ALEX SARATANI HUANG

**Development of a force actuator for hybrid model tests in
a wave tank**

São Paulo

2021

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Concentration area:
Control and Mechanic Automation
Engineering

Advisor:
Prof. Dr. Eduardo Aoun Tannuri

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ABSTRACT

HUANG, A. S. **Development of a force actuator for hybrid model tests in a wave tank.** 2021. Dissertation (Master of Sciences in Control and Mechanic Automation Engineering) – Escola Politécnica of Universidade de São Paulo, São Paulo, 2021.

One solution to overcome the limitations of small scale model testing, such as scaling conflicts or infrastructure constraints, while retaining the physical phenomena's complexity, is hybrid model testing. In hybrid model testing, the physical substructure (tested experimentally) and the numerical substructure (simulated in a computer) interact with each other through a network of sensors and actuators. Hence, to apply the desired effects in the physical substructure, precise control of the actuators is necessary. In this work, we will develop a force actuator for hybrid model testing of a vessel in a wave tank. It will be employed a robust control technique (super-twisting sliding mode) to develop the controller of the force actuator consisting in a servomotor that applies a desired force through a line attached to the vessel model. We describe the force actuator dynamics and develop the sliding mode controller considering the system uncertainties (external disturbances, unknown plant parameters or unmodeled dynamics). Then, the force actuator is validated experimentally. We analyzed a step and a sinusoid input (0.1 to 2.0 Hz), with a disturbance motion of frequencies up to 1.5 Hz. The actuator developed showed good results (mean absolute error inferior to 0.1 N in all tests), where the high frequency error spikes (up to 0.33 N) should not have a significant effect on the hybrid model tests slow dynamics. We conclude that the force actuator will be a valuable tool for hybrid model testing.

Keywords: Force control. Hybrid model testing. Super-twisting sliding mode control.

RESUMO

HUANG, A. S. **Desenvolvimento de um atuador de forças para ensaios com modelo híbrido em um tanque de ondas**. 2021. Dissertação (Mestrado em Ciências na área de Engenharia de Controle e Automação Mecânica) – Escola Politécnica da Universidade de São Paulo, São Paulo, 2021.

Uma solução para superar limitações de ensaios com modelos em escala reduzida, como por exemplo conflitos de escala ou restrições de infraestrutura, ao mesmo tempo em que a complexidade do fenômeno físico é mantida, é utilizar ensaios com modelo híbrido. No ensaio com modelo híbrido, a subestrutura física (ensaiada experimentalmente) e a subestrutura numérica (simulada no computador) interagem uma com a outra através de uma rede de sensores e atuadores. Assim, de forma a aplicar os efeitos desejados na subestrutura física, um controle preciso dos atuadores é necessário. Neste projeto, será desenvolvido um atuador de forças para ensaios com modelo híbrido de uma embarcação em um tanque de ondas. Será empregada uma técnica de controle robusto (super-twisting sliding mode) para desenvolver o controlador do atuador de forças que consiste em um servomotor que aplica uma força desejada através de uma linha conectada ao modelo de embarcação. Nós descrevemos a dinâmica do atuador de forças e desenvolvemos o controlador por modos deslizantes, levando em consideração as incertezas do sistema dinâmico (distúrbios externos, parâmetros desconhecidos ou dinâmicas não modeladas da planta). Então, o atuador de forças é validado experimentalmente. Foi analisada uma entrada degrau e uma entrada senoidal (0,1 a 2,0 Hz), com um movimento externo de distúrbio de até 1,5 Hz. O atuador desenvolvido apresentou bons resultados (erro absoluto médio inferior a 0,1 N para todos os testes), onde os erros de alta frequência (com picos de até 0,33 N) não devem apresentar um efeito significativo para a dinâmica lenta dos ensaios com modelo híbrido. Concluímos que o atuador de forças será uma ferramenta valiosa nos ensaios com modelo híbrido.

Palavras-chave: Controle de força. Ensaio com modelo híbrido. Controle por modos deslizantes super-twisting.

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LIST OF ABBREVIATIONS AND ACRONYMS

2-SM	Second-Order Sliding Mode
CFD	Computational Fluid Dynamics
DOF	Degree(s) of Freedom
FOWT	Floating Offshore Wind Turbine
HOSM	Higher-Order Sliding Mode
IMU	Inertial Measurement Unity
I/O	Input/Output
kS/s	kSample/s
NTNU	Norwegian University of Science and Technology
OCIMF	Oil Companies International Marine Forum
PC	Personal Computer
PID	Proportional-Integral-Derivative
ppr	Pulse per revolution
SMC	Sliding Mode Control
STSMC	Super-Twisting Sliding Mode Control
TPN	Tanque de Provas Numérico
USP	Universidade de São Paulo

LIST OF SYMBOLS

Roman Alphabet

$\mathbf{x}(t)$ or $\mathbf{x}$	State vector
x	Controlled output
t	Time variable
$f(t, \mathbf{x})$	Transition state function
$b(t, \mathbf{x})$ or b	Gain function
u	Control input
$\mathbf{x}_d(t)$	Desired state vector
$\tilde{\mathbf{x}}(t)$ or $\tilde{\mathbf{x}}$	Error state vector
T_A	Largest unmodeled time delay
u_{eq}	Equivalent control
$\hat{u}_{eq}$	Estimate of the equivalent control
u_{vss}	Switching control
$h(t, \mathbf{x})$ or h	Error of the equivalent control estimate
t_{reach}	Reach phase time
K	SMC controller gain
$sign()$	Sign function
$\bar{u}_{eq}$	Upper bound for error in estimate of the equivalent control
$sat()$	Saturation function
K_m	Lower bound on the uncertain gain function
K_M	Upper bound on the uncertain gain function
q	Positive parameter between 0 and 1
U_M	STSMC adaptive control term saturation

C	Upper bound on sliding variable uncertain acceleration
u_1	STSMC adaptive control term
$oddsqrt()$	Odd square root function
t_0	Time instant when $[\sigma \ \dot{\sigma}] = [0 \ \dot{\sigma}_0]$
t_c	STSMC convergence time
R	Radius of the wheel
$F(\theta, \ \mathbf{P}\)$ or $F(t)$ or F	Line force
$\mathbf{P}(t)$ or $\mathbf{P}$	Position vector of the line fixation point on the vessel
$[x_p \ y_p \ z_p]$	Coordinates of point $\mathbf{P}$
J_m	Moment of inertia of servomotor and wheel
$A(\dot{\theta})$	Servomotor friction moment
F_v	Friction viscous coefficient
F_c	Friction Coulomb coefficient
F_S	Friction Stribeck effect coefficient
k	Linear spring stiffness
f_1	Angular position transition state function
b_1	Angular position gain function
$\hat{f}_1$	Estimate of the angular position transition state function
$\hat{A}(\dot{\theta})$	Continuous static friction model
$\hat{F}$	Line force filtered measurement
$F_d(t)$ or F_d	Line force desired trajectory
$\tilde{F}$	Line force error
T_s	Controller sampling time
f_c	Butterworth filter cut-off frequency
n	Butterworth filter order

$A_S(\dot{\theta})$	Discontinuous friction model
f	Desired force frequency
D_j	Disturbance motion for run P_j
P_j	Run with motion disturbance D_j
$O_0x_0y_0, O_1x_1y_1$	Coordinate systems
$\mathbf{f}_{env}$	Resultant vector of current and wind forces
X_w, Y_w, N_w	Wind force horizontal components
X_c, Y_c, N_c	Current force horizontal components
V_w	Wind absolute speed
A_F	Vessel frontal projected area
A_L	Vessel lateral projected area
C_{wx}, C_{wy}, C_{wz}	Wind forces coefficients
V_{wr}	Wind speed relative to the vessel
L	Vessel length
U, V	Vessel absolute velocity components
V_c	Current absolute speed
T	Vessel draft
C_x, C_y, C_z	Current forces coefficients
V_{cr}	Current speed relative to the vessel
$\mathbf{X}_G$	Vessel center of mass position vector
$[x_g, y_g]$	Coordinates of point $\mathbf{X}_G$
$\mathbf{P}_i$	Position vector of the line i attachment point on the vessel
$\mathbf{M}_i$	Position vector of the servomotor i
$\mathbf{r}_i$	Vector distance between $\mathbf{P}_i$ and O_1

F_{Li}	Tension in line i
${}^A Rot_B(\quad)$	Rotation matrix from reference system B to A
$[f_x \ f_y \ M_z]$	Coordinates of vector $\mathbf{f}_{env}$
J	Cost function
$\mathbf{f}_v$	Line forces vector
B	Geometric configuration matrix
B_i	Column i of matrix B
F_{max}	Maximum force on the line constraint
F_{min}	Minimum force on the line constraint
$\hat{\mathbf{f}}_v$	Line force solution to the optimization problem
F_0	Pretension value

Greek Alphabet

σ	Sliding variable
$\lambda, \lambda_A, \lambda_R, \lambda_S$	SMC convergence time parameters
ν_R	Lowest unmodeled resonance frequency
ν_S	Sampling frequency
η	SMC parameter related to the reach phase time
ϕ	SMC Boundary layer
λ_1, λ_2	STSMC controller gains
$\sigma_{Mi}, \dot{\sigma}_{Mi}$	i th bound on the sliding variable phase plane trajectory
$\theta(t)$ or θ	Angular position of the wheel
τ_m	Servomotor torque
α, β	Static friction model coefficients
$\theta_d(t)$ or θ_d	Angular position desired trajectory

σ_1	Sliding variable of the servomotor angular position
$\tilde{\theta}$	Angular position error
σ_2	Sliding variable of the line force
τ	Time delay
$\varphi(\omega)$ or φ	Phase delay
ω	Sinusoid input frequency
$\hat{\varphi}$	Phase delay linear approximation
β_w	Wind absolute directions
ψ	Vessel heading
ψ_c	Current absolute direction
ρ_a	Air density
$\dot{\eta}$	Vessel absolute velocity vector
ρ	Water density
ψ_{cr}	Current relative direction to the vessel
α_i	Angle of line i relative to the vessel x axis

1 INTRODUCTION

The use of model testing to verify the performance of ships has a long tradition (STEEN, 2017). Around the year 1500, experiments were already performed to determine the most hydrodynamic shape for a vessel.

Nowadays, the demands of the offshore industry – be it for drilling rigs capable of extracting oil below 2000m depth or for floating offshore wind turbines (FOWT), a major development towards new renewable energy sources (Figure 1) – make physical hydrodynamic model testing in a controlled environment indispensable in the development of new structures.

Figure 1 - Floating offshore wind turbine (left), drilling rig (right)

Despite the development of sophisticated numerical models and tools, some physical phenomena still do not have satisfactory numerical models, for example: extreme wave loads (slamming, ringing and green water), viscous loads (roll and yaw damping of ship-shaped floaters, side-by-side vessels interaction), or wave-current interaction effects on floating moored structures (SAUDER et al., 2016).

Thus, in these past 20 years, complex research facilities were developed, capable of testing structures in the desired wave, wind and current condition such that the model testing results would be the most representative of the actual behavior of the vessel in a real environmental condition. Figure 2 illustrates the ocean basin of SINTEF Ocean company capable of generating wave, wind and current.

Figure 2 - Ocean basin capable of simulating wave, wind and current conditions

Source: NTNU (2014).

However, there are still cases that cannot be properly tested with a conventional model and ocean basin. In the case of FOWTs, the floater is subject mainly to hydrodynamic forces dominated by gravitational effects (Froude scaling) and the rotor and blades are subject to aerodynamic forces due to frictional effects (Reynolds scaling). Therefore, it becomes a challenge to develop a model test capable of accurately describing the structure response to both wind and waves simultaneously. It may also be the case that the laboratory's physical limits are reached, for example, the infeasibility to simulate wind gusts due to the fans response time or the tank dimension limits to model structures in very deep water.

One solution to overcome these limitations of model testing, while retaining the real physical phenomena' complexity, is to use hybrid model testing. The physical system is divided into two substructures: the physical substructure tested experimentally, connected to the numerical substructure simulated in a computer. Both substructures interact in real-time with each other through a network of sensors and actuators. Figure 3 outlines this process.

Figure 3 - Principle of model testing with hybrid model

The hybrid model test can be understood as a hardware/software-in-the-loop method. The phenomenon of interest is contained in the physical substructure tested experimentally, and secondary processes of the system are contained in the numerical substructure simulated in a computer. So, instead of modeling physically the numerical substructure, only its effects (forces or displacements) are applied to the physical substructure.

As mentioned, hybrid model tests can overcome some of the limitations of conventional model testing. For example, it solves the FOWTs Froude-Reynolds scaling conflict by modeling only the floater physically, while the aerodynamic loads on the blades and rotor are simulated numerically and directly actuated on the physical substructure.

Notice that the incorrect application of the numerical substructure effects onto the physical substructure invalidates the hybrid model test, thus a precise control of the actuators is necessary, taking into account the sensors' measurement noise, numerical substructure computational time and actuators' dynamics.

1.1 Motivation

The Numerical Offshore Tank laboratory (*Tanque de Provas Numérico – Universidade de São Paulo*, TPN-USP) has a wave tank capable of generating multidirectional waves (Figure 4), however it does not have a current or a wind generator, which limits the types of tests that can be done in the tank.

Figure 4 - TPN-USP wave tank

Therefore developing a force actuator for hybrid model testing is one possible solution to include the effects of wind and current in the laboratory experiments. Additionally, the force actuator can be used for many other types of hybrid model testing (for FOWTs or moored vessels, for example) and possibly even to test extreme model parameters, which could be impossible to test physically, improving the comprehension on the physical phenomena and their modeling.

The use of real-time hybrid models originated in Japan, in the 1970s, to study the structural effects of earthquakes in buildings (CARRION; SPENCER JR, 2007), and since then has become a standard tool in damage-mitigation analysis for structures.

In the automotive industry, the term "model-in-the-loop" is more common to refer to this method. Plummer (2006) proposed a general modeling framework for hybrid model tests, and used it to simulate the aerodynamic effects in a racing car (pitch moment and vertical force) as shown in Figure 5. The hybrid model used the 'four poster' rig, where under each wheel, a hydraulic actuator replicated the road height variation and two extra actuators applied the aerodynamic down force. Then, he compared the response of the hybrid model to that of an ideal system to assess the effects of sensor noise and actuator delays in the model test and concluded the results had an acceptable degree of realism.

Figure 5 - Race car aerodynamic hybrid model

Source: Plummer (2006).

Meanwhile, in the marine technology field, hybrid models started being used more recently. In Stansberg, Ormberg and Oritsland (2002) the interest in hybrid model testing arose from the necessity to verify new systems to explore ultra deepwater oil fields (ranging from 1000m to 3000m). By then, no ocean basin was large enough to perform model testing of the drilling rig with its mooring system within reasonable limits of model scale.

Figure 6 - Representation of a FPSO moored in 3000m depth and need for truncation in a laboratory model

Source: Stansberg, Ormberg and Oritsland (2002).

Figure 6 exemplifies this problem. The usual solution is to truncate the lines in length. The truncated system can be passive-type, where an equivalent mechanical system substitutes the full depth mooring system using springs and masses. Alternatively, in an active type (conventional hybrid model), the effect of the cut-off portion of the mooring line is calculated by a computer and applied through actuators to the physical model.

Cao and Tahchiev (2013) analyzed an active system to simulate mooring lines and risers of deepwater floating systems. The proposed hybrid system is illustrated in Figure 7. To control the actuators, they proposed a Proportional-Integral-Derivative (PID) controller and compared the results obtained from a numerical case study of the full-length mooring lines against the proposed hybrid system. Applying a sinusoidal motion on the floater in the X and Z directions, they concluded that there was a good agreement in displacement and forces between the decomposed model and the full-length mooring lines.

Figure 7 - Sketch of the active hybrid testing system

Source: Cao and Tahchiev (2013).

Chabaud, Steen and Skjetne (2013) pointed out that real-time hybrid testing has not been successful enough to be accepted as a standard and accurate method. Coupling physical and numerical models involves unwanted dynamics and time delays that must be reduced and compensated for. They analyzed a virtual hybrid model of a floating wind turbine's pitch motion focusing on the numerical substructure aspects of the hybrid model (step size of the simulation, actuators performance, controller design, sensors noise and observer project).

Hybrid models received additional attention lately due to their capacity to test floating wind turbines experimentally, since it solves the Froude-Reynolds scaling conflict in conventional model test.

Sauder et al. (2016) performed hybrid model tests in real-time to study a FOWT behavior. The aerodynamic/generator loading on the wind turbine was evaluated numerically and all the load components of importance were applied in real-time to the physical substructure, while surface waves and ocean currents were modeled physically. The oncoming turbulent wind was modeled numerically using time series generated in TurbSim v1.5 and the aerodynamic loads were calculated using the open-source code AeroDyn. Six servomotors applied these loads with pulleys connected to the physical model using thin lines (Figure 8). The authors concluded that the wind turbine hybrid model allows tests in combined wind, wave and current conditions, testing generators and rotors from different manufacturers and testing different control strategies for the wind turbine controller.

Figure 8 - Illustration of a floating wind turbine physical substructure

Source: Sauder et al. (2016).

Kanner, Yeung and Koukina (2016) used hybrid model to analyze a new concept of floating wind turbine where more than one turbine is placed in the same floating platform by using vertical axis wind turbines, optimizing the power and costs of the structure. The hybrid model actuators consisted of remote-controlled plane motors/propellers directly mounted to the model as shown in Figure 9 (applying forces F_1 and F_2). Using an Inertial Measurement Unity (IMU) and microcontrollers, the thrust force generated by the propellers could be controlled to mimic the tangential force developed by the airfoils on a vertical-axis wind turbine.

Figure 9 - Schematic of sensors and actuators on the model platform

Source: Kanner, Yeung and Koukina (2016).

Gueydon, Bayati and de Ridder (2020) compared the state of the art of two possible solutions for basin model tests of floating offshore wind turbines, the “full approach” and the “simplified approach”. In the “full approach”, the whole wind turbine is modeled and tested. In order to properly scale wind drag and lift forces on the turbine rotor, performance scaling is used to model the blades’ airfoil. Meanwhile, the “simplified approach” corresponds to the hybrid model test. Finally, they pointed that, for hybrid model testing, the force actuators are the primary constraint.

1.2 Objectives

The objective of this project is to develop a force actuator system for hybrid model tests of vessels in a wave tank. The actuator consists in a servomotor controlling the force on a line attached to the model.

First, it will be necessary to define the force actuator components and model the system dynamics. Then, the methodology and design of the actuator's controller, considering the model uncertainties (servomotor friction torque, vessel motion and system delays), will be specified. Finally, we will develop an experimental setup and numerical model to validate the proposed actuator's performance.

2 LITERATURE REVIEW

The force actuator can generate varying forces in time, following a pre-defined function. In hybrid model tests, its main purpose is to emulate the resultant forces of physical phenomena that are not possible to be modeled in reduced scale model testing. As was mentioned in chapter 1.1, the actuator can be applied in the model testing of moored structures in deep water (STANSBERG; ORMBERG; ORITSLAND, 2002 and CAO; TAHCHIEV, 2013), floating offshore wind turbines (KANNER; YEUNG; KOUKINA, 2016 and SAUDER et al., 2016) or other cases that would be unviable due to the model or laboratory restrictions.

The two types of actuators typically used in the ocean basin's hybrid model tests are fans (KANNER; YEUNG; KOUKINA, 2016) or lines/cables tensioned by a servomotor (SAUDER et al., 2016). The fan actuators have a more straightforward setup but a slower response time compared to the lines actuation. Moreover, the fan actuators can turn off the load, while the servomotor always applies some load as long as the line is connected to the model – even if the line is “fully” relieved of tension, it will still be subject to its own weight, and therefore, apply a corresponding reaction force to its support points.

Solum (2017) studied the performance of a hybrid model actuator for ocean basin model tests. He analyzed a 1 degree-of-freedom (DOF) second-order dynamic system consisting of two connected mass-spring-damper systems, where one of the masses is simulated numerically. Considering the small magnitude of the environmental loads on floating structures models, the actuator consisted of a servomotor connected to a clockspring and a wheel with a wire attached to the physical model. Figure 10 shows the actuator developed.

Figure 10 - Hybrid model force actuator

Source: Solum (2017).

The force applied to the physical substructure is controlled considering the clockspring linear relation between the wire force and the servomotor position together with a PID feedback controller to minimize force error due to model uncertainties. The time delays of the system are compensated using polynomial extrapolation, which enhances the performance of the force controller, however the optimal extrapolation time depends on the input frequency. The force measurements are filtered combining a model-based observer with a low-pass filter to minimize phase lag. The force actuator is tested experimentally using a varying frequency reference signal, he concludes that the force actuation system has a frequency limit of up to 0.3 Hz. A fully numerical model in Simulink is also developed to validate the force actuator and serve as a baseline.

Hall and Goupee (2018) consider the cable based force actuators used in hybrid model testing of vessels analogous to the cable-driven parallel manipulators (POTT, 2018). They study hybrid model tests of a wind turbine using two cables connected to the turbine nacelle in opposing directions. Servomotors following a desired speed reference signal control the tension transmitted by the cables. The hybrid model test is depicted in Figure 11.

Figure 11 – Hybrid model test configuration for FOWT with cable based force actuators

Source: Hall and Goupee (2018).

The force controller developed has three components: two feedforward terms, one to compensate the vessel motion and one to determine a desired speed based on the force reference and the cable stiffness, and the third term is a proportional error feedback to

compensate model uncertainties. To measure the system states, they use optical sensors for the floating platform motion and load cells for the cables tension. The system time delays are compensated by extrapolating a cubic polynomial fitted over a large number of measurements, also serving as a filter. Extensive tests were done with the force actuator isolated to tune its controller. The force actuator presented a root-mean-squared error of 0.1 N for inputs up to 1.8 Hz range.

Ueland, Sauder and Skjetne (2021) developed a cable based force actuator for hybrid model tests. It consisted in an actuated winch with a position controlled servomotor connected to the cable drum via a clockspring – similar setup to Sauder et al. (2016) and Solum (2017). They tested the actuator with a 1-DOF mass-spring system, Figure 12 depicts an illustration of the actuator and the experimental test setup.

Figure 12 – Representation of force actuator (left) and experimental setup (right)

Source: Ueland, Sauder and Skjetne (2021).

Unlike most controllers, their force actuator is based only in a feedforward control, which means the system cannot become unstable. In order to implement this controller, they did a precise mapping of the force transmission system, compensated time delays using third degree polynomial extrapolation and had an algorithm to adapt the system parameters online. They conclude the actuator is fit for high accuracy real-time hybrid model testing, with a 0.2 N accuracy and 1.4 Hz bandwidth.

3 THEORETICAL BACKGROUND – SLIDING MODE CONTROL

Sliding mode control (SMC) became one of the most successful methods in dealing with dynamic systems bounded uncertainties - external disturbances, unknown plant parameters or unmodeled dynamics (UTKIN; GULDNER; SHI, 2009). SMC maintains a desired dynamic behavior constraint (the sliding mode) by means of a theoretical infinite switching frequency control input.

The resulting high-frequency control chattering can cause wear and tear on the actuators, which, in most cases, needs to be avoided. The simplest solution to make the switching control continuous is by adding a boundary layer around the switching surface and forcefully make the control continuously transition inside it (SLOTINE; LI, 1991). Inside the boundary layer, the SMC's robustness is not guaranteed; therefore this solution offers a trade-off between chattering reduction and control accuracy. Another solution to smooth the control is to use higher-order sliding mode (HOSM) control (FRIDMAN; LEVANT, 2002). In this case, the discontinuous term used to compensate for the system's uncertainties is contained in the time derivative of the control signal. This solution not only removes the control chattering but also improves its accuracy. However, most of the time, higher time derivatives of the system states are needed for its implementation, which might be impractical.

Super-twisting sliding mode control (STSMC) is a HOSM, specifically a second-order sliding mode (2-SM), whose algorithm does not require more information than the conventional SMC. It maintains the higher accuracy and chattering suppression properties of a 2-SM controller.

3.1 Conventional Sliding Mode

We will consider the single-input single-output (SISO) second-order dynamic system described by

$$\ddot{x} = f(t, x) + b(t, x)u, \quad (1)$$

where x is the controlled output, u is the control input and $\ddot{x}$ represents the second-order time derivative of x . $f(t, x)$ and $b(t, x)$ are smooth not exactly known functions, but their

uncertainty is limited by an upper bound, $\mathbf{x}$ is the state vector ($\mathbf{x} = [x \dot{x}]$) and the sign of $b(t, \mathbf{x})$ is known.

The objective is to have the state vector $\mathbf{x}(t)$ following a desired trajectory $\mathbf{x}_d(t)$, or, equivalently, to have the tracking error $\tilde{\mathbf{x}}(t)$ equal to zero:

$$\tilde{\mathbf{x}}(t) = \mathbf{x}(t) - \mathbf{x}_d(t) = [\tilde{x} \ \dot{\tilde{x}}] = 0. \quad (2)$$

In order to accomplish that, we define the variable σ as a function of the state errors:

$$\sigma = \dot{\tilde{x}} + \lambda \tilde{x} \quad , \quad \lambda > 0. \quad (3)$$

Notice that satisfying the constraint $\sigma = 0$ makes the tracking error exponentially decay to zero, so our control problem can be restated as keeping the sliding variable σ at 0 (maintaining the error state vector $\tilde{\mathbf{x}}$ in the sliding surface $\sigma = 0$). We assume that all the states and their derivatives can be either directly measured or estimated through state observers.

The value of λ determines how fast the tracking error $\tilde{\mathbf{x}}$ converges to 0 when in sliding mode ($\sigma = 0$), however some limitations should be taken into account when choosing this value, Slotine and Li (1991) gives the following guidelines:

- a) the value of λ should be smaller than the lowest unmodeled resonance frequency of the system (ν_R), such that

$$\lambda \leq \lambda_R = \frac{2\pi}{3} \nu_R ; \quad (4)$$

- b) the value of λ should take into account the system largest unmodeled time delay (T_A) according to

$$\lambda \leq \lambda_A = \frac{1}{3T_A} ; \quad (5)$$

- c) the value of λ should be smaller than the sampling frequency (ν_S) according to

$$\lambda \leq \lambda_S = \frac{1}{5} \nu_S . \quad (6)$$

Therefore the value of λ will be equal to the smallest value between λ_R, λ_A and λ_S .

In order to control σ , we determine its dynamics (derive Equation (3)) which will be a function of the uncertain functions $f(t, \mathbf{x})$ and $b(t, \mathbf{x})$ The

$$\dot{\sigma} = f(t, \mathbf{x}) - \ddot{x}_d + \lambda \dot{\tilde{x}} + b(t, \mathbf{x})u . \quad (7)$$

The control law u for the sliding mode controllers can be divided into two parts ($\hat{u}_{eq}$ and u_{vss}):

$$u = \hat{u}_{eq} + u_{vss} . \quad (8)$$

First, we have the equivalent control u_{eq} . It is the control action that would be applied to maintain the constraint $\sigma = 0$ after the sliding mode is reached (can be understood as having a feedback linearization effect, ideally it would cancel all the terms in Equation (7) and guarantee $\dot{\sigma} = 0$), therefore

$$u_{eq} = -b^{-1}(t, \mathbf{x}) [f(t, \mathbf{x}) - \ddot{x}_d + \lambda \dot{\tilde{x}}] . \quad (9)$$

However, due to the system uncertainties, the equivalent control can only be estimated by $\hat{u}_{eq}$, with an error. We define the residual $h(t, \mathbf{x})$ as a function of the difference between the equivalent control (u_{eq}) and its estimate ($\hat{u}_{eq}$),

$$h(t, \mathbf{x}) = b(t, \mathbf{x})(\hat{u}_{eq} - u_{eq}) . \quad (10)$$

So, combining Equations (7), (8) and (10), we obtain the base dynamic equation to control the sliding variable σ :

$$\dot{\sigma} = h(t, \mathbf{x}) + b(t, \mathbf{x})u_{vss} . \quad (11)$$

To compensate the residual $h(t, \mathbf{x})$ and make σ evolve in a desired manner, we use the second term of the sliding mode control u_{vss} (the switching control), The method to calculate u_{vss} is what changes from the different types of sliding mode control. One condition to converge the variable σ to zero in finite time, used in the conventional sliding mode, is simply

to make σ increase when it is negative and decrease when it is positive, which is mathematically expressed as

$$\begin{cases} \dot{\sigma} > 0 & , \sigma < 0 \\ \dot{\sigma} < 0 & , \sigma > 0 \end{cases} \Rightarrow \frac{1}{2} \frac{d}{dt} \sigma^2 \leq -\eta |\sigma| \quad , \quad \eta > 0 . \quad (12)$$

Equation (12) is called the sliding condition that guarantees the error state vector $\tilde{\mathbf{x}}$ reaches the sliding surface $\sigma = 0$. It can also be shown that for $\sigma(t = 0) \neq 0$, $\tilde{\mathbf{x}}$ will reach surface $\sigma = 0$ in a finite time

$$t_{reach} < \frac{\sigma(t = 0)}{\eta} . \quad (13)$$

A switching control u_{vss} that satisfies the sliding condition is

$$u_{vss} = -K \text{sign}(\sigma) \text{sign}(b) , \quad (14)$$

The switching control is a gain K with opposite sign of σ , where the value of the gain K needs to be higher than the error of the equivalent control (equal to the known upper bound $\bar{u}_{eq}$)

$$K = \bar{u}_{eq} > |\hat{u}_{eq} - u_{eq}| . \quad (15)$$

The sliding mode dynamics is represented in the following phase plane:

Figure 13 - Phase plane of the tracking error

This control law applies a sufficiently high switching gain K to ensure that the sliding variable σ converges to zero even in the presence of model uncertainties. However this solution introduces a discontinuity in the control law in $\sigma = 0$, that will cause chattering in the control input, which is usually not a desired behavior since it can cause wear and tear in the actuators or excite unmodeled resonance frequencies of the system.

One solution to minimize this problem is to add a boundary layer ϕ around $\sigma = 0$ where the switching gain transition from $+K$ to $-K$ smoothly. This can be done, for example, by substituting the function $\text{sign}(\sigma)$ by $\text{sat}(\sigma/\phi)$,

$$u_{vss} = -K \text{sat}\left(\frac{\sigma}{\phi}\right) \text{sign}(b), \quad (16)$$

where:

$$\text{sat}\left(\frac{\sigma}{\phi}\right) = \begin{cases} \frac{\sigma}{\phi} & , \quad -1 \leq \frac{\sigma}{\phi} \leq 1 \\ -1 & , \quad \frac{\sigma}{\phi} < -1 \\ 1 & , \quad \frac{\sigma}{\phi} > 1 \end{cases} . \quad (17)$$

Note that for $|\sigma| > \phi$, control law in Equation (16) is equal to Equation (14), so the sliding condition is satisfied and σ converges to ϕ . For $|\sigma| < \phi$ the sliding condition is not satisfied and the tracking error is not guaranteed to converge to zero, however, it can be shown that if σ is bounded by a value ϕ ($|\sigma| < \phi$), the error in $\tilde{x}$ will also be bounded according to:

$$|\tilde{x}| \leq \frac{\phi}{\lambda} . \quad (18)$$

If a higher precision is required, higher-order sliding mode controller can be used. In the next section, the methodology for calculating u_{vss} using STSMC is presented.

3.2 Super-Twisting Sliding Mode Control

Assume that, for some positive constants K_m, K_M, q, U_M and C , the system in Equation (11) satisfies

$$\begin{cases} 0 \leq K_m \leq b(t, \mathbf{x}) \leq K_M \\ |h/b| < qU_M, \quad 0 < q < 1 \\ |\dot{h}| + |\dot{b}|U_M \leq C \end{cases} . \quad (19)$$

These constraints imply that:

- a) the uncertain gain $b(t, \mathbf{x})$ is bounded by K_m and K_M ;
- b) for sufficiently high control input ($u_{vss} \geq U_M$), $\dot{\sigma}$ will have the same sign as u_{vss} ($\dot{\sigma} u_{vss} > 0$);
- c) the absolute value of the time derivative of $\dot{\sigma}$ for u_{vss} constant and equal to U_M is upper bounded by C ($|d\dot{\sigma}/dt|_{u_{vss}=U_M} \leq C$).

Notice that these three constraints establish known limits for the system uncertainties. Then the super-twisting controller defined by

$$u_{vss} = -\lambda_1 |\sigma|^{1/2} \text{sign}(\sigma) + u_1, \quad \dot{u}_1 = \begin{cases} -u_{vss} & , \quad |u_{vss}| > U_M \\ -\lambda_2 \text{sign}(\sigma) & , \quad |u_{vss}| \leq U_M \end{cases}, \quad (20)$$

with λ_1 and λ_2 satisfying

$$\begin{cases} \lambda_1 > \sqrt{\frac{2}{K_m \lambda_2 - C}} \frac{(K_m \lambda_2 + C) K_M (1 + q)}{K_m^2 (1 - q)} \\ \lambda_2 > \frac{C}{K_m} \end{cases}, \quad (21)$$

will converge not only σ , but also $\dot{\sigma}$, to zero in finite time, therefore establishing a 2-SM. The control signal u_{vss} will also converge to the interval $[-U_M, U_M]$ in finite time. The proof of the STSMC convergence can be found in Shtessel et al. (2014).

The first term of u_{vss} can be interpreted as the usual SMC input smoothed by a saturation function, but instead of the $sat(\cdot)$ function, the odd square root function is used, where

$$sat(\sigma) = \begin{cases} \sigma & , \quad -1 \leq \sigma \leq 1 \\ sign(\sigma) & , \quad |\sigma| > 1 \end{cases}, \quad oddsqrt(\sigma) = |\sigma|^{1/2} sign(\sigma), \quad (22)$$

and the second term of u_{vss} can be interpreted as an adaptive control term that converges to the system uncertain dynamics, that is, as σ and $\dot{\sigma}$ tends to zero, u_1 tends to $-h/b$:

$$\sigma, \dot{\sigma} \rightarrow 0 \Rightarrow u_1 \rightarrow -\frac{h(t, \mathbf{x})}{b(t, \mathbf{x})}. \quad (23)$$

Notice that the parameters λ_1 and λ_2 , in practice, are never assigned according to Equation (21); they are tuned experimentally and/or via computer simulations, since it is not possible to estimate the uncertainties limits precisely, and values too large of λ_1 and λ_2 will make the controller more sensitive to noise and switching imperfections.

Figure 14 represents the phase plane of the sliding variable σ during the super-twisting sliding mode dynamic.

Figure 14 - Phase plane of the sliding variable σ during the super-twisting sliding mode

As the name suggests, the phase plane of the sliding variable describes a twisting trajectory around the origin, where the intersections of this trajectory with the σ and $\dot{\sigma}$ axis are upper bounded by the decreasing values of σ_{Mi} and $\dot{\sigma}_{Mi}$, respectively.

Consider that for an instant $t = t_0$, we have that $\sigma = 0$ and $\dot{\sigma} = \dot{\sigma}_0$, then it can be shown that the upper bounds on the phase plane trajectory are

$$\begin{cases} \dot{\sigma}_{M0} = \dot{\sigma}_0 & , \quad |\dot{\sigma}_{Mi}| < \frac{K_m(1-q)}{K_m(1+q)} |\dot{\sigma}_{Mi-1}| & , \quad i \in \mathbb{N}^* \\ |\sigma_{Mi}| = \frac{\dot{\sigma}_{Mi}^2}{2(K_m\lambda_2 - C)} & , \quad i \in \mathbb{N} \end{cases} \quad (24)$$

Notice that, in the super-twisting sliding mode, the values of $|\dot{\sigma}_{Mi}|$ are bounded by a decreasing geometric series and the convergence time t_c of σ and $\dot{\sigma}$ to 0 can be estimated by

$$t_c \leq t_0 + |\dot{\sigma}_0| \frac{K_m(1+q)}{(K_m\lambda_2 - C)[K_m(1+q) - K_m(1-q)]} \quad (25)$$

4 HYBRID MODEL

In this section, we will detail the hybrid model for a vessel model test in a wave tank, where the force actuators apply the wind and current effects and the waves are physically generated. Then we will present the dynamic model of the force actuator (servomotor + wheel + line) together with its super-twisting sliding mode controller.

Notice that this hybrid model test setup could be adapted to represent other external forces acting on the model, like the aerodynamic loads on a FOWT or the cables forces in a mooring system.

4.1 Hybrid Model Test Setup

The hybrid model is divided according to the substructures represented in Figure 15. Sensors measure the heading and position of the vessel, then the computer calculates the resulting forces and moments due to the wind and current effects. The actuators apply the appropriate forces to the physical model.

Figure 15 - Wave tank hybrid model substructure

The model considers the rigid body simplified dynamics in the horizontal plane with 3-DOF (surge, sway and yaw motions), the vertical motions (heave, roll and pitch) are considered independent from the horizontal motions, therefore, the force actuator would have a negligible effect on the vertical motions and, likewise, they would have a negligible effect on the vessel horizontal dynamics. Figure 16 depicts the model considered.

Figure 16 - Degrees of freedom of a vessel in the horizontal plane

We will use optic sensors to measure the heading and position of the vessel model and the effects of wind and current can be calculated as presented in Appendix A. The force actuators, that apply the calculated wind and current forces, will consist of servomotors connected by lines to the vessel model, as illustrated in Figure 17. For a given actuator configuration, a force allocation algorithm will calculate the forces to be applied by each actuator so as to replicate the wind and current effects; an example is shown in Appendix B.

Figure 17 - Hybrid model testing representation

To ensure the forces transmitted by the actuators are correct, it will be used a super-twisting sliding mode controller, where the line forces are measured with load cells and the servomotors angular position are measured with encoders.

A computer running the software Matlab R2013b will execute all the parts pertaining to the numerical model (filtering and state estimation, calculation of the wind and current effects, force allocation and servomotors control).

4.2 Line Force System Model

The system that controls the line force is represented in the figure below.

Figure 18 - Line force control system representation

The shaft of the servomotor is coupled to a wheel of radius R , whose angular position is θ . The torque applied by the servomotor is τ_m and F is the tension in the line tied to the vessel model and the wheel. The line is fixed to the vessel at point $\mathbf{P}$ of coordinates $[x_p \ y_p \ z_p]$ relative to the reference system with origin at the wheel.

The dynamic equation for the angular position of the wheel coupled to the servomotor spindle is

$$J_m \ddot{\theta} = \tau_m - RF - A(\dot{\theta}) , \quad (26)$$

where J_m is the total inertia moment of the parts and $A(\dot{\theta})$ is the friction moment. There are many empirical models for friction with the usual trade-off between algorithm accuracy and complexity. A detailed discussion on friction models can be found in Olsson et al. (1998) and Van Geffen (2021). We will consider a static model for friction represented by

$$A(\dot{\theta}) = F_v \dot{\theta} + [F_C + F_S \operatorname{sech}(\alpha \dot{\theta})] \tanh(\beta \dot{\theta}) . \quad (27)$$

The parameters F_v and F_C are the viscous and Coulomb friction coefficients, $F_S \operatorname{sech}(\alpha \dot{\theta})$ model the Stribeck effect, and $\tanh(\beta \dot{\theta})$ substitutes the sign function to keep the friction model continuous at zero velocity. These coefficients can be experimentally determined as described in Wernholt and Gunnarsson (2006).

The line is considered a linear spring of stiffness k , where

$$F = F(\theta, \|\mathbf{P}\|) , \quad \frac{dF}{d\theta} = kR , \quad \frac{dF}{d\|\mathbf{P}\|} = k . \quad (28)$$

So from Equations (26) and (28) we obtain the dynamics of the line force F as a function of the control input τ_m and the perturbations $A(\dot{\theta})$ and $\|\ddot{\mathbf{P}}\|$,

$$\ddot{F} = \frac{dF}{d\theta} \ddot{\theta} + \frac{dF}{d\|\mathbf{P}\|} \|\ddot{\mathbf{P}}\| \Rightarrow \ddot{F} = \frac{kR}{J_m} \tau_m - \frac{kR^2}{J_m} F - \frac{kR}{J_m} A(\dot{\theta}) + k\|\ddot{\mathbf{P}}\| . \quad (29)$$

Notice that the second-order line force dynamics is subject to perturbations that can't be precisely estimated. Therefore, the controller design must be sufficiently robust to deal with those uncertainties.

4.3 Force Control Actuator

The line force control algorithm consists of a position STSMC for the servomotor, which is extended to a STSMC for force control. The controller development is described in the present chapter.

4.3.1 Servomotor Position Control

The application of the STSMC for controlling the servomotor angular position is straightforward from Equation (26) and the method described in chapter 3. We want the output θ to follow the trajectory given by θ_d , so the sliding variable σ_1 is defined as

$$\sigma_1 = \dot{\tilde{\theta}} + \lambda \tilde{\theta} , \quad (30)$$

where $\tilde{\theta}$ is the trajectory error ($\theta - \theta_d$) of the servomotor angular position and $\dot{\tilde{\theta}}$, its derivative. Incremental encoders have practically zero noise and high resolution, therefore the speed of the servomotor can be accurately estimated by simply deriving the encoder signal.

The system dynamic can be rewritten as

$$\ddot{\theta} = f_1 + b_1 \tau_m , \quad (31)$$

where

$$b_1 = \frac{1}{J_m} , \quad f_1 = -\frac{R}{J_m} F - \frac{A(\dot{\theta})}{J_m} , \quad (32)$$

and the controller output (servomotor torque) is

$$\tau_m = u = -b_1^{-1} \left[\hat{f}_1 - \ddot{\theta}_d + \lambda \dot{\tilde{\theta}} \right] + u_{vss} . \quad (33)$$

The system uncertainties are due to the noise in the measurement of F and the value of the friction torque $A(\dot{\theta})$, so the estimate of f_1 is

$$\hat{f}_1 = -\frac{R}{J_m} \hat{F} - \frac{1}{J_m} \hat{A}(\dot{\theta}) , \quad (34)$$

where $\hat{A}(\dot{\theta})$ is calculated according to Equation (27) and $\hat{F}$ is the filtered measurement of F .

The value of u_{vss} is calculated as described in section 3.2.

The following figure shows the block diagram for the servomotor position control.

Figure 19 - Servomotor position control loop block diagram

To filter the high-frequency noise in the line force measurements, we will use a low-pass Butterworth filter. There are mainly two parameters to adjust, the filter cut-off frequency and the filter order. The cut-off frequency determines the filter passband, so it should be defined based on the signal's frequencies of interest. Higher filter order reduces the high frequency noise, just as lower cut-off frequencies, however in both cases there will be a trade-off between noise attenuation and signal time delay.

4.3.2 Line Force Control

To make the servomotor super-twisting position control also into a force controller, the desired trajectory θ_d will be calculated from the desired force trajectory (F_d) as a simple force feedback algorithm based on Equation (28):

$$\frac{dF}{d\theta} = kR \quad \Rightarrow \quad \theta_d(t) = \theta(t) + \frac{1}{kR} (F_d(t) - F(t)) . \quad (35)$$

One of the advantages of this implementation for the line force control, is that the derivative of the load cell signal ($\dot{F}$), which can be considerably noisy even when filtered, is not needed. Although the signal $F(t)$ is used to compute $\theta_d(t)$, the derivative of $\theta_d(t)$ can be calculated without deriving the load cell signal, from Equations (28) and (35), we have that

$$\dot{\theta}_d = \dot{\theta} + \frac{1}{kR} \left[\dot{F}_d - \left(\frac{dF}{d\theta} \dot{\theta} + \frac{dF}{d\|\mathbf{P}\|} \|\dot{\mathbf{P}}\| \right) \right] \quad \Rightarrow \quad \dot{\theta}_d = \frac{1}{kR} \dot{F}_d - \frac{1}{R} \|\dot{\mathbf{P}}\| . \quad (36)$$

Therefore, we obtain $\dot{\theta}_d$ from the derivatives of the force trajectory F_d and the distance $\|\mathbf{P}\|$. Of course, this assumes that the derivative of the measurement $\|\mathbf{P}\|/R$ is smoother than F/kR .

Now we will demonstrate that the line force dynamics will also have a super twisting sliding mode behavior. We start by substituting the servomotor position STSMC in Equations (31), (33) and (11) into the line force dynamic in Equation (29)

$$\begin{aligned} \ddot{F} &= \frac{dF}{d\theta} \ddot{\theta} + \frac{dF}{d\|\mathbf{P}\|} \|\ddot{\mathbf{P}}\| \Rightarrow \ddot{F} = kR(f_1 + b_1\tau_m) + k\|\ddot{\mathbf{P}}\| \\ \Rightarrow \ddot{F} &= kR(f_1 - \hat{f}_1 + \ddot{\theta}_d - \lambda\dot{\hat{\theta}} + b_1u_{vss}) + k\|\ddot{\mathbf{P}}\| \\ \Rightarrow \ddot{F} &= kR(\dot{\sigma}_1 + \ddot{\theta}_d - \lambda\dot{\hat{\theta}}) + k\|\ddot{\mathbf{P}}\|. \end{aligned} \quad (37)$$

Then, we substitute the force feedback algorithm results from Equations (35) and (36)

$$\begin{aligned} \Rightarrow \ddot{F} &= kR\left(\dot{\sigma}_1 + \frac{\ddot{F}_d}{kR} - \frac{\|\ddot{\mathbf{P}}\|}{R} - \lambda\frac{\dot{\tilde{F}}}{kR}\right) + k\|\ddot{\mathbf{P}}\| \\ \Rightarrow \ddot{F} &= kR\dot{\sigma}_1 + \ddot{F}_d - \lambda\dot{\tilde{F}} \\ \Rightarrow \ddot{\tilde{F}} + \lambda\dot{\tilde{F}} &= kR\dot{\sigma}_1. \end{aligned} \quad (38)$$

Finally, we define the sliding variable σ_2 , such that

$$\sigma_2 = \ddot{\tilde{F}} + \lambda\dot{\tilde{F}} \Rightarrow \dot{\sigma}_2 = \ddot{\tilde{F}} + \lambda\dot{\tilde{F}} = kR\dot{\sigma}_1. \quad (39)$$

From Equation (39), the dynamics of the sliding variable σ_2 will be equal to σ_1 multiplied by a positive constant. Since the position STSMC guarantees that σ_1 and $\dot{\sigma}_1$ converges to zero, we have that σ_2 and $\dot{\sigma}_2$ will also converge to zero. Therefore the force dynamics will also behave as a 2-SM.

One last thing to notice is that a small time delay τ was observed between the measured force dynamic and the servomotor position and position disturbance dynamics, such that

$$\dot{F}(t + \tau) = \frac{dF}{d\theta} \dot{\theta}(t) + \frac{dF}{d\|\mathbf{P}\|} \|\dot{\mathbf{P}}(t)\| . \quad (40)$$

One way to deal with this delay problem is to control the future force signal $F(t + \tau)$ instead of $F(t)$. This solution becomes clear by observing the simplified closed-loop block diagram of this compensated system:

Figure 20 – Delay compensation block diagram

It is easy to see that the red block ‘Delay Compensation’ cancels the system ‘Time Delay’ block by shifting the signal $\tilde{F}(s)$ forward in time, so the closed-loop system dynamics will be equivalent to the system with no time delay. In order to implement that, two signals will need to be predicted: $F(t + \tau)$ and $F_d(t + \tau)$. The desired force $F_d(t + \tau)$ can be predicted using a first-degree polynomial extrapolation:

$$F_d(t + \tau) = F_d(t) + \tau \dot{F}_d(t) . \quad (41)$$

The future force signal $F(t + \tau)$ can be predicted by integrating the delayed dynamic in Equation (40) from $(t - \tau)$ to t (this approach is similar to what is done in a Smith Predictor, where the system model is used to predict the output signal)

$$\begin{aligned} \int_{t-\tau}^t \dot{F}(t' + \tau) dt' &= \frac{dF}{d\theta} \int_{t-\tau}^t \dot{\theta}(t') dt' + \frac{dF}{d\|\mathbf{P}\|} \int_{t-\tau}^t \|\dot{\mathbf{P}}(t')\| dt' \\ \Rightarrow F(t + \tau) &= F(t) + kR[\theta(t) - \theta(t - \tau)] + k[\|\mathbf{P}(t)\| - \|\mathbf{P}(t - \tau)\|] . \end{aligned} \quad (42)$$

Obviously, if $\tilde{F}(t + \tau)$ converges to zero, $\tilde{F}(t)$ will also converge to zero.

A block diagram of the closed-loop system is represented in Figure 21 with the force control in cascade with the position control, for clarity, some signal routings and blocks were omitted.

Figure 21 - Force control closed-loop system block diagram

5 EXPERIMENTAL RESULTS AND NUMERICAL SIMULATION

In this chapter, we validate the proposed force actuator experimentally and compare it with numerical simulations. The system components and parameters are presented, followed by its numerical model and the tests results.

5.1 Experimental Setup and System Parameters

Before detailing the experimental setup, we will give a brief overview on each of its components.

The AC servomotor is a Delta ECMA 220 V low inertia series (Figure 22, right), motor frame size of 60mm and rated power output of 400 W. It has an incremental encoder of 12500 pulse per revolution (ppr), rated torque output of 1.27 $N.m$ and rotor inertia of $0.277 \times 10^{-4} kg.m^2$ (DELTA ELECTRONICS, 2018).

The torque applied by the servomotor is controlled by its respective servo drive, Delta ASDA-A2 (Figure 22, left). The settling time in the reference torque is of approximately $9 \mu s$ (can be neglected in the line force dynamics). The input for the torque is an analog signal ranging from -10 V to 10 V (10 V commands maximum rated torque, and voltage signal indicates torque direction).

Figure 22 - Delta ASDA-A2 servo drive (left) and AC servomotor 220V low inertia series (right)

Source: Delta Electronics (2018).

The load cell is an HBM's S2 force transducer with a rated sensitivity of 2 mV/V (Figure 23, left). It has a maximum rated load of 20 kg and accuracy class 0.05 (HBM, 2021). The load cell output signal is amplified by the Advantech ADAM-3016 signal conditioning module (Figure 23, right) to an output between ± 5 V (ADVANTECH, 1997).

Figure 23 - HBM load cell (left) and Advantec ADAM-3016 strain gauge input module (right)

Source: HBM (2021) (left) and Advantech (1997) (right).

The servomotor wheel (Figure 24, left) was 3d printed with ABS plastic. It has a radius of 3.7 cm and moment of inertia of $0.923 \times 10^{-4} \text{ kg.m}^2$. The servomotor torque is transferred by a simple fishing line (Spectra 0.5 mm) connected to a spring of stiffness 159 N/m (Figure 24, right).

Figure 24 - Servomotor wheel (left) and spring+line (right)

The National Instruments NI PCIe6321 Input/Output (I/O) board (Figure 25) was used to communicate the computer (PC) with the servo drive and sensors. The board provides 16 single ended analog input channels or 8 differential, with 16-bit resolution for an input range of ± 0.2 , 1, 5 or 10 V and a maximum sampling rate of 250 kSamples/s (kS/s) for all analog input channels. It has 2 analog output channels, the signal can be configured between -10 V to 10 V with 16-bit resolution, its typical settling time is 6 μs (negligible for the force actuator)

and has an update rate up to 840 kS/s. There are also 24 digital I/O channels and 4 counter channels with 32-bit resolution and an internal base clock of 100 MHz (NATIONAL INSTRUMENTS, 2019).

The electric diagram for the servodrive, signal conditioning module and I/O board can be found in Appendix D.

Figure 25 - NI PCIe6321 I/O board

Source: National Instruments (2019).

The PC has an Intel core i7 8700 3.2 GHz processor and 16 GB of RAM. Its operational system is a Windows 7 and the control algorithm is implemented with Matlab R2013b, the NI-DAQmx library was imported to communicate with the NI PCIe6321 I/O board.

To simulate the vessel wave motion, the servomotor was mounted on a previously developed linear guideway controlled by another servomotor (Figure 26). The guideway servomotor is programmed to perform a sinusoidal motion with a user defined frequency, amplitude and number of cycles. We chose not to mount the load cell in the linear guideway so the force measurements would not be affected by inertial forces during motion.

Therefore, the force actuator components are the PC, the I/O board, the servodrive and servomotor, and the wheel plus the fishing line and spring; while the other components are part of the experimental validation for the force actuator (load cell, signal conditioner and linear guideway).

Figure 26 - Linear guideway to perform sinusoidal motion. (A) servo drive, (B) servomotor, (C) linear guideway

Figure 27 depicts a schematic block diagram of the experimental setup with the system components (for a more detailed diagram on the control algorithms refer to chapter 4.3, Figures 19 and 21) and Figure 28 pictures the force actuator experimental test setup with its main components indicated by letters from A to E.

Figure 27 - Force actuator experimental test components block diagram

Figure 28 - Experimental setup for line force control. (A) servo drive, (B) servomotor, (C) linear guideway, (D) load cell, (E) computer

In order to estimate the time delay τ between the servomotor angular position dynamic ($\dot{\theta}$) and the measured force dynamic ($\dot{F}$), an open-loop frequency response test was performed (τ_m is a predefined sinusoidal input with increasing frequency from 0.1 to 3 Hz, the linear guideway is fixed and the measurements of θ and F are compared). To illustrate the dynamics time delay, the following graph shows the measured force (F) and the expected force from the servomotor angular position ($kR\theta$) adjusted with an offset to help compare the two signals (input frequency is around 1.6 Hz).

Figure 29 - Servomotor angular position and line force delay, open loop frequency response ~ 1.6 Hz

Figure 30 shows the phase delay φ between the servomotor angular position and the force measurement as a function of the input frequency ω , together with its linear approximation $\hat{\varphi}$. Notice that the phase delay is practically linear from 0.6 Hz onwards, so considering the relation between the phase delay φ and the time delay τ

$$\sin(\omega t + \varphi) = \sin(\omega(t + \tau)) \Rightarrow \varphi(\omega) = \omega\tau, \quad (43)$$

we conclude that the delay τ is approximately constant and equal to the slope of $\hat{\varphi}$. Table 1 presents the value of τ together with other physical parameters of the system.

Table 1- System physical parameters

Parameter	Value
J_m	$1.637 \times 10^{-4} kg.m^2$
R	$3.7 \times 10^{-2} m$
k	$159 N/m$
τ	$1.1 \times 10^{-2} s$

Figure 30 - Servomotor angular position and line force phase delay, open loop frequency response

From the open-loop frequency response test we can also estimate the servomotor friction torque $A(\dot{\theta})$ using Equation (26). The following figure shows the experimental data for the servomotor's friction moment and the fitted curve from Equation (27).

Figure 31 - Servomotor friction moment curve fit

Notice that, due to the friction uncertainty, the static friction model in Equation (27) could probably be substituted by a simpler function (the saturation function, for example) and yield similar results. The friction fitted curve parameters are indicated in Table 2.

Table 2 – Servomotor friction curve parameters

Parameter	Value
F_v	$5 \times 10^{-4} N.m/(rad/s)$
F_C	$1.7 \times 10^{-2} N.m$
F_S	$4 \times 10^{-3} N.m/(rad/s)$
α	2
β	4

The digital implementation of the STSMC reduces the accuracy level of the controller. In the case of 2-SM, the sliding variable σ will have an error order proportional to the square of the sampling time (SHTESSEL, 2014). On the other hand, the controller loop execution time (acquire sensor measurements, compute controller output, apply calculated output by the actuators) establishes a lower limit to the sampling time. Therefore, considering the controller execution time on the PC, the sampling frequency was set to 200 Hz or a sampling time T_s of 5 ms.

The frequencies of interest in a wave tank model test are in the 0 to 2 Hz range (SAUDER et al., 2016), in order to sufficiently attenuate the load cell measurements noise and introduce a minimal time delay, the Butterworth filter was set to a cut-off frequency f_c of 10 Hz and a filter order n equal to 1. Considering the criteria in chapter 3.1 – Equations (4) to (6) – to determine the time constant λ , we can calculate its value assuming that the lowest non-modeled resonance frequency ν_R is at 10 Hz and that both the sampling period $T_s = 1/\nu_S$ and the highest non modeled delay T_A are 5 ms, so

$$\begin{cases} \lambda_R = \frac{2\pi}{3} \nu_R \\ \lambda_A = \frac{1}{3T_A} \\ \lambda_S = \frac{1}{5} \nu_S \end{cases} , \quad \lambda \leq \min\{\lambda_R, \lambda_A, \lambda_S\} \Rightarrow \begin{cases} \lambda_R = 20.9 \text{ s}^{-1} \\ \lambda_A = 66.7 \text{ s}^{-1} \\ \lambda_S = 40 \text{ s}^{-1} \end{cases} , \quad \lambda = 20 \text{ s}^{-1} . \quad (44)$$

Using the methodology described in chapter 3.2, an initial estimate for the STSMC parameters U_M, λ_1 and λ_2 can be obtained, however, most of the time it is not possible to calculate the uncertainties limits on $\ddot{\sigma}$, so alternatively, they can be estimated from the conventional sliding mode gain K . Combining Equations (15), (32) and (33) together with the estimated upper bounds on the uncertainties of the measurement of F and the friction moment $A(\dot{\theta})$, we obtain the following

$$\begin{cases} K = \bar{u}_{eq} > |\hat{u}_{eq} - u_{eq}| \\ u_{eq} = -b_1^{-1} [\hat{f}_1 - \ddot{\theta}_d + \lambda \dot{\theta}] \\ b_1 = \frac{1}{J_m} \\ f_1 = -\frac{R}{J_m} F - \frac{A(\dot{\theta})}{J_m} \end{cases} , \quad \begin{cases} |F - \hat{F}| < 0.1 \text{ N} \\ |A(\dot{\theta}) - \hat{A}(\dot{\theta})| < 0.015 \text{ N.m} \end{cases} \Rightarrow \quad (45)$$

$$\Rightarrow K > R|F - \hat{F}| + |A(\dot{\theta}) - \hat{A}(\dot{\theta})| \Rightarrow K > 1.87 \times 10^{-2} \text{ N.m} .$$

Then, we use the following heuristic method to calculate the STSMC parameters from the conventional sliding mode gain K

$$\begin{cases} \lambda_1 = K \\ \lambda_2 = \frac{\lambda}{5}K \\ U_M = 3K \end{cases} \Rightarrow \begin{cases} \lambda_1 = 1.87 \times 10^{-2} N.m/(rad/s)^{0.5} \\ \lambda_2 = 7.48 \times 10^{-2} (N.m)/s \\ U_M = 5.61 \times 10^{-2} N.m \end{cases} . \quad (46)$$

Notice that the method presented is just a simple way to obtain a first estimate of the controller parameters, and as was mentioned, they should be further adjusted experimentally. Table 3 lists the final values of the STSMC parameters, the Butterworth filter parameters and the sampling time T_s .

Table 3 – Control algorithm parameters

Parameter	Value
U_M	$4.26 \times 10^{-2} N.m$
λ_1	$1.32 \times 10^{-2} N.m/(rad/s)^{0.5}$
λ_2	$9.82 \times 10^{-2} (N.m)/s$
λ	$20 s^{-1}$
n	1
f_c	10 Hz
T_s	$5 \times 10^{-3} s$

5.2 Numerical Model

The purpose of the numerical model is to give a qualitatively validation of both the mathematical model of the system and the experimental results. Even though STSMC presupposes the system model is not exactly known, numerical simulations of a simplified model can help validate some basic premises of the system. Also, the numerical model of the force actuator can be an important tool when designing hybrid model tests and evaluating different system parameters, like in the works of Qiao et al. (2021) and Solum (2017).

The numerical model is simulated using Matlab and Simulink R2016a. The Simulink Model is depicted in Figure 32.

Figure 32 – Force actuator Simulink Model

This model implements the force control actuator dynamics described in the previous chapters with some simplifications. There is no time delay in the line force dynamic and no noise in the force measurements. Although the disturbance motion is represented in the Simulink Model, the numerical model tests will not include it. The friction model used ($A_S(\dot{\theta})$) consists in a static discontinuous model, considering only the Coulomb and viscous friction coefficients (F_C and F_v , respectively), represented by

$$A_S(\dot{\theta}) = \text{sign}(\dot{\theta})(F_v|\dot{\theta}| + F_C) \quad (47)$$

From a few models tested, this simpler friction model showed the most satisfactory result when compared to the experimental test. As pointed in Figure 31, the friction torque has a high uncertainty, therefore even with more complex models it is difficult to accurately predict its behavior.

The system and controller parameters used in the numerical model are the same as the ones given in chapter 5.1. The integration step of the simulation is 0.1 ms and the system equations are integrated using the fourth order Runge-Kutta method.

The subsystem blocks in Figure 32 for ‘Force Reference’, ‘Force Feedback’, and ‘Position STSMC’ are illustrated in Appendix C

5.3 Step Response

In the first test, we analyze the force actuator's step response for the experimental setup and the numerical model. In the experimental test, we also study the actuator force keeping capacity in the presence of a motion disturbance.

5.3.1 Step Response – Experimental

In this test, the controller has to maintain a constant force of 3 N. There are no motion disturbances in the first 3 s, and then the linear guideway applies a gradually increasing sine wave motion of 2 Hz and 30 mm maximum amplitude for 5 s. The results are shown in Figure 33 and Figure 34.

Figure 33 - Line force, servomotor position, motion disturbance and torque input. Test 1, experimental

Figure 34 - Sliding variables for servomotor position (σ_1) and line force (σ_2). Test 1, experimental

Notice that the sliding variables in Figure 34 present some chattering, which also appears in the torque input signal. However, the chattering amplitude is not significant enough to cause oscillations in the servomotor position or the line force (Figure 33).

The following figure shows the first 1 s of the test focusing on the system step response.

Figure 35 - Line force and sliding variable σ_2 . Test 1, experimental, step response

We can verify that the sliding variable σ_2 takes around 0.12 s to reach the sliding surface ($\sigma_2 = 0$) and from that point onwards the system dynamic will be given by the sliding mode dynamics, therefore the settling time will be around $4/\lambda = 0.2$ s. At time $t = 0.32$ s, the line force is equal to 2.95 N (98.3% of the step value). From 0.6 s to 1.0 s, the SMC keeps the absolute force error $|\tilde{F}| < 0.02$ N (less than 1% error), while the mean absolute error is 0.0034 N (0.1% error).

The next figure focuses on the controller capacity to maintain a constant force when there is a sinusoid disturbance (time interval from 3 to 9 s).

Figure 36 - Line force, servomotor position, motion disturbance and torque input. Test 1, experimental, force keeping with motion disturbance

As expected, the force error increases proportionally and in the same frequency as the motion disturbance, but the STSMC can effectively compensate it, maintaining $|\tilde{F}| < 0.15 \text{ N}$ (5% error) with a mean absolute error of 0.025 N (less than 1% error).

The main results for this first test are presented in the following table:

Table 4 – Force error main results. Test 1, experimental

Time Interval (s)	Mean Absolute Error (N)	Max Absolute Error (N)
[0.6, 1.0]	0.0034	0.01
[3.0, 9.0]	0.025	0.13

5.3.2 Step Response – Numerical

The step response of the numerical model was also simulated with a 3 N input. The graph below shows the results for the line force and the sliding variable from 0 to 1 s.

Figure 37 - Line force and sliding variable σ_2 . Test 1, numerical, step response

Sliding variable σ_2 reaches the sliding surface ($\sigma_2 = 0$) in 0.11 s and again, we expect a settling time of $4/\lambda = 0.2$ s. At time $t = 0.31$ s, the line force is equal to 2.97 N (99% of the step value). From 0.6 s onwards, the absolute force error $\tilde{F}$ is less than 0.004 N and the mean absolute error is less than 0.003 N (0.1% error).

5.3.3 Step Response – Experimental and Numerical Comparison

The step response for the experimental test and the numerical simulation are in good agreement with an almost identical settling time, as can be observed in Figure 38.

Figure 38 - Line force and sliding variable σ_2 . Test 1, comparison, step response

In both cases, the sliding variable (σ_2) reaching time is around 0.12 s, with a settling time - when in sliding mode - of 0.2 s and stationary error inferior to 0.02 N (less than 1% error). The numerical model showed a slightly faster reaching time (0.01 s faster), but with a small overshoot and oscillation of the sliding variable. Although the representation of friction in the numerical model is simpler than the real dynamic model, its discontinuity at 0 velocity causes the numerical model to have a more oscillatory response than the actual physical system.

5.4 Force Tracking

In the second test, we evaluate the controller's force tracking capabilities with a chirp input signal, for the experimental setup and the numerical model. We also analyze the actuator performance with different motion disturbances in the experimental test.

5.4.1 Force Tracking – Experimental

The force trajectory F_d consists of a chirp signal: a sine wave of increasing frequency f from 0.1 to 2.0 Hz, amplitude 1.5 N with mean value 3.5 N and 90 s of duration. The trajectory is defined by

$$F_d = 3.5 - 1.5 \cos(2\pi \int f dt) \text{ [N]} \quad ; \quad f = 0.1 + \frac{t}{90} 1.9 \text{ [Hz]} \quad ; \quad t \in [0, 90] \text{ s} . \quad (48)$$

The next figure presents the force trajectory and the measured line force for the whole run and also details for specific frequencies of the chirp signal (0.5, 1.0, 1.5 and 2.0 Hz)

Figure 39 – Line force. Test 2, experimental, no motion disturbance

We observe that the controller is satisfactorily following the force reference, with the error amplitude increasing with the input signal frequency. This is clearly shown in Figure 40.

Figure 40 – Force error. Test 2, experimental, no motion disturbance

The error amplitude is lower than 0.07 N for input frequencies up to 1.5 Hz, however, as the chirp frequency increases, the force error considerably increase with a 0.14 N amplitude at 2.0 Hz input frequency. This force error amplitude is close to the maximum force error observed in chapter 5.3.1 (0.13 N), where the controller had to compensate a 2.0 Hz motion disturbance. This suggests that, from the controller perspective, force tracking and disturbance motion compensation have a similar dynamic behavior.

Now we will investigate the actuator performance when both following a trajectory and compensating a motion disturbance. To test different disturbance frequencies, 3 runs are done (P_1 , P_2 and P_3), each with a different disturbance motion. In all of them, the linear guideway will execute a sinusoid motion of amplitude 50 mm with a frequency of 0.5, 1.0 or 1.5 Hz. The disturbance motions (D_j) for each run (P_j) are defined as

$$D_j = 50 \sin(2\pi j 0.5t) [mm] \quad , \quad j = 1, 2, 3 \quad ; \quad (49)$$

and they are exemplified in the following Figure:

Figure 41 – Disturbance motions D_1 , D_2 and D_3

The sinusoid motion is used both to represent the expected motion of a vessel subject to waves and to characterize the frequency response of the force actuator relative to the disturbances.

Figure 42 shows the measured line force together with the servomotor angular position for these 3 runs.

Figure 42 - Line force and servomotor position. Test 2, experimental

The controller effectively compensates for the motion disturbances, with a different angular position trajectory in each run, while following the force reference signal. The next graph exhibits the force error and the total line displacement (sum of the wheel rotational displacement with the guideway motion).

Figure 43 – Line displacement and force error. Test 2, experimental

As expected, the resulting wheel plus guideway displacement resembles a sinusoid, however, due to the system uncertainties, the exact trajectory varies a little from run to run. The force error for the runs P_1 and P_2 (disturbance motions with 0.5 and 1.0 Hz, respectively) has an amplitude of approximately 0.05 N for an input frequency up to 0.5 Hz, and around 0.1 N for frequencies of 1.0 Hz or higher. Also, for higher input frequencies (1.5 and 2.0 Hz) the force error shows approximately the same frequency as the input signal with periods of 0.67 and 0.5 s, respectively. Run P_3 (disturbance motion with 1.5 Hz) presents considerably higher

force error spikes, together with its lower frequency component (at the input signal frequency).

From the previous results, it was possible to calculate the force actuator frequency response in the presence of the disturbance motions D_1, D_2 and D , and the case with no disturbance motion, referred as P_0 . The graph is shown in Figure 44.

Figure 44 – Frequency response (magnitude and phase). Test 2, experimental

We observe from the graph that, for an input frequency up to 1.0 Hz, the force actuator has a magnitude error below 4%, with a phase shift inferior to 1.5° , and for input frequencies up to 2.0 Hz, the magnitude error is below 9% with a less than 4° phase shift (considering disturbance motions with 50 mm of maximum amplitude and 1.5 Hz of maximum frequency).

Table 5 and Table 6 summarize the results, presenting the mean absolute error and the maximum force error, respectively, per frequency interval.

Table 5 – Force mean absolute error (N) per chirp and disturbance frequency intervals. Test 2, experimental

Disturbance Frequency (Hz)	Force Mean Absolute Error (N) per Chirp Frequency Interval (Hz)			
	[0.1, 0.5]	[0.5, 1.0]	[1.0, 1.5]	[1.5, 2.0]
0.0	0.02	0.03	0.04	0.07
0.5	0.02	0.02	0.03	0.05
1.0	0.02	0.03	0.04	0.05
1.5	0.04	0.04	0.05	0.06

Table 6 – Force maximum absolute error (N) per chirp and disturbance frequency intervals. Test 2, experimental

Disturbance Frequency (Hz)	Force Maximum Absolute Error (N) per Chirp Frequency Interval (Hz)			
	[0.1, 0.5]	[0.5, 1.0]	[1.0, 1.5]	[1.5, 2.0]
0.0	0.06	0.07	0.09	0.15
0.5	0.06	0.07	0.13	0.17
1.0	0.07	0.11	0.19	0.20
1.5	0.21	0.23	0.33	0.29

For all runs, the STSMC demonstrates a good performance, maintaining a mean absolute error below 0.1 N for all tested frequencies. The maximum force error has considerable spikes (up to 0.33 N) as the trajectory and disturbance frequencies increase. However, it is worth noting that the objective of the force actuator is to make a dynamic system (vessel model, for example) replicate a dynamic behavior resultant of a desired force trajectory. Since this dynamic system acts as a low-pass filter on the force input, the force error spikes should have a reduced effect on the hybrid model dynamic.

5.4.2 Force Tracking - Numerical

The force reference in the numerical model simulation consists in the same chirp signal used in the experimental tests, defined in Equation (48). Figure 45 depicts the line force and force reference during the simulation.

Figure 45 - Line force. Test 2, numerical, no motion disturbance

The controller is following the trajectory with a small phase delay as the input signal frequency increases. When there is a change of the servomotor's rotation direction (maximum and minima of the sinusoid), we observe that the force curve is flat (dead-zone behavior), which implies that servomotor has a delay to change directions. Again, this should be the effect of the discontinuity of the friction model at 0 velocity.

The following graph shows the force error during the simulation.

Figure 46 - Force error. Test 2, numerical, no motion disturbance

The numerical model has considerable error spikes even without motion disturbances and even at lower frequencies of the input signal. For an input signal at 0.5 Hz the maximum error is of 0.1 N increasing to 0.27 N for a 2.0 Hz input signal.

5.4.3 Force Tracking – Experimental and Numerical Comparison

Figure 47 presents the comparison of the force error for the force tracking tests without motion disturbance, for the experimental tests and the numerical simulation.

Figure 47 - Force error. Test 2, comparison, no motion disturbance

Both force error signals approximately follow and increase with the input signal frequency, however the numerical model simulation has force error peaks with more than double of the value obtained experimentally. Like in the step response, the numerical model force error presents a higher frequency component that is absent in the experimental test.

The next figure compares the frequency response of the experimental and numerical systems.

Figure 48 - Frequency response (magnitude and phase). Test 2, comparison

The numerical model has a slightly higher magnitude (around 0.005) than the experimental setup and close to the same magnitude for input frequencies above 1.9 Hz. However, the numerical model presents a considerably higher phase delay than the observed experimentally (7° at 2 Hz input frequency), which is enough to cause the force error spikes observed in Figure 47.

From these tests, we observe the limitations of the numerical model simulation. Although the step response had a good agreement with the experimental results, the frequency response presents a phase delay that results in more than double the maximum force error obtained experimentally. Nonetheless, the numerical simulations proved to be a fast way to obtain a qualitative estimate of the force actuator behavior, be it for testing different controller strategies or systems with new specifications.

6 CONCLUSIONS

In this work, we developed a force actuator for hybrid model tests of vessels in a wave tank, which consisted in a servomotor controlling the tension in a line. In order to achieve the desired performance, it was necessary to compensate for the force actuator model uncertainties (friction torque, motion disturbance and system time delays), therefore a super-twisting sliding mode controller was employed.

The controller consisted in a super-twisting sliding mode position control, which was extended to a force controller. Additional methods were used to minimize the errors due to the model uncertainties. We developed an experimental setup to validate the actuator and also simulated it with a simplified numerical model.

The force actuator showed a good time response (0.32 s for a step input) and satisfactory force tracking accuracy (mean absolute error of less than 0.1 N) for all the considered input and disturbance motion frequencies. Considering the actuator frequency response, we expect an amplitude error of less than 9% and phase shift of less than 5° for input frequencies up to 2.0 Hz. The force actuator numerical simulation showed discrepancies with the experimental results, mainly due to the difficulty to accurately model the friction torque, still the numerical model is a sufficient and fast way to qualitatively evaluate new system parameters. To conclude, the actuator performance was well characterized, showing promising results to be used for hybrid model tests in a wave tank.

The next step of the force actuator validation is to verify if the force tracking error spikes have a significant effect on the hybrid model dynamics or are negligible due to the low pass band frequency of the physical model. In case the performance of the actuator needs to be further improved, one could try to reduce the measurements noises by improving the state estimates (numerical methods for filters and derivatives); reduce the controller sampling time (faster hardware), or try more advanced methods to compensate for the system time delay.

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APPENDIX A – WIND AND CURRENT FORCES

We will consider the vessels dynamic in the plane (3-DOF, surge, sway and yaw directions) to determine the wind and current forces to be applied to the vessel model, as depicted in the figure below:

Figure 49 - Vessel model in the horizontal plane subject to wind and current uniform velocity fields

The wind, vessel and current directions (β_w , ψ and ψ_c , respectively) are measured relatively to the x_0 axis of the inertial frame $O_0x_0y_0$. The reference frame $O_1x_1y_1$ origin is fixed to the vessel center of mass and the axes x_1 and y_1 coincide with the vessel surge and sway directions, respectively.

The environmental forces are usually calculated relative to the body reference frame, so in our case the vector of environmental generalized forces would be

$$\mathbf{f}_{env} = \begin{bmatrix} X_w + X_c \\ Y_w + Y_c \\ N_w + N_c \end{bmatrix}, \quad (50)$$

where X_w, Y_w and N_w are the wind forces and moment components in the surge (longitudinal, axis O_1x_1), sway (lateral, axis O_1x_2) and yaw directions (rotation around vertical axis,

counterclockwise), respectively. Analogously, we have the current forces and moment components X_c , Y_c and N_c .

Considering a uniform wind velocity field of speed V_w and direction β_w , the resulting wind forces (X_w and Y_w) and moment (N_w) on the vessel can be calculated by

$$\begin{cases} X_w = \frac{1}{2} \rho_a A_F C_{wx}(\beta_{wr}) V_{wr}^2 \\ Y_w = \frac{1}{2} \rho_a A_L C_{wy}(\beta_{wr}) V_{wr}^2 \\ N_w = \frac{1}{2} \rho_a L A_L C_{wz}(\beta_{wr}) V_{wr}^2 \end{cases} , \quad (51)$$

where ρ_a is the air density, $C_{wx}(\beta_{wr})$, $C_{wy}(\beta_{wr})$ are the wind forces coefficients in the x_1 and y_1 directions, respectively, and $C_{wz}(\beta_{wr})$ is the yaw moment coefficient, L is the vessel length, A_F and A_L are the vessel emerged frontal and lateral area, V_{wr} is the wind speed relative to the vessel and β_{wr} is the relative direction of the wind to the vessel.

The wind coefficients can be obtained from model tests or estimated from the Oil Companies International Marine Forum (OCIMF) database curves. Considering the vessel velocity to be given by $\dot{\boldsymbol{\eta}} = [U \ V]$ (U in the x_0 direction and V in the y_0 direction), the wind relative speed (V_{wr}) and direction (β_{wr}) can be calculated respectively as

$$V_{wr} = \left\| V_w \begin{bmatrix} \cos(\beta_w) \\ \sin(\beta_w) \end{bmatrix} - \dot{\boldsymbol{\eta}} \right\| , \quad (52)$$

$$\beta_{wr} = \text{atan} \left(\frac{V_w \sin(\beta_w) - V}{V_w \cos(\beta_w) - U} \right) - \psi . \quad (53)$$

For the current, we also consider a uniform superficial velocity field of speed V_c and direction ψ_c , so the resulting current forces (X_c and Y_c) and moment (N_c) will be given by

$$\begin{cases} X_c = \frac{1}{2} \rho T L C_x(\psi_{cr}) V_{cr}^2 \\ Y_c = \frac{1}{2} \rho T L C_y(\psi_{cr}) V_{cr}^2 \\ N_c = \frac{1}{2} \rho T L^2 C_z(\psi_{cr}) V_{cr}^2 \end{cases} , \quad (54)$$

where ρ is the water density, T is the vessel draft, $C_x(\psi_{cr})$, $C_y(\psi_{cr})$ and $C_z(\psi_{cr})$ are the current dimensionless coefficients, V_{cr} and ψ_{cr} are the relative speed and direction of the current to the vessel.

The current coefficients can be obtained from towing tank drift tests, computational fluid dynamics (CFD) calculations or OCIMF database. Similarly to the wind, the current relative speed (V_{cr}) and direction (ψ_{cr}) are given by

$$V_{cr} = \left\| V_c \begin{bmatrix} \cos(\psi_c) \\ \sin(\psi_c) \end{bmatrix} - \dot{\mathbf{\eta}} \right\| , \quad (55)$$

$$\psi_{cr} = \text{atan} \left(\frac{V_c \sin(\psi_c) - V}{V_c \cos(\psi_c) - U} \right) - \psi . \quad (56)$$

APPENDIX B - FORCE ALLOCATION

Considering the vessel dynamics in the horizontal plane (surge, sway and yaw motions) the system model for the force allocation problem is depicted below:

Figure 50 - System model for force allocation

We'll consider two reference systems: the inertial reference system fixed at the Earth, $O_0x_0y_0$, and the reference system solidary to the body, $O_1x_1y_1$, fixed at the vessel center of mass $\mathbf{X}_g = [x_g \ y_g]^t$. The points $\mathbf{P}_1, \mathbf{P}_2, \mathbf{P}_3$ and $\mathbf{P}_4$ are the coordinates of the attachment points of the lines to the vessel model and the points $\mathbf{M}_1, \mathbf{M}_2, \mathbf{M}_3$ and $\mathbf{M}_4$ are the coordinates of the attachment points of the line to their respective servomotor wheels. The angle ψ is the heading of the vessel relative to the global reference system $O_0x_0y_0$.

The vector distance $\mathbf{r}_i$ between a point $\mathbf{P}_i$ and the origin O_1 is fixed relative to the vessel reference system and the angle of the line force F_{Li} relative to the vessel axis x_1 is given by α_i . This is represented in Figure 51.

Figure 51 - Representation of line force applied to the vessel model

The global coordinates of point $\mathbf{P}_i$ can be calculated as follow:

$${}^0\mathbf{P}_i = {}^0\mathbf{X}_g + {}^0\text{Rot}_1(\psi) {}^1\mathbf{r}_i. \quad (57)$$

Notice that the left superscript denotes relative to which reference system the coordinates are expressed, either reference system 0 ($O_0x_0y_0$) or reference system 1 ($O_1x_1y_1$). The position ${}^0\mathbf{X}_g$ and heading angle ψ are measured at each time step, the vector ${}^1\mathbf{r}_i = [{}^1r_{ix} \ {}^1r_{iy}]$ is a design parameter, and ${}^0\text{Rot}_1(\psi)$ is the rotation matrix from reference system 1 to reference system 0 given by:

$${}^0\text{Rot}_1(\psi) = \begin{bmatrix} \cos(\psi) & -\sin(\psi) \\ \sin(\psi) & \cos(\psi) \end{bmatrix}. \quad (58)$$

To decompose force F_{Li} into the vessel reference system (calculate $\cos(\alpha_i)$ and $\sin(\alpha_i)$) we simply rotate the unitary direction vector of $({}^0\mathbf{M}_i - {}^0\mathbf{P}_i)$ to reference system 1, as follow:

$${}^1\text{Rot}_0(\psi) \frac{({}^0\mathbf{M}_i - {}^0\mathbf{P}_i)}{\|({}^0\mathbf{M}_i - {}^0\mathbf{P}_i)\|} = \begin{bmatrix} \cos(\alpha_i) \\ \sin(\alpha_i) \end{bmatrix}. \quad (59)$$

Now the forces allocation problem can be stated as: given a desired forces and moment vector ${}^1\mathbf{f}_{env} = [f_x \ f_y \ M_z]^t$ to be applied to the vessel model, what value the line forces F_{Li} should have to be applied by each servomotor. Considering the case with 4 servomotors, there are usually infinite solutions. One way to solve this is to choose the solution that minimizes a desired cost function J . Therefore, we formulate the following optimization problem:

$$\min_{\mathbf{f}_v} J = \sum_{i=1}^4 (F_{Li} - F_0)^2 \quad , \quad (60)$$

such that:

$$B(\psi) \mathbf{f}_v = {}^1\mathbf{f}_{env} \quad , \quad (61)$$

$$F_{min} \leq F_{Li} \leq F_{max} \quad , \quad i = 1, \dots, 4 \quad . \quad (62)$$

The vector $\mathbf{f}_v$ contains the line forces F_{Li} , $\mathbf{f}_v = [F_{L1} \ F_{L2} \ F_{L3} \ F_{L4}]^t$.

Equation (61) represents the constraint for the resulting effect of the line forces on the vessel model to be equal to the desired forces and moments ${}^1\mathbf{f}_{env}$. $B(\psi)$ is a 3×4 matrix, where each column i is given by:

$$\mathbf{B}_i = \begin{bmatrix} \cos(\alpha_i) \\ \sin(\alpha_i) \\ {}^1r_{ix} \cdot \sin(\alpha_i) - {}^1r_{iy} \cdot \cos(\alpha_i) \end{bmatrix} . \quad (63)$$

Equation (62) is a boundary constraint on the lower and upper bound for the possible values of F_{Li} . The maximum force F_{max} depends on the maximum rated torque of the servomotor and the minimum force F_{min} is at least 0 since the lines can only apply positive forces on the vessel model.

The cost function J given by Equation (60) minimizes the distance of solution $\hat{\mathbf{f}}_v$ to a predefined tension F_0 .

This optimization problem can be solved numerically, for example using the `fmincon` function in Matlab.

APPENDIX C – SIMULINK MODEL SUBSYSTEMS

Figure 52 – ‘Position STSMC’ subsystem

Figure 53 – ‘Force Reference’ subsystem

Figure 54 – ‘Force Feedback’ subsystem

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