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Consumption predictability: the role of credit conditions

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Abstract

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School of Economics, Business Administration and Accounting at Ribeirão Preto,
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The goal of this dissertation is to investigate the relationship between consumption and credit through a dynamic structural model. With impatient consumers and low income volatility, the dependence of consumption to consumer credit is a proxy of credit supply, opposing the usual interpretation. In this case, deregulation on credit markets would enhance the predictive power of consumer credit. However, with less impatient consumers and increased income volatility, more credit supply reduce the dependence of consumption to consumer credit. This theoretical evidence shows that there is ambiguity in interpreting the predictability of aggregate consumption, and understanding dependence of consumption to credit as a measure of credit constraints might be misleading.

Keywords: Intertemporal Household Choice; Consumption; Credit.

JEL Code: D15; E21; E51.

Abstract

Caleman, B. Q. **Previsibilidade do consumo: o papel das condições de crédito.** School of Economics, Business Administration and Accounting at Ribeirão Preto, University of São Paulo, Ribeirão Preto, 2021.

O objetivo desta dissertação é investigar a relação entre consumo e crédito por meio de um modelo estrutural dinâmico. Com consumidores impacientes e baixa volatilidade da renda, a dependência do consumo em relação ao crédito para consumo é uma proxy da oferta de crédito, opondo-se a interpretação usual. Nesse caso, a desregulamentação dos mercados de crédito aumentaria o poder preditivo do crédito. No entanto, com consumidores menos impacientes e maior volatilidade da renda, mais oferta de crédito reduz a dependência do consumo em relação ao crédito para consumo. Essa evidência teórica mostra que há ambigüidade na interpretação da previsibilidade do consumo agregado, e entender a dependência do consumo em relação ao crédito como uma medida das restrições de crédito pode ser incorreto.

Keywords: Escolha doméstica intertemporal; Consumo; Crédito.

JEL Code: D15; E21; E51.

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1 Introduction

Borrowing money to finance consumption is not precisely new, but the aggregate level of debt is rising so sharply that it is reasonable to assume that the relationship of consumption and credit must also have changed in the same period. In the United States, for example, the liability of households and non-profit organizations experienced an average annual growth of 5.69% between 1990 and 2020, which represents a growth of 408% in 30 years¹. This enormous figure raises the question about the sensibility of consumption growth to credit growth.

The predictability of consumption is of interest for its potential impact on monetary and fiscal policies, and various empirical evidence shows that expected credit growth has predictive power over consumption growth. Ludvigson (1999) shows that expected credit growth is correlated with consumption growth using U.S. data. Bacheta and Gerlach (1997) use a similar framework to show that this feature of aggregate consumption is present in many countries. They also show that the correlation between consumption growth and expected income and credit growth changed over time in some of these countries, suggesting that this change is due to liberalizing reforms in credit markets. These results are very important because, essentially, they mean that expected credit growth can forecast consumption growth, which is a direct violation of the random-walk hypothesis presented by Hall (1978). In this line of thought, Brady (2008) links structural breaks that increased access to consumer credit to the lack of predictability of consumption in some U.S. data samples. It is important to point out that, according to some of his results, neither income nor consumer credit has predictive power on consumption.

Credit constraints can indeed be the reason for consumption predictability. From a theoretical point of view, Deaton (1991) proposes a structural model that shows that, in the absence of credit, consumption growth may simply follow income growth if assets are sufficiently low. Ludvigson (1999) extends his framework and shows that consumption may also follow credit growth if debt is sufficiently close to the credit limit. However, is it possible to assert that, within the scope of such models, more credit means less consumption predictability?

To test this hypothesis, we used the solution of the model to generate consumption, income, and credit histories. Regressing the linear regression proposed by Ludvigson (1999) for different sets of credit supply and credit demand. The objective is to understand exactly how credit influences consumption. From the simulations, it is possible to see that this model represents a specific type of consumer, i.e. a consumer that is almost always in debt. This was already highlighted by Ludvigson (1999), and in this model the consumer uses debt as a buffer to income variations². This feature is important to understand and

¹Board of Governors of the Federal Reserve System (US), Households and Nonprofit Organizations; Consumer Credit; Liability, Level [HCCSDODNS].

²Deaton (1991) shows that in the absence of credit, assets act like a "buffer-stock" used to smooth

contextualize the empirical results arising from the model. In particular, we show that the model is useful to understand the behavior of indebted consumers. As this must be the situation of only a part of the households, our results cast doubt on this use to interpret the behavior of US aggregate consumption.

With low income volatility, we show that the dependence of consumption on consumer credit is a proxy for credit supply, regardless of credit demand (controlled by the discount factor), opposing the usual interpretation. In this case, deregulation on credit markets will enhance the predictive power of consumer credit, which opposes Brady's (2008) interpretation. The intensity of this behavior depends on the credit demand, with higher credit demand leading to higher dependence of consumption on credit, as expected. When income volatility is higher, the results are ambiguous. Depending on the credit demand, more credit supply can either increase or decrease the predictive power of credit. Moreover, with lower credit demand the credit coefficient significance decreases with higher credit supply, a result that corroborates Brady (2008). This is theoretical possibility that liberalized consumer credit environments can lower or even eliminate the predictive power of credit to consumption in such conditions. This theoretical evidence shows that there is ambiguity in interpreting the predictability of aggregate consumption, and understanding the credit coefficient as a proxy of the measure of credit market tightness might be incorrect or even misleading because it depends on income uncertainty and preference parameters.

The rest of this work is divided as follows. Section 2 introduces the discussion regarding credit restriction and rule-of-thumb behavior. Section 3.2 shows the model proposed by Ludvigson (1999) and how to transform the model to be stated as stationary variables and what are the values calibrated for the relevant variables. In this section, we also show how to solve the model using a slightly different set of state-variables and solution method when compared to the one originally proposed by Ludvigson (1999). Section 4 shows how to simulate histories of consumption, income, and debt from the solution of the model, and also the results of the regressions using these histories. Section 5 concludes with final remarks.

2 Rule of thumb as a measure of credit constraints

The seminal work by Hall (1978) raises the famous hypothesis that aggregate consumption is a martingale, meaning that the conditional expectation of consumption is its present value, i.e. that no variable other than the first lag of consumption itself is correlated with consumption. It is important to emphasize that in his model the income is *i.i.d.*, and this feature is important to generate the random-walk consumption function (that does not even have a drift). Essentially, it means that consumption growth is unforecastable.

consumption

Hall (1988) extends this framework to include a constant relative risk aversion (CRRA) utility function, resulting in a consumption growth that is dependent on the interest rate (or, similarly, expected asset returns). However, Campbell and Mankiw (1989) find strong empirical evidence supporting that expected income growth is correlated with consumption growth³, which is, to their understanding, a violation of the permanent-income hypothesis.

From a theoretical point of view, Deaton (1991) analyzes how optimal saving and consumption behave when the consumer is liquidity constrained. The liquidity constraint imposed is that consumers cannot borrow. In that case, assets act like "buffer-stock" savings, which are used to smooth consumption over income shocks. Furthermore, for different income processes, Deaton (1991) shows that, up to a specific cash-in-hand available, which is simply income plus assets, households consume all they have, going to the next period with no assets. This result seems to be in line with the conclusion of Campbell and Mankiw (1989), suggesting that the households which consume their current income are the ones liquidity constrained, although this conclusion is not explicitly stated by him.

For one of the non-stationary income processes used by Deaton (1991), which is subsequently used by Ludvigson (1999), results show that households have a pro-cyclical saving behavior, implying that the consumption-to-income ratio decreases after positive shocks and increases after negative ones. Moreover, consumption is smoother than income, a feature that is also observed by Deaton (1991) using other processes for income. This result is particularly important because consumption smoothness is seen on actual data. Another particularly interesting result is that, in the presence of liquidity constraints, consumption growth and lagged income growth are negatively correlated, a finding similar to Hall and Mishkin (1982) and Campbell and Mankiw (1989).

Ludvigson (1999) further develops this model and analyses time-varying liquidity constraints, which will be henceforth called credit-constraints. The theoretical model proposed by her is an extension of Deaton's (1991) model, allowing consumers to borrow up to a credit ceiling, which is a fraction of income times a credit shock common to all households and independent of income. Her results resemble the ones reached by Deaton (1991), showing that if current-resources (in this case, the sum of income, assets, and the available credit) are below a certain threshold, households choose to consume all they can. In this situation, consumption is determined not only by income and assets but also by credit availability.

This theoretical result suggests that credit might also be related to consumption if credit-restrictions are in place or, in other words, that consumption can also displays *excess sensitivity* to credit. In the same article, Ludvigson (1999) finds empirical evidence in U.S. data supporting that expected credit growth is a statistically significant predictor of consumption growth. Bacchetta and Gerlach (1997) find the same evidence using data

³Flavin (1984) finds similar results and calls it an *excess sensitivity* of consumption to income.

from Japan, the U.S, the U.K., and Canada, thus supporting the hypothesis that credit is an important component of consumption in these economies. Bacchetta and Gerlach (1997) also find that consumption predictability varies over time, indicating that this could be a consequence of liberalized consumer credit markets (increased credit availability).

In more recent work, Brady (2008) analyzes the impact of structural changes in consumer credit in consumption. More specifically, he employs the empirical specification proposed by Ludvigson (1999) across different samples on U.S data from 1959 to 2005. When looking over the entire sample, his conclusion is the same as Ludvigson's (1999): consumption displays *excess sensitivity* to predictable credit growth and predictable income growth. However, when using more recent samples (particularly after the mid-1980s), the effects of predictable credit growth and predictable income growth on consumption growth are no longer statistically significant⁴. Brady (2008) suggests that the reason for this result is that consumers became less credit constrained once structural changes in consumer credit that increased credit availability took place. Therefore, according to him, the permanent-income hypothesis can no longer be rejected. Although Brady's (2008) conclusion shows that the empirical evidence found by Ludvigson (1999) is no longer valid in a liberalized credit environment, it supports the claim that liquidity constraints are important to determine whether consumption smoothing will happen or not.

3 Theoretical Model

3.1 Non-stationary Model

To understand how the impact of changes in consumer credit markets are related to consumption predictability, we will use the model proposed by Ludvigson (1999) with a few modifications. More specifically, we will use a different set of state variables and solve the problem through dynamic programming⁵. Consider a consumer that maximizes a life-time utility function given by:

$$E_0 \left(\sum_{i=0}^{\infty} \beta^i u(C_{t+i}) \right), \quad \beta = (1 + \delta)^{-1} < 1, \quad \delta > 0, \quad (3.1)$$

in which $u(C_t) = \frac{C_t^{1-\gamma}}{1-\gamma}$, C_t being the consumption of a non-durable good in period t .

⁴Brady's (2008) samples are based on structural breaks in the series for consumer credit and consumption. These breaks are consistent with regulatory and structural change in credit markets, according to the author.

⁵Ludvigson (1999) uses as state variables the resources-to-income ratio, and contemporaneous income and credit ceiling shocks, whereas we use the debt-to-income ratio, the income growth, and the contemporaneous credit ceiling shock.

The consumer is allowed to take debt, which evolves according to:

$$D_t = (1 + r)D_{t-1} + C_t - Y_t, \quad (3.2)$$

Y_t is the income at the beginning of period t , D_t is the debt at the end of period t , and r is a constant interest rate. Due to algebraic convenience, we do this slight modification of the specification proposed by Ludvigson, in which D_t is the debt at the beginning of the period t .

The labor income is exogenous and determined by a stochastic non-stationary process, following the specification proposed by Deaton (1991). It is given by $Y_t = Y_{1t}Y_{2t}$, Y_{1t} and Y_{2t} being the permanent and the transitory component, respectively. The log income, therefore, is given by $\log Y_t = \log Y_{1t} + \log Y_{2t}$. Assuming that $\log Y_{1t}$ is a white noise process and $\log Y_{2t}$ is a random walk, the income growth is given by:

$$\Delta \log Y_t = g + \eta_t - \psi \eta_{t-1}, \quad \eta_t \sim N(0, \sigma_\eta). \quad (3.3)$$

Following Ludvigson (1999), the time-varying debt limit, which is dependent of income and of income-independent shocks, is given by:

$$D_t \leq \bar{D}_t \equiv \frac{1}{\omega(1+r)} Y_t \exp(\xi_t), \quad (3.4)$$

where $\bar{D}_t$ is the maximum debt that the consumer can take in period t . Note that, in this specification, the credit-limit depends on income and a shock. This is an important feature of the model that can rationalize empirical evidence regarding consumption⁶. The parameter ω is fixed and known and ξ_t is the credit ceiling shock that is independent of income and common across households. This shock is given by a stationary positively auto-correlated process given by:

$$\xi_{t+1} = \phi \xi_t + u_{t+1}, \quad u \sim N(0, \sigma_u). \quad (3.5)$$

Replacing D_t in (3.2) for $\bar{D}_t$ and isolating C_t , we obtain the maximum expenditure in consumption, X_t , as follows:

$$\begin{aligned} X_t &= \bar{D}_t - D_{t-1}(1+r) + Y_t \\ &= Y_t \left(1 + \frac{1}{\omega(1+r)} \exp(\xi_t) \right) - D_{t-1}(1+r) \end{aligned} \quad (3.6)$$

⁶Ludvigson (1999) shows that expected credit growth has predictable power over consumption growth. If the credit limit was a just fraction of income, the model would not be able to reproduce this empirical evidence.

Table 1: Consumption dependence on credit

	High credit demand	Low credit demand
High credit supply	High dependence	Intermediate dependence
Low credit supply	Intermediate dependence	Low dependence

As detailed by Ludvigson (1999), the Euler equation of this model is given by:

$$C_t^{-\gamma} = \max[X_t^{-\gamma}, \beta(1+r)E_t C_{t+1}^{-\gamma}]. \quad (3.7)$$

The Euler equation tells us that the optimal behavior is such that the marginal utility of consumption in t must be equal to the max between the marginal utility of maximum consumption in t and the expected marginal utility of consumption in $t+1$. Given that the marginal utility is a decreasing function of C_t , the marginal utility of C_t can be no smaller than $X_t^{-\gamma}$.

One of the results of this model, which is going to be detailed in the next section, is that the consumer will simply consume all available resources if they are below a certain threshold. In this situation, consumption growth will be determined by current income growth and the current credit growth. Following this theoretical result, Ludvigson (1999) then argues that both predictable income growth and predictable consumer credit growth will have predictable power over consumption growth, given that a considerable fraction of consumers are credit-constrained (have a sufficiently low level of resources available).

The Euler equation (3.7) can give some insight into why tighter credit restrictions would make income and consumer credit more relevant predictors of consumption: less credit available means that the lower bound for marginal utility will become higher (as X_t is lower) so that the restriction is more likely to be binding. However, it is important to note that tighter credit conditions will also affect optimal consumption since it is assumed that the consumer sees how the credit ceiling is varying and takes this variation into account when forming optimal consumption plans. Furthermore, it seems to be reasonable to assume that increasing credit restrictions could raise the dependence of consumption to credit up to a point. If the credit restriction is too tight, meaning that there is very little credit available, then consumers would simply not borrow at all. Likewise, if lending is too risky, therefore rising the cost of borrowing, consumers will not take debt even though there is credit available.

To summarize our procedure to investigate the dependence of consumption on credit, Table 1 shows what would be expected when looking at the dependence of consumption on credit according to credit demand and supply. In the next section, we show how to solve the model and how to control credit supply and demand within its framework.

3.2 Model Transformation and Calibration

As there is no analytical solution to the model presented in the previous section, at least to the best of our knowledge, we will use the standard numerical procedure in dynamic programming tradition. The dynamic problem can, be stated as:

$$\begin{aligned} & \max_{\{C_t\}_{t=0}^{\infty}} E_t \left(\sum_{i=0}^{\infty} \beta^i \left(\frac{C_{t+i}^{1-\gamma}}{1-\gamma} \right) \right) \\ & \text{subject to: } \begin{cases} (3.2), (3.3), (3.4), (3.5) \\ D_0 \text{ given} \\ C_t \geq 0 \quad \forall t \end{cases} \end{aligned} \quad (3.8)$$

Equivalent to solve (3.8) is to solve the following Bellman equation:

$$\begin{aligned} & V_t(Y_t, D_{t-1}) = \max_{C_t \geq 0} \left\{ u(C_t) + \beta \mathbb{E}_t V_{t+1}(Y_{t+1}, D_t) \right\} \\ & \text{subject to: } \begin{cases} (3.2), (3.3), (3.4), (3.5) \\ D_0 \text{ given} \\ C_t \geq 0. \end{cases} \end{aligned} \quad (3.9)$$

However, as income is non-stationary, consumption and debt processes will also be non-stationary, leading to non-stationary value functions $V_t(Y_t, D_{t-1})$ and policy functions $f(Y_t, D_{t-1})$. To circumvent this issue, we will rewrite the problem in terms of the stationary ratios defined below:

$$y_{t+1} \equiv \frac{Y_{t+1}}{Y_t} = \exp(g + \eta_t - \psi \eta_{t-1}) \quad (3.10)$$

$$c_t \equiv \frac{C_t}{Y_t}, \quad x_t \equiv \frac{X_t}{Y_t}, \quad d_t \equiv \frac{D_t}{Y_t}, \quad \bar{d}_t \equiv \frac{\bar{D}_t}{Y_t} \quad (3.11)$$

To transform the model into stationary variables, we will follow Carrol (2004). Suppose for a moment that we are dealing with a finite horizon problem with the final period given by T . In such a case, it is possible to find the solution by backward induction, so that we have a series of functions $(V_0, V_1, \dots, V_T)$. The updating rule is given by:

$$V_t(Y_t, D_{t-1}) = \max_{C_t \geq 0} \left\{ u(C_t) + \beta \mathbb{E}_t V_{t+1}(Y_{t+1}, D_t) \right\}. \quad (3.12)$$

At the final period T , the consumer will spend all current resources in consumption, meaning that $V_T(Y_T, D_{T-1}) = u(X_T)$. Therefore, writing (3.12) for the second-to-last

period and rewriting C_{T-1} in terms of Y_{T-1} , D_{T-1} and D_{T-2} gives:

$$V_{T-1}(Y_{T-1}, D_{T-2}) = \max_{D_{T-1} \leq \bar{D}_{T-1}} \left\{ u \left[Y_{T-1} + D_{T-1} - (1+r)D_{T-2} \right] + \beta \mathbb{E}_{T-1} u[X_T] \right\} \quad (3.13)$$

Rewriting (3.13) in terms of the ratios:

$$\begin{aligned} V_{T-1}(Y_{T-1}, D_{T-2}) &= \max_{d_{T-1} \leq \bar{d}_{T-1}} \left\{ u \left[Y_{T-1} \left(1 + d_{T-1} - (1+r) \frac{d_{T-2}}{y_{T-1}} \right) \right] + \beta \mathbb{E}_{T-1} \left[u(Y_{T-1} y_T x_T) \right] \right\} \\ &= Y_{T-1}^{1-\gamma} \max_{d_{T-1} \leq \bar{d}_{T-1}} \left\{ u \left[1 + d_{T-1} - (1+r) \frac{d_{T-2}}{z_{T-1}} \right] + \beta \mathbb{E}_{T-1} \left[y_T^{1-\gamma} u(x_T) \right] \right\} \end{aligned}$$

Note that the maximization in the expression above satisfies the following Bellman Equation:

$$\tilde{V}_{T-1}(y_{T-1}, \xi_{T-1}, d_{T-2}) = \max_{d_{T-1} \leq \bar{d}_{T-1}} \left\{ u \left[1 + d_{T-1} - (1+r) \frac{d_{T-2}}{y_{T-1}} \right] + \beta \mathbb{E}_{T-1} \left[y_T^{1-\gamma} \tilde{V}_T(y_T, \xi_T, d_{T-1}) \right] \right\},$$

where $\tilde{V}_T(z_T, \xi_T, d_{T-1}) = u(w_T)$ as before. In this case, both the value function $\tilde{V}_T(z_T, \xi_T, d_{T-1}) = u(x_T)$ and the associated policy function $\tilde{f}_T(y_T, \xi_T, d_{T-1})$ are stationary. Therefore, (3.13) can be written as:

$$V_{T-1}(Y_{T-1}, D_{T-2}) = Y_{T-1}^{1-\gamma} \tilde{V}_{T-1}(y_{T-1}, \xi_{T-1}, d_{T-2}) \quad (3.14)$$

It is possible to use the same procedure to find all V_t in terms of Y_t and $\tilde{V}_t$. For

instance, in period $T - 2$ we have:

$$\begin{aligned}
V_{T-2}(Y_{T-2}, D_{T-3}) &= \max_{D_{T-2} \leq \bar{D}_{T-2}} \left\{ u \left[Y_{T-2} + D_{T-2} - (1+r)D_{T-3} \right] + \right. \\
&\quad \left. + \beta \mathbb{E}_{T-2} V_{T-1}(Y_{T-1}, D_{T-2}) \right\} \\
&= \max_{D_{T-2} \leq \bar{D}_{T-2}} \left\{ u \left[Y_{T-2} + D_{T-2} - (1+r)D_{T-3} \right] \right. \\
&\quad \left. + \beta \mathbb{E}_{T-2} \left[Y_{T-1}^{1-\gamma} \tilde{V}_{T-1}(y_{T-1}, \xi_{T-1}, d_{T-2}) \right] \right\} \\
&= \max_{d_{T-2} \leq \bar{d}_{T-2}} \left\{ u \left[Y_{T-2} \left(1 + d_{T-2} - (1+r) \frac{d_{T-3}}{y_{T-2}} \right) \right] \right. \\
&\quad \left. + \beta \mathbb{E}_{T-2} \left[Y_{T-2}^{1-\gamma} y_{T-1}^{1-\gamma} \tilde{V}_{T-1}(y_{T-1}, \xi_{T-1}, d_{T-2}) \right] \right\} \\
&= Y_{T-2}^{1-\gamma} \max_{d_{T-2} \leq \bar{d}_{T-2}} \left\{ u \left[\left(1 + d_{T-2} - (1+r) \frac{d_{T-3}}{y_{T-2}} \right) \right] \right. \\
&\quad \left. + \beta \mathbb{E}_{T-2} \left[y_{T-1}^{1-\gamma} \tilde{V}_{T-1}(y_{T-1}, \xi_{T-1}, d_{T-2}) \right] \right\} \\
&= Y_{T-2}^{1-\gamma} \tilde{V}_{T-2}(y_{T-2}, \xi_{T-2}, d_{T-3})
\end{aligned} \tag{3.15}$$

Therefore, the non-stationary problem (3.9) can be restated in terms of the stationary value function $\tilde{V}$ in the following form:

$$V_t(Y_t, D_{t-1}) = Y_t^{1-\gamma} \tilde{V}_t(y_t, \xi_t, d_{t-1}), \tag{3.16}$$

where:

$$\begin{aligned}
\tilde{V}_t(y_t, \xi_t, d_{t-1}) &= \max_{d_t \leq \bar{d}_t} \left\{ u \left[1 + \alpha_t - (1+r) \frac{d_{t-1}}{y_t} \right] + \beta \mathbb{E}_t y_{t+1}^{1-\gamma} \tilde{V}_{t+1}(y_{t+1}, \xi_{t+1}, d_t) \right\} \\
s.t : &\begin{cases} d_t \leq \bar{d}_t = \frac{\exp(\xi_t)}{\omega(1+r)} \\ y_t = \exp(g + \eta_t - \psi \eta_{t-1}) \\ \xi_{t+1} = \phi \xi_t + u_{t+1} \end{cases}
\end{aligned} \tag{3.17}$$

According to Deaton (1991), the solution of the finite horizon problem is unique and coincides with the infinite horizon problem if:

$$\gamma^{-1}(r - \delta) + \gamma \sigma_y^2 / 2 < g \tag{3.18}$$

Table 2: Values for $\gamma^{-1}(r - \delta) + \gamma\sigma_y^2/2$

		$\gamma = 2$			$\gamma = 4$		
		$\beta = 0.87$ ($\delta = 0.15$)	$\beta = 0.90$ ($\delta = 0.11$)	$\beta = 0.95$ ($\delta = 0.05$)	$\beta = 0.87$ ($\delta = 0.15$)	$\beta = 0.90$ ($\delta = 0.11$)	$\beta = 0.95$ ($\delta = 0.05$)
$r = 0.02$	$\sigma_\eta = 0.05$	-0.0617	-0.0426	-0.0133	-0.0264	-0.0168	-0.0022
	$\sigma_\eta = 0.10$	-0.0528	-0.0336	-0.0044	-0.0085	0.0011	0.0157
	$\sigma_\eta = 0.15$	-0.0379	-0.0187	0.0105	0.0214	0.0309	0.0456
$r = 0.03$	$\sigma_\eta = 0.05$	-0.0567	-0.0376	-0.0083	-0.0239	-0.0143	0.0003
	$\sigma_\eta = 0.10$	-0.0478	-0.0286	0.0006	-0.006	0.0036	0.0182
	$\sigma_\eta = 0.15$	-0.0329	-0.0137	0.0155	0.0239	0.0334	0.0481
$r = 0.04$	$\sigma_\eta = 0.05$	-0.0517	-0.0326	-0.0033	-0.0214	-0.0118	0.0028
	$\sigma_\eta = 0.10$	-0.0428	-0.0236	0.0056	-0.0035	0.0061	0.0207
	$\sigma_\eta = 0.15$	-0.0279	-0.0087	0.0205	0.0264	0.0359	0.0506

Notes: Bold numbers represent combinations in which the criteria is not satisfied.

The left hand side of equation (3.18) can be loosely understood as a "degree of patience", in which lower values means that the consumer is more impatient. If equation (3.18) is satisfied, then there is a unique $\tilde{V}(y_t, \xi_t, d_{t-1})$ that solves:

$$\tilde{V}(y_t, \xi_t, d_{t-1}) = u \left[1 + d_t - (1 + r) \frac{d_{t-1}}{y_t} \right] + \beta \mathbb{E}_t z_{t+1}^{1-\gamma} \tilde{V}(y_{t+1}, \xi_{t+1}, d_t), \quad (3.19)$$

with the associated policy-function $\tilde{f}(y_t, \xi_t, d_{t-1})$ for d_t . Note that both $\tilde{V}(y_t, \xi_t, d_{t-1})$ and $\tilde{f}(y_t, \xi_t, d_{t-1})$ are now stationary.

Regarding the model calibration, we follow the base-line parameter values proposed by Ludvigson (1999). For σ_η the value will be 0.05, β will be 0.87 and the auto-regressive parameter of income growth, ψ , is set to 0.44. For the credit-ceiling stochastic process, we set $\phi = 0.6$ and $\sigma_u = 0.01$. The interest rate r is 0.03 and the drift of income, g , is 0.02. Finally, we will simulate the model using $\omega \in \{8, 10, 15\}$.

Before showing some plots of the policy-functions generated by solving the model, table 2 shows the values of the LHS of equation (3.18). For some combinations, the values are bigger than $g = 0.02$, and therefore the convergence criterion is not satisfied in these situations (such cases are marked with bold numbers). Henceforth, the values on table 2 will be referred to as the degree of patience (lower values meaning more impatient consumers).

Figures 1 and 2 shows the variation of the policy function when changing some parameters of the model. The dashed 45 degrees line serves as a guide to understanding consumer behavior. When the policy function is above this line, the consumer is taking debt, meaning the opposite otherwise. Note that the policy-function is above the dashed line in almost all situations with the impatient consumer (figure 1). This happens because impatience, together with income being always positive, makes the consumer frequently prone to take debt. Furthermore, when the policy function is horizontal, it means that

the debt is at its limit or, in other words, that $\theta_t = w_t$.

A few points are worth mentioning. First, figure 1a shows that decreasing impatience (increasing β) moves the policy-function closer to the 45 degrees line, meaning that the consumption ratio will be smaller for a given level of debt ratio at $t - 1$. Moreover, the consumer will move towards the credit ceiling at higher levels of debt ratio. This happens because higher values of β make increases in consumption today, to detriment of future consumption, more costly. This is the same behavior that is verified in figures 1b and 1d, in which increased income uncertainty and risk aversion/IES⁷, respectively, induces the consumer to be more reluctant to take debt due to precautionary motives. The interest rate r plays a similar role, with higher interest rates moving the policy function closer to the 45 degrees line. However, this effect is small.

By analyzing the figure 1c, it is clear that higher levels of ω (tighter credit conditions) move the policy function closer to the 45 degrees line. Since there is less credit available, the consumer will simply take less debt. However, there is a more subtle implication of tighter credit conditions: debt reaches the ceiling of the credit limit at lower levels of debt-to-income ratios. This happens because, in this model, decreasing credit availability also decreases the volatility of the credit ceiling. Therefore, higher values of ω implies that if a bad shock happens to income when debt is at its maximum level, the impact on consumption will not be too strong. This effect induces the consumer to be less reluctant to take all credit available. This figure also reveals that credit conditions have a considerable impact on consumption, in particular when debt ratio levels are high.

Figure 2 shows what happens to the policy functions when the degree of patience is higher (meaning that the base-line parameters β , σ_η and γ are higher). Qualitatively, the results are the same as the ones obtained in Figure 1 in the sense that changes in the parameters move the policy-function in the same direction. However, in this case, there are much longer stretches of policy-functions that lie beneath the dashed line, meaning that the consumers are either saving (if $\alpha_{t-1} \leq 0$) or reducing debt (if $\alpha_{t-1} > 0$). Moreover, the policy functions are flat in fewer circumstances, meaning that the consumer is more reluctant to take debt to its limit. It is clear (by comparing Figures 1 and 2, together with Table 2) that the consumer is more prudent when the degree of patience is higher. This feature of the model has an important implication to consumption behavior: consumer's patience, in the sense defined above, is an important component of consumption smoothness and the predictability of consumption growth.

Figure 3 shows the policy function in terms of the contemporary shock to income growth η_t , where the dashed line is $d_{t-1} = 0$. Therefore, the plots show the optimal-behavior of a consumer with no debt nor assets. Furthermore, if the policy function is above the dashed line, it means that the consumer is taking debt.

⁷In CRRA utility functions, the parameter γ is both the risk aversion and the intertemporal elasticity of substitution.

Figure 1: Policy-functions as functions of d_{t-1} with impatient consumer

Base-line parameters: $\beta = 0.87, \sigma_\eta = 0.05, r = 0.03, \gamma = 2, \omega = 15, z_t = 1.05, \xi_t = 0$.

Figure 2: Policy-functions as functions of d_{t-1} with patient consumer

(a) β

(b) σ_η

(c) ω

(d) γ

(e) r

Alternative parameters: $\beta = 0.95, \sigma_\eta = 0.10, r = 0.03, \gamma = 4, \omega = 15, z_t = 1.05, \xi_t = 0$.

In every figure, the policy function is either flat or a decreasing function of η_t , the contemporaneous shock to income growth. When it is flat, the optimal behavior is to take all credit available, a situation in which the consumer would have a binding credit constraint. On the other hand, when the policy-function is a decreasing function of η_t , larger shocks to income growth in period t induces the consumer to take less debt. This behavior arises from the fact that income growth is negatively auto-correlated, meaning that positive income shocks today mean lower expected income growth tomorrow, and consequently the consumer behave more prudently. However, it is important to point out that the consumer in this model is very impatient. This can be seen by noting two features of the functions. First, all policy functions are above the dashed line, meaning that in all situations the consumer is taking debt. Second, the policy-function is flat in many situations, indicating that the consumer chooses to make the debt go from zero to the maximum in just one period. This feature of the model is even more evident in the next section, where the simulations show that the consumers constantly have binding credit constraints.

As a means of comparison, we raised the values of some parameters that increase the degree of patience. The results are presented in Figure 4. In this case, the policy function is flat in very few circumstances. Therefore the consumer does not use the available credit all at once. Nonetheless, the functions are sometimes below the dashed line, meaning that the debt-ratio is decreasing. In figure 4b in particular, the marked line when $\sigma_\eta = 0.25$ means that the patience criterion (3.18) is not being satisfied for that set of parameters. In this case, the policy function is below the dashed line for most of the time, being an illustrative example of how patient consumers take less debt or, in this case, save.

These results show that an important aspect of consumption under credit restrictions is that not only the credit restriction ω itself, but also some preferences and economic conditions have a significant impact on optimal behavior. Overall, it is expected that credit will be closely related to consumption if the consumer is in the region where consumption is at its maximum or, in other words, where the credit constraint is binding. Relaxing the credit constraint, however, has an ambiguous effect on the predictability of consumption. Depending on some deep parameters of the model, relaxing the credit constraint can either increase or decrease the dependency of consumption growth on credit growth. However, if credit conditions are too tight, consumer credit might not be very relevant to determine consumption even if the constraint is binding. This is explored in detail in the next section.

Figure 3: Policy-functions as functions of η_t with impatient consumer

Base-line parameters: $\beta = 0.87, \sigma_\eta = 0.05, r = 0.03, \gamma = 2, \omega = 15, \alpha_{t-1} = 0, \eta_{t-1} = 0, \xi_t = 0$.

Figure 4: Policy-functions as functions of η_t with patient consumer

Alternative parameters: $\beta = 0.95, \sigma_\eta = 0.10, r = 0.03, \gamma = 4, \omega = 15, \alpha_{t-1} = 0, \eta_{t-1} = 0, \xi_t = 0$.

4 Simulation and estimation

4.1 Simulation procedure

Now that it is shown that credit restrictions can significantly alter consumer behavior, the question that remains to be answered is how consumption predictability behaves if consumers are facing credit restrictions. To do so, we will use the policy functions generated when solving (3.17) to simulate credit, income, and consumption histories for a numerous set of parameters. With these, we will regress the following equation proposed by Ludvigson (1999):

$$\Delta c_t = \mu + \lambda E_{t-1} \Delta y_t + \pi E_{t-1} \Delta d_{t+1} + \epsilon_t, \quad (4.1)$$

where c_t , y_t and d_t are the aggregate log-consumption, aggregate log-income and aggregate log-debt, respectively.

The income growth process (3.3) proposed by MaCurdy (1982) is based on micro-data. Therefore, the results arising from the model must be interpreted as an individual consumer making consumption decisions, i.e. this is not a representative agent model. In order to scale these decisions up, we will simulate a number of individual histories and then explicitly aggregate them. However, if we just simulate income histories using equation (3.3) and aggregate them explicitly, the aggregate income growth would not be positively auto-correlated, a feature typically observed in actual data. To produce a positively auto-correlated aggregate income growth, we will follow Deaton (1991). Suppose that the income process of each consumer i is generated by three components: a common shock and two idiosyncratic shocks. They are given respectively by:

$$z_{1t} = \epsilon_{1t} + \chi \epsilon_{1t-1}, \quad \epsilon_{1t} \sim N(0, \sigma_1) \quad (4.2)$$

$$z_{2t}^i = \epsilon_{2t}^i, \quad \epsilon_{2t}^i \sim N(0, \sigma_2) \quad (4.3)$$

$$z_{3t}^i = \epsilon_{3t}^i - \epsilon_{3t-1}^i, \quad \epsilon_{3t}^i \sim N(0, \sigma_3) \quad (4.4)$$

$$\Delta y_t^i = g + z_{1t} + z_{2t}^i + z_{3t}^i \quad (4.5)$$

As the idiosyncratic components are assumed to be independent across individuals, summing a sufficiently large number of individual income processes will cancel out the idiosyncratic components, resulting in an aggregate income growth process that is given by $g + z_{1t}$. The individual does not see all components separately, and individual decisions are made based only on their sum, characterized by equation (3.3).

Following Deaton (1991) and Ludvigson (1999), we take $\chi = 0.5$ to match an aggregate sample auto-covariance of 0.4 and $\sigma_1 = 0.01$. The remaining parameters are given by:

$$\sigma_2^2 = (1 - \psi)^2 \sigma_\eta^2 - (1 + \beta)^2 \sigma_1^2 \quad (4.6)$$

$$\sigma_3^2 = \psi \sigma_\eta^2 + \beta \sigma_1^2 \quad (4.7)$$

Let H, T , and S be the number of households, periods, and simulations, respectively. For Tables 3, 4 and 5 we set $H = 1000$, $T = 350$ and $S = 1000$. All shock histories were held fixed throughout the simulations to ensure that the variations on the estimated coefficients are not due to shock variations. As the expected values for Δy_t and Δd_t in (4.1) are not observable, we use a 2SLS estimation. The first and second lags of income growth and credit growth, along with the constant term, are used as instruments. Consumption growth lags are not used as instruments because they are highly collinear with income growth lags, which causes numerical issues in the first stage regression.

Another variable of interest is the frequency with which households are actually credit constrained. As we solve the model using a grid of values for d_t , we interpolate the policy function between the grid-points when simulating the model. For that reason, we decided to consider situations where households are using at least 99% of their credit limit as a measure of bidding credit constraints, instead of the exact measure of 100%. To do so, define the following dummy variable for household i in period t of simulation s :

$$(CONSTR_{i,t})_s = \begin{cases} 0, & \text{if } (d_{t,i})_s < 0.99(\overline{d_{t,i}})_s \\ 1, & \text{otherwise} \end{cases} \quad (4.8)$$

The frequency of bidding credit constraints, ζ , is given by:

$$\zeta = \frac{\sum_{s=0}^S \sum_{t=0}^T \sum_{i=0}^H (CONSTR_{i,t})_s}{STH}, \quad \text{where } \zeta \in [0, 1] \quad (4.9)$$

The simulation procedure is as follows: S shocks histories with T periods and H households are drawn from the underlying distributions, and histories for z_t^i and ξ_t are constructed using $z_t^i = \exp(\Delta y_t^i)$ and equation (3.4), respectively. With that, it is possible to use the policy function $\tilde{f}(z_t, \xi_t, \alpha_{t-1})$ found by solving (3.17) and obtain a history of consumption-ratios for each consumer i . Before that, it is necessary to set an initial debt-ratio, which we will set zero for all consumers, implying that they start with no debt nor assets. Up to this point, we have consumption-ratios and debt-ratios, and to properly estimate the equation (4.1) we need the actual consumption and debt log-variations. It is possible to simply guess an initial income for each consumer. However, to prevent the bias effects of such guesses, we set an initial income of 1 to all consumers and then simulate $2T$ periods of income growths. Then, we use the last T periods of these histories in the simulations to be regressed. Having individual histories, they are explicitly aggregated over H households and the result is used to regress (4.1).

4.2 Results

Table 3 shows some features of the simulated data. It is clear from the results that consumption becomes smoother as the consumer patience or prudence increases. Moreover, for sufficiently impatient consumers, consumption growth is less smooth than income. This odd result is of particular importance given that consumption smoothness relative to income is a stylized fact of actual data. The consumption-to-income ratio is also usually lower in actual aggregated data⁸. However, though seemingly odd, these features of the model are expected given its characteristics. According to Carroll (2001), in 'buffer stock models, such as this one, consumers will engage in a target saving behavior. Once the target level of assets is attained, and the distribution of households has converged to its ergodic distribution, then consumption growth will be equal to income growth. This can explain why $C_t/Y_t \approx 1$ in all cases.

In the present model, this target level of assets would be analog to a target level of debt. However, there are some differences. First, debt is a costly precaution, while assets yield returns. Second, the desired asset level takes time to be built, whereas the debt-level can be reached in the first period as credit is readily available. This feature of the model increases the convergence between consumption and income growth (usually it takes only one or two periods), and it is important to understand the results to follow. It is also why income is smoother than consumption in some cases. If the consumer is impatient enough, consumption will reach the maximum in every period. In this situation, consumption will be subjected not only to income shocks but also to the credit ceiling shocks, hence increasing the consumption volatility relative to income.

To study the effects of credit restrictions on the estimated coefficients of equation (4.1), the methodology proposed here is to use the variable ω as control of credit supply and the discount factor β as control of credit demand. Aided by figure 1c, and by equation (3.4), it is evident that lower values of ω mean more credit available. Thus credit supply increases from bottom-up on tables 4 and 5. Likewise, the figure 1a shows that higher values of β mean less credit demand. Therefore credit demand increases from left to right on the same tables. Consequently, credit supply and demand are highest at the upper left corner, and lowest at the bottom right corner. Furthermore, the lines *Mean* and *Std* are the mean value and the standard deviation, respectively, of each coefficient across S regressions for a given set of parameters. The lines labeled *Shea's R^2 (1st stage)* are the Shea's partial R^2 statistic from the first stage of the regression (the values on the columns $\hat{\lambda}$ are related to the first stage regression of income growth, and the values on the columns $\hat{\pi}$ are related to the first stage regression of credit growth). The lines *Significance (5%)* and *Significance (10%)* are the percentage of estimated coefficients significant at 5% and 10% level of significance or better, respectively.

⁸Using U.S quarterly data from 1950 to 2019, $Var(\Delta c_t)/Var(\Delta y_t) \approx 0.36$ and the mean value of C_t/Y_t is 0.82.

Table 3: Sample correlations and variance-ratios, $\gamma = 2, r = 0.03$

$\sigma_\eta = 0.05$					
β	ω	$Corr(\Delta c_t, \Delta y_t)$	$Var(\Delta c_t)/Var(\Delta y_t)$	C_t/Y_t	ζ
0.87	8	0.9333	1.3375	0.9992	0.9977
0.87	10	0.9436	1.2706	0.9993	0.9984
0.87	15	0.9642	1.1701	0.9996	0.9995
0.91	8	0.9524	1.2849	0.9992	0.9973
0.91	10	0.9618	1.2239	0.9993	0.9974
0.91	15	0.9752	1.1449	0.9996	0.9985
0.95	8	0.9743	1.1745	0.9992	0.9703
0.95	10	0.9805	1.1349	0.9993	0.9672
0.95	15	0.9886	1.0845	0.9996	0.9631
$\sigma_\eta = 0.1$					
0.87	8	0.9516	1.2425	0.9992	0.9496
0.87	10	0.9615	1.1906	0.9994	0.9531
0.87	15	0.9773	1.1199	0.9996	0.9667
0.91	8	0.9683	1.1493	0.9992	0.8685
0.91	10	0.9747	1.1116	0.9994	0.8612
0.91	15	0.9852	1.0588	0.9996	0.8477
0.95	8	0.9781	1.0126	0.9992	0.6763
0.95	10	0.9814	0.9934	0.9994	0.6655
0.95	15	0.9869	0.9646	0.9996	0.6478

Table 4 presents the results for $\sigma_\eta = 0.05$. There are a few points worth mentioning. Firstly, $\hat{\pi}$ responds positively both to credit demand and supply, with the highest value being precisely at the best credit conditions (highest supply) and at the highest willingness to borrow (highest demand). In this case, $\hat{\pi}$ is inversely proportional to the credit constraint: higher $\hat{\pi}$ means looser constraints. Moreover, there is a trade-off between $\hat{\pi}$ and $\hat{\lambda}$: increases in one are always accompanied by decreases on the other. In other words, as consumption is more reliant on credit, consumption growth is less sensitive to expected income growth. The effect of β is on the expected direction, with higher values having a negative impact on the credit coefficient. However, varying the value of β from 0.87 to 0.91 has a very small effect. The reason is that, in this range, there is no significant change in consumer behavior. An evidence of that is the values of ζ for these parameters on table 3: there is no significant change on its value when varying β from 0.87 to 0.91 for $\sigma = 0.05$ ⁹, meaning that the consumer is always at the maximum level of debt regardless of β being 0.87 or 0.91.

The coefficients $\hat{\pi}$ and $\hat{\lambda}$ are very significant, with both coefficients having 5% or better levels of significance in more than 95% of the simulations for the majority of cases. The exception is with $\beta = 0.95$, in which case the significance of $\hat{\pi}$ drops to approximately 78% of estimations at the 5% level. An important result, in this case, is that the significance

⁹A value as low as 0.60 for β does not change the results significantly for this income uncertainty. In practice, what happens in these cases is that consumers are always consuming all they can in each period.

of $\hat{\pi}$ increases with ω , meaning that more credit supply can be the reason behind the non-significance of the credit coefficient (this effect is clearer in table 5). This effect of credit supply provides theoretical evidence to the common hypothesis that the lack of significance in such regressions is caused by liberalized credit markets, even though $\hat{\pi}$ is still inversely proportional to credit constraints. For example, Brady (2008) concludes that a liberalized consumer credit environment is the probable reason why $\hat{\pi}$ is not significant in some samples of U.S aggregate data. It is necessary to point out, however, that in this level of income uncertainty consumers are so impatient that the credit constraint is almost always binding for all households, regardless of credit constraints (check the values of ζ in table 3), a situation not likely to occur in practice.

Table 5 results, where $\sigma_\eta = 0.10$, are qualitatively similar to the ones on table 4. However, there are some exceptions. Firstly, when income is more uncertain, β has a stronger impact both on the mean value and on the significance of $\hat{\pi}$. Secondly, $\hat{\pi}$ is smaller when income uncertainty is larger for every pair of β and ω , whereas $\hat{\lambda}$ is larger in the same comparison, reflecting the trade-off aforementioned. As income becomes more uncertain, the consumer is more reluctant to take debt, diminishing the sensitivity of consumption growth to expected credit growth. The values of ζ for these parameters show this reluctance, reaching 0.6478 in the worst credit conditions.

Furthermore, for a given ω , the credit coefficient mean value and significance are smaller as β grows. In the worst case, the significance of 5% or better is achieved only in approximately 39% of the estimations. On the other hand, for a fixed value of β , the significance of $\hat{\pi}$ increases with higher values of ω . These results are similar to the ones presented in table 4, but in this case, the credit constraint is binding in a lower frequency for some sets of parameters, which is more realistic than the previous case. This result strengthens our conclusion that the lack of statistical significance of the credit coefficient can indeed mean more credit supply.

The mean value of $\hat{\pi}$ always decreases as β increases, *ceteris paribus*. However, the behavior is ambiguous when increasing ω while maintaining β constant. This is the most important conclusion of our paper: the impact of increasing credit supply on consumption predictability depends on consumers' impatience. If consumers are impatient enough, increases in credit supply will increase the dependence of consumption growth to credit growth. If, on the other hand, consumers are less impatient, increases in credit supply will have the opposite effect.

Lastly, it is important to point out that the simulated values of both the income coefficient $\hat{\lambda}$ and the credit coefficient $\hat{\pi}$ are bigger than the usual values obtained when using U.S aggregate data, a fact that was already documented by Ludvigson (1999). In fact, for some set of parameters, $\hat{\lambda}$ is close to or bigger than 1. Even though this result might suggest that consumption is not smoother than income, it happens precisely in the cases where consumption is smoother relative to income. In our understanding, the high

values of $\hat{\lambda}$ have two reasons. The first is the relatively poor results of the first stage of the regression of $\log(\Delta Y_t)$, represented by the Shea's partial R^2 . As the first stage is a projection of $\log(\Delta Y_t)$ on the sub-space of the instruments, the fitted value used on the second-stage is more flattened than actual income growth. Second, as we said before, in this model consumption growth and income growth are highly collinear. As the consumption growth must be explained by the flattened income growth, the coefficient is higher than 1.

Our conclusion is that this model represents consumers that rarely save, and that is why the coefficients are high when compared to actual data¹⁰. In this situation, consumption must be financed by either income or credit. In reality, this type of consumer must represent only a fraction of consumers, and it would be necessary to introduce some heterogeneities on the model to make it able to reproduce actual data behavior. However, it is reasonable to assume that, given the results above, the dependence of consumption growth to credit growth is produced by impatient consumers. Therefore, if either actual $\hat{\lambda}$ or $\hat{\pi}$ are high, then a considerable amount of consumers should behave as this model predicts.

¹⁰Ludvigson (1999) estimates of $\hat{\lambda}$ and $\hat{\pi}$ using U.S. aggregate data ranges from 0.265 to 0.366 and from 0.110 to 0.066, respectively.

Table 4: $\Delta \log C_t = \mu + \lambda E_{t-1} \Delta \log Y_t + \pi E_{t-1} \Delta \log D_{t+1}$, Baseline parameters : $r = 0.03, \gamma = 2, \sigma_\eta = 0.05$

		$\beta = 0.87$			$\beta = 0.91$			$\beta = 0.95$		
		$\hat{\mu}$	$\hat{\lambda}$	$\hat{\pi}$	$\hat{\mu}$	$\hat{\lambda}$	$\hat{\pi}$	$\hat{\mu}$	$\hat{\lambda}$	$\hat{\pi}$
$\omega = 8$	Mean	0.0028	0.4743	0.3870	0.0028	0.4534	0.4081	0.0001	0.8773	0.1149
	Std	0.0016	0.1365	0.0997	0.0016	0.1434	0.1099	0.0014	0.1480	0.0908
	Shea's R^2 (1 st stage)	-	0.1292	0.1261	-	0.1225	0.1191	-	0.0876	0.1318
	Significance (5%)	0.7890	0.9490	0.9990	0.7570	0.9140	0.9990	0.2160	0.9940	0.7830
	Significance (10%)	0.8710	0.9610	0.9990	0.8430	0.9400	0.9990	0.3380	0.9970	0.8100
$\omega = 10$	Mean	0.0022	0.5863	0.3014	0.0022	0.5706	0.3170	0.0001	0.8810	0.1136
	Std	0.0014	0.1118	0.0757	0.0014	0.1156	0.0820	0.0014	0.1319	0.0760
	Shea's R^2 (1 st stage)	-	0.1345	0.1289	-	0.1295	0.1228	-	0.0902	0.1279
	Significance (5%)	0.7500	0.9880	0.9990	0.7170	0.9850	0.9990	0.2100	0.9960	0.8640
	Significance (10%)	0.8300	0.9930	0.9990	0.8010	0.9920	0.9990	0.3240	0.9970	0.8850
$\omega = 15$	Mean	0.0015	0.7361	0.1897	0.0015	0.7303	0.1954	0.0000	0.9105	0.0876
	Std	0.0009	0.0657	0.0398	0.0009	0.0660	0.0416	0.0006	0.0643	0.0392
	Shea's R^2 (1 st stage)	-	0.1543	0.1508	-	0.1509	0.1457	-	0.0998	0.1498
	Significance (5%)	0.8120	0.9980	1.0000	0.7940	0.9980	1.0000	0.2000	0.9990	0.9490
	Significance (10%)	0.8950	0.9980	1.0000	0.8780	0.9980	1.0000	0.2890	0.9990	0.9560

Table 5: $\Delta \log C_t = \mu + \lambda E_{t-1} \Delta \log Y_t + \pi E_{t-1} \Delta \log D_{t+1}$, Baseline parameters : $r = 0.03, \gamma = 2, \sigma_\eta = 0.10$

		$\beta = 0.87$			$\beta = 0.91$			$\beta = 0.95$		
		$\hat{\mu}$	$\hat{\lambda}$	$\hat{\pi}$	$\hat{\mu}$	$\hat{\lambda}$	$\hat{\pi}$	$\hat{\mu}$	$\hat{\lambda}$	$\hat{\pi}$
$\omega = 8$	Mean	0.0018	0.7010	0.2063	-0.0012	1.0180	0.0414	-0.0051	1.2435	0.0109
	Std	0.0020	0.1899	0.1094	0.0018	0.1437	0.0645	0.0014	0.0788	0.0276
	Shea's R^2 (1 st stage)	-	0.0713	0.1174	-	0.0752	0.1806	-	0.1294	0.4510
	Significance (5%)	0.5480	0.9600	0.9250	0.5890	0.9990	0.5610	1.0000	1.0000	0.2190
	Significance (10%)	0.6630	0.9670	0.9440	0.6630	0.9990	0.6360	1.0000	1.0000	0.2790
$\omega = 10$	Mean	0.0016	0.7328	0.1856	-0.0011	0.9854	0.0676	-0.0050	1.2367	0.0140
	Std	0.0015	0.1504	0.0886	0.0016	0.1240	0.0532	0.0013	0.0728	0.0232
	Shea's R^2 (1 st stage)	-	0.0736	0.1132	-	0.0785	0.1729	-	0.1399	0.4383
	Significance (5%)	0.5420	0.9810	0.9620	0.6060	0.9980	0.7710	1.0000	1.0000	0.2660
	Significance (10%)	0.6620	0.9840	0.9750	0.6610	0.9980	0.8110	1.0000	1.0000	0.3180
$\omega = 15$	Mean	0.0013	0.7934	0.1421	-0.0013	1.0225	0.0417	-0.0049	1.2273	0.0167
	Std	0.0011	0.0933	0.0513	0.0010	0.0724	0.0294	0.0011	0.0536	0.0150
	Shea's R^2 (1 st stage)	-	0.0842	0.1271	-	0.0822	0.2136	-	0.1730	0.4931
	Significance (5%)	0.6390	0.9970	0.9900	0.8460	0.9990	0.7990	1.0000	1.0000	0.3890
	Significance (10%)	0.7710	0.9970	0.9970	0.8620	1.0000	0.8260	1.0000	1.0000	0.4820

5 Final Remarks

Understanding the relationship between consumption and credit is increasingly relevant in an era of liberalized credit markets. Empirical evidence supporting the fact that consumption growth can be predicted by expected consumer credit growth has been documented by Ludvigson (1999), Bacchetta and Gerlach (1997), and Brady (2009), to mention a few. What remained to be answered is how tighter credit conditions will influence the dependence of consumption on credit.

In this paper, we firstly analyzed the properties of the model proposed by Ludvigson (1999). The model shows that credit restrictions are an important component of consumption, along with preferences and income uncertainty. It suggests that tighter credit restrictions might increase the dependence of consumption on credit. However, we hypothesize that, within the scope of such models, more credit does not necessarily mean less consumption predictability.

To test this hypothesis, we used the solution of the model to generate consumption, income, and credit histories. Regressing the linear regression proposed by Ludvigson (1999) for different sets of credit supply (controlled by the parameter ω) and credit demand (controlled by β), we show that, if the consumer is sufficiently impatient, the coefficient related to expected credit growth is higher when credit conditions are more favorable, being smaller on tighter credit conditions. This result seems to be at odds with what is commonly understood when regressing similar equations, as the credit coefficient in this situation represents a measure of the credit supply instead of the measure of credit restrictions. The significance of the coefficient, on the other hand, goes in the expected direction, with less credit supply resulting in diminished significance.

However, when consumer income is more uncertain, there is not a clear behavior of the credit coefficient. The impact of increased access to credit, in this situation, depends on how impatient the consumer is. If the consumer is sufficiently patient, the mean value and the significance of the coefficient drops as credit supply increases. This theoretical result supports the hypothesis that liberalized credit markets can be the reason why credit growth is not a significant predictor of consumption growth in some samples, but at the same time shows that there is ambiguity on the behavior of consumption predictability facing swings in credit supply.

We conclude that the model can provide some useful insights and provide a theoretical basis to put in question a common hypothesis of consumption literature, at least to some degree. At the same time, the models fail to reproduce some stylized facts of actual aggregate consumption, for example, a reasonable range for both the credit and the income coefficients. This failure is expected given the nature of the model, in which it represents a consumer that is highly in debt. As this situation must represent only part of actual consumers, we believe that a further research development could be to introduce some

heterogeneity in the model, such as consumers facing different income uncertainties and different interest rates.

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