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ABASALT BAHRAMI

**Investigation of the momentum distribution of an excited
Bose-Einstein Condensate: Coupling to normal modes**

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**Investigation of the momentum distribution of an excited
Bose-Einstein Condensate: Coupling to normal modes**

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I would like to bring one of poems by Hafiz (Ghazal 178)¹ to dedicate this thesis to my family. Specially my mother, father and my wife who have shown me more love than I could ever repay.

Whoever became the confidant of his own heart, in the sacred fold of the Beloved remained and who knew not this matter, in ignorance remained. More pleasant than the sound of love's speech, naught I heard: A great token, that, in this revolving dome remained. One day, to the spectacle-place of thy tress, Hafez's heart went that it would return; but, ever, captive to thy tress, it remained. (Translation by: Behrouz Homayounfar)

¹Khawaja Shams-ud-Din Muhammad Hafez-e Shirazi, known by his pen name Hafez was a Persian poet. His collected works are regarded as a pinnacle of Persian literature and are to be found in the homes of most people in Iran, who learn his poems by heart and use them as proverbs and sayings to this day. His life and poems have been the subject of much analysis, commentary and interpretation, influencing post-fourteenth century Persian writing more than any other author. Themes of his ghazals are the beloved, faith, and exposing hypocrisy. His influence in the lives of Farsi speakers can be found in "Hafez readings" and the frequent use of his poems in Persian traditional music, visual art, and Persian calligraphy. His tomb is visited often. Adaptations, imitations and translations of Hafez' poems exist in all major languages. (Text taken from <http://en.wikipedia.org/wiki/Hafez>)

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THE REASONABLE MAN ADAPTS HIM-
SELF TO THE WORLD; THE UNREA-
SONABLE ONE PERSISTS IN TRYING
TO ADAPT THE WORLD TO HIM-
SELF. THEREFORE, ALL PROGRESS
DEPENDS ON THE UNREASONABLE
MAN.

-- George Bernard Shaw

Resumo

BAHRAMI, A. **Investigação da distribuição de momentum em um condensado de Bose-Einstein excitado: acoplamento com os modos normais.** 2014. 80 p. Dissertação (Mestrado em Ciências) - Instituto de Física de São Carlos, Universidade de São Paulo, São Carlos, 2014.

Um dos tópicos recentes das pesquisas em superfluidos atômicos é o estudo da turbulência quântica. Em fluídos, a turbulência é caracterizada pelo regime caótico no escoamento dos fluidos e aparece em muitos importantes processos na natureza. Em sistemas superfluidos, a forma mais simples da turbulência é apresentada pelo enovelamento de vórtices. Assim, o estudo de vórtices nesses sistemas torna-se um ponto de partida para estudar o fenômeno da turbulência em gases quânticos. Há alguns anos atrás, em nosso grupo de pesquisa, um condensado de Bose-Einstein de ^{87}Rb foi usado para observar e investigar a emergência de turbulência quântica. Em continuidade a esses estudos, aplicamos uma excitação oscilatória na nuvem atômica aprisionada e os vórtices são criados na interface entre o condensado e a nuvem térmica, que se propagam para o interior da nuvem, atingindo as condições ideais para o aparecimento de um regime turbulento. Uma vez que esse regime é atingido, o condensado é diagnosticado através de uma imagem de absorção obtida após a sua expansão balística em tempo de voo. O perfil de densidade obtido é usado para determinar a distribuição de momento do condensado aprisionado. Neste trabalho, observamos que os perfis de densidade dos condensados excitados possuem uma forma característica e diferente dos condensados não-excitados. Nos estudos da distribuição de momento e energia dessas nuvens excitadas, vimos uma evidência de uma lei de potência (parecida com a lei de Kolmogorov para turbulência) e, além disso, um acoplamento entre o modo quadrupolar de oscilação da nuvem e a distribuição de momentos dessa nuvem. Também discutimos algumas propriedades adicionais do sistema, por exemplo, os modos coletivos de excitação do condensado, o que tem um papel muito importante na rota para o regime de turbulência quântica. Para continuarmos com os estudos neste tópico de pesquisa, estamos melhorando nosso sistema experimental a fim de investigarmos melhor estas propriedades dinâmicas do superfluido, através dos efeitos dos modos coletivos no espectro de momentos da nuvem atômica. Para isso, pretendemos desenvolver novas técnicas e ferramentas necessárias para realizar medidas mais precisas e reprodutivas.

Palavras-chave: Condensação de Bose-Einstein. Turbulência quântica. Modos coletivos.

Abstract

BAHRAMI, A. **Investigation of the momentum distribution of an excited Bose-Einstein condensate: coupling to the normal modes.** 2014. 80 p. Thesis (Master in Science) - São Carlos Institute of Physics, Universidade de São Paulo, São Carlos, 2014.

Turbulence is a young field of research which is characterized by chaotic spinning flow regimes which appears in many important processes in nature. Vorticity, in superfluid systems, may present the simplest form of turbulence, and be a gateway to the study of this phenomenon in quantum gases. A ^{87}Rb Bose condensate was used to observe and investigate the emergence of quantum turbulence, a few years back in our group. The vortices are created on the condensed-thermal interface and propagate across the cloud, setting up the experimental conditions favorable to the emergence of turbulence. Once the turbulent regime is set, the condensate is released and expands under free fall. Then, the atomic density profile is acquired, after some time-of-flight, and used to determine the in situ momentum distribution of the BEC. In this work, we have observed that, the perturbed density profiles are characteristic and different from the standard, non-perturbed ones. We have seen evidences of power law in the studied momentum and energy distributions and also coupling of quadrupolar mode to the momentum distribution of the excited condensate which is the main part of our findings. Additional features of the system, such as the condensate's excited collective modes which plays a very important role on the roadmap to the turbulence regime, are discussed. We are currently setting up an experiment to be able to further investigate such features, and also to unfold the effects of interactions on the energy and momentum spectra associated to the density profiles. In doing so, we will further develop the tools and techniques needed to acquire more accurate and reliable results.

Keywords: Bose-Einstein condensation. Quantum turbulence. Collective modes.

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Chapter 1

Introduction

Experiments carried out at ultra-low temperatures revealed that physical systems can exhibit extraordinary properties at temperatures close to the absolute zero¹. Two well-known phenomena which can be found at ultralow temperatures, are superconductivity and superfluidity.

There is a distinctive feature stamped in the context of superfluidity, which was originally discovered in liquid ^4He (2–4): the ability of superfluids to behave like a normal fluid with zero viscosity (Section 1.1). In the context of superfluids, quantum turbulence is among the most intriguing and long-lasting unsolved problems (Chapter 2).

This chapter brings the reader an overview of superfluidity and its connection to the *Bose-Einstein condensation* (BEC) which is the underlying support of our main studies in this thesis. It also includes the Landau critical velocity and the excitation spectra. This chapter will come to the end by Gross-Pitaevskii equation describing the macroscopic occupation of the ground state in a quantum system (Section 1.1.2).

1.1 Overview of superfluids

Superfluidity is a quantum effect in which matter has vanishing viscosity when it flows at speeds not greater than the so called Landau’s critical velocity, v_L . (5) Superfluidity in liquid ^4He was independently discovered in 1938 by Kapitza (2) and back-to-back by Allen and Misener. (3)

Shortly after the discovery of superfluid ^4He , F. London (4) proposed that the new phase transition at λ -point ($T_\lambda = 2.17\text{ K}$) from normal He-I to He-II (consists of normal and superfluid component), might be closely related to the phenomenon of Bose-Einstein condensation (BEC) which was originally predicted by Einstein to occur in an ideal gas of atoms². (7)

¹ The starting point of the first studies in low-temperature physics can date back to 1883 when the main constituents of air were liquefied by Zygmunt Florenty Wróblewski. (1)

² In 1924, S. Bose achieved the Planck distribution for a collection of photons. (6) Soon after in 1925, Einstein extended the work of Bose in case of massive particles. (7) The first experimental realizations

London noticed that T_λ was close to the transition temperature of an ideal Bose gas,

$$T_c = \left[\frac{n}{\zeta(3/2)} \right]^{2/3} \frac{2\pi\hbar^2}{mk_B}, \quad (1.1.1)$$

where $\zeta(3/2) \approx 2.61$ is the Riemann zeta function, n is the particle density, m is the mass per boson, $\hbar$ is the reduced Planck constant and k_B is the Boltzmann constant. In fact, not all Bose-Einstein condensates are superfluids, and not all superfluids can be regarded as Bose-Einstein condensates. (18) However, this concept is beyond the scope of this thesis in which BEC and superfluidity are inexorably mixed.

1.1.1 Landau's critical velocity

One of the most traditional ways to describe superfluids is via Landau's critical velocity. In 1941, Landau explained that if the excitation spectrum satisfies certain criteria, the motion of the fluid will be frictionless. (5, 21)

$$v_L = \min \left[\frac{\varepsilon(\mathbf{p})}{|\mathbf{p}|} \right], \quad (1.1.2)$$

where $\varepsilon(\mathbf{p})$ and $\mathbf{p}$ are respectively excitation energy and momentum of the system. Above the Landau's critical velocity, superfluid flow breaks down and the system is heated. In the case of He, this velocity is $v_L \sim 58 \text{ m/s}$ (22) which means elementary excitations will be created only if the speed of flow exceeds this value. We may also point out that weakly interacting Bose gas at zero temperature satisfies the Landau's criteria. The Bogoliubov excitation spectrum (Section 1.1.2.2) of such a system with finite-range interaction, $g|\mathbf{r} - \mathbf{r}'|$, can be written as, (23)

$$\varepsilon(\mathbf{p}) = \left[\frac{g(\mathbf{p})n\mathbf{p}^2}{m} + \left(\frac{\mathbf{p}^2}{2m} \right)^2 \right]^{1/2}, \quad (1.1.3)$$

where n is the density and $g(\mathbf{p})$ is the Fourier transform of the interaction term, $g(\mathbf{r})$. From Equation 1.1.3, Landau's critical velocity can be readily found as the following

$$v_L = \left[\left(\frac{p_c}{2m} \right)^2 + \frac{ng(p_c)}{m} \right]^{1/2}, \quad (1.1.4)$$

where p_c is the critical momentum.

of dilute gas BEC occurred in 1995 (8–10) and since then, BECs have been formed from more than nine different elements and some of their isotopes. (11–17) Further studies can be found in. (18–20)

1.1.2 Gross-Pitaevskii equation

The dynamics of the condensate at zero temperature can be well described with a nonlinear Schrödinger equation³. (25, 26) The Gross-Pitaevskii equation (GPE) utilizes the Hartree–Fock approximation, where the total time-independent wave-function $\psi(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_N)$ of the system of N interacting bosons is taken as a product of single-particle wavefunctions $\varphi(\mathbf{r}_i)$,

$$\psi(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_N) = \prod_{i=1}^N \varphi(\mathbf{r}_i). \quad (1.1.5)$$

The general Hamiltonian that describes our system with wavefunction given in equation 1.1.5 might be written as

$$\mathcal{H} = \sum_{i=1}^N \left(-\frac{\hbar^2}{2m} \left(\frac{\partial^2}{\partial \mathbf{r}_i^2} \right) + V_{ext}(\mathbf{r}_i) \right) + \frac{g}{2} \sum_{i=1}^N \sum_{i \neq j}^N \delta(\mathbf{r}_i - \mathbf{r}_j), \quad (1.1.6)$$

where m is the mass of the boson, $V_{ext}(\mathbf{r})$ is the external potential, $g = 4\pi\hbar^2 a_s/m$ is the coupling constant and a_s is boson-boson scattering length. Using the Hamiltonian given in equation 1.1.6 in the framework of second quantization formalism (27) and Heisenberg’s time evolution equation, one finds the time-dependent Gross-Pitaevskii equation which reads, (18)

$$i\hbar \frac{\partial \psi(\mathbf{r}, t)}{\partial t} = \left(-\frac{\hbar^2}{2m} \nabla^2 + V_{ext}(\mathbf{r}) + g |\psi(\mathbf{r}, t)|^2 \right) \psi(\mathbf{r}, t). \quad (1.1.7)$$

1.1.2.1 Thomas-Fermi approximation

Gross–Pitaevskii equation given in equation 1.1.7, is a nonlinear differential equation and exact solutions are hard to find, thereby solutions have to be approximated. In a situation when the condensate arrives at the so-called Thomas-Fermi (TF) regime, the kinetic energy term can be neglected because it is much smaller than the mean-field energy⁴. In time-independent GPE, if we have $\mu \leq V(\mathbf{r}, t)$, then $\psi_{TF}(\mathbf{r}, t) = 0$ but as long as we have $\mu \geq V(\mathbf{r}, t)$ one finds the simplified solutions which reads

$$\psi_{TF}(\mathbf{r}, t) = \left[\frac{\mu - V(\mathbf{r}, t)}{Ng} \right]^{1/2}. \quad (1.1.8)$$

³ Also known as Gross-Pitaevskii (GPE) or Ginzburg-Landau equation. (24)

⁴ If the number of particles in a gas is very large, the interatomic interaction becomes large so that the kinetic energy term can be neglected from the Gross–Pitaevskii equation. Thomas-Fermi regime is not anymore valid when we are dealing with small number of atoms.

1.1.2.2 Bogoliubov approximation

Bogoliubov approximation (28) is an approach to find the elementary excitations of a Bose–Einstein condensate. To that purpose, the condensate wavefunction is approximated by a sum of the equilibrium wavefunction $\psi_0 = \sqrt{n}e^{-i\mu t}$ and a small perturbation $\delta\psi$, i.e., $\psi = \psi_0 + \delta\psi$. Then this form is inserted inside the equation 1.1.7 and after being linearized to first order in $\delta\psi$, one finds the Bogoliubov dispersion law as the following (detailed calculations might be found in, (18))

$$\varepsilon(p) = \left[\left(\frac{p^2}{2m} \right)^2 + \frac{Ng}{m} p^2 \right]^{1/2}. \quad (1.1.9)$$

For small momenta $p \ll \sqrt{mgN}$, the Bogoliubov dispersion law (equation 1.1.9) is well approximated by the phonon-like linear dispersion form $\varepsilon(p) = cp$, where $c = \sqrt{Ng/m}$. According to the Bogoliubov theory, the long wavelength (low momentum) excitations of an interacting Bose gas are sound waves. In the opposite limit $p \gg \sqrt{mgN}$, the Bogoliubov dispersion law is reduced to the free-particle form:

$$\varepsilon(p) = \frac{p^2}{2m} + gn. \quad (1.1.10)$$

The transition between the phonon-like to the free particle regime defines a characteristic interaction length ξ which is called *healing length*. The first observation of the Bogoliubov excitation spectrum was reported in 1998, using the two photon Bragg scattering spectroscopy technique in atomic BEC. (29)

1.2 Thesis layout

In this thesis, we concentrate on the investigation of a phenomenon called quantum turbulence (QT) in trapped cold atoms of ^{87}Rb . Our unique apparatus to make a BEC and consequently inject the energy into the condensate enabled our team for the first time to observe the quantum turbulence state in the ^{87}Rb cold sample. (30)

Here in this research, we particularly study collective oscillation modes (Chapter 4) and further exploring the QT phenomenon by studying its energy decay (Chapter 5). We also try to answer some of those questions which had not been studied before due to lack of data. The core questions are: How energy cascade can be studied through momentum distribution analyses of the perturbed clouds? How amplitude and time of the excitation alter the starting number of vortices generated in the cloud?

The structure of this thesis can be summarized as follows:

- Chapter 2: The goal of this chapter is to make the reader familiar with the concept of turbulence phenomenon in macroscopic framework and to introduce the basic

concepts of superfluid hydrodynamics, in particular, the two-fluid model first introduced by Tisza and Landau . Most parts of this chapter and further details can be found in the textbooks. (31–34)

- Chapter 3: This chapter gives a brief overview on the experimental apparatus used to make and observe BEC. We are not going to give an extensive description of the experimental setup, since it can be found in details in several thesis' of the group. (35–38)
- Chapter 4: This chapter starts introducing our peculiar method to excite the condensate. Excited collective modes observed in the experiment have been included as well. It can be extensively found in some of thesis' by our group. (35–38)
- Chapter 5: After the first report on the experimental investigation of momentum distribution of the turbulent condensate, (39) it got us thinking of further exploring the idea. This chapter starts reviewing the former results, and also answering the key questions raised concerning the turbulent energy decay based on our recently-taken data. The overall aim of this chapter is to answer the question of how turbulent energy spectrum is coupled to collective excited modes.
- Chapter 6: We come to a conclusion of our main goal in this thesis. This chapter includes the concluding remarks and also future directions for research which might be taken in the next step. This chapters closes with some further remarks concerning the validity of the method taken to extract the information from momentum distribution analyses. (38)

Chapter 2

Turbulence Phenomenon

Turbulence as a nonlinear multiscale phenomenon¹ is still a little-known topic, but quiet interesting which importance of its understanding is not just limited to physics, but also is ubiquitous in engineering applications. (40, 41)

In this section we will briefly review some of the main concepts of turbulence, beginning with the classical picture of turbulence and proceeding to the turbulence in superfluids (Section 2.4). Thereby, in Chapter 4 and 5 we investigate the superfluid turbulence to further understand the decay of energy in a turbulent atomic cloud which is probably one of the first problems one should tackle in the study of turbulence.

2.1 Why study turbulence ?

The understanding of turbulent behavior in spite of its widespread occurrence in daily life events remains to this day as the age-old unsolved problem of classical Physics. Turbulence exists in a wide range of contexts which is not only limited to a macroscopic level, but also in microscopic levels it can be found extensively. Turbulence can be found in the contexts of geophysical and astrophysical phenomena, motion of submarines, ships and aircrafts, pollutant dispersion in the earth's atmosphere and oceans, or interior of biological cells. (42) On the other hand, the equations of motion are known exactly (34) and can be simulated with precision². Thus, study of turbulence is motivated by the utility of its understanding in many aspects of the life.

2.2 The classical picture of turbulence

Leonardo da Vinci, who in 1507 named the phenomenon he observed in swirling flow “La turbulenza”, perhaps was the first one who noticed the turbulence phenomenon. (43)

¹ Turbulent state span a wide range of scales (Section 2.3).

² We are still far away from precise simulations. For example, even today the limits of the most powerful supercomputers still make it necessary to resort to wind tunnels to verify the design for a new airplane.

The classical turbulence (CT) in fluids is a day-to-day phenomenon (figure 2.2.1), which can be readily observed in the simple stream of water from a faucet when one opens the tap at high speed.

Figure 2.2.1 – Common examples of turbulence in classical picture. (a) Wake turbulence behind individual wind turbines can be seen in the fog, courtesy of Vattenfall wind power, Denmark. (b) Non-linear turbulent flow patterns in smoke rising from a cigarette. (c) A tornado approaching Elie, Manitoba (2007). (d) Turbulent flows at the surface of the Sun. (e) Pyroclastic flow in a volcanic eruption, Mount St. Helens, US Geological Survey (1980). (f) Water coming out of a water tap.

It is often claimed that there is no good definition of turbulence (44, 45) but for the sake of clarity and considering the most widely-used texts on turbulence, (46, 47) we provide the definition which seems to be unanimously agreed: “Turbulence is the three-dimensional time-dependent chaotic behavior of fluid flows at large Reynolds numbers, in which vortex stretching causes velocity fluctuations to spread to all wavelengths between a minimum determined by viscous forces and a maximum determined by the boundary conditions of the flow.

Turbulent flows can often be realized to arise from laminar flows (34) as the Reynolds number³, Re , is increased beyond a specific value (figure 2.2.2):

$$Re = \frac{uL}{\nu}, \quad (2.2.1)$$

where L is a characteristic length scale of the system, u a characteristic velocity and ν the fluid’s kinematic viscosity.

³ Laminar flow occurs at low Reynolds numbers, where viscous forces are dominant while the turbulent flow occurs at high Reynolds numbers and is dominated by inertial forces.

Figure 2.2.2 – Pipe-flow turbulence. Schematic representation of local instantaneous flow patterns in the Reynolds experiment in a pipe.

Source: Courtesy of Professor J. D. Jackson (31), University of Manchester

The fundamental equation governing the classical fluid dynamics is known as the non-linear Navier-Stokes equation: (48, 49)

$$\rho \left[\frac{\partial \mathbf{v}(\mathbf{r}, t)}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v}(\mathbf{r}, t) \right] = -\nabla p + \mu \nabla^2 \mathbf{v}(\mathbf{r}, t) + \mathbf{F}, \quad (2.2.2)$$

where $\mathbf{v}(\mathbf{r}, t)$ is the velocity of the fluid at position $\mathbf{r}$, ∇p the pressure force and $\mathbf{F}$ the sum of all external forces. Non-linearity of Navier-Stokes equations makes it almost impossible to be solvable analytically. Turbulent solutions of the Navier–Stokes equation (equation 2.2.2) exist only for sufficiently large Reynolds number.

2.3 Turbulence dynamics

To understand the superfluid turbulence, we might need some understanding of turbulence in a classical fluid⁴. (49) One of the significant contributions to the understanding of classical turbulence stems from Kolmogorov’s description given in 1941 (Section 2.3.1). The process of energy transfer⁵ from large scales to small scales is a significant element of

⁴ In particular, superfluid turbulence can be similar to CT on large scales compared to the spacing between individual vortex lines.

⁵ In classical turbulence energy cascade is $\delta E(k)/\delta t$ such a way that during the evolution of the turbulent system we will get $E(k_1) \rightarrow E(k_1) - \delta E$ and $E(k_2) \rightarrow E(k_2) + \delta E$. For 3D turbulence this happens when we have $k_1 \leq k_2$ which shows the cascade from large scales to the small scales.

turbulence dynamics (figure 2.3.1) . The idea that turbulent flow is composed by "eddies" of different sizes was first proposed by Richardson in 1922. (53)

Figure 2.3.1 – Energy cascade according to Kolmogorov theory. Schematic showing the transfer of energy between different scales of the flow. (a) From top to down, different scales ($l_n = l_0 2^{-n}$, $n = 0, 1, 2, \dots$) are showing the first, second and third instabilities of eddies. (b) Illustration of the breakdown of scales with a drop of dense ink in water.

Source: By Uriel Frisch (54)

The large eddies are unstable and break up successively into ever smaller eddies. These smaller eddies undergo the same process which is called self-similar Richardson cascade break up to even smaller eddies (figure 2.3.2).

Figure 2.3.2 – The self-similar cascade of eddies. This picture is schematically showing the Richardson's self-similar cascades of length scales in which large scales break down into a small scales keeping the similar process in cascade.

Source: By Javier Jimenez (55)

2.3.1 Kolmogorov theory of turbulence

In turbulence theory we often look for the so-called power law spectrum (figure 2.3.3) (50–52) which is the distribution of kinetic energy per mass across the various length scales in the Richardson cascade. (53)

Figure 2.3.3 – Energy spectrum for different turbulent flows with different boundary conditions (water jets, pipes, ducts and oceans), demonstrating the universality of the Kolmogorov law.

Source: By S. Saddoughi (58)

The well-known Kolmogorov theory of turbulence (56, 57) originally postulated for locally homogeneous, isotropic⁶ turbulence and high Reynolds number, predicts the energy distribution of turbulence and how it decays through what is called the energy cascade. For the inertial range⁷ in Kolmogorov's theory, we know that the so-called energy spectrum, $E(k)$, is a universal function that only depends on ϵ and k (figure 2.3.3).

⁶ The large scales of a flow are determined by the geometrical features of the boundaries, so they are not generally isotropic.

⁷ Inertial range is the length scale between the total length of the system (L_0) and Kolmogorov's dissipation length scale η_K , i.e., $\eta_K \leq L \leq L_0$.

Hence $E(k)$ must be of the form $E(k) = C_K \varepsilon^a k^b$, where C_K is a dimensionless universal constant called Kolmogorov constant and ε is the rate of dissipation of hydrodynamic kinetic energy per unit mass. Experiment and also numerical simulations give $C_K \sim 1.5$. (59) Through dimensional analysis it is easy to see that one must have $a = 2/3$ and $b = -5/3$. We hence obtain the famous $-5/3$ slope of the Kolmogorov “5/3” law: (60, 61)

$$E(k) = C_K \varepsilon^{2/3} k^{-5/3}. \quad (2.3.1)$$

Most of the dissipation takes place at the so-called Kolmogorov microscale $k \leq k_d$ where $k_d = \varepsilon^{1/4} \nu^{-3/4}$ (ν is kinematic viscosity). We might also be interested in the region with $k \geq k_d$. At scales smaller than the dissipation scale L_d , the spectrum falls off due to the disappearance of kinetic energy into the thermal reservoir of molecular collisions, and its shape is steeper than any power law, often taken to have a shape

$$E(k) \propto f\left(\frac{k}{k_d}\right) \exp\left[-c\left(\frac{k}{k_d}\right)^n\right], \quad (2.3.2)$$

where the direct interaction approximation together with perturbation approximation give respectively $n = 1$ and $f \propto (k/k_d)^3$. Therefore, equation 2.3.2 takes even a simpler form of $E(k) \propto k^\alpha \exp(-ck/k_d)$. The numerical calculations indicate that $\alpha \approx 3.3$ and $c \approx 7.1$ as $k \rightarrow \infty$. (62)

2.3.2 Vortices and its dynamics

Classically, when a fluid is rotated at high angular frequency, a vortex appears at the center of rotation as a consequence of the angular momentum of the fluid due to the rotation. In fact, regardless of how it is generated, the vortex will dissipate and get back to its lowest energy state when the source of the rotation is removed. Two formulations are generally available for studying the dynamics of quantized vortices, one is the vortex filament model, (63) and the other is the Gross–Pitaevskii (GP) model.

The vorticity vector Ω is defined as the curl of the velocity $\Omega \equiv \nabla \times \mathbf{u}$, and the circulation $\Gamma_C(t)$ is defined to be the line integral around an arbitrary closed curve $C(t)$,

$$\Gamma_C(t) = \oint_{C(t)} \mathbf{u} \cdot d\mathbf{l}. \quad (2.3.3)$$

For a vortex line in the center of a cylindrically symmetric system, equation 2.3.3 gives the energy of vortex as the kinetic energy of the flow

$$E_{vort} = \frac{1}{2} \rho_m \int v^2 d^3r. \quad (2.3.4)$$

Considering fluid confined in a cylindrical reservoir of radius R , it can be even possible to show that the vortex energy per unit length will be given by

$$\varepsilon = \frac{1}{2} \rho_m \left(\frac{\Gamma^2}{2\pi} \right) \ln \left(\frac{R}{\xi} \right). \quad (2.3.5)$$

2.3.2.1 Quantized vortices in superfluids

Quantized vortices first predicted by Onsager (64) and Feynman (65) are topological defects in a superfluid possessing quantized circulation⁸. Quantized vortex in liquid Helium was first visualized by Packard (67) while in atomic BEC they have been experimentally observed by use of a rotating modulation of the trap to stir the condensate. (68, 69). In the case of He-II, the critical angular velocity Ω_c for the appearance of the first vortex line has been shown to be, (66, 70)

$$\Omega_c = \frac{\kappa}{2\pi R^2} \ln \left(\frac{R}{a_0} \right), \quad (2.3.6)$$

where $a_0 \sim 10^{-8}$ cm is the size of vortex core, $\kappa = h/m$ is the quantum circulation and R is the radius of cylindrical reservoir.

Figure 2.3.4 – Absorption images after 15 ms of free expansion showing atomic clouds with different number of vortices, starting from one vortex. As the number of vortices is being increased, we are observing the tanglement of vortex lines (e) and finally, as shown in (f) we end up with reaching the granular state.

Source: By E. Henn et al. (30, 71)

One of those mostly-used methods to generate a vortex consists in using the laser beams to engineer the phase of the condensate wave function and produce the desired velocity field. (72–76) The second method referring to the magnetic induction of vortices by rotation of condensate has been also done recently in our experiment (30) which ended up with emergence of quantum turbulence phenomenon (figure 2.3.4).

⁸ Quantized vortices play an important role in the transport phenomena in the superfluid such as the critical superflow and quantum turbulence. (66)

Figure 2.3.5 – 2D simulation of our oscillatory excitation (Chapter 4) at different excitation times and $\text{TOF} = 15$ ms. Pictures are illustrating the density profiles for (a) 12.89 ms, (b) 13.65 ms, (c) 14.41 ms, (d) 15.17 ms, (e) 15.54 ms and (f) 15.92 ms of excitation time.

Source: By J. Seman et al. (78)

Considering the dissipation parameter γ in the GPE (equation 1.1.7), M. Tsubota could numerically solve the modified two-dimensional GPE presented in equation 2.3.7 (77) for a rotating trapped Bose-Einstein condensate under our external excitation potential (Section 4.1).

$$(i - \gamma) \hbar \frac{\partial \psi(\mathbf{r}, t)}{\partial t} = \left(-\frac{\hbar^2}{2m} \nabla^2 + V_{\text{ext}}(\mathbf{r}) + g |\psi(\mathbf{r}, t)|^2 \right) \psi(\mathbf{r}, t). \quad (2.3.7)$$

The dissipation term was introduced to remove the compressible excitations. As shown in figure 2.3.5, for $\gamma = 0.02$ with a given excitation parameters we see that angular momentum starts to increase which finally results in having strong fluctuations in the density profile.

Figure 2.3.6 – Formation and decay of the vortex lattice. The condensate is rotated with stirring frequency $\Omega \sim 60$ Hz for 400 ms, then left to equilibrate for different holding times (a) 100 ms, (b) 200 ms, (c) 500 ms, (d) 1 s, (e) 5 s and (f) 10 s. The cloud shown in (c) includes roughly 130 vortices and its diameter is ~ 1 mm which is being decreased to (f) due to the inelastic collisions.

Source: By Abo-Shaeer et al. (79)

If Ω is increased exceeding Ω_c , more and more vortex lines appear in the rotating condensate. Abo-Shaeer et al. at MIT⁹ succeeded in observing a vortex lattice in an atomic BEC of ^{23}Na condensate (figure 2.3.6).

⁹ Massachusetts Institute of Technology

2.4 Superfluid Turbulence

Historically, superfluid turbulence (also known as quantum turbulence¹⁰) in He-II was mentioned as a theoretical prediction by Richard Feynman in 1955, (81) who was the first to recognize that quantum turbulence can be thought of as a tangle of interacting quantized vortex lines¹¹ (figure 2.4.1).

Figure 2.4.1 – Numerical simulation illustrating a reconnection of vortex lines. (a) initially two vortices with well-defined directions are approaching and (b) interact with each other, (c) subsequently a re-connection and (d) two new vortices with different topologies are generated in different directions.

Source: By M. Tsubota (82)

The energy of quantum superfluid flow depends on the tanglement configuration of the vortex lines. During the free decay, it evolves towards lower-energy configurations until no vortex lines are left. Vortex re-connections can be regarded as a crucial process that allow the evolution of the topology of the tangle towards these lower-energy configurations (figure 2.4.2).

Figure 2.4.2 – Schematic of the dissipative process in turbulent superfluids. A large amount of energy is injected into the system, generating multiple vortices. Then, a succession of vortices of these reconnections produce large tangles. The vortex reconnections excite Kelvin waves (83) and finally energy is dissipated as phonons, and thermal excitation.

Source: By K. W. Schwarz (63)

¹⁰The term quantum turbulence (QT) was introduced by Donnelly & Swanson (1986). For further studies, reader is referred to. (80)

¹¹It is important to point out that any random distribution of vortices does not necessarily yield quantum turbulence.

The injected energy passes down to smaller length scales until reaching a sufficiently small scale such that the viscosity of the fluid can effectively dissipate the kinetic energy as phonons and thermal excitation. The energy spectrum in the quantum region $k_l < k < k_\xi$ is theoretically predicted to obey a Kolmogorov-like power law $E(k) \propto k^\eta$.

Description of quantum turbulence at finite temperature are derived from the two-fluid model¹² initially proposed by Tisza (84) and Landau. (85) We also work within a framework of the two-fluid model with friction¹³, building on ideas first introduced by Volovik (86) and Vinen. (87) Thus we have the Navier-Stokes (equation 2.2.2) and Euler Equations respectively for the normal component $\mathbf{u}_n$ and superfluid velocity $\mathbf{u}_s$:

$$\rho_s \left[\frac{\partial \mathbf{u}_s}{\partial t} + (\mathbf{u}_s \cdot \nabla) \mathbf{u}_s \right] = -\frac{\rho_n}{\rho} \nabla p + \rho_s s \nabla T - \mathbf{F}_{ns} \quad (2.4.1)$$

$$\rho_n \left[\frac{\partial \mathbf{u}_n}{\partial t} + (\mathbf{u}_n \cdot \nabla) \mathbf{u}_n \right] = -\frac{\rho_n}{\rho} \nabla p_n - \rho_s s \nabla T + \eta v \nabla^2 \mathbf{u}_n + \mathbf{F}_{ns}. \quad (2.4.2)$$

Here $\mathbf{F}_{ns} = -\rho_s \left[\alpha' (\mathbf{u}_s - \mathbf{u}_n) \times \boldsymbol{\omega} + \alpha \boldsymbol{\omega}_s \times (\mathbf{u}_s - \mathbf{u}_n) \right]$ is the mutual friction and ρ_n , ρ_s are the densities of the normal and superfluid components.

2.4.1 Experimental realization of quantum turbulence

In superfluid ^4He , the first quantum turbulence experiments were realized by heat currents (so-called counterflow turbulence), (88, 89) where the normal and superfluid components flow in opposite directions. Recently, a turbulent state was realized in atomic BECs by two methods. Weiler et al. performed a rapid quench of an ^{87}Rb gas through the BEC transition temperature. (90) The turbulent state created in the above method strongly depended on the initial uncontrollable thermal state. As a method with better control of the turbulence, Henn et al. (30) introduced an external oscillatory perturbation to an ^{87}Rb BEC (figure 2.3.4). This method is described extensively in Chapter 4.

¹²On larger scales, superfluid and normal fluids are coupled together by the mutual friction between them, and behave as a classical fluid to show the Kolmogorov energy spectrum. On small scales, turbulent flow is dissipated by the viscosity of the normal fluid, and Kelvin waves do not exist.

¹³In our experiment we are dealing with 60% pure BEC (Section 3.1). Thus considering the thermal part, two-fluid model might be a good description of turbulence in our studies.

Chapter 3

Making and Observing a BEC

We briefly describe the apparatus and methods used to create and observe the Bose-Einstein condensates of ^{87}Rb atoms in BECI experiment¹. A brief review with emphasis on a few minor modifications made to the excitation procedure (Section 4.1) during the course of my M.Sc. is presented here while a more detailed description of the experimental apparatus and methods used, can be found in. (35, 37, 38)

3.1 Overview of the apparatus

Like plenty of other BEC experiments around the world, BECI experiment also incorporates a different set of tools including Lasers and optics, UHV system and computer softwares to run and control the experiment. Our apparatus is a marriage of two magneto-optical traps called double-MOT system, (91–93) which are horizontally connected by a very narrow differential pumping tube (2 mm in diameter).

Figure 3.1.1 – MOT1. This picture is showing (a) parts of the appartus including auadropole trap and compensation coils and (b) fluorescence images of the atoms captured in the first MOT.

Source: By author

¹ The first realization of the ^{87}Rb condensate in Brazil took place in BECI experiment.

We first use the standard techniques of laser cooling and trapping inside of UHV vacuum chamber ($\sim 10^{-8}$ Torr) to obtain the first MOT (figure 3.1.1) which is running in a common configuration of three-retroreflected laser beams. The physics behind the laser cooling and trapping of neutral atoms are not discussed here, extensive explanations of the involved mechanisms can be found in. (94–99)

In addition to alkali metal dispensers² used as a source of Rb atoms in the experiment, Light-induced atom desorption (LIAD) (100) technique is also applied to get the maximally achieved loading rate.

Figure 3.1.2 – MOT2. This picture is showing (a) parts of the appartus including QUIC trap, evaporative cooling antenna and water cooling pipes and (b) fluorescence images of the atoms captured in the second MOT.

Source: By author

We selectively capture $\sim 10^8$ atoms of ^{87}Rb in the first MOT. Once the first MOT is fully loaded, a push beam which is a circularly polarized laser beam (101, 102) of power $\sim 1.22\text{ mW}$ tuned closely to the $|F = 2\rangle \rightarrow |F' = 3\rangle$ transition, forces atoms to move ballistically forward to a lower pressure ($\sim 10^{-12}$ Torr) region to be recaptured in second MOT (figure 3.1.2). Transferring atoms from first MOT to the second MOT can also be carried out by physically moving the coil pair which is called “Magnetic transfer”. (103)

About $\sim 10^8$ atoms were successfully loaded into the second MOT³, as measured by fluorescence signal. (108, 109) To have a high density atomic cloud, we compress the second MOT (110) and cool down the atoms using the standard polarization gradient

² Alkali metal dispensers (AMDs) in our experiment, purchased from SAES Getters[®] with 5 cm long, contain $\sim 3.7\text{ mg/cm}$ of Rb atoms.

³ The number of trapped atoms is governed by a capture rate (104) and collision events which leads to trap loss. (105–107) There must be balance between the MOT loading rate and also the corresponding loss rates associated to one atom and two atom loss, $L_1(N)$ and $L_2(N)$:

$$\frac{dN(t)}{dt} = R - L_1(N) - L_2(N).$$

cooling (111, 112) with three pair of counter-propagating laser beams. Finally, atoms are optically pumped (113) into the magnetically trappable states $|F = 2, m_F = 2\rangle$ and are ready to be transferred into a quadrupole magnetic trap⁴. Atoms confined in a Quadrupole magnetic trap can escape by undergoing a spin-flip Majorana transitions (114–116) due to a breakdown of the adiabatic approximation. To prevent such loss in our atomic system, evaporative cooling is instead performed in a secondary magnetic trap of the, e.g., Ioffe-Pritchard configuration (117) which does not include a vanishing magnetic field near the trap minimum⁵.

Thus in the last step, all laser fields are turned off and the atoms are transferred into a standard Quadrupole-Ioffe (QUIC) magnetic trap. (120) After transferring the captured atoms into a QUIC, atoms will undergo a magnetic evaporative cooling (Section 3.2.2) to reach the possible lowest temperatures.

3.2 BEC production

To realize the coherent state, one must look for the characteristic properties of the condensate. For example, ballistic free expansion of the condensate⁶. Moreover, after exciting the condensate one could observe the collective excited modes (Chapter 4) characterized by excitation frequencies, (121–124) which is also a signature of superfluidity. In the experiment, we fit bimodal density distribution to show the occurrence of BEC. The transition to quantum degeneracy in our experiment is reached at $T = 110\text{ nK}$ with $N_0 \sim 1.5 \times 10^5$ (figure 3.2.1).

3.2.1 Why Rubidium-87 ?

Alkali-metal atoms and specifically Rubidium atom, turned out to have intriguing properties (125) and soon came to the attention of particularly BEC experiments. More importantly, the well-know atomic structure of ^{87}Rb , which is Hydrogen-like atom, makes it favorable for laser cooling and also evaporative cooling (126) where one can neglect the $Rb - Rb$ inelastic collisions. Some of the relevant physical D2 line properties of ^{87}Rb are listed in table. 3.1

⁴ In the case of ^{87}Rb ground state, the trappable states are $|F = 2, m_F = 1, 2\rangle$ and $|F = 1, m_F = -1\rangle$. Of these three states, only the $|F = 1, m_F = -1\rangle$ and $|F = 2, m_F = 2\rangle$ states because of their relatively small inelastic loss rates, (18) are favorable for evaporative cooling stage (Section 3.2.2).

⁵ There are also several other approaches to prevent the Majorana spin-flip loss. (9, 118, 119)

⁶ The usual signature of BEC is the bimodal density distribution after a ballistic free expansion of the cloud which displays a Thomas-Fermi profile (Section 1.1.2.1) for the ground state atoms and a Gaussian distribution for the thermal atoms. (8)

Table 3.1 – D2 transition optical properties of ^{87}Rb

Property	Symbol	Value
Frequency	ω_0	$2\pi.3842304844685(62)$ THz
Wavelength (Vacuum)	λ	$780.241209686(13)$ nm
Lifetime	τ	$26.2348(77)$ ns
Natural Line Width	Γ	$38.117(11) \times 10^6 \text{ s}^{-1}$
Recoil Velocity	v_r	5.8845 mm/s
Doppler Shift	$\Delta\omega_d$	$2\pi.7.5419$ kHz
Doppler Temperature	T_D	$145.57 \mu\text{K}$

Source: By Daniel A. Steck (125)

3.2.2 Evaporative cooling

Evaporation in the QUIC trap is the final stage of our cooling process which will be carried out. In evaporative cooling or so called RF-induced evaporative cooling, (128–131) atoms with an energy higher than average are selectively removed from the trap by continuously reducing the trap depth. The remaining atoms re-equilibrate through collisions to a lower temperature. (132) Letting the remaining atoms to thermalise is very important stage to achieve a BEC (figure 3.2.1).

Figure 3.2.1 – Evaporative cooling sequence for $\text{TOF} = 25$ ms. From (a) to (f), we are increasing the RF-frequency letting most energetic atoms to scape from the trap, the remaining atoms re-equilibrate through collisions to a lower temperature.

Source: By author

In our experiment, RF-evaporative cooling ramps down from around 20 MHz to a final frequency of 1 MHz which total ramp lasts ~ 20 s, produces a condensate of ^{87}Rb with $\sim 2 \times 10^5$ atom number at temperature ~ 150 nK. The RF field is generated by an antenna which is a small one-loop copper wire coil of 30 mm diameter, placed just below the UHV vacuum cell and controlled by an arbitrary waveform generator⁷.

⁷ Model: DS345 Function/ARB Generator by Stanford Research System®

3.2.3 Absorption Imaging technique

Depending on the experiment and what is the main goal of experiment, different techniques of imaging are used which could be destructive (e.g. absorption imaging) or even non-destructive. (18) The most common observation method of the atomic clouds is absorption imaging which gives us a great amount of quantitative information, including atom number and temperature of our cold sample.

Figure 3.2.2 – Normalized absorption images. (a) is showing the reference image I_{ref} , (b) is showing the I_{atom} where image includes atoms, (c) is showing the I_b which is just an imaged of probe beam and (d) is showing the normalized image in which atoms are appeared as a dark spot in the white background.

Source: By author

In absorption imaging technique, the condensate is released from the magnetic trap and undergoes a free expansion under gravity for a period of time (In our experiment typically ranging from 3 ms to 30 ms⁸). Then, the cold atom cloud is illuminated for $\sim 50 \mu s$ with a resonant collimated beam tuned closely to the $|F = 2\rangle \rightarrow |F' = 3\rangle$ transition. The absorption of light by the atoms casts a shadow which is imaged onto a CCD camera⁹. Thus, the normalized transmission reads (figure 3.2.2)

$$T_{norm}(x, y) = \frac{I_{atom}(x, y) - I_b(x, y)}{I_{ref}(x, y) - I_b(x, y)}, \quad (3.2.1)$$

where $I_{atom}(x, y)$ is the picture including atoms, $I_b(x, y)$ is the background picture and $I_{ref}(x, y)$ is the picture without atoms. Expression 3.2.1 eliminates all interference fringes which may be caused by imaging optics or vacuum cell where probe beam is passing through.

⁸ The falling time is called *Time-of-flight* (TOF). Imaging of ultra-cold atomic gases in expansion provides a direct measurement of the momentum distribution and is therefore routinely used to extract the temperature of cold thermal samples. (133)

⁹ Charge-coupled device. Two cameras, Stingray from Allied Vision Technologies (AVT) for vertical direction and Pixelfly for horizontal direction are being used.

Chapter 4

Collective Modes of An Excited BEC

Many investigations of the condensate dynamics to discover information are mostly carried out on collective excitations, vortices generation and sound speed measurements. It has been shown that by a proper modification of the trapping potential, (134–136) one may create collective excitation in a Bose-Einstein condensate which is quite feasible experimentally (as realized in our experiment (37)).

Our experiment starts with the condensate at very low temperature where the thermal cloud of the condensate is barely present ($\sim 60\%$ pure BEC) and modulation of the external trapping potential is giving rise to the collective excited modes. There are also different methods of exciting the collective modes in the trapped BEC¹ but we will not go through it.

4.1 Excitation procedure

In our experiment the collective modes were excited by applying a small time-dependent perturbation of a given sinusoidal frequency (equation 4.1.1) upon the trapping potential. To provide the mentioned time-dependent modification of the trapping potential, a pair of anti-Helmholtz coil is employed (figure 4.1.1). AC current of the form $I_{ac}(t) = I_0 [1 - \cos(\Omega_{act})]$ with excitation frequency $f_{exc} = \Omega_{ac}/2\pi$, is running through the excitation coils which subsequently causes an external potential given by

$$V_{ac}(t) = \frac{1}{2}m \left[\Omega_x^2(t) (x' - X'_0)^2 + \Omega_y^2(t) (y' - Y'_0)^2 + \Omega_z^2(t) (z' - Z'_0)^2 \right], \quad (4.1.1)$$

where we have defined $\Omega_i^2(t) = \omega_i^2 \delta_i^2 (1 - \cos(\Omega_{act}))^2$. Here also ω_i is the frequency of the trap in i direction which in our case are ω_x and $\omega_y \simeq \omega_z = \omega_r$. Parameter δ_i is also the amplitude of the translational motion along the i direction. The coordinates $x' - X'_0$,

¹ It also has been realized that collisions between spatially separated condensates may be used to create collective excitations in the condensate. (137, 138) It has been suggested as well that a resonant Raman excitation scheme may be utilized to excite the vortex states. (139)

$y' - Y'_0$ and $z' - Z'_0$ in equation 4.1.1 are also given by

$$\begin{pmatrix} x' - X'_0 \\ y' - Y'_0 \\ z' - Z'_0 \end{pmatrix} = \begin{pmatrix} \cos \theta_0 & -\sin \theta_0 & 0 \\ \sin \theta_0 & \cos \theta_0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x - X_0 \\ y - Y_0 \\ z - Z_0 \end{pmatrix}. \quad (4.1.2)$$

We have placed the so called excitation coils with its axis slightly inclined at a particular angle ($\theta_0 \sim 5^\circ$) with respect to the symmetry axis of Ioffe coil (figure 4.1.1) such a way that we've considered $Z_0 = 0$.

Figure 4.1.1 – Excitation coils placed around the science cell. In picture (a) you see the configuration of the excitation coils with respect to the Ioffe. In (b) you see how we have placed the excitation coils around the science chamber.

Source: By J. Seman (36)

The external potential given in equation 4.1.1 together with the trapping potential² result in the total potential $V_{tot}(t)$ which reads

$$\begin{aligned} V_{tot}(t) &= V_{trap} + V_{ac}(t) \\ &= \frac{1}{2}m\omega_x^2 x^2 + \frac{1}{2}m\omega_r^2 (y^2 + z^2) \\ &\quad + \frac{1}{2}m \left[\Omega_x^2(t) (x' - X'_0)^2 + \Omega_y^2(t) (y' - Y'_0)^2 + \Omega_z^2(t) z^2 \right], \end{aligned} \quad (4.1.3)$$

Before going deep through the excitation procedure, it might be better first to make the reader familiar with some of mostly-used parameters in the experiment. T_{exc} is the excitation time in cycles (each cycle ~ 5.29 ms), A_{exc} is the amplitude of the excitation in V_{pp} ($1 V_{pp} \sim 740$ mG/cm), T_h is holding time which refers to the time between switching the excitation process off and TOF. f_{exc} is the frequency of the excitation which namely

² Our trapping potential is a harmonic potential given by $V_{trap} = \frac{1}{2}m\omega_x^2 x^2 + \frac{1}{2}m\omega_\perp^2 (y^2 + z^2)$ in which $\omega_x = 2\pi \times 21$ Hz and $\omega_y \simeq \omega_z = \omega_\perp = 2\pi \times 187$ Hz.

is around the higher frequency of the trap ~ 189 Hz.

Figure 4.1.2 – Absorption images of an excited condensate with $T_{\text{exc}} = 8$ cycles (~ 42.32 ms). Our parameters which have been unchanged during the varying TOF are $T_h = 20$ ms, $A_{\text{exc}} = 0.7 V_{\text{pp}}$, $f_{\text{exc}} = 189$ Hz. In these pictures, from (a) to (f), each picture corresponds to 6 ms time interval in TOF imaging.

Source: By author

Figure 4.1.3 – Sequence of excited condensates for different T_{exc} . These pictures show a real motion of the condensate after being excited for different T_{exc} varying from 0 ms to 64.8 ms. The excitation parameters kept fixed are $A_{\text{exc}} = 0.7 V_{\text{pp}}$, $f_{\text{exc}} = 189$ Hz and TOF = 21 ms. As has been shown, T_{exc} is being increased until reaching a specific $T_{\text{exc}} = 7$ cycles (white background) in which condensate is deformed strongly.

Source: By author

The process of observing the structures³ through absorption imaging technique (Section 3.2.3) is shown in figure 4.1.2. In our method of excitation which might be called “kicked” excitation experiment, (135) for each amplitude of excitation there are specific time of excitation which give rise to observe the structures inside the excited condensate

³ We could often observe the so-called structures instead of a clearly countable number of vortices inside the excited condensate. We believe this might happen because of not having fully control on the vortex generation method.

(figure 4.1.3). For this purpose, we are fixing one amplitude of excitation and increase the excitation time until observing a clear shape deformation (a kind of dsitortion) in the condensate.

4.2 Time evolution of the excited condensate

Another interesting observation which we make before reaching the turbulence regime is time-evolution investigation of the regular and also excited Bose-Einstein condensate (BEC) after free expansion. Aspect ratio (A.R.) is one of the key parameters for describing an expanding condensate which has its own crucial importance⁴. Following the roadmap to turbulence state, another feature which we study here is excited collective modes of the turbulent condensate (Section 4.2.2).

4.2.1 Evolution of aspect ratio

As we mentioned in Chapter 3, one way to demonstrate the quantum degeneracy in trapped ultracold gases is based on the time-of-flight expansion of the condensate atoms released from an anisotropic trap. To see how aspect ratio evolves while we have a turbulent cloud and also when there is a regular BEC, one might need to do the measurements with different TOFs. The excited condensate released from the trap, depending on the amplitude and also time of excitation keeps almost the starting aspect ratio (figure 4.2.2).

Figure 4.2.1 – Evolution of aspect ratio. These cropped pictures are demonstrating the evolution of the aspect ratio in TOF. In fact these pictures are not showing the real spatial ballistic expansion of the excited cloud. All pictures have been taken at $T_h \sim 20$ ms and $A_{exc} = 0.7 V_{pp}$ while TOF was varying from 5 ms to 25 ms.

Source: By author

For example as you see in figure 4.2.1, when we are starting to excite the cloud for a short time with specific excitation amplitude, A.R. inversion occurs for longer TOFs compared to the regular BEC which A.R. inversion time is ~ 12 ms. On the other hand, when we are increasing the time of excitation, the A.R. inversion is not even being observed

⁴ A.R. is normally defined as a ratio of the condensate widths along the two symmetric axes.

for a long TOFs. Not observing the A.R. inversion for the turbulent condensate has been recently shown numerically as well, which technically is called self-similar expansion. (140)

Figure 4.2.2 – (Color online) Inversion of aspect ratio (A.R.) for different amplitude of excitation. As long as one keeps to excite the condensate with smaller amplitude of excitations, A.R. inversion occurs for $\text{TOF} \leq 15$ ms but increasing the amplitude of excitation will not allow excited condensate to invert its A.R.

Source: By author

4.2.2 Collective modes of excitation

The collective modes in our experiment are excited by applying a small time-dependent perturbation to the trap potential (equation 4.1.1). Four modes were determined from the measured center-of-mass positions and also aspect ratio of the excited condensate in different holding time, T_h . In the case of our experiment (Chapter 3), where the confining potential is harmonic and has a axial symmetry, the collective modes are described by equation (37),

$$m\omega^2\delta n(\mathbf{r}) = -\nabla \cdot \left(\left[\mu - \frac{m}{2} (\omega_x^2 x^2 + \omega_\perp^2 (y^2 + z^2)) \right] \nabla \delta n(\mathbf{r}) \right) = 0, \quad (4.2.1)$$

where $\delta n(\mathbf{r})$ is the fluctuation in density and $\omega_\perp = \omega_y \simeq \omega_z$. Since we have axial symmetry in equation 4.2.1, we consider the specific case of $\delta n(\mathbf{r})$ expressed by $r^l Y_{lm}(\theta, \varphi)$ where $Y_{lm}(\theta, \varphi)$ are the spherical harmonics. (124) Therefore we find the shape oscillation frequencies $\omega_{lm}^2 = \omega_\perp^2$ and $\omega_{lm}^2 = (l-1)\omega_\perp^2 + \omega_x^2$ respectively for $m = \pm l$ and $m = \pm(l-1)$ where l determines the orbital angular momentum.

Figure 4.2.3 – Dipolar motion of the excited condensate. These pictures are showing absorption images of a BEC undergoing a ballistic expansion at different T_h ranging from 30 ms to 41 ms. Excitation parameters being fixed during the imaging process are $A_{\text{exc}} = 1.0 V_{\text{pp}}$, $f_{\text{exc}} = 189 \text{ Hz}$ and $\text{TOF} = 16 \text{ ms}$.

Source: By author

In the dipolar mode, only the center-of-mass oscillations are considered (figure 4.2.3). This mode occurs when we have $\delta n(\mathbf{r})$ in equation 4.2.1 to be expressed by $\delta n(\mathbf{r}) \propto r Y_{lm}(\theta, \varphi)$ corresponding to $l = 1$ ($m = 0, \pm 1$) which results in frequencies $\omega_d(m = \pm 1) = \omega_{\perp}$ and $\omega_d(m = 0) = \omega_x$. Frequencies of dipolar mode coincide with the frequencies of the trap (figure 4.2.5)⁵.

Figure 4.2.4 – Scissor motion of the excited cloud. For lower amplitude of excitation ($A_{\text{exc}} < 0.4 V_{\text{pp}}$) regardless of the time of excitation, we observe the tilting of angle which is demonstrating the scissor motion. Excitation parameters which have been taken fixed are $\text{TOF} = 21 \text{ ms}$, $A_{\text{exc}} = 0.4 V_{\text{pp}}$, $f_{\text{exc}} = 189 \text{ Hz}$ and T_h is varying. In these pictures, from (a) to (f), each picture corresponds to 30, 32, 34, 36, 38 and 40 ms of holding time.

Source: By author

Scissors modes⁶ in which the excited cloud moves into a sloping position back and forth with a well-defined amplitude is observed as well. For small amplitudes of excitation

⁵ In the experiment to find the frequencies of the trap we investigate the dipolar mode frequencies. (?)

⁶ For the first time they were predicted to occur in deformed atomic nuclei by N. LoIudice and F.

Figure 4.2.5 – (Color online) Dipolar motion frequencies for different excitation amplitudes $0.1 V_{pp}$, $0.4 V_{pp}$, $0.7 V_{pp}$ and $1.0 V_{pp}$. We have taken data along the Ioffe coil axis. The y -component motion of the center-of-mass is resulting in frequency $\omega_x = 2\pi \times 23\text{Hz}$.

Source: By author

($A_{exc} < 0.4 V_{pp}$), we observe the scissors motion of the cloud (figure 4.2.4), regardless of excitation time. For large amplitudes of excitation ($A_{exc} > 0.4 V_{pp}$) it is very likely to observe the scissors mode coupled with quadrupolar mode (figure 4.2.6).

Figure 4.2.6 – Scissor motion of the excited cloud for large amplitude of excitations. For large amplitude of excitation ($A_{exc} > 0.4 V_{pp}$), we observe that the so called scissors mode starts to be coupled to quadrupole mode. Excitation parameters which have been taken fixed are $TOF = 21\text{ ms}$, $A_{exc} = 1.0 V_{pp}$, $f_{exc} = 189\text{ Hz}$ and T_h is varying. In these pictures, from (a) to (f), each picture corresponds to 30, 32, 34, 36, 38 and 40 ms of holding time.

Source: By author

In the case of quadrupole mode which is fact shape oscillation of the excited conden-

Palumbo. (141) D. Guéry-Odelin and S. Stringari predicted similar collective excitations in Bose-Einstein condensates in magnetic traps. (142) The scissors mode can be excited by a abrupt rotation of the trapping potential through a small angle. (143)

sate, we have $\delta n(\mathbf{r}) \propto r^2 Y_{lm}(\theta, \varphi)$ corresponding to $l = 2$ ($m = 0, \pm 1, \pm 2$). Considering different magnetic quantum numbers, one finds $\omega_q(m = \pm 2) = \sqrt{2}\omega_\perp$, $\omega_q(m = \pm 1) = \sqrt{\omega_x^2 + \omega_\perp^2}$ and $\omega_q(m = 0) = \omega_\perp^2 \left(2 + \frac{3}{2}\lambda^2 - \frac{1}{2}\sqrt{9\lambda^4 - 16\lambda^2 + 16}\right)$. The breathing mode ($m = 0$ mode: m is the magnetic quantum number) is one of the most interesting collective modes which is especially affected by the inter-atom interaction (figure 4.2.7). It has been observed in experiments (135, 136) and theoretical studies explain well the experimental results at low temperatures. (147–149)

Figure 4.2.7 – Schematically illustrating the quadrupole and monopole modes of the excited condensate including a vortex. Pictures (a) has all components r_ξ (width of vortex), r_ρ (radial width) and r_z (width along z direction) in phase. In picture (b) r_ξ oscillates out of phase with r_ρ and r_z while in In picture (c) r_z oscillates out of phase with r_ρ and r_ξ . Finally in picture (d) r_ρ oscillates out of phase with r_ξ and r_z .

Source: By R. Teles (146)

Figure 4.2.8 – (Color online) Frequency of quadrupolar motion for different excitation amplitudes 0.1 V_{pp}, 0.4 V_{pp}, 0.7 V_{pp} and 1.0 V_{pp}.

Source: By author

Chapter 5

Energy Decay of Turbulent Condensate

Recent results and understanding of the turbulence of the condensate in our experiment are coming through momentum distribution analyses of the excited condensate. (39) In this study, we mostly focus on the late-time energy decay where the turbulence is almost homogeneous rather than the transient behavior where the turbulence can be strongly inhomogeneous.

5.1 Momentum distribution of the turbulent cloud

We are considering momentum distribution of a BEC which can be well studied by examining the density distribution¹ of the condensate atoms. The study of momentum distribution is the central topic of this chapter. The momentum distribution of the clouds are extracted with the same procedure performed in our previous work. (38)

5.1.1 Momentum distribution in TOF

In brief, we assume that the cloud released from the trap expands ballistically and the image after free expansion can be directly mapped in the *in situ* momentum distribution of the cloud, although this is not completely true for strongly interacting clouds. If one considers an interaction term, g , in GPE (equation 1.1.7) to be fairly negligible, then ballistically free expanding atoms at different TOFs can lead us to extract information of *in situ* momentum distribution. The position of the atoms in the turbulent regime right after releasing it from the trap can be approximate by

$$\mathbf{r}(\tau) = \mathbf{r}_0 + \mathbf{v}_{int}(t)\tau + \mathbf{v}_{vort}(t)\tau, \quad (5.1.1)$$

¹ This technique has been used to demonstrate many important physical characteristics of BEC, such as vortex lattices (Section 2.3.2.1) and collective modes (Section 4.2.2).

where τ is the specific time of imaging the condensate, $\mathbf{r}_0$ is initial position of atom, $\mathbf{v}_{int}(t)$ is average speed emerged from inter-atomic interactions and $\mathbf{v}_{vort}$ is displacement of atoms because of the presence of vortices. After being freely expanded in TOF, the position of the atoms will be approximated by

$$\mathbf{r}(t) = \mathbf{r}_0 + \mathbf{v}_{int}(t)\tau + \mathbf{v}_{vort}(t)\tau + \mathbf{v}_{kin}(t - \tau), \quad (5.1.2)$$

where $\mathbf{v}_{kin}$ is the expansion velocity. Considering the long TOFs ($t_{TOF} \geq \tau$) we can assume the kinetic energy to be the dominant energy in the system², thus equation 5.1.2 can be approximated as

$$\mathbf{r}(t) = \frac{\hbar t}{m} \mathbf{k}, \quad (5.1.3)$$

Figure 5.1.1 – Typical vortice size in the experiment compared to the size of condensate.

Source: By author

where $\hbar$ is Planck's constant, m is the atomic mass and $\mathbf{k}$ is the wavenumber. Equation 5.1.3 in turn implies that the expanded atomic density can be rewritten as a momentum distribution, $n(\mathbf{r}(t)) = n(\hbar t \mathbf{k}/m)$ which shows that atomic cloud does not have a preferred direction to be expanded in TOF. In absorption imaging (Section 3.2.3) we integrate the spatial density of the cloud expanded in one direction, i.e. projection of the spatial density of the cloud will be

² Actually interaction energy can drastically change the momentum distribution but here it is argued that the interaction is small relative to the turbulent kinetic energy. For further studies reader is referred to. (39)

$$\begin{aligned}
n'(x, \rho) &= \left[n_c(t) \left(1 - \sum_{i=1}^3 \frac{r_i^2}{R_i^2(t)} \right) \right]_x \\
&= n'_c(0) \left[1 - \left(\frac{x - x_0}{R_x} \right)^2 - \left(\frac{\rho - \rho_0}{R_\rho} \right)^2 \right],
\end{aligned} \tag{5.1.4}$$

where $n'_c(0)$ is the peak of density, x_0 and ρ_0 are the coordinates of the center of mass distribution, defined respectively by equation 5.1.5 and 5.1.6

$$x_0 = \frac{1}{N} \iint x n'(x, \rho) dx d\rho, \tag{5.1.5}$$

$$\rho_0 = \frac{1}{N} \iint \rho n'(x, \rho) dx d\rho. \tag{5.1.6}$$

5.1.2 Kinetic energy spectrum

In this work, all the experimental data were taken for different TOFs ranging from 3 ms to at most 33 ms. In the turbulent cloud, the energy contribution of the vortex to the dynamic cloud constitutes $\sim 7\%$ of the total energy, (39) therefore we suppose that the total energy of the clouds are exclusively kinetic energy which can be written as an integral of the kinetic energy of individual atoms $\hbar^2 k^2 / 2m$,

$$\begin{aligned}
E_T &= \int E(\mathbf{k}) d^3\mathbf{k} \\
&= \int n(\mathbf{k}, t) \left(\frac{\hbar^2 k^2}{2m} \right) d^3\mathbf{k}.
\end{aligned} \tag{5.1.7}$$

In our previous work done by G. Bagnato (38), it has been shown that if we consider to have a 3D homogenous turbulence condensate, we can use the Abel transformation performing on 2D momentum distribution, $n'(k')$, to recover a full momentum distribution as follows

$$n(k) \propto -k^{\alpha-1} \left(\frac{\alpha}{\pi} \right) \int_{\varphi_1}^{\varphi_2} \frac{d\varphi}{(\cos \varphi)^\alpha} = C(\varphi) k^\beta, \tag{5.1.8}$$

where $\beta = \alpha - 1$ for the case of $\alpha < 0$.

5.1.3 Momentum distribution coupled to collective modes

A typical momentum distribution obtained at this work is shown in figure 5.1.2. We observed almost a flat plato in the inertial range which gives the scaling law of linear

Figure 5.1.2 – 3D momentum distribution obtained by Abel transform **Source:** By G. Bagnato (38)

decay in energy. As we reach the tail of the plots, we see abrupt fallings which is due to the finite size of our cold sample. However, when the associate data to the mentioned momentum distribution was taken, the spectrum of images was quiet poor.

In this work, since we have not a lot of different images not only displaying the struvtures inside (figure ?) but also collective modes (Section), we could analyze the coupling behavior of momentum distribution to the collective modes. Thus, we had to find a way to eliminate these effects as much as possible. As shown in figure 5.1.3, we start with 2D absorption image of a condensate. Since dipolar motion (Section 4.2.3) of the condensate is the center-of-mass motion of the cloud, thus by sitting on the center of mass of the cloud its effect in our analyses would be eliminated.

Figure 5.1.3 – Eliminating dipole motion effect. First we are finding the center of mass of the condensate, then crop it as shown to eliminate its real spatial motion. **Source:** By author

Another normal mode which we observed affecting our momentum distribution analyses, was scissor-like mode. Since this mode is the oscilation of the axis of cloud back and forth, by rotating the cloud such a way to have its largest axis to be vertical then in

fact we have also eliminated the possible effects of the scissor-like mode (figure 5.1.4).

Figure 5.1.4 – Eliminating scissors-like mode effect. As has been illustrated in these pictures, after cropping the 2D absorption image we rotate it such a way that θ , the angle between small axis and horizontal line is $\theta = 0$. In this way in fact we are discarding the possible effects of scissor mode.

Source: By author

The only mode which we couldn't actually eliminate its effects during our analyses, was quadrupolar mode which ignited our interest in looking for a relation between this mode and also momentum distribution of the excited condensate. As shown in figure 5.1.5 we have analyzed 2D momentum distribution of the excited and also non-excited regular BEC for different holding times. We argue that after taking the procedure previously explained, there is no effect left by scissor-like and also dipolar mode and the only effect might come from the coupling of quadrupolar mode to this analyses.

Figure 5.1.5 – 2D momentum distribution for the non-excited regular BEC and excited BEC for different holding times 33.7, 34.3, 35.1, 35.7 and 36.3 ms. The amplitude of excitation for these data was $A_{\text{exc}} = 0.8 V_{\text{pp}}$.

Source: By author

5.1.3.1 Momentum distribution coupled to quadrupole mode

In fact for the homogenous 3D turbulence (in classical regime) as extensively discussed in Chapter 2, large scales at which energy is injected will cascade down to smaller scales where at the end energy can dissipate. From our analyses we could observe the fact that momentum distribution of the excited condensate resembling almost the same behavior but not for any arbitrary taken holding times, i.e. we needed to exclude some of those holding times to reach this conclusion, while for a non-excited regular BEC, regardless of the holding times we could get the same momentum distribution as for one specific holding time.

Analyzing the 2D momentum distribution for different holding times revealed that these analyses are strongly coupled to the shape of cloud. We observed that any deformation in the shape of the excited condensate is reflecting effect in the associated momentum distribution.

Figure 5.1.6 – Oscillation of the momentum distribution plots for $A_{\text{exc}} = 0.6 V_{\text{pp}}$. As shown, we have taken a specific value of k' associated to $n'(k') = 1$ and plotted it for different holding times. The filled red dots belong to non-excited regular BEC which is constant in holding time.

Source: By author

For further understanding of the behavior of momentum distribution oscillation, we thought there might be a frequency for this oscillation. Therefore, following the idea we plotted the k' with its specific value at $n'(k') = 1$ for different holding times, as shown in figure 5.1.6³. Finally we got a well-known frequency for the oscillation of $k'_{n'(k')=1}$ for the

³ Associated values of k' at $n'(k') = 1$ have been taken arbitrarily just for the sake of simplicity, i.e. one

excited condensate which was the frequency of quadrupole motion given in figure 4.2.7.

We observed that for a non-excited regular BEC, regardless of different holding times we would get the constant line. As we told before, shape oscillation of the excited condensate was the only property which could alter the associated momentum distribution. Since the figure 5.1.6 revealed the hidden quadrupole frequency coupled to the mometum distribution, we got interested in answering the question that how this oscillation changes with different excitation amplitude? For this purpose, we did the same analyses for different excitation amplitudes, as showin in figure 5.1.7.

Figure 5.1.7 – Oscillation of the momentum distribution plots for $A_{\text{exc}} = 0.8 V_{\text{pp}}$. As shown, we have taken a specific value of k' associated to $n'(k') = 1$ and plotted it for different holding times.

Source: By author

Finally we could plot the amplitude of $k'_{n'(k')=1}$ oscillation in different excitation amplitudes (figure 5.1.8). We observed that increasing the amplitude of excitation is showing its effect by increasing the oscillation amplitude of $k'_{n'(k')=1}$. However, we also noticed that exciting the condensate for amplitudes larger than $A_{\text{exc}} = 0.9 V_{\text{pp}}$ will not anymore follow our conclusion. This can be clearly seen from the last poin in figure 5.1.8.

could analyze k' oscillation for any other values of $n'(k')$.

Figure 5.1.8 – Oscillation of $k'_{n'(k')=1}$ amplitude in different excitation amplitudes.

Source: By author

This behavior reflects the same behavior observed in figure 4.2.7. When the amplitude for the quadrupole mode oscillation increases with the excitation amplitude. This is a direct evidence that both are intimately coupled and that the real characteristics of the momentum distribution are hindered behind this shape oscillation.

It must be emphasized that, although our analyses are for 2D momentum distribution but the same should be extended for the case of 3D momentum distribution. As such, it becomes the real momentum distribution can be revealed and together with it the validity of kolmogorov-like law for BEC.

Chapter 6

Conclusion and outlook

Key results detailed in this Chapter 4 and 5 together with outlook are summarized in this chapter.

6.1 Concluding remarks

The roadmap to our study in this thesis started by building up the BECI experiment to perform our main investigation on quantum turbulence phenomenon in the trapped cold atoms which was recently reported by our team. In Chapter 3 after a brief introduction to the experiment, we report the first BECs with $N_0 \sim 2 \times 10^5$ atoms of ^{87}Rb at typical temperatures of $T \sim 150 \text{ nK}$. To observe our reported cold sample of BEC, two vertical and horizontal imaging system were implemented in the experiment. After achieving the BEC, excitation coils described in Chapter 4 were imposed to rotate the condensate in two perpendicular planes. Consequently after a specific time which we named holding time we could investigate the properties of the excited condensate. Following the route to turbulence regime, depending on the amplitudes of excitation and also excitation time, we observed:

- Dipolar mode with specific frequency of $\omega_x = 2\pi \times 23 \text{ Hz}$ and $\omega_\perp = 2\pi \times 187 \text{ Hz}$.
- Quadrupolar mode (in specific mode of $m = 0$ it is called monopolar or breathing mode) through long and short axis of the stretched excited condensate. This mode has a specific frequency of $2\omega_0$.
- We observed that for a short amplitude of excitation a kind of scissors-like mode (tilting the long axis back and forth) can be observed while increasing the amplitude of excitation will couple some other modes (quadrupolar mode) to this well-known mode.
- The so-called self-similar expansion of the excited condensate confirming the theoretical simulation of the turbulent condensate where aspect ratio is keeping almost the same value after ballistic expansion in TOF. We observed that for excitation

amplitudes greater than $A_{exc} \geq 0.4 V_{pp}$ it is very unlikely to observe the A.R. inversion while one could easily observe the A.R. inversion for amplitudes of excitation $A_{exc} \leq 0.3 V_{pp}$. This fact was one of our main findings to make sure that we have got a turbulent regime after exciting the condensate.

While we had been exciting the condensate, we also observed that increasing the excitation time results in strong shape deformation in the condensate. Indeed, we got to know that excitation time must be long enough to observe this kind of deformation in the condensate, thereby we could recognize some structures inside the excited condensate. In Chapter 5, in spite of theoretical difficulties, the analyses of the energy spectrum are made for 60 % pure BEC.

In our experiment, the condensate has finite size which is produced in harmonic traps (causing an asymmetric and inhomogeneous sample). The advantages of using our method described in Chapter 5 to determine the properties of decay of energy in real time lies in the necessity of our main approximation which was considering to have an isotropic and also homogeneous turbulence. In general, it is not possible to know exactly the proper mapping of 3D profile using the Abel transformation.

To a first approximation, this problem can be solved as shown by our results given in (38) and also described briefly in Chapter 5. The difference between our results and the exact behavior of Kolmogorov's $-5/3$ power law (as predicted in classical turbulence) arises from the fact that our system is finite size, on the other hand we have been considered the case of homogenous turbulence to recover the 3D momentum distribution. However, we could still see a Kolmogorov-like power law with slope slightly different from what is predicted in classical turbulence.

Following our analyses in Chapter 5, we also noticed that depending on the different holding times, 2D momentum distribution is oscillating back and forth. Looking at the absorption images of the clouds, we concluded that these oscillations are not independent of shape oscillation of the cloud. Therefore, by taking some approaches described before we could eliminate the effects of dipolar and also scissors-like mode but there was no way to disregard the effect of quadrupole mode. Thus, we could still expect to see the oscillations of 2D momentum distributions.

Analysing the mentioned oscillations for different holding times, led us to find it out that frequency of the oscillation is in agreement with frequency of the quadrupole mode. We also analyzed the dependence of quadrupole motion of the 2D momentum distribution to the amplitude of excitation. What we concluded from this analyses also was confirming our previously achieved result in Chapter 4 where we have shown that amplitude of quadrupole oscillation is being increased while we are increasing the amplitude of excitation.

6.2 Outlook

The scope of this study was limited to homeogenous turbulent cloud which ended up with some approximated results. As describe in Chapter 5, to recover the 3D momentum distribution we have used the Abel transformation which lies in having an isotropic homeogenous consideration in our sample. As we know, considering these assumptions give us some results which might not be even close the real momentum distribution. Therefore, It is for a while that we have been thinking of having another imaging system which are experimentally feasible. The idea holds on having a sheet of probe light cutting through the atomic sample and casting the 2D absorption images onto the CCD camera.

This method can enable us to have at leats 5 images to recover the 3D momentum distribution in analogy to the tomography technique. Having these images from one atomic cloud will give us a very good approximation of the 3D momentum distribution just by juxtaposition of the discrete images. This idea has been done in some experimnet but not exactly to extract the 3D momentum distribution resembling the *in situ* momentum distribution.

Using this method, one has a good control on studying the time evolution of generated vortices to see how it decays in different holding times.

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