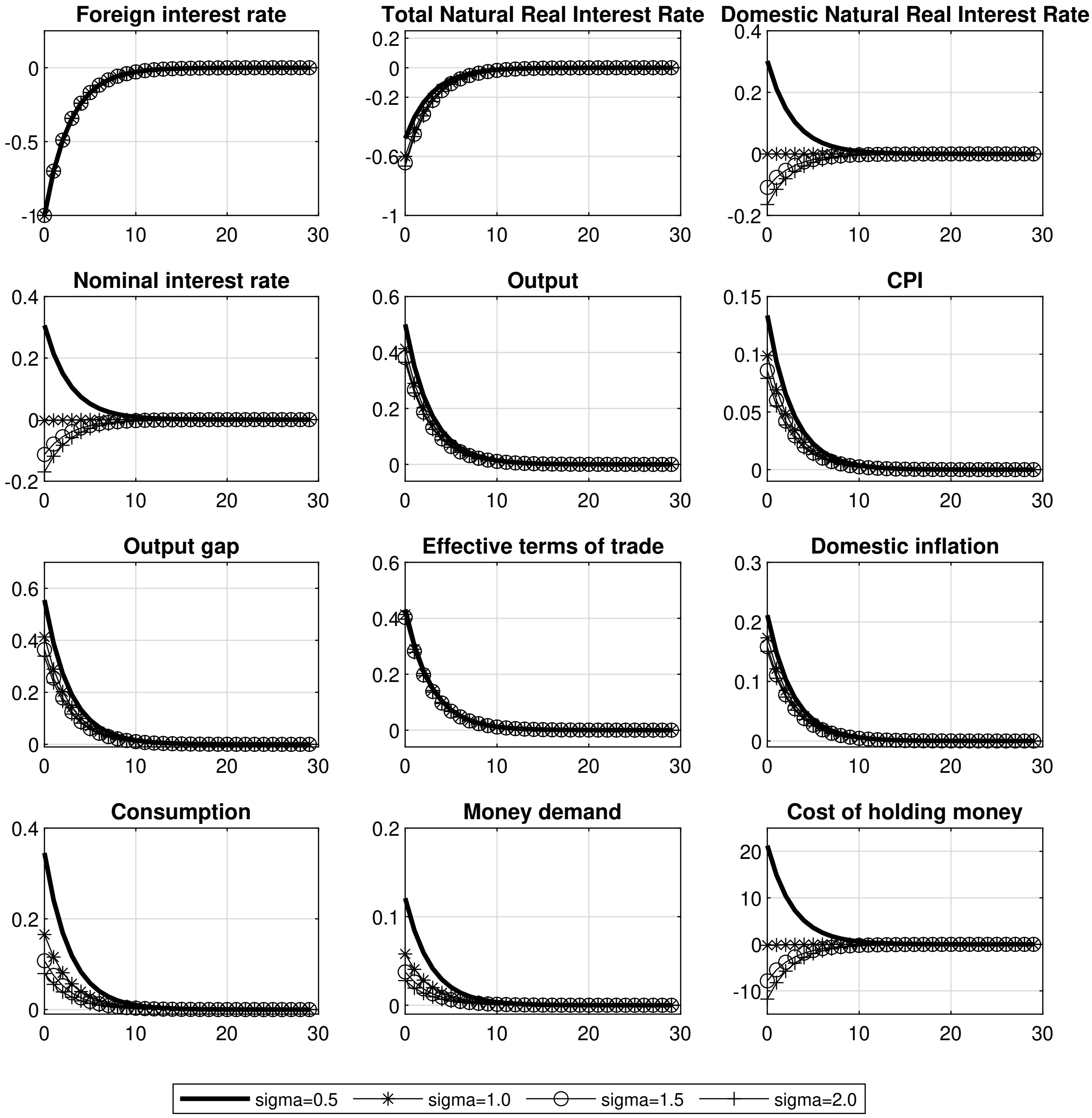


Figure 2.5.4 – CPI interest rate rule - Foreign interest rate shock

distributions as priors : β, θ, α, γ_1, ν^h ρ_r, ρ_g, ρ_z and ρ_y. Considering an annual interest rate equals 4%, we define the mean for the subjective discount as 0.99. Following (Clarida et al. 2005), the mean value of θ is 0.75 (yearly adjustment). For the degree of openness, we choose the mean value of 0.6[2.5.9], while the mean value for the fiscal parameter and the money parameter were set equal to 0.35.

For parameters that assume only positive values, we use prior gamma distributions: $\bar{h}$, σ, α_{pi} and α_y. The mean value for the labor parameter and the inter-temporal elasticity of consumption was defined as 3.0 and 2.0, respectively. For the Taylor Rule's coefficient, we choose a mean of 1.8 for the inflation deviating and 1.0 for the output gap. Finally, all the shocks' standard deviations have an inverse prior gamma distribution with means equal to 0.01. The results are present in Table 4.

Table 2.5.2 – Parameter estimation

Parameter	Prior mean	Post. mean	90% HPD interval		prior	pstdev
β	0.990	0.990	0.988	0.992	beta	0.001
θ	0.750	0.743	0.612	0.907	beta	0.1
α	0.600	0.701	0.570	0.825	beta	0.1
γ_1	0.350	0.342	0.202	0.479	beta	0.1
ν^h	0.350	0.208	0.117	0.321	beta	0.1
σ	2.000	1.859	1.735	1.979	gamma	0.1
α_{π_H}	1.800	1.792	1.616	1.929	gamma	0.1
α_y	1.000	0.9805	0.8394	1.130	gamma	0.1
ρ_g	0.730	0.732	0.718	0.750	beta	0.01
ρ_{y^*}	0.800	0.800	0.785	0.817	beta	0.01
ρ_z	0.700	0.731	0.718	0.745	beta	0.01
ρ_{r^*}	0.700	0.7267	0.714	0.744	beta	0.01
ϵz	0.010	0.029	0.026	0.030	invg	0.01
ϵ_g	0.010	0.023	0.020	0.026	invg	0.01
ϵy^*	0.010	0.012	0.010	0.013	invg	0.01
ϵ_{r^*}	0.010	0.001	0.0007	0.0008	invg	0.01
ϵ_i	0.010	0.025	0.021	0.028	invg	0.01

Considering the posterior mean estimation for the degree of openness (α = 0.701) and the inter-temporal elasticity of consumption (σ = 1.859), we have

[2.5.9] See (5).

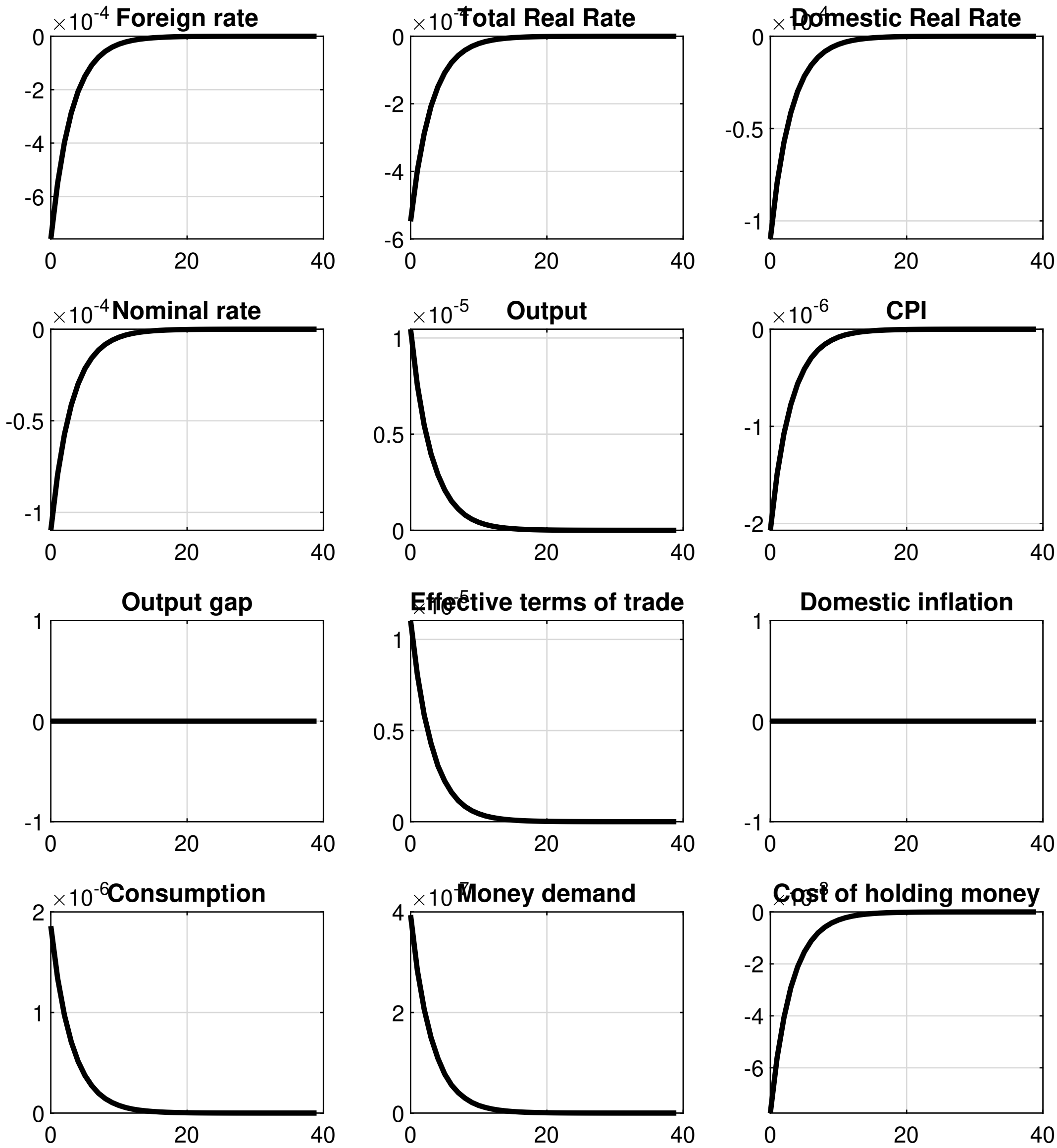


Figure 2.5.5 – Domestic interest rate rule - Foreign interest rate shock

a ω equals 2.116, thus, resulting in a positive relationship between the domestic nominal interest rate and the foreign one.

Following the previous result, with a domestic interest rate rule, the impact of the fall in the i_t in response to a negative foreign monetary policy shock leads to an increase in output and consumption. As expected, the drop in the interest rate causes a fall in the cost of holding money and a rise in the demand for money.

Finally, as highlighted in the previous section, the domestic inflation interest rate rule delivers flexible domestic price allocation. However, as the fall in the nominal interest rate is less pronounced than the drop in the foreign one, we have an exchange rate appreciation, and consequently, a fall in the consumer price index.

2.5.4 Bayesian estimation - CPI rule

This subsection estimates the model considering that the Brazilian central bank reacts to variations in the consumer price index instead of domestic inflation. Table 5 presents the main results of our parameter estimations:

Table 2.5.3 – Parameter estimation

Parameter	Prior mean	Post. mean	90% HPD interval		prior	pstdev
β	0.990	0.989	0.988	0.991	beta	0.001
θ	0.750	0.969	0.949	0.988	beta	0.1
α	0.600	0.427	0.300	0.583	beta	0.1
γ_1	0.350	0.347	0.165	0.493	beta	0.1
ν^h	0.350	0.204	0.108	0.295	beta	0.1
σ	2.000	1.882	1.758	2.015	gamma	0.1
α_{π_H}	1.800	1.799	1.650	1.940	gamma	0.1
α_y	1.000	1.088	0.8912	1.244	gamma	0.1
ρ_g	0.730	0.731	0.717	0.746	beta	0.01
ρ_{y^*}	0.800	0.798	0.777	0.816	beta	0.01
ρ_z	0.700	0.732	0.717	0.746	beta	0.01
ρ_{r^*}	0.700	0.722	0.707	0.736	beta	0.01
ϵz	0.010	0.033	0.028	0.037	invg	0.01
ϵ_g	0.010	0.026	0.020	0.025	invg	0.01
ϵy^*	0.010	0.012	0.010	0.013	invg	0.01
ϵ_{r^*}	0.010	0.001	0.0007	0.0008	invg	0.01
ϵ_i	0.010	0.033	0.0286	0.037	invg	0.01

Comparing to the previous estimation, we have a smaller value for the degree of openness ($\alpha = 0.427$) and a similar value for the inter-temporal elasticity of consumption ($\sigma = 1.882$). As in the case of the domestic Taylor Rule, the parameter ω is higher than one, resulting in a positive relation between the foreign interest rate and $\bar{r}_t$, $\bar{r}_{H,t}$ and i_t.

Figure 6 shows that domestic output and consumption expanded with a more accommodative monetary policy. In line with the calibrated model, the central bank cannot deliver flexible price allocation with a CPI interest rate rule. As a consequence of the overheated economy, domestic prices rise. At the same time, the differential between foreign and domestic rates leads to an exchange rate appreciation. In this case, the impact on the exchange rate more than compensates the rise in $\pi_{H,t}$ leading to a fall in the Consumer Price Index. With this result, the convergence of the nominal domestic interest rate to its steady-state value is slower.

2.6 Conclusion

This paper analyses the presence of international spillovers of monetary policy. Empirically, we showed that even controlling for other external variables, the Fed Funds rate and the ECB interest rate have a positive and significant impact on the nominal interest rate of other developed and emerging countries.

Thus we extended the New-Keynesian model for a small open economy with price rigidity to include the foreign interest rate in the central bank's reaction function through the real natural interest rate. The impact of the foreign rate on the domestic one depends on the main parameters of the model: with $\omega > 1$, we guarantee that correlation between the two rates is positive, and with $0 < \phi_h < 1$ we have that as the economy is more integrated more intense is the spillover effect of monetary policy.

The estimated model for Brazil showed that with the CPI-based Taylor Rule, a negative shock in the foreign interest rate leads to a fall in the real and nominal domestic interest rates, stimulating consumption and output. As a consequence of higher activity, domestic prices rise. However, the exchange rate appreciation causes a fall in the Consumer Price Index. With this result, the convergence of the nominal domestic interest rate to its steady-state value is slower.

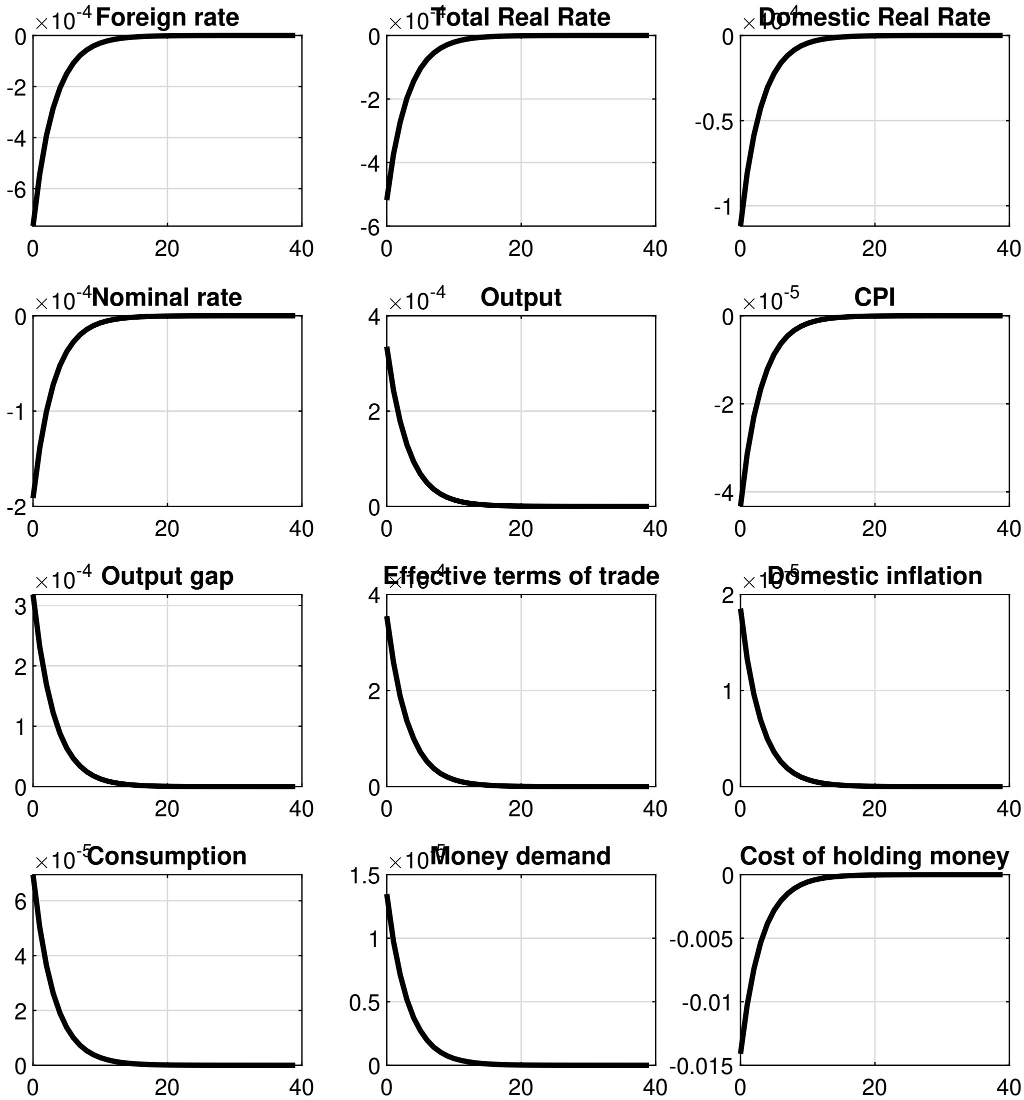


Figure 2.5.6 – CPI interest rate rule - Foreign interest rate shock

3 Welfare cost of international spillovers of monetary policy

With Marcio Nakane. [3.0.1]

3.1 Introduction

With a slower economic activity and lower interest rates, the welfare consequences of the monetary policy decisions have become a subject of high interest. This paper aims to evaluate different interest-rate rules for a small open economy with price rigidities taking into account the international spillover effects of monetary policy.

The size of the response to the external interest rate could vary between countries. To evaluate the welfare cost of these different interest-rate rules, we apply the methodology developed by (Schmitt-Grohe e Uribe 2004) that computes the second-order approximations to conditional and unconditional expected welfare.

The remainder of the paper is organized into six sections. Section 2 presents an overview of the existing literature. Section 3 provides the non-linear model. Section 4 displays the model behavior in the presence of an external interest rate shock. Section 5 computes the welfare differences between monetary policies, and Section 6 provides concluding remarks.

3.2 Bibliography Revision

In an environment of moderate economic growth combined with low-interest rates, the welfare consequences of the monetary policy decisions have become a topic of extensive research in macroeconomics. This paper departs from the literature extant evaluating distinct interest-rate rules within monetary policy spillovers.

[3.0.1] Natalia received financial support from *Coordenação de Aperfeiçoamento de Pessoal de Nível Superior - Brasil* (CAPES - Finance Code 001).

In a seminal paper, (Clarida et al. 1999) used a Dynamic Stochastic General Equilibrium (DSGE) model to analyze different monetary policy rules. According to the authors, in a closed economy New Keynesian model with sticky prices, the optimal monetary policy incorporates inflation targeting, has a coefficient on expected inflation greater than one, and accommodates shocks to potential output. Thus, the simple interest rate rule proposed by (Taylor 1993) is consistent with the main principles that they described for optimal monetary policy.

Considering a closed economy model, (Schmitt-Grohé e Uribe 2003) added the fiscal side to the analysis of optimal policies. Under stick prices a la Rotemberg, the authors highlighted the government's trade-off in choosing the path of inflation. Nevertheless, according to the paper, this trade-off is resolved in favor of price stability under a plausible assumption of price stickiness degree. Considering a degree of price stick ten times smaller than the available estimates suggest for the US economy, optimal monetary policy features relatively low inflation volatility.

For closed economy models, the case for price stability is quite robust. However, in the context of a small open economy, the conclusions are less consensual. (**?**), (Clarida et al. 2005), and (Benigno e Benigno 2003) advocated that the optimal monetary policy for the small open economy could be similar to the close economy case.

Considering a small open economy, (Clarida et al. 2001) found that the policy rule takes a standard form of a closed economy. The central bank responds to expected deviations of inflation by adjusting the nominal rate sufficiently to have a real rate move. The only significant difference between the policy objective functions is that the central bank reacts to domestic inflation in the open economy rather than the consumer price index (CPI), despite the resulting exchange variability.

The studies developed by (Clarida et al. 2005) and (Benigno e Benigno 2003) highlighted the circumstances in which the domestic inflation target emerges as the optimal policy regime. For the first authors, under a model with a log utility function and unitary elasticity of substitution, the optimal policy rule for the small open economy is isomorphic to the closed one. According to (Clarida et al. 2005), CPI inflation target and exchange rate peg are suboptimal as they involve nontrivial deviations from domestic price stability[3.2.1].

(Benigno e Benigno 2003) found that if the intra-temporal elasticity of substitution is equal to the inverse of substitution in consumption, price stability is the optimal monetary policy.

On the other hand, some papers argue that an interest rate rule that targets only output and inflation while performing well in a closed economy is sub-optimal in an open economy model. (Ball 1998) advocated that the optimal policy variable should combine real interest rate and exchange rate.

Moreover, when the New Keynesian models consider incomplete pass-through, the analysis of the monetary policy of an open economy could be fundamentally different from the closed case, as highlighted by (Corsetti e Pesenti 2001), (Clarida et al. 2005), (Smets e Wouters 2002), and (Adolfson 2001).

3.3 Model

The model described in the second part of this thesis is the starting point for our evaluation of the welfare cost of different monetary policy rules. However, in order to apply the methodology developed by (Schmitt-Grohe e Uribe 2004), based on perturbation methods, we have to make some changes in this model. This approach makes it necessary to retain the non-linear nature of the equilibrium conditions[3.3.1]. Thus we cannot restrict our analysis to linear approximations to the equilibrium conditions around a noninflationary steady-state. In particular, we cannot ignore the relative price dispersion across varieties $pd_t = (\int_0^1 \frac{P_{H,t}(j)}{P_{H,t}} dj)^{-\epsilon}$, that arises from the nature of price stickiness:

[3.2.1] However (Clarida et al. 2005) also highlighted that quantitatively the welfare losses are negligible.

[3.3.1] We present all the non-linear equilibrium conditions in Appendix.

$$pd_t = (\int_0^1 \frac{P_{H,t}(j)}{P_{H,t}} dj)^{-\epsilon} =$$
$$\theta \int_0^1 (\frac{P_{H,t-1}(j)}{P_{H,t}})^{-\epsilon} dj + (1-\theta)(\frac{\tilde{P_{H,t}}}{P_{H,t}})^{-\epsilon} = \quad (3.3.1)$$
$$\theta pd_{t-1} \pi_{H,t}^{\epsilon} + (1-\theta) \tilde{p_{H,t}}^{-\epsilon}$$

The state variable pd_t measures the resource cost induced by the inefficient price dispersing present in the Calvo-Yun model and has three main properties: it is bounded by 1.0[3.3.2]; when the non-stochastic level of inflation is equal to zero, up to the first order, the price dispersion is deterministic and follows a univariate autoregressive process; and finally, when prices are fully flexible, $d_t = 1$.

As the price dispersion entails output loss, the equilibrium condition that assures that supply must equal demand at the firm level can be rewritten as:

$$Z_t H_t(j) - \chi = (c_t + c_t^* + g_t)(\frac{P_t(j)}{P_t})^{-\epsilon} \quad (3.3.2)$$

Integrating over all firms and taking into account that labor is common across firms, we have:

$$z_t h_t - \chi = (c_t + c_t^* + g_t) d_t \quad (3.3.3)$$

Moreover, as we want to avoid making special assumptions that allow welfare to be approximated accurately up to second-order from a first-order approximation to the equilibrium conditions, the nonlinear counterpart of the domestic natural interest rate can be described by[3.3.3]:

$$r_{H,t} = (\beta \Delta s_{t+1}^{-1} \Delta \xi_{t+1}^{-1} \Delta x_{4,t+1}^{\sigma} \Delta \bar{x}_{3,t+1}^{\sigma})^{-1} \quad (3.3.4)$$

[3.3.2] See (Schmitt-Grohe e Uribe 2004).
[3.3.3] For more details see Appendix.

3.4 Results

As highlighted by (Benigno e Benigno 2003), in an open economy, the assumption of an employment subsidy that neutralizes the distortion associated with firms market power is not sufficient to render the flexible price equilibrium allocation optimal, as there is an additional factor of distortion: monetary authority can influence the terms of trade in a way beneficial to the domestic consumers.

(Clarida et al. 2005), to derive the optimal monetary policy in an open economy, restricted the analyses to a particular case where the employment subsidy exactly offsets the combined effects of market power and terms of trade distortions. In this case, the subsidy can be derived analytically, delivering the flexible price equilibrium allocation optimal.

However, this particular parameter configuration implies $\sigma = 1$, which is problematic for our model, as when the inverse elasticity of consumption is equal to one, there is no relation between foreign interest rate r_t^* and the domestic natural interest rate $r_{H,t}$[3.4.1]. Thus, we evaluate the welfare cost under different monetary policies using the parameter estimated previous paper and described in Table 1.

Table 3.4.1 – Parameters

Parameter	Value	Description
β	0.989	Subjective discount rate
θ	0.750	Calvo parameter
$\bar{h}$	3.000	Labor parameter
ϵ	6.000	Elasticity of substitution between differentiated goods
ν^h	0.204	Money constraint parameter
α_{π_H}	1.799	Taylor rule coefficient for inflation
α_y	1.088	Taylor rule coefficient for output gap
γ_1	0.347	Fiscal parameter
σ	1.882	Intra-temporal elasticity of consumption
α	0.427	Degree of openness

[3.4.1] There are other ways to guarantee the relation between foreign and domestic interest rate as considering the substitutability between domestic and foreign goods different from one or the presence of more than one foreign country.

To evaluate the model's behavior, we consider an external foreign interest rate shock. Due to an exogenous shock, the foreign central bank applies a monetary policy stimulus, starting cut its interest rate at time t=0, until the rate reaches two standard deviations below trend in quarter t+10[3.4.2].

Figure 1 displayed the behavior of the main variables of the model considering three types of interest rate rules[3.4.3]: the domestic rule that considers the natural domestic rate and the domestic inflation ($ln(i_t) = ln(\bar{r_{h,t}}) + \alpha_\pi ln(\frac{\pi_{h,t+1}}{\bar{\pi_h}}) + \alpha_y ln(\tilde{y_t})$), the CPI rule that takes into account the CPI inflation and the total natural interest rate ($ln(i_t) = ln(\bar{r}_t) + \alpha_\pi ln(\frac{\pi_{t+1}}{\bar{\pi}}) + \alpha_y ln(\tilde{y}_t) + \alpha(1-\alpha_\pi) ln(\frac{s_{t+1}}{s_t})$), and the CPI rule with $\alpha_\pi = 1$.

In line with the previous results, a negative shock in the foreign interest rate leads to a fall in the real natural domestic rate and the real total interest rate. Consequently, at t = 0, the domestic central bank cuts its nominal interest rate. This fall in the interest rate boosts domestic activity (output, consumption, and hours of work), pushing the prices. As a result, the domestic monetary authority starts to raise its rate in the following quarters.

Comparing the different interest rate rules, the domestic one has higher volatility of the nominal interest rate. In this case, at t = 0, we have a more substantial fall in the nominal interest rate, as the domestic inflation felt more than the CPI due to the exchange rate appreciation. The fall in the interest rate boosts economic activity, which presses the inflation and output gap. In this scenario, with the domestic inflation more pressured than the CPI, the central bank following the domestic rule is compelled to adopt a more aggressive monetary tightening cycle.

3.5 Measuring Welfare cost

Following (Schmitt-Grohe e Uribe 2007), we conduct the welfare cost valuation by considering the unconditional welfare measure, defined as the proportional

[3.4.2] Based on the code developed by (Schmitt-Grohe e Uribe 2004), we simulate the model for 3 million quarters and identify in which movements in the foreign interest rate is aligned to our definition of a foreign interest rate shock.

[3.4.3] We also consider a passive fiscal policy, in other words, $\tau_d = 0$.

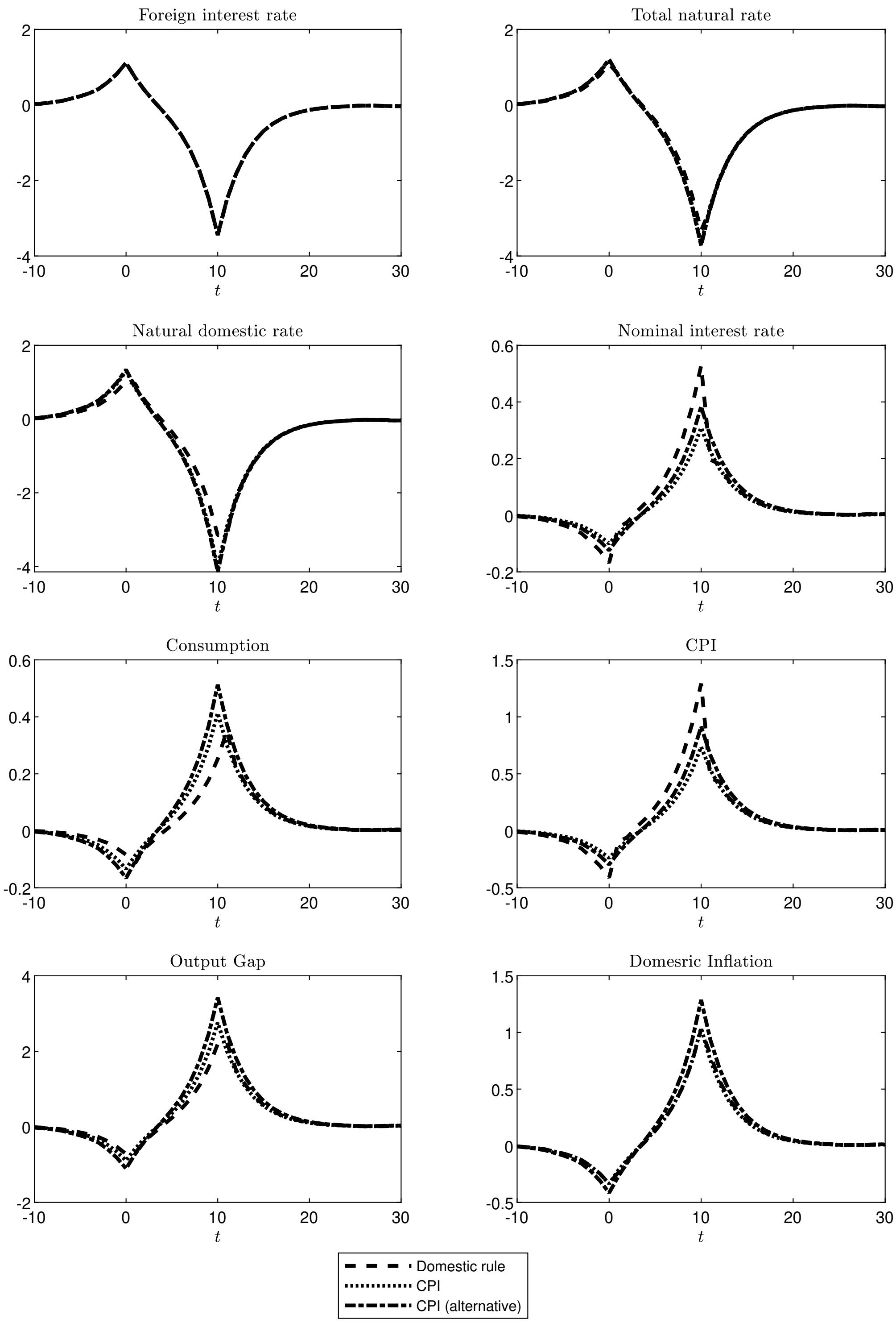


Figure 3.4.1 – External foreign interest rate shock

increase in the steam of consumption of a monetary policy with a spillover effect that makes households unconditionally indifferent between living in the economy with spillover and living in an economy without spillover.

For the economy without international monetary policy spillovers, we will use the interest rate rule that considers the domestic natural interest rate constant and equals ρ[3.5.1]. Alternatively, in an economy with international monetary policy spillovers, we will evaluate the domestic and CPI rules.

Formally, the unconditional welfare cost can be described as:

$$E\sum_{t=0}^{\infty}\beta^t[U((1+\frac{\lambda^u(\sigma_\epsilon)}{100})c_t^S - V(h_t^S)] = E\sum_{t=0}^{\infty}\beta^t[U(c_t^0) - V(h_t^0)] \qquad (3.5.1)$$

where c_t^S ans h_t^S are the consumption and hours of work in an economy that consider the spillover effect of monetary policy and c_t^0 and h_t^0 represent the economy without spillover effect.

It should be noted that the unconditional welfare cost of the economy with spillover $\lambda^u(\sigma_\epsilon)$ relies on the level of uncertainty of the economy σ_ϵ. Thus, to approximate the unconditional welfare cost up to the second order, we have to define welfare associated with each economy, v_t^0 and v_t^S as[3.5.2]:

$$v_t^0 = v^0(x_t^0, \sigma_\epsilon) = U(c_t^0) - V(h_t^0) + \beta v_{t+1}^0 \qquad (3.5.2)$$

$$v_t^S = v^S(x_t^S, \sigma_\epsilon) = U(c_t^S) - V(h_t^S) + \beta v_{t+1}^S \qquad (3.5.3)$$

Considering the following functional form for the utility function:

$$U(c_t, h_t) = \frac{c_t^{1-\sigma} - 1}{1-\sigma} + \varphi log(\bar{h} - h_t)$$

[3.5.1] $i_t = \rho + \alpha_\pi E_t \pi_{H,t+1} + \alpha_y \tilde{y}_t$.

[3.5.2] Similarly, we can also define the sub welfare functions for the no spillover economy as: $v_t^{cons} = \frac{(c_t^S)^{1-\sigma}-1}{1-\sigma} + \beta E_t v_{t+1}^{cons}$ and $v_t^h = \varphi log(\bar{h} - h_t^S) + \beta E_t v_{t+1}^h$.

We can rewrite equation (3.5.1) as:

$$Ev_t^S = (1-\lambda^u(\sigma_\epsilon)^{(1-\sigma)})Ev_t^{cons}((1-\sigma)(1-\beta)+1) - Ev_t^h((1-\sigma)(1-\beta)+1) \quad (3.5.4)$$

Isolating the unconditional welfare cost of the economy:

$$\lambda^u(\sigma_\epsilon) = [\frac{Ev_t^S + Ev_t^h((1-\sigma)(1-\beta)+1)}{Ev_t^{cons}((1-\sigma)(1-\beta)+1)}]^{\frac{1}{(1-\sigma)}} - 1 \quad (3.5.5)$$

As the unconditional expectations are independent of the state of the economy, the expression above can be described as:

$$\lambda^u(\sigma_\epsilon) = [\frac{G^S(\sigma_\epsilon) + G^h(\sigma_\epsilon)((1-\sigma)(1-\beta)+1)}{G^{cons}(\sigma_\epsilon)((1-\sigma)(1-\beta)+1)}]^{\frac{1}{(1-\sigma)}} - 1$$

Finally, the second order Taylor expansion of the $\lambda^u(\sigma_\epsilon)$ around the zero-inflation non stochastic steady state (σ_ϵ=0) is given by[3.5.3]:

$$\lambda^u(\sigma_\epsilon) \approx (\frac{G^S_{\sigma_\epsilon\sigma_\epsilon}(0)) + G^h_{\sigma_\epsilon\sigma_\epsilon}(0) - G^{cons}_{\sigma_\epsilon\sigma_\epsilon}(0)}{G^{cons}(0)(1-\sigma) + (1-\beta)^{-1}})\frac{\sigma_\epsilon^2}{2} \quad (3.5.6)$$

To compute second-order Taylor expansion of the $\lambda^u(\sigma_\epsilon)$ we are going to add the value function (3.5.2) to the set of equilibrium conditions of the economy with no spillover effects and equation (3.5.3) to the equilibrium conditions of the economy with spillover effects. Then both sets of the equations are approximated to the second order.

3.5.1 Economy with passive fiscal policy

We consider a straightforward environment with a passive fiscal policy: the government has access only to lump-sum taxes ($\tau^D = 0$). We are interested in this type of economy, similar to the canonical neo-Keynesian studies developed by

[3.5.3] For more information see (Schmitt-Grohe e Uribe 2004).

(Clarida et al. 2005).

Table 3.5.1 – Optimal Monetary policy

Panel A:	$i_t = \rho + \alpha_\pi ln(\frac{\pi_{H,t+1}}{\pi_H}) + \alpha_y ln(\tilde{y}_t)$			
	α_π	α_y	α	Welfare cost $\lambda^u(\%)$
	1.8	1.0	0.6	0.0000
	1.2	1.0	0.6	0.0014
	1.8	0.5	0.6	-0.0073
Panel B:	$ln(i_t) = ln(\bar{r_{h,t}}) + \alpha_\pi ln(\frac{\pi_{H,t+1}}{\pi_H}) + \alpha_y ln(\tilde{y}_t)$			
	α_π	α_y	α	Welfare cost $\lambda^u(\%)$
	1.8	1.0	0.6	0.1137
	1.2	1.0	0.6	0.1101
	1.8	0.5	0.6	0.3020
	1.8	1.0	0.2	0.3031
Panel C:	$ln(i_t) = ln(\bar{r_t}) + \alpha_\pi \ln(\frac{\pi_{t+1}}{\pi}) + \alpha(1-\alpha_\pi)\ln(\frac{s_{t+1}}{s_t})$			
	α_π	α_y	α	Welfare cost $\lambda^u(\%)$
	1.8	1.0	0.6	0.0911
	1.2	1.0	0.6	0.0876
	1.8	0.5	0.6	0.2703
	1.8	1.0	0.2	0.1891

Table 1 reports the welfare cost of implementing different interest rate rules. The first line represents an economy without spillover effect (a fall in the foreign interest rate has no effect on the nominal rate), considering the coefficients calibrated in the previous section ($\alpha_\pi = 1.8, \alpha_y = 1$ and $\alpha = 0.6$). The next two lines show how the welfare varies depending on the size of the coefficients of the Taylor Rule[3.5.4]. In line with the literature[3.5.5] and (Schmitt-Grohe e Uribe 2007), we find that the best interest-rate rules call for an aggressive response to inflation deviations (higher α_π). In this case, we found a result similar to (Schmitt-Grohe e Uribe 2007), that the inflation coefficient that optimizes the welfare takes the larger value allowed in our model. If we remove the upper bound for α_π, this value is significantly higher. Additionally, in alignment with (Schmitt-Grohe e Uribe 2007), stronger responses to output (higher α_y) generate a welfare cost.

[3.5.4] As we are interested in the impact of the foreign variables in the domestic ones, for this analysis we consider just the shock on y^* and r^*.

[3.5.5] See (Clarida et al. 2005), (Woodford 2001).

Panel B and C show that we have a cost of considering the spillover effect of monetary policy, and this cost depends on the coefficients' size. For $\alpha_\pi = 1.8, \alpha_y = 1$ and $\alpha = 0.6$, taking into account the foreign interest rate in the Taylor rule, through $\bar{r}_t$, generates a smaller welfare cost. Moreover, the degree of openness also impacts welfare in an economy. Thus, higher values of α lead to more significant movements in the potential output through terms of trade, which has a higher effect on the other domestic variables, such as consumption and hours of work.

3.6 Conclusion

This paper evaluates the welfare cost of international spillovers of monetary policy. Aligning with the previous results, a negative shock in the foreign interest rate leads to a fall in the domestic nominal interest rate. This fall in the interest rate boosts domestic activity, pushing the prices. Consequently, the domestic monetary authority starts to raise its rate in the following quarters.

Evaluating the welfare cost of monetary policy, we find that an optimal monetary policy calls for a solid response to inflation deviations and a more muted response to the output gap. This result is valid in an environment with or without considering the international spillover effects of monetary policy.

Comparing the different interest rate rules, we found evidence that when a domestic central bank considers the foreign interest rate in its monetary policy response, it loses some welfare. Nevertheless, two critical observations must be made: first, this loss depends on the Taylor rule's coefficient; second, the cost could be even higher when the country is less economically integrated.

Finally, as the domestic interest rate rule generates higher volatility of the nominal interest rate, this rule incurs a higher welfare cost.

3.7 Appendix A

The equilibrium condition is composed by 4 equations for the exogenous variables, g_t, z_t, r_t^* and y_t^*, one interest rate rule and the following conditions:

$$c_t^{-\sigma} = \lambda_t(1 + \nu\xi_t) \tag{3.7.1}$$

$$\frac{1}{\bar{h} - h_t} = \lambda_t w_t(1 - \tau_t^D) \tag{3.7.2}$$

$$\lambda_t = \beta i_t \frac{\lambda_{t+1}}{\pi_{h,t+1}} \tag{3.7.3}$$

$$mc_t z_t = w_t s_{t+1}^{\alpha} \tag{3.7.4}$$

$$\theta\pi_{H,t}^{\epsilon-1} + (1-\theta)\tilde{p_{h,t}}^{1-\epsilon} = 1 \tag{3.7.5}$$

$$x_{1,t} = \tilde{p_{h,t}}^{-\epsilon-1} mc_t yt + \theta\beta\frac{\lambda_{t+1}}{\lambda_t}\pi_{h,t+1}^{\epsilon}(\frac{\tilde{p_{h,t}}}{\tilde{p_{h,t+1}}})^{-1-\epsilon}x_{1,t+1} \tag{3.7.6}$$

$$x_{2,t} = \tilde{p_{h,t}}^{-\epsilon} yt + \theta\beta\frac{\lambda_{t+1}}{\lambda_t}\pi_{h,t+1}^{\epsilon-1}(\frac{\tilde{p_{h,t}}}{\tilde{p_{h,t+1}}})^{-\epsilon}x_{2,t+1} \tag{3.7.7}$$

$$x_{2,t} = \frac{\epsilon}{\epsilon - 1} \tag{3.7.8}$$

$$y_t = \frac{1}{d}(z_t h_t - \chi) \tag{3.7.9}$$

$$\tau_t = \tau_t^d y_t + \tau_t^L \tag{3.7.10}$$

$$\pi_t = \pi_{h,t}(\frac{s_{t+1}}{s_t})^{\alpha} \tag{3.7.11}$$

$$r_{h,t} = \beta^{-1}\Delta s_{t+1}\Delta\xi_{t+1}(\Delta x_{4,t+1})^{-\sigma}(\Delta x^n_{3,t+1})^{\sigma} \tag{3.7.12}$$

$$i_t = \frac{1}{1-\xi_t} \tag{3.7.13}$$

$$\bar{r}_t = r_{h,t}^{1-\alpha}(r_t^*)^{\alpha} \tag{3.7.14}$$

$$m_t = \nu c_t \tag{3.7.15}$$

$$c_t = y_t^* s_t^{\frac{1-\alpha}{\sigma}} \tag{3.7.16}$$

$$\bar{y_t} = \bar{h}z_t - \chi - \mu\frac{(y_t^*)^{\sigma}s_t(1+\nu^h\xi_t)}{(1-\tau_t^D)} \tag{3.7.17}$$

$$d_{t+1} = (1-\theta)\tilde{p_{h,t}}^{-\epsilon} + \theta\pi_{h,t}^{\epsilon}d_t \tag{3.7.18}$$

$$l_{t+1} = l_t\frac{i_t}{\pi_{h,t}} + i_t(g_t - \tau_t) - m_t(i_t - 1) \tag{3.7.19}$$

$$(\tau_t - \tau^*) = \gamma(l_t - l^*) \tag{3.7.20}$$

$$y_t = ((1-\alpha)s_t^{\alpha}c_t - \alpha s_t y_t^* + g_t)d_t \tag{3.7.21}$$

3.8 Appendix B

Deriving the non linear equation for the domestic natural real interest rate $r_{H,t}$.

Combining the Euler equation with $c_t = y_t^* * s_t^{\frac{1-\alpha}{\sigma}}$ we have:

$$(y_t^*)^{-\sigma} = \beta E_t[(y_{t+1}^*)^{-\sigma} \frac{i_t}{\pi_{h,t+1}} \frac{1}{\Delta s_{t+1} \Delta \xi_{t+1}}] \tag{3.8.1}$$

Considering Equation (38) and defining $x_{3,t} = \frac{y_t}{d_t} - g_t$:

$$x_{3,t} = y_t^* x_{4,t} \tag{3.8.2}$$

Where $x_{4,t} = (1-\alpha) s_t^{\frac{(\alpha\sigma+1-\alpha)}{\sigma}} + \alpha s_t$. Thus, combining Equations (82) and (83) we have:

$$(x_{3,t})^{-\sigma} = \beta E_t[(x_{3,t+1})^{-\sigma} \frac{i_t}{\pi_{h,t+1}} \Delta s_{t+1}^{-1} \Delta \xi_{t+1}{}^{-1} (\Delta x_{4,t+1})^{\sigma}] \tag{3.8.3}$$

Considering the marginal cost for an open economy equals to:

$$\begin{gathered} mc_t = \frac{w_t s_t^{\alpha}}{z_t} = \frac{c_t^{\sigma}(1+\nu^h \xi_t)}{(\bar{h}-h_t)(1-\tau_t^D)} \frac{s_t^{\alpha}}{z_t} = \\ \frac{(y_t^*)\sigma s_t^{1-\alpha}}{\bar{h} - \frac{y_t d_t - \chi}{z_t}} \frac{s_t^{\alpha}}{z_t} \frac{(1+\nu^h \xi_t)}{(1-\tau_t^D)} = \frac{(y_t^*)^{\sigma} s_t}{\bar{h} z_t - y_t d_t - \chi} \frac{(1+\nu^h \xi_t)}{(1-\tau_t^D)} \end{gathered} \tag{3.8.4}$$

As discussed above, under flexible prices, $mc_t = \mu^{-1}$ for all t, thus the natural level of output can be defined as:

$$y_t^n = \bar{h}z_t - \chi - \mu\frac{(y_t^*)^\sigma s_t(1+\nu^h\xi_t)}{(1-\tau_t^D)} \tag{3.8.5}$$

Using Equation (86) we can define the $x_{3,t}^n = \bar{h}z_t + \frac{1}{\mu}\frac{(y_t^*)^\sigma s_t}{(1-\tau_t^D)} - \bar{g}$. Then, Equation (84) can be derived as:

$$\tilde{x}_{3,t} = E_t[\tilde{x}_{3,t+1}[\beta\frac{i_t}{\pi_{h,t+1}}\Delta s_{t+1}^{-1}\Delta\xi_{t+1}^{-1}(\Delta x_{4,t+1})^\sigma(\Delta x_{3,t+1}^n)^{-\sigma}]]^{\frac{-1}{\sigma}} = E_t[\tilde{x}_{3,t+1}[\frac{i_t}{\pi_{h,t+1}}r_{h,t}^{-1}]]^{\frac{-1}{\sigma}} \tag{3.8.6}$$

Where the real natural domestic rate can be described as:

$$\bar{r}_{h,t} = \beta^{-1}\Delta s_{t+1}\Delta\xi_{t+1}(\Delta x_{4,t+1})^{-\sigma}(\Delta x_{3,t+1}^n)^\sigma =$$
$$\beta^{-1}\Delta s_{t+1}\Delta\xi_{t+1}[\frac{(1-\alpha)s_{t+1}^{\omega_1}+\alpha s_{t+1}}{(1-\alpha)s_t^{\omega_1}+\alpha s_t}]^{-\sigma}[\frac{\bar{h}z_{t+1}+\frac{1}{\mu}r_t^*(y_t^*)^\sigma\frac{1}{(1-\tau_{t+1}^D)}}{\bar{h}z_t+\frac{1}{\mu}(y_t^*)^\sigma\frac{1}{(1-\tau_t^D)}}]^\sigma \tag{3.8.7}$$

Where $\omega_1 = \frac{\alpha\sigma+1-\alpha}{\sigma}$. Although Equation (86) brings a more complicated relationship between the natural real domestic interest rate and the foreign variables, as opposed to its linear counterpart Equation (57), it is still possible to verify the positive correlation between the foreign interest rate and $r_{h,t}$. Other things equal, a shock in the international monetary policy has an effect with the same sign in the domestic interest rate, with the size of the impact depending mainly on the parameters μ and σ.

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