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**Topology Optimization of Flextensional Piezoelectric
Actuators with Active Control Law**

**USP – São Paulo
2019**

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Actuators with Active Control Law**

Doctoral thesis submitted to the Politechnic School
of the University of São Paulo – EP-USP, in partial
fulfillment of the requirements for the Doctor of
Science degree.

Area of Concentration:
Control and Mechanical Automation Engineering.

Advisor:
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**USP – São Paulo
2019**

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São Paulo, 25 de Fevereiro de 2019

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Catálogo-na-publicação

Moretti, Mariana

Otimização Topológica de Atuadores Piezelétricos Flexensionais com Lei de Controle Ativo / M. Moretti -- versão corr. -- São Paulo, 2019.
110 p.

Tese (Doutorado) - Escola Politécnica da Universidade de São Paulo.
Departamento de Engenharia Mecatrônica e de Sistemas Mecânicos.

1.Métodos topológicos (Otimização) 2.Atuadores piezelétricos flexensionais 3.Controle ativo 4.Sensores 5.Método dos elementos finitos (Análise dinâmica) I.Universidade de São Paulo. Escola Politécnica. Departamento de Engenharia Mecatrônica e de Sistemas Mecânicos II.t.

Dedico este trabalho à minha família.

ACKNOWLEDGEMENTS

Agradeço ao Prof. Dr. Emílio Carlos Nelli Silva pelas sugestões, ideias, apoio e constante incentivo, que foram fundamentais para a conclusão deste trabalho.

Agradeço aos meus amigos do MSOL pelos ensinamentos que contribuíram muito para o desenvolvimento desta tese.

Agradeço ao meu marido Rafael pelo imprescindível companheirismo e constante encorajamento, ao meu irmão Eduardo por se orgulhar de minhas realizações e de meu trabalho e, em especial, agradeço aos meus pais Mario e Beth, que me conduziram com tanto sucesso pelo caminho da educação.

“All the models are wrong, but some are useful”
(George Edward Pelhan Box)

RESUMO

Atuadores piezoelétricos flexensionais funcionam como amplificadores nanométricos dos deslocamentos gerados pela piezocerâmica. Em sistemas que necessitam de alta precisão de posicionamento final ou baixa energia de vibração após sofrer um impacto, como na leitura de um disco rígido ou na microscopia de força atômica, requer-se que o atuador conte com um fator de correção de posicionamento, o qual pode ser obtido através de uma lei de controle. A utilização de material piezoelétrico para o controle de vibração em dispositivos de casca e placa foi amplamente abordado na literatura, porém o projeto de sistemas piezoelétricos controlados utilizando-se do Método da Otimização Topológica (MOT) foi pouco explorado e em geral é focado na análise transiente no domínio da frequência, o qual necessita que o problema tenha que ser reduzido para que a implementação computacional torne-se viável. Tratando-se de análise transiente em atuadores piezoelétricos flexensionais pode-se considerar o emprego de um controle ativo, o qual captura informações do dispositivo através de piezo-sensores e as realimenta na forma de um sinal de entrada em piezo-atuadores para modulação do sinal de resposta. Visando aprimorar o efeito do controle ativo em atuadores piezoelétricos, este trabalho é voltado para o projeto de sua estrutura flexensional considerando o controle ativo de realimentação de velocidade, em que o ciclo envolvendo sensoreamento e atuação piezoelétricos fornece um amortecimento extra ao sistema. Deseja-se portanto que a flexibilidade do mecanismo flexensional seja distribuída no domínio de projeto pré-definido alocando vazios em regiões ao redor de onde devem estar presentes articulações da estrutura flexível, o que é obtido pelo MOT. Para encontrar a distribuição otimizada de material no domínio de projeto, o MOT emprega a análise sistemática do modelo dinâmico através do Método dos Elementos Finitos (MEF). Neste trabalho a modelagem do sistema para o MEF leva em consideração a presença dos nós intermediários das cerâmicas piezoelétricas, denominada modelagem para nós não colapsados, cuja tensão gerada ao longo da resposta dinâmica temporal influencia no controle ativo do sistema. O problema de Otimização Topológica (OT) é formulado para a atenuação da vibração do sistema em posição neutra e em posição atuada (posicionador) sujeito a restrições de volume e a valores máximo e mínimo que as variáveis de projeto assumem. A implementação do MOT é baseada no modelo de material denominado Material Isotrópico Sólido com Penalização (MISP), no cálculo da sensibilidade dinâmica adjunta, e na rotina de otimização conhecida como Programação Linear Sequencial (PLS). Para ilustrar o método, são projetados dispositivos bidimensionais e diferentes ganhos de controle de realimentação de velocidade são utilizados para obtenção da topologia otimizada, analisando-se a eficiência em cada caso.

Palavras-chave: Método da Otimização Topológica. Atuador piezoelétrico flexensional. Controle ativo. Análise transiente no domínio do tempo. Nós piezoelétricos não-colapsados.

ABSTRACT

Flextensional actuators assembled in association with piezoceramics feature the amplification of nanometric displacements generated by the ceramics energy conversion. For applications that require high precision positioning or vibration response attenuation, such as hard disc reading or atomic force microscopy, a response tracking control needs to be implemented. Shell and plate piezoactuators with vibration control have been extensively studied in literature, however the design of controlled piezoelectric systems by means of the Topology Optimization Method (TOM) has not been fully explored in literature yet, and is generally focused on the frequency domain transient analysis, which employs a model reduction method for the sake of computational implementation. Dealing with transient analysis of flextensional piezoelectric actuators, an active closed loop control design is more suited for the positioning and vibration problem, which consists on measuring the outputs of the system by the closed loop sensor layer, whose signal is modified by a control gain and eventually inputted into the actuator layer so the system response signal is modulated. Aiming to enhance the active feedback control in piezoelectric actuators (PEAs), this work targets the design of the flextensional microstructure considering an active velocity feedback control (AVFC), where the active piezoelectric sensing and actuating cycles imply in an extra damping to the system. Therefore, the flextensional mechanism compliance shall be distributed within the design domain by the allocation of void regions where there should be the flexible hinges. Such a design can be accomplished by means of the TOM, which employs a systematic analysis of the dynamic model through the finite element method (FEM). In this work, the finite element (FE) system model takes into account the piezoelectric ceramics intermediate nodes, what is denominated as non-collapsed piezoelectric nodes model, and whose induced voltage during the time domain dynamic response contributes to the active control of the system. The topology optimization (TO) problem is formulated for the system vibration suppression at the restoring position and at the actuated position (positioner) subject to material volume and design variables constraints. The TOM implemented is based on the solid isotropic material with penalization (SIMP), the dynamic adjoint sensitivity, and on the optimization solver known as sequential linear programming (SLP). To illustrate the method, bidimensional examples of optimized topologies are numerically obtained by employing different velocity feedback control gains, and the topologies efficiency are compared and contrasted.

Key-words: Topology Optimization Method. Flextensional piezoactuator. Active control law. Time-domain transient analysis. Non-collapsed piezoelectric nodes.

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LIST OF SYMBOLS

$\mathbf{E}$	Electric Field
$\mathbf{D}$	Electric Displacement
$\mathbf{H}$	Magnetic Field
$\mathbf{B}$	Magnetic Flux Density
$\mathbf{J}$	Electric Current Density
q	Volume Free Charge Density
ε	Material Permittivity
μ	Material Permeability
μ_0	Permeability Constant
c_0	Electromagnetic Waves Velocity
ε_0	Permittivity Constant
ϕ	Scalar Electric Potential
$\mathbf{T}$	Stress Tensor
$\mathbf{S}$	Strain Tensor
$\mathbf{c}$	Elastic Matrix
$\mathbf{e}$	Piezoelectric Coupling Tensor
ε	Permittivity Tensor
Ω	Solid Domain
Ω_1	Piezoelectric Domain
Ω_2	Elastic Domain
$\bar{\Omega}$	Fixed Domain
Γ	Boundary Surface
Γ_u	Displacement Boundary Condition
Γ_T	Load Boundary Condition
Γ_ϕ	Electroded Boundary Condition

Γ_D — Unelectroded Boundary

$\mathbf{n}$ — Normal Vector

$\mathbf{u}$ — Mechanical Displacement

$\bar{u}$ — Prescribed Displacement

$\bar{\phi}$ — Prescribed Voltage

$\bar{t}_j$ — Prescribed Traction

$\bar{q}_f$ — Free Charge Density per Unit Surface Area

$\mathcal{U}$ — Potential Energy on the Piezoelectric Body

μ_{st} — Mechanical Strain Energy

$\mathcal{U}_{el}$ — Electrostatic Field Energy

$\mathcal{H}$ — Electric Enthalpy

ζ — Specific Weight

t — Time

δ — First-order variation operator

L — Lagrangian

$\mathcal{T}$ — Kinetic energy

$\mathcal{W}$ — Virtual Work

$\mathbf{f}_b$ — Body Force Vector

$\bar{\mathbf{t}}$ — Surface Traction Vector

$\mathbf{f}_c$ — Concentrated Load

$\mathcal{N}_u$ — Mechanical Shape Functions Matrix

$\mathcal{N}_\phi$ — Electric Shape Functions Matrix

$\mathbf{M}_{uu}$ — Mass Matrix

$\mathbf{K}_{uu}$ — Stiffness Matrix

$\mathbf{K}_{u\phi}$ — Piezoelectric Coupling Matrix

$\mathbf{K}_{\phi\phi}$ — Dielectric Permittivity Matrix

$\mathbf{F}_u$ — External Nodal Force Vector

$\mathbf{Q}_\phi$ — Free Charge Vector

$\mathcal{B}_u$ — Mechanical Shape Function Derivative Matrix

$\mathcal{B}_\phi$ — Electric Shape Function Derivative Matrix

$\mathbf{C}_{uu}$ — Damping Matrix

$\mathbf{C}_{fb}$ — Closed-loop Damping Matrix

F_{dof} — Concentrated Mechanical Load

$\boldsymbol{\rho}$ — Design Variables Vector

p — Penalization Factor

(x_c, y_c) — Finite Element Centroid Cartesian Coordinate Pair

w_1 — Weight for static objective function

w_2 — Weight for dynamic objective function

f — Multi-objective Function

$\mathbf{U}_0$ — Initial Position Vector in Time

$\mathbf{V}_0$ — Initial Velocity Vector in Time

V_{max} — Material Volume Constraint

$\mathbf{B}$ — Degree-of-freedom target matrix

$\mathbf{F}_v$ — Load induced by electric voltage

fs — Sampling Frequency

Δt — Time Step

$\boldsymbol{\zeta}$ — Pseudo-densities Vector for Projection Filter

N^e — Neighborhood of Finite Element e

r — Projection Filter Radius

E — *Young's* Modulus

ν — *Poisson's* Ratio

E — *Young's* Modulus

ν — *Poisson's* Ratio

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INTRODUCTION

The micro and nanoscale devices have driven the modernization of electronic solutions in an accelerated pace since the beginning of this century. Applications for microactuators cover the medical domain with microrobots delivering in-body medicine (JAYARAM *et al.*, 2018) and implanted heart pacemakers (KOWARSCH *et al.*, 2017), the pharmaceutical domain with substances micromanipulation (JIVANI *et al.*, 2016), the military domain with micro unmanned aerial vehicles, and the physics and engineering fields with nanometric resolution surface characterization. Piezoelectric ceramics and flexible structures are suitable components to integrate actuators for micro and nanoscale displacements. However, for such high resolution devices, often subjected to environmental interferences, appropriate design and control are necessary. For this reason, many works in literature have focused in dynamics, characterization and control of microactuators (KHADRAOUI; RAKOTONDRABE; LUTZ, 2013; RAKOTONDRABE *et al.*, 2013). Techniques such as the performance inclusion theorem (RAKOTONDRABE; KHADRAOUI, 2013) have been applied for the design of piezoelectric actuators (PEAs) Piezo-electric Actuators, where the dimensions of a cantilevered actuator is sought based on imposed specifications that are transcribed to an interval transfer function. Vibration control techniques in frequency domain have been designed for structures actuated by smart materials, as the piezoceramic material, for the achievement of a robust control performance of Euler-Bernoulli beams and for the sliding mode controller applied to piezostack structures (CHOI; HAN, 2010).

In this work, the design of piezoactuators involves the topology optimization method (TOM) Topology Optimization Method (BENDSOE; SIGMUND, 2003) of the flexible structure with the active velocity feedback control gain (AVFC) Active Velocity Feedback Control given by the piezoelectric energy conversion property. The advantage of designing the controlled piezoelectric system through a numerical method is that it enables the modelling of the controller integrated to the structure so that the control effect within the microstructure is part of the analysis. An active vibration control scheme, defined by the extra voltage or electric charge that is supplied to the piezoceramic material (TAKEZAWA *et al.*, 2014b), has been studied for

flexible plates (TZOU; TSENG, 1990; WANG; QUEK; ANG, 2001; CARUSO; GALEANI; MENINI, 2003), for cylindrical laminated composite shells (RAY; REDDY, 2003), for beams (VASQUES; RODRIGUES, 2006; KIM; WANG; BRENNAN, 2010) and for precision positioning of hard disk drives (CHOI; KIM; PARK, 2007; CHAN; LIAO, 2008). Active control schemes are often modeled for dynamic topology optimization method designs in frequency domain analysis considering either the velocity feedback control, (ZHANG; KANG; LI, 2014) and (ZHANG; TAKEZAWA; KANG, 2018), or the control of single mode from the state space representation by using the controllability Gramian (GONÇALVES; DELEON; PERONDI, 2017) and (GONCALVES; DELEON; PERONDI, 2018).

The flexible structure designs of this work are inspired by the motion characteristics of the commercial flexure-guided and motion-amplified piezo actuator shown in figure 1, and by the positioning characteristics of the cantilever structure in multimode atomic force microscopes, highlighted in yellow in figure 2. The movement of the probe for one of the operational modes of the atomic force microscope (AFM) Atomic Force Microscope is characterized by the tip-sample interaction given by a tunneling current that has to be kept constant by a feedback circuit during the scanning process. An electronic controller guides the tip at a tip-sample distance corresponding to a constant preset tunneling current. The AFM tip moves in three directions by piezoelectric actuators, and the tip-sample distance is recorded by a computer as a function of the lateral position which is then displayed as a microscope image (MEYER; HUG; BENNEWITZ, 2013). Therefore, this work aims to design a vibration attenuated device, and a positioner with vibration attenuation given that the cantilever tip of the AFM is constantly actuated during the scanning process.

Figure 1 – Comercial motion-amplified piezo actuator.

In this work, the PEAs with vibration suppression are designed considering the velocity feedback control due to the possibility of modeling the controller in time domain. In the direct velocity feedback, the sensors measure the velocity output signals that are electronically amplified and fed back to the actuators. This method avoids the appearance of spillover effects of the uncontrolled residual modes that emerges in frequency domain (BALAS, 1979). In addition to the stability of all vibration modes provided, time domain velocity feedback implementation is,

Figure 2 – Multimode atomic force microscope (AFM)

in practice, more realistic for a real-time control action than the mathematical model of a linear quadratic-regulator.

Differently from the plate and shell actuators, where the optimized distribution of piezoceramic patches is a feasible design (WANG; TAI; QUEK, 2006; SUN *et al.*, 2009; ZHANG; KANG, 2014), for flextensional piezoactuators (FPEAs) Flextensional Piezoactuators the focus is to optimize the topology of the host layer, since the structure compliance arises from the existing hinges within the structure. Therefore, this work is a first approach on controlling vibration response of FPEAs by applying the TOM to the host layer of the structure given the locations for the sensor and actuator layers.

1.1 Active Control Strategies for Piezoelectric Structures

Regarding applications in sensing and actuation for control purposes, the piezoceramic can either operate in reception, sensing the mechanical inputs, or in electrical emission, deforming and actuating the structure. Many different assemblage configurations for piezoelectric actuators and piezoelectric sensors (PESs) Piezoelectric Sensor devices have been experimented in literature. In the first stages of development, the ceramic was embedded or bonded to the measured or actuated structure. Due to modern fabrication techniques, earlier developments started considering new designs such as piezoelectric bimorph assemblages (WANG, 2004), gradual variation of piezoceramic or its elastic properties (ZHU; MENG, 1995; ALMAJID; TAYA; HUDNUT, 2001) and a ceramic distribution for multi-actuation purposes (CARBONARI; SILVA; PAULINO, 2009). Most of the time, piezoelectric materials are embedded in isotropic elastic materials (WANG; MEGUID, 2000) found in a broad range of different designs and topologies to form the systems cited before. Such systems of piezoelectric and elastic materials are denominated piezocomposite transducers or flextensional actuators (KANG; WANG; TONG,

2011). An interesting capability of applying piezocomposite transducers to the recent electronic developments is their micro-metric scale displacements for small size designs and low energy consumption.

In the field of mechatronics engineering, researchers have fulfilled a great extent of micro-positioning devices actuated by piezoelectric ceramics. Many of these developments accounted with an optimization procedure either to enhance the displacement at a specified direction or to establish the ceramic layout to improve the actuation. However, besides the actuation itself, it might be required from the electromechanical system a certain quality of actuation when its transient response is taken into consideration. The most straightforward requirement in this case regards the vibration of the piezo-device. A high vibration amplitude or slow decaying dynamic response might damage the device and even the object that is being manipulated by it. In this context, researches have been focusing on the design of controllers applied to the dynamic response of micro and nano scale positioners and manipulators.

When the piezoelectric controlled structures are connected through a feedback architecture they are denominated intelligent structures. The reference for intelligent structures often stands for self-monitoring and control, and the necessity of such structures in microelectronics, micro-machining and other high precision applications boosted the usage of piezoelectricity at the control field in the context of distributed sensing and actuation of flexible structures. One classical example is the controlled miniaturized swing arm type actuator installed in optical disk drives for mobile electrical devices. They require highly efficient dynamic characteristics and the optimization of performance under size constraint (KIM; KIM; HAN, 1999; YU; LIU, 2003; ZHU; WANG; JIANG, 2006; PARK *et al.*, 2007; YOO; LEE, 2007; SUN *et al.*, 2009; KIM; SONG; KIM, 2012). The suspension assembly in the head disk interface must provide high reliability as a slider following capability for disk run-out, and it must also allow for good servo control during track seeking, settling and following (YU; LIU, 2003).

An active control system involves smart structures, such as piezoelectric actuators integrated with typical control systems. Effective methodologies in controlling a system featuring piezoelectric actuators involve the classic PID control, the Linear Quadratic (LQ) control, the sliding mode control, the H_∞ control, the Quantitative Feedback Theory (QFT) control, and the inverse model control (ALKHATIB; GOLNARAGHI, 2003; CHOI; HAN, 2010).

In the literature, the first work to employ the piezoelectric effect controlling structure vibrations was (BAILEY; HUBBARD, 1985). A piezoelectric polymer (poly vinylidene fluoride) was used as a distributed-parameter actuator to actively control vibrations at a cantilever beam, which is a flexible structure with infinite number of vibrational modes. Authors applied Lyapunov's second method (direct method) (FARD; SAGATUN, 2001) to design control algorithms that can easily deal with bounded inputs and distributed-parameter systems. Lyapunov's second method involves finding a Lyapunov functional of the system and the control to minimize the time rate of change of this functional at every point in time. When this control theory is used to

design a control algorithm for the active damper, it allows the possibility of controlling all modes of vibration at once, provided that the system is controllable through the actuator, and avoids problems with spillover of the uncontrolled modes. Therefore, it circumvents the difficulty of using distributed-parameter control theory with spatially discrete sensors and actuators.

A different application, on tracking control of a moving arm over a fluctuation target, proved to be efficient when applying piezoelectric cells with a PID feedback controller (JIANG; CHONAN; TANI, 1992). Feedback multiple-input and multiple-output (MIMO) Multiple-input and Multiple-output control for simply-supported plate using piezoelectric materials for structural measurements in modal analysis framework was also experimented in literature (SAUNDERS; COLE; ROBERTSHAW, 1994), with the interesting result that for the successful application of the proposed approach, it is required the simultaneous sensing and actuation of the piezoelectric sheet transducers.

Recently in literature, a study involving LQR and LQG control was developed by (PADOIN *et al.*, 2013), where mechanical vibrations of an aluminium cantilever beam are attenuated and LQR and LQG performance are compared. The development is based on fixed piezoelectric material, with a new methodology scheme to determine the correct values of weighting matrices Q and R . In the field of active vibration control of piezoelectric beams, (VASQUES; RODRIGUES, 2006) presents a numerical study comparing classical strategies with optimal control strategies (LQR, LQG).

1.2 Topology Optimization Applied to the Design of Controlled Piezo Actuators

As understood in previous section, piezoelectric materials can be employed in the distributed sensing and control of intelligent structures, obtaining an accurate response monitoring, and effective control of flexible structures (RAO; SUNAR, 1994). Smart structure layout may be optimally designed by applying a typical multidisciplinary optimization that integrated at least two disciplines, structure and control (BEGG; LIU, 2000). The topology optimization method is a type of structure optimization that allows great design freedom since during its processing it determines location of void and solid inside a given design domain (Fig. 3) being different from other structure optimization methods such as size and shape methods that deals with parameters of predefined structural configurations. In addition, TOM techniques has matured in the past two decades to include multiphysics disciplines such as combination of structures (elastic and piezoelectric for example), heat transfer, acoustics, fluid flow and so on (DEATON; GRANDHI, 2014). The material distribution (void and solid) plays an important role in the structure performance, affecting its stiffness, vibration modes and response characteristics. For these reasons, the TOM may be employed to the design of optimized structures including piezoelectric transducers under dynamic analysis.

Figure 3 – TOM procedure for the design of piezoelectric actuators with vibration suppression proposed in this thesis. The elastic material terminology is therein applied for intermediate material properties regions.

Although it is still common to design a controller for a previously designed elastic structure, it is becoming popular to integrate the design of both the structure and its controller altogether. Such a design involves a multi-objective function evaluated for a structure modeled in a finite-element (FE) environment, what refers to the TOM. In this trend, in the work (MENUZZI *et al.*, 2013), the authors analyze vibration control induced by the presence of external forces using the concept of topology optimization to minimize structural compliance and to distribute the piezoceramics. The study showed that the optimizing strategy improves the active damping of the structure. In this case, LQR optimal control index was applied and a truncated modal matrix used for the state space model. Yet working in the reduced state space control model, (SILVEIRA; FONSECA; SANTOS, 2015) also worked with vibration suppression of flexible structures by maximizing a measure of the controllability Gramian of an LQR control system. A passive linear elastic material and an active linear piezoelectric material were distributed by applying the TOM in the fixed domain. Similar study was made for the distribution of Macro Fiber Composite in (PADOIN *et al.*, 2015).

Another example still involving active vibration control but with velocity feedback controller is the work (ZHANG; ZHAN, 2014). In this article, authors optimize the actuator and sensor layers imposing a symmetric design, and the dynamic equations are solved with the complex mode superposition in the state space after a model reduction transformation. Yet, structural damping effect is not considered, as the piezoelectric layers are assumed to be very thin compared to the host layer. The same author also worked in time-domain with the same velocity feedback principle to optimize piezoelectric sensor and actuator layer topologies attached to a thin-shell structure subjected to dynamic excitations (ZHANG; KANG, 2014). Again, the damping effect of piezoelectric layers is neglected.

While working in reduced state space control model may result in a system design with uncontrolled modes, neglecting the damping effect of piezoelectric layers may masquerade a possible instability of the negative velocity feedback system (WANG; QUEK; ANG,

2001). Therefore, this work proposes the design of flextensional actuators by the fully coupled constitutive matrices applying the TOM to a time domain transient analysis problem.

1.3 Motivation

The constant demand from electronic industry for miniaturized devices and structures with micro scale displacements for fast response and rapid stabilization applications, motivates the development of innovative methodologies to the design of such devices. Until now, we have seen works focusing on the optimization of intelligent structures aiming vibration suppression and improvement of some structure parameter performance, where different approaches to that end were taken ((RUIZ *et al.*, 2013), (ALI; DHINGRA, 2014), (SILVEIRA; FONSECA, 2010), (MOLTER *et al.*, 2010), (VEEN; LANGELAAR; KEULEN, 2014)). As seen in previous sections, some authors distributed piezoceramic material associated with optimal control law in frequency domain (PADOIN *et al.*, 2015), others have worked in time domain with simpler classical control implementation, but also distributing piezoelectric material (ZHANG; KANG, 2014). However, the possibility of modifying the hosting structure stiffness (structure supporting the piezoceramics) by optimizing the distribution of elastic material in time domain analysis, opens new branches of studying intelligent structures, such as flexible actuators with vibration attenuation, and flexible structures actuated to find its steady state at an operating point.

Regarding the proposed idea of integrating a predefined control gain to the TOM of the flexible structure is supported by the fact that, in general, control strategies are adjusted in the field of application, as it happens with PID control technique (OGATA; YANG, 1970), still largely used in industry nowadays. On the contrary, what this work proposes is that the structure adapts to a predefined controller and, in addition to that, its formulation takes into account the piezoelectric intermediate nodes, which is crucial to simulate external circuits for other types of control such as the PID. The advantage of this approach is that in the configuration proposed, the ceramics regulates multiple degrees of freedom and, besides the fact that time domain models allow a practical implementation, this kind of implementation classifies in a distributed parameter control system with no model reduction, avoiding spillover of the uncontrolled modes.

1.4 Objective

The objective of this work is to design flextensional piezoactuators that operates with an active control law focusing on dynamical applications. The topology optimization method is the structural optimization procedure chosen to design the layout of the studied transducers when applying classical control techniques. The active velocity feedback control implemented and analyzed in time-domain interfere to the void and solid distribution within the domain, and therefore to the structural damping of the hole system. In this context, this work analyzes the

gain velocity value influence in the layout design and in the final FPEA performance. The gain value is chosen before the optimization procedure started.

Two types of self-controlled transducer design are considered:

- a vibration suppression device operating in restoring position, analyzed from the moment an initial disturbance is released;
- a positioning actuator with vibration attenuation around its steady state position, analyzed from the moment a permanent disturbance is applied.

In both cases the system may operate under transient electric or mechanical load. For that end, the proposed idea is to minimize the vibration energy around the transducer working position in the presence of a constant velocity feedback gain adding extra damping to the system due to the piezoelectric coupling effect. The problem is limited to a maximum desired volume to the flexible structure.

1.5 Scientific Contribution

The main scientific contribution this work presents is the development of the TOM for smart structures integrated with an active velocity feedback control in time domain to the layout design of a controlled flextensional piezoelectric structure. In general, the control strategies adopted for the design of smart structures are in frequency domain, however time domain control implementations are more realistic, and this study shows the steps taken to overcome the difficulties of setting up the parameters for the dynamic optimization problem in time. Through this work it is expected from future works to better comprehend how to apply other control strategies in time, such as the PID controller, for the design of innovative structures using structure optimization methods.

In this work, the fully piezoelectric coupling effect is analyzed, and it is shown that the intermediate piezoceramic nodes contributes to the additional damping effect that is sought to attenuate the vibration phase of the actuated flextensional transducers. In addition, it is shown that the non-collapsed nodes numerical model has different structural stiffness if compared to the collapsed nodes model, what leads the optimization problem to distinct local minimas, resulting in distinct topologies for each case. Therefore, the non-collapsed nodes consideration might contribute to obtain more realistic numerical simulation of piezoelectric transducers that were designed by numerical methods.

Additionally, an innovative transducer design by means of the TOM is experimented in this work. Inspired by applications that uses a feedback controller to attenuate vibration response of the structure at its actuated position, a positioner piezoactuator has been obtained through the TOM, and this design may contribute to involve other requirements to the TO problem

formulation aiming the design of structures better adapted to their application functioning and the proposed control scheme.

The general approach brought to discussion in this work allows the design of not only piezoelectric vibration suppression systems or positioning actuators, but also of sensors operating for dynamic measurements, in a systematic fashion.

1.6 Thesis Organization

Chapter 2 describes the isotropic piezoceramic properties and constitutive equations, followed by the piezoelectric variational problem derived from energy principles, for the dynamic finite element formulation of piezoelectric transducers. Chapter 3 introduces the FE formulation for the active velocity feedback control focusing on flextensional piezoactuators and on flextensional piezoelectric positioners with vibration suppression designs. In Chapter 4, it is described the material model implemented for the topology optimization method procedure, next the general TOM for transient problems and their sensitivity analysis are described. Chapter 5 deals with the numeric implementation of the time integration for the direct and the adjoint problems, the structure matrices differentiation necessary for the adjoint problem, the parameters set-up for the linear programming solver, and the projection filter applied to the material model guaranteeing a well-posed TO problem. In Chapter 6, a study on the TO convergence for transient problem is presented. Next, the TOM is applied for the design of bi-material piezoelectric actuators with response control and for solid-void distribution for controlled flextensional piezoactuator designs. A discussion on the results for non-collapsed and collapsed nodes models is made. Yet, the positioner piezoactuator is designed by means of the TOM for both the non-collapsed and the collapsed nodes model. Chapter 7 discusses the overall results obtained and suggests new possibilities for future works.

FE FORMULATION FOR THE DYNAMIC PIEZOELECTRIC TRANSDUCERS

In this chapter, the intrinsic dielectricity and piezoelectricity properties of piezoelectric materials, and their constitutive equations, are presented in sections 2.1, 2.2 and 2.3, respectively. These concepts are crucial to the understanding of energy conversion applied to the design of an active controller for piezoelectric transducers. Next, the continuum formulation for an electro-mechanical multiphysics media is presented based on the piezoelectric constitutive equations, disregarding the non-linear effects of the piezoelectric material and its microscopic properties. In the last section, the dynamic discrete formulation is written based on the .

2.1 Dielectricity

As part of the electromagnetic phenomena, the dielectric phenomena results from the interaction of the material with the electromagnetic fields, and is therefore related to Maxwell's equations, which governs all the macroscopic electromagnetic phenomena. A dielectric material allows the passage of the dielectric field or electric flux, however it does not permit the passage of any kind of particles, including electrons.

As it is known, Maxwell's equations imply that at any point inside a material subjected to an external electromagnetic field there exist four vector fields, being the electric field $\mathbf{E}$, the electric flux density or electric displacement $\mathbf{D}$ (displacement current, such as electromagnetic waves in free space), the magnetic field $\mathbf{H}$ and the magnetic flux density or magnetic induction

$\mathbf{B}$, valid for a time-varying current:

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \quad (2.1)$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \quad (2.2)$$

$$\nabla \cdot \mathbf{B} = 0, \quad (2.3)$$

$$\nabla \cdot \mathbf{D} = q, \quad (2.4)$$

where $\mathbf{J}$ is a vector denoting the electric current density (conduction current, such as magnetic fields in electrical machines), and q is the volume free charge density, which is usually zero.

Despite the absence of a magnetic field in the developments therein proposed, its influence is briefly explained in order to elucidate the permittivity constant of the piezoelectric media. The electric displacement and the electric field are related by the permittivity ε of the medium, equation (2.5), and the magnetic induction is linked with the magnetic field by the permeability μ of the medium, equation (2.6).

$$\mathbf{D} = \varepsilon \mathbf{E}, \quad (2.5)$$

$$\mathbf{B} = \mu \mathbf{H}. \quad (2.6)$$

Considering that a gas media is close to the so-called free space at the normal range of temperature and pressures, in free space the permeability of the media is μ_0

$$\mu_0 = 1.257 \times 10^{-6} \text{ henry m}^{-1}, \quad (2.7)$$

and according to the wave theory, the velocity of electromagnetic waves (light) is

$$c_0 = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3 \times 10^8 \text{ ms}^{-1}. \quad (2.8)$$

At these conditions, the permittivity ε_0 of the free space is

$$\varepsilon_0 = 8.854 \times 10^{-12} \text{ farad m}^{-1}. \quad (2.9)$$

For nonmagnetic materials, we can assume that the permeability is constant $\mu = \mu_0$ (KAO, 2004).

Piezoelectric materials are inherently anisotropic and their closed forms of static and dynamic solutions can not be evaluated, except for the special case of transverse isotropy (KÖGL; GAUL, 2000). At any point of a linear isotropic and homogeneous media, $\mathbf{D}$ is parallel to $\mathbf{E}$. The fourth of Maxell's equations (2.4) states the Gauss' law from which the Poisson's equation can be derived considering that the permittivity ε of the region is independent of the field:

$$\nabla \cdot \mathbf{E} = \frac{q}{\varepsilon}, \quad (2.10)$$

In the absence of a changing magnetic field $\mathbf{B}$, the curl of the electric field is defined by a scalar electric potential field ϕ

$$\mathbf{E} = -\nabla\phi, \quad (2.11)$$

and the Poisson's equation can be written in terms of scalar parameters:

$$\nabla^2\phi = -\frac{q}{\varepsilon}. \quad (2.12)$$

2.2 Piezoelectricity

The physical nature of piezoelectric materials can be exemplified by the most common piezoelectric crystal, the quartz. The intrinsic property of piezoelectric materials is the capability to transform mechanical energy into electric energy and vice versa. This phenomenon is deduced from the piezoelectric constitutive equations:

$$\begin{aligned} T_{ij} &= c_{ijkl}^E S_{kl} - e_{ijk} E_k, \\ D_i &= e_{ikl} S_{kl} + \varepsilon_{ik}^S E_k, \end{aligned} \quad (2.13)$$

where T_{ij} and S_{ij} are, respectively, the *Cauchy* stress tensor and the linear (Lagrangian) strain tensor. The physical constants matrices c_{ijkl} , e_{kij} and ε_{ik} are, respectively, the elastic matrix, the piezoelectric coupling tensor, and the dielectric (or permittivity) tensor. The superscript E indicates that the elastic matrix is evaluated under a constant electric field and the superscript S indicates that the dielectric tensor is evaluated under a constant strain, whereas the subscripts ij or kl derive from the anisotropic characteristic of a piezoelectric ceramic. The physical constants relate the direction of the applied mechanical or electric force and the directions perpendicular to this force. Axes directions X , Y and Z are represented by the subscript 1, 2 or 3, respectively, and the shear about one of these axes is represented by the subscripts 4, 5 or 6, respectively.

Considering that the mechanical stress contains only six independent components, a compact piezo-matrix notation is introduced (SHARAPOV, 2011). This notation consists of replacing the pair of indices, ik or kl by single indices, m or n assuming values from 1 to 6 as in Table 1, and thus, the elastic matrix $\mathbf{c}_{mn}^E$, the piezoelectric tensor $\mathbf{e}_{mk}$ and the permittivity tensor $\mathbf{\varepsilon}_{ik}^S$ have, respectively, 21, 18 and 6 constants, shown in appendix A by matrices (A.2) to (A.4).

Tensorial properties of crystal elements such as elasticity module and dielectric permeability, vary in crystallographic directions. While polycrystalline ceramics with its numerous randomly oriented crystal grains should be isotropic, ferroelectric ceramics such as zirconate lead titanate (PZT), preliminarily polarized by heating in electric field, is anisotropic. In this work, the symmetry type of PZT is associated with the crystallographic class $6mm$ of the hexagonal crystal system (MEITZLER *et al.*, 1988). Thus, the PZT-5A ceramic, like most piezoelectric materials used in electromechanical applications, if poled in z_3 -direction, appendix A figure

Table 1 – Piezoelectric constants notation

ij or kl	m or n
11	1
22	2
33	3
23 or 32	4
31 or 13	5
12 or 21	6

69a, with transversely isotropic material properties (plane x_1 - y_2), has the three aforementioned matrices elements reduced to 5, 3 and 2 constants, respectively (SOSA; CASTRO, 1993):

$$\mathbf{c}^E = \begin{bmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & 0 \\ c_{12} & c_{11} & c_{13} & 0 & 0 & 0 \\ c_{13} & c_{13} & c_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{66} \end{bmatrix}, \quad (2.14)$$

$$\text{where } c_{66} = \frac{c_{11} - c_{12}}{2},$$

$$\mathbf{e} = \begin{bmatrix} 0 & 0 & 0 & 0 & e_{15} & 0 \\ 0 & 0 & 0 & e_{15} & 0 & 0 \\ e_{31} & e_{31} & e_{33} & 0 & 0 & 0 \end{bmatrix}, \quad (2.15)$$

$$\boldsymbol{\epsilon}^S = \begin{bmatrix} \epsilon_{11} & 0 & 0 \\ 0 & \epsilon_{11} & 0 \\ 0 & 0 & \epsilon_{33} \end{bmatrix}. \quad (2.16)$$

The continuum piezoelectric elastic media is defined here for two spacial coordinates x and y , avoiding huge computational costs in analysis and post-processing when compared to tridimensional models. Therefore, for the load cases that are studied in this work, the *plane stress* condition is stated, where the stress components in thickness direction as well as the electric displacement component of vector $\mathbf{D}$ are equal to zero:

$$T_{zz} = T_{xz} = T_{yz} = D_z = 0. \quad (2.17)$$

After the mathematical manipulations shown in appendix A, the piezoelectric material

matrices for *plane stress* assumption becomes:

$$\mathbf{c}^E = \begin{bmatrix} c_{11} - \frac{c_{12}^2}{c_{11}} & c_{13} - \frac{c_{12}c_{13}}{c_{11}} & 0 \\ c_{13} - \frac{c_{12}c_{13}}{c_{11}} & c_{33} - \frac{c_{13}^2}{c_{11}} & 0 \\ 0 & 0 & c_{66} \end{bmatrix}, \quad (2.18)$$

$$\mathbf{e} = \begin{bmatrix} 0 & e_{31} - \frac{e_{31}c_{12}}{c_{11}} \\ 0 & e_{33} - \frac{e_{31}c_{13}}{c_{11}} \\ e_{15} & 0 \end{bmatrix}, \quad (2.19)$$

$$\boldsymbol{\epsilon}^S = \begin{bmatrix} -\epsilon_{11} & 0 \\ 0 & -\epsilon_{33} - \frac{e_{31}^2}{c_{11}} \end{bmatrix}. \quad (2.20)$$

2.3 Equilibrium equations of piezoelectric media

Assuming infinitesimal deformation and linear electric and structural fields, linear equations for piezoelectricity can be obtained from the nonlinear electroelastic equations shown in (TIERSTEN, 1971). For the transducer design, the governing equations are deduced for a solid Ω formed by a piezoelectric domain Ω_1 and an elastic domain Ω_2 ($\Omega = \Omega_1 \cup \Omega_2$) inside a fixed domain $\bar{\Omega}$ ($\Omega \subseteq \bar{\Omega}$) to which constraints and loads are defined.

Therefore, the boundary surface of the system $\bar{\Omega}$, denoted by Γ , is partitioned into Γ_u for prescribed mechanical displacement, Γ_T for prescribed traction vector, Γ_ϕ for the electroded part, where the electric potential is no more than a function of time, and Γ_D for the unelectroded part (YANG, 2010). These partitions can be written as:

$$\begin{aligned} \Gamma_u \cup \Gamma_T &= \Gamma_\phi \cup \Gamma_D = \Gamma, \\ \Gamma_u \cap \Gamma_T &= \Gamma_\phi \cap \Gamma_D = 0. \end{aligned} \quad (2.21)$$

That said, given an outward normal vector $\mathbf{n}$, the boundary conditions of this system (Fig. 4) may be stated for its partitions as bellow:

$$\begin{aligned} u_i &= \bar{u} \in \Gamma_u, \\ T_{ij}n_j &= \bar{t}_j \in \Gamma_T, \\ \phi &= \bar{\phi} \in \Gamma_\phi, \\ D_jn_j &= -\bar{q}_f \in \Gamma_D, \end{aligned} \quad (2.22)$$

where u_i is the mechanical displacement, $(\bar{u}, \bar{\phi})$ are the *Dirichlet* boundary values and $(\bar{t}_j, \bar{q}_f)$ are the *Neumann* boundary values, being $\bar{q}_f$ the free charge density per unit surface area. The electrodes are usually realized via vaporized metal layers with high conductivity leading to equipotential surfaces and, considering very thin electrodes, the mechanical effects such

as inertia and stiffness of the electrodes can be neglected (YANG, 2010). When an electrical potential $\bar{\phi}$ is prescribed over the electrode partition Γ_ϕ , ϕ does not vary spatially. Besides, on an electrode Γ_ϕ , the total free electric charge given by *Gauss's* law is:

$$Q = \int_{\Gamma_\phi} -n_i D_i d\Gamma_\phi, \quad (2.23)$$

Figure 4 – Boundary conditions for the flextensional piezoelectric transducer fixed domain $\bar{\Omega}$.

Having defined the boundary surface for piezoelectric media, the work (MINDLIN, 1961) first derived the piezoelectric constitutive equations from energy principles. Disregarding the heat flux on *Mindlin's* equation, it takes the form:

$$\dot{\mathcal{U}} = T_{ij} \dot{S}_{ij} + E_i \dot{D}_i, \quad (2.24)$$

where the potential energy $\mathcal{U}$ is caused by conservative forces on the body Ω , and therefore composed by a mechanical strain energy $\mathcal{U}_{st}$ and a electrostatic field energy $\mathcal{U}_{el}$:

$$\mathcal{U} = \mathcal{U}_{st} + \mathcal{U}_{el}. \quad (2.25)$$

The term involving the mechanical strain is given by

$$\mathcal{U}_{st} = \frac{1}{2} c_{ijkl}^E S_{ij} S_{kl}, \quad (2.26)$$

and the term involving the electrostatic field is given by

$$\mathcal{U}_{el} = \frac{1}{2} \epsilon_{ij}^S E_i E_j. \quad (2.27)$$

The electric field given by equation (2.11) is rewritten,

$$E_i = -\phi_{,i}, \quad (2.28)$$

and the strain tensor S_{ij} is related to the mechanical displacement vector u_j by

$$S_{ij} = \frac{1}{2} (u_{i,j} + u_{j,i}). \quad (2.29)$$

The electric enthalpy $\mathcal{H}$ defined in (MASON, 1950) and (TIERSTEN; MINDLIN, 1962)

$$\mathcal{H} = \frac{1}{2}T_{ij}\dot{S}_{ij} - \frac{1}{2}E_i D_i, \quad (2.30)$$

is differentiated with respect to time and, by substituting (2.24) into the time derivative of equation (2.30), gives

$$\dot{\mathcal{H}} = T_{ij}\dot{S}_{ij} - D_i\dot{E}_i. \quad (2.31)$$

Motivated by equation (2.31), given the electric displacement-electric field relation (2.5) and the stress-strain relation for linear elastic media, and by considering the linear piezoelectric effect only, the electric enthalpy $\mathcal{H}$ is obtained from the *Gibbs'* functional ((NYE, 1985), (IKEDA, 1996), (DAMJANOVIC, 1998), (ALTAY; DÖKMECI, 2017)):

$$\mathcal{H} = \frac{1}{2}c_{ijkl}^E S_{ij}S_{kl} - e_{kij}E_k S_{ij} - \frac{1}{2}\epsilon_{ik}^S E_i E_k. \quad (2.32)$$

By taking the derivative of equation (2.32) with respect to the mechanical strain and the electric potential, the piezoelectric constitutive equations (2.13) are obtained (TIERSTEN, 1967):

$$\begin{aligned} T_{ij} &= \frac{\partial \mathcal{H}}{\partial S_{ij}}, \\ D_i &= -\frac{\partial \mathcal{H}}{\partial E_i}. \end{aligned} \quad (2.33)$$

It is assumed that all S_{ij} are independent in differentiating $\mathcal{H}$, and that uniqueness in solution is achieved by defining the initial and boundary conditions, $\dot{u}_j$ and (2.22), at each point of the body $\bar{\Omega}$ at initial time t_0 .

2.4 Variational Formulation

For a dynamic system, the elastic behavior of piezoelectric media is given by *Newton's* Law for the stress equation of motion

$$T_{ij,i} = \zeta \frac{\partial^2 u_j}{\partial t^2}, \quad (2.34)$$

and the electric behavior in Γ_D is given by *Maxwell's* equation (2.4)

$$D_{i,i} = 0, \quad (2.35)$$

forming the phenomenologic equations of piezoelectricity (NAILLON; COURSAINT; BESNIER, 1983). In equation (2.34), ζ is the specific weight, u_j the displacement vector and t indicates time (KÖGL; BUCALEM, 2005). By assuming an insulating material, the electrical balance (2.35) corresponds to the charge balance expressed by *Gauss's* theorem, which is equal to zero inside the piezoceramics (LERCH, 1990).

Given the constitutive equations, boundary conditions and linear equilibrium equations, the variational formulation of a bounded piezoelectric body is derived from the *Hamilton's*

variational principle for the dynamic equations in a time interval t_1 to t_2 ((EERNISSE, 1967), (TZOU; TSENG, 1990)):

$$\delta \int_{t_1}^{t_2} (L + \mathcal{W}) dt = 0, \quad (2.36)$$

where the operator δ denotes the first-order operator of the Lagrangian L , which is defined as the excess of kinetic energy $\mathcal{T}$,

$$\mathcal{T} = \frac{1}{2} \zeta \dot{\mathbf{u}}^\top \dot{\mathbf{u}}, \quad (2.37)$$

over electric entalpy $\mathcal{H}$, written based on equation (2.31), rather than the kinetic energy minus the internal energy (TIERSTEN, 1967). The Lagrangian is then written as

$$L = \int_{\Omega} (\mathcal{T} - \mathcal{H}) d\Omega = \int_{\Omega} \left(\frac{1}{2} \zeta \dot{\mathbf{u}}^\top \dot{\mathbf{u}} - \frac{1}{2} [\mathbf{S}^\top \mathbf{T} - \mathbf{E}^\top \mathbf{D}] \right) d\Omega, \quad (2.38)$$

and

$$\begin{aligned} \delta L &= \frac{1}{2} \left[\int_{\Omega} \zeta \delta \dot{\mathbf{u}}^\top \dot{\mathbf{u}} d\Omega - \int_{\Omega} \delta \mathbf{S}^\top \mathbf{T} d\Omega + \int_{\Omega} \delta \mathbf{E}^\top \mathbf{D} d\Omega \right] = \\ &= \frac{1}{2} \left[\int_{\Omega} \zeta \delta \dot{\mathbf{u}}^\top \dot{\mathbf{u}} d\Omega - \int_{\Omega} \delta \mathbf{S}^\top (\mathbf{c}^E \mathbf{S} - \mathbf{e}^\top \mathbf{E}) d\Omega + \int_{\Omega} \delta \mathbf{E}^\top (\mathbf{e} \mathbf{S} + \boldsymbol{\epsilon}^S \mathbf{E}) d\Omega \right] \end{aligned} \quad (2.39)$$

$$\begin{aligned} \int_{t_1}^{t_2} \delta L dt &= \int_{t_1}^{t_2} \int_{\Omega} \zeta \delta \dot{\mathbf{u}}^\top \dot{\mathbf{u}} d\Omega dt - \int_{t_1}^{t_2} \int_{\Omega} \delta \mathbf{S}^\top (\mathbf{c}^E \mathbf{S} - \mathbf{e}^\top \mathbf{E}) d\Omega dt + \\ &\quad + \int_{t_1}^{t_2} \int_{\Omega} \delta \mathbf{E}^\top (\mathbf{e} \mathbf{S} + \boldsymbol{\epsilon}^S \mathbf{E}) d\Omega dt, \\ \int_{t_1}^{t_2} \delta L dt &= \int_{\Omega} \left[\underbrace{\zeta \delta \mathbf{u}^\top \dot{\mathbf{u}}}_{=0 \rightarrow t_1=t_2} \Big|_{t_1}^{t_2} - \int_{t_1}^{t_2} \zeta \delta \mathbf{u}^\top \ddot{\mathbf{u}} dt \right] d\Omega + \\ &\quad - \int_{t_1}^{t_2} \int_{\Omega} \delta \mathbf{S}^\top (\mathbf{c}^E \mathbf{S} - \mathbf{e}^\top \mathbf{E}) d\Omega dt + \\ &\quad + \int_{t_1}^{t_2} \int_{\Omega} \delta \mathbf{E}^\top (\mathbf{e} \mathbf{S} + \boldsymbol{\epsilon}^S \mathbf{E}) d\Omega dt, \\ \int_{t_1}^{t_2} \delta L dt &= - \int_{t_1}^{t_2} \int_{\Omega} \zeta \delta \mathbf{u}^\top \ddot{\mathbf{u}} d\Omega dt + \\ &\quad - \int_{t_1}^{t_2} \int_{\Omega} \delta \mathbf{S}^\top (\mathbf{c}^E \mathbf{S} - \mathbf{e}^\top \mathbf{E}) d\Omega dt + \\ &\quad + \int_{t_1}^{t_2} \int_{\Omega} \delta \mathbf{E}^\top (\mathbf{e} \mathbf{S} + \boldsymbol{\epsilon}^S \mathbf{E}) d\Omega dt. \end{aligned} \quad (2.40)$$

The virtual work $\delta \mathcal{W}$ is done by external forces, $\mathbf{f}_b$, $\bar{\mathbf{t}}$ and $\mathbf{f}_c$, and by the applied surface charge $\bar{q}$ and point charge Q_c in the presence of an electric potential on the electrodes ϕ :

$$\delta \mathcal{W} = \int_{\Omega} \delta \mathbf{u}^\top \mathbf{f}_b d\Omega + \int_{\Gamma_T} \delta \mathbf{u}^\top \bar{\mathbf{t}} d\Gamma_T + \delta \mathbf{u}^\top \mathbf{f}_c - \int_{\Gamma_\phi} \bar{q} \delta \phi d\Gamma_\phi - \delta \phi Q_c, \quad (2.41)$$

$$\begin{aligned} \int_{t_1}^{t_2} \delta \mathcal{W} dt = & \int_{t_1}^{t_2} \int_{\Omega} \delta \mathbf{u}^\top \mathbf{f}_b d\Omega dt + \int_{t_1}^{t_2} \int_{\Gamma_T} \delta \mathbf{u}^\top \bar{\mathbf{t}} d\Gamma_T dt + \int_{t_1}^{t_2} \delta \mathbf{u}^\top \mathbf{f}_c dt + \\ & - \int_{t_1}^{t_2} \int_{\Gamma_\phi} \bar{q} \delta \phi d\Gamma_\phi dt - \int_{t_1}^{t_2} \delta \phi Q_c dt \end{aligned} \quad (2.42)$$

where $\mathbf{f}_b$ is the body force vector $\bar{\mathbf{t}}$ is the surface traction vector and $\mathbf{f}$ is the concentrated load. From equation (2.36),

$$\begin{aligned} \int_{t_1}^{t_2} \delta L dt + \int_{t_1}^{t_2} \delta \mathcal{W} dt = & - \int_{t_1}^{t_2} \int_{\Omega} \zeta \delta \mathbf{u}^\top \ddot{\mathbf{u}} d\Omega dt - \int_{t_1}^{t_2} \int_{\Omega} \delta \mathbf{S}^\top (\mathbf{c}^E \mathbf{S} - \mathbf{e}^\top \mathbf{E}) d\Omega dt + \\ & + \int_{t_1}^{t_2} \int_{\Omega} \delta \mathbf{E}^\top (\mathbf{e} \mathbf{S} + \boldsymbol{\epsilon}^S \mathbf{E}) d\Omega dt + \\ & + \int_{t_1}^{t_2} \int_{\Omega} \delta \mathbf{u}^\top \mathbf{f}_b d\Omega dt + \int_{t_1}^{t_2} \int_{\Gamma_T} \delta \mathbf{u}^\top \bar{\mathbf{t}} d\Gamma_T dt + \int_{t_1}^{t_2} \delta \mathbf{u}^\top \mathbf{f}_c dt + \\ & - \int_{t_1}^{t_2} \int_{\Gamma_\phi} \bar{q} \delta \phi d\Gamma_\phi dt - \int_{t_1}^{t_2} \delta \phi Q_c dt = 0. \end{aligned} \quad (2.43)$$

Therefore,

$$\begin{aligned} \int_{t_1}^{t_2} \int_{\Omega} \zeta \delta \mathbf{u}^\top \ddot{\mathbf{u}} d\Omega dt + \int_{t_1}^{t_2} \int_{\Omega} \delta \mathbf{S}^\top (\mathbf{c}^E \mathbf{S} - \mathbf{e}^\top \mathbf{E}) d\Omega dt = & \int_{t_1}^{t_2} \int_{\Omega} \delta \mathbf{u}^\top \mathbf{f}_b d\Omega dt + \\ & + \int_{t_1}^{t_2} \int_{\Gamma_T} \delta \mathbf{u}^\top \bar{\mathbf{t}} d\Gamma_T dt + \int_{t_1}^{t_2} \delta \mathbf{u}^\top \mathbf{f}_c dt, \end{aligned} \quad (2.44)$$

$$\int_{t_1}^{t_2} \int_{\Omega} \delta \mathbf{E}^\top (\mathbf{e} \mathbf{S} + \boldsymbol{\epsilon}^S \mathbf{E}) d\Omega dt = \int_{t_1}^{t_2} \int_{\Gamma_\phi} \bar{q} \delta \phi d\Gamma_\phi dt + \int_{t_1}^{t_2} \delta \phi Q_c dt. \quad (2.45)$$

For a small virtual displacement $\delta \mathbf{u}$ of the body Ω or the surface Γ_T , the respective virtual work is $\delta \mathbf{u}^\top \mathbf{f}_b$ and $\delta \mathbf{u}^\top \bar{\mathbf{t}}$. The electrical analog of the virtual work per unit area done by $\bar{q}$ on the surface Γ_ϕ in a small virtual change in the electrical potential $\delta \phi$ is $-\bar{q} \delta \phi$. In addition, the variations $\delta \mathbf{u}$ and $\delta \phi$ vanish at t_1 and t_2 and they are arbitrary inside the volume Ω . By equating the time integrands of the right and left sides of each of the expressions (2.44) and (2.45), the piezoelectric variational formulation is obtained:

$$\int_{\Omega} \zeta \delta \mathbf{u}^\top \ddot{\mathbf{u}} d\Omega + \int_{\Omega} \delta \mathbf{S}^\top (\mathbf{c}^E \mathbf{S} - \mathbf{e}^\top \mathbf{E}) d\Omega = \int_{\Omega} \delta \mathbf{u}^\top \mathbf{f}_b d\Omega + \int_{\Gamma_T} \delta \mathbf{u}^\top \bar{\mathbf{t}} d\Gamma_T + \delta \mathbf{u}^\top \mathbf{f}_c, \quad (2.46)$$

$$\int_{\Omega} \delta \mathbf{E}^\top (\mathbf{e} \mathbf{S} + \boldsymbol{\epsilon}^S \mathbf{E}) d\Omega = \int_{\Gamma_\phi} \bar{q} \delta \phi d\Gamma_\phi + \delta \phi Q_c. \quad (2.47)$$

2.5 Discrete Formulation

In order to obtain the dynamic finite element equations from the weak form of the piezoelectric system, (2.44) and (2.45), the *Galerkin* approximation is used. This method involves

writing the differential equation in the weighted-integral form where the weight functions are chosen to be equal to the linear shape functions $\mathcal{N}_u$ and $\mathcal{N}_\phi$. Therefore, the discretization of the continuous equations relies on the interpolation of the displacement field $\mathbf{u}$ and the electric potential ϕ by means of these shape functions defined, in this work, for the $Q4$ -bilinear finite element. The convenience of applying this technique to solve the piezoelectric equations for the design of optimized transducers, relies on the fact that Topology Optimization procedure is based on a systematic analysis of the physical behavior of the system. Therefore, the FEM is ideal for a design based on computational iterations so the variational formulations of mathematical models can be solved.

After the weighted integrals are calculated and the admissible solutions are found in elementary matrix form (superscript e), the global matrices for a dynamic system are given by the mass matrix $\mathbf{M}_{uu}$, the stiffness matrix $\mathbf{K}_{uu}$, the piezoelectric coupling matrix $\mathbf{K}_{u\phi} = \mathbf{K}_{\phi u}$ and the dielectric permittivity matrix $\mathbf{K}_{\phi\phi}$ (REDDY, 1993; MOETAKEF *et al.*, 1995):

$$\mathbf{M}_{uu}^e = \int_{\Omega_e} \zeta \mathcal{N}_u^\top \mathcal{N}_u d\Omega_e, \quad \mathbf{M}_{uu} = \sum_e \mathbf{M}_{uu}^e, \quad (2.48)$$

$$\mathbf{K}_{uu}^e = \int_{\Omega_e} \mathcal{B}_u^\top \mathbf{c}^E \mathcal{B}_u d\Omega_e, \quad \mathbf{K}_{uu} = \sum_e \mathbf{K}_{uu}^e, \quad (2.49)$$

$$\mathbf{K}_{u\phi}^e = \int_{\Omega_e} \mathcal{B}_u^\top \mathbf{e}^\top \mathcal{B}_\phi d\Omega_e, \quad \mathbf{K}_{u\phi} = \sum_e \mathbf{K}_{u\phi}^e, \quad (2.50)$$

$$\mathbf{K}_{\phi\phi}^e = - \int_{\Omega_e} \mathcal{B}_\phi^\top \boldsymbol{\epsilon}^S \mathcal{B}_\phi d\Omega_e, \quad \mathbf{K}_{\phi\phi} = \sum_e \mathbf{K}_{\phi\phi}^e, \quad (2.51)$$

$$\mathbf{F}_u^e = \int_{\Omega_e} \mathcal{N}_u^\top \mathbf{f}_b d\Omega_e + \int_{\Gamma_T} \mathcal{N}_u^\top \bar{\mathbf{t}} d\Gamma_T, \quad (2.52)$$

$$\mathbf{Q}_p^e = - \int_{\Gamma_\phi} \mathcal{N}_\phi^\top \bar{q} d\Gamma_\phi. \quad (2.53)$$

The subscript uu denotes structural states, the subscript ϕ denotes electrical states, $\mathbf{F}_u$ is the external nodal force vector, $\mathbf{Q}_p$ is the free charge vector and $\mathcal{B}_u$ and $\mathcal{B}_\phi$ are the mechanical and electrical shape functions derivatives, respectively.

Assembling all elementary equations (2.48) to (2.53), the dynamic finite element equilibrium matrix equations for the piezoelectric system are written:

$$\begin{bmatrix} \mathbf{M}_{uu} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{u}} \\ \ddot{\Phi} \end{Bmatrix} + \begin{bmatrix} \mathbf{C}_{uu} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{Bmatrix} \dot{\mathbf{u}} \\ \dot{\Phi} \end{Bmatrix} + \begin{bmatrix} \mathbf{K}_{uu} & \mathbf{K}_{u\phi} \\ \mathbf{K}_{u\phi}^\top & -\mathbf{K}_{\phi\phi} \end{bmatrix} \begin{Bmatrix} \mathbf{u} \\ \Phi \end{Bmatrix} = \begin{Bmatrix} \mathbf{F}_u \\ \mathbf{Q} \end{Bmatrix}. \quad (2.54)$$

Because all structures are lightly damped, an artificial linear viscous damping matrix $\mathbf{C}_{uu}$ is added to equation (2.54), which is defined as a proportional damping,

$$\mathbf{C}_{uu} = \alpha \mathbf{M}_{uu} + \beta \mathbf{K}_{uu}, \quad (2.55)$$

where α and β are *Rayleigh's* coefficients, that are non-negative and small in the sense that the effective damping coefficients is $\ll 1$ (DONG *et al.*, 2014), and are considered to be design-independent (TCHERNIAK, 2002; ZHAO; WANG, 2017).

Now, the previous dynamic system (2.54) can be expanded so that it contemplates the elements belonging to the electroded and the non-electroded domains:

$$\begin{aligned}
 & \begin{bmatrix} \mathbf{M}_{uu} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{u}} \\ \ddot{\Phi}_f \\ \ddot{\Phi}_p \end{Bmatrix} + \begin{bmatrix} \mathbf{C}_{uu} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{Bmatrix} \dot{\mathbf{u}} \\ \dot{\Phi}_f \\ \dot{\Phi}_p \end{Bmatrix} + \\
 & + \begin{bmatrix} \mathbf{K}_{uu} & \mathbf{K}_{u\phi_f} & \mathbf{K}_{u\phi_p} \\ \mathbf{K}_{u\phi_f}^\top & -\mathbf{K}_{\phi_f\phi_f} & -\mathbf{K}_{\phi_f\phi_p} \\ \mathbf{K}_{u\phi_p}^\top & -\mathbf{K}_{\phi_f\phi_p}^\top & -\mathbf{K}_{\phi_p\phi_p} \end{bmatrix} \begin{Bmatrix} \mathbf{u} \\ \Phi_f \\ \Phi_p \end{Bmatrix} = \begin{Bmatrix} \mathbf{F}_u \\ \mathbf{Q}_f \\ \mathbf{Q}_p \end{Bmatrix}, \quad (2.56)
 \end{aligned}$$

where the subscript f refers to degrees-of-freedom on the non-electroded domain Γ_D and the subscript p refers to degrees-of-freedom on the electroded domain Γ_ϕ .

FE MODEL FOR AVFC OF PIEZOACTUATORS

The dynamic coupling between the structure and electrical networks has been studied analytically by (CRAWLEY; LUIS, 1987) and (HAGOOD; CHUNG; FLOTOW, 1990) for actuation and control, respectively, for an arbitrary elastic structure with piezoelectric elements. Since then, the increasing power of processing in computers has allowed the numerical studies on vibration of piezoelectric structures through the FE analysis (LERCH, 1990) and (FAN *et al.*, 2017). The concept of applying a control scheme for piezoelectric actuators (PEAs) with the FEM analysis has then been studied by (WANG; QUEK; ANG, 2001), which applied the negative velocity feedback for actively control vibration of smart composite plates with distributed piezoelectric sensors and actuators. Therefore in this chapter, the AVFC is formulated through the FEM for two distinct PEA designs. It is described the formulation for a controlled 2D-solid FE aiming vibration suppression of a transient response of structures vibrating around its restoring position, and for structures vibrating at an actuated position, such as positioners transducers. The FEM formulation is described for the non-collapsed piezoelectric nodes concept, which includes, within the FEM dynamic analysis, the influence of the piezoelectric nodes located between the ceramic electrodes.

3.1 Active Velocity Feedback for Vibration Suppression

The problem of vibration suppression on smart structures, addressed by performing the velocity feedback control, results in the induction of an extra damping to the dynamic equilibrium equation that may be referred to as an active damping to the structure.

Therefore, to define the coupling dynamical system with the velocity feedback control,

an expansion of the system of equations (2.56) is considered as follows:

$$\mathbf{M}_{uu}\ddot{\mathbf{u}} + \mathbf{C}_{uu}\dot{\mathbf{u}} + \mathbf{K}_{uu}\mathbf{u} + \mathbf{K}_{u\phi_f}\Phi_f + \mathbf{K}_{u\phi_p^s}\Phi_p^s + \mathbf{K}_{u\phi_p^a}\Phi_p^a = \mathbf{F}_u, \quad (3.1)$$

$$\mathbf{K}_{\phi_f u}\mathbf{u} - \mathbf{K}_{\phi_f \phi_f}\Phi_f - \mathbf{K}_{\phi_f \phi_p^s}\Phi_p^s - \mathbf{K}_{\phi_f \phi_p^a}\Phi_p^a = \mathbf{Q}_f, \quad (3.2)$$

$$\mathbf{K}_{\phi_p^s u}\mathbf{u} - \mathbf{K}_{\phi_p^s \phi_f}\Phi_f - \mathbf{K}_{\phi_p^s \phi_p^s}\Phi_p^s - \mathbf{K}_{\phi_p^s \phi_p^a}\Phi_p^a = \mathbf{Q}_p^s, \quad (3.3)$$

$$\mathbf{K}_{\phi_p^a u}\mathbf{u} - \mathbf{K}_{\phi_p^a \phi_f}\Phi_f - \mathbf{K}_{\phi_p^a \phi_p^s}\Phi_p^s - \mathbf{K}_{\phi_p^a \phi_p^a}\Phi_p^a = \mathbf{Q}_p^a. \quad (3.4)$$

where the superscripts s and a stands for the sensor and actuator layers, respectively.

The active velocity feedback control emerges from the time-domain readings of the electrical charges generated at the sensor electrodes. In order to make those readings available, a null voltage is imposed to the sensor electrodes $\Phi_p^s = 0$. A strained sensor that is only poled in the z -direction will induce an electric displacement on the sensor surfaces and the electrical charges $\mathbf{Q}_p^s(t)$ generated under a transient input load is converted to an oscillating electric current input to the actuator layer (Figure 5). Equation (3.3) is regarded to the top and bottom surfaces of the sensor layer, Γ_ϕ^{s+} and Γ_ϕ^{s-} , respectively, obtained from the *Gauss's* law (2.23).

For a fully statement of the control problem formulation, the electrodes of the sensor layer, superscript s , are set to a null electric voltage, and the input voltage (load) Φ_p^a at the actuator layer, superscript a , is prescribed by the controlled signal.

$$\phi = 0 \quad \text{on } \Gamma_{\phi_{\text{gnd}}}, \quad (3.5)$$

$$\phi = \mathcal{K}I^s \quad \text{on } \Gamma_{\phi_{\text{id}}}, \quad (3.6)$$

where (3.5) is the homogeneous *Dirichlet* boundary condition and (3.6) is the inhomogeneous *Dirichlet* boundary condition for a current amplification multiplier given by $\mathcal{K}$. Given the existence of an electric field Φ_f in the free nodes located between the electrodes, the electric charges at these internal piezoelectric nodes $\mathbf{Q}_f$ are null (insulator). Yet, as the electrodes from sensor and the actuator layers are separated from each other, the stiffness matrix elements $\mathbf{K}_{\phi_p^s \phi_p^a}$, equation (3.3), are null.

$$\mathbf{M}_{uu}\ddot{\mathbf{u}} + \mathbf{C}_{uu}\dot{\mathbf{u}} + \mathbf{K}_{uu}\mathbf{u} + \mathbf{K}_{u\phi_f}\Phi_f + \mathbf{K}_{u\phi_p^a}\Phi_p^a = \mathbf{F}_u \quad (3.7)$$

$$\mathbf{K}_{\phi_f u}\mathbf{u} - \mathbf{K}_{\phi_f \phi_f}\Phi_f - \mathbf{K}_{\phi_f \phi_p^a}\Phi_p^a = \mathbf{0} \quad (3.8)$$

$$\mathbf{K}_{\phi_p^s u}\mathbf{u} - \mathbf{K}_{\phi_p^s \phi_f}\Phi_f = \mathbf{Q}_p^s, \quad (3.9)$$

$$\mathbf{K}_{\phi_p^a u}\mathbf{u} - \mathbf{K}_{\phi_p^a \phi_f}\Phi_f - \mathbf{K}_{\phi_p^a \phi_p^a}\Phi_p^a = \mathbf{Q}_p^a, \quad (3.10)$$

To establish a velocity feedback control law, the actuator input voltage Φ_p^a is prescribed as a current amplifier (3.6). The equipotential condition at the actuator input electrode is enforced by the vector $\mathbf{I}_{\phi_p^a}$:

$$\Phi_p^a = \mathcal{K}I^s \mathbf{I}_{\phi_p^a}^\top, \quad (3.11)$$

where I^s is the measured current in the top electrode of the sensor layer. In addition to that, I^s is a scalar obtained from equation (3.9) by taking its time derivative and adding the nodal currents

by pre-multiplying it by $\mathbf{I}_{\phi_p^s}$:

$$\mathbf{I}^s = \mathbf{I}_{\phi_p^s} \dot{\mathbf{Q}}_p^s = \mathbf{I}_{\phi_p^s} \left[\mathbf{K}_{\phi_p^s u} \dot{\mathbf{u}} - \mathbf{K}_{\phi_p^s \phi_f} \dot{\Phi}_f \right] \quad (3.12)$$

given $\mathbf{I}_{\phi_p^s} = \mathbf{I}_{\phi_p^a} = \begin{Bmatrix} 1 & \dots & 1 \end{Bmatrix}$ in ϕ_p^s and ϕ_p^a dimensions, respectively. The nodal voltage input at the actuator layer Φ_p^a is then given by substituting equation (3.12) into equation (3.11):

$$\Phi_p^a = \mathcal{H} \mathbf{I}_{\phi_p^a}^\top \mathbf{I}_{\phi_p^s} \mathbf{K}_{\phi_p^s u} \dot{\mathbf{u}} - \mathcal{H} \mathbf{I}_{\phi_p^a}^\top \mathbf{I}_{\phi_p^s} \mathbf{K}_{\phi_p^s \phi_f} \dot{\Phi}_f \quad (3.13)$$

Now, substituting equation (3.13) into equations (3.7), (3.8) and (3.10), we have:

$$\begin{aligned} \mathbf{M}_{uu} \ddot{\mathbf{u}} + \left[\mathbf{C}_{uu} + \mathcal{H} \mathbf{K}_{u\phi_p^a} \mathbf{I}_{\phi_p^a}^\top \mathbf{I}_{\phi_p^s} \mathbf{K}_{\phi_p^s u} \right] \dot{\mathbf{u}} - \mathcal{H} \mathbf{K}_{u\phi_p^a} \mathbf{I}_{\phi_p^a}^\top \mathbf{I}_{\phi_p^s} \mathbf{K}_{\phi_p^s \phi_f} \dot{\Phi}_f + \\ + \mathbf{K}_{uu} \mathbf{u} + \mathbf{K}_{u\phi_f} \Phi_f = \mathbf{F}_u, \end{aligned} \quad (3.14)$$

$$\begin{aligned} - \mathcal{H} \mathbf{K}_{\phi_f \phi_p^a} \mathbf{I}_{\phi_p^a}^\top \mathbf{I}_{\phi_p^s} \mathbf{K}_{\phi_p^s u} \dot{\mathbf{u}} + \mathcal{H} \mathbf{K}_{\phi_f \phi_p^a} \mathbf{I}_{\phi_p^a}^\top \mathbf{I}_{\phi_p^s} \mathbf{K}_{\phi_p^s \phi_f} \dot{\Phi}_f + \\ + \mathbf{K}_{\phi_f u} \mathbf{u} - \mathbf{K}_{\phi_f \phi_f} \Phi_f = \mathbf{0}, \end{aligned} \quad (3.15)$$

$$\begin{aligned} - \mathcal{H} \mathbf{K}_{\phi_p^a \phi_p^a} \mathbf{I}_{\phi_p^a}^\top \mathbf{I}_{\phi_p^s} \mathbf{K}_{\phi_p^s u} \dot{\mathbf{u}} + \mathcal{H} \mathbf{K}_{\phi_p^a \phi_p^a} \mathbf{I}_{\phi_p^a}^\top \mathbf{I}_{\phi_p^s} \mathbf{K}_{\phi_p^s \phi_f} \dot{\Phi}_f + \\ + \mathbf{K}_{\phi_p^a u} \mathbf{u} - \mathbf{K}_{\phi_p^a \phi_f} \Phi_f = \mathbf{Q}_p^a. \end{aligned} \quad (3.16)$$

Equation (3.16) is the electric charge vector generated at the actuator electrode, and can be used for the sake of verification of the total electric current input at the actuator layer.

Therefore, the coupled system to be solved is given by equations (3.14) and (3.15):

$$\begin{bmatrix} \mathbf{M}_{uu} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{u}} \\ \ddot{\Phi}_f \end{Bmatrix} + \begin{bmatrix} \mathbf{C}_{uu} + \mathbf{C}_{11} & \mathbf{C}_{12} \\ \mathbf{C}_{21} & \mathbf{C}_{22} \end{bmatrix} \begin{Bmatrix} \dot{\mathbf{u}} \\ \dot{\Phi}_f \end{Bmatrix} + \begin{bmatrix} \mathbf{K}_{uu} & \mathbf{K}_{u\phi_f} \\ \mathbf{K}_{u\phi_f}^\top & -\mathbf{K}_{\phi_f \phi_f} \end{bmatrix} \begin{Bmatrix} \mathbf{u} \\ \Phi_f \end{Bmatrix} = \begin{Bmatrix} \mathbf{F}_u \\ \mathbf{0} \end{Bmatrix}, \quad (3.17)$$

where $\mathbf{C}_{uu}$ is given by equation (2.55), and

$$\mathbf{C}_{11} = \mathcal{H} \mathbf{K}_{u\phi_p^a} \mathbf{I}_{\phi_p^a}^\top \mathbf{I}_{\phi_p^s} \mathbf{K}_{\phi_p^s u}, \quad (3.18)$$

$$\mathbf{C}_{12} = -\mathcal{H} \mathbf{K}_{u\phi_p^a} \mathbf{I}_{\phi_p^a}^\top \mathbf{I}_{\phi_p^s} \mathbf{K}_{\phi_p^s \phi_f}, \quad (3.19)$$

$$\mathbf{C}_{21} = -\mathcal{H} \mathbf{K}_{\phi_f \phi_p^a} \mathbf{I}_{\phi_p^a}^\top \mathbf{I}_{\phi_p^s} \mathbf{K}_{\phi_p^s u}, \quad (3.20)$$

$$\mathbf{C}_{22} = \mathcal{H} \mathbf{K}_{\phi_f \phi_p^a} \mathbf{I}_{\phi_p^a}^\top \mathbf{I}_{\phi_p^s} \mathbf{K}_{\phi_p^s \phi_f}. \quad (3.21)$$

For the sake of simplifying the representation of the dynamic system above, equation (3.17) is rewritten as below:

$$\mathbf{M} \ddot{\mathbf{U}} + \mathbf{C}_{fb} \dot{\mathbf{U}} + \mathbf{K} \mathbf{U} = \mathbf{F}(t), \quad (3.22)$$

Figure 5 – Velocity feedback block diagram

for

$$\mathbf{M} = \begin{bmatrix} \mathbf{M}_{uu} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}, \quad (3.23)$$

$$\mathbf{C}_{fb} = \begin{bmatrix} \mathbf{C}_{uu} + \mathbf{C}_{11} & \mathbf{C}_{12} \\ \mathbf{C}_{21} & \mathbf{C}_{22} \end{bmatrix}, \quad (3.24)$$

$$\mathbf{K} = \begin{bmatrix} \mathbf{K}_{uu} & \mathbf{K}_{u\phi_f} \\ \mathbf{K}_{u\phi_f}^\top & -\mathbf{K}_{\phi_f \phi_f} \end{bmatrix}, \quad (3.25)$$

$$\mathbf{U} = \begin{Bmatrix} \mathbf{u} \\ \Phi_f \end{Bmatrix}, \quad (3.26)$$

$$\mathbf{F} = \begin{Bmatrix} \mathbf{F}_u \\ \mathbf{0} \end{Bmatrix}, \quad (3.27)$$

$$\mathbf{F}_u = \begin{Bmatrix} \mathbf{0} \\ \vdots \\ F_{\text{dof}} \\ \vdots \\ \mathbf{0} \end{Bmatrix}, \quad (3.28)$$

where $\mathbf{C}_{fb}$ is the nonsymmetric velocity feedback damping matrix and F_{dof} is the concentrated mechanical load.

3.2 Active Velocity Feedback for Positioners

Positioning based on piezoelectric actuators are widely used in high-precision positioning and tracking applications such as atomic force microscopy and scanning tunneling microscopy, figure 1. The AFM actuator mounting, shown in figure 2 have high stiffness, fast frequency

response and high resolution. The positioner designed in this section is therefore inspired by the AFM functioning schematics shown in figure 6, whose scanner tip is the flexible structure actuated by a piezoceramic (MEYER; HUG; BENNEWITZ, 2013). A positioning piezoelectric actuator (PPEA) can be driven by either voltage or current. When driven by voltage, it comes with the disadvantage of having to cope up with nonlinearities such as hysteresis. For PPEAs, hysteresis is a nonlinearity such that the output displacement depends on a combination of the currently applied voltage and some past values of the applied voltage (LIU *et al.*, 2013). Besides the fact that the PPEA designed in this section is driven by voltage, the designed model is linear and though does not take into account the hysteresis effect.

Figure 6 – AFM setup schematics

The FE model for AVFC for positioning systems developed in this section is similar to the development given in section 3.1, except for the transient excitation input. The block diagram of figure 7 shows a voltage input reference $\bar{\phi}$ that actuates the positioner by electrically exciting the actuator electrode. This design also considers the attenuation on the vibration response and the velocity feedback readings given by the sensor layer is $\mathcal{K}$ -amplified. The generated signal adds up to the reference input.

$$\phi = 0 \quad \text{on } \Gamma_{\phi_p^s} \text{ and } \Gamma_{\phi_{\text{gnd}}}, \quad (3.29)$$

$$\phi = \bar{\phi} - \mathcal{K}I^s \quad \text{on } \Gamma_{\phi_p^a}, \quad (3.30)$$

where (3.29) is the homogeneous *Dirichlet* boundary condition and (3.30) is the inhomogeneous *Dirichlet* boundary condition. As explained in section 3.1, $\mathbf{Q}_f = \mathbf{0}$ and $\mathbf{K}_{\phi_p^s \phi_p^a} = \mathbf{0}$. In this example, the transient input force is solely electrical, then $F_{\text{dof}} = 0$. Therefore, the system of equations described below is similar to the system of equations (3.7) to (3.10), except for the

input load and the expression of Φ_p^a :

$$\mathbf{M}_{uu}\ddot{\mathbf{u}} + \mathbf{C}_{uu}\dot{\mathbf{u}} + \mathbf{K}_{uu}\mathbf{u} + \mathbf{K}_{u\phi_f}\Phi_f + \mathbf{K}_{u\phi_p^a}\Phi_p^a = \mathbf{0}, \quad (3.31)$$

$$\mathbf{K}_{\phi_f u}\mathbf{u} - \mathbf{K}_{\phi_f \phi_f}\Phi_f - \mathbf{K}_{\phi_f \phi_p^a}\Phi_p^a = \mathbf{0}, \quad (3.32)$$

$$\mathbf{K}_{\phi_p^s u}\mathbf{u} - \mathbf{K}_{\phi_p^s \phi_f}\Phi_f = \mathbf{Q}_p^s, \quad (3.33)$$

$$\mathbf{K}_{\phi_p^a u}\mathbf{u} - \mathbf{K}_{\phi_p^a \phi_f}\Phi_f - \mathbf{K}_{\phi_p^a \phi_p^a}\Phi_p^a = \mathbf{Q}_p^a. \quad (3.34)$$

The velocity feedback control law in this problem also assumes the current amplifier form given by the $\mathcal{K}I^s$, boundary condition (3.30), where $\mathcal{K}$ is the amplification multiplier. Differently from the problem discussed in section 3.1, the piezoelectric positioner has its stabilization around a point that depends on the structure deformation, that by its end, depends on the input voltage $\bar{\varphi}$ and the material distribution within the given domain. The equipotential condition at Φ_p^a is enforced by the vector $\mathbf{I}_{\phi_p^a}$:

$$\Phi_p^a = \bar{\varphi}\mathbf{I}_{\phi_p^a}^\top - \mathcal{K}I^s\mathbf{I}_{\phi_p^a}^\top, \quad (3.35)$$

where I^s is the measured current in the top electrode of the sensor layer, a scalar obtained from equation (3.12)

$$I^s = \mathbf{I}_{\phi_p^s}^\top \dot{\mathbf{Q}}_p^s = \mathbf{I}_{\phi_p^s}^\top \left[\mathbf{K}_{\phi_p^s u}\dot{\mathbf{u}} - \mathbf{K}_{\phi_p^s \phi_f}\dot{\Phi}_f \right],$$

and therefore

$$\Phi_p^a = \bar{\varphi}\mathbf{I}_{\phi_p^a}^\top - \mathcal{K}\mathbf{I}_{\phi_p^a}^\top \mathbf{I}_{\phi_p^s} \mathbf{K}_{\phi_p^s u}\dot{\mathbf{u}} + \mathcal{K}\mathbf{I}_{\phi_p^a}^\top \mathbf{I}_{\phi_p^s} \mathbf{K}_{\phi_p^s \phi_f}\dot{\Phi}_f, \quad (3.36)$$

recalling that $\mathbf{I}_{\phi_p^s} = \mathbf{I}_{\phi_p^a} = \begin{Bmatrix} 1 & \dots & 1 \end{Bmatrix}$ in ϕ_p^s and ϕ_p^a dimensions, respectively.

Substituting Φ_p^a into equations (3.31) to (3.34) we get:

$$\begin{aligned} \mathbf{M}_{uu}\ddot{\mathbf{u}} + \left[\mathbf{C}_{uu} - \mathcal{K}\mathbf{K}_{u\phi_p^a}\mathbf{I}_{\phi_p^a}^\top \mathbf{I}_{\phi_p^s} \mathbf{K}_{\phi_p^s u} \right] \dot{\mathbf{u}} + \mathcal{K}\mathbf{K}_{u\phi_p^a}\mathbf{I}_{\phi_p^a}^\top \mathbf{I}_{\phi_p^s} \mathbf{K}_{\phi_p^s \phi_f}\dot{\Phi}_f + \\ + \mathbf{K}_{uu}\mathbf{u} + \mathbf{K}_{u\phi_f}\Phi_f = -\bar{\varphi}\mathbf{K}_{u\phi_p^a}\mathbf{I}_{\phi_p^a}^\top, \end{aligned} \quad (3.37)$$

$$\begin{aligned} \mathcal{K}\mathbf{K}_{\phi_f \phi_p^a}\mathbf{I}_{\phi_p^a}^\top \mathbf{I}_{\phi_p^s} \mathbf{K}_{\phi_p^s u}\dot{\mathbf{u}} - \mathcal{K}\mathbf{K}_{\phi_f \phi_p^a}\mathbf{I}_{\phi_p^a}^\top \mathbf{I}_{\phi_p^s} \mathbf{K}_{\phi_p^s \phi_f}\dot{\Phi}_f + \\ + \mathbf{K}_{\phi_f u}\mathbf{u} - \mathbf{K}_{\phi_f \phi_f}\Phi_f = \bar{\varphi}\mathbf{K}_{\phi_f \phi_p^a}\mathbf{I}_{\phi_p^a}^\top, \end{aligned} \quad (3.38)$$

$$\begin{aligned} \mathcal{K}\mathbf{K}_{\phi_p^a \phi_p^a}\mathbf{I}_{\phi_p^a}^\top \mathbf{I}_{\phi_p^s} \mathbf{K}_{\phi_p^s u}\dot{\mathbf{u}} - \mathcal{K}\mathbf{K}_{\phi_p^a \phi_p^a}\mathbf{I}_{\phi_p^a}^\top \mathbf{I}_{\phi_p^s} \mathbf{K}_{\phi_p^s \phi_f}\dot{\Phi}_f + \\ + \mathbf{K}_{\phi_p^a u}\mathbf{u} - \mathbf{K}_{\phi_p^a \phi_f}\Phi_f = \mathbf{Q}_p^a + \bar{\varphi}\mathbf{K}_{\phi_p^a \phi_p^a}\mathbf{I}_{\phi_p^a}^\top. \end{aligned} \quad (3.39)$$

Equations (3.37) and (3.38) in matrix-form become the dynamic equilibrium equation with electric input load to be solved by the topology optimization problem,

$$\begin{bmatrix} \mathbf{M}_{uu} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{u}} \\ \ddot{\Phi}_f \end{Bmatrix} + \begin{bmatrix} \mathbf{C}_{uu} + \mathbf{C}_{11} & \mathbf{C}_{12} \\ \mathbf{C}_{21} & \mathbf{C}_{22} \end{bmatrix} \begin{Bmatrix} \dot{\mathbf{u}} \\ \dot{\Phi}_f \end{Bmatrix} + \begin{bmatrix} \mathbf{K}_{uu} & \mathbf{K}_{u\phi_f} \\ \mathbf{K}_{u\phi_f}^\top & -\mathbf{K}_{\phi_f \phi_f} \end{bmatrix} \begin{Bmatrix} \mathbf{u} \\ \Phi_f \end{Bmatrix} = \begin{Bmatrix} \mathbf{F}_u \\ \mathbf{F}_{\phi_f} \end{Bmatrix}, \quad (3.40)$$

where $\mathbf{C}_{uu}$ is given by equation (2.55), and $\mathbf{C}_{11}$, $\mathbf{C}_{12}$, $\mathbf{C}_{21}$ and $\mathbf{C}_{22}$ are given by equations (3.18) to (3.21), and:

$$\mathbf{F}_u = -\bar{\varphi} \mathbf{K}_{u\phi_p^a} \mathbf{I}_{\phi_p^a}^\top \quad (3.41)$$

$$\mathbf{F}_{\phi_f} = \bar{\varphi} \mathbf{K}_{\phi_f\phi_p^a} \mathbf{I}_{\phi_p^a}^\top. \quad (3.42)$$

Equation (3.39) is the electric charge vector generated at the actuator electrode, and can be used

Figure 7 – Velocity feedback block diagram with electric input

for the sake of verification of the total electric current input at the actuator layer.

For the sake of simplifying the representation of the dynamic system above, equation (3.40) is rewritten as below:

$$\mathbf{M}\ddot{\mathbf{U}} + \mathbf{C}_{fb}\dot{\mathbf{U}} + \mathbf{K}\mathbf{U} = \mathbf{F}_{u:\phi_f}(t), \quad (3.43)$$

for $\mathbf{M}$, $\mathbf{C}_{fb}$, $\mathbf{K}$ and $\mathbf{U}$ given by equations (3.23), (3.24), (3.25) and (3.26), respectively, and

$$\mathbf{F}_{u:\phi_f} = \begin{Bmatrix} -\bar{\varphi} \mathbf{K}_{u\phi_p^a} \mathbf{I}_{\phi_p^a}^\top \\ \bar{\varphi} \mathbf{K}_{\phi_f\phi_p^a} \mathbf{I}_{\phi_p^a}^\top \end{Bmatrix}. \quad (3.44)$$

TOM FOR TRANSIENT ANALYSIS

The TOM is a powerful tool to obtain innovative structures. It employs a material model concept (ROZVANY, 2001) to distribute different material types within a design domain trying to extremize a cost function, and uses the FEM for systematic structure analysis. TOM has been extensively used on the design of multi-actuated piezoelectric structures (CARBONARI; SILVA; PAULINO, 2009), of hard disk actuator arms (KIM; SONG; KIM, 2012) and of energy harvesters (TAKEZAWA *et al.*, 2014a). Inspired by devices for applications that require a response attenuation given a time domain transient input, the dynamic objective function formulated in this work involves an energy measurement that is given by a function of the design variables $\boldsymbol{\rho}$ and the state variables $\{\mathbf{U}\}$ spanned over a time interval $[0, t_f]$. The elements of $\boldsymbol{\rho}$ are the pseudo-densities defined by the concentration of materials involved in the TOM formulation at each finite element.

4.1 Material Model

In this work, the Solid Isotropic Material with Penalization (SIMP) Solid Isotropic Material with Penalization is employed given its broad acceptance within the topology optimization scientific community because of the efficiency it presents among the density-based material models when applied to linear elastic problems (BENDSOE; SIGMUND, 2003). It allows the material properties at each element to assume intermediate values, introducing a relaxation to the TO problem. Based on SIMP, the elasticity matrix $\mathbf{c}^e(\boldsymbol{\rho})$ and the specific weight ζ on the elastic domain Ω , are functions of the design density ρ defined for each finite element e , where $\boldsymbol{\rho}$ is the design variables vector for the optimization problem.

$$\zeta^e(\boldsymbol{\rho}) = \rho^{p_1}(x_c^e, y_c^e) \zeta_{\text{mat1}} + (1 - \rho^{p_1}(x_c^e, y_c^e)) \zeta_{\text{mat2}}, \quad (4.1)$$

$$\mathbf{c}^e(\boldsymbol{\rho}) = \rho^{p_2}(x_c^e, y_c^e) \mathbf{c}_{\text{mat1}} + (1 - \rho^{p_2}(x_c^e, y_c^e)) \mathbf{c}_{\text{mat2}}, \quad (4.2)$$

where p is the penalization factor and (x_c, y_c) is the FE centroid cartesian coordinate pair. In order to illustrate the ponderation between material types one and two when p varies, figure 8 shows the elasticity percentage of each material given by equation (4.2).

Figure 8 – SIMP model for elementary design variable interpolation indicating the percentages of mat1 or mat2.

The relaxation introduced by the penalization factors p_1 and p_2 , for the specific weight ζ and the elasticity matrix $\mathbf{c}$ element properties definition, respectively, increases the number of local minima of the non-convex topology optimization problem as their value increases. To avoid spurious modes, the mass matrix penalization p_1 is smaller than the stiffness matrix penalization p_2 throughout the iterations (DEATON; GRANDHI, 2014), as it is shown in figure 9. The higher the penalization factor, the higher the quantity of possible solutions to the optimization problem. Therefore, by introducing a continuation approach (YIN; ANANTHASURESH, 2001) it helps to alleviate the non unicity solution problem by preventing premature convergence to local minima. The continuation approach presumes the penalization factor initiates with a unitary value, $p = 1$ and continuously increases at a established pace throughout the iterations of the TO algorithm until it reaches a predefined maximum value, as in figure 9.

4.2 Topology Optimization Problem

The general objective function proposed in this section solves the topology optimization (TO) problems defined for the PEAs and the PPEAs designs presented in the last chapter. Unlike the majority of the works found in literature that deals with optimized structure design in static analysis or transient analysis in frequency domains, this work considers a multi-objective function

Figure 9 – Penalty coefficients update

that involves a static function f^{st} and a dynamic function f^{dyn} in time domain:

$$f(\boldsymbol{\rho}) = w_1 f^{\text{st}} + w_2 f^{\text{dyn}}, \quad (4.3)$$

$$f^{\text{dyn}} = \int_0^{t_f} g(\mathbf{U}(t, \boldsymbol{\rho})) dt, \quad (4.4)$$

where w_1 and w_2 are the weights chosen for the static function and the dynamic function, respectively, and g is a function of the state variables in time.

The minimization of the cost function with respect to the design variables $f(\boldsymbol{\rho})$, is subjected to the static and dynamic equilibrium equations, the initial conditions $\mathbf{U}_0$ and $\mathbf{V}_0$, a volume constraint V_{max} that limits the material distribution for a lightweight flexible structure design, and a design variable constraint given by the problem stated as follows:

$$\begin{aligned} \min_{\boldsymbol{\rho}} \quad & f(\boldsymbol{\rho}) = w_1 f^{\text{st}} + w_2 \int_0^{t_f} g(\mathbf{U}(t, \boldsymbol{\rho})) dt \\ \text{s.t.} \quad & \begin{cases} \mathbf{M}(\boldsymbol{\rho})\ddot{\mathbf{U}} + \mathbf{C}_{\text{fb}}(\boldsymbol{\rho})\dot{\mathbf{U}} + \mathbf{K}(\boldsymbol{\rho})\mathbf{U} = \mathbf{F}(t) \\ \mathbf{K}\mathbf{U}_0^{\text{st}} = \mathbf{F}^{\text{st}} \\ \dot{\mathbf{U}}|_{t=0} = \mathbf{V}_0 \\ \mathbf{U}|_{t=0} = \mathbf{U}_0 \\ \sum_{n=1}^N \rho_n V_n \leq V_{\text{max}} \\ 0 \leq \rho_n \leq 1 \end{cases} \end{aligned} \quad (4.5)$$

As it has been discussed, aiming the vibration suppression of controlled piezoelectric actuators, the dynamic objective function defined in this chapter, and applied to the designs treated in this thesis, considers the minimization of the total energy of the displacement signal targeting a predefined degree-of-freedom.

Considering vibration attenuation design (section 3.2), previous works have proposed

dynamic objective functions that deals with the state variables of the system such as:

$$f^{\text{dyn}} = \int_0^{t_f} u_{\text{dof}}^2 dt, \quad (4.6)$$

$$f^{\text{dyn}} = \int_0^{t_f} v_{\text{dof}}^2 dt, \quad (4.7)$$

$$f^{\text{dyn}} = \int_0^{t_f} (u_{\text{dof}}^2 + v_{\text{dof}}^2) dt, \quad (4.8)$$

(JENSEN; NAKSHATRALA; TORTORELLI, 2014), (ZHANG; KANG, 2014). In this work, for the designs proposed in sections 3.1 and 3.2, the dynamic part of the objective function f^{dyn} is based in equation (4.6), where the integrand assumes the general form $g(\mathbf{U}(t, \boldsymbol{\rho}))$ presented in the optimization problem (4.5) for the sake of simplifying the sensitivity equations to be deduced in section 4.3. Therefore, for the design of the PEAs with vibration suppression (section 3.1), g is:

$$g_1(\mathbf{U}(t, \boldsymbol{\rho})) = \mathbf{U}^\top \mathbf{B} \mathbf{U} = u_{\text{dof}}^2(t), \quad (4.9)$$

where u_{dof} is the mechanical displacement measured at "dof", and for the PPEAs design with vibration suppression, g is:

$$g_2(\mathbf{U}(t, \boldsymbol{\rho})) = [\mathbf{U} - \mathbf{U}_{\text{cte}}]^\top \mathbf{B} [\mathbf{U} - \mathbf{U}_{\text{cte}}] = (u_{\text{dof}}(t) - u_{\text{cte}})^2, \quad (4.10)$$

where g_2 is a function that measures the displacement in time of a degree of freedom that shall stabilize around $u_{\text{dof}} = u_{\text{cte}}|_{t \rightarrow \infty}$ when time goes to infinity. The target matrix $\mathbf{B} = \mathbf{B}^\top$, has null elements except at the degree-of-freedom location addressed as "dof":

$$\mathbf{B} = \begin{bmatrix} 0 & \dots & \dots & \dots & \dots & \dots & 0 \\ \vdots & \ddots & & & & & \vdots \\ \vdots & & 0 & & & & \vdots \\ \vdots & & & 1_{\text{dof}} & & & \vdots \\ \vdots & & & & 0 & & \vdots \\ \vdots & & & & & \ddots & \vdots \\ 0 & \dots & \dots & \dots & \dots & \dots & 0 \end{bmatrix}. \quad (4.11)$$

Regarding the piezoelectric positioner design (section 3.2), a strictly dynamic objective function might not be sufficient for the optimization convergence achievement. By trying to minimize the displacement energy over time, $f^{\text{dyn}} = \int_0^{t_f} g_2(\mathbf{U}(t, \boldsymbol{\rho})) dt$, the optimizer defines a topology that brings the oscillation to a stationary point close to the restoring position (deactuated position).

In order to ensure a sufficient rigidity to the structure so the positioner operation point is achieved, a mean compliance function is considered as the static objective function f^{st} , as employed in the works (KIKUCHI *et al.*, 1998) and (CARDOSO; FONSECA, 2004) for the

mean compliant mechanism designs. The static objective functions might be as follows:

$$f^{\text{st}} = (\mathbf{U}^{\text{st}})^{\top} \mathbf{F}_u, \quad (4.12)$$

$$f^{\text{st}} = (\mathbf{U}^{\text{st}})^{\top} \mathbf{F}_v, \quad (4.13)$$

where $\mathbf{F}_u$ is a mechanical load and $\mathbf{F}_v$ is the load induced by a constant electric voltage $\bar{\varphi}$ applied to the actuator electrode $\mathbf{U}_{\phi_p^a} = \bar{\varphi}$.

However, the rigidity provided to the PPEA structure by the optimized compliance function given a mechanical load, equation (4.12), concentrates around the applied load region only, instead of being distributed around the piezoceramic and metallic layers interface. To overcome this issue, the compliance provided by the induced electrical force, equation (4.13), is more appropriate to the PPEA design.

The above discussion follows the attempts to obtain the positioner design for the multi-objective functions (4.14) to (4.18), for which the results will be discussed in chapter 6.

$$f(\boldsymbol{\rho}) = -w_1 \mathbf{U}_u^{\top} \mathbf{L} + w_2 \int_0^{t_f} u_{\text{dof}}^2 dt, \quad (4.14)$$

$$f(\boldsymbol{\rho}) = -w_1 \mathbf{U}_v^{\top} \mathbf{L} + w_2 \int_0^{t_f} u_{\text{dof}}^2 dt, \quad (4.15)$$

$$f(\boldsymbol{\rho}) = w_1 (\mathbf{U}^{\text{st}})^{\top} \mathbf{F}_v + w_2 \int_0^{t_f} u_{\text{dof}}^2 dt - w_3 \mathbf{U}_v^{\top} \mathbf{L}, \quad (4.16)$$

$$f(\boldsymbol{\rho}) = w_1 (\mathbf{U}^{\text{st}})^{\top} \mathbf{F}_v + w_2 \int_0^{t_f} v_{\text{dof}}^2 dt - w_3 \mathbf{U}_v^{\top} \mathbf{L}, \quad (4.17)$$

$$f(\boldsymbol{\rho}) = w_1 (\mathbf{U}^{\text{st}})^{\top} \mathbf{F}_v + w_2 \int_0^{t_f} (u_{\text{dof}} - u_{\text{cte}})^2 dt, \quad (4.18)$$

The functions (4.16) and (4.17) have been considered because while dealing with the dynamic equation (4.6) and to the mean compliance equation (4.13), the optimized device might become too stiff, with small actuation. For higher displacements of the positioner actuation, an intuitive suggestion is to maximize the system transduction or its displacement:

$$f^{\text{st}} = -\mathbf{U}_u^{\top} \mathbf{L}, \quad (4.19)$$

$$f^{\text{st}} = -\mathbf{U}_v^{\top} \mathbf{L}, \quad (4.20)$$

where $\mathbf{U}_u$ is the displacement vector for a mechanical input load and $\mathbf{U}_v$ is the displacement vector for an electric voltage at the actuator electrode. The $\mathbf{L}$ -vector has null elements except at the degree-of-freedom addressed as "dof".

4.3 Sensitivity Analysis

Considering the design problem as an optimization problem in the pseudo-densities $\boldsymbol{\rho}$ only, the sensitivity analysis is derived for the optimization problem proposed for the system (4.5). For problems that involve a moderate number of constraints and a large number of design

variables, the most effective method for calculating derivatives is the adjoint method, where the displacement and velocity derivatives are not calculated explicitly. The adjoint method involves the computation of an adjoint vector $\boldsymbol{\lambda}(t)$ that is chosen as the solution to the adjoint equations derived from a Lagrangian function $\mathcal{L}(\mathbf{U}, \boldsymbol{\lambda})$ (JENSEN; NAKSHATRALA; TORTORELLI, 2014):

$$\begin{aligned} \mathcal{L}(\mathbf{U}, \boldsymbol{\lambda}) = & \int_0^{t_f} g(\mathbf{U}(t, \boldsymbol{\rho})) dt + \\ & + \int_0^{t_f} \boldsymbol{\lambda}^\top(t) \left[\mathbf{M}(\boldsymbol{\rho}) \ddot{\mathbf{U}} + \mathbf{C}_{fb}(\boldsymbol{\rho}) \dot{\mathbf{U}} + \mathbf{K}(\boldsymbol{\rho}) \mathbf{U} - \mathbf{F}(t) \right] dt. \end{aligned} \quad (4.21)$$

Taking the derivative of the Lagrangian (4.21) with respect to each element of the design variables vector $\boldsymbol{\rho}$, an expression involving the derivatives of the structural matrices, the state variables and the objective function itself is obtained:

$$\begin{aligned} \frac{\partial \mathcal{L}(\mathbf{U}, \boldsymbol{\lambda})}{\partial \boldsymbol{\rho}_e} = & \int_0^{t_f} \frac{\partial}{\partial \boldsymbol{\rho}_e} [g(\mathbf{U}(t, \boldsymbol{\rho}))] dt + \\ & + \int_0^{t_f} \boldsymbol{\lambda}^\top(t) \left[\frac{\partial \mathbf{M}(\boldsymbol{\rho})}{\partial \boldsymbol{\rho}_e} \ddot{\mathbf{U}} + \frac{\partial \mathbf{C}_{fb}(\boldsymbol{\rho})}{\partial \boldsymbol{\rho}_e} \dot{\mathbf{U}} + \frac{\partial \mathbf{K}(\boldsymbol{\rho})}{\partial \boldsymbol{\rho}_e} \mathbf{U} - \frac{\partial \mathbf{F}(t)}{\partial \boldsymbol{\rho}_e} \right] dt + \\ & + \int_0^{t_f} \boldsymbol{\lambda}^\top(t) \left[\mathbf{M}(\boldsymbol{\rho}) \frac{\partial \ddot{\mathbf{U}}}{\partial \boldsymbol{\rho}_e} + \mathbf{C}_{fb}(\boldsymbol{\rho}) \frac{\partial \dot{\mathbf{U}}}{\partial \boldsymbol{\rho}_e} + \mathbf{K}(\boldsymbol{\rho}) \frac{\partial \mathbf{U}}{\partial \boldsymbol{\rho}_e} \right] dt. \end{aligned} \quad (4.22)$$

The integration by parts of each term involved in the solution of equation (4.22) are shown in equations (4.23), (4.24) and (4.25):

$$\int_0^{t_f} \frac{\partial}{\partial \boldsymbol{\rho}_e} [g(\mathbf{U}(t, \boldsymbol{\rho}))] dt = \int_0^{t_f} \frac{\partial g}{\partial \mathbf{U}} \frac{\partial \mathbf{U}}{\partial \boldsymbol{\rho}_e} dt, \quad (4.23)$$

$$\begin{aligned} \int_0^{t_f} \mathbf{M}(\boldsymbol{\rho}) \boldsymbol{\lambda}(t) \frac{\partial \ddot{\mathbf{U}}}{\partial \boldsymbol{\rho}_e} dt &= \mathbf{M}(\boldsymbol{\rho}_e) \frac{d}{dt} \left(\frac{\partial \mathbf{U}}{\partial \boldsymbol{\rho}_e} \right) \boldsymbol{\lambda}(t) \Big|_0^{t_f} - \int_0^{t_f} \mathbf{M}(\boldsymbol{\rho}) \dot{\boldsymbol{\lambda}}(t) \frac{d}{dt} \left(\frac{\partial \mathbf{U}}{\partial \boldsymbol{\rho}_e} \right) dt \\ &= \mathbf{M}(\boldsymbol{\rho}) \boldsymbol{\lambda}(t) \frac{d}{dt} \left(\frac{\partial \mathbf{U}}{\partial \boldsymbol{\rho}_e} \right) \Big|_0^{t_f} - \mathbf{M}(\boldsymbol{\rho}) \dot{\boldsymbol{\lambda}}(t) \frac{\partial \mathbf{U}}{\partial \boldsymbol{\rho}_e} \Big|_0^{t_f} + \\ &+ \int_0^{t_f} \mathbf{M}(\boldsymbol{\rho}) \ddot{\boldsymbol{\lambda}}(t) \frac{\partial \mathbf{U}}{\partial \boldsymbol{\rho}_e} dt, \end{aligned} \quad (4.24)$$

$$\int_0^{t_f} \mathbf{C}_{fb}(\boldsymbol{\rho}) \boldsymbol{\lambda}(t) \frac{\partial \dot{\mathbf{U}}}{\partial \boldsymbol{\rho}_e} dt = \mathbf{C}_{fb}(\boldsymbol{\rho}) \boldsymbol{\lambda}(t) \frac{\partial \mathbf{U}}{\partial \boldsymbol{\rho}_e} \Big|_0^{t_f} - \int_0^{t_f} \mathbf{C}_{fb}(\boldsymbol{\rho}) \dot{\boldsymbol{\lambda}}(t) \frac{\partial \mathbf{U}}{\partial \boldsymbol{\rho}_e} dt. \quad (4.25)$$

Rearranging equations (4.23) to (4.25), equation (4.22) is then rewritten:

$$\begin{aligned} \frac{\partial \mathcal{L}(\mathbf{U}, \boldsymbol{\lambda})}{\partial \rho_e} = & \int_0^{t_f} \boldsymbol{\lambda}^\top(t) \left[\frac{\partial \mathbf{M}(\boldsymbol{\rho})}{\partial \rho_e} \ddot{\mathbf{U}} + \frac{\partial \mathbf{C}_{fb}(\boldsymbol{\rho})}{\partial \rho_e} \dot{\mathbf{U}} + \frac{\partial \mathbf{K}(\boldsymbol{\rho})}{\partial \rho_e} \mathbf{U} - \frac{\partial \mathbf{F}(t)}{\partial \rho_e} \right] dt + \\ & + \int_0^{t_f} \left[\mathbf{M}(\boldsymbol{\rho}) \ddot{\boldsymbol{\lambda}}^\top(t) - \mathbf{C}_{fb}(\boldsymbol{\rho}) \dot{\boldsymbol{\lambda}}^\top(t) + \mathbf{K}(\boldsymbol{\rho}) \boldsymbol{\lambda}^\top(t) + \frac{\partial g}{\partial \mathbf{U}}(t) \right] \frac{\partial \mathbf{U}}{\partial \rho_e} dt + \\ & + \frac{\partial \mathbf{U}}{\partial \rho_e} \left[-\mathbf{M}(\boldsymbol{\rho}) \dot{\boldsymbol{\lambda}}^\top(t) + \mathbf{C}_{fb}(\boldsymbol{\rho}) \boldsymbol{\lambda}^\top(t) \right]_0^{t_f} + \\ & + \frac{\partial \dot{\mathbf{U}}}{\partial \rho_e} \left[\mathbf{M}(\boldsymbol{\rho}) \boldsymbol{\lambda}^\top(t) \right]_0^{t_f}, \end{aligned} \quad (4.26)$$

To eliminate the state variables derivatives $\frac{\partial \mathbf{U}}{\partial \rho_e}(t)$ and $\frac{\partial \dot{\mathbf{U}}}{\partial \rho_e}(t)$ from equation (4.26), we calculate the adjoint variable $\boldsymbol{\lambda}^\top$ such that it annihilates the expression in square brackets of the second term by assuming:

$$\mathbf{M}(\boldsymbol{\rho}) \ddot{\boldsymbol{\lambda}}^\top(t) - \mathbf{C}_{fb}(\boldsymbol{\rho}) \dot{\boldsymbol{\lambda}}^\top(t) + \mathbf{K}(\boldsymbol{\rho}) \boldsymbol{\lambda}^\top(t) + \frac{\partial g}{\partial \mathbf{U}}(t) = 0, \quad (4.27)$$

where the adjoint ODE conditions in time are given by the last two terms of (4.26). Note that the adjoint ODE (4.27) is similar to the primal problem except for a sign change on the adjoint velocity term $\mathbf{C}_{fb} \dot{\boldsymbol{\lambda}}$.

Considering the simplifying assumptions $\frac{\partial \mathbf{U}}{\partial \rho_e}(0) = \frac{\partial \dot{\mathbf{U}}}{\partial \rho_e}(0) = 0$, the state variables derivatives from the last two terms of (4.26) are eliminated by the expression to the final conditions for the adjoint ODE (4.27):

$$-\mathbf{M}(\boldsymbol{\rho}) \dot{\boldsymbol{\lambda}}^\top(t_f) + \mathbf{C}_{fb}(\boldsymbol{\rho}) \boldsymbol{\lambda}^\top(t_f) + \mathbf{M}(\boldsymbol{\rho}) \boldsymbol{\lambda}^\top(t_f) = 0. \quad (4.28)$$

As $\mathbf{M}$ and $\mathbf{C}_{fb}$ are not null, then $\dot{\boldsymbol{\lambda}}^\top(t_f) = \boldsymbol{\lambda}^\top(t_f) = 0$. From equations (4.27) and (4.28), the adjoint system that needs to be solved is:

$$\begin{cases} \mathbf{M} \ddot{\boldsymbol{\lambda}}^\top(t) - \mathbf{C}_{fb} \dot{\boldsymbol{\lambda}}^\top(t) + \mathbf{K} \boldsymbol{\lambda}^\top(t) = -\frac{\partial g}{\partial \mathbf{U}}(t) \\ \boldsymbol{\lambda}^\top(t_f) = \mathbf{0} \\ \dot{\boldsymbol{\lambda}}^\top(t_f) = \mathbf{0} \end{cases}. \quad (4.29)$$

In order to redefine the system (4.29) as an initial value primal problem, we apply the change of variables $\tau(t) = t_f - t$, and the adjoint system to be solved is as follows:

$$\begin{cases} \mathbf{M} \ddot{\boldsymbol{\Lambda}}^\top(\tau(t)) + \mathbf{C}_{fb} \dot{\boldsymbol{\Lambda}}^\top(\tau(t)) + \mathbf{K} \boldsymbol{\Lambda}^\top(\tau(t)) = -\frac{\partial g}{\partial \mathbf{U}}(\tau(t)) \\ \boldsymbol{\Lambda}^\top(0) = \mathbf{0} \\ \dot{\boldsymbol{\Lambda}}^\top(0) = \mathbf{0} \end{cases}, \quad (4.30)$$

where $\boldsymbol{\Lambda}(\tau(t)) = \boldsymbol{\lambda}(t)$ and $\dot{\boldsymbol{\Lambda}}(\tau(t)) = -\dot{\boldsymbol{\lambda}}(t)$. Therefore, the sensitivity expression for the transient problem is reduced to:

$$\frac{\partial \mathcal{L}(\mathbf{U}, \boldsymbol{\Lambda})}{\partial \rho_e} = \int_0^{t_f} \boldsymbol{\Lambda}^\top(t_f - t) \left[\frac{\partial \mathbf{M}(\boldsymbol{\rho})}{\partial \rho_e} \ddot{\mathbf{U}} + \frac{\partial \mathbf{C}_{fb}(\boldsymbol{\rho})}{\partial \rho_e} \dot{\mathbf{U}} + \frac{\partial \mathbf{K}(\boldsymbol{\rho})}{\partial \rho_e} \mathbf{U} - \frac{\partial \mathbf{F}(t)}{\partial \rho_e} \right] dt, \quad (4.31)$$

where the structure matrices derivatives are shown in section 5.2.

The term $\frac{\partial g}{\partial \mathbf{U}}(\tau(t))$ in the general adjoint system (4.30) is defined for each of the dynamic objective functions (4.9) and (4.10) applied to the design of the PEAs in this work as below:

$$\frac{\partial g_1}{\partial \mathbf{U}}(\tau(t)) = \mathbf{B}\mathbf{U} + \mathbf{U}^\top \mathbf{B} = \begin{Bmatrix} \mathbf{0} \\ 2u_{\text{dof}}(t) \\ \mathbf{0} \end{Bmatrix}, \quad (4.32)$$

$$\frac{\partial g_2}{\partial \mathbf{U}}(\tau(t)) = \mathbf{B}[\mathbf{U} - \mathbf{U}_{\text{cte}}] + [\mathbf{U} - \mathbf{U}_{\text{cte}}]^\top \mathbf{B} = \begin{Bmatrix} \mathbf{0} \\ 2(u_{\text{dof}}(\tau(t)) - u_{\text{cte}}) \\ \mathbf{0} \end{Bmatrix}, \quad (4.33)$$

recalling that g_1 solves the PEA with vibration suppression described in section 3.1, and g_2 solves the PPEA with vibration suppression described in section 3.2.

Yet, the load vector $\mathbf{F}$ in the sensitivity expression (4.31) is $\mathbf{F} = \mathbf{F}_u$, defined by the mechanical force vector (3.28), for the problem proposed in section 3.1, and $\mathbf{F} = \mathbf{F}_{u:\phi_f}$, defined by the electric force vector (3.44), for the problem proposed in section 3.2.

The adjoint sensitivity analysis for the minimum compliance function (4.13) of the multi-objective function (4.18) implemented for the PPEAs design is deduced given the lagrangian equation:

$$f^{\text{st}}(\mathbf{U}^{\text{st}}, \boldsymbol{\rho}) = (\mathbf{U}^{\text{st}})^\top \mathbf{F}_v + \lambda [\mathbf{K}\mathbf{U}^{\text{st}} - \mathbf{F}_v], \quad (4.34)$$

where $\mathbf{U}^{\text{st}}$ is the state variables vector composed of mechanical and electrical states given $\mathbf{F}_v$,

$$\mathbf{F}_v = -\mathbf{K}_{\phi_p^a} \mathbf{U}_{\phi_p^a}, \quad (4.35)$$

$$\mathbf{U}^{\text{st}} = \mathbf{K}^{-1} \mathbf{F}_v, \quad (4.36)$$

and ϕ_p^a are the electrode degrees-of-freedom from the actuator, that are prescribed with an electrical input.

The derivative of equation (4.34) with respect to the design variables in $\boldsymbol{\rho}$ is given by the following steps:

$$\begin{aligned} \frac{df^{\text{st}}}{d\rho_e} &= \frac{\partial f^{\text{st}}}{\partial \rho_e} + \frac{\partial f^{\text{st}}}{\partial \mathbf{U}^{\text{st}}} \frac{d\mathbf{U}^{\text{st}}}{d\rho_e} + \frac{\partial f^{\text{st}}}{\partial \mathbf{F}_v} \frac{d\mathbf{F}_v}{d\rho_e} + \frac{\partial f^{\text{st}}}{\partial \mathbf{K}} \frac{d\mathbf{K}}{d\rho_e}; \\ \frac{df^{\text{st}}}{d\rho_e} &= \frac{\partial f^{\text{st}}}{\partial \rho_e} + [\mathbf{F}_v + \lambda \mathbf{K}] \frac{d\mathbf{U}^{\text{st}}}{d\rho_e} + [(\mathbf{U}^{\text{st}})^\top - \lambda] \frac{d\mathbf{F}_v}{d\rho_e} + \lambda \frac{d\mathbf{K}}{d\rho_e} \mathbf{U}^{\text{st}}; \\ \mathbf{F}_v + \lambda \mathbf{K} &= \mathbf{0} \rightarrow \lambda = -\mathbf{K}^{-1} \mathbf{F}_v; \\ \frac{df^{\text{st}}}{d\rho_e} &= -\mathbf{K}^{-1} \mathbf{F}_v \left[\frac{d\mathbf{K}}{d\rho_e} \mathbf{U}^{\text{st}} - \frac{d\mathbf{F}_v}{d\rho_e} \right] + (\mathbf{U}^{\text{st}})^\top \frac{d\mathbf{F}_v}{d\rho_e}, \\ \frac{df^{\text{st}}}{d\rho_e} &= -\mathbf{U}^{\text{st}} \frac{d\mathbf{K}}{d\rho_e} \mathbf{U}^{\text{st}} + 2(\mathbf{U}^{\text{st}})^\top \frac{d\mathbf{F}_v}{d\rho_e}, \end{aligned} \quad (4.37)$$

for every element e within the design domain to be optimized. Note that for this minimum compliance function, $\frac{\partial f^{\text{st}}}{\partial \rho_e} = 0$.

NUMERICAL IMPLEMENTATION

The TOM implementation is based on the flowchart shown in figure 10. For the FEM analysis it is required to input the mesh data including the Cartesian coordinates for all nodes, their connectivity, load case and boundary conditions. Next, the topology design variables are assigned to each finite element of the mesh, and the projection filter technique redefines the design variables by a density function. All matrices of the equilibrium equation are assembled and the dynamic system is solved by the α -Newmark time-integration method as described in appendix B. With the measured output position, the objective function is calculated. If the solution converges according to a convergence criteria, the process is terminated. Otherwise, the gradient of the objective function, adjoint sensitivity given by equation (4.31), is calculated to solve the optimization problem with the chosen optimization algorithm, which in this case is the Sequential Linear Programming (SLP) Sequential Linear Programming. The design variables are updated and the next iteration loop initiates.

The input data regards the parameters that define the problem to be optimized. In this work they are considered to be the vibration attenuation and the positioner with vibration attenuation. For either case, it must be specified the size of the domain, its boundary conditions, which material is distributed and whether the ceramic nodes are collapsed. From the topology optimization side, it is chosen the objective functions and the penalization factor. Therein, the following sections describe the implementation procedures in MATLAB®.

5.1 Time Domain Analysis

The objective function for the topology optimization therein proposed requires the controlled equilibrium equation 3.17 to be solved for a transient input load.

The choice for modelling the problem in time-domain has to do with the computational efficiency in acquiring the response of the system comparing to the frequency-domain which

Figure 10 – Flowchart for the implemented optimization procedure

requires frequency samples across a broad bandwidth followed by a *Fourier* (or other) transform to obtain the desired result. In time-domain analysis, the bandwidth is limited only by the frequency content of the excitation and the time and space sampling used to develop the model (RAO, 1999).

In time-integration method, implemented with an implicit method, the number of time iterations required to converge to a final solution must be significantly less than the number of iterations for an explicit method since each time iteration requires a solution of linear system of equations. In mechanical engineering, researchers have used this method extensively in dynamic analysis of beams and trusses. For computation electromagnetics the Newmark implicit method is used coupled with boundary integrals to solve transient electromagnetic field coupling for a metallic enclosure by (BARKESHLI *et al.*, 1992).

In time-domain analysis the excitation shall be smooth to avoid the introduction of the excitation frequency components that exceed the highest frequency for which the model is valid. For impulsive or step-function sources, for example, the initial excitation value would be required to be less than some fraction of its peak, and the rise time would need to extend over some minimum number of time steps (RAO, 1999).

Depending on the choice for the integration parameters, the Newmark Method for time

discretization might be explicit or implicit. While the implicit form is second order accurate and unconditionally stable (any choice of parameters), the stability for the explicit form depends on the mesh size.

For linear elastodynamics problems there is no interaction between different modes during time integration, they are integrated independently of one another, the most accurate time-integration method, without numerical dissipation or artificial viscosity, should be used for the basic computations of equation 3.17, specially for a long-term integration. It means that all modes, including high-frequency modes, are integrated very accurately and the solution includes spurious high frequency oscillations (IDESMAN *et al.*, 2009). Technique for filtering spurious oscillations: first order implicit time continuous Galerkin method with large numerical dissipation (IDESMAN, 2007).

At each evaluation of the Topology Optimization problem show in the flowchart 10, it is required the calculation of the physical and the adjoint state variables for the initial-valued systems (4.5) and (4.30), respectively. The inicial displacement vector is given by either $\mathbf{KU}_0^{\text{st}} = \mathbf{F}_u$ or $\mathbf{KU}_0^{\text{st}} = \mathbf{F}_v$ (equation (4.36)) where $\mathbf{F}_u(t)$, shown in the diagram of figure 11, and $\mathbf{F}_{u:\phi_f}(t)$, induced by the electric voltage $\bar{\phi}$ shown in diagram 7, follow the load profile defined by the function $y(t)$,

$$\mathbf{F}_u(t) = \mathbf{F}_u y(t), \quad (5.1)$$

$$\mathbf{F}_{u:\phi_f}(t) = \mathbf{F}_{u:\phi_f} y(t). \quad (5.2)$$

Representing the Implicit-Newmark Time Integration (INTI) Implicit-Newmark Time Integration, described in Appendix B, by the block-diagram *INTI*, figures 11 and 12 indicate the inputs needed to obtain the referred physic and adjoint state variables.

Figure 11 – Physical states time integration

Once we have the displacement matrix in t -domain,

$$\mathbf{U}(t) = \begin{bmatrix} & t_0 & t_1 & \dots & t_f \\ u_{10} & u_{11} & \dots & u_{1f} \\ u_{20} & u_{21} & \dots & u_{2f} \\ \vdots & \vdots & \dots & \vdots \\ u_{n0} & u_{n1} & \dots & u_{nf} \end{bmatrix} \begin{matrix} \text{dof}_1 \\ \text{dof}_2 \\ \vdots \\ \text{dof}_n \end{matrix}, \quad (5.3)$$

the adjoint force is

$$\mathbf{F}_\Lambda(\tau) = -\frac{\partial g(\tau)}{\partial \mathbf{U}} = \frac{\partial u_{\text{dof}}^2(\tau)}{\partial \mathbf{U}} = \begin{Bmatrix} \mathbf{0} \\ \vdots \\ \mathbf{0} \\ 2\mathbf{u}_{\text{dof}}(\tau) \\ \mathbf{0} \\ \vdots \\ \mathbf{0} \end{Bmatrix}, \quad (5.4)$$

where

$$\mathbf{u}_{\text{dof}}(\tau) = \begin{bmatrix} t_f & \dots & t_1 & t_0 \\ u_{,f} & \dots & u_{,1} & u_{,0} \end{bmatrix} \text{ dof} \cdot \quad (5.5)$$

Now the adjoint states can be defined as shown in figure 12. Knowing that when $t = 0$ implies

Figure 12 – Adjoint states time integration

$\Lambda(t_f)$, and when $t = t_f$ implies $\Lambda(0)$, in order to numerically calculate the sensitivity given by equation (4.31), the adjoint state $\Lambda(\tau)$

$$\Lambda_{\text{dof}}(\tau) = \begin{bmatrix} \tau_0 & \tau_1 & \dots & \tau_f \\ \lambda_{,f} & \dots & \lambda_{,1} & \lambda_{,0} \end{bmatrix} \text{ dof} \quad (5.6)$$

must be flipped to the extent that it returns to the adjoint state variable λ , which reads the current time t :

$$\lambda_{\text{dof}}(t) = \begin{bmatrix} \tau_f & \dots & \tau_1 & \tau_0 \\ \lambda_{,0} & \lambda_{,1} & \dots & \lambda_{,f} \end{bmatrix} \text{ dof} \cdot \quad (5.7)$$

An estimation of the sampling frequency fs for the INTI method is based on the value of the first natural frequency ω_{n_1} (in Hz) of the structure at each optimization step. The fs used to calculate the time step Δt shall be able to resolve the highest mode that contributes to the response, and for the INTI it has been found that using twenty points per cycle of the highest frequency of interest results in a reasonably accurate solution (ANSYS, 2013), as below:

$$fs = 20\omega_{n_1}, \quad (5.8)$$

$$\Delta t = \frac{1}{fs}. \quad (5.9)$$

However, as void regions appear along the optimization procedure, the first vibration mode targeted from the first optimization step no longer represents the first natural frequency calculated and the time step, automatically updated based on equation (5.8), leads to a non accurate transient response. From the topology optimization point-of-view, this effect prevents the optimization function from convergence. In order to circumvent this problem, it is adopted a time step update criteria based on the system response in time. For the first optimization step $it = 1$, the sampling frequency is based on the outcome of the modal analysis, but for the subsequent optimization steps, the current sampling frequency it is obtained from the displacement response of the previous optimization step $it - 1$,

$$f_{sit} = 20 \frac{1}{t_{2it-1} - t_{1it-1}}, \quad (5.10)$$

where t_{1it-1} and t_{2it-1} are the time at the first and second peaks of the system response, respectively.

The frequencies ω_{n_1} resulting from the modal analysis versus the system transient response at each optimization iteration is shown in figure 13.

5.2 Structure Matrices Differentiation

In addition to the computation of the adjoint state variables in time-domain, described in the last section, the adjoint sensitivity expression (4.31) requires the calculation of the structure matrices derivatives with respect to the design variables $\boldsymbol{\rho}$. These derivatives are numerically implemented as follows:

- Mass matrix derivative

$$\frac{d\mathbf{M}}{d\rho_e} = \begin{bmatrix} \frac{d\mathbf{M}_{uu}}{d\rho_e} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}, \quad (5.11)$$

$$\frac{d\mathbf{M}_{uu}}{d\rho_e} = p_1 \rho_e^{(p_1-1)} \zeta_{\text{mat1}} - p_1 \rho_e^{(p_1-1)} \zeta_{\text{mat2}}.$$

- Stiffness matrix derivative

$$\frac{d\mathbf{K}}{d\rho_e} = \begin{bmatrix} \frac{d\mathbf{K}_{uu}}{d\rho_e} & \frac{d\mathbf{K}_{u\phi}}{d\rho_e} \\ \frac{d\mathbf{K}_{\phi u}}{d\rho_e} & -\frac{d\mathbf{K}_{\phi\phi}}{d\rho_e} \end{bmatrix}, \quad (5.12)$$

$$\frac{d\mathbf{K}_{uu}}{d\rho_e} = p_2 \rho_e^{(p_2-1)} \mathbf{c}_{\text{mat1}} - p_2 \rho_e^{(p_2-1)} \mathbf{c}_{\text{mat2}},$$

$$\frac{d\mathbf{K}_{u\phi}}{d\rho_e} = p_3 \rho_e^{(p_3-1)} \mathbf{c}_{\text{mat1}} - p_3 \rho_e^{(p_3-1)} \mathbf{c}_{\text{mat2}},$$

$$\frac{d\mathbf{K}_{\phi\phi}}{d\rho_e} = p_4 \rho_e^{(p_4-1)} \mathbf{c}_{\text{mat1}} - p_4 \rho_e^{(p_4-1)} \mathbf{c}_{\text{mat2}}.$$

- Damping matrix derivative

$$\frac{d\mathbf{C}}{d\rho_e} = \begin{bmatrix} \frac{d(\mathbf{C}_{uu} + \mathbf{C}_{11})}{d\rho_e} & \frac{d\mathbf{C}_{12}}{d\rho_e} \\ \frac{d\mathbf{C}_{21}}{d\rho_e} & \frac{d\mathbf{C}_{22}}{d\rho_e} \end{bmatrix}, \quad (5.13)$$

$$\frac{d\mathbf{C}_{uu}}{d\rho_e} = \alpha \frac{d\mathbf{K}_{uu}}{d\rho_e} + \beta \frac{d\mathbf{M}_{uu}}{d\rho_e},$$

$$\frac{d\mathbf{C}_{11}}{d\rho_e} = \mathcal{K} \left(\frac{d\mathbf{K}_{u\phi_p^a} \mathbf{I}_{\phi_p^a}^\top}{d\rho_e} \mathbf{I}_{\phi_p^s} \mathbf{K}_{\phi_p^s u} + \mathbf{K}_{u\phi_p^a} \mathbf{I}_{\phi_p^a}^\top \frac{d\mathbf{I}_{\phi_p^s} \mathbf{K}_{\phi_p^s u}}{d\rho_e} \right),$$

$$\frac{d\mathbf{C}_{12}}{d\rho_e} = -\mathcal{K} \left(\frac{d\mathbf{K}_{u\phi_p^a} \mathbf{I}_{\phi_p^a}^\top}{d\rho_e} \mathbf{I}_{\phi_p^s} \mathbf{K}_{\phi_p^s \phi_f} + \mathbf{K}_{u\phi_p^a} \mathbf{I}_{\phi_p^a}^\top \frac{d\mathbf{I}_{\phi_p^s} \mathbf{K}_{\phi_p^s \phi_f}}{d\rho_e} \right),$$

$$\frac{d\mathbf{C}_{21}}{d\rho_e} = -\mathcal{K} \left(\frac{d\mathbf{K}_{\phi_f \phi_p^a} \mathbf{I}_{\phi_p^a}^\top}{d\rho_e} \mathbf{I}_{\phi_p^s} \mathbf{K}_{\phi_p^s u} + \mathbf{K}_{\phi_f \phi_p^a} \mathbf{I}_{\phi_p^a}^\top \frac{d\mathbf{I}_{\phi_p^s} \mathbf{K}_{\phi_p^s u}}{d\rho_e} \right),$$

$$\frac{d\mathbf{C}_{22}}{d\rho_e} = \mathcal{K} \left(\frac{d\mathbf{K}_{\phi_f \phi_p^a} \mathbf{I}_{\phi_p^a}^\top}{d\rho_e} \mathbf{I}_{\phi_p^s} \mathbf{K}_{\phi_p^s \phi_f} + \mathbf{K}_{\phi_f \phi_p^a} \mathbf{I}_{\phi_p^a}^\top \frac{d\mathbf{I}_{\phi_p^s} \mathbf{K}_{\phi_p^s \phi_f}}{d\rho_e} \right).$$

- Input load derivative

$$\frac{d\mathbf{F}_u}{d\rho_e} = \mathbf{0}, \quad (5.14)$$

$$\frac{d\mathbf{F}_{u:\phi_f}}{d\rho_e} = \bar{\varphi} \left\{ \begin{array}{l} -\frac{\partial \mathbf{K}_{u\phi_p^a}}{\partial \rho_e} \mathbf{I}_{\phi_p^a}^\top \\ \frac{\partial \mathbf{K}_{\phi_f \phi_p^a}}{\partial \rho_e} \mathbf{I}_{\phi_p^a}^\top \end{array} \right\}, \quad (5.15)$$

$$\frac{d\mathbf{F}_v}{d\rho_e} = -\frac{d\mathbf{K}_{\phi_p^a}}{d\rho_e} \mathbf{U}^{\text{st}\phi_p^a}. \quad (5.16)$$

Note that when the host structure is optimized, the energized electrode nodes from the actuator layer ϕ_p^a do not share degrees of freedom with any of the metallic design variable elements e of the vector $\boldsymbol{\rho}$. For this reason, $\frac{d\mathbf{F}_v}{d\rho_e} = \mathbf{0}$.

5.3 Linear Programming

The Linear Programming (LP) Linear Programming applied for the solution of a linear or linearized problem such as the approximation of (4.4) to (4.31), consists on the problem approximation to successive linear sub-problems that are solved by means of linear programming (Simplex) (DANTZIG, 1998). The objective functions and constraints are approximated by *Taylor's* series limiting the design variables by the move limits. These move limits are adjusted at each iteration, in a way that their value is augmented if there is small variation on the value of the objective function comparing the last two iterations steps. Likewise, when there is a significant variation between them, the move limits are shortened. Equations (5.17) and (5.18) shows the move limits lower and upper bounds update, respectively.

The software was implemented in MATLAB[®] with an optimization solver based in LP, which has proved to be efficient for the kind of problem therein proposed ((YANG; CHUANG, 1994), (SILVA; KIKUCHI, 1999)). As a gradient-based mathematical programming algorithm, the LP needs the sensitivity analysis of the objective function with respect to the design variables, given by equation(4.31). The objective function and constraints are approximated by *Taylor's* series expansion and the design variables are iteratively adjusted within a percentage of their original values, what is called the moving limits and which are dependent on the objective function value history:

$$\rho_l = \begin{cases} 0.85\rho_{it}, & f(\rho_{it}) - f(\rho_{it-1}) \geq 0.6(f(\rho_{it-1}) - f(\rho_{it-2})); \\ 0.90\rho_{it}, & 0.3(f(\rho_{it-1}) - f(\rho_{it-2})) \leq f(\rho_{it}) - f(\rho_{it-1}) \leq 0.6(f(\rho_{it-1}) - f(\rho_{it-2})); \\ 0.95\rho_{it}, & f(\rho_{it}) - f(\rho_{it-1}) \leq 0.3(f(\rho_{it-1}) - f(\rho_{it-2})); \end{cases} \quad (5.17)$$

$$\rho_u = \begin{cases} 1.15\rho_{it}, & f(\rho_{it}) - f(\rho_{it-1}) \geq 0.6(f(\rho_{it-1}) - f(\rho_{it-2})); \\ 1.10\rho_{it}, & 0.3(f(\rho_{it-1}) - f(\rho_{it-2})) \leq f(\rho_{it}) - f(\rho_{it-1}) \leq 0.6(f(\rho_{it-1}) - f(\rho_{it-2})); \\ 1.05\rho_{it}, & f(\rho_{it}) - f(\rho_{it-1}) \leq 0.3(f(\rho_{it-1}) - f(\rho_{it-2})), \end{cases} \quad (5.18)$$

where ρ_l and ρ_u are, respectively, the lower and upper bounds of the design variables, and "it" is the current iteration step. After each optimization step, the design variables vector $\boldsymbol{\rho}$ is obtained and updated in the design domain.

5.4 Projection Filter

The numerical performance of the optimization problem is dependent on the discretization of the design domain. To avoid a ill-posed problem in the context of finite elements, a regularization method is necessary. Introduced by Bruns et al. (BRUNS; TORTORELLI, 2001),

the projection filter defines a material density distribution field that constrains the density distribution variation under its mask, generating a well-posed topology optimization problem. Considering this regularization technique, the structural and sensitivity analysis are based upon the filtered material density.

In this work, it is used the approach proposed by (BRUNS; TORTORELLI, 2001) to calculate the pseudo-densities of each FE by applying a linear projection filter to the design variables, which are defined for the centroid coordinate of the element (x_c^e, y_c^e) . This linear projection technique solves the undesired mesh dependency problem that arises when intermediate material densities are penalized.

The projection technique involves the definition of a density measure $\zeta^e(\rho_j \in N^e)$, a function of the design variables ρ_j , that is based on the weighted distribution of the elements densities on the neighborhood of e -element, N^e . A fixed mesh-independent projection radius r is predefined, limiting a circumference centered at the arbitrary element e , as seen in Fig. 14, and N^e is defined such that

$$N^e = \{j \mid r_j - r \leq 0\}, \quad (5.19)$$

and the distance between each element j that lies within the circumference border in N^e and the e -element is calculated by

$$r_j = \sqrt{(x_c^j - x_c^e)^2 + (y_c^j - y_c^e)^2}. \quad (5.20)$$

Figure 14 – Projection filter technique for the material densities.

A weight function ι_j for each of the j -element of N^e is defined by

$$\iota_j = \max \left(1 - \frac{r_j}{r}, 0 \right), \quad (5.21)$$

that assumes a greater value the closer the enclosed j -element is from the reference element e . Finally, the pseudo-density ζ^e at each element is defined as a weighted average of the densities in a mesh-independent neighborhood (SIGMUND, 2007):

$$\zeta^e(\rho) = \frac{\sum_{j \in N^e} \iota_j \rho_j}{\sum_{j \in N^e} \iota_j}, \quad (5.22)$$

Now, the specific weight and the elastic tensor employed in SIMP model, equations (4.1) and (4.2) respectively, are redefined for the pseudo-densities:

$$\zeta^e(\zeta(\rho)) = \zeta^{p_1}(\rho(x_c^e, y_c^e)) \zeta_{\text{mat}1} + (1 - \zeta^{p_1}(\rho(x_c^e, y_c^e))) \zeta_{\text{mat}2}, \quad (5.23)$$

$$\mathbf{c}^e(\zeta(\rho)) = \zeta^{p_2}(\rho(x_c^e, y_c^e)) \mathbf{c}_{\text{mat}1} + (1 - \zeta^{p_2}(\rho(x_c^e, y_c^e))) \mathbf{c}_{\text{mat}2}. \quad (5.24)$$

Therefore, the sensitivity expression given by equation (4.31) is rewritten in terms of ζ^e :

$$\frac{\partial \mathcal{L}(\mathbf{U}, \boldsymbol{\Lambda})}{\partial \zeta^e} = \int_0^{t_f} \boldsymbol{\Lambda}^\top (t_f - t) \left[\frac{\partial \mathbf{M}(\boldsymbol{\zeta})}{\partial \zeta^e} \ddot{\mathbf{U}} + \frac{\partial \mathbf{C}_{\text{fb}}(\boldsymbol{\zeta})}{\partial \zeta^e} \dot{\mathbf{U}} + \frac{\partial \mathbf{K}(\boldsymbol{\zeta})}{\partial \zeta^e} \mathbf{U} - \frac{\partial \mathbf{F}(t)}{\partial \zeta^e} \right] dt, \quad (5.25)$$

and

$$\frac{\partial \mathcal{L}(\mathbf{U}, \boldsymbol{\Lambda})}{\partial \rho_e} = \sum_{i \in N_k} \frac{\partial \mathcal{L}(\mathbf{U}, \boldsymbol{\Lambda})}{\partial \zeta_i} \frac{\partial \zeta_i}{\partial \rho_e}, \quad (5.26)$$

where equation (5.26) is calculated considering the set of pseudo-densities N_k that are under the influence of the design variable ρ^e ,

$$\frac{\partial \zeta_i}{\partial \rho_e} = \frac{\iota_i^e}{\left[\sum_{j \in N^e} \iota_j \right]_i}. \quad (5.27)$$

Therefore, the design variables $\boldsymbol{\rho}$ are converted into pseudo-density variables $\boldsymbol{\zeta}$ at each iteration of the optimization procedure, and these pseudo-densities are used to calculate the objective function and volume constraint values. Next, the sensitivity for each element is obtained through equation (5.26), and a new set of design variables $\boldsymbol{\rho}$ is obtained by the optimization algorithm (SIGMUND, 2007; VATANABE *et al.*, 2016).

NUMERICAL RESULTS

The material properties that constitute the structures numerically obtained in this chapter are described in table 2. The piezoelectric constants, matrices (2.14), (2.15) and (2.16), are obtained from (BERLIN COURT; CURRAN; JAFFE, 1964). The piezoelectric ceramic PZT-5A is employed in all the PEAs and PPEAs designs. The Composite material properties are obtained from (QIN *et al.*, 2017) and it is employed for a bi-material example along with Nickel. The metallic material Alumina is employed for solid-void examples and the Void properties are also given in the table below.

Table 2 – Material properties used for numerical results

PZT-5A			Nickel			Composite		
c_{11}	139	GPa	E	200	GPa	E	0.1849	GPa
c_{12}	77.84	GPa	ζ	8908	kg/m ³	ζ	92.2	kg/m ³
c_{13}	74.28	GPa	ν	0.31	-	ν	0.334	-
c_{33}	115.41	GPa	Alumina			Void		
c_{44}	25.64	GPa	E	392	GPa	E	1×10^{-15}	GPa
c_{66}	30.58	GPa	ζ	4000	kg/m ³	ζ	1×10^{-15}	kg/m ³
e_{31}	-14.8	C/m ²	ν	0.24	-	ν	1×10^{-15}	-
e_{33}	15.7835	C/m ²						
e_{15}	12.67	C/m ²						
ϵ_{11}	1474.36	pF/m						
ϵ_{33}	1299.38	pF/m						
ϵ_0	8.854188	pF/m						
ζ	7500	kg/m ³						

6.1 Study on the optimization parameters

Very few works in literature deals with the topology optimization method for time-domain transient analysis. Dynamic response topology optimization problems are tricky to converge since no model simplification is made in time-domain analysis. The final topology is sensitive to the input load profile, as reported in the work of Zhao (ZHAO; WANG, 2017), to the proportional damping coefficients, to the domain characteristics and the penalization updates, as it is demonstrated in this section.

Due to the difficulties encountered on the stability of the transient finite element analysis and on the convergence of the topology optimization for transient problems regarding the designs proposed in this work, this section is dedicated to the demonstration that the topologies obtained by the optimization procedure are dependent on the set of characteristics chosen, such as the penalization updates and the transient load profile.

For this reason, the optimization problem therein proposed is to minimize the vibration response spanned over time, as described by (4.5) for $g = g_1$, equation (4.9), for $w_1 = 0$ and $w_2 = 1$:

$$\begin{aligned} \min_{\boldsymbol{\rho}} \quad & f(\boldsymbol{\varsigma}(\boldsymbol{\rho})) = \ln \left(\int_0^{t_f} u_{\text{dof}}^2 dt \right) \\ \text{s.t.} \quad & \begin{cases} \mathbf{M}(\boldsymbol{\varsigma})\ddot{\mathbf{U}} + \mathbf{C}_{uu}(\boldsymbol{\varsigma})\dot{\mathbf{U}} + \mathbf{K}(\boldsymbol{\varsigma})\mathbf{U} = \mathbf{F}_u(t) \\ \mathbf{K}\mathbf{U}_0^{\text{st}} = \mathbf{F}_u \\ \dot{\mathbf{U}}|_{t=0} = \mathbf{V}_0 \\ \mathbf{U}|_{t=0} = \mathbf{U}_0 \\ \sum_{n=1}^N \varsigma_n V_n \leq V_{\text{max}} \\ 0 \leq \varsigma_n \leq 1 \end{cases} \end{aligned} \quad (6.1)$$

for $\mathbf{V}_0 = \mathbf{0}$, $\mathbf{U}_0 = \mathbf{U}_0^{\text{st}}$, $\mathbf{F}_u(t) = \mathbf{F}_u y(t)$ and $\mathbf{F}_u$ is given by equation (3.28). The natural logarithm of the transient function has been chosen due to the discrepancy between the order of magnitude of the objective function value and the sensitivity values. Note that the feedback controller described in chapter 3 is not taken into account in this section, $\mathbf{C}_{fb} = \mathbf{C}_{uu}$, equation (2.55), since only the elastic domain is considered.

The domain characteristics shown in figure 15 are given in table 3, with domain dimensions as described in (ZHAO; WANG, 2017). The TOM results are compared for two types of penalization update (type₁ and type₂), and for different load profiles $y(t) = y_n(t)$ for $n = \{1, 2, 3\}$, being the step input and the half sine input in two distinct time periods, as seen in tables 4 and 5. The final topologies vary significantly depending on the optimization problem setup.

From the examples stated in tables 4 and 5, the topology obtained for the load profile y_3 and the penalization update type₂ is chosen to be analyzed because of the scattered material concentration within the domain. The transient response shown in figure 17 indicates that the

Table 3 – Design domain section 6.1

Parameter	Symbol	Value	Unit
Domain size (width,height,depth)	(w_d, h_d, d_d)	(600, 300, 1)	cm
Mesh size	x_s, y_s	40, 20	-
Mechanical load amplitude	F_{dof}	1	N
Initial material volume	V	30	%
Material volume constraint	V_{max}	50	%
Proportional damping	α	0	-
	β	1×10^{-4}	-
Material 1	mat ₁ (Eqs. (4.1) and (4.2))	Alumina	-
Material 2	mat ₂ (Eqs. (4.1) and (4.2))	Void	-

Figure 15 – Boundaries and transient load for the problem proposed in section 6.1

structural damping estimated for the optimization problem is not adequate, given the sinusoidal combination characteristic present in the optimized structure response.

Therefore, the optimization problem is updated for $\beta = 6 \times 10^{-4}$ and the example (y_3, type_2) is rerun for this β -coefficient. The optimized topologies comparison is shown in figure 19, and their convergence comparison is shown in figure 20.

For a higher structural damping, the optimization problem convergence is improved to a lower minimum, as it is seen by the solid line in figure 20, and the scattered material concentration vanishes. This result is due to the fact that the sensitivity analysis is not accurate throughout the optimization steps if the penalization update curve, the proportional damping coefficient and the transient input are not properly chosen.

This section evaluated the sensitivity of the transient optimization problem with no control regulation, regarding the optimization parameters setup only. The purpose of the current analysis is to demonstrate that the optimization parameters for the subsequent problems proposed in this thesis must be carefully chosen in order to achieve convergence.

Table 4 – Results for the study on varying the optimization parameters

Figure 16 – Convergence for the examples shown in tables 4 and 5

Table 5 – Results for the study on varying the optimization parameters (cont. table 4)

Input profile	Penalization update	Topology

Figure 17 – Transient response for (y_3, type_2) optimized topology, figure 19a

6.2 Design of PEAs with Vibration Suppression

In this section, the optimized topologies of the elastic structure are obtained for different control gains $\mathcal{K}$ chosen to the vibration attenuation of piezoelectric actuators. In the first

Figure 18 – Transient response for (y_3, type_2) optimized topology with higher damping, figure 19b Figure 19 – Optimized topologies for (y_3, type_2) Figure 20 – Convergence comparison for (y_3, type_2)

subsection it is proposed a bi-material distribution in order to analyze the convergence behaviour of the optimization problem when the distribution involves stiff and soft materials. In the second

subsection, solid-void distribution topologies are obtained for another set of feedback control gains also aiming vibration suppression of the piezostructure. In both cases it is considered the non-collapsed piezoelectric nodes configuration. In the third and last subsection, the previously solid-void distribution obtained is investigated considering the collapsed piezoelectric nodes configuration. The solid-void topologies obtained for non-collapsed and collapsed piezo nodes are compared so the influence of the intermediate nodes to the optimization problem is evaluated. The transient responses of the optimized topologies obtained in this section are compared by evaluating the objective function values when these topologies are subjected to the set of feedback gains for which the optimization problem is solved.

The bi-material design for controlled piezoactuator is employed ahead of the solid-void design because the TO parameters need to be setup for a feasible optimization convergence, what is easier on bi-material distribution since the spurious modes that might appear in a solid-void distribution is not a concern. As it is discussed in section 6.1, the void regions for a $\rho_{\min} = 10^{-15}$ in transient analysis may lead the optimization problem to considerably distinct local minima, in a way that the convergence to the desired function extreme value might not be achieved.

As proposed in section 4.2, the optimization problem aims to minimize a time-domain transient function targeting the elastic material distribution for the controlled structure configuration shown in figure 21, which shall oscillate around its restoring position, as indicated in figure 22 for the presence and absence of the control gain effect $\mathcal{K} \neq 0$ and $\mathcal{K} = 0$, respectively. Therefore, the problem formulation (6.2) is such that the dynamic objective function integrand is the one proposed in equation (4.9) for the design of PEAs in this section.

Figure 21 – Transducer Model

The optimization problem proposed in section 4.2 is restated here but, differently from section 6.1, now the control action is present and, as deduced in section 3.1, the damping matrix

Figure 22 – Active velocity feedback control attenuation

is $\mathbf{C}_{fb}$, as given by equation (3.24).

$$\begin{aligned} \min_{\boldsymbol{\rho}} \quad & f(\boldsymbol{\varsigma}(\boldsymbol{\rho})) = \ln \left(\int_0^{t_f} u_{\text{dof}}^2 dt \right) \\ \text{s.t.} \quad & \begin{cases} \mathbf{M}(\boldsymbol{\varsigma})\ddot{\mathbf{U}} + \mathbf{C}_{fb}(\boldsymbol{\varsigma})\dot{\mathbf{U}} + \mathbf{K}(\boldsymbol{\varsigma})\mathbf{U} = \mathbf{F}_u(t) \\ \mathbf{K}\mathbf{U}_0 = \mathbf{F}_u \\ \dot{\mathbf{U}}|_{t=0} = \mathbf{V}_0 \\ \mathbf{U}|_{t=0} = \mathbf{U}_0 \\ \sum_{n=1}^N \varsigma_n V_n \leq V_{\max} \\ 0 \leq \varsigma_n \leq 1 \end{cases} \end{aligned} \quad (6.2)$$

where the boundaries are shown in figure 23 and the load profile $y(t)$ describing the transient mechanical excitation $\mathbf{F}_u(t) = \mathbf{F}_u y(t)$ is shown in figure 24:

Figure 23 – Boundaries and transient load for the optimization problem of section 6.2

Figure 24 – Load profile $y(t)$ for the TO design proposed in section 6.2

6.2.1 Bi-material PEA design

The bi-material piezoelectric actuator (BPEA) structure designed features non-structural functions such as the piezoelectric effect, for sensing and actuation of a controlled transducer, and optimized structural functions such as stiffness and damping of a bi-metallic host layer. Some recent applications for multifunction structures and multifunction materials such as the transducer therein described, are present in morphing aircraft wings and structurally integrated electronic components (GIBSON, 2010).

The bi-material piezoelectric actuator (BPEA) has its metallic host layer designed by the TOM, what defines the structural function. The electric function is given by two piezo-ceramic layers that act as a sensor and an actuator coupled with a constant gain active velocity feedback control (AVFC).

The penalization coefficients (p_1 and p_2) update is given by the curves shown in figure 25 and the domain characteristics are defined in table 6 for the boundaries illustrated in figure 23. The set of feedback gain values $\mathcal{K}$ applied in this bi-material design are given in table 7.

Figure 25 – Bi-material penalization coefficients update for the optimization problem of section 6.2.1

Table 6 – Design domain definitions for the vibration attenuation problem proposed in section 6.2.1

Parameter	Symbol	Value	Unit
Domain size (width,height,depth)	(w_d, h_d, d_d)	(20,4,0.2)	cm
Substrate size (width,height,depth)	(w_s, h_s, d_s)	(20,2.67,0.2)	cm
Piezoceramic size (width,height,depth)	(w_p, h_p, d_p)	(20,0.67,0.2)	cm
Substrate mesh size	x_s, y_s	120,16	-
Piezo ceramic mesh size	x_s, y_s	120,4	-
Mechanical load amplitude	F_{dof}	1	N
Initial material volume	V_{ini}	30	%
Material volume constraint	V_{max}	70	%
Projection filter radius	r	0.59	cm
Proportional damping	α	0	-
	β	1×10^{-5}	-
Material 1	mat ₁ (Eqs. (4.1) and (4.2))	Composite	-
Material 2	mat ₂ (Eqs. (4.1) and (4.2))	Nickel	-

Table 7 – Feedback gain values

	$\times 10^5$
$\mathcal{K}_1 =$	0.00
$\mathcal{K}_2 =$	0.90
$\mathcal{K}_3 =$	3.60
$\mathcal{K}_4 =$	7.20

The optimized topologies are obtained after 28 iteration steps for a non-smooth convergence, as seen in figure 26, resulting in topologies with shadowy gray scale regions and with inactive volumes constraint, set to 70%-material type 1, as seen in figure 27.

Figure 26 – Convergence comparison

The optimized piezo structure designs are shown in figures 28a, 28c, 28e and 28g in

Figure 27 – Volume constraint comparison

the design variables $\boldsymbol{\rho}$. These results are interpreted as material type 1 (black) for $\boldsymbol{\rho} \geq 0.7$ and as material type 2 (white) for $\boldsymbol{\rho} < 0.7$. The interpreted distributions are shown in figures 28b, 28d, 28f and 28h, where the ceramics are indicated by the gray regions. These post-processed results are presented to facilitate the visualization of the material distribution as the feedback gain increases. As this problem deals with non void regions only, the gray scale is not a concern and shall not be modified for the sake of validation of the optimization efficiency, although the post-processed topologies information are also presented in table 8 in order to certify the importance of the gray scale for the expected response of the system. As seen by the interpreted

Figure 28 – Optimized topologies for different feedback gain values

system in figure 28b, when $\mathcal{K} = 0$, the topology is symmetric as a result of the symmetric

dynamic system (3.22) for $\mathbf{C}_{11} = \mathbf{C}_{12} = \mathbf{C}_{21} = \mathbf{C}_{22} = \mathbf{0}$. This symmetry is though lost for $\mathcal{K} \neq 0$, as indicate figures 28b, 28f and 28h.

The transient response of the optimized topologies can be seen in figure 29, which describes that the oscillatory response is attenuated for the optimized structure actively controlled by its respective feedback gain ($\mathcal{K}_1$ to $\mathcal{K}_4$). The velocity feedback, read as the output current on the sensor layer, is amplified by those gains resulting in ϕ_p^a , and their response in time is seen in figure 30.

Figure 29 – Transient response for the optimized topologies

Figure 30 – Transient feedback voltage for the optimized topologies

In order to analyze the numerical results above, table 8 contrasts the objective function value, the material volume and the first natural frequency of the optimized structures. As $\mathcal{K}$ increases, $f(\mathfrak{S})$ decreases even though the proportion of the less stiff and lightweight composite material increases. As the problem formulation proposed for the BPEA structure imposes a constraint for the composite material, the optimizer seeks a lightweight structure better controlled by the imposed $\mathcal{K}$. The natural frequencies quantitatively shown in table 8, are in accordance with the transient curves shown in figures 29 and 30. As observed by the u_{dof} plots and $u_{\text{dof}}(t_0)$ in table 7, the initial displacements of the optimized structures indicates that the topologies are

Table 8 – Objective function values for each optimized topology

$\mathcal{K}(10^5)$	$f(\boldsymbol{\zeta})(10^{-14})$	Optimized			$f(\boldsymbol{\rho})(10^{-14})$	Post-Processed	
		$V_{\text{mat1}}(\%)$	$u_{\text{dof}}(t_0)(\mu\text{m})$	$\omega_{n_1}(\text{Hz})$		$V_{\text{mat1}}(\%)$	$\omega_{n_1}(\text{Hz})$
0.00	1.4205	46.14	-2.5007	778.0474	25.48	40.73	646.1323
0.90	1.2770	48.91	-2.5264	788.6615	20.75	42.29	640.1627
3.60	1.0231	50.90	-2.5267	790.4259	21.70	45.73	617.2293
7.20	0.9100	47.97	-2.4723	731.5900	1.98×10^7	42.50	2974.7

indeed less stiff as the control gain increases and, being also lighter in weight structures, justifies the increase in the natural frequencies. The abnormal $\mathcal{K}_4$ -data in table 7 will be explained further in this subsection.

When the optimized topologies of figure 28 are post-processed for the feedback gains shown in table 7, the objective function values shown in figure 31 are obtained. The four coloured bars for each $\mathcal{K}$ on the histogram indicate the objective function values of the topology obtained with that $\mathcal{K}$ when post-processed with each of them.

Figure 31 – Analysis of $f(\boldsymbol{\zeta})(\times 10^{-14})$ values for the optimized topologies subjected to the proposed gains $\mathcal{K}(\times 10^5)$

The histogram in figure 31 indicates that the topology obtained for $\mathcal{K}_1$ performs better in the structure response attenuation when applying any of the $\mathcal{K}$ -gains in the table 7. However, by comparing the green bars for $\mathcal{K}_1$ and $\mathcal{K}_2$ structures, it is seen that although there is an increase of 0.79% on the objective function value, the $\mathcal{K}_2$ structure is 5% lighter in weight than the $\mathcal{K}_1$ structure (verified by the material properties table 2 in the beginning of this chapter and by the volume fraction given in table 8). Similarly, $\mathcal{K}_3$ structure objective function value is less than 0.097% greater but is 3.82% lighter in weight than $\mathcal{K}_2$ structure. Therefore, the optimization procedure performed with active control gain proves to improve the structure response for the imposed constraints.

A limitation to the feedback gain value has been noted with the results obtained for $\mathcal{K}_4$.

When the feedback is too high, the actuated piezo ceramic deformation is such that the elastic host layer absorbs the strain energy instead of transmitting the deformation, as seen in figure 32b. In this case the optimizer reduces the soft material concentration, table 7, where the deformation is more critical, as it is shown in the free tip of figure 28h an increase in the concentration of the stiffer material.

Figure 32 – Optimized topology for $\mathcal{K} = 7.20 \times 10^5$ deformed at maximum displacement

6.2.2 Solid-void PEA design

The vibration suppression formulation for controlled flextensional piezoelectric actuators (FPEA) design discussed in the present section is validated by the solid-void material distribution of the substrate with non-collapsed and collapsed piezoelectric nodes. The domain characteristics are given in table 9, the material properties in table 2, and the projection filter radius is such that it encloses five elements diagonally. The boundary conditions are illustrated in figure 23 and the load profile is the same applied to the design of the bi-material PEA, figure 24.

Table 9 – Design definitions for the vibration attenuation problem proposed in section 6.2.2

Parameter	Symbol	Value	Unit
Domain size (width,height,depth)	(w_d, h_d, d_d)	(20,4,0.2)	cm
Substrate size (width,height,depth)	(w_s, h_s, d_s)	(20,2.67,0.2)	cm
Piezoceramic size (width,height,depth)	(w_p, h_p, d_p)	(20,0.67,0.2)	cm
Substrate mesh size	x_s, y_s	120,16	-
Piezo ceramic mesh size	x_s, y_s	120,4	-
Mechanical load amplitude	F_{dof}	1	N
Initial material volume	V	30	%
Material volume constraint	V_{max}	70	%
Projection filter radius	r	0.59	cm
Proportional damping	α	0	-
	β	1×10^{-5}	-
Material 1	mat ₁ (Eqs. (4.1) and (4.2))	Alumina	-
Material 2	mat ₂ (Eqs. (4.1) and (4.2))	Void	-

The optimized topologies are obtained for the assumed values of feedback gain given in table 10 and the penalization coefficients update, for equations (4.1) and (4.2), are given in

figure 33, keeping the mass matrix penalization p_1 smaller than the stiffness matrix penalization p_2 in order to avoid spurious modes.

Table 10 – Feedback gains

	$\times 10^5$
$\mathcal{K}_1 =$	0.00
$\mathcal{K}_2 =$	0.45
$\mathcal{K}_3 =$	0.90
$\mathcal{K}_4 =$	1.80
$\mathcal{K}_5 =$	3.60

Figure 33 – Penalization coefficients update

After 55 iteration steps, the topologies obtained for the constant gains $\mathcal{K}$ specified in table 10 are shown in figure 34. The convergence achieved during the optimization steps is shown in figure 35, and the volume constraint is active for the optimized topologies obtained for each of the proposed gains, as reported in table 11. It can be seen that at the first iteration step, for the initial material distribution, the higher the feedback gain, the lower the dynamic optimization function, and from table 11 it is quantitative seen that as the constant gain value increases, the objective function converges at a lower bound.

Table 11 – Objective function values for each optimized topology

$\mathcal{K}(\times 10^5)$	Optimized			Post-Processed		
	$f(\boldsymbol{\varsigma})(\times 10^{-15})$	$V_{\text{mat1}}(\%)$	$\omega_{n_1}(\text{Hz})$	$f(\boldsymbol{\rho})(\times 10^{-15})$	$V_{\text{mat1}}(\%)$	$\omega_{n_1}(\text{Hz})$
0.00	9.4657	70	937.3885	5.7073	68.00	948.9985
0.45	8.2046	70	962.2254	5.4109	68.00	964.2484
0.90	7.6558	70	975.5461	5.1363	68.30	970.6506
1.80	6.8574	70	953.0557	4.7035	68.50	963.1834
3.60	6.4525	70	956.9729	4.2977	68.54	956.9729

The transient responses in figures 36 and 37 shows that the higher is the oscillation decay, the higher the actuator transient input voltage peak ϕ_p^a , what indicates the control action.

Figure 34 – Topologies obtained from the problem proposed in section 4.2

Figure 35 – Convergence comparison

When the topologies of figure 34 are post-processed for the feedback gains shown in table 10, the objective function values shown in figure 38 are obtained. The effect of designing the PEA structure by applying the TOM with a feedback gain can be seen by comparing the objective function values of a topology obtained with a higher constant gain with the topology obtained with a lower constant gain but post-processed with the higher gain. The last statement

Figure 36 – Transient response for the optimized topologies

Figure 37 – Transient feedback voltage for the optimized topologies

can be exemplified by taking the objective function value of the $\mathcal{K}_4$ -topology post-processed with $\mathcal{K}_4$ -gain. As it should be, its value is smaller than the objective function value of $\mathcal{K}_3$ -topology post-processed with $\mathcal{K}_4$ -gain.

For the sake of comparing the optimized result for $\mathcal{K} = 0$, redrawn in figure 39d, with the solid structure, figure 39a, with the homogeneous 70%-material volume distribution, figure 39b, and with the 70%-material volume distribution obtained from the optimization for the mean compliance (MC) host structure, figure 39c, their response in time, figure 40, and table 39 are analyzed. The 70%-volume optimized topology and the 100%-volume topology have approximately the same oscillation peaks despite the greater initial displacement of the optimized topology. The advantage of an optimized design when compared with the full design is that the structure is more flexible and the host layer is 30% lighter. The MC host structure topology has been obtained by turning off the filter at the 40th iteration. Even with the checkerboard pattern regions, which creates a fictitious damping by increasing the stiffness of the structure, the dynamically optimized structure reaches a lower vibration energy value. Finally, the 70%-volume homogeneous host structure presents the highest objective function value. Among the distributions with the same material volume, by comparing their natural frequency, it is clear that

Figure 38 – Analysis of the objective function $f(\zeta)$ values for the optimized topologies subjected to the proposed gains ($\times 10^5$)

the one optimized for the objective function (6.2), figure 39d, is the stiffer topology among them, since it presents the highest natural frequency.

Figure 39 – Topologies chosen for transient response comparison for $\mathcal{K} = 0.00$

Figure 40 – Transient response comparison $\mathcal{K} = 0.00$

Table 12 – Transient response properties of figure 40

Topology	$f(\boldsymbol{\rho})(\times 10^{-15})$	$u_{\text{dof}}(t_0)(\mu m)$	$u_{\text{dof}}(t_p)(\mu m)$	$\omega_{n_1}(Hz)$
Full	4.98	-1.5431	1.1884	869.5652
70% Homogeneous	91.57	-5.3900	4.6240	555.5556
70% MC Optimized	6.25	-1.7811	1.3303	909.0909
70% Optimized	5.77	-1.7526	1.2700	952.3810

6.2.3 Solid-void PEA design considering nc and c-nodes

Once the vibration suppression is successfully achieved for a PEA considering the piezoelectric coupling effect given by the dynamic system (3.17), the $\mathcal{K} = 0.90$ gain value topology is compared with the topology obtained by considering the collapsed piezoelectric nodes. Collapsing these nodes implies in ruling out the second equation of system (3.17) along with the Φ_f -terms of the first equation, what means to reduce the PEA equation to a purely mechanical problem. Figure 41 shows the initial topology with 30% material volume for each case and the corresponding optimized topologies with 70% material volume.

As seen in figure 42, at the first iteration the objective function value is slightly higher for the non-collapsed nodes, what is contrasted in table 13. For the initial distribution, this occurs because the nc-nodes case is numerically less stiff than the collapsed nodes case, with an initial displacement of $-3.19\mu m$ against $-3.10\mu m$ of the c-nodes. In figure 43, it is noticed a faster

Figure 41 – Topologies for evaluating non-collapsed and collapsed nodes in transient response for $\mathcal{K} = 0.90$

decay response for the nc-nodes PEA, what will be explained further in the text.

Yet, figure 42 shows that both problems converges to approximately the same objective function value. As seen in table 13, the nc-nodes case has a slightly lower vibration energy, even though it continues being less stiff than the c-nodes case, with an initial displacement of $-1.77\mu m$ against $-1.73\mu m$ of the c-nodes. Therefore, the optimization procedure managed to compensate this numerical stiffness due to a lower discretization in the ceramics domain, and still achieved a lower objective function value.

Figure 42 – Convergence comparison non-collapsed versus collapsed nodes for $\mathcal{K} = 0.90$

Now, the controller efficiency is analyzed when the optimized topology in nc-nodes is subjected to the collapsed nodes domain and vice-versa, conditions proposed by the topologies in figures 44 and 45, respectively, and the results are shown in table 13. The objective function value for the nc-nodes topology in c-nodes domain is higher than when analyzed in nc-domain, 5.26 against $5.18 (\times 10^{-15})$. The collapsed nodes domain does not manage to attenuate the vibration as much as on the domain to which this topology is obtained. Contrarily, if the c-nodes topology is analyzed in the nc-nodes domain, the controller manages to bring the objective function to a lower value than it is on the c-nodes domain, 5.19 against 5.21 . These improved results obtained and the previously mentioned faster decay on the non-collapsed nodes domain are more comprehensible when the electric signal coming out from the sensor layer is analyzed.

Figure 43 – Transient response comparison non-collapsed versus collapsed nodes for $\mathcal{K} = 0.90$ Figure 44 – Topology obtained from nc-nodes optimization (Fig. 41c) analyzed in c-nodes domain for $\mathcal{K} = 0.90$ Figure 45 – Topology obtained from c-nodes optimization (Fig. 41d) analyzed in nc-nodes domain for $\mathcal{K} = 0.90$

Table 13 – Comparison results for the non-collapsed and collapsed nodes of figure 41

Design gain ($\times 10^5$)	$f(\boldsymbol{\rho})(\times 10^{-15})$	
	nc-nodes	c-nodes
Initial topology		
$\mathcal{K} = 0.90$	21.71	21.18
Optimized topology		
$\mathcal{K} = 0.90$	5.18	5.21
Nc-nodes topology in c-nodes domain		
$\mathcal{K} = 0.90$	-	5.26
C-nodes topology in nc-nodes domain		
$\mathcal{K} = 0.90$	5.19	-

Figures 46, 47 and 48 show the transient analysis of the electric signal that is amplified by

a $\mathcal{K}$ -gain and act as an extra damping to the structure when inputed on the actuator ceramic layer. This electric signal, deduced by equation (3.12), can be separated into a mechanical portion given by the first term of the equation, figure 46, and an electric portion given by the second term of the equation, figure 47. The total electric current generated by the sensor layer, I^s , is the difference between the mechanical portion and the electric portion, as stated in equation (3.12) and shown in figure 48. First of all, the mechanical portion of the electric signal discussed achieves higher peaks due to the previously discussed fact that the nc-nodes domain is numerically less stiff and therefore the piezoelectric conversion energy is higher, following the mechanical strain of the PEA. Secondly, the electric portion of I^s , that is absent on the c-nodes domain, has opposite sign with respect to the mechanical portion. In this case, recalling equation (3.12), when these two portions are added, the electric current achieves even higher peaks, beyond to the amount achieved due to the greater mechanical strain. Therefore, this is the reason why the AVFC is more effective in non-collapsed nodes domains.

Figure 46 – Transient feedback current in $\dot{u}$ comparison for $\mathcal{K} = 0.90$

Figure 47 – Transient feedback current in $\dot{\phi}$ comparison for $\mathcal{K} = 0.90$

Figure 48 – Transient feedback current I^s comparison for $\mathcal{H} = 0.90$

6.3 Design of PPEAS with Vibration Suppression

Inspired by applications that require from their actuation system a certain flexibility for targeting a location of actuation, but also need a response attenuation at this target given a transient input, such as the scanning probe microscopes schematized in figure 6, this section deals with a multi-objective function for the design of the PPEA. Therefore, this section proposes the topology design of a positioner type of PPEA that is perturbed by a transient voltage while the AVFC attenuates the oscillatory response at an operating point as illustrated by figures 49 and 50. The problem formulation (6.3) is such that the dynamic function integrand is the one proposed by equation (4.10). The TOM is used in this section to distribute the substrate material.

Figure 49 – Positioner Model

Therefore, for the PPEAs design, the topology optimization problem proposed in section 4.2 involves the minimization of the vibration response around an actuated position, and the maximization of the mean compliance for a force vector $\mathbf{F}_v$ induced by the application of a constant voltage input at the actuator electrode, objective function given by equation (4.18). The problem is subjected to the dynamic equilibrium equations given a force $\mathbf{F}_{u:\phi_f}(t)$ obtained from the electric signal of the difference between a reference input and the feedback readings, figure 7, the static equilibrium equation given a force $\mathbf{F}_v$ obtained from the electric input voltage, the

Figure 50 – Positioner with active velocity feedback control attenuation

transient initial conditions, a volume constraint and the design variables constraint, as follows:

$$\begin{aligned}
 \min_{\boldsymbol{\rho}} \quad & f(\boldsymbol{\rho}) = w_1 \mathbf{U}_{\text{st}}^T \mathbf{F}_v + w_2 \int_0^{t_f} (u_{\text{dof}} - u_{\text{cte}})^2 dt \\
 \text{s.t.} \quad & \begin{cases} \mathbf{M}(\boldsymbol{\rho}) \ddot{\mathbf{U}} + \mathbf{C}_{\text{fb}}(\boldsymbol{\rho}) \dot{\mathbf{U}} + \mathbf{K}(\boldsymbol{\rho}) \mathbf{U} = \mathbf{F}_{u:\phi_f}(t) \\ \mathbf{K} \mathbf{U}^{\text{st}} = \mathbf{F}_v \\ \dot{\mathbf{U}}|_{t=0} = \mathbf{V}_0 \\ \mathbf{U}|_{t=0} = \mathbf{U}_0 \\ \sum_{n=1}^N \rho_n V_n \leq V_{\text{max}} \\ 0 \leq \rho_n \leq 1 \end{cases} \quad (6.3)
 \end{aligned}$$

for $\mathbf{V}_0 = \mathbf{U}_0 = \mathbf{0}$ and a transient load profile $y(t)$ describing the induced excitation $\mathbf{F}_{u:\phi_f}(t) = \mathbf{F}_{u:\phi_f} y(t)$ as shown in figure 52. The boundaries are shown in figure 51.

Figure 51 – Boundaries and transient voltage input

6.3.1 Solid-void PPEA design

The domain characteristics are given in table 14 and the boundary conditions are shown in figure 51. In table 14, the objective function constant u_{dof} is estimated by a transient FEM

Figure 52 – Transient load profile $y(t)$ describing $\mathbf{F}_{u:\phi_f}$ in time

Table 14 – Design domain definitions for the vibration attenuation at an actuated position problem proposed in section 6.3.1

Parameter	Symbol	Value	Unit
Domain size (width,height,depth)	(w_d, h_d, d_d)	(10, 2, 0.2)	cm
Substrate size (width,height,depth)	(w_s, h_s, d_s)	(10, 0.33, 0.2)	cm
Piezoceramic size (width,height,depth)	(w_p, h_p, d_p)	(10, 1.33, 0.2)	cm
Substrate mesh size	x_s, y_s	90, 12	-
Piezoceramic layer mesh size	x_p, y_p	90, 3	-
Electric potential input	$\bar{\phi}$	-24	V
Initial material volume	V	30	%
Material volume constraint	$V_{\max}$	70	%
Projection filter radius	r	0.39	cm
Proportional damping	α	0	-
	β	1×10^{-5}	-
Material 1	mat ₁ (Eqs. (4.1) and (4.2))	Alumina	-
Material 2	mat ₂ (Eqs. (4.1) and (4.2))	Void	-
Objective function constant	u_{cte}	3×10^{-7}	m
Objective function weights	w_1	1000	-
	w_2	1	-

analysis of a homogeneous 30%-material volume domain and a 100%-material volume domain. The function weights w_1 and w_2 are estimated by pondering the maximum and minimum element sensitivity of the static and dynamic objective functions.

The optimized topologies are obtained for the assumed values of feedback gain given in table 15 and the penalization coefficients update, for equations (4.1) and (4.2), are given in figure 33, keeping the mass matrix penalization p_1 smaller than the stiffness matrix penalization p_2 in order to avoid spurious modes (DEATON; GRANDHI, 2014). The radius value of the projection filter applied is such that the circumference encloses five elements diagonally.

Table 15 – PPEA feedback gains

	$\times 10^5$
$\mathcal{K}_1 =$	0.00
$\mathcal{K}_2 =$	0.72
$\mathcal{K}_3 =$	3.60
$\mathcal{K}_4 =$	7.20

Figure 53 – Penalization coefficients update

After 75 iteration steps, the topologies obtained for the constant gains $\mathcal{K}$ specified in table 15 are shown in figure 54. Their respective convergence curves during the optimization steps is shown in figure 55. Table 16 shows that as the constant gain value increases, the objective function converges at a lower bound.

Figure 54 – PPEA optimized topologies

Figure 55 – Multi-objective function convergence

Table 16 – Objective function values for each optimized topology

$\mathcal{K} (\times 10^5)$	$f(\mathbf{p}) (\times 10^{-16})$	$V_{\text{mat1}} (\%)$	$\omega_{n1} (\text{Hz})$	Post-Processed		
				$f(\mathbf{p}) (\times 10^{-15})$	$V_{\text{mat1}} (\%)$	$\omega_{n1} (\text{Hz})$
0.00	1.1547	70	1231.2784	1.7132	69.35	1559.6194
0.72	1.0912	70	1300.8541	1.7104	69.72	1627.2544
3.60	1.0109	70	1296.9975	1.6908	69.63	1617.1937
7.20	0.9928	70	1367.8110	1.7002	69.54	1616.1643

The transient responses in figures 56 and 57 shows that the higher is the oscillation decay, the higher the actuator transient input voltage peak ϕ_p^a , what indicates the control action.

The post-processed columns of table 16 show higher objective function values for the post-processed topologies. The reason is that the steady state for the post-processed topologies are far from u_{cte} , as it is seen by the transient response curves for the topology obtained for $\mathcal{K} = 7.20 \times 10^5$ and for its correspondent post-processed topology, figure 58. Due to the fact

Figure 56 – Transient response for the optimized topologies

Figure 57 – Transient feedback voltage for the optimized topologies

Figure 58 – Transient response comparison for the $\mathcal{H} = 7.20$ -topology

that the oscillation decay within the time interval converges to an unknown position at each optimization iteration step, the initial setup for the constant parameter u_{cte} is an issue for the achievement of the designs shown in figure 54. If not correctly setup, u_{cte} may lead to a large range of variation for the dynamic objective function value, and a local minima may not be

reached. A similar issue occurs when the functions (4.14) to (4.17) are used to be minimized for the design of the PPEAs. However in these cases, the difficulty is to ponder the weight functions w_1 or w_3 so the displacement $\mathbf{U}_u$ or transduction $\mathbf{U}_v$ is maximized without prevailing over the minimization of the vibration energy. For this reason, the dynamic function (4.18) is applied in this section.

When the topologies of figure 54 are post-processed for the feedback gains shown in table 15, the objective function values shown in figure 38 are obtained. Once again it can be seen the influence of the control gain magnitude by comparing the objective function values of a topology obtained with a higher constant gain with the topology obtained with a lower constant gain but post-processed with the higher gain. The last statement can be exemplified by taking the objective function value of the $\mathcal{K}_3$ -topology post-processed with $\mathcal{K}_3$ -gain. As it should be, its value is smaller than the objective function value of $\mathcal{K}_2$ -topology post-processed with $\mathcal{K}_3$ -gain.

Figure 59 – Analysis of the objective function values for the optimized topologies subjected to the proposed gains ($\times 10^5$)

6.3.2 Solid-void PPEA design considering nc and c -nodes

This section focuses on the design of PPEA with collapsed piezoelectric intermediate nodes aiming to evaluate the optimized topology by comparing with the topologies obtained in the previous section 6.3.1. The first solution is attempted for the design characteristics given in section 6.3.1 and for the feedback gain $\mathcal{K} = 3.60 \times 10^5$, however the optimization problem is unstable for collapsed nodes. The topology shown in figure 60 is numerically unstable from the point of view of the time integration computation, as it is seen in figure 61. In this case, the sensitivity of the optimization procedure can not be calculated.

Therefore, by using the same domain characteristics and a slightly modified penalization update (Fig. 62), with the maximum penalization of elastic tensor and specific weight values at iterations 25 and 35 instead of 20 and 40, respectively, the optimization convergence is achieved, with the topology shown in figure 63a. However, the vibration energy function rises instead of

Figure 60 – PPEA topology with unstable response (13th iteration step)

Figure 61 – Transient responses for the topology of figure 60

achieving a minimum value, as seen in figure 64b. For the sake of analyzing the collapsed and non-collapse nodes optimized topologies in each others domains, a new optimized design for non-collapsed nodes is obtained (Fig. 63b) considering the penalization update of figure 62.

Figure 62 – Penalization update c-nodes PPEA

The controller efficiency is then analyzed when the optimized topology in nc-nodes is distributed into the collapsed nodes domain and vice-versa, conditions proposed by the topologies in figures 65 and 66, respectively. The dynamic function values for the initial and optimized topologies of figures 63a and 63, and for these topologies in switched domains, figures 65 and

Figure 63 – PPEA optimized topologies for collapsed and non-collapsed nodes

(a) Final Topology for c-nodes, $\mathcal{K} = 3.60$, it 65 (b) Final Topology for nc-nodes $\mathcal{K} = 3.60$, it 100

Figure 64 – Multi-objective function convergence c-nodes

66, are shown in table 17.

Figure 65 – Topology obtained from nc-nodes optimization (Fig. 63b) analyzed in c-nodes domain for $\mathcal{K} = 3.60$

Figure 66 – Topology obtained from c-nodes optimization (Fig. 63a) analyzed in nc-nodes domain for $\mathcal{K} = 3.60$

Table 17 – Results comparison for the non-collapsed and collapsed nodes topologies of figure 63

Design gain ($\times 10^5$)	$f^{\text{dyn}}(\boldsymbol{\rho})(\times 10^{-16})$	
	nc-nodes	c-nodes
Initial topology		
$\mathcal{K} = 3.60$	1.2822	1.2093
Optimized topology		
$\mathcal{K} = 3.60$	1.0235	2.6483
Nc-nodes topology in c-nodes domain		
$\mathcal{K} = 3.60$	-	3.4349
C-nodes topology in nc-nodes domain		
$\mathcal{K} =$	1.7490	-

As seen in table 17, the dynamic objective function values of the topology obtained for non-collapsed nodes analyzed in the collapsed nodes domain and vice-versa are both greater than the original objective function values. These phenomena are explained by the fact that the collapsed nodes domain is stiffer than the non-collapsed nodes domain, as shown in figures 64a and 64c, respectively. Therefore, the dynamic objective function $f^{\text{dyn}} = \int_0^{t_f} (u_{\text{dof}} - u_{\text{cte}})^2 dt$ should have been set up for a lower u_{cte} , since the steady state for the collapsed nodes domain is at a lower bound than for the non-collapsed nodes, as seen in figure 67. Therefore, for the piezoactuator positioner design, the optimization problem is sensitive to the feedback gains, as noticed by the different local minimas achieved by varying the $\mathcal{K}$ -value, and is sensitive to the influence of the piezoelectric intermediate nodes.

Figure 67 – Transient responses for the results presented in table 17

CONCLUSIONS

The topology optimization method has been applied to the design of piezoelectric actuators and piezoelectric positioners considering a constant active velocity feedback control gain. The feedback control in time domain aims to reduce the vibration energy of the system by adding extra damping terms to the structural damping matrix, while the optimization procedure distributes the metallic material within the design domain, so the influence of the controller is intrinsic to the dynamics of the optimized system.

It is observed that the TOM in time domain analysis is sensitive to the problem parameters setup, such as the transient input profile and the penalization update of the SIMP model, therefore a study of the problem convergence given a set of parameter combinations has been performed and it has been seen that the final topologies vary considerably depending on the set of parameters, in some cases the convergence might not even be achieved.

For the piezoelectric actuators design with vibration suppression under a mechanical load, it is considered the optimization of a bi-material distribution, and the solid-void distribution aiming to obtain a flexible structure. The TO parameters calibration is more evident for a bi-material design, and has though contributed to set up the solid-void parameters for a successful convergence of the optimization problem. In both cases, the influence of the constant feedback gain to the optimized designs is perceptible and is evaluated by post-processing the final topologies with all of the gain values considered for the optimization. The active control modeled with the dynamic system to be optimized contributes for the increased vibration attenuation of the final design.

For the piezoelectric positioner design with vibration suppression, the solid-void distribution is considered. A transient input voltage is applied to the actuator layer electrode so that the system remains actuated in steady state. The oscillation measurements are taken by having a constant displacement reference, which is defined by numerical experiments. The optimized positioner designs present better response, in terms of the vibration attenuation function, when

regulated with the control gain value to which the topology is obtained.

The PEA designs proposed in this thesis are modeled by considering the piezoelectric intermediate nodes (non-collapsed nodes), and the influence of the piezoelectric coupling effect to the feedback control signal is evaluated for the piezoactuator design case. The PEA and the PPEA designs are obtained for the non-collapsed nodes model and then compared to the design obtained for the collapsed nodes model. The designs obtained for the collapsed model are stiffer and leads the optimization problem to a distinct local minima when compared to the non-collapsed model. Therefore, the simplified piezoceramic model, that disregards the piezoelectric coupling effect, might not be realistic with the real device behavior dynamically actuated.

The controlled piezoelectric actuators designed in this thesis by means of the TOM, presented the expected response when their actuation is regulated by the control gain value for which they are designed. However, their regulation with a larger gain does not necessarily improve the attenuation response, a behavior observed through a histogram analysis for each design problem. Yet, for either the bi-material and solid-void PEA design or the PPEA design, there is the evidence of a control gain saturation to the optimized systems. The histograms show that the dynamic objective function ceases to be improved by the larger of the gain values applied to each of those three optimization models.

Therefore, this work presents a novel approach for the design of flextensional piezoactuators using the TOM considering the time domain control effect. The velocity feedback controller therein proposed is more realistic to implement for real-time regulation than frequency domain control regulators. Yet, the non-collapsed nodes model may significantly contribute to the analysis and control of a piezoelectric device application.

7.1 Future Work

From the results obtained in this work, future developments might include the implementation of different types of control in time domain, such as the PID, for the design of flextensional PEAs by means of the TOM.

Regarding the TO problem formulation, it is suggested the simultaneous distribution of elastic and piezoelectric materials, and the design of a graded multimetallic and void-multimetallic distribution, anticipating the additive manufacturing evolution. The TO problem formulation can also involve the optimization of the active velocity feedback gain simultaneously with the topology design for a better improvement of the control action.

Considering the difficulties facing the estimation of the TO problem parameters combined with the structure parameters for a time domain transient analysis, a future development might involve the machine learning methodology for the TO parameters identification.

Yet, for the positioner model, it might be considered an objective function that minimizes the vibration energy around the maximum deformation reached by a target degree-of-freedom under an electric input. In this case, the minimization of the dynamic function would not occur around a constant value, that has to be estimated according to the domain configuration, but it would depend on the design variables.

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PIEZOELECTRIC MATERIAL

Considering the mechanical tensors symmetry, the piezoelectric constitutive equations in tensors form given by equations (2.13) can be rewritten in matrix form:

$$\begin{aligned} \mathbf{T} &= \mathbf{c}^E \mathbf{S} - \mathbf{e} \mathbf{E}, \\ \mathbf{D} &= \mathbf{e} \mathbf{S} + \boldsymbol{\epsilon}^S \mathbf{E}, \end{aligned} \quad (\text{A.1})$$

$$\mathbf{c}_{pq}^E = \begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} & c_{15} & c_{16} \\ & c_{22} & c_{23} & c_{24} & c_{25} & c_{26} \\ & & c_{33} & c_{34} & c_{35} & c_{36} \\ & & & c_{44} & c_{45} & c_{46} \\ & \text{sym.} & & & c_{55} & c_{56} \\ & & & & & c_{66} \end{bmatrix}, \quad (\text{A.2})$$

$$\mathbf{e}_{kp} = \begin{bmatrix} e_{11} & e_{12} & e_{13} & e_{14} & e_{15} & e_{16} \\ e_{21} & e_{22} & e_{23} & e_{24} & e_{25} & e_{26} \\ e_{31} & e_{32} & e_{33} & e_{34} & e_{35} & e_{36} \end{bmatrix}, \quad (\text{A.3})$$

$$\boldsymbol{\epsilon}_{ik}^S = \begin{bmatrix} \epsilon_{11} & \epsilon_{12} & \epsilon_{13} \\ & \epsilon_{22} & \epsilon_{23} \\ \text{sym.} & & \epsilon_{33} \end{bmatrix}. \quad (\text{A.4})$$

For a bidimensional finite elements implementation with a hexagonal symmetry class of piezoelectric material, $6mm$ -family, the poling direction $(y, 3)$ shown in figure 69a is assumed, leading to matrices (2.14), (2.15) and (2.16) (NADER, 2002), and the system is therefore defined

as show by (A.1):

$$\begin{Bmatrix} T_{xx} \\ T_{yy} \\ T_{zz} \\ T_{yz} \\ T_{xz} \\ T_{xy} \\ D_x \\ D_y \\ D_z \end{Bmatrix} = \begin{bmatrix} c_{11}^E & c_{12}^E & c_{13}^E & 0 & 0 & 0 & 0 & 0 & e_{31} \\ c_{12}^E & c_{11}^E & c_{13}^E & 0 & 0 & 0 & 0 & 0 & e_{31} \\ c_{13}^E & c_{13}^E & c_{33}^E & 0 & 0 & 0 & 0 & 0 & e_{33} \\ 0 & 0 & 0 & c_{44}^E & 0 & 0 & 0 & e_{15} & 0 \\ 0 & 0 & 0 & 0 & c_{44}^E & 0 & e_{15} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{66}^E & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & e_{15} & 0 & -\epsilon_{11}^S & 0 & 0 \\ 0 & 0 & 0 & e_{15} & 0 & 0 & 0 & -\epsilon_{11}^S & 0 \\ e_{31} & e_{31} & e_{33} & 0 & 0 & 0 & 0 & 0 & -\epsilon_{33}^S \end{bmatrix} \begin{Bmatrix} S_{xx} \\ S_{yy} \\ S_{zz} \\ S_{yz} \\ S_{xz} \\ S_{xy} \\ -E_x \\ -E_y \\ -E_z \end{Bmatrix} \quad (\text{A.5})$$

The 2D mechanical problem solutions obtained for *plane stress* assumption takes into account the body mass width in 1,x-direction and the body mass height in 3,z-direction comparable dimensions, however, both dimensions are much larger than the 2,y-direction, as shown in figure 68. Therefore, assuming equations (2.17), the constitutive piezoelectric system (A.5) takes

Figure 68 – Body under plane stress assumption. Polarization is considered in 3,z-direction.

the form stated by matrices (2.18), (2.19) and (2.20), and the constitutive piezoelectric system is

rewritten:

$$\begin{Bmatrix} T_{xx} \\ T_{yy} \\ T_{xy} \\ D_x \\ D_y \end{Bmatrix} = \begin{bmatrix} c_{11}^E - \frac{c_{12}^E{}^2}{c_{11}^E} & c_{13}^E - \frac{c_{12}^E c_{13}^E}{c_{11}^E} & 0 & 0 & e_{31} - \frac{e_{31} c_{12}^E}{c_{11}^E} \\ c_{13}^E - \frac{c_{12}^E c_{13}^E}{c_{11}^E} & c_{33}^E - \frac{c_{13}^E{}^2}{c_{11}^E} & 0 & 0 & e_{33} - \frac{e_{31} c_{13}^E}{c_{11}^E} \\ 0 & 0 & c_{66}^E & e_{15} & 0 \\ 0 & 0 & e_{15} & -\epsilon_{11}^S & 0 \\ e_{31} - \frac{e_{31} c_{12}^E}{c_{11}^E} & e_{33} - \frac{e_{31} c_{13}^E}{c_{11}^E} & 0 & 0 & -\epsilon_{33}^S - \frac{e_{31}^2}{c_{11}^E} \end{bmatrix} \begin{Bmatrix} S_{xx} \\ S_{yy} \\ S_{xy} \\ -E_x \\ -E_y \end{Bmatrix} \quad (\text{A.6})$$

Figure 69 – Piezoelectric coordinate system

Adapted from (MEEKER, 1996)

NUMERICAL DIFFERENTIATION

For a linear elastic problem, the general equation is written:

$$\begin{aligned} \mathbf{M}\ddot{\mathbf{U}}(t) + \mathbf{C}\dot{\mathbf{U}}(t) + \mathbf{K}\mathbf{U}(t) &= \mathbf{F}(t) \quad t \in [0, t_f] \\ \mathbf{U}(0) &= \mathbf{U}_0, \\ \dot{\mathbf{U}}(0) &= \mathbf{V}_0. \end{aligned} \tag{B.1}$$

Finite difference formulas are derived from Taylor series expansions, where the mesh size influences the truncation error and though the numerical solution accuracy (PARVIZ, 2001).

Newmark's method family ((HILBER; HUGHES; TAYLOR, 1977), (CHUNG; HULBERT, 1993), (SHEARER; CESNIK, 2006)) comprises numerical time-integration methods where the equilibrium equation of the system is evaluated in discretized time-instants with sampling rates of Δt . The α -form of *Newmark's* numerical integration scheme is stated bellow.

$$\mathbf{U}_{n+1} = \mathbf{U}_n + \Delta t \dot{\mathbf{U}}_n + \frac{1}{2} \Delta t^2 (2\beta \ddot{\mathbf{U}}_{n+1} + (1 - 2\beta) \ddot{\mathbf{U}}_n), \tag{B.2}$$

$$\dot{\mathbf{U}}_{n+1} = \dot{\mathbf{U}}_n + \Delta t (\gamma \ddot{\mathbf{U}}_{n+1} + (1 - \gamma) \ddot{\mathbf{U}}_n), \tag{B.3}$$

where t_n is the present time and t_{n+1} is the incremented time ($t_n + \Delta t$).

Substituting $\mathbf{U}_{n+1}$ and $\dot{\mathbf{U}}_{n+1}$ in equation (B.1), we have:

$$\underbrace{(\mathbf{M} + \gamma \Delta t \mathbf{C} + \beta \Delta t^2 \mathbf{K})}_{\tilde{\mathbf{M}}} \ddot{\mathbf{U}}_{n+1} = \mathbf{F}_{n+1} - \mathbf{C}(\dot{\mathbf{U}}_n + \Delta t (1 - \gamma) \ddot{\mathbf{U}}_n) - \mathbf{K}(\mathbf{U}_n + \Delta t \dot{\mathbf{U}}_n + 1/2 \Delta t^2 (1 - 2\beta) \ddot{\mathbf{U}}_n). \tag{B.4}$$

Newmark- β method uses Cauchy's mean value theorem to state that the first time derivative can be solved as

$$\ddot{\mathbf{U}}_{n+1} = \dot{\mathbf{U}}_n + \Delta t \ddot{\mathbf{U}}_\gamma, \tag{B.5}$$

where

$$\ddot{\mathbf{U}}_\gamma = (1 - \gamma)\ddot{\mathbf{U}}_n + \gamma\ddot{\mathbf{U}}_{n+1}, \quad 0 \leq \gamma \leq 1. \quad (\text{B.6})$$

As acceleration also varies with time, Cauchy's mean value theorem must be extended to the second time derivative to obtain the correct displacement:

$$\mathbf{U}_{n+1} = \mathbf{U}_n + \Delta t \dot{\mathbf{U}}_n + 1/2 \Delta t^2 \ddot{\mathbf{U}}_\beta, \quad (\text{B.7})$$

where

$$\ddot{\mathbf{U}}_\beta = (1 - 2\beta)\ddot{\mathbf{U}}_n + 2\beta\ddot{\mathbf{U}}_{n+1}, \quad 0 \leq 2\beta \leq 1. \quad (\text{B.8})$$

Figure 70 shows the validation of α -Newmark method for numerical differentiation implemented in MATLAB[®] and simulated in COMSOL[®] for a transient problem. The system considered in both cases is a piezoelectric transducer of Aluminum and PZT-5A materials.

Figure 70 – Output plots of a problem solved by α -Newmark scheme implemented in MATLAB[®] and by COMSOL[®] internal code.