CELSO JACO FACCIO JÚNIOR

MODELING WIND TURBINE BLADES BY
GEOMETRICALLY-EXACT BEAM AND SHELL ELEMENTS: A
COMPARATIVE APPROACH

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“Everything should be made as simple as possible, but not simpler.”

Albert Einstein

Modelagem Estrutural de Pás de Turbinas Eólicas por Meio de Elementos de Viga e Casca: Uma Abordagem Comparativa

Resumo

A capacidade total de energia eólica instalada no mundo cresceu substancialmente nos últimos anos, principalmente devido ao número crescente de turbinas eólicas de eixo horizontal. Consequentemente, um grande esforço foi empregado com o intuito de aumentar a capacidade de produção das turbinas eólicas, que está diretamente associada ao tamanho das pás. Assim, surgiram projetos inovadores quanto à concepção de pás de turbinas eólicas levando a estruturas bastante flexíveis, susceptíveis a grandes deslocamentos, não apenas em eventos extremos, mas também em condições normais de operação. Nesse contexto, a presente dissertação tem por objetivo comparar duas abordagens de modelos estruturais geometricamente não-lineares capazes de lidar com grandes deslocamentos de pás de turbinas eólicas: elementos finitos geometricamente exatos 3D de vigas e cascas. Em relação ao modelo de viga, devido à complexidade geométrica das seções transversais típicas de pás de turbinas eólicas, adota-se uma teoria que permite a criação de seções transversais arbitrárias multicelulares. Duas geometrias de pás são testadas e comparações entre os modelos são feitas em análises estáticas e dinâmicas, sempre induzindo grandes deslocamentos e explorando os limites de precisão do modelo de viga, quando comparado ao modelo de cascas. Os resultados indicam que os modelos de viga e casca apresentam comportamento muito similar, exceto quando ocorrem violações em hipóteses do modelo de viga, tal como quando ocorre flambagem local do modelo de casca.

Palavras-chave: geometricamente exato; viga; casca; turbinas eólicas; pás de turbinas eólicas;

Abstract

The total wind power capacity installed in the world has substantially grown during the last few years, mainly due to the increasing number of horizontal axis wind turbines (HAWT). Consequently, a big effort was employed to increase HAWT's power capacity, which is directly associated to the size of blades. Then, novel designs of blades may lead to very flexible structures, susceptible to large deformation, not only during extreme events, but also for operational conditions. In this context, this thesis aims to compare two geometrically nonlinear structural modeling approaches that handle large deformation of blade structures: 3D geometrically-exact beam and shell finite element models. Regarding the beam model, due to geometric complexity of typical cross-sections of wind turbine blades it is adopted a theory that allows creation of arbitrary multicellular cross-sections. Two typical blade geometries are tested, and comparisons between the models are done in statics and dynamics, always inducing large deformation and exploring the accuracy limits of beam models, when compared to shells. Results showed that the beam and shell models present very similar behavior, except when violations occur on the beam formulation hypothesis, such as when shell local buckling phenomena takes place.

Keywords: geometrically-exact; beam; shell; HAWT; blades;

List of Figures

1.1	Global cumulative installed capacity from 2001 to 2016 extracted from [1].	14
1.2	Top ten new wind energy installed capacity in 2016 extracted from [1]. . .	15
1.3	Novel multi-rotor HAWT concept extracted from [2].	16
1.4	Blades from a 6 MW offshore wind turbine extracted from [3].	16
2.1	Cross-section notation adopted.	22
2.2	Kinematic hypothesis of the proposed beam model.	25
2.3	Kinematic description of the Mindlin-Reissner shell model adopted. . . .	30
3.1	Shear stresses distribution.	32
3.2	Thin-walled differential element with t thickness.	33
3.3	Thin-walled profile with a “cut”.	34
3.4	Multicellular closed thin-walled profile with multiple “cuts”.	37
5.1	<i>WindTurbine</i> main screen.	43
5.2	Sample file generated by the <i>WindTurbine</i> tool containing constitutive equation terms.	44
5.3	Airfoil cross-section file (*.asc) for the <i>WindTurbine</i> tool.	45
5.4	Material file (*.mip) for the <i>WindTurbine</i> tool.	46
5.5	Airfoil cross-section results at the <i>WindTurbine</i> tool.	46
6.1	S809 airfoil shape, extracted from [4].	48
6.2	S809 airfoil section including webs in green color.	49
6.3	Principal fibers coordinates positively oriented by an angle θ	50
6.4	Proposed blade geometries based on S809 airfoil.	51
6.5	Beam models discretization.	52
6.6	Shell models discretization.	53
7.1	Lift, drag and pitching moment coefficients for the S809 airfoil.	55
7.2	Equivalent realistic static loadings.	56
7.3	Static loadings proposed for nonlinear simulations.	57
7.4	L0 load case and G1 model y.	59
7.5	L1 load case and G1 geometry results.	60
7.6	L1 load case and G2 geometry results.	61
7.7	L2 load case and G1 geometry results.	62

7.8	Shell model local buckling evolution according to the end load percentage (G1L2).	62
7.9	L2 load case and G2 geometry results.	63
7.10	Shell model local buckling evolution according to the end load percentage (G2L2).	65
7.11	L3 load case and G1 geometry results.	66
7.12	Shell model local buckling evolution according to the end load percentage (G1L3).	66
7.13	Beam model in red and shell model in gray – deformed shape at 100% load (G1L3).	67
7.14	L3 load case and G2 geometry results.	67
7.15	Beam model in red and shell model in gray – deformed shape at 100% load (G2L3).	68
7.16	L4 load case and G1 geometry results.	69
7.18	L4 load case and G2 geometry results.	69
7.17	Shell model local buckling evolution according to the end load percentage (G1L4).	70
7.19	L5 load case and G1 geometry blade tip displacement results. Prescribed displacements at the root cross-section.	71
7.20	Structural response along time for the L5 load case and G1 geometry blade tip displacement results. Prescribed displacements at the root cross-section.	71
7.21	Shell model buckling at dynamic simulation (G1L5).	72
7.22	L5 load case and G2 geometry blade tip displacement results. Prescribed displacements at the root cross-section.	73
7.23	Structural response along time for the L5 load case and G2 geometry blade tip displacement results. Prescribed displacements at the root cross-section.	73
7.24	Shell model buckling of the dynamic simulation (G2L5).	74
A.1	Section 1 - box.	85
A.2	Section 2 - triangle.	88
A.3	Section 3 - arrow.	90
A.4	Section 4 - NREL S809.	93
A.5	Section 5 - NREL S805A.	96

A.6	Section 6 - NREL S807.	99
A.7	Section 7 - Generic.	101

List of Tables

1.1	Approximated scaling relations for wind turbines, adapted from [5].	17
6.1	Airfoil section absolute thickness of each S809 airfoil part.	49
6.2	Lamina information used on the calculation of the S809 equivalent material properties [6].	50
6.3	Equivalent properties for the S809 airfoil section.	51
6.4	Proposed blade geometries description.	52
7.1	Data adopted to calculate the resultant lift force.	55
7.2	Proposed static simulations and analysis for the realistic load.	56
7.3	Proposed static simulations and analysis.	57
7.4	Proposed dynamic simulations and analysis.	58
7.5	Natural periods and natural frequencies for the proposed blade geometries.	58
A.1	Cross-section definitions for comparative tests between the <i>WindTurbine</i> tool and ANSYS.	84
A.2	Section 1 geometric properties (continues).	86
A.3	Section 1 geometric properties.	87
A.4	Section 2 geometric properties.	89
A.5	Section 3 geometric properties (continues).	91
A.6	Section 3 geometric properties.	92
A.7	Section 4 geometric properties (continues).	94
A.8	Section 4 geometric properties.	95
A.9	Section 5 geometric properties (continues).	97
A.10	Section 5 geometric properties.	98
A.11	Section 6 geometric properties.	100
A.12	Section 7 geometric properties.	102

Contents

1	Introduction	14
1.1	Objective	19
1.2	Organization	20
2	Geometrically-Exact Beam and Shell Models	21
2.1	Geometrically-Exact Beam Model	21
2.1.1	Constitutive Matrix for General Beam Cross-Sections with Torsional Effects	22
2.2	Geometrically-Exact Shell Model	30
3	Shear Center and Shear Stresses on Closed Thin-Walled Beams	32
3.1	Single Cell Closed Thin-Walled Beams	34
3.2	Multicellular Closed Thin-Walled Beams	36
4	St. Venant Torsional Inertia on Closed Thin-Walled Beams	39
4.1	Single Cell Closed Thin-Walled Beams	39
4.2	Multicellular Closed Thin-Walled Beams	40
5	Computational Implementations Used For Simulations	43
5.1	WindTurbine: A computer aided design (CAD) tool for wind turbines . . .	43
5.1.1	Example	44
5.2	<i>Giraffe</i> : A nonlinear finite element solver	47
6	Wind Turbine Blade Model Description	48
6.1	Airfoil Section	48
6.2	Equivalent Material	49
6.3	Blade Span Geometry	51
6.4	Computational Models	52
7	Simulations and Discussions	54
7.1	Proposed Simulations	54
7.1.1	Realistic Simulation	54
7.1.2	Nonlinear Simulations	56

7.2	Results and Discussions	59
7.2.1	Load Case 0 – Realistic Load	59
7.2.2	Load Case 1 – Static Tension	59
7.2.3	Load Case 2 – Static Bending About Y Axis	61
7.2.4	Load Case 3 – Static Bending About X Axis	64
7.2.5	Load Case 4 – Static Torsion	68
7.2.6	Load Case 5 – Dynamic “8” Circuit	70
8	Conclusions	74
	References	76
	Appendices	83
A	<i>WindTurbine</i> Tool Verification	83

1 Introduction

The total wind power capacity installed in the world increased, only during 2016, approximately 12.6%, from 432.7 GW to 486.8 GW, as presented by the Global Wind Energy Council (GWEC) in Figure 1.1.

Figure 1.1: Global cumulative installed capacity from 2001 to 2016 extracted from [1].

Although 45.8% of the new wind energy installed capacity in 2016 has come from China, some countries, with emergent economies, such as India and Brazil, also presented significant share. Figure 1.2 presents the top ten new wind energy installed capacity in 2016. Furthermore, these numbers show that the wind energy presents itself as a very attractive resource and consequently as a world trend.

Even with new possibilities of novel conceptions for wind turbines, a very important and still most used model is the horizontal axis wind turbine (HAWT) [7]. Along history, the HAWT has significantly evolved from a power of 25 kW, with rotor diameter of 10 m, to 6 MW with rotor diameter of 126 m [5]. Most recently, there have been undergoing studies for a 10 MW offshore HAWT [8]. Figure 1.3 presents a novel HAWT concept developed by Vestas® [9] that uses multiple rotors assembled to a single tower. The superiority of the horizontal axis wind turbines when compared to other designs are mainly due to [10]:

- The blade pitching system that permits the control of the rotor speed and the power output.
- The possible optimization of wind turbine blades in order to achieve the maximum efficiency.
- The technological evolution of propeller design.

Figure 1.2: Top ten new wind energy installed capacity in 2016 extracted from [1].

Moreover, in HAWT 's the availability of power is directly related to the area covered by their blades and, therefore, related to the square of their length. Figure 1.4, for example, presents two 75 m long blades from a 6 MW offshore wind turbine developed by Alstom® [11]. The increasing size of blades length may not be an engineering straightforward process. Assumptions and models previously used for short blades may not be appropriate for the new proposals. It is possible, however, to use some approximated scale relations associated to the rotor diameter (R). As presented in [5], the scale relations may be summarized as in Table 1.1 considering the following assumptions:

- The tip speed ratio is constant.
- The number of blades, airfoil and blade materials are the same.
- Geometric similarity is maintained to the extent possible.

Figure 1.3: Novel multi-rotor HAWT concept extracted from [2].

In fact, slender and flexible wind turbine blades may experience high nonlinear structural behavior. Furthermore, aeroelastic phenomenon may play a role. Consequently, considerable research was developed in order to build realistic wind turbine blades models.

(a)

(b)

Figure 1.4: Blades from a 6 MW offshore wind turbine extracted from [3].

One may elect two main approaches to establish structural numerical models for analysis of wind turbine blades. The first one, as a high-hierarchy model, with high computational cost, is based on 3D shell finite elements. The second one, quite accurate and with low computational cost, is based on 3D beam models. Of course, one may also be interested in local structural details of such structures. For that, it is possible to compose a 3D model with solid finite elements, which is really computationally-heavy. Our interest in this work, however, is to model the wind turbine blade behavior as a whole. Thus, structural details, rather important as stress concentrators, are no more being referred in

present text.

Quantity	Scale dependence
Power, forces and moments	
Power	$\sim R^2$
Torque	$\sim R^3$
Thrust	$\sim R^2$
Rotational speed	$\sim R^{-1}$
Weight	$\sim R^3$
Aerodynamic moments	$\sim R^3$
Centrifugal forces	$\sim R^2$
Stresses	
Gravitational	$\sim R^1$
Aerodynamic moments	$\sim R^0$
Centrifugal	$\sim R^0$
Resonances	
Natural frequency	$\sim R^{-1}$
Excitation	$\sim R^0$

Table 1.1: Approximated scaling relations for wind turbines, adapted from [5].

Discretization of wind turbine blades using 3D composite shell elements may be very accurate, since the model may present thickness variation and anisotropic nonhomogeneous materials. However, high detailing of the structure discretization and simulation may take a reasonable amount of time due to all inherent model complexity. An alternative free open source tool to make 3D finite element models (FEM) meshes of wind turbine blades is the Numerical and Manufacturing Design Tool (NuMAD) developed by [12]. Moreover, the growth of computational power has been providing more developments with similar techniques. Some contributions regarding 3D shell FEM analysis on the context of wind turbines are briefly commented as follows. In [13], a coupling of a blade shell FEM model built on AbaqusTM [14] is presented, with the blade element momentum theory of the program WT_Perf [15]. In [16], the aerodynamic loads are investigated by coupling a Navier-Stokes computational fluid dynamics (CFD) solver with a FEM-based computational structural dynamics solver. In [17], a full wind turbine, in-

cluding nacelle and tower, is simulated with fully-coupled 3D fluid structure interaction. In [18], the bend-twist coupling effect is investigated regarding fatigue loads and flutter instability. In [19], the fluid structure interaction at a full-scale wind turbine blade is investigated using ANSYSTM [20]. In [21], a physics-based multi-scaled progressive damage model for predicting the durability of wind turbine blade structures is presented. In [22], a structural optimization model of wind turbine blades using finite element analysis and a genetic algorithm is developed.

Even with many contributions afore mentioned encompassing high-hierarchy models, the discretization of wind turbines into 3D beam-like structural models has proven to be computationally efficient. Indeed, the blade and the tower of wind turbines are usually slender structures. Once a beam-like model is adopted, three possible modeling ways may be taken, i. e., a modal approach, a multi-body dynamics (MBD) method and a FEM method [23].

The modal approach consists of modeling the wind turbine dynamics through a combination of previously known modal shapes. This method is computationally efficient, since it harshly reduces number of degrees of freedom (DOF's). However, the drawback is that it is limited to the available eigen-modes. Furthermore, if the structure presents nonlinear behavior, the advantageous simplicity of the method is no more kept. On MBD modeling, the structure is discretized into a simplified system composed by rigid and flexible parts, interconnected by forces, springs, dashpots and kinematic constraints. Finally, the classical FEM approach discretizes the wind turbine structure into segments or elements interconnected by nodes. Although the slightly higher computational cost, a 3D FEM wind turbine modeling, composed by beam elements, may incorporate linear or nonlinear formulations.

Concerning the beam theories adopted on wind turbine models, geometrically linear and nonlinear beam theories have been used to describe the structural behavior. Regarding wind turbine blades, classical beam theories were initially widely used to model them, particularly due to the larger stiffness of these structures when compared to helicopter blades, for example [24]. However, as commented by [24], nonlinear beam theories could become necessary if wind turbines become more flexible. Most recently, the use of geometric nonlinear beam theories on the context of slender and flexible wind turbine blades revealed not as a possibility, but as a reality and a trend [23, 25]. Some works regarding

beams nonlinear formulations on the context of wind turbines are briefly commented as follows. The HAWC2 (Horizontal Axis Wind Turbine Simulation Code 2nd generation) developed by the DTU (Technical University of Denmark) is a nonlinear aeroelastic model based on a MBD formulation and Timoshenko beam elements [26]. In [27], the authors develop a model called NAM_WTB (Nonlinear Aeroelastic Model for Wind Turbine Blades) that couples a geometrically exact beam theory formulation developed by [28] and the well-known blade element momentum (BEM) model for aerodynamic loads. In [29], the authors present a code called BeamDyn that implements the geometrically exact beam theory inside the modular framework FAST. In [30] a complete geometrically-exact beam HAWT model is built and analyzed at a dynamic simulation with the aid of the *Giraffe* [31] solver and the *WindTurbine* tool.

1.1 Objective

Among the mentioned high-hierarchy shell models and the quick-solving beam-like models, a natural question that needs to be answered is regarding the comparative modeling aspects of both approaches. Strong and weak characteristics of each method may be comparatively discussed, leading to clarify the engineering limitations of each model. In this context, the objective of this thesis is to compare the predicted physical behavior of a wind turbine flexible blade, when modeled using: (a) 3D geometrically-exact beam FEM model and (b) 3D geometrically-exact shell FEM model. The intention is to verify the accuracy range of the nonlinear beam formulation proposed, taking the shell model as a more complete and realistic reference. For that, realistic airfoil shapes (including webs), with realistic dimensions, are considered for the blades. Moreover, despite the common use of laminated composite materials on wind turbine blades, this thesis adopts an elastic linear equivalent homogeneous material properties since it focus on the comparison between the beam and shell models, concerning the nonlinear kinematics. A future work will include material nonlinearities in the models.

In order to improve the richness, discussions and applicability of the comparisons here proposed, two distinct blade geometries and two kinds of structural analysis are performed. Regarding the blade geometry, an initial analysis proposes a constant cross-section along the blade length, while a more elaborated model proposes a linear variation of the profile chord along the blade length. The structural analysis consists of static and

transient dynamics simulations and, in both cases, the structure experiences geometric nonlinear behavior. The thesis closes with a comparative discussion of all approaches concerning statics, dynamics, and influence of geometric nonlinear effects on blade structural behavior.

1.2 Organization

A brief description of each Section of this thesis is presented as follows. In Section 2, the geometrically-exact beam model formulation is introduced, with emphasis given to the constitutive matrix derivation, while the geometrically-exact shell model adopted is briefly presented. In Section 3 the process of obtaining the shear center for single and multicellular closed thin-walled cross-section is presented. In Section 4, a detailed description regarding the torsional inertia on single and multicellular closed thin-walled cross-sections is presented. In Section 5, the computational background used on the simulations is presented, including an original computer aided design (CAD) tool *WindTurbine* to evaluate the beam constitutive matrix, and the finite element solver (*Giraffe*). In Section 6, the proposed case studies are presented by comparing the two modeling approaches. In Section 7 discussions on comparative results of both models are presented. In Section 8 conclusion remarks and future works are presented.

2 Geometrically-Exact Beam and Shell Models

2.1 Geometrically-Exact Beam Model

Geometrically-exact beam structural models may be simply described as a strategy to decompose the beam deformation in two main effects: (a) a general rigid-body movement of the whole cross-section and (b) a local deformation of each cross-section (such as warping or shape changes). The effect (a) considers that the movement of each cross-section is not limited to small deflections or small rotations. With that, this will lead to a convenient technique to model parts moving and experiencing large displacements and finite rotations. The geometrically-exact beam theory may naturally describe the kinematics of mechanisms, such as helicopter blades [32], offshore risers [33, 34, 35], thin-walled beams [36, 37, 38] and wind turbines [27]. Recently, the geometrically-exact beams have been used together with joint constraints, providing a real possibility to simulate mechanisms, as an alternative to other MBD techniques (see e.g.: [39, 40, 41, 42]).

The initial developments regarding the geometrically-exact formulation adopted on this thesis were presented in [43], where the author shows a 3D beam formulation, based on the original plane formulation proposed by [44]. Both works may be seen as more general than Kirchhoff-Love's rod model [45], which does not consider shear deformation. It is remarkable to note that many researchers dedicated efforts in order to improve the geometrically-exact beam theory, by improving warping description [37] and providing a general-axis description of the theory, which is one of the basis of present work [46, 47].

Nevertheless, some important works related to theories on the context of the geometrically-exact beam theories formulations adopted on this thesis are briefly commented as follows. In [48] the author presents a full nonlinear 3D beam model that incorporates transverse shear and torsion-warping deformation. Furthermore, a class of reduced elastic constitutive equations is also presented. In [46] the rotation of 3D geometrically-exact beams is formulated using the Euler-Rodrigues formula, while in [49] the rotation is expressed using only Rodrigues parameters. In [50] the dynamic formulation of a geometrically-exact beam is used to simulate an offshore riser structures with hydrodynamic loads and with contact interaction with seabed.

2.1.1 Constitutive Matrix for General Beam Cross-Sections with Torsional Effects

The here developed constitutive matrix formulation is based on the procedure done by [48], where a class of reduced elastic constitutive equations is presented. Moreover, differently from the beam formulation from [48] – that has eight internal loads contributions, the herein adopted formulation presents six internal loads, such as done in [47].

A constitutive matrix, for a particular structural model is a mathematical entity that relates generalized stresses and generalized strains. It is desirable that the constitutive matrix be able to capture possible couplings between internal loads, due to arbitrary material distribution on cross-section. One important aspect, usually representing a concern for beam models, is the choice of the beam axis position, among numerous possibilities. According to such choice, distinct coupling terms will take place in the constitutive matrix, such as the internal loads interpretation may change. Our choice in present work is for a completely general axis position, not necessarily lying at the barycenter, shear center, or other particular point of the cross-section. This makes the derivation more elaborated, but afterwards creates a desirable generality to construct beam models with variable cross-sections along length, in situations where the beam axis does not follow, necessarily, the successive barycenter, shear center or other particular point of cross-sections. This scenario is very expected in the context of wind turbine blades.

Figure 2.1: Cross-section notation adopted.

Some geometric properties and identities are now defined, according to the notation presented in Figure 2.1. The cross-section area is

$$A = \int_A dA = \int_A dx_1 dx_2. \quad (2.1)$$

The first order moments of inertia are

$$S_1 = \int_A x_2 dA \text{ and} \quad (2.2)$$

$$S_2 = - \int_A x_1 dA. \quad (2.3)$$

The second order moments of inertia, product of inertia, and polar moment of inertia are respectively given by

$$I_1 = \int_A x_2^2 dA, \quad (2.4)$$

$$I_2 = \int_A x_1^2 dA, \quad (2.5)$$

$$I_{12} = - \int_A x_1 x_2 dA \text{ and} \quad (2.6)$$

$$I_0 = \int_A (x_1^2 + x_2^2) dA = I_1 + I_2. \quad (2.7)$$

Moreover, it is convenient to define the cross-section vector positions for points of interest, such as the centroid $\mathbf{g}$ and the shear center $\mathbf{s}$ (see Figure 2.1). The cross-section in the reference plane is defined by:

$$\mathbf{g} = g_1 \mathbf{e}_1 + g_2 \mathbf{e}_2 \text{ and} \quad (2.8)$$

$$\mathbf{s} = s_1 \mathbf{e}_1 + s_2 \mathbf{e}_2 \quad (2.9)$$

with

$$g_1 = -\frac{S_2}{A} \text{ and} \quad (2.10)$$

$$g_2 = \frac{S_1}{A}. \quad (2.11)$$

On the other hand, the process of obtaining the shear center coordinates that will rule the bend-twist coupling of the beam model may not be a straightforward process. The shear center coordinates with respect to a chosen point of interest (e.g.: the beam axis position in cross-section plane) may be expressed as

$$s_1 = \frac{1}{n_2} \int_A |\mathbf{a}^r \times \mathbf{q}| dA \text{ and} \quad (2.12)$$

$$s_2 = \frac{1}{n_1} \int_A |\mathbf{a}^r \times \mathbf{q}| dA \quad (2.13)$$

where n_1 and n_2 are parallel forces applied respectively in directions of local axis $\mathbf{e}_1$ and $\mathbf{e}_2$, $\mathbf{a}^r$ is a vector that reads each general cross-section point according to the point of

interest and $\mathbf{q}$ is the shear stress function associated to each cross-section material point. A detailed definition and discussion regarding the shear center of single and multicellular closed thin-walled beams is presented in Section 3.

Moreover, it is convenient to define first order moments of inertia with respect to the shear center with the aid of the St. Venant warping function ψ^S , also relative to the shear center, as

$$\int_A \psi_{,1}^S dA = \int_A (x_2 - s_2) dA = S_1^S = S_1 - As_2 \text{ and} \quad (2.14)$$

$$\int_A \psi_{,2}^S dA = - \int_A (x_1 - s_1) dA = S_2^S = S_2 + As_1 \quad (2.15)$$

where the superscript $(\cdot)^S$ denotes from now on calculations with respect to the shear center coordinates and the notation $(\cdot)_{,i}$ represents the derivative of a quantity with respect to coordinate x_i .

Other useful identities related to the shear center and the St. Venant warping function ψ^S as presented on [36] and [47] are

$$\int_A (x_1 \psi_{,2}^S - x_2 \psi_{,1}^S) dA = I_t^S - I_0 + A(g_1 s_1 + g_2 s_2), \quad (2.16)$$

$$\int_A (\psi_{,1}^S \psi_{,1}^S + \psi_{,2}^S \psi_{,2}^S) dA = I_0^S - I_t^S, \quad (2.17)$$

$$I_0^S = I_0 - 2A(g_1 s_1 + g_2 s_2) + A(s_1^2 + s_2^2) \text{ and} \quad (2.18)$$

$$I_t = I_t^S + A(s_1^2 + s_2^2) \quad (2.19)$$

where I_t is the torsional inertia, given by the relation between a pure torque T applied to the cross-section, a constant or homogenized shear modulus G and a rate of twist ϕ , such that

$$I_t = \frac{T}{G\phi}. \quad (2.20)$$

Moreover, a more detailed definition regarding the torsional inertia of single and multicellular closed thin-walled cross-sections is presented in Section 4.

In order to establish a constitutive matrix, one has to assume a displacement field (kinematic hypothesis) that describes the movement of each material point of the model. From this, one has to calculate the deformation gradient and the corresponding strain tensor according to the assumption regarding possible infinitesimal strains (as assumed here). Next, a material law, for example, the Hooke's law (elastic linear hypothesis) may be used to associate the stresses and strains.

Consider now that the position vector of a material point $\mathbf{x}$ of a given beam, as presented in Figure 2.2, may be expressed by

$$\mathbf{x} = \zeta \mathbf{e}_3^r + \mathbf{u} + \mathbf{Q} \mathbf{a}^r + \psi^S p \mathbf{e}_3^r \quad (2.21)$$

where $\mathbf{u}$ is the displacement of the beam axis, $\mathbf{Q}$ is the rotation tensor, describing the current orientation of cross-sections, $\mathbf{a}^r$ a vector that reads all points at the beam cross-section ($\mathbf{a}^r = x_1 \mathbf{e}_1^r + x_2 \mathbf{e}_2^r$) and the term $\psi^S p \mathbf{e}_3^r$ describes out of plane warping displacements of the beam cross-section.

Figure 2.2: Kinematic hypothesis of the proposed beam model.

Note that, at this kinematic hypothesis, the out of plane displacement is described, not only by the warping function ψ^S , but also by a new parameter p . This notation defines the out of plane displacement of a beam cross-section as a combination of a warping function ψ^S . In this context $\psi^S = \hat{\psi}(x_1, x_2)$ is the shape of the warping displacement field, and a scalar intensity factor p .

The rotation tensor may be written using Rodrigues rotation vector $\boldsymbol{\alpha} = [\alpha_1 \ \alpha_2 \ \alpha_3]^T$, where $\boldsymbol{\alpha} = \alpha \mathbf{e}$, with $\mathbf{e}$ representing the rotation axis direction experienced by each cross-section and $\alpha = 2 \tan(\theta/2)$. The scalar θ is the magnitude of the Euler rotation vector associated with the cross-section. Let one define the skew-symmetric tensor $\mathbf{A}$ by

$$\mathbf{A} = \begin{bmatrix} 0 & -\alpha_3 & \alpha_2 \\ \alpha_3 & 0 & -\alpha_1 \\ -\alpha_2 & \alpha_1 & 0 \end{bmatrix}. \quad (2.22)$$

Then, it is possible to write a linear approximation to the rotation tensor $\mathbf{Q}$, valid for small rotations, given by $\mathbf{Q} \approx \mathbf{I} + \mathbf{A}$, taking $\mathbf{I}$ as the identity matrix. With these definitions, Equation 2.21 may be rewritten as

$$\mathbf{x} = \zeta \mathbf{e}_3^r + \mathbf{u} + (\mathbf{I} + \mathbf{A})(x_1 \mathbf{e}_1^r + x_2 \mathbf{e}_2^r) + \psi^S p \mathbf{e}_3^r \quad (2.23)$$

and, by consequence, the deformation gradient $\mathbf{F}$ of $\mathbf{x}$ is given by

$$\mathbf{F} = \mathbf{x}_{,1} \otimes \mathbf{e}_1^r + \mathbf{x}_{,2} \otimes \mathbf{e}_2^r + \mathbf{x}' \otimes \mathbf{e}_3^r \quad (2.24)$$

where $(\cdot)'$ stands for partial derivatives with respect to ζ and $\otimes$ stands for tensor products. Moreover, using the Vlasov's hypothesis [51] that the warping displacement is proportional to the rotation per unit length of the beam axis ($p = \boldsymbol{\alpha}' \cdot \mathbf{e}_3^r = \alpha_3'$), it is possible to rewrite $\mathbf{F}$ as

$$\begin{aligned} \mathbf{F} = & \mathbf{I} + (\mathbf{A} \mathbf{e}_1^r + \psi_{,1}^S \alpha_3' \mathbf{e}_3^r) \otimes \mathbf{e}_1^r \\ & + (\mathbf{A} \mathbf{e}_2^r + \psi_{,2}^S \alpha_3' \mathbf{e}_3^r) \otimes \mathbf{e}_2^r + (\mathbf{u}' + \mathbf{A}' \mathbf{a}^r + \psi^S \alpha_3'' \mathbf{e}_3^r) \otimes \mathbf{e}_3^r \end{aligned} \quad (2.25)$$

From the deformation gradient $\mathbf{F}$, it is possible now to calculate a strain measure $\nabla \mathbf{S}$ by

$$\begin{aligned} \nabla \mathbf{S} = & \mathbf{F} - \mathbf{I} \\ \nabla \mathbf{S} = & (\mathbf{A} \mathbf{e}_1^r + \psi_{,1}^S \alpha_3' \mathbf{e}_3^r) \otimes \mathbf{e}_1^r \\ & + (\mathbf{A} \mathbf{e}_2^r + \psi_{,2}^S \alpha_3' \mathbf{e}_3^r) \otimes \mathbf{e}_2^r + (\mathbf{u}' + \mathbf{A}' \mathbf{a}^r + \psi^S \alpha_3'' \mathbf{e}_3^r) \otimes \mathbf{e}_3^r. \end{aligned} \quad (2.26)$$

Therefore, the strain measure $\nabla \mathbf{S}$ yields

$$\nabla \mathbf{S} = \begin{bmatrix} 0 & -\alpha_3' & u_1' - \alpha_3' x_2 \\ \alpha_3' & 0 & u_2' + \alpha_3' x_1 \\ -\alpha_2 + \psi_{,1}^S \alpha_3' & \alpha_1 + \psi_{,2}^S \alpha_3' & u_3' + \alpha_1' x_2 - \alpha_2' x_1 + \psi^S \alpha_3'' \end{bmatrix}. \quad (2.27)$$

Assuming that, for the constitutive matrix derivation purposes, only small strains take place, one may write the infinitesimal Green-Lagrange strain tensor $\mathbf{E}$, expressed as

$$\mathbf{E} = \frac{1}{2}(\nabla \mathbf{S} + \nabla \mathbf{S}^T) \quad (2.28)$$

and, by consequence:

$$\mathbf{E} = \begin{bmatrix} 0 & 0 & \frac{1}{2}(u_1' - \alpha_3' x_2 - \alpha_2 + \psi_{,1}^S \alpha_3') \\ 0 & 0 & \frac{1}{2}(u_2' + \alpha_3' x_1 + \alpha_1 + \psi_{,2}^S \alpha_3') \\ Sym. & u_3' + \alpha_1' x_2 - \alpha_2' x_1 + \psi^S \alpha_3'' \end{bmatrix}. \quad (2.29)$$

It is important to note that the current strain tensor $\mathbf{E}$ presents a degree of freedom associated to the term α_3'' . This term, that may be seen as a second order rotation with respect to the beam axis, is indeed associated to non-uniform torsion and consequently bishear and bimoment internal forces. Non-uniform torsion effects are especially relevant on beams with open thin-walled cross-sections that usually present low torsional inertia. Therefore, a reasonable hypothesis, in the context of wind turbine blades composed by multicellular closed sections, is to assume negligible non-uniform torsion effects.

From this point on, it is convenient to define two generalized strain vectors with important meanings in the context of the beam kinematics. Vector $\boldsymbol{\eta}$ is defined as

$$\boldsymbol{\eta} = \mathbf{u}' - \boldsymbol{\alpha} \times \mathbf{e}_3^r = \begin{bmatrix} u_1' - \alpha_2 \\ u_2' + \alpha_1 \\ u_3' \end{bmatrix} \quad (2.30)$$

and may be interpreted as a compact notation of the beam axial strain. Vector $\boldsymbol{\kappa}$ is defined as

$$\boldsymbol{\kappa} = \boldsymbol{\alpha}' = \begin{bmatrix} \alpha_1' \\ \alpha_2' \\ \alpha_3' \end{bmatrix} \quad (2.31)$$

and may be interpreted as the rotation derivatives, with respect to the beam axis directions, thus containing information on curvature and twist (also called specific rotation vector).

Moreover, the strain tensor $\mathbf{E}$ may be rewritten assuming negligible non-uniform torsion effects with the convenient strain notation of Equations 2.30 and 2.31 and technical notation as

$$\boldsymbol{\gamma} = \begin{bmatrix} \gamma_{13} \\ \gamma_{23} \\ \varepsilon_{33} \end{bmatrix} = \begin{bmatrix} \eta_1 - \kappa_3 x_2 + \psi_{,1} \kappa_3 \\ \eta_2 + \kappa_3 x_1 + \psi_{,2} \kappa_3 \\ \eta_3 + \kappa_1 x_2 - \kappa_2 x_1 \end{bmatrix}. \quad (2.32)$$

In addition, considering homogeneity of material distribution and the general Hooke's law, the stress-strain relation of the beam model is

$$\boldsymbol{\sigma} = \begin{bmatrix} \sigma_{13} \\ \sigma_{23} \\ \sigma_{33} \end{bmatrix} = \begin{bmatrix} G\gamma_{13} \\ G\gamma_{23} \\ E\varepsilon_{33} \end{bmatrix} \quad (2.33)$$

where G and E are the material shear and Young modulus, respectively. Equation 2.33 presents all stress components acting at any point of the beam cross-section with respect to the generalized strain vectors $\boldsymbol{\eta}$ and $\boldsymbol{\kappa}$. Therefore, the equivalent (homogenized) forces at each direction may be obtained by integration by

$$\mathbf{n} = \int_A \sigma_{13} dA \mathbf{e}_1^r + \int_A \sigma_{23} dA \mathbf{e}_2^r + \int_A \sigma_{33} dA \mathbf{e}_3^r \quad (2.34)$$

or, in an explicit way, as

$$n_1 = \int_A \sigma_{13} dA = G \int_A (\eta_1 - \kappa_3 x_2 + \psi_{,1}^S \kappa_3) dA = GA\eta_1 - GS_1\kappa_3 + GS_1^S\kappa_3, \quad (2.35)$$

$$n_2 = \int_A \sigma_{23} dA = G \int_A (\eta_2 + \kappa_3 x_1 + \psi_{,2}^S \kappa_3) dA = GA\eta_2 - GS_2\kappa_3 + GS_2^S\kappa_3, \quad (2.36)$$

$$n_3 = \int_A \sigma_{33} dA = E \int_A (\eta_3 + \kappa_1 x_2 - \kappa_2 x_1) dA = EA\eta_3 + ES_1\kappa_1 + ES_2\kappa_2. \quad (2.37)$$

Note that identities of Equations 2.14 and 2.15 are applied to Equations 2.35 and 2.36, generating the terms S_1^S and S_2^S . These, together with the term involving S_1 and S_2 , are responsible for coupling shear forces and moment in direction $\mathbf{e}_3^r$, depending on the geometry of cross-section and the chosen axis.

Now, one may also develop a vector of equivalent moments, from integration of stresses acting on cross-section, considering the axis position as the pole (torsion moment). One may defined a quantity $\mathbf{m}$ composed by the torsion moment and another contribution, such that:

$$\mathbf{m} = \int_A x_2 \sigma_{33} dA \mathbf{e}_1^r - \int_A x_1 \sigma_{33} dA \mathbf{e}_2^r + \int_A ((x_1 \sigma_{23} - x_2 \sigma_{13}) + (\psi_{,1}^S \sigma_{13} + \psi_{,2}^S \sigma_{23})) dA \mathbf{e}_3^r \quad (2.38)$$

or, in an explicit way, as

$$m_1 = \int_A x_2 \sigma_{33} dA = E \int_A (x_2 \eta_3 + \kappa_1 x_2^2 - \kappa_2 x_1 x_2) dA = ES_1\eta_3 + EI_1\kappa_1 + EI_{12}\kappa_2, \quad (2.39)$$

$$m_2 = - \int_A x_1 \sigma_{33} dA = -E \int_A (x_1 \eta_3 + \kappa_1 x_1 x_2 - \kappa_2 x_1^2) dA = ES_2\eta_3 + EI_{12}\kappa_1 + EI_2\kappa_2, \quad (2.40)$$

$$m_3 = \int_A (x_1 \sigma_{23} - x_2 \sigma_{13}) dA + \int_A (\psi_{,1}^S \sigma_{13} + \psi_{,2}^S \sigma_{23}) dA. \quad (2.41)$$

It is interesting to observe that the component m_3 is composed by a torsion moment and a new internal moment term, coming from warping, defined as bishear moment. Thus, it is convenient to split m_3 in two contributions

$$m_3 = T + Q \quad (2.42)$$

where the torsion moment T is

$$T = \int_A (x_1 \sigma_{23} - x_2 \sigma_{13}) dA \quad (2.43)$$

and the bishear moment Q is

$$Q = \int_A (\psi_{,1}^S \sigma_{13} + \psi_{,2}^S \sigma_{23}) dA \quad (2.44)$$

The expansion of Equation 2.43 and use of the identity of Equation 2.16 leads to

$$T = \int_A (x_1 \sigma_{23} - x_2 \sigma_{13}) dA = -GS_2 \eta_2 - GS_1 \eta_1 + G(I_t^S + A(g_1 s_1 + g_2 s_2)) \kappa_3 \quad (2.45)$$

While the expansion of Equation 2.44 and use of identities of Equations 2.16 and 2.17 leads to

$$Q = \int_A (\psi_{,1}^S \sigma_{13} + \psi_{,2}^S \sigma_{23}) dA = GS_1^S \eta_1 + GS_2^S \eta_2 + G(I_0^S - I_0 + A(g_1 s_1 + g_2 s_2)) \kappa_3 \quad (2.46)$$

Furthermore, it is possible to calculate explicitly the equivalent moment m_3 using the previous results of T and Q and the identities of Equations 2.18 and 2.19 such that

$$m_3 = G(S_1^S - S_1) \eta_1 + G(S_2^S - S_2) \eta_2 + GI_t \kappa_3 \quad (2.47)$$

Finally, it is possible to compose Equations 2.35-2.37, 2.39, 2.40 and 2.47 by a matrix form as

$$\begin{bmatrix} n_1 \\ n_2 \\ n_3 \\ m_1 \\ m_2 \\ m_3 \end{bmatrix} = \begin{bmatrix} GA & 0 & 0 & 0 & 0 & G(S_1^S - S_1) \\ 0 & GA & 0 & 0 & 0 & G(S_2^S - S_2) \\ 0 & 0 & EA & ES_1 & ES_2 & 0 \\ 0 & 0 & ES_1 & EI_1 & EI_{12} & 0 \\ 0 & 0 & ES_2 & EI_{12} & EI_2 & 0 \\ G(S_1^S - S_1) & G(S_2^S - S_2) & 0 & 0 & 0 & GI_t \end{bmatrix} \begin{bmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \\ \kappa_1 \\ \kappa_2 \\ \kappa_3 \end{bmatrix} \quad (2.48)$$

It is important to note that despite of all hypothesis (small displacements and small rotations, elastic linear material by the Hooke's law and assumed negligible non-uniform torsion), the obtained constitutive matrix considers consistently geometric couplings for general axis position.

Nevertheless, the full geometrically-exact beam model derivation consists on several steps as presented in [34] and [50]. The necessary steps in the geometrically-exact beam

model derivation may be summarized as: (a) assumption of a consistent rotation parameterization; (b) definition of the beam kinematics regarding unrestricted translations and unrestricted rotations; (c) calculation of the strain vector; (d) calculation of the stress vector using the First Piola-Kirchhoff tensor; (e) calculation of the internal and external load powers; (f) utilization of the Virtual Work Principle (VWP) and a tangent operator in order to establish the model equilibrium.

2.2 Geometrically-Exact Shell Model

A shell may be defined as a structure that presents one dimension (thickness) much smaller than the other two dimensions. Moreover, shell structures, from nature or manufactured, are easily found in daily life. Some few examples of shell structures are eggshells, skulls, bells, pipes, cans, wind turbine blade composites, among others. In fact, the wide use of shell structures is mainly due to their structural efficiency associated to bending-stretching coupling [52].

Figure 2.3: Kinematic description of the Mindlin-Reissner shell model adopted.

Regarding the current structural use of shell theories in engineering problems, two main theories have been widely used: the Kirchhoff-Love shell theory and the Mindlin-Reissner shell theory. The Kirchhoff-Love shell theory, often associated to the Bernoulli-Euler beam theory, was first presented by Love [53] who used the Kirchhoff's assumptions [54] for plate bending theory. One main characteristic of the Kirchhoff-Love shell theory is that the shear deformations are neglected, resulting in a shell director always normal to the deformed shell mid-surface (the shell director is a unitary vector initially orthogonal to the shell reference configuration $\mathbf{e}_3^r$, and which follows the rotation of each material point of shell mid-surface, transforming into $\mathbf{e}_3$). On the other hand, the Mindlin-Reissner

second order shell theory, often associated to the Timoshenko beam theory, presented by Reissner [55] incorporates shear deformations on the formulation. Therefore, unlike the Kirchhoff-Love shell theory, in the Mindlin-Reissner shell theory the shell director may be not orthogonal to the deformed shell mid-surface.

Nevertheless, any shell theory may be roughly categorized into geometrically linear (see for e.g.: [56, 57, 58]) or geometrically nonlinear (see for e.g.: [59, 60, 61]). In this context, the present work adopts a Mindlin-Reissner nonlinear geometrically-exact shell theory introduced by Simo in [62]. Moreover, the models are discretized according to the fully nonlinear shell finite element T6-3i as presented in [63]. Finally, this thesis adopts the dynamic geometrically-exact formulation presented in [52] for the analysis of creased shell domes. Figure 2.3 presents the kinematic description of the Mindlin-Reissner shell model adopted.

As for the geometrically-exact beam model, the full geometrically-exact shell model derivation may be summarized as: (a) assumption of a consistent rotation parameterization; (b) definition of the beam kinematics regarding unrestricted translations and unrestricted rotations; (c) calculation of the strain vector; (d) calculation of the stress vector; (e) calculation of the internal and external load powers; (f) utilization of the Virtual Work Principle (VWP) and a tangent operator in order to establish the model equilibrium. A detailed description regarding the geometrically-exact shell model is seen in [49] and [52].

3 Shear Center and Shear Stresses on Closed Thin-Walled Beams

An important information concerning the constitutive matrix presented in Equation 2.33 is the shear center $\mathbf{s}$ definition. The shear center may be seen as cross-section reference point, with respect to which the distributed shear stresses produces no resulting torsion moment. Thus, if one applies a transversal force in a given cross section and aims to produce no twist, the required point of application of the force for such task is the shear center. Despite of this simple interpretation, the process of calculating the shear center may become a laborious task involving aspects such as materials, thicknesses hypothesis and geometries. A deeper discussion regarding the shear center is found in [64] and [65].

However, as presented on Equations 2.12 and 2.13, the shear center definition is mainly ruled by the shear stresses distribution associated to shear forces applied at the cross-section plane. In this context, it is convenient to separate the shear stresses distribution in two groups. A first and more general group composed by arbitrary cross-sections with shear stresses distribution expressed by two variables such as $q = \hat{q}(x_1, x_2)$. A second and more specific group composed by thin-walled cross-sections with shear stresses distribution expressed by a single variable such as $q(s) = \hat{q}(s)$, where s denotes a path along the thin-walled cross-section. Figure 3.1 presents generically these two shear stresses distribution groups.

Figure 3.1: Shear stresses distribution.

Moreover, it is possible to calculate the shear stresses distribution of a thin-walled profile by applying the equilibrium equation in a differential element ds . Figure 3.2 presents normal and shear stresses acting on a differential ds element.

Figure 3.2: Thin-walled differential element with t thickness.

Therefore, the differential element ds is in equilibrium only if equation

$$\left(\sigma_3 + \frac{\partial \sigma_3}{\partial x_3} dx_3 \right) t ds - \sigma_3 t ds + \left(q + \frac{\partial q}{\partial s} ds \right) dx_3 - q dx_3 = 0 \quad (3.1)$$

is satisfied, or, simplifying,

$$\frac{\partial q}{\partial s} + t \frac{\partial \sigma_3}{\partial x_3} = 0. \quad (3.2)$$

It is important to note that Equation 3.2 expresses an explicit relation between a normal stress σ_3 and a shear stress distribution q . It is possible now to replace the σ_3 term in Equation 3.2 by the general bending equation from classical beam theory. The classical beam theory bending equation, assuming a linear elastic material and using the previously adopted coordinate system, is

$$\sigma_3 = \left(\frac{m_2 I_1 - m_1 I_{12}}{I_1 I_2 - I_{12}^2} \right) x_1 + \left(\frac{m_1 I_2 - m_2 I_{12}}{I_1 I_2 - I_{12}^2} \right) x_2. \quad (3.3)$$

Finally, the replacement of Equation 3.3 into Equation 3.2 leads to a shear stress distribution equals to

$$q = - \left(\frac{n_1 I_1 - n_2 I_{12}}{I_1 I_2 - I_{12}^2} \right) \int_0^s t x_1 ds - \left(\frac{n_2 I_2 - n_1 I_{12}}{I_1 I_2 - I_{12}^2} \right) \int_0^s t x_2 ds \quad (3.4)$$

which is an equation that relates the shear stress distribution of a thin-walled beam element ds associated to given shear forces n_1 and n_2 .

3.1 Single Cell Closed Thin-Walled Beams

Although one may use Equation 3.4 in order to calculate the shear stresses distribution at a thin-walled beam, the integration interval that goes from “0” to “ s ” forces the integration to start at a point of known shear stress. In practical terms, the use of Equation 3.4 is only suitable for open thin-walled profiles, once these profiles provide at their extreme points a known null shear stress.

However, on closed thin-walled beams, there is no point with previously known shear stress. Therefore, the integration interval of Equation 3.4 should start from “ q_0 ”, that is a yet unknown value, and goes along the “ s ” cross-section path. It is possible then to rewrite Equation 3.4 as

$$q = -\left(\frac{n_1 I_1 - n_2 I_{12}}{I_1 I_2 - I_{12}^2}\right) \int_0^s t x_1 ds - \left(\frac{n_2 I_2 - n_1 I_{12}}{I_1 I_2 - I_{12}^2}\right) \int_0^s t x_2 ds + q_0 \quad (3.5)$$

Despite of the similarities of Equations 3.5 and 3.4, Equation 3.5 presents an additional term, here named q_0 . This term expresses that the shear stresses distribution on a closed thin-walled section has pattern similar to an open thin-walled section, however shifted by a constant value. Therefore, one may express the shear distribution on a closed thin-walled section as

$$q = q_{op} + q_0 \quad (3.6)$$

where the q_{op} term is equal to the right side of equation 3.4.

Moreover, the q_{op} term represents a statically determined part of the problem. In order to solve this part of the shear stress distribution problem, a convenient point is chosen on the beam cross-section and a “*cut*” is made. For now, this “*cut*” represents a point of known shear stress (zero) and makes the problem a solvable open section shear flow problem. Figure 3.3 presents a thin-walled section with a “*cut*”.

Figure 3.3: Thin-walled profile with a “*cut*”.

However, an additional hypothesis associated to the cross-section rate of twist is necessary in order to find the q_0 constant shear flow. The rate of twist of a thin-walled cross-section may be expressed as

$$\frac{d\theta}{dx_3} = \phi = \frac{1}{2A} \oint \frac{q}{Gt} ds \quad (3.7)$$

where θ is the cross-section rotation in beam axis direction, A is the profile enclosed area, q is the shear stress distribution, G is the shear modulus and t is the thickness. Equation 3.7 is also known as the second Bredt-Batho equation. The additional hypothesis consists in imposing a null rate of twist based on the assumption of forces applied right at the shear center. Thus, Equation 3.7 may be rewritten as

$$0 = \oint \frac{q}{Gt} ds. \quad (3.8)$$

Moreover, it is possible now to replace the generic shear stress distribution term q by the shear stress distribution at a closed thin-walled beam as presented on Equation 3.6. The replacement of Equation 3.6 into Equation 3.8 leads to

$$0 = \oint \frac{1}{Gt} (q_{op} + q_0) ds \quad (3.9)$$

which solving for q_0 leads to

$$q_0 = -\frac{\oint q_{op} ds}{\oint ds}. \quad (3.10)$$

Equation 3.10 expresses, regarding the hypothesis adopted, the constant shear flow q_0 based on the statically determined open thin-walled shear stresses distribution q_{op} . Furthermore, it is important to note that the final shear distribution over the closed thin-walled beam is composed by the “cut” open thin-walled shear distribution q_{op} and the constant shear flow q_0 .

The shear stresses distribution on a single cell closed thin-walled beam may be summarized as

1. Choose an arbitrary point of the closed thin-walled beam cross-section and make a “cut”.
2. Calculate the open section shear stresses distribution q_{op} based on the “cut” point.
3. Use the null rate of twist condition to calculate the constant shear flow q_0 of the section, supposing the application of shear forces right at the shear center.

4. Compose the final shear stresses distribution with the open section shear distribution q_{op} and the constant shear flow q_0 .

3.2 Multicellular Closed Thin-Walled Beams

A multicellular closed thin-walled beam cross-section shear stress problem follows a pattern similar to the single cell shear stress distribution. However, multiple “cuts” should be done at the cross-section in order to produce as many statically determined open cell shear stresses distributions $q_{op,i}$. Once all open cell shear stresses distribution is defined, it is possible to use a rate of twist hypothesis (at each cell) in order to define a linear system that will lead to each cell constant shear flow $q_{0,i}$. Finally, it is possible to compose the complete profile shear stresses distribution using each open cell shear stresses distribution and their respective constant shear flow.

Similarly to the single cell closed profile, it is necessary to express each cell rate of twist on the context of a multicellular closed thin-walled profile. The rate of twist of an i cell on a multicellular closed profile may be expressed as

$$\frac{d\theta_i}{dx_3} = \phi_i = \frac{1}{2A_i} \oint_i \frac{q_i}{Gt} ds \quad (3.11)$$

where θ_i is the cell cross-section rotation in beam axis direction, A_i is the cell profile enclosed area, q_i is the cell shear stress distribution, G_i is the cell shear modulus and t_i is the cell thickness. Moreover it is possible now to impose the null rate of twist at each cell according to the assumption that loads are being applied at the shear center. Regarding this assumption, the rate of twist on each should be

$$0 = \oint_i \frac{q_i}{Gt} ds. \quad (3.12)$$

It is important to note that Equation 3.12 defines the main hypothesis regarding the shear center calculation of multicellular closed thin-walled profiles. It is now possible to replace the q_i term on Equation 3.12 by its corresponding shear stress distribution.

However, the replacement of the q_i term on Equation 3.12 is not straightforward as for the single cell shear distribution problem. In fact, the shear stress distribution q_i term of a cell from a multicellular profile is under influence of the constant shear flows from the surrounding cells (see Figure 3.4). In this case, the shear stresses distribution from a cell q_i exclusively in contact with two other cells with constant shear flows q_{i-1} and q_{i+i} may

be expressed as

$$q_i = q_{op,i} + q_{0,i} - q_{0,i+1} - q_{0,i-1}. \quad (3.13)$$

Figure 3.4: Multicellular closed thin-walled profile with multiple “cuts”.

Therefore, the general null rate twist hypothesis from a cell, as the one in the middle of Figure 3.4, may be expressed as

$$0 = \oint_i \frac{q_{op,i} + q_{0,i}}{Gt} ds - \oint_{i+1} \frac{q_{0,i+1}}{Gt} ds - \oint_{i-1} \frac{q_{0,i-1}}{Gt} ds \quad (3.14)$$

where the term $q_{op,i}$ is the already known i open cell shear stress distribution and the terms $q_{0,i}$, $q_{0,i-1}$ and $q_{0,i+1}$ are unknown constant shear flows acting at each respective cell.

Finally, it is possible to rewrite equation 3.14 taking the constant shear flows out of the integrals and separating the unknown terms at the left side as

$$q_{0,i} \oint_i \frac{1}{Gt} ds - q_{0,i+1} \oint_{i+1} \frac{1}{Gt} ds - q_{0,i-1} \oint_{i-1} \frac{1}{Gt} ds = - \oint_i \frac{q_{op,i}}{Gt} ds. \quad (3.15)$$

Note that equation 3.15 represents a linear system related to the underlying hypothesis adopted of null rotation at each cell. Therefore, the linear system solution leads to constant shear flows that satisfy the hypothesis adopted. Finally, it is possible to assemble the profile shear stresses distribution regarding each open cell shear distribution $q_{op,i}$ and its corresponding constant shear flow $q_{0,i}$.

The shear stresses distribution of multicellular closed thin-walled beam may be summarized as

1. Choose n arbitrary points of the multicellular closed thin-walled section in order to produce an statically determined problem and make that many “cuts” on it in order to produce an open profile.
2. Calculate the overall open section shear stress distribution based on the “cut” points.

3. Split the overall open section distribution into n open section shear stress distributions $q_{op,i}$.
4. Use the null rate of twist condition to calculate all n constant shear flows $q_{0,i}$ of the cross-section, supposing the application of shear force right at the shear center.
5. Compose the final shear stresses distribution with the open cross-section shear distributions $q_{op,i}$ and the constant shear flows $q_{0,i}$.

4 St. Venant Torsional Inertia on Closed Thin-Walled Beams

As presented on Equation 2.20 the torsional inertia I_t of a beam is a parameter that relates an applied torque T , a shear modulus G and a rate of twist ϕ . Additionally it is a necessary parameter on the fulfillment of the constitutive Equation 2.33 on the context of the proposed geometrically-exact beam model proposed.

4.1 Single Cell Closed Thin-Walled Beams

It is possible to prove, by equilibrium conditions, that a single cell closed thin-walled cross-section beam subject to a pure torque T , with negligible axial constraint effects, has solution only for a constant shear flow q as presented in Equation 3.7

Moreover, the relation between an applied torque T and a constant shear flow $q = \hat{q}(s)$ at a distance r orthogonal to s from an arbitrary point is, by torsional equilibrium, equals to

$$T = \oint r q ds. \quad (4.1)$$

It is also possible to prove, using geometry identities, that since q is constant and $\oint r ds = 2A$, the torque T may be written as

$$T = 2Aq. \quad (4.2)$$

Equation 4.2 is known as the first Bredt-Batho equation.

Therefore, the torsional inertia calculation of a single cell closed thin-walled profile may be summarized as:

1. Define a convenient rate of twist ϕ .
2. Calculate a resulting constant shear flow q .
3. Calculate a resulting torque T .
4. Use equation 2.19 to calculate the torsional inertia I_t .

4.2 Multicellular Closed Thin-Walled Beams

As presented in [66], the process of obtaining the torsional inertia of a multicellular closed thin-walled beam has many similarities with the single cell thin-walled beam. However, additional hypothesis are necessary since the torsional inertia of a multicellular closed thin-walled beam may not be obtained by classical torsion formulations.

A total torque T acting at a multicellular closed thin-walled section may be expressed as the sum of the torques at each cell. Therefore the total torque T may be expressed as

$$T = \sum_{i=1}^n 2A_i q_i \quad (4.3)$$

where A_i is each cell enclosed area and q_i is each cell constant shear flow.

Moreover, in order to shrink the notation and aid on the rate of twist calculations, it is convenient to define a flexural warping coefficient λ [66]. The flexural warping coefficient of a closed thin-walled cross-section is defined as

$$\lambda = \frac{1}{G} \int \frac{ds}{t} \quad (4.4)$$

where G is the shear modulus and t is the thickness of the element.

On the context of a multicellular closed thin-walled section, it is possible to define a flexural warping coefficient of an element for an i cell $\lambda_{i,i}$ as

$$\lambda_{i,i} = \frac{1}{G_i} \int \frac{ds_i}{t_i}, \quad (4.5)$$

where G_i is the shear modulus and $t_i = \hat{t}_i(s)$ is the thickness of the element.

Additionally, as a multicellular closed thin-walled cross-section presents elements at the interface between cells, it is also convenient to define a flexural warping coefficient of an i cell with an $i + 1$ cell $\lambda_{i,i+1}$ as

$$\lambda_{i,i+1} = \frac{1}{G_{i,i+1}} \int \frac{ds_{i,i+1}}{t_{i,i+1}} \quad (4.6)$$

where $G_{i,i+1}$ is the shear modulus of the interface element and $t_{i,i+1}$ is the thickness of the interface element.

Finally, regarding these flexural warping definitions in the context of multicellular closed thin-walled profile, the rate of twist of an i cell exclusively in contact with two other cells $i - 1$ and $i + 1$ may be expressed as

$$\frac{d\theta_i}{dx_3} = \phi_i = \frac{1}{2A_i} (\lambda_{i,i} q_i - \lambda_{i,i+1} q_{i+1} - \lambda_{i,i-1} q_{i-1}). \quad (4.7)$$

Equation 4.7 is in fact a linear system with a consistent formulation of a multicellular closed thin-walled torsion problem. However, Equation 4.7 presents more variables than equations which means that there are an infinite range of solutions. Thus, in order to impose a unique solution to the linear system, additional hypotheses are necessary.

As presented in [66] a possible way to circumvent this issue is the adoption of the hypothesis that each cell presents the same rate of twist. This hypothesis may be mathematically expressed as

$$\phi_1 = \phi_2 = \dots = \phi_{n-1} = \phi_n = \phi. \quad (4.8)$$

It is possible now to calculate consistently each cell constant shear flow associated to any arbitrary rate of twist. Additionally it is important to mention that, despite of the arbitrary rate of twist, the torsional inertia is invariant. This condition makes possible the assumption of any convenient rate of twist in the torsional inertia calculations.

Considering then, by convenience, that the rate of twist of the cross-section is numerically equals to one, it is possible to rewrite Equation 4.7 using a matrix notation for a generic multicellular cross-section with n cells as

$$2\phi \mathbf{A} = \mathbf{\Lambda} \mathbf{q} \quad (4.9)$$

where ϕ is

$$\phi = \phi_1 = \phi_2 = \phi_3 = \dots = \phi_{n-1} = \phi_n = 1, \quad (4.10)$$

the vector $\mathbf{A}$ is

$$\mathbf{A} = \begin{bmatrix} A_1 & A_2 & A_3 & \dots & A_{n-1} & A_n \end{bmatrix}^T,$$

the matrix $\mathbf{\Lambda}$ is

$$\mathbf{\Lambda} = \begin{bmatrix} \lambda_{1,1} & \lambda_{1,2} & 0 & 0 & 0 & 0 \\ \lambda_{2,1} & \lambda_{2,2} & \lambda_{2,3} & 0 & 0 & 0 \\ 0 & \lambda_{3,2} & \lambda_{3,3} & \lambda_{3,4} & 0 & 0 \\ 0 & 0 & \vdots & \ddots & \vdots & 0 \\ 0 & 0 & 0 & \lambda_{n-1,n-2} & \lambda_{n-1,n-1} & \lambda_{n-1,n} \\ 0 & 0 & 0 & 0 & \lambda_{n,n-1} & \lambda_{n,n} \end{bmatrix},$$

and the vector $\mathbf{q}$ is

$$\mathbf{q} = \begin{bmatrix} q_1 & q_2 & q_3 & \dots & q_{n-1} & q_n \end{bmatrix}^T.$$

Accordingly, Equation 4.9 leads to

$$\mathbf{q} = \mathbf{\Lambda}^{-1} 2\phi \mathbf{A}. \quad (4.11)$$

Therefore, the solution of Equation 4.11 for $\mathbf{q}$ provides a set of constant shear flows with respect to the imposed unitary rate of twist.

Consequently, the constant shear flow vector $\mathbf{q}$ results in a total torque T equals to

$$T = 2\mathbf{A} \cdot \mathbf{q}, \quad (4.12)$$

where $\cdot$ denotes a inner product.

Finally, once the total torque T is defined, the torsional inertia may be easily obtained through Equation 2.20.

The torsional inertia calculation of a multicellular closed thin-walled profile may be summarized as:

1. Calculate each cell enclosed area A_i .
2. Calculate each cell flexural warping coefficient $\lambda_{i,i}$ and their respective interface flexural warping $\lambda_{i,i+1}$ and $\lambda_{i,i-1}$, if they exist.
3. Assemble the rate of twist linear system.
4. Use a common unitary rate of twist hypothesis to provide a unique solution for the linear system and calculate the constant shear flow vector $\mathbf{q}$.
5. Calculate the resulting total torque T .
6. Calculate the torsional inertia I_t through the division of the resulting total torque T by the equivalent cross-section shear modulus G .

5 Computational Implementations Used For Simulations

5.1 WindTurbine: A computer aided design (CAD) tool for wind turbines

To perform our study, an originally CAD tool was developed. The main objective of the *WindTurbine* CAD tool is to aid on the calculation of geometric properties and geometry definition of wind turbine cross-sections. Figure 5.1 presents the *WindTurbine* main screen.

Figure 5.1: *WindTurbine* main screen.

In the *WindTurbine* CAD tool, a cross-section profile is simply defined by a sequence of counterclockwise coordinates, thickness values and material elastic properties. Alternatively, one may add or remove profiles to the wind turbine blade, include webs in profiles through coordinates labels, set profiles chord and twist angle, define a reference point from where all geometric properties are calculated (axis) among other possibilities. Several results used for the verification of the *WindTurbine* tool are presented in Appendix A.

Particularly, the main feature available at the *WindTurbine* tool is the possibility

of evaluation and exportation of the constitutive equation terms, as presented in Equation 2.48. Figure 5.2 presents a sample file generated by the *WindTurbine* tool containing the constitutive equation terms from an airfoil cross-section.

Figure 5.2: Sample file generated by the *WindTurbine* tool containing constitutive equation terms.

Moreover, regarding the geometric properties calculations it is important to observe that the *WindTurbine* software assumes that the cross-section profile is composed by flat thin-walled parts. Although there is no clear definition about the aspect ratio of a thin-walled structure, according to [67] the approximations are reasonable accurate for profiles with

$$\frac{t_{max}}{b} < 0.1 \quad (5.1)$$

where t_{max} is the maximum profile thickness and b is a typical profile dimension. Additionally, a main consequence of the closed multicellular thin-walled profiles hypothesis is that the shear stresses distribution may be assumed constant across each profile thickness. Therefore, it is possible to calculate the torsional inertia (see Section 4) and shear center (see Section 3) of a cross-section profile including effects of webs on the shear stresses distribution.

5.1.1 Example

The goal of this subsection is to briefly present a complete airfoil analysis with the aid of the *WindTurbine* tool. The first step for the analysis of an airfoil cross-section with the

WindTurbine tool is to set an airfoil cross-section geometry. At this point one may choose to directly input every point coordinate or to create a simple “*.asc” file containing these coordinates as presented in Figure 5.3. The main advantage of input coordinates by an “*.asc” file is that one may rapidly input a large number of coordinates.

Figure 5.3: Airfoil cross-section file (*.asc) for the *WindTurbine* tool.

Alternatively one may also add webs to the airfoil cross-section. In the *WindTurbine* tool the webs of a cross-section are defined by the indexes of two points. In order to make this task more intuitive, one may check the “*View Index Label*” option and then click the “*Refresh Plot*” button to see the point indexes in the “*Airfoil Visualization*” tab.

Although the web definition is not mandatory at airfoil cross-section, the use of webs in wind turbine blades is noteworthy. In fact, the webs in wind turbine blades not just prevent possible local buckling but also increases some other important geometry properties such as the torsional inertia and second order moments of inertia. Nevertheless, the *WindTurbine* tool presents no limits at the number of webs and consistently considers the webs effects at the cross-section geometric properties, specially the torsional inertia increment. Once the airfoil cross-section geometry is set, it is possible to visualize it by clicking the “*Refresh Plot*” button.

A next step in the analysis is the material definition. In the *WindTurbine* tool a material is defined simply with the aid of an “*.mip” file as presented in Figure 5.4. A

material file should contain at least one material and its respective Elastic Modulus, Poisson Coefficient and material density. The Shear Modulus, when necessary, is calculated assuming a homogeneous isotropic elastic linear material.

Figure 5.4: Material file (*.mip) for the *WindTurbine* tool.

After that, one may already click the “*Calculate*” button in order to obtain the airfoil geometric properties and visualize the airfoil cross-section geometry, geometric center, shear center and principal axis of inertia as presented in Figure 5.5.

Figure 5.5: Airfoil cross-section results at the *WindTurbine* tool.

Furthermore, the *WindTurbine* tool presents some useful additional features regarding common wind turbine blades such as the chord length and the angle of twist. These features may be easily set in the program and their effects in the geometric properties quickly seen in the results and in the airfoil visualization. Some other under development

features already present in the program are the possibility of creating a blade's profile list, the definition of the airfoil length in the blade and the number of divisions associated to a finite element discretization.

Finally, with all parameters set, one may export a *Giraffe*'s input's file (*.inp) containing all necessary information associated to the airfoil cross-section for a beam model simulation (see Figure 5.2). It is important to note that such cross-section information are associated to a reference point, by default (0;0). It can be modified in *WindTurbine* tool. The reference point is indeed the beam model axis that will receive the model loads (forces and moments). Alternatively, one may save a native WindTurbine tool file (*.wtr) that permits the user to share or keep working at the same project.

5.2 *Giraffe*: A nonlinear finite element solver

Generic Interface Readily Accessible For Finite Elements (*Giraffe*) is a finite element solver developed using C++ language, with the main objective of being a platform for the implementation of computational models, such as beam and shell finite element formulations, contact models and others features [31]. *Giraffe* solver has been used on robust simulations of complex engineering problems such as oil exploitation risers [34, 35, 50], mechanisms [39], woven fabrics [68], wheel-rail contact [69], creased shells [52] and others.

Regarding the proposed comparative approach in the present work, *Giraffe* solver may be seen as a suitable tool to implementation of here presented models, once it presents both (beam and shell) geometrically-exact formulations. However, it is important to note that the modeling and analysis of wind turbine blades by the geometrically-exact beam model in *Giraffe* solver is only possible once it presents the formulation described in Subsection 2.1. The geometrically-exact beam formulation of Subsection 2.1 consistently permits the definition of arbitrary cross-sections and arbitrary beam axis regarding all geometric couplings. In fact, the modeling of realistic wind turbine models through beam theories is not a trivial task since the beam model should handle complex geometries and couplings.

Additionally, as presented in [30], it is possible to use the *Giraffe* solver on a dynamic simulation of a full wind turbine, including tower, nacelle, and blades. There are, however, undergoing developments for future works in order to increase the robustness of the wind turbine model including aerodynamic loads and more elaborated geometric modeling.

6 Wind Turbine Blade Model Description

We tested the proposed structural models by considering a realistic case study. The model of a real wind turbine blade was considered, as described in the sequence.

6.1 Airfoil Section

S809 wind turbine airfoil section was adopted [4]. According to [70], S809 airfoil section was developed with the objective of achieving maximum lift coefficients (insensitive to roughness) and low drag coefficients under certain conditions. Furthermore, normalized chord “ x/c ” and “ y/c ” coordinates that define S809 airfoil section may be found in [4] and [70]. It is possible, then, to plot the airfoil shape on a graph as presented in Figure 6.1.

Figure 6.1: S809 airfoil shape, extracted from [4].

Once the airfoil shape is defined, it is necessary to define the cross-section webs. On the context of airfoil sections, webs are normally structural parts that connect the upper surface to the lower surface of an airfoil. In fact, the airfoil webs definition is crucial for the wind turbine blade structural behavior. The webs not just increase the airfoil section stiffness, but also prevent the local buckling of the section. As an approximation for the proposed blade airfoil cross-section, two webs were adopted, based on the sample input file from PreComp [6]. Therefore, for the proposed model, the webs are defined as vertical structural parts approximately at 15% and 50% of the chord length. Figure 6.2 presents S809 airfoil cross-section including the webs.

Moreover, it is necessary to define the thickness of every part of the airfoil cross-section. As for the webs, the thicknesses of PreComp airfoil sample input file were taken

as reference. Table 6.1 summarizes the airfoil absolute thickness of each airfoil section part, according to the colors of Figure 6.2.

Figure 6.2: S809 airfoil section including webs in green color.

S809	Thickness (m)
Black parts	0.0184
Red parts	0.0581
Green parts	0.03975

Table 6.1: Airfoil section absolute thickness of each S809 airfoil part.

6.2 Equivalent Material

The structural part of modern HAWT blades is often composed by a stack of laminas with distinct material properties. Additionally, even considering that each lamina is made of an orthotropic material, the load may not be aligned with material principal directions. It is possible, however, to use the Classical Laminate Theory (CLT) to calculate equivalent material properties of a laminate composed by orthotropic laminas. According to [71], CLT consists of a collection of hypothesis that permits the reduction of 3D complex problem into a 2D solvable problem. Furthermore, CLT assumes for the laminate the Kirchhoff hypothesis for plates [72] and the Kirchhoff-Love hypothesis for shells [53].

Thus, according to [71], the equivalent material properties E_x and G_{xy} of an orthotropic lamina positively oriented by an angle as presented in Figure 6.3 are defined as

$$\frac{1}{E_x} = \frac{1}{E_1} \cos^4 \theta + \left(\frac{1}{G_{12}} - \frac{2\nu_{12}}{E_1} \right) \sin^2 \theta \cos^2 \theta + \frac{1}{E_2} \sin^4 \theta \quad (6.1)$$

$$\frac{1}{G_{xy}} = 2 \left(\frac{2}{E_1} + \frac{2}{E_2} + \frac{4\nu_{12}}{E_1} - \frac{1}{G_{12}} \right) \sin^2 \theta \cos^2 \theta + \frac{1}{G_{12}} (\sin^4 \theta + \cos^4 \theta) \quad (6.2)$$

where E_1 , E_2 , G_{12} and ν_{12} are the corresponding material properties associated to fibers principal coordinates.

Once the equivalent material properties of each lamina is calculated, the laminated equivalent properties E_{eq} and G_{eq} are simply defined as

$$E_{eq} = \frac{\sum_{i=1}^n E_{x,i} t_i}{\sum_{i=1}^n t_i} \quad (6.3)$$

$$G_{eq} = \frac{\sum_{i=1}^n G_{xy,i} t_i}{\sum_{i=1}^n t_i} \quad (6.4)$$

where the i iterator represents a lamina, n is the number of laminas and t_i is the thickness of each lamina. Additionally, it is also convenient to define an equivalent density of a laminate ρ_{eq} as

$$\rho_{eq} = \frac{\sum_{i=1}^n \rho_i t_i}{\sum_{i=1}^n t_i} \quad (6.5)$$

where the i iterator follows the same pattern described above and ρ_i is the density of each lamina.

Figure 6.3: Principal fibers coordinates positively oriented by an angle θ .

Name	t (m)	θ (deg.)	E_1 (GPa)	E_2 (GPa)	G_{12} (GPa)	ν_{12}	ρ (kg/m ³)
Unid. FRP	0.03975	30	37	9	4	0.28	1860
Double-bias FRP	0.01749	20	10.3	10.3	8	0.3	1830
Gelcoat	0.000381	0	1E-8	1E-8	1E-9	0.3	1830
Nexus	0.00051	0	10.3	1E+10	8	0.3	1664

Table 6.2: Lamina information used on the calculation of the S809 equivalent material properties [6].

Nevertheless, despite of the beam formulation that incorporates only one isotropic homogeneous material, it is possible now to estimate equivalent material properties from a real laminate. For the proposed S809 airfoil cross-section material a laminate from the PreComp sample input file was taken as reference. Table 6.2 presents all lamina information used in order to calculate the equivalent material properties for the S809 airfoil.

Finally, with information from Table 6.2 and the previous equivalent material formulation, it is possible to calculate the resulting equivalent properties presented in Table 6.3.

E_{eq} (GPa)	14.0
G_{eq} (GPa)	5.6
ρ_{eq} (kg/m ³)	1849.06

Table 6.3: Equivalent properties for the S809 airfoil section.

6.3 Blade Spam Geometry

Despite of the spanwise complex geometries that a wind turbine blade may present, in this thesis two simplified geometries based on the S809 airfoil are adopted.

Figure 6.4: Proposed blade geometries based on S809 airfoil.

The first and rectangular one (G1) is defined just by a 20 m extrusion of an S809 airfoil with a 2 m chord length, thicknesses as presented in Table 6.1 and material as presented in Table 6.3.

The second and tapered one (G2) follows the same pattern of the G1 model, however, with a linear reduction on the S809 chord along length. On G2 model, the airfoil chord reduction occurs linearly from 2 m at the root to 0.6 m at the tip and takes as a path each cross-section geometric center.

Figure 6.4(a) presents the rectangular G1 model and Figure 6.4(b) presents the tapered G2 model. Table 6.4 summarizes the geometric features of the proposed blades.

It is important to mention that, regarding the formulations previously presented on this thesis, there is no limitation for the blade cross-section geometries. This feature is especially relevant on the context of modern HAWT blades.

Blade Geometry	Airfoil	Root/Tip Chord (m)	Reduction Path
G1	S809	2/2	-
G2		2/0.6	Geometric center

Table 6.4: Proposed blade geometries description.

6.4 Computational Models

G1 and G2 beam models are discretized into 20 3-node uniform mesh, with elements of length 1 m. Figure 6.5(a) and Figure 6.5(b) presents the rendered mesh for the beam models G1 and G2 respectively. This discretization of both models showed adequate for the here presented results.

Figure 6.5: Beam models discretization.

Furthermore, G1 shell model is discretized into 4884 6-node elements and the G2 shell model into 8976 6-node elements. Figure 6.6(a) and Figure 6.6(c) presents G1 shell model discretization, while Figure 6.6(b) and Figure 6.6(d) presents G2 shell model discretization.

Figure 6.6: Shell models discretization.

7 Simulations and Discussions

7.1 Proposed Simulations

In order to verify the comparative accuracy between the beam and shell models a series of static and dynamic high geometrically-nonlinear simulations are proposed. However, a previous static simulation, based in a realistic aerodynamic load is proposed. Despite this thesis focus on high geometrically-nonlinear responses with concentrated loads, a previous and more realistic scenario is proposed: a distributed load, which is important for verification of overall models' response. The proposed simulations are described as follows.

7.1.1 Realistic Simulation

To create a realistic simulation, only G1 computation model is adopted. The blade beam and shell models are defined by a single essential boundary condition: the extreme cross section at one tip is clamped (cantilever). A realistic distributed aerodynamic load is applied. As one of the most aerodynamic important loads of wind turbine blades is associated to lift forces, a realistic lift distributed load is proposed for these models.

In order to calculate the lift forces per unit length for the S809 airfoil cross-section, one may use the resultant lift force equation expressed as

$$F_l = \frac{1}{2}\rho U^2 D C_l, \quad (7.1)$$

from where ρ is the fluid density, U is the fluid velocity magnitude, D is a body characteristic dimension (in the context of wind turbine blades, the chord) and C_l is a lift coefficient [73]. Therefore, in order to define the lift force, it is necessary to define first a lift coefficient.

The lift coefficient is, in fact, a dimensionless coefficient often expressed according to the fluid angle of attack and a fixed Reynolds number. Moreover, there are also similar definitions for the drag forces (drag coefficient) and pitching moment (pitching coefficient) for wind turbine blades.

Nevertheless, the S809 airfoil lift, drag and moment coefficients obtained in [74] according to the XFOIL program [75] for a 1E+6 Reynolds number, are presented in Figure 7.1.

Figure 7.1: Lift, drag and pitching moment coefficients for the S809 airfoil.

It is now possible to combine some data as the ones of Table 7.1 to the lift coefficient of Figure 7.1(a) ($C_l = 1.0172$ at 10°) in order to calculate a resultant lift distributed load of 525 N/m (load case L0). It is important to mention that the calculated resultant lift load is applied at a point known as the aerodynamic center of the airfoil which is, by definition, at a quarter of the airfoil chord.

Data	Value	Unit
ρ (air)	1.29	kg/m ³
U (air)	20	m/s
D (chord)	2	m
C_l (10°)	1.0172	-

Table 7.1: Data adopted to calculate the resultant lift force.

As the beam model axis adopted is the geometric center, a corresponding pitching moment load is added in order to keep the mechanical equivalence between the expected aerodynamic load and the modeled one. For the shell model the distributed load is appropriately split in two distributed contributions applied at the interface between the webs

Load Case	Type	Load	Beam Model Load	Shell Model Load	Analysis
L0	Static	525 N/m	525 N/m (y) -171.42 N.m/m (z)	409.07 N/m (y) 115.8 N/m (y)	Displacement y

Table 7.2: Proposed static simulations and analysis for the realistic load.

and the airfoil lower surface in order to reduce possible concentrated stresses. Table 7.2 summarizes the beam and shell model distributed loads. Figure 7.2(a) presents the equivalent static loading for the beam model and Figure 7.2(b) presents the equivalent static loading for the shell model.

Figure 7.2: Equivalent realistic static loadings.

7.1.2 Nonlinear Simulations

At the static simulations, the structures present their root sections displacements and rotations restrained while loads are applied at the tip section, more specifically at the geometric center point calculated through the *WindTurbine* tool. As the chosen beam model axis corresponds to the locus of cross-sections geometric centers, the loads are simply applied to the tip node. However, for the shell model, a rigid node set constraint is defined for the whole tip cross-section nodes and the previously calculated geometric center node (pilot node). The objective of this constraint is to uniformly distribute the loads applied at the pilot node, through the node set constraint.

Nevertheless, the structural analysis consists in the evaluation of the pilot node displacement/rotation response and in the visualization of the deformed structures, to com-

pare their behavior. Table 7.3 summarizes the proposed static simulations and analysis. Additionally, Figure illustrates the loadings associated to each proposed static simulation.

Load Case	Type	Global Direction	Load	Magnitude G1	Magnitude G2	Analysis
L1	Static	z	Force (N)	2.0E+8	1.4E+8	Displ. x, y and z
L2		x	Force (N)	3.5E+5	2.8E+5	Displ. x, y, z and rot. z
L3		y	Force (N)	2.0E+5	1.4E+5	Displ. x, y, z and rot. z
L4		z	Moment (N.m)	6.0E+6	1.5E+6	Rotation z

Table 7.3: Proposed static simulations and analysis.

(a) L1

(b) L2

(c) L3

(d) L4

Figure 7.3: Static loadings proposed for nonlinear simulations.

At the dynamic simulations, the only load presented for the structures is a prescribed displacement time-series, and restrained rotations at root cross-sections of wind turbine blade models. The displacement time-series defines an “8” shape path, in order to excite

the blade in all directions. Taking the middle point of the “8” circuit as an origin point, the displacement starts on the direction of the Cartesian 1st quadrant (x and y positive axis) and follows to the 2nd quadrant, 4th quadrant and 3rd quadrant sequentially.

Load Case	Type	Prescribed Path	Circuit Time G1	Circuit Time G2	Analysis
L5	Dynamic	Symmetric “8” 1m Diameter	1.7 s	1.5 s	Displ. x and y along time

Table 7.4: Proposed dynamic simulations and analysis.

The “8” shape path presents equal top and bottom parts, with a diameter of 1 m. It is constructed by linear interpolated curves between points and a total circuit time of 1.7 s for the G1 model and of 1.5 s for the G2 model. The circuit time of each geometry was chosen based on the natural frequencies of the models as presented in Table 7.5 in order to ensure relevant dynamic contributions on system response.

	G1				G2			
	Beam		Shell		Beam		Shell	
Mode	T (s)	f (Hz)	T (s)	f (Hz)	T (s)	f (Hz)	T (s)	f (Hz)
1	1.67	0.60	1.67	0.60	1.40	0.72	1.43	0.70
2	0.55	1.81	0.55	1.80	0.46	2.15	0.47	2.11
3	0.27	3.74	0.27	3.70	0.35	2.85	0.35	2.86
4	0.10	10.40	0.10	10.11	0.14	6.90	0.14	6.94
5	0.09	10.92	0.09	10.92	0.12	8.49	0.12	8.49
6	0.09	11.03	0.09	11.19	0.08	12.88	0.08	12.88
7	0.05	20.23	0.05	19.08	0.05	19.63	0.05	19.82
8	0.03	29.71	0.03	29.11	0.05	20.21	0.05	20.33
9	0.03	32.38	0.03	29.68	0.05	20.80	0.05	20.57
10	0.03	33.44	0.03	31.64	0.03	30.56	0.03	29.72
11	0.03	34.40	0.03	34.42	0.03	37.00	0.03	37.07
12	0.02	48.63	0.02	40.22	0.03	38.04	0.03	38.38

Table 7.5: Natural periods and natural frequencies for the proposed blade geometries.

Additionally, it important to mention that all dynamic simulation uses the Newmark integration method with parameters $\beta = 0.3$ and $\gamma = 0.5$, according to the formulation

presented on [42] (for negligible numerical damping). The time-step adopted is variable with initially reference value of 0.01 s (also maximum) and permitting smaller values, according to difficulties faced during the nonlinear model solution.

Moreover, as this thesis aims a fair comparison between the beam and shell models, the rigid node set constraint defined for the static simulations at tip cross-section of shell models is also kept at the dynamic simulations. Therefore, the structural analysis is also made through the pilot node response and the structure visualization. Table 7.4 summarizes the proposed dynamic simulation.

7.2 Results and Discussions

7.2.1 Load Case 0 – Realistic Load

The results regarding the realistic simulation are presented in Figure 7.4. Nevertheless, the results associated to other degrees of freedom presented values in the order of 10E-3 and therefore are considered negligible when compared to the y displacement.

Figure 7.4: L0 load case and G1 model y.

The result of Figure 7.4 shows that the beam and the shell model have very similar structural responses. This result indicates that, despite the simplifying hypothesis adopted, the beam model is quite robust. Moreover, this analysis qualifies the beam model for the following high nonlinear analysis.

7.2.2 Load Case 1 – Static Tension

The results of the L1 load case and the G1 model are presented in Figure 7.5. It is seen in Figure 7.5(a) and Figure 7.5(b) that the x and y displacements present a similar pattern, however with a difference between behaviors of beam and shell models. The

reason for the differences is the distinct location of tension centers in both models. While in the beam model this position is calculated by the *WindTurbine* tool and the tip node is considered exactly there, by construction, no bending occur due to tension load. However, in the shell model, the pilot node is located at the same position of the geometric center evaluated for the beam model.

Figure 7.5: L1 load case and G1 geometry results.

Then, imprecisions due to geometric characterization of shells (e.g.: mesh refinement) or due to simplifications assumed by *WindTurbine* tool may lead to slight deviations from beam models' results. However, numerically speaking, such deviations are really small when compared to the length of the blade (maximum 0.015 m displacement in 2 m). Note that such deviations may be understood as a bending of the blade, due to application of tension load slightly out of tension center.

However, Figure 7.5(c) shows that there is a very good concordance between beam and shell models, even for a large displacement, when concerning the axial displacement.

The L1 results for the G2 model are presented at Figure 7.6. The results of Figure 7.6(a) and Figure 7.6(b) present respectively a pattern similar to the ones of Figure 7.5(a) and Figure 7.5(b), for the same previously commented reasons. Moreover,

Figure 7.6(c) shows that the beam and shell models present a very similar response at axial direction, even considering large displacements.

Figure 7.6: L1 load case and G2 geometry results.

7.2.3 Load Case 2 – Static Bending About Y Axis

The results associated to the L2 load case and G1 model are presented in Figure 7.7. It is seen in Figure 7.7(a) that the beam and shell models present a similar response, until approximately $1.8E+5$ N (approximately at 47% of the end load) and, after this point, distinct overall stiffness responses. Additionally, the shell model behaves as more flexible than the beam model for loads above $1.8E+5$ N. Indeed, the analysis showed that the shell model presents local buckling occurrence, which is the reason for such deviation, since it may affect the whole bending stiffness system.

The interesting point is that even for loads below $1.8E+5$ N, local shell buckling already can be observed. Figure 7.8 presents the shell model local buckling evolution, according to the load percentage applied to the blade tip (pilot node).

(a) x direction displacements

(b) y direction displacements

(c) z direction displacements

(d) z direction rotation

Figure 7.7: L2 load case and G1 geometry results.

(a) 33%

(b) 38%

(c) 43%

(d) 48%

(e) 53%

(f) 100%

Figure 7.8: Shell model local buckling evolution according to the end load percentage (G1L2).

Moreover, Figure 7.7(b) results show a quite different stiffness response between the beam and shell models. While the beam model presents a smooth softening nonlinear response, the shell model presents two apparently different responses with model stiffening at approximately $1.3\text{E}+5$ N (approximately at 34% of the end load).

The initial difference between the beam and shell models stiffness in this plot is likely to be due to the difference between the geometric centers evaluation, and also may be related to differences in shear center of both models, which may lead to distinct torsion-induced behavior, which can be seen in Figure 7.7(d), which shows the pilot node axial (z) rotation along load evolution. Moreover, despite of the results of Figure 7.7(a) that shows a clear divergence between the models at $1.8\text{E}+5$ N, the buckling effects starts to play a role at $1.3\text{E}+5$ N as presented in Figure 7.8(a).

In Figure 7.7(c) it is seen that the beam and shell models present a good concordance until a load around $1.8\text{E}+5$ N, from where the models present divergent responses. This response, as seen in Figure 7.7(a), looks to be associated to the local shell buckling of the structure.

Figure 7.9: L2 load case and G2 geometry results.

The results of the L2 load case with respect to the G2 model are presented in Figure 7.9.

It is seen in Figure 7.9(a) that the beam and shell models present a good concordance until approximately $2.0\text{E}+5$ N (approximately 71% of the end load). After that, the shell model stiffness decreases, indicating the same local buckling phenomena from previous analysis. Indeed, as for the G1 model, the visualization of the shell model showed a local buckling at the trailing edge of the airfoil, next to the root section. Figure 7.10 presents the local buckling evolution, according to the end load percentage applied.

Figure 7.10: Shell model local buckling evolution according to the end load percentage (G2L2).

It is seen in Figure 7.9(b) that both models present a good concordance, however divided in two phases. At phase 1, when the loads are below $2\text{E}+5$ N, the beam and shell models present very similar responses with the beam model stiffness a bit higher than the shell model. At phase 2, when the loads are above $2.0\text{E}+5$ N, both models stiffness's decreases and the shell model stiffness become higher than the beam model. Moreover, results indicate that at phase 1, there is a predominant bending behavior, while at phase 2 there is a predominant tension behavior. Additionally, the stiffness difference between the models is likely to be associated to the local shell buckling that starts even at loads

below $2.0\text{E}+5$ N as seen in Figure 7.10.

The result in Figure 7.9(c) indicates a very similar structural response between the models. Moreover, as the results of Figure 7.9(b), the structural behavior may be also divided in two phases by a load around $2.0\text{E}+5$ N. At phase 1 the structural responses of both models present a good concordance with very similar stiffness. However, at phase 2 the models present a slightly different structural response, being the shell model stiffness a bit higher than the beam model. Nevertheless, the local buckling of the shell model is likely to be associated to the stiffness difference between the models as seen in Figure 7.10.

7.2.4 Load Case 3 – Static Bending About X Axis

Results concerning the load case L3 and G1 model are presented in Figure 7.11. As seen in Figure 7.11(a), the beam and shell models presented, at this direction, moderately different structural responses, due to distinct stiffness since the beginning. The reason for that looks to be the same from previous model, due to distinct positions of shear center and consequences in differences of induced torsions, as one can see in Figure 7.11(d).

Figure 7.11: L3 load case and G1 geometry results.

The beam model shows an approximately linear behavior while the shell model starts

also with an approximately linear response and changes to a nonlinear response, thus achieving an almost infinite stiffness at the end (due to low induced torsion, which is the responsible for this effect). The analysis of this simulation showed that there is a small local buckling of the shell model near the root section as seen in Figure 7.12. Furthermore, the local buckling and the difference between the geometric centers of the models apparently played a role, especially at the small displacements at this direction.

Figure 7.12: Shell model local buckling evolution according to the end load percentage (G1L3).

Figure 7.13: Beam model in red and shell model in gray – deformed shape at 100% load (G1L3).

However, as seen in Figure 7.11(b) and Figure 7.11(c) the beam and shell models presented very similar responses at these directions, even describing the same geometrically nonlinear behavior. Therefore, it is reasonable to assume that the small local buckling of the shell model did not play a role on the overall displacements of the model, since there is no abruptly change at the structural response of the shell model, as in previous example load case. Moreover, it is important to note that the beam model could reproduce

with a very good accuracy the shell model response even for a high nonlinear response. Figure 7.13 presents the overlapping of the beam model in red and the shell model in gray at the end of the simulation.

Figure 7.14: L3 load case and G2 geometry results.

The results for the L3 load case and G2 model are presented in Figure 7.14. The result of Figure 7.14(a) presents a similar response of the beam and shell models, however with again a difference in stiffness of both models, from beginning. Again, this can be explained by the differences in induced torsion in both models, as seen in Figure 7.14(d). This response enforces the hypothesis of the difference between the geometric centers and shear centers. Moreover, unlike the result of Figure 7.11(a), the shell model response presents no abrupt change. In fact, the analysis of the shell model at this simulation shows no local buckling.

Nevertheless, the results of Figure 7.14(b) and Figure 7.14(c) presents, as the results of Figure 7.11(b) and Figure 7.11(c), a very good concordance between the beam and shell models, even for high nonlinear responses. The overlapping of the beam model in red and the shell model in gray is seen at the end of the simulation is seen in Figure 7.15.

Figure 7.15: Beam model in red and shell model in gray – deformed shape at 100% load (G2L3).

7.2.5 Load Case 4 – Static Torsion

The result of the L4 load case with respect to the G1 model is presented in Figure 7.16, which shows that beam and shell models presented very similar structural responses. However, the beam model presented an apparently linear response, while the shell model presented a slightly nonlinear response. Moreover, the analysis showed the shell model presented a complete buckling at the trailing edge. Therefore, it is reasonable to assume that the buckling of the shell model played an important role at the nonlinear response. Figure 7.17 presents the buckling evolution at the trailing edge of the shell model.

Figure 7.16: L4 load case and G1 geometry results.

Nevertheless, it is important to mention that regardless the beam model assumption of rigid cross-section, the beam model could present a good accuracy with respect to the shell model. Moreover, the torsional inertia calculated through the thin-walled hypothesis at the *WindTurbine* tool showed to be very accurate in comparison with the shell model.

Figure 7.17: Shell model local buckling evolution according to the end load percentage (G1L4).

The result of the L4 load case with respect to the G2 model is presented in Figure 7.18, which shows that both models presented an apparently linear structural response. Furthermore, the analysis of the shell model presented no local buckling.

Figure 7.18: L4 load case and G2 geometry results.

However, the torsional inertia of the models seems to be quite different since they rapidly diverge. For the G2 model, the chord of the airfoil reduces from 2 m at the root section to 0.6 m at the tip section while the thickness of each airfoil part is kept. This

condition leads, at the tip airfoil section, to a thickness ratio of

$$\frac{t_{max}}{b} = \frac{0.058}{0.126} = 0.46, \quad (7.2)$$

which strictly does not respect the thin-walled hypothesis originally assumed for the *WindTurbine* tool. Furthermore, as the chord reduces along the blade, not just the airfoil tip section, but also a substantial part of the blade does not respect the thin-walled hypothesis. Consequently, it is possible to conclude that the violation of the thin-walled hypothesis disqualifies the beam model results for this simulation.

Additionally, it is reasonable to assume that the violation of the thin-walled hypothesis plays a role in every simulation involving the G2 beam model. However, the good accuracy seen in previous analysis results shows that this hypothesis violation only plays a major role when there is relevant torsion at the model.

7.2.6 Load Case 5 – Dynamic “8” Circuit

Results regarding the dynamic simulation for the G1 model are presented in Figure 7.19, which shows that beam and shell models presented similar tip displacement shapes. However, the shell model presented an apparently smaller stiffness than the beam model, especially at the horizontal displacements. The models divergence is mainly seen at the 1st and 3rd quadrants.

Figure 7.19: L5 load case and G1 geometry blade tip displacement results. Prescribed displacements at the root cross-section.

Furthermore, a more accurately analysis between the beam and shell responses models may be done through Figure 7.20. Figure 7.20(a) presents the structural response of the

models along time for the x direction and Figure 7.20(b) presents the structural response of the models along time for the y direction.

(a) x direction displacements along time

(b) y direction displacements along time

Figure 7.20: Structural response along time for the L5 load case and G1 geometry blade tip displacement results. Prescribed displacements at the root cross-section.

Figure 7.20(a) shows that the beam and shell models have an almost analogue structural response until approximately 0.95 s of simulation. From this point, the shell model amplitude increases and virtually diverges from the beam model. Moreover, the analysis shows that the shell model presents local buckling not only at 0.95 s, but also around at 0.35s with a small intensity. Figure 7.21 presents the shell model local buckling.

(a) 0.30 s

(b) 0.32 s

(c) 0.35 s

(d) 0.86 s

(e) 0.90 s

(f) 0.95 s

Figure 7.21: Shell model buckling at dynamic simulation (G1L5).

However, Figure 7.20(b) shows that, at y direction, the beam and shell models present a very good accuracy. It is interesting to note that the local buckling of the shell model at approximately 0.95 s does not play a role at the structural response of the model at this direction. Thus, this response indicates that the shell model local buckling occurs mainly at the x direction.

The results regarding the dynamic simulation for the G2 model are presented in Figure 7.22, which shows that the beam and shell models presented similar displacement shapes. However, the shell model presents an apparently smaller stiffness than the beam model and, therefore, higher amplitudes.

Figure 7.22: L5 load case and G2 geometry blade tip displacement results. Prescribed displacements at the root cross-section.

Nevertheless, the structural response of the models may be more precisely evaluated through the results of Figure 7.23. Figure 7.23(a) and Figure 7.23(b) show the structural response of the beam and shell models with respect to the x displacements and y displacements respectively. Figure 7.23(a) shows that the beam and shell models have a good concordance until approximately 0.85 s of simulation.

From that point on, the shell model presents a substantially higher horizontal displacement and, after, a quite different structural response in comparison to the beam model. In fact, the analysis of the shell model showed that there is a local buckling not only at 0.85 s, but also at 0.35 s as seen in Figure 7.24. Moreover, the small intensity of the shell model buckling at 0.35 s apparently did not play an important role regarding the resulting displacements.

(a) x direction displacements along time

(b) y direction displacements along time

Figure 7.23: Structural response along time for the L5 load case and G2 geometry blade tip displacement results. Prescribed displacements at the root cross-section.

Furthermore, as seen in Figure 7.23(b), the beam and shell models present very similar structural responses, even regarding the high amplitudes and the shell model's buckling involved. Therefore, it is possible to conclude that the shell model buckling affects more the x than y direction displacements.

(a) 0.30 s

(b) 0.33 s

(c) 0.35 s

(d) 0.80 s

(e) 0.83 s

(f) 0.85 s

Figure 7.24: Shell model buckling of the dynamic simulation (G2L5).

8 Conclusions

The main conclusion of this thesis is that, for the proposed geometries, once respected the beam modeling hypothesis adopted, the geometrically-exact beam FEM model displacements/rotations matches with a very good accuracy the geometrically-exact shell FEM model overall structural response. On the context of this thesis two beam modeling hypothesis played a major role concerning the simulation results, the rigid cross-section hypothesis and the thin-walled hypothesis.

The rigid cross-section beam hypothesis played a very important role when there is local buckling at the corresponding shell model. In fact, as the beam model is unable to capture local buckling effects, the beam model presented higher stiffness and unreliable structural response in comparison with the corresponding shell model. However, when the shell model presents no local buckling or even small buckling, the beam and shell models presented very similar structural responses. Therefore, regarding this hypothesis, it is possible to conclude that the beam model is reliable and adequate since no local buckling is pronounced in the shell model.

The thin-walled hypothesis revealed also as a very strong hypothesis, especially on simulations when the torsional inertia plays a major role. Despite of the good accuracy of major simulations involving a variable cross-section along length, for representing a wind turbine blade, when the thin-walled hypothesis is not respected an error is made in overall structure torsional inertia behavior. In fact, the thin-walled hypothesis is highly attached to the shear flow and the torsion inertia concepts. However, when the load case in analysis presents quite low torsional effects, the beam model showed very similar structural responses. Additionally, once respected the thin-walled hypothesis, the discretization of the blade geometry into straight parts did not play a major role in the torsional inertia calculations or even at other geometric properties.

Furthermore, the combination of the geometrically-exact beam FEM formulation incorporated on *Giraffe* software and the cross-section geometric properties obtained through the *WindTurbine* tool presented very robust results in comparison to the shell geometrically-exact FEM results.

Some possible future works of this thesis are:

- Adoption of a more robust beam and/or shell formulation concerning the composite materials actually used on modern wind turbine blades.

- Adoption of a more realistic and complex wind turbine blade geometry, including an initial curvature pattern for the blade reference axis shape.
- Adoption of more realistic aerodynamic loads.
- Adoption of variable thicknesses at the blade geometry definition.

References

- [1] Wind Power Chalks Up More Strong Numbers. <http://www.gwec.net/global-figures/graphs/>, 2017. Accessed 2017-05-04.
- [2] Vestas Multi-Rotor Wind Turbine: 3 Blades Good, 12 Blades Better. <https://cleantechnica.com/2016/07/05/vestas-multi-rotor-wind-turbine-3-blades-good-12-blades-better/>, 2017. Accessed 2017-07-06.
- [3] Loading of Alstom’s 6MW Haliade at Ostend. <http://www.offshorewindindustry.com/gallery/loading-alstoms-6mw-haliade-ostend>, 2017. Accessed 2017-07-06.
- [4] J Jonkman. NREL’s S809 Airfoil Graphic and Coordinates. <http://wind.nrel.gov/airfoils/shapes/S809{ }Shape>. Accessed 2017-05-04.
- [5] James F Manwell, Jon G McGowan, and Anthony L Rogers. *Wind energy explained: theory, design and application*. John Wiley & Sons, 2010.
- [6] G Bir. User’s Guide to PreComp (Pre-Processor for Computing Composite Blade Properties). (September):31, 2006.
- [7] Ginger Gardiner. HAWTs vs. VAWTs. <http://www.compositesworld.com/articles/hawts-vs-v>, 2017.
- [8] DTU 10-MW Reference Wind Turbine. <http://www.hawc2.dk/download/hawc2-model/dtu-10-mw-reference-wind-turbine>, 2017. Accessed 2017-05-04.
- [9] Vestas. <https://www.vestas.com/>, 2017. Accessed 2017-06-07.
- [10] E. Hau and H. von Renouard. *Wind Turbines: Fundamentals, Technologies, Application, Economics*. SpringerLink : Bücher. Springer Berlin Heidelberg, 2013.
- [11] Alstom. <http://www.alstom.com/>. Accessed 2017-07-06.
- [12] J C Berg and B R Resor. Numerical manufacturing and design tool (NuMAD v2. 0) for wind turbine blades: user’s guide 2012. *SAND2012-7028*.
- [13] David Verelst. Flexible Wind turbine Blades: a BEM-FEM coupled model approach. *Delft University of Technology (TU Delft)*, (January), 2009.

- [14] ABAQUS User’s Manual - Version 6.5. *Inc., Pawtucket, RI*, 2004.
- [15] Patrick J Moriarty and A Craig Hansen. *AeroDyn theory manual*. Citeseer, 2005.
- [16] Dong Ok Yu and Oh Joon Kwon. Predicting wind turbine blade loads and aeroelastic response using a coupled CFD–CSD method. *Renewable Energy*, 70:184–196, 2014.
- [17] M. C. Hsu and Y. Bazilevs. Fluid-structure interaction modeling of wind turbines: Simulating the full machine. *Computational Mechanics*, 50(6):821–833, 2012.
- [18] Khazar Hayat, Alvaro Gorostidi Martinez de Lecea, Carlos Donazar Moriones, and Sung Kyu Ha. Flutter performance of bend-twist coupled large-scale wind turbine blades. *Journal of Sound and Vibration*, 370:149–162, 2014.
- [19] Lin Wang, Robin Quant, and Athanasios Kolios. Fluid structure interaction modelling of horizontal-axis wind turbine blades based on CFD and FEA. *Journal of Wind Engineering and Industrial Aerodynamics*, 158:11–25, 2016.
- [20] ANSYS User’s Manual. *Inc. Modeling, CFX*, 11, 2000.
- [21] John Montesano, Hao Chu, and Chandra Veer Singh. Development of a physics-based multi-scale progressive damage model for assessing the durability of wind turbine blades. *Composite Structures*, 141:50–62, 2016.
- [22] Lin Wang, Athanasios Kolios, Takafumi Nishino, Pierre Luc Delafin, and Theodore Bird. Structural optimisation of vertical-axis wind turbine composite blades based on finite element analysis and genetic algorithm. *Composite Structures*, 153(January 2015):123–138, 2016.
- [23] Lin Wang, Xiongwei Liu, and Athanasios Kolios. State of the art in the aeroelasticity of wind turbine blades: Aeroelastic modelling. *Renewable and Sustainable Energy Reviews*, 64:195–210, 2016.
- [24] M. O. L. Hansen, J. N. Sørensen, S. Voutsinas, N. Sørensen, and H. A. Madsen. State of the art in wind turbine aerodynamics and aeroelasticity. *Progress in Aerospace Sciences*, 42(4):285–330, 2006.

- [25] Pinting Zhang and Shuhong Huang. Review of aeroelasticity for wind turbine: Current status, research focus and future perspectives. *Frontiers in Energy*, 5(4):419–434, 2011.
- [26] Torben Juul Larsen and Anders Melchior Hansen. *How 2 HAWC2, the user’s manual*, volume 1597. 2007.
- [27] Lin Wang, Xiongwei Liu, Nathalie Renevier, Matthew Stables, and George M. Hall. Nonlinear aeroelastic modelling for wind turbine blades based on blade element momentum theory and geometrically exact beam theory. *Energy*, 76:487–501, 2014.
- [28] Dewey H Hodges and D Ph. Nonlinear Beam Theory for Engineers Georgia Institute of Technology. 1994.
- [29] Q Wang, N Johnson, M A Sprague, and J Jonkman. Beamdyn: A high-fidelity wind turbine blade solver in the FAST modular framework. *33rd Wind Energy Symposium 2015*, (January), 2015.
- [30] Celso Jaco Faccio Junior and Alfredo Gay Neto. Dynamics of Wind Turbine Blades Using a Geometrically-Exact Beam Formulation. In *ECCOMAS Congress 2016*, number June, pages 5–10, Crete Island, Greece, 2016.
- [31] Alfredo Gay Neto. Generic Interface Readily Accessible for Finite Elements User’s Manual v 1.0.173. pages 1–70, 2016.
- [32] O. A. Bauchau, C. L. Bottasso, and Y. G. Nikishkov. Modeling rotorcraft dynamics with finite element multibody procedures. *Mathematical and Computer Modelling*, 33(10-11):1113–1137, 2001.
- [33] Alfredo Gay Neto and Clóvis de Arruda Martins. Structural stability of flexible lines in catenary configuration under torsion. *Marine Structures*, 34:16–40, 2013.
- [34] Alfredo Gay Neto, Clóvis A. Martins, and Paulo M. Pimenta. Static analysis of offshore risers with a geometrically-exact 3D beam model subjected to unilateral contact. *Computational Mechanics*, 53(1):125–145, 2014.
- [35] Alfredo Gay Neto, Eduardo Ribeiro Malta, and Paulo M. Pimenta. Catenary riser sliding and rolling on seabed during induced lateral movement. *Marine Structures*, 41:223–243, 2015.

- [36] Eduardo M B Campello. Análise Não-Linear de Perfis Metálicos Conformados a Frio. 2000.
- [37] Eduardo M B Campello and Leonardo B. Lago. Effect of higher order constitutive terms on the elastic buckling of thin-walled rods. *Thin-Walled Structures*, 77:8–16, 2014.
- [38] David Manta and Rodrigo Gonçalves. A geometrically exact Kirchhoff beam model including torsion warping. *Computers & Structures*, 177:192–203, 2016.
- [39] Alfredo Gay Neto. Simulation of mechanisms modeled by geometrically-exact beams using Rodrigues rotation parameters. *Computational Mechanics*, 2016.
- [40] O. a. Bauchau, G. Damilano, and N.J. Theron. Numerical Integration of Nonlinear Elastic Multibody Systems, 1995.
- [41] A. Cardona, M. Geradin, and D. B. Doan. Rigid and flexible joint modelling in multibody dynamics using finite elements. *Computer Methods in Applied Mechanics and Engineering*, 89(1-3):395–418, 1991.
- [42] Adnan Ibrahimbegović and Saïd Mamouri. On rigid components and joint constraints in nonlinear dynamics of flexible multibody systems employing 3D geometrically exact beam model. *Computer Methods in Applied Mechanics and Engineering*, 188(4):805–831, 2000.
- [43] J. C. Simo. A finite strain beam formulation. The three-dimensional dynamic problem. Part I. *Computer Methods in Applied Mechanics and Engineering*, 49(1):55–70, 1985.
- [44] Eric Reissner. On one-dimensional finite-strain beam theory: The plane problem. *Zeitschrift für angewandte Mathematik und Physik ZAMP*, 23(5):795–804, 1972.
- [45] Augustus Edward Hough Love. A treatise on the mathematical theory of elasticity. 1, 1892.
- [46] Paulo M Pimenta and Takashi Yojo. Geometrically Exact Analysis of Spatial Frames. *Applied Mechanics Reviews*, 46(11S):S118–S128, 1993.

- [47] Takashi Yojo. *Análise Não-Linear Geometricamente Exata de Pórticos Espaciais*. PhD thesis, 1993.
- [48] J. C. Simo and L. Vu-Quoc. A Geometrically-exact rod model incorporating shear and torsion-warping deformation. *International Journal of Solids and Structures*, 27(3):371–393, 1991.
- [49] Paulo M. Pimenta and Eduardo M B Campello. Geometrically nonlinear analysis of thin-walled space frames. In *2nd European Congress on Computational Mechanics*, 2001.
- [50] Alfredo Gay Neto. Dynamics of offshore risers using a geometrically-exact beam model with hydrodynamic loads and contact with the seabed. *Engineering Structures*, 125:438–454, 2016.
- [51] V. Z. Vlasov. *Thin-Walled-Elastic-Beams*, 1961.
- [52] N. S N Ota, L. Wilson, A. Gay Neto, S. Pellegrino, and P. M. Pimenta. Nonlinear dynamic analysis of creased shells. *Finite Elements in Analysis and Design*, 121:64–74, 2016.
- [53] A E H Love. The Small Free Vibrations and Deformation of a Thin Elastic Shell. *Philosophical Transactions of the Royal Society of London. A*, 179:491–546, 1888.
- [54] G. Kirchhoff. Über das Gleichgewicht und die Bewegung einer elastischen Scheibe. pages 51 – 88, 1826.
- [55] E Reissner. Stress strain relations in the theory of elastic shells. *Journal of Mathematical Physics*, 31:109–119, 1952.
- [56] L. H. Donnel. Stability of Thin-Walled Tubes Under Torsion. Technical report, California Institute of Technology, Pasadena, USA, 1933.
- [57] J. L. Sanders. An Improved First-Approximation Theory for Thin Shells. pages 1–11, 1959.
- [58] Wilhelm Flügge. *General Properties of Stress Systems in Shells*, pages 1–18. Springer Berlin Heidelberg, Berlin, Heidelberg, 1960.

- [59] K M Mushtari and K Z Galimov. *Non-linear theory of thin elastic shells*. NASA technical translation. Academy of Sciences, U.S.S.R. Kazan' Branch, 1957.
- [60] J. L. Sanders Jr. Nonlinear Theories For Thin Shells. Technical report, 1961.
- [61] P M Naghdi and R P Nordgren. On the nonlinear theory of elastic shells under the Kirchhoff hypothesis. *Quarterly of Applied Mathematics*, 21:49–59, 1963.
- [62] J. C. Simo, M. S. Rifai, and D. D. Fox. On a stress resultant geometrically exact shell model. Part VI: Conserving algorithms for non-linear dynamics. *International Journal for Numerical Methods in Engineering*, 34(1):117–164, mar 1992.
- [63] E. M B Campello, P. M. Pimenta, and P. Wriggers. A triangular finite shell element based on a fully nonlinear shell formulation. *Computational Mechanics*, 31(6):505–518, 2003.
- [64] W.D. Pilkey. *Analysis and Design of Elastic Beams: Computational Methods*. Wiley, 2002.
- [65] Y.C. Fung. *An Introduction to the Theory of Aeroelasticity*. Dover Phoenix Edition: Engineering. Dover Publications, 2002.
- [66] Mohamed Shama. *Torsion and Shear Stresses in Ships*. Springer Berlin Heidelberg, Berlin, Heidelberg, 2011.
- [67] T. H. G. Megson. *Introduction to Aircraft Structural Analysis*. Butterworth-Heinemann, 2nd editio edition, 2014.
- [68] M. T. Saito and A. Gay Neto. Computational Simulation of Woven Fabrics Using Beam-to-Beam Contact Formulation. In *CILAMCE 2016*, Brasília, 2016.
- [69] T. F. M. Pereira and A. Gay Neto. Computational Model to Evaluate Actions in Railwheel Contact Interaction. In *CILAMCE 2016*, Brasília, 2016.
- [70] Dan M Somers. Design and Experimental Results for the S809 Airfoil. (January 1997):104, 1997.
- [71] Robert M Jones. *Mechanics of Composite Materials*, 1999.

- [72] Stephen P Timoshenko and Sergius Woinowsky-Krieger. *Theory of plates and shells*. McGraw-hill, 1959.
- [73] R.D. Blevins. *Flow-induced vibration*. Van Nostrand Reinhold, 1990.
- [74] NREL's S809 Airfoil (s809-nr). <http://airfoiltools.com/airfoil/details?airfoil=s809-nr>, 2017. Accessee: 2017-07-12.
- [75] Mark Drela. Xfoil: An analysis and design system for low reynolds number airfoils. In *Low Reynolds number aerodynamics*, pages 1–12. Springer, 1989.

Appendices

A *WindTurbine* Tool Verification

In order to verify the originally developed *WindTurbine* tool several comparison tests were made with the well-known ANSYS solver. Therefore, each test consisted basically in the calculation of geometric properties by both software and in the assessment of the ratio between these values. Even considering the simplifying hypotheses, the results of the *WindTurbine* tool showed a good agreement when compared to ANSYS results. Table A.1 summarizes the cross-sections adopted at the comparative tests. The results are presented as follows.

Section	Dimension	Thickness	N. of Webs	Web Thick.	Angle	Name
Box	100	5	0	5	0	S1EX1
		5	1	5	0	S1EX2
		5	2	5	0	S1EX3
		5	0	5	30	S1EX4
		5	1	5	30	S1EX5
		5	2	5	30	S1EX6
Triangle	100	5	0	0	0	S2EX1
		5	0	0	30	S2EX2
		5	0	0	60	S2EX3
		5	0	0	90	S2EX4
Arrow	100	5	0	0	0	S3EX1
		1	0	0	0	S3EX1b
		5	0	0	-30	S3EX2
		1	0	0	-30	S3EX2b
		5	0	0	-60	S3EX3
		1	0	0	-60	S3EX3b
		5	0	0	-90	S3EX4
		1	0	0	-90	S3EX4b
NREL S809	100	1	0	0	0	S4EX1
		1	1	2	0	S4EX2
		1	2	2	0	S4EX3
		1	2	2	120	S4EX4
		1	2	2	150	S4EX5
		1	2	2	270	S4EX6
NREL S805A	100	0.3	0	0	0	S5EX1
		0.3	1	0.4	0	S5EX2
		0.3	2	0.4	0	S5EX3
		0.3	2	0.4	120	S5EX4
		0.3	2	0.4	150	S5EX5
		0.3	2	0.4	270	S5EX6
NREL S807	100	1	0	0	0	S6EXA1
		0.7	0	0	0	S6EXA2
		0.5	0	0	0	S6EXA3
		0.1	0	0	0	S6EXA4
Generic	100	1	0	0	0	S7EX1
		1	2	1	0	S7EX2
		0.5	2	0.5	0	S7EX3

Table A.1: Cross-section definitions for comparative tests between the *WindTurbine* tool and ANSYS.

Figure A.1: Section 1 - box.

S1EX1				S1EX2		
	WindTurbine	ANSYS	WT/ANSYS	WindTurbine	ANSYS	WT/ANSYS
A	2000.00	2000.00	100%	2250.00	2225.00	101%
g_1	25.00	25.00	100%	22.22	22.47	99%
g_2	25.00	25.00	100%	25.00	25.00	100%
s_1	25.00	25.00	100%	19.47	19.84	98%
s_2	25.00	25.00	100%	25.00	25.00	100%
S_1	50000.00	50000.00	100%	56250.00	55625.00	101%
S_2	50000.00	50000.00	100%	50000.00	49999.98	100%
I_1	1044791.67	1050000.00	100%	1096875.00	1090000.00	101%
I_2	5626041.67	5640000.00	100%	5765451.39	5760000.00	100%
I_{12}	0.00	0.00	0%	0.00	0.00	0%
I_0	6670833.33	6690000.00	100%	6862326.39	6850000.00	100%
I_t	2812500.00	2980000.00	94%	2826086.96	2910000.00	97%
S1EX3				S1EX4		
	WindTurbine	ANSYS	WT/ANSYS	WindTurbine	ANSYS	WT/ANSYS
A	2500.00	2450.00	102%	2000	2000.00	100%
g_1	25.00	25.00	100%	9.150635095	9.15	100%
g_2	25.00	25.00	100%	34.15063509	34.15	100%
s_1	25.00	25.00	100%	9.150635095	9.15	100%
s_2	25.00	25.00	100%	34.15063509	34.15	100%
S_1	62500.00	61250.00	102%	68301.27019	68301.27	100%
S_2	62500.00	61250.00	102%	18301.27019	18301.27	100%
I_1	1148958.33	1120000.00	103%	2190104.167	2197500	100%
I_2	5939583.33	5920000.00	100%	4480729.167	4492500	100%
I_{12}	0.00	0.00	0%	1983739.441	1987528.302	100%
I_0	7088541.67	7040000.00	101%	6670833.333	6480028.30	103%
I_t	2857142.86	2950000.00	97%	2812500	2980000.00	94%

Table A.2: Section 1 geometric properties (continues).

	S1EX5			S1EX5		
	Wind Turbine	ANSYS	WT/ANSYS	Wind Turbine	ANSYS	WT/ANSYS
A	2250	2225.00	101%	2500	2450.00	102%
g_1	6.745008973	6.96	97%	9.150635095	9.15	100%
g_2	32.76174621	32.89	100%	34.15063509	34.15	100%
s_1	4.361135695	4.68	93%	9.150635095	9.15	100%
s_2	31.38541633	31.57	99%	34.15063509	34.15	100%
S_1	73713.92896	73172.65	101%	85376.58774	83669.06	102%
S_2	15176.27019	15488.75	98%	22876.58774	22419.06	102%
I_1	2264019.097	2257500.00	100%	2346614.583	2346614.58	100%
I_2	4598307.292	4592500.00	100%	4741927.083	4741927.08	100%
I_{12}	2021552.876	2022169.32	100%	2074401.475	2074401.48	100%
I_0	6862326.389	6850000.00	100%	7088541.667	7088541.67	100%
I_t	2826086.957	2910000.00	97%	2857142.857	2950000.00	97%

Table A.3: Section 1 geometric properties.

Figure A.2: Section 2 - triangle.

S2EX1				S2EX2		
	Wind Turbine	ANSYS	WT/ANSYS	Wind Turbine	ANSYS	WT/ANSYS
A	1707.11	1707.20	100%	1707.11	1707.20	100%
g_1	64.64	64.63	100%	38.31	38.31	100%
g_2	35.36	35.37	100%	62.94	62.94	100%
s_1	70.71	70.51	100%	46.59	46.59	100%
s_2	29.29	29.49	99%	60.72	60.72	100%
S_1	60355.34	60385.20	100%	107446.93	107452.79	100%
S_2	110355.34	110334.80	100%	65392.86	65396.43	100%
I_1	1891584.03	1900000.00	100%	2718322.72	2733323.42	99%
I_2	1891584.03	1900000.00	100%	1064845.35	1066676.58	100%
I_{12}	954635.60	962239.00	99%	477317.80	481119.50	99%
I_0	3783168.06	3800000.00	100%	3783168.06	3800000.00	100%
I_t	1464466.09	1550000.00	94%	1464466.09	1550000.00	94%
S2EX3				S2EX4		
	Wind Turbine	ANSYS	WT/ANSYS	Wind Turbine	ANSYS	WT/ANSYS
A	1707.11	1707.20	100%	1707.11	1707.20	100%
g_1	1.70	1.68	101%	-35.36	-35.37	100%
g_2	73.66	73.66	100%	64.64	64.63	100%
s_1	9.99	9.72	103%	-29.29	-29.49	99%
s_2	75.88	75.81	100%	70.71	70.51	100%
S_1	125748.20	125745.34	100%	110355.34	110334.80	100%
S_2	2908.41	2872.28	101%	-60355.34	-60385.20	100%
I_1	2718322.72	2733323.42	99%	1891584.03	1900000.00	100%
I_2	1064845.35	1066676.58	100%	1891584.03	1900000.00	100%
I_{12}	-477317.80	-481119.50	99%	-954635.60	-962239.00	99%
I_0	3783168.06	3800000.00	100%	3783168.06	3800000.00	100%
I_t	1464466.09	1550000.00	94%	1464466.09	1550000.00	94%

Table A.4: Section 2 geometric properties.

Figure A.3: Section 3 - arrow.

S3EX1				S3EX1b		
	Wind Turbine	ANSYS	WT/ANSYS	Wind Turbine	ANSYS	WT/ANSYS
A	1800.00	1799.23	100%	360.00	359.97	100%
g_1	61.11	61.25	100%	61.11	61.11	100%
g_2	60.00	60.00	100%	60.00	60.00	100%
s_1	37.81	38.98	97%	37.78	38.01	99%
s_2	60.00	60.00	100%	60.00	60.00	100%
S_1	107999.97	107953.80	100%	21599.99	21598.14	100%
S_2	109999.97	110193.84	100%	21999.99	21999.07	100%
I_1	1709002.37	1740000.00	98%	341351.53	341512.00	100%
I_2	1412522.59	1440000.00	98%	282233.46	282447.00	100%
I_{12}	0.00	0.00	0%	0.00	0.00	0%
I_0	3121524.96	3180000.00	98%	623584.99	623959.00	100%
I_t	568888.31	647561.00	88%	113777.66	116681.00	98%
S3EX2				S3EX2b		
	Wind Turbine	ANSYS	WT/ANSYS	Wind Turbine	ANSYS	WT/ANSYS
A	1800.00	1799.23	100%	360.00	359.97	100%
g_1	82.92	83.04	100%	82.92	82.93	100%
g_2	21.41	21.34	100%	21.41	21.40	100%
s_1	62.74	63.75	98%	62.72	62.91	100%
s_2	33.06	32.47	102%	33.07	32.96	100%
S_1	38530.74	38393.81	100%	7706.15	7705.00	100%
S_2	149262.76	149407.57	100%	29852.55	29850.83	100%
I_1	1634882.42	1665000.00	98%	326572.01	326745.75	100%
I_2	1486642.54	1515000.00	98%	297012.98	297213.25	100%
I_{12}	128379.51	129903.81	99%	25598.88	25575.90	100%
I_0	3121524.96	3180000.00	98%	623584.99	623959.00	100%
I_t	568888.31	647561.00	88%	113777.66	116681.00	98%

Table A.5: Section 3 geometric properties (continues).

S3EX3				S3EX3b		
	Wind Turbine	ANSYS	WT/ANSYS	Wind Turbine	ANSYS	WT/ANSYS
A	1800.00	1799.23	100%	360.00	359.97	100%
g_1	82.52	82.58	100%	82.52	82.52	100%
g_2	-22.92	-23.04	99%	-22.92	-22.93	100%
s_1	70.87	71.45	99%	70.85	70.96	100%
s_2	-2.74	-3.75	73%	-2.72	-2.91	93%
S_1	-41262.78	-41453.77	100%	-8252.56	-8252.69	100%
S_2	148530.71	148587.65	100%	29706.14	29704.07	100%
I_1	1486642.54	1515000.00	98%	297012.98	297213.25	100%
I_2	1634882.42	1665000.00	98%	326572.01	326745.75	100%
I_{12}	128379.51	129903.81	99%	25598.88	25575.90	100%
I_0	3121524.96	3180000.00	98%	623584.99	623959.00	100%
I_t	568888.31	647561.00	88%	113777.66	116681.00	98%
S3EX4				S3EX4b		
	Wind Turbine	ANSYS	WT/ANSYS	Wind Turbine	ANSYS	WT/ANSYS
A	1800.00	1799.23	100%	360.00	359.97	100%
g_1	60.00	60.00	100%	60.00	60.00	100%
g_2	-61.11	-61.25	100%	-61.11	-61.11	100%
s_1	60.00	60.00	100%	60.00	60.00	100%
s_2	-37.81	-38.98	97%	-37.78	-38.01	99%
S_1	-109999.97	-110193.84	100%	-21999.99	-21999.07	100%
S_2	107999.97	107953.80	100%	21599.99	21598.14	100%
I_1	1412522.59	1440000.00	98%	282233.46	282447.00	100%
I_2	1709002.37	1740000.00	98%	341351.53	341512.00	100%
I_{12}	0.00	0.00	0%	0.00	0.00	0%
I_0	3121524.96	3180000.00	98%	623584.99	623959.00	100%
I_t	568888.31	647561.00	88%	113777.66	116681.00	98%

Table A.6: Section 3 geometric properties.

Figure A.4: Section 4 - NREL S809.

S4EX1				S4EX2			
	Wind Turbine	ANSYS	WT/ANSYS	Wind Turbine	ANSYS	WT/ANSYS	
A	206.80	203.70	102%	250.69	245.32	102%	
g_1	49.20	48.52	101%	45.57	45.12	101%	
g_2	0.29	0.29	99%	0.31	0.30	101%	
s_1	27.66	31.06	89%	27.81	29.77	93%	
s_2	-0.05	-0.06	80%	-0.02	0.00	0%	
S_1	59.96	59.40	101%	76.70	74.02	104%	
S_2	10174.13	9883.76	103%	11423.18	11067.70	103%	
I_1	10100.53	10095.60	100%	11459.71	11251.80	102%	
I_2	177107.22	169749.00	104%	193097.87	184036.00	105%	
I_{12}	1671.04	1687.85	99%	2337.97	2278.13	103%	
I_0	187207.75	179844.60	104%	204557.58	195287.80	105%	
I_t	29361.11	30902.60	95%	29361.24	31123.10	94%	
S4EX3				S4EX4			
	Wind Turbine	ANSYS	WT/ANSYS	Wind Turbine	ANSYS	WT/ANSYS	
A	273.76	266.76	103%	273.76	266.76	103%	
g_1	46.76	46.29	101%	-23.62	-23.39	101%	
g_2	0.28	0.28	100%	40.35	39.94	101%	
s_1	37.57	38.09	99%	-18.89	-19.15	99%	
s_2	0.12	0.13	97%	32.47	32.93	99%	
S_1	76.97	75.08	103%	11046.52	10655.80	104%	
S_2	12799.86	12347.61	104%	-6466.59	-6238.83	104%	
I_1	11884.03	11592.50	103%	148926.76	146078.39	102%	
I_2	197340.04	188251.00	105%	60297.31	53765.11	112%	
I_{12}	2366.31	2300.18	103%	-81487.96	-75345.28	108%	
I_0	209224.07	199843.50	105%	209224.07	199843.50	105%	
I_t	33798.88	34812.30	97%	33798.88	34812.30	97%	

Table A.7: Section 4 geometric properties (continues).

S4EX5			S4EX6			
	Wind Turbine	ANSYS	WT/ANSYS	Wind Turbine	ANSYS	WT/ANSYS
A	273.76	266.76	103%	273.76	266.76	103%
g_1	-40.63	-40.23	101%	0.28	0.28	100%
g_2	23.13	22.90	101%	-46.76	-46.29	101%
s_1	-32.59	-33.05	99%	0.12	0.13	97%
s_2	18.68	18.94	99%	-37.57	-38.09	99%
S_1	6333.27	6108.78	104%	-12799.86	-12347.61	104%
S_2	-11123.49	-10730.88	104%	76.97	75.08	103%
I_1	56198.75	57749.14	97%	197340.04	188251.00	105%
I_2	153025.32	142094.36	108%	11884.03	11592.50	103%
I_{12}	-79121.65	-77645.46	102%	-2366.31	-2300.18	103%
I_0	209224.07	199843.50	105%	209224.07	199843.50	105%
I_t	33798.88	34812.30	97%	33798.88	34812.30	97%

Table A.8: Section 4 geometric properties.

Figure A.5: Section 5 - NREL S805A.

S5EX1				S5EX2		
	Wind Turbine	ANSYS	WT/ANSYS	Wind Turbine	ANSYS	WT/ANSYS
A	61.15	61.99	99%	67.54	68.22	99%
g_1	49.48	50.22	99%	47.17	47.92	98%
g_2	1.39	1.37	101%	1.48	1.46	101%
s_1	31.19	34.03	92%	29.59	31.56	94%
s_2	2.34	2.33	101%	2.42	2.43	100%
S_1	85.14	85.21	100%	100.16	99.81	100%
S_2	3025.99	3113.28	97%	3185.72	3269.03	97%
I_1	1465.01	1466.05	100%	1552.96	1548.01	100%
I_2	52135.40	54137.90	96%	55656.62	57792.50	96%
I_{12}	-459.61	-515.09	89%	-529.19	-591.76	89%
I_0	53600.41	55603.95	96%	57209.58	59340.51	96%
I_t	4642.12	4814.59	96%	4648.12	4835.02	96%
S5EX3				S5EX4		
	Wind Turbine	ANSYS	WT/ANSYS	Wind Turbine	ANSYS	WT/ANSYS
A	72.69	73.24	99%	72.69	73.24	99%
g_1	47.90	48.57	99%	-25.26	-25.58	99%
g_2	1.51	1.49	101%	40.73	41.32	99%
s_1	39.01	39.64	98%	-21.83	-22.12	99%
s_2	2.69	2.65	101%	32.44	33.00	98%
S_1	109.56	108.92	101%	2960.58	3026.37	98%
S_2	3481.84	3557.44	98%	-1835.80	-1873.05	98%
I_1	1613.93	1604.40	101%	43002.98	43555.83	99%
I_2	56178.38	58231.80	96%	14789.33	16280.37	91%
I_{12}	-537.76	-599.43	90%	-23358.22	-24820.10	94%
I_0	57792.31	59836.20	97%	57792.31	59836.20	97%
I_t	5064.42	5172.51	98%	5064.42	5172.51	98%

Table A.9: Section 5 geometric properties (continues).

	S5EX5			S5EX6		
	Wind Turbine	ANSYS	WT/ANSYS	Wind Turbine	ANSYS	WT/ANSYS
A	72.69	73.24	99%	72.69026586	73.24	99%
g_1	-42.24	-42.81	99%	1.507158136	1.49	101%
g_2	22.64	23.00	98%	-47.89962008	-48.57	99%
s_1	-35.13	-35.66	99%	2.689490677	2.65	101%
s_2	17.18	17.52	98%	-39.01015248	-39.64	98%
S_1	1646.04	1684.39	98%	-3481.836118	-3557.44	98%
S_2	-3070.14	-3135.30	98%	109.5557256	108.92	101%
I_1	15720.75	15242.13	103%	56178.38283	58231.80	96%
I_2	42071.56	44594.07	94%	1613.930518	1604.40	101%
I_{12}	-23895.98	-24220.67	99%	537.7565923	599.43	90%
I_0	57792.31	59836.20	97%	57792.31335	59836.20	97%
I_t	5064.42	5172.51	98%	5064.421771	5172.51	98%

Table A.10: Section 5 geometric properties.

Figure A.6: Section 6 - NREL S807.

S6EXA1				S6EXA2			
	Wind Turbine	ANSYS	WT/ANSYS	Wind Turbine	ANSYS	WT/ANSYS	
A	206.84	206.25	100%	144.79	144.78	100%	
g_1	48.97	48.99	100%	48.97	49.10	100%	
g_2	0.96	0.95	101%	0.96	0.95	101%	
s_1	20.57	23.91	86%	20.53	23.41	88%	
s_2	0.71	0.79	90%	0.71	0.77	92%	
S_1	198.87	196.01	101%	139.21	138.18	101%	
S_2	10129.80	10104.51	100%	7090.86	7108.94	100%	
I_1	7639.51	7641.57	100%	5341.85	5341.49	100%	
I_2	179237.72	179178.00	100%	125466.05	125980.00	100%	
I_{12}	2084.89	1952.45	107%	1459.40	1399.28	104%	
I_0	186877.23	186819.57	100%	130807.91	131321.49	100%	
I_t	22390.18	23558.20	95%	15673.13	16357.70	96%	
S6EXA3				S6EXA4			
	Wind Turbine	ANSYS	WT/ANSYS	Wind Turbine	ANSYS	WT/ANSYS	
A	103.42	103.60	100%	20.68	21.17	98%	
g_1	48.97	49.17	100%	48.97	50.20	98%	
g_2	0.96	0.96	101%	0.96	0.94	102%	
s_1	20.52	22.96	89%	20.51	22.47	91%	
s_2	0.71	0.76	93%	0.71	0.85	83%	
S_1	99.44	99.00	100%	19.89	20.00	99%	
S_2	5064.90	5094.15	99%	1012.98	1062.49	95%	
I_1	3813.66	3812.74	100%	762.34	762.21	100%	
I_2	89618.49	90280.80	99%	17923.67	19088.70	94%	
I_{12}	1042.42	1007.54	103%	208.48	190.77	109%	
I_0	93432.15	94093.54	99%	18686.02	19850.91	94%	
I_t	11195.09	11602.20	96%	2239.02	2300.24	97%	

Table A.11: Section 6 geometric properties.

Figure A.7: Section 7 - Generic.

S7EX1				S7EX2			
	Wind Turbine	ANSYS	WT/ANSYS	Wind Turbine	ANSYS	WT/ANSYS	
A	274.66	274.66	100%	339.65	339.65	100%	
g_1	50.75	50.75	100%	49.98	49.98	100%	
g_2	-0.27	-0.27	100%	1.22	1.22	100%	
s_1	62.40	62.16	100%	56.74	56.74	100%	
s_2	-5.20	-5.14	101%	-5.59	-5.59	100%	
S_1	-73.76	-73.86	100%	415.34	415.34	100%	
S_2	13938.20	13938.00	100%	16975.28	16974.98	100%	
I_1	66678.38	66712.10	100%	77585.90	77585.90	100%	
I_2	220789.18	220849.00	100%	244759.00	244759.00	100%	
I_{12}	-17442.20	-17441.00	100%	-19719.20	-19719.20	100%	
I_0	287467.56	287561.10	100%	322344.90	322344.90	100%	
I_t	96566.29	98264.30	98%	102222.00	102192.00	100%	
S7EX3							
	Wind Turbine	ANSYS	WT/ANSYS				
A	171.3279378	170.58	100%				
g_1	49.89904743	49.94	100%				
g_2	1.340248109	1.28	105%				
s_1	56.79941715	56.77	100%				
s_2	-5.905549366	-5.75	103%				
S_1	229.6219447	218.55	105%				
S_2	8549.100896	8518.31	100%				
I_1	39436.89255	39101.60	101%				
I_2	123066.5614	122713.00	100%				
I_{12}	-9971.033337	-9915.71	101%				
I_0	162503.454	161814.60	100%				
I_t	50044.76959	50570.50	99%				

Table A.12: Section 7 geometric properties.