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Hydrodynamic studies of atomic fluids in quantum regime

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Hydrodynamic studies of atomic fluids in quantum regime

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A mi amada esposa Shirley y a
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“Far better it is to dare mighty things, to win glorious triumphs - even though checkered by failure, than to take rank with those poor spirits who neither enjoy much nor suffer much, because they live in the gray twilight that knows not victory nor defeat.”

Theodore Roosevelt (1858 - 1919)

Abstract

MERCADO-GUTIÉRREZ, E. D. *Hydrodynamic studies of atomic fluids in quantum regime*. 2021. 124p. Thesis (Doctor in Science) - Instituto de Física de São Carlos, Universidade de São Paulo, São Carlos, 2021.

In this thesis, we produce a Bose-Einstein condensate of sodium atoms in a magnetic plugged trap and in an optical dipole trap. First, we investigate the critical velocity by moving an obstacle through the condensate, comparing the result obtained with the the sound velocity. We make use of the critical velocity value to determine the righth parameters to stir the condensate and nucleate quantized vortices, by introducing angular with the rotation of the srirring beam the number of vortices increases linearly until the rigid-body limit. In the second part we introduce potassium atom in the science chamber where from the begining of the MOT stage are observed lighth induced losses. In a single-specie operation we are able to cool down the potassium atoms by implementing the gray molasses technique, increasing its phase-space density in order have a good tranference efficiency to the magnetic trap. We also present the the sympathetic cooling of the K atoms due by elastic collision between the atomic species.

Keywords: Vortices. Vortex lattice. Three-body losses.

Resumo

MERCADO-GUTIÉRREZ, E. D. *Estudos hidrodinâmicos de misturas de fluidos atômicos em regime quântico degenerados*. 2021. 124p. Tese (Doutorado em Ciências) - Instituto de Física de São Carlos, Universidade de São Paulo, São Carlos, 2021.

Nesta tese, produzimos um condensado de Bose-Einstein de átomos de sódio em uma armadilha magnética plugada e em uma armadilha dipolo óptica. Primeiramente, investigamos a velocidade crítica movendo um obstáculo através do condensado, comparando o resultado obtido com a velocidade do som. Fazemos uso do valor da velocidade crítica para determinar os parâmetros adequados para agitar os vórtices quantizados condensados e nucleados, introduzindo angular com a rotação do feixe principal, o número de vórtices aumenta linearmente até o limite do corpo rígido. Na segunda parte, introduzimos o átomo de potássio na câmara de ciências, onde desde o início da fase MOT são observadas as perdas induzidas por luz. Em uma operação de uma única espécie, somos capazes de resfriar os átomos de potássio implementando a técnica do melaço cinza, aumentando sua densidade de espaço de fase para ter uma boa transferência de forma eficiente para a armadilha magnética. Apresentamos também o resfriamento simpático dos átomos de K devido à colisão elástica entre as espécies atômicas.

Palavras-chave: Vortices. Rede de Vortices. Perda de três corpos.

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Introduction

The superfluidity was discovered in 1938 by Kapitza Allen and Missener³⁻⁴ observed experimentally in ^4He liquid at a temperature below 2.18 K. At the moment, the main feature of superfluidity was that a such a fluid has no viscosity and no dissipation. In 1941, Landau showed the first model for superfluidity in which he established a critical velocity below which a fluid could exhibit superfluid behavior. As a consequence of the phenomenological description of Landau, Onsager in 1949 and Feynman in 1955, demonstrated that such quantum fluids are irrotational systems in which rotation takes place through the formation of vortices. Such vortices were defined as a holes inside the superfluid, with the phase of the wanvefunction goinf to infinite, and vanishing points density. In this case, the circulation around a close contour around the vortex is quantized in units of h/m , where h is Planck's constant and m is the mass of the fluid.

The firs experimental observation of quantized vortices in a superfluid, was maded by Viden, 1961⁵ by measuring the quantized circulation in liquif helium II in 1979. Moreover, a better direct confirmation was possible by detecting the flow of ions inside a superfluid.⁶ Regardless of the favorable orientation of the vortices, when they intertwine without orientation they give rise to a regime called quantum turbulence, which was predicted by Feynman in 1955, and observed in 1957.⁷⁻⁹

The study of superfluids changes completely since the experimental achievement of Bose-Einstein condensation in dilute gases of ^{87}Rb ,¹⁰ ^{23}Na ¹¹ and ^7Li ¹² and many others. Bose-Einstein condensation are characterized by the macroscopic quantum system with all the particle occuppying the ground state. The first path to achieve thism were initially proposed by Satyendra Nath Bose in 1924¹³ in the context of photons, later extended by Einstein to bosonic particles in 1925.¹⁴ The attempt to show that the Bose-Einstein condensate (BEC) was also a superfluid, consequently led to the experimental obtaining of vortices in these systems,^{1,15} where basically rotation was introduced by rotating the

condensate with a repulsive optical potential, commonly called as stirring beam. With this technique, not only one vortex was created, but many lines of vortices, whose quantity depended linearly on the rotation frequency, where the repulsive interaction between them caused them to organize themselves in triangular lattices, known as the Abrikosov lattices.^{16–17} Condensates with more than one hundred vortices have been produced when the rotation frequency is increased to the confinement radial frequency.^{2,18}

This thesis is organized in two parts, the first one contains three chapters where we introduce the definition and properties of Bose-Einstein condensates, governed by the Bose statistics and explained through the Gross-Pitaevskii equation (GPE) the experimental realization of a BEC as the results as well. In the chapter 1, we will present this, together with the fluids dynamic interpretation of the BEC's a formalism introduced by Madelung.¹⁹ Also, with this approach, we introduce concepts of circulations and velocity fields, important to define how vortices origins in the condensate.

In chapter 2 we present the experimental details about the production of a Bose-Einstein condensate of Sodium atoms (^{23}Na) in a magnetic and optical traps, starting from the design of the ideal environment to obtain the Sodium atoms in vacuum chamber to the evaporating cooling process, where the phase-space density of the atoms are increased until reach the condensation.

The first part ends with the results chapter. where we will present the critical velocity of our BEC, essential to determine the minimum energy we need to introduce excitations into the condensate. Also, we will present the nucleation of a triangular vortex lattice as a consequence to agitate the superfluid with an focused optical barrier.

Part 1

Bose-Einstein condensation and vortex lattices

Chapter 1

Bose-Einstein condensation theory

The notion of quantum particles was induced by the physics Satyendra Bose in 1920, deriving Planck's empirical formula considering two things: the radiation particles are indistinguishable and the discretization of phase space (x, p) into a cell of size h^3 . These results given to light a quantum character in which the light is also considered as particles,¹³ Einstein takes this idea to extend the concept to particles with mass in the paper "Quantum theory of the mono-atomic ideal gas".¹⁴

It is possible to classify the quantum particles into two types, Bosons and Fermions. The first one, obeys to the Bose-Einstein statistics, particles with integer spin, as for example ^4He , ^{39}K and ^{23}Na . Bosons are particles with the property to occupy the same quantum state simultaneously. On the other hand, Fermions are particles with half-integer spin (e.g. protons, neutrons, and electrons) following the Fermi-Dirac statistics.²⁰ Contrary to Bosons, two identical Fermions can not occupy the same quantum state simultaneously due to the Pauli exclusion principle.²¹ In this chapter we are dealing with Bosons, specifically ^{23}Na , and in the next part, we will explore the bosonic mixture of $^{23}\text{Na} - ^{39}\text{K}$.

1.1 Bose-Einstein statistics

To define some properties of the quantum particles as Bosons, we will start considering a system of N particles confined in a box potential. The mean distribution of the particles over the energy E in an ideal gas is given by the Bose-Einstein distribution function:

$$f_{BE}(E) = \frac{1}{\exp^{(E-\mu)/k_B T} - 1}, \quad (1.1.1)$$

where μ is the chemical potential, k_B the Boltzmann's constant, and T , the temperature of the particles. For each energy, the number of particles can be expressed as:

$$N(E) = f_{BE}(E)g(E) = \frac{g(E)}{e^{(E-\mu)/k_BT} - 1}, \quad (1.1.2)$$

with $g(E)$ the density of states. Then, the total number of particles N is obtained integrating the equation 1.1.2 over the total energy range of the system E . If we consider that the separation between the ground state and the excited states is higher compared with the separation between all the excited states, we can split the energy range into two branches, the energy of the ground state and all the excited states. Thus, the total atom number can be written like this:

$$N = N_0 + \int N(E)dE, \quad (1.1.3)$$

with $E \neq 0$ (ground state), and N_0 the number of particles in the ground state. Depending on where the particles are confined, the density of state is different. In the 3D box potential case, $g(E) \propto E^{1/2}$, and the integral in the equation 1.1.3 can be solved like this:

$$\int N(E)dE \propto \int \frac{E^{1/2}dE}{e^{(E-\mu)/k_BT} - 1} \propto \int \frac{x^\alpha dx}{e^x/z - 1} = \Gamma(\alpha + 1)g_{\alpha+1}(z), \quad (1.1.4)$$

with, $\alpha = \frac{1}{2}$, $x = E/k_BT$ and $z = e^{\mu/k_BT}$. Here, Γ is the gamma function²² and g a new function defined as $g_\beta(z) = \sum_{p=1}^{\infty} \frac{z^p}{p^\beta}$. With this, the integral in the equation 1.1.4 reduces to

$$\int N(E)dE = \frac{(2\pi mk_BT)^{3/2}V}{h^3}g_{3/2}(z). \quad (1.1.5)$$

Adding more particles at a constant temperature to the system, the function $g_{3/2}$ also increases, but there is a point in which this function saturates, this is for $z = 1$ ($\mu = 0$) becoming the gas saturated, and N_0 start to grow. At this point, occurs the Bose-Einstein condensation. In other words, when the inter-particle distance is to the “same” order as the thermal de Broglie wavelength the density of particles in the ground state starts to be significant, given origin to the Bose-Einstein condensate as follows:

$$n\lambda^3 = g_{3/2}(1) = 2.612,$$

where, $n = N_c/V$ and $\lambda = h/\sqrt{2\pi m k_B T}$.

In the same way, we can estimate the critical temperature in which the system becomes condensate, by taking into account equation 1.1.5 and consider that the volume and the total particle number is not constant anymore, in this case, exist a temperature in which we can see the Bose-Einstein condensate transition, rearranging for T we got:

$$T_c = \frac{h^2}{2\pi m k_B} \left(\frac{N}{g_{3/2}(1)V} \right)^{2/3}.$$

When $T < T_c$ we have fewer particles in excited states, allowing to populate the number of particle in the ground state N_0 . In the $T \rightarrow 0$ limit, all the particles will populate the ground state. This result has shown the critical number and temperature in which the Bose-Einstein condensation occurs in the case of an ideal gas in a box potential. Following a similar approach, we can obtain the critical temperature in the case of a gas trapped in harmonic trap, given by

$$T_c = 0.941 \frac{\hbar \bar{\omega}}{k_B} N^{1/3},$$

with $\bar{\omega}$ is the geometric mean of frequencies of the harmonic potential, and could be considered in analogue with the volume of the box .

1.2 Gross-Pitaevskii model of the condensate

In this part, we will show the quantum model that describes the BEC with interactions. This description is relevant in order the understand how we can see the BEC in the experiments, for example, the density distribution of the atoms and the effects of the strength of the interaction between them.

We assume that we are in a regime in which $T \ll T_c$, this means that the thermal-driven excitations of the condensate are no considered. An exact description of the Bose particles can be obtained by considering the wavefunction for the N particles of the system, $\psi(r_1, r_2, \dots r_N, t)$. But considering the complexity of the solution when the interaction and a large number of particles are considered, this approach makes impossible to solve the

problem analytically or even numerically, taking into account that in real experiments we have millions of atoms condensed.

Considering that the number of particles N is larger than one and we have only a few atoms not condensed, the many-body wavefunction ψ can be replaced by an effective single-particle wavefunction $\Psi(r, t)$, which means that effectively we can treat the condensate as a single big atom with a macroscopic matter wavefunction. In this way, we can write the single-particle Hamiltonian in second quantization formalism as

$$H = \int d^3r \Psi^\dagger H_0 \Psi + \frac{1}{2} \int d^3r \int d^3r' \Psi^\dagger(r) \Psi^\dagger(r') V_{int}(r - r') \Psi(r) \Psi(r'), \quad (1.2.1)$$

where, $H_0 = \frac{p^2}{2m} + V_{ext}(r)$ is the Hamiltonian without the interaction potential, with $V_{ext}(r)$ being the external potential, Ψ and $\Psi^\dagger$ are the bosonic creation and annihilation operators, and V_{int} is the interaction potential between two particles. The dynamics of the bosonic operators can be represented by the Heisenberg picture as follows:

$$i\hbar \partial_t \Psi(r, t) = [\Psi(r, t), H] = \left\{ H_0(t) + \int d^3r' \Psi^\dagger(r, t) V_{int}(r - r') \Psi(r, t) \right\} \Psi(r, t) \quad (1.2.2)$$

The field operators Ψ and $\Psi^\dagger$ in the equation above, are related with the single-particle eigenfunction Φ and the creation and annihilation operators $\hat{a}^\dagger$ and $\hat{a}$ and as follows:

$$\Psi(r) = \sum_i \Phi_i(r) \hat{a}_i \Psi^\dagger(r) = \sum_i \Phi_i^\dagger(r) \hat{a}_i^\dagger \quad (1.2.3)$$

The sum runs over all the atomic states ($i = 0$ corresponds to the ground state and $i \neq 0$ corresponds to all excited states). Given the fact that we are dealing with a big number of particles $N \gg 1$ we can approximate the creation and annihilation operators by its mean value, thus, $\hat{a}^\dagger \approx \hat{a} \approx \sqrt{N_0}$. This is the so-called Bogoliubov approximation.²³ Under this approximation the condensate is assumed to contain almost all the particles this means that $N - N_0 \approx N_0$, also can say that the wavefunction can be split into two parts, the first one, representing the majority of the condensed particles and the second as the excited state fluctuations of the system, this can be written as

$$\Psi(r) = \Psi_0(r) + \delta\Psi. \quad (1.2.4)$$

The mean-field approximation neglect these fluctuations, considering that in the mean value, all particles are in the ground state. In this situation, as the gas is dilute enough, the three-body interaction is negligible and the dominant interaction between the particles is governed by the Van der Waals interactions between the neutral atoms. Thus, the interaction potential $V_{int}(r - r')$ in equation 1.2.2 can be replaced by a contact interaction potential

$$V_{int}(r - r') \rightarrow g\delta(r - r') = \frac{4\pi\hbar^2 a_s}{m}\delta(r - r'), \quad (1.2.5)$$

where the interaction strength g is proportional to the s-wave scattering length a_s , usually defined as the interaction between the atoms in low energy limit.¹⁹ This value can be positive or negative, depending on the atomic species, being difficult to determine its precise value, having complicated expression that will depend on the magnetic field and other factors.²⁴ For now, the value of the scattering length for Sodium is around $a_s(Na) \approx 50 a_0$, where a_0 is the Bohr's radius.

1.2.1 Gross-Pitaevskii equation

Replacing the potential in equation 1.2.5, the Heisenberg equation (1.2.2), and also considering the mean-field approximation, we derive the equation that describes the dynamics of the Bose-Einstein condensate, the time-dependent Gross-Pitaevskii equation (GPE),

$$i\hbar\partial_t\Psi_0(r, t) = \left(-\frac{\hbar^2}{2m}\nabla^2 + V_{ext}(r) + g|\Psi_0(r, t)|^2\right)\Psi_0(r, t). \quad (1.2.6)$$

The two first terms on the right, correspond to the kinetic energy and external potential of the condensed atoms, meanwhile the last term arise from the interaction between the particles. The sign of the interaction will determine if the atoms experience repulsion ($g > 0$), or attraction ($g < 0$), the meaning of the sign in the interaction term will be discussed later.

On the other hand, the solution of the GPE can be written as the product of a time-independent amplitude and a time-dependent phase, which satisfy,

$$\Psi_0 = \psi(r)e^{-i\mu t/\hbar}, \quad (1.2.7)$$

where μ is the chemical potential and $\psi(r)$ the stationary solution. The exponential term represents how the phase of the BEC can move freely in time, while the density given by the module square of the amplitude $\psi(r)$ remains unaffected. Inserting this ansatz in the time-dependent GPE and considering that the external potential is always independent of time, we obtain the time-independent GPE equation:

$$\mu\psi = \left(-\frac{\hbar^2}{2m}\nabla^2 + V_{ext}(r) + g|\psi(r)|^2 \right) \psi(r). \quad (1.2.8)$$

We have to notice that the eigenvalue in the equation above is not the energy as in the linear Schrodinger equation, instead is the chemical potential μ which is defined as the energy per particle that the system needs to add or remove particles into the system and can be defined as,

$$\mu = \frac{1}{N} (E_{kin} + E_{pot} + 2E_{int}),$$

or in a general form $\mu = \partial E / \partial N$. Here, those energies correspond to the contribution in the energy of each term in the equation 1.2.8, like this:

$$E = \int \left[\frac{\hbar^2}{2m} |\nabla\psi|^2 + V|\psi|^2 + \frac{g}{2} |\psi|^4 \right] d^3r = E_{kin} + E_{pot} + E_{int} \quad (1.2.9)$$

The total atom number can be obtained by the normalization of the wavefunction ψ , i.e.,

$$N = \int |\psi|^2 d^3r. \quad (1.2.10)$$

In the absence of interactions, the chemical potential reduces to the total energy of the system, and the GPE becomes the Schrodinger equation.

In real experiments, there are not too many ways to trap the atoms, the two more general potentials are the box potential $V_{ext} \approx const$ or the harmonic potential $V_{ext}(r) \propto r^2$. In the particular case of harmonic potentials V_{ext} can be expressed as:

$$V_{ext} = \sum_i^3 \frac{1}{2} m\omega_i x_i^2 \quad (1.2.11)$$

with $i = x, y, z$. It's possible to solve the time-independent GPE by making some approximations or considerations, but there is not an analytical solution for this particular equation.

1.2.2 Density distribution and Thomas-Fermi approximation

At high temperatures, the inter-particle distance is larger than the de Broglie wavelength, making the interaction part in the GPE smaller compared with the kinetic term, thus, the density distribution of the atoms converges to the Maxwell-Boltzmann distribution, and the solution of the GPE equation provides a Gaussian profile $n_{thermal}(r) = \exp \left[-\sum_i^3 \frac{x_i^2}{\sigma_i} \right]$, with $\sigma_i = 2k_B T / m\omega_i^2$ with the peak density normalized to one. Close the critical temperature, the density distribution obeys the so-called Bose-enhanced Gaussian distribution, $n_{BoseEnh}(r) = \frac{1}{g_{3/2}(1)} g_{3/2} \left(\exp \left[-\sum_i^3 \frac{x_i^2}{\sigma_i} \right] \right)$, valid only close to the thermal gas - BEC transition. These two regimes, give us an idea about how different could be the density profile of the condensate, given by the solution of the GPE. But there is no analytical if we consider all the regimes, that's why it is necessary to impose some condition to have the form in which the atoms accumulate. For a large condensed atom number, the kinetic energy is small compared with the other energy terms in the GPE. The ground state of the system and the density profile can be obtained re-writing the GPE without the kinetic term as follows,

$$\mu\psi = \left(V_{ext}(r) + g |\psi(r)|^2 \right) \psi(r). \quad (1.2.12)$$

The solution is given by,

$$n(r) = |\psi(r)|^2 = [\mu - V_{ext}(r)] / g \quad (1.2.13)$$

Expression valid only for $n(r) > 0$ and $\psi = 0$ otherwise. The density profile is in this case depending on the form of the confining potential, an inverted parabola in the harmonic potential case, this is the Thomas-Fermi approximation. Here, the chemical potential can be written as a function of the experimental parameters N and ω as $\mu = \frac{15^{2/5}}{2} \frac{Na^{2/5}}{\bar{a}_{HO}} \hbar \bar{\omega}$, where $\bar{\omega} = (\omega_x \omega_y \omega_z)^{1/3}$ and $\bar{a}_{HO} = \sqrt{\hbar / m \bar{\omega}}$ the effective harmonic oscillator length. The boundary of the cloud can be estimated at the point at which the density goes to zero,

$$V(R) = \sum_i^3 \frac{1}{2} m \omega_i R_i^2 = \mu, \quad (1.2.14)$$

and the radius of the condensed cloud in the harmonic potential named Thomas-Fermi radius can be defined re-arranging the expression above for R_i as follows:

$$R_i = \sqrt{\frac{2\mu}{m\omega_i}}, \quad i = x, y, z. \quad (1.2.15)$$

1.3 Fluids dynamics interpretation

In the last section, we saw how the solution of the GPE equation has an analytical solution considering the Thomas-Fermi approximation and the time evolution dependence with the chemical potential. In this section, we will consider another approach, in which the condensate is a fluid characterized by its density and velocity distribution. Thus, the expression for the wavefunction of the BEC can be written as:

$$\Psi(r, t) = \sqrt{n(r, t)} e^{iS(r, t)}, \quad (1.3.1)$$

where, $n(r, t) = |\Psi(r, t)|^2$ is the density of the BEC with phase $S(r, t)$. The Ansatz in equation 1.3.1 is known as *Madelung transform* formulated by Erwin Madelung as an equivalent formulation of the Schrodinger equation in terms of the hydrodynamical variables. The velocity of the superfluid is related to its phase through the relation

$$v(r, t) = \frac{\hbar}{m} \nabla S(r, t). \quad (1.3.2)$$

One of the implications of defining the velocity field in this way, is that define the superfluids as irrotational, that is

$$\nabla \times v(r, t) = \frac{\hbar}{m} \nabla \times \nabla S(r, t) = 0.$$

The possible motions of the superfluid are thus much more restricted than those of a classical fluid.

Inserting this expression and the Madelung transform in the integral 1.2.9 we obtain:

$$E = \int \left[\frac{\hbar^2}{2m} \nabla^2 (\sqrt{n}) + \frac{m}{2} n v^2 + V_{ext} n + \frac{g}{2} n^2 \right] d^3 r, \quad (1.3.3)$$

the first two terms are the total kinetic energy, the first one corresponds to the quantum kinetic energy due to the zero-point motion of confined particles, vanishing for a uniform distribution of density. The second one is the kinetic energy associated with the flow of the fluid. Replacing the wavefunction given the Madelung transform into the time-dependent GPE and separating the real and imaginary parts, we obtain two equations, the imaginary component is

$$\frac{\partial n(r, t)}{\partial t} = -\frac{\hbar}{m} \nabla \cdot [n(r, t) v(r, t)], \quad (1.3.4)$$

here, we had derived the continuity equation meaning particle number conservation. On the other hand, the Real component leads to

$$-\hbar \frac{\partial S(r, t)}{\partial t} = -\frac{\hbar^2}{2m\sqrt{n}} \nabla^2 \sqrt{n} + \frac{1}{2} m v^2(r, t) + V_{ext}(r) + g n(r, t). \quad (1.3.5)$$

Taking the gradient of this equation, we obtain the velocity field dynamics of superfluid

$$m \frac{\partial v}{\partial t} = -\nabla \left(\frac{1}{2} m v^2(r, t) + V_{ext}(r) + g n(r, t) - \frac{\hbar^2}{2m\sqrt{n}} \nabla^2 \sqrt{n} \right). \quad (1.3.6)$$

Re-arranging terms, we separate the velocity terms with respect the density terms as follows:

$$m n \left(\frac{\partial v}{\partial t} + (v \cdot \nabla) v \right) = -\nabla (P + P') - n \nabla V \quad (1.3.7)$$

where $P = g n^2/2$ and $P' = (-\hbar^2/4m) n \nabla^2 \ln n$ are normal pressure and quantum pressure terms respectively. Equations 1.3.7 and 1.3.4 are known as superfluid hydrodynamic equations, equivalent to the macroscopic equation of motion for ideal fluids. The quantum pressure term comes from the superfluid property of the system and can be compared in order of magnitude with the pressure of classical fluids. If $P \sim g n^2$ and $P' \sim \hbar n/m \xi^2$, where ξ is the length scale of the variation of the density n . Then

$P'/P \sim \hbar^2/2mng\xi^2$ reveals that the quantum pressure P' becomes negligible than P in the limit of length scales larger than ξ . Furthermore, in absence of the external potential, the equation 1.3.7 becomes the Euler equation for classical fluids.²⁵ When the pressure P is comparable with the quantum pressure P' the length-scale in question is provided by the healing length defined as:

$$\xi = \frac{\hbar^2}{\sqrt{2mgn}}. \quad (1.3.8)$$

1.4 Structure of a single vortex

1.4.1 Phase defects

Now, let's consider a closed path C of arbitrary shape through a region of the condensate. The phase variation is defined as a line integral around path C of the gradient of the phase along the line element of the path as follows:

$$\Delta S = \oint_C \nabla S \cdot d\mathbf{l}. \quad (1.4.1)$$

Taking the value of the wave function as Ψ_1 and Ψ_2 as the initial and final point of path C respectively, and also taking into account that the wave function has to be single-valued, means that $\Psi_1 = \Psi_2$ in the closed path, the integral 1.4.1 is equal to $2\pi q$, where the integer q can be $q = 0, \pm 1, \pm 2, \dots$. If $q = 0$, we have the trivial solution meaning the superfluids irrotationality. If $q \neq 0$, the region enclosed by the contour C has a phase defect, a point where the phase wraps by the amount $2\pi q$. In this case, the superfluid density is reduced to zero for the wavefunction Ψ to be single-valued. Following this line of reasoning, the circulation around the closed contour C is given by

$$\Gamma = \oint_C \mathbf{v}(r, t) \cdot d\mathbf{l} = \frac{h}{m} q. \quad (1.4.2)$$

The quantization of the circulation in units of h/m shows how different flows the condensates and ordinary fluids. Quantized circulation, was proposed for the first time by Onsager and Feynman in the context of ^4He , with the last one investigating its consequences in experiments. If we consider a cylindrical symmetry for the superfluid, the line

element around the contour C in polar coordinates is $dl = r d\theta \hat{e}_\theta$, where r is the distance from the axis of the trap and $\hat{e}_\theta$ the unitary vector along the azimuthal direction. Thus, the velocity v of the superfluid around the singularity is:

$$\Gamma = \oint_C v \cdot dl = \int_0^{2\pi} r v \cdot \hat{e}_\theta d\theta = 2\pi r v_\theta \quad (1.4.3)$$

Comparing with equation 1.4.2, we can extract the velocity of the superfluid around the phase defect is,

$$v_\theta = \frac{q\hbar}{mr}. \quad (1.4.4)$$

For $q \neq 0$, the kinetic energy due to the azimuthal motion would diverge as $r \rightarrow 0$, different for ordinary fluids where the velocity field increases linearly with the distance to the rotation axis $v = r \times \Omega$. On the other hand, increasing the value of q , the flow speed increases discontinuously because q can only assume discrete values. Also, the sign of q indicates the flow direction, clockwise or anticlockwise around the singularity. The structure of the singularity into the superfluids is known as a quantized vortex line with charge q .

1.4.2 Non trapped quantized vortices

Until now, we have seen that the velocity field of superfluids is irrotational. That leads to phase defects in the wavefunction known as vortex lines, a small region where the superfluid density is zero. To have a closer view of the vortex line, we will obtain the profile of a vortex core with axial symmetry by replacing the Madelung transform in cylindrical coordinates $\Psi = A(\rho, z)e^{iq\theta}$ into GPE (ρ, θ, z) as follows:

$$\mu A = -\frac{\hbar^2}{2m} \left[\frac{1}{\rho} \frac{\partial}{\partial \rho} \left(\rho \frac{\partial A}{\partial \rho} \right) + \frac{\partial^2 A}{\partial z^2} \right] + \frac{\hbar^2 q^2}{2m\rho^2} A + V_{ext}(\rho, z)A + gA^3. \quad (1.4.5)$$

The right side of the equation shows the quantum kinetic energy, the kinetic energy due to the circulation, the external potential, and the interaction between the atoms respectively. First, we consider a homogeneous condensate like $V_{ext}(\rho, z) = 0$. In the ground state, the wave function does not depend on z , so terms with derivatives concerning

Figure 1.4.1 – Density profile of vortex core compared with the homogeneous density of a condensate at the borders in a box type confinement.

Source: Adapted from BARENGH; PARKER.²⁶

z vanish. Because of the importance of vortex with single quantum circulation, we set $q = 1$. The boundary conditions are $A(\rho) \rightarrow 0$ for $\rho \rightarrow 0$ and $A(\rho) \rightarrow \psi_0$ for $\rho \rightarrow \infty$. In the intermediate regime, the length scale of density fluctuations is governed by the healing length, which by definition in equation 1.3.8 can be related with the interaction term $\hbar^2/2m\xi^2 = gA^2 = \mu$. Re-scaling the distance from the core in term of the healing length and the amplitude of the condensed to a value A_0 far from the vortex core, we define a new variable $\rho' = \rho/\xi$ to replace into GPE 1.4.5 to get

$$-\frac{1}{\rho'} \frac{d}{d\rho'} \left(\rho' \frac{d(A/A_0)}{d\rho'} \right) + \frac{(A/A_0)}{\rho'^2} + (A/A_0)^3 - (A/A_0) = 0. \quad (1.4.6)$$

This equation has no exact solution and can be solved numerically. That solution is shown in figure 1.4.1, where density profile $n/n_0 = (A/A_0)^2$ is a function of the distance from the vortex core in units of the healing length. The region where density depletes $1/e^2$ of the background density to zero is the vortex core radius around $\rho = 5\xi$. In comparison, is plotted a static and homogeneous condensate pinned to zero density showing how the density 'heals' back to the background density, exhibit a characteristic profile $\tanh^2(\rho/\xi)$ ²⁶ showed in the figure as the dashed line.

This difference comes from the kinetic energy of the circulating flow giving rise to an outwards centrifugal force on the fluids. Then, since there is not an analytic solution for the vortex density profile, a good approximation can be,

$$n_{Vortex}(r) = n_0 \left(1 - \frac{1}{1 - \rho'^2} \right). \quad (1.4.7)$$

In trapped condensates, the difference relies in the fact that density is not uniform. In typical 2D column density integrated images, the vortex appear as a hole which profile is given by equation 1.4.7, but, in the case of a trapped BEC in the Thomas-Fermi regime, vortex density profile will be the product of the condensate profile, times vortex density in a uniform condensate as,

$$n_{vortex}(x, y, z) = n_0 \left(1 - \frac{x^2}{R_x^2} - \frac{y^2}{R_y^2} - \frac{z^2}{R_z^2} \right) \left(1 - \frac{1}{1 - \rho'^2} \right). \quad (1.4.8)$$

Another aspect to consider about a single vortex in condensates is the extra energy associated with the presence of the vortex compared with the ground state energy given by the N particles of the condensate. Analog to the equation 1.3.3, the kinetic energy per unit of length of the swirling fluid contained in a bucket of height H_0 and radius ρ_0 inside the condensate is given by

$$\epsilon_{vortex} = \frac{E_{kin}}{H_0} = 2\pi \int_{|\xi}^{R_0} \frac{\hbar^2}{2m\rho^2} A_0^2 \rho d\rho = \pi n_0 \frac{\hbar^2}{m} \ln \left(\frac{\rho_0}{\xi} \right), \quad (1.4.9)$$

where we have considered a homogeneous BEC with axial symmetry and $A_0 = n_0$. The lower limit in the integral has been introduced to avoid logarithmic divergence at $\rho = 0$ and replaced by the core radius $|q|\xi$. Despite that density goes to zero at this point, we did this for simplification assuming that the core is hollow up to the distance $\rho = \xi$, for circulation $q = 1$. The more accurate result obtained by evaluating the vortex energy per unit of length for the numerical simulation of GPE in the context of ^4He is²⁷

$$\epsilon_{vortex} = \pi n_0 \frac{\hbar^2}{m} \ln \left(1.464 \frac{\rho_0}{\xi} \right). \quad (1.4.10)$$

The vortex energy expression 1.4.9 indicates that vortices with multiple quanta of circulation are also possible when $q > 1$ and the core size for a vortex, scales with q , as $|q|\xi$. Thus, the energy contribution of a vortex with more than a single quantum of circulation is obtained by multiplying the equation 1.4.10 by q^2 and scaling the vortex core size ξ with $|q|\xi$. This shows that the energy of a vortex with circulation $q > 1$ is greater than the energy of a collection of vortices with a single quantum of circulation

and the same total circulation.

1.4.3 Trapped Vortices

In the majority of the current experiment producing condensates, atoms are in harmonic traps. We will calculate the energy of the vortex in this kind of traps with rotational invariance in the z -direction. That is important because we will find the lower angular velocity to introduce a single vortex inside the condensate. The radius of the vortex core is proportional to the healing length $\xi_0 = \hbar/\sqrt{2m\mu} = \hbar/\sqrt{2mgn(0)}$ at trap center, where the density $n(0)$ is the local density at the center of the trap in absence of rotation. Under the Thomas-Fermi approximation, the healing length is much smaller when compared with the radius of the cloud in the plane xy , $R_{xy} = \rho_0$.

The energy expression for a vortex in a harmonic trap ϵ_{vt} is similar to the case of a homogeneous potential like this,

$$\epsilon_{vt} = \pi n(0) \frac{\hbar^2}{m} \left[\ln \left(1.464 \frac{R_{xy}}{\xi} \right) - \frac{1}{2} \right] = \pi n(0) \frac{\hbar^2}{m} \ln \left(0.888 \frac{R_{xy}}{\xi} \right) \quad (1.4.11)$$

Here, the logarithmic term comes from the uniform density, while $-\pi n(0) \frac{\hbar^2}{2m}$ term is due to the reduction of the particle density caused by the harmonic confinement. That expression corresponds to condensate with axial symmetry where the variation of density in the z -direction was neglected. Now, if the size of the condensate is bigger than the healing length in this direction (3D case), the total energy is obtained by integrating the equation 1.4.11 along the z -direction,

$$E = \int_{-Z}^Z \epsilon_{vt}(z) dz. \quad (1.4.12)$$

In this case, we have to rewrite the density and the healing length as a function of z . In a harmonic potential, the density variation is $n(0) \rightarrow n(0, z) = n(0, 0)(1 - z^2/R_z)$, meanwhile the healing length change as $\xi(z) = \xi_0 [n(0, 0)/n(0, z)]^{1/2}$ and $R_{xy} = R_{xy}(1 - z^2/R_z^2)^{1/2}$. Replacing these terms, the total energy results

$$\begin{aligned}
E &= \frac{\pi \hbar^2 n(0,0)}{m} \int_{-R_z}^{R_z} dz \left(1 - \frac{z^2}{R_z^2}\right) \ln \left[0.888 \frac{R_{xy}}{\xi_0} \left(1 - \frac{z^2}{R_z^2}\right)\right] \\
E &= \frac{4\pi n(0,0)}{3} \frac{\hbar^2}{m} R_z \ln \left(0.671 \frac{R}{\xi_0}\right)
\end{aligned} \tag{1.4.13}$$

with $n(0,0)$ and ξ_0 the density and the healing length at the center of the condensate. This is the energy per vortex with single-circulation added to a condensate in a harmonic trap with axial symmetry. In the ground state, vortices with circulation bigger than one, are not stable, breaking it into a set of vortices with single circulation to minimize the energy. As we shall see, by adding more vortex to the condensate will be produced structures as vortex lattices inside of the BEC.

1.5 Rotating Condensates

1.6 Vortex Arrays

Each atom swirling around the vortex axis carries angular momentum $L_z = mv_\theta r = m(q\hbar/mr)r = q\hbar$. This means that each atom around the vortex has q units of quantized angular momentum. Then the total angular momentum is given by N units of $\hbar$ thus,

$$L = N\hbar = \hbar \int n(r)dr = \frac{8\pi}{15} n_0 R_{xy}^2 R_z \hbar. \tag{1.6.1}$$

Until now, we were considering condensates with axial symmetry, leading to the conservation of the total angular momentum on this axis. Now, we will consider an anisotropic trap rotating about some axis, case where in the a rest frame of reference, the trapping potential is time dependent. In a such case, the problem is solved by finding the equilibrium ground state of the system, transforming the rotating frame into another constant with time. If E' denotes the energy in the rotating frame and E the energy in the non-rotating frame, both are related with the rotation as follows:

$$E - E' = L \cdot \Omega. \tag{1.6.2}$$

with Ω the angular velocity of the rotating frame. From this equation we can extract

the critical angular velocity Ω_c we need to nucleate one vortex in the rotating frame. If a state with angular momentum component L along the rotation axis has energy $E = E_L$, it will be energetically favorable compared with the ground state in the rotating frame $E' - E_0$ if the angular velocity Ω_c .

$$\Omega_c = \frac{E_L - E_0}{L}. \quad (1.6.3)$$

In the last section we found the energy of the vortex estate relative to the ground state and the angular momentum given by the equations 1.4.13 and 1.6.1, resulting in a critical angular velocity given by:

$$\Omega_c = \frac{5}{2} \frac{\hbar}{m R_{xy}^2} \ln \left(0.671 \frac{R}{\xi_0} \right). \quad (1.6.4)$$

When the rotation frequency is increased above Ω_c , more vortices are introduced, first two vortices rotating around each other, then to three in a triangle and subsequently to arrays with ever more vortices, in a vortex lattice configuration.¹⁶ To have a close view of this behavior, let's consider the Hamiltonian in the rotating frame,

$$\begin{aligned} H' &= H - \Omega \cdot L \\ &= \sum_{i=1}^N \left(\frac{p_i^2}{2m} + V(r_i) - \Omega \cdot (r_i \times p_i) \right) + g \sum_{i < j} \delta(r_i - r_j) \\ &= \sum_{i=1}^N \left(\frac{(p_i - m\Omega \times r_i)^2}{2m} + V(r_i) - \Omega \cdot (r_i \times p_i) \right) + g \sum_{i < j} \delta(r_i - r_j). \end{aligned} \quad (1.6.5)$$

Here, $(p_i - m\Omega \times r_i)^2 / 2m$ is the kinetic energy in the rotating frame, vanishing for a classical system where the velocity is proportional to $v_i = p_i / m = \Omega \times r_i$, corresponding to the rigid body rotation system. The velocity field for a condensate cannot have this form, since it is the gradient of the phase given by 1.3.2. The only case in which we can approximate the velocity field with the rigid body rotation is nucleating an array of vortices aligned in the direction of the angular velocity. The number of vortices necessary to do this approximation in plane perpendicular to the rotation frequency is uniform and given by the expression:

$$N_{vortex} = \frac{2m\Omega}{h} (\pi R_{xy}^2) \quad (1.6.6)$$

Figure 1.6.1 – (left) First experimental production of vortices into a Bose-Einstein condensate.¹ (Right). Production of a vortex lattices with more than 130 vortices, proving the rigid-body limit in a rotating BEC.² Source: Adapted from MADISON *et al.*;¹ ABO-SHAER *et al.*²

the average circulation per unit area (πR_{xy}^2) in the condensate is 2Ω , the circulation for rigid-body rotation at angular velocity Ω . Here the length

$$a_\Omega = \sqrt{\hbar/m\Omega} \quad (1.6.7)$$

is a measure of the inter-vortex spacing. As we have argued, because the circulation is quantized, it is impossible for a superfluid to have a velocity field with $v = \Omega \times r$ everywhere. However, the deviations of the velocity field from that for rigid-body rotation are small for systems with many vortices.

Since the experimental realization of the Bose-Einstein condensates, there are many evidence of the nucleation of vortices^{1,15,17} and production of vortex lattices with an array of hundred of vortices.² In those experiments, an optical barrier stirs the condensate introducing angular momentum, with the number of vortices grows linear with the frequency until the rigid-body limit. An example of this, is presented on the right side in figure 1.6.1, where a Bose-Einstein condensate of Sodium atoms trapped in a magnetic trap, nucleates vortices at different rotation frequencies of the obstacle passing though the atoms nucleating up to 130 vortices. In the figure, we can also observe a plot of the equation 1.6.6, showing how the number of vortices of the system increases until the rigid-body limit.

One important thing to consider when the superfluids are rotated at higher frequencies, is the reduction of the radial frequency trapping due to the centrifugal force introduce

by the angular momentum. With this, if we write the total energy in the rotating frame and minimize the energy by assuming the average velocity field as $v = \Omega \times r$, we got:

$$E' = \int d^3r \left[\frac{1}{2}nm(v - \Omega \times r)^2 + n \left(V(r) - \frac{1}{2}m(\Omega \times r)^2 \right) + \frac{1}{2}gn^2 \right].$$

with,

$$n(r) = \frac{1}{g} \left(\mu' - V(r) + \frac{1}{2}m(\Omega \times r)^2 \right).$$

The rotation of the rotating frame, causes a parabolic increases in the coarse-grained density of the vortex lattice profile, with the effective radial trapping as a function of Ω as $\omega'_{xy} = (\omega_{xy}^2 - \Omega^2)^{1/2}$. Increasing the rotation frequency the size of the condensate also increases, by the reduction of the effective trapping frequency. When Ω is close to the radial trap frequency, it is produced an effect of anti-trapping, producing atom losses of the condensate.

In the next chapters of this first part, we shall see the production of a BEC of sodium atoms and the experimental details we need to generate vortices and arrangement of vortices in a triangular vortex lattice.

Chapter 2

Producing a BEC of Sodium atoms

This chapter will present the experimental details, and the apparatus needed to produce a Bose-Einstein condensate of Sodium in a Plugged magnetic and Dipole trap. Details about introducing the second species as Potassium atoms will be presented in the next part of this thesis. A complete description and explanation of the experimental details can be founded in references.^{28–29}

2.1 Vacuum system

To work with dilute gases, it is necessary a spotless environment. This kind of environment can be achieved in the laboratory with the help of vacuum chambers, able to maintain pressures lower than billions of times the atmospheric pressure. In our experiment, we have three vacuum chambers to contain the atoms, two of them, named 2D chambers, provide pressure up to $\sim 10^{-10} - 10^{-9}$ torr. Another one is the "science chamber" with a pressure up to 10^{-12} torr. Three ion pumps (Varian VacIon Plus) with a pump rate of 75 L/s are installed in the vacuum system to keep the pressure in each chamber. In figure 2.1.1, we present a sketch of the vacuum system; The 2D chambers (on the sides of the figure) connects with the Science chamber (SC, in the center) from opposite sides by narrow channels, which ensure a differential pumping of 10^{-3} and 10^{-4} to the 2D chambers.

Two ovens located under the 2D chambers provide the sodium (right side) and the isotope 39 of potassium atoms (left side) to work. Heating 5 grams ampoule of metallic sodium at 230°C, sodium vapor is released and taken to the chamber through a long tube from the oven. This tube is also heated to 150 °C to avoid sodium glue to the tube walls. With this, is created an atomic beam to be cooled by lasers. The potassium

Figure 2.1.1 – Sketch of the Vacuum system to produce the Bose-Einstein condensate of Sodium atoms and the Sodium-Potassium mixture. The 2D chamber, oven, and ion pump for sodium and potassium are to the right and the left of the Science chamber, respectively.

Source: By the author.

scheme configuration is similar, and we will explain this in the next part of this thesis. The following sections will focus only on a brief explanation of the experimental details about producing a Bose-Einstein condensate of sodium atoms.

2.2 Frequency scheme and optical setup

The sodium atom (Na) is an alkali metal and the sixth most abundant earth crust element. With only one isotope stable ^{23}Na , sodium atom is widely used in atomic physics to produce a large number of condensed atoms and be a coolant for other atomic species. To cool down its temperature by the laser cooling method, we must look at how the light interacts with the hyperfine energy structure. With a nuclear spin, $\mathbf{I} = 3/2$, and total spin number $\mathbf{S} = 1/2$, the hyperfine spin coupling $\mathbf{F} = \mathbf{J} + \mathbf{I}$ (with $\mathbf{J} = \mathbf{L} + \mathbf{S}$, where $\mathbf{L}$ is the total orbital angular momentum) reveals the energy structure of the atom. In the ground state ($L=0$), two levels $F=1$ and $F=2$, separated by ~ 1.7 GHz/h, will define the “Cooling” and “Repumping” transitions to manipulate the atomic state and therefore cool them. In figure 2.2.1, we show a scheme of the energy levels of the sodium atom with two highlight transitions, $3^2S_{1/2}|F=2\rangle$ to $3^2P_{3/2}|F=3\rangle$, and $3^2S_{1/2}|F=1\rangle$

Figure 2.2.1 – (Left) Hyperfine structure of sodium atoms. (Right) Optical setup for sodium, where modulators show the drifted frequency of the light coming from the laser.
Source: By the author.

to $3^2P_{3/2}|F = 2\rangle$ named as Cooling and Repumping transitions, respectively. They are responsible for reducing the temperature (Cooling transition) and repump the atoms to the excited states (Repumping transition), given the probability that the particles accumulate in the ground state $3^2S_{1/2}|F = 1\rangle$.

The laser we need to cool, imaging and manipulating the sodium atoms comes from a doubly-cavity laser (model DL-RFA-TA-SHG, from Toptica) providing 2.3 W of power and stabilized in the D2 line ($3^2S_{1/2} \rightarrow 3^2P_{3/2}$) as shown in figure 2.2.1's left side, together with the other transitions mentioned in the last paragraph. The laser table contains the laser and other optical components required to split and change the frequency of the laser light. Mirrors and polarized beam splitters separate the light into six different beams to handle atoms in the vacuum system table. Using electro-optic and acousto-optic modulators (EOMs and AOMs), we tune different laser frequencies, controls the power and the state ON or OFF for each beam. With the EOM, we generate the sidebands of ~ 1.7 GHz into the central frequency component of the beam, carrying the Cooling and

Repumping transition together. Finally, polarization-maintaining fibers deliver all the manipulated lights on the laser to the vacuum system table; In figure 2.2.1's right side, we present the layout of the optical setup for the sodium laser system.

2.3 2D and 3D MOT's

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Figure 2.3.1 – (right) It is presented the configuration of our 2D-MOT. Blue and red lines represent the magnetic field and its gradient, respectively. In combination with the Zeeman beam, the residual field in the oven region acts as a mini Zeeman slower. (Left). Sketch of 3D-MOT configuration, where the 3D beams and the quadrupole magnetic field (green lines) are presented. We can see the fluorescence of the 3D-MOT (left) and the 2D beams inside the 2D-Chamber (right) in the insights. Source: By the author.

One of these beams separated in the optical table is named Zeeman beam and is used to decelerate the atoms coming from the oven in an adapted Zeeman slower. Then, atoms are trapped in a two-dimensional magneto-optical trap (2D-MOT), composed of two perpendicular counter-propagating beams crossing at the center quadrupole magnetic field, created by a set of four stacks with five permanent magnets. The two beams are red detuned for the Cooling and Repumper transition. We use the untrapped direction (perpendicular to the beams) in the 2D-MOT to transfer the atoms to the Science chamber by adding a fourth beam, blue detuned for the Cooling transition; the push beam. On the left side of figure 2.3.1, we have a design of the 2D chamber showing the push, Zeeman,

and 2D-MOT beams. The fluorescence of the last two is capture in photography inside the 2D chamber.

We trapped the atoms in a 3D-MOT (left side of figure 2.3.1) once they arrive in the Science chamber. There, we create a magnetic field gradient by a pair of coils in an anti-Helmholtz configuration, responsible for separating the sub-energy levels of the atom outside the trap center. The 3D-MOT beams are configured differently compared with the 2D-MOT beams. All three of the counter-propagating beam contains only red detuning Cooling transition. Repumping beam in the 3D-MOT is an independent beam generated by passing through a shifter acting at ~ 1.712 GHz in the optical table. The beam is aligned only on one of three possible directions of the 3D-MOT, coaxial with the Cooling beam in a one-pass way. One of the Repumping beam particularities is a "dark spot" at the center of its transverse profile, allowing the atoms at the center of the spot to accumulate in the ground state $F = 1$, getting higher atomic densities. In the left corner of figure 2.3.1, we exhibit the photography of the 3D MOT of Sodium atoms inside the Science chamber. A photodetector outside the science chamber captures the MOT's fluorescence to count an estimated atom number captured in the trap.

2.4 Imaging System

There are different methods to observe imaging the atoms; some of them are fluorescence, absorption, and phase-contrast imaging. They offer various advantages depending on what we want to observe, for example, off-axis imaging (fluorescence), nonatomic density dependence (phase-contrast), and high signal-to-noise ratio (absorption imaging). In this experiment, we used the absorption imaging method to imaging the atomic cloud by illuminating them with a low-intensity pulse of resonant light. After passing through the atoms, the incoming light is modified by the properties of the absorption medium and can be described by the Lambert-Beer law given by:

$$I(x, y) = I_0 \exp(-OD(x, y)) \quad (2.4.1)$$

Where $I_0(x, z)$ is the incoming beam without interacting with the atoms propagating in the z -direction, $OD(x, y) = \sigma \int n(x, y, z) dz$ is the medium thickness or optical density, and $n(x, y, z)$ is the 3D density of the cloud. This beam is projected into a CCD camera to

Figure 2.4.1 – Optical setup (top) and experimental sequence (bottom) of the imaging system in the experiment. In the optical design, we can capture the atoms with two magnifications, $M_1 = 1.6$ and $M_2 = 0.5$. The lower magnification is illustrated in transparency color. Source: By the author.

extract the two-dimensional density of the cloud $n(x, z)$, perpendicular to the propagating beam axis through the relation,

$$n(x, z) = OD(x, z)/\sigma = -(1/\sigma) \ln(I/I_0). \quad (2.4.2)$$

The total atom number is obtained from the density profile by integrating the equation 2.4.2. This density distribution can be Gaussian or parabolic, depending on the cloud temperature. Taking as an example a Gaussian distribution, the number of atoms is:

$$N = \int n_0 \exp\left(-\frac{x^2}{2\sigma_x^2} - \frac{y^2}{2\sigma_y^2}\right) dx dy,$$

where n_0 is the peak density at the center of the cloud with σ_x and σ_y the Gaussian widths in full width at half maximum (FWHM). Those widths are related with atoms'

temperature; considering that in $t_0 = 0$, they are trapped with a finite size given by σ_0 and, after being released, they will expand for a time $t = t_f$, the cloud size will be σ_x and σ_y . Thus, the temperature can be approximated by the relation:

$$T_x = \frac{m}{k_B} \left(\frac{\sigma_x^2 - \sigma_0^2}{t_f^2 - t_0^2} \right); \quad T = \frac{1}{2} (T_x + T_y). \quad (2.4.3)$$

In the experiment, we released the atoms from the trap falling by the action of gravity by turning off the 3D-MOT beams and the magnetic field gradient. After some time named time-of-flight (TOF), we send a set of three pulses of resonant light through the atoms, collecting the absorption in a CCD camera. The optical image system used to prepare and manage the beam is presented in the upper part of the figure 2.4.1. The laser beam coming from the fiber is collimated and expanded to be larger than the atomic cloud size. The "shadow" that is created by the absorption of the atoms is then collected and magnified or demagnified with a set of two lenses after the science chamber. Finally, the CCD synchronized with the light pulses captures three images following the sequence shown in the lower part of the figure 2.4.1. The first pulse shines the atoms for 30 μs and contains only a resonant Repumping transition that the CCD camera does not take. We used this pulse to transfer the atoms from the $|F = 1\rangle$ state to the $|F = 2\rangle$ state. Then, the other two pulses (resonant with Cooling transition) of 80 μs width and separated by 100 ms each other, capture the beam with and without atoms with the CCD. One last image is taken without the beam to record the background noise. We obtain a normalized profile of the 2D optical density of the atoms by re-writing the equation 2.4.2 as follows:

$$n(x, z) = -(1/\sigma) \ln \left(\frac{I_{withAtoms} - I_{Dark}}{I_{WithoutAtoms} - I_{Dark}} \right) \quad (2.4.4)$$

Images are compiled and processed by a python script to fit and extract and calculate the number of atoms, temperature, optical density, size, and position of the atomic cloud.

2.5 Magnetic Trapping

2.5.1 Preparation for magnetic trapping

The Doppler temperature set the limit for the temperature in the MOT stage, which is given by:

$$T_D = \frac{\hbar\Gamma}{2k_B} = 235 \mu\text{K}, \quad (2.5.1)$$

where $\Gamma = 2\pi \times 9.794$ MHz is the natural linewidth of the Cooling transition of Sodium atoms. In the experiment, our MOT temperature is around $300 \mu\text{K}$, compatible with the expression above. To decrease this temperature, we implemented the optical molasses technique⁷ by turning off the magnetic field gradient and changing the detuning of the cooling and Repumping beams of the 3D-MOT. The atoms will expand into a "sea" of photons like a particle moves in a viscous medium where the friction force is proportional to the velocity of the atomic cloud distribution. In this order, the atoms' temperature is decreased up to $80 \mu\text{K}$ in approximately 5 ms. The total atoms number is also the same as the 3D-MOT phase, 3×10^9 atoms. The optimization to reach the minimum temperature in the molasses is made by tuning the magnetic field compensation with auxiliary coils placed in the science chamber. These coils generate a homogenous magnetic field to compensate for some spurious magnetic field at the center of the magneto-optical trap in MOT beams directions. Also, the MOT beams power balance is adjusted to decrease even more the temperature.

At this point, the atoms are distributed in the two ground states, having the possibility to go out of the magnetic, as we shall see in the next section. That is why we pre-pump the atoms to $|F = 1\rangle$ state by turning off the Repumping beam in the molasses stage for $200 \mu\text{s}$, allowing a few cycles of the cooling transitions to transfer the atoms to the lowest ground state without heating the atomic cloud.

2.5.2 Magnetic trap

After the molasses phase and pre-pumping stages, the 3D-MOT beams are switched off, and the coil currents ramped up, creating a purely magnetic trap that provides a tight confining repository for the atoms. The trapping mechanism at this moment relies on the Zeeman interaction for weak magnetic fields. Assuming that an atom's magnetic moment μ is aligned with the local magnetic field B , the magnetic energy shift will be given by

$$\Delta E = -\langle \mu \cdot B \rangle = \mu_B m_F g_F |B| \quad (2.5.2)$$

Figure 2.5.1 – Quadrupole potential of Na atoms in the $|F = 1\rangle$ state for different sub-energy levels. Solid lines correspond to quadrupole potential in the x and y -direction. The dashed line is the potential in the z -direction. Source: By the author.

with the Bohr magneton $\mu_B/k_B \approx 67\mu\text{K}/\text{G}$, the total spin projection m_F , and the Landé factor g_F . The magnetic field B is generated by a pair of coils in the anti-Helmholtz configuration as we mentioned before and has the form:

$$B(x, y, z) = B' \sqrt{x^2/4 + y^2/4 + z^2}. \quad (2.5.3)$$

with B' the magnetic field gradient along the coils axis. Depending on the combination of the Landé factor and the projection of the spin, the potential can be attractive or repulsive for the atoms. Maxwell's equations prove that it is impossible to create a local maximum in the magnetic field in free space. Thus, magnetic traps present regions of minimum magnetic field and, only the states that minimize their energy while decreasing the amplitude of the magnetic field are trappable. Then, only a few states are magnetically trappable, considering that when the atoms are transferred from the optical molasses, we keep only 2/3 over the total number of atoms, taking into account that we prepare the cloud in the $|F = 1\rangle$ state split into three sub-energy states, $|1, 1\rangle$, $|1, 0\rangle$, and $|1, -1\rangle$ after the pre-pumping stage. The states magnetically trappable are known as low field seekers. In this case, we have only one in $F=1$ state, $|1, -1\rangle$. In figure 2.5.1, we present the quadrupole potential depth in all three directions (U_x^{MT} , U_y^{MT} , U_z^{MT}) for three m_F

states in $F = 1$. For $m_F = 0$ (red solid and dashed line) and $m_F = 1$ (blue solid and dashed lines) states, there is no possible to be trapped once they are transferred to the magnetic trap.

In a semi-classical picture, the magnetic moment of the atoms precesses around the quantization axis set by the magnetic field in the quadrupole trap. If the magnetic field changes in a time scale higher than the inverse of the Larmor frequency $1/\omega_L = \hbar/\mu B$, the atom's spin can flip, unfollowing the magnetic field variations. This behavior is founded close to the center, where $B \sim 0$, where the Larmor frequency goes to zero, and $1/\omega_L$ diverges, producing changing the potential seen by the atoms, producing losses known as Majorana Losses.³⁰ The region where these losses start to be significant is the Majorana hole, founded by treating the magnetic center as an ellipsoid of mean radius ρ with absorbing boundary conditions,³¹ the radius is given by

$$\rho_{hole} = \sqrt{\frac{2v\hbar}{\mu_B g_F m_F B'}}, \quad (2.5.4)$$

where v is the root-mean-square velocity of the atoms, related with its temperature. For a given flux of atoms crossing this ellipsoid, we can obtain the Majorana loss rate, which is also given by:

$$\Gamma_M = \chi \frac{\hbar}{m} \left(\frac{\mu_B g_F m_F B'}{k_B T} \right)^2. \quad (2.5.5)$$

With χ , a numerical factor that depends on the volume of the ellipsoid. It can be calculated either by approximating the ellipsoid as a sphere, resulting in $\chi = 24^{1/3}/32 \approx 0.09$.

2.5.3 Plugging the trap

In order to suppress this mechanism of losses, diverse methods have been developed, using external magnetic fields as in the Ioffe-Pritchard trap³² or superimposing a laser beam at the center of the quadrupole trap. Our experiment, used this technique to avoid the atoms crossing the magnetic field zero points by focusing a blue detuned laser beam (Coherent Verdi V10 laser with $\lambda = 532$ nm) into the magnetic center. The following expression gives the optical potential as

Figure 2.5.2 – Plug + Magnetic trap potential for three different hyperfine states along the x-direction. The optical barrier prevents atoms pass through the low magnetic field region, avoiding Majorana losses. In the inset, we appreciate the 2D trap potential in the plane perpendicular to the propagation axis. Two minima are created when the plug beam is aligned with the magnetic center.

Source: By the author.

$$U_{Plug} = U_0 \frac{1}{1 + (x/x_R)^2} \exp \left[\frac{-2(y^2 + z^2)}{w_0^2 (1 + x/x_R)^2} \right] \quad (2.5.6)$$

with $U_0 = 3c^2\Gamma P/\omega_0^3\Delta^2w_0^2$ the potential depth, c the vacuum speed of light, P the power of the beam, Γ , and Δ the decay rate and detuning concerning the D2 line transition. w_0 the beam waist, and finally $x_R = \pi w_0^2/\lambda$ the Rayleigh length. Figure 2.5.2 plots the sum of the magnetic potential and plug potential in equations 2.5.2 and 2.5.6 in the y direction. We can see the optical barrier for a beam with 4 W of power and $w_0 = 40 \mu\text{m}$. In a horizontal beam configuration, always two minima will be generated; this is an issue when transferring the atoms to the optical dipole trap. That is why we misalign the plug beam from the center of the quadrupole to have one absolute minima potential.

In figure 2.5.2, we plotted a projection of the total potential seen by the atoms in three different hyperfine states (Solid lines). The dashed lines correspond to the magnetic potential without the plug beam at the center. A 2D dimensional plot is observed as

an insight of the figure, showing that in a perpendicular plane to the beam propagation, there are two minima with the same potential depth when the plug is aligned at the center of the magnetic potential.

The transference of the pre-pumped ^{23}Na atoms to the quadrupole trap is done by abruptly switching on the magnetic field at $B_{\text{catch}} = 94 \text{ G cm}^{-1}$. This value was optimized as the lowest magnetic field gradient that allows all trappable atoms to remain trapped into the magnetic trap. After that, the field is adiabatically ramped up to its maximum value $B_{QT} = 190 \text{ G cm}^{-1}$ during 500 ms. Once the atoms are trapped in the pure quadrupole trap, the plug beam is turned on by ramping the laser light intensity from zero to $P_{\text{Plug}} = 2.10 \text{ W}$ in 100 ms. In order to determine the available time we have in the Plug-Magnetic trap, we record the number of atoms as a function of the hold time into trapped. The result is presented in figure 2.5.3, where we saw an exponential behavior due to the collision between the atoms and the background particles. We fit an exponential decay curve to extract the time constant $\tau = 49 \text{ s}$. With a long lifetime of the atoms in the plugged magnetic trap, it is possible to go to the next step in the race to achieve the superfluid of sodium atoms.

Figure 2.5.3 – Lifetime measure of the atoms in the magnetic trap. The solid line is an exponential fit with a time constant $\tau = 49 \text{ s}$.

Source: By the author.

With the atoms in the Plug trap, forced radio-frequency (RF) evaporation is applied selectively, removing the hottest atoms.³³ A two-loop circular antenna with a diameter of 38 mm placed below the upper Quadrupole coil delivers the RF radiation to the atoms. The evaporation ramp lasts 11 s, and it is made of a series of linear ramps resembling an exponential ramp starting with $f_{RF} = 45$ MHz and final frequency between 1.0–0.3 MHz. The efficiency of the evaporation procedure can be evaluated by plotting N vs T in a log–log scale as shown in the graph of figure 2.5.4.

Figure 2.5.4 – Runaway evaporation with (green squares) and without (black dots) the plug beam into the magnetic trap. Above 30 μK , Majorana losses are despicable, and the Plug beam does not make too much difference; meanwhile, the evaporation mechanism is the same with and without the plug. Below this temperature, the curve with the optical barrier can maintain the atoms in the trap, allowing the evaporation mechanism to be efficient until it reaches enough atom number at lower temperatures to obtain the Bose-Einstein condensate.

Source: By the author.

Two different sets of data, with (green squares) and without (black dots) the addition of the plug beam shows the run-away evaporation mechanism. The Majorana temperature is identified as the temperature for which the two sets of data points start to deviate due to Majorana losses with a value around 30 μK for the sodium atoms in our system. In the

absence of the plug beam, the number of atoms decreases faster enough to avoid crossing the critical temperature, limiting the achievement of the Bose-Einstein condensation. On the other hand, the data with the plug beam has a dependence $N(T) \propto T^b$, with $b = 1.12$ as the slope in the log-log curve for $T > 3 \mu\text{K}$ and presenting lower slopes for lower temperatures, indicating an efficient evaporation process.

2.5.4 Bose-Einstein condensate a plugged magnetic trap

Below to the critical temperature $T \approx 1.2 \mu\text{K}$, we observe the Bose-Einstein condensate (BEC) of sodium atoms. A clear signature of Bose-Einstein condensate is the bimodal density profile after the time of flight. On the left side of figure 2.5.5, we observe a normalized absorption image of the atoms after 20 ms of TOF. The cloud is described by a Thomas-Fermi distribution and an inverted parabola, into a Gaussian distribution, due to the non-condensed atoms (right side in figure 2.5.5). In the experiment, BECs with $N \leq 1 \times 10^6$ atoms at $T \leq 1.2 \mu\text{K}$ are obtained by changing the final RF value in the evaporation ramp.

Figure 2.5.5 – Absorption image of a Bose-Einstein condensate in a plugged magnetic trap. (right). Density profile along the vertical and horizontal direction. Condensed atoms are distributed with an inverted parabola distribution given by the Thomas-Fermi approximation (solid blue line). Gaussian profile (dashed orange line) corresponds to the non-condensed atoms (“thermal atoms”), which velocity distribution follows a Boltzman’s distribution.

Source: By the author.

Despite the long lifetime of thermal atoms in the Plug trap, the BEC presents a tiny lifetime due to other factors that are not present when dealing with atoms at

higher temperatures. After producing the sodium condensate, we keep it into the trap for a variable time looking at the condensed atom number after releasing them in TOF. In figure 2.5.6, we present the BEC and thermal cloud normalized lifetime with a time constant $\tau_{BEC} = 175$ ms and $\tau_{thermal} = 34$ s.

Figure 2.5.6 – (Left) BEC (green squares) and total atom number lifetime (blue squares). The solid lines correspond to exponential fits to extract the time constants $T_1 = 175$ ms and 34 s for the BEC and non-condensed atoms. (Right) Atoms temperature as a function of the hold time. While the BEC atom number decays, thermal and mechanical fluctuation contribute to the heating of the atomic cloud.

Source: By the author.

The reason because of this difference corresponds to thermal excitations present in the trap, as fluctuation of the plug position and the interaction with the thermal cloud. For this reason, when we have to imprint excitations to the condensate, we need a lifetime long enough to observe the time evolution in the system. We solve this by transferring atoms to another kind of trap, free of instabilities that limits the lifetime, the optical dipole trap.

2.6 Optical Trapping

Instead of evaporating the atoms in the magnetic trap until the end, we transferred the cloud to an optical dipole trap (ODT). The optical potential seen by the atoms is like the Plug barrier case. However, the wavelength of the laser beam ($\lambda = 1064$ nm) is red-detuned for the D2 line of Na, attracting the atoms instead of expelling them. Two focused red detuned beams are crossed in the $x' - y'$ plane, at the center of the quadrupole

trap, just below a minimum of the Plug potential. With this in mind, the total potential will be the sum of potential introduced by each beam as follows:

$$U_{ODT} = U_{01} \frac{1}{1 + (x'/x_R)^2} \exp \left[\frac{-2(y'^2 + z^2)}{w_{01}^2 (1 + x'/x_R)^2} \right] + U_{02} \frac{1}{1 + (y'/y_R)^2} \exp \left[\frac{-2(x'^2 + z^2)}{w_{02}^2 (1 + y'/y_R)^2} \right] \quad (2.6.1)$$

Sub-index 1 and 2 represent the beam propagating along the x and y direction, respectively. For small displacements around the potential minimum, the equation X can be approximated as a harmonic potential in the form:

$$U_{ODT} \approx -U_0 \left(1 - \omega_x^2 x'^2 - \omega_y^2 y'^2 - \omega_z^2 z^2 \right), \quad (2.6.2)$$

With the potential frequencies equals to,

$$\omega_{x',y'} = \sqrt{\frac{2U_0}{m} \left[\frac{1}{x_R^2} + \frac{2}{w_0^2} \right]} \approx \sqrt{\frac{2U_0}{mw_0^2}}, \quad \omega_z = \sqrt{\frac{8U_0}{mw_0^2}}. \quad (2.6.3)$$

We have considered $U_{01} = U_{02} = U_0$, $w_{01} = w_{02} = w_0$, and $x_R = y_R = x_R$.

When the atoms are inside the optical dipole trap, the mechanism to cool down its temperature is slightly different from the one used in the last section, considering that the confinement of the atoms does not depend on the atomic spin. Now, to remove the hottest atoms' from the trap, the potential depth U_0 is reduced until the atoms can escape with the help of the gravitational sag due to gravity action. Thus, the effective potential feels by the atomic cloud in the vertical direction is

$$U_{ODT}(0, 0, z) \approx U_0 + \frac{1}{2}m\omega_z z^2 + mgz$$

where g is the gravity, m the sodium atomic mass, and ω_z defined in the equation 2.6.3.

Figure 2.6.1 presents plotting the optical potential in the vertical direction given by equation 2.6.1 for different intensities, with the effect of the gravitational potential and the harmonic potential approximation for small displacements. The potential minimum position z_0 is the gravitational sag $z_0 = -g/\omega_z$ depending on the beam intensity, allows the atoms to be pushed by gravity, consequently escaping from the trap.

Figure 2.6.1 – Combined potential $U_{ODT}(0,0,z) + U_{grav}$ seen by Sodium atoms with the essential parameter U_0 . The optical trap corresponds to the crossing beam propagating configuration in the $x'y'$ -plane with beam waists of $80 \mu\text{m}$ and powers $[0.25; 1; 3; 5; 6]$ W leading to trapping depths of $\sim\{2; 8; 23; 40; 48\} \mu\text{K}$. The dashed lines depict $U_{ODT}(0,0,z)$, U_{grav} , and the harmonic approximation to the optical potential.

Source: By the author.

A MEPHISTO X12 laser by coherent provides up to 40 W of power distributed in the optical table. The total power is separated into two paths, passing through two AOM's managing the total power and ON/OFF state of each beam. At maximum power, 10 W are injected into two single-mode polarization-maintaining fibers (NKT fibers LMA-PM-15) with coupling efficiency higher than 70%. We can choose two configurations to introduce the crossed dipole trap into the Science chamber. The first one is with two independent fibers with 6 W power each, as sketched on the left side of figure 2.6.2. The other option is putting all the power in one of the fibers, re-using the same beam to have a crossed ODT on the contrary direction to the incoming beam (right side of figure 2.6.2). In this case, we need to take care of the polarization of the retroreflected beam, avoiding the creation of an optical lattice at the crossing point. The ODT beams focus on the atoms' position with a lens ($f = 60 \text{ cm}$ focal length) before entering the Science chamber,

obtaining a beam waist around $80\ \mu\text{m}$.

Figure 2.6.2 – (Top). Sketch of the optical setup to manage and distribute the ODT beam. (Bottom). Different ways to crossing the ODT beams, by two independent fibers (left) and using only one fiber re-using the incoming beam afterward (right).
Source: By the author.

Figure 2.6.3 – the gravity goes to the left the horizontal axis of the images mean while the vertical direction correspond to a 45° projection of the cODT crossing in the x-y plane.
Source: By the author.

The protocol for transferring atoms to the crossed ODT (cODT) starts in the magnetic plugged trap when atomic cloud temperature is closer than $7\ \mu\text{K}$. The intensity of the ODT is ramped up to its maximum power, around 6 W in 100 ms, to load the atoms

into the trap. After that time, the magnetic field of the quadrupole trap is ramped down to zero in 500 ms without removing the plug barrier to avoid Majorana flips and transferring others states different of $|1, -1\rangle$. Finally, once the magnetic field is zero, the plug beam is also turned OFF, with all the atoms in the ODT. In figure 2.6.3, we have a clue about how this process is seen; by imaging the atoms in different steps during the whole process in the MT. On the left side, we see the atomic cloud in the absolute minima of the Plug potential with a few atoms still in the other potential minima. Placing the cODT under the potential minima, the atoms are transferred to the cODT potential, where the potential minima is more profound than in the plug potential (fig. 2.6.3 right). Decreasing the intensity exponentially for one second and decay constant of 100 ms until 0.25 W, the Bose-Einstein condensate is obtained in the ODT and observed after 20 ms of TOF. As we saw in the plug trap, a bimodal distribution can fit the density profile, exhibiting a Thomas-Fermi profile for condensed atoms and a Gaussian function for thermal atoms. With this, the number of atoms in the ODT is approximately 1×10^6 atoms with more than 80% of the condensed fraction at $T \approx 250$ nK.

Figure 2.6.4 – (Left). BEC of Sodium atoms in ODT. (Right). The lifetime of condensed atoms into the Optical dipole trap. An exponential decay function fitted the data with a time constant $\tau = 8.5$ s.
Source: By the author.

Unlike the plug trap, where the lifetime is only a hundred milliseconds, the lifetime of condensed atoms in the ODT is around 8.5 seconds (Figure X), a time long enough to extract their properties.

In theory, the trap frequencies are completely fixed through the power and waist of each beam. Yet both quantities, especially the spot size, are not exactly known at

the position of the atoms. Moreover, as the real trap frequencies can differ from day to day through imperfect relative alignment of the beams, they have to be remeasured on a regular basis. Such frequencies can be measured by exciting parametric oscillations when modulating the power of one of the ODT laser beams of the cODT. In this case, $P = P_f [1 + \Delta P \sin(2\pi f \cdot t)]$, with P_f the final power after the optical evaporation, $\Delta P = \pm 0.1 P_f$ and f , the amplitude and frequency of the modulation. The condensed fraction of the atomic cloud as a function of the modulation frequency is presented on the left side in figure 2.6.5. One can identify three peaks at different frequencies related to the ODT frequencies as $f_x \approx 67$ Hz, $f_y \approx 77$ Hz, and $f_z \approx 122$ Hz. When comparing f_x and f_y they present a slightly difference produced by beam misalignment or different waist value for each beam at the atoms's position. Moreover, taking in account this variables they are in excellent agreement with the previously calculated frequencies by the equation 2.6.3.

Figure 2.6.5 – ODT frequencies measured with the parametric heating method (left) and dipolar oscillations method (right).
Source: By the author.

In addition to the parametric excitation, we can perform a dipole excitation of the atomic cloud along the z -axis by a fast switching OFF-ON of the ODT laser beams. In our case, the trap was kept off for 2 ms when the atoms fall under the action of the gravitational force. When turning on the ODT again, the atomic cloud has acquired a center of mass velocity making the whole cloud oscillate in phase inside the potential. This oscillation will affect the position of the cloud measured in time-of-flight and displayed on the right side of figure 2.6.5. The cloud center of mass position along z and $x' - y'$

direction, presents a lovely sinusoidal behavior with frequencies $f_z = 122$ Hz and 72 Hz in the axial and radial direction with an excellent agreement related to the previous method's result.

Chapter 3

Vortex Lattice in a BEC of Na

This chapter will present how we introduce excitations in the Sodium condensate. First, we show the experimental requirements to prepare an optical obstacle moving through the condensate. Then, we measured the critical velocity in our sodium BEC, looking at what happened with its condensed fraction and temperature as we change the speed of the obstacle inside it. For a pointlike weak perturbation, the Landau criterion $v_c = \min_p (\epsilon(p)/p)$ directly connects the critical velocity v_c as a macroscopic observable and the microscopic excitations of the system with energy $\epsilon(p)$ and momentum p . We will see that the critical velocity measure here, does not match the Landau criteria, which considers the source of heating is the excitation of phonons. For these excitations, the Landau criterion predicts that the critical velocity equals the sound of sound velocity v_s into the condensate, which can be calculated within the Bogoliubov approximation for a weakly interacting Bose gas.¹⁹ The critical velocity is compared with the sound of sound velocity. Previously, X has been measured in ultracold Bose gases. Weakly interacting three-dimensional^{34–35} and two-dimensional³⁶ BECs were probed with moving repulsive obstacle potentials, with critical velocities of 10 % and 60 % of the Bogoliubov sound velocity were found. On the other hand, we also presented the nucleation of a vortex lattice in the condensate by introducing angular momentum to the cloud. It was possible to see the linear dependence between the number of vortices nucleated and the stirring frequency of the obstacle moving around the condensate.

3.1 Stirring Optical Setup

To print angular momentum to the condensate, we stir a repulsive potential on the atoms, using a focused blue detuned laser beam named *Stirring* beam. Separating a small part of the plug beam, we build an optical setup to control the position and power of the stirring beam on the atoms using a set of lenses and two AOM's as presented in figure 3.1.1.

Figure 3.1.1 – Stirring Optical Setup. Photography of the two AOM's on the optical table in the experiment.
Source: By the author.

With one of the AOM's positioned horizontally, the beam is deflected in the x -direction, where the amplitude of the movement $\theta \propto f_{RF}$ is controlled by the changing the driving frequency f_{RF} on the AOM1. The other AOM is rotated 90° in the xz plane, producing the same effect in the incoming beam from the first AOM, but this time, deflecting the beam in the vertical z -direction. Then, by crossing the two AOM's, we produce a laser beam able to deflect in any position in the xz plane. If we apply a modulation in the driving frequency in both AOM's as $f_{RF} = f_{RF} + A_{mod} \sin(2\pi f_{mod} \cdot t + \phi)$, we can create patterns by controlling the phase and amplitude of the modulation. Figure 3.1.2 presented different stirring patterns as a function of the driving frequency ratio, amplitude, and relative phase between modulation frequency. The patterns generated are

Figure 3.1.2 – Stirring Patterns as Lissajous patterns. We take the images with a CCD camera beam profiler (Thorlabs BC106N-VIS/M) with a long exposure time.

Source: By the author.

similar to Lissajous patterns.

The last independent mirror in the stirring optical setup in figure 3.1.1 is used to set the offset position of the beam (Center the beam on the condensate for the center driving frequency in the AOM's). The mirror is in a pico-motor piezo mount (New Focus 8822-AC) with an angular range of $\pm 5^\circ$ and angular resolution of $0.7 \mu\text{rad}$ on both axes. The waist w_f of the stirring beam at the position of the atoms, is given by $w_f = \lambda f / \pi w_0$, where $\lambda = 532 \text{ nm}$, $f = 7.5 \text{ cm}$, is the focal length of the last lens in the image/stirring optical setup (see figure 3.1.1), and $w_0 = 1.90 \pm 0.2 \text{ mm}$ is the incoming stirring beam waist in the last lens. With these parameters, the calculated stirring beam waist is $7 \mu\text{m}$. If we move the lens which focuses the beam on the atoms with a mechanical translator, the focal length position changes proportional to the displacement, changing the waist of the stirring beam at the position of the atoms. We can see this by making an *insitu* absorption image of the condensate along the beam direction while the *Stirring* is inside it. As the plug beam, the *Stirring* beam also creates a repulsive potential on the BEC,

Figure 3.1.3 – *In situ* absorption image of the Sodium BEC with the stirring beam on it during the image. The repulse potential repels the atoms at the beam position. The waist at the focusing position is measured fitting with a Gaussian profile the “hole” created by the beam.
Source: By the author.

expelling the atoms from the center of the beam (Figure 3.1.3). In this way, we scanned the lens position until finding the minimum waist of the stirring beam, $w_0 \sim 8.5 \mu\text{m}$.

Figure 3.1.4 – Calibration of the stirring beam displacement on the atoms for the vertical and horizontal AOM. Each Megahertz changed in the driving frequency will move $1.25 \mu\text{m}$ the beam on the condensate. When we move the beam in one direction, e.g., horizontal, the displacement in the transverse direction (vertical) created by the same AOM is negligible.
Source: By the author.

Once the stirring beam is at the center of the atoms with the right waist, the next step its movement along with the condensate. The AOM's driving frequency is controlled by an analogic voltage knob in the NI boards. Then, as the driving frequency is changed, the position of the beam changes proportionally. To find this, we scan the frequency on the horizontal and the vertical AOM, measuring the center of the *Stirring* beam on the condensate. In figure 3.1.4, we plot the displacement of the beam in each direction inside the condensate as a function of the driving frequency. We find that the calibration is approximately $1.25 \mu\text{m}/\text{MHz}$.

Figure 3.1.5 – Vertical and horizontal displacement of the stirring beam inside the sodium BEC. The stirring beam is moving along the horizontal and vertical axis of the condensate without deviates in the transverse direction. The circle represents the position of the BEC without the stirring beam on it.

Source: By the author.

The displacement in the transverse direction for each AOM is shown in the inset in figure 3.1.4, indicating that the movement of the stirring beam in the perpendicular direction is negligible. This can be appreciated in figure 3.1.5, where we have to choose a set of images of the beam moving along the horizontal and vertical direction into the BEC. The deviation is less than $4 \mu\text{m}$ with respect to the horizontal/vertical axis of the condensate. Black circles in each snapshot correspond to the condensate in the absence of the stirring beam on the atoms.

All images were taken with the maximum power set on the stirring beam, $P_{\text{stirring}} \sim 500 \mu\text{W}$ which is stabilized with a PID feedback control performed with the light reflected by a piece of uncoated glass focused in a photodetector (see figure 3.1.1).

3.2 Critical Velocity Measurement

To measure the critical velocity of a Bose-Einstein condensate, we have to perturb it to affects its superfluid state. The most common method is to move an obstacle inside the BEC with variable velocities and observe how the superfluid fraction (or density) is affected: for slow velocities, the superfluid fraction remains unperturbed, while above the critical velocity, it decreases linearly with the velocity of the obstacle. In our experiment, we use the stirring beam described in the previous section to measure the critical velocity of a BEC of sodium atoms.

Once we have control of the position and power of the stirring beam, we start our investigation on the Sodium condensate properties. The first thing that we will investigate is the critical point where excitations are introduced into the condensate. Initially, we produce a Bose-Einstein condensate in the cODT with $N \approx 1 \times 10^6$ atoms at $T = 120$ nK with 70-80% of condensed fraction. Holding the atoms in the ODT, we introduce the stirring beam at the center of the BEC. Then, the beam power is increased with a linear ramp from zero to 50 mW in 50 ms, keeping constant for 200 ms and, ramp down abruptly. Finally, without the stirring beam, we hold the cloud for thermalization in 50 ms and released it in 20 ms of time-of-flight (TOF). In figure 3.2.1 we present a sketch of the experimental sequence.

We introduce a triangular analogical modulation into the driving frequency on one of the stirring AOM's, with amplitude A and frequency f_{mod} , to move the stirring beam along with the condensate in a horizontal or in a vertical direction with constant velocity, given by $v_{stirring} = 2Af_{mod}$.

Figure 3.2.1 – Experimental sequence of the critical velocity measure.

Source: By the author.

If we take as an example the horizontal direction, we start fixing the driving frequency modulation and scanning the amplitude of the movement between 0 and the condensate radius R_{TF} . In figure 3.2.2, we present the condensed fraction N_0/N as a function of the movement amplitude for three different modulation frequencies, 20 Hz, 30 Hz, and 60 Hz.

Figure 3.2.2 – Moving the stirring beam inside the condensate to determine the critical velocity. (left) Condensed fraction as a function of the obstacle movement amplitude for three different frequencies. (Right) Condensed fraction vs. frequencies.

Source: By the author.

For small amplitude modulation and 20 MHz of modulation frequency, the condensed fraction keeps constant, around, 70%. Then, this value starts to drop down from 2.5 V equivalent to $\sim R_{TF} = 32 \mu\text{m}$ until it reaches approximately zero condensed fraction, destroying the condensate completely. When the driving frequency modulation is 30 MHz, we observe a similar behavior, but this time, the point where the condensed fraction starts to decrease is for a smaller amplitude, compared with the curve for 20 MHz. If the driving frequency increases even more, the region where the condensed fraction remains constant is reduced, falling quickly when the amplitude increases.

On the other hand, we realize the same measure, but this time, fixing the modulation amplitude and scanning the driving modulation frequency. For minor frequencies, the condensed fraction remains unchanged from its initial value when the amplitude of the movement is less than half of the Thomas-Fermi radii (1.23 V). For higher amplitudes, the threshold point for the depletion of the condensed fraction decreases with the frequency. All these results were obtained while scanning the stirring beam in the horizontal direction.

To estimate the critical velocity in our Sodium BEC, we have to plot the condensed fraction as a function of the stirring velocity defined as $v_{beam} = 2 \cdot cal \cdot A \cdot f_{mod}$. The result is presented in figure 3.2.3, where now the threshold to the condensed fraction for all curves is at the same point, corresponding to $v_c \approx 2.5$ mm/s. Before this value, the amount of energy introduced into condensate is not enough to generate elementary excitations, so the condensed fraction remains constant. If the velocity of the stirring beam is higher than the critical velocity due to the addition of energy, condensed atoms will get excited, increasing the thermal fraction. This is reflected in the fact that the total atom number does not change when the *Stirring* beam moves into the condensate with different velocities, as we can see in the insight in figure 3.2.3.

Figure 3.2.3 – Condensed fraction as a function of the stirring velocity. The vertical straight line indicates the critical velocity at 2.5 mm/s, where excitations start to enter into the BEC. In the insight, the total atom number is plotted, indicating the thermal fraction increasing.

Source: By the author.

Comparing the critical velocity founded above with the critical velocity for a superfluid given by Landau, we found that our value is lower than expected. If we calculate the sound velocity at the center of the condensate through the relationship $c_s = 2\pi f_r R_r / \sqrt{2}$ with the trap radial frequency $f_r = 80$ MHz measured in section 2.6 and the Thomas-Fermi radii $R_z \approx 35 \mu\text{m}$ calculated by equation X, we found that $c_s \approx 8.5$ mm/s, equivalent

to $v_c = 0.3c_s$. This result agrees with numerical simulation of the GPEs considering a non-homogeneous density, 3D clouds, and experiments.^{34,37–38}

3.3 Vortex Lattice Formation and its Properties

We produce a BEC in the ODT with approximately 1×10^6 atoms in the ground state with 70 percent of the condensed fraction. With the minimum energy to introduce excitations into the condensate determined, the next step is to inject angular momentum into the cloud by circularly moving the stirring beam. The driving frequency of the two AOM's f_{mod} is modulated with a sinusoidal waveform with frequency $f_{mod} = 70$ Hz and amplitude $A = 1.25$ V ($0.5R_{TF}$). To imprint a circular shape with the stirring beam, we set the phase between the modulation on each AOM to $\pi/2$ and equal modulation frequency ratio, as previously shown in figure 3.1.2.

To track the obstacle moving through the condensate, we make "*insitu*" images of the BEC with the stirring beam on it. Figure 3.3.1 presents a sequence of pictures of the hole created by the repulsive potential for different times into a period interval.

Figure 3.3.1 – Obstacle moving circularly on the condensate at Thomas-Fermi radius. Images were taking in-trap, keeping the *Stirring* beam on during the image. The frames represent in turn around the BEC.
Source: By the author.

After stirring the condensate for 500 ms, the rotation frequency of 70 Hz, and fixed amplitude of 1.25 V ($r \approx 0.5R_{TF}$), we turn off the stirring beam and release the atoms in TOF. Thus, we observe vortices appearing in the condensate as "holes," regions where the density goes to zero. Figure 3.3.2 presents the difference between a cloud with and without stirring beam on the atoms, where we nucleate more than twelve vortices organized in a vortex lattice. If we compare the size of the cloud with the size of vortices, we observe

how small they are, the order of the healing length. Even with the expansion time of up to 30 ms, we can not see a complete depletion of the cloud density at the position of the vortices. Many factors can explain this effect; one reason could be misalignment between the vortex and imaging axes, producing a blurred image of the vortices when the image integrates along the axial direction in a 3D trap geometry.

Figure 3.3.2 – Comparison between the normal BEC (left) and BEC with a vortex lattice (right) after 30 ms of TOF.

Source: By the author.

We processed the original image by frequency filtering to count vortices and their relative position in the condensate to have a clear view of the vortices. Frequency filtering is based on Fourier transform, multiplying the input image in the frequency domain by a filter function pixel-by-pixel fashion: $G(k, l) = F(k, l)H(k, l)$, where $F(k, l)$ is the input image in the Fourier domain, $H(k, l)$ the filter function, and $G(k, l)$ is the filtered image. The form of the filter determines the effect of the operator. There are three different kinds of filters: low pass, high pass, and bandpass filters. Lowpass filter acts as a smoothing filter; highpass filter enhances the smaller structures of the image, e.g., the edges. Finally, band filters are helpful to improve the borders of the image, reducing noise at the same time. We apply the bandpass filtering to the vortex lattice in figure 3.3.2 B) to extract the number of vortices in the lattice. We crop the original image, taking only the cloud without the background (figure 3.3.3A). We apply the fast Fourier transform (FFT) of A) to obtain the image in the frequency domain in B). We select the bandpass filter to cut frequency higher than 100 px and below 90 px represented in C). The product between pictures C and D corresponds to the filtered image in the frequency domain. Finally, in D, we recover the image filtered in the spatial domain, applying the inverse Fourier

transform in C.

Figure 3.3.3 – Vortex image processing method. A) Cropped Vortex lattice. B) Fast Fourier transform of the image A; $F(k, l)$. C) Bandpass filter in frequency domain $H(k, l)$. D). Inverse FFT of $G(k, l)$.
Source: By the author.

We proceed to scan the stirring frequency to observe the dependence on the number of vortices. Taking three images for each frequency value, we obtained a mean value of the number of vortices plotted in figure 3.3.4, where we can see a threshold in the frequency for the appearance of the first vortex into the condensate. This threshold is due to the gap between the ground state energy of the condensate and the energy needed to introduce excitations following the Bogoliouov spectrum of energy in equation (X). Then, the number of vortices increases with the rotation frequency until they reach a maximum, at the point where the stirring frequency is precisely the radial trap frequency. After that value, the effective trap radial trapping frequency becomes a complex number, expelling the trapped atoms, making it impossible to introduce more vortices into the condensate; that's why the number of vortices decreases to zero.

Figure 3.3.4 – Number of vortices as a function of the stirring frequency. The red dashed lines correspond to the radial trap frequency. The solid line corresponds to the rigid-body rotation limit.
Source: By the author.

Figure 3.3.5 – Vortex lattice lifetime. Setting the optimum parameters to produce the larger quantity of vortices, we keep them trapped, counting the number of vortices, as a function of the hold time.
Source: By the author.

The maximum number of vortices that we can introduce into the condensate is determined by considering a condensate as a rigid body. The straight blue line in figure 3.3.5 shows this behavior.

On the other hand, we investigate the lifetime of the vortex lattice. After nucleating the vortices, we keep the atoms trapped in the optical trap for a hold time before releasing them in time-of-flight. We scanned the hold time from 0 to 2.5 seconds, recording the mean number of vortices with three images per hold time. There is a quick drop until they reach a constant number of vortices after 1 second. We fit an exponential function to extract the decay constant $\tau = 381$ ms, revealing the vortex lattice lifetime. If we compare this lifetime with the ODT lifetime, we figure out that vortex lattice decay is due to the vortex-vortex collision...

3.4 Conclusion

In the first part of this thesis, we produce a Bose-Einstein of Sodium atoms in a plugged quadrupole trap, and in an optical dipole trap. Also. It was possible to measure the trap frequency by dipole oscillations and the parameter heating technique. Also we built an optical set up to introduce and control an optical obstacle in the BEC to measure the critical velocity and nucleate quantized vortices. By focusing a blue detuned beam on the condensate, the atoms were expelled from the beam position in the condensate due to the repulsive potential created by the stirring beam. Moving the beam horizontally or vertically with different frequencies and amplitudes on the BEC, we measure the critical velocity of the superfluid by looking at the condensed fraction behavior when for different velocities of the obstacle. We observed two regions, one in which the condensed fraction remains constant and the other where the condensate starts to dissipate energy, increasing the thermal component into the condensate. We estimate the critical velocity at the point where the condensed fraction starts to decrease, with a value of 2.5 mm/s approximately, $0.3c_s$ with c_s the sound velocity. Despite that the Landau criterium determines that critical velocity is equal to the sound velocity, this does not consider non-homogeneous density or the energy that vortices add to the condensate. Also, we nucleated the vortex lattices, where the number of vortices nucleated increases linearly with the rotation frequency of the stirring beam moving around the condensate.

The maximum number of vortices is given by the rigid-body limit, when it is reached considering the BEC as rigid-body. Finally, we measured the lifetime of the vortex lattice in the ODT of 381 ms. This short lifetime is due to...

Part 2

$^{23}\text{Na} - ^{39}\text{K}$ Mixture and Losses

Introduction

The first interest in the observation of two-component superfluids starts with the phase-separation phenomena between ^3He and ^4He by Walters and Feirbank in 1956,³⁹ many years before the achievement of the first Bose-Einstein condensate, in the 90's. In recent years there has been increased interest in producing Bose-Einstein condensate mixtures due to the high control and precision of the experimental conditions, combining different spin components of the same atomic specie^{40–41} (binary mixtures) or heteronuclear mixtures.^{42–45} Experimentally, these mixtures has been used to explores diverse topics as collectives modes^{46–47} patterns formations,⁴⁷ phase separation,^{48–49} and so many topics. In two-component condensates, the first important property to explore is how their phase will combine or separate in the miscible and immiscible regimes due to the strength of the intra- and inter-species interaction. This transition can be characterized by the miscibility parameter $\Delta = (g_{11}g_{22}/g_{12}^2) - 1$, who determines if the component will overlap in space ($\Delta > 0$) or phase separate ($\Delta < 0$). This spatial overlap has important implications on thermalization rates⁵⁰ and structures of vortex lattices.^{51–52} The possibility to access the miscibility phase diagrams relies on the control of the interaction parameter g_{12} , which is directly related to the scattering length (SL) between the atomic species. The Feshbach resonances allow manipulating the interaction between the atoms through homogeneous magnetic fields, coupling the collision energy between two atoms with a weak molecular state of their interacting potential.⁵³ Close to the resonance, the scattering length is increased to values beyond the average interparticle spacing, increasing the repulsion energy in the case of fermions, because for them, the lifetime due to three-body collisions becomes very large near a resonance, in stark constranst to bosons, where it goes to zero.⁵⁴

In the last part of this thesis, we explored concepts related to one atomic specie superfluidity, as the production of a Bose-Einstein condensate of sodium in a plugged magnetic and optical dipole trap, the measure of the critical velocity, and the nucleation

of quantized vortices into the BEC. The second part of this thesis extends the description of the Gross-Pitaevskii equation to two-component Bose-Einstein condensates, where the coupling term between them is the inter-species interaction parameter. Depending on this value, the ground state of the mixture exhibits spatial phase separation or combined phases.

Then, a brief introduction to the Feshbach resonances technique will be presented to understand how we can tune the interaction between the atoms, and how it is possible to access different miscibility regimes by exposing the atoms to homogeneous magnetic fields. Also, it is explained what are the main signatures of these resonances in the experiments, looking at different observations techniques.

In the second chapter of this part, we shall see the experimental details about the insertion of the second atomic specie, the ^{39}K atoms together with sodium atoms in order to obtain a Bose-Bose mixture of these species. Techniques such as gray molasses, are implemented to set the potassium in the right conditions to transfer into a plugged magnetic trap, taking into account that in previous stages, there are losses induced by the light that limits an efficient loading mixture. Moreover, in the plugged trap, we present the sympathetic cooling mechanics that allow cool down the temperature of the potassium atoms without any atoms losses. Here, from some critical density of both species, three-body losses start to be significant, preventing the Bose-Einstein condensation of Sodium atoms in the presence of ^{39}K atoms.

Finally, in chapter 6, we investigate the three-body losses mechanism between the two components, as a function of the amount of potassium present during the collision, extracting the three-body losses coefficient discussing the migration effect of the atoms in the double-well potential present in the plugged quadrupole trap.

Chapter 4

Two-component mixture of Bose gases

In the last part, we explore the theoretical and experimental aspects of the Bose-Einstein condensation in the case of only one component, the Sodium BEC. In that situation, the macroscopic quantum system of atoms is all represented by a wavefunction, in which the time-dependent GPE sets its dynamics. In the particular case of quantum mixtures, this can be composed of different spin components of the same atomic species, e.g., spinors BECs. For different atomic species, the mass imbalance between them provides another grade of freedom to create exotic phase diagrams that will depend on the interaction between the mass ratio. Either by mixtures with a different spin or with different atomic masses, the equation that governs its mutual dynamics is now a set of two coupled GPE. In this thesis, the two components are specifically sodium and potassium atoms. The reason why we choose these two atomic species to be combined relies on the capacity of the sodium atoms to produce BEC with large atom number condensed and the possibility of its combination with three stable isotopes of potassium, 39K, 40K, and 41K, where two of them are bosons (39K and 41K) and the other one, a fermion (40K). Another essential advantage of this choice is their favorable inter-atomic interaction conditions, allowing explore different miscibility regimes with deep understanding until these days.

4.1 Two-component GPE

In order to describe the interaction between the species quantitatively, we will start writing the coupled time-independent GPEs for the system, given by

$$\mu_1 \psi_1 = \left(-\frac{\hbar}{2m_1} \nabla^2 + V_1^{ext}(r) + g_{11} |\psi_1(r)|^2 + g_{12} |\psi_2(r)|^2 \right) \psi_2(r), \quad (4.1.1)$$

$$\mu_2 \psi_2 = \left(-\frac{\hbar}{2m_2} \nabla^2 + V_2^{ext}(r) + g_{22} |\psi_2(r)|^2 + g_{12} |\psi_1(r)|^2 \right) \psi_1(r), \quad (4.1.2)$$

The sub-index 1 and 2 represent the first and second components, respectively, and the chemical potential determines the normalization condition $\int dr |\psi_i(r)|^2 = N_i$. The coefficients $g_{11} = 4\pi\hbar^2 a_{11}/m_1$, $g_{22} = 2\pi\hbar^2 a_{22}/m_2$, are the intra-species interaction parameters, and $g_{12} = 4\pi\hbar^2 a_{12}/m_{12}$ is the inter-specie interaction parameter which couple both equations; m_{12} is the reduced mass of the system. The trivial solution is given when $g_{12} = 0$; in that case, there is no interaction between the species, and they behave like two completely independent BEC without any interference from the other meanwhile $g_{11}, g_{22} > 0$. Thus, the solution approaches the formalism presented in part 1. Things become interesting when $a_{12} \neq 0$ and depending on the sign and strength of this parameter is possible to access regimes where both species can overlap or phase-separated in space, known as miscibility regimes.

Miscibility can be characterized by the parameter Δ ,

$$\Delta = g_{11}g_{22} - g_{12}^2, \quad (4.1.3)$$

we can say the the the system is miscible if $\Delta > 0$, this correspond to $g_{11}g_{22} > g_{12}^2$ with $g_{11}, g_{22} > 0$. If $\Delta < 0$, the relation between the inter- and intra-atomic interaction parameters has to be, $g_{12}^2 > g_{11}g_{22}$. In the particular case that $g_{12} > 0$, the system is immiscible keeping, a spatial separation between the atomic species; otherwise, the mixture collapse as a Bosenova.⁵⁵ As we know, the interaction parameters are tunable by a homogenous magnetic field via Feshbach resonances, opening the possibility of studying phase transitions going from miscible to immiscible states.^{44,56–57}

One way to have a clue about these phases is considering both species trapped in a harmonic trap. In the Thomas-Fermi limit, the kinetic terms in the coupled GPEs (equations 4.1.2) are neglected, reducing to:

$$\mu_1 = V_1^{ext}(r) + g_{11} |\psi_1(r)|^2 + g_{12} |\psi_2(r)|^2 \quad (4.1.4)$$

$$\mu_2 = V_2^{ext}(r) + g_{22} |\psi_2(r)|^2 + g_{12} |\psi_1(r)|^2. \quad (4.1.5)$$

From here, we can extract the density profiles $n_1 = |\psi_1(r)|^2$ and $n_2 = |\psi_2(r)|^2$ as follows:

$$n_1 = \frac{g_{22} (\mu_1 - V_1^{ext}(r)) - g_{12} (\mu_2 - V_2^{ext}(r))}{\Delta} \quad (4.1.6)$$

and,

$$n_2 = \frac{g_{11} (\mu_2 - V_2^{ext}(r)) - g_{12} (\mu_1 - V_1^{ext}(r))}{\Delta} \quad (4.1.7)$$

For a given external potential, we can obtain the density profile of each atomic species inside the trap. If we consider the isotropic harmonic potentials $V_1 = m_1 \omega_1^2 r^2 / 2$, $V_2 = m_2 \omega_2^2 r^2 / 2 = \lambda V_1$, where $\lambda = m_2 \omega_2^2 / m_1 \omega_1^2$ and defined the chemical potential as function of the Thomas-Fermi radii $\mu_i = m_i \omega_i^2 R_i^2 / 2$, the density profiles becomes:

$$n_1 = \frac{\mu_1}{g_{11}} \frac{g_{11} g_{22}}{\Delta} \left[1 - \frac{g_{12} \mu_2}{g_{22} \mu_1} - \frac{r^2}{R_1^2} \left(1 - \frac{\lambda g_{12}}{g_{22}} \right) \right] \quad (4.1.8)$$

$$n_2 = \frac{\mu_2}{g_{22}} \frac{g_{11} g_{22}}{\Delta} \left[1 - \frac{g_{12} \mu_1}{g_{11} \mu_2} - \frac{r^2}{R_2^2} \left(1 - \frac{g_{12}}{\lambda g_{11}} \right) \right] \quad (4.1.9)$$

If there is no interaction between the atomic species, the densities vanish at $r = R_1$ and $r = R_2$ as expected for single component BEC in the Thomas-Fermi regimes. Given the competition between the atomic densities to coexist, one of them may be zero inside some region defined by the sphere $r < R_i$, in this case, the other density will occupy its place inside the potential with density $n_i = \mu_i (1 - r^2 / R_i^2) / g_{ii}$. Looking that in all cases, the density profiles will have the shape of the potential scaled by the amplitude of their interactions.

Figure 4.2.1 – Feshbach Resonances

Source: By the author.

4.2 The two channel model for Feshbach resonances

The Feshbach resonance is an effect involving multi-channels. In ultracold atoms, the channels are usually molecular potentials from a hyperfine manifold.²⁴ This molecular potential can be the Lennard-Jones potential,^{58–59} where the interaction can be repulsive, attractive, or null, depending on the interaction distance range. Nevertheless, to understand how the Feshbach resonances are connected with these molecular potentials, let us consider two of them, $V_{bg}(R)$ and $V_c(R)$. Where we can associate $V_{bg}(R)$ with the interaction potential between two atoms that no interact each other for far distances. Then, $V_c(R)$ is associated with a potential where two atoms at the same distances R have a bound state. These potentials are presented in figure 4.2.1. The solid black line corresponds to $V_{bg}(R)$, which is also named the open channel. The red curve is related to the $V_c(R)$, the closed channel, that can be coupled with the entrance channel by modifying the collision energy between the atoms.

A Feshbach resonance occurs when the bound molecular state in the closed channel energetically approaches the scattering state in the open channel. Then even weak coupling can lead to strong mixing between the two channels. The energy difference can be controlled via a magnetic field when the corresponding magnetic moments are different. This leads to a magnetically tuned Feshbach resonance.⁵³ The expression that describes the Feshbach resonance for s-wave scattering length⁶⁰ as a function of the magnetic field B is

$$a(B) = a_{bg} \left(1 - \frac{\Delta}{B - B_0} \right), \quad (4.2.1)$$

where a_{bg} is the scattering associated with the the background potential $V_{bg}(R)$ representing the off resonant value. B_0 denotes the the resonant position where a diverges ($a \rightarrow \pm\infty$) and Δ is the resonant width as presented on the right side in figure 4.2.1 (a). The energy of the weakly bound molecular state near the resonance position B_0 is shown in figure 4.2.1 (b) relative to the threshold of two free atoms with zero kinetic energy. The energy approaches threshold at $E = 0$ on the side of the resonance where a is large and positive. Away from resonance, the energy varies linearly with B with a slope given by $\delta\mu$, the difference in magnetic moments of the open and closed channels. Near resonance the coupling between the two channels mixes in entrance-channel contributions and strongly bends the molecular state. In the vicinity of the resonance position at B_0 , where the two channels are strongly coupled, the scattering length is very large. For large positive values of a , a “dressed” molecular state exists with a binding energy given by

$$E_b = \frac{\hbar^2}{2\mu a^2} \quad (4.2.2)$$

In this limit E_b depends quadratically on the magnetic detuning $B - B_0$ and results in the bend shown in the inset of figure 4.2.1. This region is of particular interest because of its universal properties; here the state can be described in terms of a single effective molecular potential having scattering length a .

In the experiments the first observation of the Feshbach resonances in cold atoms were observed in Na atoms, by Inouye et al in 1998⁶¹ and by Courteille et al in the same year using ⁸⁵Rb atoms through the enhancement of photoassociative loss induced by a probe laser.⁶² Two principals features of the Feshbach resonances are observed, first the tunability of the scattering length according to the equation 4.2.1 and the fast drop of the number of atoms (figure 4.2.2) around B_0 , attributed to strongly enhanced three-body recombination and molecule formation near a FR.⁵³ As soon as the magnetic field approximate to the resonance, the inter-atomic energy bounding creates molecules that can be detected by photoassociation, revealing a lorentzian peak, of the fluorescence of the molecules formed (rightside figure 4.2.2)

In particular in our experiment we will lead with the Bose-Bose ²³Na-³⁹K mixture

Figure 4.2.2 – Experimental observation of the Feshbach resonances .

Source: Adapted from INOUE *et al.*,⁶¹ COURTEILLE *et al.*⁶²

considering the channel created by the states $^{23}\text{Na}|1, -1\rangle + ^{39}\text{K}|1, -1\rangle$. The Feshbach resonances of this state have been observed experimentally in an optically crossed dipole trap by Shulze *et al.*⁶³ and will be our reference for tuning the inter-species scattering length. Figure 4.2.3 shows both the inter and intra-species scattering length in the region of the resonances for $^{39}\text{K}|1, -1\rangle$ (dashed red line), $^{23}\text{Na}|1, -1\rangle$ (dotted orange line), and $^{23}\text{Na}|1, -1\rangle + ^{39}\text{K}|1, -1\rangle$ (solid blue line). The resonances show many possibilities with a region where the SL of sodium remains at $52a_0$ (0-700 Gauss) allowing a good collision rate and the condensation of the sample becomes possible. In the same region, the potassium atoms three Feshbach resonances, meanwhile the mixture we can observe only two. With this rich Feshbach “spectrum” it is possible to change drastically the properties of the mixture, allowing the study of the miscibility and immiscible regimes.

4.2.1 Numerical methods

Until now, we present here a qualitative descriptions of how will looks like the ground state of the system in the Thomas-Fermi approximation, in function of a set of

Figure 4.2.3 – Feshbach Resonances NaK mixture
Source: Adapted from TORBEN *et al.*⁶³

experimental parameter as the atom number, interaction parameters and mass ratio of the atomic species in considerations. In order to have a more quantitative solution of the two-component ground state, numerical solution of the GPEs have to be implemented. In practice there are many numerical techniques to solve the equations 4.1.2, but here we will make use of the imaginary time evolution method.²⁶ To describe this method, let's consider a system governed by the hamiltonian H for which the time evolution of one of its eigen states, $\psi_n(r, t)$ with $H\psi_n(r, 0) = E_n\psi_n(r, 0)$, is easily obtained as:

$$\psi_n(r, t) = \psi_n(r, 0) \exp(-iE_nt/\hbar)$$

Then, the time evolution of an arbitrary wave function can be expressed as the linear superposition of the system eigenstates, and it's given by

$$\Psi(r, t) = \sum_n \psi_n(r, t) = \sum_n \psi_n(r, 0) \exp(-iE_nt/\hbar) \quad (4.2.3)$$

The time imaginary evolution method consists into change $t \rightarrow -i\tau$, thus, the exponential in equation will change to exponential decays function with time constant $E_n/\hbar$. By evaluating $\Psi(r, t)$ at different time steps Δt with $\tau = n_{steps}\Delta t$, the ground state of the system is founded at $\Psi(r, \tau)$. To obtaining the most accurate solution as possible, we can extend $\tau \rightarrow \infty$, however, convergence methods based on the variations of the total energy of the system are used to set an upper limit for τ . In the case of two-component condensates, each specie is assigned with a wavefunction $\Psi_i(r, t)$, where $H_i\psi(r, 0) = \mu_i\psi_i(r, 0)$. With this

for an infinitesimal change in Δt in equation 4.2.3, we get:

$$\Psi_i(r, t + \Delta t) \approx \Psi_i(r, t) - \frac{H_i}{\hbar} \Psi_i(r, t) \Delta t$$

This equation is calculated n_{step} times, normalizing at each iteration in order to preserv the total atom number N_i .

Chapter 5

Experimental Production of a Na-K Mixture

In this chapter we will describe briefly the experimental details about to introducing a second specie to be combined with the sodium atoms. As we had mentioned before in this theses, a good candidate is the potassium atom, specifically the isotope 39. One of the interesting properties in the Na – ³⁹ K combinations is the favorable Feshbach conditions, allowing the condensation of both species at the same time.

In order to combine two species simultaneously, before the magnetic trapping stage, we have the laser cooling mechanism happening in a independent way for each specie. Potassium atoms are provided by an oven in the vacuum chamber in the same way that sodium atoms, the oven is heated up to 120 °C, providing a flux of the atoms arriving into the 2D chamber, where are trapped in a 2D-MOT configuration. Different to the system implemented with the sodium atoms, the 2D-MOT of ³⁹K does not have Zeeman beam. We trap the cooling down temperature of the atoms by crossing two beams containing the Cooling ($|F = 2\rangle \rightarrow |F' = 3\rangle$) and Repumping ($|F = 1\rangle \rightarrow |F' = 2\rangle$) transitions of the D2 line ($4^2S_{1/2} \rightarrow 4^2P_{3/2}$). On the left side of figure 5.0.1, we present an sketch of energy levels of ³⁹K indicating the transitions mentioned before. In comparison with sodium atoms, where energy level structure of the excited state $3^2P_{3/2}$ is well separated (108.4 MHz), the separation between energy levels of the excited state $4^2P_{3/2}$ of potassium is 33.8 MHz, the order of the natural linewidth of the transition $\Gamma_{39K}/2\pi = 6.03$ MHz. This make more difficult to cool down its temperature below the Doppler limit, having the necessity to implement other techniques as the gray molasses to overlap this problem. Also, the small splitting energy of the excited state, makes that we need more power to maintain the atoms circling the Cooling and Repumping transition.

Figure 5.0.1 – Hyperfine energy structure of ^{39}K for D2 line (Left). The ground state energy splitting is 461 MHz, meanwhile the excite state is 33.8. (Right) Optical setup of ^{39}K atoms.
Source: By the author.

The total power for the 2D, 3D-MOT's and even for imaging, are supplied by two laser of Toptica (Toptica TA-Pro), delivering $P \sim 900\text{mW}$ each. One of them is used for the Cooling light, locking it into the D2-line transition, and the other one is treated as a slave, taking advantage of the lock from the first laser to shift its frequency in 400MHz, which is the separation of the ground state, generating the Repumping light. The total power is distributed by the optical setup presented in figure 5.0.1, where we combined Cooling and Repumping beams to inserting in two fiber for 3D and 2D MOT's respectively. In the middle of the path, we dispose of two homemade MOPA's, its function is to amplify the power before entering on each fiber. around 300 mW ($\sim 135 I_s$ with $I_s = 1.75 \text{ mW} \cdot \text{cm}^{-2}$) containing lights near the cooling ($\delta_{cool-2D} \sim -3.5 \Gamma_K$, where $\Gamma_K = 6.03 \text{ MHz}$ is the linewidth of the D2 line transitions) and the repumper ($\delta_{rep-2D} \sim -3.0 \Gamma_K$) transitions in a 50:50 ratio exits from the 2D-MOT fiber. A small part of this light ($\sim 2 \text{ mW}$) is used for the push beam while its main part is divided into two circularly polarized retro-reflected laser beams which perpendicularly cross in the center of the 2D-MOT chamber of K. The two-dimensional magnetic quadrupole field is produced by four sets of four permanent magnets each (the same model used for the 2D-MOT of

Na) producing a magnetic field gradient along the beam's propagation direction of $30 \text{ G} \cdot \text{cm}^{-1}$. The pre-cooled atoms are transferred to the Science chamber guided by the push beam. The resulting atomic flux is of around $2\text{--}4 \times 10^7 \text{ atoms/cm}$. The 3D-MOT of ^{39}K is produced with a linear gradient of magnetic field of $16 \text{ G} \cdot \text{cm}^{-1}$ and a total laser power of 320 mW ($\sim 144 I_s$) containing lights near the cooling ($\delta_{\text{cool-3D}} \sim 5\Gamma_K$) and the repumper ($\delta_{\text{rep-3D}} \sim 6.5\Gamma_K$) transitions in a 10:1 ratio. The potassium MOT also operates in a retro-reflected configuration. After a loading time of 8 s , around 1×10^8 atoms at 5.5 mK are trapped in the 3D MOT.⁶⁴

5.1 Two-Color MOT Loading

Figure 5.1.1 – Two species Na- ^{39}K MOT Loading. (a) The sodium MOT was loaded after a full K MOT loading, observing a fast drop of K fluorescence due to the light induced losses. (b) Loading K atoms after a full Na MOT loading, showing an unaltered Na fluorescence, and a very low K atoms loaded.

Source: Adapted from CASTILHO *et al.*⁶⁴

The first step in a two-species ultracold atom experiment consists in creating an efficient method to load and cool the different species prior the transference to a conservative trap. The easiest approach is to perform a two-species MOT simultaneous loading both species. However, light induced losses⁶⁵ could be a limiting factor while operating the two-species MOT. Due to the large atom number difference between the individual MOTs, the most affected species is the potassium as we can see in figure 5.1.1, where we

show the fluorescence signal of the sodium and potassium MOTs captured by independent photodiodes (sensitive to light at 767 nm or at 589 nm) for two different loading configurations. The first one corresponds when the potassium MOT is fully loaded before turning on the sodium MOT (a), showing the dramatic decrease in the potassium fluorescence; and (b) with the opposite situation, in which no difference in the sodium fluorescence was observed. The drastic drop in the K MOT fluorescence corresponds to a decrease of more than one order of magnitude in the number of atoms (from $\sim 2 \times 10^8$ atoms to $\sim 10^7$ atoms) making it difficult to perform the subsequent cooling procedures of the potassium cloud. Before to explain how we deal with this, and see how limited we are about the atoms number of potassium, we will show the gray molasses cooling technique. A cooling mechanism able to decrease the temperature down to $10 \mu\text{K}$ in only a few milliseconds.

5.2 The Gray Molasses technique in ^{39}K atoms

In the experiment, we use a different laser to generate the Cooling and Repumper transition to be used in the Gray molasses of ^{39}K . Another Toptica TA-pro locked into the D1 line transition ($4^2S_{1/2} \rightarrow 4^2P_{1/2}$) generates ~ 1 W of power to prepare two beams to be coupled into one fiber, carry on the light to the atoms with with $P_{cool-D1} \approx 240$ mW and $P_{rep-D1} \approx 40$ mW.

In the absence of Na atoms in the MOT stage, we loading the ^{39}K MOT until its maximum. After that, the atoms experiences two different pre-cooling stages in order to increase the efficiency of the gray molasses technique by the decreasing the temperature before it. After the MOT, the stages of compressed MOT (C-MOT) and Hybrid C-MOT⁶⁶ in which the magnetic field is increased, the cooling light (from the D2 line) is turned off, and the remaining D2 repumper is combined with D1 cooling light during 15 ms. After this compression step, the atomic cloud assumes a small and more symmetric shape, with a temperature reduction of 5 mK to $300 \mu\text{K}$. The gray molasses stage starts when both beams of the D1 transition are on the atoms, and its optimization started by applying a pulse of D1 lighth with a duration of 3 ms, we scan the total power of the pulse for a fixed global detuning $\Delta = 3.5\Gamma_K$ and relative detuning $\delta = 0$ (Raman condition). The result is reported in figure 5.2.1, where we observe a linear shape for the temperature as a function of the power/intensity of the pulse, in which all the atoms coming from the

Figure 5.2.1 – (Left) Global detuning Δ vs T in the gray molasses stage
Source: By the author.

Hybrid C-MOT where cooled below 70 μK . This behavior may be understood in analogy with the behaviour of bright molasses, from the reduction of the light shifts in the dressed state picture.⁶⁶

Thus, by reducing the intensity we can reach very low temperatures to the order of 8 μK for ^{39}K as reported in other group that also implement the gray molasses technique.^{66–69} To continue with the GM characterization, we performed a scan of the global and relative detuning Δ and δ to find the absolute minimum of temperature. On the left side of figure 5.2.1 we appreciate the scan of the global detuning and its relation with temperature. In the scanned range between 1Γ and 4Γ above the $F = 2 \rightarrow F' = 2$ transition (blue detuned), the atoms in the GM remains at the same temperature of 25 μK for a given intensity above the minimum with the Raman condition fulfilled.

The scan of the relative detuning δ is presented in figure 5.2.2 where we observe the narrow feature going from deep cooling in $\delta = 0.01$ revealing a typical asymmetric Fano profile, characteristic of the sisyphus mechanism involving dark state.⁷⁰ The minimum temperature achieved is 14.3 μK . It should be noted that the minimum temperature reached in the gray molasses corresponds to the better optimized parameters, as magnetic field compensation, power balance the maximum atom number loaded from the MOT. These parameters can be modified in order to find the common best conditions when combined with the sodium atoms.

Figure 5.2.2 – Scan of the raman detuning δ in the gray molasses stage. The minimum temperature is reached at $\delta = 0.01$ is approximately $14.3 \mu\text{K}$.
Source: By the author.

Despite the losses between the Sodium and Potassium atoms observed in the MOT stage, we try to transfer the atoms maintaining them together with the same conditions as in single specie operation. First, ^{39}K MOT is loaded until its maximum and right after, the Na MOT until its saturation. Immediately, the 2D-MOT beam of both species are switched off, performing the C-MOT and Hybrid C-MOT in only 15 ms. During this time, the Sodium atoms are held in the MOT stage without any perturbation induced by modifications in the magnetic field gradient in the C-MOT stages. For both species, the sub-Doppler cooling techniques as the optical molasses and gray molasses are implemented simultaneously in 3.5 ms, achieving temperatures of $80 \mu\text{K}$ and $25 \mu\text{K}$ in the case of sodium and potassium atoms respectively. A pre-pumping stage prepare the thermal atoms mixture into $|F = 1\rangle$ stage by turning off the Cooling beam of each molasses by 0.2 ms. This allows to the atoms of both species accumulate in the ground state before be transferred to the plugged magnetic trap.

5.3 Magnetic Trapping

With the atoms in the magnetic trap, the potential seen by the atoms of both species is exactly the same, remembering that the potential energy of the atoms in the presence of the magnetic field depends on the Landé factor and the magnitude of local the magnetic field, as expressed in the equation 2.5.2. The spin properties, in the case of Sodium and ^{39}K atoms in the ground state have the same numbers ($\mathbf{S} = \mathbf{1}/2$; $\mathbf{L} = \mathbf{0}$; and $\mathbf{I} = \mathbf{3}/2$), resulting in $g_{\text{Lande}}(F = 1) = -1/2$. With this, the Zeeman splitting for both species is identical, as presented in figure 5.3.1 where we plotted the shift of the sublevels energy for the two ground state ($F = 1$ and $F = 2$) in the presence of the magnetic field present in the quadrupole potential, by using the Breit-Rabi formula.

Figure 5.3.1 – Hyperfine Zeeman splitting of the ground state for Sodium (left) and Potassium atoms (right).

Source: By the author.

As we have explained previously in this theses, the transference of the atoms to the magnetic trap produces losses of 2/3th of the total number of atoms because we have only one field seeker in the $F = 1$ state. With both species into the trap, we have to take into account the collisional properties between $\text{Na}(|F = 1, m_F = -1\rangle)$ and $^{39}\text{K}(|F = 1, m_F = -1\rangle)$ states, but they will be only relevant when the phase space density of both species is increased as we shall see in the next chapter. Meanwhile, the plug potential acting like a physical barrier, prevents the crossing through the majorana hole, avoiding losses by spin change. But in this case, the optical potential in equation 2.5.6 is sensitive to the atomic specie, being less efficient in the case of K atoms. The total

Figure 5.3.2 – (Left) Plugged quadrupole potential + U_{grav} for the $|1, -1\rangle$ state for sodium (orange) and potassium atoms (red) in x (solid lines) and z (dashed lines) directions, both, perpendicular to the propagating axes; The potential minima is different for each specie, and the potential depth as well. (Right) Normalized lifetime of the atoms inside the plugged quadrupole trap. The solid lines corresponds to exponential fits with decay constants $\tau_{Na} = 49$ s and $\tau_K = 72$ s. Source: By the author.

plugged quadrupole potential in addition to the gravitational potential $U_{grav} = mgz$ for both species, is presented on the left side of figure 5.3.2. We plotted the potential profile in the perpendicular direction of the beam propagation, x (solid lines) and z (dashed lines) as a function of the distance from the center of the trap, where it is possible to appreciate the difference between the potential depth is approximately half for ^{39}K atoms (red color lines) when compared with Na atoms (Orange lines).

When we were describing the experimental details to produce the BEC of sodium atoms, we measured the lifetime of the atoms in the plugged magnetic trap, by fitting an exponential decay function founding a constant decay of $\tau_{Na} = 49$ s. To be sure about how the Potassium atoms behave in the magnetic trap, we perform a lifetime measure in the same way we did for the Na. We loaded only the ^{39}K atoms during the MOT stage, transferring the atoms to the MT keeping them by a variable hold time. Despite the negative scattering length of ^{39}K atoms at zero magnetic field, the lifetime measure was long enough to neglect any two-body or three-body collisional effect, taking into account that the atomic density at the beginning of the magnetic trap is too low. An evidence of this, is the long lifetime measured in figure 5.3.2, where we extract a constant decay $\tau_K = 72$ s, larger than τ_{Na} .

Figure 5.3.3 – Characterization of the frequency spectrum of the microwave antenna.

(Left)

Source: By the author.

To decrease the temperature of the atoms, we implement the evaporating cooling technique as we did to remove the hottest atoms from the trap by using radio frequency getting the Sodium BEC condensate. But this time, the trap depth is modified by using microwave radiation with energy $\hbar\omega_{\mu-Wave}$, on the order of the hyperfine splitting $E_{HSF}^{Na} = 1771.626$ MHz, coupling the ground states $|1, -1\rangle$ to the set $|2, -2, -1, 0\rangle$ via magnetic dipole interaction. As the microwave radiation does not couple the ground state of ^{39}K atoms, because $E_{HSF}^K \approx 400$ MHz, the potential depth remains unaffected, making this mechanism a specie-selective process. This, is different for the RF evaporation mechanism in which we coupling the transitions $|F, m_F\rangle \rightarrow |F, m'_F\rangle$ removing atoms from the both species, due to the same g-Landé factors as we mentioned before.

The microwave radiation is generated by frequency generator (Standord E15) whose signal is amplified to up to 46.5 dBm which output is coupled with RF isolator (x12) to prevents back reflections of the amplified signal. At the end of the line, a single loop antenna of diameter $2 \cdot 2.48$ mm placed at 3 cm from the atoms delivers the microwave radiation. A characterization of the frequency spectrum of the antenna can be observed in figure 5.3.3, measuring the frequency coupling changing different geometries in the antenna, as radius and shape.

At the start of a complete experimental sequence, we load a set of 6 linear ramps (right side of figure 5.3.4) into the frequency generator, waiting to be released by a trigger 50 ms after atoms are transferred to the magnetic trap. The microwave field removes the atoms by coupling the $|1, -1\rangle \rightarrow |2, -2\rangle$ states, starting at 1640 MHz, corresponding to the initial frequency to remove sodium atoms at the maximum temperature in the trap,

Figure 5.3.4 – (Left). Sketch of the μ -wave evaporating cooling. (Right). Evaporating ramp to obtain the BEC of Na atoms.
Source: By the author.

$\sim 200 \mu\text{K}$, until the maximum value at $\sim 1771 \text{ MHz}$.

5.3.1 Sympathetic Cooling

When operating in single specie operation (without loading the K MOT), The sodium atoms are evaporated by increasing the microwave frequency, in 8 seconds, following an efficient run-away evaporation rate until achieve the BEC of sodium. In figure 5.3.5 the green point corresponds to run-away evaporation points of ^{23}Na without the presence of ^{39}K atoms, following the same route when evaporating with radio frequency in figure 2.5.4.

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in figure 2.5.4.

Figure 5.3.5 – Sympathetic cooling of potassium atoms by collision with sodium atoms in the state $|1, -1\rangle$.

Source: By the author.

In dual-species operation, both Na and ^{39}K are loaded into the plugged trap. The initial atom number ratio $N_{\text{Na}}/N_{\text{K}}$ is controlled by reducing the power of the 3D MOT beam of each specie, and the initial temperature is set by the final temperature get at the end of the optical molasses and gray molasses stages. Despite the fact the temperature of each specie are different, when they are transferred to the trap, the they thermalized at the same temperature. As soon as both species reside in the trap, Na is subsequently evaporated and act as a coolant for the ^{39}K cloud mediated by interspecies collisions. Figure 5.3.5 shows the time evolution of both N_{K} (red squares) and N_{Na} (orange squares)

as a function of their respective temperatures using the evaporation protocol presented earlier. As the evaporation progresses, the ^{39}K temperature is steadily reduced, whereas the cooling efficiency of Na atoms is barely affected compared to the single-species evaporation (green squares).

Above to 10 μK , the sympathetic cooling mechanism seems to be largely efficient, with a cooling efficiency $D_K = -\log(PSD)/\log(N) = 8.2$, with $PDS = n_0 \left(\sqrt{2\pi\hbar^2/mk_B T}\right)^3$ the phase-space density. This can be observed in the insigth of the figure 5.3.5, where the PSD of K atoms increase with a higher rate than Sodium. This value is smaller that other combinations (e.g. $D = 12$ for sympathetic cooling of ^6Li by $^{41}\text{K}^{71}$), but higher than their fermionic counterpart ^{23}Na by $^{40}\text{K}^{72}$ with $D = 4.6$. Below 10 μK , we observe a fast decreasing of the total atom number of Potassium cloud, and the sympathetic cooling is not efficient anymore. The reason for this have multiple explanations, being one of them, the weaker potential depth of the plug beam for ^{39}K atoms with $U_{0\text{PlugTrap}}^K \approx 0.45U_{0\text{PlugTrap}}^{\text{Na}}$ produce Majorana losses at lower temperatures. On the other hand, the major contribution of these losses can be explained by the three-body losses (2Na-K and $\text{Na}-2\text{K}$) mechanism due to the higher densities presents below 10 μK for both species. A further descriptions these losses will be explained with more details, in the next chapter, besides, in order to suppress Majorana losses, we transfer the atoms to the ODT around the point when the losses start to be significant, being able to control the three-body losses between the species, by Feshbach resonances.

5.4 Optical trapping

Optical dipoles traps are essential tools when we need to introduce homogeneous magnetic field to control the interaction between atomic species. The potential seen by each atomic specie is given by the equation 2.6.1,

$$U_{ODT}^i = U_{01}^i \frac{1}{1 + (x'/x_R)^2} \exp\left[\frac{-2(y'^2 + z^2)}{w_{01}^2(1 + x'/x_R)^2}\right] + U_{01}^i \frac{1}{1 + (y'/y_R)^2} \exp\left[\frac{-2(x'^2 + z^2)}{w_{02}^2(1 + y'/y_R)^2}\right] \quad (5.4.1)$$

where the potential depth $U_0^i = 3c^2\Gamma_i P/\omega_{0i}^3\Delta_i^2 w_0^2$ is different for each specie. Thus, due to the smaller transition detuning Δ for ^{39}K than ^{23}Na , the potential depth $U_0^K > U_0^{\text{Na}}$ is bigger than U_0^{Na} , in $U_0^K \approx 2.8U_0^{\text{Na}}$. In real conditions, even in vacuum chambers, atoms are exposed to gravity, this is why it is necessary to add the gravitational potential

Figure 5.4.1 – cODT potential U_{ODT} plus the gravitational potential U_{grav} for ^{39}K and ^{23}Na with $P_1 = P_2 = 0.5$ W and $w_{01} = w_{02} = 80\text{ }\mu\text{m}$. They are represented with the solid red and orange lines respectively as the gravitational potential represented with dotted lines. The vertical lines correspond to the potential minima position for each specie.
Source: By the author.

$U_{grav}^i = m_i g z$ to the total optical potential in equation 5.4.1, this makes that the gravitational effects be more intense on the ^{39}K atoms, creating a gravitational sag due to their atomic masses. Also, the potential minima is different for both species, preventing that both species can overlap in space completely. In figure 5.4.1, we plot the total cODT potential plus the gravitational potential (red solid lines for K and orange solid lines for Na) for 500 mW each beam with $w_{01} = w_{02} = 80\text{ }\mu\text{m}$. The minimum position for the potential is given by the equation ??, and the center of mass separation between the clouds will be $dz = |z_{01} - z_{02}| \approx 5\text{ }\mu\text{m}$.

To transfer the atoms to the cODT, the laser beams are turned by ramping the intensity from zero to their maximum (6 W) when both species are in the plugged magnetic trap. The transference point, is chosen in a such way that we can avoid the strong three-body losses observed in the sympathetic cooling stage, e.g. temperatures below $7\text{ }\mu\text{K}$. After 100 ms, the magnetic field gradient was ramp down in another 100 ms, before switching off the

Figure 5.4.2 – (Left). Lifetime measure of the Na (yellow squares) and K (red squares) atoms in the cODT in single-species operation. The time decay constants $\tau_{Na}^{alone} = 18$ s and $\tau_K^{alone} \approx 14$ s and extracted by fitting exponential decay curve on data. (Rigth). Lifetime of the Na-K mixture in the cODT at zero magnetic field, the lifetime of ^{39}K is vastly reduced to $\tau_K^{with Na} \approx 178$ ms, meanwhile the Sodium lifetime remains unchanged. Source: By the author.

plug beam. Transferring approximately $N_{Na} \approx 5 \times 10^6$ atoms of Sodium and $N_K \approx 1 \times 10^5$ atoms of Potassium, we are able to measure the lifetime in the trap by holding the atoms in the cODT before release them in TOF. On the right side of figure 5.4.2, we present the lifetime measure in single-species configuration for ^{23}Na and ^{39}K atoms normalized by the maximum atom number transferred into the trap. In the first case, the lifetime of Sodium without the presence of Potassium atoms (Not loaded in the MOT stage) in the trap, is obtained by fitting an exponential decay curve (orange solid line) on the number of atoms a a function of the hold time (yellow squares), with time constant $\tau_{Na}^{alone} = 18$ s. On the other hand, to measure the lifetime of ^{39}K atoms without the presence of Na atoms, a resonant pulse of light with the Cooling transition of Sodium, shines the atoms for 2 ms right after the cODT transference, expelling the atoms out of trap and leaving Potassium atoms alone in the trap. With this, we measured a lifetime of $\tau_K^{alone} \approx 14$ s.

When both species are loaded in the cODT, the three-body losses affects dramatically the lifetime of the atomic specie with less atoms number, in this case the ^{39}K . the results are present on the left side of figure 5.4.2, where we observed than the number of atoms decreases exponentially with a time constant of $\tau_K^{with Na} \approx 178$ ms, contrary to what is observed with sodium atoms, where the number of atoms remains unchanged during the same time scale. To solve this problem, it is necessary to change quickly the inter-species

interaction parameter to suppress the losses and in this way obtain the superfluid mixture at the end.

5.5 ^{39}K Feshbach resonances

Since in the single-specie operation, only ^{39}K has Feshbach resonances in the magnetic field range between 0 and 1000 Gauss, we investigate the Feshbach resonances of the ^{39}K in the closed channel $|1, -1\rangle, |1, -1\rangle$, taking as reference the theoretical value calculated by D'Errico *et al.*⁷³ As in the lifetime measure, right after the transference to the cODT, a resonant pulse of light with sodium transition removes them from the trap, keeping only potassium atoms. To generate the Feshbach field and tune the interaction, we invert one of the current of quadrupole coils, transforming it into a homogeneous field. The inversion is made by implementing a H-bridge circuit, composed by four IGBT [SKM150GB12T4] connected in a such way that orientates the current in one direction or another in the lower coils of the setup (figure 5.5.1). A further description of the technical and characterization of this control can be founded in the reference.⁷⁴

Figure 5.5.1 – Scheme of the transformation of the quadrupole coils into Feshbach coils, the current direction in the lower coil is inverted, changing the magnetic field from quadrupole to homogeneous.

Source: By the author.

After 5 ms of the turned of the quadrupole field during the transference to the cODT, the polarity of the current in the lower coils is inverted, and with the ^{39}K alone in the cODT, the current is ramp up in 50 ms to $B_{feshbach}(I)$ and keep constant during 1 s. Finally, the current is suddenly turn off, before release the atoms in TOF. In figure 5.5.2 we show the number of atoms record as a function of the final $B_{feshbach}$, where it is observed

Figure 5.5.2 – Two Feshbach resonances of ^{39}K atoms measured in our experiment. Source: By the author.

a depletion in the number of atoms and an increase of the atoms temperature. The two plots corresponds a full scan between 0 and 210 Gauss where two Feshbach resonances are founded centered at $B_0 = 38.15$ Gauss and $B_0 = 166.8$ Gauss. When comparing this values with the founded in other experiment and theoretical papers,^{73,75–77} we found a great correspondence, indicating we are in the right direction. The solid lines in figure 5.5.2 corresponds to a Lorentz fit to extract the center of the depletion. The exact line-shape of the losses induced by Feshbach resonances is directly related with the enhanced three-body losses, following the equation

$$\left(\frac{dn(t)}{dt}\right)_{3\text{-body}} = -L_3 n^3(t). \quad (5.5.1)$$

where $L_3 = C(a) \frac{\hbar}{m} a^4$ is the three-body losses rate, with $C(a)$ a dimensionless constant that can also depend on a .

Despite the good agreement of the first two resonances with the values founded in literature, we can not found the third Feshbach resonance centered at 562 Gauss. The reason for this can be explained with how fast we reach high current values, without loss atoms in the two previous resonances.

5.6 ^{23}Na - ^{39}K Mixture

In the last section, we observed two Feshbach resonances of ^{39}K by scanning an homogeneous magnetic with the potassium atoms trapped in the cODT alone. We attempt to measure the inter-species Feshbach resonances without any success. Due to the

Figure 5.6.1 – BEC of Sodium atoms in the presence of ^{39}K atoms in a cODT.
Source: By the author.

short lifetime of the K atoms when both species are trapped together, we have not signal improvement of potassium atoms when scanning the Feshbach field. It is not clear why this happened. but for sure, is an important limitation to obtain the superfluid mixture. Even like this, we try to go as far as we could by performing optical evaporation in the optical trap.

After the loading of Sodium and potassium atoms in the cODT, we set the Feshbach field to $B = 80$ Gauss, a value in which the scattering length of the potassium atoms and the mixture is positive, following the theoretical map of the Feshbach resonances in figure 4.2.3. With this, we perform evaporative cooling, by reducing the intensity of the ODT beam exponentially in 4 seconds, with a time constant decay of 1 s. When the maximum amount of potassium is loaded in the MOT stage, both species are missing from the image, without any signal available. Then we start to decrease the amount of potassium loaded in the MOT stage until we obtain a signal of the BEC and Potassium atom in the absorption image. With these conditions, we obtained a small BEC of sodium with $N_{BEC} = 7 \times 10^4$ atoms at 150 nK and small dilute cloud of ^{39}K atoms with 9×10^3 atoms approximately. As we can appreciate in figure 5.6.1, it is too difficult to extract information of the potassium cloud due to its low density and contrast. Those images were taken with a TOF of 7 ms. With this signal, we try to scan the Feshbach field in order to see any improvement of the atom number of ^{39}K atom number, but we got no results. Probably, the reason for this is directly connected with the absence of the third Feshbach resonance when scanning the Feshbach field in the single-species configuration for sodium atoms.

Chapter 6

Losses and Sympathetic Cooling Between Na-K Atoms

Atoms in ultracold gases, as already discussed in the previous chapters, interact via atomic collisions. These collisions are usually treated with the mean-field formalism in which the interaction is modeled by a contact potential with the scattering length governing the interaction strength. For a single species atomic cloud at moderate densities and scattering length, only collisions between pair of atoms (two-body collisions) are relevant and, in spin-polarized atomic clouds (i.e. atomic clouds composed of atoms with a single hyperfine state), these collisions are elastic and ensure the thermal equilibrium of the sample.⁷⁸ If the atomic density or the interaction strength increases, the three-body collision between atoms is enhanced, resulting in inelastic collisions responsible for substantial atomic losses (three-body losses).⁷⁹ Effects of the increased role of three-body collisions in atomic clouds have been seen in the saturation of atom number trapped in tight optical traps and were used, for example, to characterize and identify the occurrence of Feshbach resonances once the three-body loss coefficient L_3 scales with the fourth power of the scattering length as shown in chapter four for the tuning of the Feshbach resonances of ^{39}K atoms.

When dealing with two-component atomic clouds, three-body collisions can also happen, resulting in inter-species losses. In this chapter, we are interested in study the inter-species loss mechanism in ultracold atomic clouds of sodium and potassium atoms trapped in the Plug trap at different temperatures and atom number ratios.

This chapter starts with the description of the rate equation for the atomic density in single species, followed by its extension to the two-component picture, where we derived a pair of coupled rate equations for the atomic density of each species. Later, we describe

the experimental details about the measurement of the inter-species losses measured in our experiment. Finally, we present the model we use to the atomic density dynamic with the numerical simulation of the coupled rate equations and its comparison with the experimental data.

6.1 Loses in a single-specie atomic cloud

Lets first consider a single-specie atomic cloud trapped in a conservative trap, where the rate equation that determines the evolution of its density is given by

$$\frac{dn_i(r, t)}{dt} = -L_1^i n_i(r, t) - L_2^i n_i^2(r, t) - L_3^i n_i^3(r, t), \quad (6.1.1)$$

where L_1 , L_2 , and L_3 denotes the respective one, two and three-body coefficients with the sub/super-index corresponding to each atomic specie with $g_{12} = 0$. The first coefficient L_1 is directly related to the collision between the atoms and the background particles in the vacuum environment. Typically, we can found an expression for L_1 coefficient in terms of the background environment variables as $L_1 \approx 6.8P (C_6/m_{bg}) (U_0 m)^{-1/6} / (k_B T)^{3/2}$ where P , m_{bg} and C_6 the background pressure, mass and van-der-Walls dispersion coefficient between the atomic species and the background particles respectively, mostly Hydrogen particles⁷⁵ U_0 and m are the potential depth and the atomic mass of the specie in question. For good vacuum conditions with pressures below $P \sim 10^{-10}$ Torr, $L_1 = 1/\tau$, with τ the lifetime of the atomic specie inside the trap, which is the order of tens of seconds.

If we consider the collision between two atoms, there are only two possibilities; the collision could be elastic or inelastic. In the equation 6.1.1, the L_2 term correspond to the inelastic collision coefficient, only significant when the atoms are in a non-conservative trap or non-spin polarized atomic samples, for example, in magneto-optical traps.⁸⁰ In our case, the atoms are trapped in a conservative trap, where a magnetic field polarizes the spin state. The collision between atoms with the same spin component results in two atoms with the same spin, contributing to the thermalization of the atomic sample. In this case, usually, the two-body loss coefficient is too small compared with L_1 and L_3 , resulting in $L_2 = 0$. Finally, in trapped gasses at low temperature, where the atomic density increases, the three-body losses are enhanced by increasing the atomic density,

given its proportionality to n^3 in equation 6.1.1, or also by increasing the strength of the scattering length a_s , taken into account that the three-body loss coefficient is defined in term of the scattering length as $L_3 = n_l C \hbar a_s^4 / m$, with n_l the number of atoms loss from the trap per recombination event, and C is a constant.

An expression in terms of experimental parameters for the L_3 coefficient can be extracted from the equation 6.1.1 with $L_2 = 0$. As the atomic density is not uniform in a harmonic potential, it is not a good parameter to work with. we will resolve the equation for the number of atoms, by integrating in both side as follows:

$$\int \frac{dn_i(r, t)}{dt} = \int \left[-L_1^i n_i(r, t) - L_3^i n_i^3(r, t) \right], \quad (6.1.2)$$

where the atomic density will follows a Gaussian distribution, defined as $n(r, 0) = n_0 \exp[-V_{ext}(r)/k_B T]$. From the integral of the density, we obtain that the number of atoms, $N = \int n(r, t) d^3r$ is:

$$\begin{aligned} N &= \int n(r, 0) d^3r = n_0 \int \exp[-V_{ext}(r)/k_B T] d^3r \\ N &= n_0 \prod_i^3 \int \exp\left[-\frac{1}{2} \frac{m}{k_B T} (\omega_i^2 x_i^2)\right] dx_i \\ N &= n_0 \left(\frac{\sqrt{2\pi k_B T}}{\sqrt{m\bar{\omega}}} \right)^3 \rightarrow n_0 = \left(\frac{m\bar{\omega}^2}{2\pi k_B} \right)^{3/2} \frac{N}{T^{3/2}}. \end{aligned} \quad (6.1.3)$$

with $\bar{\omega} = \sqrt{\omega_x \omega_y \omega_z}$. The second integral on the right side is practically the same, and is given by

$$\begin{aligned} -L_3 \int n^3(r, 0) d^3r &= -L_3 n_0^3 \int \exp(-3V_{ext}(r)/k_B T) d^3r \\ &= -L_3 n_0^3 \prod_i^3 \int \exp\left[-\frac{3}{2} \frac{m}{k_B T} (\omega_i^2 x_i^2)\right] dx_i \\ &= -L_3 n_0^3 \left(\frac{2\pi k_B T}{3m\bar{\omega}^2} \right)^{3/2} \\ &= -L_3 \left[\left(\frac{m\bar{\omega}^2}{2\pi k_B} \right)^{3/2} \frac{N}{T^{3/2}} \right]^3 \left(\frac{2\pi k_B T}{3m\bar{\omega}^2} \right)^{3/2} \\ &= -L_3 \left(\frac{m\bar{\omega}^2}{2\pi k_B} \right)^3 \frac{1}{\sqrt{27}} \frac{N^3}{T^3}. \end{aligned} \quad (6.1.4)$$

There, we use the value found for n_0 in equation 6.1.3. With the results of this integrals, we change the density equations rate by the atom number rates, Thus, rearranging in terms of the equations 6.1.2 we get:

$$\frac{dN_i(t)}{dt} = -\tau_i^{-1}N_i(t) - \gamma_i \frac{N_i^3(t)}{T^3}, \quad (6.1.5)$$

with $\gamma_i = L_3^i \left(\frac{m_i \bar{\omega}_i^2}{2\pi k_B} \right)^3 \frac{1}{\sqrt{27}}$. The equation 6.1.5 are expressed in pure experimental variables as N , T , and $\bar{\omega}$ opening the possibility to calculate the L_3 coefficient, the numerical solution of the rate equation.

6.2 Losses in two-component atomic clouds.

When the interaction between both species is different from zero, there is an inter-combinations between the coefficient losses due to their mutual interaction, which give origin to new terms we need to add to the equation 6.1.1. These new terms depends on crossed two and three-body coefficient or the species i and j as follows:

$$\left[\frac{dn_i(r, t)}{dt} \right]_j = -\frac{1}{2} L_2^{ij} n_i(r, t) n_j(r, t) - C L_3^{ij} n_i^2 n_j - 2C L_3^{ijj} n_i n_j^2 \quad (6.2.1)$$

The fractional coefficients are related with the contribution of each specie in the heteronuclear loss process with $C = 1/3$.⁸¹ The two-body losses coefficient, L_2^{ij} rise from the collision between the specie i and j , considered inelastic $L_2^{ij} \neq 0$ atomic species with different spin component collides. If $L_2^{ij} = 0$, the collision is fully elastic, contributing to the thermalization between both samples as explained in the section 6.1.

The three-body coefficient losses between the species is “splited” in two term, L_3^{ijj} indicating that the collision between the species can be given by two particles of the specie i with one particle of the specie j , and L_3^{ijj} when the collision is between one particle of the specie i and two particles of the specie j . The weigh of each coefficient C , is related to the number of atoms ratio and mass ratio between the species.

$$\left[\frac{dn_i(r, t)}{dt} \right]_j = -\frac{2}{3} L_3^{Na-2K} n_{Na} n_K^2 - \frac{1}{3} L_3^{2Na-K} n_{Na}^2 n_K \quad (6.2.2)$$

An important aspect in the analysis of three-body losses between sodium and potassium system is the relative strength of the two interspecies three-body recombination coefficients L_3^{Na-2K} and L_3^{2Na-K} . The high loss rate either stems from $2Na - K$ recombinations, from $Na - 2K$ ones, or from a combination of both. One can get initial insight about the physical process by setting either one of the coefficients to zero and assume that the sodium depletion as well as the temperature increase is negligible. Since two heavy and one light atom are more strongly bound than one heavy and two light atoms, the $Na - 2K$ channel is expected to be significantly stronger. However, in our experiment the atom number N_{Na} is two order of magnitude higher than N_K , hence we will consider the losses from the $Na - 2K$ channel are insignificant and are neglected from the evaluation. Then our final model will reduce to:

$$\frac{dN_{Na}(r, t)}{dT} = \frac{dN_{Na}^{alone}(r, t)}{dT} - \gamma \frac{N_{Na}^2 N_K}{\dot{T} T^3} \quad (6.2.3)$$

$$\frac{dN_K(r, t)}{dT} = \frac{dN_K^{alone}(r, t)}{dT} - \gamma \frac{N_{Na}^2 N_K}{\dot{T} T^3} \quad (6.2.4)$$

Where $\gamma = \frac{1}{3} \frac{L_3^{2Na-K}}{b_{Na} b_K}$. Here, we introduced a change of variable, $dN/dt = \frac{dN}{dT} \frac{dT}{dt}$ to express all the equation in terms of experimental variables.

It is recalled that the initially prepared atomic spin state is maintained in the magnetic trap, with both states polarized in the ground state $|1, -1\rangle$. Despite the time dependence of the density, the μ -wave evaporating process reduces the atom number and increases the density only of sodium atoms, including an extra term into $\frac{dN_{Na}^{alone}(r, t)}{dT}$ which depends on the μ -wave evaporation rate. Without the presence of potassium atoms, we can obtain the dependence of the sodium atom number with the temperature via a N vs. T plot, as is shown in figure 6.2.1. The black points correspond to the evaporation curve in the single species operation in the case of Na atoms, its behavior can be represented by fitting the function $N^{alone}(T) = \alpha T^s$ (solid line), with the parameters, $\alpha = 1.17 \times 10^{13}$ and $s = 1.18$. We can consider this as the solution of the dynamic equation in the case of sodium, giving:

$$\frac{dN_{Na}^{alone}}{dT} = \alpha \cdot s T^{s-1}$$

As expected, the variation in temperature with time is determined by the evapo-

Figure 6.2.1 – N vs T for Sodium atoms without the presence of Potassium, This set of points describes the dynamics of the losses in the magnetic trap only for sodium, including the background losses and three-body losses, and the evaporation rate.
Source: By the author.

ration rate, including the heating produced by the three-body interaction between both species. To determine how it is this variation, we calculate the numerical derivative of the temperature in function of time, for each temperature. The results is presented in on the left side figure 6.2.2, where we consider a linear rate of dT/dt with temperature. The reason behind this, is directly correlated with the μ -wave evaporating curve shape with respect time (right side if figure 6.2.2). The negatives values of dT/dt , only indicates the temperature gradient rate, from higher temperatures to lower.

As we mentioned before, during the sympathetic cooling process, we measure the number of atoms of Na and K atoms as a function of the temperature, which is given by the evaporation time. In this circumstances, where both species are loaded from the beginning, we can not produce the Bose-Einstein condensate of sodium because for a certain temperature, we observe a fast drop of the atom number induced by the collision with K atoms. We will investigate the three-body losses mechanism involved in the mixtures for temperature below 10 μK , we can observed that the number of atoms of sodium increases the loss rate as soon we increase the amount of potassium atoms from the MOT loading. Then from the loss rate equation we will extract the γ value to fit the better

Figure 6.2.2 – Temperature dependence
Source: By the author.

curves that pass for the data point. In the simulation we consider the interpolation of the number of potassium atoms, assuming that the equation rate for this species is given by the experimental data. In this circumstances, where both species are loaded from the beginning, we can not produce the Bose-Einstein condensate of sodium because for a certain temperature, we observe a fast drop of the atom number induced by the collision with K atoms.

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