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Gabriel Pino

**Fault location in transmission lines
using time-domain equations**

São Paulo
2018

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A master's thesis submitted to the Department of Energy and Electrical Automation of Polytechnic School of São Paulo for the degree of Master of Science in the subject of Power System Engineering.

Supervisor: Prof. Giovanni Manassero Junior, Ph.D.

São Paulo
2018

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Abstract

Pino, Gabriel **Fault location in transmission lines using time-domain equations**. 94 p. Master Thesis – Polytechnic School, University of São Paulo, 2018.

This thesis is a combination of the development of numerical models regarding transient simulation of transmission lines and their advantages associated with fault location methods. The transmission line models presented in this work are in time-domain, which is a new approach considering traditional methods as being the phasor and traveling wave techniques. The use of phasors for this purpose has some technical difficulties: the presence of a damped DC component, greater influence of the fault impedance and metrological equipment layout.

This work deals with single-phase AC and mono polar DC transmission lines. The proposed transmission line model has three main differentials compared to the traditional Bergeron model: full distribution of linear resistance; full distribution of leakage conductance; and just one recurrence of historical values. The first point is critical for evaluation of the exponential component of transient short circuit currents. The second point refers to the inclusion of the corona effect in the transmission line modeling. The single recurrence indicated in the third topic is given by the complete resolution of the telegrapher's equations, so there is no need of serial composition to improve waveform accuracy. The principle of fault location method is calculating the absolute difference between fault voltages seen by the transmission line ends. This technique guarantees a lower influence of the fault impedance and its electrical parameters.

Keywords: transmission lines, fault location, time-domain, electromagnetic transients.

Resumo

Pino, Gabriel **Localização de faltas em linhas de transmissão utilizando equações no domínio do tempo**. 94 p. Dissertação de mestrado – Escola Politécnica, Universidade de São Paulo, 2018.

Esta dissertação é uma combinação do desenvolvimento de modelos numéricos para simulação de transitórios eletromagnéticos em linhas de transmissão e suas benesses associadas à localização de faltas. Os modelos de linha de transmissão aqui apresentados estão no domínio do tempo, o que descaracteriza a abordagem tradicional de localização de faltas como técnicas fasoriais e ondas viajantes. A utilização de fasores para esse propósito admite algumas dificuldades técnicas: presença da componente DC amortecida, maior influência da impedância de falta e disposição dos equipamentos metrológicos.

Abordam-se linhas de transmissão de sistemas alternado monofásico e contínuo monopolar. A modelagem proposta possui três principais diferenciais frente ao modelo de Bergeron: plena distribuição da resistência linear; plena distribuição da condutância transversal; e apenas uma recorrência a valores históricos. O primeiro ponto é fundamental para avaliação da componente exponencialmente amortecida das correntes transitórias de curto circuito. O segundo ponto se refere à inclusão do efeito corona no modelamento. A recorrência unitária apontada no terceiro tópico apresenta a vantagem de não ser necessária a composição em série do modelo para aprimorar a qualidade das formas de onda.

O princípio de localização de faltas se dá pelo cálculo da diferença absoluta entre as tensões instantâneas de falta vistas pelos terminais da linha. Essa técnica garante uma menor influência de impedância de falta e dos parâmetros elétricos.

Palavras-chave: linhas de transmissão, localização de faltas, domínio do tempo, transitórios eletromagnéticos.

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Acronyms

AC alternating current

CT current transformer

DC direct current

FFT Fast Fourier Transform

HVDC high voltage direct current

ODE ordinary differential equation

PDE partial differential equation

SNR signal-to-noise ratio

TL transmission line

VT voltage transformer

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Introduction

THE TRANSMISSION LINE MODELING developed in this master's thesis dates back in 2012, when I started my undergraduate research program. At that time, I came across the first problems about voltage and current behavior under transient state in TL. The first challenge was to find out the analytical solution of the telegrapher's equations. The solution to this problem was getting closer while my interest to the theme increased. The nights spent at the Institute of Mathematics and Statistics of University of São Paulo were fundamental to this assignment.

Searching for a straightforward application to the model, I asked for a suggestion from Professor Manassero. Under his orientation during my term paper research, I had my introduction to fault location algorithms. At the time, the classic Bergeron model was implemented in the algorithms and the proposed work of time-domain fault location method was successful (PINO, 2014).

In the beginning of the master's degree course, in 2015, the question remained whether to apply this method using the time-domain TLs model with fully distributed resistance and conductance. A more mature mathematical skill and a better understanding of the wave propagation phenomena resulted in the work described in this master's thesis.

The experience acquired during the master's course influenced my professional activity when I worked at *Voith Hydro*. The numerical time-domain techniques applied to fault location in TLs were decisive for better design of electrical protection systems in hydroelectric power plants and their associated substations. Nowadays, working as a development engineer at *Enline*, I can extend these methods to adaptive protection applications and more accurate fault location algorithms for HVDC lines.

1.1 Initial considerations

The high voltage equipment installed in substation yards, such as instrumentation transformer, switchgear and circuit breaker, are manufactured and tested to withstand electrical stresses as predicted by technical standards and manufacturer recommendations.

Among the range of operating limits, there are three that summarize the equipment behavior during electrical transients: rated power frequency withstand voltage, rated lightning impulse voltage, and asymmetric short circuit peak current. The proper design of these equipment depends on a detailed study of the electromagnetic transients. The design criteria depend on the voltage and current waveforms considering adverse situations such as load rejection, energization, short circuit and atmospheric impulse.

In these studies, the consideration of TLs and their numerical models are crucial. They are usually modeled by distributed inductances and capacitances, although resistances are lumped at their ends (DOMMEL, 1969). This is pertinent to phenomena in which the propagation of voltage and current waves cannot be neglected, and is generally applied to long TLs. The parameters may have frequency dependence, a consideration that is usually recommended for studies involving atmospheric discharge phenomena.

The most popular time-domain model regarding TLs is the Bergeron model (BERGERON, 1961). It presents topographic decoupling between the TL ends and the numerical relation between them is given by current sources that depend on the historical values of voltages and currents. Besides the topographic decoupling, many works were focused on decoupling a multi-phase system in several single-phase systems, usually using the modal transform (HEDMAN, 1965; GUSTAVSEN; SEMLYEN, 1998; PRADO et al., 2007).

Electromagnetic transient studies often involve short circuit simulations (BAKER-JARVIS et al., 1992). Regarding the numerical simulation itself, the fault point is given and the terminal voltage and current are calculated. However, in a real situation the unknown variable of the problem is precisely the fault position, and it must be calculated based on the oscillographies of the electric measurements acquired at one or both ends of the TL.

Fault location in TLs is very important because the accurate location of where the short circuit occurred reduces interruption times, improves reliability and increases the operational safety of the power system. The sooner the location of a problem is determined, the lower the financial losses to the transmission company and to customers who depend on availability of electrical power supply. Fault location algorithms with accurate responses minimize operating costs by mobilizing fewer maintenance staff and for less time to meet the demand for repair and research.

1.2 Goals

The main goal of this work is to develop a fault location method for TLs using time-domain equations. The proposed model regards a single-phase TL, in which resistive, inductive, capacitive and conductive parameters are fully distributed. The model equating is also used to develop the fault location algorithm. For each time step, the routine calculates the absolute difference between the fault voltages “seen” by each terminal.

In more detail, the goals of this work are:

- ❑ Development of a mathematical tool for topographic decoupling TLs' ends;
- ❑ Modeling traveling wave phenomena in TLs considering fully distributed parameters;
- ❑ Applying the TL numerical model in time-domain, aiming at fault location.

The goals of this work are based on the following assumptions:

- ❑ The constitutive parameters of the TL are constant along the whole length;
- ❑ There are no electrical coupling elements between cables;
- ❑ The sample frequencies of the digital signals are compatible with commercial digital fault recorders;
- ❑ The space discretization is small enough to enable reasonable computing time of the fault location algorithm.

The programming routines were developed using MATLAB and SIMULINK software. The text edition and typography are based on L^AT_EX programming.

1.3 Text organization

Chapter 2 comprises a bibliographical review regarding TL time-domain numerical models, relations between voltages and currents and the corresponding equations. Through the telegrapher's and wave partial differential equations it is possible to express the classic Bergeron model. There is a discussion about some technical limitations such as the TL resistance lumping and leakage conductance neglecting. In addition, there is a review of the trend in numerical models that considers the frequency dependence of the TL parameters.

The second study topic in the chapter is different fault location methods in TLs. There is a brief equating for the main types of algorithms such as the one based on impedance and phasor measurements, the one based on traveling wave phenomena and finally those that use time-domain equations.

Chapter 3 details the proposed method. First, it presents the proposed time-domain model regarding fully distributed parameters. After the model development, it is described how to apply the proposed model for fault location purposes and also using the two-end waveform inputs.

Then, chapter 4 presents the results for AC and DC systems. The method is scrutinized with the variation of several operating parameters and corresponding sensitivity analyses. The results also come from a direct comparison of the fault location accuracy using the proposed and Bergeron models.

Chapter 5 finally concludes this master's thesis.

This chapter has some *pgfplots* L^AT_EX animated figures which are available for Acrobat Reader (version ≥ 7), PDF-XChange, Foxit Reader.

Bibliographic Review

FAULT LOCATION methods in TLs essentially depend on the numerical model used to compute the oscillographic inputs. First, this thesis provides a historical review TLs models' evolution (AMETANI, 2007). Thereafter, the main academic contributions regarding fault location methods are detailed.

2.1 Transmission line numerical models

2.1.1 Analog era

The Victorian era (1837-1901) was on of great scientific advances regarding telegraph communication technologies. In order to better understand the electrical phenomena involved in telegraph systems, several numerical models were applied. The first step of this journey was the modeling of electrical passive bipolar elements using ordinary differential equation (ODE)s. In 1853, Sir William Thomson (Lord Kelvin) the transient electrical current behavior of a capacitor charged by a Daniel battery (THOMSON, 1853). In the next year, he presented the equating for wave propagation in TLs. The voltage equation aims to relate partial derivatives of time and space for a telegraphic submarine cable. In this model, only distributed resistance and capacitance are considered (THOMSON, 1854).

In 1867, Varley presented the results of his laboratory trials of long submarine cables, especially the one that interconnected London and Sydney, 20,000 km apart. The transient measurement at the local end is emphasized when there is no load at the remote end.

In 1853, James Maxwell published his famous electricity and magnetism treatise, based on the works of Varley and Lord Kelvin. His work presents a parabolic PDE that aims to model the distributed behavior of resistance, capacitance and conductance of a telegraph cable. Maxwell's equation is similar to the one that Fourier used to describe the thermal diffusion phenomena in finite metallic bars (MAXWELL, 1873).

Oliver Heaviside introduced the concepts of inductance and impedance and presented

Figure 1 – Single-phase TL modeled by electrical circuit elements

a modern interpretation of Maxwell's electromagnetism. Regarding the three passive elements - resistance, capacitance and conductance – he added an inductive bipole to model the TL as it is known nowadays, this model is depicted in Figure 1. Heaviside's contribution to TL modeling was so ahead of its time that in just one work he presented the hyperbolic behavior of the telegrapher's equation, the concepts of propagation constant, traveling time, characteristic impedance, and used the rudiments of Laplace transform to solve PDEs (HEAVISIDE, 1893; HUNT, 2012).

The expansion of electric power systems in the early twentieth century revealed the operational adversities of large interconnected systems. One of these difficulties was understanding the transient phenomena occurring in TLs, especially those with longer lengths, in which the transient effects of voltage and current traveling waves are more prominent. Hence, a need was noted to develop of TL numerical models in order to better understand these transient phenomena. Carson's huge contribution led the way to theoretical insights of this recent theme (CARSON, 1919).

Kennelly performed experiments in reduced models of TL to record electrical disturbances through the resulting oscillographies (KENNELLY; NABESHIMA, 1920).

Bewley presented a multi-phase TL model that implements mutual inductive, capacitive and resistive elements. The main contribution of his work dating from the early 1930s is to address numerical calculation of voltages and currents using lattice diagrams (BEWLEY, 1931; BEWLEY, 1934).

Based on Bewley's work, Pipes introduced the matrix notation for voltages, currents and distributed parameters for multi-phase TLs. In his paper he discussed the constitutive differences between the matrices of the propagation times of electrical waves (PIPES, 1937). In 1941 he presented an academic contribution concerning the application of Laplace transform to the telegrapher's equations, aiming to calculate electromagnetic transients in transposed TLs (PIPES, 1941).

In 1961, Bergeron presented a graphical method to calculate traveling waves (BERGERON, 1961). The method consists of graphical analysis of the phenomenon where the

reference is an observer who moves at the same speed as the analyzed wavefront. This approach enables the calculation of transient variables in hydraulic pipes and TLs. This graphic method, although popularized and referred to as the Bergeron method, was first described by Schnyder in a treatise on hydraulic transients (SCHNYDER, 1929).

2.1.2 Digital era

After World War II, the use of digital computers became more popular in academia. The accessibility of this digital facility enabled more sophisticated researches into electromagnetic transients in power systems. Hence, there were impressive development of TL models whose mathematical principle is numerical instead of analytical. Frey et al. proposed the use of computers to calculate electromagnetic transients (FREY; ALTHAMMER, 1961).

In 1969, Dommel applied the simplified telegrapher's equation as a wave equation so he could calculate terminal voltages and currents of an ideal single-phase TL (DOMMEL, 1969). This contribution had a great repercussion because its proposal was to use electrical circuit elements to compose more complex power systems for simulations, including transient calculation for TLs. In addition, his work presented an application of modal transformation, which is used to decouple a mutual multi-phase circuit into single-phase circuits. The proposed method has countless numerical applications, so it led to the development of electromagnetic transient calculation programs such as EMTP/ATP. The model presented by Dommel, popularized as the Bergeron model, is detailed in section 2.1.3. Soon after Dommel's publication, Wedepohl presented a TL model that considered resistance and inductance frequency dependent (WEDEPOHL; MOHAMED, 1969).

During the 1970s, there were many advances regarding TL modeling. Several authors studied various mathematical tools to consider the effect of frequency variation on TL series parameters (BUDNER, 1970; AMETANI, 1973; SNELSON, 1972; AMETANI, 1976; DOMMEL; MEYER, 1974; WASLEY; SELVAVINAYAGAMOORTHY, 1974; NAKAGAWA; AMETANI; IWAMOTO, 1973; WEDEPOHL; EFTHYMIADIS, 1978; MEYER; DOMMEL, 1974; SCOTT-MEYER; BOOK, 1977). Studies were conducted of transients in insulated cables and their modeling particularities compared to overhead lines (WEDEPOHL; INDULKAR, 1975).

Since then, the improvement of EMTP programs has led to more sophisticated TL models (MARTI, 1982; NODA; NAGAOKA; AMETANI, 1996; AMETANI, 1980; SEMLYEN; DERI, 1985; DERI et al., 1981). In addition, some compendiums stand out based references for electromagnetic transient analysis (SCOTT-MEYER, 1982; DOMMEL, 1992).

In the following decade, Gustavsen improved the vector fitting technique for TLs. This technique consists of polynomial ratios as an approximation of the frequency response of the TL resistive and inductive parameters. The ratio is then transcribed and interpreted as

electrical circuit elements and these are associated with the numerical model (GUSTAVSEN; SEMLYEN, 1998; GUSTAVSEN; SEMLYEN, 1999).

In recent years there have been several contributions related to research on electromagnetic transients and TL modeling. Some authors have improved the vector fitting technique (GUSTAVSEN, 2012; SEMLYEN; GUSTAVSEN, 2012; CHINEA; TRIVERIO; GRIVET-TALOCIA, 2010). New mathematical tools have been implemented such as accelerated Fast Fourier Transform (FFT) and real-time simulation systems (CHEN; DINAHAHI, 2012). Besides these, there are contributions with improvements to the model of the earth return current (TOSSANI et al., 2016) and new considerations on matrix parameters and revision of the universal line model (MARTI; TAVIGHI, 2017; ARAUJO; SILVA; KUROKAWA, 2014).

2.1.3 Bergeron numerical model

Figure 1 summarizes the physical component model of single-phase TLs for fully distributed parameters per unit length, as originally proposed by Heaviside (HEAVISIDE, 1893). The representation of the differential section is intended to show the continuous distribution of the modeling. The aspects of the TL modeling are assigned in four passive electrical bipoles (GRAINGER; STEVENSON, 1994; FUCHS, 1977).

□ Resistance R

The linear resistance is associated with the electrical losses originated from the current flow through the TL cables. The shock between the electron flux and the conductive material atoms implies thermal dissipation, a phenomenon known as the Joule effect. The intensity of the thermal energy involved in this process depends on the nature and geometry of the conductive material and the magnitude of the electric current. The value of the electric resistance is also related to the nature of the electric signal. Unlike DC systems, AC systems have nonuniform distribution of the electric current in the conductor cross section.

This effect is known as the skin effect and is characterized by the transverse non-uniformity of the electron flux in AC currents, which implies a decrease of the conductive area and consequent increase of the apparent resistance. This phenomenon intensifies with the frequency of the alternating signal and, causing a tendency for the transverse flow to run across conductor periphery. In AC transmission installations that use conductor bundles per phase, the proximity effect occurs. This phenomenon originates from the deformation that the electric field of each sub-conductor causes in the field of the adjacent subconductor. The proximity effect increases the apparent resistance of the conductor bundle.

□ Inductance L

The electric current flow through the conductor generates a concentric magnetic field. The concentricity of the magnetic field occurs internally and externally to the conductor. The intensity of the external magnetic field depends on the current magnitude and the length of the magnetic flux line. The magnetic induction vector is therefore a function of the current and the transverse distance. Thus, the external inductance of the cylindrical conductor is a function of the ratio between the transverse distance and its radius.

On the other hand, the internal magnetic flux of the solid cylindrical conductor is independent of the conductor geometry, but dependent on the current flow. The ratio between the intensity of the total magnetic flux and the current flow is modeled as an inductance.

□ Capacitance C

When a TL is energized, there are different voltages between the conductors and the ground. Thus, there is a distinction between the density of electric charges for conductors of different phases or poles. This behavior is analogous to capacitors, so in this case the TL conductors are analogous to capacitor electrodes. The behavior of the electric field between conductors, and between conductors and the ground can be modeled by capacitors.

□ Conductance G

The presence of electrical voltage in TLs implies dissipative effects – corona effect and insulator losses. The first one is most prominent in extra high voltage installations. At this voltage level, the electric gradient is more likely to exceed the critical disruptive air gradient. When this occurs, there is a partial transfer of electrons to the atoms of the conductors' surrounding atmosphere. This ionization process consumes electrical energy so it is considered as an active element in TL modeling. The electrical circuit element that best fits this phenomenon is the leakage conductance. In addition to the presence of air ionization, there are the leakage currents flowing through the insulators. The ratio between the aforementioned losses and square voltage is given as the total conductance of the TL.

2.1.3.1 Kirchhoff's laws

It is possible to apply Kirchhoff's laws to the circuit depicted in Figure 1 in order to mathematically relate voltage and current variables as a function of time and space.

Before that, it is necessary to formally define the electric variables for the application of the circuit laws and other algebraic developments (SPIVAK, 1971).

Let f be a function of two variables $f = f(x, t) : D \rightarrow \mathbb{R}$ so it assumes, generically, the mathematical qualities of the voltage and current functions.

- x is the space variable
- t is the temporal variable
- l is the TL total length
- $v = v(x, t)$ is the voltage function
- $i = i(x, t)$ is the current function
- Δ is the difference operator

It is assumed that f satisfies a C^2 class function condition in D , i.e., the existence and continuity of the function partial differentiation of first and second order in D are presumed (CALKIN et al., 1940). The Schwartz theorem is based on these assumptions, as in (1).

$$\frac{\partial^2}{\partial x \partial t} f(x, t) = \frac{\partial^2}{\partial t \partial x} f(x, t) \quad (1)$$

2.1.3.2 Kirchhoff's voltage law

The principle of voltage conservation in the closed network shown in Fig. 1 is

$$v(x, t) = v_{\text{resistor}}(x, t) + v_{\text{inductor}}(x, t) + v(x + \Delta x, t) \quad (2)$$

The constitutive equations of the resistor and inductor are applied to (3).

$$v(x, t) = R\Delta x i(x, t) + L\Delta x \frac{\partial}{\partial t} i(x, t) + v(x + \Delta x, t) \quad (3)$$

Approaching $\Delta x \rightarrow 0$ in (3), it is possible to express the first PDE that relates the voltage and current of a TL as a function of the time and space variables.

$$-\frac{\partial}{\partial x} v(x, t) = L \frac{\partial}{\partial t} i(x, t) + R i(x, t) \quad (4)$$

2.1.3.3 Kirchhoff's current law

The principle of conservation of electric charge in the single-phase TL model implies

$$i(x, t) = i_{\text{conductance}}(x, t) + i_{\text{capacitor}}(x, t) + i(x + \Delta x, t) \quad (5)$$

The constitutive equations of the conductance and capacitor are used in (6).

$$i(x, t) = G\Delta x v(x, t) + C\Delta x \frac{\partial}{\partial t} v(x, t) + i(x + \Delta x, t) \quad (6)$$

It is analogous to adopt the infinitesimal approach of the space variable in (3). Thus, the second PDE of the TL is

$$-\frac{\partial}{\partial x}i(x, t) = C \frac{\partial}{\partial t}v(x, t) + G v(x, t) \quad (7)$$

2.1.3.4 Wave equations

The Bergeron model assumes an ideal TL in its numerical development, i.e., the distributed resistance and conductance are considered null, $R = G = 0$ (DOMMEL, 1969). This hypothesis implies simplifications in (4) and (7). Thus, the equation referred to as the Kirchhoff's voltage law assumes (8).

$$-\frac{\partial}{\partial x}v(x, t) = L \frac{\partial}{\partial t}i(x, t) \quad (8)$$

The same procedure is applied to Kirchhoff's current law in (9).

$$-\frac{\partial}{\partial x}i(x, t) = C \frac{\partial}{\partial t}v(x, t) \quad (9)$$

After the algebraic treatment of (8) and (9), it is possible to obtain the wave equations for the voltage and current functions. The general voltage expression is given by

$$\frac{\partial^2}{\partial x^2}v(x, t) = LC \frac{\partial^2}{\partial t^2}v(x, t) \quad (10)$$

Similarly, the wave equation for the current function is

$$\frac{\partial^2}{\partial x^2}i(x, t) = LC \frac{\partial^2}{\partial t^2}i(x, t) \quad (11)$$

The D'Alembert method for the resolution of the wave equation is used in (10) and (11) (D'ALEMBERT, 1747). The application of the method for this kind of PDE is related to the concept of wave propagation velocity. Regarding a single-phase TL, the propagation velocities of voltage and current are coincident, and their expression is

$$\vartheta = \frac{1}{\sqrt{LC}} \quad (12)$$

The voltage resolution equation is given by

$$v(x, t) = v_1(x - \vartheta t) + v_2(x + \vartheta t) \quad (13)$$

And the current equation is

$$i(x, t) = i_1(x - \vartheta t) + i_2(x + \vartheta t) \quad (14)$$

Let v_1 , v_2 , i_1 and i_2 be first-order differentiable functions of one variable η in the entire domain. An arbitrary and auxiliary function f assumes their mathematical characteristics, so it is defined as $f = f(\eta) : \{\eta \in \mathbb{R} \mid \eta = x - \vartheta t \geq 0\} \rightarrow \mathbb{R}$. The auxiliary functions of

Figure 2 – Quadripole configuration representing a single-phase TL

index 1 are considered as progressive waves and those of index 2 are the regressive waves. It would be interesting to describe $v(x, t)$ and $i(x, t)$ as functions of just two auxiliary functions in order to correlate the voltage and current expressions. This can be expressed by replacing (13) and (14) in (8).

$$-\frac{d}{d\eta}v_1(x - \vartheta t) - \frac{d}{d\eta}v_2(x + \vartheta t) = L \left(-\vartheta \frac{d}{d\eta}i_1(x - \vartheta t) + \vartheta \frac{d}{d\eta}i_2(x + \vartheta t) \right) \quad (15)$$

By factoring the common factor of the second side of (15), it is possible to obtain a direct relation between voltage and current functions. Therefore, the TL characteristic impedance of a lossless TL is

$$Z_c = L\vartheta = \sqrt{\frac{L}{C}} \quad (16)$$

It is possible to rewrite and simplify (15) as a function of the characteristic impedance.

$$\frac{d}{d\eta}v_1(x - \vartheta t) + \frac{d}{d\eta}v_2(x + \vartheta t) = Z_c \left(\frac{d}{d\eta}i_1(x - \vartheta t) - \frac{d}{d\eta}i_2(x + \vartheta t) \right) \quad (17)$$

The integration on both sides of (17) over the variable η simplifies the equation.

$$\int \left(\frac{d}{d\eta}v_1(x - \vartheta t) + \frac{d}{d\eta}v_2(x + \vartheta t) \right) d\eta = Z_c \int \left(\frac{d}{d\eta}i_1(x - \vartheta t) - \frac{d}{d\eta}i_2(x + \vartheta t) \right) d\eta \quad (18)$$

The integration result is

$$v_1(x - \vartheta t) + v_2(x + \vartheta t) = Z_c [i_1(x - \vartheta t) - i_2(x + \vartheta t)] \quad (19)$$

It is noteworthy that the first side of (19) matches (13).

$$v(x, t) = Z_c [i_1(x - \vartheta t) - i_2(x + \vartheta t)] \quad (20)$$

Thus, (14) and (20) are obtained as D'Alembert's general expressions for voltage and current. They are written as functions of just two auxiliary functions, i_1 and i_2 .

Figure 3 – Depiction of terminals by electrical circuit elements

2.1.3.5 Numerical development

The resulting equations from section 2.1.3.4 should be applied iteratively so that the TL terminal voltages and currents can be numerically calculated. One possibility for iterative relation is in the description of the characteristic equation of the TL. This expression is obtained through operations between (20) and (14), after multiplying the second one by Z_c , it is added to the first one. The resulting expression is

$$v(x, t) + Z_c i(x, t) = 2Z_c i_1(x - \vartheta t) \quad (21)$$

Another characteristic equation is obtained if (20) is subtracted from (14) times Z_c .

$$v(x, t) - Z_c i(x, t) = -2Z_c i_2(x + \vartheta t) \quad (22)$$

The left side of (21) is constant if $x - \vartheta t$ remains constant, and (22) is constant if $x + \vartheta t$ stays constant. Dommel noticed that a fictitious observer traveling from one end to the other of a TL of length l at a velocity ϑ , takes τ to complete the trajectory. The variable τ is defined as the propagation time.

$$\tau = \frac{l}{\vartheta} = l\sqrt{LC} \quad (23)$$

In addition, the observer at this velocity can measure the constancy of (21) during the whole path. Then, if the observer starts from the local end at $t_0 = t - \tau$, he will arrive at the remote end at $t_0 = t$ and so state that (24) is true. In order to simplify the nomenclature of the electrical variables, the index 1 is adopted for the local end and the index 2 for the remote end.

$$v_2(t) - Z_c i_2(t) = v_1(t - \tau) + Z_c i_1(t - \tau) \quad (24)$$

The inversion of the remote terminal current signal in (24) is due to the signal convention described in the quadripole of Figure 2. Analogously to (24), it is possible to write the relation between characteristic equations of a signal that leaves the remote terminal.

$$v_1(t) - Z_c i_1(t) = v_2(t - \tau) + Z_c i_2(t - \tau) \quad (25)$$

Figure 4 – Bergeron model for two-point resistance lumping

The system of equations formed by (24) and (25) can be rewritten in such a way that they can be represented and solved numerically by electrical circuit elements. This arrangement takes place through the isolation of the terminal currents.

$$\begin{cases} i_1(t) = \frac{v_1(t)}{Z_c} - \frac{v_2(t - \tau)}{Z_c} - i_2(t - \tau) \\ i_2(t) = \frac{v_2(t)}{Z_c} - \frac{v_1(t - \tau)}{Z_c} - i_1(t - \tau) \end{cases} \quad (26)$$

The system (26) can be modeled by impedances and current sources in order to use the nodal admittance matrix method, as shown in Figure 3. The current sources $h_1(t)$ and $h_2(t)$ are detailed in (27) and (28), respectively.

$$h_1(t) = \frac{v_2(t - \tau)}{Z_c} + i_2(t - \tau) \quad (27)$$

$$h_2(t) = \frac{v_1(t - \tau)}{Z_c} + i_1(t - \tau) \quad (28)$$

2.1.3.6 Two-point resistance lumping

The inclusion of the series resistance in the Bergeron model can be performed by lumping half the series resistance at each end of the TL. Figure 4 shows this approach. The voltage drop for each TL end can be separately equated.

$$\begin{cases} v_{1'}(t) = v_1(t) - \frac{R}{2}i_1(t) \\ v_{2'}(t) = v_2(t) - \frac{R}{2}i_2(t) \end{cases} \quad (29)$$

It is possible to rearrange (26) considering the voltage drops described in (29).

$$\begin{cases} i_1(t) = \frac{v_1(t)}{Z_c + \frac{R}{2}} - \left[\frac{v_2(t - \tau)}{Z_c + \frac{R}{2}} + \frac{Z_c - \frac{R}{2}}{Z_c + \frac{R}{2}} i_2(t - \tau) \right] \\ i_2(t) = \frac{v_2(t)}{Z_c + \frac{R}{2}} - \left[\frac{v_1(t - \tau)}{Z_c + \frac{R}{2}} + \frac{Z_c - \frac{R}{2}}{Z_c + \frac{R}{2}} i_1(t - \tau) \right] \end{cases} \quad (30)$$

Like (26), system (30) can be modeled by electrical circuit elements. The system rearrangement can match Figure 3 if the characteristic impedance and historical current

sources values are redefined.

$$h_2(t) = \frac{v_1(t - \tau)}{Z_{\text{eq}}} + \mu i_1(t - \tau) \quad (31)$$

$$h_1(t) = \frac{v_2(t - \tau)}{Z_{\text{eq}}} + \mu i_2(t - \tau) \quad (32)$$

Where Z_{eq} and μ are

$$\begin{cases} Z_{\text{eq}} = Z_c + \frac{R}{2} \\ \mu = \frac{Z_c - \frac{R}{2}}{Z_c + \frac{R}{2}} \end{cases} \quad (33)$$

It is interesting to notice that $R > 0 \rightarrow \mu < 1$, so μ , which is the reflection coefficient, can be interpreted as a TL attenuation factor.

2.1.3.7 Three-point resistance lumping

In order to refine the series resistance distribution, two TLs $l/2$ long with terminal resistances of $R/4$ are cascaded and modeled according to section 2.1.3.6. The resulting arrangement consists of one quarter of total resistance $R/4$ lumped at each end and $R/2$ lumped in the middle. Figure 5 shows the final arrangement. Using this final arrangement, the iterative system takes a new format, as shown in (34).

$$\begin{cases} i_1(t) = \frac{v_1(t)}{Z_c + \frac{R}{4}} - \frac{Z_c}{\left(Z_c + \frac{R}{4}\right)^2} \left[v_2(t - \tau) + \left(Z_c - \frac{R}{4}\right) i_2(t - \tau) \right] - \\ \quad - \frac{\frac{R}{4}}{\left(Z_c + \frac{R}{4}\right)^2} \left[v_1(t - \tau) + \left(Z_c - \frac{R}{4}\right) i_1(t - \tau) \right] \\ i_2(t) = \frac{v_2(t)}{Z_c + \frac{R}{4}} - \frac{Z_c}{\left(Z_c + \frac{R}{4}\right)^2} \left[v_1(t - \tau) + \left(Z_c - \frac{R}{4}\right) i_1(t - \tau) \right] - \\ \quad - \frac{\frac{R}{4}}{\left(Z_c + \frac{R}{4}\right)^2} \left[v_2(t - \tau) + \left(Z_c - \frac{R}{4}\right) i_2(t - \tau) \right] \end{cases} \quad (34)$$

This shows that the current calculation depends on the historical values of both ends. This dependency differs from the equating developed in section 2.1.3.6 because the current calculation depends just on historical values of the opposite end.

Figure 5 – Bergeron model for three-point resistance lumping

2.2 Fault location in transmission lines

The fault location algorithms covered in this bibliographic review are focused on transmission system applications, excluding TLs with lateral branches.

2.2.1 Methods based on voltage and current phasors

Fault location methods that apply voltage and current phasors are pioneers for this purpose. The usage of the phasor technique requires a previous signal treatment given to the voltage and current waveforms. This necessity is due to the diverse harmonic content of the ends' electrical signals. The filtering and calculation of the fundamental component of voltages and currents in the post-fault period are essential for this kind of fault location method. The FFT is usually used as the filtering technique. Several authors have investigated this application (TAKAGI et al., 1982; NOVOSEL et al., 1996; ZAMORA et al., 1996).

2.2.1.1 The A.T. Johns method

One of the most prominent methods regarding the phasor technique is the A.T. Johns' method (JOHNS; JAMALI, 1990). The method is applied considering single-phase TLs and both ends electrical measurements. Figure 6 shows the TL sectioning by the fault impedance – front and posterior sectioning. After the signal processing performed on the voltage and current waveforms in the post-fault period, their phasors at the industrial frequency can be estimated. If the electrical measures are known at one end, voltage and current phasors at a distance x from the referred end are calculated using the transmission matrix (GRAINGER; STEVENSON, 1994).

$$\begin{bmatrix} V(x) \\ I(x) \end{bmatrix} = \begin{bmatrix} \cosh(\gamma x) & -Z \sinh(\gamma x) \\ -\frac{\sinh(\gamma x)}{Z} & \cosh(\gamma x) \end{bmatrix} \begin{bmatrix} V_S \\ I_S \end{bmatrix} \quad (35)$$

Where $\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$, $Z = \sqrt{(R + j\omega L)/(G + j\omega C)}$ and ω is the angular electrical frequency. Taking the local end as the reference, it is possible to calculate the voltage at the fault point, $V_{F,1}$.

$$V_{F,1} = Z \cosh(\gamma x) V_1 - Z \sinh(\gamma x) I_1 \quad (36)$$

Figure 6 – TL sectioned by the fault impedance

Analogously, taking the remote end as the reference, the voltage at the fault point, $V_{F,2}$, is

$$V_{F,2} = Z \cosh(\gamma(l - x_f)) V_1 - Z \sinh(\gamma(l - x_f)) I_1 \quad (37)$$

In theory, the voltages $V_{F,1}$ and $V_{F,2}$ are equivalent and the equation $V_{F,1} = V_{F,2}$ can be solved over the variable x_f , as in (38).

$$x_f = \frac{\tanh^{-1}\left(-\frac{B}{A}\right)}{\gamma} \quad (38)$$

Where

$$A = Z \cosh(\gamma l) I_2 - \sinh(\gamma l) V_2 + Z I_1 \quad (39)$$

$$B = \cosh(\gamma l) V_2 - Z \sinh(\gamma l) I_2 + V_1 \quad (40)$$

In practice, the TL parameters may vary in time and space, so this variation may imply an imaginary component in the result presented by (38). Only the real part of x_f is considered for the final result.

2.2.2 Methods based on traveling waves

The fault location methods based on traveling wave techniques involve the calculation of the propagation time of the disturbances presented along the TL. Whenever an impedance is inserted into the transmission circuit, the originated current waves travel towards the TL ends. When a disturbance reaches one end, it is usually possible to observe an abrupt change – as a step – in the current waveforms. This event is more noticeable if some harmonic content is attenuated from the original waveform such as the fundamental component and high frequency harmonics due to aliasing error (LEWIS, 1951; LANZ; VITINS, 1977; ANDO; SCHWEITZER; BAKER, 1985). Some techniques have been developed to detect the traveling waves. The wavelet transform is one of the most

exploited (MAGNAGO; ABUR, 1998; SPOOR; ZHU, 2006; QIN et al., 2005). Applying the traveling wave principle, the equation that calculates the fault point is

$$x_f = \frac{l + |\tau_1 - \tau_2|v}{2} \quad (41)$$

Where τ_1 and τ_2 are detection travel time of the waves that reached the local and remote ends, respectively.

The traveling wave technique presents some drawbacks: the threshold of the resulting traveling waves may not be suitable to perform the fault location method; in addition, this technique does not consider the inherent differences between the power system ordinary switching operation and the short circuit occurrence.

2.2.3 Time-domain models

Fault location methods that apply time-domain models are promising and still under-explored by both academia and industry compared to the methods presented in sections 2.2.1 and 2.2.2. The contemporaneity is given by the high sample frequency requirements of the electrical signals. Time-domain techniques extend to many applications, including HVDC systems (SUONAN et al., 2010; YUANSHEG; GANG; HAIFENG, 2015; QIU et al., 2016). This approach is also used for fault location in three-phase double-circuit systems (JIALE et al., 2006). This master's thesis contributes both to time-domain model and the fault location method, which is independent of the waveforms of voltages and currents. In the next chapter, the methodology is presented.

Methodology

THIS CHAPTER COVERS THE PROPOSED METHOD of this thesis. The method description is divided into two sections. The first one deals with the development of a TL numerical model for electromagnetic transients. and the second section applies the proposed model for fault location.

3.1 Proposed transmission line model

The proposed TL model considers fully distributed parameters. This type of approach requires the original telegrapher's equations and a better understanding of their behavior as PDEs. The developed equations corresponding to the Kirchhoff's laws presented in section 2.1.3.1 are here summarized.

$$-\frac{\partial}{\partial x}v(x,t) = L \frac{\partial}{\partial t}i(x,t) + R i(x,t) \quad (42)$$

$$-\frac{\partial}{\partial x}i(x,t) = C \frac{\partial}{\partial t}v(x,t) + G v(x,t) \quad (43)$$

3.1.1 Telegrapher's equations

The telegrapher's equations are second-order PDEs which can be obtained by algebraical manipulation of expressions (42) and (43). This treatment is necessary to achieve one electrical variable per PDE. The partial differentiation over the variable x is applied to (42), and over t to (43).

$$-\frac{\partial^2}{\partial x^2}v(x,t) = L \frac{\partial^2}{\partial x \partial t}i(x,t) + R \frac{\partial}{\partial x}i(x,t) \quad (44)$$

$$-\frac{\partial^2}{\partial t \partial x}i(x,t) = C \frac{\partial^2}{\partial t^2}v(x,t) + G \frac{\partial}{\partial t}v(x,t) \quad (45)$$

Replacing (45) and (43) in (44), and using the Schwartz theorem, it is possible to obtain the voltage telegrapher's equation.

$$\frac{\partial^2}{\partial x^2}v(x,t) = LC \frac{\partial^2}{\partial t^2}v(x,t) + (RC + LG) \frac{\partial}{\partial t}v(x,t) + RG v(x,t) \quad (46)$$

Analogously, the current telegrapher's equation is

$$\frac{\partial^2}{\partial x^2} i(x, t) = LC \frac{\partial^2}{\partial t^2} i(x, t) + (RC + LG) \frac{\partial}{\partial t} i(x, t) + RG i(x, t) \quad (47)$$

As already shown in section 2.1.3.4, adopting $R = G = 0$ as an simplification indicates that the telegrapher's equation is a generalization of the wave equation.

3.1.2 Second-order PDEs

Linear second-order PDEs over the function f obey the general expression (48) (WEBER, 1869).

$$F\left(\frac{\partial^2 f}{\partial x^2}, \frac{\partial^2 f}{\partial x \partial t}, \frac{\partial^2 f}{\partial t^2}\right) = a \frac{\partial^2 f}{\partial x^2} + b \frac{\partial^2 f}{\partial x \partial t} + c \frac{\partial^2 f}{\partial t^2} + g(f(x, t), x, t, k) \quad (48)$$

Where F is an arbitrary function and $g = g(\eta)$ presents the linearity of its arguments – $f(x, t)$, x , t and $k \in \mathbb{R}$. The discriminant for second-order PDEs is defined as $\sigma = b^2 - 4ac$. The discriminant points out the behavior of the PDE solution. The following list shows the possible behaviors (HORN, 1929).

- $\sigma < 0$, elliptical – e.g., Laplace equations
- $\sigma = 0$, parabolic – e.g., heat diffusion equation
- $\sigma > 0$, hyperbolic – e.g., wave equation

When fitting (46) and (47) to the general expression (48), its coefficients are $a = LC$, $b = RC + LG$ and $c = RG$. So, the discriminant assumes $\sigma = (RC - LG)^2 > 0$, since L , C and R are real positive scalars. The telegrapher's equation, indicated as hyperbolic, has solutions related to the wave phenomena.

3.1.3 Laplace transform

It is clear that analytically solving (46) and (47) is a challenge. One possible alternative to deal with this problem is to transform the PDE into an ODE using the Laplace transform (DOETSCH, 1937). This is a linear and integral operator capable of transposing the operation domain of a function. Let $f = f(t)$ be a time-domain function. Then, its transformed pair is $F = F(s)$. The transform is well defined in (49).

$$\mathcal{L}[f(t)] = F(s) = \int_0^\infty f(t)e^{-st} dt \quad (49)$$

Where the complex number s is the transformation variable and e^{-st} is the integral kernel. The Laplace transform is a useful mathematical tool because it has the property of

transforming time-domain differentials and integrals into polynomials in the frequency-domain, as shown in (50) and (51). This procedure simplifies the resolution of ODEs and PDEs.

$$\mathcal{L} \left[\frac{d}{dt} f(t) \right] = s\mathcal{L} [f(t)] - f(0) = sF(s) - f(0) \quad (50)$$

$$\mathcal{L} \left[\frac{d^2}{dt^2} f(t) \right] = s^2\mathcal{L} [f(t)] - sf(0) - \frac{d}{dt} f(0) = s^2F(s) - sf(0) - \frac{d}{dt} f(0) \quad (51)$$

The same properties can be applied to functions of two variables, $f = f(x, t)$, with time and space dependency. The initial conditions of (52), (53) and (54) are quiescent.

$$\mathcal{L} \left[\frac{\partial}{\partial t} f(x, t) \right] = s\mathcal{L} [f(x, t)] = sF(x, s) \quad (52)$$

$$\mathcal{L} \left[\frac{\partial^2}{\partial t^2} f(x, t) \right] = s^2\mathcal{L} [f(x, t)] = s^2F(x, s) \quad (53)$$

$$\mathcal{L} \left[\frac{\partial^n}{\partial x^n} f(x, t) \right] = s^n F(x, s) \quad (54)$$

3.1.3.1 Laplace transform applied to the telegrapher's equations

Considering the purpose of expressing an analytical solution of voltage and current functions, properties (52), (53) and (54) are applied to the telegrapher's equations (46) and (47).

$$\frac{d^2}{dx^2} V(x, s) = [LCs^2 + (RC + LG)s + RG] V(x, s) \quad (55)$$

$$\frac{d^2}{dx^2} I(x, s) = [LCs^2 + (RC + LG)s + RG] I(x, s) \quad (56)$$

The definition of initial and boundary conditions is indispensable to the formal resolution of PDEs.

Boundary conditions

$$\begin{aligned} v(0, t) = v_1(t), \quad v(l, t) = v_2(t) \\ i(0, t) = i_1(t), \quad i(l, t) = i_2(t) \end{aligned} \quad \text{for } t > 0 \quad (57)$$

Initial conditions

$$i(x, 0) = v(x, 0) = 0, \quad 0 \leq x \leq l \quad (58)$$

The process implies a simplification of PDEs into second order ODEs. The resulting ODE is generally written using an arbitrary function $M = M(\eta)$, as in (59).

$$\frac{d^2}{d\eta^2} M(\eta) = r^2 M(\eta) \quad (59)$$

The solution of this type of ODE is well known as in (60).

$$M(\eta) = A_1 e^{-r\eta} + A_2 e^{+r\eta} \quad (60)$$

The use of (60) in telegrapher's equations is one way of isolating voltage and current functions in the frequency-domain.

$$V(x, s) = V_1(s) e^{-x\sqrt{as^2+bs+c}} + V_2(s) e^{+x\sqrt{as^2+bs+c}} \quad (61)$$

$$I(x, s) = I_1(s) e^{-x\sqrt{as^2+bs+c}} + I_2(s) e^{+x\sqrt{as^2+bs+c}} \quad (62)$$

The indexes of voltages and currents obey the same convention as the one adopted in the Bergeron model development, in section 2.1.3.4. It is convenient to use only voltages or currents as boundary conditions to express the general formulations (SCOTT-MEYER, 1982). This maneuver helps the development of a relation between frequency-domain voltage and current, as well as writing the TL characteristic equation. The algebraic procedure to achieve this condition starts from the replacement of (61) in (42) after the Laplace transform application.

$$I(x, s) = \frac{1}{Z_c(s)} \left[V_1(s) e^{-x\sqrt{as^2+bs+c}} - V_2(s) e^{+x\sqrt{as^2+bs+c}} \right] \quad (63)$$

Where $Z_c(s)$ is the TL characteristic impedance in frequency-domain.

$$Z_c(s) = \sqrt{\frac{R + Ls}{G + Cs}} \quad (64)$$

Based on (63) and (61), it is possible to write a system of equations in the frequency-domain that relates voltage and current as functions of two auxiliary variables.

$$\begin{cases} V(x, s) = V_1(s) e^{-x\sqrt{as^2+bs+c}} + V_2(s) e^{+x\sqrt{as^2+bs+c}} \\ Z_c(s) I(x, s) = V_1(s) e^{-x\sqrt{as^2+bs+c}} - V_2(s) e^{+x\sqrt{as^2+bs+c}} \end{cases} \quad (65)$$

The sum of equations of system (65) results in an expression capable of determining a relation between TL voltages and currents. This relation is better known as the TL characteristic equation. The relation between ends' voltage and currents is

$$V(x, s) + Z_c(s) I(x, s) = 2V_1(s) e^{-x\sqrt{as^2+bs+c}} \quad (66)$$

The local end assumes $x = 0$ as argument in (66).

$$V(0, s) + Z_c(s) I(0, s) = 2V_1(s) \quad (67)$$

On the other hand, the remote end assumes $x = l$.

$$V(l, s) + Z_c(s) I(l, s) = 2V_1(s) e^{-l\sqrt{as^2+bs+c}} \quad (68)$$

The direct replacement of (67) in (68) results in an expression capable of relating the electrical variables at both ends of the TL.

$$V(l, s) + Z_c(s) I(l, s) = [V(0, s) + Z_c(s) I(0, s)] e^{-l\sqrt{as^2+bs+c}} \quad (69)$$

Although (69) relates voltages and currents, it still remains in the frequency-domain, which precludes its usage for simulating electromagnetic phenomena in the time-domain. So, it should be anti-transformed, and one possible technique to reach a time-domain expression is to apply the convolution concept. However, the convolution of an expression is supported only if each term of the expression is in the time-domain. This restriction leads to a brief inspection at $Z_c(s)$ and $\Psi(l, s) = e^{-l\sqrt{as^2+bs+c}}$, and their Laplace anti-transforms are not trivial.

3.1.3.2 Analysis of the characteristic impedance

The frequency response of (64) demonstrates how the TL distributed resistance affects the characteristic impedance value. Figures 7 and 8 show, respectively, the frequency response of the real and imaginary parts considering $s = j\omega$, where $j = \sqrt{-1}$ and ω is the electrical angular frequency. This response analysis regards the positive sequence parameters of a typical high voltage overhead TL, where $G = 0$. The variation of the distributed resistance considerably impacts the response real part for $f < 100$ [Hz] and the imaginary part for $f < 1000$ [Hz]. It is noteworthy that the real part converges to an expression where the reactive parameters can be neglected and imaginary part converges to zero. Transient phenomena that present a frequency spectrum above 1000 [Hz], except for the fundamental component, can be simulated with a TL model that simplifies the characteristic impedance, as pointed in (70).

$$\lim_{\omega \rightarrow \infty} Z_c(j\omega) = \lim_{\omega \rightarrow \infty} \sqrt{\frac{R + j\omega L}{G + j\omega C}} = \sqrt{\frac{L}{C}} \quad (70)$$

The calculation of the Laplace anti-transform of (64) starts by using the transforming pair (71) (DOETSCH, 1958; POLYANIN, 2005).

$$\mathcal{L}^{-1} \left[\sqrt{\frac{s+q}{s}} - 1 \right] = \frac{1}{2} q e^{-\frac{1}{2}qt} \left[I_0 \left(\frac{1}{2}qt \right) + I_1 \left(\frac{1}{2}qt \right) \right] \quad (71)$$

Where $I_\nu = I_\nu(t)$ is the modified Bessel function of the first kind and order ν . Even though (71) is similar to (64), some algebraic manipulation is necessary until the immediate application. The first step consists of removing the unity element in the time-domain through the transforming pair 1 : $\delta(t)$.

$$\mathcal{L}^{-1} \left[\sqrt{\frac{s+q}{s}} \right] = \delta(t) + \frac{1}{2} q e^{-\frac{1}{2}qt} \left[I_0 \left(\frac{1}{2}qt \right) + I_1 \left(\frac{1}{2}qt \right) \right] \quad (72)$$

Figure 7 – Frequency response of the real part of $Z_c(s)$ Figure 8 – Frequency response of the imaginary part of $Z_c(s)$

Where $\delta = \delta(t)$ is Dirac's delta function. The next step consists of applying the frequency shifting property, $\mathcal{L}^{-1}[F(s+p)] = e^{-pt}f(t)$. After this application, both polynomials in the frequency-domain present linear coefficients.

$$\mathcal{L}^{-1}\left[\sqrt{\frac{s+p+q}{s+p}}\right] = e^{-pt}\left\{\delta(t) + \frac{1}{2}qe^{-\frac{1}{2}qt}\left[I_0\left(\frac{1}{2}qt\right) + I_1\left(\frac{1}{2}qt\right)\right]\right\} \quad (73)$$

Expressions (64) and (73) are algebraically similar and their coefficients can be matched over the variables p and q . The resulting system of equations is solved in (74).

$$\begin{cases} p+q = \frac{R}{L} \Rightarrow q = \frac{RC-LG}{LC} \\ p = \frac{G}{C} \end{cases} \quad (74)$$

So, the characteristic impedance expression in time-domain is

$$z_c(t) = \mathcal{L}^{-1}\left[\sqrt{\frac{R+Ls}{G+Cs}}\right] = \sqrt{\frac{L}{C}}\left\{\delta(t) + \frac{1}{2}qe^{-(p+\frac{1}{2}q)t}\left[I_0\left(\frac{1}{2}qt\right) + I_1\left(\frac{1}{2}qt\right)\right]\right\} \quad (75)$$

3.1.3.3 Analysis of the exponential factor

The exponential factor in (69) presents a non-ordinary Laplace anti-transform and its time-domain expression depends on Bessel functions (ANTIMIROV; KOLYSHKIN; VAILLANCOURT, 2007). The suitable transforming pair is described in (76).

$$\mathcal{L}^{-1}\left[e^{-p\sqrt{s^2-q^2}}\right] = \frac{pq}{\sqrt{t^2-p^2}}I_1\left(q\sqrt{t^2-p^2}\right)H(t-p) + I_0\left(q\sqrt{t^2-p^2}\right)\delta(t-p) \quad (76)$$

Where $H = H(t)$ is the Heaviside step function. In order to fit $\Psi(x, s) = e^{-x\sqrt{as^2+bs+c}}$ in the left side of (76), two auxiliary and positive variables α and λ enable matching both expressions term by term, as in (77).

$$as^2 + bs + c = a[(s + \alpha)^2 - \lambda^2] \quad (77)$$

The equivalence of the independent and first-order terms of both polynomials implies a system over the auxiliary variables α and λ .

$$\begin{cases} b = 2a\alpha \\ c = a(\alpha^2 - \lambda^2) \end{cases} \Rightarrow \begin{cases} \alpha = \frac{RC+LG}{2LC} \\ \lambda = \frac{RC-LG}{2LC} \end{cases} \quad (78)$$

Using the frequency shifting property and replacing α and λ in (76), it is possible to perform the Laplace anti-transform $\mathcal{L}^{-1}[\Psi(x, s)] = \psi(x, t)$.

$$\psi(x, t) = e^{-\alpha t}\left[\frac{x\lambda\sqrt{a}}{\sqrt{t^2-ax^2}}I_1\left(\lambda\sqrt{t^2-ax^2}\right)H(t-x\sqrt{a}) + I_0\left(\lambda\sqrt{t^2-ax^2}\right)\delta(t-x\sqrt{a})\right] \quad (79)$$

3.1.4 Convolution

Up to now the time-domain expressions of each term of (69) are individually known. However, the terms composed by the multiplication of two frequency-domain expressions must be convoluted so the whole equation can be in the time-domain. The convolution of two arbitrary functions $f(t)$ and $y(t)$ for $t > 0$ is represented by (80).

$$\mathcal{L}^{-1}[F(s)Y(s)] = f(t) * y(t) \Rightarrow \int_0^t f(t-\eta)y(\eta)d\eta = \int_0^t f(\eta)y(t-\eta)d\eta \quad (80)$$

The convolution integral can be individually applied to each composed term of (69) – characteristic impedance and exponential factor terms.

3.1.4.1 Convolution of the characteristic impedance

The calculation of the convolution between the time-domain characteristic impedance and an arbitrary function $f(t)$ is

$$\begin{aligned} \mathcal{L}^{-1}[Z_c(s)F(s)] &= \int_0^t z_c(t-\eta)f(\eta)d\eta = \int_0^t \sqrt{\frac{L}{C}} \left\{ \delta(t-\eta) \right. \\ &\quad \left. + \frac{1}{2}q e^{-(p+\frac{1}{2}q)(t-\eta)} \left[I_0\left(\frac{1}{2}q(t-\eta)\right) + I_1\left(\frac{1}{2}q(t-\eta)\right) \right] \right\} f(\eta)d\eta \end{aligned} \quad (81)$$

Expression (81) can be divided into two portions, the first one referring to Dirac's delta function and second one to the Bessel functions.

□ *First portion*

According to (STAKGOLD, 2000), the application of property (82) solves the first integral.

$$\int_{t_0+\epsilon}^{t_0-\epsilon} g(\eta)\delta(\eta-t_0)d\eta = g(t_0), \quad \epsilon > 0 \quad (82)$$

Since Dirac's delta function is even, the first portion of (81) assumes

$$\int_0^t \sqrt{\frac{L}{C}} \delta(t-\eta)f(\eta)d\eta = \sqrt{\frac{L}{C}} f(t) \quad (83)$$

□ *Second portion*

The second portion does not have an analytic solution, so its expression is called $\mu = \mu(t, f(t))$. The recovery of the variable λ in (78) is necessary to simplify (84).

$$\mu(t, f(t)) = \lambda \sqrt{\frac{L}{C}} \int_0^t e^{-\frac{1}{2}(\frac{R}{L}-\frac{G}{C})(t-\eta)} [I_0(\lambda(t-\eta)) + I_1(\lambda(t-\eta))] f(\eta)d\eta \quad (84)$$

The convolution concepts can be applied as a Laplace anti-transform. So, the convolution between the characteristic impedance and an arbitrary function $f(t)$ is generically written as in (85).

$$\mathcal{L}^{-1} [Z_c(s)F(s)] = \sqrt{\frac{L}{C}} f(t) + \mu(t, f(t)) \quad (85)$$

A lossless line, $R = G = 0$, implies $\lambda = 0$ and consequently $\mu(t, f(t)) = 0$. Overhead TLs usually present low distributed resistance and conductance, so the value assumed by $\mu(t, f(t))$ might be proportionally small. Thus, neglect of this term in (85) is reasonable.

3.1.4.2 Convolution of the exponential factor

The same method applied to the characteristic impedance convolution is also applied to the exponential factor. The convolution between $\psi(x, t)$ and an arbitrary function $f(t)$ is

$$\begin{aligned} \mathcal{L}^{-1} [\Psi(x, s)F(s)] = & \int_0^t e^{-\alpha(t-\eta)} \left[\frac{x\lambda\sqrt{a}}{\sqrt{(t-\eta)^2 - ax^2}} I_1 \left(\lambda\sqrt{(t-\eta)^2 - ax^2} \right) H(t-\eta - x\sqrt{a}) \right. \\ & \left. + I_0 \left(\lambda\sqrt{(t-\eta)^2 - ax^2} \right) \delta(t-\eta - x\sqrt{a}) \right] f(\eta) d\eta \quad (86) \end{aligned}$$

The procedure to solve the convolution integral in (86) must be separated into two portions, the first one referring to the Heaviside step function and the second to Dirac's delta function.

□ First portion

The resolution of the first portion uses the limits restriction of the integral, where one of the integrands is the Heaviside step function. Equation (87) shows this property.

$$\int_0^t g(\eta) H(\eta - t_0) d\eta = \int_{t_0}^t g(\eta) d\eta \quad (87)$$

So, the first portion is hereinafter called $d = d(x, t, f(t))$ and described in (88).

$$d(x, t, f(t)) = x \theta \int_{t-x\sqrt{a}}^t e^{-\alpha(t-\eta)} \frac{I_1 \left(\lambda\sqrt{(t-\eta)^2 - ax^2} \right)}{\sqrt{(t-\eta)^2 - ax^2}} f(\eta) d\eta \quad (88)$$

Where θ is defined as the linear distortion factor of the TL and its dimensional unity is the inverse of length. It is noteworthy that $d(x, t, f(t))$ has a linear dependence on the TL length. Parameters that satisfy the relation $RC = LG$ implies a distortionless condition of the TL.

$$\theta = \sqrt{a}\lambda = \frac{RC - LG}{2\sqrt{LC}} \quad (89)$$

□ *Second portion*

The resolution of the second portion of (86) recovers the property of Dirac's delta function previously described in (82).

$$\int_0^t e^{-\alpha(t-\eta)} I_0 \left(\lambda \sqrt{(t-\eta)^2 - ax^2} \right) \delta(t-\eta-x\sqrt{a}) f(\eta) d\eta = e^{-x\beta} f(t-x\sqrt{a}) \quad (90)$$

Where β is defined as the linear attenuation factor of the TL and is calculated according to (91). Its dimensional unity is the inverse of the length.

$$\beta = \alpha\sqrt{a} = \frac{RC + LG}{2\sqrt{LC}} \quad (91)$$

The composition of the first and second portions results in the convolution integral of the exponential factor, as shown in (92).

$$\mathcal{L}^{-1} [\psi(x, s) F(s)] = e^{-x\beta} f(t-x\sqrt{a}) + d(x, t, f(t)) \quad (92)$$

3.1.5 Characteristic equation in continuous time-domain

The original challenge of relating the electrical variables of both TL ends in the time-domain conflicted with the difficulty of applying the Laplace anti-transform in (69). This problem no longer exists due to the convolution calculation of each element in section 3.1.4. Equation (93) shows the application of the Laplace anti-transform to both sides of the equation.

$$\mathcal{L}^{-1} [V(x, s) + Z_c(s) I(x, s)] = \mathcal{L}^{-1} \left[(V(0, s) + Z_c(s) I(0, s)) e^{-x\sqrt{as^2+bs+c}} \right] \quad (93)$$

The left side of (93) is evaluated by replacing (85) and each element is separately calculated.

$$\mathcal{L}^{-1} [V(x, s) + Z_c(s) I(x, s)] = v(x, t) + \sqrt{\frac{L}{C}} i(x, t) + \mu(t, i(x, t)) \quad (94)$$

Equation (95) is the result of the expansion of (94) by using (84).

$$\begin{aligned} \mathcal{L}^{-1} [V(x, s) + Z_c(s) I(x, s)] &= v(x, t) + \sqrt{\frac{L}{C}} i(x, t) \\ &+ \lambda \sqrt{\frac{L}{C}} \int_0^t e^{-\frac{1}{2}(\frac{R}{L}-\frac{G}{C})(t-\eta)} [I_0(\lambda(t-\eta)) + I_1(\lambda(t-\eta))] i(x, \eta) d\eta \end{aligned} \quad (95)$$

The right side of equation (93) is calculated using (92).

$$\begin{aligned} \mathcal{L}^{-1} [(V(0, s) + Z_c(s) I(0, s)) \psi(x, s)] &= e^{-x\beta} \mathcal{L}^{-1} \left[(V(0, s) + Z_c(s) I(0, s)) e^{-sx\sqrt{a}} \right] \\ &+ d \left(x, t-x\sqrt{a}, \mathcal{L}^{-1} \left[(V(0, s) + Z_c(s) I(0, s)) e^{-sx\sqrt{a}} \right] \right) \end{aligned} \quad (96)$$

The effort required to expand (96) using (93) and (88) is very high, so it is necessary to implement some numerical simplifications. Two relevant propositions will be considered from now on.

- The TL parameters satisfy the distortionless condition $RC = LG \rightarrow \theta = 0$
- The calculation of the characteristic impedance only regards reactive parameters, as detailed in (70), i.e., $Z_c = \sqrt{\frac{L}{C}}$

Considering those propositions, the time-domain characteristic equation becomes much simpler to express.

$$v(x, t) + Z_c i(x, t) = e^{-x\beta} [v(0, t - x\sqrt{a}) + Z_c i(0, t - x\sqrt{a})] \quad (97)$$

The TL remote end assumes $x = l$ in (97).

$$v(l, t) + Z_c i(l, t) = e^{-l\beta} [v(0, t - l\sqrt{a}) + Z_c i(0, t - l\sqrt{a})] \quad (98)$$

The notation used to describe Bergeron model in section 2.1.3 is also used here. The local end functions have index 1 and the remote end's have index 2.

$$v_2(t) + Z_c i_2(t) = e^{-l\beta} [v_1(t - \tau) + Z_c i_1(t - \tau)] \quad (99)$$

The expression (99) shows that a wave that left the local end arrives at the remote end attenuated by $e^{-l\beta}$. The wave takes the propagation time $\tau = l\sqrt{LC}$ to complete the whole path. The propagation velocity ϑ of a TL does not depend on the active distributed parameters. This value is identical to that already mentioned in the development of the Bergeron model, as shown in (12). A wave that leaves the remote end and arrives the local end presents an analogous equation to that expressed in (99).

$$v_1(t) + Z_c i_1(t) = e^{-l\beta} [v_2(t - \tau) + Z_c i_2(t - \tau)] \quad (100)$$

The system of equations composed of (99) and (100) is iterative and can be used to simulate electromagnetic transients as well as to develop a fault location method.

$$\begin{cases} v_1(t) + Z_c i_1(t) = e^{-l\beta} [v_2(t - \tau) + Z_c i_2(t - \tau)] \\ v_2(t) + Z_c i_2(t) = e^{-l\beta} [v_1(t - \tau) + Z_c i_1(t - \tau)] \end{cases} \quad (101)$$

3.1.6 Distributed TL model by electrical circuit elements

The simulation of electromagnetic transients in power systems depends not only on the TL model but also on the other components of the simulated circuit. The feasibility of using a particular TL model is associated with the convenience of composing it with the other elements' models. The use of nodal admittance matrices in time-domain simulations is recurrent, so it is import to express (101) in such a manner that these matrices can be easily implemented. The system (101) well fits the admittance matrix formation if the current expression is isolated, so each TL end can be expressed by one conductance

and one current source. The equations of system (102) are already written obeying the current convention of Figure 2.

$$\begin{cases} i_1(t) = \frac{v_1(t)}{Z_c} - e^{-l\beta} \left[\frac{v_2(t-\tau)}{Z_c} + i_2(t-\tau) \right] \\ i_2(t) = \frac{v_2(t)}{Z_c} - e^{-l\beta} \left[\frac{v_1(t-\tau)}{Z_c} + i_1(t-\tau) \right] \end{cases} \quad (102)$$

The model represented by system (102) indicates a topographic decoupling between the TL ends. The electrical dependency between them occurs through a current source whose value is a function of historical values of voltage and current. The replacement of terms referring to the historical current source by a function $h = h(t)$ simplifies the model representation.

$$\begin{cases} i_1(t) = \frac{v_1(t)}{Z_c} - h_1(t) \\ i_2(t) = \frac{v_2(t)}{Z_c} - h_2(t) \end{cases} \quad (103)$$

Equations (104) and (105) refer to local and remote historical current sources, respectively.

$$h_1(t) = e^{-l\beta} \left[\frac{v_2(t-\tau)}{Z_c} + i_2(t-\tau) \right] \quad (104)$$

$$h_2(t) = e^{-l\beta} \left[\frac{v_1(t-\tau)}{Z_c} + i_1(t-\tau) \right] \quad (105)$$

In turn, equations (103), (104) and (105) are similar to those presented in the Bergeron model, in (26), (27) and (28). Therefore, the proposed equating can also be expressed by electric circuits as pictured in Figure 3.

3.1.7 Proposed TL model in discrete time-domain

Up to now, the numerical development presents continuous time-domain expressions regarding voltage and current functions. However, numerical simulations of electromagnetic transients are performed in discrete time-domain and generally present a constant integration time step Δt . It is necessary to adapt the developed continuous time-domain equations to their discrete form. The digitalization may lead to some numerical issues that must be analyzed and a feasible solution must be found. The discrete-time variable defined as κ is related to the time step through $\kappa = n\Delta t$. Equation (106) shows the discrete expression of the propagation time ξ .

$$\xi = \left\lfloor \frac{\tau}{\Delta t} \right\rfloor \quad (106)$$

Where the floor function is defined by

$$\lfloor \eta \rfloor = \max \{ z \in \mathbb{Z} \mid z \leq \eta \} \quad (107)$$

The system (108) shows the discrete form of (101) considering the electrical current convention in Figure 2.

$$\begin{cases} v_2(\kappa) - Z_c i_2(\kappa) = e^{-l\beta} [v_1(\kappa - \xi) + Z_c i_1(\kappa - \xi)] \\ v_1(\kappa) - Z_c i_1(\kappa) = e^{-l\beta} [v_1(\kappa - \xi) + Z_c i_2(\kappa - \xi)] \end{cases} \quad (108)$$

Terminal currents present their isolation in discrete form as in (109).

$$\begin{cases} i_2(\kappa) = \frac{v_2(\kappa)}{Z_c} - e^{-l\beta} \left[\frac{v_1(\kappa - \xi)}{Z_c} + i_1(\kappa - \xi) \right] \\ i_1(\kappa) = \frac{v_1(\kappa)}{Z_c} - e^{-l\beta} \left[\frac{v_2(\kappa - \xi)}{Z_c} + i_2(\kappa - \xi) \right] \end{cases} \quad (109)$$

3.1.7.1 Interpolation in discrete time

When using the discrete time-domain in the development of a TL numerical model, it may occur that the propagation time τ is not an integer multiple of the time step Δt . Linear interpolation is a powerful mathematical tool to deal with this issue. Equation (110) defines the interpolation factor Λ .

$$\Lambda = \frac{\tau}{\Delta t} - \left\lfloor \frac{\tau}{\Delta t} \right\rfloor \quad (110)$$

The factor Λ generally represents the proportions of a discrete signal at a given instant and the immediate previously one. Expression (111) indicates the use of Λ to determine the linear interpolation of an arbitrary discrete signal $f = f(\kappa)$.

$$f_{\text{interpolated}}(\kappa) = (1 + \Lambda)f(\kappa) - \Lambda f(\kappa - 1) \quad (111)$$

When the propagation time τ is an integer multiple of Δt the factor Λ is null and the interpolated value assumes the value of the signal at the current discrete time.

3.2 Proposed fault location method in TLs

The mathematical development presented in section 3.1.7 shows the electrical variables' relation between the TL ends. The proposed fault location method applies these equations.

The sudden input of an impedance in the transmission circuit causes an electrical disturbance. Regardless of the disturbance type – cable break, atmospheric discharge or a short circuit – it is only possible to notice these disturbances as the resulting front waves reach the TL ends. The variables related to the disturbance itself, such as the short circuit current flowing through the fault impedance, cannot be measured at the point of its occurrence. This happens because the fault point is unknown, so there is no way to measure the electrical disturbance there. The feasible option to evaluate and locate a disturbance in a TL is by measuring voltages and currents at one or two ends of the line.

Figure 9 – Sectioning of the TL

In case of a short circuit starting at t_0 , the original TL is sectioned into two segments being the first from the local end to the fault point and the second from the fault point to the remote end. Figure 9 shows the Thevenin equivalents of the terminal substations and the TL sectioning by the fault impedance at x_f . It is not possible to measure and directly calculate the partial fault currents $i_1^F(t)$ and $i_2^F(t)$ because there is no relation between them due to the lack of knowledge of the fault impedance and fault point. However, the fault voltages seen by both ends, $v_1^F(t)$ and $v_2^F(t)$, are easily related since their numerical values are ideally equivalent, regardless of the value, position and nature of the fault impedance. The algebraic procedure to suppress the partial fault currents in (108) consists of time shifting the propagation time of the arguments of the second equation. Equation (112) shows the resulting procedure.

$$\begin{cases} v_2(\kappa) - Z_c i_2(\kappa) = e^{-l\beta} [v_1(\kappa - \xi) + Z_c i_1(\kappa - \xi)] \\ v_1(\kappa + \xi) - Z_c i_1(\kappa + \xi) = e^{-l\beta} [v_2(\kappa) + Z_c i_2(\kappa)] \end{cases} \quad (112)$$

The equations of system (112) now present $i_2(\kappa)$ as a common function, which can be suppressed by replacing one in another. The resulting expression in (113) is the fault voltage seen by the local end regarding a distance x . This expression is crucial to the method development because it calculates an electrical variable at the fault point as a function of the end measurements.

$$v_2(x, \kappa) = \frac{1}{2} e^{+x\beta} \left[v_1 \left(\kappa + \left\lceil \frac{x}{v \Delta t} \right\rceil \right) - Z_c i_1 \left(\kappa + \left\lceil \frac{x}{v \Delta t} \right\rceil \right) \right] + \frac{1}{2} e^{-x\beta} \left[v_1 \left(\kappa - \left\lceil \frac{x}{v \Delta t} \right\rceil \right) + Z_c i_1 \left(\kappa - \left\lceil \frac{x}{v \Delta t} \right\rceil \right) \right] \quad (113)$$

In order to facilitate the notation, the function $v^{\text{seen}} = v^{\text{seen}}(x, \kappa)$ is defined as fault voltage

seen by one end.

$$v^{\text{seen}}(x, \kappa) = v^{\text{seen}}(v_{\text{end}}, i_{\text{end}}, \Delta t) = \frac{1}{2}e^{+x\beta} \left[v_{\text{end}} \left(\kappa + \left\lceil \frac{x}{\vartheta \Delta t} \right\rceil \right) - Z_c i_{\text{end}} \left(\kappa + \left\lceil \frac{x}{\vartheta \Delta t} \right\rceil \right) \right] + \frac{1}{2}e^{-x\beta} \left[v_{\text{end}} \left(\kappa - \left\lceil \frac{x}{\vartheta \Delta t} \right\rceil \right) + Z_c i_{\text{end}} \left(\kappa - \left\lceil \frac{x}{\vartheta \Delta t} \right\rceil \right) \right] \quad (114)$$

Regardless of the sectioning length or the reference end, some variables are constants – characteristic impedance Z_c , linear attenuation factor β and propagation velocity ϑ .

3.2.1 Fault voltage seen by sectioning

The relation between the fault voltages seen by both ends is the key feature to develop the proposed fault location method. Equation (114) assumes different arguments depending on the referential end, so it is calculated separately.

3.2.1.1 Front sectioning

Taking the local end as a reference, equation (114) assumes the arguments listed below:

- $v^{\text{seen}} = v_1^{\text{F}}$
- $v_{\text{end}} = v_1$
- $i_{\text{end}} = i_1$
- $x = x_f$

Thus, the fault voltage seen by the local end as function of local voltages and currents is

$$v_1^{\text{F}}(\kappa) = \frac{1}{2}e^{+x_f\beta} \left[v_1 \left(\kappa + \left\lceil \frac{x_f}{\vartheta \Delta t} \right\rceil \right) - Z_c i_1 \left(\kappa + \left\lceil \frac{x_f}{\vartheta \Delta t} \right\rceil \right) \right] + \frac{1}{2}e^{-x_f\beta} \left[v_1 \left(\kappa - \left\lceil \frac{x_f}{\vartheta \Delta t} \right\rceil \right) + Z_c i_1 \left(\kappa - \left\lceil \frac{x_f}{\vartheta \Delta t} \right\rceil \right) \right] \quad (115)$$

3.2.1.2 Posterior sectioning

The arguments for the posterior sectioning are

- $v^{\text{seen}} = v_2^{\text{F}}$
- $v_{\text{end}} = v_2$
- $i_{\text{end}} = i_2$
- $x = l - x_f$

Thus, the fault voltage seen by the remote end as function of remote voltages and currents is

$$v_2^F(\kappa) = \frac{1}{2}e^{+(l-x_f)\beta} \left[v_2 \left(\kappa + \left\lceil \frac{l-x_f}{\vartheta \Delta t} \right\rceil \right) - Z_c i_2 \left(\kappa + \left\lceil \frac{l-x_f}{\vartheta \Delta t} \right\rceil \right) \right] + \frac{1}{2}e^{-(l-x_f)\beta} \left[v_2 \left(\kappa - \left\lceil \frac{l-x_f}{\vartheta \Delta t} \right\rceil \right) + Z_c i_2 \left(\kappa - \left\lceil \frac{l-x_f}{\vartheta \Delta t} \right\rceil \right) \right] \quad (116)$$

3.2.2 Fault voltages' calculation using the Bergeron model

The same procedure used to calculate fault voltages using the proposed TL model, detailed in section 3.2.1, can also be applied if the Bergeron model is used. Among the three approaches of the Bergeron model, the one described in section 2.1.3.6 is the most appropriate due to its algebraic facilities and resistance lumped in both ends.

3.2.2.1 Front sectioning

The fault voltage seen by the local end is

$$v_1^F(\kappa) = \frac{Z_c - \frac{Rx_f}{2}}{2Z_c} v_1 \left(\kappa - \left\lceil \frac{x_f}{\vartheta \Delta t} \right\rceil \right) + \frac{\left(Z_c - \frac{Rx_f}{2} \right)^2}{2Z_c} i_1 \left(\kappa - \left\lceil \frac{x_f}{\vartheta \Delta t} \right\rceil \right) + \frac{Z_c + \frac{Rx_f}{2}}{2Z_c} v_1 \left(\kappa + \left\lceil \frac{x_f}{\vartheta \Delta t} \right\rceil \right) - \frac{\left(Z_c + \frac{Rx_f}{2} \right)^2}{2Z_c} i_1 \left(\kappa + \left\lceil \frac{x_f}{\vartheta \Delta t} \right\rceil \right) \quad (117)$$

3.2.2.2 Posterior sectioning

The fault voltage seen by the remote end is

$$v_2^F(\kappa) = \frac{Z_c - \frac{R(l-x_f)}{2}}{2Z_c} v_2 \left(\kappa - \left\lceil \frac{l-x_f}{\vartheta \Delta t} \right\rceil \right) + \frac{\left(Z_c - \frac{R(l-x_f)}{2} \right)^2}{2Z_c} i_2 \left(\kappa - \left\lceil \frac{l-x_f}{\vartheta \Delta t} \right\rceil \right) + \frac{Z_c + \frac{R(l-x_f)}{2}}{2Z_c} v_2 \left(\kappa + \left\lceil \frac{l-x_f}{\vartheta \Delta t} \right\rceil \right) - \frac{\left(Z_c + \frac{R(l-x_f)}{2} \right)^2}{2Z_c} i_2 \left(\kappa + \left\lceil \frac{l-x_f}{\vartheta \Delta t} \right\rceil \right) \quad (118)$$

3.2.3 Fault voltages relation

The fault point x_f is estimated by scanning the minimum value of the absolute difference between the fault voltage seen by both ends. The equating that applies the proposed TL model is based on (115) and (116), and the one that uses the Bergeron model are based on (117) and (118). It is possible to define a function $D = D(x, \kappa)$ of two variables –

space x and discrete time κ – that evaluates for both variables the absolute difference between the fault voltages seen by both ends.

$$D(x, \kappa) = f(v_1(\kappa), i_1(\kappa), v_2(\kappa), i_2(\kappa), R, L, C, G, l, \Delta x, \Delta t) = |v_1^F(x, \kappa) - v_2^F(x, \kappa)| \quad (119)$$

The scanning of $D(x, \kappa)$ for each Δt starts at $t_{\text{start}} = \kappa_0 \Delta t$ and proceeds until the protection system pick-up or the fault auto-extinguishing at $t_{\text{stop}} = \kappa_1 \Delta t$. The mean value over time of $D(x, \kappa)$ leads to a better understanding of fault voltage behavior during the whole fault period and it corresponds to the function $D_m = D_m(x)$, whose calculation is described in (120).

$$D_m(x) = \frac{1}{\Delta t (\kappa_1 - \kappa_0)} \sum_{\kappa=\kappa_0}^{\kappa_1} D(x, \kappa) \quad (120)$$

A simple search tracks the minimum value of $D_m(x)$, which is considered the final output of the proposed fault location method as pointed out in (121).

$$d = \min [D_m(x)] \rightarrow x_f = D_m^{-1}(d), \quad 0 < x < l \quad (121)$$

Figure 10 depicts a simplified flowchart of the methodology regarding the proposed fault location algorithm.

Figure 10 – Methodology flowchart for the fault location algorithm

Results

THE RESULTS obtained in this chapter covers two kinds of transmission systems: AC and DC. In both cases, the proposed TL model and the fault location method are tested and compared to the well-established Bergeron model. The simulation environment used in this section is the Matlab/Simulink. A Matlab simulation block comprises the numerical development described in section 3.1.7. The use of a TL modeling block is essential to facilitate and speed up the simulation of different scenarios. Figure 11 shows the details of the block, which consists of:

- ❑ local and remote end connectors;
- ❑ unit delay block;
- ❑ internal function *fcn*;
- ❑ current sources;
- ❑ input block for the TL constitutive parameters and the time step Δt ;
- ❑ TL characteristic impedance;
- ❑ ideal voltmeters and ammeters.

The internal function *fcn* presents six inputs: the distributed parameters R , L , C and G , the TL length l and integration step Δt . This routine initializes by calculating primary quantities such as characteristic impedance, wave propagation time and linear attenuation factor. In addition, it updates the values of the historical current sources and implements the interpolation of voltage and current signals.

Two detailed cases – one for AC and other for DC systems – show voltage and current waveforms, power system characteristics and the curves related to the fault location method. Both cases present a comparison between the proposed TL model and Bergeron's. Then there is a scrutiny of the fault location algorithm through sensitivity analyses, where many variations are implemented for an AC system.

Figure 11 – TL block in Simulink environment

4.1 AC transmission system

Figure 12 shows, generically, the analyzed AC power system. It consists of Thevenin equivalents at the substations, upstream and downstream TLs. The fault location method concerns the single-phase TL that interconnects the local and remote substations, or the positive sequence equivalent of the TL. The acquisition of the electrical variables is performed by VTs and CTs, which consider deviations regarding the transformation ratio and saturation of the magnetic core. Besides that, a delay block models the desynchronization between the TL end measurements.

The comparison between the transient responses using the proposed and Bergeron models is relevant because the resulting waveforms from the simulations are input data of the fault location algorithm. It is also possible to show how the full distribution of the linear resistance and leakage conductance impacts the voltage and current waveforms when the TL is subjected to a disturbance and how it affects the performance of the fault location method.

Figure 13 depicts the dimensions of the TL tower used in this section. Table 1 presents the TL main electromechanical data, which are used to calculate the distributed electrical parameters, as reported in Table 2. Knowing the distributed parameters of Table 2 enables calculating the linear attenuation factor β described in (91).

$$\beta = \frac{RC + LG}{2\sqrt{LC}} = 8.75 \times 10^{-5} [\text{km}^{-1}] \quad (122)$$

The exponential factor that multiplies the current sources in (105) and (104) is less than unity, $e^{-l\beta} = 0.9488433802$. An ideal and lossless TL, $R = G = 0$, implies $\beta = 0$ and consequently no attenuation of the active power signal. Although extra high voltage TLs may present a noticeable corona effect, the distributed conductance is considered to be zero so there is no discrepancy of premises, since the Bergeron model neglects this

Figure 12 – AC power system

Table 1 – Electromechanical parameters of the AC TL

Rated voltage	500 [kVAC]
Length	600 [km]
Phases	3
Ground resistivity	100 [Ω m]
Circuit	single
Phase cables	Hawk
Cables per bundle	4
Linear resistance	0.1195 [Ω /km] @ 20 [$^{\circ}$ C]
Overall diameter	21.79 [mm]
Inner diameter	8.02 [mm]
Ground wire	EHS, class B
Linear resistance	4.82 [Ω /km] @ 20 [$^{\circ}$ C]
Overall diameter	7.94 [mm]

Table 2 – Positive sequence of the 500 [kVAC] TL

resistance	0.0378 [Ω /km]
inductance	0.7356 [mH/km]
capacitance	15.735 [nF/km]
conductance	0 [μ S/km]

Figure 13 – Cross section and dimensions of the 500 [kVAC] tower

Table 3 – Thévenin equivalent parameters for 500 [kVAC] as base voltage

	local end	remote end
voltage	0.95/0° [pu]	0.90/−40° [pu]
resistance	40 [Ω]	60 [Ω]
inductance	14.5 [mH]	20.0 [mH]

Table 4 – Upstream and downstream TL parameters

	upstream	downstream
resistance	0.1 [Ω/km]	0.09 [Ω/km]
inductance	1.28 [mH/km]	1.5 [mH/km]
capacitance	15 [nF/km]	12 [nF/km]
conductance	0 [μS/km]	0 [μS/km]
length	100 [km]	5 [km]

parameter. Figure 14 shows the saturation curve of the CTs magnetic core. Table 5 details the main parameters of VTs and CTs.

4.1.1 AC case

This section presents of an AC case in which various waveforms are demonstrated. The responses of the TL models are compared for the purpose of fault location. The difference between the models is evident for the case under analysis. The location of a

Table 5 – Parameters of VT and CT

	VT	CT
burden	250 [VA]	25 [VA]
transformation ratio	$\frac{500000}{\sqrt{3}} : \frac{115}{\sqrt{3}}$	900:5
transformation ratio accuracy	+3 [%]	-2 [%]
application	protection	protection
precision class	1.2P200	10B200
secondary cables	4 [mm ²]	4 [mm ²]
cable length	900 [m]	1000 [m]

Table 6 – General parameters of the AC case

SNR	30 [dB]
sample frequency	200 [kHz]
space step	200 [m]
Fault parameters	
position	400 [km]
start	200 [ms]
stop	230 [ms]
resistance	5 [Ω]
inductance	1 [H]

Figure 14 – Saturation curve of the CT

self-extinguishing short circuit is challenging because the fault window is minimal. Such limitation requires the fault location algorithm to use voltage and current waveforms that may present intense transient components, which may be a problem during algorithm execution. The inherent disturbance of terminal voltages and currents due to the simulation initialization is irrelevant, because when the fault occurs the power system is already in sinusoidal steady state. Table 6 assigns the general parameters of the simulation of the AC case. Although the electric arcs have resistive nature, a small inductive component was considered, present in the towers' lattices for phase-ground faults.

The fault occurs once the switch specified in Figure 12 closes. The waveforms of voltages and currents waveforms change considerably after the switch closure and consequently insertion of the fault impedance into the transmission circuit. Figure 15 depicts a concomitant overview of these waveforms for both TL models. It is noteworthy that the voltage and current surges reach the terminals at different times, since the fault did not occur at the center of the TL. In addition to the presence of noise, there is an appreciable transient content in waveforms during fault and post-fault periods. Generally, the voltage waveforms for both models are similar except for a more persistent transient behavior over time for the proposed model. It is inferred that the linear resistance distribution tends to slow down the high frequency components caused by the power system disturbance.

Regarding current waveforms, there are more disparities between the models' responses

Figure 15 – Waveforms of the AC case. (a) Voltages (b) Currents

Table 7 – Saturation periods of the CTs by model and fault cycle

model	Bergeron		proposed	
cycle	1°	2°	1°	2°
local CT [ms]	3.4	3.1	5.2	3.8
remote CT [ms]	5.9	2.9	5.7	3.4

compared to the voltage waveforms. There are differences in the decay period of the exponential component and the periods in which CTs are saturated. These characteristics are more prominent for the proposed model than for Bergeron's. Table 7 summarizes the comparison between the TL models regarding saturation periods of the CTs.

4.1.2 Fault location in the AC case

The variables and waveforms presented in section 4.1.1 are used according to the methodology presented in section 3.2 for the purpose of fault location. Figure 16 exemplifies a three-dimensional image representation of $D(x, t)$. In this example, VTs and CTs are considered ideals. Figure 16 also illustrates the average curve $D_m(x, t)$, calculated according to (120). This geometric entity has a “V” profile, where its absolute minimum value indicates the output fault point.

Up to 199 [ms], the image function has an amplitude equivalent to the intensity of the signal noise. Subsequently, after the short circuit occurrence, there is a V-surface appearance along the time axis. This surface presents oscillatory behavior of 93 [Hz]. The V-profile is maintained for up to a half cycle after the fault ends. This is an interesting fact because the remaining transient behavior of electrical signals corroborates the image of the function $D(x, t)$ even after the system is cleared. Plotting $D(x, t = t_0)$ and $D_m(x)$ assists the visualization of the V-curves response to a progression over time t_0 , as shown in Figure 18. The animation indicates the evolution of the absolute minimum point of $D_m(x)$ as a function of time considering four types of input signals: proposed model and Bergeron's considering ideal and non-ideal instrumentation transformers.

There is a combination between the implemented model for the fault localization algorithm and the model used in the numerical simulation. Since there are two types of TL models, there are four possible combinations, as detailed in Figure 17 and Table 8.

4.2 DC transmission system

The evolution of power electronic technologies spread the use of HVDC systems in power transmission. The electrical characteristics of the electronic components and their configurations influence the electrical response of rectifier and inverter. These waveforms can present diverse harmonic content, ripple amplitude, dependence of the control system,

Figure 16 – 3D representation of $D(x, t)$ for the AC case

Figure 17 – V-curve profiles for different combinations of TL models

Figure 18 – V-curve profiles for a t_0 progression in AC case. (a) Instant curve $D(x, t_0)$ (b) Average curve $D_m(x)$

Table 8 – Combination diagram of TL models

		numerical simulation	
		proposed	Bergeron
fault location	proposed	D_m^1	D_m^2
algorithm	Bergeron	D_m^3	D_m^4

Table 9 – Electromechanical parameters of the DC TL

Rated voltage	500 [kVDC]
Length	1500 [km]
Ground resistivity	1000 [Ω m]
Circuit	single-pole
Pole cable	Grosbeak
Cables per pole	3
Linear resistance	0.0897 [Ω /km] @ 20 [$^{\circ}$ C]
Overall diameter	25.15 [mm]
Inner diameter	8.61 [mm]
Ground wire	EHS, class B
Linear resistance	4.82 [Ω /km] @ 20 [$^{\circ}$ C]
Overall diameter	7.94 [mm]

etc. So, voltage and current waveforms of HVDC systems may hinder fault location in the TL.

Figure 19 depicts a HVDC transmission system constituted of rectifier, inverter, smoothing reactors, passive filters and single-pole TL. The implemented system is inspired by a pioneer installation at the time, Cahora Bassa (BERIGER; HENGSEBERGER; JUETTE, 1975; GALE et al., 2001). The complex construction started in 1974 with the proposal for transmitting the energy generated in Mozambique by Cahora Bassa HPP to South Africa. The TL is 1420 [km] long and its rated voltage is 533 [kVDC].

Single-pole transmission systems are characterized by grounding one of the poles of rectifiers and inverters. This implies return path current by the ground system. This type of arrangement presents financial benefits since there is no wired connection between the negative poles and consequently there is significant structural saving in towers and in cables installed. However, the arrangement has reduced operational availability, because when the pole is unavailable the entire system is shut-down.

Figure 20 illustrates the cross section and dimensions of the HVDC tower. Table 9 shows the main electromechanical data of the installation, which are used to calculate the TL distributed parameters, as detailed in Table 10. Thus, it is possible to evaluate the linear attenuation factor in (123).

$$\beta = \frac{RC + LG}{2\sqrt{LC}} = 2.68 \times 10^{-5} [\text{km}^{-1}] \quad (123)$$

Table 10 – Electrical parameters of the 500 [kVDC] TL

resistance	0.0299 [Ω/km]
inductance	3.117 [mH/km]
capacitance	10.024 [nF/km]
conductance	0 [$\mu\text{S}/\text{km}$]

Figure 19 – HVDC 12-pulse transmission system 1000 [MW], 500 [kVDC], 50/60 [Hz]

Table 11 – Parameters of the HVDC system

	local end	remote end
AC industrial frequency	50 [Hz]	60 [Hz]
AC rated voltage	500 [kV]	345 [kV]
smoothing inductor	3 [H]	3 [H]
short circuit power	5000 [MVA]	10000 [MVA]

One of the advantages of using time-domain fault location algorithms is that their execution is independent of the nature of voltage and current inputs, such as those presented in DC systems. This section shows that the presence of control loops in electronic power circuits does not affect the performance of the fault location algorithm. In addition, the time-domain technique can be applied to any kind of waveform and fast self-extinguishing short circuits. This thesis does not address details of the control loops of the firing circuits. More information regarding these aspects are detailed in reference (CASORIA, 2018). Table 11 shows the general parameters of the HVDC system.

4.2.1 DC case

Protection and control in power electronic circuits ensures greater electrical stability against disturbances. However, the regulation of voltage and current soon after the occurrence of the fault may be unfavorable to the performance of the fault location algorithm. In this section, voltage and current waveforms are analyzed in detail. The influence of pre-fault transient state is mitigated since the fault switches when the power system is in steady state.

Figure 20 – Cross section and dimensions of the 500 [kVDC] TL

Table 12 – General parameters of the DC case

SNR	20 [dB]
sample frequency	100 [kHz]
space step	100 [m]
Fault parameters	
position	1200 [km]
start	200 [ms]
stop	215 [ms]
resistance	500 [Ω]

Table 12 shows the general simulation parameters for the DC case. Figure 21 depicts voltage and current waveforms resulting from the simulation considering the proposed model and Bergeron's.

Like in the AC case, the disturbance reaches the TL ends at different instants. This happens because the fault is not equidistant from the terminals and implies a difference of 6.7 [ms] between them. Voltage waveforms show an intense voltage sag after the fault occurs, especially at the remote end, which is closest end to the fault point. The abrupt changes in voltage and current levels clearly show the phenomenon of wave reflection between terminals and the fault impedance. The proposed and Bergeron models are very close in steady and transient states. The differences are more pronounced in the transient state due to the singularities of each model regarding linear resistance distribution.

4.2.2 Fault location in the DC case

Analogously to section 4.1.2, the electrical data are used as inputs for the fault location algorithm. Figure 22 depicts the $D(x, t)$ function image calculated using (119). The average function $D_m(x)$ calculated during the fault period presents an absolute minimum value, which indicates the output fault point. The three-dimensional representation of $D(x, t)$ correctly indicates the fault point even in the post-fault period, i.e., the remaining transient content in electrical waveforms can also be used to perform the fault location algorithm.

Figure 23 depicts the time progression of the V-curves' calculation and their averages $D_m(x)$. It is possible to observe the instantaneous minimum points when the proposed and Bergeron models are applied. In order to illustrate how function $D(x, t)$ behaves when voltage and current signals have intense noises, a white Gaussian noise component has been considered in both signals so that $\text{SNR} = 30$ [dB].

Figure 21 – Waveforms of the DC case. (a) Voltages (b) Currents

Figure 22 – 3D representation of $D(x, t)$ for the DC case

4.3 Sensitivity analysis

This section evaluates the performance of the fault location method when the algorithm is subjected to input variations deliberately included to induce errors, such as different TL distributed parameters than the ones used in the AC simulations, desynchronization of acquisitions and gaussian noise. The simulations consider the proposed and Bergeron models to provide the input signals to the fault location algorithm. The differences between the performance of the models are evaluated by calculating the relative error ϵ , as shown in (124).

$$\epsilon = \frac{100 |x - x_f|}{l} \quad (124)$$

The sensitivity analyses present simulation parameters according to Table 13. These parameters may change depending on the analysis performed. They can be classified into two groups – two-dimensional and three-dimensional. The first consists of analyzing the relative error of the algorithm response when one of the input data varies. The second refers to the variation of two inputs. There are subdivisions of both classes depending on

Figure 23 – V-curve profiles for a t_0 progression in DC case. (a) Instant curve $D(x, t_0)$ (b) Average curve $D_m(x)$

the nature of the analyzed variables – electrical parameters of the TL; computational and instrumentation imprecision; and operating parameters.

Two-dimensional analysis

The two-dimensional sensitivity analyses are grouped according to the following list:

- ❑ Electrical parameters of the TL
 - distributed linear resistance;
 - distributed linear inductance;
 - distributed transverse capacitance;
 - distributed leakage conductance;
 - total length.
- ❑ Computational and instrumentation imprecision
 - space step;
 - sample frequency;
 - CTs' burden load;
 - desynchronization between local and remote acquisitions;
 - SNR
- ❑ Operating parameters
 - fault impedance;
 - fault point;
 - fault duration.

Table 13 – Simulation parameters of the sensitivity analysis

sample frequency	300 [kHz]
space step	500 [m]
SNR	30 [dB]
fault parameters	
point	250 [km]
duration	20 [ms]
resistance	50 [Ω]
inductance	500 [mH]

4.3.1 Two-dimensional sensitivity analysis of the electrical parameters of the TL

The two-dimensional sensitivity analyses of the constitutive electrical parameters comprise five parameters – distributed resistance, inductance, capacitance, conductance, and the TL length. In the case of distributed parameters, the algorithm accuracy is calculated regarding a variation range from -75 to +75 [%] of the nominal value.

4.3.1.1 Variation of linear resistance

This thesis focuses on the distinction between the TL models mainly regarding the distribution of linear resistance. Thus, this sensitivity analysis evidences part of the relevance when partially or completely distributing this parameter. Figure 24 depicts the relative fault location error as a function of the resistance variation. In most of the domain, the proposed model is less susceptible to resistance variation, with a maximum error of 7.42 [%] at the lower limit. In turn, the Bergeron model presents a maximum error of 13.25 [%] at the lower limit.

4.3.1.2 Variation of linear inductance

Figure 25 depicts the sensitivity analysis of the distributed inductance. Both methods present substantial increase of the relative error for inductance values smaller than the nominal value. In the range of -50 to 75 [%], the proposed model performs better than Bergeron's. The models still have distinct behavior in the positive domain - while the proposed model exhibits an error not greater than 0.25 [%], the Bergeron model exhibits an increasing error with positive variation of the inductance.

4.3.1.3 Variation of the transverse capacitance

Unlike for the resistance and inductance sensitivity analyses, a small capacitance variation may imply a proportionally large variation of the relative error, as shown in Figure 26. In most of the domain, the proposed model shows better performance than Bergeron's. In the negative semi-axis there is a greater adhesion of the results for both models, but in the positive semi-axis the proposed model is less susceptible to variation of the distributed capacitance.

4.3.1.4 Variation of leakage conductance

The distributed conductance is a parameter usually neglected in fault location techniques and electromagnetic transient simulations, but it is relevant for extra high voltage TLs (MAMIS, 2003; NUCCI et al., 2000). For this analysis, the distributed conductance is set at $G = 1$ [$\mu\text{S}/\text{km}$] regarding the proposed model. Since the Bergeron model does not

Figure 24 – Fault location error as a function of variation of linear resistance

Figure 25 – Fault location error as a function of variation of the linear inductance

Figure 26 – Fault location error as a function of variation of the transverse capacitance

Figure 27 – Fault location error as a function of variation of the leakage conductance

consider this variable in its equating, it presents a constant relative error of 0.5 [%] for the whole domain, as shown in Figure 27. Otherwise, the proposed model has a similar error profile as the one presented in section 4.3.1.1. The maximum error obtained using the proposed model is 2.1 [%] at the lower limit of the domain. This sensitivity analysis shows important results because it demonstrates that incorrect premises can imply unacceptable precision of fault location methods.

4.3.1.5 Variation of the TL length

The sensitivity analysis of the TL length highlights the algorithm performance when fault point is fixed 250 [km] and TL length varies from 260 to 1500 [km]. In theory, a longer TL should present a higher transient content, which would make the fault location process more difficult. However, Figure 28 shows that the proposed model presents relative error below 0.5 [%] regarding TLs from 500 to 1500 [km] long. Although the fault window is very small and there are several sources of input imprecision, both methods present a relative error smaller than 3.5 [%] for a fault point close to the remote end. The algorithm that implements the Bergeron model presents a vertiginous error increase for TLs longer than 1100 [km]. In turn, the proposed model is, in general less susceptible to variation of the TL length.

4.3.2 Two-dimensional sensitivity analysis of computational and instrumentation imprecision

4.3.2.1 Variation of space discretization

The space discretization has great relevance for the fault location algorithm since it establishes the maximum precision of the method. Low space discretization tends to decrease the accuracy of the algorithm, but it decreases the computational efforts of running the algorithm. Figure 29 shows the sensitivity analysis of the space discretization. The implementation of both models presents relative errors up to 0.25 [%] for space discretization up to 600 [m]. There is steep growth of the relative error for discretization greater than 1000 [m] for both models. The algorithm implementing the Bergeron model performs worse in most of the domain than the one presented by the proposed model.

4.3.2.2 Variation of the sample frequency

Figure 30 illustrates the result of the sensitivity analysis for the variation of the sample frequency. Both models have similar behavior and present a slight exponential decay of the relative error as sampling frequency increases. The proposed model presents an average error lower than Bergeron's. Sample frequencies from 50 to 100 [kHz] indicate relative errors less than 1.3 [%].

Figure 28 – Fault location error as a function of variation of TL length

Figure 29 – Fault location error as a function of variation of space discretization

4.3.2.3 Variation of burden load

The sensitivity analysis of the burden load considers the CTs and secondary cables as described in Table 5. Figure 31 depicts the analysis result, indicating an identical critical burden load, $Z_{\text{critical}} = 22 \text{ } [\Omega]$ for both models. The relative error is up to 0.5 [%] for burden loads less than Z_{critical} . Above this value, there is an intense saturation of the CTs during fault and post-fault periods. This implies prominent measurement errors and consequent imprecision of the fault location algorithm.

The electrical current waveforms presented in section 4.1.1 showed differences regarding CT saturation tendency depending on the TL model implemented for the numerical simulation. The proposed model exhibits, during the transient response, a higher CT saturation tendency compared to Bergeron's. So, the relative error of the proposed model is greater than Bergeron's in most of the analysis.

4.3.2.4 Variation of desynchronization between local and remote acquisitions

The fault location algorithm uses waveform inputs acquired from both TL ends. It is noteworthy that the desynchronization between the samples can cause calculation disturbances and affect the performance of the algorithm. Figure 32 shows that a desynchronization up to 10 samples implies relative error less than 1 [%]. The implementation of the proposed model has greater accuracy compared to the Bergeron's. The relative error of both models increases linearly with the desynchronization.

4.3.2.5 Variation of the SNR

The SNR considered in this sensitivity analysis is equivalent for the voltage and current waveforms. Figure 33 illustrates an exponential decay of the relative error as the SNR increases. The proposed model exhibits a constant relative error for a SNR greater than 33 [dB], while the Bergeron model presents this behavior just for SNR greater than 52 [dB].

4.3.3 Two-dimensional sensitivity analysis of operating parameters

4.3.3.1 Variation of fault impedance

Voltage and current waveforms present higher disturbances the lower the short-circuit impedance is, so there are greater differences between the fault and pre-fault period waveforms. The performance of the fault location algorithm tends to be better for low impedance faults. The fault impedance is considered completely resistive for this sensitivity analysis. Figure 34 shows that both models have a relative error of less than 0.5

Figure 30 – Fault location error as a function of variation of sampling frequency

Figure 31 – Fault location error as a function of variation of burden load

Figure 32 – Fault location error as a function of variation of desynchronization between local and remote acquisitions

Figure 33 – Fault location error as a function of variation of SNR

Figure 34 – Fault location error as a function of fault impedance

Figure 35 – Fault location error as a function of fault point

[%] for impedances up to 20 $[\Omega]$. From this threshold there is a huge increase of relative error.

4.3.3.2 Variation of fault point

Figure 35 shows the result of the sensitivity analysis of the fault point. The Bergeron model performs better than the proposed model. This result was already predicted, given the higher CT saturation propensity of the waveforms obtained by the first model compared to the second, as also shown in section 4.3.2.3. Different fault points imply different current waveforms, which present CT saturation in a particular way.

4.3.3.3 Variation of fault duration

Figure 36 illustrates the result of the sensitivity analysis of the fault duration. The implementation of the proposed model presents less susceptibility to the fault window compared to Bergeron's. The relative error increases exponentially up to 100 [ms] for both models. It is assumed that small fault windows result in short circuit currents that do not induce CT saturation and consequently improve the performance of the algorithm in these situations. After 100 [ms] fault duration both models present very similar results.

Three-dimensional sensitivity analysis

The three-dimensional sensitivity analyzes present only the performance of the proposed model. This type of analysis is grouped according to the following classification:

- Electrical parameters of the TL
 - distributed linear inductance *versus* distributed transverse capacitance;
 - distributed linear resistance *versus* distributed leakage conductance.
- Computational and instrumentation imprecision
 - VT ratio error *versus* CT ratio error;
 - sample frequency *versus* space step.
- Operating parameters
 - fault impedance *versus* fault point.

4.3.4 Three-dimensional sensitivity analysis of the electrical parameters of the TL

The algorithm accuracy is calculated regarding a variation range from -50 to +50 [%] of the nominal value of the electrical parameters.

Figure 36 – Fault location error as a function of fault duration

Figure 37 – Fault location error as a function of distributed inductance and capacitance

4.3.4.1 Variation of distributed inductance and capacitance

TLs can present variations of their equivalent reactive parameters over time and space. This phenomenon occurs through variation of thermal and atmospheric conditions. Sections 4.3.1.2 and 4.3.1.3 deal individually with the sensitivity analyses of reactive parameters variations, while this section presents a combined analysis of these parameters.

Figure 37 presents minimum relative error threshold on domain area described by $D = \{(\Delta L, \Delta C) \in \mathbb{R}^2 \mid 0 \leq \Delta C \leq 50, \Delta L \geq -2\Delta C + 50\}$. There is a considerable increase of the relative error at third and fourth quadrant of the domain. This behavior is depicted in Figure 26. The sensitivity analysis of the reactive parameters shows that the relative error is less than 1.5 [%] for a circle centered at the Cartesian origin and radius equal 15 [%].

4.3.4.2 Variation of distributed resistance and conductance

Figure 38 shows that the relative error is more dependent on the conductance variation compared to the resistance variation. This type of information is important to the power transmission industry since most of fault location methods disregard the TL leakage conductance, whether based on phasor or traveling wave methods.

The minimum error value of 0.5 [%] is reached near the Cartesian center of the domain and extends through a strip described by the relation $\Delta R = -5\Delta G + 75$. The maximum error of 6 [%] is obtained at the extreme of the fourth quadrant of the domain.

4.3.5 Three-dimensional analysis of computational and instrumentation imprecision

4.3.5.1 Variation of VT and CT ratio error

The transformation ratio errors into VTs and CTs embed instrumentation inaccuracies to voltage and current waveforms, which are primary inputs to the proposed fault location algorithm. This sensitivity analysis considers an inaccuracy of both transformers from -5 to 5 [%]. Figure 39 depicts the main results of this analysis: the relative error is equally dependent on both transformation ratio errors. The maximum error is located at the extreme of the second quadrant of the domain.

4.3.5.2 Variation of sampling frequency and space discretization

The correlated behaviors presented in sections 4.3.2.2 and 4.3.2.1 remain in this composed sensitivity analysis. Figure 40 shows that regardless of the sampling frequency value, the relative error tends to decrease with reduction of the space discretization. Regarding the sample frequency axis, there are error spikes at every multiple of 58 [kHz].

Figure 38 – Fault location error as a function of distributed resistance and conductance

Figure 39 – Fault location error as a function of VT and CT ratio error

Figure 40 – Fault location error as a function of sampling frequency and space discretization

Figure 41 – Fault location error as a function of fault impedance and fault point

4.3.6 Three-dimensional analysis of operating parameters

4.3.6.1 Variation of fault impedance and fault point

Figure 41 depicts the result of the sensitivity analysis of the fault impedance and fault point. There is a relative error threshold less than 0.5 [%] for $Z_{\text{fault}} \leq 500 \text{ } [\Omega]$, regardless of the fault point. From this fault impedance, the relative error tends to grow steeply.

Conclusion

This master's thesis covered TL numerical models in the time-domain for electromagnetic transients and their application in fault location methods.

Regarding TL models, a single-phase model is proposed and associated with a simulation block that considers the fully distribution of the electrical parameters – resistance, inductance, capacitance and conductance. The methodology emphasizes the topographical differences between the proposed model and Bergeron's. Their simulation results exemplify the models responses for a short circuit condition in AC and DC power systems. The key feature that distinguishes the models' responses is the transient attenuation of the short circuit currents, the proposed model usually presents a higher decay time than Bergeron's.

After that, the TL models are applied to develop a fault location algorithm. Regardless of the nature of the waveforms, the time-domain fault location method presents satisfactory results even in adverse conditions such as fast self-extinguishing short circuits, low SNR signals, instrumentation transformer inaccuracy, etc.

The scrutiny performed in the sensitivity analyzes shows relations between the location error and the variables involved in the fault location process. In most the sensitivity analyses the algorithm that implements the proposed model performs better than the one that implements the Bergeron's.

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