Universidade de São Paulo
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Análise de múltiplas sondas no espaço harmônico: Contribuições para o Dark Energy Survey

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Multi-probe analysis on harmonic space: Contributions for the Dark Energy Survey

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*To Davi Charles Gomes
and Adriana Heringer Gomes.*

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“Cosmology is in an exciting state because we have a rich and growing list of problems and a growing observational base that may allow us to find a few solutions.”

P. J. E. Peebles

“Scientific inquiry is accordingly a dynamic exercise of the imagination (...) Its ideal is the discovery of coherence and meaning in that which we believe exists.”

Michael Polanyi

ABSTRACT

The combination of multiple cosmological probes is an effective way to identify problems with the theoretical framework of modern physical cosmology and to increase the precision of established results. Challenges arise when this combination requires a joint treatment of systematic effects and correlations between probes. While the Dark Energy Survey (DES) has been developing a robust method for this kind of multi-probe analysis, it has focused on configuration space functions, leaving a gap for harmonic space analysis. In this dissertation, we develop steps toward a multi-probe harmonic space analysis of DES data. In Chapter 1, we show our premises and define the concepts which are at the base of modern cosmological research. In Chapter 2, correlation functions are explored with a focus on harmonic space. We show how to arrive at the theoretical expressions for angular power spectra. In Chapter 3, we review the use of multiple probes in cosmological research, and the pioneering efforts of DES towards the analysis of galaxy clustering, galaxy-galaxy lensing and cosmic shear correlations. In Chapter 4, we describe our contribution to DES multi-probe analysis: A harmonic space galaxy clustering project with the collaboration's Year 1 data.

Keywords: Cosmology. Structure of the Universe. Galaxies. Dark Energy Survey.

RESUMO

A combinação de múltiplas sondas cosmológicas é uma maneira efetiva de identificar problemas com o arcabouço teórico da cosmologia física moderna e de aumentar a precisão dos resultados já estabelecidos. Desafios surgem quando essa combinação requer um tratamento conjunto de efeitos sistemáticos e correlações entre sondas. Enquanto o Dark Energy Survey (DES) tem desenvolvido um método robusto para esse tipo de análise de múltiplas sondas, seu foco tem sido em funções no espaço de configurações, deixando uma lacuna para a análise no espaço harmônico. Nessa dissertação, desenvolvemos passos em direção a uma análise de múltiplas sondas no espaço harmônico de dados do DES. No Capítulo 1, mostramos nossas premissas e definimos os conceitos que estão na base da pesquisa moderna em cosmologia. No Capítulo 2, funções de correlação são exploradas com um foco no espaço harmônico. Mostramos como chegar às expressões teóricas para o espectro de potências angular. No Capítulo 3, revisamos o uso de múltiplas sondas na pesquisa cosmológica, e os esforços pioneiros do DES para uma análise das correlações de aglomeramento de galáxias, galáxias e lenteamento de galáxias, e cisalhamento cósmico. No Capítulo 4, descrevemos nossa contribuição para a análise de múltiplas sondas do DES: Um projeto de aglomeramento de galáxias no espaço harmônico com dados do Ano 1 da colaboração.

Palavras-chaves: Cosmologia. Estrutura do Universo. Galáxias. Dark Energy Survey.

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INTRODUCTION

Observational cosmology relies mostly on light arriving at Earth from distant sources. Though light acts almost as a sole informer, it bestows upon us a rich and diverse set of data. When dealing with galaxies, the relation between the spectral energy distribution and distance allow us to build extensive maps of the near universe. Once we equip ourselves with the interpretative framework of General Relativity, those maps become instruments to infer the workings of the cosmos as a whole, including its yet-to-be-uncovered components, such as dark matter and dark energy.

Galaxy positions and galaxy shapes can both function as probes that help us find the value of the so-called *cosmological parameters*, which are the main free parameters of cosmological theoretical modeling. While positions map the physical distribution of galaxies, collective shape distortions are an indicator of the large scale density distribution that includes both visible and dark matter. The merging of those probes into a combined analysis has been a successful source of constraints for the possible values assumed by cosmological parameters.

The Dark Energy Survey (DES) has combination of probes as part of its central philosophy. It consolidated two decades of tentative work on a multi-probe approach towards the large-scale structure of the universe. However, data analysis in the context of DES has focused on *configuration space* functions. A complementary study of harmonic space functions is an important way of verifying the robustness of the main analysis results and scan for possible new sources of systematic error.

This dissertation describes our work towards a harmonic space analysis of galaxy position statistics with DES data. This analysis is oriented by multi-probe methodology, and intends to be a building block for a complete multi-probe analysis on harmonic space. Chapter 1 performs an overview of modern cosmology, detailing the premises and definitions from which we part in our analysis. We describe the mathematical foundations of modern cosmology, and comment on the existing probes of cosmological information, as well as on the most relevant sky surveys.

Chapter 2 brings forward the theoretical modeling of 2-point statistics on harmonic space. We also describe the process on which this modeling is combined with statistics from sky surveys to yield constraints on cosmological parameters. Chapter 3 details the antecedents to and the emergence of the $3x2pt$ analysis, which is the particular form of multi-probe inference selected and developed by DES. We then explore the methodology devised by [Krause, Eifler, et al. \(2017\)](#) for analysis of the survey’s Year 1 data.

Finally, in Chapter 4, we review the results of DES Year 1 (DES-Y1) data analysis, and discuss the relation between configuration space and harmonic space analyses. We argue that the sole use of configuration space functions leads to loss of potential cosmological information. If the reasoning for a multi-probe analysis stands on the desire to make the most out of available data, this motivation should also bring forth analyses on both configuration and harmonic space. We then present our contribution to harmonic space analysis, which took place in the context of the DES collaboration. In this project, we studied the galaxy bias of DES-Y1 redMaGiC lens galaxies, comparing our results with those from configuration space. Authoral results from this chapter, together with additional contributions from the project team, were published by [Andrade-Oliveira et al. \(2021\)](#).

BASIC CONCEPTS AND PREMISES: FROM THE BIG BANG TO MODERN SURVEY COSMOLOGY

1.1 COSMIC HISTORY

Research on physical cosmology in the context of modern-day sky surveys depends on a highly developed mathematical account of the universe's history. With such an account in hand, it is possible to make theoretical predictions of statistical properties of the universe, predictions which can be tested by comparison with sky observations. The standard version of this account includes the Big Bang, primordial nucleosynthesis, matter-radiation decoupling and structure formation. While this story is sometimes subject to revisions, such as the introduction of inflation theory and of Dark Energy, we consider its core components to form an accurate understanding of the physical development of the universe. We can't claim to comprehend every process that took hold from the beginning of space and time to the establishment of the current large-scale structure of the universe, yet the processes we do understand allow us to build robust mathematical analysis tools.

Cosmic history begins with an extremely hot and dense expanding universe. If we extrapolate enough into the past, it is possible to imagine a creation moment from which reality was brought into existence from a singularity. This sudden beginning is known as the Big Bang. The study of this utmost event is largely speculative, and can intersect with metaphysical or religious considerations. In this context, some researchers think to be more philosophically attractive those models which exclude such creation moment while leaving intact the posterior expansion. If we consider inflationary theory, it is also possible to develop a scenario in which several universes continually come into existence from a false vacuum, though it has been argued that this process can't extend indefinitely to the past. (GUTH, 2007) The absence of a unified theory of quantum gravitation is somewhat a limitation to the description of the universe's first instants of existence. The more we dive into the past, more blurry our picture becomes, and more affected by personal beliefs. Nevertheless, we can speak of the Big Bang in a more general sense, identifying it with

the first describable physical processes of the early universe.

In order to account for the observable features of today's universe, the initial hot and dense phase must have been uniform and spatially flat (as opposed to having a positive or negative curvature) up to a high precision. This fine-tuning has motivated the development of the inflation scenario, according to which the universe underwent a brief phase of exponential expansion during its first fractions of seconds of age. Many models exist which could explain the mechanism behind this inflationary phase. It is expected that research in survey cosmology will eventually show which model, if any, describes what happened appropriately.

The next major physical processes that can be identified are related to the universe's cooling down due to expansion. First, the lower temperature caused protons and neutrons to combine forming the first atomic nuclei, in a process known as the Big Bang Nucleosynthesis (BBN). Second, after further cooling took place, electrons started to be captured by atomic nuclei, and photons stopped being scattered back and forth by matter. At this moment, known as the period of recombination and of photon decoupling, light became free to travel for long distances. The imprint of this event in the sky is the Cosmic Microwave Background (CMB) radiation, which can be observed as a blackbody radiation with a temperature of $T = 2.73K$.

After decoupling, slight perturbations in the distribution of matter gave origin to a process of matter clustering. While the universe remained homogenous at large scales, gravity acted amplifying small regions of inhomogeneity. This process allowed the emergence of large gravitationally bound systems. Today, the large-scale structure of the universe can be studied as we see how galaxies and clusters of galaxies are distributed in space.

1.2 MATHEMATICAL FOUNDATIONS

While a qualitative account of the universe's history seems to leave little open problems, a turn to the quantitative foundations of this account can expose the limitations of our knowledge. The universe's rate of expansion depends on several parameters which, by what we know today, can't be determined theoretically. Thus, we can build with the same physics a myriad of possible universes, each with a distinct combination of values for those parameters. The main quest of modern cosmology is to use sky observations to

discover which of these theoretically possible universes is actually real. In other words, the quest is to determine the cosmology (e.g. combination of parameter values) of our universe. In addition to mere variations of parameter values, new parameters can also be discovered as more physical processes are explored.

The starting point for most cosmological models is the Friedmann Equation. It is a differential equation which gives us the scale factor $a(t)$ as a function of time. This factor can be defined as the ratio of the distance between two points in space at a given time t and their distance today. Thus, in a static universe, we would always have $a(t) = 1$, and in an expanding universe, we have $a = 0$ when extrapolated to its beginning and $a = 1$ today.

In order to arrive at the Friedmann Equation, we need to consider two basic assumptions. Our first assumption is that of a universe which is homogeneous and isotropic on large enough scales. There is no specific direction on the sky for which we may find special cosmological properties, and this must hold for observers anywhere in the universe. This assumption is sometimes referred to as the Cosmological Principle, and associated with the philosophical idea that we are not in a privileged place in the universe. This association, however, can be counterproductive, since it entangles a well-defined physical concept to a loose metaphysical notion. Though the latter may have played a role in the historical development of cosmological theories, the homogeneity and isotropy of the universe stand today on strong observational grounds. (GORBUNOV; RUBAKOV, 2017)

Our second assumption is that of the validity of General Relativity (GR). We consider that local geometry of space-time is related to local density, and that this relation is given by Einstein's Equations. Some cosmologies explore the effects of modified gravity models, which involve slightly different versions of Einstein's Equations. So far, however, GR cosmologies have been highly successful in predicting observable features of the universe. In the context of survey data analysis, it is justifiable to concentrate efforts on GR models and leave the alternative ones as side projects. The steps for developing modified gravity cosmologies are the same as those we will follow, though the final equations differ. Thus, the development of GR cosmologies serves also the purpose of a template for the development of more complex ones.

In order to use GR as a framework, the universe's geometry must be described in tensor formalism. The first step is to write the metric tensor $g_{\mu\nu}$, which for our purposes

can be thought of as the mathematical entity that allows distances to be measured in a curved space-time. It is defined in such a way that the quadratic differential form dS^2 becomes

$$dS^2 = g_{\mu\nu} x^\mu x^\nu, \quad (1.1)$$

where we use the Einstein notation, hiding an implicit summation over equal upper and lower indices.

The metric tensor (or simply the metric) of the expanding universe flows naturally from the assumption of its homogeneity and isotropy. [Robertson \(1935\)](#) and [Walker \(1937\)](#) showed that the line element in a homogenous and isotropic universe has always the following form:

$$dS^2 = -dt^2 + a^2(t) \left(\frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2) \right), \quad (1.2)$$

where k is a constant curvature parameter and we use $c = 1$.

This line element defines a metric, known as the Friedmann-Lemaître-Robertson-Walker (FLRW) metric. It allows a dynamic behaviour of the universe as a whole by means of the time evolution of the scale factor.

Observations have shown that the universe's overall curvature is negligible. ([GORBUNOV; RUBAKOV, 2017](#)) Though there is large theoretical and historical merit in developing the equations allowing for positive or negative curvature, such development would exceed our current research purposes. Thus, it suffices for us to take the universe's flatness as an additional premise. The flat FLRW metric assumes $k = 0$ in Eq. 1.2, and therefore becomes

$$g_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & a(t)^2 & 0 & 0 \\ 0 & 0 & a(t)^2 & 0 \\ 0 & 0 & 0 & a(t)^2 \end{pmatrix}. \quad (1.3)$$

The relation between the universe's metric and its density content is given by the Einstein's Equations:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi GT_{\mu\nu}. \quad (1.4)$$

The elements of this expression deserve individual attention. The Ricci scalar R is defined in terms of the Ricci tensor $R_{\mu\nu}$:

$$R = g^{\mu\nu} R_{\mu\nu}. \quad (1.5)$$

The Ricci tensor is defined as the following contraction of the Riemann tensor:

$$R_{\mu\nu} = R^{\lambda}_{\mu\lambda\nu}, \quad (1.6)$$

where the Riemann tensor is defined in terms of the Christoffel symbols as:

$$R^{\mu}_{\nu\rho\sigma} = \frac{\partial \Gamma^{\mu}_{\nu\sigma}}{\partial x^{\rho}} - \frac{\partial \Gamma^{\mu}_{\rho\sigma}}{\partial x^{\nu}} - \Gamma^{\mu}_{\nu\alpha} \Gamma^{\alpha}_{\rho\sigma} + \Gamma^{\mu}_{\rho\alpha} \Gamma^{\alpha}_{\nu\sigma}. \quad (1.7)$$

The Christoffel symbols are each defined in terms of the metric tensor $g_{\mu\nu}$:

$$\Gamma^{\alpha}_{\beta\gamma} = \frac{1}{2}g^{\alpha\delta} \left(\frac{\partial g_{\delta\beta}}{\partial x^{\gamma}} + \frac{\partial g_{\delta\gamma}}{\partial x^{\beta}} - \frac{\partial g_{\beta\gamma}}{\partial x^{\delta}} \right). \quad (1.8)$$

Thus, from the FLRW metric tensor and its derivatives, we can compute the left side of Eq. 1.4 in terms of the scale factor a . We find that

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \begin{pmatrix} 3\left(\frac{\dot{a}}{a}\right)^2 & 0 & 0 & 0 \\ 0 & -\dot{a}^2 - 2a\ddot{a} & 0 & 0 \\ 0 & 0 & -\dot{a}^2 - 2a\ddot{a} & 0 \\ 0 & 0 & 0 & -\dot{a}^2 - 2a\ddot{a} \end{pmatrix}. \quad (1.9)$$

The right side of Eq. 1.4 is the so-called energy-momentum tensor, which in a homogeneous and isotropic universe, modeled as a perfect fluid, is given by

$$T_{\mu\nu} = \begin{pmatrix} \rho & 0 & 0 & 0 \\ 0 & P & 0 & 0 \\ 0 & 0 & P & 0 \\ 0 & 0 & 0 & P \end{pmatrix}, \quad (1.10)$$

where ρ is the density and P the pressure.

Inserting Eqs. 1.9 and 1.10 on Eq. 1.4, we find two differential equations: one for the time-time component of the tensors, and the other for their diagonal space-space components. The former is the Friedmann Equation:

$$3 \left(\frac{\dot{a}}{a} \right)^2 = 8\pi G\rho. \quad (1.11)$$

The latter is an additional equation, which is sometimes included with the former as a set of two Friedmann Equations:

$$-\dot{a}^2 - 2a\ddot{a} = 8\pi GP. \quad (1.12)$$

The density ρ in Eq. 1.11 is dependent on the scale factor. Thus, to solve the Friedmann Equation, we need to find this dependence and rewrite the equation in terms only of a , $\dot{a}$ and constants. The first step is to note that the total density equals the sum of the densities of each different energy component of the universe. Here we ought to add as a premise the list of available components.

The trivial components are matter and radiation. In addition to them, we will assume the existence of dark energy. Data from Type Ia Supernovae show that the universe's expansion has currently a positive acceleration. (RIESS; FILIPPENKO; CHALLIS, et al., 1998) Though the physical source of this acceleration is yet unknown, it can be mathematically described and inserted in our theoretical model. Many descriptions for dark energy can be proposed and tested with observations, but so far the simpler of them seems to be the more promising: a dark energy that behaves as a cosmological constant. If this model is found to be correct, however, the physical interpretation of this constant may still require development of new physics.

By incorporating on our cosmology dark energy as a constant, we arrive at what is today the standard cosmological model, known as the Λ CDM model. It gives us for the total energy density:

$$\rho = \rho_m + \rho_r + \rho_\Lambda, \quad (1.13)$$

where the m , r and Λ subscripts refer to matter, radiation, and dark energy, respectively.

The behavior of each density component as a function of the scale factor depends on the component's equation of state, which gives its pressure as a function of its density. For matter, we have $P_m = 0$; for radiation, $P_r = \rho_r/3$; and for dark energy as a cosmological constant, $P_\Lambda = -\rho_\Lambda$. It can be shown, from the covariant conservation equation of the energy momentum tensor, that the scaling of a density factor ρ_x follows

$$\rho_x \propto a^{3(-w_x-1)}, \quad (1.14)$$

where $w_x = P_x/\rho_x$. (GORBUNOV; RUBAKOV, 2017)

Thus, we can rewrite Eq. 1.13 in terms of constants as

$$\rho = k_m a^{-3} + k_r a^{-4} + k_\Lambda. \quad (1.15)$$

It is convenient to introduce the critical density of a flat universe as $\rho_{crit} = k_m + k_r + k_\Lambda$. This makes it possible for us to define the dimensionless parameters Ω_m , Ω_r and Ω_Λ as the constants from Eq. 1.15 each divided by the critical density. These three parameters are part of the set of cosmological parameters which are central to modern research on observational cosmology.

A fourth parameter is implicit in the establishment of the critical density. Taking $a = 1$ on Eqs. 1.15 and 1.11 yields us

$$\rho_{crit} = \frac{3}{8\pi G} \left(\frac{\dot{a}}{a} \bigg|_{a=1} \right)^2. \quad (1.16)$$

The ratio, evaluated at $a = 1$, between the time derivative $\dot{a}$ and the scale factor itself is known as the Hubble constant H_0 . We can thus write the total energy density as

$$\rho = \frac{3H_0^2}{8\pi G}(\Omega_m a^{-3} + \Omega_r a^{-4} + \Omega_\Lambda). \quad (1.17)$$

This brings us to the Friedmann Equation of a flat Λ CDM universe:

$$\left(\frac{\dot{a}}{a}\right)^2 = H_0^2(\Omega_m a^{-3} + \Omega_r a^{-4} + \Omega_\Lambda), \quad (1.18)$$

with $\Omega_m + \Omega_r + \Omega_\Lambda = 1$.

Here we can view the influence of each particular cosmology on the background development of the universe. For every combination of Ω_m , Ω_r , Ω_Λ and H_0 values, the solution of Eq. 1.18 reveals a different expansion story. The night sky imprints of cosmic history, thus, can be used to determine which combination of values best describe our universe.

Two remarks are now well suited. Firstly, the Hubble constant has units of inverse time, and can be replaced by its dimensionless counterpart h , defined through

$$h = H_0 \left(\frac{1\text{Mpc}}{100\text{km/s}} \right). \quad (1.19)$$

Secondly, the value of Ω_r is well established in scientific literature due to its close relation to the CMB temperature. As reported by [Lahav and Liddle \(2019\)](#), CMB measurements made with the COBE satellite place Ω_r with high precision as

$$\Omega_r = \frac{2.47 \times 10^{-5}}{h^2}. \quad (1.20)$$

A fixed Ω_r , along with the flat-universe condition that the three density parameters sum to 1, leaves us with only two free parameters, Ω_m and h . New free parameters, however, ought to be introduced when dealing with initial conditions and physical processes other than general expansion. A possible modeling yields us six free parameters: Ω_m , h , the baryon density Ω_b , the massless neutrino density Ω_ν , the spectral amplitude A_s and the spectral number n_s .

From our expansion-only two-parameter model, to which we shall hold for now, we choose a set of nine possible cosmologies, varying Ω_m from 0.25 to 0.35 and h from

Figure 1.1 – Numerical solutions to the Friedmann Equation for nine flat Λ CDM cosmologies. The horizontal line marks $a = 1$, while the vertical lines mark the age of the universe according to each cosmology.

0.65 to 0.75. Those values were selected as to circle around the current best estimates of each parameter. Next, we numerically solve the Friedmann Equation for each of our cosmologies. Figure 1.1 shows our solutions, along with the time at which each model arrives at $a = 1$ (vertical magenta lines). Since we are today, by convention, at $a = 1$, the values we marked, which span from 12.0 to 15.2 Gigayears (Ga), correspond to the age of the universe according to each of our cosmologies.

1.3 REDSHIFT, DISTANCE AND TIME

When we observe a distant light source, we are intercepting photons that were emitted some time in the past. If we create a cosmic time scale, starting at $t = 0$ in the Big Bang, we can associate to each object a value which indicates how old the universe was when the observed light left the object towards us. These values, however, can only be inferred through indirect means. An alternative scale, on which data can be measured directly, is given by a quantity known as the redshift. The stretching of photon wavelengths due to cosmic expansion makes us receive light with shifted frequencies. The farther a source is located, in terms of distance and time, the larger this wavelength shift will be.

Redshift is defined as the ratio between the shift size and the emitted wavelength.

For a source that released light when the universe had scale factor a_{source} , observed and emitted wavelengths are related by

$$\lambda_{obs} = \frac{\lambda_{emi}}{a_{source}}. \quad (1.21)$$

Thus, the redshift z can be written in terms of the scale factor:

$$z = \frac{1}{a} - 1. \quad (1.22)$$

To each moment of the universe's history, we can associate a value for the scale factor, and therefore, a redshift value. As a consequence, it is possible to speak of redshift both as a property of specific sky objects and as a decreasing time measure, which spans from ∞ at the beginning of the universe to 0 today.

Before we consider the mathematical relations between distance, time and redshift, we need to explore the scope of definitions for these concepts. While redshift is a modern technical quantity, distance and time are ideas which have always been used in human societies. Within an Earth-centered perspective, distances can be measured by standard rulers (such as the size of an arm), or even abstract units with a fixed definition (such as the meter). The existence of cycles in nature also makes it possible to perform time measurements, and a diversity of unit systems emerges from the variety of established cycles.

When dealing with the universe as a whole, however, time and distance definitions become less trivial. The fact that the universe is expanding, for example, raises the question: Do we consider galaxies to be always at a same distance from each other or do we consider a growing distance due to cosmic expansion? Both answers are correct, and refer to distinct valid perspectives. In one sense the physical distance is growing. In another sense, the galaxies are not in physical motion, it is space itself which is stretching. The same reasoning can be applied to time. A photon traveling through an expanding space begs the question whether to incorporate the changing length of its trajectory on the travel time value.

These subtleties makes it impossible for us to have a single standard ruler for cosmic distances and time intervals. We can, nonetheless, identify the array of possible

definitions, and associate each one to a physical variable. Cosmological models can be thus studied and developed in terms of different variables, the choice between them being mostly a matter of convenience.

We start introducing two possible time measurements. The physical time t is the variable as used in the laws of relativity, being the proper time for comoving observers. This is the measure we used on our solutions to the Friedmann Equation. The conformal time η is dictated by light travel, and thus includes the effect of cosmic expansion in its definition. The relation between both is

$$\eta = \int \frac{dt'}{a(t')}. \quad (1.23)$$

Likewise, when dealing with cosmological distances, we have the physical distance d between two bodies, which grows with time, and the comoving distance χ , which remains constant under cosmic expansion. The latter is equivalent to the conformal time interval needed for light to travel between the bodies. We can also define the angular diameter distance d_A , which corresponds to the distance inferred through angular separation in the sky. In addition, there is also the distance inferred through luminosity d_L , which will differ from others due to the decreasing of photon energy caused by cosmic expansion. (DODELSON, 2003) Those distance measurements are related by

$$d_A = a\chi = a^2d_L. \quad (1.24)$$

The link between redshift and physical time flows directly from our solutions to the Friedmann Equation and from Eq. 1.22. To relate redshift and comoving distance, we evaluate the integral from Eq. 1.23 with $t(a(z))$ as lower bound and $t(a = 1)$ as upper bound. The result can be converted from Ga to Mpc units through our $c = 1$ convention.

In Figure 1.2, we present a numerical computation of the comoving distance as a function of the redshift for our nine possible universes. At $z = 2$, the difference between models is such that the comoving distance ranges from $4.6 \times 10^3 \text{Mpc}$ to $5.8 \times 10^3 \text{Mpc}$. Thus, there is a strong model dependence on the process of obtaining cosmological distances through redshifts. The probing of distance by independent measurements can

Figure 1.2 – Distance-redshift relations for flat Λ CDM cosmologies.

consequently be used to constrain cosmology, as is the case with the study of Baryon Acoustic Oscillations.

1.4 THE SIGMA 8 PARAMETER

There are several ways to parametrize our cosmological model, and so the choice of parameters is mostly a matter of convenience. One parameter that has been frequently used in the context of cosmological surveys is σ_8 . It serves as a measure of the amplitude of density perturbations at the conventional scale of $8\text{Mpc}/h$. (PRIMACK, 2005) In the context of 2-point statistics, which will be discussed in Chapter 2, it becomes useful to express cosmology in terms of σ_8 because it directly modulates the amplitude of the correlation functions.

1.5 COSMOLOGICAL PROBES

In order to determine the values of the cosmological parameters, we can resort to several types of observations. CMB radiation gives us a view of the early universe, while galaxy distribution is more related with structure formation physics. With observation of galaxy shapes, we can infer statistical data on weak lensing, which traces the clustering of all (including non-baryonic) matter. We will refer to each of those observation types as cosmological probes, or simply probes. They are sources of data that can be treated

separately or in combination to inform and constrain theoretical models. Here we present the main cosmological probes that have been explored in sky surveys.

1.5.1 CMB

CMB radiation was produced in the epoch of the decoupling between matter and radiation. It can be observed today as a blackbody signal coming from all directions of the universe. Because this radiation has been traveling free since the era of decoupling, it serves as an early universe probe.

Three major missions were held to map the small CMB temperature anisotropies, which are indicators of early perturbations on the background energy density. The first one was the Cosmic Background Explorer (COBE). It was followed by the Wilkinson Microwave Anisotropy Probe (WMAP), and finally by the Planck mission, which gave us some of the best estimates for cosmological parameters so far.

Though we have precise CMB constraints on those parameters, it is still an ongoing task to compare those results with the ones informed by other probes, such as galaxy clustering. From time to time, apparent discrepancies emerge, such as the one known as the Hubble tension, in which measurements from CMB set H_0 to a value incompatible with the one obtained by late universe probes, being the discrepancy larger than 4σ . (DI VALENTINO; MENA; PAN, et al., 2021)

1.5.2 Galaxy Clustering

Measurements of typical distances between galaxies are a way to verify the validity of our structure formation physics, which recounts the creation of the modern large-scale structure of the universe. The 3-dimensional distribution of galaxies require an establishment of both their angular positions in the sky and their distance to the Milky Way, which is found by measuring their redshifts.

While galaxy clustering measurements give a large amount of information on the distribution of visible matter, it leaves the underlying dark matter distribution unaccounted for. Thus, it can't be used alone to reconstruct the total distribution of matter on the universe. Cosmological inference through galaxy clustering, therefore, has to either resort to more specific clustering effects such as BAO (see 1.5.4) and RSD (see 1.5.7), or to a combination with weak lensing measurements (1.5.3).

1.5.3 Weak Lensing

From GR, we know that light from distant objects is deflected by the presence of mass between the observer and the object. This effect is known as Gravitational Lensing. While there are cases of galaxies which appear completely distorted due to lensing, the common scenario is somewhat subtle. Weak gravitational lensing changes slightly the shape of the objects observed, in an effect that can be identified by a statistical analysis of galaxy shapes. This signature, known as cosmic shear, functions as an imprint of the mass distribution that lies between the distorted galaxies and the observer. Mapping the effect of weak lensing on the universe allows us to go beyond galaxy clustering and uncover the large scale distribution of dark matter. (KILBINGER, 2015)

1.5.4 BAO

In structure formation models, the presence of baryons has the effect of introducing a secondary typical distance of separation between galaxies, apart from the main typical distance obtained by non-baryonic physics. This feature of the large-scale structure of the universe can be used to measure the distance-redshift relation, which gives us relevant information on our cosmology. The primordial plasma sound waves which give origin to this phenomena are known as Baryon Acoustic Oscillations (BAO). Cosmology with BAO proceeds by measuring the scale of the oscillations, and thus obtaining a determination of the angular diameter distance. (EISENSTEIN; ZEHAVI; HOGG, et al., 2005)

1.5.5 Type Ia Supernovae

Type Ia Supernovae are luminous events which happen on binary systems where a white dwarf achieves a mass limit, releasing a predictable amount of energy. The light curve received from a Type Ia Supernova can be used to determine its luminosity distance, which can then be combined with redshift data of its host galaxy to probe cosmology and evaluate dark energy models. This probe was responsible for the first observational evidence of dark energy in 1998, which added the last major ingredient to our current cosmological model. (RIESS; FILIPPENKO; CHALLIS, et al., 1998)

1.5.6 Cluster Counts

Galaxy clusters indicate the presence of large dark matter halos. The identification of those clusters, thus, can be used to measure the halo mass function. Using this measurement, different cosmological models can be tested by their predictions for the number of halos of different masses. (ALLEN; EVRARD; MANTZ, 2011)

1.5.7 Redshift space distortions

The redshift of galaxies is directly linked to their recession velocity, which is dominated by the effect of the expansion of the universe. Galaxies, however, have also individual movements, which deviate from this all-encompassing flow. Statistically, this will induce distortions on our redshift maps, which correspond to a biased determination of the redshift. (HAMILTON, 1998) Redshift space distortions (RSD) can also serve as cosmological probes. Tonegawa, Park, Zheng, et al. (2020) mention two paths toward probing cosmology with RSD. The first, which has been widely used, is to measure large scale distortions which relate to the period of structure formation. The second, which is of most interest to the authors, is the use of small scale RSD, which relate to internal movements in clusters, and produce the so-called Finger-of-God effect. (TONEGAWA; PARK; ZHENG, et al., 2020)

1.6 REDSHIFT SURVEYS

Empirical data for research in modern cosmology comes, almost in its entirety, from sky surveys. When dealing with large-scale structure, the basic required data is galaxy positions and redshifts. Cosmological models are generally indifferent to the properties of individual galaxies, but they are affected by statistical properties of the universe. Thus, the larger the galaxy sample, more potential it has to produce actual information on cosmological parameters. During the last two decades, many projects attempted to map large areas of the sky in order to extract such information.

The mapping process can be carried out with two distinct strategies. The first is to perform a simple imaging of the sky, measuring the total amount of light received through each of a limited number of filters. Projects which follow this idea are known as photometric surveys. The second is to measure the whole luminous spectra of each of the detected galaxies, in order to establish their redshift with high precision. This technique

is used in the so-called spectroscopic surveys. While the first strategy allows the survey to produce larger catalogs, it stumbles into the problem of less precise redshift values for each galaxy. Estimation of galaxy redshifts through photometry is far from trivial, but the field has seen great development following the advent of machine learning.

When introducing the neural network code ANNz, [Collister and Lahav \(2004\)](#) detailed the difference between the two main tendencies for extracting so-called photo-z (e. g. redshift values determined through photometry). The first tendency is known as *template matching*. It works by having a limited amount of pre-existing templates, of which the full spectrum and redshift value of each is known. Photometric observations of the new galaxies can then be compared to each of those templates, and a best fit can be found for the redshift through χ^2 minimization. A modern successful template matching method is the red-sequence Matched-filter Galaxy Catalog (redMaGiC) ([ROZO; RYKOFF; ABATE, et al., 2016](#)). As one of the cornerstones for the Dark Energy Survey (DES), this method will be discussed in Chapter 3.

The second tendency for photometric redshift estimation is to use *empirical methods*. This type of estimation works by finding a mathematical relation to deduce the photo-z value directly from the observed magnitude in each filter of the survey galaxies. [Collister and Lahav \(2004\)](#) follow this trend in their paper, attempting to find this relation by training and validating artificial neural networks. Similar approaches were undertaken by [Gerdes, Sypniewski, McKay, et al. \(2010\)](#), with their ArborZ code, and [Sadeh, Abdalla, and Lahav \(2016\)](#), with their updated version of ANNz.

1.6.1 Overview of past and present surveys

With the rapid evolution of redshift surveys, it became possible to obtain several tight and independent constraints on cosmological parameters. Each survey has also served to train researchers and stimulate the development of technologies for the following ones. Here we highlight some of the projects of greater importance that were held in the last years.

1.6.1.1 Pre-2000 surveys

The first redshift surveys to be conducted obtained spectroscopic measurements of only a few thousand galaxies. Among the most relevant of those projects are the

Center for Astrophysics (CfA) Survey, in the northern hemisphere, and the Southern Sky Redshift Survey (SSRS), in the southern hemisphere. (GIOVANELLI; HAYNES, 1991) With the improvement of observation technology, larger projects became possible. By the end of the 1990s, the Two Degree Field Galaxy Redshift Survey (2dFGRS) had the ambitious goal of measuring the spectra of 2.5×10^5 galaxies (COLLESSY, 1999), which was completed by the time of its final data release, in 2003.

At the turn of the century, the first successful measurements of cosmic shear were published by four independent teams (Kaiser, Wilson, and Luppino (2000), Wittman, Tyson, Kirkman, et al. (2000), Van Waerbeke, Mellier, Erben, et al. (2000), and Bacon, Refregier, and Ellis (2000)). In the context of exploring this new pathway, data from the VIRMOS-DESCART survey had a crucial role. This project, briefly described by Van Waerbeke, Mellier, Radovich, et al. (2001), used a combination of data from the VIRMOS consortium¹, and from the CFH12K camera at the Canada-France-Hawaii Telescope. A second project that contributed in this rhythm setting for cosmic shear analyses was the Red-Sequence Cluster Survey (RCS), a deep photometric survey that, starting in 1999, covered a 100deg^2 patch of the sky in search of galaxy clusters. (GLADDERS, M.; YEE, H., 2001) (GLADDERS, M. D.; YEE, H. K. C., 2005)

1.6.1.2 SDSS (2000-ongoing)

The Sloan Digital Sky Survey (SDSS) was originally intended to measure the spectra of about 10^6 galaxies and 1.5×10^5 quasar candidates, from which 65% were expected to be quasars. It would photometrically map a total of a quarter of the celestial sphere, and from those images select the objects destined for spectroscopic measuring. (YORK; ADELMAN; ANDERSON JOHN E., et al., 2000) Between 2000 and 2008, the project's two first phases SDSS-I and SDSS-II completed those objectives and also conducted a survey of Supernovae Type Ia and a survey of Milky Way stars (SEGUE). (ABAZAJI-AN; ADELMAN-MCCARTHY; AGÜEROS, et al., 2009) It was then extended to a third phase (SDSS-III), which collected data from 2008 to 2014, and a fourth phase (SDSS-IV), ranging from 2014 to 2020. Among the many features added to the project during phases III and IV, we may cite the BAO surveys BOSS and e-BOSS. (SLOAN DIGITAL SKY

¹ The VIRMOS consortium was planned to collect data from two instruments on the European Southern Observatory, the Visible Multi-Object Spectrograph (VIMOS), and the Near Infrared Multi-Object Spectrograph (NIRMOS). (LEFEVRE; VETTOLANI; MACCAGNI, et al., 1998)

[SURVEY, 2018](#)) Currently, SDSS is conducting a new series of projects known as SDSS-V, which is expected to finish by 2025. ([SLOAN DIGITAL SKY SURVEY, 2021](#))

1.6.1.3 CFHTLS (2003-2009)

The Canada-France-Hawaii Telescope Legacy Survey (CFHTLS) is a photometric survey that includes a wide component of 155 square degrees of the sky (CFHTLS Wide), and a deep survey of a smaller area, aimed at having a greater magnitude limit (CFHTLS Deep). In addition to these main components, a Supernovae Ia detection project was also conducted in the same facilities (SNLS). The results of the Supernovae survey contributed to the improvement of the photometric calibration methods of the main survey components. ([HUDELOT; GORANOVA; MELLIER, et al., 2012](#))

The period in which CFHTLS collected data was one of rising interest in weak lensing. Thus, data from the survey was soon used for several cosmic shear analyses, such as those by [Hoekstra, Mellier, Waerbeke, et al. \(2006\)](#), [Semboloni, Mellier, Van Waerbeke, et al. \(2006\)](#), and [Benjamin, Heymans, Semboloni, et al. \(2007\)](#). Eventually, a weak lensing project called CFHTLenS arose with the goal of performing a re-analysis of CFHTLS data with newer photometric redshift estimation techniques and newer strategies for dealing with the intrinsic alignment of galaxies, which is an obstacle for the transformation of galaxy shape data into actual cosmic shear data. ([ERBEN; HILDEBRANDT; MILLER, et al., 2013](#))

1.6.1.4 KiDS (2011-ongoing)

To follow the trend of cosmological analysis through weak lensing, the Kilo-Degree Survey (KiDS) is completing a multi-band photometric sky mapping which has cosmic shear as its main probe. At the time of its fourth data release, the collaboration's prospect became to observe a total of 1350 square degrees of the sky, in order to match the observing region with that of KiDS's companion infrared survey VIKING, which was completed in 2015. ([KUIJKEN, K.; HEYMANS, C.; DVORNIK, A., et al., 2019](#)) KiDS observations are made in four filters (*ugri* system), and are complemented by VIKING's five infrared bands.

1.6.1.5 HSC-SSP (2014-ongoing)

The Hyper Suprime-Cam Subaru Strategic Program is a photometric survey being conducted at the Subaru Telescope in Hawaii. Making use of five filters (*grizy* system), it combines three observing fields, each with a specific exposure time, and thus, different magnitude limit: the Wide, the Deep, and the UltraDeep. (AIHARA; ALSAYYAD; ANDO, et al., 2021) A spectroscopic counterpart of the project is being planned for the same telescope in order to provide complementary data to that of HSC. Data will be collected by the Prime Focus Spectograph (PFS), which is yet being assembled. (PRIME FOCUS SPECTROGRAPH, 2021)

1.6.1.6 HETDEX (2017-ongoing)

The Hobby-Eberly Telescope Dark Energy Experiment (HETDEX) is a spectroscopic survey intended to be complete by 2024. Its strategy is to perform spectrography of complete sky regions, instead of pre-selecting photometrically the objects of interest. HETDEX focuses on detecting Lyman- α emitter galaxies. (ZHANG; OUCHI; GEBHARDT, et al., 2021)

1.6.2 The Dark Energy Survey

The Dark Energy Survey (DES) is a photometric redshift survey which collected data from 2013 to 2019, and is now on track to complete its analysis stage. The focus of the collaboration is on the use of a diversity of probes. Cosmological issues are addressed with data from Supernovae Ia, BAO, cluster counts, galaxy clustering and weak lensing.

The survey was presented in 2004 as a proposal to the National Optical Astronomy Observatory (NOAO), which was seeking a collaborator to build a camera for the Blanco telescope at the Cerro Telolo Inter-American Observatory, in Chile. In exchange for the partnership, NOAO would award the collaboration with observing time on the telescope with the built device. (ABBOTT et al., 2005) The camera, which was called DECam, was finished by 2010. (DES, 2021) Additional spectroscopic data for DES was collected at the Anglo-Australian Telescope, in a project known as the Optical redshifts for the Dark Energy Survey (OzDES). (YUAN; LIDMAN; DAVIS, et al., 2015)

The Dark Energy Survey measured a total sky area of 5000 deg^2 and performed imaging on five filters (*grizy* system). Extensive data analysis papers have already been

published for the survey's first year of data, some of which will be discussed in Chapters 3 and 4.

1.6.3 Future surveys

1.6.3.1 DESI

The Dark Energy Spectroscopic Instrument (DESI) is a spectroscopic survey which will take place at the Mayall telescope, in the Kitt Peak National Observatory, in Arizona, and span a sky area of 14000 deg^2 . As was the case with SDSS, it will measure spectra from galaxies detected by prior photometry. For this imaging stage, it will rely on measurements from DECam, from the WISE satellite, and from two telescopes on the Kitt Peak Observatory. The survey intends to measure spectra of 3.3×10^7 objects, previously separated into four samples: the Bright Galaxy Sample (BGS), Luminous Red Galaxies (LRGs), Emission Line Galaxies (ELGs), and quasars (QSOs). DESI will have a focus on analysis of BAO and redshift space distortions. (DESI COLLABORATION, 2016)

1.6.3.2 LSST

The Vera Rubin Observatory is being assembled in Cerro Pachon, Chile, and will have as its first project the Legacy Survey of Space and Time (LSST). Equipped with a 3.2 gigapixel camera, the photometric survey will collect data on six bands (*ugrizy* system) over a sky area of 20000 deg^2 . LSST will follow the trend of DES and focus on the use of various probes, for which it is expected to catalog a number of 10^{10} galaxies. Along with its cosmological goals, the survey has also a set of astrophysical and astronomical goals, such as the detection of Near-Earth Asteroids, and the study of Active Galactic Nuclei. (COLLABORATION, 2009)

1.6.3.3 Euclid

Euclid is a survey that will be conducted by a telescope placed on the Earth-Sun Lagrange point L2. With launch currently scheduled for 2022, the Euclid equipment will enable photometric and spectroscopic measuring, covering a total sky area of 15000 deg^2 . (TUTUSAUS; MARTINELLI; CARDONE, et al., 2020) The main probes of interest for the survey will be weak lensing and BAO, and its results are expected to be as precise as to

yield insight in modified gravity, inflation and neutrino physics. (LAUREIJS; AMIAUX; ARDUINI, et al., 2011)

1.6.3.4 Roman Space Telescope

The Nancy Gray Roman Space Telescope, previously known as WFIRST, is scheduled to be launched by NASA by at most 2027. Roman’s spectroscopic survey is expected to yield complementary information to that of LSST and Euclid, in a such a way that a joint planning of the three projects could optimize each of their results. (JAIN; SPERGEL; BEAN, et al., 2015)

1.7 THE LEGACY OF DES

While expectations for the next generation of redshift surveys keep increasing, it is still necessary to explore the potential of data from previous and ongoing surveys. The contributions of DES to cosmology, for example, will certainly be surpassed by those of LSST and the space-based missions. However, the latter contributions must build over the edifice of the former. Each generation of surveys is responsible for the best up-to-date constrains, but also for the development of collective scientific experience, which involve from the dynamics of large collaborations to the refinement of methods. These aspects remain as the basis for the following generation of surveys, which usually require updated theoretical and computational models. The amount of data expected to be collected by LSST, for example, calls for a necessary optimization of data analysis techniques. Computational and practical lessons from DES data analysis stage must then serve as indispensable building blocks for those improvements.

In the context of cosmological data analysis, the multi-probe method that has been placed by DES (described in 3.5) has the potential to yield huge insights on the analysis methods for next-generation surveys. In the same way, complementary statistical analyses such as the one we describe in Chapter 4 offer a building block to future multi-probe studies to be conducted during the next decades.

HARMONIC SPACE: THE CLUSTERING OF MATTER IN THE UNIVERSE AS MEASURED BY THE ANGULAR POWER SPECTRUM

2.1 MATHEMATICAL DESCRIPTION OF MATTER CLUSTERING

When dealing with large scales, the universe can be taken as homogeneous and isotropic. Nonetheless, it is trivial that this condition breaks at small enough scales. In planetary, stellar, and even galactic levels, celestial bodies find themselves organized in structures. Thus, while the universe is subject to a background expansion, there are gravitationally bound regions which are not internally affected by the background process. Additionally, those regions interact with each other in such a way that the dense ones tend to become even denser. From any specific cosmology, it is possible to generate theoretical predictions for the expected statistical pattern of matter distribution in the universe. Measuring the actual pattern, therefore, is a path towards constraining our universe's actual cosmology.

The basic unit which is used to model the large-scale structure of the universe is the dark matter halo. Visible concentrations of matter, such as individual galaxies or galaxy clusters are understood to exist within those halos. Both measurements of galaxy clustering and weak lensing can probe such distribution of matter, however in different ways. The first is a straightforward though biased method, for it can only yield exact information on the galaxies inside the halos, but not on the halos themselves. The second is less straightforward, in the sense that it depends on converting shape measurements to actual mass distribution, but it yields unbiased results for total matter distribution. Both methods, however, rely on n -point statistics, and share the common theoretical framework of matter correlation functions. In this chapter we will review this framework with an emphasis on its application to DES data analysis of galaxy clustering and weak lensing on harmonic space.

The basic ruler for studying clustering trends is the 2-point correlation function,

which comes along with its Fourier-space counterpart function: The power spectrum. The 2-point correlation function $\xi(r)$ measures the probability for a pair of objects to be separated by a distance r . (PEEBLES, 1993) This measure is position-independent due to the homogeneity and isotropy of the universe. If n is the number density of objects, dV_1 and dV_2 volume elements distant r from each other, and dP the probability to find an object at dV_1 and another at dV_2 , the correlation function is, as described by Peebles (1993):

$$\xi(r) = \frac{1}{n^2} \frac{dP}{dV_1 dV_2} - 1. \quad (2.1)$$

By performing a Fourier transform of the correlation function, we define the power spectrum $P(k)$:

$$P(k) = \int_0^\infty 4\pi r^2 \cos 2\pi k r \xi(r) dr. \quad (2.2)$$

If we define a density contrast function δ to measure the excess density in a given point in space relative to the average density of the universe, we can write the correlation function as the average value of the pair $\delta(\mathbf{x})\delta(\mathbf{x} + \mathbf{r})$, where $\mathbf{x}$ is the position vector in 3D space, and $\mathbf{r}$ a displacement vector of size r . (MANDELBAUM; SLOSAR; BALDAUF, et al., 2013) The power spectrum, then, becomes:

$$P(k) = \int_0^\infty 4\pi r^2 \cos 2\pi k r \langle \delta(\mathbf{x})\delta(\mathbf{x} + \mathbf{r}) \rangle dr. \quad (2.3)$$

We can then define the overdensity δ_k for each Fourier mode $\mathbf{k}$, and write the power spectrum as: (PEEBLES, 1993)

$$P(k) = \langle |\delta_k|^2 \rangle. \quad (2.4)$$

2.2 MATTER CLUSTERING IN COSMIC HISTORY

Data from cosmological surveys allow us to map the universe with galaxies, galaxy clusters and radiation from CMB. Information on cosmological parameters, however, is

not sensible to the actual location of individual galaxies or anisotropies of the CMB. Changes on the parameter values would, instead, modify statistics of the probe in question. In order to take advantage of this statistical nature of cosmological research, survey data has commonly been summarized by correlation functions. Developing our theoretical models to yield predictions for those functions, thus, is an important step towards probing cosmology with real data. While it is possible to model 3-point functions, 4-point functions and beyond, an adequate amount of cosmological information comes from 2-point functions, in such a way that they suffice for our current research purposes.

From a starting point on the early universe, it is possible to model perturbations on the background density, and from these perturbations arrive at predictions for the power spectrum. Usual structure formation models start with initial conditions given by inflation. Then, from a set of random perturbations, we can model the creation and development of gravitationally stable structures and arrive at a picture of the matter distribution of the universe at any given redshift. The standard procedure is to begin with linear perturbation theory, and then enhance the model later with non-linearities.

The development of density perturbations can be approximated by a combination of two functions: the growth function $G(z)$ and the transfer function $T(k)$. The latter relates to physical processes from the era of recombination, while the former describes the growth of dense regions at late times. (DODELSON, 2003) We assume that, at early times, the universe had an initial matter power spectrum $P_{init}(k)$. The power spectrum later in time relates to that initial spectrum by

$$P(k, z) = T^2(k)G^2(z)P_{init}(k). \quad (2.5)$$

We now describe each of the ingredients for our matter power spectrum model.

2.2.1 The initial power spectrum

The early homogeneous universe is assumed to be comprised of small density perturbations, which serve as seeds for modern-day large perturbations. Its power spectrum can both be deduced from a given inflationary scenario or simply modeled through a chosen power-law expression. An argument by Harrison (1970) and Zeldovich (1972) sets what has been known as the Harrison-Zel'dovich spectrum, that has $P(k) \propto k$. A more

general power law expression is more common in the context of survey physics: (BRIDLE et al., 2003)

$$P(k) = A_s \left(\frac{k}{k_0} \right)^{n_s-1}. \quad (2.6)$$

Here A_s and n_s correspond to the spectral amplitude and spectral number, posited as cosmological parameters in Chapter 1, and k_0 is a scale of choice.

2.2.2 The growth function

The gravitational potential generated by a density field is given by the Poisson Equation. For a local density contrast δ , we write it with the derivative acting in comoving coordinates:

$$\nabla^2 \phi = 4\pi G \bar{\rho}_m a^2 \delta, \quad (2.7)$$

where $\bar{\rho}_m$ is the average matter density over local inhomogeneities.

Equation 2.7 is combined by Bonnor (1957) with the continuity equation (Eq. 2.8) and the Euler equation (Eq. 2.9) to model what is now known as the growth function. Following Peebles (1980), we write them in comoving coordinates:

$$\frac{\partial \rho}{\partial t} + \frac{1}{a} \vec{\nabla} \cdot (\rho \vec{v}) = 0 \quad (2.8)$$

and

$$\frac{\partial \vec{v}}{\partial t} + \frac{\dot{a}}{a} \vec{v} + (\vec{v} \cdot \vec{\nabla}) \vec{v} + \frac{1}{a} \vec{\nabla} \phi + \frac{1}{a\rho} \vec{\nabla} P = 0, \quad (2.9)$$

where $\vec{v}$ is the velocity of the perturbation.

The term describing the effect of the pressure P can be neglected on a modeling of late-time growth, as done by Heath (1977). We then rewrite Eq. 2.8 in terms of the density contrast:

$$\bar{\rho}_m \frac{\partial \delta}{\partial t} + \bar{\rho}_m \frac{1}{a} (\vec{\nabla} \cdot \vec{v} \delta + \vec{\nabla} \cdot \vec{v}) = 0. \quad (2.10)$$

For our linear modeling of perturbations, we neglect terms that scale with more than one power of δ or $\vec{v}$. Eqs. 2.9 and 2.10 become

$$\frac{\partial \vec{v}}{\partial t} + \frac{\dot{a}}{a} \vec{v} + \frac{1}{a} \vec{\nabla} \phi = 0 \quad (2.11)$$

and

$$\frac{\partial \delta}{\partial t} + \frac{1}{a} (\vec{\nabla} \cdot \vec{v}) = 0. \quad (2.12)$$

Now we take the divergence of Eq. 2.11:

$$\frac{\partial}{\partial t} (\vec{\nabla} \cdot \vec{v}) + \frac{\dot{a}}{a} (\vec{\nabla} \cdot \vec{v}) + \frac{1}{a} \nabla^2 \phi = 0. \quad (2.13)$$

The first and second terms can be rewritten through Eq. 2.12, and the third through Eq. 2.7. We find

$$-a \frac{\partial^2 \delta}{\partial t^2} - 2\dot{a} \frac{\partial \delta}{\partial t} + 4\pi G \bar{\rho}_m a \delta = 0. \quad (2.14)$$

Following [Bernardeau et al. \(2002\)](#), we identify two solutions and write the density contrast as the linear combination

$$\delta(\vec{x}, t) = G_1(t)A(\vec{x}) + G_2(t)B(\vec{x}). \quad (2.15)$$

The growth function $G(t)$ will then correspond to the fastest growing solution. ([BERNARDEAU et al., 2002](#)) An analytical solution is presented, in terms of the scale factor, by [Dodelson \(2003\)](#):

$$G(a) \propto H(a) \int^a \frac{da'}{(a' H(a'))^3}, \quad (2.16)$$

where $H(a) \equiv \dot{a}/a$.

Through the relation between redshift and the scale factor, given in Eq. 1.22, we therefore find the growth function $G(z)$.

2.2.3 The transfer function

The transfer function is encountered by solving the full set of equations for perturbations in matter and radiation. The following fitting formula was proposed by [Bardeen et al. \(1986\)](#) and takes into account dark matter and radiation perturbations, neglecting the effect of baryons:

$$T(k) = \frac{\ln(1 + 2.34q)}{2.34q} (1 + 3.89q + (16.1q)^2 + (5.46q)^3 + (6.71q)^4)^{-1/4}. \quad (2.17)$$

The modeling of baryonic effects was made by [Eisenstein and Hu \(1998\)](#). They split the transfer function into a dark matter component $T_c(k)$ and a baryon component $T_b(k)$ in order to model each part separately. The dark matter transfer function was found by interpolating between small-scale and large-scale limits and incorporating the effect of baryons on dark matter perturbations. The baryon transfer function was also modeled taking into account the effect of dark matter in baryon perturbations.

The total matter transfer function, in terms of its modeled contributions, is: ([EISENSTEIN; HU, 1998](#))

$$T(k) = T_c(k) \frac{\Omega_c}{\Omega_m} + T_b(k) \frac{\Omega_b}{\Omega_m}, \quad (2.18)$$

where Ω_c is the dark matter density given by $\Omega_m - \Omega_b$.

Solutions for the complete set of equations governing interaction between matter and radiation perturbations can also be found numerically. Such task is performed by the CAMB code, which we describe in 2.2.6.

2.2.4 Galaxy clustering power spectrum

To probe the large-scale structure of the universe with galaxy clustering, a model for the galaxy power spectrum becomes paramount. Perturbation theory gives us the

power spectrum of total matter, which includes visible matter concentrated in stars, galaxies and gas, but also dark matter, which is assumed to be clustered along with galaxies.

The difference between the clustering of those dark matter halos and the clustering of galaxies depends on galaxy formation processes. In order to escape from unnecessarily complex models, it has become usual to assume a linear relation between the clustering of galaxies and that of all matter. This relation defines a parameter known as the galaxy bias $b(z)$. It is known that this linear galaxy bias model breaks when small distances are considered. (KRAUSE; EIFLER, et al., 2017) A more general model for the bias allows for its scale-dependence, leading to the following relation between the power spectra:

$$P_{\text{galaxy}}(k, z) = b^2(k, z)P_{\text{matter}}(k, z). \quad (2.19)$$

2.2.5 Non-linearities in the matter power spectrum

Effects from non-linear perturbation theory can be incorporated into the matter power spectrum by the use of a halo model. COORAY and SHETH (2002) detail such procedure, that passes through a modeling of the mass function and of the density profile of dark matter halos. An alternative for the incorporation of non-linear effects is the use of fitting formulae extracted from simulations, as done by Takahashi, Sato, Nishimichi, et al. (2012).

2.2.6 CAMB

The Code for Anisotropies in the Microwave Background (CAMB) is a Fortran code written by Antony Lewis and Anthony Challinor which solves the linear perturbation equations and computes cosmological functions for the CMB and for matter distribution. It has now a full interface in Python, as well as an online interface at NASA's Legacy Archive for Microwave Background Data Analysis (LAMBDA) website¹.

From almost any given input cosmology, CAMB can compute the linear power spectrum as well as the non-linear, for which it offers a variety of modeling ways. We used the Python implementation of CAMB to compute both linear and non-linear spectra for an $\Omega_m = 0.3$ and $h = 0.7$ cosmology. The other cosmological parameters were set at

¹ <https://lambda.gsfc.nasa.gov/>

Figure 2.1 – Linear and non-linear CAMB matter power spectra for $\Omega_m = 0.3$ and $h = 0.7$. The blue dotted lines represent the values for which the non-linear spectrum starts to exceed the linear by 5%.

their CAMB default values. For the non-linear correction, we opted for the halo model described by [Mead, Heymans, Lombriser, et al. \(2016\)](#). Our spectra are shown in [Figure 2.1](#). We computed the value of k for which the non-linear spectrum starts to exceed the linear by 5%. This value, found to be $k = 7.8 \times 10^{-2} \text{Mpc}^{-1}$, is indicated, along with its correspondent linear $P(k)$, by the blue dotted lines on the figure.

We then computed with CAMB the non-linear matter power spectrum at $z = 0$ for our cosmologies used in Chapter 1. The results are shown in [Figure 2.2](#). For all cosmologies, we maintained $\Omega_b h^2$ as constant. Differing values of Ω_m , thus, modulated the dark matter density.

It is also possible to see the time evolution of the power spectra by finding its form at different redshifts for a same cosmology. For this task the spectrum is computed without the separation from [Eq. 2.5](#). We fixed our parameters at $\Omega_m = 0.3$ and $h = 0.7$ and produced CAMB spectra for $z = 0, 0.5, 1, 1.5$ and 2. Our results are shown in [Figure 2.3](#).

2.3 THE ANGULAR POWER SPECTRUM

While the 3-dimensional correlation functions suffice for a purely theoretical treatment, survey science requires the use of a larger inventory of functions. Redshift data

Figure 2.2 – Non-linear matter power spectrum at $z = 0$ for nine flat Λ CDM cosmologies. The spectra were computed by the Python implementation of CAMB.

Figure 2.3 – Non-linear matter power spectrum for different redshift values. The spectra were computed by the Python implementation of CAMB.

from photometric surveys is highly imprecise, which leads to a large uncertainty in the galaxy distance values. In contrast to this, the angular position of each galaxy can be well established. This discrepancy makes it convenient for us to introduce 2-dimensional correlation functions that measure the angular clustering of galaxies without taking into account their position in 3D space. These functions can also be used in analysis of galaxy lensing, and ultimately combined for a joint analysis of galaxy clustering and weak lensing.

Here we will construct the angular power spectra that are useful for this joint analysis, establishing their link with the usual 3D power spectrum.

2.3.1 Galaxy clustering angular power spectrum

Galaxy clustering is measured by detecting correlation patterns in galaxy positions. A function built to summarize this data must start with galaxy counts. For this purpose, we follow the usual procedure and define the density contrast function $\delta(\theta, \phi)$ in 2D space: (ANDO; BENOIT-LÉVY; KOMATSU, 2017)

$$\delta(\theta, \phi) = \frac{n(\theta, \phi) - \bar{n}}{\bar{n}}, \quad (2.20)$$

where $n(\theta, \phi)$ is the galaxy count on the given coordinate and $\bar{n}$ is the average value of n in the whole sphere.

Were we to use a single 2D function for all the galaxies observed, we would lose information by mixing low-redshift galaxies with high-redshift ones when some redshift data, though not precise, is usually available. The straightforward solution is to divide the 2D sky into a limited amount of shells, which are known as redshift bins. The density contrast of an individual redshift bin will be a projection of the 3D density contrast $\delta(z, \theta, \phi)$ along the extension of the shell. A selection function $n(z)$, which tells the probability of the galaxy having a given redshift within the shell, is used to perform the projection. Thus, the density contrast can be written in the projection form as

$$\delta(\theta, \phi) = \int_0^\infty dz n(z) \delta(z, \theta, \phi). \quad (2.21)$$

We expand this function into spherical harmonics and write

$$\delta(\theta, \phi) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{m=\ell} a_{\ell m} Y_{\ell m}(\theta, \phi). \quad (2.22)$$

Then, we define the clustering angular power spectrum C_ℓ as the variance of the expansion coefficients $a_{\ell m}$, which depends only on ℓ . (CROCCE; CABRÉ; GAZTAÑAGA, 2011) From Eqs. 2.21 and 2.22, the $a_{\ell m}$ coefficients are found to be

$$a_{\ell m} = 4\pi i^\ell \int_0^\infty dz n(z) \int_0^\infty \frac{d^3 k}{(2\pi)^3} \delta(\vec{k}, z) j_\ell(k\chi(z)) Y_{\ell m}^*(\hat{k}), \quad (2.23)$$

where $\chi(z)$ is the comoving distance to redshift z . The angular power spectrum follows:

$$C_\ell = \frac{2}{\pi} \int_0^\infty k^2 P(k) \Psi_\ell^2(k) dk, \quad (2.24)$$

where ψ_ℓ is a kernel function defined as

$$\Psi_\ell(k) \equiv \int_0^\infty n(z) b(z) G(z) j_\ell(k\chi(z)) dz. \quad (2.25)$$

In order to account for redshift space distortions, a correction term $R_\ell(k)$ can be added to the kernel. For purposes of our present study, however, the simplified equation is a valid approximation. In Eq. 2.25, we take into account the linear galaxy bias $b(z)$ in a general redshift-dependent form. Usually, however, the bias is assumed to be constant within a same redshift bin. Its effect, thus, becomes that of a b^2 multiplying factor on the spectrum.

It follows that we can start with a theoretical 3D power spectrum $P(k)$ and a projection function $n(z)$, and obtain a spectrum defined on the sphere surface. This resulting angular spectrum can be the object of comparison between data and theoretical predictions. If we have a set of galaxies from a cosmological survey, we can select subsets of them which conform to specific distributions $n^i(z)$, and thus define a redshift binning scheme. Then, the measured C_ℓ^i 's from each bin i can be compared with numerically computed ones, which may use theoretical $P(k)$ and survey-specific $n^i(z)$ functions.

2.3.2 Gravitational lensing

The angular power spectrum can also be an analysis tool for cosmic shear. Shape distortion of galaxies, when treated statistically, probe the large scale distribution of

matter. Though a more indirect form of observation, it isn't biased towards visible matter. In order to define an angular power spectrum that contains lensing information, we need to quantify the observed distortion effects. This is usually made through the parameters κ (convergence) and $\vec{\gamma}$ (shear). We can imagine the dynamic of weak lensing as the product of a layer of selected source galaxies, from which light is originated, with the matter that lies between this layer and the observer, which will bend the light rays and create shape distortion. These convergence and shear parameters relate to the observed and real positions of the source galaxies.

From Einstein's Equations, we can find a relation between the position of the source galaxy (θ_S) and the observed position of the deflected light (θ_O). (DODELSON, 2003) This relation, known as the lens equation, states that

$$\vec{\theta}_S = \vec{\theta}_O + \vec{\alpha}(\vec{\theta}), \quad (2.26)$$

where, in a flat universe, $\vec{\alpha}(\vec{\theta})$ is given in terms of the potential ϕ by

$$\vec{\alpha}(\vec{\theta}) = 2 \int_0^\chi \left(\frac{\chi - \chi'}{\chi} \right) \nabla_\perp \phi(\chi') d\chi'. \quad (2.27)$$

The equation for α includes a transverse derivative, which introduces the position shift on the direction perpendicular to that of the light travel, and an integration by the comoving distance, to account for the integrated effect of lensing along the line of sight. (KILBINGER, 2015)

The usual procedure follows by defining the matrix A_{ij} as: (DODELSON, 2003)

$$A_{ij} = \frac{\partial \theta_S^i}{\partial \theta_O^j}. \quad (2.28)$$

We conveniently separate A_{ij} into a diagonal and a traceless part. It follows from the derivation that A_{ij} is symmetric, and $A_{12} = A_{21}$. Therefore, we write

$$A = \begin{pmatrix} 1 - \kappa & 0 \\ 0 & 1 - \kappa \end{pmatrix} + \begin{pmatrix} -\gamma_1 & -\gamma_2 \\ -\gamma_2 & \gamma_1 \end{pmatrix}. \quad (2.29)$$

The first part gives us information on the magnification effect, defining the convergence parameter κ . The second gives us information on shape distortion of the source galaxies, and is a definition of the shear parameter $\vec{\gamma} = (\gamma_1, \gamma_2)$. (KILBINGER, 2015)

It is now possible to perform two-point statistics of convergence and shear. In the same way that we expanded the density contrast map into spherical harmonics to define a matter clustering angular power spectrum, we can create a convergence map $\kappa(\theta, \phi)$ and a shear map $\vec{\gamma}(\theta, \phi)$, from which lensing angular power spectra can be defined. For weak gravitational lensing, it can be shown that the convergence spectrum $C_\ell^{\kappa\kappa}$ equals the cosmic shear spectrum $C_\ell^{\gamma\gamma}$. (BARBER; TAYLOR, 2003) Therefore, though we intend to probe weak lensing by shape distortion, we can construct our functions in terms of the convergence.

In order to build our apparatus for cosmic shear analysis, we will need an expression for $C_\ell^{\kappa\kappa}$ in terms of the 3D matter power spectrum. This will make both galaxy clustering and convergence spectra flow from the same theoretical $P(k)$.

Combining Eqs. 2.26 and 2.28, it is trivial to see that

$$A_{ij} = \delta_{ij} + \frac{\partial \alpha^i}{\partial \theta_O^j}. \quad (2.30)$$

The derivative with respect to angular position θ_O^j can be transformed into a derivative with respect to the j coordinate by $x_j \approx \chi \theta_j$. The transverse derivative from Eq. 2.27 selects the direction of the shift, becoming a x_i derivative for the case of α_i . Thus, we find

$$\frac{\partial \alpha^i}{\partial \theta_O^j} = 2 \int_0^\chi \left(\frac{\chi - \chi'}{\chi} \right) \chi' \partial_i \partial_j \phi(\chi') d\chi'. \quad (2.31)$$

It is now convenient to introduce a lensing potential ψ : (KILBINGER, 2015)

$$\psi = 2 \int_0^\chi \left(\frac{\chi - \chi'}{\chi} \right) \chi' \phi(\chi') d\chi'. \quad (2.32)$$

Eq. 2.30 can be rewritten in terms of this potential:

$$A_{ij} = \delta_{ij} - \partial_i \partial_j \psi. \quad (2.33)$$

The convergence parameter is now

$$\kappa = \frac{1}{2} \partial_\alpha \partial_\alpha \psi \quad (2.34)$$

$$\Rightarrow \kappa = \int_0^\chi \left(\frac{\chi - \chi'}{\chi} \right) \chi' \partial_\alpha \partial_\alpha \phi(\chi') d\chi'. \quad (2.35)$$

We can replace $\partial_\alpha \partial_\alpha \phi$ by the full 3D Laplacian of ϕ , once the χ -direction term will become null when integrated. (BARTELMANN; MATURI, 2016) Now the newly found $\nabla^2 \phi$ can, via Eq. 2.7, be expressed in terms of the density contrast. This gives us

$$\kappa = 4\pi G \bar{\rho}_m \int_0^\chi \left(\frac{\chi - \chi'}{\chi} \right) \chi' (1 + z(\chi'))^{-2} \delta(\chi') d\chi'. \quad (2.36)$$

From Eq. 2.36, we find κ in terms of the density contrast today δ_0 , of the growth function $G(z)$, and of Ω_m :

$$\kappa = 3H_0^2 \Omega_m \int_0^\chi \left(\frac{\chi - \chi'}{\chi} \right) \chi' G(z(\chi')) (1 + z(\chi')) \delta_0(\chi') d\chi'. \quad (2.37)$$

Because we are dealing with a projection upon 2D space, we need to adjust the expression for a $z(\chi)$ that spans a full redshift distribution function. Thus, κ will be given by the following double integral in χ :

$$\kappa = 3H_0^2 \Omega_m \int_0^{\chi^{\text{lim}}} n(z(\chi)) d\chi \int_0^\chi \left(\frac{\chi - \chi'}{\chi} \right) \chi' G(z(\chi')) (1 + z(\chi')) \delta_0(\chi') d\chi', \quad (2.38)$$

where χ^{lim} is the upper limit of χ achieved by the distribution function $n(z)$.

A rearrangement of the double integral allows us to write

$$\kappa = \int_0^{\chi^{\text{lim}}} \delta_0(\chi') d\chi' \left(3H_0^2 \Omega_m \chi' (1 + z(\chi')) G(z(\chi')) \int_{\chi'}^{\chi^{\text{lim}}} d\chi n(z(\chi)) \left(\frac{\chi - \chi'}{\chi} \right) \right). \quad (2.39)$$

We define the lensing efficiency $q(\chi)$ by

$$q(\chi) = 3H_0^2\Omega_m\chi(1+z(\chi))G(z(\chi))\int_{\chi}^{\chi^{\text{lim}}}d\chi'n(z(\chi'))\left(\frac{\chi'-\chi}{\chi'}\right) \quad (2.40)$$

and write the convergence in terms of this lensing efficiency:

$$\kappa = \int_0^{\chi^{\text{lim}}} \delta_0(\chi')q(\chi')d\chi'. \quad (2.41)$$

Finally, we can expand the convergence map $\kappa(\theta, \phi)$ into spherical harmonics, following the procedure done with the density map. The variance of the $a_{\ell m}$ coefficients define the convergence angular power spectrum $C_{\ell}^{\kappa\kappa}$.

2.3.3 Galaxy-galaxy lensing spectrum

The two spectra defined in Sections 2.3.1 and 2.3.2 give information about the auto-correlations of density maps and of convergence maps. We define a third spectrum, $C_{\ell}^{\kappa\delta}$, to measure the cross-correlation between both maps. Let us use $a_{\ell m}^{\delta}$ for the coefficients of the density map expansion and $a_{\ell m}^{\kappa}$ for the ones of the convergence map expansion. The so-called galaxy-galaxy lensing spectrum is

$$C_{\ell}^{\kappa\delta} = \langle a_{\ell m}^{\kappa} a_{\ell m}^{*\delta} \rangle. \quad (2.42)$$

This function is used to detect how galaxy positions on a sample are correlated with galaxy shape distortions of a second more distant sample. The correlation arises in that the large-scale structure summarized by the former sample composes part of the distribution responsible for distorting light of the latter.

2.4 THE LIMBER APPROXIMATION

The final step towards theoretical power spectra which can be used for data analysis of galaxy clustering and weak lensing is to perform the Limber approximation. For sufficiently large values of ℓ , it allows us to take

$$k \approx \frac{\ell + 1/2}{\chi(z)}. \quad (2.43)$$

The double integration in Eqs. 2.24 and 2.25, then, becomes a single integration in χ . This alteration is useful because it eliminates the necessity to integrate highly oscillating functions, which is a computationally expensive job. (CROCCE; CABRÉ; GAZTAÑAGA, 2011)

Under the Limber approximation, the galaxy clustering angular power spectrum becomes:

$$C_{\ell}^{\delta\delta} = \int_0^{\infty} \frac{(n(z(\chi))(dz/d\chi)b(z(\chi))G(z(\chi)))^2}{\chi^2} P\left(\frac{\ell + 1/2}{\chi}\right) d\chi. \quad (2.44)$$

The derivation of the Limber approximation expression for the convergence angular power spectrum is analogous to that for the matter clustering spectrum. Starting from Eq. 2.41, we find that the product $n(z)b(z)G(z)dz/d\chi$ must be replaced by the lensing efficiency. Thus, we arrive at (KRAUSE; EIFLER, et al., 2017)

$$C_{\ell}^{\kappa\kappa} = \int_0^{\infty} \frac{q(\chi)^2}{\chi^2} P\left(\frac{\ell + 1/2}{\chi}\right) d\chi. \quad (2.45)$$

For the galaxy-galaxy lensing angular power spectrum, only one of the factors of the product $n(z)b(z)G(z)dz/d\chi$ is replaced by $q(\chi)$. Therefore, we find (KRAUSE; EIFLER, et al., 2017)

$$C_{\ell}^{\kappa\delta} = \int_0^{\infty} \frac{n(z(\chi))b(z(\chi))(dz/d\chi)G(z(\chi))q(\chi)}{\chi^2} P\left(\frac{\ell + 1/2}{\chi}\right) d\chi. \quad (2.46)$$

Therefore, from a 3D power spectrum $P(k)$ and a distribution function $n(z)$, we can find the projected angular power spectra for galaxy clustering, galaxy-galaxy lensing, and cosmic shear. These theoretical functions serve as a base for data analysis of those probes in harmonic space.

2.5 THE ANGULAR CORRELATION FUNCTION

An alternative approach to that of the angular power spectra is to perform statistics on 2D slices in configuration space. Then, the theoretical modeling makes use

Figure 2.4 – Matter clustering angular power spectra for nine flat Λ CDM cosmologies. We computed the spectra with our Python implementation of the Limber Approximation. The y-axis has a factor of $\ell(\ell + 1)$ for easier visualization

of the angular correlation functions. Such a choice has been favored in the context of DES, though much could be accomplished from the combination of both approaches. The mathematical expression for the angular correlation function in terms of the C_ℓ spectrum flows from the Legendre Addition theorem. (CROCCÉ; CABRÉ; GAZTAÑAGA, 2011)

2.6 NUMERICAL IMPLEMENTATION

We wrote a code in Python to perform the Limber approximation integral for the matter clustering power spectrum. We used CAMB generated power spectra and a gaussian redshift distribution $n(z)$ centered in $z = 0.3$, with $\sigma_z = 0.01$. For our nine considered cosmologies, the angular power spectra are given in Figure 2.4. As with the 3D matter power spectrum, we kept constant the $\Omega_b h^2$ combination. Thus, cosmologies with lower Ω_m represent those with a large baryon fraction and smaller dark matter content. We can see this effect visually by noticing the large amplitude of the wiggles on those low- Ω_m cosmologies. Those wiggles represent the BAO phenomenon, directly linked to the presence of baryonic matter.

2.7 PARAMETER FITTING

Once we are equipped with theoretical power spectra, we can proceed to constrain cosmological parameters by comparing our spectra with that retrieved from actual data. This comparison is done through Bayesian statistics, and so the end product is a joint posterior probability distribution P for the parameters of interest. In order to arrive at this distribution, we use a likelihood function $\mathcal{L}$, and when convenient a prior probability distribution π , which inserts in the calculation *a priori* information on our parameter value distribution. The relation between these functions is given by Bayes' theorem:

$$P(p|D) \propto \mathcal{L}(D|p)\pi(p), \quad (2.47)$$

where p is our set of parameters and $\mathbf{D}$ our data vector, which is constructed from the observations.

The likelihood function for a set of observations depends on its covariance matrix $\mathbf{C}$ and on the theoretical modeling. Krause, Eifler, et al. (2017) describes the construction of these ingredients in the context of DES, and writes the likelihood as follows:

$$\mathcal{L}(D|p) \propto \exp -\frac{1}{2}(\mathbf{D} - \mathbf{M}(p))^t \mathbf{C}^{-1}(\mathbf{D} - \mathbf{M}(p)), \quad (2.48)$$

where $\mathbf{M}(p)$ is a vector generated by a theoretical model with parameters p , and can be built in either harmonic or configuration space, depending on the choice of the data vector. Each choice, though extracted from the same set of observations, will resume differently the total available data.

The posterior distribution can be determined through sampling in the parameter space. Individual combinations of cosmological parameters are tested, and their likelihood is computed, along with their posterior probability. If there is an adequate criteria for the choices of parameters to be tested, the result will be a full map of the posterior probability density in parameter space. The use of Markov Chain Monte Carlo (MCMC) algorithms yields us such optimal criteria, and is thus ubiquitously used for cosmological parameter estimation.

When the parameter space undergoes this kind of mapping, it is possible to detect degeneracies between parameters. In these situations, a combination of two parameters is

constrained, but their individual values can't be resolved by the data alone. When there is little degeneracy, however, a well constrained region will appear favoring a specific combination of values for the two parameters. Independent constraints for a single parameter are found by marginalizing the posterior distribution over the set of other parameters.

The level of complexity for theoretical modeling and determination of the covariance matrix is increased by the presence of observational systematics on the data. A long phase of pre-analysis is, therefore, required by modern-day redshift surveys. During this step, systematics are modeled and incorporated in the theoretical predictions, and an analysis strategy is constructed and tested with simulated data. When it is determined that the strategy is robust, survey data analysis takes place. In this context, simulations play a role of utmost importance. Following the production of catalogs which base themselves on the current theoretical modeling, we can run parameter estimation codes and recover the parameter values used in the simulations. Success in this stage acts as a necessary part of method validation.

The procedure for data analysis of redshift surveys, therefore, is an intricate interplay between basic physical modeling, systematics modeling, simulation building and analysis, observational data gathering, and consolidation of final analysis methodology. For this reason, modern surveys aggregate a large collaboration of scientists which have to deal with some uniformity of methods. Problems arise, among other situations, when method calibration and robustness tests overlap with some knowledge of the final estimated cosmological parameters. In this situation, the previous knowledge of the researcher can in some way influence analysis choices, which is far from desirable. Science has always been prone to human bias, but that intricate structure of survey data analysis makes it paramount to design specific strategies to shield ourselves from such bias.

As explained by [Muir, Bernstein, Huterer, et al. \(2020\)](#), DES has been using a procedure called blinding, which hinders the access of the researcher to results from data until the analysis pipeline is complete. Blinding in analysis of DES Year 1 (DES-Y1) has been made at the level of the cosmological parameters, so only the final estimated posterior distributions are omitted in this stage. It is argued, however, that the collaboration should perform blinding on the level of the 2-point functions, because the current method is prone to accidental or intentional uncovering of the actual information. ([MUIR; BERNSTEIN; HUTERER, et al., 2020](#))

Once equipped with theoretical functions, data and an analysis methodology, we can start probing cosmology from survey data. There is a large arsenal of available probes for cosmology, as discussed in 1.5. With galaxy clustering, galaxy-galaxy lensing and cosmic shear angular power spectra or angular correlation functions, a multi-probe approach becomes possible. The analysis which uses the particular combination of those three functions has become known as the 3x2pt analysis, and has been a fruitful source of cosmological information. In the next chapter, we will review the general use of multi-probe approaches in cosmology and present the 3x2pt methodology devised for DES-Y1 data.

MULTI-PROBE ANALYSIS AND THE 3X2PT STRATEGY

3.1 COMBINED PROBES

The diversity of existing cosmological probes opens several investigative paths for obtaining constraints on cosmological parameters. The fact that each probe is sensitive to different epochs of the universe’s history or to different physical phenomena adds an aspect of complementarity to survey science. Thus, it has been a common strategy to combine two or more cosmological probes in a single analysis, in what may be called a multi-probe analysis. During the last two decades, this method evolved from a simple combination of final results to a comprehensive process, in which systematic effects and correlations between probes are treated in an integrated pipeline. Today, this kind of comprehensive analysis is mainly done in the context of DES.

To use a combination of uncorrelated probes can be somewhat trivial. Each probe yields an individual likelihood function for the cosmological parameters in consideration. The likelihoods can thus be combined to obtain tighter constraints on the parameters and to break possible degeneracies between them. In this case, tensions can also appear, when results from each probe appear to point toward different directions. It is also possible to test compatibility between results from different surveys, each with different methodologies, just by comparing the final likelihoods. This type of analysis happens on the level of the cosmological results, since the data has already been treated individually for each survey and probe. In the case of probes which point to the same underlying field, it becomes necessary to perform a joint analysis at the level of the data, before it is translated into cosmological constraints. (KRAUSE; EIFLER, et al., 2017)

There is also a sense in which the use of multiple probes and surveys can highlight eventual problems with the theoretical framework. When results from distinct sources are systematically inconsistent, it becomes necessary to assess whether the source of the inconsistency is a systematic effect of one specific probe or survey, or if it’s a more fundamental

process.

In this chapter we will overview some of the studies that have been done with combined probes during the last two decades, and how it culminated on the DES 3x2pt methodology.

3.2 COSMIC SHEAR AND CMB

The use of cosmic shear in survey cosmology was inaugurated in 2000 with the first reported detections of the phenomenon. During the following years, it grew to become one of the major cosmological probes. Before they became ubiquitous, however, cosmic shear studies had to deal with several challenges. The first was the fact that the effect is a weak one, and therefore requires precise survey data in order to be detected. (JOACHIMI, BENJAMIN, 2010) This explains the delay between the establishment of cosmic shear theoretical foundations and its first detections. The second challenge was that of the intrinsic alignment of galaxies. Shear is measured by correlating ellipticities of galaxies, and so any residual ellipticity trend that doesn't come from lensing will appear as a systematic error on the cosmic shear correlation function.

One of the features noticed early in cosmic shear analysis was a degeneracy between σ_8 and Ω_m . While it was theoretically proposed that the consideration of non-linear scales would break this degeneracy (JAIN; SELJAK, 1997), it wasn't as straightforward to solve the practical problem. In Van Waerbeke, Mellier, Pelló, et al. (2002), the authors comment that there is also a degeneracy between σ_8 and the redshift of the galaxies for which shape distortion is measured, and that it introduces even more ambiguity on the determination of σ_8 . The combination of probes, however, proved itself useful in this situation. The authors found that their constraints in the $\Omega_m - \sigma_8$ parameter space, which used cosmic shear data from the VIRMOS-DESCART survey, were almost orthogonal to the CMB constraints by Lahav, Bridle, Percival, et al. (2002). Therefore, the combination of cosmic shear and CMB results could limit the range of both Ω_m and σ_8 constraints and break their degeneracy.

In a cosmic shear analysis of RCS data, Hoekstra, Yee, and Gladders (2002) also found degenerate results for σ_8 and Ω_m , given by $\sigma_8 = 0.45^{+0.09}_{-0.12} \Omega_m^{-0.50}$. The authors then made use of priors from previous surveys to improve their results, and obtained $\sigma_8 = 0.46^{+0.05}_{-0.07} \Omega_m^{-0.52}$. RCS cosmic shear data was finally combined with CMB data from

WMAP and some other sources in Contaldi, Hoekstra, and Lewis (2003), which arrived at a $\Omega_m = 0.30 \pm 0.03$ constrain.

Modern day results from CMB and cosmic shear have culminated in a tension over the value of σ_8 . Di Valentino and Bridle (2018) review this tension exploring the KiDS and CFHTLenS results, which give systematically lower values for σ_8 than those found by the Planck mission, in a discrepancy of more than 2σ . A current proposed solution for this tension is the introduction of a non-physical parameter A_{lens} in CMB analysis.

3.3 WEAK LENSING AND GALAXY CLUSTERING

While cosmic shear can be combined with a variety of probes, some of the combinations will be largely uncorrelated, because they point to distinct fields. The particular combination with galaxy clustering, however, is somewhat more intriguing. Both cosmic shear and galaxy clustering trace the large scale distribution of matter in the universe. Nonetheless, galaxy clustering can't map directly the presence of dark matter, this difference being reflected by the presence of the galaxy bias in clustering correlation functions. Because of the relation between these probes, large cross-correlations arise, and so does the need for an integrated analysis.

There are many advantages in combining measurements of weak lensing with those of galaxy clustering over using only lensing information. Beyond the sole improvement of constraining power, several systematics affect differently both probes, and thus their effect can be traced by the probe combination. While the galaxy bias is introduced in the galaxy clustering power spectrum, intrinsic alignments need to be dealt with while measuring shear from statistics of galaxy shapes.

In the context of large-scale structure probes, correlations between the weak lensing signal and galaxy positions emerge as an alternative to shear or clustering auto-correlations. Described by Eq. 2.46, those correlations were detected even before the first measurements of cosmic shear. In 1996, Brainerd, Blandford, and Smail (1996) reported the detection of the galaxy-galaxy lensing signal, and used it to probe the halo mass around galaxies. The combination of galaxy-galaxy lensing with galaxy clustering was explored as a probe of dark energy by Hu and Jain (2004). They argued that the constraining power of this combination would be similar to that of cosmic shear by itself. The three functions could also be combined, and according to the authors, yield smaller

errors then cosmic shear alone by a factor of $\sqrt{2}$.

Galaxy-galaxy lensing and galaxy clustering were put together in a data analysis paper for SDSS Data Release 7 (DR7) by [Mandelbaum, Slosar, Baldauf, et al. \(2013\)](#). The authors used a non-linear model for the galaxy bias, which had two free parameters: a linear bias b and a quadratic bias b_2 . A covariance matrix was made for the galaxy clustering data, and another for the galaxy-galaxy lensing part, both through the jackknife method. When they computed the covariance matrix of the full data, they found that the terms correlating distinct probes had values compatible with zero. Therefore, they didn't attempt to perform an accurate modeling of those cross-correlations. They attributed this effect, nevertheless, to a dominance of shape noise on the uncertainties, and introduced as a caveat that their simplification would not apply to following surveys.

The results from [Mandelbaum, Slosar, Baldauf, et al. \(2013\)](#) are marked by the same $\sigma_8 - \Omega_m$ degeneracy of previous cosmic shear results. However, when marginalizing over σ_8 , the authors could obtain a reasonable constrain on the density of matter. The authors explore how this constrain is affected by the introduction of CMB data. The combined galaxy clustering and galaxy-galaxy lensing analysis found $\Omega_m = 0.269^{+0.038}_{-0.034}$. The introduction of CMB data from WMAP lowered the uncertainties by more than 50%, due to the degeneracy breaking, which restricted the allowed σ_8 values. Therefore, the authors could determine as a joint result $\Omega_m = 0.261 \pm 0.014$.

3.4 THE 3X2PT STRATEGY

While different attempts for combining lensing and galaxy clustering have been made over the last two decades, the most successful type of analysis has been that which uses the three 2-point correlation functions, as suggested by [Hu and Jain \(2004\)](#). In the context of modern-day surveys, the *3x2pt analysis* label is now consolidated, and combinations that include even more probes have been analogously labeled. Such is the case of the 5x2pt and the 6x2pt analyses, which add CMB data to the standard combination. ([DES COLLABORATION, 2019b](#))

One of the first benchmarks of the 3x2pt method development was placed by [Bernstein \(2009\)](#). From a small set of assumptions, without selecting any dark energy model, and using a generalized formalism to deal with systematics, he obtained theoretical covariances for $C_\ell^{\delta\delta}$, $C_\ell^{\kappa\kappa}$, and $C_\ell^{\delta\kappa}$. From those covariances, predictions were made for the

constraining power of generic 3x2pt surveys with the Fisher Matrix formalism. Because of the low modeling specificity, the forecasts developed by Bernstein have a broad applicability. Nevertheless, the large amount of nuisance parameters appears as an undesired consequence. (BERNSTEIN, 2009)

Following closely the methodology developed by Bernstein, predictions for the constraining strength of a DES-like survey were made by Joachimi and Bridle (2010). They found, for a sole galaxy clustering analysis, an expected uncertainty for Ω_m of 0.3620, which means virtually no information on the parameter. This happens because of the lack of prior knowledge on the galaxy bias. The constraining power grew for a sole cosmic shear analysis, for which the uncertainty on Ω_m was found to be of 0.0636. Finally, when using the full 3x2pt data, this value decreased to 0.0233.

The 3x2pt analysis method started to take place with actual data in the context of DES, in which it was incorporated as an essential part of survey methodology. By illustrating the sparsity of previous 3x2pt results, Krause, Eifler, et al. (2017) outlined how DES would offer an original contribution to multi-probe analysis. After mentioning the forecasts by Bernstein (2009) and Joachimi and Bridle (2010), the authors report some previous results on galaxy clustering and weak lensing, culminating on a 3x2pt result which combined KiDS cosmic shear data with galaxy clustering information from the Galaxies and Mass Assembly (GAMA) survey. (UITERT; JOACHIMI; JOUDAKI, et al., 2018) Nonetheless, they claim to be the first to compute and validate through simulations a configuration space analytical covariance matrix for the joint 3x2pt functions and include in it non-Gaussian terms. (KRAUSE; EIFLER, et al., 2017)

After releasing a full 3x2pt analysis of Y1 data, the DES collaboration took the multi-probe approach one step forward, publishing a joint analysis with CMB data from Planck and from the South Pole Telescope Sunyaev-Zel'dovich survey (SPT-SZ). (DES COLLABORATION, 2019b) A full multi-probe result with DES-only data was also made by combining information from the 3x2pt probes, from BAO, and from Type Ia Supernovae. (DES COLLABORATION, 2019a) In this analysis, the authors relaxed one at a time two of the Λ CDM assumptions, testing both an open universe model (relaxing the zero-curvature assumption), and a w CDM model (relaxing the assumption of dark energy as a cosmological constant). Though a lower than usual value was found for Ω_Λ in the open universe model, the constraints were significant, given that more flexibility was

Probes	Model	Ω_m
Cosmic Shear	Λ CDM	$0.260^{+0.065}_{-0.037}$
3x2pt	Λ CDM	$0.267^{+0.030}_{-0.017}$
3x2pt and CMB (external data)	Λ CDM	$0.260^{+0.029}_{-0.019}$
3x2pt, BAO, and Type Ia Supernovae	Open universe	$0.299^{+0.024}_{-0.020}$
3x2pt, BAO, and Type Ia Supernovae	w CDM	$0.300^{+0.023}_{-0.021}$

Table 3.1 – DES constraints on Ω_m from multiple probes.

allowed in parameter fitting. (DES COLLABORATION, 2019a)

The modification on constraining power by combining multiple probes beyond 3x2pt functions can be verified for the case of Ω_m . Table 3.1 shows the DES constraints when different data or models were used. While adding the CMB component didn't increase the constraining power, the joint analysis with BAO and Type Ia Supernovae yielded lower uncertainties. We include in our comparison the DES-Y1 sole cosmic shear constraints, which are introduced in 4.1.3 and carry large uncertainties.

3.5 DES-Y1 METHODOLOGY

The fundamental blocks for DES-Y1 3x2pt methodology are laid by Krause, Eifler, et al. (2017). The analysis is proposed to be done in configuration space, in a way that both theoretical modeling and data measurements are expressed by angular correlation functions. The authors describe the galaxy catalogs to be used, the sources of systematic error which need particular modeling, and some analysis choices. Here we outline the main features which need to be taken into consideration in order to undertake any DES-Y1 data analysis. In the next chapter, we will present the collaboration's 3x2pt key paper results, as well as our use of DES data for an independent harmonic space analysis.

3.5.1 Galaxy catalogs and 2-point functions

The DES 3x2pt analysis is made with two distinct sets of galaxies: one catalog of lens galaxies, which are used for galaxy clustering correlations, and a second catalog of source galaxies for analysis of shape distortion. This creates a separation between lower and higher-redshift galaxies, though with some overlap. While galaxy clustering probes the galaxy distribution of the lower redshift region, cosmic shear probes the integrated matter distribution up to the redshift of the source galaxies, including the full lower-redshift region. The lens galaxy catalog is separated in 5 redshift bins, while the source

catalog has 4 bins.

[Krause, Eifler, et al. \(2017\)](#) explains that the galaxy-galaxy lensing cross-correlations can include all combinations between the 5 lens bins and the 4 source bins, regardless of whether the source bin is actually at a higher redshift than the lens bin. Though some of those combinations are uncorrelated, they nonetheless can be used for calibration purposes. The authors mention particularly self-calibration of systematics, which is the joint constraining of cosmological and nuisance parameters, as described by [Huterer et al. \(2006\)](#).

Our galaxy-galaxy lensing angular power spectrum expression from Chapter 2 can be generalized for this cross-correlation of bins as follows. For correlations between lens galaxies with redshift distribution $n_i(z)$ and source galaxies with distribution $n_j(z)$, assuming a constant galaxy bias b_i for each lens bin, we have:

$$C_\ell^{(\kappa\delta)ij} = b_i \int_0^\infty \frac{n_i(z(\chi))(dz/d\chi)G(z(\chi))q_j(\chi)}{\chi^2} P\left(\frac{l+1/2}{\chi}\right) d\chi, \quad (3.1)$$

with $q_j(z)$ determined by $n_j(z)$ through Eq. 2.40.

Cosmic shear and galaxy clustering spectra can also cross-correlate information from different redshift bins. The spectra become:

$$C_\ell^{(\kappa\kappa)ij} = \int_0^\infty \frac{q_i(\chi)q_j(\chi)}{\chi^2} P\left(\frac{l+1/2}{\chi}\right) d\chi \quad (3.2)$$

and

$$C_\ell^{(\delta\delta)ij} = b_i b_j \int_0^\infty n_i(z(\chi))n_j(z(\chi)) \frac{((dz/d\chi)G(z(\chi)))^2}{\chi^2} P\left(\frac{l+1/2}{\chi}\right) d\chi. \quad (3.3)$$

The configuration space 2-point functions are computed by [Krause, Eifler, et al. \(2017\)](#) in terms of the C_ℓ spectra, and consist, for galaxy clustering, on a sum over ℓ indices. For the cases of galaxy-galaxy lensing and cosmic shear, the authors use the flat-sky approximation, and model the angular correlation functions as integrals over ℓ , which require an interpolation of the angular power spectra as to make them defined for non-integer ℓ values.

3.5.2 Catalog design

The catalog building process consists on taking the complete set of DES galaxies and selecting the ones most appropriate for the desired analysis, assigning a photo-z value for each. This full initial catalog, presented by [Drlica-Wagner, Sevilla-Noarbe, Rykoff, et al. \(2018\)](#), is a component of the survey’s Y1 GOLD data set. We describe now the process of sub-selection for clustering and lensing.

3.5.2.1 Creation of the lens catalog

One of the core requirements for galaxy clustering analysis is high precision on redshift determination. Because of that, as described by [Elvin-Poole, Crocce, Ross, et al. \(2018\)](#), the collaboration chose for lens catalog creation a method which yields a small yet informative galaxy set. By accepting into the catalog only Luminous Red Galaxies (LRGs), the redMaGiC algorithm proved itself the optimal choice, guaranteeing the desired precision for the photo-zs. The final size of the redMaGiC-selected lens catalog was of 6.6×10^5 galaxies, a substantially reduced portion of the total 1.37×10^8 objects present on the Y1 GOLD catalog. ([ELVIN-POOLE; CROCCE; ROSS, et al., 2018](#)) ([DRLICA-WAGNER; SEVILLA-NOARBE; RYKOFF, et al., 2018](#))

3.5.2.1.1 *redMaPPer*

The redMaGiC algorithm assigns photo-z values by template matching, and for that builds upon a previous code, known as the red-sequence Matched-filter Probabilistic Percolator (redMaPPer). ([ROZO; RYKOFF; ABATE, et al., 2016](#)) Constructed to be a finder of red sequence galaxy clusters, redMaPPer can start with an initial spectroscopic redshift estimate of a central galaxy within a cluster and arrive iteratively at a photometric redshift value for the cluster. ([RYKOFF; ROZO; BUSH, et al., 2014](#)) This provides a calibration for red sequence galaxy templates, which then serve as basis for matching in redMaGiC.

3.5.2.1.2 *redMaGiC*

Described by [Rozo, Rykoff, Abate, et al. \(2016\)](#), redMaGiC fits the input photometric data to templates drawn from redMaPPer. The algorithm requires a choice of a threshold χ^2 value and a threshold luminosity. Both values serve as cuts for the final

galaxy set, in a way that only galaxies more luminous and with a lower χ^2 than the correspondent thresholds find their way into the LRG selection. Because only red sequence templates are used, galaxies which are not LRGs fail the criteria by having high χ^2 values. (ROZO; RYKOFF; ABATE, et al., 2016) The input spectroscopic redshift for redMaPPer is given by a combination of SDSS and OzDES data. (ELVIN-POOLE; CROCCE; ROSS, et al., 2018)

3.5.2.1.3 Choice of lens redshift bins

After obtaining an LRG sample from redMaGiC, the collaboration split redshift space into five equally sized regions from $z_{min} = 0.15$ to $z_{max} = 0.9$. Each galaxy was then assigned to its proper redshift bin. The luminosity threshold was chosen to be different for each bin, and an additional maximum luminosity limit was added. These choices are detailed by Elvin-Poole, Crocce, Ross, et al. (2018).

3.5.2.2 Creation of the source catalog

For the source galaxy catalog, a less aggressive selection was made, favoring a large number of galaxies over precise photo-z values. The photo-z estimation was made through template fitting, though a different algorithm was used to allow for that broader selection. The largest cut in sample size was, therefore, made before photo-z estimation, when subsets of the Y1 GOLD data were used to form the Metacalibration and the IM3SHAPE catalogs, each with their own method for shape measuring. (ZUNTZ; SHELDON; SAMUROFF, et al., 2018)

In Hoyle, Gruen, Bernstein, et al. (2018), the authors detail the photo-z estimation process, which made use of a modified version of the Bayesian Photometric Redshift (BPZ) code, first proposed by Benítez (2000). The complete source galaxy catalog comprises 2.6×10^7 galaxies from the Metacalibration set and 1.8×10^7 from the IM3SHAPE set. All DES-Y1 lensing analyses state clearly from which of the two subsets it retrieves input data. For both Metacalibration and IM3SHAPE catalogs, the same binning was performed. The chosen boundaries for the bins lie at $z = [0.20, 0.43, 0.63, 0.90, 1.30]$. (HOYLE; GRUEN; BERNSTEIN, et al., 2018)

3.5.3 Model parameters

A robust determination of cosmological parameters imply that all relevant effects are taken into consideration. In most cosmological analyses, theoretical modeling and sources of systematic error will increase the amount of free parameters of the model. Those new parameters, however, may not be linked directly to the cosmological processes which we may want to probe. Therefore, they act as nuisance parameters in the analysis, and their final effect on the posterior is found by marginalizing over its range of values. DES-Y1 analysis uses a total of 26 parameters for the Λ CDM model and 27 for the w CDM model, which includes a multiplicative factor of $a^{-3(1+w)}$ for the Ω_Λ parameter in Eq. 1.18 to account for dark energy as more than a cosmological constant.

Of that total number, 20 are nuisance parameters. As reported by Krause, Eifler, et al. (2017), modeling of systematics yielded 5 galaxy biases, 9 photometric redshift shift parameters, 4 multiplicative shear calibration parameters, and 2 intrinsic alignment parameters. The biases b_i are assumed to be linear, which means that possible scale dependencies are neglected. The shift parameters are a way to model the uncertainty on the determination of galaxy redshifts. For each lens and source galaxy redshift bin, there is one Δz parameter. The effect of the intrinsic alignment of galaxies is measured by two nuisance parameters, A_{IA} and α_{IA} . Finally, the four shear calibration parameters m_i refer to each source redshift bin. The remainder of the cosmological parameters is composed of Ω_m , h , Ω_b , A_s , n_s , and $\Omega_\nu h^2$, with a possible addition of w .

3.5.4 Considerations on RSD and the Limber approximation

A complete theoretical modeling of the angular power spectrum requires the full integration of Eq. 2.24 and an additional RSD term on Eq. 2.25. The same need is maintained when analysis is performed with the angular correlation function. Krause, Eifler, et al. (2017), however, argues that those effects are irrelevant for DES-Y1 data. While analysis from the following years of the survey will require modeling of RSD and non-limber features of the spectra, those improvements wouldn't have any extensive effect on Y1 results.

3.5.5 Modeling of the covariance matrix

One of the greatest hardships towards a robust 3x2pt analysis is the computation of the joint covariance matrix, which involves detecting the correlation between different probes. [Krause, Eifler, et al. \(2017\)](#) describes the collaboration’s choice to use an analytical model for the covariance, which can be computed by the CosmoLike code. ([KRAUSE; EIFLER, 2017](#)) In analytical models, the covariance matrix becomes model-dependant, introducing a certain circularity on the process. It is unfeasible to recompute the covariance matrix at each step of an MCMC process, and so the selected cosmology for the covariance matrix can, at least conceptually, be imagined to bias the final results. ([EIFLER; SCHNEIDER; HARTLAP, 2009](#))

Though in practice the effect is weak, DES opted for a two-time computation of the covariance matrix. A first calculation is made assuming a certain cosmology. With this covariance, parameter fitting is performed, and the best values found for the parameters act as input to the second covariance matrix, which is actually used in analysis. ([KRAUSE; EIFLER, et al., 2017](#)) The analytical covariance is also validated by comparison with a simulation-based covariance, and a caveat is made on possible effects of the model-dependence of analytical covariances for the following years of DES data.

3.5.6 The CosmoSIS pipeline

The CosmoSIS framework is a modular code which integrates several parameter estimation techniques and cosmological calculations needed for the theoretical models. Its structure allows the construction of comprehensive pipelines, which do from the fetching of CAMB-generated power spectra to the building of angular functions, and which perform parameter estimation from input data vector and priors. CosmoSIS has a flexible nature, allowing new pieces of code to be written in different programming languages and easily integrated into the pipeline. ([ZUNTZ; PATERNO; JENNINGS, et al., 2015](#))

Two parameter estimation methods are tested by [Krause, Eifler, et al. \(2017\)](#) within the CosmoSIS framework: the emcee sampler ([FOREMAN-MACKEY et al., 2013](#)), and the MultiNest nested sampling algorithm ([FEROZ; HOBSON; BRIDGES, 2009a](#)). The authors didn’t find any significant effect on the final uncertainties when changing from one algorithm to the other, as long as adequate configurations were used.

3.5.7 Updates for DES Y3

The DES Year 3 (Y3) methodology for 3x2pt analysis builds upon that of Y1, but incorporates new modeling of effects which were negligible on the context of the latter. [Krause, Fang, Pandey, et al. \(2021\)](#) detail the improvements, which start with a non-Limber modeling of the 2-point functions and an inclusion of RSD physics. The authors also highlight which of the systematics are yet unmodeled, and then deal with them through a damage control approach. Both nonlinear galaxy bias and baryonic effects are considered important sources of possible error. A preliminary modeling of such systematics is performed and used to contaminate a set of simulations, yielding a biased parameter inference. From this test, optimal scale cuts are chosen as to minimize the difference of these results to those found through uncontaminated simulations.

GALAXY CLUSTERING IN HARMONIC SPACE WITH DES-Y1 DATA

4.1 OVERVIEW OF DES-Y1 RESULTS

The main analysis of DES-Y1 data was published by the collaboration in 2018. (DES COLLABORATION, 2018) It combines data on galaxy clustering, galaxy-galaxy lensing and cosmic shear took from a sky patch of 1321 deg^2 , and produces tight constraints on cosmological parameters using configuration space data vectors. Along with the 3x2pt analysis paper, the collaboration published individual results papers for each of the three probes. The set of three papers on galaxy clustering, galaxy-galaxy lensing and cosmic shear is entangled with the main paper, in a sense that while they inform some of the choices done for the full 3x2pt analysis, they also use in their final results the cosmology obtained by the full analysis. The methodology used in the collection of papers is that described in 3.5.

4.1.1 Galaxy clustering for 3x2pt analysis

In the galaxy clustering paper (ELVIN-POOLE; CROCCE; ROSS, et al., 2018), the authors model the angular correlation function for the sample's five redshift bins, with a linear galaxy bias parameter for each bin. Only auto-correlations (e. g. correlations between data from the same redshift bin) are used. The uncertainties in the photometric redshift estimates are also addressed, by the insertion of one Δz_i shift parameter per redshift bin. The modeled redshift distribution function $n(z)'_i$ becomes, in terms of the original $n(z)_i$:

$$n(z)'_i = n(z - \Delta z_i)_i. \quad (4.1)$$

As part of systematics modeling, the authors make use of a set of 1000 full-sky simulations, which are cut to match the sky region covered by DES-Y1. In order to create

Redshift bin		Galaxy bias
i	z_i range	b_i
1	0.15 to 0.30	1.40 ± 0.07
2	0.3 to 0.45	1.60 ± 0.05
3	0.45 to 0.60	1.60 ± 0.04
4	0.6 to 0.75	1.93 ± 0.04
5	0.75 to 0.90	1.98 ± 0.07

Table 4.1 – Constraints on the galaxy bias from the DES-Y1 configuration space analysis performed by [Elvin-Poole, Crocce, Ross, et al. \(2018\)](#)

the simulations, theoretical input angular power spectra were constructed, using the cosmological parameters inferred from the full 3x2pt analysis along with linear galaxy biases, with values given, for the five bins in consecutive order, by $b_i = [1.45, 1.55, 1.65, 1.80, 2.00]$. ([ELVIN-POOLE; CROCCE; ROSS, et al., 2018](#)) Those simulations are used to construct a data-based covariance matrix.

From the set of simulations and the covariance matrix, the analysis follows by performing contamination of the simulations with observational systematics, such as airmass and sky brightness. The effect of these systematics on the angular correlation function determines a set of weights to be applied to the actual DES-Y1 galaxies. This process is designed to yield a more accurate determination of the correlation function by assigning less relevance to the most affected galaxies.

Finally, the configuration space analysis used Y1 data to find the value of the galaxy biases. Because of a degeneracy between the biases and σ_8 , the fitted parameter was the combination $b_i\sigma_8$, from which the biases were extracted by assigning to σ_8 the value from the mean of its posterior distribution from the full 3x2pt analysis. ([ELVIN-POOLE; CROCCE; ROSS, et al., 2018](#)) The results found by the analysis team are presented in Table 4.1.

4.1.2 Galaxy-galaxy lensing for 3x2pt analysis

Modeling of the galaxy-galaxy lensing angular correlation functions was made by [Prat, Sánchez, Fang, et al. \(2018\)](#) for the 20 combinations between source and lens bins. The galaxy biases were included together with cross-correlation parameters labeled r_i , in a way that b_i in Eq. 3.1 was replaced by the $b_i r_i$ combination. This model allows for deviations to the expected scenario, for which $r_i = 1$ and the biases inferred from clustering match those inferred from galaxy-galaxy lensing.

Figure 4.1 – Redshift distribution functions for the DES-Y1 lens and source galaxy catalogs. Figure reproduced from [Prat, Sánchez, Fang, et al. \(2018\)](#)

[Prat, Sánchez, Fang, et al. \(2018\)](#) combine their measured functions with those of [Elvin-Poole, Crocce, Ross, et al. \(2018\)](#) to derive constraints on the cross-correlation parameters. As their goal was to validate the $r_i = 1$ assumption used in the full 3x2pt analysis, a methodological circularity would have emerged had they presupposed the 3x2pt output cosmology during the fitting of r_i . Aware of this risk, the authors perform validation three times, each assuming a distinct cosmology: First, they use the cosmological parameter values found by [Troxel, MacCrann, Zuntz, et al. \(2018\)](#) with cosmic shear data, which are completely independent of the data being analyzed. Second, they take the parameters obtained from a combination of cosmic shear and galaxy clustering. At last, they take the values found by the full 3x2pt analysis, regardless of the circularity. All approaches yielded values of r_i compatible with 1 for the five lens galaxy redshift bins. ([PRAT; SÁNCHEZ; FANG, et al., 2018](#))

Since galaxy-galaxy lensing analysis requires a consideration on the redshift distribution of both lens and source galaxy catalogs, [Prat, Sánchez, Fang, et al. \(2018\)](#) display the $n_i(z)$ functions for the redMaGiC lens catalog and for the Metacalibration and IM3SHAPE source catalogs. Figure 4.1, which is reproduced from the paper, shows the bin regions along with the redshift distributions. A total redshift distribution for each catalog is represented by the black curve.

4.1.3 Cosmic shear for 3x2pt analysis

Because of the absence of the galaxy bias, cosmic shear data can be used alone to obtain cosmological constraints. Troxel, MacCrann, Zuntz, et al. (2018) model the cosmic shear angular correlation functions for DES-Y1 which were used in the 3x2pt analysis. They consider both auto-correlations and cross-correlations between source galaxy redshift bins. From these functions, and with a careful modeling of the nuisance parameters, the authors find constraints on Ω_m and σ_8 . Because of the classic degeneracy between those two parameters, Troxel, MacCrann, Zuntz, et al. (2018) use along with σ_8 the better constrained combination $S_8 = \sigma_8(\Omega_m/0.3)^{0.5}$.

Cosmic shear analysis made use of the Metacalibration catalog as their main data set. Nonetheless analyses were also performed with the IM3SHAPE catalog. The authors measure constraining power of each analysis by the Figure of Merit (FoM) metric, defined for parameters p_1 and p_2 as:

$$\text{FoM} = \frac{1}{\sqrt{\det \text{Cov}(p_1, p_2)}}. \quad (4.2)$$

For the Ω_m and S_8 parameters, the Metacalibration catalog yielded constraints with $\text{FoM} = 626$, as opposed to the poorer IM3SHAPE constraints, that were found to have $\text{FoM} = 279$. The cosmic shear result from Table 3.1 is that made with the Metacalibration catalog.

4.1.4 Complete 3x2pt analysis results

The complete 3x2pt configuration space analysis of DES-Y1 data yielded constraints on Ω_m and S_8 for the Λ CDM model, as well as constraints on those same parameters and on w for the w CDM model. The so-called 3x2pt key paper (DES COLLABORATION, 2018) discuss these results and place them in the context of constraints by other probes, to the point of combining them, on the level of the likelihoods, with results from previous surveys.

For the Λ CDM model, the collaboration found $\Omega_m = 0.267^{+0.030}_{-0.017}$, while for the w CDM model, they found $\Omega_m = 0.284^{+0.033}_{-0.030}$ and $w = -0.82^{+0.21}_{-0.20}$. Since this w value is compatible with -1, no detection occurred of deviations from Λ CDM.

4.2 CONFIGURATION AND HARMONIC SPACE IN THE CONTEXT OF DES

Correlation functions can be built in both configuration and harmonic space. Mathematically, both options can be seen as interchangeable, in that they consist on different manners to resume the same information. When dealing with survey data, however, the exact equivalence easily disappears. The angular correlation function $w(\theta)$ involves a sum over the ℓ multipoles of the angular power spectrum, ranging from $\ell = 1$ to infinity. With actual data this sum is, of course, truncated at a specific ℓ_{max} , which leads to some loss of information.

The opposite is also true: the angular power spectrum is written, for each multipole, as an integral over the k modes, which relate to the angular scale θ . Scales sufficiently small are discarded in data analysis, removing information from the C_ℓ function. This scale cut process is usually aggressive to ensure the validity of the theoretical modeling on all scales considered. Therefore, the difference in information can be somewhat relevant.

Another source of divergence between both statistics can surge during the measurement process. To measure the angular power spectrum from survey sky maps require an estimation method that takes into account the effect of the survey footprint (e. g. region of the sky for which data was collected). This process is overpassed when the final statistic to be measured is the angular correlation function. Additionally, as pointed out by [Doux, Chang, Jain, et al. \(2021\)](#), systematics can affect configuration and harmonic space functions differently, further increasing the possibility of divergent constraints from the two methods.

It has been a convenient practice to choose only one of the available set of functions to perform analysis. Following this trend, the main DES-Y1 results were produced solely in configuration space. This choice betrays a presupposition that the collaboration's methods are so robust as to grant indistinguishable constraints as those that could be found in harmonic space. Nevertheless, results from alternative ongoing surveys raise alert to the weakness of this assumption.

[Doux, Chang, Jain, et al. \(2021\)](#) mention two of those discrepancies recently reported in literature: one with KiDS data, and another in the context of HSC. The divergence in KiDS results is found by comparing the cosmic shear analyses of [Hildebrandt, Viola, Heymans, et al. \(2016\)](#) and [Köhlinger, Viola, Joachimi, et al. \(2017\)](#). Both made

Analysis	Survey	S_8
CS (Hildebrandt, Viola, Heymans, et al. (2016))	KiDS-450	0.745 ± 0.039
HS (Köhlinger, Viola, Joachimi, et al. (2017))	KiDS-450	0.651 ± 0.058
CS (Hamana, Shirasaki, Miyazaki, et al. (2020))	HSC	$0.804^{+0.032}_{-0.029}$
HS (Hikage, Oguri, Hamana, et al. (2019))	HSC	$0.780^{+0.030}_{-0.033}$
CS (Asgari, Lin, Joachimi, et al. (2021))	KiDS-1000	$0.764^{+0.018}_{-0.017}$
HS (Asgari, Lin, Joachimi, et al. (2021))	KiDS-1000	$0.760^{+0.016}_{-0.038}$

Table 4.2 – Cosmic shear constraints on S_8 from configuration (CS) and harmonic space (HS) analyses of KiDS and HSC data. The Köhlinger, Viola, Joachimi, et al. (2017) constraint refers to their three-z-bins result.

use of the same data set, that was retrieved from KiDS observations of a 450 deg^2 sky patch. With independent analysis methods, they found constraints on the S_8 combination, which are shown in Table 4.2. While their main concern was the tension between their results and those from CMB, the results from both analyses have also a slight tension when compared to each other.

This same slight tension appears when comparing the HSC cosmic shear harmonic space constraints by Hikage, Oguri, Hamana, et al. (2019) with its configuration space counterpart constraints by Hamana, Shirasaki, Miyazaki, et al. (2020), both shown in Table 4.2. Though of the order of 1σ , the difference is enough to motivate an investigation by the authors of the latter. They find through tests with simulations that the HSC analyses are weakly correlated, which indicates that the angular power spectrum and the angular correlation function are carrying different information. (HAMANA; SHIRASAKI; MIYAZAKI, et al., 2020)

In light of these discrepancies, a cosmic shear analysis of the 1000 deg^2 from KiDS was undertaken in both configuration and harmonic space by Asgari, Lin, Joachimi, et al. (2021). Their results, which we also present in Table 4.2, are fully compatible with one another.

The possibility of tension between configuration and harmonic space constraints poses a new pathway for DES, in that it incites the collaboration to perform the comparison using their own data. This could be accomplished by a parallel harmonic space analysis that adapts its methods from those already developed for configuration space. If compatibility between configuration and harmonic space constraints is demonstrated, much can still be learned during the process of matching the analysis choices. Also, such comparison adds robustness to the survey’s results. If any tension is detected, exploration

of the reasons for the tension can yield insight on potential methodological improvements, or even on unconsidered systematic effects.

The fact that DES has invested much of its effort on multi-probe analysis further affirms the necessity for this complementary approach. The same reasoning applied to justify combination of probes also applies to combination of different 2-point statistics. While 3x2pt analysis makes the most of the information available through individual probes, a combined configuration and harmonic space study would also increase the total information used. The existence of redundant information in the 3x2pt functions, such as the galaxy bias, allows for internal consistency checks. Analogously, the angular power spectra and the angular correlation functions are both reductions of the same data, and share as much information as to require some level of compatibility.

4.3 HARMONIC SPACE PROJECT

While a full 3x2pt analysis of DES-Y1 data has already been performed in configuration space, the collaboration left a gap for an equivalent harmonic space analysis. This gap can only be filled through a step by step approach, in which each probe is modeled separately and with an adequate treatment of systematics. The galaxy clustering portion of this endeavour has been addressed by [Andrade-Oliveira et al. \(2021\)](#), to which we offered substantial contributions. Another significant step was placed by [Doux, Chang, Jain, et al. \(2021\)](#), who propose a methodology for a harmonic space cosmic shear analysis of DES Y3 data. In this section we describe our contributions to the DES-Y1 harmonic space galaxy clustering project.

4.3.1 The angular power spectrum in simulated data

In order to develop our methods and analysis choices, we used a set of 150 simulated full-sky lognormal maps, each one generated with the FLASK code ([XAVIER; ABDALLA; JOACHIMI, 2016](#)). Each simulation has data for five redshift bins, which correspond to those from the redMaGiC lens catalog. In order to bring our simulated data closer to survey reality, masks were applied to the full-sky maps. Each map was combined separately with eight masks, the combinations resulting in distinct partial-sky maps which recreate the DES footprint. Thus, our starting point was a set of 1200 partial-sky maps, to which we will refer as our set of mocks.

As described by [Xavier, Abdalla, and Joachimi \(2016\)](#), one of the inputs of the FLASK code is the theoretical angular power spectrum. Each of the 1200 mocks share a same underlying cosmology, which was set at $\Omega_m = 0.286$, $\Omega_b = 0.05$, $h = 0.7$, $n_s = 0.96$, $A_s = 2.232746 \times 10^{-9}$, and $\Omega_\nu h^2 = 0$. The input galaxy bias values were the same as those from the simulations by [Elvin-Poole, Crocce, Ross, et al. \(2018\)](#). We used our Limber approximation code described in 2.6 to generate reference theoretical C_ℓ spectra with such cosmological parameters and galaxy biases.

4.3.1.1 Measurements of the angular power spectrum

Our next step was to construct our simulated data vectors. We measured the angular power spectrum measurements of each of the mocks with the NaMaster code ([ALONSO; SANCHEZ; SLOSAR, 2019](#)), which uses the Pseudo- C_ℓ algorithm to to recreate the full-sky C_ℓ from a masked map. The angular power spectrum was extracted in a binned form, with 20 log-spaced ℓ bins that span from $\ell_{min} = 30$ to $\ell_{max} = 3000$. We added to the reference spectrum of each redshift bin i a theoretical shot-noise, given by $a_i = \bar{n}_i^{-1}$, where $\bar{n}_i$ is the number density of galaxies in bin i . This correction accounts for the fact that our density field is built from number counts.

The measurements are shown along with the reference theoretical spectra in Figure 4.2. The green shaded areas indicate a tentative scale cut for analysis, which we set at $\ell_{max} = \pi/\theta_{min}$, where θ_{min} is the minimum scale used in the DES-Y1 configuration space analysis. ([ELVIN-POOLE; CROCCE; ROSS, et al., 2018](#)) We will further discuss the effect of scale cut choices on parameter fitting. The average of the spectra of all mocks is represented by the black data points, with error bars given by the statistical uncertainty.

4.3.1.2 Covariance of the mocks

We measured the covariance from the set of mocks, taking into account the Hartlap correction for the finite number of maps ([HARTLAP; SIMON; SCHNEIDER, 2006](#)):

$$\text{Cov}_{new}^{-1} = \frac{n - p - 2}{n - 1} \text{Cov}^{-1}, \quad (4.3)$$

where n is the number of realizations, and p is the size of the data vector.

This measured covariance is a way to validate the theoretical CosmoLike covariance

Figure 4.2 – Measurements of the angular power spectra for our set of mocks.

Figure 4.3 – Galaxy clustering correlation matrix from FLASK measurements

that is standard in DES analyses. In 4.3.4.3, we compare the effect from both covariances on parameter fitting. Covariance information can be displayed through the correlation matrix, defined by Eq. 4.4. We show the correlation matrix measured from the mocks in Figure 4.3.

$$\text{Corr}_{ab} = \frac{\text{Cov}_{ab}}{\sqrt{\text{Cov}_{aa} \text{Cov}_{bb}}}. \quad (4.4)$$

Figure 4.4 – Results for galaxy bias on configuration space. Dark and light green regions represent 1σ and 2σ intervals for the results by [Elvin-Poole, Crocce, Ross, et al. \(2018\)](#)

4.3.2 CosmoSIS parameter estimation

The DES-Y1 3x2pt main analyses were conducted through the CosmoSIS framework, by use of a unified configuration space pipeline. An adaptation of this pipeline for harmonic space was produced to be used in our project. For all parameter fitting results, therefore, we made use of CosmoSIS. In 4.3.3, we used the collaboration’s configuration space routine, and in 4.3.4 we used the harmonic space adaptation.

4.3.3 Reproduction of configuration space results

Before proceeding to our harmonic space analysis, we made a reproduction of the galaxy bias results by [Elvin-Poole, Crocce, Ross, et al. \(2018\)](#). We used the MultiNest algorithm as implemented in the CosmoSIS framework. ([FEROZ; HOBSON; BRIDGES, 2009b](#)) Our marginalized probability distributions are shown in Figure 4.4 along with the 1σ and 2σ confidence levels of the values found by [Elvin-Poole, Crocce, Ross, et al. \(2018\)](#) (in green). We obtained similar results that lie well within a 1σ interval. Thus, we opted to proceed with our harmonic space analysis, having our configuration space chain as a baseline for comparison.

4.3.4 Harmonic space results

The next step of the project was to produce equivalent galaxy bias constraints on harmonic space. By applying similar analysis choices as those used in the configuration

space calculation, we would expect the harmonic 2-point functions to contain similar though not identical information as that of the configuration ones. The parameter constraints, thus, should be compatible with each other. One of the main factor that has been noticed to influence this compatibility is the effect of scale cuts. The 1σ discrepancy on cosmic shear HSC analyses, for example, was attributed by Hamana, Shirasaki, Miyazaki, et al. (2020) to the difference between the scale cuts of the two analyses. While Hikage, Oguri, Hamana, et al. (2019) used $\ell_{min} = 300$, Hamana, Shirasaki, Miyazaki, et al. (2020) made use of angular scales which had significant contribution from $\ell < 300$ multipoles. (HAMANA; SHIRASAKI; MIYAZAKI, et al., 2020)

4.3.4.1 Possible scale cuts

When dealing with cosmological data vectors, it is important to select only data which is adequate for use. Theoretical models usually break down at particular scales, in a way that using unallowed scales can lead to false constraints on the parameters of interest. In the DES-Y1 3x2pt analysis, the configuration space galaxy clustering vectors were cut in their lower bound at $\theta_i = 43', 27', 20', 16', 14'$, where $i = 1, 2, 3, 4, 5$ are the redshift bins. The authors for the galaxy clustering configuration space analysis explain that those values protect the results from influence of non-linear bias and baryonic feedback processes. (ELVIN-POOLE; CROCCE; ROSS, et al., 2018)

An accurate comparison of configuration and harmonic space results for the galaxy biases has to guarantee that similar information is being taken into account. The scale cut conversion, however, is not trivial. Harmonic space functions, by definition, include in each scale ℓ information of all configuration space angles θ and vice-versa. A full equivalence between spaces can only be achieved theoretically, when ℓ_{max} is taken as arbitrarily large, and θ_{min} as arbitrarily small. The task thus, becomes to select and remove the scales ℓ which have significant contribution from unallowed angles θ .

The simplest way to convert the scale cuts from configuration to harmonic space is to use $\ell_{max} = \pi/\theta_{min}$. This choice, of which we made use in 4.3.1.1, will associate the first harmonic multipole $\ell = 1$ with distances of the order of half the celestial sphere, the second multipole with those of the order of one fourth of the sphere, and so forward. It is, however, a rough estimate which can bias the final results. When applied to the DES-Y1 data set, this method yields us upper bounds given by $\ell_{max, i} = 251, 400, 540, 675, 771$.

Redshift bin		Configuration space	Harmonic space	
i	z_i range	θ_{min}	ℓ_{max} (<i>naive</i>)	ℓ_{max} (<i>physical</i>)
1	0.15 to 0.30	43'	251	80
2	0.3 to 0.45	27'	400	127
3	0.45 to 0.60	20'	540	172
4	0.6 to 0.75	16'	675	215
5	0.75 to 0.90	14'	771	246

Table 4.3 – Scale cuts for DES-Y1 analysis.

We will refer to such choice as our *naive* scale cut.

A second scale cut conversion strategy is to find the limit on the amplitude of k which is imposed by the configuration space cuts, and then translate this limit into a cut on ℓ by performing a projection on each redshift bin. Conveniently, the method selected by DES to obtain the configuration space scale cuts was exactly the imposition of a lower limit for correlations at the comoving distance $\chi_{min} = 8\text{Mpc}/h$. This translates directly to an upper limit on the amplitude of the Fourier mode by $k_{max} = 1/\chi_{min} = 1.25 \times 10^{-1} h\text{Mpc}^{-1}$. Through Eq. 2.43, using the mean redshift of each bin, we find the equivalent cuts on harmonic space. This set, to which we will refer as *physical* scale cuts, is composed of $\ell_{max,i} = 80, 127, 172, 215, 246$. Our lower bounds for ℓ were set for all bins and in both the *naive* and the *physical* scenarios as $\ell_{min} = 43$. We show our harmonic and configuration space scale cuts on Table 4.3.

4.3.4.2 Parametrization

Following the configuration space analysis by [Elvin-Poole, Crocce, Ross, et al. \(2018\)](#), we attempted to find constraints on the galaxy bias while leaving the cosmological parameters constant. We fitted the shift parameters Δz_i along with the biases, and marginalized over the former when reporting our results. We repeated this routine for the mocks, with validation purposes, and for DES-Y1 data.

4.3.4.3 Fits with different covariances

[Andrade-Oliveira et al. \(2021\)](#) verify the compatibility between our measured covariance and the analytical one from CosmoLike. Using as data vector the average C_ℓ of all mocks and performing physical scale cuts, they found virtually identical constraints on the galaxy biases with the two covariances. We reproduce their result in Figure 4.5. Once demonstrated the compatibility, we use our measured covariance matrix in all further

Figure 4.6 – Configuration and harmonic space chains for a single mock. Harmonic space chains with naive and physical scale cuts.

harmonic space, with contours of the order of those obtained with physical scale cuts. The discrepancy of the single mock constraints disappeared, suggesting that it was due to statistical fluctuations within the simulations. Our results for the average of the mocks is shown in Figure 4.7.

While it seems visually that the physical scale cuts were the ones that best recovered the configuration space constraints on the average of the mocks, we nonetheless keep both possibilities when analyzing DES-Y1 data.

4.3.4.5 Results with DES-Y1 data

Finally, we applied the methods discussed in the previous sections for DES-Y1 data. We assumed the mean 3x2pt posterior values for the cosmological parameters, and derived constraints on the galaxy bias of the redMaGiC catalog. Our results are shown in Figure 4.8.

Here a tension between configuration and harmonic space results emerge if we

Figure 4.7 – Configuration and harmonic space chains for the average of the mocks. Harmonic space chains with naive and physical scale cuts.

consider our naive scale cuts. With the physical scale cuts, however, results become compatible. This reinforces the thesis by [Doux, Chang, Jain, et al. \(2021\)](#) that scale cut differences are the main source of divergence between configuration and harmonic space results that use the same data.

Adopting the physical scale cuts for harmonic space analysis, we compare our constraints on the galaxy bias from harmonic and configuration space. Because the amplitude of each spectra is modulated by the combination $b_i\sigma_8$, we present our results in these terms. Nevertheless, since our value of σ_8 was set fix at the mean of the posterior from the 3x2pt analysis ($\sigma_8 = 0.82927$), the use of this combination becomes interchangeable with the use of the biases themselves. Our values with their uncertainties are presented in Table 4.4. They were drawn from our MultiNest samples so that their median is reported as the obtained value, and the region between the 1σ bounds comprises 68% of the samples.

Figure 4.8 – Configuration and harmonic space chains for DES-Y1 data. Harmonic space chains with naive and physical scale cuts.

i	$b_i \sigma_8$	
	Configuration space	Harmonic space
1	$1.160^{+0.066}_{-0.063}$	$1.15^{+0.10}_{-0.11}$
2	$1.331^{+0.040}_{-0.045}$	$1.315^{+0.052}_{-0.050}$
3	$1.325^{+0.034}_{-0.035}$	$1.311^{+0.035}_{-0.033}$
4	$1.593^{+0.038}_{-0.039}$	$1.620^{+0.028}_{-0.032}$
5	$1.636^{+0.058}_{-0.055}$	$1.670^{+0.055}_{-0.036}$

Table 4.4 – Constraints from configuration and harmonic space on the galaxy bias of the DES-Y1 redMaGiC galaxy catalog.

4.4 PERSPECTIVES ON A DES 3X2PT HARMONIC SPACE ANALYSIS

We demonstrated, through an analysis of the galaxy bias, that the DES-Y1 red-MaGiC galaxy clustering sample can be treated in both harmonic and configuration space with no divergence between the statistics. This step, however, has to be complemented by the harmonic space analyses of galaxy-galaxy lensing and cosmic shear. For each of those functions, robustness of analysis methods should be tested, yielding parallel results to those by [Prat, Sánchez, Fang, et al. \(2018\)](#) and [Troxel, MacCrann, Zuntz, et al. \(2018\)](#). For cosmic shear, independent constraints on Ω_m and σ_8 can be found in harmonic space. This is performed by [Camacho, Andrade-Oliveira, Troja, et al. \(2021\)](#). When all single probe analyses are complete, 3x2pt results can be found, and their compatibility with their configuration space counterpart verified.

This kind of analysis with multiple statistics is an improvement that would benefit DES not only in Y1 analysis, but also in the study of the following data that has already been gathered by the collaboration. DES has been conducting analyses on Y3 data (up to the third year of observations), and Y6 data (containing the whole survey observations). These analyses are expected to yield tighter constraints than those from Y1 data, specially due to the increase in sky area coverage. Our methods could thus be further tested and applied, and more can be learned about the relationship between configuration and harmonic space descriptions of sky data.

The perspective in the collaboration is to apply the 3x2pt methodology with improved modeling and perform combinations with more probes such as CMB. As the multi-probe approach is a way to take most advantage of the available information, however, so is the combination of statistics. Such can be seen in [Figure 4.8](#), on which probability contours indicate full compatibility between configuration and harmonic space (with physical scale cuts), but also point to the existence of complementary information in each function. Therefore, DES analyses should increasingly incorporate the use of harmonic space functions.

FINAL REMARKS

Survey cosmology has long benefited from having a variety of probes and methods. While information from distant galaxies is all encoded on the light we observe from them, it has been possible to infer detailed data on the distribution patterns, not only of visible matter, but of all matter in the universe. The particular combination of statistics with galaxy positions and galaxy shapes, which yields us three 2-point correlation functions (galaxy clustering, galaxy-galaxy lensing, and cosmic shear) offers many advantages over separate analyses of the data.

Because each of those functions contain a different factor of the galaxy bias, the degeneracy of the cosmological parameters with the bias can be broken, and cosmological information from galaxy clustering that was previously unavailable can become useful. In addition to this reasoning, forecasts made during the last two decades also point to an increased constraining power when using our combination of probes. (JOACHIMI; BRIDLE, 2010)

The Dark Energy Survey multi-probe methodology, outlined for Year 1 by Krause, Eifler, et al. (2017), and for Year 3 by Krause, Fang, Pandey, et al. (2021), has been yielding fruitful results, with constraints on Ω_m and σ_8 surpassing in quality those from a sole cosmic shear analysis. The multi-probe strategy has also provoked a better understanding of systematics, since they had to be jointly modeled.

In the context of devising improved methods for DES, we bring light to the recent analyses of KiDS and HSC data, which questioned the identity between constraints from configuration space and harmonic space. As reported by Hamana, Shirasaki, Miyazaki, et al. (2020) when discussing the divergence in HSC results, the discrepancies point to the fact that complementary information was used in the concurrent analyses.

Our DES-Y1 harmonic space galaxy bias results further enhances the thesis of complementary information, while highlighting the importance of adequate scale cuts. A naive translation from θ_{min} cuts to ℓ_{max} ones would have given us a $1\sigma - 2\sigma$ discrepancy relative to the configuration space results. However, the use of physical scale cuts guaran-

teed compatibility between results, while not suggesting an exaggerated redundancy. We conclude that DES, and even the next generation of large surveys, would largely benefit from multi-probe approaches that also gather inferences from different 2-point statistics.

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