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Efeito Hall de spin em nanoestruturas semicondutoras: rumo à novos dispositivos de spintrônica

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Spin Hall effect in semiconductor nanostructures: towards novel spintronic devices

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Abstract

This thesis presents electronic transport properties of two-dimensional topological insulators (TI) based on HgTe/CdTe quantum wells. These heterostructures, in the band inverted regime, host a novel state known as the quantum spin Hall insulator. This state is identified as insulator in the bulk, but exhibits gapless conducting states at their edges which can be verified in transport experiments. Four-terminal resistance close to the quantized value has been observed in mesoscopic samples. However, for samples longer than $1\text{ }\mu\text{m}$, the resistance might be much higher than $h/2e^2$ due to the presence of spin dephasing, inhomogeneity or disorder in the sample. This thesis addresses the problem of non-quantized resistance observed in macroscopic samples of dimensions longer than a few microns.

We report on the observation and a systematic investigation of local and nonlocal transport in HgTe quantum wells (8.0-8.3 nm) with inverted band structure corresponding to the quantum spin Hall insulating (QSHI) phase. The device MCT1 consists of three $4\text{ }\mu\text{m}$ wide consecutive segments of different length ($2\text{ }\mu\text{m}$, $8\text{ }\mu\text{m}$, $32\text{ }\mu\text{m}$), and seven voltage probes. The device MCT2 was fabricated with a lithographic length $6\text{ }\mu\text{m}$ and width $5\text{ }\mu\text{m}$. Both devices are equipped with a top gate which allows tuning the carrier density of the device. Applying gate bias changes the carrier density transforming the quantum well conductivity from n-type to p-type via an intermediate phase, called the charge neutrality point (CNP). Non-universal ($R \gg h/2e^2$) peaks in both local and nonlocal resistivity were observed near the CNP which decreases rapidly going away from CNP. Such a behavior near CNP can be explained using the edge plus bulk transport model, which includes both the edge states and bulk contribution to the total current.

Deviation of the four-terminal resistance from quantization ($R \gg h/2e^2$) in macroscopic samples, with dimensions above a few microns, is one of the major issues in the field of topological insulators. Recently a model was proposed by Vayrynen et al., where influence of charge puddles, resulting from inhomogeneous charge distribution in 2d topological insulators, on its helical edge conductance has been considered. The edge states are tunnel coupled to these metallic puddles or quantum dots. Electron's dwelling in the quantum dot may lead to significant inelastic backscattering within the edge and modifies the ballistic transport. Therefore ballistic coherent transport is expected only in the region between the puddles, and the total four-terminal resistance exceeds the quantized value.

Introducing electron-electron interactions in one-dimensional systems results in a Luttinger liquid (LL). The helical edge states in 2d topological insulator, can be treated as ideal Luttinger liquid, since it naturally appears in HgTe quantum wells. Among the various specific signatures of the LL behavior, such as temperature dependence, it is important to focus on non-equilibrium properties of LL. In contrast to conventional Fermi liquids, none of the excited states will decay to equilibrium state, characterized by temperature, in the absence of disorder. Electron-heating measurements can be used to understand the physics governing relaxation processes in LL. We have performed non-linear transport measurements at the CNP in HgTe based 2d topological insulators. This method together with temperature dependence of resistance can be used to determine the energy relaxation mechanism of the helical edge modes in QSHI. Our experiments fail to confirm the specific signatures of Luttinger liquid behavior. However, electron heating effect can be described by conventional energy relaxation mechanism, expected for electron-phonon interactions.

Resumo

Este trabalho apresenta as propriedades de transporte eletrônico de isolantes topológicos bidimensionais (TI) baseados em poços quânticos de HgTe/CdTe. Estas heteroestruturas, no regime de bandas invertido, contem um novo estado conhecido como isolante de spin Hall quântico (QSHI). Este estado apresenta um comportamento de isolante no corpo (*bulk*), mas exibe estados condutores sem lacunas nas bordas (*edges*), as quais podem ser verificadas em medidas de transporte. Medidas de resistência de quatro terminais foram observadas perto do valor quantizado em amostras mesoscópicas. No entanto, para amostras com mais de um μm , a resistência pode ser muito maiores que $h/2e^2$ devido à presença de defasagem de spin, não homogeneidade ou desordem na amostra. Esta tese aborda o problema da resistência não quantizada observada em amostras macroscópicas de dimensões maiores a algum μm .

Nós relatamos observação e investigação sistemática de transporte local e não local em poços quânticos de HgTe (8.0-8.3 nm) com estrutura de banda invertida correspondente à fase de isolante de spin Hall quântico. O dispositivo MCT1 consiste de três segmentos consecutivos de largura 4 μm e de comprimentos diferentes (2 μm , 8 μm , 32 μm), e sete sondas de tensão. O dispositivo MCT2 foi fabricado com um comprimento litográfico de 6 μm e largura 5 μm . Ambos dispositivos estão equipados com uma porta superior (*top gate*), que permite ajustar a densidade de portadores do dispositivo. A aplicação de uma tensão de porta muda a densidade de portadores, transformando a condutividade do poço quântico de tipo n para tipo p através de uma fase intermediária chamada de ponto de neutralidade de carga (CNP). Picos acentuados não universais ($R \gg h/2e^2$) em ambas as resistividades, local e não local, foram observados próximos ao CNP os quais diminuem rapidamente a medida que se afasta do CNP. Tal comportamento próximo ao CNP pode ser explicado usando o modelo de transporte de bordas (*edge*) e corpo (*bulk*), que inclui tanto os estados de borda como o corpo para a contribuição à corrente.

O desvio dos valores da resistência de quatro terminais do valor quantizado ($R \gg h/2e^2$) em amostras macroscópicas com dimensões acima de algum μm é um dos principais problemas no campo dos isolantes topológicos. Recentemente foi proposto um modelo por Vayrynen et al., onde tem sido considerado a influência de poças de carga, resultantes de distribuições de carga não homogêneas em isolantes topológicos 2d, na condutância de estados de borda helicoidal. Os estados de borda são acoplados por tunelamento a essas poças metálicas ou pontos quânticos. A permanência dos elétrons em pontos quânticos pode levar a um retroespalhamento inelástico significativo dentro da borda e modifica o transporte balístico. Portanto transporte balístico coerente é esperado somente na região entre poças, e o total de resistência de quatro terminais excede o valor quantizado.

Introduzindo as interações elétron-elétron em sistemas de uma dimensão resulta em um líquido de Luttinger (LL). Os estados de borda helicoidais em isolantes topológicos 2d, podem ser tratados como um líquido de Luttinger ideal, uma vez que, naturalmente, aparecem em poços quânticos de HgTe. Entre as várias assinaturas específicas do comportamento do LL, como a dependência da temperatura, é importante se concentrar nas propriedades de não equilíbrio do LL. Em contraste com os líquidos de Fermi

convencionais, nenhum estado excitado decairá ao estado de equilíbrio, caracterizado pela temperatura, na ausência de desordem.

Medidas de elétron-aquecimento podem ser usadas para entender a física que governa os processos de relaxamento em LL. Nós temos realizado medidas de transporte não linear no CNP em isolantes topológicos 2d de HgTe. Este método, juntamente com a dependência da resistência com a temperatura, pode ser utilizado para determinar o mecanismo de relaxação da energia dos estados de borda helicoidais em QSHI. Nosso experimento falhou em confirmar as assinaturas específicas do comportamento do líquido de Luttinger. No entanto, o efeito de aquecimento de elétron pode ser descrito pelo mecanismo convencional de relaxamento de energia, esperado para espalhamento elétron-fônon.

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List of symbols

Symbol	Meaning	Symbol	Meaning
$\hbar$	Reduce Planck's constant	E_g	Band gap
m^*	Effective mass of carriers	E_F	Fermi energy
$D(\epsilon)$	Density of states	V_g	Gate potential
n	Electron's concentration	n_d	Donor density
p	Holes concentration	n_0	Cross over density
$f(\epsilon)$	Fermi-Dirac distribution function	n_p	Density of puddles
K_B	Boltzmann's constant	φ	Chemical potential of edge states
μ_f	Chemical potential	ψ	Chemical potential of bulk states
g_s	Spin degeneracy	γ	edge-edge coupling constant
g_v	Valley degeneracy	g	Bulk-edge coupling constant
δ	Dirac delta function	L	Edge channel length
Γ	Broadening parameter	L_φ	Mean free path
τ_q	Quantum life time	τ_ϵ	Energy relaxation time
ν	Filling factor	α	Phonon dimensionality
λ	de-Broglie wavelength	T_L	Lattice temperature
ω_c	cyclotron frequency	T_e	Electron temperature
μ	Mobility of carriers	Δ	Activation energy
J	Current density	P	Power dissipated
σ	Conductivity	N	Number of electrons
ρ	Resistivity	a_B	Bohr radius
T	Temperature	d_c	Critical thickness of QW
λ_c	Coupling constant of conduction band		

Chapter 1

Fundamental concepts

1.1 Two-Dimensional Electron Gas Systems

Since the discovery of the Quantum Hall effect by Von Klitzing *et al.* [4], the transport properties of a two dimensional electron gas (2DEG) has received a great deal of attention. Early work on the quantization of electrons was initiated in the 1960s, when electrons in the inversion layer of the MOSFET (Metal-Oxide-Semiconductor Field Effect Transistor) were found to be quantized at the interface and to exhibit two-dimensional properties. Another attempt to fabricate quantum structures, which has led to the most important applications in semiconductor physics and devices, was initiated in 1960 by Esaki and Tsu [5], who introduced the molecular beam epitaxial (MBE) technology to grow layered structures, called superlattices. Since then MBE growth techniques have been used to tailor various types of quantum structures where the electron transport and optical properties arising from the quantization effect have been observed [6].

Two-Dimensional Electron Gas in MOS Inversion Layer The two-dimensional characteristics of electrons were first observed in MOSFET and various types of quantization effects have also been investigated in MOSFETs. Therefore, MOSFETs can be considered as a prototype for a detailed understanding of quantum effects in hetero-structures.

A schematic representation of a Si-MOSFET is shown in Fig. 1.1 (a) [7]. The lightly p-doped Si substrate is coated with SiO_2 layer that serves as an insulator between the Si surface and a metallic gate electrode (Al). The contact area (source S and drain D) is heavily n-doped to favor the formation of an ohmic contact to the 2DEG. The 2DEG realization is represented in the band diagram of Fig. 1.1 (b). When a positive gate voltage $+V_g$ is applied, the chemical potential in the metal is lowered, and hence one introduces holes in the metal that attract, via the electric field effect, electrons from the semiconductor to the semiconductor-insulator

Figure 1.1: Silicon-MOSFET; (a) Cross section of Device, (b) Schematic energy diagram.

interface. Consequently, this field induces a bending of the valence and conduction bands. As the substrate is p-doped, electrons of the valence band will populate the level of the acceptors and leave holes in the valence band. The electrons attracted to the interface first fill these holes. However, if the gate voltage is high enough, the bottom of the conduction band can be shifted below the Fermi level E_F , allowing the electrons to fill the bottom of the conduction band. This is called the inversion layer: the bottom of the conduction band is below the top of the valence band, reversing the usual order [7]. The 2DEG is located within this inversion layer which is usually around 3–5 nm wide. This width is smaller than the de-Broglie wavelength, hence the electron gas can be considered as two-dimensional: the motion along the z axis is fully quantized. One thus obtains a confinement potential of triangular shape for the electrons in the conduction band, and, therefore, the dynamics of electrons is quantized into discrete electronic sub-bands in the z -direction. In general, the electronic wave functions are three dimensional, but in typical MOSFETs only the lowest sub-band is filled, such that the electrons are purely 2D from a dynamical point of view, i.e., there is no electronic motion in the z -direction.

2DEG in GaAs-AlGaAs Hetrostructures Another important system with 2DEG includes modulation-doped GaAs-AlGaAs hetero-structures. This system has the advantage of relatively higher mobility over Si-MOSFETs. The drawback with the metal-oxide-semiconductor junction is that there are occupied dopant states in the oxide-semiconductor interface region exactly where the two-dimensional electron gas (2DEG) forms. In addition, the amorphous oxide barrier is not smooth and contains a high density of trapped charges resulting in inhomogeneous charge density. The GaAs-AlGaAs hetero-structures are grown by molecular beam epitaxy and the interface region, where the two-dimensional electron gas forms, is flat to within a mono-layer with no trapped charges. In such structures modulation doping is used to place dopants hundreds of Angstroms away from the interface to reduce their ability

Figure 1.2: GaAs Hetrostructures (a) Cross section showing the formation of 2DEG, (b) Energy band diagram

to scatter. In the case of GaAs-AlGaAs, the two semiconductors do not possess the same band gap – indeed the band gap of intrinsic GaAs is smaller than that of AlGaAs, which is chemically doped by donor ions at a certain distance from the interface between GaAs and AlGaAs (Fig.1.2 (a) [7]). The Fermi energy is pinned by these donor levels in AlGaAs, which may have a higher energy than the originally unoccupied conduction band in the GaAs, such that it becomes energetically favorable for the electrons in the donor levels to occupy the GaAs conduction band in the vicinity of the interface. As a consequence, the energy bands of AlGaAs are bent upwards, whereas those of GaAs are bent downwards. Similarly, to the above-mentioned MOSFET, one thus obtains a 2DEG at the interface on the GaAs side, with a triangular confinement potential.

1.1.1 Basic properties of 2DEG Systems

As discussed above, a two dimensional electron gas (2DEG) is achieved by confining electron motion along the crystal growth direction (z-direction for instance) and so electrons can only move freely in the xy plane. Starting from the time-independent Schrodinger equation in three dimensions

$$\left[-\frac{\hbar^2}{2m^*}\nabla^2 + V(z)\right]\psi(x, y, z) = \epsilon\psi(x, y, z) \quad (1.1)$$

where m^* is the effective mass of the conduction band electrons ($m^* = 0.067m_e$ for GaAs) and $V(z)$ is the potential of the 2DEG perpendicular to the interface layer. As the confinement potential is in the Z-direction, suggesting plane wave solutions along x- and y-directions and hence the wave function can be written as follows [8]

$$\psi(x, y, z) = \varphi_n(z) \exp[ik_x x] \exp[ik_y y] \quad (1.2)$$

and the total energy of each electron is expressed as

$$E_n(k_x, k_y) = \epsilon_n + \frac{\hbar^2}{2m^*} (k_x^2 + k_y^2) \quad (1.3)$$

showing that the motion in the xy-plane is not influenced by the potential $V(z)$ and hence electrons in the xy-plane are characterized by a continuous parabolic energy spectrum with effective mass m^* , while ϵ_n describe energy in the direction of quantization [9]. Each of these parabolas, for a given value of n , is called a sub-band starting at $\epsilon_n = E_n(k = 0)$.

The shape of the density of states $D(E)$ changes due to sub-bands such that for a given sub-band (fixed n), the energy represents the energy of a free two dimensional electron gas with the bottom of the band shifted to ϵ_n . The density of states is, therefore, a step function of height $g_s g_v m^* / 2\pi \hbar^2$, starting at ϵ_n (g_s, g_v are the spin and valley degeneracy, respectively). Each sub-band contributes a step, so the total density of states (DOS) looks like a staircase having jumps at energies of the sub-band [8]. We can express it as follows

$$D(\epsilon) = D_0 \sum_j \Theta(\epsilon - \epsilon_j), \quad (1.4)$$

where $D_0 = g_s g_v m^* / 2\pi \hbar^2$ for each sub-band as mentioned above and it appears energy-independent within a given sub-band and Θ is the Heaviside step function. The density of electrons per unit area is then

$$n_s = \int D(\epsilon) f(\epsilon) d\epsilon, \quad (1.5)$$

where $f(\epsilon)$ is the Fermi-Dirac distribution function, describing the probability of an electron which occupies a band state with energy ϵ at temperature T , and is given by

$$f(\epsilon) = \frac{1}{\exp(\frac{\epsilon - \mu_f}{K_B T}) + 1}, \quad (1.6)$$

with K_B is the Boltzmann constant and μ_f is the chemical potential.

1.1.2 2DEG in a perpendicular magnetic field: Landau quantization

Let us consider an electron in 2DEG to move only in xy-plane, by definition, under a magnetic field pointing along the z-direction, perpendicular to the plane of the 2DEG, i.e., $\vec{B} = (0, 0, B)$. The Hamiltonian for such a system is given as

$$\mathcal{H} = \frac{1}{2m^*}(\vec{p} - e\vec{A})^2,$$

where $\vec{p} = -i\hbar\vec{\nabla}$ is the generalized momentum of the particle and $\vec{A}$ is the vector potential associated with the magnetic field $\vec{B}$, $\vec{B} = \vec{\nabla} \times \vec{A}$. Under gauge choice, we are free to choose $\vec{A}$ such that $\vec{B}$ remains unchanged. One common choice preserves the translational invariance along y-axis, $\vec{A} = (A_x, A_y, A_z) = B(0, x, 0)$, while other preserves the translational invariance along x-axis $\vec{A} = (A_x, A_y, A_z) = B(-y, 0, 0)$, and yet another preserves the rotational invariance (Symmetric gauge), $\vec{A} = (A_x, A_y, A_z) = B/2(-y, x, 0)$. Then the Schrodinger equation of the electron in 2DEG has form: using $q = -e$,

$$[\frac{1}{2m^*}(\vec{p} + e\vec{A}) + V(z)]\psi(x, y) = \epsilon\psi(x, y). \quad (1.7)$$

For instance, if we choose Landau gauge, $\vec{A} = B(0, x, 0)$, the Schrodinger equation becomes

$$\{\frac{1}{2m^*}[p_x^2 + (p_y + eBx)^2] + V(z)\}\psi(x, y) = \epsilon\psi(x, y). \quad (1.8)$$

Since the vector potential does not depend on y, which suggests that the wave function should be a product of a plane wave of y with some unknown function of x, $u(x)\exp(iky)$. Substituting this in the Schrodinger equation, using $\vec{p} = -i\hbar\vec{\nabla}$, yields

$$[-\frac{\hbar^2}{2m^*}\frac{d^2}{dx^2} + \frac{1}{2}m^*\omega_c^2(x - \frac{\hbar k_y}{eB})^2]u(x) = \epsilon u(x),$$

$$[-\frac{\hbar^2}{2m^*}\frac{d^2}{dx^2} + \frac{1}{2}m^*\omega_c^2(x - X_k)^2]u(x) = \epsilon u(x), \quad (1.9)$$

which is similar to the Schrodinger equation for one-dimensional harmonic oscillator with $\omega_c = \frac{eB}{m^*}$, the cyclotron frequency. The solution of the above equation is [8]

$$u(x) = \phi_n(x - X_k) = (\frac{1}{2^n n! \sqrt{\pi} l_B})^{1/2} \exp[-\frac{(x - X_k)^2}{2l_B^2}] H_n(\frac{x - X_k}{l_B}), \quad (1.10)$$

where $n = 1, 2, 3, \dots$, $X_k = \frac{\hbar k_y}{eB} = l_B^2 k_y$ is the center of the cyclotron orbit, $l_B = \sqrt{\frac{\hbar}{eB}} = \sqrt{\frac{\hbar}{m^* \omega_c}}$ is the magnetic length, and $H_n(x)$ are the Hermite polynomials. We can, therefore, write down the energies and wave functions for motion in the xy-plane

$$E_n = (n + \frac{1}{2})\hbar\omega_c, \quad (1.11)$$

$$\psi_n(x, y) = \phi_n(x - X_k) \exp(iky). \quad (1.12)$$

Figure 1.3: (a) The magnetic field application causes electron energy states to collapse onto concentric circular orbits in k space. (b) Increasing the magnetic field increases the size of the LL, so that for a fixed electron density, fewer and fewer LLs are occupied.

As the electrons are moving in cyclotron orbits, the density of states (DOS) in the xy -plane obeys an additional quantization giving rise to a sequence of δ -like energy levels (see Fig. 1.3), separated by the energy $\hbar\omega_c$. These energy levels are called Landau levels (LLs). The DOS and energy spectrum are obtained as

$$D(\epsilon) = \frac{g_s}{2\pi l_B^2} \sum_{i,n} \delta(\epsilon - \epsilon_{i,n}), \quad (1.13)$$

and

$$\epsilon_{i,n} = \epsilon_i + \left(n + \frac{1}{2}\right) \hbar\omega_c, \quad (1.14)$$

where $i, n = 0, 1, 2, 3, \dots$, i = sub-band index, n = Landau quantum number, g_s = spin degeneracy and δ is the Dirac delta function.

However, in real systems, there are always scattering events which broaden the Landau levels and the DOS is given by;

$$D(\epsilon) = \frac{g_s}{2\pi l_B^2} \sum_{i,n} \Gamma(\epsilon - \epsilon_{i,n}), \quad (1.15)$$

where the disorder induced broadening, Γ , of the Landau level is defined as $\Gamma = \frac{\hbar}{2\tau_q}$; τ_q is the quantum scattering time.

As the energy of a Landau level only depends on index n , all the states with different wave vectors k_y will be degenerate. The degeneracy of a Landau level, n_L , which is defined as the allowed number of states in each Landau level per unit area, is

$$n_L = \frac{\Phi}{\Phi_0} = \frac{AB}{\Phi_0} = \frac{eB}{h} = \frac{1}{2\pi l_B^2}, \quad (1.16)$$

where $\Phi_0 = \frac{h}{e}$ is the magnetic flux quantum and $\Phi = AB$ is the total applied flux.

Using this definition of n_L equation 1.15 can be written as,

$$D(\epsilon) = g_s n_L \sum_{i,n} \Gamma(\epsilon - \epsilon_{i,n}). \quad (1.17)$$

This form will be helpful in understanding the concepts of magneto-resistance oscillations. From Fig. 1.3 (b), one can see that as the magnetic field is increased from zero, the separation between the landau levels, $\hbar\omega_c$, becomes larger and the number of states in each landau level increases. In a 2DEG, the density of electrons is constant, for most of the experimental purposes, and so the number of occupied landau levels must change by changing the magnetic field. If n_s is the total density of carriers, one can estimate the filling factor ν , which gives the number of occupied landau levels:

$$\nu = \frac{n_s}{n_L} = \frac{\hbar n_s}{eB} = \frac{\Phi_0 n_s}{B} = 2\pi l_B^2 n_s. \quad (1.18)$$

It is obvious from the above relation, when n_s is kept constant, the filling factor, ν , decreases with raising the magnetic field $\vec{B}$. Generally, the filling factor, ν , needs not to be an integer. At zero temperature, there will be n LLs fully occupied, where n is the largest integer below ν , and the top level will be partially occupied. When $\nu = n$, the top level becomes empty. A further increase in $\vec{B}$ will lower n by 1 and the next level begins to empty and so on.

1.2 Electronic transport in Low-dimensional systems

The application of bias voltages to semiconductors causes the net flow of electrons and holes, known as electrical current, and transport is the process that describes the motion of the charged particles. The two major transport processes are the drift and diffusion mechanisms. The drift mechanism is basically the movement of charged carriers under the influence of applied electric fields, and the diffusion mechanism is the flow of charged particles due to the density variation. Transport properties in semiconductors may be complicated, depending on the actual size of the samples. Therefore, it is very important to discuss the classical and quantum limitations and regimes of the system. In order to define the limits of various transport regimes, one may scale the size of the sample against the de-Broglie wavelength, λ , which is defined as:

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2m^*E}} = \lambda_0 \sqrt{\frac{m}{m^*}},$$

where h = Planck's constant, p = momentum, E = energy, m^* = Effective mass of electron in semiconductor and λ_0 = de-Broglie wavelength of a free electron.

The room temperature de-Broglie wavelength of a free electron is $\sim 76\text{\AA}$, and that of an electron in GaAs is $295\text{\AA}$ [10]. For low temperatures, such as 4.2K, which is working temperature for most of recent research in this field, the de-Broglie wavelength increases to a fraction of micron. This implies that the wavelength is comparable with the size of semiconductor devices in the nanoscale limit. Hence, a quantum mechanical treatment of the transport properties in microscopic semiconductors should be considered. When electrons in semiconductors loose their wavelike behavior, they can be treated classically. This could happen when the electron scattering from impurities and imperfections of the host crystal is dominant. Another reason, why electrons loose their wavelike behavior, is related to finite temperature and electron statistics. The electron scattering process in semiconductors and hetero-junctions is dominated by scattering from impurities (including dopants), native defects, phonons, and interfaces.

1.2.1 Classical approach: The Drude Model

In the classical approach, electron transport in a solid is described by the Drude model where electrons are considered as classical particles. The interaction between electrons and the periodic lattice potential is included in the effective electron mass m^* . The collisions of electrons in the relaxation approach are included by a scattering time τ , i.e., the average time between two collisions. The Drude model is only valid for small magnetic fields and overlapping Landau levels ($\omega_c\tau \ll 1$), where the Landau quantization is neglected. The motion of electrons, in the presence of electric- ($\vec{E}$) and magnetic- ($\vec{B}$) fields, can be described by Newton's equation of motion [11]

$$m^* \frac{d\vec{v}}{dt} = -e(\vec{E} + \vec{v} \times \vec{B}) - m^* \frac{\vec{v}}{\tau},$$

where $m^* \frac{\vec{v}}{\tau} = \frac{P(t)}{\tau}$ is the correction due to individual electron collision and it, basically, introduces a frictional damping term to the equations of motion. This equation is sometime written in the following form,

$$m^* \frac{d\vec{v}}{dt} + m^* \frac{\vec{v}}{\tau} = -e(\vec{E} + \vec{v} \times \vec{B}), \quad (1.19)$$

where $\vec{v}$ is the drift velocity and τ is the scattering time, also known as relaxation time.

As $\vec{v} \times \vec{B} = v_y B \hat{i} - v_x B \hat{j}$, the above equation can be written as

$$m^* \frac{dv_x}{dt} = -eE_x - ev_y B - m^* \frac{v_x}{\tau},$$

$$m^* \frac{dv_y}{dt} = -eE_y + ev_x B - m^* \frac{v_y}{\tau}.$$

In steady state, the velocity of electron shall not be changed with time, $\frac{d\vec{v}}{dt} = 0$, so that the expressions for v_x and v_y are

$$v_x = -\mu(E_x - \omega_c \tau E_y) / (1 + \omega_c^2 \tau^2),$$

$$v_y = -\mu(E_y + \omega_c \tau E_x) / (1 + \omega_c^2 \tau^2),$$

where $\mu = \frac{e\tau}{m^*}$ is the mobility and $\omega_c = \frac{eB}{m^*}$ is the cyclotron frequency.

Using the relations, $\vec{j} = -en\vec{v}$, and $\vec{j} = \sigma \vec{E}$, for current density, we obtain the expression for conductivity tensor, σ ,

$$\sigma = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{pmatrix} = \frac{\sigma_0}{(1 + \omega_c^2 \tau^2)} \begin{pmatrix} 1 & -\omega_c \tau \\ \omega_c \tau & 1 \end{pmatrix}, \quad (1.20)$$

where $\sigma_0 = en_s \mu$ is the DC conductivity at zero magnetic field.

In an isotropic system the tensor components can be written as $\sigma_{xx} = \sigma_{yy}$ and $\sigma_{xy} = -\sigma_{yx}$. The resistivity tensor $\hat{\rho}$ is obtained by a simple tensor inversion of $\hat{\sigma}$

$$\begin{pmatrix} \rho_{xx} & \rho_{xy} \\ -\rho_{xy} & \rho_{xx} \end{pmatrix} = \frac{1}{\sigma_{xx}^2 + \sigma_{xy}^2} \begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ -\sigma_{xy} & \sigma_{xx} \end{pmatrix}, \quad (1.21)$$

with the tensor components ρ_{xx} and ρ_{xy} given by,

$$\rho_{xx} = \frac{\sigma_{xx}}{\sigma_{xx}^2 + \sigma_{xy}^2} = \frac{1}{en_s \mu}, \quad (1.22)$$

$$\rho_{xy} = \frac{\sigma_{xy}}{\sigma_{xx}^2 + \sigma_{xy}^2} = \frac{B}{en_s}. \quad (1.23)$$

In a standard magneto-transport measurements made on a Hall bar sample, with length L and width W , the above components can be written as,

$$\rho_{xx} = R_{xx} \left(\frac{W}{L} \right),$$

and

$$\rho_{xy} = R_{xy},$$

where R_{xx} and R_{xy} are the measured longitudinal and transverse resistances, respectively.

1.2.2 Semi-classical transport: Boltzmann equation

In the limit of weak magnetic field, the electronic transport in a 2DEG can be treated semi-classically, using the Boltzmann transport equation, without invoking the Landau quantization. In Boltzmann theory, the (possibly) non-equilibrium state of a system is described by a distribution function $f(\vec{r}, \vec{k}, t)$. A distribution-function, $f(\vec{r}, \vec{k}, t)$, is the probability of occupation of an electron at time t , at position r , with wave vectors lying between $k, k + dk$ in the momentum space (k-space). Under equilibrium ($\vec{E} = \vec{B} = \vec{\nabla}_r f = \vec{\nabla}_k f = 0$, i.e. no external electric- ($\vec{E}$) or magnetic- ($\vec{B}$) field and no spatial and thermal gradients), the distribution function is found from quantum-statistical analysis to be given by the Fermi-Dirac function for fermions:

$$f_0(\epsilon) = \frac{1}{1 + \exp(\frac{\epsilon_k - E_F}{K_B T})} \quad (1.24)$$

where ϵ_k is the energy of the electron, E_F is the Fermi energy, and K_B is the Boltzmann constant. The external forces act to drive the distribution function away from equilibrium while the collisions between particles act to restore equilibrium.

The idea behind the Boltzmann transport equation is to divide the interactions of the particles into two parts: One part, due to macroscopic forces and potentials, is described by a Hamiltonian. The other part is due to the microscopic interactions between particles—the collisions. The “external” interactions satisfy Liouville’s theorem. The collisions “create” and “destroy” particles i.e., a particle undergoing a collision will “suddenly” disappear from its volume of phase space (it’s been destroyed) and reappear (be created) in a different volume of phase space. In this case one can write [12]

$$\frac{df}{dt} = \left[\frac{\partial}{\partial t} + \vec{v} \cdot \vec{\nabla}_r + \frac{\vec{F}}{\hbar} \cdot \vec{\nabla}_k \right] f(\vec{r}, \vec{k}, t) = \left(\frac{\partial f}{\partial t} \right)_{coll}, \quad (1.25)$$

where the collision term, $(\frac{\partial f}{\partial t})_{coll}$, also called collision integral, is supposed to represent the net creation/destruction of particle density per unit time at a particular point in phase space.

Usually, the Boltzmann equation is solved numerically or under the relaxation-time approximation which is a good approximation for isotropic scattering and elastic scattering for low-field transport. Within relaxation-time approximation, Boltzmann equation for electrons takes the following form:

$$\left[\frac{\partial}{\partial t} + \vec{v} \cdot \vec{\nabla}_r + \frac{\vec{F}}{\hbar} \cdot \nabla_k \right] f = -\frac{(f - f_0)}{\tau}, \quad (1.26)$$

where f and f_0 are the distribution functions with and without external fields, respectively, τ is the relaxation time, and $\vec{v}$ is the drift velocity given as

$$\vec{v} = \hbar \vec{\nabla}_k \epsilon(k)$$

and the minus sign assures that particles leave the volume, if the actual distribution is larger than the equilibrium density. The relation for calculating current density of 2DEG system is given by

$$\vec{j}(\vec{r}, t) = \frac{1}{(2\pi)^2} \int d^2k (-e\vec{v})(f - f_0) \quad (1.27)$$

such that the conductivity tensor can be estimated using the relation $\vec{j} = \sigma \vec{E}$.

1.2.3 Quantum transport in 2DEG

In a strong magnetic field such that $\mu B = \omega_c \tau \gg 1$, the Landau levels are well resolved. Such energy quantization by the magnetic field drastically modifies the energy spectrum and the density of states. Although a Fermi level can still be well defined, there is no Fermi surface in the momentum space. In this situation, the magnetic field can no longer be taken as a perturbation but has to be incorporated into the system at the starting point. The electrical transport in this regime is thus called quantum transport since the Landau quantization plays a central role and quantum mechanical treatments are essential. The condition, $\omega_c \tau \gg 1$, implies that the carriers are collisionless. By applying this condition to Eq.1.20, one can obtain $\sigma_{xx} = 0$ and $\sigma_{xy} = -en_s/B$. We can see that for an ideal 2DES under strong magnetic field, the diagonal conductivity is zero and the off-diagonal conductivity is the Hall conductivity with the exact classic value. In real systems, scatterers are always present and the scattering potential can be perturbatively treated through quantum mechanical approaches, knowing that its value is much smaller than the external potential applied to the system. It is worth noting that the presence of scattering potential may lead to nonzero diagonal conductivity as well as diagonal resistivity, it means there is no effect on the Hall conductivity.

Another important property of quantum transport to be emphasized is that, even when diagonal conductivity is nonzero due to scattering, it is still much smaller than the off-diagonal one, that is $\sigma_{xy} \gg \sigma_{xx}, \sigma_{yy}$, which will therefore lead:

Figure 1.4: Typical form of the Shubnikov-de Haas oscillations in 2D electron systems

$$\rho_{xx} = \frac{\sigma_{xx}}{\sigma_{xx}\sigma_{yy} + \sigma_{xy}^2} \approx \frac{\sigma_{xx}}{\sigma_{xy}^2}, \quad (1.28)$$

showing that diagonal resistivity is directly proportional to diagonal conductivity which is contrary to common idea on the reverse relation between resistivity and conductivity at weak or zero magnetic field.

1.2.3.1 Shubnikov-de Haas effect

The classical result for the resistivity tensor stipulates that the longitudinal resistivity $\rho_{xx} = \frac{1}{\sigma_0} = \frac{1}{en_s\mu}$ (and also longitudinal resistance) is independent of the magnetic field. L. Shubnikov and W.J. de Haas [13] found that above some characteristic magnetic field, the longitudinal resistance oscillates as a function of the magnetic field. This phenomena is known as Shubnikov-de Haas oscillations (SdH), and is schematically shown in Fig. 1.4. In contrast to this oscillation in the longitudinal resistance, the Hall resistance remains linear in the magnetic field, in agreement with the classical result from the Drude model, $\rho_{xy} = \frac{B}{en_s}$.

The origin of SdH oscillations results from the fact that the density of states oscillates with the filling factor $\nu = \frac{n_s}{n_L} = \frac{h}{eB}n_s$. If the magnetic field is swept for a given n_s , the filling factor will continuously change, which is equivalent to a continuous motion of the Fermi level across the Landau levels. Maxima in the density of states pass through the Fermi energy, E_f , with increasing magnetic field, resulting in resistance oscillations, periodic in the inverse magnetic field $1/B$. The experimental observation of SdH oscillations requires the following conditions:

(i) The broadening of the Landau levels, Γ , has to be smaller than the separation between adjacent Landau levels $\hbar\omega_c$: $\Gamma \ll \hbar\omega_c$ or $\omega_c\tau \gg 1$.

(ii) The thermal energy of the system must be small compared to the separation of Landau levels and the separation of Landau levels must be less than the Fermi energy:

$$K_B T \ll \hbar \omega_c \ll E_f .$$

The conclusion of the conditions given above is that one requires low temperatures and magnetic field values which satisfy the condition $(\frac{\hbar e B}{m^*}) < E_f$. The analysis of the SdH oscillations can be used to extract some important parameters of a 2D system like period, effective g-factor, effective mass m^* , quantum life time of electrons and the Dingle temperature, etc.

The Shubnikov-de Haas oscillations can be usually described by the Dingle formula [14]

$$\frac{\Delta \rho_{xx}}{\rho_{xx}} \propto C \exp\left(\frac{-\pi}{\omega_c \tau_q}\right) \frac{\xi}{\sinh \xi} \cos\left(\frac{2\pi E_f}{\hbar \omega_c} + \varphi\right) \quad (1.29)$$

or we can write as

$$\rho_{xx}(B) = \rho_{xx}(0) + C \exp\left(\frac{-\pi}{\omega_c \tau_q}\right) \frac{\xi}{\sinh \xi} \cos\left(\frac{2\pi E_f}{\hbar \omega_c} + \varphi\right) \quad (1.30)$$

where C and φ are fitting constants, ω_c is cyclotron frequency, τ_q is the quantum lifetime and

$$\xi = \frac{2\pi^2 K_B T}{\hbar \omega_c}.$$

From the argument of the cosine term in above equation, $\frac{2\pi E_f}{\hbar \omega_c} = \frac{2\pi E_f m^*}{\hbar e} \frac{1}{B}$, we can calculate the 2D density of states, using the fact that the complete interval between two consecutive oscillation is 2π :

$$\frac{2\pi m^* E_f}{\hbar e} \left(\frac{1}{B_1} - \frac{1}{B_2}\right) = 2\pi,$$

using

$$n_s = \left(\frac{m^*}{\pi \hbar^2}\right) E_f,$$

we obtain

$$n_s = \frac{2e}{h \Delta\left(\frac{1}{B}\right)}, \quad (1.31)$$

where $\Delta\left(\frac{1}{B}\right) = \left|\frac{1}{B_1} - \frac{1}{B_2}\right|$ is the period of oscillation.

The quantum lifetime, τ_q , can also be determined from Shubnikov-de Haas oscillations at sufficiently low temperatures, where the τ_q dependent Dingle term dominates over the thermal amplitude reduction [14]. With this conditions Eq. 1.29 becomes

$$\frac{\Delta \rho_{xx}}{\rho_{xx}} = \pm \exp\left(\frac{-\pi}{\omega_c \tau_q}\right) \frac{\xi}{\sinh \xi} = \pm \exp\left(\frac{-\pi}{\omega_c \tau_q}\right) f(B, T). \quad (1.32)$$

By plotting the quantity

$$\ln\left(\frac{\Delta\rho_{xx}}{\rho_{xx}} \frac{1}{f(B,T)}\right) = \frac{-\pi m^*}{e\tau_q} \frac{1}{B} + \text{const}, \quad (1.33)$$

versus $1/B$ one can extract τ_q from the slope of the resulting straight line.

1.2.3.2 The Integer Quantum Hall effect

The integer quantum Hall effect (QHE) arises in the Hall conductivity or Hall resistivity of a two-dimensional electron gas at high magnetic field. At small magnetic field below about 1 tesla, the Hall resistivity shows a linear increase with magnetic field, expected from the classical Drude model. As the magnetic field is increased beyond the range considered in Fig. 1.4, the sinusoidal characteristic of the magneto-resistance becomes distorted and drops to zero between the peaks. The Hall resistance shows a step like increase with well-defined plateaus as shown in Fig. 1.5 [1]. This effect was discovered by Klaus von Klitzing about 100 years after the discovery of the classical Hall effect. The effect is very pronounced, if at large magnetic field, the Landau level separation, $\hbar\omega_c$, is large compared to the Landau level broadening, i.e.,

$$\omega_c\tau_q \gg 1, \quad (1.34)$$

which demands the high quality of the samples. Another condition to observe this effect clearly, is to work at temperatures, where the thermal broadening of oscillations in the magneto-transport coefficients, $K_B T$, is smaller than the Landau gap $\hbar\omega_c$ [1]:

$$\hbar\omega_c > K_B T.$$

The plateau values of the Hall resistivity are given by the relation

$$\rho_{xy} = \frac{h}{ie^2}, \quad (1.35)$$

where i is an integer. The constant, $R_K = \frac{h}{e^2} = 25812.807449\Omega$ is called the von Klitzing constant, or the resistance quantum. In the magnetic field regions where plateaus occur in ρ_{xy} , the longitudinal magneto-resistance, ρ_{xx} , shows well-pronounced minimas, which implies no scattering. This behavior of the system can be explained in the following way: At high magnetic field, due to presence of impurities in 2DEG system, most of the electrons' states are localized except the states at the center of Landau levels. This means that only the center of each Landau level contains extended states and the energy span between two adjacent

Figure 1.5: Measurement of the longitudinal resistivity and the Hall resistivity of a two-dimensional electron gas in a GaAs/AlGaAs heterostructure as a function of the magnetic field. The measurement temperature was $T = 100\text{mK}$. Figure taken from [1].

extended states area is the mobility gap, as shown in Fig. 1.6. During transport, only the extended states contribute to conduction. For integer filling factor, $i = 1, 2, 3, \dots$ i.e., at exact filling of the Landau levels (LLs), the Fermi level lies within the gap between Landau levels (the mobility gap), where the density of states (DOS) is zero. All states around the Fermi level are localized and contribute nothing to the conduction. The QHE can thus be considered as a transition from localized to de-localized states, as the Fermi level moves across the density of states.

Finally, the above description of the QHE assumes an ideal 2D sample of infinite extent in the x and y directions. In real systems, the boundaries effect causes the Landau levels (LLs) to bend up at the edges. This gives rise to the edge state picture which interprets the quantum Hall effect in terms of a sequence of incompressible stripes crossing the Fermi level at the edge, see Fig. 1.7.

In the edge state picture of the QHE, when the fermi level is located in the bulk gap, all states are localized inside the bulk 2DEG and electrons only can travel through edge states. The number of edge states at the Fermi level is the number of occupied Landau levels, that is, the filling factor ν . Therefore the edge states transport can be described in terms of Landauer Buttiker formalism [15]. All edge states, at a particular sample edge, have electrons moving in the same direction while electrons moving in opposite directions are spatially separated. As a consequence, back-scattering is completely suppressed in the vicinity of integer filling factors and voltage contacts on the same side of the Hall bar are connected via the edge states that

Figure 1.6: Diagram showing the region of localized and extended states in LL's broadened by disorder. The Zeeman splitting of LL's is also illustrated.

Figure 1.7: Energy spectrum of a 2DEG as a function of finite width, W , of the sample. The boundary confinement bend LL's upward. All states below the Fermi level are occupied (Blue circles) while the transport edge states exist at the intersection of LL's with the Fermi level (Red circles).

constitute ideal dissipation-less one-dimensional wires. That is why the longitudinal voltage, and with it the longitudinal resistance, vanishes near integer filling factors [1]. In contrast, contacts at opposite edges of the sample are electronically completely separated such that the Hall voltage can build up. At half integer filling factors, the Fermi energy coincides with the energy of percolating states in the interior of the sample, where backscattering is possible, and ρ_{xx} is finite. Therefore edge states for the highest Landau level disappears.

1.3 Spin-dependent transport in Nanostructures

1.3.1 Spin Hall Effect

Spin-dependent transport phenomena in semiconductor nanostructures are of great interest not only they leads to the emergence of new phenomena but also in their possible potential applications to spin-electronic devices. Efficient spin injection, spin accumulation and spin detection are key factors in utilizing the spin degree of freedom of electron as a new functionality in spin-electronic devices [16]. Spin injection and spin valve behavior have already been realized in magnetic semiconductors [17], yet another effect, spin Hall effect (SHE) based on the spin-orbit interaction, has been discussed as a candidate for spin polarization in semiconductors. The SHE results from the spin-orbit interaction, when electrical current passes through the sample, inducing spin accumulation near the lateral edges, with opposite polarization at the opposing edges of the sample [18]. This effect was proposed 40 years ago by Dyakonov and Perel [19, 20] that passing an un-polarized electrical current can generate a transverse spin current in systems with relativistic spin-orbit coupling. However, this prediction was scarcely noticed until Hirsch [21] and Zhang [22] rediscovered it and introduced the experimental detection of the phenomena. Hirsch proposed a device in which a spin current is generated by SHE in one part of the device and injected into another part where it is detected by the inverse spin Hall effect. In inverse SHE, a transverse charge current can be generated by a spin current, and when accumulated at the edges of the sample, the charge can be detected electrically [21]. Zhang proposal is based on the electrical detection of the edge spin accumulation by SHE using a ferromagnetic probe attached to one side of the conductor where spin accumulates [22].

The experimental discovery of the SHE was prompted by theoretical debate, mostly focusing on the intrinsic nature of the effect in semiconductors, where the effect may originate from the relativistic band structure without involving Mott scattering. Therefore, it was suggested that the optical activity of semiconductors can be utilized to detect SHE, particularly, the circularly polarized electroluminescence and magneto-optical Kerr effect [23, 24].

Figure 1.8: Schematic picture illustrating the Skew scattering mechanism. Labels 'u' and 'd' denotes the spin-up and spin-down electrons respectively.

The first experimental realization of SHE used these methods where Kato *et al.*, [25] employed a magneto-optical Kerr microscope to probe the spin polarization across the channel. While, Wunderlich *et al.*, [26] detected circularly polarized electroluminescence at opposite edge of the channel using the coplanar p-n diodes. Remarkably, Kato *et al.*, ascribed the observed signal to the extrinsic SHE whereas Wunderlich *et al.*, to the intrinsic SHE. Following the discovery of SHE, experiments with optical spin detection have explored the basic phenomenology of the various mechanism of SHE. Therefore it is necessary to describe different mechanisms driving the SHE. Based on the role played by external impurities, two distinct physical mechanisms have been identified for the SHE. The extrinsic SHE, which includes skew scattering and side jump mechanism, arise from the effect of the spin-orbit interaction on electron-impurity collisions [27]. The skew scattering is identified by the asymmetry of the electron-impurity scattering in the presence of spin-orbit interactions. Electrons that are drifting in the +x direction under the action of an electric field are more likely to be scattered to the left than to the right, if say their spin is up, while the opposite is true for the spin-down electrons. This results in generation of a net z-spin current in the y direction (Fig.1.8 [27]).

The second mechanism is called 'side jump', as it can be derived semi-classically from a lateral shift in the position of a wave-packet during collision with impurities (Fig.1.9, taken from [28]). This effect is more subtle and is caused by the anomalous relationship between the physical and canonical position operator ([27, 28]).

$$\vec{r}_{phys} = \vec{r} + \frac{\lambda^2}{4\hbar} \vec{P} \times \vec{\sigma}, \quad (1.36)$$

where $\vec{r}_{phys}$ is the physical position operator in the positive energy subspace, while $\vec{r}$ is the

Figure 1.9: Semi-classical picture of side-jump in a head-on collision with an impurity. The center of the electron wave packet is shifted by $\nabla \vec{r}_{phys}$ in a direction perpendicular to the change of momentum.

canonical position operator. Taking the derivative of this equation with respect to time we get

$$\vec{v} = \frac{\hbar \vec{k}}{m^*} + \frac{2\lambda_c^2}{\hbar} \vec{\nabla} V_{imp}(\vec{r}) \times \vec{\sigma}, \quad (1.37)$$

with λ_c is the coupling constant of the conduction band and $V(\vec{r})$ is the impurity potential acting on electron. This anomalous velocity, given by Eq. 1.37 usually ignored in standard scattering theory, while included in Boltzmann equation, and this is the origin of the so-called side jump contribution.

Another mechanism, which is called intrinsic SHE, as it does not depend explicitly on impurities and associated with the spin dependent band structure of the material. For example, the Dresselhaus term in bulk zinc-blend semiconductors, or, the Rashba term in 2D GaAs quantum well [24]. The essential mechanism of this effect lies in the precession of spins about the K-dependent effective magnetic field $B(K)$ which characterizes the band structure. The application of external electric field changes K , hence, $B(K)$ and thus forces the electrons out of alignment with the effective magnetic field. While attempting to regain alignment, the spins tilt away from their original orientations and the tilting goes in opposite direction on the opposite sides of the Fermi surface generating a spin current (Fig. 1.10 [24]).

From the above discussion it is clear that, in general, the SHE can not be ascribed to a single mechanism, but rather several different mechanisms operate simultaneously. Now the question arise, which mechanism dominates over others for a given situation? Ironically, the extrinsic skew-scattering dominates at very high mobilities, while, going towards lower mobilities, one may enter the side jump regime or the proper intrinsic regime. Which of these two mechanisms sets in first depends on whether $10^3 \lambda_c^2$ is larger or smaller than $\frac{a^4}{\lambda_c^2}$, where

Figure 1.10: Schematic diagram illustrating the intrinsic mechanism. The green arrows represent the momentum of electron while the red arrows represent its spin.

λ_c is the coupling constant of conduction band and a is the effective Bohr radius [27]. For a semiconductor, such as GaAs, where $a = 100\text{\AA}$, we have $a^4/\lambda_c^2 \gg 10^3\lambda_c^2$ and, therefore, a direct transition from the skew-scattering regime to the intrinsic regime will occur. On the other hand, in metals, for instance Platinum, $a = 0.5\text{\AA}$ and $\lambda_c^2 = 0.03\text{\AA}^2$, we have $a^4/\lambda_c^2 \ll 10^3\lambda_c^2$ and hence there will be transition to the side jump regime first and then to the intrinsic regime with further decrease of mobility.

1.3.2 The Quantum Spin Hall Effect

In recent years, a novel state of quantum matter has emerged, known as topological insulators which are materials that can be categorized neither metal nor insulator [29, 30, 31, 32, 33]. The salient feature of such materials is that they are insulating in the bulk, but exhibits gap-less conducting states at their edges (2D TIs) or surfaces (3D TIs). The edge states of 2D topological insulators are termed helical; each edge consists of two counter propagating channels of opposite spin polarities. In the absence of magnetic impurity or inelastic scattering, edge states are protected from back-scattering by the time reversal symmetry [34, 35]. The 2D topological insulators (2D TIs) are also known as quantum spin hall insulators (QSHIs), because they host the quantum spin hall (QSH) effect. The QSH effect shares many similarities with the well known QH effect but it differs in some aspects, particularly, the QH effect requires a magnetic field and thus breaks time reversal symmetry (TRS), while, the QSH effect does not require magnetic field and preserves TRS.

The QSH effect was first predicted by Kane and Mele in graphene [30], and independently by Bernevig and Zhang in strained semiconductors [32]. But none of these proposal turn out to be realistic because of the small spin-orbit gap in these materials, specifically, graphene

has an extremely small gap. Bernevig, Hughes, and Zhang [31], shortly after this, started the search for semiconductors with an “inverted” band-structure to exhibit QSH state and predicted a quantum phase transition in type-III quantum wells (QW) of HgTe/CdTe, where a transition between a normal insulating phase and a QSH phase is governed by the thickness of the quantum well. For QW having a thickness below a critical thickness, d_c , the system behaves like an ordinary insulator, but, if the thickness of the QW is increased beyond d_c , a quantum spin Hall insulating (QSHI) phase can be obtained, where the bulk of the material is insulating but has gap-less edge states that conducts. Several other systems have been proposed theoretically to exhibit QSH effect, such as GaAs under shear strain [32], 2D bismuth [36], or inverted Type-II InAs/GaSb/AlSb quantum wells [37], etc. Experimentally, the QSH effect was first observed in inverted HgTe quantum wells [38, 39], where the band structure can be tuned by fabricating quantum wells of different thicknesses.

As mentioned above that QSH state shares similarities with the QH state, one of them is the edge state conduction, though, it is important to mention that different mechanisms are involved governing this edge states transport. The QH effect occurs when a strong magnetic field is applied to a 2D gas of electrons in a semiconductor. At low temperature and high magnetic field, electrons travel only along the edge of the semiconductor, and the two counter-flows of electrons are spatially separated into different “lanes” located at the sample’s top and the bottom edges. The top edge of a QH system, compared with a one-dimensional system of spin less electrons (Figure 1.11, top left, taken from [40]), thus contains only half the degrees of freedom (Figure 1.11, bottom left). This spatial separation results in chiral traffic rule, which suppresses electrons scattering by the impurity, when encounters an impurity, electrons simply takes a detour and continue to move in the same direction. (Figure 1.11, bottom left, blue dot). The edge state structure of the QSH state (Figure 1.11, bottom right) can thus be described pictorially by superposing two copies of QH edge states (Figure 1.11, bottom left), with opposite chirality for each spin. Compared with a spinful one-dimensional system (Figure 1.11, top right), the top edge of a QSH system contains only half the degrees of freedom. The resulting edge states are termed helical, because spin is correlated with the direction of propagation. Although electrons are now allowed to travel both forward and backward on the same edge, there is a new “traffic rule” that suppresses back-scattering: To backscatter, an electron needs to flip its spin, which requires the breaking of time reversal symmetry (TRS) . If TRS is preserved, as is the case for nonmagnetic impurities, no backscattering is allowed [40].

Now the question is what mechanism allows this spatial separation. In the case of the QH effect, the separation results by applying an external magnetic field, while, in the case of QSH effect, the separation is achieved through the TR invariant spin-orbit coupling, which is

Figure 1.11: A spinless one-dimensional system (top left) has both right-moving and left-moving degrees of freedom which are spatially separated in a QH system (bottom left), right movers at the top edge while left mover at the bottom edge. Owing to the twofold spin degeneracy, a spinful 1D system (top right) has twice as many degrees of freedom as the spinless system. Those four degrees of freedom are separated in a time-reversal (TR) invariant way in a QSH system (bottom right). The top edge has a right-mover with spin up (red dot) and a left-mover with spin down (blue cross), and conversely for the bottom edge.

why strong spin-orbit coupling is at the heart of semiconductors possessing QSH insulating state.

Another possible question one may ask; why back-scattering by nonmagnetic impurities is forbidden even though, a QSH edge consists of both backward and forward movers? To answer this question, one may take help of an analogy from daily life: Anti-reflection coating is used in most eyeglasses and camera lenses to achieve perfect transmission. As shown in Fig. 1.12 (a) [41], the destructive interference between the reflected light from the top and the bottom surfaces of the lens leads to zero net reflection and thus, perfect transmission [41]. However, it is important to notice that this effect is not robust, as it depends on the matching between the optical wavelength and the thickness of the coating.

An electron can be reflected by an impurity, in a fashion similar to the reflection of light by lens' surface, and electrons following different reflection paths also interfere with each other. As shown in Fig. 1.12 (b), an electron in a QSH edge state can take either clockwise or counter-clockwise turn around the impurity, and during that turn the spin rotates by an angle of π or $-\pi$ to the opposite-direction. As a result, the two paths which are related by time reversal symmetry, differ by a full π - ($-\pi$) = 2π rotation of the electron spin. Based on principle of quantum mechanics, electron being a spin 1/2 particle and so its wave-function acquires a negative sign upon a full 2π rotation. Consequently, the two back-scattering paths always interfere destructively, leading to perfect transmission. Note, that this discussion is valid only for non magnetic impurities. If the impurity carries a magnetic moment, the TR

Figure 1.12: (a) Destructive interference between light waves reflected from the top (blue line) and bottom (red line) surfaces of a lens with antireflection coating leads to perfect transmission (b) Two possible ways of scattering of a quantum spin Hall edge state off the impurity (nonmagnetic). Going clockwise along the blue curve, the spin rotates by π , counter-clockwise along the red curve by $-\pi$. A phase difference of 2π invokes a quantum phase factor of -1 and thus leads to destructive interference of the two paths.

symmetry is broken, and the two reflected waves no longer interfere destructively. In that sense the robustness of the QSH edge state is protected by the TR symmetry.

It is important to mention here that above picture applies only to the case of single pair of QSH edge states. If there are two pairs at a given edge, i.e., two forward movers and two backward movers at each edge, then an electron can be scattered from a forward to a backward-moving channel without reversing its spin and without the perfect destructive interference, and thus there is dissipation. To sum up, for the QSH state to be robust, the edge states must consist of an odd number of forward movers and an odd number of backward movers. This even odd effect, characterized by a so called Z_2 topological invariant, is the heart of the QSH state and this is why a QSH insulator is synonymously referred to as a topological insulator.

Chapter 2

Quantum Spin Hall effect and HgTe-based topological insulators

2.1 Topological band theory and the Quantum Spin Hall State

One of the most salient features of the band theory of solids is to explain the difference between metals, semiconductors and insulators. Both insulators and semiconductors have an energy band, the valence band (where all states are occupied), and a band of higher energy, the conduction band (where all states are empty). The presence of electrons in the conduction band is crucial to the conduction process. Insulators and semiconductors can be differentiated by the size of the band gap between the valence and the conduction band. Due to the large band gap, electrons in an insulator, normally, do not have energy enough to move up to the conduction band. The band gap in a semiconductor is relatively smaller, making it possible for electrons to jump to the conduction band.

From solid state physics, the eigenvalues of the Bloch Hamiltonian define the energy bands of the band structure. Therefore, the two states of matter (Insulators and Semiconductors) are topologically equivalent in the sense that the Hamiltonian, encoding the dispersion relation leading to the band structure, can be continuously deformed from one of the two to the other, without closing this gap. As a first example of a topologically non-trivial state with a band gap, consider the quantum Hall state which occurs when electrons confined to two dimensions are placed in a strong perpendicular magnetic field. The electrons start to move in cyclotron orbits, and arrange themselves, energetically, in Landau levels with certain energies. If n Landau levels are filled and the rest are empty, then the energy gap separates the occupied and empty states just like in an insulator. But unlike an ordinary insulator,

the application of a finite electric field causes the cyclotron orbits to drift resulting in a Hall current characterized by the quantized Hall conductivity

$$\sigma_{xy} = n \frac{e^2}{h}.$$

The search then began to understand how such precise quantization could arise independent of the geometry or purity of the system. Thouless *et al.*, [42] discovered a connection between the Hall conductivity and a topological invariant, called Chern number. The roots of the Chern invariant goes to the mathematical theory of fiber bundles [43].

The Chern number c is an example of a topological invariant, so it can not be changed through continuous deformations of the Hamiltonian. The Chern number basically distinguishes between ordinary and non-trivial insulators; $c = 0$ for trivial insulators while $c = 1$ for non-trivial insulators. If we want to change c , the gap has to go to zero.

Since the Hall conductivity is odd under time reversal, topologically non-trivial states described above can occur only when time reversal (TR) symmetry is broken. However, it is possible to use spin-orbit interaction to obtain a class of time-reversal symmetric topologically nontrivial gaped systems [30]. The key to understand this new topological class is to examine the role of TR symmetry for spin 1/2 particles. The TR symmetry is represented by an anti-unitary operator $\Theta = \exp(i\pi S_y/\hbar)K$ where S_y is the spin operator and K is the complex conjugation [33]. For spin 1/2 particles $\Theta^2 = -1$, leads to Kramer theorem [44], which states that all eigenstates of a time-reversal invariant Hamiltonian commuting with Θ must be at least two-fold degenerate. This is automatically fulfilled, in systems without spin-orbit interaction, since states with different spins are degenerate in the absence of a magnetic field; the magnetic field then break TR symmetry.

In the presence of spin-orbit interactions, however, the energy levels of different spin will be shifted. If there are electronic states inside the band gap (bound to the surface of the system or the edge in the case of a 2D topological insulator), they have to be degenerate. This becomes interesting at the points in the Brillouin zone where $k_x = 0$ and $k_x = \pi/a$, since these points transform to themselves under time reversal. Away from these special points, a spin-orbit interaction will remove the degeneracy. There are two ways to connect the states at $k_x = 0$ and $k_x = \pi/a$. For instance, in Fig. 2.1 (a) [33], they are connected pairwise. In this case, the edge states can be eliminated by pushing all of the bound states out of the gap. Between $k_x = 0$ and $k_x = \pi/a$, the bands intersect Fermi level an even number of times. In contrast, in Fig. 2.1 (b), the edge states cannot be eliminated and so the bands intersect Fermi level an odd number of times. These two alternatives depend on the topological class of the bulk structure. There is thus a $\mathbb{Z}_2$ topological invariant, which tells us if the system is trivial insulator or a topologically non-trivial insulator with conducting edge states.

Figure 2.1: Electronic dispersion between two boundary Kramer's degenerate points $\Gamma_a = 0$ and $\Gamma_b = \pi/a$. The number of surface states crossing the Fermi energy E_F is even in (a), whereas it is odd in (b). An odd number of crossings leads to topologically protected metallic edge states.

Comparing with the integer quantum Hall case, the Chern number is $c = 0$ in these systems, but we now have a new topological invariant, κ , to consider, with the values 0 or 1, corresponding to an even or an odd number of Kramer's pairs, respectively. We can now form a "bulk boundary correspondence" for the Kramer's pairs,

$$N_k = \Delta\kappa, \text{mod} 2$$

where N_k is the number of Kramer's pairs intersecting the Fermi level and $\Delta\kappa$ is the change in $\mathbb{Z}_2$ topological invariant across the interface.

2.2 HgTe-based 2D topological insulators

2.2.1 Band structure of bulk HgTe and CdTe

In this section, our major interest is to describe the electronic band structures of mercury telluride (HgTe) and cadmium telluride (CdTe) bulk materials. HgTe and CdTe are binary II-VI semiconductors, where cadmium (Cd) and mercury (Hg) belong to group-II in the periodic table with atomic numbers $Z = 48$ and $Z = 80$, respectively, while tellurium (Te) is a group-VI element with atomic number $Z = 52$. Furthermore, both CdTe and HgTe, as bulk materials, crystallize in the zincblende structure. The lattice geometry is the same as the diamond lattice, i.e., two interpenetrating FCC lattices shifted along the body diagonal, but with the difference that each sublattice has a different atom. For instance, Hg atom on one sublattice, while Te atom on other sublattice. However, in case of diamond there is a carbon

Figure 2.2: The crystal structure of the mercury telluride (HgTe) lattice showing the four-fold coordination of each of Hg and Te atom

atom everywhere. The presence of two different atoms on each sublattice breaks the inversion symmetry of the diamond lattice and reduces the point group symmetry from O_h (cubic) to T_d (tetrahedral) [39]. In other words, both CdTe and HgTe are tetrahedrally bonded compounds in which each atom is four-fold coordinated, i.e., each atom is surrounded by four nearest-neighbor atoms forming a tetrahedron (see Fig.2.2)¹. Even though, the inversion symmetry is explicitly broken, this has only a negligible effect on the physics of the quantum spin Hall (QSH) state [45], hence, the bulk inversion asymmetry is ignored except otherwise specified.

For both HgTe and CdTe, the important bands near the Fermi level are close to the Γ -point in the Brillouin zone. Because, the energy bands have a simple parabolic form in the vicinity of Γ -point, therefore, can be indexed according to the Γ -point representations of the cubic group. At the Γ -point there are four important bands, namely, Γ -6, Γ -7, Γ -8 (light hole), Γ -8 (heavy hole) [46]. The Γ -6 band is two-fold degenerate, stems from the $|j = \frac{1}{2}, m_j = \pm\frac{1}{2}, l = 0\rangle$ states, and shows an s-orbital like behavior. Moreover, in normal semiconductors, the Γ -6 band is usually referred to as electron-like (E) band, since, it has positive curvature at the Γ -point with effective mass $m_{\text{eff}} > 0$, i.e., $\xi_{\Gamma 6}(k) = \frac{k^2}{2m_{\text{eff}}}$. The Γ -7 band is also two-fold degenerate, but stems from $|j = \frac{1}{2}, m_j = \pm\frac{1}{2}, l = 1\rangle$ states and, thus, shows a p-orbital like character. Due to the spin-orbit splitting between the Γ -7 and Γ -8 bands, the Γ -7 band is usually called the spin-orbit split-off band. The Γ -8 band is, however, four-fold degenerate, because it originates from the $|j = \frac{3}{2}, m_j = \pm\frac{3}{2}, l = 1\rangle$ and $|j = \frac{3}{2}, m_j = \pm\frac{1}{2}, l = 1\rangle$ states show also p-orbital like behavior. As the Γ -8 bands, have the p-like valence bands, therefore, negative effective masses $m_{\text{eff}} < 0$ describing hole-like excitations. In addition, the $|j = \frac{3}{2}, m_j = \pm\frac{3}{2}, l = 1\rangle$ states are known as heavy-hole (HH)

¹www.webelements.com

Figure 2.3: Schematic picture of the bulk band structure of HgTe and CdTe close to the Fermi level

bands, while the $|j = \frac{3}{2}, mj = \pm\frac{1}{2}, l = 1\rangle$ states are referred to as light-hole (LH) bands due to their different effective masses.

As shown in Fig. 2.3 (right), the band ordering of CdTe resembles that of GaAs, with an s-type (Γ -6) conduction band and p-type (Γ -7, Γ -8) valence bands separated from the conduction band by a large direct energy gap of ~ 1.6 eV. This is what we call a normal band ordering. In contrast, HgTe, as a bulk material, can be regarded as a symmetry-induced semi-metal (Fig. 2.3, left) [39]. As evident from the negative energy gap of -300 meV in HgTe, the Γ -8 band that usually forms the valence band, lies above the Γ -6 band. The light-hole (LH) bulk sub-band of the Γ -8 band becomes the conduction band, while, the heavy-hole (HH) Γ -8 sub-band becomes the first valence band, and the s-type Γ -6 band is pushed below the Fermi level to lie between the HH sub-band and the spin-orbit split-off Γ -7 band [40]. Because of this peculiar sequence of states, such a band structure with the associated negative band gap is termed as inverted band structure. Also, due to the degeneracy between the heavy- and light-hole bands at the Γ -point, HgTe is a zero-gap semiconductor.

2.2.2 CdTe/HgTe/CdTe quantum well heterostructures

Over the past two decades, fabrication techniques for quantum well heterostructures have greatly improved, and ultimately these achievements have led to the experimental discovery of the quantum spin Hall effect (QSHE) and topological edge states in CdTe/HgTe/CdTe quantum wells. While growing HgTe-based quantum well (QW) structures, utilization of the eccentric properties of the well material (HgTe) can be made to engineer the bandstructure in a controlled fashion. More precisely, we discuss the nature of the two-dimensional subbands for

Figure 2.4: (a) Bulk bandstructure for three-dimensional HgTe (left) and CdTe (right); (b) schematic picture of the type-III quantum well geometry and lowest-lying subbands for the trivial insulator state with $d < d_c$ (left) and the nontrivial QSH insulator state with $d > d_c$ (right).

the propagation of electrons and holes in the plane perpendicular to the growth axis, which is denoted by Z . The particular QW structure of our interest are grown by sandwiching a HgTe layer of thickness ' d ' between two layers of CdTe of equal thicknesses (see Fig. 2.4 (b)). The resulting quantum well heterostructures will then have either a normal or an inverted band ordering, depending on the thickness of the sandwiched HgTe layer. In wide QW layers (large d), quantum confinement effects are weak and the bandstructure remains “inverted,” i.e., the band ordering is similar as in HgTe. In other words, conduction subbands have Γ_8 character and the valence subbands have Γ_6 character. In case of very thin QW layers (small d), the confinement energy increases which shift the energy states leading to a normal band ordering. Thus, the physics is dominated by the CdTe barriers for thin quantum wells where the conduction subbands have mostly Γ_6 character and the valence subbands have mostly Γ_8 character. Therefore, in these heterostructures, one expects a critical thickness ' d_c ' intermediate to these two regimes, where the Γ_8 and Γ_6 subbands cross and the band ordering changes.

The shift of the energy levels with the QW thickness is shown in Fig. 2.5 [39], where the

Figure 2.5: Energy of various levels in HgTe QW as a function of QW thickness d_{QW}

subband structure has been obtained from self-consistent Hartree calculations using an 8×8 k.p model [39, 47]. One can notice that the notations H (heavy hole-like) and E (electron-like) used for the subbands are based on the properties of the respective wave functions. The light hole-like subbands are far away in energy ($E < -100 \text{ meV}$) and are thus not shown in Fig. 2.5. The transition from a normal band ordering to an inverted one can be seen clearly in this figure.

For a thin QW layer $d < d_c$, where $d_c = 6.3 \text{ nm}$, quantum confinement gives rise to a normal subband sequence: the valence band is formed by H-like subbands while the conduction band is formed by the E-like subbands, such that the subband gap, defined as $E_g = E_{E1} - E_{H1}$, is positive. For a thick QW layer $d > d_c$, the subband sequence is inverted: the H-like subband now forms the conduction band while the E-like subband is lower in energy and forms the valence band and hence E_g is negative. Under such circumstances the CdTe/HgTe/CdTe quantum well heterostructures represent the topologically non-trivial state or quantum spin hall (QSH) insulating state, where the bulk is insulating and transport is only through the edge channels of the system.

2.3 Previous experimental work on HgTe-based 2D Topological insulators

In this section, we briefly review some of the experimental works on the transport properties of CdTe/HgTe/CdTe quantum wells grown by molecular beam epitaxy [38, 48, 49, 50, 51]. HgTe samples with various QW thickness were prepared to confirm the band inversion in

Figure 2.6: Landau fan chart for HgTe/CdTe QW subject to a perpendicular magnetic field: (a) Trivial insulator, $d < d_c$, No LL crossing as a function of B if E_F is kept fixed in the $B=0$ gap, $\sigma_{xy} = 0$ (b) QSH insulator, $d > d_c$, crossing of LL at $B=B_c$ for a fixed E_F in the $B=0$ gap, giving rise to re-entrant QHE with $\sigma_{xy} = \pm e^2/h$

these superlattices. Since the inverted band structure is a pre-requisite for the topologically non trivial state (QSH state) in HgTe QW, one needs to verify the existence of band inversion in HgTe/CdTe. A manifestation of this is the so-called re-entrant QH effect that has been experimentally observed [39]. The peculiar band structure of inverted HgTe QWs gives rise to a unique dispersion of Landau levels (LLs). For a normal band ordering, $d < d_c$, all LL are shifted to higher energies for increasing magnetic field (see Fig.2.6 (a) [40]). This usual behavior is commonly observed for a variety of semiconductors. When the band structure of the HgTe QW is inverted, $d > d_c$, an unusual dispersion of LL can be observed (see Fig. 2.6 (b)).

Because of the mixing of the electron-like and hole-like states, one of the states of the H_1 conduction subband is a pure heavy-hole state ($m_j = -3/2$), and thus, the energy of the corresponding LL decreases with increasing magnetic field. On the other hand, one of the E_1 valence subband has mainly electron like character and, therefore, shifts to higher energies with increasing magnetic field. This leads to a crossing of these two peculiar LLs for a finite magnetic field. Experimentally, the crossing of the conduction and valence band Landau levels can be verified using quantum Hall experiments [?]. In an experiment by Konig *et al.*, when the Fermi energy E_F is adjusted in the gap, that is, $-1.9V < V_g < -1.4V$ (see Fig. 2.7 [39]), the Hall resistance shows an insulating behavior at $B = 0$. However, at higher magnetic fields the Hall resistance for these gate voltages exhibits a re-entrance of the $\nu = 1$

Figure 2.7: (a) Hall resistance, R_{xy} , of a 6.5 nm HgTe QW structure with different carrier concentrations obtained for different gate voltages V_g in the range of -1V to -2 V. Decreasing V_g will decrease the concentration of n-type carrier and a transition to a p-type conductor is observed, passing through an insulating regime between -1.4V and -1.9V at $B = 0$ T. (b) Landau fan chart of a 6.5 nm quantum well obtained from an eight band k.p calculations.

Figure 2.8: Longitudinal resistance R_{xx} as a function of normalized gate voltage for two devices with $L = 1 \mu\text{m}$. The width W is $1 \mu\text{m}$ {solid (black) and dotted (blue)} and $0.5 \mu\text{m}$ {dashed (red)}, respectively. The solid and dashed traces were obtained at 1.8K and the dotted one at 4.2K.

quantum Hall plateau. This is a direct consequence of the crossing of the Fermi level with the conduction-band derived LL for the green trace ($V_g = -1.4V$) and with the valence band LL, for instance, for the red trace where $V_g = -1.7V$. The observation of a re-entrant quantum Hall state is a clear indication of the non-trivial insulating behavior, which is a prerequisite for the existence of the QSH state.

As discussed in section 1.3.2, the most striking feature of the QSH state is the existence of gapless edge states, however an experimental verification of the edge state conduction is desired. Because, the spin-orbit coupling destroys spin conservation, we do not expect any quantized spin hall conductance. The most suitable natural experiment would be the observation of charge transport by these edge states. However, as the QSH effect does exist without applying external magnetic field, one cannot perform any measurement of the Hall conductance, which would be zero. The simplest way is to measure the longitudinal conductance of CdTe/HgTe/CdTe QW heterostructures in the Hall bar geometry. From quantum transport theory [15], it is well known that a single quantum channel has a longitudinal conductance of e^2/h . Because a single helical edge state per edge is predicted by BHZ model [31], and a strip has two edges, a longitudinal conductance of $2e^2/h$ is expected, independent of the aspect ratio of sample. Figure 2.8 [39] shows the experimental results obtained by König *et al.*, [38, 39] for CdTe/HgTe/CdTe QW in the inverted regime, $d > d_c$. The two devices with length $L = 1 \mu\text{m}$ and width $W = 1 \mu\text{m}$ and $0.5 \mu\text{m}$, were fabricated from the same wafer. The longitudinal resistance reaches a plateau $R_{xx} = h/2e^2$, when the Fermi level stays inside the gap which can be adjusted by the gate voltage V_g . This shows that

the longitudinal resistance is independent of sample width, provided that the sample size is smaller than the phase coherence length, assumed by Kramer's theorem [44]. The resistance quantization observed to be independent of temperature as long as $K_B T$ is much smaller than the 2D bulk energy gap.

The edge state transport in multiterminal HgTe devices was confirmed by Roth *et al.*, [52] by making non local transport measurements. In such a kind of measurements the current is not applied through the regular current channel but rather between any pair of contacts, say i, j , of the device and detecting voltage between any other pair of contacts, say k, l , so that the non local resistance for a given configuration would be $R_{ij;kl}$. The results of their measurements were interpreted in terms of the Landauer-Buttiker formalism of phase-coherent transport:

$$I_i = \frac{e^2}{h} \sum_j (T_{ji} V_i - T_{ij} V_j) \quad (2.1)$$

where I_i is the current flowing out of the i th contact into the sample region, V_i is the voltage on the i th contact, and T_{ij} is the transmission matrix giving the transmission probability for an electron from the i th to the j th contact. For a general two dimensional sample, the number of transmission channels scales with the width of the sample, such that the transmission matrix T_{ij} is complicated and non-universal. However, a tremendous simplification arises, if the quantum transport is dominated by edge states. Consider labeling consecutive terminals of an N -terminal device such as a Hall bar by consecutive integers $i = 1, 2, \dots, N$. Since there is a single chiral edge state at a given edge in the QH state, the transmission matrix for the QH state reads $T_{i+1,i}^{QH} = 1$ for all i and is zero otherwise. In the QSH state, the edge states are helical and consist of counter-propagating Kramer's partners, which can be considered as two copies of chiral edge states related by TR symmetry, and thus, the transmission matrix follows as $T^{QSH} = T^{QH} + (T^{QH})^\dagger$. Hence, for the QSH state, the only non-vanishing matrix elements are given by $T_{i+1,i}^{QSH} = T_{i,i+1}^{QSH} = 1$.

Figure 2.9, after Roth *et al.* [52], shows the four- and two-terminal resistances for the six terminal Hall bar devices of different dimensions mentioned in the figure. Strikingly, the experimental results are in close agreement with the theoretical predictions of Landauer-Buttiker formalism. Solving the Landauer-Buttiker equations for the four- and two-probe resistances one can obtain $R_{14,23} = h/2e^2$ and $R_{14,14} = 3h/2e^2$. These and other nonlocal transport measurements in various multiterminal geometries [52] can be taken as definitive evidences for the existence of edge channel transport in the QSH regime.

Magneto-transport measurements near charge neutrality point (CNP) were made in 20 nm quantum wells realized in undoped CdTe/HgTe/CdTe hetrostructures [48]. When the

Figure 2.9: Showing two-terminal ($R_{14,14}$) (top two traces) and four-terminal ($R_{14,23}$) (bottom traces) resistance as a function of normalized gate voltage for the Hall bar devices with different dimensions as indicated. The dotted blue lines indicate the resistance values expected from the Landauer-Büttiker formalism.

gate voltage was swept through the CNP, the longitudinal resistivity (ρ_{xx}) shows a maxima at CNP while the Hall resistivity (ρ_{xy}) gradually passes through zero from the positive (negative) quantized value ($\pm \frac{h}{e^2}$) on electron side to negative (positive) quantized value ($\mp \frac{h}{e^2}$) on hole side. A strong dependence on the magnetic field was observed for both the resistivity maxima and the QH plateau at the CNP. The transport behavior at the CNP was attributed to a percolation of the snake-type trajectories along the $\nu = 0$ lines.

Nonlocal transport measurements were performed in 8 nm HgTe QW, in inverted regime, in the absence of magnetic field [49]. No significant difference was observed between local and nonlocal transport measurements in these 2d- topological insulator samples. However, both local and nonlocal resistances are much higher than the quantized value, $\frac{h}{2e^2}$. These high resistance values were attributed to spin-flip scattering between the pair of the helical edge states induced by two-particle back-scattering, which breaks the TR symmetry. The quantized value is expected as long as the TR symmetry is protected. The application of an in-plane magnetic field results in strong reduction of local resistance while a complete suppression of nonlocal signal.

Recently a large nonlocal signal was found near the CNP in 20 nm HgTe quantum wells in the presence of a perpendicular magnetic field [50]. An increase in both local and nonlocal

resistances was observed with the increase in magnetic field. The nonlocal resistance grows from zero at $B=0$ to $1400\text{ k}\Omega$ at $B=11\text{ T}$ and becomes comparable with the local resistance at the CNP. The local resistance peak R_{xx} at the CNP also increases from few $\text{k}\Omega$ to $1800\text{ k}\Omega$ at $B=11\text{ T}$. The temperature dependence of both local and nonlocal responses near the CNP was also checked keeping B fixed at 11 T . It was found that the nonlocal resistance is more sensitive to temperature than the local one, and the peak of the nonlocal signal completely disappear above 60 K . The increase in local and nonlocal resistances with magnetic field is completely opposite to the one given in reference [49]. This is because the wider 20 nm HgTe quantum wells can be treated as a semi-metal rather than a topological insulator. In a 2d semi-metal, the magnetic field may result in enhancement of the nonlocal resistance due to the edge state contribution to the transport.

Quantized transport in inverted HgTe quantum wells was reported in 8 nm HgTe/CdTe heterostructures with a local top gate which can be used for density variation across the gap for each region allowing a n-p-n junction to be formed at the interfaces [51]. Fractional quantum Hall effect plateau, $R = 2\frac{h}{e^2}$, in the n-p-n regime was observed which was attributed to the mixing of counter-circulating modes at the p-n interface. By varying the voltage on the local gate, QH transport studies were done in the n-2DTI-n regime, where two counter-propagating helical modes circulate along the junction interface. A quantized resistance value of $R = 1.3\frac{h}{e^2}$, is obtained in the n-2DTI-n regime which clearly demonstrates the existence of counter-circulating edge states in the bulk gap region.

A transition from insulating to metallic phase, by the application of an in-plane magnetic field, was recently observed in 8 nm HgTe quantum wells [53]. Three devices with different aspect ratios and gate dimension were studied. It was found that the resistance peak at the CNP decreases monotonically rapid for magnetic fields larger than 5 T in all three samples. This behavior was attributed to transition from insulating to gap-less state. Parallel field dependence of the nonlocal signal was also studied. The experimental finding shows a strong suppression of local resistance and complete destruction of nonlocal signal.

Temperature dependence of resistance was studied in HgTe based 2d topological insulators [54]. At low temperatures, the resistance at the CNP was found to be temperature independent, in striking contrast to a strong temperature dependence expected for such an insulating state. However, at high temperatures ($T \geq 25\text{ K}$), the resistance shows thermally activated behavior. Absence of resistance quantization in such 2DTI has been explained by considering the tunneling between the edge states and charge carrier puddles in the bulk.

Very recently, electron transport studies were made in wide (14 nm) HgTe samples [55]. Edge channel transport was observed in long (1 mm) sample confirming the persistence of two-dimensional topological insulator phase that was previously reported for narrow (8 nm)

quantum wells. The observed behavior of the local and nonlocal resistances was found to be qualitatively similar to that for 2d topological insulators.

Chapter 3

Samples and Experimental Techniques

This chapter presents an overview of growth and device fabrication technology of HgTe-based topological insulator samples and the measurement techniques used to investigate their properties. Topics will range from sample preparation to cryogenics, ac lock-in techniques, data acquisition and measurements procedures to characterize these samples.

3.1 HgTe/CdTe Quantum well samples

3.1.1 Wafer preparation

The samples studied in this work were grown by the molecular beam epitaxy (MBE) at the Institute of Semiconductor Physics Novosibirsk, Russia. HgTe quantum wells were grown with well width of 8.0–8.3 nm on GaAs substrate with (013) surface orientations [56, 57, 58]. The authors [57] claim that growing samples on substrates with (013) orientation is a good choice for the growth of nearly perfect heterostructures. Based on thermodynamic analysis, the MBE growth of HgCdTe layers (MCT) occurs under conditions, when two phases of MCT and Te are stable. There exists two possibilities for diatomic telluride molecules striking the surface: dissociation of telluride molecules and growth of a perfect MCT film, and secondly, crystallization of telluride as an independent phase, when there is insufficient time for dissociation to take place. In the latter case, the growth of MCT is interrupted, bringing on deformations and irreversible degradation of the surface and structure of MCT. It is known that the dissociation of telluride molecules is energetically favorable at a step. Hence, it is clear that the use of substrates, inclined to singular orientations, may lead to the growth of better quality heterostructures [57]. A schematic diagram showing the epitaxial layers structure of HgTe/CdTe is illustrated in Fig. 3.1.

GaAs substrate with (013) orientation was chemically etched and thermally annealed

Figure 3.1: Schematic of the epitaxial layers showing MBE growth of HgTe QW structures

in arsenic vapors. Buffer layers of ZnTe (50 nm) and CdTe (5-7 μm) were then grown, consecutively, under optimal technological conditions in ultra-high vacuum. Then the growth of the HgTe QW, with wide-gap mercury cadmium telluride (MCT) layers, was carried out on the top of CdTe buffer layer at temperatures of 180-190 $^{\circ}\text{C}$. The central part of the wide-gap MCT layer was doped from a source of indium to an electron concentration of 10^{15}cm^{-3} . The growth rate was 0.15 $\mu\text{m}/\text{h}$. The MCT composition and thickness of the grown layers was controlled by an LEF-755 fast-response single-beam ellipsometer.

It is important to notice that the layers thickness control, while growing III-V semiconductors, is generally achieved by measuring the reflection high energy electron diffraction (RHEED) oscillations. However, RHEED can not be used at the growth of HgCdTe-based QWs for two reasons. First, the highly energetic electron beam (with energy greater than 15 keV) essentially influences the growth process, resulting variations in the surface morphology and composition of the growing epitaxial film. Second, the RHEED oscillations are observed only for sample growth on substrate with singular (001) orientations while the substrate used for efficient growth of HgTe QW has an inclined surface orientation [57]. Hence the ellipsometry is used to control the growth and composition of HgTe-based hetrostructures.

Figure 3.2: HgTe devices fabricated with a top gate electrode covering the entire sample. Blue and red lines shows the edge states counter-propagating at a given edge.

3.1.2 Device Fabrication

Lithography techniques are most commonly used to fabricate micro-electronic devices. The basic technique in semiconductor industry is photo-lithography, where patterns are transferred to the sample with the help of masks and resists that are sensitive to UV-light. The resolution is crucially limited by the wavelength of the light used. To fabricate even smaller scale design, electron beam (e-beam) lithography is required. The samples studied in this work were prepared using the standard photo-lithographic techniques (see Fig. 3.2). The ohmic contacts to the 2DEG were made by brazing-in of indium. To prepare the gate, a dielectric layer containing 100 nm of SiO_2 and 200 nm of Si_3Ni_4 was first grown on the structure using the plasmochemical method. Then a TiAu gate of the desired size was deposited.

3.2 Equipments for low temperature transport measurements

3.2.1 Cryogenics

Many of the transport properties of interest can be seen only at low temperatures (e.g., QHE and weak localization) so it is necessary to perform measurements using a low temperature cryostat. All the measurements in this thesis are performed using a ^4He cryostat (Oxford Instruments) capable of a minimum temperature of 1.5 K. The ^4He cryostat, at LNMS, is equipped with a superconducting magnet (12T) and a variable temperature insert (VTI) for measurements between 1.5 K and 300 K. A schematic of the ^4He system is shown in Figure 3.3 (a). The outer vacuum chamber and the liquid He main bath, insulate the insert from

Figure 3.3: (a) Schematic of a ^4He cryostat with a variable temperature insert for measurements down to 1.5 K (b) Snap shot of the ^4He cryostat at LNMS. The superconducting magnet can produce field up to 12 T.

the surroundings at room temperature. The insert itself is thermally decoupled from the He main bath by the inner vacuum chamber. An intermediate layer containing liquid N_2 helps to decrease the temperature to 77 K. The main bath temperature is 4.2 K which is the liquid He temperature. From thermodynamics the He temperature can be further decreased by decreasing its pressure. By opening a needle valve in a tube connecting the main bath with the insert, and simultaneously pumping the insert with a rotary pump, the sample can be cooled down to 1.5 K, by evaporative cooling.

3.2.2 Temperature measurement and control

As discussed above, variable temperature inserts are used in bath cryostat to adjust the temperature of a sample without affecting the helium reservoir. The inner parts of the insert are vacuum insulated from the liquid helium. The sample temperature can be controlled at any point in the specified range. Lower temperatures can often be achieved in single shot mode: the sample space is filled with liquid and the needle valve is closed to allow the pump to reduce the vapor pressure above the liquid to the lowest possible level. To go to temperatures higher than 4.2 K, the system has a heating system (a calibrated resistor), at

the bottom of the cryostat, which allows to heat the helium bath in the chamber containing sample. The resistance, corresponding to a given value of sample chamber temperature, can be read directly from the digital voltmeter connected to the heating system.

3.2.3 Magnetic field control

To generate magnetic field, superconducting coils are used, which are located in the chamber with the main ^4He bath. Since this chamber is free of any kind of pumping system, the temperature is 4.2K. Superconductivity allows the flow of current through coils without energy dissipation and generate a magnetic field. The coils are made of niobium and titanium wires wrapped in a copper matrix. The superconducting coil of our cryostat, at LNMS, allows to generate field nominally up to 15 T, with the VTI system and standard cooling of ^4He . For safety measures; to avoid quenching and to save helium for cooling purposes, the superconducting coils are limited to produce field up to 12 T for most of the experiments performed in our group. Both perpendicular and in-plane magnetic fields can be applied to the sample, depending on the choice of the sample probe used for experiment.

3.2.4 Data acquisition

During the preparations and the cool down of the sample, all relevant values may be read from the lock-in and the temperature controller. The experimental data are collected with a LabView program. LabView- interface software allows the computer to control and monitor external devices, e.g., multimeter, lock-in amplifier, electromagnet, temperature sensor, etc. Only the parameters for the magnetic field and the sweep rate are entered. The readings from the lock-in and magnet's current source are automated.

3.2.5 Ac lock-in techniques

A lock-in is designed to measure a signal in a very narrow frequency range and suppress any contribution outside of it. At first glance, this technique appears very simple. Using a band-pass filter before feeding the signal to the measurement device seldom yields the desired results. Noise suppression, speed and accuracy of a good lock-in outperform simple signal filtering techniques by orders of magnitude. Lock-ins are able to measure tiny AC-signals, even though they might be obscured by larger noise components. The key figure is the maximum signal to noise ratio which a lock-in will tolerate without going astray by more than 5%. In order to perform this well, the lock-in needs to receive a very clean reference signal with the same frequency as the measurement signal. At the core of the lock-in is a

measurement technique called phase sensitive detection. The input signal is multiplied with a reference signal of known frequency and constant amplitude, which is phase locked with the measurement signal.

3.3 Sample characterization

The devices studied in this thesis, are fabricated using HgTe based quantum well hetero-structures described in section 3.1.1. These quantum wells, of a particular thickness d greater than the critical thickness d_c ($d_c \sim 6.3nm$), are expected to be two-dimensional topological insulators. Therefore, the first measurement need to confirm the edge channel conduction in such devices, when the Fermi level is pinned into the bulk gap. Since the hetero-structures are n-type, an appropriate gate potential is needed to pin the Fermi level into the bulk forbidden gap. Hence, we measure the sample resistance as a function of gate potential at zero magnetic field ($B = 0$). Extremely small current, usually 1-10 nA, is applied to the sample, to avoid heating effects.

The measurements are performed as follows. The sample, loaded to the sample probe, is inserted into the sample chamber of the ^4He cryostat. A low frequency (3-13 Hz) AC current from the lock-in amplifier is passed through the current channel of the sample and potential difference, between a given pair of contacts, is measured and sent back to lock-in, which carries out the actual measurement. From there, the signal is sent to digital-analog converter, that send it to computer via LabView interface. The LabView software automatically record all the measured data and information.

The cryostat is also equipped with additional equipments, such as, helium and nitrogen level sensors and also the temperature sensor, which is located close to the sample position for a more reliable measurements of the sample temperature.

Chapter 4

Transport measurements on band-inverted CdTe/HgTe/CdTe quantum well hetrostructures

4.1 Introduction

In this chapter, we describe the transport properties of CdTe/HgTe/CdTe quantum well heterostructures with inverted band structure which are expected to be two-dimensional (2d) topological insulators. The 2d topological insulators are materials which are insulating in the bulk, but exhibits gap-less conducting states at their edges [29, 30, 32, 59]. A well known example of 2d topological insulators is the quantum Hall (QH) insulating state, which exists in a strong magnetic field, perpendicular to the plane of the sample, and is characterized by chiral edge states. Another novel 2d topological system is the quantum spin Hall (QSH) insulator state, which results from the strong spin-orbit interactions, intrinsic to the system, and hence requires no magnetic field. Such systems thus preserve time reversal symmetry (TRS) in contrast to QH systems where the time reversal symmetry is broken by the applied magnetic field. The edge states of QSH insulators are termed helical; that is, the electron's spin is locked to its momentum. Each edge consists of two counter-propagating channels of opposite spin polarities. In the absence of magnetic impurity or inelastic scattering, edge states are protected from back-scattering by the time reversal symmetry [34, 35].

A remarkable consequence of the presence of the edge states in ballistic 2d- QSH insulators is the quantized longitudinal resistance $R_{xx} = \frac{h}{2e^2}$, which has been observed in HgTe Hall bars with sub-micron dimensions [38, 60]. This quantized resistance is expected, as contribution from the edge states, when the Fermi energy is located in the bulk insulating regime. Note, however, that the quantized resistance plateau is destroyed if the sample size exceed a value of

about a few microns [39]. The resistance of samples longer than 1 μm might be much higher than $\frac{h}{2e^2}$ due to the presence of the spin dephasing, i.e., electrons spin-flip back-scattering on each boundary [61]. The backscattering mechanisms are new appealing tasks for theoreticians and is a matter of ongoing debate. In this thesis we uncover the transport properties of HgTe devices with dimensions larger than the ballistic 2d QSH samples studied previously [38]. The observed resistances are much higher than the quantized value $\frac{h}{2e^2}$. Despite the deviation of resistance from its quantized value, large non local signals are observed in the absence of magnetic field confirming the edge state conduction in these heterostructures.

4.2 Resistance peak and transport near charge neutrality point

Electronic transport near charge neutrality point (CNP) remains a subject of hot debate since the realization of graphene and HgTe-based field effect devices. Our HgTe quantum well devices were designed with a top gate electrode as described in chapter 3. The gate voltage induces charge density variations, transforming the quantum well conductivity from n-type to p-type via an intermediate state, where electrons and holes coexist with comparable densities. This intermediate state is called the charge neutrality point (CNP) or the minimum conductivity point. The CNP is usually identified as the maxima of resistivity or the conductivity minima in carrier density dependent transport measurements. In what follows, we present the experimental data showing the gate voltage dependence of local resistances of two different types of HgTe devices which are representative of several devices with the same geometry and configuration.

Device A, (hereafter called as MCT1), consists of three narrow (4 μm wide) consecutive segments of different length (2, 8, 32 μm) and seven voltage probes, is designed for multi-terminal transport measurements (see Fig. 4.1 (a)). Device B, (hereafter called as MCT2), is a six-probe Hall bar fabricated with a lithographic length 6 μm and width 5 μm (see Fig. 4.3 (a)). The dimensions of gate electrode are $62 \times 8 \mu\text{m}^2$ for device MCT1 and $18 \times 10 \mu\text{m}^2$ for device MCT2. Both devices are based on $\text{Cd}_{0.65}\text{Hg}_{0.35}\text{Te}/\text{HgTe}/\text{Cd}_{0.65}\text{Hg}_{0.35}\text{Te}$ quantum wells grown by molecular beam epitaxy (MBE) with (013) surface orientations. The quantum well width d of both type of devices is 8.0-8.3 nm, which is greater than the critical thickness $d_c = 6.3 \text{ nm}$, and are, therefore, expected to be QSH insulators.

As mentioned above, the most striking feature of 2d topological insulating phase is the existence of gapless edge states, therefore, an experimental verification of the edge state conduction is desired. The spin-orbit coupling destroys spin conservation and we don't

expect any quantized spin hall conductance. Observation of charge transport by the edge states would be the most suitable natural experiment. However, as the QSH effect does exist without applying any external magnetic field, one cannot perform a measurement of the Hall conductance, which would be zero. The simplest way is to measure the longitudinal resistance in the Hall bar geometry. The transport measurements, performed on these sample, are divided in two categories: The first one is called “local transport measurements” where current is applied between the regular channel (source and drain) of the Hall bar structure. The resulting potential difference between a given pair of voltage probes on a given side of the Hall bar will give a longitudinal resistance $R_{xx} = \frac{V_{xx}}{I}$. We may also use the term “local resistance” for this longitudinal resistance. The second category describes measurements called as “nonlocal transport measurements”, where current is not applied through regular current channel but, rather, between any pair of contacts. The salient features of such a kind of measurements is that the measured potential difference (if any) between another pair of contacts in the device, far way from the bulk current path, results from the edge channel conduction only because the interior of the sample is insulating. The resistance measured by using this method will be referred as “nonlocal resistance”. This terminology will be followed here and in the subsequent chapters of this thesis.

We begin by measuring the longitudinal or local resistances in one of the device called MCT1. The local resistances for three different configurations (representative configurations) of the device MCT1 (Fig. 4.1 (a)) are calculated by applying current through the contacts 1-5 and measuring the potential differences for the pairs of contacts (9-8), (8-7) and (7-6) such that:

$$R_{I=1-5;V=9-8} = \frac{V_{9-8}}{I_{1-5}},$$

$$R_{I=1-5;V=8-7} = \frac{V_{8-7}}{I_{1-5}},$$

$$R_{I=1-5;V=7-6} = \frac{V_{7-6}}{I_{1-5}}.$$

The contacts 1-5 are labeled as source and drain or regular channel. The spacing between voltage probes is $2\mu m$, $8\mu m$ and $32\mu m$, respectively for probes pairs (9-8), (8-7) and (7-6) which can be considered as the edge channel lengths for these segments. It is worth noting that the contacts to the QSH insulators are assumed to be thermal reservoirs, where the electron states with opposite spins are mixed. In contrast to the QHE, where the mixing of the edge states occurs within metallic Ohmic contacts, in our samples this takes place in the 2D electron gas region outside the metallic gate due to a finite bulk conductivity.

Figure 4.1: (a) Schematics showing the local transport measurements on multi-terminal device MCT1. Current is applied between contacts 1-5 and the potential difference is measured in consecutive pairs (9-8), (8-7), (7-6) corresponding to different segments of the device. (b) Device MCT1 with a top gate electrode covering the entire sample. Blue and red lines shows the edge states counter-propagating at a given edge.

Therefore, the effective length of the 1D channels exceeds the distance between the probes of the Hall bar by 3–4 μm . Resistances were also measured for other local configurations. For example, passing current between contacts 1-7 and measuring the potential difference in contact pair 9-8 (wine line) and 2-3 (violet line) in Fig. 4.2. Figure 4.2 presents the experimental data of local resistances measured as a function of normalized gate voltage, $V_g - V_{\text{CNP}}$, for several different local configurations of the device MCT1. V_{CNP} is the gate voltage for which the resistance is maximum and is known as the charge neutrality point (CNP). The HgTe quantum wells studied in this work are n-doped as already mentioned in chapter 3. Since the devices are equipped with a top gate electrode which allow for carrier density variation in the quantum well via electric field effect. For a certain value of the gate potential, electrons and holes coexist with close densities. This point is known as the charge neutrality point (CNP) and in the measurements of resistance or conductance versus gate voltage, V_{CNP} correspond to this point. Note that at the CNP, electrons and holes have finite densities. A small low frequency (3-13 Hz) current of about 1 nA was applied to avoid overheating effect. The measurement temperature was 4.2 K. Schematic diagram showing the local transport measurements on the smaller device MCT2 is shown in Fig. 4.3 (a). The current is applied between contacts 1-4, which is the regular current channel of the device and the potential difference is measured between the pair of contacts 5-6. The resulting longitudinal resistance is:

Figure 4.2: Longitudinal (Local) resistances R_{xx} as a function of normalized gate potential $V_g - V_{CNP}$ for various configurations of the device. V_{CNP} is the gate voltage for which the resistance is maximum. The blue curve should be multiplied by 1.5; $I=1\text{nA}$ and $T=4.2\text{K}$.

$$R_{I=1-4;V=5-6} = \frac{V_{5-6}}{I_{1-4}}$$

Figure 4.4 shows the local resistance, measured in configuration $R_{I=1-4;V=5-6} = \frac{V_{5-6}}{I_{1-4}}$, plotted as a function of normalized gate voltage. As can be seen that, by sweeping the gate voltage from positive to negative, we arrive at CNP where the resistance shows its maxima. At CNP the Fermi-level is assumed to be in the bulk insulating gap and transport is dominated by edge states. One, therefore, expects a quantized longitudinal resistance of $h/2e^2$ for 2d topological insulators, in the ballistic regime. Since our samples are longer than ballistic samples ($1\mu\text{m} \times 1\mu\text{m}$) used before [38, 39], the resistance in our case is an order of magnitude higher than its quantized value. This means that the transport in our samples is diffusive, rather than ballistic one. The effective length of 1d channel, even in case of the short sample (MCT2), is about $10\mu\text{m}$. The deviation of longitudinal resistance from its quantized value, in macroscopic samples, is discussed in section 4.4.

Figure 4.3: (a) Schematics of local transport measurements on smaller device MCT2. Current is applied between contacts 1-4 and the potential difference is measured in pair of contacts 2-3. (b) Device MCT2 with a top gate electrode making it possible to change the quantum well polarity from n-type to p-type. Blue and red lines is a manifestation of the helical edge states which counterpropagate at a given edge of the sample.

Figure 4.4: Local resistance $R_{I=1-4;V=5-6}(=V_{5-6}/I_{1-4})$ plotted as a function of normalized gate voltage $V_g - V_{CNP}$, when the current is applied in contacts 1 and 4 and potential difference measured between contacts 5-6. V_{CNP} is the minimum conductance point.

4.3 Nonlocal transport: Unequivocal evidence for edge states conduction

Bulk transport in conventional diffusive electronics is governed by Ohm's law. The resistance of a conductor is directly proportional to the length and inversely proportional to the cross-sectional area, which intimate the existence of a local resistivity or conductivity tensor. The presence of edge states, however, leads to nonlocal transport which undermines the concept of local resistivity. Nonlocal effects appears due to the formation of edge channels that are isolated from the bulk and can carry current to classically non accessible regions. Applying current between a pair of the probes creates a net current along the sample edge and can be detected by another pair of voltage probes away from the dissipative bulk current path. Such nonlocal transport has been observed, experimentally, in the quantum Hall (QH) systems in the presence of a magnetic field and these measurements are now widely acknowledged as constituting definitive experimental evidence for the existence of edge states in the QH regime [62]. It is worth noting that the physics of the non-locality is significantly different in the QH and the QSH systems. In the QH systems, the nonlocal resistance arises from the suppression of electron scattering between the edge channels and the bulk states [62]. The non-locality in QSH insulators stems from the unique features of helical edge modes which exists in the absence of a magnetic field and is an intrinsic property of this novel class of materials [52].

In this section we present experimental results of nonlocal transport measurements on two different devices, with layouts as outlined below. Device MCT1 specially designed for multi-terminal measurements consists of three narrow ($4\text{ }\mu\text{m}$ wide) consecutive segments of different length ($2, 8, 32\text{ }\mu\text{m}$) and seven voltage probes (see Fig.4.5). The device MCT2, is a six-probe Hall bar fabricated with a lithographic length $6\text{ }\mu\text{m}$ and width $5\text{ }\mu\text{m}$ (Fig. 4.7). The dimensions of gate electrode are $62\text{ }\mu\text{m}\times 8\text{ }\mu\text{m}$ for device MCT1 and $18\text{ }\mu\text{m}\times 10\text{ }\mu\text{m}$ for device MCT2.

Figure 4.6 shows the nonlocal resistances as a function of gate voltage measured for several different configurations of the device MCT1. For instance; in one configuration (red line) the current is applied through contacts 9-2 and voltage is measured between contacts pair 3-8; i.e., $R_{I=9-2;V=3-8}(=V_{3-8}/I_{2-9})$ while in other configuration (Magenta line) the current passes through contacts 4-7 and voltage is measured between contacts 2-9, i.e., $R_{I=4-7;V=2-9}(=V_{2-9}/I_{4-7})$.

Resistances were measured in several other nonlocal configurations. As an example, consider the configuration when the current is applied between contacts 7-6 and the voltage is measured in contact pairs (1-2) and (2-3). The resulting nonlocal resistances (cyan and olive lines, respectively) are shown in Fig. 4.6. The figure shows the resistances measured

(a)

(b)

Figure 4.5: Schematic diagrams showing the nonlocal transport measurements on multi-terminal device MCT1. (a) Current is applied between contacts 2,9 and the potential difference is measured in pair of contacts 3-8. (b) Another nonlocal configuration where current is applied between contacts 4,7 and the potential difference is measured in pair of contacts 2-9 far away from the current path.

Figure 4.6: Nonlocal resistances plotted as a function of normalized gate potential $V_g - V_{\text{CNP}}$ for different configurations of the device. The red and blue curves should be multiplied by 2; $I = 1 \text{ nA}$ and $T = 4.2 \text{ K}$

in various nonlocal configurations. One can notice that the peaks are narrower compared to the local resistances measured for the same device. Because the samples are macroscopic, we don't see any quantization or applying the Landauir-Buttiker formalism as done before for mesoscopic samples [52]. We assume that in large macroscopic samples, of several microns length, the bulk contribution can not be ignored. Including bulk contribution to edge state conduction, we can roughly scale the observed resistances with probe spacing and spacing from the current contacts. The experimental findings are discussed in section 4.4.

Figure 4.7 shows the schematic picture of nonlocal transport measurements in device MCT2. The current is applied in the left leg (contacts 2-6) and the potential difference is measured in the right leg (contacts 3-5) of the sample. The resulting nonlocal resistance is given as $R_{I=2-6;V=3-5} = \frac{V_{3-5}}{I_{2-6}}$ and is shown in Fig. 4.8 (dotted magenta line). The figure also shows the comparison of local and nonlocal resistances of the same device with same probe spacing (solid violet and dotted magenta lines). One can see that the nonlocal signal (dotted magenta line) is three times smaller than the local one (solid violet line). Though it is important to mention that both the local and nonlocal resistances are still higher than the quantized value but comparatively smaller than the corresponding values for the longer device (MCT1). The nonlocal signal here is about $70 \text{ k}\Omega$ which is four times larger than the

Figure 4.7: Showing nonlocal configuration measurements for the device MCT2. The current flows through contacts 2,6 and voltage is measured in pair of contacts 3,5 away from the bulk current path.

resistance quantum.

The sharp peak in the local and nonlocal resistivity near the CNP with a coexistence of electrons and holes can be explained in terms of a model including simultaneously the edge states and bulk transport, and taking into account the back-scattering both between the counter-propagating edge states, at a given edge described by a single phenomenological parameter γ , and the bulk-edge coupling described by another phenomenological parameter g . The detailed description of this model and how we apply this model to simulate our experimental findings is given in chapter 5.

Figure 4.8: Local (Violet solid line) and nonlocal (Magenta dotted line) resistances plotted as function of normalized gate voltage for the device MCT2

4.4 Discussions

According to theoretical predictions electron transport along the edge of a 2d topological insulator is reflection-less, provided the time reversal symmetry (TRS) is preserved, leading to quantized edge conductance if the Fermi level is placed in the bulk energy gap [30]. Conductance closed to the quantized value $G_0 = \frac{2e^2}{h}$ per edge was observed for very small samples ($1\mu m \times 1\mu m$) of HgTe based 2d topological insulators [38, 39, 52]. However, in samples larger than few microns, the edge channel conductance deviates from its quantized value [39, 49, 63, 64]. Several theoretical models were proposed to explain the deviation of conductance from quantized value in macroscopic samples. Out of these models inelastic mechanisms were predicted to have a strong temperature dependence. Conductance deviation scales with temperature as $(\frac{T}{E_g})^4$ [65] or $(\frac{T}{E_g})^6$ [66] unless TRS is broken. Recently influence of charge puddles, resulting from inhomogeneous charge distribution in 2d topological insulators, on its helical edges conductance, has been considered [2]. Since the structures are doped, donor or acceptor charged impurities, which are inevitably present in these heterostructures, results in non-uniform potential landscape for electrons. Also the quantum spin Hall insulators are narrow-gap semiconductors with a typical gap of 10-20 meV [38, 49]. An appropriate gate potential is required in order to place the Fermi level in the band gap

Figure 4.9: Electrons moving along a helical edge tunnel in and out of puddles created by the inhomogeneous charge distribution in the heterostructure. Figure taken from Ref.[2].

so that the gate charge compensate the uncompensated donor or acceptor charge density. The combined effect of gate potential and ionized dopant may results in the formation of electrons and holes puddles in the quantum well (see Fig. 4.9[2]).

A single puddle is modeled by a quantum dot tunnel-coupled to the helical edge. Electron's dwelling in the dot may lead to significant inelastic backscattering within the edge. A crossover value n_0 of the donor density n_d is established below which the puddles are rare. This characteristic value is given as

$$n_0 = \frac{E_g^2 \kappa^2}{8\pi e^4 l_n \left[\frac{l_g^2}{(2l_g - l_d)l_d} \right]} \quad (4.1)$$

where E_g is the gap energy, $\kappa = 13$ is the dielectric constant $l_g = 340 \text{ nm}$ is the distance to the gate and $l_d = 8 \text{ nm}$ is the distance to the donors. The density of puddles can be estimated as

$$n_P = \left(\frac{1}{l_g a_B} \right) \left(\frac{n_0}{n_d} \right) \exp\left(-\frac{n_0}{n_d}\right) \quad (4.2)$$

where $a_B = \frac{2\hbar v}{\alpha E_g}$ is the effective Bohr radius, $\alpha = e^2/\kappa\hbar v$, v being the electron velocity. Using the parameters for our samples: $l_g = 340 \text{ nm}$, $l_d = 8 \text{ nm}$, $n_d = 2 \times 10^{11} \text{ cm}^{-2}$, $\alpha = 0.3$ and $a_B = 120 \text{ nm}$ we can calculate n_0 and n_p from equations (4.1) and (4.2), $n_0 \simeq 4 \times 10^{10} \text{ cm}^{-2}$ and $n_p = 4 \times 10^9 \text{ cm}^{-2}$.

A characteristic length $n_p^{-1/2}$ is defined such that for $L \leq n_p^{-1/2}$ only few puddles occur in the vicinity of the edge. This may leads to mesoscopic fluctuations of conductance with

the Fermi level E_F in small samples. For $L \gg n_p^{-1/2}$ the self averaged resistance is given by

$$R = \frac{h}{e^2} \frac{1}{g^2} n_p \lambda \left(\frac{T}{\delta} \right)^3 L \quad (4.3)$$

where $\lambda = \frac{\hbar v}{E_g}$ is the electron penetration depth into the puddles, $v = 5.5 \times 10^7 \text{ cm/s}$ is the electron velocity and $E_g \simeq 20 \text{ meV}$ is the forbidden gap. δ is the mean level spacing within the dot, L is the probe spacing (length of the edge channel), g is the dimensionless conductance within the dot and can be estimated as $g \sim \sqrt{N} \simeq 1 - 2$ where N is the number of electrons in the puddle, $N \sim a_B n_d^{1/2} \approx 2 - 5$. Equation 4.3 can be rewritten as $R = \rho_{edge} L$ where

$$\rho_{edge} = \frac{h}{e^2} \rho_0 = \frac{h}{e^2} \frac{1}{g^2} n_p \lambda \left(\frac{T}{\delta} \right)^3 \quad (4.4)$$

Using all the above given parameters we calculate $\rho_0 = 2 \times 10^3 \left(\frac{T}{\delta} \right)^3 e^2/h/\text{cm}$ for $g \sim 1$. The mean level spacing within the dot is estimated as $\delta \sim \alpha^2 E_g \approx 1 - 2 \text{ meV}$. At relatively high temperature, $T \approx 10 \text{ K}$, $T \sim \delta$ and it is expected that the temperature dependence is saturated. At $n_d \ll n_0$ the charge puddles are small and rare. In case $n_d \gg n_0$ large puddles are formed separated by p-n junction and the bulk conductivity could shunt the edge state contribution.

To summarize, we have measured local and nonlocal resistances in HgTe-based two-dimensional topological insulators, in the absence of magnetic field. Even though, the resistance values at the CNP is higher than the resistance quantum h/e^2 and large nonlocal signals at the CNP, in the absence of magnetic field, is a clear evidence of edge state conduction in these heterostructures. One can notice that the peak resistance values at the CNP for the relatively short sample (MCT2, $L = 6 \mu\text{m}$) is smaller than that of the longer sample (MCT1, $L = 62 \mu\text{m}$). Deviation from quantized resistance may result from the relaxation processes in the contact regions. The contacts are assumed to be thermal reservoirs [67], where the mixing of electron states with different spins will occur. The high resistance values at the CNP is more likely the result of backscattering between counter propagating edge channels. Elastic processes are prohibited as long as the time reversal symmetry (TRS) is preserved. Inelastic backscattering within the helical edge, resulting from electron's dwelling in the puddles, is the most reasonable explanation ([2]). In our samples, the condition $n_d \gg n_0$ implies the formation of large size puddles which are tunnel coupled to the edge states. Based on this model, we may argue that in short sample, $L \sim n_p^{-1/2}$ and hence the self averaged resistance peak at the CNP is smaller than that of the longer sample.

Chapter 5

Model, simulations and comparison with experiment

5.1 Introduction

In this chapter we present the numerical simulations we have performed to model our results using a transport model developed originally for graphene [68]. It was found that the transport, near the Dirac point, in graphene in a high magnetic field is dominated by a pair of counter-propagating chiral edge modes. It was argued that such modes, intrinsic to Dirac fermions, manifest themselves in a large longitudinal resistivity $\rho_{xx} \gtrsim \frac{h}{e^2}$ near the Dirac point also known as the charge neutrality point (CNP). This behavior is in striking contrast to the standard quantum hall systems where the longitudinal resistivity goes to zero whenever the hall resistivity passes through a plateau. Such a dissipative QHE state was modeled by the so-called edge plus bulk transport model which, simultaneously, takes into account a combination of both edge states transport and transport through the bulk. The edge states contributes at the charge neutrality point (CNP) while away from the CNP the bulk currents short circuit the edge states transport. We have used this simple model for our HgTe quantum well structures to describe the transport properties near the CNP. The model reproduces many of the key features of the data, in particular the density dependence of the local and nonlocal resistivity. The main difference between graphene and our system is that in QSH insulators the counter-propagating edge states arises from the spin orbit interactions, intrinsic to the system, and hence requires no magnetic field. In our self-consistent calculations the solution $\psi_{1,2}$ of Laplace equation in two-dimensions (2D) is obtained by solving the partial differential equation (PDE) problem using Finite Element Method (FEM) while the solutions $\varphi_{1,2}$ on the edges of the hall bar are obtained by solving the boundary value problem for a system of ordinary differential equations (ODE). The details of how to set up

this PDE and ODE problems and the program for self consistent calculations are given in section 5.3.

5.2 The edge plus bulk transport model

The non-universal ($R \gg h/2e^2$) peak in the local and nonlocal resistivity near the CNP with a coexistence of electrons and holes (experimental results of chapter 4) can be explained in terms of a model including both the edge-state and bulk transport contributions, taking into account the backscattering within one edge described by a single phenomenological parameter γ and the bulk-edge coupling described by another phenomenological parameter g . The local and nonlocal transport coefficients arise from the edge state contribution and short-circuiting of the edge transport by the bulk contribution, the latter is more pronounced away from the CNP.

The edge transport model including spin flip back-scattering is described by the coupled equations for particle density in the two spin-polarized modes [68, 69]:

$$\begin{aligned}\partial_t n_1 + \partial_x \varphi_1 &= \gamma(\varphi_2 - \varphi_1) \\ \partial_t n_2 - \partial_x \varphi_2 &= \gamma(\varphi_1 - \varphi_2)\end{aligned}\tag{5.1}$$

where $n_i = \nu_i \varphi_i$,

ν_i = compressibility of each mode,

φ_i = local chemical potential of electron-like and hole-like edge modes,

and $\gamma^{-1} = l$ is the mean free path for 1d- backscattering between modes 1 and 2.

In a stationary state, equation (5.1) have an integral $\tilde{I} = \varphi_1 - \varphi_2$ which expresses the conservation of currents $I = \frac{e^2}{h} \tilde{I}$. The general solution in the stationary current-carrying state is $\varphi_{1,2} = \varphi_{1,2}^* - \gamma \tilde{I} x$. Taking into account that $\tilde{I}$ is the current in one edge, we calculate the total current as $I = 2\tilde{I}$. In the absence of transport through the bulk the longitudinal resistance is

$$R_{xx} = (\gamma L + 1) \frac{h}{2e^2}$$

and

$$\rho_{xx} = \left(\frac{w}{L}\right) R_{xx}$$

with w/L the aspect ratio. For sake of convenience, in the rest of the text we will work in the units of $h/e^2 = 1$.

It is crucial that the edge transport model (equation 5.1) treats both edges of a Hall bar

in an identical way, thus predicting a zero Hall effect. In order to understand the observed density dependence of the Hall coefficients [68, 34], which changes sign smoothly at $\nu = 0$ without exhibiting a plateau, and of ρ_{xx} which has a sharp peak at $\nu = 0$, Abanin *et.al.*, incorporated the transport in the bulk in their model [68]. This model is called the edge plus bulk transport model. In the full edge plus bulk model, the density dependence of the transport coefficients arises from bulk currents short-circuiting edge currents away from the CNP. This explains in a quite better way the results of transport measurements of graphene as well as the HgTe based 2D topological insulators.

The transport properties in the bulk can be described by the current-potential relation:

$$J_i(r) = -\hat{\sigma}_i \nabla \psi_i(r),$$

$$\hat{\sigma}_i = \begin{pmatrix} \sigma_{xx}^{(i)} & \sigma_{xy}^{(i)} \\ -\sigma_{xy}^{(i)} & \sigma_{xx}^{(i)} \end{pmatrix} \quad (5.2)$$

where $i = 1, 2$ represents electronic states for up- and down spin, ψ_i is the electrochemical potentials for electrons in the bulk and $r = (x, y)$ is the 2D coordinate. As we consider the isotropic conduction, the non-diagonal terms of the conductivity tensor appears only at non-zero magnetic field. Assuming the components of the conductivity tensor as coordinate-independent parameters, we can solve the problem by solving the Laplace equation for the potentials, $\nabla^2 \psi_i(r) = 0$, since the charge continuity conditions requires $\nabla J_i = 0$. A solution to Laplace's equation is uniquely determined if the value of the function is specified on all boundaries. In our case, the boundary conditions are modified by the bulk-edge leakage current. In order to describe the transport properties in the presence of the edge states we introduce two phenomenological constants γ and g , which represent edge to edge and bulk to edge inverse scattering length, respectively. In the presence of bulk-edge coupling, the boundary conditions, expressing zero current normal to the boundary, are given by

$$nJ_i = g(\psi_i - \varphi_i) \quad (5.3)$$

where n is a normal vector to the boundary and $g(\psi_i - \varphi_i)$ represent the edge-bulk leakage current density.

Equation for the edge state transport can be obtained by adding the bulk-edge leakage term to equation (5.1), giving

$$\partial_x \varphi_1 = \gamma(\varphi_2 - \varphi_1) + g(\psi_1 - \varphi_1) \quad (5.4)$$

$$-\partial_x \varphi_2 = \gamma(\varphi_1 - \varphi_2) + g(\psi_2 - \varphi_2) \quad (5.5)$$

along with a similar pair of equations for φ'_1, φ'_2 at the opposite edges.

The general solution of this problem, therefore, includes solution of a 2D Laplace equation for $\psi_{1,2}(x, y)$ together with equations (5.3), (5.4), (5.5) describing the scattering between up- and down spin modes and between edge and bulk states. The current can be calculated from this solution as a sum of the contribution from the bulk and both edge states.

The general relation between current and current density is:

$$I = \int j dA$$

which in 2D, for local configuration of this problem, turns out to be;

$$I_x = \int_0^w j_x dy$$

where w is the width of the Hall bar geometry of the sample. For spin-up mode

$$I_x^{(1)} = \int_0^w j_x^{(1)} dy = - \int_0^w \sigma_{xx}^{(1)} \frac{\partial \psi^{(1)}}{\partial x} dy$$

while for spin-down mode

$$I_x^{(2)} = \int_0^w j_x^{(2)} dy = - \int_0^w \sigma_{xx}^{(2)} \frac{\partial \psi^{(2)}}{\partial x} dy$$

So that the bulk current becomes

$$I_b = - \int_0^w \sigma_{xx}^{(1)} \frac{\partial \psi^{(1)}}{\partial x} dy - \int_0^w \sigma_{xx}^{(2)} \frac{\partial \psi^{(2)}}{\partial x} dy$$

For the nonlocal configuration the currents are calculated in a similar way, but the variables for derivative and integration in the equation for I_b are interchanged such that:

$$I_b = - \int_0^L \sigma_{xx}^{(1)} \frac{\partial \psi^{(1)}}{\partial y} dx - \int_0^L \sigma_{xx}^{(2)} \frac{\partial \psi^{(2)}}{\partial y} dx$$

where L is the length of the Hall bar segments. The conductivities $\sigma_{xx}^{(1,2)}$ can be calculated using Drude picture for diffusive transport;

$$\sigma_{xx}^{(1)} = e\mu_p p$$

$$\sigma_{xx}^{(2)} = e\mu_n n$$

where $e = 1.6 \times 10^{-19}C$ is the electronic charge, n , p is the respective electrons and holes concentrations and μ_n , μ_p are the mobilities of electrons and holes, respectively.

The n and p can be calculated using formulas

$$n = \int D_0 \exp\left(-\frac{E^2}{2\Gamma_n^2}\right) f(E) dE$$

$$p = \int D_0 \exp\left(-\frac{E^2}{2\Gamma_p^2}\right) f(E) dE$$

Note that, the sharp band edges are smeared according to a Gaussian law with broadening energies $\Gamma_{n,p}$ and $D_0 = 4\pi m^*/h^2$ is the energy independent density of states (DOS) in 2D. $f(E) = \frac{1}{e^{(E-E_f)/K_B T} + 1}$ is the Fermi-Dirac distribution function with E_f being the Fermi level, T is the temperature and K_B is the Boltzmann constant. In the expression for D_0 , h is the Planck's constant and m^* is the effective mass of electrons and holes. In the case of HgTe quantum well, we use $m^* = 0.024m_e$ for electrons and $m^* = 0.15m_e$ for holes.

It is important to mention that the mobility gap can be smaller than the energy gap due to disorder. We may assume that the random potential due to charged impurities can result in tails states in the band gap for both electrons and holes which decreases the band gap. It is worth noting that the disorder parameter, which can reduce the band gap in QSHI, is related to the deviations of the HgTe quantum well thickness from its average value [70] rather than to the random potential due to charged impurities. The energy gap is known to be 30 meV for 8 nm HgTe quantum wells with (013) surface orientation [71]. Fig. 5.1 shows qualitative picture of the smearing of the band edges due to disorder. In the absence of disorder, the band edge are sharply defined as E_c and E_v such that the mobility gap is the same as the band gap. In the presence of disorder, the band edges smears out so that the mobility gap is smaller than the actual band gap.

The edge current can be calculated using the relation:

$$I_{ed} = \frac{e^2}{h} (\varphi_1 - \varphi_2 + \varphi'_2 - \varphi'_1),$$

where the primed and unprimed φ are potentials at the bottom and top edges of the Hall bar respectively. The total current is the sum of the edge and bulk currents. i.e

$$I_t = I_{ed} + I_b,$$

and finally the resistance can be calculated as

Figure 5.1: Schematic diagram showing disorder induced smearing of the band edges. Random potential due to fluctuations in the quantum well thickness causes Gaussian like tails in band gap. E_v and E_c defines the sharp band edges for ideal clean samples in the absence of disorder.

$$R_{xx} = \frac{V_{xx}}{I_t}.$$

5.3 Numerical Methods

Self-consistent calculations were performed to find the solution $\psi_{1,2}$ of Laplace equation in two-dimensions on the rectangular grid and $\varphi_{1,2}$ solutions of equations (5.4), (5.5) on the edges of the Hall bar. The desired geometry of the device, to be simulated, was drawn using the graphical user interface (GUI) in Matlab. This geometry is then decomposed into minimal regions and the equations for $\psi_{1,2}$ are discretized by the Finite Element Method. Dirichlet boundary conditions are set at the $62\mu m$ long metal contacts located at left and right side of the bar for the local case or $8\mu m$ wide contacts around $y = 0$ at the top and the bottom edge of the bar for the non local case (see Fig. 5.2). The generalized Neumann boundary conditions (5.3) are set everywhere outside the metal contacts. The contacts are assumed to be thermal reservoirs, where full mixing of electron spin states and bulk states occurs [69]. Note that, in contrast to the standard QHE, where mixing of the edge states occurs within metallic Ohmic contacts, in our samples potential mixing is provided by 2D electron gas in the region outside of the metallic gate.

To solve the boundary value problem for a system of ordinary differential equations (5.4) and (5.5) we use a finite difference code that implements the 3-stage Lobatto IIIa formula. The boundary conditions are set to $\varphi_{1,2} = \psi_{1,2}$ inside the metal contacts. The solution $\psi_{1,2}$ of Laplace equation is feed to calculate $\varphi_{1,2}$ which is feed back to PDE problem to get $\psi_{1,2}$. The loop is repeated so many times and thus we have an iterative solution.

Figure 5.2: Solution of Laplace equation on 2D plane of the sample MCT1. The left figure shows the nonlocal configuration when the current is applied between contacts 2-9, while the right figure shows the situation when the current flows in contact pair 4-7. The maximum potential amplitude is determined by Dirichlet boundary conditions.

5.4 Simulation results and comparison with experiment

In this section we present the results obtained from simulating HgTe devices used in this work. Simulations were performed on geometries drawn very close to the real devices in both local and nonlocal configurations. We compare the results obtained from simulations with those measured experimentally on CdTe/HgTe/CdTe quantum well heterostructures. The modeled behavior of local and nonlocal resistances as a function of carrier density for both type of devices, MCT1 and MCT2, is shown in figures 5.3 and 5.4. We assume that the electrostatic potential of charged impurities, which are inevitably introduced during device fabrication, acts as a source of disorder. Since the charges are randomly distributed the disorder is modeled by a Gaussian distribution of carrier densities. Several values of the standard deviation ΔN_s (disorder strength) were tried to get a best fit of the experimental data. Results close to the experimental data were obtained for $\Delta N_s = 3.5 \times 10^{10} \text{cm}^{-2}$ for the device MCT1 and $\Delta N_s = 3.0 \times 10^{10} \text{cm}^{-2}$ for the device MCT2. The best agreement between the experiment and theory is reached for the value of phenomenological parameters $\gamma = 3.0$ and $g = 0.03$ for the sample MCT1 while $\gamma = 1.4$ and $g = 0.05$ for sample MCT2.

The comparison shown in figures 5.3 is the representative behavior of couple of local and nonlocal configurations in device MCT1. Because this sample is specially designed for multi-terminal measurements, several possible local and nonlocal resistances were measured and simulated. The comparison of the experimental and simulated values is given in table 5.1 for the local case, while table 5.2 present the experimental and simulation data for the nonlocal configurations of the same device.

Figure 5.3: (Top) Comparison of local resistances for three segments of device MCT1: $L=2\mu\text{m}$ (red solid line), $8\mu\text{m}$ (cyan solid line) and $32\mu\text{m}$ (blue solid line). Dashed traces show resistances for all the three segments predicted by the edge plus bulk transport model. (Bottom) Nonlocal resistances for two different configurations of device MCT1. Red solid line is the nonlocal response when current is applied between contacts 2-9 and potential is measured between 3-8. Blue solid line represent the nonlocal signal when current flows in contacts 4-7 and voltage is measured in pair of contacts 2-9. The dashed traces are corresponding signals predicted by theory.

Figure 5.4: Local (blue solid line) and nonlocal (red solid line) resistances measured as a function of normalized carrier density for device MCT2. The dotted lines are the corresponding simulation results using the edge plus bulk transport model.

Device configuration	Experimental $R_{\text{CNP}}(\text{K}\Omega)$	Simulation results $R_{\text{CNP}}(\text{K}\Omega)$
	290	280
	596	630
	1150	1180
	315	220
	624	430
	330	450
	310	400
	265	398
	412	450
	585	406
	660	480
	616	417
	685	466
	510	405

Table 5.1: Table showing various local configurations of the device MCT1 with simulated and experimental value given for each configuration. The black circles denote contacts where current is applied while green squares shows contacts for voltage measurements.

Device configuration	Experimental $R_{\text{CNP}}(\text{K}\Omega)$	Simulation results $R_{\text{CNP}}(\text{K}\Omega)$
	450	510
	120	98
	395	426
	244	210
	100	75
	210	230
	24	22
	16	7
	20	7
	100	75
	130	97
	20	19
	96	81
	68	88
	73	75
	120	90
	76	40

Table 5.2: Table showing various nonlocal configurations of the device MCT1 with simulated and experimental value given for each configuration. The black circles denote contacts where current is applied while green squares shows contacts for voltage measurements.

5.5 Discussion

Deviation of edge channel conductance from its quantized value is one of the major issue in QSHI sample longer than few microns. We have addressed this issue by using a model which, simultaneously, takes into account a combination of edge states transport and transport through the bulk. The metallic contacts in the sample are assumed to be thermal reservoirs, where full mixing of electron spin states and bulk states occurs. This was accomplished by mixing the chemical potential of edge states and bulk states in the contact region. The model reproduces the key features of the data, in particular, the density dependence of the local and nonlocal resistances. The resistance peak values at the CNP are calculated for several local and nonlocal configurations shown in tables 5.1 and 5.2. We can see that, for some of these configurations (not all), the peak values are very close or comparable.

Chapter 6

Kirchhoff's Rules and edge channel transport in macroscopic multi-probe Hall bars

6.1 Introduction

Topological insulators are materials characterized by gapless electron states on the surface while gaped ones in the bulk. In two-dimensional topological insulators (quantum spin hall insulators) the conducting states propagate along the edges of the device. These states are protected by time reversal symmetry (TRS) and can therefore propagate without backscattering as long as the TRS is not broken. In such a case a quantized conductance of $2e^2/h$ is expected for edge channels if the chemical potential is pinned into the bulk gap. This quantized conductance is experimentally verified in small micron size samples of 2D topological insulators based on HgTe quantum wells where the transport occurs in the ballistic regime [38]. The conduction due to edge channels is particularly demonstrated in nonlocal transport measurements in these samples [52]. The Landauer-Buttiker formalism [72] better explain the experimental observations in these ballistic transport conductors. Recently Protogenov *et.al.* [3], has extended the ballistic Landauer-Buttiker approach to general N-terminal scheme for studying nonlocal transport in N-terminal devices. In the device shown in figure 6.1 if the current flows through 1 and N, the voltage on all other terminals can be obtained from [3]

$$AV = I \tag{6.1}$$

where the matrix A equals $A_{ij} = 2\delta_{ij} - \delta_{i,j+1} - \delta_{i,j-1} - \delta_{i,1}\delta_{j,N} - \delta_{i,N}\delta_{j,1}$

$\delta_{i,j}$ is the Kronecker delta, $1 \leq i, j \leq N$, $V = V_1, V_2, \dots, V_N$ and $I = I_{1N}$. The

Figure 6.1: Edge channels in two dimensional N-terminal insulators. Figure taken from [3].

solution of equation 6.1, for arbitrary N, has the form $V_i = I_{1N}(1 - \frac{i}{N})$.

The nonlocal resistance $R_{kl,ij}$ for $k = 1$ and $l = N$ when measuring the voltage between terminals i and j is

$$R_{1N,ij} = \frac{j-i}{N} \quad (6.2)$$

Note that equation (6.2) is valid only if the current is applied between the first and the last terminals. In case the current flows between terminals 1 and k ($k \leq N$) the exact solution of equation (6.1) using $I = I_{1k}$ is of the form

$$V_i = I_{1k}(1 - \frac{i}{N}(1 - k + N)) \quad (6.3)$$

if $1 \leq i \leq k$ and

$$V_i = I_{1k}(1 - \frac{i}{N})(1 - k) \quad (6.4)$$

if $k \leq i \leq N$

The nonlocal resistance in this general case for the quantum spin hall insulators, when the Fermi level is located in the bulk forbidden gap, is given by

$$R_{1k,ij} = \begin{cases} \frac{j-i}{N}(1 - k + N), & 1 \leq i, j \leq k \\ \frac{j-i}{N}(1 - k), & k \leq i, j \leq N \\ \frac{j-i}{N}(1 - k) + (k - i), & 1 \leq i \leq k, k \leq j \leq N. \end{cases} \quad (6.5)$$

Applying equation (6.5) to our device shown in figure 6.2 with $N = 9$ we obtain resistances for various nonlocal configurations shown in table 6.1.

Figure 6.2: Schematic diagram showing nonlocal transport measurements in multi-terminal device with a configuration where current is applied through contacts 4-7 and the potential difference is measured in pair of contacts 2-9 and 3-8 far away from the dissipative path.

Device configuration	Calculated using eq. 6.5 $R(h/e^2)$	Experimental results $R(h/e^2)$
	1.11	17
	0.66	4.6
	1.11	15.2
	1.33	9.4
	0.66	3.85
	1.33	8

Table 6.1: Table showing comparison of experimental values of resistances and those calculated using equation (6.5) for various nonlocal configurations. The black circles denote contacts where current is applied while green squares show contacts for voltage measurements.

Figure 6.3: Effective circuit model describing edge channel transport in macroscopic device with several voltage probes. Current applied between contacts 1-5 is divided into two paths: 1-2-3-4-5 and 1-9-8-7-6-5. The current ratio along the two paths is governed by Kirchhoff rules.

The table shows the comparison between the calculated and experimental values. The experimental values are much higher than the quantized values for a given configuration. This is because our devices are of large size so that we don't expect quantized conductance. In macroscopic samples with $L \gg L_\varphi$, where L_φ is the phase coherence length, the transport is no longer ballistic but rather diffusive. Resistances higher than quantized values are expected if the edge channel length L (probe separation between a given pair of contacts) is longer than L_φ ($L \gg L_\varphi$). The current and voltage distributions in such macroscopic devices can be understood using the effective circuit model as shown in figure 6.3, where the system can be described by a network of channel resistances r_{ij} ($i=1, 2, \dots, \text{mod}9, j = i+1$). In diffusive case the resistance of an edge channel is given by [68]

$$r_{ij} = \frac{h}{2e^2} \frac{L_{ij}}{L_\varphi} \quad (6.6)$$

where L_{ij} is the length of a given edge channel and L_φ is the mean free path for 1d back-scattering between counter-propagating edge modes. In other words two contacts to the helical channels are connected by a series of quantum resistors with a total resistance value given by equation (6.6). For the configuration shown in figure 6.3 the contacts used as source and drain are 1 and 5, respectively. According to Kirchhoff rules the injected current is partitioned into two paths along the sample edges connecting the source and drain contacts in the clockwise (i_1) and anti-clockwise (i_2) directions. The potential difference between a given pair of contacts (say 9-8) equals the potential drop across the channel resistor which is the fractional current (i_2) through the resistor times the resistance r_{ij} .

contact pairs: i,j	Edge channel length: $L_{i,j}$	Channel resistance: $r_{i,j}$
1-2	4 μ m	130k Ω
2-3	6 μ m	195k Ω
3-4	12 μ m	390k Ω
4-5	40 μ m	1300k Ω
5-6	4 μ m	130k Ω
6-7	36 μ m	1170k Ω
7-8	12 μ m	390k Ω
8-9	6 μ m	195k Ω
9-1	4 μ m	130k Ω

Table 6.2: Table showing edge lengths and channel resistances for various contact pairs of the device shown in Figure 6.2.

6.2 Local configurations

Let start with the local configuration that is by applying current through the regular current channel (contacts 1-5) and measure the potential difference between other pair of contacts (see figure 6.3). As the top and bottom edges are of equal length ($L = 62 \mu\text{m}$) the current equally divides between the upper and lower branches of the network such that the current through each branch is $I/2$. The potential drop across a given pair of contacts (say 7-6 for instance) is $\frac{I}{2}r_{7-6}$ such that a rough estimation or guess of L_φ can be made by comparison of equation (6.6) with the experimental value of resistance R_{7-6} as follows

$$L_\varphi = \frac{h}{2e^2} \frac{L_{7-6}}{R_{7-6}} \quad (6.7)$$

using sample and experimental values we estimated $L_\varphi = 0.4 \mu\text{m}$. Since from equation (6.6) the resistance of a given edge channel is proportional to its length L_{ij} we can calculate the channel resistances knowing L_{ij} and L_φ . Table 6.2 shows these resistances corresponding to the edge channel lengths.

As can be seen in figure 6.3, equal channel resistances in the upper and lower branches make the current equally divide between the branches so that current $I/2$ is flowing in each branch, I being the current applied. The potential difference between a given pair of contacts is simply the drop across that resistor $\frac{I}{2}r_{i,i+1}$. The resistance is then calculated using Ohm's law.

Other local configurations are also possible for example applying current between contacts pair 1-6; 1-7; 1-4 etc. and measuring potential difference between other pairs of contacts in a similar way. Using $L_\varphi = 0.4 \mu\text{m}$ and edge length L_{ij} values from table 6.2 we can obtain resistance values for various configurations given in table 6.3.

Device configuration	Experimental $R_{\text{CNP}}(\text{K}\Omega)$	Kirchhoff's calculations $R_{\text{CNP}}(\text{K}\Omega)$
	290	195
	596	390
	1150	1170
	315	207
	624	622
	330	321
	310	70
	265	70
	412	321
	585	547
	660	415
	616	365
	685	440
	510	340

Table 6.3: Table showing various local configurations of the device with calculated and experimental value given for each configuration. The black circles denote contacts where current is applied while green squares shows contacts for voltage measurements.

6.3 Nonlocal configurations

In nonlocal configuration the current is not applied through regular current channel but rather between a pair of probes moving along the sample edges and can be detected by another pair of probes away from the dissipative bulk path. Three nonlocal configurations are presented here, though other nonlocal configurations are also possible and measured. In first type of configuration the current is applied between contacts 2-9 and the potential difference across pair of contacts 3-8 and 4-7 is measured (see figure 6.4). The same procedure of section 6.2 is applied here. To move from source 2 to drain 9 the current is divided into two paths: 2-1-9 (counter-clockwise) and 2-3-4-5-6-7-8-9 (clockwise). The current ratio through each branch depends on the resistance of the branch. From Kirchhoff's calculations 93.5% of the applied current flow through the short path 2-1-9 while 6.5% flows through path 2-3-4-5-6-7-8-9. Let's label the clockwise current I_1 while counter-clockwise current as I_2 , then the potential difference between contacts 3-8 is given as

$$V_{3-8} = I_1(r_{34} + r_{45} + r_{56} + r_{67} + r_{78}).$$

similarly the potential difference between contacts 4-7 can be calculated as

$$V_{4-7} = I_1(r_{45} + r_{56} + r_{67}).$$

using the channel resistances values from table 6.2 we can compute these potential drops. The resistances are then simply given by Ohm's law.

Figure 6.4: Schematic diagram showing the nonlocal configuration of the device when current is applied between contacts 2-9 and the potential difference is measured in pairs of contacts 3-8 and 4-7. In diffusive transport ($L \gg L_\phi$) the edge channels are represented by a resistor network.

Figure 6.5: Schematic diagram showing the nonlocal configuration of the device when current is applied between contacts 3-8 and the potential difference is measured in pairs of contacts 2-9 and 4-7. In diffusive transport ($L \gg L_\varphi$) the edge channels are represented by a resistor network.

Figure 6.6: Schematic diagram showing the nonlocal configuration of the device when current is applied between contacts 4-7 and the potential difference is measured in pairs of contacts 3-8 and 2-9. In diffusive transport ($L \gg L_\varphi$) the edge channels are represented by a resistor network.

Figures 6.5 and 6.6 show the other two nonlocal configuration presented here. Figure 6.5 shows the scheme when the current is applied between pair of contacts 3-8 and potential difference is measured across other pairs of contacts. The distribution of current through the paths: 3-2-1-9-8 and 3-4-5-6-7-8 follows the Kirchhoff rules. The configuration of figure 6.6 shows the scheme when the current flows from the source 4 to drain 7. Again the current divides according to the resistances of the network branches. The path followed are 4-3-2-1-9-8-7 and 4-5-6-7. The potential drops are calculated in a similar way as discussed above. The results calculated for all three nonlocal configurations are shown in table 6.4 along with experimental results for comparison. The three nonlocal schemes shown here are for pedagogical purposes, though other nonlocal schemes have been measured and studied in this project. They are shown in table 6.4.

We derive a more general form of Kirchhoff formulation that can be used for all nonlocal schemes studied here. Let's label the clockwise current as I_{mn} flowing through the channel branch L_{mn} while anti-clockwise current as I_{kl} flowing through the branch L_{kl} . The current ratio is inversely proportional to the length ratio:

$$\frac{I_{mn}}{I_{kl}} = \frac{L_{kl}}{L_{mn}}$$

$$I = I_{mn} + I_{kl} = I_{kl} \left(\frac{L_{kl}}{L_{mn}} + 1 \right) = I_{kl} \left(\frac{L}{L_{mn}} \right)$$

$$I_{kl} = I \left(\frac{L_{mn}}{L} \right)$$

such that the potential drop along the branch L_{kl} is

$$V_{kl} = I_{kl} \left(\frac{L_{kl}}{L_\varphi} \right) \left(\frac{h}{e^2} \right) = I \left(\frac{L_{mn} L_{kl}}{L L_\varphi} \right) \left(\frac{h}{e^2} \right)$$

giving the resistance value

$$R_{kl} = \frac{L_{mn} L_{kl}}{L L_\varphi} \left(\frac{h}{e^2} \right) \quad (6.8)$$

or for a given segment in the branch L_{kl}

$$R^{ij} = \frac{L_{mn} L^{ij}}{L L_\varphi} \left(\frac{h}{e^2} \right) \quad (6.9)$$

where L is the perimeter of the sample and L^{ij} is the length of the segment in branch L_{kl} .

The mean free path can be obtained as

$$L_\varphi = \frac{L_{mn} L^{ij}}{L R_{ij}} \left(\frac{h}{e^2} \right) \quad (6.10)$$

where R_{ij} is the resistance measured from experiment. Using R_{ij} values from experiment for different configurations we have extracted the mean free path (MFP) given in table 6.4.

From table 6.4 we see discrepancy between calculations and experiment for some of the nonlocal configurations. This may arise from the bulk conductivity short circuiting the edge channel conduction. The inhomogeneous charge distribution creates electron hole puddles which can results in tunneling probability in the bulk.

Device configuration	Experimental $R_{\text{CNP}}(\text{K}\Omega)$	Mean Free Path $L_\varphi(\mu\text{m})$
	450	0.4
	120	1.12
	395	0.43
	244	1.3
	100	1.34
	210	1.6
	24	1.88
	16	1.88
	100	0.1
	130	0.08
	20	2.2
	96	0.1
	68	0.14
	73	0.14
	120	0.08
	76	0.13

Table 6.4: Table showing various nonlocal configurations of the device with experimental values given for each configuration. The black circles denote contacts where current is applied while green squares shows contacts for voltage measurements. The mean free path calculated for each configuration, using Eq. 6.10, is also given.

6.4 Discussion

In mesoscopic samples of two-dimensional topological insulators, the four-terminal longitudinal resistance was found very close to the quantized value ($h/2e^2$), expected for edge state conduction if the bulk is made insulating by pinning the Fermi level in the forbidden gap [38]. The small dimensions ($1\mu m \times 1\mu m$) of such devices allow the carriers to traverse ballistically across the device without dissipation. The channel resistance between a given pair of contacts in the device equals the resistance quantum. However, in macroscopic samples longer than few microns, the four-terminal resistance significantly deviates from its quantized value ($R \gg h/2e^2$). In such samples with $L \gg L_\varphi$, where L_φ is the dephasing length, the transport is diffusive. The edge channels, in this case, can be represented by a resistors network where the resistance of each channel resistor is proportional to L/L_φ . Kirchhoff's rules can be used to investigate the current and potential distribution in such macroscopic samples. Note that, in such kind of approach, we do not consider transport across the bulk of the device as done before in chapter 5. Except for few configurations, resistances calculated by this method are far away from their experimental values. This implies that the edge plus bulk model is a good approach to explain our experimental findings. The mean free path L_φ calculated for various configurations, using equation (6.10), is also given in table 6.4. We see that for some of these configurations L_φ is closer while for other configurations it is quite different, reflecting strong inhomogeneity of the sample.

Chapter 7

Heating effects and electron-phonon interactions in HgTe-based 2d topological insulators

7.1 Introduction

Recent progress in the fabrication technology of two-dimensional electron gases have led to the development of effectively one-dimensional systems, where electrons are confined within a narrow channel, called quantum wires. These systems can be called one-dimensional when their width is smaller than the Fermi wavelength of electron. Edge states formed at the boundaries of the two-dimensional electron gas, when placed in a strong magnetic field in the quantum Hall regime, provide another important example of a one-dimensional electron system. Carbon nanotubes have also proved to be amongst systems exhibiting one-dimensional physics. Helical edge channels formed at the boundaries of two-dimensional (2D) topological insulators can be treated as ideal quantum wires. This is because two-dimensional topological insulators are characterized by a bulk insulating gap and conducting channels at the boundaries. By applying an appropriate gate potential to place the Fermi level in the bulk gap, a quantized edge conductance of $G = 2e^2/h$ is expected in short clean samples. A one-dimensional quantum wire is appropriately characterized by a conductance, which can be measured in a transport experiment. In case of no interactions, the conductance of an ideal clean quantum wire is quantized and is independent of temperature. Introducing electron-electron interactions in one-dimensional systems results in a Luttinger liquid (LL). A LL is a one-dimensional interacting electrons state, characterized by a parameter β that measures the strength of the interaction between electrons. For sufficiently weak disorder, theories predict a small correction to conductance, which scales with temperature as $T^{(\beta-1)}$. On the other

hand, strong temperature dependence of conductance is expected, for systems containing strong disorder. For helical Luttinger liquid, in 2D topological insulators, the conductance deviation is predicted to scale with temperature as T^4 or T^6 .

The helical liquid theory is the extension of the conventional spinless or spinfull LL theory [73]. It provides a complete low-energy description of one-dimensional (1D) metallic system even for strong interaction and predicts the stability to arbitrary disorder [74]. One of the experimental evidences pointing towards the existence of the Luttinger liquids, in strongly interacting one dimensional electrons system, could be the temperature dependence of resistance. First theoretical model developed within the framework of the clean LL without impurities, in the presence of a single strong barrier, predicted power-law dependence of the temperature [75, 76]. In the system containing many impurities, the resistance is dominated by combined effects of interaction and disorder potential, and should exhibit stronger temperature dependence [77, 78]. The Luttinger-liquid behavior via temperature dependence has been observed in many mesoscopic 1D systems, such as single-wall carbon nanotubes [79], cleaved-edge [80], split-gate [81] and V-groove semiconductor wires [82].

The helical edge states in 2D topological insulator is the most prominent example of the ideal Luttinger liquids, since it naturally appears in HgTe quantum wells with inverted band structure. Further, it does not require manufacturing nano-techniques and allows the formation of very long (of the millimeter size) 1D wires [49].

Among the various specific signatures of the Luttinger liquid behavior, such as the temperature dependence, it is important to emphasize on non-equilibrium properties of LL. In the absence of the disorder any arbitrary excited state will never decay to the equilibrium state characterized by temperature in contrast to conventional Fermi liquids. To understand the physics governing relaxation processes in LL, one can use the electron-heating measurements. In particular, the Fermi liquid can be driven out of equilibrium by a large current. Electrons accelerated by the electric field are scattered primarily with other electrons, and electronic temperature is established within thermalization time, as a results of electron-electron collisions. Hot electrons then relax to low temperatures by phonon emission.

Electron heating experiments have been widely used to study the nature of electron-phonon interactions in metallic films [83] and in semiconductors [84, 85]. When electrons are heated above the equilibrium lattice temperature, they relax by imparting their excess energy to the lattice. Emission of acoustic phonons is the dominant relaxation mechanism at low temperatures. As the temperature is increased, the electrons are capable of emitting higher energy phonons with larger wave vectors. Electron heating measurements can be used to determine the energy-relaxation time (τ_ϵ), which is helpful in understanding the energy relaxation processes of the system when it is driven out of equilibrium by a large current. From many

experimental and theoretical studies, the energy relaxation time due to phonon emission at temperature (T) follows a power-law dependence on temperature:

$$\tau_\varepsilon \propto T^{-\alpha} \quad (7.1)$$

The exponent $\alpha = 3$, has been reported in GaAs-based two-dimensional electron system where the energy-relaxation is dominated by three-dimensional electron-phonon scattering [86]. A similar dependence $\tau_\varepsilon \sim T^{-3}$ was observed for GaAs multiple quantum wells [87]. Thus, the exponent α simply reflects the dimensionality of phonons. Energy relaxations were also studied in strained quantum wells of $Ge/Si_{0.4}Ge_{0.6}$ and $Si/Si_{0.7}Ge_{0.3}$ where the large lattice mismatch at the interface was argued to give rise to two-dimensional electron-phonon scattering [88]. Recently, electron-phonon interactions and energy relaxations have been investigated in an etched InGaAs quantum wire, where, the current dependence of the electron temperature, and the temperature dependence of the energy-relaxation time was found to be consistent with energy relaxation processes dominated by scattering of electrons with three-dimensional phonons [89].

In this thesis, we study energy-relaxation and electron-phonon interactions in HgTe-based two dimensional topological insulators. Temperature-dependent resistance of the sample is used as thermometer to probe the electron-phonon interactions.

7.2 Hot carriers transport and Nonlinear effects

Deviations from Ohm's law in semiconductors leads to nonlinear effects. The electrical conductivity of semiconductors strongly depends on the applied electric field. This is because, in semiconductors, the conductivity depends strongly on the average energy of the charge carriers, which in turn is a function of electric field E . In semiconductors typically $E \sim 10^4 V/cm$, and higher mobility results in significant power input to the crystal and hence electrons can gain a significant amount of energy when subject to external electric field [90]. To establish a steady state the electrons must lose this energy. The main energy loss of electrons is due to the excitation of lattice vibrations. The energy supplied to the carrier gas raises the carrier temperature to a value T_e which is above the lattice temperature T_L . Under steady state conditions, the rate of input energy from the field to carries is balanced by the rate of energy loss by carries to the lattice.

The free carriers and the semiconductor lattice are two separate systems which exchange energy via phonon scattering. Under low fields, the carriers and the lattice are in equilibrium. The application of strong electric fields can give rise to states in semiconductors which are far from thermodynamic equilibrium. Hot electrons in semiconductors are a typical example

Figure 7.1: Schematics showing the local transport measurements on multi-terminal device MCT1. Current is applied between contacts 1, 5 and the potential difference is measured in consecutive pairs (9-8), (8-7), (7-6) corresponding to different segments of the device.

of a non-equilibrium state of carriers in which their average kinetic energy is increased by an external electric field so that it can be described by an effective temperature T_e which exceeds the lattice temperature T_L . Such heating of carriers by an electric field considerably alters many physical properties of semiconductors and gives rise to new effects, in particular, to a dependence of the electrical conductivity on the electric field (non-linearity), hot electron diffusion, and non-linear galvano-magnetic effects [12].

In this work, we investigate the field (bias current) dependence of resistance for various segments of the sample shown in Fig. 7.1. The sample consists of three narrow ($4 \mu\text{m}$ wide) consecutive segments of different length ($2, 8, 32 \mu\text{m}$) and seven voltage probes. The bias current (field) is applied between contacts 1-5 used as source and drain, respectively. The potential drops in the three segments can be measured using pair of contacts; 9-8, 8-7, and 7-6, respectively for the 2 -, 8 - and $32 \mu\text{m}$ segments.

Figure 7.2 shows the evolution of the resistance peak with current for the device segment ($L = 2 \mu\text{m}$), measured using contact pair 9-8. We find that increasing the drive current at the fixed lattice temperature, $T_L = 4.2 \text{ K}$, reduces peak of the resistance at the charge neutrality point (CNP). The current is varied from 1 nA to $0.7 \mu\text{A}$. As can be seen in Fig. 7.3, the resistance does not vary significantly for currents up to 30 nA . In other words, at low fields ($I = 1 \text{ nA} - 30 \text{ nA}$) the transport behavior is linear as expected (black circles). Increasing the bias current above this, one can see the nonlinear effects. It is worth noting that the

Figure 7.2: Current dependence of resistance in the local configuration, $R_{I=1-5;V=9-8}(=V_{9-8}/I_{1-5})$, when the current is applied through contacts 1-5 and voltage is measured between contacts 9-8. The current is varied from 1 nA to 0.7 μ A. $T=4.2$ K.

non-linearity is prominent for $V_g - V_{CNP} = 0$ V which corresponds to the charge neutrality point. Moving away from the CNP, we enter into either the electrons or holes regimes. For $V_g - V_{CNP} = +2$ V the transport is dominated by electrons while for $V_g - V_{CNP} = -1.3$ V, holes are the majority carriers. Loosely speaking, no significant non-linearity is observed in both regimes (red and blue circles).

Figure 7.4 shows the four-terminal resistance plotted as a function of gate voltage V_g and bias current for the device segment ($L = 8 \mu m$). One can note that the resistance peak value at the CNP is higher than that of the $L = 2 \mu m$ segment. This issue is already addressed in chapters 4 and 5. Here we only concentrate on the non-linearity. The nonlinear behavior is similar to the small segment described above. Again, most prominent effects can be seen at the CNP where electrons and holes coexist with comparable densities. Going to either of the extremes, i.e., electrons and holes sides, the non-linearity has no significance. We have also measured the field-dependent resistivity in the longer segment $L = 32 \mu m$. The resistance value at the CNP is about 1.2 $M\Omega$. The behavior is qualitatively similar to the other two segments discussed above. The non-linearity is obvious at the CNP while for both extremes, i.e., electrons or holes sides, the current dependence seems to be Ohmic (see Fig. 7.6). The nonlinear effects observed in our samples are combined with the temperature dependence measurements made on the same sample and discussed in sections 7.4 and 7.5.

Figure 7.3: $R_{I=1-5;V=9-8}(=V_{9-8}/I_{1-5})$ plotted as a function of current for three different gate potential $V_g - V_{\text{CNP}} = 0$ V, +2 V and -1.3 V. $V_g - V_{\text{CNP}} = 0$ corresponds to the charge neutrality point (CNP) while $V_g - V_{\text{CNP}} = +2$ and -1.3 corresponds to electrons and holes sides respectively.

Figure 7.4: Current dependence of resistance $R_{I=1-5;V=7-8}(=V_{7-8}/I_{1-5})$ plotted as a function of gate potential $V_g - V_{\text{CNP}}$. The current is varied from 1 nA to 0.7 μA . $T = 4.2$ K.

Figure 7.5: $R_{I=1-5;V=7-8}(=V_{7-8}/I_{1-5})$ plotted as a function of current for three different gate potential $V_g - V_{\text{CNP}} = 0$ V, +2 V and -1.3 V. $V_g - V_{\text{CNP}} = 0$ corresponds to the charge neutrality point (CNP) while $V_g - V_{\text{CNP}} = +1.8$ and -1.4 corresponds to electrons and holes sides respectively.

Figure 7.6: Resistance $R_{I=1-5;V=7-6}(=V_{7-6}/I_{1-5})$ plotted as function of gate potential for various current values. The current is varied from 1 nA to 0.7 μA . $T=4.2$ K.

Figure 7.7: $R_{I=1-5;V=7-6}(=V_{7-6}/I_{1-5})$ plotted as a function of current for three different gate potential $V_g - V_{CNP} = 0V$, $+2V$ and $-1.3V$. $V_g - V_{CNP} = 0$ corresponds to the charge neutrality point (CNP) while $V_g - V_{CNP} = +1.6$ and -1.8 corresponds to electrons and holes sides respectively.

7.3 Temperature-dependent transport

The typical dependence of the four-terminal resistances of device MCT1 as a function of normalized gate voltage, $V_g - V_{CNP}$, is shown in Figure 7.8 for the $L = 2\mu m$ segment having the configuration $R_{I=1-5;V=9-8}(=V_{9-8}/I_{1-5})$ (see Fig. 7.1). One can see the evolution of resistance-voltage profile with temperature. The figure shows that the electronic part is weakly dependent on temperature in the accessible range of temperature, while the hole part shows a strong temperature-dependence with saturation at low temperature. Figure 7.9 shows the four terminal resistance plotted as a function of temperature for three different gate voltages $V_g - V_{CNP} = 0$, $+2$ and -1.4 V, corresponding to the CNP, electrons and holes sides respectively. For $T > 15$ K a sharp decrease in resistance is observed while for T less than 10 K the resistance saturates. The high temperature behavior can be fitted well by the activation law $R = R_0 \exp(\frac{\Delta}{2k_B T})$ where Δ is the activation gap. The value of Δ estimated from the fitting is $\Delta \approx 20 meV$. The thermally activated behavior of resistance above 15 K corresponds to the gap of 20 meV between electrons and holes in HgTe wells. Recent theoretical calculations based on the effective 6×6 matrix Hamiltonian predicted a gap of ≈ 30 meV for 8 nm HgTe wells with [013] orientation [71]. The mobility gap can be

Figure 7.8: Temperature dependence of resistance $R_{I=1-5; V=9-8}(=V_{9-8}/I_{1-5})$ versus gate potential $V_g - V_{CNP}$. Temperature range from $T=4.2$ K to $T=60$ K, $I=1$ nA

smaller than the energy gap due to disorder. It is worth noting that the disorder parameter, which can reduce the energy gap in quantum spin hall insulators (QSHI), is related to the deviations of the HgTe quantum well thickness from its average value [70] rather than to the random potential due to charged impurities. The saturation of resistance at low temperature is completely unexpected because the electrons are supposedly in a strongly localized regime, where the electrical resistivity of the system is two orders of magnitude greater than the quantum of resistance $h/2e^2$.

Figure 7.10 shows the four-terminal resistance plotted as a function of gate voltage $V_g - V_{CNP}$ and base temperature T for the device segment ($L = 8\mu m$). Except for the resistance peak value higher than that of the $L = 2\mu m$ segment, the resistance versus temperature profile is similar. The resistance plotted as a function of temperature for three different gate voltages $V_g - V_{CNP} = 0, +1.8$ and -1.4 V, corresponding to the CNP, electrons and holes sides respectively is shown in figure 7.11. Again, the hole part shows a strong temperature dependence. Activation behavior can be seen for $T \geq 20$ K while for low temperatures, the resistance seems to saturate. Measurements were also made to investigate the temperature dependence of the $L = 32\mu m$ segment of the device shown in Fig. 7.1. The four terminal resistance as a function of $V_g - V_{CNP}$ for different base temperatures is plotted in Fig. 7.12. The resistance peak at the CNP is about $1.2 M\Omega$ at $T = 4.2$ K, which drops to about $70 k\Omega$

Figure 7.9: Resistance $R_{I=1-5;V=9-8}(=V_{9-8}/I_{1-5})$ as a function of temperature for different gate potentials $V_g - V_{\text{CNP}} = 0, +2$ and -1.4 V. $V_g - V_{\text{CNP}} = 0$ corresponds to the charge neutrality point.

Figure 7.10: Resistance $R_{I=1-5;V=7-8}(=V_{7-8}/I_{1-5})$ plotted as a function of gate potential $V_g - V_{\text{CNP}}$ for different temperature ranging from $T = 4.2$ K to $T = 60$ K, $I = 1$ nA

Figure 7.11: Resistance $R_{I=1-5;V=7-8}(=V_{7-8}/I_{1-5})$ as a function of inverse temperature for different gate potentials $V_g - V_{\text{CNP}} = 0, +1.8$ and -1.4 V. $V_g - V_{\text{CNP}} = 0$ corresponds to the charge neutrality point.

at $T = 60$ K. Saturation can be obviously seen at lower temperatures ($T \leq 15$ K). The high temperature profile can be fit to Arrhenius function giving an activation energy of 20 meV.

Figure 7.12: Temperature dependence of resistance $R_{I=1-5;V=7-6}(=V_{7-6}/I_{1-5})$ versus gate potential V_g . Temperature ranges from $T=4.2$ K to $T=60$ K, $I=1$ nA

Figure 7.13: Resistance $R_{I=1-5;V=7-6}(=V_{7-6}/I_{1-5})$ as a function of inverse temperature for different gate potentials $V_g - V_{CNP} = 0, +1.6$ and -1.5 V. $V_g - V_{CNP} = 0$ corresponds to the charge neutrality point.

7.4 Electron-phonon scattering and energy relaxations

The temperature dependence of the resistance can be used for calibration of the electronic temperature for different AC currents. In the present study we utilize electron heating, which is sensitive to energy relaxation mechanism and allows us to obtain information on the electron-phonon relaxation process in 2D topological insulators. We suppose that the energy relaxation mechanism occurs via the thermalization in multiple puddles created by the inhomogeneous charge distribution. The fluctuation of the local insulating gap width induced by smooth inhomogeneities, which can be represented as metallic puddles or dots. According to the model [2], charge carrier puddles coupled to a coherent conductor result in incoherent inelastic processes and modifies the ballistic transport. Therefore, metallic puddles can lead to spin dephasing since an electron entering the puddle is thermalized by dissipation, scattered back into the helical edge state. Relaxation of the energy occurs due to emission of phonons in the puddles.

Heating in an electric field is characterized by an energy relaxation time τ_e , determined from the balance of the energy received by carriers from the field and of the energy transferred from the carriers to the phonon subsystem:

$$\frac{P}{N} = \frac{k_B(T_e(I) - T_L)}{\tau_e} \quad (7.2)$$

where P is the electric power dissipated in 1D channel by N electrons, T_e is the electron temperature, T_L is the lattice temperature, τ_e is the energy relaxation time due to the phonon emission.

By combining the data plotted in Figures 7.2 and 7.8, it is possible to determine the variation of the electron temperature with currents. The results of such comparison for the $L = 2\mu m$ segment is shown in Fig. 7.14. In Fig. 7.15 we plot the temperature dependence of the energy relaxation time, multiplied by electron number N , which is determined by applying equation (7.2) to our data. The solid line indicates the T^{-3} variation, expected for the energy relaxation due to the scattering by three dimensional phonons [89].

A similar analysis is made for the $L = 8\mu m$ and $L = 32\mu m$ segments of the device shown in figure 7.1. The plot of electron temperature versus current (see Fig. 7.16) can be extracted by combining the data given in figures 7.10 and 7.11 for the $L = 8\mu m$ segment. The resulting energy relaxation time multiplied by N versus temperature profile is shown in Fig. 7.17. The same procedure is followed for the $L = 32\mu m$ segment. The T_e versus I plot is shown in figure 7.18. The N times energy relaxation time, determined by using equation (7.2), is plotted as a function of electron temperature in Fig. 7.19. The temperature dependence follow a T^{-3} behavior, which indicate the relaxation mechanism of electron's scattering by

Figure 7.14: The variation of effective electron temperature with current inferred from experiment. The solid (red) and dashed (blue) lines indicate different power law variations.

Figure 7.15: Energy relaxation time plotted as a function of electron temperature. The red line indicates the T^{-3} variation expected for three dimensional electron-phonon scattering.

Figure 7.16: The variation of effective electron temperature with current inferred from experiment. The solid (red) and dashed (blue) lines indicate different power law variations.

three-dimensional phonons.

Figure 7.17: Energy relaxation time plotted as a function of electron temperature. The solid line (red) indicates the T^{-3} variation expected for three dimensional electron-phonon scattering.

Figure 7.18: The variation of effective electron temperature with current inferred from experiment. The solid (red) and dashed (blue) lines indicate different power law variations.

Figure 7.19: Energy relaxation time plotted as a function of electron temperature. The solid line (red) indicates the T^{-3} variation expected for three dimensional electron-phonon scattering.

7.5 Discussion

The absence of resistance quantization in samples with dimensions above a few microns is another unsolved problem. The helical edge states are protected by time reversal symmetry, and any kind of elastic scattering between these states are forbidden. One of the possible explanation is the fluctuations of the local insulating gap width induced by smooth inhomogeneities, which can be represented as metallic puddles or dots. The well-localized metallic regions along the edge have been found using scanning gate microscopy [63] and also in microwave experiments [91]. According to the model [2], charge carrier puddles coupled to a coherent conductor result in incoherent inelastic processes and modify the ballistic transport. Therefore ballistic coherent transport is expected only in the region between the puddles, and the total 4-terminal resistance exceeds the quantized value. Self-averaging resistance of the sample with edge state dominated contribution to transport is given by.

$$R = \frac{h}{e^2} \frac{1}{g^2} n_p \lambda \left(\frac{T}{\delta} \right)^3 L \quad (7.3)$$

where n_p is the density of puddles, $\lambda = \frac{\hbar v}{E_g} = 18nm$ is the electron penetration depth into the puddles ($v = 5.5 \times 10^7 cm/s$ is the electron velocity, $E_g \simeq 20 meV$ is the forbidden gap), g

Figure 7.20: Schematics of helical edge state propagation along a boundary in 2DTI. Electrons, accelerated by external electric field, tunnel in and out the quantum dot (puddle) created by inhomogeneous charge distribution and emit phonons.

is the dimensionless conductance within the dot (puddle), δ is the mean level spacing within the dot, L is the spacing between probes (length of the edge channel). It has been found that equation (7.3) gives a satisfactory explanation of the high resistance value, obtained in experiments [48, 50]. Combining all parameters we calculate $\rho_0 = R/(\frac{h}{e^2}L) = 8 \times 10^3 (\frac{T}{\delta})^3 (\frac{e^2}{h})/cm$, which is comparable with the experimental value $\rho_0 = 15 \times 10^3 (\frac{e^2}{h})/cm$. The density of the puddles increases and the size of the puddles is growing, when the Fermi level lies closer to the conduction band.

Figure 7.20 illustrates the helical edge states in a disordered 2D TI in the presence of quantum dots created by smooth inhomogeneity in the sample. Electrons are accelerated by the strong external electric field. Note, however, that in 1D system quasi-particle interaction does not involve momentum transfer between quasiparticles. Therefore the decay of excitations into quasi-particle is not possible. We propose the following mechanism for the energy relaxation in our systems. The hot electrons can tunnel into the quantum dot (puddle). Assuming the size of the puddle about 100 nm, we consider single puddle as small metallic 2D lake. In this case, hot electrons in the puddles are scattered by other electrons and electronic temperature is established within relaxation time. After that electrons emit phonons, lost the energy, and scattered back to the edge (see figure 7.20). This mechanism explains T^{-3} dependence of the energy relaxation time, which is observed in our sample (figure 7.15).

To sum up, we have performed non-Ohmic measurements of the resistance at charge neutrality point in 2D topological insulator based on HgTe quantum well. We used this method for determination of the mechanism of energy relaxation of the helical edge modes with opposite spin polarization propagating along the HgTe quantum well. Our experiments fail to confirm the specific signatures of Luttinger liquid behavior. In contrast, the electron heating effect can be described by conventional energy relaxation mechanism, expected for

electron-phonon scattering. We suppose that the energy relaxation occurs via thermalization in the metallic puddles created by the inhomogeneous charge distribution in HgTe quantum well resulting in emission of phonons.

Chapter 8

Conclusions

This thesis presents the experimental studies of transport properties of CdTe/HgTe/CdTe quantum well heterostructures, with inverted band structure, which are expected to be quantum spin Hall (QSH) insulators. The striking feature of QSH insulators is that they conduct via edge states while the bulk is insulating. A remarkable consequence of the presence of the edge states in QSH insulators is the quantized longitudinal resistance $R_{xx} = h/2e^2$, which has been observed in mesoscopic Hall bars of HgTe quantum wells. However, in macroscopic samples, with dimensions longer than few microns, the longitudinal resistance deviates significantly from its quantized value ($R \gg h/2e^2$). This is one of the major unsolved problem in this field these days. We have investigated this issue by measuring local and nonlocal resistances in HgTe quantum wells, in zero magnetic field. The measurements were performed in two different devices with quantum well thickness 8.0-8.3 nm. The device MCT1 consists of three 4 μm wide consecutive segments of different length (2 μm , 8 μm , 32 μm), and seven voltage probes. The device MCT2 was fabricated with a lithographic length 6 μm and width 5 μm . Both devices are equipped with a top gate electrode whose dimensions are $62 \times 8 \mu\text{m}^2$ for device MCT1 and $18 \times 10 \mu\text{m}^2$ for device MCT2 which allows to tune the carrier density of the device. The application of a gate bias induces charge density variations transforming the quantum well conductivity from n-type to p-type via an intermediate state, called the charge neutrality point (CNP), where electrons and holes coexist with comparable densities. Non-universal ($R \gg h/2e^2$) peaks in both local and nonlocal resistivity were observed near the CNP. Such a behavior near the CNP can be explained in terms of a model, including simultaneously the edge states and bulk contribution to the total current, and taking into account the backscattering both between the edge states described by a phenomenological parameter γ and the bulk-edge coupling described by another phenomenological parameter g . In this model the density dependence of the local and nonlocal transport coefficients arises from bulk conductivity short circuiting the edge current away from the charge neu-

trality point. The model reproduces the key features of the data, in particular, the density dependence of the local and nonlocal resistivity.

Recently a model was proposed, which considers how edge state transport is influenced by electron and holes puddles, resulting from inhomogeneous charge distribution in 2d topological insulators [2]. Since the structures are doped, donor or acceptor charged impurities, which are inevitably present in these heterostructures, result in non-uniform potential landscape for electrons. Also the QSH insulators are narrow-gap semiconductors with a typical gap of 10-20 meV[38, 49]. In order to place the Fermi level in the forbidden gap, an appropriate gate voltage is required, such that, the gate charge compensates the uncompensated donor or acceptor charge density. The combined effect of gate potential and ionized dopant may result in the formation of electrons and holes puddles (quantum dots) in the quantum well. These puddles are tunnel coupled to the helical edge states. Electron's dwelling in the quantum dot results in inelastic processes which modifies the ballistic transport. One may, therefore, expect ballistic coherent transport only in the region between the puddles, and the total four-terminal resistance exceeds the quantized value.

In mesoscopic samples of two-dimensional topological insulators, the four-terminal longitudinal resistance was found very close to the quantized value ($h/2e^2$), indicating that the transport is ballistic with no backscattering [38]. However, in macroscopic samples, longer than few microns, the resistance deviation from its quantized value ($R \gg h/2e^2$) invokes the presence of scattering processes. In macroscopic samples, with $L \gg L_\varphi$ the transport is diffusive, where L is the length of the edge channel and L_φ is the mean free path for 1d-backscattering between the edge states. The edge channels, in this case, can be represented by a resistors network, where the resistance of each channel resistor is proportional to L/L_φ . Kirchhoff's rules can be used to investigate the current and potential distribution in such macroscopic samples.

In the presence of electron-electron interactions, one-dimensional systems result in a state called Luttinger liquid (LL). Helical edge channels formed at the boundaries of two-dimensional (2D) topological insulators is the most prominent example of the ideal Luttinger liquid, since it naturally appears in HgTe quantum wells with inverted band structure. Among the various specific signature of the Luttinger liquid behavior, such as the temperature dependence, non-equilibrium properties of LL need considerable attention. In the absence of the disorder, none of the arbitrary excited state will decay to equilibrium state characterized by temperature, in contrast to conventional Fermi liquids. To understand the physics governing relaxation processes in LL, one can use the electron-heating experiments. In particular, the Fermi liquid can be driven out of equilibrium by a large current. Electrons accelerated by the electric field are scattered primarily with other electrons, and electronic temperature is

established within thermalization time, as a results of electron-electron collisions. Hot electrons then relax to low temperatures by phonon emission. We have performed non-Ohmic measurements of the resistance at charge neutrality point in 2D topological insulator based on HgTe quantum well. We used this method for determination of the mechanism of energy relaxation of helical edge modes with opposite spin polarization propagating along the edges of HgTe quantum well Hall bar structures. Our experiments fail to confirm the specific signatures of Luttinger liquid behavior. In contrast, the electron heating effect can be described by conventional energy relaxation mechanism, expected for electron-phonon scattering. We assume that the energy relaxation occurs via thermalization in the metallic puddles (quantum dots) created by the inhomogeneous charge distribution in HgTe quantum well that results in the emission of phonons.

List of publications

1. Transferring Few-Layer Graphene Sheets on Hexagonal Boron Nitride Substrates for Fabrication of Graphene Devices, Graphene, 3, 25-35, 2014.
2. Heating effect and electron-phonon interaction in 2D Topological insulator based on HgTe quantum well. (In preparation)[see attached appendix A]
3. Scaling of the local and nonlocal resistances in 2D Topological insulator based on HgTe quantum well. (In preparation)

List of conference presentations

1. 16th Brazilian workshop on semiconductor Physics, Nonlocal transport near charge neutrality point in two-dimensional topological insulator, May-2013, Sao Paulo, Brazil.
2. 20th International conference on electronic properties of 2d systems/16th international conference on modulated semiconductors structures, Nonlocal transport near charge neutrality point in two-dimensional topological insulator, July-2013, Wroclaw, Poland.
3. 37th National meeting on condensed matter Physics, Nonlinear transport near charge neutrality point in two-dimensional topological insulator, May-2014, Bahia, Brazil.
4. The 21st International Conference on High Magnetic Fields in Semiconductor Physics, Scaling of the local and nonlocal resistances in 2D topological insulators based on HgTe quantum wells, Aug-2014, Florida, USA.
5. 32nd International Conference on the Physics of Semiconductors, Nonlinear transport near charge neutrality point in two-dimensional topological insulators, Aug-2014, Austin, Texas, USA.

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Appendix A-Manuscript to be submitted

Heating effect and electron-phonon interaction in 2D Topological insulator based on HgTe quantum well.

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We report on the observation and a systematic investigation of a non-Ohmic behaviour of the resistance in HgTe quantum wells with inverted band structure corresponding to the two-dimensional topological insulator (2D TI) phase. The electric power transferred to the one-dimensional electron subsystem increased the energy of thermal motion of carriers; it was then transferred to the lattice by the electron-phonon interaction. The energy relaxation time is governed by the scattering of electrons to the puddles formed by inhomogeneous charge distributions, and then by scattering on the phonons.

I. INTRODUCTION

The two-dimensional topological insulators (2DTI) are characterized by a bulk energy gap and gapless boundary modes that are robust to impurity scattering and electron-electron interactions¹⁻³. 2DTI have been realized in HgTe quantum wells with inverted band structure^{4,5}. This novel state is driven by the intrinsic spin-orbit interaction, which leads to the formation of the helical edge modes with opposite spin polarization counter-propagating at a given edge.

In the presence of electro-electron interactions, the edge states of 2D topological insulator can be modeled as helical Luttinger liquids (LL)⁶⁻⁸.

The helical liquid theory is extension of the conventional spinless or spinfull LL theory⁹. It provides a complete low-energy description of one-dimensional (1D) metallic system even for strong interaction and predicts the stability to arbitrary disorder⁶. The one experimental evidence pointing towards the existence of the Luttinger liquids in strongly interacting one dimensional electrons could be temperature dependence of the resistance. First theoretical model developed within the framework of the clean LL without impurities in the presence of a single strong barrier predicted power dependence of the temperature¹⁰. In the system containing many impurities the resistance is dominated by combined effects of interaction and disordered potential and should exhibit stronger temperature dependence¹¹. The Luttinger-liquid behaviour via temperature dependence has been observed in many mesoscopic 1D systems, such as single-wall carbon nanotubes¹², cleaved-edge¹³, split-gate¹⁴ and V-groove¹⁵ semiconductor wires.

The helical edge states in 2D topological insulator is most prominent example of the ideal Luttinger liquids, since it is naturally appeared in HgTe wells with inverted band structure. Further it does not require manufacturing nano-techniques and allows very long (of the millimeter size) 1D wire have been developed¹⁶. The electronic conductance of the 2DTI is quantized in units of the universal value $2e^2/h$, as was observed in the short and clean

FIG. 1: (Color online) The top insert shows schematics view of the sample. The resistance $R_{1,5;8,7}$ ($I=1,5$; $V=8,7$) as a function of the gate voltage for different temperatures $T(K)$: 4.2-6 (step 0.1K), 6-39 (step 1K), 42-60 (step 4K), $I=10^{-9}$ A. The insert shows the resistance $R_{1,5;7,6}$ as a function of the inverse temperature for $V_g - V_{CNP} = 0, +1$ and -1 V. The solid line is fit of the data with Arrhenius functions where $\Delta = 120$ K.

micrometer-scale Hall bars⁵. However, the quantized ballistic transport has not been seen in the sample with dimensions above a few microns^{4,5,17}.

Among the various specific signature of the Luttinger liquid behaviour, such as the temperature dependence, it should be emphasized non-equilibrium properties of LL. In the absence of the disorder any arbitrary excited state will never decay to the equilibrium state characterized by temperature in contrast to conventional Fermi liquids. To understand the physics governing relaxation processes in LL one can use the electron-heating measurements. In particular, the Fermi liquid can be driven out of equilibrium by a large current. Electrons accelerated by electric field scattered primarily with other elec-

trons, and electronic temperature is established within thermalization time, as a results of electron-electron collisions. Hot electrons relax to the low temperature by the phonon emission.

In the present study we utilize electron heating, which is sensitive to energy relaxation mechanism and allows us to obtain information on the electron-phonon relaxation process in 2D topological insulators. We suppose that the energy relaxation mechanism occurs via the thermalization in multiple puddles created by the inhomogeneous charge distribution. the fluctuation of the local insulating gap width induced by smooth inhomogeneities, which can be represented as metallic puddles or dots. According to the model¹⁸, charge carrier puddles coupled to a coherent conductor result in incoherent inelastic processes and modifies the ballistic transport. Therefore, metallic puddles can lead to spin dephasing since an electron entering the puddle is thermalized by dissipation, scattered back into the helical edge state. Relaxation of the energy occurs due to emission of the phonons in the puddles.

II. EXPERIMENT

The $Cd_{0.65}Hg_{0.35}Te/HgTe/Cd_{0.65}Hg_{0.35}Te$ quantum wells with (013) surface orientations and the width d of 8-8.3 nm were prepared by molecular beam epitaxy. A detailed description of the sample structure has been given in²⁰⁻²². The device consists of three $4\mu m$ wide consecutive segments of different length (2, 8, $32\mu m$), and 7 voltage probes (Figure 1, top panel). The ohmic contacts to the two-dimensional gas were formed by the in-burning of indium. To prepare the gate, a dielectric layer containing 100 nm SiO_2 and 200 nm Si_3Ni_4 was first grown on the structure using the plasmachemical method. Then, the TiAu gate with the sizes $62 \times 8\mu m^2$ (device A) and $18 \times 10\mu m^2$ (device B) were deposited. Several devices with the same configuration have been studied. The density variation with gate voltage was $1.09 \times 10^{15} m^{-2} V^{-1}$. The transport measurements in the described structures were performed in an variable temperature insert (VTI) cryostat (the temperature range 1.4-60 K). We used a standard four point circuit with a 3-13 Hz ac current of 0.1-100 nA through the sample.

The carriers density in HgTe quantum wells can be electrically manipulated with gate voltage V_g . The typical dependence of the four-terminal R resistances of the sample as a function of V_g is shown in Figure 1. One can see the evolution of resistance-voltage profile with temperature. Figure shows that the electronic part of the dependence is weakly dependent on temperature in the accessible range of the temperature, while the hole part shows a strong T-dependence with saturation at low T. The Hall coefficient (not shown) reverses its sign and $R_{xy} \approx 0$ when R approaches its maximum value¹⁹, which can be identified as the charge neutrality point (CNP). The insert shows the resistance as a function of inverse temperature for different gate voltages, including CNP.

We see that resistance shows a sharp decrease above 15 K, and below 10 K resistivity exhibits a tendency toward saturation. We find that the profile of the $R_{1,5;7,6}$ temperature dependencies above $T > 15K$ fits very well the activation law $\sim \exp(\Delta/2kT)$, where Δ is the activation gap.

The thermally activated behaviour of resistance above 15 K corresponds to the gap in 9 meV between electrons and holes in HgTe wells. Recent theoretical calculations based on the effective 6×6 matrix Hamiltonian predicted a gap of $\approx 30meV$ for 8 nm HgTe wells with [013] interfaces²³. The mobility gap can be smaller than the energy gap due to disorder. It is worth noting that the disorder parameter, which can reduce the energy gap in QSHI, is related to the deviations of the HgTe quantum well thickness from its average value²⁴ rather than to the random potential due to charged impurities. The saturation of resistance at low temperature is completely unexpected because the electrons are in a supposedly strongly localized regime, where the electrical resistivity of the system is two orders of magnitude greater than the quantum of resistance $h/2e^2$.

FIG. 2: (Color online) The resistance $R_{1,5;8,7}$ ($I=1,5$; $V=8,7$) as a function of the gate voltage for different currents I (nA): 4.2-6 (step 0.1K), 6-39 (step 1K), 42-60 (step 4K), $T=4.2K$. The insert shows the resistance $R_{1,5;7,6}$ as a function of the inverse temperature for $V_g - V_{CNP} = 0, +1$ and $-1V$. The solid line is fit of the data with Arrhenius functions where $\Delta = 120$ K.

The temperature dependence of the resistance can be used for calibration of the electronic temperature for different AC currents. Figure 2 shows evolution of the resistance peak with current. We find that increasing of the driven current at the fixed lattice temperature, reduces peak of the resistance. By combining the data plotted in Figures 1 and 2, it is possible to determine the variation of the electron temperature with currents. The results of such comparison is shown in Figure 3.

Heating in an electric field is characterized by an en-

FIG. 3: (Color online) The variation of the electron temperature with the current. Solid and dashed lines shows the different power-law dependencies, lattice temperature $T=4.2\text{K}$

ergy relaxation time T , determined from the balance of the energy received by carriers from the field and of the energy transferred from the carriers to the phonon subsystem²⁵:

$$\frac{P}{N} = \frac{k_B(T_e(I) - T_L)}{\tau_\varepsilon} \quad (1)$$

where P is the electric power dissipated in 1D channel by N electrons, T_e is the electron temperature, T_L is the lattice temperature, τ_ε is the energy relaxation time due to the phonon emission.

In figure 4 we plot the temperature dependence of the energy relaxation time, multiplied by electron numbers, which is determined by applying Eq (1) to the our data. The solid line indicates the T^{-3} variation, expected for the energy relaxation due to the scattering by three dimensional phonons.

III. DISCUSSION

Absence of the resistance quantization in samples with dimensions above a few microns^{4,5,16} is another unsolved problem. The helical edge states are protected by the time reversal symmetry, and any scattering between states are forbidden. One of the possible explanation is the fluctuations of the local insulating gap width induced by smooth inhomogeneities, which can be represented as metallic puddles or dots. The well-localized metallic regions along the edge have been found using scanning gate microscopy²⁶ and in microwaves experiments²⁷. According to the model¹⁸ charge carrier puddles coupled to a coherent conductor results in incoherent inelastic processes and modifies the ballistic transport. Therefore ballistic

FIG. 4: (Color online) The temperature dependence of the energy relaxation time multiplied by the numbers of electrons, determined from equation 1. The dotted line corresponds to the T^{-3} variation, expected for scattering of electrons by three dimensional phonon.

coherent transport is expected only in the region between the puddles, and total 4-terminal resistance exceeds the quantized value. Self-averaging resistance of the sample with edge state dominated contribution to transport is given by¹⁸:

$$R \sim \frac{h}{e^2} \frac{1}{g^2} n_p \lambda \left(\frac{T}{\delta} \right)^3 L \quad (2)$$

where n_p is the density of the puddles, $\lambda = \hbar v / E_g \approx 18\text{nm}$ is the electron penetration depth into the puddles ($v \approx 5.5 \times 10^7 \text{cm/s}$ is the electron velocity, $E_g \approx 20\text{meV}$ is the forbidden gap), g is the dimensionless conductance within the dot (puddle), δ is the mean level spacing within the dot, L is the distance between probes (length of the edge states). It has been found that the equation 1 gives a satisfactory explanation of the high resistance value, obtained in experiments²². Combining all parameters we calculate $\rho_0 = R / (\frac{h}{e^2} L) = 8 \times 10^3 \left(\frac{T}{\delta} \right)^3 (e^2/h)/\text{cm}$, which is comparable with experimental value $\rho_0 = 15 \times 10^3 (e^2/h)/\text{cm}$. The density of the puddles increases, and the size of the puddles is growing, when the Fermi level lies nearer to the conductive band.

Figure 5 illustrates the helical edge states in a disordered 2D TI in the presence of the quantum dot created by smooth inhomogeneity in the sample. Electrons are accelerated by the strong external electric field. Note, however, that in 1D system quasi-particle interaction does not involve momentum transfer between quasi-particles. Therefore the decay of the excitations into quasi-particle is not possible. We propose the following mechanism for the energy relaxation in our systems. The hot electrons can tunnel into the quantum dot (puddle).

FIG. 5: (Color online) Schematics of helical edge state propagation along a boundary in 2DTI. Electrons, accelerated by external electric field, tunnel in and out quantum dot (puddle) created by inhomogeneous charge distribution and emit phonons.

Assuming the size of the puddle $100nm$, we consider single puddle as small metallic 2D lake. In this case hot electron in the puddles are scattered by other electrons and electronic temperature is established within relax-

ation time. After that electrons emit phonons, lost the energy, and scatter back to the edge (figure 5). This mechanism explains T^{-3} dependence of the energy relaxation time, which is observed in our sample (figure 4).

IV. CONCLUSION

We performed non-Ohmic measurements of the resistance at charge neutrality point in 2D topological insulator based on HgTe quantum well. We used this method for determination of the mechanism of the energy relaxation of the helical edge modes with opposite spin polarization propagating along the HgTe quantum well. Our experiments fail to confirm the specific signatures of Luttinger liquid behaviour. In contrast, the electron heating effect can be described by conventional energy relaxation mechanism, expected for electron-phonon scattering. We suppose that the energy relaxation occurs via the thermalization in the metallic puddles created by the inhomogeneous charge distribution in HgTe quantum well.

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