**UNIVERSIDADE DE SÃO PAULO
INSTITUTO DE FÍSICA DE SÃO CARLOS**

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**Effects of Lorentz invariance violation on the ultra-high
energy cosmic rays spectrum**

São Carlos

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energy cosmic rays spectrum**

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Concentration area: Basic Physics

Supervisor: Prof. Dr. Luiz Vitor de Souza Filho

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*“An expert is a person who has found out by his own painful experience all the mistakes
that one can make in a very narrow field.”*

Niels Bohr

ABSTRACT

LANG, R. G. **Effects of Lorentz invariance violation on the ultra-high energy cosmic rays spectrum.** 2017. 104p. Dissertation (Master in Science) - Instituto de Física de São Carlos, Universidade de São Paulo, São Carlos, 2017.

Relativity is one of the most important and well tested theories and Lorentz invariance is one of its pillars. Lorentz invariance violation (LIV), however, has been discussed in several quantum gravity and high energy models. For this reason, it is crucial to test it. Several tests, both terrestrial and astrophysical, have been performed in the last years and provide limits on the violation. This work takes part in these efforts and discuss the possibility of testing LIV with ultra-high energy cosmic rays (UHECRs). The effects of LIV in their propagation and the resulting changes in the spectrum of UHECRs are obtained and compared to the experimental data from the Pierre Auger Observatory. An analytical calculation for the inelasticity in the laboratory frame with LIV of any $a + b \rightarrow c + d$ interaction is presented and used to obtain the phase space and the energy losses of the pion production for protons, the photodisintegration for nuclei and the pair production for photons with LIV. A parametrization for the threshold energy of the photodisintegration with LIV is also proposed. The main effect seen is a decrease in the phase space and a resulting decrease in the energy loss. These changes have been implemented in Monte Carlo propagation codes and the resulting spectra of protons, nuclei and photons on Earth have been obtained and fitted to the data from the Pierre Auger Observatory. It is shown that upper limits on the photon LIV coefficient can be derived from the upper limits on the photon flux from the Pierre Auger Observatory.

Keywords: Lorentz invariance violation. Ultra-high energy cosmic rays. Propagation. UHECR spectrum. Pierre Auger Observatory.

RESUMO

LANG, R. G. **Efeitos da violação da invariância de Lorentz no espectro de raios cósmicos de altíssima energia.** 2017. 104p. Dissertação (Mestrado em Ciências) - Instituto de Física de São Carlos, Universidade de São Paulo, São Carlos, 2017.

Relatividade é uma das mais importantes e bem testadas teorias e a invariância de Lorentz é um de seus pilares. A violação da invariância de Lorentz (VIL), todavia, tem sido discutida em diversos modelos de gravidade quântica e altas energias. Por tal motivo, é crucial testá-la. Diversos testes, tanto terrestres quanto astrofísicos, foram realizados nos últimos anos e fornecem limites na violação. Este trabalho se insere nesses esforços e discute a possibilidade de testar VIL com raios cósmicos de altíssima energia. Os efeitos da VIL em sua propagação e as consequentes mudanças no espectro de raios cósmicos de altíssima energia são obtidos e comparados com os dados experimentais do Observatório Pierre Auger. Um cálculo analítico para a inelasticidade no referencial do laboratório com VIL para qualquer interação da forma $a + b \rightarrow c + d$ é apresentado e usado para obter o espaço de fase e as perdas de energia para a produção de píons para prótons, a fotodesintegração para núcleos e a produção de pares para fótons com VIL. Uma parametrização para o limiar de energia da fotodesintegração com VIL também é proposta. O principal efeito observado é uma diminuição no espaço de fase e uma consequente diminuição nas perdas de energia. Tais mudanças foram implementadas em códigos de Monte Carlo para a propagação e os espectros resultantes para prótons, núcleos e fótons na Terra foram obtidos e ajustados aos dados do Observatório Pierre Auger. É mostrado que limites superiores nos coeficientes de VIL para o fóton podem ser deduzidos dos limites superiores para o fluxo de fótons do Observatório Pierre Auger.

Palavras-chave: Violação da invariância de Lorentz. Raios cósmicos de altíssima energia. Propagação. Espectro de raios cósmicos de altíssima energia. Observatório Pierre Auger.

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1 INTRODUCTION

In 1912, the Austrian physicist Victor Franz Hess was interested in the ionization of the atmosphere. Using an electroscope, he measured it at different altitudes going as high as 5 km in balloon flights. His results have shown that, other than expected, the degree of ionization increases with the altitude (1). He interpreted it as due to an ionizing radiation reaching Earth from space. Later, in 1925, the term *cosmic ray* was introduced by Robert Andrews Millikan (2). Hess was awarded the Nobel Prize of Physics of 1936 for this discovery.

The introduction of the study of cosmic rays was really important for particle physics and, before the construction of particle accelerators, cosmic rays were the main source for discovering new elementary particles. The muon and the pion are examples of particles discovered by studying them. The first one was discovered by Carl Anderson and Seth Neddermeyer in 1937 (3), while the later was discovered by César Lattes, Cecil Powell and Giuseppe Occhialini in 1947. (4)

Cosmic rays not only provide a powerful tool to understanding astrophysical questions, such as acceleration processes, propagation and dark matter, but are also important for particle and theoretical physics since particles with energies higher than those achieved in laboratory can be studied. These energetic particles could provide information on, for example, hadronic interactions and new physical scenarios that could be suppressed and have effects only at really large energies, such as Lorentz Invariance Violation (LIV).

The cosmic ray flux, however, decreases with the energy and at some point it becomes impossible to measure them directly. In 1939, nevertheless, the French physicist Pierre Victor Auger found simultaneous events in two different ionization detectors (5), which meant that these simultaneous events were both related to the same primary cosmic ray. It was then understood that energetic primary cosmic rays interact with the atmosphere creating a cascade of secondary particles called extensive air shower (EAS). This made possible the detection of ultra-high energy cosmic rays (UHECRs), primaries with $E > 10^{18}$ eV.

Even though the expected flux for these particles with energies above 10^{18} eV is really low (about 1 particle per km^2 per year), they can be studied detecting the secondary particles in the EAS with experiments that cover a large area (thousands of km^2) and operate for a long time (twenty years).

Nowadays, there are several experiments interested in UHECRs. The largest of them is the Pierre Auger Observatory (6), an hybrid experiment in operation since 2004 in

Argentina. This experiment, described in details in Chapter 2, is composed of 1660 surface detectors and 27 fluorescence detectors spread over 3000 km².

The observatory is designed to detect particles with energies ranging from 10¹⁷ eV to 10²⁰ eV, which is about 3 orders of magnitude larger than the energy achieved at the Large Hadron Collider (LHC), the largest particle accelerator so far*. Therefore, it can be used to study physical effects that could be suppressed in energies lower than 10¹⁹ or 10²⁰ eV.

The Lorentz Invariance Violation (LIV), described in more details in Chapter 3, can be one of these effects. It has been investigated in different quantum gravity models and proposed by other high energy models of spacetime structure. (7) Its effects are suppressed by the Planck Scale and can be investigated by cosmic rays with energies around 10²⁰ eV. (8)

In this work, we are interested in the effects of LIV in the propagation of cosmic rays through the universe as well as how these effects influence their spectra measured on Earth.

In Chapter 4, cosmic ray propagation is described as well as how LIV affects it. Cosmic rays undergo energy losses due to the expansion of the universe and due to interactions with the photon background. Nuclei interact mainly with the cosmic microwave background (CMB) and the infrared background (IRB) via pair production, photopion production and photodisintegration. In the energy range of about 10²⁰ eV the photopion production and the photodisintegration are dominant over the others and, therefore, the effects of LIV are only studied in these interactions. Photons, on the other hand, interact not only with the CMB and the IRB, but also with the radio background (RB) via pair production. The main effect of LIV seen is an increase in the mean free path of such interactions, which allows cosmic rays to travel farther without interacting.

In Chapter 5, the spectra that should be measured on Earth taking into account the LIV effects in the propagation are calculated and fitted to the Auger data, in order to understand how these effects could be measured by the experiment.

Finally, in Chapter 6 the conclusions of this work are presented.

* The maximum energy achieved in LHC is 14×10^{12} eV. This energy, however, is in the center of mass of the interaction reference frame and, therefore, cannot be compared to the energies measured from UHECRs, which are in the laboratory reference frame. In order to have a fair comparison it is needed to obtain the center of mass energy of the cosmic ray-air interaction. This results in about 5×10^{14} eV for the highest primaries measured by Auger.

2 PIERRE AUGER OBSERVATORY

The Pierre Auger Observatory is a state of the art experiment in the province of Mendoza, Argentina designed to detect cosmic rays with energies above 10^{17} eV. Now a partnership of 18 countries, including Brazil, its construction began in 2002 and was completed in 2008, even though it started collecting data in 2004.

It is the largest cosmic ray experiment at the moment with an area of over 3000 km². It uses hybrid detection, while the lateral profile of the air shower is detected by the surface detector (SD), the longitudinal profile is detected by the fluorescence detector (FD).

Even though both SD and FD are used to measure the same properties, such as energy, mass composition and direction, they have very different systematics. Therefore, the hybrid design is important in order to have complementary detection, providing cross-check and redundancy. Besides the Pierre Auger Observatory, currently the Telescope Array experiment (9) also uses hybrid detection for cosmic rays above 10^{18} eV.

2.1 Design

Figure 1 – The Pierre Auger Observatory layout. The SD stations are represented by the red dots and the field of view of the FD telescopes is represented by the green lines.

Source: THE PIERRE AUGER COLLABORATION. (6)

Figure 1 shows the observatory design. The SD is composed of 1660 stations on a grid separated by 1.5 km and a smaller array with stations separated by 750 m. The FD is composed of 27 fluorescence telescopes distributed over 4 sites located at the borders of the SD array. The area is mostly flat, with altitude ranging from 1340 m to 1610 m, averaging around 1400 m.

2.1.1 Surface Detector

The surface detector consists of 1660 water Cherenkov stations (shown in Figure 2). Each station is a water tank of 3.6 m diameter and 1.2 m height with 12000 liters of pure water. Once an energetic charged particle from the shower crosses the water tank, it emits Cherenkov light if its speed is larger than the speed of light in water. This light is reflected until it reaches the three photomultiplier tubes (PMTs) located on the top of the tank (shown in Figure 3).

As the only light measured is that inside the station, the Sun light or that of the stars are not background for the SD. Therefore, it has a duty cycle close to 100%.

The main goal of the SD is to measure the lateral distribution function (LDF) of the shower, which means, the number of particles as a function of the distance to the shower core. From the LDF it is possible to reconstruct the primary particle energy. (6,10)

Figure 2 – Station from the Surface Detector.

Source: PIERRE AUGER OBSERVATORY. (11)

Figure 3 – Photomultiplier tube (PMT) from the station.

Source: By the author.

2.1.2 Fluorescence Detector

The fluorescence detector was initially built with 24 fluorescence telescopes in four different sites: Los Leones, Los Morados, Loma Amarilla and Coihueco (the Coihueco site can be seen in Figure 4). At each site there are 6 of them, each with a field of view of 30° in azimuth, summing 180° . All telescopes face the array, so each site has a complete view of it, as shown in Figure 1. The field of view in elevation for each site is also 30° , starting at 1.5° above the horizon level.

Figure 4 – The fluorescence telescope at the Coihueco site.

Source: By the author.

In 2009, an extension for the FD was built with 3 more telescopes installed in Coihueco, with an elevated field of view from 30° to 60° . This extension was called High Elevation Auger Telescopes (HEAT) and aimed to detect showers of lower energies (from 10^{17} eV to $10^{18.5}$ eV). The HEAT building can be seen in Figure 5.

The telescopes are composed by mirrors based on Schmidt optics with corrector lenses (shown in Figure 6) covering an area of about 13 m^2 , a camera with 440 PMTs (shown in Figure 7), electronics, aperture system (curtains and shutter) and an UV filter.

Charged particles in the shower induce the nitrogen molecules in the atmosphere to emit fluorescence light isotropically in the UV region ($300 - 430 \text{ nm}$). This light is then measured by the telescopes. Due to the low intensity when compared to the Moon light, the FD can only take data in dark and clean nights, when the Moon is not in the field of view of the telescopes. It also cannot take data in rainy or windy weather. Consequently, it has a duty cycle of about 15%.

The FD directly measures the longitudinal profile of the shower, through which both the energy of the primary and the depth of maximum number of particles, X_{max} , can

Figure 5 – HEAT. Each building hosts one of the three telescopes.

Source: By the author.

Figure 6 – Mirrors from the fluorescence telescope.

Source: By the author.

be obtained. (12, 13) The $X_{\max}$ measurements can be used to estimate the primary mass, as discussed later.

2.2 Scientific Highlights

In the 13 years since the first data were taken, the Pierre Auger Collaboration has made key measurements and discoveries. Some of them are highlighted and briefly explained in this section.

Figure 7 – Camera from the fluorescence telescope with 440 PMTs.

Source: By the author.

2.2.1 Spectrum of Cosmic Rays above 3×10^{17} eV

The flux of UHECRs is essential to understanding both the origin and the propagation of cosmic rays. It has been measured with unprecedented precision and statistics by the Pierre Auger Observatory for energies above 3×10^{17} eV. (10)

The data is divided in four data sets, which take into account the zenith angle (θ) and the technique with which they were obtained (SD-750, SD-1500 or Hybrid).

In relation to the zenith angle, showers are divided into vertical ($\theta < 60^\circ$) and inclined ($60^\circ < \theta < 80^\circ$). This is important due to the amount of atmosphere traversed by each one. Inclined showers traverse a larger amount of atmosphere and, therefore, interact more and are more attenuated than vertical ones.

In relation to the detection technique, on the other hand, showers are divided into three different groups: SD-750, SD-1500 and Hybrid. The first two use events detected by the SD, while SD-750 uses data from the 750 m array and is designed for the lower energies ($< 10^{18.4}$ eV), the SD-1500 uses data from the 1500 m array and is designed for the highest energies. The hybrid technique, on the other hand, uses events detected by both SD and FD.

Using only SD events is advantageous for the large statistics as the duty cycle of the SD is 100%. Using hybrid events, nevertheless, has a much better precision as two complementary techniques with different systematic errors are used.

There are different methods for the energy reconstruction. For vertical events in the SD the LDF is used, parametrized as a Nishimura-Kamata-Greisen-like (NKG-like) (14,15)

function:

$$S(r) = k \left(\frac{r}{r_s} \right)^{-\beta} \left(1 + \frac{r}{r_s} \right)^{-\beta}, \quad (2.1)$$

where $\beta = \beta(\theta)$ is a function of the zenith angle and r_s is a constant for the scale factor of the showers. It has been shown that for $r = 1000$ m the fluctuations caused by the shape of the LDF are minimized in the 1500 m array and for $r = 450$ m in the 750 m array. (16) Therefore, the energy estimators $S(1000)$ and $S(450)$ are defined as the value of the LDF at $r = 1000$ m and $r = 450$ m and are used to estimate the primary energy. However, for a given energy, $S(1000)$ and $S(450)$ decrease with the zenith angle as the shower is attenuated. For that reason, the Constant Intensity Cut method (17) is used to compensate this effect and the S_{38} and S_{35} estimators are calculated as the signal that the shower would have produced if they had arrived at $\theta = 38^\circ$ or $\theta = 35^\circ$.

In inclined showers, muons dominate the SD signal. And as muons are deflected by the geomagnetic field, they produce asymmetric signals. Consequently, for these events the N_{19} estimator is defined as the normalisation of the muon content relative to a distribution obtained by Monte Carlo (MC) simulations of air showers. (18)

Finally, the primary energy for FD events, E_{FD} , is estimated by integrating the longitudinal profile. It is also necessary to add the energy carried away by muons and neutrinos as those do not excite the nitrogen molecules and, therefore, cannot be detected by the fluorescence detectors. The hybrid events are those observed by the FD and at least one SD station. There are also quality cuts for those events in order to obtain a reliable energy reconstruction. (19) The estimated energy has a total systematic uncertainty of 14%. (20)

Figure 8 shows the correlation between the SD estimators and the FD reconstructed energy, E_{FD} . It can be well described as a power-law function:

$$E_{FD} = AS^B, \quad (2.2)$$

where S can be S_{38} , S_{35} or N_{19} . As the main uncertainties for the SD energy estimation come from this relation, the SD also has a systematic uncertainty for the energy of 14%.

Finally it is necessary to determine the exposure for the observations. Figure 9 shows the integrated exposure for the detectors. The SD exposure is based on the geometrical aperture of the array for a given zenith angle interval and observation time. Hybrid exposure, however, is obtained in more complex MC simulations that use the data taking conditions and the response of the hybrid detector. (19)

Figure 10 shows the resulting spectrum for each data set. And then, lastly, Figure 11 shows the combined spectrum, which is obtained taking into account the individual

Figure 8 – Correlation between the SD estimators and the FD energy. The blue points represent the S_{38} estimator given in VEM*, the gray ones represent the S_{35} also in VEM and the red ones represent the N_{19} , a dimensionless factor. The lines represent the fitted functions using Eq. 2.2.

Source: VALIÑO (10)

Figure 9 – Integrated exposure as a function of the energy. Each color represents a different data set: SD-1500 (black), SD inclined (red), hybrid (green) and SD-750 (blue).

Source: SCHULZ (21)

systematic uncertainties.

* Vertical Equivalent Muon, or VEM, is a unit defined as the signal produced by a vertical

Figure 10 – Left: energy spectrum for each data set, error bars represent statistical uncertainties[†]. Right: fraction between the Auger spectra and a power-law function with $\alpha = 3.26$.

Source: VALIÑO. (10)

Figure 11 – Combined spectrum measured by the Pierre Auger Observatory. The black dots represent the data, the error bars represent the statistical uncertainty and the black line represents a fitted broken power-law with a suppression. The total number of events in each bin is shown above the points.

Source: VALIÑO. (10)

The spectrum at these energies can be described by a broken power-law function ($dN/dE \propto E^{-\alpha_i}$) with two main structures: an ankle at about $10^{18.7} \text{ eV}$ and a strong suppression at about $10^{19.5} \text{ eV}$.

muon traversing the detector through its center

[†] The spectra are multiplied by E^3 in order to highlight their structures.

The ankle can be explained mainly by two different models. The first one, called the dip model (22) assumes a cosmic ray spectrum composed only of protons and the ankle would naturally appear due to the interaction of propagating protons with the low energy photons from the CMB via pair-production (discussed in details later in this work). The second model assumes a cosmic ray spectrum composed by different nuclei and explains the ankle as a transition from galactic to extragalactic sources. (23)

The dip model, however, is almost discarded as many experiments show results that indicate mixed mass composition (discussed in details in the next section).

The last structure of the spectrum is the suppression at the highest energies. This is usually explained either by the GZK effect or by a limitation in the maximum energy at the sources or even by a combination of both.

Greisen (24) and Zatsepin and Kuzmin (25) showed independently that ultra energetic protons and nuclei should interact with the CMB by both photopion production and photodisintegration (for nuclei). Stecker (26) then calculated the energy loss of protons due to photopion production and showed that a proton with an energy larger than 10^{20} eV would not travel farther than 100 Mpc. As a result, the proton spectrum should be suppressed at these energies. This suppression is usually referred to as the GZK suppression. These interactions are widely explained later in this work.

Other reason for the suppression could be a natural limit in the acceleration energy at the sources, which could also be combined with the GZK effect.

2.2.2 Depth of Shower Maximum and Mass Composition

Measuring the mass composition is needed for testing the models for both the ankle and the suppression. A change from a heavier composition to a lighter one at the ankle would be an evidence of a transition from galactic to extragalactic sources. Also the composition in the highest energies can bring information about the suppression (also discussed later in this work).

As the primary mass cannot be directly measured, the distribution of the depth of shower maximum, $X_{\max}$, measured by the FD is used to study it. Figure 12 shows an event detected in coincidence by one fluorescence detector at HEAT and one at Coihueco. The longitudinal profile is then fitted using a Gaisser-Hillas function (27), which is given by:

$$\frac{dE}{dX}(X) = \left(\frac{dE}{dX} \right)_{\max} \left(\frac{X - X_0}{X_{\max} - X_0} \right)^{\frac{X_{\max} - X_0}{\lambda}} e^{-\frac{X_{\max} - X}{\lambda}}, \quad (2.3)$$

where $\left(\frac{dE}{dX} \right)_{\max}$, $X_{\max}$, X_0 and λ are free parameters.

The distribution of $X_{\max}$ is then parametrized by a Gumbel function. (29) As a first approximation, a primary of mass A and energy E can be approximated as A protons

Figure 12 – Left: trace of the event in the camera. The timing scale goes from violet (early) to red (late). Right: longitudinal profile. The black dots are HEAT data, the blue points are Coihueco data and the red line is the fitted Gaisser-Hillas function. The red dot in both images show the $X_{\max}$ position.

Source: PORCELLI. (28)

with energy E/A . Thus, the first moment of the distribution, $\langle X_{\max} \rangle$, can be approximated as a linear function of $\ln(E/A)$. The second moment, $\sigma^2(X_{\max})$, should decrease with larger mass number. They can be related to the first two moments of the distribution of the logarithm of the mass by (12):

$$\begin{aligned} \langle X_{\max} \rangle &= \langle X_{\max} \rangle_p + f_E \langle \ln A \rangle, \\ \sigma^2(X_{\max}) &= \langle \sigma_{sh}^2 \rangle + f_E^2 \sigma^2(\ln A), \end{aligned} \quad (2.4)$$

where $\langle X_{\max} \rangle_p$ is the average $X_{\max}$ for protons and $\langle \sigma_{sh}^2 \rangle = \sum_i f_i \sigma_i^2(X_{\max})$ is the composition-averaged fluctuation, and f_E is a parameter that contains the information about hadronic interactions.

Figure 13 shows the first and second moments of the distribution of $X_{\max}$ measured by the Pierre Auger Observatory for each energy. The moments are compared to Monte Carlo simulations of air showers using a pure proton composition (in red) and a pure iron composition (in blue) in three different models for the hadronic interactions: EPOS-LHC (30), QGSJetII-04 (31) and Sibyll 2.1 (32). Figure 14 shows the moments for the distribution of the logarithm of mass using EPOS-LHC and QGSJetII-04.

It can be seen in both distributions that between 10^{17} and $10^{18.3}$ eV the composition becomes lighter, then after $10^{18.3}$ eV it starts to get heavier again. The results are compatible with a mixed composition at the ankle and challenge the dip model. (33)

The problem in this analysis is that it relies on hadronic interactions models. These models use extrapolated data from accelerators, such as LHC, even though the validity of these extrapolations cannot be tested directly at the Auger energy range.

Figure 13 – The first two moments of the $X_{\max}$ distributions. They are compared to pure proton (red) and pure iron (blue) scenarios using EPOS-LHC, QGSJetII-04 and Sibyll 2.1.

Source: PORCELLI. (28)

Figure 14 – The first two moments of the $\ln(A)$ distributions on top using EPOS-LHC and on bottom using QSGJet-II.

Source: PORCELLI. (28)

2.2.3 Photon and neutrino fluxes limits

Protons and nuclei interacting with the CMB produce charged pions, muons and neutrons, which decay into neutrinos and also produce neutral pions, which decay into UHE photons. At these energies, photons can propagate a few tens of Mpc before being absorbed. (34) Neutrinos also reach Earth without interacting or being deflected. Therefore,

studying these fluxes is important for understanding UHECR origins and propagation. (35)

The small cross-sections for neutrinos are the key for identifying showers initiated by them. There are two strategies: looking for showers with large zenith angle ($\theta > 60^\circ$), called Downward-Going (DG) showers and looking for showers coming from beneath ($90^\circ < \theta < 95^\circ$), called Earth-Skimming (ES) showers.

At zenith angles larger than 60° the atmosphere is thick enough to absorb almost completely the electromagnetic component of a shower initiated by a nucleus. At these angles, however, neutrinos might initiate showers deep in the atmosphere, which would reach the detectors with a considerable amount of electromagnetic component remaining. This allows the differentiation between showers induced by neutrinos and protons with ($\theta > 60^\circ$).

The other possible option is that tau-neutrinos coming from beneath may interact close to the surface, producing a τ , which can decay in the atmosphere close to the detectors.

For photon showers a more complex analysis is needed. These showers have a lower number of muons and a larger $\langle X_{\max} \rangle$. The $\langle X_{\max} \rangle$ for a primary photon of 10^{19} eV is larger than the atmospheric depth at the site, and consequently only showers with zenith angle larger than 30° are analyzed. In this inclination, the $\langle X_{\max} \rangle$ is smaller than the depth of the observatory and it is guaranteed that the majority of the showers has reached its maximum at the detectors.

The search for photon events is made by using $30^\circ < \theta < 60^\circ$ events and applying several cuts, which can be found in reference. (35)

After applying these cuts, no neutrino event has been selected for the period between 01/01/2004 and 20/06/2013. For photons, only 4 events were selected between 01/01/2004 and 15/05/2013.

Upper limits at 90% confidence level (CL) were then obtained for the diffuse flux of neutrinos and at 95% CL for the diffuse flux of photons and are shown in Figure 15 in comparison with limits obtained by other experiments and with predictions from theoretical models.

For neutrinos, the upper limits are yet above the theoretical predictions. For photons, on the other hand, the limits imposed by the Pierre Auger Observatory are the strongest so far above 10^{18} eV and rule out top-down models of photon production from the decay of heavy primordial particles. (47, 48) Cosmogenic models using a spectral index of -2 and maximum energy at the sources of 10^{21} eV (39), cannot be ruled out with the current exposure.

Figure 15 – Left: Upper limits to the integral (horizontal lines) and differential diffuse flux of neutrinos. Auger limits (red) are compared to IceCube (36) and ANITA (37) limits and predictions from cosmogenic models (38–40) and the Waxman-Bahcall bound (41). Right: Upper limits to the integral diffuse flux of photons. Auger hybrid (blue) and SD (black) limits are compared to limits from Telescope Array (42) (green), Yakutsk (43) (dark red), Haverah Park (44) (red), AGASA (45) (orange), older Auger limits (46) (gray) and predictions from top-down (47, 48) and cosmogenic (39, 47) model.

Source: BLEVE. (35)

2.2.4 Other Important Results

There are several other important results from the Pierre Auger Collaboration that are not directly related to this work which include: search for dipolar modulations in right ascension (49), correlation between arrival directions and AGNs position (50, 51) and measurement of proton-air cross-section at 57 TeV. (52)

3 LORENTZ INVARIANCE VIOLATION

The concept of invariance has been present in physics since the 17th century. It is based on Inertial Frames (IFs), which are frames in which the physical laws are the same. For almost three centuries, physics was taken to be Galilean invariant, i.e., IFs were related to one another via Galilean transformations of their coordinates, given by:

$$\begin{aligned}x' &= x - vt, \\y' &= y, \\z' &= z, \\t' &= t.\end{aligned}\tag{3.1}$$

In the beginning of last century, however, Einstein proposed the Special Relativity (SR), in which time is no longer absolute and all the relative velocities between IFs are smaller than the speed of light. For that reason, physics had to be no longer Galilean invariant, but Lorentz invariant, meaning that the IFs were related by:

$$\begin{aligned}x' &= \gamma(x - vt), \\y' &= y, \\z' &= z, \\t' &= \gamma\left(t - \frac{vx}{c^2}\right), \\ \gamma &= \frac{1}{\sqrt{1-v^2/c^2}}.\end{aligned}\tag{3.2}$$

Since then, relativity has been one of the most important and reliable theories of the last century. (7) That would be enough reason for testing Lorentz invariance, which is one of its pillars. But as Lorentz transformations are reduced to the Galilean transformations in the non-relativistic limit (Eq. 3.2 becomes Eq. 3.1 if $v \ll c$), they could have a regime in which they are valid, e.g., for low energies. That means that for higher energies there could be a Lorentz invariance violation (LIV)*.

LIV has been investigated in quantum gravity models, such as string theory (53), warped brane world (54) and loop quantum gravity (55), and is incorporated in several high energy models, e.g., non-commutative fields (56), emergent gauge bosons (57), varying speed of light cosmologies (58), among others. Therefore, testing LIV is essential to theoretical and high-energy physics.

* Actually, for quantum gravity models this energy is supposed to be the Planck energy ($E_{Pl} \approx 10^{19}$ GeV)

3.1 Framework

Most phenomenological frameworks for LIV converge into a perturbative theory around Lorentz invariance suppressed by the Planck scale. (59–61) Starting with the Lorentz invariant dispersion relation[†]

$$E_a^2 = p_a^2 + m_a^2, \quad (3.3)$$

a perturbative expansion is proposed:

$$E_a^2 = p_a^2 + m_a^2 + \sum_{n=0} \frac{\eta_{a,n}}{M_{Pl}^n} p^{(n+2)}, \quad (3.4)$$

where a denotes the particle (since there is no reason *a priori* for this expansion to be the same for every particle), n denotes the order of the expansion, M_{Pl} denotes the Planck mass and $\eta_{a,n}$ denotes the LIV coefficient for n and a . For a cleaner notation it will be used

$$\delta_{a,n} = \frac{\eta_{a,n}}{M_{Pl}^n}, \quad (3.5)$$

as the LIV coefficient and δ_a for $n = 0$.

In a phenomenological approach, the LIV coefficients are free parameters that are not necessarily derived from the theory and must be experimentally limited separately. As these coefficients modulate the effect at each order, it is not necessarily true that higher orders will have weaker effects.

Eq. ?? can be seen as a sum of two effects. For $n > 0$, the dispersion relation is expanded in terms of $p^{(n+2)}/M_{Pl}^n$, which means that the usual dispersion relation, given in Eq. 3.3, is only an approximation for energies much lower than the Planck mass. For $n = 0$, however, there is no suppression by the Planck scale. In this case, Eq. 3.4 can be rewritten defining[‡] $c_a^2 = (1 + \delta_a)c^2$:

$$E_a^2 = p_a^2 c^2 (1 + \delta_a) + m_a^2 c^4 = p_a^2 c_a^2 + m_a^2 \frac{c_a^4}{(1 + \delta)^2} \approx p_a^2 c_a^2 + m_a^2 c_a^4, \quad (3.6)$$

which means that this term can be seen as a shift in the maximum attainable velocity of the particle. In simple words, each particle would have its “own speed of light”.

[†] Throughout this work natural units are used, in which $c = 1$.

[‡] Only this equation will be shown in International System (IS) units in order to highlight the speed of light.

As it will be discussed later, most detectable effects of LIV are related to a shift in the energy threshold of particle interactions. These thresholds are intrinsically related to the center of mass system (CMS) energy of the particles, given by:

$$s_a = E_a^2 - p_a^2. \quad (3.7)$$

Since the effects of LIV should stop being negligible only at very high energies, where $E \gg m$, then the CMS energy of a particle with LIV is given by:

$$s_a = E_a^2 - p_a^2 = m_a^2 + \sum_{n=0} \delta_{a,n} p_a^{(n+2)} \approx m_a^2 + \sum_{n=0} \delta_{a,n} E_a^{(n+2)}. \quad (3.8)$$

3.2 Tests of LIV

Several experiments have been performed in order to obtain constraints on LIV. As its effects are heavily suppressed at low energies, these experiments must have either very high energy or very high precision. They can be organized in mainly two groups: terrestrial and astrophysical.

3.2.1 Terrestrial

Terrestrial experiments have limited energy and even the most energetic accelerator, the LHC, cannot achieve enough energy to measure some effect of LIV. Therefore, such experiments must rely on very precise measurements.

Some examples are using the relation between the cyclotron frequency, w_c , and the precession frequency, w_s , in Penning traps (62), using atomic clocks moving in different directions (63) or measuring the Doppler shift of lithium ions moving at high velocity. (64)

Since terrestrial test of LIV are not in the scope of this work, they are not explicitly discussed and are only mentioned for reference.

3.2.2 Astrophysical

Astrophysics provides the most suitable tools for testing LIV. Cosmic rays are detected with energies up to three decades higher than those achieved in accelerators and travel for very long distances, through which small effects could be integrated and become non-negligible.

3.2.2.1 Time of Flight

The velocity of a particle is given by $v = \partial E / \partial p$. Therefore, using Eq. 3.4, the velocity of a photon with LIV will be given by:

$$v = \frac{\partial E}{\partial p} = \frac{2p + \sum_{n=0} \delta_{\gamma,n}(n+2)p^{(n+1)}}{2\sqrt{p^2 + \sum_{n=0} \delta_{\gamma,n}p^{(n+2)}}} \approx 1 + \frac{1}{2} \sum_{n=0} \delta_{\gamma,n}(n+2)p^n \approx 1 + \frac{1}{2} \sum_{n=0} \delta_{\gamma,n}(n+2)E^n. \quad (3.9)$$

The velocity of a photon is, thus, shifted and, for $n > 0$, it becomes a function of the energy. Even though this effect is very small, it can be amplified by long travels. Two photons emitted at the same time, but with different energies, E_1 and E_2 travelling over a time T would reach Earth with a difference in time given by:

$$\Delta t = \Delta v T = \frac{T}{2} \sum_{n=0} \delta_{\gamma,n}(n+2) (E_1^n - E_2^n). \quad (3.10)$$

Therefore, large values of T could compensate for small values of $\delta_{\gamma,n}$.

Fig. 16 shows the time of arrival of photons detected by the MAGIC telescope with energies ranging from 0.25 TeV to 10 TeV. These photons were emitted by rapid flares from Markarian 501 in 2005. (65)

The uncertainty for this analysis comes from making sure that the photons were all emitted at the same time, which is a strong hypothesis. Nevertheless, this problem can be solved by looking at different gamma-ray bursts (GRBs) for sources at different redshifts. Since the time of flight is different for such sources the effects from different emission times could be separated from the effects from LIV.

Even though this has already been studied (66, 67), a better sensitivity is still needed. This will be achieved with the construction of the Cherenkov Telescope Array (CTA) (68), which will have a sensitivity one order of magnitude better than the present experiments, providing potential for studying this effect. (69)

3.2.2.2 Neutrinos

Neutrinos have the smallest mass of all known particles and, consequently, can be very powerful for testing LIV as even at low energies the shift in CMS energy becomes significant. This shift can be treated as an effective mass, η_{ν_i} , and has no reason to be the same for each flavor

$$s = m_{\nu_i}^2 + \sum_{n=0} \delta_{\nu_i,n} p_{\nu_i}^{(n+2)} := \eta_{\nu_i}^2(E). \quad (3.11)$$

Figure 16 – Measurements of the Markarian 501 flare from the MAGIC telescope. The energies are separated in 4 bands, from the top to the bottom, 0.15 – 0.25 TeV, 0.25 – 0.6 TeV, 0.6 – 1.2 TeV, 1.2 – 10 TeV.

Source: ALBERT. (65)

Neutrino oscillations are directly related to the mass of the neutrinos of each flavor and, consequently, are affected by the effective mass. The probability of a neutrino produced in a flavor i be detected in a flavor j after traveling a distance L is given by[§]:

$$P_{ij} = \delta_{ij} - \sum_{i,j>i} 4F_{ij} \sin^2 \left(\frac{(\eta_{\nu_i}^2 - \eta_{\nu_j}^2) L}{4E} \right) + 2G_{ij} \sin \left(\frac{(\eta_{\nu_i}^2 - \eta_{\nu_j}^2) L}{2E} \right). \quad (3.12)$$

Therefore, with LIV, neutrinos could oscillate even if they had null mass. (70) Some problems in neutrino physics could also be explained by shifting their masses. (71)

3.2.2.3 Vacuum Cherenkov

Charged particles traveling faster than the speed of light in a medium produce Cherenkov radiation. In a Lorentz invariant scenario, vacuum Cherenkov emission is forbidden, since no particle can be faster than the speed of light in the vacuum. As shown

[§] δ_{ij} is only the Kronecker Delta, not the LIV coefficient.

in Section 3.1, however, with LIV each particle has its own maximum attainable velocity and, therefore, vacuum Cherenkov could become kinematically allowed.

This process is abrupt and, thus, any charged cosmic ray detected by experiments must be below the emission threshold. Therefore, the most energetic events detected so far are used to impose upper limits on the LIV coefficients. (72–74)

3.2.2.4 Threshold of Interactions

Ultra-high energy cosmic rays lose energy due to several interactions with the photon background, e. g., pair or pion production. The threshold of these interactions is directly related to the CMS energy of the particles. Therefore, the shift in the CMS energy caused by LIV will imply in a shift in the threshold of the interactions, increasing or decreasing the energy losses of UHECR. This would change both the spectrum and the composition measured by UHECR experiments, like Auger or Telescope Array.

The modifications in the kinematics of the most energetic interactions have been studied by several authors, including pion production for protons (8, 61, 75–78), photodisintegration for nuclei (79, 80) and pair production for photons. (81, 82)

This work takes part in these efforts and presents a more complete view, studying the modification for the kinematics of any interaction $a + b \rightarrow c + d$, discussing the effects of LIV on the propagation of protons, nuclei and photons and calculating the detectability of those by the Pierre Auger Observatory.

4 UHECR PROPAGATION WITH LORENTZ INVARIANCE VIOLATION

Ultra-high energy cosmic rays are accelerated in astrophysical sources and lose energy while propagating through the Universe. These energy losses modulate both the spectrum and the composition measured by experiments and since several results in the field derive from these measurements, their understanding is crucial.

In this chapter the theory for the propagation of cosmic rays including LIV is presented. The analytical calculation for the inelasticity of any interaction $a + b \rightarrow c + d$ with LIV is shown and its results in the propagation of protons, nuclei and photons are discussed.

4.1 Theory

Propagating cosmic rays lose energy mainly due to the expansion of the universe and due to interactions with the photon background.

4.1.1 Adiabatic Losses

As the universe itself expands, the space through which the cosmic rays are propagating is expanding and this can be seen as a continuous decrease in its energy. Since this energy is not lost by increasing the energy of other particles, this is called an adiabatic loss. For a flat Λ CDM universe, this adiabatic loss can be expressed as a function of the particle's redshift, z , the Hubble constant, H_0 , and the densities of matter (both ordinary and dark), radiation and dark energy, Ω_m , Ω_r and Ω_Λ , respectively. (77)

$$\left(-\frac{1}{E} \frac{dE}{dt} \right)_{\text{adiabatic}} = H_0 \sqrt{\Omega_m(1+z)^3 + \Omega_r(1+z)^4 + \Omega_\Lambda}. \quad (4.1)$$

The values used for the constants were: $H_0 = 70 \text{ km Mpc}^{-1} \text{ s}^{-1}$, $\Omega_m = 0.3$ and $\Omega_\Lambda = 0.7$. Ω_r is really small ($\sim 10^{-4}$) and should only be significant at very large redshifts, when the universe was very young. Even for the largest redshift used in this work, 2.5, Ω_r is still negligible.

This energy loss is supposed not to be modified by LIV.

4.1.2 Interactions with the Photon Background

The energy loss per interval of time by of the interaction of a cosmic ray with the photon background is given by:

$$\left(-\frac{1}{E} \frac{dE}{dt}\right)_{\text{interaction}} = c \int_{-1}^1 \frac{d \cos \theta}{2} \int_{\epsilon_{th}(E, \theta)}^{\infty} d\epsilon n(\epsilon) \sigma(E, \epsilon, \theta) K(E, \epsilon, \theta), \quad (4.2)$$

where E is the cosmic ray energy, ϵ is the background photon energy and θ is the angle between the cosmic ray momentum and photon momentum. The laboratory frame (LF) is defined as the one in which the cosmic microwave background radiation (CMB) is isotropic and, therefore, in this frame the probability of finding the photon in any direction is the same. $n(\epsilon)d\epsilon$ is the spatial density of background photons and is proportional to the probability of finding a photon with that energy. $\sigma(E, \epsilon, \theta)$ is the cross section and is proportional to the probability of a cosmic ray interacting with a photon in a given kinematic configuration. $K(E, \epsilon, \theta)$ is the inelasticity and quantifies the fraction of energy lost in the interaction. Finally, ϵ_{th} is the threshold energy, which is obtained from the inelasticity in the LIV case.

In the following subsections, each term of Eq. 4.2 is discussed separately.

4.1.2.1 Photon Background Density

The universe is populated by photons in a wide range of wavelengths, cosmic rays can interact with photons ranging from infrared ($\lambda \sim 1 \mu\text{m} \implies \epsilon \sim 1 \text{ eV}$) to radio ($\lambda \sim 10^4 \text{ m} \implies \epsilon \sim 10^{-10} \text{ eV}$).

At the center of the background spectrum is the well known cosmic microwave background radiation (CMB). It has been predicted in 1948 (83,84) and was formed after the decoupling stage, right after the universe was cool enough to form the first hydrogen atoms and, therefore, it contains meaningful cosmological informations about the early universe. It was measured by the first time in 1965 (85) and is now measured with high precision. (86) It is described by a blackbody spectrum, following a Planck distribution with $T = 2.7255 \pm 0.0006 \text{ K}$:

$$n(\epsilon)d\epsilon = \frac{1}{\pi^2 \hbar^3 c^3} \frac{\epsilon^2}{e^{\epsilon/k_B T} - 1} d\epsilon, \quad (4.3)$$

where k_B is the Boltzmann constant, $\hbar$ is the reduced Planck constant and c is the speed of light.

At higher energies there is the infrared background radiation (IRB). It is related to the formation of stars and is used to study the formation of galaxies. (87) It is obtained by considering both the evolution of the sources and their luminosity as well as interstellar

dust. (88) It contains a lot of uncertainties and several models in the literature propose parametrizations for its distribution (shown in Fig. 17). A more complete review on IRB can be found in (89).

Figure 17 – Parametrizations for the infrared background radiation.

Source: ANJOS. (89)

Figure 18 – Comparison between the IRB and the CMB. On the left is the photon density, while on the right is the energy density. The red line shows the CMB described by a Planck distribution with $T = 2.7$ K and the black line shows the CMB distribution from the Dominguez model. (87)

Source: By the author.

Fig. 18 shows a comparison between the IRB and the CMB. The density of photons is several orders of magnitude larger for the CMB. This difference, however, is smaller for

the energy density, since the IRB photons are more energetic.

Finally, at the lowest energies, there is the radio background radiation (RB). Extragalactic radio sources contribute to the diffuse background at these energies and it can be parametrized as a Planck distribution but with an effective temperature that depends on the frequency, $\nu = \epsilon/h$. (90) There can also be a cut on the frequency at around 1 MHz.

$$n_{RB}(\epsilon) = \begin{cases} \frac{1}{\pi^2 \hbar^3 c^3} \frac{\epsilon^2}{e^{\epsilon/k_B T_{eff}} - 1} & , \text{ if } \epsilon > E_{cut}, \\ 0 & , \text{ if } \epsilon \leq E_{cut}, \end{cases} \quad (4.4)$$

$$T_{eff}(\nu) = T_0 \left(\frac{\nu}{\nu_0} \right)^{\gamma_0} + T_1 \left(\frac{\nu}{\nu_0} \right)^{\gamma_1}, \quad (4.5)$$

where T_0 , T_1 , ν_0 , γ_0 and γ_1 are free parameters fitted to the measured brightness of the universe.

Even the most energetic photons from the background of interest ($\sim 10^1$ eV) still are several orders of magnitude below the energy at which the effects of LIV would appear ($\sim 10^{20}$ eV), therefore, no LIV is considered on the photon background.

4.1.2.2 Cross Section

The cross section is an individual calculation for each interaction and can either be derived directly from the theory or fitted to experiments. The cross sections used in this work are described in Appendix A.

The cross section is also supposed not to be affected by LIV.

4.1.2.3 Inelasticity

The inelasticity quantifies the fraction of energy lost in each interaction and contains the kinematics of the process. The effects of LIV on it have been discussed in several works. (61, 76–79, 91, 92) Nevertheless, they are usually discussed for a single interaction (mostly for the pion production) and in the nucleus reference frame (NRF). In this work the calculations for the inelasticity of any $a + b \rightarrow c + d$ interaction with LIV under some assumptions are presented in both the LF and the NRF. This is advantageous as this framework can be used for studying the effects of LIV in any interaction with the photon background*.

The energy distribution of cosmic rays is measured in the LF and, therefore, any calculation made in the NRF must contain a boost from the NRF to the LF in order

* The only UHE cosmic ray interaction that is not in the form $a + b \rightarrow c + d$ is the proton pair production ($p + \gamma \rightarrow p + e^- + e^+$). However, it can be reduced to this form by considering the pair a single particle with two times the mass.

to compare with the experimental results. This boost, however, is based on Lorentz transformations and, therefore, the assumption that the violation is not strong enough to change Lorentz transformations is needed. A more elegant way is presented in this section, in which all the calculations are performed in the LF, not needing, thus, this assumption. The calculations in the NRF are shown in Appendix B[†].

Consider an interaction of the form $a + b \rightarrow c + d$, in which E_m and $\vec{p}_m$ are, respectively, the energy and the momentum of each particle in the laboratory frame and where θ_i is the angle between a and b and θ_f the angle between c and d . If $E_a \gg E_b$ and $p_m \gg m_m$, for $m = a, c, d$ (which is valid for any interaction of a cosmic ray with the background, being a the cosmic ray and b the background photon), then the total energy and the momentum before (i) and after (f) the interaction are given by[‡]:

$$\begin{cases} E_{total,i} = E_a + E_b \approx E_a, \\ E_{total,f} = E_c + E_d, \end{cases} \quad (4.6)$$

$$\begin{cases} \vec{p}_{total,i} = \vec{p}_a + \vec{p}_b \approx \vec{p}_a \approx E_a \hat{x}, \\ \vec{p}_{total,f} = \vec{p}_c + \vec{p}_d \approx \left(\sqrt{E_c^2 + E_d^2 + 2E_c E_d \cos \theta_f} \right) \hat{r}_f. \end{cases} \quad (4.7)$$

Imposing energy and momentum conservation in Eqs. 4.6 and 4.7:

$$\begin{cases} \hat{r}_f = \hat{x} \\ E_a = E_c + E_d \end{cases} \implies \theta_f = 0. \quad (4.8)$$

It is, then, possible to define the inelasticity, K , of the interaction as the fraction of energy lost by a due to the production of c :

$$K = \frac{E_c}{E_a}. \quad (4.9)$$

From Eqs. 4.8 and 4.9, E_c and E_d , which are unknown, can be expressed as a function of the inelasticity and E_a :

$$\begin{cases} E_c = K E_a, \\ E_d = (1 - K) E_a. \end{cases} \quad (4.10)$$

[†] Unfortunately, the NRF calculation is needed in some cases, such as the photodisintegration.

[‡] The direction of $\vec{p}_a$ can be taken, without loss of generality, to be $\hat{x}$.

It is then necessary to look at the initial and final total rest energy, $\sqrt{s_{total}} = \sqrt{E_{total}^2 - p_{total}^2}$:

$$\begin{cases} s_i = (E_a + E_b)^2 - (\vec{p}_a + \vec{p}_b)^2 = E_a^2 - p_a^2 + E_b^2 - p_b^2 + 2E_a E_b - 2p_a p_b \cos \theta_i, \\ s_f = (E_c + E_d)^2 - (\vec{p}_c + \vec{p}_d)^2 = E_c^2 - p_c^2 + E_d^2 - p_d^2 + 2E_c E_d - 2p_c p_d. \end{cases} \quad (4.11)$$

Eq. 4.11 can be rewritten using the CMS energy for each particle, $s_m = E_m^2 - p_m^2$. Considering that $E_m^2 \gg s_m$:

$$\begin{cases} s_i = s_a + s_b + 2E_a E_b \left(1 - \cos \theta_i + \cos \theta_i \left(\frac{s_a}{2E_a^2} + \frac{s_b}{2E_b^2} + \mathcal{O}(2) \right) \right), \\ s_f = s_c + s_d + 2E_c E_d \left(\frac{s_c}{2E_c^2} + \frac{s_d}{2E_d^2} + \mathcal{O}(2) \right). \end{cases} \quad (4.12)$$

Using the inelasticity from Eq. 4.10 and the CMS energy of the particle with LIV[§] from Eq. 3.8, Eq. 4.12 becomes only a function of the known variables, E_a , E_b , s_a , s_b and θ_i , of the LIV coefficients and of K :

$$\begin{cases} s_i = s_a + s_b + 2E_a E_b \left(1 - \cos \theta_i + \cos \theta_i \left(\frac{s_a}{2E_a^2} + \frac{s_b}{2E_b^2} \right) \right), \\ s_f = s_c + s_d + K(1 - K) \left(\frac{s_c}{K^2} + \frac{s_d}{(1-K)^2} \right), \end{cases} \quad (4.13)$$

with

$$\begin{cases} s_a = m_a^2 + \sum_{n=0} \delta_{a,n} E_a^{(n+2)}, \\ s_b = m_b^2 + \sum_{n=0} \delta_{b,n} E_b^{(n+2)}, \\ s_c = m_c^2 + \sum_{n=0} \delta_{c,n} (K E_a)^{(n+2)}, \\ s_d = m_d^2 + \sum_{n=0} \delta_{d,n} ((1 - K) E_a)^{(n+2)}. \end{cases} \quad (4.14)$$

Lastly, it is necessary to impose the conservation of the total rest energy, i.e., $s_i = s_f$ and multiply the equation by $K(1 - K)$ [¶]. This results in a polynomial of order $(n + 3)$ for K , where n is the LIV expansion order. The calculation to obtain it is long and contains no physics, therefore, it is described in Appendix C.

[§] It is used only one order of the expansion each time, since they have to be restricted independently.

[¶] This is only possible as it is known that $K \neq 0$ and $K \neq 1$, otherwise it would result in a particle with null energy ($E_c = 0$ or $E_d = 0$).

The inelasticity is then obtained by averaging the polynomial roots that are physically possible:

$$K = \sum_i \frac{K_i}{N}, \text{ for } \begin{cases} K_i \in (0, 1) \\ K_i \in \mathcal{R} \end{cases}. \quad (4.15)$$

If no root fulfills these criteria, then the interaction is kinematically forbidden ($K = 0$).

4.1.3 Other physical quantities

The attenuation length, the mean free path, the survival probability and the horizon are physical quantities related to the energy loss that give a more tangible view of the processes.

The attenuation length, ℓ , is the average length that a cosmic ray can travel before losing $1/e$ of its energy. It is used to understand up to which distance the universe is transparent for an interaction at a given energy, i.e., from how far a cosmic ray can reach us before losing energy due to that interaction. It is given by:

$$\ell = \left(-\frac{1}{E} \frac{dE}{dx} \right)^{-1} = \left(-\frac{\frac{1}{E} \frac{dE}{dt}}{c} \right)^{-1}. \quad (4.16)$$

The mean free path, λ , is the average length a cosmic ray can travel before interacting and does not take into account the inelasticity. It is useful for interactions as the photodisintegration, where the primary cosmic ray is lost, generating another particle. The mean free path is:

$$\lambda = \left(\int_{-1}^1 \frac{d \cos \theta}{2} \int_{\epsilon_{th}(E, \theta)}^{\infty} d\epsilon n(\epsilon) \sigma(E, \epsilon, \theta) \right)^{-1}. \quad (4.17)$$

The survival probability, τ , is defined in such way that a cosmic ray emitted with energy E at a redshift z has a probability $e^{-\tau}$ of reaching Earth before interacting:

$$\tau(z) = \int_0^z dz' \frac{c}{H_0(1+z) \sqrt{\Omega_m(1+z)^3 + \Omega_\Lambda}} \int_{-1}^1 \frac{d \cos \theta}{2} \times \int_{\epsilon_{th}(E, \theta, z')}^{\infty} d\epsilon n(\epsilon, z') \sigma(E, \epsilon, \theta, z') K(E, \epsilon, \theta, z'). \quad (4.18)$$

Finally, the horizon, D_z , is the distance at which a cosmic ray must be emitted in order to have a survival probability of 1 and, consequently, a probability $1/e$ of reaching Earth without interacting. It is also used to understand the transparency of the universe

for a interaction. For small redshifts ($1 + z \sim 1$), where the cosmological corrections are negligible, the mean free path and the survival probability are related via:

$$\lambda = \frac{cz}{H_0 \tau}. \quad (4.19)$$

4.2 Results

4.2.1 Protons

Protons interact via pair production ($p + \gamma \rightarrow p + e^+ + e^-$) and via pion production ($p + \gamma \rightarrow p + \pi$). Fig. 19 shows the energy losses for both processes together with the adiabatic losses. At the energies for which the effects of LIV are expected ($E > 10^{20}$ eV), the pion production is very dominant over the others and, therefore, the effects of LIV are studied only in this interaction.

Figure 19 – Energy losses for a propagating ultra-high energy proton. The red line shows the adiabatic loss, the green line shows the loss due to pair production, and the blue line the one due to pion production. The black line shows the total energy loss.

Source: By the author.

The inelasticity in the laboratory frame of the pion production was obtained using the analytical calculation presented in Section 4.1.2.3. The LI scenario and scenarios

considering LIV with $n = 0$ for the proton ($\delta_p = 10^{-23}$) and for the pion ($\delta_\pi = 10^{-23}$) were considered. The results are shown in Figs. 20 and 21.

Figure 20 – Inelasticity of the pion production in a LI scenario. The x axis represents the initial proton energy in the LF and the y axis represents the background photon energy in the LF.

Source: By the author.

From the inelasticity it is possible to obtain both the phase space and the threshold energy of the interaction. The phase space is the region where the interaction is kinematically allowed, i.e, the region where $K > 0$. The threshold energy, $\epsilon_{th}(E_p)$, on the other hand, is the lowest background photon energy from the phase space. It depends on the initial proton energy and can be obtained as the first $\epsilon(E_p)$ for which $K > 0$.

In the LI scenario, the threshold energy depends on the proton energy as $\epsilon_{th}(E) \propto E_p^{-1}$ and its typical values are close to the CMB peak, at $\sim 10^{-4}$ eV.

With LIV, on the other hand, there is a strong modification in the phase space and, consequently in the threshold energy. The effect becomes more significant as the initial proton energy increases. A change in the LIV coefficient would result in a change in the energy at which LIV becomes significant.

For the proton, at some point, the threshold energy shifts down from the LI threshold and, therefore, the phase space becomes larger. The interaction, thus, happens more often than it would without LIV. For the pion, the opposite happens and, consequently, the interaction happens less often.

The LIV term in the CMS energy (Eq. 3.8) can be seen as shift in the mass, the particle would have a mass that becomes larger with the energy. As the pion is only present in the final product of the interaction, this would mean that more energy would

Figure 21 – Inelasticity of the pion production in LIV scenarios with $n = 0$. The x axis represents the initial proton energy in the LF and the y axis represents the background photon energy in the LF. The left panel shows a scenario with LIV for the proton with $\delta_p = 10^{-23}$ and the right panel shows a scenario with LIV for the pion with $\delta_\pi = 10^{-23}$.

Source: By the author.

be necessary to create it, shifting the threshold up. Nevertheless, the proton is present both before and after the interaction, and it is more energetic before it. Therefore, the shift in the mass is stronger for the initial proton, shifting the threshold up.

These changes have direct effects on the pion production attenuation length. In order to obtain it, it is necessary to look not only at the inelasticity, but also at the cross section and at the photon density. The cross section is discussed in Appendix A and the only photon background used is the CMB, which is the most significant for this interaction.

As shown in Fig. 22, the effects are seen more clearly in the attenuation length. Stronger LIV coefficients for the pion result in a larger attenuation length, allowing the proton to travel farther. Stronger LIV coefficients for the proton, however, result in the opposite. The effects are more substantial for the pion, for which the attenuation length quickly becomes over two orders of magnitude larger than in the LI scenario.

4.2.2 Nuclei

Nuclei lose energy mainly due to photodisintegration^{||}. An initial nucleus of mass A interacts with the background and emits one nucleon, becoming a nucleus of mass $A - 1$ ($N_A + \gamma \rightarrow N_{A-1} + N_1$). It is also possible to emit two or more nucleons, however, this is

^{||} A nucleus of mass A and energy E is treated as A protons of energy A/E for the pair and pion production.

Figure 22 – Attenuation length of the pion production in LIV scenarios with $n = 0$ as a function of the initial proton energy in the LF. The left panel shows a scenario with LIV for the proton and the right panel shows a scenario with LIV for the pion. The black lines show the LI scenario, while the colored lines show the LIV scenario with different LIV coefficients: $\delta = 5 \times 10^{-24}$, $\delta = 1 \times 10^{-23}$, $\delta = 5 \times 10^{-23}$ and $\delta = 1 \times 10^{-22}$.

Source: By the author.

much less probable.

This interaction is more complex and demands changes in the calculations for the inelasticity and, consequently, for the threshold energy. It also contains uncertainties in the determination of both the cross section and the photon background density. As the threshold energy without LIV is needed in the calculations and it is usually given in the NRF, the inelasticity for this interaction with LIV is obtained in the NRF, as discussed in Appendix B. The cross section can be described by three independent models shown in Appendix A. Finally, the significant photon background for this interaction is not only the CMB but also the IRB, which can be described by several models (see Fig. 17).

This interaction is usually implemented in propagation codes by obtaining separately the threshold energy, the cross section model and the IRB model. This allows choosing the different models as an input. As a consequence, it is better to study the effects of LIV on the threshold energy separately.

It would be necessary to study the effects independently for each nucleus. However, as cosmic rays are composed of several species of nuclei, assuming a different δ_{N_A} for each species would lead to a huge number of free parameters and, therefore, it is desirable to relate every δ_{N_A} . This can be achieved by assuming that the total energy and momentum of the nucleus is the sum of the energy and momentum of its constituents. (79) Therefore,

$E_{N_A} \approx AE_p$, $p_{N_A} \approx Ap_p$ and $m_{N_A} \approx Am_p$, leading to the dispersion relation:

$$\begin{aligned}
 E_{N_A}^2 &= p_{N_A}^2 + m_{N_A}^2 + \sum_{n=0} \delta_{N_A,n} E_{N_A}^{(n+2)} \\
 \implies A^2 E_p^2 &= A^2 p_p^2 + A^2 m_p^2 + A^2 \sum_{n=0} \delta_{N_A,n} A^n E_p^{(n+2)} \\
 \implies E_p^2 &= p_p^2 + m_p^2 + \sum_{n=0} \delta_{N_A,n} A^n E_p^{(n+2)}.
 \end{aligned} \tag{4.20}$$

Eq. 4.20, however, becomes the dispersion relation for the proton by setting $\delta_{N_A,n} = \delta_{p,n}/A^n$. And, thus, for the case of $n = 0$, $\delta_{N_A} = \delta_p$ for any nucleus. It is also usual to use only 4 different nuclei, hydrogen ($A = 1$), helium ($A = 4$), nitrogen ($A = 14$) and iron ($A = 56$), which are almost equally separated in $\ln A$.

Figure 23 – Threshold energy in the NRF for the photodisintegration of nuclei as a function of the initial nucleus energy in the LF. The left panel shows the LI scenario and the right panel shows the LIV scenario with $\delta_p = 10^{-22}$. The blue line represents the interaction for a initial nucleus of helium, the green line represents a nucleus of nitrogen and the red line a nucleus of iron.

Source: By the author.

Fig. 23 shows the threshold energy for the photodisintegration for helium, nitrogen and iron. In the LI scenario, it depends only on the species of the initial nucleus (this comes from the binding energy of each nucleus, as discussed in Appendix B), but not on its energy. With LIV, however, this dependence appears. The effects are stronger the larger the initial energy and the lighter the mass. This is expected, since, as discussed before, the LIV term can be seen as a shift in the mass. Therefore, heavier masses suffer less significant effects. Once again, the effect of changing the value of δ_p is changing the energy at which the LIV effects become significant.

As the photodisintegration is usually simulated in Monte Carlo codes, it is important to obtain a parametrization of the effects of LIV. Monte Carlo simulations demand a high computational cost and parametrizations are preferred to make the code run faster. This, however, is not found in the literature and is first proposed in this work.

Figure 24 – Parametrization for the threshold energy in the NRF with LIV. The black dots represent the threshold energies obtained using the calculations described in Appendix B and the red lines represent the parametrized function. The right panel is just a zoom of the left panel, highlighting the most important points.

Source: By the author.

The proposed parametrization, shown in Fig. 24, is a suppressed exponential with three parameters, a , b and c :

$$\left(\epsilon_{th}^{LIV}\right)' = \left(\epsilon_{th}^{LI}\right)' + \frac{a \log(E_N/\text{eV})}{1 + e^{c(\log(E_N/\text{eV}) - b)}}, \quad (4.21)$$

where a gives a global normalization to the effect and is parametrized as a function of the nucleus mass and the LIV coefficient and b gives the energy at which the effect becomes significant and is also parametrized as a function of A and δ_p . Finally c gives how fast the threshold energy grows and is parametrized as a function of A .

$$\begin{cases} a = e^{\alpha_0(A) + \alpha_1(A) \ln \delta_p} \times \text{MeV}, \\ b = \beta_0(A) + \beta_1(A) \ln \delta_p, \\ c = \gamma_0 + \gamma_1 \ln(A), \end{cases} \quad (4.22)$$

where α_0 , α_1 , β_0 and β_1 are parametrized as linear functions of A . It is important to note that in the LI scenario $\delta_p = 0$, therefore $\ln \delta_p \rightarrow -\infty \implies a \rightarrow 0$, i.e., the parametrization function is built in such way that $\delta_p = 0$ recovers the LI result.

The parameters were obtained for the emission of 1 and 2 nucleons and their values and plots are shown in Appendix D.

As it can be seen in Fig. 24, the parametrized function describes very well the threshold energy. Only at very high energies ($E > 10^{22}$ eV) there is a small difference. This, however, is not a problem as at these energies the cosmic ray flux is heavily suppressed either by the maximum energy or by the energy distribution at the sources. In the region of most interest, (10^{19} eV $< E < 10^{22}$ eV) the effects are nicely fitted.

Figure 25 – Mean free path for the photodisintegration as a function of the initial nucleus energy. The full lines show the analytical calculation and the dashed lines show the calculation using the parametrized function (Eq. 4.21). The black lines show the LI scenario and the red and green lines show the LIV scenario for $\delta_p = 10^{-23}$ and $\delta_p = 10^{-22}$, respectively. The left panel shows the results for helium and the right panel shows the results for iron. The Dominguez model (87) was used for the IRB distribution and the parametrizations from Rachen (93) were used for the cross section.

Source: By the author.

Fig. 25 shows the resulting mean free path using the threshold energy obtained both from the calculations described in Appendix B and from the parametrization. Both results agree very well, validating the proposed parametrization. The effects of LIV on the mean free path of the photodisintegration are similar to those obtained for the attenuation length of the pion production. It becomes larger than in the LI scenario, meaning that the interaction will happen less often and the cosmic ray can travel farther. It is also noteworthy that, as discussed before, LIV starts to get significant effects for lower energies for lighter masses.

Fig. 26 shows the mean free path for an iron nucleus emitting 1 and 2 nucleons. For the LI scenario, the emission of 2 nucleons represents about 33% of the process and it

Figure 26 – Mean free path for the photodisintegration of an iron nucleus. The black lines show the LI scenario and the red and green lines show the LIV scenario for $\delta_p = 10^{-23}$ and $\delta_p = 10^{-22}$, respectively. The full lines show the mean free paths for the emission of 1 nucleon and the dashed lines show the ones for the emission of 2 nucleons. The Dominguez model (87) was used for the IRB distribution and the parametrizations from Rachen (93) were used for the cross section.

Source: By the author.

becomes more important for $E > 10^{21.5}$ eV. With LIV, on the other hand, it becomes less significant in relation to the emission of 1 nucleon.

4.2.3 Photons

Very energetic photons interact with low energy photons from the background via pair production ($\gamma + \gamma \rightarrow e^+ + e^-$). This interaction is used to estimate the transparency of the universe to gamma-rays. (34) Its phase space including LIV can be determined as the region where the interaction is kinematically allowed in a three-dimensional space, with the axes being the energetic photon energy, E_γ , the background photon energy, ϵ , and the LIV coefficient, $\delta_{\gamma,n}$ **.

Figs. 27 and 28 show cuts in this three-dimensional space with the phase space obtained using the calculations for the inelasticity in the LF discussed in Section 4.1.2.3.

The first shows the phase space as a function of E_γ and ϵ for different negative LIV coefficients. The results are similar to those presented for the pion production. The LI threshold energy depends on the initial photon energy as $\epsilon_{th} \propto E_\gamma^{-1}$. With LIV, the threshold energy becomes larger than in the LI case and this effect is stronger the higher the energy. The LIV coefficients once more are important to determine the point where

** This calculation was done in collaboration with Humberto Martínez.

Figure 27 – Phase space for the photon pair production as a function of the initial energy in LIV scenarios with $n = 0$. The colored areas show the phase space of the interaction for different LIV coefficients, ranging from $\delta_\gamma = 0$ to $\delta_\gamma = -10^{-16}$. The regions are inclusive. The red dashed line shows the threshold energy for the LI scenario.

Source: LANG. (94)

this effect becomes significant.

The later, on the other hand, shows an innovative way of looking at the phase space and represents it as a function of δ_γ and E_γ for different values of ϵ . Positive values of δ_γ increase the phase space, while negative values decrease it. Also for negative values there is a threshold δ_γ for each (E_γ, ϵ) combination. LIV coefficients lower than this threshold result in a null phase space. As the effect is much more significative for negative values of δ_γ , only this case is discussed.

It might be misleading to look only at the phase space, since these threshold coefficients are only for a given combination of (E_γ, ϵ) . As the background distribution is a continuous distribution, even though certain δ_γ would result in a null phase space for (E_γ, ϵ) , that is not necessarily true for higher values of ϵ .

A complementary information comes from the mean free path, in which this phase space is convoluted with the background density and with the cross section. The dominant backgrounds for this interaction are the IRB for $E_\gamma < 10^{14.5}$ eV, the CMB for $10^{14.5}$ eV $< E_\gamma < 10^{19}$ eV and the RB for $E_\gamma > 10^{19}$ eV. Fig. 29 shows the resulting mean free path without violation for different models of IRB and RB compared to the result from (34). In the LIV calculations the Gilmore model (95) is used for the IRB and the Gervasi model (90) with a frequency cut at 1 MHz is used for the RB.

The mean free path for the interaction and the photon horizon were obtained for

Figure 28 – Phase space for the photon pair production in LIV scenarios with $n = 0$ as a function of the LIV coefficient. The colored areas show the phase space of the interaction for 4 different background photon energy, corresponding to $T = 2.7, 2, 1.5$ and 1 K. The left panel shows the scenario for $\delta_\gamma < 0$, the right for $\delta_\gamma > 0$ and the central for the LI scenario. The red dashed line shows the threshold energy for the LI scenario and the blue dots show the threshold LIV coefficient.

Source: LANG. (94)

the first time considering LIV.

Fig. 30 shows the mean free path for LIV scenarios with $n = 0$, $n = 1$ and $n = 2$. The effects are very similar in all the cases and are very similar to those presented before for protons and nuclei. It is important to note that, as discussed before, a higher n does not imply in a weaker effect, since $\delta_{\gamma,n}$ is a free parameter that must be limited experimentally and, therefore, can modulate the effect of each n .

Finally, Fig. 31 shows the photon horizon with LIV. In the presence of LIV, the horizon increases and, therefore, the fluxes of sources farther than in the LI scenario would be detectable.

Figure 29 – Mean free path for the photon pair production in a LI scenario for different background models. The black line shows the result from (34). The blue lines represent the IRB models of Gilmore (95), Malkan & Stecker (88) and Dominguez (87). The red line represents the CMB with $T = 2.7$ K. The green lines represent the RB model from Gervasi (90) with no frequency cut and cuts at 1, 2 and 10 MHz.

Source: By the author.

Figure 30 – Mean free path for the photon pair production in LIV scenarios as a function of the initial photon energy in the LF. The panels show LIV scenarios with $n = 0$, $n = 1$ and $n = 2$, respectively. The black lines show the LI scenario, while the colored lines show the LIV scenario with different LIV coefficients.

Source: LANG. (94)

Figure 31 – Horizon for the photon pair production in LIV scenarios as a function of the initial photon energy in the LF. The black lines show the LI scenario, while the colored lines show the LIV scenario with different LIV coefficients. The left axis represents the distance in redshift, while the right axis represents the equivalent distance in Mpc.

Source: LANG. (94)

5 COMPARISON WITH THE RESULTS FROM THE PIERRE AUGER OBSERVATORY

In the last chapter, the propagation of UHECRs with LIV was discussed. It was shown that the phase space and, consequently, the energy losses of several interactions can be modified by LIV. Nevertheless, they cannot be directly measured by experiments. Therefore, in order to use experimental results to test LIV in the astrophysical scenario, it is necessary to determine how these changes affect the measurements, such as the spectrum and the composition.

In this chapter the spectra of protons, nuclei and photons with LIV are obtained and fitted to the data from the Pierre Auger Observatory. The sensitivity of these measurements to effects of LIV and the possibility of using the data from Auger to impose upper limits on the LIV coefficients are discussed.

For protons the analytical spectrum with LIV is presented and fitted to the Auger spectrum. For nuclei the spectrum and the composition with LIV are obtained via Monte Carlo simulations and are also fitted to the Auger data. Finally, for photons, the flux of GZK photons with LIV resulting from Monte Carlo simulations is compared to the upper limits on the photon flux from Auger.

5.1 Protons

In both pion and pair production, an incoming proton interacts with a photon from the background and, despite losing energy by emitting a pion or a pair, it remains a proton. For this reason, it is possible to treat both interactions as continuous. Using this assumption, the proton spectrum can be obtained analytically. This has been done by a few authors. (8, 77, 78) This calculation, however, depends on unknown features that are usually left free and fitted to experimental data, such as the spectral index and the maximum energy at the sources. Still, in (8, 77, 78) these features are set by hand and not fitted in the LIV scenario. In this section, an analytical calculation for the proton spectrum fitting the spectrum index and maximum energy for LI and LIV scenarios is presented. The latest data from the Pierre Auger Observatory (10) are used in the fit.

Sources distributed spatially following a redshift evolution, $q(z) \propto (1+z)^\xi$ and emitting protons following an energy distribution, dN/dE_s are supposed.

$$\frac{dN}{dE_s} = \begin{cases} E_s^{-\Gamma} & , \text{ for } E_s < E_{Max} \\ E_s^{-\Gamma} e^{(1-E_s/E_{Max})} & , \text{ for } E_s \geq E_{Max} \end{cases} , \quad (5.1)$$

where E_s is the energy at the source and the source evolution term, ξ , the spectral index,

Γ , and the maximum energy at the source, E_{Max} , are free parameters.

The spectrum on Earth is, then, given by:

$$J(E) = \frac{3cK(0)}{8\pi H_0} E^{-\Gamma} \int_0^{z_{max}} dz \frac{(1+z)^{(\xi-1)}}{\sqrt{\Omega_m(1+z)^3 + \Omega_\Lambda}} \times \left(\frac{E_s}{E}\right)^{-\Gamma} e^{(\Theta(E-E_{Max})(1-E/E_{Max}))} \frac{dE_s}{dE}, \quad (5.2)$$

where $K(0)$ is the normalization, another free parameter, and E is the energy of the proton on Earth.

It is necessary to consider the cosmological effects. For increasing z , the space shrinks with $(1+z)$ and, consequently, the CMB density increases with $(1+z)^3$. Also, the energy increases with $(1+z)$. Therefore, the energy loss at a redshift z is given by:

$$\left(\frac{1}{E} \frac{dE}{dt}\right)(E_z) = (1+z)^3 \left(\frac{1}{E} \frac{dE}{dt}\right)((1+z)E_{z=0}), \quad (5.3)$$

where the energy losses at $z = 0$ are the ones discussed in Chapter 4, considering both pair and pion production. Adiabatic losses are also taken into account, as given by 4.1. Using Eq. 5.3, the relation between E , E_s and z can be obtained using numerical methods for solving ordinary differential equations. This is exemplified in Fig. 32.

Figure 32 – Energy at the source as a function of the redshift and of the energy at Earth. The x axis represents the energy on Earth, the y axis represents the redshift and the color scale represents the energy at the source.

Source: By the author.

Fig. 33 shows the influence of each free parameter in the spectrum. Γ is, as expected, related to the slope of the spectrum. For $E_{Max} < 10^{19.8}$ eV, the maximum energy at the

Figure 33 – Influence of the free parameters in the spectrum. The black dots represent the spectrum from Auger (10), while the colored lines represent the spectra for different values of the free parameters. The first panel shows different values of Γ for a fixed $E_{Max} = 10^{19.5}$ eV and $\xi = 3.6$. The second panel shows different values of E_{Max} for a fixed $\Gamma = 2$ and $\xi = 3.6$. The third panel shows different values of ξ for a fixed $\Gamma = 2$ and $E_{Max} = 10^{19.5}$ eV. The spectra are multiplied by E^3 to highlight their structures.

Source: By the author.

sources is responsible for the beginning of the suppression. For $E_{Max} > 10^{19.8}$ eV, however, the changes only appear at the end of the spectrum, since in these cases the suppression is dominated by pion production. ξ does not change significantly the spectrum. UHECRs are provenient, in their vast majority, from close sources ($1 + z \sim 1$) and, therefore, the source distribution does not play an important role in the spectrum. Finally, the normalization has only the effect of shifting the spectrum up or down.

The final result is obtained by fitting the free parameters to the data from the Pierre Auger Observatory for $E > 10^{18.7}$ eV*, which gives both the values of the and how well the model describes the data. A χ^2 fit was used. This method aims to minimize the value of χ_{red}^2 , given by:

$$\chi_{red}^2 = \sum_i \left(\frac{y(x_i) - f(x_i, \vec{\Theta})}{\sigma_i} \right)^2 / N_{dof}, \quad (5.4)$$

where $y(x_i)$ and σ_i are, respectively, the experimental data and corresponding uncertainties, $f(x_i, \vec{\Theta})$ is the value of the model for a given point and values of the free parameters, and N_{dof} is the number of degrees of freedom given by the difference between the number of data points and free parameters. A model that properly fits the data has $\chi_{red}^2 \approx 1$, showing that the difference from the model to the data is similar to the experimental uncertainties.

The spectrum was obtained for values of δ_p and δ_π ranging from 0 to 10^{-22} in intervals of 5×10^{-24} , using Γ , E_{Max} and $K(0)$ as free parameters. As shown before, the source evolution has almost no influence in the spectrum. Its value, therefore, was fixed at $\xi = 3.6$, which corresponds to the star formation rate. A maximum redshift of $z_{max} = 2.5$ was used.

Figure 34 – Mapping of the spectrum fit. The x axis represents the values of Γ , the y axis represents the values of E_{Max} and the color scale represents the χ^2 . The red points are the smallest values of χ^2 . The left panel show a LI scenario, while the right panels shows a LIV scenario with $\delta_\pi = 10^{-22}$.

Source: By the author.

* Only the energies above the ankle are used, since at this point a transition from galactic from extragalactic sources is expected. If energies both below and above the ankle were considered, it would be necessary to use two different assumptions for the sources.

Fig. 34 shows the consistency of the fit. A mapping of the values of χ^2 as a function of Γ and E_{Max} is shown for a LI scenario and for a scenario with LIV for the pion. In both cases the minimum is well localized, showing that the fit is consistent.

Figure 35 – Fitted parameters as a function of the LIV coefficient. The x axis represents the values of the LIV coefficient, while the y axis represents the values of the fitted parameters. The black dots represent the LIV scenario for the pion and the red dots for the proton. The left panel shows the spectrum index, Γ , and the right one shows the maximum energy at the sources, E_{Max} .

Source: By the author.

Figure 36 – Goodness of the fit as a function of the LIV coefficient. The x axis represents the values of the LIV coefficient, while the y axis represents the values of χ^2_{red} . The black dots represent the LIV scenario for the pion and the red dots for the proton.

Source: By the author.

Fig. 35 shows the fitted parameters as a function of the LIV coefficient. Both parameters are insensitive to LIV. This is because the best fit is found for $E_{Max} \approx 10^{19.5}$ eV and, therefore, the maximum energy at the sources is lower than the energy at which the protons interact via pion production. Consequently, most protons would not be accelerated to the energy where the effects of LIV are significant. Therefore, in this fit the suppression of the spectrum would be explained not by the GZK effect but by the maximum energy at the sources.

Fig. 36 shows the values of χ_{red}^2 as a function of the LIV coefficients. Despite of the parameters, χ_{red}^2 decreases for larger values of δ_π , going from ~ 8 to ~ 5 . This can be understood in Fig. 37 that shows the spectra for the LI scenario and for the LIV scenario with the strongest LIV coefficients used for the proton ($\delta_p = 10^{-22}$) and for the pion ($\delta_\pi = 10^{-22}$). There is almost no difference between the LI spectrum and the spectrum with LIV for the proton. With LIV for the pion, on the other hand, the spectrum shows a small recovery at the highest energies. These points, however have a large statistical uncertainty and, therefore, have a small contribution to χ_{red}^2 . This explains the decrease in χ_{red}^2 for the pion case. Nevertheless, even though, the goodness of the fit is sensitive to δ_π , even the smallest value of χ_{red}^2 is still very large.

Figure 37 – Proton spectrum with LIV. The black dots represent the data from Auger. (10) The black line represents the LI scenario, the red line the scenario with LIV for the pion ($\delta_\pi = 10^{-22}$), and the blue line the scenario with LIV for the proton ($\delta_p = 10^{-22}$). The spectra are multiplied by E^3 to highlight their structures.

Source: By the author.

Another important feature that must be taken into account is the systematic uncertainty of the data. As discussed in Chapter 2, the observatory has an energy resolution of 14%, which can be treated in a first order approximation by introducing an energy

scaling to the spectrum. The fit was performed for energy scales in the uncertainty range, from -14% to 14% in intervals of 2% and the best one was obtained for -14% .

Figure 38 – Fitted parameters as a function of the LIV coefficient with an energy scale of -14% . The x axis represents the values of the LIV coefficient, while the y axis represents the values of the fitted parameters. The black dots represent the LIV scenario for the pion and the red dots that for the proton. The first panel shows the spectrum index, Γ , and the second shows the maximum energy at the sources, E_{Max} .

Source: By the author.

Figure 39 – Goodness of the fit as a function of the LIV coefficient with an energy scale of -14% . The x axis represents the values of the LIV coefficient, while the y axis represents the values of χ^2_{red} . The black dots represent the LIV scenario for the pion and the red dots that for the proton.

Source: By the author.

Figs. 38 and 39 show the resulting fitted parameters and χ_{red}^2 using the energy scaling. Once more the parameters show no dependence on the LIV coefficients. There is a large decrease in the χ_{red}^2 and its dependence on δ_π gets much weaker. The values of χ_{red}^2 , however, are still large.

It can be concluded that, if cosmic rays were composed purely of protons, the Pierre Auger Observatory would have almost no sensitivity to the effects of LIV in the models discussed. Additionally, a model as the one described, considering the assumptions for the composition, the propagation and the sources would not describe the measured data well. The composition of cosmic rays, however, is believed to be mixed. (12, 13) This case is discussed in the next section.

5.2 Nuclei

In contrast with the pion production, in the photodisintegration the resulting cosmic ray is different from the initial one. An initial nucleus of mass A emits a nucleon and becomes a nucleus of mass $A - 1$. Therefore, it is very complex to treat the propagation analytically and, thus, Monte Carlo simulations are preferred. Several codes have been developed to treat this problem. (96–98) Using these codes it is possible to obtain the flux of each species of nuclei on Earth and, consequently, both the spectrum and the composition of cosmic rays. None of these codes, however, contains an implementation of LIV effects.

In this work, both the pion production losses with LIV and the parametrization for the threshold energy of the photodisintegration with LIV, described in Chapter 4, were implemented in the SimProp v2r3 code. (97) Assumptions similar to those used in (99) were used to obtain the resulting spectra: sources homogeneously distributed in a comoving volume, injecting hydrogen, helium, nitrogen and iron with an energy distribution given by:

$$\frac{dN}{dE_s} = \begin{cases} E_s^{-\Gamma} & , \text{ for } R_s < R_{Max} \\ E_s^{-\Gamma} e^{(1-R_s/R_{Max})} & , \text{ for } R_s \geq R_{Max} \end{cases}, \quad (5.5)$$

where $R = p/Z \approx E/Z$ is the rigidity. The injected fraction of each nucleus (H, He, N and Fe), p_i , is defined as the fraction of cosmic rays of each species emitted at $E = 10^{18}$ eV[†], in such a way that $\sum p_i = 1$. The pion production was treated as continuous, while the photodisintegration was treated stochastically. The model used for the IRB distribution was the Gilmore model (95) and the model used for the cross section was the PSB-Salamon model. (85, 100) No magnetic fields were considered, resulting in a

[†] It is important to define it at an energy below the energies where the suppression has effects. Since lower masses are suppressed first, the fraction for higher energies is not the same.

one-dimensional propagation. This is usual as UHECRs have a large rigidity and, therefore, are almost not deflected by the magnetic field.

Monte Carlo simulations have a high computational cost and, consequently, it would be impossible to simulate each combination of free parameters. The solution is to simulate a given combination of parameters and later weight the events for each of the others combinations.

Four different scenarios were simulated, the one with LI ($\delta_p = 0$) and three with LIV ($\delta_p = 5 \times 10^{-24}$, $\delta_p = 1 \times 10^{-23}$ and $\delta_p = 1 \times 10^{-22}$). For each scenario, 1.4×10^7 events were simulated following a flat distribution in $\log E_s$ from $\log E_s = 18.7$ to $\log E_s = 22$:

$$\frac{dN}{d \log E_s} \propto E_s^0 \implies \frac{dN}{dE_s} \propto E_s^{-1}. \quad (5.6)$$

The events were equally distributed in 7 asymmetrical redshift intervals: [0.00, 0.01), [0.01, 0.05), [0.05, 0.10), [0.10, 0.20), [0.20, 0.30), [0.30, 0.50), and [0.50, 2.50), resulting in 2×10^6 events per redshift interval. They were also equally distributed in the initial masses, resulting in 3.5×10^6 events of each initial nucleus, H, He, N and Fe.

The flat distribution and the asymmetrical intervals are advantageous to avoid large statistical uncertainties at the highest energies. With a flat distribution, there is more chance of randomly generating an event with high energy at the sources. Also, for asymmetrical intervals, the probability of generating an event close to Earth increases and these events are expected to compose the end of the spectrum. If those tools were not used, an even larger number of simulated events would have been necessary in order to obtain a few events reaching Earth at the highest energies.

The SimProp code results in a file containing the crucial information for each event, such as the initial energy, redshift, mass and atomic number and the final energy, mass and atomic number. The resulting masses are divided in four groups: proton-like ($A = 1$), helium-like ($2 \leq A \leq 4$), nitrogen-like ($5 \leq A \leq 26$) and iron-like ($27 \leq A \leq 56$). For each combination of free parameters, histograms in $\log E$ binned in intervals of 0.1 are filled with events. Each one is weighted by the energy distribution (Eq. 5.5), by the size of the redshift interval (this recovers a flat distribution in redshift), by $dz/dt^\ddagger$ and by the corresponding fraction of its injected mass, p_i . Binning the histogram in intervals of $\log E$ introduces an energy dependent term in the spectrum:

$$\frac{dN}{d \log E} = \frac{dN}{dE} E = EJ. \quad (5.7)$$

[‡] This is necessary as SimProp generates, for each interval, events homogeneously distributed in z . Multiplying by $dz/dt = (1+z)H_0\sqrt{\Omega_m(1+z)^3 + \Omega_\Lambda}$ will result in events homogeneously distributed in a comoving volume.

Finally, the resulting spectra are obtained together with the mean and RMS of the $X_{\max}$ distribution, using the parametrization described in Appendix E.

Fig. 40 shows the resulting spectra with $\Gamma = 0.94$, $R_{Max} = 10^{18.67}$ V, $pH = 0$, $pHe = 0.62$ and $pN = 0.372$ for both the LI scenario and the LIV scenario with $\delta_p = 10^{-22}$.

Figure 40 – Spectra of nuclei for $\Gamma = 1.3$, $R_{Max} = 10^{19.15}$ V, $pH = 0.22$, $pHe = 0.2$ and $pN = 0.56$. The black continuous lines show the total spectrum and the colored dashed lines show the spectra for proton-like ($A = 1$), helium-like ($2 \leq A \leq 4$), nitrogen-like ($5 \leq A \leq 26$) and iron-line ($27 \leq A \leq 56$). The black dots show the data from Auger. (10). The left panel shows a scenario with LI, while the right panel shows a LIV scenario with $\delta_p = 10^{-22}$.

Source: By the author.

No strong effects are seen. Nevertheless, to better quantify the effects a fit similar to that made in the proton case is necessary and will later be performed using a procedure similar to the one used for the combined fit of the observatory data. (99)

5.3 Photons

Propagating protons and nuclei interact with the background photons and can produce neutral pions, as discussed in Chapter 4. These pions decay into photons with energies of the order of EeV, which are called GZK photons. They propagate and can be absorbed by the background radiation, emitting a pair.

As discussed in Chapter 2, the Pierre Auger Collaboration has imposed upper limits on the flux of EeV photons on Earth. Also, as shown in Chapter 4 the effect of LIV in the photon propagation is an increase in the photon horizon, i.e., in the distance a photon can be emitted in order to be detected on Earth before interacting. Therefore, it is expected that the flux of photons would increase with LIV and, for some LIV coefficient,

it could be larger than the upper limits imposed by Auger. Consequently, the upper limits on the photon flux from Auger can be used to impose upper limits on the LIV coefficients for the photon.

Like nuclei, the propagation of photons is treated in Monte Carlo simulations and none contains the implementation of LIV effects. The energy losses for the photon pair production with LIV have been implemented in the EleCa code (101) and for the first time the spectrum of GZK photons considering LIV was obtained. Protons and nuclei propagation have been simulated using CRPropa 3. (96) The resulting secondary photons from the propagation were used as an input for EleCa, which simulates the photon propagation and gives the resulting spectrum of GZK photons on Earth.

As the injected spectrum for GZK photons comes directly from the propagation of protons and nuclei, it is highly dependent on the assumptions made for it, such as the energy distribution and composition at the sources, the IRB and RB models and the cross section for the photodisintegration.

The spectrum of photons has been obtained using the same assumptions used in (101). A spectral index of $\Gamma = 2.7$, with a maximum rigidity of $R_{Max} = 10^{21}$ V and a pure composition were used for the sources and the default settings were used in CRPropa 3. The nuclei spectrum is normalized to the Auger data at $E = 10^{18.85}$ eV and the integral spectrum is used for photons in order to decrease the statistical uncertainties, since only few photons are expected to reach Earth.

Fig. 41 shows the integral spectrum of GZK photons on Earth for a LI scenario with a pure composition of either protons or irons. The spectrum has a strong dependence on the initial mass. A pure proton composition gives a spectrum almost two orders of magnitude larger than a pure iron one. This is expected since the only interaction directly related to the emission of GZK photons is the pion production and, as discussed in Chapter 4, a nucleus of mass A and energy E is treated as A protons of energy E/A for the pion production. Using this assumption, many nuclei will be treated as protons with energies below the pion production threshold, resulting in less GZK photons. In both cases the spectrum is below the upper limits from Auger.

Fig. 42 shows the integral spectrum for a pure proton composition considering both LI and LIV. As expected, the flux grows with LIV, since photons can travel longer without interacting. For a given LIV coefficient, the flux becomes larger than the upper limits from Auger. Under these assumptions for both the sources and the propagation (sources emitting purely proton with $\Gamma = 2.7$, $R_{Max} = 10^{21}$ V, propagation using default settings for CRPropa 3 and EleCa), a limit of $\delta_\gamma \geq -10^{-23}$ is imposed for the zeroth order LIV coefficient in the photon sector using the upper limits of Auger for the photon flux with 95% confidence level. For heavier compositions, however, only stronger coefficients would be restricted, since the spectrum is expected to be smaller.

Figure 41 – Integral spectrum of GZK photons in a LI scenario for a energy distribution with $\Gamma = 2.7$, $R_{Max} = 10^{21}$ V and pure composition. The black line shows the spectrum for a pure proton composition at the sources, while the blue line shows the spectrum for a pure iron composition. The red arrows show the upper limits imposed by the Pierre Auger Observatory (35), the first four points are limits from the hybrid data, while the last three are limits from the SD data.

Source: By the author.

As it has been shown, upper limits on the LIV coefficients for the photons can be derived from the upper limits on the photon flux measured by the Pierre Auger Observatory. This, however, relies on simulations that contain a lot of uncertainties due to several features, such as the source assumptions, the codes used for the simulations and the models for the photon background and for the cross sections. Therefore, it is very important to understand very well these sources of uncertainties before imposing a reliable limit on LIV. It has also been shown that the lighter composition the stronger will be the imposed limit. Since the Pierre Auger Observatory is still collecting data, its limits will become stronger, which will also result in stronger limits for the LIV.

Figure 42 – Integral spectrum of GZK photons in a LIV scenario for a energy distribution of $\Gamma = 2.7$ and a pure proton composition. The black line shows the spectrum for the LI scenario, while the colored lines show the spectrum for LIV scenarios with different LIV coefficients. The red arrows show the upper limits imposed by the Pierre Auger Observatory (35), the first four points are limits from the hybrid data, while the last three are limits from the SD data.

Source: By the author.

6 CONCLUSIONS

In this work the effects of Lorentz invariance violation on the propagation of ultra-high energy cosmic rays and the possibility of detecting it with the Pierre Auger Observatory have been discussed.

Relativity is one the most solid and well tested theories of the last century and Lorentz invariance is one of its pillars. It explains physics at both large scales and large energies. In the last decades, however, the search for a theory that explains physics at both large energies and small scales, combining both quantum mechanics and relativity, has been intensified. Many of these theories discuss the possibility of violating Lorentz invariance at very high energies. Therefore, testing Lorentz invariance violation is essential for both high-energy and theoretical physics.

In this work a commonly used perturbative framework for LIV has been presented, in which the effects would be suppressed by the Planck scale and, consequently, be detectable only at very high energies. For this reason, astrophysics provides a fruitful scenario for these tests, since cosmic rays are detected with energies almost three orders of magnitude higher than those achieved at terrestrial accelerators.

The most energetic cosmic rays are studied at the Pierre Auger Observatory, the largest cosmic ray observatory so far. Its design and scientific highlights have been presented, emphasizing the spectrum of cosmic rays above 3×10^{17} eV, the distribution of the depth of shower maximum (and consequently the mass composition) and the upper limits in the photon flux. The main goal of this work was to describe how these three measurements are changed by LIV and, thus, discuss the sensitivity of the observatory to LIV and the possibility of using its measurements to impose upper limits on LIV.

For the propagation of UHECRs, it has been discussed that the only expected effect of LIV is a change in the inelasticity of the interactions. Unlike most works in the literature, for which the inelasticity with LIV is usually obtained for a single interaction and in the nucleus reference frame, an analytical calculation for the inelasticity with LIV in the laboratory frame has been presented for any $a + b \rightarrow c + d$ following the assumptions that $E_a \gg E_b$ and $p_n \gg m_n$ for $n = a, c, d$. This is very important as it provides a framework for treating the effect of LIV in any of the interactions present in the propagation. The calculation in the laboratory frame proposed here is more elegant as it is not necessary to impose that Lorentz transformations are preserved in spite of LIV. The calculation shows that the kinematics and the geometry of the interaction and the LIV effects can be reduced to a single polynomial of order $n + 3$, where n is the order of LIV.

This framework has been used to discuss the propagation of protons, nuclei and

photons.

For protons, the inelasticity as a function of the initial proton and the background photon energies with LIV for the proton and for the pion have been presented. The main effect of LIV is a shift in the phase space and, consequently, in the threshold energy. This change only becomes significant at some initial energy (which is given by the LIV coefficient) and becomes stronger the higher the initial proton energy. For LIV for the proton, the phase space becomes larger and the opposite happens for the pion. The attenuation lengths for this interaction in both LIV scenarios have also been shown. The effects become more clear as the attenuation length is increased in the pion case, representing that the pion can travel farther than in the LI scenario and the opposite happens for the proton.

For nuclei, a parametrization for the threshold energy of the photodisintegration considering LIV effects has been proposed here for the first time. The parametrized function depends on both the initial nucleus energy and mass. It is important to obtain it since nuclei propagation is usually performed with Monte Carlo simulations, which demand a high computational cost. The mean free path for the photodisintegration of a helium and of an iron nucleus have been obtained using both the parametrization and the analytical calculation for the threshold energy. The results obtained with the parametrization are very similar to those obtained with the analytical calculation, which validates the proposed parametrization. The effects of LIV on the mean free path are similar to those obtained for the pion production. With LIV, the mean free path becomes larger, meaning that a nucleus can travel farther before interacting.

Finally, for photons the phase space as a function of the initial energies and of the LIV coefficient is discussed in an innovative way. The main effect is a decrease for negative values of the LIV coefficient. The mean free path and the horizon considering LIV with $n = 0$, $n = 1$ and $n = 2$ have also been presented for the first time. The effects are once more similar, resulting in larger mean free paths and horizons with LIV. The effects of a larger LIV order is not necessarily weaker, since the LIV coefficients are free parameters not derived from the theory and must be constrained independently.

In summary, it is concluded in this work that LIV have strong effects on the propagation of ultra-high energy cosmic rays, which can be seen directly in the phase space, the attenuation length, the mean free path, and the horizon. Photons are neutral and, consequently, not deflected by magnetic fields. For that reason, from the photon horizon, it is possible to determine from which distance a source could be detected on Earth for a given sensitivity. This could be used by the next generation of GeV-TeV gamma-ray astronomy observatories, such as CTA, to test LIV. The other quantities, however, cannot be directly measured and, thus, it is essential to study how they affect the measurements performed on Earth.

The resulting effects in the propagation have been used in an analytical calculation

of the proton spectrum and have been unprecedentedly implemented in two different Monte Carlo propagation codes, SimProp v2r3 and EleCa. It has been shown how the spectrum of UHECRs is changed for a LIV scenario considering both a pure proton and a mixed composition. For the first time, the proton spectrum fitted to the Pierre Auger Observatory data and the integral spectrum of GZK photons are presented for a LIV scenario. The possibility of using the upper limits on the photon flux from the Pierre Auger Observatory to impose upper limits on the photon LIV coefficient has also been widely discussed for the first time.

For a pure proton composition the fit parameters have almost no dependence in the LIV coefficient and, even though there is a small decrease in the χ_{red}^2 for large values of δ_p , its values are still very large. For that reason, it can be concluded that, if the cosmic rays were composed only by protons, the Pierre Auger Observatory would have almost no sensitivity to LIV in the proton and in the pion scenario. The main reason is that the measured suppression on the spectrum would be best described by a maximum energy at the sources smaller than the energies at which the pion production is dominant. There are, however, several evidences that disagree with a pure proton composition scenario.

For a mixed composition, the spectrum and the composition must be obtained via Monte Carlo simulations. The results for a singular case have been shown. Nevertheless, a more complete result must be taken from a combined fit considering LIV effects. This has never been done and will be performed using the results from this work and a procedure similar to that used in (99). There are, however, large uncertainties in this calculation coming from the code used, the models for the background distribution, the models for the cross section of the photodisintegration, the fit algorithm, among others. This is still an open topic even for the LI case. (99)

For photons, the integral spectrum of GZK photons with LIV have been treated for the first time in the literature. It has been shown that negative values for the LIV coefficient of the photon result in a more intense spectrum and, therefore, for a given LIV coefficient it would be higher than the upper limit from the Pierre Auger Observatory. Using this, it is possible to obtain upper limits on the LIV coefficients given the flux upper limits. For the first time, limits for the zeroth order LIV coefficient for the photon ($\delta_\gamma \geq -10^{-23}$) were obtained for a particular case: pure proton composition with $\Gamma = 2.7$, $R_{Max} = 10^{21}$ V, propagation using the default settings from CRPropa 3 and EleCa. Nevertheless, these fluxes of GZK photons are highly dependent on the propagation of the nuclei and protons that generated them and, therefore, the same uncertainties from the nuclei case apply. New limits will be obtained using the results from the combined fit for nuclei.

It has been shown that the effects of LIV in the proton and nuclei spectra are more subtle than in the flux of GZK photons and, therefore, stronger limits can be derived for photons. Nevertheless, it is still necessary to understand better such uncertainties for all

three cases in order to derive a final result for the tests of LIV using UHECRs. Those are open questions that are still under discussion even for the LI scenario. This work, however, provides a wide and useful framework for testing LIV in the astrophysical scenario, presenting several new results in the field, ranging from the calculation of the changes in the kinematics of the interactions in the UHECR propagation up to the resulting spectra and composition on Earth and the comparison with the experimental data from the Pierre Auger Observatory.

This work can be used to study the effects of LIV on interactions not treated here such as the pair production for protons or the inverse Compton scattering for electrons. It can also be adapted to treat the effects of LIV in the development of extensive air showers (EASs), which would change the measured composition. (80) A more complete analysis can also be discussed considering LIV in a multi-messenger approach. Changes in propagation of protons, nuclei, photons, electrons and neutrinos and in the development of EASs can be considered and the result of different experimental techniques and energies can be used for a more robust result.

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Appendix

APPENDIX A – CROSS SECTIONS

A.1 Pion Production

The pion production interaction ($p + \gamma \rightarrow p + \pi$) can happen through several channels, the $\Delta(1232)$ resonance being the most important. The cross sections used in this work are parametrizations taken from the SOPHIA code (102) for each channel.

Figure 43 – Cross section for each of the pion production channels as a function of the photon energy in the NRF. The red dashed line represents the direct channel, the blue dashed line represents the resonant channel, the green dashed line represents the multipion production channel and the black continuous line represents the total cross section.

Source: By the author.

Fig. 43 shows the importance of each channel in the cross section. In the region where the interaction is most important ($\epsilon' \sim 0.3$ GeV), the dominant channel is the resonant one. The cross section for the direct channel is almost one order of magnitude lower and this channel is expected only at the same energies where the resonant channel is also expected. Finally, the multipion production channel is important only at very high energies ($\epsilon' > 3$ GeV).

Another parametrization for the cross section can be found in (93).

A.2 Photodisintegration

The cross section for the photodisintegration is parametrized by different models. The two most used ones are the PSB model (85) using the Stecker-Salamon thresholds (100)

and TALYS (103). Arbitrary functions can also be used to fit the data such as Gaussian and Breit-Wigner functions. A simple parametrization can also be found in (93).

Figure 44 – Cross section for each of the photodisintegration channels for an iron nucleus as a function of the photon energy in the NRF using the parametrization from (93). The red dashed line represents the giant dipole resonance channel, the green dashed line represents the quasi-deuteron, the blue dashed line represents the baryon resonance excitation channel, the pink dashed line represents the fragmentation channel and the black continuous line represents the total cross section.

Source: By the author.

Fig. 44 shows the cross section for each channel using the parametrizations from (93). Each channel is dominant for an energy. The dominant channels are: the giant dipole resonance (GDR) for $6 \text{ MeV} < \epsilon' < 25 \text{ MeV}$, the quasi-deuteron (QD) for $25 \text{ MeV} < \epsilon' < 160 \text{ MeV}$, the baryon resonance excitation (BRE) for $160 \text{ MeV} < \epsilon' < 1600 \text{ MeV}$ and the fragmentation for $\epsilon' > 1600 \text{ MeV}$.

A.3 Photon pair production

The Breit-Wheeler (104) cross-section was used for photon pair production:

$$\sigma(E, \epsilon, \theta) = \frac{2\pi\alpha^2}{3m_e^2} W(\beta), \quad (\text{A.1})$$

with

$$W(\beta) = (1 - \beta^2) \times \left[2\beta (\beta^2 - 2) + (3 - \beta^2) \ln \left(\frac{1 + \beta}{1 - \beta} \right) \right], \quad (\text{A.2})$$

where β is the speed of the electron-positron pair in the center-of-mass reference frame and is given by:

$$\beta(E, \epsilon, \theta) = \left(1 - \frac{2m_e^2 c^4}{E\epsilon(1 - \cos\theta)}\right)^{1/2}. \quad (\text{A.3})$$

APPENDIX B – NRF INELASTICITY

These calculations are based on those presented in (77).

Considering the same situation and the same assumptions presented in Section 4.1.2.3, the initial total rest energy, $s_{total} = \sqrt{E_{total}^2 - p_{total}^2}$, is given by:

$$s_i := s_{total,i} = (E_a + E_b)^2 - (\vec{p}_a + \vec{p}_b)^2. \quad (\text{B.1})$$

Assuming that LIV is not strong enough, this quantity is the same in any frame. Specifically looking at the nucleus reference frame (NRF)*, where $(\vec{p}_a)' = 0 \implies E'_a = \sqrt{s_a}$:

$$\begin{aligned} s_i := s_{total,i} &= (E_a + E_b)^2 - (\vec{p}_a + \vec{p}_b)^2 = (E'_a + E'_b)^2 - ((\vec{p}_a)' + (\vec{p}_b)')^2 = \\ &= (\sqrt{s_a} + E'_b)^2 - (\vec{p}_b')^2. \end{aligned} \quad (\text{B.2})$$

Now looking at the center of mass reference frame (CMF), where $(\vec{p}_c)^* = -(\vec{p}_d)^*$:

$$s_f := s_{total,f} = (E_c + E_d)^2 - (\vec{p}_c + \vec{p}_d)^2 = (E_c^* + E_d^*)^2 - ((\vec{p}_c)^* + (\vec{p}_d)^*)^2 = (E_c^* + E_d^*)^2. \quad (\text{B.3})$$

Imposing the conservation of the total rest energy ($s = s_i = s_f$), this leads to:

$$\begin{cases} E_c^* = \sqrt{s} - E_d^*, \\ (p_c^*)^2 = (p_d^*)^2. \end{cases} \quad (\text{B.4})$$

Therefore, it is possible to relate s_c , s_d and s :

$$\begin{aligned} s_c &= (E_c^*)^2 - (p_c^*)^2 = (\sqrt{s} - E_d^*)^2 - (p_d^*)^2 \\ &\implies s + 2\sqrt{s}E_d^* + (E_d^*)^2 - (p_d^*)^2 = s_c \\ &\implies E_d^* = \frac{s + s_d - s_c}{2\sqrt{s}}. \end{aligned} \quad (\text{B.5})$$

It is then necessary to use the Lorentz transformations to obtain the relation between E_d and E_d^* :

$$E_d = \gamma (E_d^* + \beta (\vec{p}_d)^* \cos \phi) = \gamma \left(E_d^* + \beta \sqrt{(E_d^*)^2 - s_d} \cos \phi \right), \quad (\text{B.6})$$

* All quantities in the NRF will be denoted with a ', while the quantities in the CMF will be denoted with a * and in the LF they will have no superscript.

where ϕ is the angle between the momentum of d in the CMF and in the LF. β is the velocity of the frame, and as $E_a \gg E_b$, $\beta \approx 1$. γ is the Lorentz factor and is given by $\gamma = E_a/\sqrt{s}$.

Finally, using $E_d = (1 - K_\phi)E_a$ together with Eqs. B.5 and B.6:

$$(1 - K_\phi) = \frac{s + s_d - s_c}{2s} + \sqrt{\left(\frac{s + s_d - s_c}{2s}\right)^2 - \frac{s_d}{s}} \cos \phi. \quad (\text{B.7})$$

K_ϕ must be solved numerically and, lastly, K is given by the average of K_ϕ :

$$K = \int_{\mathcal{C}} \frac{d\phi}{\mathcal{C}} K_\phi, \quad (\text{B.8})$$

where $\mathcal{C}$ is the region where K_ϕ fulfills the physical criteria:

$$\begin{cases} K_\theta \in (0, 1), \\ K_\theta \in \mathcal{R}. \end{cases} \quad (\text{B.9})$$

In the usual case where b is a background photon with energy ϵ , Eq. B.2 becomes:

$$s = (s_a + \epsilon')^2 - (\epsilon')^2 = s_a + 2\sqrt{s_a}\epsilon', \quad (\text{B.10})$$

and, therefore K can be obtained as a function of the background photon energy in the NRF. This is usual as many cross sections are obtained in this frame.

For the photodisintegration ($N_A + \gamma \rightarrow N_{A-1} + N_1$), however, it is also necessary to take into account the binding energy of both the initial and the final nuclei. This is achieved by using the threshold energy without LIV, which is usually given in the NRF. A shift in the energy of the background photon in the relation dispersion is put by hand, resulting in:

$$s = s_N + 2\sqrt{s_N} \left(\epsilon' - \left(\epsilon_{th}^{LI} \right)' \right). \quad (\text{B.11})$$

The values of $\left(\epsilon_{th}^{LI} \right)'$ used in this work are taken from the CRPropa code. (96)

APPENDIX C – LF INELASTICITY POLYNOMIAL

Define A as:

$$A = s_a + s_b + 2E_a E_b \left(1 - \cos \theta_i + \cos \theta_i \left(\frac{s_a}{2E_a^2} + \frac{s_b}{2E_b^2} \right) \right) - m_c^2 - m_d^2 = \quad (C.1)$$

$$= s_i - m_c^2 - m_d^2.$$

Starting from Eqs. 4.13 and 4.14 and imposing $s_i = s_f$

$$A - \delta_{c,n}(KE_a)^{(n+2)} - \delta_{d,n}((1-K)E_a)^{(n+2)} - \left(\frac{(1-K)s_c}{K} + \frac{Ks_c}{(1-K)} \right) = 0. \quad (C.2)$$

Multiplying Eq. C.2 by $K(1-K)$:

$$K(1-K)A - \delta_{c,n}E_a^{(n+2)}K^{(n+3)}(1-K) - \delta_{d,n}E_a^{(n+2)}(1-K)^{(n+3)}K + \quad (C.3)$$

$$- \left((1-K)^2 s_c + K^2 s_d \right) = 0.$$

Writing the complete form of s_c and s_d :

$$K(1-K)A - \delta_{c,n}E_a^{(n+2)}K^{(n+3)}(1-K) - \delta_{d,n}E_a^{(n+2)}(1-K)^{(n+3)}K + \quad (C.4)$$

$$- (1-K)^2 m^2 - K^2 m_d - \delta_{c,n}E_a^{(n+2)}K^{(n+2)}(1-K)^2 +$$

$$- \delta_{d,n}E_a^{(n+2)}(1-K)^{(n+2)}K^2 = 0.$$

Writing $(1-K)^m$ in the binomial form:

$$(1-K)^m = \sum_{i=0}^m \left(\binom{m}{i} (-1)^i K^i \right), \quad (C.5)$$

$$(K-K^2)A - \delta_{c,n}E_a^{(n+2)}K^{(n+3)} + \delta_{c,n}E_a^{(n+2)}K^{(n+4)} + \quad (C.6)$$

$$- \delta_{d,n}E_a^{(n+2)} \sum_{i=0}^{(n+3)} \left(\binom{n+3}{i} (-1)^i K^{i+1} \right) - (1-2K+K^2)m^2 +$$

$$- K^2 m_d - \delta_{c,n}E_a^{(n+2)}(K^{(n+2)} - 2K^{(n+3)} + K^{(n+4)}) +$$

$$- \delta_{d,n}E_a^{(n+2)} \sum_{i=0}^{(n+2)} \left(\binom{n+2}{i} (-1)^i K^{i+2} \right) = 0.$$

Finally, arranging Eq. C.6 in terms of K :

$$\begin{aligned}
0 &= K^0 \left[-m_c^2 \right] \\
&+ K^1 \left[A + 2m_c^2 - \delta_{d,n} E_a^{(n+2)} \right] \\
&+ K^2 \left[-A - m_c^2 - m_d^2 \right] \\
&+ K^{(n+2)} \left[-\delta_{d,n} E_a^{(n+2)} \right] \\
&+ K^{(n+3)} \left[\delta_{d,n} E_a^{(n+2)} \right] \\
&- \delta_{d,n} E_a^{(n+2)} \left(\sum_{i=0}^{(n+3)} \left(\binom{n+3}{i} (-1)^i K^{i+1} \right) + \sum_{i=0}^{(n+2)} \left(\binom{n+2}{i} (-1)^i K^{i+2} \right) \right).
\end{aligned} \tag{C.7}$$

APPENDIX D – PARAMETRIZATION OF THE PHOTODISINTEGRATION THRESHOLD ENERGY WITH LIV

The proposed parametrization is a suppressed exponential with three parameters, a , b and c :

$$\left(\epsilon_{th}^{LIV}\right)' = \left(\epsilon_{th}^{LI}\right)' + \frac{a \log(E_N/\text{eV})}{1 + e^{c(\log(E_N/\text{eV})-b)}}. \quad (\text{D.1})$$

Here a and b are parametrized as functions of A and δ_p , and c is parametrized as a function of A :

$$\begin{cases} a = e^{\alpha_0(A) + \alpha_1(A) \ln \delta_p} \times \text{MeV}, \\ b = \beta_0(A) + \beta_1(A) \ln \delta_p, \\ c = \gamma_0 + \gamma_1 \ln(A). \end{cases} \quad (\text{D.2})$$

Finally, α_0 , α_1 , β_0 and β_1 are parametrized as $p_0 + p_1 \ln A$.

Fig. 45 shows the fitted a , b and c , while Fig. 46 shows the fitted α_0 , α_1 , β_0 and β_1 , both for the emission of 1 nucleon.

Table 1 contains the values for the fitted parameters for the emission of 1 and 2 nucleons.

Table 1 – Fitted parameters for the emission of 1 and 2 nucleons.

Parameter	1n emission		2n emission	
γ_0	-4.48		-4.42	
γ_1	0.043		0.023	
	p_0	p_1	p_0	p_1
α_0	3.994	0.063	4.65	0.072
α_1	0.0053	0.00162	0.0048	0.00169
β_0	8.88	0.512	8.74	0.55
β_1	-0.21898	0.00061	-0.21923	0.00066

Source: By the author.

Figure 45 – Parameters for the emission of 1 nucleon. The three panels show the parametrization for a , b and c , respectively.

Source: By the author.

Figure 46 – Parameters for the emission of 1 nucleon. The four panels show the parametrization for α_0 , α_1 , β_0 and β_1 , respectively.

Source: By the author.

APPENDIX E – PARAMETRIZATION OF THE $X_{\max}$ DISTRIBUTION

In this work, the same parametrizations for the mean and RMS of the $X_{\max}$ distribution from (12) and (13) were used. These parametrizations were obtained using air showers simulated with the Conex code (105) and considering three hadronic interaction models: EPOS-LHC (30), QGSJetII-04 (31) and Sibyll 2.1 (32). These models contain a large uncertainty, since they are obtained by extrapolating data from accelerators, which are orders of magnitude less energetic than cosmic rays.

The parametrizations are presented for each model $m = 1, 2, 3$ and for four primary masses $A = 1, 4, 14, 56$. The mean and the standard deviation of the distribution for each mass (A) and each model (m) are given by:

$$\langle X_{\max} \rangle_{m,A} = a_{m,A} + b_{m,A} \log \left(\frac{E}{10^{18} \text{ eV}} \right) + c_{m,A} \log \left(\frac{E}{10^{18} \text{ eV}} \right)^2, \quad (\text{E.1})$$

$$\sigma(X_{\max})_{m,A} = d_{m,A} + e_{m,A} \log \left(\frac{E}{10^{18} \text{ eV}} \right) + f_{m,A} \log \left(\frac{E}{10^{18} \text{ eV}} \right)^2, \quad (\text{E.2})$$

where $a_{m,A}$, $b_{m,A}$, $c_{m,A}$, $d_{m,A}$, $e_{m,A}$ and $f_{m,A}$ are fitted parameters given in Tables 2-4.

Finally, for a mixed composition with mass fractions, p_H , p_{He} , p_N and p_{Fe} , the mean and the RMS are given by:

$$\langle X_{\max} \rangle_m = \sum_i p_i \langle X_{\max} \rangle_{m,i}, \quad (\text{E.3})$$

$$RMS(X_{\max})_m = \sqrt{\sum_i p_i \left(\sigma(\langle X_{\max} \rangle_{m,i})^2 + (\langle X_{\max} \rangle_{m,i} - \langle X_{\max} \rangle_m)^2 \right)}, \quad (\text{E.4})$$

with $i = 1, 4, 14, 56$.

Table 2 – Fitted parameters for the EPOS-LHC model (30) from the parametrizations used in (12) and (13).

Parameter	EPOS-LHC			
	H	He	N	Fe
a	746.744	712.257	677.766	640.074
b	61.5764	61.9081	63.6418	65.686
c	0.683626	-0.157953	-0.614843	-1.49584
d	59.5088	40.8119	27.2669	19.1397
e	-3.06679	-2.64365	-1.92396	-1.30599
f	1.62307	0.160232	0.471025	0.0294594

Source: By the author.

Table 3 – Fitted parameters for the Sybill 2.1 model (32) from the parametrizations used in (12) and (13).

Parameter	Sybill 2.1			
	H	He	N	Fe
a	736.145	703.476	671.147	635.505
b	57.9241	58.1001	57.8896	58.9037
c	0.368459	-0.21767	-0.00581617	-0.467717
d	57.4096	42.0575	33.5773	22.9498
e	-5.11702	-4.22864	-3.49536	-2.17181
f	1.1529	0.490269	0.0920901	0.144377

Source: By the author.

Table 4 – Fitted parameters for the QGSJetII-04 model (31) from the parametrizations used in (12) and (13).

Parameter	QGSJetII-04			
	H	He	N	Fe
a	734.338	704.898	674.68	643.132
b	49.1772	50.843	53.9165	56.8632
c	-1.68727	-1.61006	-1.71672	-2.34459
d	59.946	43.0545	32.0093	22.7121
e	-4.38379	-3.57018	-2.02791	-1.41324
f	0.896223	0.971876	0.494634	0.168233

Source: By the author.