RODRIGO SAURI LAVIERI

IMAGE-BASED WAVE FEED-FORWARD FOR DYNAMIC
POSITIONING SYSTEM

SÃO PAULO
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So, even when we are absorbed in profound thoughts, the majority of our brain is actually busy trying to make sense of information coming from our eyes.

Ma et al. (2004)

RESUMO

Os sistemas atuais de Posicionamento Dinâmico (DP) empregam técnicas de controle baseadas na realimentação da posição e do aproamento, associadas à compensação antecipada das cargas de vento (*wind feed-forward*) para manter a posição da embarcação. Esta tecnologia melhora a eficiência do DP, pois antecipa a ação do vento. Entretanto, não há tecnologia consolidada para a pré-compensação de forças ondas (*wave feed-forward*) e o maior desafio reside na medição desses agentes ambientais. A presente tese tem como objetivo preencher essa lacuna tecnológica propondo uma abordagem alternativa para a medição das ondas próxima ao casco. Métodos de medição baseados em imagem são não invasivos e produzem descrições espaciais e temporais da superfície analisada, tornando-os particularmente adequados à medição de superfícies líquidas. Entretanto, os métodos comumente empregados, principalmente aqueles baseados em imagens estéreo, são incapazes de medir ondas produzidas em ambiente de laboratório, pois, nestas condições, a superfície da água é lisa, apresenta transparência e comporta-se como um espelho. Por outro lado, métodos baseados na intensidade de luz são capazes de lidar com tais características, se aplicados em condições adequadas e quando empregam modelos apropriados. Neste trabalho, um modelo de refletância amplamente conhecido é empregado para extrair os parâmetros principais de ondas regulares produzidas em um tanque de provas *offshore*. Inicialmente, propõe-se um arranjo experimental que permita a aplicação do modelo de refletância construído, mesmo sob as condições de iluminação naturais do laboratório. Posteriormente, um segundo conjunto de experimentos com um modelo de embarcação DP é proposto, no qual se aplica o método baseado em imagem, testado anteriormente, como sistema de pré-compensação de forças de onda. Três métodos de controle são avaliados, quais sejam: PID (Proporcional-Integral-Derivativo), PD (Proporcional-Derivativo) e o PD-WFF (PD associado ao *wave feed-forward*). Os resultados demonstram que a presença pré-compensação de forças de onda reduz o erro em regime do controlador PD. Estes são os primeiros passos em direção da incorporação da pré-compensação de forças de onda nos sistemas DP reais e diversos aspectos técnicos ainda estão pendentes. Entretanto, os resultados promissores e discussões acerca dos futuros passos da pesquisa definem a contribuição do presente trabalho no campo do Posicionamento Dinâmico.

Palavras-chave: Posicionamento Dinâmico, wave feed-forward, medição de ondas baseada em imagem, superfície translúcida, experimental.

ABSTRACT

The Dynamic Positioning (DP) systems currently available employ a feedback controller based on the position and heading error, associated with a wind feed-forward control action to keep the vessel position. This last technology improves the efficiency of the DP by anticipating wind loads. However, there is no consolidated technology to feed the controller with wave loads information and the major issue is related to the wave measurement process. The present thesis aims at filling this technological gap by proposing an alternative approach for the measuring of waves near the vessel. Image-based measurement methods are noninvasive and can produce a spatial and temporal description of the surface, making them suitable for recovering the geometry of liquid surfaces. Nevertheless, these surface reconstruction methods, particularly those applying stereoscopic approaches, have difficulty in measuring waves produced in laboratory facilities, mainly because, in such conditions, the water surface is smooth, translucent and highly specular. Alternatively, intensity based image methods are capable of dealing with these surface characteristics, if employed under controlled conditions and if a suitable reflectance model is selected. In the present study, a well-known reflectance model is applied to recover the parameters of regular waves produced in an offshore basin. Firstly, an experimental setup is proposed, designed to grant appropriate conditions for the application of the reflectance model, even under conventional laboratory illumination. Later, the second set of experiments with a model scale DP vessel are presented, which applied the image-based method developed before as a wave feed-forward system. Three control methods are evaluated, namely: PID, PD (Proportional-Derivative) and the PD with the wave feed-forward. Results demonstrated that the presence of the wave feed-forward reduced the steady error of the PD controller. These are the first steps towards the practical use of the wave feed-forward, and several aspects remain pending. However, the promising results and discussions regarding the future steps define the contribution of this work.

Keywords: Dynamic Positioning, wave feed-forward, image-based wave measurement, translucent surface, experimental.

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List of Symbols

α	Angle between the surface normal vector and the bisector of the incoming and outgoing light ray; Rotational movement of the floating body
β	Bisector angle between the incoming and outgoing light ray
δ	Dirac's function; Error or deviation
ξ	Translational movements of the floating body
ζ	Damping coefficient; Wave runup
θ	Spherical coordinate related to the inclination angle
λ	Wavelength
ρ	Water density
σ	Depth factor
φ	Spherical coordinate related to the azimuth angle
ϕ	Wave phase; Velocity potential
ω	Wave angular frequency
ω_n	System natural frequency
Π	Projective Matrix
Ω	Image domain
$\forall$	Body displacement

d	Diameter of the aperture/lenses
f	Focal length
g	Gravity Acceleration
k	Wave number
m	Body mass
n	Surface normal vector or vector component
p	Surface gradient in x direction
q	Surface gradient in y direction
A	Wave amplitude
A_{wp}	Water plane area
F	Force
H	Wave height; Second order rotation matrix; Homography
I	Function describing the Image; Irradiance at a certain patch of the image
L	Radiance emitted by a certain surface patch
K	Matrix of of intrinsic parameters
K_p	Proportional gain
K_i	Integral gain
K_d	Derivative gain
M	Moment
M_a	Added mass matrix
R	Bidirectional Reflectance Distribution Function; Rotation matrix (extrinsic parameters)
T	Translation vector (extrinsic parameters); Wave period
T_p	Wave peak period

Acronyms and Abbreviations

<i>DP</i>	Dynamic Positioning
<i>PID</i>	Proportional-Integral-Derivative
<i>PD</i>	Proportional-Derivative
<i>WFF</i>	Wave feed-forward
<i>WP</i>	Wave Probe
<i>FPO</i>	Floating Production and Offloading unit
<i>FPSO</i>	Floating Production Storage and Offloading unit
<i>PSV</i>	Platform Supply Vessel
<i>AHTS</i>	Anchor Handling Tug Supply
<i>TLP</i>	Tension Leg Platform
<i>GPS</i>	Global Positioning System
<i>DGPS</i>	Differential Global Positioning System
<i>IMU</i>	Inertial Measurement Unit
<i>SBL</i>	Short Base Line
<i>PC</i>	Personal Computer
<i>RGB</i>	Red, Green, Blue color space
<i>FFT</i>	Fast Fourier Transformation
<i>CCD</i>	Charge-coupled device
<i>TCP/IP</i>	Transmission Control Protocol/ Internet Protocol
<i>fps</i>	Frames per second

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Chapter 1

Introduction

1.1 Industry Perspective

Offshore oil exploitation began in Brazil in the mid-1970s, after the first wells were drilled in the Garoupa field at Campos Basin started to produce petroleum. These wells were located at relatively low water depth, from 120m to 160m, allowing the use of fixed production platforms such as Jack-ups.

At the end of that decade, after 1977, the Brazilian government, aiming at making the country self-sufficient in petroleum, pushed the national oil company to move to deeper waters and, for that, the first floating production units were applied. Since then, new oil reservoirs were discovered at crescent water depth culminating with the pre-salt fields preliminary tests, in 2009. The major efforts of oil companies are currently to move their production units to the pre-salt areas, located at least at 200km from the coast and with a mean water depth of 2000m.

Such water depth imposes the use of floating production units (FPOs) or a variation of that with storage capacity, the FPSOs, and floating drilling units (semi-submersibles or ships). These kinds of structures require some station-keeping system, such as mooring lines or Dynamic Positioning (DP). Whatever the system chosen, it must be capable of dealing with the environmental forces produced by winds, currents and waves. The most common method to keep the location of a floating structure is to moor it to the sea floor by one or more mooring lines. The primary mooring line arrangements applied are: Spread mooring, Single buoy or Turret and Tension Leg Platform (TLP). All these systems require installation processes the complexity of which increases together with the water depth. A spread mooring for an FPSO typically applies from 16 to 20 mooring lines. Each line needs 1 or 2 AHTS(Anchor Handling Tug Supply) and near 24 hours to

be deployed. All this effort is plausible for production structures that will remain in the same location for several years. However, there are a considerable number of activities on offshore oil and gas exploitation that utilizes short period station keeping. Drilling ships or platforms usually remain in place for a few months; shuttle tankers that offload oil from production platforms stay close to them for near one day; Platform Supply Vessels (PSVs) remain near the platform for a couple of hours. These are only a few examples of short-term station keeping and none of them would justify employing the mooring system by the time it would take to be installed, and the related costs. In this context an active station-keeping system, Dynamic Positioning, is suitable.

A DP vessel or platform has some transverse or steerable thrusters that are controlled by a computer which maintain the ship position counteracting the environmental forces. The first ship equipped with steerable thrusters starts its operation in 1961, by 1990s more than 400 vessels already had DP systems, according to Bray (1998), and currently no one knows the exact number of DP vessels in operation, but knowledgeable people guess more than 2000, Shatto (2011).

Although the DP system requires that the thrusters work uninterruptedly, increasing the fuel consumption, the absence of mooring saves time and costs related to the anchoring process as well as the risk of line clashing or damage to underwater structures such as manifolds, flowlines, Christmas tree, etc.

1.2 Feed Forward in DP Systems

A DP computer applies a proportional-integral-derivative (PID) controller and an extended Kalman Filter (EKF) to estimate the vessel state. A set of sensors, such as GPS, DGPS, Gyros, Inertial Measurement Units (IMUs), Short Base Line (SBL), among others, feeds the estimator with data, and estimates the position of the vessel. This position is then compared with a set point and an error is calculated. The controller tries to eliminate this error by the action of the thrusters. The sensors will continually keep track of the error and the controller will try to fix it. This was a very simple explanation of a system with *feed-back*. More recently, DP systems have incorporated a new element in this loop, based on feed-forward logic. Besides providing the position of the vessel, the estimator also receives information about the environmental forces and feeds it to the controller, which tries to counterbalance the forces before it actually moves the unit. Today, the wind feed forward is a reality and equips most DP systems. DP vessels are set up with anemometers that measure the wind speed and direction at different positions, then by applying a look-up table or a mathematical model (wind model), the controller converts

the wind information into forces and moments. This calculation is based on wind tunnel, analytical/empirical models, e.g. Isherwood (1973), or numerical approach such as CFD (Computational Fluid Dynamics) codes. Additionally, these models vary from vessel to vessel, as their geometries are distinct. Precisely evaluating the wind feed forward performance is a difficult task, as described by Stephens (2011). Dynamic simulations are alternatives as all the parameters can be controlled. However, a large number of simulations would be necessary, applying several types of ships to produce the full picture of the system performance. Figure 1.1 presents the differences between a simulation with and without the wind feed forward, both for a barge shape vessel with a 20knot gust wind acting transversely. The simulation was executed using the numerical model presented in Tannuri and Morishita (2006) and indicates a substantial reduction in the offset of the unit once the wind feed forward is present.

Figure 1.1: Offset of dynamic simulations with and without wind feed-forward.

Movements produced by waves on a floating structure can be divided into two broad groups, see, for example Faltinsen (1998), namely: movements in the same frequency as the incident waves (first order movements); movements the frequency of which differs from that of incident waves, called second order movements. The latter movements can be divided into lower frequency and higher frequency movements.

First order movements, as mentioned, have the same frequency as incident waves and are related to large forces, proportional to the structure displacements, whereas second order movements are linked to much lower forces, which the thruster of the DP system can handle.

The wave feed forward idea was first introduced by Pinkster (1978) as an alternative to reducing the horizontal motions of the vessel caused by second order wave drifting forces in irregular waves. The paper presents the calculation to predict the second order forces, at least their major component, and the effect of inputting such forces on the DP system using measures of wave height around the hull, by model test experiments. Promising results confirmed the concept.

In the present work, an additional benefit produced by the wave feed forward is explored. Although the mean drift, which is a constant force, can be well counterbalanced by the regular feed-back system, the use of feed forward can at least partially replace the integral term of the controller thus reduces effects such as system overshoot. Therefore, the wave feed forward can enhance the DP performance even under regular waves. This conclusion is also discussed by Waals (2002), which points out the improvements produced by the wave feed forward when the ship heading changes under regular waves.

Even though the concept of the wave feed-forward has been proved, the system has not been incorporated to commercial DP due to some technical issues related to wave measurements. The model tests incorporating wave feed-forward applied wire wave probes attached to the hull to measure wave height. These are fragile instruments, mainly employed in laboratories, under controlled situation, and require proper care. Using such devices in a real scenario, e.g. a platform, shuttle tanker or PSV, seems to be unfeasible. Besides the risk of collision and of damage to the instruments, the lack of maintenance or regular calibration would make them produce erroneous measurements.

A non-invasive robust measure method would allow the field application of wave feed forward. Additionally, the measurement method should provide spatial and time information of the wave height not only at the hull surface but also in the region around it; the hydrodynamic forces could then be estimated ahead of their effect.

Some non-invasive measurement technologies are available, such as microwave-based water elevation and X-band marine radars. These pieces of equipment can provide spatial and temporal information on water elevation ¹ even for relatively short waves (15m long). Computer vision techniques are explored herein to develop a measurement system that meets the requirements previously mentioned.

¹The radar readings must be post-processed in order to extract wave height and period, OceanWaveS GmbH ().

1.3 The use of Computer Vision

Images contain significant amounts of information about the scene, although an entire dimension is lost. Every small patch of a picture behaves as a sensor of light or of other sources of energy, and from a set of them, it is possible to infer shapes and materials present in the scene. These patches on digital images are called pixels (picture elements) and they encode the amount of energy perceived as integer numbers. An array of these sensors produces a matrix of integers, which can be post-processed in several ways.

CCDs (Charge-coupled devices) are the main technology capable of recording digital images; they are currently cheap and popular pieces of hardware, which equip from mobile phones to cars. Additionally, the processing capacity of computers has increased making it possible to transfer full-resolution images into the memory and to execute operations with large matrices almost in real time. Finally, the completely understood and systematic explanations of vision geometry have been a reality since the beginning of the 1990s, Ma et al. (2004).

A sequence of frames can provide both a spatial and a temporal description of the water surface, and it is possible to investigate an area relatively far from the hull, thus giving some extra time for the DP system to make decisions and allocate the thrust.

1.4 Research Objectives and Contribution

The present research aims to confirm the concept of using image-based methods as a measurement source for the wave feed forward in DP systems, by introducing a framework of algorithms and experimental methods.

In order to create an experiment using ship model in a wave tank, the intensity-based image method had to be used since the optical characteristics of water surface and the surrounding scenario required so. The stereoscopic approach, on the other hand, is suitable for measuring wave parameters in a real outdoor situation, as recent works have demonstrated. This second method is briefly discussed in the Appendix C of the present thesis.

1.5 Structure of the Thesis

This thesis is divided into two main parts, the first describes the image-based wave measuring method, and the second presents the integration of this system to the DP. Each of them has its bibliographic review, presents a brief theoretical framework and describes an experiment. We have adopted such separation because the aim is merging two very distinct and extensive fields of science.

The text comprises four chapters: an introduction, a chapter describing the image-based method, a chapter describing the wave feed forward and its integration with the DP system, besides a final chapter with the conclusions. Additionally, some Appendices were included to make the text more self-contained, describing theoretical tools related to measuring surfaces by using images. Finally, the complete set of experimental time series is included in the Appendix section.

Chapter 2

Image-Based Wave Measurement

2.1 Historical and Bibliographic Review

Images are rich sources of information and have thus been used to measure physical quantities since the beginning of photography. Nevertheless, an entire dimension is lost during the imaging process, encumbering the reconstruction of three-dimensional geometries such as bodies' surfaces.

Jähne, Waas and Klinke (1992) mentioned that the primary cues from which depth information can be extracted are: shape from shading and shape from stereo. The latter class of methods apply two or more images and find the displacement between corresponding points. In this case, the correspondence problem represents the major challenge in the task, and it is the topic in which most algorithms differ from each other. Koschan (1993) mentioned that, by the beginning of the 1990s, more than 150 stereo vision methods were available, all of them applying different strategies to solve the correspondence problem.

Humans solve correspondence problems effortlessly; however, a large quantity of information and complex connections are used for that. Chase et al. (1957) measured waves using stereo images from aerial photography and all matching points were manually identified. Regular computational routine, on the other hand, basically looks for features (points, corners, edges and blobs) in the arrays of intensities representing digital images. Thus, if there is complete absence of textures (features) or if such structures do not look similar in both images, due to the characteristics of the surface, the correspondence problem cannot be solved by a computer.

Water surface in the open sea or large outdoor areas can present the required texture, particularly when exposed to the wind and when observed from an adequate distance.

Additionally, surface textures will present similar shapes in two or more images from distinct viewpoints when illuminated by a uniform extended source of light, such as in a foggy day or a completely clear sky. In such a scenario, a specular surface presents the same reflectance characteristics as a Lambertian (perfectly diffuse) surface, Horn and Sjoberg (1979).

Shemdin (1990) made an experiment using a stereo rig mounted on a lighthouse. The images were captured in a 70mm format film and further digitized. This work describes both a manually matching method and a digital stereo-correlation approach, representing the earlier stages of an automatic approach to the correlation problem for wave images. Later, Gracian et al. (1992) presented a similar experiment, except that the rig was much closer to the water, as his objective was to measure wavelengths between 3cm and 7cm. Quite dense maps were automatically obtained, although the algorithm was designed for satellite image processing.

More recently, stereo image methods have been successfully applied to measure ocean waves by Benetazzo (2006), Benetazzo et al. (2012), Wanek and Wu (2006). Even applying simple matching metric and cost aggregation, these studies produced good results regarding elevation measurements by taking advantages of epipolar constraints. In order to make the present text more self-contained, the reader may find a brief explanation about perspective and epipolar geometry in Appendix A.

Gallego et al. (2008), Gallego (2011) proposed a method that determines the surface geometry that minimizes the cost functional based on intensity correlation between two images. These are global approaches to the problem since a PDE (Partial Differential Equation) modifies the whole surface, which produces better results even for poorly textured images (smoother surfaces). Even though such methods are more robust regarding surface texture, they are still based on the hypothesis that images of the same surface region look similar from distinct viewpoints or, in other words, behave as a Lambertian surface. Nevertheless, in a wave tank, the water does not behave as a diffuse surface and the methods previously described cannot be applied. In Appendix C, the reader can find a stereoscopic method, similar to that proposed by Benetazzo, which is capable of recovering diffuse, texture-rich surfaces, such as the wind-generated waves in outdoor areas, by stereoscopic means.

Aiming at testing the concept in an offshore basin, where waves can be generated with the desired characteristics and precisely measured, a distinct computer vision approach should be explored.

In a laboratory facility, the light comes from point sources and reflects on all sorts of objects before reaching the water surface, creating an extended heterogeneous source of

light. Furthermore, the waves are generated by flaps or wedge mechanism, which make their surface smooth and highly specular; consequently, most of the features observed on the surface are reflections from the surrounding scenario. The parallax of objects reflected by a glossy convex surface leads to a depth estimate behind the real surface, whereas a concave one will produce a depth in front of it. For this reason, even if a particularly reflected feature were identified in both images, the triangulation process would produce an erroneous surface estimate. Finally, the clear water and the relative small depth make the bottom of the basin visible from the surface, adding undesired features to the image.

In order to apply the stereoscopic approach in a wave basin, Mozumi, Waseda and Chabchoub (2015) used a floating marker-net and focused on the correspondence between these floating elements to allow the stereo reconstruction and to deal with the surface characteristics previously mentioned. After identifying the targets in both images, they can be tracked in the following frames, reducing the computational effort to solve correspondence. However, this method requires some sort of physical contact with the water surface and we intent to avoid such a solution here.

The method presented herein explores a distinct paradigm in which surface information is recovered from the light intensity rather than from corresponding features. Following the taxonomy proposed by Jähne, Waas and Klinké (1992), the present method is a variation of the *Shape from Shading*, introduced by Horn (1970), for specular and translucent surfaces. The radiance of a patch on a diffuse surface is related to the relative orientation between its normal and the light source. On a specular and translucent surface, such as water, under a uniform extended light source, this dependence is related to the angle between the surface normal and the viewer, and the radiance variation is due to the reflection properties of the interface. This relation was firstly explored by Stilwell (1969) and later demonstrated by Jähne (1993) who named the method *Shape from Reflection*. Here, a surface from reflection method is used to estimate the steepness of regular waves.

A widely known reflectance model proposed by Schlick (1994) was applied to derive the relation between water surface orientation and image irradiance; then, an experiment with regular waves was carried out applying industrial grade cameras. Waves steepness measured by wire waves probes were compared to those estimated using images irradiance and a good correlation between both measurements was observed.

The next section provides an overview of the photometric description of the problem.

2.2 Image Formation Overview

2.2.1 Image Representation

Images are created whenever an array of sensors is excited by a source of energy. Hence, images can be produced by ultrasound, X-ray, microwave, infra-red, ultraviolet among other sources. Gonzalez and Woods (2008) apply the term *illumination* to characterize a generic energy capable of impinging sensors and producing images.

The human eye, as well as the digital camera, are visible light sensors. When light reaches the human retina, it disturbs a tissue of discrete receptors (cones and rods) that convert the intensity of light into electric pulses. These pulses travel through nerves up to the brain where this information is interpreted. Likewise, digital cameras acquire images applying a 2D array of photometric sensors, the CCDs. These individual devices produce an electrical response proportional to the integral of light energy projected over its surface for a period of time. As cameras commonly use rectangular grids of sensors, the result is a matrix containing positive numbers, proportional to the light intensity. Therefore, a digital image can be described as a function $I(x, y)$ that maps 2D coordinates from a domain Ω (related to the *Spatial Resolution*) into positive numbers.

$$I : \Omega \subset \mathbb{R}^2 \rightarrow \mathbb{R}_+; (x, y) \mapsto I(x, y) \quad (2.1)$$

Digital sensors produce integer output in a particular range. 8-bit encoding allows 256 levels of intensity; output $\mathbb{R}_+$ must thus be approximated by the subset $[0, 255] \subset \mathbb{Z}_+$, this is called *Intensity Resolution*.

Grayscale images are characterized by a single matrix where each element or pixel can assume values from 0, black, to 255, white. Colour images, in the digital context, are commonly described by three matrices with the same size, each of them corresponding to a grey-scale image. These matrices are related to the colours red, green and blue, composing what is called *colour space*. RGB colour space is the most common and it was designed to be projected on monitors; it is thus an additive colour model or, in other words, the sum of the three channels producing brighter points.

2.2.2 Basic Photometry

Initially, two important photometric quantities should be defined: *Irradiance* I , represents the light power deposited in a unit area (W/m^2) of a surface, and *Radiance* L , is the

amount of light radiated from a surface into a given solid angle¹ per unit area (W/m^2sr).

Considering the image formation inside a camera², the relation between a light source Radiance and the Irradiance detected by a patch of the image plane (pixel) is defined by equation 2.2,

Figure 2.1: Illustration of the radiance emitted by a certain patch of the surface and the irradiance perceived by the imaging system.

$$I = L \frac{\pi}{4} \left(\frac{d}{f} \right)^2 \cos^4 \epsilon \quad (2.2)$$

where d and f are the lens diameter and focal length of the camera respectively and ϵ is the angle between the vector connecting the emitter point and the pixel at the image plane and the normal vector of the image plane.

Once how the camera will perceive the incoming light is established, it is necessary to describe how much of the light produced by a bright source, such as the sun or a bulb, will reflect on objects and reach each pixel. The *Bidirectional Reflectance Distribution function* or *BRDF* describes what portion of the incoming light is reflected onto another specific direction and it is represented by equation 2.3. It depends on the nature of the material as well as on the directions of the light source and on the observer. Two types of surfaces are traditionally distinguished by the structure of the BRDF, namely Diffuse and Specular surfaces. On the first one, the light is reflected in all directions and, if the phenomenon occurs uniformly, the surface is called Lambertian. In this case, the BRDF does not depend on the incoming or outgoing angles, being represented by a constant; then, equation 2.4. The second surface type reflects light only in a restricted area

¹A solid angle is defined to be the area of a cone cut out on a unit sphere, it is analogous to an arc length on a 2D circle but in 3D space. The solid angle unit is *sr* or *steradian*

²The imaging device is assumed correctly focused; thus, rays originating from a particular point on the object which goes through the aperture are deflected to meet a single point in the image plane.

symmetrically located with respect to the surface patch normal. On a perfectly secular surface, the BRDF becomes a Dirac function. A general surface presents a combination of both behaviours, whose mathematical description, or Reflectance Models, can be obtained by empirical or theoretical means. Figure 2.2 exemplifies the main reflection patterns.

$$R(\theta, \varphi, \theta', \varphi') = \frac{dL(\theta', \varphi')}{dI(\theta, \varphi)} \quad (2.3)$$

The function arguments are the incident and emitted light beam directions, here represented by spherical coordinates θ' , φ' , θ and φ , respectively. Derivatives of equation 2.3 indicate that integration must be promoted to obtain a finite amount of light energy. The simplest case of *BRDF* is that where β is a constant, physically meaning that the light is equally reflected in all directions. This is called *Diffuse Reflection* and objects that present such property are named *Lambertian* surfaces. The *BRDF* equations for a Lambertian surface is given by,

$$R_L = \frac{\rho(\lambda)}{\pi} \quad (2.4)$$

where $\rho(\lambda)$ is called *reflecting coefficient* and describes the fraction of reflected radiance.

Figure 2.2: Reflection patterns, Petty (2006)

2.3 Water Surface Radiance Model

The radiance of the water surface is a combination of surface reflection and light coming from beneath the water. Because the present study aims to measure wave parameters,

the relevant information, regarding the air/water interface is expected to come from the reflected light. Jähne (1993) proposed an alternative approach for wave measurement called Surface from Refraction that uses information from light crossing the two means. However, such experimental set-up includes a camera inside the tank or beneath it which is inconvenient in an offshore basin and therefore was not investigated here.

The portion of light coming from beneath the water and refracted towards the camera presents diminished intensity. Additionally, its effect on the image can be further reduced by applying a low-pass filter (i.e. pass-orange) or, particularly for colour cameras, using only the red channel. Those light wavelengths have low penetration into the water and are therefore absorbed by the liquid. By excluding the shorter wavelengths, the primary source of light will come from the reflected long wavelengths. These aspects will be discussed further in the experimental section herein.

By neglecting the light coming from beneath the water, the wave surface radiance depends on the amount of light reaching a certain area and the fraction of it that is reflected.

Figure 2.3 presents a geometrical description of the problem.

Figure 2.3: Definition of angles and vectors

Schlick (1994) proposed a *reflectance model*, intermediary between empiricism and theory, with intuitive parameters which distinguishes materials with homogeneous optical properties, such as clear water, from those with heterogeneous properties, which combine opacity with translucence and specularly, such as muddy water. Expression 2.5 is applicable for

V'	Outgoing direction of light.
V	Incoming direction of light.
N	Surface normal vector.
T	Surface tangent vector.
H	Bisector vector of V and V' .
$\bar{V}$	Projection of vector V on surface patch.
$\bar{V}'$	Projection of vector V' on surface patch.

the first type of material, which is suitable for the present study.

$$R(\alpha, \beta, \theta, \theta', \varphi, \varphi') = S(\beta)D(\alpha, \theta, \theta', \varphi, \varphi') \quad (2.5)$$

In the expression above, R is the BRDF, S is called Spectral factor which encodes the rate of light reflected and absorbed (refracted); D is called Directional factor, which defines how diffusive or specular the surface is. The Fresnel Equations are here directly applied to calculate the spectral factor instead of the model shown in Schlick (1994) to increase precision. The expression for S becomes,

$$S(\beta) = \frac{1}{2} \frac{(a - u)^2 + b^2}{(a + u)^2 + b^2} \left[\frac{(a + u - 1/u)^2 + b^2}{(a - u + 1/u)^2 + b^2} + 1 \right] \quad (2.6)$$

where,

$$a = \frac{1}{2} \left(\sqrt{(n^2 - k^2 + u^2 - 1)^2 + 4n^2k^2} + n^2 - k^2 + u^2 - 1 \right)$$

$$b = \frac{1}{2} \left(\sqrt{(n^2 - k^2 + u^2 - 1)^2 + 4n^2k^2} - n^2 + k^2 - u^2 + 1 \right)$$

$u = \cos\beta$, n is the refractive index of water, assumed constant and equal to 1.333, and k is the extinction coefficient, assumed constant and equal to 1.9×10^{-9} . The dielectric properties of water were considered constant because they present small variation in the range of electromagnets wavelengths of visible light.

For an ideal specular surface, the Directional factor is represented by Dirac functions, expression 2.7, since the light arriving from direction (θ', φ') is uniquely reflected into the

direction $(\theta', \varphi' + \pi)$.

$$D(\alpha, \theta, \theta', \varphi, \varphi') = \frac{1}{\sin\theta' \cos\theta'} \delta(\alpha) \delta(\varphi - \varphi' - \pi) \quad (2.7)$$

Finally, the radiance of the surface $L(\theta, \varphi)$ is the product of the BRDF and the function that represents the light source $E(\theta', \varphi')$. For an extended light source, the radiance is obtained by integrating the previously mentioned product along all the directions of the upper hemisphere centred at each surface patch, described by expression 2.8.

$$L(\theta, \varphi) = \int_{-\pi}^{\pi} \int_0^{\pi/2} R(\alpha, \beta, \theta, \theta', \varphi, \varphi') E(\theta', \varphi') \sin\theta' \cos\theta' d\theta' d\varphi' \quad (2.8)$$

Expression 2.9 presents the solution of this integral, since $\alpha = \theta - \theta'$ and $\beta = (\theta + \theta')/2$, see figure 2.3.

$$L(\theta, \varphi) = S(\theta) E(\theta, \varphi - \pi) \quad (2.9)$$

Expression $S(\theta)$ is the same as that of 2.6, except that u , which originally equals $\cos\beta$, was replaced by $\cos\theta$.

The radiance of the water surface is equal to the radiance of the reflected piece of the extended source weighted by the reflectivity of that particular area focused by the camera. To solve this function, a complete spherical description of the extended light source (the surrounding scenario) is needed. The origin of this spherical description coincides with the centre of each surface patch; therefore, a function describing the surrounding scenario is needed for every position investigated.

By assuming that the light source is uniform, E becomes constant for every coordinate (θ, φ) and the problem is greatly simplified. However, this hypothesis goes against the primary objective of the present study which is measuring indoor waves. The solution chosen to deal with it was to ensure that, for a certain range of waves and over a restricted area, the orientation of the surfaces reflects a known piece of the scenario which presents a homogeneous aspect. For planar, regular waves, the reflection will come from a certain range of azimuths and inclinations, which are a function of the camera position and the steepness of the set of wave tested as illustrated in figure 2.4.

As detailed in the experimental section, a large white wall at the back of the basin facility was chosen as the homogeneous extended source.

Figure 2.4: Section of the surrounding scenario reflected into the camera.

Expression 2.9 relates the surface radiance to the angle between the viewer and the normal vector of a particular patch, θ . However, it is convenient to segregate the surface slope from the camera orientation to recover the surface parameters. Taking the origin at the centre of a surface patch and considering a coordinate system where z -axis is pointing vertically, x -axis is parallel to vector $\bar{V}$ and y -axis is perpendicular to the two previous ones. The surface elevation is defined by function $\eta(x, y)$, which allows rewriting the vectors in terms of the surface gradients as,

$$\begin{aligned} N &= (-p, -q, 1) \\ V &= (-p_v, -q_v, 1) = (l_c, 0, h_c) \end{aligned} \tag{2.10}$$

where $p = \frac{\partial \eta}{\partial x}$ and $q = \frac{\partial \eta}{\partial y}$. Similarly, p_v and q_v describe the orientation of the surface patch faced towards the camera. Rewriting $\cos \theta$ using relation 2.10, we obtain expression 2.11, which can be replaced in 2.8. In the same way, expression 2.9 can be rewritten as 2.12.

$$\cos \theta = \frac{pp_v + qq_v + 1}{\sqrt{p^2 + q^2 + 1} \sqrt{p_v^2 + q_v^2 + 1}} \tag{2.11}$$

$$L(p, q) = S(p, q)C \quad (2.12)$$

Note that the expression for L is related to surface radiance up to a scale factor C .

Expression 2.12 is also known as Reflectance Map and establishes a direct relation between surface orientation and brightness. The Reflectance map is usually represented graphically as contours or a surface on the pq -plane, which is called gradient space. Figure 2.5 presents L in the gradient space, considering the camera incidence angle of 45° ($p_v = 1$).

Figure 2.5: Graphical representation of the Radiance model on gradient space for the camera point of view of 45° ($p_v = 1$ and $q_v = 0$).

From the figures, it becomes clear that the Reflectance map is not linear in the wave slope. Positive slopes present slight variation whereas negative slopes produce large changes on the surface radiance. Jähne points out that this is the major problem when using water reflection as a source of information to reconstruct the complete surface. All the faces of the waves the normals of which point in the camera direction present very low brightness whereas those pointing in the other direction are much brighter, and the same is applicable for the derivative of this quantity. However, even without the complete reconstruction of the water surface, some information can still be recovered from the light intensity coming from the brighter faces of the wave and, therefore, this method is still suitable for measurement purposes.

The amount of light reaching a patch of the image, the *Image Irradiance*, in an 8-bit digital monochromatic image, corresponds to an integer number between 0 and 255 in a cell of a matrix. The Irradiance is proportional to Scene Radiance, and the factor of proportionality depends on the characteristics of the lenses and the CCD. For a given set of lenses, particularly telephoto ones, the proportionality factor between the image

Irradiance I and scene Radiance L is approximately constant throughout the image area; additionally, we assume here that the CCD behaves linearly to the changes on the scene radiance. By accepting such hypothesis, one may incorporate this proportionality factor in the global scale factor C . Although C includes the light source E , which was assumed constant, the letter was changed to make clear that this new constant encodes other losses associated with the imaging process. Therefore, from this point onwards, expression 2.12, with constant C , will be directly applied to evaluate the images of the waves.

2.4 Obtaining Regular Wave Parameters

The Reflectance map described in the previous section relates the surface slope to the image irradiance; however, the major interest lies in regular wave parameters, namely its amplitude A , period T and phase ϕ . The main relations regarding the linear wave theory in deep water are:

$$\begin{aligned}\eta(x, t) &= A \cos(\omega t - kx + \phi) \\ k &= \frac{2\pi}{\lambda} \\ \lambda &= \frac{gT^2}{2\pi} \quad ,\end{aligned}\tag{2.13}$$

where $\eta(x, t)$ is the surface elevation, which is a function of the space coordinate x parallel to the wave propagation direction and t is the time variable, ω is the wave angular frequency ($2\pi/T$), k is the wave number, λ is the wavelength, and g is the acceleration of the gravity. Assuming that the wave propagation direction is parallel to the x -axis defined in the previous topic, the derivative of η in terms of x leads to the surface slope p , expression 2.14, and implies that $q = 0$.

$$p(x, t) = kA \sin(\omega t - kx + \phi)\tag{2.14}$$

The wave period and phase could be measured by picking one or more pixels of the image and analysing the brightness fluctuation during the time. This concept was experimentally evaluated by Rapizo et al. (2014) which also applied optical flow approach to estimate the propagation direction. However, information about the wave period is encoded both in the ωt and kx . Since an image of the wave contains the spatial description of the surface

for a particular instant t , then the most direct way of obtaining such information is from the spatial term kx .

The method proposed herein extracts both the wavelength and phase directly from the spatial distribution of brightness. To do that, the image must be mapped into a metric space so that the intensity peaks distant from each other by the wavelength.

The projection matrix, Π , represents the linear transformation that maps three-dimensional coordinates of a point on the surface onto bi-dimensional coordinates of the image plane. It incorporates all the linear aspects that characterize the camera during the imaging process in the 3×4 matrix:

$$\Pi = [KR, KT] \quad (2.15)$$

The calibration process aims at recovering the intrinsic, K , and extrinsic, $[R, T]$, parameters of the camera. The most simple and popular method to recover the camera parameters was proposed by Zhang (1999) and uses planar checkerboards that can be printed and attached to a planar surface. The reader may find more details regarding the camera calibration process in Appendix B.

By applying transformation Π to 3-Dimensional world coordinates of a surface point, expression 2.16, one may obtain the position of that point on the image and, repeating this process for the corners of a known planar area, it is possible to identify it on the image, as presented in 2.6(a). Since Π is a 3×4 matrix, both scene and image points are in their homogeneous form in equation 2.16 and thus have an additional coordinate. The image coordinates (c, r) can be recovered by dividing the left side of equation 2.16 by its third coordinate, depth factor σ . This method is widely known, and can be found in detail in Ma et al. (2004).

$$\sigma \begin{bmatrix} c \\ r \\ 1 \end{bmatrix} = \Pi \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \quad (2.16)$$

Once the corners of the image area are defined, (c_i, r_i) considering $i = 1, 2, 3, 4$, which corresponds to a scene area defined by the corners (x_i, y_i) , transformation 2.17 can be calculated to map the image quadrilateral into the metric one, which produces the resulting image 2.6(b). The linear transformation that maps a 2-Dimensional image into another image is called homography and is applied here to create an image where pixel

coordinates (c, r) are properly replaced by metric coordinates (x, y) ³. This approach is called *forward mapping* and may present problems because two or more pixels in the input image can be transformed to the same location in the output image or, on the contrary, some areas in the output image may remain empty. The most common approach to this problem is to apply the *inverse mapping*, which computes the correspondent pixel location in the input image for every pixel location of the output image (see Gonzalez and Woods (2008) for details regarding these techniques). Here we applied the forward mapping followed by bi-cubical interpolation to produce a dense output image.

Figure 2.6: Calibration image using a checkerboard with 50mm squares .(a) Calibration grid floating on the flat water surface and the analysed portion of the image; (b) Highlighted area of the image (a) mapped into metric coordinates using homography 2.17.

$$\begin{bmatrix} c_1 & r_1 & 1 & 0 & 0 & 0 & -x_1c_1 & -x_1r_1 \\ c_2 & r_2 & 1 & 0 & 0 & 0 & -x_2c_2 & -x_2r_2 \\ c_3 & r_3 & 1 & 0 & 0 & 0 & -x_3c_3 & -x_3r_3 \\ c_4 & r_4 & 1 & 0 & 0 & 0 & -x_4c_4 & -x_4r_4 \\ 0 & 0 & 0 & c_1 & r_1 & 1 & -y_1c_1 & -y_1r_1 \\ 0 & 0 & 0 & c_2 & r_2 & 1 & -y_2c_2 & -y_2r_2 \\ 0 & 0 & 0 & c_3 & r_3 & 1 & -y_3c_3 & -y_3r_3 \\ 0 & 0 & 0 & c_4 & r_4 & 1 & -y_4c_4 & -y_4r_4 \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \\ h_5 \\ h_6 \\ h_7 \\ h_8 \end{bmatrix} = \begin{bmatrix} x_1 \\ y_1 \\ x_2 \\ y_2 \\ x_3 \\ y_3 \\ x_4 \\ y_4 \end{bmatrix} \quad (2.17)$$

Figure 2.7 presents the results of the metric transformation previously described for a regular wave. In figure 2.7(a), the analysis area was carefully chosen to avoid large specularities as those produced by the light bulbs of the basin facility. Since the camera was

³This transformation is suitable here because the mapped surface is nearly planar.

placed perpendicularly to the wave crests and the same alignment was kept when the analysis area was defined, no brightness variation through y -axis in figure 2.7(b) was expected and, thus the profile of that intensity can be plotted as a curve, exemplified in figure 2.7(c).

Figure 2.7: (a) Image of a regular wave the period of which is $0.55s$ and its amplitude $7.0mm$; (b) Highlighted area of the image (a) mapped into metric coordinates using homography 2.17; (c) Brightness profile of the image analyzed obtained by calculating the mean value of each row of image (b).

The distance between the peaks presented in figure 2.7(c) is the wavelength. However, note that these peaks do not correspond to the maximum elevation but the brightest areas of the image, which are the patches with minimum slopes (in figure 2.5, it is possible to notice that negative slopes are associated to higher radiances). Several techniques can be applied to extract the wavelength and the phase of the oscillation, such as directly identifying the peaks or by fitting a sine to the signal. An FFT (Fast Fourier Transform) technique can also be applied although it will not produce satisfactory results if the number of cycles in the image is reduced.

Once the wavelength is known, wave amplitude A can be obtained from surface slopes p . In the previous section, the derivation of the expression that relates the image Irradiance to the wave slopes assumed an extended homogeneous light source, which is valid just for a particular range of surface slopes. The brighter areas in image 2.7(a) correspond to the wave facets pointing at the white wall, on the back of the basin installation, which present homogeneous characteristics. The darker areas correspond to the facets oriented towards the camera and reflect the light coming from the ceiling of the basing facility, which presents a distinct radiance than the white wall on the back. Therefore, it is not possible to apply the same function $L(\theta, \varphi)$ to describe all the surface slopes since the scene

radiance function is distinct through the different areas of the wave surface. However, the maximum surface slopes of regular waves are easily calculated and, by choosing the range of wavelengths and amplitudes, it is possible to ensure that the areas where the slope is maximum will reflect the desired region of the surrounding scenario.

Figure 2.8: Flow chart of the steps to recover the wave amplitude of regular waves based on the peak of brightness.

The amplitude of regular waves can be estimated based on the maximum wave slope, which is kA , and corresponds to the peaks of brightness as described in figure 2.8. Accordingly, in the present study, only the maximum wave slope will be estimated using the peaks of the brightness profile.

A last aspect to be discussed is the method to estimate constant C , which encodes the scene radiance and losses related to the lenses. For this, an image of the flat surface is used since the geometry of the surface is known (p and q) as well as the camera orientation (p_v and q_v). However, the image of the flat water surface must reflect the same portion of the scenario as that reflected when waves are present. This condition can be ensured by a suitable experimental setup detailed in the next section.

2.5 Experimental Setup

The primary concern regarding the experimental setup is to ensure that the wave surface reflects the homogeneous area of the scenario. By adjusting the camera height and inclination and considering the simple geometric relations illustrated in figure 2.4, it is possible to define a range of surface slopes which respect the extended and uniform light source hypothesis. Figure 2.9 and Table 2.1 describe the relevant dimensions of the tank facility whereas expressions 2.18 defines the range of wave slopes which ensures that the reflected light is coming from the white wall.

Figure 2.9: Illustrations of the main dimensions related to the experimental setup.

$$\frac{1}{2} \left[\frac{h_c}{l_c} - \frac{h_w}{(l_w - l_c)} \right] < |p| < \frac{1}{2} \left(\frac{h_c}{l_c} - \frac{h_o}{l_o} \right) \quad (2.18)$$

Table 2.1: Main dimensions of the tank facility and the camera arrangement.

Camera Height	h_c	2.0(m)
Closest Obstacle Height	h_o	1.8(m)
Wall Height	h_w	7.5(m)
Horiz. Distance Camera to Analysed Area	l_c	5.0(m)
Horiz. Distance Analysed Area to Obstacle	l_o	12.6(m)
Horiz. Distance Camera to Wall	l_w	22.7(m)

The higher the camera, the larger the absolute slope the system can evaluate. However, the camera will receive light from lower regions of the scenario and, possibly, from the

region beneath the water level and thus outside the homogeneous light source area. In the present study, the maximum absolute wave slope was limited by the closest (and higher) obstacle, as shown in figure 2.9.

Even though the maximum wave slope, together with the wavelength and phase, is all the information needed to describe a regular wave, images of the flat water surface reflecting the same extended source of light are necessary for the calibration process, as discussed in the previous section. Therefore, the camera height and distance to the analysed area were chosen to ensure that both the maximum wave slope and the flat surface still reflect the white wall. The set of waves tested during the experiments is described in table 2.2.

A set of 22 waves was evaluated with a maximum absolute slope (p_{max}) spanning from 0.035 to 0.133, that is, waves whose steepness ranged from 1.1% to 4.2%. The largest steepness measured exceeds the maximum theoretical limit ($<1\%$) for the linear theory, although these models are applied to higher values in practical terms. All the waves were measured by the wave probe system of the offshore tank which are wire, or capacitive, wave probes. A detailed description of the generations process will be presented in section 3.5.

Table 2.2: Set of regular waves tested. The amplitudes and wavelengths presented in the table were measured by wire wave probes.

Wave ID	A (mm)	λ (mm)	p_{max}
1	4.7	474.5	0.062
2	7.1	474.5	0.094
3	9.4	473.6	0.125
4	4.9	565.5	0.054
5	7.3	566.5	0.081
6	9.5	564.8	0.106
7	12.0	565.1	0.133
8	4.8	662.4	0.046
9	7.1	662.4	0.067
10	9.6	663.7	0.091
11	12.1	661.1	0.115
12	4.7	768.8	0.038
13	6.9	769.5	0.056
14	9.4	767.8	0.077
15	11.7	767.3	0.096
16	14.1	767.1	0.116
17	4.9	886.7	0.035
18	7.2	885.3	0.051
19	9.6	885.1	0.068
20	12.0	879.3	0.086
21	14.5	879.8	0.104
22	17.2	880.7	0.123

A video camera recorded each wave, and the elevation was also measured by wire wave probes for further comparisons.

Preliminary experiments were carried out using off-the-shelf web cams, see Lavieri and Tannuri (2016). Although promising results were obtained, the web cams have poor optical characteristics and almost any parameter, such as shutter speed or aperture, is configurable. Thus, an industrial-grade camera was employed here to allow control over all image acquisition parameters and to improve precision. Characteristics of the camera and lenses used during the experiments are presented in tables 2.3 and 2.4. The combination of the size of the CCD (Charge Coupled Device), the focal distance of the lenses and the attitude of the camera defines the field of view. The image covers an area of approximately $1.4m \times 0.6m$ but just a region of it is used for the analysis, as illustrated in figure 2.7. The analysis area chosen was rectangular with $1.1m \times 0.1m$ placed in a region of the image free from major specularities produced by the light bulbs.

The videos were recorded for 10s, using 10fps (frames per second) and thus, a single cycle of the shortest wave (0.55s) could be described by five images. From each of the 100

images obtained in the experiments estimates of wavelength, phase, and amplitude were calculated.

Table 2.3: Characteristics of the camera.

Resolution	1392(h) $\times$ 1040(v)(<i>pixels</i>)
Pixel Size	4.65(μm) (square)
CCD Size	1/2"
Maximum Frame Rate	30 <i>fps</i>
Used Frame Rate	10 <i>fps</i>
Output Image	8-bit Monochrome

Table 2.4: Characteristics of the lenses.

Focal Length	50(<i>mm</i>)
Filter Diameter	29.5(<i>mm</i>)
Aperture Range	F2.3-F22
Field Angle (Horizontal)	10°03'
Field Angle (Vertical)	7°33'

Tests with IR-pass filters were performed, as suggested by Jähne, Waas and Klinke (1992); however, the mercury gas or fluorescent light bulbs that can be found in wave tanks facilities produce light whose spectrum concentrates on shorter waves. Therefore, IR-pass filters produce a drastically drop in the amount of light perceived by the camera, which deteriorates the performance of the method.

Figure 2.10 presents the camera rig used for the experiments. Two cameras with the same inclination (22° about horizontal) placed side by side we employed for the images acquisition. However, the second camera was only applied to increase the accuracy of the calibration process ⁴, and the results presented in the further sections were obtained using images from the left camera alone.

⁴After calculating the calibration parameters of each camera, a global optimization can be performed over a minimal set of unknown parameters. In particular, only one pose unknown (6DOF) is considered for the location of the calibration of the stereo pair. This assumption ensures the global rigidity of the structure going from left view to right view. Therefore, the uncertainties on the intrinsic parameters (especially the focal lengths) for both cameras became smaller. (NIOLA; ROSSI, 2007)

(a)

(b)

Figure 2.10: Pictures of the experimental setup; (a) Details of the camera on its base with inclination measurement device; (b) Picture of the complete camera rig, including the Ethernet switch and power source.

In order to calculate constant C , which relates the reflectance map to the image irradiance, the mean value along x of brightness profiles associated with the flat water surface was used. The flat surface must reflect a homogeneous area and significant brightness variations as those observed in the two situations illustrated in figure 2.11(f) are avoided. Moving the camera sideways or turning off some of the bulbs can produce a uniform profile as the one presented in 2.11(c).

Figure 2.11: Calibration images of the flat water surface; (a) Image produced by the left camera without significant brightness fluctuation; (b) Image in metric space; (c) Homogeneous intensity profile; (d) Image produced by the right camera in which brightness fluctuation can be observed; (e) Image in metric space; (f) Non-homogeneous brightness profile.

2.6 Experimental Results

In this section, the experimental results are presented. Firstly, the intensity profiles are analysed to verify all the hypothesis previously discussed; later, the wave parameters estimation is assessed. By selecting specific frames of the videos, it is possible to align the brightness peaks for each set of wavelength, as presented in figure 2.12, and compare the maximum for each wave. Similar wavelengths were grouped in Figures 2.12(a) to 2.12(e) and compared to the brightness of the flat surface.

Note that the larger the slopes, the greater the brightness peaks. The peaks observed

are coming from the wave faces pointing at the white wall, i.e. the slope is negative considering the defined reference frame, and produce greater brightness accordingly to the curve in figure 2.5.

Figure 2.12: Brightness profile; (a) Waves with approximately 474mm in length and amplitudes from 4.7mm to 9.4mm ; (b) Waves with approximately 565mm in length and amplitudes from 4.9mm to 12.0mm ; (c) Waves with approximately 662mm in length and amplitudes from 4.8mm to 12.1mm ; (d) Waves with approximately 768mm in length and amplitudes from 4.7mm to 14.1mm ; (e) Waves with approximately 883mm in length and amplitudes from 4.9mm to 17.2mm .

In a single image, peaks closer to the camera are lower than those away from it. This happens because the angle between the surface normal and the reflected light is reduced, in other words, for the same p , p_v is reduced.

Waves with the largest steepness (IDs 3, 6, 7, 11, 16 and 22) present cut off at the peaks of brightness, which can be verified in all graphs. The maximum slopes of theses waves are very close to the geometric limits defined in expression 2.18 and, therefore, at the maximum slope, the surface probably reflected part of the obstacle, producing the abrupt drop observed.

The minimum slope which still reflects the white wall is zero and thus no consideration regarding the troughs in figure 2.12 can be made. However, smaller variations in that region are expected, once the sensitivity of the model to positive slopes is equally smaller.

For each frame of the wave video, the peaks of the brightness profile were extracted, and the mean values were adopted as the brightness measurement. It is possible to use the mean values in the present experiment because the imaged area is reduced (the angle of view of the lenses is relatively small). However, one may divide the image into sections and compute the peaks for each p_v .

The mean value of the flat surface image, the profile of which is presented in figure 2.11, was used to calculate constant C and the curve presented in figure 2.13. By plotting the brightness in the same graph, close trends can be verified between experiments and model, particularly for $p > -0.1$. Deviations for $p < -0.1$ were expected since the brightness peaks of largest steepness waves could not be recorded correctly.

The maximum slopes associated with the waves were implicitly calculated using the mean values of brightness peaks identified in each frame. Figure 2.14 presents the comparisons between the slopes estimates produced by the imaging system and those measured by the wire wave probe system. A linear regression with $r^2 = 0.943$, figure 2.14, indicates a clear linear relationship between the two measurement methods. By excluding erroneous records, the estimated bias with respect to the direct relationship is 0.012 and the scale factor is 1.001.

Figure 2.13: Comparison between the brightness readings for each wave slope and the model described by expression 2.12.

Figure 2.14: Comparison between the image-based wave slope estimate and those measured by wire wave probes. Erroneous measurements related to greater slopes were excluded for the linear fit.

In order to recover the wave amplitude from the estimated slope, the wavelength must be known. By applying the method presented in section 2.4, the wavelength can be calculated using a single frame. Figure 2.15 presents the comparison between the image-based estimates of wavelengths versus those measured by the wire wave probes. Each cross in the figure represents the mean value of the 100 frames estimates, for each of the 22 waves tested.

Figure 2.15: Comparison between the image based wave length estimates and those provided by the wire wave probes.

The precision of the image-based method decreases as the waves become longer. It was an expected effect, since the method presented in section 2.4 calculates the wavelength using the brightness peaks and, for longer waves, many frames exhibit a single peak. A larger analysis area (i.e. by applying wider field angle lenses) would increase the precision of the method for longer waves.

Finally, by using both the wave slope and wavelength information, the wave amplitude can be estimated. Comparisons between the amplitudes measured by the image-based system and those measured by the wire wave probes are presented in figure 2.16. Erroneous slope measures were plotted in light gray, as in figure 2.14.

Figure 2.16: Comparison between the image-based wave amplitude estimates and those provided by the wire wave probes.

The bias present in wave slope estimates can also be noticed in figure 2.16, producing underestimated amplitude measurements.

2.6.1 Real Time Estimation

All the results previously presented were post-processed, but the primary interest is to measure the waves in real time. Therefore, the method was adapted to run while the frames are recorded.

Some distinct techniques were implemented to compute the wave length, which is not only used to calculate the wave period but also is multiplied by the wave slope p to calculate the amplitude, thus representing a key element to the method performance.

It were evaluated the use of FFT (*Fast Fourier Transform*), a sine function fitting using least squares method and by simply finding the horizontal distance between peaks. The first method is very fast but requires a greater number of complete cycles in the same frame. Although more robust regarding the number of cycles in the image, the second method is computationally costly, leading to the choosing of the third method, which presented satisfactory results and reduced processing time.

Figures 2.17 and 2.18 present the time series of the image-based amplitude estimation and the wave probe register. The final image-based measurements frequency was 1Hz and a moving average of 5 points was used to smooth the signal.

(a)

(b)

(c)

Figure 2.17: Real time estimation of wave amplitude. (a) 4.3mm wave amplitude and period 0.65s, (b) 6.7mm wave amplitude and period 0.65s and (a) 8.7mm wave amplitude and period 0.65s.

(a)

(b)

Figure 2.18: Real time estimation of wave amplitude. (a) 6.7mm wave amplitude and period 0.60s and (b) 9.4mm wave amplitude and period 0.60s.

Chapter 3

Wave Feed-Forward

Fluid mechanics is present in almost all Maritime Engineering problems. Among these problems, the *seakeeping* plays an important role, particularly for the design of offshore structures since it deals with the interaction between floating bodies and gravity waves.

It is well known that the most complete¹ model for describing the dynamics of fluids is the Navier-Stokes equation, however, only recently this equation managed to be solved² by numerical means and spending significant computational effort. Nevertheless, the seakeeping problem allows some simplifying hypothesis that makes possible the application of a more expedited approach. Water can be considered incompressible, thus *continuity* is preserved. Additionally, viscous effects can be neglected once inertial forces rule the movements of large floating structures. The thin *boundary layer*, compared to the scales of the problem, is a physical evidence of such fact.

In the absence of viscosity and hence shearing forces, the curl of the vector field is null, and the problem can be treated as a *potential flow*.

Albeit the potential approach simplifies the solution of the flow around structures, there are still several non-linear boundary conditions, introduced by the free surface and the floating body, requiring additional approximations. Through the use of a perturbation technique, the original non-linear problem can be transformed into a sequence of linked linear ones with distinct orders of magnitude.

Floating bodies in waves are subjected to large forces and moments which are linearly proportional to wave height and have the same oscillation period. These are called *first*

¹For Newtonian fluids.

²Even the so-called *full Navier-Stokes* CFD codes apply several models and simplifications. DNS (*Direct Numerical Solution*) could only be done for few simple cases, even though taking weeks of processing time.

order forces and are hardly influenced by the mooring lines. However, as purely oscillatory forces, the average position of the system remains the same after a full period.

Besides the first order effects, stationary floating systems are also subjected to small mean and low frequency³ forces and moments which are proportional to the square of the wave height, Maruo (1960). These are called *second order* forces and its horizontal components are also known as *wave drift*.

Since the first observation of wave drifts phenomena, its physical cause has been related to the destruction of the wave structure by reflection due to the presence of the ship and its movements, Suyehiro (1924). Consequently, the relative length of the vessel and the incoming waves shall dictate the magnitude of drift forces. A hull may appear a wall for short waves while very long waves shall cross it almost unaltered⁴. Therefore, particular interest shall be given to short waves and a system capable of measuring them.

Dynamic Positioning systems use thrusters to keep the vessel position and heading while it is exposed to environmental forces. Just like all control systems, DP applies a control rule, usually a PID (*Proportional Integral Derivative*), that take decisions based on position error and hence this method is called *feed-back*. Dynamic Positioning systems can not counteract wave frequency forces as these are too large and its variations too fast. The controller neglects those forces by filtering the position and movements readings.

In order to improve the speed and efficiency of the controller, one can feed it not only with the position errors but also with the perturbation forces. In a perfect scenario, forces could be counteracted before any offset takes place. It is the basis of the *feed-forward*.

Since 1970's DP systems have incorporated the *wind feed forward*, demonstrating significant efficiency⁵ advancement, Aalbers, Tap and Pinkster (2001). Wave feed-forward, on the other hand, was initially proposed by Pinkster (1978) and have only been tested in model scale. As previously mentioned, one of the main reasons for such system still be restricted to laboratory tests is the fact that no practical mean of measure the incoming waves, Aalbers and Nienhuis (1987).

3.1 Historical and Bibliographic Review

Albeit man, since he watches the sea, has the intuitive notion that floating objects tend to drift away when left untied, apparently, Suyehiro (1924) made the first laboratory

³As will be shown further, a high-frequency component is also noticed among second order forces.

⁴Formal mathematical derivation for such statement will be presented in further topics.

⁵Lower offsets and energy consumption.

observation of such phenomenon. He experimentally identified a side force acting on a ship model rolling due to beam waves. Such force was assigned to the reflection produced by the hull, but some preliminary mathematical explanation came later with Watanabe (1938) work, which pointed that it was a second-order phenomenon.

Differently from these two quoted works, most of the research concerning second order effects produced before 1970 decade focused on the added resistance produced by waves. Mainly because in the first half of twenty century very few structures were moored offshore, particularly under bad weather, Hsu and Blenkarn (1970) and Remery and Hermans (1971). Havelock (1940) and Havelock (1942) are examples of studies regarding second order wave forces under vessels with forward speed.

Maruo (1960) presented the first formulation for both longitudinal and transverse components of mean second order drift force for a moored vessel. The average *yaw* moment was later incorporated by Newman (1967) that re-derived Maruo's formulation. Following the taxonomy proposed by Pinkster (1980), these latter works differ from those previously mentioned not only by the scope of the formulation but also by the way the problem was faced. Maruo (1960) and Newman (1967) applied momentum conservation in order to estimate drift forces whereas Watanabe (1938), Havelock (1940) and Havelock (1942) do so by the direct integration of the pressure over the hull.

All these works dealt with mean drift, low-frequency second order forces were further studied by Hsu and Blenkarn (1970) and Remery and Hermans (1971), through the evaluation of experiments with a barge shape vessel model under irregular waves. These works aimed attention at the interaction between second order wave forces and the mooring system, highlighting the relevance of such forces for tension level in mooring lines. Among the major contributions of Remery and Hermans (1971) work is the relations established between wave group period and the slow drift frequency.

It was observed that an irregular wave train can be thought of as a succession of half-periods of regular waves and hence slow drift force was the time variation of mean drift related to regular slow variation of wave trains. Such effect is illustrated in figure 3.1, reproduced from his paper.

Pinkster (1979)⁶ introduced the formulation of the second order forces through the pressure integration on the hull, under arbitrary wave condition and for all six degrees of freedom. All primary quantities of the hydrodynamic problem were expanded in power series until second order, producing complete expressions for the pressure. Those were

⁶The first derivation were originally presented in the previous paper (1976), but this seems to be unavailable on-line nowadays. However, the quoted paper present part of the derivation, numerical and experimental results as well as explanations concerning each term of the forces.

Figure 3.1: Wave drift force from wave record, Remery and Hermans (1971).

numerically computed and compared with experimental results.

Pinkster (1980) presents the detailed derivation of low-frequency second order forces previously mentioned and discusses the magnitude of each component. By numerical computation of each component and by the confrontation with experimental results, it was found that the overall behavior of second order forces is ruled by the component proportional to the square of the *relative* free surface elevation. Therefore, the drift forces and moments could be calculated by the integration of wave height around the hull water line. Such result was fundamental for the proposition of the wave feed-forward technique for the DP systems, as will be presented in the next chapter.

Numerical computation of forces applying the second order velocity potential is time expensive, thus an approximation was presented that relies only on the first order potential to produce good estimate of second order forces.

The frequency domain approach, commonly applied to solve the first order hydrodynamic forces, was borrowed to the second-order velocity potential computation, bringing forth the definition of a quadratic transfer function, so-called *QTF*. Such function relates wave frequencies in an irregular⁷ sea two by two, encoding mean drift, slow drift and high frequencies effects. However, the calculation of the full matrix can be computationally costly, particularly some decades ago. Therefore, Newman (1974) proposed a simplification⁸ relating the cross terms with the diagonal terms, what drastically reduced the computational cost whereas such terms do not require the calculation of the second-order velocity potential.

Although less straightforward, the QTF can be applied to predict second order drift forces

⁷Regular sea is also comprehended as it is represented by the situation where both frequencies are the same.

⁸So-called Newman's approximation.

in real time situation as presented by Knoop and Taylor (1991). This work describes a method to produce time history register of undisturbed waves, deconvoluting relative elevation measurements with the knowledge of the system impulse response. Then, the quadratic transfer function is applied to compute non-linear drift forces. Experiments carried out with a tanker model equipped with a wave probe on its bow and force measurement, confirmed the effectiveness of the method considering that good agreement between measurement and calculated drift forces was obtained.

The wave feed-forward (WFF) concept was first introduced by Pinkster (1978) after he has established a simple and reliable approximation to compute the second order drift forces based on the relative wave elevation along the water line, as discussed in the previous chapter. In order to confirm the effectiveness of the method, experiments with a 1:82.5 scale tanker model equipped with eight wave probes attached to the hull were carried out under irregular waves. Results for the surge motion are presented in figure 3.2, reproduced from the original paper. The model was attached to servo units to emulate the thruster action.

Figure 3.2: Surge motion in irregular head waves, Pinkster (1978).

Aalbers and Nienhuis (1987) applied the same approximation to estimate second order forces on each side of the hull and hence calculate an optimum heading. Wind effects were also considered as the *weathervane* problem must take into account all environmental forces.

Model of a drilling ship was tested as well, but the only the heading was controlled. A *soft springs* mooring arrangement restrained all the other DOFs, calibrated to mimics DP

system. These tests indicated that it was feasible to evaluate the wave direction and to obtain a reasonable estimate of the wave drift force from relative elevation signals since rolling motions remain low.

The mathematical model was incorporated to a dynamic simulator where three other vessels were tested, a large tanker, a drilling ship and a smaller supply vessel. The direction feed-forward system produced improvement for all the three ships, although the supply vessel presented similar results just with the wind feed-forward.

Aalbers, Tap and Pinkster (2001) implemented and compared these latter methods. Experiments were carried out with a tanker model equipped with azimuthal thrusters. Six DOF position and forces on each thruster were measured. It was figured out that better results could be obtained using the first method, applying an array of sensors all around the hull waterline. In spite of coarse results in terms of wave drift forces estimations, the second method managed to produce good measurement of wave direction.

Linfoot and Wright (1995) performed test with a DP ship model moored in turret configuration. It was investigated the effect of the DP in the reduction of the mooring tensions. The work explored other control rules than PID. LQG (*Linear Quadratic Gaussian*) and a minimum-variance control were tested together with self-tuning techniques.

Aalbers, Waals and Tap (2004) evaluated the wave feed-forward on the operation of a shuttle tanker by dynamic simulation. The complete offloading operation was also evaluated by means of a Monte Carlo type simulation with a routine to take decision based on forecast and present weather conditions. The study indicated that an improve of 3% to 4% on the uptime could be reached by the inclusion of the wave feed-forward.

Recent developments on the technology are presented by Hughes et al. (2009) and Quadvlieg et al. (2011). These works present investigation on the performance of wave feed-forward during side-by-side operations. Model tests with two large vessels distant about 10m^9 from each other were carried on. Only one of the vessels was equipped with wave probes and hence has the wave feed-forward enabled.

No significant improvement was observed and the pointed reason was that if none of the vessels had the wave feed-forward system, they would move together with the drift forces, and the distance between them would remain the same. No further tests were performed to prove such statement.

⁹Real scale length.

3.2 WFF in DP Control

Figure 3.3 shows a general scheme of the DP control. The components of the scheme are explained below.

Figure 3.3: DP control scheme including feed-forward.

The *Set-point* is the reference position, described in the three DOFs of the plane, used to calculate the errors. These parameters are defined in terms of a fixed reference system which, in a real operation, is represented by latitude, longitude and heading about the North.

The *DP Control* is typically a PID controller, which calculates forces proportional to the error between the vessel's position and the Set-point, as well as to the integration of this error over time and to the rate that it changes. Forces proportional to the error counteracts the external forces, like a spring, but depend on the presence of error to exist. By including a term related to the integral of the error, the permanent offset is reduced as the time passes. Finally, the so-called derivative term operates like a damping to the system, reducing excessive oscillation.

The output of the DP control are forces and moments in surge, sway, and yaw, then, the *Thrust Allocator* is responsible for calculating the proper rotation of the propellers and azimuth of the steerable thrusters, capable of producing the desired set of forces and moments. This algorithm should take into account the efficiency of each thruster as well as the cross-influence between them.

Commands calculated by the Allocator are sent to the *Thruster* which produces forces that are input to the *Dynamic of the Vessel*, resulting in a new position. It is called

Reference Position System any device capable of measuring the position of the vessel on the global coordinate system, such as GPS, DGPS, Compasses, Taut Wire, among others.

Under the action of waves, the vessel will present first order movements which may be detected by the reference position system but can not be counteracted by the DP control. Therefore, these high-frequency movements have to be filtered from the position readings and, to do that, a *Wave Filter* is employed. An Extended Kalman Filter (EKF) is typically used, which, generally speaking, filter the signal by upcoming the next state (position/velocity) of the vessel using a non-linear dynamic model of it. From time to time, this state estimate is corrected using an external reading.

Finally, the presence of *Wind* and *Wave* forces estimators allow the DP controller to anticipate such loads and deal with them more efficiently. By subtracting the estimated force, a forward counteraction is produced, which avoids the vessel to gain kinetic energy. As mentioned in the introductory sections, the wind feed-forward is already implemented in real ships, and the focus of the present work is to develop the wave system.

3.3 DP Controller Tuning

A DP system $C(s)$ typically applies a PID controller for each degree of freedom, which are the surge, sway, and yaw movements, described in figure 3.4. In the present work, PD controller was also evaluated as well as the PD with the WFF since, in the absence of the integral term, the system will show a steady position error related to the wave drift force which shall be reduced by applying the WFF.

The gains of both controllers were tuned using a conventional pole placement technique, considering the closed loop natural frequency ω_n and a damping coefficient ζ_c for each degree of freedom. In order to calculate the pole placement, it was considered a simplified dynamic model of the ship $G_i(s)$ without restoring or dissipative forces, as presented in expression 3.1,

$$G_i(s) = \frac{1}{(m + Ma_{ii})s^2} \quad (3.1)$$

where m is the mass (or moment of inertia) of the model and Ma_{ii} is the diagonal term of the i th degree of freedom of the Added Mass Matrix ¹⁰. By neglecting the propulsion, the block diagrams of the closed loop systems with PID and PD controllers are shown in

¹⁰The Added Mass Matrix is calculated when the hydrodynamic radiation problem is solved, using the same technique applied in section 3.4.

Figure 3.4: Description of main axis and movements to be controlled.

figure 3.5.

Figure 3.5: Block diagram of the closed loop system, (a) PID controller and (b) PD controller.

In the diagram, Kp_i , Kd_i and Ki_i are the proportional, derivative and integral gains respectively. Expressions 3.2 and 3.3 are the closed loop transfer functions for the systems

previously presented.

$$P_i^{PID}(s) = \frac{Kd_i s^2 + Kp_i s + Ki_i}{(m + Ma_{ii})s^3 + Kd_i s^2 + Kp_i s + Ki_i} \quad (3.2)$$

$$P_i^{PD}(s) = \frac{Kd_i s + Kp_i}{(m + Ma_{ii})s^2 + Kd_i s + Kp_i} \quad (3.3)$$

The dominant poles are defined as $-\omega_n \zeta_c \pm \omega_n j \sqrt{1 - \zeta_c^2}$ and the non-dominant pole for the system with PID control are represented by $-\alpha \omega_n$, therefore gains of the controllers can be calculated using expressions from 3.4 to 3.8.

$$Kp_i^{PID} = (m + Ma_{ii})(1 + 2\alpha \zeta_c^2)\omega_n^2 \quad (3.4)$$

$$Ki_i^{PID} = (m + Ma_{ii})\alpha \zeta_c \omega_n^3 \quad (3.5)$$

$$Kd_i^{PID} = (m + Ma_{ii})(2 + \alpha)\zeta_c \omega_n \quad (3.6)$$

$$Kp_i^{PD} = (m + Ma_{ii})\omega_n^2 \quad (3.7)$$

$$Kd_i^{PD} = 2(m + Ma_{ii})\zeta_c \omega_n \quad (3.8)$$

3.4 Drift Force Computation

Drift forces and moments are the main issues related to the DP system, and these are second order effects. In order to calculate such effects, the product and the square of first-order results, as well as purely second order terms, must be considered. Pinkster (1980) points out that, for many practical terms, the contributions arising from components dependent on first order quantities tend to be dominant. The formulation presented in this section concentrates on these second order force components which apply first order quantities, obtained after the regular numerical solution and was extracted from WAMIT (2015).

The mean forces and moments are the time average of hydrodynamic forces acting on

a floating structure, which is the integral of the pressure over the wetted surface of the body. Being P the pressure and $\vec{N}$ the surface normal at a particular point of the hull, the second order forces $\vec{F}^{(2)}$ and moments $\vec{M}^{(2)}$ are calculated as presented below.

$$\vec{F}^{(2)} = \iint_{\tilde{S}_b} \vec{N} P ds \quad (3.9)$$

$$\vec{M}^{(2)} = \iint_{\tilde{S}_b} (\vec{X} \times \vec{N}) P ds \quad (3.10)$$

The position vector on inertial coordinate system $\vec{X}$ is given by equation 3.11.

$$\vec{X} = \vec{x} + \vec{\xi} + \vec{\alpha} \times \vec{x} + H\vec{x} \quad (3.11)$$

Both the surface points $\vec{X}$ and normal vectors $\vec{N}$ must be described in an inertial coordinate system, thus the transformation given by equation 3.12 is used. Quantities $\vec{\alpha}$ are the rotational movements whereas $\vec{\xi}$ are the translational movements of the body in the body fixed reference system.

$$\vec{N} = \vec{n} + \vec{\alpha} \times \vec{n} + H\vec{n} \quad (3.12)$$

Where $\vec{n}$ is the surface normal described in the body fixed reference system and H is the second order transformation matrix, that converts the position on the moving body reference frame $\vec{x}$ to the inertial reference frame, neglecting the purely second order terms, expression 3.13.

$$H = \frac{1}{2} \begin{pmatrix} -(\alpha_2^2 + \alpha_3^2) & 0 & 0 \\ 2\alpha_1\alpha_2 & -(\alpha_1^2 + \alpha_3^2) & 0 \\ 2\alpha_1\alpha_3 & 2\alpha_2\alpha_3 & -(\alpha_1^2 + \alpha_3^2) \end{pmatrix} \quad (3.13)$$

Being ϕ the velocity potential¹¹ of the flow, the pressure at point $\vec{X}$ is given by equation 3.14.

$$\begin{aligned} P = & -\rho[g(z + Z_0) + (\phi_t + g(\xi_3 + \alpha_1 y - \alpha_2 x)) \\ & + (\frac{1}{2} \nabla \phi \cdot \nabla \phi + (\vec{\xi} + \vec{\alpha} \times \vec{x}) \cdot \nabla \phi_t + hH\vec{x} \cdot \nabla z] \end{aligned} \quad (3.14)$$

¹¹The velocity potential is a scalar field whose gradient describes the velocity at every point of the fluid. The reader may find complete explanation in Newman (1977).

Where Z_0 is the elevation of each particular point with respect to the mean water level, ϕ_t is the time derivative of the velocity potential, g is the acceleration of the gravity and ρ is the water density. Applying equations 3.9, 3.14 and 3.13, the second order forces can be written as,

$$\begin{aligned}
\vec{F}^{(2)} = & \frac{1}{2}\rho g \int_{WL} \vec{n}' [\zeta^2 - 2\zeta(\xi_3 + \alpha_1 y - \alpha_2 x)] dl \\
& + \frac{1}{2}\rho g \vec{k} \int_{WL} \frac{n_z}{\sqrt{1-n_z^2}} [\zeta - (\xi_3 + \alpha_1 y - \alpha_2 x)]^2 dl \\
& - \rho \iint_{S_b} \vec{n} \left(\frac{1}{2} \nabla \phi \cdot \nabla \phi + (\vec{\xi} + \vec{\alpha} \times \vec{x}) \cdot \nabla \phi_t \right) ds \\
& + \vec{\alpha} \times \left(-\rho \iint_{S_b} \vec{n} \phi_t ds \right) \\
& + \vec{F}_S^{(2)}
\end{aligned} \tag{3.15}$$

here ζ is the first order runup (the water elevation at the body surface). Likewise, the second order moments can be calculated applying equations 3.10, 3.14 and 3.13, resulting in 3.16.

$$\begin{aligned}
\vec{M}^{(2)} = & \frac{1}{2}\rho g \int_{WL} (\vec{x} \times \vec{n}') [\zeta^2 - 2\zeta(\xi_3 + \alpha_1 y - \alpha_2 x)] dl \\
& + \frac{1}{2}\rho g \int_{WL} \frac{n_z}{\sqrt{1-n_z^2}} (\vec{x} \times \vec{k}) [\zeta - (\xi_3 + \alpha_1 y - \alpha_2 x)]^2 dl \\
& - \rho \iint_{S_b} (\vec{x} \times \vec{n}) \left(\frac{1}{2} \nabla \phi \cdot \nabla \phi + (\vec{\xi} + \vec{\alpha} \times \vec{x}) \cdot \nabla \phi_t \right) ds \\
& - \rho \vec{\alpha} \times \iint_{S_b} (\vec{x} \times \vec{n}) \phi_t ds \\
& + \vec{\xi} \times \left(-\rho \iint_{S_b} \vec{n} \phi_t ds \right) \\
& + \vec{M}_S^{(2)}
\end{aligned} \tag{3.16}$$

Where $\vec{F}_S^{(2)}$ and $\vec{M}_S^{(2)}$ are the second order hydrostatic components and can be calculated as follows.

$$\vec{F}_S^{(2)} = -\rho g A_{wp} \{ \alpha_1 \alpha_2 + \alpha_2 \alpha_3 y_f + \frac{1}{2} (\alpha_1^2 + \alpha_2^2) Z_0 \} \vec{k} \tag{3.17}$$

$$\begin{aligned}
\vec{M}_S^{(2)} = \rho g \{ & [(\xi_2\alpha_2 - \xi_3\alpha_3)A_{wp}x_f - \xi_2\alpha_1A_{wp}y_f + \alpha_1\alpha_2A_{wp}x_fZ_0 \\
& - (\frac{3}{2}\alpha_1^2 + \frac{1}{2}\alpha_2^2)A_{wp}y_fZ_0 - \xi_2\xi_3A_{wp} - \xi_3\alpha_1A_{wp}Z_0 \\
& + \alpha_2\alpha_3(L_{11} - L_{22}) - 2\alpha_1\alpha_2L_{12} + \alpha_1\alpha_2\forall x_b - \frac{1}{2}(\alpha_1^2 + \alpha_3^2)\forall y_b] \vec{i} \\
& [-\xi_1\alpha_2A_{wp}x_f + (\xi_1\alpha_1 - \xi_3\alpha_3)A_{wp}y_f + (\frac{1}{2}\alpha_1^2 + \frac{3}{2}\alpha_2^2)A_{wp}x_fZ_0 \\
& - \alpha_1\alpha_2A_{wp}y_fZ_0 + \xi_1\xi_3A_{wp} - \xi_3\alpha_2A_{wp}Z_0 + \alpha_1\alpha_3(L_{11} - L_{22}) \\
& + 2\alpha_2\alpha_3L_{12} + \frac{1}{2}(\alpha_2^2 + \alpha_3^2)\forall x_b] \vec{j} \}
\end{aligned} \tag{3.18}$$

Quantity A_{wp} is the water plane area, $\forall$ is the volume of the body, (x_f, y_f) are the coordinates of the center of flotation, (x_b, y_b, z_b) are the coordinates of the center of buoyancy and L_{ij} are the moments of water plane area.

As Pinkster (1979) demonstrated, the first terms of expressions 3.15 and 3.16 are dominant and, therefore, were the basis of the first developments on the wave feed forward. As described in section 3.1, Pinkster proposed experiments employing wave probes attached to the ship hull to measure the wave runup ζ and estimate the drift forces. On the other hand, the approach presented herein uses information from waves far from the floating body. Thus, the water elevation on the hull, as well as the other terms in equations presented above, were previously calculated and incorporated in the QTFs. These quadratic transfer functions relate the forces and moments with incident waves, according to the angle, wave period and the square of the wave amplitude.

In the present study, the mean drift forces were calculated using the software WAMIT[®], which apply panel method to solve the diffraction and radiation problem and then compute the quantities described earlier. It requires the surface of the body to be previously discretized into panels, as presented in figure 3.6.

The orientation assumed to describe the incident wave angles is shown in figure 3.7. During the experiments, waves reached the portside of the model, thus, the incident angles were close to 270° . Figure 3.8 presents the numerical results of the mean drift forces and moments for the incident wave angles evaluated during experiments ¹².

The values presented in figure 3.8 are normalized mean drift forces and moments for 1m wave amplitude and must be multiplied by A^2 , being A the wave amplitude¹³. All the

¹²Mean drift forces were calculated for all directions and, during the experiments, the values were interpolated to match the exact incident angle.

¹³This procedure is only valid for regular waves. When dealing with irregular waves, the full response spectrum must be calculated by integral means.

Figure 3.6: Panel mesh of the PSV hull.

Figure 3.7: Orientation of incident waves directions.

calculations were done in real scale, thus during the experiments, forces and moments were converted to model scale using the scale factor cubed and raised to the fourth power.

Figure 3.8: Normalized mean drift forces and moment curves calculated for PSV. (a) surge mean drift force curves, (b) sway mean drift force curves and (c) yaw mean drift force curves.

3.5 Experimental Setup

Experiments were carried out in the Hydrodynamic Calibrator of the Numerical Offshore Tank (HC-TPN) of Escola Politécnica of the University of São Paulo, the same facility where the image-based wave measurement system was tested. The HC-TPN is a square offshore basin with 14m wide and 4.1m depth capable of generating and absorbs waves in

all directions using 148 independent flaps distributed along the four side walls of the tank. A detailed description of the wave generations and absorption system can be found in Mello (2012). The wave generation system is capable of creating waves with frequencies between 0.4 Hz and 2.0 Hz and a maximum height of 0.4m, at 0.75Hz. Figure 3.9(a) presents, in grey, the operational envelope of the wave generator and it was highlighted the region of waves tested during the current study.

Figure 3.9: Graph (a) presents the operational envelope of the wave generations system, and graph (b) presents experimental curve of the reflection coefficient Cr , defined as the rate between the amplitudes of incident and reflected waves.

The envelope is defined by the operational limits of the mechanical components (stroke amplitude and screw velocity), as well as by the physical limitation for the wave steepness. These boundaries are identified in the legend of figure 3.9.

Every flap is equipped with an ultrasonic wave probe which readings are used for absorption. The absorption performance varies according to the wave frequency, from about 95% for waves close to 0.5Hz to approximately 85% for 2.0Hz waves, as presented in figure 3.9(b). For the tested waves, the absorption performance should be about 90%.

The upper limit of wave slopes which the image-based system is capable of measuring was presented in section 2.6. Keeping such limit in mind and aiming at producing the largest drift forces, a set of waves was defined and presented in table 3.1.

Figure 3.10: Wave tank scheme showing the direction of wave propagation with respect to the model.

Figure 3.10 shows the location of the model, the wire wave probe and the image system, as well as the direction in which the waves were generated. The incident angles presented in table 3.1 were obtained by changing the model heading while the wave direction was kept constant, as illustrated in figure 3.10.

Table 3.1: List of tested waves.

Case ID	<i>Nominal</i>					<i>Real</i>				
	Incid.	Model Scale		Real Scale		Incid.	Model Scale		Real Scale	
		A	T	H	T		A	T	H	T
	(°)	(mm)	(s)	(m)	(s)	(°)	(mm)	(s)	(m)	(s)
1	270	10.00	0.65	1.40	5.44	270	8.49	0.65	1.19	5.42
2	270	7.50	0.65	1.05	5.44	270	6.45	0.65	0.90	5.43
3	270	5.00	0.65	0.70	5.44	270	4.13	0.65	0.58	5.44
4	270	10.00	0.60	1.40	5.02	270	8.50	0.60	1.19	5.02
5	270	7.50	0.60	1.05	5.02	270	5.73	0.60	0.80	5.02
6	270	5.00	0.60	0.70	5.02	270	4.47	0.60	0.63	5.02
7	225	10.00	0.65	1.40	5.44	225	8.46	0.65	1.18	5.44
8	225	7.50	0.65	1.05	5.44	225	6.50	0.65	0.91	5.44
9	225	5.00	0.65	0.70	5.44	225	4.39	0.65	0.61	5.43
10	225	10.00	0.60	1.40	5.02	225	8.15	0.60	1.14	5.02
11	225	7.50	0.60	1.05	5.02	225	5.78	0.60	0.81	5.02
12	225	5.00	0.60	0.70	5.02	225	4.21	0.60	0.59	5.02
13	247.5	12.50	0.70	1.75	5.86	240.00	10.11	0.70	1.42	5.86
14	247.5	10.00	0.70	1.40	5.86	243.00	8.12	0.70	1.14	5.87
15	247.5	10.00	0.65	1.40	5.44	241.00	8.49	0.65	1.19	5.44
16	247.5	7.50	0.65	1.05	5.44	243.00	6.38	0.65	0.89	5.44
17	247.5	10.00	0.60	1.40	5.02	241.00	9.19	0.60	1.29	5.02
18	247.5	7.50	0.60	1.05	5.02	244.00	6.87	0.60	0.96	5.02
19	247.5	7.50	0.55	1.05	4.60	244.00	7.27	0.55	1.02	4.60

The *Nominal* values presented in the table are the inputs, which may be related to those values in table 2.2. On the right side of the table, the values read by the wire wave probe are presented. One may notice that these values are related to the beginning of the experiment before any reflected wave reaches the wire wave probe like shown in figure 3.11.

Waves which produce the greatest drifting forces are mainly diffracted when reaching ship model; therefore, the presence of this floating structure necessarily distorts the elevation records. Additionally, near 10% of wave amplitude is reflected by the tank walls as mentioned previously, contributing to a non-uniform elevation profile, as presented in the second highlighted region of figure 3.11. However, it is expected that the initial incident wave remains practically the same during the whole experiment, whereas the wave probe will register amplitude fluctuations due to the interaction with the reflected waves. Such effect was mainly observed in experiments in which wave reach the model transversely (270° or *beam seas*).

Figure 3.11: Amplitude profile measured using a wire wave probe placed near the image-based wave measurement system focus area. The time series presented is related to **CASE ID 1**.

By changing the ship heading (*Yaw* angle) to 45° (wave incident angle to 225°), the amplitude of the diffracted waves at the wave probe position is significantly reduced and the overall wave amplitude profile become uniform. The complete set of wave elevation records is presented in Appendix D.

The WFF experiments were done with a 1:70 scale model of a PSV ship which particulars are presented in table 3.2. The model received appropriate ballast to reproduce the weight and moments of inertia of a real vessel, and these properties were verified using bifilar suspension technique.

The model has two steerable thrusters at the stern and a tunnel thruster at the bow as shown in figure 3.3. Previous experiments demonstrated that the performance of the DP, considering the particular kind of test we carried out where waves come from a single direction at a time, would be increased if its azimuth angle was kept constant, figure 3.3 left.

The maximum thrust force produced by the steerable thrusters was 1.13N whereas the bow thruster could deliver 0.61N maximum force. In full scale, these forces correspond to 2500KW thrusters, considering the typical efficiency of 0.18kN/kW and 0.14kN/kW for the steerable and bow thrusters respectively.

Table 3.2: Ship model characteristics.

	Model Scale (1:70)	Real Scale
LOA (<i>Length Overall</i>)	1.279m	89.5m
LBP (<i>Length Between Perpendiculars</i>)	1.197m	83.8m
B (<i>Beam</i>)	0.228m	16m
d (<i>Draft</i>)	0.089m	6.2m
D (<i>Depth</i>)	0.111m	7.8m
Displacement (<i>Fresh Water</i>)	20.18kg	6923.8ton
LCG (<i>Longitudinal posit. Center of Gravity</i>)	-0.030m	-2.12m
VCG (<i>Vertical posit. Center of Gravity</i>)	0.069m	4.82m
I_{xx}	0.145(kg.m ²)	2.44×10^5 (ton.m ²)
I_{yy}	1.797(kg.m ²)	3.02×10^6 (ton.m ²)
I_{zz}	1.815(kg.m ²)	3.05×10^6 (ton.m ²)
GM (<i>Metacentric Height</i>)	0.027m	1.9m

Table 3.3: Thrusters details.

The position of the model was monitored in real time using a commercial optical system (Qualisys®). This system uses stereoscopic approach to tracking reflexive targets stick to the body which movements want to be registered. In order to isolate the targets from the rest of the image, these objects are IR reflective, and the camera system uses IR-pass filters. A total of four reflective targets were stuck to ship model, and three cameras were used for the experiments described herein.

All the control processing was made by a PC and the commands to the thruster were transmitted via radio to the ship model.

As presented in section 3.3, the gains of the controller were estimated using polo placement technique. For both the PID and PD controllers tests, the natural frequency ω_n of the system was chosen to produce similar proportional gains Kp . Additionally, the gains for yaw movements were increased in order to grant reduced heading variations during the experiments. Therefore, assuming $\alpha^{PID} = 1$, $\zeta_c^{PID} = 1$, $\omega_n^{PID} = 0.24rad/s$ and $\zeta_c^{PD} = 1$, $\omega_n^{PD} = 0.80rad/s$, for the translational degrees of freedom and $\omega_n^{PID} = 0.40rad/s$, $\omega_n^{PD} = 0.69rad/s$ for yaw, the calculated gains are presented in table 3.4.

Table 3.4: Gains of the PID and PD controllers.

Gain)	Surge	Sway	Yaw
Kp^{PID} (<i>Proportional</i>)	3.65 (N/m)	6.64 (N/m)	1.94 (N.m)
Ki^{PID} (<i>Integral</i>)	0.29 (N/m.s)	0.52 (N/m.s)	0.26 (N.m/s)
Kd^{PID} (<i>Differential</i>)	15.53 (N.s/m)	28.24 (N.s/m)	4.84 (N.m.s)
Kp^{PD} (<i>Proportional</i>)	3.52 (N/m)	6.40 (N/m)	1.92(N.m)
Kd^{PD} (<i>Differential</i>)	17.60 (N.s/m)	32.00 (N.s/m)	5.56 (N.m.s)

In order to verify the performance of the controller, the dynamic model presented in section 3.3 was compared to experimental results. Tests with step inputs were carried out and the results are presented in figures 3.12 and 3.13. Step changes on set-points of 0.3m, 0.3m and 15° were applied during one minute on x, y and around z axis respectively, according to the frame presented in figure 3.4.

(a)

(b)

(c)

Figure 3.12: Comparisons between the dynamic model and the experimental behaviour of the system with PID controller.

(a)

(b)

(c)

Figure 3.13: Comparisons between the dynamic model and the experimental behaviour of the system with PD controller.

Good agreement was verified between the dynamic model and the experimental results in both cases and similar stabilization time was observed in both PID and PD controllers.

Note that the PD controller does not present overshooting for the same stabilization time. However, this control method will present steady error in the presence of external disturbance if no feed-forward system is implemented.

Figure 3.14 shows a picture of the experimental setup, identifying all the measurements

devices as well as the model position and the wave directions. All the 19 waves presented (cases) in table 3.1 were repeated for the PID controller, PD controller and PD controller with the image-based wave feed forward system (PD+WFF). Except for the first case, which took five minutes long, the duration of the other tests was two and a half minutes.

Figure 3.14: Experimental Setup.

Finally, figure 3.15 summarizes all the distinct systems used during the tests and the flow of information. A total of four independent computers was applied for the experiments, besides the wave generation system.

Before the start of wave generation strokes, the DP was engaged, using the feed-back positioning system, to move the ship model to the initial position, including the required heading. Then, the tank wave generation system was activated and the water elevations registered by the wire wave probe. During the experiments including the WFF, information of the wave amplitude and period were sent to the DP control computer once per second via TCP/IP communication.

Figure 3.15: Flow of information during the experiment.

3.6 Experimental Results

In the present section, the main results of the 19 experiments integrating the image-based wave measurement to the feed-forward of the DP system are presented and compared. In order to allow the direct comparison between the experiments, the mean values are used. However, the full set of time series recorded during the experiments are presented in Appendix D.

As illustrated in figure 3.15, each experiment employed three distinct acquisition/control systems, namely: the wire wave probe record, the DP system record and the wave measurements produced by the image-based system. All the records were synchronized to ease the comparison, as exemplified in figure 3.16.

Figure 3.16: Example of synchronized time series (**CASE ID 5**) and the samples used to calculate the mean values. From the top to the bottom: the wire wave probe elevation record, the wave amplitude estimated by the image-based system, the real sway force delivered by the control system and the records of the offsets.

Once defined the beginning of the record, when the waves reach the model, a sample was defined (between 120s and 150s time length) in which the mean value was calculated. As previously mentioned, a shorter sample was used to define the incident wave amplitude which was extracted from the beginning of the wire wave probe record (see the graph at the top of figure 3.16), before any reflected wave reaches the sensor. Additionally, both the wire wave probe record and the image-based measurement were synchronized with the DP data, even though these two measurement systems were placed ahead of the model and thus the waves reached them first (the two graphs at the top compared with the two graphs at the bottom of figure 3.16).

Some experiments presented oscillatory behaviours instead of the constant offset, figure 3.17. Even though, the same strategy was used to calculate the mean values.

Figure 3.17: Example of experiments in which sway offset presented distinct behaviours. On the top of the figure is presented the offset time series of experiment **11**, which presented uniform displacements. On the bottom the time series of experiment **2**, which offsets presented oscillatory behaviour. In both cases the mean values were calculated in the same way and are represented by continuous black lines.

Three controllers were evaluated for each wave case, namely: PID, PD and PD+WFF. Since the mean drift force is constant, the integral term of PID controller should eliminate the position error while the PD controller shall present a steady position error in the presence of a disturbing force, figure 3.18. The addition of the WFF should eliminate, partially or totally, the position error of the PD controller, although a initial positive error is verified since the system anticipates the wave forces.

Figure 3.18: Example of the differences on the three control methods used in the experiments.

Although the major offsets are expected to be related to the sway DOF, the model also presented movements in surge and yaw. The footprint graph, exemplified in figure 3.19, shows the complete trajectory of the model during the experiment for the three control methods. It is possible to observe in the figure that the mean position of the model with PD+WFF control is closer to the mean position of the experiment using PID control.

Figure 3.19: Displacement on the plane (footprint) of experiments with the three distinct control methods. This data is related to **CASE ID 11**

The trajectory is related to the center of gravity of the ship model with respect to a fixed global reference frame and the cross markers in the figure represent the mean position of the records. All the footprint graphs are presented in Appendix D.

The following sections present the results and comparisons considering the mean values of the time series.

3.6.1 Wave Parameters Estimation

The wave amplitude and period were estimated using the image-based system during experiments with PD+WFF control method. These parameters were recorded and the mean values are presented in table 3.5.

Table 3.5: Mean values of wave parameters during experiments.

CASE	Wire WP	Image-based	Error	Wire WP	Image-based	Error
ID	A (mm)	A (mm)	(%)	T (s)	T(s)	(%)
1	8.49	8.05	-5.26%	0.65	0.58	-10.15%
2	6.45	8.11	25.79%	0.65	0.61	-5.97%
3	4.13	5.08	22.81%	0.65	0.58	-10.29%
4	8.50	7.81	-8.07%	0.60	0.55	-9.10%
5	5.73	7.26	26.64%	0.60	0.56	-6.00%
6	4.47	5.44	21.71%	0.60	0.58	-3.71%
7	8.46	9.35	10.62%	0.65	0.63	-2.84%
8	6.50	6.56	0.99%	0.65	0.62	-4.31%
9	4.39	4.35	-0.82%	0.65	0.60	-7.71%
10	8.15	6.80	-16.54%	0.60	0.56	-6.91%
11	5.78	6.06	4.84%	0.60	0.56	-6.33%
12	4.21	4.12	-2.18%	0.60	0.56	-6.31%
13	10.11	10.97	8.50%	0.70	0.66	-5.83%
14	8.12	10.35	27.47%	0.70	0.67	-4.69%
15	8.49	10.18	19.88%	0.65	0.63	-3.07%
16	6.38	8.20	28.63%	0.65	0.63	-2.50%
17	9.19	7.98	-13.15%	0.60	0.54	-9.45%
18	6.87	7.89	14.82%	0.60	0.56	-6.01%
19	7.27	7.47	2.64%	0.55	0.53	-3.19%

Errors up to 29% were verified for the amplitude estimate, corresponding to absolute values of approximately 2mm. Wave periods were systematically underestimated and larger error was 10.29%, corresponding to 0.2s.

3.6.2 Forces Estimation

Forces and moments were estimated during the experiments using the amplitude and period information obtained from the image-based measurement system and the curves of nondimensional coefficients of mean drift forces, previously calculated, presented in figure 3.8. Additionally, it was kept records of the total forces and moments employed by the DP system. Comparisons between the estimated forces and moments and those applied by the DP system, identified as *Real*, are presented in tables 3.6, 3.7 and 3.8. In the last column of the tables, it is given the force estimate based on the mean value of the wire wave probe readings. All the forces and moments presented in the tables are mean values.

Table 3.6: Mean sway forces.

CASE ID	Incid. ($^{\circ}$)	Wire WP A (mm)	Wire WP T (s)	Real F. <i>sway</i> (N)	Image WFF F. <i>sway</i> (N)	Wire WP F. <i>sway</i> (N)
1	270	8.49	0.65	0.34	0.3	0.36
2	270	6.45	0.65	0.24	0.29	0.21
3	270	4.13	0.65	0.12	0.09	0.09
4	270	8.50	0.60	0.27	0.31	0.36
5	270	5.73	0.60	0.21	0.26	0.16
6	270	4.47	0.60	-	-	-
7	225	8.46	0.65	0.30	0.21	0.20
8	225	6.50	0.65	-	-	-
9	225	4.39	0.65	0.07	0.06	0.05
10	225	8.15	0.60	0.30	0.14	0.21
11	225	5.78	0.60	0.19	0.12	0.11
12	225	4.21	0.60	0.08	0.06	0.08
13	240	10.11	0.70	0.85	0.32	0.25
14	243	8.12	0.70	0.58	0.28	0.17
15	241	8.49	0.65	0.70	0.34	0.26
16	243	6.38	0.65	0.41	0.23	0.15
17	241	9.19	0.60	0.65	0.25	0.34
18	244	6.87	0.60	0.37	0.24	0.19
19	244	7.27	0.55	0.36	0.22	0.23

The missing values in the table are related to bad experiments, which time series are presented in Appendix D. In these experiments, some of the systems presented a problem or stopped working, producing truncated signal or complete loss of position.

Even under regular waves, the magnitude of drift forces are small and consistent records could only be observed on the sway degree of freedom. It is possible to find better agreement between the estimated and real forces for the experiments with incident wave angle of 270° and 245° . Additionally, in most cases the image-based system underestimated the force, but the same was observed when such estimates were based on the wire wave probe readings.

The image-based method presented an average deviation about the real forces of 28% and when using the wire wave probe, this difference is reduced to 18%.

Table 3.7: Mean surge forces.

CASE	Incid.	Wire WP	Wire WP	Real	Image WFF	Wire WP
ID	(°)	A (mm)	T (s)	F. <i>surge</i> (N)	F. <i>surge</i> (N)	F. <i>surge</i> (N)
1	270	8.49	0.65	0.03	0.01	0.02
2	270	6.45	0.65	0.04	0.01	0.01
3	270	4.13	0.65	0.02	0.00	0.01
4	270	8.50	0.60	0.03	0.01	0.02
5	270	5.73	0.60	0.03	0.01	0.01
6	270	4.47	0.60	-	-	-
7	225	8.46	0.65	0.01	0.05	0.05
8	225	6.50	0.65	-	-	-
9	225	4.39	0.65	0.00	0.01	0.01
10	225	8.15	0.60	0.02	0.03	0.04
11	225	5.78	0.60	0.00	0.02	0.02
12	225	4.21	0.60	0.00	0.01	0.01
13	240	10.11	0.70	0.05	0.08	0.05
14	243	8.12	0.70	0.09	0.07	0.07
15	241	8.49	0.65	0.05	0.08	0.05
16	243	6.38	0.65	0.06	0.05	0.06
17	241	9.19	0.60	0.02	0.04	0.04
18	244	6.87	0.60	0.04	0.04	0.07
19	244	7.27	0.55	0.00	0.03	0.04

Table 3.8: Mean yaw moments.

CASE	Incid.	Wire WP	Wire WP	Real	Image WFF	Wire WP
ID	($^{\circ}$)	A (mm)	T (s)	M. <i>yaw</i> (N.m)	M. <i>yaw</i> (N.m)	M. <i>yaw</i> (N.m)
1	270	8.49	0.65	0.02	0.01	0.01
2	270	6.45	0.65	0.01	0.01	0.01
3	270	4.13	0.65	0.01	0.00	0.00
4	270	8.50	0.60	0.05	0.01	0.01
5	270	5.73	0.60	0.05	0.01	0.01
6	270	4.47	0.60	-	-	-
7	225	8.46	0.65	0.02	0.01	0.01
8	225	6.50	0.65	-	-	-
9	225	4.39	0.65	0.00	0.00	0.00
10	225	8.15	0.60	0.02	0.00	0.00
11	225	5.78	0.60	0.00	0.00	0.00
12	225	4.21	0.60	0.00	0.00	0.00
13	240	10.11	0.70	0.23	0.02	0.02
14	243	8.12	0.70	0.19	0.02	0.01
15	241	8.49	0.65	0.21	0.02	0.02
16	243	6.38	0.65	0.08	0.01	0.01
17	241	9.19	0.60	0.16	0.01	0.02
18	244	6.87	0.60	0.10	0.02	0.01
19	244	7.27	0.55	0.10	0.01	0.01

Figure 3.20 presents the comparison between the real force and the estimates produced by the image-based method. Lines representing the direct relation were plotted on the graphs to make easier the evaluation of the results. As previously discussed, no relation was observed for surge and yaw, but a correlation was verified for the sway force, particularly for the experiments which model headings were 270° and 225° .

Figure 3.20: Comparison between the estimate and real forces. (a) surge force, (b) sway force and (c) yaw moment.

3.6.3 Offsets

In this section, the position errors are presented. Only the sway is presented herein because, similarly to the forces, displacements on the surge and yaw are very small. However, the complete set of results can be found in Appendix D.

The results were organized according to the wave incident angle, thus, figure 3.21(a) present results of experiments with waves coming from 270° (beam seas), 3.21(b) present

results of experiments with waves coming from 225° and 3.21(c) the waves incident angle of 247.5° .

(a)

(b)

(c)

Figure 3.21: Mean sway offsets, (a).

All the experiments presented an offset reduction due to the WFF, which ranges from 40% to 96%. Table 3.9 contains the results shown in figure 3.21 and the respective offset reduction.

Table 3.9: Mean sway offsets for the PID, PD and PD+WFF controller.

CASE ID	PID offset	PD offset	PD+WFF offset	Offset Reduction
1	0.00	58.04	18.69	68%
2	0.96	39.18	18.76	52%
3	2.09	18.37	1.92	90%
4	0.26	44.60	1.93	96%
5	3.17	34.44	14.22	59%
6	-	-	-	-
7	2.20	45.77	15.43	66%
8	-	-	-	-
9	0.43	10.07	3.40	66%
10	3.82	44.53	25.11	44%
11	2.13	27.74	8.48	69%
12	1.12	11.81	2.11	82%
13	8.88	137.83	83.06	40%
14	5.06	92.77	39.38	58%
15	11.15	106.13	51.04	52%
16	4.91	63.15	23.76	62%
17	6.49	107.06	60.08	44%
18	4.45	56.26	20.64	63%
19	3.77	54.87	17.65	68%

3.6.4 Results Analysis

In this final section a comparison between the errors observed in each of the previous sections, from the image-based measurement to the model DP offset, is presented. Percent errors were calculated using the module of the difference between the estimated and paradigm divided by this last quantity. The paradigms, or reference values, are the wire wave probe measurement, the DP force records and the offset observed in the experiments with the PD controller respectively. All the values were extracted from tables 3.5, 3.6 and 3.9 and the errors are summarized in table 3.10.

Table 3.10: Mean errors between the estimates and the reference values of wave amplitude, drift forces and offsets.

CASE ID	Heading (°)	Wave. A Error (%)	Sway F Error (%)	Offset Error (%)
1	270	5.26%	18.92%	67.80%
2	270	25.79%	16.00%	52.11%
3	270	22.81%	25.00%	89.55%
4	270	8.07%	6.90%	95.67%
5	270	26.64%	18.18%	58.70%
6	270	-	-	-
7	225	10.62%	27.59%	66.29%
8	225	-	-	-
9	225	0.82%	0.00%	66.23%
10	225	16.54%	51.72%	43.62%
11	225	4.84%	33.33%	69.42%
12	225	2.18%	25.00%	82.1%
13	≈240	8.50%	63.64%	39.74%
14	≈240	27.47%	52.54%	57.55%
15	≈240	19.88%	50.00%	51.90%
16	≈240	28.63%	42.50%	62.38%
17	≈240	13.15%	63.77%	43.88%
18	≈240	14.82%	33.33%	63.32%
19	≈240	2.64%	37.14%	67.84%

Initially, the wave amplitude estimate should be compared to the force estimate. Equation 3.19 presents the relation between the error of quantities Q and x if they are related by a power function of exact exponent n such as $Q = x^n$.

$$\frac{\delta Q}{|Q|} = |n| \frac{\delta x}{|x|} \quad (3.19)$$

The drift force is proportional to the square of the wave amplitude, see section 3.4. Thus a percent error in the wave amplitude produces a doubled percent error in the force.

Figure 3.22 presents the scatter of the errors on the estimated amplitude and forces. The experiments were grouped by their headings and the theoretical relation between the errors was also plotted in the figure as a continuous line.

Figure 3.22: Comparison between Amplitude estimates errors and Sway drift forces errors.

Poor correlation was observed between the errors and the theoretical relation, since the maximum $r^2 = 0.3$, calculated for experiments with wave incident angle of 225° . Such result indicates that other aspects rule the definition of the magnitude of errors on forces estimates than the wave amplitude measurement.

By assessing the yaw time series of the experiments, it is possible to notice that the tests with incident angle of approximately 240° presented greater fluctuation in heading, figure 3.23. Additionally, the larger errors were verified for the longer wave periods, which were 0.7s and 0.65s in model scale (5.9s and 5.4s in real scale). As shown in figure 3.23, much greater mean yaw moments are expected at this range of wave periods for incidences close to 240° . This fact may explains the difficult of the DP system in keeping the model heading.

Figure 3.23: Normalized mean yaw curve for the three incident wave angles evaluates. In gray the range of wave periods tested. On the right the yaw time series related to each incident angles.

Fluctuations in the incident angle may produce errors on force estimates, particularly when mean values are used.

The errors in the wave period measurements, particularly for the larger periods previously mentioned, also produce erroneous force estimate. It is possible to verify in figure 3.24 that the curves present quite steady behaviours up to 5.2s, but significant changes for larger values. Therefore, by underestimating the wave period by 10% near 5.8s, the sway force modulus may present errors up to 30%.

Figure 3.24: Normalized sway drift force curves. The range of wave periods evaluated was defined by the vertical lines.

Additional sources of error are errors on the numerical calculation of the mean drift forces curves.

Figure 3.25 shows the comparison between the errors on force estimates and the offset reduction. A direct relation between the drift force and the offset is expected since these quantities are related by the proportional gain of the controller, which is constant. As the previous figure, a weak correlation was observed, denoting the existence of additional sources of error. The presence of reflected waves may change the resultant force on the ship model, leading to the increase of offset errors.

Figure 3.25: Comparison between Amplitude estimates errors and Sway drift forces errors.

In conclusion, although appreciable reductions on the model offsets were observed it is not possible to precisely identify the major sources of error just considering the monitored parameters. Besides the possible reasons for deviations discussed above, the presence of reflected waves acting on the model are probably important sources of error. This cause may be mitigated by placing the model in the centre of the wave tank, as far as possible from the walls, or carrying out experiments in a larger tank.

Chapter 4

Conclusions

The DP systems currently available employ a feedback controller based on the position and heading error, associated with a wind feed-forward control action. This last technology improves the efficiency of the DP by anticipating wind loads. However, there is no consolidated technology to feed the controller with wave loads information and the major issue is related to the wave measurement process. We demonstrated here using model scale experiments that the computer vision is a promising approach to enable the wave feed-forward in DP systems.

A review of the main image-based wave measurement techniques was presented, pointing its advantages and limitations. It was discussed that, although the stereoscopic techniques have been successfully applied to reconstruct irregular ocean surface for more than a decade now, this class of algorithms relies not only on the Lambertian surface hypothesis, which we emulated by defining a particular experimental set-up but also on the presence of surface textures. Thus, these techniques could not be used in a wave feed-forward scale model test. In order to make the experiment possible, intensity-based algorithms were explored, particularly the so-called shape from reflection. Such method does not require the solution of the correspondence problem and, as a matter of fact can be implemented with a single camera. However, it does need a uniform source of light. This condition was satisfied by carefully defining a range of waves as well as the camera position.

Experiments were carried out to evaluate the performance of the image-based measurement method and to define a range of waves which parameters could be estimated in further tests with the DP system. The results presented some bias on the wire wave probe measurement. However, the image based methods exhibited linear relation on it, as well as scale factors close to one. These facts indicate that the applied reflectance model can indeed reproduce the optical characteristics of indoor water waves. Satisfactory results were verified for wave steepness up to 3%, which comprises the linear wave

theory. Additionally, some effort was made to make the algorithm expedite and allow its integration with the control of DP system.

The idea behind the feed forward is simple, the forces and moments estimates are subtracted from those calculated by the controller. Therefore, the focus was given at force estimation which requires, not only the wave parameters measurement but also the calculation of hydrodynamic of the floating body. These forces are second order effects, which magnitudes are much lower than those related to the wave frequency forces but still capable of producing noticeable position errors on a DP vessel. Forces and moments coefficients were previously calculated for a PSV ship geometry using boundary elements method and stored in a database.

An experiment with a ship model equipped with steerable thrusters and DP control was proposed, which incorporates the hydrodynamic forces estimation using the image-based system. In order to verify the performance of the wave feed-forward, a PID, a PD and a PD controller with the wave feed-forward were evaluated for each wave condition. Since regular waves produce constant drift forces, a well-tuned PID controller shall present no position error after some time whereas the PD controller should show some offset. The introduction of the WFF aimed at reducing the steady error of the PD controller. Such reduction should be as good as the estimate of incoming waves and the forces induced by them. Experiments conducted in this research presented reductions between 40% and 96% of the sway position error when incorporating the WFF, demonstrating the potential of the method.

Many issues still remain pending before the proposed concept becomes a useful technology. The major limitation is that the method, as proposed, is only capable of dealing with regular waves, which represents a small aspect of maritime and ocean research as well as no considerable obstacle for the real PID DP systems. The assumption of a regular wave was necessary in two steps of the method derivation. First to allow a direct relationship between the maximum wave slope and the wave amplitude. Second to allow a precise determination of the piece of the surrounding scenario that would be reflected by that particular patch of the water surface. These are two issues mostly related to indoor experiments since under an open sky the uniform light source hypothesis would be valid. Under a large extended uniform light source, the surface could be recovered either by solving the PDE defined by the reflectance map proposed herein, or by stereoscopic means.

A second important limitation is the fact that the camera rig was fixed, outside the ship or onshore. In a real situation, it is evident that the camera system would be on the ship and move with it. In order to deal with this issue, the movements of the ship would have to be known and the transformations discussed in section 2.4 updated in real time.

Many ships are equipped with an IMU (Inertial Measurement Unit) capable of measuring movements in 6DOF and, therefore, it is possible to make corrections on the projective transformations. A third limitation is the fact that any stationkeeping system must work 24 hours and, thus operates during the night. Tests with infra-red cameras would be necessary to evaluate the amount of light available and if its variation would be enough to describe a large range of slopes.

Finally, the water elevation near the vessel is a superposition of incoming, diffracted and irradiated waves, therefore, a more elaborated hydrodynamic analysis would be necessary to increase the effectiveness of the proposed method.

4.1 Future Works

Based on the conclusions of this thesis, the following tasks are considered relevant future works for the improving of the method and its applicability:

- More experiments with the model placed at the centre of the wave tank would provide better information regarding the effect of reflected waves on the performance of the system. Additionally, by applying camera lenses with wider view angle, it would be possible to evaluate larger wave periods and consequently distinct hydrodynamic responses. All these actions would allow the precise identification of sources of error.
- The image method presented herein relies on several hypotheses which, in the future, could be generalized or even eliminated. Regarding the mathematical model, it was assumed a linear relation between the surface radiance L and the image irradiance I . The constant C not only incorporated the intensity of the uniform extended light source but also any losses related to the lenses or electronic imaging process. For instance, the CCDs are not perfectly linear on the brightness changes and this kind of effect should be incorporated in the model. Furthermore, the white wall, which was assumed to be an extended uniform light source, is a diffuse surface that receives light from several point sources. Therefore, fluctuations of the brightness intensity are expected and should be incorporated in the mathematical model. These are some actions which can improve the precision and the robustness of the model.
- It is important to improve the shape from reflection technique used in the present work, to make it capable of reconstructing irregular waves surface. Since the major issue regarding such method is the presence of a large extended light source, experiments should be carried out with a curved white screen close to the water. It would not be necessary to cover a large are of the tank facility but a narrow region

above the analysis area on the water surface. Such screen would allow the camera to be placed higher and thus register larger wave slopes, up to the limit where all possible surface orientations could be measured. However, it must be noticed that there is a trade-off between the maximum wave slope measured and the amount of light reflected since the reflectivity coefficient is drastically reduced when the optical axis of the camera gets closer to the normal of the water surface.

- Finally, in order to produce a useful technology, the whole system would have to be mounted on the moving vessel. Therefore, the image algorithm should be adapted to consider "moving extrinsic parameters". These parameters would be updated using information from a movement sensor installed on the ship, such as an IMU.

References

AALBERS, A.; NIENHUIS, U. Wave direction feed-forward on basis of relative motion measurements to improve dynamic positioning performance. In: **19th Annual Offshore Technology Conference**. Houston, Texas: [s.n.], April 1987.

AALBERS, A.; TAP, R.; PINKSTER, J. An application of dynamic positioning control using wave feed forward. **International Journal of Robust and Nonlinear Control**, v. 11, p. 1207–1237, August 2001.

AALBERS, A.; WAALS, O.; TAP, R. Wave feed forward dp and analysis of the effect on shuttle tanker operation. In: **Dynamic Positioning Conference**. September 2004.

BENETAZZO, A. Measurement of short water waves using stereo matched image sequences. **Coastal Engineering**, v. 53, p. 1013–1032, 2006.

BENETAZZO, A.; FEDELE, F.; GALLEGU, G.; SHIH, P.; YEZZI, A. Offshore stereo measurement of gravity waves. **Coastal Engineering**, v. 64, p. 127–138, 2012.

BERG, M.; CHEONG O. ABD KREVELD, M.; OVERMARS, M. **Computational Geometry: Algorithms and Applications**. : Springer, 2008.

BOUGUET, J. **Camera Calibration Toolbox for Matlab**. 2016. Available from: http://www.vision.caltech.edu/bouguetj/calib_doc/.

BRAY, D. **Dynamic Positioning**. : Oilfield Publications Ltd. (OPL), 1998. (The Oilfield Seamanship Series, v. 9).

CHASE, J.; COTE, L.; MARKS, W.; MEHR, E.; PIERSON, W.; RONNE, F.; STEPHENSON, G.; VETTER, R.; WALDEN, G. **The Disrectional Spectrum of a Wind Generated Sea: As determined from data obtained by the Stereo Wave**

Observation Project. : New York College of Engineering: Department of Meteorology and Oceanography and Engineering Statistics Group, (1957).

FALTINSEN, O. **Sea Loads on Ships and Offshore Structures.** : Cambridge University Press, 1998.

FAUGERAS, O. **Three-Dimensional Computer Vision: A Geometry Viewpoint.** : The MIT Press, 1993.

FUSIELLO, A.; IRSARA, L. Quasi-euclidean uncalibrated epipolar rectification. In: **19th International Conference on Pattern Recognition, 2008. ICPR 2008.** Tampa, FL: [s.n.], December 2008.

FUSIELLO, A.; TRUCCO, E.; VERRI, A. A compact algorithm for rectification of stereo pairs. **Machine Vision and Application**, v. 12, p. 16–22, 2000.

GALLEGO, G. **Variational Image Processing Algorithms for the StereoStereo Space-Time Reconstruction of Water Waves.** 2011. Thesis (PhD) — Georgia Institute of Technology, 2011.

GALLEGO, G.; BENETAZZO, A.; YEZZI, A.; FEDELE, F. Wave statistics and spectra via a variational wave acquisition stereo system. In: **Proceedings of ASME 27th International Conference on Offshore Mechanics and Arctic Engineering, June 15-20, 2008, Estoril, Portugal.** 2008.

GOMES, M. **Recuperação de Informações Tridimensionais a partir de Múltiplas Imagens.** 2012. Dissertation (Master) — Universidade Estadual de Campinas - Faculdade de Engenharia Elétrica e de Computação - Departamento de Computação e Automação Industrial, 2012.

GONZALEZ, R.; WOODS, R. **Digital Image Processing.** : Prentice Hall, 2008.

GRACIAN, A.; MULLER, J.-P.; CHELLENOR, P.; SROKOSZ, M. Automated 3d spatial measurement of short gravity-capillary waves. **International Society for Photogrammetry and Remote Sensing**, v. 29, p. 667–673, 1992.

HARTLEY, R.; ZISSERMAN, A. **Multiple View Geometry in computer vision.** : Cambridge, 2003.

HAVELOCK, T. The pressure of water waves upon a fixed ostacle. **Proceedings of Royal Sociery of London. Series A**, v. 1750, p. 409–421, 1940.

HAVELOCK, T. Xlvii. the drifting force on a ship among waves. **Philosophical Maga-zine**, v. 33, n. 221, p. 467–475, 1942. Philosophical Magazine Series 7, Published online: 29 Apr 2009.

HORN. **Shape from Shading: A Method for Obtaining the shape of a Smooth Opaque Object from one View.** : Massachusetts Institute of Technology, November, (1970. 232).

HORN, B. K. P.; SJOBERG, R. W. Calculating the reflectance map. **Appied Optics**, v. 18, n. 11, p. 1770 – 1779, June 1979.

HSU, F.; BLENKARN, K. Analysis of peak mooring force caused by slow vessel drift oscillation in random seas. In: **Offshore Technology Conference**. 6200 North Central Expressway, Dallas, Texas 75206: [s.n.], p. 135–146, April 1970.

HUGHES, G.; HARRIS, R.; QUADVLIEG, F.; HALLMANN, R. Close-in precision dp using wave feed forward stlvast phase 2 & 3. In: **Dynamic Positioning Conference**. October 2009.

ISHERWOOD, R. Wind resistance of merchant ships. **Proceedings of Royal Institute of Naval**, v. 115, p. 327–338, 1973.

JäHNE, B. Spatio-temporal image processing. **Lecture Notes in Computer Science**, v. 751, p. 205, 1993.

JäHNE, B.; WAAS, S.; KLINKE, J. A critical theoretical review of optical techniques for short ocean wave measurements. In: **Proc. SPIE 1749, Optics of the Air-Sea Interface.** : SPIE, v. 1749, p. 204 – 215, December 1992.

KNOOP, J.; TAYLOR, R. Real time estimation of waves and drift forces. **International Journal of Offshore and Polar Engineering**, v. 1, n. 1, p. 1–7, March 1991.

KOSCHAN, A. A framework for area based and feature based stereo vision. **Machine Graphics & Vision**, v. 2, p. 285–308, 1993.

LAVIERI, R. S.; TANNURI, E. A. Image based method to estimate regular waves parameters. In: **ASME. Proceedings of the ASME 2016 35th Int. Conf. on Ocean, Offshore and Arctic Engineering**. 2016.

LINFOOT, B.; WRIGHT, V. Wave feedforfeed control of floating production systems. **Control Engineering Prattice**, v. 3, n. 5, p. 687–694, January 1995.

MA, Y.; SOATTO, S.; KOSECKA, J.; SASTRY, S. **An Invitation to 3-D Vision: From Iimage to Geometric Models**. : Springer, 2004.

MARUO, H. The drift of a body floating on waves. **Journal of Ship Research**, v. 4, n. 3, p. 1–10, December 1960.

MELLO, P. C. **SISTEMA DE AUTOMAÇÃO E CONTROLE PARA TANQUES OCEÂNICOS COM MULTIPLOS ATUADORES**. 2012. Thesis (PhD) — Escola Politécnica da Universidade de São Paulo, 2012.

MOZUMI, K.; WASEDA, T.; CHABCHOUB, A. 3D Stereo Imaging of Abnormal Waves in a Wave Basin. In: **Proceedings of the ASME 2015 34th International Conference on Ocean, Offshore and Arctic Engineering**, May 31-June 5, 2015, St. John's, Newfoundland, Canada. : ASME, May 2015.

NEWMAN, J. The drift force and moment on ship in waves. **Journal of Ship Research**, v. 11, p. 51–60, March 1967.

NEWMAN, J. Second order, slowly varying forces on vessel in irregulr waves. In: BISHOP, R.; PRICE, W. (Ed.). **Proceedings of International Symposium of Dynamics of Marine Vehicles and Structures**. : Mechanical Engineering Publications Ltd., p. 182, 1974.

NEWMAN, J. **Marine Hydrodynamics**. : MIT Press, 1977.

NIOLA, V.; ROSSI, C. **Industrial RobRobot: Programming, Simulation and Applications**. Mammendorf, Germany: pIV pro literatur Verlag Robert Mayer-Scholz, 2007.

OCEANWAVES GMBH. **WaMoS® - The Wave and Surface Current Monitoring System**. Vor dem Bardowicker Tore 6b - D-21339 Lüneburg - Germany. Available from: <http://www.oceanwaves.de/start.html>.

PETTY, G. **A First Course in Atmospheric Radiation.** : Sundog Publishing, 2006.

PINKSTER, J. Wave feed-forward as a means to improve dynamic positioning. In: **Offshore Technology Conference.** 1978.

PINKSTER, J. Mean and low frequency wav drifting forces on floating structures. **Ocean Engineering**, v. 6, p. 593–615, 1979.

PINKSTER, J. **Low Frequency Second Order Wave Exciting Forces on Floating Structures.** 1980. Thesis (PhD) — T.U.Delft, 1980.

QUADVLIEG, F.; HALLMANN, R.; HUGHES, G.; HARRIS, R. Improved dynamic positioning using wave feed forward. In: ASME. **Proceedings of the ASME 2011 30th International Conference on Ocean, Offshore and Arctic Engineering, OMAE2011.** Rotterdam, The Netherlands, June 2011.

RAPIZO, H.; D'ÁVILA, V.; CARVALHO N., V.; PINHO, U.; PARENTE, E. Identification of Period and Direction of Monochromatic Waves Through the Processing of Optical Images. **Anuário do Instituto de Geociências - UFRJ**, v. 37, p. 36 – 45, 2014.

REMERY, G.; HERMANS, A. The slow drift oscillations of moored object in random seas. In: **Offshore Technology Conference.** 6200 North Central Expressway, Dallas, Texas: [s.n.], April 1971.

SCHLICK, C. An inexpensive BRDF model for physically-based rendering. In: **Proc. Eurographics 94, Computer Graphics Forum.** v. 13, n. 3, p. 233 – 246, 1994.

SHATTO, H. **History of DP - 2011 - The year in which Dynamic Positioning celebrated its fiftieth anniversary!** 2011. Electronic. Available from: <http://dynamic-positioning.com/history-of-dp/>.

SHEMDIN, O. H. Measurement of short surface waves with stereophotography. **Ocean Research and Engineering**, -, p. 568–571, 1990.

STEPHENS, R. Wind feedforward: blowing away the myths. In: DYNAMIC POSITIONING COMMITTEE. **Dynamic Positioning Conference.** : MTS, October 2011.

STILWELL, D. Directional energy spectra of the sea from photographs. **Jornal of Geophysical Research**, v. 74, n. 8, p. 1974–1986, April 1969.

SU, H.; HE, B. A simple rectification method of stereo image pairs with calibrated cameras. In: **Information Engineering and Computer Science (ICIECS), 2010 2nd International Conference on Information Engineering and Computer Science**. 2010.

SUYEHIRO, K. On the drift of ships caused by rolling among waves. **Transactions of INA**, v. 66, p. –, 1924. Available in RINA Transactions and Annual Report.

TANNURI, E. A.; MORISHITA, H. M. Experimental and numerical evaluation of a typical dynamic positioning system. **Applied Ocean Research**, v. 28, p. 133–146, 2006.

WAALS, O. J. **Wave Feed Forward control in Dynamic Positioning**. 2002. Dissertation (Final Report) — TUDelft, May 2002.

WAMIT. **WAMIT User Manual**. Version 7.1. 822 Boylston St. - Suite 202 Chestnut Hill, MA 02467-2504 USA, 2015.

WANEK, J.; WU, C. Automated trinocular stereo imaging system for three-dimensional surface wave measurements. **Ocean Engineering**, v. 33, p. 723–747, 2006.

WATANABE, Y. Some contribution to the theory of rolling. **Transactions of INA**, v. 80, p. 408–432, 1938.

ZHANG, Z. Flexible camera calibration by view a plane from unknown orientations. In: **The Proceedings of the Seventh IEEE International Conference on Computer Vision**. v. 2, September 1999.

Appendix A

Three-Dimensional View Geometry

This appendix briefly presents the fundamentals of perspective through the so-called pinhole camera model and introduction to two views geometry.

A.1 Pinhole Camera Model

The ideal *pinhole* camera is a dark box with a infinitesimal hole on one of its faces, figure A.1. Light beams coming from an illuminated object in the scene go inside the box, through the hole, and creates an inverted image at the opposite face. Once the hole has an infinitesimal diameter, a point on the 3D scene is related to a single point on the image plane. Although it is an idealization, a well-focused camera behaves similar and is well decied by this model or.

Figure A.1: Pinhole camera model.

The *image plane* or *retinal plane* is the plane where the image will be formed and, in practical terms is where the sensitive film or the array of sensors is. The *optical axis* is a line, normal to the image plane, crossing the image referential frame origin. Along the optical axis, at a distance f from the image plane is the *optical center* O , where the aperture of the camera is located. Distance f is called *focal length*.

In order to eliminate the inverted image, one can place the image plane in front of the optical center, at the same distance f . This frontal pinhole camera is a more common form of presenting such model, and figure A.2 illustrates it.

Figure A.2: Inverted Pinhole camera model.

Points $\mathbf{x} = [x, y]^T$ and $\mathbf{X} = [X, Y, Z]^T$, written in terms of camera reference frame, $Oxyz$, are related by the *perspective projection*, equation A.1.

$$x = f \frac{X}{Z}, \quad y = f \frac{Y}{Z} \quad \text{or}$$

$$\mathbf{x} = \begin{bmatrix} x \\ y \end{bmatrix} = \frac{f}{Z} \begin{bmatrix} X \\ Y \end{bmatrix} \quad (\text{A.1})$$

A.2 Projective space

Before further derivations, it is proper to present some concepts related to projective geometry since it provides mathematical representation for image elements. Complete explanation about the theme can be found in Hartley and Zisserman (2003) and Faugeras

(1993).

A line on Euclidean plane $\mathbb{R}^2$ can be represented by the equation $ax + by + c = 0$ or simply by its coefficients $[a, b, c]$, as a $\mathbb{R}^3$ vector. Similarly, a point on the plane $\mathbb{R}^2$, commonly described by a coordinate pair $p = [x, y]$, can also be represented by a 3D vector $\tilde{p} = [\lambda x, \lambda y, \lambda]$ since $\lambda x / \lambda = x$ and $\lambda y / \lambda = y$. This fact becomes evident if one interprets a point on the plane as the intersection of two lines, $[\lambda x, \lambda y, \lambda][a, b, c] = 0$.

This point representation is called *homogeneous* notation and the set of such vectors constitute the 2D projective space, $\mathbb{P}^2$. Usually, the most direct way to write a point $p = [x, y]$ is in a normalized homogeneous form just adding a final coordinate 1, $\tilde{p} = [x, y, 1]$.

Along this lines, a point on the plane can be represented by a whole class of vectors on space. Figure A.3 illustrates¹ this conclusion.

Figure A.3: Points with distinct depth projected on a single image point.

As a matter of fact, the projective space $\mathbb{P}^2$ is formed by vectors in $\mathbb{R}^3 - [0, 0, 0]$, since $[0, 0, 0]$ is not related to any line. This correspondence between the spaces makes projective geometry an efficient tool to deal with images of 3D scenes.

Additionally, homogeneous notation allows the representation of points at the infinite. Vector $\tilde{p} = [x, y, 0]$ is an element of projective plane but it does not have a *inhomogeneous* representation as it would be $p = [x/0, y/0]$. In fact, this is the point where two parallel lines intersect, and the image of such point is called *vanishing point*.

¹Topological models for the projective space are presented in Ma et al. (2004)

A.3 Projection Matrix

The *projection matrix*, Π , is the linear transformation that maps 3D coordinates of a point on the scene into 2D coordinates of the image plane ². Thus, it incorporates all the linear effects that characterize the camera during the imaging process ³.

Rewriting equation A.1 in homogeneous for leads to the following expression.

$$Z \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} \quad (\text{A.2})$$

While Z is the main information lost during the imaging process, it can be replaced by an arbitrary positive scalar λ . Besides, one may choose a distinct referential from the camera frame, what introduced a rigid body transformation to the previous equation, and the scene coordinates become $[X_0, Y_0, Z_0]^T$.

$$\lambda \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} R & T \\ \mathbf{0}^{1 \times 3} & 1 \end{bmatrix} \begin{bmatrix} X_0 \\ Y_0 \\ Z_0 \\ 1_0 \end{bmatrix} \quad (\text{A.3})$$

This equation describes the *ideal camera* model. Defining the three matrices as,

$$K_f \doteq \begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix} \in \mathbb{R}^{3 \times 3}, \quad \Pi_0 \doteq \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \in \mathbb{R}^{3 \times 4}, \quad g \doteq \begin{bmatrix} R & T \\ \mathbf{0}^{1 \times 3} & 1 \end{bmatrix} \in \mathbb{R}^{4 \times 4}$$

one may rewrite equation A.3.

$$\lambda \mathbf{x} = K_f \Pi_0 g \mathbf{X}_0 \quad (\text{A.4})$$

K_f is a particular case of the camera *Intrinsic Parameters* matrix, Π_0 is called *canonical* Projection matrix and g contains the *Extrinsic Parameters* of the camera.

²Actually, the transformation takes a 4D vector of $\mathbb{P}^3$ to 3D vector of $\mathbb{P}^2$.

³Nonlinear effects such as radial distortion must be corrected previously.

A.3.1 Intrinsic Parameters

The origin of this pixel array is commonly placed at the upper-left corner of the sensor plate. Additionally, pixels have dimensions and may not be perfectly square. These aspects must be taken into account once matrix K (general form of K_f) maps metric units into pixels coordinates. Equation A.5 presents matrix K , also called *calibration matrix*.

$$K \doteq \begin{bmatrix} fs_x & fs_\theta & o_x \\ 0 & fs_y & o_y \\ 0 & 0 & 1 \end{bmatrix} \quad (\text{A.5})$$

The origin of image coordinates is represented by o_x, o_y , described in pixel units. As mentioned before, pixels may not be square, and s_x and s_y characterize both the dimension in metric units and the aspect ratio of such structures, used to correct focal length horizontally and vertically. Finally, in case that pixels are not rectangular, quantity s_θ , called *skew factor*⁴, incorporates such correction. Intrinsic parameters, as well as radial distortions, are obtained through a calibration process.

A.3.2 Extrinsic Parameters

For a single camera, the extrinsic parameters describe a referential frame transformation. For a rig of two or more cameras, these parameters define the pose (position and orientation) of one camera with respect to another.

As presented in equation A.3, matrix g comprises a 3D rotation matrix $R \in \mathbb{R}^{3 \times 3}$, a 3D translation vector $T \in \mathbb{R}^{3 \times 1}$ and a row vector $[0, 0, 0, 1]$.

Assuming the global reference frame to be one of the cameras, matrix g can be recovered just using information available on the pair of images.

In conclusion, the general *perspective projection matrix*, Π , is calculated as,

$$\lambda \mathbf{x}' = K \Pi_0 g \mathbf{X}_0 = [KR, KT] \mathbf{X}_0 = \Pi \mathbf{X}_0 \quad (\text{A.6})$$

and $\mathbf{x}$ has been replaced by $\mathbf{x}'$ because this last vector is given in pixel coordinates and incorporates camera characteristics ($\mathbf{x}' = K\mathbf{x}$).

⁴Skew factor is proportional to $\cot(\theta)$, where θ is the angle between the horizontal and the vertical edge of the pixel. This angle is typically close to 90° so s_θ is close to zero.

A.4 Two Views Geometry

The previous topic presented a brief overview of the imaging process related to a single camera. Here some geometric properties of two views of the same scene will be presented. These concepts are applied both for recovering of camera poses and 3D point positions on space, as to improve the efficiency of correspondent points searching.

When a point on the scene is identified in both images, thus is described in terms two images coordinates $\mathbf{x}_1$ and $\mathbf{x}_2$, a geometric constraint is established. Once enough correspondent points are identified, a linear system can be solved, and parameters of the camera rig and the scene are recovered. The algebraic model of these constraints is presented next.

A.5 Epipolar Geometry

Epipolar geometry is the geometry between two views of the same scene and incorporates all its relations. These, mainly, raise from the fact that a plane can be defined between the two camera centers and the point in the space, the *epipolar plane*. Figure A.4 is a classical representation of this plane and also identifies key elements of Epipolar geometry.

Figure A.4: Epipolar relations.

The figure illustrates the two cameras and respective image planes as well as a point p in the scene. Point e_1 is the projection of optical center O_2 of camera 2 on image plane 1, while e_2 is the reciprocal for image plane 2. These points are called *epipoles*. The line l_1 , connecting the image point x_1 to the epipole e_1 , is called *epipolar line* and corresponds to the intersection of the epipolar plane with image plane 1. Line l_2 is just the analogous for

the image plane 2. For every point on the 3D scene there is an associated epipolar plane.

A.5.1 Epipolar Constraint and the Fundamental Matrix

Assuming the global frame at one of the camera, e.g. O_1xyz , and being (R, T) the relative pose between the cameras, the named *epipolar constraint* is given by equation A.7.

$$\langle \mathbf{x}_2, T \times R\mathbf{x}_1 \rangle = 0, \quad \text{or} \quad \mathbf{x}_2^T \hat{T} R \mathbf{x}_1 = 0 \quad (\text{A.7})$$

It is the form presented in (MA et al., 2004), and it has a simple derivation that can be found in his book. The right-hand expression in A.7 applies the hat operator that is an alternative way to express the cross product.

$$\hat{T}R = \begin{bmatrix} 0 & -t_3 & t_2 \\ t_3 & 0 & -t_1 \\ -t_2 & t_1 & 0 \end{bmatrix} \begin{bmatrix} r_{1,1} & r_{1,2} & r_{1,3} \\ r_{2,1} & r_{2,2} & r_{2,3} \\ r_{3,1} & r_{3,2} & r_{3,3} \end{bmatrix} \doteq E \quad (\text{A.8})$$

Equation A.7 is a triple product stating that vectors starting from two optical centers pointing at p and the baseline are contained in the same plane.

Matrix E , defined in A.8, condense the relation between the image points of two views that, in this case, is the relative pose between the cameras. This is called *Essential Matrix* and it belongs to a set of matrices in $\mathbb{R}^{3 \times 3}$ named *essential space*.

One may notice that the present constraint depends only on the relative pose of the cameras and not on point p . The constraint, as presented above, presumes a calibrated camera or, in practical terms, that image points have been previously multiplied by matrix K . However, in many cases, the calibration matrices of the cameras are not available. Even though, some information can be obtained from the image as the same epipolar constraint is valid, equation A.9.

$$\mathbf{x}_2'^T K^{-T} \hat{T} K R K^{-1} \mathbf{x}_1' = 0 \quad (\text{A.9})$$

The geometry of a pair of uncalibrated images is encoded by the *fundamental matrix*, defined as follows.

$$F = K^{-T} \hat{T} K R K^{-1} \quad (\text{A.10})$$

Matrix F maps image points $\mathbf{x}_1'$ onto the line $l_2 \doteq F\mathbf{x}_1'$ on the second image. This is

an epipolar line, as illustrated in figure A.4, and the relation is known as the *Epipolar Matching Lemma*. The analogous can be done for the essential matrix as it is a particular case of the fundamental matrix when $K = I$ (I representing the identity matrix).

The epipolar matching lemma is particularly useful for establishing correspondences across images as the search for features can be restricted to an epipolar line. This fact will be explored and illustrated further in Appendix C.

Once the epipolar constraint is presented, the computation of the fundamental matrix can be done just by establishing a sufficient number of correspondent points. Although $F \in \mathbb{R}^{3 \times 3}$ has nine elements, the transformation satisfies the constraint $\det F = 0$ which removes one degree of freedom. Thus, given at least eight correspondent points it is possible to solve linearly the entries of F up to a scale. A complete derivation of these constraints can be found in Gomes (2012).

Real images are corrupted by noise so the set of constraints should be solved as an error minimization problem, usually applying the least square method. Therefore, more equations (more correspondent points) lead to smaller errors⁵.

In order to perform the structure reconstruction, the relative pose (rotation and translation) of the cameras must be known. These parameters can be calculated from the Essential matrix E or, their analogous uncalibrated form can be retrieved from Fundamental matrix F .

A.6 Image Rectification

The matching lemma presented before is particularly useful to improve the speed and accuracy of the search for correspondent points on a pair of stereo images. Instead of searching over the entire second image, one may restrict the exploration over a single line. This problem can be further simplified if the images undergo a particular transformation that makes epipolar lines horizontal and collinear before the correspondence process starts. These transformations are called *Image Rectification* and are new rotations and translation that emulates another stereo rig, where both cameras differ only by their optical center position that is separated along the x axis. As cameras can be represented by their projection matrices, the rectification process can also produces a new pair of transformations Π_{n1} and Π_{n2} that maps 3D points onto the image plane.

Several image rectification algorithms are available both for calibrated or uncalibrated images. While this work deals mostly with calibrated images, algorithms such as the

⁵The system can, eventually, becomes ill-conditioned making the numerical solution difficult. Normalization techniques are required in these cases.

proposed by Su and He (2010) and Fusiello, Trucco and Verri (2000) shall be explored.

In order to make the epipolar lines parallel on the tow images, both epipoles must be mapped to infinite, just rotating both focal planes⁶ until they become parallel. At the same time, to obtain horizontal epipolar lines, one must make the baseline parallel to the new x axis of both cameras. Finally, it is desired that the y coordinate of both lines are the same, and this is achieved by making the intrinsic parameters of both cameras the same. Matrix R which gives the new camera's pose is the same for both Π_{n1} and Π_{n2} and can be written as follows.

$$R = \begin{bmatrix} \mathbf{r}_1^T \\ \mathbf{r}_2^T \\ \mathbf{r}_3^T \end{bmatrix} \quad (\text{A.11})$$

Each row of the matrix represents a vector of new reference frame, in terms of word coordinates.

$$\mathbf{r}_1 = (\mathbf{o}_1 - \mathbf{o}_2) / \|\mathbf{o}_1 - \mathbf{o}_2\|$$

$$\mathbf{r}_2 = \mathbf{k} \times \mathbf{r}_1 \quad (\text{A.12})$$

$$\mathbf{r}_3 = \mathbf{r}_1 \times \mathbf{r}_2$$

For the second direction calculation, one may pick an arbitrary unit vector $\mathbf{k}$, belonging to the optical plane and orthogonal to $\mathbf{r}_1^T$. Recovering equation A.6, the new projection matrices can be written as,

$$\Pi_{n1} = K [R, -R\mathbf{o}_1], \quad \Pi_{n2} = K [R, -R\mathbf{o}_2] \quad (\text{A.13})$$

where $\mathbf{o}_1$ and $\mathbf{o}_2$ are kept the same for the new projection transformations. According to Fusiello, Trucco and Verri (2000), this algorithm fails when the baseline is parallel to the optical axis or, in other words, when the tow images are taken from a camera translating forwards.

Eventually, the rectification can be performed directly between two images. Thus a image transformation, mapping image points into another image plane, must be calculated from

⁶Plane parallel to the image plane that contains the optical center.

the new projection matrices.

Appendix B

Camera Calibration

The calibration process aims at recovering the intrinsic and extrinsic parameters of the camera. Besides the linear parameters, calibration procedures can also quantify radial distortions. Since the objective is to develop a measurement system, intrinsic parameters must be known to the reconstruction to contain metric quantities.

Several methods are available, but all require some knowledge of the scene. Ma et al. (2004), Hartley and Zisserman (2003) and Faugeras (1993) are some examples of textbooks that contain detailed explanations of calibration methods.

Following the division presented in Ma et al. (2004), a calibration can be performed based on a *partial knowledge of the scene* or by applying a *calibration rig* which can be planar or nonplanar. The most simple and popular method proposed by Zhang (1999) uses planar checkerboards, easier to build than the nonplanar calibration rigs. In fact, it is mentioned that the pattern can be printed on a laser printer and attached to a "reasonable" planar surface. A description of the main concepts concerned with the method is presented below.

Assuming that the calibration grid is planar and, without loss of generality, choosing the world reference frame attached to it, oriented as illustrated in figure B.1, one may write equation B.1,

$$\lambda \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = K[r_1, r_2, T] \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}, \quad H \doteq K[r_1, r_2, T] \quad (\text{B.1})$$

where r_1 and r_2 are the first two columns of the rotation matrix R . A point on the checkerboard at 3D homogeneous coordinates is given by $\mathbf{X} = [X, Y, 0, 1]^T$, but it can

Figure B.1: Planar pattern coordinate system.

also be described at 2D "checkerboard homogeneous coordinates" as $\mathbf{X} = [X, Y, 1]^T$. Thus, a homography¹ H can be established between a point on the pattern and the image plane. Multiplying both sides of equation B.1 by $\widehat{\mathbf{x}'}$ it is possible to eliminate the depth length λ , highlighting that $\widehat{\mathbf{x}'}\mathbf{x}' = 0$.

$$\widehat{\mathbf{x}'}H[X, Y, 1]^T = 0 \quad (\text{B.2})$$

Point $\mathbf{x}'$ is known from the image and $\mathbf{X}$ is known from the checkerboard layout. If enough² points of the planar pattern are selected, H can be solved up to a scale factor. Since H is obtained, the calibration parameters of matrix K can be recovered. Once $[h_1, h_2] \sim K[r_1, r_2]$ or $K^{-1}[h_1, h_2] \sim [r_1, r_2]$, $K^{-1}h_1$ and $K^{-1}h_2$ are orthonormal vectors as well as $[r_1, r_2]$. Therefore, K must satisfy the following constraints,

$$h_1^T K^{-T} K^{-1} h_2 = 0, \quad h_1^T K^{-T} K^{-1} h_1 = h_2^T K^{-T} K^{-1} h_2 \quad (\text{B.3})$$

where h_1 and h_2 are the first two columns of the homography matrix. The five intrinsic parameters cannot be directly extracted from the above equations. Matrix $S = K^{-T}K^{-1}$ is nonlinear in terms of the intrinsic parameters as can be observed in B.4. However,

¹ $H \in \mathbb{R}^{3 \times 3}$ and maps a 3D point onto another 3D point.

²At least four points are required, Ma et al. (2004).

terms of matrix S can be solved using the least square minimization.

$$S = \begin{bmatrix} \frac{1}{fs_x^2} & -\frac{fs_\theta}{fs_x^2 fs_y} & \frac{fs_\theta o_y - o_x fs_y}{fs_x^2 fs_y} \\ -\frac{fs_\theta}{fs_x^2 fs_y} & \frac{fs_\theta^2}{fs_x^2 fs_y^2} + \frac{1}{fs_y^2} & -\frac{fs_\theta(fs_\theta o_y - o_x fs_y)}{fs_x^2 fs_y^2} - \frac{o_y}{fs_y^2} \\ \frac{fs_\theta o_y - o_x fs_y}{fs_x^2 fs_y} & -\frac{fs_\theta(fs_\theta o_y - o_x fs_y)}{fs_x^2 fs_y^2} - \frac{o_y}{fs_y^2} & \frac{(fs_\theta o_y - o_x fs_y)^2}{fs_x^2 fs_y^2} + \frac{o_y^2}{fs_y^2} + 1 \end{bmatrix} \quad (\text{B.4})$$

Each selected point produces two equations, B.3, thus at least three points are necessary to determine the intrinsic parameters. A factorization method can be applied to recover the elements from matrix K . Ma et al. (2004) mention that this procedure is sensitive to noise and, in practice, a refinement of the solution based on nonlinear optimization³ techniques is necessary.

Radial distortion can be described in simply and effectively by two parameters as B.5, Zhang (1999), Ma et al. (2004).

$$\begin{aligned} x &= x_d(1 + a_1 r^2 + a_2 r^4) \\ y &= y_d(1 + a_1 r^2 + a_2 r^4) \end{aligned} \quad (\text{B.5})$$

where x_d and y_d are coordinates of the distorted points and $r^2 = x_d^2 + y_d^2$. Parameters a_1 and a_2 can be solved by a linear least-square method applied on a system of stacked equations B.5 obtained for each selected point; then, it iteratively updates the estimate of the intrinsic parameters previously calculated. However, best results are obtained when the distortion model can also be incorporated to the nonlinear minimization problem.

Toolboxes containing this routines are available in C (OpenCV library) and Matlab[®], e.g.,

http://www.vision.caltech.edu/bouguetj/calib_doc/ and links therein, Bouguet (2016).

³Zhang (1999) applies the Levenberg-Marquardt algorithm.

Appendix C

Stereo Reconstruction Method

The method herein employed was based on that presented by Benetazzo (2006). As previously mentioned, it is a local-based matching technique, which takes advantage of epipolar constraints, and some prior hypothesis concerning the shape of the surface.

The flowchart presented in figure C.1 summarizes the steps of the algorithm, and each of its blocks is explained in details in further items. One may notice that calibration parameters are formerly known at the beginning of the flowchart. These parameters were obtained by applying the calibration toolbox made available by Bouguet (2016), described earlier.

Figure C.1: Reconstruction routine flowchart.

C.1 Stereo Calibration Parameters and Projection Matrices

Before actually dealing with the images, some important parameters must be computed. In fact, the first three blocks of the flowchart C.1 can be crossed during the reconstruction of a whole sequence of frames. The calibration parameters recorded during the calibration

process are Intrinsic parameters matrices K_l and K_r ; Rotation between two cameras R and Translation between two cameras (baseline) T .

In addition to these parameters, the poses of each camera with respect to the horizontal plane are also recorded. This information is used during the initialization of the matching process and it will be further detailed.

Three coordinate frames must be defined, a world coordinate frame on the left camera in the optical center position, used to describe points on 3D space $\mathbf{X} = [X, Y, Z, 1]^T$, left and right image coordinate frames, placed at each optical center with z axis pointing normally to image plane, $\mathbf{x}_l = [x_l, y_l, 1]^T$ and $\mathbf{x}_r = [x_r, y_r, 1]^T$.

At this point, the world reference frame is coincident with the left camera reference frame; therefore, Π_l is build just by intrinsic parameters and Π_r encodes the rotation and the baseline between cameras, equation C.1.

$$\begin{aligned}\Pi_l &= K_l[I, 0] \\ \Pi_r &= K_r[R, T]\end{aligned}\tag{C.1}$$

C.2 Rectified Projection Matrices and Images Transformations

To incorporate the epipolar constraint into the matching algorithm, the pair of stereo images must be rectified. The method proposed by Fusiello, Trucco and Verri (2000) was applied and briefly described in item A.6. The algorithm uses the relative pose between cameras to compute new projection matrices. The intrinsic and extrinsic parameters can be directly used provided that are known from calibration process; otherwise, an pair of projection matrices can be factorized into matrices K , R and T .

New projection matrices are used to map 3D scene points onto rectified image plane coordinates. Besides, a image transformation is also calculated as presented in equation C.2,

$$H = [K_n R_n][K R]^{-1}\tag{C.2}$$

where K_n and R_n are the new rectified matrices. The new intrinsic parameters are arbitrarily selected as their only restriction is that they must be the same for both cameras, in this case, the left camera parameters were assumed. Subscript n will be used to identify the rectified projection matrices and homographies.

C.3 Image Acquisition and Rectification

All the methods herein presented deals with gray scale images, particularly 8-bit gray scale. Therefore, images are represented by matrices with integers between 0 and 255.

Preliminary tests were performed by applying a pair of web cams that produce 8-bit RGB images with a 763 x 1317 resolution. In order to convert coloured images into gray scale, the weighted sum of each channel was calculated, as shown in equation C.3.

$$I = 0.2989R + 0.5870G + 0.1140B \quad (\text{C.3})$$

Once the pair of images is captured and converted, they must be rectified, so that the corresponding points lay on the same row on both images. The rectification homographies were calculated as presented on the previous topic, and they are applied to the original images as regular projective transformations, C.5.

$$H = \begin{bmatrix} h_1 & h_4 & h_7 \\ h_2 & h_5 & h_8 \\ h_3 & h_6 & h_9 \end{bmatrix} \quad (\text{C.4})$$

$$\begin{cases} \hat{x} = \frac{xh_1 + yh_2 + h_3}{xh_7 + yh_8 + h_9} \\ \hat{y} = \frac{xh_4 + yh_5 + h_6}{xh_7 + yh_8 + h_9} \end{cases} \quad (\text{C.5})$$

Figure C.2 illustrates a pair of stereo images after the rectification process. Some corners were highlighted and can be identified in the same row.

Figure C.2: A pair of rectified images. Red lines highlight corresponding points in the same row.

C.4 Initialization Process

Before matching, few assumptions concerning the structure of the scene can be made. Benetazzo (2006) mentioned that only during the first step, the search is performed all over the epipolar line. In his work, an initial grid is defined and matched with a point on the second image while they overcome some correlation threshold value. A slightly different approach is used herein, as long as an assumption of the initial shape is made.

During the calibration process, the position of the horizontal plane, a rotation matrix and a translation were recorded. In fact, such information was recorded for both cameras. Being R_{cl} and T_{cl} the parameters that describe a rigid body transformation between a horizontal reference plane on the scene with respect to the left camera (and R_{cr} , T_{cr} the right camera analogous), figure C.3 illustrates the transformation used to project a planar grid over the scene.

Figure C.3: Sequence of transformations to project a rectangular grid on the image.

A grid is defined in its own reference frame, scene reference frame; then, their points are mapped onto a 3D world coordinate frame, which was defined to be coincident with the left camera reference frame, by matrices R_{cl} , R_{cr} and respective translations. Points mapped on the right camera frame must also be transformed into the world reference frame, by the relative poses R and T . Finally, these points can be projected on image planes by the projection matrices. Additional translation and rotation can be performed before all the coordinate frame transformations fit the plane into the desired area.

The assumption that corresponding points will be near the grid points on both images equals considering the scene shape close to a plane. Especially for low steepness waves it is an acceptable assumption.

In this case, the searching for correspondence will be confined to a part of the epipolar line, near the grid point. The size of the searching area was chosen to be a fraction of the base size of the grid and the fraction is a calibration parameter.

The yellow points represent the planar grid and the squares are the searching areas near each node, while the red rounded markers represent the matching points. Besides a horizontal length for the searching area, a vertical clearance is also defined to deal with imprecisions on the epipolar rectification. Details concerning the matching process will be presented in next topic. Since searching areas do not overlap horizontally, an *Ordering* constraint, Faugeras (1993), is respected.

The first matching process must be precise and grant a spread set of points because they will define the range of the reconstruction. If the points on the boundaries of the grid are matched, the overall searching area will decrease. Therefore, larger templates and searching areas can be defined, even though it may increase computational effort.

Figure C.4: Initial rectangular grid of points, the respective searching areas and the matchings (red circles).

C.5 Matching

Template matching technique was applied to find corresponding points over the images. The size of the template depends on the characteristics of the image as well as on the metric applied to calculate matching. Three metrics were implemented, SAD (Sum of Absolute Differences), SSD (Sum of Squared Differences) and NCC (Normalized Cross Correlation).

Four parameters were used to control the matching process: the size of the template window, the size of the searching area, the metric to evaluate the correlation between areas and a threshold. Larger templates produce fewer ambiguities; however, they require more significant computational effort as well as tending to neglect smaller details of the scene. Small windows will be easily matched and produce a detailed description of the surface; nevertheless, several false matchings shall arise.

The size of the template depends on the nature of the scene, e.g. smooth surfaces such as waves on the absence of wind require larger templates as its texture is only perceived on larger scales. Rich texture surfaces, on the other hand, can be dealt with by using smaller templates. The optimal size for the foam objects scene presented in figure C.4 vary from 25×25 to 31×31 pixels. These sizes produce good results for the three metrics evaluated, although NCC produced consistent results up to the 13×13 templates.

The template window travels through the searching area and a correlation quantity is defined as presented in equations C.6, C.7 and C.8. All valid pixels¹ of the searching area are evaluated and sorted to find the best matching. By the application of a threshold, only pixels that exceed some determined value are considered good matches, otherwise, that pair of points is eliminated. Particularly during the first matching of the initial planar grid, smaller threshold with larger templates were applied in furtherance of producing matches all over the area.

$$SSD = \sum_{u,v} (I_1(u, v) - I_2(u + d, v))^2 \quad (C.6)$$

$$SAD = \sum_{u,v} |I_1(u, v) - I_2(u + d, v)| \quad (C.7)$$

$$NCC = \frac{\sum_{u,v} (I_1(u, v) - \bar{I}_1)(I_2(u + d, v) - \bar{I}_2)}{\sqrt{(\sum_{u,v} (I_1(u, v) - \bar{I}_1)^2)(\sum_{u,v} (I_2(u + d, v) - \bar{I}_2)^2)}} \quad (C.8)$$

¹Pixels close to the borders cannot be correctly evaluated once some positions of the template may fall outside the image.

C.6 Triangulation

Triangulation is used to transform points on both image planes onto the 3D points in space. Besides the position $\mathbf{x}_1$ and $\mathbf{x}_2$, the relative pose between cameras must be known and this information is encoded by projection matrices Π_1 and Π_2 . The method applied is presented in Hartley and Zisserman (2003), called DLT (*Direct Linear Transformation*) algorithm, and is it used to solve equations of the form $A\mathbf{x} = 0$. Points on image planes are related to points in the scene by equation $\mathbf{x}_1 = \Pi_1\mathbf{X}$ and $\mathbf{x}_2 = \Pi_2\mathbf{X}$ which can be rewritten in the homogeneous form $\mathbf{x}_1 \times \Pi_1\mathbf{X} = 0$ and $\mathbf{x}_2 \times \Pi_2\mathbf{X} = 0$. Each of these equations, related to each image, produces the following three equations, of which two are linearly independent.

$$\begin{cases} x(\mathbf{p}_3^T\mathbf{X}) - (\mathbf{p}_1^T\mathbf{X}) = 0 \\ y(\mathbf{p}_3^T\mathbf{X}) - (\mathbf{p}_2^T\mathbf{X}) = 0 \\ x(\mathbf{p}_2^T\mathbf{X}) - y(\mathbf{p}_1^T\mathbf{X}) = 0 \end{cases} \quad (\text{C.9})$$

where $\mathbf{p}_i^T$ concerns the rows of the projective matrices. Defining,

$$A \doteq \begin{bmatrix} \mathbf{x}_1\mathbf{p}_3^{1T} - \mathbf{p}_1^{1T} \\ \mathbf{x}_1\mathbf{p}_3^{2T} - \mathbf{p}_2^{1T} \\ \mathbf{x}_2\mathbf{p}_3^{2T} - \mathbf{p}_1^{2T} \\ \mathbf{x}_2\mathbf{p}_3^{1T} - \mathbf{p}_2^{2T} \end{bmatrix} \quad (\text{C.10})$$

where the superscript numbers identify the projective matrices which the rows belong to, an equation of the form $A\mathbf{X} = 0$ is obtained, which is linear in components of $\mathbf{X}$.

The least square method is applied to solve the set of equations presented above. Using the SVD (*Single Value Decomposition*) this solution² is directly obtained from the last column of matrix V .

$$A = U\Sigma V^T \quad (\text{C.11})$$

In the factorization C.11, A is a rectangular $m \times n$ matrix, U is an orthonormal $m \times m$ matrix containing eigenvectors of AA^T , V is a $n \times n$ matrix containing the orthonormal eigenvectors of $A^T A$ in its columns and Σ is a diagonal matrix containing the square roots of eigenvalues from U or V in descending order.

²The least square solution for homogeneous equations can be stated as the minimization of $\|A\mathbf{X}\|$ subject to $\|\mathbf{X}\| = 1$.

C.7 Surface Update and New Matching Candidates

After the initial matching is computed, a set of 3D points belonging to the surface is available (red round markers on C.5). Then, a Delaunay triangulation³ is applied to define a connectivity array representing all the triangular planes. Two parameters are computed for each plane, its barycenter, and reference lengths. The barycenter, identified by blue markers on image C.5 are be used as a guess for the next matching iteration while the reference length determines the size of the searching area.

Figure C.5: After one step, the planar grid has been replaced by a surface of triangular planar faces (green). In blue is the new matching candidates corresponding to the barycentre of the faces.

The dimensions of the triangular face are used to estimate the length of the searching area over the epipolar line. A reference dimension w is obtained computing the maximum horizontal distance between the vertices of the triangular surface, as presented in figure C.6. A scale factor s was multiplied by w in order to improve the control during the matching process.

Rectification process should make corresponding points belong to the same row on both images, hence, the template would basically have to travel horizontally through the searching area. However, there are some imprecisions related to the rectification⁴ and a vertical clearance is necessary. Figure C.6 illustrate such height clearance h_c .

In summary, for every new matching candidate at the barycenter of the triangular face, a width w and a height h_c is calculated. These parameters are inputs for the matching

³Details concerning the method can be found in Berg, Cheong O. abd Kreveld and Overmars (2008).

⁴As presented by Fusiello and Irsara (2008), an error characterized by the root mean square of the distance (in pixels) from each matched point to the epipolar line can be used to evaluate rectification methods.

Figure C.6: The new searching area represented by the yellow square, centred at the matching candidate (yellow star). The template is illustrated by the orange rectangle.

routine, defining the position and size of the searching area.

For every new iteration, $2n - 2 - k$ new points are generated, where n is the number of points in the previous step and k is the number of points in the perimeter of the convex hull described by the scattering of points. In the absence of a threshold, which means that there is the elimination of points, the number of corresponding points would grow exponentially with respect to the number of iterations.

C.8 Iterative 3D Surface Improving

The iterative process comprehends the matching process and the improvement of the triangular mesh, consequently the grid of points. This loop carries on until the maximum iteration number is reached. As aforementioned, the number of points grows exponentially at every iteration, likewise, the number of template matchings required and, consequently, the computational time.

Several aspects rule the selection of the optimal number of points to describe the surface and, consequently, the number of required iterations. At first, to produce a continuous surface, some sort of interpolation method must be applied and the selection of this method is related to the number and distribution of recovered points. Secondly, the wider the evaluated area, the larger the number of recovered points must be to keep the same spatial resolution. Finally, characteristics of the structure being reconstructed also determine the required resolution. Corner rich structures, such as the scenes C.3, need more points than a near flat structure such as a long water wave.

Figures C.7 and C.8 illustrate the evolution of the reconstructed surface during the iterative process. The computational time was recorded for each case, although it is merely comparative, since no particular care has been given to improving performance.

In this example, the initial grid had 25 points (a rectangular grid presented in figure C.3) spread over an area of approximately 0.225m^2 . During the iterative process, NCC was applied as the matching metric with a 13×13 template window and a threshold value of 0.35. For the initialization process, a larger template (61×61) and a lower threshold of 0.2 were used .

Figure C.7: First steps of the algorithm. Red dots on the images indicate the matchings. The reconstructed surface is shown on the right side.

Figure C.8: Last steps of the algorithm.

It can be noticed that after six iterations, the resulting surface is close to the real object. Further iterations do not substantially increase the quality of the reconstructed surface as can be verified at the bottom of figure C.8. Particularly, although no subpixel treatment was implemented, the accuracy of the description is also limited by image resolution.

Appendix D

DP Experiments Time Series

D.1 Wave probe Records

Figure D.1: **Case ID - 1**, water elevation record.

Figure D.2: **Case ID - 2**, water elevation record.

Figure D.3: **Case ID - 3**, water elevation record.

Figure D.4: **Case ID - 4**, water elevation record.

Figure D.5: **Case ID - 5**, water elevation record.

Figure D.6: **Case ID - 6**, water elevation record.

Figure D.7: **Case ID - 7**, water elevation record.

Figure D.8: **Case ID - 8**, water elevation record.

Figure D.9: **Case ID - 9**, water elevation record.

Figure D.10: **Case ID - 10**, water elevation record.

Figure D.11: **Case ID - 11**, water elevation record.

Figure D.12: **Case ID - 12**, water elevation record.

Figure D.13: **Case ID - 13**, water elevation record.

Figure D.14: **Case ID - 14**, water elevation record.

Figure D.15: **Case ID - 15**, water elevation record.

Figure D.16: **Case ID - 16**, water elevation record.

Figure D.17: **Case ID - 17**, water elevation record.

Figure D.18: **Case ID - 18**, water elevation record.

Figure D.19: **Case ID - 19**, water elevation record.

D.2 Ship Model Offsets

D.2.1 Beam Seas - Incidence 270°

Figure D.20: **Case ID - 1**, mean incident wave amplitude 8.49mm and period 0.65s, total duration 300s.(a) surge offset and (b) sway offset.

Figure D.21: **Case ID - 2**, mean incident wave amplitude 6.45mm and period 0.65s, total duration 150s.(a) surge offset and (b) sway offset.

Figure D.22: **Case ID - 3**, mean incident wave amplitude 4.13mm and period 0.65s, total duration 150s.(a) surge offset and (b) sway offset.

(a)

(b)

Figure D.23: **Case ID - 4**, mean incident wave amplitude 8.50mm and period 0.60s, total duration 150s.(a) surge offset and (b) sway offset.

(a)

(b)

Figure D.24: **Case ID - 5**, mean incident wave amplitude 5.73mm and period 0.60s, total duration 150s.(a) surge offset and (b) sway offset.

(a)

(b)

Figure D.25: **Case ID - 6**, mean incident wave amplitude 4.47mm and period 0.60s, total duration 150s.(a) surge offset and (b) sway offset.

D.2.2 Quarter Seas - Incidence 225°

Figure D.26: **Case ID - 7**, mean incident wave amplitude 8.46mm and period 0.65s, total duration 150s.(a) surge offset and (b) sway offset.

Figure D.27: **Case ID - 8**, mean incident wave amplitude 6.50mm and period 0.65s, total duration 150s.(a) surge offset and (b) sway offset.

Figure D.28: **Case ID - 9**, mean incident wave amplitude 4.39mm and period 0.65s, total duration 150s.(a) surge offset and (b) sway offset.

Figure D.29: **Case ID - 10**, mean incident wave amplitude 8.15mm and period 0.60s, total duration 150s.(a) surge offset and (b) sway offset.

Figure D.30: **Case ID - 11**, mean incident wave amplitude 5.78mm and period 0.60s, total duration 2.5 minutes.(a) surge offset and (b) sway offset.

Figure D.31: **Case ID - 12**, mean incident wave amplitude 4.21mm and period 0.60s, total duration 150s.(a) surge offset and (b) sway offset. The PD+WFF experiment was suddenly interrupted.

D.2.3 Quarter Seas - Incidence 247.5°

Figure D.32: **Case ID - 13**, mean incident wave amplitude 10.11mm and period 0.70s, total duration 150s.(a) surge offset and (b) sway offset.

Figure D.33: **Case ID - 14**, mean incident wave amplitude 8.12mm and period 0.70s, total duration 150s.(a) surge offset and (b) sway offset.

Figure D.34: **Case ID - 15**, mean incident wave amplitude 8.49m and period 0.65s, total duration 150s.(a) surge offset and (b) sway offset.

(a)

(b)

Figure D.35: **Case ID - 16**, mean incident wave amplitude 6.38mm and period 0.65s, total duration 150s.(a) surge offset and (b) sway offset.

(a)

(b)

Figure D.36: **Case ID - 17**, mean incident wave amplitude 9.19mm and period 0.60s, total duration 150s.(a) surge offset and (b) sway offset.

(a)

(b)

Figure D.37: **Case ID - 18**, mean incident wave amplitude 6.87mm and period 0.60s, total duration 150s.(a) surge offset and (b) sway offset.

Figure D.38: **Case ID - 19**, mean incident wave amplitude 7.27mm and period 0.55s, total duration 150s.(a) surge offset and (b) sway offset.

D.3 Ship Model Footprint

In this section, the displacements of the ship on the plane (footprints) are presented for all 19 experiments. The crosses in the graphs represent the mean position of the vessel during the experiment.

D.3.1 Beam Seas - Incidence 270°

Figure D.39: **Case ID - 1**, ship model footprint.

Figure D.40: **Case ID - 2**, ship model footprint.

Figure D.41: **Case ID - 3**, ship model footprint.

Figure D.42: **Case ID - 4**, ship model footprint.

Figure D.43: **Case ID - 5**, ship model footprint.

Figure D.44: **Case ID - 6**, ship model footprint.

D.3.2 Quarter Seas - Incidence 225°

Figure D.45: **Case ID - 7**, ship model footprint.

Figure D.46: **Case ID - 8**, ship model footprint.

Figure D.47: **Case ID - 9**, ship model footprint..

Figure D.48: **Case ID - 10**, ship model footprint.

Figure D.49: **Case ID - 11**, ship model footprint.

Figure D.50: **Case ID - 12**, ship model footprint.

D.3.3 Quarter Seas - Incidence 247.5°

Figure D.51: **Case ID - 13**, ship model footprint.

Figure D.52: **Case ID - 14**, ship model footprint.

Figure D.53: **Case ID - 15**, ship model footprint.

Figure D.54: **Case ID - 16**, ship model footprint.

Figure D.55: **Case ID - 17**, ship model footprint.

Figure D.56: **Case ID - 18**, ship model footprint.

Figure D.57: **Case ID - 19**, ship model footprint.