**UNIVERSIDADE DE SÃO PAULO
INSTITUTO DE FÍSICA DE SÃO CARLOS**

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**Measuring the shower maximum of extensive air showers
using imaging atmospheric Cherenkov telescopes**

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**Measuring the shower maximum of extensive air showers
using imaging atmospheric Cherenkov telescopes**

Dissertation presented to the Graduate Program in Physics at the Instituto de Física de São Carlos, Universidade de São Paulo, to obtain the degree of Master in Science.

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Supervisor: Prof. Dr. Luiz Vitor de Souza Filho

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*“The game of science can accurately be described
as a never-ending insult to human intelligence.”*

João Magueijo

ABSTRACT

DELGADO GILER, A. G. **Measuring the shower maximum of extensive air showers using imaging atmospheric Cherenkov telescopes.** 2019. 85p.

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Cosmic rays are at the foundation of astroparticle physics and the extensive air showers (EAS) is one indirect way to detect them. Air showers, however, have been used to infer information not just of cosmic rays particles, but also to localize gamma rays sources. The shower maximum of an EAS, defined as the position at the atmosphere where the maximum quantity of charged particles is reached, is an observable of air showers that can permit to infer the mass composition of cosmic rays. For this reason, it is important to propose methods to measure it. Several methods to determine the shower maximum have been implemented in the last decades with the development of different kinds of telescopes. This work discusses the possibility of determining the maximum of air showers using imaging atmospheric Cherenkov telescopes (IACT). The Cherenkov telescopes can detect the Cherenkov radiation produced by the interaction of charged particles with the atmosphere. Those Cherenkov photons are projected back into the plane containing the longitudinal development of the air shower. Each plane is saved as a 2D histogram with the longitudinal and lateral development in the vertical and horizontal axis, respectively. A detailed analysis of each 2D histogram is presented and used to obtain the depth of the maximum of the Cherenkov profile. The main effect seen is a decrease in the shower maximum of Cherenkov photons as a function of the telescope position from the shower axis to 150 m. After 150m from the shower axis, there is a constant behavior that is correlated to the real depth of the maximum of an EAS. Based on this constant behavior after 150 m, the shower maximum is reconstructed and it is shown the resolution of the method as a function of the energy, which is around 55 g/cm² considering just one telescope, and 15 g/cm² for the best case considering zenith angle of 20 degrees. Moreover, the method is tested with some simulations took from Very Energetic Radiation Imaging Telescope Array System (VERITAS) experiment to compare with the results of our simulations. The resolution of the reconstruction of the shower maximum for proton and iron showers was also done which ranges around 80 g/cm² for proton and around 30 g/cm² for iron in the case of 20° of zenith angle.

Keywords: Extensive air shower. Cosmic rays. Depth of shower maximum. Longitudinal development. Imaging atmospheric Cherenkov telescopes.

RESUMO

DELGADO GILER, A. G. **Medição do máximo desenvolvimento de chuueiros atmosféricos extensos usando telescópios de imagem atmosférica Cherenkov.** 2019. 85p. Dissertação (Mestrado em Ciências) - Instituto de Física de São Carlos, Universidade de São Paulo, São Carlos, 2019.

Os raios cósmicos estão na base da física das astropartículas e os chuueiros atmosféricos extensos (EAS pela sigla em inglês) são uma maneira indireta de detectá-los. Os chuueiros atmosféricos, no entanto, têm sido usados para inferir informações não apenas sobre partículas de raios cósmicos, mas também para localizar fontes de raios gama. A profundidade de máximo num EAS, definido como a profundidade atmosférica onde a quantidade máxima de partículas carregadas é atingida, é um observável de EAS que permite inferir a composição de massa dos raios cósmicos. Por esse motivo, é importante propor métodos para medi-lo. Vários métodos para determinar a profundidade de máximo foram implementados nas últimas décadas com o desenvolvimento de diferentes tipos de telescópios. Este trabalho discute a possibilidade de determinar a profundidade de máximo de chuueiros atmosféricos utilizando os telescópios atmosféricos Cherenkov (IACT). Os telescópios Cherenkov podem detectar a radiação Cherenkov produzida pela interação de partículas carregadas com a atmosfera. Esses fótons Cherenkov são projetados de volta ao plano que contém o desenvolvimento longitudinal do chuueiro. Cada plano é salvo num histograma 2D com o desenvolvimento longitudinal e lateral no eixo vertical e horizontal, respectivamente. Uma análise detalhada de cada histograma 2D é apresentada e usada para obter a profundidade máxima do perfil de emissão de luz Cherenkov. O principal efeito visto é uma diminuição na profundidade de máximo dos fótons Cherenkov do chuueiro como função da posição do telescópio a partir do eixo do chuueiro até 150 m. A partir de 150 m do eixo do chuueiro, há um comportamento constante que está correlacionado com a profundidade real de máximo do EAS. Com base nesse comportamento constante após 150 m, o máximo do chuueiro é reconstruído e é mostrada a resolução do método em função da energia, que é cerca de 55 g/cm² considerando apenas um telescópio, e 15 g/cm² para o melhor caso, considerando o ângulo zenital de 20 graus. Além disso, o método é testado com algumas simulações cedidas pelo experimento VERITAS (Very Energetic Radiation Imaging Telescope Array System, pela sigla em inglês) para comparar com os resultados de nossas simulações. Também foi feita a resolução da reconstrução da profundidade de máximo para chuueiros atmosféricos de prótons e ferro, que varia em torno de 80 g/cm² para prótons e em torno de 30 g/cm² para ferro no caso chuueiros inclinados a um ângulo de 20°.

Palavras-chave: Chuueiros atmosféricos extensos. Raios cósmicos. Profundidade do máximo desenvolvimento. Desenvolvimento longitudinal. Telescópios de imagem Cherenkov atmosférica.

LIST OF FIGURES

Figure 1 – Energy spectra of cosmic rays from various experiments.	21
Figure 2 – Schematic representation of the main parts of an air shower.	23
Figure 3 – Side view of air shower initiated by a photon, a proton and an iron of 100 TeV each. Electromagnetic particles is shown in red, hadrons particles are black and muons are green.	25
Figure 4 – Longitudinal development of a proton initiated shower. It shows the three components in a EAS: electromagnetic (blue), hadronic (red) and muonic (green).	26
Figure 5 – Heitler model representation of an air shower. (a) An electromagnetic-induced shower. (b) A hadron-induced shower; solid blue lines represent $\pi^\pm$, and dashed red lines represent neutral pions π^0	28
Figure 6 – Mean depth of shower maximum X_{max} as function of energy, compared with the predictions of CONEX ¹ program for proton and iron initiated showers. .	30
Figure 7 – Longitudinal development of different showers initiated by primaries of 10 TeV simulated with CONEX. ² The fluctuations on the X_{max} from shower-to-shower is due to the stochastic nature of cascades.	31
Figure 8 – The first two moments of the X_{max} distributions data of Pierre Auger. They are compared to proton and iron using QGSJETII-04, Sibyll2.3, and EPOS-LHC.	32
Figure 9 – The first two moments of the $\ln A$ distributions using QGSJETII-04, Sibyll2.3, and EPOS-LHC.	33
Figure 10 – Geometry of Cherenkov light phenomenon.	35
Figure 11 – Main differences in the detection of gamma and cosmic ray showers.	38
Figure 12 – Hillas parameters of air shower Cherenkov image analysis to select gamma ray initiated showers and discriminate against proton showers. The X-Y coordinate system corresponds to the telescope. The solid ellipse shows the Cherenkov image where C is the centroid of the image and M is the center of the field of view. The main parameters are the major and minor axis of the ellipse that is labeled as <i>Length</i> and <i>Width</i> , the <i>Distance</i> between C and M, the <i>Miss</i> that is the offset of the image and the <i>Azwidth</i> that is the azimuthal width of the image. The dashed ellipse, labeled <i>On-Source Gamma Ray Image</i> shows the typical form for a gamma ray shower detection when the telescopes axis is pointing at the source.	39

Figure 13 – (a) Cherenkov photon density, Q , as a function of core distance, r , measured at sea level by 10^{17} eV proton initiated showers. (b) The ratio, R , of the photon density at $r_1 = 100$ m, $Q(100)$, to that at $r_1 = 600$ m from the shower core, $Q(600)$, as a function of depth of maximum development, X_{max} , of 10^{17} eV proton initiated showers as shown 13a.	44
Figure 14 – X_{max} measured by Yakutsk (triangles) array compared to data obtained at MSU (diamonds) and LOPES (dots) arrays. Simulation of X_{max} using the model QGSJET II-04 for a pure mass composition of protons (solid line), nuclei of carbon (dash line) and nuclei of iron (dot line), is also shown. . . .	45
Figure 15 – Reconstruction geometry of X_{max} of the Pierre Auger Observatory. Shower event 15346477 with $\theta = 58.3^\circ$ and $\phi = 324.7^\circ$	46
Figure 16 – Fitting of Gaisser-Hillas function of the event 15346477 with $\theta = 58.3^\circ$ and $\phi = 324.7^\circ$	47
Figure 17 – Shower maximum resolution as a function of the energy shower. The bands are the systematic uncertainties.	47
Figure 18 – Left side column: Comparison of longitudinal development between Cherenkov photons and charged particles profiles of three (solid, dash and dot lines) showers of different mass composition and energy. Right side column: distribution of the shower maximum, X_{max} , for charged particles and Cherenkov photons profiles generated by showers of different mass composition and energy.	50
Figure 19 – Scheme of geometry used to study the longitudinal development of air shower.	51
Figure 20 – Shower face plane of 300 TeV gamma induced shower for a telescope located at 140 m from shower core. In the horizontal axis is the lateral distance measured in meters and the vertical axis is the longitudinal development measured in g/cm^2	52
Figure 21 – Shower face plane of different initiated primary showers reconstructed for telescopes located at 50 m (up) and 200 m (low) from the shower core. . . .	53
Figure 22 – Left side column: histogram 2D of the longitudinal and lateral distribution of Cherenkov photons projected on the shower plane of an induced gamma shower of 300 TeV for each telescope along to axis-x at (from top to bottom) 10 m, 50 m, 130 m, and 250 m. Right side column: Fitting of a second degree polynomial around of the maximum bin in the longitudinal distribution, to get the $X_{max}^{Cherenkov}$ for each position of the telescope, respectively.	56
Figure 23 – Charged particle profile as a function of the slant depth of (a) 30 TeV iron and (b) 300 TeV gamma shower at 20° (black curve), and Gaisser-Hillas function fitted to the charged particle profile (red line).	58

Figure 24 – $X_{max}^{Cherenkov}$ of gamma, proton and iron shower of 300 TeV as a function of the telescope position. The $X_{max}^{Cherenkov}$ was obtained from second-degree polynomial fitting of the longitudinal development. Three different air showers were plotted (markers) with their corresponding $X_{max}^{charged}$ (dash lines) for each primary particle. Field of view angle = 10°	59
Figure 25 – Mean of $X_{max}^{Cherenkov}$ as a function of the distance to telescope from shower core for gamma, proton and iron showers of (a) 300 TeV and (b) 30 TeV of 20° of zenith angle. The curves are plotted for different fields of view of the telescope.	61
Figure 26 – Correlation between $X_{max}^{charged}$ and $X_{max}^{apparent}$ for gamma (a), proton (b) and iron (c) showers of 10, 30, 100 and 300 TeV of energy with $\theta = 20^\circ$. Telescopes with a field of view angle of 10°	63
Figure 27 – (a) Correlation of $X_{max}^{charged}$ vs $X_{max}^{apparent}$ for $X_{max}^{Cherenkov}$ adjusted in telescopes from 150m to 200m of gammas (black), protons (red) and irons (green) air showers. (b) Resolution calculated from the subtraction of X_{max}^{rec} minus $X_{max}^{charged}$	65
Figure 28 – Resolution in the reconstruction of the $X_{max}^{charged}$ as a function of the energy for a field of view of 10° (a) and 3.5° (b).	66
Figure 29 – Layout of the VERITAS telescopes.	68
Figure 30 – (a) Event simulation detected by the camera of VERITAS telescope. (b) Projection of the pixel in the shower face plane. The color of the ellipse indicates the corresponding pixel in the camera.	69
Figure 31 – Mean of $X_{max}^{Cherenkov}$ as function of the position of the telescope. $X_{max}^{Cherenkov}$ was obtained from second-degree polynomial fitting to the longitudinal development profile. The blue and red curve belong to iron and proton showers of (a) [9-11] TeV, (b) [20-40] TeV and (c) [80-120] TeV for $\theta = 0^\circ$, respectively.	71
Figure 32 – (a) Correlation of $X_{max}^{charged}$ vs $X_{max}^{apparent}$ for proton showers of zenith angle = 20° with energy 13.25 in $\log_{10}(E/\text{eV})$, using the configuration of four telescopes from VERITAS experiment for proton showers. (b) Resolution calculated from the subtraction of X_{max}^{rec} minus $X_{max}^{charged}$	73
Figure 33 – Energy evolution of the resolution that it was obtained, on the basis of the $X_{max}^{apparent}$, when it was reconstructed $X_{max}^{charged}$ from the linear correlation between $X_{max}^{apparent}$ and $X_{max}^{charged}$ for showers simulations of VERITAS observatory of (a) 0 and (b) 20 degrees of zenith angle.	74

LIST OF TABLES

Table 1	–	Parameters of the AT616 atmosphere (June 16, 1993).	54
Table 2	–	Number of total showers and total triggered telescopes (trig tel) for each energy interval for the curve of proton of (a) 0 and (b) 20 degrees of zenith angle and curve of iron of (c) 0 and (d) 20 degrees of zenith angle of the plots of Fig. 31	70
Table 3	–	Selection efficiencies for proton and iron Monte Carlo showers of (a) 0 and (b) 20 degrees of zenith angle as a function of energy.	72

CONTENTS

1	INTRODUCTION	19
2	EXTENSIVE AIR SHOWERS	21
2.1	Introduction	21
2.1.1	Structure of an air shower	22
2.2	Development of air shower	24
2.2.1	Electromagnetic processes	24
2.2.2	Hadronic interactions	27
2.3	Depth of the maximum development	28
2.3.1	Heitler model	28
2.3.2	Gaisser-Hillas function	30
2.4	Importance of the X_{max}	30
3	IMAGING ATMOSPHERIC CHERENKOV TELESCOPE	35
3.1	Introduction	35
3.2	Cherenkov light	35
3.2.1	Fundamental physics process	35
3.2.2	Radiation Yield Distribution	36
3.3	Cherenkov imaging technique	37
3.3.1	Hillas parameters	39
3.3.2	Image parametrization	40
4	METHODS OF X_{max} DETERMINATION	43
4.1	Cherenkov Photon Lateral Distribution Function	43
4.1.1	Correlation between radio signals and X_{max}	44
4.1.2	X_{max} results of Yakutsk experiment	45
4.2	X_{max} by air fluorescence shower	45
5	PROPOSAL OF A METHOD TO RECONSTRUCT X_{max} WITH	
	IACS	49
5.1	Description of the method	49
5.1.1	Shower face plane	52
5.2	Simulation Setup	54
5.3	Analysis of Cherenkov light distribution	55
5.4	Results	59
5.4.1	$X_{max}^{Cherenkov}$ as function of telescope position	59
5.4.2	Correlation between $X_{max}^{charged}$ and $X_{max}^{apparent}$ for the same primary particle	62

5.4.3	X_{max} reconstruction	64
6	ANALISYS OF EVENTS SIMULATED FOR THE VERITAS OB-	
	SERVATORY	67
6.1	VERITAS	67
6.1.1	Specifications	67
6.1.2	Parameters extracted of shower simulations	67
6.2	Analyze of event simulations	68
6.2.1	Selection cuts	68
6.2.2	Mean $X_{max}^{Cherenkov}$ as function of the telescope position	70
6.2.3	Resolution curves	72
7	CONCLUSIONS	75
	REFERENCES	79

1 INTRODUCTION

Charged particles that are continuously hitting the Earth from space are known as *cosmic rays*. 90% of cosmic rays are protons and 10% correspond to heavier nuclei, for instance, iron nucleus.³ The energy range of these particles goes from less than 1 GeV up to 10^{20} eV. Cosmic rays provide information about the sources, propagation and acceleration processes which are fundamental challenges in astrophysics.

On the one hand, high energy gamma rays are photons of $E \geq 1$ TeV that can be the result of hadronic parents or electromagnetic processes. Gamma rays are not deflected by magnetic fields in the galaxy, therefore they can indicate the direction of the source. For this reason, there are various instruments detect gamma ray and provide an understanding of the origin of them.⁴

When a proton or a gamma ray hit the earth it interacts with the components of the atmosphere generating a cascade of secondary particles called an *extensive air shower*.⁵ The quantity and energy of these sub-particles give information about the energy and nature of the primary particle. In Chapter 2 the theory and the importance of extensive air showers are described.

The detection of cosmic rays and gamma rays of high energy is done by detectors located at the ground. These detectors cover large areas because of the flux of primary particles decreases with energy. There are three forms to detect and study extensive air showers: detecting directly the secondary charged particles, detecting the fluorescence light produced by electrons/positrons in inelastic collisions with air molecules⁶ or detecting the Cherenkov light emitted by the interaction of charged particles with the components of the atmosphere.

In the last case, imaging atmospheric Cherenkov telescopes are used to measure the Cherenkov radiation emitted by charged particles that move faster than the phase velocity of light in the air.⁷ The imaging atmospheric Cherenkov technique, described in detail in Chapter 3, is usually based on the ellipsoidal fitting to the image formed in the telescopes. The Hillas parameters are obtained from the parametrization of the ellipse. The Hillas parameters permit discriminating gamma initiated showers from proton initiated showers.

One important observable of the EAS is the depth (or height), measured in g/cm^2 (or km) along to the atmosphere, where the maximum quantity of charged particles is reached. The depth of the maximum development, known as shower maximum, depends on the nature and energy of the primary particle. Knowing the shower maximum of extensive air showers is important because of the mass composition of cosmic rays can be determined from it.⁸

Some methods used to measure the shower maximum, described in more detail in Chapter 4, depend on the way EAS is detected. Different techniques have been investigated different techniques to determine the position of shower maximum and it was proposed that the shower maximum is related to the position of the Cherenkov photons emitted.

In this work, we are interested in measuring the shower maximum of extensive air showers using imaging atmospheric Cherenkov telescopes.

In Chapter 5, the proposed technique used is described as well as how simulations were done. The maximum quantity of Cherenkov photons is generated approximately where the maximum quantity of charged particles is produced in an air shower. Those Cherenkov photons are detected by photomultiplier tubes localized in the imaging atmospheric Cherenkov telescopes. The orientation of the telescopes and the position of the center of the shower in the ground, permit to project Cherenkov photons back in a plane along with the longitudinal development of the shower. This plane is used to study the position of the maximum Cherenkov photon emission as a function of the distance of the telescope to shower axis and compare with the shower maximum of the EAS.

In Chapter 6, some simulations of air showers of the VERITAS⁹ experiment were used to test our technique, to get the resolution in the reconstruction of the shower maximum of proton and iron started showers.

And, finally, in Chapter 7 the conclusions and final remarks of this work are presented.

2 EXTENSIVE AIR SHOWERS

In this chapter will be described the theory of the extensive air showers. It starts with the definition and the main parts of extensive air showers, followed by the processes involved in their longitudinal development. Heitler's model for electromagnetic and hadron initiated air showers is also described to understand the depth of the maximum development, X_{max} , dependence with the energy of the primary particle. Moreover, the differences between gamma and cosmic rays air shower will be established.

2.1 Introduction

Although cosmic rays (CR) composition depend on the energy, they consist roughly of 90% protons, 9% alpha particles and 1% heavier nuclei. The CR spectrum extends from some MeV to 10^{20} eV energy as can be seen in Fig. 1. From $\sim$ MeV to $\sim$ GeV, these particles come from the solar wind and the planets, and from $\sim$ TeV they have an origin outside of the solar system.¹⁰ The flux of CR decreases with energy, going from 1 particle per square meter per second at the lowest energy to 1 particle per square kilometer per year at the highest energy. Also, CRs are deflected from their source due to the magnetic fields present in the own source, interstellar medium and Milky way.¹¹

Figure 1 – Energy spectra of cosmic rays from various experiments.

Source: BIETENHOLZ.¹²

On the other hand, gamma rays are not deflected by magnetic fields in the Galaxy, which permit to obtain the direction of their source. The energy range of gamma-rays correspond to photons from some $\sim$ MeV to the most energetic photons detected of some hundreds of TeV.¹³

When a cosmic ray or a gamma ray enters the atmosphere, it produces a large number of secondaries particles (like photons (γ), electrons and positrons (e^- , e^+), muons ($\mu^\pm$), charged and neutral pions ($\pi^\pm$, π^0), neutrinos (ν_e, ν_μ) and others hadrons) due to successive collisions with target nuclei of the atmosphere constituents (mainly N_2 , O_2 , Ar) along their trajectories.¹⁴ This cascade of particles is called extensive air shower (EAS) or air showers.

2.1.1 Structure of an air shower

Fig. 2 shows the main parts in the lateral and longitudinal development of an air shower. The primary particle hits the atmosphere forming an angle, θ , from the normal of the atmosphere called, zenith angle. Depending on θ , part of the shower particles, which carry away a fraction of primary energy, will penetrate the Earth continuing its path throughout space. In the case of inclined showers, with $\theta > 60^\circ$, the atmospheric depth traversed is much larger than in the case of vertical showers, producing an earlier longitudinal development in the atmosphere. Then, just energetic muons will be detected at the ground, including a smaller fraction of electrons and photons that are going to be absorbed by the atmosphere before they reach the ground.¹⁵ These charged particles can also be deflected because of the magnetic field of the Earth affecting the characteristic cylindrical symmetry of air showers.¹⁶ For inclined showers, the distortion in the density of charged particles of an EAS measured at ground becomes more appreciable from $\theta > 20^\circ$.¹⁷

The shower axis is the prolongation of the initial momentum vector of the incident primary particle in the direction of cascade propagation. Its interception in the plane of observation is reconstructed with the lateral density distribution of shower particles. The point of this interception is known as shower core. This central region has a high particle density and drops with increasing the distance to the shower axis. Experiments use the central density of the shower core to know the mass composition of CR.^{18,19} There also exist shower events with multi-core structures. This can be due to nuclear fragments from fragmented heavy primaries with high energy and large transverse momentum or energetic secondaries generated at high altitude. This sub-cores are used to study interaction mechanisms and their original particles.²⁰

The direction of the shower axis (zenith and azimuthal angles) is determined by the arrival time of the shower front of particles on the plane of observation.²¹ The shower front has a disk-form with a slight curvature for larger events, as shown in Fig. 2. The thickness

Figure 2 –Schematic representation of the main parts of an air shower.

Source: GREIDER.¹⁴

of the front shower near from core is around $1 \sim 3$ m. At larger distances from the center, front broadens due to the increase of particle scattering, and because of lower Lorentz factors of the parent nucleons, responsible for the local sub-showers. In the case of the time profile of the particles, the bulk of the particles arrives at a range of few nanoseconds near from shower core to some decades of nanoseconds at larger distances from the shower axis.

The total number of particles that are produced in the shower at a particular level in the atmosphere is called shower size. It is mainly a function of the energy, the zenith angle, and the height of the first interaction of the primary that triggers the event in the atmosphere. The shower size does not depend strongly on primary mass, A , and commonly includes only charged particles.

The altitude or height in the atmosphere depends on the atmospheric pressure or depth. For that reason is common to use the atmospheric depth, measured in g/cm^2 , to describe the propagation of the shower particles. The vertical atmospheric depth, X , is defined as the integral in height of the atmospheric density for the observation level h ²²:

$$X(h) = \int_0^\infty \rho(h') dh', \quad (2.1)$$

and can be express in its most simple form for a standard isothermal exponential atmosphere as:

$$X(h) = X_0 e^{-\frac{h}{h_0}}, \quad (2.2)$$

where X_0 is the atmospheric depth at sea level and h_0 is the scaled height. In the case for $0 < \theta < 60^\circ$, when the Earth surface can be still considered flat, the atmospheric depth

is scaled with $1/\cos \theta$, giving the slant depth. Then, the slant atmospheric depth can be simplified as²³

$$X(h, \theta) \approx \frac{X(h)}{\cos(\theta)}. \quad (2.3)$$

2.2 Development of air shower

On average, the interaction length for a primary proton in the atmosphere is about 80 g/cm^2 compared to 38 g/cm^2 for gamma ray.²⁴ Therefore, proton initiated showers will develop further in the atmosphere compared to gamma ray showers. CR particles interact with the components of the atmosphere at a typical height of 15 to 35 km. The resulting cascade propagates longitudinally along of the direction of the incident primary, but in hadron initiated showers, the cascade spread out laterally due to the transverse momentum of the secondaries. On the other hand, gamma ray showers are more compact, electrons and photons are beamed in the forward direction and closer to the shower axis. These characteristics can be observed in Fig. 3, where the photons, electrons and positrons are shown in red, muons are green and hadrons are black.

Most of the secondaries hadrons produced in EAS are charged and neutral pions, including kaons in less quantity.

Charged pions have a slightly higher lifetime ($c\tau = 7.8 \text{ m}$) than charged kaons ($c\tau = 3.7 \text{ m}$) that are also present in the shower. Charged pions usually interact before decaying ($\pi^\pm \rightarrow \mu^\pm + \nu_\mu/\bar{\nu}_\mu$) once $E_{\pi^\pm} \leq 30 \text{ GeV}$, and charged kaons decay at higher energies.²⁷ These secondary hadrons (including other baryons) form the hadronic shower core.

Neutral pions decay quickly ($c\tau = 25 \text{ nm}$) into two photons forming the dominant source of the electromagnetic shower component. New photons are produced by photoproduction and a very small fraction of muons can be produced through muon pair production. 90 % of muons are produced in the hadronic cascade. Muons lose small fractions of energy throughout the atmosphere and can reach the surface of the Earth.

Muons and electromagnetic particles are the only particles that can be detected at the ground. Fig. 4 shows the electromagnetic (blue), hadronic (red) and muonic (green) components present in the development of a proton initiated shower.

2.2.1 Electromagnetic processes

The most relevant electromagnetic processes for the development of electromagnetic component in air showers are electron pair production by photons and bremsstrahlung by electrons in the Coulomb field of a nucleus:

Figure 3 –Side view of air shower initiated by a photon, a proton and an iron of 100 TeV each. Electromagnetic particles are shown in red, hadrons particles are black and muons are green.

Source: GREIDER.²⁵

- **electron pair production:**

$$\gamma \longrightarrow e^+ + e^-. \quad (2.4)$$

Occurs when the energy of a photon exceeds the pair creation threshold of 1.052 MeV. Below this threshold energy, Compton scattering and the photoelectric effect continue to contribute low energy electrons to the air showers. The electron pair production is the main energy loss mechanism of photons at energies above the critical energy^{*}, $E_{crit} = 24$ MeV (in the air).

- **Bremsstrahlung by Electrons:** correspond to radiation emitted by the deceleration of an electric charge $e^\pm$ in the presence of a nucleus of charge Z at a given distance of approach. The intensity of the radiation is proportional to

$$\left(\frac{N_A Z^2}{A m^2} \right) E, \quad (2.5)$$

* For gamma rays the critical energy, E_{crit} , is where Compton scattering begins to be more important than pair production.

Figure 4 –Longitudinal development of a proton initiated shower. It shows the three components in a EAS: electromagnetic (blue), hadronic (red) and muonic (green).

Source: PRADO.²⁶

where N_A is Avogadro's number, Z is the mass number, m is the mass of the projectile particle and E its energy. From equation (2.5) it is evident that in most cases the emission of bremsstrahlung by heavy particles can be neglected.

- **Coulomb Scattering of Electrons:** is responsible for the lateral spread of the electrons and, therefore, of the photons, in the development of an air shower. The number of deflections, k , that undergo an electron, when traverses a layer of thickness 1 g/cm^2 , is approximately given by

$$k \approx \frac{N_A}{A} d^2, \quad (2.6)$$

where N_A is Avogadro's number, A is the atomic number of the components of the medium and d is the diameter of the atom (scattering center). This means, that in the air there are roughly 10^6 scattering events per g/cm^2 .

There are other processes like Compton scattering that play an important role at the later stage of the shower development at locations away from the shower axis. Ionization losses are also present when charged particles are propagating in air.²⁸ The photo-electric effect is also considered in the interaction of photons but this process is only important at very low energy in air showers physics. At higher energies, the photoelectric cross section is small compared to other processes.²⁹

2.2.2 Hadronic interactions

As outlined in the previous subsections an EAS consists of the superposition of two kinds of cascades: electromagnetic and hadronic cascades. The electromagnetic cascade is chiefly the result of the decay of a neutral pion unless the primary particle is a gamma ray or an electron. Consequently, the electromagnetic cascade is built up through the repetitive of the electron pair production and bremsstrahlung processes. Therefore an EAS can be understood as the superposition of a huge number of electromagnetic sub-showers that result from different hadronic interactions along the atmosphere.

There are three specific hadronic interactions in air showers: nucleon-nucleus ($N - A$), nucleus-nucleus ($A - A$) and pion-nucleus ($\pi^\pm - A$).³⁰ To model the development of hadronic cascade, each of these interactions have also to consider the probability of a given projectile particle that undergoes an inelastic collision with a nucleus in the air. This probability is quantified by the inelastic cross section, σ_{inel} . Therefore in hadronic cascades, the inelastic cross section mostly determines the rate of growth of the cascade.³¹

The energy dependence of the inelastic cross section is important because of the enormous energy range of air showers, up to 10^{12} GeV, in the laboratory frame of reference. For that reason experiments of detection of EAS are important to extra information about unknown parameters and observables of high energy particle physics. Experiments that contribute with the proton-proton cross-section at Large Hadron Collider (LHC) at CERN in Geneva have been reported by ALICE³² (2.76 and 7 TeV), ATLAS³³⁻³⁵ (7, 8 and 13 TeV), CMS^{36,37} (7 and 13 TeV), LHCb^{38,39} (7 and 13 TeV) and TOTEM⁴⁰⁻⁴² (7, 8 and 13 TeV)[†].

Experiments like Large Hadron Collider (LHC) at CERN in Geneva contribute with cross section until a certain value of energy. For instance from collider experiments, above about 1 TeV the pp and $\bar{p}p$ cross sections are essentially identical, and the $\pi^\pm p$ cross sections are about 2/3 of the pp cross section:

$$\sigma_{inel}^{\pi p} \simeq \frac{2}{3} \sigma_{inel}^{pp}. \quad (2.7)$$

In cosmic ray physic, it is common to use the interaction mean free path, λ_{int} , instead of cross section. The reason for this is that the interaction mean free path express is practical to compute the location of interactions of a particle undergo when traversing a certain distance in the atmosphere. Cross section and the interaction mean free path are related by:

$$\lambda_{int} = \frac{A}{N_A \sigma_{inel}} \quad [\text{gcm}^{-2}], \quad (2.8)$$

where A is the mass number of the target nucleus and N_A is Avogadro's number.

[†] These energies dependence in the inelastic proton-proton cross-section, however, is in the center of mass of the interaction reference frame and, therefore, cannot be compared to the energies measured from energies of cosmic rays, which are in the laboratory reference frame.

2.3 Depth of the maximum development

The depth of maximum development or the shower maximum, X_{max} , is the location in g cm^{-2} where the maximum number of charged particles are generated. It depends on the process that governs the longitudinal development of the shower and is a function of the energy and the mass of the primary particle.

2.3.1 Heitler model

For electromagnetic showers, the X_{max} dependence in the energy of the primary can be understood in terms of the Heitler model⁴³ which is outlined in Fig. 5 a. In this model, one particle (electron) with energy E traverses a depth λ_e and any interaction leads to new two particles with energy $E/2$. The number of generations or consecutive interactions is denoted as n . Therefore, the number of particles, N , generated at the depth $X = n\lambda_e$ is given by

$$N(X) = 2^n = 2^{\frac{X}{\lambda_e}}, \quad (2.9)$$

and the energy E of the particle of the generation n is

$$E(X) = \frac{E_0}{2^{\frac{X}{\lambda_e}}}, \quad (2.10)$$

where E_0 is the energy of the primary particle with $n = 0$. The creation of new particles continues until the ionization-energy losses dominate over radiative losses at critical energy $E = E_c$. Replacing the maximum number of particles, given by

Figure 5 –Heitler model representation of an air shower. (a) An electromagnetic-induced shower. (b) A hadron-induced shower; solid blue lines represent $\pi^\pm$, and dashed red lines represent neutral pions π^0 .

$$N_{max} = \frac{E}{E_c}, \quad (2.11)$$

in eq. (2.9), we can get the maximum development:

$$X_{max}^{(EM)}(E_0) = \lambda_e \log_2 \left(\frac{E_0}{E_c} \right). \quad (2.12)$$

For hadronic showers, the need to treat particle decays does not allow getting analytical expressions for the longitudinal development. Nevertheless, Matthews⁴⁵ generalized the Heitler model including the hadronic component. In this case, a hadron can interact to produce n_{total} new particles with energy E/n_{total} . 2/3 of the new particles correspond charged pions and 1/3 to neutral pions. The total energy ($E_{had} + E_{EM}$) of the shower for each component at the generation n is given by:

$$E_{had} = \left(\frac{2}{3} \right)^n E_0, \quad E_{EM} = \left[1 - \left(\frac{2}{3} \right)^n \right] E_0. \quad (2.13)$$

With $n \approx 6$, 91% of the initial shower energy is carried by the electromagnetic component. Finally, the depth of the shower maximum is dominated by electromagnetic particles, so one can write:

$$X_{max}^{(had)}(E_0) \approx \lambda_{ine} + X_{max}^{(EM)} \left[\frac{E_0}{2n_{total}} \right], \quad (2.14)$$

where λ_{ine} is the hadronic interaction length.

Fig. 6 shows the mean depth of the shower maximum as a function of the energy of photon, proton, and iron initiated showers, with the measurements of some experiments and the predictions of simulations programs. Both, the measurements and numerical simulations confirm the predictions of the Heitler model that the depth of the shower maximum depends logarithmically on the primary energy E_0 . Moreover, the mean of the X_{max} is different for each primary particle, because is also a function to the primary mass. In general, for a primary particle of mass A and total energy E_0 , the mean depth of the maximum is given by⁴⁶

$$\langle X \rangle = X_0 \ln \left(\frac{E_0}{A} \right) + \lambda_A, \quad (2.15)$$

where λ_A is the interaction length of the primary.

The position of the X_{max} is an important quantity because it gives information about the mass composition of CR. To determine the mass A and the energy E_0 that originated the air shower, two independent quantities have to be measured: the shower size, N_e , and the X_{max} . The muon shower, N_μ , can be also used instead of the X_{max} . However, it is impossible to determine the mass composition and the energy on the shower-to-shower basis, because of the intrinsic fluctuations of the X_{max} (or the N_μ) of each cascade. Those fluctuations of shower-to-shower come from the stochastic nature of the interaction process, as the atmospheric depth at which the first interaction occurs. The experimental limitation also arises from those fluctuations. The Fig. 7 shows the fluctuations in the longitudinal development of air shower generated for different particles.

Figure 6 – Mean depth of shower maximum X_{max} as function of energy, compared with the predictions of CONEX¹ program for proton and iron initiated showers.

Source: RIEHN.⁴⁷

2.3.2 Gaisser-Hillas function

The X_{max} can be obtained from Fig. 7 by fitting a Gaisser-Hillas function.⁴⁸ The number of charged particles, N , generated at an atmospheric slant depth X in the development of an air shower could be parametrized by the Gaisser-Hillas function, which is given by:

$$N(X) = N_{max} \left(\frac{X - X_0}{X_{max} - X_0} \right)^{\frac{X_{max} - X_0}{\lambda}} e^{\frac{X_{max} - X}{\lambda}} \quad (2.16)$$

where N_{max} is the maximum number of particles at the maximum shower X_{max} , X_0 is the depth of the first interaction and λ is the elongation parameter. Others functions that parametrize the longitudinal shower profile can be found in.⁴⁹

2.4 Importance of the X_{max}

The mass and the energy of the primary particle (in an air shower) are important quantities because they are correlated with the nature of the accelerating astrophysical source.^{50,51} Therefore, cosmic ray composition at high energies is a fundamental observable to test and constraint theories concerning the origin and dynamics site properties of such radiation. At least two observables have to be measured to estimate both energy and mass of the primary cosmic ray. Those two shower observables are the depth of the shower

Figure 7 – Longitudinal development of different showers initiated by primaries of 10 TeV simulated with CONEX.² The fluctuations on the X_{max} from shower-to-shower is due to the stochastic nature of cascades.

Source: By the author

maximum, X_{max} , and the number of the muons at ground level, N_μ . These two quantities exhibit the strongest correlation with the mass of primary.⁵²

The overall distribution of X_{max} is composed of a mixture distribution $f_i(X_{max})$ produced by a nucleus of mass A_i :

$$f(X_{max}) = \sum_i p_i f_i(X_{max}), \quad (2.17)$$

where p_i is the fraction of the primary particle of type i . As a first approximation, primary mass A and energy E of a nucleus can be approximated as A protons with energy E/A . Then, the first two moments of $f(X_{max})$, i.e., its mean $\langle X_{max} \rangle$ and variance $\sigma^2 \langle X_{max} \rangle$, can be approximated as^{53,54}:

$$\langle X_{max} \rangle = \langle X_{max}^p \rangle - D_p \langle \ln A \rangle, \quad (2.18)$$

$$\sigma^2(X_{max}) = \langle \sigma_i^2 \rangle + D_p^2 (\langle \ln^2 A \rangle - \langle \ln A \rangle^2), \quad (2.19)$$

where $\langle X_{max}^p \rangle$ is the average of the shower maximum for protons, D_p is the elongation rate parameter for protons and its numerical value from air showers simulations is about 25 g/cm^2 , and $\langle \sigma_i^2 \rangle = \sum_i f_i \sigma_i^2(X_{max})$ is the composition averaged fluctuation. These equations explicitly demonstrate the relation of $\langle X_{max} \rangle$ to the average logarithmic mass of the cosmic ray composition and that $\sigma^2(X_{max})$ is proportional to the variance of $\ln A$.

Fig. 8 shows the first two moments of the X_{max} distribution measured by the Pierre Auger Observatory⁵⁵ from 10^{17} to 10^{20} eV. The mean of the X_{max} and the $\sigma(X_{max})$ are compared to proton (red) and iron (blue) pure compositions simulated with QGSJETII-04, Sibyll2.3, and EPOS-LHC. Fig. 9 shows the mean of the $\ln A$ (upper) and the variance (lower) of $\ln A$ compared to pure compositions of proton, helium, nitrogen, and iron, using QGSJETII-04, Sibyll2.3 and EPOS-LHC models.

The X_{max} between $10^{17.2}$ and $10^{18.33}$ eV (Fig. 8 left) demonstrates that the mean of the mass composition becomes lighter with increasing energy. Above $10^{18.33}$ eV the composition becomes heavier. The fluctuations of the shower maximum (Fig. 8 right) after $10^{18.33}$ eV also shows a heavier composition. In the case of the mean of $\ln A$ (Fig. 9) the interpretation of the mass composition becomes dependent on the validity of the hadronic interaction models.

Figure 8 – The first two moments of the X_{max} distributions data of Pierre Auger. They are compared to proton and iron using QGSJETII-04, Sibyll2.3, and EPOS-LHC.

Source: PIERRE AUGER COLLABORATION.⁵⁶

Figure 9 –The first two moments of the $\ln A$ distributions using QGSJETII-04, Sibyll2.3, and EPOS-LHC.

Source: PIERRE AUGER COLLABORATION.⁵⁶

3 IMAGING ATMOSPHERIC CHERENKOV TELESCOPE

3.1 Introduction

In this chapter, we are going to describe the imaging atmospheric Cherenkov technique used to study an extensive air shower. To understand this technique it is important to know about the Cherenkov radiation produced by an air shower in the atmosphere.

3.2 Cherenkov light

3.2.1 Fundamental physics process

In 1934, Pavel Cherenkov proposed the existence of Cherenkov radiation⁵⁷ as well as in the subsequent publications.⁵⁸ This effect is produced by ultra-relativistic charged particles moving a speed faster than the speed of light in a dielectric medium. The geometry of the phenomenon is shown in Fig. 10. As a result of the polarization of the medium, the electromagnetic radiation emitted by the dipole transitions spread over a band of frequencies interfering constructively under a particular emission angle, θ , measured from the direction of the motion and combine to form a plane wave beyond to certain distance from their origin. The θ angle defines the Cherenkov cone.

Figure 10 –Geometry of Cherenkov light phenomenon.

Source: GRIEDER.⁵⁹

If n is the index of refraction of the medium and $\beta = v/c$ characterizes the velocity of the charged particle, v , respect that of the light in vacuum, c , the coherence condition can be obtained from momentum and energy conservation the trigonometric relationship

as⁶⁰:

$$\cos(\theta) = \frac{1}{\beta n} + \frac{\hbar k}{2p} \left(1 - \frac{1}{n^2}\right), \quad (3.1)$$

where $\hbar k$ is the momentum of the photon, p is the momentum of charged particle and λ is the wavelength of the emitted photon. The second term is due to the photon recoil on the moving charged particle causing a tiny deflection on the charged particle. For the following conclusions we are going to consider that

$$\cos(\theta) = \frac{1}{\beta n}, \quad (3.2)$$

due to the second term in eq. 3.1 $\frac{\hbar k}{2p} \left(1 - \frac{1}{n^2}\right)$ is negligible. In a medium with a fixed refractive index η there is a threshold velocity $\beta_{th} = 1/n$ below which no radiation Cherenkov is generated, i.e., the angle of emission θ is zero. There also exists a maximum angle of emission, θ_{max} , called Cherenkov angle, that occurs at $\beta = 1$ given by

$$\theta_{max} = \arccos\left(\frac{1}{n}\right). \quad (3.3)$$

3.2.2 Radiation Yield Distribution

The rate of energy loss of charged particle due to Cherenkov light when traversing a dielectric medium is given by Frank-Tamm relation⁶¹:

$$\left(\frac{dE}{dl}\right) = \left(\frac{ze}{c}\right)^2 \int_{\beta\eta > 1} \left(1 - \frac{1}{\beta^2\eta^2}\right) \omega d\omega \quad [\text{eV/cm}], \quad (3.4)$$

where z is the charge of the particle, and ω is the frequency of the emitted radiation. It is possible to simplify the Frank-Tamm relation if we know the frequency ω dependence on the refractive index. To get the number of Cherenkov photons, N_{ph} , from Eq.(3.4) we can make the connection with the density of photons, $n_\gamma(\omega, l)$ per unit path length and unit frequency of light, where:

$$N_{ph} = \int \int d\omega \, dl \, n_\gamma(\omega, l), \quad (3.5)$$

and the total energy E of the Cherenkov radiation is scaled by the photon energy given by $\hbar\omega$:

$$E = \int \int d\omega \, dl \, \hbar\omega n_\gamma(\omega, l). \quad (3.6)$$

Now both equations can be connected by differential descriptions:

$$d^2 N_{ph} = n_\gamma(\omega, l) d\omega \, dl, \quad (3.7)$$

$$d^2 E = \hbar\omega n_\gamma(\omega, l) d\omega \, dl, \quad (3.8)$$

to get:

$$\frac{d^2 N_{ph}}{d\omega \, dl} = \frac{1}{\hbar\omega} \frac{d^2 E}{d\omega \, dl}. \quad (3.9)$$

Using the latter equation combine with (3.4) we can get the number of Cherenkov photons radiated along a path of length l . We then obtain

$$N_{ph} = \frac{2\pi (ze)^2 l}{\hbar c} \left(1 - \frac{1}{\beta^2 n^2}\right) \left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right). \quad (3.10)$$

For the same dielectric medium, the Cherenkov threshold energy depends on the mass charged particle. As an example in the air at sea level, where the refraction index is $n = 1.00029$, one obtains for the Cherenkov threshold energy of an electron, muon, proton, and iron is $E_{th}^e = 21.22$ MeV, $E_{th}^\mu = 4.38$ GeV, $E_{th}^p = 38.96$ GeV and $E_{th}^{Fe} \approx 1$ TeV. The maximum Cherenkov emission angle in the air at sea level is $\theta = 1.3^\circ$. The number of Cherenkov photons of an electron of energy above threshold in this medium is approximately 0.3 photons/cm.

3.3 Cherenkov imaging technique

The Cherenkov imaging technique is based on the detection of the Cherenkov light emitted by shower charged particles. All contribution of each particle Cherenkov light emission produces a cone of this radiation that is detected by imaging atmospheric Cherenkov telescopes (IACT) located at the ground. The objective of IACTs is to discriminate the Cherenkov light of gamma-ray air showers from the background of light pulses of the numerous cosmic ray showers, to study potential TeV gamma sources. For this reason, Monte Carlo simulations of the shower development and Cherenkov light emission were necessary to establish differences that could permit to select gamma ray showers. In Fig. 11 these differences are shown and explained below.

The lateral distribution of the Cherenkov light intensity from gamma ray showers is more uniform than that from cosmic ray showers. For gamma ray showers, the intensity is roughly flat from shower core until more than ~ 100 m and then decreases with distance. The peak around 100 m is because of the focussing effect resulting from the changing angle of Cherenkov emission with atmospheric depth.⁶²

The time duration of the pulse can be longer for cosmic ray showers due to contains more penetrating particles that survive to level detector (mostly muons). Finally, the angular spread of an image of Cherenkov light has a more regular distribution from gamma ray showers and is more compact and smaller than in the case of hadron showers. For that reason this prove was the most powerful discriminant based in the shape of the image light.

Gamma ray initiated showers have Cherenkov light patterns on an imaging detector that show an elongated elliptic shape and are much more compact than proton showers. Proton showers image on detectors show light spots distributed in irregular patches giving a less elliptic region. This technique is an efficiency and confidence way to identify gamma showers. The typical pixel pattern of a Cherenkov light detection of a gamma ray shower

has an elliptic shape as is illustrated in Fig. 12. The solid ellipse in the upper right of the figure represents a pixel pattern observed by the camera of the telescopes. Its shape and orientation can be identified by the Hillas parameters.

Figure 11 –Main differences in the detection of gamma and cosmic ray showers.

Source: WEEKES.⁷

Figure 12 – Hillas parameters of air shower Cherenkov image analysis to select gamma ray initiated showers and discriminate against proton showers. The X-Y coordinate system corresponds to the telescope. The solid ellipse shows the Cherenkov image where C is the centroid of the image and M is the center of the field of view. The main parameters are the major and minor axis of the ellipse that is labeled as *Length* and *Width*, the *Distance* between C and M, the *Miss* that is the offset of the image and the *Azwidth* that is the azimuthal width of the image. The dashed ellipse, labeled *On-Source Gamma Ray Image* shows the typical form for a gamma ray shower detection when the telescopes axis is pointing at the source.

Source: GRIEDER.⁵⁹

3.3.1 Hillas parameters

In 1985, Hillas proposed to distinguish between background cosmic ray showers and gamma ray showers from a point source based on some parameters that characterize the Cherenkov light images of the air showers.⁶³ These relevant parameters, known as Hillas parameters, are six: centroid, length, width, distance, miss, and azwidth (an abbreviation of azimuthal-width). Fig. 12 shows the Hillas parameters where C is the centroid of the ellipse and M is the center of the field of view, i.e. the source where the telescope is pointing. The X-Y coordinate system corresponds to the telescope. The definitions of parameters are listed below²⁴:

- **Length and width** are the major and minor axis of the ellipse, labeled with the corresponding names. These parameters mark the r.m.s of the spread light and represent the longitudinal and lateral development of the shower.
- **Miss** is the offset, i.e. the perpendicular distance between the extension of the major axis of the ellipse and M.

- **Distance** is the distance between C and M.
- **Azwidth** is the r.m.s spread of light perpendicular to the line to the line connecting C with M.
- ϕ is the angle between the major axis respect to the X-axis of the X-Y coordinate system.
- α is the angle between the major axis of the ellipse and the line that connects C with M. The angle α is being used in place of the parameter miss. It can be calculated from $\alpha = \arcsin(\text{miss}/\text{distance})$.

There are other shower image parameter definitions like:

- **Size** is the total integrated light content of the shower.
- **Frac2** is the degree of light concentration as determined from the ratio of the two largest tube signals to the sum of all signals.
- **Asym** is a measure of how asymmetric the image is.

3.3.2 Image parametrization

The parametrization of shower image properties is based on a moment fitting approach. Due to the elliptical nature of the shower images, the parametrization is performed in terms of the moment analysis of the record pixel signal amplitudes. The moments are calculated with the ADC (analog to digital converter) counts and the pixel coordinates. The position of each pixel is given by the coordinates x and y , and the number of ADC counts after image cleaning is given by n . Then, the moments are defined as²⁴:

$$\begin{aligned} \langle x \rangle &= \frac{\sum n_i x_i}{\sum n_i}, & \langle x^2 \rangle &= \frac{\sum n_i x_i^2}{\sum n_i}, & \langle xy \rangle &= \frac{\sum n_i x_i y_i}{\sum n_i}, \\ \langle y \rangle &= \frac{\sum n_i y_i}{\sum n_i}, & \langle y^2 \rangle &= \frac{\sum n_i y_i^2}{\sum n_i}, \end{aligned} \quad (3.11)$$

and the spreads of the images in different directions are defined in terms of the moments as follows:

$$\begin{aligned} \sigma_{x^2} &= \langle x^2 \rangle - \langle x \rangle^2, \\ \sigma_{y^2} &= \langle y^2 \rangle - \langle y \rangle^2, \\ \sigma_{xy} &= \langle xy \rangle - \langle x \rangle \langle y \rangle. \end{aligned} \quad (3.12)$$

Then images parameters can be calculated from the moments and the image spreads as follows:

$$\text{distance} = \sqrt{\langle x \rangle^2 + \langle y \rangle^2}, \quad (3.13)$$

$$length = \sqrt{\frac{\sigma_{x^2} + \sigma_{y^2} + s}{2}}, \quad (3.14)$$

$$width = \sqrt{\frac{\sigma_{x^2} + \sigma_{y^2} - s}{2}}, \quad (3.15)$$

$$miss = \sqrt{\frac{1}{2} (u \langle x \rangle^2 + v \langle y \rangle^2) - \left(\frac{2\sigma_{xy} \langle x \rangle \langle y \rangle}{s} \right)}, \quad (3.16)$$

$$\sin(\alpha) = \frac{miss}{distance}, \quad (3.17)$$

$$azwidth = \sqrt{\frac{\langle x^2 \rangle \langle y^2 \rangle - 2 \langle x \rangle \langle y \rangle \langle xy \rangle \langle x \rangle^2 \langle y \rangle^2}{(distance)^2}}, \quad (3.18)$$

where $d = \sigma_{y^2} - \sigma_{x^2}$, $s = \sqrt{d^2 + 4(\sigma_{xy}^2)}$, $u = (1 + \frac{d}{s})$ and $v = 2 - u$.

4 METHODS OF X_{max} DETERMINATION

The methods to determine the X_{max} depend on what signature measure the experiment: Cherenkov light, secondary particles or air fluorescence. In this chapter is presented two methods to measure the X_{max} : the first with an air Cherenkov signature based on the Cherenkov lateral distribution and the second based on the fluorescence light induced by the EAS in the atmosphere. This later is roughly near to our work.

4.1 Cherenkov Photon Lateral Distribution Function

In 1973, Smith and Turver⁶⁴ suggested based on their theoretical work that the measurement of the ratio of the Cherenkov photon density, $Q(r)$, at small and large core shower distances, r , may indicate the stage of development shower. They simulated 10^{17} eV proton initiated showers with different X_{max} and plotted the Cherenkov photon density as a function of the shower core distance as shown in Fig. 13 a. To describe the photon lateral density distribution, $Q(r)$, they used a power law of the form

$$Q(r) = Cr^{-\gamma}, \quad (4.1)$$

where C and γ are free parameters. The ratio, R , of the photon density at $r_1 = 100$ m, $Q(100)$, to that at $r_1 = 600$ m from the shower core, $Q(600)$, is a function of the X_{max} as shown in Fig. 13 b.

In 1979, M. N. Dyakonov et al.⁶⁵ presented experimental data obtained at Yakutsk array⁶⁶ to determine the depth of the shower maximum. The observed values of $Q(r)$ at $100 < r < 800$ m and $0^\circ < \theta < 45^\circ$ were approximated by expression:

$$Q(r) = 2^{\beta-\alpha} Q_{400} \left(\frac{r}{400} \right)^{-\alpha} \left(1 + \frac{r}{400} \right)^{-\beta+\alpha}, \quad (4.2)$$

where α and β are given by:

$$\alpha = 0.9 + 0.15 \log \left(\frac{Q_{400}}{10^7} \right) - 0.80 (\sec \theta - 1), \quad (4.3)$$

$$\beta = 4.5 + 0.15 \log \left(\frac{Q_{400}}{10^7} \right) - 3.33 (\sec \theta - 1). \quad (4.4)$$

The accuracy of the method can be improved by choosing an appropriate r_1 and r_2 for each array, in this case, 200 and 550 m, as in.⁶⁵ The parameter $\log(Q_{200}/Q_{550})$ is the ratio light flux density measured at distances 200 and 550 m from the shower core of an EAS observed at ground level. According to their calculations, this parameter is correlated to the X_{max} and it has a very weak dependence on the shape the EAS development. For EAS with $\theta = 0^\circ$, it was found that:

$$\log \left(\frac{Q_{200}}{Q_{550}} \right) = 0.54 + 6.64 \times 10^{-4} X_{max}. \quad (4.5)$$

Figure 13 –(a) Cherenkov photon density, Q , as a function of core distance, r , measured at sea level by 10^{17} eV proton initiated showers. (b) The ratio, R , of the photon density at $r_1 = 100$ m, $Q(100)$, to that at $r_1 = 600$ m from the shower core, $Q(600)$, as a function of depth of maximum development, X_{max} , of 10^{17} eV proton initiated showers as shown 13a.

Source: SMITH.⁶⁴

This relationship is valid for lateral density fitting with a correlation coefficient* greater than 0.9. Therefore, the values of maximum development, X_{max} , can be estimated using the eq. (4.5). They also obtained the following dependence of X_{max} and the primary particle energy, E_0 :

$$X_{\text{max}} = (700 \pm 30) + (120 \pm 75) \times \log \left(\frac{E_0}{10^{18}} \right) [\text{g/cm}^2]. \quad (4.6)$$

4.1.1 Correlation between radio signals and X_{max}

From 1972 to 1989 Yakutsk array added experiments to detect radio signals emitted by EAS. The radio array of Yakutsk recorded EAS radio emissions between 2009 and 2013, and showers with energies above 10^{19} eV. They used the radio pulse amplitudes detected at 175 m and 750 m from the shower axis (instead of the Cherenkov photon density), to establish a correlation between radio signals at 32 MHz, A , and the maximum shower, X_{max} .⁶⁷ The relationship that they obtained can be analytically represented by the formula:

$$X_{\text{max}} = a \left(\ln \left[b \frac{A_{175}}{A_{750}} \right] \right)^c [\text{g/cm}^2], \quad (4.7)$$

* The correlation coefficient corresponds to the correlation between the flux density ratio and the X_{max} .

where a , b , and c are constant coefficients.⁶⁸

4.1.2 X_{max} results of Yakutsk experiment

Fig. 14 shows the result of X_{max} measured by Yakutsk (triangles) array using the technique described above. They are compared to data obtained measuring X_{max} at MSU⁶⁹ (diamonds) and LOPES⁷⁰ (dots) arrays. Fig. 14 also shows the simulation of X_{max} using the model QGSJET II-04 for a pure mass composition of protons (solid line), nuclei of carbon (dash line) and nuclei of iron (dot line).

For $10^{17} - 10^{18}$ eV, the X_{max} shows an increasing trend with big statistical uncertainty. Yakutsk results demonstrate that radio emission and Cherenkov measurements can be used to calculate the X_{max} (therefore, the mass composition) as an independent method. In the range of $10^{17} - 10^{18}$ eV, the estimation of $\langle \ln A \rangle$ indicated a change of the cosmic ray mass composition from heavy to light nuclei above 3×10^{17} eV.

Figure 14 – X_{max} measured by Yakutsk (triangles) array compared to data obtained at MSU (diamonds) and LOPES (dots) arrays. Simulation of X_{max} using the model QGSJET II-04 for a pure mass composition of protons (solid line), nuclei of carbon (dash line) and nuclei of iron (dot line), is also shown.

Source: PETROV.⁷¹

4.2 X_{max} by air fluorescence shower

Fluorescence light is produced by the excitation of nitrogen molecules due to the passage of the charged particles generated in the air shower⁷² and thus measure the longitudinal profile of EAS. This technique has been used by some experiments like HiRes,⁷³ Telescope array⁷⁴ and Pierre Auger.⁷⁵ In 2015, the Pierre Auger Collaboration

reported a study of the distribution of X_{max} of EAS profiles with energies above $10^{17.8}$ eV observed with fluorescence telescope of the observatory. The fluorescence telescopes of the observatory consist of a Fluorescence Detector (FD) that allows the detection of ultraviolet fluorescence light during the nights and under favorable meteorological conditions.

The event reconstruction is performed as follows: each detection of the PMT pixel in the camera is validated if it belongs to the shower image using Hough-transformation⁷⁶ and it is used to reconstruct the plane spanned as a function of the axis of the air shower and the telescope position. With the plane spanned, the shower arrival direction is reconstructed from the arrival times of the shower light considering the viewing angle. The arrival time is a function of the surface-detector station closest to the shower axis. The geometry of the reconstruction is shown in Fig. 15.

Figure 15 –Reconstruction geometry of X_{max} of the Pierre Auger Observatory. Shower event 15346477 with $\theta = 58.3^\circ$ and $\phi = 324.7^\circ$.

Source: PIERRE AUGER COLLABORATION.⁷⁶

Each signal in the PMTs is converted to a general time-trace of light that depends on the calibration of the optical system. At each time t_i , all the previous signals are sum up within an opening angle, ζ_{opt} , that is 1.3° on average. Then, every time bin is projected to a portion of the length, Δl_i , on the shower axis centered at height h_i . The photo-electrons of each bin is proportional to the energy deposit by the charged particles. The projection is taking account of the attenuation of light due to Rayleigh and Mie scattering. h_i is transformed to slant depth, X_i , by integrated the atmospheric density through a curved atmosphere. A detailed description of the process can be found in.⁷⁶ This process is also taking account of the emission of Cherenkov photons in the shower, and it is estimated following,⁷⁷ to calculate the contribution of direct and scattered Cherenkov light to the shower profiles. In Fig. 16 is shown the final step of the reconstruction. The data of energy deposit as a function of the slant depth is fitted using the Gaisser-Hillas function to obtain the shower maximum, X_{max} , and the total energy of the shower, E .

The resolution of the method determines the broadening of the original distribution due to statistical fluctuations of the X_{max}^{rec} around the true shower maximum. In Fig. 17 is shown the resolution of the X_{max} reconstruction where the bands denote its systematic uncertainty. The resolution decreases with the energy and it is about 15 g/cm² above 10¹⁹ eV. The X_{max} resolution is better than 25 g/cm² around 10¹⁸ eV.

Figure 16 – Fitting of Gaisser-Hillas function of the event 15346477 with $\theta = 58.3^\circ$ and $\phi = 324.7^\circ$.

Source: PIERRE AUGER COLLABORATION.⁷⁶

Figure 17 – Shower maximum resolution as a function of the energy shower. The bands are the systematic uncertainties.

Source: PIERRE AUGER COLLABORATION.⁷⁶

5 PROPOSAL OF A METHOD TO RECONSTRUCT X_{max} WITH IACTS

In this chapter the method for the determination of the X_{max} is presented. The simulations of gamma, protons, and iron initiated air showers and the process to obtain the *shower face plane* used to project the Cherenkov photons are explained.

The 2D histograms with the longitudinal and lateral development of the projection of Cherenkov photons is analyzed. The curves of the $X_{max}^{Cherenkov}$ as a function of the distance from the telescope to shower core are discussed. Finally, the resolution in the X_{max} reconstruction is presented.

5.1 Description of the method

In an air shower, the interaction of the secondary charged particles with the atmosphere produces Cherenkov photons. The left side of Fig. 18 shows individual profiles of the number of charged particles (blue lines) and production of Cherenkov photons (black lines) along the atmosphere for gamma, proton and iron showers of 3 TeV, 100 TeV, and 30 TeV, respectively.

For the same air shower, the depth of the maximum quantity of charged particles and Cherenkov photons coincides roughly in the same position in the atmosphere. then, the position where the maximum quantity of charged particles was reached is correlated to the position where the maximum quantity of Cherenkov photons has to be produced.

This latter conclusion could be observed in the distributions of the shower maximum (the right side of Fig. 18) obtained from charged particles (blue lines) and Cherenkov photons (black lines) profiles of more than 2000 air showers of the same primary composition and energy described above, i.e. the mean value of X_{max} coincide for both charged particles and Cherenkov photons distributions. For 100 TeV proton, 30 TeV iron and 3 TeV gamma showers both distributions match approximately at 490 g/cm², 300 g/cm² and 377 g/cm².

Therefore, the measurement of the maximum development for Cherenkov photons could give information about the X_{max} of an EAS.

Figure 18 – Left side column: Comparison of longitudinal development between Cherenkov photons and charged particles profiles of three (solid, dash and dot lines) showers of different mass composition and energy. Right side column: distribution of the shower maximum, X_{max} , for charged particles and Cherenkov photons profiles generated by showers of different mass composition and energy.

Source: By the author

Taking into account the previous discussion, the geometry of Fig. 19 was designed and used to study air showers as follows: the air shower (red) produces a cascade of particles, which produce a Cherenkov photon (blue) that is detected by the Cherenkov telescopes (dash circles) located at the ground. Each telescope contains mirrors where the Cherenkov photons are reflected toward a camera, which contains PMTs* as pixels, located in the focus of the telescope. Cherenkov photons are just detected if they hit inside of the field of view (FOV) of the Cherenkov telescope. The orientation of the telescopes allows to associate a depth (or height) in the atmosphere for each pixel of the optical sensor, so for each Cherenkov photon we can associate a depth in the atmosphere from where it comes.

Figure 19 –Scheme of geometry used to study the longitudinal development of air shower.

Source: By the author

The first approximation to simulate the geometry described above was to not consider physical telescopes in the simulations but to collect all photons hitting the detector area, drawn as a circle in the observation level.

The detection of Cherenkov photons was considered if they just traversed the circle solid that represents the telescope. The computational package of CORSIKA was used to simulate air showers with different primary particles and energies. The output file of CORSIKA gives information about the number of photons that hit at a specific position, direction, and height where they were generated. With this information was possible to project the Cherenkov photons back in a plane called *shower face plane*.

* Photomultiplier tube.

5.1.1 Shower face plane

The shower face plane contains the direction of the primary particle, specified by the vector $\vec{l}$, and is perpendicular to the vector that connects the shower core to each telescope, $\vec{d}_{tel}$. In the system of reference of Fig. 19, the shower face plane can be expressed as the set of points $\vec{p}$ for which

$$(\vec{p} - \vec{p}_0) \cdot \vec{n} = 0, \quad (5.1)$$

where $\vec{n}$ is a normal vector to the shower face plane, and $\vec{p}_0$ is a point of the plane, in this case, it is the shower core which is $\vec{p}_0 = (0, 0, 0)$. $\vec{n}$ can be found using the cross product between $\vec{l} \times (\vec{d}_{tel} \times \vec{l})$. Each point $\vec{p}$ of the shower face plane can be found by the line-plane intersection. Defining $\vec{p}$ as

$$\vec{p} = \vec{l}_0 + d\vec{l}_{ph}, \quad (5.2)$$

where $\vec{l}_0$ is the photon impact position, d is a scalar and $\vec{l}_{ph}$ is the direction of the photon. Then, using equations 5.2 and 5.1 the value of d is obtained:

$$d = \frac{(\vec{p}_0 - \vec{l}_0) \cdot \vec{n}}{\vec{l}_{ph} \cdot \vec{n}}. \quad (5.3)$$

The scalar d can be replaced in Eq. 5.2 to get each point of the shower face plane. This technique involves projecting each Cherenkov photon detected back into the shower face plane. The points in the shower face plane are stored in a 2D histogram. An example of the Cherenkov photons projected in the shower face plane is shown in the Fig. 20.

Figure 20 – Shower face plane of 300 TeV gamma induced shower for a telescope located at 140 m from shower core. In the horizontal axis is the lateral distance measured in meters and the vertical axis is the longitudinal development measured in g/cm^2 .

Source: By the author

This histogram corresponds to a 300 TeV gamma shower reconstructed from a telescope located at 140 m from shower core. The X and Y-axis of the 2D histogram

represent the lateral spread (in meters) and the longitudinal development (in g/cm^2) of the air shower based on the projection of Cherenkov photons.

It is important to note that the shower face plane has its system of coordinate and it depends on the telescope position, $\vec{d}_{tel}$. For that reason, each telescope has its shower face plane as can be seen in Fig. 21.

Figure 21 – Shower face plane of different initiated primary showers reconstructed for telescopes located at 50 m (up) and 200 m (low) from the shower core.

Source: By the author

5.2 Simulation Setup

Large samples of air shower were simulated using the software CORSIKA.⁷⁸ Three primary particles were considered: gamma, proton ($A = 1$) and iron ($A = 56$) with energies of 10 TeV, 30 TeV, 100 TeV and 300 TeV. The energy values were determined so that they match the range covered by CTA and VERITAS observatory. The incident zenith and azimuthal angle of the primary particles were set to 20° and randomly, respectively. The zenith angle of 20° was chosen because of considering inclined air showers, which exemplify the observations of gamma ray sources. The cutoff energy for secondary particles of the air shower was: 300 MeV for hadrons (without π^0 's), 100 MeV for muons, 20 MeV for electrons and 20 MeV for photons (including π^0 's).

The atmosphere adopted by CORSIKA contained 78.1% of N_2 , 21.0% O_2 and 0.9% of Ar . The density of the atmosphere, T , measured in $[g\ cm^{-2}]$, is modeled by 5 layers. The first lower fourth layers follow an exponential dependence on the altitude:

$$T(h) = a_i + b_i \cdot \exp^{-h/c_i} \quad i = 1, \dots, 4 \quad . \quad (5.4)$$

The density of the atmosphere at fifth layer decreases linearly with altitude:

$$T(h) = a_5 - b_5 \cdot \frac{h}{c_5} \quad . \quad (5.5)$$

The model parameters of the atmosphere used were of June 16, 1993, of middle Europe listed in table 1. The level of observation was 2150 m above sea level. The telescopes were set along the axis of the frame system of CORSIKA, from the shower core (0 m) to 300 m in steps of 10 m, as it is shown in the Fig. 19. The radius of the telescopes was set at 2.8 m. The field of view (fov) angle of the telescope was set to 10° . For lower fov angles, the histograms 2D of $fov = 10^\circ$ are recycled by making a geometric cut. The high and low hadronic interaction models considered were EPOS-LHC⁷⁹ and FLUKA,⁸⁰ respectively. For each combination of primary mass and energy, 10^5 air showers were simulated.

Table 1 – Parameters of the AT616 atmosphere (June 16, 1993).

Layer i	Altitude h (km)	$a_i(g/cm^{-2})$	$b_i(g/cm^{-2})$	$c_i(g/cm^{-2})$
1	$0 \dots 4$	-208.12899	1251.474	1032310.
2	$4 \dots 10$	-120.26179	1173.321	925528.
3	$10 \dots 40$	0.31167036	1307.826	645330.
4	$40 \dots 100$	5.591489×10^{-4}	763.1139	720851.4
5	> 100	0.01128292	1	10^9

Source: HECK.⁷⁸

5.3 Analysis of Cherenkov light distribution

Around 10^4 showers were analyzed for each composition of primary and energy. The 2D histograms were analyzed within the ROOT[†] framework. The left side of Fig. 22 shows the histograms for four different positions: 10 m, 50m, 130m, and 250m. It can be seen that the number of Cherenkov photons detected decreases with distance from the shower core, as it is expected. Each histogram was converted into a profile along the vertical axis to analyze the longitudinal development, as can be seen on the right side of Fig. 22. These profiles represent the longitudinal development and each one has a position in slant depth where the maximum quantity of the projection of Cherenkov photons were produced, called as $X_{max}^{Cherenkov}$.

To get the $X_{max}^{Cherenkov}$, a second-degree polynomial was fitted:

$$N(X) = p_0 + p_1X + p_2X^2, \quad (5.6)$$

where p_i are the free parameters of the fit. Then, the $X_{max}^{Cherenkov}$ can be obtained by the derivative of Eq. 5.6

$$\left. \frac{dN(X)}{dX} \right|_{X=X_{max}^{Ch}} = 0 \implies X_{max}^{Cherenkov} = -\frac{p_1}{2p_2}. \quad (5.7)$$

The error in the X_{max} was calculated by the error propagation which is

$$\Delta X_{max}^{Cherenkov} = \frac{p_1}{2p_2} \sqrt{\left(\frac{\Delta p_1}{p_1}\right)^2 + \left(\frac{\Delta p_2}{p_2}\right)^2 - 2\frac{\rho_{12}}{p_1 p_2}}, \quad (5.8)$$

where ρ_{12} is the element of the covariance matrix in the polynomial fitting.

In order to optimize the quality of our fitting profiles the following cuts were applied:

- **Shape cut:** parameters of the fitting must be real numbers and the parameter p_2 must be negative to guarantee the parabola is facing down
- **Fit range cut:** The X_{max}^{Ch} must be inside of the range of fitting, otherwise it will not be taken into account. Also, if the X_{max}^{Ch} fitting is near from the cut of the profile by 50 g/cm², it will not be taken into account.
- **Fit quality:** Only events with a good longitudinal profile fit ($\chi^2/ndf < 10$) to a "pol2" function are accepted.

[†] ROOT is a modular scientific software toolkit. More information can be found in <https://root.cern.ch/>.

Figure 22 – Left side column: histogram 2D of the longitudinal and lateral distribution of Cherenkov photons projected on the shower plane of an induced gamma shower of 300 TeV for each telescope along to axis-x at (from top to bottom) 10 m, 50 m, 130 m, and 250 m. Right side column: Fitting of a second degree polynomial around of the maximum bin in the longitudinal distribution, to get the $X_{max}^{Cherenkov}$ for each position of the telescope, respectively.

Source: By the author

The 2D histograms of Fig. 22 also reflect the properties of different kinds of air showers. The lateral dispersion in the histograms for gamma showers is lower when compare to hadronic showers of the same energy, due to the compactness of gamma showers. For showers with higher energies and for heaviest nuclei, the number of Cherenkov photons in the histograms is much larger.

In the case of hadronic showers, some energetic muons generate single Cherenkov rings, which distort the Cherenkov light distribution of the shower in the 2D histograms. This kind of contamination was more frequent for hadronic showers of lower energies.

The 2D histograms of Fig. 22 also have a central region where the maximum number of Cherenkov photons is being projected. This region is correlated with the position where they were produced. Cherenkov photons that are produced near the ground are projected to a depth of around $\sim 1000 \text{ g/cm}^2$. This peak of the X_{max} can be seen in the two first profiles on the left side of Fig. 22 and the quantity of photons is large. As telescope detections go further away from the shower core, the projection is higher in the atmosphere and near 0 g/cm^2 . A second peak appears because this region corresponds to the maximum production of charged particles and Cherenkov photons.

The charged particles distribution of each shower was also saved and fitted with the Gaisser-Hillas function,

$$N(X) = N_{max} \left(\frac{X - X_0}{X_{max} - X_0} \right)^{\frac{X_{max} - X_0}{\lambda}} e^{\frac{X_{max} - X}{\lambda}}, \quad (5.9)$$

described in Chapter 2, to get the real X_{max} of the air shower. The Fig. 23 shows the Gaisser-Hillas fitting for charged particles distribution of a 30 TeV iron (23a) and 300 TeV gamma (23b) shower with 20° of zenith angle. In this fitting, the χ^2/ndf does not have a statistical meaning due to the absence of error bars in the charged particle profiles. It was considered, however, a cut in the χ^2/ndf to avoid anomalous longitudinal profiles in the case of proton showers.⁸¹

(a)

(b)

Figure 23 –Charged particle profile as a function of the slant depth of (a) 30 TeV iron and (b) 300 TeV gamma shower at 20° (black curve), and Gaisser-Hillas function fitted to the charged particle profile (red line).

Source: By the author

5.4 Results

5.4.1 $X_{max}^{Cherenkov}$ as function of telescope position

Plots of Fig. 24 show $X_{max}^{Cherenkov}$ obtained from the quadratic fitting of the Cherenkov light distribution of a gamma, proton and iron shower of 300 TeV as a function of the telescope position to the shower core. Each plot contains three curves with showers of 20° of zenith angle and random azimuthal angle. The X_{max} is labeled as $X_{max}^{charged}$ (dash lines). It can be seen in all three curves a descent from the shower core to about 120 m. This is the depth where Cherenkov photons are being projected decreases with an increasing distance to the telescope. After ~ 120 m, there is a plateau. Similar behavior was observed for 100, 30 and 10 TeV of primary particle energy of the same composition.

Figure 24 – $X_{max}^{Cherenkov}$ of gamma, proton and iron shower of 300 TeV as a function of the telescope position. The $X_{max}^{Cherenkov}$ was obtained from second-degree polynomial fitting of the longitudinal development. Three different air showers were plotted (markers) with their corresponding $X_{max}^{charged}$ (dash lines) for each primary particle. Field of view angle = 10° .

Source: By the author

The interpretation of the curve can be understood as follows: the Cherenkov angle increases from $\sim 0.2^\circ$ at an altitude of ~ 30 km to $\sim 1.5^\circ$ at sea level (reference). This variation causes the formation of a ring of light at a distance of ~ 120 m from the shower core. The region of the light annulus contains the majority Cherenkov light of the shower particles. As a result, there is a disk of light at ~ 120 m called Cherenkov ring. Inside of the light annulus, the Cherenkov light comes from particles of the last stage of the longitudinal development of the air shower. This behavior can be seen in the plots of Fig. 24 in which the most of Cherenkov photons near from the shower core are generated near the ground (~ 1000 g/cm²) and then the altitude of Cherenkov photon production increases as the distance from the telescope to shower core.

Outside of the Cherenkov ring (i.e. ~ 120 m) the quantity of Cherenkov light decrease with the lateral distance. There is, however, lateral scattering particles in the air shower that can produce Cherenkov light outside of light annulus. There is a high probability that such particles come from the region of the X_{max} . Therefore, the constant-like behavior after ~ 120 m could be correlated to the X_{max} . This constant behavior after 120 m was called *apparent shower maximum* ($X_{max}^{apparent}$). The $X_{max}^{apparent}$ shows a correlation with the real shower maximum, $X_{max}^{charged}$, as is shown in the plots of Fig. 24.

The difference between the $X_{max}^{apparent}$ and $X_{max}^{charged}$ is different for each air shower. This difference can be understood in the sense of projection geometry. The thickness of the air shower around the shower maximum is some tens of meters. Therefore the detected Cherenkov photons at the ground is a contribution of the Cherenkov radiation produced along the line of view that traverses the air shower. Taking account that the shower maximum is above ~ 10 km and the telescopes are located till 300 m, the projection of the Cherenkov photons back in the shower face plane does not reflect the real position where they were produced. This difference is evident between $X_{max}^{charged}$ and $X_{max}^{apparent}$ in plots of Fig. 24 and it depends on the development of the air shower.

It was also plotted the mean of $X_{max}^{Cherenkov}$ as a function of the distance from the telescope to shower axis for different fields of view, as is shown in Fig. 25, for energy value of 300 TeV (Fig. 25a) and 30 TeV (Fig. 25b). From the shower core to 120 m, the mean of $X_{max}^{Cherenkov}$ decreases as the position of the telescope, which explains the majority production of Cherenkov photons (and charged particles) comes from the latter development of the air shower. From 120 m, it begins to appear the constant like behavior that could be correlated to the $X_{max}^{charged}$. This means that the location where the maximum quantity of charged particles is being produced, then the maximum quantity of Cherenkov photons have to be generated. It is important to note that a small field of view angles limits the capacity to measure the full Cherenkov light distribution from telescopes after 150 m.

Figure 25 – Mean of $X_{max}^{Cherenkov}$ as a function of the distance to telescope from shower core for gamma, proton and iron showers of (a) 300 TeV and (b) 30 TeV of 20° of zenith angle. The curves are plotted for different fields of view of the telescope.

Source: By the author

5.4.2 Correlation between $X_{max}^{charged}$ and $X_{max}^{apparent}$ for the same primary particle

Due to the constant like behavior after ~ 120 m showed in plots of Fig. 24, the $X_{max}^{apparent}$ was correlated with the real shower maximum ($X_{max}^{charged}$). The $X_{max}^{apparent}$ was obtained as a result of a constant fitting considering different quantity of telescopes. Four configurations were used:

- **6 telescopes:** at 150 m, 160 m, 170 m, 180 m, 190 m, and 200 m.
- **3 telescopes:** at 150 m, 170 m, and 190 m.
- **1 telescope:** at 200 m.
- **1 telescope:** at 150 m.

Plots of Fig. 26 correspond to the configuration of 6 telescopes and show the correlation between $X_{max}^{charged}$ and $X_{max}^{apparent}$ for three different primary particles: gamma, proton and iron of 10 TeV, 30 TeV, 100 TeV, and 300 TeV of energy. Around $\sim 10^4$ air showers with $\text{fov} = 10^\circ$ were used to this analysis. The results show a linear correlation between $X_{max}^{charged}$ and $X_{max}^{apparent}$ as was expected.

The points of Fig. 26a and Fig. 26b are more concentrated in comparison to Fig. 26c, which are more scattered with decreasing energy. In the case of iron initiated showers of low energies, the Cherenkov light distribution does not follow a well-known behavior compared to gamma and proton showers. This is due to the Cherenkov radiation comes from charged particles distributed along with the air shower. Then the $X_{max}^{Cherenkov}$ is difficult to extract and the $X_{max}^{apparent}$ could not be constant in the range [150 - 200] m. Moreover, the cuts of quality reduce below 50% of the sample. For this reason the $X_{max}^{apparent}$ in Fig. 26c has spread around of $\approx 300 \text{ g/cm}^2$ for 30 and 10 TeV iron showers. For 300 and 100 TeV, the dispersion reduces markedly.

It is important to note that in the plots of Fig. 26 the mean and the dispersion of the real X_{max} can be extracted directly from the $X_{max}^{apparent}$. This offers the possibility to reconstruct the $X_{max}^{charged}$ using the $X_{max}^{apparent}$ or even used the $X_{max}^{apparent}$ as new observable to characterize the air showers. On the other hand, Fig. 26a shows outliers below of $X_{max}^{apparent} = 400 \text{ g/cm}^2$ for 10 TeV. This is due to errors in the fitting reconstruction of the $X_{max}^{Cherenkov}$ and $X_{max}^{apparent}$. The cuts in the $X_{max}^{apparent}$ until 800 g/cm^2 in the plots of Fig. 26 is due to the value of fov angle (10°) in the simulations..

(a)

(b)

(c)

Figure 26 –Correlation between $X_{max}^{charged}$ and $X_{max}^{apparent}$ for gamma (a), proton (b) and iron (c) showers of 10, 30, 100 and 300 TeV of energy with $\theta = 20^\circ$. Telescopes with a field of view angle of 10° .

Source: By the author

5.4.3 X_{max} reconstruction

As it was shown before, there is a linear correlation between $X_{max}^{charged}$ and $X_{max}^{apparent}$ for the same primary particle. Therefore it was proposed to reconstruct the real shower maximum ($X_{max}^{charged}$) through a linear fitting between $X_{max}^{charged}$ and $X_{max}^{apparent}$ considering the configurations of telescopes mentioned above. In this case, the correlation is based on a mixed composition of gamma, proton, and iron initiated air shower at the same energy value. Fig. 27a shows a linear fitting for the correlation between the $X_{max}^{charged}$ and $X_{max}^{apparent}$ for 300 TeV air showers generated by gamma, proton and iron particle, using the configuration of 6 telescopes.

From the linear fitting, it gets the shower maximum reconstructed (X_{max}^{rec}). The X_{max}^{rec} was calculated for different values of primary particle energy. Fig. 27b shows the difference between X_{max}^{rec} and $X_{max}^{charged}$ for the fitting of Fig. 27a. This difference could be approximated by a Gaussian function,

$$f(X_{max}^{rec} - X_{max}^{charged}) = \frac{1}{\sigma\sqrt{2\pi}} \exp^{-\frac{1}{2}\left[\frac{(X_{max}^{rec} - X_{max}^{charged}) - \mu}{\sigma}\right]^2}, \quad (5.10)$$

where μ is the mean value and σ is the standard deviation (Std Dev). In this case, σ is taken as the X_{max} resolution of our technique. The smaller the standard deviation, the better the reconstruction of X_{max} will be.

The reconstruction was done for each value of energy simulated. The standard deviation was plotted as the X_{max} resolution as a function of the energy as can be seen in plots of Fig. 28. The different configurations of telescopes were taking account and are labeled with the telescope positions.

The field of view of the telescopes was also considered in the resolution of the X_{max} reconstruction. Fig. 28a and 28b are the X_{max} resolution curves for the field of view of 10° and 3.5° , respectively. Both curves are essentially the same, with the difference of the green curve that uses one telescope in the reconstruction located at 200 m from the shower core. The interpretation of such difference at high energies is caused by the small field of view of the telescopes that cuts part of the Cherenkov light profiles and does not allow an optimal reconstruction of the $X_{max}^{charged}$.

Figure 27 –(a) Correlation of $X_{max}^{charged}$ vs $X_{max}^{apparent}$ for $X_{max}^{Cherenkov}$ adjusted in telescopes from 150m to 200m of gammas (black), protons (red) and iron (green) air showers. (b) Resolution calculated from the subtraction of X_{max}^{rec} minus $X_{max}^{charged}$.

Source: By the author

(a)

(b)

Figure 28 – Resolution in the reconstruction of the $X_{max}^{charged}$ as a function of the energy for a field of view of 10° (a) and 3.5° (b).

Source: By the author

6 ANALISYS OF EVENTS SIMULATED FOR THE VERITAS OBSERVATORY

In the previous chapter, the reconstruction technique of X_{max} using IACT was proposed and explained. It was shown that IACT could measure the $X_{max}^{apparent}$ after 150 m from the shower core and, consequently, the $X_{max}^{apparent}$ can be correlated to $X_{max}^{charged}$ to reconstruct the real shower maximum. Therefore, to use experimental data, it is necessary to determine how IACTs reproduce the bidimensional histograms for the $X_{max}^{Cherenkov}$ longitudinal and lateral development profiles of air showers.

In this chapter, air showers simulations granted by the VERITAS collaboration are analyzed. The simulations contain air showers of proton and iron for 0° and 20° of the zenith angle. The layout of the simulation and the parameters of the shower extracted from the simulations are also presented. For proton and iron showers, the procedure used in the last chapter is applied to shower simulations of the VERITAS observatory, including the simulation of telescope responses. Finally, the resolution in the X_{max} reconstruction for proton and iron is presented.

6.1 VERITAS

6.1.1 Specifications

The Very Energetic Radiation Imaging Telescope Array System (VERITAS) is an array of four imaging atmospheric Cherenkov telescopes (IACTs) located at Fred Lawrence Whipple Observatory in southern Arizona, USA (see Fig. 29). The four 12 m diameter telescopes are located at an altitude of 1268 m. The mirror area is approximately 106 m^2 and contains 499 pixel camera on each telescope.

The system works in an energy range from 85 GeV to 30 TeV. The observatory is designed to study gamma ray emission from astrophysical sources in the very high energy (50 GeV - 50 TeV). Its construction was based on the design of the Whipple 10 m telescope and the first light with VERITAS was detected in 2007.

6.1.2 Parameters extracted of shower simulations

Air showers simulations were granted by the VERITAS collaboration to be analyzed. VERITAS uses the VEGAS⁸³ software package for the processing of telescope data and event simulations. The array simulated is illustrated in Fig. 29. The field of view of the telescope was 3.5 degrees. The simulations contain 500 recycled air showers of 0° and 20° each one of proton and iron with energies from $5 \times 10^{10} \text{ eV}$ to $5 \times 10^{14} \text{ eV}$. For our analysis was only taking account air showers with energies above to 1 TeV.

Figure 29 –Layout of the VERITAS telescopes.

Source: The VERITAS Collaboration.⁸²

The parameters extracted from the air shower simulations were: all parameters of the second-degree polynomial fitting in order to get $X_{max}^{Cherenkov}$, upper and lower limits of fitting (in order to reject $X_{max}^{Cherenkov}$ out of limits), the real shower maximum, primary particle energy, direction of the shower (zenith and azimuth), distance from core shower to telescope detection, and the position of image in the camera. The process to reconstruct the X_{max} is similar to the described in Chapter 5. The pixels (PMTs) of the camera (see Fig. 30a) are projected in the shower face plane (see Fig. 30b).

The color in the ellipses of Fig. 30b indicates the corresponding pixel in the camera of Fig. 30a. In this case, the points of the shower face plane are limited to the quantity of pixels (PMTs) in the telescope. Therefore the $X_{max}^{Cherenkov}$ is obtained from the second-degree polynomial fitting with 5 or 6 points. This, however, guarantees that the region with the maximum number of Cherenkov photons is considered.

6.2 Analyze of event simulations

6.2.1 Selection cuts

To optimize the quality of the reconstruction of the profiles the following cuts were applied:

- **Shape cut:** parameters of the second degree polynomial must converge and the slope of the parabola must be negative to guarantee the parabola is facing down.
- **Energy cut:** Since the points in the shower face plane histogram are Cherenkov photons detected dependent, we have observed that in the events with energies below 10 TeV the fitting of the second-degree polynomial does not exist or is very bad, giving a poor reconstruction of the X_{max} . On the other hand, there are few events

Figure 30 –(a) Event simulation detected by the camera of VERITAS telescope. (b) Projection of the pixel in the shower face plane. The color of the ellipse indicates the corresponding pixel in the camera.

Source: SOUZA.⁸⁴

above 200 TeV. Therefore we restricted our analysis to events with energy larger than 10 TeV and lower than 200 TeV.

- **Fit quality:** only events with a good second-degree polynomial fit ($\chi^2/ndf \leq 2$) to shower face profile at maximum Cherenkov photons projection are accepted.
- **Image shape:** When the image formed in the telescope is near to the edges of the camera, it is just taking account a part of the longitudinal development of the air shower. So, only events with images well centered in the camera with $r < 1.5^\circ$ are considered in our analysis.

6.2.2 Mean $X_{max}^{Cherenkov}$ as function of the telescope position

Plots of the mean of the $X_{max}^{Cherenkov}$ as a function of the distance of telescope to shower axis were done as can be seen in plots of Fig. 31. Iron (blue) and proton (red) showers of $\theta = 0^\circ$ and $\theta = 20^\circ$ were plotted for three different energy intervals: [9-11], [20-40] and [80-120] TeV. It is important to note that each shower can trigger four telescopes as a maximum. The surviving events and the total number of telescopes triggered are shown at tables 2a and 2b for proton of $\theta = 0^\circ$ and $\theta = 20^\circ$ and tables 2c and 2d for iron of $\theta = 0^\circ$ and $\theta = 20^\circ$, respectively.

Plots of Fig. 31 present, as expected, a similar behavior to the plots described in the last chapter. For telescope positions ≤ 150 m, the curves have a similar slope and describe the final stage in the development of the air showers. For telescope positions > 150 m, however, the curves seem to be slightly constant. Nevertheless, telescope detections after 150 m still could be associated with the X_{max} as was already explained. Using this assumption, the $X_{max}^{apparent}$ can be obtained from the telescopes that were triggered after 150 m.

Another important feature that must be taken into account is the difference between proton and iron curves. As discussed in Chapter 2, the shower maximum depends on the mass and energy primary particle. If the $X_{max}^{apparent}$ is a direct measurement of the X_{max} , the difference between proton and iron curves would have almost to indicate the difference between the mean of X_{max} . Despite showing a very small difference in the plots of Fig. 31, it is a difference that does exist.

Table 2 – Number of total showers and total triggered telescopes (trig tel) for each energy interval for the curve of proton of (a) 0 and (b) 20 degrees of zenith angle and curve of iron of (c) 0 and (d) 20 degrees of zenith angle of the plots of Fig. 31

E [TeV]	N° of showers	N° trig tel
9 - 11	171	294
20 - 40	856	590
80 - 120	119	171

(a)

E [TeV]	N° of showers	N° trig tel
9 - 11	35	62
20 - 40	71	121
80 - 120	31	54

(b)

E [TeV]	N° of showers	N° trig tel
9 - 11	411	534
20 - 40	1487	2135
80 - 120	484	783

(c)

E [TeV]	N° of showers	N° trig tel
9 - 11	78	104
20 - 40	295	447
80 - 120	159	251

(d)

Source: By the author.

Figure 31 – Mean of $X_{\text{Cherenkov}}^{\text{max}}$ as function of the position of the telescope. $X_{\text{Cherenkov}}^{\text{max}}$ was obtained from second-degree polynomial fitting to the longitudinal development profile. The blue and red curve belong to iron and proton showers of (a) [9-11] TeV, (b) [20-40] TeV and (c) [80-120] TeV for $\theta = 0^\circ$, respectively.

Source: By the author

6.2.3 Resolution curves

The events selected have been grouped into 6 bins of energy. Each bin has a width of 0.2 in $\log_{10}$ (E/eV). The linear correlation and fitting between $X_{max}^{charged}$ and $X_{max}^{apparent}$ were done for each bin of energy as it is shown in Fig. 32a for proton showers of zenith angle = 20° with energy 13.25 in $\log_{10}$ (E/eV) for proton showers of zenith angle = 20° with energy 13.25 in $\log_{10}$ (E/eV). In this case, the $X_{max}^{apparent}$ was obtained with the configuration of four telescopes (VERITAS array).

Then the reconstruction of the X_{max} was done calculating the X_{max} resolution from the difference between X_{max}^{rec} and $X_{max}^{charged}$ (see Fig. 32b). Considering the cut selection explained above, the overall event selection efficiencies are lower (around 50%) in the case of protons, and slightly discreet (around 30%) in the case of iron as can be seen in Tables 3a and 3b for 0 and 20 degrees of zenith angle, respectively.

The resolution ranges from about 80 g/cm² for the proton curve to less than 30 g/cm² for the iron curve of 0 degrees of zenith angle. In the case of $\theta = 20^\circ$, the resolution is slightly similar for the iron curve, and for the proton curve. It ranges from 80 g/cm² at lower energies to less than 30 g/cm² at the highest energy (see Fig. 33a and 33b). We note in Fig. 33b that the last three points at 13.75, 14.00 and 14.25 in $\log_{10}$ (E/eV) were obtained with few event simulated (some tens of air showers).

Table 3 – Selection efficiencies for proton and iron Monte Carlo showers of (a) 0 and (b) 20 degrees of zenith angle as a function of energy.

$\log_{10}(\text{E/eV})$	$\epsilon_p(\%)$	$\epsilon_{Fe}(\%)$
13.00	61	37
13.25	52	35
13.50	51	33
13.75	44	33
14.00	43	33
14.25	36	30

(a)

$\log_{10}(\text{E/eV})$	$\epsilon_p(\%)$	$\epsilon_{Fe}(\%)$
13.00	57	40
13.25	61	35
13.50	59	31
13.75	53	30
14.00	63	35
14.25	47	29

(b)

Source: By the author.

(a)

(b)

Figure 32 – (a) Correlation of $X_{max}^{charged}$ vs $X_{max}^{apparent}$ for proton showers of zenith angle = 20° with energy 13.25 in $\log_{10}(E/\text{eV})$, using the configuration of four telescopes from VERITAS experiment for proton showers. (b) Resolution calculated from the subtraction of X_{max}^{rec} minus $X_{max}^{charged}$.

Source: By the author

(a)

(b)

Figure 33 –Energy evolution of the resolution that it was obtained, on the basis of the $X_{\max}^{\text{apparent}}$, when it was reconstructed $X_{\max}^{\text{charged}}$ from the linear correlation between $X_{\max}^{\text{apparent}}$ and $X_{\max}^{\text{charged}}$ for showers simulations of VERITAS observatory of (a) 0 and (b) 20 degrees of zenith angle.

Source: By the author

7 CONCLUSIONS

In this work, the possibility of measuring the depth of the maximum development of extensive air showers using imaging atmospheric Cherenkov telescopes have been studied.

Extensive air shower is on the most well-studied branch in the phenomenology of cosmic rays. It helps to detect cosmic rays and gamma rays at large energies using detectors located at the ground. In the last decades, many authors have presented studies about the development of air shower and how to use them to infer information about the primary particle. For instance, many of these works use the shower maximum, X_{max} and the number of muons (N_μ) to infer the mass, A , and the energy, E , of the primary particle. Therefore, knowing the shower's observables is essential in order to determine the energy and mass composition of cosmic rays.

In this work, the use of Cherenkov photons generated by secondaries charged particles to study the longitudinal development of the air shower has been presented. Although the cosmic rays shower detections by IACT's are rejected due to the fact that they act as a background and contaminate the study of a source of gamma rays, they could give information about some parameters of the air shower. For this reason, the Cherenkov telescopes as VERITAS or CTA provides a fruitful scenario to test this work, since both gamma rays and cosmic rays are detected by IACTs.

The main goal of this work was to study the $X_{max}^{Cherenkov}$ as a function of the position of the telescope respect to the center of the shower, using the photons Cherenkov detected by an array of IACT's, and, thus, measure the shower maximum X_{max} .

For the development of air showers, it has been discussed that the electromagnetic component is dominant for gamma and cosmic rays initiated shower at high energies. This permit the generation of huge quantity of Cherenkov photons (for charge particles above Cherenkov threshold energy) at the altitude where the maximum quantity of secondary charged particles was generated. This is very important because it provides the connection between the $X_{max}^{Cherenkov}$ and $X_{max}^{Charged}$, where $X_{max}^{Charged}$ is the real shower maximum of the air shower. The calculation proposed here establish that at a certain distance, after of the Cherenkov ring, approximately from 120 m and till 250 m, the projection of the Cherenkov photons in a spanned plane on the shower, called as *shower face plane*, would give direct information of the X_{max} of the air shower. The simulations showed that the X_{max} can be reconstructed from different configurations of telescopes between 150 m and 250 m from the center of the shower.

In this work air showers initiated by gamma ray, proton and iron of 10 TeV, 30 TeV, 100 TeV and 300 TeV of 20 degrees of zenith angle were simulated.

For gamma ray showers, the curve of the $X_{max}^{Cherenkov}$ as a function of the telescope position shows a constant value after the Cherenkov ring, 120 m approximately, and therefore, it presents a correlation with the X_{max} . Nevertheless, due to the projection of the Cherenkov photons in the shower face plane, X_{max} is not equal to the $X_{max}^{Cherenkov}$. Cherenkov photons are produced about 100 m around the shower axis at an altitude of approximately 10 km, and their projections in the shower face plane from the telescope located some meters from shower core at ground, would give a different depth for both measurements, $X_{max}^{Cherenkov}$ and $X_{max}^{Charged}$, of the same shower.

For protons and irons, the effect describes above would be a little major, because of the characteristic of not compactness of hadronic showers. Nevertheless, the plots of $X_{max}^{Cherenkov}$ as a function of the distance of the telescope to shower axis show also a constant behavior after 150 m approximately, and, thus, it is a strong evidence to correlate the $X_{max}^{Cherenkov}$ with the $X_{max}^{Charged}$.

As a result of the above mentioned, the $X_{max}^{Cherenkov}$ was fitted using a different number of telescopes that detected Cherenkov photons in the region between 150 m and 200 m, called as $X_{max}^{apparent}$. The correlation between the $X_{max}^{apparent}$ and $X_{max}^{Charged}$ for the same primary particle and different values of energies was done. This showed a linear correlation between $X_{max}^{apparent}$ and $X_{max}^{Charged}$. For this reason, it was proposed a linear correlation between $X_{max}^{apparent}$ and $X_{max}^{Charged}$ for a mixed composition of gamma, proton, and iron initiated air showers with the same value of energy. The linear fitting permitted to reconstruct the real shower maximum and the residuals were adjusted using a Gaussian function where the standard deviation is the resolution of the technique.

Finally, the reconstruction of the X_{max} was done considering $X_{max}^{apparent}$ weighted up for 1, 3 and 6 telescopes. The resolution of X_{max} reconstruction has also been presented for the first time with this technique. It was not observed a big difference in the resolution for 10 and 3.5 degrees of field of view of the telescope. The main reason is that the telescope axis was equal to the shower axis, in such a way that it can still observe the total longitudinal development of the air shower. In the worst case, the resolution is around 55 g/cm² considering just one telescope and 15 g/cm² for the best.

In summary, it is concluded in this work that the total longitudinal development of an air shower can be studied using imaging atmospheric Cherenkov telescopes and the $X_{max}^{Cherenkov}$ between 150 m and 200 m have strong constant behavior, which can be correlated directly with the $X_{max}^{charged}$. From the projection of the Cherenkov photons in the shower face plane, it is possible to reconstruct all the longitudinal and lateral development of the air shower. For that reason, a gamma ray astronomy observatory, such as VERITAS, was used to test this method.

The VERITAS observatory contains four Cherenkov telescopes to study gamma ray emissions in the energy range from 50 GeV to 50 TeV. It was used the database

of air shower simulations of proton and iron of 0 and 20 degrees of zenith angle. The technique proposed in this work was applied to the database simulations of VERITAS. The $X_{max}^{Cherenkov}$ was obtained from the second-degree polynomial for each telescope. For the first time, the mean of $X_{max}^{Cherenkov}$ as a function of the distance of the telescope to the shower core is presented for the VERITAS observatory. It has been shown that for proton and iron showers of 0 and 20 degrees of zenith angle, the curve $\langle X_{max}^{Cherenkov} \rangle$ as a function of telescope position seems to be slightly constant from 150 m.

It has been shown that in the case of 20 degrees of zenith angle, the resolution curves on the shower maximum reconstruction ranges around 80 g/cm² for proton and around 30 g/cm² for iron. Both iron curves at 0 and 20 degrees are similar. Both proton curves at 0 and 20 degrees range from 80 g/cm² at lower energies to less than 30 g/cm² at the highest energy. Nevertheless, it is still necessary to mention that a small number (around 500 showers) of air shower events were used to do the resolution curve of $X_{max}^{charged}$ reconstruction.

This work, however, provides a wide and useful framework for study the longitudinal development of the extensive air showers in the scenario of cosmic and gamma rays that can be detected with Cherenkov telescopes. It also can be adapted to the new generation of IACTs as it could be the case of CTA.

REFERENCES

- 1 BERGMANN, T. et al. One-dimensional hybrid approach to extensive air shower simulation. **Astroparticle Physics**, v. 26, n. 6, p. 420–432, 2007. doi: [10.1016/j.astropartphys.2006.08.005](https://doi.org/10.1016/j.astropartphys.2006.08.005).
- 2 ALEKSEEVA, M. et al. Extensive air shower simulation program CONEX: matching Monte Carlo and numerical methods. In: INTERNATIONAL COSMIC RAY CONFERENCE (ICRC), 29, 2005. **Proceedings...** Pune, India: ICRC, 2005. p. 139. Available from: <http://cds.cern.ch/record/965307/files/17139-ger-ostapchenko-S-abs2-he14-poster.pdf>. Accessible at: 11 June 2019.
- 3 GAISSER, T. K.; ENGEL, R.; RESCONI, E. **Cosmic rays and particle physics**. 2nd. ed. Cambridge: Cambridge University Press, 2016. doi: [10.1017/CBO9781139192194](https://doi.org/10.1017/CBO9781139192194).
- 4 MONTARULI, T. Gamma-rays and their future. **Nuclear Physics B - proceedings supplements**, 2019. Available from: <https://arxiv.org/pdf/1902.10484.pdf>. Accessible at: 11 June 2019.
- 5 AUGER, P. et al. Extensive cosmic-ray showers. **Reviews of Modern Physics**, v. 11, n. 3-4, p. 288–291, 1939. doi: [10.1103/RevModPhys.11.288](https://doi.org/10.1103/RevModPhys.11.288).
- 6 KEILHAUER, B. et al. Nitrogen fluorescence in air for observing extensive air showers. **European Physical Journal Web of Conferences**, v. 53, p. 01010, 2013. doi: [10.1051/epjconf/20135301010](https://doi.org/10.1051/epjconf/20135301010).
- 7 WEEKES, T. C. The Atmospheric Cerenkov imaging technique for very high energy gamma-ray astronomy. 2005. Available from: <https://arxiv.org/pdf/astro-ph/0508253.pdf>. Accessible at 11 June 2019.
- 8 KNURENKO, S. P.; SABOUROV, A. The depth of maximum shower development and its fluctuations: cosmic ray mass composition at $E_0 \geq 10^{17}$ eV. **Astrophysics and Space Sciences Transactions**, v. 7, n. 3, p. 251–255, 2011. doi: [10.5194/astra-7-251-2011](https://doi.org/10.5194/astra-7-251-2011).
- 9 WEEKES, T. C. et al. VERITAS: the very energetic radiation imaging telescope array system. **Astroparticle Physics**, v. 17, p. 221–243, 2002. doi: [10.1016/S0927-6505\(01\)00152-9](https://doi.org/10.1016/S0927-6505(01)00152-9).
- 10 BLANDFORD, R.; SIMEON, P.; YUAN, Y. Cosmic ray origins: an introduction. **Nuclear Physics B - proceedings supplements**, v. 256-257, p. 9 – 22, 2014. doi: [10.1016/j.nuclphysbps.2014.10.002](https://doi.org/10.1016/j.nuclphysbps.2014.10.002).
- 11 BECK, R.; WIELEBINSKI, R. Magnetic fields in the milky way and in galaxies. 2013. Available from: <https://arxiv.org/pdf/1302.5663.pdf>. Accessible at: 11 June 2019.
- 12 BIETENHOLZ, W. The most powerful particles in the Universe: a cosmic smash. 2013. Available from: <https://arxiv.org/pdf/1305.1346.pdf>. Accessible at: 11 June 2019.
- 13 DEGRANGE, B.; FONTAINE, G. Introduction to high-energy gamma-ray astronomy. **Comptes Rendus Physique**, v. 16, p. 587–599, 2015. doi: [10.1016/j.crhy.2015.07.003](https://doi.org/10.1016/j.crhy.2015.07.003).

- 14 GRIEDER, P. K. F. **Extensive air showers: high energy phenomena and astrophysical aspects - a tutorial, reference manual and data book**. Berlin, Heidelberg: Springer-Verlag, 2010. doi: [10.1007/978-3-540-76941-5](https://doi.org/10.1007/978-3-540-76941-5).
- 15 THE PIERRE AUGER COLLABORATION. Reconstruction of inclined air showers detected with the Pierre Auger Observatory. **Journal of Cosmology and Astroparticle Physics**, v. 1408, n. 08, p. 019, 2014. doi: [10.1088/1475-7516/2014/08/019](https://doi.org/10.1088/1475-7516/2014/08/019).
- 16 BAHMANABADI, M. et al. A study of the effect of geomagnetic field on extensive air showers with small arrays. **Experimental Astronomy**, v. 13, n. 1, p. 39–57, 2002. doi: [10.1023/A:1022038816555](https://doi.org/10.1023/A:1022038816555).
- 17 IVANOV, A. A. Azimuthal effect on extensive air showers of cosmic rays. In: INTERNATIONAL COSMIC RAY CONFERENCE (ICRC), 26, 1999. Salt Lake City. **Proceedings...** Utah, USA: ICRC, 1999. p. 403. Available from: <http://articles.adsabs.harvard.edu/pdf/1999ICRC....1..403I>. Accessible at: 11 June 2019.
- 18 LIU, J.-S. et al. Development of Yangbajing air shower core detector for a new EAS hybrid experiment. **Chinese Physics C**, v. 39, p. 086004, 2015. doi: [10.1088/1674-1137/39/8/086004](https://doi.org/10.1088/1674-1137/39/8/086004).
- 19 PETERS, B. Primary cosmic radiation and extensive air showers. **Il Nuovo Cimento (1955-1965)**, v. 22, n. 4, p. 800–819, 1961. doi: [10.1007/BF02783106](https://doi.org/10.1007/BF02783106).
- 20 LUO, Y. et al. Simulation study on origin of multi-core events in cosmic rays extensive air showers. 2013. Available from: <https://arxiv.org/pdf/1309.3973.pdf>. Accessible at: 11 June 2019.
- 21 REBEL, H. et al. Arrival time distributions of muons from extensive air showers as signature of the mass composition of cosmic rays. **Journal of Physics G: nuclear and particle physics**, v. 21, n. 3, p. 451–472, 1995. doi: [10.1088/0954-3899/21/3/019](https://doi.org/10.1088/0954-3899/21/3/019).
- 22 CECCHINI, S.; SPURIO, M. Atmospheric muons: experimental aspects. **Geoscientific Instrumentation, Methods and Data Systems Discussions**, v. 2, p. 603–641, 2012. doi: [10.5194/gid-2-603-2012](https://doi.org/10.5194/gid-2-603-2012).
- 23 BRETZ, T. Zenith angle dependence of the cosmic ray rate as measured with imaging air-Cherenkov telescopes. **Astroparticle Physics**, v. 111, p. 72–86, 2019. doi: [10.1016/j.astropartphys.2019.02.004](https://doi.org/10.1016/j.astropartphys.2019.02.004).
- 24 FEGAN, D. J. Hadron separation at tev energies. **Journal of Physics G: nuclear and particle physics**, v. 23, n. 9, p. 1013–1060, 1997. doi: [10.1088/0954-3899/23/9/004](https://doi.org/10.1088/0954-3899/23/9/004).
- 25 GRIEDER, P. K. Introduction, facts and phenomenology. In: _____. **High energy phenomena and astrophysical aspects - a tutorial, reference manual and data book**. Berlin, Heidelberg: Springer-Verlag, 2010. cap. 1, p. 3–32. doi: [10.1007/978-3-540-76941-5_1](https://doi.org/10.1007/978-3-540-76941-5_1).
- 26 PRADO, R. R. **Estudo da composição de raios cósmicos de altas energias através da análise de dados medidos pelo Observatório Pierre Auger**. 2014. Dissertação (Mestrado em Ciências) — Instituto de Física de São Carlos, Universidade de São Paulo, São Carlos, 2014. doi: [10.11606/D.76.2014.tde-23042014-155508](https://doi.org/10.11606/D.76.2014.tde-23042014-155508).

-
- 27 ENGEL, R.; HECK, D.; PIEROG, T. Extensive air showers and hadronic interactions at high energy. **Annual Review of Nuclear and Particle Science**, v. 61, n. 1, p. 467–489, 2011. doi: [10.1146/annurev.nucl.012809.104544](https://doi.org/10.1146/annurev.nucl.012809.104544).
- 28 GRIEDER, P. K. F. Electromagnetic interactions and photon–electron cascades. In: _____. **Extensive air showers: high energy phenomena and astrophysical aspects - a tutorial, reference anual and data book**. Berlin, Heidelberg: Springer-Verlag, 2010. cap. 4, p. 147–203. doi: [10.1007/978-3-540-76941-5_4](https://doi.org/10.1007/978-3-540-76941-5_4).
- 29 RAO, M. V. S.; SREEKANTAN, B. V. Phenomenology. In: _____. **Extensive air showers**. Singapore: World Scientific Publishing Co. Pte. Ltd., 1998. cap. 2, p. 9–32. doi: [10.1142/9789812817211_0002](https://doi.org/10.1142/9789812817211_0002).
- 30 GRIEDER, P. K. Hadronic interactions and cascades. In: _____. **Extensive air showers: high energy phenomena and astrophysical aspects - a tutorial, reference anual and data book**. Berlin, Heidelberg: Springer-Verlag, 2010. cap. 3, p. 77–146. doi: [10.1007/978-3-540-76941-5_3](https://doi.org/10.1007/978-3-540-76941-5_3).
- 31 ULRICH, R.; ENGEL, R.; UNGER, M. Hadronic multiparticle production at ultra-high energies and extensive air showers. **Physical Review D**, v. 83, p. 054026, 2011. doi: [10.1103/PhysRevD.83.054026](https://doi.org/10.1103/PhysRevD.83.054026).
- 32 ABELEV, B. et al. Measurement of inelastic, single- and double-diffraction cross sections in proton–proton collisions at the LHC with ALICE. **The European Physical Journal C**, v. 73, n. 6, p. 2456, 2013. doi: [10.1140/epjc/s10052-013-2456-0](https://doi.org/10.1140/epjc/s10052-013-2456-0).
- 33 AAD, G. et al. Measurement of the inelastic proton-proton cross-section at $\sqrt{s} = 7$ TeV with the ATLAS detector. **Nature Communications**, v. 2, p. 463, 2011. doi: [10.1038/ncomms1472](https://doi.org/10.1038/ncomms1472).
- 34 AABOUD, M. et al. Measurement of the total cross section from elastic scattering in pp collisions at $\sqrt{s} = 8$ TeV with the ATLAS detector. **Physics Letters B**, v. 761, p. 158–178, 2016. doi: [10.1016/j.physletb.2016.08.020](https://doi.org/10.1016/j.physletb.2016.08.020).
- 35 _____. Measurement of the inelastic proton-proton cross section at $\sqrt{s} = 13$ TeV with the ATLAS detector at the LHC. **Physical Review Letters**, v. 117, n. 18, p. 182002, 2016. doi: [10.1103/PhysRevLett.117.182002](https://doi.org/10.1103/PhysRevLett.117.182002).
- 36 CHATRCHYAN, S. et al. Measurement of the inelastic proton-proton cross section at $\sqrt{s} = 7$ TeV. **Physics Letters B**, v. 722, p. 5–27, 2013. doi: [10.1016/j.physletb.2013.03.024](https://doi.org/10.1016/j.physletb.2013.03.024).
- 37 SIRUNYAN, A. M. et al. Measurement of the inelastic proton-proton cross section at $\sqrt{s} = 13$ TeV. **Journal of High Energy Physics**, v. 07, p. 161, 2018. doi: [10.1007/JHEP07\(2018\)161](https://doi.org/10.1007/JHEP07(2018)161).
- 38 AAIJ, R. et al. Measurement of the inelastic pp cross-section at a centre-of-mass energy of $\sqrt{s} = 7$ TeV. **Journal of High Energy Physics**, v. 02, p. 129, 2015. doi: [10.1007/JHEP02\(2015\)129](https://doi.org/10.1007/JHEP02(2015)129).
- 39 _____. Measurement of the inelastic pp cross-section at a centre-of-mass energy of 13 TeV. **Journal of High Energy Physics**, v. 06, p. 100, 2018. doi: [10.1007/JHEP06\(2018\)100](https://doi.org/10.1007/JHEP06(2018)100).

- 40 ANTICHEV, G. et al. First measurement of the total proton-proton cross section at the LHC energy of $\sqrt{s} = 7$ TeV. **Europhysics Letters**, v. 96, n. 2, p. 21002, 2011. doi: [10.1209/0295-5075/96/21002](https://doi.org/10.1209/0295-5075/96/21002).
- 41 _____. Luminosity-independent measurement of the proton-proton total cross section at $\sqrt{s} = 8$ TeV. **Physical Review Letters**, v. 111, p. 012001, 2013. doi: [10.1103/PhysRevLett.111.012001](https://doi.org/10.1103/PhysRevLett.111.012001).
- 42 _____. First measurement of elastic, inelastic and total cross-section at $\sqrt{s} = 13$ TeV by TOTEM and overview of cross-section data at LHC energies. **The European Physical Journal C**, v. 79, n. 2, p. 103, 2019. doi: [10.1140/epjc/s10052-019-6567-0](https://doi.org/10.1140/epjc/s10052-019-6567-0).
- 43 HEITLER, W. Cosmic rays. In: _____. **The quantum theory of radiation**. Oxford: Oxford University Press, 1944. cap. 24, p. 232–240.
- 44 ENGEL, R.; HECK, D.; PIEROG, T. Extensive air showers and hadronic interactions at high energy. **Annual Review of Nuclear and Particle Science**, v. 61, n. 1, p. 467–489, 2011. doi: [10.1146/annurev.nucl.012809.104544](https://doi.org/10.1146/annurev.nucl.012809.104544).
- 45 MATTHEWS, J. A heitler model of extensive air showers. **Astroparticle Physics**, v. 22, n. 5, p. 387 – 397, 2005. doi: [10.1016/j.astropartphys.2004.09.003](https://doi.org/10.1016/j.astropartphys.2004.09.003).
- 46 NIZ, J. A.-M. et al. Hybrid simulations of extensive air showers. **Physical Review D**, v. 66, p. 033011, 2002. doi: [10.1103/PhysRevD.66.033011](https://doi.org/10.1103/PhysRevD.66.033011).
- 47 RIEHN, F. et al. A new version of the event generator Sibyll. In: INTERNATIONAL COSMIC RAY CONFERENCE (ICRC), 34, 2005. The Hague. **Proceedings...** The Netherlands: ICRC, 2015. p. 558. doi: [10.22323/1.236.0558](https://doi.org/10.22323/1.236.0558).
- 48 GAISSER, T. K.; HILLAS, A. M. Reliability of the method of constant intensity cuts for reconstructing the average development of vertical showers. In: INTERNATIONAL COSMIC RAY CONFERENCE (ICRC), 15, 1977. Plovdiv. **Proceedings...** Bulgaria: ICRC, 1977. p. 353–357. Available from: <http://adsabs.harvard.edu/abs/1977ICRC....8..353G>. Accessible at: 11 June 2019.
- 49 MATTHEWS, J. A. J. et al. A parameterization of cosmic ray shower profiles based on shower width. **Journal of Physics G: nuclear and particle physics**, v. 37, n. 2, p. 025202, 2010. doi: [10.1088/0954-3899/37/2/025202](https://doi.org/10.1088/0954-3899/37/2/025202).
- 50 HOERANDEL, J. R. Cosmic-ray composition and its relation to shock acceleration by supernova remnants. **Advances in Space Research**, v. 41, p. 442–463, 2008. doi: [10.1016/j.asr.2007.06.008](https://doi.org/10.1016/j.asr.2007.06.008).
- 51 CAPRIOLI, D. An original mechanism for the acceleration of ultra-high-energy cosmic rays. **Nuclear and Particle Physics Proceedings**, v. 297-299, p. 226–233, 2018. doi: [10.1016/j.nuclphysbps.2018.07.032](https://doi.org/10.1016/j.nuclphysbps.2018.07.032).
- 52 DOMENICO, M. D. et al. Reinterpreting the development of extensive air showers initiated by nuclei and photons. **Journal of Cosmology and Astroparticle Physics**, v. 2013, n. 07, p. 050–050, 2013. doi: [10.1088/1475-7516/2013/07/050](https://doi.org/10.1088/1475-7516/2013/07/050).
- 53 KAMPERT, K.-H.; UNGER, M. Measurements of the cosmic ray composition with air shower experiments. **Astroparticle Physics**, v. 35, p. 660–678, 2012. doi: [10.1016/j.astropartphys.2012.02.004](https://doi.org/10.1016/j.astropartphys.2012.02.004).

- 54 ABRAHAM, J. et al. Measurement of the depth of maximum of extensive air showers above 10^{18} eV. **Physical Review Letters**, v. 104, p. 091101, 2010. doi: [10.1103/PhysRevLett.104.091101](https://doi.org/10.1103/PhysRevLett.104.091101).
- 55 _____. Properties and performance of the prototype instrument for the Pierre Auger Observatory. **Nuclear Instruments and Methods in Physics Research Section A**, v. 523, p. 50–95, 2004. doi: [10.1016/j.nima.2003.12.012](https://doi.org/10.1016/j.nima.2003.12.012).
- 56 BELLIDO, J. Depth of maximum of air-shower profiles at the Pierre Auger Observatory: measurements above $10^{17.2}$ eV and composition implications. In: INTERNATIONAL COSMIC RAY CONFERENCE (ICRC), 35, 2017. Bexco. **Proceedings...** Busan, Korea: ICRC, 2017. p. 506. doi: [10.22323/1.301.0506](https://doi.org/10.22323/1.301.0506).
- 57 CHERENKOV, P. A. Visible luminescence of pure liquids under the influence of γ -radiation. **Proceedings of the USSR Academy of Sciences**, v. 2, n. 8, p. 451–454, 1934. doi: [10.3367/UFNr.0093.196710n.0385](https://doi.org/10.3367/UFNr.0093.196710n.0385).
- 58 CERENKOV, P. A. Visible radiation produced by electrons moving in a medium with velocities exceeding that of light. **Physical Review**, v. 52, p. 378–379, 1937. doi: [10.1103/PhysRev.52.378](https://doi.org/10.1103/PhysRev.52.378).
- 59 GRIEDER, P. K. F. Atmospheric cherenkov radiation. In: _____. **Extensive air showers: high energy phenomena and astrophysical aspects - a tutorial, reference manual and data book**. Berlin, Heidelberg: Springer-Verlag, 2010. cap. 16, p. 835–878. doi: [10.1007/978-3-540-76941-5_16](https://doi.org/10.1007/978-3-540-76941-5_16).
- 60 COX, R. T. Momentum and energy of photon and electron in the Čerenkov radiation. **Physical Review**, v. 66, p. 106–107, 1944. doi: [10.1103/PhysRev.66.106](https://doi.org/10.1103/PhysRev.66.106).
- 61 FRANK, I.; TAMM, I. Coherent visible radiation of fast electrons passing through matter. In: BOLOTOVSKII, B. M.; FRENKEL, V. Y.; PEIERLS, R. (Ed.). **Selected papers**. Berlin, Heidelberg: Springer, 1991. p. 29–35. doi: [10.1007/978-3-642-74626-0_2](https://doi.org/10.1007/978-3-642-74626-0_2).
- 62 HOLDER, J. Atmospheric Cherenkov Gamma-ray Telescopes. 2015. Available from: <https://arxiv.org/pdf/1510.05675.pdf>. Accessible at 11 June 2019.
- 63 HILLAS, A. M. Cerenkov light images of EAS produced by primary gamma. **International Cosmic Ray Conference**, v. 3, p. 445, 1985. Available from: <http://articles.adsabs.harvard.edu/pdf/1985ICRC....3..445H>. Accessible at 11 June 2019.
- 64 SMITH, G. J.; TURVER, K. E. Optical cherenkov emission from large cosmic ray air showers (computer simulation). **Journal of Physics A: mathematical, nuclear and general**, v. 6, n. 8, p. L121–L126, 1973. doi: [10.1088/0305-4470/6/8/004](https://doi.org/10.1088/0305-4470/6/8/004).
- 65 DYAKONOV, M. N. et al. Some features of large EAS with fixed atmospheric Cerenkov light densities. In: INTERNATIONAL COSMIC RAY CONFERENCE (ICRC), 16, 1979. Tokyo. **Proceedings...** Japan: ICRC, 1979. p. 174. Available from: <http://articles.adsabs.harvard.edu/pdf/1979ICRC....8..174D>.
- 66 ANATOLY, I. The Yakutsk array experiment: main results and future directions. **European Physical Journal**, v. 53, p. 04003, 2013. doi: [10.1051/epjconf/20135304003](https://doi.org/10.1051/epjconf/20135304003).

- 67 HUEGE, T.; ULRICH, R.; ENGEL, R. Dependence of geosynchrotron radio emission on the energy and depth of maximum of cosmic ray showers. **Astroparticle Physics**, v. 30, n. 2, p. 96 – 104, 2008. doi: [10.1016/j.astropartphys.2008.07.003](https://doi.org/10.1016/j.astropartphys.2008.07.003).
- 68 KNURENKO, S. P.; PETROV, I. S. Correlation between radio signals at a frequency of 32 mhz and the parameters of extensive air showers, using data from measurements at the yakutsk eas array. **Bulletin of the Russian Academy of Sciences: physics**, v. 79, n. 3, p. 410–412, 2015. doi: [10.3103/S1062873815030302](https://doi.org/10.3103/S1062873815030302).
- 69 KALMYKOV, N. N.; KONSTANTINOV, A. A.; VEDENEEV, O. V. Estimating primary mass composition at energies 10^{17} – 10^{18} eV from EAS radio emission lateral distribution. **Journal of Physics: conference series**, v. 409, p. 012071, 2013. doi: [10.1088/1742-6596/409/1/012071](https://doi.org/10.1088/1742-6596/409/1/012071).
- 70 APEL, W. D. et al. Reconstruction of the energy and depth of maximum of cosmic-ray air-showers from LOPES radio measurements. **Physical Review D**, v. 90, n. 6, p. 062001, 2014. doi: [10.1103/PhysRevD.90.062001](https://doi.org/10.1103/PhysRevD.90.062001).
- 71 PETROV, I.; KNURENKO, S.; PETROV, Z. Depth of maximum development of extensive air showers by radio emission data at Yakutsk EAS array. In: INTERNATIONAL COSMIC RAY CONFERENCE (ICRC), 34, 2015. Bexco. **Proceedings...** Busan, Korea: ICRC, 2015. p. 255. doi: [10.22323/1.236.0255](https://doi.org/10.22323/1.236.0255).
- 72 SOUZA, V. de; BARBOSA, H.; DOBRIGKEIT, C. A Monte Carlo method to generate fluorescence light in extensive air showers. **Astroparticle Physics**, v. 22, n. 3, p. 263 – 273, 2004. doi: [10.1016/j.astropartphys.2004.07.006](https://doi.org/10.1016/j.astropartphys.2004.07.006).
- 73 SOKOLSKY, P.; COLLABORATION, H. Final results from the high resolution Fly's Eye (HiRes) experiment. **Nuclear Physics B - proceedings supplements**, v. 212, p. 74–78, 2011. doi: [10.1016/j.nuclphysbps.2011.03.010](https://doi.org/10.1016/j.nuclphysbps.2011.03.010).
- 74 THOMSON, G. Results from the Telescope Array Experiment. p. 448, 2010. Available from: <https://arxiv.org/pdf/1010.5528.pdf>.
- 75 COLLABORATION, T. P. A. The Pierre Auger Cosmic Ray Observatory. 2015. Available from: <https://arxiv.org/pdf/1502.01323.pdf>. Accessible at: 11 June 2019.
- 76 AAB, A. et al. Depth of maximum of air-shower profiles at the Pierre Auger Observatory. I. Measurements at energies above $10^{17.8}$ eV. **Physical Review D**, v. 90, n. 12, p. 122005, 2014. doi: [10.1103/PhysRevD.90.122005](https://doi.org/10.1103/PhysRevD.90.122005).
- 77 NERLING, F. et al. Universality of electron distributions in high-energy air showers—Description of Cherenkov light production. **Astroparticle Physics**, v. 24, n. 6, p. 421–437, 2006. doi: [10.1016/j.astropartphys.2005.09.002](https://doi.org/10.1016/j.astropartphys.2005.09.002).
- 78 HECK, D. et al. CORSIKA: a Monte Carlo code to simulate extensive air showers. 1998. Available from: <https://ui.adsabs.harvard.edu/abs/1998cmcc.book.....H>. Accessible at: 11 June 2019.
- 79 PIEROG, T. et al. EPOS LHC : test of collective hadronization with LHC data. 2013. Available from: <https://arxiv.org/pdf/1306.0121.pdf>. Accessible at: 11 June 2019.

- 80 FASSO, A. et al. The FLUKA code: present applications and future developments. 2003. Available from: <https://arxiv.org/pdf/physics/0306162.pdf>. Accessible at: 11 June 2019.
- 81 BAUS, C. Anomalous longitudinal shower profiles and hadronic interactions. In: INTERNATIONAL COSMIC RAY CONFERENCE (ICRC), 32, 2011. Beijing. **Proceedings...** China: ICRC, 2011. p. 206. Available from: <https://arxiv.org/pdf/1111.0504.pdf>. Accessible at: 12 June 2019.
- 82 PERKINS, J. S.; MAIER, G.; COLLABORATION, T. V. VERITAS telescope 1 relocation: details and improvements. 2009. Available from: <https://arxiv.org/pdf/0912.3841.pdf>. Accessible at: 11 June 2019.
- 83 COGAN, P. VEGAS, the VERITAS Gamma-ray Analysis Suite. **International Cosmic Ray Conference**, v. 3, p. 1385–1388, 2008. Available from: <https://arxiv.org/pdf/0709.4233.pdf>. Accessible at: 11 June 2019.
- 84 SOUZA, V. 2008. Private communication.