Models and methods for the direct simulation of
rough and micropatterned surfaces

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Hugo Marcial Checo Silva

**Modelos e métodos para a simulação direta de superfícies
rugosas e micro-texturizadas**

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Abstract

Friction in hydrodynamic bearings are a major source of losses in car engines ([69]). The extreme loading conditions in those bearings lead to contact between the matching surfaces. In such conditions not only the overall geometry of the bearing is relevant, but also the small-scale topography of the surface determines the bearing performance. The possibility of shaping the surface of lubricated bearings down to the micrometer ([57]) opened the question of whether friction can be reduced by mean of micro-textures, with mixed results. This work focuses in the development of efficient numerical methods to solve thin film (lubrication) problems down to the roughness scale of measured surfaces.

Due to the high velocities and the convergent-divergent geometries of hydrodynamic bearings, cavitation takes place. To treat cavitation in the lubrication problem the Elrod-Adams model is used, a mass-conserving model which has proven in careful numerical ([12]) and experimental ([119]) tests to be essential to obtain physically meaningful results. Another relevant aspect of the modeling is that the bearing inertial effects are considered, which is necessary to correctly simulate moving textures.

As an application, the effects of micro-texturing the moving surface of the bearing were studied. Realistic values are assumed for the physical parameters defining the problems. Extensive fundamental studies were carried out in the hydrodynamic lubrication regime. Mesh-converged simulations considering the topography of real measured surfaces were also run, and the validity of the lubrication approximation was assessed for such rough surfaces.

Keywords: measured surfaces, Elrod-Adams model, cavitation, multigrid, numerical simulation.

Resumo

O atrito em mancais hidrodinâmicos é uma fonte importante de perdas em motores de combustão ([69]). As condições extremas de carga induzem contato entre as superfícies dos mancais. Em tais condições não somente a macro-geometria do mancal é relevante, mas também são as escalas menores da superfície as que determinam o desempenho do mancal. A possibilidade de fabricar superfícies com detalhes na escala do micrometro ([57]) deixou em aberto a questão de se o atrito pode ser reduzido por meio de micro-texturas, até agora com resultados mistos. Este trabalho centra-se no desenvolvimento de métodos numéricos eficientes para resolver problemas de lubrificação na escala da rugosidade das superfícies.

Devido às altas velocidades e a forma convergente-divergente dos mancais hidrodinâmicos o fluido cavita. Para tratar o fenômeno de cavitação empregamos o modelo de Elrod-Adams, um modelo conservativo que tem demonstrado em cuidadosos testes numéricos ([12]) e experimentais ([119]) ser essencial para obter resultados físicos significativos. Outro aspecto revelante do modelado é que os efeitos inerciais do mancal são considerados, o que é necessário para simular corretamente texturas em movimento.

Como aplicação, os efeitos de micro-texturizar a superfície móvel do mancal foram estudados. Valores realistas são assumidos nos parâmetros físicos que definem o problema. Foram realizados extensivos estudos no regime de lubrificação hidrodinâmica. Também foram executadas simulações convergidas em malha, levando em conta a topografia real de superfícies medidas, e as hipóteses de lubrificação para superfícies rugosas foram avaliadas.

Palavras-chave: superfícies texturizadas, modelo de Elrod-Adams, cavitação, multigrid, simulação numérica.

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Symbols and acronyms

Symbol	Description
α	crank angle
β	asperity mean radius of curvature
η	asperity density
θ	fluid fraction
θ_s	threshold for the onset of friction in the Couette term of the friction force
λ	period of textures
μ	dynamic viscosity
μ_c	boundary friction coefficient
ν_l^d	number of relaxation sweeps in grid l , descending in a V-cycle
ν_l^u	number of relaxation sweeps in grid l , ascending in a V-cycle
ρ	fluid density
σ	composite standard deviation
σ	stress tensor
ϕ_r	fraction of p_c acting on the back part of ring r
ω^h	solution vector at multigrid mesh h
Γ	boundary of the domain Ω
Σ	cavitation boundary
Ω_0	cavitated region
Ω_+	pressurized region
a, b	initial and final position of the ring in the x_1 direction
d	depth of circular pockets
$\hat{e}_i$	unitary vector pointing in the i -th direction
f	instantaneous friction coefficient

$\bar{f}$	time-averaged friction coefficient
g	function controlling the onset of friction in the Couette term
h	clearance between matching surfaces (in the x_3 direction)
h_t, h_L	function describing the texture shape
h_p	function characterizing the ring profile
h_b, h_U	function characterizing the upper surface
$\hat{i}$	versor in the x_1 direction
i, j	finite volume cell position (i in x_1 , j in the x_2 direction)
l	radius of circular pockets
ℓ_r	ring width
m	ring linear mass per unit width
n	number of time step
$\hat{n}$	unitary normal to a surface
p	hydrodynamic pressure
p_{con}	contact pressure
q	finite volume aspect ratio $\Delta x_1 / \Delta x_2$
$r_{1,2}$	radius of the shaft (1) or the housing (2) of a journal bearing
$\mathbf{r}^h$	residue at mesh h
t	time
Δt	discretization in the time variable
u	relative velocity between the piston (ring) and the liner
$\mathbf{v}$	fluid velocity
v_i	i -th component of the fluid velocity vector $\mathbf{v}$
x_1, x_2, x_3	coordinate directions
x_{1l}, x_{1r}	initial and final position of the computational domain (x_1 direction)
$\Delta x_1, \Delta x_2$	finite volume cell size in the x_1 and x_2 directions
A	amplitude
B	domain width (x_2 direction)
C	minimum clearance
E'	composite modulus of elasticity of ring and liner
$\mathbf{F}$	total force exerted by a fluid on a surface
F_L, F_U	friction force exerted by the fluid on the lower (upper) surface
F^{cou}	Couette term of the hydrodynamic friction force
F^{poi}	Poiseuille term of the hydrodynamic friction force
F^{pre}	Pressure term of the hydrodynamic friction force

H	non-dimensionalization length for x_3 , Z and e
$\mathcal{T}_h^H$	mesh transfer operator (fine to coarse mesh)
$\mathcal{T}_H^h$	mesh transfer operator (coarse to fine mesh)
$\mathbf{J}$	flux vector
L	non-dimensionalization length for x_1 and x_2
N_p	number of parallel processes
M	number of multigrid levels in each V-cycle
P	power loss
R	ring profile curvature radius
S	Stribeck number
$\mathbf{V}$	relative velocity of the upper surface with respect to the lower
V_i	i -th component of the $\mathbf{V}$ velocity
V_Σ	velocity of the cavitation boundary Σ
U	velocity non-dimensionalization
W^c	contact force per unit width acting on the ring
W^{gp}	force per unit width acting on the back part of the rings due to the gas pressure
W^h	hydrodynamic force per unit width acting on the ring
W^{ps}	ring pre-stress force per unit width
W^a	applied forces on the ring pre-stress, per unit width
Z	ring position in the x_3 direction
Z_0	initial condition for the ring position Z
V_0	initial condition for the velocity of the ring along x_3
Z^n	ring position at $t = t_n$
a.e.	almost everywhere
rpm	revolutions per minute
BDC	Bottom Dead Center
CFL	Courant-Friedrich-Levy number
FAS	Full Approximation Scheme
JFO	Jakobsson-Floberg-Olsson
LHS	left hand side
LST	Laser Surface Texturing
RHS	right hand side
TDC	Top Dead Center

Introduction

I Motivation

Fluid-dynamic bearings are present in many key high-load, high-speed precision applications where other mechanisms like ball bearings would not perform as well due to noise and vibrations, or because it is even impossible to apply another solution. In car engines they are present in the piston assembly (ring/liner contact, connecting rod in both the big-end and the small-end), crankshaft bearings, valves, valve seats, camshaft contacts and seals in general. The piston assembly accounts for 50% of the engine friction losses, which is approximately 10% of the total fuel consumption. Half of that takes place in the piston ring/liner contact [69], thus being one of the major contributors to the mechanical power losses. Thus, it is important to understand the key factors influencing film formation and friction in fluid film bearings.

All the mentioned engine components work under severe loading conditions. A complete separation of both bearing surfaces (*hydrodynamic* lubrication regime) is not assured, and partial solid-solid lubrication (*mixed* lubrication regime) or mainly solid-solid contact (*boundary* lubrication regime) takes place. Under these conditions the real topography of the surfaces is a determining factor in the bearing performance.

It is known that some surface finishing (or texture) is needed to avoid stiction between the ring pack and the liner. Diverse solutions such as coatings on the rings, surface finishings on both the ring and the liner are commonly practiced in order to improve performance. Honed liners have been used for many decades for that purpose. Surface

texturing has attracted much attention in the last years as a new mean to improve the tribological performance of lubricated contacts, especially since surface microengineering techniques have become available. Honing, machining, ion beam texturing, etching and laser surface texturing (LST) are some of the techniques used [57]. Among them LST distinguishes because of its excellent control of shape and size of the textures and short processing times [60].

Determining the best texture in terms of friction and wear for each working condition by means of experimental procedures is a expensive process. Furthermore, accurate measurements of point-wise quantities like film thickness or pressure are extremely difficult. A faster and cheaper approach is the mathematical modeling and numerical simulation of lubricated contacts, which is not free from challenges, from the different scales involved in the problem to the accurate modeling of the relevant physical phenomena. Micropatterned LST surfaces usually consist of pockets (dimples) with dimensions in the order of tens or hundreds of microns, honed grooves are on the tens of microns and the surface roughness level is even smaller, in the order of the micron or even lower. With a domain in the centimeters (size of the ring pack, journal bearing, seal, etc), this enforces the use of very fine meshes which is traduced in a greater computational cost. The scope of this thesis is to develop efficient numerical methods to solve thin film (lubrication) problems down to the roughness scale. As an application, these methods will help with the assessment of rough surfaces in the performance of fluid lubricated bearings.

A model which takes into account the bearing dynamics, involving the hydrodynamic and the forces coming from the solid-solid contact between surfaces is presented. Due to the geometry of the contacts and the high velocities involved *cavitation* takes place. It has been shown by Ausas et al [12] that a mass-conserving model is of foremost importance, which is confirmed by several authors, among them Zhang & Meng [119] who performed a careful comparison with experiments. Here we employ the Elrod-Adams model [55], which includes into the Reynolds equation conservative boundary conditions for cavitation.

An efficient numerical solution for the Elrod-Adams model is by no means trivial, as the classical Newton-Raphson method is not suitable for this problem. The best approach so far is to solve it with a Jacobi or Gauss-Seidel type algorithm, as done by Ausas et al [11]. However, it becomes computationally expensive for large problems, due to the elliptic problem being solved for the pressure. Multigrid methods arise like a natural choice to accelerate results, yet its implementation for the Elrod-Adams model is not straightforward. In this work, an efficient and robust multigrid implementation is presented, which to the extent of our knowledge has never been published.

With this tool at hand, studying hundreds of micro-patterned surfaces for varying operational conditions became possible in reasonable time. It also allowed converged solutions of the problem at the roughness scale. Surface roughness is usually treated in lubrication problems by means of stochastic models, averaged models or homogenization methods. The stochastic models ([110, 45]), compute expected values of pressure, friction force and load carrying capacity. The approach of averaged models (Patir and Cheng [89, 90]) is quite different, as in this case an average Reynolds equation is developed in terms of empirical flow factor functions based in a probability distribution for the surface roughness. Homogenization methods ([25, 26, 70, 31, 34]) assume periodicity of the surface and develop an average equation with coefficients computed from solutions of local problems. In this work, the uncertainties associated with such methods are avoided using the measured topography of the surface.

Textures were set on the moving surface of the lubricated pair. Moving textures are more challenging from the numerical point of view since the problem becomes intrinsically transient. Then, the dynamics of the pad must be solved coupled with the hydrodynamics and other forces involved. If we think in terms of the piston-ring/liner problem, this amounts to consider textured liners. This is specially useful as the moving surface undergoes less wear ([109]) and the topography of the textured surface is more likely to be preserved. The vast majority of the numerical studies found in the literature deal only with textures on the fixed surface (e.g., the ring), stationary with respect to the pad, and thus the simulations reach a steady state.

The large scale of the bearing's geometry is a decisive factor in whether texturing improves the bearing performance. This was already acknowledged by Etsion ([60]). The performance of very flat bearings, which have limited load carrying capacity, seem to benefit from the convergent-divergent geometry introduced by the surface texture, as seen in many experimental works ([119, 115, 118, 94, 77, 84]). Still, in bearings with a pronounced wedge experimental results are more ambiguous ([76, 5, 80, 73, 92, 91]). Numerical simulations of lubricated contacts aim to shed light on the physical mechanisms responsible for friction reduction in cavitated microtextured contacts.

II Outline

This work is structured as follows: in Chapter 2 a general model for fluid lubricated contacts is presented, leaving the details of the Elrod-Adams model and a thorough discussion of mathematical results to Chapter 3.

Chapter 4 presents the finite volume method used in the discretization of the problem.

Details of the multigrid acceleration by means of the *Full Approximation Method* [61] and the parallel implementation with *OpenMP* are addressed in Chapter 5.

Results of numerical simulations are given in chapters 6 and 7. In Chapter 6 a comprehensive study is carried out with one and two-dimensional textures for different loads and ring masses. Certain configurations are chosen to explain the results shown in friction and clearance charts. The chapter ends with a simulation of a three-ring pack sliding over a textured liner with realistic conditions of pressure and velocity of a four-stroke engine. Chapter 7 is dedicated to direct simulations performed with measured data from honed surfaces representative of automotive cylinder liners. To our knowledge, these are the first mesh-converged simulations that consider the unsteadiness that results from the motion of the rough surface through the computational domain. The validity of the lubrication approximation is analyzed for these rough surfaces by direct comparison against Navier-Stokes results. Also, the friction force models usually found in the literature are compared to a thin film approximation of the friction force, which is found to represent more accurately the Navier-Stokes results. Computational times were greatly improved by smoothing the surfaces and removing atypically large asperities, at the expense of introducing small differences. The chapter concludes with simulations in real engine conditions. The novelty is that the liner surface is differently textured in the regions of higher and lower piston velocities.

Theoretical advances in a new cavitation model are presented in Chapter 8. Finally, conclusions are drawn and ideas for future research are presented in Chapter 9.

III Contributions

This thesis original contributions concerns the modeling, numerical methods and results of two-phase flows in thin films, which as an application, helped to understand the main factors influencing the performance of fluid lubricated bearings considering cavitation phenomena. These are outlined in the following points:

- An efficient and robust multigrid algorithm to solve the Elrod-Adams model coupled with the pad dynamics (Chapter 5).
- Mass-conserving dynamical simulations of lubricated contacts with idealized, mathematically defined surfaces, which led to important contributions to the understanding of the effect of textures on lubrication devices were presented in Chapter 6. These contributions appeared in two articles ([38, 42]) in renowned international journals, one of them as invited contribution in a special issue.

- A lubrication approximation of the friction force for rough surfaces was obtained through asymptotic expansions, which proved to be more accurate than other models found in the literature when compared with Navier-Stokes simulations. This was done in Chapter 7. Results were published in [44].
- Direct simulation of measured surfaces, where the inertial effects of the pad are taken into account (Chapter 7).
- Advances were made in the development of a new cavitation model (Chapter 8). An inaccuracy in the transport velocity in the cavitated regions was identified by both Organisciak [86] and Ausas [14], the straightforward solution leading to an ill-posed problem. We have implemented a more versatile model in 1D and re-interpreted it as resulting from the dynamic coupling of Rankine-Hugoniot conditions (mass conservation) and a film-rupture law that should arise from physical considerations. This allowed us to extend the model to situations with non-positive pressure fields, which is crucial to arrive at a numerical approximation.

Modeling of lubricated contacts

In this Chapter and the following one we aim to introduce a model for fluid lubricated contacts.

The first part of this Chapter is dedicated to the description of the parts and the dynamics of the connecting rod-crank mechanism common to most combustion engines, since the numerical results in this work are focused on the study of the piston ring/liner contact with measured and micropatterned surfaces on the liner. However, the outcoming model presented in the second part remains valid for other lubricated devices.

I The piston-ring-liner lubricated mechanism

I.1 The ring pack

The main function of the piston rings are to:

- Seal the combustion chamber;
- Distribute and control the oil;
- Stabilize the piston.

A ring pack is composed of 2-5 rings, usually including 2-4 *compression* rings and 0-3 *oil control* rings. These numbers depends on the engine type. For example, fast speed four-stroke diesel engines have 2-3 compression rings and a single oil control ring: figure

Figure 2.1: Piston and ring pack of a common fast speed four-stroke engine.

2.1 shows this typical configuration. In addition there could be *scraper* rings, also used as second compression rings. All of them are manufactured nearly circular, with a gap that has several functions: ease of placement in the piston grooves and higher conformability to a deformed cylinder [9]. The rings form along with the piston grooves a labyrinth seal so that pressure drops from the one in the combustion chamber to the one in the crankcase.

I.2 Piston and ring dynamics

The movement of the piston inside the liner is determined by the rod-crank mechanism it is attached to. If r_c is the radius of the crank and l_r is the length of the connecting rod then the piston velocity is:

$$v_p \simeq r_c \omega (\sin(\alpha) + \frac{r_c}{2l_r} \sin(2\alpha)), \quad (2.1)$$

where ω is the rotational velocity of the crankshaft. This formula is valid for small connecting rod ratios r_c/l_r . A example for a engine running at $\omega=1900$ rpm, with $r_c=7.75$ cm and $l_r=25.1$ cm is shown in figure 2.3.

The piston movement is also the primary motion of the piston rings. It moves between

Figure 2.2: Connecting rod-crank mechanism common to most combustion engines.

Figure 2.3: Gas pressure (top) and piston velocity (bottom) as functions of the crank angle.

two extreme positions, one closer the combustion chamber called the Top Dead Center (TDC) and the other one adjacent to the crankcase, which is named Bottom Dead Center (BDC). "Top" and "Bottom" come from the usual vertical position of the bore in car engines. The *stroke length*, that is the BDC-TDC distance can be easily deduced to be $2r_c$.

The piston's head faces the combustion chamber. In that side of the liner, the pressure changes along the four engine strokes: *compression*, *power*, *exhaust* and *admission*, as seen in figure 2.3. On the crankcase side it stays close to the atmospheric pressure.

Apart from the reciprocating movement, secondary movements inside the groove are allowed. Clearances in the axial direction are of the order of $10\ \mu\text{m}$ - $100\ \mu\text{m}$ ([17]). In the radial direction, despite the rings sitting tight against the liner, there is a gap between them and the piston. This gives so far two degrees of freedom for movement inside the groove, one in the axial and the other one in the radial direction.

These secondary movements of the ring depend on the forces and moments acting on it. At the ring/liner conjunction the friction and the contact force are due to contact between the asperities of both surfaces. Also, due to the usual convergent-divergent geometry of the ring there is a net pressure generation in the lubricating oil, creating a lifting force that contributes to separate the ring and the liner along with a mainly viscous fluid friction force. To conclude, there are the forces exerted on the ring by the groove flank. The imbalance of torques of these forces generates the so called *ring twist*, that is, a rotation of the cross-section of the ring. This counts as a third secondary movement of the ring. However, this deflection is normally small, about 1° ([17]). Another more complex behavior are the in-plane deformations of the ring as a result of the non-uniform distribution of forces ([16]). This has various origins: the ring gap (depicted in fig. 2.1), the piston side-thrust, bore distortion, thermal effects among others ([9]).

1.3 The cylinder liner

The cylinder liner is the central working part of a reciprocating engine. Its primary functions are heat transfer, retain lubricant and together with the rings to form a sliding surface to seal the space between the piston and itself. Its macro form is, as the name self-explains, that of a cylinder with a certain nominal radius and length larger than $2r_c$. In practice, the liner is not perfectly cylindrical and certain bore distortions due to manufacturing tolerances, engine assembly and thermal effects. This induces loss of conformity between the rings and the liner, which leads to a increase in oil consumption and blow-by ([9]).

It is known that the surface finish of a liner has influence on the scuffing resistance. A rise in friction, leading to scuffing is associated with polished liners ([63]), therefore some texture on it is always needed. At the micro scale, the trend in the bore of cast-iron cylinders surfaces finish is a abrasive machining process known as *honing*. The cutting marks of the honing form a pattern of diagonal valleys on the liner known as *honing grooves*, and between them there is a flat plateau ([9]). Two $1.2\text{mm} \times 0.9\text{mm}$ samples of

measured honed liners are shown in figure 2.4: notice the different scales in the x_1 - x_2 plane and in the orthogonal direction. The objective of this machining process is to minimize oil consumption and wear.

Figure 2.4: Samples of a honed liner surfaces. Figures (a) and (b) are from a 120° honing, the liner movement direction being x_1 . (c), (d) correspond to a 150° honing.

Tailor-made surfaces for liners have attracted much attention in the last years as a new mean to improve the tribological performance of lubricated contacts, especially since surface microengineering techniques have become available. Honing, machining, ion beam texturing, etching and laser surface texturing (LST) are some of the techniques used ([57]). Among them LST distinguishes because of its excellent control of shape and size of the textures and short processing times ([60]). It is used to create patterned microstructures commonly in the form of pockets or dimples, aiming to improve the load capacity, wear rate and reduce friction of the lubricated pair. Some examples of those laser-produced textures are seen in figure 2.5.

The importance of considering textured *liners*, instead of textured rings (as done for example in [97],[75],[98],[59]), arises from several practical considerations: (a) All actual

Figure 2.5: Textures produced by Laser Surface Texturing. Figures (a) and (b) were extracted from [76] and [102], respectively.

cylinder liners are, in fact, textured in some way. (b) The wear of the liner is smaller ([109]), and more evenly distributed, than that of the ring; the texture can thus be designed with more confidence that it will preserve its shape for long operating times. (c) Since the rings contact the same axial position of the liner always at the same crank angle, texturing the liner allows one to optimize the texture that contacts the ring independently for each instant of the engine cycle. (d) Also, in account of the increase of the curvature radius of the ring with wear, the texture (if undamaged) becomes more beneficial as time evolves ([38]).

II Modeling fluid bearings

II.1 Geometrical model

In this section we aim to develop a model for fluid lubricated bearings. It is clear that a single model cannot encompass all lubricated devices, but we aspire to keep it as comprehensive as possible.

A fluid bearing has two surfaces in relative movement with a lubricating fluid filling the gap between them. Without loss of generality, we can consider that one of those surfaces is in the x_1 - x_2 plane, while the other one is given by a function $h = h(x_1, x_2, t) > 0 \forall (x_1, x_2, t) \in \Omega$, being $\Omega \subset \mathbb{R}^2$ the computational domain. This is true for journal bearings such as the one depicted in figure 2.6, where the domain Ω can be mapped to a rectangle $\Omega = [x_{1\ell}, x_{1r}] \times [0, B]$ in $\mathbb{R}^2$. In the case of the piston ring/liner contact, if we further assume:

- No bore deformations;

- The piston axis remains aligned at every instant t with the liner axis, and;
- The effects of the ring gap (fig. 2.1) are negligible;

Then due to the problem symmetry, the computational domain is also a very thin rectangular strip $\Omega = [x_{1\ell}, x_{1r}] \times [0, B]$ with its largest side in the x_1 direction (shown in fig. 2.1). As B is much smaller than the bore radius, the curvature along x_2 is neglected. If ring twist is neglected, then the piston ring/liner contact problem has been reduced to that of a pad bearing such as the one shown in figure 2.6.

Figure 2.6: Examples of lubricated devices.

If the frame of reference is fixed in the x_1 - x_2 plane, the relative velocity of the upper surface is $\mathbf{V} = (V_1, V_2, V_3)$. Figure 2.6 shows that assuming $V_2 = 0$ is consistent with journal and pad bearings. They are not compatible though with mechanisms such as the tilting pad bearing, as they have one more degree of freedom: a rotation around the x_2 axis.

Figure 2.7: Section of a pad with $\min_{(x_1, x_2) \in \Omega} h_p(x_1, x_2) = 0$. Its analytical expression is the one corresponding to an arc of circumference of radius $R=64$ mm.

For the purpose of mathematical modeling, the function h can be split in components $h_b(x_1, x_2, t)$ describing the main shape of the upper surface and $h_t(x_1, x_2, t) = h_t(x_1 - V_1 t, x_2)$, describing the lower surface. Thus:

$$h(x_1, x_2, t) = h_b(x_1, x_2, t) - h_t(x_1 - V_1 t, x_2), \quad (2.2)$$

It is clear that $h_t = 0$ for a untextured liner. In the particular case of the journal bearing h_b assumes the form:

$$h_b(x_1, x_2, t) = r_2 - r_1 + X(t) \cos(2\pi x_1) + Y(t) \sin(2\pi x_1), \quad (2.3)$$

In all the domain. Here r_2, r_1 are the housing and shaft radius and $X(t), Y(t)$ are the shaft positions relative to the housing center. Unlike the shaft of the journal bearing, the pad bearing occupies a limited region $x_{1\ell} < a \leq x_1 \leq b < x_{1r}$, then:

$$h_b(x_1, x_2, t) = \begin{cases} h_p(x_1, x_2) + Z(t) & \text{if } a \leq x_1 \leq b \\ e & \text{otherwise} \end{cases}, \quad (2.4)$$

with $\min_{(x_1, x_2) \in \Omega} h_p(x_1, x_2) = 0$, a example of this being illustrated in figure 2.7. As $h_p = h_p(x_1, x_2)$, it is being considered that there is no axial movement of the pad relative to the piston. $Z(t)$ parametrizes the pad's position, and is chosen so that $Z = 0$ corresponds to the ring touching the x_1 - x_2 plane. Coherent with the assumption that the piston axis coincides with the bore axis, e is a gap of uniform thickness outside the ring position, as shown in figure 2.6.

II.2 Bearing dynamics

The simulation of a fluid bearing with textures on the runner constitutes a scientific challenge from the numerical perspective. The dynamical behavior of the system is completely different depending on whether the texture is on the shaft/pad or on the runner. In the former case, the texture is stationary with respect to the contact, and thus bearing stabilizes at some position at which force equilibrium takes place. This position only changes with the time scale of variations in the relative velocity or oil-feeding conditions. If the texture is on the runner, the texture cells enter the contact from one side, move under the bearing, and leave the contact at the opposite side. In this case the ring exhibits a superposed oscillatory motion with the time scale λ/V_1 , where λ is the size of the texture cells (whether they are dimples or honed grooves) and V_1 the relative speed. The complexity grows substantially, since the simulation must be transient and must incorporate the rigid-body dynamics of the bearing, which is subject to time-varying forces.

The dynamics of the lubricated device are governed by the forces acting on it. Newton's second law of motion expresses:

$$m \frac{d^2 \mathbf{Z}}{dt^2} = \mathbf{W}(t) + \mathbf{W}^h(t) + \mathbf{W}^c(t), \quad (2.5)$$

where $\mathbf{Z}(t) = (X(t), Y(t))$ for the journal bearing and simply $\mathbf{Z}(t) = Z(t)$ for the pad bearing from figure 2.6. m is the linear mass of the rigid mobile body: shaft or pad, according to the considered device. The forces imposed are given by $\mathbf{W}(t)$, the contact force between the matching surfaces $\mathbf{W}^c(t)$ and the hydrodynamic force $\mathbf{W}^h(t)$ originated from the pressure $p(x_1, x_2, t)$ that develops in the oil film between the moving surfaces:

$$W_X^h(t) = \int_{\Omega} p(x_1, x_2, t) \cos(2\pi x_1) d\Omega, \quad (2.6a)$$

$$W_Y^h(t) = \int_{\Omega} p(x_1, x_2, t) \sin(2\pi x_1) d\Omega, \quad (2.6b)$$

for the journal bearing and simply

$$W^h(t) = \int_{\Omega} p(x_1, x_2, t) d\Omega, \quad (2.7)$$

for the pad bearing. It is clear that initial conditions are required for $\mathbf{Z}$ and $\mathbf{Z}'$ at $t = 0$.

$\mathbf{W}(t)$ is the known load imposed on the shaft of a journal bearing. For the piston ring/liner arrangement this force can be split in two components:

$$W(t) = W^{ps} + W^{gp}(t), \quad (2.8)$$

where

- The *pre-stress force* per unit width W^{ps} comes from the elastic response of the ring to the deformation needed to make it fit into its placement. It points outwards (i.e.; along $-x_3$) and can be assumed constant (ring's deformation in the x_3 direction are in the order of the microns).
- The *gas pressure force* $W^{gp}(t)$ is the force produced by the pressure p_c of the combustion gases acting on the back part of the rings. This force is time-dependent (a function of the crank angle), and its value per unit width is assumed to be:

$$W^{gp}(t) = -\phi_r p_c(t), \quad (2.9)$$

Where $0 \leq \phi_r \leq 1$ is a parameter representing the fraction of the pressure p_c acting on the back part of each ring. Each of these values is assumed to be constant and decreasing from the compression to the oil control ring.

The *contact force* is the one exerted by one surface to the other. At each contact point between the liner and the ring a contact pressure p_{con} acts on both surfaces and thus the contact force force per unit width on the ring is:

$$W^c(t) = \frac{1}{B} \int_{\Omega} p_{con}(x_1, x_2, t) d\Omega. \quad (2.10)$$

This contact pressure is the result of elastic or even plastic deformation of the asperities. Therefore, it is a function of the geometry and the properties of the materials of both surfaces. Here we adopt the Greenwood-Tripp model [68] to compute p_{con} :

$$p_{con}(h) = K E' F_{5/2} \left(\frac{h}{\sigma} \right) \quad (2.11)$$

where E' is the composite modulus of elasticity, while K is defined as:

$$K = \frac{16\sqrt{2}}{15} \pi (\eta \beta \sigma)^2 \sqrt{\frac{\sigma}{\beta}} \quad (2.12)$$

being η , β and σ are the asperity density, radius of curvature and composite standard

Figure 2.8: Forces acting on a piston ring.

deviation of asperity height. The integral

$$F_{5/2}(h/\sigma) = \frac{1}{\sqrt{2\pi}} \int_{h/\sigma}^{\infty} \left(u - \frac{h}{\sigma}\right)^{5/2} e^{-\frac{u^2}{2}} du \quad (2.13)$$

does not have an straightforward analytical solution and it has to be evaluated numerically or approximated by some function. Due to its simplicity and excellent fit, the Panayi-Schock approximation [88] is employed.

Reynolds equation and the Elrod-Adams model of cavitation

In the previous chapter we started to present a encompassing model for lubricated contacts. What remains to be explained is the assessment of the fluid pressure p .

This section deals with fluid bearing problems where cavitation phenomena is treated with the Elrod-Adams model, a mass-conserving model ([12]) which has proven experimentally ([119]) to give a accurate position of the cavitation boundary.

I The governing equations in hydrodynamic lubrication

Figure 3.1: Scheme of the problem.

Let us consider two surfaces in relative motion with a gap $h(x_1, x_2, t)$ between them. Without loss of generality we can consider one of those surfaces on the x_1 - x_2 plane. The projection of both on $x_3 = 0$ is the domain $\Omega \subset \mathbb{R}^2$, as can be seen in figure 3.1.

With an Eulerian frame set on the lower surface, the relative velocity of the upper one with respect to the former is $\mathbf{V} = (V_1, V_2, V_3)$. As shown in figure 2.6, when $V_2 = 0$ these assumptions are consistent with several devices, such as journal and pad bearings (e.g., piston ring/liner contact). Neglecting the variations of density ρ in the lubricant filling the gap between both surfaces and furtherly assuming it Newtonian with viscosity μ , the problem is governed by the Navier-Stokes equations for a incompressible Newtonian fluid:

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} - \frac{1}{\rho} \nabla \cdot (\mu \nabla \mathbf{v}^T) + \frac{1}{\rho} \nabla p - \mathbf{f} = 0 \quad (3.1a)$$

$$\nabla \cdot \mathbf{v} = 0 \quad (3.1b)$$

Where equation 3.1a represents the momentum and 3.1b the mass conservation. $\mathbf{v} = \mathbf{v}(x_1, x_2, x_3, t) = (v_1, v_2, v_3)$ and $p = p(x_1, x_2, x_3, t)$ are the fluid velocity and the hydrodynamic pressure at (x_1, x_2, x_3, t) . $\mathbf{f} = (f_1, f_2, f_3)$ is a term including the body forces. In terms of the components in x_1 to x_3 we have:

$$\begin{aligned} \frac{\partial v_1}{\partial t} + v_1 \frac{\partial v_1}{\partial x_1} + v_2 \frac{\partial v_1}{\partial x_2} + v_3 \frac{\partial v_1}{\partial x_3} \\ - \frac{1}{\rho} \left(\frac{\partial}{\partial x_1} \left(\mu \frac{\partial v_1}{\partial x_1} \right) + \frac{\partial}{\partial x_2} \left(\mu \frac{\partial v_1}{\partial x_2} \right) + \frac{\partial}{\partial x_3} \left(\mu \frac{\partial v_1}{\partial x_3} \right) \right) \\ + \frac{1}{\rho} \frac{\partial p}{\partial x_1} - f_1 = 0 \end{aligned} \quad (3.2a)$$

$$\begin{aligned} \frac{\partial v_2}{\partial t} + v_1 \frac{\partial v_2}{\partial x_1} + v_2 \frac{\partial v_2}{\partial x_2} + v_3 \frac{\partial v_2}{\partial x_3} \\ - \frac{1}{\rho} \left(\frac{\partial}{\partial x_1} \left(\mu \frac{\partial v_2}{\partial x_1} \right) + \frac{\partial}{\partial x_2} \left(\mu \frac{\partial v_2}{\partial x_2} \right) + \frac{\partial}{\partial x_3} \left(\mu \frac{\partial v_2}{\partial x_3} \right) \right) \\ + \frac{1}{\rho} \frac{\partial p}{\partial x_2} - f_2 = 0 \end{aligned} \quad (3.2b)$$

$$\begin{aligned} \frac{\partial v_3}{\partial t} + v_1 \frac{\partial v_3}{\partial x_1} + v_2 \frac{\partial v_3}{\partial x_2} + v_3 \frac{\partial v_3}{\partial x_3} \\ - \frac{1}{\rho} \left(\frac{\partial}{\partial x_1} \left(\mu \frac{\partial v_3}{\partial x_1} \right) + \frac{\partial}{\partial x_2} \left(\mu \frac{\partial v_3}{\partial x_2} \right) + \frac{\partial}{\partial x_3} \left(\mu \frac{\partial v_3}{\partial x_3} \right) \right) \\ + \frac{1}{\rho} \frac{\partial p}{\partial x_3} - f_3 = 0 \end{aligned} \quad (3.2c)$$

$$\frac{\partial v_1}{\partial x_1} + \frac{\partial v_2}{\partial x_2} + \frac{\partial v_3}{\partial x_3} = 0 \quad (3.2d)$$

Until this point we have not introduced the most relevant trait of the lubrication problem: the breach where the lubricant flows is much smaller than the dimensions in the orthogonal plane. If L is the characteristic length in the x_1 and x_2 dimensions and H in x_3 , we have that $h(x_1, x_2, t) \simeq H \ll L$.

At this point we can consider a change of variables $x_3 = \epsilon z$, where $\epsilon > 0$ is a small quantity. As done in [23], p and v_i ($i = 1, 2, 3$) are written in power series of ϵ :

$$v_i = v_i^0 + \epsilon v_i^1 + \epsilon^2 v_i^2 + \dots \quad (3.3a)$$

$$p = \frac{1}{\epsilon^2} p^0 + \frac{1}{\epsilon} p^1 + p^2 + \dots \quad (3.3b)$$

The dominant terms once equations 3.3 have been replaced in 3.2 are those in ϵ^{-3} , ϵ^{-2} , ϵ^{-1} and ϵ^0 , which must be zero:

$$- \frac{\partial}{\partial z} \left(\mu \frac{\partial v_1^0}{\partial z} \right) + \frac{\partial p^0}{\partial x_1} = 0; \text{ term in } \epsilon^{-2} \text{ from equation 3.2a} \quad (3.4)$$

$$- \frac{\partial}{\partial z} \left(\mu \frac{\partial v_2^0}{\partial z} \right) + \frac{\partial p^0}{\partial x_2} = 0; \text{ term in } \epsilon^{-2} \text{ from equation 3.2b} \quad (3.5)$$

$$\frac{\partial p^0}{\partial z} = 0; \text{ term in } \epsilon^{-3} \text{ from equation 3.2c} \quad (3.6)$$

$$\frac{\partial v_3^0}{\partial z} = 0; \text{ term in } \epsilon^{-1} \text{ from equation 3.2d} \quad (3.7)$$

$$\frac{\partial v_1^0}{\partial x_1} + \frac{\partial v_2^0}{\partial x_2} + \frac{\partial v_3^1}{\partial z} = 0; \text{ term in } \epsilon^0 \text{ from equation 3.2d} \quad (3.8)$$

Equation 3.6 leads to $p^0 = p^0(x_1, x_2, t)$, that is, the pressure is not a function of z . If we further assume no variations of temperature through the gap h , then μ remains constant

along z . With this in mind, we take the integrals of equations 3.4 and 3.5 from $z = 0$ to $z = h$ and assuming the boundary conditions:

$$v_1^0(x_1, x_2, 0, t) = 0, \quad v_1^0(x_1, x_2, h, t) = V_1, \quad (3.9)$$

$$v_2^0(x_1, x_2, 0, t) = 0, \quad v_2^0(x_1, x_2, h, t) = V_2, \quad (3.10)$$

Thus, the following expressions for v_1^0 and v_2^0 are attained:

$$v_1^0 = \frac{1}{2\mu} \frac{\partial p^0}{\partial x_1} (z^2 - zh) + \frac{z}{h} V_1 \quad (3.11)$$

$$v_2^0 = \frac{1}{2\mu} \frac{\partial p^0}{\partial x_2} (z^2 - zh) + \frac{z}{h} V_2 \quad (3.12)$$

To arrive at a equation independent of the fluid velocities v_1 to v_3 we integrate equation 5.1 along z :

$$\int_0^h \left(\frac{\partial v_1^0}{\partial x_1} + \frac{\partial v_2^0}{\partial x_2} + \frac{\partial v_3^1}{\partial z} \right) dz = 0 \quad (3.13)$$

The first two terms can be rearranged according to the Leibniz rule (supposing continuity of the partial derivatives of v_1^0 , v_2^0 and v_3^1):

$$\int_0^h \left(\frac{\partial v_1^0}{\partial x_1} \right) dz = \frac{\partial}{\partial x_1} \int_0^h v_1^0 dz - v_1^0(h) \frac{\partial h}{\partial x_1} = \frac{\partial}{\partial x_1} \left(-\frac{h^3}{12\mu} \frac{\partial p^0}{\partial x_1} + \frac{V_1}{2} h \right) - V_1 \frac{\partial h}{\partial x_1} \quad (3.14a)$$

$$\int_0^h \left(\frac{\partial v_2^0}{\partial x_2} \right) dz = \frac{\partial}{\partial x_2} \int_0^h v_2^0 dz - v_2^0(h) \frac{\partial h}{\partial x_2} = \frac{\partial}{\partial x_2} \left(-\frac{h^3}{12\mu} \frac{\partial p^0}{\partial x_2} + \frac{V_2}{2} h \right) - V_2 \frac{\partial h}{\partial x_2} \quad (3.14b)$$

And assuming boundary conditions:

$$v_3^1(x_1, x_2, 0, t) = 0, \quad v_3^1(x_1, x_2, h, t) = \frac{dh}{dt} = \frac{\partial h}{\partial t} + V_1 \frac{\partial h}{\partial x_1} + V_2 \frac{\partial h}{\partial x_2} \quad (3.15)$$

we arrive at:

$$\int_0^h \left(\frac{\partial v_3^1}{\partial z} \right) dz = \frac{\partial h}{\partial t} + V_1 \frac{\partial h}{\partial x_1} + V_2 \frac{\partial h}{\partial x_2}, \quad (3.16)$$

which leads to the dimensional form of the Reynolds equation:

$$\frac{\partial h}{\partial t} + \frac{\partial}{\partial x_1} \left(-\frac{h^3}{12\mu} \frac{\partial p^0}{\partial x_1} + \frac{V_1}{2} h \right) + \frac{\partial}{\partial x_2} \left(-\frac{h^3}{12\mu} \frac{\partial p^0}{\partial x_2} + \frac{V_2}{2} h \right) = 0 \quad (3.17)$$

or (disregarding the superindexes):

$$\frac{\partial h}{\partial t} + \nabla \cdot \left(-\frac{h^3}{12\mu} \nabla p + \frac{\mathbf{V}}{2} h \right) = 0 \quad (3.18)$$

It has to be noticed that the body forces $\mathbf{f}$ have no influence on the two first terms of the asymptotic expansions and therefore they do not appear in the Reynolds equation. The rigorous mathematical justification of the Reynolds equation was given by Bayada and Chambat ([23]) with boundary conditions $v_3^1(h) = v_3^1(0) = 0$ (instead of conditions 3.15) for the weak convergence of the linearized Stokes flow to the Reynolds flow.

We can identify that equation 3.18 is a conservation law ([79]). It is written in the form $\partial_t h + \nabla \cdot \mathbf{J} = 0$, where

$$\mathbf{J} = -\frac{h^3}{12\mu} \nabla p + \frac{\mathbf{V}}{2} h \quad (3.19)$$

is the volumetric flux of lubricant. The conserved quantity is $h(x_1, x_2, t)$, that is, the amount of oil present at (x_1, x_2, t) .

II Cavitation and the mass-conservation problem with the Reynolds equation

With appropriate conditions equation 3.18 allows negative pressures to take place. This can be seen in a simple example where the Reynolds equation is solved in an unidimensional domain $\Omega = [0, 1]$ where:

$$h(x) = 1 + (2x - 1)^2, \quad (3.20)$$

with boundary conditions

$$p(0) = 0, \quad p(1) = 0 \quad (3.21)$$

In figure 3.2 the frame of reference is fixed on the upper surface. The lower surface is moving to the right with velocity $V_1=1$, $V_2=0$. $V_3=0$ as h does not depend on time. Pressure at the boundaries of the domain are set to $p(0) = p(1) = 0$, which generates

Figure 3.2: From top to bottom: geometry of upper surface, pressure given by the Reynolds equation, Gumbel boundary conditions, solution with Reynolds boundary conditions.

a symmetric pressure profile around the position of the upper surface's apex, $x = 0.5$. On the convergent part of the bearing ($0 \leq x \leq 0.5$) $p > 0$, while on the divergent part $p < 0$. It is known that if the lubricant pressure falls below certain values, gaseous and/or vaporous cavitation may take place ([52]). Vapor bubbles at the vapor pressure will be generated, hence, inside this cavitated regions pressure must remain constant. Without loss of generality we will set $p_{cav} = 0$.

Several models have been proposed to correct this problem with the Reynolds equation. A straightforward film rupture condition was proposed by Gumbel ([52]) in which the Reynolds solution is forced to $p = 0$ where $p < 0$ (third figure from top to bottom in 3.2). This leads to a non-physical situation as the flux $\mathbf{J}$ is not balanced at the cavitation boundary:

$$(\mathbf{J}^+ - \mathbf{J}^-) \cdot \hat{n} = \frac{V_1}{2} h^+ (\hat{e}_1 \cdot \hat{n}) - \left(-\frac{(h^-)^3}{12\mu} (\nabla p^- \cdot \hat{n}) + \frac{V_1}{2} h^- (\hat{e}_1 \cdot \hat{n}) \right), \quad (3.22)$$

and supposing h continuous at Σ :

$$(\mathbf{J}^+ - \mathbf{J}^-) \cdot \hat{n} = \frac{h^3}{12\mu} \partial_{x_1} p^- \neq 0, \quad (3.23)$$

as $\frac{\partial p^-}{\partial n} \neq 0$ at σ , in fact $\frac{\partial p^-}{\partial n} < 0$.

The first attempt at a mass-conserving film rupture condition was given by Swift(1931) and Stieber (1933), although called the Reynolds boundary condition, as Reynolds (1886) first suggested it ([52]). The considerations made for this condition also come from flux continuity at the boundary. Going back to equation 3.23 we can see that, with continuous h , what is required for mass-conservation is:

$$\frac{\partial p^-}{\partial n} = \frac{\partial p^+}{\partial n} = \frac{\partial p}{\partial n} = 0 \quad (3.24)$$

More generally, the expression for the condition imposed by the Reynolds' model at a zero-pressure free boundary is ([12]):

$$(\mathbf{J}^+ - \mathbf{J}^-) \cdot \hat{n} \geq 0, \quad (3.25)$$

which with continuity in h returns the condition given in equation 3.24. This can be seen also in figure 3.2 (last figure from top to bottom). However, for a discontinuous h the Reynolds condition does not guarantee mass-conservation and therefore a correct computation of the pressure field.

II.1 The Rankine-Hugoniot boundary conditions and the Elrod-Adams model

The lack of mass-conservation in the Reynolds model can be corrected introducing a new scalar field $\theta = \theta(x_1, x_2, t)$, which can be viewed as saturation function ([22]), a fractional film content ([55]) or the results of the homogenization of the oil fingers through the height of the gap ([71]). This field takes values $0 \leq \theta \leq 1$ such that $\theta = 1$ in the full film region Ω_+ and $\theta < 1$ in the cavitated region Ω_0 . In this model the flux reads:

$$\mathbf{J} = -\frac{h^3}{12\mu} \nabla p + \left(\frac{\mathbf{V}}{2}\right) h \theta \quad (3.26)$$

Which leads in a steady state problem to the following mathematical formulation of the conservation law $\nabla \cdot \mathbf{J} = 0$:

$$\nabla \cdot \left(\frac{h^3}{12\mu} \nabla p \right) = \nabla \cdot \left(\frac{\mathbf{V}}{2} h \theta \right) \quad (3.27)$$

$$p > 0 \quad \theta = 1 \quad \text{in } \Omega_+ \quad \text{and} \quad p = 0 \quad 0 \leq \theta < 1 \quad \text{in } \Omega_0 \quad (3.28)$$

This is known as the Elrod-Adams model ([55]) for a steady state problem. Now,

the condition satisfied on Σ corresponds to the strict mass-conservation, that is, the Rankine-Hugoniot conditions at the interface:

$$(\mathbf{J}^+ - \mathbf{J}^-) \cdot \hat{n} = 0 \quad \text{in } \Sigma \quad (3.29)$$

This condition is also named *JFO boundary condition* in honor to Jacobsson and Floberg ([71]) and Olsson ([85]). If h is continuous the condition 3.29 coincides with the Reynolds condition $\frac{\partial p}{\partial n} = 0$. In fact, we have that at film rupture:

$$\mathbf{V} \cdot \hat{n} > 0, \quad \theta^- < 1, \quad \left(\frac{\partial p}{\partial n} \right)^- = 0 \quad (3.30)$$

which combined with equation 3.29 gives:

$$\frac{h^3}{12\mu} \left(\frac{\partial p}{\partial n} \right)^- = h(1 - \theta^+) \left(\frac{\mathbf{V}}{2} \cdot \hat{n} \right) \geq 0 \quad (3.31)$$

Since $\left(\frac{\partial p}{\partial n} \right)^- \leq 0$ as a consequence of the restriction $p \geq 0$, $\left(\frac{\partial p}{\partial n} \right)^- = 0$, that is the Reynolds condition.

In film reformation the situation changes: a discontinuity in θ and in $\frac{\partial p}{\partial n}$ develops, which leads to a mass-conserving solution in this model that does not coincide with that of Reynolds' model. Ausas et al ([12]) show a comparison of both solutions as well as the existence of nonphysical noncavitated areas.

The conserved quantity here can be identified as $h\theta$, that is, the quantity of oil present at a certain (x_1, x_2, t) . The conservation law for a transient problem reads:

$$\frac{\partial h\theta}{\partial t} + \nabla \cdot \mathbf{J} = 0 \quad (3.32)$$

from which we can deduce:

$$\frac{\partial h\theta}{\partial t} + \nabla \cdot \left(-\frac{h^3}{12\mu} \nabla p + \frac{\mathbf{V}}{2} h\theta \right) = 0 \quad (3.33)$$

that under the complementarity conditions

$$\begin{cases} p > 0 & \Rightarrow & \theta = 1 \\ \theta < 1 & \Rightarrow & p = 0 \\ 0 \leq \theta \leq 1 \end{cases} \quad (3.34)$$

constitutes the Elrod-Adams model for a transient problem. At each instant the domain spontaneously divides into two regions (fig. 3.1):

- A *pressurized region* Ω^+ , where $p > 0$ and $\theta = 1$, where a elliptic equation for p is solved

$$\frac{\partial h}{\partial t} + \nabla \cdot \left(-\frac{h^3}{12\mu} \nabla p + \frac{\mathbf{V}}{2} h \right) = 0 \quad (3.35)$$

- A *cavitated region*, Ω^0 , where the film is not full and a hyperbolic equation of θ is solved:

$$\frac{\partial h\theta}{\partial t} + \frac{\mathbf{V}}{2} \cdot \nabla (h\theta) = 0 \quad (3.36)$$

It is worth noting that an interface condition at Σ is implicitly expressed by equation 3.33 which performs as the classical Rankine-Hugoniot condition for shocks in fluid mechanics problems ([24]). Therefore, this incorporates into a single formulation the Reynolds equation for the pressurized region and the Jacobsson-Floberg-Olsson boundary conditions for the transient case:

$$(\mathbf{J}^+ - \mathbf{J}^-) \cdot \hat{n} = \left(h^+ \theta^+ \left(\frac{\mathbf{V}}{2} \cdot \hat{n} \right) \right) - \left(-\frac{(h^-)^3}{12\mu} \frac{\partial p}{\partial n} + h^- \theta^- \left(\frac{\mathbf{V}}{2} \cdot \hat{n} \right) \right) = (h^+ \theta^+ - h^-) (\mathbf{V}_{\Sigma} \cdot \hat{n}) \quad (3.37)$$

This model is mass conserving, which as shown by [12] is essential for obtaining physically meaningful results in lubrication problems involving textured surfaces.

Of course, the previous model 3.33-3.34 must be supplemented with appropriate boundary and initial conditions. A initial condition $\theta(x_1, x_2, t = 0) = \theta_I(x_1, x_2)$ with $(x_1, x_2) \in \Omega$ is given.

For both the journal and the pad bearings the domain boundary $\partial\Omega$ can be divided into three parts according to the boundary conditions:

- $p(x_1, x_2, t) = 0$ on Γ_0 , the part of the boundary where the lubricant leaves the domain;
- $V_1 h \theta - h^3 \frac{\partial p}{\partial n} = \eta(x_2, t)$ on Γ_1 , in the feeding part of the domain, and;
- periodic boundary conditions in some parts of the domain.

The boundary condition on Γ_1 is of a forced flux of lubricant η . The periodic boundary conditions on the flux and p depend on the lubricated device:

- *Journal bearings*: periodicity in the x_1 direction ,

- *Pad bearings (e.g., piston ring/liner contact)*: periodicity in x_2 ,

as shown in figure 2.6.

III Non-dimensionalization and final equations

We consider a non-dimensionalization of the equations based on the following fundamental scales:

Quantity	Scale
Velocity	U
Length	L
Gap thickness	H

These scales, together with the viscosity of the oil μ , lead to the following derived scales for the different quantities

Quantity	Scale	Name
t	$\frac{L}{U}$	time
h	H	gap thickness
Z	H	ring's radial position
p	$\frac{6\mu UL}{H^2}$	pressure
W, W^h, W^c	$\frac{6\mu UL^2}{H^2}$	radial forces per unit width
m	$\frac{6L^4\mu}{H^3U}$	mass per unit width

Therefore, we have:

$$\hat{x}_i = \frac{x_i}{L}, \quad \hat{V}_1 = \frac{V_1}{U}, \quad \hat{h} = \frac{h}{H}, \quad \hat{Z} = \frac{Z}{H}, \quad \hat{t} = \frac{t}{L/U}, \quad (3.38)$$

$$\hat{p} = \frac{p}{\frac{6\mu UL}{H^2}}, \quad \hat{p}_{con} = \frac{p_{con}}{\frac{6\mu UL}{H^2}}, \quad \hat{W} = \frac{W}{\frac{6\mu UL^2}{H^2}}, \quad \hat{m} = \frac{m}{\frac{6L^4\mu}{H^3U}}. \quad (3.39)$$

Upon non-dimensionalization of all variables, and omitting for simplicity all carets evidencing non-dimensional variables, let us collect the complete mathematical problem to be solved:

“Find function $Z(t)$ and fields $p(x_1, x_2, t)$ and $\theta(x_1, x_2, t)$ solution of

$$\begin{cases} m \frac{d^2 Z}{dt^2} = W(t) + W^h(t) + W^c(t) & t \in [0, T] \end{cases} \quad (3.40)$$

$$\begin{cases} Z(0) = z_0, \end{cases} \quad (3.41)$$

$$\begin{cases} Z'(0) = v_0, \end{cases} \quad (3.42)$$

where m is the linear mass of the bearing, T is a given positive number, $W(t)$ the time-implicitly known load and

$$W^h(t) = \frac{1}{B} \int_a^b \int_0^B p(x_1, x_2, t) dx_1 dx_2, \quad (3.43)$$

$$W^c(t) = \frac{1}{B} \int_a^b \int_0^B p_{con}(x_1, x_2, t) dx_1 dx_2 \quad (3.44)$$

are the hydrodynamic lift and the contact force, respectively. p_{con} is a function of the distance $h(x_1, x_2, t)$ between the surfaces

$$p_{con} = p_{con}(h(x_1, x_2, t)), \quad (3.45)$$

while fields $p(x_1, x_2, t)$, $\theta(x_1, x_2, t)$, defined on $\Omega = (x_{1\ell}, x_{1r}) \times (0, B)$ and periodic in x_2 satisfy

$$\begin{cases} \nabla \cdot \left(\frac{h^3}{2} \nabla p \right) = \frac{V_1}{2} \frac{\partial h \theta}{\partial x_1} + \frac{\partial h \theta}{\partial t} & \text{in } (\Omega) \times]0, T[\end{cases} \quad (3.46)$$

$$\begin{cases} p > 0 & \Rightarrow & \theta = 1 \end{cases} \quad (3.47)$$

$$\begin{cases} \theta < 1 & \Rightarrow & p = 0 \end{cases} \quad (3.48)$$

$$\begin{cases} 0 \leq \theta \leq 1 \end{cases} \quad (3.49)$$

with boundary conditions and initial conditions:

$$\begin{cases} V_1 h \theta - h^3 \frac{\partial p}{\partial x_1} = \eta(x_2, t) & \forall x_2 \in \Gamma_0 = \{x_{1\ell}\} \times [0, B] \end{cases} \quad (3.50)$$

$$\begin{cases} p = 0 & \forall x_2 \in \Gamma_1 = \{x_{1r}\} \times [0, B] \end{cases} \quad (3.51)$$

$$\begin{cases} \theta(x_1, x_2, t = 0) = \theta_I(x_1, x_2), & (x_1, x_2) \in \Omega \end{cases} \quad (3.52)$$

Function h is given by

$$h(x_1, x_2, t) = \begin{cases} Z(t) + h_p(x_1, x_2) - h_t(x_1 - V_1 t, x_2) & \text{if } a \leq x_1 \leq b \\ e - h_t(x_1 - V_1 t, x_2) & \text{otherwise} \end{cases}, (3.53)$$

where h_b and h_t are known functions, a and b are such that $x_{1\ell} < a < b < x_{1r}$ and e is a constant ”.

IV Review of mathematical results on the Elrod-Adams model

Since Elrod & Adams published the model in [55] the mathematical study of the problem has been considered by several authors with increasing grade of complexity. Existence and uniqueness (under some assumptions) of the steady state problem have been treated in [21, 22]. The unsteady case was studied in [53, 8] for journal and face seals boundary conditions. Proof of the existence of the solution of the transient problem for a journal bearing (that is, periodic boundary conditions in x_1) are given in [54].

The closest mathematical results to the problem stated in the previous section was given by Buscaglia et al ([33]). The purely hydrodynamic regime ($p_{con} \equiv 0$) was studied through a semi-discretization in time, while two boundary conditions in the x_2 direction were considered:

1. $p = 0$ in $\partial\Omega \setminus \Sigma$;
2. periodic in x_2 , $p = 0$ in Γ_1 ,

with a predefined flux $\eta = \eta(x_2, t)$ in Γ_0 for both cases. In this section we will recall those results, specifically the one for the piston-ring-like boundary conditions, which with minor changes make for the journal bearing or the mechanical seal cases. Prior to presenting these existence results some notation is needed.

IV.1 Notation

Definition IV.1: $L^1(\Omega)$ is the space of integrable functions from Ω to $\mathbb{R}$.

Definition IV.2: Let $p \in \mathbb{R}$ with $1 < p < \infty$; we set

$$L^p(\Omega) = \{ f : \Omega \rightarrow \mathbb{R}; f \text{ is measurable and } |f|^p \in L^1(\Omega) \}$$

equipped with the norm

$$\|f\|_{L^p} = \|f\|_p = \left[\int_{\Omega} |f(x)|^p d\Omega \right]^{\frac{1}{p}}.$$

Definition IV.3:

$$L^\infty(\Omega) = \left\{ f : \Omega \rightarrow \mathbb{R} \left| \begin{array}{l} f \text{ is measurable and there is a constant } C \\ \text{such that } |f(x)| \leq C \text{ a.e. on } \Omega \end{array} \right. \right\},$$

equipped with the norm

$$\|f\|_{L^\infty} = \|f\|_\infty = \inf C; |f(x)| \leq C \text{ a.e. on } \Omega.$$

Definition IV.4: Let V be a Banach space, $0 < T < \infty$, $1 \leq p \leq \infty$. We denote by $L^p(0, T; V)$ the set of measurable functions $f : (0, T) \rightarrow V$ such that $\|f\|_V^p$ is integrable.

Definition IV.5: Let V be a Banach space, $0 < T < \infty$. We denote by $C^k(0, T; V)$ the set of measurable functions $f : (0, T) \rightarrow V$ whose first k derivatives are continuous.

IV.2 Existence result for the moving pad in purely hydrodynamic conditions

The result of the existence of a solution of problem 3.40-3.53 is given by theorem 3.1. It must be observed that the proof of this theorem did not involve the force W^e , that is, it was proved for $p_{con}(x_1, x_2, t)=0$.

Theorem 3.1: Let an applied load $W(t)$ in $L^\infty(0, +\infty)$, a flux η in $L^\infty(0, +\infty; L^2(\Gamma_0))$, an initial condition θ_I satisfying

$$\theta_I \in L^\infty(\Omega), \tag{3.54}$$

$$0 \leq \theta_I \leq 1 \tag{3.55}$$

and two real numbers z_0, v_0 be given. V_1 is in $L^\infty(\Omega \times (0, +\infty))$. It is also assumed that functions h_b, h_t satisfy

$$h_b \in L^\infty(\Omega), \tag{3.56}$$

$$h_t \in C^1(0, +\infty; L^\infty(\Omega)) \tag{3.57}$$

and they are such that there exists h_m, h_M satisfying

$$0 < h_m \leq z_0 + h_b(x) - h_t(x_1, x_2, t) \leq h_M, \quad \forall t \geq 0 \quad (3.58)$$

then there exists a small enough positive number T such that problem 3.40-3.45 with $p_{con} = 0$ admits at least one solution $Z(t), t \in [0, T]$ with p a solution of problem 3.46-3.53.

Proof. The theorem is proven in [33]. □

Existence might not be guaranteed by Theorem 3.1 for all of the simulations run in this work. The assumptions being met in all the simulations are

$$\left\{ \begin{array}{l} W(t) \in L^\infty(0, +\infty), \end{array} \right. \quad (3.59)$$

$$\theta_I \in L^\infty(\Omega), \quad 0 \leq \theta_I \leq 1, \quad (3.60)$$

$$\eta \in L^\infty(0, +\infty, L^2(\Gamma_0)), \quad (3.61)$$

$$V_1 \in L^\infty(\Omega \times (0, +\infty)), \quad (3.62)$$

$$h_b \in L^\infty(\Omega), \quad (3.63)$$

$$\left\{ \begin{array}{l} \exists h_m, h_M \text{ such that } 0 < h_m \leq z_0 + h_b(x) - h_t(x_1, x_2, t) \leq h_M \quad \forall t \geq 0. \end{array} \right. \quad (3.64)$$

However, hypothesis 3.57 conflicts with some simulations run in this thesis. For the dimpled geometries defined in Section II of Chapter 6 we have that $h_t \in C^0(0, +\infty; L^\infty(\Omega))$ but $h_t \notin C^1(0, +\infty; L^\infty(\Omega))$. The same is true for the measured surfaces of Chapter 7. Notice that the requirement for the pad is less restrictive, as condition 3.56 allows discontinuous geometries.

Another shortcoming is the lack of existence (and uniqueness) results in the literature for problem 3.40-3.53 when a non-zero $W^c(t)$ must be considered. The applications of Section IV, Chapter 6 and Section IV, Chapter 7 both consider contact forces.

V Elrod-Adams-like models

Various types of models have been proposed to deal with the lubrication problem with cavitation. Bayada & Chupin ([27]) point out that they can be classified (roughly) into two categories:

1. The main feature of the cavitation phenomenon is the appearance of two distinct areas: the cavitated region and the full film zone, which constitutes a class of free boundary problems, and;
2. The existence of a macrobubble of cavitation is not assumed *a priori*, and it is deduced from the solution of the constitutive equations. Thus, a detailed description of the flow at a mixture level is needed.

As an example of models in the latter category we could mention the works of Tao et al ([107]) and Xing et al ([117]), where the authors resort to solving the problem with the Navier-Stokes equations and a fixed gas volume fraction (no change of phase was allowed). In this section we will focus in the first group of models, to which the Elrod-Adams model belongs.

V.1 Vijayaraghavan & Keith model

While in Elrod-Adams model [55] the fluid is treated as incompressible, the mathematical model developed by Vijayaraghavan & Keith ([113]) assumes a constant bulk modulus β :

$$\frac{\partial \rho_c h \theta}{\partial t} + \frac{\partial}{\partial x_1} \left(\frac{\rho_c h V_1}{2} \theta - \frac{\rho_c \beta h^3 g}{12\mu} \frac{\partial \theta}{\partial x} \right) + \frac{\partial}{\partial x_2} \left(-\frac{\rho_c \beta h^3 g}{12\mu} \frac{\partial \theta}{\partial x_2} \right) = 0, \quad (3.65)$$

where ρ_c is the density of the lubricant at the cavitation pressure and g is a switch function defined by:

$$g = \begin{cases} 1 & \text{in the full film region} \\ 0 & \text{in the cavitated region} \end{cases} \quad (3.66)$$

Notice two important features of this model: first, only θ needs to be solved in equation 3.65. It also has a slightly different meaning in the cavitated and in the full film region. In the former $\theta \leq 1$ it still is (as in the Elrod's model) the fractional film content, while in the latter represents the lubricant's compression, with $\theta > 1$. Second, the model requires a extra parameter compared to the Elrod's model (the bulk modulus).

Once θ has been solve, the pressure is obtained with the expression:

$$p = p_{cav} + g\beta \ln(\theta), \quad (3.67)$$

More general models, with minor modifications to this one were proposed by [99]

V.2 Kumar & Booker model

In Kumar & Booker's model ([78]) the cavitated region is treated as compressible, but with bulk modulus $\beta=0$. The notion of *biphase* is characterized by the following density and viscosity relationships:

$$0 \leq \rho \leq \rho_{liq}, \quad 0 \leq \mu \leq \mu_{liq}, \quad \text{where } \frac{\rho}{\rho_{liq}} = \frac{\mu}{\mu_{liq}}$$

The film is complete if $\rho = \rho_{liq}$ and $p \geq p_{cav}$. A incomplete film is given by the conditions $p = p_{cav}$ with $\rho \leq \rho_{liq}$. Then, p and ρ (and therefore μ) are solved from:

$$\nabla \cdot \left(\frac{\rho h^3}{12\mu} \nabla p \right) = \nabla \cdot (\rho h \mathbf{V}) + \frac{\partial \rho h}{\partial t} \quad (3.68)$$

V.3 Complementarity formulations

In the complementarity formulations the variables of the lubrication equation are redefined yielding a description in terms of a linear complementarity problem (LCP). The advantage of this is that standard techniques for LCP problems can be applied, such as the Lemke's algorithm ([48]), which allows to solve the problem in a finite number of steps, avoiding issues related to iterative processes. Several methods of doing so have been proposed over the years.

Strozzi [105] presented the pressure and the residual of the Reynolds equation as complementary variables in the cavitation problem. Bayada et al ([28]) chose p and the void fraction $1 - \theta$ from the Elrod-Adams model to introduce the strong formulation of the problem. Giacomini et al ([65]) reformulated the density ρ in equation 3.68 into:

$$r = 1 - \frac{\rho}{\rho_{liq}}$$

This with the conditions $p \geq 0$, $r \geq 0$ (from the definition). In the full film region $p > 0$ and the density equals that of the liquid ρ_{liq} , so the void fraction $r = 0$. In the cavitated region $p = 0$ and the density falls below that of the liquid thus giving $0 \leq r < 1$. It can be noticed that with this redefinition of ρ into r , at every point in the domain it is:

$$pr = 0,$$

which is the complementarity equation. Even though this change of variables does not make equation 3.68 turn into a linear problem, the authors in [65] point out that the nonlinear term:

$$\nabla \cdot \left(\frac{rh^3}{12\mu} \nabla p \right)$$

can be omitted as it is zero almost everywhere in the domain, the exception being at the interface Σ . In a different approach Almqvist et al ([6]) reached a LCP without resorting to the previous assumption. This was done by introducing a convenient expression for the density as a function of pressure:

$$\rho(p) = \rho_c \begin{cases} e^{(p-p_c)/\beta}, & \text{if } p > p_c \\ \delta, & \text{if } p = p_c \end{cases}$$

which in equation 3.68 leaves a linear complementarity problem in the variables $\eta = 1 - \delta$ and $u = e^{(p-p_c)/\beta}$. As $\beta \rightarrow \infty$ this model can be used to obtain the solution of the one proposed by Giacomini et al ([65]).

V.4 Other models

There is plenty in the literature on models discussing the lubrication problem with cavitation. More complex models usually have to recourse to additional physical parameters as input data. Such is for example the case of [19], where a constant subcavitation pressure is admitted to take place. For this to happen at least two more parameters are required: the subcavitation pressure and density.

VI Revisiting the Elrod-Adams model

As presented previously in this Chapter, the volumetric flux in the Elrod-Adams model is given by:

$$\mathbf{J} = -\frac{h^3}{12\mu} \nabla p + aV_1 h \theta \hat{e}_1, \quad (3.69)$$

where $a = 1/2$, which holds for both the pressurized and the cavitated regions. This is true in the parts of the domain where $\theta = 1$ or if the fluid in the cavitated areas form *streamers* ([58]). Otherwise, the transport velocity for the fluid film is just half of the real physical value in some situations. In the piston ring/liner problem if the fluid in the cavitated region is adhered to the liner, then the correct value is $a = 1$.

In that case, for a start we could simply redefine the flux as:

$$\mathbf{J} = -\frac{h^3}{12\mu} \nabla p + \frac{\beta(x_1, x_2, t)}{2} h \theta \hat{e}_1, \quad (3.70)$$

where:

$$\beta(x_1, x_2, t) = \begin{cases} 2V_1 & \text{if } \theta(x_1, x_2, t) < 1 \\ V_1 & \text{if } \theta(x_1, x_2, t) = 1 \end{cases} \quad (3.71)$$

The differences between the two models can be appreciated in a simple example with a fixed ring position $Z(t)$, no texture on the runner (the problem becoming then one-dimensional) and constant velocity $V_1 > 0$. This is shown in figure 3.3 for four time steps. The film thickness values $h\theta$ are depicted for both the Elrod-Adams model and the modification introduced by equation 3.71.

Due to the higher transport velocity the quantity of oil amassed at the left of the ring is larger than with the unmodified Elrod-Adams model. A notorious change is also seen at the right cavitation boundary: $\theta(x, t)$ now is discontinuous, being its value at the cavitated region half it is immediately to the left. This ensures that this implementation of the modified Elrod-Adams is mass-conserving.

Figure 3.3: A ring sliding with constant velocity over a untextured runner. Film thickness $h\theta$ is shown for both the E-A model and the modification introduced by equation 3.71: (a) $t=0.0004$, (b) $t=0.0032$, (c) $t=0.0064$ and (d) $t=0.0096$.

Spurious oscillations are present in the first time steps (figs. 3.3(a), 3.3(b) and 3.3(c)). If the velocity V_1 is changed afterwards, they reappear as they are seen in fig. 3.3(a). This kind of behavior suggests loss of uniqueness in the solution.

VI.1 A new lubrication model with cavitation

Recently, a new cavitation model [37] that considers the proper transport velocity was proposed, and applied to the piston ring/liner problem, but its numerical implementation and other details must be studied:

- In the one-dimensional case a front-tracking method was already proposed. The number of problems that can actually be treated with front-tracking methods is limited.
- The extension for the bidimensional case is not trivial.

Solving this problem will be the central subject of Chapter 8.

VII Lubrication approximation of the hydrodynamic friction force

The total force exerted by an incompressible Newtonian fluid on a surface S is given by

$$\mathbf{F} = \int_S \boldsymbol{\sigma} \cdot \hat{\mathbf{n}} \, dS = \int_S [-p \mathbb{I} + \mu (\nabla \mathbf{v} + \nabla \mathbf{v}^T)] \cdot \hat{\mathbf{n}} \, dS. \quad (3.72)$$

The component of $\mathbf{F}$ along the direction of motion (i.e., the first component) is the *friction force* F . Computing F on the lower surface, since the normal is given by

$$\hat{\mathbf{n}} = \frac{(-\partial h_t / \partial x_1, -\partial h_t / \partial x_2, 1)}{\sqrt{1 + (\partial h_t / \partial x_1)^2 + (\partial h_t / \partial x_2)^2}}, \quad (3.73)$$

one gets

$$F_L = \int_{\Omega} \left[-\frac{\partial h_t}{\partial x_1} \left(-p + 2\mu \frac{\partial v_1}{\partial x_1} \right) - \frac{\partial h_t}{\partial x_2} \mu \left(\frac{\partial v_1}{\partial x_2} + \frac{\partial v_2}{\partial x_1} \right) + \mu \left(\frac{\partial v_1}{\partial x_3} + \frac{\partial v_3}{\partial x_1} \right) \right] dx_1 \, dx_2. \quad (3.74)$$

If we now consider the change of variables $z = x_3/\epsilon$ and the expansions (3.3) of

section I, substitution into (3.74) produces

$$F_L = \int_{\Omega} \left[\epsilon^{-1} \left(p^{(0)} \frac{\partial h_t}{\partial x_1} + \mu \frac{\partial v_1^{(0)}}{\partial z} \right) + \epsilon^0 \left(p^{(1)} \frac{\partial h_t}{\partial x_1} + \mu \frac{\partial v_1^{(1)}}{\partial z} \right) + \epsilon(\dots) + \dots \right] dx_1 dx_2. \quad (3.75)$$

Knowing that $v_1^{(0)}$ satisfies equation (3.9), and going back to the original variable x_3 , equation (3.75) becomes

$$F_L = \int_{\Omega} \left[\left(-\frac{\mu V_1}{h} - \frac{1}{2} h \frac{\partial p^{(0)}}{\partial x_1} + p^{(0)} \frac{\partial h_t}{\partial x_1} \right) + \epsilon \left(p^{(1)} \frac{\partial h_t}{\partial x_1} + \mu \frac{\partial v_1^{(1)}}{\partial x_3} \right) + \epsilon^2(\dots) + \dots \right] dx_1 dx_2. \quad (3.76)$$

The leading order asymptotic approximation of the friction force as computed on the lower surface is thus given by

$$F_L(t) \simeq - \int_{\Omega} \left(\frac{\mu V_1}{h} + \frac{1}{2} h \frac{\partial p}{\partial x_1} - p \frac{\partial h_t}{\partial x_1} \right) dx_1 dx_2. \quad (3.77)$$

Similarly, for the upper surface,

$$F_U(t) \simeq - \int_{\Omega} \left(-\frac{\mu V_1}{h} + \frac{1}{2} h \frac{\partial p}{\partial x_1} + p \frac{\partial h_b}{\partial x_1} \right) dx_1 dx_2. \quad (3.78)$$

The previous approximations, having started from (3.72) and thus considering a single-phase incompressible fluid between the surfaces, do not consider cavitation effects. Friction models in cavitated areas are controversial. We adopt here a slight modification of the formulas above, in which the first term is multiplied by a heuristic factor $g(\theta)$, which in this work we take as

$$g(\theta) = \begin{cases} \theta & \text{if } \theta > \theta_s \\ 0 & \text{otherwise.} \end{cases} \quad (3.79)$$

The parameter θ_s is a threshold for the onset of friction, which can be interpreted as the minimum oil fraction for shear forces to be transmitted from one surface to the other. The other two terms are left unmodified, and thus are equal to zero in the cavitated region. The

final heuristically modified model arising from the asymptotic expansion reads

$$F_L(t) = - \int_{\Omega} \left(\mu \frac{V_1 g(\theta)}{h} + \frac{1}{2} h \frac{\partial p}{\partial x_1} - p \frac{\partial h_t}{\partial x_1} \right) dx_1 dx_2, \quad (3.80a)$$

$$F_U(t) = - \int_{\Omega} \left(-\mu \frac{V_1 g(\theta)}{h} + \frac{1}{2} h \frac{\partial p}{\partial x_1} + p \frac{\partial h_b}{\partial x_1} \right) dx_1 dx_2. \quad (3.80b)$$

Or in terms of the non-dimensional variables:

$$F_L(t) = - \int_{\Omega} \left(\mu \frac{V_1 g(\theta)}{h} + 3h \frac{\partial p}{\partial x_1} - 6p \frac{\partial h_t}{\partial x_1} \right) dx_1 dx_2, \quad (3.81a)$$

$$F_U(t) = - \int_{\Omega} \left(-\mu \frac{V_1 g(\theta)}{h} + 3h \frac{\partial p}{\partial x_1} + 6p \frac{\partial h_b}{\partial x_1} \right) dx_1 dx_2. \quad (3.81b)$$

Three terms are identified in the previous equations. For the lower surface they are the Couette term

$$F_L^{cou}(t) = - \int_{\Omega} \left(\mu \frac{V g(\theta)}{h} \right) dx_1 dx_2, \quad (3.82)$$

the Poiseuille term

$$F_L^{poi}(t) = - \int_{\Omega} \left(\frac{1}{2} h \frac{\partial p}{\partial x_1} \right) dx_1 dx_2, \quad (3.83)$$

and the pressure term

$$F_L^{pre}(t) = \int_{\Omega} \left(p \frac{\partial h_t}{\partial x_1} \right) dx_1 dx_2. \quad (3.84)$$

Some elucidation might be necessary for the pressure term of the friction force $p \frac{\partial h_t}{\partial x_1}$. In the first place it is not a shear force. Instead, it corresponds to the projection of the pressure force along x_1 when the normal to the runner is not along x_3 . For example, if the runner is moving to the right $\frac{\partial h_t}{\partial x_1} < 0$ corresponds to a converging wedge which will produce a pressure force with a negative x_1 component, thus opposing the motion. Similarly, $\frac{\partial h_t}{\partial x_1} > 0$ corresponds to a diverging wedge which will produce a pressure force with a positive x_1 component, thus assisting the motion. Since in general the pressure in converging wedges is higher than in diverging ones, this term is always negative and thus *always contributes positively to the friction*. Notice that this term is omitted in the literature, which mostly considers textures on the stationary surface.

VII.1 Friction force model for the mixed lubrication regime

If we include the friction force due to the contact of asperities (section II.2), then we have

$$F_L(t) = -\frac{1}{B} \int_a^b \int_0^B \left(\frac{\mu V_1 g(\theta)}{h} + \frac{1}{2} h \frac{\partial p}{\partial x_1} - p \frac{\partial h_t}{\partial x_1} + \mu_c p_{con} \right) dx_1 dx_2, \quad (3.85a)$$

$$F_U(t) = -\frac{1}{B} \int_a^b \int_0^B \left(-\mu \frac{V_1 g(\theta)}{h} + \frac{1}{2} h \frac{\partial p}{\partial x_1} + p \frac{\partial h_b}{\partial x_1} - \mu_c p_{con} \right) dx_1 dx_2, \quad (3.85b)$$

where μ_c is the boundary friction coefficient, that is, the "dry" friction coefficient, as if the surfaces in contact where not lubricated.

It is to be noticed that the previous equations satisfy that the forces on the upper and lower surfaces add up to zero. This can be verified as follows:

$$F_L(t) + F_U(t) = - \int_{\Omega^+} \left(h \frac{\partial p}{\partial x_1} - p \frac{\partial h_t}{\partial x_1} + p \frac{\partial h_b}{\partial x_1} \right) d\Omega \quad (3.86)$$

$$= - \int_{\Omega^+} \left(h \frac{\partial p}{\partial x_1} + p \frac{\partial (h_b - h_t)}{\partial x_1} \right) d\Omega \quad (3.87)$$

$$= - \int_{\Omega^+} \left(h \frac{\partial p}{\partial x_1} + p \frac{\partial h}{\partial x_1} \right) d\Omega \quad (3.88)$$

$$= - \int_{\Omega^+} \frac{\partial (ph)}{\partial x_1} d\Omega = - \int_{\Sigma} ph \mathbf{\check{n}} \cdot \mathbf{\check{e}}_1 d\Gamma = 0. \quad (3.89)$$

The instantaneous friction coefficient for the pad then results from

$$f(t) = \frac{-F(t)}{W(t)}. \quad (3.90)$$

If we use the non-dimensionalizations of section III and the ones from the following table:

Quantity	Scale	Name
F	$\frac{\mu UL}{H}$	friction force per unit width

then, in non-dimensional variables equation 3.85a becomes:

$$F_L(t) = -\frac{1}{B} \int_a^b \int_0^B \left(\frac{\mu V_1 g(\theta)}{h} + 3h \frac{\partial p}{\partial x_1} - 6p \frac{\partial h_t}{\partial x_1} + \frac{6L}{H} \mu_c p_{con} \right) dx_1 dx_2 . \quad (3.91)$$

Transient mass-conserving finite volume method

In this chapter we present the numerical methods to solve the lubrication problem with cavitation and pad dynamics for our non-multigrid algorithms. Multigrid methods and algorithms will be introduced in the next chapter.

First a fixed point problem is stated for the finite volume discretization of the Elrod-Adams equation. Initially, the pad dynamics are disregarded and two algorithms are proposed in order to solve that simplified problem. Afterwards, an algorithm to solve the coupled system is presented.

I Finite Volume discretization

The rectangle $\Omega = [x_{1\ell}, x_{1r}] \times [0, B]$ is divided into finite volumes in a uniform grid composed of $N_1 \times N_2$ nodes. The constant distance between contiguous nodes is $\Delta x_1 = (x_{1r} - x_{1\ell}) / (N_1 - 1)$ in x_1 and $\Delta x_2 = B / (N_2 - 1)$ in x_2 . Then, the non-dimensional flux:

$$\mathbf{J} = \frac{V_1}{2} h \theta \hat{e}_1 - \frac{h^3}{2} \nabla p, \quad (4.1)$$

in each facet of a finite volume must balance the variation of oil inside of it:

Figure 4.1: Finite volume mesh.

$$\left(\frac{h_{i,j}^{n+1} \theta_{i,j}^{n+1} - h_{i,j}^n \theta_{i,j}^n}{\Delta t} \right) \Delta x_1 \Delta x_2 =$$

$$(f_{i-1/2,j} - f_{i+1/2,j}) \Delta x_2 + (f_{i,j-1/2} - f_{i,j+1/2}) \Delta x_1, \quad (4.2)$$

Here the time variable has been discretized into uniform time steps of length Δt , with a first order approximation for the derivative $\partial_t(h\theta)$. In turn, the Poiseuille term of the flux is treated with centered approximations, while the Couette term is treated with one-sided approximations depending of the sign of the transport velocity, being

$$f_{i-1/2,j} \simeq \frac{V_1^{n+1}}{2} h_{i-1,j}^{n+1} \theta_{i-1,j}^{n+1} - \left(\frac{(h_{i-1,j}^{n+1})^3 + (h_{i,j}^{n+1})^3}{4} \right) \left(\frac{p_{i,j}^{n+1} - p_{i-1,j}^{n+1}}{\Delta x_1} \right), \quad (4.3)$$

$$f_{i+1/2,j} \simeq \frac{V_1^{n+1}}{2} h_{i,j}^{n+1} \theta_{i,j}^{n+1} - \left(\frac{(h_{i+1,j}^{n+1})^3 + (h_{i,j}^{n+1})^3}{4} \right) \left(\frac{p_{i+1,j}^{n+1} - p_{i,j}^{n+1}}{\Delta x_1} \right), \quad (4.4)$$

$$f_{i,j-1/2} \simeq - \left(\frac{(h_{i,j}^{n+1})^3 + (h_{i,j-1}^{n+1})^3}{4} \right) \left(\frac{p_{i,j}^{n+1} - p_{i,j-1}^{n+1}}{\Delta x_2} \right), \quad (4.5)$$

$$f_{i,j+1/2} \simeq - \left(\frac{(h_{i,j+1}^{n+1})^3 + (h_{i,j}^{n+1})^3}{4} \right) \left(\frac{p_{i,j+1}^{n+1} - p_{i,j}^{n+1}}{\Delta x_2} \right), \quad (4.6)$$

for $V_1 \geq 0$ and

$$\begin{cases} f_{i-1/2,j} \simeq \frac{V_1^{n+1}}{2} h_{i,j}^{n+1} \theta_{i,j}^{n+1} - \left(\frac{(h_{i-1,j}^{n+1})^3 + h_{i,j}^3}{4} \right) \left(\frac{p_{i,j}^{n+1} - p_{i-1,j}^{n+1}}{\Delta x_1} \right), \\ f_{i+1/2,j} \simeq \frac{V_1^{n+1}}{2} h_{i+1,j}^{n+1} \theta_{i+1,j}^{n+1} - \left(\frac{(h_{i+1,j}^{n+1})^3 + (h_{i,j}^{n+1})^3}{4} \right) \left(\frac{p_{i+1,j}^{n+1} - p_{i,j}^{n+1}}{\Delta x_1} \right), \end{cases} \quad (4.7)$$

for $V_1 < 0$. All fluxes normal to the facets of each finite volume are taken implicitly to avoid stability issues and to allow bigger time steps. Putting equations 4.2 to 4.6 together we have:

$$\begin{aligned} q_2 h_{i,j}^{n+1} \theta_{i,j}^{n+1} + V_1^{n+1} \Delta x_1 (h_{i,j}^{n+1} \theta_{i,j}^{n+1} - h_{i-1,j}^{n+1} \theta_{i-1,j}^{n+1}) + \\ (s_{i-1,j} + s_{i+1,j} + q_1^2 (s_{i,j-1} + s_{i,j+1})) p_{i,j}^{n+1} - s_{i-1,j} p_{i-1,j}^{n+1} - s_{i+1,j} p_{i+1,j}^{n+1} - \\ q_1^2 (s_{i,j-1} p_{i,j-1}^{n+1} + s_{i,j+1} p_{i,j+1}^{n+1}) = q_2 h_{i,j}^n \theta_{i,j}^n. \end{aligned} \quad (4.9)$$

This is true for $V_1 \geq 0$, while for $V_1 < 0$ the term $h_{i,j}^{n+1} \theta_{i,j}^{n+1} - h_{i-1,j}^{n+1} \theta_{i-1,j}^{n+1}$ must be replaced by $h_{i+1,j}^{n+1} \theta_{i+1,j}^{n+1} - h_{i,j}^{n+1} \theta_{i,j}^{n+1}$ from equations 4.7-4.8. Here $s_{i+1,j}$, q_1 , q_2 are characterized as:

$$s_{i+1,j} = \frac{(h_{i+1,j}^{n+1})^3 + (h_{i,j}^{n+1})^3}{2}, \quad q_1 = \frac{\Delta x_1}{\Delta x_2}, \quad q_2 = 2 \frac{\Delta x_1^2}{\Delta t},$$

and $s_{i-1,j}$, $s_{i,j-1}$, $s_{i,j+1}$ are defined analogously. The variables θ and p in equation 4.9 must also satisfy the complementarity conditions at every cell (i, j) :

$$\begin{cases} p_{i,j}^{n+1} \geq 0, 0 \leq \theta_{i,j}^{n+1} \leq 1, \\ p_{i,j}^{n+1} > 0 \Leftrightarrow \theta_{i,j}^{n+1} = 1. \end{cases} \quad (4.10)$$

I.1 Boundary conditions

Boundary conditions are set at $i = 1, j = 1, \dots, N_2$ and $i = N_1, j = 1, \dots, N_2$ by imposing pressure p , a flux η (as presented in Section III of the previous Chapter) or the saturation θ . For the piston ring arrangement a constant thickness of oil d_{oil} is set above the lower surface at both the leftmost ($d_{oil,\ell}$) and rightmost ($d_{oil,r}$) parts of the domain:

$$\begin{cases} p_{1,j} = p_\ell, & p_{N_1,j} = p_r & \forall j = 1, \dots, N_2, \\ \theta_{1,j} = \frac{d_{oil,\ell}}{h_{1,j}}, & \theta_{N_1,j} = \frac{d_{oil,r}}{h_{N_1,j}} & \forall j = 1, \dots, N_2. \end{cases} \quad (4.11)$$

$$\begin{cases} p_{1,j} = p_\ell, & p_{N_1,j} = p_r & \forall j = 1, \dots, N_2, \\ \theta_{1,j} = \frac{d_{oil,\ell}}{h_{1,j}}, & \theta_{N_1,j} = \frac{d_{oil,r}}{h_{N_1,j}} & \forall j = 1, \dots, N_2. \end{cases} \quad (4.12)$$

The complementarity conditions must be satisfied, and so $p_\ell = p_r = 0$ as $\theta_{1,j}, \theta_{N_1,j} < 1$. Depending on the sign of V_1 , outflow boundary conditions can also be used at the cells with $i = 1$ or $i = N_1$. Depending on the symmetry of the problem, two different type of conditions are enforced along the lower and upper borders of the domain. If there is periodicity in the x_2 direction, then:

$$\begin{cases} p_{i,1} = p_{i,N_2-1}, & p_{i,N_2} = p_{i,2} & \forall i = 2, \dots, N_1 - 1, \\ \theta_{i,1} = \theta_{i,N_2-1}, & \theta_{i,N_2} = \theta_{i,2} & \forall i = 2, \dots, N_1 - 1. \end{cases} \quad (4.13)$$

$$\begin{cases} p_{i,1} = p_{i,N_2-1}, & p_{i,N_2} = p_{i,2} & \forall i = 2, \dots, N_1 - 1, \\ \theta_{i,1} = \theta_{i,N_2-1}, & \theta_{i,N_2} = \theta_{i,2} & \forall i = 2, \dots, N_1 - 1. \end{cases} \quad (4.14)$$

On the other hand, if the symmetry is a mirror-like reflection at both the lower and upper borders, then the boundary conditions in the discrete case are:

$$\begin{cases} p_{i,1} = p_{i,3}, & p_{i,N_2} = p_{i,N_2-2} & \forall i = 2, \dots, N_1 - 1, \\ \theta_{i,1} = \theta_{i,3}, & \theta_{i,N_2} = \theta_{i,N_2-2} & \forall i = 2, \dots, N_1 - 1. \end{cases} \quad (4.15)$$

$$\begin{cases} p_{i,1} = p_{i,3}, & p_{i,N_2} = p_{i,N_2-2} & \forall i = 2, \dots, N_1 - 1, \\ \theta_{i,1} = \theta_{i,3}, & \theta_{i,N_2} = \theta_{i,N_2-2} & \forall i = 2, \dots, N_1 - 1. \end{cases} \quad (4.16)$$

I.2 Friction losses

Equation 3.91 is discretized according to:

$$\begin{aligned} F_L(t) \simeq & -\frac{1}{B} \sum_{i \in [a,b]} \sum_{j \in [0,B]} \left(\frac{\mu V_1 g(\theta_{i,j})}{h_{i,j}} + 3 \left(\frac{h_{i+1,j} + h_{i,j}}{2} \right) \left(\frac{p_{i+1,j} - p_{i,j}}{\Delta x_1} \right) - \right. \\ & \left. 6 \left(\frac{p_{i+1,j} + p_{i,j}}{2} \right) \left(\frac{h_{t_{i+1,j}} - h_{t_{i,j}}}{\Delta x_1} \right) + \mu_c(p_{con})_{i,j} \right) \Delta x_1 \Delta x_2 \quad (4.17) \end{aligned}$$

Similarly, for the force in the upper surface:

$$F_U(t) \simeq -\frac{1}{B} \sum_{i \in [a,b]} \sum_{j \in [0,B]} \left(-\frac{\mu V_1 g(\theta_{i,j})}{h_{i,j}} + 3 \left(\frac{h_{i+1,j} + h_{i,j}}{2} \right) \left(\frac{p_{i+1,j} - p_{i,j}}{\Delta x_1} \right) + \right. \\ \left. 6 \left(\frac{p_{i+1,j} + p_{i,j}}{2} \right) \left(\frac{h_{b_{i+1,j}} - h_{b_{i,j}}}{\Delta x_1} \right) - \mu_c(p_{con})_{i,j} \right) \Delta x_1 \Delta x_2 . \quad (4.18)$$

It is to be observed that this particular discretization ensures the balance $F_L + F_U = 0$ at the discrete level.

II The fixed point problem for the non-moving pad

Equation 4.9 can be rearranged as

$$a_{i,j} \theta_{i,j} + b_{i,j} p_{i,j} - c_{i,j}(p, \theta) = l_{i,j} , \quad (4.19)$$

where

$$a_{i,j} = \begin{cases} (q_2 + V_1 \Delta x_1) h_{i,j}^{n+1} & \text{if } V_1 \geq 0 , \\ (q_2 - V_1 \Delta x_1) h_{i,j}^{n+1} & \text{if } V_1 < 0 , \end{cases} \quad (4.20)$$

$$b_{i,j} = s_{i-1,j} + s_{i+1,j} + q_1^2 (s_{i,j-1} + s_{i,j+1}) , \quad (4.21)$$

$$c_{i,j}(p, \theta) = \begin{cases} s_{i-1,j} p_{i-1,j}^{n+1} + s_{i+1,j} p_{i+1,j}^{n+1} + q_1^2 (s_{i,j-1} p_{i,j-1}^{n+1} + s_{i,j+1} p_{i,j+1}^{n+1}) + V_1 \Delta x_1 h_{i-1,j}^{n+1} \theta_{i-1,j}^{n+1} & \text{if } V_1 \geq 0 , \\ s_{i-1,j} p_{i-1,j}^{n+1} + s_{i+1,j} p_{i+1,j}^{n+1} + q_1^2 (s_{i,j-1} p_{i,j-1}^{n+1} + s_{i,j+1} p_{i,j+1}^{n+1}) - V_1 \Delta x_1 h_{i+1,j}^{n+1} \theta_{i+1,j}^{n+1} & \text{if } V_1 < 0 , \end{cases} \quad (4.22)$$

and

$$l_{i,j} = q_2 h_{i,j}^n \theta_{i,j}^n . \quad (4.23)$$

We can introduce at this point a new variable ω :

$$\omega_{i,j} = p_{i,j} + \theta_{i,j} \geq 0 . \quad (4.24)$$

It is easy to verify that $p_{i,j}$ and $\theta_{i,j}$ can be recovered uniquely from $\omega_{i,j} \geq 0$ with:

$$p(\omega_{i,j}) = \max\{\omega_{i,j} - 1, 0\}, \quad \theta(\omega_{i,j}) = \min\{\omega_{i,j}, 1\}. \quad (4.25)$$

Given a certain ordering of the cell centers (i, j) , for instance $i' = i + (j - 1)N_1$, we can write then in vector form:

$$\boldsymbol{\omega} = \mathbf{p} + \boldsymbol{\theta}, \quad \boldsymbol{\omega} \in \mathbb{R}^{N=N_1 \times N_2}, \quad (4.26)$$

where $\omega_{i'} = \omega_{i,j}$. At this point we can drop the prime superindexes on i' .

All the points i in the domain belong to a set that we will call I . The points in the Dirichlet part of the boundary belong to $I_1 \neq \{\emptyset\}$ and those in the reflection/periodic boundaries are in $I_2 \neq \{\emptyset\}$, both subsets of I such that $I_1 \cap I_2 = \{\emptyset\}$.

Thus for all points $i \in I \setminus I_1 \setminus I_2$

$$\mathbf{a}_i \boldsymbol{\theta}_i + \mathbf{b}_i \mathbf{p}_i - \mathbf{c}_i(\boldsymbol{\omega}) = \mathbf{l}_i, \quad (4.27)$$

must be satisfied, while conditions

$$\omega_i = \bar{\omega}_i = p_{\ell/r} + \frac{d_{oil,\ell/r}}{h_{\{1,N_1\},j}}, \quad (4.28)$$

must be enforced for all $i \in I_1$. Subindexes ℓ and r have the meaning of Subsection I.1. For points $i \in I_2$ we define an injective operator:

$$\mathcal{I} : I_2 \longrightarrow I \setminus I_2, \quad (4.29)$$

that associates to every i in I_2 the corresponding j in $I \setminus I_2$ such that $j = \mathcal{I}(i)$ is the corresponding cell in the reflection/periodic boundary conditions, as pointed out in Subsection I.1.

We can introduce now the operators

$$\mathcal{R}_1(\omega_i) = \begin{cases} \frac{\mathbf{l}_i + \mathbf{c}_i - \mathbf{a}_i}{\mathbf{b}_i} + 1 & \text{if } \omega_i \geq 1 \text{ and } \mathbf{l}_i + \mathbf{c}_i - \mathbf{a}_i \geq 0, \\ \theta_i(\omega_i) & \text{if } \omega_i \geq 1 \text{ and } \mathbf{l}_i + \mathbf{c}_i - \mathbf{a}_i < 0, \end{cases} \quad (4.30)$$

and

$$\mathcal{R}_2(\omega_i) = \begin{cases} \frac{\mathbf{l}_i + \mathbf{c}_i}{\mathbf{a}_i} & \text{if } \omega_i < 1 \text{ and } \frac{\mathbf{l}_i + \mathbf{c}_i}{\mathbf{a}_i} < 1, \\ 1 & \text{if } \omega_i < 1 \text{ and } \frac{\mathbf{l}_i + \mathbf{c}_i}{\mathbf{a}_i} \geq 1. \end{cases} \quad (4.31)$$

Then we define

$$\mathcal{R} = \mathcal{R}_2 \circ \mathcal{R}_1 . \quad (4.32)$$

So the fixed point problem to be solved is:

Problem 1: “Find $\omega \in \mathbb{R}_+^N := \{\mathbf{v} \in \mathbb{R}^N | \mathbf{v}_i \geq 0 \ \forall i \in I\}$ such that $\omega = \mathbf{A}(\omega)$ where $\mathbf{A} : \mathbb{R}_+^N \longrightarrow \mathbb{R}_+^N$ is defined by:

$$(\mathbf{A}(\omega))_i = \begin{cases} \mathcal{R}(\omega_i) & i \in I \setminus I_1 \setminus I_2 \\ \bar{\omega}_i & i \in I_1 \\ \omega_{\mathcal{I}(i)} & i \in I_2 \end{cases} , \quad (4.33)$$

where $\mathcal{R}$ is defined by equation 4.32, $\bar{\omega}$ is defined by 4.28 and the operator $\mathcal{I}$ by equation 4.29”.

Remark: $\mathbf{A}$ is indeed an operator from $\mathbb{R}_+^N$ to $\mathbb{R}_+^N$. It is easy to show from equations 4.19 to 4.23 that $\mathbf{a}_i > 0$, $\mathbf{b}_i > 0$, $\mathbf{c}_i \geq 0$ and also $\mathbf{l}_i \geq 0$. Then

1. If $i \in I_1$ it is $(\mathbf{A}(\omega))_i = \bar{\omega}_i > 0$;

2. If $i \in I \setminus I_1 \setminus I_2$ and:

(a) $0 \leq \omega_i < 1$, then $(\mathbf{A}(\omega))_i = \mathcal{R}_2 \circ \mathcal{R}_1(\omega_i) = \mathcal{R}_2(\omega_i)$, then as

$$\frac{\mathbf{l}_i + \mathbf{c}_i}{\mathbf{a}_i} \geq 0 ,$$

$\mathcal{R}_2(\omega_i) \geq 0$ is guaranteed.

(b) $\omega_i \geq 1$: the only case of concern is when $\mathbf{l}_i + \mathbf{c}_i - \mathbf{a}_i < 0$. But then $(\mathbf{A}(\omega))_i = \mathcal{R}_2 \circ \mathcal{R}_1(\omega_i) = \mathcal{R}_2(0 \leq \theta_i(\omega_i) < 1) \geq 0$ in any case.

3. If $i \in I_2$ then $(\mathbf{A}(\omega))_i \geq 0$ as a consequence of the previous item.

Therefore $(\mathbf{A}(\omega))_i \geq 0$ for all i in I .

II.1 A Jacobi algorithm

If we apply the operator defined in equation 4.33 iteratively with a initial guess ω^0 we have a Jacobi-type algorithm:

$$\omega^{k+1} = \mathbf{A}(\omega^k) , \quad (4.34)$$

for $k = 0, 1, \dots$. The stopping criterion for the algorithm is whether the norm of the residue

$$\mathbf{r}_i^k = \mathbf{l}_i - \mathbf{a}_i \theta(\boldsymbol{\omega}_i^k) - \mathbf{b}_i p(\boldsymbol{\omega}_i^k) + \mathbf{c}_i(\boldsymbol{\omega}^k), \quad (4.35)$$

is smaller than a certain tolerance:

$$\|\mathbf{r}^k\| \leq \epsilon_r. \quad (4.36)$$

This is a way to characterize convergence based on mass-imbalance at the cell level. Another stopping criterion can be the difference between two consecutive solutions:

$$\|\boldsymbol{\omega}^{k+1} - \boldsymbol{\omega}^k\| \leq \epsilon_r. \quad (4.37)$$

II.2 A Gauss-Seidel algorithm

We define now an operator

$$\mathbf{B} : \mathbb{R}_+^N \longrightarrow \mathbb{R}_+^N, \quad \boldsymbol{\omega} \rightarrow \tilde{\boldsymbol{\omega}} = \mathbf{B}(\boldsymbol{\omega}), \quad (4.38)$$

$$\tilde{\omega}_i = (\mathbf{B}(\boldsymbol{\omega}))_i := (\mathbf{A}(\mathbf{z}^{(i)}))_i \quad \forall i \in I, \quad (4.39)$$

where $\mathbf{z}^{(i)} \in \mathbb{R}_+^N$ is given by

$$\mathbf{z}^{(1)} = \boldsymbol{\omega}; \quad \text{for } i > 1, \quad z_j^{(i)} = \begin{cases} \tilde{\omega}_j, & j < i, \\ \tilde{\omega}_j, & j \geq i, \end{cases} \quad \forall j \in I. \quad (4.40)$$

Then

$$\boldsymbol{\omega}^{k+1} = \mathbf{B}(\boldsymbol{\omega}^k), \quad (4.41)$$

constitutes a Gauss-Seidel algorithm to solve Problem 1. These two algorithms have the advantage of being a matrix-free approach to solve potentially very large problems, and that allow to enforce the complementarity conditions a node at a time through the operator $\mathcal{R}$.

III Introducing the pad movement: Newmark marching scheme for the pad dynamic's

The Newmark method was originally introduced for the solution of second order differential equations arising from dynamic equilibrium in structural dynamics. Here it is assumed that:

$$\frac{dZ^{n+1}}{dt} = \frac{dZ^n}{dt} + \Delta t \left((1 - \delta_1) \frac{d^2 Z^n}{dt^2} + \delta_1 \frac{d^2 Z^{n+1}}{dt^2} \right), \quad (4.42)$$

$$Z^{n+1} = Z^n + \Delta t \frac{dZ^n}{dt} + \Delta t^2 \left(\left(\frac{1}{2} - \delta_2 \right) \frac{d^2 Z^n}{dt^2} + \delta_2 \frac{d^2 Z^{n+1}}{dt^2} \right), \quad (4.43)$$

where $Z^{n+1} = Z(t_{n+1})$ and δ_1, δ_2 are parameters that can be determined to obtain integration accuracy and stability. When $\delta_1 = 1/2$ and $\delta_2 = 1/6$, relations 4.42-4.43 correspond to the linear acceleration method, while it coincides with the trapezoidal rule for $\delta_1 = 1/2$ and $\delta_2 = 1/4$ ([18]).

This method is unconditionally stable as long as

$$\begin{cases} \delta_1 \geq \frac{1}{2} \\ \delta_2 \geq \frac{1}{4}(\frac{1}{2} + \delta_1)^2 \end{cases}$$

Here we choose $\delta_1 = 1$, $\delta_2 = \frac{1}{2}$ and with this, equations 4.42 and 4.43 yield:

$$Z^{n+1} = Z^n + \Delta t \frac{dZ^n}{dt} + \frac{\Delta t^2}{2} \frac{d^2 Z^{n+1}}{dt^2}, \quad (4.44)$$

$$\frac{dZ^{n+1}}{dt} = \frac{dZ^n}{dt} + \Delta t \frac{d^2 Z^{n+1}}{dt^2}. \quad (4.45)$$

However, recalling equation 3.40 such a choice would imply knowing

$$\frac{d^2 Z^{n+1}}{dt^2} = \frac{1}{m} \left(W^{n+1} + (W^h)^{n+1} + (W^c)^{n+1} \right),$$

which is not a problem with W^{n+1} as it can be recalled from Chapter 2 that it is a known function of time. However, $(W^h)^{n+1}$ and $(W^c)^{n+1}$ are functions of $h(\mathbf{x}_i)^{n+1}$, which in time is function of Z^{n+1} . Therefore:

$$Z^{n+1} = Z^n + \Delta t \frac{dZ^n}{dt} + \frac{\Delta t^2}{2m} ((W)^{n+1} + W^h(Z^{n+1}) + W^c(Z^{n+1})) . \quad (4.46)$$

Let it be $I_3 \subset I$ the set of points i of the domain under the pad, for example the ones in the rectangle $[a, b] \times [0, B]$ of equations 3.43 and 3.44. Then, we can state at this point the following fixed point problem:

Problem 2: “Given $Z^n, \frac{dZ^n}{dt}$ and $(W)^{n+1}$ in $\mathbb{R}$, find $Z^{n+1} \in \mathbb{R}$ such that

$$Z^{n+1} = Z^n + \Delta t \frac{dZ^n}{dt} + \frac{\Delta t^2}{2m} ((W)^{n+1} + (W^h(Z^{n+1}) + (W^c(Z^{n+1}))) , \quad (4.47)$$

where

$$W^h(Z^{n+1}) = \int_{i \in I_3} p(\omega_i(Z^{n+1})) d\Omega , \quad (4.48)$$

$$W^c(Z^{n+1}) = \int_{i \in I_3} p_{con}(Z^{n+1}) d\Omega , \quad (4.49)$$

and ω is the solution of Problem 1”.

This problem can be solved taking successive approximations:

$$(Z^{n+1})^{q+1} = Z^n + \Delta t \frac{dZ^n}{dt} + \frac{\Delta t^2}{2m} ((W)^{n+1} + W^h((Z^{n+1})^q) + W^c((Z^{n+1})^q)) \quad (4.50)$$

where $q = 0, 1, 2, \dots$, starting from $(Z^{n+1})^0 = Z^n$. This is done until a defined tolerance on

$$|(Z^{n+1})^{q+1} - (Z^{n+1})^q| \leq \epsilon_Z , \quad (4.51)$$

is reached. At each step q , $(Z^{n+1})^q$ must be updated and hence the operator **A** (or **B**), in order to solve ω to afterwards compute $W^h((Z^{n+1})^q)$. $W^c((Z^{n+1})^q)$ depends solely on $(Z^{n+1})^q$, as seen in equation 2.13. This leads us to the following algorithm:

Algorithm 1 Jacobi (Gauss-Seidel) algorithm to solve problem [3.40-3.52](#)

```

1: Provide initial values  $Z^0 = z_0, (Z')^0 = v_0, \omega_i = (\theta_I)_i \forall i \in I$ 
2: for  $n = 1 \rightarrow N$  do
3:    $q = 0$ 
4:   Compute  $W^{n+1}, Z^q = Z^{n-1}$ 
5:   while  $|Z^{q+1} - Z^q| > \epsilon_Z$  and  $\|\mathbf{r}\| > \epsilon_r$  do
6:     Compute  $W^c(Z^q)$ 
7:     Perform  $K$  Jacobi (or G-S) sweeps  $\omega^{k+K} = \mathbf{A}^K(\omega^k)$  (or  $\omega^{k+K} = \mathbf{B}^K(\omega^k)$ )
8:     Compute  $W^h(Z^q)$ 
9:     Find  $Z^{q+1}$  according to eq. 4.50, update  $h(\mathbf{x}_i)^q$  and  $\mathbf{A}$  (or  $\mathbf{B}$ )
10:     $q \rightarrow q + 1$ 
11:  end while
12:  Compute  $(Z')^n$  according to eq. 4.45
13:   $Z^n = Z^q$ 
14:   $\omega^n = \omega^k$ 
15: end for

```

Notice that the proposed algorithm updates dynamically the variable $Z(t)$, simultaneously with the pressure and fluid fraction fields. Here $K = 1, 2, \dots$. If $K = 1$ then Z is updated at each Jacobi or Gauss-Seidel sweep.

Multigrid acceleration and parallel implementation

Multigrid methods solve efficiently a large class of problems. Among those problems we found the elliptic partial differential equations, particularly elliptic boundary-value problems [61].

The method is based in approximating a problem on multiple overlapping grids with widely varying mesh sizes and cycling between these approximations, using relaxation to reduce the error on each grid.

Algorithm 1 has proven to be robust and efficient [11]. However, it is expensive from a computational point of view. In the Elrod-Adams model, an elliptic equation is solved in Ω_+ , as explained in subsection II.1 of Chapter 3. Fine meshes are required to capture the effect of the smaller scale features such as surface roughness and micro-textures and therefore the computational cost is high. This makes the problem solved in this work suitable for a multigrid implementation.

The aim of this chapter is first to explain the *Full Approximation Scheme*-FAS (a multigrid method for nonlinear problems) and the technicalities involved. Then a study of accuracy and mesh convergence is presented. The latter part is dedicated to the presentation of the numerical results showing performance for both the multigrid and parallel implementations.

I Full Approximation Scheme

Let us suppose that we want to solve the following nonlinear problem:

$$\mathcal{N}(\omega) = \ell . \quad (5.1)$$

Let

$$\mathcal{N}^h(\omega^h) = \ell^h , \quad (5.2)$$

be the discrete associated problem, with h being the size of the mesh elements. Then, the residue is:

$$\mathbf{r}^h(\omega^h) = \ell^h - \mathcal{N}^h(\omega^h), \quad (5.3)$$

which is $\mathbf{r}^h = 0$ if ω^h is the solution of equation 5.2.

Introducing at this point a coarser mesh of size H , where problem 5.1 is also being solved. In this case $\mathcal{N}^H(\omega^H) = \ell^H$, then

$$\mathcal{N}^H((\mathcal{T}_\omega)_h^H \omega^h) = (\mathcal{T}_r)_h^H(\mathbf{r}^h(\omega^h)) + \mathcal{N}^H((\mathcal{T}_\omega)_h^H \omega^h), \quad (5.4)$$

remains true if ω^h is the solution of 5.2. Here $(\mathcal{T}_\omega)_h^H, (\mathcal{T}_r)_h^H$ represents the restriction operators for the solution ω and the residue $\mathbf{r}$, here taken as the injection and the full-weighting operators ([61]). In what follows, whenever subindexes ω and $\mathbf{r}$ are redundant they will be dropped.

The *Full Approximation Scheme* ([61]) is based on solving the following problem:

$$\mathcal{N}^H(\omega^H) = \mathcal{T}_h^H(\mathbf{r}^h(\omega^h)) + \mathcal{N}^H(\mathcal{T}_h^H \omega^h) . \quad (5.5)$$

It is to be noticed that the right hand side of the problem defined in equation 5.5 is not equal to ℓ^H . The solution ω^H of equation 5.5 is afterwards transferred to the finer mesh, adding the short-wavelength components:

$$\tilde{\omega}^h = \mathcal{T}_H^h \omega^H + (\omega^h - \mathcal{T}_H^h \mathcal{T}_h^H \omega^h) , \quad (5.6)$$

where $\mathcal{T}_H^h$ is the linear interpolation operator ([61]), as seen in Figure 5.1. This new approximation $\tilde{\omega}^h$ contains less error in the long wavelength components, and therefore converges faster in the finest mesh. The operation $\omega^h \rightarrow \omega^H \rightarrow \tilde{\omega}^h$ is called a V-cycle.

Figure 5.1: Mesh transfer operators.

Recalling Problem 1 of Chapter 4, we can identify:

$$(\mathcal{N}^h(\omega^h))_i = \mathbf{a}_i^h \theta(\omega_i^h) + \mathbf{b}_i^h p(\omega_i^h) - \mathbf{c}_i^h(\omega^h), \quad (5.7)$$

$$(\ell^h)_i = \mathbf{l}_i^h. \quad (5.8)$$

Here h is associated with the discretization Δx_1 , Δx_2 and Δt , and H with the coarser discretization $2\Delta x_1$, $2\Delta x_2$ and $2\Delta t$. From these two discretizations operators $\mathbf{A}^h$ and $\mathbf{A}^H$ arise, with $\mathbf{A}$ being defined by equation 4.33. If we follow the notation of Section II, Chapter 4, then for each point $i \in I$ we can define the following two-grid operator:

$$\mathbf{C}(\omega^h) = \begin{cases} \mathbf{A}^h((\omega^h)^{(1)}), \\ \text{where } (\omega^h)^{(1)} = (\mathcal{T}_\omega)_H^h \omega^H + \omega_d^h - (\mathcal{T}_\omega)_H^h (\mathcal{T}_\omega)_H^h \omega_d^h, \\ \omega^H = \mathbf{A}^H((\mathcal{T}_\omega)_h^H \omega_d^h, \mathbf{l}^H), \\ \omega_d^h = \mathbf{A}^h(\omega^h), \\ \mathbf{l}^H = (\mathcal{T}_r)_h^H \mathbf{r}^h(\omega_d^h) + \mathcal{N}^H((\mathcal{T}_\omega)_h^H \omega_d^h), \\ \text{and } \mathbf{r}^h(\omega_d^h) = \mathbf{l}^h - \mathcal{N}^h(\omega_d^h). \end{cases} \quad (5.9)$$

Here $\mathbf{l}^H$ is the right hand side of the modified problem being solved in the coarse mesh H . This two-level operator can be extended easily to a multilevel operator if equations 5.5-5.6 are used recursively. Here we establish multigrid levels $l = 0, 1, 2, \dots, M-1$, being $l = 0$ the finest and $l = M-1$ the coarsest. This multigrid operator reads:

$$\mathbf{M}(\omega^h) = \begin{cases} \omega_u^h, \text{ where:} \\ \omega_u^{2^l h} = \mathbf{A}^{2^l h} \left(\left(\omega_u^{2^l h} \right)^{(1)}, \mathbf{l}^{2^l h} \right), & \text{with} \\ \left(\omega_u^{2^l h} \right)^{(1)} = \mathcal{T}_{2^{l+1}h}^{2^l h} \omega_u^{2^{l+1}h} + \omega_d^{2^l h} - \mathcal{T}_{2^{l+1}h}^{2^l h} \mathcal{T}_{2^l h}^{2^{l+1}h} \omega_d^{2^l h} & \text{for } l = 0, \dots, M-2; \\ \omega_u^{2^l h} = \omega_d^{2^l h} & \text{for } l = M-1; \\ \omega_d^{2^l h} = \mathbf{A}^{2^l h} \left(\mathcal{T}_{2^{l-1}h}^{2^l h} \omega_d^{2^{l-1}h}, \mathbf{l}^{2^l h} \right) & \text{for } l = 1, \dots, M-1; \\ \omega_d^{2^l h} = \mathbf{A}^{2^l h} \left(\omega^h, \mathbf{l}^{2^l h} \right) & \text{for } l = 0; \\ \mathbf{l}^{2^l h} = \mathbf{l}^h & \text{for } l = 0; \\ \mathbf{l}^{2^l h} = (\mathcal{T}_{\mathbf{r}})_{2^{l-1}h}^{2^l h} (\mathbf{r}^{2^{l-1}h}) + \mathcal{N}^{2^l h} ((\mathcal{T}_{\omega})_{2^{l-1}h}^{2^l h} \omega_d^{2^{l-1}h}) & \text{for } l = 1, \dots, M-1, \\ \mathbf{r}^{2^l h} = \mathbf{l}^{2^l h} - \mathcal{N}^{2^l h} (\omega_d^{2^l h}) & \text{for } l = 0, \dots, M-1. \end{cases} \quad (5.10)$$

The solution vectors $\omega_d^{2^l h}$ and $\omega_u^{2^l h}$ correspond to the solution of the problems defined at each level l in the descent (*down*) and in the ascent (*up*) of a V-Cycle. With this already defined, we can state the following multigrid fixed point version of Problem 1:

Problem 3: “Find $\omega \in \mathbb{R}_+^N := \{\mathbf{v} \in \mathbb{R}^N | \mathbf{v}_i \geq 0 \ \forall i \in I\}$ such that $\omega = \mathbf{M}(\omega)$ where $\mathbf{M} : \mathbb{R}_+^N \longrightarrow \mathbb{R}^N$ is given by equation 5.10”.

Remark: Notice that $\mathbf{M}$ is not an operator such that $\mathbf{M} : \mathbb{R}_+^N \longrightarrow \mathbb{R}_+^N$ because of the integrid transfers. Equation 5.6 does not guarantee $\tilde{\omega}^h \geq 0$.

I.1 Jacobi and Gauss-Seidel-type multigrid algorithms for the non-moving pad

Problem 3 leads to a Jacobi and Gauss-Seidel-type algorithms to solve it. Lets define two vectors ν^u and ν^d in $(\mathbb{Z}^+)^M$. Lets redefine operator $\mathbf{M}$ as $\mathbf{M}^A$:

$$\mathbf{M}^A(\omega^h) = \begin{cases} \omega_u^h, \text{ where:} \\ \omega_u^{2^l h} = \left(\mathbf{A}^{2^l h} \right)^{\nu_l^u} \left(\left(\omega_u^{2^l h} \right)^{(1)}, \mathbf{l}^{2^l h} \right), & \text{with} \\ \left(\omega_u^{2^l h} \right)^{(1)} = \mathcal{T}_{2^{l+1}h}^{2^l h} \omega_u^{2^{l+1}h} + \omega_d^{2^l h} - \mathcal{T}_{2^{l+1}h}^{2^l h} \mathcal{T}_{2^l h}^{2^{l+1}h} \omega_d^{2^l h} & \text{for } l = 0, \dots, M-2; \\ \omega_u^{2^l h} = \omega_d^{2^l h} & \text{for } l = M-1; \\ \omega_d^{2^l h} = \left(\mathbf{A}^{2^l h} \right)^{\nu_l^d} \left(\mathcal{T}_{2^{l-1}h}^{2^l h} \omega_d^{2^{l-1}h}, \mathbf{l}^{2^l h} \right) & \text{for } l = 0, \dots, M-1; \\ \mathbf{l}^{2^l h} = \mathbf{l}^h & \text{for } l = 0; \\ \mathbf{l}^{2^l h} = (\mathcal{T}_{\mathbf{r}})_{2^{l-1}h}^{2^l h} (\mathbf{r}^{2^{l-1}h}) + \mathcal{N}^{2^l h} ((\mathcal{T}_{\omega})_{2^{l-1}h}^{2^l h} \omega_d^{2^{l-1}h}) & \text{for } l = 1, \dots, M-1, \\ \mathbf{r}^{2^l h} = \mathbf{l}^{2^l h} - \mathcal{N}^{2^l h} (\omega_d^{2^l h}) & \text{for } l = 0, \dots, M-1. \end{cases} \quad (5.11)$$

Here $\left(\mathbf{A}^{2^l h} \right)^{\nu_l^{u,d}}$ implies the application of the operator succesively $\nu_l^{u,d}$ times. Simi-

Figure 5.2: A multigrid V-cycle.

larly we can define a Gauss-Seidel-type algorithm by defining operator $\mathbf{M}^B$:

$$\mathbf{M}^B(\omega^h) = \begin{cases} \omega_u^h, \text{ where:} \\ \omega_u^{2^l h} = (\mathbf{B}^{2^l h})^{\nu_u^l} \left((\omega_u^{2^l h})^{(1)}, \mathbf{l}^{2^l h} \right), & \text{with} \\ (\omega_u^{2^l h})^{(1)} = \mathcal{T}_{2^{l+1}h}^{2^l h} \omega_u^{2^{l+1}h} + \omega_d^{2^l h} - \mathcal{T}_{2^{l+1}h}^{2^l h} \mathcal{T}_{2^l h}^{2^{l+1}h} \omega_d^{2^l h} & \text{for } l = 0, \dots, M-2; \\ \omega_u^{2^l h} = \omega_d^{2^l h} & \text{for } l = M-1; \\ \omega_d^{2^l h} = (\mathbf{B}^{2^l h})^{\nu_d^l} \left(\mathcal{T}_{2^{l-1}h}^{2^l h} \omega_d^{2^{l-1}h}, \mathbf{l}^{2^l h} \right) & \text{for } l = 0, \dots, M-1; \\ \mathbf{l}^{2^l h} = \mathbf{l}^h & \text{for } l = 0; \\ \mathbf{l}^{2^l h} = (\mathcal{T}_r)_{2^{l-1}h}^{2^l h} (\mathbf{r}^{2^{l-1}h}) + \mathcal{N}^{2^l h}((\mathcal{T}_\omega)_{2^{l-1}h}^{2^l h} \omega_d^{2^{l-1}h}) & \text{for } l = 1, \dots, M-1, \\ \mathbf{r}^{2^l h} = \mathbf{l}^{2^l h} - \mathcal{N}^{2^l h}(\omega_d^{2^l h}) & \text{for } l = 0, \dots, M-1. \end{cases} \quad (5.12)$$

Here operator $\mathbf{B}$ is the one defined in equations 4.39-4.40. If we apply operators 5.11 or 5.12 iteratively we have:

$$\omega_{k+1}^h = \mathbf{M}^{A,B}(\omega_k^h), \quad (5.13)$$

for $k = 0, 1, \dots$ until $\|(\mathbf{r}^h)^k\| \leq \epsilon_r$. Notice that the Jacobi/Gauss-Seidel sweeps are performed at each multigrid level.

This is depicted in figure 5.2. At each multigrid level l , ν_l^d relaxations are done in the way "down" and ν_l^u in way "up" a multigrid V-cycle. For linear problems these values are usually taken as $\nu_l^d = 2$, $\nu_l^u = 1 \forall l = 0, \dots, M-1$.

I.2 Jacobi and Gauss-Seidel-type multigrid algorithms for the moving pad

In the same manner as in the previous Chapter, we can state the following problem:

Problem 4: “Given $Z^n, \frac{dZ^n}{dt}$ and $(W)^{n+1}$ in $\mathbb{R}$, find $Z^{n+1} \in \mathbb{R}$ such that

$$Z^{n+1} = Z^n + \Delta t \frac{dZ^n}{dt} + \frac{\Delta t^2}{2m} ((W)^{n+1} + (W^h(Z^{n+1}) + (W^c(Z^{n+1}))) , \quad (5.14)$$

where

$$W^h(Z^{n+1}) = \int_{i \in I_3} p(\omega_i(Z^{n+1})) d\Omega , \quad (5.15)$$

$$W^c(Z^{n+1}) = \int_{i \in I_3} p_{con}(Z^{n+1}) d\Omega , \quad (5.16)$$

And ω is the solution of Problem 3”.

This leads to the following multigrid algorithm:

Algorithm 2 Multigrid algorithm to solve problem 3.40-3.52

- 1: Provide initial values and vectors $Z^0 = z_0, (Z')^0 = v_0, \omega_i = (\theta_I)_i \forall i \in I, \nu^u, \nu^d$
 - 2: **for** $n = 1 \rightarrow N$ **do**
 - 3: $q = 0$
 - 4: Compute $W^{n+1}, Z^q = Z^{n-1}$
 - 5: **while** $|Z^{q+1} - Z^q| > \epsilon_Z$ and $\|\mathbf{r}\| > \epsilon_r$ **do**
 - 6: Compute $W^c(Z^q)$
 - 7: Perform K V-cycles: $\omega^{k+K} = (\mathbf{M}^A)^K (\omega^k)$ (or $\omega^{k+K} = (\mathbf{M}^B)^K (\omega^k)$)
 - 8: Compute $W^h(Z^q)$
 - 9: Find Z^{q+1} according to eq. 4.50, update $h(\mathbf{x}_i)^q$ and $\mathbf{M}^A$ (or $\mathbf{M}^B$)
 - 10: $q \rightarrow q + 1$
 - 11: **end while**
 - 12: Compute $(Z')^n$ according to eq. 4.45
 - 13: $Z^n = Z^q$
 - 14: $\omega^n = \omega^k$
 - 15: **end for**
-

I.3 The cavitation boundary issue with the mesh transfers

The major drawback of the multigrid method with the Elrod-Adams model is the shifting of the cavitation boundary done in the coarsest meshes. This situation is depicted in figure 5.3: a ring sliding over an untextured liner generates a pressure field in Ω_+ , which is shown in the upper left corner. A zoom showing the finite volume centers occupies the upper right corner.

Figure 5.3: The cavitation boundary changes its position through the way down and up in a V-cycle.

The position of the cavitation boundary changes as the problem is passed to the coarser meshes (fig. 5.3, bottom). The result is that the correction returned to the finest mesh will not resemble the problem solved in the finest mesh. Therefore the residue of equation 4.9 increases at the end of a V-cycle or at least stalls.

An effective solution for this problem is to *maintain* the cavitation boundary position in the finest mesh. This is done by redefining the operator $\mathcal{R}$ (equation 4.32) in which our methods are based, for the coarser meshes. The operators $\mathcal{R}_1$ and $\mathcal{R}_2$ (equations 4.30-4.31) in the multigrid setting are defined in a straightforward manner:

$$\mathcal{R}_1^{2^k h}(\omega_i^{2^k h}) = \begin{cases} \frac{l_i^{2^k h} + c_i^{2^k h} - a_i^{2^k h}}{b_i^{2^k h}} + 1 & \text{if } \omega_i^{2^k h} \geq 1 \text{ and } l_i^{2^k h} + c_i^{2^k h} - a_i^{2^k h} \geq 0, \\ \theta_i(\omega_i^{2^k h}) & \text{if } \omega_i^{2^k h} \geq 1 \text{ and } l_i^{2^k h} + c_i^{2^k h} - a_i^{2^k h} < 0, \end{cases} \quad (5.17)$$

and

$$\mathcal{R}_2^{2^k h}(\omega_i^{2^k h}) = \begin{cases} \frac{l_i^{2^k h} + c_i^{2^k h}}{a_i^{2^k h}} & \text{if } \omega_i^{2^k h} < 1 \text{ and } \frac{l_i^{2^k h} + c_i^{2^k h}}{a_i^{2^k h}} < 1, \\ 1 & \text{if } \omega_i^{2^k h} < 1 \text{ and } \frac{l_i^{2^k h} + c_i^{2^k h}}{a_i^{2^k h}} \geq 1. \end{cases} \quad (5.18)$$

Then we define a new $\mathcal{R}$ for all multigrid $k = 0, 1, \dots, M - 1$ levels as:

$$\mathcal{R}_\Sigma^{2^k h} = \begin{cases} \mathcal{R}_2^{2^k h} \circ \mathcal{R}_1^{2^k h} & \text{if } k = 0, \\ \mathcal{R}_1^{2^k h} & \text{if } k = 1, \dots, M - 1. \end{cases} \quad (5.19)$$

Compare this to our previous definition of $\mathcal{R}$:

$$\mathcal{R}^{2^k h} = \mathcal{R}_2^{2^k h} \circ \mathcal{R}_1^{2^k h}, \quad \forall k = 0, 1, \dots, M - 1. \quad (5.20)$$

The main difference is that with operator $\mathcal{R}_\Sigma^{2^k h}$ only pressure is computed in the coarser meshes, and at the cells which are already pressurized according to the information coming from the finest mesh. If a cell is depressurized, it sets the pressure to zero but keeps marking it with a saturation value of 1.

This defines also a new operator $\mathbf{A}_\Sigma^{2^k h} : \mathbb{R}_+^N \longrightarrow \mathbb{R}_+^N$ at multigrid level k :

$$\left(\mathbf{A}_\Sigma^{2^k h}(\omega^{2^k h}) \right)_i = \begin{cases} \mathcal{R}_\Sigma^{2^k h}(\omega_i^{2^k h}) & i \in I^{2^k h} \setminus I_1^{2^k h} \setminus I_2^{2^k h} \\ \bar{\omega}_i^{2^k h} & i \in I_1^{2^k h} \\ \omega_{\mathcal{I}(i)}^{2^k h} & i \in I_2^{2^k h} \end{cases}. \quad (5.21)$$

Also, a new operator $\mathbf{B}_\Sigma^{2^k h}$, analogous to the one defined in Subsection II.2 of the previous chapter can be defined based in $\mathbf{A}_\Sigma^{2^k h}$. The multigrid operators of equations 5.11 and 5.12 can be rewritten in terms of $\mathbf{A}_\Sigma^{2^k h}$ and $\mathbf{B}_\Sigma^{2^k h}$, which brings about operators $\mathbf{M}^{\mathbf{A}_\Sigma}$ and $\mathbf{M}^{\mathbf{B}_\Sigma}$, and Algorithm 3:

Algorithm 3 Multigrid algorithm to solve Problem 3.40-3.52 preserving the cavitation boundary in the coarser meshes

```

1: Provide initial values and vectors  $Z^0 = z_0, (Z')^0 = v_0, \omega_i = (\theta_I)_i \forall i \in I, \nu^u, \nu^d$ 
2: for  $n = 1 \rightarrow N$  do
3:    $q = 0$ 
4:   Compute  $W^{n+1}, Z^q = Z^{n-1}$ 
5:   while  $|Z^{q+1} - Z^q| > \epsilon_Z$  and  $\|\mathbf{r}\| > \epsilon_r$  do
6:     Compute  $W^c(Z^q)$ 
7:     Perform  $K$  V-cycles:  $\omega^{k+K} = (\mathbf{M}^{\mathbf{A}\Sigma})^K (\omega^k)$  (or  $\omega^{k+K} = (\mathbf{M}^{\mathbf{B}\Sigma})^K (\omega^k)$ )
8:     Compute  $W^h(Z^q)$ 
9:     Find  $Z^{q+1}$  according to eq. 4.50, update  $h(\mathbf{x}_i)^q$  and  $\mathbf{M}^{\mathbf{A}\Sigma}$  (or  $\mathbf{M}^{\mathbf{B}\Sigma}$ )
10:     $q \rightarrow q + 1$ 
11:  end while
12:  Compute  $(Z')^n$  according to eq. 4.45
13:   $Z^n = Z^q$ 
14:   $\omega^n = \omega^k$ 
15: end for

```

A comparison between results with Algorithms 2 and 3 can be seen in figure 5.4. The norm of the residue $\|\mathbf{r}^h\|$ in the finest mesh is shown for a problem solved maintaining the position of the cavitation boundary at the coarser meshes ($\mathbf{M}^{\mathbf{B}\Sigma}$), and with the straightforward implementation of the multigrid ($\mathbf{M}^{\mathbf{B}}$). These are the red and blue lines, respectively. Each dot corresponds to one relaxation iteration in the finest mesh, here $\nu_{0,d} = \nu_{0,u} = 5$, thus 10 relaxation iterations are performed in the finest mesh per V-cycle. After the first $\nu_{0,d}$ iterations are made, the problem is solved in the coarser meshes, and then $\nu_{0,u}$ iterations relaxations are performed again, as depicted in Figure 5.2. The residue grows at the beginning of each group of $\nu_{0,u}$ iterations, as the solution has been modified in the coarser meshes. These are the spikes in both curves of Figure 5.4. Notice how these jumps in the residue are controlled by not modifying the position of the cavitation boundary computed in the finest mesh, and the overall residue is effectively reduced.

Figure 5.4: Norm of the residue $\|\mathbf{r}^h\|$ in the finest mesh relative to the residue also in the finest mesh prior to the iterative solution. Each iteration in the fine mesh is represented by a circle. The black arrows indicate the beginning and end of a V-cycle in the finest mesh.

Figure 5.5 shows how the reduction in the residue against the number of relaxation sweeps carried out in the finest mesh. It takes more than 500.000 iterations in the finest mesh for algorithm 1 to reduce the error five orders of magnitude, while the multigrid algorithm with $M=5$ makes the same reduction in less than 250 iterations.

Figure 5.5: Reduction in the residue with the multigrid Algorithm 3 for the two-dimensional problem with measured surfaces of Section II. Results were computed with $M=1$ (no multigrid) to $M=5$ meshes in a V-cycle. The domain was discretized into 2000×400 finite volumes.

II Convergence analysis

A convergence analysis for the numerical method presented in Chapter 4 was done by Jaramillo [72]. The convergence rate was measured in the L^2 and the H^1 norms

$$\|f\|_{L^2} = \sqrt{\int_{\Omega} f^2 d\Omega}, \quad (5.22)$$

$$\|f\|_{H^1} = \sqrt{\|f\|_{L^2}^2 + \sum_i \left\| \frac{\partial f}{\partial x_i} \right\|_{L^2}^2}, \quad (5.23)$$

for the errors

$$\|e_p\|_{H^1} = \|p - p_a\|_{H^1}, \quad (5.24)$$

$$\|e_{\theta}\|_{L^2} = \|\theta - \theta_a\|_{L^2}, \quad (5.25)$$

where $(p_a(x_1, x_2, t), \theta_a(x_1, x_2, t))$ is the analytical solution of the problem. Two one-dimensional problems were assessed: a pure squeeze motion and a flat pad with a traveling pocket. It was shown that $\|e_p\|_{H^1} \propto \Delta x_1^{1.9}$ and $\|e_{\theta}\|_{L^2} \propto \Delta x_1^{0.5}$. Analytical solutions of two-dimensional problems are harder to obtain. Instead of the analytical

solution the errors are computed with a numerical solution in a very fine mesh. The two-dimensional problem of Section III, Chapter 7, where h_t is given by a measured surface, is solved in a domain $\Omega = (0, 1) \times (0, 0.2)$ with meshes

Mesh	N_1	N_2
1	500	100
2	1000	200
3	2000	400
4	4000	800

A Courant number $C = (u/2)\Delta t/\Delta x_1=1$ is fixed in every simulation. Results are compared with those obtained with a very fine mesh of $N_1, N_2=8000 \times 1600$ finite volumes. A typical result for time t is shown in Figure 5.6. For this two dimensional problem it was found that $\|e_p\|_{H^1} \propto \Delta x_1^{1.23}$ and $\|e_\theta\|_{L^2} \propto \Delta x_1^{0.403}$. A lower rate of convergence of θ is expected due to the discontinuities present in the solution.

Figure 5.6: Rate of convergence for (a) pressure in the H^1 norm and (b) fluid fraction θ for the L^2 norm.

III Parallel implementation

The code was parallelized with the API (*Application Programming Interface*) OpenMP (<http://openmp.org/wp/>), in the *shared memory* paradigm. Details of the efficiency are given in the next section.

III.1 Multigrid and parallel performance

The capabilities of both the multigrid acceleration and the parallel implementation were tested on a ring of width $L=1$ mm, curvature radius $R=64$ mm with a linear mass $m=0.048$ kg/m. In the liner surface a square array of pockets with dimensions $l=50\mu\text{m}$,

$d=10 \mu\text{m}$ and period $\lambda=200 \mu\text{m}$ (19.6% of textured surface) is taken. The ring slides with a constant velocity $u=10 \text{ m/s}$ and the lubricant viscosity is set at $\mu=4 \text{ mPa.s}$. The conditions upstream of the ring are $d_{oil}=20 \mu\text{m}$ which with a Stribeck number $S=10^{-3}$ ensures fully flooded conditions. This grants no contact between the surfaces.

Domain dimensions are $x_{1l}-x_{1r}=2 \text{ mm}$ and $B=0.1 \text{ mm}$. Meshes of 512×32 , 1024×64 and 2048×128 elements lead to cell sizes of 4, 2 and 1 microns. Courant number is fixed to 1, while 1000, 50 and 10 time steps are computed in each mesh respectively.

Tolerance in the residue is set to 10^{-5} of the residue in the first relaxation at each time step. Enough relaxation sweeps/V-cycles are taken to achieve this precision. The number of relaxation sweeps at every mesh in the way down and up a V-cycle are detailed in the following table:

Multigrid level l	ν_l^u	ν_l^d
0	5	5
1	5	5
2	10	10
3	10	10
4	20	20
5	20	20

where number of meshes are ordered from finest (1) to coarsest (6).

III.2 Results

The goals of the multigrid and parallel implementations is to be able to run the code with satisfying precision and low computational times. With this in mind, two tests were performed:

1. With fixed number of parallel processes ($N_p=4$) Gauss-Seidel relaxation was compared to *FAS* results.
2. With fixed number of multigrid meshes ($M=4$) the number of parallel processes were varied to appraise the parallel implementation performance.

The first test was carried along in a Intel Xeon 5430 machine (8 cores of up to 2.66 GHz, 12 Mb of cache) with 32 Gb of RAM. The coarsest mesh used in every case was of 64×4 elements, thus giving 4, 5 and 6 meshes used in every V-cycle.

The multigrid speedup (quotient between the runtime without and with multigrid) and parallel speedup (quotient between the execution time of the algorithm with one process

and with several processes) are probably the most representative metrics to measure performance. The former is shown in table 5.1: execution times with Gauss-Seidel relaxation (column 2) and *FAS* (column 5) depict the benefits of the multigrid. The speedup grows with the problem size, reaching the value 345.2 for the finest mesh.

The best computational times were attained taking the coarsest mesh possible (64×4 elements) as the lowest level of every V-cycle. It was stated previously in this Chapter that very "deep" V-cycles lead to a increase in the residue and therefore more iterations to achieve a certain precision. This may seem to be disproven by these results, however, it does happen in a fraction of the time steps, but the overall result is still the best for the chosen configuration.

Mesh	Runtime with $M=1$ (s)	Multigrid with $M=$	Runtime with multigrid (s)	Multigrid speedup
512×32	30244	4	211	143.3
1024×64	4972	5	33	150.7
2048×128	20024	6	58	345.2

Table 5.1: Multigrid acceleration.

For the second test three different architectures were employed up to the maximum number of cores available:

1. Intel Xeon E5345 2.33 GHz 16Gb RAM (8-cored)
2. Intel Xeon E5430 2.66 GHz 32Gb RAM (8-cored)
3. Intel Xeon E5690 3.46 GHz 32Gb RAM (12-cored)

Results can be seen in fig. 5.7. Speedup values are fair for the mesh with 512×32 elements, decreasing past $N_p=5$ or 6 depending on the architecture, due to the small size of the problem.

As expected, efficiency increases with the problem size for the same N_p . The same trend is shared among the three machines. For the 1024×128 mesh speedup values are quasi-optimal (close to the $y = x$ line), thus proving the quality of the implementation.

Figure 5.7: Assessment of the efficiency of code parallelization with several architectures: (a) Intel Xeon E5345, (b) Intel Xeon E5430 and (c) Intel Xeon E5690. Speedup is plotted against number of processes N_p .

Understanding the performance of micropatterned surfaces

The purpose of this chapter is to study of the effects in friction and minimum oil film of micro-textured surfaces running against piston-ring-like barrel-shaped pads. In the first part of this chapter, the influence of the shape of the pad in friction and clearance is assessed by means of mesh-converged two dimensional simulations for 72 dimple configurations and 10 different pad shapes. Three representative cases among those were analyzed, leading to a better understanding of the mechanisms of friction reduction with moving textures. These results were published in [38], as well as in two conferences ([40], [39]).

These previous results gave a better understanding of the role of the shape of the pad and its interaction with textured runners. However, a more fundamental and comprehensive research of the optimum texture dimensions for varying operational conditions (loads on the pad and pad masses, among others) was required. A broad study with one-dimensional textures (grooves on the runner) was carried out in the second part of this chapter. This work produced another journal publication ([42]) and another conference paper ([43]).

To finalize this chapter an application is presented. Realistic engine conditions are considered in a three-ring pack simulation over a fully textured liner, and the effects of texturing are analyzed in clearance, friction and power loss. Here not only the effects of texturing are studied but also the interaction between rings. The latter results were presented in [41].

I Review of the literature

Textured tribological surfaces have attracted much attention of the research community lately. After a significant number of experimental and theoretical studies, the possibility of reducing friction by means of microtextures has been established, together with a basic understanding of why this happens (at least in the hydrodynamic regime).

That certain textures reduce friction in some hydrodynamic bearings is by no means an obvious phenomenon. Buscaglia et al [36, 32] performed asymptotic analyses of general smooth (i.e., untextured) surfaces by introducing short-wavelength periodic perturbations of arbitrary shape and obtained that the untextured shape *always* maximizes the load carrying capacity and minimizes the friction coefficient. This implies that, for friction reduction to take place, there exist two possibilities: It can be a consequence of a *finite* perturbation (outside the validity of asymptotic theory), *or* involve physical mechanisms which were not considered in the aforementioned mathematical studies, such as *cavitation*.

Most of the numerical studies have only considered the texture to be on the pad, and thus stationary with respect to the contact. Kligerman et al [75] solved the problem for flat textured and partially textured rings using the Reynolds equation and Reynolds boundary condition, which is known not to conserve mass. They concluded that surface texture reduces the friction between the surfaces. On the other hand, Tomanik [109], with the same cavitation model, numerically compared barrel shaped rings (compression rings), flat rings (oil control rings) and flat partially textured rings, also incorporating experimental data in the analysis. His results (both numerical and experimental) showed that the untextured barrel-shaped rings performed better than their flat, partially textured counterparts, both in hydrodynamic support and in friction coefficient. This finding coincides with recent numerical results of Gadeschi et al [62] assessing the effect of Laser Surface Texture (LST) pockets on barrel-shaped rings. These authors adopted the Reynolds equation with Gumbel boundary conditions, which are non mass-conservative.

Numerical simulations of hydrodynamic ring/liner contact with a *mass-conservative cavitation model* and *considering the texture as being on the liner* have only been reported up to now by Yin et al ([29]), though with Reynolds cavitation condition (non-mass conservative), and by Tanaka & Ichimaru ([106]), who considered purely transversal textures, rendering the geometry one-dimensional, and adopted Reynolds cavitation condition.

Experimental studies of textured surfaces have mainly focused on conformal contacts (nominally flat pin or ring on nominally plane disk) [119, 115, 118, 94, 77, 84]. Experiments in non-conformal contact configurations have yielded mixed results up to now. In a recent article, Kovalchenko et al ([76]) found that dimpled disks accelerated the wear of

the contacting ball and only as a result thereof some friction reduction could be attained. Their explanation focused on a change of regime from boundary to mixed lubrication. Ali et al ([5]) performed ball-on-disk experiments and found that dents did not reduce friction in fully flooded conditions, whereas significant reduction was attained under severe starvation. Similar experiments by Li et al ([80]) on PDMS disks lead them to conclude that texturing only reduces friction at low sliding velocities, consistent with a boundary or mixed lubrication regime. Other experimental studies of non-conformal contacts in the boundary lubrication regime have been reported by Kim et al [73], Peterson & Jacobson ([92] and [91]).

The hydrodynamic lubrication regime was studied by Costa & Hutchins ([47]) by means of cylinder-on-plane experiments, in which the texture was on the plane. For a cylinder diameter of 16 mm they found that all textured surfaces performed worse than the untextured one. For a cylinder diameter of 200 mm, on the other hand, they observed that the *average* film thickness, as measured by capacitance techniques, was improved by some non-zero textures. Unfortunately, they did not measure the *minimum* film thickness or the friction coefficient. This picture coincides with the one recently put forward by Gadeschi et al [62] for non-moving textures (texture on the ring) on the basis of a non-mass-conservative model, that for a given load the minimal friction is in general obtained with a moderate R/L ratio (of the order of 10), being R the curvature radius of the bearing and L its length. Further, their results indicate that moderate-conformity contacts cannot be improved by texturing the surfaces. Their friction cannot be reduced to the point of becoming smaller than that of smooth bearings with moderate conformity, but nevertheless significant reduction appears to be achievable. These findings are consistent with more recent studies by Kovalchenko et al [76], Yin et al [29], Tomanik [108, 109], Scaraggi et al [101], among others[67, 118, 66, 49, 119].

This phenomena, friction reduction only occurring in high-conformity bearings in which the surfaces are so parallel that a texture of some suitable size and depth manages to produce local cavitation has already been advanced by Etsion ([60]). However, though the general picture seems to be coming to a consensus, little is still known about the actual physical mechanisms that take place at a cavitating microtextured contact. Detailed quantitative assessments that allow for the practical selection of a suitable texture for a specific device are also very scarce.

II Two-dimensional simulations of micro-pocketed surfaces

Several values of R are considered (shown in fig. 6.1) for pads with a circular profile, ranging from 2 millimeters to nominally conformal contact (large R values) with the objective of studying the effects of the conformity degree between the matching surfaces.

Figure 6.1: Pad profiles for curvature radius ranging from $R=0.2$ to $R=102.4$ centimeters, in powers of 2.

The runner velocity is assumed constant. Although in practical devices it changes periodically (shaft rpm's in a journal bearing, piston velocity in a combustion engine) and the friction coefficient also changes accordingly, this assumption still allows to study what happens at one determined instant.

A viscosity $\mu=0.004$ Pa.s is assumed. The scales chosen for the non-dimensionalization are:

Quantity	Scale	Scale value adopted
Velocity	V	10 m/s
Length	L	1 cm
Gap thickness	H	1 μm

Which lead to the derived quantities:

Quantity	Scale	Scale value
x_1, x_2	L	10^{-2} m
R	L	10^{-2} m
t	$\frac{L}{V}$	10^{-3} s
h	H	10^{-6} m
Z	H	10^{-6} m
p	$\frac{6\mu VL}{H^2}$	2.4×10^9 Pa
W, W^h	$\frac{6\mu VL^2}{H^2}$	2.4×10^7 N/m
F	$\frac{\mu VL}{H}$	4×10^2 N/m
m	$\frac{6L^4\mu}{H^3V}$	2.4×10^7 kg/m

A commonly used dimensionless number in tribology is the Stribeck number, defined by:

$$S = \frac{\mu u}{W} \quad (6.1)$$

where μ , u and W are dimensional. $W(t)$ is the load per unit width on the pad, which in our case it is constant in time and equal to $W(t) = W$. In terms of the non-dimensional variables:

$$S = \frac{H^2}{6 L^2} \frac{\hat{u}}{\hat{W}} \quad (6.2)$$

An example of a Stribeck curve for an untextured runner is seen in fig. 6.2. It can be divided in three lubrication regimes:

1. The *hydrodynamic* lubrication regime: where no contact among the asperities in both surfaces occur;
2. The *mixed* lubrication regime: friction is due partially to contact between the asperities;
3. The *boundary* lubrication regime: the load between the surfaces is carried mostly by the asperities.

All simulations are set to study the hydrodynamic regime. The value chosen for W lead to a Stribeck number of $S = 10^{-3}$. Fully flooded conditions are established by providing enough oil ($d_{oil}=20$) upstream of the pad position.

Figure 6.2: Stribeck curve computed for an untextured runner: the different lubrication regimes are shown.

The textures on the runner are those inspired in LST (Laser Surface Texturing) and consist of circular pockets with semi-ellipsoidal shape, in which both semi-axis of size l are oriented in the x_1 and x_2 dimension and the third semi-axis is the texture depth d (Figure 6.3).

Figure 6.3: Pockets on the runner surface, resembling the ones produced with Laser Surface Texturing. λ , l and d are the texture's period, radius and depth.

Let us define at this point the main tribological quantities that will be extracted from the numerical simulations. The instantaneous friction coefficient $f(t)$ and clearance $C(t)$ depend periodically on time once the bearing has reached its periodic regime. The

tribological quantities that will be considered representative of the periodic regime are the *average friction coefficient*

$$\bar{f} = \frac{1}{\lambda/V_1} \int_T^{T+\lambda/V_1} f(t) dt \quad (6.3)$$

which characterizes the power lost to friction, and the *minimum clearance*

$$C_{\min} = \min_{T \leq t \leq T+\lambda/V_1} C(t) \quad (6.4)$$

which can be used to characterize wear.

II.1 Mesh convergence

A mesh convergence study is needed to prove the soundness of the numerical method under moving-texture conditions. Consider the case of a pocketed runner with parameters $\lambda = 0.06$, $l = 0.01$, $d = 5.0$, $u = 1.0$ and $W = 1.66 \times 10^{-6}$, corresponding to $S = 10^{-3}$, with a linear pad mass of 2.0×10^{-9} (which is a realistic value for compression piston rings). The simulation starts with the pad separated $2.5 \mu m$ from the runner's plateau ($Z(0) = 2.5$), with zero velocity ($Z'(0) = 0$). The domain is 0.2 cm in the axial direction (x_1) and $300 \mu m$ along x_2 (just half the domain is simulated, with reflection boundary conditions).

A 512×32 spatial mesh is considered as a starting point for the refinement, with time step $\Delta t = 4 \times 10^{-4}$, for 1.6 seconds of simulation. The refined meshes have 1024×64 cells (run with $\Delta t = 2 \times 10^{-4}$) and 2048×128 cells (run with $\Delta t = 10^{-4}$), respectively. Fully flooded conditions are taken (fluid film of $20 \mu m$ imposed at the left boundary).

Figure 6.4 shows the position x_1 of the cavitation boundary along the line $x_2 = \lambda/2$ (corresponding to the longitudinal line passing through the dimples' centers) as a function of time. Though the cavitation boundary is a very sensitive variable, it can be seen that its position for each time converges steadily (with roughly first order in the mesh size).

Let us now turn to the convergence of the friction force, since it will be the focus of the results in the next sections. In Figure 6.5(b) the total friction force versus time, and a detail thereof, are shown. As mentioned, there is an adjustment period of about 1 time unit until the pad reaches its stable motion, which is periodic in time with period λ/u . Convergence of the friction force is observed in the figure, both in the initial transient and in the periodic regime. The relative errors in the average friction force in the periodic regime on the meshes with 512×32 and 1024×64 cells, as compared to those computed on the 2048×128 mesh, are of 0.72% and 0.27%, respectively. The numerical errors for

Figure 6.4: Position x_1 of the cavitation boundaries on the line $x_2 = \lambda/2$, which passes through the dimples' centers. Shown are the results for the 512×32 , 1024×64 and 2048×128 meshes, exhibiting convergent behavior of first order.

the computations made on the 512×32 mesh, therefore, can be estimated at 1%. It is this mesh that is employed in all the computations presented heretoforth, though some of them have been computed in all three meshes to check that the claimed conclusions are not consequences of numerical artifacts.

II.2 Effect of surface texturing on friction for different pad profiles

This study considers the cylindrical pad profiles shown in Fig. 6.1. The dimensionless velocity is set to $V_1 = 1$ and a linear pad mass of 2.0×10^{-9} , as before. A range of the parameters that define the texture shape was explored, simulating for each texture shape the transient lubrication problem for 10 different pad shapes (non-dimensional curvature radii from $R = 0.2$ to $R = 102.4$). Specifically, all combinations of the values of the three parameters l , λ and d were computed for ten different values of R . This yields a total of $8 \times 6 \times 5 \times 10 = 2400$ cases. Excluding those combinations which are geometrically incompatible (because $2l > \lambda$) and those which do not satisfy the thin film hypothesis (because $l < 5d$), there remain 720 cases.

Figure 6.5: (a) Pad position and (b) Friction force in the x_1 direction for the 512×32 , 1024×64 and 2048×128 meshes.

$l_1 = l_2 = l$	λ	d
0.001	0.005	2
0.0025	0.005	5
0.005	0.01	10
0.01	0.02	20
0.02	0.03	30
0.03	0.04	
	0.05	
	0.06	

(a) Best textures in terms of friction							
small R				large R			
#	l	λ	d	#	l	λ	d
1	0.0025	0.06	2	1	0.03	0.06	10
2	0.0025	0.05	2	2	0.03	0.06	5
3	0.0025	0.04	2	3	0.03	0.06	20
4	0.0025	0.03	2	4	0.02	0.04	10
5	0.005	0.06	2	5	0.02	0.04	5

(b) Best textures in terms of clearance							
small R				large R			
#	l	λ	d	#	l	λ	d
1	0.0025	0.06	2	1	0.03	0.06	5
2	0.0025	0.05	2	2	0.02	0.05	5
3	0.0025	0.04	2	3	0.02	0.06	5
4	0.0025	0.06	5	4	0.03	0.06	10
5	0.0025	0.05	5	5	0.03	0.06	2

Table 6.1: Tables showing the best textures in terms of the average friction coefficient (a) or of the minimum film thickness (b). For each criterion, the best five textures for pads with small R are shown on the left, while the best five for pads with large R are shown on the right.

These cases were organized as 72 curves giving the average friction coefficient $\bar{f}$ and minimum clearance $C_{\min}$ of each texture as a function of the curvature radius of the pad. The curves are all plotted simultaneously in Fig. 6.6, in black dotted lines with a cross at each actual simulation result. The friction and clearance obtained with a perfectly flat runner (untextured) is displayed as a thick red line with a triangle indicating each actual simulation.

It is observed in Fig. 6.6 that none of the simulated textures attains lower friction than the untextured runners when $R < 1.6$ (notice that this value corresponds to 1.6 centimeters). The lowest overall friction is obtained with a pad having $R = 0.8$ sliding on an untextured runner. This configuration also maximizes the clearance between pad and runner. For flatter pads, those with $R = 3.2$ or greater, there are textures that improve both the friction and the clearance.

One can thus divide the results into two general classes, corresponding to either small values of R or large values of R . For each class, one can find the textures that yield the lowest average friction (Table 6.1(a)) and those that yield the largest minimum clearance (Table 6.1(b)). The best runner for small- R pads is, as mentioned, one with untextured

Figure 6.6: All the 72 curves for (a) friction and (b) minimum clearance with the envelope of those and the untextured case ($S = 1.0 \times 10^{-3}$).

surface. Considering the explored ranges of l , λ and d , one observes that the lowest friction and largest clearance are obtained with the smallest values of l and d , together with the greatest values of λ . In other words, the best textures correspond to runner surfaces as planar as possible.

The best runner for large- R pads, on the other hand, exhibits a non-zero texture with $l = 0.03$, $\lambda = 0.06$ and $d = 10$ (or $d = 5$ if one wants the maximal clearance). This corresponds to pockets touching one another, i.e., dimple length and width equal to the texture-cell size.

To provide some insight into the detailed physics taking place during the passage of each texture cell under the pad, three cases are reported in detail in the next section. These cases are labeled Case I, Case II and Case III in Fig. 6.6. In fact, Case III corresponds to

the texture that yields lowest friction for large- R pads.

Let us however insert a few comments before going into the detailed case analyses:

- The radius of curvature of actual piston compression rings is of about 5 centimeters [104], corresponding to $R = 5$ and thus falling into the large- R class, in which texturing the cylinder liner may be beneficial (at least for the Stribeck number considered here, $S = 10^{-3}$).
- There could exist a width-to-length ratio l_2/l_1 (being the l_1 and l_2 semiaxis of an ellipse) that improves the friction for small- R pads. Though not illustrated here, we explored other pocket shapes, with $l_2/l_1 = 0.25, 0.5, 2$ and 4 , and none of them yielded better performances than circular pockets.

II.3 Influence of parameter θ_s on the previous results

Above, all computations of the friction coefficient have adopted the model parameter $\theta_s = 0.95$. Similar results are obtained for other values of θ_s . In fact, Figure 6.7 shows the lower envelopes of the friction curves for $\theta_s = 0.05, 0.25, 0.50$ and 0.75 . All of them are qualitatively consistent with those of $\theta_s = 0.95$, showing that for small- R pads ($R < 1.6$) the untextured runner yields minimal friction, while for large- R pads there exist friction-reducing textures.

Figure 6.7: Lower envelopes of the friction curves as obtained taking different values of θ_s , namely 0.05, 0.25, 0.5 and 0.75. Also shown are their corresponding untextured cases for comparison.

II.4 Analysis of three cases

Here we analyze and compare the following three cases:

- **Case I:** A textured runner with $l = 0.01$, $\lambda = 0.06$ and $d = 5.0$, combined with a pad having $R = 0.8$. In this configuration, the textured runner yields higher friction and lower clearance than the untextured one. The textured area fraction is 8.7%.
- **Case II:** The same textured runner ($l = 0.01$, $\lambda = 0.06$ and $d = 5.0$), now combined with a large- R pad ($R = 51.2$). In this configuration the texture is beneficial both in friction and in clearance. The textured area fraction is 8.7%.
- **Case III:** A textured runner with $l = 0.03$, $\lambda = 0.06$ and $d = 10.0$, combined with a pad having $R = 3.2$. This corresponds to the texture yielding the lowest friction for the given value of R . The textured area fraction is 78.5%.

In Fig. 6.8, the first line from the top (parts (a) and (b)) corresponds to Case I, while the second and third correspond to Cases II and III, respectively. The pad position is highest for Case I, being 6.41 (microns) above the runner if untextured and about 6.25 if textured. The corresponding friction coefficients are, accordingly, the lowest of the three cases (0.00754 for the untextured runner and 0.00763 for the textured one, remarkably similar considering that 8.7% of the area consists of pockets of depth 5 microns). One also observes that the friction is dominated by the Couette term, which is increased (in mean) by the texture. Figure 6.9 shows the pressure profile and the film thickness at several instants along the passage of one individual pocket under the pad. At instants (c) to (f), the pressure generated under the pad is greater than the one generated on the untextured runner. The pocket creates additional lift, thus, when it is centered under the pad or slightly towards the diverging side of it (up to one pocket diameter to the right). At instants (h) to (a) the pocket is detrimental for pressure generation, corresponding to the pocket being upstream of the pad's apex. Notice that in Case I, since the contact is strongly non-conformal (small- R pad), the pressure field always exhibits a main pressurized region (positive p) on the left (converging) half of the pad.

Figure 6.8: Pad position Z ((a), (c), (e)) and instantaneous friction force terms ((b), (d), (f)) for Cases I-III. Points “a” to “j” in the insert of parts (a) and (b) correspond to the instants shown in Figure 6.9 (Case I). Points “a” to “j” in the insert of parts (c) and (d) correspond to the instants shown in Figure 6.10 (Case II). Points “a” to “j” in the insert of parts (e) and (f) correspond to the instants shown in Figure 6.11 (Case III).

Parts (c) and (d) of Fig. 6.8 show that for Case II the textured runner performs much better than the untextured one. There is a gain of 0.35 microns in the pad's position (from 2.70 to 3.05), and a 15.7% decrease in the friction coefficient (from 0.0287 to 0.0242). As in Case I, the friction is dominated by the Couette term, which this time is significantly *decreased* by the texture. The corresponding instantaneous profiles of pressure and film thickness are shown in Fig. 6.10. The contact in this case is conformal (large- R pad), and one observes that the main pressurization takes place at the left end of the pocket, irrespective of its position under the pad. Each pocket “carries” its own pressure field with it as it moves to the right. The lift force generated in this way pushes the pad further away from the runner, thus reducing the average friction force (dominated by the Couette term proportional to $1/h$).

Finally, the pad position and friction coefficient of Case III are plotted versus time in parts (e) and (f) of Fig. 6.8. This configuration exhibits some striking features. The textured runner produces a friction that is lower than that of the untextured one, but not, as in Case II, by lifting the pad farther away. In fact, the minimum clearance of the textured runner is 4.44, while that of the untextured one it is 5.71. The friction coefficient, in turn, is lower for the textured runner (0.0087) than for the untextured one (0.0101). The instantaneous profiles of pressure and film thickness that are shown in Fig. 6.11 help understand this apparent paradox. On the right half of each pocket, the diverging geometry generates a vast cavitating zone. In this zone, the saturation variable θ falls below θ_s thus making the Couette contribution to the friction to vanish.

Figure 6.9: Instantaneous pressure and film thickness profiles for **Case I** at times: (a) $t = 0.9920$ (b) $t = 0.9984$ (c) $t = 1.0048$ (d) $t = 1.0112$ (e) $t = 1.0176$ (f) $t = 1.0240$ (g) $t = 1.0304$ (h) $t = 1.0368$ (i) $t = 1.0432$ (j) $t = 1.0496$ (Points “a” through “j”, respectively, in the inserts of Figures 6.8(a) and 6.8(b)).

Figure 6.10: Instantaneous pressure and film thickness profiles for **Case II** at times: (a) $t = 5.9008$ (b) $t = 5.9104$ (c) $t = 5.9168$ (d) $t = 5.9232$ (e) $t = 5.9296$ (f) $t = 5.9360$ (g) $t = 5.9424$ (h) $t = 5.9488$ (i) $t = 5.9552$ (j) $t = 5.9616$ (Points “a” through “j”, respectively, in the inserts of Figures 6.8(c) and 6.8(d)).

Figure 6.11: Instantaneous pressure and film thickness profiles for **Case III** at times: (a) $t = 0.9920$ (b) $t = 0.9984$ (c) $t = 1.0048$ (d) $t = 1.0112$ (e) $t = 1.0176$ (f) $t = 1.0240$ (g) $t = 1.0304$ (h) $t = 1.0368$ (i) $t = 1.0432$ (j) $t = 1.0496$ (Points “a” through “j”, respectively, in the inserts of Figures 6.8(e) and 6.8(f)).

Figure 6.12: Pressure profiles with fixed pad position $Z=2.5$ at time (a) $t=1.092$ and (b) $t=1.200$ ($R=51.2$).

The pressure generation mechanism that takes place in close-packed square arrays of pockets (such as in Case III) of different radii is shown in Fig. 6.12. There, the pad position is fixed at $Z = 2.5$ microns, and we examine the pressure field (along $x_2 = \lambda/2$) at two different instants for 10- μm -deep pockets of radii 100, 200 and 400 microns. It is clear from the picture that there is a pressurized region along the left part of each pocket, which acts as a texture-cell-level wedge. The pressurized region moves with the pocket but is otherwise quite constant. The peak pressure and the load carrying capacity increase with the pocket size, so that the optimal texture is a closed-pack square array of pockets approximately the size of the pad width. The optimal depth as suggested by our study is 10 microns, though it is probably load-dependent.

III A further study with one-dimensional textures

This section focuses on one-dimensional problems to allow a more encompassing study, comprising 2500 texture configurations, two curvature radius of the pad, three loads and three pad masses. As previously done, as the textures are on the runner surface each simulation must be run in unsteady mode, with a small enough time step to capture the vertical oscillations of the pad as textures pass under it.

III.1 Description of numerical tests

Again, an effort has been made in this section to refine the space and time discretizations until the results become discretization-independent. Very fine spatial meshes are needed for this, because each texture cell needs to be discretized into one hundred or more finite volumes. The coarsest spatial mesh used consisted of 512 cells along x , and some simulations were double-checked with up to 4096 cells. Although the method is fully-implicit, a unit-CFL condition was enforced, leading to time steps that ranged from $\Delta t = 4 \times 10^{-3}$ to $\Delta t = 5 \times 10^{-4}$.

The typical running time of one simulation on the 512-cells mesh is about five minutes in a state-of-the-art desktop, while about three hours are needed for the finest mesh.

The numerical experiments consider velocity and applied load as constant. The runner's texture is assumed periodic, with period λ , i.e.;

$$h_t(x) = h_t(x + \lambda) \quad \forall x$$

An initial transient takes place as the pad adjusts its average vertical position. This transient lasts between 1 and 8 time units. After it, the pad reaches a periodic vertical motion, with time period λ (notice that the non-dimensional velocity is $V_1 \equiv 1$), and all quantities become periodic. It is for $T > 8$ that the tribological quantities from equations 6.3 and 6.4 are computed.

The contacting surface of the pad is assumed to be the same ones of the previous section. Since the length of the ring is 1, radii R greater than about 128 correspond to essentially conformal contacts.

Let us analyze on what non-dimensional parameters the solution of the problem depends. Since we focus on the periodic regime, the initial conditions for θ , Z and Z' are irrelevant. Starvation effects are not addressed. For this purpose, d_{oil} was chosen large enough to assure fully-flooded conditions, thus making this parameter also irrelevant. There only remain two parameters in the model: The mass per unit width m and the applied

load W .

The default values that we select for these parameters are

$$m_0 = 2 \times 10^{-5} \quad \text{and} \quad W_0 = 1.666 \times 10^{-4}$$

Along the experiments, other values were tested so as to assess the problem's sensitivity to the data.

When considering realistic scale values such as $U = 10$ m/s, $L = 10^{-3}$ m, $H = 1$ micron, $\mu = 4 \times 10^{-3}$ Pa-s, the value selected for m_0 corresponds to 0.048 kg/m.

The value selected for W_0 , with the same scales as above, corresponds to an applied load of 40 N/m, corresponding to a Stribeck number $S_0 = 10^{-3}$. This value is relatively low, but it was selected so as to ensure hydrodynamic lubrication conditions.

III.2 Effects of the runner's texture

A large set of experiments was conducted in the simplified setting of one-dimensional transverse textures. Further, the runner was assumed to be sinusoidal of depth d , namely

$$h_t(x) = d \frac{\cos(2\pi x/\lambda) - 1}{2} \quad (6.5)$$

(notice that h_t is a “height” (positive upwards), so that it is maximal at crests and minimal at troughs). Such a runner was selected so as to observe the dynamics of the system under maximal smoothness of the contacting surfaces. The applied load was fixed at W_0 corresponding to a Stribeck number $S = 10^{-3}$. At this load, the equilibrium clearance of the pad for an untextured runner depends on R as shown in Table 6.2, which also shows the friction coefficient for each R . The minimal friction is obtained for $R = 8$, while the maximum clearance takes place for $R = 16$. Notice that for an untextured runner the equilibrium values of f and C do not depend on m .

General description of the intervening phenomena

The study was conducted for different values of the texture parameters d and λ , also considering different values of the pad's curvature radius R . The mass of the pad was fixed at $m = m_0$. Some selected cases are discussed below (see Table 6.3 for details of each case). Notice that some cases have rather large values of λ , of the order of the pad's length. Though this could be viewed as a “waviness” of the runner instead of as a texture, we keep the word “texture” for all values of λ .

Case 1: Consider first a pad with a moderate value of $R = 32$, sliding on a runner with a texture of period equal to the pad length ($\lambda = 1$) and depth $d = 5$. The system behavior

R	f	$C_{\min}$
4	7.607×10^{-2}	6.369
8	7.449×10^{-2}	7.809
16	8.122×10^{-2}	8.021
32	9.562×10^{-2}	7.408
64	1.189×10^{-1}	6.363
128	1.536×10^{-1}	5.231
256	2.030×10^{-1}	4.325
512	2.612×10^{-1}	3.695
1024	2.871×10^{-1}	3.324

Table 6.2: Friction coefficient f and clearance $C_{\min}$ for pad's with several values of R once they reached the equilibrium position over an *untextured* runner. The computations were made for a load $W = W_0$.

Case #	W^a	R	λ	d	$\bar{f}$	$C_{\min}$	V_f	V_C
1	1.66×10^{-4}	32	1	5	0.082	6.89	-14%	-7%
2	1.66×10^{-4}	32	0.1	5	0.114	5.66	+19%	-24%
3	1.66×10^{-4}	256	0.1	5	0.193	2.52	-5%	-42%
4	1.66×10^{-4}	256	1	5	0.084	6.33	-59%	+46%

Table 6.3: Details of the cases discussed in Sections III.2. W stands for the applied load, m for the linear mass, R for the curvature radius of the pad, λ for the texture's period (wavelength), d for its depth, $\bar{f}$ for the numerically obtained average friction coefficient and $C_{\min}$ for the numerically obtained minimum clearance. V_f and V_C stand for the relative variations of $\bar{f}$ and $C_{\min}$ with respect to the untextured case. All quantities are non-dimensional. Notice that Case 4 is introduced in subsection III.3.

is then periodic in time with period equal to one. Redefining $t = 0$ as the instant at which the texture crest is exactly under the left edge of the pad, profiles of p and θ are shown in Fig. 6.13 for $t = 0, 0.25, 0.5$ and 0.75 .

Figure 6.13: Instantaneous profiles of pressure p (in red) and saturation θ (in blue) for a bearing with $R = 32$, $d = 5$ and $\lambda = 1$ (Case 1 in the text) at $t = 0, 0.25, 0.50$ and 0.75 (from top to bottom) once the periodic regime has been attained. The steady pressure profile corresponding to the untextured runner is also plotted for comparison (in green).

At $t = 0$ the crests of the texture coincide with the left ($x = 0.5$) and right ($x = 1.5$) edges of the pad, while the trough coincides with the pad's centerline ($x = 1$). Notice that, *since the texture is moving with the runner*, its *convergent* wedge corresponds to the sector between a crest and its neighboring trough *to its right*, where the upward normal is tilted *to the right*. This is the opposite to what occurs on the pad's surface, in which the *convergent* wedge corresponds to the left half of the pad, at which the (downward) normal

is tilted to the left. Analogously, if the upward normal to the runner is tilted to the left this corresponds to a *divergent* wedge.

At $t = 0$, thus, the convergent wedge of the texture is located under the left half of the pad, coincident with the convergent wedge of the pad's surface. This leads to a pressure profile similar to that of the untextured case, but much larger in value. At $t = 0.25$ the pressure peak has moved to the right, accompanying the texture's convergent wedge, and the net effect is still that of increasing the average pressure under the pad. At $t = 0.5$ the texture's crest is under the pad's center, leaving the left half of the pad over the texture's divergent wedge and consequently at zero pressure. A cavitation bubble develops there, which is evident from the θ profile ($\theta < 1$ implies cavitation). This cavitation bubble, which first appears at $t \simeq 0.304$ and $x \simeq 0.514$, grows and moves to the right with the divergent wedge that generates it, passing under the pad's center and eventually leaving the pad to the right. In fact, this same cavitated region but corresponding to the previous texture cell can be observed leaving the pad at $t = 0.5$ (just notice the region with $\theta < 1$ near $x = 1.5$). The cavitated region is always located at the *divergent* wedge of the texture and travels with it. Its right boundary is a *rupture* boundary, and as expected θ is continuous there and $\partial p / \partial x = 0$. The left boundary of the cavitated region is a *reformation* boundary, and as such the θ profile is very steep there, and $\partial p / \partial x$ exhibits a jump.

Remark: It is worth pointing out that the profiles at $t = 0$ show a cavitation boundary at $x \simeq 1.15$ for both the textured and the untextured cases. This boundary is a *rupture* boundary in the untextured runner and a *reformation* boundary in the textured one.

Case 2: With the same pad ($R = 32$) and the same texture depth ($d = 5$), it is interesting to consider a texture size much smaller than the pad's length. Figure 6.14 shows profiles of p and θ at four instants of the periodic regime obtained with $\lambda = 0.1$. Since the time period is equal to λ , and again defining $t = 0$ when a crest is at the left edge of the pad, the instants shown in Fig. 6.14 correspond to $t = 0, 0.025, 0.05$ and 0.075 .

In this case the convergent wedge of the pad dominates the pressure field in the left part of the contact. No cavitation bubbles form there, and the effect of the texture is merely a modulation of the pressure field with local maxima/minima at the convergent/divergent wedges of the runner, respectively. The right part of the contact, on the other hand, exhibits moving pressurized regions (local pressure peaks) that are absent in the untextured case. The texture's convergent wedges generate locally pressurized regions, with cavitation bubbles between them, and all this structure moves to the right with the runner.

Remark: In fact, a careful analysis of the θ field near the left edge of the pad shows that a tiny cavitation bubble appears there when the divergent wedge of the texture enters

the contact, but it collapses soon afterwards and has no significant effect on the pressure field. Cavitation bubbles that appear near the left edge and collapse (or not) under the pad are discussed further later on.

Figure 6.14: Instantaneous profiles of pressure p (in red) and saturation θ (in blue) for a bearing with $R = 32$, $d = 5$ and $\lambda = 0.1$ (Case 2 in the text) at $t = 0, 0.025, 0.050$ and 0.075 (from top to bottom) once the periodic regime has been attained. The steady pressure profile corresponding to the untextured runner is also plotted for comparison (in green).

Case 3: The last case we consider in this general description corresponds to the same texture as before ($\lambda = 0.1$, $d = 5$), but now with a high-conformity pad ($R = 256$). The corresponding profiles of p and θ at times $t = 0, 0.025, 0.05$ and 0.075 are shown in Fig. 6.15. One observes that a cavitation bubble develops whenever the divergent wedge of a texture enters the contact, and that this bubble then travels with the texture until leaving the

pad at its right edge. Also, a locally pressurized region develops at the convergent wedges of each texture and travels with it. The height of the pressure peaks are slightly modulated by the pad's shape, but there is no doubt that the overall solution is governed by the runner's texture. The cavitation bubbles generated at each texture impose a zero-pressure boundary condition for the pressurized region that develop at the convergent texture wedges. The global pressure field simply consists of the juxtaposition of these otherwise independent local pressure peaks.

Figure 6.15: Instantaneous profiles of pressure p (in red) and saturation θ (in blue) for a bearing with $R = 256$, $d = 5$ and $\lambda = 0.1$ (Case 3 in the text) at $t = 0, 0.025, 0.050$ and 0.075 (from top to bottom) once the periodic regime has been attained. The pressure profile corresponding to the untextured runner is also plotted for comparison (in green).

Figure 6.16: Relative differences (a) V_f and (b) V_C (expressed in percentages) with respect to the untextured runner, as functions of the texture parameters d and λ for a ring with $R=32$. The colorbars (as the isolines) indicate these percentages. Parts (c) and (d) of the Figure show analogous plots for $R=256$. All simulations computed with $W = W_0$ and $m = m_0$. The specific cases 1 to 4 discussed in the text are shown as white dots.

III.3 Friction and clearance charts

In this section the parameter space is explored for the selected configuration. Thousands of runs were made varying R between 4 and 1024, λ between 0.1 and 2, d between 0 and 10, and $S = 10^{-3}$ or $S = 0.5 \times 10^{-3}$.

R=32

Very little or no friction reduction appears for $R = 16$ or smaller. A bearing of $R = 32$ can be thus considered of moderate conformity for this configuration. Taking $m = m_0$ and $W = W_0$ the parameters are those of which one specific example was discussed as Case 1 of Section III.2. For the specific depth $d = 5$ and the specific period $\lambda = 1$ we observed in Fig. 6.13 the pressure and saturation fields. The average friction coefficient $\bar{f}$ is 14% smaller for Case 1 than for an untextured liner. This significant improvement is however accompanied by an 7% loss in minimum clearance, suggesting an increase in wear.

It is interesting to see these values in the context of values obtained for all other possible textures, at least within some ranges. For this purpose, we performed 2500 simulations spanning the whole ranges of d and λ and plotted two-dimensional contours in the $d - \lambda$ plane in Fig. 6.16. The quantities plotted are:

- The relative variation V_f of the average friction coefficient, defined as

$$V_f(d, \lambda) = \frac{\bar{f}(d, \lambda) - f_{\text{untextured}}}{f_{\text{untextured}}}$$

- The relative variation V_C of the minimal clearance, defined as

$$V_C(d, \lambda) = \frac{C_{\min}(d, \lambda) - C_{\min, \text{untextured}}}{C_{\min, \text{untextured}}}$$

Please notice that the untextured case corresponds to the vertical axis of the plots ($d = 0$).

In Fig. 6.16(a) a contour map of V_f is shown. The axis $d = 0$ has obviously $V_f = 0$, and one observes that, depending on whether $\lambda > 0.5$ or not, V_f becomes negative or positive when the texture depth d is increased. For $\lambda > 0.5$ the friction diminishes as the texture is made deeper. Relative diminutions of about 30% can be seen at the top right corner of the chart, corresponding to $d = 10$ and $\lambda = 2$. Interestingly, for some given depth there exists an optimum period in terms of friction. It is roughly of value $\lambda = 1$ (texture's wavelength equal to the pad's length), but it increases with d to about 1.7 for $d = 10$. Notice that Case 1 discussed in Section III.2 lies close to the optimum period for $d = 5$. From the position on the diagram one may conclude that Case 1 is indeed representative of the friction-reducing textures for pads with $R = 32$.

The contour map of V_C shown in Fig. 6.16(b), in turn, shows that all textures have minimal clearances smaller than that obtained with an untextured runner. Only negative values of V_C are observed in the chart.

An interesting phenomenon occurs at the bottom right sector of Figs. 6.16(a) and (b). Steep changes in $C_{\min}$ when d is varied around $d = 8$ are evident in the sector $\lambda < 0.2$. Significant variations of $\bar{f}$ are also observed there. The physics behind this phenomenon can be understood starting from Case 2 in Section III.2, which has $d = 5$ and $\lambda = 0.1$. The texture-generated oscillations in the pressure field (see Fig. 6.14) are superposed on the baseline pressurization curve generated by the pad's geometry, of which the pressure profile generated on the untextured liner provides an estimate. As d is increased keeping the other parameters fixed at the values of Case 2, some of the pressure oscillations touch the cavitation pressure and bubbles appear near the left edge of the pad. For $d = 8.15$ the

p and θ profiles are as shown in Figure 6.17 (top graph). A bubble is seen to have grown on the left side of the pad. This cavitation bubble travels to the right with the divergent microwedge that generated it, but does not succeed to pass under the pad's center. Along its travel to the right, at some instant it gets pressurized and collapses (the value of θ goes back to 1 and the pressure becomes positive when the microwedge reaches $x \simeq 0.7$). One other bubble is generated at the same divergent wedge when it gets to $x \simeq 1.15$, as can be seen in the figure, which then travels until the right edge of the pad.

Figure 6.17: Profiles of p and θ in the periodic regime corresponding to bearings with $\lambda=0.1$, $R=32$, $m = m_0$ and $W = W_0$. The depth for the top graph is $d=8.15$ and for bottom one it is $d=8.20$.

Between $d = 8.15$ and $d = 8.2$ a catastrophic event (in the mathematical sense) takes place. Increasing the texture depth makes the divergent microwedges steeper and the cavitation bubbles created near the inlet now travel under the pad without collapsing until the right edge. The pressure field undergoes a major change, since the relatively large pressurization region that existed between $x = 0.7$ and $x = 1.15$ now has disappeared. The pad loses lift and only re-encounters equilibrium at a much lower position. All these features can be seen in Fig. 6.17 (bottom graph). In Fig. 6.18 we plot the vertical movement of the pad as a function of time when the pad starts from the position $Z(0) = 4$. For $d \leq 8.15$ it climbs to values of Z of about 4.5, but for $d \geq 8.2$ it descends to about 2 units from the x -axis.

This interesting phenomenon, which does not depend on the model adopted for the friction force, suggests caution against carelessly increasing the depth of the texture, since

Figure 6.18: $Z(t)$ for textured bearings with $\lambda=0.1$, $R=32$, $m = m_0$ and $W = W_0$. Noticed the dramatic change in behavior when d changes from 8.15 to 8.20.

tribological performance deteriorates substantially if this transition occurs. Experimental investigations could bring light onto the actual physical occurrence of this prediction of the Elrod-Adams model. Cases of high sensitivity of the friction coefficient to the texture depth have already been experimentally detected by Scaraggi *et al* [100].

R=256

Case 3 of Section III.2 has $R = 256$, $d = 5$ and $\lambda = 0.1$. It corresponds to the central point of the horizontal axis in the graphs of V_f and V_C in Figs. 6.16 (c) and (d). Again, 2500 simulations were performed to compute the data for these graphs.

The value $R = 256$ corresponds to a high-conformity contact, in which the pressure field consists of a train of local pressurized regions (one at each convergent microwedge) traveling under the pad (see Fig. 6.15). Case 3 is an example of a poorly designed texture, because increasing the period λ a little (to 0.4 for example) would reduce the friction by about 40% while keeping the minimum clearance at a value similar to that of the untextured bearing. The best predicted performances in terms of both friction and wear correspond to $\lambda \simeq 1.5$ and $d = 10$.

In order not to depart too much from Cases 1-3 while investigating the friction-reducing sector of the charts, let us consider **Case 4**, which has the same runner as Case 1 but with the pad of Case 3; i.e.,

$$\textbf{Case 4: } R = 256, \quad d = 5, \quad \lambda = 1$$

For this bearing, the texture brings more than 60% improvement in friction together with

Figure 6.19: Instantaneous profiles of pressure p (in red) and saturation θ (in blue) for a bearing with $R = 256$, $d = 5$ and $\lambda = 1$ (Case 4 in the text) at $t = 0, 0.025, 0.050$ and 0.075 (from top to bottom) once the periodic regime has been attained. The pressure profile corresponding to the untextured runner is also plotted for comparison (in green).

40% increase in minimum clearance, as compared to the untextured results. Instantaneous plots of pressure and saturation at $t = 0, 0.25, 0.5$ and 0.75 are shown in Fig. 6.19. Instead of the train of ten bubbles and ten pressure peaks translating under the pad as it was in Case 3, one now essentially observes a single cavitation bubble and a single pressurized region during most of the period. The larger size of the instantaneous pressurized region allows the pressure (and consequently the hydrodynamic force) to attain values comparable to those of the untextured case without diminishing (in fact, increasing) the clearance with respect to the untextured value. A comparison of $Z(t)$ and $F(t)$ for Cases 1 and 4 can be found in Fig. 6.20, where the complete evolution from an initial position $Z(0) = 4$ is shown (the instantaneous friction force is depicted just for one period). Notice that both cases, though having widely different values of R , have similar average F .

III.4 Increasing the load

A high sensitivity to the load is not a positive characteristic for a bearing. Even if the bearing works under constant *dimensional* load and at constant velocity, the *non-dimensional* load W is likely to vary in time because of its dependence on the viscosity of the lubricant.

To explore this issue, additional charts of V_f and V_C , analog to those of Fig. 6.16, were computed with twice the load, i.e., with $W = 2W_0$. The results can be found in Fig. 6.21. Let us compare part (a) of the figure, corresponding to V_f contours for $R = 32$, with its reference-load counterpart of Fig. 6.16(a). One sees that doubling the load makes the friction-reduction region in the $d - \lambda$ plane to shrink a little, but the global qualitative and quantitative trends are preserved. The abrupt transition discussed in the previous sections changes its location slightly, but otherwise the bearing performance is not severely affected by the change in load. Similar remarks can be made about the minimum clearance examining part (b) of Fig. 6.21.

The weak sensitivity of $V_f(d, \lambda)$ to W is also evident from the chart corresponding to $R = 256$, which is found in Fig. 6.21(c). The chart of V_C , in turn (Fig. 6.21(d)), in turn, exhibits some more sensitivity to the load. The maximum relative gain in V_C , which is more than 40% for the reference load, reduces to about 25% for $W = 2W_0$. The beneficial effect of the texture of Cases 1 and 4 is nevertheless maintained at this larger load.

III.5 Discussion of results

For low-conformity bearings (here considered as defined by $R/\ell \leq 16$) the results above confirm that no beneficial effects in either friction or wear are predicted by the model, which is consistent with previous findings [62, 38, 60].

For moderate and high-conformity bearings (here represented by the cases $R/\ell = 32$

Figure 6.20: Ring position $Z(t)$ and friction force $F(t)$ (detail in figure) for pads with a curvature radius R of (a) 32 (b) 256. The texture parameters are $\lambda=1.0$ and $d=5$. Notice that (a) corresponds to Case 1 in the text, while (b) corresponds to Case 4.

and $R/\ell = 256$, respectively), on the other hand, the two-dimensional contour plots of friction and clearance together with the specific simulations (cases 1-4) selected for detailed scrutiny, revealed some *general* trends and underlying physical mechanisms:

1. Friction coefficients of high-conformity bearings are greater than those of moderate-conformity ones. In both cases, textures of period comparable to the pad's length and sufficiently deep seem to be optimal in terms of friction for the ranges considered. The mathematical model predicts a relative reduction of friction of up to 40% for moderate-conformity bearings and up to 75% for high-conformity ones.
2. The optimal period (in terms of friction and wear) for each depth d is predicted to be comparable to the pad's length ($\lambda \simeq 1$) and a slowly increasing function of d .
3. Though textures can indeed reduce the friction of moderate-conformity bearings, all textured runner produce pad-to-runner clearances that are smaller than that of the untextured runner.

Figure 6.21: Same organization of results as in Fig. 6.16, but changing the applied load to $W = 2W_0$.

4. Suitable textures significantly improve the clearance of high-conformity bearings (by 20% or more), thus predicting reductions in wear.
5. The basic mechanism of friction reduction is a local pressurization of the convergent microwedges at each texture cell, accompanied by local cavitation at the divergent microwedges. The cavitation bubble that forms at each texture cell prevents the appearance of local negative pressure peaks that would cancel out the positive lift force generated at the convergent microwedges. The extra lift generated in this way increases the clearance and reduces friction.
6. This explains why longer texture periods (wavelengths of order L) are predicted to be more efficient than shorter ones, since the capacity of a wedge to generate lift increases with length.
7. Best performance is achieved when the depth is of the order of twice the pad's fly height Z , so that the convergent microwedge (which has film thickness $Z + d$ at the trough and Z at the crest, approximately) has taper ratio $(Z + d)/Z$ in the range

2.5 – 3.5. Taper ratios in this range are known to be effective not only for generating load capacity (the Rayleigh step, which is optimal, has a slightly smaller taper ratio of 1.866) but also for minimizing friction[95].

Figure 6.22: Texture depth corresponding to minimal friction $d_f(\lambda)$ as a function of the period λ . Shown are numerical data corresponding to $R = 32$ and $R = 256$, in each case considering loads $W = W_0$, $W = 2W_0$ and $W = 3W_0$.

8. To prove the previous assertion, let us present the results in Figs. 6.16 and 6.21 in another way. For each period λ there is a depth $d_f(\lambda)$ that minimizes friction, which is shown in Fig. 6.22. For $R = 256$, the minimal-friction depth $d_f(\lambda)$ is zero, corresponding to the untextured liner, only for periods smaller than 0.15. For $\lambda > 0.15$ one observes $d_f(\lambda)$ increasing steadily with λ . A similar trend is observed for $R = 32$, though in this case the untextured liner is optimal until λ is about 0.5 (depending on the charge). There is also a corresponding minimal clearance, $C_{\min}(d_f(\lambda), \lambda)$, with can be taken as a representative value for the ring's fly height. One can thus plot the taper ratios corresponding to minimal friction for each λ , which is given by

$$t_f(\lambda) = \frac{C_{\min}(d_f(\lambda), \lambda) + d_f(\lambda)}{C_{\min}(d_f(\lambda), \lambda)} \quad (6.6)$$

This is done in Fig. 6.23 for $R = 32$ and $R = 256$ and for loads $W = W_0$, $W = 2W_0$ and $W = 3W_0$. Consider first the curve corresponding to $R = 256$ and $W = W_0$. The taper ratio that yields minimal friction is remarkably constant from $\lambda = 0.2$ up to $\lambda = 1.4$, taking values in the range 2.5–3.5. As the load is

Figure 6.23: Taper ratio corresponding to minimal friction $t_f(\lambda)$ as a function of the period λ . Shown are numerical data corresponding to $R = 32$ and $R = 256$, in each case considering loads $W = W_0$, $W = 2W_0$ and $W = 3W_0$.

increased, and although $d_f(\lambda)$ decreases accordingly (see Fig. 6.22), the optimal taper ratio remains in the same range (2.5–3.5) but up to a smaller value of λ (up to $\lambda = 1.2$ for $W = 2W_0$ and up to $\lambda = 1$ for $W = 3W_0$). One thus concludes that for highly-conformal bearings the Elrod-Adams model suggests selecting the texture depth so that the taper ratio is between 2.5–3.5, which corresponds to d in the range between $1.5 C_{\min}$ and $2.5 C_{\min}$.

9. For the case $R = 32$ one observes that the untextured liner is optimal up to $\lambda \simeq 0.5$ (in fact, 0.45 for $W = W_0$ and 0.55 for $W = 3W_0$). At $\lambda \simeq 0.5$ the optimal taper ratio $t_f(\lambda)$ undergoes a jump to a value of about 2.5 and from there increases steadily with λ , remaining below 3.5 up to $\lambda = 1$. The model thus again suggests to take d in the range $1.5 C_{\min} - 2.5 C_{\min}$, though only for texture periods larger than half the pad's length this time.
10. Since $d/C_{\min}$ must be $\simeq 1.5 - 2.5$ to be effective, larger loads require shallower textures. This puts forward the challenge in designing textures for variable loads.
11. An interesting behavior is predicted for textures with short wavelength ($\lambda < 0.25$). As the depth d is increased, cavitation bubbles begin to form at the left edge of the pad over each divergent microwedge. Each of these bubbles travels with the wedge that generated it, but it collapses under the pressurization effect of the converging

part of the pad. When the wedge gets to the divergent part of the pad, a new bubble forms on it and travels with it until reaching the right edge of the pad. This leaves a central portion of the pad pressurized.

12. For moderate-conformity bearings, the previous behavior persists until the depth reaches a limit value of about $d = 8$. If d is further increased, the bubbles no longer collapse under the pad and the central pressurized portion of the pad no longer exists. As a consequence, the clearance is severely reduced (by up to 50%!). This sudden transition should be further investigated by experimental techniques, since our results are model dependent.
13. For high-conformity bearings a similar transition occurs, but at a much lower value of d .

IV An application: full engine cycle simulation with mathematically defined textures on the liner

We can take advantage of the knowledge gained in the preceding sections of this chapter to analyze the effects of pocketing in a concrete application: a three-ring pack sliding against a textured liner, with realistic velocities and loading conditions. Plausible improvements of texturing the whole stroke length are studied. Notice that all values here are dimensional, in agreement with the conference publication [41]. A three-ring pack composed of a compression, a wiper and a oil control ring is set up as in figure 6.24. Dimensions, as well as relevant data for the simulations are furnished in table 6.4.

Figure 6.24: Ring's arrangement in the computational domain. From left to right: oil control, wiper and compression rings. The piston's head is facing to the right.

Each ring endures changing conditions along the cycle. These changing conditions are originated from the variations in piston velocity and gas pressure. For the case analyzed here, engine data for a rotating speed of 2000 rpm is provided in Figure 6.25. The parameters for the Greenwood-Tripp contact model are defined by $\sigma=0.04 \mu\text{m}$, $K=0.05$ and $E'=100 \text{ GPa}$. The oil viscosity is considered constant and equal to $\mu=0.004 \text{ Pa.s}$.

From the results of section II.2 improvements in friction are expected when the conformity degree of the lubricated pair is high, that is, when both surfaces are nearly parallel. Looking at fig. 6.24, we could expect that a gain is more likely to be obtained with the oil control ring. Whats more, better results are expected with dimples of about the size of the contact. This is also shared by the findings in section III, however, the dimple optimal depth depends on the minimum clearance, which is also dependent upon the load and piston velocity. Thus, a single dimple arrangement along the stroke's length will not be optimal for varying conditions, moreover, it could be pernicious in certain parts of the engine cycle. Even though, the results presented here are restricted to a comparison between an untextured and a single-texture textured liner.

Ring r	W_r^{ps} (N/m)	m_r (kg/m)	ℓ_r (mm)	ϕ_r	a_r (mm)	b_r (mm)
1: compression	-325	0.03	1.7	0.9	12.0	13.7
2: wiper	-290	0.05	2.1	0.25	5.5	7.6
3: oil control	-704	0.05	2.6	0.0	1.0	3.6

Table 6.4: Ring data.

IV.1 Numerical tests

Two simulations were performed: the first with an untextured liner and the second with the previously defined square arrangement of semi-ellipsoidal dimples. For the latter $l = 0.075$ mm, $d = 10$ μ m and $\lambda = 0.2$ mm thus giving a total textured area of 44.2%. The initial film thickness as the lubricant feeding conditions are set to $d_{oil}=5.0$ μ m.

All rings start at $Z(0)_i=2.0$ with $Z'(0)_i=0$. The domain size is such that $x_\ell=0$, $x_r=14.5$ mm and is discretized in 2048×16 finite volumes with a time step such that the maximum Courant number is 1.56 at the instant of maximum piston velocity. Tolerance in the residue is set to 10^{-5} of the residue in the first relaxation at each time step (computed in the Euclidean norm). Enough relaxation sweeps/V-cycles in the multigrid are taken to achieve this precision.

The instantaneous quantities $C_r(t)$ (*minimum clearance*) and $F_r(t)$ (*friction force per unit width*) are computed for rings $r=1,2,3$, once the transients have passed and a periodic regime has been established. In this study it required two engine cycles.

Results for C (in microns) are seen in figure 6.26, with the clearance for the fully textured liner shown with a full line. The dotted lines are reserved for the untextured liner. This is kept for all following figures.

Clearance

A logical way to assess the outcome of the simulations is to study the effects of texturing in clearance for the four engine strokes: intake, compression, power and exhaust; then the texturing effects in friction force can be easily explained.

A relevant aspect determining the oil film thickness along the engine cycle is the notorious asymmetry of the profiles of both compression and wiper rings. From TDC to

Figure 6.25: (a) Piston velocity (b) Combustion chamber pressure.

BDC (intake, power) they have reduced load carrying capacity and thus experience much lower mean clearances. On the other hand, from BDC to TDC (compression, exhaust) they have a moderate wedge that grants larger clearances. This trend is common for both rings as seen in figure 6.26.

The minimum clearance curve of the oil control ring accompanies the piston ring velocity curve, as it is almost symmetrical in the x_1 direction and does not withstand the gas pressure forces ($\phi_3 = 0$). In consequence, clearances are quite similar in the down and upstrokes. This is true for both the textured and untextured liners, although the textures have a detrimental effect on C , which is diminished from 0.64 (untextured) to 0.30 (both average values of the complete engine cycle).

Now that this broader overview of the results has been set, we can focus on the compression and wiper rings and grasp finer details of the ring's dynamics:

- *Intake* (0 to 180 degrees): In the interval $0 \leq \alpha \leq 65$ the compression ring clearance increases due to the recently deposited oil during the exhaust. At approximately $\alpha=65$ for the untextured liner and $\alpha=55$ for the textured one, the starvation effect of the little oil left by the wiper ring is felt, with a significant drop-off in C . From this point on, the compression ring for the untextured liner stays in the mixed to

boundary lubrication regime, the load being held mostly by the contact force, while for the textured liner there is enough oil to generate some carrying capacity and separate the surfaces. This "extra" oil comes from two sources: a) the oil trapped in the dimples and afterwards; b) from the oil supplied in behalf of the oscillating movement of the wiper ring, from $\alpha \simeq 70$ to $\alpha \simeq 135$.

- *Compression* (180 to 360 degrees): As the sides with larger load carrying capacity face upwards for the compression and wiper rings, clearances increase with velocity. When the TDC (top dead center) is approached, the pressure in the combustion chamber grows rapidly. This causes a severe drop in C , which is faster for the compression ring. The textures have a deleterious effect on clearance, being C much lower for the textured liner.
- *Power* (360 to 540 degrees): Results can be appraised realizing that the power stroke is similar to the intake with a large load decreasing from TDC to BDC (bottom dead center). By that reason clearances are much lower, although they still present the same behavior as in the intake stroke. A brisk decline is seen again due to starvation at $\alpha \simeq 428$ for the compression ring with the untextured liner, while for the textured a slight increase in C is observed from approximately that same point until $\alpha \simeq 530$.
- *Exhaust* (540 to 720 degrees): During the exhaust, the piston experiences the same dynamics as in the compression event but the pressure in the combustion chamber is lower (atmospheric or nearly) and virtually constant along the stroke. This leads to an analogous disposition of the clearances, even though they are larger and more widespread for both compression and wiper rings by virtue of the lower loads.

Friction force

Figure 6.27 portrays the dimensionless friction force for each ring. Maximum friction is attained coincident with lower velocities and particularly for the compression and wiper rings, with higher gas pressures.

The compression ring endures the highest friction forces (844.8 N/m), followed by the wiper ring (331.6 N/m) and the oil control ring (78.0 N/m). For the first two, this is attained at the TDC immediately after the power stroke begins ($\alpha = 360$). The friction at this point is significantly higher than for the rest of the engine cycle, and the maximum value is not influenced by the liner texturing. Close to the TDC during the compression-power stages textures have a detrimental effect, increasing friction.

Figure 6.26: Minimum clearance for the three rings, from top to bottom: compression, wiper and oil control ring.

Despite this situation, at mid-stroke where higher velocities takes place significant reductions in friction can be achieved. This is seen for the compression ring in the intervals $65 \leq \alpha \leq 200$ and $429 \leq \alpha \leq 560$. The intervals where this happens for the wiper ring are more numerous. The pocketed liner induces more friction for the oil control ring (figure 6.26), specially at and near dead centers. Little reduction is seen at mid-stroke. These results hint about not texturing close to the dead centers, specially near the TDC, thereby avoiding the regions where texturing increases the friction.

The friction force can be separated in its contact and hydrodynamic parts, as is shown in figure 6.28. It can be noticed that the hydrodynamic components for the three rings are similar in magnitude and slightly higher for the untextured liner all along the engine cycle, while the largest friction values are due mainly to the contact force. The reductions in friction discussed in the previous paragraph were possible as a result of a marked decrease in the contact force.

Figure 6.27: Total friction force in each ring.

Power losses

Power losses are proportional to the piston velocity and the friction force. Maximum values in the friction force were identified near dead centers, however at those points the piston velocity is low or null. These lower velocities mitigate the high friction, specially at TDC. As no reduction in friction was achieved with texturing near both TDC and BDC, neither is in power loss. Results for P are depicted in figure 6.29: around $\alpha = 360$ the power loss is increased for the textured liner for the compression and wiper rings, and also around $\alpha=0,180$ and 540 for the oil control ring.

On the other hand, friction attains moderately high values at mid-stroke, where it is effectively reduced by the textured liner. This greatly reduces the power loss around $\alpha=100,450$ for the compression and wiper rings. Only minor cutbacks are observed in the results for the oil control ring around $\alpha=100,250,470$ and 610 .

Average values of power loss are presented for the three rings and the whole pack in table 6.5, which summarizes the previous discussion. Results for the untextured and textured liners are compared, and the relative variation with respect to the former is

Figure 6.28: Friction force separated in its hydrodynamic and contact components, from top to bottom: compression, wiper and oil control ring.

displayed in the last column.

Reductions of 33.5% and 13.9% are possible for the first two rings with a textured liner. The oil control ring contributes negatively, increasing the energy lost to friction in 34.1% with respect to the untextured liner simulations, yet the net effect for the entire ring pack is a drop of 12% in power loss.

V Summary of the chapter

In the first part of this chapter we have studied the consequences of texturing the runner surface on the tribological performance of a generic pad/runner contact (observations are then applicable to other devices, such as thrust bearings and lip seals) with fixed loads and fixed runner velocity, that corresponds to hydrodynamic lubrication conditions.

The study was carried on through hundreds of two dimensional mesh-converged runs, each of which involved a transient dynamically-coupled calculation. This was done using the methods described in Chapters 4 and 5.

Ring	Textured	Untextured	% Variation
Compression	174.8	263.2	-33.5
Wiper	185.2	215.2	-13.9
Oil control	176.4	131.6	+34.1
Whole pack	536.4	609.6	-12.0

Table 6.5: Power loss (W/m) per ring and total for the ring pack.

It was observed that :

- The ring profile determines whether pocketing the liner is beneficial or not;
- The more conformal the contact, the greater the reduction that can be obtained in the friction by texturing the liner with pockets of properly chosen parameters;
- For flatter pads ($R \leq 5$ centimeters) and the specific conditions of the study a close-packed square array of circular pockets seems to be a good choice in terms of clearance and friction;
- Up to 73% reduction in the friction coefficient and up to 86% increase in minimum film thickness were numerically obtained;
- These results are interesting if it is realized that in applications such as the piston-ring/liner contact the curvature radius of the rings of the pack increase as they undergo wear, and thus the benefits of texturing increase with time (see, e.g., [51]);
- Close-packed textures may be structurally fragile, since the walls between pockets are very thin. Physical restrictions on the acceptable textures should be incorporated before choosing an optimal one.

The time-varying conditions of actual devices, such as the ring/liner contact in internal combustion engines, in which the piston velocity and the load on the rings change along the engine cycle and starved conditions take place at several instants, make the results reported here not directly applicable to the practical selection of a cylinder liner texture. To guide

Figure 6.29: Power loss in each ring.

such selection, a broader study encompassing a wider range of texture parameters, multiple loads and pad masses was needed. This was done in Section III. To allow the thousands of simulations required for such endeavor, the problem was simplified to one-dimensional transverse sinusoidal textures. Based in the preceding results, two R values were selected, portraying the conformal and non-conformal contact cases.

The following *general* trends and underlying physical mechanisms were reached after a careful analysis of the results:

- For low-conformity bearings ($R/L \leq 16$) the results confirm that no beneficial effects in either friction or wear are predicted by the model;
- Friction coefficients of high-conformity bearings are greater than those of moderate-conformity ones. In both cases, textures of period comparable to the pad's length and sufficiently deep ($d \simeq 10$) seem to be optimal in terms of friction for the ranges considered. The relative reduction of friction predicted is up to 40% for moderate-conformity bearings and up to 75% for high-conformity ones.

- The optimal period (in terms of friction and wear) for each depth d seems to be a slowly increasing function of d .
- Though friction can indeed be reduced for moderate-conformity bearings, all textured runners produce pad-to-runner clearances that are smaller than that of the untextured runner.
- Suitable textures ($\lambda \simeq 1$, $d \geq 3$) significantly improve the clearance of high-conformity bearings (by 20% or more), thus predicting reductions in wear.
- The basic mechanism of friction reduction is a local pressurization of the convergent microwedges at each texture cell, accompanied by local cavitation at the divergent microwedges. The cavitation bubble that forms at each texture cell prevents the appearance of local negative pressure peaks that would cancel out the positive lift force generated at the convergent microwedges.
- As a consequence, longer texture periods (wavelengths) are predicted to be more efficient than shorter ones, since the capacity of a wedge to generate lift increases with length.
- An interesting behavior is predicted for textures with short wavelength ($\lambda < 0.25$). As the depth d is increased cavitation bubbles begin to form at the left edge of the pad over each divergent microwedge. Each of these bubbles travels with the wedge that generated it, but it collapses under the pressurization effect of the converging part of the pad. When the wedge gets to the divergent part of the pad, a new bubble forms on it and travels with it until reaching the right edge of the pad. This leaves a central portion of the pad pressurized.
- For moderate-conformity bearings, the previous behavior persists until the depth reaches a limit value of about $d = 8$. If d is further increased, the bubbles no longer collapse under the pad and the central pressurized portion of the pad no longer exists. As a consequence, the clearance is severely reduced (by up to 50%!). This sudden transition should be further investigated by experimental techniques, since our results are model dependent.
- For high-conformity bearings a similar transition occurs, but at a much lower value of d .

The sensitivity of the above conclusions to the load applied on the pad, an equivalent (5000 simulations) were run with twice the load. The differences observed in the friction

and clearance charts are rather small and show that our conclusions are quite insensitive to changes in the applied load. Further investigations extending the ranges of the study reported here can of course unveil new phenomena.

Within the limitations of the model here adopted, the aim of this generic study was to provide a guideline for the selection of textures for actual lubricated devices, such as the piston-ring/liner problem. With the former results in mind, a texture was selected for the numerical simulation of a three-ring pack enduring a complete cycle of a typical diesel engine running at 2000 rpm. It was observed that highest friction takes place for the compression and wiper rings nearby the top dead center (TDC), in the transition from the compression to the power stroke. No improvements were seen with the textured liner in this part of the engine cycle. However, the textures had a beneficial effect in clearance, friction and power loss at mid-stroke, where the piston endures high velocities and the hydrodynamic effect of the pocketed surfaces are important. A drop of 12% in power loss is predicted for the textured surface.

These numerical experiments predict that in order to maximize the benefits of texturing, the pocketed portion should be placed at the intervals of maximum piston velocity, leaving the regions close to the dead centers untextured.

Direct simulation of measured surfaces

Direct, deterministic or measured-surface simulations [30] aim to solve lubrication problems down to the roughness scale. It means using the measured topography of the surface without resorting to averaged or stochastic models. This avoids ambiguities in the definition of average coefficients at the expense of solving Reynolds equation with a discretization finer than the resolution of the measurements.

To be able to solve these problems in acceptable time, numerical and computational techniques must be used. Besides that, some care must be taken, as the short wavelength of the surface roughness and the low clearances put the Reynolds equation to its limits of validity. To assess this issue, a comparison is made with results obtained with the Navier-Stokes equations.

The organization of this chapter is the ensuing: first, a review of the methods used to treat surface roughness in numerical simulations is made. In Section II, the Reynolds equation results and the friction force computed with the lubrication approximation developed in Chapter 3 are validated against single-phase Navier-Stokes simulations. These single-phase results are used to make a further comparison of the friction formula against friction force models commonly found in the literature. Results with the Elrod-Adams model in simulations of honed surfaces are given in Section III. The chapter concludes with simulations made in collaboration with researchers from the R&D department of **Mahle GmbH** Brazil, a automotive parts manufacturer based in Stuttgart, Germany ([1]).

Topographies of actual honed liners were supplied to perform full engine cycle simulations, which are shown and discussed in Section IV.

I A brief review of methods assessing rough surfaces

Theoretical and numerical studies of lubricated devices often represent the contacting surfaces as smooth. While this approximation stands under some conditions, this is not the case when the film thickness is comparable to the roughness amplitude, as in heavily loaded journal bearings, seals, piston ring/cylinder liner contacts, among others. The introduction of surface roughness into lubrication models can be traced back to the stochastic model of Tzeng and Saibel [110]. In their work, a probability distribution is adopted for the surface roughness and formulas of the expected values of pressure, friction force and load-carrying capacity are deduced for a one-dimensional problem. A more encompassing model was given by Christensen [45], who presented a modified (stochastic) Reynolds equation in which the coefficients were expected values. These results were limited to transverse or longitudinally oriented variations in roughness.

Later, Patir and Cheng [89, 90] introduced a new approach. An average Reynolds equation for rough surfaces was defined in terms of pressure and shear flow factors, which were empirical functions of the non-dimensional roughness. Elrod [56] later extended this model to account for roughness anisotropy.

All of these techniques are based on the intuitive idea of decoupling the global length scale, corresponding to the whole bearing, from the local length scale of the roughness. Homogenization theory formalizes this intuition. It develops an average equation valid throughout the domain, with its coefficients computed from solutions of local problems. The homogenization method has been studied in depth both for incompressible ([25, 26]) and compressible ([70, 31, 34]) fluids. It exhibits good accuracy when the roughness is periodic in space and its period is much smaller than the bearing size. Unfortunately, no rigorous homogenization model considers general roughness shapes with cavitation effects.

To avoid such uncertainties, in this work we resort to the direct simulation of measured surfaces.

II Hydrodynamic friction force models

A central result of the simulations is the hydrodynamic force exerted by the fluid, defined by

$$\mathbf{F} = \int_S \boldsymbol{\sigma} \cdot \mathbf{\check{n}} dS, \quad (7.1)$$

of which the component parallel to the movement is the *friction force*. Different formulae for the lubrication approximation of (7.1) appear in the literature. Patir and Cheng [90] identified the existence of the three terms F^{cou} , F^{poi} and F^{pre} in the lubrication approximation of the friction force. While some authors consider all three terms ([38, 42, 83]), there exist publications in which only F^{cou} is considered (thus not requiring knowledge of the pressure [97, 75, 103, 96, 74]) and others in which F^{pre} is neglected [93, 4].

We consider three models for the friction force. The first model is the one obtained from the asymptotic expansion of Chapter 3:

$$F_L^I = F_L^{cou} + F_L^{poi} + F_L^{pre}. \quad (7.2)$$

The second one [93, 4],

$$F_L^{II} = F_L^{cou} + F_L^{poi}, \quad (7.3)$$

neglects the normal contribution of the pressure, while the third model [97, 75, 103, 96, 74],

$$F_L^{III} = F_L^{cou}, \quad (7.4)$$

computes the friction force with just the Couette term. Analogous definitions can be made for F_U^I , F_U^{II} and F_U^{III} .

It is to be noticed that only models F^I and F^{III} satisfy that the forces on the upper and lower surfaces add up to zero. For model F^{II} this is obvious, while for model F^I it is

verified as follows:

$$F_L^I(t) + F_U^I(t) = - \int_{\Omega^+} \left(h \frac{\partial p}{\partial x_1} - p \frac{\partial h_L}{\partial x_1} + p \frac{\partial h_U}{\partial x_1} \right) d\Omega \quad (7.5)$$

$$= - \int_{\Omega^+} \left(h \frac{\partial p}{\partial x_1} + p \frac{\partial (h_U - h_L)}{\partial x_1} \right) d\Omega \quad (7.6)$$

$$= - \int_{\Omega^+} \left(h \frac{\partial p}{\partial x_1} + p \frac{\partial h}{\partial x_1} \right) d\Omega \quad (7.7)$$

$$= - \int_{\Omega^+} \frac{\partial (ph)}{\partial x_1} d\Omega = - \int_{\partial\Omega^+} ph \check{\mathbf{n}} \cdot \check{\mathbf{e}}_1 d\Gamma = 0, \quad (7.8)$$

where $\partial\Omega_+$ is the cavitation boundary.

II.1 Assessment of friction force formulae against single-phase Navier-Stokes simulations

In the previous section we have presented three models for the calculation of the friction force from the results (p and θ) of the Elrod-Adams method. The first one, denoted by F^I , was obtained from an asymptotic expansion, while the other two (F^{II} and F^{III}) omit terms that appear in F^I and have been taken from the literature. In this section we assess the accuracy of the three models against accurate numerical solutions of the incompressible Navier-Stokes equations. A remark must be made here: the comparison presented in this section neglects both inertia and cavitation effects. Neglecting cavitation implies $\theta \equiv 1$, so that the Elrod-Adams result is, in fact, the solution of the Reynolds equation. To deal with the cavitation issue more sophisticated CFD simulations involving moving boundaries and phase change would be necessary.

To further simplify the comparison, we consider all variables independent of x_2 and solve the flow at the single instant $t = 0$ in the two-dimensional domain

$$\{(x_1, x_3) \mid 0 \leq x_1 \leq \lambda, h_L(x_1) \leq x_3 \leq h_U(x_1)\}, \quad (7.9)$$

using a finite element in-house code which has been extensively validated (see [35, 13, 46]). The unknowns are the velocity field $\mathbf{u}_h$ and the pressure field p_h . The boundary conditions are: (a) $\mathbf{u}_h = (0, 0)$ on the upper surface. (b) $\mathbf{u}_h = (V, 0)$ on the lower surface. (c) periodic boundary conditions at $x_1 = 0$ and $x_1 = \lambda$. The pressure is thus determined up to an additive constant, which has no effect on the value of the friction force. Once $\mathbf{u}_h$ and p_h

Figure 7.1: (a) View of the honed measured surface with depth in micrometers, (b) Abbott-Firestone curve for the surface shown in Figure (a) and (c) fractional distribution of the measured surface according to the norm of its gradient $\|\nabla h_L\|$.

have been solved for, a nodal approximation of

$$\boldsymbol{\sigma}_h = -p_h \mathbb{I} + \mu (\nabla \mathbf{u}_h + \nabla \mathbf{u}_h^T) \quad (7.10)$$

is built. The friction force on a surface S is then computed from

$$F^{NS} = \int_S (\boldsymbol{\sigma}_h \cdot \mathbf{\tilde{n}}) \cdot \mathbf{\tilde{e}}_1 dS \quad (7.11)$$

assuming $\boldsymbol{\sigma}_h$ independent of x_2 . Above, $\mathbf{\tilde{e}}_1$ denotes the horizontal unit vector $(1, 0, 0)$. In all simulations it was checked that mesh convergence of the Navier-Stokes solver had been attained, with discretization error not larger than 1% in all relevant variables.

The numerical experiments consider sinusoidal profiles for both the upper and lower surfaces:

$$h_U(x_1) = A [1 + \alpha \cos(2\pi x_1/\lambda)], \quad h_L(x_1) = -A [1 + \alpha \cos(4\pi x_1/\lambda)] \quad (7.12)$$

where $2A$ is the mean distance between the surfaces and αA the waviness amplitude for both surfaces. The wavelength in the upper surface is λ , while that of the lower surface is $\lambda/2$. A scheme of the geometrical setting is shown in Figure 7.2.

Figure 7.2: Scheme of the geometrical setting to compare Reynolds and Navier-Stokes results.

The simulations consider $A = 0.3 \mu\text{m}$, $\lambda = 200 \mu\text{m}$, and several values of α between 10^{-3} and 0.75. From the definitions (7.12) one observes that

$$\frac{h_{\max}}{\lambda} = \frac{2(1+\alpha)A}{\lambda} \quad \text{and} \quad \|\nabla h\|_{\max} = \frac{17.2 \alpha A}{\lambda}.$$

Figure 7.3: Comparison of the pressure profiles for different values of the relative waviness amplitude α as obtained by numerically solving the Reynolds equation (continuous lines) and the Navier-Stokes equations (circles). In the latter case, the pressure profile corresponds to $x_3 = 0$.

For our values $A/\lambda = 1.5 \times 10^{-3}$ the *lubrication hypotheses*

$$\frac{h}{\ell} \ll 1, \quad \|\nabla h\| \ll 1, \quad (7.13)$$

are satisfied (here ℓ is the length scale of the problem in the $x_1 - x_2$ plane) and good agreement between the Navier-Stokes solution and the Reynolds solution is expected.

The Navier-Stokes solver was run on a very fine mesh consisting of 160,000 linear triangular elements. Its results were compared with those of the Reynolds solver run on a mesh of 2,000 one-dimensional cells. Figure 7.3 shows that both models predict exactly the same pressure field.

Let us now compare the three friction models F^I , F^{II} and F^{III} , as computed from the Reynolds pressure solution, with the force F^{NS} obtained from the Navier-Stokes solution. This is plotted in Figure 7.4. One observes that model F^I agrees with F^{NS} in the whole range of the relative amplitude α . The other two models, though coincident with F^{NS} when $\alpha = 0$ (parallel bearing), completely fail to reproduce the dependence of the friction force with α .

In simulations of measured surfaces, however, the lubrication hypotheses (7.13) are close to their limit of validity, especially the second one. Figure 7.1 illustrates a specific

Figure 7.4: Total horizontal force as a function of the relative waviness amplitude α . The Navier-Stokes reference values F^{NS} are plotted with circles (for $\alpha = 0.25, 0.5$ and 0.75), and they are seen to coincide with the results of the first model F^I . The other two models fail to reproduce the dependence of the friction force with α .

topography, corresponding to a honed cylinder, which is an example of the kind of measured data for the function h_L . Part (b) of the figure shows the fractional distribution $a(\|\nabla h_L\|)$ on the surface. For any number s , the value $a(s)$ is the fraction of the surface which has $\|\nabla h_L\| > s$. The gradient is computed by discrete differentiation of the matrix of measured points. By direct inspection of Fig. 7.1(c), one observes that there are points where the hypothesis $\|\nabla h_L\| \ll 1$ is not satisfied.

It is thus also interesting to use our two-dimensional toy example to evaluate the accuracy of lubrication models against Navier-Stokes predictions for values of $\|\nabla h_L\|$ of order unity, focusing on the accuracy of friction predictions.

Fixing $A = 0.3 \mu\text{m}$ and $\alpha = 0.75$, simulations with smaller values of λ were performed, as shown in Table 7.1. For $\lambda = 2 \mu\text{m}$ the maximum $\|\nabla h_L\|$ is 1.935, in clear violation of (7.13) and one observes that all lubrication friction formulas provide poor predictions. The pressure and velocity fields produced by the Navier-Stokes solver are shown in Fig. 7.5. Vertical gradients of the pressure field are evident, a clear sign that the lubrication approximation does not hold. For $\lambda \geq 10 \mu\text{m}$, which corresponds to $\|\nabla h_L\|_{\max} \leq 0.387$, Table 7.1 shows that the lubrication approximation with friction model F^I predicts the

reference value F^{NS} with good accuracy, while models F^{II} and F^{III} fail.

Figure 7.5: Contours of pressure (p , top line) and velocity components (u_1 , u_3 , middle and bottom lines) corresponding to the Navier-Stokes simulations with $\lambda = 2$ (left column), 10 (center column) and 20 μm (right column).

Going back to our surface of interest with honed finishing, one observes in Fig. 7.1(c) that 99.9% of its area has $\|\nabla h_L\| < 0.387$. Though with some caveats, this suggests also that using model F^I the lubrication approximation may provide a reasonable estimate for the friction in a measured honed surface.

On the basis of these results, we conclude that the friction force model F^I , which can be obtained from an asymptotic expansion, is the best formula for approximating the friction force in lubrication models based on the Reynolds equation. This model, which consists of three terms (Couette, Poiseuille and Pressure terms), exhibits good accuracy when applied to surfaces with characteristics similar to those of honed surfaces and will be considered hereafter as *correct*. In the following section we solve the Elrod-Adams model using the measured honed surface data so as to estimate the importance of each term in the

friction formula under realistic conditions.

λ (μm)	$\ \nabla h\ _{\max}$	F_L^{NS}	F_L^I	F_L^{II}	F_L^{III}
2	1.935	-0.890	-0.603 (32%)	-0.337 (62%)	-0.378 (57%)
10	0.387	-3.198	-3.016 (5.7%)	-1.687 (47%)	-1.891 (41%)
20	0.193	-6.148	-6.032 (1.9%)	-3.374 (45%)	-3.782 (38%)
200	0.019	-60.36	-60.32 (<0.1%)	-33.74 (44%)	-37.81 (37%)

Table 7.1: Comparison of the friction lubrication models F_L^I , F_L^{II} and F_L^{III} with the Navier-Stokes result F_L^{NS} for short waviness wavelengths. For all cases $\mu = 0.01$ Pa-s, $V = 10$ m/s, $A = 300$ μm and $\alpha=0.75$. The second column is $\max_{x_1} |\partial h / \partial x_1|$. The friction force per unit width (along x_2), integrated over one wavelength λ in x_1 , is given in Newton/meter. Between parentheses the relative error of each model as compared to F_L^{NS} . Analogous results are obtained if the integration is performed on the upper surface.

III Numerical simulations of measured surfaces

We consider as upper surface a single barrel-shaped pad, inspired in the shape of compression rings of car engines. Its shape corresponds to an arc of circumference of radius R . Its height h_U with respect to the reference plane is:

$$h_U(x_1, x_2) = \begin{cases} Z + R - \sqrt{R^2 - (x_1 - \frac{L}{2})^2} & \text{if } a \leq x_1 \leq b \\ e & \text{otherwise} \end{cases} \quad (7.14)$$

The parameter Z is the minimal distance between the surface and the $x_1 - x_2$ plane and e is a constant much larger than Z . This pad slides with constant velocity $-V$ over the measured topography of a cross-hatched finished plane (honed runner). This finish is typical of cylinder liners of car engines. The measured surface is the one shown in Figure 7.1(a). A proper way to characterize a honed surface is by means of its Abbott-Firestone curve [3]. This is given in Figure 7.1(b) for the surface under study. Its related roughness parameters R_k (plateau height), R_{pk} (reduced peak height), R_{vk} (reduced valley height), M_{r1} (percentage of R_{pk} peaks) and M_{r2} ($1-M_{r2}$ can be understood as the percentage of R_{vk} valleys) are shown in Table 7.2, along with the more conventional parameters S_a (roughness average), S_q (root mean square roughness), S_{sk} (skewness) and S_{ku} (kurtosis).

The measured surface dimensions are $L_s \times L_s$ with $L_s = 1.8$ mm, sampled each 2 μm

Parameter	Value
S_a	0.15 μm
S_q	0.25 μm
S_{sk}	-6.56
S_{ku}	81.8
R_{pk}	0.18 μm
R_k	0.2 μm
R_{vk}	6.47 μm
M_{r1}	6.6%
M_{r2}	75.9%

Table 7.2: Surface-finish parameters for the surface of Figure 7.1(a).

in both directions. In order to decrease the computational effort, instead of solving the problem on the whole surface, it is solved on three strips S_1 , S_2 and S_3 of width $B=0.2$ mm, as indicated in the same figure.

The measured points provide a function $x_3 = h_s(x_1, x_2)$. The reference plane $(x_1 - x_2)$ is defined as the average height of the measured points, so that

$$\int_0^{L_s} \int_0^B h_s(x_1, x_2) dx_1 dx_2 = 0 . \quad (7.15)$$

The domain of this function along x_1 is just the interval $[0, L_s]$, which is too small to conduct simulations. We extended it periodically to tackle this difficulty. Incorporating also the movement of the runner with respect to the simulation frame of reference (fixed to the pad) one gets the function $h_L(x_1, x_2, t)$ as follows: Let n be an integer (negative in general) such that $2nL_s \leq x_1 - Vt < 2(n+1)L_s$, then

$$h_L(x_1, x_2, t) = \begin{cases} h_s(x_1 - Vt - 2nL_s, x_2) & \text{if } 0 \leq x_1 - Vt - 2nL_s < L_s \\ h_s(2L_s - (x_1 - Vt - 2nL_s), x_2) & \text{if } L_s \leq x_1 - Vt - 2nL_s \leq 2L_s \end{cases} . \quad (7.16)$$

The resulting function h_L is depicted at the top of Fig. 7.6, in which the sector covered by the computational domain at time $t = 0.8 \times 10^{-4}$ s is indicated. The remaining physical parameters defining the domain size, pad dimensions, fluid viscosity and surface velocity are given in table 7.3. Reflection boundary conditions are imposed at $x_2 = 0$ and $x_2 = B$. A constant film height d is assumed at the entrance, which amounts to enforce $\theta = d/h$

at $x_1 = x_{1\ell}$. The chosen d value ensured fully-flooded conditions for all the simulations performed.

Parameter	Value	Units	Description
μ	0.01	Pa.s	fluid viscosity
a	0.2	mm	leftmost position of pad in x_1
b	0.8	mm	rightmost position of pad in x_1
d	20	μm	fluid film height at the entrance $x_1 = x_{1\ell}$
e	500	μm	clearance for $x_1 < a$ and $x_1 > b$
V	10	m/s	sliding velocity
$x_{1\ell}$	0	mm	left boundary of computational domain
x_{1r}	1.0	mm	right boundary of computational domain
B	0.2	mm	width of computational domain
L	1.0	mm	length of computational domain
R	8, 128	mm	radii of pad
Z	0.5, 1.0, 2.0, 4.0	μm	distance from pad to the reference plane $x_1 - x_2$

Table 7.3: Physical parameters defining the problem.

III.1 Mesh convergence study

The complex topography of the runner surface compels the use of a fine mesh, but it is unclear how fine they should be as compared to the spacing of the measured points (which is $2 \mu\text{m}$ in our case). A mesh convergence study is reported here to determine the level of discretization required to decrease the error in the solution to acceptable levels.

The study was performed on the strip S_1 with parameters $Z=2 \mu\text{m}$ and $R=128 \text{ mm}$. A typical snapshot of the solution is shown in Fig. 7.6. Part (a) shows the pressure over the computational domain, while part (b) shows a cross-section along $x_2 = B/2$. Parts (c) and (d) show the saturation field θ , in which regions with $\theta < 1$ identify cavitated zones. Finally, parts (e) and (f) show the friction force density f , defined as

$$f = \mu \frac{V}{h} g(\theta) + \frac{1}{2} h \frac{\partial p}{\partial x_1} - p \frac{\partial h_L}{\partial x_1}, \quad (7.17)$$

which is observed to be highly oscillatory in both space and time.

Figure 7.6: Fields (a) p , (c) θ and (e) f at $t=1.6$ for strip S_1 . This solution corresponds to parameters $Z=2.0$ and $R=128$. The right column shows cross-sections of the fields along $x_2 = B/2$.

The computational domain Ω was discretized using 500×100 , 1000×200 , 2000×400 and 4000×800 finite volumes. This corresponds to $\Delta x_1 = \Delta x_2 = 2, 1, 0.5$ and $0.25 \mu\text{m}$, respectively. A recent measured surface simulation by Bouassida [30] used $\Delta x_1 = \Delta x_2 = 0.78 \mu\text{m}$ and a mesh of 1400×512 cells. The time step was taken such that the Courant number $C = (V/2)(\Delta t/\Delta x_1)$ equals unity for all meshes, that is $\Delta t = 0.4 \times 10^{-6}, 0.2 \times 10^{-6}, 0.1 \times 10^{-6}$ and $0.05 \times 10^{-6} \mu\text{s}$ and 1500, 3000, 6000 and 12000 time steps for each mesh, respectively.

Convergence was numerically verified for all variables. Figures 7.7 (a) and (b) show the hydrodynamic lift,

$$L(t) = \int_a^b \int_0^B p(x_1, x_2, t) dx_1 dx_2, \quad (7.18)$$

and the friction force $F_L^I(t)$ as functions of time for the four meshes. Clearly, the difference between the two finest meshes is negligible for practical purposes. Part (c) of the figure contains a cross-section of p along $x_2 = B/2$ at an arbitrary instant ($t = 1$), showing that pointwise convergence is also (almost) verified with the same refinement.

This study suggests that the 2000×400 mesh, with mesh spacing taken as one fourth of the measurement spacing, is practically converged and correctly represents the exact solution between the pad and the measured topography. This mesh is thus adopted in all results that follow.

It is also interesting to point out that the choice $B=0.2\text{mm}$ makes the results representative of the whole surface sample of width $L_s=1.8\text{mm}$, as seen in Table 7.4. The whole sample was solved in a solved with a 2000×3596 mesh, which took 57 hours of computation in a 12-cored Intel Xeon X5690. The surface is then divided in nine strips of width $B=0.2\text{mm}$. Differences in hydrodynamic lift and friction force with the whole surface results are 2.1% and 0.2% respectively, which justifies the chosen width of each strip.

Figure 7.7: (a) Hydrodynamic lift, (b) friction force and (c) pressure at $t=1.0$ in $x_2 = 0.1$ computed with the four meshes of the convergence study.

Strip	$\overline{L(t)}$ (N)	$\overline{F(t)}$ (10^{-3} N)
1	1.905	21.10
2	2.138	21.35
3	2.009	21.50
4	2.089	21.26
5	2.073	21.49
6	2.148	21.59
7	2.167	21.73
8	2.226	21.47
9	2.178	21.43
Sum	18.993	192.92
Whole surface	18.6	192.53
Difference	2.1 %	0.2%

Table 7.4: Average hydrodynamic lift $\overline{L(t)}$ and friction force $\overline{F(t)}$ for the nine contiguous strips of width 0.2 mm in which the surface of figure 7.1(a) is subdivided, and for the whole surface. $Z=0.5$ stands for all nine simulations.

III.2 Term-by-term contribution to friction

Results were obtained for the three strips S_1 , S_2 and S_3 , two different curvature radii $R=8$ and 128 mm, and for four separations between the pad and the runner: $Z = 0.5, 1.0, 2.0$ and $4.0 \mu\text{m}$, totaling 24 runs of the code. Each run takes about 3.0 hours on a six-cored Intel i7-4930K desktop computer with a 3.4 GHz clock rate. In the simulated conditions there is no contact between the surfaces, since $\max_{(x_1, x_2) \in \Omega} h_L(x_1, x_2, t) = 0.37 \mu\text{m}$ for all t .

The relative contribution of each of the three terms of the friction force (model F_L^I) are calculated as

$$\frac{F_L^{cou}(t)}{F_L^I(t)}, \quad \frac{F_L^{poi}(t)}{F_L^I(t)} \quad \text{and} \quad \frac{F_L^{pre}(t)}{F_L^I(t)}. \quad (7.19)$$

Notice that the sign of each term is kept, so that though the relative contributions add up to one the individual terms can take any sign. They are shown as functions of time for strip S_1 in Figure 7.8. Parts (a) and (b) correspond to the Couette term for $R=8$ and 128 mm, respectively. The general trend is that the Couette term becomes increasingly dominant as Z is increased, and also less oscillatory in time due to the higher average clearance. For the higher curvature pad, $R = 8$ mm, the time-averaged contribution of the Couette term is 68% for $Z = 0.5 \mu\text{m}$, growing up to 89% for $Z = 4 \mu\text{m}$. For the lower curvature pad, $R = 128 \mu\text{m}$, it is 89% for $Z = 0.5 \mu\text{m}$ and 99% for $Z = 4 \mu\text{m}$.

Figure 7.8: Friction force terms F_L^{cou} , F_L^{poi} and F_L^{pre} relative to the friction force F_L for strip S_1 , $R = 8$ (figures (a), (c), (e)) and $R=128$ (figures (b), (d), (f)).

Parts (c) and (d) of Figure 7.8 correspond to the Poiseuille term. Its contribution to the total friction force diminishes as Z increases, opposite to the trend seen with the Couette term. This term's relative contribution changes significantly with time at low Z . Interestingly, it is positive for the higher curvature pad and negative for the lower curvature one, which can be considered a conformal contact. For the rough surface chosen for this study, the Poiseuille term is thus seen to compensate to some extent the Couette term in conformal contacts at low clearance, reducing the friction force computed by the model.

The pressure term, on the other hand, is mostly positive (see parts (e) and (f) of the figure), as explained in [38]. Furthermore, it is consistently larger than the Poiseuille term for the low curvature pad, at all values of Z , while for the $R=8$ pad it is much smaller. The

net effect of $F_L^{poi} + F_L^{pre}$ is a *positive* contribution to the friction, which can be confirmed by noticing that the curves of F_L^{cou}/F_L^I never exceed unity. Results of the term-by-term analysis of strips S_2 and S_3 showed the same general trends as S_1 .

From the relative contribution of each term one can quantify the error resulting from adopting models F^{II} or F^{III} :

$$e^{II} = \left| \frac{F^{II} - F^I}{F^I} \right|, \quad (7.20a)$$

$$e^{III} = \left| \frac{F^{III} - F^I}{F^I} \right|. \quad (7.20b)$$

The average values of these errors for the twenty-four conditions considered are shown in Table 7.5. Results for the three strips present similar values and tendencies while varying the parameters R and Z .

The error of F^{II} for the results computed with $R=128$ (first column) are the lowest in the table, which can be explained by the small pressure terms, as seen in Fig. 7.8(e). The second lowest are the F^{III} errors for the results computed with $R=128$ (last column). This can be explained by: a) the aforementioned subtraction of the Poiseuille and pressure terms for $R=128$ and b) lower average Poiseuille and pressure terms (Figures 7.8(d) and 7.8(f)) at low Z values. For the same model the errors computed for $R=8$ are the largest in the table. While the pressure term drops rapidly with Z (Figure 7.8(e)) the Poiseuille term remains relatively high (approximately 10% for $Z=4.0$), thus making e^{III} of about the same order of the Poiseuille term for high Z values.

The error e^{II} represents the omission of the pressure term in F^I . The contribution of the pressure term decreases with increasing Z , the errors are below 4% and also comparable in size for both R values and $Z \geq 2.0$. Larger errors are observed for lower Z values.

In general, omitting terms in the friction force has a significant impact on friction evaluation, especially at low clearances. For $Z = 0.5$ the errors are of about 10% or as large as 30%.

Strip	Z	% e^{II} ($R=8$)	% e^{II} ($R=128$)	% e^{III} ($R=8$)	% e^{III} ($R=128$)
S_1	4.0	1.1	0.72	10.97	0.7
	2.0	1.8	3.1	18.6	1.9
	1.0	3.4	6.8	26.2	4.7
	0.5	7.2	14.6	32.2	10.8
S_2	4.0	0.0	0.0	10.6	0.0
	2.0	1.2	1.9	18.6	1.5
	1.0	2.0	5.2	26.1	4.1
	0.5	6.2	12.6	32.3	10.3
S_3	4.0	1.1	0.7	10.8	0.7
	2.0	1.2	1.9	19.0	1.5
	1.0	2.4	4.8	26.0	3.9
	0.5	5.6	11.6	32.2	9.9

Table 7.5: Percentage errors e^{II} and e^{III} , averaged in time over a passing of the honed runner.

If Z is large enough compared with the surface's roughness it is expected that the problem solution will be governed by the length scale of the bearing. Thus, it is interesting to compute the difference between the results of the average friction force F^I extracted from the fully deterministic simulations and F_{untext}^I , computed neglecting the surface roughness ($h_L(x_1, x_2, t) \equiv 0$):

$$e_{untext} = \left| \frac{F_{untext}^I - F^I}{F^I} \right|. \quad (7.21)$$

This is shown in Table 7.6. For $Z=0.5$, e_{untext} is above 10% and as high as 24%. However, for $Z \geq 2.0$ the errors are lower or equal to 7%. The later results are remarkable due to the low errors attained and to the fact that a computationally inexpensive problem is

being solved instead of a complex measured-surface simulation. This was already pointed out by Bouassida [30] while comparing results of deterministic simulations of various honed surfaces with one-dimensional untextured-runner simulations.

Strip	R	Z			
		0.5	1.0	2.0	4.0
S_1	8	10.5	4.7	2.3	1.2
	128	23.1	12.4	5.3	1.1
S_2	8	11.2	5.0	2.3	1.2
	128	23.9	13.1	6.0	0.2
S_3	8	11.5	5.3	2.9	1.3
	128	24.3	13.4	6.4	0.16

Table 7.6: Percentage error e_{untext} committed by neglecting the surface roughness.

III.3 Influence of the cavitation model

Surface roughness leads to several cavitated regions, such as the ones seen in Figures 7.6(c) and 7.6(d). In this scenario, mass-conservation is extremely important. We can compare results computed with the Elrod-Adams model, with Reynolds (Swift-Stieber) boundary conditions and with the Reynolds equation. This is done in figure 7.9 for strip S_1 , $Z=2.0$, $R=128$ and the 2000×400 mesh for a complete passing of the texture under the pad.

Figure 7.9: (a) Hydrodynamic lift, (b) friction force and (c) pressure at $t=1.4$ in $x_2 = 0.1$ for two cavitation models (Elrod-Adams and Swift-Stieber) and the Reynolds equation results, which disregard cavitation.

The Swift-Stieber boundary conditions leads to higher values of $L(t)$. This can be explained with Figure 7.9(c), where the pressure profile $p(x_1, x_2, t)$ at $x_2 = B/2$, $t = 1.4 \times 10^{-4}$ is shown. Pressurized regions are clearly larger than with the Elrod-Adams, which induces higher pressure build-ups. This is due to lack of mass-conservation, as already pointed out by Ausas et al [12]. The Reynolds model predicts large negative pressurized regions, mostly in the divergent part of the bearing, thus giving a much lower load carrying capacity.

The friction force is depicted in Figure 7.9(b). The one computed with the Reynolds equation results (no cavitation model at all) is the largest due to: a) a dominant Couette component of the friction force, and b) that is integrated all under the pad, that is in $a \leq x_1 \leq b$. For the same reason, the Swift-Stieber results are the second largest: the model predicts fewer cavitated regions than the Elrod-Adams model. The large difference in friction force is noteworthy.

III.4 Dynamic simulations on other measured surfaces

Table 7.8 presents six surface samples. $\mathcal{A}$, $\mathcal{B}$, $\mathcal{D}$, $\mathcal{E}$ and $\mathcal{F}$ of are honed surfaces while $\mathcal{C}$ is a surface obtained with another surface finish process, which creates a coating in the surface of the bore (liner) by a thermal spraying process. Each sample's dimension and measuring resolution are given in Table 7.7. The one used in the simulations of the previous sections of this Chapter is $\mathcal{E}$.

Surface	L_s (mm)	Resolution (μm)
$\mathcal{A}$	1.241	0.982
$\mathcal{B}$	1.241	0.892
$\mathcal{C}$	0.84	0.822
$\mathcal{D}$	0.84	0.822
$\mathcal{E}$	1.8	2.0
$\mathcal{F}$	1.0	1.0

Table 7.7: Surface sample dimensions and resolution.

Texture	$x_1[\text{mm}] \times x_2[\text{mm}] \times h_L[\mu\text{m}]$	Abbott-Firestone parameters	Texture	$x_1[\text{mm}] \times x_2[\text{mm}] \times h_L[\mu\text{m}]$	Abbott-Firestone parameters
$\mathcal{A}$		<ul style="list-style-type: none"> • $R_{pk}=2.64$ • $R_k=0.91$ • $R_{vk}=7.97$ • $M_{r1}=5\%$ • $M_{r2}=74\%$ 	$\mathcal{D}$		<ul style="list-style-type: none"> • $R_{pk}=0.85$ • $R_k=0.86$ • $R_{vk}=1.57$ • $M_{r1}=9\%$ • $M_{r2}=86\%$
$\mathcal{B}$		<ul style="list-style-type: none"> • $R_{pk}=0.84$ • $R_k=0.62$ • $R_{vk}=2.94$ • $M_{r1}=7\%$ • $M_{r2}=79\%$ 	$\mathcal{E}$		<ul style="list-style-type: none"> • $R_{pk}=0.18$ • $R_k=0.21$ • $R_{vk}=6.46$ • $M_{r1}=3\%$ • $M_{r2}=77\%$
$\mathcal{C}$		<ul style="list-style-type: none"> • $R_{pk}=0.71$ • $R_k=0.27$ • $R_{vk}=9.94$ • $M_{r1}=8\%$ • $M_{r2}=85\%$ 	$\mathcal{F}$		<ul style="list-style-type: none"> • $R_{pk}=0.14$ • $R_k=0.21$ • $R_{vk}=10.6$ • $M_{r1}=5\%$ • $M_{r2}=71\%$

Table 7.8: Measured surfaces and their Abbott-Firestone parameters

To compare performance in terms of friction and clearance, a fixed load $W^{ps} = -0.6$ N was imposed on the pad. This load was chosen large enough so that the small clearance enforces a deterministic simulation, but not as high as to originate contact between the pad and the runner. For the same reason, surface $\mathcal{A}$ was omitted from this group of simulations. It is a surface with large peaks as its parameter $R_{pk} = 2.64$ μm shows, and consequently contact takes place for the chosen load. Surface data pre-processing (peak removal and surface smoothing) will be treated in the next section.

Surface	$R=8$		$R=128$	
	$\overline{F(t)}$ (10^{-3}N)	C_{min} (μm)	$\overline{F(t)}$ (10^{-3}N)	C_{min} (μm)
$\mathcal{B}$	5.21	0.13	9.45	0.11
$\mathcal{C}$	4.92	0.35	8.50	0.32
$\mathcal{D}$	5.45	0.02	9.74	0.04
$\mathcal{E}$	4.95	0.84	8.34	0.87
$\mathcal{F}$	5.2	0.81	11.47	0.56

Table 7.9: Friction and minimum clearance for the measured surfaces of Table 7.8.

The pad linear mass was set to $m = 0.01$ kg/m, and its initial position for all cases is $Z(t=0) = 1.5$ μm . 6000 time steps are solved with $\Delta t = 0.1$ μs . For the simulations here performed the lowermost strip of width $B = 0.2\text{mm}$ ($(x_1, x_2) \in [0, L_s] \times [0, 0.2]$) is chosen from each of these surfaces. This choice of domain size is a good tradeoff between computational time and representativity of the whole sample, as seen in Table 7.4. The domain is discretized in 2000×400 finite volumes, with the same parameters of Table 7.3.

Minimum clearance $C_{min}(t)$ and friction force $F(t)$ are shown in Figure 7.11 for a periodical solution in each measured surface. It seems proper to recall at this point that each measured-surface is extended periodically as set forth by equation 7.16, and so the period is $2L_s$.

For both R values (8 and 128), the surfaces with lowest minimum clearance, listed from smaller to larger values are: $\mathcal{D}$, $\mathcal{B}$, $\mathcal{C}$, $\mathcal{F}$ and $\mathcal{E}$. This can be understood taking a look at its respective R_{pk} values: 0.85, 0.84, 0.71, 0.14 and 0.18 μm . Surfaces with higher asperities have a lower minimum clearance C_{min} . A detail of what takes place is seen in Figure 7.10(a). The upper figure shows the hydrodynamic pressure, while the lower one the measured surface $\mathcal{D}$. The white boxes show the pad position in the domain. At this particular time t , the highest asperity is directly below the pad, leaving a clearance of 0.1 μm . This small gap generates the pressure spike indicated by the arrow in the figure. A

detail of the texture and pressure in the region bound by the black box is shown in Figures 7.10(b) and 7.10(c).

C_{min} and the averaged friction force $\overline{F(t)}$ over each period $2L_s$ are given in Table 7.7. In spite of the minimum clearance being closely related to wear, it might not always correlate well with hydrodynamic friction. Surface $\mathcal{D}$ presents the largest friction force for $R=8$ but the second largest for $R=128$. This makes surface $\mathcal{F}$ particularly interesting, given that it shows a higher C_{min} with higher friction. This can be explained by looking at $Z(t)$ for each surface, as in Figure 7.12(b). It is much lower for $\mathcal{F}$, leading to a lower average h which means higher shear stresses. The high peaks of surface $\mathcal{D}$ ($R_{pk}=0.85$) induce its small minimum clearance.

(a)

(b)

(c)

Figure 7.10: (a) Pressure (above) and surface $\mathcal{D}$ (below), for $R=128$ and $t = 1.183 \times 10^{-4}$. Figures (b) and (c) show the flux and a detail of the surface profile and pressure at the box marked in Fig. (a).

Figure 7.11: Minimum clearance $C_{min}(t)$ and friction force $F(t)$ for five measured surfaces of Table 7.8. Only the last period of the periodic solution is shown for each surface. Figures (a) and (b) display results for $R=8$, while Figures (b) and (d) do the same for $R=128$.

Figure 7.12: $Z(t)$ for (a) $R=8$ and (b) $R=128$ for the five measured surfaces of Table 7.8.

IV Measured-surface simulation of a complete engine cycle

As mentioned in the beginning of this chapter, this section is dedicated to the work done in association with researcher Dr. Eduardo Tomanik from *Mahle*. Most of the results are not shown here for a matter of confidentiality.

The aim of these simulations was to assess the numerical formulation presented in Chapter 4 in liners with different honings along the length of the stroke.

IV.1 Surface sample pre-processing

One of the assumptions made in Chapter 2 is that curvature of the ring and the liner in the x_2 direction is neglected. Also measurement techniques introduce some macro-curvature ([30]) and the raw data is not horizontally positioned, that is, parallel to the $x_1 - x_2$ plane. These forms are removed usually for honed bores by fitting the surface with a 2D quadratic polynomial ([82, 50]). This was done for every surface in Table 7.8.

Figure 7.13: Section of the domain showing the ring and liner arrangement used in every simulation in this section.

A 3-dimensional view of the whole surface $\mathcal{A}$ is shown in Figure 7.14(a). The honed grooves and asperities can be observed. Among the latter, the one indicated by the black arrow protrudes more than the others. This asperity is most likely to be eliminated by wear during the running-in of the engine. Leaving the surface as it is will lead to results in the quantities of interest (friction, clearance) that might not be representative of the engine performance in the long run. Besides, from the numerical point of view it will generate some violent behavior in the simulations which can be really hard to capture. To deal with this, peaks in surface $\mathcal{A}$ were removed. Each height $h_s(x_1, x_2) \geq 2.0 \mu\text{m}$ was set to $h_s(x_1, x_2) = 2.0$. Figure 7.14(b) shows (top, left) the original surface and the same after

peak-removal (top, right). It can be seen that the surface characteristics are maintained, aside from the atypical spikes.

(a)

(b)

Figure 7.14: (a) Surface $\mathcal{A}$ without processing. (b) Successive stages in pre-processing: (top-left) unprocessed; (top-right) the highest peaks have been removed; (below-left) surface smoothed 3 times, and (below-right) surface smoothed 20 times. In all figures the most prominent asperity is pinpointed by a black arrow.

Another measured-surface data pre-processing technique with the only objective to render simulations computationally more amenable is surface smoothing. Each measured point $h_{s_{i,j}}$ is now taken as the average of the nine points $i, j = -1, 0, 1$. This is done in several smoothing sweeps across the measured surface. Both bottom images in Figure 7.14(b) show the outcome for 3 (bottom, left) and 20 (bottom, right) sweeps. As an example, in

figure 7.15 a longitudinal section of the $\mathcal{A}$ texture is shown to better appreciate the effect of the smoothing procedure. The red line corresponds to the original texture profile plotted with high resolution, the green line is the profile without the smoothing on the actual computational grid and the blue line is the smoothed profile on the computational grid. As noticed, the main features of the texture were well preserved, only some pronounced peaks were eliminated.

Figure 7.15: Longitudinal section of the texture $\mathcal{A}$ to better illustrate the effect of the smoothing procedure.

To compare the differences in the results for each of the four surfaces of Figure 7.14(b), simulations are run in a section of the domain, corresponding to the Strip 2 of Figure 7.17. The ring shown in Figure 7.13 shows the ring employed in every simulation in this section. A constant load $\mathbf{W}(t) = -500 \text{ N/m}$ is set on the ring. It starts at $Z(0) = 2.5 \text{ } \mu\text{m}$, and the rest of the parameters are the ones in Table 7.12. Table 7.10 shows the computational time in a 4-cored Intel i7-4810MQ processor for each run. As can be seen, removing peaks reduces the computational in only 1.4%, however the smoothing procedure decreases it greatly. Solving the problem with the 20-sweep surface takes just one third of the time than solving it on the original surface.

But how about the errors committed by modifying the original $\mathcal{A}$ surface?. Figure 7.16 shows the results. For the surface with peaks removed and three smoothing sweeps the

Surface	Time (s)	Decrease in time (%)
As received	1010.5	–
Only peaks removed	996.4	1.4
3 averaging sweeps	601.2	40.5
20 averaging sweeps	326.7	67.7

Table 7.10: Computational time savings with surface data pre-processing.

ring displacement $Z(t)$ did not change more than 5%, while the difference in the average friction force $\overline{F(t)}$ over the last passage of the texture under the ring is less than 7%. Friction, as expected, tends to decrease with this surface pre-processing, specially those spikes seen in Figure 7.16(c) at $t = 2 \times 10^{-4}$ and $t = 4.5 \times 10^{-4}$, caused by the higher summits of the untreated surface.

Surface	Error in $Z(t)$	Error in $\overline{F(t)}$
Only peaks removed	2.5%	4.1%
3 averaging sweeps	4.1%	6.5%
20 averaging sweeps	5.1%	7.7%

Table 7.11: Percentage errors of the modified surfaces with peaks removed and with peaks removed and smoothing.

Another problem related with high peaks is surface penetration. A closer look at Figure 7.16(b) shows some negative C_{min} values, which are indicated by the black arrows. These negative values take place due to: a) a high asperity moving towards the ring already at a low $Z(t)$ position; b) insufficient hydrodynamic and contact force support to hold the load imposed on the ring, and c) a large time step Δt , not able to capture the rapid dynamics of the ring. For the load used in this tests, a) is the most probable cause of penetration as C_{min} is not only positive, but also much higher after peak-removal. None of the other three cases shown in Figure 7.16(b) displayed this problem.

Figure 7.16: (a) Ring position, (b) minimum clearance and (c) friction force for Strip 2 of figure 7.17 without modifications, with peaks removed, and with peaks removed and some smoothing.

IV.2 Single-ring engine cycle simulations

Deterministic simulations of actual liner surfaces face a wide variety of challenges. Ring, piston and fluid dynamics, contact forces among others must be modeled. Once this has been dealt with, there remains the difficulties associated with the numerics. Supposing a domain large enough to hold all three rings of the pack, of 15 mm in the movement direction and 0.913 mm in the radial direction (the complete width of surfaces $\mathcal{A}$ and $\mathcal{B}$), a structured mesh large enough to capture the level of detail of surfaces $\mathcal{A}$ and $\mathcal{B}$ given in Table 7.7 would have approximately *17 million finite volumes*. If a Courant number of 1 is to be kept with a constant time step Δt , more than *half a million* iterations in time will be necessary to simulate just one engine cycle. This, in a single desktop machine will take **months** to compute, even with the efficient multigrid and parallel implementation presented in Chapter 5. Here we solve the problem for only the compression ring, and thus in a domain of reduced size. Two complete cycles were simulated for several liner configurations. These simulations took approximately 20 hours on a desktop computer with a six-cored Intel i7-4930K processor at 3.4GHz.

Figure 7.17: $\mathcal{A}$ (left) and $\mathcal{B}$ (right) surfaces divided into the three strips considered to assess the numerical formulation.

Surfaces $\mathcal{A}$ and $\mathcal{B}$ are used in the simulations in this section. As the samples sizes are 1.241mm in x_1 and 0.913 in the x_2 direction it must be repeated somehow all along the stroke, which for the provided engine data is 156mm. The election here was the most straightforward: they were repeated contiguously until the 156mm of the stroke were fulfilled.

The ring geometry employed was the same as the compression ring used in the preceding section, while the engine data is given in figure 7.18, corresponding to a rotational speed of 1900rpm. The stroke's length is 156mm and the bore diameter 132mm. This constitutes a challenging simulation, as the piston velocity and pressure coming from the combustion chamber are really large.

Different liner configurations were considered. Each configuration is made of three regions as illustrated in figure 7.19 for which the $\mathcal{A}$ and the $\mathcal{B}$ textures are alternatively used. The three cases actually considered are given in the table below

Liner	Region I (bottom)	Region II (middle)	Region III (top)
1	$\mathcal{A}$	$\mathcal{B}$	$\mathcal{A}$
2	$\mathcal{B}$	$\mathcal{A}$	$\mathcal{A}$
3	$\mathcal{B}$	$\mathcal{A}$	$\mathcal{B}$

Figure 7.18: Gas pressure (top) and piston velocity (bottom) as functions of the Crank angle.

Figure 7.19: Schematic of a liner divided into regions I, II and III.

The sizes of the different regions are also indicated in the figure as well as the position of the top dead center (TDC) and the bottom dead center (BDC). In behalf of this and figure 7.18, the ring's transition between regions is given by the following table

Crank angle	From region:	To region:	Stroke
38.2	III	II	Power
140.0	II	I	Power
219.5	I	II	Exhaust
321.8	II	III	Exhaust
398.2	III	II	Admission
500.1	II	I	Admission
579.5	I	II	Compression
681.8	II	III	Compression

Figure 7.20: Surface patterns used on the liner. The left column correspond to the original texture plotted on a grid with the same resolution as the one in the data provided. The left column corresponds to the texture after the smoothing procedure plotted on the computational grid actually used.

Table 7.12 presents several physical and numerical parameters adopted in the simulations. Notice the scales of the variables adopted for the plots, which are all non-dimensional.

Table 7.12: Physical and numerical parameters adopted in the simulations.

Parameter	Symbol	Value
Problem setting		
Domain size	L_x	2700 μm
	L_y	337.5 μm
Initial radial clearance	h_0	4.5 μm
Film thickness @ inlet	h_ℓ	5 μm
Ring width	L_r	1.2 mm
Linear ring mass	m	0.03 kg/m
Oil properties:		
Specifications		SAE30, 110°C
Viscosity	μ	0.008 Pa s
Contact force parameters:		
Young modulus	E	100 GPa
Standard deviation of asperities	σ_A	0.78 μm
	σ_B	0.48 μm
Friction coefficient	μ_s	0.11
Discretization:		
Mesh	$N_x \times N_y$	1024 $\times$ 128
Number of time steps	N_t	287, 400
Maximum Courant number	$CFL(= u_{\max} \Delta t / \Delta x)$	1
Scales:		
Length	L	1 mm
Film thickness	H	1 μm
Velocity	U	10 m/s
Friction force	$\mu U L / H$	80 N/m
Friction power		800 W/m = 331.7 W/cylinder

To further reduce the computational burden, peaks were removed and both surfaces $\mathcal{A}$, $\mathcal{B}$ were also smoothed. Also these surfaces were split into three strips, thus reducing the computational domain to one third in each case. This is shown by the thick black lines in figure 7.17. The results with these strips were compared on a $\mathcal{A}$ - $\mathcal{B}$ - $\mathcal{A}$ liner.

Some results in minimum clearance are shown in figure 7.21. $C_{\min}$ is plotted as a function of the Crank angle for each strip. Liners made with strips 1 and 3 exhibit more violent contact dynamics, so that stripe 2 was selected for the simulations (see the black arrows in figure 7.21). Transitions between textures can be appreciated by sudden jumps in clearance. Some filtering of this data has been done in order to enhance visualization.

Some quantities, like the $C_{\min}$ reported in figure 7.21, are very noisy in nature due to

the highly oscillatory shape of the textures under study. To enhance visualization these quantities are filtered taking the average every 100 time steps. Notice that even with this filtering results still exhibit oscillations that are more evident near the dead centers.

Figure 7.21: Minimum clearance $C_{\min}$ as a function of the crank angle for the simulations considering the three stripes. Notice some interpenetration near ignition angle.

Results for three liner arrangements

Results for the $\mathcal{A}$ - $\mathcal{B}$ - $\mathcal{A}$ liner are shown in figure 7.22. The plotted variables are the minimum clearance, the contact friction force, the total friction force and the total friction power as functions of the crank angle. The variables are non-dimensional. The region over which the ring is sliding at each angle is marked on the horizontal axis of the top figure. Jumps in the solutions are due to the differences in roughness between surfaces $\mathcal{A}$ and $\mathcal{B}$.

The maxima in clearance are reached close to the midstrokes, simultaneously with the highest piston velocities. The contact force drops to zero at midstroke during the exhaust, admission and compression phases, but does take place during the power stroke. However, the total friction force is mainly hydrodynamic, as seen by comparing the contact and the total friction force curves.

Clearance reaches its minimum points at the TDC and BDC, being its global minimum at the TDC during the power stroke. Notice that some surface penetration occurs. This is due to the highest loads on the ring taking place during the explosion in the combustion chamber. As piston velocity is null close to the dead centers, friction force is mostly due to contact.

Figure 7.22: Minimum clearance, friction contact force, total friction force and friction power loss as functions of the crank angle for the $\mathcal{A}\text{-}\mathcal{B}\text{-}\mathcal{A}$ liner.

The behavior of the other three liner arrangements is similar. This is shown compared in Fig. 7.23. In general the $\mathcal{B}$ texture exhibits less friction at each location. Averaging the friction power over the four-stroke cycle we obtained:

Liner	Average Power loss
1: $\mathcal{A}\text{-}\mathcal{B}\text{-}\mathcal{A}$	1065 W/m = 442 W/cylinder
2: $\mathcal{B}\text{-}\mathcal{A}\text{-}\mathcal{A}$	1251 W/m = 519 W/cylinder
3: $\mathcal{B}\text{-}\mathcal{A}\text{-}\mathcal{B}$	1154 W/m = 479 W/cylinder

The least friction losses correspond to Liner 1, which has texture $\mathcal{B}$ at Region II (middle) and $\mathcal{A}$ at Region III (top). The intermediate friction losses correspond to Liner 3, which has texture $\mathcal{A}$ at Region II and $\mathcal{B}$ at Region III. In the current configuration the region in which the smoother texture $\mathcal{B}$ is more beneficial is Region II (middle).

Figure 7.23: Minimum clearance, total friction force and friction power loss as functions of the crank angle for the three different liner configurations.

V Appraising surface penetration

Surface penetration, as seen in figures 7.16(c) or 7.11 leads to an ill posed problem. Existence and uniqueness of the solution of the Reynolds equation depend on the coercivity of the bilinear form

$$B(h; u, v) = \int_{\Omega} h^3 \nabla u \nabla v \, d\Omega . \quad (7.22)$$

One of the necessary conditions for it to be coercive is

$$0 < a \leq h(x_1, x_2, t) \leq b < +\infty \text{ a.e. on } \Omega \, \forall t \in [0, +\infty) . \quad (7.23)$$

Therefore, a solution of the problem is not assured if $h(x_1, x_2, t) \leq 0$ but in a measure zero set. Some proposals have been made to deal with this problem, for example the ones presented by Li et al [81].

In [81] the approach is the following: if there is a region Ω_p with $h \leq 0$, as in a asperity summit penetrating the ring, h cannot be set to zero in Ω_p because it violates condition 7.23. Li et al suggest a straightforward solution: setting $h(x_1, x_2, t)$ to an arbitrarily small value h_0 . They recognize that this choice induces an ill matrix, as $h^3 = h_0^3$ in Ω_p will be very small, resulting in more computational time. However, this solution presents more fundamental problems.

Figure 7.24: A discontinuity moving to the right.

To simplify let's consider the one-dimensional problem of Figure 7.24. Here, the lower surface moves to the right with velocity V while the upper one remains steady. The

clearance to the left of node i is set arbitrarily to h_0 , while the one on immediate node to the right is H . To further simplify things, we will consider that the gap is full of fluid. A balance of fluxes at node i reads:

$$\left(\frac{h_0 - H}{\Delta t} \right) \Delta x_1 \Delta x_2 = f_1 \Delta x_2 - f_2 \Delta x_2, \quad (7.24)$$

with

$$f_1 = \frac{V}{2} h_0 - h_0^3 \left(\frac{p_i - p_{i-1}}{\Delta x_1} \right), \quad (7.25)$$

$$f_2 = \frac{V}{2} h_0 - \frac{h_0^3 + H^3}{2} \left(\frac{p_{i+1} - p_i}{\Delta x_1} \right). \quad (7.26)$$

Hence, the following expression for p_i is obtained:

$$p_i = \left(\frac{H^3 + h_0^3}{H^3 + 3h_0^3} \right) p_{i+1} + \left(\frac{2h_0^3}{H^3 + 3h_0^3} \right) p_{i-1} + 2 \left(\frac{H - h_0}{H^3 + 3h_0^3} \right) \frac{\Delta x_1^2}{\Delta t}. \quad (7.27)$$

In the particular case that $h_0 \ll H$, then

$$p_i = p_{i+1} + \frac{V \Delta x_1}{C H^2}, \quad (7.28)$$

where C is the Courant number. A series of simulations were performed, for continuous and discontinuous geometries at node i , all keeping $C=1$. With a large H or very fine meshes the pressure at node i is expected to be continuous. This is shown in Figures 7.25(a) and 7.25(b) for a large discontinuity ($H=10 \mu\text{m}$) traveling to the right. Both figures show almost no sensitivity to h_0 and mesh refinement, as $H^2 \gg \Delta x_1$. Small differences are seen for the first figure (detail in Fig. 7.25(a)), although they seem to disappear rapidly for $h_0 > 0.01 \mu\text{m}$.

However, in a more realistic surface h is continuous at the contacts, and then $H \simeq h_0$. This creates a stronger dependency of the results on the choice of h_0 . Rewriting H as

$$H = h_0 + \frac{H - h_0}{\Delta x_1} \Delta x_1, \quad (7.29)$$

and supposing $H - h_0 \ll h_0$, back in equation 7.27 we arrive at the following expression for p_i :

$$p_i = \frac{p_{i+1} + p_{i-1}}{2} + \frac{2(H - h_0)}{4h_0^3} \frac{\Delta x_1^2}{\Delta t} = \frac{p_{i+1} + p_{i-1}}{2} + \frac{(H - h_0)V}{4h_0^3 C} \Delta x_1. \quad (7.30)$$

For a fixed mesh (fixed Δx_1), the jump in pressure at node i is expected to grow as h_0 decreases. This is shown in Fig. 7.25(c). On the other hand, results also are very sensitive to mesh refinement, as depicted in Fig. 7.25(d), where $h_0=0.1 \mu\text{m}$. Notice the large differences between the results for the 1000×10 , 2000×20 and 4000×40 meshes. This corresponds to $\Delta x_1=1.0, 0.5$ and $0.25 \mu\text{m}$, which are very reasonable meshes for the measured-surface textures of the previous section.

Another approach is to define a boundary condition at $\partial\Omega_p$, such as zero flux to the interior of Ω_p , and avoid computing the hydrodynamic pressure inside Ω_p . Unfortunately, this method induces the same problems explained above. So far there is no consensus in how to treat surface penetration and remains an open problem.

Figure 7.25: Hydrodynamic pressure for the moving geometry of Figure 7.24. The two upper figures correspond to a discontinuous geometry, while the figure at the bottom to a continuous one. In Figures (a) and (c) h_0 is varied for a constant $C=1$ and the 1000×10 mesh. In both figures (b) and (d) the mesh is refined for a fixed values $h_0=0.01 \mu\text{m}$ and $h_0=0.1 \mu\text{m}$, respectively.

VI Summary of the chapter

In the first part of this chapter we have discussed two important issues in lubrication analysis: (i) models for the friction force for non-planar surfaces, and (ii) high-resolution simulations of measured surfaces.

The first issue was addressed by direct comparisons of lubrication models with two-dimensional Navier-Stokes results (inertia and cavitation effects were neglected). It was found that the three-term formulas (3.81a) and (3.81b) are the most accurate ones, providing a reasonable estimate of the hydrodynamic friction of rough surfaces at sub-micron clearances, even when the lubrication hypotheses (7.13) are not being met.

Regarding the second issue, simulations of a measured honed surface were reported using the conservative time-dependent Elrod-Adams model with spatial resolutions as fine as $0.25\ \mu\text{m}$ per cell (4000×800 mesh). This allowed us to confirm the mesh convergence of the numerical method for the challenging topographies of measured data, and suggested cell sizes of a quarter or even half of the measurement resolution (here $0.5\ \mu\text{m}$ and $1\ \mu\text{m}$ respectively) as a reasonable compromise of accuracy and cost for the simulation of honed runners. It was also seen that for large clearances good accuracy can be achieved by performing low-cost simulations where the surface roughness is neglected. Also, the significance of each term of the friction formula (3.81a) was computed, so as to assess the error involved in neglecting terms in it, as done in some of the models found in the literature. When the clearance is much larger than the surface roughness, it was seen that the Couette term of the friction force is dominant and can be taken as an acceptable approximation. However, from the studied friction models only formulas (3.81a) and (3.81b) granted accurate results when the clearance is of the order or lower than the surface roughness, such as in highly loaded lubricated devices.

Later, dynamic simulations were run for the different measured surfaces of Table 7.8. The same load was set on the ring for each simulation, so as to compare performance in terms of friction and clearance for each surface. It was seen that surfaces with large peaks induce lower minimum clearances, although not necessarily higher friction.

The second part of this chapter is concerned with direct simulations in real engine conditions. Additionally, the liner is divided in three regions (top, bottom and mid-stroke) with different measured surfaces. The main challenges for this simulations are (a) the size of the problem in terms of unknowns and (b) the small time steps required to capture both the ring dynamics and the high piston velocities. Peak removal and surface smoothing techniques were proposed in order to mitigate the violent ring dynamics. The latter improved the computational time in 68%, with only a 7% difference in friction compared to

the results obtained with the unmodified surface. The purpose of removing the abnormally high summits is to reduce surface penetration, yet it can not be completely avoided with the model presented in Chapter 2. A method adopted by the research group of Tian Tian ([81]) to treat this problem was assessed, but it was found to be too sensitive to the defining parameters and also to mesh refinement.

A proposal for improving the cavitation model

The most frequently used models for cavitation in bearings are the Reynolds model and the Elrod-Adams model ([55]), which was already presented in Chapter 2.

Special care must be taken with the piston ring/liner contact when using these models. The gap between the liner and the piston is not fully flooded of oil in all the piston extension due to starved lubrication conditions or cavitation. Whats more, it is expected that the oil will be adhered to one of the surfaces: the liner, trapped in the cross-hatched finish left by the honing process, at least far away from the ring/liner contact. This is not an issue in devices such as journal or thrust bearings, were the oil does not remain necessarily attached to one or another surface, see for example the experimental data and corresponding theory on streamers ([58],[52]).

In the part of the domain Ω_0 where the fluid film is not complete and attached to the liner, a transport velocity of $V/2$ is not correct from the mathematical point of view, being just half of its actual value. This is taken into account by none of both models, hence a new mathematical model dealing with differentiated velocities at pressurized and non-pressurized regions is required.

This deficiency in the aforementioned models was first pointed out by Organisciak ([87]). He also found that a straightforward modification of the model (simply changing

Figure 8.1: Problem domain and main variables. In the non-pressurized regions the fluid is assumed to be attached to the runner's surface.

the translation speed in the non fully flooded regions) led to loss of uniqueness of the solution.

Let us formalize this problem in the next section:

I Physical problem

The problem's scheme is shown in figure 8.1. The frame of reference is fixed on the upper surface. The lower surface moves with velocity V . A continuous differentiable function $h = h(x, t) \geq h_{min} > 0$ gives the clearance between both of them. An incompressible lubricant flows between the two surfaces, occupying a fraction $\theta(x, t)$ of the clearance ($0 \leq \theta \leq 1$). Regions Ω_0 and Ω_+ are defined as in Chapter 2: $0 \leq \theta(x, t) < 1$ defines the former and $\theta(x, t) = 1$ the latter. A single pressurized region is considered, with $\alpha(t)$ and $\beta(t)$ being the cavitation boundary $\Sigma(t)$. This is a problem unknown.

In $\Omega_+(t)$ a non-negative pressure field $p(x, t)$ develops obeying the Reynolds equation (here in non-dimensional form):

$$\partial_x (h^3 \partial_x p) = \frac{V}{2} \partial_x h . \quad (8.1)$$

This is also a problem unknown in Ω_+ , with $p = 0$ in both $x = \alpha$ and $x = \beta$. In Ω_0 (the cavitated zone) what has to be solved is the transport equation:

$$\partial_t (h\theta) + V_T \partial_x (h\theta) = 0 . \quad (8.2)$$

Where V_T is the constant transport velocity, to be defined later. The initial condition for the fluid fraction is $\theta(x, t = 0) = \theta_0(x)$. As boundary condition, an specific lubricant thickness $h(0)\theta(0, t) = d_{oil}$ is set at the left entrance. As V is assumed to be positive, no boundary condition is needed for θ at the domain's left boundary.

Mass conservation must be guaranteed at the cavitation boundaries. In the pressurized region the lubricant flow is:

$$J = -h^3 \partial_x p + \frac{Vh}{2} , \quad (8.3)$$

while in Ω_0 :

$$J = V_T h \theta , \quad (8.4)$$

so that the Rankine-Hugoniot conservation conditions read:

$$\left\{ V_T \theta(\alpha(t)^-, t) h(\alpha(t)) \right\} - \left\{ -h(\alpha(t))^3 \partial_x p(\alpha(t)^+, t) + \frac{Vh(\alpha(t))}{2} \right\} = \alpha' h(\alpha(t)) \left\{ \theta(\alpha(t)^-, t) - 1 \right\} , \quad (8.5)$$

$$\left\{ -h(\beta(t))^3 \partial_x p(\beta(t)^-, t) + \frac{Vh(\beta(t))}{2} \right\} - \left\{ V_T \theta(\beta(t)^+, t) h(\beta(t)) \right\} = \beta' h(\beta(t)) \left\{ 1 - \theta(\beta(t)^+, t) \right\} . \quad (8.6)$$

These conditions are known as JFO boundary conditions (Jacobson and Floberg [71] and Olsson [85]). If $V_T = V/2$, the model admits a unique solution ([20],[24]). It could also be reassembled into a single equation by defining $p = 0$ in the cavitated region. This leads to the Elrod-Adams model, which in a weak sense, automatically incorporates the conservation conditions.

The aim is to develop a well-posed model of the form:

$$\partial_x \left(\frac{h(x)^3}{2} \partial_x p(x, t) - V_T (\theta) \theta(x, t) h(x) \right) = \partial_t [\theta(x, t) h(x)] , \quad \text{with } x \in \Omega, t \geq 0 , \quad (8.7)$$

where

$$V_T(\theta) = \begin{cases} \frac{V}{2} & \text{if } \theta = 1 \\ aV & \text{if } \theta < 1 \end{cases}, \quad (8.8)$$

with constant $a \in [\frac{1}{2}, 1]$, taken this way for the sake of generality. This is also supplemented by the complementarity conditions:

$$p \geq 0, \quad 0 \leq \theta \leq 1, \quad \theta \leq 1 \Rightarrow p(x, t) = 0, \quad p(x, t) > 0 \Rightarrow \theta(x, t) = 1. \quad (8.9)$$

Moreover, the boundary and initial conditions are:

$$p(0, t) = p(1, t) = 0, \quad (8.10a)$$

$$\theta(0, t) = \theta_{in}(t) = \frac{d_{oil}}{h(0)}, \quad (8.10b)$$

$$\theta(x, 0) = \theta_0(x), \quad (8.10c)$$

with θ_0 and θ_{in} known.

To further increase the simplifications on this problem, let's assume that $\Omega_+(t) = (\alpha(t), \beta(t))$, i.e., there is only one pressurized region. There, the Reynolds equation must be satisfied:

$$\partial_x (h(x)^3 \partial_x p(x, t)) = \frac{V}{2} \partial_x h(x) \quad \text{on } \Omega_+(t), \quad (8.11)$$

with boundary conditions:

$$p(\alpha(t), t) = p(\beta(t), t) = 0. \quad (8.12)$$

Besides, $\alpha(t)$ and $\beta(t)$ must satisfy $p(x, t) \geq 0, \forall x \in \Omega_+(t)$. Making use of the results found in [37] we have that:

$$h(x)^3 \partial_x p(x, t) = \frac{V}{2} (h(x) - G(\alpha(t), \beta(t))) , \quad (8.13)$$

where

$$G(\alpha(t), \beta(t)) = \frac{\int_{\alpha}^{\beta} \frac{1}{h^2(x)} dx}{\int_{\alpha}^{\beta} \frac{1}{h^3(x)} dx}. \quad (8.14)$$

In Ω_0 we have the transport equation for $h(x)\theta(x, t)$:

$$\partial_t(h(x)\theta(x, t)) + aV\partial_x(h(x)\theta(x, t)) = 0, \forall x \in \Omega_0(t). \quad (8.15)$$

As we are assuming $V > 0$ and a film thickness of d_{oil} at the left boundary of the domain, then we have that due to continuity in $h(x)$:

$$\theta(\alpha(t)^-, t) = \frac{d_{oil}}{h(\alpha(t))}. \quad (8.16)$$

That is, $\alpha(t)$ is a reformation boundary. With these results we can rewrite the JFO boundary conditions:

$$\alpha'(t) = \frac{V}{2} \frac{G(\alpha(t), \beta(t)) - 2ad_{oil}}{h(\alpha(t)) - d_{oil}}. \quad (8.17)$$

$$\beta'(t) = \frac{V}{2[1 - \theta(\beta^+(t), t)]} \left(\frac{G(\alpha(t), \beta(t))}{h(\beta(t))} - 2a\theta(\beta^+(t), t) \right) \text{ if } \theta(\beta^+(t), t) < 1, \quad (8.18)$$

I.1 Loss of uniqueness

In equation 8.17 it can be seen that the evolution of $\alpha(t)$ is already determined given that $\beta(t)$ is known. However, $\beta'(t)$ still depends of a unknown value, $\theta(\beta^+(t), t)$, which in time depends on the velocity of the cavitation boundary $\beta(t)$:

1. If $\beta(t)' \geq aV$, $x = \beta(t)$ is a *reformation boundary*. In that case, $\theta(\beta^+(t), t)$ must be found tracking back in the characteristic lines of equation 8.2.
2. If $\beta(t)' < aV$, $x = \beta(t)$ is a *rupture boundary*. In this case, the only condition restraining $\beta(t)'$ is:

$$\partial_x p(\beta^-(t), t) \leq 0, \quad (8.19)$$

because $p \geq 0$ on $\Omega_+(t)$. Substituting equation 8.19 in 8.18:

$$\theta(\beta^+(t), t) \geq \frac{\frac{V}{2} - \beta'(t)}{aV - \beta'(t)} = F(\beta'(t)). \quad (8.20)$$

This condition implies that:

$$\max(0, F(\beta'(t))) \leq \theta(\beta^+(t), t) \leq 1. \quad (8.21)$$

Notice that in the case of the Elrod-Adams model $a \rightarrow \frac{1}{2}$, $\theta(\beta^+(t), t) \rightarrow 1$, that is $\theta(\beta^+(t), t)$ tend to the unique value $\theta(\beta^+(t), t) = 1$.

In [86],[14] the authors resorted to the trivial substitution $V_T = V$ in the cavitated zone and both reported wild oscillations of the θ field at the rupture boundary. Condition 8.20 explains the oscillations observed as lack of uniqueness in the solution.

I.2 Recovering uniqueness

In a recent paper, Buscaglia et al [37] proposed a model dealing with this issue. Existence and uniqueness of the solution were proven. A finite volume implementation of the same model showing its adequacy to solve the piston ring/liner problem was given by Ausas et al [15]. A front-tracking algorithm was used. Another relevant characteristic of the model is that the choice of $\theta(\beta^+(t), t)$ depended on $\beta'(t)$, which in turn depends on $\theta(\beta^+(t), t)$. To solve this a correction scheme was proposed.

In this section we aim to study the possibilities that are left open by the selection of the $\theta(\beta^+(t), t)$ value, even granting negative pressures close to the rupture boundary. This allows us to consider also subambient pressure models for rupture, such as the *separation boundary condition* ([52]).

Taking into account the continuity of $h(x)$, we can rewrite equation 8.6 as:

$$\beta'(t) = \frac{-h^2(\beta^-(t))\partial_x p(\beta^-(t), t) + \frac{V}{2}(1 - 2a\theta^+)}{1 - \theta^+}. \quad (8.22)$$

In figure 8.2 we plot $\beta'(t)$ vs. $\theta(\beta^+(t), t)$ for this equation. Lines of constant $-h^2(x)p(x, t)$ are also plotted.

It should be noticed that for a pair $\alpha(t), \beta(t)$, the pressure field $p(x, t)$ is already determined by solving the ordinary differential equation 8.13 with boundary conditions 8.12. Therefore, $\partial_x p(\beta^-(t), t)$ is defined given $\alpha(t)$ and $\beta(t)$. This means that for a certain time t all $[\beta'(t), \theta^+(\beta(t), t)]$ pairs are on a curve on the $\beta'(t), \theta^+(\beta(t), t)$ plane. This is shown in figure 8.2. The curve of constant $\partial_x p = 0$ divides the space of feasible solutions in two: the light-shaded blue region, where $\partial_x p(\beta^-(t), t) < 0$, and the red region below the $\partial_x p = 0$ curve, where $\partial_x p(\beta^-(t), t) > 0$. Without further information, any $0 \leq \theta^+(\beta(t), t) < 1$ is a valid choice on a $h^2(\beta^-(t))\partial_x p(\beta^-(t), t)$ constant line, however, depending on the choice the dynamics of the rupture boundary $x = \beta(t)$ can change drastically.

In order to recover uniqueness, a choice must be made. Several rupture conditions can be adopted [52] to make it physically meaningful:

- The *Reynolds condition*: $\partial_x p(\beta^-(t), t) = 0$,

Figure 8.2: Plot of the $\mathcal{L}(\theta)$ function given in equation 8.23.

- the *separation boundary condition*: $h^2(\beta^-(t))\partial_x p(\beta^-(t), t) = 2\mu V$,

among others. We could try to meet, for example, the requirements of the Reynolds rupture conditions. For that, what is needed is a relationship between $\beta'(t)$ and $\theta(\beta^+(t), t)$.

An almost-Reynolds rupture condition

A initial condition would be any $\alpha(0) = \alpha_0, \beta(0) = \beta_0$ pair, and from that point we would like to approach the $\partial_x p(\beta^-(t), t) = 0$ condition as much as possible, but allowing small negative or positive pressures at $x = \beta^-(t)$. One way of doing this is by defining a $\beta'(t), \theta(\beta^+(t), t)$ dependency as depicted by the dotted line $\mathcal{L}(\theta)$ in figure 8.2:

$$\mathcal{L}(\theta) = \frac{V}{2} \frac{(1 - 2a\theta)(2 - \theta - \frac{1}{2a})}{1 - \theta^2}. \quad (8.23)$$

This line intersects the lines of constant $-h^2(\beta^-(t))\partial_x p(\beta^-(t), t) \leq \frac{V}{2}(1 - \frac{1}{2a})$ just once in the range $0 \leq \theta < 1$, and for those with $-h^2(\beta^-(t))\partial_x p(\beta^-(t), t) > \frac{V}{2}(1 - \frac{1}{2a})$, $\theta^+ = 0$ is taken.

This choice leads to a recovering velocity $\beta'(t)$ towards the position of the cavitation boundary $x = \beta(t)$ where the pressure gradient is null. For positive pressure gradients at $x = \beta(t)$ it returns a $\beta'(t) < 0$, for negative gradients a $\beta'(t) > 0$ and for null gradients $\beta'(t) = 0$. If the goal is to maintain $x = \beta(t)$ as close as possible to the position of $\partial_x p = 0$, a increase in the recovering velocity would be desirable. This could be achieved modifying equation 8.23 in the following manner:

$$\mathcal{L}(\theta) = \frac{V}{2} \frac{(1 - 2a\theta)(2 - \theta - \frac{1}{2a} - (1 - \delta)(1 - \frac{1}{2a}))}{1 - \theta^2}, \quad (8.24)$$

with constant $0 < \delta \leq 1$. This effectively increases the recovering velocity. As $\delta \rightarrow 0$, $\mathcal{L}(\theta)$ approaches the curve of constant $\partial_x p = 0$, leading to a larger $\beta'(t)$ in the direction of the $\partial_x p = 0$ position. For this $\mathcal{L}(\theta)$, $\theta(\beta^+(t), t)$ becomes:

$$\theta(\beta^+(t), t) = \frac{h^2(\beta^-(t))\partial_x p(\beta^-(t), t) + \frac{V}{2}(1 - \frac{1}{2a})\delta}{h^2(\beta^-(t))\partial_x p(\beta^-(t), t) + aV(1 - \frac{1}{2a})\delta}. \quad (8.25)$$

To test this, a simple program in Octave ([2]) was developed to solve the ordinary differential equation 8.18 under the following conditions:

- $h(x, t) = 1 + (2x - 1)^2 + A \sin(2\pi t/T)$, $A=0.15$, $T=2.0$;
- $V=1.0$;
- $a=1.0$;
- $\eta=0.52$;
- $\alpha_0=-1000$, $\beta_0=0.7$;

$\alpha(t)$ was fixed at $\alpha(t) = \alpha_0$. $\beta(t)$ and $\partial_x p(\beta^-(t), t)$ are shown in figure 8.3 for various $\delta \rightarrow 0$ values. As δ tends to zero, $\beta(t)$ converges faster to the point x where $\partial_x p(x, t) = 0$ at time t .

The result shown here could become a starting point for a new numerical approximation.

Figure 8.3: (a) Position of the rupture boundary and (b) pressure gradient at the rupture boundary for several δ values. The dotted red line depicts the position x where $\partial_x(x, t) = 0$.

II Summary of the Chapter

In this chapter first we have recalled a problem common to some of the most widely used cavitation models in lubrication. An erroneous transport velocity of the oil is set in the non-pressurized regions. This was initially recognized by Organisciak [87]. Then, the lack of unicity of the straightforward solution (forcing the correct value of the transport velocity in the cavitated region) is explained.

Previous efforts have been made to recover the unicity of the solution. Buscaglia et al [37] proposed a generalization of the Elrod-Adams model in a simplified mathematical setting of a one-dimensional problem with a single pressurized region. The key element was the selection of the value of the fluid saturation θ at the rupture boundary or the velocity of the rupture boundary β' , these two being related by equation (8.22). Following a similar line of work, the last section of this chapter explored other rupture models, which can constitute the basis for new conservative cavitation models.

Conclusions

I Conclusions

In this work we have presented the modeling, an efficient numerical treatment and numerical studies of textured moving surfaces in fluid lubricated contacts. The problem of a pad sliding over a surface is solved considering its dynamics and taking into account cavitation in the hydrodynamics.

The modeling incorporated one degree of freedom for the pad: a vertical movement, orthogonal to the slider surface. Rotations and horizontal translations were omitted. This amounts to consider a rigid ring fitting tightly in the ring groove. Also, some geometrical simplifications were introduced: bore curvature was neglected and perfect alignment of the piston axis was assumed. The pad dynamics are governed by the hydrodynamic forces and the solid-solid forces arising from the contact of both the pad and the slider. In this work we adopt the Greenwood-Trip [68] model to compute such forces.

The operational conditions and convergent-divergent geometry of devices such as journal and piston-ring/liner bearings give rise to cavitation. Additionally, several cavitated regions are expected to appear due to the unevenness of measured surfaces. Ausas et al [12] showed the importance of a mass-conservation model while solving micro-textured surfaces, in order to correctly predict cavitated areas, and therefore to correctly predict the hydrodynamic pressure. Lubrication models dealing with cavitation were assessed, and the Elrod-Adams model was chosen because it fulfills the mass-conserving condition and also due to its simplicity (the only parameter to be defined is the cavitation pressure).

The Elrod-Adams model was solved with the Finite Volume Method, using a upwind-ing discretization of the Couette flux and centered discretization of the Poiseuille flux. Cavitated nodes are checked through the complementarity conditions at every iteration of the Gauss-Seidel algorithm employed to solve the Elrod-Adams model. The pad dynamics are solved coupled with the hydrodynamic using a Newmark marching scheme, built in the overall iterative process.

The first contribution of this thesis is the multigrid algorithm presented in Chapter 5. The *Full Approximation Scheme*, a multigrid method for nonlinear problems, was adopted. However, the straightforward implementation was far from being efficient. The cavitation boundary is not known *a priori* and changes its position between Gauss-Seidel iterations. This slows down convergence, and is quite unavoidable. During a multigrid *V-cycle*, information is passed from fine to coarse meshes on the way down the *V-cycle*, and viceversa on the way up. Here it must be acknowledged that the best approximation for the cavitation boundary comes from the finest mesh, and that that information must be kept for all coarser meshes. This was already pointed out by Venner [112] for the Reynolds equation with Swift-Stieber boundary conditions. In addition, the definition of the transfer operators was critical to obtain good results. To the extent of our knowledge, it is the first time that a efficient multigrid algorithm to solve the Elrod-Adams model is presented.

A second contribution is the systematic study of moving micro-textured surfaces with a mass-conservative cavitation model, and the understanding of the mechanisms responsible for friction reduction in the hydrodynamic lubrication regime. This study was made feasible in great manner due to the multigrid implementation and a shared memory parallelization. Hundreds of two-dimensional simulations were performed for dimpled surfaces with a fixed load on the pad. The influence of the bearing macro-shape was also analyzed by varying the curvature radius of the fixed length pad. Although most of the work is focused in the piston-ring/liner contact in the form of a pad-slider geometry, the modeling and the conclusions reached with the results obtained remains valid for other lubricated contacts, such as journal bearings, thrust bearings and others. It was found that micro-textures improve performance for moderately to highly flat bearings, providing the hydrodynamic support they lack. The improvement grows with surface conformity. These results were confirmed later with a more encompassing study (more than 10.000 transient simulations) of one-dimensional sinusoidal textures with several pad masses and loads. The best performance is achieved with textures lengths comparable with the size of the contact, and depths similar to the pad-to-slider clearance. As the clearance depends on the operational conditions of the bearing, so does the optimal texture. This suggests that

texturing the sliding surface (e.g., the cylinder liner of combustion engines) can be more efficient than texturing the pad (piston ring).

Another major outcome was the simulation of lubricated contacts considering the measured topography of some surface finishings, such as the honed and coated bores of cylinder liners of car engines. A lubrication approximation of the friction force was developed, and its soundness was assessed comparing results with Navier-Stokes simulations. Also, the friction force formula developed was compared with friction models usually found in the literature, the former results being more accurate than the latter in any case. It has to be mentioned that although this is not the first time that a comparison between the Reynolds equation and Stokes flow was made ([111, 7]), it is the first time is made for the friction force. The relevance of each of the shear force terms (Couette and Poiseuille) and of the pressure term of the friction force were analyzed. It was seen that despite most of the friction force being due to the Couette flow, neglecting the other terms induces significant errors when the clearances of the order of the surface roughness. If the clearance is much larger than the roughness, the Couette term of the friction force can be taken as a reasonable approximation.

The discretization required to obtain converged solutions for surfaces with measurements in the order of the micron was also examined, suggesting cell sizes of a quarter or even half of the measurement resolution. Another lesser contribution of this thesis is the pre-processing of the measured surfaces. Large, atypical asperities are present in the measured samples provided, but no more than a few per sample, which are certain to be removed in the long run by wear. In the dynamic simulations these large summits induce penetration of the pad and the slider, leading to an ill posed problem. Removing them reduced drastically the number of points where penetration took place. A smoothing procedure on the rough surfaces was also studied. This greatly reduced the computational time with only a small difference with the results of the simulations performed with the original data.

The last contribution of this thesis was a progress in the development of a new cavitation model in lubrication. It is not adequate to solve lubrication problems where oil sticks to one of the surfaces, as happens for example in the piston ring/liner problem (oil in cylinder walls). The Elrod-Adams model, and other cavitation models based in the Reynolds equation assign an erroneous transport velocity is assigned to that film, equal to one half of the real transport velocity. Efforts made to take this situation into account lead to bad posed problems, as reported by Organisciak [86] and Ausas [14]. Some advances have been made in this direction. We have implemented an improved, more versatile model in 1D and

re-interpreted it as resulting from the dynamic coupling of Rankine-Hugoniot conditions (mass conservation) and a film-rupture law that should arise from physical considerations. This allowed us to extend the model to situations with non-positive pressure fields, which is crucial to arrive at a numerical approximation. The one-dimensional implementation of this method is already showing interesting results.

II Future work

This work also opens the possibility of expanding current research and of new research subjects.

1) Modeling of lubricated contacts:

- Include the rheological properties of the lubricant, such as variations of viscosity with temperature, pressure and shear stress and of density with pressure. Also, the dissipated energy by viscous effects can be important ([114]) and the Elrod-Adams equation should be solved coupled with the energy equation.
- The complementarity conditions in the Elrod-Adams model enforce cavitation pressure when the fluid film is not complete. Concomitantly, this imposes cavitation pressure at the boundaries when the gap is not full of oil. These boundary conditions are not realistic for some lubricated devices, such as the piston-ring/liner contact. The pressure at each of the inter-ring spacings is different due to *blow-by* ([]). This should be taken into account in the modeling.
- Consider tribofilm formation ([64]).
- When measured surfaces are assessed statistic contact models such as Greenwood-Tripp lose their significance. Then surface deformations should be considered, and an elastohydrodynamic lubrication (EHL) problem must be solved. This can also constitute a solution for the solid-solid penetration issue.
- Extend the results of Chapter 8 to two dimensions and develop a numerical method for the new cavitation model. The possibility of front-tracking and front-capturing methods to reconstruct the cavitation interface should be considered. Once this is achieved, a next step is coupling this model with the equation of the pad dynamics.

2) Existence and uniqueness of solutions:

- Extend the results of Buscaglia et al [33] to the mixed lubrication problem, that is, when the function $W^c = W^c(Z(t))$ is a function of the separation $Z(t)$ between the upper and lower surfaces.

3) Numerical methods:

- A better initial approximation decreases the number of iterations required for the multigrid to solve the problem. A *Full Multigrid Method* ([61]) could improve convergence in this sense.
- Lubrication problems imply solving a elliptic equation in part or in the whole domain, the pressure flux being governed by the coefficient

$$\frac{h^3}{12\mu}.$$

For rough surfaces h can be as small, or even smaller than $0.1 \mu\text{m}$ at the summit of the asperities and as large as $10 \mu\text{m}$ at the bottom of the grooves of honed surfaces. As the conductivity-like coefficient depends on the cube of h , at least six ($10^3/0.1^3 = 10^6$) orders of magnitude are expected for the values of h^3 . This makes this problem specially suitable for multiscale methods ([116]), which look like a very promising approach.

- The discontinuities that arise at cavitation boundaries in the Elrod-Adams model preclude the use of standard high order methods. Discontinuous Galerkin methods then emerge as a possibility for improving the order of convergence for the type of problems treated in this work ([10]).

4) Study of micro-patterned and rough surfaces:

- Extend numerical studies for other micro-texture shapes. The validity of the conclusions reached in Chapter 6 should be checked for a broader range of loads on the pad, pad shapes and feeding conditions (fully flooded vs. starved). Here thermal and lubricant rheology effects can also be studied.

Appendices

Publications in conferences, workshops and journals

Three journal publications ([38, 42, 44]) and four conference papers ([40],[39],[43],[41]) were published:

Journal publications:

1. A journal article named "*Moving textures: simulation of a ring sliding on a textured liner*" ([38]) (DOI: 10.1016/j.triboint.2013.12.013), studying the effects of a textured liner on friction coefficient and minimum oil film thickness was published in *Tribology International* (ISSN: 0301-679X).
2. One journal article analyzing the influence of cavitation bubbles in friction reduction entitled *Texture-induced cavitation bubbles and friction reduction in the Elrod-Adams model* ([42]) was published in *Journal of Engineering Tribology* (ISSN: 2041-305X).
3. A journal article where the lubrication approximation of the hydrodynamic friction force for rough surfaces is provided and assessed, named *The lubrication approximation of the friction force for the simulation of measured surfaces* ([44]) (DOI: 10.1016/j.triboint.2015.12.030) was submitted and accepted in *Tribology International* (ISSN: 0301-679X).

Conference papers:

1. Mass-conserving simulation of friction reduction in a hydrodynamic thrust bearing. In *Proceedings of the 12th EDF-PPrime Workshop: Solutions for performance improvement and friction reduction of journal and thrust bearings*, 2013.
2. Mass-conserving numerical simulation of ring-textured liner contact in the hydrodynamic lubrication regime. In *Anais do congresso de Matemática Aplicada e Computacional CMAC Sudeste 2013*, pages 464-470, 2013.
3. Tribological performance of textured surfaces in the piston ring/liner contact using the Elrod-Adams model. In *Mecânica Computacional*, pages 129-145, 2014.
4. Effect of liner texture on ring/liner tribological performance. In *Anais do Second International Conference on Tribology*, 2014.

The first page of each publication is provided in the following pages, in the same order as in the previous list.

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Moving textures: Simulation of a ring sliding on a textured liner

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ABSTRACT

Numerical simulations of the ring/liner contact in which the liner exhibits a periodic texture (pockets) are reported. The mass-conservative Elrod–Adams model is used to treat cavitation, and the dynamics of the ring is considered with a linear mass that corresponds to actual engine compression rings. The results, computed at a Stribeck number of 10^{-3} and thus in the hydrodynamic lubrication regime, show that the ring profile determines whether pocketing is beneficial or not. For strongly non-conformal contacts pocketing is detrimental, but for quasi-conformal contacts friction reductions of up to 73% are predicted. The largest reduction in friction was obtained for textures consisting of close-packed arrays of circular pockets of diameter comparable to the size of the contact.

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1. Introduction

Texturing of contact surfaces to reduce friction and wear has attracted much attention in the last years, especially since surface microengineering techniques have become available. One important application is the piston-ring/cylinder-liner contact in combustion engines, which accounts for about 5% of fuel consumption [1].

It is known that some texture is needed to avoid stiction between the ring pack and the liner, and honed liners have been used for many decades for that purpose. Current investigations are aimed at determining the best texture in terms of friction and wear for each working condition.

Numerical and experimental studies on the ring/liner lubrication problem with textured surfaces are recent and quite scarce.

The numerical studies have only considered the texture to be on the ring, and thus stationary with respect to the contact. Kligerman et al. [2] solved the problem for flat textured and partially textured rings using the Reynolds equation and Reynolds boundary condition, which is known not to conserve mass. They concluded that surface texture reduces the friction between the surfaces. On the other hand, Tomanik [3], with the same cavitation model, numerically compared barrel shaped rings (compression

rings), flat rings (oil control rings) and flat partially textured rings, also incorporating experimental data in the analysis. His results (both numerical and experimental) showed that the untextured barrel-shaped rings performed better than their flat, partially textured counterparts, both in hydrodynamic support and in friction coefficient. This finding coincides with recent numerical results of Gadeschi et al. [4] assessing the effect of Laser Surface Texture (LST) pockets on barrel-shaped rings. These authors adopted the Reynolds equation with Gumbel boundary conditions, which are non mass-conservative.

The goal of this paper is to present numerical simulations of hydrodynamic ring/liner contact with a *mass-conservative cavitation model* and *considering the texture as being on the liner* and thus moving through the computational domain, as sketched in Fig. 1. Similar simulations have only been reported up to now by Yin et al. [5], though with Reynolds cavitation condition (non-mass conservative), and by Tanaka and Ichimaru [6], who considered purely transversal textures, rendering the geometry one-dimensional, and adopted Reynolds cavitation condition.

The crucial importance of mass-conserving boundary conditions at cavitation boundaries has been established by Ausas et al. [7] and confirmed by several authors, among them Zhang and Meng [8] who performed a careful comparison with experiments.

The importance of considering textured *liners*, instead of textured rings, arises from several practical considerations: (a) All actual cylinder liners are, in fact, textured in some way. (b) The wear of the liner is smaller, and more evenly distributed, than that of the ring; the texture can thus be designed with more confidence in that it will preserve its shape for long operating

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Texture-induced cavitation bubbles and friction reduction in the Elrod–Adams model

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Abstract

A thrust bearing consisting of an infinitely wide pad, subject to a constant load and sliding at constant speed on a runner with transverse sinusoidal textures is considered. The analysis method consists of time- and mesh-resolved simulations with a finite volume approximation of the Elrod–Adams model. Friction and clearance contours as functions of the texture depth and wavelength are built by performing more than 10,000 simulations. Conclusions are drawn for bearings of low, moderate and high conformity, unveiling basic mechanisms of friction reduction and global quantitative trends that are useful for texture selection.

Keywords

Textured bearings, Elrod–Adams model, friction reduction, cavitation, numerical simulation

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Introduction

Textured tribological surfaces have attracted much attention of the research community lately. After a significant number of experimental and theoretical studies, the possibility of reducing friction by means of microtextures has been established, together with a basic understanding of why this happens (at least in the hydrodynamic regime).^{1–4}

That certain textures reduce friction in some hydrodynamic bearings is by no means an obvious phenomenon. Buscaglia et al.^{5,6} performed asymptotic analyses of general smooth (i.e., untextured) surfaces by introducing short-wavelength periodic perturbations of arbitrary shape and obtained that the untextured shape *always* (in the hydrodynamic regime) maximizes the load carrying capacity and minimizes the friction coefficient. This implies that, for friction reduction to take place, there exist two possibilities: It can be a consequence of a *finite* perturbation (outside the validity of asymptotic theory), or involve physical mechanisms which were not considered in the aforementioned mathematical studies, such as *cavitation*.

In a recent study, Checo et al.⁴ discussed several hundred numerical simulations and concluded that in fact *both* of the previous possibilities hold true in

textured bearings that exhibit less friction than their untextured counterparts. As had already been advanced by Etsion,³ friction reduction only occurs in high-conformity bearings, in which the surfaces are so parallel that a texture of some suitable size and depth manages to produce local cavitation.

High-conformity bearings are not infrequent in technology. Assuming a bearing of length L with a nominally planar surface opposing a surface with curvature radius R , the degree of conformity can be measured by the quotient R/L . For the compression ring of an internal combustion engine R/L is already quite high (up to 52 in diesel engines, as discussed by Gadeschi et al.²), and it is much higher for other piston rings (oil rings in particular) and for other contacts such as seals.

It was numerically shown by Gadeschi et al.² with a non-mass-conserving model, and later by Checo et al.⁴ with a mass-conserving model, that for a

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The lubrication approximation of the friction force for the simulation of measured surfaces

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ABSTRACT

Models of the friction force are assessed by direct comparison with two-dimensional Navier–Stokes results. A three-term formula obtained from asymptotic expansion provides a reasonable estimate of the hydrodynamic friction of rough runners even at sub-micron clearances.

Simulations of a measured honed surface are then reported using the conservative time-dependent Elrod–Adams model with spatial resolutions as fine as 0.25 μm per cell (4000 $\times$ 800 mesh). Mesh convergence of the numerical method is observed. Cell sizes between 0.5 μm and 1 μm appear as a reasonable compromise of accuracy and cost for the simulation of honed runners. The significance of each term of the friction formula is discussed, so as to assess the error involved in neglecting terms of the friction formula.

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1. Introduction

Theoretical and numerical studies of lubricated devices often represent the contacting surfaces as smooth. While this approximation stands under some conditions, this is not the case when the film thickness is comparable to the roughness amplitude, as in heavily loaded journal bearings, seals, piston ring/cylinder liner contacts, among others. The introduction of surface roughness into lubrication models can be traced back to the stochastic model of Tzeng and Saibel [1]. In their work, a probability distribution is adopted for the surface roughness and formulas of the expected values of pressure, friction force and load-carrying capacity are deduced for a one-dimensional problem. A more encompassing model was given by Christensen [2], who presented a modified (stochastic) Reynolds equation in which the coefficients were expected values. These results were limited to transverse or longitudinally oriented variations in roughness.

Later, Patir and Cheng [3,4] introduced a new approach. An average Reynolds equation for rough surfaces was defined in terms of pressure and shear flow factors, which were empirical functions of the non-dimensional roughness. Elrod [5] later extended this model to account for roughness anisotropy.

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All of these techniques are based on the intuitive idea of decoupling the global length scale, corresponding to the whole bearing, from the local length scale of the roughness. Homogenization theory formalizes this intuition. It develops an average equation valid throughout the domain, with its coefficients computed from solutions of local problems. The homogenization method has been studied in depth both for incompressible [6,7] and compressible [8–10] fluids. It exhibits good accuracy when the roughness is periodic in space and its period is much smaller than the bearing size. Unfortunately, no rigorous homogenization model considers general roughness shapes with cavitation effects.

A current trend is to study the hydrodynamics of lubrication devices resolving all scales of the problem down to the roughness scale, using the measured topography and without resorting to averaged or stochastic models. These so-called *deterministic* or *measured-surface* simulations [11–14] avoid ambiguities in the definition of average coefficients at the expense of solving Reynolds equation with a discretization finer than the resolution of the measurements. A central result of the simulations is the hydrodynamic force exerted by the fluid, defined by

$$\mathbf{F} = \int_S \boldsymbol{\sigma} \cdot \mathbf{n} \, dS, \quad (1)$$

of which the component parallel to the movement is the *friction force*. Different formulae for the lubrication approximation of Eq. (1) appear in the literature. Patir and Cheng [4] identify the existence of three terms in this approximation, which we denote here as Couette term F^{cou} , Poiseuille term F^{poi} and pressure term F^{pre} ,

Mass-conserving simulation of friction reduction in a hydrodynamic thrust bearing**Simulation conservative de la réduction du frottement dans une butée hydrodynamique texturée**

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Keywords: Friction reduction, textured lubricated contacts, hydrodynamic thrust bearing, Elrod-Adams model.

Mots clés: Réduction du frottement, contact lubrifié texturé, butée hydrodynamique, modèle d'Elrod-Adams.

Friction reduction in the hydrodynamic thrust bearing contact by texturing of the surface is addressed by means of numerical simulations. Salient features of our study are: (a) Mass-conserving cavitation modeling (Elrod-Adams method); (b) Fixed load on the thrust bearing sufficiently low to correspond to the hydrodynamic regime, with a Stribeck number of 10^{-3} ; (c) Fully flooded conditions; (d) The texture is on the runner surface, and thus moving through the computational domain; (e) The dynamics of the thrust bearing, which is not trivial because of the passage of textures under it, is solved simultaneously with the hydrodynamics of the film; (f) The study considers thrust bearings of cylindrical shape with curvature radius ranging from 0.2 to 102.4 cm; (g) The study analyses pocketed textures with different sizes, aspect ratios, depths and coverage fractions, totalling more than three hundred liner topographies. The results show a clear dependence of the effect of the runner texture on the curvature radius of the thrust bearing. For low curvature radius, all textures lead to friction coefficients that are higher than that of the untextured runner surface. For high enough curvature radius, however, it is possible to identify pocketed surfaces which significantly reduce the friction (up to 80%). The reported set of results, arguably the first to deal with texture on the moving surface (which is the standard case in experimental studies), provides both an explanation for mixed experimental findings available in the literature and a guidance on surface texture selection for low-load thrust bearing contact.

Nous traitons dans ce travail, par des simulations numériques, la réduction du frottement dans le contact d'une butée cylindrique. Les éléments importants de notre étude sont : (a) Le modèle de cavitation conservatif d'Elrod-Adams ; (b) Une charge fixe appliquée à la butée suffisamment petite pour que le régime soit purement hydrodynamique ; (c) Le film d'huile est plein ; (d) La texture est présente sur le collet, ce qui entraîne un domaine de calcul en mouvement ; (e) La dynamique du segment est résolue en couplage avec l'hydrodynamique du film ; (f) Les rayons de courbure des butées varient entre 0.2 et 51.2 cm ; (g) L'étude analyse des textures sous forme de poche en variant la taille, le rapport largeur/longueur, la profondeur et la zone de couverture des stries. La variation de tous ces paramètres totalise plus de 300 topographies du collet. Les résultats montrent une dépendance claire de l'effet de la texture du collet sur le rayon de courbure de la butée. Pour des faibles rayons de courbure, toutes les textures donnent un coefficient de frottement plus grand que celui du collet non texturé. En revanche, pour des butées avec de grands rayons de courbures il est possible d'identifier des collets admettant des textures de type poches qui réduisent significativement le frottement (jusqu'à 80 %). Les résultats présentés dans ce travail sont probablement les premiers à traiter des surfaces texturées en mouvement (qui est le cas standard dans l'expérimentation) et donnent une explication des résultats expérimentaux existant en littérature et aussi une sélection des surfaces texturées dans le cas de contact avec des petites charges appliquées.

1 INTRODUCTION

Texturing of contact surfaces to reduce friction and wear has attracted much attention in the last years, especially since surface microengineering techniques have become available. The goal is to use

Mass-conserving numerical simulation of ring-textured liner contact in the hydrodynamic lubrication regime

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Abstract: *Friction reduction in the ring/liner contact by texturing of the surface of the liner is addressed by means of numerical simulations with a mass-conserving cavitation algorithm for fully flooded conditions. The dynamics of the ring is solved simultaneously with the hydrodynamics of the film. Results are obtained in the hydrodynamic lubrication regime for a fixed Stribeck number (10^{-3}), studying the effect of the textures (totalling more than three hundred liner topographies) in friction coefficient and minimum film clearance for several ring curvatures. The results show a clear dependence of the effect of the textures on the curvature radius of the ring.*

Palavras-chave: *Friction reduction, textured lubricated contacts, ring/liner contact, Elrod-Adams model.*

1 Introduction

Texturing of contact surfaces to reduce friction and wear has attracted much attention in the last years, especially since surface microengineering techniques have become available. The goal is to use surface texturing on several applications such as thrust bearings, mechanical seals, lip seals, piston ring/liner contact among others.

In the piston ring/liner application recent works have shown experimentally ([6]) and numerically ([9], [11]) the dependence on the ring curvature of the benefits of texturing, which will be studied here. However, the experimental works perform a very limited number of experiments. The numerical works solve the problem with non mass-conserving algorithms and most of them solve the problem with the texture *on the ring*, which is a problem much less involved than solving the texture on the liner.

In this article, we will study the piston ring/liner lubrication problem with a cylindrical ring moving against a moving textured surface. The conditions are of fixed relative velocity and with atmospheric pressure (assumed equal to the cavitation pressure) at the inlet and outlet of the pad. The texture is on the runner surface, and thus moving through the simulation domain. The texture's geometry corresponds to pockets inspired in those produced by Laser Surface Texturing (LST) ([8]).

Notably, numerical simulations of textured lubricated contacts in which the texture moves through the simulation domain are practically non-existent in the literature. Some attempts at using mass-conserving models have been reported by Organisciak [10] and Ausas [3]. In assessing textured lubricated contacts, the crucial importance of mass-conserving boundary conditions at cavitation boundaries has been established by Ausas et al ([2]) and confirmed by several authors, among them Zhang & Meng ([13]) who performed a careful comparison with experiments.

TRIBOLOGICAL PERFORMANCE OF TEXTURED SURFACES IN THE PISTON RING/LINER CONTACT USING THE ELROD-ADAMS MODEL

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Keywords: Friction reduction, textured surfaces, piston ring, Elrod-Adams model.

Abstract. The possibility of improving the performance of lubricated surfaces through surface texturing has been a topic of intense research in the latest years. Considerable efforts in both experimental and numerical works have been made to study the effects of micro-textures in load capacity, friction and wear. In the industry it has been known for a long time that some texturing is required in the liners of combustion engines to avoid stiction with the piston rings. The numerical simulation of that problem poses a challenge in modeling the intervening phenomena (cavitation, starvation, ring dynamics) and solving it efficiently.

In this work we present numerical simulations of the piston ring/liner problem in the hydrodynamic lubrication regime using the Elrod-Adams model and a mass-conserving algorithm. Realistic values are assumed for the parameters defining the problem. The formation of cavitation bubbles and its relation with the load-carrying capacity, friction and clearance is analyzed by considering different texture configurations and shapes in one-dimensional tests. Afterwards, two-dimensional dimpled surfaces are simulated for more than a hundred different texture configurations in the hydrodynamic lubrication regime for several Stribeck numbers. A robust code accelerated by means of a multigrid implementation allowed the realization of this extensive study. Results show that friction and wear reduction are attainable by virtue of dimples of the size of the contact, although the gain decreases or disappears as the mixed lubrication regime is approached.

Effect of liner texture on ring/liner tribological performance

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Abstract

Friction-reducing textures for lubricated mechanisms have received much attention since the advent of microtexturing techniques. In this work we consider the piston ring/liner contact, numerically simulating the rings as they slide against a textured liner. This situation has not been sufficiently analyzed because its numerical complexity is much greater than that of texturing the rings. Further, we consider the actual typical geometries of oil-control, wiper and compression rings, under realistic loads and velocities. The results show the possibility of friction reduction when the texture on the liner is properly designed, identifying starvation alleviation as a crucial intervening mechanism.

Keywords: laser texture, cylinder liner, piston ring, mass-conservative model, friction, fourstroke cycle.

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Bibliography

- [1] Mahle GmbH. <http://http://www.br.mahle.com>. Accessed: 2015-02-2.
- [2] Octave. <https://www.gnu.org/software/octave/>. Accessed: 2015-02-10.
- [3] E. Abbott and F. Firestone. Specifying Surface Quality - A Method Based on Accurate Measurements and Comparison. *Journal of Mechanical Engineering*, 55:569–572, 1933.
- [4] A. Akchurin, R. Bosman, P. Lugt, and M. van Drogen. On a Model for the Prediction of the Friction Coefficient in Mixed Lubrication Based on a Load-Sharing Concept with Measured Surface Roughness. *Tribology Letters*, 59:19, 2015.
- [5] Fadi Ali, Ivan Křupka, and Martin Hartle. Analytical and experimental investigation of friction of non-conformal point contacts under starved lubrication. *Meccanica*, October 2012.
- [6] A. Almqvist, J. Fabricius, R. Larsson, and P. Wall. A new approach for studying cavitation in lubrication. *ASME Journal of Tribology*, 136:011706–1–011706–7, 2014.
- [7] T. Almqvist and R. Larsson. Some Remarks on the Validity of Reynolds Equations in the Modeling of Lubricant Film Flows on the Surface Roughness Scale. *ASME Journal of Tribology*, 126:703–710, 2004.
- [8] S. Alvarez. *Problemas de frontera libre en teoría de lubricación*. PhD thesis, Universidad Complutense de Madrid, Spain, 1986.

- [9] P. Andersson, J. Tamminen, and Sandström. Piston ring tribology. Technical report, VTT Industrial Systems, Finland, 2012.
- [10] Douglas N Arnold, Franco Brezzi, Bernardo Cockburn, and L Donatella Marini. Unified analysis of discontinuous galerkin methods for elliptic problems. *SIAM journal on numerical analysis*, 39(5):1749–1779, 2002.
- [11] R. Ausas, M. Jai, and G. Buscaglia. A Mass-Conserving Algorithm for Dynamical Lubrication Problems With Cavitation. *ASME Journal of Tribology*, 131:031702 (7 pages), 2009.
- [12] R. Ausas, P. Ragot, J. Leiva, G. Jai, M. Bayada, and G. Buscaglia. The impact of the Cavitation model in the Analysis of Micro-Textured Lubricated Journal bearings. *ASME Journal of Tribology*, 129:868–875, 2009.
- [13] R. Ausas, F. Sousa, and G. Buscaglia. An improved finite element space for discontinuous pressures. *Comput. Methods Appl. Mech. Engrg.*, 199:1019–1031, 2010.
- [14] R.F. Ausas. *Simulación Numérica en Flujo de dos Fases Inmiscibles con aplicaciones en Lubricación Hidrodinámica*. PhD thesis, Instituto Balseiro, Argentina, 2010.
- [15] Roberto F. Ausas, Mohammed Jai, Ionel S. Ciuperca, and Gustavo C. Buscaglia. Conservative one-dimensional finite volume discretization of a new cavitation model for piston-ring lubrication. *Tribology International*, 57:54–66, 2013.
- [16] C. Baker, S. Theodossiades, H. Rahnejat, and B. Fitzimons. Influence of in-plane dynamics of thin compression rings on friction in internal combustion engines. *ASME Journal of Engineering for Gas Turbines and Power*, 134:092801–1–092801–11, 2012.
- [17] D.J.W. Barrell, M Priest, and C.M. Taylor. The axial motion of the piston ring in the top ring groove of a gasoline engine. *Proceedings of the 27th Leeds-Lyon Symposium on Tribology*, 39:641–649, 2001.
- [18] K.J. Bathe. *Finite Element Procedures*. Prentice-Hall, 1996.
- [19] G. Bayada. From a compressible fluid model to new mass conserving cavitation algorithms. *Tribology International*, 71:38–49, 2014.

- [20] G. Bayada and M. Chambat. Analysis of a free boundary problem in partial lubrication. *Quart. Applied Math.*, 4:369–375, 1983.
- [21] G. Bayada and M. Chambat. Existence and uniqueness for a lubrication problem with non regular conditions on the free boundary. *Boll. U.M.I.*, 6(3b):543–557, 1984.
- [22] G. Bayada and M. Chambat. Sur quelque modelisation de la zone de cavitation en lubrification hydrodynamique. *Journal de mécanique théorique et appliqué*, 5:703–729, 1986.
- [23] G. Bayada and M. Chambat. The transition between the Stokes equations and the Reynolds equation: A mathematical proof. *Applied Mathematics & Optimization*, 14(1):73–93, 1986.
- [24] G. Bayada, M. Chambat, and M. El Alaoui. Variational Formulations and Finite Element Algorithms for Cavitation Problems. *ASME Journal of Tribology*, 112:398–403, 1990.
- [25] G. Bayada, M. Chambat, and J.B. Faure. Some effects of the boundary roughness in a thin film. Boundary control and boundary variations. *Lecture Notes in Compu Sci.*, 100:96–115, 1988.
- [26] G. Bayada, M. Chambat, and J.B. Faure. A Double Scale Analysis Approach of the Reynolds Roughness. Comments and Application to the Journal Bearing. *ASME Journal of Tribology*, 111:323–330, 1989.
- [27] G. Bayada and L. Chupin. Compressible fluid model for hydrodynamic lubrication with cavitation. *ASME Journal of Tribology*, 135:041702–1–041702–13, 2013.
- [28] G. Bayada, S. Martin, and C. Vázquez. Two-scale homogenization of a hydrodynamic Elrod-Adams model. *Asymptotic Analysis*, 44:75–110, 2005.
- [29] Yin Bifeng, Xiaodong Li, Yonghong Fu, and Wang Yun. Effect of laser textured dimples on the lubrication performance of cylinder liner in diesel engine. *Lubrication Science*, 24:293–312, 2012.
- [30] H. Bouassida. *Lubricated piston ring cylinder liner contact: Influence of the liner microgeometry*. PhD thesis, INSA de Lyon, 2014.

- [31] G. Buscaglia, I. Ciuperca, and M. Jai. Homogenization of the transient Reynolds equation . *Asymptotic Analysis*, 32:131–152, 2002.
- [32] G. Buscaglia, I. Ciuperca, and M. Jai. On the optimization of surface textures for lubricated contacts. *Journal of Mathematical Analysis and Applications*, 335:1309–1327, 2007.
- [33] G. Buscaglia, M. El Alaoui Talibi, and M. Jai. Mass-conserving cavitation model for dynamical lubrication problems. Part I: Mathematical analysis. *Math. Comput. Simulation*, 2015.
- [34] G. Buscaglia and M. Jai. Homogenization of the Generalized Reynolds Equation for Ultra-Thin Gas Films and Its Resolution by FEM . *ASME Journal of Tribology*, 126:547, 2004.
- [35] G. C. Buscaglia and R. F. Ausas. Variational formulations for surface tension, capillarity and wetting. *Computer Methods in Applied Mechanics and Engineering*, 70:829–850, 2012.
- [36] G. C. Buscaglia, I. Ciuperca, and M. Jai. The effect of periodic textures on the static characteristics of thrust bearings. *ASME Journal of Tribology*, 127:899–902, 2005.
- [37] Gustavo C. Buscaglia, Ionel Ciuperca, Erick Dalissier, and Jai Mohammed. A new cavitation model in lubrication: the case of two-zone cavitation. *Journal of Engineering Mathematics*, n/a:n/a–n/a, 2013.
- [38] H. Checo, R. Ausas, M. Jai, J.-P. Cadalen, F. Choukroun, and G. Buscaglia. Moving textures: Simulation of a ring sliding on a textured liner. *Tribology International*, 72:131–142, 2014.
- [39] H. Checo, G. Buscaglia, and M. Jai. Mass-conserving numerical simulation of ring-textured liner contact in the hydrodynamic lubrication regime. In *Anais do Congresso de Matemática Aplicada e Computacional CMAC Sudeste 2013*, pages 464–470, 2013.
- [40] H. Checo, M. Jai, J.-P. Cadalen, F. Choukroun, and G. Buscaglia. Mass-conserving simulation of friction reduction in a hydrodynamic thrust bearing. In *Proceedings of the 12th EDF-PPprime Workshop: Solutions for performance improvement and friction reduction of journal and thrust bearings*, page 777, 2013.

- [41] H. Checo, A. Jaramillo, M. Jai, and G. Buscaglia. Effect of liner texture on ring/liner tribological performance. In *Anais do Second International Conference on Tribology*, 2014.
- [42] H. Checo, A. Jaramillo, M. Jai, and G. Buscaglia. Texture-induced cavitation bubbles and friction reduction in the Elrod–Adams model. *Proc IMechE Part J: J Engineering Tribology*, pages 1–15, 2014.
- [43] H. Checo, A. Jaramillo, M. Jai, and G. Buscaglia. Tribological performance of textured surfaces in the piston ring/liner contact using the Elrod-Adams model. In *Mecánica Computacional*, pages 129–145, 2014.
- [44] H. Checo, A. Jaramillo, M. Jai, and G. Buscaglia. The lubrication approximation of the friction force for the simulation of measured surfaces. *Tribology International*, 2015 (10.1016/j.triboint.2015.12.030).
- [45] H. Christensen. Stochastic Models of Hydrodynamic Lubrication of Rough Surfaces. *Proc, Int. Mech. Eng*, 55:1013, 1969.
- [46] R. Codina, J. Blasco, G. Buscaglia, and A. Huerta. Implementation of a stabilized finite element formulation for the incompressible navier-stokes equations based on a pressure gradient projection. *Int. J. Numer. Meth. in Fluids*, 37:410–444, 2001.
- [47] H.L. Costa and I.M. Hutchings. Hydrodynamic lubrication of textured steel surfaces under reciprocating sliding conditions. *Tribology International*, 40:1227–1238, 2007.
- [48] R.W. Cottle, J.S. Pang, and R.E. Stone. *The linear complementarity problem*. Report No. 60 SIAM Philadelphia, PA, 2009.
- [49] Andrew T. Cross, Farshid Sadeghi, Richard G. Rateick, and Scott Rowan. Hydrodynamic pressure generation in a pocketed thrust bearing. *Tribology Transactions*, 56:652–662, 2013.
- [50] Leonardo De Chiffre, P Lonardo, H Trumpold, DA Lucca, G Goch, CA Brown, J Raja, and Hans Nørgaard Hansen. Quantitative characterisation of surface texture. *CIRP Annals-Manufacturing Technology*, 49(2):635–652, 2000.
- [51] M. Dobrica and M. Fillon. About the validity of Reynolds equation and inertia effects in textured sliders on infinite width. *Proc. IMechE*, 223:69–78, 2008.

- [52] D. Dowson and C. Taylor. Cavitation in bearings. *Annu. Rev. Fluid Mech.*, 11:35–66, 1979.
- [53] A. El Alaoui. *Sur un problème à frontière libre en mécanique des films minces*. PhD thesis, Univ. Claude Bernard Lyon I, Lyon, France, 1986.
- [54] M. El Alaoui Talibi and M. Jai. Existence and numerical results of a transient lubrication problem with cavitation. *Journal of Mathematical Analysis and Applications*, 394:260–275, 2012.
- [55] H. G. Elrod and M. Adams. A computer program for cavitation. Technical report 190. *1st LEEDS LYON Symposium on Cavitation and Related Phenomena in Lubrication, I.M.E.*, 103:354, 1974.
- [56] H.G. Elrod. A General Theory for Laminar Lubrication with Reynolds Roughness. *Transactions of the ASME*, 101:8–14, 1979.
- [57] I. Etsion. State of the art in laser surface texturing. *J. Tribol.: Trans. ASME*, 127:248–253, 2005.
- [58] I. Etsion and O. Pinkus. Analysis of Short Journal Bearings with New Upstream Boundary Conditions. *Journal of Lubrication Technology, Trans. ASME*, 96:489–496, 1974.
- [59] I. Etsion and E. Sher. Improving fuel efficiency with laser surface textured piston rings. *Tribology International*, 42:542–547, 2009.
- [60] Izhak Etsion. Modeling of surface texturing in hydrodynamic lubrication. *Friction*, 3:195–209, 2013.
- [61] Scott R. Fulton, Paul E. Ciesielsky, and Wayne H. Schubert. Multigrid methods for elliptic problems: a review. *Mon. Wea. Rev.*, 114:943–959, 1986.
- [62] Gonzalo Brito Gadeschi, Katja Backhaus, and Gunther Knoll. Numerical Analysis of Laser-Textured Piston-Rings in the Hydrodynamic Lubrication Regime. *Journal of Tribology*, 134:041702–1–041702–8, October 2012.
- [63] J. Galligan, A.A. Torrance, and G. Liraut. A scuffing test for piston ring/bore combinations: Pt. II. Formulated motor lubrication. *Wear*, 236:210–220, 1999.

- [64] A. Ghanbarzadeh, M. Wilson, A. Morina, D. Dowson, and A. Neville. Development of a new mechano-chemical model in boundary lubrication. *Tribology International*, 93:573–582, 2015.
- [65] M. Giacomini, M. Fowell, D. Dini, and A. Strozzi. A mass-conserving complementarity formulation to study lubricant films in the presence of cavitation. *ASME Journal of Tribology*, 132:041702–1–041702–12, 2010.
- [66] Wieslaw Grabon, Waldemar Koszela, Pawel Pawlus, and Slawomir Ochwat. Improving tribological behavior of piston ring-cylinder liner frictional pair by liner surface texturing. *Tribology International*, 61:102–108, 2013.
- [67] Wieslaw Grabon, Pawel Pawlus, and Jaroslaw Sep. Tribological characteristics of one-process and two-process cylinder liner honed surfaces under reciprocating sliding conditions. *Tribology International*, 43:1882–1892, 2010.
- [68] J. Greenwood and J. Williamson. Contact of Nominally Flat Surfaces. *Proceedings of the Royal Society of London. Series A, Mathematical and Physical Sciences*, 295:300–319, 1966.
- [69] Kenneth Holmberg, Peter Andersson, and Ali Erdemir. Global energy consumption due to friction in passenger cars. *Tribology International*, 47:221–234, 2012.
- [70] M. Jai. Homogenization and two-scale convergence of the compressible Reynolds lubrication equation modeling the flying characteristics of a rough magnetic head over a rough rigid-disk surface. *Math. Modeling and Numer. Anal.*, 29:199–233, 1995.
- [71] B. Jakobson and L. Floberg. The Finite Journal Bearing Considering Vaporization. *Tran. Chalmers University of Technology*, 190:1–119, 1957.
- [72] A. Jaramillo. *Mathematical modeling of micro-textured lubricated contacts*. PhD thesis, Universidade de São Paulo, 2015.
- [73] Bom Sok Kim, Woo Yooung Chung, Meung-Ho Rhee, and Sang-Yul Lee. Studies on the application of laser surface texturing to improve the tribological performance of AlCrSiN-coated surfaces. *Met. Mater. Int.*, 18(6):1023–1027, 2012.
- [74] Y. Kligerman and A. Shinkarenko. Analysis of friction in surface textured components of reciprocating mechanism. *Proc IMechE Part J: J Engineering Tribology*, page 14, 2014.

- [75] Y. and Etsion I. Kligerman and A. Shinkarenko. Improving tribological performance of piston rings by partial surface texturing. *Journal of Tribology*, 127(3):632–638, 2005.
- [76] Andriy Kovalchenko, Oyelayo Ajayi, Ali Erdemir, and George Fenske. Friction and wear behavior of laser textured surface under lubricated initial point contact. *Wear*, 271:1719–1725, 2011.
- [77] Andriy Kovalchenko, Oyelayo Ajayi, Ali Erdemir, George Fenske, and Izhak Etsion. The effect of laser surface texturing on transitions in lubrication regimes during unidirectional sliding contacts. *Tribology International*, 38:219–225, 2005.
- [78] A. Kumar and J.F. Booker. A finite element cavitation algorithm. *ASME Journal of Tribology*, 113:276–286, 1991.
- [79] R. LeVeque. *Finite Volume Methods for Hyperbolic Problems*. Cambridge University Press, 2004.
- [80] Jinfeng Li, Fei Zhou, and Xiaolei Wang. Modify the friction between steel ball and PDMS disk under water lubrication by surface texturing. *Meccanica*, 46:499–507, 2011.
- [81] Y. Li, H. Chen, and T. Tian. A deterministic model for lubricant transport within complex geometry under sliding contact and its application in the interaction between the oil control ring and rough liner in internal combustion engines. *SAE Technical Paper*, (2008-01-1615), 2008.
- [82] PM Lonardo, H Trumpold, and Leonardo De Chiffre. Progress in 3d surface microtopography characterization. *CIRP Annals-Manufacturing Technology*, 45(2):589–598, 1996.
- [83] S. Medina, M. Fowell, S. Vladescu, T. Reddyhoff, I. Pegg, A. Olver, and D. Dini. Transient effects on lubricated textured bearings. *Proc IMechE Part J: J Engineering Tribology*, 2015.
- [84] Surya P. Mishra and Andreas A. Polycarpou. Tribological studies of unpolished laser surface textures under starved lubrication conditions for use in air-conditioning and refrigeration compressors. *Tribology International*, 44:1890–1901, 2011.
- [85] K. Olsson. Cavitation in dynamically loaded bearings. *Tran. Chalmers University of Technology*, 308, 1965.

- [86] M. Organisciak and G. and Lubrecht A.A. Cavallaro. Starved hydrodynamic lubrication of the piston ring cylinder liner contact: Preliminary study of the influence of surface texturing. *Tribology and Interface Engineering Series*, 48:573–583, 2005.
- [87] Michel Organisciak. *Optimisation de la microgeometrie des chemises de moteurs a combustion interne*. PhD thesis, l’Institut National des Sciences Appliquées de Lyon, 2007.
- [88] A. Panayi and H. Schock. Approximation of the integral of the asperity height distribution for the Greenwood–Tripp asperity contact model. *Proc IMechE Part J: J Engineering Tribology*, 222:165–169, 2008.
- [89] N. Patir and H.S. Cheng. An Average Flow Model for Determining Effects of Three-Dimensional Roughness on Partial Hydrodynamic Lubrication. *Transactions of the ASME*, 100:12–17, 1978.
- [90] N. Patir and H.S. Cheng. Application of Average Flow Model to Lubrication Between Rough Sliding Surfaces. *Transactions of the ASME*, 101:220–229, 1979.
- [91] U. Pettersson and S. Jacobson. Textured surfaces for improved lubrication at high pressure and low sliding speed of roller/piston in hydraulic motors. *Tribology International*, 40:355–359, 2007.
- [92] Ulrika Pettersson and Staffan Jacobson. Influence of surface texture on boundary lubricated sliding contacts. *Tribology International*, 36:857–864, 2003.
- [93] F. Profito, E. Tomanik, and D. Zachariadis. Effect of cylinder liner wear on the mixed lubrication regime of TLOCs. *Tribology International*, 93:723–732, 2015.
- [94] Y. Qiu and M. Khonsari. Experimental investigation of tribological performance of laser textured stainless steel rings. *Tribology International*, 44:635–644, 2011.
- [95] R. Rahmani, A. Shirvani, and H. Shirvani. Analytical analysis and optimization of the Rayleigh step slider bearing. *Tribology International*, 42:666–674, 2009.
- [96] Ashwin Ramesh, Wasim Akram, Surya P. Mishra, Andrew H. Cannon, Andreas A. Polycarpou, and William P. King. Friction characteristics of microtextured surfaces under mixed and hydrodynamic lubrication. *Tribology International*, 57:170–176, 2013.

- [97] A. Ronen, I. Etsion, and . Kligerman. Friction-reducing surface-texturing in reciprocating automotive components. *Tribology Transactions*, 45:359–366, 2001.
- [98] G. Ryk and I. Etsion. Testing piston rings with partial laser surface texturing for friction reduction. *Wear*, 261:792–796, 2006.
- [99] F. Sahlin, A. Almqvist, R. Larsson, and S. Glavatskih. A cavitation algorithm for arbitrary lubricant compressibility. *Tribology International*, 40:1294–1300, 2007.
- [100] M. Scaraggi, F. Mezzapesa, G. Carbone, A. Ancona, D. Sorgente, and P. Lugarà. Minimize friction of lubricated laser-microtextured-surfaces by tuning microholes depth. *Tribology International*, 75:123–127, 2014.
- [101] Michele Scaraggi, Francesco P. Mezzapesa, Giuseppe Carbone, Antonio Ancona, and Luigi Tricarico. Friction properties of lubricated laser-microtextured-surfaces: an experimental study from boundary to hydrodynamic lubrication. *Tribology Letters*, 49:117–125, 2013.
- [102] D.Z. Segu, S.G. Choi, J. Choi, and S.S. Kim. The effect of multi-scale laser textured surface on lubrication regime. *Applied Surface Science*, 270:58–63, 2013.
- [103] A. Shinkarenko, Y. Kligerman, and Etsion I. The effect of surface texturing in soft elasto-hydrodynamic lubrication. *Tribology International*, 42:284–292, 2009.
- [104] Edward H. Smith. Optimising the design of a piston-ring pack using DoE methods. *Tribology International*, 44:29–41, 2010.
- [105] A. Strozzi. Formulation of three lubrication problems in terms of complementarity. *Wear*, 104:103–119, 1985.
- [106] Hiroyoshi Tanaka and Kazunori Ichimaru. Numerical simulation of oil-film behavior on cylinder liners in sliding contacts with piston rings. *Transient Processes in Tribology*, pages 283–290, 2004.
- [107] L. Tao, S. Diaz, L. San Andrés, and K.R. Rajagopal. Analysis of squeeze film dampers operating with bubbly lubricants. *ASME Journal of Tribology*, 122:205–210, 2000.
- [108] Eduardo Tomanik. Friction and wear bench test of different engine liner surface finishes. *Tribology International*, 41:1032–1038, 2008.

- [109] Eduardo Tomanik. Modelling the hydrodynamic support of cylinder bore and piston rings with laser textured surfaces. *Tribology International*, 59:90–96, 2013.
- [110] S.T. Tzeng and Saibel E. Surface Roughness Effect on Slider Bearing Lubrication. *ASLE Transactions*, 10:334–338, 1967.
- [111] D. van Odyck and C. Venner. Stokes Flow in Thin Films. *ASME Journal of Tribology*, 125:121–134, 2003.
- [112] C.H. Venner and A.A. Lubrecht. *Multilevel methods in lubrication*. Elsevier, 2000.
- [113] D. Vijayaraghavan and T. Keith. Development and evaluation of a cavitation algorithm. *Tribology Transactions*, 32:225–233, 1989.
- [114] X. Wang, K. Zhu, and C. Gui. A study of a journal bearing lubricated by couple stress fluids considering thermal and cavitation effects. *Proc IMechE Part J: J Engineering Tribology*, 216:293–305, 2002.
- [115] Xiaolei Wang, Koshi Adachi, Katsunori Otsuka, and Koji Kato. Optimization of the surface texture for silicon carbide sliding in water. *Applied Surface Science*, 253:1282–1286, 2006.
- [116] E. Weinan. *Principles of multiscale modeling*. Cambridge University Press, 2011.
- [117] C. Xing, M. Braun, and H. Li. A three-dimensional Navier-Stokes-based numerical model for squeeze film dampers. Part 1: Effects of gaseous cavitation on pressure distribution and damping coefficients without consideration of inertia. *Tribology Transactions*, 52:680–694, 2009.
- [118] Sihuan Yuan, Wei Huang, and Xiaolei Wang. Orientation effects of micro-grooves on sliding surfaces. *Tribology International*, 44:1047–1054, 2011.
- [119] Jinyu Zhang and Yonggang Meng. Direct observation of cavitation phenomenon and hydrodynamic lubrication analysis of textured surfaces. *Tribology Letters*, 46:147–158, 2012.