ANDRÉ SEIJI SANDES IANAGUI

**ROBUST SYSTEM DESIGN FOR CONSENSUS CONTROL IN
DYNAMICALLY POSITIONED VESSEL FLEET**

**São Paulo
2019**

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Research area:
Mechanical Engineering
Control and Automation

Advisor:
Prof. Dr. Eduardo Aoun Tannuri

**São Paulo
2019**

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Dedicated to Viviane and Auri.

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ABSTRACT

As offshore operations grow larger and more complex, the requirements for a larger number of agents – from vessels to equipment – working simultaneously and cooperatively become evident. In this scenario, the ability to perform such tasks safely coordinating all these elements will eventually reach human limits, bounding also the complexity that can be achieved. Enhanced levels of autonomy emerge as a response to these requirements, giving margin to safer, larger and possibly more cost-effective operations. This work presents the development of cooperative control algorithms applied in Dynamic Positioned (DP) vessels. In this approach, DP vessels are treated as “Drone ships”, which can not only perform station keeping but also trajectory tracking tasks collectively. The proposed methods encompass the use of robust nonlinear techniques designed in the form of cooperative protocols, such as Sliding Mode Control (SMC) and Super Twisting Algorithm (STA), to solve a multi-agent consensus problem. Such methods need to account for effects from geometric and dynamic nonlinearities and for environmental disturbances that affect DP systems. By considering a fixed network topology, but prone to communication breakdowns, the use of a parallel Kalman Filter for neighbor agents’ states estimation is proposed to enhance the system robustness. Finally, a consensus filter is considered for the improvement of measurements and estimations of environmental data, benefiting from the multiple vessels in the network. All methods were validated through a set of study cases, tested in simulation and in small scale models.

Keywords: Cooperative Control, Dynamic Positioning Systems, Robust Nonlinear Control.

RESUMO

Com o aumento da extensão e da complexidade de operações *offshore*, os requisitos para o uso de um grande número de agentes – de embarcações a equipamentos – operando simultaneamente e cooperativamente se tornam evidentes. Em tal cenário, a possibilidade de realização de tais tarefas de maneira segura coordenando todos esses elementos eventualmente atingirá os limites da capacidade humana, impondo assim barreiras à complexidade que pode ser atingida. Níveis elevados de automatização surgem como resposta a essas demandas, provendo margens para operações mais seguras, maiores e possivelmente reduzindo custos. Este trabalho apresenta o desenvolvimento de algoritmos de controle cooperativo aplicados a embarcações equipadas com sistemas de posicionamento dinâmico (DP). Nesta abordagem, os navios DP são tratados como “navios robóticos”, que podem realizar não apenas tarefas de manutenção de posição como também rastreamento de trajetórias conjuntamente. Os métodos propostos englobam o uso de técnicas de controle robusto não linear na forma de protocolos de controle cooperativo, tais como controle por modos deslizantes (SMC) e o controle *super-twisting* (STA), para resolver um problema de consenso de múltiplos agentes. Esses métodos devem levar em consideração efeitos decorrentes de não linearidades geométricas e dinâmicas, além de distúrbios ambientais que afetam os sistemas DP. Ao considerar a topologia de comunicação fixa, mas suscetível a falhas de comunicação, propõe-se o uso de filtros de Kalman rodando em paralelo para a estimação de estados de agentes vizinhos, de forma a aumentar a robustez do sistema. Por fim, um filtro de consenso é considerado para a melhoria das estimativas e medidas de dados ambientais, beneficiando-se dos múltiplos navios em rede. Todos os métodos foram validados através de um conjunto de estudos de caso, testados via simulação e em modelos de escala reduzida.

Palavras-chave: Controle Cooperativo, Sistema de Posicionamento Dinâmico, Controle Robusto Não-Linear.

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LIST OF ABBREVIATIONS

AIS.	<i>Automatic Identification System</i>
ASV.	<i>Autonomous Surface Vehicles</i>
AUV.	<i>Autonomous Underwater Vehicles</i>
CDP.	<i>Coordinated Dynamic Positioning</i>
COM.	<i>Center of Mass</i>
DOF.	<i>Degree of Freedom</i>
DPS.	<i>Dynamic Positioning Systems</i>
EKF.	<i>Extended Kalman Filter</i>
FPWSO.	<i>Floating, Production, Working, Storage and Offloading</i>
GNSS.	<i>Global Satellite Navigation System</i>
GPS.	<i>Global Positioning System</i>
GRF.	<i>Global Reference Frame</i>
HOSMC.	<i>High Order Sliding Mode Control</i>
IMU.	<i>Inertial Measurement Unit</i>
ISSC.	<i>International Ship Structures Congress</i>
ITTC.	<i>International Towing Tank Conference</i>
JONSWAP.	<i>Joint North Sea Wave Project</i>
KF.	<i>Kalman Filter</i>
LAT.	<i>Latitude</i>
LON.	<i>Longitude</i>
LRF.	<i>Local Reference Frame</i>
NSB.	<i>Null-Space Behavioral, Null-Space Behavioral</i>
PD.	<i>Proportional-Derivative</i>
PID.	<i>Proportional-Integral-Derivative</i>
PM.	<i>Pierson-Moskowitz</i>
RAO.	<i>Response Amplitude Operator</i>
SMC.	<i>Sliding Mode Control</i>
SNAME.	<i>Society of Naval Architects And Marine Engineers</i>
STA.	<i>Super-Twisting Algorithm</i>
STC.	<i>Super-Twisting Controller</i>
TPN.	<i>Tanque de Provas Numérico (Numerical Offshore Tank)</i>
UAV.	<i>Unmanned Aerial Vehicle</i>

UKF. *Unscented Kalman Filter*

USP. *Universidade de São Paulo*

USV. *Unmanned Surface Vessels*

UTM. *Universal Transverse Mercator*

VSC. *Variable Structure Control*

NE. *Northeast*

SE. *Southeast*

LIST OF SYMBOLS

In general, vector quantities are represented in bold. Matrix quantities are in capital letters. Time derivatives are expressed through the dot notation [$\dot{*}$]. Letters i, j and k are used as indices counting.

o_L	LRF origin
x_L	LRF axis, pointing to the vessel bow
y_L	LRF axis, pointing to the vessel port side
z_L	LRF axis, vertical, normal to the $x_L y_L$ plan
o	GRF origin
$\boldsymbol{\eta}$	Vessel midship position vector in the GRF
x	Vessel midship Easting in the GRF
y	Vessel midship Northing in the GRF
ψ	Vessel Yaw angle
$\boldsymbol{p}^L$	Vector in local reference frame
$\boldsymbol{p}^G$	Vector in global reference frame
R_L^G	Local to global reference transform matrix
R_G^L	Global to local reference transform matrix
S_L^G	Skew-symmetric kinematic transform matrix (from Global to Local)
S_G^L	Skew-symmetric kinematic transform matrix (from Local to Global)
m	Vessel mass
$\boldsymbol{v}$	Vessel midship velocity vector described in the LRF
u	Vessel midship velocity x_L component described in the LRF (surge)
v	Vessel midship velocity y_L component described in the LRF (sway)
r	Vessel midship angular velocity component (yaw velocity) described in the LRF (rate-of-turn)
$\dot{\boldsymbol{v}}$	Vessel midship acceleration vector described in the LRF
$\dot{u}$	Vessel midship acceleration x_L component described in the LRF (surge)
$\dot{v}$	Vessel midship acceleration y_L component described in the LRF (sway)
$\dot{r}$	Vessel midship angular acceleration component (yaw acceleration) described in the LRF

I_z	Vessel moment of inertia around the LRF vertical axis (z_L).
x_G	x_L component position of the COM relative to the vessel midship
$\mathbf{F}^L$	Generalized Force vector in the LRF applied to the vessel midship
F_u	Generalized Force x_L component in the LRF applied to the vessel midship
F_v	Generalized Force y_L component in the LRF applied to the vessel midship
F_r	Generalized Moment in the GRF applied around the vessel midship
$\mathbf{F}_{thrust}$	Thruster actuation forces
$\mathbf{F}_{hydro}$	Vessel hydrodynamic forces
$\mathbf{F}_{oper}$	Operational forces
$\mathbf{F}_{curr}$	Oceanic current forces
$\mathbf{F}_{wind}$	Wind forces
$\mathbf{F}_{wave}$	Wave forces
M_a	Added mass matrix
$X_{\ddot{u}}$	Surge added mass
$Y_{\ddot{v}}$	Sway added mass
$N_{\ddot{r}}$	Yaw added inertia
$Y_{\ddot{r}}$	Coupled added inertia (yaw effect in sway)
$\mathbf{v}_r$	Relative velocity vector in LRF (Speed Through Water)
u_r	x_L component of the relative velocity vector in LRF
v_r	y_L component of the relative velocity vector in LRF
r_r	Rotational component of the relative velocity vector in LRF
$\mathbf{v}_c$	Current velocity vector in LRF (Speed Through Water)
u_c	x_L component of the current velocity in LRF
v_c	y_L component of the current velocity in LRF
r_c	Rotational component of the current velocity in LRF
M	Virtual Inertia Matrix
C_{cor}	Coriolis and centripetal forces matrix
C_a	Current Inertial hydrodynamic effects matrix
V_c	Absolute current velocity
ψ_c	Global current angle
$\boldsymbol{\tau}_{thr}$	Thruster output force vector

$\mathbf{u}_{thr}$	Commanded thrust vector
A_{thr}	Thruster dynamic system matrix
$T_{thr,n}$	Thruster time constant
$T_{3 \times n}$	Thruster configuration matrix
α	Vector of thrusters' orientation angles
J_0	Advance velocity nondimensional parameter
V_{thr}	Water input velocity
K_T	Propeller's thrust constant
K_Q	Propeller's torque constant
Q_{thr}	Propeller torque
ρ	Water density
D_{thr}	Propeller diameter
ρ_{air}	Air density
V_{wr}	Instantaneous relative wind velocity
A_{front}	Frontal wind area
A_{lat}	Lateral wind area
C_{uw}	Dimensionless wind drag coefficient in surge
C_{vw}	Dimensionless wind drag coefficient in sway
C_{rw}	Dimensionless wind drag coefficient in yaw
ψ_w	Global wind direction
V_w	Global wind velocity
L_{pp}	Vessel hull length between perpendiculars
T	Vessel draft
C_u	Non-dimensional drag coefficient for surge
C_v	Non-dimensional drag coefficient for sway
C_r	Non-dimensional drag coefficient for yaw
V_r	Current relative velocity modulus
S_ζ	Wave spectral density
m_k	Wave spectral moment
H_s	Wave spectrum significant height
T_1	Wave spectrum central period
T_2	Wave spectrum mean period between zeros

ω_p	Wave spectrum peak frequency
k_1, k_2	Wave spectrum constants
$F_i(j\omega)$	Force RAO
$H_i(j\omega)$	Force-to-motion RAO
ξ_i	Wave motion state component in time domain
ϕ_w	Random wave phase
K_i^{RAO}	Wave motion transfer function gain constant
ω_0	Wave motion transfer function natural frequency
λ	Wave motion transfer function damping factor
K_w	Wave motion transfer function gain
σ	Wave motion transfer function constant
ξ	High frequency wave motion state vector
$\mathbf{w}_w$	Wave noise excitation
A_w	High frequency wave motion state space system matrix
E_w	High frequency wave motion state space input matrix
$\mathbf{F}_{md}$	Mean wave drift force vector
$\mathbf{F}_{sd}$	Slow wave drift force vector
$\mathbf{x}$	State space vector
A	State space system matrix
B	State space input matrix
C	State space observation matrix
D	Linear damping matrix
X_u	Surge damping
Y_v	Sway damping
Y_r	Coupled yaw damping in sway
N_v	Coupled sway damping in yaw
N_r	Yaw damping
$\mathbf{x}_0$	Stationary state vector
x_0	Stationary x position in GRF
y_0	Stationary y position in GRF
ψ_0	Stationary yaw position
u_0	Stationary surge velocity
v_0	Stationary sway velocity

r_0	Stationary yaw velocity
δx	Small perturbation vector
J	Jacobian matrix
f	Generic functions vector
b	Bias vector
T_b	Bias time constant
$w_{b1,2,3}$	Bias random noise
H	Extended Observation Matrix
E	Noise input matrix
w	Random process noise vector
v	Random measurement noises
$\tilde{\eta}$	Position error vector in GRF
η_d	Desired position vector in GRF
x_d	Desired x position in GRF
y_d	Desired y position in GRF
ψ_d	Desired heading
e^L	Local error vector
$\tilde{v}$	Velocity error in LRF
v_d	Desired velocity in LRF
η_0	Fixed position in GRF
$\dot{e}^L$	Local error vector velocity
δe^L	Small perturbation of error vector in LRF
$\delta \tilde{\eta}$	Small perturbation of position vector in GRF
$\delta \dot{e}^L$	Small perturbation of error vector velocity in LRF
$\delta \tilde{v}$	Small perturbation of velocity in LRF
K_P	Proportional Gain Matrix
K_D	Derivative Gain Matrix
K_I	Integral Gain Matrix
$k_{Px,y,\psi}$	Proportional Gain in x , y and ψ
$k_{Dx,y,\psi}$	Derivative Gain in x , y and ψ
$k_{Ix,y,\psi}$	Integral Gain in x , y and ψ
s	Laplace variable

$G_{x,y,\psi}(s)$	Transfer function for each degree of freedom
$G_{cx,y,\psi}(s)$	PID controller transfer function
$G_{CLx,y,\psi}$	DPS Closed Loop Transfer Function
ω_n	DPS natural frequency
ζ	DPS damping factor
$\hat{\mathbf{F}}_{wind}$	Filtered estimation of mean wind forces
$\Theta_{n \times 3}$	Optimal allocation matrix
W	Allocation weight matrix
λ	Lagrange multipliers vector
$L(\tau_{thr}, \lambda)$	Lagrangian function
$T^{\dagger}$	Moore-Penrose pseudo-inverse matrix
M_o	Observability Matrix
Φ	State Transition Matrix
Δ	Input matrix discretization
Γ	Input noise matrix discretization
h	Sample time
k	KF Step counter
$\hat{\mathbf{x}}$	State vector estimate
$\mathbf{x}^-$	A priori state vector estimate
P^-	A priori error covariance matrix estimate
$\hat{P}$	Error covariance matrix estimate
K_{kal}	Kalman gain
R	Measurement noise covariance matrix
Q	Process noise covariance matrix
$\mathcal{G}$	Graph pair
V	Vertex set
E	Edge set
$v_1, v_2, \dots, v_n$	Graph Vertices
N_i	Neighbor set
a_{ij}	Edge weight factor
A	Adjacency matrix
d_i	In-degree of node

d_i^o	Out-degree of node
D	In-degree matrix
L	Laplacian matrix
$\lambda_{1,2,3}$	Eigenvalues of Laplacian matrix
c_{ss}	Steady State consensus value
d_{max}	Maximum in-degree
J_i	Neighbors set with agent itself
x_i	Agent i state
u_i	Agent i input
J	Jordan Canonical Form
T	Non-singular matrix transformation
v_j	Left eigenvector
w_j	Right eigenvector
$\tilde{x}$	Error vector
u	System input vector
c	Consensus synchronization gain
K	Gain matrix
e	Error vector
P	Solution to ARE
Q, R	Weighting Matrices
$f(x), b(x)$	Vector functions
v	Virtual control input
S	Sliding manifold
σ	Sliding variable
Λ	Sliding variable gain matrix
λ_i	Sliding variable scalar gain
Φ	Boundary layer
Δ_i	Sliding mode constant
F	Modeling error extension
$\hat{u}$	Best estimate of Equivalent control
K_{sm}	Sliding mode switching gain
φ	Super twisting bounding vector

K_m	Super twisting bounding vector
K_M	Super twisting bounding vector
K_1	Super twisting gain
K_2	Super twisting gain
$\tilde{u}_i$	Acceleration input vector for agent i
U_i	Feedforward input vector for agent i
$\bar{f}_i$	Best estimate of unknown bounded disturbances
$\tilde{f}_i$	Unknown bounded disturbances
z_i	Generalized error state
σ_i	Sliding Manifold Vector
η_i	Vessel Agent i position in the GRF
c_{ss}	Consensus value vector
σ_{ik}	Sliding variable of agent i in degree-of-freedom k
$\tilde{f}_{ik}$	Best estimate of unknown bounded disturbances of agent i in degree-of-freedom k
$\bar{f}_{ik}$	Unknown bounded disturbances of agent i in degree-of-freedom k
k_k	Sliding gain of degree of freedom k
Δ_{ik}	Overestimation of bounded disturbances of agent i in degree-of-freedom k
F_{ik}	Lumped modelling error boundary of agent i in degree-of-freedom k
Γ_{ik}	Modelling error boundary of agent i in degree-of-freedom k
δ_{ij}	Commanded Relative position reference between agents i and j .
φ_{ik}	Super twisting bounding vector of agent i in degree-of-freedom k .
Γ_{ia}	Modelling error derivative bound $\bar{e}_{c,k}$ of agent i in degree-of-freedom k
e_{ij}	Vector Consensus Error
$e_{ij,x}, e_{ij,y}, e_{ij,\psi}$	Vector Consensus Error components
$\bar{e}_{c,k}$	Normalized Consensus Error
J_{Err}	Formation Error Performance index
W_η	diagonal weighting matrix in positioning
W_v	diagonal weighting matrix in velocity
J_L	Tracking Error Performance index

J_c	Total consensus performance index
$\bar{x}$	Vessel center of gravity Easting in the GRF
$\bar{y}$	Vessel center of gravity Northing in the GRF
$\ddot{\bar{x}}$	Vessel COM acceleration x component represented in the GRF
$\ddot{\bar{y}}$	Vessel COM acceleration y component represented in the GRF
$\ddot{\bar{\psi}}$	Vessel angular acceleration around the COM represented in the GRF
$\bar{F}_x$	Generalized Force x component in the GRF applied to the vessel COM
$\bar{F}_y$	Generalized Force y component in the GRF applied to the vessel COM
$\bar{M}_z$	Generalized Moment around z in the GRF applied around the vessel COM
$\bar{\mathbf{F}}^G$	Generalized Force vector in the GRF applied to the vessel COM
$\bar{\mathbf{M}}$	Vessel mass matrix relative to the COM
$\bar{I}_z$	Vessel moment of inertia around the axis passing through its COM and parallel to the LRF vertical axis.
$\ddot{\bar{\eta}}$	Vessel acceleration vector described in the GRF
$\bar{\mathbf{v}}$	Vector of vessel COM velocity described in the LRF
$\bar{u}$	Vessel COM x_L component (described in the LRF)
$\bar{v}$	Vessel COM y_L component (described in the LRF)
$\bar{r}$	Vessel yaw velocity (or rate-of-turn) around the COM
$\dot{\bar{\eta}}$	Vessel velocity vector described in the GRF
$\dot{\bar{x}}$	Vessel COM velocity x component represented in the GRF
$\dot{\bar{y}}$	Vessel COM velocity y component represented in the GRF
$\dot{\bar{\psi}}$	Vessel yaw velocity (rate-of-turn) around the COM
$\bar{\mathbf{F}}^L$	Generalized Force vector in the LRF applied to the vessel COM
$\bar{F}_u$	Generalized Force x_L component in the LRF applied to the vessel COM
$\bar{F}_v$	Generalized Force y_L component in the LRF applied to the vessel COM
$\bar{F}_r$	Generalized Moment in the GRF applied around the vessel COM
S_w	Wind spectrum

ω_w	Wind frequency
C_w	Roughness coefficients
$\bar{V}_w$	Mean wind velocity
$\omega_{w1,2,\dots}$	Wind frequency component

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1. INTRODUCTION

Dynamic Positioning Systems (DP Systems or simply DPS) have become a standard technology and essential tool in general offshore operations. Since its first development and trials in the 1960s until the introduction of modern digital systems, the DPS have shown to be a well proven concept and one of first and most successful examples of industrial cybernetics applications. As an established technology, its general working method and the engineering tools used in its design and application have changed little, with most advances coming from the introduction of advanced digital electronics, satellite based and inertial positioning systems and newer feedback methods. For many offshore operations, such as oil offloading, deep-water coring and oil extraction, cable laying systems and general subsea installations, availability of DP systems are now one of the most basic prerequisites.

As such marine operations grow larger and more complex, the requirements for a growing number of agents – from vessels to equipment – working simultaneously and cooperatively become evident. In this scenario, the ability to perform such tasks safely coordinating all these elements may eventually reach human limits, bounding also the complexity that can be achieved. Enhanced levels of autonomy emerge as a response to these requirements, giving margin to safer, larger and possibly more cost-effective operations. Current state-of-the-art technologies in robotics and autonomous systems along with great advances in bandwidth availability and in reliability of wireless communication systems have permitted the development of large-scale multi-robot and multi-autonomous-vehicle architectures, able to solve cooperatively tasks that would not be possible with a single agent. In (MURRAY, 2007) a list of several devised applications for multi-vehicle operations is presented, including examples for autonomous cars, unmanned aerial vehicles (UAVs), satellite arrays and mixed vehicles. Given such examples, it is easy to think also of sets of marine vehicles such as autonomous underwater vehicles (AUVs) and autonomous surface vehicles (ASVs) that could perform cooperative tasks and enable large and complex offshore or even restricted waters operations. DP vessels, which can be thought of as actual “drone ships” are interesting candidates for study and development of marine multi-agent tasks. Their flexibility and already highly automated operational capacity might be enhanced with the introduction of cooperative systems concepts.

It was not until very recently that unmanned surface vessels (USVs) have been discussed as a target for application of a cooperative control concept. Some works such as (SKJETNE; MOI; FOSSEN, 2002), (KYRKJEBØ; PETTERSEN, 2003), (IHLE; JOUFFROY; FOSSEN, 2005), and (ARRICHIELLO; CHIAVERINI; FOSSEN, 2006) exploit path following and trajectory tracking missions of marine vessels working in coordination. Researches in cooperative control applied to DP systems are even scarcer. In (QUEIROZ FILHO; TANNURI, 2013) an extensive experimental analysis of a consensus-based control law applied to DP vessels was presented. (PENG; WANG; WANG, 2016) showed a cooperative DP control system using their so-called Dynamic Surface Control (DSC) combined with an adaptive algorithm that observes and corrects the control law to respond to oceanic disturbances.

Among the several applications of cooperative control in oceanic systems that are devised are subsea installation operations, joint materials hoisting, multi-agent offloading operations, rescues, multiple tugboats operations, large cargo transportation and underway replenishment. When devised as multiple agent tasks, these operations can be advantageous by reducing overall costs, enhancing operational safety and reliability due to redundancy and by enabling large configuration flexibility.

The subject “cooperative multi-agent systems” is rather broad and involve notions from computer science to mechatronics. The most studied topics in this area for application in multi-vehicle systems are the cooperative control methods. These methods involve decision making, cooperative motion planning and actual control theory required to perform the cooperative tasks. To accomplish such objectives, the vehicles may be required to collectively adopt several behaviors, such as (but not restricted to) swarming, flocking, setting a formation and navigating in it, tracking a leader and avoiding collisions. These behaviors can be achieved through the application of collective control laws, referred in literature as cooperative protocols, which can be calculated either in a centralized fashion or distributed among the agents. While centralized control is easier and can be designed through standard control methods, its architecture lacks robustness (as a failure in the centralized controller might impair the mission), flexibility and scalability to be applied in larger scale systems. Within this set of requirements, distributed cooperative methods are usually preferable for application in multi-agent control.

While not vastly different from the control structures that are applied to other vehicles, the cooperative protocols needed for Dynamic Positioned vessels have to address some specific challenges, such as persistent and/or random punctual disturbances from waves, winds and currents. The communication topologies used should also be correctly addressed in order to keep the system safe, dependable and available. Within this scenario, this work intends to provide novel protocols and a general system architecture that can fulfill such requirements. Due to the modeling uncertainties and the nonlinearities of this system, sliding mode control techniques are good candidates to be used as basis for the cooperative controllers of DP vessels. These methods are robust to modeling errors and uncertain disturbances, therefore are suitable to deal with the proposed problem. The sliding modes approach can also simplify the problem approach, as it can reduce the problem of controller design to a lower dimensional space when the appropriate sliding manifold is selected.

In the present development the standard 1st order Sliding Mode Control and a 2nd order sliding mode method named Super Twisting Control will be designed as distributed control protocols and applied to sets of DP vessels working cooperatively. A structure using Kalman Filters for neighbor agents' state estimations will be proposed as a method to enhance robustness communication failures and delays and a collective sensor fusion technique will be investigated as a method to enhance environmental data estimation. The proposed architecture is capable of providing consensus in a heterogeneous group of agents by canceling its individual dynamic behaviors. Nevertheless, it enables an increased level of redundancy by keeping individual DP systems operational within the inner loop.

An extensive historical background on researches on DP systems and cooperative control methods is presented, followed by the set of theoretical knowledge – from modeling to control methods – required to develop and simulate adequately the proposed methods. Finally, the proposed methods are tested in a series of case studies ran through numerical simulations and model-scale experiments, with most of the important dynamic effects modeled to verify performance and robustness.

1.1. Motivation

This study intends to develop technologies that permit the use of cooperative multi-agent robotics concepts in marine operations. Thought of as “robotic ships”, DP

vessels are perfect candidates for application of such concepts, enabling novel and complex operational possibilities.

1.2. Objectives

The main objective of this work is to study the application and feasibility of new cooperative control methods in Dynamic Positioned Vessels. In specific, a Sliding Mode Cooperative Controller and a Super Twisting Controller are to be proposed and applied in both simulation and experimental environments. State estimation techniques, such as a dead reckoning capable discrete Kalman Filter are introduced to enhance robustness of cooperative tasks. Based on these methods, a cooperative architecture suitable for DP systems is proposed.

As secondary objectives, the advantages of multi-agent DP operations are to be analyzed and discussed through the evaluation of case studies. Finally, an assessment on how the introduction of shared agent data knowledge can enhance the quality of sensing and estimation of operational data is to be performed.

1.3. Main Contributions

The novelty of this work relays on the innovative approaches to design and solve a cooperative consensus problem adequate to the specific case of DP vessels, accounting for its peculiarities and disturbances. Its main contributions are listed as follows:

- i. A complete system architecture involving the high level decentralized cooperative controllers and estimators for a DP system is proposed, adding robustness to failure to the overall structure due to the added redundancy layer (page 107);
- ii. A simple method for cancellation the agent's dynamics is presented, permitting usage of a heterogeneous fleet (page 109);
- iii. A framework for the application of standard and super-twisting sliding mode controllers, adequate for usage in presence of modeling errors and disturbances is elaborated and finite-time convergence is shown for the fleet consensus problem. Determination of controller based on the extent of modelling errors is discussed (pages 111 and 118);

- iv. A distributed filter for estimation of environmental parameters is proposed (page 123);
- v. The proposed methods are simulated and, when possible, experimentally evaluated (section 5).

1.4. Structure of the Text

First, a comprehensive literature review on Dynamic Positioning Systems and on Cooperative Control is presented, approaching both historical perspectives and current state-of-the art technologies. Following the review, important theoretical concepts are introduced, configuring the basis of the methods proposed. Kinematic and dynamic modeling, DPS control systems, Cooperative Control and Robust Nonlinear control techniques are discussed in this chapter. The next chapter discusses the proposed methods in this research, introducing the novel applications and methods of cooperative sliding modes control and cooperative estimation on DP systems. Finally, case studies with application examples of the technology, presenting both simulated and experimental results are presented and discussed. In the closing chapter, conclusions and proposals of further work are discoursed.

2. LITERATURE REVIEW

In the following section, history and state-of-the-art research on the topics discussed in this work are presented in a brief manner.

2.1. Dynamic Positioning Systems

A Dynamic Positioning system is the set of technologies put together to perform automatic stationkeeping of a floating vessel through exclusive use of propellers and thrusters. By using these actuators, sensors and an integrated control system, it automatically works to counteract environmental efforts. DP Systems are a prolific research topic in both academy and industry. While its basic architecture has been set for more than forty years, refinements in its inner system blocks still provide a broad field for innovations.

A comprehensive history on dynamic positioning systems can be found in (JENSSEN, 2016) and in (BREIVIK; KVAAL; ØSTBY, 2015). The DP concept as an automatic control system was proposed and designed by Howard Shatto, an electrical engineer from Shell's R&D department often referred to as the "Father of Dynamic Positioning". In the early 1960s Shell had requirements for a drillship that could perform deep-water coring with low deviation from the reference position. In 1961, the CUSS 1 barge had already successfully performed experimental drilling in such conditions, with its position kept through manual thruster control by two operators who commanded the vessel by using visual references and a sonar screen displaying relative position to four deep-water buoys. Due to more flexible requirements for later drillings, Shatto was convinced that manual control would not work and, inspired by his earlier experience in petrochemical plants, proposed the use of electronic PID controllers which had recently come available to the market. As a result, the first Dynamically Positioned vessel was developed and launched in 1961, Eureka. It worked with PID controllers for each horizontal degree of freedom and used a kind of an archaic taut-wire sensor and a gyrocompass for referencing; thruster allocation was performed by means of a patented electromechanical unit.

Later, in 1968, the Glomar Challenge was released for Deepwater coring at depths of 7000m. This vessel's DP system is remarkable due to its digital control system, which was the first to use wave filtering to remove 1st order wave-motion. It also introduced

a new position sensing method, through Short Baseline Hydroacoustics. Three years later, the first drillship for oil exploration was released, defining the basic structure for the standard DP systems that remain up to now. The Sedco 445 had an advanced wave filtering system, wind feedforward compensation, redundant position sensing through hydroacoustics, taut-wire and riser tilt measurement and a redundant control computer, to enable high reliability. As a result, more than 150 days of continuous operation was achieved, and a new industry standard for DP architecture was defined.

Since then, much of the basic building blocks of the DP system have been kept the same, being most of the technologies only refined. One important advance was the introduction of optimal control theory and the brand new (at the time) Kalman Filter (BALCHEN et al., 1980), as results of the work of a cybernetics research group in Norwegian Institute of Technology (today called NTNU) leaded by Jens Glad Balchen. Revolutionary for its time, this system is a perfect example of how academy and industry could build products together with state-of-the-art technology. This new filter could eliminate the wave-induced responses from the control system and at the same time reduce the phase lag associated with traditional filters. Besides that, it provided novel resources such as sensor fusion, dead reckoning, enhanced signal processing and fault tolerance capacity.

The 1990's brought the introduction of new sensing technologies, as the Global Positioning System (GPS), which was promptly adopted by industry and made possible the development of georeferenced DP systems, which could be used by cruise liners and cable laying vessels. Later, the advanced digital computers permitted also the application of the full Extended Kalman Filter, the nonlinear counterpart of the original filter which provided more robust and accurate results.

Such a prolific topic has rendered much of academy attention since its first years. Around the world, marine cybernetics research groups such as Sorensen & Fossen (Norway), Grimbly (Scotland), Yamamoto (Japan) among much others have produced (and still produce) innumerable projects focusing in DP systems and subsystems. State-of-the-art technologies cited from now on will focus on the DP system's control logic, but much attention has also been given to thruster allocation methods, guidance systems and power plant management.

On the signal processing and filtering methods topic, alternatives to the Kalman Filter have been proposed and studied, such as the Nonlinear Passive Observer (FOSSEN; STRAND, 1999), which provided a simpler tuning framework and less online calculation requirements. Despite its lack of complexity, it could be proven Lyapunov Stable by passivity analysis. Variations of the original KF and EKF were also proposed for this task, such as the Unscented Kalman Filter, UKF, (JULIER; UHLMANN, 1997) which introduced the concept of particle filtering enhancing the results obtained by the Extended Kalman Filter. The UKF was proposed and simulated for a DP system in (SHI et al., 2011) and in (WIRTENSOHN; SCHUSTER; REUTER, 2016). Such extensions keep, though, some of the drawbacks of the KF, such as the intense computational requirements and the challenging task of tuning the filter covariance matrices.

Advanced and nonlinear control methods have also been vastly studied in DP systems through the last two decades. In (AARSET; STRAND; FOSSEN, 1998), in (STRAND; FOSSEN, 1998) and in (MORISHITA; SOUZA, 2014) Vectorial Backstepping was studied. A feedback linearization for an under actuated craft was proposed in (BERTIN et al., 2000). Sliding Mode Control in DP was studied in (TANNURI; DONHA; PESCE, 2001) and in (AGOSTINHO et al., 2009). Experimental evaluations of this method were performed in (TANNURI et al., 2010).

2.2. Cooperative Control

The fast pace development of digital embedded systems and wireless communication technologies have permitted innovative approaches to solve problems of logistics, people and cargo transportation, equipment commissioning, ambient data logging and military missions. The possibility of using multiple vehicles to perform these tasks, breaking the mission to smaller parts to be executed by individual agents in coordinated manner introduces new operational methods, reduce costs and enhance robustness. Even though some topics on distributed computing have been studied since the 1960's, researches in cooperative control of sets of robotic agents are relatively new, such as stated in (PARKER, 2000), dating from the late 1980's, when questions about application and control of multiple mobile and reconfiguring robots were raised and discussed. In (ASAMA; MATSUMOTO; ISHIDA, 1989), practical aspects of the architecture of a multiple robot system, centered in hardware and

software requisites were presented. Early notions of cooperative navigation strategies, describing leader-follower and formation control can be found in (WANG, 1989).

Since then, the cooperative control technologies have received much attention in academy. The overall idea of cooperative control systems is to coordinate the actions of multiple agents (robots, vehicles) to perform one or various joint tasks. The common objective can be translated in terms of single or collective behaviors, such as swarming and flocking, area surveillance, stabilizing and maneuvering formations. System architectures required to organize and decide how behaviors are chosen and prioritized can be found in several papers (KUBE; HONG ZHANG, 1994), (BALCH; ARKIN, 1998), (PIRJANIAN et al., 2000) and (BEARD; LAWTON; HADAEGH, 2000). While most of these works deal with software architecture, (ANTONELLI; ARRICHIELLO; CHIAVERINI, 2008) provide a formal mathematical approach to prioritize competing tasks, using the so-called Null Space Behavioral (NSB) method. An application of the NSB to a group of navigating vessels is commented in section 2.2.2.

Different approaches for solving the required cooperative tasks are described in literature. A set of methods based on potential field functions is particularly recurrent, able to solve problems of cooperative navigation, leader-following, formation setting, swarming and collision avoidance. In (LEONARD; FIORELLI, 2001) a decentralized controller based on artificial potentials was proposed. The vehicles were described as point-masses, and potential functions for flocking, schooling, formation control and collision avoidance were presented. Leader followed behavior was introduced through moving reference points referred as artificial beacons. Stability of schooling motion was addressed through Lyapunov analysis. Potential field methods enable mixed, non-concurrent behaviors, such as formation and collision avoidance, which can be applied simultaneously. The technique is, though, subjected to falling in local minimum points, especially when multiple objectives are mixed. In another potential fields based approach, a formation control algorithm using a potential function obtained through structural constraints was proposed in (OLFATI-SABER; MURRAY, 2002). The method results in a collision-free distributed control law. A drawback on the communication topology of both aforementioned works is the need of two-way communication between the agents.

Most of the control theory related literature is focused on the formation control and leader-following missions. The most common way to solve this kind of problem is to monitor relative distances between the members and introduce desired behaviors based on linear functions of these measurements. The so-called virtual-structure approach tries to solve both the formation setting and navigation in formation by forcing the vessels to keep relative positions of a craft group in a tight structure. All motion is controlled jointly, which means that velocities are also matched. In (REN; BEARD, 2004) this concept was explored in a decentralized fashion with the assignment of a virtual dynamics behavior for the structure (using the concepts of virtual mass and inertia matrices and virtual forces applied in the structure). In (LAWTON; BEARD; YOUNG, 2003) three decentralized approaches to the formation control of nonholonomic differential robots were proposed. These methods are mostly based on weighting the effects of two desired behaviors: a (possibly) time-varying formation keeping behavior during travel and the desired transitions between the general positions of the formation structure. Other formation control methods that can be mentioned are (STIPANOVIĆ et al., 2004), which studied the stability of a decentralized method applied to aerial vehicles; (KEVICZKY et al., 2006), which solved the distributed control laws through Model Predictive Control by optimizing of a cost function representing the individual formation error between a vehicle and its neighbors. In (PILZ; POPOV; WERNER, 2011), the introduction of information flow filters designed through standard robust control techniques was evaluated for multi-agent systems with varying communication structures and random communication delays. The objective was to add robustness to the group control by making it insensitive to structural changes in communication.

The so-called Consensus methods try to provide a general framework on how some of the cooperative control problems – such as the ones related to formation, swarming and rendezvous are handled. According to (MURRAY, 2007), the consensus problem is proposed as having a group of vehicles (or more general agents) reaching a common assessment or decision based on distributed information and communication protocols. This value may be defined in terms of system states, sensor readings, parameters estimations, etc. Most of the coordination tasks described above can be thought of as particular cases under the consensus techniques.

Consensus in multi-robot systems have been introduced with such description as a control problem in (REN; BEARD; MCLAIN, 2004), providing analysis on single-integrator linear systems. The work from (OLFATI-SABER; MURRAY, 2004) is also worth mentioning, in which the stability of linear consensus protocols are explored for networks with switching topologies and time-delays. Important requirements about the connectivity of graph network structure for stability analysis and the relation of rate of convergence with the eigenvalues of the graph Laplacian matrix were derived in it.

A more comprehensive approach, in which second order dynamics are taken in account and stability of the group dynamics, was studied in (FAX; MURRAY, 2002) as a linear control problem using algebraic graph theory. An information flow filter, such as the one in (PILZ; POPOV; WERNER, 2011) was used to guarantee stability even under an evolving network structure. Review articles, containing general basis for linear consensus theory can be found in (WEI REN; BEARD, 2005) and in (REN; BEARD; ATKINS, 2007).

While many of the aforementioned works deal with robustness of the protocols under switching network topologies, other approaches try to handle communication problems with the introduction of parallel observers, such as (HADAEGH; SMITH, 2007). In this approach, effects from network connection failure are mitigated and a general observation vector from all the agents can be estimated from a single one, enabling more effective control design. The problem is that this method also brings possible destabilization effects, requiring adequate network topology to guarantee convergence.

Distributed consensus estimators can also be applied to monitor joint variables in wireless sensor networks. This technique fuses the data of multiple sensor agents, enabling more accurate and reliable measurements. In (OLFATI-SABER; SHAMMA, 2005), a linear, first order consensus protocol is introduced as sensor fusion method for a distributed sensor system. Extending this idea, (OLFATI-SABER, 2005) applies this consensus protocol in decentralized Kalman Filter (named micro-Kalman Filter) to be embedded in distributed sensors. The architecture of the system, using both the distributed filter and the linear consensus protocols to estimate sensor and linear data, is shown in Figure 1.

Research in more advanced consensus methods can be seen in (LIN; JIA; LI, 2008), in which a distributed H_∞ controller is designed to cope with model uncertainties, communication delays and switching network topologies; the work was extended in (LI; DUAN; CHEN, 2011) to introduce the concept of H_∞ performance region in order to evaluate robustness properties of communication topologies.

Figure 1 - Architecture of the Distributed Kalman Filter

Source: (OLFATI-SABER, 2005)

2.2.1. Sliding Mode Cooperative Control

Literature is very extensive in both linear and nonlinear control methods for cooperative control. Yet, sliding modes control has not received much attention. Most of the papers that mention or use sliding mode techniques are presented next. Several approaches, either using standard or modified SMC methods for leader-following, formation control and swarm aggregations have been studied. A key aspect of the application of SMC in cooperative control is an adequate definition of the sliding surfaces.

In (GAZI, 2005), the concept of potential field functions for swarm aggregations, formation control and collision avoidance was used and presented in the form of a Kinematic model, in which no dynamics of the agents is considered. In this kinematic approach, the control law for N agents is given as $-\nabla_x J(x)$, for $i = 1, \dots, N$. The function $J(x)$ is a potential function of the relative positions of the agents that can incorporate the repulsive and attractive behavior required for swarm aggregation,

formation control and collision avoidance. Through the introduction of a sliding mode control law, the dynamics of agents can be, then, considered, with the advantage of the robust characteristics of this kind of controller, accounting for bounded modeling uncertainties. The potential field is incorporated through the sliding manifold variable definition for each agent:

$$\sigma^i = \dot{x}^i + \nabla_{x^i} J(x) = 0, \quad i = 1, \dots, N$$

One interesting characteristic of this approach is the flexibility to use any potential function that provides the desired behavior characteristic, including obstacle avoidance and navigation. Their following work, (JINGYI; ORDONEZ; GAZI, 2007), provides an extension of the concept, introducing a target tracking behavior. An important drawback on the usage of potential fields is the difficulty of defining such functions, especially when multiple objectives are integrated, which may give rise to local minimum points.

A more standard approach, formally defining a directed communication graph structure for leader-follower behavior and defining a decentralized control law based on local error measurements was given in (FERRARA; FERRARI-TRECCATE; VECCHIO, 2007). The sliding manifold vector was calculated as the local error (the weighted sum of the relative position errors between an agent and its neighbors, without error velocities introduced). Stability proof and definition of control parameters through the concept of Input-State Stability is provided and simple case studies with simulation results are shown. As stated by the authors, the paper does not address perturbations affecting the agents' behavior.

The work from (DEFOORT et al., 2008) provides an applied SMC leader-follower and formation control method for nonholonomic constrained two wheeled robots. The coordination scheme is based on the relative distance between a follower robot and the leader (which means now relative positioning between followers is taken in account). As the controlled variables are the individual relative positions of each agent to the leader, standard sliding mode implementation is straightforward. In order to reduce information flow between the agents, an alternate algorithm is proposed using second order sliding modes. The concept of higher order sliding modes will be given in details in section 3.4.4. The overall idea is to use the high order derivatives of the sliding manifold to stabilize the system. The method in this paper uses the twisting

algorithm (LEVANT, 1993), while modifying the sliding variable so that it does not require the derivative of the relative positioning error (thus, diminishing the need of velocity in the wireless information flow).

A variation of standard approaches for Cooperative Sliding Mode is given in (KHOO; XIE; MAN, 2009). The method was also applied to two wheeled robots and has very similar characteristics to the work from (DEFOORT et al., 2008) in the modeling and error definition, but, as opposed to most SMC methods, it defined a nonlinear integral sliding variable for each agent i :

$$\sigma_i = e_i(t) - e_i(t_0) + \int_{t_0}^t \left[\frac{\alpha}{2} e_i(t) + \frac{\beta}{2^\gamma} e_i^\gamma(t) \right] dt$$

The error variable e_i measures the relative distance between the follower agent and the leader. Parameters α , β and γ are real numbers with $0 < \gamma < 1$ and can be tuned to obtain adequate time response.

In (CAO; REN, 2011), several distributed consensus algorithms with variable structure control (VSC) were proposed for formation and leader following control and swarm tracking. The Variable Structure Control is a set of methods in which discontinuous control laws are introduced and of which SMC is part of. This VSC consensus law is based largely on linear consensus methods, with the advantage of being able to cope with varying communication structures. For first order systems, with controlled variable r_i and with α , β and a_{ij} control parameters, each agent control law is given by

$$u_i = -\alpha \sum_{j=0}^n a_{ij}(r_i - r_j) - \beta \text{sign} \left[\sum_{j=0}^n a_{ij}(r_i - r_j) \right]$$

For second order systems, with v_i being the agents' velocities and γ an extra control gain,

$$u_i = -\sum_{j=0}^n a_{ij}[(r_i - r_j) + \alpha(v_i - v_j)] - \beta \text{sign} \left[\sum_{j=0}^n a_{ij}[\gamma(r_i - r_j) + \alpha(v_i - v_j)] \right]$$

Note that, even though not referred such as this, the input terms of the *signum* function can be thought of as the sliding variables of the controllers. An algorithm for swarm tracking, based on potential functions and very similar to the one from (GAZI, 2005)

was also proposed. This paper provides means to prove Lyapunov stability of the consensus algorithms even under a varying or switching network topology, showing how the introduction of SMC or other VSC algorithms and the analysis of systems under such methods can enhance the cooperative systems robustness.

The work from (GHASEMI; NERSESOV, 2013) proposed a sliding controller that works in two surfaces, one used when the error is bounded through a certain measurement and one when it is not. The authors introduce a generalized error measurement for the cooperative agents, which allows a decentralized scheme. The controller is applied to a group of underactuated wheeled robots, with stability proof being given only for this case. The results for the so-called finite time sliding mode controller were compared to standard sliding mode applied to this group, showing a slight advantage of their method energy-wise.

One of the latest works in sliding mode methods applied in cooperative control is given in (LIU et al., 2015), in which a second-order super twisting sliding mode control algorithm was used (LEVANT, 1993). This is a technique that works in the second derivative of the sliding variable without the need of high order derivatives of the system states and provides continuous control, reducing the chattering phenomenon that is common in SMC. The application of the method is straightforward, but in this paper, it was required that all agents had information on the leader's states.

2.2.2. Cooperative control in Surface Vessels

The concept of cooperative control methods has been largely explored in a diversity of autonomous machines, such as planes, helicopters, cars and wheeled mobile robots, as exemplified in (MURRAY, 2007). It was not until very recently that unmanned surface vessels (USVs) have been discussed as a target for application of a cooperative control concept. Most applications have been focused on formation control of underactuated vessels, category in which most of standard ships fall. Some DPS applications are more recent and shall be discussed in the next section.

In (SKJETNE; MOI; FOSSEN, 2002), a Nonlinear Formation Control method based in Backstepping procedure was presented. The method consists in enforcing the vessel agents to accomplish two parallel tasks, a Geometric Task and a Dynamic Task. The first is implemented through the definition of a formation reference point which will

follow a desired path and then deriving the paths for the individual vessels based on the formation geometry required. The vessels are then forced to be kept in the neighborhood of the desired path. The Dynamic Task is implemented by forcing each agent's speed to be kept in the neighborhood of the assigned speed. The method has the drawback of needing a centralizing update law for the speed, which needs full state information of all vessels in formation. In (IHLE; SKJETNE; FOSSEN, 2004), this problem was solved through the introduction of a synchronization constraint function for the path parameterization variables and the velocity variables. This approach permitted the realization of decentralized controllers. A similar method was used in (KYRKJEBØ; PETTERSEN, 2003), in which the technique was applied to a rendezvous task and under-way replenishment of a leader-following vessel scheme. A recent work from (PENG et al., 2019) proposes an nonlinear output-feedback cooperative formation maneuvering controller. In this work, competing tasks are also solved by splitting the task in a geometric and dynamic one, while collision avoidance (not only among agents, but also between agents and obstacles) and connectivity preservation behaviors are addressed.

The work in (IHLE; JOUFFROY; FOSSEN, 2006) takes a rather complex path, through the introduction of a Lagrangian multiplier method which enforces constraints to the motion control of each system. The constraints are defined in terms of constraint functions, kinematic relations that are chosen for a required task. For example, when simply controlling the relative distance between the members, the constraint function is set as a circle geometric equation. If controlling fixed formation positions, the constraint function is set as the relative position of each agent.

In (ARRICHIELLO; CHIAVERINI; FOSSEN, 2006) a behavior-based approach was used to solve a formation problem when navigating under an unknown environment without the need of low-level path planning. The coordination of multiple required tasks – such as running in a rigid formation, obstacle avoidance and moving the barycenter of a platoon – was solved through the use of the so called Null-Space Behavioral (NSB) method, which work as a centralized guidance system that provides references for the individual control system of the agents. The NSB controller prioritizes the multiple tasks according to the desired structure, which means that it ensures there is no conflict between those. An obvious drawback of this method is the centralization of the NSB controller.

A Sliding Mode method controlling the heading and relative position of an underactuated vessel squad was studied in (FAHIMI, 2007). A geometric transformation from the standard vessel motion equation to input-output equations relating only relative distance and heading between vessels is applied and a standard sliding mode controller is introduced to ensure robust tracking. The system was simulated for six vessels with uncertain parameters and disturbance in motion. In this work, stability is only proved for certain limited operational conditions, which, as recognized by the authors, limits the safe options for practical applications.

2.2.3. DPS Cooperative Control

Research in cooperative control applied to Dynamic Positioned systems is recent, and very scarce in literature. Requirements for usage of multiple vessels in DP mode are often related as relative positioning tasks, mostly restricted to the use of two vessels. Among the most practical uses of relative positioning systems are offloading operations with relative positioning measuring. Even though this may not be considered a decentralized, scalable cooperative control system, it relies on shared data – position information of both vessels is necessary for the control system to run.

One of the first mentions in cooperative DP control is given in (OSHIRO et al., 2012). In this work, relative positioning of two Dynamic Positioned Supply Vessels is controlled with the final objective of assessing the so-called Y subsea launching method and preventing undesirable effects, such as cable slackening. The control system presented in this paper modulates the vertical phase difference between the two connection points of the launching vessels $\tilde{\theta} = \theta - 180^\circ$. The instantaneous phase difference is calculated through a Hilbert transformation of the motion signal of the connection points and a saturated proportional-derivative control action is enforced in the velocity reference for the longitudinal set-point position of one of the vessels. General results for a simulated environment show that the control system can indeed reduce cable slackening. This method requires additional equipment, such as accelerometers installed in the cable connection points and a controlled cable releasing system.

Note that, even though there are two agents involved, the actual feedback law is only applied in one of them, which limits the possible escalation of the method for use in a larger number of vessels. The system can only be considered cooperative in the sense

that more than one agent is controlled to perform a single task. A relevant part of this work is the example of a practical operation in which cooperative methods are advantageous or even a necessity. A scalable method, such as the one presented in the current work, may be used to increase the number agents, while keeping the slackening constraints in the formation definition.

In (MORATELLI et al., 2013) and in (MORATELLI; MORISHITA, 2015), a relative positioning approach was developed for an offloading operation involving a dynamic positioned FPWSO (Floating, Production, Working, Storage and Offloading) and a Shuttle Tanker. The positioning is enforced by means of application of constraints in references and applied in nonlinear joint control laws (a Lagrangian-based controller in the first and a geometric controller in the second). This is used as the constraint definition for the controller. The control laws are based on the overall system dynamics, which means that, in both cases, they are not decentralized. Once again, this control method can be considered cooperative in the sense of usage of multiple agents with a single task, but it does not relate to most modern applications of cooperative robotics, which require scalability and decentralization.

A fully decentralized and nonlinear cooperative DP controller for formation keeping tasks was presented in (FU; JIAO, 2013). This paper addresses the formation control problem of general motion of surface vessels, using and hybrid (in the sense of multiple control tasks) approach which weights the control output between formation tasks, path following tasks and dynamic positioning tasks, keeping relative position between the agents. While it is not the main focus of this work, it provides a Coordinated Dynamic Positioning (CDP) algorithm based on the Backstepping Method. The problem is addressed as a Consensus problem, and this nonlinear method is applied to cope with the geometric nonlinearities introduced with the vessel model transformation matrix. Results for simulations of the system are presented, including when the vessels are in DP mode are presented and discussed. One important feature lacking in the system studied was the presence of environmental disturbances and the adequate action to counteract it, which would probably affect performance.

In (QUEIROZ FILHO; TANNURI, 2013) an extensive numerical and experimental analysis of a linear consensus-based control law applied to DP vessels was presented. The system architecture uses an inner Dynamic Positioning controller loop, adding an

external loop and a feed-forward thruster control to perform the cooperative positioning (Figure 2). In their further work (QUEIROZ FILHO, 2015), this architecture is changed slightly, with the cooperative algorithm being responsible for providing the set point references for the inner DP loops of each vessel.

Figure 2 - Cooperative DP control Architecture

Source: (QUEIROZ FILHO; TANNURI, 2013)

As opposed to most cooperative methods (which are usually developed in the time domain), the system is tuned and analyzed in the frequency domain. This permitted the application of well-established classical control concepts, such as pole placement and disturbance rejection analysis. The cooperative algorithm was based on standard PI control with a coordinate conversion to cope with the transformations between local and global reference systems. Disturbance from wind and waves were introduced in order to evaluate robustness to environmental loads, but in the experimental tests wave motion was shown to affect energy usage.

The linearity assumptions required for the frequency domain control design may limit the widespread application of this architecture. Due to the geometric nonlinearities of the vessel's equations of motion, some control applications use nonlinear methods. In (PENG; WANG; WANG, 2016), a cooperative control method based on the Dynamic Surface Control (SWAROOP et al., 1997) and an adaptive law to calculate environmental disturbance forces from wind, current and waves was studied. The

Dynamic Surface Control is an extension to the Backstepping Method, with the introduction of filters between the virtual control inputs in order to prevent the problem of “explosion of terms” that rise due to repeated differentiation of virtual control inputs. The formation error is defined in the body fixed frame, and cooperative control is solved through state feedback. The overall stability of the system, including the adaptive term, is proved and simulated results are presented. As most of the CDP works cited so far, this paper does not address wave filtering, modeling errors and robustness to communication failure. Further work, introducing a disturbance observer and accounting for input saturation, was presented in (XIA et al., 2019).

Other nonlinear CDP method, based on sliding modes control, is provided in (IANAGUI; TANNURI, 2015) and in (IANAGUI; QUEIROZ FILHO; TANNURI, 2016), in which numerical and experimental analysis of the sliding modes CDP algorithm proposed in the current work was performed. The method was able to handle multiple vessels with different dynamics among each other and was based on an architecture in which each vessel already had an inner loop DP system, and the cooperative distributed controller provided set points for the inner loop. Its main drawback was the lack of wave filtering and large control activity.

Figure 3 - Energy Efficient DP Vessels Formation

Source: (BABIĆ et al., 2016)

The usage of formation control in order to achieve advantages over single-controlled systems was also briefly explored in academy. In (BABIĆ et al., 2016), a linear consensus formation CDP system was applied to floating robots. A virtual force algorithm, to introduce collision avoidance behavior was also introduced. The study was part of a larger project, involving a heterogeneous group of surface and underwater robots working in cooperation for various tasks (SUBCULTRON, 2017).

The main focus of the paper was to show how a smart formation decision-making algorithm can reduce energy usage. The results show how a formation aligned to the oceanic current, with a leader in front of it, can take advantage of the drag weakening effect and reduce thruster usage of the agents that are behind the leader (Figure 3).

Using a linear consensus decentralized protocol and a high level PID controller for a virtual leader set-point generation, (IANAGUI; TANNURI, 2019) have proposed an application in which a floating unactuated load (a drilling platform) was held and maneuvered under environmental loads. The paper shows how the application of the coordination protocol is able to facilitate the load maneuvering task when multiple agents are operating. In a novel approach, it also provides means to control position and heading of the load by controlling the overall formation position. Figure 4 illustrates the effect of the controller introduction when a northeast environmental load is affecting the platform and the offset in its position is corrected.

Figure 4 – Cooperative Hold Back Maneuver (a) Before Control Activation (b) After control Activation.

Source: (IANAGUI; TANNURI, 2019) - modified

3. PRELIMINARY CONCEPTS

In this section, the theoretical concepts used throughout the work are presented.

3.1. System Modeling

To facilitate understanding, the system model is presented in a divided manner. First, vessel rigid body dynamics and hull interaction hydrodynamics are presented. Then, environmental forces from waves, currents and wind acting on a vessel are incorporated. The model developed will be the basis for the development of the DPS control system, the filtering and basic dynamic analysis. It may also be used for simple simulations, even though simulation and validation of the methods proposed were performed in the Numerical Offshore Tank fast-time simulator, which aggregates more complex effects.

3.1.1. Vessel

REFERENCE FRAMES

The vessel's equations of motion used are valid for a 3 degree of freedom (DOF) model of a free-floating body, in which positions for the Easting, Northing and Attitude Angle referenced to an Earth Fixed coordinate system represented by xyz are used as variables. This coordinate system will also be called Inertial or Global Reference Frame (GRF) from now on. Even though the Earth is a moving body in space, it is reasonable to assume the inertial hypothesis as, for this analysis purposes, Earth's motion does not provoke considerable forces in bodies on its surface.

Accurate position of an object on the surface of the planet can be described using a Geographic Coordinate (or Georeferenced) System, *i.e.* through Latitude (LAT) and Longitude (LON) angles. Modern geolocation systems, such as the Global Positioning System (GPS), describe a body's position using these variables. For the objectives of this work, most of the errors that arise from the assumption of a plane surface in the region of the DPS maneuvers can be ignored, as motion occur in a very restricted area (radius minor than 2000 m). For this reason, the GRF will be considered a plane, with the x -axis representing the Easting and the y -axis representing the Northing. The graphical description of this plane with relation to Earth's geoid can be done in several ways, being the most common for engineering purposes the Universal Transverse

Mercator (UTM) coordinate system. A more comprehensive discuss on the conversion relations between the Geographic Coordinate System and the most used reference plans can be found in (OSBORNE, 2013) and in (ASHCRAFT, 1999).

It is also convenient to define a moving frame, $o_L x_L y_L z_L$ attached to each vessel and centered in its midship position. This frame moves and rotate around z_L axis with the body and will be called Local Reference Frame (LRF). Figure 5 displays the geometry of the reference frames in the plane.

Figure 5 - Reference Frames

Source: Author

In the GRF, the location of a vessel midship and its attitude will be given by vector $\boldsymbol{\eta} = [x \ y \ \psi]^T$. In this representation, x is the easting, y is the northing and ψ is the yaw angle.

A vector $\mathbf{p} \in \mathbb{R}^3$ fixed to the LRF and represented in the same frame (noted as $\mathbf{p}^L$) can be represented in the GRF (noted as $\mathbf{p}^G$) through a rotation matrix transformation R described by equation (1).

$$\mathbf{p}^G = \begin{bmatrix} \cos(\psi) & -\sin(\psi) & 0 \\ \sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{bmatrix} \mathbf{p}^L = R_L^G(\psi) \mathbf{p}^L \quad (1)$$

Subscript L and superscript G indicate that this is a transformation of a vector described in the Local Reference Frame to a representation in the Global Reference Frame. The inverse transform is equivalent to the transpose of the $R_L^G(\psi)$ matrix and is given by equation (2).

$$\mathbf{p}^L = R_G^L(\psi) \mathbf{p}^G = R_L^{G^{-1}} \mathbf{p}^G = R_L^{G^T} \mathbf{p}^G \quad (2)$$

Subscript G and superscript L indicate that this is a transformation of a vector described in the Global Reference Frame to the description in the Local Reference Frame. To simplify the notation, from now on, heading variable ψ will be omitted from the transformations but may appear in the required contexts.

The time derivatives of the transformation matrices are given by equation(3).

$$\dot{R}_L^G = -\dot{\psi} \begin{bmatrix} \sin(\psi) & \cos(\psi) & 0 \\ -\cos(\psi) & \sin(\psi) & 0 \\ 0 & 0 & 0 \end{bmatrix}; \dot{R}_G^L = \dot{R}_L^{G^T} \quad (3)$$

Finally, it is also useful to calculate the transformation of the time derivative of a vector represented in LRF to a GRF representation and the inverse transform, as shown in differential equations (4), (5) and (6).

$$\dot{\mathbf{p}}^G = \dot{R}_L^G \mathbf{p}^L + R_L^G \dot{\mathbf{p}}^L = \dot{R}_L^G R_G^L \mathbf{p}^G + R_L^G \dot{\mathbf{p}}^L = S_G \mathbf{p}^G + R_L^G \dot{\mathbf{p}}^L \quad (4)$$

$$\dot{\mathbf{p}}^L = \dot{R}_G^L \mathbf{p}^G + R_G^L \dot{\mathbf{p}}^G = \dot{R}_G^L R_L^G \mathbf{p}^L + R_G^L \dot{\mathbf{p}}^G = S_L \mathbf{p}^L + R_G^L \dot{\mathbf{p}}^G \quad (5)$$

$$S_G = \dot{R}_L^G R_G^L = \begin{bmatrix} 0 & -\dot{\psi} & 0 \\ \dot{\psi} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}; S_L = S_G^T \quad (6)$$

The matrices $S_G = \dot{R}_L^G R_G^L$ and $S_L = \dot{R}_G^L R_L^G$ are skew-symmetric and permit the calculation of the accelerations in both reference frames. In fact, a matrix multiplication $S_G \mathbf{p}$ represents the cross-product operator between the angular velocities vector and the arbitrary vector $\mathbf{p}$.

EQUATIONS OF MOTION

Derivation of the vessel midship equations of motion used in this work is presented in APPENDIX A, summarized by equation (7):

$$\begin{bmatrix} m & 0 & 0 \\ 0 & m & mx_G \\ 0 & mx_G & I_z \end{bmatrix} \begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{r} \end{bmatrix} - \begin{bmatrix} 0 & mr & mx_G r \\ -mr & 0 & 0 \\ -mx_G r & 0 & 0 \end{bmatrix} \begin{bmatrix} u \\ v \\ r \end{bmatrix} = \begin{bmatrix} F_u \\ F_v \\ F_r \end{bmatrix} \quad (7)$$

The vector $\mathbf{v} = [u \ v \ r]^T$ is the vessel midship velocity represented in the LRF; $\dot{\mathbf{v}} = [\dot{u} \ \dot{v} \ \dot{r}]^T$ is the vessel acceleration vector in the same frame and $\mathbf{F}^L = [F_u \ F_v \ F_r]^T$ lumps the generalized external forces described in the LRF. Parameter m is the vessel rigid body mass, I_z is the moment of inertial around z_L , and x_G is the location of the center of mass with respect to the vessel midship position described in the LRF.

It is convenient to declare the usual nomenclature of some of the motion variables. The vessel midship velocity component in the x_L axis, described in the LRF and relative to the GRF, u , is called *surge velocity*. The midship velocity component in the y_L axis, described in the LRF and relative to the GRF, v , is called *sway velocity*. The yaw velocity, relative to the GRF (and independent of the frame's description), r is called *rate-of-turn*.

The generalized forces in equation (7) are composed of multiple factors, as they are the sum of all external forces applied to the floating body. External influences such as hydrodynamic forces, operational forces and environmental action are accounted. The actuator forces, the manipulated variables in the DP system, are also included. In equation (8), the accounted forces in the model developed in the current work are evinced. While it does not account for all the possible external forces, it is reasonable to assume that these covers most of the factors affecting a DP controlled vessel.

$$\mathbf{F}^L = \mathbf{F}_{thrust} + \mathbf{F}_{hydro} + \mathbf{F}_{oper} + \mathbf{F}_{curr} + \mathbf{F}_{wind} + \mathbf{F}_{wave} \quad (8)$$

Here, $\mathbf{F}_{thrust}$ is the force vector of actuator forces, which shall be calculated by the control system, $\mathbf{F}_{hydro}$ is the vector of forces arising from the hull interaction with the water, $\mathbf{F}_{oper}$ is the vector of operational loads, which may come from cables, mooring and hull-to-hull interactions and for most operations will be set to zero, $\mathbf{F}_{curr}$, $\mathbf{F}_{wind}$ and $\mathbf{F}_{wave}$ are the vectors of environmental forces due to oceanic current, wind and first and second order wave interactions.

HYDRODYNAMIC FORCES

Hydrodynamic forces arise from the interactions between the vessel hull and the fluid around it. Potential flow theory is one of the most helpful tools to perform marine hydrodynamics analysis, providing means to describe linear frequency domain models for a vessel's motion in fluid. While the wave motion models presented in section 3.1.2 are also due to the potential flow model, the focus for now will be on inertial and damping effects. The main assumptions required for the application of potential flow theory is the absence of viscosity and compressibility in the fluid and the irrotationality hypothesis (NEWMAN, 1977).

The added mass coefficients are the most relevant parameters for a fluidic inertial reaction to a vessel's movement. The name comes for the analogy with the mass of a

rigid body for its inertial response, as some of its properties are shared. With the correct assumptions, it can be seen as a virtual mass added to the vessel, due to the motion of the fluid surrounding an accelerating body. Yet, the added mass values depend on the geometry of the hull, which makes the analogy limited once there is geometrical asymmetry in some of the degrees of freedom c.

For a body floating on a fluidic free surface, the added mass coefficients are a function of the motion frequency and usually are calculated with hydrodynamic damping coefficients, known as potential damping, which are also functions of hull geometry and frequency. Actual nature of added mass and potential damping effects come from fluidic radiation forces. For most marine motion control systems applications, added mass coefficients and potential damping are considered constant and calculated for low frequency. This abridgement cancels out the potential damping terms. This assumption is valid within the usual natural periods of the considered feedback control systems, once natural periods are of the order of 100-200s and drag forces along with viscous damping dominate the potential damping terms (FOSSEN, 2011).

The constant added mass matrix for the studied vessel will be given by equation (9). Its coefficients can be calculated by the usage of numeric hydrodynamic software, such as WAMIT®. The assumption of hull symmetry in the $x_L z_L$ plane allows the canceling of cross-coupling terms between some of the components. In (NEWMAN, 1977), the added mass matrix is shown to be symmetric.

$$M_A = - \begin{bmatrix} X_{\dot{u}} & 0 & 0 \\ 0 & Y_{\dot{v}} & Y_{\dot{r}} \\ 0 & Y_{\dot{r}} & N_{\dot{r}} \end{bmatrix} \quad (9)$$

The variables nomenclature follows the SNAME (Society of Naval Architects and Marine Engineers) notation (SNAME, 1950). The added mass matrix M_A is composed of the hydrodynamic derivatives $X_{\dot{u}}$, $Y_{\dot{v}}$, $Y_{\dot{r}}$ and $N_{\dot{r}}$.

The inertial hydrodynamic forces will be given by equation (10) (DE KAT; WICHERS, 1991):

$$\mathbf{F}_{hydro} = - \begin{bmatrix} X_{\dot{u}} & 0 & 0 \\ 0 & Y_{\dot{v}} & Y_{\dot{r}} \\ 0 & Y_{\dot{r}} & N_{\dot{r}} \end{bmatrix} \begin{bmatrix} \dot{u}_r \\ \dot{v}_r \\ \dot{r}_r \end{bmatrix} + \begin{bmatrix} 0 & 0 & Y_{\dot{v}}v_r + Y_{\dot{r}}r_r \\ 0 & 0 & -X_{\dot{u}}u_r \\ -Y_{\dot{v}}v_r - Y_{\dot{r}}r_r & X_{\dot{u}}u_r & 0 \end{bmatrix} \begin{bmatrix} u_r \\ v_r \\ r_r \end{bmatrix} \quad (10)$$

Here, $\mathbf{v}_r = [u_r \ v_r \ r_r]^T$ is the vector of velocities of the fluid motion relative to the vessel, expressed in the LRF. Hydrodynamic forces due to drag and viscous damping

are not accounted in F_{hydr} . The term $(X_{\dot{u}} - Y_{\dot{v}})u_r v_r$ in the right hand side of the yaw inertial equation is referred in literature as *Munk Moment* and has destabilizing effects in yaw during surge motion. It is the only nonzero term for an ideal fluid. The Munk Moment is usually lumped with effects from hull drag resistance forces (which will later be introduced in F_{curr}), as it is impossible to separate drag and potential flow effects from towing tank test data.

The application of equation (10) and equation (8) to the rigid body motion equation (7) provides the complete motion equation with hydrodynamic effects, equation (11).

$$\begin{aligned}
 & \begin{bmatrix} m & 0 & 0 \\ 0 & m & mx_G \\ 0 & mx_G & I_z \end{bmatrix} \begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{r} \end{bmatrix} + \begin{bmatrix} X_{\dot{u}} & 0 & 0 \\ 0 & Y_{\dot{v}} & Y_{\dot{r}} \\ 0 & Y_{\dot{r}} & N_{\dot{r}} \end{bmatrix} \begin{bmatrix} \dot{u}_r \\ \dot{v}_r \\ \dot{r}_r \end{bmatrix} - \begin{bmatrix} 0 & mr & mx_G r \\ -mr & 0 & 0 \\ -mx_G r & 0 & 0 \end{bmatrix} \begin{bmatrix} u \\ v \\ r \end{bmatrix} \\
 & - \begin{bmatrix} 0 & 0 & Y_{\dot{v}} v_r + Y_{\dot{r}} r_r \\ 0 & 0 & -X_{\dot{u}} u_r \\ -Y_{\dot{v}} v_r - Y_{\dot{r}} r_r & X_{\dot{u}} u_r & 0 \end{bmatrix} \begin{bmatrix} u_r \\ v_r \\ r_r \end{bmatrix} \\
 & = \mathbf{F}_{thrust} + \mathbf{F}_{oper} + \mathbf{F}_{curr} + \mathbf{F}_{wind} + \mathbf{F}_{wave}
 \end{aligned} \tag{11}$$

The relative velocity vector $\mathbf{v}_r$ can be calculated by the difference between the rigid body's velocity and the current velocity vector $\mathbf{v}_c$, as illustrated in Figure 6 and equation (12), which gives the slowly varying current velocities decomposed in the LRF.

$$\mathbf{v}_r = \mathbf{v} - \mathbf{v}_c \tag{12}$$

The relation between the absolute current velocity V_c , the global current angle ψ_c and the current velocity vector $\mathbf{v}_c$ is given by equation (13). Note that the irrotationality hypothesis makes $r_c = 0$.

$$\mathbf{v}_c = \begin{bmatrix} u_c \\ v_c \\ r_c \end{bmatrix} = \begin{bmatrix} V_c \cos(\psi_c - \psi) \\ V_c \sin(\psi_c - \psi) \\ 0 \end{bmatrix} \tag{13}$$

Figure 6 - Current Velocity Vector

Source: Author

The derivative of equation (13) yields

$$\dot{\mathbf{v}}_r = \dot{\mathbf{v}} - \dot{\mathbf{v}}_c = \begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{r} \end{bmatrix} - \begin{bmatrix} 0 & r & 0 \\ -r & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} u_c \\ v_c \\ r_c \end{bmatrix} = \dot{\mathbf{v}} - S_L \mathbf{v}_c \quad (14)$$

The application of equations (12) and (14) to equation (11) with the adequate cancelations finally gives the complete low frequency model motion equation, accounting for hydrodynamic inertial effects:

$$\begin{aligned} & \begin{bmatrix} m + X_{\dot{u}} & 0 & 0 \\ 0 & m + Y_{\dot{v}} & mx_G + Y_{\dot{r}} \\ 0 & mx_G + Y_{\dot{r}} & I_z + N_{\dot{r}} \end{bmatrix} \begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{r} \end{bmatrix} \\ & + \begin{bmatrix} 0 & -mr & -mx_G r - Y_{\dot{v}} v - Y_{\dot{r}} r \\ mr & 0 & X_{\dot{u}} u \\ mx_G r + Y_{\dot{v}} v + Y_{\dot{r}} r & -X_{\dot{u}} u & 0 \end{bmatrix} \begin{bmatrix} u \\ v \\ r \end{bmatrix} \\ & + (Y_{\dot{v}} - X_{\dot{u}}) \begin{bmatrix} 0 & r & 0 \\ r & 0 & 0 \\ -v & u_c - u & 0 \end{bmatrix} \begin{bmatrix} u_c \\ v_c \\ r_c \end{bmatrix} = \mathbf{F}_{thrust} + \mathbf{F}_{oper} + \mathbf{F}_{curr} + \mathbf{F}_{wind} + \mathbf{F}_{wave} \end{aligned} \quad (15)$$

In vector form, equation (15) becomes

$$M \dot{\mathbf{v}} + C_{cor}(\mathbf{v}) \mathbf{v} + C_{cur}(\mathbf{v}, \mathbf{v}_c) \mathbf{v}_c = \mathbf{F}_{thrust} + \mathbf{F}_{oper} + \mathbf{F}_{curr} + \mathbf{F}_{wind} + \mathbf{F}_{wave} \quad (16)$$

In which

$$\begin{aligned} M &= \begin{bmatrix} m + X_{\dot{u}} & 0 & 0 \\ 0 & m + Y_{\dot{v}} & mx_G + Y_{\dot{r}} \\ 0 & mx_G + Y_{\dot{r}} & I_z + N_{\dot{r}} \end{bmatrix} \\ C_{cor}(\mathbf{v}) &= \begin{bmatrix} 0 & -mr & -mx_G r - Y_{\dot{v}} v - Y_{\dot{r}} r \\ mr & 0 & X_{\dot{u}} u \\ mx_G r + Y_{\dot{v}} v + Y_{\dot{r}} r & -X_{\dot{u}} u & 0 \end{bmatrix} \\ C_{cur}(\mathbf{v}, \mathbf{v}_c) &= (Y_{\dot{v}} - X_{\dot{u}}) \begin{bmatrix} 0 & r & 0 \\ r & 0 & 0 \\ -v & u_c - u & 0 \end{bmatrix} \end{aligned} \quad (17)$$

The matrix C_{cur} refers to “inertial” hydrodynamic effects that rise from the potential flow theory, while the vector $\mathbf{F}_{curr}$ lumps all current drag effects.

ACTUATION MODEL

For DP control design purposes, perfect response is usually considered between the commanded thrust force and its liquid output. This assumption is due to the vessel's much slower dynamics than of the servo-actuators used for the propulsion. Phase lag, amplitude reduction and nonlinear effects due to dead band and reduced efficiency can reduce the DP system's performance when the commands disturbances frequencies increase. Operational loads, such as towing strains, may reach these

higher frequencies and, therefore, at least a simple dynamic model for the actuators shall be introduced for simulation purposes.

A first order model for a vessel with n actuators will be given by (18). While servo and electric motors dynamics do not have this exactly characteristic, the assumption of a motor speed and/or torque controller in the loop permit this approximation.

$$\dot{\mathbf{\tau}}_{thr} = -A_{thr}(\mathbf{\tau}_{thr} + \mathbf{u}_{thr}) \quad (18)$$

The actuator output force vector $\mathbf{\tau}_{thr} = [\tau_1 \ \tau_2 \ \dots \ \tau_n]^T$ is the state vector, $\mathbf{u}_{thr} = [u_1 \ u_2 \ \dots \ u_n]^T$ is the commanded thrust vector and the system matrix is given by

$A_{thr} = diag\left(\left[\frac{1}{T_{thr,1}} \ \frac{1}{T_{thr,2}} \ \dots \ \frac{1}{T_{thr,n}}\right]\right)$. Each propulsion unit has time constant $T_{thr,n}$ and this value depends largely on its type and size. Due to its inertial characteristics, Fixed-Speed/Controllable-Pitch Propellers have faster time constants when compared to Controllable-Rotation/Fixed-Pitch Propellers.

The relation between the thrust output vector $\mathbf{\tau}_{thr} \in \mathbb{R}^n$ and the control vector $\mathbf{F}_{thrust} \in \mathbb{R}^3$ is considered to be purely geometrical and depends on the position configuration of the actuators along the vessel hull. The contribution of each actuator thrust to the control vector is calculated by decomposing each vector to the LRF basis and building a configuration matrix $T_{3 \times n}(\boldsymbol{\alpha})$, in which $\boldsymbol{\alpha}$ is the vector of thrusters' orientation angles α_i relative to the LRF's x_L axis in the plane $o_L x_L y_L$. Note that, depending on the propulsion system type, α_i may be fixed to a direction (90° for tunnel thrusters and 0° for main thrusters) or varying, such as in an azimuth thruster. Each propeller's block can then be built by

$$\mathbf{F}_{thr,i} = \begin{bmatrix} \cos(\alpha_i) \\ \sin(\alpha_i) \\ -a_i \cos(\alpha_i) + b_i \sin(\alpha_i) \end{bmatrix} \tau_i = \mathbf{T}_{i3 \times 1}(\alpha_i) \tau_i \quad (19)$$

In which a_i and b_i are the displacement between the actuator axis and the vessel centerline axis (x_L) and the vessel midship axis (y_L), respectively. The thruster configuration matrix is then built as the column sequence of vector $\mathbf{T}_{i3 \times 1}$, that is $T_{3 \times n}(\boldsymbol{\alpha}) = [\mathbf{T}_{13 \times 1}(\alpha_1) \ \mathbf{T}_{23 \times 1}(\alpha_2) \ \dots \ \mathbf{T}_{n3 \times 1}(\alpha_n)]$. Figure 7 exemplifies the construction of a configuration matrix of a vessel with two main thrusters, two tunnel thrusters and one azimuth thruster.

Figure 7 - Thruster Configuration Example

Source: Author

The thruster forces will then be given by

$$\mathbf{F}_{thrust} = T_{3 \times n}(\alpha) \boldsymbol{\tau}_{thr} \quad (20)$$

The usual DP control system approach calculates the required forces $\mathbf{F}_{thrust}$ to maintain a vessel position. The distribution of thruster output forces $\boldsymbol{\tau}_{thr}$ that satisfy equation (20) has infinite solutions, and optimal thrust allocation algorithms are required to solve it efficiently, energy-wise. Section 3.2.3 provides a brief explanation to basic thrust allocation technique.

Even though not present in the model used for control design purposes, it is important to remark that in most DP systems the commanded variable is actually the propeller's angular velocity or its pitch. This means that a nonlinear transformation, usually performed through a lookup table is necessary to map the required forces τ_{thr} to the commanded angular velocity n_{thr} or the propeller pitch angle α_{thr} . Figure 8 shows an example of thruster mapping. For modeling purposes, equations (21), (22) and (23) provide the power and torque as functions of geometry, vessel advance velocity and table constants.

$$J_0 = \frac{V_{thr}}{n_{thr} D_{thr}} \quad (21)$$

$$\tau_{thr} = K_T \rho |n_{thr}| n_{thr} D_{thr}^4 \quad (22)$$

$$Q_{thr} = K_Q \rho |n_{thr}| n_{thr} D_{thr}^5 \quad (23)$$

In which J_0 is an advance velocity nondimensional parameter, V_{thr} is the water input velocity, K_T and K_Q are the propeller's thrust and torque constants, Q_{thr} is the propeller torque, ρ is the water density and D_{thr} is the propeller diameter.

Figure 8 - (a) Thruster Constants as function of advance speed J_0 . (b) Thruster Force as function of Power Output.

Source: (TANNURI, 2002)

Note that the liquid output thrust varies largely with geometry, and possible disagreements in the diameter can cause large load differences, which mean that experimental data needs to be raised to minimize errors. An alternative approach, that provides an almost linear mapping, is to use electronic motor inverters that can perform direct torque control (applicable only on electric motors). When this method is used, it is important to address eventual spin effects due to ventilation and in-and-out-of-the-water effects. More on thruster control techniques can be found in (SØRENSEN, 2013).

3.1.2. Environmental Action

The external uncontrolled loads considered for the model will be the hull interaction effects with wind gusts (F_{wind}), oceanic currents drag (F_{curr}) and wave forces (F_{wave}). The wave motion loads will be split into two separate effects, referred in literature as first order motion and second order drift forces. As all environmental interaction depends strongly on the vessel's complex geometry, parametric data necessary to the calculations is usually obtained through numeric integration software and/or experimental analysis. A thorough explanation on the various environmental loads affecting a vessel can be found in (DE KAT; WICHERS, 1991) and in (FOSSEN, 2011).

WIND FORCES

Wind forces will be calculated through the pressure integration over the frontal and lateral projected areas of the vessel exposed to the air (windage areas).

$$\mathbf{F}_{wind} = 0.5\rho_{air}V_{wr}^2 \begin{bmatrix} A_{front}C_{uw}(\psi_w - \psi) \\ A_{lat}C_{vw}(\psi_w - \psi) \\ A_{lat}L_{pp}C_{rw}(\psi_w - \psi) \end{bmatrix} \quad (24)$$

Here, ρ_{air} is the air density; V_{wr} is the instantaneous relative wind velocity; A_{front} and A_{lat} frontal and lateral windage areas; L_{pp} is vessel hull length between perpendiculars C_{uw} , C_{vw} and C_{rw} are the nondimensional wind drag coefficients, as function of the relative angle between the global wind direction (ψ_w) and the vessel yaw angle (ψ). The angle $\psi_w - \psi$ is referred to as wind attack angle (see Figure 9). As the global wind velocity and its effects are much higher than the vessel velocities' effects when operating in DP, only the wind velocity shall be used in expression, making $V_{wr} = V_w$, in which V_w is the global wind velocity.

Figure 9 - Wind relative and global velocities related to the vessel

Source: Author

The drag coefficients are obtained either through experiment in the wind tunnel or thorough databases of and/or previous calculations available in the literature. In (OCIMF, 1977) a study on the parametrization of loads in Tanker vessels was performed, raising wind coefficients for this kind of ships. In (ISHERWOOD, 1972) and in (OWENS; PALO, 1982) the parameterization of merchant ships using regression techniques and geometric characteristics of the vessels was performed and tabled. Figure 10 displays an example of nondimensional wind parameters for a barge as function of the angle of attack.

Note that equation (24) is a function of instantaneous wind velocities, which can be represented through combination of mean wind bias and a wind spectrum. A brief review of the most common wind spectra used for simulation purposes is given in APPENDIX B.

Figure 10 - BGL1 Barge experimental wind coefficients

Source: (TANNURI, 2002)

OCEANIC CURRENT FORCES

Oceanic currents are considered to be slow varying fields. For the purpose of this work, this means they are virtually constant. The current load calculation is similar to the wind. The formulation presented here is shown as used by (WITCHERS, 1987). The resulting forces are given by

$$\mathbf{F}_{curr} = 0.5 \rho V_r^2 L_{pp} T \begin{bmatrix} C_u(\psi_c - \psi) \\ C_v(\psi_c - \psi) \\ L_{pp} C_r(\psi_c - \psi) \end{bmatrix} \quad (25)$$

Here, ρ is the water density, T is the vessel draft, L_{pp} is the vessel hull length, C_u , C_v and C_r are non-dimensional coefficients as function of the current angle of attack relative to the vessel heading, $\psi_c - \psi$ (see Figure 6) and V_r is the current relative velocity modulus:

$$V_r = \sqrt{(u - u_c)^2 + (v - v_c)^2} \quad (26)$$

The most complex task in defining the static current forces is to determine the coefficient functions, $C_u(\psi_c - \psi)$, $C_v(\psi_c - \psi)$ and $C_r(\psi_c - \psi)$. These parameters are usually tabled and, just as the wind drag parameters, obtained either empirically or by semi-empirical methods. For oil tankers, (LEITE et al., 1998) produced a semi-empirical method based on the low aspect ratio wing theory, having its results extended by (SIMOS, 2001) in order to address yaw rotation effects. For other hull geometries, ITTC (International Towing Tank Conference) reports and cross flow drag principles are commonly used (FOSSEN, 2011). Figure 11 displays an example of nondimensional current parameters for a barge as function of the angle of attack.

Figure 11- BGL1 Barge experimental current coefficients

Source: (TANNURI, 2002)

WAVE EXCITATION

The most common sea waves are formed through the interaction between the wind over the free surface and the ocean water. These are important on the context of marine engineering, once they are in the frequency range that is capable to excite significantly the vessels dynamics. Shorter waves usually grow until instability and “break”, in such way that energy in high frequencies waves is dissipated. The wind generated waves may reach an equilibrium state if there is enough fetch and time for the wind to act over the surfaces. The sea in this condition is called *fully developed sea*. If the fetch is limited and the peak energy frequencies are higher, the condition is called *developing sea*.

For DP Systems, the effects of this excitation are two-fold. The high frequency oscillating motion that in average performs no work, modeled here as first order motion, need to be filtered out from sensor readings so that it is not compensated by the control system (which would otherwise spend too much energy without need). The second order effects, which produce mean loads that create the so-called mean and slow wave drift, need to be compensated by control action or feedforward techniques.

Wind waves are randomly created and spread, and their effect will be modeled here by spectral density functions of frequency $S_{\zeta}(\omega_w)$, characterized by statistical parameters called “spectral moments of order k ”, m_k , defined by equation (27):

$$m_k = \int_0^{\infty} \omega_w^k S_{\zeta}(\omega_w) d\omega_w \quad (27)$$

The spectral moment values may be obtained through sea state sampling, by taking measurements of other statistical parameters, such as the significant height H_s , which is the mean value of the 1/3 highest waves:

$$H_s = 4\sqrt{m_0} \quad (28)$$

The central period and the mean period between zeros, respectively T_1 and T_2 are given by equation (29).

$$T_1 = 2\pi \frac{m_0}{m_1}; \quad T_2 = 2\pi \sqrt{\frac{m_0}{m_2}} \quad (29)$$

These periods are the basis to determine the spectrum peak frequency ω_p (or the peak period T_p), related to T_1 and T_2 by equation (30).

$$\omega_p = \frac{2\pi}{T_p} = \frac{2\pi}{k_1 T_1} = \frac{2\pi}{k_2 T_2} \quad (30)$$

Constants k_1 and k_2 depend on the standard spectrum adopted. A brief review of the most common wave spectra used for simulation purposes is given in APPENDIX B. In the scope of this work, the JONSWAP spectrum will be used for simulations and model testing, as it has been used in most engineering design applications (SIMOS, 2014).

FIRST ORDER WAVE MOTION

The calculation of a floating body's motion when excited by waves characterized by the spectra described in the last section can be performed through a spectral crossing between the adopted wave spectrum $S_\zeta(\omega_w)$ and first order functions that model the free surface pressure effects over the hull. These transfer functions, usually described in the frequency domain, are a classical tool in marine engineering and naval architecture to describe a vessel motion under wave incidence and are called Response Amplitude Operators (RAO). Adequate care must be taken as it considers the hypothesis of relatively small amplitude motion (permitting linearity simplifications) and, in this modeling, only first order effects from potential field theory are applied. The total motion of the i^{th} degree of freedom ($i = u, v, r$) calculated through spectral crossing is given by equation (31) (NEWMAN, 1977).

$$S_i(\omega_w) = |H_i(j\omega)F_i(j\omega)|^2 S_\zeta(\omega_w) \quad (31)$$

The function $F_i(j\omega)$ is called force RAO and represents the frequency response of the pressure induced forces from the incident waves' elevation. It depends mainly on Hull's geometry and the waves' angle of attack $\beta_w = \psi_{wave} - \psi$. The function $H_i(j\omega)$ is called force-to-motion RAO and is also a function of the hull's geometry and the body's mass distribution (FOSSEN; PEREZ, 2009). Figure 12 shows a response plot for $H_i(j\omega)F_i(j\omega)$ of an oil tanker vessel, obtained through WAMIT®. Note that there are multiple plots for each degree of freedom, as the response varies with different wave incidence angles. To get the motion response in the time domain, the inverse discrete Fourier transform is applied (just as with the wind case) for n frequencies:

$$\xi_i = \sum_{j=1}^n \sqrt{2|H_i(j\omega)F_i(j\omega)|^2 S_\zeta(\omega_{wj}) \Delta\omega} \cos(\omega_{wj}t + \angle(H_i(j\omega)F_i(j\omega)) + \phi_{wj}) \quad (32)$$

The operator $\angle(\cdot)$ expresses the RAO Phase and ϕ_w is the random wave phase. The behavior of the RAO's is quite similar to that of a low-pass filter. Indeed, from the control engineering point of view, the total motion of the vessel can be approximated by a signal spectrum obtained using a linear filter $G_i(j\omega)$ excited by a Gaussian noise w with uniform spectrum S_w through the frequencies of interest. In this case,

$$S_i(\omega) = |G_i(j\omega)|^2 S_w \quad (33)$$

For the task of control design, the functions in equation (33) may be simplified by excitation of a second order filter driven by noise input, as in equation (34).

$$G_i(s) = K_i^{RAO} \cdot \frac{K_w s}{s^2 + 2\lambda\omega_0 s + \omega_0^2} = \frac{w_i}{\xi_i}, i = u, v, r \quad (34)$$

In a simplified sense, the wave motion is modeled as a linear damped oscillator. The constant K_i^{RAO} simplifies the first order response of the force and force-to-motion RAO's, and depends on the RAO dynamics and the wave incidence direction (FOSSEN; PEREZ, 2009). While it is reasonable to modulate K_i^{RAO} through a rotation function of the angle of attack, most control applications consider it a constant for all frequencies and angles.

The natural frequency ω_0 is estimated as the peak spectrum frequency $\omega_p = 2\pi/T_p$. If required for filtering purposes, its value can be obtained through the sampling of vertical movements of the vessel center of mass, obtaining the main spectrum frequencies through a Fast Fourier Transform. Other methods to calculate ω_p on line are described in (BALCHEN et al., 1980), and in (FOSSEN; PEREZ, 2009). A

particularly straightforward method to estimate ω_0 is to set it as a slowly varying parameter with time derivative equal to zero and use it as a state to be estimated in a Kalman Filter, such as one of the techniques described in (TANNURI et al., 2005).

Figure 12 - RAO Amplitude and Phase plot for a Tanker

Source: Author

The damping factor λ has a value between 0.01 and 0.1. It is mainly dependant of the sea spectrum that characterizes the sea state. For the JONSWAP spectrum, it can be considered constant in 0.1. The gain K_w is calculated through the wave spectrum relations according to equation (35) (FOSSEN, 2011).

$$K_w = 2\lambda\omega_0\sigma \quad (35)$$

The constant σ is calculated by the peak value of the wave spectrum $\sigma = \sqrt{S_\zeta(\omega_0)}$.

SECOND ORDER WAVE DRIFT FORCES

Second order wave interaction effects not modelled through RAO's are also of significant importance for Dynamic Positioning and moored marine systems. These effects induce motion through low frequency drift forces which, differently from the first order motion, need to be compensated in DP Systems. The second order wave forces are divided in two parcels, one that is constant (the so-called mean wave drift) and one slowly variable (the slow wave drift):

$$\mathbf{F}_{wave} = \mathbf{F}_{md} + \mathbf{F}_{sd} \quad (36)$$

The mean wave drift forces F_{md} result from the wave reflection on a floating body's surface. The large energy associated with the inversion of the wave momentum when the reflection occurs provokes a liquid pressure induced drift force. The larger the vessel, when compared to the wave length, the larger the forces associated.

The slow wave drift forces F_{sd} rise from the interaction between waves with different frequencies originated from random seas. The random wave generated pressure fields on the vessel hull excite the system in frequencies corresponding to the sum and the difference between the frequencies of the waves. While summing frequencies do not create significant forces (they are in a larger frequency range), the difference frequencies are in a lower span that can affect the natural frequencies of the vessel.

Mathematical modeling of the parcels F_{sd} and F_{md} are not in the scope of the current work. These are mentioned as important hydrodynamic effects that create drift forces along with the current, but in most marine systems are hard to measure separately. Yet they justify the introduction of integral action in the motion controller, or the addition of feedforward bias when available from observers.

More on wave drift forces can be found in (DE KAT; WICHERS, 1991), (CHIANG, 1992) and (ARANHA; FERNANDES, 1995).

3.2. Dynamic Positioning Systems

Although new state of the art technology is researched for Dynamic Positioning Systems every year, the basic aspects of DPS are mostly settled. The system architecture has remained unchanged for the last three decades, as most of the technology developments concentrate in the inner blocks of the system, or in redundancy requirements for safety and robustness enhancement.

The following sections will detail the most relevant theoretical aspects of the DP control system. Figure 13 shows a typical DPS block diagram. The vessel block corresponds to the plant. The sensors block corresponds to the position measuring system, which may be composed by a GNSS aided by an inertial navigation system. A wave filter block is composed either by a linear notch filter, a Kalman Filter or any estimator that can isolate the low-frequency motion estimates required by the controller from the direct sensor measurements. The controller block can be composed by any control technique, being usually implemented as a PID or a state-feedback optimal controller.

The wind feedforward block converts the wind velocity measurements to a wind drag force estimation. Finally, the Thrust Allocation block converts the three-dimensional forces output from the controller to the actual actuator commands.

Each block is designed independently from the others, which, in the case of the Controller and Filter block, is feasible under the separation principle.

Figure 13 - DPS Architecture

Source: Author

3.2.1. Plant Models

In order to obtain equations adequate for control design and for simulation purposes, the presented models are presented in the forms that will be used throughout the text.

COMPLETE NONLINEAR MODEL

By defining as states the global position (x, y, ψ) and the vessel velocities described in LRF (u, v, r) the vessel nonlinear model is given by equation (37).

$$\begin{aligned} \dot{x} &= A(x)x + B\{F_{thrust} + F_{oper} + F_{curr} + F_{wind} + F_{wave}\} \\ y &= Cx \end{aligned} \quad (37)$$

In which $x = [x \ y \ \psi \ u \ v \ r]^T$ is the state vector. The system matrices are

$$A(x) = \begin{bmatrix} 0_{3 \times 3} & R_L^G(x) \\ 0_{3 \times 3} & -M^{-1}C_{cor}(x) \end{bmatrix}; \quad B = \begin{bmatrix} 0_{3 \times 3} \\ M^{-1} \end{bmatrix}; \quad C = [I_{3 \times 3} \quad 0_{3 \times 3}] \quad (38)$$

Note that the matrix B and C are not functions of the state variables. The inertial fluid effects from $C_{cur}v_c$ are lumped into the drag vector F_{curr} . An alternate version of this model can be found in (FOSSEN, 2011), in which a linear damping term is added to model slow-speed viscous drag and thus ensuring exponential convergence at low speeds, making the system matrix $A(x)$ as in equation

$$A(\mathbf{x}) = \begin{bmatrix} 0_{3 \times 3} & R_L^G(\mathbf{x}) \\ 0_{3 \times 3} & -M^{-1}[D + C_{cor}(\mathbf{x})] \end{bmatrix} \quad (39)$$

In which D is the linear damping matrix:

$$D = \begin{bmatrix} -X_u & 0 & 0 \\ 0 & -Y_v & -Y_r \\ 0 & -N_v & -N_r \end{bmatrix} \quad (40)$$

The damping parameters X_u, Y_u, Y_v, Y_r, N_v and N_r produce a very small effect when compared to current drag forces and may be neglected in maneuvering models.

LINEARIZED MODEL FOR CONTROL DESIGN

The assumption of small angles and velocities and no external current forces allows a linearization of equation (37) through the calculation of the Jacobian Matrix (x) in the working point states. For control design purposes, some assumptions are also taken on the external forces:

- i. The wind velocity and direction can be measured, and its corresponding forces F_{wind} can be estimated and cancelled via feedforward control;
- ii. The operational forces F_{oper} , such as towing cable loads, can be measured and cancelled via feedforward control;
- iii. Drag forces from current and 2nd order wave effects $F_{curr} + F_{wave}$ are considered as slow varying unmeasured disturbances that can be estimated or compensated via integral action.

If the vessel stationary values are given by $\mathbf{x}_0 = [x_0 \ y_0 \ \psi_0 \ u_0 \ v_0 \ r_0]^T$, the states can be approximated by $\mathbf{x} = \mathbf{x}_0 + \delta\mathbf{x}$ and the small perturbation model of the system dynamics becomes

$$\begin{aligned} \delta\dot{\mathbf{x}} &= J(\mathbf{x}_0)\delta\mathbf{x} + B\mathbf{F}_{thrust} \\ \delta\mathbf{y} &= C\delta\mathbf{x} \end{aligned} \quad (41)$$

The Jacobian Matrix is calculated through equation (42), with its input functions vector $\mathbf{f} = [f_1 \ f_2 \ f_3 \ f_4 \ f_5 \ f_6]^T$ being the given by the nonlinear system model $\mathbf{f} = A(\mathbf{x})\mathbf{x}$.

$$J(\mathbf{x}) = \nabla \mathbf{f} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \cdots & \frac{\partial f_2}{\partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f_m}{\partial x_1} & \frac{\partial f_m}{\partial x_2} & \cdots & \frac{\partial f_m}{\partial x_n} \end{bmatrix} \quad (42)$$

The Jacobian Matrix for the nonlinear model (37) with no external forces is given by:

$$J(\mathbf{x}) = \begin{bmatrix} J_1 & R_L^G \\ 0_{3 \times 3} & J_2 \end{bmatrix} \quad (43)$$

With

$$J_1 = \begin{bmatrix} 0 & 0 & -u \sin(\psi) - v \cos(\psi) \\ 0 & 0 & u \cos(\psi) - v \sin(\psi) \\ 0 & 0 & 0 \end{bmatrix}$$

$$J_2 = -M^{-1} \begin{bmatrix} 0 & -(m + Y_{\dot{v}})r & -(m + Y_{\dot{v}})v - 2(mx_G + Y_{\dot{r}})r \\ (m + X_{\dot{u}})r & 0 & (m + X_{\dot{u}})u \\ (mx_G + Y_{\dot{r}})r + (Y_{\dot{v}} - X_{\dot{u}})v & (Y_{\dot{v}} - X_{\dot{u}})u & (mx_G + Y_{\dot{r}})u \end{bmatrix} \quad (44)$$

When considering a static situation with $\psi = \psi_0$ and $u_0 = v_0 = r_0 = 0$, the Jacobian Matrix will be cancelled almost completely, keeping only the geometric coupling between the local velocities and global positions. The linearized model will be described by

$$\delta \dot{\mathbf{x}} = A_0 \delta \mathbf{x} + B \{ \mathbf{F}_{thrust} + \mathbf{F}_{oper} + \mathbf{F}_{curr} + \mathbf{F}_{wind} + \mathbf{F}_{wave} \}$$

$$\delta \mathbf{y} = C \delta \quad (45)$$

With

$$A_0 = \begin{bmatrix} 0_{3 \times 3} & R_L^G(\psi_0) \\ 0_{3 \times 3} & 0_{3 \times 3} \end{bmatrix} \left(\text{alternatively } A_0 = \begin{bmatrix} 0_{3 \times 3} & R_L^G(\psi_0) \\ 0_{3 \times 3} & -M^{-1}D \end{bmatrix} \right) \quad (46)$$

When considering the linear dumper (equation (39)) an alternative option is also given in (46).

STATE OBSERVER MODEL

In order to construct state observers, the model will be adapted to be able to account for first order wave effects and current and wave drift forces. In this model, the state vector is expanded by the introduction of the high frequency wave motion state vector ξ and a set of bias variables $\mathbf{b} = [b_1 \ b_2 \ b_3]^T$ that represent the slow varying external forces.

The filter described by equation (34) can be put in the state space form with the state variables $\xi = [\xi_1 \ \xi_2 \ \xi_3 \ \xi_4 \ \xi_5 \ \xi_6]^T$, the high frequency wave excited motion in surge, sway and yaw and white noise excitation $w_w = [w_1 \ w_2 \ w_3]^T$:

$$\dot{\xi} = A_w \xi + E_w w_w \quad (47)$$

in which

$$A_w = \begin{bmatrix} 0_{3 \times 3} & I_{3 \times 3} \\ -\omega_0^2 I_{3 \times 3} & -2\lambda\omega_0 I_{3 \times 3} \end{bmatrix}; E_w = \begin{bmatrix} I_{3 \times 3} \\ -2\lambda\omega_0 I_{3 \times 3} \end{bmatrix} \begin{bmatrix} K_u^{RAO} & 0 & 0 \\ 0 & K_v^{RAO} & 0 \\ 0 & 0 & K_r^{RAO} \end{bmatrix} K_w \quad (48)$$

It is assumed that the global position and heading of the vessel are measured, via Global Satellite Navigation Systems (GNSS) or Inertial Measurement Units (IMU). The wave motion system matrices are considered to be constant but may be recalculated for varying sea states. The contribution of the high frequency motion to the overall motion is modeled as an output with added disturbance, making the measured vector y a linear composition of the global position states and the wave disturbances:

$$y = \begin{bmatrix} x + \xi_4 \\ y + \xi_5 \\ \psi + \xi_6 \end{bmatrix} \quad (49)$$

The bias variables are environmentally sourced external forces which lump nonlinear unmodeled effects, second order wave loads and current forces. These effects are considered to be constant (or slowly varying) when described in the GRF, which means this term must be rotated so it can be introduced in the motion equation. Even though the bias values usually carry no dynamics, being only updated through the integration of random noise (the so-called random walk), some filter formulations introduce a time constant matrix T_b , accounting for slow changes in the bias values.

$$\begin{bmatrix} \dot{b}_1 \\ \dot{b}_2 \\ \dot{b}_3 \end{bmatrix} = \mathbf{0}_{3 \times 1} \left(\text{or} \begin{bmatrix} \dot{b}_1 \\ \dot{b}_2 \\ \dot{b}_3 \end{bmatrix} = -T_b^{-1} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \right) \quad (50)$$

DP operators commonly name the estimated drift specific force bias $[b_1, b_2, b_3]^T$ as DP current. It is a non-physical value that produces the motion that would be expected by the incidence of a drift current.

The extended state vector $x = [x, y, \psi, u, v, r, \xi_1, \xi_2, \xi_3, \xi_4, \xi_5, \xi_6, b_1, b_2, b_3]^T$ produces the observer model (51), with system matrices given in equation (52)

$$\begin{aligned}\dot{\mathbf{x}} &= A_o(\mathbf{x})\mathbf{x} + B\mathbf{F}_{thrust} + E\mathbf{w} \\ \mathbf{y} &= H\mathbf{x} + \mathbf{v}\end{aligned}\quad (51)$$

$$\begin{aligned}A(\mathbf{x}) &= \begin{bmatrix} 0_{3 \times 3} & R_L^G(\mathbf{x}) & 0_{3 \times 6} & 0_{3 \times 3} \\ 0_{3 \times 3} & -M^{-1}C_{cor}(\mathbf{x}) & 0_{3 \times 6} & M^{-1}R_G^L(\mathbf{x}) \\ 0_{6 \times 3} & 0_{6 \times 3} & A_w(\omega_0) & 0_{3 \times 3} \\ 0_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 6} & -T_b^{-1} \end{bmatrix}; B = \begin{bmatrix} 0_{3 \times 3} \\ M^{-1} \end{bmatrix}; \\ E &= \begin{bmatrix} I_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} \\ 0_{3 \times 3} & M^{-1} & 0_{3 \times 3} & 0_{3 \times 3} \\ 0_{3 \times 3} & 0_{3 \times 3} & E_w & 0_{3 \times 3} \\ 0_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} & I_{3 \times 3} \end{bmatrix}; H = \begin{bmatrix} I_{3 \times 3} & 0_{3 \times 6} & I_{3 \times 3} & 0_{3 \times 3} \end{bmatrix}\end{aligned}\quad (52)$$

in which $\mathbf{w} = [w_x, w_y, w_\psi, w_u, w_v, w_r, w_1, w_2, w_3, w_{b1}, w_{b2}, w_{b3}]^T$ is the vector of random process noises and $\mathbf{v} = [v_1, v_2, v_3]^T$ is the vector of random noises in the measurements.

This model assumes prior knowledge of the mean wave frequency ω_0 , obtained through any of the methods described in section *First Order Wave Motion*. When relevant throughout the text, the estimation of this parameter will be performed by setting it as a slowly varying state with variation driven by random gaussian noise, that is $\dot{\omega}_0 = w_\omega$, such as described in (TANNURI et al., 2005). In this case the state vector is extended to $\mathbf{x} = [x, y, \psi, u, v, r, \xi_1, \xi_2, \xi_3, \xi_4, \xi_5, \xi_6, b_1, b_2, b_3, \omega_0]^T$.

3.2.2. Control Algorithm

The controller block contains the function that maps the low frequency positioning error signal vector $\tilde{\boldsymbol{\eta}}$ to the commanded forces $\mathbf{F}_{thrust}$ in order to track the desired set point position vector $\boldsymbol{\eta}_d$ (control law). It is important to note this signal should be already filtered from high frequency motion (such as waves induced first order motion).

The error vector is defined in the GRF as the difference between the estimated low frequency position and the desired position (equation (53)).

$$\tilde{\boldsymbol{\eta}} = \hat{\boldsymbol{\eta}} - \boldsymbol{\eta}_d = \begin{bmatrix} \hat{x} - x_d \\ \hat{y} - y_d \\ \hat{\psi} - \psi_d \end{bmatrix}\quad (53)$$

As the thruster matrix is expressed in the body's reference frame, the control laws must be calculated in the LRF. The application of the matrix transformation R_G^L provides the local error vector $\mathbf{e}^L$.

$$\mathbf{e}^L = R_G^L \tilde{\boldsymbol{\eta}} \quad (54)$$

The derivative of the error vector is obtained with transformation (5), evidencing the state vector local velocity variable $\mathbf{v}$. By defining the local velocity error $\tilde{\mathbf{v}} = \mathbf{v} - \mathbf{v}_d$, the derivative of equation (54) becomes

$$\dot{\mathbf{e}}^L = S_L \mathbf{e}^L + R_G^L \dot{\tilde{\boldsymbol{\eta}}} = S_L R_G^L \tilde{\boldsymbol{\eta}} + \tilde{\mathbf{v}} \quad (55)$$

The simplest method to design a control law for DP systems is to consider that it is kept in a fixed position $\boldsymbol{\eta}_0$ with small velocities. This cancels out the centripetal and Coriolis forces and fixes the rotation matrices R_G^L and R_L^G . The assumption of a fixed rotation matrix is referred in literature as Vessel Parallel Coordinate System. Environmental forces are considered a disturbance and are not accounted for in the control synthesis phase. By adopting the small perturbation linearized model from section 3.2.1 and considering a fixed angle ψ_0 , the error vector and the derivatives of the error vectors becomes

$$\begin{cases} \delta \mathbf{e}^L = R_G^L(\psi_0) \delta \tilde{\boldsymbol{\eta}} \\ \delta \dot{\mathbf{e}}^L = \delta \tilde{\mathbf{v}} \end{cases} \quad (56)$$

The control algorithm most used by industry is the PID (Proportional-Integral-Derivative), given by equation (57):

$$\mathbf{F}_{thrust} = -K_P \delta \mathbf{e}^L - K_D \delta \tilde{\mathbf{v}} - K_I \int_0^t \delta \mathbf{e}^L d\tau \quad (57)$$

The positive definite gain matrices $K_P = \text{diag}(k_{Px}, k_{Py}, k_{P\psi})$, $K_D = \text{diag}(k_{Dx}, k_{Dy}, k_{D\psi})$ and $K_I = \text{diag}(k_{Ix}, k_{Iy}, k_{I\psi})$ are the design variables to be tuned. As the rotation matrix $R_G^L(\psi_0)$ provides only geometrical coupling, the control gains are tuned setting $\psi_0 = 0$ and $R_G^L = I_{3 \times 3}$, in which $I_{3 \times 3}$ is the identity matrix.

The above simplification decouples the vessel motions and allows the application of classic SISO control theory in the frequency domain to calculate the gains for each degree of freedom. Applying the Laplace transform in the decoupled linearized model from equation (45), using s as the Laplace variable yields the second order transfer functions from equation (58).

$$\begin{cases} G_x(s) = \frac{1}{(m + X_u)s^2} \\ G_y(s) = \frac{1}{(m + Y_v)s^2} \\ G_\psi(s) = \frac{1}{(I_z + N_r)s^2} \end{cases} \quad (58)$$

The transfer functions for the PID controller are given in equation (59)

$$G_{ci}(s) = -k_{pi} - sk_{di} - \frac{1}{s}k_{ii} ; \quad i = x, y, \psi \quad (59)$$

Which yields the following closed loop transfer function

$$G_{cli}(s) = \frac{G_{ci}(s)G_i(s)}{G_{ci}(s)G_i(s) - 1} ; \quad i = x, y, \psi \quad (60)$$

The simplest tuning method is through pole placement, as described in (MAZZILLI; TANNURI, 2010). Taking, for example, the heading transfer functions, the closed loop equation (60) becomes

$$G_{CL\psi}(s) = \frac{k_{D\psi}s^2 + k_{P\psi}s + k_{I\psi}}{(I_z + N_r)s^3 + k_{D\psi}s^2 + k_{P\psi}s + k_{I\psi}} \quad (61)$$

The surge and sway transfer functions take the same form, with adequate gains and mass variables for each case. The closed loop poles should be chosen according to the design requirements, like overshoot limit, settling time and steady state error. Note that equation (61) has three poles. A straightforward method is to choose two of the poles to be dominant and set the third pole over the negative real axis far from the origin. This allows the transfer function to be related to a second order transfer function, in which damping and natural frequency values are well related to the poles' locations.

By choosing a natural frequency $\omega_n \in \mathbb{R}^+$ and relative damping factor $\zeta \in \mathbb{R} \mid 0 \leq \zeta \leq 1$, the dominant poles can be chosen as a complex conjugate pair $p_1 = -\omega_n(\zeta + j\sqrt{1 - \zeta^2})$ and $p_2 = -\omega_n(\zeta - j\sqrt{1 - \zeta^2})$, which will give the system response a damped oscillator characteristic. The third pole is chosen as a negative real number, with enough distance from the dominant poles. For example, $p_3 = -k\zeta\omega_n$, in which k is a positive real number. The choice of these poles for the heading control provides

$$\begin{cases} k_{P\psi} = (I_z + N_r)\omega_n^2(1 + 2k\zeta^2) \\ k_{I\psi} = (I_z + N_r)k\zeta\omega_n^3 \\ k_{D\psi} = (I_z + N_r)\zeta\omega_n(k + 2) \end{cases} \quad (62)$$

The procedure is similar for the other degrees of freedom. Care should be taken when choosing the natural frequency and damping factor. In marine systems, ζ is usually chosen as a value between 0.6 and 0.9 (FOSSEN, 2011). The natural frequency ω_n should be chosen outside the first order wave disturbance range. Gain factor k cannot be too large, otherwise will produce large gains and may saturate actuators.

An alternative control tuning method for the linearized system, using a Linear Quadratic approach in the time domain can be found in (BALCHEN et al., 1980). Modern control theory approaches calculate gains for a PD (Proportional-Derivative) controller and either extends the results for the PID or use other methods to eliminate steady state error, as feedforward compensation of measured or estimated loads.

In most DP systems relative wind speed and direction measurements are also available. When adequately filtered to eliminate high frequency wind gusts, the mean values can be used to introduce feedforward compensation. It is straightforward to obtain wind forces from the application of equation (24) along with the drag coefficient tables. The resulting control law for the linearized system becomes

$$\mathbf{F}_{thrust} = -\hat{\mathbf{F}}_{wind} - K_P \delta \mathbf{e}^L - K_D \delta \dot{\mathbf{v}} - K_I \int_0^t \delta \mathbf{e}^L d\tau \quad (63)$$

in which $\hat{\mathbf{F}}_{wind}$ is the filtered estimate of the mean wind forces.

It is important to remind at this point that assumption of linearity may limit the operational characteristics of the DP system. The geometric nonlinearity introduced by the coordinate system transformations introduce coupling between the motion modes and may affect the closed loop dynamic response and stability when positions and yaw angle are changed simultaneously.

3.2.3. Thruster Allocation

To map the calculated control forces $\mathbf{F}_{thrust}$ to the required thrust/rotation and azimuth angle for each propeller ($\boldsymbol{\tau}_{thr}$), a thruster allocation algorithm is required. Take back equation (20):

$$\mathbf{F}_{thrust} = T_{3 \times n}(\alpha) \boldsymbol{\tau}_{thr}$$

In many DP systems the number of propellers degrees of freedom is larger than the number of planar degrees of freedom ($n > 3$), characterizing an over-actuated

problem. In this case, the thruster vector distribution that solves the equation above has infinite solutions. The thruster allocation algorithms are methods to find the optimal matrix $\Theta_{n \times 3}$ that solves the force distribution problem:

$$\boldsymbol{\tau}_{thr} = \Theta_{n \times 3} \mathbf{F}_{thrust} \quad (64)$$

If the problem is considered unconstrained, an optimal solution may be found algebraically via application of Lagrange multipliers as described in (DE WIT, 2009). Consider the following optimization problem:

$$\boxed{\begin{array}{l} \min_{\boldsymbol{\tau}_{thr}} \boldsymbol{\tau}_{thr}^T W \boldsymbol{\tau}_{thr} \\ \text{subject to } \mathbf{F}_{thrust} - T_{3 \times n} \boldsymbol{\tau}_{thr} = 0 \end{array}}$$

In which W is a $n \times n$ positive definite weighting matrix, usually diagonal. The application of the Lagrange Multipliers $\boldsymbol{\lambda}$ to the equality constraint provides the Lagrangian (65):

$$L(\boldsymbol{\tau}_{thr}, \boldsymbol{\lambda}) = \boldsymbol{\tau}_{thr}^T W \boldsymbol{\tau}_{thr} + \boldsymbol{\lambda}^T (\mathbf{F}_{thrust} - T_{3 \times n} \boldsymbol{\tau}_{thr}) \quad (65)$$

The calculation of the stationary point $\frac{\partial L(\boldsymbol{\tau}_{thr}, \boldsymbol{\lambda})}{\partial \boldsymbol{\tau}_{thr}} = 0$ in combination with the equality constraint yields the solution for equation (64):

$$\Theta_{n \times 3} = W^{-1} T_{3 \times n}^T (T_{3 \times n} W^{-1} T_{3 \times n}^T)^{-1} \quad (66)$$

The matrix $\Theta_{n \times 3}$ is called the generalized inverse. If the weighting matrix W is the identity matrix, the solution reduces to

$$\Theta_{n \times 3} = T^\dagger = T_{3 \times n}^T (T_{3 \times n} T_{3 \times n}^T)^{-1} \quad (67)$$

In which $T^\dagger$ is called Moore-Penrose pseudo-inverse matrix.

The calculation of azimuth angles α for variable azimuth propellers is done by the decomposition of the thruster vectors $\boldsymbol{\tau}_{thr}$ in the orthogonal directions (and adequate definition of the thruster matrix $T_{3 \times (n+a)}$, where a is the number of azimuth propellers), followed by the solution described above and the recalculation of modulus and angle of the decomposed thrust vector.

In practice, several actuation constraints (actuator saturation, maximum azimuth angle and rate of turn, etc.) limits the pure pseudo-inverse solution, requiring either recalculation of the weighting matrices every time a saturation occurs or using an online numerical constrained optimization method, such as linear or quadratic

programming. An survey on various state-of-the-art available methods for the optimization problem, is found in (DE WIT, 2009) and in (JOHANSEN; FOSSEN, 2013).

3.2.4. *Wave Filtering*

The control system of marine systems cannot respond to the high frequency excitation from the first order wave forces, as the amount of energy required to compensate this mode would make it operationally unfeasible. This means that the motion information from the feedback system needs to be filtered, so that the controller receives as input only the low frequency response of the Vessel.

The standard solution for the wave filtering algorithm is to use either low pass filters or the so-called cascaded Notch Filter, in which three band pass filters are cascaded with pre-defined central frequencies.

Frequency-domain designed filters will introduce phase lag. Since the introduction of the Kalman Filter (KALMAN, 1960) and its DP application in the Linear Quadratic Gaussian Regulator proposed by (BALCHEN et al., 1980), industrial DP systems have dropped the usage of notch filter in favor of the most modern approach.

The Kalman Filter (KF) is a recursive optimal state estimator for dynamic systems that treats the state variables as random variables and the dynamic system equations as a stochastic process driven by Gaussian noise. It is optimal in the sense that it minimizes the mean square error of the estimated parameters if the noise is Gaussian. In digital systems this filter is usually implemented in its discrete-time form. As shall be seen later, this brings an extra advantage to navigation systems, once it allows the implementation of Dead Reckoning functions.

The following formulation implements the discrete KF in two phases, the error and state propagation (or prediction) cycle and the update cycle. The only requirement for the application of this filter to a linear dynamic system is the same of any observer design: the system needs to be observable, i.e. the rank of its observability matrix needs to be equal to the number of its rows. Consider the linear dynamic system with extended Gaussian noise inputs given by (68)

$$\begin{cases} \dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} + \mathbf{E}\mathbf{w} \\ \mathbf{y} = \mathbf{C}\mathbf{x} + \mathbf{v} \end{cases} \quad (68)$$

The standard state-space representation is extended with the input process Gaussian noise vector $\mathbf{w}$, which is modulated by noise input matrix $\mathbf{E}$. The process noise has covariance matrix $\mathbf{Q} > 0$. The system observation vector $\mathbf{y}$ is also extended with measurement Gaussian noise vector $\mathbf{v}$. This vector has covariance matrix $\mathbf{R}$, which corresponds to the measurement equipment standard deviations. $\mathbf{Q}$ and $\mathbf{R}$ are parameters chosen by the designer according to the system characteristics.

The discrete-time Kalman Filter requires the discretization of equation (68). The discrete time representation of the extended state-space is given by

$$\begin{cases} \mathbf{x}(k+1) = \Phi\mathbf{x}(k) + \Delta\mathbf{u} + \Gamma\mathbf{w} \\ \mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{v} \end{cases} \quad (69)$$

In which

$$\begin{cases} \Phi = e^{\mathbf{A}h} = \left[\mathbf{I} + \mathbf{A}h + \frac{1}{2}\mathbf{A}^2h^2 + \dots \right] \\ \Delta = \mathbf{A}^{-1}(\Phi - \mathbf{I})\mathbf{B} \\ \Gamma = \mathbf{A}^{-1}(\Phi - \mathbf{I})\mathbf{E} \end{cases} \quad (70)$$

The matrix Φ is called State Transition Matrix. It is a function of the dynamic system matrix $\mathbf{A}$ and the time discretization $h = \Delta t$. For GNSS readings, for example, h have values between 0.5 and 2s. By following the above transformations, the vessel linearized dynamic equation (45) can be easily put in the discrete form.

The propagation cycle for the Kalman filter consists in the integration of the discrete time state space equation and the error covariance, by using the information from the last update cycle

$$\mathbf{x}^-(k+1) = \Phi\hat{\mathbf{x}}(k) + \Delta\mathbf{u}(k) \quad (71)$$

In this notation, the superscripted vector $(\cdot)^-$ indicates that this variable is calculated prior to the update measurement. The hat notation $(\hat{\cdot})$ indicates that the variable is an estimate of the real variables. The error covariance integration will be given by

$$\mathbf{P}^-(k+1) = \Phi\hat{\mathbf{P}}(k)\Phi^T + \Gamma\mathbf{Q}(k)\Gamma^T \quad (72)$$

The matrix $\mathbf{P}$ is the error covariance matrix and $\mathbf{Q}(k) = \mathbf{Q}^T(k) > 0$ is the process noise covariance matrix, usually chosen diagonal and constant. Note that the integrations

from equations (71) and (72) require the definition of initial conditions for $k = 0$, $\hat{P}(0)$ and $\hat{x}(0)$.

The update cycle proceeds with equations for the measurement, the Kalman gain and the error covariance updates.

$$\begin{cases} K_{kal}(k+1) = P^-(k+1)H^T[HP^-(k+1)H^T + R(k)]^{-1} \\ \hat{x}(k+1) = x^-(k+1) + K_{kal}(k+1)[y(k+1) - Hx^-(k+1)] \\ \hat{P}(k+1) = P^-(k+1) - K_{kal}(k+1)HP^-(k+1) \end{cases} \quad (73)$$

The matrix $R(k) = R^T(k) > 0$ is the measurement noise covariance matrix, usually chosen constant and diagonal (which means one measurement noise does not affect the others). The elements of R are taken as the standard deviation σ^2 of each sensor.

EXTENDED KALMAN FILTER

The most embracing case of the filtering algorithm for DP systems requires the usage of the full nonlinear state space equation. In this case, the Extended Kalman Filter (EKF) is the standard and most used solution in industry. Differently from its linear counterpart, the EKF does not provide necessarily an optimal solution (from the point of view of Gaussian error). Even though the stability of the EKF is not formally proven, the extensive use of this tool has shown it to be a well proven concept by experience.

The formulation applies to a nonlinear system of the form

$$\begin{cases} \dot{x} = f(x, u) + Ew \\ y = Hx + v \end{cases} \quad (74)$$

In dynamic positioning systems, the observer model from equation (51) is used:

$$\begin{aligned} \dot{x} &= A_o(x)x + BF_{thrust} + Ew \\ y &= Hx + v \end{aligned}$$

This allows the estimation of the low and high frequency motion separately, in such way that only low frequency motion is compensated.

The state propagation for the extended Kalman Filter is calculated through the integration of the motion equation.

$$\dot{x}^-(t) = f(x^-(t), u) \begin{cases} \text{initial condition } \hat{x}(k) \\ t(k) < t < t(k+1) \end{cases} \quad (75)$$

Similarly, the error covariance matrix is propagated through integration of the matrix function:

$$\begin{aligned} \dot{P}^-(t) &= J(\mathbf{x}^-, t)P(t) + P(t)J(\mathbf{x}^-, t)^T \\ &+ E(t)Q(t)E(t)^T \begin{cases} \text{initial condition } \hat{P}(k) \\ t(k) < t < t(k+1) \end{cases} \end{aligned} \quad (76)$$

The error covariance dynamics uses the Jacobian of the system dynamics, $J(\mathbf{x}^-, t)$ and the same process covariance matrix Q defined in the standard Kalman Filter. The integration is performed over the continuous form of the nonlinear dynamic equations, by means of Euler Integration, *Runge-Kutta* or any other numeric technique. If Euler integration is used in a single step with size h the propagation become:

$$\begin{cases} \mathbf{x}^-(k+1) = \hat{\mathbf{x}}(k) + hf(\mathbf{x}, \mathbf{u}) \\ P^-(k+1) = \hat{P}(k) + h(J(k)\hat{P}(k) + \hat{P}(k)J(k)^T + EQE^T) \end{cases} \quad (77)$$

The update cycle is similar to that of the standard Kalman Filter:

$$\begin{cases} K_{kal}(k+1) = P^-(k+1)H^T[HP^-(k+1)H^T + R(k)]^{-1} \\ \hat{\mathbf{x}}(k+1) = \mathbf{x}^-(k+1) + K_{kal}(k+1)[\mathbf{y}(k+1) - H\mathbf{x}^-(k+1)] \\ \hat{P}(k+1) = P^-(k+1) - K_{kal}(k+1)HP^-(k+1) \end{cases} \quad (78)$$

DEAD RECKONING

When used in the propagation/update form as described above, the Kalman Filter can perform the so-called Dead Reckoning. When the sensor signal is lost for a period of time, the solution is to use the model integration without updates. The propagation/update is also useful when different or low measuring rates apply. Implementation is simple, provided that the information on availability of measurement is given. Equation (79) gives the adaptation on the gain formula that provides dead reckoning.

$$\begin{aligned} &K_{kal}(k+1) \\ &= \begin{cases} P^-(k+1)H^T[HP^-(k+1)H^T + R(k)]^{-1} & \text{if measurement is available} \\ 0 & \text{if measurement is not available} \end{cases} \end{aligned} \quad (79)$$

Note that this means that, when measurement is not available, the state and error covariance estimate is composed only by the propagation data.

As will be covered later, the Dead Reckoning feature can be used to enhance robustness under signal transmission loss between agents of a cooperative system.

3.3. Linear Cooperative Control

This section presents the basics of current technology on linear cooperative control algorithms. In the context of multi-agent systems, such algorithms are called *protocols*.

The strategies presented are mostly based on the definition of a consensus problem, in which multiple agents' outputs are required to reach a consensus value in a coordination variable. By defining this basic problem, standard cooperative tasks such as formation maintenance and leader-following behavior can be described. A thorough discussion on linear consensus theory can be found in (REN; BEARD; ATKINS, 2007).

3.3.1. Algebraic Graph Theory

Graphs and graph matrices are powerful tools to model the interactions between elements in a multi-agent system. Before proceeding to the cooperative protocols, basic concepts on Algebraic Graph Theory are presented and discussed. Most definitions will be applied to directed graphs but, except when explicit throughout the text, can also be applied to undirected graphs. The definitions presented in this section can be found in (BONDY; MURTY, 2008) and in (LEWIS et al., 2014).

A graph is defined as a pair $\mathcal{G} = (V, E)$, where V is a set $\{v_1, v_2, \dots, v_n\}$ called vertices set or nodes set and E is a set of pairs $\{v_i, v_j\}$ called edges set. A simple graph with two elements is shown in Figure 14. In a multi-agent communication structure, each vertex represents a single agent and each edge represents the availability of information between edges.

Figure 14 - A simple graph

Source: Author

In a standard or undirected edge, $(v_i, v_j) = (v_j, v_i)$. This means, from the multi-agent network design point of view, that information is shared bi-directionally, i.e. the agent nodes connected through this edge have access to each other's states. In an oriented edge, $(v_i, v_j) \neq (v_j, v_i)$, which means that information is shared in only one direction and only one agent has access to the other's states. Graphically, an oriented edge is represented through an arrow in which the point enters v_j and the tail leaves v_i .

Figure 15 represents a simple oriented graph. An oriented or directed graph is also called a digraph.

The in-degree of a node v_i is the number of edges pointing to v_i . The out-degree of a node v_i is the number of edges with tails leaving v_i . If the out-degree is equal to the in-degree of a graph, it is said to be balanced. Note that in an undirected graph, the in-degree and out-degree of every node are equal, so every undirected graph is balanced. The (in) neighbors of a node are the set $N_i = \{v_j: (v_j, v_i) \in E\}$, i.e., it is the set of nodes with edges pointing to v_i . The neighbor set number of elements $|N_i|$ is equal to the (in) degree of v_i .

Figure 15 – A simple digraph

Source: Author

Each edge (v_j, v_i) can be associated with a weighting factor $a_{ij} > 0$. When this is considered, the neighbor set can be defined as $N_i = \{v_j: a_{ij} > 0\}$. In multi-agent systems, the weighting factor can be considered as an interaction force between neighbors. In Figure 16, a digraph is presented with some weights. By the definitions above, the set (v_j, v_k) is the neighbor set N_i of the node v_i .

Figure 16 - A digraph with weights.

Source: Author

A directed path is a sequence of edges $v_0, v_1, \dots, v_r: (v_i, v_{i+1}) \in E$. If there is a directed path from v_i to v_j , it is said that v_i is connected to v_j . In Figure 16, for example, there are two directed paths from v_l to v_i , one passing through v_j and other not. The distance between two connected nodes is the path with the minimum number of edges between them.

A graph is said to be strongly connected if any pair $(v_i, v_j) \in V \times V, i \neq j$ is connected.

In undirected graphs, if there is a directed path between v_i and v_j , then there is a directed path between v_j and v_i and the graph is simply called connected.

The diameter of a graph is the largest distance between any two of its nodes. The diameter of a disconnected graph is infinite.

A (directed) tree is a digraph in which every node, except one called root node, has in-degree equal to one. A spanning tree is a directed tree in which all nodes are connected (Figure 17).

A graph has a spanning tree if a subset of its edges forms a spanning tree. This is equivalent to say that all the vertices of this graph are reachable if starting from a single vertex and following the edges arrows.

A graph can have multiple spanning trees. A set of root nodes or leader set is the set of all roots of all spanning trees of a graph.

If a graph is strongly connected, it has at least one spanning tree and all vertices are roots.

Figure 17 - A spanning tree

Source: Author

GRAPH MATRICES

For a graph with edges weighted through a_{ij} , the adjacency matrix A is defined as

$$A = [a_{ij}] \begin{cases} a_{ij} > 0 & \text{if } (v_j, v_i) \in E \\ a_{ij} = 0 & \text{if } (v_j, v_i) \notin E \end{cases} \quad (80)$$

Observe that $a_{ii} = 0$ and that in an undirected graph, $A = A^T$.

Take, for example, the graphs from Figure 16 and Figure 17, considering, when applicable, i, j, k, l equal to 1,2,3,4 and all nonzero weights equal to 1. The adjacency matrices will be, respectively

$$A = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \text{ and } A = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Note that in digraphs, nodes that do not have any in-neighbors have all of their rows' elements equal to zero. As shall be seen later, when applied to multi-agent systems these nodes are referenced as leader nodes.

The weighted in-degree (d_i) of a node v_i is the sum of the elements of the i – th row of the adjacency matrix:

$$d_i = \sum_{j=1}^N a_{ij} \quad (81)$$

Analogously, the weighted out-degree is the sum of the elements of the i – th column of the adjacency matrix:

$$d_i^o = \sum_{j=1}^N a_{ji} \quad (82)$$

The (in) Volume is the sum of the (in) degrees:

$$Vol(\mathcal{G}) = \sum_{i=1}^N d_i \quad (83)$$

A graph is considered weight balanced if its weighted in-degree is equal to its weighted out-degree; i.e. $d_i = d_i^o$. If all weights $a_{ij} \neq 0$ are equal to 1, the graph is balanced. Note that an undirected graph is weight balanced, as $A = A^T$ and the number of rows and columns are equal.

The in-degree matrix D is defined as

$$D = \text{diag}(d_i) \quad (84)$$

Taking again graph examples of Figure 16 and Figure 17, the in-degree matrices will be, respectively,

$$D = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \text{ and } D = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

With the in-degree definition, the Laplacian matrix can also be defined as

$$L = D - A \quad (85)$$

The example graphs from Figure 16 and Figure 17 provide the following Laplacian Matrices:

$$L = \begin{bmatrix} 2 & -1 & -1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \text{ and } L = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

The Laplacian matrix can be normalized by its in-degree matrix. In this case, the normalized Laplacian matrix L' will be.

$$L' = D^{-1}L = D^{-1}D - D^{-1}A = I - D^{-1}A \quad (86)$$

Note that, for both the Laplacian matrix and the Normalized Laplacian matrix, the sum of the elements of each row vector is equal to zero. That is easily observed by taking a single line sum:

$$\sum_j l_{ij} = d_i - \underbrace{\sum_j a_{ij}}_{d_i} = 0 \quad (87)$$

The eigenvalue structure of a Laplacian Matrix has direct influence on a network dynamic behavior. To understand its properties, first note that equation (87) allows the determination of at least one eigenvector of the Laplacian matrix:

$$L[\mathbf{1}]_{N \times 1} c_{ss} = 0 \quad (88)$$

The vector $[\mathbf{1}]_{N \times 1} c_{ss} = [1 \ 1 \ \dots \ 1] c_{ss}$ is an eigenvector of L , for any constant c_{ss} . The corresponding eigenvalue of $[\mathbf{1}]_{N \times 1} c_{ss}$ is $\lambda_1 = 0$. This means that $[\mathbf{1}]_{N \times 1} c_{ss}$ is in the null space of L , $N(L)$. If the dimension of $N(L)$ is equal to 1, i.e. the rank of L is equal to $N - 1$, then $\lambda_1 = 0$ does not repeat (is a simple eigenvalue) and $[\mathbf{1}]_{N \times 1} c_{ss}$ is the only direction in $N(L)$.

For an undirected graph, $\lambda_1 = 0$ is a simple eigenvalue if and only if the graph is connected. For a directed graph, $\lambda_1 = 0$ is a simple eigenvalue if the graph is strongly connected (FAX; MURRAY, 2002). In a stricter sense, if, and only if, the graph $\mathcal{G}$ has a spanning tree, then $\lambda_1 = 0$ is a simple eigenvalue. Suppose the eigenvalues of L are ordered ($|\lambda_1| \leq |\lambda_2| \leq \dots \leq |\lambda_N|$). The last condition provides the following results.

- i) If the graph has a spanning tree, $|\lambda_2| > 0$.
- ii) If the graph is strongly connected, then it has a spanning tree and L has rank $N - 1$.

The signal of the real part of the eigenvalues of L can be analyzed through the application of the Gershgorin's Circle Criteria (CALLIOLI; DOMINGUES; COSTA, 1990). It states that all the eigenvalues of a square matrix $E = [e_{ij}]$ of dimension $N \times N$ are located in the union of N discs:

$$\bigcup_{i=1}^N \left\{ z \in \mathbb{C} : |z - e_{ii}| \leq \sum_{j \neq i} |e_{ij}| \right\}$$

The i -th disc is drawn with a center in the diagonal e_{ii} and radius equal to the sum of the modulus of the non-diagonal elements of the row i . For a Laplacian Matrix $L = D - A$ it is clear that all discs have centers in d_i (the in-degree of the graph), and that all have radius equal to d_i . The largest Gershgorin circle of a Graph Laplacian Matrix is given by a circle with a center in $d_{\max}$ and with radius $d_{\max}$. This circle contains all the eigenvalues of L (see Figure 18). Observe that, if the graph possesses a spanning tree, then there is a unique eigenvalue in $\lambda_1 = 0$ and all other eigenvalues have positive real parts, being inside a circle with center in $d_{\max}$ and radius $d_{\max}$.

Figure 18 - Gershgorin's Discs

Source: Author

As shall be seen later, the second eigenvalue, λ_2 sometimes called Fiedler Eigenvalue (LEWIS et al., 2014) or, in the case of undirected graphs, algebraic connectivity (REN; BEARD; ATKINS, 2007), determines the rate of convergence of the consensus algorithms.

3.3.2. Consensus Control of Linear Systems

Take now a first order system in which each agent i has scalar dynamics given by

$$\dot{x}_i = u_i \quad (89)$$

With $\dot{x}_i$ being the velocity of the state and u_i the control input.

The simplest and most common protocol to control this system is given by:

$$u_i = \sum_{j \in N_i} a_{ij}(x_j - x_i) \quad (90)$$

Here, N_i is the set of neighbors of the i – th agent and a_{ij} are the weight entrances of the Adjacency Matrix. In closed loop, each agent will have dynamics ruled by

$$\dot{x}_i = \sum_{j \in N_i} a_{ij}(x_j - x_i) = \sum_{j \in N_i} a_{ij}x_j - x_i \sum_{j \in N_i} a_{ij} = -d_i x_i + \sum_{j \in N_i} a_{ij}x_j \quad (91)$$

In which d_i is the in-degree. If the global state vector x is formed by $[x_1, x_2, \dots, x_N]^T$, then the global dynamics will be

$$\dot{x} = -Dx + Ax = -(D - A)x \quad (92)$$

With D and A being the In-Degree matrix and the Adjacency Matrix of the graph. By the definition of the Laplacian Matrix, the closed Loop Dynamics of the Global System will be given by

$$\dot{x} = -Lx \quad (93)$$

And the Global Input Vector will be given by

$$u = -Lx \quad (94)$$

The system matrix of equation (93) is $-L$, and hence all of its eigenvalues are in the left half-plane inside the largest Gershgorin's disc. In steady state, equation (93) becomes

$$Lx = [0]_{N \times 1} \quad (95)$$

This means that the steady state is in the Null space of L . According to the results from last section, if L has rank $N - 1$ then $[\mathbf{1}]_{N \times 1} c_{ss}$ is the only vector in the null space of L and the steady state value of the system will be

$$\mathbf{x} = [\mathbf{1}]_{N \times 1} c_{ss} \quad (96)$$

Under protocol (90), consensus is reached in steady state with value c_{ss} . To calculate this steady state value as described in (LEWIS et al., 2014) take first the time solution of the system (93):

$$\mathbf{x} = e^{-Lt} \mathbf{x}(0) \quad (97)$$

If matrix L can be put in the Jordan Canonical Form J (CALLIOLI; DOMINGUES; COSTA, 1990) by a non-singular matrix transformation T such that

$$J = T^{-1} L T \quad (98)$$

Then e^{-Lt} can be decomposed (see OGATA, 2002) by

$$e^{-Lt} = T e^{-Jt} T^{-1} \quad (99)$$

The left and right eigenvectors $\mathbf{v}_j$ and $\mathbf{w}_j$ each associated with eigenvalue λ_j of L can be chosen as the rows and columns of the transformation matrix, such that

$$\mathbf{x} = e^{-Lt} \mathbf{x}(0) = T e^{-Jt} \mathbf{x}(0) T^{-1} = \sum_{j=1}^N \mathbf{v}_j^T \mathbf{x}(0) e^{-\lambda_j t} \mathbf{w}_j \quad (100)$$

Observe that if the graph has a spanning tree, $\lambda_1 = 0$ is a simple eigenvalue and all other eigenvalues of L are positive. As $t \rightarrow \infty$, equation (100) becomes

$$\mathbf{x} \rightarrow \mathbf{w}_2 e^{-\lambda_2 t} \mathbf{v}_2^T \mathbf{x}(0) + \mathbf{w}_1 \mathbf{v}_1^T \mathbf{x}(0) \quad (101)$$

The term $\mathbf{w}_2 e^{-\lambda_2 t} \mathbf{v}_2^T \mathbf{x}(0)$ is the transient part of the response and $\mathbf{w}_1 \mathbf{v}_1^T \mathbf{x}(0)$ is the steady state value. If the left and right eigenvectors are normalized such that $\mathbf{v}_i^T \mathbf{w}_i = 1$, and taking from equation (88) $\mathbf{w}_1 = [\mathbf{1}]_{N \times 1}$, then the value of constant c_{ss} from steady state equation (96), referred as consensus value, is the weighted average of the initial states:

$$c_{ss} = \sum_{j=1}^N p_j x_j(0) \quad (102)$$

in which $\mathbf{v}_1 = [p_1, p_2, \dots, p_N]^T$ is the normalized left eigenvector of L . Moreover, the Fiedler Eigenvalue dominates the dynamics of the consensus reaching. Global time constant is given by:

$$\tau = \frac{1}{\lambda_2} \quad (103)$$

All results are valid for directed graphs if it contains at least one spanning tree. If the graph is weight balanced, (LEWIS et al., 2014) also show that

$$c_{ss} = \frac{1}{N} \sum_{j=1}^N x_j(0) \quad (104)$$

Now suppose the first order system of equation (89) is represented as a multivariable state vector. In this case, the model will be given by

$$\dot{\mathbf{x}}_i = \mathbf{u}_i \quad (105)$$

in which $\mathbf{x}_i = [x_{i1}, x_{i2}, \dots, x_{in}]^T$ and $\mathbf{u}_i = [u_{i1}, u_{i2}, \dots, u_{in}]^T$ are respectively the $n \times 1$ state vector and input vector of the i – th agent. To represent the global closed loop and the control vector, the Kroenecker Product operator $\otimes$ is introduced. For a matrix $A = [a_{ij}]_{p \times q}$ and a matrix B of any size, the Kroenecker product is defined as

$$A \otimes B = \begin{bmatrix} a_{11}B & \dots & a_{1q}B \\ \vdots & \ddots & \vdots \\ a_{p1}B & \dots & a_{pq}B \end{bmatrix} \quad (106)$$

Note that there is no commutative property in this operation.

Using the definition above, the global control vector and the global closed loop dynamics for a n sized system will be given by equations (107) and (108), respectively. As the operation introduces only sparse system blocks to the overall matrixs, the stability properties of the system hold when the Kroenecker product is introduced.

$$\mathbf{u} = -(L \otimes I_n) \mathbf{x} \quad (107)$$

$$\dot{\mathbf{x}} = -(L \otimes I_n) \mathbf{x} \quad (108)$$

Take now a full state space system with dynamics of the form

$$\dot{\mathbf{x}}_i = A \mathbf{x}_i + B \mathbf{u}_i \quad (109)$$

The linear consensus control law for this system is given as (LEWIS et al., 2014):

$$\mathbf{u}_i = cK \sum_{j \in N_i} a_{ij} (x_j - x_i) \quad (110)$$

in which c and K are positive gains to be defined. In closed loop, the vertex dynamics will be

$$\dot{x}_i = Ax_i + cBK \sum_{j \in N_i} a_{ij}(x_j - x_i) = Ax_i + cBK \left(-d_i x_i + \sum_{j \in N_i} a_{ij} x_j \right) \quad (111)$$

The global dynamics is then

$$\dot{x} = [(I_N \otimes A) - cL \otimes BK]x \quad (112)$$

Stability for system (112) is assessed as follows. First, to verify that a consensus value is reached, take again v , the normalized left eigenvector of L . The consensus global error is defined by

$$e = [(I_N - \mathbf{1}_{N \times 1} v^T) \otimes I_n]x \quad (113)$$

From equation (113), if $e = 0 \Leftrightarrow x_1 = x_2 = \dots = x_N$ (LI; DUAN, 2015) and e has the same dynamics of x :

$$\dot{e} = [(I_N \otimes A) - cL \otimes BK]e \quad (114)$$

If the Laplacian Matrix can be put in the Jordan Canonical Form (as in equation (99)) through a non-singular matrix transformation T , define a state transformation $\tilde{e} = (T^{-1} \otimes I_n)e$. Then the system (114) can be represented by

$$\dot{\tilde{e}} = [(I_N \otimes A) - cJ \otimes BK]\tilde{e} \quad (115)$$

All the elements of the state matrix of (115) are either block diagonal or block upper-triangular. Then, the stability properties of the global closed loop system (112) is equivalent of the stability properties of the N systems

$$\dot{\tilde{e}}_i = (A - c\lambda_i BK)\tilde{e}_i, \quad \text{for } i = 1, 2, \dots, N \quad (116)$$

in which λ_i are the eigenvalues of the Laplacian matrix. Calculation of the gains c and K required to reach stability are an extensive topic; their values depend on the graph topology. Stability attendance for the gains can be found through standard linear theory applied in each block of equation (116), for example, the Routh Stability Criteria (see OGATA, 2002). Optimal definition of gains for consensus problems are given in (LEWIS et al., 2014) and in (LI; DUAN, 2015). The former proposes, for example, that K_i can be calculated independently from the graph topology:

$$K_i = R^{-1}B_i^T P \quad (117)$$

In which P is a positive definite solution of the control algebraic Riccati equation with error and control action weighted by Q and R , respectively:

$$A_i^T P + P A_i + Q - P B_i R^{-1} B_i^T P = 0 \quad (118)$$

Throughout the text, it will be assumed that the graphs have fixed structure and at least one spanning tree. Under these conditions, the gain calculated provides asymptotical stability as long as the coupling gain c satisfies

$$c \geq \frac{1}{2 \min(\operatorname{Re}(\lambda_i))}, \text{ for all } \lambda_i > 0 \quad (119)$$

3.3.3. Consensus Filters

An alternate version of the consensus algorithm to be applied in sensor networks has been proposed in (OLFATI-SABER; SHAMMA, 2005). Take a network measurement model given by

$$u_i(t) = r(t) + v_i(t), \quad i = 1, \dots, n \quad (120)$$

In which r is the measuring signal and v_i is a zero-mean white Gaussian noise. For a low-pass behavior, one dynamic consensus algorithm for a distributed filter is given by

$$\dot{x}_i(t) = \sum_{j \in N_i} a_{ij} [x_j(t) - x_i(t)] + \sum_{j \in J_i} a_{ij} [u_j(t) - x_i(t)] \quad (121)$$

where $J_i = N_i \cup \{i\}$ is the set of neighbors of agent i joined by own agent, x_i is the local state estimate and a_{ij} are the entrances of the adjacency matrix. In discrete form,

$$\begin{aligned} x_i(k+1) \\ = x_i(k) + h \left\{ \sum_{j \in N_i} a_{ij} [x_j(k) - x_i(k)] + \sum_{j \in J_i} a_{ij} [u_j(k) - x_i(k)] \right\} \end{aligned} \quad (122)$$

in which h is the step size. Equation (121) can be rewritten as

$$\begin{aligned} \dot{x}_i &= \sum_{j \in N_i} a_{ij} [x_j - x_i] + \sum_{j \in J_i} a_{ij} [u_j - x_i - u_i + u_i] \\ &= \sum_{j \in N_i} a_{ij} [x_j - x_i] + \sum_{j \in J_i} a_{ij} [u_j - u_i] + |J_i| (u_i - x_i) \end{aligned} \quad (123)$$

But note that $|J_i| = d_i + 1$, the in-degree of the node agent added by the own node. From the Laplacian matrix definition, the collective dynamics of the filter will be given by

$$\dot{\mathbf{x}} = -L\mathbf{x} - L\mathbf{u} + (D + I_n)(\mathbf{u} - \mathbf{x}) = -(D + L + I_n)\mathbf{x} + (D - L + I_n)\mathbf{u} \quad (124)$$

This is an LTI system with system matrix $-(D + L + I_n)$ and input matrix $D - L + I_n$. Stability properties of the system can be evaluated through the eigenvalues of the system matrix and can be proven through the Gershgorin criteria.

3.3.4. Formation control and Leader-following behavior

Even though the standard consensus theory can provide control over the relative states of multiple agents, it is rather limited (in the way it was presented) to enforce some desired behaviors on the agents that are of operational practice. The protocol and graph topology are now discussed in order to provide Formation control and Leader Following Behavior, which are more applicable in the operations of marine vehicles.

Formation control can be achieved through the introduction of reference relative position signals δ_{ij} between the agents in the control inputs. For the most general case,

$$u_i = cK \sum_{j \in N_i} a_{ij} (x_j - x_i - \delta_{ji}) \quad (125)$$

The reference signals need not to be constant along time, which means that a trajectory generation law can be input to enforce a dynamically varying formation.

The consensus algorithm does not provide means to control the general location of the agents, and the formation function from equation (125) controls only relative position. Full controllability of the cooperative positioning behavior can be achieved through the introduction of a leader agent. The consensus leader-following is referred in literature as *consensus tracking*.

Consensus tracking can only be introduced in digraphs, by the usage of one root node of a spanning tree that has in-degree equal to zero. In this case, the row of the adjacency, the in-degree and the Laplacian Matrices corresponding to this node has all elements equal to zero. The overall result is that the consensus value reached for every agent, except the leader, is the initial leader state (LEWIS et al., 2014). One important drawback, though, is that the weight of the leader node in the topology affects the time to reach consensus with the leader included. Due to the averaging characteristic of the consensus protocols, the agents will reach consensus among each other in less time than they will align values with the leader, as the former does not act to reach the consensus. This is translated by a smaller Fiedler eigenvalue in the Laplacian Matrix. Note that the leader node is not required to be real; it can be defined as only a sequence of signals representing a desired dynamic behavior. In fact, this setup is even desirable, as the eventual failure of an agent defined as the leader system could handle the cooperative system uncontrollable.

3.4. Nonlinear Control

The linear control methods described either to perform control of a single standard DP vessel or to realize control of multiple agents can have its performance limited in the presence of nonlinearities that may rise during the operation. In order to cancel or cope with these nonlinearities and to add robustness to the system, basic aspects of some nonlinear control methods and concepts are now discussed.

3.4.1. Preliminary Definitions

Before proceeding with the basis of the nonlinear methods used, a set of operator functions will be defined. For a vector $\mathbf{p} \in \mathbb{R}^n$ of decoupled states, the modulus operator will be defined as the absolute value of each element of the vector:

$$|\mathbf{p}| = [|p_1| \quad |p_2| \quad \cdots \quad |p_n|]^T \quad (126)$$

Similarly, the vector power function will be given by

$$(\mathbf{p})^m = [(p_1)^m \quad (p_2)^m \quad \cdots \quad (p_n)^m]^T \quad (127)$$

The vector signum function will be defined as

$$\text{sign}(\mathbf{p}) = [\text{sign}(p_1) \quad \text{sign}(p_2) \quad \cdots \quad \text{sign}(p_n)]^T \quad (128)$$

with function $\text{sign}(p_i)$ is defined by

$$\text{sign}(p_i) = \begin{cases} -1, & \text{if } p_i < 0 \\ 0, & \text{if } p_i = 0 \\ 1, & \text{if } p_i > 0 \end{cases} \quad (129)$$

The vector saturation function will be defined as

$$\text{sat}(\mathbf{p}) = [\text{sat}(p_1) \quad \text{sat}(p_2) \quad \cdots \quad \text{sat}(p_n)]^T \quad (130)$$

The saturation function sat is defined by equation (131).

$$\text{sat}(p_i) = \begin{cases} -1 & \text{if } p_i < -1 \\ p_i, & \text{if } -1 \leq p_i \leq 1 \\ 1 & \text{if } p_i > 1 \end{cases} \quad (131)$$

3.4.2. Input-Output Feedback Linearization

The feedback linearization method consists in using the system states to create a control law that cancels out the plant's nonlinear terms, forcing the closed loop system to behave linearly. Take a nonlinear system given by the following equation:

$$\dot{x} = f(x) + b(x)u \quad (132)$$

in which x is the state vector, u is the input vector and $f(x)$ and $b(x)$ are known vector functions of x . A feedback linearization law that drives the tracking error $\tilde{x} = x - x_d$ to zero is given by

$$u = b(x)^{-1}[-K\tilde{x} + \dot{x}_d - f(x)] \quad (133)$$

The gain matrix K is chosen so that all the closed loop poles of the system are kept in the open left half plane of the complex plane. In closed loop,

$$\dot{\tilde{x}} = -K\tilde{x} \quad (134)$$

The Feedback Linearization technique is largely used in the control of nonlinear systems and, in particular, as a step in the application of Cooperative Control Methods. By defining a virtual control input v , the control law (133) can be modified to

$$u = b(x)^{-1}[v - f(x)] \quad (135)$$

The closed loop system will be, then, described by

$$\dot{x} = v \quad (136)$$

This is the same system used in equation (105) to describe a single integrator agent's motion.

While this method works well for systems with a well-known dynamic model, it fails to provide guaranteed performance when the model or the parameters are uncertain. In this case, a robust alternative to the feedback linearization is the sliding mode algorithm.

3.4.3. Sliding Mode Control

Suppose now that equation (132) has uncertainties in the vector functions $f(x)$ and $b(x)$. These uncertainties may come from simplifications in the system modeling, the presence of a random variable in the model or even the lack of complete knowledge of the system under all possible conditions. Although these effects may not arise near the open loop cut-off frequency, they may be significant under closed loop control in which higher frequencies can be achieved.

Take now the error vector $\tilde{x} = x - x_d$ and initial states $x(0)$. The control problem is to design a control algorithm that keeps the tracking error zero even in the presence of

disturbances or modeling uncertainties. Sliding Mode Control (SMC) can perform that task under the condition that uncertainties are finite and bounded.

Define a sliding manifold $\mathcal{S}(t)$ with dynamics chosen by the designer and in which all trajectories converge to the desired values. In the scalar case, $\mathcal{S}(t)$ is called sliding surface. The SMC method consists in forcing all trajectories of the system states to be driven to and kept in the manifold.

Take a sliding manifold $\mathcal{S}(t)$ defined in $\mathbb{R}^r$ through a vector equation $\sigma(x, t) = [\sigma_1, \sigma_2, \dots, \sigma_n]^T = [\mathbf{0}]_{n \times 1}$. In (SLOTINE; LI, 1991), the sliding manifold is defined as the binomial expansion (137):

$$\sigma(x, t) = \sum_{j=0}^{r-1} \binom{r-1}{j} \Lambda^{r-j-1} \frac{d^{(j)}}{dt^{(j)}} \tilde{x} \quad (137)$$

in which

$$\Lambda_{n \times n} = \text{diag}(\lambda_i); \lambda_i \in \mathbb{R}^+ \text{ and } i = 1, 2, \dots, n \quad (138)$$

The order of the system is r and n is the length of the controlled output vector. For example, if the system has 3 controlled states and order 3, the sliding vector will be defined by

$$\sigma(x, t) = \ddot{\tilde{x}} + 2\Lambda_{3 \times 3} \dot{\tilde{x}} + \Lambda_{3 \times 3}^2 \tilde{x} \quad (139)$$

Figure 19 - Graphical Interpretation of a sliding manifold with order 2

Source: (SLOTINE; LI, 1991)

When defined through equation (137), $\tilde{x}$ becomes the output of a sequence of low-pass filters with Λ equal to its cut-off frequency. The tracking problem is reduced to keep the trajectories on the sliding manifold $\mathcal{S}$, by keeping $\sigma = [\mathbf{0}]$ for all $t > 0$. Indeed,

$\sigma = [0]$ is a differential equation for which a solution is $\tilde{x} = [0]$. Figure 19 represents the graphical interpretation of the sliding manifold of a single variable with order 2.

As the error vector and the sliding manifold vector are related through a series of low-pass filters, its bounds can also be related. In (SLOTINE; LI, 1991) the relation between the sliding manifold vector σ bounds (referred here as Φ , the vector of the manifold bounds) and the error vector and error vector derivatives bounds is shown to be

$$|\sigma(t)| < \Phi \Rightarrow \left| \frac{d^{(j)}\tilde{x}(t)}{dt^{(j)}} \right| < 2^j \Lambda^{-(n-j-1)} \Phi, \quad (140)$$

for $\forall \Phi$ and $j = 0, 1, 2 \dots n - 1$

This bounds the tracking error of the system. For example, for a second order, the tracking error will be given by

$$\tilde{x} < \Lambda^{-1} \Phi \quad (141)$$

To enforce all the trajectories to be taken to and trapped in the sliding manifold, the so-called sliding condition must be met for each degree of freedom:

$$\frac{1}{2} \frac{d}{dt} \sigma_i^2 \leq -\Delta_i |\sigma_i| \quad (142)$$

in which Δ_i is a strictly positive constant, related to the velocity with which the trajectory converges to the sliding manifold. Equation (142) imposes that the quadratic distance to the surfaces, measured here by σ_i^2 diminish along all the system trajectories with velocities Δ_i , enforcing all the trajectories to point to the sliding manifold, as can be seen in Figure 20. Furthermore, the attendance of equation (142) makes the surface an invariant set.

Figure 20 - Sliding Condition

Source: Author

The times required for the trajectories to reach the sliding manifold when $x(0) \neq x_d(0) \Leftrightarrow \sigma(0) \neq [0]$ will be given by

$$t_{reach} = \Delta^{-1} |\sigma(0)| \quad (143)$$

in which $\Delta = \text{diag}(\delta_1, \delta_2, \dots, \delta_m)$ and t_{reach} is the vector of reaching times. Once the sliding condition is reached, the tracking error tends exponentially to zero with same dynamics of σ and time constants given by Λ^{-1} .

In order to ease the understanding, a simple example follows. Take the second order system given by equation (144):

$$\ddot{x} = f(x) + b(x)u \quad (144)$$

Consider f a vector function not exactly known. A control algorithm that enforces the sliding condition will be derived. Assume the extension of the error of f is known and bounded, in such way that

$$|f - \hat{f}| < F \quad (145)$$

In which F is the extension of the modeling error and $\hat{f}$ is the estimate of the f function. The sliding manifold variable will be, according to equation (137),

$$\sigma(x, t) = \tilde{x} + \Lambda \tilde{x} \quad (146)$$

Take now the first time derivative of σ :

$$\dot{\sigma}(x, t) = \ddot{\tilde{x}} + \Lambda \dot{\tilde{x}} = f(x) + b(x)u - \ddot{x}_d + \Lambda \dot{\tilde{x}} \quad (147)$$

In the first order sliding mode control technique, in order to track $\sigma = 0$ it is enough to enforce $\dot{\sigma} = 0$. In the absence of modeling errors, the best estimate of the so-called equivalent control, that enforces this condition would be calculated by

$$\hat{u} = b(x)^{-1}(-\hat{f}(x) + \ddot{x}_d - \Lambda \dot{\tilde{x}}) \quad (148)$$

When uncertainties are considered, the control law is extended by a discontinuous switching term, designed to cope with the bounded uncertainties. With this effect accounted, the overall control law will become

$$u = \hat{u} - K_{sm} \text{sign}(\sigma) \quad (149)$$

The strictly positive diagonal gain matrix K_{sm} is designed to cope with the sliding condition and not necessarily constant. For each degree-of-freedom the sliding condition is verified by

$$\frac{1}{2} \frac{d}{dt} \sigma_i^2 = \dot{\sigma}_i \sigma_i = [f_i(x_i) - \hat{f}_i(x_i)] \sigma_i - k_i |\sigma_i| \leq -\Delta_i |\sigma_i| \quad (150)$$

The last inequality is complied with if

$$k_i \geq F_i + \Delta_i \quad (151)$$

i.e. each gain k_i must be as big as the modeling error of its corresponding degree of freedom.

The introduction of the switching term $K_{sm} \text{sign}(\sigma)$ assumes perfect control action. In reality, it can introduce a high control effort when the trajectory of the state variables is next to the sliding manifold. This effort is translated to a high frequency switching, which can excite unmodeled vibration modes and cause undesired effects, provoking unacceptable actuator action and premature wear. This phenomenon is referred in literature as *chattering* and becomes more pronounced with larger modeling uncertainties and perturbations. Figure 21 shows the geometrical interpretation of the chattering effect.

Figure 21 - Chattering effect

In (SLOTINE; LI, 1991) a method to reduce the chattering is proposed by modifying the switching function to a smoothed version, at the cost of lower precision. The sliding manifold is closed by a boundary layer into which there is a transition of the control signal, according to the new control output

$$u = u_{eq} - K_{sm} \text{sat}(\Phi^{-1} \sigma) \quad (152)$$

The Boundary Layer is defined by the positive diagonal matrix $\Phi = \text{diag}(\phi_1, \phi_2, \dots, \phi_m)$. The control action is kept inside the boundary layer

where $\text{sat}\left(\frac{\sigma_i}{\phi_i}\right) = \frac{\sigma_i}{\phi_i}$. When outside it, the control vector $\mathbf{u}$ satisfies the sliding condition and the trajectories point towards the inside of the boundary layer, with tracking error limited by the width of it, as described by equation (153). Figure 22 illustrates the effect of the addition of the saturation function to the control action.

$$|\tilde{x}_i(t)| = \frac{\phi_i}{\lambda_i^{(n-1)}} \quad (153)$$

The boundary layer introduction is equivalent of introducing a low pass filter in the dynamics of the sliding vector σ , eliminating, thus, the chattering oscillations.

The matrix parameters Λ , Φ and K_{sm} , are tuned according to the extension of the modelling error (in the case of K_{sm}), the admissible tracking error (in the case of Φ) and the desired sliding dynamics (in the case of Λ). More specifically, λ_i is related to the velocity of convergence of the system when over the sliding manifold. While from the performance point of view it may seem interesting to make this variable high, it cannot be enhanced indefinitely, as it is related to the passing band of the system. The following criteria should be applied when choosing it:

- i. λ_i must be smaller than the first unmodeled resonance frequency of the system (referred as ν_r), according to the relation $\lambda_i < \frac{2\pi}{3} \nu_r$;
- ii. λ_i must be smaller than the inverse of the largest delay time of the system T_d such that $\lambda_i < \frac{1}{3T_d}$;
- iii. λ_i must be smaller than the sampling rate ν_s , such that $\lambda_i < \frac{1}{5} \nu_s$;
- iv. λ_i is chosen as the smaller of the three criteria above.

Figure 22 - Boundary Layer Illustration

3.4.4. Super Twisting Sliding Mode

The first order sliding mode method is simple to implement and effective to control both linear and nonlinear uncertain systems. The downside is the induction of high frequency switching and chattering, which can be attenuated through the smoothing boundary layer at the cost of a lower precision. From early 90's, a different approach has been introduced to reduce chattering, through the incorporation of the switched structure in higher order derivatives of the sliding variable. The technique is referred in literature as High Order Sliding Mode Control (HOSMC), but more specifically, it has mostly been applied on the second derivative of the sliding variable $\ddot{\sigma}$.

When the switched structure control is applied in higher orders, the discontinuity is eliminated from first order and consequently chattering is reduced. Overall, the objective is to keep (LEVANT, 2003):

$$\sigma(t) = \dot{\sigma}(t) = \ddot{\sigma}(t) = \frac{d^3\sigma(t)}{dt^3} = \frac{d^4\sigma(t)}{dt^4} = \dots = \frac{d^{r-1}\sigma(t)}{dt^{r-1}} = [0] \quad (154)$$

in which r is the order of the sliding mode.

Several approaches have been presented since then, including the Twisting Algorithm and the Super-Twisting Algorithm (LEVANT, 1993) and the Sub-Optimal Algorithm (BARTOLINI; FERRARA; USAI, 1998).

By looking over equation (154) it is clear that the high order implementations require information that is unavailable in most of the applications, the high order derivatives of the state variables. This limits the application of most of these algorithms or requires the implementation of other devices such as the robust exact differentiator (LEVANT, 1998), which is based on the Super-Twisting Algorithm (STA). The STA works in the second order derivatives of the sliding manifold, but it does not need information on the derivative of the sliding variable. Due to its simplicity, a lot of attention has been given to the STA in recent years, by its application in the form of controllers and observers. Basic formulation of the STA is given as follows. Take again the nonlinear state equation (132):

$$\dot{x} = f(x) + b(x)u$$

The sliding manifold variable $\sigma(x, t)$ can be defined, for example, by equation (137). Suppose the equation has relative degree $r = 1$, i.e. that the first derivative of the

sliding variable is a direct function of the control variable input. This means that $\frac{\partial \sigma}{\partial u} \neq 0$. This assumption makes possible the introduction of a discontinuous auxiliary control input $\dot{u}$, which can steer the sliding variable to zero in both first and second order sliding modes. The plant control u is kept continuous, reducing chattering. Conditions for robust solution are given next. Take the first and second derivative of the sliding variable. In a general form,

$$\dot{\sigma}(x, t) = \frac{\partial}{\partial t} \sigma(x, t) + \frac{\partial}{\partial x} \sigma(x, t) \dot{x} = \frac{\partial}{\partial t} \sigma(x, t) + \frac{\partial}{\partial x} \sigma(x, t) [f(x) + b(x)u] \quad (155)$$

And

$$\ddot{\sigma}(x, t) = \frac{\partial}{\partial t} \dot{\sigma}(x, t) + \frac{\partial}{\partial x} \dot{\sigma}(x, t) [f(x) + b(x)u] + \frac{\partial}{\partial x} \sigma(x, t) b(x) \dot{u} \quad (156)$$

Note that the relative degree equal to 1 pre-condition means that the derivative of the control input $\dot{u}$ affects the second time derivative of the manifold. If the control signal u is continuous (even though its derivative is not) the chattering is avoided. Consider now the following conditions hold (LEVANT, 1993):

- i. The modulus $|u(t)|$ of the control input is bounded by a constant $U_M > 1$ and the solution of the system is continuous for all $t > 0$ provided that $u(t)$ is continuous;
- ii. There exists $u_1 \in (0,1)$ such that for any continuous function $u(t)$ with $|u(t)| > u_1$, there is t_1 , such that $\sigma(t)u(t) > 0$ for all $t > t_1$. Therefore, exists $u(t) = -\text{sign}(\sigma(t_0))$, where t_0 is the initial value of time, provides hitting the manifold $\sigma = 0$ in finite time
- iii. Suppose there exists vectors φ , K_m and K_M with positive valued elements such that

$$\begin{cases} \left| \frac{\partial}{\partial t} \dot{\sigma}(x, t) + \frac{\partial}{\partial x} \dot{\sigma}(x, t) [f(x) + b(x)u] \right| \leq \varphi \\ 0 \leq K_m \leq \left| \frac{\partial}{\partial u} \dot{\sigma}(x, t) \right| \leq K_M \end{cases} \quad (157)$$

The above conditions provide bounding requirements for the sliding manifolds derivatives and the control input variables. The super twisting algorithm will be given by equation (158) (SHTESSSEL et al., 2014).

$$\begin{cases} \mathbf{u} = \mathbf{u}_{eq} - K_1 |\sigma|^{\frac{1}{2}} \text{sign}(\sigma) + \mathbf{u}_a \\ \dot{\mathbf{u}}_a = -K_2 \text{sign}(\sigma) \end{cases} \quad (158)$$

Note that the term $\mathbf{u}_a$ is continuous in the first order sliding manifold and the term $-K_1 |\sigma|^{\frac{1}{2}} \text{sign}(\sigma)$ decreases with σ , making the overall control $\mathbf{u}$ chattering free. For the bounds described in equation (157), the positive gains K_1 and K_2 are related as

$$\begin{cases} K_2 > \frac{\varphi}{K_m} \\ K_1^2 \geq \frac{4\varphi K_M(K_2 + \varphi)}{K_m^2 K_m(K_2 - \varphi)} \end{cases} \quad (159)$$

The phase portrait of the manifold variable is shown in Figure 23. The trajectories are twisted towards the origin.

Figure 23 - Phase portrait of the Sliding Manifold in the STA

Source: Author

As an example, take again a second order system given by equation (144):

$$\ddot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{b}(\mathbf{x})\mathbf{u}$$

Its sliding manifold will be given by equation (146), and its first time derivative will be given by (147).

$$\begin{cases} \sigma(\mathbf{x}, t) = \tilde{\mathbf{x}} + \Lambda \tilde{\mathbf{x}} \\ \dot{\sigma}(\mathbf{x}, t) = \mathbf{f}(\mathbf{x}) + \mathbf{b}(\mathbf{x})\mathbf{u} - \ddot{\mathbf{x}}_d + \Lambda \dot{\tilde{\mathbf{x}}} \end{cases}$$

Finally, the second time derivative of the sliding manifold variable will be

$$\ddot{\sigma}(\mathbf{x}, t) = \dot{\mathbf{f}}(\mathbf{x}) + \Lambda \mathbf{f}(\mathbf{x}) + [\dot{\mathbf{b}}(\mathbf{x}) + \Lambda \mathbf{b}(\mathbf{x})]\mathbf{u} - \ddot{\mathbf{x}}_d + \mathbf{b}(\mathbf{x})\dot{\mathbf{u}} \quad (160)$$

The required bounds will be

$$\begin{cases} |\dot{f}(x) + \Lambda f(x) + [\dot{b}(x) + \Lambda b(x)]u - \ddot{x}_d| \leq \varphi \\ 0 \leq K_m \leq |b(x)| \leq K_M \end{cases} \quad (161)$$

Note that the bounding condition of equation (145), can be mapped to this one, provided that there is also bounding in the derivatives of the functions f and b .

Stability proof for the STA is complex and usually solved by geometric means, through the interpretation of the dynamics as a discontinuous set, in the so-called Filippov sense (SLOTINE; LI, 1991). The overall idea is that the bounding requirements from equation (157) can be mapped to a bounded differential inclusion. The following stability arguments are extracted from (LEVANT, 1993), applied for scalar states. Consider the following inequalities

$$\begin{cases} |C| \leq \varphi \\ 0 \leq K_m \leq |K| \leq K_M \end{cases} \quad (162)$$

in which $C = \frac{\partial}{\partial t} \dot{\sigma}(x, t) + \frac{\partial}{\partial x} \dot{\sigma}(x, t)[f(x) + b(x)u]$ and $K = \frac{\partial}{\partial u} \dot{\sigma}(x, t)$. Application of the super twisting algorithm to the second time derivative of the sliding variable provides

$$\ddot{\sigma} = C + K\dot{u}_a - \frac{1}{2}K_1K \frac{\dot{\sigma}}{|\sigma|^{\frac{1}{2}}} \quad (163)$$

The states trajectory lies inside the set of solution points of the differential inclusion

$$\ddot{\sigma} \in -[K_m K_2 - \varphi, K_M K_2 + \varphi] \text{sign}(\sigma) - \frac{1}{2}K_1[K_m, K_M] \frac{\dot{\sigma}}{|\sigma|^{\frac{1}{2}}} \quad (164)$$

It follows from the sequence $\{\dot{\sigma}_i\}$ of intersection points with the axis $\sigma = 0$ satisfies $|\dot{\sigma}_{i+1}/\dot{\sigma}_i| \leq q < 1$, which implies $|\dot{\sigma}_i| \rightarrow 0$ as $i \rightarrow \infty$. The sum of the encircling time series is estimated by a geometric series.

A consistent Lyapunov approach to the gains calculation was presented in (MORENO; OSORIO, 2012) based on which most applications of the Super Twisting Controller are proved stable. Yet, a popular, simple and usual tuning rule, given in (LEVANT, 1998) can be applied. If the closed loop sliding vector can be written as

$$\begin{cases} \dot{\sigma} = -K_1 |\sigma|^{\frac{1}{2}} \text{sign}(\sigma) + \omega \\ \dot{\omega} = -K_2 \text{sign}(\sigma) + \delta_F \end{cases} \quad (165)$$

with disturbances bounded by

$$|\delta_F| \leq \varphi \quad (166)$$

then the gains can be calculated by

$$\begin{cases} K_1 = 1.5|\boldsymbol{\varphi}|^{\frac{1}{2}} \\ K_2 = 1.1\boldsymbol{\varphi} \end{cases} \quad (167)$$

A recent work by (SEEBER; HORN, 2017) has shown that such rule provide finite time stability to the STA.

4. PROPOSED METHODS

In this chapter, the proposed methods for this work are presented. First, a general structure for the cooperative DP system is discussed, tying up the control and estimation blocks that are to be proposed. Then, a framework for application of both first and second order sliding mode control applied to the cooperative system is deduced and discussed. Finally, taking advantage of the general data structure enabled by wireless communication, methods for enhancing estimated data and robustness of the system are proposed.

4.1. An architecture for cooperative DP systems

The architecture proposed for the cooperative DP system is presented in Figure 24. The control is considered to be decentralized, so the wireless communication structure illustrated in the figure is not a hardware implementation, but a connection topology, described by a graph. The actual hardware implementation is performed through any wireless technology, such as radio, wi-fi or 5G. The Automatic Identification System (AIS), an International Maritime Organization standard broadcast signal present in all large ships, for instance, could be used. Each vessel sends its position and environmental data measurements or estimate, such as DP Current, sea states or wind measurements. Relative position references are considered to be broadcasted.

Figure 24 - General Architecture for the Cooperative DP

Source: Author

Figure 25 shows the internal blocks of each agent depicted in the last figure. The system considers that a standard, local DP system is running in each single vessel. Wind feedforward compensation is not shown but should be present.

Figure 25 - Control blocks for a single vessel

Source: Author

Note that there are three estimation filters running. In the local DP system, an Extended Kalman Filter for the vessel velocity, environmental action (such as DP current) and filtered positions is implemented. This filter sends data for the cooperative controller Kalman Filter, which estimates the data of all neighbor agents. This enhances the overall system robustness, as in the event of a network shortage the neighbor agents' positions can be integrated (dead reckoning). Finally, a consensus filter can be implemented to enhance environmental data estimates, in a method similar to a distributed wireless sensor network. The environmental action may be composed of wind velocity, sea state and, under adequate assumptions, DP current estimates.

The cooperative controller block sees the junction DPS + Vessel as a single plant and sends position references to it. This controller can be seen, in a way, as a guidance system. A standard approach for its implementation would be to use the linear consensus controller from equation (110):

$$\boldsymbol{\eta}_{di} = \boldsymbol{\eta}_i + c \sum_{j \in N_i} a_{ij} [k_1(\boldsymbol{\eta}_j - \boldsymbol{\eta}_i) + k_2(\dot{\boldsymbol{\eta}}_j - \dot{\boldsymbol{\eta}}_i)] \quad (168)$$

For each vessel i , $\boldsymbol{\eta}_i = [x_i, y_i, \psi_i]^T$ is the positions vector in the plane and the yaw angle. The set point for each local DP system is given by $\boldsymbol{\eta}_{di} = [x_{di}, y_{di}, \psi_{di}]^T$. Gains k_1 , k_2 and c are the consensus gains that can be calculated through equations (117), (118) and (119). When thinking about a DP system mass-spring-damper analogy

(Figure 26-a), the introduction of such law corresponds to adding extra spring-damper sets between the vessels' commanded set points, as can be seen in Figure 26-b.

Figure 26 - a) Mass-Spring-Damper DP Analogy (b) Mass-Spring DP+Cooperative Protocol Analogy

In the next sections, a cooperative sliding mode controller and a cooperative super-twisting controller will be presented and discussed.

Figure 27 summarizes the control sequence to be run in each agent according to the architecture proposed.

Figure 27 - Control loop sequence

1. Update measurements and local DPS Extended Kalman Filter states, producing the agent's own states and, when available, environmental data estimations;
2. Transmit own states and data to the neighbors;
3. Receive neighbors' states and environmental data, when available;
4. Update local Kalman Filter for neighbors;
5. Update consensus filter using neighbors' environmental estimations;
6. Calculate DPS references via cooperative controller;
7. Calculate feedforward compensation from estimated environmental data;
8. Calculate own DPS control action and allocation via any available method.

Source: Author

4.2. Agent Anonymization

A framework for the definition of cooperative sliding modes for the DP system is presented next. The basis of this method have been addressed in (IANAGUI; TANNURI, 2015) and in (IANAGUI; QUEIROZ FILHO; TANNURI, 2016).

The first step of the technique involves the cancellation of the DP closed loop dynamics in order to represent each agent of the system in the form of a double integrator. This enables the cooperative control law to be designed independently of each agent's dynamic behavior, permitting the application of the method in heterogeneous systems.

Suppose a PID controller is running on a DP vessel with linearized dynamics. The equations are solved in the LRF, but the resulting transfer function is given in the GRF, as described in section 3.2.2. Closed loop DPS plant transfer function for each DOF, according to equation (60) will be given by

$$G_{CLk}(s) = \frac{k_{Dk}s^2 + k_{Pk}s + k_{Ik}}{m_{kk}s^3 + k_{Dk}s^2 + k_{Pk}s + k_{Ik}}, k = x, y, \psi \quad (169)$$

Note that this is a linearized system that does not address disturbances and neither high velocity dynamics. While adequate for station keeping, this model's adherence may be reduced in maneuvering situations.

The sliding mode cooperative control system proposed is designed in the state space. The third order system in (169) has one non-dominant pole that can be cut-out. The remaining states can be mapped to positions and velocities. The resulting transfer function will be a second order system with a zero in the left side of the complex plane:

$$G_k(s) = \frac{b_{1k}s + b_{0k}}{s^2 + a_{1k}s + a_{0k}}, k = x, y, \psi \quad (170)$$

Parameters b_{1k} , b_{0k} , a_{1k} and a_{0k} , for each degree of freedom $k = x, y, \psi$ are dependent on the system's pole. In state space representation the system is given by

$$\begin{aligned} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} &= \begin{bmatrix} 0 & I_{3 \times 3} \\ -A_0 & -A_1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} B_1 \\ B_0 - A_1 B_1 \end{bmatrix} u \\ y_k &= [1 \quad 0] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \end{aligned} \quad (171)$$

In which $u = \eta_d$, $x_1 = \eta$ and $x_2 = \dot{\eta} - B_1 \eta_d$. The state space variables can be directly mapped into the vessel position and velocity in the GRF and

$$\begin{aligned} A_1 &:= \text{diag}([a_{1x} \quad a_{1y} \quad a_{1\psi}]); A_0 := \text{diag}([a_{0x} \quad a_{0y} \quad a_{0\psi}]); \\ B_1 &:= \text{diag}([b_{1x} \quad b_{1y} \quad b_{1\psi}]); B_0 := \text{diag}([b_{0x} \quad b_{0y} \quad b_{0\psi}]) \end{aligned} \quad (172)$$

Consider a group of n vessels with the described dynamics put in the input-output form:

$$\ddot{\eta}_i = -A_{0i}\eta_i - A_{1i}\dot{\eta}_i + B_{0i}u_i + B_{1i}\dot{u}_i, \text{ for } i = \{1, 2, \dots, n\} \quad (173)$$

The input vector $\mathbf{u}_i$ is the set point for the DP system, given by $\boldsymbol{\eta}_{di}$. In order to simplify the control signal output generation and design a cooperative controller with dynamic characteristics independent of each agent's dynamic behavior, the input vector dependent on u_i and $\dot{u}_i$ is replaced by the aggregating vector $U_i = B_{0i}u_i + B_{1i}\dot{u}_i$. The inverse transformation can be achieved by applying a filter in each degree of freedom, as described in equation (174).

$$u_{ki} = \frac{1}{b_{1ki}s + b_{0ki}} U_{ki}; \quad \begin{cases} k = x, y, \psi \\ i = 1, 2, \dots, n \end{cases} \quad (174)$$

The dynamics of ships $1, 2, \dots, n$ can be feedforward canceled such that their acceleration becomes a function of a new input $\tilde{\mathbf{u}}$, as in (175).

$$\ddot{\boldsymbol{\eta}}_i = \tilde{\mathbf{u}}_i + \tilde{\mathbf{f}}_i, i = 1, 2, \dots, n \quad (175)$$

This is achieved by making

$$\mathbf{U}_i := A_{0i}\boldsymbol{\eta}_i + A_{1i}\dot{\boldsymbol{\eta}}_i + \tilde{\mathbf{u}}_i \quad (176)$$

Each function $\tilde{\mathbf{f}}_i$ represents a lumped term of bounded errors that account for modeling, disturbances and measurement errors. The above cancelation is introduced so as not to force each agent to have previous information about its neighbors' dynamics. Note that, if not fully canceled, the dynamics could also be replaced by a desired closed loop behavior, as long as this is equal and known by all agents (maintaining the convenience of low information quantities exchanged). Finally, this cancellation can also be applied to nonlinear systems by using feedback linearization techniques. The filter from equation (174) is implemented along with the cancellation equation (176) through the differential equation:

$$\dot{\mathbf{u}}_i = B_{1i}^{-1}\mathbf{U}_i - B_{1i}^{-1}B_{0i}\mathbf{u}_i = B_{1i}^{-1}A_{0i}\boldsymbol{\eta}_i + B_{1i}^{-1}A_{1i}\dot{\boldsymbol{\eta}}_i + B_{1i}^{-1}\tilde{\mathbf{u}}_i - B_{1i}^{-1}B_{0i}\mathbf{u}_i \quad (177)$$

Digital implementation of Equation (177) can be performed by any discretization method. If, for example, forward difference is used, considering $\boldsymbol{\eta}_i(k)$ and $\dot{\boldsymbol{\eta}}_i(k)$ known states and $\tilde{\mathbf{u}}(k)$ the control algorithm output, for a sample time h it becomes

$$\begin{aligned} \mathbf{u}_i(k+1) &= hB_{1i}^{-1}A_{0i}\boldsymbol{\eta}_i(k) + hB_{1i}^{-1}A_{1i}\dot{\boldsymbol{\eta}}_i(k) + hB_{1i}^{-1}\tilde{\mathbf{u}}_i(k) \\ &\quad + (I_{3 \times 3} - hB_{1i}^{-1}B_{0i})\mathbf{u}_i(k) \end{aligned} \quad (178)$$

4.3. Cooperative Sliding Mode Control

Suppose the n agents with dynamics described by (175) are related through a communication topology described by a connected digraph with no leader node.

Define an agent consensus error $\mathbf{z}_i$ as the weighted relative position errors of an agent and its neighbors:

$$\mathbf{z}_i = \sum_{j=1}^n a_{ij}(\boldsymbol{\eta}_i - \boldsymbol{\eta}_j) = \sum_{j=1}^n a_{ij}\boldsymbol{\eta}_i - \sum_{j=1}^n a_{ij}\boldsymbol{\eta}_j = d_i\boldsymbol{\eta}_i - \sum_{j=1}^n a_{ij}\boldsymbol{\eta}_j \quad (179)$$

Recall that a_{ij} is the entrance i, j of the weighted adjacency matrix. Variable d_i is the in-degree value of agent i , which is the diagonal entrance of the Laplacian matrix L .

Take now each agent's error vector as an input to the sliding manifold vector $\boldsymbol{\sigma}_i$:

$$\boldsymbol{\sigma}_i = \dot{\mathbf{z}}_i + \Lambda \mathbf{z}_i = \sum_{j=1}^n a_{ij}(\dot{\boldsymbol{\eta}}_i - \dot{\boldsymbol{\eta}}_j) + \Lambda \sum_{j=1}^n a_{ij}(\boldsymbol{\eta}_i - \boldsymbol{\eta}_j) \quad (180)$$

In which Λ is a 3×3 positive definite diagonal matrix. Alternatively,

$$\boldsymbol{\sigma}_i = d_i(\dot{\boldsymbol{\eta}}_i + \Lambda \boldsymbol{\eta}_i) - \sum_{j=1}^n a_{ij}(\dot{\boldsymbol{\eta}}_j + \Lambda \boldsymbol{\eta}_j) \quad (181)$$

The complete sliding vector characterizing the agent's behaviors when under consensus is given by

$$\boldsymbol{\sigma} = L[\dot{\boldsymbol{\eta}} + (I_{n \times n} \otimes \Lambda)\boldsymbol{\eta}] \quad (182)$$

Theorem 1. Assuming the graph that characterizes the communication topology has a spanning tree and is weight balanced, the cooperative sliding manifold $S(t)$ dynamic behavior, defined by the sliding vector (182), is asymptotically stable, guaranteeing, that, when in sliding condition, the error converges asymptotically to zero.

Proof. The system dynamics will lay over the sliding manifold when $\boldsymbol{\sigma} = \mathbf{0}$, resulting in the first order low pass dynamics given by

$$L[\dot{\boldsymbol{\eta}} + (I_{n \times n} \otimes \Lambda)\boldsymbol{\eta}] = [\mathbf{0}]_{3n \times 1} \quad (183)$$

If L is strongly connected and has a spanning tree, it has rank $n - 1$ and the only vector in the null-space of L is $I_{3n \times 3n} \mathbf{c}_{ss}$, such as stated in section 3.3.1. The resulting dynamics are then given by

$$\dot{\boldsymbol{\eta}} = -(I_{n \times n} \otimes \Lambda)\boldsymbol{\eta} + I_{3n \times 3n} \mathbf{c}_{ss} \quad (184)$$

The dynamic system above is clearly stable for a positive definite matrix Λ , and asymptotically converges to the common vector $\boldsymbol{\eta}_{i_{ss}} = \Lambda^{-1} \mathbf{c}_{ss}$ ■.

This result states that, when over the sliding manifold the system dynamic behavior is not affected by the network topology (as long as the graph maintains its connectivity).

The consensus vector $c_{ss} \in \mathbb{R}^3$ calculated as average value of the initial conditions of each agent $\eta_i(0)$ weighted by matrix Λ . If the graph is weight balanced, the steady state vector is calculated as a form of equation (102):

$$c_{ss} = \frac{1}{n} \sum_{i=1}^n \Lambda \eta_i(0) \quad (185)$$

As an overall note, observe that the sliding condition gain Λ determines the system dynamics when in sliding motion. As the system's own dynamics have been canceled, Λ must be adequately chosen so as not only to satisfy conditions i-iv given in section 3.4.3, but also not to change too quickly the DP system's set point, as otherwise that vessel would not be able to track it.

The following theorem states the conditions for the sliding motion to be achieved.

Theorem 2. Consider the agent's dynamics given by equation (175). Suppose a vector $\bar{f}_i$ can be defined as the best approximation of the unknown bounded disturbances $\tilde{f}_i$. If the communication topology graph is connected, then the sliding motion of the formation will be achieved, and the sliding manifold will be reached in finite time by application of the following control law:

$$\tilde{u}_i = -\bar{f}_i - \Lambda \dot{\eta}_i + \frac{1}{d_i} \left[\sum_{j=1}^n a_{ij} (\tilde{u}_j + \bar{f}_j + \Lambda \dot{\eta}_j) - K_{sm} \text{sgn}(\sigma_i) \right] \quad (186)$$

In which $K_{sm} \in \mathbb{R}^{3 \times 3}$ is a positive definite diagonal gain matrix and $\text{sgn}(\sigma_i)$ is the vector of sign functions of each entrance.

Proof. As seen in section 3.4.3, sliding motion is achieved by enforcing $\dot{\sigma} = 0$. For each agent, the time derivative of the sliding manifold will be given by

$$\dot{\sigma}_i = d_i (\ddot{\eta}_i + \Lambda \dot{\eta}_i) - \sum_{j=1}^n a_{ij} (\ddot{\eta}_j + \Lambda \dot{\eta}_j) \quad (187)$$

Equation (187) can be rewritten in terms of the input-output dynamics from (175):

$$\dot{\sigma}_i = d_i (\tilde{u}_i + \bar{f}_i + \Lambda \dot{\eta}_i) - \sum_{j=1}^n a_{ij} (\tilde{u}_j + \bar{f}_j + \Lambda \dot{\eta}_j) \quad (188)$$

The application of (186) in (188) yields:

$$\dot{\sigma}_i = d_i (\tilde{f}_i - \bar{f}_i) - \sum_{j=1}^n a_{ij} (\tilde{f}_j - \bar{f}_j) - K_{sm} \text{sgn}(\sigma_i) \quad (189)$$

The generalized vector of derivatives of $\sigma_{3n \times 1}$ will then, be

$$\dot{\sigma}_{3n \times 1} = (L_{n \times n} \otimes I_{3 \times 3})(\tilde{f} - \bar{f})_{3n \times 1} - (I_{n \times n} \otimes K_{3 \times 3})\text{sgn}(\sigma)_{3n \times 1} \quad (190)$$

Vector and matrices dimensions are evidenced for better understanding. Define

$$L_{3n} = (L_{n \times n} \otimes I_{3 \times 3}) \quad (191)$$

$$K_{3n} = (I_{n \times n} \otimes K_{sm_{3 \times 3}}) \quad (192)$$

And consider the scalar function

$$\sigma^T \dot{\sigma} = \sigma^T [L_{3n}(\tilde{f} - \bar{f}) - K_{3n}\text{sgn}(\sigma)] = \sigma^T L_{3n}(\tilde{f} - \bar{f}) - \sigma^T K_{3n}\text{sgn}(\sigma) \quad (193)$$

As the gain matrices are diagonal and the system can be considered decoupled, it can be rewritten as

$$\begin{aligned} \sigma^T \dot{\sigma} &= \sum_{i=1}^n \left\{ \sigma_i^T \left[d_i(\tilde{f}_i - \bar{f}_i) - \sum_{j=1}^n a_{ij}(\tilde{f}_j - \bar{f}_j) \right] - \sigma_i^T K_{sm} \text{sgn}(\sigma_i) \right\} \\ &= \sum_{k=x,y,\psi} \sum_{i=1}^n \left\{ d_i \sigma_{ik}(\tilde{f}_{ik} - \bar{f}_{ik}) - \sigma_{ik} \sum_{j=1}^n a_{ij}(\tilde{f}_{jk} - \bar{f}_{jk}) - k_k |\sigma_{ik}| \right\} \end{aligned} \quad (194)$$

For each degree of freedom k of each agent i , the sliding condition (142) will be satisfied if:

$$\left[d_i(\tilde{f}_{ik} - \bar{f}_{ik}) - \sum_{j=1}^n a_{ij}(\tilde{f}_{jk} - \bar{f}_{jk}) \right] \sigma_{ik} - k_{ik} |\sigma_{ik}| \leq \Delta_{ik} |\sigma_{ik}| \quad (195)$$

If $k_{ik} = F_{ik} + \Delta_{ik}$, where F_{ik} represents a lumped error boundary, then the requirements for the sliding condition will be

$$F_{ik} \geq d_i(\tilde{f}_{ik} - \bar{f}_{ik}) - \sum_{j=1}^n a_{ij}(\tilde{f}_{jk} - \bar{f}_{jk}) \quad (196)$$

Note that the modelling error $\tilde{f}_{ik} - \bar{f}_{ik}$ is bounded but unknown. Suppose that, for each degree of freedom, the following inequality holds.

$$|\tilde{f}_{ik} - \bar{f}_{ik}| \leq \Gamma_{ik} \quad (197)$$

Consider this condition holds for all agents. Then, if

$$d_i |\tilde{f}_{ik} - \bar{f}_{ik}| + \sum_{j=1}^n a_{ij} |\tilde{f}_{jk} - \bar{f}_{jk}| \leq 2d_i \Gamma_{ik} \leq F_{ik} \quad (198)$$

The sliding condition will be satisfied, and the group will reach the sliding manifold in finite time ■.

Recall that, for smoothing purposes, the sign function can be replaced by the sat function. In equation (186) the connectivity assumption for the graph is required, as the

in-degree value of all agents needs to be non-zero (and thus, positive). Figure 28 shows the algorithm summary.

This result states that, despite not affecting sliding motion, the graph topology affects the conditions for it to be reached.

Figure 28 - Sliding Mode Cooperative Control Algorithm Summary

1. Receive the neighbors' current states and action $\eta_j, \dot{\eta}_j$ and $\tilde{u}_j$;
2. Calculate the agent's sliding vector from equation (181):

$$\sigma_i = d_i(\dot{\eta}_i + \Lambda\eta_i) - \sum_{j=1}^n a_{ij}(\dot{\eta}_j + \Lambda\eta_j)$$
3. Calculate the control action from equation (186):

$$\tilde{u}_i = -\bar{f}_i - \Lambda\dot{\eta}_i + \frac{1}{d_i} \left[\sum_{j=1}^n a_{ij}(\tilde{u}_j + \bar{f}_j + \Lambda\dot{\eta}_j) - K \text{sgn}(\sigma_i) \right]$$
4. Calculate the local DPS reference from filter (178):

$$\mathbf{u}_i(k+1) = hB_{1i}^{-1}A_{0i}\eta_i(k) + hB_{1i}^{-1}A_{1i}\dot{\eta}_i(k) + hB_{1i}^{-1}\tilde{u}_i(k) + (I_{3 \times 3} - hB_{1i}^{-1}B_{0i})\mathbf{u}_i(k)$$

Source: Author

4.3.1. Formation Control

The formation behavior can be introduced in the sliding modes consensus through the modification of the error variable (179), such that

$$\mathbf{z}_i = \sum_{j=1}^n a_{ij}(\eta_i - \eta_j - \delta_{ij}) \quad (199)$$

Vector $\delta_{ij}(t) = -\delta_{ji}(t)$ is the smooth, time varying relative position reference required for the desired formation. Stability follows from the same arguments from last section, with sliding variable and control output given, respectively by

$$\sigma_i = d_i(\dot{\eta}_i + \Lambda\eta_i) - \sum_{j=1}^n a_{ij}[\dot{\eta}_j + \dot{\delta}_{ij} + \Lambda(\eta_j + \delta_{ij})] \quad (200)$$

And

$$\tilde{u}_i = -\bar{f}_i - \Lambda\dot{\eta}_i + \frac{1}{d_i} \left\{ \sum_{j=1}^n a_{ij}[\tilde{u}_j + \bar{f}_j + \dot{\delta}_{ij} + \Lambda(\dot{\eta}_j + \dot{\delta}_{ij})] - K \text{sgn}(\sigma_i) \right\} \quad (201)$$

Rules for definition of relative position reference will depend on the required mission objectives.

4.3.2. Consensus tracking

The consensus tracking problem is considered with the introduction of a Leader vessel, which can be a real or a virtual agent. If a virtual leader is used, adequate care with its dynamics must be taken, as following agents will need to be able to synchronize to it. A leader vessel is introduced in a digraph through a spanning tree, with the leader as its root node.

In the following simulations and experiments, the virtual leader dynamics will be modelled as a second transfer function for each degree of freedom with responses similar to that of the single DP agents, as in equation (171).

The Leader's states and inputs are considered to be known by the follower agents.

4.3.3. Extent of modeling errors and disturbances

For an adequate application of the sliding mode technique, an estimate of the size of the modeling error $|\tilde{f}_{ik} - \bar{f}_{ik}|$ needs to be addressed. In a DP vessel, such errors may come from linearization assumptions, inaccurate environmental forces estimations and non-measured disturbances.

Take for example the linearized DP model from section 3.2.1, used for control design in chapter 3.2.2. Even though it is valid for most DP applications in which the main objective is station keeping, the slow speed and small angles assumption may affect controllability in more dynamically changing maneuvers and in the presence of varying disturbances, a situation that cannot be discarded for the devised cooperative applications. The following steps are proposed as a method for estimating such inaccuracies and determine error boundaries for the determination of sliding gains.

Consider the input-output linearized model of the closed loop controlled vessel with a proportional derivative controller (tuned according to the method from section 3.2.2, and given by

$$\ddot{\eta} = -M^{-1}K_P\eta - M^{-1}K_D\dot{\eta} + M^{-1}K_P\eta_d + M^{-1}K_D\dot{\eta}_d \quad (202)$$

The estimations of $\tilde{f} - \bar{f}$ can be thought as the deviations of the above dynamics. Take for example the same control law applied to the full nonlinear model given in equation (16), with operational forces considered null and feedforward cancellations:

$$M\dot{\mathbf{v}} + C_{cor}(\mathbf{v})\mathbf{v} + K_D\mathbf{v} + K_P R_G^L \boldsymbol{\eta} = K_P R_G^L \boldsymbol{\eta}_d + K_D \mathbf{v}_d + (\mathbf{F}_{curr} - C_{cur}(\mathbf{v}, \mathbf{v}_c)\mathbf{v}_c + \mathbf{F}_{wave} - \hat{\mathbf{F}}_{drift}) + (\mathbf{F}_{wind} - \hat{\mathbf{F}}_{wind}) \quad (203)$$

The feedforward estimates $\hat{\mathbf{F}}_{wind}$ and $\hat{\mathbf{F}}_{drift}$ are respectively the estimates of wind forces (usually based on speed measurements) and lumped current and second order wave forces, output from the Kalman Filter bias term:

$$\hat{\mathbf{F}}_{drift} = M R_G^L \hat{\mathbf{b}} \quad (204)$$

By applying the adequate transformations between frames and isolating the acceleration vector, equation (203) in the GRF will be given by

$$\begin{aligned} \ddot{\boldsymbol{\eta}} = & -R_L^G [M^{-1}C_{cor}(\mathbf{v}) + M^{-1}K_D - R_G^L \dot{R}_L^G] R_G^L \dot{\boldsymbol{\eta}} - R_L^G M^{-1}K_P R_G^L \boldsymbol{\eta} + R_L^G M^{-1}K_P R_G^L \boldsymbol{\eta}_d \\ & + R_L^G (\mathbf{F}_{curr} - R_L^G M^{-1}C_{cur}(\mathbf{v}, \mathbf{v}_c)\mathbf{v}_c + \mathbf{F}_{wave} - \hat{\mathbf{F}}_{drift}) \\ & + R_L^G M^{-1}K_D R_G^L \dot{\boldsymbol{\eta}}_d + R_L^G (\mathbf{F}_{wind} - \hat{\mathbf{F}}_{wind}) \end{aligned} \quad (205)$$

To evaluate the direct comparison of equations (202) and (205), consider a simplification in which the transformation matrices are accurately known and used to solve the control output at each step. In this case, the double transformations $R_L^G[*]R_G^L$ will be dropped:

$$\begin{aligned} \ddot{\boldsymbol{\eta}} = & -[M^{-1}C_{cor}(\mathbf{v}) + M^{-1}K_D - R_G^L \dot{R}_L^G] \dot{\boldsymbol{\eta}} - M^{-1}K_P \boldsymbol{\eta} + M^{-1}K_P \boldsymbol{\eta}_d + M^{-1}K_D \dot{\boldsymbol{\eta}}_d \\ & + R_L^G (\mathbf{F}_{curr} - R_L^G M^{-1}C_{cur}(\mathbf{v}, \mathbf{v}_c)\mathbf{v}_c + \mathbf{F}_{wave} - \hat{\mathbf{F}}_{drift}) + R_L^G (\mathbf{F}_{wind} - \hat{\mathbf{F}}_{wind}) \end{aligned} \quad (206)$$

Subtract (202) from (206) to obtain

$$\begin{aligned} \tilde{\mathbf{f}} - \bar{\mathbf{f}} = & R_L^G (\mathbf{F}_{curr} - R_L^G M^{-1}C_{cur}(\mathbf{v}, \mathbf{v}_c)\mathbf{v}_c + \mathbf{F}_{wave} - \hat{\mathbf{F}}_{drift}) \\ & - [M^{-1}C_{cor}(\mathbf{v}) - R_G^L \dot{R}_L^G] \dot{\boldsymbol{\eta}} + R_L^G (\mathbf{F}_{wind} - \hat{\mathbf{F}}_{wind}) \end{aligned} \quad (207)$$

Equation (207) is composed of three terms. If the measurement is accurate and the mean wind component is compensated by $\hat{\mathbf{F}}_{wind}$, the wind force boundary can be given by adequate scaling of the standard deviation of one of the standard spectrums (see APPENDIX B):

$$|\mathbf{F}_{wind} - \hat{\mathbf{F}}_{wind}| \leq \delta_{wind} = 0.5\rho_{air} \left[\sum_{j=1}^n \sqrt{2S_W(\omega_{wj})\Delta\omega_w} \right]^2 \quad (208)$$

The rotation matrix can be kept so as to modulate the forces by the attitude angle. The static current and wave drift force error is bounded and almost fully corrected in the Kalman Filter as deviation from the expected dynamics (the slowly varying Bias term):

$$|\mathbf{F}_{curr} - R_L^G M^{-1} C_{cur}(\mathbf{v}, \mathbf{v}_c) \mathbf{v}_c + \mathbf{F}_{wave} - \hat{\mathbf{F}}_{drift}| \leq \delta_{drift} \quad (209)$$

As each vessel is a mechanical system, it is feasible to determine a boundary in the magnitude of equation (207) as a function of the local velocity states, which can be defined simply through the maximum values of velocities in each DOF:

$$|-[M^{-1} C_{cor}(\mathbf{v}) - R_G^L \dot{R}_L^G] \mathbf{v}| \leq \delta_{din} = |-[M^{-1} C_{cor}(\mathbf{v}_{max}) - R_G^L \dot{R}_L^G] \mathbf{v}_{max}| \quad (210)$$

The bounds defined above produce rather conservative results, which may conduct to large switching gains for the sliding algorithm. Limits in $\mathbf{v}_{max}$ can be further reduced by noticing that in normal operation vessel will hardly simultaneously reach u_{max} , v_{max} and r_{max} . The determination of such values in the form of $(uv)_{max}$, $(ur)_{max}$ and $(vr)_{max}$ may be done through the use of simulation tools.

Finally, note that equation (210) is continuous and can be differentiated. As shall be seen in next section, boundaries in $|\dot{\tilde{\mathbf{f}}} - \dot{\tilde{\mathbf{f}}}|$ may need to be determined for the application of the super Twisting Algorithm. Time differentiation of $-[M^{-1} C_{cor}(\mathbf{v}) - R_G^L \dot{R}_L^G] \mathbf{v}$ delivers:

$$-M^{-1} \dot{C}_{cor}(\mathbf{v}) \mathbf{v} - M^{-1} C_{cor}(\mathbf{v}) \dot{\mathbf{v}} + (\dot{R}_G^L \dot{R}_L^G + R_G^L \ddot{R}_L^G) \mathbf{v} + R_G^L \dot{R}_L^G \dot{\mathbf{v}} \quad (211)$$

One important remark on the definition of boundaries for the dynamic part of the deviations explained above is that those could be simply feedback cancelled in the control law $\tilde{\mathbf{u}}$. While this could lead to better dynamic results and much smaller gains, each agent in formation would need to know the dynamic properties of its neighbors to perform adequate cancellations.

4.4. Super Twisting Cooperative Control

Next, the Cooperative Sliding Control method will be extended to the application of the Super Twisting Algorithm.

Theorem 3. Consider the same sliding manifold from equation (181) and its time derivative from equation (188) along the agent's dynamics given by equation (175)

$$\begin{cases} \sigma_i = \sum_{j=1}^n a_{ij}(\dot{\boldsymbol{\eta}}_i - \dot{\boldsymbol{\eta}}_j) + \Lambda \sum_{j=1}^n a_{ij}(\boldsymbol{\eta}_i - \boldsymbol{\eta}_j) \\ \dot{\sigma}_i = d_i(\tilde{\mathbf{u}}_i + \tilde{\mathbf{f}}_i + \Lambda \dot{\boldsymbol{\eta}}_i) - \sum_{j=1}^n a_{ij}(\tilde{\mathbf{u}}_j + \tilde{\mathbf{f}}_j + \Lambda \dot{\boldsymbol{\eta}}_j) \end{cases}$$

Suppose a vector $\dot{\tilde{f}}_i$ can be defined as the best approximation of the derivatives of the unknown bounded disturbances $\dot{\tilde{f}}_i$. If the communication topology graph is connected, then the sliding motion of the formation will be achieved, and consensus will be reached in finite time by application of the following control law:

$$\begin{cases} \ddot{\tilde{u}}_i = -\dot{\tilde{f}}_i - \Lambda \dot{\tilde{u}}_i + \frac{1}{d_i} \left[\sum_{j=1}^n a_{ij} (\ddot{\tilde{u}}_j + \dot{\tilde{f}}_j + \Lambda \dot{\tilde{u}}_j) - K_1 |\sigma_i|^{\frac{1}{2}} \text{sign}(\sigma_i) + \mathbf{u}_a \right] \\ \dot{\mathbf{u}}_a = -K_2 \text{sign}(\sigma_i) \end{cases} \quad (212)$$

In which $K_1 \in \mathbb{R}^{3 \times 3}$ and $K_2 \in \mathbb{R}^{3 \times 3}$ are positive definite diagonal gain matrices and $\text{sgn}(\sigma_i)$ is the vector of sign functions of each entrance.

Proof. Calculate the second time derivative of the sliding manifold

$$\begin{aligned} \ddot{\sigma}_i &= d_i (\ddot{\tilde{u}}_i + \dot{\tilde{f}}_i + \Lambda \dot{\tilde{u}}_i) - \sum_{j=1}^n a_{ij} (\ddot{\tilde{u}}_j + \dot{\tilde{f}}_j + \Lambda \dot{\tilde{u}}_j) \\ &= d_i [\ddot{\tilde{u}}_i + \dot{\tilde{f}}_i + \Lambda (\tilde{u}_i + \tilde{f}_i)] - \sum_{j=1}^n a_{ij} [\ddot{\tilde{u}}_j + \dot{\tilde{f}}_j + \Lambda (\tilde{u}_j + \tilde{f}_j)] \end{aligned} \quad (213)$$

Take the derivative of the input variable for each degree of freedom (as the system is decoupled, this is done without loss of generality):

$$\begin{aligned} \ddot{\tilde{u}}_{ik} &= -\dot{\tilde{f}}_{ik} - \Lambda \dot{\tilde{u}}_{ik} \\ &+ \frac{1}{d_i} \left[\sum_{j=1}^n a_{ij} (\ddot{\tilde{u}}_{jk} + \dot{\tilde{f}}_{jk} + \Lambda \dot{\tilde{u}}_{jk}) - \frac{1}{2} k_{1k} \frac{\dot{\sigma}_{ik}}{|\sigma_{ik}|^{\frac{1}{2}}} - k_{2k} \text{sign}(\sigma_{ik}) \right] \end{aligned} \quad (214)$$

Substitution of (214) in (213) yields

$$\ddot{\sigma}_{ik} = d_i (\dot{\tilde{f}}_{ik} - \dot{\tilde{f}}_{ik}) - \sum_{j=1}^n a_{ij} (\dot{\tilde{f}}_{jk} - \dot{\tilde{f}}_{jk}) - k_{2k} \text{sign}(\sigma_{ik}) - \frac{1}{2} k_{1k} \frac{\dot{\sigma}_{ik}}{|\sigma_{ik}|^{\frac{1}{2}}} \quad (215)$$

Note that this has the same form of equation (163). Bounding requirements for 2nd order sliding motion follows:

$$\begin{cases} |\dot{\tilde{f}}_{ik} - \dot{\tilde{f}}_{ik}| \leq \Gamma_{ia} \\ 2d_i \Gamma_{ia} \leq \varphi_{ik} \end{cases} \quad (216)$$

In this case, the differential inclusion (164) holds and 2nd order sliding motion occurs ■.

Gain requirements are, for each degree of freedom $k = x, y, \psi$:

$$\begin{cases} k_{2k} > \varphi_i \\ k_{1k}^2 \geq 4\varphi_{ik} \frac{(k_{2k} + \varphi_i)}{(k_{2k} - \varphi_i)} \end{cases} \quad (217)$$

An alternate solution to facilitate the calculation of gains is presented as follows. The insertion of the cooperative super twisting control law (212) in the derivative of the sliding vector yields:

$$\begin{cases} \dot{\sigma}_i = d_i(\tilde{f}_i - \bar{f}_i) - \sum_{j=1}^n a_{ij}(\tilde{f}_i - \bar{f}_i) - K_1|\sigma_i|^{\frac{1}{2}} \text{sign}(\sigma_i) + \mathbf{u}_a \\ \dot{\mathbf{u}}_a = -K_2 \text{sign}(\sigma_i) \end{cases} \quad (218)$$

Equation (218) can be rewritten as

$$\begin{cases} \dot{\sigma}_i = -K_1|\sigma_i|^{\frac{1}{2}} \text{sign}(\sigma_i) + \omega_i \\ \dot{\omega}_i = d_i(\dot{\tilde{f}}_i - \dot{\bar{f}}_i) - \sum_{j=1}^n a_{ij}(\dot{\tilde{f}}_i - \dot{\bar{f}}_i) - K_2 \text{sign}(\sigma_i) \end{cases} \quad (219)$$

In which $\omega_i = d_i(\dot{\tilde{f}}_i - \dot{\bar{f}}_i) - \sum_{j=1}^n a_{ij}(\dot{\tilde{f}}_i - \dot{\bar{f}}_i) + \mathbf{u}_a$. For each degree of freedom, the error boundary can be calculated as

$$\varphi_{ik} \geq d_i |\dot{\tilde{f}}_{ik} - \dot{\bar{f}}_{ik}| + \sum_{j=1}^n a_{ij} |\dot{\tilde{f}}_{ik} - \dot{\bar{f}}_{ik}| = 2d_i \Gamma_{ik} \quad (220)$$

And the gains can be calculated as

$$\begin{cases} k_{1k} = 1.5|\varphi_{ik}|^{\frac{1}{2}} \\ k_{2k} = 1.1\varphi_{ik} \end{cases} \quad (221)$$

The Super-Twisting Cooperative Controller algorithm is summarized in Figure 29.

Figure 29 - Super-Twisting Cooperative Control Algorithm Summary

Source: Author

4.5. Neighbor State Estimation

In a cooperative multi-agent system, the wireless transmitted data may be subject to communication delays and connection shortages. While most cooperative control and consensus methods are designed to hold stability under these circumstances and even with other requirements (such as a varying communication structure), the devised idea in this work is to perform cooperative tasks under a fixed communication structure. In order to achieve robustness, communication delays and shortages are managed through a dead-reckoning capable observer, designed in a corrector-predictor form from section 3.2.4. If the communication structure is held constant, a linear Kalman Filter that predicts an agent's neighbor's positions and velocities is a suitable candidate for such observer. The main drawback in this method is the requirement of prior knowledge of the neighbors' dynamics. This can be restricted if all agents are equal or their local controllers are tuned to provide the same response. Consider each DP-controlled vessel low-frequency dynamics as a second order system with state space equation

$$\begin{cases} \begin{bmatrix} \dot{x}_{1i} \\ \dot{x}_{2i} \end{bmatrix} = \underbrace{\begin{bmatrix} 0_{3 \times 3} & I_{3 \times 3} \\ -A_{0i} & -A_{1i} \end{bmatrix}}_{A_i} \begin{bmatrix} x_{1i} \\ x_{2i} \end{bmatrix} + \underbrace{\begin{bmatrix} B_{1i} \\ B_{0i} - A_{1i}B_{1i} \end{bmatrix}}_{B_i} u_{di} + E_i w_i \\ y_i = \underbrace{\begin{bmatrix} I_{3 \times 3} & 0_{3 \times 3} \end{bmatrix}}_{C_i} \begin{bmatrix} x_{1i} \\ x_{2i} \end{bmatrix} + v_i \\ \begin{bmatrix} x_{1i} \\ x_{2i} \end{bmatrix} = \begin{bmatrix} \eta_i \\ \dot{\eta}_i - B_{1i} u_{di} \end{bmatrix}; u_{di} = \eta_{di} \end{cases} \quad (222)$$

Each agent already knows its own states through the output of its local extended Kalman Filter. Its positions and inputs are shared to this agent's out-neighbors. The set of an agent's in-neighbors $N_i = \{v_j: a_{ij} > 0\}$ also produces the neighbors' positions and inputs. The task is to design a Kalman Filter that estimates these data in the event of communication failure.

One way to implement this is to create a joint system, with a block diagonal system matrix composed of each neighbor's dynamics and state vector as the in-neighbors states concatenated:

$$\begin{cases} \dot{x}_{N_i} = \begin{bmatrix} A_j & \dots & 0_{6 \times 6} \\ \vdots & \ddots & \vdots \\ 0_{6 \times 6} & \dots & A_{j+d_i-1} \end{bmatrix} x_{N_i} + \begin{bmatrix} B_j \\ \vdots \\ B_{j+d_i-1} \end{bmatrix} x_{N_i d} + \begin{bmatrix} E_j \\ \vdots \\ E_{j+d_i-1} \end{bmatrix} w_{N_i} \\ y_{N_i} = [C_j \quad \dots \quad C_{j+d_i-1}] x_{N_i} + v_{N_i} \end{cases} \quad (223)$$

While this approach is simple to implement, it can have numerical problems, the system matrices are large (with dimension $6d_i \times 6d_i$, where d_i is the in-degree of the agent) and sparse. The required matrix inversions would also present computational costs and numerical problems. Lots of operations with known results (multiplication by zero) during the numerical integration could be avoided.

To solve the problems described above, a possible method would be to separately run a filter for each neighbor. The prediction integrations and corrections can be run in the same code loop, which can reduce computational cost. The result is d_i filters running simultaneously, with equations (71)-(73) and system matrices as described in equation (222) (see Figure 30).

Figure 30 - Parallel Filter structure for 3 neighbors

The remaining issue for dead reckoning purposes is the filter driving inputs. To accurately predict each neighbors' states, an agent would have to know its inputs, which are calculated locally by the cooperative controller. In a communication failure event, this information would not be available. For this situation, the input η_{dj} of each neighbor can be either considered static, frozen in its last known value, or taken as the

estimating agent's own input reference η_{di} (in the simple consensus case) which, in the formation control case becomes

$$\hat{\eta}_{dj} = \eta_{di} + \delta_{ij} \quad (224)$$

Observe that this is far from an accurate estimation but may be used in a limited communication shortage event.

4.6. Joint Estimation of Environmental disturbances

As the number of agents grows, information on the general environmental conditions is expected to be more available. By using the idea of wireless sensor networks, a multi-agent system method can be devised to evaluate such conditions with higher accuracy. As an obvious example, take the wind velocity measurements from multiple ships in a determined area. This variable estimation can be performed jointly through the use of the consensus filter from equation (121) (or (122), in the discrete time):

$$\dot{x}_i(t) = \sum_{j \in N_i} a_{ij} [x_j(t) - x_i(t)] + \sum_{j \in J_i} a_{ij} [u_j(t) - x_i(t)]$$

$$x_i(k+1) = x_i(k) + h \left\{ \sum_{j \in N_i} a_{ij} [x_j(k) - x_i(k)] + \sum_{j \in J_i} a_{ij} [u_j(k) - x_i(k)] \right\}$$

Figure 31 displays an example in which a 15 m/s wind modeled through a Harris spectrum run along 1000s is filtered by four sensors. A measurement delay of 10s is introduced between the sensors measurements and a 1s sample rate is used. The low pass characteristic of the filter is evidenced, and all signals are driven to consensus in less than 60s.

Figure 31 - Consensus Filters Applied to Wind Measurements

Source: Author

The algorithm described above and the distributed Kalman Filter from (OLFATI-SABER, 2005) is suited for systems in which direct measurements of the variables needed to reach consensus are available. The objective in this section is to propose a fusion filter for variables which are not measured directly but can be estimated. For a DP system, for example, wave frequency and, under the adequate circumstances, DP Current direction and velocities are possible candidates for this filtering system.

The overall idea is that, as more information is available through multiple agents, output values tend to be more accurate simply based on oversampling. The filtering algorithm will be an extension of a state output from the Kalman filter, using this variable as the input drive of a consensus filter:

$$\dot{x}_i(t) = \sum_{j \in N_i} a_{ij} [x_j(t) - x_i(t)] + q \sum_{j \in J_i} a_{ij} [\hat{x}_j(t) - x_i(t)] \quad (225)$$

The variables $\hat{x}_j$ represent the state output from the Kalman filter, a wave frequency value, for example. The positive gain q is introduced as a relative weight between the new measurements from an agent and the estimates from the other agents. Note that its introduction does not affect the stability of the filter from section 3.3.3.

In discrete time, the filter will be given by

$$x_i(k+1) = x_i(k) + h \left\{ \sum_{j \in N_i} a_{ij} [x_j(k) - x_i(k)] + q \sum_{j \in J_i} a_{ij} [\hat{x}_j(k) - x_i(k)] \right\} \quad (226)$$

At each Kalman filter step, the slowly varying value estimated is filtered through the consensus algorithm, taking in account the outputs from other agents. Note that the filtered variable must be normalized to a common value for all agents. In case of an oceanic current vector, for example, the estimated loads in the vessel cannot be input straight to the filter. This way, an adequate mapping from the bias force output from the KF and the current velocity and angle must be introduced, as distinct positions and attitudes from the agent vessels produce different loads from the same current velocity and direction. For wave frequency estimations and KF implementations that estimate directly the current velocity field this procedure is not necessary, as these are the direct environmental field variables and common to all agents.

An example follows to make the method clearer and illustrate a case in which the measured variable must be normalized. Take a group of six agents represented by 1

DOF mass-spring-damper systems, each one subjected to a force input (kept constant, but can be variable and used for control feedback) and an external field that act upon each system according to a geometric variable, such as frontal area. Figure 32 illustrates a single agent system.

Figure 32 - Diagram of a single agent's model

Source: Author

Table 1 - Mass-Spring-Damper Agents Parameters

	$m(kg)$	$k(N/m)$	$c(Ns/m)$	$A(m^2)$
Agent 1	1	0.2	0.1	1
Agent 2	1	0.4	0.1	1
Agent 3	2	0.2	0.1	2
Agent 4	1.5	0.1	0.1	1.75
Agent 5	2	0.5	0.2	1.5
Agent 6	1	0.4	0.1	1.2

Source: Author

The motion equation of each agent will be given by

$$\ddot{x}_i = \frac{1}{m_i} (-k_i x_i - c_i \dot{x}_i + F_i^{in} + F_i^{ext})$$

With $F_i^{ext} = \frac{\phi}{A_i}$ is the force resulting from a pressure field affecting all agents and with a priori value unknown. Parameters used for the simulation of each agent are presented in Table 1. A Kalman filter is run for each agent with the following state space model:

$$\begin{cases} \dot{\mathbf{x}}_i = \begin{bmatrix} 0 & 1 & 0 \\ -\frac{k_i}{m_i} & -\frac{c_i}{m_i} & \frac{1}{m_i} \\ 0 & 0 & 0 \end{bmatrix} \mathbf{x}_i + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \mathbf{u}_i + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \mathbf{w}_i \\ \mathbf{y}_i = [1 \quad 0 \quad 0] \mathbf{x}_i + \mathbf{v}_i \end{cases}$$

In which the state vector $\mathbf{x}_i$ is given by $[x_i, \dot{x}_i, F_i^{ext}]^T$, $\mathbf{u}_i = F_i^{in}$; $\mathbf{w}_i$ and $\mathbf{v}_i$ are the process and sensor noise, respectively. The relative weight q used is set to 1 and the adjacency matrix is given by

$$A_{ij} = \{a_{ij}\} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

The system is simulated through 4th Order Runge Kutta method with time discretization of 0.01s and process, sensor and pressure field white Gaussian noises with variance given respectively by 0.01, 0.05 and 0.2. A constant force of 5N is applied in each agent.

A standard Kalman Filter for each agent, with 0.05s sampling rate (equations (71)-(73)), $R = 0.0025$ and process covariance matrix given by

$$Q = \text{diag}([10^{-4} \quad 10^{-4} \quad 10])$$

Note that the pressure related process covariance matrix entrance is large, which accelerates the estimated value convergence but contribute to a noisy output. After each KF step, the following difference equation is run:

$$\hat{\phi}_i(k+1) = \hat{\phi}_i(k) + h \left\{ \sum_{j \in N_i} a_{ij} [\hat{\phi}_j(k) - \hat{\phi}_i(k)] + q \sum_{j \in J_i} a_{ij} \left[\frac{\hat{F}_j^{ext}(k)}{A_j} - \hat{\phi}_i(k) \right] \right\}$$

The resulting measurements for all six filters compared to the field actual values and the Kalman Filter estimations can be seen in Figure 33. The first thing to notice is that the consensus filter estimations are clearly smoother. Indeed, variance data for these outputs are of magnitude at least one order lower than the ones from the single KF. Table 2 displays the comparison of mean values and variance for the single KF estimation and the consensus filter estimations. There is not much change in the mean value, but variance of the consensus filter output is much lower. If this data is used in the control algorithm, such as a feedforward current compensation, this lower variance

means a smaller output variation and less energy usage. The drawback is a slower output response, which can lead to a less responsive control.

Figure 33 - Consensus Filter Response for the Mass-Spring-Damper

Source: Author

Table 2 - Mean and variance values for the KF Pressure Field estimate and the Consensus Filter estimate

KF Output (single Agent)		Consensus Filter Output	
Mean	Variance	Mean	Variance
5.000055	0.041494	4.999973	0.001353
4.999877	0.041701	4.999939	0.000958
4.999743	0.004512	4.999908	0.001056
4.999868	0.008216	4.999886	0.000386
4.999655	0.006596	5.000009	0.000835
5.000379	0.029320	4.999922	0.000929

Source: Author

This simplified system results indicate that this is a suitable method for estimation of joint environmental variables; yet, further results are to be analyzed from the actual DP system data from the next chapter case studies.

5. VALIDATION

In this chapter, the applications of the proposed methods are evaluated through simulation and, when possible, experimentally in model scale tests at the TPN wave basin.

5.1. Resources and Methodology

The physical resources for the evaluation of the case studies were provided by the *Numerical Offshore Tank of the University of São Paulo* (TPN-USP). Its facilities are described in (TPN, 2016).

Mathematical analysis, basic simulations and algorithm implementations were performed using the Matlab® and Simulink® computing platform. Optimization and the Control System toolboxes were used in order to simplify the coding tasks.

Complete simulations and operational real-time simulations were performed using the numeric simulator (TANNURI et al., 2014) from TPN. This simulator allows full control over the scenario (winds, waves, currents and other dynamic elements), making it ideal to verify the system behavior under different environmental and operational conditions. The control and observation algorithms applied to it were implemented and interfaced via Matlab®. Figure 34 shows the Full Mission Real Time Simulation Environment at the TPN.

Figure 34 - Real Time Simulation Environment at TPN-USP

Source: Author

Figure 35 – Wave basin at TPN-USP.

Source:(MELLO, 2012).

To create more realistic conditions, subjected to implementation issues such as sampling, sensor noise and modelling errors, small scale model experimental tests were performed in the Hydrodynamic Calibrator wave basin of the Numerical Offshore Tank (Figure 35). Within this facility, scaled model tests of offshore operations under action of regular or irregular waves with tight parametric control can be performed. A detailed description of TPN wave basin is given in (MELLO et al., 2013). 3D motion tracking was performed through IR Qualisys® Cameras (Figure 36b) and actuation commands were performed via radio link (Figure 36a). Control and observer solutions were calculated through a fixed computer in the control room. Embedded software in the subject model is only responsible for motor commands interpretation from the radio communication protocol. Real time control and observation algorithms and the operation interface (Figure 37) were implemented in Matlab®, providing flexibility for algorithm changes and ease of operation. The Cooperative Control Interface (Figure 38) ran the cooperative algorithms studied in parallel with the and individual DP interfaces and was linked to them via TCP connection.

Figure 36 - Control System Experimental Setup (a). Qualisys Measuring Infrared Camera (b).

a) Source: (TANNURI; MORISHITA, 2006)

b) Source: Author

The complete experimental setup (for each ship) is illustrated in Figure 36a. A detailed description of the hardware setup used for these DP tests is provided in (MORISHITA et al., 2009).

Figure 37 - DP Control Interface

Source: Author

Figure 38 - Cooperative Control Interface

Source: Author

5.1.1. System Description and Input Data

Due to the large thrust capacity and maneuverability, the target vessels for both the simulations and experiments were oceanic tugboats. Network topology, environmental action and protocol gains are described for each case study proposed in the corresponding section.

SIMULATION MODEL

In the simulations tests each vessel agent is a Platform Supply Vessel (PSV). All PSVs from the formation have equal dynamic parameters and actuator properties. The parametric data of the simulated PSVs is given in APPENDIX D, section D.1.1.

The DP controller is a nonlinear PD from equation (63) (with suppressed integral action) designed to yield a closed loop natural frequency of 0.01 rad/s for all degrees of freedom and a relative damping of 0.7 in surge and sway and 1.2 in yaw. This renders, in surge and sway, a settling time $T_s = 4/(\zeta\omega_n) = 571 \text{ s}$, with maximum overshoot $P.O. = 100e^{-\zeta\pi/\sqrt{1-\zeta^2}} = 4.6\%$ and for yaw a settling time of 333 s and no overshoot. As shall be verified, this performance is changed due to the presence of zeros in the model and due to drag effects. Controller gains are given in Table 3. Saturation for the overall control outputs of the system is considered as the sum of all the possible thruster action in each DOF.

Table 3 – Simulated Control Gains

Gain	Surge	Sway	Yaw
$K_P (N.m^{-1} N.m.rad^{-1})$	4.64×10^3	8.05×10^3	6.64×10^6
$K_D (N.s.m^{-1} N.m.s.rad^{-1})$	3.86×10^5	6.69×10^5	3.62×10^8

Source: Author

Control allocation is performed through the pseudo-inverse of the force allocation matrix. In order to simplify implementation, the azimuth thruster is considered fixed in 90° . If the thruster allocation results in a value larger than that of a given thruster, it is kept in full capacity and eliminated from the allocation algorithm iteratively, until the requested action is reached.

A local Extended Kalman Filter provides wave filtering, velocity state estimations and drift intrinsic forces estimations. Process and sensor covariance matrices are given in Table 4.

Table 4 – Simulated Extended Kalman Filter Design Matrices

Matrix	Value
Process Covariance	$Q_1 = \text{diag}([0.01 \quad 0.01 \quad 4 \times 10^{-8} \quad 1 \quad 1 \quad 4 \times 10^{-6}]);$
	$Q_2 = \text{diag}([10 \quad 20 \quad 50 \quad 10 \quad 20 \quad 50]);$
	$Q_3 = 1 \times 10^7 \text{diag}([1 \quad 1 \quad 100]);$
	$Q = \text{blkdiag}([Q_1 \quad Q_2 \quad Q_3])$
Noise Covariance	$R = 1 \times 10^{-2} \text{diag}([1 \quad 1 \quad 4 \times 10^{-6}])$

Source: Author

Figure 39 presents the step responses for the PSV running the complete DP system. In the same figure the plot from the linearized ideal model response is presented, along with a plot of the actual responses if no Kalman Filter is used. The differences between responses of actual system without KF and the linearized system rely on the current drag, which produces damping in the real system, not only making it slower but also reducing the overshoot. Note that actual DP response with Kalman Filtering is more oscillatory than the expected when compared to the linearized system (especially in yaw). The settling time is of 500s in the complete system using the KF, for all degrees of freedom.

Figure 39 - 10m/10° step responses for Maersk Handler DP System

Source: Author

The deviations in dynamic behavior of the plant may be large when compared to the designed behavior, and designed performance will differ if the consensus controller is not robust.

SCALE MODEL

The small-scale tests were performed using three vessels, being two identical 1:42 PSVs with characteristics close to that of a Maersk Handler, from now on referred to as M510A and M510B and one smaller 1:70 PSV Model, referred to as M539. The parametric data of the model scale PSVs is given in APPENDIX D, section D.2.1 and D.2.2, respectively.

The DP controller is a nonlinear PID from equation (63) designed to yield a closed loop natural frequency of 0.162 rad/s for all degrees of freedom and a relative damping of 0.7 in surge and sway and 1.2 in yaw. This renders, in surge and sway, a settling time $T_s = 4/\zeta \omega_n = 35.3 \text{ s}$, with maximum overshoot $P.O. = 100e^{-\zeta / \sqrt{1-\zeta^2}} = 4.6\%$ and for yaw a settling time of 20.6 s and no overshoot. Controller Gains are given in Table 5. Saturation for the overall control outputs of the system is considered as the sum of all the possible thruster actions in each DOF.

Table 5 – Experimental Control Gains

Gain	Surge	Sway	Yaw
$K_p (N.m^{-1} N.m.rad^{-1})$	8.65	13.58	6.11
$K_D (N.s.m^{-1} N.m.s.rad^{-1})$	44.32	69.59	20.57
$K_I (N.s^{-1}.m^{-1} N.m.s^{-1}.rad^{-1})$	0.83	1.31	0.39

Source: Author

Control allocation is also performed through the pseudo-inverse of the force allocation matrix. In order to simplify implementation, the azimuth thruster is considered fixed in 90° . If the thruster allocation results in a value larger than that of a given thruster, it is kept in full capacity and eliminated from the allocation algorithm iteratively, until the requested action is reached.

A local Extended Kalman Filter provides wave filtering, velocity state estimations, drift intrinsic forces estimations and wave frequency estimations. Process and sensor covariance matrices are given in Table 6.

Table 6 – Experimental Extended Kalman Filter Design Matrices

Matrix	Value
Process Covariance	$Q_1 = \text{diag}([0.5 \quad 0.5 \quad 4 \times 10^{-7} \quad 200 \quad 200 \quad 8 \times 10^{-4}]);$
	$Q_2 = \text{diag}([75 \quad 75 \quad 3 \quad 75 \quad 75 \quad 3]);$
	$Q_3 = 1 \times 10^7 \text{diag}([1 \quad 1 \quad 0.01])$
	$Q = \text{blkdiag}([Q_1 \quad Q_2 \quad Q_3])$
Noise Covariance	$R = 1 \times 10^{-3} \text{diag}([1 \quad 1 \quad 4 \times 10^{-5}])$

Source: Author

Due to implementation constraints, all reference inputs for the scale models was smoothed by a second order filter, with natural frequency of 0.1 rad/s and relative damping of 1.0. The step responses for the M510A and B, with filtered input, are given in Figure 40. Note that the responses are remarkably similar for both vessels, with minor differences relying on thruster particularities. The same responses for the M539 are given in Figure 41. At this point it is interesting to remark that, even though the DP systems for all vessels are designed to yield the same dynamic response, the M539 and the M510 vessels have a different behavior for the same filtered input.

Figure 40 - $1\text{m}/10^\circ$ step responses for M510A and M510B DP System

Source: Author

Figure 41 - 1m/10° step responses for M539 DP System

Source: Author

5.1.2. Auxiliary analysis indices

To ease the analysis, some performance indices will be defined as follows. Consider a set of DP vessel agents composed by a Leader vessel indexed by L and n follower vessels.

The Vector Consensus Error $\mathbf{e}_{ij} = [e_{ij,x} \ e_{ij,y} \ e_{ij,\psi}]^T$ will be defined as the error between two agents' current positions and their required position within the formation:

$$\mathbf{e}_{ij} := \mathbf{e}_i - \mathbf{e}_j - \delta_{ij} \quad (227)$$

Note that $\mathbf{e}_{ij} = -\mathbf{e}_{ji}$.

The Normalized Consensus Error will be defined, for each degree of freedom, as the norm of all the errors $e_{ij,k}$, including the leader:

$$\bar{e}_{c,k} := \left(\sum_{i=1}^{n+1} \sum_{j=1}^{n+1} \frac{1}{2} |e_{ij,k}|^2 \right)^{\frac{1}{2}}, \quad k = x, y, \psi \quad (228)$$

This is a measure of total deviation and can be used to evaluate convergence performance of the system in a given DOF. When the Normalized Error does not account for errors with respect to the Leader, it will be called Formation Error.

The Formation Error Performance index J_{Err} is an integral scalar index accounting for total deviations between the agents' positions and velocities vectors during a maneuver, except for the leader:

$$J_{Err} := \int_0^T \sum_{i=1}^n \sum_{j=1}^n \left(\mathbf{e}_{ij}^T \mathbf{W}_\eta \mathbf{e}_{ij} + \dot{\mathbf{e}}_{ij}^T \mathbf{W}_v \dot{\mathbf{e}}_{ij} \right) dt \quad (229)$$

The diagonal weighting matrices W_η and W_v are calculated as the squared inverse of the maximum deviations in position and velocity allowed for each degree of freedom.

The Tracking Error Performance index is similar to the Formation Error index, but account only for the deviations from the leader:

$$J_L := \int_0^T \sum_{i=1}^n (e_{iL}^T W_\eta e_{iL} + \dot{e}_{iL}^T W_v \dot{e}_{iL}) dt \quad (230)$$

As the quantities have similar scales, one can always define a total consensus performance index as

$$J_C := J_{Err} + J_L \quad (231)$$

The performance indices will be normalized based on the highest value of a standard case run for each controller. The normalizing factors used were different for each protocol, as the main interest is to compare the effects of changes in conditions and not pure dynamic performance.

5.2. Current Estimation through Consensus Filter

The first case proposes the evaluation of an environmental variable that is common to all agents using the consensus filter. In the following validation study, the chosen parameters to estimate here were the oceanic current velocity and direction. These values are not available through direct sensor measurements. The simplest method to obtain them is through the estimation of the so-called DP current, a virtual current value that provides the slowly varying drift forces calculated by the Kalman Filter. It is important to remark, though, that the actual vessel drift forces are caused not only by current drag, but also by second order wave drift forces. This limits the actual application of the method, especially in large hulls more susceptible to wave drift. The assessment of sea state by other means, though, can be used to calculate the wave drift. Note that even the sea state values could be “consensus filtered”, see, e.g., (NIELSEN; BRODTKORB; SØRENSEN, 2019). Finally, care must be taken when performing joint estimation of current values. If the vessel agents are close enough, effects from hull interaction may affect the local drag forces and the consensus filter output will provide erroneous measurements.

The bias forces of each agent ship must be mapped to a global current velocity and angle values so the consensus filter can estimate comparable values for all vessels. A

relative current velocity and relative angle pair can be mapped directly to drag forces (i.e. $\mathbf{F}_{curr} = f(V_c, \psi_c)$), but there is no unique inverse function that can map the forces to current velocity and angle values. There are two viable solutions for this problem. The first is to formulate the Kalman Filter to output directly the DP current values, as described in (JENSSEN, 2006). This approach may augment the complexity of the EKF formulation and increase instability due to modeling errors. The method applied in the following study comprises an online optimization of an error function between the estimated drift force and the output of the drag force function:

$$\min_{V_c, \psi_c} \left(\hat{\mathbf{F}}_{drift} - 0.5 \rho \{ [u - V_c \cos(\psi_c - \psi)]^2 + [v - V_c \cos(\psi_c - \psi)]^2 \} L_{pp} T \begin{bmatrix} C_u(\psi_c - \psi) \\ C_v(\psi_c - \psi) \\ L_{pp} C_r(\psi_c - \psi) \end{bmatrix} \right)^2$$

$$\text{subject to } \begin{cases} -2 \leq V_c \leq 2 \\ 0 \leq \psi_c \leq 2\pi \end{cases}$$

The optimization problem is to be solved at each step through Sequential Quadratic Programming, using as initial condition the last result.

In the study case, a stationkeeping simulation is run at 0.5 seconds step in which no cooperative control protocol is applied at first, except for the consensus filter estimations. Two environmental conditions with different directions and intensities are assessed, one in which the environmental vectors come from Northeast (NE) and other in which the environmental vectors come from Southeast (SE). Standard PD control with characteristics described in the last section is considered. In this scenario all vessels have two way communications (see Figure 42 and Table 7). Laplacian Matrix eigenvalues are $[0, 2, 2, 4]$. Note that the Fiedler eigenvalue is 2, rendering a negligible time to reach consensus, as there is no dynamics considered in the sensor. Observe that this graph is connected. The weighting gain q from equation (225) is set to 0.05.

Figure 42 - Wireless communication structure for consensus filter

Source: Author

Table 7 – Cooperative control design matrices for consensus filters

Matrix	Value
Adjacency A	$A = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$
In-Degree D	$D = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}$
Laplacian $L = D - A$	$L = \begin{bmatrix} 2 & -1 & 0 & -1 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ -1 & 0 & -1 & 2 \end{bmatrix}$

Source: Author

CASE 1: NE MILD INTENSITY CONDITION

In this case, the vessels are positioned initially with attitudes of 354, 85, 162 and 91 degrees relative to North axis (y). At $t = 2000s$ they are commanded to reposition themselves and change heading collectively to 45 degrees, using a linear synchronization protocol. Lack of integral action prevents the vessels to reach alignment, so that attitude obtained are of 44, 70, 77 and 70 degrees by the end of the simulation. Figure 43 displays the position and heading configurations of the ships.

Figure 43 - Vessels distribution for the NE current estimation. (a) Initial position. (b) Final position.

(a) Source: Author

(b) Source: Author

A current direction of 255° (going to, referred from North, equivalent to 195° relative to East) is considered, with velocities of 0.3 m/s . Wind is coming from NE but measured and counterbalanced by feedforward. Simulation is run without significant wave disturbance. Figure 44 summarizes the climate conditions applied. The current initially hits vessel 1 by its starboard side, vessel 2 by its port-bow side, vessel 3 by its port-

stern side and vessel 4 by its port-bow side. After the change in heading, vessel 1 is hit by its starboard-bow side, while the remaining vessels are aligned with the current.

Figure 44 – NE Mild Condition for current estimation.

Source: Author

In the initial situation the independent current estimates from vessels 1 and 3 are expected to be smoother, while the ones from vessels 2 and 4 are more prone to have variations. After the change in heading, vessel 1 estimations tend to have smoother readings than the rest. This is due to the magnitude of drift forces induced by current, which are much lower when the ship is aligned to the current vectors. In this case the individual bias estimations and consequently the optimization outputs are more sensitive to variations in heading. Indeed, as can be seen by Figure 45 (left), the independent estimates from 2 and 4 (red and purple lines) are noisier; vessel 2 in special produces an oscillatory behavior, which is probably due to small oscillations of the vessel itself. After the change in heading, all estimations but the first get noisy outputs. A large transient response can be verified for vessels 2 and 4 during the first 800 seconds, as opposed to smaller variations for vessels 1 and 3. During the change in heading, the large oscillations are verified for vessels 2,3 and 4. Note that once the consensus is reached, it is not lost during the transients. The times to reach steady state parameter values is also of 800 s after the change. The angle filtered estimates, in particular, are much less sensitive to the changes in heading.

The right side of Figure 45 shows the response of the consensus filtered values. They are smoother in both transient response and steady state for velocity and angle. Even though consensus is maintained throughout the whole simulation, note that the consensus average in steady state is actually not reached for all ships, but only in pairs (1 and 3; 2 and 4). This reflects the weight given for the neighbor's measurements in the consensus algorithm when compared to the weight of an agent's own measurements.

Figure 45 – Current velocity and angle estimates for 195° - 0.3 m/s.

Source: Author

A better notion on the accuracy of steady state values is given by Table 8 and Table 9. In both cases it is clear that a larger deviation on velocity and angle estimates is present for the aligned ships. The consensus filter outputs are in the mean values between the independent outputs. Standard deviation for the proposed filter output is much smaller when compared to individual measurements. Interestingly, consensus outputs for vessels 2 and 4 are better than the ones for vessels 1 and 3.

Table 8 - Steady state estimate statistics for current of 0.3 m/s at 195° from East before change in heading.

Vessel	Velocity (m/s)				Angle (degrees)			
	Independent		Consensus		Independent		Consensus	
	Mean	STD	Mean	STD	Mean	STD	Mean	STD
1	0.3076	0.0013	0.3201	0.0005	197.81	0.19	194.67	0.04
2	0.3337	0.0100	0.3186	0.0004	194.34	0.71	194.76	0.03
3	0.2993	0.0014	0.3201	0.0005	193.67	0.13	194.67	0.04
4	0.3375	0.0029	0.3186	0.0004	193.21	0.41	194.76	0.03

Source: Author

Table 9 - Steady state estimate statistics for current of 0.3 m/s at 195° from East after change in heading.

Vessel	Velocity (m/s)				Angle (degrees)			
	Independent		Consensus		Independent		Consensus	
	Mean	STD	Mean	STD	Mean	STD	Mean	STD
1	0.3505	0.0039	0.3126	0.0018	200.93	0.81	196.31	0.14
2	0.3037	0.0204	0.3135	0.0019	194.21	0.73	196.51	0.15
3	0.2957	0.0356	0.3126	0.0018	195.57	1.20	196.31	0.14
4	0.3042	0.0203	0.3125	0.0019	194.24	0.72	196.51	0.15

Source: Author

CASE 2: SE MEDIUM INTENSITY CONDITION

In this case, the vessels are positioned again with attitudes of 354° , 85° , 162° and 91° degrees relative to North axis (y). At $t = 2000s$ they are commanded to reposition themselves and change heading collectively to 45° degrees, using a linear synchronization protocol. Lack of integral action prevents the vessels to reach alignment, so that attitude obtained are of 50° , 30° , 50° and 30° degrees by the end of the simulation. Figure 46 displays the position and heading configurations of the ships.

Figure 46 - Vessels distribution for the SE current estimation. (a) Initial position. (b) Final position.

(a) Source: Author

(b) Source: Author

A current direction of 300° (going to, referred from North, equivalent to 150° referred to East) is considered, with velocity of 0.6 m/s . Wind is coming from SE but measured and counterbalanced by feedforward. Once again, no significant wave disturbance was considered. Figure 47 displays the summarized environmental condition for the SE case. The current hits vessel 1 by its stern-bow side, vessel 2 by its starboard-bow

side, vessel 3 by its port-bow side and vessel 4 by its starboard-bow side. After the change in heading, all vessels are hit by their starboard side.

Figure 47 – SE Medium Condition for current estimation.

Source: Author

Figure 48 displays results for the independent estimations (left) and the consensus filters outputs. It should be noticed that vessel 1 estimates have failed to find a solution from the optimization algorithm during the initial phase, yielding a zero output for both velocity and angle estimates. The robustness of the setup provided by the consensus filter can compensate this error by rejecting failed inputs. As a result, the consensus filter can output an estimation even for the vessel with failed individual results.

Figure 48 – Current velocity and angle estimates for 150° - 0.6 m/s.

Source: Author

Table 10 and Table 11 provide statistics for the steady state estimates of the current. Before the change in heading, independent current estimations are all a bit larger than that of the actual current, resulting also in an offset of the consensus estimate. Individual angle estimations are also offset by five and eight degrees in both directions. The consensus estimates in this case present an increase in accuracy by reaching the

mean estimates. Vessel 1 individual outputs are ignored, nevertheless its consensus filter output reaches an accurate value. After the change in heading, all vessels find solutions and get better values individually. As a result, the consensus values are also more accurate.

Table 10 - Steady state estimate statistics for current of 0.6 *m/s* at 150° from East, before change in heading.

Vessel	Velocity (m/s)				Angle (degrees)			
	Independent		Consensus		Independent		Consensus	
	Mean	STD	Mean	STD	Mean	STD	Mean	STD
1	—	—	0.6783	0.0002	—	—	151.62	0.05
2	0.6848	0.0006	0.6780	0.0002	157.16	0.11	151.14	0.05
3	0.6814	0.0008	0.6783	0.0002	141.62	0.19	151.62	0.05
4	0.6836	0.0012	0.6780	0.0002	155.54	0.13	151.14	0.05

Source: Author

Table 11 - Steady state estimate statistics for current of 0.6 *m/s* at 150° from East, after change in heading.

Vessel	Velocity (m/s)				Angle (degrees)			
	Independent		Consensus		Independent		Consensus	
	Mean	STD	Mean	STD	Mean	STD	Mean	STD
1	0.6110	0.0120	0.6029	0.0025	153.82	2.52	151.84	0.62
2	0.5981	0.0007	0.6034	0.0028	150.50	0.04	151.97	0.68
3	0.6063	0.0006	0.6029	0.0025	152.70	0.07	151.84	0.62
4	0.6836	0.0008	0.6034	0.0028	150.50	0.04	151.97	0.68

Source: Author

DISCUSSION

The results validate the application of the consensus filter algorithm for indirect measurements, providing slight advantages over the direct usage of independent estimations. Other indirect measured variables may also be filtered such as, for example, wave frequency and other environmental related parameters. When the

inputs affect the agents in diverse ways – such as the current angle affects the vessels' drifts differently – a consensus measurement contributes to homogenize and correct large errors that may rise from an independent measurement. This was made clear when verifying results from case 2, in which vessel 1 had erroneous measurements due to the failed optimization. The robustness of this setup is evidenced this way.

It is interesting to notice that filter outputs from vessels 1 and 3 reached perfect consensus between them, while outputs from vessels 2 and 4 reached equal values among themselves. The values between the sets 1, 3 and 2, 4 are slightly different, though. When observing the communication topology used in these simulations, one can note that the vessels that reached exact consensus values did not have direct communication between them.

This is actually related to consensus filter algorithm. Take again equation (225):

$$\dot{x}_i(t) = \sum_{j \in N_i} a_{ij} [x_j(t) - x_i(t)] + q \sum_{j \in J_i} a_{ij} [\hat{x}_j(t) - x_i(t)]$$

The neighbor states and measurements have a heavy weight on the resulting dynamics of the filter, especially if compared with the weight of the filters' own measurements. The application of the weighting factor q contributes to the reduction of differences between the pairs. For the topology used, the results from this equation are almost the same for the non-neighbor vessels.

5.3. Formation Maintenance and Leader-Following

In this case, the formation maintenance and leader-following behavior is assessed. A group of four PSVs and a virtual leader are simulated using the protocols proposed and a linear consensus protocol. Then, experimental results of the same methods are presented using three heterogeneous PSV models. This case study is of interest as it permits the evaluation of the properties of the proposed methods without external operational loads.

5.3.1. Numerical Evaluation

In this section, maneuvering results for numerical simulation of linear consensus, first order sliding modes and super twisting consensus methods are presented. In this simulation the vessels are required to enter in formation and keep it while tracking a

virtual leader. The objective is to demonstrate how changes in communication topology, environmental action and transmission failures affect the performance of the proposed system. The simulations will be run with a Linear Consensus Controller, the proposed First Order Sliding Mode Cooperative Controller and the Super Twisting Cooperative Controller. The evaluations were performed in three environmental conditions, with two possible network topologies and two different transmission failure rates.

NETWORK TOPOLOGY

The communication structure is composed by a virtual leader and four follower vessels. The two topologies evaluated are presented in Figure 49. Note that both graphs are not only connected but they also have a spanning tree.

Table 12 displays the graph matrices related to both topologies, named, hereinafter A and B. Observe that the Fiedler Eigenvalue have values of 1 and 0.382, respectively. Within linear consensus theory, this change in topology should affect the time constants of the group's dynamic behavior. For the sliding modes consensus controllers, such change in topology should not affect the group's time constants.

Figure 49 - Wireless Communication Structure Graphs (a) Complete Topology A. (b) Reduced Topology B.

(a) Source: Author

(b) Source: Author

For the application of the proposed control methods, remember that the output of each vessels' cooperative control algorithm must be filtered according to equation (178).

For the linearized Proportional Derivative controlled vessel, the parameters for this equation will be $A_{0i} = B_{0i} = M_i^{-1}K_{Pi}$ and $A_{1i} = B_{1i} = M_i^{-1}K_{Di}$. Note that, in this specific case of PD control, a simplification will occur. Replacing the parameters in the equation yields

$$\begin{aligned} \mathbf{u}_i(k+1) = & hK_{Di}^{-1}K_{Pi}\boldsymbol{\eta}_i(k) + h\dot{\boldsymbol{\eta}}_i(k) + (I_{3 \times 3} - hK_{Di}^{-1}K_{Pi})\mathbf{u}_i(k) \\ & + hK_{Di}^{-1}M_i\tilde{\mathbf{u}}_i(k) \end{aligned} \quad (232)$$

The consensus formation and leader step maneuvers were tested along with the local neighbor estimation Kalman Filters to assess the robustness of the system in the event of communication failure.

Table 12 – Cooperative Control Design Matrices.

Matrix	Value 1	Value 2
Adjacency A	$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \end{bmatrix}$	$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \end{bmatrix}$
In-Degree D	$D = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 3 \end{bmatrix}$	$D = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 3 \end{bmatrix}$
Laplacian L	$L = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ -1 & 3 & -1 & 0 & -1 \\ -1 & -1 & 3 & -1 & 0 \\ -1 & 0 & -1 & 3 & -1 \\ -1 & -1 & 0 & -1 & 3 \end{bmatrix}$	$L = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ -1 & 3 & -1 & 0 & -1 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 \\ -1 & -1 & 0 & -1 & 3 \end{bmatrix}$
Eigenvalues of L	$[0, 1, 3, 3, 5]^T$	$[0, 0.382, 2.618, 2.382, 4.618]^T$
Fiedler Eigenvalue	1	0.382

Source: Author

NEIGHBOR FILTER

In the simulations, all communications have a random rate of at least 20% of transmission failure. A condition with 50% of transmission failures was also evaluated. As the system structure incorporates a neighbor filter, these failures should have reduced effect. The model used for the estimator was of a second order system calibrated to give response close to the DP vessels' closed loop dynamics. State space matrices were taken as

$$\begin{aligned} \begin{bmatrix} \dot{\mathbf{x}}_1 \\ \dot{\mathbf{x}}_2 \end{bmatrix} &= \begin{bmatrix} 0_{3 \times 3} & I_{3 \times 3} \\ -A_0 & -A_1 \end{bmatrix} \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \end{bmatrix} + \begin{bmatrix} 0_{3 \times 3} \\ A_0 \end{bmatrix} \mathbf{u} \\ \mathbf{y}_k &= \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \end{bmatrix} \end{aligned}$$

And

$$A_1 := \text{diag}([2\zeta_x\omega_{nx} \quad 2\zeta_y\omega_{ny} \quad 2\zeta_\psi\omega_{n\psi}]); A_0 := \text{diag}([\omega_{nx}^2 \quad \omega_{ny}^2 \quad \omega_{n\psi}^2]);$$

The states are the vessels' positions and velocities in the GRF. Values for relative damping and natural frequencies are the same used for control design. Filter design matrices in Table 13.

Table 13 - Neighbor KF Design Matrices.

Matrix	Value
Process Covariance Q	$Q_1 = \text{diag}([1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1]);$
Noise Covariance R	$R = 1 \times 10^{-3} \text{diag}([1 \quad 1 \quad 1 \times 10^{-3}])$

Source: Author

LINEAR CONSENSUS

The linear consensus algorithm evaluated in the proposed study case had controller gain optimally calculated via equations (117) and (118), yielding gains from Table 14. Synchronization gain c was set to 0.8. Note that, due to the nature of the linear consensus design method, Northing and Easting dynamic behavior is expected to be a bit different than that of the heading dynamic behavior, as the DP system is already tuning is distinct for these degrees of freedom. Moreover, one should also expect dissimilar behaviors even between surge and sway movements, as drag forces are vastly different in those motion modes. Based on equation (116), the expected closed loop behavior of the formation error should be ruled by system matrix $(A - c\lambda_i BK)$ for each agent i .

Given the Fiedler eigenvalue of the Laplacian matrices for topology A ($\lambda_2 = 1$), one should expect the resulting natural frequency of 0.0155 rad/s for all degrees of freedom and relative damping of 0.69 for Easting and Northing and 0.94 for Heading. These values would render a settling time of 371 s (92.8 s time constant) with a 4.9% overshoot for Easting and Northing and 274 s (68.5 s time constant) with virtually zero overshoot for Heading.

In topology B, $\lambda_2 = 0.382$ and the resulting natural frequency is of 0.0124 rad/s for all degrees of freedom and relative damping of 0.68 for Easting and Northing and 1.04 for Heading. These values would render a settling time of 474 s (118 s time constant) with

a 5.4% overshoot for Easting and Northing and 308s (77 s time constant) with zero overshoot for Heading. Designed responses for both topologies are plotted in Figure 50.

Table 14 - Linear Consensus control gains.

Matrix	Value
Consensus control Gain	$K = \begin{bmatrix} 1.77 & 0 & 0 & 94.5 & 0 & 0 \\ 0 & 1.77 & 0 & 0 & 94.5 & 0 \\ 0 & 0 & 1.77 & 0 & 0 & 64.9 \end{bmatrix}$

Source: Author

Figure 50 - Designed decay for linear consensus protocol.

Source: Author

FIRST ORDER SLIDING MODES

The first order sliding mode consensus control algorithm for formation from equation (201) is defined next. The sliding vector is designed with a gain of 0.018rad/s for all degrees of freedom. Once the system has reached the manifold, it should decay with a time constant of 55.6s (see Figure 51). As opposed to the linear consensus method discussed above, if the switching gains are tuned adequately with relation to modeling errors in each degree of freedom, motion will lay over the sliding manifold and all modes should have a similar dynamic behavior.

The derivatives of the formation references are considered equal to zero, and so are the estimates of the non-cancelled dynamics $\bar{f}$. A boundary layer is added to smooth out the control output:

$$\tilde{\mathbf{u}}_i = -\Lambda\dot{\boldsymbol{\eta}}_i + \frac{1}{d_i} \left[\sum_{j=1}^n a_{ij} (\tilde{\mathbf{u}}_j + \Lambda\dot{\boldsymbol{\eta}}_j) - K\text{sat}(\Phi^{-1}\boldsymbol{\sigma}_i) \right]$$

Figure 51 - Designed Error decay for sliding modes.

Source: Author

The estimate of dynamics error was defined by setting maximum velocities expected during the maneuver. An estimation of such values can be made by analyzing the dynamics set for the virtual leader. With a natural frequency of 0.005 rad/s and relative damping set to 1, maximum velocities reached in the operation would be of 0.46 m/s for surge and sway and of 0.0014 rad/s for yaw. For these values, using the results from section 4.3.3 one can set the modelling error boundaries as $|\tilde{\mathbf{f}}| = [0.0012, 0.0012, 0.0003]^T$. For this error, with a value for $\Delta = [0.0108, 0.0108, 0.0105]$, the time to reach the sliding surface from a deviation of 1 m/s and of $1^\circ/\text{s}$ would be of at most 93 seconds and 95 seconds, respectively. Note that this is only a rough estimate, as during distinct phases of operation and different disturbance conditions, the modeling error could be vastly different. The gains calculated are given in Table 15. The boundary layer was defined by trial-and-error to give maximum dynamic performance with minimal chattering. It renders a precision of 10m in Easting and Northing and of 10 degrees in Heading.

Table 15 - SM Consensus control gains

Parameter	x	y	ψ
λ	0.018 rad/s	0.018 rad/s	0.018 rad/s
φ	0.18 m/s	0.18 m/s	0.0031 rad/s
k	0.012 m/s^2	0.012 m/s^2	0.0108 rad/s^2

Source: Author

SUPER TWISTING ALGORITHM

The Super Twisting controller from equation (212) was also assessed. Once again, the derivatives of the formation references are considered equal to zero, and so are the estimates of the non-cancelled dynamics $\bar{f}$:

$$\begin{cases} \ddot{\tilde{u}}_i = -\Lambda \dot{\tilde{u}}_i + \frac{1}{d_i} \left[\sum_{j=1}^n a_{ij} (\ddot{\tilde{u}}_j + \Lambda \dot{\tilde{u}}_j) - K_1 |\sigma_i|^{\frac{1}{2}} \text{sign}(\sigma_i) + \ddot{u}_a \right] \\ \dot{u}_a = -K_2 \text{sign}(\sigma_i) \end{cases}$$

The sliding constants $\Lambda_i = \text{diag}[\lambda_x, \lambda_y, \lambda_\psi]$ were the same used in SM control. The estimate of dynamics error based on the expected operation accelerations was set to $|\dot{\tilde{f}}| = [0.15, 0.15, 0.006]^T \times 10^{-3}$. For this error, the gains calculated through equation (221) are given in Table 16.

Table 16 - ST Consensus control gains

Parameter	x	y	ψ
λ	0.018 rad/s	0.018 rad/s	0.018 rad/s
k_1	0.0184 m/s	0.0184 m/s	0.0012 rad/s
k_2	$0.165 \times 10^{-3} \text{ m/s}^2$	$0.165 \times 10^{-3} \text{ m/s}^2$	$6.88 \times 10^{-7} \text{ rad/s}^2$

Source: Author

ENVIRONMENTAL CONDITIONS

Three environmental conditions were tested. In the first, no significant disturbances were applied, setting then the standard for the motion performance.

Figure 52 - Simulated Environmental Conditions. (Left) NE Condition. (Right) SE Condition

(a)Source: Author

(b)Source: Author

Then a mild intensity NE condition, in which a 10 kn wind, a 0.6 kn current and a 1.5m/7.5s sea come from the Northeast Quadrant (see Figure 52 - Left) was

evaluated. Finally, a medium intensity SE condition was applied, in which a 10 *kn* wind, a 0.6 *kn* current and a 1.5*m*/7.5*s* sea come from the Southeast Quadrant (see Figure 52 - Right).

Simulated wind was generated using Harris spectrum, waves used Jonswap Spectrum.

SUMMARY OF SIMULATION CONDITIONS

The simulations were run for the three controllers (Linear, Sliding Modes, Super-Twisting), at first with no environmental disturbances, with the topology A and only 20% of message failure rate (that is, only 80% of the messages reach the final destination). This sets the performance paradigm for each controller in the analysis, and the case will be referred to as *Standard Case*.

A second situation, similar to that of the Standard Case, with no environmental disturbances, 20% message failure rate and a change to topology B, will be referred to as *Reduced Connectivity Case*.

The third situation, similar to that of the Standard Case, with no environmental disturbances, topology A, and a rate of messages failure of 50% will be referred to as *Communication Shortage Case*.

Finally, the two environmental disturbances will be run for topology A with 20% message failure rate. These shall be referred to as *NE Case* and *SE Case*.

Table 17 summarized the simulated conditions.

Table 17 - Summary of Simulation Conditions

Condition	Environmental Disturbances	Topology	Communication Failure Rate
Standard	None	A	20%
Reduced Connectivity	None	B	20%
Communication Shortage	None	A	50%
NE	NE	A	20%
SE	SE	A	20%

Source: Author

FORMATION SETTING RESULTS AND DISCUSSION

The first part of the maneuver consisted in the formation setting. The vessels start in scattered initial positions with different attitudes and positions (see Table 18 and Figure 53 - Right). As the instantaneous power for each vessel is limited and thrusters are subject to saturation, the initial positions could not be too far from the final formation setup. During the first 400s, each vessel ran its own DP systems, without any cooperative control algorithm.

After the initial stabilizing period required for the DP system to estimate and compensate the DP current, the cooperative protocol was turned on and the vessels were commanded to enter formation. The pattern is illustrated in Figure 53 (Right). Observe that the required leader position is offset by 50 meters in Easting and 20 meters in Northing from the agents' initial position average coordinate. The change in formation was smoothed by a first order filter with 120s time constant to avoid saturation. Note that this produces an alteration in the designed decay dynamics.

$$\begin{aligned}\delta_{1L} &= \begin{bmatrix} -98.5 \\ 0 \\ 0 \end{bmatrix}; & \delta_{2L} &= \begin{bmatrix} 0 \\ -59.25 \\ 0 \end{bmatrix}; & \delta_{3L} &= \begin{bmatrix} 98.5 \\ 0 \\ 0 \end{bmatrix}; & \delta_{4L} &= \begin{bmatrix} 0 \\ 59.25 \\ 0 \end{bmatrix} \\ \delta_{12} &= -\delta_{21} = \begin{bmatrix} 98.5 \\ 59.25 \\ 0 \end{bmatrix}; & \delta_{23} &= -\delta_{32} = \begin{bmatrix} -98.5 \\ -59.25 \\ 0 \end{bmatrix}; \\ \delta_{34} &= -\delta_{43} = \begin{bmatrix} 98.5 \\ -59.25 \\ 0 \end{bmatrix}; & \delta_{14} &= -\delta_{41} = \begin{bmatrix} -98.5 \\ -59.25 \\ 0 \end{bmatrix}\end{aligned}$$

Table 18 - Vessel Initial Positions for Formation Simulation

Vessel	x (m)	y (m)	ψ (°)
L	0.0	0.0	0
1	-179.82	16.32	6
2	-19.34	-91.58	5
3	155.82	22.66	334
4	43.34	52.60	359

Source: Author

Figure 53 - (a) Initial condition for the cooperative maneuvering. (b) Stabilized Formation.

(a) Source: Author

(b) Source: Author

Figure 54 displays the formation setting under the Standard Case for the three controllers. A slight difference between protocols can be verified when the footprints are analyzed. Vessel 1's track, for instance has a larger variation in SM and ST controllers for Easting in the beginning of the motion. As shall be seen, this is related to the reaching phase of the sliding motion.

Figure 54 - Formation Setting Footprints for the three controllers (a) Linear. (b) SM. (c) ST.

(a) Source: Author

(b) Source: Author

(c) Source: Author

Figure 55 shows the relevant relative Vector Consensus Error from the starting instant of all protocols (from top to bottom, Linear, SM and ST). The wrapping curves are quite similar for all controllers. After 600s from the protocol start, all errors are set to zero and the formation is settled.

There is not much sense in comparing dynamic performance between the linear and both the sliding mode controllers, as tuning of the designed behaviors are a bit different. The comparison is fulfilled, then, on how different disturbance and topology scenarios affect the system's behavior.

Figure 55 - Standard Case Formation Error time series for all controllers.

Source: Author

Figure 56 - Normalized Error for Linear Consensus - Standard, Reduced Connectivity and Communication Shortage

Source: Author

Consider the effects of changes in the communication structure. Figure 56 provides the normalized consensus error of the Linear Consensus protocol observed during the formation setting, for each degree of freedom. In the same axes, the error decay is shown for the Standard Case, the Reduced Connectivity Case, the Communication Shortage Case and the designed performances. Note that designed performances account for the first order filter introduced in the formation setpoint change, making the response slower than what would be expected in a direct step input.

The first thing to notice is that the standard and the reduced connectivity decays are quite different from the designed ones. Actual behavior is faster than that the predicted, indicating that the linear model inaccuracies affect the response.

Due to the slower motion produced by the input filter, the reduced graph connectivity has much less effect in the final dynamic response than what would be expected. The zoomed axes highlight the points in which the differences are larger. This condition delays the dynamic response for up to 85 seconds, depending on the degree of freedom. The Formation Error performance index (Figure 57) show, though, that some differences do occur. The index in reduced connectivity is about 4% larger than that of the standard topology.

Figure 57 – Normalized Error Index for Linear Consensus - Standard, Reduced Connectivity and Communication Shortage.

Source: Author

The Normalized Error responses for the Communication Shortage scenario and the standard scenario are virtually the same. This can also be verified by the formation and tracking error performance index. Such insensitivity to the reduced communication rate stems from the usage of the Neighbor Kalman Filter. An example of this filter's

estimation in both conditions is shown in Figure 58. As the neighbor estimation filter uses the target vessel's own filter output, the resulting estimate will be only as good as the received signal. The zoomed detail of the figure shows the effect of a 5 seconds shortage in communication. During the event, the filter integrates the model in a different direction than that of the “actual” signal. Yet, errors are kept under 0.1 degree.

Figure 58 - Example Estimation of Neighbor Filter.

Source: Author

Observe now the normalized error decay for the Sliding Mode Consensus Protocol in Figure 59 (top) and the Super Twisting Protocol (bottom). The designed decay from the sliding surface is very adherent to the obtained responses and the dynamic behavior is extremely consistent for all three cases. The difference between settling times for the standard and the reduced connectivity case is under 3s for the SM and under 7s for the ST protocol. For both controllers, though, there is small but noticeable change between the standard and the Communication Shortage series, larger than what was seen in the linear consensus case. Even though the neighbor Kalman filter is running, the lack of the neighbor control input information $\tilde{u}_j$ in the consensus law provokes this small behavior change.

The performance indices for both controllers are shown in Figure 60. The change in topology actually reduces the index, in 2% for the SM controller and in 1% for the ST controller. The Communication Shortages, on the other hand, increases the index in 2% and 1%, respectively.

Figure 59 - Normalized Error for SM (Top) and for ST(Bottom) - Standard, Reduced Connectivity and Communication Shortage.

Source: Author

Figure 60 - Error Indices for SM (Left) and ST(Right) Consensus - Standard, Reduced Connectivity and Communication Shortage.

Source: Author

The observation of the sliding vectors helps to explain not only the slight differences between the SM controller and the ST controller, but also on how the changes in communication conditions affect the system behavior. Figure 61 displays the sliding variable in the Easting axis for the SM controller running in all vessels. The first thing to notice is that there is a longer deviation and consequent transient for vessel 1 in all cases, explaining the Easting displacement noticed in the track of Figure 54. Observe

also that vessel 2's easting position starts close to the final one, which translates in this vessel not even leaving the Boundary Layer for this axis.

Figure 61 – Easting Sliding Variables for SM Control for Standard Case (Left), Reduced Connectivity (Middle) and Communication Shortage (Right).

Source: Author

From the sliding vector transient behavior one can estimate, for this degree of freedom, how large is the parameter Δ . Vessel 4's variable falls from 1 m/s to the Boundary Layer within 77s. This render $\Delta = 0.013$. Considering that the designed value is of 0.0108, the response seems coherent, as the velocities reached are actually smaller than that of the design. Once the vectors reached the Boundary Layer, they converge to zero at a slower rate and remain.

Compare now the behavior of Vessel 3's Sliding Variable in the Standard and the Reduced Connectivity scenarios. In the standard connectivity, the in-degree of the vessel is of 3. In the reduced connectivity, this value is reduced to 2. By considering the bounds of the modelling error according to equation (198) ($F_{ik} \leq 2d_i\Gamma_{ik}$) we verify that indeed the topology affects the transient behavior of the sliding variable. Note that, for vessel 4, almost no change can be verified, as the in-degree value is equal in this vessel for both topologies. The same can be said about vessel 1.

The sliding vectors for the Super-Twisting protocol (Figure 62) permit drawing the same conclusions. Even though not used in the ST algorithm, the Boundary Layer is plotted to permit some behavior comparisons. Notice that first that the values decay faster than the ones of the pure sliding modes. Vessel 4's reaching time is under 60s, and all vessels' values are settled over the sliding manifold under 150s. Notice that, as there is no usage of the boundary layer, there is no difference in the convergence rate when these values are crossed.

Figure 62 - Easting Sliding Variables for ST Control for Standard Case (Left), Reduced Connectivity (Middle) and Communication Shortage (Right).

Source: Author

Through the next results we shall analyze the effects of the presence of environmental disturbances in the cooperative system dynamics. The conditions are mild for a pure stationkeeping task, but for a dynamic maneuver such as the formation settling will be pose an increased challenge.

Figure 63 shows the normalized formation error for the linear protocol, in both environmental conditions. The system is shown to be highly affected by these disturbances, not even reaching the set points and holding a steady state error. This can be related to both the lack of integral action in the DP inner loop and the lack of any integral action in the consensus loop. Addition of integral action in the outer (cooperative) loop might enhance results, at a cost of increased instability and the introduction of extra states in the controller synthesis. When the normalized error is plotted without the errors related to the leader (Figure 64), one can verify that synchronization is reached among the real agents.

Figure 63 – Normalized Error for Linear Consensus - Standard, NE and SE Cases

Source: Author

Figure 64 - Normalized Error for Linear Consensus without Leader- Standard, NE and SE Cases

Source: Author

The footprints of this motion in the worst case (Southeast Condition) is shown for all controllers in Figure 65. Observe that all reach the formation among themselves, but the linear one fails to track the leader.

Figure 65 - SE Formation Setting Footprints for the three controllers (a) Linear. (b) SM. (c) ST.

(a) Source: Author

(b) Source: Author

(c) Source: Author

The normalized formation error for the Sliding Mode and the Super-Twisting are shown in Figure 66, on top and bottom, respectively. Those account for the Leader error. For the Northeast condition, that is weaker, both controllers reach the formation with small steady state error. For the Southeast condition, that is stronger, the first order Sliding protocol reach a larger steady error, while the Super- Twisting protocol retains its performance.

The difference between the sliding controllers is explained by the reduction of accuracy introduced by the boundary layer. In Figure 67 the Northing sliding variable is depicted for the SM protocol for the three environmental conditions. The zooming axes evidence that the variable is not exactly zero in the disturbed cases. In particular, note that the weaker NE condition provokes a drift in the negative direction (middle figure) and the stronger SE condition provokes a drift in the positive direction (right figure).

This is where the advantages of the Super-Twisting algorithm rise when compared to the standard Sliding Modes with Boundary Layer. The disturbances affect the reaching

time, as they decrease the difference between the gains and the bounds of the modelling errors, but once over the Sliding Manifold the accuracy is retained.

Figure 66 - Normalized Error for SM (Top) and for ST(Bottom) - Standard, NE and SE Condition.

Source: Author

Figure 67 - Northing Sliding Variables for SM Control for Standard Case (Left), NE (Middle) and SE(Right).

Source: Author

Figure 68 - Northing Sliding Variables for ST Control for Standard Case (Left), NE (Middle) and SE(Right).

Source: Author

Figure 69 - Standard, SE and NE Performance Indices, for Linear Controller (top), SM Controller (Middle) and ST Controller (Bottom).

Source: Author

The effects of environmental changes in the three controllers can be summarized by the Formation Error and the Tracking error performance Indices. These values are plotted in Figure 69. The indices time curves evidence that, for all controllers, the formation error among controlled agents is settled to zero. The linear controller tracking performance, is greatly affected by environmental changes (a constant inclination in the performance curve indicates that there is steady state error; the larger the slope, the larger the error), while the sliding mode controller is less affected (though an inclination can be verified) and the super twisting controller eliminates the steady error.

Table 19 - Linear Consensus Performance For all Conditions during Formation Setting

Condition	$T_{s,x}$	$T_{s,y}$	$T_{s,\psi}$	J_{err}	J_L	J_C
Standard	559	564	517	1.00	1.00	2.00
Reduced Connectivity	592	649	513	1.04	0.97	2.01
Communication Shortage	559	564	517	1.00	1.00	2.00
NE	—	—	—	0.98	1.24	2.22
SE	—	—	—	1.06	1.94	3.00

Source: Author

Table 20 - SM Consensus Performance For all Conditions during Formation Setting

Condition	$T_{s,x}$	$T_{s,y}$	$T_{s,\psi}$	J_{err}	J_L	J_C
Standard	546	550	527	1.00	1.00	2.00
Reduced Connectivity	543	547	524	0.98	1.05	2.03
Communication Shortage	546	550	522	1.02	1.02	2.04
NE	587	669	530	1.11	1.12	2.23
SE	639	—	—	1.06	1.33	2.39

Source: Author

Table 21 - ST Consensus Performance For all Conditions during Formation Setting

Condition	$T_{s,x}$	$T_{s,y}$	$T_{s,\psi}$	J_{err}	J_L	J_C
Standard	529	539	546	1.00	1.00	2.00
Reduced Connectivity	537	535	539	0.99	1.03	2.03
Communication Shortage	535	537	536	1.01	1.02	2.03
NE	532	525	514	0.97	1.04	2.01
SE	550	580	451	1.04	1.12	2.16

Source: Author

The performance results for all conditions tested in each controller is summarized in Table 19 (Linear Protocol), Table 20 (SM Protocol) and Table 21 (ST Protocol). The following conclusions can be drawn from the formation setting results:

- i. The Linear Protocol controlled system is clearly affected by the reduction of connectivity, even though less than expected. The Northing Settling Time in this scenario is increased in almost 15% (84s). The environmental disturbances affect little the formation performance, increasing its value in, at most, 6% by the end of the maneuver. Its Leader Tracking performance, on the other hand, is greatly impaired; in the SE scenario it is almost doubled by the end of the maneuver;
- ii. The SM and the ST controlled systems present an extremely consistent behavior in the scenario with reduced connectivity. This is consistent with the theory for the sliding motion, in which the sliding manifold dynamics is not affected by the communication topology;
- iii. The SM controller Formation Performance is affected a bit with the environmental disturbances, with an increase of 11% for this index in the NE case. Its Leader Tracking index is much less affected than the linear case. The loss of precision in both requirements is related to the boundary layer;
- iv. The ST controller is the one less affected by any of the disturbances. Its settling time is kept consistent even under the environmental disturbances, just as its Formation Performance Index and its leader Tracking index, with an increase of 4% and 12% respectively in the Southeast scenario.

- v. All controllers are robust to the Communication Shortage scenario. The SM and ST controllers are slightly more affected due to their reliance in the neighbor control input information.

TRACKING RESULTS AND DISCUSSION

After formation is assembled, the leader vessel is commanded to a sequence of set-points in a square maneuver (Figure 70) referred to in the literature as 4-corner test (SKJETNE et al., 2017), waiting for stabilization after each command (see Table 22). The leader is designed as a second order critically damped system, with natural frequency of 0.005rad/s for all degrees of freedom. This maneuver permits the evaluation of the controller performance under a more dynamic situation, where coupling effects drag disturbances are increased.

Table 22 - Leader Vessel Set-Point Sequence

Step	$x(m)$	$y(m)$	$\psi(^{\circ})$	Time(s)
1	0	0	0	0
2	250	0	0	1500
3	250	250	0	2500
4	250	250	45	3500
5	0	250	45	4500
6	0	0	0	5500

Source: Author

Figure 70 - Square Maneuver

Source: Author

Figure 71 exemplifies the footprints of a Square maneuver ran in the Standard case, using the Super-Twisting controller. Under such conditions, there is not much difference in the footprints produced by the other controllers.

The time series for positions and heading of all controllers are plotted in Figure 72. The heading response is more sensitive to the coupling with sway motions. A small deviation in the heading series of all vessels can be seen at about 2500 s, during phase 2 to 3. Also, during phase 4 to 5 of the motion, at about 4500s, in which the motion has a coupling effect from the misaligned velocity vector, the headings are particularly affected. Note though, that even in this situation the vessels keep formation. The observable deviations are all in relation to the virtual leader. This disturbance effect is reduced in the sliding modes and in the super-twisting controller.

Figure 71 - Footprints of Square Maneuver for ST controller in Standard Case.

Source: Author

The normalized consensus error for the linear controller in all cases is shown in Figure 73. The first thing to notice is that all of these generalized error responses, even in the standard case, are much affected at each transient, despite the application of a smooth step in the virtual leader. The communication shortage is once again of negligible effect, not producing observable changes in relation to the standard case.

There are significant changes in performance in the other scenarios. The Reduced Connectivity scenario produces much larger error transients, demonstrating coherence, then, with the expected results from the change in topology. The environmentally disturbed cases are in line with the results from the formation setting case, producing steady state error in both NE and SE conditions.

Figure 72 - Position and Heading time series for: (Top) Linear Protocol. (Middle) SM Protocol. (Bottom) ST Protocol.

Source: Author

Figure 73 - Linear Consensus Error Norm for Standard, Reduced Connectivity, Communication Shortage, NE and SE Cases.

Source: Author

Figure 74 help describing the nature of such errors. The Formation Error, in which the errors in relation to the leader are not accounted for, is presented for all cases, except the Reduced Connectivity. Note that Easting and Northing values are kept under 0.6 m for all cases plotted, and heading is kept within the 1.7° boundary. When analyzed with Figure 73, this indicate that the main source of deviations come from the virtual leader tracking, while the formation of the controlled ships is kept assembled within tight margins.

Figure 74 - Normalized Formation Error for Linear Protocol in Standard, Severed Communication, NE and SE Cases

Source: Author

The Reduced Connectivity case is an exception to this. Note from Figure 75 that the formation errors are also quite large, especially if compared to other cases. This result, put together with the output from Figure 73, shows that the Reduced Connectivity scenario affects not only the tracking performance, but also the formation setting in a dynamic condition. Observe, though, that the formation error settles once the vessels are stopped.

Figure 75 - Normalized Formation Error for Linear Protocol in Reduced Connectivity Case.

Source: Author

For the overall motion control results, this represents a lack of synchronization, as can be seen in the heading response of Figure 76. The pair that has larger connectivity (vessel 1 and vessel 4) produces an aligned motion among themselves, while the pair that has a lower connectivity (vessels 2 and 3) synchronizes their own motion. This result resembles the outputs from the Consensus Filter, which is natural as both the controller and the filter are Linear Consensus algorithms.

Figure 76 - Position and Heading time series for Linear Protocol, in Reduced Connectivity Scenario.

Source: Author

The Normalized Consensus Error results for the Sliding Mode controller are presented in Figure 77. The error amplitudes and dynamic behavior are consistent for all cases. The Reduced Connectivity and the Severed Communications alter the amplitude of errors observed in the standard case, but their decay rate is maintained. The SE disturbed case, though, presents a large steady error, compatible with the results from the formation setting section.

Figure 77 - SM Consensus Error Norm for Standard, Reduced Connectivity, Communication Shortage, NE and SE Cases.

Source: Author

The Formation Error is, though, mildly affected, as can be seen in Figure 78. All formation errors are kept within a 1.5 m and 2° boundary, indicating that the main source of deviations is from the leader tracking, in all cases. It should be remarked, though, that the Reduced Connectivity scenario does affect in some extent the behavior of the formation when compared to the standard case.

Figure 78 - Normalized Formation Error for SM in Standard, Reduced Connectivity, Severed Communication, NE and SE Cases

Source: Author

Figure 79 - Northing Sliding Variable for SM control in Standard, Reduced Connectivity and Southeast Cases.

Source: Author

To understand the source of the deviations, the sliding vector Northing Variable is presented in Figure 79 for the standard and the two most critical cases. Observe first that even in the standard situation disturbance from the transient motion can be verified in the variable. However, the Boundary layer is neither surpassed in this case or in the Reduced Connectivity Case. As for the former, notice how the vessels 1 and 4 are affected differently than vessels 2 and 3. The communication with the leader kept by the first pair produce a visible disturbance on the sliding variable, while the ones connected only to other agents deviate less. This leads to a small lack of

synchronization among the pairs, but in a much-reduced amplitude when compared to the linear controlled case.

The SE case is the most critical, and the presence of boundary layer produces, as expected, a small positive offset in the mean value of the Northing sliding variable, translating to a loss of position accuracy in this mode. As all vessels are subject to the same disturbance, this deviation is only related to the leader tracking. The final motion phase (from position 5 to 1) that is started at 5500s poses the most challenging task, as it is a coupled motion receiving athwart disturbances. It produces a deviation from the boundary layer that is corrected, at most, in 50 seconds, once again coherent with the designed reaching time. Figure 80 present the effects of the reduced connectivity scenario in the position and heading time series. The zoomed axis evidence the small deviation in synchronization. Figure 81 shows the effects of the SE disturbances in the motion. The most noticeable characteristic is the steady state error in the northing and in the heading, as predicted by the sliding vector analysis.

Figure 80 - Position and Heading time series for SM Protocol, in Reduced Connectivity Scenario.

Source: Author

Figure 81 - Position and Heading time series for SM Protocol, in Southeast Scenario.

Source: Author

Figure 82 shows the consensus error norm for the Super-Twisting protocol in all cases, except the Southeast one, which presented an abnormal behavior and will be discussed next. For the remaining cases, the error amplitudes and dynamic behavior are consistent. The amplitudes are smaller than the ones observed in other controllers, reaching at most $10m$ and 8° (against $15m$ and 12° for the SM and $42m$ and 12° for the Linear). The changes in error amplitude and decay of each case compared to the standard are negligible.

Figure 82 - ST Consensus Error Norm for Standard, Reduced Connectivity, Communication Shortage and NE Cases.

Source: Author

As can be seen in Figure 83, all formation errors are kept within a $1.0m$ and 3° boundary, indicating that in this controller also the main source of deviations is from the leader tracking.

Figure 83 - Normalized Formation Error for ST in Standard, Reduced Connectivity, Severed Communication and NE Cases

Source: Author

Observe now the Consensus Formation error in the Southeast Case from Figure 84. While the values obtained in Easting and Heading axis are coherent with most the results of the other cases, the Easting axis presents a rather large deviation during the

last transient (even though its behavior is also coherent to the other cases through the remaining periods of the maneuver). Note that this occurs in the most challenging part of the motion, in which the environmental actions work strongly against the sideboard of the ships. Such anomaly can also be verified in the Formation error (Figure 84), indicating that it affects not only the tracking performance but also the formation setting.

Figure 84 - ST Consensus Error Norm for SE Case.

Source: Author

Figure 85 - Normalized Formation Error for ST in SE Cases.

Source: Author

To try to understand this behavior the sliding variable in the Northing direction is plotted in Figure 86. The first plot, on the left, presents the sliding variable in the standard case. It presents a larger oscillating behavior around zero when compared to the standard SM with boundary layer, a behavior that can be considered a low amplitude/low frequency chattering. The super twisting controller, despite reducing considerably chattering when compared to pure First Order Sliding Modes, does not eliminate it completely, as actuator dynamics and discretization effects are still present.

The middle graph of Figure 86 is plotted as means to compare the effects of the reduced connectivity in the SM controller. Note that this case provides almost no difference to the Standard Case.

Figure 86 - Northing Sliding Variable for ST control in Standard, Reduced Connectivity and Southeast Cases.

Source: Author

Finally, in the right side of the figure, the Northing sliding variable for the SE case is shown. Note, first, that there is no visible steady state error such as the one observed in the SM controlled system for this case. The large errors in formation and tracking observed in the last transient of are translated in a large variation of the sliding variable for all vessels. Starting at 5600s the deviation is slowly reversed and start to decrease. Note, though, that it continues to grow to the opposite side after the sliding manifold is reached. This behavior resembles that of integral windup in a PID controller provoked by saturation. Indeed, as the ST controller structure incorporates the integral of the signum function in the first derivative of the sliding vector, a similar effect may be in action. Note, though, that the system recovers from this state and at approximately 6400s the sliding variables reaches zero again. Figure 87 confirms this suspect by showing that the commanded control output indeed saturates for all vessels.

The effects of this perceived “windup” effect can be seen in the times series of the motion for this controller in the Southeast case (Figure 88). Observe that motion is synchronized for all degrees of freedom and the leader tracking produces no steady error. The saturation issues are apparent in the final part of the motion, creating an overshoot in all vessels’ Northing response.

Figure 87 – Smoothed Control Action in Sway for the SE Case in the Super Twisting Controller.

Source: Author

Figure 88 - Position and Heading time series for ST Protocol, in Southeast Scenario.

Source: Author

Figure 89 – (a) Thruster Usage Example. (b) Consensus Control Outputs.

(a) Source: Author

(b) Source: Author

Even though not being the main focus of this work, a brief discussion should be raised at this point about thruster usage. The advantages of the sliding control come with a price: there is, evidently, a raise in power spent to keep such a tight precision. The high actuation activity from these controllers can be seen in Figure 89 - a, in which the

rotation output of one of vessel 2's thrusters is plotted for the standard case and for the three controllers. Observe that the oscillations amplitudes from the ST controller are larger and more frequent. A value of 20 to 30rpm in thruster is equivalent to approximately 1 to 2 tons in thrust. The correspondent time frame of the consensus controllers is shown in Figure 89 – b. As these controllers act on an upper level, producing set-points for the inner DP loop, some measures can be taken to reduce these effects, such as optimizing tuning and using input filters. A tuning method that reduces oscillatory effects from the ST algorithm is proposed in (PÉREZ-VENTURA; FRIDMAN, 2019).

As in the formation setting discussion, the performance results for all conditions tested in each controller is summarized in Table 23 (Linear Protocol), Table 24 (SM Protocol) and Table 25 (ST Protocol). The following conclusions are drawn from the Tracking results:

- i. In the tracking scenario the Linear Protocol controlled system is hugely affected by the reduction of connectivity. This situation changes completely the system dynamics and should be avoided by the linear controller. The Environmental disturbances increase the indices considerably.
- ii. The SM controller Formation Performance is affected both by the reduced connectivity (although in an extremely smaller scale than the Linear) and the environmental action. For the latter case, one can verify an overall performance similar to that of the Linear Controller. The presence of the Boundary Layer, indeed, reduces accuracy and approximate the system's behavior to that of a linear one. Note that, for the boundary layer and the sliding variable gains used, one should expect an accuracy of up to $10\text{m}/10^\circ$ in steady state, values coherent to the observed results;
- iii. The ST controller has consistent behavior in all conditions except in the already discussed Southeast Scenario, in which the saturation occurred. The past results indicate, though, that the controller precision when not in this anomalous condition is kept;
- iv. Once again, all controllers are robust to the Communication Shortage scenario. The SM and ST controllers are slightly more affected due to their reliance in the neighbor control input information.

Table 23 - Linear Consensus Performance For all Conditions during Tracking Maneuver.

Condition	J_{err}	J_L	J_c
Standard	1.00	1.00	2.00
Reduced Connectivity	1.46×10^4	6.61	1.46×10^4
Communication Shortage	0.99	0.97	1.96
NE	2.75	1.58	4.33
SE	4.49	5.38	9.87

Source: Author

Table 24 - SM Consensus Performance For all Conditions during Tracking Maneuver.

Condition	J_{err}	J_L	J_c
Standard	1.00	1.00	2.00
Reduced Connectivity	14.1	1.32	15.4
Communication Shortage	1.41	1.41	2.82
NE	2.16	1.08	3.24
SE	7.58	2.84	10.4

Source: Author

Table 25 - ST Consensus Performance For all Conditions during Tracking Maneuver.

Condition	J_{err}	J_L	J_c
Standard	1.00	1.00	2.00
Reduced Connectivity	2.61	1.11	3.72
Communication Shortage	1.04	1.17	2.21
NE	1.36	1.12	2.48
SE	50.93	16.0	66.9

Source: Author

5.3.2. Experimental Evaluation

In this section, maneuvering results for a small scaled model test involving three vessels in an undisturbed scenario and in two sea conditions are presented. The first

order sliding modes and super twisting consensus methods were tested. The three vessels were required to keep formation while tracking a virtual leader. The main objective was to evaluate practical implementation of the methods under a disturbed condition.

NETWORK TOPOLOGY

In the experiments, vessels have a communication structure composed by a virtual leader and three follower vessels arranged according to Figure 90. The vessel identified by 1 is the M510A, the vessel identified by 2 is the M510B and the vessel identified by 3 is the M539. Table 26 displays the graph matrices related to this topology. Note that the graph is not only connected but also has a spanning tree. During tests there are no communication failure assumptions.

Figure 90 - Wireless communication structure for scale model tests

Source: Author

Table 26 – Cooperative Control Design Matrices for scale model tests

Matrix	Value
Adjacency A	$A = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$
In-Degree D	$D = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}$
Laplacian $L = D - A$	$L = \begin{bmatrix} 0 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ -1 & -1 & -1 & 3 \end{bmatrix}$

Source: Author

FIRST ORDER SLIDING MODES

For the experimental evaluation, the first order sliding mode gains are given in Table 27. The gains were calculated based on modelling error boundaries of $|\tilde{\mathbf{f}}| = [0.0052, 0.0052, 0.018]^T$, calculated considering the maximum velocities expected to be imposed on the virtual leader. For this error, with a value for $\Delta = [0.011, 0.011, 0.022]$ the reaching time from a 1 m/s and a 1°/s disturbances would be of about 90 seconds and 45 seconds, respectively. Observe that the sliding vector gain λ has different values for heading than for Easting and Northing.

Table 27 - SM Consensus control gains

Parameter	x	y	ψ
λ	0.285 rad/s	0.285 rad/s	0.98 rad/s
φ	0.05 m/s	0.05 m/s	0.05 rad/s
k	0.0162 m/s ²	0.0162 m/s ²	0.04 rad/s ²

Source: Author

SUPER TWISTING ALGORITHM

For the Super Twisting controller, the sliding constants were the same used in SM control. The estimate of dynamics error was set to $|\dot{\tilde{\mathbf{f}}}| = [4.0, 4.0, 1.25]^T \times 10^{-4}$. For this error, the gains calculated through equation (221) are given in Table 28.

Table 28 - ST Consensus control gains

Parameter	x	y	ψ
λ	0.285 rad/s	0.285 rad/s	0.98 rad/s
k_1	0.0212 m/s	0.0212 m/s	0.017 rad/s
k_2	2.2×10^{-4} m/s ²	2.2×10^{-4} m/s ²	1.3×10^{-4} rad/s ²

Source: Author

WAVE CONDITIONS

The experiments were run in a standard, non-disturbed condition, named from hereafter Standard Case and in two wave conditions representing sea states from East of 8s and 12s respectively for the 1:42 model. As the models have different scales, the

smaller one (1:70) will be disturbed by a much harsher condition, an equivalent of a $2m/10.3\text{ s}$ and a $2.5m/15.5\text{ s}$ wave. Table 29 summarizes the conditions.

Table 29 - Wave conditions for the Formation Experimental Tests

Wave Spectrum	$H_s(m)(1:42/1:70/1:1)$	$T_p(s)(1:42/1:70/1:1)$	Dir. (°)
None	0.0	0.0	0.0
JONSWAP ($\gamma = 3.3$)	1.2/2.0/0.029	8.0/10.33/1.23	0.0
JONSWAP ($\gamma = 3.3$)	1.5/2.5/0.036	12.0/15.49/1.85	0.0

Source: Author

RESULTS

The formation pattern used throughout the maneuver is shown in Figure 91, following the set-points given as:

$$\delta_{1L} = \begin{bmatrix} -0.5 \\ -1.0 \\ 0.0 \end{bmatrix}; \quad \delta_{2L} = \begin{bmatrix} -0.5 \\ 1.0 \\ 0.0 \end{bmatrix}; \quad \delta_{3L} = \begin{bmatrix} 1.5 \\ 0.0 \\ 0.0 \end{bmatrix};$$

$$\delta_{12} = -\delta_{21} = \begin{bmatrix} 0.0 \\ -2.0 \\ 0.0 \end{bmatrix}; \quad \delta_{23} = -\delta_{32} = \begin{bmatrix} -1.0 \\ 1.0 \\ 0.0 \end{bmatrix}; \quad \delta_{13} = -\delta_{31} = \begin{bmatrix} -1.0 \\ -1.0 \\ 0.0 \end{bmatrix}$$

Figure 91 – Formation Pattern (Left) Commanded. (Right) Obtained.

Source: Author

With the formation assembled and stabilized, the virtual leader vessel is commanded to perform the square maneuver according to Figure 92 and Table 30. The leader is designed as a second order underdamped system, with natural frequency of 0.1 rad/s and relative damping of 0.7 for all degrees of freedom.

Table 30 - Leader Vessel Set-Point Sequence

Step	$x(m)$	$y(m)$	$\psi(^{\circ})$	Time(s)
1	0	0	0	0
2	0	2	0	60
3	2	2	0	260
4	2	2	45	460
5	2	0	45	660
6	0	0	0	860

Source: Author

Figure 92 – Experimental Square Maneuver

Source: Author

Figure 93 - Motion Footprints for Experimental Square Maneuver under 12s wave for SM protocol.

Source: Author

Figure 94 - Time Series for SM control motion in 8s wave.

Source: Author

The experiments were run for the three conditions (Standard, 8s wave and 12 s) and the two controllers. Figure 93 presents the motion footprints for the 12s wave maneuver, considering the SM controller. There is a noticeable deviation in the easting

direction during the coupled part of the motion (phases 4 to 5). The time series for the same motion, taken from the 8s wave experiment, are shown in Figure 94.

Observe first that the yaw motion of vessel M539 (Vessel 3) is more affected by the wave disturbance than the others due to its smaller scale. This difference, though, does not seem to affect the cooperative motion, as expected by the methodology proposed. Observe that the Easting and Northing positioning curves present a synchronization that makes deviations from the formation almost imperceptible, even during the coupled motion phase (4-5), in which the leader tracking performance is impaired. Indeed, as can be seen from Figure 95, the formation error norm for the SM controller is kept under 0.15 m for positioning, and under 10 degrees for heading. The wave motion plays a significant role in the increase of this value during steady state, as the wave-disturbed curves make clear (especially in Easting, that is the direction from which the waves are coming), and for the heading after 460 s , which is when the waves start hitting the vessels by their port-bow side.

Figure 95 - Formation Error Norm for SM controller - Standard, 8s and 12s wave.

Source: Author.

Figure 96 - Consensus Error Norm for SM controller - Standard, 8s and 12s wave.

Source: Author

The tracking characteristics can be verified by the consensus error norm plot (Figure 96). The peaks during transient motion reach up to 75% of the commanded displacement. As the formation errors are kept well below these values, the main source of deviations come from leader tracking errors. Note that the tracking performance is almost unaffected by the presence of wave disturbances. A more detailed analysis of the tracking performance can be made through the analysis of the sliding vector dynamics. For each degree of freedom, its plot is shown for the standard case in Figure 97.

The first noticeable characteristic is that the vessels that have a higher in-degree (in particular, the ones connected directly to the leader) have a higher deviation from zero during the tracking task. That is in line with the results predicted from theory and the simulations, in which the topology does not change the system dynamics when relaying on the sliding manifold, but it does affect the boundaries of modeling errors, pushing the states out of the manifold and requiring the switched action to take place.

From the reaching phase of the motion one can estimate the size of vector Δ by taking the reaching times and an initial value of σ . $\Delta = [0.0067, 0.0034, 0.0205]^T$. These are close to the designed values, indicating that the modeling error boundaries used are not far from the actual ones.

Figure 97 - Sliding Vector for SM controller - Standard Case.

Source: Author

The effects of wave disturbances over the sliding vector can be seen in Figure 98, where the variable is plotted for the Easting direction. Note that the reaching dynamics is not affected by the disturbances. Vessel 3's steady state is particularly affected by the 1st order motion, but despite the noise, the mean sliding variable value in this situation is still zero.

Figure 98 - Easting Sliding Variable for SM controller - Standard, 8s and 12s wave.

Source: Author

Next, the results for the Super-Twisting controller tracking maneuvers are presented. In Figure 99 the motion footprints of the vessels under the 12s wave are shown. The first noticeable characteristic is that, when compared to the footprints produced by the SM controller, is that the latter has stationary points sharper than the ones observed for the Super-Twisting. This is related to a larger overshoot observed in the super-twisting controller series, present in all experiments three experiments.

Figure 99 - Motion Footprints for Experimental Square Maneuver under 12s wave for ST protocol.

Source: Author

The time series of the motion under 8s waves for this controller are plotted in Figure 100. The overshoots present also a longer decay time. As shall be seen in the sliding vector plots, this is related to a poor definition of the bounds of modeling errors and tuning of the ST algorithm, affecting the integral action of the controller.

Figure 100 - Time Series for ST control motion in 8s wave.

Source: Author

The overshoot behavior is observed only in the leader tracking performance. Notice that the motion is synchronized between agents even during the transient deviations from the commanded motion. The Formation Error results, plotted in Figure 101, produce the same level of error amplitudes than that of the Sliding Mode. There is one point for the standard case at approximately 700s in which an abnormal raise in the positioning errors can be verified. This is due to a spurious reading from vessel 3 and can be disregarded.

Figure 101 - Formation Error Norm for ST controller - Standard, 8s and 12s wave.

Source: Author

The tracking errors amplitudes are also close to the results obtained with the sliding mode controller (Figure 102). The curves present, though, a second, smaller peak after the larger ones observed in the leader transient, related to the already mentioned overshoot. Once again, wave action does not seem to affect the dynamic performance, producing only zero-mean 1st order disturbances.

Figure 102 - Consensus Error Norm for ST controller - Standard, 8s and 12s wave.

Source: Author

The sliding variable plots for the ST controller are shown in Figure 103. The boundary layer is plotted to provide means of comparison, but not used in the controller. The behavior related to the topology of the communication verified in the SM is repeated here, with vessel 2 sliding variable remaining almost insensitive to the leader vessel's changes. The dynamic behavior of vessel 1's and 3's sliding variables demonstrate the source of the overshoot verified in the motion time series. It reflects a possible poor choice of parameters, producing an effect similar to the windup observed in the simulated Southeast case, except that, in the present data, no disturbance is applied.

Figure 103 - Sliding Vector for ST controller - Standard Case.

Source: Author

Finally, the plots for the easting sliding variable under the different disturbances scenarios is shown in Figure 104. Despite the low level, zero-mean noise from the 1st order wave motion, the wave effects are negligible.

Figure 104 - Easting Sliding Variable for ST controller - Standard, 8s and 12s wave.

Source: Author

GENERAL DISCUSSION

The controller performances verified in the experiments seem to be much more sensitive to the coupling effects and drag disturbances raised during the motion than that of what was seen in the simulations. The dynamic behavior for the formation settling, though, is consistent. The Normalized Formation Error and the Tracking error performance indices graphs, plotted in Figure 105, also demonstrate that the introduction of the wave external disturbances do not alter radically the system performances for both controllers. The ST controller, in particular, seem less sensitive to such variations. It is worth mentioning, though, that when the vessels port-bow side are hit by the waves (from approximately 460s on), the formation error is a bit affected, probably due to the difference in scale from the smaller vessel, which is much more affected by the wave motion than the others, explaining the detachment of the Formation Error index seen in both controllers.

Figure 105 - Experimental Performance Indices for the SM (Left) and the ST(Right).

Source: Author

Considering such consistent behavior produced by both controllers (evidencing once again their robustness properties), a possible explanation to the sensitivity verified is that a poor choice of parameters was made for the controllers (especially the super-

twisting) and possibly for the virtual leader dynamics, which produced larger tracking errors. Actual tuning of the controllers, both in the experiments and the numerical evaluation, requires dealing application constraints that may be hard to balance with performance, such as control saturation. Finding the extent of modeling errors may require a more detailed methodology for a more dynamically challenging task than pure stationkeeping.

5.4. Towing Maneuver

In the last case study, the SM and the ST algorithms are put to test in a situation of operational loads are present. The maneuver, tested in small scale model, simulates a change of the overall formation heading (not only the individual agents') while a floating unactuated load connected to them is pulled. This type of maneuver could be used in an automated change of heading during an offloading operation, for instance. A simulation study presenting automated hold-back and positioning of an unactuated Drilling Rig by four oceanic tugs was presented in (IANAGUI; TANNURI, 2019).

PHYSICAL SETUP

For the experiments proposed, the M510A and the M539 are set to act as tugboats, while the M510B, disconnected from the network and unactuated, is considered as the load. The virtual leader is responsible for the tracking generation of the setpoints.

The proposed setup is that of a convoy, with M539 in the front, acting as a hold-back tug, and M510A in the back, working on the load heading. The M510B's bow is tied to the M539's stern; accordingly, M510A bow is tied to M510B's stern.

Figure 106 - Physical setup for towing maneuver.

Source: Author

A set of springs ((Figure 107 – a), with stiffness 50.83 N/m , scaled to reflect that of a steel cable catenary (at about 90 kN/m) are used to link the vessels, according to Figure 107 – b.

Figure 107 - Spring Connection used in the experiment (a) Springs Used. (b) Connection Illustration.

(a) Source: Author

(b) Source: Author

NETWORK TOPOLOGY

In the towing experiments, vessels have a communication structure composed by a virtual leader and two follower vessels arranged according to Figure 108. The vessel identified by 1 is the M510A, the vessel identified by 2 is the M539. Note that M510B is not on the topology. Table 31 displays the graph matrices related to this topology.

Figure 108 - Wireless communication structure for scale model towing tests

Source: Author

Table 31 – Cooperative Control Design Matrices for scale model towing tests

Matrix	Value
Adjacency A	$A = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$
In-Degree D	$D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$
Laplacian $L = D - A$	$L = \begin{bmatrix} 0 & 0 & 0 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}$

Source: Author

FIRST ORDER SLIDING MODES

For this experimental evaluation, the first order sliding mode gains are given in Table 32.

Table 32 - SM Consensus control gains

Parameter	x	y	ψ
λ	0.98 rad/s	0.98 rad/s	0.98 rad/s
φ	0.5 m/s	0.5 m/s	0.05 rad/s
k	0.065 m/s^2	0.065 m/s^2	0.004 rad/s^2

Source: Author

SUPER TWISTING ALGORITHM

For the Super Twisting controller, the gains are given in Table 33.

Table 33 - ST Consensus control gains

Parameter	x	y	ψ
λ	0.98 rad/s	0.98 rad/s	0.98 rad/s
k_1	0.0367 m/s	0.0367 m/s	0.017 rad/s
k_2	$6.6 \times 10^{-4} \text{ m/s}^2$	$6.6 \times 10^{-4} \text{ m/s}^2$	$1.4 \times 10^{-4} \text{ rad/s}^2$

Source: Author

WAVE CONDITIONS

The experiments were run in one wave conditions representing sea states from East of 8s for the 1:42 model. As the models have different scales, the smaller one (1:70) will be disturbed by a much harsher condition, an equivalent of a $2m/10.3\text{ s}$. Table 34 summarizes the condition.

Table 34 - Wave conditions for the Towing Experimental Tests

Wave Spectrum	$H_s(m)(1:42/1:70/1:1)$	$T_p(s)(1:42/1:70/1:1)$	Dir.(°)
JONSWAP ($\gamma = 3.3$)	1.2/2.0/0.029	8.0/10.33/1.23	0.0

Source: Author

OPERATION DESCRIPTION

The vessels start linked and in formation, with formation heading set to 210° from the North (240° from the East). Figure 109 (Left) shows the initial setup. The formation set points are given in the virtual leader vessel's axis, according to the following displacements:

$$\delta_{1L}^L = \begin{bmatrix} -4.5 \\ 0 \\ 0.0 \end{bmatrix}; \quad \delta_{2L}^L = \begin{bmatrix} 0.0 \\ 0.0 \\ 0.0 \end{bmatrix}; \quad \delta_{12}^L = -\delta_{21}^L = \begin{bmatrix} -4.5 \\ 0.0 \\ 0.0 \end{bmatrix}$$

To change the overall formation setpoint, a Rotation transformation must be applied to the relative set-points at each instant, using the leader's heading as input:

$$\delta_{ij}^G(t) = R_L^G(\psi_L)\delta_{ij}^L$$

Figure 109 - Formation Set Points Before (Left) and after (Right) the maneuver

Source: Author

With the formation stabilized, the wave generator was turned on, with wave hitting the vessels by their open port-aft side (see Figure 110 – a). The leader vessel is then commanded to change its heading to 0.0° (relative to East), slowly changing the formation's heading to its final position (Figure 110 – b). To achieve a slow and controlled response, avoiding saturations, the virtual leader's natural frequency is set to 0.01 rad/s in all degrees of freedom, with relative damping maintained in 0.7.

Figure 110 – Formation Setting (a) Before Heading Change (b) After Heading Change.

(a) Source: Author

(b) Source: Author

RESULTS AND DISCUSSION

The motion footprints for the maneuver run with the SM controller is shown in Figure 111. Observe that during the initial phase of the experiment, when the change in heading had not been applied, there is sharper footprint to the portside of the M510B (the load). This is the effect of the second order wave drag, pushing the load away from the alignment. The unactuated vessel is held from drifting away by the towing lines attached to the hold back vessels.

Figure 111 - Motion Footprint for Towing maneuver with SM controller.

Source: Author

It is also noticeable that there is a noticeable footprint around the holding vessel (M539), indicating that it has struggled to maintain positioning during the maneuver, as its commanded position was set to be kept constant, changing only the heading. Due to such oscillations, the turning radius of the Load vessel and the M510A is not held constant. The positioning and heading time series for this vessel is presented in Figure 112. Note that, as the change in the leader's heading is smooth, the tracking performance for this motion is almost perfect, and the changes in position of the M510A are also smooth. Observe also that there is, indeed, a deviation in the holding vessel's position for the position axes.

Figure 112 - Position and Heading Time Series for SM towing maneuver.

Source: Author

As the Northing and Easting formation setpoints vary in time, the Consensus error norm is calculated for the two-dimensional position as the horizontal distance between

the vessels' centers. This measurement is plotted in Figure 113 for the SM control maneuver. Note that the transient is of less than 200s, which indicates that the formation is being able to track the virtual leader. Note, though, that a small steady state error is held. The Formation Error Norm, shown in Figure 114, indicate that the steady error is not related to tracking, but appears in the formation setting. The peak formation error during the transient is of about $0.48m$ (around $20m$ in the 1:42 scale), with a steady state value of $0.05m$ (around $2.1m$ in real scale). Given the vessels' size and the set point distances, such values seem reasonable.

Figure 113 - Consensus Error Norm for towing maneuver using SM control.

Source: Author

Figure 114 - Formation Error Norm for towing maneuver using for SM control.

Source: Author

The source of the steady error can be verified through analysis of the sliding vectors (Figure 115). There is indeed an offset in both of the vessel's Easting sliding variable, which lays within the boundary layer in this steady condition. It is evident, then, that the accuracy is within the designed bounds for the sliding motion. The Northing sliding variable, whose measurements are perpendicular to the direction of the wave disturbances do not even cross the boundary layer during the transient. The Heading

variables stay on the limit of the layer through most of the maneuver, only settling after the virtual leader motion is settled.

Figure 115 - Sliding vector for towing maneuver using for SM control.

Source: Author

The result of the Second Order SM (ST) strategy is presented first through the motion footprints produced by such controller during the operation, in Figure 116. Compared to the SM controller, there are two noticeable behaviors, actually related. First, the positioning accuracy of the hold-back vessel (M539) seems higher, as it produces a smaller bounding footprint. This results in the smoother track observed from vessel M510A, that is closer to an expected circle section.

Figure 116 - Motion Footprint for Towing maneuver with ST controller.

Source: Author

The time series for this motion are shown next, in Figure 117. In this controller too there is a smooth tracking of the leader, noticeable not only in the heading series but also in the positioning axes. Just as the results observed in the SM controller, there is a disturbance in the position of the M539 during the motion transient. The Consensus Error Norm (Figure 118) shows, though, that the error amplitudes produced during the transient are less than half of the ones seen in the first order controller (about $0.45m$ at most, corresponding to a maximum error of $18.5m$ in the tracking). Notice also that there is no steady error (even though small oscillations remain at this state). The heading Error norm also reduces faster in this controller, although the peak transient values are the same.

Figure 117 - Position and Heading Time Series for SM towing maneuver.

Source: Author

Figure 118 - Consensus Error Norm for towing maneuver using ST control.

Source: Author

The Formation Error Performance (Figure 119) reaches about $0.18m$ during the transient, an equivalent of about $7.6m$ in real scale and only 4% of the set-point distance. In steady state, the error is of less the $0.05m$.

Figure 119 - Formation Error Norm for towing maneuver using ST control.

Source: Author

The sliding vector, shown in Figure 120 explains both the increase in accuracy observed in comparison to the SM controller and the steady state oscillations, which appear in the variables as a zero-mean limit cycle.

Figure 120 - Sliding vector for towing maneuver using for ST control.

Source: Author

DISCUSSION

In this case the tuning of the controllers seems more adequate than the parameterization used for the formation tests. The slow motion imposed by the leader also helped in the overall performance. As a result, accuracy of both controllers as kept within very tight values, with a slight advantage for the ST controller, observed once again due to its lack of the smoothing boundary layer. The total maneuver time is of about 800s (1 hour and 26 minutes in real scale), which is also reasonable given the magnitude of forces and distances involved in the task.

6. CONCLUSIONS

This work has presented a methodology for design and implementation of a reliable scheme for the cooperative control of Dynamically Positioned Vessels. The approach involves not only the choice and synthesis of adequate controllers for the task, but also a whole architecture involving filtering and estimation of the fleet positions.

A first level of redundancy, increasing the overall operational safety, is added by keeping each vessel's own DP system operational as the inner control loop. This permits each vessel to safely hold control in an event of total failure of the wireless network.

As opposed to most conventional cooperative control methods, which mostly rely on convergence for dynamically changing network topologies (at a cost of performance changes), the proposed system structure adds a redundancy layer, more adequate for the envisioned DP Vessel Operations, through the introduction of the Kalman Filter for neighbor state estimation. The simulated study case has shown that such structure does add some insensitivity in the cooperative dynamic behavior to failures in data transmission between agents. Note, though, that even in a scenario of prolonged connection failure between a set of agents in the network (rendering the dead-reckoning too inaccurate to be used), the cooperative task can be achieved, as long as a minimal topology that keeps the overall network connectivity is held. The cooperative tracking simulation has shown that the Linear Consensus Protocol implemented in the cooperative control loop maintains convergence in a scenario with modifications in topology. The dynamic properties of the formation setting task and the leader tracking task are, though, changed, as predicted by the linear consensus theory. The cooperative dynamics, in this case, are not only highly dependent on the network topology, but also on the agent's own dynamic behavior, which makes its application limited in tasks with heterogeneous agents.

As this is a likely condition for DP Vessel operations, measures for "agent anonymization", were proposed, making the inner loop dynamics transparent for the cooperative control protocols. To increase the dynamic performance robustness of the cooperative protocol block, two nonlinear robust consensus protocols, based on first and second order sliding modes theory were developed. Theoretical results have shown that, under such controllers and adequate definition of the sliding vector, the

cooperative system becomes insensitive to the network topology when laying over the Sliding Manifold. The simulated study provides evidence that this is indeed achieved.

Conditions for reaching the Sliding Manifold were formalized based on the modeling error (for the first order SMC) and their derivatives (for the Super Twisting) boundaries. As such limits applied to each agent's modeling error contribute for the total deviation of the dynamics of the sliding vector, this is the only phase of the motion that is affected by the network topology; in specific, it depends on the in-degree of each agent. A practical method for calculation of these boundaries in Dynamically Positioned Vessels was presented, generating conservative but applicable parametric values for the controllers. Additional study may permit broadening of such prerequisites and the development of better tuning methods, which may guarantee not only stability but also bound to designed requirements the performance of convergence to the sliding motion.

The proposed case studies have produced interesting results for actual application of these protocols in DP systems. When well-tuned, the controlled behaviors were in line with the theoretical results, showing a relative insensitivity not only to network alterations but also to environmental disturbances, a key requirement to DP systems. The performances were evaluated for formation settling characteristics and leader tracking. Both controllers have shown superior robustness characteristics when compared to the Linear controller. The Super-Twisting controller was also shown to be more accurate than the First Order Sliding modes under heavy disturbances, due to the lack of the smoothing boundary layer in its algorithm. It is prone, though, to integration windup and care must be taken not to reach such situation.

The towing maneuver case has produced an interesting illustration of the application of cooperative control methods to an actual operation, producing reasonable accuracy and timing figures. The results set-up a possibility of a broader study, introducing critical environmental and operational scenarios observed in actual offloading operations, for instance. It also opens the possibility of automated decision on the formation heading based on weathervaning methods.

The superior performance of the proposed controllers has, of course, a cost. Despite not being the main focus of this work, energy usage under these methods may be exceedingly high, depending on how tight the tuning is parametrized. Sliding controllers tend to produce high fluctuations in actuation due to chattering, even when

smoothing techniques and high order methods are applied. While searching for balance between performance and actuation in practical tuning during the simulation and the experimental tests, some general lessons were learned. First, lots of care must be taken to respect the actual dynamics of the closed loop DP vessels. This also involves defining adequate input and input rate limits, which will permit avoidance of actuator saturation conditions. Defining both an appropriate Virtual Leader dynamic behavior and a compatible sliding vector will also contribute to avoid saturation during tracking.

Another point of attention is on how the Extended Kalman Filter affects the overall behavior. When the estimated force drift (DP current) is used to compensate steady state errors, the closed loop dynamics of the DP system changes. To obtain best results, an identification procedure must be performed to accurately define its linear model representation.

The consensus fusion filter provided interesting results the case of the oceanic current estimations. The filtered outputs are smooth and accurate, being found to be rather insensitive to dynamic situations. Performance is promising but testing with actual feedforward usage of this information was not performed, as this is still a developing topic, requiring further investigations on how other drift forces, such as second order wave effects and unmatched wind disturbances affect these measurements.

In a practical usage, a network of floating robots with hulls designed specifically for this task could be devised. An adequately designed hull could reduce effects from wind and wave drag, permitting the direct application of the method without the need of other variables.

6.1. Future topics

While the objectives of this work have been achieved, some investigations remain open to research.

A deeper study on the definitions of parameters for the controllers may be performed, focusing either on a structured method for definitions of modeling error boundaries or even in the development of auto-tuning and adaptive laws for such controllers, based on methods proposed in (DAVILA; MORENO; FRIDMAN, 2010) and in (BARTH et al., 2015). In another approach, optimality of parameterization may be studied, such as in

(ZHANG; REGER, 2018). As thruster usage is always a large restriction in DP systems, further studies should also include a deeper analysis on energy optimization in these methods.

Effects of failure in the low-level DP controller of one or more agents were not discussed within the scope of this text and may be further investigated. By acknowledging that a single-agent's failure (and consequent drift) may cause drifting of the whole formation, means to detect and correct the formation behavior while keeping safe distances from the failing agent should be addressed.

The consensus fusion filter provided interesting results and remains to be tested with other indirect measurements, such as sea state. Online calculation of these variables is a task being currently explored by academy (see, e.g., (BRODTKORB; NIELSEN; SØRENSEN, 2018)) and the proposed method seems suitable for this task.

Finally, a broad range of applications studies involving multiple DP vessels is left open. The case for cooperative towing is of particular interest and could be tested in a much more dynamic situation, like towage in restricted waters. For the operations studied, cooperative weathervaning with the generation of optimal formation heading solutions for a given condition could be applied using, for instance, the consensus fusion filter outputs.

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APPENDIX A – DERIVATION OF RIGID BODY MOTION EQUATIONS

It is straightforward to derive the rigid body equations of motion for a 3 degree of freedom vessel. As there are no motion constraints, the application of Newton's laws to the center of mass (COM) of the vessel described in the GRF, given by $(\bar{x}, \bar{y})$ provides vector equation (233):

$$\begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & \bar{I}_z \end{bmatrix} \begin{bmatrix} \ddot{\bar{x}} \\ \ddot{\bar{y}} \\ \ddot{\bar{\psi}} \end{bmatrix} = \begin{bmatrix} \bar{F}_x \\ \bar{F}_y \\ \bar{M}_z \end{bmatrix} \Leftrightarrow \bar{M} \ddot{\bar{\eta}} = \bar{F}^G \quad (233)$$

The vector $\bar{F}^G = [\bar{F}_x \ \bar{F}_y \ \bar{M}_z]^T$ gives the generalized forces (in global reference frame) applied to the vessel's COM. The variables m and $\bar{I}_z$ are the mass and the moment of inertia the axis passing through the vessel's COM and parallel to the z_L axis, respectively, and form the COM mass matrix, $\bar{M}$. The vector $\ddot{\bar{\eta}} = [\ddot{\bar{x}} \ \ddot{\bar{y}} \ \ddot{\bar{\psi}}]^T$ gives the acceleration of the vessel's COM in the GRF.

By defining the vessel's center of mass velocity vector represented in the LRF as $\bar{\mathbf{v}} = [\bar{u} \ \bar{v} \ \bar{r}]^T$, the application of equation (1) gives the relation between the velocities representations in both reference frames, as in equations (234) and (235).

$$\bar{\mathbf{v}} = R_G^L \dot{\bar{\eta}} \quad (234)$$

$$\dot{\bar{\eta}} = R_L^G \bar{\mathbf{v}} \quad (235)$$

The vector $\dot{\bar{\eta}} = [\dot{\bar{x}} \ \dot{\bar{y}} \ \dot{\bar{\psi}}]^T$ is the vessel's center of mass velocity represented in the GRF.

By taking the time derivative of equation (234), the transformation from equation (5), and using equation (233), the equation of the vessel center of mass motion, calculated in the Local Reference Frame yields equation (236).

$$\dot{\bar{\mathbf{v}}} = S_L \bar{\mathbf{v}} + \bar{M}^{-1} \bar{F}^L \quad (236)$$

Note that the generalized velocity vector was already transformed to its LRF counterpart, $\bar{F}^L = [\bar{F}_u \ \bar{F}_v \ \bar{F}_r]^T = R_G^L \bar{F}^G$.

The above equation describes the vessel center of mass motion in the LRF. It is more convenient to use as its state variables the positions, velocities and accelerations of the vessel's midship position as opposed to its center of mass, as the vessel's COM changes according to its loading condition. Supposing the vessel is symmetric with

respect to the $x_L z_L$ plane, the following relations between the kinematic variables in the COM and in the vessel's midship can be applied:

$$\begin{aligned}
 \begin{bmatrix} \bar{x} \\ \bar{y} \\ \bar{\psi} \end{bmatrix} &= \begin{bmatrix} x \\ y \\ \psi \end{bmatrix} + \begin{bmatrix} x_G \\ 0 \\ 0 \end{bmatrix} \\
 \begin{bmatrix} \bar{u} \\ \bar{v} \\ \bar{r} \end{bmatrix} &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & x_G \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u \\ v \\ r \end{bmatrix} \Leftrightarrow \begin{bmatrix} \dot{\bar{u}} \\ \dot{\bar{v}} \\ \dot{\bar{r}} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & x_G \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{r} \end{bmatrix} \\
 \begin{bmatrix} \bar{F}_u \\ \bar{F}_v \\ \bar{F}_L \end{bmatrix} &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -x_G & 1 \end{bmatrix} \begin{bmatrix} F_u \\ F_v \\ F_L \end{bmatrix} \\
 \bar{I}_z &= I_z - mx_G^2
 \end{aligned} \tag{237}$$

The new vectors, $\boldsymbol{\eta} = [x \ y \ \psi]^T$, $\boldsymbol{v} = [u \ v \ r]^T$, $\dot{\boldsymbol{v}} = [\dot{u} \ \dot{v} \ \dot{r}]^T$ and $\boldsymbol{F}^L = [F_u \ F_v \ F_r]^T$ are the midship kinematic vectors for their COM counterparts. Parameter I_z is the moment of inertia around z_L , translated through the Steiner theorem, and x_G is the location of the center of mass with respect to the vessel's midship position described in the LRF.

The application of the relations described in (237) to the vessel's COM motion equation (236) provides the equation of motion for the vessel's midship.

$$\begin{bmatrix} m & 0 & 0 \\ 0 & m & mx_G \\ 0 & mx_G & I_z \end{bmatrix} \begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{r} \end{bmatrix} - \begin{bmatrix} 0 & mr & mx_G r \\ -mr & 0 & 0 \\ -mx_G r & 0 & 0 \end{bmatrix} \begin{bmatrix} u \\ v \\ r \end{bmatrix} = \begin{bmatrix} F_u \\ F_v \\ F_r \end{bmatrix} \tag{238}$$

APPENDIX B – LIST OF WIND SPECTRA

Some of the most used wind spectrums can be found in (FEIKEMA; WICHERS, 1991), in which the formulations of Harris (equation (239)), Davenport (equation (240)), Ochi and Shin (equation (241)) and Willis (equation (242)) are presented.

$$S_W(\omega_w) = \frac{7200}{2\pi} C_w \bar{V}_w \left[1 + \left(\frac{286\omega_w}{\bar{V}_w} \right)^2 \right]^{-\frac{5}{6}} \quad (239)$$

$$S_W(\omega_w) = \frac{916700}{2\pi} C_w \omega \left[1 + \left(\frac{191\omega_w}{\bar{V}_w} \right)^2 \right]^{-\frac{4}{3}} \quad (240)$$

$$S_W(\omega_w) = \frac{(750 + 69V_w)10^{-6}}{\omega_w} \bar{V}_w^2 F_g$$

$$F_g = \begin{cases} 583x, & 0 \leq x \leq 0.003 \\ \frac{420x^{0.70}}{(1 + x^{0.35})^{11.5}}, & 0.003 \leq x \leq 0.0999 \\ \frac{838x}{(1 + x^{0.35})^{11.5}}, & x \geq 0.0999 \end{cases} \quad (241)$$

$$x = \frac{10\omega_w}{2\pi\bar{V}_w}$$

$$S_W(\omega_w) = \frac{3672}{2\pi} C_w \bar{V}_w \left[x^{0.15} + \frac{9}{8}x \right]^{-\frac{5}{3}} \quad (242)$$

$$x = \frac{286.5\omega_w}{V_w}$$

Here, $S_W(\omega_w)$ is the spectrum as function of the frequency ω_w , C_w are roughness coefficients described by each spectrum used and $\bar{V}_w$ is the mean wind velocity. When applying Harris and Davenport Spectrums, C_w can be chosen equal to 0.002 for rough seas and 0.0015 for moderate seas. As for the Willis Spectrum, C_w is usually chosen equal to 0.003. Figure 121 presents a spectral density plot and time domain wind speeds for the four wind spectrums described at a mean velocity of 30 m/s. The Ochi and Shin spectrum is the only one that is based on sea measurements (DE KAT; WICHERS, 1991).

To calculate the wind velocities in time domain for equation (243), the discrete inverse Fourier transform is applied over a discrete frequency range $\omega_w = \{\omega_{w1}, \omega_{w2}, \dots, \omega_n\}$:

$$V_w = \bar{V}_w + \sum_{j=1}^n \sqrt{2S_W(\omega_{wj})\Delta\omega_w} \cos(\omega_{wj}t + \phi_{wj}) \quad (243)$$

Figure 121 - Wind spectrum example for 30 m/s mean speed in frequency and time domain

Source: Author

APPENDIX C – LIST OF WAVE SPECTRA

There are several standard spectra described in literature, as results from empirical and semi-empirical studies carried through the 1980's and 1990's. Some of the most used ones will be described next, as collected in (FOSSEN, 2011).

The Pierson-Moskowitz (PM) Spectrum (equation (244)) was developed by Pierson and Moskowitz in 1963 based on wind-generated sea measurements in the North Atlantic Ocean. Note that it is based only in one parameter, the significant height, and is better used if representing fully developed seas.

$$S_{\zeta}(\omega_w) = 0.0081g^2\omega_w^{-5}e^{\left\{-\frac{3.11}{H_s^2}\omega_w^{-4}\right\}} \quad (244)$$

In which $g = 9.81 \text{ m/s}^2$ is the gravity constant.

In order to extend the PM spectrum, the ISSC (*International Ship Structures Congress*) recommended in 1967 the adoption of a two-parameter spectrum (equation (245)), by using the significant height H_s and the central period T_1 .

$$S_{\zeta}(\omega_w) = \frac{173H_s^2}{T_1^4}\omega_w^{-5}e^{\left\{-\frac{692}{T_1^4}\omega_w^{-4}\right\}} \quad (245)$$

In this formulation, $k_1 = 1.296$ and $k_2 = 1.407$. This spectrum is known as the Bretschneider and is one of the most used forms of the PM spectrum.

Between 1968 and 1969 an extensive measuring program took place in the North Sea, near the German coast. The monitoring program was called *Joint North Sea Wave Project* (JONSWAP) and its resultant spectrum is the most used to model local, fetch limited seas:

$$S_{\zeta}(\omega_w) = \frac{320H_1^2}{T_p^4}\omega_w^{-5}e^{\left\{-\frac{1950}{T_p^4}\omega_w^{-4}\right\}} 3.3^Y \quad (246)$$

$$Y = e^{\left\{-\left[\frac{\left(\frac{\omega_w}{\omega_p}-1\right)}{\sigma\sqrt{2}}\right]^2\right\}}$$

$$\sigma = \begin{cases} 0.07 & \text{if } \omega_w < \omega_p \\ 0.09 & \text{if } \omega_w > \omega_p \end{cases}$$

In the JONSWAP spectrum, the moment constants k_1 and k_2 are respectively 1.296 and 1.407.

Figure 122 - Comparison between the most used spectra with $T_p = 6s$, $8s$ and $10s$. The significant wave height used is $H_s = 1.5m$.

Source: Author

Figure 122 presents a comparison between the three standard spectra described. The first thing to notice is that the JONSWAP spectrum produces higher peaks, which means that the mean wave inclination is higher for this representation (as expected in a developing sea).

APPENDIX D – MODEL PARAMETERIZATION

D.1. Simulation Models

D.1.1. PSV Model

The subject vessel for the simulation tests has parametric data taken from the *Maersk Handler* (Figure 123) and given in Table 35. Propellers' power, thrust, type and positions related to the midship section are given in Table 36. Current drag curves are shown in Figure 124. Motion RAOs are shown in Figure 125.

Figure 123 - Maersk Handler

Source: (SHIPBUCKET, 2017)

Table 35 - Maersk Handler Data

Parameter	Value
m (kg)	7.24000×10^6
I_z (kg.m ²)	2.75000×10^9
$X_{\dot{u}}$ (kg)	6.40970×10^5
$Y_{\dot{v}}$ (kg)	6.40380×10^6
$N_{\dot{r}}$ (kg.m ²)	1.56150×10^9
$N_{\dot{v}} = Y_{\dot{r}}$ (kg.m)	7.97240×10^6
x_G	0.0
LOA (m)	80.0
LPP (m)	69.3
BEAM (m)	18.0
Draft (m)	6.6

Source: Author

Table 36 - Maersk Handler Propeller Data

Propeller	Power (kW)	Max. Thrust (kN)	Pos. X (m)	Pos. Y (m)	KT
Tunnel Bow	883	117.6	31.2	0.0	0.0236
Azimuth Bow	883	147.0	24.7	0.0	0.0326
Tunnel Stern	883	117.6	-29.3	0.0	0.0236
Main 1	6440	965.3	-34.7	-5.2	0.0345
Main 2	6440	965.3	-34.7	5.2	0.0345

Source: Author

Table 37 – Maersk Handler Control Saturation Values

DOF	Max
Surge Force F_u (kN)	1930.6
Sway Force F_v (kN)	382.2
Yaw Moment F_r (kN.m)	20788.98

Source: Author

Figure 124 - Maersk Handler Current Drag Coefficients

Source: Author

Figure 125 - Maersk Handler Motion RAOs

Source: Author

D.2. Experimental models

D.2.1. 1:42 PSV Model

Two of the subject vessels for the scale model tests are 1:42 scale models (namely M510A and M510B) of a typical Platform Supply Vessel (*Figure 127*) with parametric properties based on the Maersk Handler and indicated in *Table 38*. The models were weighted and measured, with moment of inertia about the vertical axis taken from a bifilar pendulum test and ballast position distribution. Added masses and RAOs were obtained through a hydrodynamic simulation software (Wamit®). All the damping was considered from the relative current drag, which translates to a zero linear damping matrix. Cross added mass $Y_{\dot{r}}$ and the position of center of gravity x_{CG} was considered negligible.

The vessels are equipped with two main thrusters, two tunnel thrusters (one at the bow and one at the stern), and a bow azimuth thruster fixed and pointing to the port side. Dimensional properties and thrust capacities are given in *Table 39*. Parametric constant K_t was obtained via static bollard pull test. For control purposes, saturation for each degree of freedom is presented in *Table 40*.

Figure 126 – 1:42 PSV Model.

Source: Author

Figure 127 – 1:42 PSV Model.

Source: Author

Table 38 – 1:42 PSV Model Dimensional Properties.

Parameter	Value
<i>Scale</i>	1: 42
<i>LOA (m)</i>	1.90
<i>LPP (m)</i>	1.65
<i>BEAM (m)</i>	0.43
<i>Draft (m)</i>	0.115
x_G	0.0
$m \text{ (kg)}$	55.25
$I_z \text{ (kg.m}^2 \text{)}$	9.34
$X_{\dot{u}} \text{ (kg)}$	3.58
$Y_{\dot{v}} \text{ (kg)}$	35.41
$N_{\dot{r}} \text{ (kg.m}^2 \text{)}$	5.78

Source: Author

Table 39 - 1:42 PSV Thrusters characterization.

Propeller	Max. Thrust (N)	Min. Thrust (N)	Pos. X(m)	Pos. Y(m)	KT(+)	KT(-)
Tunnel Bow	1.7	-1.6	0.7	0.0	0.23	-0.20
Azimuth Bow	2.2	-1.6	0.6	0.0	0.32	-0.26
Tunnel Stern	1.7	-1.6	-0.7	0.0	0.20	-0.27
Main 1	8.5	-5.0	-0.75	0.15	0.5	-0.28
Main 2	8.5	-5.0	-0.75	-0.15	0.42	-0.30

Source: Author

Table 40 - 1:42 PSV Actuator Saturation Values

DOF	Max. Thrust (N)	Min. Thrust (N)
Surge	19.2	-12.2
Sway	3.4	-3.8
Yaw	4.335	-4.695

Source: Author

D.2.2. 1:70 PSV Model

The third subject vessel for the scale model tests is a 1:70 scale model (namely M539) of a typical Platform Supply Vessel (*Figure 127*) with parametric properties indicated in *Table 38*. The models were weighted and measured, with inertia moment about the vertical axis taken from a bifilar pendulum test and ballast position distribution. Added masses and RAOs were obtained through a hydrodynamic simulation software (Wamit®). All the damping was considered from the relative current drag, which translates to a zero linear damping matrix. Cross added mass Y_r and the position of center of gravity x_{CG} was considered negligible.

The vessels are equipped with two main azimuth thrusters and one tunnel thruster at the bow. Dimensional properties and thrust capacities are given in *Table 39*. Parametric constant K_t was obtained via static bollard pull test. For control purposes, saturation for each degree of freedom is presented in *Table 40*.

Figure 128 – 1:70 PSV Model.

Source: Author

Table 41 – 1:70 PSV Model Dimensional Properties.

Parameter	Value
<i>Scale</i>	1: 42
<i>LOA (m)</i>	1.24
<i>BEAM (m)</i>	0.3450
<i>Draft (m)</i>	0.0820
x_G	0.0
m (kg)	22.15
I_z (kg.m ²)	2.4
$X_{\dot{u}}$ (kg)	1.61
$Y_{\dot{v}}$ (kg)	20.7
$N_{\dot{r}}$ (kg.m ²)	1.44

Source: Author

Table 42 - 1:70 PSV Thrusters characterization.

Propeller	Max. Thrust (N)	Min. Thrust (N)	Pos. X(m)	Pos. Y(m)	KT(+)	KT(-)
Tunnel Bow	1.5	-1.5	0.4245	0.0	0.25	-0.22
Main Azim. 1	1.5	-1.5	-0.528	-0.0675	0.33	-0.19
Main Azim. 2	1.5	-1.5	-0.528	0.0675	0.33	-0.17

Source: Author

Table 43 - 1:70 PSV Actuator Saturation Values

DOF	Max. Thrust (N)	Min. Thrust (N)
Surge	3.0	-3.0
Sway	3.25	-3.25
Yaw	1.665	-1.665

Source: Author

APPENDIX E – CONTROLLER AND FILTER MATLAB CODES

```

classdef MyController < handle
% Generic Controller Parent Class
properties
    h
    integralValue = [0 0 0]';
    saturated = [false false false];
    resetOnSPChange
end

methods
function u = saturateController(u,controller,vessel)
    % Saturation
    overMax = u>vessel.Famax';
    underMin = u<vessel.Famin';
    controller.saturated = (overMax|underMin)';
    u(overMax) = vessel.Famax(overMax);
    u(underMin) = vessel.Famin(underMin);
end;

function controller = resetIntegralValue(controller)
    controller.integralValue = [0 0 0]';
end;
end
end

classdef MyCoopController < handle
% Generic Cooperative controller parent class
properties
    h
    nAgents;
    maxOut = [150 150 pi];
    minOut = -[150 150 pi];
    resetOnSPChange = false;
    saturated;
    A;
    D;
    L;
    deltaX;
    deltaY;
    deltaPsi;
end
methods
function u = saturateController(controller,u)
    % Saturation
    overMax = u>controller.maxOut';
    underMin = u<controller.minOut';
    controller.saturated = (overMax|underMin)';
    u(overMax) = controller.maxOut(overMax);
    u(underMin) = controller.minOut(underMin);
end;

function controller = updateMatrices(controller,Adj)
    controller.A = Adj;
    controller.D = zeros(controller.nAgents);
    for k = 1:controller.nAgents;
        controller.D(k,k) = sum(controller.A(k,:));
    end;
    controller.L = controller.D-controller.A;
end;
end
end

```

```

classdef pidController < MyController
% PID Controller implementation
properties
    Kp
    Ki
    Kd
end
methods
function controller = pidController(controllerFile)
    if nargin == 0
        [file,path] = uigetfile('*.mat','Select a Controller File');
        controllerFile = [path,file];
    end;
    data = load(controllerFile);
    controller = updateProperties(controller,data);
end;

function controller = updateProperties(controller,data)
    controller.h = data.h;
    controller.Kp = data.Kp;
    controller.Kd = data.Kd;
    controller.Ki = data.Ki;
    controller.integralValue = [0 0 0]';
    controller.saturated = [false false false];
    controller.resetOnSPChange = true;
end;

function u = controlOutput(controller,x,xd,vessel)
% Error Definition
error = rgl(x(3))*(xd(1:3,1)-x(1:3));
error(3) = wrapToPi(error(3));
% Integration & Anti-Windup
controller.integralValue = controller.integralValue + controller.h*diag(controller.saturated)*error;
% PID Calculation
P = controller.Kp*error;
D = -controller.Kd*x(4:6);
I = diag(controller.saturated)*controller.Ki*controller.integralValue;
u = P+I+D;
u = saturateController(u,controller,vessel);
end;
end
end

```

```

classdef EKF < MyObserver
%Kalman Filter implementation
properties
    Q
    R
    G
    H
    Pest
    GQG
    P_
    x_
    numOfStates
end

methods
function observer = EKF(observerFile)
    if nargin == 0
        [file,path] = uigetfile('*.mat','Select an Observer File');
        observerFile = [path,file];
    end;
    data = load(observerFile);
    observer = updateProperties(observer,data);
end;

```

```

function observer = updateProperties(observer,data)
    observer.h = data.h;
    observer.Aw = data.Aw;
    observer.Ac = data.Ac;
    observer.numOfStates = data.numOfStates;
    observer.Q = data.Q;
    observer.R = data.R;
    observer.G = data.G;
    observer.H = data.H;
    observer.GQG = data.G*data.Q*data.G';
    observer.Pest = zeros(data.numOfStates,data.numOfStates);
end;

function observer = setState(observer,value)
    observer.xest = [value;zeros(observer.numOfStates-length(value),1)];
end

function xest = observerOutput(observer,y,input,vessel)
    % Kalman Error Propagation
    observer = kalmanPropagation(observer,input,vessel);
    % Kalman Update
    observer = kalmanUpdate(observer,y);
    xest = observer.xest;
end;

function observer = kalmanPropagation(observer,input,vessel)
    h = 0.1;
    observer.x_ = observer.xest;
    observer.P_ = observer.Pest;
    for i = 0:h:observer.h
        df = observerModel(observer,input,vessel);
        observer.x_ = observer.x_+h*df;
        F = odeJacobian(observer,vessel);
        observer.P_ = observer.P_ + ...
            h*(F*observer.P_ + observer.P_*F' + observer.GQG);
    end;
end;

function J = odeJacobian(observer,vessel)
    x = observer.x_;
    xG = 0;
    Rlg = rlg(x(3));
    Rgl = Rlg';
    sp = Rgl(2,2);
    cp = Rlg(1,1);

    m1 = vessel.m+vessel.mxx;
    m2 = vessel.m+vessel.myy;
    m4 = vessel.m*xG+vessel.myp;
    m5 = vessel.myy-vessel.mxx;

    J1 = [0 0 -x(4)*sp-x(5)*cp;
          0 0 x(4)*cp-x(5)*sp;
          0 0 0];

    J2 = -vessel.Minv*[0 -m2*x(6) -m2*x(5)-2*m4*x(6);
                      m1*x(6) 0 m1*x(4);
                      m4*x(6)+m5*x(5) m5*x(4) m4*x(4)];

    Js = [J1 Rlg;
          zeros(3,3) J2];

    Abias = [zeros(3,3);
             Rgl];

    J = [Js, zeros(6,6),Abias;
         zeros(6,6), observer.Aw,zeros(6,3);
         zeros(3,6),zeros(3,6),observer.Ac;
         ];
end

```

```

function dx = observerModel(observer,u,vessel)
    x = observer.x_;
    % x: x(1) = x
    % x: x(2) = y
    % x: x(3) = psi
    % x: x(4) = u
    % x: x(5) = v
    % x: x(6) = r
    % x: x(7) = ksi1
    % x: x(8) = ksi2
    % x: x(9) = ksi3
    % x: x(10) = ksi4
    % x: x(11) = ksi5
    % x: x(12) = ksi6
    % x: x(13) = b1
    % x: x(14) = b2
    % x: x(15) = b3
    % b1|b2|b3 : Bias Forces
    % u: u(1) = Fu
    % u: u(2) = Fv
    % u: u(3) = Fr

    Rlg = rlg(x(3));
    Rgl = Rlg';
    % Coriolis
    Ccor = [0, -vessel.m*x(6), -vessel.myy*x(5)-vessel.myp*x(6);
            vessel.m*x(6), 0, vessel.mxx*x(4);
            vessel.myy*x(5)+vessel.myp*x(6), -vessel.mxx*x(4), 0;
            ];

    A = [zeros(3,3) rlg(x(3));
         zeros(3,3) -vessel.Minv*(Ccor+vessel.D);
         ];
    B = [zeros(3,3);
         vessel.Minv;
         ];

    Abias = [zeros(3,3);
             Rgl];

    Ao = [A, zeros(6,6), Abias;
          zeros(6,6), observer.Aw, zeros(6,3);
          zeros(3,6), zeros(3,6),observer.Ac;
          ];
    Bo = [B;
          zeros(6,3);
          zeros(3,3)];

    dx = Ao*x + Bo*u;
end

function observer = kalmanUpdate(observer,y)
    % Kalman Gain
    S = observer.H*observer.P_ *observer.H'+observer.R;
    Kkal = (observer.P_ *observer.H')/S;
    % Mean Value Update
    innovation = (y-observer.H*observer.x_);
    innovation(3) = wrapToPi(innovation(3));
    observer.xest = observer.x_ +Kkal*innovation;
    % Error Covariance Estimation Update
    observer.Pest = observer.P_ -Kkal*observer.H*observer.P_;
end
end
end

```

```

classdef linCoopController < MyCoopController
% Linear Cooperative Controller implementation
properties
    c
    K
end

methods
function controller = linCoopController(controllerFile)
    if nargin == 0
        [file,path] = uigetfile('*.mat','Select a Controller File');
        controllerFile = [path,file];
    end;
    data = load(controllerFile);
    controller = updateProperties(controller,data);
end;

function controller = updateProperties(controller,data)
    controller.h = data.h;
    controller.c = data.c;
    controller.K = data.K;
    controller.nAgents = size(data.A,1);
    controller.updateMatrices(data.A);
    controller.saturated = [false false false];
end;

function controller = setFormation(dX,dY,dPsi)
    controller.deltaX = dX;
    controller.deltaY = dY;
    controller.deltaPsi = dPsi;
end;

function u = controlOutput(controller,input,x,i)
    a = controller.A;
    veli = x(4:6,i);

    summation = [0 0 0 0 0 0]';
    for j = 1:length(a(i,:))
        delta = input(:,j);
        distance = x(1:3,j)-x(1:3,i) + delta;
        distance(3) = wrapToPi(distance(3));
        relVel = x(4:6,j)-veli;
        summation = summation + a(i,j)*[distance;relVel];
    end;
    summation(3) = wrapToPi(summation(3));
    u = controller.c*controller.K*summation + x(1:3,i);
end;
end
end

classdef smCoopController < MyCoopController
% Sliding Modes Cooperative Controller implementation
properties
    K
    Lambda
    Phi
    s
    u_
    utilde
    A0
    A1
    B0
    B1
end

methods
function controller = smCoopController(controllerFile)
    if nargin == 0
        [file,path] = uigetfile('*.mat','Select a Controller File');
        controllerFile = [path,file];
    end;
    data = load(controllerFile);
    controller = updateProperties(controller,data);
end;

```

```

function controller = updateProperties(controller,data)
    controller.h = data.h;
    controller.Phi = data.Phi;
    controller.K = data.Ksm;
    controller.Lambda = data.Lambda;
    controller.nAgents = size(data.A,1);
    controller.updateMatrices(data.A);
    controller.saturated = [false false false];
    controller.u_ = zeros(3,controller.nAgents);
    controller.utilde = zeros(3,controller.nAgents);
    controller.A0 = data.A0;
    controller.A1 = data.A1;
    controller.B0 = data.B0;
    controller.B1 = data.B1;
end;

function controller = setFormation(dX,dY,dPsi)
    controller.deltaX = dX;
    controller.deltaY = dY;
    controller.deltaPsi = dPsi;
end;

function controller = setReferences(controller,input)
    controller.utilde = input;
end

function u = controlOutput(controller,input,x,i)
    a = controller.A;
    d = controller.D(i,i);

    veli = x(4:6,i);
    sumvel = [0 0 0]';
    sumpos = [0 0 0]';
    sumu = [0 0 0]';
    u = [0 0 0]';
    for j = 1:length(a(i,:))
        if a(i,j)~=0
            delta = input(:,j);
            sumvel = sumvel + a(i,j)*(x(4:6,j));
            sumpos = sumpos + a(i,j)*(x(1:3,j) + delta);
        end;
    end;
    sumpos = d*x(1:3,i)-sumpos;
    sumpos(3) = wrapToPi(sumpos(3));
    sumvel = d*veli - sumvel;
    sigma = sumvel + controller.Lambda*sumpos;

    for j = 1:length(a(i,:))
        if a(i,j)~=0
            sumu = sumu + a(i,j)*(controller.utilde(:,j) + controller.Lambda*(x(4:6,j)));
        end;
    end;
    k0 = sigma./controller.Phi;
    satu = sat(k0,[1 1 1]');
    % Saturated switching gain
    Ksat = controller.K*satu;
    % Control output
    ui = - controller.Lambda*veli + (1/d)*(sumu - Ksat);
    controller.utilde(:,i) = ui;

    % PD Dynamic cancellation
    U = controller.A0*x(1:3,i) + controller.A1*veli + ui;
    u = controller.B1*U +controller.B0*controller.u_(:,i);
    controller.s = sigma;
    controller.u_(:,i) = u;
end;
end
end

```

```

classdef stCoopController < MyCoopController
properties
    K1
    K2
    Lambda
    s
    u_
    utilde
    w
    A0
    A1
    B0
    B1
end

methods
function controller = stCoopController(controllerFile)
    if nargin == 0
        [file,path] = uigetfile('*.mat','Select a Controller File');
        controllerFile = [path,file];
    end;
    data = load(controllerFile);
    controller = updateProperties(controller,data);
end;

function controller = updateProperties(controller,data)
    controller.h = data.h;
    controller.K1 = data.K1;
    controller.K2 = data.K2;
    controller.Lambda = data.Lambda;
    controller.nAgents = size(data.A,1);
    controller.updateMatrices(data.A);
    controller.saturated = [false false false];
    controller.u_ = zeros(3,controller.nAgents);
    controller.utilde = zeros(3,controller.nAgents);
    controller.w = zeros(3,controller.nAgents);
    controller.A0 = data.A0;
    controller.A1 = data.A1;
    controller.B0 = data.B0;
    controller.B1 = data.B1;
end;

function controller = setFormation(dX,dY,dPsi)
    controller.deltaX = dX;
    controller.deltaY = dY;
    controller.deltaPsi = dPsi;
end;

function controller = setReferences(controller,input)

    controller.utilde = input;
end

function u = controlOutput(controller,input,x,i)
    a = controller.A;
    d = controller.D(i,i)
    veli = x(4:6,i);
    sumvel = [0 0 0]';
    sumpos = [0 0 0]';
    sumu = [0 0 0]';
    u = [0 0 0]';
    for j = 1:length(a(i,:))
        if a(i,j)~=0
            delta = input(:,j);
            sumvel = sumvel + a(i,j)*(x(4:6,j));
            sumpos = sumpos + a(i,j)*(x(1:3,j) + delta);
        end;
    end;
    sumpos = d*x(1:3,i)-sumpos;
    sumpos(3) = wrapToPi(sumpos(3));
    sumvel = d*veli - sumvel;
    sigma = sumvel + controller.Lambda*sumpos;

```

```

for j = 1:length(a(i,:))
    if a(i,j)~=0
        sumu = sumu + a(i,j)*(controller.utilde(:,j) + controller.Lambda*(x(4:6,j)));
    end;
end;
% Saturated switching gain
controller.w(:,i) = controller.w(:,i) - controller.h*controller.K2*sign(sigma);
% Control Output
ui = -controller.Lambda*veli + (1/d)*(sumu - controller.K1*diag(sqrt(abs(sigma))))*sign(sigma) + controller.w(:,i);
controller.utilde(:,i) = ui;
% PD Dynamic cancellation
U = controller.A0*x(1:3,i) + controller.A1*veli + ui;
u = controller.B1*U + controller.B0*controller.u_(:,i);
controller.s = sigma;
controller.u_(:,i) = u;
end;
end
end

```