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Study of Excitations in a Bose-Einstein Condensate

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Study of Excitations in a Bose-Einstein Condensate

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*"One doesn't discover new lands without consenting
to lose sight of the shore for a very long time."*

— ANDRÉ GIDE (1869 - 1951)

*"Physics is like sex: sure, it may give some practical results,
but that's not why we do it."*

— RICHARD P. FEYNMAN (1918 - 1988)

*"And, in the end, the love you take
is equal to the love you make."*

— PAUL MCCARTNEY (1942 -)

RESUMO

SEMAN, J. A. **Estudo de excitações em condensados de Bose-Einstein**. 2011. Thesis (Doutorado) - Instituto de Física de São Carlos, Universidad de São Paulo, São Carlos, 2011.

Neste trabalho, estudamos um condensado de Bose-Einstein de átomos de ^{87}Rb sob os efeitos de uma excitação oscilatória. O condensado é produzido por meio de resfriamento evaporativo por radiofrequência em uma armadilha magnética harmônica. A excitação é gerada por um campo quadrupolar oscilatório sobreposto ao potencial de aprisionamento. Para um valor fixo da frequência de excitação, observamos a produção de diferentes regimes no condensado como função de dois parâmetros da excitação, a saber, o tempo e a amplitude. Para os valores mais baixos destes parâmetros observamos a inclinação do eixo principal do condensado, isto demonstra que a excitação transfere momento angular à amostra. Ao aumentar o tempo ou a amplitude da excitação observamos a nucleação de um número crescente de vórtices quantizados. Se incrementarmos ainda mais o valor dos parâmetros da excitação, os vórtices evoluem para um novo regime que identificamos como *turbulência quântica*. Neste regime, os vórtices se encontram emaranhados entre si, dando origem a um arranjo altamente irregular. Para os valores mais altos da excitação o condensado se quebra em pedaços rodeados por uma nuvem térmica. Isto constitui um novo regime que identificamos como a *granulação* do condensado. Apresentamos simulações numéricas junto com outras considerações teóricas que nos permitem interpretar as nossas observações. Nesta tese, apresentamos ainda a descrição da montagem de um segundo sistema experimental cujo objetivo é o de estudar propriedades magnéticas de um condensado de Bose-Einstein de ^{87}Rb . Neste novo sistema o condensado é produzido em uma armadilha híbrida composta por uma armadilha magnética junto com uma armadilha óptica de dipolo. A condensação de Bose-Einstein foi já observada neste novo sistema, os experimentos serão realizados no futuro próximo.

Palavras-chave: Condensação de Bose-Einstein. Superfluidez. Turbulência quântica.

Abstract

SEMAN, J. A. **Study of Excitations in a Bose-Einstein Condensate**. 2011. Thesis (Doctorate) - Instituto de Física de São Carlos, Universidad de São Paulo, São Carlos, 2011.

In this work we study a Bose–Einstein condensate of ^{87}Rb under the effects of an oscillatory excitation. The condensate is produced through forced evaporative cooling by radio–frequency in a harmonic magnetic trap. The excitation is generated by an oscillatory quadrupole field superimposed on the trapping potential. For a fixed value of the frequency of the excitation we observe the production of different regimes in the condensate as a function of two parameters of the excitation: the time and the amplitude. For the lowest values of these parameters we observe a bending of the main axis of the condensate. This demonstrates that the excitation is able to transfer angular momentum into the sample. By increasing the time or the amplitude of the excitation we observe the nucleation of an increasing number of quantized vortices. If the value of the parameters of the excitation is increased even further the vortices evolve into a different regime which we have identified as *quantum turbulence*. In this regime, the vortices are tangled among each other, generating a highly irregular array. For the highest values of the excitation the condensate breaks into pieces surrounded by a thermal cloud. This constitutes a different regime which we have identified as *granulation*. We present numerical simulations together with other theoretical considerations which allow us to interpret our observations. In this thesis we also describe the construction of a second experimental setup whose objective is to study magnetic properties of a Bose–Einstein condensate of ^{87}Rb . In this new system the condensate is produced in a hybrid trap which combines a magnetic trap with an optical dipole trap. Bose–Einstein condensation has been already achieved in the new apparatus; experiments will be performed in the near future.

Keywords: Bose–Einstein condensation. Superfluidity. Quantum turbulence.

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1 Introduction

1.1 General Remarks

After its prediction in 1924 by Satyendra N. Bose and Albert Einstein (1, 2), the Bose–Einstein condensation was simply an interesting textbook example of a macroscopic quantum degenerate system. Later, in 1938, P. Kapitza, J. F. Allen and D. Misener, observed for the first time the phenomenon of superfluidity in liquid helium at a temperature below 2.18 K (3, 4). Subsequent theories developed by F. London (5) indicated that the phenomenon of superfluidity might be a consequence of Bose–Einstein condensation of the helium atoms.

The fact that Bose-Einstein condensation (BEC) in dilute gases occurs at much lower temperatures, of the order of 10^2 nK, made superfluid helium the only Bose–condensed system available during almost six decades. With the advent of laser cooling and trapping techniques it was possible to reach temperatures below the milli–Kelvin scale (6). Finally, in 1995 the production of Bose–Einstein condensates in dilute atomic fluids became reality (7–9). Initially, the main challenge was to demonstrate the phenomenon but there were no perspectives for many advances. However, science sometimes takes unexpected paths and, very soon, a large variety of fundamental questions and interesting effects appeared around the BEC. In this way, BEC became one of the most rapidly growing research topics of modern physics.

There are few physical systems in nature that provide a level of control as high as the one offered by BECs. In this system it is possible to control independently and almost at will all the parameters of the system. This includes the external potential, the number of particles, the density, the temperature and the dimensionality of the system. Even the interatomic interacti-

ons can be externally manipulated using Feshbach resonances (10). Consequently, BECs are excellent model systems which link different areas of physics.

For example, in thermodynamics and statistical mechanics, the condensation represents an important quantum phase transition where the occupation in phase-space can be controlled (11). In the same way, in quantum field theory, condensates constitute an interesting demonstration of spontaneous symmetry breaking (12).

A very important example is the intersection between BEC and condensed matter physics, that has been extensively explored during the last years (13). Using a stationary laser light wave, it is possible to create a periodic potential in which the atoms accumulate in the minima of the stationary wave. By loading a BEC in such an optical periodic potential it is possible to create an artificial perfect crystal. This possibility opened a vast research area with condensates in which solid state systems can be modeled with an unprecedented degree of control. Exploring exotic phase transition (14) or the role of disorder and randomness in the lattice (15, 16) are examples of two very novel research topics.

Many-body physics is also directly related to Bose-Einstein condensation (17). Superfluidity itself is probably the most remarkable many-body effect present in BECs. Many of the earliest works on condensation concerned the formation and study of quantized vortices in the sample (18, 19). In fact, this was the first experimental demonstration of the superfluid character of atomic BECs.

Few years later, a quantum degenerate Fermi gas was also produced using similar techniques (20). This achievement has a very deep and important consequence in condensed matter physics and many-body physics. Using Feshbach resonances it is possible to finely tune the interaction between the fermions of the degenerate gas, forming Cooper pairs in a very controlled way. Superfluid behavior was observed in the produced gas of Cooper pairs, demonstrating that superconductivity and superfluidity are, in essence, the same phenomenon (21).

In the present Thesis we explore a very interesting consequence of superfluidity: the possibility of having turbulence. Superfluid turbulence, just as its classical counterpart, is characterized

by a very disordered flux. Since superfluids experience important quantum limitations, turbulence in these systems is known as *Quantum Turbulence*. Quantum turbulence was idealized for the first time by Richard P. Feynman (22) in 1955 and, shortly after, observed in superfluid ^4He by W. F. Vinen and H. E. Hall (23–27). In the present thesis we report on the first observation of this phenomenon in a Bose–Einstein condensate. This is a very important result because turbulence was never generated in a system as controllable as a BEC, opening new and exciting possibilities of research and understanding of this subject.

In our experiment, the turbulent state is generated by an oscillatory magnetic excitation. Depending on the parameters of this excitation, different regimes besides the turbulent one are also generated. For a low strength excitation, it is possible to nucleate quantized vortices. These vortices can evolve to the turbulent regime if the strength of the excitation is increased. A different regime, which we identify as *Granulation*, can be produced with the strongest excitations. In the granulated regime the condensate breaks into small pieces surrounded by a non–condensed cloud. As we will see, all our results can be summarized in a diagram which shows how the parameters of the excitation must be combined in order to produce a certain regime.

This diagram is a very important and novel result. First, it clarifies the route to produce nontrivial states in a BEC, such as quantum turbulence and granulation. Second, the diagram is peculiar of atomic Bose–Einstein condensates and it is not present in bulk superfluids, such as superfluid helium.

1.2 This Thesis

In this thesis we will present the work performed in two different experimental setups, which we call through out the thesis “*BEC–I*” and “*BEC–II*” systems.

We start by revisiting the basic concepts of Bose–Einstein condensation and superfluidity in Chapter 2. In Sections 2.1 and 2.2 we present the non–interacting and interacting Bose gas and how the description of both systems is done. Next, In Section 2.3 we introduce the Bogoliubov approximation and show how this theory predicts the existence of superfluidity in

an interacting BEC. We end this chapter discussing the basic concepts of turbulence in both situations, classical and quantum, giving emphasis to the similarities between them.

In Chapter 3 we present the main results obtained in the BEC–I system. In Section 3.1 we provide a brief description of the experimental apparatus and explain how the excitation is applied. Next, in Section 3.2 we discuss the properties of the external fields used to excite the condensate. Later, in Section 3.3 we present our main results, explaining each of the different excited regimes produced in the sample. This includes the generation of quantized vortices in the sample and their subsequent evolution to quantum turbulence as the strength of the excitation increases. For the strongest excitations we observe a new phase which we have identified as the *Granulation* of the condensate. Finally, in Section 3.4 we discuss the presented results and provide theoretical results very useful to discuss and interpret our experimental observations.

In Chapter 4 we describe our second generation setup and our motivations to construct it. Next, we explain all the steps necessary to produce a Bose condensed sample of ^{87}Rb . This includes the mounting of the vacuum system in which the sample is produced and the experiments are performed (Section 4.2); the laser setup used to produce the light in the proper conditions to manipulate and cool down the atoms (Section 4.3); the magneto–optical trap where the atoms are initially captured and cooled down (Section 4.4); the diagnosis system based on absorption imaging (Section 4.5); the mode matching process through which the atoms are transferred from the magneto–optical trap to a pure magnetic trap (Section 4.6) to be subsequently transferred into a hybrid trap (Section 4.7). This hybrid trap is a combination of magnetic and laser fields that generate a harmonic potential where the atoms are evaporatively cooled down below the phase transition temperature. This Chapter ends with Section 4.8 with a summary of the whole experimental sequence to produce the BEC.

We finally present our conclusion and discuss our future plans in Chapter 5.

2 *Bose–Einstein Condensation and Superfluidity*

In this Chapter we introduce the main concepts of Bose–Einstein condensation and superfluidity, that are very important for understanding many of the results presented in this Thesis.

In Section 2.1 we start by describing the ideal Bose gas in a box and in a harmonic potential. Next, in Section 2.2, we explain how interactions are taken into account and deduce the Gross–Pitaevskii equation, which is an excellent model to describe a weakly interacting Bose gas at zero temperature.

In Section 2.3 we go beyond the Gross–Pitaevskii model and present the Bogoliubov approximation, which will allow us to explain one of the most remarkable phenomena at low temperatures: *superfluidity*. In particular we describe quantized vortices, which represent a very interesting effect of superfluidity.

Finally, in Section 2.4, we introduce the concept of *Turbulence* for both, classical and quantum fluids. Quantized vortices and quantum turbulence will be the central subjects in Chapter 3.

2.1 The non–interacting Bose gas

The following discussion can be found in standard textbooks, see for example Reference (11).

In a system of N identical bosons, the phenomenon of Bose–Einstein condensation consists of the macroscopic population of the single particle ground state. For certain conditions of

density and temperature of the system, all particles occupy exactly the same quantum level. The system becomes quantum degenerate. In order to understand how this phenomenon occurs, we need to look at the particle statistics.

Consider that the gas of N bosons is confined in an external potential $U(\mathbf{r})$ where the energy of the n -th level is ϵ_n , then the occupation number of the state $|i\rangle$ is given by the Bose–Einstein distribution function,

$$f_{BE}(\epsilon_i) = \frac{1}{e^{\beta(\epsilon_i - \mu)} - 1}, \quad (2.1)$$

here we define $\beta = 1/k_B T$, where k_B is the Boltzmann constant, T the temperature of the gas and μ the chemical potential. The chemical potential can be understood as a measure of how much the free energy of a system changes by adding or removing a particle while all other thermodynamical variables remain constant. Note that for keeping $f_{BE}(\epsilon_i)$ positive and finite, this equation requires that $\mu < \epsilon_0$, where ϵ_0 is the energy of the ground state ($f_{BE}(\epsilon_0)$ would diverge if $\mu = \epsilon_0$). To simplify calculations, it is very common to set $\epsilon_0 = 0$ and then this condition becomes $\mu < 0$.

The total number of particles in the system is the sum of $f_{BE}(\epsilon_i)$ over all states i . The sum can also be performed over all energies ϵ_i but in this case we need to consider the degeneracy of each energy level g_i , therefore

$$N = \sum_j f_{BE}(\epsilon_j) = \sum_j \frac{1}{z^{-1} e^{\beta \epsilon_j} - 1} = \sum_{\epsilon_j} \frac{g_j}{z^{-1} e^{\beta \epsilon_j} - 1}, \quad (2.2)$$

where we have defined the *fugacity* as $z = \exp(\beta \mu)$.

In the thermodynamic limit in which the volume and the number of atoms tend to infinity but the density keeps constant, $V \rightarrow \infty$, $N \rightarrow \infty$ and $n = N/V = \text{constant}$, the spacing between two consecutive energy levels is much smaller than the typical energy scale of the system $\epsilon_{i+1} - \epsilon_i \ll k_B T$. Under this limit, our distribution of states becomes continuous and the difference between energy levels becomes infinitesimal. In this case we can substitute the sums of Equation (2.2)

by integrals and replace the degeneracy of states g_j by the density of states which is given by

$$\rho(\varepsilon) = \frac{2\pi(2m)^{3/2}}{h^3} \int_{V^*(\varepsilon)} \sqrt{\varepsilon - U(\mathbf{r})} d^3r, \quad (2.3)$$

where $V^*(\varepsilon)$ is the available volume in the ε -space for particles with energy ε . The physical meaning of the density of states is clear, $\rho(\varepsilon)d\varepsilon$ is the number of states with energy between ε and $\varepsilon + d\varepsilon$.

Considering this, the Equation (2.2) is rewritten as

$$N = N_0 + \int_0^\infty f_{BE}(\varepsilon) \rho(\varepsilon) d\varepsilon, \quad (2.4)$$

where we have explicitly separated the population of the ground state N_0 . The reason for doing this is that it can be shown that for most cases of interest $\rho(\varepsilon) \propto \varepsilon^\alpha$ with $\alpha > 0$, which means that $\rho(\varepsilon) \rightarrow 0$ when $\varepsilon \rightarrow 0$ and, therefore, $N_0 \rightarrow 0$. Typically, the population of the ground state is very small, excepting the special case of Bose–Einstein condensation in which it becomes macroscopic, hence, it is convenient to study separately the term for N_0 .

To evaluate the integral of Equation (2.4) we need to know the explicit form of the potential $U(\mathbf{r})$ in which the bosons are confined. In the following we do it for two specific cases: the box and the harmonic trap.

2.1.1 Non-interacting Bose gas in a box

Consider the case of free particles trapped in a three-dimensional box with volume V . Using Equation (2.3) it is easy to show that the density of states is given by

$$\rho(\varepsilon) = V \frac{2\pi(2m)^{3/2}}{h^3} \sqrt{\varepsilon}. \quad (2.5)$$

Using Equations (2.2) and (2.4) we can obtain an expression for the density n of the gas (11),

$$n = \frac{N}{V} = \frac{1}{V} \frac{z}{1-z} + \frac{1}{V} \int_0^\infty \frac{\rho(\varepsilon)}{z^{-1}e^{\beta\varepsilon} - 1} d\varepsilon = \frac{1}{V} \frac{z}{1-z} + \frac{g_{3/2}(z)}{\lambda_{dB}^3} = n_0 + n_{ex}, \quad (2.6)$$

where n_0 and n_{ex} are, respectively, the density of the particles in the ground state and in all

excited states. Here we have introduced the *thermal de Broglie wavelength* λ_{dB} and the Bose functions respectively given by

$$\lambda_{dB} = \frac{h}{\sqrt{2\pi m k_B T}} \quad (2.7)$$

$$g_\alpha(z) = \sum_{m=1}^{\infty} \frac{z^m}{m^\alpha}. \quad (2.8)$$

Now, let us add particles to the system keeping T and V constant, this will increase the density n of the gas. If we increase n , the right-side of Equation (2.6) must also increase. In fact, the chemical potential μ continuously increases up to the ground state energy ε_0 that we have set to be zero. So, as $\mu \rightarrow 0$ the fugacity $z \rightarrow 1$. Remember, in order to keep $f_{BE}(\varepsilon)$ positive, μ cannot be larger than zero. As the fugacity approaches to unity the density of particles in the excited states saturates to a maximum value given by

$$n_{ex}^{max} = g_{3/2}(1)/\lambda_{dB}^3. \quad (2.9)$$

Thus, if we keep adding particles to the gas, the population of the excited states cannot increase anymore and the ground state gets macroscopically populated giving rise to the Bose–Einstein condensation.

A very important quantity is the *phase-space density*, defined as $\varpi = n\lambda_{dB}^3$. It provides a measure of the typical occupancy of single-particle states in the 6-dimensional $(\mathbf{x}, \mathbf{p})$ phase-space. From Equation (2.9) we can define a critical phase-space density for the BEC phase transition to occur, namely

$$\varpi_c = n\lambda_{dB}^3 = \zeta(3/2) \approx 2.612\dots \quad (2.10)$$

where we have used the Bose functions property $g_\alpha(z=1) = \zeta(\alpha)$, where $\zeta(\alpha)$ is the Riemann zeta function. When $\varpi = \varpi_c$ the population of the excited states saturates and when $\varpi > \varpi_c$ the occupation of the ground state starts to increase.

Instead of increasing the number of particles at T and V constant, we could also decrease the temperature to achieve BEC. In this case, Equation (2.10) can be used to obtain an expression for

the critical temperature below which the macroscopic occupation of the ground state happens, namely

$$T_c = \frac{h^2}{2\pi m k_B} \left(\frac{n}{\zeta(3/2)} \right)^{2/3}. \quad (2.11)$$

Finally, using Equations (2.6), (2.10) and (2.11) we can obtain an expression for the fraction of bosons in the ground state as a function of the temperature

$$n_0(T) = 1 - \left(\frac{T}{T_c} \right)^{3/2}. \quad (2.12)$$

2.1.2 Non-interacting Bose gas in a harmonic potential

A general discussion considering an arbitrary polynomial potential can be found in Reference (28), here we only consider the specific case of a three-dimensional harmonic potential. This case is very important because most of experiments with ultracold gases use this kind of potential to trap the atoms.

Let us consider an ideal Bose gas confined in an anisotropic harmonic potential given by

$$U(x, y, z) = \frac{1}{2}m(\omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2), \quad (2.13)$$

where ω_i is the frequency of the oscillator along the i -direction. These frequencies characterize the confinement of the potential; the higher the frequency the greater the confinement. The energy levels of this potential are given by

$$\varepsilon(n_x, n_y, n_z) = \left(n_x + \frac{1}{2}\right)\hbar\omega_x + \left(n_y + \frac{1}{2}\right)\hbar\omega_y + \left(n_z + \frac{1}{2}\right)\hbar\omega_z. \quad (2.14)$$

Using Equation (2.3) it can be shown that the density of states for the harmonic potential is given by

$$\rho(\varepsilon) = \frac{\varepsilon^2}{2\hbar^3 \omega_x \omega_y \omega_z}. \quad (2.15)$$

Substituting Equation (2.15) into Equation (2.4) and solving the integral we obtain an ex-

pression for the number of atoms in the excited states,

$$N_{ex} = N - N_0 = g_3(z) \left(\frac{k_B T}{\hbar \bar{\omega}} \right)^3, \quad (2.16)$$

where $\bar{\omega} = (\omega_x \omega_y \omega_z)^{1/3}$ is the geometric mean of the frequencies of the trap (not to be confused with the phase-space density $\bar{\omega}$).

In this case, the quantum degeneracy occurs when the chemical potential approaches to the energy of the ground state: $\mu \rightarrow (\omega_x + \omega_y + \omega_z)\hbar/2$. However, for simplicity we have set this energy to zero and consider the critical point at $\mu \rightarrow 0$. In this situation we see the saturation of the population of the excited states given by

$$N_{ex}^{max} = N - N_0 = \zeta(3) \left(\frac{k_B T}{\hbar \bar{\omega}} \right)^3. \quad (2.17)$$

Supposing that at the critical point $N_0 \ll N$ we can obtain the critical temperature from Equation (2.17),

$$T_c = \frac{\hbar \bar{\omega}}{k_B} \left(\frac{N}{\zeta(3)} \right)^{1/3} \approx 0.94 \frac{\hbar \bar{\omega}}{k_B} N^{1/3}. \quad (2.18)$$

Using this expression we finally can find the ground state population as a function of temperature

$$n_0(T) = \frac{N_0(T)}{N} = 1 - \left(\frac{T}{T_c} \right)^3. \quad (2.19)$$

This Equation shows that, as T is lowered, the macroscopic population of the ground state in the harmonic trap occurs more rapidly than in the box.

2.2 Weakly interacting Bose gas

The previous discussion is quite useful to understand the physics of the phenomenon of Bose–Einstein condensation. It also provides intuition about the value of important quantities such as critical temperature and density. However, in real gases, the constituent particles always interact with each other. As a consequence, to properly describe the BEC it is important to consider the internal interactions of the system.

In this Section we briefly recall the quantum theory of scattering at low energies and introduce the concept of scattering length. Next, we introduce the Gross–Pitaevskii equation as a proper model to describe the quantum gas.

2.2.1 Quantum scattering at low energies

The quantum theory of scattering can be found in any standard quantum mechanics textbook (see, for instance, Reference (29)), here we just derive the important concepts useful for the further discussions.

Let us consider the elastic scattering of two particles with no internal degrees of freedom and masses m_1 and m_2 , approaching each other along the z -direction. Neglecting spin–spin and spin–orbit interactions, the Schrödinger equation written in the coordinate system of the center of mass of the particles is

$$\left(-\frac{\hbar^2}{2m^*}\nabla^2 + V(r)\right)\psi(\mathbf{r}) = E\psi(\mathbf{r}), \quad (2.20)$$

where $\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$ is the interatomic separation, $r = |\mathbf{r}|$ and $m^* = m_1 m_2 / (m_1 + m_2)$ is the reduced mass of the particles. Here we have assumed that the interatomic potential, $V(r)$, is spherically symmetric. In the asymptotic limit for large interatomic distances¹, the solution of Equation (2.20) can be seen as the sum of an incoming plane wave and a scattered spherical wave modulated with a certain amplitude,

$$\psi(\mathbf{r}) = e^{ikz} + f(\theta) \frac{e^{ikr}}{r}, \quad (2.21)$$

where $k = \sqrt{2m^*E/\hbar^2}$ is the amplitude of the wave vector of the incoming and scattered waves, and θ is the angle between $\mathbf{r}$ and the z -axis. The function $f(\theta)$ is called *scattering amplitude* and determines the scattering cross section of the collision σ through the expression

$$\frac{d\sigma}{d\Omega} = |f(\theta)|^2, \quad (2.22)$$

where $d\Omega = \sin\theta d\theta d\phi$ is the element of solid angle.

¹ This means that $r \gg r_0$, where r_0 is the range of the potential $V(r)$.

To calculate the scattering amplitude we propose a wavefunction in terms of an expansion in the different components of angular momentum, l , that is

$$\psi(r) = \sum_{l=0}^{\infty} A_l P_l(\cos \theta) R_{kl}(r). \quad (2.23)$$

Using this *ansatz* it is possible to show that in the asymptotic limit the radial wavefunction $R_{kl}(r)$, the scattering amplitude and cross section are expressed in terms of a *phase shift* δ_l , namely

$$R_{kl}(r) = \frac{1}{kr} \sin(kr - l\pi/2 + \delta_l), \quad (2.24)$$

$$f(\theta) = \frac{1}{2ik} \sum_{l=0}^{\infty} (2l+1) \left(e^{i2\delta_l} - 1 \right) P_l(\cos \theta), \quad (2.25)$$

$$\sigma = \frac{4\pi}{k^2} \sum_{l=0}^{\infty} (2l+1) \sin^2 \delta_l. \quad (2.26)$$

For a finite range potential, that is, a potential that decays faster than r^{-3} (interatomic potentials typically behave as r^{-6} or r^{-7}) the phase shift satisfies $\delta_l \propto k^{2l+1}$ for small k . In an ultracold gas, the energy of the collisions is very low and $k \rightarrow 0$, thus the scattering will be dominated by terms with $l = 0$ (the so-called s-wave scattering). In this limit, Equations (2.24), (2.25) and (2.26) can be approximated as

$$R_{k0}(r) \simeq c_1 \frac{\sin kr}{kr} + c_2 \frac{\cos kr}{r}, \quad (2.27)$$

$$f(\theta) \simeq \frac{\delta_0}{k} \quad \text{and} \quad \sigma \simeq \frac{4\pi}{k^2} \delta_0^2, \quad (2.28)$$

where c_1 and c_2 are constant coefficients related to the phase shift through the expression

$$\tan \delta_0 = k \frac{c_2}{c_1}. \quad (2.29)$$

As mentioned above, in this approximation ($k \rightarrow 0$), the phase shift satisfies $\delta_l \propto k^{2l+1}$; for $l = 0$ we define the proportionality constant $\delta_0 = -a_s k$, where a_s is known as *scattering length*. By taking the limit $k \rightarrow 0$ in Equation (2.29) we obtain an expression for the scattering length

in terms of the coefficients c_1 and c_2 ,

$$a_s = - \left. \frac{c_2}{c_1} \right|_{k \rightarrow 0} \quad (2.30)$$

Therefore, the scattering amplitude and cross section at very low temperatures in the asymptotic limit are given by

$$f(\theta) = -a_s \quad \text{and} \quad \sigma = 4\pi a_s^2. \quad (2.31)$$

The scattering process can be understood in the following way: during the collision, the wavefunction of the system suffers a phase shift δ_0 which can be positive or negative, depending on the sign of a_s . If $a_s < 0$ the phase is “delayed” with respect to the situation in which there is no scattering. This is equivalent to having an attractive interaction. In opposition, if $a_s > 0$ the phase is “advanced” and the interaction is repulsive. Evidently, the intensity of the interaction is proportional to the value of $|a_s|$.

The expression for σ in Equation (2.31) indicates that the atoms behave as hard spheres with radius $|a_s|$. The specific value of a_s will depend on the interaction potential, however, the details of the potential become unimportant and all the information of the collision is contained in a_s . As a consequence, in the low energies limit, we can suppose that the collision is mediated by an effective potential $U_e(\mathbf{r})$ which has the property

$$\int U_e(\mathbf{r}) d^3r = \frac{4\pi\hbar^2}{m} a_s \equiv U_0, \quad (2.32)$$

therefore, the effective interaction among two particles at positions $\mathbf{r}$ and $\mathbf{r}'$ can be considered as a contact interaction given by

$$U_e(\mathbf{r}, \mathbf{r}') = U_0 \delta(\mathbf{r} - \mathbf{r}'). \quad (2.33)$$

2.2.2 Gross–Pitaevskii Equation

In an atomic Bose–Einstein condensate the density is so high that the atomic interactions become important. However, the density is still low enough to neglect the effect of collisions

between more than two atoms, so we only need to consider binary interactions. In consequence, the theory presented in the last Section turns out to be very appropriate for our system. In the following we will deduce the Gross–Pitaevskii equation, which constitutes an excellent description for a zero temperature BEC. The following discussion is based on References (30–32) an additional reference are the lecture notes of Professor Víctor Romero from Universidad nacional Autónoma de México, these notes are an excellent and didactic introduction to many–body physics (33).

A gas of N interacting bosons trapped in an external potential $U(\mathbf{r}, t)$ can be correctly described using the second quantization scheme. In this formalism the state of the system is expressed using number–particle states in which the number of particles in a determinate one–particle state is explicitly indicated. The i -th single–particle state is represented by a quantum number k_i which contains all the quantum numbers necessary to represent the state. In this notation, the state

$$|n_{k_0}, n_{k_1}, \dots, n_{k_i}, \dots, n_{k_\infty}\rangle = |n_{k_0}\rangle \otimes |n_{k_1}\rangle \otimes \dots \otimes |n_{k_\infty}\rangle, \quad (2.34)$$

is a many–body state with n_{k_0} particles in the single–particle ground state, n_{k_i} particles in the single–particle state with quantum numbers k_i , etc. Since we are studying a system of N bosons, $n_{k_i} = 0, 1, 2, 3, \dots, N$; *i. e.* n_{k_i} can take any value. For the same reason, when this state is projected onto the real space, $\langle \mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_N | n_{k_0}, n_{k_1}, \dots, n_{k_i}, \dots, n_{k_\infty} \rangle$, we must obtain a symmetrized combination of wavefunctions. These number–particle states must also obey the number conservation, orthogonality and completeness conditions:

$$N = \sum_k n_k \quad (\text{number conservation}) \quad (2.35)$$

$$\langle n_{k_0}, n_{k_1}, \dots, n_{k_\infty} | n_{k_0}, n_{k_1}, \dots, n_{k_\infty} \rangle = \delta_{n'_{k_0} n_{k_0}} \delta_{n'_{k_1} n_{k_1}} \dots \delta_{n'_{k_\infty} n_{k_\infty}} \quad (\text{orthogonality}) \quad (2.36)$$

$$\sum_{n_{k_0}, n_{k_1}, \dots, n_{k_\infty}} |n_{k_0}, n_{k_1}, \dots, n_{k_\infty}\rangle \langle n_{k_0}, n_{k_1}, \dots, n_{k_\infty}| = \hat{1} \quad (\text{completeness}). \quad (2.37)$$

The many-body Hamiltonian is given by

$$\begin{aligned}\hat{H} = & \int d^3r \hat{\psi}^\dagger(\mathbf{r}) \left(-\frac{\hbar^2}{2m} \nabla^2 + U(\mathbf{r}, t) \right) \hat{\psi}(\mathbf{r}) + \\ & + \frac{1}{2} \int d^3r \int d^3r' \hat{\psi}^\dagger(\mathbf{r}) \hat{\psi}^\dagger(\mathbf{r}') U_e(\mathbf{r}, \mathbf{r}') \hat{\psi}(\mathbf{r}') \hat{\psi}(\mathbf{r}),\end{aligned}\quad (2.38)$$

where we are considering binary interactions between the bosons through the potential $U_e(\mathbf{r}, \mathbf{r}')$. Here $\hat{\psi}^\dagger(\mathbf{r})$ and $\hat{\psi}(\mathbf{r})$ are the so-called *field operators* and are defined as the following linear superpositions

$$\hat{\psi}(\mathbf{r}) = \sum_{\mathbf{k}} \Phi_{\mathbf{k}}(\mathbf{r}) \hat{b}_{\mathbf{k}}, \quad (2.39)$$

$$\hat{\psi}^\dagger(\mathbf{r}) = \sum_{\mathbf{k}} \Phi_{\mathbf{k}}^*(\mathbf{r}) \hat{b}_{\mathbf{k}}^\dagger, \quad (2.40)$$

where the vector $\mathbf{k}$ indicates the state of the particle and $\Phi_{\mathbf{k}}(\mathbf{r})$ is the wavefunction of a particle in state $\mathbf{k}$. The operators $\hat{b}_{\mathbf{k}}$ and $\hat{b}_{\mathbf{k}}^\dagger$ are, respectively, the bosonic creation and annihilation operators which satisfy the following commutation rules

$$[\hat{b}_{\mathbf{k}}, \hat{b}_{\mathbf{k}'}^\dagger] = \delta_{\mathbf{k}\mathbf{k}'} \quad \text{and} \quad [\hat{b}_{\mathbf{k}}, \hat{b}_{\mathbf{k}'}] = [\hat{b}_{\mathbf{k}}^\dagger, \hat{b}_{\mathbf{k}'}^\dagger] = 0, \quad (2.41)$$

and act on the number states as follows

$$\hat{b}_{\mathbf{k}} |n_{\mathbf{k}}\rangle = \sqrt{n_{\mathbf{k}}} |n_{\mathbf{k}} - 1\rangle, \quad (2.42)$$

$$\hat{b}_{\mathbf{k}}^\dagger |n_{\mathbf{k}}\rangle = \sqrt{n_{\mathbf{k}} + 1} |n_{\mathbf{k}} + 1\rangle, \quad (2.43)$$

$$\hat{b}_{\mathbf{k}}^\dagger \hat{b}_{\mathbf{k}} |n_{\mathbf{k}}\rangle = n_{\mathbf{k}} |n_{\mathbf{k}}\rangle. \quad (2.44)$$

The *field* operators defined in Equations (2.39) and (2.40) are so called simply because they are operators that depend on the position $\mathbf{r}$. It is not difficult to prove that they obey the following commutation relations

$$[\hat{\psi}(\mathbf{r}), \hat{\psi}^\dagger(\mathbf{r}')] = \delta(\mathbf{r} - \mathbf{r}'), \quad (2.45)$$

$$[\hat{\psi}(\mathbf{r}), \hat{\psi}(\mathbf{r}')] = [\hat{\psi}^\dagger(\mathbf{r}), \hat{\psi}^\dagger(\mathbf{r}')] = 0. \quad (2.46)$$

Now we obtain the evolution of these fields using the Heisenberg equation,

$$i\hbar \frac{\partial}{\partial t} \hat{\psi}(\mathbf{r}, t) = [\hat{\psi}(\mathbf{r}, t), \hat{H}]. \quad (2.47)$$

Substituting the Hamiltonian of Equation (2.38) into Equation (2.47), using the commutation relations (2.45) and (2.46), and using the interatomic potential of Equation (2.33) we get the Heisenberg equation for $\hat{\psi}(\mathbf{r})$

$$i\hbar \frac{\partial}{\partial t} \hat{\psi}(\mathbf{r}, t) = \left(-\frac{\hbar^2}{2m} \nabla^2 + U(\mathbf{r}, t) \right) \hat{\psi}(\mathbf{r}, t) + U_0 \hat{\psi}^\dagger(\mathbf{r}, t) \hat{\psi}(\mathbf{r}, t) \hat{\psi}(\mathbf{r}, t). \quad (2.48)$$

Now we consider the gas to be at zero temperature. In this case, we would expect most of the bosons to be in the ground state of the potential, that is, most of the particles are in the state with $\mathbf{k} = 0$, which corresponds to the zero momentum state. In this case, the field operators of Equations (2.39) and (2.40) can be approximated as

$$\hat{\psi}(\mathbf{r}, t) \simeq \Phi_0(\mathbf{r}, t) \hat{b}_0, \quad (2.49)$$

$$\hat{\psi}^\dagger(\mathbf{r}, t) \simeq \Phi_0^*(\mathbf{r}, t) \hat{b}_0^\dagger. \quad (2.50)$$

Substituting these field operators into Equation (2.48) we obtain

$$i\hbar \frac{\partial}{\partial t} \Phi_0(\mathbf{r}, t) = \left(-\frac{\hbar^2}{2m} \nabla^2 + U(\mathbf{r}, t) + U_0 |\Phi_0(\mathbf{r}, t)|^2 \right) \Phi_0(\mathbf{r}, t). \quad (2.51)$$

This is the Gross–Pitaevskii equation for the condensate wavefunction $\Phi_0(\mathbf{r}, t)$. It is a non-linear equation with great mathematical richness; as we will see, very interesting phenomena such as vorticity and quantum turbulence can be described with it. Of course, if the external potential is time-independent, $U(\mathbf{r}, t) = U(\mathbf{r})$, we can obtain the time-independent form of the Gross–Pitaevskii equation by substituting the wavefunction $\Phi_0(\mathbf{r}, t) = \varphi(\mathbf{r}) e^{-i\mu t/\hbar}$, where μ is the chemical potential (31),

$$\left(-\frac{\hbar^2}{2m} \nabla^2 + U(\mathbf{r}) + U_0 |\varphi(\mathbf{r})|^2 \right) \varphi(\mathbf{r}) = \mu \varphi(\mathbf{r}). \quad (2.52)$$

The Thomas–Fermi approximation

The simplest solution of Equation (2.52) accounts for the case in which the kinetic energy is much smaller than the interaction energy. Since $|\varphi(\mathbf{r})|^2$ can be interpreted as the density of the condensate, this approximation is valid for sufficiently large clouds. In this case we neglect the kinetic term obtaining the following algebraic equation:

$$\left(U(\mathbf{r}) + U_0 |\varphi(\mathbf{r})|^2 \right) \varphi(\mathbf{r}) = \mu \varphi(\mathbf{r}), \quad (2.53)$$

whose solution is given by

$$|\varphi(\mathbf{r})|^2 = \frac{1}{U_0} [\mu - U(\mathbf{r})], \quad (2.54)$$

for the case in which $\mu \leq U(\mathbf{r})$ and $|\varphi(\mathbf{r})|^2 = 0$ otherwise. Therefore, the density in the center of the cloud (the “peak density”) is $n(0) = \mu/U_0$ and the boundary of the cloud is given by $\mu = U(\mathbf{r})$.

This approximation is called *Thomas–Fermi approximation* and shows that if we know the trapping potential then we know the density profile of the condensate. For the harmonic potential of Equation (2.13) the extension of the cloud along the three directions is

$$R_i = \sqrt{\frac{2\mu}{m\omega_i^2}}, \quad i = x, y, z. \quad (2.55)$$

Using the condition $N = \int |\varphi(\mathbf{r})|^2 d^3r$ we can obtain an expression for the chemical potential

$$\mu = \frac{\hbar\bar{\omega}}{2} \left(\frac{15Na_s}{\bar{a}} \right)^{2/5}, \quad (2.56)$$

where $\bar{a} = \sqrt{\hbar/m\bar{\omega}}$ and $\bar{\omega} = (\omega_x\omega_y\omega_z)^{1/3}$.

Using the definition $\mu = \partial E / \partial N$ and Equation (2.56) the energy per particle can be obtained (31)

$$\frac{E}{N} = \frac{5}{7}\mu. \quad (2.57)$$

The Thomas–Fermi approximation, although very simple, provides one of the main signatures of the onset of the condensation: a thermal cloud has a Gaussian density profile, but as

we decrease the temperature below the critical point, the appearance of a parabolic peak can be observed, indicating the presence of a condensed fraction.

Healing length

The Thomas–Fermi approximation is a very good approximation to describe a BEC trapped in a potential which smoothly varies in space. In the case of a BEC trapped in a box, the potential increases abruptly from zero to infinity at the walls. The prediction under the Thomas–Fermi approximation is that the condensate wavefunction is constant everywhere inside the box, namely

$$|\varphi(\mathbf{r})|^2 = \frac{\mu}{U_0}. \quad (2.58)$$

This is a good description far away from the walls, however close to them it is not possible to neglect the kinetic energy term anymore. To understand the behavior of the BEC let us consider a potential that vanishes for $x \geq 0$ and is infinite if $x < 0$. In this case, the wavefunction $\varphi(\mathbf{r})$ is uniform in the y and z -directions and the Gross–Pitaevskii equation reduces to a 1D equation,

$$-\frac{\hbar^2}{2m} \frac{d^2 \varphi(x)}{dx^2} + U_0 |\varphi(x)|^2 \varphi(x) = \mu \varphi(x). \quad (2.59)$$

From Equation (2.58) we can obtain an approximate expression for the chemical potential: $\mu = U_0 |\varphi_0|^2$, where φ_0 is the wavefunction far from the wall. Thus, $|\varphi_0|^2 \equiv \rho_0$ can be interpreted as the density of the bulk condensate. The boundary conditions are $\varphi(0) = 0$ and $\varphi(x) \rightarrow \varphi_0$ as $x \rightarrow \infty$. Under this considerations, the Equation (2.59) has analytical solution given by

$$\varphi(x) = \varphi_0 \tanh\left(\frac{x}{\sqrt{2}\xi}\right), \quad (2.60)$$

where ξ is given by

$$\xi = \left(\frac{\hbar^2}{2m\rho_0 U_0}\right)^{1/2} = \left(\frac{1}{8\pi\rho_0 a_s}\right)^{1/2}. \quad (2.61)$$

ξ is called *healing length* and defines the distance over which the condensate wavefunction tends to its bulk value when it is subjected to a local perturbation, such as the wall presented

in this case or at the limits of a harmonically trapped BEC. In other words, if we are able to produce a local perturbation in the BEC its size will be of the order of the healing length. Note that since $\xi \sim \rho_0^{-1/2}$, the healing length of a dilute BEC is much bigger than in other ultracold Bose systems, such as superfluid helium which is much denser. In fact, while $\xi \sim 0.1$ nm for ^4He and $\xi \sim 70$ nm for ^3He , for a typical ^{87}Rb BEC $\xi \sim 1$ μm . This constitutes one of the main advantages of studying superfluidity in BECs when compared with superfluid helium: the size of the perturbations, such as vortices, is much bigger and, hence, easier to observe.

2.3 Superfluidity

Superfluidity is one of the most remarkable phenomena at ultra low temperatures. It consists of the capacity of the fluid to flow without viscosity. To explain this phenomenon, sophisticated full-quantum theories were required to account for interactions in ultracold systems of bosons, showing that both, interactions and macroscopic population of a single quantum level are essential ingredients. Yet, the intuitive idea of superfluidity is very simple: if energy is pumped to the system below a certain threshold value, it shall not be able to generate excitations in the superfluid, instead, all energy will be employed to flow. Since no excitations were generated the energy is conserved and continuously used for the fluid to flow with no resistance nor dissipation. This represents the superfluid state of the system.

In the following we will present the important steps to theoretically understand superfluidity by means of the so-called Bogoliubov approximation. A deep development of the theory can be found in many references, for instance (31, 34, 35).

2.3.1 Bogoliubov Approximation

Let us consider the case of N interacting bosons contained in a box of volume V . In this case, the wavefunctions $\Phi_{\mathbf{k}}(\mathbf{r})$ of Equation (2.40) are free waves, and, for instance, the annihilation

field operator is rewritten as

$$\hat{\psi}(\mathbf{r}) = \frac{1}{V^{1/2}} \sum_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{r}} \hat{b}_{\mathbf{k}} = \frac{V^{1/2}}{(2\pi)^3 \hbar^2} \int d\mathbf{k} e^{i\mathbf{k}\cdot\mathbf{r}} \hat{b}_{\mathbf{k}}, \quad (2.62)$$

where we have supposed that the spacing between $\mathbf{k}$ -levels is small and, thus, substituted the sum by an integral. Note that the integral of the left side of Equation (2.62) has the mathematical form of a Fourier transform, so we can invert it, thus

$$\hat{b}_{\mathbf{k}} = \frac{1}{V^{1/2}} \int d\mathbf{r} e^{-i\mathbf{k}\cdot\mathbf{r}} \hat{\psi}(\mathbf{r}). \quad (2.63)$$

Substituting Equation (2.62) into Equation (2.38) and using the contact interaction of Equation (2.33), the many-body Hamiltonian becomes

$$\hat{H} = \sum_{\mathbf{k}} \frac{\hbar^2 k^2}{2m} \hat{b}_{\mathbf{k}}^\dagger \hat{b}_{\mathbf{k}} + \frac{U_0}{2} \sum_{\mathbf{k}, \mathbf{k}', \mathbf{q}} \hat{b}_{\mathbf{k}+\mathbf{q}}^\dagger \hat{b}_{\mathbf{k}'-\mathbf{q}}^\dagger \hat{b}_{\mathbf{k}} \hat{b}_{\mathbf{k}'}. \quad (2.64)$$

At this point we must understand that, due to the presence of interactions, the number state of Equation (2.34) is not an eigenstate of the Hamiltonian of Equation (2.64), and the N -body ground state cannot be $n_{k_0} = N_0 = N$ and $n_{k_i} = 0$ for $k_i \neq 0$. The Bogoliubov approximation consists in assuming that there are many particles in the states with $k_i \neq 0$ but still we have a very big condensed fraction ($N_0 \sim N$). Now we shall see how we must account for the presence of particles in the excited single-particle states.

Now two important suppositions: first, since $\hat{b}_0^\dagger \hat{b}_0 |n_{k_0}, n_{k_1}, \dots, n_{k_\infty}\rangle = N_0 |n_{k_0}, n_{k_1}, \dots, n_{k_\infty}\rangle$ and $N_0 \gg n_{k_i}$ for $k_i \neq 0$, we approximate the creation and annihilation operators of the single-particle ground state by numbers

$$\hat{b}_0^\dagger \approx \sqrt{N_0} \quad \text{and} \quad \hat{b}_0 \approx \sqrt{N_0}. \quad (2.65)$$

Next, we note that the total number of particles is $N = N_0 + \sum_{\mathbf{k} \neq 0} \hat{b}_{\mathbf{k}}^\dagger \hat{b}_{\mathbf{k}}$, but we neglect terms of second order in $\hat{b}_{\mathbf{k}}^\dagger \hat{b}_{\mathbf{k}}$, that is

$$N^2 \approx N_0^2 + 2N_0 \sum_{\mathbf{k} \neq 0} \hat{b}_{\mathbf{k}}^\dagger \hat{b}_{\mathbf{k}}. \quad (2.66)$$

Using the approximations of Equations (2.65) and (2.66) in the Hamiltonian of Equation (2.64) we obtain Bogoliubov's Hamiltonian:

$$\hat{H} = \frac{N^2 U_0}{2V} + \sum_{\mathbf{k} \neq 0} \left(\epsilon_{\mathbf{k}}^0 + \frac{NU_0}{V} \right) \hat{b}_{\mathbf{k}}^\dagger \hat{b}_{\mathbf{k}} + \frac{NU_0}{2V} \sum_{\mathbf{k} \neq 0} \left(\hat{b}_{\mathbf{k}}^\dagger \hat{b}_{-\mathbf{k}}^\dagger + \hat{b}_{\mathbf{k}} \hat{b}_{-\mathbf{k}} \right), \quad (2.67)$$

where we have defined $\epsilon_{\mathbf{k}}^0 = \hbar^2 k^2 / 2m$. The solution for this Hamiltonian was also provided by Bogoliubov and has a very deep and beautiful physical interpretation.

The Bogoliubov transformation

Bogoliubov's proposal (36) consists in introducing new operators $\hat{\alpha}_{\mathbf{k}}$ and $\hat{\alpha}_{-\mathbf{k}}$ in the following way,

$$\begin{aligned} \hat{\alpha}_{\mathbf{k}} &= u_{\mathbf{k}} \hat{b}_{\mathbf{k}} + v_{\mathbf{k}} \hat{b}_{-\mathbf{k}}^\dagger \\ \hat{\alpha}_{-\mathbf{k}} &= u_{\mathbf{k}} \hat{b}_{-\mathbf{k}} + v_{\mathbf{k}} \hat{b}_{\mathbf{k}}^\dagger, \end{aligned} \quad (2.68)$$

where $u_{\mathbf{k}}$ and $v_{\mathbf{k}}$ are real coefficients with the property $u_{\mathbf{k}} = u_{-\mathbf{k}}$ and $v_{\mathbf{k}} = v_{-\mathbf{k}}$. If we impose the condition $u_{\mathbf{k}}^2 - v_{\mathbf{k}}^2 = 1$, then the commutation rule $[\hat{\alpha}_{\mathbf{k}}, \hat{\alpha}_{\mathbf{k}'}^\dagger] = \delta_{\mathbf{k}\mathbf{k}'}$ is fulfilled, and hence these operators are bosonic creation and annihilation operators. Then, the inverse transformation is given by

$$\begin{aligned} \hat{b}_{\mathbf{k}} &= u_{\mathbf{k}} \hat{\alpha}_{\mathbf{k}} - v_{\mathbf{k}} \hat{\alpha}_{-\mathbf{k}}^\dagger \\ \hat{b}_{-\mathbf{k}} &= u_{\mathbf{k}} \hat{\alpha}_{-\mathbf{k}} - v_{\mathbf{k}} \hat{\alpha}_{\mathbf{k}}^\dagger. \end{aligned} \quad (2.69)$$

The Bogoliubov Hamiltonian of Equation (2.67) can be rewritten in a very simple way by choosing the coefficients $u_{\mathbf{k}}$ and $v_{\mathbf{k}}$ in the following way

$$u_{\mathbf{k}}^2 = \frac{1}{2} \left(\frac{\epsilon_{\mathbf{k}}}{\sqrt{(\epsilon_{\mathbf{k}}^0)^2 + 2\frac{NU_0}{V}\epsilon_{\mathbf{k}}^0}} + 1 \right) \quad \text{and} \quad v_{\mathbf{k}}^2 = \frac{1}{2} \left(\frac{\epsilon_{\mathbf{k}}}{\sqrt{(\epsilon_{\mathbf{k}}^0)^2 + 2\frac{NU_0}{V}\epsilon_{\mathbf{k}}^0}} - 1 \right), \quad (2.70)$$

where we define $\epsilon_{\mathbf{k}} \equiv \epsilon_{\mathbf{k}}^0 + \frac{NU_0}{V}$. Now we substitute the transformations of (2.69) with the

coefficients of (2.70) in the Hamiltonian of Equation (2.67) and obtain

$$\hat{H} = \frac{N^2 U_0}{2V} + \sum_{\mathbf{k} > 0} \left(\sqrt{(\epsilon_{\mathbf{k}}^0)^2 + 2 \frac{NU_0}{V} \epsilon_{\mathbf{k}}^0} - \epsilon_{\mathbf{k}}^0 - \frac{NU_0}{V} \right) + \sum_{\mathbf{k} \neq 0} \sqrt{(\epsilon_{\mathbf{k}}^0)^2 + 2 \frac{NU_0}{V} \epsilon_{\mathbf{k}}^0} \hat{\alpha}_{\mathbf{k}}^\dagger \hat{\alpha}_{\mathbf{k}}. \quad (2.71)$$

Note that this Hamiltonian has the following form

$$\hat{H} = \mathcal{A} + \sum_{\mathbf{k} \neq 0} \mathcal{E}_{\mathbf{k}} \hat{\alpha}_{\mathbf{k}}^\dagger \hat{\alpha}_{\mathbf{k}}, \quad (2.72)$$

where $\mathcal{A}$ is a constant. If we compare this Hamiltonian with the Hamiltonian of Equation (2.64) we can see that, except for a constant, it corresponds to the many-body Hamiltonian of a non interacting gas constituted *not* by particles, but by *quasi-particles*. These quasi-particles are created and annihilated by the operators $\hat{\alpha}_{\mathbf{k}}^\dagger$ and $\hat{\alpha}_{\mathbf{k}}$, having momentum $\mathbf{p} = \hbar \mathbf{k}$ and energy $\mathcal{E}_{\mathbf{k}}$, given by

$$\mathcal{E}_{\mathbf{k}} = \sqrt{(\epsilon_{\mathbf{k}}^0)^2 + 2 \frac{NU_0}{V} \epsilon_{\mathbf{k}}^0}. \quad (2.73)$$

This is Bogoliubov energy spectrum of quasi-particles in the condensate. Looking at the definition of Equation (2.68) we see that the eigenstates of the quasi-particle operators $\hat{\alpha}_{\mathbf{k}}^\dagger$ and $\hat{\alpha}_{\mathbf{k}}$ are superpositions of particle-number states with different number of particles in the states $\mathbf{k}$ and $-\mathbf{k}$. For this reason, the quasi-particles are also known as *elementary excitations*, because they correspond to excitation of particles from the ground state to states with $\mathbf{k} \neq 0$. Additionally, we can find a basis of states for the operators $\hat{\alpha}_{\mathbf{k}}^\dagger$ and $\hat{\alpha}_{\mathbf{k}}$ in terms of occupation-number states, namely

$$\left| n_{\mathbf{k}_1}^{(\alpha)}, n_{\mathbf{k}_2}^{(\alpha)}, \dots, n_{\mathbf{k}_\infty}^{(\alpha)} \right\rangle \quad (2.74)$$

having the same rules as for creation/annihilation operators, this is

$$\begin{aligned} \hat{\alpha}_{\mathbf{k}} \left| n_{\mathbf{k}}^{(\alpha)} \right\rangle &= \sqrt{n_{\mathbf{k}}^{(\alpha)}} \left| n_{\mathbf{k}}^{(\alpha)} - 1 \right\rangle \\ \hat{\alpha}_{\mathbf{k}}^\dagger \left| n_{\mathbf{k}}^{(\alpha)} \right\rangle &= \sqrt{n_{\mathbf{k}}^{(\alpha)} + 1} \left| n_{\mathbf{k}}^{(\alpha)} + 1 \right\rangle. \end{aligned} \quad (2.75)$$

The ground state of the system corresponds to $n_{\mathbf{k}}^{(\alpha)} = 0$ for all $\mathbf{k}$, and we denote it as $|0\rangle$.

We can obtain the ground state energy by evaluating $\langle 0 | \hat{H} | 0 \rangle = E_0$, which turns out to be

$$E_0 = \frac{N^2 U_0}{2V} \left[1 + \frac{128}{15\pi^{1/2}} \left(\frac{Na_s^3}{V} \right)^{1/2} \right]. \quad (2.76)$$

This result was obtained by the first time by Lee and Yang (37, 38). To be valid, the gas must be weakly interacting, *i. e.* the condition $Na_s^3/V \ll 1$ must be fulfilled. We can see that the true ground state of the N-body interacting system consists of ‘a lot’ of particles ($N_0 \sim N$) in the single-particle ground state and ‘few’ particles ($N_{ex} \ll N_0$) in all single-particle excited states. This means that, due to interactions, the state

$$|N, 0, 0, \dots, 0\rangle, \quad (2.77)$$

is *not* the ground state of the many-body system. Instead, the true ground state of the system has the following form

$$|N_0, N_1, N_2, \dots, N_\infty\rangle, \quad (2.78)$$

where $N_0 \approx N \gg N_1 + N_2 + \dots + N_\infty$. As a consequence, if somehow we were able to produce the state given by Equation (2.77), it would eventually decay to the true ground state of Equation (2.78).

2.3.2 Landau critical velocity

Now we show how the Bogoliubov spectrum of Equation (2.73) leads to the phenomenon of superfluidity.

Let us consider a bosonic fluid at zero temperature moving with velocity $\mathbf{v}$ through a tube. We expect that elementary excitations are produced in the fluid as it interacts with the walls of the tube. As a consequence, a fraction of the kinetic energy of the fluid shall be used to produce these excitations and the fluid should slow down. In other words, the excitations will generate viscosity, that is, the quantum origin of viscosity. If the excitation has energy $\mathcal{E}(\mathbf{p})$

and momentum $\mathbf{p} = \hbar \mathbf{k}$, then the total energy of the fluid is

$$\begin{aligned} E &= \mathcal{E}(\mathbf{p}) + \frac{(\mathbf{p} + M\mathbf{v})^2}{2M} = \mathcal{E}(\mathbf{p}) + \mathbf{p} \cdot \mathbf{v} + \frac{1}{2}Mv^2 + \frac{p^2}{2M} \\ &\approx \mathcal{E}(\mathbf{p}) + \mathbf{p} \cdot \mathbf{v} + \frac{1}{2}Mv^2 \end{aligned} \quad (2.79)$$

where M is the mass of the whole superfluid, since it is a macroscopic quantity while $\mathbf{p}$ is the microscopic momentum of elementary excitations we can neglect the term $p^2/2M$. The term $Mv^2/2$ is the initial kinetic energy of the fluid and, thus $\mathcal{E}(\mathbf{p}) + \mathbf{p} \cdot \mathbf{v}$ represents the energy of the excitation. Since the kinetic energy of the superfluid must decrease due to the excitation, this last term must be negative, $\mathcal{E}(\mathbf{p}) + \mathbf{p} \cdot \mathbf{v} < 0$. We know that $\mathcal{E}(\mathbf{p})$ is always positive, therefore, the condition to generate elementary excitations is

$$\mathcal{E}(\mathbf{p}) < p v, \quad (2.80)$$

where $\mathbf{p}$ and $\mathbf{v}$ should be antiparallel. In other words, we can define a critical fluid's velocity above which elementary excitation will be produced, namely

$$v_c = \min \left(\frac{\mathcal{E}(\mathbf{p})}{p} \right), \quad (2.81)$$

which is known as *Landau critical velocity*. For velocities below v_c it is impossible to generate excitations and there are no mechanisms to decrease the kinetic energy of the fluid, in consequence, the system will exhibit *superfluidity*. A general theory of the hydrodynamics of superfluids can be found in Reference (39).

Superfluidity in Bose–Einstein condensates

In the case of a weakly interacting BEC, the energy spectrum of the elementary excitations is given by the Bogoliubov spectrum of Equation (2.73). For small values of the momentum $p = \hbar k$, this spectrum becomes

$$\mathcal{E}(\mathbf{p}) \approx \left(\frac{NU_0}{mV} \right)^{1/2} \hbar k = \left(\frac{4\pi\rho\hbar^2 a_s}{m^2} \right)^{1/2} p = pc_s, \quad (2.82)$$

Figure 2.1 – The red curve represents the elementary excitation spectrum for (a) a weakly interacting gas and (b) an ideal gas, v is the velocity of the fluid. In (a) the black curve does not intersect the spectrum if $v < c_s$ and, thus, the system presents superfluidity. In (b) the black curve always intersects the spectrum, hence, an ideal gas is not a superfluid.

where $\rho = N/V$ is the density of the fluid and c_s is known as the *speed of sound* and is defined as

$$c_s \equiv \frac{\hbar}{m} \sqrt{4\pi\rho a_s}. \quad (2.83)$$

Note that the spectrum of Equation (2.82) is that of acoustic phonons. Therefore, at low energies, the elementary excitations of an interacting BEC are sound waves. Moreover, we can see that the condition for having superfluidity in an interacting BEC simply is $v < c_s$, Figure 2.1(a) sketches this situation.

A remarkable situation is that of the ideal BEC, in which we have no interactions ($a_s = 0$). In this case, the Bogoliubov spectrum reduces to the free particle spectrum

$$\mathcal{E}(\mathbf{p}) = \frac{\hbar^2 k^2}{2m} = \frac{p^2}{2m}. \quad (2.84)$$

Note that for this spectrum $\mathcal{E}(\mathbf{p}) > pv$ always, as shown in Figure 2.1(b). In consequence, no matter how small the fluid velocity is, there will be elementary excitations always. Therefore, an ideal BEC cannot be a superfluid and, as Landau first noticed, superfluidity and Bose–Einstein condensation are not the same phenomenon.

2.3.3 Quantized Vortices

To understand the phenomenon of vorticity in superfluids let us return to the time-dependent Gross-Pitaevskii equation (2.51) at zero temperature. We consider the general case in which the external potential $U(\mathbf{r}, t)$ is time-dependent. In this case, the wavefunction can be written as

$$\Phi_0(\mathbf{r}, t) = \sqrt{n_0(\mathbf{r}, t)} e^{-i\vartheta(\mathbf{r}, t)}, \quad (2.85)$$

where $\vartheta(\mathbf{r}, t)$ is the *phase* of the condensate which, as we will see in the following, turns out to be a very important quantity.

We now calculate the probability current of the quantum system, given by

$$\begin{aligned} \mathbf{j}(\mathbf{r}, t) &= \frac{\hbar}{2mi} (\Phi_0^*(\mathbf{r}, t) \nabla \Phi_0(\mathbf{r}, t) - \Phi_0(\mathbf{r}, t) \nabla \Phi_0^*(\mathbf{r}, t)) \\ &= \frac{\hbar}{m} n_0(\mathbf{r}, t) \nabla \vartheta(\mathbf{r}, t). \end{aligned} \quad (2.86)$$

From the previous discussion we know that the interacting BEC, in fact, is able to flow, so we can associate the probability current of Equation (2.86) with the actual current of the fluid, namely $\mathbf{j} = n_0 \mathbf{v}_s$. Then we can identify the condensate velocity field as

$$\mathbf{v}_s(\mathbf{r}, t) = \frac{\hbar}{m} \nabla \vartheta(\mathbf{r}, t). \quad (2.87)$$

As we can see, the hydrodynamic behavior of the BEC strongly depends on the quantum phase ϑ .

Note that since we are considering the zero temperature case the whole fluid is a superfluid². Moreover, in this Gross-Pitaevskii approximation the whole fluid is in the condensate. Then $\mathbf{v}_s(\mathbf{r}, t)$ is the superfluid velocity flow. From Equation (2.87) it is easy to see that the superfluid is irrotational, that is

$$\nabla \times \mathbf{v}_s(\mathbf{r}, t) = 0. \quad (2.88)$$

If the fluid is irrotational, how can vortices be formed? The answer is that Equation (2.88)

² This situation is not possible in strongly interacting superfluids, such as superfluid helium, where the normal fluid fraction is considerable. However, for a weakly interacting BEC it is a good approximation.

is always satisfied except when the phase has a singularity, in this case $\nabla \times \mathbf{v}_s(\mathbf{r}, t) \neq 0$. Singularities in the phase can be introduced in many ways, for instance, using time-alternating trapping potentials, as explained in Chapter 3. Now consider a closed contour $\mathcal{C}$ around the singularity, since the condensate wavefunction is single-valued, the change of the phase $\Delta\vartheta$ around the contour must be a multiple of 2π , that is

$$\Delta\vartheta = \oint_{\mathcal{C}} \nabla\vartheta \cdot d\mathbf{l} = 2\pi\ell, \quad (2.89)$$

where ℓ is an integer. Hence, we can calculate the *circulation* Γ around a closed loop, namely

$$\Gamma = \oint_{\mathcal{C}} \mathbf{v}_s \cdot d\mathbf{l} = \frac{h}{m}\ell, \quad (2.90)$$

therefore, the circulation of a superfluid is quantized in units of h/m .

Simple example: single straight vortex

As a simple example, let us consider a superfluid contained in an infinite cylindrical vessel of radius R such that $R \gg \xi$, with ξ the healing length of the superfluid. This example is explored in detail in Reference (31). Suppose that the superfluid contains a straight vortex along z -direction centered at $r = 0$. From symmetry arguments, the streamlines around the filament are concentric circles centered around the vortex line, therefore $\mathbf{v}_s(\mathbf{r}) = v_s(r) \hat{\theta}$. Choosing the contour $\mathcal{C}$ to be a circle of radius r centered in the filament we obtain

$$\begin{aligned} \Gamma &= \oint_{\mathcal{C}} \mathbf{v}_s(\mathbf{r}) \cdot d\mathbf{l} = \int_0^{2\pi} v_s(r) \hat{\theta} \cdot (\hat{\theta} r d\theta) \\ &= 2\pi r v_s(r) = \frac{h}{m}\ell. \end{aligned} \quad (2.91)$$

Therefore, the velocity field of the superfluid with a quantized vortex is

$$\mathbf{v}_s(\mathbf{r}) = \ell \frac{\hbar}{mr} \hat{\theta}. \quad (2.92)$$

To have an idea of how different the rotation of superfluids is, consider the case of a vortex in a classical fluid, which in most of situations satisfies the rotational field $\mathbf{v} = \mathbf{\Omega} \times \mathbf{r}$, where $\mathbf{\Omega}$ is the angular velocity. So, while in a classical fluid the modulus of the velocity field increases

with r , in a superfluid we have exactly the opposite behavior.

The parameter ℓ is called “vortex charge” and tells us how many quanta of angular momentum (in $\hbar$ units) the vortex has. Note that at $\nabla \times \mathbf{v}_s(\mathbf{r}, t) = 0$ everywhere excepting at $r = 0$ where it diverges. In fact, it can be shown that for this simple example

$$\nabla \times \mathbf{v}_s(\mathbf{r}, t) = \hat{\mathbf{z}} \frac{\ell \hbar}{m} \delta^2(\mathbf{r}), \quad (2.93)$$

where $\delta^2(\mathbf{r}) = \delta(x)\delta(y)$ is the two-dimensional Dirac delta function in the xy -plane and $\mathbf{r} = (x, y)$.

The kinetic energy per unit length of the vortex can be estimated using a semi-classical approach. First we define ρ_m as the mass density of the superfluid; if n is the particle density then $\rho_m = nm$. The kinetic energy of a streamline at radius r is

$$\epsilon_{kin}(r) = \frac{1}{2} \rho_m v_s^2 = \frac{\hbar^2 \ell^2}{2m} \frac{n}{r^2}. \quad (2.94)$$

To obtain the kinetic energy per length unit we simply integrate expression (2.94) across a plane perpendicular to the vortex axis. However, note that the velocity field $v_s \sim r^{-1}$ and thus we cannot integrate from zero, instead we choose the healing length ξ which, in this case, represents a measure of the vortex core size. Then, the kinetic energy per length unit is

$$E_{semi} = \int_0^{2\pi} \int_{\xi}^R \epsilon_{kin}(\mathbf{r}) r dr d\theta = \pi n \frac{\hbar^2 \ell^2}{m} \ln \left(\frac{R}{\xi} \right). \quad (2.95)$$

Note that a multiply-charged vortex with $\ell = \ell_0$ is energetically less favorable than ℓ_0 vortices with unitary charge ($\ell = 1$). Therefore, a multiply-charged vortex is unstable and might decay into several single-charged vortices.

To exactly calculate the vortex energy the following *ansatz* is proposed

$$\Phi_0(\mathbf{r}) = \varphi(r, z) e^{i\ell\vartheta}. \quad (2.96)$$

Next it is necessary to numerically solve the Gross–Pitevskii equation (2.51) and calculate

the expectation value of the energy of the vortex through the expression

$$E = \langle \Phi_0 | \hat{H} | \Phi_0 \rangle = \int d\mathbf{r} \left[\frac{\hbar^2}{2m} |\nabla \Phi_0(r, z)|^2 + U(\mathbf{r}) |\varphi(r, z)|^2 + U_0 |\varphi(r, z)|^4 \right], \quad (2.97)$$

where $\hat{H}$ is the Gross-Pitaevskii Hamiltonian.

Using this exact calculation it is found that the energy per unit length of a single-charged vortex in a uniform cylindrical condensate is given by (31)

$$E_{unif} = \pi n \frac{\hbar^2}{m} \ln \left(1.464 \frac{R}{\xi} \right), \quad (2.98)$$

which is, actually, very close to the prediction of our semi-classical calculation. Although the wavefunction $\varphi(r, z)$ has no analytic form, it can be demonstrated proposing a trial function and using variational theory that the following expression

$$\varphi(r, z) = \frac{nr}{\sqrt{2\ell^2 \xi^2 + r^2}} \quad (2.99)$$

is a good approximation (31). Note that the healing length ξ characterizes the size of the vortex. Figure 2.2 shows both, the numerical solution and the approximation of Equation 2.99, of the wavefunction of the condensate with a single-charged vortex.

For a harmonically trapped superfluid we can, actually, calculate the total energy of vortex because the fluid is confined in all directions. In this case, the exact calculation gives

$$E_{tot} = \frac{4\pi n_0 \hbar^2}{3m} R_z \ln \left(0.671 \frac{R_r}{\xi_0} \right), \quad (2.100)$$

where n_0 and ξ_0 are respectively the density and the healing length at the center of the fluid; R_z and R_r are, respectively, the extensions of the cloud along the axial and the radial directions which in the Thomas-Fermi approximation are given by Equation (2.55).

Being one of the most important manifestations of superfluidity, quantized vortices constitute a very vast research topic. They have been deeply investigated theoretically and experimentally in both, superfluid helium and atomic BECs. The first direct observation of vortices in superfluid helium was carried out by E. J. Yarmchuk *et al.* in 1979 (40), while the first production of quantized vortices in BECs was performed, separately, by the group of Eric Cornell

Figure 2.2 – (Solid line) Wavefunction of a BEC with a single–charged vortex and (dashed line) the approximate wavefunction of Equation (2.99). Image taken from (31).

at NIST in Boulder (18), and by the group of Jean Dalibard at ENS in Paris (19). As mentioned before, because the healing length of a BEC is much larger than in superfluid helium, the vortices are much easier to visualize. For this reason, condensates are an ideal system to study vorticity.

In the very simple model presented above we have described a single straight vortex centered in an axially symmetric superfluid. However the dynamics of a vortex can be very complex, it could be off–center or present a complicated geometry. Theoretical investigation of this situation requires more sophisticated methods. On the experimental side, for instance, the group of J. Dalibard at ENS in Paris, has produced a BEC with a single vortex in it and studied its geometry and dynamics by imaging the BEC along two perpendicular directions (41). The authors show that the vortex is not a straight static line but a curved line which evolves in time. In Figure 2.3 we show absorption images of this condensate at different evolution times.

It is also possible to have many vortices in the sample by transferring many quanta of angular momentum to the fluid. If the angular momentum is transferred along a single direction the

Figure 2.3 – (a) Schematics of the imaging systems: two perpendicular beams image simultaneously a BEC which contains a single vortex. (b)–(c) Simultaneous images of the condensate after (b) 4 s, (c) 7.5 s and (d) 5 s of evolution time. Image taken from (41).

Figure 2.4 – Abrikosov vortex lattice in a BEC containing (A) 16, (B) 32, (C) 80 and (D) 130 vortices. Image taken from (42).

vortices may arrange in a periodic lattice known as *Abrikosov lattice*. This fact is a consequence of the quantum nature of the system and can be understood by considering the vortices as being able to repel each other. The first vortex lattices in BECs were observed by K. W. Madison *et al.* (19). However, the most spectacular sample was produced by the Wolfgang Ketterle's group at MIT (42), being able to produce more than one hundred vortices in the sample, as shown in Figure 2.4.

The dynamics of a collection of vortices is a very novel research topic. A particularly important effect of the dynamics of many vortices is the emergence of turbulence in the superfluid, which in this case is known as *Quantum Turbulence*. We offer some theoretical aspects about it in the next section. Finally, we must say that the production of vortices and their subsequent evolution to quantum turbulence constitutes one of the research lines of our group and one of the main results of this thesis, as described in Chapter 3.

2.4 Turbulence

It is well-known that turbulence is one of the most important open problems in Physics. Nobel laureate Richard P. Feynman put this fact in words as follows: “[...turbulence represents] the most important unsolved problem of classical physics”. Moreover, Feynman himself was the first to conceive the idea of turbulence in superfluids in 1955 (22). Due to quantum restrictions to which the superfluid flow is constrained, turbulence in superfluids has been named *Quantum Turbulence* by Russell J. Donnelly (43).

While our understanding of turbulence is very limited, the amount of work devoted to this topic is enormous. In this section we will only define the concepts of classical turbulence (CT) and quantum turbulence (QT) and introduce the basic ideas to understand the results presented in Chapter 3.

2.4.1 Classical Turbulence

There are many introductory references to the topic of classical turbulence, two excellent examples are the References (44, 45). In the following we only introduce the very basic notions.

The motion of a classical fluid can be described by the Navier–Stokes equation for the velocity field $\mathbf{u} = \mathbf{u}(\mathbf{r}, t)$, and it is given by

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} = -\frac{1}{\rho} \nabla P + \nu \nabla^2 \mathbf{u}, \quad (2.101)$$

where ρ and P are the density and pressure of the fluid, respectively, and we define the *kinematic viscosity* as $\nu = \eta/\rho$ with η the viscosity of the fluid. The terms on the left–side are related to the acceleration of the fluid; particularly important is the term $(\mathbf{u} \cdot \nabla) \mathbf{u}$ which is known as *nonlinear inertial* term. In the right–side the term $-\nabla P/\rho$ accounts for pressure gradients while the term $\nu \nabla^2 \mathbf{u}$ describes the effect of viscosity.

We can rewrite the Navier–Stokes equation in a very suitable form by realizing that there are always a characteristic length r_0 and velocity u_0 in the bulk fluid. For instance, if we consider the case of an obstacle moving through the fluid, r_0 would be the size of the object and u_0 its velocity. We define dimensionless variables in the following way

$$\mathbf{r}' = \frac{\mathbf{r}}{r_0}; \quad \mathbf{u}' = \frac{\mathbf{u}}{u_0}; \quad t' = \frac{tu_0}{r_0}; \quad P' = \frac{P}{\rho u_0^2}, \quad (2.102)$$

with these definitions the Navier–Stokes equation becomes

$$\frac{\partial \mathbf{u}'}{\partial t'} + (\mathbf{u}' \cdot \nabla') \mathbf{u}' = -\nabla' P' + \frac{1}{Re} \nabla'^2 \mathbf{u}', \quad (2.103)$$

where Re is the Reynolds number defined as $Re = r_0 u_0 / \nu$. This number is very important because it defines which kind of flow we will have. If Re is small it means that the viscous term is dominant and the nonlinear inertial term can be neglected. In this case we will have a *laminar* flow. At large values of Re the viscosity is negligible and the nonlinear term dominates. In this case the laminar flow becomes unstable and the flow might develop turbulence.

Figure 2.5 shows some pictures of different turbulent flows. In these pictures we can see two

Figure 2.5 – Turbulent flow produced by (a) a fluid passing around a cylindrical obstacle, (b) a jet of water, (c) and (d) a fluid passing through a mesh. (e) Numerical simulation of a homogeneously turbulent fluid. Images (c), (d) and (e) are examples of homogeneous turbulence. Image (a) taken from (46). Images (b) and (d) taken from (47). Figure (c) taken from (45). Figure (e) taken from (48).

of the main features of turbulent flows: (i) the formation of many “eddies”, and (ii) these eddies occur in a broad range of length scales. Evidently, as we look into the different scale lengths, the corresponding Reynolds number changes (because the characteristic length r_0 depends on the scale length). However, as long as Re remains large, the nonlinear term will dominate and viscosity will not affect the flow.

In a classical fluid the evolution to turbulence can be understood as follows. As the Reynolds number increases, the laminar flow is interrupted and large eddies, or vortices, are formed. These vortices become unstable and break up into smaller vortices. Thus the initial energy of the largest eddies is divided into the smaller vortices. These new vortices experience the same process, dividing into even smaller eddies among which the energy from the predecessors is distributed. This process repeats over and over again until a very small scale length in which $Re \sim 1$ is reached. At this point the viscous flow predominates, the energy is dissipated and no more vortices are able to form. At these small scales, the kinetic energy of the turbulent flow is converted into heat. The range of length scales in which Re remains large and there is no viscous dissipation is known as the *inertial range*.

This process, known as *Richardson cascade*, is extremely complex and no analytical expressions for the flow can be obtained. Nonetheless, it is possible to understand the transfer of energy among the different length scales if we assume that the turbulence is homogeneous and isotropic. That means that we will observe, statistically speaking, the same behavior of the fluid independently of the position and direction of observation, Figures 2.5(c)–(e) are examples of this kind of flow.

Kolmogorov theory

During the 1930’s and 40’s the mathematician Andrey N. Kolmogorov studied the problem of turbulence and made major contributions to the area (49, 50). In the following we briefly explain some of them.

As explained before, we must look at the different length scales of the system. To do

so, we should examine the Fourier transform of the velocity field in the k -space, $\mathbf{v}(\mathbf{k}) = \int \mathbf{v}(\mathbf{r}) e^{-i\mathbf{k}\cdot\mathbf{r}} d\mathbf{r}$, where k^{-1} defines a length scale.

Let us consider an incompressible fluid in which homogeneous turbulence has been produced. This can be done in many ways, a particularly efficient one is to force a laminar flow to pass through a grid, as shown in Figure 2.5(c). Far away from the grid the fluid is homogeneously turbulent³.

Now, let us consider $D \sim k_D^{-1}$ to be the largest length scale of the system. Then at this scale no dissipation at all occurs and the energy is transferred to the smaller length scales up to the limit in which viscosity plays an important role and energy is dissipated. Let us define the wave number k_K corresponding to this limit. In literature k_K^{-1} is known as *Kolmogorov dissipation length*. We also define ε as the rate of energy transfer per mass unit between the different scales. Therefore, the fully developed Richardson cascade occurs with an energy transfer rate of ε , in the range $k_D \ll k \ll k_K$, which corresponds to the inertial range mentioned above. Remember that in this range the nonlinear inertial term $(\mathbf{u} \cdot \nabla) \mathbf{u}$ governs the dynamic of the fluid.

Finally, we define the energy spectrum $E(k)$ such that the average amount of energy in the range dk is given by $E(k) dk$. Therefore the total kinetic energy distributed among all scales of the system is given by

$$E_T = \int_0^\infty E(k) dk. \quad (2.104)$$

Kolmogorov demonstrated that within the inertial range, the energy spectrum only depends on k and ε through the following expression

$$E(k) = C\varepsilon^{2/3} k^{-5/3}, \quad (2.105)$$

where C is a universal constant that, according to experiments, is ~ 1.5 (51). This is the so-called *Kolmogorov spectrum* and expresses how the energy distributes along all the scale lengths in a turbulent flow. Note that most of the energy is contained in the smallest values of k , that is, in the biggest eddies whose size is of the order of D . The range of length scales larger

³ In this context “far away” means a distance from the grid much greater than the mesh of the grid.

than D is known as the *energy containing range* because all the energy that will produce the turbulent flow (which is distributed among all scales) is macroscopically pumped at this scale ($k \ll k_D$).

Now an expression for k_K in terms of the fluid parameters ε and ν is required. A very simple deduction is as follows. As mentioned, k_K^{-1} corresponds to the typical size of the smallest possible vortex before viscosity dissipates the kinetic energy into heat. Let u_K the typical velocity of this vortex, then, the typical time scale is

$$\tau_K = (k_K u_K)^{-1}. \quad (2.106)$$

At the same time, at this small scale the Reynolds number must be of the order of unity,

$$Re_K = \frac{k_K^{-1} u_K}{\nu} = 1 \quad \Rightarrow \quad u_K = \frac{\nu}{k_K}. \quad (2.107)$$

At this scale, the energy transfer rate per unit of mass is of the order of the kinetic energy of the smallest eddie divided by its typical time, namely

$$\varepsilon \sim \frac{u_K^2}{\tau_K} = u_K^3 k_K \quad \Rightarrow \quad u_K \sim \left(\frac{\varepsilon}{k_K} \right)^{1/3}. \quad (2.108)$$

Using Equations (2.107) and (2.108) we obtain an expression for the Kolmogorov dissipation length

$$k_K^{-1} = \left(\frac{\nu^3}{\varepsilon} \right)^{1/4}. \quad (2.109)$$

Additionally, we can obtain other important scales acting at the smallest eddies regime. From Equations (2.108) and (2.109) we obtain the *Kolmogorov velocity scale*,

$$u_K = (\varepsilon \nu)^{1/4}, \quad (2.110)$$

and from Equations (2.106) and (2.110) we obtain the *Kolmogorov time scale*

$$\tau_K = \left(\frac{\nu}{\varepsilon} \right)^{1/2}. \quad (2.111)$$

Figure 2.6 – Normalized energy spectrum of different turbulent flows, such as boundary layers, wakes, grids, ducts, pipes, jets and oceans demonstrating the universality of Kolmogorov spectrum. Here, η corresponds to the Kolmogorov dissipation length, that is $\eta = k_K^{-1}$ (image taken from (52)).

The Kolmogorov spectrum from Equation (2.105) has two very important properties: first, it is universal, which means that it is independent of the boundary conditions or of the mean flow field since it only depends on k and ε ; second, it shows that, within the inertial range, turbulence is self-similar along all scales (Richardson cascade is self-similar). Note that ν simply serves to define where the inertial range ends. The graph of Figure 2.6 is a compilation of several measurements of the normalized energy spectrum of different turbulent flows (taken from (52)). It includes measurements of boundary layers, wakes, grids, ducts, pipes, jets and even oceans, demonstrating the universality of Kolmogorov spectrum. This graph exhibits all the considered ranges: the energy containing range (large length scales), the inertial range and dissipation range (for the smaller length scales).

2.4.2 Quantum Turbulence

Now that we have introduced the main concepts of classical turbulence, we can start to discuss its quantum counterpart. Part of the discussion presented below is based on the review article of W.F. Vinen and J.J. Niemela (53). Reference (54) is also a very good introduction to this subject.

As it can be seen from the previous section, a very important ingredient to get a turbulent flow is the rotational motion across different length scales, up to the point in which it is dissipated by viscosity.

In a superfluid, the flow is strongly restricted by quantum effects, as demonstrated in Section 2.3. However, it is possible to have rotational motion through quantized vortices. Richard Feynman was, evidently, aware of this when he first proposed the possibility of having turbulence in superfluids. The proposed turbulent state consists in a configuration of vortices in which the vortex lines are spatially tangled (22). Between 1956 and 1958 William Vinen and Henry Hall demonstrated experimentally the Feynman's hypothesis by producing quantum turbulence (QT) in superfluid Helium (23–27). In this case, the turbulent flow was produced by thermal counterflow in which the normal and the superfluid fractions flow in opposite directions. In the

Vinen experiments it is shown that the dissipation of energy occurs due to the friction between the quantized vortices and the normal fraction of the liquid.

Later, experiments in Paris by Jean Maurer and Patrick Tabeling (55) and, simultaneously, experiments at the University of Oregon by S.R. Stalp *et al.* (56) showed that the superfluid turbulence in ^4He , under a certain range of length scales, also satisfies the Kolmogorov spectrum of Equation (2.105) independently of the temperature of the fluid. This is a surprising result considering how different the classical and the quantum flows are. In the Paris experiments turbulence was generated by two parallel counterrotating disks. In the Oregon experiment a grid oscillated within the superfluid. The energy spectrum was measured using pressure sensors distributed in the liquid. In these cases, the largest length scale D is given by the diameter of the disks or by the size of the mesh of the grid. The smallest scale length is given by the mean spacing between the vortex-lines ℓ . So the range $\ell \ll r \ll D$ is equivalent to the inertial range in which the Kolmogorov law is fulfilled. In other words, in the inertial range, the behavior of the superfluid flow is quasi-classical.

Equivalently, we will have a Richardson cascade that distributes the energy transferred to the system at length scales larger than D among all other scales with a rate per mass unit of ε . At the length scale smaller than ℓ the cascade ends and energy is dissipated. At this point arises one of the most important and still open questions concerning QT. How is the energy dissipated? In a classical fluid viscosity does the job, but in a superfluid there are no obvious dissipation mechanisms. One could think that the normal (not superfluid) component of the system could dissipate the energy, and it does; but it is not enough because even at very low temperatures (with normal component less than 2%) the Kolmogorov law still applies.

The most accepted hypothesis concerns a quantum effect with no equivalence in classical fluids known as *vortex reconnection*. Figure 2.7 shows a numerical simulation of the reconnection process. The simulation was performed by M. Kobayashi and M. Tsubota at Osaka City University (57). It can happen that two straight vortices (Fig. 2.7(a)) approach each other close enough and reconnect (Fig. 2.7(b) and (c)). Reconnection leaves two new vortices that are not

Figure 2.7 – Reconnection of two quantized vortices. (a) Initially two straight vortices that (b) approach each other and (c) reconnect. (d) After the reconnection emerge two kinked vortices. Image taken from (57).

straight anymore and, instead, are sharply kinked (Fig. 2.7(e)). The role of reconnections was identified by the first time by Boris Svistunov, from the University of Massachusetts (58). If the superfluid contains many vortices densely distributed, reconnections will occur very frequently. After a while each vortex will have suffered many reconnections and will be completely twisted, forming a big tangle with all other vortices. This is the turbulent scheme that Feynman imagined.

As reconnections keep occurring, a specific excitation may be generated in the vortex line known as *Kelvin wave*. It consists of a helical deformation of the vortex line which propagates along it. At high temperatures⁴ the normal fraction of the fluid damps the Kelvin wave by mutual friction, dissipating in this way the energy. At low temperatures this damping does not occur. Numerical simulations have shown that if the Kelvin wave reaches a sufficiently high frequency it radiates phonons. Phonons are the element that, ultimately, could dissipate the energy.

This represents the current picture of the energy dissipation cascade in superfluids. Figure 2.8 illustrates the whole process. It is still very speculative and much deeper research is required to fully understand it.

To theoretically describe QT there are two principal approaches. This first one is known as the *filament model* (59). In this model a vortex is described as structureless filament mo-

⁴ In this context, “high temperature” means a temperature such that the normal fluid fraction is of the order of or greater than the superfluid fraction.

Figure 2.8 – Scheme of the energy dissipation process in turbulent superfluids. A macroscopic amount of energy is pumped into the system, generating a great number of vortices. Subsequently, the vortices reconnect several times and a vortex tangle is generated. Next, Kelvin wave excitations are produced in the vortex. Finally, energy is dissipated as phonons and thermal excitations.

ving through the superfluid whose direction is defined by its corresponding vorticity. The flow around the filament is expressed by a Biot–Savart type expression (that is, the description of the superfluid flow around the vortex filament is analogous to that of a magnetic field around a conducting wire). This model is very suitable for describing QT in liquid helium because in this system the vortex core is very small (of the order of $1 \text{ \AA}$). The second much simpler method consists in numerically solving the Gross–Pitaevskii equation. This method is not appropriate to describe liquid helium because it is not a weakly interacting system, however it is very suitable in the case of atomic Bose–Einstein condensates (see, for instance, (60)). For this reason, this method will be relevant in this thesis, and we focus on it in the following.

Quantum Turbulence in a BEC

The discussion below is based on the review article of Makoto Tsubota (61) and Reference (60)

So far we have discussed how QT occurs in general in superfluids. The specific case of a BEC is interesting because it is described by the Gross-Pitaevskii equation, making the calculations easier. Also, BECs have the advantage of having a much bigger healing length, hence, the vortices can be visualized using optical means.

Here we simply sketch the numerical method employed by Makoto Tsubota and Michikazu

Figure 2.9 – (a) Scheme to generate quantum turbulence in a trapped BEC. It consists in stirring the cloud around two perpendicular directions. (b) Energy spectrum of the quantum turbulent state in a BEC. The points correspond to the numerical calculation while the solid line refers to the Kolmogorov spectrum. Images taken from (60).

Kobayashi to address this problem.

Consider a weakly interacting BEC confined in an arbitrary potential $U(\mathbf{r}, t)$. If $\Phi(\mathbf{r}, t) = \varphi(\mathbf{r}, t) e^{i\vartheta(\mathbf{r}, t)}$ is the condensate wavefunction, with $\varphi(\mathbf{r}, t)$ and $\vartheta(\mathbf{r}, t)$ real functions, we can calculate the total energy through Equation (2.97), which in this case is rewritten as

$$E(t) = \int d\mathbf{r} \left\{ \frac{\hbar^2}{2m} [\varphi(\mathbf{r}, t) \nabla \vartheta(\mathbf{r}, t)]^2 + \frac{\hbar^2}{2m} [\nabla \varphi(\mathbf{r}, t)]^2 + U(\mathbf{r}, t) \varphi(\mathbf{r}, t)^2 + U_0 \varphi(\mathbf{r}, t)^4 \right\}. \quad (2.112)$$

The second term of this equation is known as *quantum energy* and has no classical analog. The last two terms are, respectively, the potential and the interaction energies. The first term is the most important for us because it corresponds to the kinetic energy of the superfluid. This is easy to see in the following way. We interpret $n(\mathbf{r}, t) = \varphi(\mathbf{r}, t)^2$ as the density of the superfluid and, from Equation (2.87) we know that $\nabla \vartheta(\mathbf{r}, t)$ is proportional to the superfluid velocity field. Then, this term can be written as

$$E_k(t) = \frac{\hbar^2}{2m} \int [\varphi(\mathbf{r}, t) \nabla \vartheta(\mathbf{r}, t)]^2 d\mathbf{r} = \frac{m}{2} \int n(\mathbf{r}, t) v_s^2(\mathbf{r}, t) d\mathbf{r}, \quad (2.113)$$

which has the proper form of the kinetic energy.

The proposal of Michikazu Kobayashi and Makoto Tsubota to generate quantum turbulence consists in rotating a harmonically trapped BEC around two orthogonal directions (60), as illustrated in Figure 2.9(a). This will generate two perpendicular vortex lattices that will tangle, producing the turbulent state. In this case, the potential $U(\mathbf{r}, t)$ is given by a static harmonic potential plus the rotating field. Therefore, the Gross–Pitaevskii equation of the system is given by

$$[i - \gamma(\mathbf{r})]\hbar \frac{\partial}{\partial t} \Phi(\mathbf{r}, t) = \left(-\frac{\hbar^2}{2m} \nabla^2 + U_0 |\Phi(\mathbf{r}, t)|^2 + U(\mathbf{r}) - \boldsymbol{\Omega}(t) \cdot \hat{\mathbf{L}}(\mathbf{r}) \right) \Phi(\mathbf{r}, t). \quad (2.114)$$

In Equation (2.114) we consider the following

- The harmonic potential is given $U(\mathbf{r}) = m\omega^2 (0.95x^2 + y^2 + 1.025z^2) / 2$, with $\omega = 2\pi \times 150$ Hz the oscillator frequency. That is, the trapping potential is weakly elliptical.
- $\hat{\mathbf{L}} = \hat{\mathbf{r}} \times \hat{\mathbf{p}}$ is the angular momentum operator
- $\boldsymbol{\Omega}(t) = (\Omega_x, \Omega_z \sin \Omega_x t, \Omega_z \cos \Omega_x t)$ is the rotation vector, where Ω_x and Ω_z are the frequencies of rotation around the x and the z -directions.
- Therefore, the term $\boldsymbol{\Omega}(t) \cdot \hat{\mathbf{L}}(\mathbf{r})$ accounts for the rotations. Note that it is analog to the Coriolis force term in classical fluid mechanics.
- The term $\gamma(\mathbf{r})$ is a phenomenological function which describes dissipation. The function is constructed numerically in such a way that dissipation exclusively occurs at the smallest length scales, below the inertial range.

The Gross–Pitaevskii equation is solved numerically and the wavefunction $\Phi(\mathbf{r}, t)$ is obtained. The kinetic energy of Equation (2.113) can be now computed. To compare with the Kolmogorov spectrum it will be necessary to transform all quantities to the wavenumber space, which can be done by simply performing the Fourier transform of the wavefunction and of the kinetic energy. If R_{TF} is the Thomas–Fermi radius (given by Equation (2.55)) and ξ is the healing length of the condensate at the center of the trap, the numerical simulations show that within the inertial range given by $2\pi/R_{TF} < k < 2\pi/\xi$, the kinetic energy $E_{kin}(k)$ satisfies the

Figure 2.10 – (a) Vortex tangle of a turbulent BEC in a box. (b) The squares correspond to the numerical calculation of the energy spectrum of the QT. The solid line is the Kolmogorov spectrum. Images taken from (61).

Kolmogorov spectrum, as shown in Figure 2.9(b).

These numerical calculations were also performed for a homogeneous BEC confined in a box of size L , producing similar results (61). Figure 2.10(a) shows the distribution of vortices in the sample, exhibiting the vortex tangle which characterizes the turbulent state. Figure 2.10(b) is a plot of the energy spectrum which obeys the Kolmogorov law. In this case, the inertial range is given by $2\pi/L < k < 2\pi/\xi$.

These are remarkable results showing that the semi-classical Kolmogorov law is satisfied also in confined quantum systems. The only difference found between the classical and the quantum fluids is the value of the constant C . While $C \approx 1.5$ in classical fluids, $C \approx 0.25$ in a harmonically trapped BEC and $C \approx 0.55$ in a homogeneous condensate.

Quantum turbulence in BECs was observed by the first time in our laboratories very recently. The tangle arrangement of vortices is observed, however, the Kolmogorov spectrum was not possible to measure. This experiment constitutes the central matter of the next Chapter.

3 *Route to Turbulence in a BEC by oscillatory fields*

In this Chapter we will describe the first studies performed by our group with a ^{87}Rb condensate. This research was carried out in our first experimental system, which we will call “*BEC-I*” system. The goal of this experiment is to produce and study the phenomenon of Quantum Turbulence (QT). In Section 2.4.2 we described this phenomenon and presented the proposal of Makoto Tsubota’s group to produce it. In this proposal, angular momentum is transferred to the BEC along two perpendicular directions in order to create a disordered tangle of quantized vortices in the sample. This tangled configuration is the principal signature of the presence of QT. This kind of composed rotation may be produced, for instance, by stirring the condensate with two orthogonal blue detuned laser beams.

In our laboratory we did not have the possibility of using the lasers stirring scheme proposed by M. Tsubota, instead, we have decided to transfer angular momentum using magnetic means by superimposing an external oscillating magnetic potential to the trap. We have observed that, depending on the value of its parameters, the excitation is able not only to produce QT but other different regimes. These regimes are regular quantized vortices, with no turbulent behavior, or another complex regime that we have identified as the *Granulation* phenomenon.

We find out that it is possible to summarize all results in a single diagram of amplitude versus duration of the excitation where different domains can be identified, each corresponding to one of the different produced regimes. This diagram is important because it allows us to understand the route from a given regime to another. Specifically, it exhibits the condition to produce QT.

The structure of the present Chapter is as follows. In Section 3.1 we offer a brief description of the experimental sequence employed for producing and exciting the condensate. A detailed description of this sequence can be found in Chapter 4, where we provide a deep description of our new experimental system, which we call “*BEC-II*”. Both setups are almost identical, except for the trapping system in which we produce the quantum gas. We have a pure magnetic trap for *BEC-I* and a hybrid optical–magnetic trap for *BEC-II*. The motivation to have a second experimental system is explained in detail, later, in Section 4.1.

In Section 3.2 we present an approximate analytic expression for the trapping and excitation potentials which contains all the effects necessary to understand our results. Next, in Section 3.3 we expose our principal results. We present all the different states produced in the BEC as a function of the parameters of the excitation. We show a diagram of amplitude versus time of excitation in which all our results are summarized. Finally, in Section 3.4 we discuss our observations and present theoretical models useful to qualitatively understand our findings.

3.1 The *BEC-I* Experimental Setup

The basic steps to produce a Bose-Einstein condensate will be explained in detail in Chapter 4. Reference (62) also describes our setup. In the following we only provide a brief description of the experimental system and explain the procedure to excite the condensate.

This experimental setup has a vacuum system and a laser scheme essentially identical to the *BEC-II* system described in Chapter 4. About 5×10^8 atoms are collected in a magneto–optical trap and transferred into a pure magnetic trap. Initially, the magnetic trap is a magnetic quadrupole produced by two coils in anti–Helmholtz configuration. The transfer process from the magneto–optical trap to the magnetic quadrupole is described in Section 4.6 and it has an efficiency above 95%. Once the atoms are in the magnetic quadrupole they are transferred to a purely harmonic potential where they are further cooled. This harmonic potential is produced by superimposing the field of a third coil perpendicular to the quadrupole, this third coils is named “Ioffe coil” and the system of three coils is known as “Quadrupole and Ioffe configuration”

Figure 3.1 – (a) Top and (b) side views of the trapping region of the BEC–I system. The orange coils correspond to the QUIC trap, the gray coils represent the ac–coils showing its tilt between the axes. Also, the direction of the imaging beam is shown.

magnetic trap (QUIC trap). The physics of magneto–optical traps and pure magnetic trapping is described in Appendix A.

Once the atoms are in the harmonic magnetic trap we apply radio–frequency evaporative cooling to cool the sample to quantum degeneracy. This technique selectively removes the most energetic atoms and is able to produce very cold samples. After the evaporation we end up with a Bose–Einstein condensate containing $(1 - 2) \times 10^5$ atoms, the critical temperature of the sample is $T_c = 250$ nK. The frequencies of the trap are $\omega_y = \omega_z \equiv \omega_r = 2\pi \times 210$ Hz and $\omega_x = 2\pi \times 23$ Hz so we have a cigar–shaped condensate.

After reaching BEC, an extra field, produced by a pair of anti–Helmholtz coils is superimposed on the magnetic trapping field. The center of the extra field, defined by the zero–field amplitude position, does not match with the QUIC trap minimum nor is the axis of the extra coils aligned with the trap axis, rather the angle between them is $\theta_0 \sim 5^\circ$. We call these extra coils *ac–coils* or *excitation coils*. To excite the condensate, an oscillatory current of the form $I_{ac}(t) = I_0 [1 - \cos(\Omega_{act})]$, is applied to the ac–coils, here Ω_{ac} is the excitation frequency. Note that it always has the same sign and always starts from zero, so we do not give an abrupt kick to the condensate in the beginning of the excitation phase. In Section 3.2 we show that this field rotates along two orthogonal directions. Figure 3.1 shows the schematics of our system.

To summarize, the sequence of the experiment is as follows: we load the atoms in a har-

monic magnetic trap and cool them down below the critical temperature by means of radio-frequency evaporative cooling. Next, the excitation field is turned on. The frequency, the duration and the amplitude of the oscillatory excitation are parameters that we can vary. After the end of the oscillatory stage, atoms are left trapped for an extra 20 ms before being released and observed in free expansion by a standard absorption imaging technique after 15 ms of time-of-flight.

3.2 Trapping and excitation fields

Before presenting our results it is worth studying the different fields involved in our experiment.

As already mentioned, the QUIC trap generates a harmonic potential in which the condensate is produced. Strictly speaking, this potential has not an analytical expression and is only harmonic around its minimum. However, since the atoms to be condensed are very cold they remain in the bottom of the trap; therefore the harmonic description is a good approximation for our potential.

The total potential that interacts with the atoms has two components, a static component V_{trap} which corresponds to the QUIC magnetic trap and an oscillating potential V_{ac} that corresponds to the excitation, thus the total potential is the sum of them: $V(t) = V_{trap} + V_{ac}(t)$.

The static trapping potential is given by

$$V_{trap} = \frac{1}{2}m\omega_x^2x^2 + \frac{1}{2}m\omega_r^2(y^2 + z^2), \quad (3.1)$$

where $\omega_x = 2\pi \times 23\text{Hz}$ and $\omega_r = 2\pi \times 210\text{Hz}$.

The exact form of the excitation field is unknown because it has not been possible to precisely evaluate the tilting between the ac-coils and the QUIC coils. However a good approximation that reproduces well our observations has been proposed by Professor Kasamatsu and

collaborators and is given by:

$$V_{ac}(t) = \frac{1}{2}m\Omega_x^2(t)(x' - X'_0)^2 + \frac{1}{2}m\Omega_y^2(t)(y' - Y'_0)^2 + \frac{1}{2}m\Omega_z^2(t)(z - Z_0)^2, \quad (3.2)$$

where the coordinates $(x' - X'_0, y' - Y'_0)$ are given by

$$\begin{pmatrix} x' - X'_0 \\ y' - Y'_0 \end{pmatrix} = \begin{pmatrix} \cos \theta_0 & -\sin \theta_0 \\ \sin \theta_0 & \cos \theta_0 \end{pmatrix} \begin{pmatrix} x - X_0 \\ y - Y_0 \end{pmatrix}. \quad (3.3)$$

In Equations (3.2) and (3.3) we consider the following:

- The angle between the QUIC and the ac-coils axes is $\theta_0 = 5^\circ$, this misalignment occurs only in the xy -plane
- (X_0, Y_0, Z_0) represents the relative shift of the minimum of V_{ac} from that of V_{trap} .
- We consider $Z_0 = 0$.
- We have defined $\Omega_i^2(t) = \omega_i^2 \delta_i^2 (1 - \cos \Omega_{ac} t)^2$, ($i = x, y, z$), remembering that $\omega_y = \omega_z \equiv \omega_r$, and δ_i is the amplitude of the translation along the i -direction.

Therefore, the total potential $V(t) = V_{trap} + V_{ac}(t)$ is given by

$$\begin{aligned} V(t) = & \frac{1}{2}m\omega_x^2 [x^2 + \delta_x^2 (1 - \cos \Omega_{ac} t)^2 (x' - X'_0)^2] + \\ & + \frac{1}{2}m\omega_r^2 [y^2 + \delta_y^2 (1 - \cos \Omega_{ac} t)^2 (y' - Y'_0)^2 + z^2 + \delta_z^2 (1 - \cos \Omega_{ac} t)^2 z^2]. \end{aligned} \quad (3.4)$$

Note that at $t = 0$, $V = V_{trap}$, and the maximum translational shift of the minimum takes place at $t = \pi/\Omega_{ac}$.

In Figure 3.2 we show graphs of the equipotential lines of Equation (3.4) for different times, showing that the excitation produces a combination of rotation, translation and deformation on the atoms. Of particular importance is to note that the rotation occurs along two orthogonal directions.

Figure 3.2 – Equipotential lines of Equation (3.4) for three different times. In (a), (b) and (c) are shown the equipotential lines in the xy -plane, while in (d), (e) and (f) those of the xz -plane. The red dashed axes show the position of the minimum when $t = 0$.

3.3 Diagram of Oscillatory Excitations

Now that it is clear which is the action of the external potential we can discuss the excitations that it produces in the BEC.

The excitation is produced by passing a current of the form $I_{ac}(t) = I_0[1 - \cos(\Omega_{ac}t)]$. This means that we can only control three parameters: the frequency (Ω_{ac}), the amplitude (I_0) and the duration (t) of the excitation. This has the disadvantage of not allowing to control independently the three effects of the excitation: translation, rotation and deformation. This disadvantage makes the theoretical interpretation of the experiment a difficult task.

We start by investigating the effect of the variation of the frequency of the excitation. We observe that, besides translation, no other excitations are produced in the quantum sample except in a very narrow range of frequencies. We find that for frequencies close to the largest trapping frequency the desired effects of the excitation are produced. After several measurements we have determined that the best value for this frequency is $\Omega_{ac} = 2\pi \times 200$ Hz. Using this frequency we observe that the formation of the structures that we will describe in the fol-

lowing is maximized and, for this reason, the frequency will be fixed in this value in all our measurements.

Next we vary the amplitude and the time of the excitation and observe the effect produced in the quantum fluid. We have considered a broad range of times and amplitudes. For the duration of excitation we have explored the range $t \in [0, 55]$ ms which contains several periods of the excitation. For the amplitude we have used the corresponding gradient of the magnetic field produced by the ac-coils along its axial direction (corresponding to the x' -direction according to the notation of Section 3.2). In this case, the range is $\partial_{x'} B_{x'} \in [0, 170]$ mG/cm. This means that, if we suppose a separation between the QUIC field and the ac-field of $X_0 = 5$ mm, when the maximum amplitude of excitation is applied (*i. e.* 170 mG/cm) the variation of the magnetic field value in the bottom of the trap is of about 8%.

Depending on the combination of time and amplitude, the excitation is able to generate four different kinds of behavior in the BEC, namely:

1. Bending of the main axis of the cloud.
2. Nucleation of regular vortices.
3. Generation of quantum turbulence.
4. Granulation of the superfluid.

In the following Sections we will describe each one of these regimes. It is important to mention that independently of the value of any of the parameters of the excitation, we always observe translation of the sample, thus, our external oscillations move the atomic cloud.

3.3.1 Bending of the cloud

For small amplitudes of excitation, $\partial_{x'} B_{x'} < 40$ mG/cm, and independently of the time of excitation, we observe a bending of the main axis of the cloud (63, 64). This effect turns out to be a collective mode of the quantum system known as *scissors mode*. It has been previously reported in literature (65, 66), and it is consequence of the superfluid nature of the sample.

Figure 3.3 – Pictures of the bended condensate, the dashed line indicates the inclination of the axis of the cloud in relation to the vertical direction.

The main conclusion that we can obtain from this observation is the capability of the sample to mechanically transfer angular momentum to the sample. As we will see later, this angular momentum transfer is able to produce vortices and, for the proper conditions, to generate QT in the sample. Figure 3.3 shows two typical images of the bending of the cloud, both taken after 15 ms of time-of-flight. We find that for a given time of excitation the bending angle is always the same, therefore, this regime is still deterministic.

3.3.2 Regular vortices

After observing the bending of the cloud, increasing the amplitude of the excitation we observe an increasing number of dips in the density profile of the cloud ($\partial_{x'} B_{x'} \geq 40$ mG/cm). Absorption imaging technique does not allow us to discriminate the phase of the condensate, however, we know that these dips are quantized vortices for two reasons: (i) we have seen that our excitation rotates the sample and, in consequence, transfers angular momentum; (ii) during the free expansion the dip is not “filled up” again, indicating that the atoms are rotating around the core of the dip (63, 64, 67).

At this point, increasing both, the time or the amplitude of the excitation produces a monotonic increase in the average number of vortices in the sample, although there is a big variation in the number of vortices when the same conditions are employed. Figure 3.4 shows typical images of the quantum cloud with different number of vortices, all images were taken after 15 ms of free expansion. The average number of vortices as a function of the amplitude of the

Figure 3.4 – Absorption images of the excited condensate with (a) one, (b) two, (c) three and (d)–(e) many vortices.

excitation for three different times is shown in Figure 3.5(a), equivalently we can fix the amplitude and vary the time of excitation. Figure 3.5(b) shows the average number of vortices as a function of excitation time for three different amplitudes (68). The main conclusion that we obtain from this is that increasing the strength of the excitation or, in other words, increasing the quanta of angular momentum transferred to the cloud we can nucleate a bigger number of vortices.

An important observation is the fact that the distribution of vortices in our sample does not correspond to any regular pattern. Instead, the vortices seem to be distributed in a random way and both, number and position, present a big fluctuation when the same experimental conditions are employed. It is important to mention that in our measurements we assume that the *in-situ* configuration keeps its geometry after the time-of-flight expansion. In fact, this is a widely used assumption in experimental vortex studies. Also all vortices are approximately perpendicular to the xy -plane.

Many of the works reported in literature concerning formation of vortices exhibit very clear patterns that correspond to triangular lattices of vortices (19, 42). The formation of this kind of lattices, known as *Abrikosov lattices*, is a consequence of the superfluidity of the sample. An

Figure 3.5 – Average number of vortices observed in the cloud as a function of (a) the amplitude for three different excitation times and (b) as a function of the excitation time for three different amplitudes. Lines are guides for eyes. The error bars show the standard deviation of the mean value of the number of vortices.

Figure 3.6 – Absorption images showing configurations of vortices forming (a) an equilateral triangle, or (b) a linear array. Images were taken after 15 ms of free expansion. (c) Sketch of the BEC with three vortices and the largest internal angle α .

Abrikosov lattice constitutes an equilibrium configuration of an arrangement of parallel vortices with the same sign of circulation. In Section 2.3.3 we have mentioned this kind of configurations and have shown typical images.

We believe that the fact of not observing the “standard” configurations is a consequence of our vortex formation mechanism, in which it is possible to nucleate vortices but also *anti-vortices*, that is, vortices with the opposite circulation sign. Observing configurations of three vortices provides evidence of this affirmation, as we explain in the following.

Three-vortex configurations

Three-vortex configurations are very interesting because, as we will see, they indicate a very important feature of our excitation: the capability of generating vortices with opposite signs of circulation (67).

When considering exclusively configurations with three vortices we observe predominantly two types of distributions. The vortices are distributed as a near-equilateral triangle or as a near-linear array. Figures 3.6(a) and (b) respectively show an example of each configuration. To quantify the frequency of these configurations we measure the largest internal angle α of the triangle whose vertices are the position of the vortices. Figure 3.6(c) sketches this angle.

In the histogram of Figure 3.7 we summarize our results for more than 60 measurements. We have grouped our data in three intervals. We consider an *equilateral-type* configuration

Figure 3.7 – Observed relative frequency of 3-vortex configurations as a function of the angle α . The inset shows the expected distribution of α when the vortices are distributed at random positions in a two-dimensional cloud.

when $\alpha \in [60^\circ, 100^\circ]$, a *linear* configuration when $\alpha \in [140^\circ, 180^\circ]$ and an *intermediate* configuration when $\alpha \in (100^\circ, 140^\circ)$. We can clearly see that the two more stable configurations are the equilateral and the linear arrays.

To interpret this results we have carried out some simple calculations considering a two-dimensional condensate. The reason of this is that the amount of published papers concerning dynamics of configurations of few vortices in three-dimensional systems is quite scarce. Actually, performing calculations in a 3D system is considerably more difficult and requires much higher computational power.

In the following discussion we assume that the formation of a vortex is equally probable at any position within the cloud.

Initially we have considered an elliptical area with an aspect ratio of 1.5, which corresponds to that of the measured samples. Next, we randomly distribute three points in it and measure the corresponding angle α of the formed triangle. Then, we repeat this process for 400 thousand random triangles and, finally, do the statistics. In the inset of Figure 3.7 we present the histogram of our counting, showing that triangles with the intermediate configuration, $\alpha \in (100^\circ, 140^\circ)$,

Figure 3.8 – Evolution of the largest angle α , in Gross-Pitaevskii simulations starting from various three-vortex configurations in a circularly trapped 2D BEC. Initial configurations are shown on right.

are almost as numerous as the equilateral arrangement. This shows that the vortices in the cloud must have an internal dynamics that causes certain equilibrium configurations to be more probable.

In order to have some insight about the dynamics of the vortices in the cloud, we have performed more sophisticated simulations in collaboration with Professor Masudul Haque, from Max-Planck Institute for the Physics of Complex Systems (MPI-PKS) in Dresden, Germany. In this simulation we consider a 2D non-rotating BEC and place on it three vortices with the same sign of rotation in an initial well-determined position. Next, we solve the time-dependent Gross-Pitaevskii equation to obtain the dynamics of the vortices. The results for four different initial states is shown in the graph of Figure 3.8.

When we start with near-equilateral configurations, *i. e.* $\alpha \in [60^\circ, 100^\circ]$, the simulation shows that the vortex cluster precesses around its center but the triangle slightly modifies its shape. Therefore, if initially the vortices form an equilateral triangle, they will keep its shape no matter at what time we perform the measurement. In consequence, in the case of three vortices with the same circulation sign, the equilateral-type arrangement constitutes an equilibrium configuration. We can interpret this as a reminiscent of the Abrikosov lattice. This is the case

Figure 3.9 – Schematics of the (a) equilateral and (b) tripole configurations of vortices, arrows indicate the vortex circulation direction.

of the configurations illustrated in Figures 3.8(A) and (B).

However, if the initial configuration is such that $\alpha \in (100^\circ, 140^\circ)$ (Figure 3.8(C)), we observe that, besides the precession of the cluster, there is a change in its shape. As the vortices precess the angle α oscillates in a bigger range of $\alpha \in [60^\circ, 140^\circ]$. This means that, depending on the moment in which we observe the cluster we could observe an equilateral-type configuration even when the initial configuration was not. In the same way, if the initial configuration is the linear array (Figure 3.8(D)), we observe precession of the vortices together with a very dramatic shape oscillation. In this case, the oscillation of α covers all the possible range, $\alpha \in [60^\circ, 180^\circ]$.

Consequently, we would expect a preponderance of observations of the equilateral-type configuration, secondly, the intermediate configuration and less frequently the linear array. We obtain the same result from the random distribution of triangles described above. However, our experiment shows that the linear array is more frequent than we would expect from simulations. To explain this we offer the following hypothesis: while in the equilateral array all vortices have the same circulation sign as illustrated in Figure 3.9(a), in the linear configurations the vortex in the middle has opposite circulation, in other words, it is an *anti-vortex*. The latter configuration is known as *tripole* and is sketched in Figure 3.9(b).

To support this hypothesis we use the results recently published by the group of Mikko Möttönen from Finland (69, 70). The authors have demonstrated that, at least in the 2D case, the tripole can be a stable configuration if the interactions between the atoms are strong enough.

In the $2D$ system the interaction strength corresponds to the coupling parameter $\tilde{g}$, defined as

$$\tilde{g} = \sqrt{8\pi} \frac{Na}{a_{\perp}}, \quad (3.5)$$

where a is the scattering length, N is the number of particles and $a_{\perp}$ is the oscillator length in the direction parallel to the vortex cores. Following Reference (70) the tripole configuration is stable if $\tilde{g} \geq 108$. In our experimental geometry, the vortex cores are oriented along one of the radial directions, thus $a_{\perp} = a_r = \sqrt{\hbar/m\omega_r}$. For our system we obtain that $\tilde{g} \approx 200$. Thus, from $2D$ arguments, we expect the linear tripole to be stable in our setup. This supports our hypothesis about the predominance of linear configurations due to the presence of one anti-vortex.

Therefore, a very possible explanation for the non regular distributions of vortices observed in our BEC is to assume the presence of anti-vortices in the sample.

There is another point that deserves to be clarified. If the probability of forming a vortex is the same than that of forming an anti-vortex, then we would expect the linear array to be more frequent than the equilateral-type configuration. However, we must consider the fact that configurations of vortices and anti-vortices have a much more complex dynamics, having several decay mechanisms. For example, there exist the probability of a vortex/anti-vortex pair to annihilate. Also, one of the vortices may migrate to the borders and escape from the BEC during its precession dynamics. All these decay mechanisms occur in a timescale of the order of the inverse of the mean frequency of the trap (70), that is $\tau \sim (\omega_x \omega_y \omega_z)^{-1/3} = (\omega_z \omega_r^2)^{-1/3}$, which in our case is of 10 ms. Since we wait 20 ms of equilibration time between the excitation and the release of the atoms from the trap, we certainly underestimate the number of tripoles formed during the excitation period because many of them could have decayed during this equilibration time. In fact, the observed fluctuations in the number and spatial distribution of vortices could be understood in terms of decay mechanisms due to the complex dynamics of configurations with vortices and anti-vortices.

Summarizing, in this section we demonstrate that our excitation is capable to nucleate

vortices. The number of vortices depends on the strength of the excitation. Observing the spatial distributions of vortices, particularly the distributions of three vortices, we realize that there are distributions that only can be explained by invoking the presence of anti-vortices, that is, vortices with the opposite sign of circulation. This makes sense because our excitation is not a rotation but an oscillation around an equilibrium point. At a given phase, the oscillation transfers angular momentum in one direction and when it comes back it transfers angular momentum in the opposite direction. Since vortices and anti-vortices annihilate each other with a certain probability, the presence of anti-vortices also explains the big fluctuations observed in the number of vortices generated for the same experimental conditions. Even in the absence of regularity, the states where the number of vortices in the sample can be well identified will be called *regular*.

3.3.3 Quantum Turbulence

As the parameters of the excitation increase, when about twenty vortices are formed in the BEC, a very dramatic change on the behavior of the cloud is observed. The most noticeable effect is a very different distribution of vortices across the cloud. The vortex cores are not oriented along a single direction, instead the absorption images suggest that the vortices are distributed along several directions. The vortex lines seem also to have curvy patterns (64, 68, 71, 72). This type of highly irregular configurations is known as “*vortex tangle configuration*” and constitutes one of the main features of the presence of Quantum Turbulence in the sample. Figure 3.10 shows three typical images of the turbulent condensate under the same experimental conditions. Figure 3.11(b) is a sketch of the inferred distribution of vortices in the turbulent cloud of Figure 3.11(a).

The observed images present large fluctuations for the same experimental conditions, indicating the chaotic nature of QT but also the random nature of our excitation. Also, as we showed in the graph of Figure 3.2, the potential oscillates in the xy —plane and along the xz —plane, as well, but with a smaller amplitude in such a way that longer excitation times or higher excitations amplitudes are needed to nucleate vortices along the other directions. Recall that

Figure 3.10 – Typical images of a turbulent condensed cloud after 15 ms of free expansion. All images were taken under the same experimental conditions.

Figure 3.11 – (a) Turbulent cloud after 15 ms of free expansion. (b) Sketch of the inferred distribution of vortices in picture (a).

according to Tsubota's proposal, QT can be produced by nucleating vortices along two perpendicular directions. Therefore, our vortex formation mechanism, although very different from that of Tsubota's proposal, has the same effect.

The graphs of Figures 3.5(a) and (b) show a clear connection between the time and the amplitude of excitation. Excitations with a big amplitude require shorter times to reach the turbulent condition.

Figure 3.12 – (a) Absorption images of a thermal cloud, a regular BEC and a turbulent BEC for three different expansion times. (b) Aspect ratio as a function of the expansion time for the different clouds. Lines are guides for eyes.

We have observed a very interesting feature which we consider a very important indication of the turbulent regime, it consists of a very different expansion dynamics of the cloud when it is released from the trap. It is well known that a thermal cloud expands isotropically, this means that it does not matter how anisotropic is the potential that contains the atoms, after some time of expansion the cloud becomes spherical. In contrast, a BEC undergoes anisotropic expansion, expanding faster along the direction that was more tightly confined in the trap. This is a consequence of the wave behavior of the quantum gas. The way of quantifying the expansion is by measuring the aspect ratio, which is the ratio of the width of the cloud to its length. Hence, in a thermal cloud the aspect ratio tends to unity as the expansion time increases. In a BEC, the aspect ratio starts below unity and, after certain time it exceeds unity; this phenomenon is known as “aspect ratio inversion” and the inversion time depends on the initial confinement of the cloud. In a turbulent BEC the aspect ratio remains constant during the whole expansion (71). It does not evolve to unity as in a thermal cloud, revealing that the system is not classical, but neither presents aspect ratio inversion, showing that a very complex dynamics is happening inside the condensate. Figure 3.12(a) shows side-to-side snapshots for different expansion times of a thermal cloud, a regular BEC and a turbulent condensate. Figure 3.12(b) shows a graph of the aspect ratio evolution of these three clouds. We believe that this remarkable characteristic is a new effect in atomic superfluids, possibly containing signatures of the emergence of QT in this system. Nevertheless, this fact is still not understood and presently is under investigation. Somehow, the presence of vortices must modify the hydrodynamics of the superfluid.

3.3.4 Granulation

After reaching the quantum turbulent regime, increasing even further the value of the parameters of the excitation, a new phase appears in the condensate. We observe that the condensate breaks into pieces. The resulting state is a thermal cloud with little grains of condensate scattered within it (68). The grains persist if the flow of energy from the excitation is kept flowing into the sample. In Figure 3.13 we can see a picture of this state, showing small bright spots distributed inside the cloud. These spots correspond to the condensed grains.

Figure 3.13 – Absorption image of a granulated cloud after 15 ms of free expansion.

We have identified the observed state with the prediction of V. I. Yukalov. In References (73, 74, 77) Yukalov *et al.* demonstrate that, under proper conditions, an oscillatory excitation is able to break the condensate into droplets distributed inside a non-condensed cloud. This is a heterogeneous phase which is known as *Granulation* and constitutes a non-equilibrium state, it is dynamical in the sense that the droplets do not remain fixed in space. In Section 3.4.3 we provide a deeper discussion.

Many important questions arise at this point. Are the grains still superfluid? Do the spatial, size and shape distributions of the grains obey any statistical law? How do such distributions depend on the parameters of the excitation? To answer this questions our current imaging technique is not sufficient. Our granulated cloud is a three-dimensional system, while the absorption imaging is a destructive technique that provides only a two-dimensional projection of the cloud. As a consequence we cannot study the spatial distribution nor the dynamics of the system. To overcome this limitations, we have adopted two future strategies. The first one is to implement a non-destructive imaging diagnosis which will allow us to investigate the dynamics of the cloud. Second, through a collaboration with the group of Professor Randall G. Hulet from the Rice University at Texas in the United States, we have started to study granulation in a unidimensional condensate of ^7Li . In this case we can easily observe the distribution of the grains in the sample using optical techniques.

3.3.5 Diagram of excitations

To summarize all our observations we have plotted our data in a diagram of Amplitude versus Time of excitation, which we present in Figure 3.14. This diagram shows very clear domains, each corresponding to one of the observed regimes. This diagram is a very important and novel result because it exhibits the route in which the parameters of the excitation must be varied in order to achieve a specific state (68). In particular we can understand the conditions to produce QT in the condensate.

The border lines between the regimes are just guides for eyes and have not been obtained experimentally. However, their shape indicates that, very likely, the important quantity related to the route to the observed states is the product of the amplitude and the time of excitation. In other words, we could consider the total pumped energy into the cloud to characterize the threshold behavior between the domains.

Another very interesting feature of this diagram is that it is not expected in bulk superfluids such as ^4He and ^3He . The finite size of the condensate is a very important difference with respect to superfluid Helium. Because of the healing length in a condensate being only one order of magnitude smaller than the system itself, the number of vortices that the BEC can contain is limited. In fact, we observed that in our system QT is produced after nucleating about 20 vortices. In Reference (78) it is shown that finite size criteria can be used to predict the border line between the regular vortex and the QT regimes.

3.4 Discussion

In the next subsections we present a theoretical analysis and discuss our results. We initially focus in the turbulent regime and next we revise the granulation. Finally we discuss some hypothesis about the physical mechanism to nucleate vortices.

Figure 3.14 – Diagram showing the domains of parameters associated with the observed regimes of the condensate. Figures on the top correspond to typical observations. For the region (b) of regular vortices, the number of vortices varies with the parameters as presented in Figures 3.5(a) and (b). Gray lines are guides for eyes, separating the domains of different observations.

3.4.1 Numerical calculations for the turbulent regime

To better understand our observations we have performed a numerical simulation of our experiment. The calculations were carried out by the groups of Professors Makoto Tsubota, Michikazu Kobayashi and Kenichi Kasamatsu, respectively from the Osaka City University, the University of Tokio and Kinki University, in Japan (68). The main goal is to numerically solve the Gross-Pitaevskii equation (GPE), given by

$$i\hbar \frac{\partial}{\partial t} \Psi(\vec{r}, t) = \left(-\frac{\hbar^2 \nabla^2}{2m} + V(\vec{r}, t) + g |\Psi(\vec{r}, t)|^2 \right) \Psi(\vec{r}, t), \quad (3.6)$$

where $g = 4\pi a_s \hbar^2 N/m$ is the coupling parameter and represents the strength of the interaction among the atoms and a_s is the scattering length. The potential $V(\vec{r}, t)$ is given by Equation (3.4).

We must consider that the sample is not a pure condensate and a thermal cloud is present. In an oscillating condensate the existence of a thermal cloud may affect its dynamics. Different dissipative effects could be present as well. In order to account for dissipation a very simple and widely used model is to add a phenomenological constant, γ , into Equation (3.6), namely

$$(i - \gamma)\hbar \frac{\partial}{\partial t} \Psi(\vec{r}, t) = \left(-\frac{\hbar^2 \nabla^2}{2m} + V(\vec{r}, t) + g |\Psi(\vec{r}, t)|^2 \right) \Psi(\vec{r}, t). \quad (3.7)$$

In absence of dissipation $\gamma = 0$ and, as we will see, it is necessary that $\gamma \neq 0$ in order to nucleate vortices and produce QT.

Equation (3.7) describes the dynamics of our system and if we were able to solve it and obtain $\Psi(\vec{r}, t)$ we would have all the information of the system. Unfortunately, solving this 3D equation with such a complex potential is very difficult and would demand computational resources that currently we do not have.

However, we can perform a 2D simulation by decomposing the wavefunction as $\Psi(x, y, z) = \psi(y, z)\phi(x)$ and solving the GPE for $\psi(y, z)$. This approximation is far from being an accurate description of our system, nevertheless it might provide some quantitative insight of the pro-

blem. Under this consideration, the Gross–Pitaevskii equation reads

$$(i - \gamma)\hbar \frac{\partial \psi}{\partial t} = \left(-\frac{\hbar^2 \nabla^2}{2m} + V_{2D}(t) + g_{2D} |\psi|^2 \right) \psi, \quad (3.8)$$

where $g_{2D} = (1/4\pi l_x^2)^{1/2} g$ is the 2D coupling parameter, with $l_x = (\hbar/2m\omega_x)^{1/2}$ the oscillator length along the x -direction.

$V_{2D}(t)$ represents the two-dimensional potential, it must contain the static harmonic trap and also the excitation. The 2D excitation that we propose contains rotation around the x -axis and translation along the y -direction. Rewriting Equation (3.8) in a reference frame co-moving with the oscillating potential we obtain

$$(i - \gamma)\hbar \frac{\partial \psi}{\partial t} = \left(-\frac{\hbar^2 \nabla^2}{2m} + V_0 + g_{2D} |\psi|^2 - \boldsymbol{\Omega}(t) \cdot \hat{\mathbf{L}} - \mathbf{v}(t) \cdot \hat{\mathbf{p}} \right) \psi. \quad (3.9)$$

In Equation (3.9) we consider the following:

- The linear and angular momentum operators are defined as: $\hat{\mathbf{p}} = -i\hbar \nabla$ and $\hat{\mathbf{L}} = \hat{\mathbf{r}} \times \hat{\mathbf{p}}$.
- The unperturbed harmonic potential is given by $V_0 = m\omega_r^2(y^2 + z^2)/2$.
- We define $T = \pi/\Omega_{ac}$ as the half of the oscillation period, where Ω_{ac} is the frequency of the excitation.
- The angular velocity of the rotation is given by $\boldsymbol{\Omega}(t) = (\Omega_x, 0, 0) \sin(\Omega_{ac}t)$, where we estimate $\Omega_x \simeq 2\theta_0/T = (2\theta_0/\pi)\Omega_{ac}$ and $\theta_0 = 5^\circ = \pi/36$ rad. θ_0 corresponds to the misalignment of the ac-coils. Therefore, Ω_x is a very small quantity.
- We assume that the translation occurs only along the y -direction, consequently $\mathbf{v}(t) = (0, v_y, 0) \sin(\Omega_{ac}t)$, where we estimate $v_y \simeq 2\delta_y/T = (2\delta_y/\pi)\Omega_{ac}$. Here, δ_y is the maximum displacement of the minimum of the potential and we estimate $\delta_y = \alpha \times (5\mu m)$. The dimensionless parameter α will be used as a variable parameter that represents the amplitude of the center-of-mass oscillation.
- Recall: γ is a phenomenological parameter representing dissipation of energy in the system. We have the possibility of varying its value in our numerical calculations.

Taking into account all these considerations, the equation to be simulated is

$$(i - \gamma)\hbar \frac{\partial \psi}{\partial t} = \left(-\frac{\hbar^2 \nabla^2}{2m} + V_0 + g_{2D} |\psi|^2 - \Omega_x \sin(\Omega_{act}) \cdot \hat{L}_x - v_y \sin(\Omega_{act}) \cdot \hat{p}_y \right) \psi. \quad (3.10)$$

The numerical method used for solving Equation (3.10) is based on the Crank–Nicolson method which will not be described here (see, for example, Reference (79)). Now we discuss the results of the simulation.

Numerical results

In our numerical calculations two very important quantities are obtained: (i) The evolution of the two-dimensional wavefunction $\psi(y, z, t)$ obtained from Equation (3.10), and (ii) the mean angular momentum per atom, $\langle L_x \rangle = \int dr \psi^* \hat{L}_x \psi$, as a function of the excitation time. We obtain the following results.

Figure 3.15 shows snapshots of the density profile $|\psi|^2$ for different excitation times ranging from 13 to 17 ms, in this calculation $\alpha = 1.6$ and $\gamma = 0.02$. Figure 3.16 is the graph of the mean angular momentum $\langle L_x \rangle$ as a function of time. For these specific values of α and γ we can see that $\langle L_x \rangle$ initially oscillates around zero and after ~ 15 ms of excitation it blows up, presenting an abrupt growth and very big fluctuations. This behavior indicates that something is happening in the condensate; indeed, in the corresponding snapshot of Figure 3.15 we can observe the formation of wavy patterns which develop to dark solitary waves which subsequently decay into several vortex/anti-vortex pairs. The decay from dark solitons to vortex pairs occurs via the snake instability (80). Next, a much more complex dynamics takes place involving the formation of an undetermined number of vortices and anti-vortices in the sample. This final state characterizes the emergence of quantum turbulence in the superfluid.

Summarizing, the evolution of the cloud is as follows: after a certain time of excitation the first event in the cloud is the formation of a dark soliton¹. Then, a certain number of vortices

¹ In a nonlinear medium, a soliton is an isolated wave which propagates keeping its shape. This is possible because the nonlinear and the dispersive effects cancel each other as the wave moves. Due to its nonlinearity, the Gross–Pitaevskii equation admits soliton-like solutions. In a BEC, a *dark soliton* consists of a dip in the density profile propagating without losing its shape. Its size is of the order of the healing length of the superfluid.

Figure 3.15 – Snapshots of the BEC after different times of excitation. The left and the right columns show the 2D and 3D plots of the density profile, respectively. The colors range from red (high density) to blue (low density).

Figure 3.16 – Mean angular momentum per atom as a function of the excitation time with parameters $\alpha = 1.6$ and $\gamma = 0.02$. Image courtesy of K. Kasamatsu, M. Kobayashi and M. Tsubota.

are formed as a consequence of the decay of this soliton, this corresponds to the regular vortices regime shown in Figure 3.14. As the time of the excitation increases, these vortices become much more numerous and their dynamics much more complex, this indicates the presence of QT in the superfluid.

It is very important to understand the effect of the parameters α and γ in the observed dynamics. We have varied the value of γ and realized that the dynamics of the BEC strongly depends on this parameter. Figure 3.17 presents the evolution of $\langle L_x \rangle$ for two different values of γ . We find that the formation of vortices only happens if $\gamma \in [0.015, 0.025]$. Below this interval no instabilities occur and the soliton does not decay into vortices. Above this interval the dissipative effects are so strong that the oscillation of the BEC is rapidly damped and vortex formation does not occur. This suggests that our vortex formation mechanism is associated with dissipative effects, we discuss this point in Section 3.4.2.

Fixing γ at $\gamma = 0.02$, we vary the value of α . Figure 3.18 presents the evolution of $\langle L_x \rangle$ for several values of α . It is found that below a threshold value ($\alpha = 1.3$) vortex formation does not occur, and only the oscillatory motion of the cloud is observed. Increasing the value of α the

Figure 3.17 – Mean angular momentum per atom as a function of the excitation time for two different values of the dissipation γ . Here $\alpha = 1.6$ for both curves. Image courtesy of K. Kasamatsu, M. Kobayashi and M. Tsubota.

Figure 3.18 – Mean angular momentum per atom as a function of the excitation time for different values of α . Here $\gamma = 0.02$ for all curves. Arrows indicate the onset of vortex nucleation. Image courtesy of K. Kasamatsu, M. Kobayashi and M. Tsubota.

mean angular momentum $\langle L_x \rangle$ exhibits a complicated behavior, indicating the onset of vortex formation. This onset occurs for shorter times with a faster evolution to QT as α increases. This fact supports the observations presented in the diagram of Figure 3.14. Note that the time scale of the vortex formation is of the order of 10 ms, consistently with the observed times in the experiment.

The presented simulations cannot reproduce the whole observed diagram, since our system is a three-dimensional gas. Nevertheless, good qualitative agreement with the experiment has been achieved, these simulations have been useful to understand the processes involved in the formation of vortices and the route to turbulence.

3.4.2 On the vortex formation mechanism

Now we devote few words about the actual mechanism of the formation of vortices in our experiment.

From the discussion presented above and the experimental observations it seems that there are three main ingredients to produce vortices:

1. Translation of the center of mass of the superfluid.
2. Rotation of the cloud.
3. Dissipation of energy in the system.

The first two ingredients are properties of the excitation field, the third one depends on the initial state of the condensate.

It is known that in an oscillating BEC the oscillation modes of the pure condensed fraction may be different from those of the thermal fraction. This is the situation of our experiment in which we have a finite temperature condensate subjected to an oscillation. In fact, as discussed in Reference (63), we consider that the relative movement of the condensed and thermal components subjected to the external field is related to the mechanism of formation of the vortices.

Figure 3.19 – (a) Absorption imaging of the atomic cloud from Figure 3.4(e) with a different contrast. In (b) the red arrows show round structures around the condensed component which correspond to quantized vortices.

This phenomenon, known as Kelvin–Helmholtz instability (47), occurs in the interface between two fluids that have a relative velocity. This instability is able to nucleate vortices in the interface between the fluids. One of the evidences that we have to support this hypothesis appears when modifying the contrast of our images to observe clearer the interface between the condensate and the thermal cloud. Figure 3.19(a) shows the same cloud as in Figure 3.4(e) where only the contrast has been modified. Vortices can be seen distributed around the condensed cloud, as indicated by the red arrows in Figure 3.19(b). This experimental observation suggests that the vortices are initially produced at the interface of the condensate and the thermal cloud; eventually some of them will migrate into the condensate.

With these considerations we can interpret more precisely the physical meaning of the constant γ as being the contribution of the thermal cloud to the dynamics of the condensate. However, to better understand this aspect, either more sophisticated measurements or more accurate finite–temperature simulations of a three–dimensional cloud are required.

3.4.3 Theoretical considerations about Granulation

The granular state is still under theoretical and experimental investigation. As mentioned before, it is very difficult to analyse the properties of this non–equilibrium 3D system by means

of absorption images. However, we have established a collaboration with Professor Vyacheslav I. Yukalov, from the Joint Institute of Nuclear Research in Dubna, Russia, to theoretically understand our results.

In recent work (73), Professor Yukalov and collaborators have demonstrated that the action of an external alternating field is equivalent, on average, to the action of an external spatially random potential. Actually, granulation of a condensate has been initially predicted for a static condensate trapped in an spatially random potential (74). This static state, also known as *Bose glass* phase can be reached if the random potential fulfills certain conditions. Let us first understand these conditions and then we present the analogy to the time-oscillating case.

Consider a BEC subjected to a spatially random potential given by $\xi(\mathbf{r})$ which satisfies the condition of being bounded, *i. e.* $|\xi(\mathbf{r})| \leq V_R$, where V_R is a constant. The interatomic distance is a and the typical size of the unperturbed cloud is L . It is well-known that spatial disorder has the property of localizing the atomic motion within a certain length called “localization length”, given by

$$l_{loc} = \frac{4\pi\hbar^4}{7m^2V_R^2l_R^3}, \quad (3.11)$$

where l_R is the correlation length of the disordered potential (see, for instance, References (75, 76)).

It can be shown that if the system satisfies the condition $a \ll l_{loc} \ll L$, then the BEC fragments into multiple pieces separated by the normal fluid phase. This is called *Bose glass* or *granular condensate* (73).

Now consider a BEC trapped in a harmonic potential of frequency ω_0 , thus the oscillator length is given by $l_0 = \sqrt{\hbar/m\omega_0}$. The condensate is then subjected to an external oscillating field $V(\mathbf{r}, t) \sim V_0$, where V_0 is the amplitude of the oscillation. In their article, V.I. Yukalov and collaborators demonstrate that in the time-averaged situation, this system is analogous to the BEC in an external spatially random potential. In this situation, the amplitude of the oscillation is analogous to the amplitude of the random potential, $V_0 \rightarrow V_R$, and the oscillator length plays the role of the disorder correlation length, $l_0 \rightarrow l_R$. In this case, it is shown that the localization

length is given by

$$l_{loc} \sim \left(\frac{\hbar\omega_0}{V_0} \right)^2 l_0. \quad (3.12)$$

When the energy pumped into the system by the excitation is small (*i. e.* $V_0 \ll \hbar\omega_0$), there is a single condensate filling the trap. This condensate could contain any kind of excitations, such as vortices or quantum turbulence, but still fills the whole trap. If the amplitude and/or the time of the excitation are increased, the condensate reaches the condition in which $l_{loc} \sim l_0$. In consequence the condensate granulates into pieces. The condition for having the granular phase can be written as

$$\hbar\omega_0 \leq V_0 \leq \hbar\omega_0 \sqrt{\frac{l_0}{a}}. \quad (3.13)$$

For our experimental conditions this condition is fulfilled. The upper limit was not reached in our experiment, however this condition has an interesting consequence. Having $V_0 \sim \hbar\omega_0 \sqrt{\frac{l_0}{a}}$, implies that $l_{loc} \sim a$ and we would expect a complete destruction of the granular condensate. This state could correspond to a non-equilibrium normal fluid in a chaotic regime, as predicted by V. I. Yukalov (77). This is an interesting trend to follow in future experiments.

4 *Construction of a New Experimental Setup*

In Chapter 3 we have described the experiments performed in a ^{87}Rb BEC, however we have not provided the details about the production of the condensate. The goal of this Chapter is to describe all the steps for the construction of a system for producing BECs of ^{87}Rb . The setup described in the present Chapter, which we call *BEC-II system*, is our second generation apparatus and it is essentially identical to the BEC-I setup except for the trapping system and the configuration of the vacuum apparatus.

This Chapter is structured as follows. In Section 4.1 we discuss our motivation to construct a second setup. Next, we describe the vacuum system in Section 4.1 and explain how to achieve the required vacuum regime. In Section 4.3 we present our laser setup and discuss some important techniques such as the saturated absorption spectroscopy that we use to lock the lasers. Then, in Section 4.4 we characterize our magneto-optical trap. Later, in Section 4.5 we present our imaging system, which represents the main tool to probe and study our atomic samples. Section 4.6 is devoted to the processes required to transfer the atoms from the magneto-optical trap to the pure magnetic trap. This stage is one of the most critical and the success in the production of the BEC strongly depends on it. Next, in Section 4.7 we describe the hybrid trapping stage and the subsequent evaporative cooling technique which cools the atoms down to the transition temperature. At the end of this Section we summarize all the previous stages and provide a general vision of the whole sequence to produce the quantum gas. Finally, in Section 4.8.1 we provide some details about the control programs employed to synchronize all the processes of the experiment.

4.1 Motivation

The reason for having a second system able to produce condensates of ^{87}Rb is that we are interested in performing an experiment impossible to carry out in the BEC–I system due to its intrinsic construction.

This experiment consists in measuring the interaction of a BEC with an external conducting coil. Specifically, the idea is to produce a spin-polarized BEC (hence, having a global magnetization) and drop it through a closed loop. The condensate will induce a current on the loop that will depend on the global magnetic structure of the quantum system. No analogous experiments or theoretical predictions are available in literature, therefore, this experiment should yield very novel and interesting knowledge on quantum magnetism. Since the BEC is a mesoscopic system with no more than 1×10^6 atoms, the induction signal is expected to be very small. However, its intensity depends on the velocity at which the condensate passes through the loop. Therefore, the faster the condensate crosses the coil, the higher the intensity of the induction signal. In consequence, it is desirable to have a vacuum chamber that allows us to drop the condensate a long distance in order to acquire a higher velocity. In the BEC–I system, the glass cell in which the sample is produced is 7 cm long and 3 cm wide, and it is oriented along the horizontal direction. Therefore, the gas has less than 3 cm of falling distance, making this system very unsuitable for loop induction measurements. As will be shown in Section 4.2, in the BEC–II system we use a 15 cm long glass cell oriented along the vertical direction, having more than 10 cm of free fall distance, which we estimate to be enough for our purposes.

The BEC–II also presents other advantages. First, the trapping system is not a pure magnetic trap, but a hybrid of a magnetic quadrupole and a optical-dipole trap. These kind of traps are very flexible, offering control in more parameters than in a QUIC trap. Second, the optical access to the experiment region is much greater. This will allow us to perform a larger variety of experiments in an easier way (for example, the implementation of an optical lattice) and obtain images with higher resolution.

4.2 Vacuum System

A very important characteristic that an experimental setup to produce BEC must fulfill is the possibility of trapping many atoms (of the order of 10^9) in a ultra-high vacuum ambient (this means, a pressure $\leq 10^{-9}$ torr). A small number of atoms will produce a very small condensate, and the presence of impurities can rapidly destroy it or even avoid its production.

In our laboratory we have adopted a strategy known as “double-MOT configuration” which was proposed and implemented by C. J. Myatt *et al.*, in the 90’s (81). The main idea is the following: initially a magneto-optical trap (MOT) is produced in a first glass cell which has been filled with a dilute rubidium vapor. The vapor comes from the emission produced when a current circulates through rubidium filaments previously installed inside the cell. The current heats up the filaments, provoking their outgassing. The pressure in this cell, of 2×10^{-9} torr, is low enough for producing a MOT but too high for achieving Bose-Einstein condensation. Using a thin tube it is possible to connect the first cell to a second one which has the proper vacuum conditions for BEC. If the tube is thin enough it is possible to have differential pumping between the two cells, obtaining a pressure lower than 10^{-11} torr in the second cell. In this case we use a laser beam to push the atoms from the MOT of the first cell through the tube up to the second cell. In the second cell we produce a second MOT with the atoms pushed from the first one without increasing substantially the cell pressure. These atoms are then submitted to the processes necessary for achieving the quantum degeneracy. As we can see, the first cell serves as a source of atoms and the second cell serves as a “science chamber” where the BEC will be produced and the experiments performed.

For convenience we will adopt the following notation: the MOT produced in the first cell will be called “MOT-1” and that of the second cell will be named “MOT-2”.

The MOT-1 glass cell was manufactured in the glassware shop of our Institute and it is made of borosilicate (also known as Pyrex[®]). It is a rectangular cell with dimensions $30 \times 30 \times 150$ mm³. Inside this cell we have installed rubidium filaments, known as *dispensers*, through a 5-pins feedthrough. When an electric current circulates through these dispensers a

dilute vapor of Rb is emitted, becoming our source of atoms for loading the MOT (82). This cell is continuously pumped by a 55 l/s ion pump from Varian[®] (model VacIon Plus 55).

For the MOT-2 we use a commercial quartz cell with dimensions $30 \times 30 \times 150 \text{ mm}^3$ from Hellma[®] Analytics of very high optical quality. We need a better quality glass cell for the MOT-2 just for guaranteeing good quality imaging of the atoms, without optical distortions caused by the cell. The pumping is done with a 300 l/s ion pump, also from Varian[®] (model VacIon Plus 300).

Both cells are connected by a tube $\sim 550 \text{ mm}$ long and 4 mm internal diameter. However, the cells are also connected by an all-metal valve (from MDC[®]) that is only open during the first stage of pumping and remains closed once the ultra-high vacuum (UHV) regime is achieved.

Near to the MOT-2 cell we have a titanium sublimation pump. It consists of a *L*-shaped tube, approximately 15 cm in diameter, which has been machined in our institute's workshop. Inside of it we have a titanium filament that is sublimated during the final stage of pumping. In Section 4.2 we describe all the pumping process.

Figures 4.1(a) and (b) show respectively a scheme and a photograph of our system. As can be seen, we have mounted the glass cells along the vertical direction. As discussed in Section 4.1 we need a system which allows a long free fall of the sample, therefore, our configuration is optimal for this objective.

How to achieve the ultra-high vacuum regime?

The main steps necessary for achieving the UHV regime are listed in the following:

1. **Mounting.** All pieces are mounted and the system is closed. For doing this all the components have a flange which can be attached to another one. Between to flanges we put a copper gasket that guarantees good sealing among the parts.
2. **Turbomolecular pumping.** We initiate the pumping of the system using a turbomolecular pump connected to the system by a Gate valve. At this moment the all-metal valve

Figure 4.1 – (a) Scheme and (b) picture of the vacuum system.

that connects the two glass cells is completely open.

3. **Baking.** During the turbomolecular pumping all the system is heated up to temperatures that vary from 150°C to 250°C. The glass cells must be at the lower temperature. This process, known as “*baking*” has the objective of ejecting water and other substances that could be adsorbed on the internal walls of the system. For warming we use heating tapes placed around the system and then completely wrap it with a layer of fiberglass, useful for preserving the heat. Optionally, a layer of aluminum foil can be used to wrap the whole system to avoid that fiberglass pieces disperse in the laboratory. This process is applied for several days, in our case two weeks, until the internal pressure reaches approximately 10^{-8} torr. At this point the heating is gradually decreased, then the fiber glass and the heating tapes are removed and the tubomolecular pump is switched off and disconnected from the system.
4. **Ion pumping.** When the tubomolecular pumping stage is over, the ion pumps are turned on. After few days, in our case four days, the pressure in the MOT-1 cell is of 2×10^{-9} torr and in the MOT-2 cell is of the order of 10^{-10} torr. At this situation the All-metal valve is definitely closed and the final stage of pumping starts.
5. **Titanium sublimation pumping.** This kind of pumping deposits in the internal walls of the system a thin layer of titanium which has the property of adsorbing any particle that collides with it. Thus, such a particle no longer contributes to the pressure in the system. For carrying out this sublimation we have a titanium filament inside an L-shaped tube with a big internal surface. On this surface the titanium gas is adsorbed, and hence it should be as large as possible. Using this pumping technique we are able to reach a pressure below 5×10^{-11} torr, which is suitable for our purposes.

Reference (83) is an excellent source of information on UHV fundamentals and techniques.

4.3 Laser setup

Before transferring the atoms to the final harmonic potential where the condensate will be produced a pre-cooling stage by means of magneto-optical trapping is necessary. It is also very important to be able to control the internal state of the atoms. Finally, it is fundamental to have an imaging technique for studying the sample. All these processes use laser light as the main tool, therefore, it is mandatory to have a very well designed laser system.

For producing all the laser beams required in our experiment we have three high power diode lasers from TOPTICA[®] Photonics (model DLX-110L). Diode lasers are an excellent tool due to their high stability and very narrow linewidth (below 1 MHz) that permits the excitation of individual hyperfine levels. These lasers have external electronics that allows to control the parameters of the diode such as temperature and current passing through it. To lock the laser at the atomic hyperfine frequencies we use saturated absorption spectroscopy (SAS) technique (84) in a vapor cell. This technique suppresses the Doppler broadening effect on the spectrum, making possible to resolve the hyperfine levels of the atom. The SAS signal shows an intensity peak at the position of each hyperfine transition and also at the midway of any two transitions, the so called “crossovers”. The control circuits of the laser have a **lock-in regulator**. This regulator has the function of locking the laser at a fixed frequency. In brief, the way it works is the following:

1. The frequency of the laser light is slightly modulated. In consequence, the saturated absorption signal will also be modulated.
2. The modulated absorption signal is sent to the lock-in regulator circuit where it is mixed with the modulation itself. This mixing generates a new signal with two components, a DC component and a AC component.
3. Using a low-pass filter we can eliminate the AC signal, keeping only the DC signal, which turns out to be proportional to the derivative of the original signal and sensitive to the phase of the modulation with respect to the response of the circuit. This DC signal

Figure 4.2 – Example of an absorption peak (top) and its corresponding dispersion signal (bottom).

represents the lock-in output that is used to fix the frequency of the laser and it is called “*dispersion signal*”.

4. Note that the position of a peak in the saturated absorption signal corresponds to a zero point with a certain slope in the dispersion signal. The idea is to lock the laser frequency keeping the dispersion signal fixed at one of these zero points. This is achieved using a PID-regulator which already is contained in the lock-in regulator circuit of the laser. The top of Figure 4.2 represents a typical absorption peak of the SAS of a hyperfine transition. The bottom of Figure 4.2 shows the corresponding dispersion signal, the vertical dashed line exhibits the correspondence between the SAS peak and the zero point in the dispersion signal.

The isotope of ^{87}Rb is an alkaline atom with a relatively simple structure. We use the transition between the fine levels $5^2S_{1/2} \longrightarrow 5^2P_{3/2}$ (known as the rubidium D_2 line) to perform all the necessary processes in our experiment. These two levels present a hyperfine splitting due to the nuclear spin interaction. Figure 4.3 shows the SAS of the D_2 line of rubidium. For obtaining this spectrum we use glass cells with rubidium vapor heated at $\sim 40^\circ\text{C}$. In this case

Figure 4.3 – Saturated absorption spectrum of the D_2 line of ^{85}Rb and ^{87}Rb isotopes.

the sample is not pure, instead we have a mixture of ^{85}Rb and ^{87}Rb isotopes. Each big dip of the spectrum of Figure 4.3 comes from the transitions from one of the hyperfine levels of the ground state ($5^2S_{1/2}$ state) to the hyperfine levels of the $5^2P_{3/2}$ state.

In our experiment we use a total of six different frequencies. We need two frequencies to produce each of the two MOTs (trapping and repumper frequencies) and one *push* frequency for transferring the atoms from the MOT-1 to the MOT-2. Also, two frequencies are required for controlling the internal state of the atoms (optical pumping frequencies). Finally, an extra beam for performing the imaging of the atoms is necessary. Figure 4.4 shows the energy levels of the D_2 line and the different frequencies employed in the experiment. To produce all these frequencies we have three diode lasers at 780 nm. One of them, which we call “Trapping Laser 1”, is used exclusively to produce the trapping light of the MOT-1. The second laser, named “Repumper Laser” is used to generate the repumper light of both MOTs and one of the optical pumping frequencies. Finally, the third laser, the “Trapping Laser 2”, produces the trapping

Figure 4.4 – D_2 line of ^{87}Rb together with the frequencies employed in the experiment.

light of the MOT–2, the push beam and the second optical pumping beam.

During the course of the experiment it will be necessary to vary the value of the frequency of a specific beam or to abruptly switch it off. For this reason, across their paths, the different beams pass through Acousto–Optic Modulators (AOM). An AOM is an opto–mechanical device that uses the acousto–optic effect to diffract and shift the frequency of a light beam that passes through it (85). It contains a crystal in which a radio–frequency (RF) acoustic wave is induced, when the light interacts with this wave it is diffracted and its frequency is shifted. Considering that the wavelength and frequency of the light are, respectively, λ and f , and those of the acoustic wave are Λ and F , we have that the diffracted angle and the frequency shift in the light are given respectively by

$$\sin \theta = \frac{n\lambda}{2\Lambda} \quad \text{and} \quad f \rightarrow f + nF, \quad (4.1)$$

where $n = \{\dots -2, -1, 0, 1, 2, \dots\}$ is the order of diffraction. In our system we only use the first diffracted order $m = \pm 1$. From these equations it is clear that shifting the frequency of the light using an AOM will also change the angle of the diffracted beam. In an optical setup, where everything is finely aligned, this would cause undesired misalignment. However, the capability of changing the frequency is fundamental in our experiment. The strategy adopted to

overcome this problem is to align the AOM in the so-called *double-pass* configuration. In this configuration, the laser beam passes across the AOM, getting a deflection of θ and a frequency shift of $\Delta f_1 = F$. Next, the diffracted order is retro-reflected along its own path. When the retro-reflected beam passes by the second time through the AOM it is diffracted again, getting a new deflection of $-\theta$ and a new frequency shift of $\Delta f_2 = F$. Therefore, after the double-pass through the AOM, the beam will have a total frequency shift of $\Delta f = 2F$ but no deflection (or a very small one) with respect to the initial beam. In consequence, using an AOM in double-pass configuration we can vary the frequency of the beam with minimal misalignment.

AOMs are useful not only to control the frequency of a beam but they are also very fast switches. The RF induced in the crystal can be rapidly switched off, extinguishing the diffracted order in few microseconds. In our system there are in total ten AOMs, in some cases they are used to finely tune the frequency of a beam, in some others they function simply as a fast switch. However, there is always a small fraction of light that is still diffracted and for this reason we also have mechanical shutters that completely block the light in specific places.

In the following we describe the optical setup for each laser. Figure 4.5 is a drawing of this scheme, where lenses and wave plates were remove for clarity.

Repumper Laser

This laser produces the repumper light for both MOTs and also one of the optical pumping beams, having a total power of ~ 250 mW. Initially, we extract from the beam a small portion that passes through an AOM (at -80 MHz) and the diffracted beam is used in the SAS lock-in system. The laser is locked at the $5^2S_{1/2}(F = 1) \rightarrow 5^2P_{3/2}(F' = 1)$ transition; however, due to the AOM in the SAS system, the outgoing light is 80 MHz above the lock-in point.

The beam then passes through an AOM in double-pass configuration (at $+77.9$ MHz) which will be used to control the frequency of the light. Next, a small fraction is separated and passes through an AOM (at -78.7 MHz) to produce one of the optical pump beams, whose frequency is $5^2S_{1/2}(F = 1) \rightarrow 5^2P_{3/2}(F' = 2)$. This beam is mixed with the second optical pum-

Figure 4.5 – General laser setup. Lenses and wave plates were removed for clarity.

ping beam (resonant with $5^2S_{1/2}(F = 1) \rightarrow 5^2P_{3/2}(F' = 2)$) and then both beams are coupled in a polarization maintaining (PM) optical fiber that takes the light to the experiment. These two frequencies will be used in the process of optical pumping which will be described later in Section 4.6. The rest of the light is again divided into two identical beams. One of the beams is mixed with the trapping light of the MOT-1 and the second beam is mixed with the trapping light of the MOT-2. Each mixture passes through an AOM (at -78.7 MHz) which serves as a switch, and then reaches the respective glass cell by means of PM optical fibers. The repumper light of the MOTs is resonant with the transition $5^2S_{1/2}(F = 1) \rightarrow 5^2P_{3/2}(F' = 2)$.

The path of the light produced by this laser is represented by the purple line in Figure 4.5.

Trapping Laser 1

This laser is used exclusively to produce the trapping light for the MOT-1 and generates about 350 mW of light power.

It is locked at the $5^2S_{1/2}(F = 2) \rightarrow 5^2P_{3/2}(F = 3)$ transition but, due to the AOM at the SAS system, the outgoing light is shifted 53.2 MHz above the lock-in frequency. Next, the light is mixed with the repumper light and then both beams pass through an AOM (at -78.7 MHz) used to switch off the light. Finally, both frequencies are coupled in a PM optical fiber which takes the light to the MOT-1 trapping region. At this point, the trapping light is detuned to red by ~ 25 MHz from the $5^2S_{1/2}(F = 2) \rightarrow 5^2P_{3/2}(F' = 3)$ hyperfine transition, later we will see that this detuning is fundamental for producing the MOT.

The path of the light produced by this laser is represented by the green line in Figure 4.5.

Trapping Laser 2

This laser generates one of the optical pumping frequencies, the imaging beam, the push beam and the trapping light for the MOT-2. The total power is ~ 550 mW. It is locked at the crossover between the transitions $5^2S_{1/2}(F = 2) \rightarrow 5^2P_{3/2}(F' = 1)$ and $5^2S_{1/2}(F = 2) \rightarrow 5^2P_{3/2}(F' = 3)$. We denote this crossover as *CO 1-3*. Due to the AOM in the SAS system, the

frequency is shifted 83.2 MHz to the blue from this crossover.

Initially, a small part of the beam is separated and sent to an AOM (at +138 MHz) to produce the second optical pumping frequency, resonant with the $5^2S_{1/2}(F=2) \rightarrow 5^2P_{3/2}(F'=2)$ transition. This light is then mixed with the one coming from the Repumper Laser and coupled into the PM optical fiber to be sent to the experiment.

The rest of the beam passes across an AOM (at +93.5 MHz) in double-pass configuration. This AOM is very important because it controls several processes: (i) it shifts to the red the frequency of the trapping light in the sub-Doppler cooling stage, during this period the frequency change is larger than 30 MHz; (ii) it pulses the imaging beam during the diagnosis stage and shifts its frequency from 93.5 MHz to 105 MHz for making the light resonant with the $5^2S_{1/2}(F=2) \rightarrow 5^2P_{3/2}(F'=2)$ transition; (iii) finally, it operates as a general switch to block all the light going to the experiment when it is necessary.

After the double-pass, the beam is divided again in two beams, one of them will be used as the trapping light of the MOT-2, the other one will be used to generate the imaging and push beams. The trapping beam mixes with the repumper light and passes through an AOM (at -78.7 MHz), obtaining a frequency shifted to the red by ~ 20 MHz from the $5^2S_{1/2}(F=2) \rightarrow 5^2P_{3/2}(F'=3)$ transition. The repumper and trapping beams arrive to the MOT-2 region through a PM optical fiber. Finally, the second beam passes through an AOM (at -82 MHz) and then it is divided in the imaging and the push beams, each reaches the experiment region through PM optical fibers. The push beam is red-shifted by ~ 22 MHz from the $5^2S_{1/2}(F=2) \rightarrow 5^2P_{3/2}(F'=3)$ transition, while the imaging beam, when pulsed, will be taken to resonance with this transition using the double-pass AOM.

The path of the light produced by this laser is represented by the red line in Figure 4.5.

We have in total five PM optical fibers which take the light to the experiment. In front of each one we have a mechanical shutter that can block completely the light that couples into the fiber when it is necessary. The polarization of the light that reaches the experiment needs to be very stable. In a regular optical fiber external perturbations such as temperature changes or

Table 4.1 – Frequencies and powers of the beams outgoing from the fibers

Optical Fiber	Frequency	Power
MOT-1	Trapping: -25 MHz from $(F = 2) \rightarrow (F' = 3)$ Repumper: Resonant with $(F = 1) \rightarrow (F' = 2)$	85 mW 13 mW
MOT-2	Trapping: -20 MHz from $(F = 2) \rightarrow (F' = 3)$ Repumper: Resonant with $(F = 1) \rightarrow (F' = 2)$	100 mW 15 mW
Optical Pumping	Resonant with $(F = 1) \rightarrow (F' = 2)$ Resonant with $(F = 2) \rightarrow (F' = 2)$	$300 \mu\text{W}$ $300 \mu\text{W}$
Push	-22 MHz from $(F = 2) \rightarrow (F' = 3)$	3 mW
Imaging	Resonant with $(F = 2) \rightarrow (F' = 3)$	$700 \mu\text{W}$

mechanical tensions along the fiber can produce variations on the polarization of the outgoing light. For this reason all our fibers are polarization maintainers. In this case the light must be coupled with the correct linear polarization, hence we have a half-wave plate before every fiber that allows us to control the polarization of the input light. Finally, for warranting a maximum stability of the fiber we thermally isolate them and avoid mechanical torsions across their way to the region of interest. To summarize, Table 4.1 shows the frequency and power of the beam coming out of every fiber.

4.4 Magneto-optical trapping

Magneto-optical trapping is a technique that combines inhomogeneous magnetic fields and radiation pressure to cool and confine a sample of atoms. It is actually the first stage of the experiment and allows us to collect a sample of atoms at low temperatures, of the order of $100 \mu\text{K}$, which subsequently can be transferred into a harmonic trap and be condensed.

The principle behind it can be seen in References (6, 87–89); we also provide a detailed description in Appendix A. A brief explanation is as follows. Absorption of light by an atom causes a momentum transfer along the photon direction. This can be used to change the kinetic energy of the absorbing atom. Therefore, radiation pressure can be used to decrease the temperature of a sample of atoms. The way of doing that is to align three pairs of counter-propagating beams along three perpendicular directions. The intersection of these beams becomes a region in which an atom decreases its velocity by scattering photons, no matter which was its initial direction. For this reason, such a region is known as *Optical Molasses* because it acts as a viscous medium in which the atoms slow down, and consequently the temperature of the sample decreases. This process is known as *laser cooling* and it is very useful.

Nevertheless, an optical molasses does not collect the atoms in a determined region, it simply slows down the atoms that pass through it. To confine a large number of atoms it is possible to use an inhomogeneous magnetic field in which the splitting of the Zeeman levels of the atoms is spatially dependent. Therefore, the scattering of photons, which certainly depends on the internal structure of the atom, will also depend on the position of the atom. By applying a linear magnetic field in the optical molasses region it is possible to produce a net force that always points toward the center of the trap (*i. e.* toward the zero point of the magnetic field). In consequence, we can trap a large number of very cold atoms: this is a magneto-optical trap (MOT).

We have chosen the $5^2S_{1/2}(F = 2) \leftrightarrow 5^2P_{3/2}(F' = 3)$ hyperfine transition to apply the laser cooling technique, with a red detuning of ~ 20 MHz. Due to selection rules, this transition is ideal because the state $F' = 3$ only decays to the hyperfine ground state $F = 2$, making this transition very stable. Nonetheless, due to non-resonant scattering of photons, the hyperfine ground state $F = 1$ can also be populated. Atoms in $F = 1$ escape from the cooling cycle and consequently are lost from the MOT. To avoid this “escape” of atoms, we also have a repumper frequency resonant with the transition $(F = 1) \rightarrow (F' = 2)$.

The linear magnetic field is produced by a pair of coils in anti-Helmholtz configuration¹ that, during the MOT stage, generates a magnetic gradient of about 20 G/cm. Later, these coils will also produce the field of the magnetic trap with a much higher gradient.

As mentioned before, in our system we have two MOTs. The MOT-2 is produced with the atoms pushed from the MOT-1. The MOT-1 is produced by three retroreflected beams, and all its parameters (such as alignment and gradient field) are adjusted to maximize the transfer of atoms to the second cell.

The MOT-2 must be much more carefully prepared. We use six independent beams whose alignment, power and polarization can be separately adjusted. Additionally, the MOT-2 has three pairs of coils in Helmholtz configuration whose axes are mutually perpendicular. Since the magnetic field of each pair of coils is homogeneous, they are used to compensate spurious fields in the MOT region.

To know if our MOTs and the transference among them are properly optimized, there are some diagnostics that can be used. The most simple diagnostic is to measure the fluorescence emitted by the cloud. This measurement allows us to know the number of atoms, the transfer rate between the MOTs and also to estimate the quality of the vacuum inside the system. Figures 4.6(a) and (b) are, respectively, pictures of the MOT-1 and MOT-2. What we are actually seeing is their fluorescence.

To measure the fluorescence of the MOT we simply place a lens of focal length f at certain distance d from the MOT (obviously, $d > f$). The lens collects a fraction of the light emitted by the atoms and focuses it on a photodiode. The photodiode generates a voltage proportional to the power of the detected light. Figure 4.6(c) shows our scheme for measuring the fluorescence of the MOTs.

The voltage V produced by the photodiode is linearly proportional to the power of the light P emitted by the atoms that reaches the photodiode: $P = A \cdot V$, where A is the proportionality

¹ The anti-Helmholtz configuration consists of two identical coils placed along a common axis, separated by a distance equal to the radius of the coils. The electrical current in each coil is the same, but it circulates along opposite directions.

Figure 4.6 – Pictures of the (a) MOT-1 and (b) MOT-2, the red circles indicate the position of the MOTs. (c) Scheme to measure the fluorescence of the MOT.

factor and has units of Watts/Volts. At the same time, P is proportional to: (1) the number of atoms of the cloud N ; (2) the energy of the emitted photons $\varepsilon = hc/\lambda$ (where $\lambda = 780$ nm is the wavelength of the photons); (3) the solid angle subtended by the lens $\Omega = r^2/4d^2$ (where r is the radius of the lens and d its distance to the atoms), and inversely proportional to the mean lifetime $\tau_{Rb} = 26.2$ ns of the transition. Finally, we must consider that the light of the MOT passes through some glass surfaces that, in the case of $\lambda = 780$ nm, absorb about 4% of the light. This can be modeled by a factor of the form $(0.96)^\alpha$, where α is the number of surfaces that the light crosses. Taking into account all these considerations, we can obtain an expression for the measured power of the light emitted by the atoms; from it we can obtain the number of atoms in the sample:

$$P = A \cdot V = N \frac{hc}{\lambda} \frac{(0.96)^\alpha}{2\tau_{Rb}} \frac{r^2}{4d^2} \implies N = \frac{8\lambda\tau_{Rb}d^2A}{hcr^2(0.96)^\alpha} V. \quad (4.2)$$

In our case we have $N_1 \approx 3 \times 10^8$ atoms in the MOT-1 and $N_2 \approx 5 \times 10^8$ atoms in the MOT-2. These numbers are a good starting point for obtaining a reasonably large BEC. The fluorescence signal can also be used to estimate the flux of atoms between the MOTs, to do this we simply measure the loading of atoms in the MOT-2 as a function of time. This process is an exponential growth of the type $S_I(t) = S_0[1 - \exp(-t/\tau_I)]$, and the loading time can be estimated by the time constant τ_I . The mean flux of the atoms between the MOTs can be defined as $\phi = N_2/\tau_I$, where N_2 is the total number of atoms loaded in the MOT-2. Switching off the push beam will cause the flux of atoms between the MOTs to stop and therefore the number of

Figure 4.7 – Loading and decay of the MOT-2 (black line). The red curve is an exponential fitting for the loading process and the blue curve for the decay process.

atoms in the MOT-2 will start to decay due to collisions with the background vapor; therefore, observing this decay can provide information about the quality of the vacuum inside the glass cell. This decay is also exponential, having the form $S_d(t) = S_0 \exp(-t/\tau_d)$, in this case we interpret the time constant τ_d as the lifetime of the MOT inside the vacuum. Empirically, it is known that the lifetime of the atoms in the magnetic trap is $\tau_m \approx 2\tau_d$ and the pressure inside the cell is approximately $P [\text{torr}] \sim 3 \times 10^{-10} / \tau_m [\text{s}]$. Figure 4.7 shows the fluorescence signal during the loading of the MOT-2 and, once the MOT is completely loaded, it shows the decay after the push beam is switched off. From our fittings, we obtain a loading time of $\tau_l = 16.5$ s and a flux of atoms of $\phi = N_2/\tau_l = 3.0 \times 10^7$ atoms/s. The decay time is $\tau_d = 30.6$ s, giving an approximate pressure of $P = 4.9 \times 10^{-12}$ torr.

There is an additional diagnostic to know if there are spurious magnetic fields in the MOT region or to know if the MOT beams are properly balance. The idea is to switch off the MOT's magnetic field, slightly detune to the red the trapping light and, in this condition, to observe the expansion of the cloud in the light field (*i. e.* we will see the movement of the atoms in the optical molasses). In ideal conditions, the cloud will expand isotropically in the molasses region. In presence of spurious fields or trapping light imbalance the cloud will move rapidly away from the trapping region. This observation is very simple and it can be done using an infra-red sensitive camera connected to a TV.

Once we have produced a MOT-2 with a large number of atoms the next step is to transfer the atoms to a purely magnetic trap. The properties of the cloud in the MOT in terms of temperature, geometry and internal state of the atoms, are very different from that of a cloud trapped in a pure magnetic potential. We cannot simply switch off the MOT and turn on the magnetic trap (MT) because we would lose most of the atoms. We need a series of processes that successfully lead to an efficient transference of the atoms from the MOT to the MT. In Section 4.6 we describe all these processes. However, the diagnostics techniques described above are no longer applicable at this stage, and more sophisticated imaging techniques are required. In the next section we describe the absorption imaging technique.

4.5 Imaging System

The most useful method to study our sample is the imaging by optical absorption. It consists in illuminating the sample with a collimated laser beam resonant with one of the electronic transitions of the atoms. We use the $5^2S_{1/2}(F = 2) \leftrightarrow 5^2P_{3/2}(F' = 3)$ transition. The cloud absorbs some of the photons of the beam and immediately scatters them, leaving a dark “shadow” in the beam. Afterwards, the beam passes through a lens system that forms an image of the shadow. This shadow corresponds to the absorption profile of the gas which is proportional to the density profile. Therefore, this technique allows us to count the number of atoms in the sample and to measure the temperature, the dimensions and the geometry of the cloud.

To analyze the image we use the Beer–Lambert law, that states that the intensity $I(x, y)$ of a beam that propagates along the z direction through an absorptive medium of density $n(x, y, z)$ is given by

$$I(x, y) = I_0(x, y) \exp \left(-\sigma \int n(x, y, z) dz \right), \quad (4.3)$$

in this case $n(x, y, z)$ is the density profile of the gas, σ is the absorption cross-section of the photons and $I_0(x, y)$ is the initial intensity of the beam. From Equation (4.3) we obtain

$$\rho(x, y) \equiv \int n(x, y, z) dz = -\frac{1}{\sigma} \ln \frac{I(x, y)}{I_0(x, y)}. \quad (4.4)$$

Equation (4.4) is telling us that by comparing the absorbed beam with the original beam we can obtain the density profile integrated along the beam propagation direction, $\rho(x, y)$; this is known as *normalized* absorption imaging. Since the trapped gases usually have a very well-defined symmetry we actually can obtain most of the physical information of the cloud from this measurement.

The imaging sequence is as follows: we first obtain an image of the absorbed beam by the atoms, *i. e.* we obtain $I(x, y)$; next we obtain an image of the beam with no atoms, $I_0(x, y)$, and finally we obtain an image without any light to account for the intrinsic noise of the camera and also for the spurious ambient light, it is known as “dark” or “bias” image, $I_d(x, y)$. Hence, the 2D density profile is given by

$$\rho(x, y) = -\frac{1}{\sigma} \ln \left(\frac{I(x, y) - I_d(x, y)}{I_0(x, y) - I_d(x, y)} \right) = \int n(x, y, z) dz. \quad (4.5)$$

From Equation (4.5) we can obtain many physical parameters of the cloud. The total number of atoms in the sample is simply the integrated 2D profile,

$$N_T = \int \rho(x, y) dx dy. \quad (4.6)$$

The quantity $OD \equiv \sigma \int n(x, y, z) dz$, known as *Optical Density*, is a very important measurement. In fact, for the evaporative cooling to be successful the OD must increase during the process and, actually, it is the quantity that we optimize during the evaporation. We will discuss this point in Section 4.7.4.

Analyzing the profile $\rho(x, y)$ also provides the dimensions and the temperature of the cloud. The density profile of a thermal cloud can be properly approximated by a gaussian distribution. Thus the size of the cloud can be defined as the width ω of this distribution. Next, we release the cloud from the trap and image it after different expansion times. The velocity of the expansion is related to the temperature of the sample through the expression

$$\frac{3}{2}k_B T = \frac{1}{2}mv^2, \quad (4.7)$$

where m is the mass of the atoms and v is the expansion velocity. The imaging of the cloud will provide the profile $\rho(x, y)$ for each expansion time t_{exp} . By fitting a gaussian distribution to this profile, we can obtain the width as a function of the expansion time $\omega = \omega(t_{exp})$. Therefore the velocity of the expansion will be given by

$$v = \frac{d\omega(t_{exp})}{dt_{exp}}. \quad (4.8)$$

From Equation 4.7 we see that the temperature of the cloud is given by

$$T = \frac{m}{3k_B} \left(\frac{d\omega(t_{exp})}{dt_{exp}} \right)^2. \quad (4.9)$$

Since the cloud expands freely, the expansion velocity is constant. Therefore, at a certain expansion time t_{exp} , the velocity of expansion is $v = (\omega - \omega_0) / t_{exp}$, where ω_0 is the initial width of the cloud. For a long enough expansion time (which is of the order of 10 ms) we can assume that $\omega \gg \omega_0$. Then we can extract the temperature of the cloud with a single image through the expression

$$T = \frac{m}{3k_B} \left(\frac{\omega}{t_{exp}} \right)^2. \quad (4.10)$$

To produce the image in the experiment, we will use a standard telescope. A lens with focal length f_1 , placed at a distance equal to f_1 from the sample, collects the absorption beam. A second lens with focal length f_2 forms the image in a camera placed at a distance equal to f_2 from this second lens. The magnification of the system will be $M = f_2/f_1$. If R is the radius of the first lens, the maximum numerical aperture of the imaging system is given by $NA = R/(R^2 + f_1^2)^{1/2}$. Therefore, the maximum optical resolution (also known as *resolving power*) of the imaging system, is given by $OR = 0.61\lambda/NA$, where λ is the wavelength of the imaging beam. In other words, the minimum separation between two spots that the imaging system can resolve is given by OR (86).

We have mounted two imaging system along two perpendicular directions. This arrangement gives us the possibility of studying the sample along several directions, and is particularly

Figure 4.8 – Scheme of the two imaging axes.

useful if we are interested in producing topological excitations in a BEC because some features can only be seen along a certain direction. A scheme of these imaging systems can be seen in Figure 4.8. One of the absorption beams is mixed with the MOT beam that goes along the magnetic quadrupole axis. For this imaging axis the telescope lenses have focal lengths of $f_1 = 18.5$ cm and $f_2 = 40$ cm, and therefore the magnification is $M = 2.16$. The first lens has a diameter of 5.1 cm, therefore the maximum optical resolution is $OR = 4.48\lambda = 3.5 \mu\text{m}$. The second imaging beam is perpendicular to the first one, along this direction we have good optical access, allowing to put a collecting lens very close to the atoms. In this case, $f_1 = 5$ cm with a diameter of 5 cm, $f_2 = 25$ cm and $M = 5$, which allows us to produce a high resolution image with $OR = 2.13\lambda = 1.66 \mu\text{m}$.

Figure 4.9 – Image processing to obtain the normalized absorption image of the atoms.

The images are produced in a CCD camera (Charged–Coupled Device) which is very sensitive, produces low noise and digitizes the images to be processed. Both images are formed in the same CCD camera, with the possibility of choosing any of the axes by placing or removing a single mirror mounted in a magnetic holder that always fits in the same position. Our camera is a CCD *pixelfly* from *pco.imaging*[®], model 270XS, which is very compact. The chip is composed by an arrangement of 1024×1024 pixels with dimensions of $6.45 \times 6.45 \mu\text{m}^2$ each.

For acquiring and processing the images produced in the CCD we use a program that has

Figure 4.10 – Main window of the image acquisition program

been developed in our group. This program was written in the programming environment LabVIEW, that we will discuss in more detail in Section 4.8.1. The program acquires the three pictures mentioned above and produces the normalized absorption imaging through the process illustrated in Figure 4.9. A screen shot of the user interface of this program is shown in Figure 4.10, showing the normalized absorption imaging of a typical cold cloud. The program is also able to analyze the resulting image and obtain the physical information from it. To do this we have an extension written in Python programming language. This extension is a contribution from collaborators from the European Laboratory for Non-linear Spectroscopy (LENS) from the University of Florence.

4.6 Transference from the MOT to the Magnetic Trap

Once we have collected the atoms in the MOT we must transfer them into a conservative potential where evaporative cooling can be applied. The first step is to transfer the atoms to a

pure magnetic quadrupole trap (MT). The magnetic quadrupole is not a conservative potential, however, it has a big capture volume. Once the atoms are trapped in the quadrupole they can be efficiently transferred into a conservative harmonic potential. Another advantage of the quadrupole is that it is produced with the same coils used to produce the magnetic field for the MOT, where the only difference is the magnitude of the magnetic gradients involved in each kind of trap.

In the route to the BEC, the transference from the MOT to the magnetic trap (MT) is certainly the most critical stage of the whole experiment. There are four reasons for this process to be so delicate, we list them in the following.

1. **Position.** In a MT the position of the sample matches the potential minimum position. Nevertheless, in a MOT even a small imbalance in the intensity of the beams has as a consequence that the cloud position is not the same as the minimum of the quadrupole position. This mismatch can cause unnecessary heating of the atoms and even compromise the whole transference process.
2. **Geometry.** Since the gradient of the magnetic field in the MT is about ten times higher than that of the MOT, the capture volumes and the trapping geometries can be very different in both traps. While a cloud in a MOT has a typical radius of 2.5 mm, a magnetically trapped gas does not exceed 0.5 mm of radius. Also, due to the light forces involved in the MOT formation, the shape of the cloud can be very irregular, while the cloud in a MT is approximately an ellipsoid whose aspect ratio depends on the magnetic gradient along the radial and the axial directions.
3. **Temperature.** Because we increase the magnetic gradient when the MT is switched on, the temperature of the sample noticeably increases. This fact can induce atom losses and even compromise the efficiency of the evaporative cooling process.
4. **Internal state.** Our magneto-optical trap is able to confine atoms in all the Zeeman levels of the hyperfine state $5^2S_{1/2}(F = 2)$. In contrast, as explained in Appendix A, the magnetic trap can only contain the states $|F = 2, m_F = 2\rangle$ and $|F = 2, m_F = 1\rangle$.

Therefore, before transferring the atoms to the MT we first need to perform a good spatial *mode-match*, in which the MOT cloud becomes as similar as possible to the magnetically trapped cloud. To do so, we first need to precisely control the MOT position. We take advantage of the possibility of imaging along two perpendicular directions. Taking absorption images of the atoms in the MOT and in the MT, we can know their relative positions. Smoothly changing the balance of the intensity of the MOT beams, we can vary the MOT position until it matches the MT position.

Next it is necessary to submit the sample to a compression and cooling processes in order to decrease both the size and the temperature of the gas. We call these processes, respectively, *MOT compression* or *C-MOT* and *sub-Doppler cooling* or *Polarization gradient cooling*. Finally, all the atoms must be pumped into a single Zeeman state by means of the *optical pumping* process. In the following we describe all these procedures.

4.6.1 MOT compression

We start with a MOT with a temperature of approximately $180\ \mu\text{K}$. The MOT compression process has the goal of decreasing the size of the cloud. During this stage of 1 ms, we shift the trapping light to the red, going from the initial detuning of 20 MHz to ~ 40 MHz. At the same time, the power of the trapping light falls to 65% of its original value. Consequently, the scattering of photons decreases and the atoms accumulate in the center of the trap. Additionally, it is possible to change the gradient of the magnetic quadrupole, we have observed that decreasing this gradient from 20 G/cm to 10 G/cm improves the matching with the MT. At the end of this stage we have a denser cloud with a lower temperature of $\sim 140\ \mu\text{K}$.

4.6.2 Sub-Doppler cooling

In the next step we completely turn off the MOT's magnetic field and allow the cloud to expand in a red detuned light field during 6 ms. In this case the detuning is of almost 60 MHz from the frequency of the MOT (that is, ~ 80 MHz from the resonance). This is the maximum

value that our AOM allows us to shift. The intensity of the trapping light is kept at 65% of its original value. The effect of this process is to cool down the atomic sample below the Doppler limit, and this is why it is also known as “sub-Doppler cooling technique”.

The reason for this substantial cooling of the cloud is the following: first, this larger detuning implies that the fastest atoms of the already cooled sample interact with the trapping beams (due to the Doppler effect) slowing them down. This would decrease the temperature of the cloud up to the Doppler limit. This limit is reached when the cooling rate due to the absorption of photons is equal to the heating rate due to spontaneous emission. Nevertheless, there is a very interesting additional effect due to the polarization of the beams. We have three pairs of counter-propagating beams; each beam of the pair has orthogonal circular polarization. Due to the interference of the beams of the pair, the atoms see a standing wave potential where the polarization is spatially dependent. Recall that the atom-light interaction is very sensitive to the polarization of the light. Then, when an atom moves through this standing wave it can go up of a “hill”, losing a part of its kinetic energy. Once the atom reaches the top of the hill, instead of “rolling down” it is optically pumped to the bottom of the hill without gaining any kinetic energy back, thus it decreases its velocity. This optical pump occurs due to the spatially dependent polarization gradient of the light beams. After many cycles of this process, the cloud cools down well beyond the Doppler limit. Using this technique we have achieved temperatures as low as $30\text{ }\mu\text{K}$. The sub-Doppler cooling technique is also known as *polarization gradient cooling* or *Sisyphus cooling* (6).

4.6.3 Optical pumping

As discussed before, in the MOT stage the atoms are distributed in all Zeeman levels of the state $5^2S_{1/2}(F=2)$, and also there is a small probability of finding them in any of the Zeeman levels of the state $5^2S_{1/2}(F=1)$.

However, only the states $|F=2, m_F=2\rangle$, $|F=2, m_F=1\rangle$ and $|F=1, m_F=-1\rangle$ are magnetically trappable. The optical pumping procedure is used to pump all the atoms of the sample

Figure 4.11 – Scheme of the optical pumping beams. OP $2 \rightarrow 2'$ represents the $(F = 2) \rightarrow (F' = 2)$ transition while OP $1 \rightarrow 2'$ denotes the $(F = 1) \rightarrow (F' = 2)$ transition.

to the $|F = 2, m_F = 2\rangle$ state. It is performed in two stages, one named “Hyperfine Pumping” and a second one called “Spin Polarization”.

In the **hyperfine pumping** stage we completely switch off the trapping light and 1.3 ms later we switch of the repumper light. Consequently, all the atoms are transferred to the state $5^2S_{1/2}(F = 2)$. At the same time we apply a weak homogeneous magnetic field, of the order of 1 Gauss, using an extra pair of coils in Helmholtz configuration. This field serves to split the Zeeman levels of the $5^2S_{1/2}(F = 2)$ manifold and is switched off at the end of the Spin Polarization process.

Once that the Zeeman levels are split we **spin-polarize** the sample using an optical pumping pulse. This pulse is right-circularly polarized in such a way that only the transitions of the type $|F, m_F\rangle \rightarrow |F', m_F + 1\rangle$ are promoted. As already described in Section 4.3, the optical pumping pulse contains the frequencies $(F = 1) \rightarrow (F' = 2)$ and $(F = 2) \rightarrow (F' = 2)$ as shown in Figure 4.11. Each frequency has a power of $300\mu\text{W}$, and the pulse has a duration of $500\mu\text{s}$. After absorbing some photons of the pulse, the atoms, initially in the state $|F, m_F\rangle$, will be transferred to the state $|F' = 2, m_F + 1\rangle$ (that is, due to the right-circular polarization, the

Figure 4.12 – Scheme of the optical pumping process. Initially, the atoms are distributed in all Zeeman levels of the ground state. After some optical pumping cycles the atoms are completely transferred to the $|2, 2\rangle$ state.

selection rule is $\Delta m_F = +1$). The atoms can reemit any kind of photons, then the selection rule will be $\Delta m_F = 0, \pm 1$, which, in average, can be considered $\Delta m_F = 0$. Figure 4.12 illustrates this process, in this sketch initially we consider the situation in which all Zeeman levels of the states $F = 1$ and $F = 2$ are populated. When the atoms absorb the light they are optically pumped to $|F' = 2, m_F + 1\rangle$. When the atoms reemit, in average, they conserve their m_F number. Therefore, after few absorption and reemission cycles, the atoms are forced to populate the state $|2, 2\rangle$.

The efficiency of optical pumping procedure is very high, having more than 95% of the atoms of the MOT in the $|2, 2\rangle$ state.

4.7 Hybrid Trapping and evaporative cooling

As already mentioned, the trap in which the BEC will be produced is a combination of a magnetic quadrupole, and single-beam optical dipole trap. The superposition of these two potentials generates a total harmonic potential where the condensate is produced. This configuration is known as Hybrid Trap, it was implemented by the first time in Ian B. Spielman's group at NIST and constitutes a very versatile potential that can be easily manipulated (90).

The idea is to use a quadrupole magnetic field superimposed with an optical dipole trap whose minimum is slightly dislocated from the quadrupole's zero-point. In a optical dipole trap we have strong confinement along the radial direction but very weak confinement along the axial direction, generating very long quasi 1D samples. The addition of the magnetic field slightly modifies the field along the radial direction but provides good confinement along the axial one. The resulting cloud is elongated but still three-dimensional.

At the start of the hybrid trapping, when the atoms have not been evaporated yet and, hence, the sample is still hot, the magnetic component completely dominates the trapping dynamics. For this reason, the previous stages were optimized to efficiently transfer the atoms to the magnetic trap. In the following Sections we will describe both, the magnetic quadrupole and the optical dipole trap.

Figure 4.13 – (a) Sketch of the quadrupole coil showing their relative position with the glass cell. (b) Absolute value of the magnetic field produced by the quadrupole coil during the magnetic trapping stage.

4.7.1 Magnetic trap

After the mode-matching processes described in Section 4.6 we obtain a denser and colder cloud ($T \approx 30 \mu\text{K}$) with approximately 5×10^8 atoms in the $|2, 2\rangle$ magnetically trappable state. The atoms are ready to be transferred into a magnetic trap. To do so, we abruptly turn on the magnetic quadrupole at certain axial gradient, called “catching gradient”. Immediately after that, we ramp adiabatically the magnetic field to its final trapping value, generating the quadrupole magnetic trap. At this point no resonant light at all should be present, therefore we switch off the AOMs and block the entrance of all the optical fibers using fast mechanical shutters.

The catching gradient is chosen to maximize the transference of atoms from the MOT to the MT. In our experiment we found that a value of 95 G/cm is optimum. At the end of the ramp the final value of the axial gradient of the MT is 160 G/cm. This value needs to be high enough to increase the density of the cloud for the evaporative cooling to be efficient. By increasing the gradient we also compress the cloud, in consequence the gas heats up. To minimize the heating, the ramp of the magnetic field has to be slow enough. In our case, our ramp has a duration of 150 ms. At this point, the temperature of the sample is of $300 \mu\text{K}$.

The principles of magnetic trapping are explained in Reference (91). We have presented a short review in Appendix A. To produce the magnetic quadrupole trap we use the MOT coils but with much higher currents. The coils are rolled with 2 mm isolated copper wire. Each coil has 99 turns, nine along the radial direction and eleven along the axial one. The separation between the coils is 36 mm. The maximum gradient used in the experiment (160 G/cm) is generated by a current of 20 A. Each coil is rolled around an aluminum reel with 24 mm of internal diameter. The reel is water cooled and it has a longitudinal groove that avoids the formation of eddy currents on it. Figure 4.13(a) is a sketch of our trapping coils system and Figure 4.13(b) shows the absolute value of the magnetic field along the axial direction of the quadrupole during the magnetic trapping stage, we can see that close to the minimum the field is linear. To produce the electric current we use a power supply from **DELTAELECTRONIKA**[®], model SM45–70 D, which is a very stable low-noise power supply. It can be used as a current supply, with a range of 0 to 45 A, or like a voltage supply with a range of 0 to 70 V, having a total power of 3 150 W. In our case we use it as a current supply and the output current can be very precisely controlled with an external analog signal. To turn off the current we use a MOSFET (acronym of metal-oxide-semiconductor field-effect transistor) in series with the quadrupole coils, the MOSFET is switched with an external digital signal.

Once the atoms are trapped in the magnetic quadrupole it is very important to measure the lifetime of the sample to know if the pressure is low enough to continue with the subsequent stages of the experiment. To do this we hold the atoms in the MT and measure the number of atoms as a function of the trapping time. The decay of the number of atoms is an exponential process and the lifetime of the trap corresponds to the time constant of this decay. In ideal conditions, the atom losses are caused exclusively by the collisions between the trapped sample with the background vapor. Therefore, measuring the trap lifetime is also an useful indication to know if there are undesired effects, such as spurious fields or resonant light leaks. Figure 4.14 shows the measurement of number of atoms versus time of trapping, we have more than one minute of lifetime, which is a very satisfactory result. This indicates us that the pressure is low enough and that there are not additional loss mechanisms.

Figure 4.14 – Measurement of the number of atoms as a function of the trapping time (black circles). The red curve is an exponential fitting with a decay constant of about 63 s.

4.7.2 rf-Evaporative cooling

Evaporative cooling is the only known technique that allows cooling a trapped sample below the critical temperature of the quantum phase transition (92). This temperature is of the order of 10^2 nK, that is, a thousand times lower than the typical temperatures of the sample in the magnetic trap. This technique consists in selectively removing the most energetic atoms of the sample, initially at certain temperature. Next, the sample will thermalize at a lower temperature. Again, the most energetic atoms from this new and colder sample are selectively removed, producing an even cooler sample. The repetition of this process leads the sample to the sub-microKelvin regime necessary for the BEC to occur.

The thermalization of the sample occurs because the energy is distributed between the atoms of the sample through elastic collisions. Therefore, the time of thermalization depends on the collision rate of the sample. However, the collisions between the trapped atoms and the background vapor can heat up the sample. For these two reasons it is so important to have a long enough trap lifetime.

In our experiment we have two stages of evaporative cooling, each mediated by different physical effects. The first one occurs during the stage in which the magnetic component of the hybrid trap is dominant. The second stage is applied when both, the optical and the magnetic components are important, and will be described later.

For the first evaporation stage we use radio–frequency radiation (93) to remove the hottest atoms of the sample. The idea is to use the fact that in an inhomogeneous magnetic field the separation of the Zeeman levels of the atoms is spatially dependent. The intensity of the quadrupole field increases linearly from the center of the trap. The most energetic atoms will have a larger kinetic energy that allows them to reach regions in which the magnetic field is higher. This means that a “hot” atom will have a bigger Zeeman splitting than a “cold” one. With radio–frequency (rf) radiation we can induce transitions between the Zeeman levels, transferring an atom from a magnetically trapped state to non–trappable state, hence such an atom would be expelled from the trap. In consequence, a high value of rf will remove only the most energetic atoms from the trap, while the less energetic do not interact with the radiation. Ramping down the value of the rf subsequently removes atoms with lower temperatures. In Figure 4.15 the rf–evaporative cooling process is illustrated.

However, the quantum degeneracy cannot be achieved in a pure magnetic quadrupole because this potential contains a zero–field point. In this point the atom can suffer transitions to non–trappable states, and in this case the coldest atoms would be expelled from the trap. This loss process is known as Majorana transition (94) and constitutes the main disadvantage of quadrupole traps.

To produce the radio–frequency radiation we use a function generator, from *Stanford Research Systems*® model DS345, coupled with a small antenna. This antenna is a two–loop coil of 25 mm of diameter and is made with copper wire of 1 mm of diameter. The antenna is placed between one of the quadrupole coils and the glass cell, as close to the atoms as possible.

To guarantee a good coupling of the rf, the antenna is connected in series with a 50Ω resistor. To evaluate this coupling we use a directional coupler (from *MINI CIRCUITS*®, model

Figure 4.15 – Sketch of the rf-evaporative cooling process showing that the splitting of the Zeeman levels of the atoms decreases as atoms approach to the center of the magnetic trapping potential.

ZFDC-20-5+) to measure the radiation that is reflected by the antenna, the lower the power of the reflected radiation, the better the antenna coupling. The graph of Figure 4.16(a) shows a measurement of the reflected power as a function of the frequency of our antenna with and without the $50\,\Omega$ resistor. Also, for comparison we show the reflected power for the $50\,\Omega$ resistor alone, which has the maximum expected coupling, and for the case in which there is nothing coupled to the function generator, where we would expect a maximum in the reflexion. We can see that the antenna plus the resistor couple the rf much better than simply the antenna. We have measured, as well, the reflected power for antennas with different geometries (changing shape, size and number of loops), a picture of the best design is shown in Figure 4.16(b).

In our experiment, during the magnetic trapping we apply two rf linear ramps, each one with a duration of 3 s. The first ramp from 20 MHz to 9 MHz and the second one from 9 MHz to 3.5 MHz. Evaporating beyond this value starts to induce Majorana losses, which means that atoms are removed from the sample without any further cooling. During the rf-evaporation, the temperature decreases from $300\,\mu\text{K}$ to $30\,\mu\text{K}$ and the number of atoms falls from 5×10^8 to

Figure 4.16 – (a) Graph of the power of the reflected power as a function of the frequency for different situations. (b) Picture of the antenna with the best rf coupling.

4×10^6 . Figure 4.17 shows a series of absorption imaging of the atomic cloud for different final values of the evaporation ramp after 9 ms of time-of-flight. Note that below 3.5 MHz the decrease of the temperature is very small, even increasing when the final frequency of evaporation is 2 MHz. This inefficient cooling is a consequence of the Majorana transitions.

4.7.3 Transference to the hybrid trap

The physics of the optical dipole traps (ODT) and of the hybrid trapping is explained in References (90, 95); we also give details in Appendix A. Here we just provide the experimental description.

After the first rf-ramp, the atoms of the MT are ready to be loaded into an single-beam optical dipole trap. To avoid Majorana losses, the minimum of the ODT must be displaced from the zero-field point of the magnetic quadrupole. However, if the separation of the minima is too large the transference of atoms will be inefficient. The ideal offset between the center of the traps is approximately a beam waist, which in our case we calculate to be of 70 μm .

Figure 4.17 – Series of absorption images of the atomic cloud for different final values of the rf-evaporation ramp. After 9 ms of free expansion time. The corresponding rf-frequency, temperature and number of atoms is indicated below each image.

Figure 4.18 – (a) Side and (b) top view of the magnetic quadrupole, the optical trap and the glass cell. The black cross indicates the position of the minimum of the magnetic trap. The dimensions have been exaggerated for the sake of clarity.

Figure 4.19 – Optical setup of the optical dipole trap.

Figures 4.18(a) and (b) show, respectively, a side and a top view of the hybrid trap.

Figure 4.19 presents the optical setup of the optical trap. To produce the light we use an Ytterbium fiber laser from **IPG PHOTONICS®** at 1064 nm. This laser produces up to 20 W of single-frequency linearly-polarized light. It is very important that the intensity of the light be constant, otherwise the atoms inside the trap could be heated. To determine the stability of the laser we have measured the power of a small fraction of the beam using a photodiode. Next, we analyze the Fourier transform of the signal of the photodiode. Any oscillation on the amplitude of this signal will be translated as a peak in the Fourier transform, indicating the presence of noise at certain frequency. Fortunately, the internal stabilization mechanism of the laser is good enough for our purposes and no significant noise was found.

To control the intensity of the light that reaches the experiment the laser beam initially passes through a 110 MHz acousto-optic modulator and we keep the first diffracted order. As explained in Section 4.3, the AOMs are a very useful tool to control the intensity of the light. Next, the beam is expanded and collimated to an approximate diameter of 7.5 mm, and finally focused on the trapping region using a lens with a focal length of $f = 75$ cm. The measured

Figure 4.20 – Calculated hybrid potential for our experiment along (a) coils axis direction, (b) gravity direction and (c) ODT direction.

beam waist is $70 \mu\text{m}$ and the offset between the MT and ODT minima is of $\sim 90 \mu\text{m}$. This focusing lens is mounted in a xyz -translator which allows a very precise control on the position of the ODT's minimum. The ODT propagates parallel to one of our imaging beams. To do so, we use a dichroic plate that transmits the 780 nm wavelength of the imaging laser and reflects the 1064 nm wavelength of the optical trap.

In our experiment, the ODT is switched on together with the magnetic trap through a linear ramp of 150 ms . The power of the laser beam on the atoms is of $\sim 5.6 \text{ W}$, whose depth is $U_0/k_B = 94 \mu\text{K}$. Evidently, during the initial stages of the magnetic trapping, the ODT has no effect on the hot atoms, however, at the end of the second rf-ramp the ODT increases the density of the cloud making the rf-evaporation more efficient. When the second rf-ramp finishes the intensity of the magnetic field is ramped down through two linear ramps, one from an axial gradient of 160 G/cm to 65 G/cm during 2 s , and a second one from 65 G/cm to 42 G/cm during 0.8 s . During the magnetic ramping we also apply an extra rf-ramp during 2 s going from 3.5 MHz to 2 MHz , and then we keep it constant at 2 MHz during 0.8 s . At this point the rf is switched off and the atoms have been completely transferred to the hybrid trap. During this magnetic decompression process, the ODT is at its maximum value of 5.6 W . At this point we have produced a sample with a temperature of $17 \mu\text{K}$ and a number of 2.5×10^6 atoms.

Figure 4.20 shows the final potential that the atoms feel in the hybrid trap before being optically evaporated. The measured frequencies of this potential are $\omega_y \simeq 2\pi \times (63 \pm 3) \text{ Hz}$

along the beam direction, $\omega_x \simeq 2\pi \times (424 \pm 6)$ Hz along the quadrupole axis direction and $\omega_z \simeq 2\pi \times (342 \pm 10)$ Hz along the gravity direction. Notice that the x and z -directions correspond to radial directions of the ODT and are expected to be equal, however, $\omega_x > \omega_z$. This difference is due two reasons: first, the gravity, which goes along z -direction, weakens the trap confinement along this direction. Second, the magnetic gradient along the x -direction is larger than along the z -direction, proving stronger confinement in the x -direction.

To measure the frequencies of the trap it is necessary to “kick” the cloud in order to produce an oscillation of its center of mass inside the trap. Next, we hold the cloud in the trap during a variable time. Then, we image the cloud and measure the position of its center of mass along the x , y and z -directions as a function of the holding time. The oscillation of the atoms along each direction is properly fitted by a sinusoidal curve of the type $i(t) = i_0 + A_i \sin(2\pi\nu_i t + \phi_i)$ with $i = x, y, z$. Here, $2\pi\nu_i = \omega_i$ corresponds to the frequency of the trap along the i -direction. The way in which we “kick” the condensate is by pulsing the gradient of the magnetic quadrupole. This pulse consists in abruptly decreasing the axial magnetic gradient from its final value of 42 G/cm to approximately 30 G/cm and then abruptly increase it to its original value of 42 G/cm. The duration of the pulse is about 1 ms.

The transference to the hybrid trap is a very critical stage. Just as in the case of the transference from the MOT to the MT, a good mode-match is necessary to properly transfer the atoms from the MT to the hybrid trap. There are two very important parameters that must be carefully optimized to guarantee the success of this stage. First, the waist of the ODT must be appropriate: a too large waist does not provide the proper confinement, causing an inefficient evaporative cooling. In contrast, a too tight waist can produce three-body losses and decrease the lifetime of the trap. Second, the relative position between the ODT and the magnetic quadrupole must be correct: if the separation is too small, Majorana losses can play an important role, if it is too big the transference from the MT to the hybrid trap is very inefficient. In our case, a beam waist of $\sim 70\mu\text{m}$ and a separation between the MT and the ODT of $90\mu\text{m}$ have worked satisfactorily.

Figure 4.21 – Typical in-situ images of the atoms in the pure magnetic trap, in the pure optical trap and in the hybrid trap along (a) the y -direction and (b) the x -direction.

To measure the separation between both traps we produce an image of the atoms in the pure magnetic trap and an image of the atoms in the pure optical trap. Then we simply measure the distance between the position of the centers of mass of both clouds. We can do this in a very precise way because, as explained in Section 4.5, we can do the imaging along two orthogonal directions. To set the relative position between the ODT and the MT we adjust the position of the optical trap. The lens that focuses the ODT beam on the atoms is mounted in xyz -translator which can be adjusted with micrometric precision. Figures 4.21(a) and (b) show in-situ pictures of the atoms in the pure MT, in the pure ODT and the hybrid trap. Figure 4.21(a) corresponds to images taken along the y -direction (ODT axis direction) while Figure 4.21(b) shows the images along the x -direction (quadrupole axis direction). In these figures we can notice the different relative positions between the traps, as well as the difference between their geometries.

To measure the waist of the optical trap beam we take advantage of two facts. First, as shown in Figure 4.19, the ODT is aligned parallel to one of the imaging beams, hence, it will be aligned with the imaging system and, consequently, we can perform an image of the beam. Second, the center of the ODT is on the atoms, since the atoms are on focus in the imaging system, the position of the center of the ODT is also on focus. Therefore, the image of the

ODT beam corresponds to the image of the focus of the beam. Consequently, we can extract the waist of the beam by simply fitting a gaussian curve on this image. The waist of the beam corresponds to the width of the gaussian curve.

4.7.4 Optical Evaporative cooling

An ODT is not spin selective, which means that we cannot use rf-evaporation anymore to selectively remove the most energetic atoms. The idea of evaporative cooling is, however, essentially the same. Again, the most energetic atoms are able to “climb” to the highest regions of the hybrid potential, therefore, to remove them we simply reduce the depth of the hybrid potential. To do this we ramp down both, the magnetic gradient and the power of the laser beam of the ODT. This is done with a collection of several linear ramps. The magnetic axial gradient is slightly reduced from 42 G/cm to 37.7 G/cm. The power of the optical trap, in opposition, is significantly decreased from the initial 5.6 W to a variable value of tens to few hundreds of milliwatts, which is equivalent to decrease the trap depth from $U_0/k_B = 94\mu\text{K}$ to few μK . The total process takes approximately 11.3 s and the evaporation ramps are described in detail in Section 4.8.

It is very important to evaluate the efficiency of the evaporative cooling process in order to know if it is optimal. The important quantity that we need to analyze is the elastic collision rate $\gamma_{el} = n\sigma_{el}\langle v \rangle$, where n is the density of the cloud at its center (peak density), σ_{el} is elastic collision cross section and $\langle v \rangle$ is the mean velocity of the atoms in the cloud. γ_{el} must be higher than the inelastic losses rate due to, for example, collisions with the background vapor. Thus, γ_{el} should not decrease as the evaporative cooling is applied. This condition warrants an efficient evaporative cooling and is named *run-away evaporation*. Note that as the temperature of the atoms decreases, $\langle v \rangle$ also decreases, therefore, in order to keep constant or even increase γ_{el} , the density n must increase as the process occurs. In a three-dimensional harmonic trap we know that $n \propto NT^{-3/2}$, and from Equation (4.7) we know that $\langle v \rangle \propto T^{1/2}$, therefore, $\gamma_{el} \propto N/T$. Consequently, in order to achieve run-away evaporation the temperature must decrease in a faster rate than the atom losses. This condition can be written as $N \propto T^s$, with $s \leq 1$.

Figure 4.22 – Number of atoms as a function of the temperature of the sample as the evaporative cooling process is applied.

Figure 4.22 shows a graph of $\log(N)$ vs $\log(T)$ at different stages of the evaporative cooling process, including both, rf and optical evaporation. This result shows that for the initial rf-evaporation stage $N \propto T^{0.61 \pm 0.03}$, which is already below the criterion above. During the optical evaporation the efficiency of the process improves even further, getting $N \propto T^{0.21 \pm 0.02}$. Finally, at the end of the optical evaporation the behavior changes again and becomes $N \propto T^{0.34 \pm 0.03}$. This means that the whole evaporation process satisfies the criterion above, indicating that our evaporative cooling stage is efficient.

Observation of Bose–Einstein condensation

At the end of the evaporative cooling stage we are able to reach the quantum degeneracy. For a final optical trap depth of $1.5 \mu\text{K}$ (corresponding to a power of 91 mW) and a final magnetic gradient of 37.7 G/cm, we produce a Bose–Einstein condensate with a temperature of ~ 210 nK, a total number of atoms of 1.2×10^5 atoms and a condensed fraction of 15%. Eva-

porating even further, up to a trap depth of $0.5\mu\text{K}$, we produce an almost pure² BEC with 3.5×10^4 atoms and a temperature below 50 nK.

We have measured the frequencies of the hybrid trap for a cloud slightly above the transition temperature. This measurement has been done using the procedure described in Section 4.7.3. We found that the frequency along the magnetic quadrupole axis is $\omega_x \simeq 2\pi \times (96 \pm 3)$ Hz, along the gravity direction is $\omega_z \simeq 2\pi \times (34 \pm 2)$ Hz and along the ODT beam is $\omega_y \simeq 2\pi \times (58 \pm 4)$ Hz.

The main signature of the occurrence of the Bose–Einstein condensation is an abrupt change in the density profile of the cloud when it undergoes the phase transition. This change is a very important evidence of the quantum degeneracy of the sample. It is also a very useful way of characterizing the sample because it allows us to easily distinguish the thermal from the condense fractions. According to the Thomas–Fermi approximation discussed in Section 2.2.2, the density profile of the condensed cloud reflects the trapping potential (in our case, the density profile is a parabolic peak), while the thermal cloud presents a gaussian profile. In fact, the way of measuring the temperature of the atoms is by measuring the temperature of the thermal cloud using Equation (4.9) or (4.10). Figure 4.24 shows pictures of the cloud and its density profile for different temperatures. When the sample is completely thermal its density profile is well fitted by a gaussian curve. When the temperature decreases below the critical point we observe a sharp parabolic peak surrounded by broader gaussian “wings”. The parabolic peak corresponds to the condensed component while the gaussian wings are the gaussian distribution of the thermal component. This distribution is known as *bimodal distribution* and allows us to distinguish the two components of the sample. Finally, when the temperature is much lower than the transition temperature we only observe the condense fraction with no thermal component. In Figure 4.24 we show three–dimensional densitie profiles of the cloud for different temperatures, also showing the bimodal and parabolic distributions when the temperature is below the critical temperature.

Figure 4.25(a) shows a series of images of the condensed cloud at different expansion times. It exhibits another very important signature of Bose–Einstein condensation: the inversion of the

² That is, we are not able to measure any thermal fraction.

Figure 4.23 – Density profile of the atomic cloud for different temperatures above and below the critical point. Clearly, the profile changes from the gaussian distribution of a thermal cloud to a parabolic peak for a pure condensate. For intermediate temperatures the cloud presents a bimodal distribution where both gaussian and parabolic profiles are observed. Pictures taken after 19 ms of time-of-flight.

Figure 4.24 – Three-dimensional density profile of the atomic cloud for different temperatures above and below the transition temperature T_C . When $T > T_C$ a broad gaussian profile is observed. When $T < T_C$ the sample presents a bimodal distribution. For $T \ll T_C$ the cloud is completely condensed and the density profile is parabolic.

Figure 4.25 – Absorption images at different expansion times for (a) a BEC and (b) a thermal cloud.

Figure 4.26 – Evolution of the aspect ratio of (a) the BEC and (b) the thermal cloud. Lines are guides for eyes.

aspect ratio. In Figure 4.25(b) we have a similar sequence of images of a thermal cloud just above the critical temperature and it clearly shows isotropic expansion. Figure 4.26(a) is a graph of the evolution of the aspect ratio of the BEC and Figure 4.26(b) exhibits the expansion dynamics of a thermal cloud. These figures show that the former clearly undergoes aspect ratio inversion while the later tends to unity. This observation constitutes a very strong evidence of the achievement of the Bose–Einstein condensation.

4.8 Summarizing: the experimental sequence

To summarize this Chapter, we present the experimental sequence to produce the ultracold samples. The temporal sequence of the important parameters of the *mode–matching* process, described in Section 4.6 is shown in Figure 4.27. Recall, this process is used to transfer the atoms from the MOT to the magnetic trap. In this graph we show the evolution of the power and detuning of the trapping laser, the power of the repumper laser and the magnetic trap gradient.

Figure 4.28 illustrates the temporal sequence during the magnetic and hybrid trapping sequences. In this Figure we can see the evolution of the magnetic field, the rf–evaporation ramps and optical dipole depth.

4.8.1 Control Programs

It is worthwhile to notice that while the MOT loading takes more than 30 seconds and the evaporation processes, as we have seen, last around 10 seconds, the whole mode–matching procedure, described in Section 4.6 has a duration of ~ 10 ms. The complexity of our experiment clearly manifests at this point, in which a very critical and precise stage, composed of three very different phases, lasts a very small fraction of the total sequence. Consequently we have to control all the different components of the experiment at very different scales of time with a perfect synchrony.

To program the experimental sequence described above we use two acquisition boards from National Instruments. We have an analog board (model PCI 6713) and a multifunction board

Figure 4.27 – Temporal sequence of the power and detuning of the trapping laser, the power of the repumper laser and the magnetic trap gradient during the transference from the MOT to the magnetic trap.

Figure 4.28 – Temporal sequence of the magnetic field, the rf-evaporation ramps and the optical dipole trap depth during the magnetic and hybrid trapping processes.

(model PCI 6259) which allow us to control and coordinate our experiment.

The analog board has eight output channels that can take any value between -10 to 10 V. These outputs are useful to control external equipment whose function requires continuous change. Examples of these equipments are the current power supply or the rf value of the AOMs. Besides the analog outputs, the board contains some synchronizer channels, which include an internal clock and counters. The internal clock generates a periodic signal very useful to synchronize the board with external equipment or with other boards. The counters have many functions. For instance, they can be used to count pulses, to measure the width or the frequency of an external signal or even to generate a pulse chain with an specific duration. Finally, we have a set of special channels, known as *Programmable Function Interfaces* (PFI) which can be configured to function as input or output. These channels are very useful for triggering purposes.

The multifunction board contains 32 digital outputs which can take the values zero or 5 V. These outputs are very useful to control equipment with just two states (on/off or open/closed) like mechanical shutters. Also they serve as a trigger which indicates the moment in which a certain equipment must start to work, an example is the frequency generator used to produce the rf ramps during the evaporative cooling. This board has four analog outputs and 32 analog inputs, very useful to acquire an analog signal. Finally, the multifunction board also has synchrony channels (clock and counters) and several PFIs. When both boards, analog and multifunction, are synchronized it is possible to obtain a precision of one part in 10^5 in each of the 40 employed channels.

The experimental sequence that the acquisition boards will execute is compiled in a program written exclusively for this purpose. This control program was written in LabVIEW from National Instruments, a very versatile programming environment. Figure 4.29 shows the main window of the program, in which it is possible to write individually the sequence of any of the stages described in this Chapter.

Figure 4.29 – Main window of the program in which the experimental temporal sequence is compiled.

5 *Conclusions*

In this Section we present our main conclusions and future perspectives. Since we have described two different experiments with different objectives, it is appropriate to divide our conclusions in two parts.

5.1 Summary of Chapter 3

In Chapter 3 we have exposed the experiments performed in the BEC–I system and the main results obtained. We summarize as follows:

1. We have described the experimental apparatus and the experimental sequence to produce and excite a Bose–Einstein condensate.
2. We apply an oscillatory excitation in a magnetically trapped condensate which is able to rotate, translate and deform the sample.
3. Depending on the combinations of time and amplitude of the excitation, it is possible to produce four different regimes in the condensate, namely: *(i)* Bending of the cloud, *(ii)* formation of quantized vortices, *(iii)* generation of quantum turbulence and *(iv)* granulation of the superfluid.
4. All our results can be analyzed in a diagram of amplitude versus time of excitation. This diagram exhibits domains corresponding to the different states realized. The gradual evolution from a bended BEC to the regular vortex state, to the turbulent regime, and, finally, to granulated condensate is presented.

5. The obtained diagram serves as a guide to demonstrate the parameters that are necessary to experimentally produce different nontrivial non-equilibrium states of trapped atoms, such as turbulent condensates and granular condensates.
6. We have presented numerical simulations that allow us to qualitatively explain the observations and to identify the requirements for realizing this or that regime. This simulation, together with the experimental observations, indicates that our vortex formation mechanism is related with the relative movement of the condensate and its thermal cloud.

All the presented results have been published in various Journals (63, 64, 67, 68, 71, 72, 78), these results have opened new questions that remain to be answered. The future trends of our laboratory are:

- To develop more powerful imaging techniques that allow us to perform non-destructive images to study the dynamics of the system. Also, new techniques are required to reconstruct the 3D structure of the superfluid.
- To better understand the mechanism of formation of vortices.
- To study deeply the phenomenon of quantum turbulence. In particular the dependence on the temperature and the decay mechanisms are two very interesting questions that are still open. Also, it is very important to demonstrate that the turbulent cloud obeys the Kolmogorov spectrum.
- To better characterize the granular phase and to understand its relaxation processes after the end of the excitation.

5.2 Summary of Chapter 4

In Chapter 4 we have described in detail the BEC-II system. We list the most important remarks:

1. We have described all the important components in the construction of an apparatus to

produce a Bose–Einstein condensate of ^{87}Rb atoms. Particularly important is the implementation of the hybrid trap because it represents a novel and versatile trapping system. The BEC–II setup has been designed to be a more versatile and robust experimental apparatus than the first generation BEC–I system.

2. We have characterized all the stages of the experimental sequence.
3. We have succeeded in the process of obtaining the condensate.

The future steps with these experiments are:

- Optimization of the whole sequence in order to produce larger and more robust condensates.
- Perform the experiments depicted in Section 4.1 concerning the measurement of the magnetic induction produced by the condensate in an external conducting loop.
- A possible future trend includes continuing the investigation of quantum turbulence. Due to the good optical access available in the BEC–II it is feasible to implement the Kobayashi & Tsubota’s proposal to generate turbulence (60). According to it, the turbulent state can be produced with two perpendicular stirring lasers that generate a tangle of vortices in the sample. This method allows more control than the magnetic excitation employed in this thesis.

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APPENDIX A – Trapping techniques for neutral atoms

In this appendix we discuss the different techniques to trap neutral atoms employed in this thesis. This includes magnetic trapping, magneto–optical trapping, optical dipole traps and hybrid traps.

A.1 Magnetic Trapping

The main physical effect behind magnetic trapping is the Zeeman effect. Since the internal energy of an atom depends on an external field it is possible to create a spatially varying potential in which an atom, with the proper spin projection, can be confined. References (31, 91) review this subject.

Let us consider a ^{87}Rb atom in any of the hyperfine energy levels of its ground state. As we can see in Figure 4.4, the ground state of ^{87}Rb has two hyperfine levels with $F = 1$ and $F = 2$, where $\vec{F} = \vec{S} + \vec{L} + \vec{I} = \vec{J} + \vec{I}$ is the total angular momentum of the atom. Each hyperfine F level contains $2F + 1$ magnetic sublevels. In the absence of external magnetic fields, these sublevels are degenerate. However, when an external magnetic field $\vec{B}$ is applied, their degeneracy is broken. The Hamiltonian describing the atom interacting with the magnetic field is

$$H_Z = -\frac{\mu_B}{\hbar} \left(g_S \vec{S} + g_L \vec{L} + g_I \vec{I} \right) \cdot \vec{B} = -\frac{\mu_B}{\hbar} (g_S S_z + g_L L_z + g_I I_z) B, \quad (\text{A.1})$$

where $\mu_B = e\hbar/2m_e$ is the Bohr magneton and we have considered the magnetic field to be along the quantization axis of the atom (z–axis) and, thus, $B = |\vec{B}|$ is the magnitude of the field.

Figure A.1 – Hyperfine structure of the ground state of the ^{87}Rb atom in presence of a magnetic field.

The quantities g_S , g_L and g_I account, respectively, for the electron spin, electron orbital, and nuclear Landé “g-factors”.

It can be shown that if the splitting of the energy levels due to the external magnetic field is small compared to the fine and hyperfine splittings, then F is a good quantum number and the Hamiltonian becomes

$$H_Z = -\frac{\mu_B}{\hbar} g_F F_z B, \quad (\text{A.2})$$

where g_F is the hyperfine Landé factor. The correction to the energy due to the magnetic field can be found using perturbation theory. For a weak field we can keep only the lowest order correction, and the energy shift of an state $|F, m_F\rangle$ is found to be

$$E_{F, m_F} = \mu_B g_F m_F B. \quad (\text{A.3})$$

In the case of the ground state of the ^{87}Rb atom, it can be shown that, in good approximation, the hyperfine g-factors are

$$g_F = -1/2, \text{ for } F = 1 \text{ and } g_F = 1/2, \text{ for } F = 2. \quad (\text{A.4})$$

In Figure A.1 we can see the splitting of the energy levels of the ground state of the ^{87}Rb

atom. Note that the states $|F = 2, m_F = 2\rangle$, $|F = 2, m_F = 1\rangle$ and $|F = 1, m_F = -1\rangle$ increase their energy as the field increases. For this reason these states are known as “*low-field seekers*” and can be trapped in the minimum of a space-varying magnetic field. The states whose energy decreases as the magnetic field increases are known as “*high-field seekers*” and only can be trapped in the maximum of a field. However, we know that in a region free of currents it is only possible to create a minimum of the field. Therefore, magnetic traps are able to confine only “*low-field seekers*” states.

There are many types of magnetic traps, here we describe just those that we use in our experiments, namely the *Quadrupole trap* and the *QUIC-trap*.

A.1.1 Quadrupole and QUIC traps

A magnetic quadrupole is a field which, near the minimum, increases linearly in all directions and vanishes at the origin. This field is always symmetric along one axis, if we consider it to be axially symmetric and if B' is the gradient along the radial direction, the quadrupole field and its magnitude can be written as

$$\vec{B} = B' (x, y, -2z) \quad \text{and} \quad B = B' \sqrt{x^2 + y^2 + 4z^2} \quad (\text{A.5})$$

This field increases linearly from the minimum but with a different gradient, depending on the direction.

The quadrupole is the most commonly used magnetic field for trapping cold neutral atoms and, typically, it is produced by two coils in anti-Helmholtz configuration. This configuration consists of two identical coils placed along a common axis, separated by a distance equal to the radius of the coils. The electrical current in each coil is the same, but it circulates along opposite directions. However, the quadrupole field has a serious disadvantage. Implicitly, we have assumed that the hyperfine state of the atom is conserved as it interacts with the magnetic field. Nevertheless, this is not necessarily true. The separation among the Zeeman sublevels and, therefore among a low-field seeker and high-field seeker states, is of the order of $\mu_B B$. At

Figure A.2 – Magnetic field along the Ioffe axis direction for different values of the ratio $I_{\text{ioffe}}/I_{\text{quad}}$.

the origin this separation is zero and, in consequence, an atom moving through the minimum can suffer a transition from a magnetically trappable state to a non-trappable state. In this case, the atom would escape from the trap. Literally, the quadrupole field has a leak where the atoms can escape. For a hot cloud this is not a problem, but as we approach to lower temperatures the atomic losses become significant, avoiding the Bose–Einstein condensation to happen.

On the other hand, the main advantage of this trap is that it possesses a large trapping volume. For this reason our strategy is to trap a big amount of atoms in a quadrupole and then transfer them into a trap with no vanishing points. There are many options of non-vanishing traps, such as the QUIC trap, the optical-dipole trap and hybrid trap which we describe in the next sections.

In the BEC–I system described in Chapter 3 we use a third coil placed perpendicularly to the magnetic quadrupole, as illustrated in Figure 3.1. This coil is named *Ioffe coil* and the whole set is known as *Quadrupole and Ioffe configuration* (QUIC). The action of the Ioffe coils is to compensate the zero point of the magnetic quadrupole by adding an extra field. When a current is ramped in the Ioffe coil, the minimum position dislocates along the direction of the symmetry axis of the Ioffe coils. For a high enough current circulating through the Ioffe coil the zero field region disappears and the field is compensated. Although there is not an analytical

expression for the QUIC field, its bottom is harmonic in good approximation. Figure A.2 shows the magnetic potential along the axis of the Ioffe coil for several values of the ratio between the currents passing through the Ioffe and the quadrupole coils (I_{ioffe}/I_{quad}).

A.2 Magneto–optical trapping

As mentioned in Section 4.4, a magneto–optical trap (MOT) uses a combination of magnetic and laser fields to cool down and confine a sample of atoms. This topic is deeply described in References (6, 96). In our experiments, this trap represents the starting point in the route to Bose–Einstein condensation.

Let us consider a two–level atom whose energy levels are separated by $\hbar\omega_0$. We indicate the transition decay rate as Γ . Consider that the atom is interacting with a monochromatic electromagnetic wave with frequency ω and wave vector $\mathbf{k}$, namely

$$\mathbf{E}(\mathbf{r}, t) = \mathbf{E}_0 \exp(\omega t - \mathbf{k} \cdot \mathbf{r}). \quad (\text{A.6})$$

Then, it is possible to demonstrate that the atom experiences two different kind of forces, a conservative dipole force given by

$$\mathbf{F}_C = -\frac{\epsilon_0 \nabla E_0^2}{4} \left[\frac{\mu_{12}^2}{3\epsilon_0 \hbar} \left(\frac{\Delta\omega}{(\Delta\omega)^2 + (\Gamma/2)^2 + \Omega_0^2/2} \right) \right], \quad (\text{A.7})$$

and a dissipative force, due to absorption and emission of photons, given by

$$\mathbf{F}_D = \frac{\epsilon_0 E_0^2 \mathbf{k}}{4} \left[\frac{\mu_{12}^2}{3\epsilon_0 \hbar} \left(\frac{\Gamma/2}{(\Delta\omega)^2 + (\Gamma/2)^2 + \Omega_0^2/2} \right) \right], \quad (\text{A.8})$$

where ϵ_0 is the vacuum permittivity, μ_{12} is the transition dipole moment, $\Delta\omega = \omega - \omega_0$ is the detuning between the atomic transition and the frequency of the light, and $\Omega_0 = \mu_{12}E_0/\hbar$ is the Rabi frequency.

The force of Equation (A.8) is also known as *radiation pressure* because the direction of the force is along the light propagation. This force plays a fundamental role in laser cooling tech-

Figure A.3 – (a) Sketch of a magneto–optical trap in one dimension. (b) Relevant transitions for the production of a MOT.

niques, particularly in a magneto–optical trapping. The conservative force of Equation (A.7) points along the direction of gradient of the field, in the following discussion we will consider plane waves, so $\mathbf{F}_C = 0$, however, the dipole force is fundamental for optical traps, as discussed in Section A.3.

To understand how the MOT operates, let us consider the simplified situation illustrated in Figure A.3(a), in which we have a magnetic quadrupole and two counter–propagating beams with opposite circular polarization. Let us assume that these beams are red detuned. Now, assume that the ground state of the two–level atom has total angular momentum $F = 0$ and the excited state has $F = 1$. As shown in Figure A.3(b), the linear magnetic field produced by the quadrupole splits the Zeeman levels of excited state of the atom and this splitting is spatially dependent. Therefore, at the right side of the zero point of the magnetic quadrupole the energy of the state with $m_F = -1$ is smaller than the energy of the state with $m_F = +1$, in the left side occurs the opposite situation. Therefore, the atoms moving to the left side will absorb with a much higher probability the beam with polarization σ^- , experiencing a force, due to radiation pressure, toward the center of the quadrupole. Using the same arguments, we conclude that the atoms moving to the right will be pushed back to the zero point of the magnetic field.

In consequence, the center of the magnetic quadrupole becomes a confining region in which

Figure A.4 – Sketch of a magneto–optical trap in three dimensions.

atoms can trapped. Since the radiation pressure is not a conservative force, the atoms loose kinetic energy when interact with the laser beams, therefore, the sample besides being trapped is also cooled down. If instead of using a sing pair of red–detuned laser beams use three pair of counter–propagating beams along three orthogonal directions we can produce a three–dimensional trap, as illustrated in Figure A.4.

In a real MOT of ^{87}Rb , as those described in Chapters 3 and 4, the hyperfine structure of the atom is more complex. The ground state is split into two hyperfine levels $F = 1$ and $F = 2$, while the excited state has four components $F' = 0, 1, 2, 3$. The employed MOT transition is $F = 2 \rightarrow F' = 3$, however, some atoms will be non resonantly excited to the $F' = 2$ state having the possibility of decaying into the ground hyperfine state $F = 1$. This atom cannot be trapped in the MOT anymore. For this reason, we also use a *repumper* frequency between the states $F = 1 \rightarrow F' = 2$ which warranties that all the atoms will remain in the MOT.

A.3 Optical–dipole trap

The review article (95) provides an excellent general view on this subject. The main physical concept behind the optical–dipole trap (ODT) is contained in the conservative force of

Figure A.5 – Sketch of an optical dipole trap using (a) a single beam and (b) two crossed beams.

Equation (A.7).

Again, let us consider a two-level atom with energy separation $\hbar\omega_0$ and decay rate Γ interacting with a monochromatic laser beam with intensity $I(x, y, z)$ propagating along z -direction. Using Equation (A.7) it is possible to demonstrate that the light generates a potential given by

$$U(\vec{r}) = \frac{3\pi c^2}{2\omega_0^3} \frac{\Gamma}{\Delta\omega} I(x, y, z). \quad (\text{A.9})$$

Note that if the laser is red-detuned ($\Delta\omega < 0$) the potential is attractive and it is blue-detuned ($\Delta\omega > 0$) it will be repulsive. As described in Section 4.7, our optical trap is generated by focused gaussian beam, so we will only consider that case. It is very important to have a very large red-detuning ($|\Delta\omega| \gg \Gamma$ and $|\Delta\omega| \gg \Omega_0$), otherwise undesired photon scattering processes become important. In the focus of the gaussian beam the intensity is higher, therefore, this will be the trapping region. In this case, the intensity is given by

$$I(r, z) = \frac{I_0}{1 + (z/z_R)^2} \exp \left[-\frac{2r^2}{w_0^2} \frac{1}{1 + (z/z_R)^2} \right], \quad (\text{A.10})$$

where we are considering cylindrical symmetry along z -direction ($r = \sqrt{x^2 + y^2}$) and w_0 is the beam waist at the focus position. $z_R = \pi w_0^2 / \lambda$ is known as *Rayleigh length*, where λ is the wavelength of the light. Figure A.5(a) illustrates his potential.

For a very cold atomic sample ($T \sim 1\mu\text{K}$), the kinetic energy of the atoms is much smaller

that the depth of the potential of Equation (A.9). Additionally, the extension of the sample is much smaller than w_0 and z_R . Hence, the potential that the atoms interact with can be properly approximated by an axially symmetric harmonic oscillator. Substituting Equation (A.10) into Equation (A.9), the harmonic approximation of the potential is given by

$$U(r, z) \simeq -U_0 + \frac{m}{2} (\omega_r^2 r^2 + \omega_z^2 z^2), \quad (\text{A.11})$$

where $U_0 = (3\pi c^2 \Gamma / 2\omega_0^3 \Delta \omega) I_0$ is the potential depth. the radial and axial frequencies of the harmonic oscillator, ω_r and ω_z , are given by

$$\omega_r = \sqrt{\frac{4U_0}{mw_0^2}} \quad \text{and} \quad \omega_z = \sqrt{\frac{2U_0}{mz_R^2}}, \quad (\text{A.12})$$

where the frequencies along the x and y -directions satisfy $\omega_x = \omega_y \equiv \omega_r$.

Usually, the beam waist w_0 is of the order of tens of microns. In our experiments, the wavelength of the beam is $\lambda = 1064$ nm, hence z_R is of the order of few millimeters. Thus $w_0 \ll z_R$ and, consequently, the trap is much more confining along the radial direction ($\omega_r \gg \omega_z$). This generates very elongated samples. There are several ways of increasing the confinement along the axial direction, the most common technique consists in using a second focused gaussian beam propagating perpendicularly with respect to the first beam. The beams intersect in their foci. This creates a region with approximately the same confinement along all directions. This configuration is known as *crossed-beam dipole trap* and is illustrated in Figure A.5(b).

However, in our experiment, instead of using a second focused beam we will use a magnetic field which provides confinement along the weak direction of the ODT. This technique was proposed and used by the first time by the group of Ian Spielman at NIST. We briefly describe it in the next section.

A.4 Hybrid trap

The following discussion is based on Reference (90). The hybrid trap is composed by a quadrupole magnetic trap and a single-beam optical dipole trap, we have discussed both in

Figure A.6 – (a) Side and (b) top view of the hybrid trap. The black cross indicates the position of the minimum of the magnetic trap. The dimensions have been exaggerated for the sake of clarity.

Sections A.1 and A.3.

Let us consider a magnetic quadrupole, whose symmetry axis is parallel to the x -direction. Consider as well a single-beam optical dipole trap with waist w_0 propagating along the y -direction. There is an offset z_0 along the z -direction from the center of the magnetic quadrupole. These two fields compose the hybrid trap, Figures A.6(a) and (b) are side and top views of this system. Considering the gravitational potential and using Equations (A.5) and (A.9) we obtain an expression for the hybrid potential,

$$U(\mathbf{r}) = \frac{\mu B'}{2} \sqrt{4x^2 + y^2 + z^2} - U_0 \exp \left[-2 \frac{x^2 + (z - z_0)^2}{w_0^2} \right] + mgz, \quad (\text{A.13})$$

where B' is the gradient of the magnetic field along x -direction and we have neglected the optical confinement along the y -direction. Figure A.7 shows graphs of this potential along the z and y -directions for different values of the magnetic gradient.

Just as in the case of the pure ODT, we can only consider the case of very low temperatures in which the potential of Equation (A.13) can be correctly approximated as a harmonic potential, namely

$$U(\mathbf{r}) \simeq \frac{m}{2} \left(\omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 (z - z_m)^2 \right), \quad (\text{A.14})$$

Figure A.7 – Hybrid potential for several values of the magnetic gradient along (a) gravity direction and (b) dipole beam direction. Image taken from (90).

where z_m is the position of the minimum of the trap and the frequencies are given by

$$\omega_x = \omega_z = \sqrt{\frac{4U_0}{mw_0^2}} \quad \text{and} \quad \omega_y = \sqrt{\frac{\mu B'}{4mz_m}}, \quad (\text{A.15})$$

as we can see, the confinement along x and z -directions is given by the optical trap while the confinement along the y -direction is dominated by the magnetic trap. More detail concerning the experimental operation of this trap are given in Section 4.7.