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The vehicle routing problem with drones

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O problema de roteamento de veiculos com drones

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*To Julianny,
the brightest star in the sky.*

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*“All we have to decide is what to do
with the time that is given us.”
(J. R. R. Tolkien)*

ABSTRACT

COSTA, J. G. C. **The vehicle routing problem with drones**. 2019. 124 p. Dissertação (Mestrado em Ciências – Ciências de Computação e Matemática Computacional) – Instituto de Ciências Matemáticas e de Computação, Universidade de São Paulo, São Carlos – SP, 2019.

In this Dissertation, the Vehicle Routing Problem with Drones (VRPD), motivated by the growing interest on Unmanned Aerial Vehicles (UAVs, or Drones) by the industry and their applications in logistics is studied. A pioneer work by (MURRAY; CHU, 2015) shows a combination between UAV and a truck to deliver products, presenting an adaptation to the Traveling Salesman Problem (TSP). After a literature review, an extension of the model from Murray and Chu (2015) we present a model for the problem with multiple vehicles. This model is developed as a Mixed Integer Linear Programming (MILP) problem and solved with the solver CPLEX. A heuristic based on a Hybrid Genetic Algorithm (HGA) is also developed and presented. Our results show that the use of drones reduces the total mileage of the trucks by a significant percentage.

Keywords: Vehicle Routing Problem, Drones, Mixed Integer Linear Programming, Hybrid Genetic Algorithm, Logistics.

RESUMO

COSTA, J. G. C. **O problema de roteamento de veículos com drones**. 2019. 124 p. Dissertação (Mestrado em Ciências – Ciências de Computação e Matemática Computacional) – Instituto de Ciências Matemáticas e de Computação, Universidade de São Paulo, São Carlos – SP, 2019.

Nessa monografia estuda-se o Problema do Roteamento de Veículos com Drones (PRVD), motivado pelo crescente interesse da indústria em Veículos Aéreos Não Tripulados (VANTs) e suas aplicações em logística. O trabalho pioneiro de (MURRAY; CHU, 2015) mostra uma combinação entre VANT e um caminhão para realização de entregas de produtos, no qual foi proposta uma adaptação do Problema do Caixeiro Viajante (PCV). Após uma revisão de literatura, apresenta-se uma extensão do modelo de Murray and Chu (2015) para o problema com múltiplos veículos. Desenvolveu-se um modelo de Programação Linear Inteira Mista que foi resolvido com o *solver* CPLEX. Uma heurística baseada em um Algoritmo Genético Híbrido também foi desenvolvido e é apresentada. Resultados mostram que a utilização dos VANTs reduzem a quilometragem dos caminhões significativamente.

Palavras-chave: Problema do Roteamento de Veículos, VANTs, Programação Linear Inteira Mista, Algoritmo Genético Híbrido, Logística.

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INTRODUCTION

Modern customers' eagerness to receive everything faster, whether it being information through the Internet, or services in general, has compelled the market and industry to invest in technologies to meet these customers new demands. One of these new technologies is civil unmanned aerial vehicles (UAV), which are commonly known as drones.

Recent advances in drone technology allow applications that were real only in science fiction to become a reality. Including video capture for personal use, surveillance, inspection of cables and structures, or even using drones as auxiliary video capture units in the Winter Olympics 2018 (INTEL, 2018). The drones can also be used for saving lives. Drones are being used for disaster reliefs, where they can transport water, blood, medicine and equipment for rescue teams (MEIER, 2018),(CHOWDHURY *et al.*, 2017), (PULVER; WEI, 2018); or even organ deliveries (SEILER, 2019).

The focus of this work, however, is the use of drones in the transportation or in delivery business, which can include medical applications, but are not limited to it. Some companies are interested in applying drones to their transportation markets, such as DHL (KUHLMANN, 2016), UPS (LITHIA, 2017), Amazon (CBS, 2013), Dominos (SCHOLZ, 2016), and Google (XCOMPANY, 2017). Several small or test applications are already being developed as well: 7-Eleven is testing small products being delivered by autonomous drones in America (SHAW, 2016); DHL, in Germany, is delivering pharmaceuticals to an island (DHL, 2014); Zipline's drones are being used to transport blood and medicine to remote locations in Rwanda (SIMMONS, 2016).

All these applications under development or even in execution show that the drone industry is promising. Market trends reported by the Business Insider magazine suggest a growth in drone sales that will reach \$12 billion dollars by 2021 (MEOLA, 2017). To illustrate this growth Figure 1 shows drone values in different industries in 2015, where, for

example, \$13 billion dollars were invested on drone transportation solutions. Apart from delivery speed, accessibility is a strong appeal for the use of drones, which helps to justify the investments. For example: locations that are hard to access by regular vehicles because they lack paved roads (or have no roads at all); islands or other isolated areas (due to landslides, fires and other disasters); highly dense urban areas, where heavy traffic or vehicle limitations are present; or even rural areas, where the distance between residences can be significant. This way, new markets (more clients) can be reached, and/or time (and therefore money) can be saved. Another recurrent concern in the transportation industry is the emission of greenhouse gases, which adds an extra incentive to use drones. As most drones are battery-powered, they can lower up to 54% of emissions (STOLAROFF; SAMARAS, 2018).

Figure 1 – Drone value in different industries.

Source: (STOLAROFF; SAMARAS, 2018)

A successful application with drone in transportation and delivery is the Icelandic company AHA, which is using Flytrex's drones to deliver products to customers who are located at the other side of a bay in Reykjavík (O'KANE, 2017). As seen in Figure 2, the distance reduced by the use of the drones is noticeable. This smaller route cuts delivery times and costs significantly. The company employs drones to fly over the bay and a deliveryman would wait on the other side collecting the drone's cargo. Then, the deliveryman would serve the customers on that side of the bay. We explored this idea of a drone transporting parcels to a deliveryman at another location in a paper that was published in a

national operations research conference (COSTA *et al.*, 2018). The paper can be seen in the [Appendix C](#).

Figure 2 – A comparison between a drone’s route (green) and a delivery vehicle’s (red) in Reykjavik.

Source: Flytrex, found in ([GILCHRIST, 2017](#)).

Although the use of drones seems to be very promising, many challenges arise in real applications, such as the legislation on drone usage (privacy issues, congested airspace and threat to other aircraft, and even safety issues), the engineering difficulty for autonomous flight, the logistics implications of using vehicles with spatial and weight limitations, and the current batteries’ duration. DHL has written a report on these problems and the expectations for drones in ([KÜCKELHAUS; NIEZGODA; ENDRIß, 2014](#)). Some papers have explored a couple of these adversities, such as ([HONG; KUBY; MURRAY, 2018](#)) with recharging stations, and ([EREN; ÇELİK, 2018](#)) with risk assessment. Given the current adversities and their implication has lead to manufacturers, logistics and transportation companies to research and invest into solving them.

This dissertation focuses on the logistics aspect, creating solutions methods for drone delivery. As most companies are in a stage of research and legislation challenges, some assumptions have to be made on how applications are going to work, and, therefore, results may be overly theoretical.

One of the main problems in logistics is defining routes for vehicles. The classical Traveling Salesman Problem (TSP) is the basic problem in the field. When several vehicles are involved, and adding a limited capacity, an extension of the TSP is born as the Vehicle Routing Problem (VRP). The addition of Drones in this process is the Vehicle Routing

Problem with Drones (VRPD). The main differences from VRP using drones to the classic VRP, or even to other variants, are: the reduced capacity, the trend being to deliver one package per flight; the distance covered (or flight endurance) when compared to other vehicles is very small; and not all deliveries or customers are able to be served with a drone, either because of weight or lack of landing sites.

An alternative, so as to reduce some of these difficulties and while benefiting from the advantages offered by drones, is the use of trucks (or other medium to large vehicles) with a drone in tandem. This way the drone flies from one point (a customer served by the truck) to the client it has to deliver to, then meets the truck in another customer's location. Thus, long distance trips can be executed by the truck (as well delivering packages which are not drone-eligible) and short distance trips (or areas with heavy traffic) can be covered by the drone, especially in context of last-mile delivery. This idea is being tested by UPS ([LITHIA, 2017](#)) and was first introduced in the scientific field (as far as the authors are aware) by ([MURRAY; CHU, 2015](#)), where only one truck-drone pair is used. Their work will be further explored in [Chapter 2](#). On the other hand, in several real-life applications, using only one truck (and one drone) is not enough. Due to the large number of clients and/or the limited time for delivery, most logistic companies use several vehicles to fulfill their demands. The VRP, which is used for such cases, is not covered by the aforementioned model. The use of this truck-drone system as a VRP is explored in ([WANG; POIKONEN; GOLDEN, 2016](#)), where the authors provide mathematical proof that the time for delivery is indeed reduced when using drones. However, the authors do not provide any mathematical model nor solution method for the problem.

Hence, the purpose of this dissertation is to provide a mathematical Mixed Integer Linear Problem (MILP) model based on the formulation from ([MURRAY; CHU, 2015](#)), extending it to a VRPD, adding constraints and/or variables and adapting them to fit the original model to the idea of using several truck-drone pairs. A solution method based on a Hybrid Genetic Algorithm (HGA) in combination with Local Search (LS) is also introduced as an realistic (time-wise) technique to solve the VRPD.

It is important to note that this work, despite being inspired by the UAV technology and its applications, can also be applied for other types of autonomous vehicles such as wheeled robots (which can also be referred as drones), since the principles of other autonomy vehicles are very similar to the ones applied to UAVs. The differences would mainly be about parameters adjustment.

The following chapters are divided as follows: in [Chapter 2](#) a literature review on the VRP and drone problems related to this work is presented; [Chapter 3](#) presents scenarios for the problem and its description as well as the MILP model; [Chapter 4](#) presents our HGA and the LS used; The results are discussed in [Chapter 5](#); Finally, we summarize the work in [Chapter 6](#), as well as present some research perspectives.

LITERATURE REVIEW

In this chapter, a preliminary review of the literature on the subject of the Vehicle Routing Problem with Drones is presented. First, the origins of the VRP and some types of VRP are given to provide some context. Afterwards, related TSP and VRP work in logistics with drones are addressed. Finally, a comparison of this work with all the other works presented is shown.

2.1 An Introduction to VRP

Formally, the Vehicle Routing Problem (VRP) in its classical formulation is defined by a set of m vehicles with capacity Q each, which have to visit a set of customers with demand q_i (being either delivery or pick-up). The world model is represented by a directed graph $G = (V, A)$, where the Vertices $V = \{0, 1, \dots, n\}$ are the customers (with 0 being the depot), and the Arcs $A = \{(i, j) : i, j \in V, i \neq j\}$ being the distance from node i to node j (in time or distance units). The VRP is a generalization of the Traveling Salesman Problem (TSP) with several vehicles and limited capacity. In the TSP a single salesman wants to visit every city (from a set of cities) in such a way that he visits all cities only once, reducing the total distance traveled. In the VRP, however, m routes are computed, one for each vehicle, every one of them starting and ending in the depot, with every customer being visited exactly once, and no route demand exceeds the capacity Q , i.e. to find the shortest feasible routes. Both the TSP and the VRP are of NP-Hard complexity, and are therefore very costly to solve (LENSTRA; KAN, 1981). Figure 3 shows a visualization of the problem.

The first work published for the VRP was presented by (DANTZIG; RAMSER, 1959). The truck dispatching problem, as it is called in their work, tries to find an optimal solution to a gasoline delivery service, from one single distribution center (a depot), containing several trucks and customers. The objective is to find the minimal route for

Figure 3 – An instance of the VRP and one solution.

Source: ([NETWORKING; OPTIMIZATION, 2013](#))

every truck in a way where each customer is visited exactly once, and their demand is satisfied. The trucks also present the same capacity (homogeneous fleet). Later, ([CLARKE; WRIGHT, 1964](#)) wrote a generalization of [Dantzig and Ramser \(1959\)](#) as a linear optimization problem, and proposed the Savings Algorithm heuristic to solve it.

Both of these works were later specified as a Capacitated VRP, since they acknowledge that the vehicles have a transportation capacity. As the VRP grew in popularity the following decades, several variations were introduced to represent more realistic situations. For example, the addition of time constraints was introduced as VRP with Time Windows (VRPTW), where customers have a specific time window they can be served. Another variant is the pick-up and delivery, where cargo is to either be picked-up or delivered during the course, taking into account the order in which the customers will be visited so the vehicle can be more efficiently filled or emptied. More variants are related to the dynamism of the problem, whether by customers being unknown before execution or following stochastic demands, or even the travel times, for example, taking into account traffic or detours. For more on the VRP and its variations check ([BRAEKERS; RAMAEKERS; NIEUWENHUYSE, 2016](#)), ([LAPORTE, 2009](#)), ([EKSIOGLU; VURAL; REISMAN, 2009](#)).

In the classical literature for the VRP, the problem that is comparable to the approach of truck and drone (VRPD) is the VRP with multiple deliverymen (VRPMD), with only one deliveryman, since delivery drones have a relatively large size, so it is impractical to fit more than one in a single truck. In the VRPMD a vehicle is driven to a parking location, and then the servicemen (deliverymen) walk to the customers, returning afterwards to the parked vehicle to continue to the next parking point (for more on the VRPMD read ([PUREZA; MORABITO; REIMANN, 2012](#))). However, this problem is not similar enough to VRPD, as the drones have batteries which need to be taken into account and the truck does not need to stay parked, waiting for the drone. The VRPD

could also be characterized as a Green Vehicle Routing Problem (GVRP), as it utilizes a sustainable transportation method (LIN *et al.*, 2014) in the form of a battery-powered drone and implicitly reduces the total mileage covered by the truck. In the next section, works related to that of the VRPD are presented.

2.2 Use of Drones in Logistics

Using drones for package delivery is a very recent development, and there was a boom in the last two years (2017-2018) of several works in literature being published. (MURRAY; CHU, 2015) wrote one of the first papers on delivering with drones (using the UAV notation), called the Flying Sidekick Traveling Salesman Problem (FSTSP), referring to the truck with its UAV partner. They present two different Mixed Integer Linear Programming (MILP) approaches for two types of delivery: the first one is a combination of one truck and one UAV delivering to customers outside the flying range from the depot, and a second one where several UAVs deliver directly from the depot and a truck delivers to customers who are out of range (called the PDSTSP, or Parallel Drone Scheduling TSP); in both cases they seek to minimize the time the last vehicle (UAV or truck) returns to the depot. Their work introduces the idea of having two different types of customers, those to which an UAV can deliver, and those to which it cannot, either because of the product's weight, because the site does not allow clear landing, and others. Murray and Chu (2015) solve the first problem (truck and an UAV) with a TSP heuristic that finds a route for the truck, which gets improved by checking whether delivering to a client with an UAV is viable (reduces total delivery time) and then adjusting the truck's route accordingly. The second problem is solved by initially using a TSP heuristic for the truck, including all customers that are out of range and whose demand is not UAV-suitable, and the remaining customers are scheduled to the UAVs by using a parallel identical machine scheduling problem heuristic, where a customer is a job, the flight time is the processing time, and the objective is to have a minimal makespan. That initial solution is improved by checking if swapping the customers from one route to another is viable to the time cost (of course, respecting the UAV-suitable restrictions). Their results, which compare a truck-only TSP with their models, show an improvement to the solutions. The model for the FSTSP, which is the base for the model in this Dissertation, is available in Appendix A.

The study of (AGATZ; BOUMAN; SCHMIDT, 2015) is very similar to the one of (MURRAY; CHU, 2015). They named it as the Traveling Salesman Problem with Drone (TSP-D) and developed an Integer Programming (IP) model based on operations for the problem. They also propose a heuristic approach to solve it where they find a truck-only route, and then improve it by adding customers to the drone route, based on a fast greedy heuristic and an exact partitioning algorithm.

([HA et al., 2015](#)) propose two heuristics to solve the TSP-D, both based on clustering techniques to find the customers which will be served by the drone. They compare their results with the heuristic proposed in the FSTSP ([MURRAY; CHU, 2015](#)), and achieve better results in some test cases, but most of their results do not.

([FERRANDEZ et al., 2016](#)) utilize a combination of a truck and one or more drones. They use a K-Means algorithm clustering the clients to find parking points where a truck could stop and send the drones in tandem to deliver packages. They then apply a genetic algorithm to solve the TSP between parking spots. The authors show results in terms of time and energy, where the first one is improved when compared to the standalone TSP, but is largely affected by the speed and/or the number of drones, and the second one vastly surpasses the first one when compared to the truck alone, claiming the drones are more efficient energy-wise. The algorithm developed was able to solve instances of more than 200 customers, in around two minutes.

([DORLING et al., 2017](#)) elaborate a VRP using drones only, where one drone can deliver to multiple customers. From a starting depot, the drones are routed to serve all customers and they are allowed to return to the depot multiple times. [Dorling et al. \(2017\)](#) write an MILP to the problem, in which not only routing and re-routing are done, but also each drone's battery weight and consumption are measured, that way they can optimize the weight and the number of routes a drone can fly, with a single charge. They propose this model either to minimize delivery time or to minimize the number of drones used and energy consumption. They solve the model with a simulated annealing heuristic. They also did a linearization of the power consumption equation for drones, and a practical test for the average voltage, current and power consumed in some flight modes.

([WANG; POIKONEN; GOLDEN, 2016](#)) write an analysis of worst-case scenarios for the Vehicle Routing Problem with Drones (VRPD), and are the first to use this terminology. The paper is for theoretical results only. The authors show mathematical relations for different scenarios of drone usage in combination with trucks, where each drone carries a single package, where the goal is to minimize time of completion, proving both upper and lower bounds for all cases, varying the number of trucks, the speed of the drones and the number of drones in each truck. The work of ([WANG; POIKONEN; GOLDEN, 2016](#)) is very relevant to any posterior work in the VRPD, since the results show the relation between the regular TSP or VRP with the VRPD. For example, one particular case with m trucks and 2 drones each will have the completion time reduced by at most 75% when compared to a VRP also with m trucks, considering the drones travel 50% faster than the truck.

The authors from ([WANG; POIKONEN; GOLDEN, 2016](#)) also wrote another work on the subject, ([POIKONEN; WANG; GOLDEN, 2017](#)), where they expand their scenarios to more realistic situations with more mathematical proof, taking into account

the battery life, different metrics for drones and trucks, and the costs of utilizing drones. They also provide proof that the Close-Enough Vehicle Routing Problem (CEVRP), where trucks do not need to visit exactly the location of a client, but instead must be within some distance $R_i > 0$, is a bound to VRPD.

(PUGLIESE; GUERRIERO, 2017) propose a MILP model for the VDRPTW, Vehicle and Drone Routing Problem with Time Windows, where several trucks with drones in tandem are used to perform parcel deliveries. Their model has similarities with the one of (MURRAY; CHU, 2015), whereas a drone can only deliver one package before returning to its truck, at a client location, and the truck will make deliveries whilst the drone is flying. The synchronization process is similar as well, although they disregard time to set-up launch or landing, and take into account the service time. The main difference between the models, besides several vehicles and the set-up time, and therefore their main contribution, is that (PUGLIESE; GUERRIERO, 2017) also add time windows for customers visits, the trucks' capacity and the objective function, which aims to minimize the total transportation cost for both drones and trucks. They test the model for small samples (5 and 10 customers, and a percentage of that is drone-eligible) with 2 vehicles. Their results, however, show that the use of drones is very costly even in those small instances. For example, with the same cost of transportation for trucks and drones, the model resulted in no drones being used in the optimal solution. The drones are used only when the transportation cost of a truck is more expensive. When comparing operational time, however, the drones do reduce the completion time, as expected and as pointed in the work of (WANG; POIKONEN; GOLDEN, 2016).

(SCHERMER; MOEINI; WENDT, 2018) propose two heuristics to solve the VRPD, and test them using 750 large-scale instances, from the *TSPLIB* dataset¹. Their assumptions on how the drone and truck combination work are based on (WANG; POIKONEN; GOLDEN, 2016). The two heuristics developed are based on a route-first cluster-second approach, where a single route is created with a nearest neighbor heuristic, the route is equally divided between the m trucks and are then improved. In the first heuristic they improve the trucks route before adding drones. In the second one, the drones are considered from the beginning. On their tests, the authors show that the first heuristic presents better results when compared to the second one, concluding that finding better VRP routes before adding the drones will result in better VRPD routes.

(FREITAS; PENNA, 2018) present a Randomized Variable Neighborhood Descent (RVND) heuristic to solve the flying sidekick traveling salesman problem (FSTSP) (MURRAY; CHU, 2015). On the proposed heuristics, first they solve the regular TSP optimally with the solver Concorde plus CPLEX. Then, a initial solution for the FSTSP is created by removing customers from the truck's route and adding them to the drone's route.

¹ <<https://www.iwr.uni-heidelberg.de/groups/comopt/software/TSPLIB95/index.html>>

Finally, this route is improved with the RVND where, similarly to the VND, from the starting route, neighbor routes are explored and the best one is set as the new route and then moving to a new neighborhood, the difference being that there is a shuffle in the neighborhoods in the RVND. The neighborhoods utilized in their work are based on: changing one truck customer location; permutation between two truck customers; swapping two consecutive customers and an additional one; 2-OPT; and, one customer from the truck's route is relocated to the drone's route. To test their results, the authors utilize 11 instances of the TSPLib dataset, containing between 51 to 100 nodes. Although in the best cases the GAP between the regular TSP and their heuristic is an average of 7.2% improvement, the overall gap considering the 10 runs is only 0.057%, for about 1 second of execution time. The best result, improved delivery time by almost 20%.

([LUO; LIU; SHI, 2017](#)) write a paper on Two-echelon Cooperated Routing Problem where, similarly to the TSPD, they use one ground vehicle and one UAV. However, the problem approach is quite different, as all customers are served by the UAV, which can serve multiple customers before returning to the vehicle. Additionally, the UAV can only rendezvous with the vehicle in predetermined parking spots. They solve this problem by proposing an IP formulation, and two constructive heuristics. The first one builds a single route with all nodes and splits it by adding the parking points, while the second one builds a route for the vehicle first (with the parking spots) and then assigns the UAV routes based on that. They create one dataset with up to 200 customers and 80 parking spots, randomly distributed, resulting in 600 instances. They compare the IP formulation with the heuristics in the 30 instances that the model could solve (with times of up to 12000s), and prove that the heuristics are competitive. For larger instances, they compare both heuristics with each other, where the first one is more accurate, but the second one is faster.

([CARLSSON; SONG, 2018](#)) aims to find how efficient is the FSTSP with a mathematical approach. The authors use asymptotic theoretical analysis in the Euclidean plane, with computational experiments, to demonstrate that the use of the UAV will improve the efficiency of deliveries on a proportion equal to the square root of the ratio between the truck's speed and the UAV's speed.

([BOUMAN; AGATZ; SCHMIDT, 2017](#)) propose an exact approach for the TSP-D by using Dynamic Programming (DP). To summarize, the DP consists of three main steps: enumerating all shortest paths for the truck; a combination of truck paths with the drone nodes; and the computation of the optimal path given previous steps. The authors also apply an A^* algorithm to guide the proposed DP, to try to solve it faster. The results show that the DP does succeed in finding exact solutions faster than the IP model of the same authors [Agatz, Bouman and Schmidt \(2015\)](#), and that the addition of the A^* technique also helps solve some larger instances, while also saving CPU time.

(CHUNG, 2018) propose a heuristic method for the FSTSP, here called Truck and Drone Parcel Delivery Problem (TDRP). The heuristic, named Two Step Ahead Tour Growing Heuristic, builds the truck route constructively based on a saving heuristics. The savings is built for all combination of three customers, with one of them being served by the drone. Then, starting from the depot, the least costing route is added to the current one. The author create an small toy problem with 9 customers to test this heuristic, saving up to 33% in completion time.

(HAM, 2018) propose an extension to the PDSTSP (where drones serve directly from the depot and go back) containing: pick-up and delivery, multiple trips, multiple depots, and time-windows. The drones can leave the depot with one parcel, drop it at the desired customer location, and then fly to another customer to perform a parcel pick-up, then returning to one of the depots. The trucks will work similarly, however they can serve (deliver or pick-up) several customers before returning. The author utilize two distinct Constraint Programming models to solve the problem. He tests the model with instances of 20, 50 and 100 customers, finding optimal values to most instances with 20 – 50 customers within 300s of execution using IBM ILOG CP OPTIMIZER.

The authors of Ha *et al.* (2015) write another paper on TSP-D, (HA *et al.*, 2018), where they apply two new heuristics and a new MILP model for the problem, aiming to minimize operational costs with added waiting time variables (which accounts for the time the truck or the drone have to wait for each other). The first heuristic is a GRASP with adapted local search algorithms to better fit the drone usage. The second heuristic is based on (MURRAY; CHU, 2015), consisting of constructing a truck route and adding local search to find drone routes. The authors test their solutions in instances of 10, 50 and 100 customers, running a total of 1080 tests. The exact solution can only solve the ones with 10 customers within 1h, and therefore cannot be compared with the heuristics for other instances. In those which can, the GRASP performs a lot better than the second heuristic, finding optimal solutions in all cases. For the larger instances, the GRASP still outperforms the other, but the average execution time is higher. When compared to the heuristic of (MURRAY; CHU, 2015), the GRASP also finds better or equal solutions for most compared cases.

(CHANG; LEE, 2018) solve the TSP-D with a nonlinear programming model approach. They find clusters of customers using K-means, and a TSP where the trucks visits all clusters centers to where the drones will fly from. They also shift the clusters centers to minimize the truck distance, yet without any customer getting out of range for the drones and compare the two models. They test the models with random instances ranging from 10 to 100 customers, showing that by shifting the clusters centers, which increases the use of drones, is more effective since in their experiments the drones are faster than the trucks.

The dissertation of (PONZA, 2016) does an study on the FSTSP. She analyzes the problem numerically, investigating the trade-offs of faster drones or drones with longer flight autonomy. She also develops a Simulated Annealing (SA) heuristic to solve the problem. To test the algorithm and to collect results, she develops her own instances ranging from 5 to 200 customers (not regularly). She compares the SA result to the regular TSP, and finds the drones improve all instances when comparing distance. She also shows that a faster drone is actually more influential than one that flies for longer periods, even though both are not that significant. The first case improves about at average 3.5% when improving the speed by 50%, and the second case average around 2.5% when the endurance is increased in approximately 60%. However, these tests are made only to a very small sample, and may lack statistical significance.

(MATHEW; SMITH; WASLANDER, 2015) use a directed graph in this problem similar to the TSP-D. The difference here is that the truck follow the street network, while the drone fly directly to the customer. They formulate the problem as an optimal path planning problem, and solve it by using a novel reduction to the Generalized TSP. The authors prove that the problem is of NP-hardness complexity.

Similar to the last paper, (OTHMAN *et al.*, 2017) propose a problem more similar to the FSTSP, where only the drones deliver the parcels, but with a pre-determined route for the truck. They model the problem in a graph of special structure. The authors also prove that this problem is of NP-hardness complexity. They solve it by finding minimum-cost paths in the special graph.

(ULMER; THOMAS, 2018) write a paper on the VRP with drones for Same Day Delivery (SDD), and heterogeneous vehicles, with stochastic demands. They use a policy function approximation method to solve the problem, and find that the use of drones may reduce operational costs.

2.3 Literature Summary

Table 1 summarizes the works mentioned and identifies their contribution, whether it is a truck + drone approach, what kind of exact method (if any) and type of heuristic are used. Although there are a MILP formulation and some variations have a non-exact solution method for the VRPD, we deem our work is more complete and more competitive than the ones listed here. It is also important to note that not all work related to the use of drones in logistics are listed, since there is several already published. However, we tried to list the most relevant and the most related to this work. In the next chapter 3, we present the MILP formulation and the overall definition of the VRPD. After that, Chapter 4 present our HGA and LS procedures.

Table 1 – Current work on Truck and Drone problems.

Work	Truck + Drone	Issue	Exact Method	Heuristic
(MURRAY; CHU, 2015)	✓	TSP	MILP	Construct + Savings
(AGATZ; BOUMAN; SCHMIDT, 2015)	✓	TSP	IP	Greedy + Partitioning
(HA <i>et al.</i> , 2015)	✓	TSP	-	Clustering
(FERRANDEZ <i>et al.</i> , 2016)	✓	TSP	-	K-Means + Genetic
(DORLING <i>et al.</i> , 2017)	-	VRP	MILP	Simulated Annealing
(WANG; POIKONEN; GOLDEN, 2016)	✓	VRP	-	-
(POIKONEN; WANG; GOLDEN, 2017)	✓	VRP	-	-
(PUGLIESE; GUERRIERO, 2017)	✓	VRP	MILP	-
(SCHERMER; MOEINI; WENDT, 2018)	✓	VRP	-	Route-first Cluster-second
(FREITAS; PENNA, 2018)	✓	TSP	-	Random Variable Neighborhood Descent
(LUO; LIU; SHI, 2017)	✓	TSP	IP	Two Constructive Heuristics
(CARLSSON; SONG, 2018)	✓	TSP	-	-
(BOUMAN; AGATZ; SCHMIDT, 2017)	✓	TSP	DP	-
(CHUNG, 2018)	✓	TSP	-	Savings-based
(HAM, 2018)	✓	mTSP	CP	-
(HA <i>et al.</i> , 2018)	✓	TSP	MILP	GRASP and Constructive + Local Search
(CHANG; LEE, 2018)	✓	TSP	NL	-
(PONZA, 2016)	✓	TSP	-	Simulated Annealing
(MATHEW; SMITH; WASLANDER, 2015)	✓	TSP	OPP	-
(OTHMAN <i>et al.</i> , 2017)	✓	TSP	-	-
(ULMER; THOMAS, 2018)	✓	VRP	-	Policy Function Approximation
This dissertation	✓	VRP	MILP	Hybrid Genetic Algorithm

Source – Authors.

PROBLEM DEFINITION AND MODEL

In this chapter the problem and how it is going to be approached in this dissertation is discussed. Firstly, two scenarios are presented to exemplify how the use of drone is advantageous, and why such research is relevant. Afterwards, we present the Mixed-Integer Linear Programming (MILP) formulation to the problem.

3.1 Scenario I

Large cities' downtown areas are heavily crowded with traffic and people, which make a delivery truck's access difficult or even prohibited (outside work hours, usually). An alternative to solve, or at least minimize these problems could be delivering packages with drones. The truck would navigate around these traffic heavy regions, and drones could fly into downtown and deliver packages to roofs of buildings or special sites set-up for this task, avoiding traffic, and speeding up delivery time. Additionally, if their use reduced the number of vehicles in central areas, pollution would decrease and air quality would improve.

Figure 4 shows an example of this case, where the trucks are surrounding the city center, serving clients that are not eligible for drone delivery (red and orange marks), and the drones start flying towards delivery points (green and blue marks). It is worth noting that all drone-eligible clients are also truck-eligible, since all the shipment is within the truck's cargo. Since a drone's weight capacity is limited, possible applications could be mail, documents, food delivery and other small packages with low weight.

3.2 Scenario II

Rural areas and small cities tend to have large distances between one delivery point to another, or some access roads are not paved (specially in underdeveloped countries).

Figure 4 – First scenario of drone usage in logistics: delivery in downtown areas.

Source: Authors, Based on Designed by Freepik

Drone usage in this case could help to heavily reduce the truck's mileage and avoid riskier roads. A truck attending one client can send the drone to another one that stands outside the main road, and meet the truck back at the next client's location. Rural areas also have more space where a drone can safely land.

Figure 5 shows an example of rural area where a drone can save time, and the trucks can keep delivering along the main road, while the drones deliver to the side roads. Also in this case, aside from any road condition, some packages will be delivered by truck, since not all packages are drone-eligible.

3.3 Problem Discussion

Given the above scenarios, the first one is somewhat complicated to turn into reality in today's world, since several problems are present: privacy, for one, since modern drones use cameras to process their flight path; the risk of failure, where the drone can fall

Figure 5 – Second scenario of drone usage in logistics: delivery in rural areas

Source: Authors, Based on Designed by Freepik

to the ground and hurt a pedestrian or damage a property; air space shared with other aircrafts (planes, helicopters) increasing the risk of accidents.

The second scenario is more likely to happen within a few years, since most of the problems from the first one are a lot less probable. Large areas with no buildings or people, and the large distances allow drones to be used to their full potential, though security should not be disregarded. This scenario would save a lot of time for the truck, allowing more packages to be delivered within the same number of hours or to have reduced mileage (resulting in fewer emissions of greenhouse gases). Both of these scenarios were reported by DHL to be the most promising for logistic companies in the next years (KÜCKELHAUS; NIEZGODA; ENDRIß, 2014). Despite the complications, the advantages are overwhelming, reducing costs, time and pollution.

However, these scenarios will only be efficient after answering the following question: **How to decide which clients are served by the truck and which are served by the drone in an optimal way?** The answer to this question could be achieved by solving a VRP, where the routes contain which clients are to be served by which vehicle. Since the number of customers is expected to be very large (from logistics companies' standpoint), a TSP solution would not suffice, therefore a VRP approach needs to be proposed. However, it is not an easy task to achieve, because the problem has several particularities that need to be taken into account. Hence, before extending the model, some assumptions and considerations are required:

- Each fleet type is homogeneous, all the drones have the same capacity Q^v , endurance e and speed v_d ; similarly, all the trucks have the same capacity C_t and speed v_t , the trucks' endurance is ignored due to it being sufficiently large.
- The drone can only take-off and land while the truck is at a client location or the depot, no extra stops are allowed.
- A drone is linked to only one truck, meaning the drones cannot depart from one truck and arrive at another.
- If a drone has to wait for its truck, the drone cannot land so as to preserve energy, and will have to hover.
- Not all clients can be served by a drone, meaning some packages are required to be delivered by truck.
- Each drone can only carry one package, even if its capacity allows it for more, meaning it needs to return to its truck before it makes another delivery.
- No drone nor truck can visit the same client more than once.
- All customers have time-windows and service time pre-established. However, the service time is only taken into account for the truck deliveries. We assume that the drones will just deliver the product and leave, while the deliveryman has to do extra work other than deliver, such as unload the truck, get the customer's signature, or some other time consuming task.
- The time to make the package flight-ready is incorporated in the prep time for the drone launch. Additionally, any time related to a returning drone is incorporated to the landing time.
- The depot has a landing site which support the number of available drones.

Based on these considerations we present in the next section the MILP for the VRPD.

3.4 The VRPD Mixed-Integer Linear Programming Model

The mathematical programming model for the Vehicle Routing Problem with Drones is presented. In this model, based on (MURRAY; CHU, 2015), some adjustments were made to either make the model less computationally expensive, or to encourage the drone usage, for example, by relaxing the requirement that a drone needs to arrive at the final depot together with its truck.

The MILP model has several sets, parameters and variables that are explained next. All constraints are grouped up by their goal with a briefly description, and are presented afterwards.

Set:

- C : set of clients;
- C' : set of clients drone-eligible;
- N : set of nodes with starting point 0 and ending point $c + 1$ being the depot;
- N_0 : set of nodes which vehicles may depart from, i.e. $0, 1, \dots, c$;
- N_+ : set of nodes which vehicles may arrive at, i.e. $1, 2, \dots, c + 1$;
- P : set of nodes which combined can form a drone route, i.e. (i, j, k) such that $i \in N_0, j \in C', k \in N_+$;
- M : set of vehicles that can be used.

Data:

- d_{ij} : the distance between vertices i and j , for trucks;
- d'_{ij} : the distance between vertices i and j , for drones;
- q_i : the demand of client i ;
- Q^t : the truck's load capacity;
- a_i : the start of the time windows for client i ;
- b_i : the end of the time windows for client i ;
- s_i : the service's duration for client i (note: 0 for drone deliveries);
- v_d : the drones' average flight speed;
- v_t : the trucks' average speed;
- e : the drones' flight endurance;
- S_L : set-up time for launching a drone from one truck;
- S_R : set-up time for returning a drone from one truck;
- T_{max} : maximum operational time (closing window of depot);
- K : a large enough number.

Decision Variables:

$$\begin{aligned}
x_{ij}^v: & \begin{cases} 1 & \text{if the truck } v \text{ leaves } i \text{ and goes to } j; \\ 0 & \text{otherwise.} \end{cases} \\
y_{ijk}^{vd}: & \begin{cases} 1 & \text{if the drone } d \text{ (associated with truck } v) \text{ leaves from } i, \text{ serves } j \text{ and} \\ & \text{meet the truck in } k; \\ 0 & \text{otherwise.} \end{cases} \\
p_{ij}^v: & \begin{cases} 1 & \text{if client } i \text{ on the route of truck } v \text{ is visited before } j \text{ } (j \neq i); \\ 0 & \text{otherwise.} \end{cases} \\
t_i: & \text{instant in which a truck arrives at vertex } i; \\
\tau_i: & \text{instant in which a drone arrives at vertex } i; \\
u_i^v: & \text{auxiliary variable to avoid subtours}
\end{aligned}$$

The objective function is the classical VRP objective, which aims to reduce the total distance, and will be used to compare this model with the classical known results.

Objective Function *I* (Equation 3.1): to minimize the total distance traversed by the trucks (i.e., maximizing the distance traveled by the drones).

$$\text{Min } Z = \sum_{i \in N_0} \sum_{\substack{j \in N_+ \\ i \neq j}} \sum_{v \in M} d_{ij} x_{ij}^v \quad (3.1)$$

Constraints 3.2 ensure every customer j is visited by either a truck or a drone.

$$\text{s.t. } \sum_{\substack{i \in N_0 \\ i \neq j}} \sum_{v \in M} x_{ij}^v + \sum_{\substack{i \in N_0 \\ i \neq j}} \sum_{\substack{k \in N_+ \\ (i,j,k) \in P}} \sum_{(v,d) \in M} y_{ijk}^{vd} = 1 \quad \forall j \in C \quad (3.2)$$

Constraints 3.3 and 3.4 ensure that every truck leaves the depot and returns to the depot. Here the trucks are allowed to leave from the starting depot to the ending depot because some customers could be served by drones directly from the depot.

$$\sum_{j \in N_+} x_{0j}^v = 1 \quad \forall v \in M \quad (3.3)$$

$$\sum_{i \in N_0} x_{i(c+1)}^v = 1 \quad \forall v \in M \quad (3.4)$$

Constraints 3.5 prevents sub-cycles for each vehicle v .

$$u_i^v - u_j^v + 1 \leq (c+2)(1 - x_{ij}^v) \quad \forall i \in C, j \in \{N_+ : j \neq i\}, v \in M \quad (3.5)$$

The flow constraints 3.6 ensures that a truck v will arrive and leave the customer j .

$$\sum_{\substack{i \in N_0 \\ i \neq j}} x_{ij}^v = \sum_{\substack{k \in N_+ \\ k \neq j}} x_{jk}^v \quad \forall j \in C, v \in M \quad (3.6)$$

Constraints 3.7 and 3.8 limits a drone d to arrive and leave a customer at most once.

$$\sum_{\substack{j \in C' \\ j \neq i}} \sum_{\substack{k \in N_+ \\ (i,j,k) \in P}} y_{ijk}^{vd} \leq 1 \quad \forall i \in N_0, (v,d) \in M \quad (3.7)$$

$$\sum_{\substack{i \in N_0 \\ i \neq k}} \sum_{\substack{j \in C' \\ (i,j,k) \in P}} y_{ijk}^{vd} \leq 1 \quad \forall k \in N_+, (v,d) \in M \quad (3.8)$$

Constraints 3.9 ensure that the truck v will visit customer i and k if the correspondent drone d does the route $i \rightarrow j \rightarrow k$. Constraints 3.10 do the same when $i = 0$ (so the truck only needs to visit k).

$$2y_{ijk}^{vd} \leq \sum_{\substack{h \in N_0 \\ h \neq i}} x_{hi}^v + \sum_{\substack{l \in C \\ l \neq k}} x_{lk}^v \quad \forall i \in C, j \in \{C : j \neq i\}, k \in \{N_+ : (i,j,k) \in P\}, (v,d) \in M \quad (3.9)$$

$$y_{0jk}^{vd} \leq \sum_{\substack{h \in N_0 \\ h \neq k}} x_{hk}^v \quad \forall j \in C, k \in \{N_+ : (0,j,k) \in P\}, (v,d) \in M \quad (3.10)$$

Constraints 3.11 ensure that the truck v will have i before k in its route, if the correspondent drone d does the route $i \rightarrow j \rightarrow k$.

$$u_k^v - u_i^v \geq 1 - (c+2) \left(1 - \sum_{\substack{j \in C' \\ (i,j,k) \in P}} y_{ijk}^{vd} \right) \quad \forall i \in C, k \in \{N_+ : k \neq i\}, (v,d) \in M \quad (3.11)$$

The next set of constraints ensure that a truck and its drone will have a synchronous time in both take-off and landing customers. The first two (3.12 and 3.13) synchronize the arrival at customer i , whilst the next two 3.14 and 3.15 synchronize the arrival at customer k . Here, the "big M" K equals to T_{max} , since the biggest difference (for non bounding cases) in any time schedule for the VRPTW is less than the maximum operational time.

$$\tau_i \geq t_i - K \left(1 - \sum_{(v,d) \in M} \sum_{\substack{j \in C \\ j \neq i}} \sum_{\substack{k \in N_+ \\ (i,j,k) \in P}} y_{ijk}^{vd} \right) \quad \forall i \in C \quad (3.12)$$

$$\tau_i \leq t_i + K \left(1 - \sum_{(v,d) \in M} \sum_{\substack{j \in C \\ j \neq i}} \sum_{\substack{k \in N_+ \\ (i,j,k) \in P}} y_{ijk}^{vd} \right) \quad \forall i \in C \quad (3.13)$$

$$\tau_k \geq t_k - K \left(1 - \sum_{(v,d) \in M} \sum_{\substack{i \in N_0 \\ i \neq k}} \sum_{\substack{j \in C \\ (i,j,k) \in P}} y_{ijk}^{vd} \right) \quad \forall k \in C \quad (3.14)$$

$$\tau_k \leq t_k + K \left(1 - \sum_{(v,d) \in M} \sum_{\substack{i \in N_0 \\ i \neq k}} \sum_{\substack{j \in C \\ (i,j,k) \in P}} y_{ijk}^{vd} \right) \quad \forall k \in C \quad (3.15)$$

$$K = T_{max}$$

Constraints 3.16 and 3.17 are related to the truck v arrival time at customer k , and the depot $c+1$, respectively. They account the travel time from h to k (or $c+1$), the service time at customer h (in case h is the depot, the service time is 0) and possible take-off and landing times (S_L and S_R , respectively) from the drone d . Here K also incorporates the travel time, service time and take-off (for the first case) and landing times.

$$t_k \geq t_h + \frac{d_{hk}}{v_t} + s_h + S_L \left(\sum_{\substack{l \in C \\ l \neq k}} \sum_{\substack{m \in N_+ \\ (k,l,m) \in P}} y_{klm}^{vd} \right) + S_R \left(\sum_{\substack{i \in N_0 \\ i \neq k}} \sum_{\substack{j \in C \\ (i,j,k) \in P}} y_{ijk}^{vd} \right) - K(1 - x_{hk}^v) \\ \forall h \in N_0, k \in \{C : k \neq h\}, (v,d) \in M \quad (3.16)$$

$$K = T_{max} + \frac{d_{hk}}{v_t} + S_L + S_R + s_h$$

$$t_{(c+1)} \geq t_h + \frac{d_{h(c+1)}}{v_t} + s_h + S_R \left(\sum_{i \in N_0} \sum_{\substack{j \in C \\ (i,j,c+1) \in P}} y_{ij(c+1)}^{vd} \right) - K(1 - x_{h(c+1)}^v) \\ \forall h \in N_0, (v,d) \in M \quad (3.17)$$

$$K = T_{max} + \frac{d_{h(c+1)}}{v_t} + S_R + s_h$$

Constraints 3.18 ensure that the time of visit for customer j , served by the drone d , accounts the travel time from i to j and the time the drone left i . This constraint is bounding when the drone does travel from i to j and then to some k , otherwise, the "big M" K is equal to T_{max} plus the travel time from i to j .

$$\tau_j \geq \tau_i + \frac{d'_{ij}}{v_d} - K \left(1 - \sum_{\substack{k \in N_+ \\ (i,j,k) \in P}} y_{ijk}^{vd} \right) \\ \forall j \in C', i \in \{N_0 : i \neq j\}, (v,d) \in M \quad (3.18)$$

$$K = T_{max} + \frac{d'_{ij}}{v_d}$$

Constraints 3.19 and 3.20 are similar to the above, but for the customer k or depot $c+1$, respectively, if the drone lands there ($i \rightarrow j \rightarrow k$). The main difference is the landing time S_R that needs to be added, as well as for K .

$$\tau_k \geq \tau_j + \frac{d'_{jk}}{v_d} + S_R - K \left(1 - \sum_{\substack{i \in N_0 \\ (i,j,k) \in P}} y_{ijk}^{vd} \right) \quad \forall j \in C', k \in \{C : k \neq j\}, (v,d) \in M \quad (3.19)$$

$$K = T_{max} + \frac{d'_{jk}}{v_d} + S_R$$

$$\tau_{c+1} \geq \tau_j + \frac{d'_{j(c+1)}}{v_d} + S_R - K \left(1 - \sum_{\substack{i \in N_0 \\ (i,j,c+1) \in P}} y_{ij(c+1)}^{vd} \right) \quad \forall j \in C', (v,d) \in M \quad (3.20)$$

$$K = T_{max} + \frac{d'_{j(c+1)}}{v_d} + S_R$$

Constraints 3.21 ensure that the drone endurance e is not exceeded. This endurance is not cumulative, which means that the batteries are either fully recharged or swapped after each trip. Here the "big M" cannot be larger than the longest possible waiting time reduced by the endurance.

$$\tau_k - \tau_i \leq e + K \left(1 - \sum_{\substack{j \in C \\ (i,j,k) \in P}} y_{ijk}^{vd} \right) \quad \forall k \in C, i \in C, (v,d) \in M \quad (3.21)$$

$$K = T_{max} - e$$

Constraints 3.22, 3.23 and 3.24 set the variables u_i and p_{ij} for each truck v .

$$u_i^v - u_j^v \geq 1 - (c+2)p_{ij}^v \quad \forall i \in C, j \in \{C : j \neq i\}, v \in M \quad (3.22)$$

$$u_i^v - u_j^v \leq -1 + (c+2)(1 - p_{ij}^v) \quad \forall i \in C, j \in \{C : j \neq i\}, v \in M \quad (3.23)$$

$$p_{ij}^v + p_{ji}^v \leq 1 \quad \forall i \in C, j \in \{C : j \neq i\}, v \in M \quad (3.24)$$

Constraints 3.25 cover the drone not leaving from another customer while it is still in the middle of a delivery. Figure 6 shows the unfeasible cases that are avoided by these constraints.

$$\tau_l \geq \tau_k - K \left(3 - \sum_{\substack{j \in C \\ (i,j,k) \in P \\ j \neq l}} y_{ijk}^{vd} - \sum_{\substack{m \in C \\ m \neq i \\ m \neq k \\ m \neq l}} \sum_{\substack{n \in N_+ \\ (l,m,n) \in P \\ n \neq i \\ n \neq k}} y_{lmn}^{vd} - p_{il}^v \right) \quad \forall i \in N_0, k \in \{N_+ : k \neq i\}, l \in \{C : l \neq i, l \neq k\}, (v, d) \in M \quad (3.25)$$

$$K = T_{max}$$

Constraints 3.26 ensure the vehicles load capacities are respected.

$$\sum_{i \in N_0} q_i \sum_{\substack{j \in C \\ j \neq i}} x_{ij}^v + \sum_{j \in C} q_j \sum_{\substack{i \in N_0 \\ i \neq j}} \sum_{\substack{k \in N_+ \\ (i,j,k) \in P}} y_{ijk}^{vd} \leq Q^v \quad \forall v \in M \quad (3.26)$$

Constraints 3.27 and 3.28 are symmetry breaking constraints. The first one ensures the trucks are used in order, while the second one assures the trucks visit order is ascending, i.e. a truck v ($v > 1$) can only start its route by visiting a customer which has greater index than the truck's $v-1$ first customer.

$$x_{0j}^v \leq x_{0j}^{v-1} \quad \forall j \in C, v \in M - \{0\} \quad (3.27)$$

$$x_{0j}^v \leq \sum_{\substack{l \in C \\ l < j}} x_{0l}^{v-1} \quad \forall j \in C, v \in M - \{0\} \quad (3.28)$$

The next constraints determines the start value and the domains of the variables.

$$t_0 = 0 \quad (3.29)$$

$$\tau_0 = 0 \quad (3.30)$$

$$p_{0j}^v = 1 \quad \forall j \in C, v \in M \quad (3.31)$$

$$x_{ij}^v \in \{0, 1\} \quad \forall i \in N_0, j \in \{N_+ : j \neq i\}, v \in M \quad (3.32)$$

$$y_{ijk}^{vd} \in \{0, 1\}$$

$$\forall i \in N_0, j \in \{C : j \neq i\}, k \in \{N_+ : (i, j, k) \in P\}, (v, d) \in M \quad (3.33)$$

$$1 \leq u_i^v \leq c + 2 \quad \forall i \in N_+, v \in M \quad (3.34)$$

$$a_i \leq t_i \leq b_i \quad \forall i \in N \quad (3.35)$$

$$a_i \leq \tau_i \leq b_i \quad \forall i \in N \quad (3.36)$$

$$p_{ij}^v \in \{0, 1\} \quad \forall i \in N_0, j \in \{C : j \neq i\}, v \in M \quad (3.37)$$

3.5 Additional Notes on the Model

The MILP developed has some characteristics that require further notes. For instance, the addition of classical symmetry breaking constraints (3.27 and 3.28) are possible only for the trucks' routes, since they are the only independent source. And the possible drones symmetry problems are very narrow, the main case is when there are different drones serving customers directly from the depot and returning.

The variables p_{ij}^v and u_i^v , unlike the classical models from literature, are added the vehicle index because the order of visit for each customer is relevant for the drone d linked with the truck v . The constraints 3.25 use the variable p to check whether the truck v visits i after l . The original version of this constraint works because in the FSTSP all customers are visited by the same vehicle (if not visited by drone, obviously), therefore, the variable always makes sense. However, in the VRPD, customer i and l may be from different routes and, thus, losing its original intent. By adding the vehicle index, the value of p is now guaranteed to have both customers i and l in the same route, if the case is so. This restriction is very important to the problem since it avoid the possibility of mixed drone routes, where the single drone would be launched more than once before returning to the truck (as shown in Figure 6). This change implicates in the constraint 3.24 being less than equal, rather than equal to 1, since i is not necessarily before j (or vice-versa).

Unlike the original model, a decision was made to allow the drone to return to the depot by itself, as seen in constraints 3.14 and 3.15 where the synchronous return point k contains only the customer set C . Considering that a drone which is not going to delivery to any other customer, then it should be okay for it to return to the depot. This way the

Figure 6 – Avoided scenarios

Source: Authors

drone could have its battery charged for the next operation and its basic maintenance taken care as soon as possible, whilst also allowing for more drone routes, as both the trucks and drones will not depend on the next customers' time-windows to synchronize arrivals.

Similarly to the original model, the objective function can be changed to minimize the time of return to the depot. Objective Function *II*: to minimize the time of return to the depot, hence minimizing all delivery times. This is the function used by (MURRAY; CHU, 2015).

$$\text{Min } t_{(c+1)} \quad (3.38)$$

In the next chapter (4), we present the HGA approach.

HYBRID GENETIC ALGORITHM

This chapter focuses on the Hybrid Genetic Algorithm (HGA) developed for the Vehicle Routing Problem with Drones (VRPD). Our HGA utilizes a combination of a Genetic Algorithm (GA) and a Local Search (LS). The GA is an evolutionary algorithm that start with several solutions which, as the name says, evolve into a better fit population; while the LS is a method in which one solution has one or more of its neighborhoods explored to find better results. The combination of GA and LS was first successfully used in solving a VRP by (PRINS, 2004). The version of the HGA done in this dissertation, however, is more similar and was inspired by the work of (VIDAL *et al.*, 2014), although they share similarities.

The HGA used follows the general guideline of a traditional GA: a initial population is created (each individual is represented by a chromosome); a selection within the population where two parents are chosen occurs; these parents are combined in a crossover operation; and finally the children may be altered in some way as part of the mutation process and are returned to the population. However, as in (VIDAL *et al.*, 2014), instead of a possible mutation step there is an education step. In the education step, the LS procedures are applied to the offspring in order to improve the next generation in a more efficient way. The education also contains the drone route decision method, a heuristic to find a good route for the drone.

4.1 HGA

[Algorithm 1](#) describes the structure of the HGA. The parameters for the algorithm are the size of the population n , the maximum number of iterations (number of generations and/or execution time) and the maximum size of the population, as it is of variable size. The algorithm start by generating the initial population, which is then evaluated (lines 3 and 4). The evaluation step temporarily stores in the memory the fitness of all the

chromosomes in a way that the selection process does not need to compute it again. Using that temporary memory, the selection process occurs (line 7) and two distinct parents are selected to perform the Crossover process (line 8). The crossover will return at most two offspring (already evaluated in the process), which will be passed to the Education step (line 9). The children then are added to the population. If the maximum size of the population is reached, the population is trimmed (line 12), i.e. the least fit individuals are removed from the population. The best individual is updated if needed in the final step (line 15). The procedure does this while the time limit or the maximum number of generations is not reached, returning the best individual. The main steps of the HGA are further elaborated in the next sections.

Algorithm 1 – Hybrid Genetic Algorithm structure

```

1: procedure GA( $n, MaxIter, MaxLen$ )
2:    $t \leftarrow 0$ 
3:    $population \leftarrow InitialPopulation(n)$ 
4:    $Evaluate(population)$ 
5:    $best \leftarrow GetBest(population)$ 
6:   while  $t \leq MaxIter$  do
7:      $parents \leftarrow Selection(population)$ 
8:      $children \leftarrow Crossover(parents)$ 
9:      $Education(children)$  ▷ Here the drone route is also selected
10:     $population \leftarrow children$ 
11:    if  $length(population) > MaxLen$  then
12:       $RemoveWorst(population)$ 
13:    end if
14:    if  $GetBest(population) \neq best$  then
15:       $best \leftarrow GetBest(population)$ 
16:    end if
17:     $t \leftarrow t + 1$ 
18:  end while
19:  return  $best$  ▷ Return the best individual
20: end procedure

```

The chromosome structure, which is a mix of the traditional single route without trip delimiters used in TSP problems combined with a second array with the drone information, is explained in the next section.

4.2 Chromosome Representation

One of the main concerns of any computational procedure is representing a real world solution computationally without loss of information. Solutions from mathematical models are very similar to real solutions. However, this is not the case for other types of solutions. For instance, in metaheuristics, usually there are generic procedures that

can be applied to several types of problems. These generic procedures, therefore, need to be adapted to the problem being solved. And with adaptation, usually comes a loss of information or precision in the data. The goal would be to minimize this loss of information without making it too difficult to use the metaheuristic's original generic format. This struggle was present when trying to represent VRPD solutions.

Traditional GA representations for VRP solutions such as binary values or routes with delimiters fails to account for drone deliveries. However, as in the work of (VIDAL *et al.*, 2012), the traditional schemes utilized could be altered in a way that fits the problem's particularities.

The chromosome for the VRPD's HGA is represented with a two by n matrix ($2 \times n$, n being the number of customers) where: the first row is a single route without delimiters, containing each customer in the order they should be visited; and the second row is a binary vector to determine if a customer is served by truck or drone. The corresponding customer from the first row will have the value 1 if it is served by the drone and 0 otherwise (as seen in Figure 7).

The single route without delimiters, although initially used for the TSP, can be used for VRP variants because it can be split into different routes. For example, in (PRINS, 2004) the author utilizes an algorithm of order $O(n^2)$ to split the routes by building an auxiliary graph and returning minimum cost paths. Based on that approach we built one split algorithm for the VRPD which is detailed next.

Figure 7 – Chromosome representation.

6	4	3	2	5	1	7	Route
0	1	0	1	0	0	1	Drone status

Source: Authors.

4.2.1 Split Procedure

We illustrate this split with Figure 8, where each route shows its own section of a single route chromosome. The customers from where drones launch or land are colored blue (just for customers' locations, the depot in the example is also a launch and landing point) and are served by the truck. The green circles represent the customers served by the drones. Black circles are supplied by the truck. The routes although disconnected here, can be joined with any permutation of those routes. This can lead to different chromosomes having the same fitness value, as they are the same routes, only in different order. The Split procedure recovers the trips from the single route chromosome by utilizing some characteristics from the problem's instance to improve computational performance. Firstly, we build a graph G given the problem input, the customers and their time-windows.

Figure 8 – Chromosome translation and split example.

Source: Authors.

G contains each customer as nodes, where the edges between them are present if, and only if, it is possible for going from one customer to another. For example, the time windows of two customers may be overlapping, and it might be impossible to visit them in sequence because of that. To calculate whether it is possible, we use the following inequation:

$$a_i + \text{travel\_time}(i, j) + S_i \leq b_j \quad (4.1)$$

If the Inequation 4.1 is false, then it is not possible to visit j after visiting customer i , because even if arriving at the earliest possible at i , the shortest time possible to arrive at j would be after its time window closes.

The graph G is built with every customer pair using the Inequation 4.1. This graph is independent of current solutions, therefore it is calculated before the start of the HGA. The graph G is the input, along the chromosome for the Split procedure, as seen in Algorithm 2. The algorithm consists of forming new routes as long as the next customer in the chromosome does not break the load capacity or the time window (line 11), including waiting for the window to open if necessary. When checking the customer or when storing the customer's identifier (lines 8, 10, 13, 14, 24), we refer to the first row of the chromosome matrix. However, both rows will be saved in the other cases. This algorithm has $O(n)$ complexity.

Algorithm 2 – Chromosome Split Algorithm

```

1: procedure SPLIT(chromosome, G, n)
2:   trips  $\leftarrow \{\}$ 
3:   trip  $\leftarrow \{0\}$ 
4:   current  $\leftarrow$  chromosome[0]
5:   load  $\leftarrow q_{\text{current}}$ 
6:   time  $\leftarrow$  travel_time(0, current)
7:   for each customer  $\in$  chromosome do
8:     if feasible(trip, customer) and G[current][customer] = 1 then
9:       trip  $\leftarrow$  customer
10:      load  $\leftarrow$  load + qcustomer
11:      time  $\leftarrow$  time + travel_time(current, customer)
12:      current  $\leftarrow$  customer
13:     else
14:       trip  $\leftarrow$  0 ▷ Current trip gets the ending depot
15:       trips  $\leftarrow$  trip
16:       trip  $\leftarrow \{0\}$ 
17:       trip  $\leftarrow$  customer
18:       time  $\leftarrow$  travel_time(0, customer)
19:       load  $\leftarrow q_{\text{customer}}$ 
20:       current  $\leftarrow$  customer
21:     end if
22:   end for
23:   return trips ▷ Return each route.
24: end procedure

```

This algorithm is greedy, thus it does not optimally split the chromosome. For example, in a case where the route of a drone consists in visiting a customer directly from the depot and back, the split is unlikely to return that as a solution. Therefore the HGA utilizes another auxiliary structure which saves the routes separately. This will happen when the Local Search finds better routes, and will be discussed in the Education section. Next, we specify how the initial population is created.

4.3 Initial Population

The efficiency of most heuristics is heavily dependent on the initial solution (or solutions, for population based heuristics). Since the algorithms explore the characteristics of current solutions, the better they are, the better the performance overall. To better exploit this concept, we created one initial population based on the sweep heuristic, completed with random solutions to add more variability.

We start the population by adding chromosomes generated by an adapted sweep heuristic, as seen in [Algorithm 3](#). The sweep heuristic was proposed by (WREN; HOLLIDAY, 1972) and it works by adding customers based on its angle when compared to the

depot. This aims to put customers that are around the same region in the same route, since customer with similar angles are supposedly closer to each other. It is a very simple heuristics that can return one route with less computational effort. The sweep used here does not take the drones into account, so it builds truck-only routes for the starting population. Since we want several routes, we change the starting angle for each route, avoiding duplicates.

Algorithm 3 – Sweep Algorithm

```

1: procedure SWEEP( $n$ )
2:    $angles \leftarrow calculate\_angles()$ 
3:    $sorted\_angles \leftarrow sort(angles)$ 
4:    $population \leftarrow \{\}$ 
5:    $start \leftarrow 0$ 
6:    $it \leftarrow 0$ 
7:   while  $it \leq InitSize$  do
8:      $route \leftarrow \{\}$ 
9:      $visited \leftarrow \{\}$ 
10:     $route \leftarrow sorted\_angles[start]$ 
11:     $visited \leftarrow route[0]$ 
12:     $load \leftarrow q_{route[0]}$ 
13:     $current \leftarrow start$ 
14:     $time \leftarrow travel\_time(depot, start)$ 
15:     $next \leftarrow current + 1$ 
16:    while  $length(visited) \leq n$  do
17:      if  $next \notin visited$  then
18:        if  $time + travel\_time(current, next) \leq b_{next}$  and  $load + q_{next} \leq Q$  then
19:           $time \leftarrow time + Max(travel\_time(current, next), a_{next})$ 
20:           $load \leftarrow load + q_{next}$ 
21:           $route \leftarrow next$ 
22:           $current \leftarrow next$ 
23:           $visited \leftarrow current$ 
24:        end if
25:      end if
26:      if  $next \leq length(sorted\_angles)$  then
27:         $next \leftarrow next + 1$ 
28:      else
29:         $next \leftarrow 0$ 
30:      end if
31:    end while
32:     $population \leftarrow route$ 
33:     $start \leftarrow start + 1$ 
34:     $it \leftarrow it + 1$ 
35:  end while
36:  return  $population$ 
37: end procedure

```

▷ Return the initial population

Since we want to diversify our population, some customers are generated by the

random algorithm 4. It simply returns non-duplicates of random permutation of customers. The difference here is that we allow drones to serve customers. The parameter *drone_served* is the probability in which each drone-eligible customer has of being served by a drone. Given this initial population, the next step consists of selecting the individuals to perform a crossover.

Algorithm 4 – Random Chromosome Algorithm

```

1: procedure RANDOM(max, n, population, drone_served)
2:   while length(population) < max do
3:     random  $\leftarrow$  random_permutation(n, drone_served)
4:     if random  $\notin$  population then
5:       population  $\leftarrow$  random
6:     end if
7:   end while
8:   return population    ▷ Return the initial population added with random routes.
9: end procedure

```

4.4 Crossover and Selector Operators

The crossover step is very important for the performance of a GA. If the crossover function is too simple, the population might take longer to evolve. And if it is too complex, it might take too much time to compute or could make the population converge too quickly. However, it is maybe even more crucial to the growth (fitness-wise) of the population to select which parents will breed. Therefore, before presenting the crossover method, we introduce the selection process.

There are several selection methods utilized in GA literature: Tournament, roulette wheel, best fit, to name a few. The selection method we chose is the traditional tournament without replicates. This method selects *n* individuals from the population at random, and the best two of these are selected to become the parents. In our runs, the value of *n* is 10% of the population size.

Having both parents selected, we can cross them with the chosen method of crossing: order crossover (OX). This method is well suited for TSP and VRP solutions, because it keeps some of the current route formation intact, meaning good routes can be passed on to new generations. It consists of isolating one random continuous sub-string out of one parent, ranging from 2 to the length of the string minus 1 (*length(parent)* – 1), and completing this string with the rest of the other parent positions circularly, starting at the same position as the first parent’s sub-string end. An example can be seen in [Figure 9](#). This results in 2 children being created, each with one exact copy of a sub-string of one parent.

Figure 9 – OX Crossover example.

					i=4		j=6					
					↓		↓					
P1	:	1	3	2		6	4	5		9	7	8
P2	:	3	7	8		1	4	9		2	5	6
C1	:	8	1	9	6	4	5	2	3	7		
C2	:	2	6	5	1	4	9	7	8	3		

Source: (PRINS, 2004).

The offspring may not always be feasible, since the crossover can lead to two different customers who are served by drone to be adjacent to each other. To fix unfeasible solutions and as well as to improve each child, both children are sent to the Education step.

4.5 Education

In this section the Education process is described. Here unfeasible routes are fixed, a new heuristic for determining a drone launch and landing point is applied, and a local search is performed to improve quality. These steps are not done one after another though, as one local move can make drones routes infeasible, therefore, they all occur several times.

Traditional mutation, for example simple customer swaps or similar moves, is not good enough for more complex structures such as VRP routes, especially with the added complexity of time windows and drone service. As shown in (PRINS, 2004), the addition of local search improves greatly the quality of GAs. Our education step contains LS algorithms that improve both truck's and drone's routes, as well as inter and intra routes.

The education step can be summarized with Algorithm 5. This step is performed for all the children, but the algorithm is invoked individually for each one. First, Split (Algorithm 2) is called to recover the routes from the single route chromosome (line 2). Then the child is evaluated to get its current fitness (line 3) and is duplicated for later comparison (line 4). Afterwards, the first inter route neighborhoods are applied in random order: route merge and route split (line 5). We call it the strong inter routes LS because they alter a route dramatically. The route merge algorithm tries to combine two different routes into one single new route by concatenating them (as seen in Figure 11). The split route algorithm tries to separate one route into two different ones (Figure 12). However, all route changes (including the next LS neighborhoods) are only applied if they improve the current solution, and if so, the current solution value is updated. Figure 10 illustrates this concept.

Figure 10 – Flow graph for the LS process.

Source: Authors.

Algorithm 5 – Education Step

```

1: procedure EDUCATION(child)
2:   routes  $\leftarrow$  Split(child)
3:   current_fitness  $\leftarrow$  Evaluate(child)
4:   current  $\leftarrow$  current_fitness
5:   routes, current  $\leftarrow$  strong_inter_LS(routes, current)
6:   for each Route R  $\in$  routes do
7:     R, current  $\leftarrow$  drone_neighborhood(R, current)
8:     for each Route S  $\in$  routes, S  $\neq$  R do
9:       S, current  $\leftarrow$  soft_inter_LS(R, S, current)
10:    end for
11:    R, current  $\leftarrow$  intra_route_LS(R, current)
12:  end for
13:  if current  $\geq$  current_fitness then
14:    child, routes  $\leftarrow$  string_neighborhood(child, current_fitness)
15:  end if
16:  save_route_in_memory(child, routes)
17:  return child ▷ The child can return without any modifications.
18: end procedure
  
```

In the next step, we iterate over the routes (line 7) and try to add more drone-served customers (line 8) with a simple LS that tries to serve drone-eligible customers (example in Figure 13), before trying to explore the other inter route neighborhood. The next set of inter route LS try to improve quality by performing simpler swaps. We apply three searches also in random order: the first one, the two-swap*, swaps two consecutive customers from two different routes (Figure 14); the second one, two-swap-exchange*,

Figure 11 – Merge routes example. Here the route 726 is added to the end of the route 5314.

Source: Authors.

Figure 12 – Split routes example. The first route 5314726 is split into two: 531 and 4726

Source: Authors.

is very similar to the two-swap but also changes the customers' order (Figure 15); the third one simply tries to swap two customers between the routes, also known as two-opt* (Figure 16). In the next step the intra route neighborhood is explored: the classical two-opt, where two customers are swapped (illustrated in Figure 17); a two-swap move, where two consecutive customers swap places with another two consecutive customers (Figure 18); and the last one, two-swap-exchange, works the same way the two-swap, but additionally the customers' have their sequence swapped (Figure 19).

Finally, with all the neighborhoods explored we check whether the new solution is worse than the initial one (line 13). If there was no improvement, i.e. a local minimum, the string neighborhood (line 14) will perform the same changes (also in random order) that the intra-route algorithms did, but considering the whole chromosome instead of just a single route. This is a more expensive (computationally) LS, since we need to check the

Figure 13 – Drone Local Search example, where the customer 7 is changed from truck-served to drone-served.

Source: Authors.

Figure 14 – An example of the two-swap* move.

Source: Authors.

feasibility within all routes after any move, but it is a good way to escape local minima and diversify the population when needed. Even so, since most chromosomes will not enter this case, the extra computational cost is barely relevant, and if the child comes out of this step unchanged, then there is likely a chance we have a very strong individual, potentially a global optimum.

The next step for both cases is saving the routes and the child in the local memory. This way, even if the Split algorithm does not return the same routes, we do not lose them. The return of the algorithm is the resulting child chromosome, which may or not have been altered, and can be added to the population in the next steps of the HGA.

Now we present the method which calculates the launching and landing points for

Figure 15 – An example of the two-swap-exchange* move.

Source: Authors.

Figure 16 – An example of a two-opt* move.

Source: Authors.

each drone.

4.5.1 Drone Route Method

Here we introduce our heuristic to find good launching and landing points for the drones. Since we have a chromosome that does not contain that information, each time we evaluate a route we need to calculate those points. And since it is done several times within an execution, it needs to be fast and reliable.

The chosen approach, as shown in Algorithm 6 and illustrated in Figure 20, takes one route as input (line 1) and explore all drone-served customers (line 2). The first

Figure 17 – An example of a two-opt move.

Source: Authors.

Figure 18 – An example of the two-swap move.

Source: Authors.

Figure 19 – An example of the two-swap-exchange move.

Source: Authors.

customer (highlighted in red in the figure) will have all possible launch and landing sites explored (line 4). First, we calculate if it is feasible for the drone to leave from the previous nodes. In the example from Figure 20, the depot and customer 4 are analyzed. Then, we verify whether the drone can land in any of the later nodes which are not drone-served, customer 3 in the figure. We choose the best feasible route among those to determine the launch and land sites, and since we want to minimize the truck's route, the longest drone's route will reduce the truck's route the most. In the example, we picked the second pair

($6 \rightarrow 4 \rightarrow 3$), and temporarily set the statuses of those two customers as drone launch (line 12) and land (line 13), represented as green zeros in the figure. We repeat this process for the next drone-served customer, in example 2. Both pairs of customers are then explored, (3,5) and (3,1), with the latter being chosen. For the last drone-served customer 7, the only possibility remaining is the pair (1,0) because the drone cannot depart from client 5 since it already lands on customer 1. If none of the pairs are feasible, we remove the customer from the drone-served status (line 15).

Algorithm 6 – Drone Route Heuristic

```

1: procedure DRONE ROUTE(route)
2:   for each Drone-Served Customer  $C \in route$  do
3:      $best \leftarrow 0$ 
4:     for each Route Pair  $P = \{c_1, c_2\}$  (where  $c_1$  is any node visited before  $C$  but
       after any previous selected drone launch or landing point, and  $c_2$  is any node visited
       after  $C$  but before any customer  $c_3$  which is Drone Served) do
5:       if  $best \geq route\_length(c_1, c_2)$  and  $route(c_1, c_2)$  is feasible then
6:          $best \leftarrow route\_length(c_1, c_2)$ 
7:          $launch \leftarrow c_1$ 
8:          $land \leftarrow c_2$ 
9:       end if
10:    end for
11:    if  $best \neq 0$  then
12:       $route[launch] \leftarrow drone\_launch\_status$ 
13:       $route[land] \leftarrow drone\_landing\_status$ 
14:    else
15:       $route[C] \leftarrow remove(drone\_status)$   $\triangleright$  The customer is no longer served by
        drone.
16:    end if
17:  end for
18:  return  $length(route)$   $\triangleright$  Return the length of the route with existing drone
    services.
19: end procedure

```

Figure 20 – Drone Heuristic visualization.

Source: Authors.

RESULTS

In this section, the computational experiments with their results are shown. The tests are run in a computer cluster with the following configuration: two Intel Xeon E5-2680v2 2.8 GHz processors and 128 GB of RAM, using a total of up to 40 threads. The MILP model was implemented using Python version 3.5.4 and the IBM CPLEX Python API version 12.8. The HGA is also implemented with the same Python version.

5.1 Test Instances

In order to test both the MILP model and the Genetic Algorithm, the Solomon's dataset ([SOLOMON, 1987](#)) instances were used. The Solomon's dataset is suitable for testing VRP algorithms that have time-windows, since it varies the length of the time-window from really tight to more relaxed ones, which are harder to solve. The dataset also has clustered, non-clustered and mixed (both clustered and non-clustered) geographical locations for customers being the classes C, R and RC, respectively, which helps to see how do the algorithms behave with such types of customer's location. The class C has a total of 17 instances, the class R has a total of 23 and the RC has 16, each divided into two sub-groups, where the first (aka C1, R1 and RC1) has a shorter time window, whereas the second group (C2, R2 and RC2) has a long schedule horizon.

For all the dataset the number of customers is set to 100. However, it is very common to divide these into subsets of 25 and 50 customers (taking the first 25 or first 50 customers from the original one, respectively), which work better for more complex versions of the VRPTW. We will be utilizing the 25 subset for testing our proposed MILP, and both the 25 and original 100 for testing the HGA. The next subsections elaborates on the parameters changed overall and the ones specifics to each solution methods.

5.1.1 Overall Parameters

Here the parameters utilized for both the MILP and the HGA are described. All tests are executed with a time limit of 1 hour, not including pre and post processing.

One of the most relevant parameter for testing the VRPD would be the set of drone-eligible customers. Since the Solomon's instances have different weights metrics between each instance type, it was developed the [Algorithm 7](#) to select customers rather than modify all of the instances to fit the particularities of the VRPD (the drones lower weight capacity). On this procedure, the number of customers is not taken into account directly, but rather the number of weights. The procedure takes the list of customers and the chosen percentage in which the calculations will be based upon (line 1). We remove all duplicates weights from the customer list (line 2). And then, by sorting the unique weights of all customers (line 3), each position of the ordered list will cover several customers. The position for each instance is based on the *percentage* parameter, and by choosing one of these positions (line 4), the set of drone-eligible customers can be defined by selecting every customer that has a parcel with weight less or equal than the one chosen (lines 7 – 9). For example, for a list of 6 customers' weights $L = [2, 2, 4, 6, 11, 20]$, the list without duplicates would be $L' = [2, 4, 6, 11, 20]$, and for a 50% drone-eligible customer rate, then the procedure from [Algorithm 7](#) would return a list $D = [2, 4, 6]$, since these are the customers that have a weight equal to or lower than the one in the 50% position in the list L' (rounded-up), in this case, every customer with weight less than or equal to 6 will be selected. This way, the procedure does not depend on the number of customers. In all our tests we use three different drone-eligible values: 25%, 50% and 75%.

Algorithm 7 – Weight Classification Algorithm

```

1: procedure WEIGHT_CLASSIFICATION(customers, percentage)
2:   unique_weights  $\leftarrow$  remove_duplicates(customers)
3:   ordered_list  $\leftarrow$  sort(unique_weights)
4:   position  $\leftarrow$   $\lceil \text{length}(\text{ordered\_list}) \times \text{percentage} \rceil$ 
5:   i  $\leftarrow$  0
6:   drone_eligible  $\leftarrow$  {}
7:   while i < length(customers) do
8:     if customer[i]  $\leq$  ordered_list[position] then
9:       drone_eligible  $\leftarrow$  customer[i]
10:    end if
11:    i  $\leftarrow$  i + 1
12:  end while
13:  return drone_eligible ▷ Return the drone-eligible customers
14: end procedure

```

Other important parameters are the travel speed of both the drone and truck. As a general rule, when using Solomon's dataset, the truck speed is set to 1, and all major works utilize that norm. Therefore we also set the truck's speed to 1 and changed the

drone's speed as a relation to the truck's: half the speed, or 0.5; same speed, also 1; and half as fast, with 1.5.

As for the drone endurance, since there are different instances types with varying distances between customers, we chose to not use a single endurance value, even though drone models usually have fix flight times. What we did instead was to come up with a formula relating the average distance between customers with an addition to the standard deviation for the distance average, shown in Equation 5.1. Therefore the endurance is dependent on the instance, and can cover a significant amount of customers. This formula when applied to the dataset, had an approximate value to what most commercial drones have which is around 30 minutes of flight, whereas our tests result in value between 25 to 40 units of time, depending on the instance. Additionally, the preparation time for launching (S_L) and for landing (S_R) are set to 10% of the endurance value.

$$endurance = \mu_{distances} + \sigma_{distances} \quad (5.1)$$

The last parameter is the distance metrics for each type of vehicle. Traditionally, studies use Euclidean distance for the vehicles, since it is a simplification of the real world. And for the sake of comparison, both drones and trucks will be using Euclidean on our tests.

Table 2 – General parameters for the tests. Default parameters are in bold.

Parameter	Values
Time limit	3600s
Drone eligible weight	25%, 50% , 75%
Drone travel speed	50%, 100% , 150%
Drone endurance	$\mu_{distances} + \sigma_{distances}$
Distance metrics	Euclidean

Source – Authors

Table 2 summarizes all test parameters. Since this combination of parameters would require a minimum of 9 runs for all instances (more for the HGA, since we want an average), we defined a default parameter configuration to compare the results. Except for the results of the MILP + HGA, which are run in all possible cases, all other use the following values: 50% for drone eligible weight and 100% for drone speed. In the MILP we also use inequation 4.1 to avoid creating unnecessary variables.

5.1.2 CPLEX Parameters

The parameters used for solving the MILP were not set to the default values and, hence, we explain them here. The first parameter set was the number of threads used

for computation. As we had available a robust cluster, we set the number of threads to be used for the MILP calculations to be 40. To improve the high amount of threads usage, we also set an parameter of CPLEX that exploits better concurrency use, i.e. the Opportunistic and Concurrent strategy.

Since the problem was initially taking too much time on node 0, we also changed the parameter that tries to spend less time on each node by branching based on a pseudo cost algorithm, rather than the exact solution. Additionally, the frequency in which the heuristics were applied was set to 0, so that to leave the starting node was quicker.

Finally, since to get any feasible solution was hard for a lot of instances, we set the emphasis in feasibility, rather than optimality.

5.1.3 Hybrid Genetic Parameters

As described in [Chapter 4](#), the HGA contains few parameters that can be changed. The main ones are the number of generations and the population size. We set these as a function of the number of customers, being the generations six times the amount, and the population with the same size. We summarize the parameters and their value in [Table 3](#).

Table 3 – Parameters for the HGA.

Parameter	Value
Time limit	3600s
Population Size	n
Max Population Size	$n \times 2$
Number of Generations	$n \times 6$
Drone served (Random Chromosome)	50%
Stop Criteria	<i>TL or Generations Limit</i>

Source – Authors

The other parameter present in [Table 3](#) is the chance in which the customers have to be served by drone, that our [Algorithm 4](#) uses. Unlike other GAs, there is no mutation parameter, as we always perform the education step for the offspring.

5.2 MILP Results

In this section we present the results for the MILP formulation. There were two approaches to find feasible solutions to the problem. The first one utilizes the Sweep algorithm ([Algorithm 3](#)) to find truck-only routes and then CPLEX would improve the routes and add the drones. The second approach was to utilize the result of the HGA ([Algorithm 1](#)) to start with a good solution as a boost for the solver. It is important to

note that without the initial solution given, the CPLEX would reach time limit without any feasible solutions for most instances.

We present the results in the following scheme: the instance name, which are the original Solomon instance names added by a 25 after the instance identifier, since all MILP results are only available to these instances; the objective function [Equation 3.1](#); the GAP given by CPLEX; the dual objective; execution time in seconds; number of explored nodes during execution; the number of variables in the problem; and the number of constraints generated.

Table 4 – Results for the class of instances C, using the Sweep Algorithm as a starting solution.

Instance	Objective Function	Dual	GAP (%)	Exec. Time	Nodes	Variables	Constraints
c25101	181.46	181.45	0.0	105.48	12672	52255	522001
c25102	637.95	107.57	83	TL	1	70555	543926
c25103	502.13	79.80	84	TL	2	75805	551551
c25104	389.05	79.64	80	TL	0	81555	558176
c25105	183.64	139.18	24	TL	95215	57705	528126
c25106	177.90	177.90	0.0	TL	26929	53805	523651
c25107	418.45	107.37	74	TL	44975	58780	529701
c25108	469.70	26.00	94	TL	6841	64480	536776
c25109	436.62	22.77	95	TL	1	69380	542951
c25201	201.87	201.87	0.0	297.93	65329	53905	524251
c25202	589.37	96.01	84	TL	9	63405	537376
c25203	550.19	53.52	90	TL	1	71830	547951
c25204	461.24	46.91	90	TL	0	75505	552576
c25205	547.23	97.57	82	TL	49671	59705	531001
c25206	438.65	46.10	89	TL	0	62105	534001
c25207	445.57	54.72	88	TL	1	66880	539976
c25208	398.32	46.10	88	TL	0	64630	537101
Average	-	-	67	-	-	64840.29	537711.29

5.2.1 MILP + Sweep

The first results are from the MILP execution with the Sweep algorithm providing the initial solution. [Table 4](#), [Table 5](#) and [Table 6](#) summarize the data from the 3 classes, C, R and RC, respectively. As seen in the tables, the number of variables and constraints is large for an instance of just 25 customers, averaging around 60000 variables and 530000 constraints, which reflects in the large gaps present. The excess of "Big M" constraints also complement to the great discrepancy in the dual bound. Overall, the model is hard to solve, with many instances not leaving the root node in the time limit of 1 hour.

[Table 7](#), [Table 8](#) and [Table 9](#) compares the results from the MILP, with the Sweep algorithm providing an initial solution, with the optimal value from the VRPTW. As seen, the sweep is not a good starting point for the VRPD. Most values are far worse than the VRPTW optima, where the solutions from classes C and RC are on average more than the double of the optimal. Although this comparison is unfair, since the VRPD are

Table 5 – Results for the class of instances R, using the Sweep Algorithm as a starting solution.

Instance	Objective Function	Dual	GAP (%)	Exec. Time	Nodes	Variables	Constraints
r25101	515.19	515.19	0.0	59.77	34658	31680	490126
r25102	773.97	202.38	74	TL	102	42180	505426
r25103	678.39	182.06	73	TL	12	46730	512751
r25104	582.78	167.97	71	TL	1	49730	517126
r25105	390.39	390.38	0.0	1055.45	27032	37130	497501
r25106	782.37	202.38	74	TL	5	45030	509501
r25107	617.44	182.06	71	TL	1	47880	514551
r25108	594.70	167.97	72	TL	1	50230	518101
r25109	689.29	227.88	67	TL	4477	41030	504176
r25110	563.54	175.23	69	TL	6	47455	513551
r25111	653.14	182.06	72	TL	5	46780	512851
r25112	522.93	167.97	68	TL	2	51680	520776
r25201	449.79	333.13	26	TL	60396	39505	502326
r25202	635.44	202.38	68	TL	224	46505	512401
r25203	612.00	182.06	70	TL	45	49080	516926
r25204	536.87	167.97	69	TL	87	51005	519626
r25205	565.43	228.94	60	TL	752	43405	508526
r25206	579.00	178.29	69	TL	10	48130	515201
r25207	548.74	167.97	69	TL	32	49655	517826
r25208	533.49	167.97	69	TL	4	51180	519901
r25209	512.67	192.50	62	TL	218	46830	513401
r25210	605.60	209.00	65	TL	147	47380	514301
r25211	475.78	167.97	65	TL	3	51705	520826
Average	-	-	65	-	-	46170.22	514419.75

mostly not in their optimal and all VRPTW are. Nevertheless, some instances still have an improvement, showing the potential of the application. Given the poor performance with a bad initial solution, we can assume that such simplistic heuristics is not enough for such a complex model. Hence a more elaborated method, such as our proposed HGA, is necessary to find good solutions. We show those results next.

5.2.2 MILP + HGA

The next set of results, [Table 10](#), [Table 11](#) and [Table 12](#), show the same tests aforementioned, but with the HGA providing the initial solution. Even though there is still a large gap to the lower bound (dual), overall solutions improved, which becomes more noticeable when comparing to the VRPTW. This comparison is shown in [Table 13](#), [Table 14](#) and [Table 15](#), where we see that in fact there is a reduction in cost (distance), averaging a decrease of around 7%, despite several of the VRPD solutions not being the optimal. It is important to note that since we are minimizing total distance traveled by the trucks, by using drones to serve some customers, then it is expected that the VRPD will outperform the classical VRPTW. Our goal is to show by how much it does.

The improvements for these instances are also less significant because there is little room for improvement with fewer customers and fewer possible routes. In the next section

Table 6 – Results for the class of instances RC, using the Sweep Algorithm as a starting solution.

Instance	Objective Function	Dual	GAP (%)	Exec. Time	Nodes	Variables	Constraints
rc25101	869.38	151.38	83	TL	4	49880	515826
rc25102	764.73	47.07	94	TL	2	61055	531101
rc25103	622.29	38.05	94	TL	1	67955	540626
rc25104	573.94	29.73	95	TL	0	71705	545526
rc25105	875.21	48.59	94	TL	1	58630	526951
rc25106	695.34	45.93	93	TL	1	59580	528326
rc25107	747.24	29.68	96	TL	0	69805	541701
rc25108	587.71	29.68	95	TL	0	73455	547926
rc25201	722.92	187.89	74	TL	26791	53230	522301
rc25202	794.62	47.08	94	TL	11	62905	535026
rc25203	738.73	39.54	95	TL	1	69680	543651
rc25204	519.67	29.74	94	TL	3	72655	547401
rc25205	738.90	125.18	83	TL	2	61180	532326
rc25206	583.21	53.54	91	TL	2	61130	532501
rc25207	539.85	33.54	94	TL	0	68830	541926
rc25208	454.11	29.68	93	TL	0	74505	549826
Average	-	91	-	-	-	64761.25	536433.81

Table 7 – A comparison between the result of the MILP with Sweep as initial solution and the optima from the VRPTW for the class C.

Instance	MILP + Sweep	VRPTW Optimal	GAP (%)
c25101	181.46	191.3	-5.14
c25102	637.95	190.3	235.23
c25103	502.13	190.3	163.86
c25104	389.05	186.9	108.16
c25105	183.64	191.3	-4.00
c25106	177.9	191.3	-7.00
c25107	418.45	191.3	118.74
c25108	469.7	191.3	145.53
c25109	436.62	191.3	128.24
c25201	201.87	214.7	-5.98
c25202	589.37	214.7	174.51
c25203	550.19	214.7	156.26
c25204	461.24	213.1	116.44
c25205	547.23	214.7	154.88
c25206	438.65	214.7	104.31
c25207	445.57	214.5	107.72
c25208	398.32	214.5	85.70
Average	-	-	104.56

we show that with more customers, the improvements are more substantial. Table 16 compares how the change in speed affects the performance of the model. We see that with greater speeds better results, which is expected, since faster drones can cross the same distances in less time, allowing for more possible routes. While the other way around works just the opposite: a slower drone will barely benefit the overall distance. It is good to remember that these results are not from optimal values, therefore, in the worst best case (where we have all optimum), the optimal route would have a gap of 0%, since the trucks would perform the same route if the drones would not improve anything. This is

Table 8 – A comparison between the result of the MILP with Sweep as initial solution and the optima from the VRPTW for the class R.

Instance	MILP + Sweep	VRPTW Optimal	GAP (%)
r25101	515.19	617.1	-16.51
r25102	773.97	547.1	41.47
r25103	678.39	454.6	49.23
r25104	582.78	416.9	39.79
r25105	390.39	530.5	-26.41
r25106	782.37	465.4	68.11
r25107	617.44	424.3	45.52
r25108	594.7	397.3	49.69
r25109	689.29	441.3	56.20
r25110	563.54	444.1	26.89
r25111	653.14	428.8	52.32
r25112	522.93	393	33.06
r25201	449.79	463.3	-2.92
r25202	635.44	410.5	54.80
r25203	612	391.4	56.36
r25204	536.87	355	51.23
r25205	565.43	393	43.88
r25206	579	374.4	54.65
r25207	548.74	361.6	51.75
r25208	533.49	328.2	62.55
r25209	512.67	370.7	38.30
r25210	605.6	404.6	49.68
r25211	475.78	350.9	35.59
Average	-	-	39.79

true even for slower drones. The drawback from the increase in speed, is a significant increase in model complexity, as it will have more variables.

On the other hand the drone eligible weight parameter does not seem to change the results by a significant amount, as seen in [Table 17](#). Even though the addition of new customers may also add new route possibilities for the drones, the problem also becomes more hard to solve due to the addition of new variables, so the optimal values could have an slight improvement with higher percentage of customers. For all other instance configurations (shown in [Table 2](#)) the results are available at [Appendix B](#). The instances with 100 customers, however, are not solvable by the MILP model, since the system run out of memory beforehand. In the next section we present the results for such instances by using our HGA.

5.3 HGA Results

This section presents the results obtained with our Hybrid Genetic Algorithm. All Solomon instances were run in their original size, with 100 customers, and the designed default parameters to the VRPD (presented previously in [subsection 5.1.1](#): 50% drone eligible weight; and 100% speed of drone when compared to the truck). [Table 18](#), [Table 19](#)

Table 9 – A comparison between the result of the MILP with Sweep as initial solution and the optima from the VRPTW for the class RC.

Instance	MILP + Sweep	VRPTW Optimal	GAP (%)
rc25101	869.38	461.1	88.54
rc25102	764.73	351.8	117.38
rc25103	622.29	333.1	86.82
rc25104	573.94	306.6	87.20
rc25105	875.21	411.3	112.79
rc25106	695.34	345.5	101.26
rc25107	747.24	298.3	150.50
rc25108	587.71	294.5	99.56
rc25201	722.92	360.2	100.70
rc25202	794.62	338	135.09
rc25203	738.73	326.9	125.98
rc25204	519.67	299.7	73.40
rc25205	738.9	338	118.61
rc25206	583.21	324	80.00
rc25207	539.85	298.3	80.98
rc25208	454.11	269.1	68.75
Average	-	-	101.72

Table 10 – Results for the class of instances C, using the HGA as a starting solution.

Instance	Objective Function	Dual	GAP (%)	Exec. Time	Nodes	Variables	Constraints
c25101	181.46	181.46	0.0	257.58	21039	52255	521377
c25102	189.99	107.47	43	TL	5	70555	543302
c25103	188.12	79.61	58	TL	0	75805	550927
c25104	182.73	79.61	56	TL	0	81555	557552
c25105	183.50	133.05	27	TL	9246	57705	527502
c25106	177.90	177.90	0.0	785.24	20137	53805	523027
c25107	184.10	106.66	42	TL	91	58780	529077
c25108	184.10	31.00	83	TL	3	64480	536152
c25109	184.10	22.77	88	TL	1	69380	542327
c25201	201.87	201.87	0.0	44.92	1231	53905	523627
c25202	205.46	96.19	53	TL	58	63405	536752
c25203	207.47	53.52	74	TL	2	71830	547327
c25204	206.57	46.91	77	TL	0	75505	551952
c25205	207.47	97.25	53	TL	4	59705	530377
c25206	205.11	46.83	77	TL	2	62105	533377
c25207	205.73	54.30	74	TL	1	66880	539352
c25208	204.94	46.47	77	TL	1	64630	536477
Average	-	-	52	-	-	64840.29	537087.29

and Table 20 show, for each of the three classes of instances, C, R and RC, respectively, the results from a total of 5 runs. The tables present: the average fitness for all the initial and the final populations, which show that the convergence happens within close grasp from the best solution; the average best solution and how it is compared to the best known solution for the VRPTW ¹; the best solution found in the 5 runs and again how it compares to the VRPTW; and, finally, the average execution time in seconds.

¹ Available at <https://www.sintef.no/projectweb/top/vrptw/solomon-benchmark/100-customers/>

Table 11 – Results for the class of instances R, using the HGA as a starting solution.

Instance	Objective Function	Dual	GAP (%)	Exec. Time	Nodes	Variables	Constraints
r25101	515.19	515.19	0.0	20.39	490	31680	489502
r25102	492.87	208.23	58	TL	792	42180	504802
r25103	430.94	182.06	58	TL	1	46730	512127
r25104	391.60	167.97	57	TL	1	49730	516502
r25105	398.99	346.28	13	TL	60980	37130	496877
r25106	439.75	202.38	54	TL	2	45030	508877
r25107	385.65	182.06	53	TL	1	47880	513927
r25108	361.20	167.97	53	TL	3	50230	517477
r25109	379.48	223.52	41	TL	2087	41030	503552
r25110	361.28	175.23	51	TL	1	47455	512927
r25111	385.65	182.06	53	TL	2	46780	512227
r25112	335.70	169.03	50	TL	4	51680	520152
r25201	379.52	345.00	09	TL	74931	39505	501702
r25202	382.74	202.38	47	TL	8	46505	511777
r25203	353.81	182.06	49	TL	1	49080	516302
r25204	334.21	167.97	50	TL	2	51005	519002
r25205	358.38	239.08	33	TL	6449	43405	507902
r25206	329.48	178.29	46	TL	1	48130	514577
r25207	314.74	167.97	47	TL	6	49655	517202
r25208	298.05	167.97	47	TL	0	51180	519277
r25209	316.44	192.50	39	TL	2	46830	512777
r25210	364.39	209.00	43	TL	9	47380	513677
r25211	313.44	167.97	46	TL	1	51705	520202
Average	-	-	42	-	-	46170.22	537087.29

From the results, we can observe that the improvement is more significant, with around 20% decrease in the total traveled distance. This is very motivational for companies that are interested in drone applications, as a reduction of this caliber to a large fleet can have a tremendous impact economically. This also implicates in significantly less greenhouse gas emissions and probably even for customer satisfaction, with faster deliveries.

Figure 21, Figure 22 and Figure 23 illustrates the convergence of the HGA with one example from each class (C, R and RC). The part (a) of each figure is the initial population, where we can clearly see the half sweep based and the half random based; in part (b) we see how the final population converged, with the first individuals being the better ones; part (c) is a box plot of the convergence, where the variance is clearly decreasing with each generation; and finally, a line plot (d) that shows the evolution of the average and the best individual. Specially in the part (d) of the graphs, we can see that the HGA evolves the most within the first 100 generations. Therefore, it is possible to have a reasonable to good solution even for cases where there is limited execution time.

Table 12 – Results for the class of instances RC, using the HGA as a starting solution.

Instance	Objective Function	Dual	GAP (%)	Exec. Time	Nodes	Variables	Constraints
rc25101	427.12	143.25	66	TL	4	49880	515202
rc25102	368.94	47.07	87	TL	0	61055	530477
rc25103	297.69	38.05	87	TL	1	67955	540002
rc25104	276.32	29.73	89	TL	0	71705	544902
rc25105	452.32	48.59	89	TL	2	58630	526327
rc25106	269.41	45.93	83	TL	1	59580	527702
rc25107	257.94	29.68	88	TL	0	69805	541077
rc25108	260.16	29.68	89	TL	0	73455	547302
rc25201	351.48	226.79	35	TL	2416	53230	521677
rc25202	329.98	47.29	86	TL	9	62905	534402
rc25203	309.62	38.05	88	TL	1	69680	543027
rc25204	282.72	29.74	89	TL	0	72655	546777
rc25205	376.56	121.57	68	TL	1	61180	531702
rc25206	302.47	53.54	82	TL	1	61130	531877
rc25207	275.61	33.54	88	TL	0	68830	541302
rc25208	234.03	29.69	87	TL	0	74505	549202
Average	-	-	81	-	-	64761.25	535809.81

Table 13 – A comparison between the result of the MILP with HGA as initial solution and the optima from the VRPTW for the class C.

Instance	MILP + HGA	VRPTW Optimal	GAP (%)
c25101	181.46	191.30	-5.1
c25102	189.99	190.30	-2.0
c25103	188.12	190.30	-1.1
c25104	182.73	186.90	-2.2
c25105	183.50	191.30	-4.0
c25106	177.90	191.30	-7.0
c25107	184.10	191.30	-3.8
c25108	184.10	191.30	-3.8
c25109	184.10	191.30	-3.8
c25201	201.87	214.70	-6.0
c25202	205.46	214.70	-4.3
c25203	207.47	214.70	-3.4
c25204	206.57	213.10	-3.1
c25205	207.47	214.70	-3.4
c25206	205.11	214.70	-4.5
c25207	205.73	214.50	-4.1
c25208	204.94	214.50	-4.5
Average	-	-	-3.89

Table 14 – A comparison between the result of the MILP with HGA as initial solution and the optima from the VRPTW for the class R.

Instance	MILP + HGA	VRPTW Optimal	GAP (%)
r25101	515.19	617.10	-16.51
r25102	492.87	547.10	-9.91
r25103	430.94	454.60	-5.20
r25104	391.60	416.90	-6.07
r25105	398.99	530.50	-24.79
r25106	439.75	465.40	-5.51
r25107	385.65	424.30	-9.11
r25108	361.20	397.30	-9.09
r25109	379.48	441.30	-14.01
r25110	361.28	444.10	-18.65
r25111	385.65	428.80	-10.06
r25112	335.70	393.00	-14.58
r25201	379.52	463.30	-18.08
r25202	382.74	410.50	-6.76
r25203	353.81	391.40	-9.60
r25204	334.21	355.00	-5.86
r25205	358.38	393.00	-8.81
r25206	329.48	374.40	-12.00
r25207	314.74	361.60	-12.96
r25208	298.05	328.20	-9.19
r25209	316.44	370.70	-14.64
r25210	364.39	404.60	-9.94
r25211	313.44	350.90	-10.68
Average	-	-	-11.39

Table 15 – A comparison between the result of the MILP with HGA as initial solution and the optima from the VRPTW for the class RC.

Instance	MILP + HGA	VRPTW Optimal	GAP (%)
rc25101	427.12	461.10	-7.4
rc25102	368.94	351.80	4.87
rc25103	297.69	333.10	-10.6
rc25104	276.32	306.60	-9.9
rc25105	452.32	411.30	10.0
rc25106	269.41	345.50	-22.0
rc25107	257.94	298.30	-13.5
rc25108	260.16	294.50	-11.7
rc25201	351.48	360.20	-2.4
rc25202	329.98	338.00	-2.4
rc25203	309.62	326.90	-5.3
rc25204	282.72	299.7	-5.67
rc25205	376.56	338.0	11.4
rc25206	302.47	324.0	-6.6
rc25207	275.61	298.3	-7.6
rc25208	234.03	269.1	-13.03
Average	-	-	-5.74

Table 16 – The effect of changes in speed in the result of the MILP + HGA.

Instance Class	Speed 50% GAP to VRPTW	Speed 100% GAP to VRPTW	Speed 150% GAP to VRPTW
C	0.61%	-3.89%	-5.49%
R	1.43%	-11.39%	-18.60%
RC	-4.96%	-5.74%	-7.11%

Table 17 – The effect of changes in drone eligible weight in the result of the MILP + HGA.

Instance Class	Drone Eligible W. 25% GAP to VRPTW	Drone Eligible W. 50% GAP to VRPTW	Drone Eligible W. 75% GAP to VRPTW
C	-2.56%	-3.89%	-3.81%
R	-4.08%	-11.39%	-13.30%
RC	2.76%	-5.74%	-6.97%

Table 18 – The HGA average for 5 runs and the best of these are compared to the VRPTW best known solutions of the class C. The average initial and final population fitness, and execution time are also shown.

Instance	Avg. Initial Population	Avg. Final Population	Avg. Best	GAP (%) to VRPTW BKS	Overall Best	GAP (%)to VRPTW BKS	Exec. Time
c101	4049.22	795.59	697.44	-15.86%	694.43	-16.23%	TL
c102	1603.07	388.73	355.88	-57.07%	355.33	-57.13%	520.92
c103	3431.80	845.87	754.36	-8.90%	718.36	-13.25%	TL
c104	3079.21	874.77	766.87	-7.02%	741.70	-10.07%	TL
c105	4032.10	812.86	726.49	-12.36%	699.50	-15.62%	TL
c106	3790.64	806.73	711.40	-14.18%	697.12	-15.90%	TL
c107	3996.90	823.48	736.24	-11.18%	715.36	-13.70%	TL
c108	3604.33	811.67	715.73	-13.66%	704.73	-14.98%	TL
c109	3346.49	836.91	746.59	-9.93%	724.92	-12.55%	TL
c201	4390.46	688.64	563.85	-4.68%	521.23	-11.89%	TL
c202	1661.88	364.47	336.04	-43.19%	332.65	-43.77%	709.36
c203	3481.79	749.31	611.45	3.43%	588.77	-0.41%	TL
c204	3107.49	822.06	688.03	16.50%	640.29	8.41%	TL
c205	3844.24	654.45	537.18	-8.78%	526.38	-10.61%	TL
c206	3660.49	672.95	570.09	-3.13%	536.74	-8.79%	TL
c207	3591.80	671.65	537.41	-8.65%	496.68	-15.57%	TL
c208	3517.30	665.71	557.88	-5.17%	522.88	-11.12%	TL
Average				-11.99%		-15.48%	-

Table 19 – The HGA average for 5 runs and the best of these are compared to the VRPTW best known solutions of the class R. The average initial and final population fitness, and execution time are also shown.

Instance	Avg. Initial Population	Avg. Final Population	Avg. Best	GAP (%) to VRPTW BKS	Overall Best	GAP (%)to VRPTW BKS	Exec. Time
r101	3800.53	1450.587	1378.90	-16.47%	1365.15	-17.30%	TL
r102	1947.18	822.6307	813.53	-45.26%	806.51	-45.73%	338.59
r103	3190.85	1094.622	1007.83	-22.04%	987.98	-23.57%	TL
r104	2877.30	893.5116	824.19	-18.18%	813.45	-19.24%	TL
r105	3576.28	1181.89	1070.65	-22.25%	1053.52	-23.50%	TL
r106	3398.87	1127.614	1016.70	-18.80%	1000.64	-20.08%	TL
r107	3062.30	977.0588	895.88	-18.90%	884.95	-19.89%	TL
r108	2832.98	863.0353	794.32	-17.33%	763.44	-20.55%	TL
r109	3247.62	1026.56	921.73	-22.85%	886.15	-25.83%	TL
r110	3055.64	947.32	854.18	-23.66%	830.02	-25.81%	TL
r111	3039.66	958.61	848.43	-22.64%	822.21	-25.03%	TL
r112	2856.07	878.48	802.51	-18.29%	776.49	-20.94%	TL
r201	1819.86	734.77	722.83	-42.28%	713.66	-43.02%	416.39
r202	3166.85	1012.62	930.44	-21.92%	912.71	-23.41%	TL
r203	2941.52	887.33	800.88	-14.75%	787.32	-16.20%	TL
r204	2717.57	749.48	649.37	-21.34%	636.72	-22.87%	TL
r205	2974.62	952.95	877.25	-11.78%	848.55	-14.67%	TL
r206	2845.67	887.60	801.98	-11.49%	777.21	-14.23%	TL
r207	2749.56	820.94	734.59	-17.52%	722.84	-18.84%	TL
r208	2552.39	702.52	601.01	-17.31%	555.30	-23.60%	TL
r209	2752.94	862.21	726.42	-16.40%	625.58	-19.97%	TL
r210	2872.91	880.22	785.91	-16.34%	761.01	-18.99%	TL
r211	2692.67	823.82	720.81	-17.71%	654.79	-26.07%	TL
Average				-20.67%		-23.01%	-

Table 20 – The HGA average for 5 runs and the best of these are compared to the VRPTW best known solutions of the class RC. The average initial and final population fitness, and execution time are also shown.

Instance	Avg. Initial Population	Avg. Final Population	Avg. Best	GAP (%) to VRPTW BKS	Overall Best	GAP (%)to VRPTW BKS	Exec. Time
rc101	4520.94	1532.99	1439.48	-15.17%	1414.99	-16.62%	TL
rc102	2667.74	793.32	690.55	-55.58%	634.83	-59.17%	528.91
rc103	4045.26	1230.17	1115.57	-11.58%	1089.46	-13.65%	TL
rc104	3730.75	1099.66	1004.83	-11.51%	982.76	-13.45%	TL
rc105	4487.25	1417.07	1257.35	-22.84%	1195.46	-26.63%	TL
rc106	4031.00	1261.29	1150.41	-19.25%	1116.38	-21.64%	TL
rc107	3886.94	1179.04	1060.01	-13.85%	1047.37	-14.88%	TL
rc108	3817.71	1079.67	964.30	-15.40%	938.21	-17.69%	TL
rc201	4211.00	1246.49	1151.74	-18.14%	1128.36	-19.80%	TL
rc202	2386.24	674.57	611.15	-55.25%	578.12	-57.67%	629.98
rc203	3741.70	992.38	878.43	-16.31%	838.76	-20.09%	TL
rc204	3450.02	870.62	751.56	-5.87%	712.64	-10.75%	TL
rc205	3928.10	1140.34	1047.60	-19.27%	1020.84	-21.33%	TL
rc206	3909.84	1072.14	975.96	-14.86%	944.61	-17.60%	TL
rc207	3666.33	1016.87	927.54	-12.59%	874.05	-17.63%	TL
rc208	3323.67	857.54	745.81	-9.94%	730.65	-11.77%	TL
Average				-19.84%		-22.52%	-

- (a) Initial population of the HGA. The left side is generated with the Sweep algorithm, and the right side by a random permutation.
- (b) Result of the convergence in the population by the end of execution.

- (c) Box plot of the population by generations.

- (d) Evolution of population and best individual through generations.

Figure 21 – Illustration of the HGA in execution. Example with instance C101 from (SOLOMON, 1987).

Figure 22 – Illustration of the HGA in execution. Example with instance R206 from (SOLOMON, 1987).

- (a) Initial population of the HGA. The left side is generated with the Sweep algorithm, and the right side by a random permutation.
- (b) Result of the convergence in the population by the end of execution.

- (c) Box plot of the population by generations.

- (d) Evolution of population and best individual through generations.

Figure 23 – Illustration of the HGA in execution. Example with instance RC107 from (SOLOMON, 1987).

FINAL REMARKS

In this dissertation, the Vehicle Routing Problem with Drones is discussed. The problem is considered a very topical discussion, not only in the scientific field but in politics and in general society as well. Several companies are already developing techniques to apply drones to their markets. Although there are complications implied in their use, drones can speed up delivery time and could directly help reduce traffic and greenhouse gas emissions.

We presented a literature review showing the most relevant work in the field of logistics applications considering drones as a form of delivery vehicle. Specifically, we focus on the work that also combines the drones with regular vehicles, in order to exploit the best of the two worlds: the truck's autonomy with the drone's agility.

To solve the problem we proposed two approaches: a Mixed Integer Linear Programming model, although this approach is more theoretical and to serve as a mathematical basis of comparison and structural definition; and a Hybrid Genetic Algorithm, which combines the benefits of a population-based algorithm, i.e exploration, with a local search that tries to find immediate neighbors with better solutions, i.e exploitation. The first approach was based on the model in (MURRAY; CHU, 2015) (discussed in Chapter 3), which was extended for allowing vehicle limited capacity and customers time windows, as well as the additional vehicles. The HGA was based on the idea of (VIDAL *et al.*, 2014). We developed a new chromosome, split and drone procedures, and some neighborhoods for the VRPD, which resulted in a consistent output to a complex problem.

By using the dataset of (SOLOMON, 1987), we tested both approaches to verify their correctness and to measure the benefits of employing drones on the delivery network. The results show that the drones can reduce the distance traversed by trucks to astounding 50% in some test instances, and on average the reduction is around 20%. This improvement is more notable in the larger tests, however, the small instances still have an

average gain of 7%. We also aimed to compare our model with the one in (PUGLIESE; GUERRIERO, 2017), since it is the only other MILP model for the VRPD. However, the model was unable to execute any of the 25 customer instances due to the excessive amount of memory consumption.

The results show the potential of drone driven applications. They are relatively cheap and given the opportunity can benefit a lot of parties: customers with faster deliveries; companies with less expenses; and the environment with fewer gas emission. There is still a lot to discuss and to study before the drones become a reality. We discuss some next steps on this matter next.

6.1 Future Research

The application of drones has a wide open field of possibilities. Considering only logistics and the approaches discussed in this work, some are very evident. For example, finding better and more sophisticated neighborhoods for drones routes, as well as a more refined way to determine their routes. One possibility is to find the optimal take-off and landing locations, based on the already fixed truck route. This would require a robust model that can be solved very quickly, but could improve the results significantly.

One obvious addition to the model is to have more than one drone per truck. Our model can be adapted to this, but since it is already hard to solve, adding more indexes to the variables or even more constraints may hinder it to a point where it becomes unsolvable. The proposed HGA can also be adapted, where each additional drone would add one more layer to our chromosome. This can become too complex to handle the local searches, but it is an interesting possibility.

Other research perspectives include: the drones visiting more than one customer before returning to the truck; consider weather to determine the use of drones or other types of stochasticity; more realistic scenarios, such as addition of no-fly zones.

For this work, we want to explore other parameters, such as different distance metrics for each vehicle type; compare to other solution methods in the literature and test with other datasets; evaluate more deeply the impact of each parameter, each neighborhood, and different initial algorithms; do a statistical analysis of the results; and, add energy and battery consumption constraints.

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MILP MODEL FOR FSTSP

Following, it is the Flying Sidekick Traveling Salesman Problem MILP model from (MURRAY; CHU, 2015).

$$\text{Min } t_{c+1} \quad (\text{A.1})$$

$$\text{s.t. } \sum_{\substack{i \in N_0 \\ i \neq j}} x_{ij} + \sum_{\substack{i \in N_0 \\ i \neq j}} \sum_{\substack{k \in N_+ \\ (i,j,k) \in P}} y_{ijk} = 1 \quad \forall j \in C \quad (\text{A.2})$$

$$\sum_{j \in N_+} x_{0j} = 1 \quad (\text{A.3})$$

$$\sum_{i \in N_0} x_{i,c+1} = 1 \quad (\text{A.4})$$

$$\sum_{i \in N_0} x_{i,c+1} = 1 \quad (\text{A.5})$$

$$u_i - u_j + 1 \leq (c+2)(1 - x_{ij}) \quad \forall i \in C, j \in \{N_+ : j \neq i\} \quad (\text{A.6})$$

$$\sum_{\substack{i \in N_0 \\ i \neq j}} x_{ij} = \sum_{\substack{k \in N_+ \\ k \neq j}} x_{kj} \quad \forall j \in C \quad (\text{A.7})$$

$$\sum_{\substack{j \in C \\ j \neq i}} \sum_{\substack{k \in N_+ \\ (i,j,k) \in P}} y_{ijk} \leq 1 \quad \forall i \in N_0 \quad (\text{A.8})$$

$$\sum_{\substack{i \in N_0 \\ i \neq k}} \sum_{\substack{j \in C \\ (i,j,k) \in P}} y_{ijk} \leq 1 \quad \forall k \in N_+ \quad (\text{A.9})$$

$$2y_{ijk} \leq \sum_{\substack{h \in N_0 \\ h \neq i}} x_{hi} + \sum_{\substack{l \in C \\ l \neq k}} x_{lk} \quad \forall i \in C, j \in \{C : j \neq i\}, k \in \{N_+ : (i,j,k) \in P\} \quad (\text{A.10})$$

$$y_{0jk} \leq \sum_{\substack{h \in N_0 \\ h \neq k}} x_{hk} \quad \forall j \in C, k \in \{N_+ : (0,j,k) \in P\} \quad (\text{A.11})$$

$$u_k - u_i \geq 1 - (c+2) \left(1 - \sum_{\substack{j \in C \\ (i,j,k) \in P}} y_{ijk} \right) \quad \forall i \in C, k \in \{N_+ : k \neq i\} \quad (\text{A.12})$$

$$t'_i \geq t_i - M \left(1 - \sum_{\substack{j \in C \\ j \neq i}} \sum_{\substack{k \in N_+ \\ (i,j,k) \in P}} y_{ijk} \right) \quad \forall i \in C \quad (\text{A.13})$$

$$t'_i \leq t_i + M \left(1 - \sum_{\substack{j \in C \\ j \neq i}} \sum_{\substack{k \in N_+ \\ (i,j,k) \in P}} y_{ijk} \right) \quad \forall i \in C \quad (\text{A.14})$$

$$t'_k \geq t_k - M \left(1 - \sum_{\substack{i \in N_0 \\ i \neq k}} \sum_{\substack{j \in C \\ (i,j,k) \in P}} y_{ijk} \right) \quad \forall k \in N_+ \quad (\text{A.15})$$

$$t'_k \leq t_k + M \left(1 - \sum_{\substack{i \in N_0 \\ i \neq k}} \sum_{\substack{j \in C \\ (i,j,k) \in P}} y_{ijk} \right) \quad \forall k \in N_+ \quad (\text{A.16})$$

$$t_k \geq t_h + \tau_{hk} + S_L \left(\sum_{\substack{l \in C \\ l \neq k}} \sum_{\substack{m \in N_+ \\ (k,l,m) \in P}} y_{klm} \right) + S_R \left(\sum_{\substack{i \in N_0 \\ i \neq k}} \sum_{\substack{j \in C \\ (i,j,k) \in P}} y_{ijk} \right) - M(1 - X_{hk}) \quad \forall h \in N_0, k \in \{N_+ : k \neq h\} \quad (\text{A.17})$$

$$t'_j \geq t'_i + \tau'_{ij} - M \left(1 - \sum_{\substack{k \in N_+ \\ (i,j,k) \in P}} y_{ijk} \right) \quad \forall j \in C', i \in \{N_0 : i \neq j\} \quad (\text{A.18})$$

$$t'_k \geq t'_j + \tau'_{jk} + S_R - M \left(1 - \sum_{\substack{i \in N_0 \\ (i,j,k) \in P}} y_{ijk} \right) \quad \forall j \in C', k \in \{N_+ : k \neq j\} \quad (\text{A.19})$$

$$t'_k - (t'_j - \tau'_{ij}) \leq e + M(1 - y_{ijk}) \quad \forall k \in N_+, j \in \{C : j \neq k\}, i \in \{N_0 : (i, j, k) \in P\} \quad (\text{A.20})$$

$$u_i - u_j \geq 1 - (c + 2)p_{ij} \quad \forall i \in C, j \in \{C : j \neq i\} \quad (\text{A.21})$$

$$u_i - u_j \leq -1 + (c + 2)(1 - p_{ij}) \quad \forall i \in C, j \in \{C : j \neq i\} \quad (\text{A.22})$$

$$p_{ij} + p_{ji} = 1 \quad \forall i \in C, j \in \{C : j \neq i\} \quad (\text{A.23})$$

$$t'_l \geq t'_k - M \left(3 - \sum_{\substack{j \in C \\ (i,j,k) \in P \\ j \neq l}} y_{ijk} - \sum_{\substack{m \in C \\ m \neq i \\ m \neq k \\ m \neq l}} \sum_{\substack{n \in N_+ \\ (l,m,n) \in P \\ n \neq i \\ n \neq k}} y_{lmn} - p_{il} \right) \quad \forall i \in N_0, k \in \{N_+ : k \neq i\}, l \in \{C : l \neq i, l \neq k\} \quad (\text{A.24})$$

$$t_0 = 0 \quad (\text{A.25})$$

$$t'_0 = 0 \quad (\text{A.26})$$

$$p_{0j} = 1 \quad \forall j \in C \quad (\text{A.27})$$

$$x_{ij} \in \{0, 1\} \quad \forall i \in N_0, j \in \{N_+ : j \neq i\} \quad (\text{A.28})$$

$$y_{ijk} \in \{0, 1\} \quad \forall i \in N_0, j \in \{C : j \neq i\}, k \in \{N_+ : (i, j, k) \in P\} \quad (\text{A.29})$$

$$1 \leq u_i \leq c + 2 \quad \forall i \in N_+ \quad (\text{A.30})$$

$$t_i \geq 0 \quad \forall i \in N \quad (\text{A.31})$$

$$t'_i \geq 0 \quad \forall i \in N \quad (\text{A.32})$$

$$p_{ij} \in \{0, 1\} \quad \forall i \in N_0, j \in \{C : j \neq i\} \quad (\text{A.33})$$

ADDITIONAL RESULTS AND TABLES

Here we present additional results according to the specifications in [Chapter 5](#).

B.1 MILP + Sweep (cont.)

The first set of results are using the Sweep algorithm as a start solution. We test the model with the other two drone eligible weight parameter values: 25% and 75%. The main additional results observed are the lower gaps in the 25% family, as well as the greater gaps in the 75%, as mentioned in [Chapter 5](#).

Table 21 – Results for the class of instances C, with 100% drone speed and 25% drone eligible weight, using the Sweep as a starting solution.

Instance	Objective Function	GAP	Dual	Exec. Time	Nodes	Variables	Constraints
c25101	181.50	0.00	181.50	21.03	2604	41405	501301
c25102	637.95	0.82	115.93	3600.61	3	54705	518251
c25103	502.13	0.79	103.26	3601.61	1	58805	524751
c25104	389.05	0.73	103.26	3601.61	2	62855	529676
c25105	177.94	0.00	177.94	656.16	72798	44955	505526
c25106	177.94	0.00	177.94	27.91	885	42455	502451
c25107	323.87	0.51	157.51	3622.14	34469	46030	507101
c25108	469.70	0.89	52.13	3845.14	10422	50455	512926
c25109	436.62	0.89	48.90	3601.28	5	53855	517626
c25201	209.89	0.00	209.89	59.43	16940	41455	501951
c25202	589.37	0.79	125.16	3601.83	15156	49155	513301
c25203	550.19	0.81	104.36	3602.91	72	55530	521851
c25204	461.24	0.77	104.36	3603.72	173	57180	524451
c25205	547.23	0.76	133.08	3601.16	3739	45680	507151
c25206	438.65	0.75	110.89	3601.60	973	47280	509376
c25207	445.57	0.76	106.50	3602.74	57	50155	513451
c25208	398.32	0.74	104.42	3601.28	30	48755	511426

Table 22 – Results for the class of instances C, with 100% drone speed and 75% drone eligible weight, using the Sweep as a starting solution.

Instance	Objective Function	GAP	Dual	Exec. Time	Nodes	Variables	Constraints
c25101	181.5	0.0	181.5	130.3	14793	54305	526501
c25102	637.9	0.9	46.1	3600.7	5	73005	548826
c25103	502.1	1.0	15.1	3602.2	1	78930	557126
c25104	389.0	1.0	14.5	3600.1	0	85055	564126
c25105	202.8	0.4	114.8	3600.7	69690	60480	533376
c25106	177.9	0.0	177.9	395.6	29453	55855	528151
c25107	661.2	0.9	86.6	3600.6	2	61555	534951
c25108	469.7	1.0	17.3	3601.1	1	67480	542226
c25109	436.6	1.0	15.3	3602.4	1	72955	548976
c25201	201.9	0.0	201.9	82.7	12808	57630	530426
c25202	589.4	0.9	80.5	3601.3	41	68355	544776
c25203	550.2	0.9	39.1	3602.6	1	76780	555351
c25204	461.2	0.9	33.4	3600.4	0	80705	560226
c25205	547.2	0.8	88.8	3600.8	421	64005	537751
c25206	438.7	0.9	39.2	3609.6	93	66655	541001
c25207	445.6	0.9	49.6	3603.0	1	71730	547276
c25208	398.3	0.9	36.4	3600.8	0	69480	544401

Table 23 – Results for the class of instances R, with 100% drone speed and 25% drone eligible weight, using the Sweep as a starting solution.

Instance	Objective Function	GAP	Dual	Exec. Time	Nodes	Variables	Constraints
r25101	595.04	0.00	595.04	12.73	8596	27030	476776
r25102	773.97	0.58	323.58	3600.55	10471	35055	489651
r25103	678.39	0.65	237.06	3601.40	245	38005	495401
r25104	582.78	0.62	221.34	3601.63	531	40380	499151
r25105	469.19	0.16	396.02	3607.27	66410	30530	482226
r25106	656.17	0.61	255.88	3606.60	7600	36830	492676
r25107	617.44	0.62	237.06	3601.19	39	39030	497076
r25108	594.70	0.63	221.34	3601.72	221	40880	500126
r25109	689.29	0.60	272.29	3604.93	13804	33755	488201
r25110	563.54	0.58	239.24	3601.59	958	38405	495826
r25111	653.14	0.64	237.06	3601.64	81	38355	495801
r25112	522.93	0.58	221.34	3601.40	1414	42105	502576
r25201	435.36	0.00	435.33	1851.36	142571	32980	487126
r25202	635.44	0.62	243.94	3600.88	1018	38255	495526
r25203	612.00	0.61	237.06	3601.36	547	40205	499426
r25204	536.87	0.59	221.34	3601.66	1585	41630	501626
r25205	565.29	0.49	286.09	3608.67	14205	35830	492276
r25206	579.00	0.61	226.07	3601.26	1261	39455	497901
r25207	548.74	0.59	226.07	3615.47	2204	40555	500101
r25208	533.49	0.59	221.34	3601.79	1560	41730	501826
r25209	512.67	0.53	243.06	3601.78	110	38555	496476
r25210	605.60	0.56	265.94	3607.66	3934	39105	497401
r25211	475.78	0.53	221.34	3601.91	1223	42130	502626

Table 24 – Results for the class of instances R, with 100% drone speed and 75% drone eligible weight, using the Sweep as a starting solution.

Instance	Objective Function	GAP	Dual	Exec. Time	Nodes	Variables	Constraints
r25101	478.69	0.00	478.69	39.87	10951	34305	498876
r25102	773.97	0.81	147.20	3600.47	8	45730	515101
r25103	678.39	0.82	119.12	3601.24	0	50905	523051
r25104	582.78	0.81	112.49	3602.33	1	54330	527851
r25105	480.53	0.45	264.30	3601.03	145729	40005	506501
r25106	782.37	0.84	127.74	3601.57	0	48730	519326
r25107	617.44	0.81	119.00	3601.97	1	52105	524901
r25108	594.70	0.81	112.49	3601.13	0	54980	528976
r25109	689.29	0.77	158.03	3601.46	2	44755	514026
r25110	563.54	0.79	117.06	3601.43	0	51880	524101
r25111	653.14	0.82	119.04	3602.18	1	51030	523226
r25112	522.93	0.79	107.97	3602.39	1	56430	531651
r25201	523.06	0.48	272.37	3600.41	23444	42430	511376
r25202	635.44	0.79	133.61	3601.25	1	50255	522276
r25203	612.00	0.80	119.50	3601.82	1	53355	527326
r25204	536.87	0.79	112.49	3603.04	1	55755	530501
r25205	565.43	0.68	178.51	3611.17	1754	47430	518676
r25206	579.00	0.80	114.95	3602.05	2	52280	525476
r25207	548.74	0.79	113.33	3602.01	3	53980	528276
r25208	533.49	0.79	112.40	3602.78	1	55930	530776
r25209	512.67	0.76	123.53	3601.05	2	51005	523701
r25210	605.60	0.75	151.68	3601.19	2	51605	524651
r25211	475.78	0.77	107.97	3601.34	0	56455	531701

Table 25 – Results for the class of instances RC, with 100% drone speed and 25% drone eligible weight, using the Sweep as a starting solution.

Instance	Objective Function	GAP	Dual	Exec. Time	Nodes	Variables	Constraints
rc25101	394.08	0.40	238.13	3600.41	60916	33255	485676
rc25102	764.73	0.86	103.89	3607.86	1231	39355	495901
rc25103	622.29	0.86	84.70	3602.25	4317	44530	503726
rc25104	573.94	0.86	82.31	3602.77	609	45905	506251
rc25105	875.21	0.80	176.33	3698.49	8167	38155	493001
rc25106	695.34	0.84	113.08	3601.65	1071	38505	493776
rc25107	747.24	0.90	78.31	3602.49	659	43955	502376
rc25108	587.71	0.87	78.31	3603.03	241	46780	507776
rc25201	336.95	0.32	228.47	3600.58	245369	35580	491151
rc25202	794.62	0.88	96.88	3609.65	4055	40780	499426
rc25203	738.73	0.89	84.70	3602.12	634	45830	506326
rc25204	519.67	0.84	82.31	3602.68	207	46830	508101
rc25205	738.90	0.76	176.63	3601.30	1020	40755	498426
rc25206	583.21	0.81	113.08	3601.20	593	40930	498826
rc25207	539.85	0.84	87.88	3602.15	471	44455	504051
rc25208	454.11	0.83	78.31	3603.43	351	47705	509551

Table 26 – Results for the class of instances RC, with 100% drone speed and 75% drone eligible weight, using the Sweep as a starting solution.

Instance	Objective Function	GAP	Dual	Exec. Time	Nodes	Variables	Constraints
rc25101	869.38	0.92	65.97	3601.17	187	54030	523651
rc25102	764.73	0.95	41.00	3601.50	1	67580	541301
rc25103	622.29	0.95	31.01	3600.14	0	75005	551351
rc25104	573.94	0.96	24.95	3600.27	0	79555	557051
rc25105	875.21	0.96	38.58	3601.80	0	64605	536601
rc25106	695.34	0.95	31.98	3614.04	0	65805	538226
rc25107	747.24	0.97	24.88	3600.40	0	77205	552776
rc25108	587.71	0.96	24.84	3601.33	0	81555	559701
rc25201	804.39	0.84	130.67	3600.78	568	57630	530376
rc25202	794.62	0.95	42.14	3601.04	4	69355	545151
rc25203	738.73	0.96	31.02	3601.09	0	76730	554376
rc25204	519.67	0.95	24.95	3600.23	0	80505	558926
rc25205	738.90	0.92	56.08	3600.65	0	67080	541901
rc25206	583.21	0.94	35.86	3601.52	1	67680	542726
rc25207	539.85	0.95	25.04	3601.70	0	75780	552551
rc25208	454.11	0.95	24.84	3601.42	0	82605	561601

Table 27 – Results for the class of instances C, with 50% drone speed and 25% drone eligible weight, using the HGA as a starting solution.

Instance	Objective Function	GAP	Dual	Exec. Time	Nodes	Variables	Constraints
c25101	191.81	0.00	191.81	5.10	0	30655	489777
c25102	190.74	0.22	148.77	3600.37	63	38405	501152
c25103	190.74	0.42	110.61	3601.30	283	41530	506702
c25104	187.45	0.45	103.41	3600.88	5	43255	509302
c25105	191.81	0.00	191.81	8.43	0	32705	492502
c25106	191.81	0.00	191.81	5.65	0	31255	490477
c25107	191.81	0.00	191.80	747.67	45860	33530	493827
c25108	191.81	0.52	92.59	3600.89	4751	36130	497827
c25109	191.81	0.72	53.37	3601.32	106	38005	500977
c25201	214.88	0.00	214.88	4.87	0	26680	486327
c25202	214.88	0.31	149.17	3600.49	39948	30530	493827
c25203	213.19	0.36	135.91	3617.12	2394	33280	498827
c25204	218.64	0.41	128.67	3600.70	5887	34230	500727
c25205	214.88	0.26	159.54	3600.38	248684	27930	488627
c25206	214.88	0.36	136.80	3632.69	22728	28705	490027
c25207	214.67	0.26	158.05	3600.81	12520	30105	492627
c25208	214.88	0.39	130.72	3606.91	9641	29480	491377

B.2 MILP + HGA (cont.)

The second set of results are using the HGA algorithm as a start solution. Since this is our main contribution, we test the model with all other parameter values: drone eligible weight of 25%, 50% and 75%; drone speed of 50%, the default of 100% and the faster 150%. Resulting in a combination of 9 combinations per instance class (the default parameter results are in the [Chapter 5](#)). The results in this section corroborate the increase in drone eligible customers being more detrimental to the model’s solvability .

Table 28 – Results for the class of instances C, with 50% drone speed and 50% drone eligible weight, using the HGA as a starting solution.

Instance	Objective Function	GAP	Dual	Exec. Time	Nodes	Variables	Constraints
c25101	191.81	0.00	191.81	6.53	0	34455	503527
c25102	190.74	0.26	141.34	3600.45	4	44780	517477
c25103	190.74	0.53	89.29	3600.45	0	48105	523177
c25104	187.45	0.57	80.27	3600.69	0	50580	526527
c25105	191.81	0.00	191.81	19.08	0	37380	507127
c25106	191.81	0.00	191.81	7.47	0	35280	504452
c25107	191.81	0.36	122.19	3600.87	3885	38205	508452
c25108	191.81	0.81	35.60	3601.54	2214	41580	513202
c25109	191.81	0.85	28.59	3601.18	5	44180	517077
c25201	214.88	0.00	214.88	68.93	12704	28280	497927
c25202	213.19	0.39	129.04	3851.21	30295	32280	505552
c25203	213.19	0.46	114.93	3601.28	691	35380	510827
c25204	214.67	0.50	106.48	3601.35	629	36480	512877
c25205	214.88	0.36	138.11	3600.68	16042	29705	500327
c25206	232.55	0.48	121.74	3600.69	6129	30480	501702
c25207	214.67	0.37	135.96	3602.05	1734	32155	504577
c25208	214.88	0.48	110.70	3600.92	1722	31480	503277

Table 29 – Results for the class of instances C, with 50% drone speed and 75% drone eligible weight, using the HGA as a starting solution.

Instance	Objective Function	GAP	Dual	Exec. Time	Nodes	Variables	Constraints
c25101	191.81	0.00	191.81	6.98	0	35180	506702
c25102	190.74	0.28	137.87	3600.43	5	45680	520827
c25103	192.54	0.82	35.42	3601.40	43	49030	526552
c25104	187.45	0.90	18.33	3601.51	1	51555	529952
c25105	191.81	0.00	191.81	29.96	0	38330	510552
c25106	191.81	0.00	191.81	7.44	0	36005	507627
c25107	191.81	0.33	128.69	3600.88	812	39180	511902
c25108	191.81	0.86	26.99	3600.99	3168	42555	516627
c25109	191.81	0.89	20.20	3600.89	4	45180	520527
c25201	214.88	0.03	215.54	19.32	0	28880	501002
c25202	214.88	0.41	126.46	3600.65	43520	32880	508627
c25203	214.88	0.53	99.86	3605.83	284	35980	513877
c25204	218.88	0.58	91.88	3601.36	536	37080	515927
c25205	232.55	0.43	133.40	3600.57	4059	30355	503427
c25206	232.55	0.49	119.47	3614.68	9854	31130	504802
c25207	214.88	0.41	126.66	3601.25	2431	32805	507677
c25208	215.54	0.49	110.57	3601.04	884	32130	506377

Table 30 – Results for the class of instances C, with 100% drone speed and 25% drone eligible weight, using the HGA as a starting solution.

Instance	Objective Function	GAP	Dual	Exec. Time	Nodes	Variables	Constraints
c25101	181.49	<0.01	181.49	75.21	15839	41405	500677
c25102	190.72	0.40	114.88	3601.02	3	54705	517627
c25103	188.26	0.45	103.26	3601.70	2	58805	524127
c25104	184.06	0.43	104.33	3601.49	5	62855	529052
c25105	177.93	<0.01	177.92	96.36	12460	44955	504902
c25106	177.93	0.00	177.93	21.78	5	42455	501827
c25107	177.93	0.06	166.77	3600.59	90196	46030	506477
c25108	186.82	0.72	52.13	3601.47	1394	50455	512302
c25109	184.13	0.73	48.90	3601.24	6	53855	517002
c25201	209.89	0.00	209.89	24.58	1223	41455	501327
c25202	210.83	0.39	128.49	3601.27	6031	49155	512677
c25203	212.81	0.50	106.30	3601.55	8	55530	521227
c25204	212.96	0.51	104.36	3602.30	2	57180	523827
c25205	212.52	0.37	133.43	3600.70	9918	45680	506527
c25206	212.52	0.47	111.80	3601.18	4508	47280	508752
c25207	212.32	0.50	106.50	3602.58	2572	50155	512827
c25208	212.52	0.50	104.73	3600.89	6	48755	510802

Table 31 – Results for the class of instances C, with 100% drone speed and 75% drone eligible weight, using the HGA as a starting solution.

Instance	Objective Function	GAP	Dual	Exec. Time	Nodes	Variables	Constraints
c25101	181.46	0.00	181.45	334.54	27555	54305	525877
c25102	188.98	0.76	46.07	3601.93	12	73005	548202
c25103	188.12	0.92	15.15	3602.25	1	78930	556502
c25104	182.73	0.92	14.51	3603.55	0	85055	563502
c25105	184.10	0.34	122.04	3600.65	1365	60480	532752
c25106	177.90	0.00	177.90	1069.93	29012	55855	527527
c25107	186.47	0.55	83.42	3602.92	18	61555	534327
c25108	184.10	0.91	17.27	3601.42	4	67480	541602
c25109	184.10	0.92	15.32	3602.07	1	72955	548352
c25201	201.88	0.00	201.87	44.31	222	57630	529802
c25202	207.12	0.61	80.26	3601.03	15	68355	544152
c25203	205.73	0.81	39.09	3601.97	0	76780	554727
c25204	205.38	0.84	33.36	3601.44	0	80705	559602
c25205	205.11	0.57	89.10	3600.64	4	64005	537127
c25206	205.11	0.81	39.87	3601.48	10	66655	540377
c25207	205.25	0.76	49.56	3602.97	1	71730	546652
c25208	205.46	0.81	38.45	3602.60	0	69480	543777

Table 32 – Results for the class of instances C, with 150% drone speed and 25% drone eligible weight, using the HGA as a starting solution.

Instance	Objective Function	GAP	Dual	Exec. Time	Nodes	Variables	Constraints
c25101	173.06	0.00	173.06	27.06	0	72705	532077
c25102	189.39	0.39	114.89	3601.48	4	101505	564452
c25103	187.52	0.45	103.60	3605.90	127	111555	576877
c25104	183.29	0.44	103.26	3601.39	0	118355	584552
c25105	179.14	0.07	166.64	3600.98	27752	80505	540502
c25106	173.06	0.00	173.06	36.51	1	74280	533752
c25107	184.13	0.10	165.59	3601.11	3	83430	543927
c25108	186.81	0.70	56.13	3602.92	355	91930	553802
c25109	182.26	0.73	48.90	3603.31	2	99030	562177
c25201	197.00	0.00	197.00	502.34	19769	75355	535252
c25202	220.44	0.43	125.16	3601.87	10	94830	558377
c25203	220.73	0.52	105.89	3604.15	7	110605	576302
c25204	217.11	0.52	104.36	3601.64	0	113180	579827
c25205	210.37	0.35	136.86	3602.11	29728	83405	544277
c25206	212.52	0.47	112.35	3601.75	12434	86705	548177
c25207	212.52	0.38	130.88	3604.31	299	93705	556377
c25208	212.52	0.51	104.42	3602.44	1722	89655	551702

Table 33 – Results for the class of instances C, with 150% drone speed and 50% drone eligible weight, using the HGA as a starting solution.

Instance	Objective Function	GAP	Dual	Exec. Time	Nodes	Variables	Constraints
c25101	159.65	0.00	159.65	69.83	8281	102805	572002
c25102	189.24	0.43	107.12	3602.81	1	143605	616352
c25103	187.38	0.58	79.61	3600.82	0	158180	633302
c25104	183.05	0.57	79.61	3602.21	0	169455	645452
c25105	186.80	0.34	122.78	3810.11	13779	114905	584727
c25106	159.65	0.00	159.65	498.48	24984	105330	574627
c25107	186.47	0.43	106.35	3601.56	1	118380	588702
c25108	184.10	0.86	26.00	3602.67	1	130505	602177
c25109	182.25	0.88	22.77	3601.35	0	142105	615052
c25201	186.15	0.41	110.60	3600.63	11276	111855	581577
c25202	205.11	0.70	60.94	3602.69	3	137330	610677
c25203	205.38	0.76	49.93	3603.42	0	157780	633277
c25204	206.36	0.78	44.51	3601.45	0	166055	642502
c25205	205.11	0.75	51.31	3601.13	3	123430	594102
c25206	205.46	0.79	43.22	3601.73	2	129080	600352
c25207	205.11	0.76	48.81	3601.09	0	140205	612677
c25208	205.11	0.78	44.81	3602.56	1	133780	605627

Table 34 – Results for the class of instances C, with 150% drone speed and 75% drone eligible weight, using the HGA as a starting solution.

Instance	Objective Function	GAP	Dual	Exec. Time	Nodes	Variables	Constraints
c25101	157.79	0.00	157.79	85.67	5525	110030	581677
c25102	188.00	0.76	44.67	3603.22	1	152880	628077
c25103	188.54	0.93	14.07	3602.32	0	169630	647202
c25104	183.64	1.00	0.00	4000.09	0	182105	660552
c25105	186.77	0.51	91.48	3601.19	6	123880	596152
c25106	157.79	0.00	157.78	899.62	21373	112555	584302
c25107	184.10	0.59	75.86	3601.88	2	127980	600752
c25108	184.09	0.92	14.48	3602.65	1	140905	615027
c25109	182.25	0.92	14.29	3600.16	0	154155	629552
c25201	201.36	0.50	100.26	3602.99	9530	121155	593327
c25202	207.12	0.77	48.34	3603.21	1	152530	628327
c25203	205.38	0.83	34.26	3606.14	0	172980	650927
c25204	206.68	0.85	30.18	3601.47	0	181705	660602
c25205	205.11	0.81	38.08	3601.20	4	133705	606827
c25206	205.11	0.87	27.61	3601.87	3	139805	613527
c25207	205.46	0.85	29.86	3601.03	0	151680	626602
c25208	206.88	0.86	29.07	3602.34	1	145255	619552

Table 35 – Results for the class of instances R, with 50% drone speed and 25% drone eligible weight, using the HGA as a starting solution.

Instance	Objective Function	GAP	Dual	Exec. Time	Nodes	Variables	Constraints
r25101	612.62	0.00	612.62	24.35	2314	22005	470127
r25102	548.11	0.32	375.16	3605.75	481825	26755	479652
r25103	464.83	0.36	296.89	3611.98	459436	29555	485252
r25104	439.76	0.34	292.27	3611.86	551877	31005	488152
r25105	525.83	0.00	525.79	87.27	29803	23905	473927
r25106	470.33	0.33	315.61	3611.22	462802	28005	482152
r25107	431.81	0.30	300.20	3609.04	484477	30205	486552
r25108	398.94	0.29	282.65	3612.51	513980	31480	489102
r25109	433.66	0.00	433.62	680.22	200485	26680	479477
r25110	441.17	0.35	284.89	3609.95	484022	29705	485552
r25111	408.76	0.25	305.74	3602.84	637250	29605	485352
r25112	401.86	0.30	280.75	3633.41	408871	32705	491552
r25201	454.61	0.00	454.61	111.24	29739	26380	478877
r25202	441.83	0.32	298.88	3611.07	645111	29430	485002
r25203	392.33	0.26	289.06	3614.02	470152	31380	488902
r25204	356.57	0.21	280.93	3608.47	503661	32230	490602
r25205	377.51	0.00	377.49	228.49	50212	28655	483427
r25206	375.48	0.24	285.00	3607.37	479607	30605	487352
r25207	362.63	0.22	282.37	3623.68	473070	31705	489552
r25208	340.53	0.18	279.83	3609.01	479394	32330	490802
r25209	375.97	0.21	297.10	3606.36	619063	30155	486452
r25210	405.48	0.27	296.57	3607.79	486405	30480	487102
r25211	352.50	0.21	279.31	3614.55	419113	32730	491602

Table 36 – Results for the class of instances R, with 50% drone speed and 50% drone eligible weight, using the HGA as a starting solution.

Instance	Objective Function	GAP	Dual	Exec. Time	Nodes	Variables	Constraints
r25101	612.62	0.00	612.62	16.57	1	22005	478702
r25102	548.11	0.32	374.25	3605.24	202328	26805	488377
r25103	462.93	0.33	308.23	3616.03	464687	29630	494052
r25104	436.52	0.36	278.63	3606.30	405455	31105	497027
r25105	525.79	0.00	525.74	38.99	13086	23930	482577
r25106	468.68	0.33	312.66	3606.86	430404	28080	490952
r25107	431.81	0.32	293.48	3606.72	454842	30280	495352
r25108	451.10	0.39	274.13	3606.42	364196	31580	497977
r25109	430.40	0.00	430.36	530.91	152242	26730	488202
r25110	448.25	0.36	287.52	3699.84	255028	29805	494427
r25111	440.05	0.36	281.57	3619.82	275579	29680	494152
r25112	404.60	0.34	265.53	3612.38	383572	32805	500427
r25201	454.61	0.00	454.60	104.57	30115	26405	487527
r25202	411.49	0.27	300.86	3609.53	512960	29505	493802
r25203	394.70	0.27	289.56	3613.52	475923	31455	497702
r25204	366.95	0.28	265.20	3607.71	425110	32330	499477
r25205	377.51	0.13	328.06	3614.46	584204	28705	492152
r25206	375.48	0.24	284.09	3608.76	535266	30680	496152
r25207	362.63	0.23	279.96	3612.92	618728	31780	498352
r25208	347.80	0.24	265.53	3614.34	452830	32430	499677
r25209	373.06	0.17	309.55	3600.73	851888	30205	495177
r25210	412.80	0.24	315.46	3609.01	353067	30555	495902
r25211	350.44	0.25	263.53	3606.94	389855	32830	500477

Table 37 – Results for the class of instances R, with 50% drone speed and 75% drone eligible weight, using the HGA as a starting solution.

Instance	Objective Function	GAP	Dual	Exec. Time	Nodes	Variables	Constraints
r25101	612.62	0.00	612.62	21.25	3	22030	484902
r25102	548.11	0.32	370.58	3606.38	477836	26855	494652
r25103	455.70	0.34	302.01	3608.15	554220	29680	500327
r25104	422.34	0.37	266.77	3620.17	441764	31155	503302
r25105	522.54	0.00	522.54	46.83	27723	23980	488852
r25106	494.41	0.37	313.31	3612.52	447825	28130	497227
r25107	431.81	0.32	292.26	3608.98	529671	30330	501627
r25108	398.94	0.32	270.18	3618.35	383311	31630	504252
r25109	430.40	0.00	430.36	975.83	280650	26780	494477
r25110	447.47	0.38	278.36	3668.23	193969	29855	500702
r25111	440.05	0.36	281.57	3645.98	330001	29730	500427
r25112	395.41	0.36	254.94	3600.01	289541	32855	506702
r25201	452.90	0.00	452.90	46.57	14777	26455	493802
r25202	411.49	0.27	299.54	3611.50	550056	29555	500077
r25203	392.33	0.26	289.66	3614.91	504403	31505	503977
r25204	355.14	0.27	258.40	3613.44	425830	32380	505752
r25205	374.26	0.00	374.23	355.36	105927	28755	498427
r25206	375.48	0.25	282.64	3611.47	362634	30730	502427
r25207	361.61	0.23	278.86	3618.00	536119	31830	504627
r25208	352.79	0.29	251.92	3616.95	471969	32480	505952
r25209	370.48	0.17	309.32	3602.20	864048	30255	501452
r25210	405.48	0.25	305.97	3609.28	468854	30605	502177
r25211	352.45	0.27	256.44	3621.34	294696	32880	506752

Table 38 – Results for the class of instances R, with 100% drone speed and 25% drone eligible weight, using the HGA as a starting solution.

Instance	Objective Function	GAP	Dual	Exec. Time	Nodes	Variables	Constraints
r25101	595.04	0.00	595.04	5.22	0	27030	476152
r25102	540.37	0.40	326.86	3619.86	10656	35055	489027
r25103	434.57	0.43	247.82	3601.62	4995	38005	494777
r25104	411.65	0.46	221.34	3601.11	2	40380	498527
r25105	485.48	0.23	373.01	3607.85	41506	30530	481602
r25106	461.03	0.43	262.86	3601.20	1358	36830	492052
r25107	445.93	0.47	237.06	3601.46	5239	39030	496452
r25108	380.40	0.42	221.34	3601.86	276	40880	499502
r25109	428.14	0.35	277.49	3615.30	3658	33755	487577
r25110	413.54	0.40	248.17	3601.49	1412	38405	495202
r25111	395.03	0.38	243.84	3602.77	2671	38355	495177
r25112	372.74	0.39	228.56	3601.42	1794	42105	501952
r25201	435.36	0.15	370.24	3613.19	311358	32980	486502
r25202	390.08	0.34	255.80	3601.04	3576	38255	494902
r25203	372.23	0.34	243.94	3601.45	211	40205	498802
r25204	355.62	0.35	230.10	3601.66	911	41630	501002
r25205	371.26	0.22	290.27	3609.76	17913	35830	491652
r25206	340.01	0.32	230.67	3601.05	4069	39455	497277
r25207	341.97	0.34	226.82	3601.27	2914	40555	499477
r25208	323.94	0.31	222.13	3602.28	351	41730	501202
r25209	362.09	0.33	243.06	3601.42	709	38555	495852
r25210	382.81	0.30	267.53	3601.36	2088	39105	496777
r25211	335.97	0.32	229.17	3601.94	1678	42130	502002

Table 39 – Results for the class of instances R, with 100% drone speed and 75% drone eligible weight, using the HGA as a starting solution.

Instance	Objective Function	GAP	Dual	Exec. Time	Nodes	Variables	Constraints
r25101	478.69	0.00	478.69	123.89	16636	34305	498252
r25102	487.52	0.70	141.43	3600.84	1	45730	514477
r25103	393.07	0.70	119.12	3601.92	1	50905	522427
r25104	370.39	0.70	112.65	3602.38	1	54330	527227
r25105	446.31	0.47	236.24	3600.42	8	40005	505877
r25106	425.68	0.70	127.74	3601.69	1	48730	518702
r25107	378.70	0.69	119.00	3602.28	1	52105	524277
r25108	340.30	0.67	112.49	3601.09	0	54980	528352
r25109	349.11	0.56	154.55	3601.50	1	44755	513402
r25110	368.16	0.68	117.06	3602.05	1	51880	523477
r25111	371.74	0.68	119.04	3602.17	1	51030	522602
r25112	303.47	0.64	107.97	3601.05	0	56430	531027
r25201	411.25	0.32	279.05	3633.29	36287	42430	510752
r25202	365.85	0.63	133.61	3601.31	2	50255	521652
r25203	348.47	0.66	119.50	3602.07	2	53355	526702
r25204	327.51	0.66	112.49	3602.69	1	55755	529877
r25205	354.47	0.49	178.70	3601.05	3	47430	518052
r25206	324.13	0.65	114.95	3601.98	1	52280	524852
r25207	326.87	0.65	113.33	3602.41	1	53980	527652
r25208	290.39	0.61	112.40	3602.22	1	55930	530152
r25209	306.62	0.57	131.34	3601.83	4	51005	523077
r25210	363.76	0.58	152.12	3601.83	2	51605	524027
r25211	315.78	0.66	107.97	3601.11	0	56455	531077

Table 40 – Results for the class of instances R, with 150% drone speed and 25% drone eligible weight, using the HGA as a starting solution.

Instance	Objective Function	GAP	Dual	Exec. Time	Nodes	Variables	Constraints
r25101	542.28	0.00	542.28	35.66	829	41755	491002
r25102	507.20	0.38	312.91	3602.78	4539	59505	513552
r25103	423.62	0.43	239.47	3602.07	15	63080	519902
r25104	374.25	0.39	229.63	3605.17	267	67255	525452
r25105	454.12	0.21	359.35	3600.77	19269	48305	499502
r25106	428.56	0.39	260.50	3601.71	18	62030	517302
r25107	400.46	0.39	243.76	3602.62	16	64680	522152
r25108	363.98	0.37	229.63	3604.70	357	68880	526552
r25109	427.65	0.35	276.53	3603.06	3359	55030	508977
r25110	399.80	0.38	247.44	3604.12	793	62905	519802
r25111	394.71	0.40	237.06	3604.10	3	64455	521327
r25112	351.62	0.33	234.63	3606.03	595	69105	529002
r25201	394.50	0.08	362.00	3600.45	81693	50905	504552
r25202	390.08	0.36	248.76	3602.95	245	63555	520252
r25203	372.01	0.36	237.06	3603.08	1	65855	524502
r25204	395.06	0.44	222.37	3605.81	947	68630	528052
r25205	334.25	0.20	266.72	3601.26	16772	56730	512652
r25206	337.63	0.31	233.29	3604.17	495	65280	523152
r25207	337.49	0.33	226.55	3604.60	1021	66555	525527
r25208	323.86	0.30	226.74	3607.10	1116	68730	528252
r25209	368.33	0.31	252.38	3604.03	160	61855	519227
r25210	382.81	0.38	237.06	3601.97	7	64305	522052
r25211	308.63	0.27	224.01	3607.58	543	69130	529052

Table 41 – Results for the class of instances R, with 150% drone speed and 50% drone eligible weight, using the HGA as a starting solution.

Instance	Objective Function	GAP	Dual	Exec. Time	Nodes	Variables	Constraints
r25101	330.34	0.00	330.34	193.37	9926	62355	520177
r25102	426.92	0.53	202.38	3602.37	2	90505	553127
r25103	388.43	0.53	182.06	3601.52	0	99255	564652
r25104	342.58	0.51	167.97	3601.77	0	105830	572602
r25105	360.16	0.13	312.06	3600.84	2038	74455	534252
r25106	383.09	0.47	202.38	3601.20	0	95230	559077
r25107	375.21	0.51	182.06	3605.80	1	101480	567527
r25108	338.26	0.50	167.97	3601.23	0	106755	574002
r25109	370.95	0.42	216.49	3602.69	8	85730	548277
r25110	338.21	0.48	175.23	3602.76	0	99880	565352
r25111	359.79	0.49	182.06	3605.31	1	99780	565227
r25112	293.79	0.38	180.81	3610.35	80	109505	577977
r25201	324.10	0.09	294.26	3601.18	55390	77730	539977
r25202	371.44	0.46	202.38	3603.45	1	97280	562552
r25203	351.78	0.48	182.06	3600.60	0	102705	569927
r25204	316.87	0.47	167.97	3601.22	0	107530	575527
r25205	350.16	0.40	209.62	3603.63	768	88930	553452
r25206	329.10	0.46	178.29	3606.81	112	101480	567927
r25207	311.37	0.46	168.76	3608.59	213	104605	572152
r25208	280.72	0.40	168.64	3605.08	4	108055	576152
r25209	317.58	0.39	193.01	3605.46	144	96780	562727
r25210	364.39	0.50	182.06	3601.34	0	99230	565552
r25211	278.00	0.39	167.97	3602.31	0	109530	578027

Table 42 – Results for the class of instances R, with 150% drone speed and 75% drone eligible weight, using the HGA as a starting solution.

Instance	Objective Function	GAP	Dual	Exec. Time	Nodes	Variables	Constraints
r25101	272.16	0.14	232.76	3603.55	85686	73430	537377
r25102	407.35	0.70	122.45	3603.19	0	106505	575252
r25103	374.28	0.69	115.21	3604.90	1	118555	590077
r25104	340.41	0.68	107.97	3602.05	0	127180	600077
r25105	382.35	0.55	172.43	3601.58	1	88580	554502
r25106	378.47	0.68	122.45	3604.93	2	112480	582452
r25107	365.45	0.68	115.21	3601.68	0	121030	593202
r25108	334.38	0.68	107.97	3601.68	0	128105	601477
r25109	366.29	0.64	132.21	3600.85	0	103280	571952
r25110	334.61	0.67	110.98	3601.19	0	120005	591602
r25111	336.38	0.66	115.21	3602.06	0	119305	590877
r25112	258.47	0.59	106.34	3601.77	0	131080	605677
r25201	442.21	0.61	170.96	3600.44	17281	90730	559102
r25202	356.26	0.66	122.45	3603.29	1	114380	585777
r25203	350.88	0.67	115.21	3600.98	0	122380	595727
r25204	301.11	0.64	107.97	3607.22	1	128905	603027
r25205	346.34	0.62	132.21	3602.69	2	106905	577552
r25206	308.96	0.65	107.97	3604.85	1	120455	593027
r25207	304.01	0.64	107.97	3601.94	0	124755	598427
r25208	265.33	0.59	107.97	3601.73	0	129430	603652
r25209	328.76	0.65	115.24	3604.31	1	115580	587652
r25210	367.06	0.69	115.21	3604.17	2	118380	590827
r25211	275.77	0.61	106.34	3601.44	0	131105	605727

Table 43 – Results for the class of instances RC, with 50% drone speed and 25% drone eligible weight, using the HGA as a starting solution.

Instance	Objective Function	GAP	Dual	Exec. Time	Nodes	Variables	Constraints
rc25101	406.84	0.24	309.63	3600.39	148168	28255	479927
rc25102	441.77	0.72	125.54	3631.48	6964	33705	489577
rc25103	358.59	0.76	86.11	3608.60	15199	37855	496377
rc25104	303.42	0.72	84.41	3601.60	1314	39005	498677
rc25105	513.32	0.65	179.86	3600.43	11582	32330	486452
rc25106	331.78	0.65	116.39	3625.40	5699	32780	487352
rc25107	287.15	0.73	78.86	3601.23	2308	36930	494677
rc25108	287.15	0.73	78.66	3601.62	2564	39805	500127
rc25201	360.35	0.00	360.34	28.64	7943	31080	485852
rc25202	338.68	0.65	117.56	3614.36	12682	35630	493577
rc25203	326.84	0.74	86.42	3601.22	4760	39155	498977
rc25204	304.08	0.72	84.53	3603.37	1931	39930	500527
rc25205	337.40	0.46	180.72	3600.94	10644	35180	492152
rc25206	325.07	0.63	119.72	3604.92	17478	35255	492452
rc25207	295.65	0.70	88.28	3601.43	4579	37655	496577
rc25208	260.22	0.70	78.40	3602.07	702	40655	501827

Table 44 – Results for the class of instances RC, with 50% drone speed and 50% drone eligible weight, using the HGA as a starting solution.

Instance	Objective Function	GAP	Dual	Exec. Time	Nodes	Variables	Constraints
rc25101	340.31	0.45	188.05	3600.66	41939	36130	501402
rc25102	331.04	0.83	57.14	3601.12	4	43830	513202
rc25103	306.04	0.88	38.14	3601.45	1	49130	521127
rc25104	287.27	0.90	30.04	3602.12	1	51030	524177
rc25105	450.41	0.72	124.37	3600.79	5	41630	509252
rc25106	305.81	0.84	47.26	3601.24	4	42355	510427
rc25107	276.40	0.88	31.95	3601.51	3	48830	520052
rc25108	272.51	0.89	29.95	3601.68	1	52355	526152
rc25201	348.73	0.23	267.85	3600.41	189965	39105	507477
rc25202	334.93	0.81	63.95	3601.93	258	45755	517202
rc25203	320.90	0.88	38.47	3601.19	5	50430	523727
rc25204	297.04	0.89	31.87	3602.26	2	51955	526027
rc25205	333.99	0.54	154.39	3601.96	207	44655	515127
rc25206	318.90	0.82	57.33	3601.57	1038	44380	515077
rc25207	290.32	0.88	33.74	3602.44	34	48905	521327
rc25208	252.22	0.88	29.95	3601.05	0	53205	527852

Table 45 – Results for the class of instances RC, with 50% drone speed and 75% drone eligible weight, using the HGA as a starting solution.

Instance	Objective Function	GAP	Dual	Exec. Time	Nodes	Variables	Constraints
rc25101	450.81	0.66	154.21	3600.77	70739	37630	506577
rc25102	378.69	0.88	46.81	3600.45	0	46230	519277
rc25103	311.90	0.90	31.37	3601.59	1	51980	527652
rc25104	286.12	0.91	25.30	3602.26	2	54380	531202
rc25105	444.98	0.75	112.55	3600.84	3	43880	515177
rc25106	361.77	0.91	33.41	3601.23	3	44955	516702
rc25107	277.21	0.90	27.04	3601.84	1	52030	526927
rc25108	271.23	0.91	25.13	3601.79	3	55855	533327
rc25201	350.48	0.24	264.63	3600.39	97080	40605	512652
rc25202	334.93	0.80	65.81	3620.15	2771	48155	523277
rc25203	321.86	0.90	32.80	3634.34	62	53280	530252
rc25204	295.15	0.91	25.34	3602.00	2	55305	533052
rc25205	331.00	0.57	143.03	3601.58	648	46905	521052
rc25206	384.27	0.88	46.76	3601.18	20	46830	521202
rc25207	287.28	0.91	25.75	3600.70	0	51655	527752
rc25208	246.14	0.90	25.12	3602.66	1	56705	535027

Table 46 – Results for the class of instances RC, with 100% drone speed and 25% drone eligible weight, using the HGA as a starting solution.

Instance	Objective Function	GAP	Dual	Exec. Time	Nodes	Variables	Constraints
rc25101	392.82	0.33	262.15	3600.78	45323	33255	485052
rc25102	441.16	0.70	130.28	3601.44	2530	39355	495277
rc25103	373.61	0.76	88.37	3601.52	8	44530	503102
rc25104	355.12	0.76	83.93	3602.57	425	45905	505627
rc25105	488.03	0.64	177.99	3600.96	9751	38155	492377
rc25106	394.68	0.71	113.08	3601.41	5813	38505	493152
rc25107	286.93	0.73	78.31	3616.08	1459	43955	501752
rc25108	286.93	0.73	78.66	3602.59	853	46780	507152
rc25201	336.95	0.14	289.77	3675.45	131371	35580	490527
rc25202	335.81	0.65	117.55	3600.70	12654	40780	498802
rc25203	320.39	0.73	86.58	3602.20	2121	45830	505702
rc25204	302.09	0.72	83.09	3602.42	559	46830	507477
rc25205	337.18	0.47	177.04	3600.67	17171	40755	497802
rc25206	308.70	0.62	118.85	3600.65	9390	40930	498202
rc25207	291.31	0.69	90.05	3602.00	1253	44455	503427
rc25208	254.87	0.69	78.61	3603.17	575	47705	508927

Table 47 – Results for the class of instances RC, with 100% drone speed and 75% drone eligible weight, using the HGA as a starting solution.

Instance	Objective Function	GAP	Dual	Exec. Time	Nodes	Variables	Constraints
rc25101	377.09	0.82	69.17	3601.05	1	54030	523027
rc25102	361.33	0.89	41.00	3600.76	0	67580	540677
rc25103	300.84	0.90	31.01	3600.11	0	75005	550727
rc25104	275.68	0.91	24.95	3603.90	0	79555	556427
rc25105	393.97	0.90	38.21	3600.74	0	64605	535977
rc25106	286.73	0.89	31.98	3602.17	1	65805	537602
rc25107	264.54	0.91	24.88	3601.27	0	77205	552152
rc25108	260.32	0.90	24.84	3600.17	0	81555	559077
rc25201	402.08	0.54	184.30	3601.11	1173	57630	529752
rc25202	328.37	0.87	42.61	3601.54	3	69355	544527
rc25203	306.59	0.90	31.02	3602.61	1	76730	553752
rc25204	284.20	0.91	24.95	3601.24	0	80505	558302
rc25205	325.21	0.83	56.08	3601.35	7	67080	541277
rc25206	308.30	0.88	36.82	3601.51	1	67680	542102
rc25207	278.37	0.91	25.04	3601.18	0	75780	551927
rc25208	231.11	0.89	24.84	3601.37	0	82605	560977

Table 48 – Results for the class of instances RC, with 150% drone speed and 25% drone eligible weight, using the HGA as a starting solution.

Instance	Objective Function	GAP	Dual	Exec. Time	Nodes	Variables	Constraints
rc25101	450.54	0.43	256.66	3601.13	36158	49355	501177
rc25102	428.65	0.76	103.89	3601.32	2	59830	515777
rc25103	320.32	0.73	84.70	3604.56	328	68780	527352
rc25104	302.88	0.73	82.31	3605.10	132	70455	530177
rc25105	452.16	0.61	177.55	3602.46	2179	59530	513752
rc25106	389.78	0.71	113.08	3603.21	382	59730	514377
rc25107	286.93	0.72	80.46	3603.51	1	67905	525702
rc25108	286.93	0.73	78.31	3601.93	2	72255	532627
rc25201	341.14	0.28	244.11	3600.44	73539	52105	507102
rc25202	329.42	0.69	103.07	3601.99	390	61180	519202
rc25203	320.39	0.74	84.70	3603.19	3	70630	530502
rc25204	302.24	0.72	83.93	3605.03	436	71755	532402
rc25205	370.63	0.52	176.63	3601.92	3492	61555	518602
rc25206	308.70	0.62	117.08	3601.98	717	61530	518802
rc25207	291.31	0.70	87.88	3603.90	132	67530	526527
rc25208	255.09	0.69	78.55	3606.02	561	73105	534327

Table 49 – Results for the class of instances RC, with 150% drone speed and 50% drone eligible weight, using the HGA as a starting solution.

Instance	Objective Function	GAP	Dual	Exec. Time	Nodes	Variables	Constraints
rc25101	365.58	0.63	136.03	3601.68	2	91180	556527
rc25102	364.03	0.87	46.94	3602.25	1	116055	585477
rc25103	294.23	0.87	37.93	3601.81	0	131905	603952
rc25104	271.78	0.89	29.54	3602.74	0	138130	611327
rc25105	368.41	0.88	45.93	3601.39	0	112405	580102
rc25106	277.57	0.83	45.93	3603.75	2	116155	584277
rc25107	263.37	0.89	29.54	3601.15	0	135455	606727
rc25108	256.03	0.88	29.54	3602.23	0	143130	616977
rc25201	378.99	0.66	127.23	3601.35	11933	97480	565952
rc25202	397.86	0.88	46.94	3601.94	0	118855	590352
rc25203	311.83	0.88	37.93	3602.58	0	134955	608302
rc25204	279.43	0.89	29.54	3602.05	0	140030	614152
rc25205	324.89	0.69	100.69	3602.40	1	114505	585027
rc25206	305.70	0.83	51.93	3602.40	2	114730	585477
rc25207	275.15	0.88	33.54	3601.78	0	130905	603377
rc25208	234.18	0.87	29.54	3602.46	0	144180	618877

Table 50 – Results for the class of instances RC, with 150% drone speed and 75% drone eligible weight, using the HGA as a starting solution.

Instance	Objective Function	GAP	Dual	Exec. Time	Nodes	Variables	Constraints
rc25101	363.55	0.84	56.45	3601.84	1	100830	569852
rc25102	348.75	0.88	40.77	3601.22	0	130405	603502
rc25103	287.38	0.89	30.80	3602.84	0	147655	623377
rc25104	272.66	0.91	24.80	3602.77	0	155630	632502
rc25105	427.61	0.93	31.67	3603.74	1	125030	596402
rc25106	271.50	0.88	33.24	3601.54	0	130455	602252
rc25107	260.45	0.90	24.78	3602.08	0	152580	627527
rc25108	257.81	0.90	24.78	3600.41	0	161380	638902
rc25201	352.71	0.87	46.14	3601.03	22	107280	579427
rc25202	323.31	0.87	40.77	3601.20	0	132580	607752
rc25203	308.97	0.90	30.81	3601.99	0	150105	627127
rc25204	282.22	0.91	24.80	3602.81	0	157530	635327
rc25205	328.99	0.90	32.16	3603.04	1	127055	601252
rc25206	306.11	0.90	31.67	3601.17	0	129030	603452
rc25207	274.81	0.91	24.82	3602.18	0	146780	622927
rc25208	228.84	0.89	24.78	3603.48	0	162430	640802

A MIXED INTEGER LINEAR PROGRAMMING MODEL FOR THE DRONE-ASSISTED TSP

Here we present the paper published in the *Simpósio Brasileiro de Pesquisa Operacional, 2018* ([COSTA et al., 2018](#)). This work was derived from an early brainstorm on drones and logistics applications, as this real case of drone usage in Iceland ([O'KANE, 2017](#)) made us wonder if this type of application was more likely to be expanded. However, since the circumstances in which this problem is more relevant are less general, i.e. the physical constraints such as lakes, islands, mountains, etc. are not applicable to every site, we decided to follow a more general possibility of the VRPD.

A Mixed Integer Linear Programming Model for the Drone-Assisted TSP

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ABSTRACT

In this article, the Drone-Assisted Traveling Salesman Problem (DATSP), a new context in which drones are used to extend a classic TSP, is explored. Drones travel unimpeded by traffic and can reach areas of difficult access, whereas other vehicles might not be able to do so. In this new paradigm, an obstacle, such as a river or a mountain, would hinder a company's delivery time to the other side of that obstacle. The drone would then fly repeatedly across it, carrying packages which are then delivered to the clients by a deliveryman. This research is motivated by a real life application, where an Icelandic company started using drones to deliver food across a bay. This operational problem inspired the DATSP, which is modeled mathematically as a Mixed Integer Linear Programming (MILP) problem in this article. Finally, instances were generated and solved with the solver GUROBI and their solutions were investigated.

KEYWORDS. Drones. Traveling Salesman Problem. Mathematical Model.

Paper topics (L&T - Logística e Transportes)

1. Introduction

Drones have increasingly become part of modern society. Many companies have sought to streamline their services using drones in the context of delivering goods to customers. Icelandic company AHA, for instance, is already using Flytrex's drones to fly over a bay in Reykjavik, cutting costs and delivery times significantly [Gilchrist, 2017], as seen in Figure 1. Zipline, a company in Silicon Valley, has teamed up with the Rwandan health department and is saving lives using drones to deliver blood and medicine to hospitals in remote regions [Simmons, 2016]. DHL, the German postal service, trialled delivery drones in combination with fixed stations between which the drones fly, saving around 20 minutes in delivery time during winter [DHL, 2016]. Amazon announced plans in 2013 to make deliveries using drones 30 minutes after customers purchase an item on their website [CBS, 2013]. Dominos is testing pizza deliveries using drones in New Zealand directly to customers [Scholz, 2016]. UPS successfully tested using drones in tandem with a truck to deliver products to clients in rural areas [Lithia, 2017].

Figure 1: A comparison between a drone's route (green) and a delivery vehicle's (red) in Reykjavik.

Source: Flytrex, found in [Gilchrist, 2017].

This sudden interest in this type of application is due to the advantages drones provide, such as not following traffic, being able to reach areas of difficult access, being efficient energy-wise [Stolaroff et al., 2018], and being greener alternatives, which could reduce greenhouse gas emissions by almost 54% [Stolaroff and Samaras, 2018] when compared to trucks. Due to these factors, drones are already being studied in the academic field. For instance, [Murray and Chu, 2015] use a drone in tandem with a truck in the context of last-mile delivery, similarly to what has been tested by UPS. The authors propose the use of a drone to deliver to clients while the truck delivers to other clients and serves as a base for the drone, reducing the total delivery time when compared to a classic Traveling Salesman Problem (TSP) model. The work of [Dorling et al., 2017] uses drones to make deliveries within a radius around the depot on their own, allowing for multiple trips and multiple clients in the same route.

Despite the benefits granted from using drones, there are still many hindrances that need to be assessed. The issue of privacy is among the problems being discussed, since drones use cameras for their guidance systems. The risk of damage to private property or to people in case of accidents is also taken into account. The re-use of drones and the toll on battery life are also issues

that hinder the continuous use of drones in the context of delivering goods, since modern delivery drone technology does not allow for long flight times. Moreover, modern cargo drone models have load capacities limited to only a few kilos. In spite of these issues, the authors deem pertinent the use of drones in logistics, since there are many potential benefits to using drones. Amazon's CEO, for instance, claimed in an interview that 86% of all deliveries the company makes weigh less than 3 kilos [Rose, 2015], which would more than justify their investments in drone technology.

This paper proposes and solves the Drone-Assisted Traveling Salesman Problem (DATSP), where a drone is employed to gradually transport products (such as food, medicine, or any products light enough) to a deliveryman, aiming to minimize the operational time. Unlike the other works mentioned before, the drone does not itself deliver to the customers, its function is to transport those products to the deliveryman who is then responsible for delivering them to the customers. In fact, the authors could not find a similar problem that was explored in the literature.

The rest of this paper is organized as follows: Section 2 explains the DATSP; Section 3 presents the mathematical model based on Mixed Integer Linear Programming techniques; Next, Section 4 shows the computational results obtained using the solver GUROBI. Finally, Section 5 concludes the work and presents research perspectives.

2. The Drone-Assisted TSP

The problem to be discussed in this section is, to the authors' best knowledge, not one that has been treated in the literature. The goal of the Drone-Assisted Traveling Salesman Problem (DATSP) is to utilize a drone to help in the delivery of goods by flying over areas of difficult access, such as rivers, lakes, bays, forests or mountains. The scenario involved comprises a drone, the obstacle, and the deliveryman waiting across from the obstacle. The drone flies over and drops off the goods to the deliveryman who delivers them to the clients on the other side. Since the total cargo to be delivered exceeds the drone's load capacity, multiple trips are necessary until every product has been delivered. After the drone unloads the products, the deliveryman executes the route and meets the drone at the drop-off point to receive a new batch of goods that needs to be delivered. The deliveryman's route is described by the solution to a TSP. Thus, the decisions of the DATSP are determining how many visits the drone makes, which client is served in which visit, and the deliveryman's routes, minimizing total operational time. A similar mode of operation being used by Icelandic company AHA demonstrates that using drones to help deliveries manages to reduce time and costs to the company [Gilchrist, 2017].

Figure 2 illustrates the DATSP, where two landmasses separated by a body of water can only be crossed using a boat or a ferry (there is no bridge nearby). On one side there is a company that delivers products locally. On the other side there are customers to whom delivering products might not be feasible without an alternative mode of transportation. The presence of an obstacle hence limits the number of possible customers to that company or discourages clients from using their services. Therefore, a drone can fly across the water and the deliveryman waiting on the other side can deliver to those customers. In the example, the route shown visits both customers with a green mark, whereas the other ones will be served during later visits.

Companies that deliver food, medicine or other light objects would benefit from this type of application by increasing their client base, reducing delivery times, improving market competitiveness and customer satisfaction. Given that this type of delivery is time-sensitive, the DATSP model could be used to schedule deliveries in short periods of time. For instance, in the context of food delivery, the customer does not expect to wait more than two hours, thus a possible strategy could be to solve the DATSP hourly or whenever a certain amount of orders is placed.

3. The Mathematical Model for the DATSP

The problem can be represented by a graph in which $V = \{0, 1, \dots, n\}$ is the set of vertices (0 being the drone's drop-off point) and $E = \{(i, j) | i, j \in V\}$ is the arcs set, in which (i, j) is an

Figure 2: An example of a DATSP application: A company utilizes the drone to cross the river.

Source: Authors, based on "Designed by Freepik" artwork.

arc that represents the deliveryman's transit from i to j . Each client i also has a demand that must be served by the deliveryman.

The main decisions of the DATSP are determining the number of trips the drone makes, the clients that deliveryman serves after each trip and the routes the deliveryman travels, aiming to minimize the total operational time. All demands are known at the start of the operation, and, since the total weight of the demands exceeds the drone's load capacity, many trips between the depot and the drone's drop-off point are necessary. For every time the drone unloads goods at the drop-off point, the route that must be executed by the deliveryman must be determined in a way that the deliveryman will always return to the drop-off point before the drone arrives with more products to be delivered. There is no priority between clients. The mathematical model for the DATSP is presented with its parameters and variables as follows.

Parameters:

- V : vertex set;
- C : client set ($C = V \setminus \{0\}$);
- M : set of the possible number of times the drone unloads at the drop-off point;
- l_i : the demand of client i ;
- d_{ij} : the distance between clients i and j ;
- Q_d : the drone's load capacity;
- Q_e : the deliveryman's load capacity;
- v_d : the drone's average flight speed;
- v_e : the deliveryman's average speed;
- τ : average total time the drone takes to take off and land;
- h : the distance between the depot and the drone's drop-off point;
- T_{max} : maximum operational time;
- K : a large enough number.

Variables:

x_{ijm} :	$\begin{cases} 1 & \text{if the deliveryman leaves } i \text{ and goes to } j \text{ on the drone's } m\text{-th visit;} \\ 0 & \text{otherwise.} \end{cases}$
p_{im} :	$\begin{cases} 1 & \text{if client } i \text{ is served on the drone's } m\text{-th visit;} \\ 0 & \text{otherwise.} \end{cases}$
s_{im} :	$\begin{cases} 1 & \text{if client } i \text{ has already been served on the drone's } m\text{-th visit or before;} \\ 0 & \text{otherwise.} \end{cases}$
y_m :	$\begin{cases} 1 & \text{if an } m\text{-th drone visit happens;} \\ 0 & \text{otherwise.} \end{cases}$
t_i :	instant in which the deliveryman leaves client i ;
a_{im} :	instant in which the deliveryman returns to the depot on the drone's m -th visit if client i was the last client in this particular visit;
T :	instant in which the deliveryman returns to the drop-off point after serving the last client overall;
γ_m :	instant in which the drone makes its m -th visit;
q_i :	total load delivered by the deliveryman after leaving client i .

- Flow constraints

$$\sum_{m \in M} p_{im} = 1, \quad \forall i \in C; \quad (1)$$

$$\sum_{\substack{i \in V \\ i \neq j}} x_{ijm} = p_{jm}, \quad \forall j \in C, \forall m \in M; \quad (2)$$

$$\sum_{j \in C} x_{0jm} = y_m, \quad \forall m \in M; \quad (3)$$

$$\sum_{\substack{i \in V \\ i \neq j}} x_{ijm} = \sum_{\substack{i \in V \\ i \neq j}} x_{jim}, \quad \forall j \in V, \forall m \in M; \quad (4)$$

Constraints (1) guarantee that every client is served by the end of the operation. Constraints (2) link the visit variables with the flow variables, where if a client j is served on the m -th visit, then the deliveryman necessarily serves j exactly once on that route. Constraints (3) demand that deliveries only happen in the m -th visit if and only if the drone makes that visit. Constraints (4) are responsible for ensuring the flow of the routes, meaning if the deliveryman arrives at client j 's location on the m -th visit, then he must also leave j on that same visit.

- Visit constraints

$$y_{m-1} \geq y_m, \quad \forall m \in M \setminus \{1\}; \quad (5)$$

$$p_{im} \leq s_{im}, \quad \forall i \in C, \forall m \in M; \quad (6)$$

$$s_{i,(m-1)} \leq s_{im}, \quad \forall j \in C, \forall m \in M \setminus \{1\}; \quad (7)$$

$$\sum_{i \in C} (1 - s_{i,(m-1)}) \leq |C| y_m, \quad \forall m \in M \setminus \{1\}; \quad (8)$$

$$\sum_{i \in C} (1 - s_{i,(m-1)}) \geq y_m, \quad \forall m \in M \setminus \{1\}; \quad (9)$$

Constraints (5) guarantee that the m -th visit will happen if and only if the previous visit occurs as well. Constraints (6) link the variable that determines if client i is served on visit m , p_{im} , to the variable that memorizes the clients that have already been served, s_{im} . For instance, if client 4 is served on visit number 3, $p_{4,3} = 1$ and, therefore, $s_{4,3} = 1$. Constraints (7) ensure that, if client i has already been served, then this information is passed on to future visits. Constraints (8) and (9) establish the drone's stopping point, that is, if all the clients have already been served on the m -th visit, then there are no more visits from the drone.

- Load constraints

$$q_j \leq q_i + l_j + K(1 - \sum_{m \in M} x_{ijm}), \quad \forall i \in V, j \in C, i \neq j; \quad (10)$$

$$q_j \geq q_i + l_j - K(1 - \sum_{m \in M} x_{ijm}), \quad \forall i \in V, j \in C, i \neq j; \quad (11)$$

$$l_i \leq q_i \leq \min\{Q_e, Q_d\}, \quad \forall i \in C; \quad (12)$$

$$q_0 = 0; \quad (13)$$

Constraints (10) and (11) determine the load that has already been delivered by the deliveryman after serving client j , preceded by client i , if the route goes from i to j . Constraints (12) guarantee that the load capacity will never be exceeded. Constraint (13) indicates the total cargo delivered at the start of every route.

- Time constraints

$$t_j \leq t_i + \frac{d_{ij}}{v_e} + K(1 - \sum_{m \in M} x_{ijm}), \quad \forall i \in C, j \in C, i \neq j; \quad (14)$$

$$t_j \geq t_i + \frac{d_{ij}}{v_e} - K(1 - \sum_{m \in M} x_{ijm}), \quad \forall i \in C, j \in C, i \neq j; \quad (15)$$

$$t_j \leq \gamma_m + \frac{d_{0j}}{v_e} + K(1 - x_{0jm}), \quad \forall j \in C, \forall m \in M; \quad (16)$$

$$t_j \geq \gamma_m + \frac{d_{0j}}{v_e} - K(1 - x_{0jm}), \quad \forall j \in C, \forall m \in M; \quad (17)$$

Constraints (14) and (15) refer to the delivery times, meaning the time when the deliveryman starts serving client j , if he serves client i before, is the time when he arrived on client i plus the time to travel between i and j . Constraints (16) and (17) refer to the times of the first deliveries of each route, in which the first client to be served in the m -th visit depends on the time the drone arrives at the drop-off point, and the time to travel from the drop-off point to client j .

- Time constraints for the last deliveries in each route

$$a_{im} \leq t_i + \frac{d_{i0}}{v_e} + K(1 - x_{i0m}), \quad \forall i \in C, \forall m \in M; \quad (18)$$

$$a_{im} \geq t_i + \frac{d_{i0}}{v_e} - K(1 - x_{i0m}), \quad \forall i \in C, \forall m \in M; \quad (19)$$

$$a_{im} \leq Kx_{i0m}, \quad \forall i \in C, \forall m \in M; \quad (20)$$

$$T \geq a_{im}, \quad \forall i \in C, \forall m \in M; \quad (21)$$

$$T \leq T_{max}; \quad (22)$$

Constraints (18) and (19) determine the time when the deliveryman returns to the drone's drop-off point in each visit m , similarly to constraints (14) and (15). Constraints (20) guarantee that the time a_{im} will be zero if i is not the last client to be served in the m -th visit. Constraints (21) define the floor for the time of last delivery T . Constraint (22) limits the time for the last delivery to the maximum delivery time.

- The drone's time constraints

$$\gamma_m \geq 2\left(\tau + \frac{h}{v_d}\right) + \gamma_{m-1} - K(1 - y_m), \quad \forall m \in M \setminus \{1\}; \quad (23)$$

$$\gamma_1 = \tau + \frac{h}{v_d}; \quad (24)$$

Constraints (23) determine the minimum intervals between each of the drone's visits, which are the time to travel back and forth between the depot and the drop-off point if the m -th visit happens. Constraint (24) establishes the time of the first visit.

- Constraints synchronizing the drone and the deliveryman

$$a_{im} \geq \gamma_m - K(1 - x_{i0m}), \quad \forall i \in C, \forall m \in M; \quad (25)$$

$$a_{i(m-1)} \leq \gamma_m + K(1 - x_{i0(m-1)}) \quad \forall i \in C, \forall m \in M \setminus \{1\}; \quad (26)$$

Constraints (25) ensure that the deliveryman's m -th route ends after the drone's m -th visit, if visit m happens. Constraints (26) guarantee that the deliveryman will be at the drone's drop-off point when the drone arrives.

$$x_{ijm} \in \{0, 1\}, \quad \forall i \in V, \forall j \in V, \forall m \in M; \quad (27)$$

$$p_{im} \in \{0, 1\}, \quad \forall i \in V, \forall m \in M; \quad (28)$$

$$s_{im} \in \{0, 1\}, \quad \forall i \in V, \forall m \in M; \quad (29)$$

$$y_m \in \{0, 1\}, \quad \forall m \in M; \quad (30)$$

$$t_i \geq 0, \quad \forall i \in C; \quad (31)$$

$$a_{im} \geq 0, \quad \forall i \in C, \forall m \in M; \quad (32)$$

$$\gamma_m \geq 0, \quad \forall m \in M. \quad (33)$$

The domain of the variables is described in (27) - (33)

- Objective function

$$\text{Minimize } T \quad (34)$$

The objective function (34) aims to minimize the total operational time T . Thus, the model for the DATSP can be written as such:

$$\begin{aligned} &\text{Minimize } T \\ &\text{subject to: } (1) - (33) \end{aligned}$$

4. Computational experiments

In this section the computational experiments and their results are presented.

4.1. Test environment

The model was implemented using the programming language Python v.2.7.3 and the results were obtained using the optimization solver GUROBI v.7.0.1. The tests were executed in a computer with an Intel(R) Xeon(R) CPU E5-2620 0 @ 2.00GHz processor and 70 GB of RAM in a Linux Ubuntu v.3.5.0 x64 operating system.

4.2. Test instances data

Five test instances were developed for the problem, containing 4, 9, 16, 25 and 32 customers (see Table 2), randomly distributed on a $10km$ by $10km$ field, where the depot is located at the bottom-left corner. The demands of these clients were generated randomly between 1, 2, and 3 units of weight, and the load capacities were of 4 units of weight for the drone and 5 units of weight for the deliveryman. The average speeds were defined as $60km/h$ for the drone and $40km/h$ for the deliveryman. The distance between the depot and the drone's drop-off point is $2km$, while the time for the drone to take off and land was defined as 1 minute. Moreover, M was defined as a set $\{1, \dots, n\}$, in which n is the number of clients in a given instance, since n is the maximum number of visits necessary to serve every client. The maximum operational time was defined as $T_{max} = 30h$. The values considered for K in each constraint are described in Table 1.

Table 1: Calculation of the value given to K in each constraint.

No.	Constraint	Value
(10)	$q_j \leq q_i + l_j + K(1 - \sum_{m \in M} x_{ijm})$	$K = \min\{Qe, Qd\}$
(11)	$q_j \geq q_i + l_j - K(1 - \sum_{m \in M} x_{ijm})$	$K = \min\{Qe, Qd\} + \max_{i \in C}\{l_i\}$
(14)	$t_j \leq t_i + \frac{d_{ij}}{v_e} + K(1 - \sum_{m \in M} x_{ijm})$	$K = T_{max}$
(15)	$t_j \geq t_i + \frac{d_{ij}}{v_e} - K(1 - \sum_{m \in M} x_{ijm})$	$K = T_{max} + \max_{i,j \in C}\{\frac{d_{i,j}}{v_e}\}$
(16)	$t_j \leq \gamma_m + \frac{d_{0j}}{v_e} + K(1 - x_{0jm})$	$K = T_{max}$
(17)	$t_j \geq \gamma_m + \frac{d_{0j}}{v_e} - K(1 - x_{0jm})$	$K = T_{max} + \max_{j \in C}\{\frac{d_{0,j}}{v_e}\}$
(18)	$a_{im} \leq t_i + \frac{d_{i0}}{v_e} + K(1 - x_{i0m})$	$K = T_{max}$
(19)	$a_{im} \geq t_i + \frac{d_{i0}}{v_e} - K(1 - x_{i0m})$	$K = T_{max} + \max_{i \in C}\{\frac{d_{i,0}}{v_e}\}$
(20)	$a_{im} \leq Kx_{i0m}$	$K = T_{max}$
(23)	$\gamma_m \geq 2(\tau + \frac{h}{v_d}) + \gamma_{m-1} - K(1 - y_m)$	$K = 2(\tau + \frac{h}{v_d}) + T_{max}$
(25)	$a_{im} \geq \gamma_m - K(1 - x_{i0m})$	$K = T_{max}$
(26)	$a_{i,m-1} \leq \gamma_m + K(1 - x_{i0(m-1)})$	$K = T_{max}$

4.3. Results

Table 2 presents the results obtained using the model and test instances. The maximum execution time was defined as 3600 seconds. The first two columns of the table indicate the instance number and the number of clients. The solutions for each instance are in column *O.F.* (Objective Function). The fourth column, *Bound*, presents the values of the lower bounds. Column *Visits*

shows how many visits the drone made to the drop-off point in each instance. In *Gap* the relative gap values are presented. Column *Execution time* displays the execution times in seconds. The last two columns present the number of variables and constraints generated in each instance.

Table 2: Results for the instances obtained using the solver GUROBI.

Inst.	Clients	Model					Dimensions	
		O.F. (h)	Bound	Visits	Gap	Execution time (s)	Variables	Constraints
1	4	1.04	1.04	2	0.00	< 1	146	297
2	8	2.01	2.01	4	0.00	77	802	1142
3	15	3.63	0.70	9	0.81	3600	4337	3928
4	24	6.11	0.30	13	0.95	3600	16226	9958
5	32	7.56	0.06	17	0.99	3600	36994	17630

Analyzing the results obtained, it is clear that the dimensions of the problem grow immensely when the number of clients is increased. It is also worth noting that, for smaller instances, the solver found the optimal solutions quickly, but cannot do so for larger instances. One aspect that might be responsible for the poor quality of the dual bounds of the problem is the amount of constraints that include the parameter K .

Figure 3 displays the order of events in the solution for instance 1. Firstly, the drone arrives at the drop-off point (γ_1 , that is, $m = 1$), bringing with it the products for customers 2 and 3. The deliveryman then travels a route defined by the TSP between the two, going first to client 3 (time t_3), and then to client 2 (time t_2), returning to the drop-off point at time a_{21} . The drone's second visit (γ_2 , $m = 2$) occurs at the same time the deliveryman returns from his first route. The drone does not wait at the drop-off point, as expected. The deliveryman then serves the remaining customers, 4 (time t_4) and 1 (time t_1), returning to the drop-off point at time a_{12} . As there are no more clients left, the drone does not make any new trips, marking the end of the operation.

Figure 3: Timeline of events for Instance 1.

Source: Authors.

Figure 4 shows the complete route for instance 1 in the same order as presented in Figure 3. Figure 5 illustrates the routes taken by the deliveryman during each of the drone's visits.

Figure 4: The complete route traveled by the deliveryman.

Source: Authors.

Figure 5: Individual routes taken by the deliveryman in each visit.

Source: Authors.

5. Concluding Remarks

In this article a preliminary study on the Drone-Assisted Traveling Salesman Problem (DATSP) was introduced. The DATSP is a new type of problem wherein a drone flies over an obstacle carrying products to a deliveryman. The authors establish that there is commercial relevance in this type of application, given that several companies have shown interest in using drones. A mathematical model elaborated as a Mixed Integer Linear Programming (MILP) problem was developed so as to represent the DATSP.

In order to test the model, five different instances were created and then solved using the solver GUROBI. The results showed that the mathematical model behaves as expected, providing optimal solutions for small instances. However, the optimal solution is not found for larger instances within the time limit, due both to the poor quality of the dual bounds and to the problem's NP-Hard nature, since the complexity of the problem grows exponentially as the number of clients increases.

Since the model does not provide optimal solutions for instances of medium and large scales, the study of heuristics or other solution methods is the starting point for any future research. Investigations concerning the mathematical model can be made so as to bring it closer to reality, as

the current model does not consider the drone's battery consumption, which may change depending on the weight being carried, the total flight time, and even the drone's model. Another perspective could be to expand the current model for multiple drones or multiple deliverymen.

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