UNIVERSIDADE DE SÃO PAULO

Instituto de Ciências Matemáticas e de Computação

Mathematical modeling of ripple- and oscillation-mark formation in the casting of steel

Marcos Zambrano Fernández

Tese de Doutorado do Programa de Pós-Graduação em Ciências de Computação e Matemática Computacional (PPG-CCMC)

SERVIÇO DE PÓS-GRADUAÇÃO DO ICMC-USP

Data de Depósito:

Assinatura: _____

Marcos Zambrano Fernández

Mathematical modeling of ripple- and oscillation-mark formation in the casting of steel

Doctoral dissertation submitted to the Institute of
Mathematics and Computer Sciences – ICMC-USP, in
partial fulfillment of the requirements for the degree of
the Doctorate Program in Computer Science and
Computational Mathematics. *FINAL VERSION*

Concentration Area: Computer Science and
Computational Mathematics

Advisor: Prof. Dr Jose Alberto Cuminato

USP – São Carlos
November 2018

Ficha catalográfica elaborada pela Biblioteca Prof. Achille Bassi
e Seção Técnica de Informática, ICMC/USP,
com os dados inseridos pelo(a) autor(a)

Z24m Zambrano Fernandez, Marcos
 Mathematical modeling of ripple- and oscillation-
 mark formation in the casting of steel / Marcos
 Zambrano Fernandez; orientador Jose Alberto
 Cuminato. -- São Carlos, 2018.
 118 p.

 Tese (Doutorado - Programa de Pós-Graduação em
 Ciências de Computação e Matemática Computacional) --
 Instituto de Ciências Matemáticas e de Computação,
 Universidade de São Paulo, 2018.

 1. Mathematical modeling. 2. Casting of steel.
 3. Asymptotic methods. I. Cuminato, Jose Alberto,
 orient. II. Título.

Marcos Zambrano Fernández

**Modelagem matemática da formação de marcas ondulantes
e oscilantes em lingotamento de aço**

Tese apresentada ao Instituto de Ciências
Matemáticas e de Computação – ICMC-USP,
como parte dos requisitos para obtenção do título
de Doutor em Ciências – Ciências de Computação e
Matemática Computacional. *VERSÃO REVISADA*

Área de Concentração: Ciências de Computação e
Matemática Computacional

Orientador: Prof. Dr Jose Alberto Cuminato

USP – São Carlos
Novembro de 2018

In memory of my brother Rafael Zambrano Fernández.

ACKNOWLEDGEMENTS

First, I want to express my gratitude to Dr. Michael Vynnycky for the continuous support and guidance to the conclusion of this thesis related to the mathematical modeling of processes of the casting of steel.

Second, I would like to thank Prof. Cuminato for accepting to be my advisor during these four years and for the support that I received from him.

Third, I would like to thank the University of São Paulo for accepting me as a doctoral student and the Instituto de Computação e Matemática Computational (ICMC) for allowing me to start my research in this very interesting area of applied mathematics: computational fluid dynamics and heat transfer processes.

Also, I would like to thank CAPES and CNPq for two fellowships (reference PROEX-8769350/D and SWE-CNPq-208697/2015-2, respectively) which allowed me to develop the objectives proposed in every step of these four years, during which I was a PHD student at ICMC.

This study was financed in part by the Coordenação de Aperfeiçoamento de Pessoal de Nível Superior - Brasil (CAPES) - Finance Code 001.

*“I do not want to live in a
world where everything I do and say is recorded.
That is not something I am willing to support
or live under.”
(Edward Snowden)*

ABSTRACT

ZAMBRANO M. **Mathematical modeling of ripple- and oscillation-mark formation in the casting of steel**. 2018. 118 p. Tese (Doutorado em Ciências – Ciências de Computação e Matemática Computacional) – Instituto de Ciências Matemáticas e de Computação, Universidade de São Paulo, São Carlos – SP, 2018.

Ripple marks and oscillation marks are undesirable defects which occur on the surface of solidified steel produced industrially in the ingot and the continuous casting processes, respectively; these defects are characterized by more or less evenly spaced indentations on the metal surface. Although the mechanisms for their formation are thought to be qualitatively understood, there is still considerable scope for improvement as regards quantitative mathematical modeling. In this thesis, models for the two processes are developed. For the case of ripple marks, transient two-dimensional (2D) momentum and heat transfer in ingot casting is considered, and a criterion is derived, in terms of the process parameters, that can help to inform how to avoid such marks. For the case of oscillation marks in continuous casting, a novel numerical formulation for a transient 2D model is developed with the aim of tracking the spatial location of the first point of molten steel to solidify, since this determines the profile of the final oscillation mark. In both cases, the models are nondimensionalized, and the sizes of the dimensionless parameters that appear are used to derive asymptotically reduced models, with a view to not only clarifying the qualitative behavior, but also as a means to reducing the computational expense; both finite-difference and finite-element methods are used to solve the resulting model equations. One of the conclusions is that, although experimentalists and metallurgists have, in the past, treated the two cases as being linked, the present modeling approach shows quite clearly, and perhaps for the first time, how they quantitatively differ.

Keywords: Mathematical modeling, ripple marks, oscillation marks, first point of molten steel.

RESUMO

ZAMBRANO M. **Modelagem matemática da formação de marcas ondulantes e oscilantes em lingotamento de aço**. 2018. 118 p. Tese (Doutorado em Ciências – Ciências de Computação e Matemática Computacional) – Instituto de Ciências Matemáticas e de Computação, Universidade de São Paulo, São Carlos – SP, 2018.

Marcas de ondulação e marcas de oscilação são defeitos indesejáveis que ocorrem na superfície do lingote de aço solidificado produzido industrialmente; esses defeitos são caracterizados por recortes mais ou menos uniformemente espaçados na superfície do metal. Embora se acredite que os mecanismos para sua formação sejam entendidos qualitativamente, ainda há considerável espaço para melhorias no que diz respeito à modelagem matemática quantitativa. Nesta tese, os modelos para os dois processos são desenvolvidos. Para o caso de marcas de ondulação, considera-se a transferência bidimensional e transitória (2D) de calor e de momento no lingotamento, e um critério é derivado, em termos dos parâmetros do processo, que pode ajudar a informar como evitar tais marcas. Para o caso de marcas de oscilação em lingotamento contínuo, uma nova formulação numérica para um modelo 2D transiente é desenvolvida com o objetivo de rastrear a localização espacial do primeiro ponto de aço fundido para solidificar, pois isso determina o perfil da marca final de oscilação. Em ambos os casos, os modelos são adimensionalizados, e os tamanhos dos parâmetros adimensionais que aparecem são usados para derivar modelos assintoticamente reduzidos, visando não apenas esclarecer o comportamento qualitativo, mas também como meio de reduzir o gasto computacional; ambos os métodos de diferenças finitas e elementos finitos são usados para resolver as equações dos modelos resultantes. Uma das conclusões é que, embora os experimentalistas e metalúrgicos tenham, no passado, tratado os dois casos como estando ligados, a presente abordagem de modelagem mostra claramente, e talvez pela primeira vez, como eles diferem quantitativamente.

Palavras-chave: Modelagem matemática, marcas ondulantes, marcas de oscilação, primeiro ponto de aço fundido.

LIST OF FIGURES

Figure 1 – A cast ingot sample.	25
Figure 2 – Square billets.	26
Figure 3 – Round billets.	26
Figure 4 – A sample of a slab in the steelmaking industry	26
Figure 5 – Schematic of a downhill casting process.	27
Figure 6 – Schematic of an uphill casting process.	28
Figure 7 – Schematic diagram of the continuous casting process	28
Figure 8 – Schematic diagram of the onset of solidification of steel.	29
Figure 9 – Negative strip time.	30
Figure 10 – Surface marks produced in the ingot casting of steel.	30
Figure 11 – Appearance of oscillation marks produced during continuous casting.	31
Figure 12 – Schematic representation of the formation of a ripple mark.	36
Figure 13 – Schematic of the mold used by Jacobi and Schwerdtfeger to study the mark formation at the surface of a solidifying static steel ingot.	36
Figure 14 – (a) The overflow mechanism; and (b) the folding mechanism.	37
Figure 15 – Prediction of the location of oscillation marks on a cast shell surface via heat transfer measurements.	38
Figure 16 – The strand surface with oscillation marks for a portion of a slice at the center of the broad face for incoloy alloy 825 continuously cast bloom. (CD - Casting direction).	39
Figure 17 – Experimental results showing the number of marks associated with a given pitch. The vertical dashed line shows the average value.	40
Figure 18 – Two adjacent oscillation marks and typically observed microstructure under- neath them.	40
Figure 19 – Experimental results showing the number of marks associated with a given pitch.	41
Figure 20 – Oscillation-mark formation during an oscillation period t_0	42
Figure 21 – (a) Oscillation marks obtained from the model developed by Hill <i>et al.</i> ; and (b) effect of the amplitude of the mold oscillation on the depth of the oscillation marks.	43
Figure 22 – Mold velocity, casting speed and mold displacement in one oscillation cycle.	43
Figure 23 – Temperature field, meniscus and velocity vectors in one oscillation cycle. All dimensions are in millimeters.	44

Figure 24 – Schematic of oscillation mark formation.	45
Figure 25 – Profiles of the solid flux/molten flux, molten flux/solid steel and solid steel/- molten steel interfaces, given by $x = s$, $x = s + h$ and $x = s + h + f$, respectively. 46	
Figure 26 – Comparison of the oscillation mark profiles measured experimentally and theoretically.	47
Figure 27 – Schematic for uphill casting.	50
Figure 28 – Schematic of modeling for a rising meniscus.	50
Figure 29 – Schematic of the regions for the shape of the meniscus.	56
Figure 30 – Structure of the meniscus profile.	59
Figure 31 – Geometry and boundary conditions for equation (3.83).	63
Figure 32 – θ_{tip} vs τ at $\tilde{X} = 0$ using $B = B_{crit}$	66
Figure 33 – θ on the surface of the molten steel for $\tau = 0.0, 1.0, 2.0, 3.0$ and $\tau = \tau^*$	66
Figure 34 – Solution θ in the whole region of the thermal boundary layer at $\tau = \tau^*$. . .	67
Figure 35 – V_c as a function of $T_c - T_m$ (dashed line) and regions where the solidification of steel takes place as given by inequality (3.104).	68
Figure 36 – Schematic diagram showing the process of oscillation-mark formation . . .	70
Figure 37 – V as a function of τ (solid curve), as well the curve $V = 1$ (dashed line). Each interval of τ for which $V > 1$ is commonly referred to as the “negative strip time”.	77
Figure 38 – Typical mesh to compute the fluid flow for the molten flux in the upper zone. 86	
Figure 39 – $Q_U^{(l)}$ as a function of τ using equation (4.96) for for different values of X_∞ . .	87
Figure 40 – $Q_U^{(l)}$ as a function of τ using equation (4.96) for X_0 for different values of N 87	
Figure 41 – $Q_U^{(l)}$ as a function of τ using equation (4.96) for X_0 , using the true value of $\delta (= 0.6)$ and $\delta = 0$ in equations (4.73) and (4.74)	88
Figure 42 – X_0 as a function of τ , determined using equation (4.94), with $\delta = 0$ and for four different values for H_0^* , using the weakly quasi-steady approach. For these runs, $Z_0 = 1$	90
Figure 43 – Magnitude of the velocity of the fluid flow at $\tau = 0$ when $X_0 = 0.2$	92
Figure 44 – X_0 as a function of τ using $\delta = 0$ and the weakly quasi-steady approach, and with $Z_0 = 0.3, 0.6, 0.9$	92
Figure 45 – X_0^{max} vs. Z_0	93
Figure 46 – Asymptotic structure near the meniscus tip once solidification starts.	98
Figure 47 – Asymptotic structure near the meniscus tip once solidification starts.	99
Figure A.1– θ vs. η at $X = 1$ for $\tau = \tau^*$ when solving (A7) for $Pe = 200, 400, 800$	106
Figure A.2– θ vs. X at $\eta = 1$ for $Pe = 200, 400$ and 800	107
Figure B.1– X_0 vs. τ showing the formation of oscillation marks as result of the numerical solution to equations (B1)-(B3) for $Re_f = 2, 4, 8, 16$	110
Figure B.2–Maximum value of X_0 by computing the depth of an oscillation mark	111

LIST OF ALGORITHMS

Algorithm 1 – Computation of B_{crit}	64
Algorithm 2 – Computation of X_0	91

LIST OF TABLES

Table 1 – Model parameters from Saleem (SALEEM, 2016) and Vynnycky <i>et al.</i> (VYN- NYCKY <i>et al.</i> , 2017) *Mills <i>et al.</i> (MILLS <i>et al.</i> , 2016b)	76
Table 2 – Values of dimensionless parameters using data from Hill <i>et al.</i> (HILL; WU; WIWATANAPATAPHEE, 1999) and Saleem (SALEEM, 2016)	80

LIST OF SYMBOLS

B — dimensionless constant, $\lambda Pe^{-1/2}$

B_{crit} — critical value of B

Bi — Biot number, $mW/k_f^{(s)}(1 + Rm)$

Bo — Bond number, $\rho g W^2 / \gamma_{LG}$

Ca — Capillary number, $\mu_f V_c / \sigma_{fs}$

c_p — specific heat capacity [$\text{J kg}^{-1} \text{K}^{-1}$]

Fr — Froude number, V_c^2 / gW

$\hat{g}$ — gravitational acceleration [m s^{-2}]

h — meniscus height [m]

H — dimensionless meniscus height

$\bar{H}$ — scaled dimensionless meniscus height

h_0 — initial meniscus height [m]

H_0 — initial dimensionless meniscus height

k — thermal conductivity [$\text{W m}^{-1} \text{K}^{-1}$]

L_e — latent heat of vaporization [J kg^{-1}]

M — molecular weight [kg mol^{-1}]

p — pressure [Pa]

P — dimensionless pressure

p_a — atmospheric pressure [Pa]

Pe — Péclet number, $\rho c_p V_c W / k$

Q — mold-wall heat flux profile [W m^{-2}]

$\mathcal{Q}$ — dimensionless mold-wall heat flux profile

$[Q]$ — characteristic scale for Q [W m^{-2}]

Re — Reynolds number, $\rho W V_c / \mu$

t — time [s]

T — temperature [K]

T_{amb} — ambient temperature [K]

T_c — casting temperature [K]

T_m — melting temperature [K]

u — x -direction velocity component [ms^{-1}]

U — dimensionless x -direction velocity component

v — y -direction velocity component [ms^{-1}]

V — dimensionless y -direction velocity component

V_c — casting velocity [ms^{-1}]

V_0 — initial volume of melt per unit length [m^2]

W — half-width of the mold region [m]

x — horizontal coordinate [m]

X — dimensionless horizontal coordinate

y — vertical coordinate [m]

Y — dimensionless vertical coordinate

Greek symbols —

ε_0 — radiation heat transfer emissivity of the molten metal surface

η — transformation variable, $Y/(Y_0 + \tau)$

θ — dimensionless temperature

λ — dimensionless parameter, $[Q]W/k(T_c - T_m)$

μ — dynamic viscosity of molten metal [$\text{kg m}^{-1}\text{s}^{-1}$]

ρ — molten metal density [kg m^{-3}]

$\tilde{\sigma}$ — Stefan-Boltzmann constant, $5.6704 \times 10^{-8} \text{W m}^{-2} \text{K}^{-4}$

γ_{LG} — liquid-gas surface tension coefficient [N m^{-1}]

γ_{SG} — solid-gas surface tension coefficient [N m^{-1}]

γ_{SL} — solid-liquid surface tension coefficient [N m^{-1}]

τ — dimensionless time, $t/(W/V_c)$

ϕ_c — contact angle [$^\circ$]

σ_{fs} — surface tension between molten flux and molten steel [N m^{-1}]

CONTENTS

1	INTRODUCTION	25
1.1	Background to the casting of steel	25
1.1.1	<i>Ingot casting</i>	27
1.1.2	<i>Continuous casting</i>	27
1.2	Problems in the steelmaking industry	30
1.3	Contributions	31
1.4	Outline	32
2	LITERATURE REVIEW	35
2.1	Ripple marks	35
2.2	Oscillation marks	37
2.2.1	<i>Experimental</i>	37
2.2.1.1	<i>Tomono (TOMONO, 1979)</i>	37
2.2.1.2	<i>Badri et al. (BADRI et al., 2005)</i>	38
2.2.1.3	<i>Saleem and co-workers (SALEEM, 2016; SALEEM; VYNNYCKY; FREDRIKSSON, 2016)</i>	39
2.2.2	<i>Modeling</i>	40
2.2.2.1	<i>Steinrück et al. (STEINRÜCK; RUDISCHER; SCHNEIDER, 1997)</i>	40
2.2.2.2	<i>Hill et al. (HILL; WU; WIWATANAPATAPHEE, 1999)</i>	41
2.2.2.3	<i>Jonayat and Thomas (JONAYAT; THOMAS, 2014)</i>	43
2.2.2.4	<i>Vynnycky et al. (VYNNYCKY et al., 2017)</i>	44
3	RIPPLE MARKS	49
3.1	Introduction	49
3.2	Model equations	50
3.3	Nondimensionalization	53
3.4	Analysis	55
3.4.1	$1 - X \sim O(1)$	56
3.4.2	$1 - X \sim O(Bo^{-1/2})$	57
3.4.3	$1 - X \sim O(Pe^{-1/2})$	58
3.5	Numerics for the thermal boundary layer	62
3.6	Results	65
4	OSCILLATION MARKS	69

4.1	Introduction	69
4.2	Modeling considerations	71
4.3	Problem formulation	72
4.3.1	<i>Governing equations</i>	72
4.3.2	<i>Boundary, interfacial and initial conditions</i>	73
4.4	Nondimensionalization	76
4.4.1	<i>Governing equations</i>	77
4.4.2	<i>Boundary conditions</i>	78
4.4.3	<i>Nondimensional parameters</i>	79
4.5	Reduced model	80
4.5.1	<i>Sub-problem I: velocity field in the molten flux</i>	81
4.5.2	<i>Sub-problem II: velocity field in the molten steel</i>	81
4.5.3	<i>Sub-problem III: temperature field in the molten steel</i>	82
4.5.4	<i>Sub-problem IV: temperature field in the solid and molten flux</i>	82
4.5.5	<i>Caveats</i>	82
4.6	Analysis	83
4.7	Numerical implementation and experimentation	85
4.8	Results	91
5	CONCLUSION AND FUTURE WORK	95
5.1	Summary of the work	95
5.2	Originality	96
5.3	Significance	96
5.4	Unifying ripple and oscillation marks	97
5.5	Future work	97
	BIBLIOGRAPHY	101
APPENDIX A	BENCHMARKING FOR RIPPLE MARKS	105
APPENDIX B	BENCHMARKING FOR OSCILLATION MARKS	109
APPENDIX C	SOME TECHNICAL TERMS IN PORTUGUESE LANGUAGE RELATED TO CASTING OF METALS	113
ANNEX A	PUBLICATIONS IN JOURNALS	115

INTRODUCTION

1.1 Background to the casting of steel

Steel production worldwide is on the increase: for instance, in 2017, between the months of January and October, 1.4 billion tons of steel were produced, up from 1.34 billion tons in the corresponding months in 2016 ([BRASIL, 2017](#)). Also, in Brazil, steel production has been increasing in recent years: between the months of January and October in 2016, just over 26 million tons were produced and around 28.5 million tons were produced between the same months in 2017 ([BRASIL, 2017](#)). Most of this activity results in the manufacture of steel in several forms: ingots, billets, blooms and slabs.

An example of an ingot produced in the steelmaking industry is shown in figure 1; typically, an ingot may weigh between 5.7 and 7.2 tons and have cross-sectional dimensions

Figure 1 – A cast ingot sample.

Source: ([OUTOKUMPU, 2016](#)).

between $629 \times 629 \text{ mm}^2$ and $790 \times 790 \text{ mm}^2$.

In figure 2, three samples of square billets are shown; their dimensions vary between 3.8

to 12 meters in length and between $127 \times 127 \text{mm}^2$ and $200 \times 200 \text{mm}^2$ in cross-section. Blooms are similar to these, although they have non-square cross-sections.

Figure 2 – Square billets.

Source: ([OUTOKUMPU, 2016](#)).

Also, there exist round billets, whose dimensions vary between 3 and 12 meters in length and have a diameter between 75 and 400mm; they are shown in figure 3.

Figure 3 – Round billets.

Source: ([OUTOKUMPU, 2016](#)).

Finally, another important product of the steelmaking industry is the slab, which is shown in figure 4. Their dimensions vary in thickness between 170 and 300mm, in width between 700 and 1575mm and in length between 3 and 12 meters.

Figure 4 – A sample of a slab in the steelmaking industry

Source: ([OUTOKUMPU, 2016](#)).

In the steelmaking industry, two main processes are used to make these forms: conventional or ingot casting, and continuous casting. These are explained in more detail next.

1.1.1 Ingot casting

In conventional or ingot casting, molten steel is tapped from a furnace into ladles of up to 300 tonne capacity; the ladle is then moved to a teeming platform where molten metal is poured into molds and allowed to solidify into ingots. These ingots are subsequently reheated and rolled to produce semi-finished shapes such as slabs, blooms and billets.

In the case of ingot casting, the mold wall which surrounds the molten steel is static and the process of casting is characterized by the fact that once the mold region is filled with molten steel, the next step is to wait until the solidification of the metal is complete.

The process of steelmaking for ingot casting considers the teeming of molten steel in two ways. Either the melt is teemed from above into the mold as shown in figure 5, which is usually known as *downhill casting*, or the melt is supplied from below, as shown in figure 6; this is usually known as *uphill casting* (FREDRIKSSON; ÅKERLIND, 2006).

Figure 5 – Schematic of a downhill casting process.

Source: Elaborated by the author.

In chapter 3, the process of uphill casting is investigated and there, some results of the present work are presented.

1.1.2 Continuous casting

In the last four decades, a process known as continuous casting has emerged with success around the world. In this process, which is as depicted in figure 7, the molten steel is cast directly into semi-finished steel shapes. The major components of the process are:

Figure 6 – Schematic of an uphill casting process.

Source: Elaborated by the author.

Figure 7 – Schematic diagram of the continuous casting process

Source: Elaborated by the author.

- a ladle;
- a tundish;
- a water-cooled, oscillating, open-ended copper mold;
- a secondary cooling or spray cooling;
- withdrawal rollers.

In this process, the molten delivered from the ladle to the tundish; then, it is delivered to

the mold region, wherein it starts to solidify. Both ends of the mold are open and the withdrawing rollers extract the partially solidified strand from the mold as shown in figure 7.

During this process of cooling, a mold powder is added at the top of the mold, which sinters and then becomes liquid to act as a lubricant, usually known as slag or *molten flux*. If lubricant were not used, solidified steel would simply stick to the mold, with detrimental results. Note also that the molten flux may re-solidify, leading to what is referred to as *rim*. However, the oscillation of the mold helps to prevent over-agglomeration of rim.

At the same time, inside of the mold there exists a point where the solidification of the steel starts, denoted by (x_0, z_0) in figure 8, and determines two zones: *the upper zone* which is upwards of this point and *the lower zone* where the steel shell starts to form downwards. This point is usually referred to as *the triple point*.

As shown in figure 8, in the upper zone, there exists an interface between the molten flux (or liquid slag) and the molten steel which is called *meniscus* and has its origin at the triple point.

During the process of solidification of steel, below the triple point, a shell of solid steel is forms and leaves the mold at velocity V_c , usually known as the *casting velocity*, while the mold is oscillating vertically at velocity V_m , usually known as the *mold velocity*.

Figure 8 – Schematic diagram of the onset of solidification of steel.

Source: Elaborated by the author.

The mold velocity can be expressed by

$$V_m = V_0 \cos(\omega t), \quad (1.1)$$

where V_0 is the maximum amplitude of the oscillation and ω is its angular frequency. The period of time in which, $V_m > V_c$ is referred to as the *negative strip time* (NST) and is shown in figure 9, where the value of t_0 is obtained from $V_m = V_c$ and (1.1), and is given by

$$t_0 = \frac{1}{\omega} \cos^{-1}\left(\frac{V_c}{V_0}\right), \quad (1.2)$$

being the half of the NST.

Figure 9 – Negative strip time.

Source: Elaborated by the author.

1.2 Problems in the steelmaking industry

In both ingot and continuous casting, it has been observed that the final product has surface depressions or indentations; these are considered as defects which must be minimized or avoided in order to obtain better quality steel. Figure 10 shows surface marks, known as *ripple*

Figure 10 – Surface marks produced in the ingot casting of steel.

Source: (JACOBI; SCHWERDTFEGER, 2013).

marks, that appear in ingot casting. They usually have a spacing of the order of millimeters and a depth of the order of tens of micrometers, and lie parallel to the surface of the cast (VYNNYCKY; ZAMBRANO; CUMINATO, 2015).

Similar marks have also been observed on the surfaces of products made by continuous casting, as shown in figure 11; these marks are perpendicular to the casting direction, i.e. the direction in which the steel is leaving the mold, and are known as *oscillation marks*.

Figure 11 – Appearance of oscillation marks produced during continuous casting.

Source: (TAKEUCHI; BRIMACOMBE, 1984).

Whether such defects can be tolerated depends, in part, on the customer specifications, and often there is some leeway in tolerating marks that have a depth of fractions of a millimetre. However, such marks are often the source of surface cracks that penetrate deeper into the steel sample. Also, since steel is a multicomponent alloy, marks are the sites of significant segregation, meaning that the composition is not uniform there; this also leads to quality issues. Often, industry works around the problems of oscillation and ripple marks by grinding the surface of the steel after it has cooled down; however, this consumes a lot of energy. Therefore, an alternative is to obtain a better insight into the physical phenomena and the mechanisms involved by means of mathematical modeling. However, even this is not entirely straightforward, since mark-formation is clearly a three-dimensional and time-dependent process. Moreover, there is vast disparity of scales: whereas the marks themselves are associated with a length scale of a millimetres, the casting process itself occurs over metres.

1.3 Contributions

The present thesis has two main contributions, which are related to the casting of steel. They are the mathematical modeling of the onset of the formation of ripple marks and the modeling of a mechanism for the formation of oscillation marks, presented, respectively, in chapters 3 and 4; these are explained in more detail below.

1. For ingot casting, a two-dimensional (2D) mathematical model is formulated for the fluid flow and heat transfer of molten steel, confined in a mold, with a rising meniscus. Then, by nondimensionalizing the variables, the key parameters are identified and thereafter, a

reduced asymptotic model is developed, which leads to a moving boundary problem that is solved numerically. The solution of this problem allows us to establish a criterion for controlling the onset of the solidification of steel and hence the ripple mark formation.

2. In continuous casting, using the ideas of Steinrück *et al.* (STEINRÜCK; RUDISCHER; SCHNEIDER, 1997) related to the meniscus and the ideas of Hill *et al.* (HILL; WU; WIWATANAPATAPHEE, 1999) for the molten flux/solid steel interface, a 2D mathematical model that describes the formation of oscillation marks, through the fluid flow of the molten flux and the oscillatory motion of the mold, is formulated and solved numerically.
3. While Steinrück *et al.* (STEINRÜCK; RUDISCHER; SCHNEIDER, 1997) proposed a model in which the solidification point remains in a fixed position, the model developed in this work considers a solidification point, or triple point, located at $(x_0(t), z_0(t))$, that is free to move in space and with time, i.e. a moving point of solidification. Moreover, whereas Vynnycky *et al.* (VYNNYCKY *et al.*, 2017) and Hill *et al.* (HILL; WU; WIWATANAPATAPHEE, 1999) proposed a model for the formation of oscillation marks, wherein the upper zone has a flat meniscus, in the present work, a more realistic profile of the meniscus is considered in this upper zone. The shape of this profile depends on the interfacial tension between the molten flux and the molten steel and is expressed mathematically by the capillary number.
4. A novel formulation for describing the motion of the triple point is developed. At this point, three interfaces - molten flux/molten steel, molten steel/solid steel and solid steel/molten flux - meet, and knowledge of its motion makes it possible to predict the shape of the oscillation marks.

1.4 Outline

The work is structured as follows:

- In chapter 2, a literature review is presented in two sections: first for ripple marks and then for oscillation marks.

To begin with, the work of Tomono *et al.* (TOMONO; KURZ; HEINEMANN, 1981), wherein a mechanism for the formation of ripple marks was proposed for the first time, is reviewed. After this, relevant aspects related to the formation of the meniscus are explained.

Then, the experimental work on ripple marks by Jacobi and Schwerdtfeger (JACOBI; SCHWERDTFEGER, 2013) is touched upon. Here, a mechanism for the formation of ripple marks, based on their experiments carried out over a period of thirty years, is mentioned.

Next, an overview of the literature on oscillation marks is given. It starts with the work of Tomono (TOMONO, 1979), which proposes two mechanisms for the formation of oscillation marks, and is followed by the experimental work of Badri *et al.* (BADRI *et al.*, 2005), wherein the significance of the heat flux at the mold wall is described.

Then, from the work of Saleem and co-workers (SALEEM; VYNNYCKY; FREDRIKSSON, 2016; SALEEM, 2016), a sample of oscillation marks obtained in his experiments is shown.

After that, the mathematical modeling of oscillation marks is explained briefly. The modeling of Steinrück *et al.* (STEINRÜCK; RUDISCHER; SCHNEIDER, 1997) for oscillation marks is presented; here, it is shown that the interfacial tension determines the shape of the meniscus between the molten flux and molten steel, and some results of their mathematical model are commented on.

Thereafter, some aspects of the analysis of Hill *et al.* (HILL; WU; WIWATANAPATAPHEE, 1999) are mentioned and their results are shown, principally, those wherein the depth of an oscillation mark and its dependence on the mold oscillation and casting velocity are mentioned.

The work of Jonayat and Thomas (JONAYAT; THOMAS, 2014), wherein a model coupling the temperature field and the fluid flow of the molten flux was employed, is explained and some selected results are shown.

Finally, an overview of the ideas and formulation of a model for the formation of oscillation marks of fold-type, expressed by Vynnycky *et al.* (VYNNYCKY *et al.*, 2017), are given. This model produced results which were compared with experimental data.

- In chapter 3, a mathematical model for the onset of the formation of ripple marks in uphill casting is proposed. Then, a nondimensionalization is applied to the governing equations for the conservation of momentum, mass and heat. Thereafter, an asymptotic analysis allows us to simplify the nondimensionalized equations by identifying key parameters, leading towards a reduced asymptotic model.

This treatment allows to recognize the relation between the momentum boundary layer and the thermal boundary layer that exists adjacent to the mold wall. Then, numerical methods are used to obtain the temperature field of the molten steel adjacent to the mold.

- In chapter 4, an mathematical model for the formation of oscillation marks, via the motion of the triple point, is developed. Here, the onset of the solidification of the steel defines a triple point where three interfaces meet: molten flux-molten steel, molten flux-solid steel and molten steel-solid steel. Thus, this triple point defines the location of the start of the meniscus that determines the boundary between the molten flux and molten steel.

This chapter carries out an analysis of the molten flux in the region located above the triple point and the interaction between the molten flux and the molten steel using the Navier

Stokes equations in full form to show that this interaction is ruled by the interfacial tension, as explained by Steinrück *et al.* (STEINRÜCK; RUDISCHER; SCHNEIDER, 1997), thus determining the shape of the meniscus.

After this, a treatment of the Navier Stokes equations for the flow of the molten flux in the upper zone is developed, with the purpose of obtaining the mass of flux, which is to be equated to the mass of flux obtained by Vynnycky *et al.* (VYNNYCKY *et al.*, 2017) in the lower zone. This comparison allows to express an equation, wherein the time-dependent position of the triple point is obtained.

In addition, the motion of this triple point modifies the the region of the molten flux and makes it necessary to compute the solution of the Navier Stokes equations numerically on a mesh that moves in time. This motion is driven by the periodically oscillating mold wall.

- In chapter 5, a unified conclusion is drawn for chapters 3 and 4, and some ideas for future research related to ripple and oscillation marks are presented.

LITERATURE REVIEW

2.1 Ripple marks

Tomono *et al.* (TOMONO; KURZ; HEINEMANN, 1981) pointed out the importance of the first seconds of the solidification process in ingot casting, which determines the quality of the surface of the ingot. They observed periodic marks formed on the cast steel surface during experiments carried out and explained that the depth and the vertical spacing between two marks, usually known as the *pitch*, could be related to the cooling intensity at the mold wall. These experiments were carried out in a cylinder as mold, wherein there was an equilibrium as a result of the balance between the inner pressure given by $\rho \hat{g}y$ and the capillary pressure σ/R expressed by

$$\rho \hat{g}y = \frac{\sigma}{R}, \quad (2.1)$$

where $\rho, \hat{g}, y, \sigma, R$ are, respectively, the density of the molten metal, the acceleration due to the gravity, the vertical coordinate, the surface tension coefficient between the molten metal and the gas inside the mold and the radius of curvature. The principle of this balance that determines the shape of the meniscus also can be found in (BIKERMAN, 1970) and equation (2.1) is usually referred to as *the Bikerman equation*.

Tomono *et al.* (TOMONO; KURZ; HEINEMANN, 1981) also suggested that the molten steel at the meniscus might be mechanically supported by a solidified layer of steel, as shown in figure 12, and after some time the liquid steel will overflow this layer. Thus, the depth and the pitch of a ripple mark depend on the shape of the meniscus, related to the surface tension of the molten steel in contact with the gas inside of the mold.

Jacobi and Schwerdtfeger (JACOBI; SCHWERDTFEGER, 2013) described ripple marks as defects on the vertical surface of the ingot that occur due to that the molten steel at the meniscus solidifies, forming a solid contour that later, the molten steel arriving from below will flow over; hence, there will be an overflow, and each overflow determines a new ripple mark.

Figure 12 – Schematic representation of the formation of a ripple mark.

Source: (TOMONO; KURZ; HEINEMANN, 1981).

The experiments mentioned in (JACOBI; SCHWERDTFEGER, 2013), were carried out over a period of thirty years, in a mold as depicted in figure 13. There, the molten steel was added into the feeding channel via a small tundish placed above the upper opening of the feeding channel, then entered into the interior mold space through a hole at the center of the bottom plate.

Figure 13 – Schematic of the mold used by Jacobi and Schwerdtfeger to study the mark formation at the surface of a solidifying static steel ingot.

Source: (JACOBI; SCHWERDTFEGER, 2013).

2.2 Oscillation marks

2.2.1 Experimental

Although there exist numerous experimental works which document oscillation marks obtained under different conditions, here we review only those which can be said to have significantly advanced knowledge about how they are formed.

2.2.1.1 Tomono (TOMONO, 1979)

Arguably the earliest work to shed light on oscillation-mark formation in continuous casting was by Tomono (TOMONO, 1979). He proposed that one oscillation mark occurs in every oscillation cycle of the mold wall, and that each mark is formed due to the pressure caused by the mold wall and the shell of the solidified steel, thereby resulting in the deformation of the meniscus. However, as shown in figure 14, one of two alternatives may occur:

1. the formed shell is strong enough to withstand the pressure of the liquid steel and overflow occurs (figure 14(a));
2. the formed shell bends back under the pressure of the liquid and a folded mark is formed (figure 14(b)).

When one of last two alternatives finishes, a new cycle of the oscillation of the mold wall starts. It should also be pointed out that these figures are based on experiments using an organic solvent that is teemed into a mold region; consequently, in both cases, the level of the meniscus rises according to the numerical sequence given in each sub-figure. Thus, although the configuration is not exactly identical to that in continuous casting, the observations have nevertheless been used to posit mechanisms for oscillation-mark formation in this process.

Figure 14 – (a) The overflow mechanism; and (b) the folding mechanism.

Source: (TOMONO, 1979).

2.2.1.2 Badri et al. (BADRI et al., 2005)

Much later, in an effort to understand the link between oscillation-mark formation and heat transfer, Badri *et al.* (BADRI *et al.*, 2005) constructed a laboratory-scale simulator and pointed out that each oscillation mark can be related to an increase in the heat flux. Thus, an increase of heat flux during the negative strip time indicates the partial solidification of the liquid steel at the meniscus; this constitutes the first part of the formation of the mark. The final part of such formation depends on the strength of the shell. Thus, positive peaks of heat flux can be thought of as indications of the formation of oscillation marks. This analysis of the heat flux correlates quite well with the physical location of oscillation marks, as shown in figure 15.

Figure 15 – Prediction of the location of oscillation marks on a cast shell surface via heat transfer measurements.

Source: (BADRI *et al.*, 2005).

Local conditions will determine the type of the mark. On a frozen and strong meniscus, the marks will be the result of an overflow while if meniscus lacks strength, the rising molten steel can force the shell back to the mold. These criteria are proposed by Badri *et al.* based upon experimental evidence on the formation of oscillation marks found in (BADRI *et al.*, 2005).

2.2.1.3 Saleem and co-workers (SALEEM, 2016; SALEEM; VYNNYCKY; FREDRIKSSON, 2016)

Whilst the above works focus on how oscillation marks might form, they do not consider what oscillation marks actually look like. However, this has been dealt with in detail recently by Saleem and co-workers (SALEEM, 2016; SALEEM; VYNNYCKY; FREDRIKSSON, 2016), who conducted metallographic studies on several different continuously cast samples, obtained from a number of industrial sites. From these studies, it becomes apparent that there is a great variety in the character and appearance of oscillation marks, as might be inferred from Tomono (TOMONO, 1979) and Badri *et al.* (BADRI *et al.*, 2005).

For example, figure 16 shows a series of micrographs for a small portion of the bloom of a steel grade known as *incoloy* 825. Therein, two types of oscillation marks were observed: hook (overflow marks) and non-hook (folded marks), wherein, **1** is a fold and **2** is a hook.

Figure 16 – The strand surface with oscillation marks for a portion of a slice at the center of the broad face for incoloy alloy 825 continuously cast bloom. (CD - Casting direction).

Source: (SALEEM; VYNNYCKY; FREDRIKSSON, 2016)

Although figure 16 shows only four oscillation marks, a total of 52 were collected. The data for these is shown in figure 17, wherein the number of marks associated with a given pitch can be observed and also the corresponding type of oscillation mark: overflow mark or folded mark.

Whilst figures 16 and 17 indicate the presence of both types of marks, Saleem (SALEEM, 2016) also collected data where only folded marks were present. Figure 18 shows the observed microstructure of two adjacent oscillation marks of fold-type. The results of these experiments were collected to obtain a distribution of the number of the marks and the corresponding pitch. The entire dataset of the spacing of measured oscillation mark is shown in figure 19; here, we see a total of 13 marks, which were analyzed and the average value of the pitch obtained was close to the theoretical value of $2\pi V_{cast}/\omega$ ($f = \omega/2\pi$). In particular, the marks shown in figure 18 correspond to two of the marks at 8mm in figure 19 (SALEEM, 2016). Note that in figure 19, as in figure 17, not all of the marks are spaced according to the theoretical value. There are numerous possible reasons for this: for example, the molten steel flow is turbulent; although the meniscus is monitored very carefully, it can nevertheless rise or fall by up to a millimetre. Moreover, the theoretical value is somewhat idealized, since it does not take into account thermal effects.

Figure 17 – Experimental results showing the number of marks associated with a given pitch. The vertical dashed line shows the average value.

Source: (SALEEM; VYNNYCKY; FREDRIKSSON, 2016)

Figure 18 – Two adjacent oscillation marks and typically observed microstructure underneath them.

Source: (SALEEM, 2016).

2.2.2 Modeling

In this section, some of the literature relating to the modeling of oscillation marks is examined in detail. A shorter review containing more references, but with less detail, is given in section 4.1.

2.2.2.1 Steinrück et al. (STEINRÜCK; RUDISCHER; SCHNEIDER, 1997)

Steinrück *et al.* (STEINRÜCK; RUDISCHER; SCHNEIDER, 1997) developed a 2D model for the formation of oscillation marks by considering a fixed point on the meniscus, where the onset of the solidification of steel starts. In their work was used the *capillary number* Ca given by

$$Ca = \frac{\mu V_c}{\sigma}, \quad (2.2)$$

where μ , V_c , σ are the viscosity of the molten steel, casting speed and the interfacial tension coefficient between the molten steel and molten flux, respectively. Also, in their work was used

Figure 19 – Experimental results showing the number of marks associated with a given pitch.

Source: (SALEEM, 2016).

the capillary length, given by

$$L = \sqrt{\frac{\sigma}{\hat{g}(\rho_s - \rho_f)}}, \quad (2.3)$$

where $\hat{g}, \rho_s, \rho_f$ are, respectively, the acceleration due to gravity, the density of the steel and the density of the flux. This value of L was used to nondimensionalize the variables of the mass and momentum conservation equations and thus to obtain a mathematical expression for the shape of the meniscus, which was

$$\bar{y}(\bar{x}, \bar{t}) = \sqrt{2} - \sqrt{4 - \bar{x}^2} + \ln \left(\frac{2 + \sqrt{4 - \bar{x}^2}}{(1 + \sqrt{2})\bar{x}} \right), \quad (2.4)$$

where $\bar{x}, \bar{y}, \bar{t}$ are, respectively, the vertical coordinate, the horizontal coordinate and time in nondimensionalized variables.

Figure 20 shows oscillation marks obtained using experimental parameters at four different times during one period of oscillation of the mold wall.

The most severe restriction in (STEINRÜCK; RUDISCHER; SCHNEIDER, 1997) is that the coupling with the heat transfer is not considered in the model. Thus, this isothermal model focuses its attention on obtaining the shape of the meniscus, fixing the solidification point of the steel where the oscillation marks begin to form.

2.2.2.2 Hill et al. (HILL; WU; WIWATANAPATAPHEE, 1999)

Hill *et al.* (HILL; WU; WIWATANAPATAPHEE, 1999), using an iterative procedure, obtained numerically the approximation of the thickness of the flow channel for the molten flux from the solidification point of steel downwards and the depth of the oscillation marks on the surface of the cast steel. To achieve these results, the flux flow equations were coupled with the heat transfer equation in the their model.

Figure 20 – Oscillation-mark formation during an oscillation period t_0 .

Source: (STEINRÜCK; RUDISCHER; SCHNEIDER, 1997).

The analysis performed by Hill *et al.* (HILL; WU; WIWATANAPATAPHEE, 1999) was based on a temperature-dependent viscosity and the solidification point of the steel was allowed to depend on time according to the evolution of the temperature field.

The mechanism of the formation of oscillation marks proposed by Hill *et al.* (HILL; WU; WIWATANAPATAPHEE, 1999) is summarized as follows: during the downward motion of the mold wall, molten flux is dragged into region between the mold wall and the molten steel; in turn, the molten steel is pushed away from the mold wall by the pressure exerted on the fluid flow in the region between the mold wall and the meniscus. Thereafter during the upward motion of the mold wall, the suction forces within the flux layer reduce its width to zero and the molten steel and molten flux fill the upper zone, i.e., from the triple point upwards. Also, as the cast steel moves downwards, the steel shell increases in thickness and a shrinkage of the steel takes place.

Another important feature of the model developed by Hill *et al.* (HILL; WU; WIWATANAPATAPHEE, 1999) is that the lubrication equation governs the flow of the molten flux, as considered by Fowkes *et al.* (FOWKES; WOODS, 1989); this is different from the model developed by Steinrück *et al.* (STEINRÜCK; RUDISCHER; SCHNEIDER, 1997), where the fluid flow of the flux between the mold wall and meniscus is governed by the full Navier Stokes equations.

Figure 21(a) shows the profile of the oscillation marks and the depth obtained by Hill *et al.* (HILL; WU; WIWATANAPATAPHEE, 1999), where the oscillation marks were pointed out as regularly spaced depressions with depth about 0.9mm and having the shape of hemispherical-type notches. In figure 21(b), we can observe the effect of the amplitude of the mold oscillation on the depth of the oscillation marks when this amplitude is varied from 2.5mm to 7.5mm.

Figure 21 – (a) Oscillation marks obtained from the model developed by Hill *et al.*; and (b) effect of the amplitude of the mold oscillation on the depth of the oscillation marks.

Source: (HILL; WU; WIWATANAPATAPHEE, 1999).

2.2.2.3 Jonayat and Thomas(JONAYAT; THOMAS, 2014).

Jonayat and Thomas (JONAYAT; THOMAS, 2014) coupled the transient energy equation with the Navier Stokes equations in two dimensions for the incompressible fluid flow of the molten flux; the equations were discretized and solved using the finite volume method on a fixed structured grid. The computations were carried out using the commercial software Ansys Fluent, where the model was implemented using the shape of the meniscus obtained from the Bikerman equation, as presented in (BIKERMAN, 1970).

Figure 22 shows the velocity of the mold, casting speed and displacement during one-cycle of the oscillation of the mold wall; here, the period in which the mold moves downward faster than the casting speed, called the negative strip time (NST), can be observed.

Figure 22 – Mold velocity, casting speed and mold displacement in one oscillation cycle.

Source: (JONAYAT; THOMAS, 2014)

In figure 23 are shown the temperature fields for the mold, the flux and the steel in six

Figure 23 – Temperature field, meniscus and velocity vectors in one oscillation cycle. All dimensions are in millimeters.

Source: (JONAYAT; THOMAS, 2014)

times of the simulation and the fluid flow of the molten flux or slag is indicated using arrows. Also, the shape of the meniscus changing from figure 23(a) through (f) corresponds to the points (1)-(6) denoted in figure 22 respectively. In Figure 23(a) is shown a rising rim that pulls the meniscus upwards. Then during the downward motion of the mold wall, the rim starts to squeeze the gap between the rim and the meniscus, as shown in figures 23(b) and (c).

2.2.2.4 Vynnycky et al. (VYNNYCKY et al., 2017).

Vynnycky et al. (VYNNYCKY et al., 2017) recapped the model previously formulated by Hill et al. (HILL; WU; WIWATANAPATAPHEE, 1999) for the oscillation marks using figure 24, where the solidification point, located at $z = z_0(t)$, allowed them to establish two zones: the upper zone ($z < z_0(t)$) and the lower zone ($z > z_0(t)$).

In the lower zone, the heat transfer equation in $0 < x < s(z, t)$, the solid flux region, is given by

$$\rho_f^{(s)} c_f^{(s)} \left(\frac{\partial T}{\partial t} + V_m \frac{\partial T}{\partial z} \right) = k_f^{(s)} \frac{\partial^2 T}{\partial x^2}, \quad (2.5)$$

where $T, c_f^{(s)}, k_f^{(s)}, \rho_f^{(s)}, V_m$ are, respectively, the temperature of the solid flux, the specific heat

where $c_s^{(s)}$, $k_s^{(s)}$ and $\rho_s^{(s)}$ are, respectively, the specific capacity, the thermal conductivity and the density of the solid steel.

Vynnycky *et al.* (VYNNYCKY *et al.*, 2017), using the equations mentioned above, computed the flow rate of flux in the lower zone, Q_L , and in the upper zone, Q^U . Hence, by the continuity of the flow rate of flux at $z = z_0$,

$$Q^U = Q_L \quad \text{at} \quad z = z_0(t); \quad (2.11)$$

it was then possible to obtain analytical expressions for f , h and s . The interfaces $x = s$, $x = s + h$ and $x = s + h + f$ are shown in figure 25.

Figure 25 – Profiles of the solid flux/molten flux, molten flux/solid steel and solid steel/molten steel interfaces, given by $x = s$, $x = s + h$ and $x = s + h + f$, respectively.

Source: (VYNNYCKY *et al.*, 2017).

Also, one achievement of the model developed by Vynnycky *et al.* (VYNNYCKY *et al.*, 2017) is the computation of the maximum depth of a oscillation mark, given by

$$\left(\frac{3[\mu_f](V_0 - V_c)}{(\rho_s - \rho_f)\hat{g}} \right)^{1/2}, \quad (2.12)$$

where $[\mu_f]$, ρ_s , ρ_f are, respectively: the viscosity of the flux, density of the steel, density of the flux.

Using the fold-type oscillation-mark profile obtained experimentally by Saleem (SALEEM, 2016) and already shown in figure 18, a comparison was made with the oscillation marks obtained analytically in (VYNNYCKY *et al.*, 2017), as shown in figure 26. The noteworthy feature here is that there is a good agreement for the maximum depth of the oscillation mark, and indeed for the steel surface profile as a whole.

Figure 26 – Comparison of the oscillation mark profiles measured experimentally and theoretically.

RIPPLE MARKS

3.1 Introduction

Ripple marks, also known as striations or grooves, are defects on the surface of ingots during the process of the traditional casting in the steelmaking industry. With the purpose to improve the understanding about the formation of ripplemarks and how they can be avoid, a mathematical model as function of process parameters can be of scientific interest ([VYNNYCKY; ZAMBRANO; CUMINATO, 2015](#)).

In this chapter, a mathematical model will be developed using the equations of momentum conservation and heat transfer, with the purpose of preventing the formation of ripple marks that appear at the so-called *pointed tip* or simply *tip*, at which the onset of the solidification of the steel takes place, adjacent to the mold wall.

We focus on the configuration given by Tomono *et al.* ([TOMONO; KURZ; HEINE-MANN, 1981](#)) and Jacobi and Schwerdtfeger ([JACOBI; SCHWERDTFEGER, 2013](#)), which is depicted in figure 27. Here, the molten steel enters into the mold through a hole at the bottom and subsequently rises. As the mold is cooled, the molten steel inside the mold also cools, leading eventually to the onset of solidification.

Ripple marks can form in one of two ways: if the shell is strong enough to avoid deformation, the molten steel at the meniscus overflows the tip; otherwise, if the shell is too weak, its tip bends back by the pressure of the molten steel, and hence a folding mechanism takes place.

The goal of this chapter is to determine how this situation can be avoided and the result of the analysis obtained in this chapter can become the starting point for future mathematical modeling of ripple-mark formation.

Figure 27 – Schematic for uphill casting.

Source: (VYNNYCKY; ZAMBRANO; CUMINATO, 2015).

3.2 Model equations

A vertical channel is considered as the mold region, wherein the interaction of momentum and heat transfer take place on a rising and possibly deforming molten steel meniscus; this situation is depicted in figure 28.

Figure 28 – Schematic of modeling for a rising meniscus.

Source: (VYNNYCKY; ZAMBRANO; CUMINATO, 2015).

The molten steel, initially at rest inside the mold whose walls are a distance $2W$ apart, is at a temperature T_c , which is greater than the melting temperature T_m . The surface of the molten

steel describes a meniscus profile given by $y = h_0(x)$, with x and y as the horizontal and vertical coordinates, respectively.

Subsequently, more molten steel is introduced into the mold at $y = 0$ and at a speed V_c and temperature T_c ; in this way, a profile $y = h(x, t)$, where t denotes a time, of the surface of molten steel is formed. Meanwhile, heat is extracted through the mold walls at $x = \pm W$, implying that the temperature of the meniscus may decrease to T_m , and that the meniscus freezes at the mold wall. This work focuses on how to control T_c and V_c , so as to prevent this situation from occurring.

Inside the mold, the governing equations for the fluid flow and heat transfer are given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (3.1)$$

$$\rho \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = -\frac{\partial p}{\partial x} + \mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right), \quad (3.2)$$

$$\rho \left(\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = -\frac{\partial p}{\partial y} + \mu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) - \rho \hat{g}, \quad (3.3)$$

$$\rho c_p \left(\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right), \quad (3.4)$$

where T is the molten steel temperature, the x - and y - components of the velocities are u and v respectively, p denotes the pressure, $\hat{g}$ is the acceleration due to the gravity, ρ the density of the molten steel, μ is the viscosity, c_p the heat capacity and k the thermal conductivity. It will be assumed that ρ, μ, c_p and k are constant.

Equation (3.1) implies the incompressibility of the fluid, i.e. mass conservation. Equations (3.2) and (3.3) express the conservation of the momentum; in addition, the flow is assumed to be laminar. Equation (3.4) is the heat transfer equation of the molten steel.

Since the configuration is symmetric about $x = 0$, the discussion will be limited to the region $x \geq 0$; thus the boundary conditions are given as follows.

At the inlet $y = 0$,

$$u = 0, \quad v = V_c, \quad T = T_c, \quad (3.5)$$

whereas at the axis of symmetry $x = 0$, we have

$$u = 0, \quad \frac{\partial v}{\partial x} = 0, \quad \frac{\partial T}{\partial x} = 0, \quad \frac{\partial h}{\partial x} = 0; \quad (3.6)$$

also, at the mold wall $x = W$, we have

$$u = 0, \quad v = 0, \quad k \frac{\partial T}{\partial x} = -Q(y), \quad (3.7)$$

where $Q > 0$ is the heat flux. Observe here the significance of the third equation in (3.7). If we assume that the mold is at a uniform constant temperature, then there is no problem to solve for: if the temperature is lower than the melting temperature of the molten metal, solidification

would begin immediately, and there is no possibility of preventing ripple marks; on the other hand, if the temperature is higher, solidification would not occur at all. On the other hand, a heat flux condition is more realistic for casting problems, since it is never possible to control the temperature of a mold directly. In ingot casting, all that can be said is that the outer surface of the mold is cooled by the natural convection of the surrounding air; hence not even the outer surface of the mold is directly controlled. Continuing with the other boundary conditions, we assume no slip and no normal flow for the components of the velocity. In addition we shall need to prescribe a condition for either h or $\partial h/\partial x$ at the mold wall. Typically in problems containing contact lines, as is the case here, it is usual to prescribe the contact angle via the Young equilibrium equation given by

$$\gamma_{SG} - \gamma_{LG} \cos(\phi_c) = \gamma_{SL}, \quad (3.8)$$

where γ_{SG} , γ_{LG} , and γ_{SL} are the surface tension coefficients for the solid mold wall-gas, molten steel-gas and solid mold wall-molten steel interfaces, respectively, and ϕ_c is the contact angle as shown in figure 28. Consequently, since $\partial h/\partial x = \cot(\phi_c)$ at $x = W$, we have

$$\frac{\partial h}{\partial x} = \tilde{\gamma}, \quad (3.9)$$

where

$$\tilde{\gamma} = \frac{\gamma_{SG} - \gamma_{SL}}{\sqrt{\gamma_{LG}^2 - (\gamma_{SG} - \gamma_{SL})^2}}. \quad (3.10)$$

For the moving boundary at $y = h(x, t)$, we have:

- a kinematic condition for the motion of the meniscus given by

$$v = \frac{\partial h}{\partial t} + u \frac{\partial h}{\partial x}; \quad (3.11)$$

- zero tangential stress, given by

$$2\mu \left(\frac{\partial v}{\partial y} - \frac{\partial u}{\partial x} \right) \frac{\partial h}{\partial x} + \mu \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \left\{ 1 - \left(\frac{\partial h}{\partial x} \right)^2 \right\} = 0; \quad (3.12)$$

- a normal stress condition given by

$$\begin{aligned} -p + \frac{2\mu}{1 + (\frac{\partial h}{\partial x})^2} \left[\frac{\partial u}{\partial x} \left(\frac{\partial h}{\partial x} \right)^2 + \frac{\partial v}{\partial y} - \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \left(\frac{\partial h}{\partial x} \right) \right] \\ = -p_a + \frac{\gamma_{LG} \partial^2 h / \partial x^2}{[1 + (\partial h / \partial x)^2]^{3/2}}, \end{aligned} \quad (3.13)$$

where p_a is the atmospheric pressure above the molten steel;

- negligible heat flux from the melt to the air, expressed by

$$-\frac{\partial h}{\partial x} \frac{\partial T}{\partial x} + \frac{\partial T}{\partial y} = 0, \quad (3.14)$$

wherein we neglect any possible heat loss due to radiation or evaporation, although these assumptions are evaluated later.

Furthermore, since filling occurs at a rate V_c , then conservation of mass implies

$$WV_c t = \int_0^W (h(x, t) - h_0(x)) dx. \quad (3.15)$$

For the initial conditions at $t = 0$, when there is no fluid flow, we have, for $0 \leq y \leq h_0(x)$,

$$u = 0, \quad v = 0, \quad T = T_c. \quad (3.16)$$

Equations (3.2), (3.3) and (3.16) then imply a hydrostatic pressure given by

$$p = C - \rho g y, \quad (3.17)$$

where C is a constant to be determined and equation (3.13) leads to

$$p_a - C + \rho g h_0 = \frac{\gamma_{LG} d^2 h_0 / dx^2}{(1 + (dh_0/dx)^2)^{3/2}}. \quad (3.18)$$

From the fourth equality of (3.6), we should expect, at $x = 0$,

$$\frac{dh_0}{dx} = 0, \quad (3.19)$$

whereas equation (3.9) gives at $x = W$

$$\frac{dh_0}{dx} = \tilde{\gamma}. \quad (3.20)$$

Lastly, and in order to specify the problem completely, and hence determine the constant C , we must prescribe the initial volume of molten steel per unit length V_0 , given by

$$V_0 = \int_0^W h_0(x) dx. \quad (3.21)$$

3.3 Nondimensionalization

Proceeding to a formal nondimensionalization, we set

$$X = \frac{x}{W}, \quad Y = \frac{y}{W}, \quad U = \frac{u}{V_c}, \quad V = \frac{v}{V_c}, \quad P = \frac{p - p_a}{\rho g W}, \quad (3.22)$$

$$\theta = \frac{T - T_m}{T_c - T_m}, \quad \tau = \frac{t}{W/V_c}, \quad H = \frac{h}{W}, \quad H_0 = \frac{h_0}{W}, \quad \mathcal{Q} = \frac{Q}{[Q]}, \quad (3.23)$$

where $[Q]$ is the characteristic scale for $Q(y)$. Equations (3.1)-(3.4) give, respectively

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0, \quad (3.24)$$

$$Fr \left(\frac{\partial U}{\partial \tau} + U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} \right) = -\frac{\partial P}{\partial X} + \frac{Fr}{Re} \left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right), \quad (3.25)$$

$$Fr \left(\frac{\partial V}{\partial \tau} + U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} \right) = -\frac{\partial P}{\partial Y} + \frac{Fr}{Re} \left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right) - 1, \quad (3.26)$$

$$\frac{\partial \theta}{\partial \tau} + U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} = \frac{1}{Pe} \left(\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} \right), \quad (3.27)$$

where, Fr , Re and Pe denote the Froude, Reynolds and Peclet numbers respectively, and are given by

$$Fr = \frac{V_c^2}{gW}, \quad Re = \frac{\rho V_c W}{\mu}, \quad Pe = \frac{\rho c_p V_c W}{k}. \quad (3.28)$$

The boundary conditions (3.5)-(3.7) becomes respectively

$$U = 0, \quad V = 1, \quad \theta = 1 \quad \text{at} \quad Y = 0, \quad (3.29)$$

$$U = 0, \quad \frac{\partial V}{\partial X} = 0, \quad \frac{\partial \theta}{\partial X} = 0, \quad \frac{\partial H}{\partial X} = 0 \quad \text{at} \quad X = 0, \quad (3.30)$$

$$U = 0, \quad V = 0, \quad \frac{\partial \theta}{\partial X} = -\lambda \mathcal{Q}(Y) \quad \text{at} \quad X = 1, \quad (3.31)$$

where λ is a dimensionless parameter given by

$$\lambda = \frac{[Q]W}{k(T_{cast} - T_{melt})} \quad (3.32)$$

and Q was nondimensionalized so that $\mathcal{Q} \equiv 1$.

At the moving boundary $Y = H(X, \tau)$, we obtain from (3.11)-(3.14), respectively

$$V = \frac{\partial H}{\partial \tau} + U \frac{\partial H}{\partial X}, \quad (3.33)$$

$$2 \left(\frac{\partial V}{\partial Y} - \frac{\partial U}{\partial X} \right) \frac{\partial H}{\partial X} + \left(\frac{\partial U}{\partial Y} + \frac{\partial V}{\partial X} \right) \left[1 - \left(\frac{\partial H}{\partial X} \right)^2 \right] = 0, \quad (3.34)$$

$$\begin{aligned} & Bo \left(-P + \frac{2}{1 + (\frac{\partial H}{\partial X})^2} \frac{Fr}{Re} \left\{ \frac{\partial U}{\partial X} \left(\frac{\partial H}{\partial X} \right)^2 + \frac{\partial V}{\partial Y} - \left(\frac{\partial U}{\partial Y} + \frac{\partial V}{\partial X} \right) \left(\frac{\partial H}{\partial X} \right) \right\} \right) \\ &= \frac{\partial^2 H / \partial X^2}{(1 + (\partial H / \partial X)^2)^{3/2}}, \end{aligned} \quad (3.35)$$

$$-\frac{\partial H}{\partial X} \frac{\partial \theta}{\partial X} + \frac{\partial \theta}{\partial Y} = 0, \quad (3.36)$$

whereas, equation (3.15) becomes

$$\tau = \int_0^1 (H(X, \tau) - H_0(X)) dX. \quad (3.37)$$

For the initial conditions, at $\tau = 0$, we have for $0 \leq Y \leq H_0(X)$,

$$U = 0, \quad V = 0, \quad \theta = 1, \quad (3.38)$$

whereas equation (3.17) becomes

$$P = C^* - Y, \quad (3.39)$$

where C^* is a constant to be determined and equation (3.18) leads to

$$Bo(-C^* + H_0) = \frac{d^2 H_0 / dX^2}{(1 + (dH_0 / dX)^2)^{3/2}}; \quad (3.40)$$

here, $C^* = (C - p_a) / \rho g W$ and Bo , denotes the Bond number, which is given by

$$Bo = \frac{\rho g W}{\gamma_{LG}}. \quad (3.41)$$

Equations (3.19) and (3.20) lead to

$$\frac{dH_0}{dX} = 0 \quad \text{at} \quad X = 0, \quad (3.42)$$

$$\frac{dH_0}{dX} = \tilde{\gamma} \quad \text{at} \quad X = 1. \quad (3.43)$$

3.4 Analysis

To proceed, we use the physical properties for molten steel, for which we have

$$k \sim 20 \text{ W m}^{-1} \text{ K}^{-1}, \quad \rho \sim 8000 \text{ kg m}^{-3}, \quad c_p \sim 500 \text{ J kg K}^{-1},$$

$$\mu \sim 0.0067 \text{ N s m}^{-2}, \quad \gamma_{LG} \sim 1880 \text{ N m}^{-1}, \quad T_c - T_m \sim 50 \text{ K}.$$

For V_c and W , we use characteristic parameter values from (JACOBI; SCHWERTDFEGER, 2013),

$$V_c \sim 0.01 \text{ m s}^{-1}, \quad W \sim 0.1 \text{ m},$$

which lead to

$$Fr \sim 10^{-4}, \quad Re \sim 10^3, \quad Pe \sim 200, \quad Bo \sim 800; \quad (3.44)$$

for the time being, we can proceed with the analysis without knowing a characteristic value for λ . The estimates in (3.44) lead us to consider the governing equations for the regime

$$Fr \ll 1, \quad Re \gg 1, \quad Pe \gg 1, \quad Bo \gg 1.$$

Then, to investigate the shape of the meniscus, the analysis can be considered separately for three regions related to:

1. the central region of the molten steel;

2. the deviation of the meniscus with respect to the mold wall, where the Bond number will be used;
3. the thermal boundary layer where the Peclet number plays a important role.

These regions can be seen in figure 29.

Figure 29 – Schematic of the regions for the shape of the meniscus.

Source: Elaborated by the author.

3.4.1 $1 - X \sim O(1)$

First of all, equations (3.25) and (3.26) reduce, respectively, to

$$\frac{\partial P}{\partial X} = 0, \quad (3.45)$$

$$\frac{\partial P}{\partial Y} = -1, \quad (3.46)$$

whence

$$P = -Y + f(\tau), \quad (3.47)$$

where f is a function to be determined.

Since $Fr/Re \ll 1$, equation (3.35) becomes, on using (3.47) at $Y = H$,

$$Bo(H - f(\tau)) = \frac{\partial^2 H / \partial X^2}{(1 + (\partial H / \partial X)^2)^{3/2}}, \quad (3.48)$$

with H subject to:

$$\frac{\partial H}{\partial X} = 0 \quad \text{at} \quad X = 0, \quad (3.49)$$

$$\frac{\partial H}{\partial X} = \tilde{\gamma} \quad \text{at} \quad X = 1. \quad (3.50)$$

Since $Bo \gg 1$, we now note that (3.48) reduces simply to

$$H \approx f(\tau). \quad (3.51)$$

3.4.2 $1 - X \sim O(Bo^{-1/2})$

Now, to take into account the effects of the right hand side term of (3.48), we rescale using

$$H - f(\tau) = Bo^{-1/2} \bar{H}(\bar{X}, \tau), \quad X = 1 - Bo^{-1/2} \bar{X}, \quad (3.52)$$

to obtain

$$\bar{H} = \frac{\partial^2 \bar{H} / \partial \bar{X}^2}{(1 + (\partial \bar{H} / \partial \bar{X})^2)^{3/2}}, \quad (3.53)$$

subject to

$$\frac{\partial \bar{H}}{\partial \bar{X}} = 0 \quad \text{as} \quad \bar{X} \rightarrow \infty, \quad (3.54)$$

$$\frac{\partial \bar{H}}{\partial \bar{X}} = -\tilde{\gamma} \quad \text{at} \quad \bar{X} = 0. \quad (3.55)$$

The solution for the shape of the meniscus can be obtained analytically as

$$\bar{X} - \bar{X}_0 = -\sqrt{4 - \bar{H}^2} + \ln \left(\frac{2 + \sqrt{4 - \bar{H}^2}}{\bar{H}} \right), \quad (3.56)$$

where $\bar{X}_0$ is, in general, a function of τ to be determined; using (3.55) and (3.56), we find that

$$\bar{X}_0 = \sqrt{4 - \bar{H}_0^2} - \ln \left(\frac{2 + \sqrt{4 - \bar{H}_0^2}}{\bar{H}_0} \right), \quad (3.57)$$

where

$$\bar{H}_0 = \bar{H}(0, \tau) = \sqrt{2(1 - \sin \phi_c)}, \quad (3.58)$$

so that $\bar{H}_0$ and $\bar{X}_0$ are both constant. In fact, equation (3.56) is akin to that obtained by Bikerman (BIKERMAN, 1970) for the case of a stationary, semi-infinite meniscus; here, we have shown in a systematic way how it is relevant to the case of a moving finite meniscus.

The above implies that the meniscus is flat at leading order, apart from an $O(Bo^{-1/2})$ correction within a region of width $O(Bo^{-1/2})$ adjacent to the mold wall.

Now, using equations (3.37) and (3.51), we have

$$\tau = f(\tau) - Y_0, \quad (3.59)$$

where

$$Y_0 = \int_0^1 H_0(X) dX.$$

3.4.3 $1 - X \sim O(Pe^{-1/2})$

The fact that $Pe \gg 1$ and $Re \gg 1$ implies that there will be momentum and thermal boundary layers, of thickness $Re^{-1/2}$ and $Pe^{-1/2}$, respectively, near $X = 1$. However, since $Pe \ll Re$, the thermal boundary layer effectively sees the inviscid velocity, which is given by

$$U \approx 0, \quad V = 1, \quad (3.60)$$

and equation (3.27) reduces to

$$\frac{\partial \theta}{\partial \tau} + \frac{\partial \theta}{\partial Y} = \frac{1}{Pe} \left(\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} \right). \quad (3.61)$$

Now, we use a rescaling for this boundary layer with

$$X = 1 - Pe^{-1/2} \tilde{X}, \quad (3.62)$$

so that equation (3.61) becomes

$$\frac{\partial \theta}{\partial \tau} + \frac{\partial \theta}{\partial Y} = \frac{\partial^2 \theta}{\partial \tilde{X}^2} + \frac{1}{Pe} \frac{\partial^2 \theta}{\partial Y^2}; \quad (3.63)$$

then, since

$$\frac{1}{Pe} \frac{\partial^2 \theta}{\partial Y^2} \ll 1, \quad (3.64)$$

equation (3.63) at leading order is given by

$$\frac{\partial \theta}{\partial \tau} + \frac{\partial \theta}{\partial Y} = \frac{\partial^2 \theta}{\partial \tilde{X}^2}, \quad (3.65)$$

and is valid for $0 \leq \tilde{X} < \infty, \tau \geq 0$.

It is important to notice that, from (3.62),

$$\tilde{X} = Pe^{1/2}(1 - X), \quad (3.66)$$

thus

$$\tilde{X} \rightarrow \infty \quad \text{as} \quad X \rightarrow 0 \quad (3.67)$$

and

$$\tilde{X} \rightarrow 0 \quad \text{as} \quad X \rightarrow 1. \quad (3.68)$$

In figure 30 is shown a schematic of the tip at the meniscus region; this has been drawn taking into account the hierarchy suggested by the analysis, namely

$$Re \gg Bo \gg Pe \gg 1.$$

Now, if we consider the region where

$$Pe^{-1/2} \gg Bo^{-1/2}, \quad (3.69)$$

Figure 30 – Structure of the meniscus profile.

Source: Elaborated by the author.

then the thermal boundary layer is now much wider than the region across which the correction to the meniscus occurs; thus, with respect to the thermal boundary layer, the meniscus is planar and consequently equation (3.65) will be valid for $0 \leq Y \leq Y_0 + \tau$.

Turning now to the boundary conditions for equation (3.65), these are

$$\frac{\partial \theta}{\partial \tilde{X}} = \lambda Pe^{-1/2} \quad \text{at} \quad \tilde{X} = 0, \quad (3.70)$$

$$\theta \rightarrow 1 \quad \text{as} \quad \tilde{X} \rightarrow \infty, \quad (3.71)$$

$$\theta = 1 \quad \text{at} \quad Y = 0. \quad (3.72)$$

In addition, we have the following initial condition at $\tau = 0$:

$$\theta = 1, \quad 0 \leq \tilde{X} < \infty, \quad 0 \leq Y \leq Y_0. \quad (3.73)$$

The problem will now be correctly scaled, meaning that

$$\lambda Pe^{-1/2} \sim O(1),$$

or, by using (3.32),

$$[Q] \times \left\{ \frac{W Pe^{-1/2}}{k(T_{cast} - T_{melt})} \right\} \sim O(1),$$

and since

$$\frac{W Pe^{-1/2}}{k(T_{cast} - T_{melt})} \sim O(10^{-5}),$$

we have

$$[Q] \sim 10^5 \text{ W m}^{-2}.$$

In fact, now that $[Q]$ has been estimated, we are in a position to determine the validity of neglecting radiation and evaporation from the molten metal surface in equation (3.14).

We consider the radiation first; in particular, the right-hand side of this equation would have contained the term

$$\varepsilon_0 \tilde{\sigma}(T^4 - T_{amb}^4),$$

where $\tilde{\sigma}$ is the Stefan-Boltzmann constant, T_{amb} is the ambient temperature and ε_0 is the emissivity of molten steel. If we take $T \sim 1600\text{K}$ and $T_{amb} \sim 300\text{K}$, we have

$$\tilde{\sigma}(T^4 - T_{amb}^4) \sim 10^5 \text{Wm}^{-2},$$

which is the same order magnitude as $[Q]$; thus, whether radiation at the upper surface can be neglected depends on the value of ε_0 , where $0 \leq \varepsilon_0 \leq 1$. Thus, at worst, radiation might not be negligible by comparison with heat loss at the mold wall, although this does not take away from the fact that solidification occurs first at the mold wall, rather than at some location on the meniscus away from the wall. Since it is the former that we are investigating here, we have chosen to neglect the effect of the radiation.

As for evaporation, the right-hand side of equation (3.14) would have contained the term

$$\rho_g L_e \frac{\partial h}{\partial t}$$

(VYNNYCKY; MAENO., 2012; VYNNYCKY; MAENO, 2012), where ρ_g is the density of the metal vapor and L_e is the latent heat of vaporization. Using the ideal gas law for ρ_g , so that

$$\rho_g = \frac{p_a M}{\mathcal{R} T},$$

where M is the molecular weight and $\mathcal{R}$ is the ideal gas constant ($8.314 \text{JK}^{-1} \text{mol}^{-1}$), we insert the appropriate values for iron,

$$M = 0.056 \text{kg mol}^{-1} \quad \text{and} \quad L_e \sim 6.0 \times 10^6 \text{J kg}^{-1}, \quad (3.74)$$

as well as

$$T \sim 1800\text{K} \quad \text{and} \quad \frac{\partial h}{\partial t} \sim V_c, \quad (3.75)$$

to obtain

$$\rho_g \sim 0.4 \text{kg m}^{-3} \quad \text{and} \quad \rho_g L_e \frac{\partial h}{\partial t} \sim 2 \times 10^4 \text{W m}^{-2}. \quad (3.76)$$

Consequently, this is smaller than both the heat loss at the wall and that due to radiation. Furthermore, setting $B = \lambda Pe^{-1/2}$, it is clear that the onset of solidification will not begin for $Y \leq L/W$ if $B < B_{crit}$, where B_{crit} is some critical value that we will be able to determine.

The remaining numerical task is to solve, for $0 \leq \tilde{X} < \infty$, $0 \leq Y \leq Y_0 + \tau$, $\tau \geq 0$,

$$\frac{\partial \theta}{\partial \tau} + \frac{\partial \theta}{\partial Y} = \frac{\partial^2 \theta}{\partial \tilde{X}^2} \quad (3.77)$$

subject to the boundary conditions

$$\frac{\partial \theta}{\partial \tilde{X}} = B \quad \text{at} \quad \tilde{X} = 0, \quad (3.78)$$

$$\theta \rightarrow 1 \quad \text{as} \quad \tilde{X} \rightarrow \infty, \quad (3.79)$$

$$\theta = 1 \quad \text{at} \quad Y = 0 \quad (3.80)$$

and the initial condition

$$\theta = 1, \quad \text{for} \quad 0 \leq \tilde{X} < \infty, \quad 0 \leq Y \leq Y_0, \quad \tau = 0. \quad (3.81)$$

Here, we mention two important facts to solve (3.77), first, we have a constant heat flux at the mold wall given by (3.78) and second, a moving boundary problem at $Y = H(X, \tau)$. There is no obvious analytical solution to equations (3.77) -(3.81), which must therefore be solved numerically.

In fact, it proves useful to employ a boundary immobilization method to solve the equations although such a method is convenient for one-dimensional time-dependent problems (MITCHELL; VYNNYCKY, 2009; MITCHELL; VYNNYCKY, 2012; MITCHELL *et al.*, 2011; MITCHELL; VYNNYCKY, 2014). Nevertheless, setting

$$\eta = \frac{Y}{Y_0 + \tau}, \quad (3.82)$$

we have, for $0 \leq \tilde{X} < \infty$, $0 \leq \eta \leq 1$, $\tau \geq 0$,

$$\frac{\partial \theta}{\partial \tau} + \left(\frac{1 - \eta}{Y_0 + \tau} \right) \frac{\partial \theta}{\partial \eta} = \frac{\partial^2 \theta}{\partial \tilde{X}^2}, \quad (3.83)$$

subject to the boundary conditions

$$\frac{\partial \theta}{\partial \tilde{X}} = B \quad \text{at} \quad \tilde{X} = 0, \quad (3.84)$$

$$\theta \rightarrow 1 \quad \text{as} \quad \tilde{X} \rightarrow \infty, \quad (3.85)$$

$$\theta = 1 \quad \text{at} \quad \eta = 0, \quad (3.86)$$

and the initial condition

$$\theta = 1, \quad \text{for} \quad 0 \leq \tilde{X} < \infty, \quad 0 \leq \eta \leq 1 \quad \text{at} \quad \tau = 0. \quad (3.87)$$

An unexpected analytical result arises if we consider equation (3.83) at $\eta = 1$; we obtain

$$\frac{\partial \theta}{\partial \tau} = \frac{\partial^2 \theta}{\partial \tilde{X}^2}, \quad (3.88)$$

subject to the boundary conditions

$$\frac{\partial \theta}{\partial \tilde{X}} = B \quad \text{at} \quad \tilde{X} = 0, \quad (3.89)$$

$$\theta \rightarrow 1 \quad \text{as} \quad \tilde{X} \rightarrow \infty, \quad (3.90)$$

and the initial condition

$$\theta = 1, \quad \text{for} \quad 0 \leq \tilde{X} < \infty \quad \text{at} \quad \tau = 0, \quad (3.91)$$

whose solution is given by

$$\theta(\tilde{X}, \tau) = 1 + B \left[\tilde{X} \operatorname{erfc} \left(\frac{\tilde{X}}{2\tau^{1/2}} \right) - 2 \frac{\tau^{1/2}}{\sqrt{\pi}} \exp \left(-\frac{\tilde{X}^2}{4\tau} \right) \right]. \quad (3.92)$$

Hence, we need to have the onset the solidification of steel, $\theta = 0$, beginning at the top of the mold, i.e. when

$$y = L \quad \text{or} \quad Y = \frac{L}{W}, \quad (3.93)$$

and at the mold wall $\tilde{X} = 0$. This implies

$$\theta \left(0, \frac{L}{W}, \tau \right) = 0 \quad \text{or} \quad 1 - \frac{2B\tau^{1/2}}{\pi^{1/2}} = 0 \quad (3.94)$$

and thus

$$B = \frac{\pi^{1/2}}{2\tau^{1/2}}. \quad (3.95)$$

At the same time, the meniscus must reach the top of mold, which means

$$\frac{L}{W} = H = Y_0 + \tau \quad \text{or} \quad \tau = \frac{L}{W} - Y_0. \quad (3.96)$$

Therefore, the value of B_{crit} is determined from (3.95), using (3.96), by

$$B_{crit} = \frac{\pi^{1/2}}{2 \left(\frac{L}{W} - Y_0 \right)^{1/2}}. \quad (3.97)$$

3.5 Numerics for the thermal boundary layer

A variety of numerical methods exist for solving either equations (3.77)-(3.81) as they stand or solving (3.83)-(3.87); here, we apply a Crank-Nicolson scheme (PLETCHER; TANNERHILL; ANDERSON, 1997) to equations (3.83)-(3.87) in the region depicted in figure 31.

A primary issue that must be resolved is where to set the outer limit $\tilde{X}_\infty$ of the computational domain in the $\tilde{X}$ – direction. Equation (3.92) indicates that θ will decay exponentially as $\tilde{X} \rightarrow \infty$, so that $\tilde{X}_\infty$ need not be much greater than 14; thus, we take $\tilde{X}_\infty = 14$.

Figure 31 – Geometry and boundary conditions for equation (3.83).

Source: Elaborated by the author.

On the other hand, the integration in time of (3.83) will be up to τ^* , given by

$$\tau^* = \frac{L}{W} - Y_0 \approx 3.5.$$

For the discretization of (3.83), a quadrilateral mesh was used, where the integer numbers $N_{\tilde{X}_\infty}$ and N_η define the spacings

$$\Delta \tilde{X} = \frac{\tilde{X}_\infty}{N_{\tilde{X}_\infty}} \quad \text{and} \quad \Delta \eta = \frac{1}{N_\eta}$$

of the nodes, given by

$$\tilde{X}_i = (i-1)\Delta \tilde{X} \quad \text{for} \quad i = 1, \dots, N_{\tilde{X}_\infty} \quad \text{and} \quad \tilde{\eta}_j = (j-1)\Delta \eta \quad \text{for} \quad j = 1, \dots, N_\eta,$$

and for an integer N_τ , we have

$$\tau^n = n\Delta \tau \quad \text{for} \quad n = 0, \dots, N_\tau,$$

where $\Delta \tau = \tau^*/N_\tau$.

Thus, the discretized expression of

$$\frac{\partial \theta}{\partial \tau} + a(\eta, \tau) \frac{\partial \theta}{\partial \eta} = \frac{\partial^2 \theta}{\partial \tilde{X}^2}, \quad (3.98)$$

where $a(\eta, \tau) = (1 - \eta)/(Y_0 + \tau)$, turns out to be, for $i = 1, \dots, N_{\tilde{X}} - 1$ and $j = 1, \dots, N_\eta$,

$$\begin{aligned} \frac{\theta_{i,j}^{n+1} - \theta_{i,j}^n}{\Delta \tau} &+ \frac{1}{2} \left(a_j^{n+1} \frac{\theta_{i,j}^{n+1} - \theta_{i,j-1}^{n+1}}{\Delta \eta} + a_j^n \frac{\theta_{i,j}^n - \theta_{i,j-1}^n}{\Delta \eta} \right) \\ &= \frac{1}{2} \left(\frac{\theta_{i-1,j}^{n+1} - 2\theta_{i,j}^{n+1} + \theta_{i+1,j}^{n+1}}{\Delta \tilde{X}^2} + \frac{\theta_{i-1,j}^n - 2\theta_{i,j}^n + \theta_{i+1,j}^n}{\Delta \tilde{X}^2} \right), \end{aligned}$$

where $\theta_{i,j}^n$ is the numerical approximation to the value $\theta(\tilde{X}_i, \eta_j, \tau^n)$ and

$$a_j^n = \frac{1 - \eta_j}{Y_0 + \tau^n}.$$

On the other hand, the boundary conditions (3.84)-(3.85) and (3.86) are discretized as

$$\frac{1}{2} \left(\frac{\theta_{1,j}^{n+1} - \theta_{-1,j}^{n+1}}{2\Delta\tilde{X}} + \frac{\theta_{1,j}^n - \theta_{-1,j}^n}{2\Delta\tilde{X}} \right) = B, \quad \text{for } j = 1, \dots, N_\eta, \quad (3.99)$$

$$\frac{1}{2} \left(\theta_{N_{\tilde{X}},j}^{n+1} + \theta_{N_{\tilde{X}},j}^n \right) = 1 \quad \text{for } j = 1, \dots, N_\eta, \quad (3.100)$$

$$\frac{1}{2} \left(\theta_{i,0}^{n+1} + \theta_{i,0}^n \right) = 1 \quad \text{for } i = 1, \dots, N_{\tilde{X}} \quad (3.101)$$

and the initial condition is discretized as

$$\theta_{i,j} = 1 \quad \text{for } i = 1, \dots, N_{\tilde{X}} - 1 \quad \text{and } j = 1, \dots, N_\eta. \quad (3.102)$$

It was found that the choices $N_\tau = 160$, $N_{\tilde{X}} = 80$ and $N_\eta = 160$ gave mesh-independent results for the solution and provided good agreement with the analytical expression derived earlier.

In algorithm 1 is explained the iterative procedure to find B_{crit} , which will allow us to obtain $\theta_{tip} = 0$ depicted in figure 31 that corresponds to the onset of the solidification of the steel.

Algorithm 1 – Computation of B_{crit}

```

1: procedure FINDERBCRIT( $B_0, B_1$ ) ▷  $B_0 \leq B_{crit} \leq B_1$ 
2:    $0.0001 \rightarrow TOL$ .
3:   Solve (3.83) using  $B = B_0$  at  $\tilde{X} = 0$ .
4:   Assign the value of  $\theta_{tip}$  of the solution as  $\theta_0$ .
5:   while  $|B_0 - B_1| \geq TOL$  do ▷ Main loop.
6:      $\frac{B_0 + B_1}{2} \rightarrow B_m$ 
7:     Solve (3.83) using  $B = B_m$  at  $\tilde{X} = 0$ .
8:     Assign the value of  $\theta_{tip}$  of the solution as  $\theta_m$ .
9:     if  $\theta_0 \times \theta_m < 0$  then
10:       $B_m \rightarrow B_1$ .
11:     else
12:       $B_m \rightarrow B_0$  and  $\theta_m \rightarrow \theta_0$ .
13:     end if
14:   end while
15:    $B_m \rightarrow B_{crit}$ .
16:   return  $B_{crit}$ .
17: end procedure

```

In algorithm 1, we must provide two numerical values B_0 and B_1 , large enough such that the value of B_{crit} satisfies $B_0 < B_{crit} < B_1$. Every time we obtain a new value of B_m , equation

(3.83) must be solved and the value of θ_{tip} must be extracted; call it θ_m . When the value of θ_m is small enough, close to zero, we have obtained the desired value of B_{crit} .

To implement computationally the algorithm 1, the free programming software *GNU-Octave*, provided by the *Free Software Foundation* (FSF), was used. The programming and execution of algorithm 1 was developed on the distribution *Debian* version 9 of the operating system *GNU-Linux*.

Prior to generating the results, which are documented next, expensive benchmarking of the program was carried out; this is documented in Appendix A.

3.6 Results

Using the numerical scheme outlined in the previous section, computations were carried out based on the physical parameters given in section 3.4. To enable some fixing of ideas, we have taken $L = 0.35\text{m}$, based on the mold height in (JACOBI; SCHWERDTFEGER, 2013) and $H_0 = 0.01$ is set to an arbitrary small value, rather than zero, simply to avoid the difficulty associated with solving for a liquid region that initially would not be present.

The onset of solidification of steel starts at the point where $\theta_{tip} = 0$ as shown in figure 31; there, $\tilde{X} = 0$. To calculate θ_{tip} we use expression (3.92) which represents the temperature at $Y = H(X, \tau)$, i.e. on the surface of the molten steel, and $B = B_{crit}$. Thus, the analytical expression for θ_{tip} is given by

$$\theta_{tip} = 1 - \frac{\tau^{1/2}}{\left(\frac{L}{W} - Y_0\right)^{1/2}}. \quad (3.103)$$

On the other hand, after a number of computations for different values for B , using the computational implementation of algorithm 1, it was found that $B_{crit} \approx 0.48$. This value has been used for figure 32, which shows θ at $\tilde{X} = 0$, using the analytical expression (3.103) and the numerical approximation to the solution of (3.83).

Figure 32 shows excellent agreement between the analytical and numerical solutions for θ_{tip} as a function of τ , indicating that the numerical work has been carried out correctly. Note also that, in dimensional terms, this figure implies that the meniscus reaches the top of the mold after around 35 seconds, at which point solidification would start at that position also.

The profiles of temperature obtained as a result of computations by solving (3.83) numerically, using the implementation of the algorithm 1, for values $\tau = 1.0, 2.0, 3.0$ and $\tau = \tau^*$ are shown in figure 33, where the value of θ is decreasing towards $\theta = 0$ at $\tilde{X} = 0$.

The numerical solution in the whole region of the thermal boundary layer at $\tau = \tau^*$ was obtained from (3.83) and is shown in figure 34, wherein θ reaches, clearly, the value of 0 that indicates the onset of the solidification of steel.

Figure 32 – θ_{tip} vs τ at $\tilde{X} = 0$ using $B = B_{crit}$.

Source: Elaborated by the author.

Figure 33 – θ on the surface of the molten steel for $\tau = 0.0, 1.0, 2.0, 3.0$ and $\tau = \tau^*$.

Source: Elaborated by the author.

Finally, once B_{crit} has been determined, the onset of solidification will not begin at the top of mold if $B < B_{crit}$ is employed in the boundary condition at $\tilde{X} = 0$, which means that

$$\left(\frac{\lambda Pe^{-1/2}}{B_{crit}} \right)^2 < 1,$$

or

$$\left(\frac{\lambda}{B_{crit}} \right)^2 < Pe = \frac{V_c W \rho c_p}{k};$$

in terms of the process variables, this can be written as

$$V_c > \frac{[Q]^2 W}{k(T_c - T_m)^2 \rho c_p B_{crit}^2}, \quad (3.104)$$

Figure 34 – Solution θ in the whole region of the thermal boundary layer at $\tau = \tau^*$

Source: Elaborated by the author.

and in terms of the function $\theta(\tilde{X}, Y, \tau)$, the tip of the meniscus will not solidify provided that

$$\theta(0, Y_0 + \tau, \tau) > 0, \quad \text{for} \quad \tau \leq \frac{L}{W} - Y_0. \quad (3.105)$$

Figure 35 shows the region, shaded region, in which there exists solidification at the meniscus since $\theta < 0$ that is equivalent to $T < T_m$. On the other hand, the region where there exists no solidification at the meniscus, $\theta > 0$, is pointed out and is defined by inequality (3.105). The dashed line indicates the $(V_c, T_c - T_m)$ -combinations for which we can expect the solidification on the tip of the meniscus when it reaches the top of the mold.

It would be convenient at this point to be able to compare the theoretical results with available experimental results. The only possibility would be to use data for which ripple marks have formed and verify that that data does not satisfy inequality (3.104). Although the work by Jacobi and Schwerdtfeger (JACOBI; SCHWERDTFEGER, 2013) would have been a suitable candidate for this, it is unfortunate that they do not document the heat flux at the mold wall; had they done so, a comparison could have been made.

Figure 35 – V_c as a function of $T_c - T_m$ (dashed line) and regions where the solidification of steel takes place as given by inequality (3.104).

Source: Elaborated by the author.

OSCILLATION MARKS

4.1 Introduction

In the continuous casting of steel, the solidification of the molten metal is initiated by a cooling mold that oscillates in the axis of the casting direction; furthermore, in order to avoid sticking of the solid shell to the mold, mold powder, often also termed as flux, is added at the top of the mold, melts and forms a lubricating layer in the gap between the steel and the mold walls, as a meniscus forms between the molten flux and the molten steel. However, mold oscillations lead inevitably to the formation of regularly spaced indentations along the slab width, known as oscillation marks. A schematic of the situation is shown in figure 36.

The nature of these marks has long been a subject of modeling in the continuous casting literature: see, for instance (TAKEUCHI; BRIMACOMBE, 1984) and a comprehensive recent review by Jonayat and Thomas (JONAYAT; THOMAS, 2014). The modeling of oscillation-mark formation generally takes one of two forms: either involving computational fluid dynamics (CFD) which tries to take into account all of the known physics (JONAYAT; THOMAS, 2014; RAMIREZ-LOPEZ *et al.*, 2012) or the use of asymptotic methods in order to try to capture the essential mechanisms via controlling dimensionless parameters (HILL; WU; WIWATANAPAT-APHEE, 1999; BLAND, 1984; KING *et al.*, 1993; FOWKES; WOODS, 1989; FOWKES *et al.*, 1988; VYNNYCKY *et al.*, 2017).

In the context of the latter, Vynnycky *et al.* (VYNNYCKY *et al.*, 2017) recently revisited an earlier model for oscillation-mark formation by Hill *et al.* (HILL; WU; WIWATANAPAT-APHEE, 1999) who had built on yet earlier work by Bland (BLAND, 1984), King *et al.* (KING *et al.*, 1993) and Fowkes and co-workers (FOWKES; WOODS, 1989; FOWKES *et al.*, 1988) by using lubrication theory coupled to heat conduction, so as to predict the solid and liquid slag thickness and the oscillation-mark shape.

Moreover, a comparison with experimentally measured oscillation marks indicated that

Figure 36 – Schematic diagram showing the process of oscillation-mark formation

Source: Elaborated by the author.

the model was able to predict fold-type marks (VYNNYCKY *et al.*, 2017). However, all of these works completely neglect the possible role of the molten steel/molten flux meniscus in oscillation-mark formation, whereas there is substantial experimental evidence which indicates that its effects cannot necessarily be neglected in general: for example, other experimental data obtained by Saleem *et al.* (SALEEM; VYNNYCKY; FREDRIKSSON, 2016) appears to show overflow-type marks, whose profiles cannot be predicted by the model of Vynnycky *et al.* (VYNNYCKY *et al.*, 2017) at all.

Even though there is much qualitative discussion about how solidification occurs in the presence of a meniscus (FREDRIKSSON; ELFSBERG, 2002; SALEEM; VYNNYCKY; FREDRIKSSON, 2016), there is arguably as yet no transparent quantitative model that takes into account all of the known physics for how this occurs in a systematic rational way, although the CFD model by Ramirez-Lopez *et al.* (RAMIREZ-LOPEZ *et al.*, 2012) comes closest to providing a complete mathematical description showing how it may happen, as do, to a lesser extent, earlier works by Steinrück and co-workers (STEINRÜCK; RUDISCHER; SCHNEIDER, 1997; STEINRÜCK; RUDISCHER; SCHNEIDER, 1998; STEINRÜCK; RUDISCHER, 2002).

Against this background, the purpose of this chapter is to develop a framework for a model that takes account of the meniscus, although based around the use of asymptotic methods, so as to be able to elucidate more clearly the barest modeling essentials required to predict mark formation.

4.2 Modeling considerations

From a modeling perspective, a key difficulty in the problem is the fact that four phases are present: solid flux, molten flux, molten steel and solid steel. Whilst this can be handled in CFD solvers using the volume-of-fluid (VOF) method (JONAYAT; THOMAS, 2014; RAMIREZ-LOPEZ *et al.*, 2012), so that each phase is identified by its corresponding volume fraction, here we will choose an alternative approach, which is to actually resolve each phase separately.

Thus, we have a traditional moving boundary problem (CRANK, 1984) although with two complications. The first is that, as is evident in figure 36, we have four moving boundaries: the solid-molten boundary for the flux; the molten flux-solid steel boundary; the solid-molten steel boundary; the molten steel-molten flux boundary. The second is the presence of an idealized triple point, labelled (x_0, z_0) in figure 36, at which molten flux, solid steel and molten steel meet, and that it is necessary to be able to track the location of this point, since its motion determines the ultimate shape of the oscillation mark.

In this sense, we assume that there is no further deformation of the shell once solidification has occurred. To the best of our knowledge, the solution to such a problem, analytical or numerical, is more or less unheard of. The closest is perhaps the moving contact line problem in fluid mechanics (HUH; SCRIVEN, 1971; DUSSAN; DAVIS, 1974; BENNEY; TIMSON, 1980; BENILOV; VYNNYCKY, 2013; PISMEN; NIR, 1982), but even there the contact line moves along a planar surface. In addition, that problem is treated isothermally, whereas here we are dealing with the interaction of fluid mechanics and heat transfer.

Note also that we have called the triple point “idealized” because, since steel is an alloy, there will in practice be a mushy zone that separates fully molten flux and fully solid steel; the focus of attention would then be the tracking of the point that is at the coherence temperature, i.e. the temperature below which the material behaves mechanically as a solid (KRISTIANSSON, 1982). Hence, including the mushy zone adds nothing conceptually new to the problem.

A key issue is then to have a strategy for determining the location of the triple point. In what follows, we employ the idea used by Hill *et al.* (HILL; WU; WIWATANAPATAPHEE, 1999) and Vynnycky *et al.* (VYNNYCKY *et al.*, 2017) of considering the problem in terms of two regions: one above the triple point and one below. However, the novel feature is that the region above the triple point is treated more comprehensively, so as to account for the effects of the meniscus.

In fact, our treatment for the upper region turns out to have similarities with that given by Steinrück *et al.* (STEINRÜCK; RUDISCHER; SCHNEIDER, 1997), except that it allows for more generality: whereas they considered a meniscus that was stationary and approached the oscillating mold wall tangentially (STEINRÜCK; RUDISCHER; SCHNEIDER, 1997), we consider the possibility that the meniscus moves in time and space, and that it can approach the mold wall at any angle.

4.3 Problem formulation

We consider a time-dependent 2D formulation, a schematic for which is shown in figure 36. For $x < 0$, there is a mold, cooled by water at temperature T_w that flows through horizontally oriented pipes, that oscillates in the vertical z -plane with speed $V_m(t)$, where t is the time. The mold cools a layer of molten flux powder that is fed vertically down adjacent to it; some of the flux solidifies, occupying the region $0 < x < S(z, t)$, whereas the rest remains molten and occupies $S(z, t) < x < S(z, t) + H(z, t)$; the interface at $x = S$ is taken to be at the melting temperature of the flux, $T_{m,f}$. Further out, the region $x > S(z, t) + H(z, t) + F(z, t)$ is occupied by molten steel, which starts to solidify at a distance $z_0(t)$ below the meniscus. The interface at $x = S + H + F$ separates the solid and molten steel and is assumed to be at the melting temperature of steel, $T_{m,s}$; moreover, the solid steel, which occupies the region $S + H < x < S + H + F$ is withdrawn vertically down in the z -direction with a uniform speed, V_c .

Note also that the solidified steel shell can also be subject to thermal contraction, although we neglect this effect here, as it seemed to make little contribution to the results of Hill *et al.* (HILL; WU; WIWATANAPATAPHEE, 1999).

4.3.1 Governing equations

For $0 < x < S(z, t)$, heat transfer is governed by

$$\rho_f^{(s)} c_f^{(s)} \left(\frac{\partial T}{\partial t} + V_m \frac{\partial T}{\partial z} \right) = k_f^{(s)} \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial z^2} \right), \quad (4.1)$$

where T is the temperature and $c_f^{(s)}$, $k_f^{(s)}$ and $\rho_f^{(s)}$ are, respectively, the specific heat capacity, the thermal conductivity and the density of the solid flux; typically, the mold oscillates according to

$$V_m(t) = V_0 \cos \omega t \quad (4.2)$$

and the solid flux is thus assumed to oscillate with it; we note, however, that non-sinusoidal oscillation is also possible (SHIN *et al.*, 2006; SUZUKI *et al.*, 1991; LIU *et al.*, 2002) and that, as in earlier work (VYNNYCKY *et al.*, 2017), the foregoing analysis goes through for this case also.

The conservation of mass, x -direction momentum, z -direction momentum and heat are expressed, respectively, by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial z} = 0, \quad (4.3)$$

$$\rho_i^{(l)} \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial z} \right) = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial x} \left(\mu_i \frac{\partial u}{\partial x} \right) + \frac{\partial}{\partial z} \left(\mu_i \frac{\partial u}{\partial z} \right), \quad (4.4)$$

$$\rho_i^{(l)} \left(\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial z} \right) = -\frac{\partial p}{\partial z} + \frac{\partial}{\partial x} \left(\mu_i \frac{\partial v}{\partial x} \right) + \frac{\partial}{\partial z} \left(\mu_i \frac{\partial v}{\partial z} \right) + \rho_i^{(l)} \hat{g}, \quad (4.5)$$

$$\rho_i^{(l)} c_i^{(l)} \left(\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial z} \right) = k_i^{(l)} \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial z^2} \right), \quad (4.6)$$

where $c_i^{(l)}, k_i^{(l)}, \rho_i^{(l)}$ and μ_i are respectively, the specific heat capacity, the thermal conductivity, the density and viscosity of molten material i , where $i = f$ (flux) or s (steel); all of these will be assumed to be constant. In addition, u and v are the velocity components in the x - and z -directions, respectively, p is the pressure and $\hat{g}$ is the acceleration due to gravity. The equations are for $i = f$ when $S(z, t) < x < G(z, t)$ and for $i = s$ when $x > G(z, t)$.

4.3.2 Boundary, interfacial and initial conditions

A heat balance at $x = 0$ gives

$$k_f^{(s)} \frac{\partial T}{\partial x} = \frac{m}{1 + mR_{mf}} (T - T_w), \quad (4.7)$$

where R_{mf} is the interface thermal contact resistance between the solid flux and the mold and m is the heat transfer coefficient linking the temperature at the outer surface of the solid flux and T_w ; the form of equation (4.7) is thus similar to that for Newton's law of cooling. Equation (4.7) is appropriate if there is a layer of solid flux at the mold wall; however, it is possible that during the oscillation cycle the temperature is high enough that there is no flux layer in solid phase, but rather in liquid phase; in this case, (4.7) would need to be replaced by

$$k_f^{(l)} \frac{\partial T}{\partial x} = m(T - T_w); \quad (4.8)$$

here, the interface thermal contact resistance has been removed, since one can expect perfect contact between molten flux and the mold wall.

Similarly, we have the heat flux at this surface, q , which is defined by

$$q = \begin{cases} \left(-k_f^{(s)} \frac{\partial T}{\partial x} \right)_{x=0}, & \text{if } (T)_{x=0} < T_{m,f}, \\ \left(-k_f^{(l)} \frac{\partial T}{\partial x} \right)_{x=0}, & \text{if } T_{m,f} \leq (T)_{x=0} \text{ and } S = 0, H > 0. \end{cases} \quad (4.9)$$

At $x = S(z, t)$, we set

$$v = V_m(t), \quad (4.10)$$

$$u = \frac{\partial S}{\partial t} + v \frac{\partial S}{\partial z}, \quad (4.11)$$

$$T = T_{m,f}, \quad (4.12)$$

$$k_f^{(s)} (\nabla T \cdot \mathbf{n}_S)_- - k_f^{(l)} (\nabla T \cdot \mathbf{n}_S)_+ = \rho_f^{(l)} \Delta H_f \left(\frac{\partial S}{\partial t} + V_m \frac{\partial S}{\partial z} \right), \quad (4.13)$$

where $()_{\pm}$ denotes the value of a function in the limit as x tends to the boundary in question from above and below, respectively, and ΔH_f is the latent heat of fusion for the flux; also, $\mathbf{n}_S$ denotes the unit normal to $x = S(z, t)$ and is given by

$$\mathbf{n}_S = \frac{1}{(1 + S_z^2)^{1/2}} (1, -S_z).$$

Physically, equations (4.10)-(4.13) represent, respectively: the continuity of the z -component of velocity, so that the molten flux moves with the speed of the solid flux, which is in turn assumed to move with the speed of the mold wall, i.e. no slip; the continuity of the x -component of the velocity; the temperature is equal to that of the melting temperature of the flux; conservation of heat, relating the differences in heat flux to the latent heat released due to phase change.

At the molten flux/ molten steel interface, $x = G(z, t)$, we have:

- continuity of tangential and normal velocity, leading to

$$[u]_{-}^{+} = 0, \quad [v]_{-}^{+} = 0, \quad (4.14)$$

with $[]_{-}^{+}$ denoting the difference in the value of a function above and below the boundary in question;

- a kinematic condition for the motion of the meniscus, given by

$$v = \frac{\partial \bar{G}}{\partial t} + u \frac{\partial \bar{G}}{\partial x}, \quad (4.15)$$

where $\bar{G}$ is the inverse function of G , so that $z = \bar{G}(x, t)$;

- continuity of tangential stress, given by

$$\left[2\mu_i \left(\frac{\partial v}{\partial z} - \frac{\partial u}{\partial x} \right) \frac{\partial \bar{G}}{\partial x} + \mu_i \left(\frac{\partial u}{\partial z} + \frac{\partial v}{\partial x} \right) \left\{ 1 - \left(\frac{\partial G}{\partial x} \right)^2 \right\} \right]_{-}^{+} = 0; \quad (4.16)$$

- a normal stress condition, given by

$$\left[-p + \frac{2\mu_i}{1 + \left(\frac{\partial \bar{G}}{\partial x} \right)^2} \left\{ \frac{\partial u}{\partial x} \left(\frac{\partial \bar{G}}{\partial x} \right)^2 + \frac{\partial v}{\partial z} - \left(\frac{\partial u}{\partial z} + \frac{\partial v}{\partial x} \right) \left(\frac{\partial \bar{G}}{\partial x} \right) \right\} \right]_{-}^{+} = \frac{\sigma_{fs} \partial^2 \bar{G} / \partial x^2}{\left(1 + \left(\partial \bar{G} / \partial x \right)^2 \right)^{3/2}}, \quad (4.17)$$

where σ_{fs} denotes the interfacial tension between the molten flux and the molten steel;

- continuity of temperature, given by

$$[T]_{-}^{+} = 0; \quad (4.18)$$

- continuity of heat flux, given by

$$\left[k_i^{(1)} \nabla T \cdot \mathbf{n}_G \right]_{-}^{+} = 0, \quad (4.19)$$

where

$$\mathbf{n}_G = \frac{1}{(1 + G_z^2)^{1/2}} (1, -G_z). \quad (4.20)$$

At $z = 0$, we have

$$p = p_a \quad \text{at } z = 0, \quad (4.21)$$

$$T = T_{m,f} \quad \text{at } z = 0, \quad (4.22)$$

with p_a as the atmospheric pressure. The thinking behind the first of these is that the flux bed on top of the meniscus will be no more than a centimetre in height as seen, for example, in the detailed computations of Ramirez-Lopez *et al.* (RAMIREZ-LOPEZ *et al.*, 2012) and will therefore contribute a pressure head no greater than $2930 \times 10 \times 0.01 \approx 293$ Pa. Compared to the atmospheric pressure, 10^5 Pa, this is clearly negligible. As regards (4.22), it is quite natural to assume that the bottom of the flux bed should be at the melting temperature of the flux.

Boundary conditions should be required at the right-hand end of the domain, but we leave these unspecified for the time being. In addition, boundary conditions should be set at $z = z_0(t)$. In some sense, this is an artificial boundary which has been chosen to delineate the upper region, where there is no solid steel, and a lower one where there is. Since, in the model that we will solve later on, we only require boundary conditions for $S_0(t) < x < G_0(t)$, where

$$S_0(t) := S(z_0(t), t), \quad G_0(t) := G(z_0(t), t),$$

we set the same condition that was used there by Vynnycky in *et al.* (VYNNYCKY *et al.*, 2017), i.e.

$$p = p_a + \rho_f^{(l)} \hat{g} z_0(t) \quad \text{at } z = z_0(t); \quad (4.23)$$

we should also, in general, expect continuity of temperature and heat flux on going from the upper to the lower zone, but we return to this point at the end of section 4.6.

The molten flux/molten steel interface only extends until the point of solidification, $(x_0(t), z_0(t))$, where $x_0(t) = G_0(t)$. An unclear issue at this stage is whether further constraints are required, for example, on the gradient of $x = G(z, t)$ at this point, or even the angles that the three phases make relative to each other. Some existing alternatives are:

- Steinruck *et al.* (STEINRÜCK; RUDISCHER; SCHNEIDER, 1997; STEINRÜCK; RUDISCHER; SCHNEIDER, 1998; STEINRÜCK; RUDISCHER, 2002) consider that the meniscus does not move in time and that solidification begins at the point when the meniscus is practically tangential to the mold wall;
- Fredriksson and Elfsberg (FREDRIKSSON; ELFSBERG, 2002) and Elfsberg (ELFSBERG, 2010) propose that the Dupré equation and the cosine theorem should be used, so that the angles subtended by each phase are related to σ_{fs} and the corresponding interfacial tensions for the molten flux/solid steel and solid steel/molten steel interfaces.

The problem also formally requires an initial condition, but since we will be considering the periodic behavior of the system that is eventually established, there is no need to specify these explicitly here.

Symbol	Value	Unit
$c_f^{(l)}$	1260	J kg ⁻¹ K ⁻¹
$c_f^{(s)}$	1260	J kg ⁻¹ K ⁻¹
$c_s^{(s)}$	670	J kg ⁻¹ K ⁻¹
$k_f^{(l)}$	1.5	W m ⁻¹ K ⁻¹
$k_f^{(s)}$	2.25	W m ⁻¹ K ⁻¹
$k_s^{(l)}$	30*	W m ⁻¹ K ⁻¹
$k_s^{(s)}$	30*	W m ⁻¹ K ⁻¹
$\hat{g}$	9.81	m s ⁻²
m	8.5×10^3	W m ⁻² K ⁻¹
R	2×10^{-4}	m ² K W ⁻¹
$T_{f,s}$	1373	K
$T_{m,s}$	1773	K
T_w	300	K
V_c	0.013	m s ⁻¹
V_0	0.029	m s ⁻¹
ΔH_s	272,000	J kg ⁻¹
μ_f	0.1	Pa s
μ_s	0.006	Pa s
$\rho_f^{(l)}$	2930	kg m ⁻³
$\rho_s^{(l)}$	7800	kg m ⁻³
σ_{fs}	1.3	N m ⁻¹
ω	$19\pi/6$	rad s ⁻¹

Table 1 – Model parameters from Saleem (SALEEM, 2016) and Vynnycky *et al.* (VYNNYCKY *et al.*, 2017) *Mills *et al.* (MILLS *et al.*, 2016b)

4.4 Nondimensionalization

We nondimensionalize using

$$X = \frac{x}{W}, \quad Z = \frac{z}{W}, \quad \tau = \frac{t}{[t]}, \quad U = \frac{u}{V_c}, \quad V = \frac{v}{V_c}, \quad \theta = \frac{T - T_{m,f}}{T_{m,s} - T_w},$$

$$P = \frac{p - p_a}{\mu_f V_c / L}, \quad \mathcal{G} = \frac{G}{L}, \quad \bar{\mathcal{G}} = \frac{\bar{G}}{L}, \quad \mathcal{S} = \frac{S}{L},$$

where L is termed as *the capillary length* and is given by

$$L = \sqrt{\frac{\sigma_{fs}}{(\rho_s^{(l)} - \rho_f^{(l)}) \hat{g}}}$$

and $[t] = 2\pi/\omega$. The numerical values for all model parameters are as given in Table 1; these consist of the data from Saleem (SALEEM, 2016) as well as supplementary data where required from Mills *et al.* (MILLS *et al.*, 2016a; MILLS *et al.*, 2016b). Note that L has been chosen as the appropriate length scale in anticipation of the importance of surface tension in this region;

Figure 37 – V as a function of τ (solid curve), as well the curve $V = 1$ (dashed line). Each interval of τ for which $V > 1$ is commonly referred to as the “negative strip time”.

Source: Elaborated by the author.

the expression is consistent with that used by Steinrück *et al.* (STEINRÜCK; RUDISCHER; SCHNEIDER, 1997). Using the values in Table 1, we find that $L \sim 5$ mm, which should be considered in the context of cast blooms, billets and slabs that have cross-sectional length scales of the order of tens of centimetres; consequently, the length scale for the surface-tension-dominated region is much greater than that of typically observed oscillation-mark depths, but still much smaller than that of the overall casting geometry.

4.4.1 Governing equations

So, for $0 < X < \mathcal{S}(Z, \tau)$, equation (4.1) gives

$$\delta \frac{\partial \theta}{\partial \tau} + \mathcal{V}_m \frac{\partial \theta}{\partial Z} = \frac{\alpha_f}{Pe_f} \left(\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Z^2} \right), \quad (4.24)$$

where

$$\mathcal{V}_m(\tau) = \mathcal{V}_0 \cos 2\pi\tau, \quad (4.25)$$

with

$$\mathcal{V}_0 = \frac{V_0}{V_c}, \quad \delta = \frac{\omega L}{2\pi V_c}, \quad \alpha_f = \frac{k_f^{(s)} / \rho_s^{(s)} c_s^{(s)}}{k_f^{(l)} / \rho_s^{(l)} c_s^{(l)}};$$

Pe_f is defined further down in equation (4.30) and, for ease of orientation later on, a plot of V as a function of τ is given in figure 37.

Equations (4.3)-(4.5) give, for $X > \mathcal{S}(Z, \tau)$ and $i = f, s$,

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Z} = 0, \quad (4.26)$$

$$Re_i \left(\delta \frac{\partial U}{\partial \tau} + U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Z} \right) = -\lambda_i \frac{\partial P}{\partial X} + \frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Z^2}, \quad (4.27)$$

$$Re_i \left(\delta \frac{\partial V}{\partial \tau} + U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Z} \right) = -\lambda_i \frac{\partial P}{\partial Z} + \frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Z^2} + \frac{\lambda_i \rho_i^{(1)}}{Ca \left(\rho_s^{(1)} - \rho_f^{(1)} \right)}, \quad (4.28)$$

$$\delta \frac{\partial \theta}{\partial \tau} + U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Z} = \frac{1}{Pe_i} \left(\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Z^2} \right), \quad (4.29)$$

where, for $i = f, s$,

$$Re_i = \frac{\rho_i^{(1)} V_c W}{\mu_i}, \quad Pe_i = \frac{\rho_i^{(1)} c_i^{(1)} V_c L}{k_i^{(1)}}, \quad \lambda_i = \frac{\mu_i}{\mu_f}, \quad (4.30)$$

and

$$Ca = \frac{\mu_f V_c}{\sigma_{fs}}; \quad (4.31)$$

moreover, in (4.30), Re_i and Pe_i are the Reynolds and Péclet numbers for material i , respectively, and in (4.31) Ca is the capillary number.

4.4.2 Boundary conditions

At $X = 0$, if solid flux contacts the mold, we have

$$\frac{\partial \theta}{\partial X} = Bi_{ms} \theta, \quad (4.32)$$

where Bi_{ms} is the Biot number and is given by

$$Bi_{ms} = mW / k_f^{(s)} (1 + Rm). \quad (4.33)$$

If molten flux contacts the mold, then

$$K_{ff} \frac{\partial \theta}{\partial X} = Bi_{ml} \theta, \quad (4.34)$$

where $K_{ff} = k_f^{(l)} / k_f^{(s)}$ and $Bi_{ml} = mW / k_f^{(s)}$.

At $X = \mathcal{S}(Z, \tau)$,

$$V = \mathcal{V}_m, \quad (4.35)$$

$$U = \delta \frac{\partial \mathcal{S}}{\partial \tau} + V \frac{\partial \mathcal{S}}{\partial Z}, \quad (4.36)$$

$$\theta = \theta_{m,f}, \quad (4.37)$$

$$\left(\frac{\partial \theta}{\partial X} \right)_- = K_{ff} \left(\frac{\partial \theta}{\partial X} \right)_+, \quad (4.38)$$

where $\theta_{m,f} = (T_{m,f} - T_w)/(T_{m,s} - T_w)$; note that, in (4.38), we assume that the latent heat release is small enough that there is no need to consider it, as was done in (HILL; WU; WIWATANAP-ATAPHEE, 1999).

As for boundary conditions (4.14)-(4.23), these become, respectively, as follows: at $X = \mathcal{G}(Z, \tau)$,

$$[U]_-^+ = 0, \quad [V]_-^+ = 0, \quad (4.39)$$

$$V = \delta \frac{\partial \mathcal{G}}{\partial \tau} + U \frac{\partial \mathcal{G}}{\partial X}, \quad (4.40)$$

$$\left[\frac{1}{\lambda_i} \left\{ 2 \left(\frac{\partial V}{\partial Z} - \frac{\partial U}{\partial X} \right) \frac{\partial \mathcal{G}}{\partial X} + \left(\frac{\partial U}{\partial Z} + \frac{\partial U}{\partial X} \right) \left\{ 1 - \left(\frac{\partial \mathcal{G}}{\partial X} \right)^2 \right\} \right\} \right]_-^+ = 0, \quad (4.41)$$

$$\begin{aligned} Ca[-P]_-^+ + Ca \left[\frac{2}{\lambda_i \left[1 + \left(\frac{\partial \mathcal{G}}{\partial X} \right)^2 \right]} \left\{ \frac{\partial U}{\partial X} \left(\frac{\partial \mathcal{G}}{\partial X} \right)^2 + \frac{\partial V}{\partial Z} - \left(\frac{\partial U}{\partial Z} + \frac{\partial V}{\partial X} \right) \left(\frac{\partial \mathcal{G}}{\partial X} \right) \right\} \right]_-^+ \\ = \frac{\partial^2 \mathcal{G} / \partial X^2}{\left(1 + \left(\partial \mathcal{G} / \partial X \right)^2 \right)^{3/2}}, \end{aligned} \quad (4.42)$$

$$[\theta]_-^+ = 0, \quad (4.43)$$

$$(\nabla \theta \cdot \mathbf{n}_{\mathcal{G}})_- = K_{sf} (\nabla \theta \cdot \mathbf{n}_{\mathcal{G}})_+, \quad (4.44)$$

where $K_{sf} = k_s^{(l)} / k_f^{(l)}$ and

$$\mathbf{n}_{\mathcal{G}} = \frac{1}{(1 + \mathcal{G}_Z^2)^{1/2}} (1, -\mathcal{G}_Z). \quad (4.45)$$

At $Z = 0$, we have

$$P = 0, \quad (4.46)$$

$$\theta = \theta_{m,f}, \quad (4.47)$$

and, at $Z = Z_0(\tau)$, equation (4.23) gives

$$P = \frac{1}{Ca(\rho - 1)} Z_0, \quad (4.48)$$

where $\rho = \rho_s^{(l)} / \rho_f^{(l)}$.

4.4.3 Nondimensional parameters

Taking, for simplicity, $Bi_{ml} = Bi_{ms} = Bi$, there are, by this stage, 15 nondimensional parameters:

$$Bi, Ca, K_{ff}, K_{fs}, K_{sf}, Pe_f, Pe_s, Re_f, Re_s, \alpha_f, \delta, \lambda_s, \tilde{V}_0, \theta_{m,f}, \rho;$$

their values are given in Table 2.

Parameter	Value
Bi	10
Ca	10^{-3}
K_{ff}	1.5
K_{fs}	0.075
K_{sf}	13
Pe_f	11
Pe_s	110
Re_f	2
Re_s	88
α_f	0.7
δ	0.6
λ_s	0.06
$\tilde{V}_0$	2.22
$\theta_{m,f}$	0.723
ρ	2.66

Table 2 – Values of dimensionless parameters using data from Hill *et al.* (HILL; WU; WIWATANAPAT-APHEE, 1999) and Saleem (SALEEM, 2016)

4.5 Reduced model

We now proceed by proposing a reduced model that makes use of the sizes of the dimensionless parameters to simplify the governing equations, as appropriate. Although the majority of the remaining dimensionless are neither small enough nor large enough to justify simplification, the fact that $Ca, K_{fs}, \lambda_s \ll 1$ leads to a number of simplifications. Principally, we see that because $\lambda_s \ll 1$, boundary condition (4.41) decouples: at leading order, it states that the quantity on the flux side is zero, to leading order. In other words, the molten flux experiences zero shear stress at this boundary.

Also, we will later find that, because $Ca \ll 1$, (4.42) gives the same ordinary differential equation that leads to the Bikerman profile (BIKERMAN, 1970) with the second term on the left being negligible. However, the fact that $K_{sf} \gg 1$ means that, at leading order, the molten steel sees a thermally insulated boundary. We have then the following hierarchy of problems, which can, in principle, be treated sequentially:

I velocity field in the flux;

II velocity field in the molten steel;

III temperature field in the molten steel;

IV temperature field in the solid and molten flux.

4.5.1 Sub-problem I: velocity field in the molten flux

Equations (4.3)-(4.5) give, for $\mathcal{S}(Z, \tau) < X < \mathcal{G}(Z, \tau)$,

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Z} = 0, \quad (4.49)$$

$$Re_f \left(\delta \frac{\partial U}{\partial \tau} + U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Z} \right) = -\lambda_f \frac{\partial P}{\partial X} + \frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Z^2}, \quad (4.50)$$

$$Re_f \left(\delta \frac{\partial V}{\partial \tau} + U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Z} \right) = -\lambda_f \frac{\partial P}{\partial Z} + \frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Z^2} + \frac{\lambda_f}{Ca(\rho - 1)}, \quad (4.51)$$

subject to

$$V = \mathcal{V}_m \quad \text{at } X = \mathcal{S}(Z, \tau), \quad (4.52)$$

$$U = \delta \frac{\partial \mathcal{S}}{\partial \tau} + V \frac{\partial \mathcal{S}}{\partial Z} \quad \text{at } X = \mathcal{S}(Z, \tau), \quad (4.53)$$

$$V = \delta \frac{\partial \mathcal{G}}{\partial \tau} + U \frac{\partial \mathcal{G}}{\partial X} \quad \text{at } X = \mathcal{G}(Z, \tau), \quad (4.54)$$

$$\left[\frac{1}{\lambda_i} \left\{ 2 \left(\frac{\partial V}{\partial Z} - \frac{\partial U}{\partial X} \right) \frac{\partial \mathcal{G}}{\partial X} + \left(\frac{\partial U}{\partial Z} + \frac{\partial V}{\partial X} \right) \left\{ 1 - \left(\frac{\partial \mathcal{G}}{\partial X} \right)^2 \right\} \right\} \right]_{-}^{+} = 0, \quad (4.55)$$

$$Ca[-P]_{-}^{+} + Ca \left[\frac{2}{\lambda_i \left[1 + \left(\frac{\partial \mathcal{G}}{\partial X} \right)^2 \right]} \left\{ \frac{\partial U}{\partial X} \left(\frac{\partial \mathcal{G}}{\partial X} \right)^2 + \frac{\partial V}{\partial Z} - \left(\frac{\partial U}{\partial Z} + \frac{\partial V}{\partial X} \right) \left(\frac{\partial \mathcal{G}}{\partial X} \right) \right\} \right]_{-}^{+} \\ = \frac{\partial^2 \mathcal{G} / \partial X^2}{\left(1 + \left(\partial \mathcal{G} / \partial X \right)^2 \right)^{3/2}}, \quad (4.56)$$

$$P = 0 \quad \text{at } Z = 0, \quad (4.57)$$

$$P = \frac{Z_0}{Ca(\rho - 1)} \quad \text{at } Z = Z_0(\tau). \quad (4.58)$$

4.5.2 Sub-problem II: velocity field in the molten steel

For $X > \mathcal{G}(Z, \tau)$,

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Z} = 0, \quad (4.59)$$

$$Re_s \left(\delta \frac{\partial U}{\partial \tau} + U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Z} \right) = -\lambda_s \frac{\partial P}{\partial X} + \frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Z^2}, \quad (4.60)$$

$$Re_s \left(\delta \frac{\partial V}{\partial \tau} + U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Z} \right) = -\lambda_s \frac{\partial P}{\partial Z} + \frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Z^2} + \frac{\lambda_s}{Ca(\rho - 1)}, \quad (4.61)$$

subject to

$$U = U^*, \quad V = V^* \quad \text{at } X = \mathcal{G}(Z, \tau), \quad (4.62)$$

where U^* and V^* are the profiles of U and V , respectively, obtained at $X = \mathcal{G}(Z, \tau)$ from solving sub-problem I.

4.5.3 Sub-problem III: temperature field in the molten steel

For $X > \mathcal{G}(Z, \tau)$,

$$\delta \frac{\partial \theta}{\partial \tau} + U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Z} = \frac{1}{Pe_s} \left(\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Z^2} \right), \quad (4.63)$$

subject to

$$\nabla \theta \cdot \mathbf{n}_{\mathcal{G}} = 0 \quad \text{at } X = \mathcal{G}(Z, \tau). \quad (4.64)$$

4.5.4 Sub-problem IV: temperature field in the solid and molten flux

For $0 < X < \mathcal{S}(Z, \tau)$,

$$\delta \frac{\partial \theta}{\partial \tau} + V_m \frac{\partial \theta}{\partial Z} = \frac{\alpha_f}{Pe_f} \left(\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Z^2} \right), \quad (4.65)$$

and for $\mathcal{S}(Z, \tau) < X < \mathcal{G}(Z, \tau)$,

$$\delta \frac{\partial \theta}{\partial \tau} + U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Z} = \frac{1}{Pe_f} \left(\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Z^2} \right), \quad (4.66)$$

subject to

$$\theta = \theta_{m,f} \quad \text{at } Z = 0, \quad (4.67)$$

$$\theta = \theta^* \quad \text{at } X = \mathcal{G}(Z, \tau), \quad (4.68)$$

$$\frac{\partial \theta}{\partial X} = \begin{cases} Bi_{ms} \theta & \text{if } \mathcal{S} > 0 \\ Bi_{ml} \theta / \kappa & \text{if } \mathcal{S} = 0 \end{cases} \quad \text{at } X = 0, \quad (4.69)$$

$$\theta = \theta_{m,f} \quad \text{at } X = \mathcal{S}(Z, \tau), \quad (4.70)$$

$$(\nabla \theta \cdot \mathbf{n}_S)_- = \kappa (\nabla \theta \cdot \mathbf{n}_S)_+ \quad \text{at } X = \mathcal{S}(Z, \tau). \quad (4.71)$$

where θ^* is the profile of θ obtained at $X = \mathcal{G}(Z, \tau)$ from solving sub-problem III.

4.5.5 Caveats

Whilst the above gives a strategy for solving the overall problem, there are a number of obstacles:

- all of the physical properties $\mu_f, \mu_s, k_f, k_s, c_p, \sigma_{fs}$ are in general temperature-dependent; however, we take all of them to be constant.
- the solution to sub-problem I depends on the location of $\mathcal{S}$, which of course depends on the temperature, and hence the other sub-problems. However, the simplification that $\mathcal{S} = 0$ removes this coupling; indeed, this was found by Vynnycky *et al.* (VYNNYCKY *et al.*, 2017) to be appropriate for one set of data, but was also found to be of relevance to other recent work (YANG *et al.*, 2017).

- here, we have considered the sub-problems in the context of what happens above $Z = Z_0(\tau)$, but it is transparent that sub-problems II and III can also affect what happens for $Z > Z_0(\tau)$, in particular the location of the solid/molten steel interface.

At this stage, we can note that sub-problems II and III are clearly lacking outer boundary conditions, which would come, loosely speaking, from matching to the outer flow. However, in what follows, we will focus on sub-problem I with $\mathcal{S} = 0$, which means that we do not need to consider sub-problems II and III at all.

4.6 Analysis

For the rest of the work, we focus on the solution of sub-problem I, assuming that there is no solid flux. Since $Ca \ll 1$, we can consider a regular perturbation problem by setting, for $\varphi = (U_X, U_Z, \mathcal{G}, \bar{\mathcal{G}}, Z_0)$,

$$\varphi = \varphi^{(0)} + Ca\varphi^{(1)} + \dots,$$

and

$$P = \frac{Z}{Ca(\rho - 1)} + P^{(1)} + \dots$$

Thus, at $O(1)$, (4.49)-(4.51) give

$$\frac{\partial U^{(0)}}{\partial X} + \frac{\partial V^{(0)}}{\partial Z} = 0, \quad (4.72)$$

$$Re_f \left(\delta \frac{\partial U^{(0)}}{\partial \tau} + U^{(0)} \frac{\partial U^{(0)}}{\partial X} + V^{(0)} \frac{\partial U^{(0)}}{\partial Z} \right) = -\frac{\partial P^{(1)}}{\partial X} + \frac{\partial^2 U^{(0)}}{\partial X^2} + \frac{\partial^2 U^{(0)}}{\partial Z^2}, \quad (4.73)$$

$$Re_f \left(\delta \frac{\partial V^{(0)}}{\partial \tau} + U^{(0)} \frac{\partial V^{(0)}}{\partial X} + V^{(0)} \frac{\partial V^{(0)}}{\partial Z} \right) = -\frac{\partial P^{(1)}}{\partial Z} + \frac{\partial^2 V^{(0)}}{\partial X^2} + \frac{\partial^2 V^{(0)}}{\partial Z^2}, \quad (4.74)$$

subject to

$$P^{(1)} = 0 \quad \text{at } Z = 0, \quad (4.75)$$

$$P^{(1)} = 0 \quad \text{at } Z = Z_0^{(0)}(\tau), \quad (4.76)$$

$$U^{(0)} = 0 \quad \text{at } X = 0, \quad (4.77)$$

$$V^{(0)} = \mathcal{V}_m \quad \text{at } X = 0, \quad (4.78)$$

and, at $X = \mathcal{G}^{(0)}(Z, \tau)$,

$$2 \left(\frac{\partial V^{(0)}}{\partial Z} - \frac{\partial U^{(0)}}{\partial X} \right) \frac{\partial \bar{\mathcal{G}}^{(0)}}{\partial X} + \left(\frac{\partial U^{(0)}}{\partial Z} + \frac{\partial V^{(0)}}{\partial X} \right) \left\{ 1 - \left(\frac{\partial \bar{\mathcal{G}}^{(0)}}{\partial X} \right)^2 \right\} \bigg|_f = 0, \quad (4.79)$$

$$U^{(0)} = \delta \frac{\partial \bar{\mathcal{G}}^{(0)}}{\partial \tau} + V^{(0)} \frac{\partial \bar{\mathcal{G}}^{(0)}}{\partial Z}. \quad (4.80)$$

In addition, (4.58) becomes

$$\frac{\partial^2 \bar{\mathcal{G}}^{(0)} / \partial X^2}{\left(1 + (\partial \bar{\mathcal{G}}^{(0)} / \partial X)^2\right)^{3/2}} = \bar{\mathcal{G}}^{(0)}, \quad (4.81)$$

with

$$\bar{\mathcal{G}}^{(0)} \rightarrow 0 \quad \text{as} \quad X \rightarrow \infty. \quad (4.82)$$

Now, we drop the ⁽⁰⁾ and ⁽¹⁾ superscripts. Integrating (4.81) and applying (4.82), we have

$$-\frac{1}{\left(1 + (\partial \bar{\mathcal{G}} / \partial X)^2\right)^{1/2}} = \frac{1}{2} \bar{\mathcal{G}}^2 - 1, \quad (4.83)$$

whence

$$\frac{\partial \bar{\mathcal{G}}}{\partial X} = \frac{\bar{\mathcal{G}} (4 - \bar{\mathcal{G}}^2)^{1/2}}{\bar{\mathcal{G}}^2 - 2}. \quad (4.84)$$

Thus,

$$X - C_0(\tau) = -\sqrt{4 - \bar{\mathcal{G}}^2} + \ln \left(\frac{2 + \sqrt{4 - \bar{\mathcal{G}}^2}}{\bar{\mathcal{G}}} \right), \quad (4.85)$$

where $C_0(\tau)$ is a function of τ that is to be determined. However, if the triple point is at $(X_0(\tau), Z_0(\tau))$, then

$$C_0(\tau) = X_0 + \sqrt{4 - Z_0^2} - \ln \left(\frac{2 + \sqrt{4 - Z_0^2}}{Z_0} \right), \quad (4.86)$$

although $X_0(\tau)$ and $Z_0(\tau)$ are both still undetermined at this stage. Note also that, as $X \rightarrow \infty$,

$$X \sim -\ln \bar{\mathcal{G}}, \quad (4.87)$$

so that $\bar{\mathcal{G}} \sim \exp(-X)$; this is an important observation for the numerical work that follows later. Moreover, although (4.85) may give the impression that $\bar{\mathcal{G}} < 2$, note that (4.83) implies that $\bar{\mathcal{G}} \leq \sqrt{2}$, which means that we can only have $Z_0(\tau) < \sqrt{2}$. To find $\partial \mathcal{G} / \partial \tau$ for later use in equation (4.80), we differentiate (4.85) with respect to τ to obtain

$$-\frac{dC_0}{d\tau} = \frac{\bar{\mathcal{G}}^2 - 2}{\bar{\mathcal{G}} \sqrt{4 - \bar{\mathcal{G}}^2}} \frac{\partial \bar{\mathcal{G}}}{\partial \tau}, \quad (4.88)$$

with, from (4.86),

$$\frac{dC_0}{d\tau} = \frac{dX_0}{d\tau} - \frac{Z_0^2 - 2}{Z_0 \sqrt{4 - Z_0^2}} \frac{dZ_0}{d\tau}, \quad (4.89)$$

whence

$$\frac{\partial \bar{\mathcal{G}}}{\partial \tau} = \frac{\bar{\mathcal{G}} \sqrt{4 - \bar{\mathcal{G}}^2}}{\bar{\mathcal{G}}^2 - 2} \left\{ \frac{Z_0^2 - 2}{Z_0 \sqrt{4 - Z_0^2}} \frac{dZ_0}{d\tau} - \frac{dX_0}{d\tau} \right\}. \quad (4.90)$$

It still remains, however, to determine X_0 and Z_0 , and this must come from matching to the lower zone at $Z = Z_0(\tau)$. We can argue as follows. The mass of molten flux exiting the upper zone, $Q_U^{(l)}$, must equal that entering the lower zone, $Q_L^{(l)}$; similarly the total heat transported out of the upper zone must equal to that entering the lower zone. Thus, these constitute two conditions that can be used to determine X_0 and Z_0 ; this idea is in the spirit of Hill *et al.* (HILL; WU; WIWATANAPATAPHEE, 1999) and Vynnycky *et al.* (VYNNYCKY *et al.*, 2017). We narrow down this idea further by positing that if only one of these two conditions is satisfied, then only X_0 or Z_0 can be determined. To ascertain whether this is feasible, we will suppose that Z_0 is fixed and that

$$Q_L^{(l)} = Q_U^{(l)} \quad \text{at } Z = Z_0. \quad (4.91)$$

In dimensional terms, we have

$$Q_U^{(l)}(\tau) = \rho_f^{(l)} V_c L \int_0^{X_0(\tau)} (V)_{Z=Z_0(\tau)} dX, \quad (4.92)$$

and, from Vynnycky *et al.* (VYNNYCKY *et al.*, 2017),

$$Q_L^{(l)}(\tau) = \rho_f^{(l)} V_c [x_L] \left\{ \frac{6(\tilde{V} - 1) \int_0^{Z_L} \frac{dZ}{H_L^2} + \Lambda Z_L}{12 \int_0^{Z_L} \frac{dZ}{H_L^3}} + H_L \right\}, \quad (4.93)$$

where

$$[x_L] = \left(\frac{k_f^{(s)}[t]}{\rho_f^{(s)} c_f^{(s)}} \right)^{1/2}, \quad \Lambda = \frac{\rho_f^{(l)} g [x_L]^2}{\mu_f V_c}, \quad Z_L = \frac{\tilde{L}}{V_c [t]},$$

with $\tilde{L}$ as the length of the mold. Moreover, it was found earlier that that $H_L = H_0(\tau - Z_L)$ (VYNNYCKY *et al.*, 2017), where $Z_L = \delta Z$. Note now that $X_0 = [x_L] H_L / L$ and that $X_0 = H_0$, so that (4.91) gives the following nondimensional equation for $X_0(\tau)$:

$$\int_0^{X_0(\tau)} (V)_{Z=Z_0} dX = \frac{6(\gamma_m - 1) \int_{\tau-Z_L}^{\tau} \frac{d\zeta}{X_0^2(\zeta)} + \tilde{\Lambda} Z_L}{12 \int_{\tau-Z_L}^{\tau} \frac{d\zeta}{X_0^3(\zeta)}} + X_0(\tau), \quad (4.94)$$

where $\tilde{\Lambda} = \Lambda W^2 / [x_L]^2$. Moreover, from the earlier work, it was found that the right-hand side of (4.94) effectively reduced to just $X_0(\tau)$, as a consequence of the fact that X_0 vanished for some value of τ , reducing the first term on the right-hand side of (4.94) to zero, because its denominator is more singular than its numerator (VYNNYCKY *et al.*, 2017); however, there are no grounds for making this assumption *a priori* here.

4.7 Numerical implementation and experimentation

The remaining numerical task is then to solve (4.72)-(4.80) and (4.94). For this, we turned to the transient solver in the finite element-based software, Comsol Multiphysics, coupled to MATLAB.

The discretization of the region of molten flux used Lagrangian P2-P1 triangular elements, as well as the software's *moving mesh* feature.

To give reliability to the computations, in every step of the iteration in time, performed by Comsol Multiphysics, the convergence criterion employed was given by

$$\left(\frac{1}{N_{dof}} \sum_{i=1}^{N_{dof}} \left(\frac{|E_i|}{A_i + \mathcal{R}|U_i|} \right)^2 \right)^{\frac{1}{2}} < 1, \quad (4.95)$$

where N_{dof} is the number of degrees of freedom, E_i is the estimated error in the latest approximation to the i^{th} component of the true solution vector, U_i , at each time step, A_i is the absolute tolerance for the i^{th} degree of freedom, and $\mathcal{R}$ is the relative tolerance; for the computations, $\mathcal{R} = 10^{-2}$, $A_i = 10^{-3}$ for $i = 1, \dots, N_{dof}$ were used.

Figure 38 shows a typical and suitable computational mesh that was used to obtain the fluid flow velocity. In this mesh, there are 854 elements, for which $N_{dof} = 6333$.

Figure 38 – Typical mesh to compute the fluid flow for the molten flux in the upper zone.

Source: Elaborated by the author.

There are a number of issues to resolve:

1. How large should the extent of the computational domain in the X -direction, which we shall denote by X_∞ , be?
2. How many elements are required for mesh-independent results?
3. How many elements should be taken across $Z = Z_0(\tau)$?

To answers these questions, we first prescribed a profile for $X_0(\tau)$ and just solved equations (4.72)-(4.80); we took

$$X_0 = X^* + \bar{X}^* \sin 2\pi\tau, \quad (4.96)$$

where X^* and $\bar{X}^*$ are strictly positive constants, and with $X^* > \bar{X}^*$. Here, X^* was fixed and $\bar{X}^*$ was changing through a lot of trials performed computationally, always using the fact that $X_0(\tau) > 0$ in every moment of the computations; eventually, the values $X^* = 0.11$ and $\bar{X}^* = 0.09$ were chosen.

It was found that $Q_U^{(1)}(\tau)$ was mesh-independent if we took $X_\infty = 2.5$ as shown in figure 39, wherein the mass of flux has the same value when compared with the mass of flux for larger values of X_∞ . Thus, the shape of the meniscus is defined from X_0 to $X_\infty = 2.5$.

Figure 39 – $Q_U^{(1)}$ as a function of τ using equation (4.96) for different values of X_∞

Source: Elaborated by the author.

Now, the number of elements $N = 4$ across $Z = Z_0$ was chosen because the mass of flux behaves with no variations during the computations; this result is summarized in figure 40.

Also, we see that the periodicity for $Q_U^{(1)}$ in all cases is evident after only two periods,

Figure 40 – $Q_U^{(1)}$ as a function of τ using equation (4.96) for X_0 for different values of N

Source: Elaborated by the author.

which is perhaps not so surprising since the arbitrarily-chosen X_0 profile has the same period of oscillation as the mold wall velocity.

A more stringent test is to consider what happens as we decrease the minimum value of X_0 , i.e. $X^* - \bar{X}^*$, in particular as this value approaches zero, which is the case when the molten flux gap would close up completely; we can suspect beforehand that this eventuality would lead to numerical problems.

Using (4.96) and the 6333-element mesh described earlier, we did indeed find that we were not able to obtain solutions when $X^* - \bar{X}^*$ became as small as 5×10^{-4} ; even for values higher than this, there were problems with the convergence of the solution pointed out by the solver of Comsol Multiphysics, indicating that the method would always run into problems in this way.

Thus, whether there was a way to bypass this situation was considered. Since the value of δ in equations (4.73) and (4.74) is only around 0.6, we considered simply setting it to zero. Doing this and using (4.96), we found that $Q_U^{(1)}$ was, to all intents and purposes, unchanged; this is shown in figure 41.

Figure 41 – $Q_U^{(1)}$ as a function of τ using equation (4.96) for X_0 , using the true value of $\delta (= 0.6)$ and $\delta = 0$ in equations (4.73) and (4.74)

Source: (VYNNYCKY; ZAMBRANO, 2018).

This means that it is possible to solve the problem in a quasi-steady fashion, i.e. we can remove the time derivatives in equations (4.73) and (4.74), yet the time-dependence of the problem still enters through (4.78) and (4.80), in the latter of which the true value of δ is retained. It needs to be pointed out that, even with this adjustment, the problem only becomes weakly quasi-steady because (4.80) ensures that a time-derivative is still retained, albeit in a boundary condition rather than in a partial differential equation. Nevertheless, we update the geometry

externally in MATLAB, outside of the call to Comsol Multiphysics. Note that this approach requires us to use Comsol's steady-state solver, rather than the transient one used earlier. For this, the convergence criterion in (B9) is replaced by

$$\left(\frac{1}{N_{dof}} \sum_{i=1}^{N_{dof}} |E_i|^2 \right)^{1/2} < \varepsilon, \quad (4.97)$$

where $\varepsilon = 10^{-6}$; lower values of ε were also tried, but the results were practically indistinguishable.

Consequently, this weakly quasi-steady approach was deemed more suitable for tackling the actual problem when X_0 has to be determined by the solution to (4.94), and where there may be the real possibility that $X_0 = 0$ for substantial intervals in τ . Thus, based on the experience of this experimentation, we proceed to the solution of the full system of equations, i.e. (4.72)-(4.80) and (4.94). Also, we note that all computations were carried out on an ICORE I5 Notebook with a 2.5 GHz processor and 8GB RAM, with a computation corresponding to one oscillation period requiring around one minute of CPU time.

For the actual problem, a preliminary issue is how to begin the computation at $\tau = 0$. We note that $\tau = 0$ corresponds to the instant of time when the difference between the mold velocity and the casting speed is at its greatest; we would therefore expect there to be a flux gap and that $X_0 > 0$. However, $Q_L^{(1)}$ requires the value of H for $Z_0 < Z < Z_L$, and we must clearly make a guess for this. However, since $H = H_0(\tau - \delta Z)$, this effectively means having to prescribe H_0 for $-Z_L \leq \tau \leq 0$; for this, it is simplest to take H_0 to be a constant, which we shall call H_0^* . Different values were tried for H_0^* , but in each case it was always found that X_0 reached zero. This is exemplified in figure 42, which shows solutions for X_0 , computed using $Z_0 = 1$. Nevertheless, the reasons why X_0 reaches zero are slightly different, depending on the value of H_0^* , and are worthy of explanation. First, we note that $X_0(0)$ satisfies

$$\int_0^{X_0(0)} (V)_{Z=Z_0} dX = \frac{6C(\mathcal{V}_m(0) - 1)H_0^* + \tilde{\Lambda}H_0^{*3}}{12} + X_0(0);$$

thus, the larger we take H_0^* , the smaller will be the value for $X_0(0)$, as is seen in figure 42. Thereafter, for $\tau > 0$, it follows that $X_0(\tau)$ should reach zero faster for higher values of H_0^* , since the first term on the right-hand side of (4.94) will be larger; this can certainly be said for $H_0^* = 0.07$ and 0.1 . However, for $H_0^* = 0.001$ and 0.03 , we find X_0 reaches zero more or less when $\mathcal{V}_m - 1$ reaches zero, i.e. when

$$\tau = \frac{1}{2\pi} \cos^{-1} \left(\frac{1}{\mathcal{V}_0} \right); \quad (4.98)$$

using the data in Table 1, this gives $\tau \approx 0.176$. In turn, the fact that X_0 vanishes suggests that equation (4.94) can be ultimately reduced to just

$$\int_0^{X_0(\tau)} (V)_{Z=Z_0} dX = X_0(\tau), \quad (4.99)$$

Figure 42 – X_0 as a function of τ , determined using equation (4.94), with $\delta = 0$ and for four different values for H_0^* , using the weakly quasi-steady approach. For these runs, $Z_0 = 1$.

Source: (VYNNYCKY; ZAMBRANO, 2018).

as was the case in the earlier work (VYNNYCKY *et al.*, 2017). We may also add that using a non-constant profile for H_0 for $-Z_L \leq \tau \leq 0$ will not alter this conclusion. Thus, from now on, we shall use equation (4.99).

To compute the solution of (4.99), it is necessary to consider the interval $[\varepsilon, M]$ where ε and M are two positive real numbers such that $\varepsilon \leq X_0 \leq M$. Since X_0 is changing in time, we consider several values of τ

$$0 = \tau_0 < \tau_1 < \dots < \tau_n < \dots, \quad (4.100)$$

to compute the values

$$X_0(\tau_0), X_0(\tau_1), \dots, X_0(\tau_n), \dots \quad (4.101)$$

In algorithm 2 is shown a procedure to compute the value of X_0 at any value of τ . The numerical value Q^U corresponds the mass of flux in the upper zone. in every computation of X_0 one of two situations can occur:

1. there exists a numerical value X_m , with $\varepsilon < X_m < M$, such that

$$Q^U(X_m) = X_m,$$

which implies that the mass of flux in upper zone and determined by the meniscus starting at $X = X_m$ equals to the mass of flux in the lower zone;

2. or there is no numerical solution to the equation

$$Q^U(X) = X,$$

for X in the interval $[\varepsilon, M]$. Therefore, the trivial solution $X = 0$ is employed and we consider $X_0 = 0$.

Algorithm 2 – Computation of X_0

```

1: procedure FINDERX0( $\varepsilon, M$ )  $\triangleright \varepsilon \leq X_0 \leq M$ 
2:    $0.0001 \rightarrow TOL$ .
3:    $\varepsilon \rightarrow X_a$  and  $M \rightarrow X_b$ .
4:   Compute  $Q^U(X_a)$  and  $Q^U(X_a) \rightarrow Q_a$ 
5:   Compute  $Q^U(X_b)$  and  $Q^U(X_b) \rightarrow Q_b$ 
6:   if  $(Q_a - X_a) \times (Q_b - X_b) < 0$  then
7:     while  $|X_a - X_b| > TOL$  do  $\triangleright$  Main loop.
8:        $\frac{X_a + X_b}{2} \rightarrow X_m$ .
9:       Compute  $Q^U(X_m)$  and  $Q^U(X_m) \rightarrow Q_m$ .
10:      if  $(Q_a - X_a) \times (Q_m - X_m) < 0$  then
11:         $X_m \rightarrow X_b$ .
12:      else
13:         $X_m \rightarrow X_a$  and  $Q_m \rightarrow Q_a$ .
14:      end if
15:    end while
16:     $X_m \rightarrow X_0$ .
17:    return  $X_0$ .
18:  else
19:    return  $0 \rightarrow X_0$ .
20:  end if
21: end procedure

```

The set of numerical values computed this way will define the profile of the oscillation marks.

4.8 Results

Figure 43 shows the magnitude of the velocity and the velocity vectors, in nondimensionalized coordinates, of the fluid flow of the molten flux in the upper zone at $\tau = 0$; for this simulation, $Z_0 = 1.0$.

Figure 44 shows X_0 as a function of τ for three different values of Z_0 . To obtain these results, simulations were started at $\tau = 0$; in all cases, X_0 reached zero at around $\tau = 0.176$, and then remained zero until approximately

$$\tau = 1 - \frac{1}{2\pi} \cos^{-1} \left(\frac{1}{\gamma_0} \right) \approx 0.824.$$

The sum total of these observations might be that we only need to solve the problem numerically for

$$1 - \frac{1}{2\pi} \cos^{-1} \left(\frac{1}{\gamma_0} \right) < \tau < 1 + \frac{1}{2\pi} \cos^{-1} \left(\frac{1}{\gamma_0} \right),$$

in order to determine the solution for all time. However, we see that there is a slight complication to this: for some values of Z_0 , e.g. $Z_0 = 1$, X_0 starts to become positive slightly before the start

Figure 43 – Magnitude of the velocity of the fluid flow at $\tau = 0$ when $X_0 = 0.2$

Source: Elaborated by the author.

Figure 44 – X_0 as a function of τ using $\delta = 0$ and the weakly quasi-steady approach, and with $Z_0 = 0.3, 0.6, 0.9$

Source: Elaborated by the author.

of the negative strip time, i.e. when the velocity of the mold downwards is greater than the casting velocity, and eventually becomes zero slightly after it ends; for other values of Z_0 , X_0 only become strictly positive after the start of the negative strip time. Consequently, there is a need to solve over an entire period to find the complete solution. Thus, figure 44 shows the oscillation marks calculated as result of the mechanism developed in this work.

Next, figure 45 shows the maximum value of X_0 , which we shall denote as $X_0^{\max}$, as a function of Z_0 for $0.1 \leq Z_0 \leq 1$. From this, we see that that $X_0^{\max}$ increases with Z_0 . Although we could have carried on to higher values of Z_0 , we choose not to do so, but rather to examine

Figure 45 – $X_0^{\max}$ vs. Z_0

Source: Elaborated by the author.

how the solutions obtained would compare with available experimental data for oscillation-mark profiles. On the one hand, this model so far predicts the same oscillation-mark spacing as the model of Vynnycky *et al.* (VYNNYCKY *et al.*, 2017). However, noting that the actual maximum depth of a mark in the current model would be given by $LX_0^{\max}$, it is clear from Table 1 and figure 45 that the model would severely overestimate the experimental value of 0.3 mm (VYNNYCKY *et al.*, 2017; SALEEM, 2016).

On the other hand, the values of $X_0^{\max}$ obtained for smaller values of Z_0 do approach the upper end of the interval 0.5 mm-2 mm reported in much earlier work (KING *et al.*, 1993; HILL; WU; WIWATANAPATAPHEE, 1999; PLEASE; DEWYNNE, 1990; CRAMB; MANNION, 1985).

Even so, this preliminary comparison does serve a purpose. Since what was solved so far was essentially an isothermal model, it is clear that the temperature of the triple point was not taken into account. When this *is* taken into account, the cooling of the mold should ensure that the triple point will never be as far away from the mold as the values obtained here for $LX_0^{\max}$ suggest. Thus, there is considerable hope that when the current model is added to, the predictions will be in better agreement with the experimental data.

Note also that, in the work presented above, a single value of the viscosity, and hence Re_f , was used. However, further results for different values of Re_f are documented in Appendix B. From those, it is evident that the value of Re_f can strongly affect X_0 , and hence $X_0^{\max}$.

CONCLUSION AND FUTURE WORK

5.1 Summary of the work

This thesis has considered the related topics of ripple marks and oscillation marks in the ingot and continuous casting of steel, respectively, from a theoretical perspective. Following a general introduction to the casting of steel in chapter 1, a literature review was conducted in chapter 2. In chapter 3, which was related to ripple marks in ingot casting:

1. a mathematical model to describe the process of cooling inside a mold containing liquid steel was developed. This model was based around the conservation equations for mass and momentum and heat;
2. asymptotic methods were used to reduce systematically equations for the fluid flow of the molten steel and temperature and so to obtain just one partial differential equation for temperature;
3. the analysis developed managed to uncover three length scales
 - $W Re^{-1/2} \sim 2.9 \times 10^{-3} \text{m}$;
 - $W Pe^{-1/2} \sim 7.1 \times 10^{-3} \text{m}$;
 - $W Bo^{-1/2} \sim 3.6 \times 10^{-3} \text{m}$.

These correspond, respectively, to the width of the momentum boundary layer, the width of thermal boundary layer and the width across which the meniscus deviates from the horizontal;

4. a criterion was derived, in terms of the steel superheat and the casting speed, which indicated how ripple marks could be avoided in a mold of given height.

In chapter 4, which was related to oscillation marks in continuous casting:

5. using the conservation equations for mass, momentum and heat, a model was developed to explain the formation of oscillation marks;
6. the interaction of molten flux and molten steel was analyzed to determine that the capillary number was of primary importance for defining the shape of the meniscus;
7. asymptotic methods were used to reduce the Navier Stokes equations for the fluid flow of the molten flux and to compute the mass of flux. This reduction was possible due to the small value of the capillary number, which led to a quasi-steady problem in which the time-derivative term in the Navier Stokes equation could be safely dropped, although the time-dependence of the problem was still present through an oscillating mold-wall boundary condition.

5.2 Originality

The work constituted several original contributions:

- in the context of ripple marks, there are no models in the literature that describe their formation, and although the work presented here does not do this either, the criterion obtained for how to avoid them is certainly original and its exact form, as given in equation (3.104), would not be obvious on physical grounds alone. It is also noteworthy that this criterion appears as a comparatively simple analytical expression;
- in the context of oscillation marks, which is certainly a topic for which there exist far more models, the idea of developing a moving-point methodology to determine the location where the molten steel first solidifies is highly original, and in direct contrast to many other research efforts which use the volume-of-fluid method implemented in commercial CFD software. Although there is still some way to go in developing this methodology in its complete form, the fact that the model can be solved within such short computational times, compared to earlier work, is also a novelty.

5.3 Significance

Both of the developed models have significance for the industrial applications that motivated them. In the case of the work in chapter 3, it is clear that it suggests a way to avoid the ripple-mark defect completely; hence, modeling the formation of ripple marks constitutes challenging scientific research work, but is of limited technological interest.

On the other hand, in the case of the work in chapter 4, it is apparent that oscillation marks really cannot be avoided, since the use of a molten flux and an oscillating mold are deemed to be essential to the continuous casting process. This being the case, a theoretical

understanding of how they form is essential. However, the fact that any model for the process has so many model parameters, and that computational times have always proved to be prohibitive, has meant that it has been difficult to conduct detailed parameter studies; thus, it is of significance that the methods developed here seem able to allow this.

5.4 Unifying ripple and oscillation marks

It is perhaps unusual that this thesis has chosen to consider ripple and oscillation marks simultaneously. In fact, this approach was motivated by the fact that the original explanations for these marks were suggested based on the same types of experiments conducted by Tomono (TOMONO, 1979) and Tomono *et al.* (TOMONO; KURZ; HEINEMANN, 1981), which were for an ingot casting-type configuration; indeed, the results of these experiments have been cited subsequently many times as motivation for model development.

This background may suggest that a theoretical model for the formation of ripple marks can also be used for the formation of oscillation marks; for example, the Bikerman profile, which was first suggested in the context of ingot casting, often appears in the modeling of oscillation marks. Hence, one corollary of the present work is to identify whether it really is the case that one model would describe both situations. In fact, it turns out to be straightforward to show that this is not the case.

Starting with the model with model in chapter 3, we recall that, far from the mold wall, the meniscus rises at constant cast speed, whereas in the model in chapter 4, the meniscus far from the wall is stationary. Thus, one configuration can be transformed to the other using the translational transformation $x' \rightarrow x - V_c t$, with both menisci now being stationary. This now means that the mold wall in the ripple-mark model translates down with speed V_c , which clearly does not correspond to the oscillating mold wall in the model for continuous casting. Hence, it is evident that the the configurations cannot be unified by the same model.

5.5 Future work

1. To develop a more detailed analysis at the meniscus tip in ingot casting is of crucial importance for determining the evolution of the location of the triple point - marked P in figure 46 - where melt, solid and gas phases meet, since this, in tandem with the evolution of the meniscus profile as the molten steel fills the mold region, would be responsible for ripple-mark formation and the overflow/folding mechanisms discussed earlier.
2. Related to this is the issue of how such a model would be computed numerically. Whereas a technique such as the volume-of-fluid (VOF) method could determine the motion of the meniscus, i.e. the air/melt interface, prior to the onset of solidification, this would not be straightforward once the meniscus begins to solidify; this is because it becomes

necessary to model three interfaces - air/melt, air/solid and melt/solid - as well as the motion of the triple point that connects them. Since it is the locus of this triple point that will determine the profile of the ripple mark, it is evident that this would need a careful numerical treatment; it would seem unlikely that VOF-implementations in existing computational fluid dynamics (CFD) packages would be able to reproduce the ripple mark profiles accurately enough to enable meaningful comparison with experimentally-obtained profiles.

3. Whilst this development would require more analysis, in order to derive a self-consistent asymptotic problem, there would be advantages: a better qualitative understanding of the solution behavior in the tip region, without neglecting any of the essential physics; reduced computational expense as compared with the numerical solution of the full problem.

Figure 46 – Asymptotic structure near the meniscus tip once solidification starts.

Source: Elaborated by the author.

4. In the modeling of continuous casting, the inclusion of the thermal model is crucial. Whilst this may not seem too surprising, it should be noted that the models of Steinrück *et al.* (STEINRÜCK; RUDISCHER; SCHNEIDER, 1997) and Vynnycky *et al.* (VYNNYCKY *et al.*, 2017) predict oscillation marks having a realistic depth under isothermal conditions; however, both have neglected key aspects of the meniscus region that have been included here.
5. The inclusion of the thermal model will provide a control over temperature; thus, the solidification point not only will move horizontally, but vertically. This way, the triple point will be changing its position with respect to time in a plane, rather than just a line.
6. In this work, a mechanism to describe the formation of oscillation marks was analyzed taking into account values of Z_0 , such that $0 < Z_0 < \sqrt{2}$. This restriction is related to

the fact that we formulate to reproduce oscillation marks following a mechanism of folding. Nevertheless, the values of Z_0 such that $\sqrt{2} < Z_0 < 2$, are found in the lowest part of the shape of the meniscus if we consider the equation that allowed us to obtain the mathematical expression of this shape. It would imply that the meniscus will already have collapsed, leading to an overflow mechanism to produce oscillation marks.

7. What happens when X_0 and Z_0 are both allowed to vary can only be determined by solving the thermal model together with the fluid flow model that was solved here; nevertheless, the solution of the model given here will serve to inform the even more complex behavior that can be expected in the coupled model.
8. The boundary conditions required to solve the thermal model in the upper zone of the molten flux region are depicted in figure 47. These are: the extraction of heat at the oscillating mold wall; the temperature at the top boundary $T_{m,f}$; the continuity of the temperature denoted by $[T]_{-}^{+} = 0$ at the meniscus; the continuity of heat flux at the meniscus; the temperature at $Z = Z_0$, which initially can be prescribed as a linear function of x with the condition that, at the triple point, the temperature is equal to the solidification temperature of steel.

Figure 47 – Asymptotic structure near the meniscus tip once solidification starts.

Source: Elaborated by the author.

9. In this work, the depth of oscillation marks were related to the increase in the Reynolds number for the molten flux. Thus, in future work, the Reynolds number for the fluid flow of the molten flux and the Peclet number for the temperature in the upper zone can be useful parameters when investigating their effects on the depth of the oscillation marks.

10. For future work, it is suitable to develop the implementation of the mathematical model related to the mechanism of formation of oscillation marks in a freely distributed software. For instance, in chapter 3, to solve the temperature field in the thermal boundary layer, the software *GNU-Octave* was used and the results obtained were compared to other ones obtained using the proprietary software as *Comsol Multiphysics*. The agreement between these results gives confidence for using open source software in future research. In addition, in the open source community, there are several software packages with the capabilities to solve problems where heat transfer and Navier Stokes equations are involved. In this context, it is worth mentioning that *OpenFoam* is a reliable software for computing the Navier Stokes and heat transfer equations in a region such as that depicted in figure 47.

BIBLIOGRAPHY

BADRI, A.; NATARAJAN, T. T.; SNYDER, C. C.; POWERS, K. D.; MANNION, F. J.; CRAMB, A. W. A mold simulator for continuous casting of steel: Part II. The formation of oscillation marks during the continuous casting of low carbon steel. **Metall. Mater. Trans. B**, v. 36B, p. 373–383, 2005. Citations on pages [23](#), [33](#), [38](#), and [39](#).

BENILOV, E. S.; VYNNYCKY, M. Contact lines with a 180° contact angle. **J. Fluid Mech.**, 718, p. 481–506, 2013. Citation on page [71](#).

BENNEY, D. J.; TIMSON, W. J. The rolling motion of a viscous-fluid on and off a rigid surface. **Stud. Appl. Maths**, 63, p. 93–98, 1980. Citation on page [71](#).

BIKERMAN, J. J. **Physical Surfaces**. [S.l.]: Academic Press, New York, 1970. Citations on pages [35](#), [43](#), [57](#), and [80](#).

BLAND, D. R. Flux and the continuous casting of steel. **IMAJ. Appl. Maths**, v. 32, p. 89–112, 1984. Citation on page [69](#).

BRASIL, I.-A. co. Estatística preliminar. **Instituto Aço Brasil**, 2017. Citation on page [25](#).

CRAMB, A. W.; MANNION, F. J. The measurement of meniscus marks at Bethlehem Steel's Burns Harbor slab caster. In: **Steelmak. Conf. Proc.** [S.l.: s.n.], 1985. v. 68, p. 349–359. Citation on page [93](#).

CRANK, J. **Free and Moving Boundary Problems**. [S.l.]: Oxford Science Publications, 1984. Citation on page [71](#).

DUSSAN, V. E.; DAVIS, S. H. On the motion of a fluid–fluid interface along a solid surface. **J. Fluid Mech.**, v. 65, p. 71–95, 1974. Citation on page [71](#).

ELFSBERG, J. **Influence of Mould Flux Properties on Early Solidification in Continuous Casting**. Phd Thesis (PhD Thesis) — Royal Institute of Technology, Stockholm, Sweden, 2010. Citation on page [75](#).

FOWKES, N.; WOODS, A. **The Flux of Flux in a Continuous Steel Caster**. 1989. Dep. Math. Preprint, University of Western Australia, 18pp. Citations on pages [42](#) and [69](#).

FOWKES, N.; WOODS, A.; HINCH, E. J.; PLEASE, C. P. **Flux consumption in continuous casting**. 1988. Oxford Study Group with Industry Report. Oxford, UK, pp. 1-5. Citation on page [69](#).

FREDRIKSSON, H.; ÅKERLIND, U. **Materials Processing during Casting**. [S.l.]: Wiley, 2006. Citation on page [27](#).

FREDRIKSSON, H.; ELFSBERG, J. Thoughts about the initial solidification process during continuous casting of steel. **Scand. J. Metall.**, v. 31, p. 292–297, 2002. Citations on pages [70](#) and [75](#).

HILL, J. M.; WU, Y. H.; WIWATANAPATAPHEE, B. Analysis of flux flow and the formation of oscillation marks in the continuous caster. **J. Engng Maths**, v. 36, p. 311–326, 1999. Citations on pages [19](#), [23](#), [32](#), [33](#), [41](#), [42](#), [43](#), [44](#), [69](#), [71](#), [72](#), [79](#), [80](#), [85](#), [93](#), and [115](#).

HUH, C.; SCRIVEN, L. E. Hydrodynamic model of steady movement of a solid/liquid/fluid contact line. **J. Colloid Interface Sci.**, v. 35, p. 85–101, 1971. Citation on page [71](#).

JACOBI, H.; SCHWERDTFEGGER, K. Ripple marks on cast steel surfaces. **ISIJ Intl.**, v. 53, p. 1180–1196, 2013. Citations on pages [30](#), [32](#), [35](#), [36](#), [49](#), [55](#), [65](#), and [67](#).

JONAYAT, A. S. M.; THOMAS, B. G. Transient thermo-fluid model of meniscus behavior and slag consumption in steel continuous casting. **Metall. Mater. Trans. A**, v. 45, p. 1842–1864, 2014. Citations on pages [23](#), [33](#), [43](#), [44](#), [69](#), and [71](#).

KING, J. R.; LACEY, A. A.; PLEASE, C. P.; WILMOTT, P.; ZORYK, A. The formation of oscillation marks on continuously cast steel. **Math. Engng. Ind.**, v. 4, p. 91–106, 1993. Citations on pages [69](#) and [93](#).

KRISTIANSSON, J.-O. Thermal stresses in the early stage of the solidification of steel. **J. Thermal Stresses**, v. 5, p. 315–330, 1982. Citation on page [71](#).

LIU, H.; QUI, S.; GAN, Y.; ZHANG, H.; DE, J. Development of mechanical drive type non-sinusoidal oscillator for continuous casting of steel. **Ironmaking Steelmaking.**, v. 29, p. 180–184, 2002. Citation on page [72](#).

MILLS, K. C.; KARAGADDE, S.; LEE, P. D.; YUAN, L.; SHAHBAZIAN, F. Calculation of physical properties for use in models of continuous casting process-part 1: mould slags. **ISIJ Intl.**, 56, p. 264–273, 2016. Citation on page [76](#).

_____. Calculation of physical properties for use in models of continuous casting process-part 1: steels. **ISIJ Intl.**, 56, p. 274–281, 2016. Citations on pages [19](#) and [76](#).

MITCHELL, S. L.; VYNNYCKY, M. Finite-difference methods with increased accuracy and correct initialization for one-dimensional Stefan problems. **App. Math. Comp.**, v. 215, p. 1609–1621, 2009. Citation on page [61](#).

_____. An accurate finite-difference method for ablation-type Stefan problems. **J. Comp. Appl. Math.**, v. 236, p. 4181–4192, 2012. Citation on page [61](#).

_____. On the numerical solution of two-phase stefan problems with heat-flux boundary conditions. **J. Comp. Appl. Maths**, v. 264, p. 49–64, 2014. Citation on page [61](#).

MITCHELL, S. L.; VYNNYCKY, M.; GUSEV, I. G.; SAZHIN, S. S. An accurate numerical solution for the transient heating of an evaporating droplet. **Appl. Math. Comput.**, v. 217, p. 9219–9233, 2011. Citation on page [61](#).

OUTOKUMPU. Outokumpu - high performance stainless steel. **Outokumpu.**, 2016. Citations on pages [25](#) and [26](#).

PISMEN, L. M.; NIR, A. Motion of a contact line. **Phys. Fluids**, 25, p. 3–7, 1982. Citation on page [71](#).

PLEASE, C. P.; DEWYNNE, J. N. **Oscillation mark formation in continuous casting**. 1990. European Study Group with Industry and Training Course in Mathematical Modeling. Department of Mathematics, University of Southampton, UK, 18pp. Citation on page [93](#).

PLETCHER, R.; TANNEHILL, J. C.; ANDERSON, D. A. **Computational Fluid Mechanics and Heat Transfer**. 3rd. ed. [S.l.]: CRC Press, 1997. Citation on page 62.

RAMIREZ-LOPEZ, P. E.; MILLS, K. C.; LEE, P. D.; SANTILLANA, B. A unified mechanism for the formation of oscillation marks. **Metall. Mater. Trans. B**, v. 43B, p. 109–122, 2012. Citations on pages 69, 70, 71, and 75.

SALEEM, S. **On the surface quality of continuously cast steels and phosphor bronzes**. Phd Thesis (PhD Thesis) — KTH Royal Institute of Technology, Stockholm, Sweden, 2016. Citations on pages 19, 23, 33, 39, 40, 41, 46, 76, 80, and 93.

SALEEM, S.; VYNNYCKY, M.; FREDRIKSSON, H. A study of the oscillation marks' characteristics of continuously cast Incoloy alloy 825 blooms. **Metall. Mater. Trans. A**, v. 47A, p. 4068–4079, 2016. Citations on pages 23, 33, 39, 40, and 70.

SHIN, H.-J.; KIM, S.-H.; THOMAS, B. G.; LEE, G.-G.; PARK, J.-M.; SENGUPTA, J. Measurement and prediction of lubrication, powder consumption, and oscillation mark profiles in ultra-low carbon steel slabs. **ISIJ Intl.**, v. 11, p. 1635–1644, 2006. Citation on page 72.

STEINRÜCK, H.; RUDISCHER, C. Numerical investigation of the entrainment of flux into the lubrication gap in continuous casting of steel. In: **Fifth World Congress on Computational Mechanics, Vienna, Austria**. [S.l.: s.n.], 2002. p. 1–15. Citations on pages 70 and 75.

STEINRÜCK, H.; RUDISCHER, C.; SCHNEIDER, W. Modelling of continuous casting processes. **Nonlinear Analysis, Theory, Methods & Applications**, v. 30, p. 4915–4925, 1997. Citations on pages 23, 32, 33, 34, 40, 41, 42, 70, 71, 75, 77, 98, and 115.

STEINRÜCK, H.; RUDISCHER, C.; SCHNEIDER, W. The formation of oscillation marks in continuous casting of steel. In: **Modelling of Casting, Welding and Advanced Solidification Processes VIII**. [S.l.: s.n.], 1998. p. 639–646. Citations on pages 70 and 75.

SUZUKI, M.; MIZUKAMI, H.; KITAGAWA, T.; KAWAKAMI, K.; UCHIDA, S.; KOMATSU, M. Development of a new mold oscillation mode for high-speed continuous-casting of steel slabs. **ISIJ Intl.**, v. 31, p. 254–261, 1991. Citation on page 72.

TAKEUCHI, E.; BRIMACOMBE, J. K. The formation of oscillation marks in the continuous casting of steel slabs. **Metall. Mater. Trans. B**, v. 15B, p. 493–509, 1984. Citations on pages 31 and 69.

TOMONO, H. **Elements of oscillation mark formation and their effect on transverse fine cracks in continuous casting of steel**. Phd Thesis (PhD Thesis) — École Polytechnique Fédérale de Lausanne, Lausanne, Switzerland, 1979. Citations on pages 23, 33, 37, 39, and 97.

TOMONO, H.; KURZ, W.; HEINEMANN, W. The liquid steel meniscus in molds and its relevance to the surface quality of castings. **Metall. Mater. Trans. B**, v. 12B, p. 409–411, 1981. Citations on pages 32, 35, 36, 49, and 97.

VYNNYCKY, M.; MAENO, N. Axisymmetric natural convection-driven evaporation of hot water and the Mpemba effect. **Int. J. Heat Mass Transfer**, v. 55, p. 7297–7311, 2012. Citation on page 60.

VYNNYCKY, M.; MAENO, N. Axisymmetric natural convection-driven evaporation of water: analysis and numerical solution. **Int. J. Heat Mass Transfer**, v. 55, p. 6238–6249, 2012. Citation on page 60.

VYNNYCKY, M.; SALEEM, S.; DEVINE, K. M.; FLORIO, B. J.; MITCHELL, S. L.; O'BRIEN, S. B. G. On the formation of fold-type oscillation marks in the continuous casting of steel. **Royal Soc. Open Sci.**, v. 4, 2017. Citations on pages [19](#), [23](#), [32](#), [33](#), [34](#), [44](#), [45](#), [46](#), [47](#), [69](#), [70](#), [71](#), [72](#), [75](#), [76](#), [82](#), [85](#), [90](#), [93](#), and [98](#).

VYNNYCKY, M.; ZAMBRANO, M. Towards a 'moving point' formulation for the modelling of oscillation-mark formation in the continuous casting of steel. **Applied Mathematical Modelling.**, v. 63, p. 243–265, 2018. Citations on pages [88](#), [90](#), and [115](#).

VYNNYCKY, M.; ZAMBRANO, M.; CUMINATO, J. A. On the avoidance of ripple marks on cast metal surfaces. **Int. J. Heat Mass Transfer**, v. 86, p. 43–54, 2015. Citations on pages [30](#), [49](#), [50](#), and [115](#).

YANG, J.; MENG, X.; WANG, M.; ZHU, M. Oscillation-mark formation and liquid-slag consumption in continuous casting mold. **Metall. Mater. Trans. B**, 48B, p. 1230–253, 2017. Citation on page [82](#).

BENCHMARKING FOR RIPPLE MARKS

In order to provide a benchmark for the solution found in Section 3.5, we consider, for $0 \leq X \leq 1, 0 \leq Y \leq Y_0 + \tau$,

$$\frac{\partial \theta}{\partial \tau} + \frac{\partial \theta}{\partial Y} = \frac{1}{Pe} \left(\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} \right), \quad (\text{A1})$$

subject to

$$\theta = 1 \quad \text{at } Y = 0, \quad (\text{A2})$$

$$\frac{\partial \theta}{\partial X} = 0 \quad \text{at } X = 0, \quad (\text{A3})$$

$$\frac{\partial \theta}{\partial X} = -\lambda \quad \text{at } X = 1, \quad (\text{A4})$$

$$\frac{\partial \theta}{\partial Y} = 0 \quad \text{at } Y = Y_0 + \tau, \quad (\text{A5})$$

and the initial condition

$$\theta = 1, \quad 0 \leq X < 1, \quad 0 \leq Y \leq Y_0. \quad (\text{A6})$$

Using transformation (3.82), equations (A1)-(A6) become

$$\frac{\partial \theta}{\partial \tau} + \frac{1 - \eta}{Y_0 + \tau} \frac{\partial \theta}{\partial \eta} = \frac{1}{Pe} \left(\frac{\partial^2 \theta}{\partial X^2} + \frac{1}{(Y_0 + \tau)^2} \frac{\partial^2 \theta}{\partial \eta^2} \right), \quad (\text{A7})$$

subject to

$$\theta = 1 \quad \text{at } \eta = 0, \quad (\text{A8})$$

$$\frac{\partial \theta}{\partial X} = 0 \quad \text{at } X = 0, \quad (\text{A9})$$

$$\frac{\partial \theta}{\partial X} = -\lambda \quad \text{at } X = 1, \quad (\text{A10})$$

$$\frac{\partial \theta}{\partial \eta} = 0 \quad \text{at } \eta = 1, \quad (\text{A11})$$

and the initial condition

$$\theta = 1, \quad 0 \leq X < 1, \quad 0 \leq \eta \leq 1. \quad (\text{A12})$$

In particular, setting $\lambda = B_{crit} Pe^{1/2}$, several simulations were run for different values of Pe . These values are increasing that is equivalent to increase the value of the casting velocity V_c , since

$$Pe = \frac{\rho c_p V_c W}{k},$$

where the values of ρ, c_p, k and W are constant.

In figure A.1 is shown θ vs. η for $Pe = 200, 400$ and 800 . This increment in the value of Pe , will imply that the mold will be filled with molten steel more quickly and a delay in the onset of the solidification of steel will take place.

Figure A.1 – θ vs. η at $X = 1$ for $\tau = \tau^*$ when solving (A7) for $Pe = 200, 400, 800$.

Source: Author.

The computational mesh that was used, also was able to resolve the temperature field in the thermal boundary layer for Pe as high as 200 - the value given in (3.44) or even a larger value as shown in figure A.1; we did not undertake a formal grid independence study, but the fact that we obtain good agreement with the numerical solution to the results of the asymptotically reduced problem, as given above, shows that we can be sure that we have computed accurately enough.

Figure A.2 compares the profile for θ as a function of X at $\eta = 1$ when $\tau = \tau^*$, obtained via the numerical solution to equation (A7) for $Pe = 200, 400$ and 800 . In this case, increasing the Peclet number allows us to delay the onset of the solidification of steel and shows that the lowest value of temperature is located adjacent to the mold wall.

Figure A.2 – θ vs. X at $\eta = 1$ for $Pe = 200, 400$ and 800 .

Source: Elaborated by the author.

BENCHMARKING FOR OSCILLATION MARKS

In this appendix, we provide a benchmark of the solutions of the Navier Stokes equations for the fluid flow of the molten flux in the upper zone. The purpose of this benchmark is to compute the oscillation marks for several values of the Reynolds number of the molten flux.

The equations to solve are given by

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0, \quad (\text{B1})$$

$$Re_f \left(U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Z} \right) = -\frac{\partial P}{\partial X} + \frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Z^2}, \quad (\text{B2})$$

$$Re_f \left(U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Z} \right) = -\frac{\partial P}{\partial Y} + \frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Z^2}, \quad (\text{B3})$$

With respect to the boundary conditions, they are given by

$$U = 0, \quad V = \mathcal{V}_m(\tau) \quad \text{at} \quad X = 0, \quad (\text{B4})$$

$$P = 0 \quad \text{at} \quad Z = 0, \quad (\text{B5})$$

$$P = 0 \quad \text{at} \quad Z = Z_0(\tau) \quad (\text{B6})$$

and at the meniscus, the condition of zero-shear stress is set as given in (4.79) and (4.80).

Figure B.1 compares the profile for θ as a function of τ as obtained via a numerical solution to equations (B1)-(B3) for $Re_f = 2, 4, 8, 16$, since

$$Re_f = \frac{\rho_f^{(l)} V_c L}{\mu_f},$$

this increasing in the value of Re_f is equivalent to increase of the value of the casting velocity V_c . Thus, with the mechanism of formation of oscillation mark developed in this work, we can observe that the depth of the oscillation marks decreases when the casting velocity increases.

In addition, in figure B.1 are shown oscillation marks as result of applying the algorithm 2, starting from $\tau = 0$, as can be seen, precisely at this value of τ is obtained the maximum depth of the oscillation mark due to largest numerical value of $V_m - V_c$, the difference between the mold wall velocity and the casting velocity, this situation physically occurs when the motion of the mold wall downwards reaches its maximum amplitude.

Figure B.1 – X_0 vs. τ showing the formation of oscillation marks as result of the numerical solution to equations (B1)-(B3) for $Re_f = 2, 4, 8, 16$.

Source: Author.

Focusing our attention to the depth of the oscillation marks obtained in this work, it is suitable to know how this depth depends on the value of Re_f .

Thus, intensive numerical work through simulations was performed, in which the value of Re_f was increasing continuously to see the effect on the depth. Thus, equation to be solved is given by

$$\int_0^{X_0(0)} V_{(Z=Z_0)} dX = X_0(0), \quad (\text{B7})$$

which comes from (4.99). In addition, for an integer n , the following inequalities

$$X_0(\tau_0) > X_0(\tau_1) > \dots > X_0(\tau_n) = 0$$

hold while

$$0 = \tau_0 < \tau_1 < \dots < \tau_n = \tilde{\tau}.$$

where

$$\tilde{\tau} = \frac{1}{2\pi} \cos^{-1} \left(\frac{1}{\gamma_0} \right) \approx 0.176. \quad (\text{B8})$$

Figure B.2 shows the maximum depth of oscillation marks versus the value of Re_f . As can be observed, the maximum depth decreases as the value of Re_f increases.

Figure B.2 – Maximum value of X_0 by computing the depth of an oscillation mark

Source: Author.

To solve equations (B1)-(B3), we have used the finite element-based software Comsol Multiphysics, where second-order triangular elements for the velocity and first order triangular elements for the pressure were used on a triangular mesh having 6300 elements corresponding to around 20,000 degrees of freedom, N_{dof} . The convergence criterion at each time step of the computation was taken as

$$\left(\frac{1}{N_{dof}} \sum_{i=1}^{N_{dof}} \left(\frac{|E_i|}{A_i + R|U_i|} \right)^2 \right)^{\frac{1}{2}} < 1, \quad (\text{B9})$$

where (U_i) is the solution vector corresponding to the solution at each time step, A_i is the absolute tolerance for the i th degree of freedom, and R is the relative tolerance; for the computations, $R = 10^{-2}$, $A_i = 10^{-3}$ for $i = 1, \dots, N_{dof}$ were used.

SOME TECHNICAL TERMS IN PORTUGUESE LANGUAGE RELATED TO CASTING OF METALS

English	Portuguese
Boundary layer	Camada limite
Capillary number	Número de capilaridade
Continuous casting	Lingotamento contínuo
Downhill casting	Lingotamento em declive
Ingot casting	Lingotamento convencional
Meniscus	Menisco
Oscillation marks	Marcas de oscilação
Ripple marks	Marcas de ondulação
Steelmaking	Fabricação do aço
Uphill casting	Lingotamento ascendente

PUBLICATIONS IN JOURNALS

To date, as result of the work during the period of the Ph.D. studies, two articles have been published: the first was related to chapter 3, where a mathematical model to avoid the formation of ripple marks in ingot casting processes was developed and; in the second, related to chapter 4, a mathematical model describing the formation of oscillation marks in continuous casting processes was formulated.

1. Corresponding to the formation of ripple marks (VYNNYCKY; ZAMBRANO; CUMINATO, 2015), a paper entitled **“On the avoidance of ripple marks on cast metal surfaces”** was published in *International Journal of Heat and Mass Transfer*.
2. Corresponding to the formation of oscillation marks (VYNNYCKY; ZAMBRANO, 2018), a paper entitled **“Towards a ‘moving point’ formulation for the modelling of oscillation-mark formation in the continuous casting of steel”**, was published in the journal *Applied Mathematical Modelling*.

The first publication was the result of working in mathematical modeling of problems related to ingot casting during the years 2014-2015, the first two year of the Ph.D. studies. After these two years, the research activities were performed in Stockholm, Sweeden.

In Stockholm, the activities of research were developed at KTH *Royal Institute of Technology* from April 2016 to December 2016. The problem to investigate was the formation of oscillation marks and our objective was focused on developing a mathematical model to explain their formation. Several alternatives were analyzed to elaborate a mechanism to reproduce oscillation marks, but finally the chosen alternative was that which deveoped some fundamental ideas already presented by Steinrück (STEINRÜCK; RUDISCHER; SCHNEIDER, 1997) and Hill (HILL; WU; WIWATANAPATAPHEE, 1999).

During 2017, back in Brazil, up to the beginning of 2018, computationally intensive work to reproduce the oscillation marks took place and gave as result the publication of the second paper, where the formation of oscillation marks have been achieved.

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

On the avoidance of ripple marks on cast metal surfaces

M. Vynnycky^{a,*}, M. Zambrano^b, J.A. Cuminato^b^aDivision of Casting of Metals, Department of Materials Science and Engineering, Royal Institute of Technology, Brinellvägen 23, 100 44 Stockholm, Sweden^bDepartment of Applied Mathematics and Statistics, Institute of Mathematical and Computer Sciences, University of São Paulo at São Carlos, PO Box 668, 13560-970 São Carlos, São Paulo, Brazil

ARTICLE INFO

Article history:

Received 19 September 2014

Received in revised form 19 February 2015

Accepted 21 February 2015

Available online 13 March 2015

Keywords:

Ripple marks

Casting

Asymptotics

ABSTRACT

A mathematical model is derived for the purposes of predicting how to avoid unwanted defects, known as ripple marks, in the casting of metal ingots; the model is based around the momentum and heat transfer that occurs when a cooling molten metal meniscus rises between two parallel and vertical chill-mould walls. By using asymptotic techniques, the model is reduced systematically to a form that requires the numerical solution of a moving boundary problem involving just one partial differential equation. Numerical results are presented, and the significance of the model for predicting the depth and spacing of ripple marks in the casting of ingots and oscillation marks in continuous casting are discussed.

© 2015 Elsevier Ltd. All rights reserved.

1. Introduction

The casting of metals is a well-established application of heat and mass transfer [1–7], the understanding of which is essential for avoiding or limiting defects that occur in industrial casting processes. Ripple marks are one example of such defects. Also called surface marks, wrinkles, striations or grooves, they are defects that develop on the surface of static ingots during casting; in general, they have a certain spacing, typically of the order of millimetres, and depth, typically of the order of tens of micrometres, and lie parallel to the meniscus surface. Although interest in ripple marks had subsided considerably since the early 1980s [8–10], there is now renewed interest in them [11], because the mechanism for their formation is thought to be related to that for the formation of oscillation marks in the much more prevalent continuous casting process [12–15]. In both ingot and continuous casting, there is a need to either minimise or avoid such marks completely, and mathematical modelling represents one way to gain insight into how this can be achieved.

In considering the formation of such marks, a knowledge of what happens in the region where the molten metal meniscus meets the cold mould wall is thought to be critical. However, the situation prevalent in continuous casting, where there is the combination of a solidifying meniscus and an oscillating mould, makes for a challenging modelling problem; in this context, the case of ingot casting, wherein the mould is stationary, represents a more

suitable starting point. Even so, there appear to be no models available for this problem. Moreover, whereas a model predicting the depth and separation of ripple marks as a function of process parameters would be of scientific interest, it is of technical interest to know rather how ripple marks can be avoided; this will be the premise of this paper.

Although there are other ways to generate ripple marks [9], we focus here on the uphill casting-type configurations considered by Tomono et al. [8] and Jacobi and Schwerdtfeger [11]. A schematic is given in Fig. 1, which shows melt passing down from a tundish through the upper opening of a feeding channel, entering into an interior mould space through a hole at the bottom and subsequently rising. As the mould is cooled, the melt will also cool, leading eventually to the onset of solidification at the mould walls. Ripple marks are believed to occur via either an overflow or a flow-back mechanism. In both cases, the shell solidifies along the curved meniscus profile to form a pointed tip. If the shell is strong enough to avoid deformation, the melt meniscus overflows the tip; otherwise, if the shell is too weak, its tip bends back under the rim pressure, i.e. a folding mechanism. As stated in the previous paragraph, our goal will be to determine how this situation can be avoided; however, the results of this analysis can nevertheless become the starting point for future modelling of actual ripple mark formation.

The layout of the paper is as follows. In Section 2, we formulate a mathematical model for the fluid flow and heat transfer in a melt with a rising meniscus; in Section 3, we nondimensionalize the model and identify the key dimensionless parameters. In Section 4, based on the characteristic values of these parameters, we formulate a reduced asymptotic model that leads to a moving

* Corresponding author.

E-mail address: michaelv@kth.se (M. Vynnycky).

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

Towards a “moving-point” formulation for the modelling of oscillation-mark formation in the continuous casting of steel

M. Vynnycky^{a,*}, M. Zambrano^b^a*Division of Processes, Department of Materials Science and Engineering, Royal Institute of Technology, Brinellvägen 23, Stockholm 100 44, Sweden*^b*Department of Applied Mathematics and Statistics, Institute of Mathematical and Computer Sciences, University of São Paulo at São Carlos, 13560-970 São Carlos, São Paulo, PO Box 668, Brazil*

ARTICLE INFO

Article history:

Received 8 March 2018

Revised 23 May 2018

Accepted 14 June 2018

Available online 22 June 2018

Keywords:

Continuous casting

Oscillation marks

Asymptotic analysis

ABSTRACT

In the continuous casting of steel, solidification begins at a triple point where solid steel, molten steel and molten flux meet; the motion of this point determines how surface defects known as oscillation marks (OSMs) are formed. Here, under a number of simplifying assumptions, we derive an asymptotic model in 15 dimensionless parameters that describes the relevant momentum and heat transfer for the process, and involves both surface tension at the meniscus between molten flux and molten steel and solidification; further development couples an earlier lubrication-theory model for the region below the triple point to a reduced model for the region above that is based around a regular perturbation treatment of the Navier Stokes equations in terms of the capillary number. The resulting problem is then broken up into a hierarchy of four sub-problems; the first one – for the velocity field in the molten flux – is considered in depth. A numerical algorithm is developed for an isothermal situation in which the triple point moves only perpendicular to the casting direction; this involves the solution of a novel “moving point” problem to determine motion of the triple point. Comparison of model and experimental results indicates that this reduced model could produce OSMs having the observed depth if the triple point were not too far below the top of the meniscus; otherwise, the computed marks would be too deep. The analysis also tentatively relates the location of the triple point to the fold- and overflow-type OSMs that generally form in practice.

© 2018 Elsevier Inc. All rights reserved.

1. Introduction

In the continuous casting of steel, the solidification of the molten metal is initiated by a cooling mould that oscillates in the axis of the casting direction; furthermore, to prevent the solid shell from sticking to the mould, mould powder, often also termed flux, is added at the top of the mould, melts and forms a lubricating layer in the gap between the steel and the mould walls, as well as a meniscus with the molten steel. However, the oscillations inevitably result in the formation of more or less evenly spaced indentations along the slab width; these are commonly referred to as oscillation marks. A schematic of the situation is shown in Fig. 1.

* Corresponding author.

E-mail address: michaelyv@kth.se (M. Vynnycky).