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Instituto de Física

Desenvolvimento de Método de Análise de Materiais
Equivalentes ao Tecido Humano por Simulação
Monte Carlo

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Orientador: Prof. Dr. Paulo Roberto Costa

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Development of Monte Carlo Simulations Analysis Method for Tissue Equivalent Materials

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To Luma, for she is the reason I've done it.
To my parents, for they are the reason I could do it.

One Art

BY ELIZABETH BISHOP

The art of losing isn't hard to master;
so many things seem filled with the intent
to be lost that their loss is no disaster.

Lose something every day. Accept the fluster
of lost door keys, the hour badly spent.
The art of losing isn't hard to master.

Then practice losing farther, losing faster:
places, and names, and where it was you meant
to travel. None of these will bring disaster.

I lost my mother's watch. And look! my last, or
next-to-last, of three loved houses went.
The art of losing isn't hard to master.

I lost two cities, lovely ones. And, vaster,
some realms I owned, two rivers, a continent.
I miss them, but it wasn't a disaster.

-Even losing you (the joking voice, a gesture
I love) I shan't have lied. It's evident
the art of losing's not too hard to master
though it may look like (*Write it!*) like disaster.

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“A força da alienação vem dessa fragilidade dos indivíduos que apenas conseguem identificar o que os separa, e não o que os une. ”

- Milton Santos

RESUMO

Materiais radiologicamente equivalentes ao tecido humano (Tissue Equivalent Material - TEM) têm a finalidade de evitar exposição injustificável à radiação de pacientes e são amplamente utilizados no controle de qualidade de equipamentos de diagnóstico por imagem. Esses materiais devem ser caracterizados para que se possa confiar em sua semelhança, em termos de suas propriedades de interação com a matéria, aos tecidos que substituem. Uma das maneiras de caracterizar os materiais é verificando se o espectro transmitido através deles se assemelha ao transmitido através do tecido que substituem. O método Monte Carlo (MC) é uma ferramenta útil no processo de caracterização dos TEM pois pode evitar o processo custoso de realizar experimentos de transmissão de raios-X. Esse trabalho investigou a aplicabilidade do método MC à caracterização de TEM de tecido mamário (Breast Tissue Equivalent Material - bTEM) utilizados no controle de qualidade de equipamentos de mamografia. Para verificar a aplicabilidade do método MC, uma série de resultados de simulações foi comparada a resultados experimentais. Espectros de raios X transmitidos foram comparados diretamente através da média de seus resíduos reduzidos (Mean Weighted Squared Residuals - MWSR). Comparações foram feitas através de grandezas derivadas dos espectros. Essas grandezas foram: as camadas semi-redutoras (primeira e segunda), a energia média e a energia efetiva. Foi realizada uma discussão acerca da eficiência de cada uma dessas comparações, através da estimativa do poder de cada teste de hipótese. Os experimentos de transmissão de radiação foram realizados em duas instalações, no Laboratório de Dosimetria das Radiações e Física Medida da Universidade de São Paulo, onde foi utilizado um tubo de raios X com anodo de tungstênio adaptado para qualidades de feixe utilizadas em aplicações mamográficas, e no Centro de Desenvolvimento de Tecnologia Nuclear da Comissão Nacional de Energia Nuclear, equipado com um mamógrafo clínico com anodos de tungstênio e molibdênio. Diversas condições experimentais foram variadas para assegurar a robustez das conclusões, tais como as combinações anodo/filtro, os materiais constituintes dos bTEM, suas glandularidades, espessuras e as tensões de pico. Os espectros sem nenhuma atenuação (0 mm) também foram medidos e utilizados nas comparações. Os espectros foram medidos com um detector comercial de CdTe. Dosímetros termoluminescentes foram utilizados para estimar a dose depositada em diversas regiões do bTEM, e esses resultados foram também comparados às simulações. Além da estimativa do nível de exatidão alcançado pelo código de MC nas referidas condições, também se concluiu que o teste de hipótese do MWSR teve o maior poder estatístico, de 0,996. O MWSR foi o teste que demonstrou a compatibilidade dos espectros medidos o maior número de vezes. Esse teste aceitou 48% dos pares de espectros contra 40% de aceitação do teste da primeira camada semi-redutora, que foi o segundo teste com maior aceitação. A comparação de dose medida via TLD mostrou mais do que 90% de compatibilidade entre dados experimentais e simulados.

Palavras-chave: Materiais tecido equivalentes, espectrometria de raios X, mamografia, método Monte Carlo, transporte de radiação, dosimetria, PENELOPE.

ABSTRACT

Human radiologically tissue equivalent materials (TEM) have the purpose of avoiding unjustifiable irradiation of patients; they are largely used in the quality control of image diagnostic equipment. These materials must be characterized so that their similarity to the tissues they simulate can be relied upon, regarding their properties of interaction with radiation. One way of characterizing the materials is by checking the resemblance between their transmitted spectrum to the one of tissue they simulate. The Monte Carlo (MC) method is a useful tool in the TEM characterization process, since it may avoid the realization of costly experiments of transmitted X-ray spectrometry. MC may even dismiss preliminary experiments. This work investigated the applicability of the MC method to the characterization of breast tissue TEM (bTEM) used in the quality control of mammography equipment. To evaluate the applicability of the MC method, a series of simulation results was compared to experimental data. Transmitted spectra were directly compared through their mean weighted squared residues (MWSR), and by the comparisons of spectra derived quantities, as it is commonly done in the literature. These quantities were: the half value layers (first and second), the mean energy and the effective energy. A discussion about the efficiency of each one of these comparisons was made by estimating the statistical power of each hypothesis test. The radiation transmission experiments were carried out in two facilities: at the Laboratory of Radiation Dosimetry and Medical Physics of the University of São Paulo, where a tungsten anode X-ray tube adapted to mammographic applications was used. The rest of the experiments was done at the Nuclear Technology Development Center of the National Commission of Nuclear Energy, equipped with a clinical mammographic equipment with anodes composed by tungsten and molybdenum. Several experimental conditions were varied to ensure the robustness of the conclusions, such as the anode/filter combination, the bTEM constituent materials, their glandularities, thicknesses and peak voltages. Spectra with no attenuation (0 mm) were also measured and used in the comparisons. The spectral measurements were done with a commercial CdTe detector. Thermo-luminescent dosimeters were used to estimate the dose deposited at several regions inside the bTEM, and these results were also compared to simulations. In addition to estimating the level of accuracy achieved by the MC code in the mentioned conditions, it was also concluded that the highest statistical power was scored by the MWSR and it was of 0.996. The MWSR was also the test which attested this compatibility of the measured spectra the most. It accepted 48% of the spectra pair against 40% acceptance of the first half value layer test, which was the second test with most acceptance. The TLD dose comparison showed over 90% compatibility between experimental and simulated data.

Key words: Equivalent tissue materials, X-ray spectrometry, mammography, Monte Carlo method, radiation transport, dosimetry, PENELOPE.

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LIST OF ABBREVIATIONS

bTEM – Breast Tissue-equivalent Material
CDF - Cumulative Distribution Function
CDTN - Centro de Desenvolvimento de Tecnologia Nuclear
CI – Confidence Interval
CNEN – Comissão Nacional de Energia Nuclear
CPE – Charged Particle Equilibrium
CS – Cross-Section
ED - Energy Deposition
EE - Effective Energy
GDRFM - Grupo de Dosimetria das Radiações e Física Médica
HVL – Half-Value Layer
INCA - Instituto Nacional de Câncer
IUPAC - International Union of Pure and Applied Chemistry
MC - Monte Carlo
MCA – Multichannel Analyzer
MCMC - Monte Carlo Markov Chain
ME - Mean Energy
MWSR - Mean Weighted Squared Residuals
NA - Nuclear Associates
NEA - Nuclear Energy Agency
NIST - National Institute of Standards and Technology
PDF - Probability Density Function
PHS - Pulse Height Spectrum
PUR - Pile-up Rejection
STP - Standard Temperature and Pressure
TEM – Tissue-equivalent Material
TLD - Thermoluminescent Dosimetry
WHO - World Health Organization

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1. INTRODUCTION

1.1. Motivation

One of the research programs of the Radiation Dosimetry and Medical Physics Group of the Physics Institute of the University of Sao Paulo (GDRFM) aims to improve the knowledge of radiological tissue-equivalent material manufacture. The formulation of the material's composition and the guarantee of the manufactured composition's accuracy are important steps to master the tissue equivalent material (TEM) manufacture. Another key step is to ensure that the radiation transport characteristics are as similar as possible to those of human tissue.

A branch of this work consists of the evaluation of breast tissue-equivalent materials (bTEM). The determination of the physical properties of this type of material is particularly important because many mammographic examinations are done every day all over the world. Nowadays mammography is recommended for breast cancer screening by national and international health organizations all over the world, one of them is the World Health Organization (WHO).

The WHO defines three women age groups, 40-49, 50-69 and 70-75 years old (Chestnov et al. 2014). For the 40-49 and 70-75 groups, screening is recommended only when well-resourced health-care system is available, and if there are scientific studies being carried out, which would provide extreme risk control for those women. For the 50-69 age group, mammography for breast cancer screening is recommended once every two years. The screening procedure aims to increase the rates of early breast cancer detection. Many studies showed it increases patients diagnosed with breast cancer survival rate (DeSantis et al. 2014; Kerlikowske et al. 1995; Tabár et al. 2014).

In Brazil, breast cancer is responsible for 30% of female cancer incidence. In 2016, 57,960 new cases were documented (Corrêa et al. 2016). Therefore, screening exams in Brazil are recommended by the Ministry of Health for women between 50 and 69 years old. The Brazilian recommendations

are the same as those adopted by WHO. The number of performed mammograms in Brazil has increased from 1,869,285 in 2002 to 4,713,530 in 2014 (da Silva et al. 2013).

This culminates in a public health necessity of many medical facilities equipped with mammography equipment. There are 5,509 registered mammography equipment installed in Brazil, 2,671 of them form part of the Unified Public Health-system (Secretaria de Atenção à Saúde 2016).

X-ray equipment must be submitted to periodic tests, which are supposed to guarantee the quality of the acquired images and to control the dose delivered to the patient. Some physical parameters are used to evaluate image quality, e.g. contrast, spatial resolution and noise. Organ dose is also an important parameter to be periodically accessed. According to Brazilian requirements, protocols are established by the nº 1016/2006 regulation (Agência Nacional de Vigilância Sanitária 2006). Besides that, the nº 453/98 regulation determines that all individual's exposure to ionizing radiation must be justified, and the patients must be submitted to the minimal dose required to perform diagnostically useful images.

Mammography equipment are no exception and must be calibrated to ensure that the guidelines on patient exposure to ionizing radiation are being observed (AAPM 1990; Lopez et al. 2006; M. Oresegun and Rehani 2005). Specific calibration protocols used to perform the quality assurance of mammography equipment demand the use of radiological tissue-equivalent materials.

Tissue-equivalent materials are necessary for a vast number of purposes regarding radiation interaction (Argo et al. 2004; Boone et al. 2000). Equipment calibrations, quality control, the design of therapeutic or diagnostic rooms, are some of the tasks that require these materials.

Empirical approaches and clinical quality control procedures, e.g. calibration of mammography equipment's automatic exposure control, suggest the use of unsophisticated plastic bTEM for simple image quality evaluation. These bTEM are normally composed by poly-methyl-methacrylate (PMMA) (AAPM 1990; Lopez et al. 2006; Pernicka and McLean 2007; Sociedad Española de Física Médica 2011). However, more realistic compositions are required for accurate dosimetry purposes (Carton

et al. 2011; White et al. 1977). In these cases, the bTEM compositions should be as similar as possible to the female human breast or, at least, to present similar radiation transmission properties as those of the real tissue.

Different methodologies for material characterization in terms of its response under the incidence of a radiation beam can be found in the literature (Bindu et al. 2004; Byng et al. 1998; Cubukcu and Yücel 2016; Geraldelli et al. 2013). For dosimetry purposes, the attenuation of specific radiation beams is the main aspect to be observed when verifying if the material is an adequate TEM. An almost complete procedure to perform the characterization of these bTEM materials is detecting the impinging and transmitted X-ray energy spectrum through a given thickness of the material (Byng et al. 1998; Geraldelli et al. 2013).

The compatibility between the same incident spectrum transmitted through a given bTEM, and through the real human breast, indicate that the bTEM successfully mimics the human breast radiation transport, i.e. transmitted spectra comparison leads to validation of the bTEM. The validation can also be done against a previously validated bTEM, as shown in Fig. 1. This is advantageous, for it does not require irradiation of real tissue.

Fig. 1. Scheme of TEM characterization through X-ray spectrometry.

MC method has been used in Medical Physics for over 50 years, the first MC related paper published in one of the most important medical physics journals, namely Physics in Medicine and Biology, dates from 1967 and was by Bentley et al. (Bentley et al. 1967). In the Physics in Medicine and Biology and Medical Physics journals, there has been a doubling of the number of MC papers every five years from 1967 to 2000 (Rogers 2006). The number of papers after 2000 still raises every year, but the exponential growth gives place to a smoother growing pattern (Fig. 2).

MC method has wide applications in Medical Physics, ranging from commercial treatment planning systems to evaluation of spectrometers' efficiency curve. As already described, TEM characterization can be done by detecting the impinging and transmitted X-ray spectrum through a given thickness of the material. However, awaiting the manufacture of a bTEM to perform the characterization experiments is time consuming and costly. Therefore, determining the transmitted spectra before the bTEM's manufacture is advantageous. As an X-ray spectrum can be simulated through the MC method, the MC simulations can be used to access radiation transport experiments. Another advantage on simulating is that it usually demands less physical resources and are highly reproducible.

Fig. 2. Number of papers published in Physics in Medicine and Biology (PMB) or Medical Physics with the term 'Monte Carlo' in the abstract or title (after Nahum (Nahum and Rogers 1988) and Rogers (Rogers 2006)).

On the other hand, MC simulations use atomic characteristics of material's, its composition, and theoretical models of physical phenomena to simulate the radiation interaction with the material. They are influenced by several approximations, which can lead to inaccuracies. The approximations are especially relevant to low energies, such as mammography'. This motivates the verification of its capacity to represent the actual radiation transmission process, i.e. experiments must be done to assure that simulation and experimental results are similar considering the inherent uncertainties in each method.

Provided that the simulation results have at least once been validated, the simulation can be relied on to predict the experimental results. Validations are only valid for a certain energy range, and for the specific outcomes tested, e.g. radiation spectra, attenuation, energy imparted, etc. A change of these parameters must be followed by new validations.

1.2. Objectives

The purpose of this work was to evaluate mammographic X-ray spectra transmission through various commercially available breast tissue-equivalent material by means of MC simulation and experimental measurements. This study quantitatively compared spectra obtained through experimental approaches and MC simulations.

To bring robustness to the validation, the comparisons were performed with a vast variety of experimental parameters, such as the beam's energies, different tissue-equivalent material composition and manufacturers, different attenuation thicknesses, and a different set of X-ray tubes and beam filtration. All these options are commonly used in dosimetry and quality control practice in mammography. Specific objectives were:

- The study of experimental methods for characterization of tissue-equivalent materials, mainly those that can be used in mammography dosimetry techniques.
- To develop computational geometries representative of experiments that evaluate attenuation and scattering properties of breast-equivalent materials.

- To determine the transmission and scattering properties of breast-equivalent materials applying computational simulations using the MC method, more specifically using the PENELOPE code.
- To conduct simulations equivalent to the experiments, to characterize breast-equivalent materials.
- To validate the adopted MC code regarding radiation interaction with breast tissue-equivalent material in the energy range used in mammography.
- To improve the accuracy of uncertainty estimates, determining the statistical validity limit for compatibility between measured and simulated results.
- To estimate the dose deposited in TLDs allocated inside a breast tissue-equivalent block using the PENELOPE MC code and to compare the results with experimental dose estimations.

1.3. A brief history of mammography and the state of the art in bTEM development

The discovery of X-rays, in the late 1890s, represented an advance in medical diagnoses, but it was only in 1925 that the first radiologic protection rules appeared (Silva 2002). Since then, a growth in the use of radiographic technics for supporting clinical diagnose has been observed, because of the permanent technological advances in the field of imageology. The technological development resulted in an increase of clinical examination frequency, and in the diversification of procedures. Hence, this implied in the need of special attention to the doses received by patients and to the quality of the images.

Mammography is a specific branch of radiation imaging, in which anthropomorphic and non-anthropomorphic breast TEM are widely used for dosimetry and image quality applications (Dance and Sechopoulos 2016; López-Pineda et al. 2014). Despite the great necessity of bTEM for mammography applications, their formulation is challenging, mainly due to the low energy range used in mammography imaging (10-39 keV). In low energies, the influence of composition on linear attenuation coefficients is significantly large. Another complication factor is the diverse composition

of breast tissue among women, since the adipose to glandular tissue proportion (breast glandularity) and the breast density vary greatly (Ng and Lau 2015). This culminates in the necessity of different bTEM materials for each glandularity (Argo et al. 2004).

A thorough review of the history of mammography was done by Gold et al. (Gold et al. 1990). The first study regarding the use of radiation imaging techniques to examine breast tissue is dated from the year 1913 (Salomon 1913). Albert Salomon used radiography techniques and mastectomy tissue to investigate cancer metastasis to the axillary lymph nodes. But it was only in 1949 that the concept of screening, i.e. the periodic radiological examination of breast tissue of non-symptomatic women, was introduced. This approach, that aims to detect breast cancer early manifestations, was firstly conceived by Raul Leborgne. Leborgne identified that the most common manifestation of breast cancer, which is the formation of microcalcifications, was present in 30% of cancerous breast tissues. This percentage was found to be greater in Leborgne's further works from the years of 1951 and 1953.

Breast cancer screening through mammography was firstly done in a systematic way from 1963 to 1966 in New York City, by Louis Venet and Sam Shapiro. They were able to provide regular mammograms to a significant number of women using the city's health insurance system (Gold et al. 1990). Their study proved that the screening reduces the mortality rates of women when compared to a control group, this was the decisive argument which demonstrated that the periodical radiation exposure was justified.

One of the first works in the pursuit of TEM is from the year 1906 (Kienböck 1906). Since then these materials have been developed and improved for use in radiologic diagnosis and therapy (Carton et al. 2011; Hermann et al. 1986; Hermann et al. 1985; Kiarashi et al. 2013; White et al. 1977). There are several commercially available TEM (phantoms) which are applied to all variety of ionizing and non-ionizing imaging techniques. These phantoms are usually formulated based on polymers or

epoxy resins which allow mimicking the attenuation characteristics of a large variety of human tissues (DeWerd and Kissick 2014).

Specific characteristics of a given TEM are necessary for each application, e.g. accepted energy range, the contrast of embedded structures, and its geometry. Each application may raise one or more necessities. Novel bTEM were recently developed with the capability of reproducing different breast glandularities (Argo et al. 2004), and breast anatomy (Carton et al. 2011; Kiarashi et al. 2013). These phantoms are regarded as the state of the art in bTEM development. Tomal (Tomal 2014) presents a thorough description of breast phantoms for dosimetry applications.

The technological advancements regarding phantom development culminate in elevated costs of radiological breast tissue-equivalent materials. According to prices estimates from March 2017 in the Brazilian market, the simplest mammographic phantom costs approximately 1,300.00 BRL, and the price rises with an increase in bTEM complexity, up to 17,300.00 BRL for a set dedicated to the investigation in breast dosimetry. The elevated costs are partially due to importation taxes, among other factors, what hinders the access of many mammography performing facilities to them. This impairs the proper maintenance and quality assurance of some mammography equipment.

2. THEORY

2.1. Photon interaction with matter

There are many known ways by which ionizing radiation interacts with matter. The interaction depends on the type of radiation that impinges upon the matter, on the energy of the radiation, and on the properties of the irradiated media. Some of them are coherent scattering, photoelectric effect, Compton effect, pair and triplet production. Regarding mammographic energies, the most relevant X-ray interactions for dosimetry purposes are the Compton effect, photoelectric effect, and coherent scattering. These interactions, except from coherent scattering, can set charged particles in motion, which will be responsible for the energy deposited in the matter. Each one of these interactions probabilities depends on the atomic number Z of the target atom and of the impinging photon's energy $h\nu$. Fig. 3 shows a diagram of the interaction dominance relative to the relevant quantities, atomic number (Z) and photon energy ($h\nu$), with highlight to the mammography energy range and atomic numbers. The coherent scattering is not shown in the plot because of the vast energy range considered. The coherent scattering is more important for low energies, which is the case in mammography.

Fig. 3. Relative importance of the three types of X-ray interactions: Photoelectric and Compton effects and Pair production. The curves show the values of Z and $h\nu$ for which two types of effects are equal. Adapted from Attix (Attix 2004)

The occurrence probability (cross-section) of the radiation interaction processes for a given material is energy-dependent. Due to the energy range and the materials studied in this work, only the photoelectric effect, Compton effect, and coherent scattering are relevant. These will be therefore explained in the following subsections, and are schematized in Fig. 4.

Fig. 4. Classic Bohr's model of the atom with three electronic shells shown, two inner and one outer shell, 4 different radiation interaction events are shown; 1 is the photoelectric effect exemplifying the ionization of an inner-shell electron, 2 is the Compton effect with an ejected electron and a photon emission; number 3 is emission characteristic of atomic relaxation, which results in Auger electron and fluorescence photon emission; 4 shows a *Bremsstrahlung* event, with acceleration of an electron and radiation emission caused by it; 5 shows a coherent scattering event occasioned by the atom's potential influence.

2.1.1. Cross-section

Collision events in Physics are quantified by cross-sections (CS). These quantities are defined as the occurrence probability of particle interactions, such as scattering and absorption phenomena. If there are N particles impinging on a target and $N' < N$ incident particles are scattered by matter, the total cross-section (σ) is proportional to

$$\sigma \approx \frac{N'}{N} \quad (1)$$

thus σ represents the probability that an impinging particle will undergo an interaction process.

Aside from the total cross-section concept, the differential cross-section is also an important quantity in the physical study of collisional processes. A particle that undergoes a scattering event can scatter in various directions, as seen in Fig. 5 (Bransden and Joachain 2003), and with varied energies.

Fig. 5. The geometry of scattering, the detector D subtends the element of solid angle $d\Omega$ at the scattering center O. The polar of OD are (θ, ϕ) . The incident beam is directed along the Z axis so that θ and ϕ are the scattering angles. Extracted from Bransden and Joachain (Bransden and Joachain 2003).

The differential cross-section is the quantity that expresses the fraction of the scattered particles (dN') per unit of scattering solid angle ($d\Omega$) and per energy unit (dE) (Bransden and Joachain 2003):

$$\frac{d\sigma}{d\Omega dE} = \frac{dN'}{N} \frac{1}{d\Omega dE} \quad (2)$$

i.e., in the limit of $d\Omega$ and dE being infinitesimal, it is the fraction of all scattered particles which have a determined energy and are travelling away from the scattering center towards the same direction.

The differential cross-section can be experimentally determined by adopting rigorously controlled experimental setup, and a well-characterized detector of known detection area. In such experiments, the detected particles can be related to the incidents. The experiment can be done for a variety of solid angles and energies, which results in cross-section curves.

Cross-sections can still be evaluated through Theoretical Physics combining quantum mechanics and electrodynamics. This approach is used to predict the behavior of interacting particles. Sophisticated elements such as effective potentials, perturbation theories and, more elegantly, quantum fluctuations of the fields are employed in such calculations (Bransden and Joachain 2003).

The interaction phenomena of which the cross sections are usually calculated and measured, and which have a particular interest in the framework of the present work, are described in sections 2.1.2 to 2.1.4.

2.1.2. Photoelectric Effect

The photoelectric effect was explained by Albert Einstein in 1905. It was announced in one of his *annus mirabilis'* papers (ter Haar 1967), and he became a Nobel Prize laureate for this theory.

It consists of the interaction of a photon with a strongly bound electron of the matter. The ionization of the atom occurs in the process, that is, an electron bound to the nucleus's Coulomb potential is emitted. This electron gets free from the atoms potential influence, and the incident photon is completely absorbed. This means that the photon energy is completely transmitted to the atom. The energy is then divided into three parts: the energy required to eject an electron, the ejected electron's kinetic energy and a small part go to the atom to preserve momentum. This allows the electron to get pass the potential barrier (Okuno and Yoshimura 2010).

The most accurate differential cross-section for this effect was estimated following Scofield's formalism (Scofield 1973) and was thoroughly tabulated by Cullen et al. and by Sabbatucci and Salvat (Cullen et al. 1997; Sabbatucci and Salvat 2016). The historical development of the photoelectric effect cross-section estimation was published by Hubbell (Hubbell 2006).

2.1.3. Compton Effect

In 1923, Arthur Compton used the recently developed theory of special relativity and also the new concept of quantization of the electromagnetic field (light quanta) to develop his work (Compton 1923). The introduced corpuscular characteristic of electromagnetic radiation, which interacts with another particle, the electron, induced the development of an inelastic shock theory. Compton received a Nobel Prize in 1927 for the theoretical deduction and for the experimental confirmation of the predicted results.

In Compton effect, there is an electron ejection and the emergence of a scattered photon in a characteristic angle θ with respect to the original incident photon's direction. The energy of the scattered photon and of the incident photon are different (assuming $\theta \neq 0$). The energy of the scattered photon is lower because some of the original energy was transmitted to the recoil electron (Okuno and Yoshimura 2010).

Assuming that the incident photons are unpolarized, Compton effect has azimuthal symmetry, as frequently observed in scattering phenomena, i.e. there is no dependence with respect to azimuthal angle ϕ in the differential cross-section.

It is common that photons undergoing Compton effect interact with weakly bound electrons. Considering that the electron is at rest and free of any potential bound and using the energy conservation principle with relativistic considerations, the energy of the emerging photon is given by equation (3) (Attix 2004)

$$h\nu' = \frac{h\nu}{1 + \alpha[1 - \cos(\theta)]} \quad (3)$$

where

$$\alpha = \frac{h\nu}{m_e c^2} \quad (4)$$

and $h\nu'$ and $h\nu$ are the energy of the photon emitted after the interaction, and the energy of the impinging photon, respectively. In equation (4), m_e is the electron's rest mass, c is light's velocity in vacuum. In equation (3), θ is the angle between the directions of the impinging and emitted photons.

The process is illustrated in Fig. 6

Fig. 6. Illustration of the Compton effect. $h\nu'$ and $h\nu$ are the energy of the photon emitted after the interaction, and the energy of the impinging photon, respectively and θ is the angle between the directions of the impinging and emitted photons.

Equation (4) uniquely correlates a scattering angle with a scattering energy and determines a maximal and a minimal energy for the emitted photon

$$h\nu'_{max} = h\nu \quad (5)$$

$$h\nu'_{min} = \frac{h\nu}{1 + 2\alpha} \quad (6)$$

Which correspond respectively to $\theta = 0$ and $\theta = \pi$.

It is evident that as the incident photons interact with matter, they interact with electrons that are subject to the nuclei Coulomb potentials, and moreover have momentum. It means that the

considered approximations of a free and in rest electron generate inaccuracies in the theoretical estimation of the Compton's effect cross-section. Nevertheless, the differential cross-section which arises from these approximations (σ_{KN}) is given by the Klein-Nishina equation (Klein and Nishina 1929)

$$\frac{d\sigma_{KN}}{d\Omega} = \frac{r_e^2}{2} \left(\frac{h\nu'}{h\nu} \right)^2 \left(\frac{h\nu}{h\nu'} + \frac{h\nu'}{h\nu} - \sin^2 \theta \right) \quad (7)$$

where r_e is the classic electron's radius given by

$$r_e \approx \frac{e^2}{m_e c^2} \quad (8)$$

where e is the electron's electrical charge.

Klein-Nishina cross-section is widely used because, besides being in general a good approximation, more precise calculations are extremely difficult to carry out. One of the approaches to consider electrons bound state and its momentum is the impulse approximation theory (Eisenberger and Reed 1974; Ribberfors 1975). Although it dates from the 1970's, novel contributions are being made, e.g. (Stutz 2014), and it may be the next gold standard for calculation of Compton effect cross-section.

2.1.4. Coherent Scattering

A photon impinging on the matter can interact by coherent or Rayleigh scattering. Coherent scattering occurs when a photon interacts with the atom as a whole. In other words, the interaction cannot be claimed to happen in any specific atomic site. It is, therefore, a global effect.

When a photon is coherently scattered by a single free electron, the interaction is called Thomson scattering, which has the following classically derived cross section (Bransden and Joachain 2003).

$$\frac{d\sigma_T}{d\Omega} = r_e^2 \frac{1 + \cos^2 \theta}{2} \quad (9)$$

In coherent scattering, the electronic density $\rho(r)$ plays an important role, because of the interaction's global nature, where all electrons influence the scattering. So, an atomic form factor function $F(q, Z)$ is added to the cross section, to account for the electron density

$$\frac{d\sigma_C}{d\Omega} = \frac{d\sigma_T}{d\Omega} [F(q, Z)]^2 \quad (10)$$

The form factor is obtained by the Fourier transform of the electronic density

$$F(q, Z) = \int_0^\infty \rho(r) \frac{\sin\left(\frac{qr}{\hbar}\right)}{\frac{qr}{\hbar}} 4\pi r^2 dr \quad (11)$$

where q is the module of the interaction's momentum transfer vector

$$q = 2 \frac{E}{c} \sin \frac{\theta}{2} \quad (12)$$

Coherent or Rayleigh scattering is less relevant to dose deposition than Compton and photoelectric effects because there is no ionization related to them. The atom as a whole is responsible for radiation scattering. The atom gains just as much energy as it is required for momentum conservation. Because of its great mass, compared to photon's energy, the energy gained by the atom is negligible (Attix 2004).

2.2. Monte Carlo method

2.2.1. General Functioning of radiation transport MC codes

The Monte Carlo (MC) method consists of using randomly sampled numbers to simulate physical processes, solve mathematical problems, among other applications (Sobol 1974). It was firstly introduced by Metropolis and Ulam in 1949 (Metropolis and Ulam 1949). The method became popular at the end of the 20th century due to advancements in computerized data processing and facilitated access to computers.

Many MC code systems are used in Medical Physics for radiation transport simulation purposes. Some examples are PENELOPE (Salvat et al. 2015), EGS4 (Bielajew et al. 1994), GEANT4 (Agostinelli

et al. 2003) and MCNP (Booth et al. 1986). A detailed description of the role of MC codes in medical physics can be found in a review paper by D. Rogers (Rogers 2006).

Radiation transport MC codes simulation occurs history-by-history. In other words, the code simulates particle-matter interaction one particle at a time. When the simulation of a particle is finished, i.e. when the particle's energy becomes smaller than a preset cutoff energy or when the particle leaves the simulation region, another particle's history begins.

These codes associate the MC method framework to the stochastic nature of the particle-matter interaction, using as probability density functions (PDF) the tabulated cross-sections published in the literature, e.g. (Berger et al. 2010; Hubbell 2006). Out of the tabulated cross-sections, the MC codes determine which type of interaction will happen. They also determine parameters following the interaction, such as the energy of secondary particles and angular deflections.

As a particle enters a matter filled volume, the code chooses how long it shall remain in its original trajectory until the first interaction takes place, based on the particle mean free path or on its range. The particle is transported to the interaction point, then the program selects which type of interaction will happen, and chooses the secondary particle's parameters following the cross-sections. A secondary particle is any particle that was originated in an interaction. The secondary particles' original coordinates and energy are stored in the computer's memory and, when the simulation of the original particle is finished, the simulation of the secondary particles starts.

The simulation steps are summarized as:

- i. Calculation of the distance until the next interaction
- ii. Determination of the interaction mechanism
- iii. Determination the scattering angle and the energy loss
- iv. Storage of the initial state of any secondary particle
- v. Repetition of the process for each secondary particle

2.2.2. The PENEPOLE MC code system

The MC code used in this work was the PENELOPE (v2011) (Salvat et al. 2015) with the PenEasy v20120601 (Sempau et al. 2011) interface. PENELOPE is an acronym for “*Penetration and ENERGY LOss of Positrons and Electrons*”. This simulation tool is distributed by the Nuclear Energy Agency (NEA), and it is able to simulate radiation transport through a vast number of materials and within a broad energy range, namely from 50 eV up to 1 GeV (Salvat et al. 2015).

As the name states, the code simulates transport of electrons and positrons, but also of photons. The ability to simulate photons is crucial for accurately describing electron transport. This is because the interactions of a charged particle with matter generate photons as secondary particles, and vice-versa. There are few MC studies of radiation interaction using PENELOPE regarding breast or bTEM reported (Ghammraoui et al. 2014; Panettieri et al. 2009). This motivated this work’s choice for PENELOPE.

In MC codes, the necessity of random number sampling is fulfilled by computer’s pseudo random number generators. The random number generator used by PENELOPE is the RANECU (L’Ecuyer 1988).

2.2.3. Definition of PenEasy Tallies

The PENELOPE code system was used through an auxiliary program called PenEasy (Sempau et al. 2011), its main characteristics are organizing the output possibilities of the simulation in the form of tallies, and then letting the user easily set the tallies of interest for a given simulation.

The tallies used in this work were the pulse height spectrum (PHS) and the energy deposition (ED). In PHS configuration, the user provides information of the material in which the spectrum shall be measured, the energy range of the spectrum, and the size of the spectrum’s bins. When the ED tally is turned on, PenEasy records and writes in a text file the total energy deposited in all defined materials. If more than one body is composed of the same material, the energy deposition result will

be the sum of the energy deposited in all these bodies. A way to discriminate the energy deposition result for different bodies containing the same material is to create equal materials with different file names and assign each material file to the body, or group of bodies, where the deposited energy is of interest.

In PENELOPE, deposited energy is defined as all energy that was deposited and remains inside the volume of the object (body). This means that all forms of energy propagation towards the exterior of the volume of interest are discounted, including radiative emission and charged particles transport.

2.2.4. PENELOPE Parameter's

Some of the simulation parameters must be defined by PENELOPE's user in the simulation's input file. The following parameters are the most important for this work:

- i. the radiation source's location,
- ii. the radiation source's geometry,
- iii. the initial particles' propagation direction distribution,
- iv. the initial particles' energy distribution,
- v. the number of initial particles,
- vi. the geometry of the propagation space,
- vii. the material of the bodies occupying the propagation space,
- viii. the electron absorption energy,
- ix. the photon absorption energy,

The initial particle generation position is defined according to the radiation source's location and geometry. The generated particle propagates in the direction defined by the initial propagation vector with an energy defined by the initial particle energy. The initial propagation vector and energy are sampled from user defined distributions. This process is repeated for as many particles as the user has defined; PENELOPE is limited to generating maximal 1×10^{15} histories per simulation.

To carry out a radiation transport MC simulation, the user must define the geometry through which the radiation will be transported. In PENELOPE, this is done through the PENGEOm sub-

routine. The definition of the geometries is done by superimposing conical surfaces, e.g. planes, cones, hyperboloids, spheres, and ellipses. The intersection of the surfaces defines a volume; PENGEOm names these volumes as bodies.

Bodies can be translated and rotated by the change of parameters in the definition file. The user can define a module to enclose many bodies. Rotation and translation can be applied to entire modules. This basic surfaces and operations enable the definition of a vast number of geometries. For simplicity, bodies and modules can be cloned, rotated and translated; what facilitates the definition of bodies with regularly repeated patterns.

The user must indicate the material that fills the volume defined by each declared body. PENELOPE has an embedded subroutine to create material definition files. The material definition file consists of the compilation of the cross-section tables of all elements that compose the material being defined; this file is the one used by the main program during the simulation when sampling interaction probabilities.

The inputs of the material subroutine are the material's composing elements, the fraction by weight of these elements and the material's density. The fraction by weight of each element in a material defines the probability of a particle interacting with this element. The material's density is used by PENELOPE to calculate the number of scattering centers enclosed by a body. The user can also define the material of whole modules; the module material fills the volume surrounding the bodies enclosed by the module.

A substantial number of materials are standardized in PENELOPE. The standard materials are those of interest in the Medical Physics field, among other fields. Each standard material has a reference number; to define the standard materials the user can simply inform their reference number to the material subroutine. The reference numbers can be consulted in PENELOPE's manual, or in the "pdconpos" file.

The absorption energy of electrons and photons, respectively EABS(e-) and EABS(ph), must be defined by the user; they are the energy below which the propagation of the particle (electron or photon) is terminated. These values are set for each material in the simulation. The higher they are set, the quicker the simulation ends; but setting them to unrealistically high values may distort the simulation's output. They must be set to an energy in which the probability of the particle propagating further and depositing its energy elsewhere is neglectable.

2.2.5. MC Simulation Limitations

As commonly observed in theoretical prediction of physical events, the MC radiation transport simulation codes are susceptible to inaccuracies; these inaccuracies are occasioned by approximations employed in the calculations. Besides the approximations used in the cross-sections calculations, there are others to which PENELOPE is subject to (Salvat et al. 2015). The matter is homogeneously defined, which is not the truth for many radiation transport situations. Also, the molecular structural conformation is neglected. This can play a significant role in radiation interaction; the approximation that introduced inaccuracies are greater for low particles' energy. For very low energies (less than 1 keV) the accuracy of the cross-sections noticeably decreases, making PENELOPE's (and any MC code) results merely qualitative.

2.3. Dosimetric quantities

2.3.1. Attenuation and linear mass attenuation coefficients

X-ray attenuation and scattering are the main phenomena considered in the evaluation of the materials tested in this project. The attenuation curve for a monoenergetic beam is obtained by dividing the radiation intensity transmitted through a given thickness of the material $I(d)$ by the intensity I_0 before the transmission. For a narrow monoenergetic beam, the attenuation is given by:

$$I(x) = I_0 e^{-\mu x} \quad (13)$$

where μ is the material's linear attenuation coefficient for the considered energy, and x is its thickness. Attenuated radiation is composed by the particles which interacted inside the targeted specimen, i.e. only the photons which traversed the target volume without interacting contribute to the intensity $I(x)$.

The beams generated by the clinical X-ray emitters are not monoenergetic. Therefore, these beams present $N(E)$ photons for each energy, E . Considering this kind of photon distribution, the energy dependence of the attenuation coefficient, $\mu(E)$, must be considered. Hence the knowledge of the incident beam's energy distribution is important to the determination of the beam's attenuation. For the polyenergetic case, equation (13) is generalized to

$$I(x, E) = \int_0^{E_{max}} N(E) e^{-\mu(E)x} dE \quad (14)$$

Despite the knowledge of the input data $N(E)$ and $\mu(E)$, and despite the simple analytical form of (14), the equation is valid only for narrow beams, whose **scatter effects** can be neglected in the experimental measurement. For broad beams, the transmission curve prediction by an analytical equation becomes difficult and less accurate, it should thus be obtained by numerical methods, such as the MC method or by semi-empirical approaches such as Archer's model (Archer 1983). The significance of the scatter effects was assessed in Appendix A.

The linear attenuation coefficient considers all possible radiation-matter interaction. Therefore, it is sometimes called **total** linear attenuation coefficient. It is related to the total macroscopic cross-section σ , which is evaluated for specific radiation and matter. The relationship between them is

$$\mu = \eta \cdot \sigma \quad (15)$$

where η is the number of target units per volume of the material, i.e. the number of targets per unit volume, given by

$$\eta = \frac{N_A \rho}{A} \quad (16)$$

where N_A is the Avogadro's constant and A is the molar mass of the material.

Fig. 7. Plot of Mo and Rh total attenuation coefficient curve in the energy range used in mammography highlighting the discontinuities regarding energy, data from Berger et al. (Berger et al. 2010).

The only macroscopic parameter of which the attenuation coefficient depends is the mass density ρ . The linear attenuation coefficient (μ) is usually expressed in units of the material's mass density. This cancels out the density dependence in (17) and defines the mass attenuation coefficient ($\frac{\mu}{\rho}$):

$$\frac{\mu}{\rho} = \frac{\eta \sigma}{\rho} = \frac{\frac{N_A \rho}{A} \sigma}{\rho} = \frac{N_A}{A} \sigma \quad (17)$$

Fig. 7 illustrates linear mass attenuation coefficients of relevant elements used for radiation filtering in the mammography framework. The discontinuities occur due to the energy dependence of the possibility of ionization of an electronic shell. The filtration aims to remove low energy photons

of the beam, favoring the optimal energy region for breast imaging (Jennings 1981; Young et al. 2006).

2.3.2. Mass energy-transfer coefficient

The mass attenuation coefficient accounts for all photon which interacted with matter, but only a fraction of the energy of the original interacting photon stays in the volume in the form of electron's kinetic energy. This fraction accounts for the transferred energy, and hence for the kerma and the dose. The energy which escapes from the volume by radiative emission (*bremsstrahlung*) is also considered in energy transfer (Attix 2004).

In the photoelectric effect, there are two contributions to electronic kinetic energy: the kinetic energy of the ionized electron (E_e) and the kinetic energy of the Auger electrons generated from atomic relaxation. The kinetic energy of the ejected electron is

$$E_e = h\nu - E_b \quad (18)$$

where E_b is the binding energy of the shell from which the electron was ejected. The ionized atom will then perform the relaxation process, in which it can emit radiation with the characteristic energy from any shell whose energy is greater than the energy of the ionized shell, or it will relax by Auger electron emission. Accounting only for the emission of characteristic radiation of electrons occupying the k and the l shells (greater emission probability), the equation which provides the energy transfer cross-section $\left(\frac{\sigma_{PH}}{\rho}\right)_{tr}$, already accounting for the mean energy of the Auger electrons is (Attix 2004)

$$\left(\frac{\sigma_{PH}}{\rho}\right)_{tr} = \frac{\sigma_{PH}}{\rho} \left[\frac{h\nu - P_k y_k h\bar{\nu}_k - (1 - P_k) P_l y_l h\bar{\nu}_l}{h\nu} \right] \quad (19)$$

where P_i is the probability of an electron occupying the ionized i^{th} shell, y_i is the yield of characteristic radiation emission in a transition to the i^{th} shell, and $h\bar{\nu}_i$ is the average energy of the fluorescence emission for an electron transition to the i^{th} shell. Summing up, the term in between

brackets accounts for the probability of the energy originated from the atomic relaxation process to be emitted in the form of Auger electrons.

In Compton effect, the kinetic energy imparted to the electrons is essentially the difference between the energy of the impinging photon and the scattered photon. The energy transfer cross-section is thus (Attix 2004)

$$\begin{aligned} \left(\frac{d\sigma_{KN}}{d\Omega} \right)_{tr} &= \frac{d\sigma_{KN}}{d\Omega} \cdot \left(\frac{h\nu - h\nu'}{h\nu} \right) = \\ &= \frac{r_e^2}{2} \left(\frac{h\nu'}{h\nu} \right)^2 \left(\frac{h\nu}{h\nu'} + \frac{h\nu'}{h\nu} - \sin^2 \theta \right) \left(\frac{h\nu - h\nu'}{h\nu} \right). \end{aligned} \quad (20)$$

The Klein-Nishina cross-section assumes that the radiation is scattered by a free electron, therefore there is no Auger electron emission, and this is to be considered as one of the approximations of the theoretical calculation of cross-sections and attenuation coefficients. As mentioned before, coherent scattering events do not contribute for the kerma or for the dose.

2.3.3. Half-Value Layer

An X-ray beam attenuation curve consists of a plot of the relative beam intensity as a function of the attenuation layer thickness. In this kind of curve, there are two important quantities named first and second half-value layers, *HVL1* and *HVL2*, respectively. The *HVL1* is defined as the attenuation thickness required to reduce the relative beam intensity from one to 0.5. The *HVL2* is defined as the attenuation thickness which is necessary to result in a reduction of the relative beam intensity from 0.5 to 0.25.

The adequate evaluation of the first and second half-value layers requires that the corresponding attenuation curve for an specific combination of X-ray tube target, applied voltage and beam filtration must be measured in specific conditions, which are: (i) narrow beam geometry and (ii) pure absorption material (aluminum or copper, for example) filtration (Attix 2004). The *HVL1* and *HVL2* characterize the X-ray spectra and are relatively easy to measure.

2.3.4. Mean Energy and effective energy

The mean energy (ME) of an X-ray energy spectrum is defined as the mean of the photon's energies weighted by the intensity of the energy's channel.

$$ME = \frac{1}{\sum_{i=1}^{N_{ch}} C_i} \cdot \sum_{i=1}^{N_{ch}} E_i \cdot C_i \quad (21)$$

where N_{ch} is the total number of channels in the spectra, E_i and C_i are the i^{th} channel's energy and number of counts respectively (Attix 2004; Johns and Cunningham 1983). It is an obvious conclusion that identical spectra have identical ME values. Therefore, this quantity can be used to assess the similarity between spectra.

In the X-ray spectrum framework, effective energy (EE) is defined as the energy of a monoenergetic beam that has the same HVL as the spectrum. As equation (13) applies to monoenergetic X-ray beams, the relationship between the EE and the spectrum's HVL is expressed by (Attix 2004; Johns and Cunningham 1983):

$$HVL = \frac{\ln(2)}{\mu(EE)} \quad (22)$$

2.4. Experimental measurement techniques

2.4.1. X-ray Spectrometer

X-ray energy spectrum, in the present work's context, is composed of the combination of *Bremsstrahlung* radiation resulting from the radiative interaction of electrons with a high-Z metal anode and by the characteristic fluorescence radiation. These are typical X-ray tube spectra. Excepting the backscatter radiation, the study of transmitted X-ray spectra provides the complete information regarding attenuation and energy deposition inside the material.

One way of performing X-ray spectrometry using clinical radiation beams is with solid-state detectors biased by an electrical field and linked to a Multi-Channel Analyzer (MCA) (Abbene et al. 2012; Bottigli et al. 2006; Kurková and Judas 2015; Miyajima 2003; Miyajima et al. 2002; Santos et al. 2016). Care must be taken because the correct quantitative determination of the measured spectra can only be achieved after careful corrections according to the response curve of the detector are performed (García-Alvarez et al. 2016; LeClair et al. 2006; Nunn et al. 2008; Tomal et al. 2012; Tomal et al. 2015).

Time resolved spectrometer, the equipment used in the present work, functions by very rapidly (25-100 *ns*) measuring the radiation field's stimulus done to a constraint volume of matter. The time-resolution must be high enough to measure the stimulus caused on the medium by one single particle, so it can distinguish the energy of a particle from another's (Jenkins et al. 1995).

The most recent photon spectrometry equipment available use solid state materials as the medium to be stimulated by radiation (sensitive element). Germanium, Silicon with Lithium, and Cadmium Telluride are examples of materials used to measure the ionization done by a particle. These materials are chosen for being semiconductors, and also for their stopping-powers and mass attenuation coefficients. Each material combination optimizes the measurement of photons of a certain energy range.

The sensitive element is usually mounted over a cooling device to diminish the thermal energy of its electrons and to reduce noise in the pre-amplifier's circuits. The cooling is sometimes done by liquid nitrogen immersion or by mounting the crystal on top of Peltier plates. Fig. 8 shows a scheme of the electronic devices which compose a modern time-resolved X-ray spectrometer (XR-100T, Amptek).

The sensitive element is electrically biased with an electrical field (~ 1000 V), forming a capacitor, and generating charge collection. The charge collection time is very small, around 25-100 *ns*. The number of freed electrons, N , by an impinging photon is proportional to the photon's energy. The

proportionality factor, ϵ , is the mean energy required to eject an electron. The capture of the free electrons generates charge accumulation in the pre-amplifier's capacitor, the charging and discharging of the capacitor generates the pulse pattern.

Fig. 8. On the left hand side, a scheme of the whole time-resolved X-ray spectrometry apparatus, adapted from (Jenkins et al. 1995). On the right hand side, a representation of a commercially available CdTe portable model (XR-100T-CdTe, Amptek, Bedford, Ma) (Amptek 2017). It shows the spectrometer detection parts, the Be window, the cooling device, and the sensitive element of the detector.

The discharge takes a relatively large time interval, therefore pulse shaping by the preamplifier is necessary. The detected pulse is pre-amplified to a 0 to 2 V range and then shaped to shorten its duration, without altering its height. This enhances the spectrometer's ability to discriminate between the pulse height of one photon and the next photon influence.

After pulse shaping, the pulse is digitalized by a multichannel analyzer (MCA) (Schumann and McMahon 1956; Wilkinson 1950). The MCA process is also exhibited in Fig. 9.

For the pulse analysis process to be successful, several detection parameters must be correctly set. Pulse shaping is controlled by setting the peaking time and the flat top time (trapezoidal pulse shaping). The relation between the MCA channels and the energy range is controlled by the course gain.

Despite pulse shaping, it is possible that more than one photon arrives at the crystal within a non-discriminable time interval, generating piled-up pulses. Some spectrometers have an embedded pile up rejection (PUR) system. When PUR is active and another photon arrives during the peaking time period, pile-up is flagged by the PUR and disregarded. Nevertheless, when PUR is turned on, detection may be greatly impaired, depending on the field's photon fluency and of the peaking time.

Fig. 9. Signals occurring during the pulse height measurement process in the multichannel analyzer (Gedke 1972). In (a) there is the amplifier output pulse. (b) shows the lower level discrimination output. The signal on the rundown capacitor is exhibited in (c). Letter (d) shows the address clock. The memory cycle can be seen in (e). (f) shows the time the linear gate is closed. (g) indicates the deadtime gate.

When pile-up is flagged, the system still needs to discharge below the threshold to start a new acquisition. This can be time-consuming and cause elevated dead times and poor detection efficiency. Therefore, correct setting of peaking time and correct collimation of the field is crucial for satisfactory X-ray spectrometry.

2.4.2. Ionization Chamber

The ionization of gases by incident radiant energy is one of the first methods developed to measure the radioactivity and is still the gold standard of radiation measurement. Ionization Chambers are gas filled chambers designed to be sensitive to the passage or radiation (Knoll 2010).

The passage of electromagnetic radiation through gas culminates in the formation of free electrons and ionized atoms, which are called ion pairs. The ionization chambers function by charge collection through an electrically biased capacitor. They establish a relation between radiant energy and electrical potential.

The biasing electrical field is commonly a DC field, and the ratio of energy per number of ion pair collected, the W -value, is a smooth function of the energy of the impinging particle (Knoll 2010). Therefore, ionization chambers are good measurement equipment for dosimetric quantities such as exposure and air-kerma.

The accurate measurement of radiation exposure must consider the energy of all charged particles put in movement by the incident radiation (Attix 2004). The low atomic number of air generates a difficulty involving electrons range. The range is the expected value of the path each ion travels inside the gas volume. The range of electrons in air considering typical diagnostic energies is a few centimeters (~ 3 cm for electrons of 40 keV). The size of the radiation chamber becomes thus impractical for almost any application. This problem is solved by achieving charge particle equilibrium (CPE) by using cavity chambers, which are ionization chambers enclosed by an air equivalent wall.

CPE condition is achieved inside a given volume when the energy carried by the electrons leaving the volume is equal to the energy brought in the detection volume by electrons entering it. For achieving CPE, an air equivalent wall must be similar to air regarding electron ionization cross-section, and regarding electron stopping-power. To meet this condition, it is sufficient that the atomic number of the material composing the chamber walls is close to air's. This is met by aluminum and many polymers (Knoll 2010).

A cavity chamber is equivalent to placing the chamber in the center of a very large room filled with air. Furthermore, equivalence also means that all the air of this imaginary room would be homogeneously irradiated.

Finally, the exposure measured by the Ionization Chamber, when all the requirements are met, is given by (Knoll 2010)

$$R = I \cdot \left(\rho_{air} \cdot V \cdot \frac{P}{P_0} \cdot \frac{T_0}{T} \right)^{-1} \quad (23)$$

where R is the exposure rate, ρ_{air} is the air's density, I is the measured ionization current, V is the chamber's volume, P and T indicate respectively pressure and temperature, and the subscript 0 denotes standard temperature and pressure (STP) values.

2.4.3. Thermoluminescent dosimetry

Thermoluminescent dosimetry is the technique of measuring the absorbed dose in specific materials. A part of this technique, applied to this work, consists of placing thermoluminescent dosimeters (TLD) in the used radiation fields. The TLD material has physical characteristics that enable maintaining information which relates to the dose deposited by the field. The stored information can be read from the dosimeter after its irradiation.

TLDs consist of solid materials, organized in crystalline structure, commonly containing a small weight fraction of some other element added to the crystalline lattice. The process of adding small parts of impurities to the crystal is called doping, and it is an important part of the mechanism that allows these materials to be used in dosimetry applications. Some examples of TLD materials are CaSO_4 doped with Mn, LiF doped with Mg and Ti, CaF_2 and CaF_2 doped with Mn (Knoll 2010).

The information retained by the TLD when exposed to a radiation field occurs through the electron-hole trapping mechanism, schematized in Fig. 10. The trapping process of an electron will be hereafter explained, and the trapping of a hole can be inferred from this explanation.

When radiation impinges on the doped crystal, it may eject its electrons, freeing them from the crystal lattice electric attraction, and providing kinetic energy. The doping elements added to the crystal establish different potential levels, because of having different electronic shells than those of

the main element, these regions are called traps. The ejected electrons have a probability of binding to the trap region. This gives the TLDs “memory” to retain the energy deposition information.

Fig. 10. Scheme of thermally stimulated emission process of electron thermal stimulation (a) and hole thermal stimulation (b), E_e , E_H , E_{ph} and kT are the respectively the energy level of the trapped electrons in relation to the conduction band, the energy level of the trapped hole in relation to the valence band, the energy of the emitted photon, and the thermal energy.

Even though electron trapping is stochastic, the high number of impinging photons causes a constant trapping rate. Therefore, the number of trapped electrons is related to the total ionization of the crystal, hence it is a measure of the radiant energy exposure of the beam (Knoll 2010).

The process of retrieving the TLDs exposure is a heating process, which increases the thermal energy, kT , of the particles and enables a trapped electron to escape from the potential bind to a free state. As the temperature raise is gradual, the electron has almost no kinetic energy when it leaves the trap and it is promptly captured by one of the holes from the lattice. This results in the emission of a fluorescence photon of energy $h\nu$.

The heating process is conducted in a dark chamber, and the fluorescence photons are captured by a photomultiplier, generating an electrical signal. As the heating occurs over time, the recorded results are represented in a diagram of signal intensity by time, this is called the glow curve.

The correlation between thermoluminescence and dosimetric characteristics of the TLD is obtained by calibration of the glow curve. It is calibrated by simultaneously irradiating TLDs and a

calibrated ionization chamber (or another calibrated detector). There is a linear relationship between the ionization chamber measurement and the integral of the glow curve is linear.

The reading of a TLD provides the energy that was deposited in it (DE). This quantity can be translated to dose (D) by

$$D = \frac{DE}{m_{TLD}} \quad (24)$$

where m_{TLD} is the TLD's mass.

2.5. Hypothesis testing and the Z-test

In statistical sciences, a hypothesis is a very simple and direct statement about a set of data. These statement is always accompanied by its alternative statement. For example: One may hypothesize that the mean of a dataset (population) is equal to, **not different than**, a given number. The alternative hypothesis would be that the mean of the population is different than this number. The “no difference” hypothesis is called null hypothesis and it can be represented by the symbol H_0 . The alternative hypothesis is usually represented by H_1 and postulates non-zero difference (Everitt 2006).

Hypothesis testing is defined as the procedure of assessing whether sample data is consistent or not with statements made about the population (Altman 1991). After the test is complete, the experimenter is able to decide if he or she will retain or deny the hypothesis.

The hypothesis test used in this work was the Z-test. The Z-test is a hypothesis test for population means (Everitt 2006; Vanin et al. 2005). It is represented as

$$Z = \frac{x_{calc} - x_{ref}}{\sigma} \quad (25)$$

where x_{calc} is a statistic of a population, and x_{ref} is a value to which this statistic is being compared. A statistic, in this context, is defined as a numerical characteristic of a population or sample. For

example a population's mean (Everitt 2006). The variable x_{ref} can be a value with no associated variance, or a statistic (a numerical characteristic of a population or sample). The variable σ is the standard deviation (uncertainty) of x_{calc} , or the combination of the uncertainties of x_{calc} and x_{ref} , whenever the x_{ref} has an associated variance.

The result provided by equation (25) is called the Z-score or standard score. It means the variable value transformed to zero mean and unit variance (Everitt 2006). The Z-score is used to decide if H_0 should be retained or rejected. Despite the numerical nature of the Z-score, deciding if the hypothesis will or will not be accepted is ultimately an arbitrary decision.

There are three bounded quantities which are used to set the rejection decision threshold. They are the critical value, the significance level and the confidence interval. Determining one of them determines the other two. It is simpler to understand their meaning by observing Fig. 11, where these concepts are applied for a normal distribution.

The upper image in Fig. 11 represents a two-tailed normal distribution of null mean and unity variance and unity area (as any PDF). The confidence interval (CI) is the area highlighted by blue horizontal hatches. This area is smaller than unity. The confidence interval determines a range of values that is believed to contain the true value of the estimated quantity with a certain probability (Everitt 2006). The sum of the area marked by green diagonal hatches is the significance level α . It is interpreted as the level of probability at which it is agreed that the null hypothesis will be rejected.

Finally, using the definitions above, the sum of the values of the confidence interval and the significance level must result in the unity, as represented in equation (26).

$$\alpha + CI = 1 \quad (26)$$

The critical values are marked by a black solid line and are symmetric with relation to zero. When they are symmetric, we simply refer to it as a unique critical value, represented by its absolute value. Critical value and cut-off are synonyms in the hypothesis test context. The acceptance region is

indicated by a red dashed line and the orange dashed line indicates the critical region. If the Z-score is in the acceptance value, H_0 is retained. If it is in the critical region, H_0 is rejected.

The lower part of the figure shows distributions for two-tailed tests with symmetric and asymmetric critical values, and a one-tailed test distribution. Which one of these cases to choose depends on the specific problem being analyzed. In this work, only two-tailed symmetric tests were used. Therefore, this was the highlighted case in Fig. 11.

Fig. 11. The upper image represents a two-tailed normal distribution of null mean and unity variance. The confidence interval is the area highlighted by blue horizontal hatches. The sum of the area marked by green diagonal hatches is the significance level α . The critical values are marked by a black solid line and are symmetric with relation to zero. The acceptance region is indicated by a red dashed line. The orange dashed line indicates the critical region. The lower part of the figure shows distributions for two-tailed tests with symmetric and asymmetric critical values, and a one-tailed test distribution. The case explored in the upper figure is marked by an orange rectangle.

The critical value, the significance level and the confidence interval are bounded quantities, determining one of them determines the other two. The determination of these quantities is

arbitrary, i.e. it is chosen by the experimenter. The choice must be done based on the application on the text. According to Altman (Altman 1991), the significance level, α , is “conventionally set at 0.05.” (Altman 1991). Altman clearly expresses the arbitrary nature of the significance level definition:

“Although the choice of the cut-off (critical value) is arbitrary, in practice in most cases we use (a significance level of) 0.05. In other words, an outcome that could occur less than one time in 20 when the null hypothesis is true would lead to the rejection of the null hypothesis.” (Altman 1991)

“However there is no particular reason for choosing 95% confidence interval other than convention, and levels of 80%, 90% and 99% are sometimes used” (Altman 1991)

The arbitrary nature of this choice is a vast subject that is not in this work’s scope, further readings are indicated as references (Cowles and Davis 1982; Nuzzo 2014).

The hypothesis tests are subject to errors. The type I error occurs when the null hypothesis is falsely rejected. The type II error, when the null hypothesis is falsely accepted. To assess if a test failed, one should know a priori the result the test should have (Everitt 2006).

Another important quantity when realizing a hypothesis test is the p-value. Its definition is similar as the one from the significance level, i.e. it is the integral of the area further always from H_0 acceptance. But the value which defines the point beneath which the p-value area will be integrated is not arbitrarily defined, such as the critical value. Instead, it is the outcome of the data being tested. If the data being tested leads to a statistic which equals the critical value, then p-value equals α .

The correct interpretation of the p-value is that it defines the probability of the observed data (or data showing a more extreme difference from the null hypothesis) when the null hypothesis is true. Some common wrong interpretations of this value are that it is the probability of the null hypothesis or that it is the probability of the data having arisen by chance (Everitt 2006).

3. METHODS AND MATERIALS

The experimental procedures on radiation transmission and beam spectrometry conducted in the present work were divided into two experiments. The first one (Experiment A) was done in the Radiation Dosimetry and Medical Physics Group of the Institute of Physics of University of Sao Paulo (GDRFM). The radiation beam used in this experiment was generated by a tungsten-target X-ray emitter originally designed to industrial applications which were configured for representing mammography conditions such as applied voltage, beam filtration, and geometry. The second experiment (Experiment B) was done in the Nuclear Technology Development Center (CDTN) of the National Commission of Nuclear Energy (CNEN). This data collection was performed in a commercial mammography system. Experiment B was done by Josilene C. Santos, one of the Ph.D. students from GDRFM.

The motivation for doing two different sets of experiments was to increase this work's robustness, i.e. to increase the number of experimental conditions in which the conclusions are valid. Each measurement approaches will be explained in the following sections. A diagram summarizing the major features of experiments A and B is exhibited in Fig. 12.

Fig. 12. Diagram summarizing the major features of experiments A and B. Experiment A used an industrial tungsten-target X-ray emitter configured for representing mammography conditions. Experiment B used a commercial mammography system. Experiment B was done by the PhD student Josilene C. Santos.

The MC radiation transport simulation, which results were compared to the experimental data, aimed to be as similar as possible to the experimental assemblies. When differences occurred, considerations and preliminary tests were made to justify that the influence on the results would be neglectable.

The experiments adopted in the present work estimated two quantities in general: the transmitted energy spectra and the energy deposited after attenuation by the bTEM. Regarding spectrometry, both experiments assessed the transmitted energy spectra using the same detection equipment and X-ray spectrometry method described in section 3.2.3. Energy deposited after attenuation by the bTEM was evaluated only in Experiment B, using thermoluminescent dosimetry.

3.1. Tissue equivalent material characteristics

Two different sets of phantoms were studied:

- i. BR12 kit by Nuclear Associates (NA) (Cleveland, OH), today Fluke Biomedical Corporation, in Experiment A
- ii. CIRS 012A® (Norfolk, USA) in Experiment B.

3.1.1. Dimensions and compositions

Both studied bTEMs consisted of six individual slabs: one with thicknesses equal to 0.5, two of 1.0, and three of 2.0 cm. The surface dimensions of all slabs are $10.00 \times 12.55 \text{ cm}^2$. One example of the slabs is shown in, the CIRS 012A® (Norfolk, USA).

Fig. 13. Picture of one of the Breast tissue equivalent material (bTEM) used in this work. Research set model 012A by CIRS (Norfolk, USA). Adapted from CIRS' website (CIRS).

The slabs are manufactured using an epoxy resin based tissue substitute material, composed of various low atomic number materials, homogeneously combined (White et al. 1977). These materials are composed of the following chemical elements: H, C, N, O, Cl, Ca distributed homogeneously. No structures, such as microcalcification simulators, were present in the middle of the slabs. The transmission properties were studied through seventeen different thicknesses in Experiment A, ranging from 0 to 85 mm in 5 mm steps. In Experiment B, fewer thicknesses were used. They were 0, 5, 10, 20, 30, and 40 mm. These thicknesses were achieved by superimposing the bTEM plates, creating blocks.

The NA BR12 kit material was developed by White (White 1977). It simulates the composition and density of a 50/50 glandularity breast. This material is identified as W5050 in Table 1. Nowadays, the CIRS 012A is one of the phantoms to simulate different breast tissue glandularities. In the present work, CIRS 012A bTEM of 50/50 and 30/70 glandularities were used. The compositions of these bTEM were provided by two distinct sources: one is CIRS 012A's manual, the other one is a paper from Poletti et al. (Poletti et al. 2002b). This two different composition sources were used to evaluate the dimension of the uncertainties associated with the bTEM slab's atomic composition. This will be further discussed in section 3.4.3.

In Table 1, the compositions provided by CIRS' manual are identified as C5050 and C3070. The compositions provided by Poletti et al. are indicated as P5050 and P3070. All sources' compositions,

of bTEM of both manufacturers and glandularities, can be found in Table 1. The associated uncertainties were not informed by all sources.

Table 1. Composition of the bTEM slabs gathered from different sources such as the material's manuals, and published papers (Poletti et al. 2002a; White et al. 1977). When the values do not present uncertainties, it is because they were not informed by the source. Poletti's paper estimated the composition of the CIRS 012A bTEM, and White's from NA BR12 kit.

Code	Source	Manuf.	Glandu.	H (%)	C (%)	N (%)	O (%)	Cl (%)	Ca (%)	Density (g/cm ³)
W5050	White 1977	NA	50/50	8.68	69.95	2.37	17.91	0.14	0.95	0.970
P3070	Poletti et al. 2002	CIRS	30/70	11.78 (6)	75.12 (7)	0.66 (3)	12.14 (24)	0.00	0.30 (8)	0.970 (1)
C3070	CIRS' Manual	CIRS	30/70	11.73	75.51	1.23	9.96	1.18	0.35	0.960
P5050	Poletti et al. 2002	CIRS	50/50	11.10 (5)	72.74 (9)	1.04 (4)	14.82 (26)	0.00	0.30 (8)	0.980 (1)
C5050	CIRS' Manual	CIRS	50/50	11.60	75.07	1.23	10.16	1.17	0.67	0.982

This section exhibits the cross-sections dependence with the energy of radiation interaction events in the breast tissue-equivalent materials used in the present work. Cross-section data from NIST database (Berger et al. 2010) are shown in Fig. 14. The presented data considers only White's bTEM's composition, which is representative of the entire set. It was chosen as reference because it is defined as the standard bTEM composition by the ICRU report number 44 (White et al. 1989).

Fig. 14. Coherent scatter, Compton effect and photoelectric effect mass cross-sections of the bTEM manufactured by CIRS (Norfolk, USA), The cross-sections were calculated by XCOM (Berger et al. 2010) based on the composition measured by Poletti et al. (Poletti et al. 2002b).

3.2. Experimental description

3.2.1. X-ray sources

The X-ray equipment used in Experiment A was an industrial unit model MG450 by Philips (Amsterdam, Netherlands) coupled with an MGC30 control unit, as seen in Fig. 15. This tube's beams are submitted to an inherent Be filtration of 2.2 mm. The X-ray tube anode was composed of tungsten, and the beam was filtered by a 0.030(1) mm thick Mo filter.

Fig. 15. Tungsten anode X-ray equipment. Philips MG45 (Amsterdam, Netherland) filtered by a 0.030(1) mm thick Mo filter. Image is taken from L. Mariano Ph.D. thesis (Mariano 2017)

Two different voltages in the range of mammographic applications were used: 28 and 31 kV (Pernicka and McLean 2007). Different current-time products (mAs) in the range of 4.5 to 165 mAs were used to control the photon fluence and to reduce uncertainties originated by the x-ray spectra detection system (pile-up effect). These current-time products were also adopted to prevent damage by charge saturation on the sensitive element of the spectrometer. A circular collimator was placed in the X-ray emitter output to obtain a beam of 106.4(3) mm diameter cross-section at 650(10) mm distance from the focal spot to the bTEM.

In Experiment B, the X-ray emitter was Mammomat 3000 Nova mammography equipment (Fig. 16) manufactured by Siemens (Germany, Munich). This tube has a 1 mm Be inherent filtration.

Fig. 16. Image of the Siemens Mammomat 3000 Nova used in Experiment B.

The measurements were conducted using voltages of 26 and 28 kV. The current-time product was of 100 or 200 mAs, depending on the thickness of the bTEM. Three different anode filter combinations were used. The composition of the anode, the filter material, and the thickness of the filter for these combinations are shown in Table 2. The set-up used in Experiment A is also shown in Table 2.

Table 2. Composition of the anodes and filters used in Experiments A and B, and the thickness of the filters used. The mammography X-ray equipment's manufacturer doesn't present uncertainties for the thicknesses, the presented uncertainty was estimated at the GDRFM.

Experiment	Anode	Voltage (kV)	Filter	Filter's Thickness (μm)
A	Tungsten	28 and 31	Molybdenum	30 (1)
B	Molybdenum	26 and 28	Molybdenum	30
B	Molybdenum	26 and 28	Rhodium	25
B	Tungsten	26 and 28	Rhodium	50

Experiments A and B set-ups main difference was the X-ray emitter; this caused another difference in the experimental set-ups. The commercial mammography X-ray equipment used in

experiment B imposed bTEM placement distances. Improving the robustness of the comparisons was the motivation to perform two different experiments.

3.2.2. Experimental Geometry

Schemes of Experiment A's set up is shown in Fig. 17 and of Experiment B's set up in Fig. 18. The distal surface (surface further away from the X-ray beam emitting point) of the BR12 block was positioned 650(10) mm away from the X-ray emitter focal spot in Experiment A. This distance was 330(10) mm in Experiment B. The primary spectra measured with no BR12 attenuation were also assessed. The spectrometer's sensor was placed 685(10) mm and 425(10) mm away from the X-ray tube focal spot for Experiments A and B, respectively. When measured from the bTEM block distal point, the afore mentioned distances where of 35(10) and 95(10) mm respectively.

Fig. 17. Experimental set up of the Experiment A. All the distances are scaled.

Fig. 18. Scheme of Experiment B, done in the mammography X-ray equipment.

The dose deposition in Experiment B was assessed via a set of LiF TLDs TLD-100 (Harshaw Chemical Company, OH, USA) placed in between the bTEM slabs (inside the block) in a cross format, as shown in Fig. 19.

Fig. 19. Scheme of the TLDs used in the irradiation, this figure was adapted from Santos (Santos 2015). The TLDs were placed in a cross format. Each dosimeter in a layer was numbered from 1 to 9 as seen in the figure. Five layers were used altogether. Vertical sections of the block are indicated by yellow and greenish dashed lines and exhibited in the right-hand side.

A dosimeter layer was defined by a group of nine dosimeters forming a cross. This enabled the assessment of the dose homogeneity along the x and y axes. Five different layers were positioned in between each bTEM slab, this means there were layers positioned at 0, 5, 15, 25, and 45 mm of depth inside the bTEM block.

The positioning of the TLDs occasioned a spacing of 1 mm between the bTEM slabs, which was filled by air. To avoid 5 layers of air between slabs during a single irradiation, each TLD irradiation was performed separately, according to the following procedure:

- i. The first TLD layer was set up between two bTEM slabs;
- ii. The bTEM slabs and TLDs were irradiated;
- iii. The irradiated TLDs were removed from within the bTEM block and properly stored;
- iv. A distinct set of TLDs was arranged in a new depth;
- v. The irradiation of the new layer was conducted, following from step ii).

This procedure was repeated until all 5 TLD layers were irradiated.

3.2.3. X-ray Spectrometry

The spectrometer equipment used in this work is shown in Fig. 20. The transmitted X-ray spectra were measured, in both experiments, with a detection system which consisted of an Amptek (Bedford, MA) XR-100T spectrometer, coupled with an Amptek PX4 MCA using 1024 channels. The sensitive element of the spectrometer was a 9 mm³ CdTe crystal, protected by a 1 mm thick Beryllium window. The voltage which biased the detection crystal was of 500 V. The spectrometer alignment with the radiation beam was obtained following the method described by Santos et al. (Santos et al. 2017)

Fig. 20. Picture of the used spectrometry acquisition system, an Amptek XR-100T coupled with an Amptek PX4 MCA.

The Amptek ADMCA acquisition software was used to register the spectra. The pulse shaping was trapezoidal and done digitally. The user can control the shaping through two parameters, the peaking time and the flat top. They were equally set for both experiments as 3.2 μ s and 0.4 μ s respectively. The coarse gain was of 91.86, which produces a spectral energy bin of approximately 0.06 keV/channel. Specific care was taken to prevent high counting rate in the spectrometer, avoiding thus dead time losses and pile-up distortions (Bottigli et al. 2006). The higher relative dead time was of 3.58% of the total irradiation time. The pile-up rejection option was turned on.

The energy calibration was performed through the measurement of ^{241}Am and ^{152}Eu calibration sources. Detection parameters maintained during calibration measurements were the same as the ones used to measure the emitter's mammography spectra (Bé et al. 2004; Lépy et al. 1994). For illustration purposes, Fig. 21 shows examples of the spectra measured in this work

Fig. 21. Example of typical mammography spectra generated from W (solid line) and Mo (dashed line) anodes, which respectively undergo 30 μm Mo and 25 μm Rh filtration, before impinging upon the breast. These spectra were measured with a CdTe XR-100T AmpTek spectrometry system.

3.2.4. Dose Deposition Assessment

In Experiment B, the energy transmission was assessed using 45 TLD-100. These dosimeters were positioned in between the bTEM plates, to estimate the dose delivered to the bTEM in different thicknesses, as showed in Fig. 19.

The thermoluminescent dosimeters (TLD) were read on a Risø (DTU Nutech. Inc., Roskilde, Denmark) dosimeter reader, model DA-20. The TLDs were heated from room temperature ($\sim 25^\circ\text{C}$) to 350°C . The luminescence response curve was integrated to estimate a dosimeter read out, which

was converted to exposure through a previously constructed calibration curve. A series of dosimeters was submitted to the same conditions as all other TLDs, but was not directly irradiated by the mammography equipment. These dosimeters were used to perform the background subtraction.

3.3. Monte Carlo simulation description

3.3.1. Input Spectra Characteristics

The input spectra used for the simulations were the ones measured without attenuation by any tissue-equivalent material. These input spectra were corrected for the detection distortion effects (Di Castro et al. 1984; Kurková and Judas 2015). The present work used Di Castro's et al. stripping method, as described by Santos et al. (Santos et al. 2016). The used detection efficiency curves and escape factors are specific for the used CdTe detector and for mammographic energy range and were estimated by Tomal et al. (Tomal et al. 2012; Tomal et al. 2015). Detection distortion effects are expected to be the same for experimental and simulated transmission spectra (except for pile-up effect). Therefore, the simulated and experimental transmitted spectra were not corrected, as the correction would not impact the comparative analysis. In other words, if a comparative test indicates compatibility between the experimental and the simulated spectra, this compatibility will hold after both spectra are corrected. The opposite is also true, i.e. incompatibility is also expected to hold.

The disregard of the pile-up effect is a reasonable approximation, because the dead time was low (3.58% in the worst case), and because PUR was turned on.

3.3.2. Simulation's Geometry and Material Composition

The bTEM, the TLDs and the CdTe detector used in this work were constructed following the process explained in section 2.2.4. Their definition followed the geometric characteristics used in the experiment, described in 3.2.2.

Another important geometric feature is the initial radiation beam pattern and its possible initial propagation directions. In this work's MC simulations, the photons were generated in the vertex of a cone, through which the radiation beam propagated. The semi-aperture angle θ_0 of the cone was set so that its vertical section had a radius of enough size to enclose the bTEM surface. Better understanding of the geometry is achieved by observation of Fig. 22.

Fig. 22. 3D representation of the radiation beam impinging upon the bTEM slabs, this representation is not scaled and is valid for the simulations of the experiments A and B.

Besides the bTEM, three other materials had to be defined in the simulations: the CdTe of the spectrometer detector, the Be of the spectrometer window, and the LiF of the TLDs. All constitutions of these materials were taken from PENELOPE's embedded table of standard materials.

The PHS tally was defined to get the spectra measured in the CdTe crystal. The ED tally was used to get the energy deposited in the LiF parallelepiped which simulated the dosimeters in Experiment B.

Even though the TLD layers of Experiment B were individually irradiated in the real experiment (as described in section 3.2.2), all TLD layers were irradiated at once in the simulation. This was done to enhance simulation time, without loss of generality on the results. This approximation was possible because of the similarity of the radiation attenuation inherent to the LiF TLDs and to human breast tissues (Knoll 2010).

3.3.3. Number of Simulated Histories and Simulation Uncertainties

In general, mainly two distinct simulation types were done in this work: one to assess the transmitted radiation spectra and another to assess the deposited energy within some volume of TLD. In each simulation, the number of histories was chosen so that the relative uncertainty of the simulated quantity was smaller than the uncertainty of the corresponding experimental measuring procedure. The simulated quantities were the energy deposition or the counts in a spectrum's channel.

To assure that the statistical (type A) uncertainties of the simulated spectrum were smaller than those of the experimental, it sufficed to impose that the channels in the simulated spectra had higher counts than those of the experimental. This occurs because the relative uncertainty associated with Poisson's distribution is inversely proportional to the number of counts. Other simulated spectra's uncertainty sources were considered. This will be explored in section 3.4.

3.3.4. Cutoff energies

In the spectrometry simulations, the EABS(ph) was set to 10 keV at the bTEM materials. This was done to reduce the simulation time by avoiding to transport photons below this energy. These photons had very little probability of reaching the detector, given the range of photons in BR12. Therefore, this selection of threshold energy introduces no systematic error.

In MC radiation transport simulations, the electron transport events are the most time-consuming. This occurs because electrons interact mainly by means of collisional interactions, where a small fraction of the electron energy is lost per interaction and there is a small angular deflection (Salvat et al. 2015). Considering the energies and materials adopted in this work, the electron range was sufficiently small to consider that it deposited all its kinetic energy in the ionization position.

Therefore, the EABS(e-) of all materials was set to 100 keV, so that secondary electrons accelerated by photon interaction would deposit its energy locally, which greatly reduced the simulation time.

3.4. Monte Carlo Markov Chain (MCMC) variance propagation

MC codes may provide not only the counts in a given channel of the spectra but also this channel's count uncertainty. The provided uncertainty is usually the statistical Poisson's distribution uncertainty $\left(N^{-\frac{1}{2}}\right)$ (Sobol 1974) ever, other uncertainty sources can contribute to the simulated results' uncertainties (Aguirre et al. 2016). The present work considered two other uncertainty sources in the spectra counts besides Poisson's. They were the uncertainties of the tissue equivalent materials compositions and of the simulations' input spectra.

The input spectra adopted for the MC simulations were experimentally obtained. Therefore, they have uncertainties which should propagate to the transmitted simulated spectra. The same is valid for the bTEM' compositions. These two uncertainty sources are the most relevant ones when compared, for instance, to the uncertainties in the cross-sections ($\sim 1\%$) (Aguirre et al. 2016) or to the uncertainties of the bTEM dimensions ($< 0.2\%$), which were numerically propagated but disregarded.

3.4.1. Geometry adaptation to a pencil beam

The geometry of the simulations used in the MCMC uncertainty propagations was different from the one described in section 3.3.2. The cone aperture was greatly reduced. For the MCMC propagation, the semi-aperture angle θ_0 was defined so that its vertical section had a radius of enough size to enclose only the detector's crystal (1.5 mm). Therefore, the beam used in the MCMC propagation was a pencil beam. The number of simulated histories was diminished proportionally to the decrease in the cone aperture. This assured that the photon fluence ϕ was kept constant, i.e.

was the same of the simulations that used the total cone aperture. The photon fluency (Φ) of the simulation is defined by the total number of simulated photons (N) and by the area traversed by the photon beam (S)

$$\Phi = \frac{N}{S} \quad (27)$$

Where S is defined in terms of the solid angle (Ω) and by the distance from the radiation emission source to the bTEM surface (d).

$$S = d^2 \Omega = 2\pi d^2 (1 - \cos \theta_0) \quad (28)$$

Ω is defined by the cone semi-aperture angle (θ_0).

The cone aperture reduction was implemented because a substantial number of spectra were necessary to estimate the MCMC uncertainty. Altogether there were 25 spectra for each of the 108 comparisons, hence 2,700 spectra. As this technique did not alter the mean individual history duration (T) the simulation's efficiency ϵ was improved (Bielajew and Rogers 1988).

$$\epsilon = \frac{1}{T \cdot \sigma^2} \quad (29)$$

where σ^2 is the variance of the simulations output, which was reduced by this technique.

Nevertheless, this technique almost completely disregarded the scattering phenomena, what alters the accuracy of the simulations' output spectra. The MCMC propagation objective was to estimate the variance in each channel of the spectra. Therefore, it was considered that the inaccuracy in determining the channels' counts have low influence on the variance determination. A more accurate determination of the variance would result from simulating with broad beam.

The complete set of MCMC variance propagation simulations took about 6×10^5 s (around one week) to be performed using 11 cores of an Intel® Xeon® E5-2420 v2 2.20GHz computer. The standard deviation of the simulation results was used to estimate the input spectrum's component of the simulated spectra's uncertainty. The importance of the consideration of this uncertainty sources will be discussed in section 4.3.

3.4.2. Uncertainties regarding simulations' input spectra

The primary spectra used as simulations inputs have uncertainties associated with their channels. These uncertainties were propagated to the simulation. The propagation was done by the MCMC propagation method (Sobol 1974). In this method, twenty-five different entry spectra were randomly generated following Poisson's PDF. The distribution parameter for each channel was the number of counts in the channel of the primary spectra. The simulation of transmitted spectra was then performed for each one of the 25 spectra, as described in section 3.3. with the geometric adaptation explained in section 3.4.1.

3.4.3. Uncertainties related to tissue equivalent materials compositions

The MCMC propagation method was also used to propagate the bTEM composition's uncertainties. Normal distributions were assumed for the PDFs of each chemical elements' fraction by weight present in the studied bTEM's. The weight fraction values exhibited in Table 1 were used as the means of the normal PDFs. The bTEM's manufactured by CIRS had two informed composition, from Poletti et al. (Poletti et al. 2002a) and by CIRS' manual. In these cases, the mean between values informed by the two sources was used.

The standard deviations of the normal distributions were defined as the uncertainties presented in Table 3. This table shows the relative variation between Poletti's estimation and CIRS' manual data of the fraction by weight of each element in the CIRS bTEM's composition.

Random values of the fraction by weight of each element were then sampled from the normal distributions. When the random sample generated negative values for the fractions by weight, what mostly happened to chemical element Chlorine, the composition of this element was set to zero. This introduced a systematic error, which was neglected. A set of 25 different bTEM's compositions was created by randomly varying each element's composition. These 25 compositions were used to estimate the variance due to this effect.

The uncertainties in the elements fractions of the compositions were estimated through the values exhibited in Table 1. These values were determined as half of the difference between P5050 and C5050 estimations of the bTEM's compositions. One relative uncertainty was estimated for each element in the mixture, as exhibited in Table 3.

Table 3. Relative variation between Poletti's estimation and CIRS manual data of the fraction by weight of each element in the CIRS bTEM's composition.

Glandularity	ΔH (%)	ΔC (%)	ΔN (%)	ΔO (%)	ΔCl (%)	ΔCa (%)
30/70	0.21	0.26	23.17	10.94	50.00	7.14
50/50	2.16	1.55	7.72	22.93	50.00	27.61

Because White's estimation composition was used by NA in the W5050 manual, no composition difference was found between the manufacturer's manual and White's estimation, as expected. As no composition uncertainty could be estimated thereby, the same composition uncertainties estimated for CIRS' 50/50 glandularity bTEM were considered to NA's BR12 kit. Therefore, the herein estimated uncertainties are expected to be less accurate for the NA's bTEM.

The MCMC propagation method was done for every set of parameters, such as anode/filter combination, voltage, bTEM thickness, and bTEM glandularity. Each propagation generated a variance vector, which related to that transmitted spectra. This variance was then combined with the spectra uncertainty, as follows:

$$\sigma_{sim}^2 = (\sigma_{input}'^2 - \sigma_{poisson}^2) + (\sigma_{composition}'^2 - \sigma_{poisson}^2) + \sigma_{poisson}^2. \quad (30)$$

The Poisson's variance presence is indicated by the prime symbol in each uncertainty term of equation (30), where σ_{sim}^2 indicates the variance of the simulated transmitted spectra, and the right-hand side variances account for each uncertainty source being considered. The variance $\sigma_{poisson}^2$ accounts for the intrinsic variance associated with any MC method spectrum estimation. The variances $\sigma_{input}'^2$ and $\sigma_{composition}'^2$ represent, respectively, the variances estimated by MCMC due to the input spectra variance, and the variance due to bTEM's composition. The Poisson's variance was present in both MCMC estimations. Therefore, it was subtracted to account for covariance. Hence, equation (30) simplifies to

$$\sigma_{sim}^2 = \sigma_{input}^2 + \sigma_{composition}^2 + \sigma_{poisson}^2 \quad (31)$$

where the absence of the prime indicates that Poisson's uncertainties were already subtracted from the MCMC estimates. Equation (31) highlights the three composing terms of the assessed uncertainty.

Altogether, 5400 spectra were generated. The standard deviation calculation was done with sets of 25 spectra. This calculation yielded 108 pairs of uncertainty vectors. One vector related to the input's and the other to the composition's uncertainties. The uncertainty vectors were also formatted to be used in the comparison, as explained in section 3.5.1.

3.5. X-ray Spectra analysis

3.5.1. Correspondence of energy and intensity scales for comparison of the simulated and experimental spectra

Energy scales

The experimental acquisition system organizes the experimental spectra in a text file that records the number of counts in each energy channel. PENELOPE records: the energy from the beginning and from the middle of each energy channel, the number of counts of each channel divided by the bin size and by the number of simulated histories, and the Poisson's standard deviation of the simulated spectra.

The energy range of the experimental spectra was 0 to approximately 57 keV divided into 1024 channels. The simulated spectra's energy range was from 5 to approximately 33 keV distributed in 526 channels. The very low ($< 10 \text{ keV}$) and very high ($> 33 \text{ keV}$) energy bins had none, or a very small number of counts (mainly attributed to the electronic noise in experiments). As these energies were of no interest for the comparison, they were disregarded. The considered interval for the analysis was limited by the upper and the lower energies of interest. The superior energy was determined as 1.5 keV above the spectrum maximum energy; which was defined by the voltage

applied in the X-ray tube (see Table 2). It was graphically observed that the additional 1.5 keV added accounted for fluctuations in the expected maximal energy of the spectra. The inferior energy was graphically determined for each spectrum as the point below which only noise was identified on the experimental spectra.

The simulation parameters were set to assure that the experimental and the simulated spectra would be similar. For instance, the energy channel width of the simulated spectra was chosen with the same width of the experimental ones, which was of 0.0555(2) keV.

intensity scales

Each spectra pair (experimental and simulated) had a different height and hence area. Therefore, it was necessary to determine a numerical relationship between both spectra in a pair, to equalize their fluences. This procedure is called normalization of the spectra. The normalization was done by assuming a multiplicative scalar constant $\widehat{\alpha}_0$ that, multiplied by the simulated spectrum counts vector $\vec{C}_s$, would normalize the pair.

To estimate $\widehat{\alpha}_0$, first a vector $\vec{\alpha}$ was calculated by

$$\vec{\alpha} = \vec{C}_e \oslash \vec{C}_s \quad (32)$$

where $\vec{C}_e$ is the vector of the counts of the experimental spectrum and $\oslash$ denotes the element-wise division of the vectors. Then, the expected value of $\vec{\alpha}$ was estimated by the mean

$$\langle \vec{\alpha} \rangle = \alpha_0 = \frac{1}{N} \sum_{i=1}^N \vec{\alpha} \quad (33)$$

where N is the number of channel in the spectra, hence the number of elements in $\vec{\alpha}$.

The goal of this procedure was to find the condition for which the differences between the experimental and simulated spectra areas was minimal. Therefore $\widehat{\alpha}_0$ was obtained, by solving the following equation

$$\frac{d}{d\alpha_0} \left[\sum_{i=1}^N (C_{e_i} - \alpha_0 C_{s_i})^2 \right]_{\widehat{\alpha}_0} = 0 \quad (34)$$

The behavior of the sum due to α_0 variation was parabolic. The minimum of this sum was numerically found for each spectra pair. The minimum of the parabola defined $\widehat{\alpha}_0$. It is important to notice that α_0 is different from $\widehat{\alpha}_0$. While the first one is the result of the averaging procedure described in equation (33), the second one is a numerical optimization, that uses α_0 as the initial guess.

Because of the transformation in the simulated spectra area, this work's spectral comparisons are limited to spectra's morphology assessment. However, a quantitative comparison between the spectra can be done through the comparison of $\widehat{\alpha}_0$. The $\widehat{\alpha}_0$ constant can be interpreted as a relation between the radiation yield of the emitter and the number of histories in the simulation. This analysis was not in this work's scope.

The herein described steps were considered sufficient to prepare the spectra pairs to the comparison. All analysis hereafter described were done after the described formatting. Altogether, 108 spectra pairs were formatted and compared

3.5.2. Spectra derived quantities

The simulated and experimental X-ray spectra of this work were compared using 5 different quantities:

- i. Mean Energy
- ii. 1st Half-Value Layer
- iii. 2nd Half-Value Layer
- iv. Effective Energy
- v. Mean Weighted squared residuals

The first four ones are spectra derived quantities. This section explains the methodology used when comparing spectra by means of these quantities. The following section uses the MWSR to perform the comparison, which is a measure of the spectra similarity considering every channel in the spectra.

Three spectra derived quantities were assessed for each spectrum: the mean energy (ME), the effective energy (EE) and the aluminum half value layers (HVL). These quantities were chosen

because of being recommended to assess x-ray beam quality in calibration procedures (Pernicka and McLean 2007). It is assumed that, regarding dosimetry purposes, the similarity between these quantities implies a similarity between the spectra. The similarity was assessed through the Z hypothesis test; a significance level of 0.01 was adopted (Vanin et al. 2005).

The ME was estimated through equation (21). To calculate the HVL from the measured spectra, it was first necessary to calculate the air-kerma, which was estimated as follows (Johns and Cunningham 1983):

$$K = \sum_{i=1}^{N_{ch}} E_i \cdot \Phi_i \cdot \left(\frac{\mu_{tr_i}}{\rho} \right)_{air} \quad (35)$$

where Φ is the photon fluence and $\left(\frac{\mu_{tr}}{\rho} \right)_{air}$ is the mass energy-transfer coefficient of air, as described in section 2.3.2. The photon fluency (Φ) is related to the photon counts in the spectrometer (C) by a constant with area dimension (A) by $\Phi = C/A$. Therefore,

$$K = \frac{1}{A} \sum_{i=1}^{N_{ch}} E_i \cdot C_i \cdot \left(\frac{\mu_{tr_i}}{\rho} \right)_{air} \quad (36)$$

The HVL characterizes the radiation spectra with a set of two values: the 1st and 2nd HVL . They are henceforth referred to as $HVL1$ and $HVL2$ and were calculated by numerically solving equation (37).

$$\frac{1}{O_r} K = \sum_{i=1}^{N_{ch}} E_i \cdot C_i \cdot \left(\frac{\mu_{tr(E_i)_i}}{\rho} \right)_{air} \cdot e^{-\left(\frac{\mu_{(E_i)_i}}{\rho} \right)_{Al} \cdot \rho_{Al} \cdot HVL} \quad (37)$$

where O_r is 2 when estimating $HVL1$, and $\left(\mu_{(E_i)} / \rho \right)_{Al}$ is aluminum's total attenuation coefficient for energy E_i . When estimating $HVL2$, O_r was set to 4 in equation (37), and the value of the $HVL1$ was discounted from the obtained result. The attenuation and energy-transmission coefficients curves were linear interpolations of values extracted from the National Institute of Standards and Technology (NIST) XCOM software (Berger et al. 2010; Hubbell and Seltzer 1996).

The spectra's effective energies (EE) were estimated using the calculated HVL . The estimation consisted of calculating values of HVL for monoenergetic beams in the energy range of interest. The considered calculation ranged from 14 to 36 keV in a 0.05 step and their HVL s were estimated by the afore described method. The HVL estimation yielded HVL s of 0.03 to 0.36 mm of Al. the effective energy related to the spectrum was estimated by linear interpolation of the curve of HVL by energy.

The uncertainties of the ME , HVL and EE were numerically propagated. Each of these estimations was performed 100 times with variation of the input data. The input data for the HVL and the ME calculations was the energy spectrum, and the varied quantity were the counts in each channel. For the EE calculation, the input data was the spectrum's HVL . The input data varied within their uncertainties, following normal distributions. The standard deviation of the output was considered as the uncertainty.

3.5.3. Mean Weighted Squared residuals

It is well known and documented that the HVL is a quantity representative of the spectrum. However, the HVL has certain limitations to fully characterize the spectrum. The limitation was listed by Attix (Attix 2004):

“(i) Pure aluminum or copper must be used as the attenuating medium, Al being preferred. (ii) Narrow-beam geometry is required, i.e. scattered rays from the attenuator must not reach the detector. (iii) Lastly, the detector must be air-equivalent, i.e. must give a constant response per unit of exposure, independent of photon energy.” Attix's statement was slightly adapted.

Experiments commonly do not meet these requirements, as is this work's case. The first requirement does not apply as the HVL was not experimentally determined, but calculated from spectra. The third requirement was not fulfilled because a spectrometer was used as radiation sensor and not an ionization chamber. The second one failed because broad beam geometry was used. This leads us to believe that there are inaccuracies when using HVL to represent the beam.

As Attix also stated, “specification of the spectrum is the most rigorous mean of describing the X-ray beam quality”. Therefore, the mean weighted squared residual (*MWSR*) was used as a spectra comparison metric. This metric compares all channels of a given spectrum with the correspondent one measured or simulated. It was considered as a more rigorous mean of X-ray spectra characterization. The definition of the *MWSR* is very similar to the definition of the function Q^2 (Vanin et al. 2005). This terminology was, however, not adopted because, Q^2 is commonly used when comparing an experimental set of data and an analytical function, and this work’s comparison was done between two discrete set of data: pairs of experimental and simulated x-ray spectra. In this work, the simulated set was interpreted as the theoretic model to which the experimental data is compared. The *MWSR* was defined as

$$MWSR = \frac{1}{N} \sum_{i=1}^N \frac{(C_{ei} - C_{si})^2}{\sigma_{exp}^2 + \sigma_{sim}^2} \quad (38)$$

where σ_{exp} is the uncertainty of the experimental spectrum and σ_{sim} is the uncertainty of the simulated spectrum. The uncertainties of the simulated spectra were estimated through Monte Carlo uncertainty propagation technique, which was explained in section 3.4. The experimental spectra considered uncertainties were the Poisson distribution standard uncertainties, as each count number in each channel of a spectrum follows the Poisson probability distribution (Vanin et al. 2005).

3.6. Thermoluminescent dosimeter dose estimation method

The TLDs used in Experiment B were arranged in cross configured layers to assess field inhomogeneity. This configuration is shown in Fig. 19. Each TLD provided a dose (D) estimate. It was found that the dose deposition in one layer was homogeneous. Therefore, the dose deposited in each layer of the bTEM block (D_L) was estimated by averaging over the 9 dosimeters of the layer.

$$D_L = \frac{1}{9} \sum_{i=1}^9 D_i \quad (39)$$

There were five TLD layers, each one in a different depth. Therefore, the five D_L values determined an attenuation curve (depth dose profile) with respect to bTEM thickness.

The energy deposited in the TLD can be translated to dose by knowing the TLD dimensions and its density. This conversion was done for the experimental data, as explained in section 2.4.3. The simulated doses were also normalized as described in section 3.5.1.

3.7. Statistical analysis of the results

3.7.1. Z hypothesis test

The Z hypothesis test was used in this work to assess compatibility between random variables related to X-ray dosimetry. These random variables were related to the X-ray spectra, and thus were used to assess spectra compatibility. The test was performed for all five statistics estimated from the spectra, i.e. ME , EE , $HVL1$, $HVL2$ and $MWSR$. For the $MWSR$, the reference value, x_{ref} , was the expected value, which is equal to 1. for all other statistics, the simulated data were tested against the experimental data. Every time the Z test was used, the null hypothesis (H_0) was that the tested spectra were compatible. The Z scores for each quantity are denoted: Z_{MWSR} , Z_{ME} , Z_{HVL1} , Z_{HVL2} and Z_{EE} .

As explored in 2.5, the determination of the critical value is arbitrary. Also, spectra comparison in the literature does not contain any hypothesis testing. Therefore, section 5.3 presents a discussion about which critical value to choose.

3.7.2. Correlation coefficient

The correlation coefficient (ρ) is an index which quantifies the linear relationship between a pair of variables (Everitt 2006). It can be estimated for n observations of two variables, a and b , as:

$$\rho(a, b) = \frac{\sum_{i=1}^n (a_i - \bar{a}) \cdot (b_i - \bar{b})}{\sqrt{\sum_{i=1}^n (a_i - \bar{a})^2 \sum_{i=1}^n (b_i - \bar{b})^2}} \quad (40)$$

where the bar sign denotes the variable's mean.

Correlation coefficient was used in this work to determine the relationship between the five different estimated statistics. A correlation matrix was constructed using the five statistic tests. ρ takes values in the range of -1 to 1. When the module of the correlation coefficient between two tests is elevated, it means that the tests are considered equivalent. In other words, correlated tests give the same information about a tested spectra pair.

3.7.3. Jarque-Bera test

The Jarque-Bera (JB) test (Jarque and Bera 1987) consists of performing statistical hypothesis tests with kurtosis and skewness (Everitt 2006; Vanin et al. 2005), which are parameters estimated from an arbitrary distribution, to assess the probability of this distribution being normal. In this work, the JB test was used to assess the probability that the Z-test results are distributed according to a normal PDF. If the Z-test results distribute normally, it means the variables involved in the test also distribute according to a normal PDF. The JB test was implemented through a native function of the MatLab software (Mathworks, Natick, Massachusetts, EUA).

3.7.4. Power of the test

In the hypothesis test's framework, the concept of power of the test (H) is defined as: the probability of rejecting the null hypothesis when it is wrong (Everitt 2006). It is represented as

$$H = 1 - \beta \quad (41)$$

where β is the probability of committing a type-II error (Vanin et al. 2005).

The null hypothesis adopted in this work was that the tested spectra pair is compatible. In this context, power of the test means: the probability of concluding that the spectra are not the same

when they truly are not. For example, when comparing two spectra generated with different voltages, e.g. 26 and 31 kV, the probability of concluding that they are not compatible is the power of the test.

The power of the used tests was assessed by means of random testing of a great number of spectra. To do this, a set of 108 auxiliary spectra was constructed. The experimental spectra were used as a basis, and the counts were varied in each channel of each spectrum within their uncertainty range, following a normal distribution. By construction, it is expected that each spectrum in the set of auxiliary spectra is compatible with its relative spectrum from the original experimental spectra set. The test consisted of randomly selecting two spectra and testing for compatibility. If the random sample resulted in the selection of the corresponding spectra of each set, then the null hypothesis should be retained. For all other pairing possibilities, H_0 should be rejected. This procedure was repeated for 1000 times.

If the null hypothesis was correctly rejected, 1 point was credited to the test. If it the hypothesis wrongfully retained (Type II error, false positive), then 1 point was discounted from the test. No point was given or taken from the statistic when the $|Z|$ value lied between 2.8 and 3.2.

4. RESULTS

4.1. X-ray spectra analysis

4.1.1. A representative set of the obtained spectra

This work generated a great number of spectra, either by MC method or by direct measurement. Not all spectra will be exhibited in the present document. Nevertheless, all spectra can be found [online \(https://1drv.ms/f/s!AqdQuA6yL3yhiJByObg8WMLBA1mJjg\)](https://1drv.ms/f/s!AqdQuA6yL3yhiJByObg8WMLBA1mJjg). A representative set of all the generated spectra was chosen and is exhibited in Fig. 24. The diagram of Fig. 23 shows which combination of experimental/simulated conditions was chosen for each of the representative spectra. All used voltages, bTEMs and anode/filter combination are represented in Fig. 24. Not all used bTEM attenuation thicknesses are represented, but the represented set is representative of all the spectra, because the chosen thicknesses range from the lowest to the highest and are distributed as equally as possible.

Tension (kV)	26	28	31															
bTEM	W5050	C5050	C3070															
Anode/Filter	Mo/Mo	Mo/Rh	W/Rh	W/Mo														
Thickness (mm)	0	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85

Fig. 23. Diagram showing the chosen experimental/simulated conditions for each of the representative spectra of Fig. 24. All used tensions, bTEMs and anode/filter combination were used. Not all used bTEM attenuation thicknesses are represented, but the represented set is representative of all the spectra.

It is noticeable that the anode/filter combination has deep influences in the spectra morphology. This is due to variations in fluorescence emission from the anode, and in attenuation discontinuities occasioned by the filter's element absorbing edge.

Fig. 24. A representative set of all the generated spectra. The diagram of Fig. 21 shows which combination of experimental/simulated conditions was chosen for each of this figure's spectra. All used tensions, bTEMs and anode/filter combination are represented. Not all used bTEM attenuation thicknesses are represented, but the represented set ranges from the lowest to the highest attenuation thickness which are distributed as equally as possible. The experimental data are plotted in black, and the simulated as a red line. The experimental and simulated uncertainties were combined and are plotted as black bars associated to the experimental points. The lower subplot of each spectra comparison exhibits the reduced residuals of the comparison.

Another noticeable feature is that the channel's counts associated uncertainties are relatively larger for the spectra which had the greatest attenuations, this occurred because the number of simulated histories was kept constant between simulations.

4.1.2. Monte Carlo variance propagation

The uncertainty of the counts in a channel was determined by three different contributions: the one propagated from the uncertainties in the input spectrum's channels (input), the one propagated from the uncertainties of the bTEM's compositions (composition) and the Poisson's uncertainty inherent from any spectrum simulation. The relative contribution of each uncertainty component ($\dot{\sigma}_k$) was evaluated for all spectra. It consisted of the sum of all channels' uncertainties divided by the sum of the spectrum's counts.

$$\dot{\sigma}_k = \frac{\sum_{i=1}^N \sigma_k}{\sum_{i=1}^N C_i} \quad (42)$$

where the sub-index k is a placeholder for one of the three components stressed out in equation (31) and N is the number of channels in the spectrum and C_i is its i^{th} channel.

Fig. 25 and Fig. 26 show the results showed that the Poisson uncertainty contribution was the smallest for every case. As the number of simulated histories was fixed, Poisson's uncertainty ($\sqrt{N}$) grew with the increase in attenuation. For more than 10 mm of attenuation by the bTEM, the uncertainty relative to the composition was dominant. As expected, the composition component was null when there was no bTEM attenuation. The uncertainty propagated from the input spectra was relevant in all cases, and practically constant. This was true for all simulation conditions, such as bTEM manufacturer, radiation source, anode and filter compositions.

Fig. 25. Relative uncertainties evaluated for the 50/50 NA BR12 kit, irradiated by a WMo beam, and with variate bTEM attenuations.

Fig. 26. Relative uncertainties evaluated for the 30/70 glandularity CIRS 012a bTEM, irradiated by the MoRh beam, and with variate bTEM attenuations.

4.1.1. Input Spectra comparison

The input spectra used in this work were compared, by means of the Z-test, with the MC generated spectra. In these simulations, no bTEM attenuation was used. The results of the Z-scores are exhibited in Fig. 27. The dashed line represents the expected value of the Z-scores. There were eight different input spectra, one for each voltage/anode/filter combination. A legend can be seen in the lower portion of the figure, containing the configurations for each spectrum. The x axis contains labels that indicate which quantity was used in the hypothesis test.

Fig. 27. Representation of the Z-scores obtained when testing the input spectra. The dashed line represents the expected value of the Z-scores. There were one spectra for each voltage/anode/filter combination. The legend indicates the configurations for each spectrum. The labels in the x axis indicate which quantity was used in the hypothesis test.

4.1.2. Mean Energy

The mean energy of each transmitted spectrum was estimated as described in equation (21). A pair of ME values was estimated for each spectra pair comparison, for all used bTEM, voltages and attenuation thicknesses. These values are exhibited in Fig. 28. As the radiation beam is hardened by the attenuation of the bTEM, the mean energy of the transmitted spectrum grows. The ME values of the simulated and experimental spectra are expected to overlap. This was not always observed, what can indicate a certain level of incompatibilities. The quantitative assessment of this data is going to be thoroughly performed in the next subsections.

4.1.3. Half Value Layers

Other quantities derived from this work's transmitted spectra were the $HVL1$ and $HVL2$. They were derived solving equation (37). Both $HVL1$ and $HVL2$ also increase with beam hardening and are exhibited in Fig. 29 and Fig. 30 respectively. In some of the plots, all simulated quantities were higher than the experimental. The opposite happened for some other plots. This shows that there were no global tendencies in the data, neither for the $HVL1$ nor for the $HVL2$. Despite no global tendencies were noted, local tendencies occurred. This means, for some plots, all simulated points were higher than the experimental, or vice-versa.

4.1.1. Effective Energy

The effective energy was assessed as described in section 3.5.2 for all transmitted spectra pairs. Fig. 31 shows all effective energies calculated in this work. Two points were generated for each comparison. Assuming identical spectra, it is expected that the two points will overlap. Not all pairs of compared data are compatible. The effective energy, as the mean energy, increases with the radiation beam's hardening. The cases where the experimental effective energy was higher than the simulated EE were more common. Nonetheless, some exceptions occurred, where the simulated EE was higher than the experimental.

Fig. 28. Results of the mean energy comparison. The yellow markers represent the simulated results and the green markers represent the experimental results. 31 kV data is represented by triangles pointing up, while 28 kV data is represented by circles and 26 kV data is denoted as triangles pointing down. The yellow and green markers may be superimposed where simulated and experimental data have great similarity. The uncertainties are smaller than the size of the markers.

Fig. 29. Results of the HVL1 comparison. The yellow markers represent the simulated results and the green markers represent the experimental results. 31 kV data is represented by triangles pointing up, while 28 kV data is represented by circles and 26 kV data is denoted as triangles pointing down. The yellow and green markers may be superimposed where simulated and experimental data have great similarity. The uncertainties are smaller than the size of the markers.

Fig. 30. Results of the HVL2 comparison. The yellow markers represent the simulated results and the green markers represent the experimental results. 31 kV data is represented by triangles pointing up, while 28 kV data is represented by circles and 26 kV data is denoted as triangles pointing down. The yellow and green markers may be superimposed where simulated and experimental data have great similarity. The uncertainties are smaller than the size of the markers.

Fig. 31. Results of the effective energy comparison. The yellow markers represent the simulated results and the green markers represent the experimental results. 31 kV data is represented by triangles pointing up, while 28 kV data is represented by circles and 26 kV data is denoted as triangles pointing down. The yellow and green markers may be superimposed where simulated and experimental data have great similarity. The uncertainties are smaller than the size of the markers.

4.1.2. Mean Weighted Squared Residuals

One mean weighted squared residual value was assessed for each spectra pair by (33). The results are shown in Fig. 32. The dashed line in the plots represents the expected value of this quantity. Some points clearly stand out as outliers. The outlier points were removed from the plots for better visualization of the rest of the points. The new visualization is exhibited in Fig. 33. All the spectra pairs related to the outlier data are exhibited in

Fig. 48 and a thorough discussion about them is performed in Appendix B. Furthermore, the outlier data is exhibited separately at Fig. 34.

The estimation of the $MWSR$ is strongly dependent of the considered uncertainties. As stated in section 3.4 the uncertainties estimated were expected to be less accurate for the NA's BR12 kit. This can be observed in Fig. 32 plot (g). It is noticeable that there is a pattern in the distribution of the data points around the expected value. Furthermore, the data points are more distant from the expected value than those of CIRS 012A bTEM.

To analyze the pattern of the $MWSR$ distribution, two histograms of all $MWSR$ results were plotted (Fig. 35). The histograms were constructed by using all the 108 $MWSR$ values calculated from the measured and simulated spectra pairs. Two uncertainty scenarios were considered when constructing the histograms: using only Poisson's uncertainty for the spectra channel, and using the uncertainties assessed by this work with MCMC procedures, as described in section 3.4. In the histograms, the mean was assessed with and without the outlier data. The result improved without using outliers.

Fig. 32. Results of all the estimated $MWSRs$. The dashed line represents the quantity's expected value. 31 kV data is represented by yellow triangles pointing up, while 28 kV data is represented by green circles and 26 kV data is denoted as blue triangles pointing down. The uncertainties not shown are smaller than the size of the markers. The outlier points are labeled by lowercase letters from a to h.

Fig. 33. Results of the estimated $MWSRs$, excluding the outliers. The dashed line represents the quantity's expected value. 31 kV data is represented by yellow triangles pointing up, while 28 kV data is represented by green circles and 26 kV data is denoted as blue triangles pointing down. The uncertainties not shown are smaller than the size of the markers.

Fig. 34. Results of outlier *MWSR*s data. 31 kV data is represented by yellow triangles pointing up, while 28 kV data is represented by green circles. The points are labeled by lowercase letters from a) to h) following the same denomination of Fig. 32.

Fig. 35. Histogram of all *MWSR* calculated in this work. The estimation of the left-hand side histogram was performed with Poisson's uncertainty. The estimation of the right-hand side histogram was performed with the herein described MCMC variance propagation method. The mean and mean uncertainty were assessed with and without the outlier data. They are indicated respectively by a dashed and a solid line. Their values are written inside a text box.

4.2. Dose deposition analysis

In experiment B, dose in between the bTEM block was assessed via TLD measurements. The measurement results are exhibited in Fig. 36. It is noticeable that no tendency was observed in the deposited energy with respect to the x (TLDs number 4, 8 and 9) or y (dosimeters 1-7) axes. Therefore, the radiation field of experiment B was considered homogenous within the analyzed area, which was approximately of the size of the bTEM plates' surface.

From the Fig. 36, it is noticeable that the layers are visibly apart from each other. This means that experimental procedure and the MC simulation were able to distinguish the dose deposited in the TLDs from each layer. Furthermore, the variance of the experimental measurement was higher than that of the simulated. The dose in each layer, and its uncertainty were estimated from data exhibited in Fig. 36 by means of average and standard deviation, respectively. The attenuation curves which resulted from this data are exhibited in Fig. 37. From the reduced residuals, it can be noticed that experimental and simulated data agreed within the considered uncertainties.

XY Dose Distribution

Fig. 36. Dose assessed in each of the 9 TLDs irradiated in experiment B. The XY Distribution of the dose deposited in each dosimeter can be observed. The five different TLD layers are indicated by blue arrows in plot (B) and can be clearly observed in all plots. The dosimeter numbering refers to the numbering showed in Fig. 19.

Fig. 37. Plots of dose deposition exhibiting bTEM thickness attenuation, measured by the TLD method. The reduced residuals for each case are exhibited below each plot. The uncertainty bars are of the size of the data points.

4.3. Results from the statistical analysis

4.3.1. Z-test assessment of normality – TLD dose measurement

The use of the Z hypothesis test to compare the data exhibited in Fig. 37 yielded the histogram shown in Fig. 38.

Fig. 38. Distribution of the Z-Tests performed with the five different quantities used to assess transmitted spectra compatibility. The module of the Z-test was not calculated. The mean ($\bar{Z}$) is indicated as a red line. The standard deviation (σZ) and the total number of data (N) are also shown.

The mean ($\bar{Z}$) is indicated as a red line. The standard deviation (σZ) and the total number of data (N) are also shown in the figure. The compatibility analysis of these values was done and will be presented and discussed in section 5.3.

4.3.2. Z-test assessment of normality – Transmitted Spectra

The distribution of the Z-Tests relative to the 108 spectra pairs is exhibited in Fig. 39 (including the input spectra). These distributions are similar to normal distributions. The p-value (section 2.5) of the JB test is denoted as p in Fig. 39. It is interpreted as the minimum significance level (α) which leads to normality conclusion considering a JB test (Jarque and Bera 1987). For instance, the value $p = 0.06$ in plot (a) means that the hypothesis that the distribution is gaussian can be accepted with

a confidence interval of $1 - 0.06 = 94\%$. The module of the Z-tests was not calculated for the construction of the histograms. Therefore, two tailed distributions are observed.

Fig. 39. Distribution of the Z-Tests performed with the five different quantities used to assess transmitted spectra compatibility. The p-value, p , for hypothesis acceptance in a JB test is indicated within each plot. The module of the Z-test was not calculated.

4.3.3. Estimated Powers of the tests

The method described in section 3.7.4 aims to assess the power of the tests given the different used variables. The sum of all the 1000 results obtained from the procedure of section 3.7.4 are shown in Table 4.

Table 4. Results from the 1000 tests done with the a priori assumption that matching spectra pairs should pass the test, and all other should not. The total possible score was 1. The estimated significance level by the test is shown in the second row.

	zMWSR	zME	zEE	zHVL1	zHVL2
H	0.998	0.982	0.977	0.977	0.973
$\hat{\alpha}$	0.002	0.018	0.023	0.023	0.027

The results presented in Table 4 show that the variable *MWSR* was the one with which the test failed the smallest number of times. All other tests resulted in more frequent type-II errors, i.e. they

more frequently retained the hypothesis where it should be rejected. We therefore conclude that the Z-test done from the MWSR quantity has got the highest power.

5. DISCUSSIONS

5.1. MCMC uncertainty estimation

The herein described MCMC uncertainty evaluation method was of major importance to the compatibility of the comparative results. This importance could be observed by the differences between the two histograms presented in Fig. 35. When only Poisson's uncertainty was considered, the uncertainty of the simulated spectra's channels was greatly underestimated. The underestimation resulted in a *MWSR* value 230% higher than its expected value, whilst the mean calculated with full uncertainty consideration was only 4.2% higher.

Regarding uncertainty estimation, the experimental spectra uncertainties were limited to Poisson's uncertainty for each channel's counts. Some preliminary results of the GDRFM suggest that Poisson's uncertainty underestimates the uncertainty of the experimental spectra's channels by approximately 20%. This probably happens due to other uncertainty sources such as the spectrometer alignment with the radiation field.

5.2. MWSR outliers

In Fig. 32 some outlier data are observed. Several reasons explain the elevated *MWSR* of the outliers. In plot (h), the fluctuation observed in the experimental spectrum endpoint occasions the elevated *MWSR* value. For plots (a) to (g), there is a systematic phenomenon responsible for the elevated *MWSR*. In these plots, there is a greater number of counts in the final portion of the experimental spectra. This phenomenon is observed in many spectra measured by the GDRFM. It is currently subject of a joint study of the group. Preliminary results suggest that it may be explained by the scattering of photons from the primary radiation field by the metal plates that enclose the spectrometer's electronics (Fig. 20). Further investigation is necessary to draw any reliable conclusions. These investigations are outside of the scope of this work.

Excluding the outlier (h), all other outliers are relative to the CIRS 012A 30/70 glandularity bTEM. Also, they are relative to the greatest bTEM thicknesses (20, 30 and 40 mm). This leads to the hypothesis that the 30/70 glandularity composition may be inaccurately informed (Table 1). Because, as the attenuation thickness increased, so did the discrepancy between simulated and experimental spectra. In fact, 61% of the CIRS 012A 50/50 glandularity comparisons was accepted, against only 50% of the 30/70 glandularity.

Another hypothesis can be formulated the voltage of the outlier spectra (a) to (g) were the same, namely 28 kV. The hypothesis is that the phenomenon that elevates the final portion of the experimental spectra is positively correlated with the channel's energy. This means, it increases proportionally to the energy. This kind of behavior is compatible with scattering due to Compton effect (7). Therefore, the increase of the energy implies in an endpoint's increase, leading to a more noticeable discrepancy. Thus, observation of this phenomenon was more evident for the higher endpoints of the 28 kV spectra.

5.3. Comparing variables: critical value, power, correlation and hypothesis acceptance

The choice of the critical value was arbitrary. Therefore, Fig. 40 supports a discussion about the behavior of the acceptance percentage depending on the critical value's choice for every five variables used to perform the hypothesis tests of this work.

Fig. 40 shows that the increase in the acceptance percentage is approximately linear for critical values from 1 to 5. After 5, the acceptance increase starts to get smoother. Before 1, it tends to be steeper. This is because of the Gaussian nature of the Z-tests distributions, as seen in Fig. 39. Therefore, we conclude that it is reasonable to set the critical value between 1 and 4. Any number lower than one would result on a highly rigorous test. Any choice bigger than five results on a too permissive test, i.e. almost all spectra pair would be accepted for all five tests (~75%).

Fig. 40. behavior of the acceptance percentage depending on the critical value's choice for every five variables used to perform the hypothesis tests of this work. The specific test is denoted inside the plot area.

It seems reasonable, because of the shape of the curves in Fig. 40, to set the acceptance region as follows: if the absolute value of Z ($|Z|$) scored less than 2.8, then the spectra were considered compatible and H_0 was accepted (Z_{acc}). If $|Z|$ scored more than 3.2, they were considered incompatible and therefore H_0 is rejected (Z_{rej}). If $|Z|$ scored between 2.8 and 3.2, no conclusion was drawn from the test (Z_{nc}).

$$Z_{acc} < 2.8 < Z_{nc} < 3.2 < Z_{rej} \quad (43)$$

it is important to stress that any critical value within the 1 - 5 interval is reasonable to perform a comparison between the variables used in the tests. This is because the shapes of all curves in Fig. 40 are similar.

In this work, when a Z-test was used, it was considered that the variable of interest was distributed normally. This hypothesis was also tested (Fig. 39) by means of the Jarque-Bera test (section 3.7.3). By considering a normal distribution, our confidence interval ($-2.8 < Z < 2.8$) was of 99.49%.

Regarding the power of the used variable, the results presented in Table 4 show that the *MWSR* was the variable that led to the smallest number of times when the test failed. Furthermore, the correlation of the Z-test results using *MWSR* with Z-test results using other variables was low. This means that the *MWSR* cannot be substituted by (is not equivalent to) another variable, regarding the Z-test. The correlation coefficients are showed in Table 5. They were estimated using the 108 Z-test values of all compared spectra. Correlation between Z-tests using *HVL1*, *HVL2*, *ME* and *EE* was elevated, indicating those variables are almost equivalent. A correlation matrix was constructed (Table 5) using the five statistic tests. ρ takes values in the range of -1 to 1. When the module of the correlation coefficient was elevated, the tests were considered equivalent.

Table 5. Assessment of the correlation between the used statistical tests Z value for all spectra pairs.

	zMWSR	zHVL1	zHVL2	zME	zEE
zMWSR	1	0.2375	0.435	0.4849	0.241
zHVL1	0.2375	1	0.9334	0.9109	0.9974
zHVL2	0.435	0.9334	1	0.9729	0.9321
zME	0.4849	0.9109	0.9729	1	0.9112
zEE	0.241	0.9974	0.9321	0.9112	1

Fig. 41. Bar plot of the percentage of acceptances and rejections of the null hypothesis by each one of the used variables in the hypothesis test. The vertical black line separates the results relative to transmitted spectra from the results relative to TLD measurements.

Fig. 41 exhibits the percentage of accepted hypothesis H_0 , rejected hypothesis, and inconclusive tests. The *ME* variable was the one with which the test resulted in the least acceptance rate. It is

noticeable that the behavior of the tests done with the *HVL1* and the *EE* variables was very similar. This is to be expected because of the correlation in the definition of these two quantities. The *HVL2* variable showed a greater number of hypothesis acceptance, but the variable which made the test differed greatly from the others was that from the *MWSR* test. The test done with the *MWSR* variable was also the one which lead to a greater number of accepted hypothesis tests (48%). If one considers that this variable was also the one that resulted in the greatest statistical power for the hypothesis test, the fact that this variable led to the most hypothesis acceptance shows that the other tests underestimate the quality of the MC method in reproducing transmitted spectra in this work's conditions. Furthermore, It can be noticed that the dose measurements via TLD led to greater acceptance, when compared to all tests done with transmitted spectra. The acceptance rate was of 92%.

5.4. Experimental/simulated conditions relevance

The *MWSR* test results were used to assess if there were any trends in the acceptance regarding the experimental/simulational conditions: Btem's glandularity, anode/filter combination, tension and bTEM's thicknesses. The obtained results are shown in Fig. 42. Each bar in Fig. 42's plot represents the accepted spectra percentage out of the total spectra for which the parameter was applied. For instance, only 18 of all 108 assessed spectra pairs were generated or measured with a 31 kV voltage condition. 9 spectra pairs, out of these 18, were compatible. Therefore, the bar plot of the 31 kV parameter shows 50% compatibility. The uncertainty was estimated following a binomial distribution expected variance

$$\sigma_{Bin} = \sqrt{Np(1 - p)} \quad (44)$$

where N is the total number of observations and the probability p was considered to be the calculated value. This means that for the 31 kV example, p was considered as 0.5.

Fig. 42. Bar plot of the compatibility percentage for the *MWSR* test discriminated by the used parameters: glandularity in green, anode filter combination in yellow, voltage in gray and thickness in blue. The uncertainty was estimated following a binomial distribution expected variance.

5.4.1. bTEM thicknesses

Regarding the attenuation thicknesses, the compatibility percentage was smaller ($\sim 25\%$) for high bTEM attenuation thicknesses than for small attenuation thicknesses ($\sim 46\%$). This means that the accuracy of the MC code diminishes as the quantity of TEM attenuation it must simulate rises. Also, the percentage of input spectra (0 mm) that was compatible is remarkably low ($\sim 7\%$). This is due to overestimation of the uncertainties. This conclusion is drawn from the low values of the MWSR points in Fig. 27. For all the input spectra used in experiment B, the underestimation probably occurred because the correlation between the Poisson uncertainty and the MCMC uncertainty propagated from the input spectra was significant in these cases. This correlation was not considered in this work. This estimation is suggested as a possible improvement for the developed method, mainly when there is special interest in the simulation of low attenuation transmitted spectra. For the two input spectra of experiment A, the Z-score are elevated, in contrast with the other values.

This indicates that the compatibility between the spectra simulates and the ones generated by the industrial tube was smaller than for the mammography equipment.

5.4.2. Anode/filter

A standard deviation of $\sim 5\%$ was seen regarding W/Rh, Mo/Mo and Mo/Rh anode filter combinations. When anode filter combination of W/Mo is taken into account, these values raises to 10%, showing a considerable increase. The W/Mo had a compatibility percentage value much smaller than the other combinations, namely 41% smaller than the mean of the other anode filter combinations. This occurred because this composition was the one used specifically in experiment A. As already discussed, lower compatibility was expected for experiment A, since the uncertainty estimation was less accurate for this condition. The lower compatibility percentage can be observed in Fig. 43 and occurred in this case because the uncertainty was probably underestimated.

Fig. 43. Null hypothesis acceptance, rejection and indecision percentage discriminated by experiments A and B.

Also, it is possible that the greater discrepancies happened because experiment A used an industrial tube. Although this tube was adapted to resemble a mammography equipment. It is

expected that it is less accurate, because of being designed for applications that require less accuracy.

5.4.3. Applied voltage

The compatibility percentages had no dependence on the voltage, i.e. the variation of the compatibility percentage due to voltage showed no trend. The lowest percentage acceptance value was observed for 28 kV, while the highest one was observed for 26 kV. This can also be explained by the lower accuracy of experiment A. Only 12% of the 28 kV spectra obtained via experiment A were compatible, and they compose 33 % of the 28 kV overall results. Therefore, it is believed that experiment's A results are biasing the 28 kV acceptance percentage down.

5.4.4. Glandularity

Regarding bTEM glandularity, no difference was exhibited between 50/50 and 30/70. However, because of the more accurate uncertainty estimation, it was expected that the 50/50 glandularity bTEMs would occasion a greater level of compatibility. This unexpected compatibility percentage equality between spectra obtained using 30/70 and 50/50 glandularity bTEMs result can again be explained by the biasing introduced by experiment A. This happened because experiment A was entirely done with 50/50 glandularity bTEM, corresponding to 33% of all 50/50 glandularity results.

5.4.5. Reevaluation: experiment B only

As already noticed by observation of the other parameters, experiment A had less frequency of compatibility (33%) than experiment B (56%). Analyzing the hypothesis acceptance dependency with glandularity, voltage and anode filter combination, it was noticed that experiment's A results are biasing the general results. Therefore, the same analysis was repeated, this time only with results from experiment B, which are shown in Fig. 44.

The main observed changes noticed comparing Fig. 42 and Fig. 44 occur in the glandularity and in the voltage parameters. As expected, the compatibility percentage raised almost 30%, from 47 to

61%, for the 50/50 glandularity. This happened because data from experiment A were biasing this quantity, lowering its estimation. Furthermore, the 28 kV voltage compatibility percentage raised 26%, from 35% to 44%. Confirming thus the hypothesis that the consideration of experiment A data was biasing it downwards. Despite its growth, the 28 kV compatibility percentage, 44%, remained lower than that of 26 and 31 kV voltage, respectively 62 and 50%.

Fig. 44. Bar plot of the compatibility percentage for the *MWSR* test discriminated by the used parameters: glandularity in green, anode filter combination in yellow, voltage in gray and thickness in blue. The uncertainty was estimated following a binomial distribution expected variance. Only data from experiment B was used in this comparison.

6. CONCLUSIONS

Spectrometry parameters, such as gain, rising time and collimation, were defined during the measurements; the defined parameters optimized the quality of X-ray spectra measurement in mammographic energy range. In addition to that, PENELOPE MC simulation optimal parameters were also determined. With these parameters, the simulation provides the most accurate results with the least time consumption. Concerning simulation time, the high electron absorption energy is stressed out as the most important approximation done.

Summarizing, 108 experimental spectra were generated. They were generated using different conditions, such as anode filter combination, bTEM attenuation, and peak voltage. All experimental spectra were paired to the equivalent MC simulated spectrum. A novel approach used in this work within the MC simulation framework was the implementation of a corrected experimental spectra as input. Considering the obtained X-ray spectra comparison results, using the experimental spectrum as input proved to be an appropriate simulation method. It also saved a great amount of computation time, because it dismissed the simulation of the *Bremsstrahlung* X-ray production of the X-ray emitter, which is a laborious task.

The present work developed a computational method to use the PENELOPE MC code to indirectly assess the similarity between bTEM and breast tissue by means of transmitted spectra. The results generated in this work set a validity limit for mammographic spectra generated with the MC code PENELOPE. i.e., they define a standard to guide future bTEM investigations using PENELOPE MC method code.

It was noticed that, regarding transmitted X-ray spectra compatibility PENELOPE's compatibility rate is expected to be around 50%. This is expected when considering this work's conditions, such as voltages and anode/filters combinations. This work also verified that the PENELOPE MC code is accurate when simulating the dose deposition inside breast tissue. Simulated results were

compatible with results measured with TLDs. For TLD dose measurement, in this work's conditions, PENELOPE is expected to have a compatibility percentage of over 90% with experimental data.

PENELOPE will be used by the GDRFM for the development of tissue equivalent material in the future. In future works, the simulations will aim to predict the X-ray transmission through materials which compositions were developed in the group. These materials will be candidates to manufacture of equivalent tissue prototypes. The composition of these materials shall be determined following the process described by Mariano (Mariano 2017). The simulation will use the estimated compositions to assess if the spectra transmitted through the prototype are compatible with the expected spectra for breast tissue. These simulations will avoid thus the manufacture of materials whose compositions fail to mimic X-ray attenuation of breast tissue. The herein described tests will be performed with the simulated results.

In the process of comparing the spectra pairs, a more accurate estimation of uncertainties related to the spectra counts was performed. It used MCMC processes to propagate uncertainties associated with the simulation's input spectra and to the composition of the attenuation materials. These uncertainties were the most relevant ones. Other uncertainty sources, such as the uncertainty in the bTEM's dimensions, were small when compared to the other most relevant sources.

Five statistics were used as discriminators to assess spectra compatibility: *ME*, *EE*, *HVL1*, *HVL2* and *MWSR*. Both, *HVL* and *EE*, carry less information about the spectra than the *MWSR*. The *MWSR* is a statistical test that assesses the similarity of all channels in the spectra. It is possible that a spectra pair has compatible *HVLs* or *EE* values, but still is not representative of two similar X-ray beams (Attix 2004). This can't happen with the *MWSR*.

Furthermore, we concluded that the hypothesis test using the *MWSR* statistic is the most powerful one, among the tested statistics. Therefore, this method should be preferred to others (Everitt 2006).

As a final conclusion, we would like to stress that the use of statistical tests within the medical physics related spectra framework is of major importance, even when compatibility is not to be

expected. The statistical comparison establishes a metric that quantifies the level of similarity between the compared quantities. This allows the quantification of possible similarity improvements of the comparison development by future works.

REFERENCES

- AAPM. Equipment requirements and quality control for mammography. New York Am. Inst. Physics, Rep. 1990;29(29):1–72.
- Abbene L, Gerardi G, Principato F, Del Sordo S, Raso G. Direct measurement of mammographic X-Ray spectra with a digital CdTe detection system. *Sensors (Switzerland)*. 2012;12(6):8390–404.
- Agência Nacional de Vigilância Sanitária. RESOLUÇÃO - RE Nº. 1016, DE 3 DE ABRIL DE 2006. 2006.
- Agostinelli S, Allison J, Amako K, Apostolakis J, Araujo H, Arce P, et al. GEANT4 - A simulation toolkit. *Nucl. Instruments Methods Phys. Res. Sect. A Accel. Spectrometers, Detect. Assoc. Equip.* 2003;506(3):250–303.
- Aguirre E, David M, DeAlmeida CE, Bernal MA. Impact of photon cross section uncertainties on Monte Carlo-determined depth-dose distributions. *Phys. Medica. Associazione Italiana di Fisica Medica*; 2016;32(9):1065–71.
- Altman DG. *Practical Statistics for Medical Research*. 1st ed. London: Chapman and Hall; 1991.
- Amptek. XR-100T-CdTe X-Ray & Gamma Ray Detector [Internet]. 2017 [cited 2017 Aug 11]. Available from: <http://amptek.com/products/xr-100t-cdte-x-ray-and-gamma-ray-detector/>
- Archer BR. Diagnostic X-ray shielding design based on an empirical model of photon attenuation. 1983;44(5):507–17.
- Argo WP, Hintenlang K, Hintenlang DE. A tissue-equivalent phantom series for mammography dosimetry. *J. Appl. Clin. Med. Phys. [electronic Resour. / Am. Coll. Med. Physics]*. 2004;5(4):112–9.
- Attix FH. *Introduction to radiological and radiation dosimetry*. 2nd ed. Weinheim: WILEY-VCH Verlag GmbH & Co. KGaA; 2004.
- Bé MM, Chisté V, Dulieu C, Browne E, Chechev V, Kuzmenko N, et al. Table of radionuclides (Vol. 2-A= 151 to 242). *Bur. Int. des poids Mes.* 2004;2:1–282.
- Bentley RE, Jones J c., Lillicrap S c. X-Ray Spectra from Accelerators in the range 2 to 6 MeV. *Phys. Med. Biol.* 1967;12(3):301–14.
- Berger MJ, Hubbell JH, Seltzer SM, Chang J, Coursey JS, Sukumar R, et al. XCOM: Photon Cross Sections Database. NIST Stand. Ref. Database 8 [Internet]. 2010; Available from: <http://www.nist.gov/pml/data/xcom/index.cfm>
- Bielajew AF, Hirayama H, Nelson WR, Rogers DWO. History, overview and recent improvements of EGS4. *Radiat. Transp. Calc. Using EGS4*. Capri, Italy; 1994.
- Bielajew AF, Rogers DWO. *Variance Reduction Techniques. Monte Carlo Transp. Electrons Photons*. Springer US; 1988. p. 407–19.
- Bindu G, Lonappan A, Thomas V, Hamsakkutty V, Aanandan CK, Mathew KT. Microwave characterization of breast-phantom materials. *Microw. Opt. Technol. Lett.* 2004;43(6):506–8.
- Boone JM, Lindfors KK, Cooper VN, Seibert J a. Scatter/primary in mammography: comprehensive results. *Med. Phys.* 2000;27(10):2408–16.
- Booth TE, J. F. Briesmeister, Collins O. G., Devaney JJ, Estes GP, Fisher HM, et al. MCNP-A General Monte Carlo Code for Neutron and Photon Transport. 3rd ed. Briesmeister JF, editor. Los Alamos National Laboratory; 1986.
- Bottigli U, Golosio B, Masala GL, Oliva P, Stumbo S, Delogu P, et al. Comparison of two portable solid state detectors with an improved collimation and alignment device for mammographic x-ray spectroscopy. *Med. Phys.* 2006;33(9):3469–77.
- Bransden BH, Joachain CJ. *Physics of Atoms and Molecules (2nd Edition)*. Longman; 2003.
- Byng JW, Mainprize JG, Yaffe MJ. X-ray characterization of breast phantom materials. *Phys. Med. Biol.* 1998;43(5):1367–77.
- Carton A-K, Bakic P, Ullberg C, Derand H, Maidment ADA. Development of a physical 3D anthropomorphic breast phantom. *Med. Phys.* 2011;38(2):891.
- Di Castro E, Pani R, Pellegrini R, Bacci C. The use of cadmium telluride detectors for the qualitative analysis of diagnostic x-ray spectra. *Phys. Med. Biol.* 1984;29(9):1117–31.
- Chestnov O, Sepúlveda C, Murad H, Heneghan C, Gartlehner G, Bramley D, et al. WHO Position Paper on Mammography Screening. WHO. 2014.
- CIRS. Mammography research set model 012A [Internet]. [cited 2017 May 5]. Available from:

<http://www.cirsinc.com/products/modality/43/mammography-research-set/>

Compton AH. A Quantum Theory of the Scattering of X-rays by Light Elements. *Phys. Rev.* 1923. p. 483–502.

Corrêa AO, Schilithz, Lima FC da S de, Andrade JHV, Oliveira JFP, Santos M de O, et al. Estimativa 2016 - Incidência de câncer no Brasil [Internet]. INCA. 2016. Available from: <http://www.inca.gov.br/estimativa/2014/sintese-de-resultados-comentarios.asp>

Cowles M, Davis C. On the origins of the .05 level of statistical significance. *Am. Psychol.* 1982;37(5):553–8.

Cubukcu S, Yücel H. Characterization of paraffin based breast tissue equivalent phantom using a CdTe detector pulse height analysis. *Australas. Phys. Eng. Sci. Med.* [Internet]. 2016 Dec 7;39(4):877–84. Available from: <http://link.springer.com/10.1007/s13246-016-0487-1>

Cullen DE, Hubbell JH, Kissel L, Laboratory LLN. EPDL97: the evaluated photo data library '97 version. Uclr--50400. 1997.

Dance DR, Sechopoulos I. Dosimetry in x-ray-based breast imaging. *Phys. Med. Biol.* IOP Publishing; 2016 Oct 7;61(19):R271–304.

DeSantis C, Ma J, Bryan L, Jemal A. Breast cancer statistics, 2013. *CA Cancer J Clin* [Internet]. 2014;64(1):52–62. Available from: <http://0-dx.doi.org.innopac.wits.ac.za/10.3322/caac.21203>

DeWerd LA, Kissick M. The Phantoms of Medical and Health Physics. DeWerd LA, Kissick M, editors. New York, NY: Springer New York; 2014.

Eisenberger P, Reed WA. Relationship of the relativistic Compton cross section to the electron's velocity distribution. *Phys. Rev. B.* 1974;9(8):3237–41.

Everitt BS. The Cambridge Dictionary of Statistics. 3rd ed. J. Chem. Inf. Model. New York: Cambridge University Press; 2006.

García-Alvarez JA, Maidana NL, Vanin VR, Fernández-Varea JM. Analytical response function for planar Ge detectors. *Radiat. Phys. Chem.* [Internet]. Elsevier; 2016 Apr;121:23–34. Available from: <http://linkinghub.elsevier.com/retrieve/pii/S0969806X15301237>

Gedke DA. The Si (Li) X-Ray energy analysis system: operating principles and performance. *X-Ray Spectrom.* 1972;1:129–41.

Geraldelli W, Tomal A, Poletti ME. Characterization of tissue-equivalent materials through measurements of the linear attenuation coefficient and scattering profiles obtained With Polyenergetic Beams. *IEEE Trans. Nucl. Sci.* 2013;60(2):566–71.

Ghammraoui B, Peng R, Suarez I, Bettolo C, Badal A. Including the effect of molecular interference in the coherent x-ray scattering modeling in MC-GPU and PENELOPE for the study of novel breast imaging modalities. In: Whiting BR, Hoeschen C, editors. International Society for Optics and Photonics; 2014. p. 90334N.

Gold RH, Bassett LW, Widoff BE. Highlights from the history of mammography. *Radiographics.* 1990;10(6):1111–31.

ter Haar D. The Old Quantum Theory. *Am. J. Phys.* 1967;35(11):1098.

Hermann KP, Geworski L, Hatzky T, Lietz R, Harder D. Muscle- and fat-equivalent polyethylene-based phantom materials for x-ray dosimetry at tube voltages below 100 kV. *Phys. Med. Biol.* 1986 Sep;31(9):1041–6.

Hermann KP, Geworski L, Muth M, Harder D. Polyethylene-based water-equivalent phantom material for x-ray dosimetry at tube voltages from 10 to 100 kV. *Phys. Med. Biol.* 1985 Nov;30(11):1195–200.

Hubbell JH. Review and history of photon cross section calculations. *Phys. Med. Biol.* 2006;51(13):R245–62.

Hubbell JH, Seltzer SM. Tables of X-Ray Mass Attenuation Coefficients and Mass Energy-Absorption Coefficients from 1 keV to 20 MeV for Elements Z = 1 to 92 and 48 Additional Substances of Dosimetric Interest [Internet]. 1996 [cited 2016 Sep 21]. Available from: <https://www.nist.gov/pml/x-ray-mass-attenuation-coefficients>

Jarque CM, Bera AK. A Test for Normality of Observations and Regression Residuals. *Int. Stat. Rev.* 1987;55(2):163.

Jenkins R, Gould RW, Gedcke D. Quantitative X-Ray Spectrometry. 2nd ed. Marcel Dekker; 1995.

Jennings RJ. Optimal x-ray spectra for screen-film mammography. *Med. Phys.* 1981;8(5):629.

Johns HE, Cunningham JR. Physics of Radiology. 4th ed. Charles C Thomas Pub Ltd; 1983.

Kerlikowske K, Grady D, Rubin SM, Sandrock C, Ernster VL. Efficacy of screening mammography: A

meta-analysis. *J. Am. Med. Assoc.* 1995;273(2):149–54.

Kiarashi N, Sturgeon GM, Nolte LW, Lo JY, Dobbins JT, Segars WP, et al. Development of matched virtual and physical breast phantoms based on patient data. *SPIE Med. Imaging Phys. Med. Imaging.* 2013;8668:866805.

Kienböck R. On the quantimetric method. *Arch. Roentgen Ray.* 1906;11(17).

Klein O, Nishina T. Über die Streuung von Strahlung durch freie Elektronen nach der neuen relativistischen Quantendynamik von Dirac. *Zeitschrift für Phys.* 1929;52(11–12):853–68.

Knoll G. *Radiation Detection and Measurement*. 2nd ed. John Wiley & Sons; 2010.

Kurková D, Judas L. X-ray tube spectra measurement and correction using a CdTe detector and an analytic response matrix for photon energies up to 160 keV. *Radiat. Meas.* 2015;85:64–72.

L'Ecuyer P. Efficient and portable combined random number generators. *Commun. ACM.* 1988;31(6):742–51.

LeClair RJ, Wang Y, Zhao P, Boileau M, Wang L, Fleuret F. An analytic model for the response of a CZT detector in diagnostic energy dispersive x-ray spectroscopy. *Med. Phys.* 2006;33(2006):1329–37.

Lépy MC, Duchemin B, Morel J. Comparison of experimental and theoretical L X - ray emission probabilities of ²⁴¹Am , ²³⁹ Pu and ²⁴⁰ Pu. 1994;353:10–5.

Lopez PO, Pernicka F, Zednik M. Control de Calidad en Mamografía. IAEA-TECDOC-1517. Viena, Austria; 2006.

López-Pineda E, Ruiz-Trejo C, Brandan ME. A mammographic phantom to measure mean glandular dose by thermoluminescent dosimetry. *Radiat. Meas.* Elsevier Ltd; 2014;71:297–9.

M. Oresgun, Rehani MM. Optimization of the radiological protection of patients: Image quality and dose in mammography (coordinated research in Europe). IAEA-TECDOC-1447. 2005.

Mariano L. Desenvolvimento de uma metodologia para formulação de materiais radiologicamente equivalentes ao tecido humano [P.h.D. Thesis]. Universidade de São Paulo; 2017.

Metropolis N, Ulam S. The Monte Carlo Method. *J. Am. Stat. Assoc.* 1949;44(247):335–41.

Miyajima S. Thin CdTe detector in diagnostic x-ray spectroscopy. *Med. Phys.* 2003;30(5):771.

Miyajima S, Imagawa K, Matsumoto M. CdZnTe detector in diagnostic x-ray spectroscopy. *Med. Phys.* 2002;29(7):1421–9.

Nahum AE, Rogers DWO. *Monte Carlo Transport of Electrons and Photons*. 1st ed. Jenkins TM, Nelson WR, Rindi A, editors. New York and London: Plenum Press; 1988.

Ng K-H, Lau S. Vision 20/20: Mammographic breast density and its clinical applications. *Med. Phys.* 2015;42(12):7059–77.

Nunn AA, Davis SD, Micka JA, DeWerd LA. LiF:Mg,Ti TLD response as a function of photon energy for moderately filtered x-ray spectra in the range of 20-250 kVp relative to ⁶⁰Co. *Med. Phys.* 2008;35(5):1859–69.

Nuzzo R. Scientific method: Statistical errors [Internet]. *Nat. | News Featur.* 2014 [cited 2017 Aug 31]. Available from: <http://www.nature.com/news/scientific-method-statistical-errors-1.14700>

Okuno E, Yoshimura EM. *Física das Radiações. Fis. das Radiações*. São Paulo: Cia. das Letras; 2010.

Panettieri V, Barsoum P, Westermarck M, Brualla L, Lax I. AAA and PBC calculation accuracy in the surface build-up region in tangential beam treatments. Phantom and breast case study with the Monte Carlo code penelope. *Radiother. Oncol.* 2009;93(1):94–101.

Pernicka F, McLean ID. Dosimetry in diagnostic radiology: an international code of practice. *Tech. REPORTS Ser. No. 457*. Viena; 2007.

Poletti M, Gonçalves O, Mazzaro I. X-ray scattering from human breast tissues and breast-equivalent materials. *Phys. Med. Biol.* 2002a;47:47–63.

Poletti ME, Gonçalves OD, Mazzaro I. X-ray scattering from human breast tissues and breast-equivalent materials. *Phys. Med. Biol. Phys. Med. Biol.* 2002b;47(47):47–63.

Ribberfors R. Relationship of the relativistic Compton cross section to the momentum distribution of bound electron states. II. Effects of anisotropy and polarization. *Phys. Rev. B.* 1975;12(8):3136–41.

Rogers DWO. Fifty years of Monte Carlo simulations for medical physics. *Phys. Med. Biol.* 2006;51:287–301.

Sabbatucci L, Salvat F. Theory and calculation of the atomic photoeffect. *Radiat. Phys. Chem.* Elsevier; 2016;121:122–40.

Salomon A. Contributions to the Pathology and Clinical Picture of Carcinoma of the Breast. *Langenbecks Arch. Fur Klin. Chir. Ver. Mit Dtsch. Zeitschrift Fur Chir.* 1913;101:563–78.

- Salvat F, Sempau J, Fernandez-Varea JM, Josep S. PENELOPE-2014: A code system for Monte Carlo simulation of electron and photon transport. Work. Proc. Barcelona; 2015.
- Santos JC. Determinação experimental da distribuição de dose absorvida em diferentes qualidades de feixes mamográficos [P.h.D. qualification]. University of Sao Paulo; 2015.
- Santos JC, Mariano L, Tomal A, Costa PR. Evaluation of conversion coefficients relating air-kerma to $H^*(10)$ using primary and transmitted x-ray spectra in the diagnostic radiology energy range. J. Radiol. Prot. IOP Publishing; 2016;36(1):117–32.
- Santos JC, Tomal A, Furquim TA, Fausto AMF, Nogueira MS, Costa PR. Direct measurement of clinical mammographic x-ray spectra using a CdTe spectrometer. Med. Phys. 2017;Volume 44(4):3504–11.
- Schumann RW, McMahon JP. Argonne 256 Channel Pulse Height Analyzer. Rev. Sci. Instrum. 1956;27(9):675–85.
- Scofield JH. Theoretical photoionization cross sections from 1 to 1500 keV. Lawrence Livermore Lab. Rep. No. UCRL-51326. 1973;
- Secretaria de Atenção à Saúde. Cadastro Nacional de Estabelecimentos de Saúde [Internet]. 2016 [cited 2017 Jan 17]. Available from: http://cnes2.datasus.gov.br/Mod_Ind_Equipamento.asp?VEstado=00
- Sempau J, Badal A, Brualla L. A PENELOPE based system for the automated Monte Carlo simulation of clinics and voxelized geometries, application to far from axis fields. Med Phys. 2011;38(2011):5887.
- Silva NC da. Histórico das biociências nucleares. Alasbimn J. 2002;14(4):1–5.
- da Silva RCF, Assis M de, Migowsky A. Direito à saúde e incorporação de tecnologias: o caso do rastreamento mamográfico no brasil. Rev. APS. 2013;15(2):234–6.
- Sobol IM. The Monte Carlo Method. 1974.
- Sociedad Española de Física Médica. Protocolo Español de Control de Calidad en Radiodiagnóstico. Senda Ed. S.A. 2011.
- Stutz GE. Compton scattering cross section for inner-shell electrons in the relativistic impulse approximation. Nucl. Instruments Methods Phys. Res. Sect. B Beam Interact. with Mater. Atoms. Elsevier B.V.; 2014;319:8–16.
- Tabár L, Dean PB, Chen TH-H, Yen AM-F, Chiu SY-H, Tibor Tot RAS, et al. The Impact of Mammography Screening on the Diagnosis and Management of Early-Phase Breast Cancer. Breast Cancer. New York: Springer; 2014. p. 31–78.
- Tomal A. Mammography Phantoms. In: DeWerd LA, Kissick M, editors. phantoms Med. Heal. Phys. New York, NY: Springer; 2014. p. 143–57.
- Tomal a., Santos JC, Costa PR, Lopez Gonzales AH, Poletti ME. Monte Carlo simulation of the response functions of CdTe detectors to be applied in x-ray spectroscopy. Appl. Radiat. Isot. Elsevier; 2015;100:32–7.
- Tomal A, Cunha DM, Antoniassi M, Poletti ME. Response functions of Si(Li), SDD and CdTe detectors for mammographic x-ray spectroscopy. Appl. Radiat. Isot. Elsevier; 2012;70(7):1355–9.
- Vanin VR, Gouffon P, Helene O. Análise estatística de medidas em ciências exatas. Unpublished; 2005.
- White DR. The formulation of tissue substitute materials using basic interaction data. Phys. Med. Biol. 1977;22(5):889–99.
- White DR, Booz J, Griffith R V, Spokas JJ, Wilson IJ. Report 44. J. Int. Comm. Radiat. Units Meas. [Internet]. 1989 Jan 15;os23(1). Available from: <https://academic.oup.com/jicru/article/2923846/Report>
- White DR, Martin RJ, Darlison R. Epoxy resin based tissue substitutes. Br. J. Radiol. 1977;50(599):814–21.
- Wilkinson DH. A Stable Ninety-Nine Channel Pulse Amplitude Analyzer for Slow Counting Pulse-amplitude Analyser. Math. Proc. Cambridge Philos. Soc. 1950;43(3):508–18.
- Young KC, Oduko JM, Bosmans H, Nijs K, Martinez L. Optimal beam quality selection in digital mammography. Br. J. Radiol. 2006;79(948):981–90.

Appendix A: scatter effects importance assessment

A.1 Evaluation of the dependence of energy deposited on the CdTe crystal with the beam size

In this work's simulations, the radiation beam used was wide enough to impinge upon the entire bTEM slab surface, like shown in Fig. 22. The large width was used to accurately simulate the scattering photons which may arrive at the detector. In this section of the work, it is shown that this was an important consideration. For this purpose, the radiation field's cone aperture angle θ_0 was varied. Hence, the diameter Θ of the beam when it impinged on the bTEM block was varied. They relate by

$$\Theta = 2 \cdot \tan \theta_0 \cdot d \quad (45)$$

Θ was varied from 1 to 16 cm because the diagonal of the bTEM's plates measured 15.6 cm. The beam fluency was kept constant, as described in section 3.4. Hence, the number of simulated histories varied from approximately 6×10^6 to approximately 2×10^{10} .

The aperture variation was performed to assess the relevance of the contribution of the scattering photons. If the scattered photons' contribution was neglectable, a narrow beam would produce the same results as a wide beam. The quantity used to assess the influence of the cone aperture variation was the ratio between the energy deposited (ED) in the CdTe crystal after 40 mm of bTEM attenuation and the ED without any attenuation was assessed for every Θ . The relationship between the Θ and the ED ratio is seen in Fig. 45.

The 40 mm thickness was chosen because it is common to all experiments and simulations, and represented an intermediate condition. For simulations greater than 1×10^{10} histories (Θ of approximately 12 cm), the energy deposited on the spectrometer crystal remains approximately constant. In other words, the effect of the photons scattered from regions 6 cm further from the center of the beam is neglectable.

Fig. 45. Plot of the ratio between the energy deposited in the CdTe crystal after 40 mm of bTEM attenuation and the energy deposited without any attenuation. The dependence of this quantity with the diameter of the beam Θ was assessed. The diameter is considered as the beam impinging upon the bTEM block. Squares represent a 31 kV beam and circles a 28 kV beam.

A.2 Analytical evaluation of the low detection probability of photons scattering from peripheral areas of the bTEM

Regarding this work used energy range, three physical processes can interact with the bTEM to produce secondary scattered photons that may reach the detector. The scattered photons can come from fluorescent relaxation, coherent scattering or Compton effect. The highest possible energy of fluorescence radiation photons scattering from the bTEM is 3.7 keV. This energy is relative to the K_{α_1} transition from Ca. Photons with this energy or lower will be completely absorbed in the Be window or in the protective metal shielding that involves the spectrometer's electronics (Fig. 20). Therefore, they can be neglected for spectrometry and exposure measurements purposes.

Photons coherently scattered from peripheral areas of the bTEM slabs can also be neglected since their deflection angles are very small. Another factor is that the coherent scattering contribution to the considered materials' total interaction cross-section is small, at most 12.4%

(Berger et al. 2010). Therefore, Compton scattering is the dominant component regarding scattering of the breast tissue-equivalent materials.

The probability of secondary photons reaching the detector was analytically estimated. This estimation was done for the photons scattered by Compton effect of the bTEM surface. The equation that determines the angular cross section of the secondary Compton photon is Klein-Nishina's (7). It is important to stress that this analysis disregarded the probability of the Compton event occurring, i.e. it is assumed that Compton event already underwent.

We evaluated the probability of a secondary Compton photon reaching the detector based on equation (7). To do this, we determined the polar (θ) and azimuthal (ϕ) scattering angles that implied in the emitted photon impinging on the detector. This determination was done for any point P that layed upon the surface of the bTEM. All calculation was performed considering that the photons scattered of the infinitesimal plane laying on the bTEM surface more approximated to the detector. This approach was assumed since scattered photons have the smallest probability of being self-attenuated by the scattering material. This process is better illustrated in figure Fig. 46

Although Fig. 46 is not scaled, the equation was evaluated using the geometrical considerations of the experimental set-up. The CdTe sensitive element was centralized with respect to the bTEM's surface (Fig. 22) and was much smaller than the bTEM dimensions. The dimensions of the detector's sensitive element were 3 x 3 mm² and the tissue equivalent material considered as the scatter medium was 50 x 60 mm² in area and from 5 to 85 mm thick. The analytical calculation considered that the x-ray field impinged on the whole surface of the bTEM. The geometry considered for this calculation was the same as the exhibited in Fig. 22.

Fig. 46. Illustration of the determination of the polar (θ) and azimuthal (ϕ) scattering angles that implied in the emitted photon impinging on the detector. The large radiation beam cone is depicted in yellow, and in orange, there is an example of a reduced radiation beam cone. The figure is out of scale, mostly the detector, which is greatly enlarged for visualization purpose. The color pattern of the infinitesimal layer is explained in Fig. 47.

Once these angles were determined, we used them as integration limits of equation (7). This provided the partial cross section, named σ_{KN} .

$$\sigma_{KN} = \frac{\pi r_e^2}{\alpha} \{ \phi_1 - \phi_2 \} \cdot \left\{ -\frac{1}{2} \left[\frac{1}{\mu^2} \right]_{\mu_2}^{\mu_1} - \frac{1 + 2\alpha}{\alpha^2} \left[\frac{1}{\mu} \right]_{\mu_2}^{\mu_1} + \frac{\alpha^2 - 2\alpha - 2}{\alpha^2} [\ln(\mu)]_{\mu_2}^{\mu_1} + \frac{1}{\alpha^2} [\mu_1 - \mu_2] \right\} \quad (46)$$

with

$$\mu_i = 1 + \alpha [1 - \cos(\theta_i)] \quad (47)$$

The integration in the entire solid angle, i.e. in all possible scattering directions, results in the total cross section ($\sigma_{KN,T}$). The division

$$P_c = \frac{\sigma_{KN}}{\sigma_{KN,T}} \quad (48)$$

is interpreted as the probability of a photon scattered by Compton effect to reach the detector. This quantity distributes as a probability, i.e. in the $[0,1]$ interval. The result of equation (48) evaluation for every point P on the surface of the bTEM is seen in Fig. 47.

All the integrations were done with monoenergetic photons. The chosen energy was the maximum energy used in the experiments of the present work, namely 31 keV. This energy was chosen because it maximizes the probabilities on average.

Fig. 47. The probability of a photon scattered of the bTEM surface by Compton effect reaching the CdTe detector. The image area represents the bTEM's surface. The red square in the center of the figure represents the perpendicular projection of the detector area over the bTEM surface. The black circle represents the regions that would be illuminated by the beam.

The area delimited by the black curves represents a circle of 12 cm in diameter. The black line encloses 94% of the bTEM block surface area. This area is responsible for 99% of the Compton scattered photon probability of reaching the detector. This means that unless one considers 99% of the photon scattering of the bTEM surface, one will be sensibly disregarding scattering effects. This proves that the photon scattering is an important phenomenon to account for.

Appendix B: a study of the MWSR outliers

Visual inspection of the outlier spectra, mainly of their reduced residuals, reinforces the conclusion that these spectra should not be compatible. Therefore, they should fail a compatibility test. The knowledge that these spectra are not similar was used to assess the quality of the different quantities used to compare the spectra. This was done by noting that a test that assesses spectra compatibility satisfactorily, should indicate that the eight outlier spectra pairs are not compatible.

A decision matrix was constructed with the Z-tests related to the outlier spectra, to decide for the most efficient test. The same scoring method described in section 3.7.4 was applied, i.e. if the null hypothesis was correctly rejected, 1 point was credited to the test. If it the hypothesis wrongfully retained (Type II error, false positive), then 1 point was discounted from the test. No point was given or taken from the statistic when the $|Z|$ value lied between 2.8 and 3.2. Results are shown in Table 6.

Table 6. Comparison of the different statistics used, regarding the ability to discriminate spectra that should not pass a statistical test. The identification letters refer to the ones in Fig. 48.

ID	<i>HVL1</i>	<i>HVL2</i>	<i>ME</i>	<i>EE</i>	<i>MWSR</i>
(a)	1	1	1	1	1
(b)	1	1	1	1	1
(c)	1	1	1	1	1
(d)	-1	1	1	-1	1
(e)	1	1	1	1	1
(f)	1	-1	-1	1	1
(g)	-1	-1	1	-1	1
(h)	-1	-1	1	-1	1
SUM	2	2	6	2	8

In addition to the higher score in the described test, the *MWSR* results were weakly correlated to the estimated uncertainty. This means that the spectra which showed greater *MWSR* were not necessarily the ones with the lowest associated relative uncertainties. This shows that the statistic test was not biased by the uncertainty estimation. The correlation of the *MWSR* with the uncertainty was calculated by assessing the correlation coefficient between the *MWSR* values with the mean uncertainty of the counts in each spectrum. The coefficient was of -0.32.

It may be argued that the decision matrix was biased by choosing the outliers from the observation of the *MWSR* results in Fig. 32. However, visual examination of the spectra exhibited in Fig. 48 and of their reduced residuals shows they are dissimilar. Hence, the *MWSR* test correctly stressed out spectra that have clearly observable discrepancies. Therefore, the hypothesis that they should fail the tests is not unreasonable and the other statistics failed at some tests.

Fig. 48. Experimental and simulated spectra pair of the greater *MWSR* values. The experimental data are plotted in black, and the simulated as a red line. The experimental and simulated uncertainties were combined and are plotted as black bars associated to the experimental points. The lower subplot of each spectra comparison exhibits the reduced residuals of the comparison