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Física do Relaxion: uma nova solução para o problema da hierarquia

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Relaxion Physics: a new solution to the hierarchy problem

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Abstract

The electroweak hierarchy problem is one of the most important puzzles of particle physics that remains without conclusive answer nowadays. One of the most recent new class of solutions to this problem is presented in this thesis, *i.e.*, the cosmological relaxation of the electroweak scale. In this framework, we postulate the existence of a new particle, the relaxion, which drives the Higgs mass to values much smaller than the cutoff of the theory during inflation. As tools to develop this subject, this work presents a resume of chiral perturbation theory, the strong CP problem, axions and the η' particle. Finally, we will describe the most simple model of cosmological relaxation of the electroweak scale and the non-QCD model, where a new strong group $SU(N)$ forms a condensate that interacts with the relaxion.

Keywords: Particle Physics. Hierarchy Problem. Cosmological Relaxation of the Electroweak Scale. Relaxion. Chiral Perturbation Theory. Axion. η' **.**

Resumo

O problema da hierarquia eletrofraco é um dos enigmas mais importantes da física de partículas que continua sem uma solução conclusiva hoje em dia. Uma nova classe de soluções, dentre as mais recentes, para este problema é apresentado nessa dissertação, *i.e.*, o relaxamento cosmológico da escala eletrofraca. Neste quadro, nós postulamos a existência de uma nova partícula, o relaxion, que conduz a massa do bóson de Higgs para valores muito menores que o cutoff da teoria durante a inflação. Como ferramentas para desenvolver este assunto, este trabalho apresenta um resumo de teoria de perturbação quiral, o problema CP forte, axions e a partícula η'. Finalmente, iremos descrever o modelo mais simples de relaxamento cosmológico da escala eletrofraca e o modelo sem-QCD, onde um novo grupo de interação forte $SU(N)$ forma um condensado que interage com o relaxion.

Palavras-Chave: Física de Partículas. Problema da Hierarquia. Relaxamento Cosmológico da Escala Eletrofraca. Relaxion. Teoria de Perturbação Quiral. Axion. η' .

Contents

Chapter 1

Introduction

The Standard Model of particle physics (SM), as developed in the 1960's after the work of Glashow, Weinberg and Salam [1][2], is an extremely successful theory, whose predictions have been experimentally confirmed to the permil level. As we are going to explain, however, the presence of a fundamental scalar particle makes the theory unstable under radiative corrections, introducing in principle a huge tuning to explain the lightness of the Higgs mass. This "fine tuning" (or naturalness) problem has been the focus of most of the theoretical activities over the past decades. The attempts to solve it include compositeness for the Higgs, supersymmetry, extra-dimensions, quantum gravity at the electroweak scale and anthropics. Since the last one assumes the existence of a multiverse, while the others have collider and indirect constraints that force these models into fine-tuned regions of their parameter spaces, we are still lacking a concrete and conclusive solution. In this thesis we will study a new class of solutions, the so-called relaxion framework, one of the most recent proposals for the solution of the hierarchy problem of particle physics. As we will see, in this proposal an axion-like particle will scan the Higgs mass during inflation, making the Higgs mass technically natural, solving the hierarchy problem with no multiverse assumption or fine-tuned regions on the parameter space.

This thesis is organized as follows: in the remainder of the introduction we will discuss in detail the hierarchy problem of the SM. Since, as already mentioned, the relaxion is an axion-like particle (ALP), in Chapters 2 and 3 we will study the QCD axion and chiral perturbation theory. The results of these chapters will then be used in Chapter 4 to discuss the minimal relaxion framework as well as a simple alternative dubbed "non-QCD" model.

1.1 Naturalness

Naturalness, roughly speaking, is the idea that all parameters of a fundamental theory should be of order one. The most primitive way of defining naturalness is due to Dirac, and states that for an operator $\mathcal{A}$ in the Lagrangian we must have that the corresponding Wilson coefficient $c_{\mathcal{A}}$ has the form

$$c_{\mathcal{A}} = \mathcal{O}(1) \times \Lambda^{4-\Delta_{\mathcal{A}}}, \tag{1.1}$$

where Λ is the fundamental scale of the theory and $\Delta_{\mathcal{A}}$ is the dimension of the operator $\mathcal{A}$.

Another possibility, that takes into account the notion that in addition to scales and interactions in QFT there are also symmetries, is the so called 't Hooft criterium for technical naturalness. It states that if the theory has an enhanced symmetry when a parameter is zero, then the quantum corrections of the parameter will be proportional to the parameter itself. Thus, if the parameter is small, it will remain small after radiative corrections are considered, and we say that the parameter is "protected" by the symmetry.

As an example we consider the electron mass. The self-energy graph, given by the diagram in Figure 1.1, is

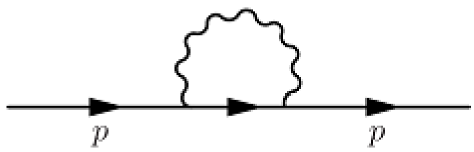

Figure 1.1: Diagram of electron self-energy.

$$i\Sigma_2(\not{p}) = -i\frac{e^2}{8\pi^2}\int_0^1 dx(2m_e - x\not{p})\left(\frac{2}{\epsilon} + \log\frac{\tilde{\mu}^2}{(1-x)(m_e^2 - p^2x) + xm_\gamma^2}\right), \tag{1.2}$$

where e is the electron charge, m_e is the electron mass, $\tilde{\mu} \equiv 4\pi e^{-\gamma_E}\mu$, μ is the arbitrary parameter of dimension 1 of dimensional regularization, and m_γ is a fictitious photon mass used for the regularization (that will be set to zero at the end of the computation).

Notice that, from Eq. (1.2), the QED correction is proportional to the electron mass at the pole $\Sigma_2(\not{p} = m_e)$. This happens because the electron mass is protected by the chiral symmetry, which is a global symmetry where right-handed and left-handed electrons have opposite charges $\psi_L \to e^{-i\alpha}\psi_L$ and $\psi_R \to e^{i\alpha}\psi_R$. This transformation can be written as

$$\psi \to e^{i\alpha\gamma_5}\psi,$$

and leaves the QED kinetic term invariant

$$\bar{\psi}\not{D}\psi \to \psi^\dagger e^{-i\alpha\gamma_5^\dagger}\gamma_0\not{D}e^{i\alpha\gamma_5}\psi = \bar{\psi}\not{D}\psi,$$

where in the last equation we used $\gamma_5^\dagger = \gamma_5$ and $[\gamma_5, \gamma_0\gamma_\mu] = 0$. Similarly one can show that the QED interaction term is also invariant. The mass term however is not invariant:

$$m_e\bar{\psi}\psi \to m_e\bar{\psi}e^{2i\alpha\gamma_5}\psi \neq m_e\bar{\psi}\psi.$$

This shows that we have the chiral symmetry as an exact symmetry only if we set $m_e = 0$, and in this case m_e will stay 0 to all orders in perturbation theory. For $m_e \neq 0$ we treat the mass as an interaction term in such a way that every diagram that violates chiral symmetry, including corrections to the mass itself and the diagram of Figure 1.1, must be proportional to m_e. Notice however that even if a small parameter is technically natural it is still not Dirac-natural.

We now investigate examples of parameters that are not protected by any symmetry. Problems of non-naturalness fall in the class of problems that consist of a conflict between the theoretical expectations for the size of the parameters and the actual size of parameters we see in nature. Examples of such problems are the strong CP problem (that will be considered in Chapter 3), the cosmological constant problem and the hierarchy problem that we will now explain. The present discussion follow Refs. [3] and [4].

1.2 The hierarchy problem

The hierarchy problem is often described as the problem related to the fact that radiative corrections to the Higgs boson mass are quadratically divergent when one applies regularization by a sharp momentum cutoff. But this is not the best way to define the naturalness problem for two reasons:

First, the physics must never depend on the regularization procedure chosen to avoid the infinities in the loop integrals that will be absorbed by the counterterms, and the quadratic dependence on the cutoff only appears in sharp momentum cutoff regularization. When one uses dimensional regularization for the radiative loop correction (see below) this quadratic dependence disappears and another kind of divergence arises. Second, a fermion loop on the photon self-energy radiative correction is also quadratically divergent, but there is no naturalness problem for the vanishing mass of the photon.

1.2.1 Statement of the problem

A more profound statement of the problem is that the mass of a scalar particle, which is not protected by any symmetry, receives, from any particle or interaction, radiative corrections of the order of the energy scale of this particle or interaction. So the scalar mass is not Dirac natural (because of the large radiative corrections) and not even technically natural.

Now, we have several reasons to believe that the Standard Model (SM) is not the ultimate theory of Nature, among which:

1. Gravity is not described in the SM and it becomes important at the Planck mass scale $M_{Pl} \sim 10^{19}$ GeV;

2. We have no candidate for a dark matter particle in the SM;

3. When we simulate QED in the lattice above the Landau pole (at 10^{286} GeV), we find that the only consistent non-perturbative theory obtained has a vanishing coupling constant ($e = 0$), which is a contradiction since we are at a large coupling regime.

Since the SM has a gauge $SU(3)_c \times SU(2)_L \times U(1)_Y$ symmetry, even if we ignore gravity at M_{Pl}, we have to consider the same Landau pole problem of item 3 above for the g' coupling of $U(1)_Y$ symmetry, so we expect the SM to be an effective theory. Beyond that we must consider gravity at M_{Pl}, so anyway, either at M_{Pl} or above the Landau pole we expect New Physics (NP), *i.e.*, new degrees of freedom. If the Higgs boson of the SM is an elementary scalar, we expect it to be sensitive to NP either via the graviton or, since it has a hypercharge, via the boson B_μ. In both ways, the scalar mass which has no symmetry protecting it, will receive huge radiative corrections of order of at least the Planck mass scale, which makes very difficult to understand why the Higgs mass is only of 125 GeV, 16 orders of magnitude smaller of M_{Pl} (see Figure 1.2).

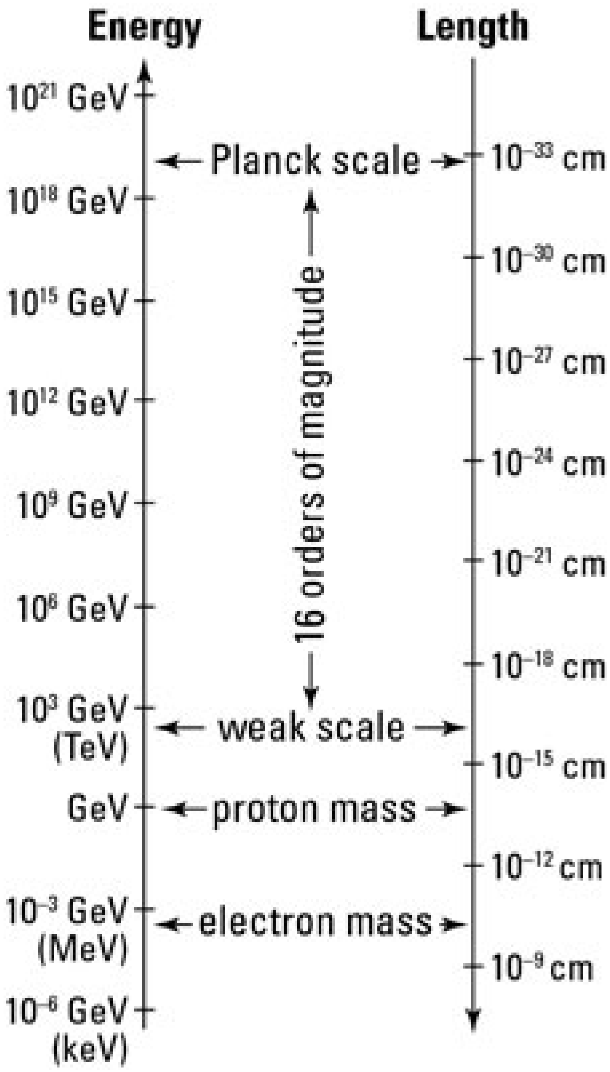

Figure 1.2: Hierarchy between the scales.

1.2.2 The fermion loop

As an example, we will show that the radiative corrections to the Higgs boson mass from a fermion with a large mass will be proportional to the mass of the fermion. We will do the computation in dimensional regularization to show that the main problem does not depend on the quadratic dependence of cutoff regularization.

First, we add to the Lagrangian a Yukawa coupling with the new heavy fermion f

$$\mathcal{L}_Y = -yh\bar{f}_L f_R + h.c.,$$

where y is the Yukawa coupling, h is the scalar field Higgs boson and $f_{L/R}$ are the left/right handed fermion fields.

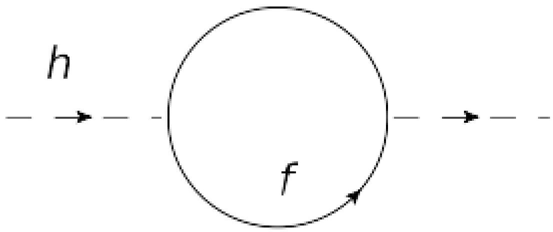

Figure 1.3: Fermion contribution to the Higgs self-energy.

The one-loop contribution from the fermion to the Higgs boson self-energy from the diagram of Figure 1.3, gives

$$i\mathcal{A} = -\mu^{4-d}(-iy)^2 \int \frac{d^d p}{(2\pi)^d} \mathrm{Tr}\left[\frac{i}{\not{k}+\not{p}-M} \cdot \frac{i}{\not{p}-M}\right], \tag{1.3}$$

where μ is the arbitrary parameter of dimension 1 of dimensional regularization, k is the momentum of the incoming and outgoing Higgs boson, $k+p$ and p are the momenta of the virtual fermions in the loop and M is the mass of the heavy fermion.

By computing (see Appendix B) the amplitude of Eq. (1.3) we get

$$i\mathcal{A} = -\frac{iy^2}{4\pi^2}\int_0^1 dx 3(-xk^2+x^2k^2+M^2)\left(\frac{2}{\epsilon}-\gamma+\log\left(\frac{\mu^2}{M^2-x(1-x)k^2}\right)+\cdots\right). \tag{1.4}$$

It is opportune now to compare Eq. (1.4) with Eq.(1.2). We notice that in Eq.(1.2) the electron self-energy is proportional to the electron mass, while in Eq. (1.4) the self-energy of the Higgs boson is not proportional to its own mass but rather it has a dominant contribution from M^2. This is a direct consequence of the fact that electron mass is technically natural, while the Higgs boson mass is not. Also, since the amplitude depends quadratically on the fermion mass, there is a large ultraviolet (UV) sensitivity.

We obtain the renormalization group equation (RGE) for the Higgs mass squared (see Eq. (B.13))

$$\mu\frac{dm_h^2}{d\mu} = -\frac{3y^2M^2}{2\pi^2} + \cdots \tag{1.5}$$

where the $\cdots$ represents the possible contribution from other loops that are not shown.

For a fermion mass, the RGE is always proportional to itself because it is protected by the chiral symmetry. That is not the case for Eq.(1.5), instead the RGE of the Higgs mass is proportional to the large fermion mass. This shows that the running of the Higgs mass takes contributions from any fermion it couples to. This is an example of how the naturalness problem appears.

1.2.3 Fine tuning

Now suppose that, as we already stressed, we have good reasons to believe that the SM is a low energy theory of a more complete one. If this more complete theory has a input scale Λ_{in} where its parameters are generated, the hierarchy problem will demand a high precision tuning for specifying the value of $m_h^2(\Lambda_{in})$ and run it down to find the right value of $m_h^2(\Lambda_{SM})$. This can be seen if we change $m_h^2(\Lambda_{in})$ by a small value ϵ and see how it affects $m_h^2(\Lambda_{SM})$. This can be expressed in the following way:

$$m_h^2(\Lambda_{in}) \to (1+\epsilon)m_h^2(\Lambda_{in}) \Rightarrow m_h^2(\Lambda_{SM}) \to (1+\Delta\epsilon)m_h^2(\Lambda_{SM})$$

where Δ is the low energy effect of the change ϵ at high energy.

After calling ${m'_h}^2(\Lambda_{in}) = (1+\epsilon)m_h^2(\Lambda_{in})$ and ${m'_h}^2(\Lambda_{SM}) = (1+\Delta\epsilon)m_h^2(\Lambda_{SM})$ and demanding $m_h'^2(\Lambda_{in}) - m_h^2(\Lambda_{in}) = \delta m_h^2(\Lambda_{in}) = \epsilon m_h^2(\Lambda_{in})$, while $m_h'^2(\Lambda_{SM}) - m_h^2(\Lambda_{SM}) = \delta m_h^2(\Lambda_{SM}) = \Delta\epsilon m_h^2(\Lambda_{SM})$, and finally eliminating ϵ, we and obtain (see Appendix B)

$$\Delta = \frac{d \log m_h^2(\Lambda_{SM})}{d \log m_h^2(\Lambda_{in})}. \tag{1.6}$$

Notice that from Eq. (1.5) we obtain

$$m_h^2(\Lambda_{SM}) \simeq m_h^2(\Lambda_{in}) + \frac{3y^2}{2\pi^2} M^2 \log\left(\frac{\Lambda_{in}}{\Lambda_{SM}}\right), \tag{1.7}$$

from which we already see that we need a very large fine tuning so that the cancellation of the right hand side of Eq. (1.7) gives a small number for $m_h^2(\Lambda_{SM})$. From Eq. (1.6) we obtain

$$\Delta = \frac{m_h^2(\Lambda_{in})}{m_h^2(\Lambda_{SM})} \frac{dm_h^2(\Lambda_{SM})}{dm_h^2(\Lambda_{in})} \approx \frac{m_h^2(\Lambda_{SM}) - \frac{3y^2}{2\pi^2} M^2 \log\left(\frac{\Lambda_{in}}{\Lambda_{SM}}\right)}{m_h^2(\Lambda_{SM})} \sim -\frac{M^2}{m_h^2(\Lambda_{SM})}, \tag{1.8}$$

where we used that $\frac{dm_h^2(\Lambda_{SM})}{dm_h^2(\Lambda_{in})} = 1$ from Eq. (1.7). Identifying $M \sim \Lambda_{in}$ we have

$$\Delta \sim -\frac{\Lambda_{in}^2}{m_h^2(\Lambda_{SM})}. \tag{1.9}$$

Now, in order to obtain the value of $m_h(\Lambda_{SM}) \sim 10^2$ GeV with an input scale of order of at least the Planck scale $\Lambda_{in} \sim 10^{19}$ GeV we obtain $\Delta \sim -10^{34}$. This means that a small deviation of order 1 at high energy causes a deviation of order 10^{34} at low energies, requiring a very precise tuning of the high energy parameter $m_h^2(\Lambda_{in})$ in order to reproduce the right value of $m_h^2(\Lambda_{SM})$ at low energy. One can argue that this is in fact just an aesthetic problem, since it is not a problem of prediction of the theory, and it is, in fact, an aesthetic problem but one that challenges our intuition since we expect from effective theories that the theory would depend very mildly on the UV.

1.3 Structure of this dissertation

After this presentation of the hierarchy problem we shall study chiral perturbation theory in Chapter 2, which is an effective field theory used for the study of mesons. This theory will be used in Chapter 3 for the computation of the neutron electric dipole moment and will be the basis for the treatment of the non-QCD model of the relaxion mechanism, where we have a condensate of fermions just like the condensate of quarks that form the mesons.

In Chapter 3 we will begin describing the θ angle in the QCD Lagrangian, then we will compute the neutron electric dipole moment culminating in the definition of the strong CP problem. We will present the most studied solution of the strong CP problem which is the axion, which will be essential for the definition of the relaxion (the relaxion is an axion-like particle) and will motivate the relaxion mechanism in the sense that both (axion and relaxion) solve the strong CP problem and the hierarchy problem, respectively, by means of a dynamical mechanism. At the end of Chapter 3 we will study the η' particle which will have an analogue in the fermion condensate in the non-QCD relaxion model.

Finally in Chapter 4 we will introduce the relaxion mechanism by first describing the minimal model where the relaxion is simply the axion coupled with the Higgs during inflation and then describing the non-QCD model which will solve the hierarchy problem without spoiling the solution to the strong CP problem. Appendix A is a brief summary of inflation used to apply the constraints on the parameter space of the relaxion and the Appendix B is devoted to the computations of Chapter 1.

Chapter 2

Chiral Perturbation Theory

We shall study the theory of mesons, which are condensates of pairs of quarks and anti-quarks. This is important for the present work because the relaxion will interact with condensates of fermions, and these fermions will form condensates just like the condensates of quarks that we will study now.

2.1 Running of α_s and Λ_{QCD}

At low energy, QCD becomes non-perturbative, because the coupling of the theory becomes small at high energies and large at low energies due to the negative sign of the QCD beta function. This can be calculated by using the Callan-Symanzik equation and using the counterterms for the gauge boson self energy, fermion self energy and fermions-gauge boson vertex, for any non-belian gauge theory. This is done in [5] and gives, at 1-loop, the result

$$\beta(g) = -\frac{g^3}{(4\pi)^2}[\frac{11}{3}C_2(G) - \frac{4}{3}n_f C(r)], \tag{2.1}$$

where g is the coupling constant, $C_2(G)$ is the quadratic Casimir operator of the non-abelian group G, $C(r)$ is the Casimir operator of the representation r and n_f is the number of species of fermions. The beta function is defined as

$$\beta(g) \equiv \mu \frac{\partial g}{\partial \mu},$$

where μ is the energy scale where g is fixed.

Going to QCD case, we have $G = SU(3)_c$, $C_2(G) = 3$ and $C(r) = 1/2$, so (2.1) is of the form

$$\mu\frac{d}{d\mu}\alpha_s = -\frac{\alpha_s^2}{2\pi}(11 - \frac{2n_f}{3}),$$

where $\alpha_s = \frac{g^2}{4\pi}$. The above equation means that, unless there are more then 16 flavours of quarks (there are six), the beta function is negative. Solving this equation and using $n_f = 6$, we have

$$\alpha_s(\mu) = \frac{2\pi}{7}\frac{1}{\ln\frac{\mu}{\Lambda_{QCD}}}, \qquad (2.2)$$

where Λ_{QCD} is the Landau pole of QCD, *i.e.*, the location where the coupling blows up, and can be found by measuring α_s at any scale, giving the result $\Lambda_{QCD} = 218 \pm 24$ MeV. Equation (2.2) makes clear that the coupling grows as μ becomes smaller, and it is valid only at $\mu > \Lambda_{QCD}$, since under this scale the coupling becomes too strong and undefined so that confinement happens. The plot of Equation (2.2) is given in Figure 2.1.

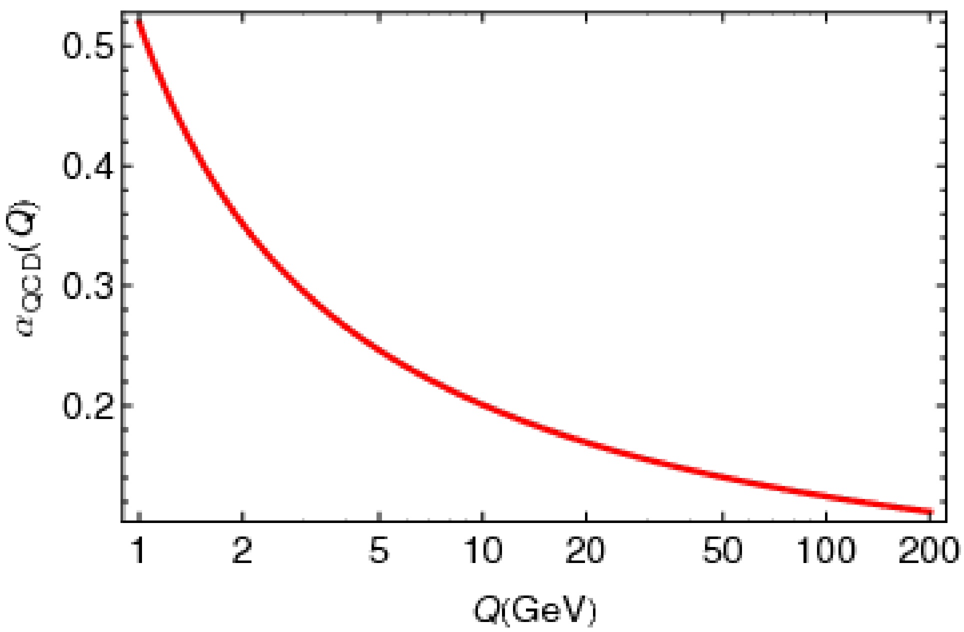

Figure 2.1: Running of QCD's coupling constant α_{QCD}.

This means that usual perturbation theory is unreliable at low energy in QCD, motivating us to construct an effective field theory at low energies trying to describe the behavior of the degrees of freedom we observe, namely, mesons and baryons. This theory is called Chiral Perturbation Theory (ChPT).

After confinement happens we don't observe anymore free quarks q and gluons G_μ states, but rather condensates which are the nucleons $\epsilon^{abc}q_aq_bq_c$ and the mesons $\bar{q}q$. In the present work we will study only the theory of mesons, the theory of nucleons can be found on [6]. To do this, we shall first describe a little bit what is and how to write an effective field theory.

2.2 Effective field theories

The basic idea of constructing Effective Field Theories (EFT) is to demand that the parameters of our model don't depend on the parameters that belong to scales much different than the scale that we are working on. In Quantum Field Theory we can construct models starting from this heuristic fact.

This can be done in two ways. First, if we have a general and full theory, but we want a simpler and more "effective" theory to calculate observables at lower energy. This can be done by fixing a scale Λ that separates between light fields l_i, with mass $m_{li} < \Lambda$, and heavy fields h_j, with mass $m_{hj} > \Lambda$, and integrate out the heavy fields solving their equations of motion. This will leave us with an effective lagrangian that will only depend of the light degrees of freedom. In other words

$$\mathcal{L}(l_i, h_j) \to \mathcal{L}_{eff}(l_i). \tag{2.3}$$

The new effective lagrangian has a tower of operators suppressed by increasing powers of m_{hj} and it must agree in the IR with the full theory. This "top-down" way of writing an EFT has some examples as Non Relativistic QED, Non Relativistic QCD and Effective Theory for Heavy Quarks.

In the second way, dubbed as the "bottom up", we do not know the general and full theory to be used, so we follow the path of equation (2.3) by writing down the most general possible operators/interactions consistent with all symmetries of the full theory we want. The couplings can be fit by experiment. This follows the "Weinberg's conjecture" that

"Quantum Field Theory has no content besides unitarity, analyticity, cluster decomposition and symmetries." [7].

It is needed to say that this conjecture is also valid to the first "top down" case. Some examples of the second way of constructing EFT is the Standard Model itself and Chiral Perturbation Theory.

2.3 How to treat the mesons

Since we do not know how to compute the QCD Green's functions at low energies, we must follow the second way described above, and try to write a theory with the same symmetries as QCD. To do so, we must look closely to the QCD's lagrangian and study its symmetries.

By decomposing the quark fields in chiral components, we have

$$q = \frac{1}{2}(1-\gamma_5)q + \frac{1}{2}(1+\gamma_5)q = P_L q + P_R q = q_L + q_R,$$

where $q = \begin{pmatrix} u \\ d \\ s \end{pmatrix}$ and $q_{L/R} = \begin{pmatrix} u_{L/R} \\ d_{L/R} \\ s_{L/R} \end{pmatrix}$ and we integrate out the heaviest quarks, considering only the lightest quarks at low energies.

The QCD lagrangian is then

$$\mathcal{L} = i\bar{q}\not{D}q - \bar{q}Mq - \frac{1}{4}G^a_{\mu\nu}G^{\mu\nu,a} = i\bar{q}_L\not{D}q_L + i\bar{q}_R\not{D}q_R - \bar{q}_L M q_R - \bar{q}_R M^\dagger q_L - \frac{1}{4}G^a_{\mu\nu}G^{\mu\nu,a},$$

where M is the quark mass matrix, $G^a_{\mu\nu}$ is the gluon field strength and $\not{D}$ is the QCD covariant derivative.

In the limit

$$M \to 0 \tag{2.4}$$

the QCD lagrangian will be invariant under the chiral symmetry $U(3)_L \times U(3)_R$, so that

$$(q_L, q_R) \mapsto (Lq_L, Rq_R),\ \ L, R \in U(3)_{L/R}.$$

The limit (2.4) is called chiral limit, and it is a good low energy approximation because $m_{u,d} \sim 1$ MeV and $m_s \sim 100$ MeV, which are much lighter than Λ_{QCD}. Thus, the chiral symmetry is explicitly broken when the quarks mass terms are different than 0, and the chiral limit is just a first order approximation.

We can rewrite the symmetry as

$$U(3)_L \times U(3)_R = SU(3)_L \times SU(3)_R \times U(1)_V \times U(1)_A, \tag{2.5}$$

where $V = L + R$ are the vector and $A = L - R$ are the axial-vector transformations. For the moment, we are going to forget the $U(1)_V \times U(1)_A$ of the symmetry, and only consider $SU(3)_L \times SU(3)_R$ as the full chiral symmetry. The reasons will become clear later, in chapter 3.

Now, we must remember that, at low energies when a condensation forms, such that $\langle \bar{q}q \rangle \neq 0$, we have a spontaneous symmetry breaking of the chiral symmetry, because if

$$(q_L, q_R) \to (U_L q_L, U_R q_R),\ U_{L/R} \in SU(3)_{L/R},$$

then

$$\langle \bar{q}q \rangle \to \langle \bar{q}_L U_L^\dagger U_R q_R + h.c. \rangle,$$

and

$$\langle \bar{q}_L U_L^\dagger U_R q_R + h.c. \rangle = \langle \bar{q}_L q_R + h.c. \rangle = \langle \bar{q}q \rangle \Leftrightarrow U_L = U_R \Leftrightarrow SU(3)_L = SU(3)_R = SU(3)_V,$$

where $SU(3)_V$ is a vectorial SU(3).

We have then that the quark condensate spontaneously breaks

$$SU(3)_L \times SU(3)_R \to SU(3)_V, \tag{2.6}$$

which gives us 8 NGBs in our theory. The central idea of ChPT is to treat the mesons as Pseudo Nambu Goldstone Bosons (PNGBs) (pseudo because the chiral symmetry is explicitly broken by the quark mass terms). To construct such theory with NGBs and invariant lagrangian, we must use the Coleman-Callan-Wess-Zumino (CCWZ) construction [8].

2.4 CCWZ construction

We should start by considering a set of fields Φ transforming under a group $\mathcal{G}$. Now suppose the fields acquire a non-zero expectation value in the vacuum $\mid \Omega \rangle$, such that $\langle \Omega \mid \Phi \mid \Omega \rangle = F$. If this vacuum configuration is left invariant by a subgroup $\mathcal{H} \subset \mathcal{G}$ we have the Spontaneous Symmetry Breaking (SSB) $\mathcal{G} \to \mathcal{H}$.

2.4.1 Separating the NGBs

To find the NGBs we can use the ansatz

$$\Phi(x) = \exp\left(\frac{i\sqrt{2}}{F_0}\theta_A(x)T^A\right)F, \tag{2.7}$$

where T^A are the generators of $\mathcal{G}$, $\theta_A(x)$ are scalar fields, and F_0 is a dimensionful constant such that $[F_0] = 1$. And we defined in such a way that $\langle\Omega \mid \theta_A(x) \mid \Omega\rangle = 0$.

Now, note we can expand every element $g \in \mathcal{G}$ as [8]

$$g = \exp(i\alpha_A T^A) = \exp(if_{\hat{a}}[\alpha]\hat{T}^{\hat{a}})\exp(if_a[\alpha]T^a), \tag{2.8}$$

where $f_{\hat{a}}[\alpha] = \alpha_{\hat{a}} + \mathcal{O}(\alpha^2)$, $f_a[\alpha] = \alpha_a + \mathcal{O}(\alpha^2)$, T^a are the generators of $\mathcal{H} \subset \mathcal{G}$ (called unbroken), while $\hat{T}^{\hat{a}}$ are the remaining generators of $\mathcal{G}$ (called broken).

Using (2.8) in (2.7), and remembering that the invariance of F under $\mathcal{H}$ implies that $T^a F = 0$, we have

$$\begin{aligned}\Phi(x) =& \exp\left(\frac{i\sqrt{2}}{F_0}\theta_A(x)T^A\right)F = \exp\left(\frac{i\sqrt{2}}{F_0}\Pi_{\hat{a}}\hat{T}^{\hat{a}}\right)\exp(i\xi(x)_a T^a)F \\ =& \exp\left(\frac{i\sqrt{2}}{F_0}\Pi_{\hat{a}}\hat{T}^{\hat{a}}\right)F,\end{aligned} \tag{2.9}$$

where the $\Pi_{\hat{a}}$ are now identified as the NGBs (one for each broken generator).

As a consequence, (2.9) implies that

$$\Phi(x) = U[\Pi]F, \tag{2.10}$$

where $U[\Pi] = \exp\left(\frac{i\sqrt{2}}{F_0}\Pi_{\hat{a}}\hat{T}^{\hat{a}}\right)$.

Making use of equation (2.8), *i.e.*, decomposing a generic group element into broken and unbroken generators dependent part, we can study the action of an element g of the group $\mathcal{G}$ on $\Phi(x)$, such that

$$gU[\Pi] = \exp(i\alpha_A T^A)\exp\left(\frac{i\sqrt{2}}{F_0}\Pi_{\hat{a}}\hat{T}^{\hat{a}}\right) \equiv U[\Pi^{(g)}]h[\Pi, g],$$

where $U[\Pi^{(g)}]$ is the broken generators dependent part and $h[\Pi, g]$ is the unbroken generator dependent part of $\exp(i\alpha_A T^A)\exp\left(\frac{i\sqrt{2}}{F_0}\Pi_{\hat{a}}\hat{T}^{\hat{a}}\right)$. So we obtain

$$g\Phi(x) = gU[\Pi]F = U[\Pi^{(g)}]h[\Pi, g]F = U[\Pi^{(g)}]F,$$

where in the last equation we used that $h[\Pi, g]F = F$ because $h[\Pi, g] \in \mathcal{H}$. So we have

$$gU[\Pi] = U[\Pi^{(g)}]h[\Pi, g] \Rightarrow U[\Pi^{(g)}] = gU[\Pi]h^{-1}[\Pi, g].$$

But, by equation (2.8), we have that $h[\Pi, g]$ is hermitian, so $h^{-1}[\Pi, g] = h^\dagger[\Pi, g]$, and then

$$U[\Pi] \to gU[\Pi]h^\dagger[\Pi, g]. \tag{2.11}$$

2.4.2 Finding the Lagrangian

Let us now try to write down a Lagrangian invariant under the group $\mathcal{G}$. To do so, we can construct an object that transforms covariantly under $\mathcal{H}$, which is

$$-iU^\dagger[\Pi]\partial_\mu U[\Pi] \equiv d_\mu + e_\mu, \tag{2.12}$$

where $d_\mu \equiv d_\mu^{\hat{a}}\hat{T}^{\hat{a}}$ and $e_\mu \equiv e_\mu^a T^a$.

Now, noting that, calling $c = \sqrt{2}/F_0$ and expanding U in c, we have

$$\begin{aligned}-iU^\dagger\partial_\mu U =& -i(1 - ic\Pi_{\hat{a}}\hat{T}^{\hat{a}} + \cdots)\partial_\mu(1 + ic\Pi_{\hat{a}}\hat{T}^{\hat{a}} + \cdots) = -i(1 - ic\Pi_{\hat{a}}\hat{T}^{\hat{a}} + \cdots)(ic\partial_\mu\Pi_{\hat{a}}\hat{T}^{\hat{a}} + \cdots)\\ =& c\partial_\mu\Pi_{\hat{a}}\hat{T}^{\hat{a}} + \cdots,\end{aligned}$$

where the $\cdots$ means $\mathcal{O}(c^2)$ or T^a dependent terms. So we identify

$$d_\mu^{\hat{a}} = \frac{\sqrt{2}}{F_0}\partial_\mu\Pi_{\hat{a}} + \cdots. \tag{2.13}$$

By noting that $h^\dagger$ depends on $\Pi(x)$, we have that, using Eq. (2.11)

$$\begin{aligned}-iU^\dagger\partial_\mu U \to -ihU^\dagger g^\dagger\partial_\mu(gUh^\dagger) =& -ihU^\dagger g^\dagger(g(\partial_\mu U)h^\dagger + gU(\partial_\mu h^\dagger))\\ =& -ihU^\dagger(\partial_\mu U)h^\dagger - ih(\partial_\mu h^\dagger).\end{aligned} \tag{2.14}$$

Writing (2.14) in terms of d_μ and e_μ, by Eq. (2.12)

$$\begin{aligned} d_\mu + e_\mu \to & h(d_\mu + e_\mu)h^\dagger - ih\partial_\mu h^\dagger = hd_\mu h^\dagger + h(e_\mu - i\partial_\mu)h^\dagger \\ \Rightarrow d_\mu \to & h[\Pi, g] d_\mu h^\dagger[\Pi, g] \qquad (2.15) \\ \Rightarrow e_\mu \to & h[\Pi, g](e_\mu - i\partial_\mu)h^\dagger[\Pi, g]. \qquad (2.16) \end{aligned}$$

Where we let the derivative term be associated with e_μ and not with d_μ because we want the unbroken part to behave like a gauge field. Let us observe that e_μ transforms just like a gauge field, while with d_μ we can easily construct an invariant kinetic term

$$\mathcal{L} = \frac{F_0^2}{4}\,\mathrm{Tr}[d_\mu d^\mu], \qquad (2.17)$$

because it transforms like

$$\mathrm{Tr}[d_\mu d^\mu] \to \mathrm{Tr}[hd_\mu h^\dagger h d^\mu h^\dagger] = \mathrm{Tr}[hd_\mu d^\mu h^\dagger] = \mathrm{Tr}[d_\mu d^\mu h^\dagger h] = \mathrm{Tr}[d_\mu d^\mu].$$

The factor F_0^2 ensures the correct dimensions, while the factor $\frac{1}{4}$ is a convenient normalization.

Using (2.13) in (2.17) gives

$$\mathcal{L}^{(2)} = \frac{F_0^2}{4}\,\mathrm{Tr}[d_\mu d^\mu] = \frac{1}{2}(\partial_\mu \Pi_{\hat{a}})(\partial^\mu \Pi_{\hat{a}}) + \cdots . \qquad (2.18)$$

As we have just shown, the CCWZ procedure allows us not only to identify the NGBs with the broken generators, but gives also a procedure to write down an invariant Lagrangian. We will now see how to proceed in the case of non-simple groups, as the one use in ChPT.

2.4.3 CCWZ in ChPT

In QCD we have that the group $\mathcal{G}$ is non-simple, $SU(3)_L \times SU(3)_R$. So we must generalize the CCWZ procedure for groups where $\mathcal{G}_1 \times \mathcal{G}_2 \times \cdots \times \mathcal{G}_N \to \mathcal{H}$. In order to do so, we proceed as follows:

1. For each $\mathcal{G}_k$ we define a matrix σ_k such that

$$\sigma_k \to g_k \sigma_k h^\dagger; \tag{2.19}$$

2. From the matrix σ_k, we construct covariant derivatives according to

$$\alpha_\mu^k = -i\sigma_k^\dagger \partial_\mu \sigma_k. \tag{2.20}$$

   This covariant derivative transforms as

$$\alpha_\mu^k \to h\alpha_\mu^k h^\dagger - ih\partial_\mu h^\dagger; \tag{2.21}$$

3. We now construct derivative operators that transform as in (2.15), allowing us to write down the invariant kinetic term. To do that, we construct the differences

$$\alpha_\mu^{kj} = \alpha_\mu^k - \alpha_\mu^j, \tag{2.22}$$

   which transform like

$$\alpha_\mu^{kj} \to h\alpha_\mu^{kj} h^\dagger, \tag{2.23}$$

   having $N-1$ independent α_μ^{kj};

4. We build invariants as in (2.17), i.e., $\mathrm{Tr}[\alpha_\mu^{kj}\alpha_\mu^{kj}]$ for each independent α_μ^{kj};

5. Now, thinking on $SU(3)_L \times SU(3)_R$, we can separate an element $g \in \mathcal{G}$ in $g = (R, L)$, $R/L \in SU(3)_{R/L}$. Thus we can rewrite

$$g = (R, L) = (\mathbb{I}, LR^\dagger)(R, R),$$

   since $(R, R) \in \mathcal{H}$. The equation above is very similar to (2.9), so we can recognize $LR^\dagger = U$ as the unitary matrix made of the NGBs along the broken directions. Taking now $\tilde{R} \in SU(3)_R$ and $\tilde{L} \in SU(3)_L$, the transformation of U under a generic element $g = (\tilde{R}, \tilde{L}) \in \mathcal{G}$ is

$$(\tilde{R}, \tilde{L})(\mathbb{I}, U)F = (\mathbb{I}, \tilde{L}U\tilde{R}^\dagger)(\tilde{R}, \tilde{R})F = (\mathbb{I}, \tilde{L}U\tilde{R}^\dagger)F,$$

or

$$U \to LUR^\dagger. \tag{2.24}$$

Let us now make contact with the formalism developed for the case of non-simple groups. To this end, let us consider two matrices U_L and U_R (each analog to σ_k of Eq.(2.19))

$$\begin{aligned} U_L \to & LU_L h^\dagger, \\ U_R \to & RU_R h^\dagger, \end{aligned}$$

where $L, R \in SU(3)_{L/R}$. Equation (2.24) then demands U to be of the form

$$U = U_L U_R^\dagger. \tag{2.25}$$

Finally, all the discussion leads us to identify the objects that describe the NGBs as

$$U^{kj} \equiv \sigma_k \sigma_j^\dagger. \tag{2.26}$$

That transforms, under a element of $\mathcal{G}_1 \times \cdots \times \mathcal{G}_N$, as

$$U^{kj} \to g_k \sigma_k h^\dagger h \sigma_j^\dagger g_j^\dagger = g_k \sigma_k \sigma_j^\dagger g_j^\dagger = g_k U^{kj} g_j^\dagger, \tag{2.27}$$

where $g_{k/j} \in \mathcal{G}_{k/j}$. Note that equations (2.26) and (2.25) are the same, as well as (2.27) and (2.24).

6. Remembering the invariant $\text{Tr}[\alpha_\mu^{kj} \alpha_\mu^{kj}]$ of item 4, we shall now relate it to U^{kj}. To do this, we must first compute

$$\partial_\mu U^{kj} = (\partial_\mu \sigma_k)\sigma_j^\dagger + \sigma_k(\partial_\mu \sigma_j^\dagger),$$

multiply it by $\sigma_k^\dagger$ from the left and by σ_j from the right, obtaining

$$\begin{aligned} \sigma_k^\dagger \partial_\mu U^{kj} \sigma_j = & \sigma_k^\dagger((\partial_\mu \sigma_k)\sigma_j^\dagger + \sigma_k(\partial_\mu \sigma_j^\dagger))\sigma_j = \sigma_k^\dagger(\partial_\mu \sigma_k) + (\partial_\mu \sigma_j^\dagger)\sigma_j \\ = & \sigma_k^\dagger \partial_\mu \sigma_k - \sigma_j^\dagger \partial_\mu \sigma_j = i\alpha_\mu^{kj}, \end{aligned}$$

where in the third equality we used $\partial_\mu(\sigma_j^\dagger\sigma_j) = 0$, and in the last one we used equation (2.20). Using last equation, we can write the invariant as

$$\begin{aligned} Tr[\alpha_\mu^{kj}\alpha_\mu^{kj}] =& - Tr[\sigma_k^\dagger\partial_\mu U^{kj}\sigma_j\sigma_k^\dagger\partial_\mu U^{kj}\sigma_j] = -Tr[\sigma_j\sigma_k^\dagger\partial_\mu U^{kj}\sigma_j\sigma_k^\dagger\partial_\mu U^{kj}] \\ =& - Tr[(U^{kj})^\dagger\partial_\mu U^{kj}(U^{kj})^\dagger\partial_\mu U^{kj}] = Tr[(d_\mu^{kj})^2], \end{aligned} \tag{2.28}$$

where in the last equality we used the definition

$$d_\mu^{kj} \equiv -i(U^{kj})^\dagger\partial_\mu U^{kj}. \tag{2.29}$$

Equation (2.28) shows that we can construct an invariant object $\mathrm{Tr}[(d_\mu^{kj})^2]$, made out of d_μ^{kj}, which is like the covariant derivative of U^{kj}.

Finally we can use the above list to construct the ChPT for mesons. The invariant lagrangian describing the NGBs encoded on the matrix U transforming like

$$U \to LUR^\dagger, \tag{2.30}$$

is

$$\mathcal{L} = \frac{f^2}{4}Tr[d_\mu d_\mu], \tag{2.31}$$

where

$$d_\mu = -iU^\dagger\partial_\mu U, \tag{2.32}$$

f is the pion decay constant$\approx$ 92.4 MeV [6].

Now, since U is made of NGBs, it can be written as $\exp\left(\frac{i\sqrt{2}}{f}\Pi_{\hat{a}}\hat{T}^{\hat{a}}\right)$, just like in equation (2.10), where $\Pi_{\hat{a}}$ are the NGBs and $\hat{T}^{\hat{a}}$ are the broken generators. A full $SU(3)$ symmetry was broken, so the broken generators can be chosen as the Gell-Mann matrices. Explicitly, we can write

$$U = e^{i\frac{\sqrt{2}}{f}\hat{\Phi}}, \tag{2.33}$$

where

$$\hat{\Phi} = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\eta}{\sqrt{6}} \end{pmatrix}. \tag{2.34}$$

Here π^0 is the neutral pion, $\pi^\pm$ are the chaged pions, $K^0, \bar{K}^0$ are the neutral and antineutral kaons, $K^{\pm}$ are the charged kaons and η is the eta particle.

Inserting Eqs. (2.33) and (2.34) in Eq. (2.31) we obtain the Lagrangian for massless mesons. Expanding U in powers of $\hat{\Phi}$ we can get the interactions between mesons. But in the real world the quarks have masses, breaking explicitly chiral symmetry and giving masses to the mesons. We need complete the theory to encompass this correction, and we also need to describe the gauge interactions to the mesons.

2.5 Explicit chiral breaking and gauge interactions

As already mentioned, the light quarks masses break the $SU(3)_L \times SU(3)_R$ symmetry explicitly. This is because the quark mass term in the Lagrangian has the form

$$\mathcal{L}_{M_q} = \bar{q}_L M_q q_R + h.c., \tag{2.35}$$

where $q = \begin{pmatrix} u \\ d \\ s \end{pmatrix}$ and M_q is the mass matrix $M_q = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix}$. Under a $SU(3)_L \times SU(3)_R$ transformation we have

$$\begin{aligned} q_L \to & g_L q_L, \\ q_L \to & g_R q_R, \end{aligned}$$

where $g_{L,R} \in SU(3)_{L,R}$, and this transformations clearly do not leave the lagrangian (2.35) invariant.

To properly include the mass effects in the low energy theory we use the **spurion technique**, which consists in noting that $\mathcal{L}_{M_q}$ would be invariant under $SU(3)_L \times SU(3)_R$ if

$$M_q \to g_L M_q g_R^{\dagger}.$$

The idea now is to include M_q as a new degree of freedom of the low energy theory, construct invariants using it, and then fix its numerical value to the experimentally observed one.

The simplest invariant we can construct out of U and M_q is

$$\mathrm{Tr}[UM_q^\dagger + h.c.] \to \mathrm{Tr}[g_L U g_R^\dagger g_R M_q^\dagger g_L^\dagger + h.c.] = \mathrm{Tr}[UM_q^\dagger + h.c.].$$

Thus we can write the Lagrangian

$$\mathcal{L} = \frac{f^2}{4}\,\mathrm{Tr}[d_\mu d_\mu] + \frac{1}{2} f^2 \mu\,\mathrm{Tr}[UM_q^\dagger + h.c.] \tag{2.36}$$

where μ is a mass scale that must be fixed to reproduce the mesons mass, which may be defined as $\mu = cf$, where c is some dimensionless constant. It is important to notice that μ was defined in this particular way by historical reasons.

Since experimentally we mesure interactions between the photon and the mesons, we need to include these interactions in our low energy theory. The way to do this is by considering part of the $SU(3)_L \times SU(3)_R$ as a local symmetry, and couple it to the gauge fields. We first suppose that in some part of the lagrangian of the high energy theory we have gauge couplings described like

$$\mathcal{L} \supset \bar{q}_L \gamma^\mu L_\mu q_L + \bar{q}_R \gamma^\mu R_\mu q_R, \tag{2.37}$$

where L_μ and R_μ are gauge fields and transform like

$$\begin{aligned} L_\mu \to & g_L L_\mu g_L^\dagger - i(\partial_\mu g_L) g_L^\dagger, \\ R_\mu \to & g_R R_\mu g_R^\dagger - i(\partial_\mu g_R) g_R^\dagger, \end{aligned}$$

where $g_{L,R} \in SU(3)_{L,R}$.

Since

$$U \to g_L U g_R^\dagger \tag{2.38}$$

we can construct compensators $V_L(y,x)$ and $V_R(y,x)$ such that

$$\begin{aligned} V_L(y,x) \to & g_L(y) V_L(y,x) g_L^\dagger(x), \\ V_R(y,x) \to & g_R(y) V_R(y,x) g_R^\dagger(x). \end{aligned}$$

using this with (2.38) we obtain

$$\begin{aligned}V_L(y,x)U(x)V_R^\dagger(y,x) \to & g_L(y)V_L(y,x)g_L^\dagger(x)g_L(x)U(x)g_R^\dagger(x)g_R(x)V_R^\dagger(y,x)g_R^\dagger(y)\\ =& g_L(y)V_L(y,x)U(x)V_R^\dagger(y,x)g_R^\dagger(y).\end{aligned}$$

The last expression allows us to construct a covariant derivative D_μ, such that

$$\begin{aligned}n^\mu D_\mu U(x) =& \lim_{\epsilon\to\ 0}\frac{1}{\epsilon}[U(x+\epsilon n) - V_L(x+\epsilon n,x)UV_R^\dagger(x+\epsilon n,x)]\\ =& \lim_{\epsilon\to\ 0}\frac{1}{\epsilon}[U(x+\epsilon n) - (\mathbb{I}+i\epsilon n^\mu L_\mu)U(x)(\mathbb{I}-i\epsilon n^\mu R_\mu)]\\ =& n^\mu\partial_\mu U(x) - in^\mu L_\mu U(x) + in^\mu U(x)R_\mu\\ =& n^\mu(\partial_\mu U(x) - iL_\mu U(x) + iU(x)R_\mu)\end{aligned} \tag{2.39}$$

where n^μ is the vector that points the direction of the derivative and in the second equality we expand $V_L(x+\epsilon n,x) = \mathbb{I} + i\epsilon n^\mu L_\mu + \mathcal{O}(\epsilon^2)$ and $V_R(x+\epsilon n,x) = \mathbb{I} + i\epsilon n^\mu R_\mu + \mathcal{O}(\epsilon^2)$.

As an example, let us look at the part of the initial lagrangian concerning to the photon interaction

$$\mathcal{L}_q = e\bar{q}_L\gamma^\mu A_\mu Q q_L + e\bar{q}_R\gamma^\mu A_\mu Q q_R,$$

with

$$Q = \begin{pmatrix} 2/3 & 0 & 0\\ 0 & -1/3 & 0\\ 0 & 0 & -1/3\end{pmatrix}.$$

Comparing the above lagrangian with (2.37) we find that L_μ and R_μ are equal to

$$L_\mu = eQA_\mu,\ \ R_\mu = eQA_\mu.$$

Making use of equation (2.39), the covariant derivative is

$$D_\mu U = \partial_\mu U - ieA_\mu QU + ieA_\mu UQ = \partial_\mu U - ieA_\mu[Q,U]. \tag{2.40}$$

Exchanging the partial derivative by the covariant derivative D_μ in d_μ, we insert the gauge interactions in our theory. We have also to add to our lagrangian the kinetic term of our new degree of freedom A_μ to make it dynamical. Then, the lagrangian takes the form

$$\mathcal{L} = \frac{f^2}{4}\,\mathrm{Tr}[d_\mu d_\mu] + \frac{1}{2} f^2 \mu\,\mathrm{Tr}[U M_q^\dagger + h.c.] - \frac{1}{4} F^{\mu\nu} F_{\mu\nu}$$

where $F^{\mu\nu}$ is the field strength of the photon and now $d_\mu = -iU^\dagger D_\mu U$.

At the end we have a theory that describe the dynamics and interaction of the mesons, include the chiral breaking which gives mass to them, and describe their interactions it with the gauge bosons at low energy. The only ingredient still missing, the physics of the η' meson, will be described in the next chapter.

Chapter 3

Axions and the η' meson

This chapter will be dedicated to the study of the strong CP problem, the axion particle and the η' particle. The same procedure will be used after in the construction of relaxion models. In such models we will work with a very similar particle to the η'. But before introducing the axion, we must show what the θ angle and the Strong CP Problem are.

3.1 The θ angle

3.1.1 Effects of the winding number in the functional integral

When we impose boundary conditions to extended field configurations, such fields acquire topological configurations that can't be changed. These are described by the winding number ν, which is an integer, with one value for each topological configuration. In a Yang-Mills theory, this number is given by [9]

$$\nu = -\frac{1}{64\pi^2}\int d^4x F^A_{\mu\nu}\tilde{F}^{\mu\nu A}, \tag{3.1}$$

where $F^A_{\mu\nu}$ is the field strength for the gauge boson A (for every gauge boson within the theory), $\tilde{F}^{\mu\nu A} = \epsilon^{\mu\nu\rho\sigma}F^A_{\rho\sigma}$ is the dual field strength and $\epsilon^{\mu\nu\rho\sigma}$ is the Levi-Civita tensor.

The effects of these topological configurations must be included on the path integral due to instantons [9][10]. To compute these effects, we may be very general and consider that the $\nu's$ affect the observable with a weight factor of $f(\nu)$. In this way the mean value of an observable O in a Minkowski space Ω is

$$\langle O \rangle_\Omega = \frac{\sum_\nu f(\nu) \int_\nu [d\phi] e^{iS_\Omega[\phi]} O[\phi]}{\sum_\nu f(\nu) \int_\nu [d\phi] e^{iS_\Omega[\phi]}}, \tag{3.2}$$

where ϕ represents all the fields of the theory, $\int_\nu [d\phi]$ indicates that we are integrating only over the fields with field configurations that have winding a number ν, and $S_\Omega[\phi]$ represents the action in all space Ω.

If we now divide Ω in two parts, Ω_1 and Ω_2, with O in volume Ω_1, we have that the integral $\int_\nu [d\phi]$ may be divided as the integral over all fields with winding number ν_1 in Ω_1 and ν_2 in Ω_2, with $\nu = \nu_1 + \nu_2$, so that (3.2) becomes

$$\langle O \rangle_\Omega = \frac{\sum_{\nu_1,\nu_2} f(\nu_1+\nu_2) \int_{\nu_1} [d\phi] e^{iS_{\Omega_1}[\phi]} O[\phi] \int_{\nu_2} [d\phi] e^{iS_{\Omega_2}[\phi]}}{\sum_{\nu_1,\nu_2} f(\nu_1+\nu_2) \int_{\nu_1} [d\phi] e^{iS_{\Omega_1}[\phi]} \int_{\nu_2} [d\phi] e^{iS_{\Omega_2}[\phi]}}.$$

But locality implies that $\langle O \rangle_\Omega$ should be independent of Ω_2, *i.e.*

$$\langle O \rangle_\Omega = \frac{\sum_{\nu_1} f(\nu_1) \int_{\nu_1} [d\phi] e^{iS_{\Omega_1}[\phi]} O[\phi]}{\sum_{\nu_1} f(\nu_1) \int_{\nu_1} [d\phi] e^{iS_{\Omega_1}[\phi]}}.$$

This is possible only if we can factorize the weight function as

$$f(\nu_1 + \nu_2) = f(\nu_1) f(\nu_2).$$

This implies

$$f(\nu) = e^{i\theta\nu}, \tag{3.3}$$

where θ is an arbitrary variable.

Using this form of $f(\nu)$ in equation (3.2), we see that the non trivial topological configurations can be accounted by adding a term $\theta\nu$ in the action S_Ω. Using (3.1) we have

$$\mathcal{L}_\theta = -\frac{\theta F^A_{\mu\nu} \tilde{F}^{\mu\nu A}}{64\pi^2}, \tag{3.4}$$

where $\mathcal{L}_\theta$ is the new term, θ dependent, in the Lagrangian.

3.1.2 Consequences of $\mathcal{L}_\theta$

One important consequence is that $\mathcal{L}_\theta$ is not invariant under P and CP transformations, unlike what happens in QCD, where P and CP are exact symmetries. To see this, note that for the electromagnetic case, $\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma} \sim \vec{E}\cdot\vec{B}$ with, $\vec{E} \to -\vec{E}$, $\vec{B} \to \vec{B}$ when P is applied and $\vec{E} \to \vec{E}$, $\vec{B} \to -\vec{B}$ when CP is applied. This makes $\vec{E}\cdot\vec{B}$ not invariant under P and CP.

Let us now show an interesting connection between chiral transformations and $\mathcal{L}_\theta$. Applying to the fermions of the theory a chiral rotation

$$\psi_f \to e^{i\gamma_5\alpha_f}\psi_f, \tag{3.5}$$

where f stands for different flavours, we have a change in the jacobian of the functional integral. This is called chiral anomaly, and the change is given by [9]

$$[d\psi][d\bar{\psi}] \to e^{-\frac{i}{32\pi^2}\int d^4x\epsilon^{\mu\nu\rho\sigma}F^A_{\mu\nu}F^A_{\rho\sigma}\sum_f\alpha_f}[d\psi][d\bar{\psi}]. \tag{3.6}$$

This amounts to a shift of the Lagrangian by

$$\mathcal{L} \to \mathcal{L} - \frac{1}{32\pi^2}F^A_{\mu\nu}\tilde{F}^{\mu\nu A}\sum_f\alpha_f,$$

and, comparing with Eq. (3.4) we see that the shift amounts to change θ by

$$\theta \to \theta + 2\sum_f\alpha_f. \tag{3.7}$$

Now, since this is just a field redefinition, it cannot affect physics, so the θ angle per-se can not be physical. But note that this transformation also changes the mass term, as can be seen looking directly to the terms

$$\mathcal{L}_M = -\sum_f M_f\bar{\psi}_{fL}\psi_{fR} - \sum_f M_f^*\bar{\psi}_{fR}\psi_{fL}.$$

and noting that, under the transformation in (3.5),

$$\psi_{fL} \to e^{-i\alpha_f}\psi_{fL} \quad\text{and}\quad \psi_{fR} \to e^{i\alpha_f}\psi_{fR},$$

we have that the mass term transforms like

$$M_f \to e^{2i\alpha_f}M_f. \tag{3.8}$$

Since $\mathcal{L}_M$ can be written as

$$\mathcal{L}_M = -\psi_L M \psi_R + h.c.$$

where $\psi_L = (\psi_{1L}, \cdots, \psi_{fL}, \cdots)^T, \psi_R = (\psi_{1R}, \cdots, \psi_{fR}, \cdots)^T$ and $M = \mathrm{diag}(M_1, \cdots, M_f, \cdots)$, we can see that, using (3.8), we have

$$\det M \to \det M e^{i2\sum_f \alpha_f}. \tag{3.9}$$

Comparing Eqs. (3.7) and (3.9) we see that the combination $\det[M]e^{-i\theta}$ is invariant under chiral transformations. Alternatively, we can define an effective angle

$$\bar{\theta} \equiv \theta - \arg[\det M], \tag{3.10}$$

which is now invariant and can have physical interpretation. This physical effect will be seen in the neutron electric dipole moment. At first sight, it seems that the breaking of CP due to the term in Eq. (3.4) is not physical, since this term can be eliminated from the Lagrangian using a chiral transformation. However, also complex masses break the CP invariance in $\mathcal{L}$, in such a way that the explicit breaking of CP is physical, as we are now going to see.

3.1.3 The vacuum energy as a function of $\bar{\theta}$

We shall study the potential

$$V(U) = -\frac{1}{2} f^2 \mu \,\mathrm{Tr}(UM^\dagger + h.c.), \tag{3.11}$$

and its $\bar{\theta}$ dependence at the minimum.

The first interesting information is that we can get μ from expanding $U = e^{i\frac{\sqrt{2}}{f}\hat{\phi}}$ in second order (with $\hat{\phi}$ given by Equation (2.34)), then

$$\mathcal{L} = \frac{1}{2} f^2 \mu \,\mathrm{Tr}[M(1 + i\frac{\sqrt{2}}{f}\hat{\phi} - \frac{1}{2}i^2\frac{2}{f^2}\hat{\phi}^2 + \cdots) + M(1 + i\frac{\sqrt{2}}{f}\hat{\phi} + \frac{1}{2}i^2\frac{2}{f^2}\hat{\phi}^2 + \cdots)], \tag{3.12}$$

and, evaluating the terms quadratic in ϕ, we obtain the masses of the mesons. Doing this, and focusing only at the quadratic π^0 terms, we now obtain

$$\mathcal{L} \supset -\mu(\frac{(\pi^0)^2}{2}m_d + \frac{(\pi^0)^2}{2}m_u + \cdots) + \cdots = -\frac{\mu(m_u + m_d)}{2}(\pi^0)^2 + \cdots ,$$

this gives us a π^0 mass of $m_{\pi^0}^2 = \mu(m_d + m_u)$ and, fixing the experimental value of $m_{\pi^0} \approx 135\text{MeV}$, we get $\mu = \frac{m_{\pi^0}^2}{m_u + m_d}$.

Now, as explained earlier, we can eliminate completely the θ term from the Lagrangian, via a chiral transformation. This gives a phase to the mass matrix in such a way that

$$M = \begin{pmatrix} m_u & 0 \\ 0 & m_d \end{pmatrix} e^{-i\frac{\bar{\theta}}{2}}. \tag{3.13}$$

This way, from now on, all the information about $\bar{\theta}$ is carried by the phase of the mass matrix M.

If we have the minimum of Eq. (3.11) at $U = U_0$ (note that we could have the minimum at $U = I$ but it is not possible since $\bar{\theta} \neq 0$), we demand that U_0 is diagonal, to minimize with M which is also diagonal, and that has unit determinant because U is generated by elements that belong to a special group. Then, the most general form for U_0 is

$$U_0 = \begin{pmatrix} e^{i\phi} & 0 \\ 0 & e^{-i\phi} \end{pmatrix}.$$

The potential becomes

$$V(U_0) = -f^2\mu\Big[m_u \cos\Big(\phi - \frac{1}{2}\bar{\theta}\Big) + m_d \cos\Big(\phi + \frac{1}{2}\bar{\theta}\Big)\Big], \tag{3.14}$$

which, after using the value of μ and some factorization, becomes

$$V(U_0) = -m_\pi^2 f^2 \sqrt{1 - \frac{4m_u m_d}{(m_u + m_d)^2}\sin^2\Big(\frac{\bar{\theta}}{2}\Big)}\cos(\phi - \bar{\phi}) \tag{3.15}$$

where

$$\tan\bar{\phi} = \frac{m_u - m_d}{m_u + m_d}\tan\Big(\frac{1}{2}\bar{\theta}\Big).$$

3.1.4 The neutron electric dipole moment

The strongest effect of the CP violating $\bar{\theta}$ term is seen in the measurements of the electric dipole moment of the neutron. To calculate this quantity, we must first use chiral perturbation theory to study the coupling of the pions to the nucleons.

This can be done using the chiral techniques of the previous chapter, but instead of having the breaking of the chiral symmetry $SU(3)_L \times SU(3)_R \to SU(3)_V$, we will construct a simpler model using only $q = \begin{pmatrix} u \\ d \end{pmatrix}$, integrating out the s quark. Now we have the breaking $SU(2)_L \times SU(2)_R \to SU(2)_V$ and the main change in the construction of the previous chapter is that this time we have 3 PNGBs (3 pions) and the broken generators of Eq. (2.9) are no longer Gell-Man matrices but are the Pauli matrices (the standard generators of $SU(2)$).

With these considerations, we now introduce the nucleon as a isospin doublet $N = \begin{pmatrix} p \\ n \end{pmatrix}$ of $SU(2)_V$ such that

$$N \to VN \tag{3.16}$$

where $V \in SU(2)_V$.

In the last chapter we built our ChPT in terms of U which transform like $U \to g_L U g_R^\dagger$ under $SU(2)_L \times SU(2)_R$ and it was constructed via CCWZ from

$$\begin{aligned} &u_L \to g_L u_L V^\dagger \\ &u_R \to g_R u_R V^\dagger \\ &\Rightarrow U \equiv u_L u_R^\dagger \to g_L U g_R^\dagger . \end{aligned}$$

Now, because there is a parity mapping g_L generators to g_R generators we have $u_L = u_R^\dagger$ [11]. This way we can call

$$u = u_L = u_R^\dagger \to g_L u V^\dagger = V u g_R^\dagger, \tag{3.17}$$

$$u^\dagger = u_R = u_L^\dagger \to g_R u^\dagger V^\dagger = V u^\dagger g_L^\dagger, \tag{3.18}$$

$$\Rightarrow U = u^2. \tag{3.19}$$

Since N transforms via V while U transforms via g_L, g_R, we can use u to construct the invariant Lagrangian terms for the nucleon field. We shall construct such terms with the spurions under $SU(2)_L \times SU(2)_R$ which are in the original QCD Lagrangian as

$$\mathcal{L} \supset \bar{q}_L M_q q_R + \text{h.c.} + \bar{q}_L \gamma^\mu L_\mu q_L + \bar{q}_R \gamma^\mu R_\mu q_R,$$

where

$$M_q \to g_L M_q g_R^\dagger, \tag{3.20}$$

$$L_\mu \to g_L L_\mu g_L^\dagger - i(\partial_\mu g_L) g_L^\dagger, \tag{3.21}$$

$$R_\mu \to g_R R_\mu g_R^\dagger - i(\partial_\mu g_R) g_R^\dagger. \tag{3.22}$$

We can include the vector spurions promoting

$$g_L \to g_L(x); \quad g_R \to g_R(x),$$

and promoting the derivative to a covariant derivative

$$\partial_\mu u^\dagger \to (\partial_\mu - iR_\mu) u^\dagger.$$

We can check how it transforms by using (3.18) and (3.22)

$$\begin{aligned} \partial_\mu u^\dagger &\to (\partial_\mu g_R) u^\dagger V^\dagger + g_R \partial_\mu (u^\dagger V^\dagger), \\ -iR_\mu u^\dagger &\to -i(g_R R_\mu g_R^\dagger - i(\partial_\mu g_R) g_R^\dagger) g_R u^\dagger V^\dagger = -i g_R R_\mu u^\dagger V^\dagger - (\partial_\mu g_R) u^\dagger V^\dagger, \\ \Rightarrow (\partial_\mu - iR_\mu) u &\to g_R[\partial_\mu (u^\dagger V^\dagger) - iR_\mu u^\dagger V^\dagger] = g_R(\partial_\mu - iR_\mu) u^\dagger V^\dagger + g_R u^\dagger \partial_\mu V^\dagger, \end{aligned}$$

applying u from the left, we have from (3.17)

$$\begin{aligned} u(\partial_\mu - iR_\mu) u^\dagger \to & V u g_R^\dagger [g_R(\partial_\mu - iR_\mu) u^\dagger V^\dagger + g_R u^\dagger \partial_\mu V^\dagger] \\ = & V[u(\partial_\mu - iR_\mu) u^\dagger] V^\dagger + V \partial_\mu V^\dagger, \end{aligned} \tag{3.23}$$

in a similar way

$$u^\dagger (\partial_\mu - iL_\mu) u \to V[u^\dagger (\partial_\mu - iL_\mu) u] V^\dagger + V \partial_\mu V^\dagger. \tag{3.24}$$

At this moment it is convenient to define

$$\begin{aligned} a_\mu \equiv& i[u^\dagger(\partial_\mu - iL_\mu)u - u(\partial_\mu - iR_\mu)u^\dagger], \\ v_\mu \equiv& \frac{u^\dagger(\partial_\mu - iL_\mu)u + u(\partial_\mu - iR_\mu)u^\dagger}{2}, \end{aligned}$$

that due to (3.23) and (3.24) transform as

$$a_\mu \to V a_\mu V^\dagger, \tag{3.25}$$

$$v_\mu \to V v_\mu V^\dagger + V\partial_\mu V^\dagger. \tag{3.26}$$

These equations closely remember Eq. (2.12), but they are now written in terms of u and not of U.

Now we can construct invariant kinetic terms using these objects. Using that $N \to VN$ and (3.26) we get

$$\begin{aligned} (\partial_\mu + v_\mu)N \to& (\partial_\mu + V v_\mu V^\dagger + V\partial_\mu V^\dagger)VN \\ =& (\partial_\mu V)N + V\partial_\mu N + V v_\mu N + V(\partial_\mu V^\dagger)VN \\ =& V(\partial_\mu + v_\mu)N + (\partial_\mu V + V(\partial_\mu V^\dagger)V)N = V(\partial_\mu + v_\mu)N, \end{aligned} \tag{3.27}$$

where in the last step we used $VV^\dagger = 1 \Rightarrow (\partial_\mu V)V^\dagger + V\partial_\mu V^\dagger = 0 \Rightarrow V(\partial_\mu V^\dagger)V = -\partial_\mu V$. From (3.27) we can construct an invariant kinetic term

$$\mathcal{L}_{kin} = \bar{N} i\gamma^\mu(\partial_\mu + v_\mu)N, \tag{3.28}$$

which effectively acts like a covariant derivative.

Now, we should also be able to construct a term involving a_μ and the nucleon field. A term like

$$\bar{N}\gamma^\mu a_\mu N \to \bar{N}\gamma^\mu V^\dagger V a_\mu V^\dagger V N = \bar{N}\gamma^\mu a_\mu N, \tag{3.29}$$

is invariant under $SU(2)_L \times SU(2)_R$ but is not invariant under CP. In fact under CP $a_\mu \to -a_\mu$. This happens because u is made on pseudoscalar mesons, such that

$$u = e^{i\pi^a T^a/(2f)}, \tag{3.30}$$

and under CP $\pi^a \to -\pi^a \Rightarrow u \to u^\dagger$. Using this fact we have that under CP

$$a_\mu = i[u^\dagger(\partial_\mu - iL_\mu)u - u(\partial_\mu - iR_\mu)u^\dagger] \to i[u(\partial_\mu - iR_\mu)u^\dagger - u \dagger (\partial_\mu - iL_\mu)u] = -a_\mu \quad \text{and} \tag{3.31}$$

$$v_\mu = \frac{u^\dagger(\partial_\mu - iL_\mu)u + u(\partial_\mu - iR_\mu)u^\dagger}{2} \to \frac{u(\partial_\mu - iR_\mu)u^\dagger + u^\dagger(\partial_\mu - iL_\mu)u}{2} = v_\mu. \tag{3.32}$$

This means that we have to couple the term in (3.29) with a CP odd term, and this term is

$$\bar{N}\gamma^\mu\gamma_5 N \xrightarrow{CP} -\bar{N}\gamma^\mu\gamma_5 N,$$

and thus we can construct the Lagrangian

$$\mathcal{L} = \bar{N}\gamma^\mu[i(\partial_\mu + v_\mu) - g_A a_\mu \gamma_5]N \tag{3.33}$$

where $g_A = 1.27$ is the axial vector coupling, determined by the neutron decay rate via weak interaction [10].

Now we are going to construct mass terms. To do that we need to "dress" the mass spurion term $M \to g_L M g_R^\dagger$ with u to obtain an object that transforms under $SU(2)_V$. There are two possibilities, the first one is

$$u^\dagger M u^\dagger \to V u^\dagger g_L^\dagger g_L M g_R^\dagger g_R u^\dagger V^\dagger = V u^\dagger M u^\dagger V^\dagger, \tag{3.34}$$

and the second is

$$u M^\dagger u \to V u g_R^\dagger g_R M^\dagger g_L^\dagger g_L u V^\dagger = V u M^\dagger u V^\dagger. \tag{3.35}$$

By noting that if there is a θ term the mass matrix becomes complex and under CP $\theta \xrightarrow{CP} -\theta$ we conclude that $M \xrightarrow{CP} M^\dagger$. This way we have two possible combinations to construct the Lagrangian:

$$u^\dagger M u^\dagger + u M^\dagger u,$$

which is hermitian and CP invariant, and the combination

$$u^\dagger M u^\dagger - u M^\dagger u,$$

which is antihermitian. In addition, this term is CP-odd, in such a way that it must be coupled with $\bar{N}\gamma_5 N$. Thus, the mass Lagrangian is

$$\mathcal{L}_M = c_+\bar{N}(u^\dagger M u^\dagger + uM^\dagger u)N + ic_-\bar{N}(u^\dagger M u^\dagger - uM^\dagger u)\gamma_5 N, \tag{3.36}$$

where c_+ and c_- are constants.

We can also create terms involving the spurion M by multiplying some invariant by $\bar{N}N$ or $\bar{N}\gamma_5 N$, which are also invariant. The candidates are

$$\begin{aligned}\mathrm{Tr}[u^\dagger M u^\dagger] &= \mathrm{Tr}[MU^\dagger],\\ \mathrm{Tr}[uM^\dagger u] &= \mathrm{Tr}[M^\dagger U],\end{aligned}$$

making two possible combinations as before, and leaving the Lagrangian as

$$\begin{aligned}\mathcal{L}_M =& c_+\bar{N}(u^\dagger M u^\dagger + uM^\dagger u)N + ic_-\bar{N}(u^\dagger M u^\dagger - uM^\dagger u)\gamma_5 N\\ & c_1\,\mathrm{Tr}[MU^\dagger + M^\dagger U]\bar{N}N + ic_2\,\mathrm{Tr}[MU^\dagger - M^\dagger U]\bar{N}\gamma_5 N,\end{aligned} \tag{3.37}$$

where c_1 and c_2 are constants.

Using now that $\mathrm{Tr}(U - U^\dagger)$ is 0 for two light flavors, and Equation (3.37), we get the θ dependent terms in the Lagrangian to be

$$\mathcal{L}_\theta = -i\bar{\theta}\tilde{m}[-\frac{1}{2}c_+\bar{N}(U - U^\dagger)N + \frac{1}{2}c_-\bar{N}(U + U^\dagger)\gamma_5 N + c_2\,\mathrm{Tr}(U + U^\dagger)\bar{N}\gamma_5 N],$$

where $\tilde{m} = \frac{m_u m_d}{m_u + m_d}$. Expanding in powers of π^a we have

$$\mathcal{L}_\theta = -i\bar{\theta}\tilde{m}(c_- + 4c_2)\bar{N}\gamma_5 N - (\bar{\theta}c_+\tilde{m}/f)\pi^a\bar{N}\sigma^a N + \cdots . \tag{3.38}$$

By making a field redefinition of the form $N \to e^{-i\alpha\gamma_5}N$ we can eliminate the first term in (3.38). This creates new terms in (3.37), but we can safely neglect these terms due their dependence on, at least, two factors of quark masses. The second term in (3.38) creates a pion-nucleon coupling that breaks P and CP. We can estimate the value of c_+ by the difference of masses of baryons, which yields $c_+ = 1.7$ [10].

The strongest interaction between pion and nucleon conserving P and CP is the last term of Equation (3.33), and is equal to

$$\mathcal{L}_{\pi\bar{N}N} = (g_A/f)\partial_\mu \pi^a \bar{N}\sigma^a\gamma^\mu\gamma_5 N. \tag{3.39}$$

Integrating by parts, putting the derivative on N and using the Dirac equation (because the nucleons are on-shell) we have

$$\mathcal{L}_{\pi\bar{N}N} = -i(g_A m_N/f)\pi^a \bar{N}\sigma^a\gamma_5 N. \tag{3.40}$$

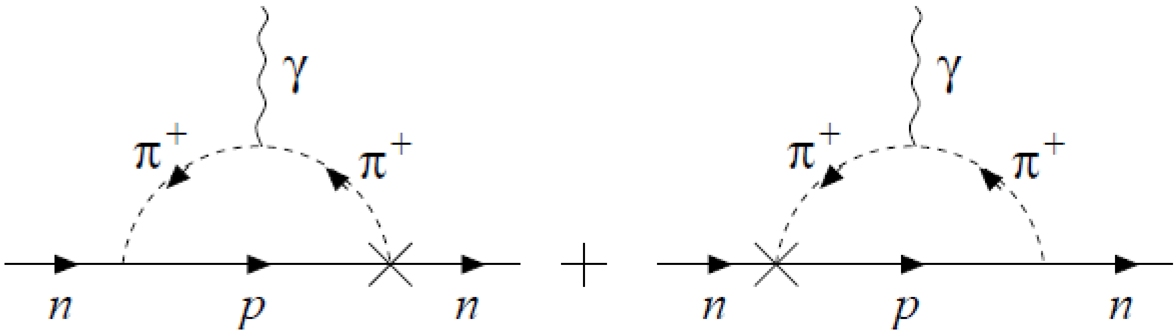

Figure 3.1: Diagrams that contribute to the electric dipole moment of the neutron.

We finally compute the electric dipole moment of the neutron, whose contributions come from the diagrams of Figure 3.1, where the vertex that violates CP is denoted with a cross.

The amplitude associated with the diagrams of Figure 3.1 is given by

$$\mathcal{T} = -2iD(q^2)\epsilon^*_\mu(q)\bar{u}_{s'}(p')\sigma^{\mu\nu}q_\nu i\gamma_5 u_s(p) \tag{3.41}$$

where p' is the outgoing momentum of the nucleons, p is the incoming momentum and $q = p' - p$ is the momentum of the photon. In the limit $q \to 0$, $D(0)$ is the electric dipole moment of the neutron d_n.

By using

$$\pi^a\sigma^a = \begin{pmatrix} \pi^0 & \sqrt{2}\pi^+ \\ \sqrt{2}\pi^- & -\pi^0 \end{pmatrix},$$

expanding N in function of n and p and using (3.38) and (3.40), we have the useful vertices

$$\mathcal{L}_{\pi\bar{N}N} = -i\sqrt{2}(g_A m_N/f)(\pi^+\bar{p}\gamma_5 n + \pi^-\bar{n}\gamma_5 p),$$
$$\mathcal{L}_{\bar{\theta}\pi\bar{N}N} = -\sqrt{2}(\bar{\theta}c_+\tilde{m}/f)(\pi^+\bar{p}n + \pi^-\bar{n}p).$$

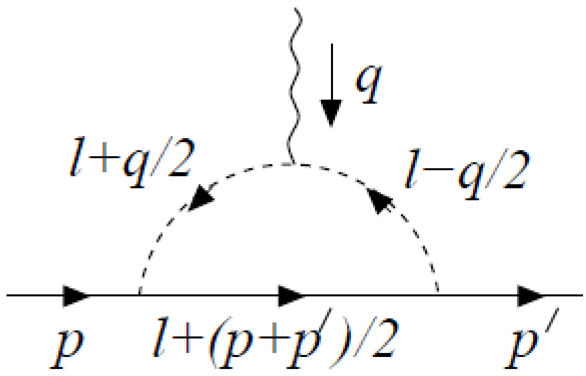

Figure 3.2: Momentum flow for the diagrams of Figure 3.1.

Using the Feynman rules coming from these Lagrangians, we get the amplitude for the Feynman diagram of Figure 3.2

$$i\mathcal{T} =(\frac{1}{i})^3(ie)(\sqrt{2}g_A m_N/f)(-i\sqrt{2}\bar{\theta}c_+\tilde{m}/f)\epsilon^*_\mu\int_0^\Lambda\frac{d^4l}{(2\pi)^4}$$
$$\times\frac{(2l^\mu)\bar{u'}[(-\not{l}-\not{p}+m_N)\gamma_5+\gamma_5(-\not{l}-\not{p}+m_N)]u}{((l+\bar{p})^2+m_N^2)((l+\frac{1}{2}q)^2+m_\pi^2)((l-\frac{1}{2}q)^2+m^2\pi)},$$

where l is the internal momentum, $\Lambda \sim 4\pi f$ is the cutoff of the effective theory and $\bar{p} = \frac{1}{2}(p'+p)$. Using $\{\gamma^\mu,\gamma_5\} = 0$ we obtain that the spinor part of the numerator is $2m_N\bar{u'}\gamma_5 u$. Using $p \gg l$ we get $(l+p)^2+m_N^2 \approx 2p\cdot l$, and the amplitude simplifies

$$\mathcal{T} = 4(e\bar{\theta}g_A c_+\tilde{m}m_N^2/f^2)\epsilon^*_\mu\int_0^\Lambda\frac{d^4l}{(2\pi)^4}\frac{2l^\mu\bar{u'}\gamma_5 u}{(2p\cdot l)(l^2+m_\pi^2)^2}.$$

We now make the replacement $\frac{l^\mu}{p\cdot l} \to \frac{p^\mu}{p^2} = -\frac{p^\mu}{m_N^2}$ in the integral, and use the Gordon identity $p^\mu\bar{u'}\gamma_5 u = \bar{u'}\sigma\mu\nu q_\nu i\gamma_5 u + \mathcal{O}(q^2)$ [12]. This way, the amplitude becomes

$$\mathcal{T} = -4(e\bar{\theta}g_A c_+\tilde{m}/f^2)\epsilon^*_\mu\bar{u'}\sigma^{\mu\nu}q_\nu i\gamma_5 u\int_0^\Lambda\frac{d^4l}{(2\pi)^4}\frac{1}{(l^2+m_\pi^2)^2},$$

where the integral gives $(i/16\pi^2)\ln\Lambda^2/m_\pi^2$, so

$$\mathcal{T} = -i\frac{(e\bar{\theta}g_A c_+ \tilde{m}/f^2)\epsilon_\mu^* \bar{u}'\sigma^{\mu\nu}q_\nu i\gamma_5 u}{4\pi^2}\ln(\Lambda^2/m_\pi^2). \tag{3.42}$$

Comparing (3.42) with (3.41) gives

$$d_n = \frac{e\bar{\theta}g_A c_+ \tilde{m}}{8\pi^2 f^2}\ln\Lambda^2/m_\pi^2.$$

Using the numerical values $g_A = 1.27, c_+ = 1.7, f = 94.2\text{MeV}, \tilde{m} = 1.2\text{MeV}$ and $m_\pi = 139.5\text{MeV}$, we have

$$d_n = 3.2 \times 10^{-16}\bar{\theta}e\text{cm}.$$

Since experimental measurements give the upper bound $| d_n |< 6.3 \times 10^{-26}$ [10], we finally get

$$| \bar{\theta} |< 2 \times 10^{-10}.$$

The fact that the $\bar{\theta}$ term is so close to 0 is called strong CP problem.

Notice that $\bar{\theta}$ is technically natural due to the fact that the theory obtain CP symmetry when $\bar{\theta} \to 0$, this explains why $\bar{\theta}$ is so small. But the difficulty lies in explaining the fine tuning between θ and $\arg[\det M]$ so that $\bar{\theta} = \theta - \arg[\det M]$ be so close to 0.

Now that we know that the $\bar{\theta}$ angle must be extremely small to be phenomenologically viable, we must search for explanations for such smallness. One possible idea is to note that the invariant physical term is $e^{-i\theta}\det[M]$, that implies that if at least one quark has vanishing mass, this invariant term would be 0 and we would have no CP breaking. Unfortunately, there is evidence that all six quark flavours have non-vanishing masses, making this solution unviable [13]. Another possibility would be spontaneous CP breaking, which postulates that in the original underlying theory CP is conserved, and CP violation arises spontaneously. The available models are somewhat contrived, and we will not study them here. We will instead focus on the more popular solution of the Strong CP Problem, the axion.

3.2 The axion

As we saw, the strong CP problem can be solved in various ways. In this section we will focus on the most studied solution, the axion, in which the parameter $\bar{\theta}$ is made dynamical and relaxed to zero via dynamics.

3.2.1 Peccei-Quinn symmetry

A way to make $\bar{\theta}$ dynamical is to construct a global symmetry that is spontaneously broken and whose NGB couple to the anomaly. We can do this by considering adding to the Standard Model a massless quark, given by the pair of Weyl fermions Q_L and $Q_R^\dagger$ in the 3 and $\bar{3}$ representations of $SU(3)$, respectively. We also add a complex scalar Φ in the singlet representation of $SU(3)$. Then we assume that these fields have a Yukawa interaction

$$\mathcal{L}_Y = y\Phi Q_L Q_R^\dagger + h.c.$$

where y is the Yukawa coupling constant.

Since we need a new NGB to appear in our theory, we define a new global symmetry that acts only on Q_L, Q_R and Φ

$$\begin{aligned} Q_L &\to e^{i\alpha} Q_L, \\ Q_R^\dagger &\to e^{i\alpha} Q_R^\dagger, \\ \Phi &\to e^{-2i\alpha}\Phi, \end{aligned} \tag{3.43}$$

that leaves $\mathcal{L}_Y$ invariant. This symmetry is called Peccei-Quinn (PQ) symmetry [14], and is denoted by $U(1)_{PQ}$.

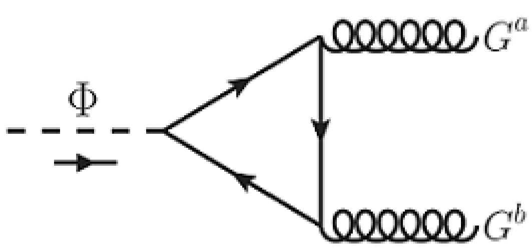

Figure 3.3: Triangle diagram from the anomaly.

From the diagram of Figure 3.3 we obtain the anomaly coefficient

$$2\,\mathrm{Tr}[\{T^a,T^b\}X_{PQ}] = \mathrm{Tr}[\{T^a,T^b\}] = \mathrm{Tr}\left[\left\{2d_{abc}T^c + \frac{4}{3}\delta^{ab}\mathbb{I}\right\}\right] \neq 0,$$

where $T^{a,b}$ are the Gell-Man matrix correspondent to the gluons G^a, G^b and $X_{PQ} = \mathbb{I}_3$ is the Peccei-Quinn generator. The fact that the anomaly coefficient is not 0 shows that the PQ symmetry is an anomalous symmetry.

Since the PQ symmetry is anomalous, and acts like a chiral transformation on the new quarks Q, we know from the discussion around Eq. (3.6) that $\mathcal{L}$ is not invariant, but it rather shifts as

$$-\frac{\alpha}{32\pi^2}\epsilon^{\mu\nu\rho\sigma}F^A_{\mu\nu}F^A_{\rho\sigma}. \tag{3.44}$$

Suppose now that the PQ is spontaneously broken by $\langle\Phi\rangle = F/\sqrt{2}$ with $F \gg f$. Using the polar parametrization $\Phi = \frac{1}{\sqrt{2}}(F+\rho)e^{ia/F}$ we clearly see that the effect of a PQ transformation is to shift the NGB a according to $\frac{a}{F} \to \frac{a}{F} - 2\alpha$. Once we integrate Q_L, Q_R and ρ out, we can include the effect of the anomalous PQ transformation writing a Lagrangian

$$\mathcal{L} = \mathcal{L}_{SM} + \frac{\bar{\theta}}{64\pi^2}\epsilon^{\mu\nu\rho\sigma}F^A_{\mu\nu}F^A_{\rho\sigma} + \frac{1}{2}\left(\partial_\mu a\right)^2 + \mathcal{L}_{int} + \frac{a}{F}\frac{1}{64\pi^2}\epsilon^{\mu\nu\rho\sigma}F^A_{\mu\nu}F^A_{\rho\sigma}. \tag{3.45}$$

Notice that the last term is exactly what is needed to reproduce the anomalous transformation (3.44).

The a field is the axion, and it substitutes $\bar{\theta}$ by $\bar{\theta} + \frac{a}{F}$ in the Lagrangian. This way we can say that we made $\bar{\theta}$ dynamical. By noting that the potential in (3.15) has its minimum when $\phi = \bar{\phi}$ and substituting $\bar{\theta}$ by $\bar{\theta} + \frac{a}{F}$ we get

$$V(a) = -m_\pi^2 f^2 \sqrt{1 - \frac{4m_u m_d}{(m_u+m_d)^2}\sin^2\left(\frac{\bar{\theta}+\frac{a}{F}}{2}\right)}. \tag{3.46}$$

This potential is minimized when $a = -\bar{\theta}F$. Expanding a around the minimum $a = -\bar{\theta}F + \tilde{a}$ we see that the θ dependence in the Lagrangian completely vanishes, solving the Strong CP Problem, leaving only the interaction term

$$\mathcal{L} \supset -\frac{\tilde{a}\epsilon^{\mu\nu\rho\sigma}F^A_{\mu\nu}F^A_{\rho\sigma}}{F64\pi^2}.$$

For convenience we will drop the tilde and simply denote by a the physical axion field from now on.

Now the potential (3.46) becomes

$$V(a) = -m_\pi^2 f^2 \sqrt{1 - \frac{4m_u m_d}{(m_u + m_d)^2} \sin^2\left(\frac{a}{2F}\right)}, \tag{3.47}$$

which is the axion potential. Expanding up to quadratic order we get the axion mass

$$m_a^2 = \frac{m_u m_d}{(m_u + m_d)^2} \frac{m_\pi^2 f^2}{F^2}.$$

3.2.2 Axion interactions at low energies

Depending on the specific model, other interactions involving the axion and Standard Model fields can appear in the Lagrangian in addition to Eq. (3.45). Since the axion is the NGB of the Peccei-Quinn symmetry, it shifts by $a \to a - 2F\alpha$ under a transformation (3.43). So, if we want interactions invariant under this shift, we must couple the derivative of the axion to gauge invariant operators. We can also have anomalous couplings to $G\tilde{G}$, $W\tilde{W}$ and $Y\tilde{Y}$ where G is the field strength of $SU(3)_c$, W is the field strength of $SU(2)$, Y is the field strength of $U(1)_Y$ and $\tilde{F} = \epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}$ is the dual field strength such that $F\tilde{F} = 1/2\epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma}$.

With these types of interactions, we construct the more general Lagrangian [15] that above the weak scale is

$$\begin{aligned}\mathcal{L}_a =& \frac{1}{2}\partial^\mu a \partial_\mu a + \frac{\partial^\mu a}{F}(x_H H^\dagger i \overleftrightarrow{D}_\mu H + \sum_\psi \bar{\psi}_L \gamma_\mu X_\psi \psi_L) \\ & - \frac{a}{F}[\frac{g_3^2}{32\pi^2} G\tilde{G} + C_{aWW}\frac{g_2^2}{32\pi^2} W\tilde{W} + C_{aYY}\frac{g_1^2}{32\pi^2} Y\tilde{Y}],\end{aligned} \tag{3.48}$$

where H is the Higgs doublet, ψ are the five types of left handed fermions: the quark doublets q, the charge $-\frac{2}{3}$ antiquarks u^c, the charge $\frac{1}{3}$ antiquarks d^c, the lepton doublets L and the charge $+1$ antileptons l. Each one of these is a triplet in flavor space and so the different X_ψ are 3×3 matrices in flavor space, $\overleftrightarrow{D} \equiv (D_\mu H)^\dagger H - H^\dagger (D_\mu H)$ is the $SU(2) \times U(1)_Y$ gauge covariant derivative and g_1, g_2 and g_3 are the $U(1)_Y$, $SU(2)$ and $SU(3)_c$ gauge couplings, respectively.

We now go down in energy. The first important effect to be taken into account is electroweak symmetry breaking, *i.e.* the scale at wich H gets its vacuum expectation value (vev). At this scale we can integrate out the physical Higgs particle (and the top quark). Then the first derivative term in (3.48) gets an interaction between the axion and the NGB eaten by the Z. To avoid this problem, we make an axion dependent $U(1)_Y$ rotation where

$$H \to e^{2ix_H aY/F} H \quad \text{and} \quad \psi_L \to e^{2ix_H aY/F} \psi_L,$$

where Y are the Higgs and fermions hypercharge, respectively. These transformations change the values of x_H and X_ψ by

$$\begin{aligned} &x_H \to x_H - 2x_H Y_H = 0, \text{ since } Y_H = 1/2, \\ &X_\psi \to X_\psi - 2x_H Y_H \end{aligned} \tag{3.49}$$

where the X_ψ changes because of the contribution due to kinetic terms.

Going to scales below the electroweak scale, we can integrate out the W and Z bosons, so that we have only the photon surviving as gauge field, and the anomalous term becomes

$$\mathcal{L} \supset -\frac{a}{F}\frac{e^2}{32\pi^2}(C_{aWW} + C_{aYY})F\tilde{F} - \frac{a}{F}\frac{g_3^2}{32\pi^2}G\tilde{G}, \tag{3.50}$$

where F is the photon field strength.

The next relevant scale is Λ_{QCD}, where the coupling g_3 becomes large and quarks and gluons are no longer the relevant degrees of freedom. We eliminate the gluon coupling in (3.50) by a chiral rotation, putting all the axion dependence on the quark mass matrix. We shall make the most general transformation as follows

$$q \to e^{-i\frac{a}{F}(Q_V + Q_A\gamma_5)} q, \text{ with } \mathrm{Tr}[Q_A] = 1/2, \tag{3.51}$$

where $q = \begin{pmatrix} u \\ d \\ s \end{pmatrix}$, Q_A and Q_V are 3×3 matrices and we made the restriction $\mathrm{Tr}[Q_A] = 1/2$ because Q_A appears quadratically in the Lagrangian. The lagrangian now takes the form

$$\begin{aligned} \mathcal{L} =& \frac{1}{2}\partial^\mu a \partial_\mu a - \bar{q_R} e^{iaQ_A/F} M e^{iaQ_A/F} q_L + h.c. + \frac{\partial_\mu a}{F}\bar{q}\gamma^\mu[(X_V + Q_V) + (X_A + Q_A)\gamma_5]q \\ & - \frac{a}{F}\frac{e^2}{32\pi^2} C_{a\gamma\gamma} F\tilde{F} + \text{leptonic terms} \end{aligned} \tag{3.52}$$

where the new terms involving the derivative coupling come from contributions of the kinetic term after transformation (3.51) and the terms X_V and X_A come from the X term in Eq. (3.48). The term $C_{a\gamma\gamma} \equiv C_{aWW} + C_{aYY} - \mathrm{Tr}[Q_A Q_E Q_E]$, where Q_E is the electric charge matrix of the quarks, appears to take into account the electromagnetic anomaly that transformation (3.51) causes.

Finally we translate Eq. (3.52) and write the corresponding ChPT, using the tools developed in chapter 2. The chiral lagrangian is given by (see Eq. (2.36))

$$\mathcal{L} = \frac{f^2}{4}\,\mathrm{Tr}[D^\mu U D_\mu U^\dagger] + \frac{1}{2} f^2 \mu\, \mathrm{Tr}[M U^\dagger + h.c.] \tag{3.53}$$

where D_μ is given in Equation (2.40) and M is the spurion mass, that according to Equation (3.52) must be replaced by

$$M \to e^{iaQ_A/F} M e^{iaQ_A/F}. \tag{3.54}$$

Notice that $\partial_\mu a$ in Eq. (3.52) can be interpreted as the interaction between the axion and an hadronic current, and we thus search for a meson current with the same transformation properties. It happens that those currents are

$$\begin{aligned} j^\mu_{Va} &= i\frac{1}{2} f^2\, \mathrm{Tr}[T_a(U D^\mu U^\dagger + U^\dagger D^\mu U)], \\ j^\mu_{Aa} &= i\frac{1}{2} f^2\, \mathrm{Tr}[T_a(U D^\mu U^\dagger - U^\dagger D^\mu U)], \end{aligned}$$

where A stands for axial, V stands for vectorial, T_a are the Gell-Mann matrices and the index a goes from 1 to 8.

Putting all together we obtain the Lagrangian

$$\begin{aligned} \mathcal{L} = &- \frac{a}{F}\frac{e^2}{32\pi^2} C_{a\gamma\gamma} F\tilde{F} + 2\frac{\partial_\mu a}{F} \sum_{a=1}^{8} \left\{ j^\mu_{Va}\, \mathrm{Tr}[T_a(X_V + Q_V)] + j^\mu_{Aa}\, \mathrm{Tr}[T_a(X_A + Q_A)] \right\} \\ &\frac{1}{2} f^2 \mu \Big[i\frac{a}{F}\, \mathrm{Tr}[\{M, Q_A\} U] - (\frac{a}{F})^2\, \mathrm{Tr}[\{\{M, Q_A\}, Q_A\} U] + \mathcal{O}[(\frac{a}{F})^3] + h.c. \Big] \end{aligned}$$

Note that in the second line of the above Lagrangian the term proportional to a generates a mass mixing between the axion and the mesons. In order to eliminate this mass mixing we demand that $\mathrm{Tr}[\{M, Q_A\} U]\frac{a}{F} = \mathrm{Tr}\left[\{M, Q_A\}\left(\mathbb{I} + i\frac{\pi^a \lambda^a}{\sqrt{2} f} + \cdots\right)\right]\frac{a}{F}$ has the mass mixing term equal to 0. We solve this by demanding $\{M, Q_A\}$ to be proportional to identity. A simple solution is $Q_A = \frac{1}{2}M^{-1}$, and to make Q_A dimensionless, we divide by $\mathrm{Tr}[M^{-1}]$ obtaining

$$Q_A = \frac{1}{2}\frac{M^{-1}}{\mathrm{Tr}[M^{-1}]}. \quad (3.55)$$

The final Lagrangian is

$$\mathcal{L} = -\frac{a}{F}\frac{e^2}{32\pi^2}C_{a\gamma\gamma}F\tilde{F} + 2\frac{\partial_\mu a}{F}\sum_{a=1}^{8}\{j^\mu_{Va}\,\mathrm{Tr}[T_a(X_V + Q_V)] + j^\mu_{Aa}\,\mathrm{Tr}[T_a(X_A + \frac{1}{2}\frac{M^{-1}}{\mathrm{Tr}[M^{-1}]})]\}$$
$$\frac{1}{2}f^2\mu\Big[-(\frac{a}{F})^2\frac{\mathrm{Tr}[M^{-1}U]}{\mathrm{Tr}[M^{-1}]^2} + \mathcal{O}[(\frac{a}{F})^3] + h.c.\Big] \quad (3.56)$$

This Lagrangian provides interactions between mesons and the axion.

3.3 The η' meson

In writing the chiral theory we have until now considered the breaking $SU(3)_L \times SU(3)_R \to SU(3)_V$. But, as we stressed in chapter 2, the full symmetry group is actually $U(3)_L \times U(3)_R = SU(3)_L \times SU(3)_R \times U(1)_V \times U(1)_A$ spontaneously broken to $SU(3)_V \times U(1)_V$ where $SU(3)_V$ stands for vectorial $SU(3)$ and $U(1)_V$ is identified as the Baryon number. What happens to the degree of freedom associated with the $U(1)_A$ that is also broken, should not it be a Goldstone boson? Phenomenologically we know that while pions and kaons have masses well below the GeV, this is not true for the 9^{th} pseudoscalar meson, η', which has a mass of $m_{\eta'} = 958$ MeV [13]. The absence of the ninth pseudoscalar meson has been dubbed "$U(1)_A$" problem.

The solution of the problem resides in the fact that $U(1)_A$ is an anomalous symmetry, with no associated NGB. We will now see how to include the anomaly, and the η', in our Lagrangian.

3.3.1 The chiral Lagrangian including the $U(1)_A$ anomaly

In order to properly include the anomaly effects and the η' in the Lagrangian we need a result from large-N QCD [16]: the coefficient of the $U(1)_A$ anomaly in the Lagrangian vanishes as $1/N$ as the number of colors N gets large. This means that in the $N \to \infty$ QCD with L light flavours undergoes the spontaneous symmetry breaking pattern $U(L)_L \times U(L)_R \to U(L)_V$, and we can describe the NGB's considering an $L \times L$ matrix

U containing L^2 pseudoscalar mesons. In this section, we will follow the analysis of [17].

Being $U(1)_A$ anomalous, once we perform an axial transformation the action gets shifted by

$$\int d^4x \mathcal{L}_{QCD}(x) \to \int d^4x \mathcal{L}_{QCD}(x) + L\alpha \int d^4x q(x), \tag{3.57}$$

where

$$\begin{aligned} q(x) &= \frac{g^2}{64\pi^2}\epsilon_{\mu\nu\rho\sigma}F^a_{\mu\nu}F^a_{\rho\sigma} = \frac{1}{4}\epsilon_{\mu\nu\rho\sigma}F_{\mu\nu\rho\sigma}, \\ F_{\mu\nu\rho\sigma} &= \partial_\mu A_{\nu\rho\sigma} - \partial_\sigma A_{\mu\nu\rho} + \partial_\rho A_{\sigma\mu\nu} - \partial_\nu A_{\rho\sigma\mu}, \\ A_{\nu\rho\sigma} &= \frac{g^2}{96\pi^2}[A^a_\mu \overleftrightarrow{\partial}_\rho A^a_\sigma - A^a_\rho \overleftrightarrow{\partial}_\nu A^a_\sigma - A^a_\nu \overleftrightarrow{\partial}_\sigma A^a_\rho + 2f_{abc}A^a_\nu A^b_\rho A^c_\sigma], \end{aligned}$$

and α is the parameter of the chiral transformation, A^a_μ are the gauge fields (each a for one generator of $SU(N)$), f_{abc} are the structure constants of $SU(N)$, $A_{\nu\rho\sigma}$ is an abelian totally antisymmetric gauge field and $F_{\mu\nu\rho\sigma}$ is a gauge invariant field tensor [17].

At low energy we cannot have quarks and gluons degrees of freedom in our Lagrangian, so we have only the dependence of $F_{\mu\nu\rho\sigma}$ and $A_{\nu\rho\sigma}$. We start by constructing the following kinetic term for the field $A_{\nu\rho\sigma}$

$$\mathcal{L}_{kin} = -cF_{\mu\nu\rho\sigma}F^{\mu\nu\rho\sigma}, \tag{3.58}$$

where c is a positive constant.

Now, we need also the Lagrangian to reproduce the shift in the action given by Eq. (3.57). To construct such part in the Lagrangian we must note that, under a $U(1)$ axial rotation, the U field transforms as

$$U \to Ue^{-i\alpha},$$

which means that

$$\log U \to \log U - i\alpha,$$

where α is the parameter of the chiral transformation. This way, the Lagrangian term that reproduces (3.57) is given by

$$\frac{i}{2}q\,\mathrm{Tr}(\log U - \log U^\dagger) \to \frac{i}{2}q\,\mathrm{Tr}(\log U - \log U^\dagger) + L\alpha q.$$

This implies that, in order to correctly reproduce the $U(1)_A$ anomaly, we need to add to the Lagrangian a term

$$\mathcal{L}_{anomaly} = \frac{1}{8}\epsilon_{\mu\nu\rho\sigma}F^{\mu\nu\rho\sigma} i\,\mathrm{Tr}(\log U - \log U^{\dagger}). \tag{3.59}$$

Considering that $F_{\mu\nu\rho\sigma}$ is gauge invariant and $\epsilon_{\mu\nu\rho\sigma}F^{\mu\nu\rho\sigma}$ is $SU(L) \times SU(L)$ invariant, and taking into account the large-N power counting according to Ref. [17], the final Lagrangian is

$$\begin{aligned}\mathcal{L} =& \mathcal{L}_0 + \frac{1}{8}\epsilon_{\mu\nu\rho\sigma}F^{\mu\nu\rho\sigma} i\,\mathrm{Tr}(\log U - \log U^{\dagger}) \\ & - cF_{\mu\nu\rho\sigma}F^{\mu\nu\rho\sigma} - \theta\frac{1}{4}\epsilon_{\mu\nu\rho\sigma}F^{\mu\nu\rho\sigma}.\end{aligned} \tag{3.60}$$

Now we may use the field $q(x) = \frac{1}{4}\epsilon_{\mu\nu\rho\sigma}F_{\mu\nu\rho\sigma}$ as relevant degree of freedom, since it is the only gluon dependence in the Lagrangian and acts as a scalar. Also adding to the Lagrangian the chiral mass term like in chapter 2, we obtain

$$\begin{aligned}\mathcal{L} =& \mathcal{L}_0 + \frac{1}{2} iq(x)\,\mathrm{Tr}[\log U - \log U^{\dagger}] \\ & \frac{N}{af^2}q^2(x) - \bar{\theta}q(x) + \frac{f^2\mu}{2}\,\mathrm{Tr}[UM^{\dagger} + MU^{\dagger}],\end{aligned} \tag{3.61}$$

where the $q^2(x)$ term comes from the kinetic term, a is conveniently defined associated with c of Eq. (3.60) and $M_{ij} = m_{q_i}\delta_{ij}$ is the quark mass matrix. We also make a chiral transformation such that $\arg\det M = 0$.

Now, we can see from the Lagrangian that the $q(x)$ field does not have dynamics. This way, we can integrate it out using its equations of motion

$$\frac{\delta\mathcal{L}}{\delta q} = 0 \Rightarrow q = \frac{af^2}{2N}\Big(\bar{\theta} - \frac{1}{2} i\,\mathrm{Tr}[\log U - \log U^{\dagger}]\Big),$$

and using Eq(3.61) the Lagrangian becomes

$$\mathcal{L} = \mathcal{L}_0 + \frac{f^2\mu}{2}\,\mathrm{Tr}[UM^{\dagger} + MU^{\dagger}] - \frac{af^2}{4N}\Big[\bar{\theta} - \frac{1}{2} i\,\mathrm{Tr}[\log U - \log U^{\dagger}]\Big]^2. \tag{3.62}$$

3.3.2 The η' mass and its interactions

Equation (3.62) is interesting because it shows how the vacuum energy is modified by the inclusion of the η' particle. We will now repeat the analysis of Section 3.1.3 in order to find the vacuum energy and, ultimately, the η' mass. From the fact that U is unitary, we have that the most general $\langle U \rangle$ is

$$\langle U_{ij} \rangle = \delta_{ij} e^{-i\phi_i}. \tag{3.63}$$

By analyzing the Lagrangian of Eq. (3.62) we note that the enegy density is

$$E = -\frac{f^2\mu}{2} \operatorname{Tr}[UM^\dagger + MU^\dagger] + \frac{af^2}{4N}\Big[\bar{\theta} - \frac{1}{2} i \operatorname{Tr}[\log U - \log U^\dagger]\Big]^2. \tag{3.64}$$

Using equation (3.63) in (3.64) we find

$$\begin{aligned} E =& -\frac{f^2\mu}{2} \operatorname{Tr}\Big[\operatorname{diag}(m_{q_1}, \cdots, m_{q_L})\operatorname{diag}(e^{-i\phi_1}, \cdots, e^{-i\phi_L}) + h.c.\Big] \\ &+ \frac{af^2}{4N}\Big[\bar{\theta} - \frac{1}{2} i \operatorname{Tr}\Big[\log(\operatorname{diag}(e^{-i\phi_1}, \cdots, e^{-i\phi_L})) - \log(\operatorname{diag}(e^{i\phi_1}, \cdots, e^{i\phi_L}))\Big]\Big]^2 \\ =& -\frac{f^2\mu}{2} \sum_{i=1}^{L} m_{q_i}(e^{-i\phi_i} + e^{i\phi_i}) + \frac{af^2}{4N}\Big[\bar{\theta} - \frac{1}{2} i \sum_{i=1}^{L}(-2i\phi_i)\Big]^2 \\ =& -f^2\mu \sum_{i=1}^{L} m_{q_i} \cos\phi_i + \frac{af^2}{4N}\Big[\bar{\theta} - \sum_{i=1}^{L}\phi_i\Big]^2, \end{aligned}$$

and minimizing as a function of ϕ_j

$$\frac{\partial E}{\partial \phi_j} = 0, \quad \text{for each } j \ \Rightarrow \mu m_{q_j} \sin\phi_j = \frac{a}{2N}\Big(\bar{\theta} - \sum_{i=1}^{L}\phi_i\Big). \tag{3.65}$$

We now redefine the field U using a $U(L) \times U(L)$ transformation according to

$$V = AUB^\dagger, \text{ where } A_{\alpha\beta} = B^\dagger_{\alpha\beta} = \delta_{\alpha\beta} e^{i\phi_\alpha/2}. \tag{3.66}$$

The new dynamical field V has $\langle V \rangle = \mathbb{I}$ and includes the effect of the non-trivial vacuum phases.

By noticing that $U = A^\dagger V B = \operatorname{diag}(e^{-i\phi_1/2}, \cdots, e^{-i\phi_L/2}) V \operatorname{diag}(e^{-i\phi_1/2}, \cdots, e^{-i\phi_L/2})$

and using it in the Lagrangian of (3.62), we have

$$\begin{aligned}
\mathcal{L} =&\mathcal{L}_0(V) + \frac{f^2\mu}{2}\operatorname{Tr}[\operatorname{diag}(m_{q_1},\cdots,m_{q_L})\operatorname{diag}(e^{-i\phi_1/2},\cdots,e^{-i\phi_L/2})V\operatorname{diag}(e^{-i\phi_1/2},\cdots,e^{-i\phi_L/2})\\
&+\operatorname{diag}(m_{q_1},\cdots,m_{q_L})\operatorname{diag}(e^{i\phi_1/2},\cdots,e^{i\phi_L/2})V^\dagger\operatorname{diag}(e^{i\phi_1/2},\cdots,e^{i\phi_L/2})]\\
&-\frac{af^2}{4N}\Big[\bar\theta-\frac{1}{2}i\Big(-2\sum_j\phi_j+\operatorname{Tr}[logV-logV^\dagger\Big)\Big]^2\\
=&\mathcal{L}_0(V) + \frac{f^2\mu}{2}\operatorname{Tr}[\operatorname{diag}(m_{q_1}e^{-i\phi_1},\cdots,m_{q_L}e^{-i\phi_L})V+\operatorname{diag}(m_{q_1}e^{i\phi_1},\cdots,m_{q_L}e^{i\phi_L})V^\dagger]\\
&-\frac{af^2}{4N}\Big[\text{const.}-2\bar\theta\frac{1}{2}i\Big(\text{const.}+\operatorname{Tr}[\log V-\log V^\dagger]\Big)-\frac{1}{4}\Big(-2\sum_j\phi_j+\operatorname{Tr}[\log V-\log V^\dagger]\Big)^2\Big],
\end{aligned}$$

and using the Euler formula for complex exponential

$$\begin{aligned}
\mathcal{L} =&\mathcal{L}_0(V) + \frac{f^2\mu}{2}\operatorname{Tr}[m_{q_i}(\cos\phi_i-i\sin\phi_i)\delta_{ij}V_{jk}+m_{q_i}(\cos\phi_i+i\sin\phi_i)\delta_{ij}V^\dagger_{jk}]\\
&-\frac{af^2}{4N}\Big(-\bar\theta i\operatorname{Tr}[\log V-\log V^\dagger]+\text{const.}-\frac{1}{4}\Big(-4\sum_j\phi_j\operatorname{Tr}[\log V-\log V^\dagger]\\
&+(\operatorname{Tr}[\log V-\log V^\dagger])^2\Big)\Big)\\
=&\mathcal{L}_0(V)+\frac{af^2}{16N}(\operatorname{Tr}[\log V-\log V^\dagger])^2+\frac{f^2}{2}\operatorname{Tr}[M(\theta)(V+V^\dagger)]\\
&\frac{f^2}{4}\frac{ia}{N}\Big(\bar\theta-\sum_{j=1}^L\phi_j\Big)[-\operatorname{Tr}[V-V^\dagger]+\operatorname{Tr}[\log V-\log V^\dagger]]+\text{const.},
\end{aligned}$$

where in the last equation we used Eq.(3.65) and defined $M_{ij}(\theta)=\mu m_{q_i}\cos\phi_i\delta_{ij}$. By summing and subtracting a constant in the Lagrangian we finally have

$$\begin{aligned}
\mathcal{L} =&\mathcal{L}_0(V)+\frac{af^2}{16N}(\operatorname{Tr}[\log V-\log V^\dagger])^2+\frac{f^2}{2}\operatorname{Tr}[M(\theta)(V+V^\dagger)]\\
&\frac{f^2}{4}\frac{ia}{N}\Big(\bar\theta-\sum_{j=1}^L\phi_j\Big)[-\operatorname{Tr}[V-V^\dagger]+\operatorname{Tr}[\log V-\log V^\dagger]].
\end{aligned}\tag{3.67}$$

Now that we have our physical field V and the Lagrangian described as a function of it, we can use the CCWZ construction on V, in such a way that

$$V=e^{\frac{i\sqrt{2}}{f}\Phi},\tag{3.68}$$

where $\Phi=\pi^iT^i+S/\sqrt{L}$, π^i are the respective pions fields, T^i are the generators of $SU(L)$ and S is the field of the pion corresponding to the $U(1)$ axial symmetry (notice that this symmetry has the identity matrix as generator and we are considering that this particle may be a Goldstone boson).

Applying Eq. (3.68) in (3.67) we get

$$\mathcal{L} = \frac{f^2}{4}\,\mathrm{Tr}[\partial_\mu V^\dagger \partial_\mu V] - \frac{a}{2N}(\mathrm{Tr}[\Phi])^2 + f^2\,\mathrm{Tr}\left[M(\theta)\cos\left(\frac{\sqrt{2}}{f}\Phi\right)\right] \\ + \frac{a}{2N} f^2 \big(\bar{\theta} - \sum_j \phi_j\big)\,\mathrm{Tr}\left[\sin\left(\frac{\sqrt{2}}{f}\Phi\right) - \frac{\sqrt{2}}{f}\Phi\right], \tag{3.69}$$

this Lagrangian gives us quadratic, cubic and higher-order terms for the field Φ. By expanding $\cos(\Phi) = 1 - \frac{\Phi^2}{2!} + \frac{\Phi^4}{4!} + \cdots$ we get the second order terms

$$\mathcal{L}_2 = \frac{1}{2}\,\mathrm{Tr}[\partial_\mu \Phi \partial_\mu \Phi] - \frac{a}{2N}(\mathrm{Tr}\,\Phi)^2 - \mathrm{Tr}[M(\theta)\Phi^2],$$

and we search for the mass of S, by looking at

$$\begin{aligned}\mathcal{L}_{Mass} &= -\frac{a}{2N}(\mathrm{Tr}\,\Phi)^2 - \frac{1}{2}\,\mathrm{Tr}[M(\theta)\Phi^2] \\ &= -\frac{a}{2N}\big(\cdots + L\frac{S}{\sqrt{L}}\big)^2 - \mathrm{Tr}\left[\mathrm{diag}(\mu m_{q_1}\cos\phi_1, \cdots, \mu m_{q_L}\cos\phi_L)\big(\cdots + (\cdots)\frac{S}{\sqrt{L}} + \frac{S^2}{L}\big)\right] \\ &= -\frac{aL}{2N}S^2 - \frac{1}{L}\sum_i \mu m_{q_i}\cos\phi_i S^2 + \cdots,\end{aligned}$$

so we find that

$$m_S^2 = \frac{La}{N} + \frac{2}{L}\sum_i \mu m_{q_i}\cos\phi_i, \tag{3.70}$$

where m_S is the mass of the S particle.

From Eq. (3.70) we can conclude that at the chiral limit (massless quark) and large N limit we have that the S particle, associated to the $U(1)$ axial symmetry, is massless, being a true Goldstone boson. But in the real QCD we have $N = 3$, $L = 3$ and $m_S^2 = a \neq 0$ (at the chiral limit), this particle S is actually the η', which is not a Goldstone boson and has the mass fixed only by the parameter a (to be defined experimentally), which permits the mass of this particle to be larger than the mass of any other meson. Associating S with the η' we can get the η' interaction with the other mesons by Eq.(3.69) and, by using the methods of Section 2.5 and promoting the derivative of Eq.(3.69) to a covariant derivative, we can add the gauge fields interactions to the η'.

Chapter 4

The relaxation of the electroweak scale

In this chapter we will describe the physics behind the "relaxation of the electroweak scale" and we will apply the machinery introduced in the previous chapter to this scenario. As already mentioned, the relaxation of the EW scale consists in making the Higgs squared mass parameter dynamical (very much like the θ term is made dynamical by the axion field), and to use the evolution in the early universe to break the electroweak symmetry and obtain a small EW scale [18]. These models make the weak scale technically natural and we can judge the effectiveness of the model by how much they naturally raise the cutoff of the Higgs.

4.1 The Minimal Model

4.1.1 The central idea

The simplest model of relaxion is the one where we consider the relaxion as simply the QCD axion coupled with the Higgs in such a way that the Lagrangian has the terms

$$\mathcal{L} \supset -(-M^2 + g\phi) \mid H \mid^2 - V(g\phi) - \frac{\phi G_{\mu\nu}\tilde{G}^{\mu\nu}}{64\pi^2 F}, \tag{4.1}$$

where H is the Higgs doublet, g is a dimensionful coupling, M is the cutoff of the theory, $G^{\mu\nu}$ is the QCD field strength, ϕ is the axion (now relaxion) field and F is the scale where the Peccei-Quinn symmetry breaks. The relaxion has a very large field range, can assume values much larger than F, and we see from Eq.(4.1) that the effective mass squared of the Higgs, $m_H^2 = -M^2 + g\phi$, depends on the evolution of the relaxion and has thus be promoted to a dynamical quantity.

Notice that when we set $g \to 0$ the Lagrangian in Eq. (4.1) acquires a shift symmetry $\phi \to \phi + 4\pi F$. In this way small values of g are technically natural. Going to energies below the QCD scale, we get the axion potential just like in Eq. (3.47). Approximating such potential as $\Lambda^4 \cos\left(\frac{\phi}{2F}\right)$ with the overall scale Λ given in terms of the up and down Yukawa couplings $y_{u,d}$ and the Higgs vev as

$$\Lambda^4 \sim m_\pi^2 f^2 \sim v(y_d + y_u)\mu f^2, \tag{4.2}$$

we obtain the total potential

$$V(H) = (-M^2 + g\phi) \mid H \mid^2 + (gM^2\phi + g^2\phi^2 + \cdots) + \Lambda^4 \cos\left(\frac{\phi}{2F}\right), \tag{4.3}$$

where the ellipsis represents the higher order terms in $g\phi/M^2$. Notice that the term with g breaks explicitly the shift symmetry and we must add an UV explanation for such term.

Now, if we take the initial value of ϕ such that the Higgs squared mass parameter m_H^2 is positive, during the evolution of the universe the field ϕ starts to roll over the potential and scans m_H^2, decreasing its value, because of the slope of $(gM^2\phi + g^2\phi^2 + \cdots)$. At a certain point, $\phi \approx M^2/g$, the mass squared of the Higgs turns negative, and the Higgs acquires a vev. This makes the height of the bumps Λ^4 in the last term of (4.3) grow, in principle building up potential barriers that make ϕ stop rolling when $m_H^2 \approx m_{EW}^2 \ll M^2$, solving the Hierarchy Problem (see Figure 4.1).

In order for the relaxion evolution to stop to small values of the Higgs vev and avoid overshooting the electroweak range vacua, we demand the process to happen during inflation so that the field ϕ slow-rolls with the Hubble friction given by (see Eq.(A.23))

$$3H_I\dot{\phi} = -\frac{\partial V}{\partial \phi}, \tag{4.4}$$

where H_I is the Hubble scale during inflation.

We are now going to analyze under which conditions the relaxation of the EW scale is a viable solution of the hierarchy problem.

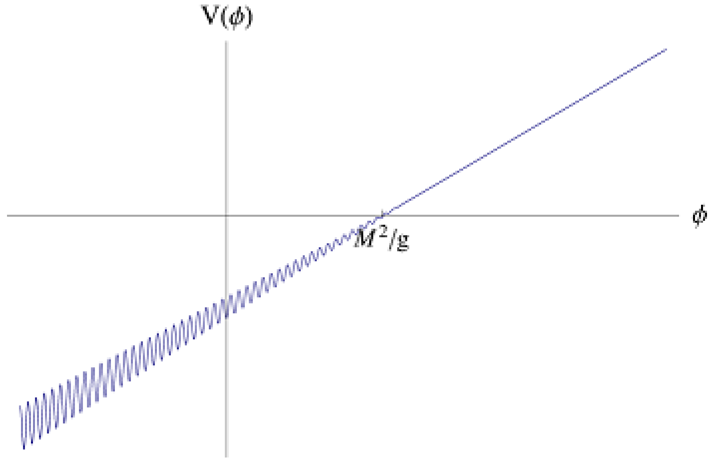

Figure 4.1: A characterization of the ϕ's potential where the Higgs gets a vev and the barriers starts to grow.

4.1.2 Constraints on the parameter space

In order to provide a natural solution to the Hierarchy Problem, *i.e.*, to dynamically provide a stable separation between the weak scale and the high energy scale M, we need to impose the following requirements on the relaxion mechanism:

1. The Higgs field must be the only one responsible for stopping ϕ from sliding any longer. At the minimum of the potential we have $\partial V/\partial\phi = 0$, which implies

$$gM^2 \sim \frac{\Lambda^4}{F} \Rightarrow gM^2F \sim \Lambda^4. \tag{4.5}$$

   From here we also see that since $\Lambda^4 \sim v(y_u + y_d)\mu f^2$, the Higgs vev scales like $v \sim gM^2F/[(y_u + y_d)\mu f^2]$ in such a way that small technically natural values of g make also v technically natural.

2. Inflation is independent of ϕ evolution. We thus demand that the typical energy density carried by ϕ, namely M^4, remain smaller then the inflation scale. Using Eq.(A.5) this means that

$$\frac{H_I^2}{G} > M^4 \Rightarrow H_I > M^2\sqrt{G} \sim \frac{M^2}{M_{Pl}} \Rightarrow H_I > \frac{M^2}{M_{Pl}}, \tag{4.6}$$

   where G is the Newton gravitational constant and M_{Pl} is the reduced Planck mass.

3. Assuming that the cosmological evolution of ϕ is dominated by classical physics, it is essential that quantum fluctuations, of order of H_I (see Eqs.(A.31) and (A.33)),

remain smaller than the classical field displacements in one Hubble time

$$H_I < \Delta\phi \sim \Delta T \frac{d\phi}{dt} \sim H_I^{-1} \frac{d\phi}{dt}.$$

Using Eq.(4.4) we have

$$H_I^{-1} \frac{d\phi}{dt} \sim H_I^{-2} \frac{dV}{d\phi} \Rightarrow H_I < \frac{gM^2}{H_I^2} \Rightarrow H_I < (gM^2)^{1/3}. \tag{4.7}$$

4. Inflation must last long enough for ϕ to scan the entire range. We require then that during inflation $\Delta\phi \geq M^2/g$ where M^2/g is an $\mathcal{O}(1)$ fraction of the full range. Now, from the definition of e-folds that we have in the Appendix A, we obtain

$$N = \int H_I dt = \int \frac{H_I}{\dot{\phi}} d\phi \sim \Delta\phi \frac{H_I}{\dot{\phi}} \Rightarrow \Delta\phi \sim \frac{N\dot{\phi}}{H_I},$$

and using Eq. (4.4) we get

$$\Delta\phi \sim \frac{V'_\phi}{H_I^2} N \sim \frac{gM^2}{H_I^2} N \geq \frac{M^2}{g} \Rightarrow N \geq \frac{H_I^2}{g^2}. \tag{4.8}$$

5. In order for the barrier to form, we must have that the Hubble scale during inflation is lower than the QCD scale

$$H_I < \Lambda_{QCD}. \tag{4.9}$$

Putting together Eqs. (4.5), (4.6) and (4.7) we get

$$M < \left(\frac{\Lambda^4 M_{Pl}^3}{F}\right)^{\frac{1}{6}} \sim 10^7 \text{ GeV} \times \left(\frac{10^9\text{GeV}}{F}\right)^{\frac{1}{6}}, \tag{4.10}$$

where we approximate $\Lambda_{QCD} \sim \Lambda$. Scaling F by its lower bound of 10^9 GeV [19], we get $M < 10^7$ GeV, and we obtain a constraint on the cutoff M.

We have also, from Eqs. (4.6) and (4.9), a finite range for the Hubble scale of

$$\Lambda_{QCD} > H_I > \frac{M^2}{M_{Pl}}.$$

Going now back to the expression of the Higgs vev and writing $g = \epsilon M$, we obtain

$$\frac{v}{10^2\mathrm{GeV}} \sim \left(\frac{\epsilon}{10^{-26}}\right)\left(\frac{M}{10^7\mathrm{GeV}}\right)^3\left(\frac{F}{10^9\mathrm{GeV}}\right).$$

This last equation tells us that, in order to obtain a Higgs vev of order of 100 GeV and having $M \sim 10^7$ GeV and $F \sim 10^9$ GeV, we must have an extremely small value for ϵ, $\epsilon \sim 10^{-26}$.

We are now in a position to estimate the number of e-folds that our inflation needs in order to provide the separation between the weak scale and the cutoff M. We begin by noticing that if we approximate the potential to the dominating term, $V(\phi) \approx gM^2\phi$, we have the number of e-folds (see Eq.(A.30))

$$N \approx -\frac{1}{M_{Pl}^2}\int_{\phi_i}^{\phi_f}\frac{gM^2\phi}{gM^2}d\phi = \frac{\phi_i^2}{2M_{Pl}^2} - \frac{\phi_f^2}{2M_{Pl}^2}, \tag{4.11}$$

where for finding ϕ_f we impose that $\epsilon_V(\phi_f) = 1$ at the end of inflation, which implies that (see Eq. (A.25))

$$\epsilon_V(\phi_f) = \frac{M_{Pl}^2}{2}\left(\frac{gM^2}{gM^2\phi_f}\right)^2 = 1 \Rightarrow \phi_f = \frac{M_{Pl}}{\sqrt{2}},$$

and for ϕ_i we suppose $\phi_i \sim \frac{M^2}{g}$ so that the Higgs starts with positive mass squared, then we obtain, from Eq. (4.11)

$$N \approx \frac{M^4}{2g^2M_{Pl}^2} - \frac{1}{4},$$

where replacing $g = \epsilon M$, $\epsilon \sim 10^{-26}$, $M \sim 10^7$ GeV and $M_{Pl} \sim 10^{18}$ GeV, we have

$$N \sim 10^{30}.$$

This means that in order for the relaxation of the EW scale to work as a solution to the hierarchy problem we need an extremely small parameter, $\epsilon \sim 10^{-26}$, and a very long period of inflation, $N \sim 10^{30}$.

The Minimal Model is discarded by the Strong CP Problem because Eq. (4.5) predicts that the local minimum of ϕ is displaced from the minimum of QCD part of the potential by $\mathcal{O}(F)$. Remembering that if ϕ is the QCD axion its vev is given by $\langle a \rangle = -\bar{\theta}F$ then a displacement from the minimum of $\mathcal{O}(F)$ means $\bar{\theta} \sim 1$.

4.2 Non-QCD Model

We concluded the previous section showing that the relaxion does not solve the strong CP problem.

One way to avoid this is to consider that the potential barriers for ϕ arise from a new strong group, different from QCD. Instead of creating the potential barriers with the axion, we will postulate another strong sector whose purpose is to generate the barrier with an Higgs vev dependence.

4.2.1 A new strong group

Let us now consider the new fermions that will couple to the new strong group to be doublets under $SU(2)_L$ with hypercharge $-\frac{1}{2}$: $L_L = \begin{pmatrix} N_{1L} \\ E_L \end{pmatrix}$ and $L_R = \begin{pmatrix} N_{1R} \\ E_R \end{pmatrix}$, and singlets under $SU(2)_L$ with hypercharge 0: N_{2L} and N_{2R}. All of them are singlets under $SU(3)_c$ and transform under the fundamental representation of $SU(N)$, which is the new strong group. We call this model of L+N, or 2+1.

The most general Lagrangian is

$$\begin{aligned}\mathcal{L} =& i\bar{L}_L \not{D} L_L + i\bar{L}_R \not{D} L_R + i\bar{N}_{2L}\not{D} N_{2L} + i\bar{N}_{2R}\not{D} N_{2R} - m_L(\bar{L}_L L_R + h.c.) - m_N(\bar{N}_{2L} N_{2R} + h.c.) \\ &- (y\epsilon_{ab}\bar{N}_{2R} H^a L_L^b + h.c.) - (\tilde{y}\epsilon_{ab}\bar{N}_{2L} H^a L_R^b + h.c.),\end{aligned} \tag{4.12}$$

where the covariant derivatives are such that

$$\bar{L}_L \not{D} L_L = \bar{L}_L(\not{\partial} - ig\not{W}^a\tau^a - ig' Y_L \not{B} - ig''\not{G}^a T_F^a) L_L \text{ and } \bar{N}_{2L}\not{D} N_{2L} = \bar{N}_{2L}(\not{\partial} - ig''\not{G}^a T_F^a) N_{2L},$$

defined similarly for the $\bar{L}_R \not{D} L_R$ and $\bar{N}_{2R}\not{D} N_{2R}$. Here τ^a are the generators of $SU(2)_L$, Y_L is the hypercharge generator and T_F^a are the generators of $SU(N)$.

Separating the mass dependent part of the Lagrangian in (4.12), we have

$$\begin{aligned}\mathcal{L}_m =& - m_L(\bar{L}_L L_R + h.c.) - m_N(\bar{N}_{2L} N_{2R} + h.c.) - y\epsilon_{ab}\bar{N}_{2R} H^a L_L^b \\ &- y^*\epsilon_{ab}\bar{L}_L^b H^{\dagger a} N_{2R} - \tilde{y}\epsilon_{ab}\bar{N}_{2L} H^a L_R^b - \tilde{y}^*\epsilon_{ab}\bar{L}_R^b H^{\dagger a} N_{2L}.\end{aligned} \tag{4.13}$$

Expanding the Higgs around the vev $H = \begin{pmatrix} 0 \\ \frac{h+v}{\sqrt{2}} \end{pmatrix}$, we have that (4.13) takes the form

$$\begin{aligned}\mathcal{L}_m =& - m_L(\bar{N}_{1L}N_{1R} + \bar{E}_L E_R + h.c.) - m_N(\bar{N}_{2L}N_{2R} + h.c.) + \frac{yv}{\sqrt{2}}\bar{N}_{2R}N_{1L} \\ &+ \frac{y^*v}{\sqrt{2}}\bar{N}_{1L}N_{2R} + \frac{\tilde{y}v}{\sqrt{2}}\bar{N}_{2L}N_{1R} + \frac{\tilde{y}^*v}{\sqrt{2}}\bar{N}_{1R}N_{2L},\end{aligned} \tag{4.14}$$

which can be written in a more compact form as

$$\mathcal{L}_m = -\bar{\Psi}_L M \Psi_R + h.c., \tag{4.15}$$

where $\Psi_R = \begin{pmatrix} N_{1R} \\ N_{2R} \\ E_R \end{pmatrix}$, $\Psi_L = \begin{pmatrix} N_{1L} \\ N_{2L} \\ E_L \end{pmatrix}$ and $M = \begin{pmatrix} m_L & -\frac{y^*v}{\sqrt{2}} & 0 \\ -\frac{\tilde{y}v}{\sqrt{2}} & m_N & 0 \\ 0 & 0 & m_L \end{pmatrix}$.

The mass matrix M can be diagonalized noting that $MM^\dagger$ is hermitian, so that there is a unitary operator U such that

$$MM^\dagger = UM_d^2U^\dagger, \tag{4.16}$$

with M_d^2 the diagonal matrix made of the eigenvalues of $MM^\dagger$. This is equivalent of saying that there is a unitary matrix K such that

$$M = UM_dK^\dagger, \tag{4.17}$$

and we get M_d as the diagonalized mass matrix, whose diagonal elements are the masses of the fermions in the mass basis.

By using this procedure we obtain the diagonal elements of M_d as

$$\begin{aligned}&m_1 = m_L, \\ &m_2 = \Big(\frac{1}{2}m_L^2 + \frac{1}{2}m_N^2 + \frac{1}{4}\tilde{y}^2v^2 + \frac{1}{4}y^2v^2 - \frac{1}{4}\Big((-2m_L^2 - 2m_N^2 - \tilde{y}^2v^2 - y^2v^2)^2 \\ &- 4(4m_L^2m_N^2 - 2m_Lm_N\tilde{y}^*yv^2 - 2m_Lm_N\tilde{y}y^*v^2 + \tilde{y}^2y^2v^4)\Big)^{1/2}\Big)^{1/2}, \\ &m_3 = \Big(\frac{1}{2}m_L^2 + \frac{1}{2}m_N^2 + \frac{1}{4}\tilde{y}^2v^2 + \frac{1}{4}y^2v^2 + \frac{1}{4}\Big((-2m_L^2 - 2m_N^2 - \tilde{y}^2v^2 - y^2v^2)^2 \\ &- 4(4m_L^2m_N^2 - 2m_Lm_N\tilde{y}^*yv^2 - 2m_Lm_N\tilde{y}y^*v^2 + \tilde{y}^2y^2v^4)\Big)^{1/2}\Big)^{1/2}.\end{aligned} \tag{4.18}$$

4.2.2 ChPT of the non-QCD model

Collider and other constraints require m_L to be greater than the weak scale [20], but no such constraint exists on m_N, so we integrate out the heaviest particles, *i.e.*, E_L, E_R, N_{1L} and N_{1R} by solving the equations of motion at the Lagrangian in (4.14), and we get

$$\mathcal{L}_2 = \left(\frac{y^*\tilde{y}}{2m_L}(v+h)^2 - m_N\right)\bar{N}_{2L}N_{2R} + h.c. \tag{4.19}$$

The same result can be obtained from Eq. (4.18) in the $m_L \to \infty$ limit.

Notice that, from the Lagrangian of Eq. (4.19) we obtain a new double Higgs interaction with the light fermion $\frac{y^*\tilde{y}}{2m_L}h^2\bar{N}_{2L}N_{2R} + h.c.$ and a complex mass for the light fermion, which we can parameterize as

$$M_N \equiv \frac{y^*\tilde{y}v^2}{2m_L} - m_N = \mid M_N \mid e^{i\theta_N}, \tag{4.20}$$

where $\mid M_N \mid$ is the modulus of M_N and will be now the physical mass and θ_N will be a phase.

We will now repeat the same kind of reasoning we applied in Section 3.3, now using the new interacting strong group $SU(N)$ in place of large-N QCD and with one light flavor which is the fermion N_2 that condensates in $\bar{N}_2N_2$. The spontaneous symmetry breaking pattern is $U(1)_L \times U(1)_R \to U(1)_V$. The matrix U now will contain only one meson, the dark analog of the η', since the broken symmetry is the anomalous $U(1)_A$.

Following the steps of Subsection 3.3.1 we obtain a Lagrangian analogous to Eq. (3.62)

$$\mathcal{L} = \mathcal{L}_0 + \frac{f'^2\mu'}{2}\,\mathrm{Tr}[UM_N^\dagger + M_NU^\dagger] - \frac{a'f'^2}{4N}\left[\bar{\theta}' - \frac{1}{2}i\,\mathrm{Tr}[\log U - \log U^\dagger]\right]^2,$$

where f' is the chiral symmetry breaking scale of the new strong group, μ' is the dark analogous of μ from ChPT, a' is the dark analogous of a from Subsection 3.3.1 and $\bar{\theta}'$ is the dark theta angle from the new strong group.

The procedure of Subsection 3.3.2 allows us to find the vacuum energy, and we obtain

$$E = -f'^2\mu' \mid M_N \mid \cos(\xi - \theta_N) + \frac{a'f'^2}{4N}[\bar{\theta}' - \xi]^2,$$

where ξ is the vev of the dark meson, given by

$$\mu' \mid M_N \mid \sin(\xi - \theta_N) = \frac{a'}{2N}(\bar{\theta}' - \xi). \tag{4.21}$$

Redefining the field U to the dynamical field V which includes the effect of the non trivial vacuum phase, $V = e^{i\xi/2} U e^{i\xi/2}$, and using the CCWZ construction such that

$$V = e^{\frac{i\sqrt{2}}{f'}\eta'}, \tag{4.22}$$

where η' is the dark meson, we obtain the Lagrangian analogous to the Eq. (3.69)

$$\begin{aligned}\mathcal{L} =& \frac{f'^2}{4}\,\mathrm{Tr}[\partial_\mu V^\dagger \partial_\mu V] - \frac{a'}{2N}\eta'^2 + f'^2\mu' \mid M_N \mid \cos(\xi - \theta_N)\cos\left(\frac{\sqrt{2}}{f'}\eta'\right) \\ &+ \frac{a'}{2N} f'^2(\bar{\theta}' - \xi)\left[\sin\left(\frac{\sqrt{2}}{f'}\eta'\right) - \frac{\sqrt{2}}{f'}\eta'\right].\end{aligned}$$

Now, since the relaxion will be the correspondent axion of the new strong group, we can make the substitution $\bar{\theta}' \to \bar{\theta}' + \frac{\phi}{F'}$, where ϕ is the relaxion and F' is the dark analogous of F from Peccei-Quinn and we must have that F' is at least of order of the cutoff M. This modifies the above Lagrangian and Eq. (4.21) in such a way that the ξ angle has a relaxion dependence

$$\xi = -\frac{2N}{a'}\mu' \mid M_N \mid \sin(\xi - \theta_N) + \bar{\theta}' + \frac{\phi}{F'},$$

and the Lagrangian becomes

$$\begin{aligned}\mathcal{L} =& \frac{f'^2}{4}\,\mathrm{Tr}[\partial_\mu V^\dagger \partial_\mu V] - \frac{a'}{2N}\eta'^2 + f'^2\mu' \mid M_N \mid \cos\Big(- \frac{2N}{a'}\mu' \mid M_N \mid \sin(\xi - \theta_N) + \bar{\theta}' \\ &+ \frac{\phi}{F'} - \theta_N\Big)\cos\left(\frac{\sqrt{2}}{f'}\eta'\right) + \frac{a'}{2N}f'^2\left(\bar{\theta}' + \frac{\phi}{F'} - \xi\right)\left(-\frac{(2)^{3/2}}{3!}\eta'^3 + \frac{(2)^{5/2}}{5!}\eta'^5 + \cdots\right).\end{aligned} \tag{4.23}$$

This Lagrangian contains the dynamics of the dark η' and interactions between the relaxion and the dark η', and it will also give the quadratic terms and thus the mass of the dark η'. Notice that from the third term of the Lagrangian, from expanding $\cos\left(\frac{\sqrt{2}}{f'}\eta'\right) = 1 - \frac{1}{2!}\left(\frac{\sqrt{2}}{f'}\eta'\right)^2 + \frac{1}{4!}\left(\frac{\sqrt{2}}{f'}\eta'\right)^4 + \cdots$, we obtain the relaxion dependent periodic potential

$$V(\phi) = -f'^2\mu' \mid M_N \mid \cos\left(\frac{\phi}{F'} + \alpha(\phi)\right) \tag{4.24}$$

where $\alpha(\phi) = -\frac{2N}{a'}\mu' \mid M_N \mid \sin(\xi - \theta_N) + \bar{\theta}' - \theta_N$.

The potential has the height of the bump defined by

$$\Lambda^4 = f'^2\mu' \mid M_N \mid, \tag{4.25}$$

which shows that the periodic barriers grow as the Higgs vev grows, since $\mid M_N \mid$ depends on the vev by Eq. (4.20).

From following the same steps as the end of Subsection 3.3.2 we obtain

$$\mathcal{L}_{mass} = -\frac{a'}{2N}\eta'^2 - \mu' \mid M_N \mid \cos(\xi - \theta_N)\eta'^2,$$

in first order approximation we have $\phi = 0$ which implies

$$m_{\eta'}^2 = \frac{a'}{N} + 2\mu' \mid M_N \mid \cos(\xi' - \theta_N), \tag{4.26}$$

where this ξ' satisfies

$$\xi' = -\frac{2N}{a'}\mu' \mid M_N \mid \sin(\xi' - \theta_N) + \bar{\theta}'.$$

This completes our treatment of the condensation of the light fermions in the L+N model of the new strong group. We have found the mass of the degree of freedom that arises at low energies after the condensate of $\bar{N}_2 N_2$ in Eq. (4.26), and the interactions of this degree of freedom with the relaxion given in the Lagrangian of Eq. (4.23) which causes a periodic potential barrier (see Eq.(4.25)) that depends on the Higgs vev and stops the rolling of the relaxion at the correct weak scale in the same way as in the Minimal Model, solving the hierarchy problem without jepardizing the solution of the strong CP problem.

Now, depending of the NP at the F' scale, we may have operators such as in the Lagrangian of Eq (3.48) of Subsection 3.2.2 interacting with the relaxion instead of the axion. All the reasoning outlined in that Subsection may be repeated for the case of the relaxion.

4.2.3 Constraints on the parameter space in the non-QCD model

As already mentioned, this model solves the hierarchy problem with the same idea as the minimal model, but now with an essential participation of M_N in the generation of the barriers that stop the rolling of the relaxion. For this mechanism to work we need again to apply constraints on the parameter space, starting with constraints on the radiative corrections on the mass of N_2.

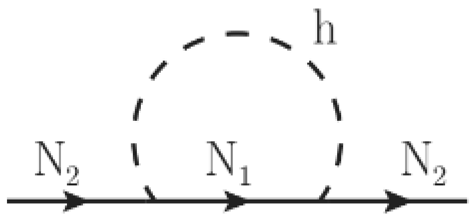

Figure 4.2: Feynman diagram contributing for the radiative corrections of m_N from the Yukawa couplings.

Notice that we have radiative corrections for the light fermion mass coming from the diagram of Figure 4.2, whose vertices comes from the Yukawa coupling of the Lagrangian from Eq. (4.13), that gives the amplitude

$$\begin{aligned} i\mathcal{A}_{N_2hN_1} =& \frac{iyi\tilde{y}}{2}\int \frac{d^4k}{(2\pi)^4}\frac{i(\not{k}+m_L)}{k^2-m_L^2}\frac{i}{(p-k)^2-m_L^2} \\ =& \frac{y\tilde{y}}{2}\int \frac{d^4k}{(2\pi)^4}\int_0^1 dx \frac{\not{k}+m_L}{[x((p-k)^2-m_h^2)+(1-x)(k^2-m_L^2)]^2} \\ =& \frac{y\tilde{y}}{2}\int_0^1 dx \int_0^M \frac{d^4l}{(2\pi)^4}\frac{x\not{p}+m_L}{[l^2-a^2]^2}, \end{aligned}$$

where $a = x^2p^2 - xp^2 + m_L^2 + x(m_h^2 - m_L^2)$, m_h is the Higgs boson mass, p is the momentum of the incoming and outgoing fermion N_2, k is the momentum of the virtual fermion N_1 and in the last equation we made the substitution $k \to l + xp$. Now, computing the last integral with sharp cutoff regularization using the cutoff of the theory M, we obtain

$$i\mathcal{A}_{N_2hN_1} = \frac{y\tilde{y}}{2}\int_0^1 dx \frac{i(x\not{p}+m_L)}{(4\pi)^2}\left(\log\left(\frac{M^2+a^2}{a^2}\right)+\frac{a^2}{M^2+a^2}-1\right) \sim i\frac{y\tilde{y}}{32\pi^2}m_L\log\left(\frac{M^2}{m_L^2}\right), \tag{4.27}$$

where in the last step we used that we expect $m_L \gg m_h$, $m_L \gg p$ and $M \gg m_L$.

Similarly we have the radiative corrections coming from the Higgs loop whose vertex

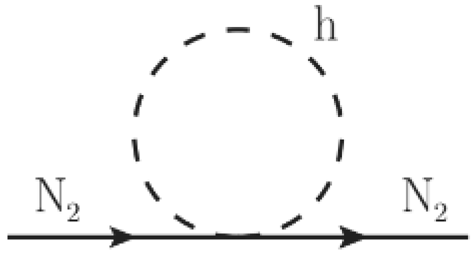

Figure 4.3: Feynman diagram contributing for the radiative corrections of m_N from the Higgs loop.

come from the new interaction in the effective Lagrangian in Eq. (4.19) (see Figure 4.3), this gives the amplitude

$$i\mathcal{A}_{N_2h^2N_2} = \frac{iy^*\tilde{y}}{2m_L}\int_0^{4\pi f'}\frac{d^4k}{(2\pi)^4}\frac{i}{k^2-m_h^2} \sim i\frac{y\tilde{y}}{2m_L}\frac{(4\pi f')^2}{(4\pi)^2}, \tag{4.28}$$

where we evaluate this integral with sharp cutoff regularization where we used the cutoff of the effective theory as $4\pi f'$.

Now, we want these quantum corrections not to spoil our mechanism, *i.e.*, we require these terms not to be too large so the relaxion does not stop rolling long before the Higgs reaches its correct expectation value. This implies that we want our radiative corrections to be smaller than $\mid M_N \mid \sim \frac{y\tilde{y}}{2m_L}v^2$ (notice that this cannot be done for the case where $v = 0$, for which $\mid M_N \mid = m_N$, this can cause troubles in the potential before EWSB and is considered in [21]). Then, imposing this in the radiative corrections of Eqs. (4.27) and (4.28) we obtain

$$m_L \leq \frac{4\pi v}{\sqrt{\log(M^2/m_L^2)}} \quad \text{and} \quad f' \leq v.$$

By applying the same constraints listed in Subsection 4.1.2 for the minimal model plus the constraints over the radiative corrections, we obtain the following constraints:

1. The Higgs field must be the only one responsible for stopping ϕ from sliding any longer. This implies

$$gM^2F' \sim \Lambda^4, \tag{4.29}$$

   where now Λ^4 is given by Eq. (4.25);

2. Inflation is independent of ϕ evolution, implying

$$H_I > \frac{M^2}{M_{Pl}}; \tag{4.30}$$

3. Assume that the cosmological evolution of ϕ is dominated by classical physics results in

$$H_I < (gM^2)^{1/3}; \tag{4.31}$$

4. Inflation must last long enough for ϕ to scan the entire range, so we have

$$N \geq \frac{H_I^2}{g^2}; \tag{4.32}$$

5. In order for the barrier to form, we must have that the Hubble scale during inflation is lower than the new strong group scale

$$H_I < 4\pi f'; \tag{4.33}$$

6. The constraints over the radiative corrections, as explained above

$$m_L \leq \frac{4\pi v}{\sqrt{\log(M^2/m_L^2)}} \quad \text{and} \quad f' \leq v. \tag{4.34}$$

Using the list above we guarantee a stable separation between the weak scale and the cutoff of the theory in the non-QCD model.

Now, using Eqs. (4.29), (4.30) and (4.31) we obtain

$$M^2 < M_{Pl}\left(\frac{\Lambda^4}{F'}\right)^{1/3} \leq M_{Pl}\left(\frac{\Lambda^4}{M}\right)^{1/3},$$

where in the last step we used that $F' \geq M$. This implies

$$M < 10^9 \text{ GeV}\left(\frac{\Lambda}{10^3 \text{ GeV}}\right)^{4/7}. \tag{4.35}$$

With the estimates $| M_N | \sim \frac{y\tilde{y}}{2m_L}v^2$, $\mu' \sim 4\pi f'$ and $y, \tilde{y}$ of $\mathcal{O}(1)$, we will have that $\Lambda^4 \sim 4\pi f'^3 \frac{v^2}{m_L}$. Using the constraints of Eq. (4.34) we will have that $\Lambda < 10^3$ GeV. In this way, Eq. (4.35) tells us that we have an upper bound of 10^9 GeV for M in non-QCD model.

Notice that from Eq. (4.29) we obtain

$$gM^2F' \sim f'^2\mu'\frac{y\tilde{y}}{2m_L}v^2 \Rightarrow v \sim \sqrt{g\frac{M^2F'm_L}{f'^2\mu'y\tilde{y}}}, \tag{4.36}$$

so that technically natural values of g turn v technically natural.

Conclusions

Although the SM is an extremely successful theory, it cannot be the final theory of particle physics. After the discovery of the Higgs boson in 2012 [22][23], one of the problems of the SM - the so-called hierarchy problem - has become even more acute. The hierarchy problem stems from the observation that, whenever a fundamental scalar is present in a theory (like the Higgs boson in the SM), any heavy particle interacting with such scalar will generate a quadratic sensitivity of the scalar mass on its threshold. This undermines our idea of effective field theory, and introduces a huge tuning in the theory. Classical solutions to this problem are supersymmetry, models with a composite Higgs, technicolor or anthropics, which are either untestable (anthropics), experimentally excluded (technicolor in its simpler form) or cornered by experiments in tuned regions of parameter space (supersymmetry and composite Higgs models).

Recently, a new class of solutions have been proposed, involving the dynamical relaxation of the EW scale. The framework consists in making the Higgs mass parameter dynamical coupling the Higgs boson to an axion-like particle (the relaxion). During inflation, the Higgs mass parameter is scanned by the evolving relaxion field, until EWSB occurs and the evolution is stopped by a Higgs-vev-dependent barrier.

In this thesis we focused on the study of the relaxion frameworks, paying attention to the preliminary knowledge (chiral perturbation theory, strong CP problem and axion physics) needed to correctly write a relaxion theory. More specifically, we have first studied the theory of mesons and chiral Lagrangians in Chapter 2. We have then focused on the strong CP problem and one of its solutions, the axion, studying in detail its physics and its natural connections with the theory of mesons. Finally, at the end of Chapter 3 we have focused on the $U(1)_A$ problem and the η' meson.

All the techniques developed in Chapters 2 and 3 have been used in Chapter 4 to construct the relaxion Lagrangian in two cases: (i) when the relaxion is the QCD axion, a case in which we lose the solution to the strong CP problem, and (ii) a non-QCD model in which the Higgs-vev-dependent barriers that stop the relaxion evolution are generated by a new strongly interacting group. In the last case, we considered the so-called L+N model and write down the correct Lagrangian for the dark analog of the η' meson, which was the original computation of this work.

The work done in theis thesis can be extended in several directions: (i) considering non-QCD models other than the L+N possibility; (ii) writing a UV complete relaxion model; (iii) considering in detail what happens varying the scale of condensation of the new strongly interacting group; (iv) studying in detail the phenomenology of such models, along the lines of references [21],[24] and [20].

Appendix A

This Appendix will be devoted to a brief study of Inflation. We shall start by showing the classic Big Bang theory. The discussion is based on [25].

A.1 FRW metric and the Friedmann equations

If we assume the homogeneity and isotropy of the universe at large scales, we are led to the Friedmann-Robertson-Walker (FRW) metric for the universe [26]

$$ds^2 = dt^2 - a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right), \tag{A.1}$$

where $a(t)$ is the scale factor that characterizes the relative size of spacelike hypersurfaces Ω and is the only time dependent parameter at the spatial part of the metric and k is the curvature parameter which is +1 for positively curved Ω, 0 for flat Ω and -1 for negatively curved Ω.

The dynamics of the universe is given by the Einstein field equation

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}, \tag{A.2}$$

where $G_{\mu\nu}$ is the Einstein tensor, G is the Newton gravitational constant and $T_{\mu\nu}$ is the energy-momentum tensor of the universe. Now, solving the Einstein equation by assuming that matter behaves as a perfect fluid in the energy-momentum tensor and using the FRW metric we obtain the following differential equations for the scale factor a (that now characterize the dynamics of the universe) [26]:

$$H_I^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\rho \tag{A.3}$$

and

$$\dot{H}_I + H_I^2 = \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p), \tag{A.4}$$

where ρ is the energy density, p is the pressure and $H_I \equiv \frac{\dot{a}}{a}$ is the Hubble parameter which is very important since it sets the Hubble time, $t \sim H_I^{-1}$ which is the characteristic time-scale of the homogeneous universe, and sets the Hubble distance $d \sim H_I^{-1}$ which sets the size of the observable universe. Eqs. (A.3) and (A.4) are the Friedmann equations, and determine the evolution of $a(t)$ given the the relation between ρ and p.

One can define the critical density ρ_c such that Eq. (A.3) takes the form

$$H_I^2 = \frac{8\pi G}{3}\rho_c, \tag{A.5}$$

which is very useful because we can write $\rho = \Omega\rho_c$ where Ω is a parameter that depends on time and is equal to 1 when the universe is flat. We will have that $\Omega = 1$ for the case of the inflaton (see section A.2).

Now, combining the two Friedmann equations we can obtain the continuity equation [26]

$$\frac{d\rho}{dt} + 3H_I(\rho + p) = 0. \tag{A.6}$$

Defining the equation of state parameter

$$w \equiv \frac{p}{\rho}, \tag{A.7}$$

we obtain

$$\frac{d\log\rho}{d\log a} = -3(1+w),$$

which may be integrated to give

$$\rho \sim a^{-3(1+w)}, \tag{A.8}$$

which, together with Eq. (A.3) gives the time evolution of the scale factor

$$a(t) \sim t^{2/3(1+w)} \text{ for } w \neq -1 \text{ and } a(t) \sim e^{H_I t} \text{ for } w = -1. \tag{A.9}$$

These results can be used to characterize the dynamics of the universe in case of a flat universe dominated by non-relativistic matter ($w = 0$) which gives $a(t) \sim t^{2/3}$, radiation or relativistic matter ($w = 1/3$) which gives $a(t) \sim t^{1/2}$ and a cosmological constant ($w = -1$) which gives $a(t) \sim e^{H_I t}$.

A.2 Inflation

A.2.1 Definition of inflation

In the above section we described the standard Big Bang theory. This theory carries serious fine tuning problems in its initial conditions. Examples of such problems are the horizon problem, the flatness problem and the monopole problem [27]. These problems arise essentialy from the fact that, in the standard Big Bang Theory, the comoving Hubble radius, $(aH_I)^{-1}$, is strictly increasing [25]. The natural solution to this problem is to state that the comoving Hubble radius should shrink.

For this reason we define inflation as the regime where the universe behaves respecting the following equation

$$\frac{d}{dt}\left(\frac{1}{aH_I}\right) < 0. \tag{A.10}$$

Now, from the relation

$$\frac{d}{dt}(aH_I)^{-1} = \frac{-\ddot{a}}{(aH_I)^2},$$

we conclude that Eq. (A.10) is equivalent to

$$\frac{d^2a}{dt^2} > 0, \tag{A.11}$$

which means an accelerated expansion of the universe.

If we use Eq. (A.11) in Eq. (A.4) we must conclude that

$$p < -\frac{1}{3}\rho, \tag{A.12}$$

which means that negative pressure is the responsible for the accelerated expansion. As a summary, we write the definition of inflation as

$$\frac{d}{dt}\left(\frac{1}{aH_I}\right) < 0 \Leftrightarrow \ddot{a} > 0 \Leftrightarrow w < -\frac{1}{3}. \tag{A.13}$$

Even though these conditions seem unnatural, this can be done if we postulate a scalar field responsible for it, *i.e.*, the inflaton.

A.2.2 The inflaton

In General Relativity the gravity plus matter action is given by [26]

$$S = \int d^4x\sqrt{-g}\left(-\frac{R}{16\pi G} + \mathcal{L}_m\right), \tag{A.14}$$

where g is the determinant of the metric tensor, $\mathcal{L}_m$ is the lagrangian for any matter field and R is the Ricci scalar. The Euler-Lagrange equation from the action of Eq. (A.14) gives the Einstein field equation (A.2) if the stress-energy tensor is

$$T^{\mu\nu} = -2\frac{\partial \mathcal{L}_m}{\partial g_{\mu\nu}} - g^{\mu\nu}\mathcal{L}_m. \tag{A.15}$$

We will not specify the nature of the scalar field inflaton, ϕ, we will use it as an order parameter to parametrize the time-evolution of the inflationary energy density and set its Lagrangian as a minimally-coupled scalar field so that

$$\mathcal{L}_m = \mathcal{L}_\phi = \frac{1}{2}g^{\mu'\nu'}\partial_{\mu'}\phi\partial_{\nu'}\phi - V(\phi). \tag{A.16}$$

Using Eq. (A.16) in Eq. (A.15) and using the identity [27]

$$\frac{\partial g^{\mu'\nu'}}{\partial g_{\mu\nu}} = -g^{\nu\nu'}g^{\mu\mu'},$$

we obtain

$$T^{\mu}_{\nu} = g^{\mu\rho}\partial_\rho\phi\partial_\nu\phi - \delta^{\mu}_{\nu}\left(\frac{1}{2}\partial_\rho\phi\partial^\rho\phi - V(\phi)\right). \tag{A.17}$$

The density and pressure can be obtained for an isotropic and homogeneous universe from $\rho = T^0_0$ and $T^i_j = -P\delta^i_j$. For a homogeneous field, Eq.(A.16) becomes

$$\mathcal{L}_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi). \tag{A.18}$$

Using Eq. (A.18) in Eq. (A.17) we obtain

$$\begin{aligned} \rho_\phi &= \frac{1}{2}\dot{\phi}^2 + V(\phi) \\ P_\phi &= \frac{1}{2}\dot{\phi}^2 - V(\phi), \end{aligned} \tag{A.19}$$

which implies

$$w_\phi = \frac{\frac{1}{2}\dot{\phi}^2 - V}{\frac{1}{2}\dot{\phi}^2 + V}. \tag{A.20}$$

Eq. (A.20) shows us that in order to satisfy the conditions of Eq. (A.13) (creating negative pressure and accelerating expansion) all the inflaton needs is that the potential energy dominates V over the kinetic energy $\frac{1}{2}\dot{\phi}^2$, so that $w_\phi < -1/3$.

Now, by solving the Euler-Lagrange equation for the action of Eq. (A.14) with $\mathcal{L}_m = \mathcal{L}_\phi$ given by Eq. (A.18) we find [26] again the Friedmann equations plus the couple Klein-Gordon equation for the scalar field:

$$H_I^2 = \frac{1}{3M_{Pl}^2}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right), \qquad \dot{H} = \frac{1}{M_{Pl}}\left(-\frac{1}{2}\dot{\phi}^2\right) \tag{A.21}$$

and

$$\ddot{\phi} + 3H_I\dot{\phi} = -\frac{dV(\phi)}{d\phi}, \tag{A.22}$$

where M_{Pl} is the reduced Planck mass $M_{Pl} = m_P/\sqrt{8\pi} = 1\sqrt{8\pi G}$.

In Eq. (A.22) we have an acceleration term, $\ddot{\phi}$, a friction term proportional to H_I and a force given by $-\frac{dV(\phi)}{d\phi}$. If we imagine our field as a ball rolling down a potential hill with a friction, we can impose that this ball rolls slowly and its acceleration is small in comparison with the other terms in the equation. This makes Eq. (A.22) turn into

$$3H_I\dot{\phi} \approx -\frac{dV(\phi)}{d\phi}, \tag{A.23}$$

and the Friedmann equations to

$$H_I^2 \approx \frac{1}{3M_{Pl}^2}V(\phi) \qquad \text{and} \qquad H \approx \text{const.}. \tag{A.24}$$

The above approximations are called *slow-roll approximations.*

It is useful to define the slow-roll parameter

$$\epsilon_V(\phi) \equiv \frac{M_{Pl}^2}{2}\left(\frac{V_{,\phi}}{V}\right)^2, \tag{A.25}$$

where $_{,\phi}$ represents a derivative with respect to ϕ. Using Eqs. (A.23), (A.24) and (A.25) we have

$$\dot{\phi}^2 \approx \frac{V_{,\phi}^2}{9H_I^2} \approx M_{Pl}^2 \frac{V_{,\phi}^2}{3V} \approx \frac{2}{3}\epsilon_V V, \tag{A.26}$$

which, in first order in ϵ_V gives

$$P = \left(\frac{2}{3}\epsilon_V - 1\right)\rho. \tag{A.27}$$

Eq. (A.27) means that we can exchange our condition $w_\phi < -1/3$ by $\epsilon_V < 1$ so that inflation ends at $\epsilon_V = 1$.

A.2.3 The amount of inflation

The amount of inflation is given by the number of e-foldings N which is defined as

$$\frac{a(t_f)}{a(t_i)} = e^N, \tag{A.28}$$

where t_f is the time at the end of inflation and t_i is the time at the beginning of inflation.

By noticing that

$$d\log a = \frac{1}{a}\frac{da}{dt}dt = H_I dt,$$

we have

$$N = \int_{a(t_i)}^{a(t_f)} d\log a = \int_{t_i}^{t_f} H_I dt. \tag{A.29}$$

Using Eqs. (A.23) and (A.24) we find

$$H_I dt = H_I \frac{dt}{d\phi}d\phi = H_I \frac{d\phi}{\dot{\phi}} = -\frac{3H_I^2 d\phi}{V_{,\phi}} = -\frac{1}{M_{Pl}^2}\frac{V}{V_{,\phi}}d\phi,$$

so that we obtain

$$N = -\frac{1}{M_{Pl}^2}\int_{\phi_i}^{\phi_f} \frac{V}{V_{,\phi}}d\phi. \tag{A.30}$$

A.2.4 Quantum fluctuations during inflation

Besides solving the horizon, the flatness and the monopole problem, inflation can explain the primordial fluctuations that are the seeds for the large scale structure of the universe and also the anisotropies in the CMB.

These fluctuations can be calculated by expanding the action of the inflaton to second order in the fluctuations (on all length scales, *i.e.* with a spectrum of wave numbers k) in terms of the gauge-invariant curvature perturbation, $\mathcal{R}$, derive the equations of motion from it and show that this has the form of a harmonic oscillator, make various approximate solutions valid during slow-roll, promote the field $\mathcal{R}$ to a quantum operator and quantize it, define the vacuum state by matching the solutions to the Minkowski vacuum when the mode is deep inside the horizon (Hubble radius $k \gg aH_I$) and finally compute the power spectrum of the curvature fluctuations at horizon crossing ($k = aH_I$). This is done in [25] and in [27] (where it was done using spatially-flat gauge), giving

$$\mid w(k) \mid^2 = \frac{H_I^2}{2k^3}, \tag{A.31}$$

where w is the amplitude of the oscillation of the inflaton, $\delta\phi$, and k is the wave number.

The power spectrum of a field $\chi(\vec{x}, t)$ is defined so that

$$\langle \chi(\vec{k}, t) \chi(\vec{k}', t) \rangle = \frac{2\pi^2}{k^3} \mathcal{P}_\chi(k, t) \delta(\vec{k} + \vec{k}'), \tag{A.32}$$

where $\mid \vec{k} \mid = k$ and $\mathcal{P}_\chi$ is the power spectrum of $\chi(\vec{x}, t)$. With this definition one can show that the power spectrum of $\delta\phi$ is

$$\mathcal{P}_{\delta\phi}(k) \approx \left(\frac{H_I}{2\pi} \right)^2. \tag{A.33}$$

Appendix B

In this Appendix we will show more details on the computations of Chapter 1.

B.1 Fermion loop

Let us start with the computation of the amplitude in Eq. (1.3).

We begin by noticing that

$$\mathrm{Tr}\left[\frac{i}{\not{k}+\not{p}-M}\cdot\frac{i}{\not{p}-M}\right]=\mathrm{Tr}\left[\frac{(\not{k}+\not{p}+M)}{(k+p)^2-M^2}\cdot\frac{(\not{p}+M)}{p^2-M^2}\right]. \tag{B.1}$$

The trace of the spinor structures in the numerator is

$$\begin{aligned}&\mathrm{Tr}[(\not{k}+\not{p}+M)(\not{p}+M)]=\mathrm{Tr}[\not{k}\not{p}+\not{p}\not{p}+M\not{k}+2M\not{p}+M^2]\\&=(k_\mu p_\nu+p_\mu p_\nu)\,\mathrm{Tr}[\gamma^\mu\gamma^\nu]+4M^2=4(k_\mu p^\mu+p_\mu p^\nu+M^2),\end{aligned} \tag{B.2}$$

where we used in the second equation that $\mathrm{Tr}[1]=4$ and $\mathrm{Tr}[\gamma^\mu]=0$, and in the last equation we used $\mathrm{Tr}[\gamma^\mu\gamma^\nu]=4g^{\mu\nu}$.

Substituting Eq. (B.2) in Eq.(B.1) and using the Feynman parameterization we obtain

$$\begin{aligned}&\mathrm{Tr}\left[\frac{i}{\not{k}+\not{p}-M}\cdot\frac{i}{\not{p}-M}\right]=\int_0^1\frac{dx4(k\cdot p+p^2+M^2)}{[x((k+p)^2-M^2)+(1-x)(p^2-M^2)]^2}\\&=\int_0^1\frac{dx4(k\cdot p+p^2+M^2)}{[xk^2+2xk\cdot p+p^2-M^2]^2}=\int_0^1\frac{dx4(k\cdot(l-kx)+l^2-2xk\cdot l+x^2k^2+M^2)}{[xk^2-x^2k^2+l^2-M^2]^2}\end{aligned} \tag{B.3}$$

where we redefine $l=p+xk$ which implies $p^2=l^2-2xk\cdot l+x^2k^2$ in the last equation. Now using Eq. (B.3) in Eq. (1.3) and dropping the linear terms in l we get

$$i\mathcal{A}=\mu^{4-d}(-iy)^2 4\int_0^1 dx\int\frac{d^dl}{(2\pi)^d}\frac{l^2-xk^2+x^2k^2+M^2}{(l^2-a^2)^2}, \tag{B.4}$$

where we defined $a^2=M^2-x(1-x)k^2$. Now, making a Wick rotation with $l_0=il_4$ and $l_E^2=-l^2$, we obtain

$$
\begin{aligned}
i\mathcal{A} &= i\mu^{4-d}(-iy)^2 4\int_0^1 dx \int \Omega_d \int_0^\infty \frac{dl_E}{(2\pi)^d} \frac{l_E^{d-1}(-l_E^2 - xk^2 + x^2k^2 + M^2)}{(l_E^2+a^2)^2} \\
&= i\mu^{4-d}(-iy)^2 4\int_0^1 dx \int \Omega_d \left(\int_0^\infty \frac{dl_E}{(2\pi)^d} \frac{l_E^{d-1}(-xk^2 + x^2k^2 + M^2)}{(l_E^2+a^2)^2} - \int_0^\infty \frac{dl_E}{(2\pi)^d} \frac{l_E^{d+1}}{(l_E^2+a^2)^2} \right) \\
&= i\mu^{4-d}(-iy)^2 4\frac{1}{2}\int_0^1 dx \int \Omega_d \left(\int_0^\infty \frac{dl_E^2}{(2\pi)^d} \frac{(l_E^2)^{\frac{d}{2}-1}(-xk^2 + x^2k^2 + M^2)}{(l_E^2+a^2)^2} - \int_0^\infty \frac{dl_E^2}{(2\pi)^d} \frac{(l_E^2)^{\frac{d}{2}}}{(l_E^2+a^2)^2} \right)
\end{aligned}
\tag{B.5}
$$

where $\int \Omega_d$ is the d-dimensional solid angle and has the value $\int \Omega_d = \frac{2\pi^{d/2}}{\Gamma(\frac{d}{2})}$ and $\Gamma(n)$ is the Gamma function of n. Using this value of the solid angle and the identity [5]

$$
\int_0^\infty \frac{dt\ t^{m-1}}{(t+a^2)^n} = \frac{1}{(a^2)^{n-m}} \frac{\Gamma(m)\Gamma(n-m)}{\Gamma(n)},
$$

we get

$$
\begin{aligned}
i\mathcal{A} =& -\frac{i\mu^{4-d}4y^2}{(2\pi)^d} \frac{\pi^{d/2}}{\Gamma\left(\frac{d}{2}\right)} \frac{1}{2} \int_0^1 dx \Big(\frac{1}{(a^2)^{2-\frac{d}{2}}} \frac{\Gamma\left(\frac{d}{2}\right)\Gamma\left(2-\frac{d}{2}\right)}{\Gamma(2)} (-xk^2 + x^2k^2 + M^2) \\
& - \frac{1}{(a^2)^{1-\frac{d}{2}}} \frac{\Gamma\left(\frac{d}{2}+1\right)\Gamma\left(1-\frac{d}{2}\right)}{\Gamma(2)} \Big) \\
=& -\frac{i\mu^{4-d}4y^2}{(2\pi)^d} \frac{\pi^{d/2}}{\Gamma\left(\frac{d}{2}\right)} \frac{1}{2} \int_0^1 dx \Big(\frac{1}{(a^2)^{2-\frac{d}{2}}} \Gamma\left(2-\frac{d}{2}\right) (-xk^2 + x^2k^2 + M^2) \\
& - \frac{a^2}{(a^2)^{2-\frac{d}{2}}} \frac{d}{2(1-d/2)} \Gamma\left(2-\frac{d}{2}\right) \Big)
\end{aligned}
\tag{B.6}
$$

where in the last equation we used $\Gamma(n+1) = n\Gamma(n)$.

Now letting $d = 4-\epsilon$, where $\epsilon \to 0$ and using $\frac{1}{(a^2)^{\epsilon/2}} \approx 1 - \frac{\epsilon}{2}\log a^2 + \mathcal{O}(\epsilon)$ and $\Gamma\left(\frac{\epsilon}{2}\right) \approx \frac{2}{\epsilon} - \gamma + \mathcal{O}(\epsilon)$ we finally obtain

$$
i\mathcal{A} = -\frac{iy^2}{4\pi^2} \int_0^1 dx 3(-xk^2 + x^2k^2 + M^2) \left(\frac{2}{\epsilon} - \gamma + \log\left(\frac{\mu^2}{M^2 - x(1-x)k^2} \right) + \cdots \right). \tag{B.7}
$$

Let us now compute the RGE shown in Eq. (1.5). Let h_0 be the bare Higgs field, h the renormalized Higgs field, m_{h0}^2 the bare Higgs mass and m_h^2 the renormalized Higgs mass, such that $h_0 = Z_1^{1/2} h$ with $Z_1 = 1 + \delta Z_1$ and $m_{h0}^2 Z_1 = m_h^2 + \delta m_h^2$ where δZ_1 and δm_h^2 are the counterterms. Using the minimal subtraction (MS) scheme [12], we have that

$$
\delta Z_1 = -\frac{y^2}{24\pi^2}\frac{2}{\epsilon} \quad \text{and} \quad \delta m_h^2 = -\frac{3y^2M^2}{4\pi^2}\frac{2}{\epsilon}. \tag{B.8}
$$

Finally we obtain the counterterms for the radiative corrections from the fermions loops contributing to the self-energy of the Higgs boson. Now we can use it to find the renormalization group equation (RGE) for the running of the Higgs mass parameter.

To obtain the RGE we can start by noticing that the bare mass, by definition, doesn't depend on the free parameter μ, so we have

$$\mu\frac{dm_{h0}^2}{d\mu} = \mu\frac{dm_h^2}{d\mu} + \mu\frac{d\delta m_h^2}{d\mu} = 0,$$

which using Eq.(B.8) implies

$$\mu\frac{dm_h^2}{d\mu} = -\mu\frac{d\delta m_h^2}{d\mu} = \mu\frac{d}{d\mu}\left(\frac{3y^2M^2}{4\pi^2}\frac{2}{\epsilon}\right). \tag{B.9}$$

Now, notice that if we define the bare Yukawa coupling constant as y_0, it will have dimension $[y_0] = \frac{4-d}{2}$ in dimensional regularization, so if we want a dimensionless renormalized coupling y we must rescale it by

$$y = \frac{1}{Z_y}\mu^{\frac{d-4}{2}}y_0,$$

where μ is again the arbitrary scale of dimensional regularization and Z_y includes the counterterms of the Yukawa coupling. Using again that the bare parameter doesn't depend on the parameter μ and making $\epsilon = 4 - d$ we have

$$\mu\frac{dy_0}{d\mu} = \mu\frac{d}{d\mu}(Z_y\mu^{\epsilon/2}y) = \mu\left(\frac{dZ_y}{d\mu} + \frac{\epsilon}{2}Z_y\mu^{\frac{\epsilon}{2}-1} + Z_y\mu^{\frac{\epsilon}{2}}\frac{dy}{d\mu}\right) \Rightarrow \mu\frac{dy}{d\mu} = -\frac{\epsilon}{2}y, \tag{B.10}$$

where in the last step we used that $Z_y = 1$ at leading order. Similarly we can define the bare mass of the fermion M_0 which has dimension 1 and thus the renormalized fermion mass must be $M = \frac{1}{Z_M}M_0$ and we have that

$$\mu\frac{dM_0}{d\mu} = \mu\frac{dM}{d\mu} + \mu\frac{dZ_M}{d\mu} = 0 \Rightarrow \mu\frac{dM}{d\mu} = 0, \tag{B.11}$$

where we used that $Z_M = 1$ at leading order.

From Eq. (B.9) we have

$$\mu\frac{dm_h^2}{d\mu} = \mu\left(\frac{2yM^2}{8\pi^2}\frac{2}{\epsilon}\frac{dy}{d\mu} + y^2 2M\frac{dM}{d\mu}\frac{2}{\epsilon}\right), \tag{B.12}$$

and using Eqs. (B.10) and (B.11) in (B.12) we finally obtain

$$\mu\frac{dm_h^2}{d\mu} = -\frac{3y^2M^2}{2\pi^2}, \tag{B.13}$$

which is the RGE used in Chapter 1.

B.2 Fine tuning

We now elaborate on the definition of the fine-tuning measure given in Eq. (1.6).

If we call $m_h'^2(\Lambda_{in}) = (1+\epsilon)m_h^2(\Lambda_{in})$ and $m_h'^2(\Lambda_{SM}) = (1+\Delta\epsilon)m_h^2(\Lambda_{SM})$ we have

$$m_h'^2(\Lambda_{in}) - m_h^2(\Lambda_{in}) = \delta m_h^2(\Lambda_{in}) = \epsilon m_h^2(\Lambda_{in}), \tag{B.14}$$

while

$$m_h'^2(\Lambda_{SM}) - m_h^2(\Lambda_{SM}) = \delta m_h^2(\Lambda_{SM}) = \Delta\epsilon m_h^2(\Lambda_{SM}). \tag{B.15}$$

From Eqs. (B.14) and (B.15) we get

$$\Delta\epsilon = \frac{\delta m_h^2(\Lambda_{SM})}{m_h^2(\Lambda_{SM})} \text{ and } \epsilon = \frac{\delta m_h^2(\Lambda_{in})}{m_h^2(\Lambda_{in})},$$

and mixing these two equations together and eliminating the ϵ we obtain

$$\Delta = \frac{\delta m_h^2(\Lambda_{SM})/m_h^2(\Lambda_{SM})}{\delta m_h^2(\Lambda_{in})/m_h^2(\Lambda_{in})}$$

which, when we take the infinitesimal limit, becomes

$$\Delta = \frac{d \log m_h^2(\Lambda_{SM})}{d \log m_h^2(\Lambda_{in})}, \tag{B.16}$$

which is the expression for Δ used in Chapter 1.

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