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Control chart for monitoring the ratio of two Poisson rates

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Abstract

In this study we are concerning in monitoring the ratio ρ of two Poisson rates by the control chart. Let X and Y be two independent Poisson random variables with means λ_1 and $\lambda_2 = \lambda_1/\rho$ respectively. The study considers that only individual observations X_i and Y_i are available at each sampling time i . The performance in detecting shifts on the ratio ρ by several statistics, some based on normalized transformations are evaluated by an extensive simulation study. Two types of control charts, Shewhart and EWMA are considered. The one-sided control limit with UCL is applied so that we are focusing on detecting when the ratio ρ shifted to higher rate in this dissertation. Our focus is on monitoring the ratio ρ and not on the inference of means of λ_1 and λ_2 . The results pointed out that EWMA control chart is better alternative. Some guidelines indicating which statistics yield best performance are proposed for the practitioners. In general, the combination of statistics W_0 and Z_4^{**} are recommended once EWMA control chart is applied. In case the difference between λ_1 and λ_2 is small, the statistic W_4^* could be used instead of W_0 . Numerical example illustrates an application.

Keywords: Maximum likelihood estimator; conditioned maximum likelihood estimator; normal transformations; EWMA; Shewhart control charts; Monte Carlo simulation.

Resumo

Neste estudo estamos interessados no monitoramento da razão ρ de duas variáveis de Poisson por meio de gráfico de controle. Definimos X e Y como variáveis aleatórias independentes Poisson de parâmetros λ_1 e $\lambda_2 = \lambda_1/\rho$ respectivamente. O estudo considera que apenas observações individuais X_i e Y_i estão disponíveis de cada tempo i . Os desempenhos de detecção de mudanças da razão ρ são feitas empregando várias estatísticas, algumas são baseadas em transformações normais e foram avaliadas através uma extensa simulação. Dois tipos de gráfico de controle, Shewhart e EWMA são considerados. O limite de controle unilateral LCS é determinado visto que estamos focando na detecção da razão ρ para uma taxa maior nesta dissertação. Nosso foco é no monitoramento de razão ρ e não na inferência de médias λ_1 e λ_2 . Os resultados apontaram que o gráfico de controle de EWMA é a melhor alternativa. Algumas orientações com indicação de estatísticas que têm os melhores desempenhos são propostas para praticantes. Em geral, a combinação de das estatísticas W_0 e Z_4^{**} é recomendada quando o gráfico de controle de EWMA é utilizado. No caso da diferença entre λ_1 e λ_2 for pequena, a estatística W_4^* poderia ser utilizada ao invés de W_0 . Um exemplo numérico ilustra uma aplicação.

Palavras-chaves: Estimador de máxima verossimilhança; Estimador condicionado de máxima verossimilhança; Transformações normais; EWMA; Gráfico de controle de Shewhart; Simulação de Monte Carlo.

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1 Introduction

Many companies employ indexes to monitor their activities and some of these indexes are ratio of two independent counts. The decision if an activity is stable is based on a threshold usually chosen without statistical criterion but only considering historical data and managers' expertise.

Several examples illustrate the necessity of this type of monitoring. For example, a printing machine sales company is used to planning the purchase order amount of consumables (toner, developer, drum etc.), by the monthly indicator which is estimated by the ratio between the demand of specific consumable (the amount of delivery of specific consumable from warehouse) and the MIF (the amount of printing "Machine In Field"). This indicator estimates the demand of specific consumable per a unit of machine and it should be stable if there is no extraordinary situation such as the hike (reduction) of the print volume of customer's machine, new sales of machine to the customer which has intensive production and so on. If a company could not perceive a shift of indicator, the purchase order amount would be wrong and there is the risk for the lack or excess of stock. So the monitoring of this indicator is crucial and there is little meaning to monitor individually the demand and the MIF as it is normal that the demand of the products increases (decreases) when MIF increases (decreases).

A second example is the monitoring of quality of product like cloth (linen, silk, etc). Two independent types of non-conformity are counted. Important to mention that the presence of the non-conformity does not mean that the product is inappropriate for use or prejudice the visual appearance. An increase in the ratio may harm the visual appearance like non-conformity of type 1 being more standing out than type 2.

The two previous cases are examples which two quality characteristics are monitored simultaneously on the sample unit. Now the third example, which quality characteristic is taken on independent sample units. Let us consider two brands of a product and the brand A is the reference one. The producer of brand A would like to affirm that competitor brand B is $(1 - \rho)\%$ worse than A in some quality characteristic of interest that follows Poisson distribution (like the number of breaks, faults). The company of the reference brand has interest in evaluating if the ratio of the number of defects (breaks, faults, etc) in samples of two brands remains stable.

Here we are considering that the quality characteristics $X \sim P(\lambda_1)$ and $Y \sim P(\lambda_2 = \lambda_1/\rho)$ are independent Poisson random variables. Moreover, the most interest concerns is to monitor if the ratio ρ remains stable at a fixed value ρ_0 , but not rather than their means. It is also assumed that at every inspection time i , only one observation

of X_i and Y_i are available and based on them, the person in charge has to decide if the ratio ρ is stable or not.

Two types of control chart are considered: Shewhart and EWMA. The performance of several statistics are compared in terms of the speed to detect shifts on parameter of interest, based on extensive simulation study, since the development of the probability distribution function of the monitored statistics are quite complicated.

This dissertation is organized as follows. In Chapter 2, a brief review of the main concepts related to control chart is presented as also the monitored statistics. Chapter 3 presents the methodology of Monte Carlo simulation and simulation conditions. Chapter 4 shows the results of the simulation and the performances of each control chart are discussed. Chapter 5 illustrates the proposal by a numerical example. At the last the conclusion, final remarks and some issues for future work are described.

The results of this dissertation are parts of a manuscript submitted for publication at Quality and Reliability Engineering International on June 24th 2019 and accepted on August 22nd 2019.

2 A brief review of the main concepts

In this Chapter 2, a brief review of the main concepts related to control chart is presented. The Section 2.1 introduces the statistical process control and the control charts, Shewhart control chart and EWMA control chart. In the Section 2.2, the Poisson random variables are defined and several statistics of the ratio of Poisson rates are presented. The section 2.3 shows adjustments for some statistics. Finally some normal transformations are presented in the Section 2.4 in order to apply them for Poisson rate monitoring.

2.1 Statistical process control and control charts

Statistical process control (SPC) is a method to improve the manufacturing process performance and reduce variability in key parameters with statistical tools, basically histogram, check sheet, Pareto chart, cause-and-effect diagram, defect concentration diagram, scatter diagram, DOE (design of experiments) and then control chart (MONTGOMERY, 2005). Statistical methods and their application in quality improvement have developed since 1930s. In 1924, Walter A. Shewhart (SHEWHART, 1924) of Bell Laboratories introduced the statistical control-chart concept, which is considered the formal beginning of statistical quality control. The control chart has been considered as the most technically sophisticated type of online statistical process control tool to monitor the variability and to detect the shift of the key parameters such as mean and variance. After that, the acceptance sampling methodology is developed and refined as an alternative to 100 percent inspection by H.F. Dodge and H.G. Romig at same laboratory (DODGE H. F.; ROMIG, 1929). The use of quality improvement techniques, which includes control chart has promoted this tool for all type of products and services.

The control chart contains a center line that represents the average value of the key parameters and two horizontal lines, the upper control limit (UCL) and the lower control limit (LCL) as it is shown in Figure 1.

As long as the points fall within the control limits, we assume that the process is in control and no action is necessary. On the other hand, once a point falls outside of the control limits we interpret that the process is out of control and some action to find and eliminate the assignable cause should be done.

Let X^* be a statistic, which its mean and standard deviation are respectively μ_{X^*} and σ_{X^*} , then the center line, the upper control limit and the lower control limit are defined as:

Figure 1 – Shewhart Control Chart

$$\begin{aligned}
 UCL &= \mu_{X^*} + L\sigma_{X^*} \\
 CL &= \mu_{X^*} \\
 LCL &= \mu_{X^*} - L\sigma_{X^*}
 \end{aligned} \tag{2.1}$$

where L is the distance of control limits from center line by standard deviation units. Thus, whenever $X^* > UCL$ or $X^* < LCL$, the process is declared out-of-control and the search for assignable cases is started. L is fixed with the consideration of type I error which is the risk of a point falling beyond the control limits when no assignable case is present and type II error which is the risk of a point falling between the control limits when the process is out of control. This primary control chart is named Shewhart control chart.

Shewhart control chart is efficient to detect large shifts in the monitored parameters. However, Shewhart control chart is based on the current value of the statistic, meaning that it uses only the information of the last sample observation. So it ignores any information given by the entire sequence of points. This fact makes Shewhart control chart insensitive to small shifts.

In 1960s, other types of control charts were introduced to cover this disadvantage of Shewhart control chart. E. S. Page developed the cumulative sum (CUSUM) control chart (PAGE, 1954) and S. Roberts introduced the exponentially weighted moving average (EWMA) control chart (ROBERTS, 1959).

Both control charts can be alternatives to the Shewhart control chart when we are interested in detecting small shifts. The performance of the CUSUM control chart and EWMA control chart are similar but EWMA control chart is considered easier to set up and operate.

The exponentially weighted moving average (EWMA) is defined with sample statistic X_j^* as the following expression, j is number of observation sequence.

$$Z_j = \theta X_j^* + (1 - \theta)Z_{j-1} \quad (2.2)$$

where $0 < \theta < 1$ is a constant and $Z_0 = \mu_0$ or $Z_0 = \bar{X}$. The center line is μ_0 and upper and lower control limits are as follows:

$$UCL_j = \mu_0 + L\sigma\sqrt{\frac{\theta}{(2-\theta)}}[1 - (1-\theta)^{2j}] \quad (2.3)$$

$$LCL_j = \mu_0 - L\sigma\sqrt{\frac{\theta}{(2-\theta)}}[1 - (1-\theta)^{2j}] \quad (2.4)$$

Whenever $Z_j > UCL_j$ or $Z_j < LCL$ the process is declared out-of-control. The factor L in (2.3)-(2.4) is the distance of control limits from center line by standard deviation units. The constant θ and the factor L should be selected by optimization to achieve the desired average run length (ARL).

The performance of control charts is described by the shift detection speed. This speed is usually expressed by average run length (ARL) which is the average number of points that must be plotted before a point indicates an out-of-control condition. When the monitored statistics X^* are independent random variables, the ARL can be calculated from:

$$ARL = \frac{1}{p} \quad (2.5)$$

where p is the probability that any point exceeds the control limits.

The in-control ARL_0 is expressed as the following, with the risk α of Type I error, which is the risk in a statistical test that a null hypothesis will be rejected when no difference actually existed. That is

$$ARL_0 = \frac{1}{\alpha} \quad (2.6)$$

So it is better for the control chart to be set up with higher ARL_0 , it means that there is lower α , so that it is able to diminish false alarms on the control chart. On the other hand, the out-of-control ARL_1 is defined with the risk β of Type II error as:

$$ARL_1 = \frac{1}{1 - \beta} \quad (2.7)$$

β is the risk that the data on a control chart indicate the process is in control but in reality the process is out of control. The power of control chart is defined as $Power = 1 - \beta$. Then ARL_1 is better to be lower as the risk β is lower. However when the monitored statistics

X^* are not independent (for example like (2.2)), ARL can be achieved by Markov Chain approach as proposed by Brook and Evans (BROOK; EVANS, 1972) or by simulation of the run length distribution.

However there is a trade-off between α and β , as lower α increases β and lower β increases α . So our purpose is to look for sufficiently high ARL_0 in order not to detect false alarm of Type I error and at the same time low ARL_1 to detect the shift as possible as rapid.

In process monitoring, there are phases I and II to apply control charts. Phase I is the first step to bring the process into a state of statistical control. In phase I, set of process data is gathered and analyzed and then trial control limits are constructed to determine if the process has been in control over the period of time where the data were collected and to see if reliable control limits can be established to monitor future process.

In phase II the control chart is applied to monitor the process by comparing the sample statistic for each successive sample. The process is assumed to be reasonable stable. ARL is a valid basis for evaluating the performance of a control chart in phase II.

The focus of this study is on phase II to monitor the process by the ratio of two Poisson rates.

2.2 Ratio of two Poisson rates

Poisson distribution is the probability distribution that expresses the probability of the number of random occurrences of events in a fixed interval of time or space.

Let λ be the average rate of event, n ($0 \leq n < \infty$) the number of events and t ($0 \leq t < \infty$) the length of the interval time, the probability to observe n events is:

$$p_n(t) = \frac{(\lambda t)^n \cdot e^{-\lambda t}}{n!}$$

In this study a time interval t is considered as unity, so the expression is simplified with:

$$p_n = \frac{(\lambda)^n \cdot e^{-\lambda}}{n!} \quad (2.8)$$

The counting process $\{N(t), t \in [0, \infty)\}$ is called a Poisson process with rates λ with the following conditions:

1. $N(0) = 0$
2. $N(t)$ has independent increments

3. The number of events has Poisson distribution

Let two independent Poisson random variables X and Y be the corresponding number of events $X \sim \text{Poisson}(\lambda_1)$ and $Y \sim \text{Poisson}(\lambda_2)$ with time interval $t = 1$.

Our interest is one-sided hypotheses, the ratio of Poisson rate is equal to a specified average rate ρ_0 against greater than ρ_0 :

$$H_0 : \lambda_1/\lambda_2 = \rho_0 \text{ against } H_1 : \lambda_1/\lambda_2 > \rho_0 \quad (2.9)$$

The applied control chart in this study is to detect the shift of hypotheses H_1 . The maximum likelihood estimate (MLE) of λ_1 and λ_2 given single observations of X_t and Y_t are respectively:

$$\hat{\lambda}_1 = \frac{X_t}{t} \text{ and } \hat{\lambda}_2 = \frac{Y_t}{t} \quad (2.10)$$

where t is the interval of time. Thus, the MLE of ρ with individual observations X and Y is:

$$\hat{\rho} = \frac{X}{Y} \quad (2.11)$$

As the determination of the distribution (2.11) is quite complicated, the first candidate that comes to our mind is to consider

$$W_0 = \hat{\lambda}_1 - \hat{\lambda}_2 = X - Y \quad (2.12)$$

as a monitored statistic to detect shifts in the ratio ρ .

According to Gu et al. (GU K.; NG, 2008), the constrained maximum likelihood estimates (CMLE) of λ_1 and λ_2 can be shown to be:

$$\hat{\lambda}_1 = \frac{X + Y}{1 + 1/\rho_0} \text{ and } \hat{\lambda}_2 = \frac{X + Y}{1 + \rho_0}. \quad (2.13)$$

Additionally, they showed that the null hypothesis of (2.9) can be re-expressed as $H_0 : \lambda_1 - \rho_0\lambda_2 = 0$ so the test statistics based on the statistic

$$W_0^1 = \hat{\lambda}_1 - \rho_0\hat{\lambda}_2 \quad (2.14)$$

can be a monitored statistic. The variance of the statistic W_0^1 is given by:

$$\sigma_W^2 = \frac{\lambda_1}{t} + \frac{\rho_0^2\lambda_2}{t}$$

Considering $t = 1$, the test statistic W_0^1/σ_W for H_0 , replaced by MLEs of (2.10), we get:

$$W_1 = \frac{X - \rho_0 Y}{\sqrt{\lambda_1 + \rho_0^2 \lambda_2}} \quad (2.15)$$

In case of λ_1 and λ_2 unknown, σ_W^2 can be estimated by:

$$s_W^2 = \hat{\lambda}_1 + \rho_0^2 \hat{\lambda}_2$$

and the test statistic W_0^1/s_W for H_0 , replaced by MLEs of (2.10) becomes:

$$W_1^* = \frac{X - \rho_0 Y}{\sqrt{X + \rho_0^2 Y}} \quad (2.16)$$

Similarly, replacing CMLE's of (2.13), we obtain:

$$W_2^* = \frac{X - \rho_0 Y}{\sqrt{(X + Y)\rho_0}} \quad (2.17)$$

And a new statistic can be derived from by a replacement of expected rate of X and Y in the denominator:

$$W_2 = \frac{X - \rho_0 Y}{\sqrt{(\lambda_1 + \lambda_2)\rho_0}} \quad (2.18)$$

Furthermore, Gu et al. (GU K.; NG, 2008) rewrote the null hypothesis (2.9) as $H_0 : \ln(\lambda_1/\lambda_2) - \ln(\rho_0) = 0$. The test statistic based on the statistic $U = \ln(\hat{\lambda}_1/\hat{\lambda}_2) - \ln(\rho_0) = \ln(X/Y) - \ln(\rho_0)$ can be established replacing by MLEs. The variance of U is approximated by:

$$\sigma_U^2 \approx \frac{1}{\lambda_1} + \frac{1}{\lambda_2}$$

Hence, the following statistic can be used to monitor ρ

$$W_3 = U/\sigma_U = \frac{\ln(X/Y) - \ln(\rho_0)}{\sqrt{1/\lambda_1 + 1/\lambda_2}} \quad (2.19)$$

When σ_U^2 are unknown it can be estimated as

$$s_U^2 \approx \frac{1}{\hat{\lambda}_1} + \frac{1}{\hat{\lambda}_2} \quad (2.20)$$

Thus, the test statistic U/s_U are proposed by Gu et al. (GU K.; NG, 2008). After replacing by their MLEs it results:

$$W_3^* = \frac{\ln(X/Y) - \ln(\rho_0)}{\sqrt{1/X + 1/Y}} \quad (2.21)$$

Or replacing by CMLEs, it yields:

$$W_4^* = \frac{\ln(X/Y) - \ln(\rho_0)}{\sqrt{(2 + 1/\rho_0 + \rho_0)/(X + Y)}} \quad (2.22)$$

Again new statistic can be proposed from (2.22) replacing X and Y by their expected rates in the denominator as

$$W_4 = \frac{\ln(X/Y) - \ln(\rho_0)}{\sqrt{(2 + 1/\rho_0 + \rho_0)/(\lambda_1 + \lambda_2)}} \quad (2.23)$$

2.3 Some adjustments

Some statistics used to monitor are presented in the previous section. To avoid division by zero, the addition of 1 for denominator is considered. For example the statistic (2.11) is reexpressed as

$$\hat{\rho}_{sd}^* = \frac{X}{Y + 1} \quad (2.24)$$

In this case our interest indicator, the ratio of Poisson rate ρ_0 is transformed to another ratio of Poisson rate ρ_0^* and the hypotheses test (2.9) is re-expressed as below:

$$H_0 : \lambda_1/(\lambda_2 + 1) = \rho_0^* \text{ against } H_1 : \lambda_1/(\lambda_2 + 1) > \rho_0^* \quad (2.25)$$

Some adjustments are made for the other statistics as following: For the statistic (2.16),

$$W_1^{**} = \frac{X - \rho_0 Y}{\sqrt{X + \rho_0^2 Y + 1}} \quad (2.26)$$

For the statistic (2.2),

$$W_2^{**} = \frac{X - \rho_0 Y}{\sqrt{(X + Y)\rho_0 + 1}} \quad (2.27)$$

For the statistic (2.19) and (2.21),

$$W_3^* = \frac{\ln[(X + 1)/(Y + 1)] - \ln(\rho_0)}{\sqrt{1/\lambda_1 + 1/\lambda_2}} \quad (2.28)$$

$$W_3^{**} = \frac{\ln[(X + 1)/(Y + 1)] - \ln(\rho_0)}{\sqrt{1/(X + 1) + 1/(Y + 1)}} \quad (2.29)$$

And finally for the statistic (2.22) and (2.23),

$$W_4^* = \frac{\ln[(X + 1)/(Y + 1)] - \ln(\rho_0)}{\sqrt{(2 + 1/\rho_0 + \rho_0)/(\lambda_1 + \lambda_2)}} \quad (2.30)$$

$$W_4^{**} = \frac{\ln[(X + 1)/(Y + 1)] - \ln(\rho_0)}{\sqrt{(2 + 1/\rho_0 + \rho_0)/(X + Y + 1)}} \quad (2.31)$$

2.4 Normal transformation of Poisson variables

In the literature many transformations have been proposed for Poisson variables in order to get approximately normal variables. On the purpose of the control chart performance improvement, the following normal transformations are applied separately on each the Poisson variables. They are applied on (2.11) or on (2.24). Here we list the transformations considered in this study

(a) Bartlett's Transformation ([BARTLETT, 1936](#))

The square root transformation is applied on X and Y for the statistic (2.24) in order avoid the denominator be zero. So the ratio with this transformation in this study is expressed as below:

$$\hat{\rho}_{sq}^* = \frac{\sqrt{X}}{\sqrt{Y+1}} \quad (2.32)$$

As $\sqrt{X}$ and $\sqrt{Y}$ are $\approx N(0; 1/2)$, this transformation can be applied for the statistic (2.14). After some manipulations, the final statistic can be written as.

$$Z_1 = \frac{2[\sqrt{X} - \sqrt{\rho_0 Y}]}{\sqrt{1 + \rho_0}} \approx N(0, 1) \quad (2.33)$$

(b) Anscombe's Transformation ([ANSCOMBE, 1948](#))

The transformation proposed by Anscombe is also a square root plus a constant equal to $3/8$ and is applied on (2.11). Then the transformation results in:

$$\hat{\rho}_{as} = \sqrt{\frac{X + \frac{3}{8}}{Y + \frac{3}{8}}} \quad (2.34)$$

Huffman ([HUFFMAN, 1984](#)) proposed to apply Anscombe's transformation on statistics $W_0^1 = \hat{\lambda}_1 - \rho_0 \hat{\lambda}_2$ in order to reduce the variance of the statistic. This statistic is expressed by:

$$Z_2 = \frac{2[\sqrt{X + 3/8} - \sqrt{\rho_0(Y + 3/8)}]}{\sqrt{1 + \rho_0}} \quad (2.35)$$

(c) Freeman - Tukey's Transformation ([FREEMAN MURRAY F.; TUKEY, 1950](#))

This transformation is based on a sum of two square roots and was applied on (2.11)

$$\hat{\rho}_{ft} = \frac{\sqrt{X+1} + \sqrt{X}}{\sqrt{Y+1} + \sqrt{Y}} \quad (2.36)$$

Similarly, Freeman-Tukey's transformation can also be applied for the test statistic based on the statistic $W_0^1 = \hat{\lambda}_1 - \rho_0 \hat{\lambda}_2$. The expression of the statistic is deduced as below.

$$Z_3 = \frac{\sqrt{X+1} + \sqrt{X} - \rho_0 [\sqrt{Y+1} + \sqrt{Y}]}{\sqrt{1 + \rho_0^2}} \quad (2.37)$$

(d) Rossi-Lampugnani's Transformation ([ROSSI G.; LAMPUGNANI, 1950](#))

Let W be a generic Poisson random variable. Consider the two transformations $Q_1 = \frac{W - \lambda}{\sqrt{\lambda}}$; $Q_2 = 2\sqrt{W}$. As Q_1 and Q_2 are asymptotically standardized normally distributed, Rossi and Lampugnani ([ROSSI G.; LAMPUGNANI, 1950](#)) proposed to use the average of these two transformations which results $\frac{W + 2\sqrt{W\lambda}}{2\sqrt{\lambda}}$. Applying on (2.11), it yields:

$$\hat{\rho}_{rs} = \frac{X + 2\sqrt{X\lambda_{10}}}{Y + 2\sqrt{Y\lambda_{20}}} \sqrt{\frac{1}{\rho_0}}$$

Due the possibility of the denominator be zero, an adjustment is required by the addition of 1 to the denominator.

$$\hat{\rho}_{rs}^* = \frac{X + 2\sqrt{X\lambda_{10}}}{Y + 2\sqrt{Y\lambda_{20}} + 1} \sqrt{\frac{1}{\rho_0}} \quad (2.38)$$

Replacing λ_1 and λ_2 in (2.38) by their MLE's, a new statistic can be created:

$$\hat{\rho}_{rs}^{**} = \frac{3X}{3Y + 1} \sqrt{\frac{1}{\rho_0}} \quad (2.39)$$

Similarly, Rossi's transformation can be applied for statistic (2.14). The expression of the statistic is deduced as below.

$$Z_4 = \frac{X + 2\sqrt{X\lambda_{10}} - \rho_0(Y + 2\sqrt{Y\lambda_{20}})}{2\sqrt{\lambda_{10} + \lambda_{20}\rho_0^2}} \quad (2.40)$$

In case of λ_1 and λ_2 unknown, the statistic can be expressed:

$$Z_4^* = \frac{3X - 3\rho_0 Y}{2\sqrt{X + \rho_0^2 Y}}$$

To avoid division by zero the addition of 1 to the denominator is used and then the statistics is transformed as the following:

$$Z_4^{**} = \frac{3W - 3\rho_0 Z}{2\sqrt{W + \rho_0^2 Z + 1}} \quad (2.41)$$

And finally the last normalized transformation. According to Mood et al. (MOOD; GRAYBILL; DUANE, 1974), the mean and variance of rate of two random variables can be approximated as following:

$$E \left[\frac{X}{Y} \right] \approx \frac{E(X)}{E(Y)} - \frac{1}{E(Y)^2} Cov[X, Y] + \frac{E(X)}{E(Y)^3} Var[Y] \quad (2.42)$$

$$Var \left[\frac{X}{Y} \right] \approx \left(\frac{E(X)}{E(Y)} \right)^2 \left(\frac{Var[X]}{E(X)^2} + \frac{Var[Y]}{E(Y)^2} - \frac{2Cov[X, Y]}{E(X)E(Y)} \right) \quad (2.43)$$

So as our variables are Poisson's, $E[X]$, $Var[X]$, $E[X^2]$ and $E[X^3]$ are respectively

$$\begin{aligned} E[X] &= Var[X] = \lambda_1 \\ E[X^2] &= \lambda_1 + \lambda_1^2 \\ E[X^3] &= \lambda_1 + 3\lambda_1^2 + \lambda_1^3 \end{aligned}$$

Furthermore, as X and Y are independent so $Cov[X, Y]$ is zero. Therefore, (2.42) and (2.43) can be reexpressed as

$$E \left[\frac{X}{Y} \right] \approx \frac{\lambda_{10}}{\lambda_{20}} + \frac{\lambda_{10}}{1 + 3\lambda_{20} + \lambda_{20}^2} \quad (2.44)$$

$$Var \left[\frac{X}{Y} \right] \approx \left(\frac{\lambda_{10}}{\lambda_{20}} \right)^2 \left(\frac{1}{1 + \lambda_{10}} + \frac{1}{1 + \lambda_{20}} \right) \quad (2.45)$$

So the standardized statistic of $\frac{X}{Y}$ is expressed as following.

$$Z_5^0 = \frac{(X/Y) - [\lambda_{10}/\lambda_{20} + \lambda_{10}/(1 + 3\lambda_{20} + \lambda_{20}^2)]}{(\lambda_{10}/\lambda_{20})\sqrt{1/(1 + \lambda_{10}) + 1/(1 + \lambda_{20})}}$$

In order to avoid division by zero in $\frac{X}{Y}$, the addition of one in its denominator was made

$$Z_5 = \frac{[X/(Y + 1)] - [\lambda_{10}/\lambda_{20} + \lambda_{10}/(1 + 3\lambda_{20} + \lambda_{20}^2)]}{(\lambda_{10}/\lambda_{20})\sqrt{1/(1 + \lambda_{10}) + 1/(1 + \lambda_{20})}} \quad (2.46)$$

The performance of control chart with this statistic Z_5 is evaluated in this study as well. And again substituting λ_1 and λ_2 in (2.46) by their MLE's it results in:

$$Z_5^* = -\frac{(1 + 4Y + Y^2)\sqrt{1 + X}}{(1 + 3Y + Y^2)\sqrt{(1 + Y)(X + Y + 2)}}. \quad (2.47)$$

3 Methodology

In order to evaluate the performance of several statistics presented in the previous chapter, an extensive Monte Carlo simulation study is conducted as the determination of the exact distribution of the statistics is quite complicated.

3.1 Monte Carlo simulation study

Monte Carlo simulation is a method to obtain numerical results that might be deterministic in principle with repeated sampling. Monte Carlo simulation is widely used in physical and mathematical problems when it is difficult to solve by calculation formula. In this study the performances of control charts are evaluated in term of ARL. However the control limits with fixed ARL_0 and ARL_1 of control charts for the ratio of two Poisson rates have not expressed by mathematical formula yet. Because of that, Monte Carlo simulation is adopted for this study. The simulation was coded in R ([R Core Team, 2017](#)) and available upon request.

3.2 Simulation study's conditions

We consider $ARL_0 = 170$, which means approximately $\alpha = 0.0059$ according to the equation (2.6); the in-control ratios $\rho_0 = 0.4, 0.7$ with in-control Poisson rates equal to $(\lambda_{10}, \lambda_{20}) = (4, 10), (16, 40)$ for $\rho_0 = 0.4$ and $(\lambda_{10}, \lambda_{20}) = (7, 10), (28, 40)$ for $\rho = 0.7$. The UCLs are set up with this condition by Monte Carlo simulation in the first stage.

To evaluate the performances of control charts, we calculate ARL_1 with UCL set up in stage 1. The out of control condition is made by the shifts on the ratio ρ and on the means λ_1 and λ_2 . The shifts of ρ_0 are expressed as $\rho_1 = \tau \rho_0$, $\rho_0 = \frac{\lambda_{10}}{\lambda_{20}}$ with $\tau = (1.02, 1.05, 1.10, 1.20, 1.30, 1.40, 1.50)$, and the shifts of means λ_{10} and λ_{20} are $\lambda_{11} = \delta_X \lambda_{10}$, $\lambda_{21} = \frac{\lambda_{11}}{\rho_1} = \frac{\delta_x}{\tau} \lambda_{20}$ with $\delta_X = (0.8, 0.9, 1.0, 1.1, 1.2)$. The parameter δ_x makes the shift of λ_1 as it is shown in (3.2). At the same time, the shift of λ_2 is defined because it depends on ρ and λ_1 (refer (3.3)). So the parameter δ_x makes λ_1 and λ_2 increase (decrease) together under fixed ρ .

$$\rho_1 = \tau \rho_0 \text{ where } \rho_0 = \frac{\lambda_{10}}{\lambda_{20}} \quad (3.1)$$

$$\lambda_{11} = \delta_X \lambda_{10} \quad (3.2)$$

$$\lambda_{21} = \frac{\lambda_{11}}{\rho_1} \quad (3.3)$$

3.3 Simulation flow chart

Two simulations are established. One is for the stage I which defines UCL with desired ARL_0 and another is for stage II to get ARL_1 .

The flow chart of simulation for stage I is presented in the Figure 2. Firstly the means λ_{10} and λ_{20} are defined and then the data sets of random variables X and Y are generated with the defined means. As a consequence, the statistics are estimated with the data sets of X and Y. After that, we define the initial UCL and simulate 5,000 in-control run lengths. If the ARL achieves ARL_0 , then stops. Otherwise choose another UCL (by increments of $\epsilon = 0.00001$ and repeat until the ARL_0 of 170 is reached. The ARL_0 is estimated by 5,000 in-control run lengths.

The flow chart of simulation for stage II is shown in the Figure 3. At first the means λ_{10} , λ_{20} and UCL, which is fixed in stage I, are set up. Furthermore, the parameters τ and δ_X are selected. In next step, the data set of X and Y are generated with the selected parameters and the data sets of statistics are created. Then 5,000 out-of-control run lengths are simulated and ARL_1 is calculated.

Figure 2 – Simulation Flow Chart for stage I

Figure 3 – Simulation Flow Chart for stage II

4 Results from simulation study

In this chapter the results from simulation studies are presented. In Section 4.1 the results with Shewhart control charts are shown and we describe the analysis of them. The results with EWMA type control charts are discussed in Section 4.2. Finally we give an overview of results in Section 4.3. The evaluated statistics are summarized in Table 1.

Table 1 – A summary of the statistics

Statistics	Ref. No.	Statistics	Ref. No.
$\hat{\rho}_{sd}^* = \frac{X}{Y+1}$	(2.24)	$\hat{\rho}_{sq}^* = \frac{\sqrt{X}}{\sqrt{Y+1}}$	(2.32)
$\hat{\rho}_{as} = \sqrt{\frac{X + \frac{3}{8}}{Y + \frac{3}{8}}}$	(2.34)	$\hat{\rho}_{ft} = \frac{\sqrt{X+1} + \sqrt{X}}{\sqrt{Y+1} + \sqrt{Y}}$	(2.36)
$\hat{\rho}_{rs}^* = \frac{X + 2\sqrt{X\lambda_{10}}}{Y + 2\sqrt{Y\lambda_{20}} + 1} \sqrt{\frac{1}{\rho_0}}$	(2.38)	$\hat{\rho}_{rs}^{**} = \frac{3X}{3Y+1} \sqrt{\frac{1}{\rho_0}}$	(2.39)
$W_0 = X - Y$	(2.12)	$W_1 = \frac{X - \rho_0 Y}{\sqrt{\lambda_1 + \rho_0^2 \lambda_2}}$	(2.15)
$W_1^{**} = \frac{X - \rho_0 Y}{\sqrt{X + \rho_0^2 Y + 1}}$	(2.26)	$W_2 = \frac{X - \rho_0 Y}{\sqrt{(\lambda_1 + \lambda_2)\rho_0}}$	(2.17)
$W_2^{**} = \frac{X - \rho_0 Y}{\sqrt{(X+Y)\rho_0 + 1}}$	(2.27)	$W_3^* = \frac{\ln[(X+1)/(Y+1)] - \ln(\rho_0)}{\sqrt{1/\lambda_1 + 1/\lambda_2}}$	(2.28)
$W_3^{**} = \frac{\ln[(X+1)/(Y+1)] - \ln(\rho_0)}{\sqrt{1/(X+1) + 1/(Y+1)}}$	(2.29)	$W_4^* = \frac{\ln[(X+1)/(Y+1)] - \ln(\rho_0)}{\sqrt{(2+1/\rho_0 + \rho_0)/(\lambda_1 + \lambda_2)}}$	(2.30)
$W_4^{**} = \frac{\ln[(X+1)/(Y+1)] - \ln(\rho_0)}{\sqrt{(2+1/\rho_0 + \rho_0)/(X+Y+1)}}$	(2.31)	$Z_1 = \frac{2[\sqrt{X} - \sqrt{\rho_0 Y}]}{\sqrt{1 + \rho_0}}$	(2.33)
$Z_2 = \frac{2[\sqrt{X + 3/8} - \sqrt{\rho_0(Y + 3/8)}]}{\sqrt{1 + \rho_0}}$	(2.35)	$Z_3 = \frac{\sqrt{X+1} + \sqrt{X} - \rho_0[\sqrt{Y+1} + \sqrt{Y}]}{\sqrt{1 + \rho_0^2}}$	(2.37)
$Z_4 = \frac{X + 2\sqrt{X\lambda_{10}} - \rho_0(Y + 2\sqrt{Y\lambda_{20}})}{2\sqrt{\lambda_{10} + \lambda_{20}\rho_0^2}}$	(2.40)	$Z_4^{**} = \frac{3X - 3\rho_0 Y}{2\sqrt{X + \rho_0^2 Y + 1}}$	(2.41)
$Z_5 = \frac{[X/(Y+1)] - [\lambda_{10}/\lambda_{20} + \lambda_{10}/(1 + 3\lambda_{20} + \lambda_{20}^2)]}{(\lambda_{10}/\lambda_{20})\sqrt{1/(1 + \lambda_{10}) + 1/(1 + \lambda_{20})}}$	(2.46)	$Z_5^* = -\frac{(1 + 4Y + Y^2)\sqrt{1 + X}}{(1 + 3Y + Y^2)\sqrt{(1 + Y)(X + Y + 2)}}$	(2.47)

4.1 Shewhart control chart's performances

4.1.1 Case of $\lambda_{10} = 7$ and $\lambda_{20} = 10$

Under the same conditions of τ and δ_X , "equal" performance is considered if $ARL_1 - \min(ARL_1's) \leq 0.5$. Tables 2 - 7 present the results related to $\rho_0 = 0.7$ and ARL_1 values which satisfy such condition are bold. In order to simplify the explanation, we consider that $\tau \leq 1.20$ are low shifts and $\tau > 1.20$ are high shifts. And we consider that $\delta_X \leq 1.00$ are low ranges and $\delta_X > 1.00$ are high ranges.

Tables 2 shows the results of the simulation for the Shewhart-type chart about the case of $\lambda_{10} = 7$ and $\lambda_{20} = 10$.

For larger increases in the average ($\delta_x \geq 1.10$) and lower shifts in the ratio ($\tau \leq 1.10$) the statistic Z_4^{**} presents a best performance. For other combinations, in general the statistics $\hat{\rho}_{as}$, $\hat{\rho}_{ft}$ and $\hat{\rho}_{rs}^*$ can be used as they provides good performance. For a very few combinations of τ and δ_x , the statistics W_2^* and W_4^{**} can also used employed.

4.1.2 Case of $\lambda_{10} = 28$ and $\lambda_{20} = 40$

Table 3 shows the results for the case $\lambda_{10} = 28$ and $\lambda_{20} = 40$, the tendency of results is same observed for the case of $\lambda_{10} = 7$ and $\lambda_{20} = 10$ and they will not repeat here. What differs is for higher shifts in the ratio ($\tau \geq 1.30$), other statistics besides $\hat{\rho}_{as}$, $\hat{\rho}_{ft}$ and $\hat{\rho}_{rs}^*$ yield "equal" same performance in terms of ARL_1 . It is worth to note that the values of in-control λ_{i0} , $i = 1, 2$ play an important role in the speed to detect the shifts. A noticeable reduction in ARL_1 is observed from the former case (smaller in-control rates) to this case.

4.1.3 Case of $\lambda_{10} = 4$ and $\lambda_{20} = 10$

As for the case of $\lambda_{10} = 4$ and $\lambda_{20} = 10$, Z_3 presents the best performance for larger increases in the average ($\delta_x \geq 1.10$) and lower shifts in the ratio ($\tau \leq 1.10$) instead of the statistic Z_4^{**} which performs well in case of $\lambda_{10} = 7$ and $\lambda_{20} = 10$. On the other hand, $\hat{\rho}_{as}$, $\hat{\rho}_{ft}$ and $\hat{\rho}_{as}$ have good performance for the other combinations, similarly to the case of $\lambda_{10} = 7$ and $\lambda_{20} = 10$. Z_1 and Z_2 can be used to complement few combination of τ and δ_x .

4.1.4 Case of $\lambda_{10} = 16$ and $\lambda_{20} = 40$

For larger increases in the average ($\delta_x \geq 1.10$) and lower shifts in the ratio ($\tau \leq 1.10$) the statistic Z_4^{**} has the best performances as the cases of $\rho_0 = 0.7$. What differentiate from the other results of Shewhart control chart is that W_0 became the best

Table 2 – Shewhart Control chart - $\lambda_{10} = 7$ and $\lambda_{20} = 10$

τ	δ_X	$\hat{\rho}_{sd}^*$	$\hat{\rho}_{sd}^*$	$\hat{\rho}_{sd}$	$\hat{\rho}_{rs}$	$\hat{\rho}_{rs}$	W_0	W_1	W_1^*	W_2	W_2^*	W_3	W_3^*	W_4	W_4^*	Z_1	Z_2	Z_3	Z_4	Z_4^{**}	Z_5	Z_5^*
1.02	1.20	279.85	279.85	297.57	297.57	297.57	136.77	101.48	131.95	101.48	137.71	287.08	131.62	287.08	137.71	152.42	152.42	133.07	117.10	63.05	279.85	148.39
	1.10	215.65	215.65	216.84	216.84	216.84	151.43	128.37	150.41	128.37	148.12	217.84	150.13	217.84	148.10	150.31	150.31	145.88	135.59	99.95	215.65	172.66
	1.00	158.58	158.58	152.00	152.00	152.00	174.32	166.84	166.34	166.84	151.09	158.21	166.65	157.81	150.84	152.10	152.32	161.94	162.91	163.67	158.58	206.17
	0.90	118.55	118.55	105.97	105.97	105.97	214.10	243.99	188.25	243.99	158.87	116.56	189.82	116.56	158.57	157.04	157.58	187.20	200.84	272.09	118.55	244.27
	0.80	92.37	92.37	75.45	75.45	75.45	274.36	371.08	209.04	371.08	161.89	88.11	213.79	88.11	159.75	162.30	164.63	212.63	248.30	491.81	92.37	272.77
1.05	1.20	225.26	225.26	237.06	237.06	237.06	115.52	86.08	115.09	86.08	119.61	230.58	114.95	230.58	119.59	125.54	125.63	112.26	99.40	59.98	225.26	180.35
	1.10	176.40	176.40	178.33	178.33	178.33	135.53	116.75	128.78	116.75	124.44	178.21	128.86	178.21	124.32	133.13	133.13	131.06	123.64	92.64	176.40	211.72
	1.00	132.72	132.72	125.09	125.09	125.09	156.46	155.33	141.02	155.33	128.64	132.25	141.31	132.25	128.51	133.62	133.78	146.93	147.45	147.48	132.72	248.72
	0.90	100.03	100.03	89.56	89.56	89.56	184.61	213.81	165.50	213.81	139.38	97.82	167.45	97.82	138.72	134.62	135.40	159.41	172.97	257.35	100.03	282.58
	0.80	78.23	78.23	64.29	64.29	64.29	241.63	327.68	181.64	327.68	141.83	74.88	186.99	74.88	140.20	138.02	140.99	181.91	213.53	467.96	78.23	313.49
1.10	1.20	165.99	165.99	172.28	172.28	172.28	93.72	74.35	90.08	74.35	91.95	168.97	90.09	168.97	91.95	99.28	99.34	91.54	82.77	52.99	165.99	249.97
	1.10	127.82	127.82	126.97	126.97	126.97	104.64	96.10	101.08	96.10	97.75	128.12	101.21	128.12	97.65	99.82	99.87	100.60	96.95	81.32	127.82	294.51
	1.00	100.72	100.72	95.81	95.81	95.81	124.44	130.41	115.34	130.41	102.29	100.49	115.65	100.49	102.09	106.03	106.29	115.08	117.87	134.88	100.72	339.19
	0.90	76.99	76.99	67.69	67.69	67.69	148.32	180.84	128.51	180.84	107.01	75.12	129.34	75.12	106.55	106.64	107.02	126.73	139.59	219.97	76.99	374.27
	0.80	60.33	60.33	49.69	49.69	49.69	195.21	275.51	147.07	275.51	113.26	57.72	151.40	57.72	111.94	111.22	113.55	143.32	172.35	408.99	60.33	420.03
1.20	1.20	87.64	87.64	89.67	89.67	89.67	60.06	50.52	60.23	50.52	58.90	88.44	60.17	88.44	58.90	60.84	60.84	58.42	54.12	42.78	87.64	481.56
	1.10	74.54	74.54	73.02	73.02	73.02	70.87	69.15	68.26	69.15	63.50	74.59	68.45	74.59	63.49	65.80	65.91	67.39	66.34	66.20	74.54	522.79
	1.00	60.81	60.81	56.85	56.85	56.85	86.29	93.98	76.47	93.98	66.86	60.03	77.05	60.03	66.65	69.12	69.29	78.69	82.12	104.98	60.81	550.45
	0.90	48.61	48.61	41.75	41.75	41.75	105.43	134.72	87.37	134.72	71.32	47.22	88.73	47.22	70.94	72.04	72.60	86.45	97.29	175.11	48.61	601.79
	0.80	40.09	40.09	32.90	32.90	32.90	138.89	205.20	100.79	205.20	77.22	38.21	104.72	38.21	75.83	76.17	77.99	99.46	120.11	326.12	40.09	644.22
1.30	1.20	55.39	55.39	56.21	56.21	56.21	43.00	40.02	42.08	40.02	40.48	55.83	42.10	55.83	40.46	41.89	41.89	41.62	39.98	35.16	55.39	774.18
	1.10	46.11	46.11	45.32	45.32	45.32	50.22	51.82	47.92	51.82	42.87	46.17	47.96	46.17	42.86	44.33	44.38	47.21	47.83	53.74	46.11	807.69
	1.00	38.23	38.23	35.36	35.36	35.36	62.45	71.31	56.23	71.31	48.84	37.82	56.70	37.82	48.68	47.91	47.97	55.63	59.65	86.76	38.23	835.34
	0.90	33.08	33.08	28.94	28.94	28.94	79.39	105.55	65.15	105.55	52.40	31.94	66.17	31.94	51.99	51.67	52.24	63.49	72.62	148.38	33.08	861.12
	0.80	28.35	28.35	22.97	22.97	22.97	103.18	164.10	74.70	164.10	56.59	26.96	78.04	26.96	55.27	53.69	55.34	70.95	86.10	273.90	28.35	894.72
1.40	1.20	36.35	36.35	36.48	36.48	36.48	32.11	30.99	30.77	30.99	29.04	36.57	30.79	36.57	29.01	30.43	30.45	30.87	30.45	30.54	36.35	1090.60
	1.10	32.07	32.07	31.00	31.00	31.00	38.70	41.84	36.22	41.84	32.91	31.96	36.33	31.96	32.90	32.55	32.59	35.72	36.96	45.23	32.07	1092.53
	1.00	28.13	28.13	25.93	25.93	25.93	47.41	57.35	41.42	57.35	35.23	27.77	41.88	27.77	35.07	35.64	35.77	41.70	45.00	72.74	28.13	1105.74
	0.90	23.76	23.76	20.65	20.65	20.65	60.84	84.88	47.80	84.88	38.60	22.88	48.81	22.88	38.31	39.18	39.70	48.67	55.56	122.25	23.76	1122.74
	0.80	21.36	21.36	17.17	17.17	17.17	83.37	137.35	56.33	137.35	42.72	19.97	59.68	19.97	41.69	41.88	43.46	55.83	68.21	222.67	21.36	1141.57
1.50	1.20	26.15	26.15	26.06	26.06	26.06	25.19	25.63	24.43	25.63	22.72	26.27	24.44	26.27	22.71	22.98	22.99	24.09	24.15	26.93	26.15	1335.29
	1.10	23.41	23.41	22.28	22.28	22.28	31.09	34.76	28.77	34.76	25.38	23.38	28.84	23.38	25.35	25.60	25.63	28.28	29.74	40.61	23.41	1348.61
	1.00	20.50	20.50	18.70	18.70	18.70	37.97	47.20	32.81	47.20	27.75	20.10	33.15	20.10	27.59	27.05	27.14	32.38	35.86	62.44	20.50	1335.16
	0.90	18.14	18.14	15.50	15.50	15.50	48.60	71.18	37.47	71.18	30.50	17.53	38.76	17.53	30.13	29.25	29.72	36.72	43.40	104.79	18.14	1322.35
	0.80	16.87	16.87	13.45	13.45	13.45	67.32	114.67	45.24	114.67	34.77	15.72	48.23	15.72	33.67	32.02	33.36	43.70	54.09	189.22	16.87	1337.15
UCL		2.20	1.48	1.61	1.60	1.60	7.00	2.64	2.24	2.64	2.44	2.50	2.34	2.50	2.43	2.52	2.49	3.17	2.52	3.87	4.45	-0.11
ARL0		181.62	181.62	172.48	172.48	172.48	192.37	185.42	184.44	185.42	170.30	184.19	184.83	184.63	171.45	174.75	174.45	184.20	179.09	171.29	188.80	180.49

Table 3 – Shewhart Control chart - $\lambda_{10} = 28$ and $\lambda_{20} = 40$

τ	δ_X	$\hat{\rho}_{sd}^*$	$\hat{\rho}_{sq}^*$	$\hat{\rho}_{ss}^*$	$\hat{\rho}_{\mu}^*$	$\hat{\rho}_{rs}^*$	$\hat{\rho}_{rs}^*$	W_0	W_1	W_1^*	W_2	W_2^*	W_3^*	W_3^{**}	W_4^*	W_4^{**}	Z_1	Z_2	Z_3	Z_4	Z_4^{**}	Z_5	Z_5^*
1.02	1.20	243.45	243.45	250.45	250.45	228.96	265.07	196.56	79.14	131.70	79.14	136.86	257.18	134.16	257.18	136.76	137.88	137.88	94.27	100.85	27.48	243.45	58.45
	1.10	180.88	180.88	179.33	179.33	175.60	182.00	185.51	103.53	134.25	103.53	136.78	183.98	135.65	183.98	136.13	133.67	133.67	113.17	116.53	60.04	180.88	115.67
	1.00	136.54	136.54	132.16	132.16	136.25	135.23	180.49	143.13	141.87	143.13	142.46	133.94	142.21	133.94	138.94	136.71	136.75	139.17	139.45	158.11	136.54	237.88
	0.90	100.58	100.58	96.88	96.88	103.47	95.40	177.50	210.26	148.39	210.26	148.35	97.23	148.45	97.23	143.04	147.42	148.01	174.13	170.40	486.64	100.58	453.52
	0.80	71.18	71.18	68.05	68.05	75.51	68.42	180.73	335.93	167.28	335.93	165.08	68.40	167.28	68.40	158.75	160.61	162.97	218.88	210.23	1243.17	71.18	816.91
1.05	1.20	161.58	161.58	163.58	163.58	152.12	164.82	136.18	59.72	92.80	59.72	96.91	169.05	94.99	169.05	96.81	97.87	97.87	69.80	74.06	24.08	161.58	92.79
	1.10	124.39	124.39	123.56	123.56	121.40	123.22	132.46	79.02	94.36	79.02	95.93	126.15	95.20	126.15	94.96	96.70	96.70	84.47	86.38	52.24	124.39	186.32
	1.00	94.87	94.87	91.87	91.87	94.98	90.82	129.10	110.22	99.71	110.22	99.95	92.77	99.81	92.77	97.67	99.38	99.54	104.90	104.51	138.05	94.87	368.18
	0.90	71.16	71.16	68.66	68.66	73.88	68.58	127.98	161.26	108.67	161.26	108.70	68.84	108.73	68.84	104.10	108.07	108.53	129.81	126.24	423.22	71.16	689.80
	0.80	53.76	53.76	50.97	50.97	56.80	50.82	131.48	260.50	123.43	260.50	121.49	51.48	123.39	51.48	116.66	118.07	119.98	165.38	156.66	1181.37	53.76	1085.53
1.10	1.20	86.27	86.27	87.20	87.20	81.95	90.02	77.34	39.11	54.24	39.11	55.49	89.36	54.90	89.36	55.20	56.84	56.84	43.75	45.83	19.77	86.27	201.62
	1.10	70.13	70.13	69.10	69.10	69.26	68.56	77.53	53.41	58.97	53.41	59.54	70.34	59.28	70.34	59.05	58.92	58.92	54.91	55.78	43.48	70.13	400.53
	1.00	55.35	55.35	53.39	53.39	56.14	51.59	79.70	75.41	62.84	75.41	62.96	53.82	62.88	53.82	61.35	63.14	63.14	69.06	68.61	107.35	55.35	733.28
	0.90	43.49	43.49	41.62	41.62	45.09	43.01	80.21	110.40	71.57	110.40	71.46	41.71	71.57	41.71	68.59	70.35	70.63	85.30	82.07	331.67	43.49	1113.47
	0.80	33.99	33.99	32.47	32.47	35.83	33.46	86.30	182.48	81.58	182.48	80.09	32.75	81.55	32.75	76.90	76.94	78.57	110.63	105.54	1031.54	33.99	1441.65
1.20	1.20	31.32	31.32	31.21	31.21	30.67	33.01	31.54	20.41	22.84	20.41	23.24	31.92	23.03	31.92	23.14	24.94	24.94	21.53	22.07	14.67	31.32	825.03
	1.10	27.33	27.33	26.77	26.77	27.18	26.91	32.59	26.50	26.39	26.50	26.46	27.18	26.44	27.18	26.11	26.03	26.03	25.98	26.14	31.76	27.33	1242.39
	1.00	22.93	22.93	22.17	22.17	23.30	22.46	34.21	37.39	29.46	37.39	29.47	22.21	29.47	22.21	28.56	28.98	29.04	32.28	31.72	76.64	22.93	1537.50
	0.90	19.78	19.78	18.94	18.94	20.58	18.66	37.42	58.52	33.87	58.52	33.57	19.02	33.86	19.02	32.47	33.68	34.06	42.40	40.92	226.34	19.78	1772.20
	0.80	16.79	16.79	15.92	15.92	17.77	16.32	41.27	95.96	39.68	95.96	38.13	16.09	39.61	16.09	36.97	37.01	38.17	55.26	53.15	745.27	16.79	1843.22
1.30	1.20	14.77	14.77	14.71	14.71	14.62	14.67	15.96	12.19	12.86	12.19	12.95	14.96	12.93	14.96	12.86	13.03	13.03	12.30	12.42	11.67	14.77	1620.51
	1.10	13.35	13.35	13.03	13.03	13.42	12.90	16.99	16.16	14.17	16.16	14.18	13.15	14.17	13.15	13.91	14.20	14.21	15.00	14.90	23.41	13.35	1772.50
	1.00	11.96	11.96	11.61	11.61	12.18	11.76	18.30	21.87	16.37	21.87	16.35	11.62	16.37	11.62	15.87	16.02	16.07	18.49	18.14	56.18	11.96	1887.82
	0.90	10.65	10.65	10.29	10.29	11.06	10.09	20.64	34.20	19.02	34.20	18.87	10.32	19.02	10.32	18.35	19.03	19.33	24.06	23.19	154.54	10.65	1927.60
	0.80	9.61	9.61	9.05	9.05	10.24	9.07	23.48	56.97	22.39	56.97	21.27	9.16	22.33	9.16	20.65	21.05	21.69	30.92	29.95	561.55	9.61	1950.94
1.40	1.20	8.40	8.40	8.30	8.30	8.36	8.31	9.40	7.90	7.73	7.90	7.77	8.38	7.75	8.38	7.68	7.98	7.98	7.88	7.93	9.41	8.40	1896.92
	1.10	7.58	7.58	7.39	7.39	7.66	7.52	9.98	10.43	8.67	10.43	8.68	7.42	8.67	7.42	8.52	8.66	8.67	9.37	9.30	18.25	7.58	1945.18
	1.00	7.15	7.15	6.99	6.99	7.36	6.96	11.02	14.64	9.97	14.64	9.97	7.00	9.97	7.00	9.70	10.07	10.12	11.60	11.24	42.81	7.15	1969.60
	0.90	6.80	6.80	6.52	6.52	7.05	6.27	12.69	22.42	12.10	22.42	11.92	6.57	12.10	6.57	11.59	11.74	11.93	15.27	14.74	118.60	6.80	1977.39
	0.80	6.22	6.22	5.89	5.89	6.50	6.04	14.39	37.39	14.20	37.39	13.44	5.94	14.16	5.94	12.96	12.60	13.00	19.50	18.83	417.32	6.22	1987.25
1.50	1.20	5.31	5.31	5.23	5.23	5.31	5.04	6.13	5.75	5.23	5.75	5.24	5.28	5.24	5.28	5.18	5.31	5.31	5.50	5.48	7.78	5.31	1975.22
	1.10	5.11	5.11	4.99	4.99	5.16	4.90	6.70	7.52	5.91	7.52	5.91	5.00	5.91	5.00	5.78	6.05	6.08	6.59	6.51	14.99	5.11	1985.46
	1.00	4.81	4.81	4.69	4.69	4.92	4.76	7.36	10.41	7.13	10.41	7.10	4.70	7.13	4.70	6.96	6.92	6.96	8.13	7.88	33.42	4.81	1991.30
	0.90	4.72	4.72	4.57	4.57	4.90	4.53	8.52	15.74	8.18	15.74	8.05	4.59	8.18	4.59	7.85	7.96	8.09	10.43	10.09	91.22	4.72	1993.68
	0.80	4.50	4.50	4.28	4.28	4.70	4.32	9.91	25.54	9.70	25.54	9.02	4.35	9.65	4.35	8.79	8.51	8.77	13.36	12.88	310.59	4.50	1996.36
UCL	1.26	1.12	1.14	1.14	1.14	1.25	1.54	9.00	2.57	2.40	2.57	2.54	2.49	2.53	2.49	2.49	2.53	2.53	3.92	2.53	6.87	3.20	-0.07
ARL0	174.46	174.46	170.07	170.07	170.07	172.32	171.20	228.17	172.01	170.38	172.01	171.60	170.93	171.23	171.46	171.01	171.86	170.68	170.99	170.87	170.33	173.48	170.63

Table 4 – Shewhart Control chart - $\lambda_{10} = 4$ and $\lambda_{20} = 10$

τ	δ_X	$\hat{\rho}_{sd}^*$	$\hat{\rho}_{sq}^*$	$\hat{\rho}_{as}$	$\hat{\rho}_{ft}$	$\hat{\rho}_{rs}$	$\hat{\rho}_{rs}^*$	W_0	W_1	W_1^*	W_2	W_2^*	W_3	W_3^*	W_4	W_4^*	Z_1	Z_2	Z_3	Z_4	Z_4^*	Z_5	Z_5^*
1.02	1.2	272.85	272.85	318.89	318.89	265.13	301.18	254.60	115.27	138.00	115.27	166.44	307.55	145.05	307.55	166.44	145.76	145.76	109.86	129.61	813.03	272.85	154.76
	1.1	212.42	212.42	228.18	228.18	201.73	224.73	228.87	142.77	147.85	142.77	173.46	224.02	150.59	224.02	173.46	144.79	144.79	129.15	141.54	935.15	212.42	182.28
	1	170.78	170.78	164.79	164.79	156.12	166.14	220.07	196.31	170.31	196.31	179.15	165.60	162.27	165.60	178.78	155.04	155.04	167.19	167.96	1,088.43	170.78	218.38
	0.9	129.88	129.88	114.15	114.15	112.47	117.41	196.00	264.31	196.85	264.31	175.93	115.87	175.49	115.87	175.15	155.86	155.86	204.12	195.78	1,244.84	129.88	282.34
	0.8	99.22	99.22	78.29	78.29	81.52	84.62	185.23	393.63	243.65	393.63	182.43	82.71	206.25	82.71	180.56	159.23	159.23	257.43	238.47	1,378.00	99.22	397.01
1.05	1.2	221.82	221.82	254.23	254.23	214.48	246.49	210.17	102.57	120.45	102.57	143.57	246.98	125.65	246.98	143.56	125.60	125.60	96.89	112.87	728.57	221.82	181.30
	1.1	177.20	177.20	188.36	188.36	168.96	184.49	194.72	131.19	134.14	131.19	153.31	185.76	135.68	185.76	153.29	129.11	129.11	118.38	127.21	875.65	177.20	213.12
	1	140.81	140.81	136.25	136.25	130.26	137.24	178.46	173.19	150.52	173.19	157.45	137.35	145.34	137.35	157.10	130.44	130.44	146.89	146.38	998.86	140.81	255.48
	0.9	110.69	110.69	96.83	96.83	95.84	100.51	169.40	246.26	177.67	246.26	155.91	98.88	157.00	98.88	154.77	138.58	138.58	187.13	175.95	1,171.91	110.69	334.03
	0.8	85.99	85.99	66.76	66.76	69.27	69.00	159.88	362.68	218.34	362.68	155.75	70.02	179.73	70.02	153.85	140.33	140.33	228.70	211.56	1,321.58	85.99	483.09
1.10	1.2	161.58	161.58	183.56	183.56	155.42	181.19	159.62	88.26	95.16	88.26	113.38	178.07	99.36	178.07	113.25	99.62	99.62	81.89	92.88	611.59	161.58	238.63
	1.1	131.76	131.76	139.39	139.39	124.98	135.92	148.29	113.37	107.39	113.37	118.88	137.88	107.41	137.88	118.66	102.17	102.17	101.42	103.89	731.91	131.76	289.84
	1	107.24	107.24	101.85	101.85	97.94	100.50	139.36	152.77	121.74	152.77	122.52	102.71	114.36	102.71	122.31	105.29	105.29	125.74	121.49	889.41	107.24	338.86
	0.9	84.77	84.77	73.64	73.64	73.86	75.96	134.42	211.26	149.18	211.26	125.41	75.47	129.42	75.47	124.65	112.11	112.11	157.35	148.19	1,043.43	84.77	451.49
	0.8	67.21	67.21	52.62	52.62	55.23	55.97	129.15	312.30	181.00	312.30	127.17	55.83	151.77	55.83	125.20	114.71	114.71	192.84	182.27	1,200.99	67.21	632.93
1.20	1.2	93.86	93.86	101.08	101.08	89.97	99.35	99.48	68.60	67.22	68.60	76.24	98.79	68.48	98.79	76.23	66.41	66.41	62.31	65.99	426.70	93.86	405.94
	1.1	78.41	78.41	78.55	78.55	73.63	80.76	93.28	87.01	78.43	87.01	82.48	78.65	76.08	78.65	82.40	69.24	69.24	74.88	75.17	520.53	78.41	465.35
	1	66.56	66.56	61.59	61.59	60.22	62.70	90.81	116.11	87.50	116.11	82.16	62.29	79.94	62.29	81.96	71.69	71.69	91.33	86.97	654.65	66.56	559.49
	0.9	54.94	54.94	46.55	46.55	47.19	47.99	90.23	169.13	106.56	169.13	85.19	48.08	92.43	48.08	84.50	77.22	77.22	116.01	108.87	826.02	54.94	732.68
	0.8	45.39	45.39	34.40	34.40	36.39	36.13	88.43	249.51	134.10	249.51	88.34	36.65	110.98	36.65	86.66	80.23	80.23	142.78	134.26	1,010.41	45.39	993.71
1.30	1.2	60.54	60.54	64.40	64.40	58.29	61.68	66.97	55.17	50.77	55.17	54.39	63.18	49.71	63.18	54.34	47.27	47.27	49.01	48.60	314.22	60.54	631.15
	1.1	51.44	51.44	50.39	50.39	47.99	50.71	64.62	71.03	58.26	71.03	56.96	50.57	54.33	50.57	56.79	50.03	50.03	58.62	56.62	385.03	51.44	720.76
	1	45.28	45.28	40.87	40.87	40.42	41.58	64.43	94.42	68.89	94.42	59.74	41.53	61.42	41.53	59.43	53.73	53.73	71.14	67.80	501.39	45.28	843.43
	0.9	38.53	38.53	32.19	32.19	32.75	32.94	64.41	138.43	83.14	138.43	62.74	33.38	71.71	33.38	61.73	57.03	57.03	89.92	83.83	638.41	38.53	1,059.77
	0.8	31.97	31.97	23.83	23.83	25.49	25.97	65.52	204.83	108.98	204.83	66.60	25.58	88.05	25.58	65.05	59.98	59.98	109.74	103.48	843.17	31.97	1,328.55
1.40	1.2	40.99	40.99	42.43	42.43	39.04	41.23	46.94	44.59	38.36	44.59	40.45	41.90	37.12	41.90	40.41	35.45	35.45	38.00	37.42	227.74	40.99	903.66
	1.1	36.37	36.37	34.83	34.83	33.57	35.54	47.30	59.28	45.44	59.28	43.23	35.05	41.44	35.05	43.11	38.11	38.11	47.51	45.17	288.39	36.37	1,030.65
	1	32.43	32.43	28.93	28.93	28.84	29.48	47.59	78.77	54.90	78.77	45.51	29.64	47.77	29.64	45.17	41.03	41.03	56.69	54.11	392.53	32.43	1,160.11
	0.9	27.69	27.69	22.63	22.63	23.47	23.51	48.74	115.68	67.48	115.68	47.69	23.61	57.17	23.61	46.88	44.08	44.08	72.12	67.55	506.81	27.69	1,352.66
	0.8	24.17	24.17	18.37	18.37	19.75	19.77	51.66	179.11	87.80	179.11	52.83	19.81	72.32	19.81	51.27	47.82	47.82	92.45	88.19	687.71	24.17	1,571.76
1.50	1.2	29.87	29.87	29.95	29.95	28.09	30.73	35.20	37.65	32.25	37.65	32.03	29.91	30.41	29.91	32.01	27.76	27.76	31.72	31.20	178.92	29.87	1,164.07
	1.1	26.34	26.34	24.77	24.77	23.98	25.50	35.92	50.19	37.28	50.19	33.65	24.96	33.72	24.96	33.51	30.39	30.39	38.80	36.73	226.08	26.34	1,287.28
	1	24.32	24.32	21.12	21.12	21.32	20.98	37.34	69.93	44.34	69.93	35.22	21.69	38.86	21.69	34.84	33.14	33.14	47.95	45.34	301.69	24.32	1,413.76
	0.9	21.50	21.50	17.37	17.37	17.98	18.37	38.45	99.02	54.84	99.02	38.65	18.18	47.26	18.18	37.86	35.47	35.47	59.22	56.03	413.55	21.50	1,585.46
	0.8	19.27	19.27	14.06	14.06	15.28	15.65	39.98	155.68	72.25	155.68	40.88	15.32	60.03	15.32	39.33	37.14	37.14	75.06	72.27	572.40	19.27	1,727.20
	UCL	1.4	1.1832	1.2984	1.2923	1.5752	2.5542	3	2.789	1.99	2.789	2.4946	2.3433	2.5658	2.3433	2.3003	2.399	2.4276	3.8292	2.5975	3.3546	4.4936	-0.0783
	ARL0	192.89	192.89	189.43	189.43	177.94	187.73	243.04	208.48	182.25	208.48	192.68	189.62	175.89	189.62	192.32	170.03	170.03	178.82	181.41	173.94	192.89	197.61

statistic in general except previous combination of τ and δ_x . W_1^* , W_2^* , W_3^* and W_4^* present the best performance only for the combination of $\tau = 1.30$ and $\delta_x = 1.2$.

4.2 EWMA control chart's performance

EWMA control chart performances are obtained with the constant $\theta = 0.2$ and the same previous conditions are considered. Tables 6, 7 show the results.

4.2.1 Case of $\lambda_{10} = 7$ and $\lambda_{20} = 10$

For the case $\lambda_{10} = 7$ and $\lambda_{20} = 10$, the statistic Z_4^{**} also presents good performance for large shift $\delta_x = 1.20$ and $\tau \leq 1.30$ and when $\delta_x = 1.10$ and $\tau \leq 1.05$. For other combinations of τ and δ_x , the statistic W_4 is the best options as it presents the lowest ARL_1 in most of combinations.

4.2.2 Case of $\lambda_{10} = 28$ and $\lambda_{20} = 40$

For the case $\lambda_{10} = 28$ and $\lambda_{20} = 40$, the performance of the statistic Z_4^{**} follows closely as the previous case and whenever the statistic Z_4^{**} does not perform well, the W_0 is the statistic with a better performance. This behavior is observed for $\tau \leq 1.20$. For $\tau \geq 1.30$, several statistics yield similar performance.

4.2.3 Case of $\lambda_{10} = 4$ and $\lambda_{20} = 10$

Tendency is the same as previous cases. Z_4^{**} has the best performances for larger increases in the average and lower shifts in the ratio and W_0 is the best statistic in general.

4.2.4 Case of $\lambda_{10} = 16$ and $\lambda_{20} = 40$

We could observed also that the case have same character of the previous case as Z_4^{**} and W_0 cover almost every combination of τ and δ_x . However for $\tau \geq 1.30$, several statistics yield similar performance to W_0 as we could observed the results of case of $\lambda_{10} = 28$ and $\lambda_{20} = 40$.

4.3 Overview of results

We have "three classes" of statistics: $\rho_{(.)}$, $W_{(.)}$ and $Z_{(.)}$. Some general features are observed: as the shift of the ratio increases ($\tau \uparrow$), ARL_1 decreases as expected. As δ_X increases, ARL_1 may decrease or increase depending on the nature of the statistics. For the statistics of the type $\rho_{(.)}$, as $\delta_X \uparrow$, $ARL_1 \uparrow$. For the type $W_{(.)}$, different behaviors are observed: as $\delta_X \uparrow$, $ARL_1 \downarrow$ for W_1 and W_2 ; but for W_0 , W_1^* , W_2^* , W_3^* and W_4^* , the

Table 5 – Shewhart Control chart - $\lambda_{10} = 16$ and $\lambda_{20} = 40$

τ	δ_X	$\hat{\rho}_{sd}^*$	$\hat{\rho}_{sq}^*$	$\hat{\rho}_{as}$	$\hat{\rho}_{ft}$	$\hat{\rho}_{rs}$	$\hat{\rho}_{rs}^*$	W_0	W_1	W_1^*	W_2	W_2^*	W_3^*	W_3^{**}	W_4^*	W_4^{**}	Z_1	Z_2	Z_3	Z_4	Z_4^{**}	Z_5	Z_5^*
1.02	1.2	243.38	243.38	266.78	266.78	234.64	247.89	386.13	86.09	122.47	86.09	132.31	267.00	133.76	267.00	130.52	133.95	133.95	71.69	105.39	34.92	243.38	87.04
	1.1	178.55	178.55	188.80	188.80	177.01	180.14	225.72	107.87	127.00	107.87	132.93	188.80	133.05	188.80	132.82	134.15	134.15	98.94	120.40	70.73	178.55	140.35
	1	137.21	137.21	137.09	137.09	138.26	134.17	133.67	148.05	140.30	148.05	139.93	137.09	140.01	137.09	139.93	139.61	139.61	145.85	142.29	164.34	137.21	225.72
	0.9	98.89	98.89	94.41	94.41	102.08	103.12	74.64	210.74	155.66	210.74	145.08	94.41	145.08	94.41	145.08	140.78	140.78	222.02	164.04	449.47	98.89	338.60
	0.8	73.92	73.92	67.08	67.08	79.19	74.35	44.11	330.59	164.75	330.59	144.13	67.08	144.13	67.08	144.51	144.17	144.17	363.92	202.83	1,112.88	73.92	493.21
1.05	1.2	165.10	165.10	179.48	179.48	160.43	171.80	250.72	67.11	92.70	67.11	98.58	179.55	99.54	179.55	98.13	99.09	99.09	57.25	80.86	31.97	165.10	129.44
	1.1	127.48	127.48	133.75	133.75	126.14	130.85	152.20	88.58	99.79	88.58	103.53	133.75	103.86	133.75	103.38	101.03	101.03	82.92	94.88	64.86	127.48	202.79
	1	96.98	96.98	96.10	96.10	98.12	99.52	93.00	117.25	111.46	117.25	109.49	96.10	109.49	96.10	109.42	105.30	105.30	117.98	109.47	152.88	96.98	329.48
	0.9	74.87	74.87	70.99	70.99	77.24	73.57	55.18	172.75	117.86	172.75	109.36	70.99	109.36	70.99	109.37	110.56	110.56	182.04	130.80	400.87	74.87	484.44
	0.8	56.09	56.09	51.38	51.38	60.28	56.04	32.95	270.64	129.79	270.64	113.64	51.38	113.64	51.38	113.99	109.43	109.43	297.35	158.75	1,054.30	56.09	631.22
1.10	1.2	94.45	94.45	100.18	100.18	92.53	96.69	128.74	47.88	61.53	47.88	64.61	100.19	64.88	100.19	64.38	63.60	63.60	43.15	54.71	28.99	94.45	241.30
	1.1	74.66	74.66	76.34	76.34	74.17	76.26	81.87	61.99	64.89	61.99	66.51	76.34	66.59	76.34	66.47	65.61	65.61	59.79	63.44	56.00	74.66	375.07
	1	60.64	60.64	59.73	59.73	61.83	59.18	54.31	88.07	71.79	88.07	69.52	59.73	69.53	59.73	69.51	71.01	71.01	89.76	76.34	129.30	60.64	578.71
	0.9	48.77	48.77	45.58	45.58	50.82	46.35	34.93	125.67	80.70	125.67	73.58	45.58	73.58	45.58	73.60	73.77	73.77	134.13	91.06	358.95	48.77	759.13
	0.8	38.42	38.42	35.02	35.02	41.17	36.56	22.27	200.69	89.06	200.69	76.27	35.02	76.27	35.02	76.55	77.95	77.95	223.87	114.71	951.74	38.42	941.98
1.20	1.2	37.10	37.10	38.66	38.66	36.70	38.25	44.83	27.17	29.89	27.17	31.07	38.66	31.10	38.66	31.02	30.50	30.50	25.70	28.79	23.03	37.10	765.43
	1.1	32.02	32.02	32.41	32.41	32.30	31.55	32.27	35.91	33.60	35.91	33.16	32.41	33.16	32.41	33.12	32.85	32.85	35.94	33.66	46.34	32.02	1,005.77
	1	26.83	26.83	25.88	25.88	27.71	27.66	22.53	50.68	38.63	50.68	36.33	25.88	36.33	25.88	36.33	35.74	35.74	53.37	40.77	105.35	26.83	1,232.45
	0.9	23.14	23.14	21.36	21.36	24.33	22.78	15.91	73.12	43.07	73.12	38.35	21.36	38.35	21.36	38.38	37.60	37.60	80.33	49.27	286.14	23.14	1,354.95
	0.8	19.54	19.54	17.70	17.70	21.07	18.81	11.08	115.12	47.99	115.12	41.27	17.70	41.27	17.70	41.64	40.56	40.56	128.39	62.70	817.94	19.54	1,454.24
1.30	1.2	18.68	18.68	18.99	18.99	18.67	18.26	19.89	17.67	16.86	17.67	16.89	18.99	16.90	18.99	16.89	17.43	17.43	17.45	17.45	19.55	18.68	1,431.74
	1.1	16.71	16.71	16.49	16.49	16.83	16.39	15.41	23.37	19.24	23.37	18.81	16.49	18.81	16.49	18.81	19.05	19.05	23.98	20.39	37.60	16.71	1,563.81
	1	14.54	14.54	13.95	13.95	15.00	14.54	11.69	31.25	22.51	31.25	20.90	13.95	20.90	13.95	20.91	20.35	20.35	33.17	24.05	87.06	14.54	1,667.93
	0.9	12.88	12.88	11.94	11.94	13.67	12.72	8.76	47.21	25.44	47.21	22.63	11.94	22.63	11.94	22.64	22.05	22.05	51.76	30.01	228.39	12.88	1,711.02
	0.8	11.50	11.50	10.44	10.44	12.41	11.10	6.59	78.21	28.80	78.21	24.47	10.44	24.47	10.44	24.82	24.82	24.82	86.93	40.20	674.34	11.50	1,740.99
1.40	1.2	10.54	10.54	10.49	10.49	10.59	10.65	10.51	11.99	11.29	11.99	11.09	10.49	11.09	10.49	11.09	10.94	10.94	12.10	11.27	17.45	10.54	1,789.76
	1.1	9.80	9.80	9.58	9.58	9.98	9.81	8.67	16.28	12.56	16.28	12.01	9.58	12.01	9.58	12.01	12.26	12.26	16.89	13.51	32.08	9.80	1,832.44
	1	8.93	8.93	8.42	8.42	9.27	8.91	6.88	22.30	14.13	22.30	13.02	8.42	13.02	8.42	13.02	13.11	13.11	24.03	15.93	70.72	8.93	1,854.43
	0.9	8.37	8.37	7.81	7.81	8.89	8.18	5.58	33.43	16.58	33.43	14.70	7.81	14.70	7.81	14.75	14.84	14.84	36.67	20.90	190.24	8.37	1,875.52
	0.8	7.76	7.76	7.00	7.00	8.35	7.59	4.50	55.40	19.84	55.40	16.60	7.00	16.60	7.00	16.82	16.51	16.51	61.26	27.38	564.78	7.76	1,885.62
1.50	1.2	6.88	6.88	6.83	6.83	6.92	6.81	6.58	9.34	7.86	9.34	7.64	6.83	7.64	6.83	7.64	7.64	7.64	9.61	8.23	14.90	6.88	1,921.99
	1.1	6.55	6.55	6.36	6.36	6.71	6.44	5.57	12.13	8.93	12.13	8.43	6.36	8.43	6.36	8.43	8.48	8.48	12.81	9.61	28.83	6.55	1,930.50
	1	6.20	6.20	5.82	5.82	6.48	6.18	4.66	16.73	10.24	16.73	9.44	5.82	9.44	5.82	9.45	9.56	9.56	17.89	11.74	63.64	6.20	1,936.66
	0.9	5.92	5.92	5.52	5.52	6.33	5.73	3.99	24.28	11.78	24.28	10.40	5.52	10.40	5.52	10.48	10.20	10.20	26.50	14.70	158.80	5.92	1,943.52
	0.8	5.38	5.38	4.94	4.94	5.80	5.46	3.29	40.83	13.89	40.83	11.74	4.94	11.74	4.94	12.03	11.51	11.51	44.51	19.28	492.19	5.38	1,946.54
UCL		0.7778	0.8819	0.8993	0.8997	1.0034	1.2609	-6	2.62	2.23	2.62	2.5508	2.3957	2.6347	2.3957	2.3709	2.4496	2.4777	5.1961	2.5441	5.866	3.1934	-0.0563
ARL0		171.39	171.39	172.71	172.71	172.58	171.4	228.17	176.55	170.90	176.55	171.05	172.71	171.23	172.71	171.05	171.05	171.05	171.05	170.98	175.67	171.39	171.32

Table 6 – EWMA Control chart - $\lambda_0 = 7$ and $\lambda_{20} = 10$

τ	δx	$\hat{\rho}_{sd}^*$	$\hat{\rho}_{sv}^*$	$\hat{\rho}_{ss}^*$	$\hat{\rho}_{rt}^*$	$\hat{\rho}_{rs}^*$	$\hat{\rho}_{rs}^*$	W_0	W_1	W_1^*	W_2	W_2^*	W_3	W_3^*	W_4	W_4^*	Z_1	Z_2	Z_3	Z_4	Z_4^*	Z_5	Z_5^*
1.02	1.20	213.19	180.64	251.15	250.33	220.47	299.28	190.07	89.76	122.70	89.76	127.57	203.06	128.11	174.58	127.35	131.74	160.41	89.11	107.72	29.13	214.36	36.06
	1.10	167.26	152.31	177.44	176.51	166.64	217.15	152.94	104.98	125.61	104.98	129.49	159.23	130.39	138.35	128.97	126.90	159.22	104.90	115.54	57.49	168.63	82.62
	1.00	139.30	132.65	134.82	134.81	135.68	151.26	128.63	134.63	131.11	134.63	128.02	129.28	127.45	112.26	126.33	133.77	166.22	134.19	134.71	148.67	139.88	260.63
	0.90	115.59	119.20	102.47	102.96	108.98	107.86	107.35	176.55	139.52	176.55	132.84	106.56	132.19	94.37	129.16	136.19	172.59	172.65	154.59	533.97	116.38	899.00
	0.80	94.70	104.71	76.64	77.54	85.13	75.32	87.26	252.55	147.30	252.55	142.76	83.67	143.62	76.03	135.86	137.06	180.07	231.09	185.21	1474.79	95.17	1711.91
1.05	1.20	152.30	126.03	167.78	166.85	152.07	242.50	123.14	65.03	85.50	65.03	86.18	136.13	86.35	115.24	86.58	90.05	109.48	64.79	75.86	26.00	153.20	55.64
	1.10	123.39	109.82	126.06	125.97	121.03	174.78	103.34	78.02	88.70	78.02	89.72	108.43	89.41	94.62	88.89	91.32	112.42	78.78	84.36	49.53	124.00	144.71
	1.00	102.66	95.37	94.62	94.50	97.05	124.77	85.90	98.59	92.46	98.59	93.50	87.61	93.24	77.75	91.91	91.64	113.85	96.22	93.88	126.55	103.35	464.95
	0.90	85.98	85.69	73.66	73.94	80.19	88.48	75.29	130.60	101.42	130.60	98.78	73.94	98.37	65.98	95.85	96.99	120.43	124.95	112.43	436.06	86.34	1319.36
	0.80	72.27	78.04	56.90	57.25	63.85	64.16	65.11	183.13	106.05	183.13	105.51	61.88	104.81	55.76	99.84	99.34	127.28	165.63	131.42	1356.61	72.61	1862.04
1.10	1.20	85.71	69.30	88.82	88.34	82.74	170.21	65.75	41.87	51.24	41.87	51.69	69.85	51.44	61.44	51.70	53.04	62.69	41.47	46.55	22.24	86.14	124.07
	1.10	75.93	66.40	73.96	73.63	72.93	127.38	60.07	51.39	54.27	51.39	54.73	63.83	54.51	57.01	53.84	56.07	67.08	51.78	53.84	41.16	76.33	379.12
	1.00	66.57	60.90	59.07	59.09	61.77	94.16	52.88	64.65	58.48	64.65	58.26	54.17	58.20	48.84	57.49	58.76	71.69	63.85	61.71	100.38	66.92	1137.81
	0.90	58.43	56.22	48.35	48.56	52.65	68.48	47.73	84.10	61.92	84.10	62.63	46.85	62.24	42.68	60.33	60.64	75.29	79.65	70.90	333.82	58.70	1784.50
	0.80	50.52	51.05	37.99	38.29	43.52	49.29	41.16	118.39	70.30	118.39	66.75	39.47	65.27	36.01	63.35	61.39	78.24	104.63	81.98	1139.93	50.65	1962.92
1.20	1.20	40.31	32.30	37.79	37.60	37.38	92.02	28.52	23.10	24.51	23.10	24.70	30.49	24.49	27.86	24.45	25.97	29.86	23.26	24.43	17.09	40.51	847.32
	1.10	36.77	31.14	33.46	33.27	34.21	73.16	26.82	27.05	26.64	27.05	28.00	28.13	27.78	25.74	27.57	26.96	31.15	26.82	27.01	30.74	36.94	1561.31
	1.00	33.55	29.74	28.63	28.60	30.44	54.25	25.56	33.34	28.93	33.34	28.77	25.94	28.65	23.99	28.08	28.69	33.11	32.15	30.63	64.95	33.66	1907.78
	0.90	31.51	29.09	25.16	25.17	27.95	42.17	24.53	43.49	31.88	43.49	31.43	23.62	31.28	21.83	30.48	30.40	36.11	40.27	35.79	196.21	31.69	1984.93
	0.80	29.80	28.91	22.38	22.50	25.42	32.75	23.66	60.08	36.22	60.08	34.45	22.12	33.85	20.81	33.04	32.78	39.97	52.07	42.69	795.16	29.84	1998.92
1.30	1.20	22.65	18.66	20.64	20.51	20.97	55.70	16.30	15.28	14.89	15.28	15.59	17.04	15.57	16.04	15.36	15.91	17.52	15.19	15.51	14.53	22.78	1835.23
	1.10	21.35	18.56	18.88	18.81	19.68	44.70	16.01	17.69	16.36	17.69	16.85	16.48	16.73	15.44	16.53	16.61	18.80	17.33	17.09	23.57	21.43	1961.25
	1.00	20.71	18.51	17.49	17.49	18.79	36.86	15.96	21.66	17.92	21.66	17.77	15.82	17.66	14.84	17.31	17.85	20.37	20.61	19.49	46.79	20.76	1991.96
	0.90	19.83	18.17	15.94	15.94	17.51	29.42	15.70	27.00	19.56	27.00	19.55	14.89	19.32	14.02	18.87	18.87	21.60	24.57	22.07	128.35	19.89	1999.11
	0.80	19.22	18.25	14.71	14.78	16.50	23.22	16.12	36.79	22.35	36.79	21.99	14.39	21.74	13.55	21.09	20.71	24.68	31.57	26.54	511.34	19.26	1998.80
1.40	1.20	14.79	12.55	13.47	13.39	13.80	37.01	11.01	11.19	10.99	11.19	11.23	11.44	11.20	10.79	11.02	10.99	12.09	10.97	11.06	12.16	14.83	1985.69
	1.10	14.60	12.73	12.80	12.74	13.43	31.47	11.46	12.87	11.72	12.87	12.05	11.19	11.96	10.61	11.77	11.76	12.92	12.52	12.22	19.52	14.67	1996.76
	1.00	14.42	12.93	12.11	12.12	13.08	24.94	11.42	15.66	12.68	15.66	13.16	10.99	13.05	10.38	12.74	12.75	14.30	14.86	14.01	37.39	14.44	1999.25
	0.90	14.41	13.26	11.52	11.56	12.72	20.92	11.75	19.62	14.27	19.62	14.05	10.93	13.91	10.37	13.58	13.70	15.65	17.78	16.08	90.29	14.45	2000.00
	0.80	13.97	13.50	10.87	10.92	12.16	16.74	12.18	25.51	15.90	25.51	15.58	10.66	15.48	10.13	14.90	14.98	17.31	21.63	18.89	332.68	14.00	1999.60
1.50	1.20	11.07	9.74	10.08	10.04	10.35	25.18	8.63	9.11	8.49	9.11	9.33	8.77	8.63	8.32	8.45	8.69	9.49	8.88	8.85	10.67	11.10	1999.09
	1.10	10.88	9.76	9.62	9.62	10.12	22.37	8.83	10.40	9.31	10.40	9.33	8.74	9.33	8.30	9.11	9.23	10.15	9.96	9.73	16.33	10.92	2000.00
	1.00	10.92	9.89	9.30	9.31	9.97	18.69	9.12	12.40	10.10	12.40	10.03	8.54	10.00	8.18	9.77	9.88	10.93	11.41	10.90	29.52	10.95	1999.60
	0.90	11.10	10.36	9.08	9.08	9.98	15.88	9.44	15.19	11.05	15.19	11.03	8.50	10.96	8.08	10.70	10.76	12.12	13.46	12.50	67.54	11.12	2000.00
	0.80	10.68	10.26	8.47	8.50	9.44	13.25	9.68	19.50	12.34	19.50	12.16	8.32	12.10	7.95	11.73	11.41	13.10	16.18	14.29	230.21	10.70	2000.00
UCL	1.05	0.98	1.05	1.05	1.05	1.11	3.03	0.11	0.77	0.68	0.77	0.73	0.77	0.78	0.77	0.76	0.75	0.77	1.48	0.76	2.42	0.91	-0.18
ARL0	170.04	170.04	170.05	170.07	170.01	172.54	170.01	170.04	170.05	170.04	170.01	170.01	170.01	170.02	170.05	170.03	170.01	170.01	170.05	170.06	170.05	170.08	170.02

Table 7 – EWMA Control chart - $\lambda_{10} = 28$ and $\lambda_{20} = 40$

τ	δ_X	$\hat{\rho}_{ed}^*$	$\hat{\rho}_{en}^*$	$\hat{\rho}_{as}^*$	$\hat{\rho}_{ft}^*$	$\hat{\rho}_{rs}^*$	$\hat{\rho}_{rs}^*$	W_0	W_1	W_1^*	W_2	W_2^*	W_3	W_3^*	W_4	W_{4+}^*	Z_1	Z_2	Z_3	Z_4	Z_4^*	Z_5	Z_5^*
1.02	1.20	154.43	143.12	163.75	163.98	152.44	170.46	334.68	56.89	94.83	66.21	95.66	177.05	96.16	146.37	97.53	97.44	108.32	50.37	80.12	10.34	163.54	14.87
	1.10	125.97	120.90	127.17	127.30	126.54	133.10	175.06	67.70	99.99	80.57	99.07	139.78	99.09	118.92	100.71	100.58	111.15	70.79	91.64	25.48	133.26	46.57
	1.00	107.26	106.80	105.93	105.79	110.16	105.60	99.27	85.56	102.90	103.47	102.64	117.65	102.22	101.30	103.99	107.29	119.57	109.37	108.04	138.09	112.29	376.97
	0.90	84.11	86.70	80.26	80.17	87.89	80.31	54.70	104.01	104.88	128.40	104.41	89.86	102.93	78.80	105.41	103.56	116.51	165.91	117.45	1422.70	87.57	1800.84
	0.80	71.80	74.94	64.74	64.69	74.54	64.07	33.12	144.23	112.50	183.75	107.36	73.91	105.33	65.76	107.78	109.20	123.96	301.48	142.94	1992.85	73.93	1997.66
1.05	1.20	74.50	68.22	75.08	75.09	72.17	78.63	123.27	32.45	49.72	37.01	50.70	77.73	51.01	67.25	51.59	49.71	54.16	30.35	43.38	9.25	77.84	25.33
	1.10	63.71	60.93	63.49	63.51	63.50	67.95	75.77	37.91	51.28	43.90	53.58	67.56	53.58	58.71	54.21	51.96	56.66	40.80	48.53	21.76	66.69	119.29
	1.00	56.53	55.32	54.52	54.44	57.02	56.25	48.28	47.88	54.96	55.71	56.66	58.25	56.21	51.60	57.13	55.07	60.43	59.22	56.50	104.56	58.59	1147.02
	0.90	48.05	48.46	44.85	44.78	49.24	48.36	31.24	59.39	59.53	71.64	58.17	48.37	57.62	42.98	58.68	57.02	62.92	93.19	64.80	1186.40	49.99	1973.26
	0.80	43.42	44.35	38.83	38.69	44.43	39.19	21.48	79.51	63.99	98.55	61.63	42.93	60.55	38.48	61.82	60.54	67.26	153.96	77.12	1984.90	44.51	1999.45
1.10	1.20	30.71	28.47	30.12	30.13	29.61	31.70	38.73	17.08	22.57	19.05	22.95	30.68	23.00	27.64	23.05	22.97	24.52	16.99	21.08	8.10	31.56	92.82
	1.10	28.84	27.37	27.83	27.81	28.34	29.20	29.17	20.28	23.88	22.84	23.93	28.56	23.83	25.84	24.00	24.64	26.33	22.27	24.00	16.96	29.74	857.48
	1.00	26.32	25.69	25.22	25.22	26.43	26.32	21.71	24.21	25.79	27.53	25.76	26.14	25.60	23.87	25.81	26.17	28.15	29.40	27.21	68.05	27.07	1927.65
	0.90	24.95	24.61	23.37	23.33	25.05	24.12	17.07	30.30	28.10	34.81	27.74	24.31	27.43	22.48	27.75	28.15	30.18	42.69	31.61	803.88	25.71	1999.34
	0.80	23.24	23.59	21.16	21.10	23.56	21.62	13.46	39.34	30.69	46.51	30.76	22.43	30.26	20.61	30.69	30.46	32.57	66.38	37.35	1968.72	23.82	2000.00
1.20	1.20	10.91	10.30	10.55	10.55	10.59	11.19	10.88	7.86	9.13	8.47	9.50	10.53	9.50	9.88	9.49	9.19	9.59	8.25	8.89	6.61	11.15	1764.57
	1.10	10.97	10.49	10.49	10.49	10.85	10.99	10.02	9.19	9.92	10.00	9.87	10.54	9.86	9.91	9.85	10.04	10.49	10.09	10.10	11.82	11.22	1994.44
	1.00	10.62	10.31	10.09	10.09	10.60	10.47	8.89	10.57	10.47	11.53	10.53	10.15	10.49	9.59	10.48	10.62	11.15	12.30	11.13	36.01	10.87	2000.00
	0.90	10.42	10.18	9.71	9.70	10.43	10.11	8.01	12.78	11.39	14.13	11.47	9.94	11.36	9.37	11.40	11.49	12.12	16.14	12.82	312.71	10.62	2000.00
	0.80	10.33	10.22	9.49	9.48	10.32	9.83	7.36	16.09	12.68	18.05	12.68	9.73	12.53	9.19	12.55	12.55	13.24	22.71	15.04	1864.93	10.46	2000.00
1.30	1.20	6.36	6.13	6.18	6.18	6.26	6.43	6.08	5.27	5.64	5.63	5.77	6.18	5.77	5.89	5.75	5.79	5.97	5.61	5.73	5.56	6.46	1999.67
	1.10	6.32	6.13	6.11	6.10	6.27	6.24	5.86	5.89	6.10	6.25	6.18	6.12	6.16	5.86	6.14	6.10	6.30	6.35	6.21	9.34	6.41	2000.00
	1.00	6.39	6.24	6.11	6.11	6.37	6.20	5.65	6.76	6.42	7.27	6.54	6.17	6.52	5.87	6.50	6.58	6.82	7.61	6.94	22.95	6.48	2000.00
	0.90	6.37	6.25	6.03	6.03	6.39	6.07	5.47	7.92	7.05	8.62	7.10	6.12	7.09	5.84	7.06	7.05	7.35	9.32	7.80	144.24	6.47	2000.00
	0.80	6.39	6.35	6.00	5.99	6.43	6.04	5.27	9.49	7.58	10.41	7.64	6.15	7.59	5.88	7.58	7.64	7.93	11.79	8.83	1606.53	6.47	2000.00
1.40	1.20	4.52	4.40	4.42	4.41	4.48	4.46	4.35	4.05	4.29	4.27	4.30	4.45	4.30	4.27	4.28	4.30	4.43	4.29	4.31	4.86	4.56	2000.00
	1.10	4.56	4.47	4.43	4.43	4.54	4.44	4.30	4.53	4.50	4.78	4.52	4.48	4.52	4.32	4.51	4.56	4.71	4.84	4.70	7.55	4.62	2000.00
	1.00	4.60	4.52	4.41	4.41	4.59	4.41	4.28	5.11	4.84	5.43	4.83	4.48	4.82	4.29	4.79	4.83	4.98	5.56	5.13	17.13	4.65	2000.00
	0.90	4.65	4.57	4.43	4.42	4.66	4.40	4.25	5.91	5.21	6.30	5.15	4.49	5.15	4.31	5.12	5.20	5.39	6.55	5.74	75.76	4.70	2000.00
	0.80	4.58	4.54	4.31	4.31	4.62	4.40	4.17	6.91	5.59	7.46	5.63	4.41	5.61	4.24	5.58	5.54	5.79	7.95	6.40	1160.92	4.63	2000.00
1.50	1.20	3.57	3.52	3.52	3.52	3.57	3.48	3.49	3.40	3.51	3.56	3.52	3.56	3.53	3.44	3.51	3.51	3.62	3.58	3.56	4.41	3.61	2000.00
	1.10	3.60	3.55	3.51	3.51	3.60	3.46	3.51	3.73	3.68	3.92	3.72	3.57	3.72	3.45	3.70	3.69	3.80	3.94	3.82	6.64	3.63	2000.00
	1.00	3.60	3.56	3.49	3.49	3.62	3.51	3.51	4.12	3.94	4.34	3.91	3.55	3.93	3.45	3.90	3.86	3.98	4.38	4.11	13.21	3.63	2000.00
	0.90	3.67	3.64	3.53	3.54	3.70	3.44	3.56	4.76	4.19	5.05	4.18	3.61	4.18	3.49	4.15	4.16	4.30	5.11	4.56	48.64	3.70	2000.00
	0.80	3.69	3.69	3.53	3.53	3.74	3.53	3.57	5.53	4.50	5.89	4.48	3.61	4.48	3.50	4.45	4.41	4.57	5.99	5.07	690.10	3.73	2000.00
UCL		0.85	0.91	0.93	0.93	0.95	1.03	-5.80	0.72	0.72	0.75	0.75	0.78	0.78	0.77	0.77	0.75	0.76	2.18	0.75	5.39	0.77	-0.09
ARL0		170.09	170.01	170.13	170.00	170.05	170.01	170.13	170.01	170.02	170.00	170.00	170.00	170.00	170.02	170.04	170.03	170.00	170.00	170.04	170.04	170.05	170.39

Table 8 – EWMA Control chart - $\lambda_0 = 4$ and $\lambda_{20} = 10$

τ	δ_X	$\hat{\rho}_{sd}^*$	$\hat{\rho}_{sq}^*$	$\hat{\rho}_{ss}^*$	$\hat{\rho}_{\mu t}^*$	$\hat{\rho}_{rs}^*$	$\hat{\rho}_{rs}^*$	W_0	W_1	W_1^*	W_2	W_2^*	W_3	W_3^*	W_4	W_4^*	Z_1	Z_2	Z_3	Z_4	Z_4^*	Z_5	Z_5^*
1.02	1.20	202.65	171.64	242.36	242.46	200.66	256.75	629.24	93.82	117.48	93.82	125.44	234.68	137.99	234.68	132.21	127.89	134.07	65.60	108.77	65.30	202.62	41.66
	1.10	163.01	149.62	176.91	177.22	162.27	189.78	272.43	109.81	121.57	109.81	126.18	171.94	131.78	171.94	128.20	129.90	132.55	92.05	119.41	65.42	163.01	90.08
	1.00	139.16	135.64	135.00	134.88	136.68	139.40	127.16	138.29	131.69	138.29	132.15	130.68	131.43	130.68	130.35	134.69	134.80	142.28	136.79	154.69	139.09	247.48
	0.90	110.42	119.23	97.76	97.81	106.28	102.96	62.09	177.44	146.56	177.44	142.91	99.98	133.08	99.98	133.80	140.90	136.29	230.74	159.29	460.10	110.42	810.75
	0.80	90.06	104.48	71.27	71.28	84.94	77.57	33.22	246.20	160.95	246.20	151.43	72.46	131.09	72.46	133.92	144.07	134.67	416.59	189.25	1,273.96	90.04	1,639.73
1.05	1.20	146.25	121.69	162.97	163.01	142.65	180.51	347.83	70.90	86.45	70.90	92.93	148.34	100.42	148.34	95.63	92.87	96.03	52.62	79.52	32.33	146.25	62.87
	1.10	122.47	108.03	125.00	124.61	117.65	137.77	164.57	86.72	93.07	86.72	95.97	118.81	98.77	118.81	96.47	95.49	96.59	75.57	89.68	59.53	122.47	152.70
	1.00	104.18	99.02	96.93	97.24	100.09	104.45	83.13	107.85	96.77	107.85	99.86	92.74	97.57	92.74	96.68	100.17	99.34	115.14	104.20	138.30	104.11	438.87
	0.90	85.13	88.59	72.97	72.73	80.73	80.46	43.73	138.46	112.63	138.46	108.89	71.62	102.23	71.62	101.84	104.21	101.14	186.74	120.04	422.63	85.10	1,217.22
	0.80	72.87	81.06	55.89	56.24	67.10	64.06	26.22	191.76	122.86	191.76	116.93	56.09	100.85	56.09	103.13	108.96	103.09	328.07	142.98	1,195.13	72.85	1,818.97
1.10	1.20	90.91	74.44	93.32	92.94	85.76	113.06	151.36	50.07	57.09	50.07	60.47	84.86	63.73	84.86	61.29	58.97	60.17	40.58	54.06	29.22	90.91	135.85
	1.10	79.75	68.82	75.81	75.41	74.44	85.84	80.74	59.66	60.05	59.66	62.77	68.47	63.53	68.47	61.75	61.07	61.35	55.21	59.61	53.48	79.73	361.97
	1.00	70.74	65.35	62.02	61.82	65.49	71.51	47.63	74.58	65.67	74.58	67.55	57.77	65.08	57.77	64.08	65.35	64.31	82.01	69.99	117.38	70.74	1,001.47
	0.90	59.80	59.19	48.87	48.80	55.12	57.58	29.04	93.40	72.97	93.40	71.75	46.22	66.23	46.22	66.19	68.73	66.45	131.44	79.76	342.19	59.80	1,688.29
	0.80	52.27	56.48	38.80	39.00	47.17	45.10	18.46	133.37	82.14	133.37	80.49	37.06	69.04	37.06	69.89	74.33	69.54	234.07	98.11	1,075.94	52.26	1,942.75
1.20	1.20	43.77	35.86	41.06	40.85	40.20	50.34	45.60	28.17	29.49	28.17	31.72	35.69	31.99	35.69	30.73	30.08	30.43	25.84	28.81	23.93	43.77	760.63
	1.10	41.11	35.04	36.51	36.22	37.77	42.77	31.39	34.44	32.09	34.44	33.25	31.93	32.85	31.93	31.84	32.01	32.15	34.64	33.10	42.35	41.11	1,449.53
	1.00	36.52	33.45	30.37	30.32	33.30	36.58	21.12	41.91	35.66	41.91	36.49	27.62	34.70	27.62	34.24	34.13	33.59	49.00	37.57	86.55	36.52	1,850.06
	0.90	34.01	33.07	26.85	26.83	30.91	31.06	15.97	54.63	39.85	54.63	39.89	25.20	36.70	25.20	36.03	37.59	36.21	75.86	45.39	236.82	34.01	1,972.06
	0.80	31.43	32.73	23.25	23.35	28.17	26.91	12.24	74.99	44.42	74.99	44.01	21.53	37.84	21.53	37.63	40.55	38.46	126.19	53.91	814.95	31.43	1,995.72
1.30	1.20	25.47	21.18	22.93	22.84	23.34	27.85	21.18	19.08	18.67	19.08	19.92	19.64	19.83	19.64	18.93	18.64	18.80	18.58	18.69	20.48	25.47	1,754.31
	1.10	24.30	21.25	20.94	20.82	22.28	25.76	16.87	23.09	20.46	23.09	21.45	18.54	20.84	18.54	20.04	20.36	20.32	24.75	21.83	34.62	24.30	1,930.82
	1.00	22.96	20.94	19.07	19.00	21.09	22.85	13.33	27.56	22.18	27.56	22.96	17.03	21.54	17.03	21.07	21.52	21.21	33.05	24.24	68.03	22.96	1,985.24
	0.90	22.32	21.13	17.26	17.26	19.99	20.88	10.69	36.34	24.91	36.34	25.74	15.65	23.24	15.65	22.62	23.51	22.76	49.82	28.78	177.52	22.32	1,994.93
	0.80	20.95	21.25	15.45	15.49	18.65	18.52	9.00	48.57	28.97	48.57	28.68	14.07	24.82	14.07	24.76	26.13	24.82	79.99	34.40	631.61	20.95	1,999.37
1.40	1.20	17.30	14.72	15.22	15.15	15.87	18.66	13.36	14.75	13.48	14.75	14.35	13.42	14.10	13.42	13.57	13.62	13.61	14.83	14.11	17.83	17.30	1,970.48
	1.10	16.95	15.07	14.52	14.45	15.57	17.62	11.46	17.48	14.64	17.48	15.42	13.09	14.82	13.09	14.30	14.81	14.68	19.00	15.93	28.66	16.95	1,994.78
	1.00	16.46	14.95	13.45	13.40	14.88	15.94	9.68	20.90	16.57	20.90	17.09	12.10	16.11	12.10	15.57	15.83	15.51	25.04	18.15	57.34	16.46	1,998.18
	0.90	16.34	15.59	12.78	12.75	14.75	14.86	8.38	26.80	18.29	26.80	18.87	11.53	16.99	11.53	16.56	17.59	16.98	35.83	21.08	138.67	16.34	1,999.80
	0.80	15.74	15.91	11.63	11.67	13.85	13.44	7.27	35.08	20.86	35.08	21.00	10.72	18.33	10.72	18.08	19.12	18.41	55.52	25.12	469.55	15.74	2,000.00
1.50	1.20	13.06	11.22	11.34	11.28	11.93	13.40	9.73	12.06	10.48	12.06	11.12	10.05	10.85	10.05	10.39	10.63	10.65	12.38	11.23	15.96	13.06	1,998.49
	1.10	12.98	11.43	10.98	10.95	11.84	12.84	8.62	13.99	11.69	13.99	12.34	9.79	11.78	9.79	11.27	11.45	11.42	15.31	12.61	25.55	12.98	1,998.98
	1.00	12.77	11.84	10.48	10.46	11.69	12.81	7.74	16.86	12.82	16.86	13.21	9.44	12.43	9.44	12.04	12.66	12.42	20.10	14.38	48.46	12.77	2,000.00
	0.90	12.53	11.99	9.74	9.73	11.26	11.51	6.84	20.63	14.28	20.63	14.63	8.93	13.35	8.93	12.96	13.47	13.13	27.67	16.34	113.55	12.53	2,000.00
	0.80	12.43	12.40	9.33	9.40	11.05	10.73	6.21	27.46	16.21	27.46	16.21	8.63	14.43	8.63	14.03	14.67	14.19	40.83	19.21	367.14	12.43	2,000.00
UCL		0.63	0.76	0.82	0.82	0.87	1.11	-3.21	0.78	0.58	0.78	0.72	0.81	0.93	0.81	0.81	0.70	0.77	2.17	0.73	1.98	0.93	-0.15
ARL0		170.05	170.08	170.04	170.01	170.07	170.12	170.01	170.05	70.05	170.05	170.00	170.00	170.05	170.00	170.08	170.05	170.11	170.11	170.02	170.12	170.05	170.06

Table 9 – EWMA Control chart - $\lambda_{10} = 16$ and $\lambda_{20} = 40$

τ	δ_X	$\hat{\rho}_{sd}^*$	$\hat{\rho}_{sq}^*$	$\hat{\rho}_{as}$	$\hat{\rho}_{ft}$	$\hat{\rho}_{rs}$	$\hat{\rho}_{rs}^*$	W_0	W_1	W_1^*	W_2	W_2^*	W_3^*	W_3^{**}	W_4^*	W_4^{**}	Z_1	Z_2	Z_3	Z_4	Z_4^{**}	Z_5	Z_5^*	
1.02	1.20	166.62	153.05	175.86	178.06	153.49	175.98	1,725.63	76.91	98.69	76.91	103.20	173.15	109.08	173.15	104.88	106.28	107.82	32.56	88.35	13.35	166.62	20.18	
	1.10	137.75	132.37	141.92	142.90	133.87	142.47	595.17	93.57	106.53	93.57	108.96	138.51	111.53	138.51	109.43	109.85	111.21	58.42	100.66	32.56	137.70	59.88	
	1.00	111.55	110.74	110.15	110.04	110.91	110.84	91.73	112.98	110.21	112.98	111.37	106.71	110.58	106.71	109.38	110.33	110.19	123.15	112.13	150.10	111.55	350.53	
	0.90	91.43	94.47	86.71	86.16	93.84	87.96	24.84	151.00	116.37	151.00	115.05	84.79	110.22	84.79	110.26	113.73	112.61	337.85	130.15	1,230.51	91.43	1,617.93	
	0.80	73.58	78.72	66.47	65.86	76.47	70.89	10.86	210.18	128.59	210.18	122.18	65.33	113.30	65.33	116.50	117.85	114.77	988.23	155.51	1,972.00	73.58	1,977.63	
1.05	1.20	87.48	80.43	89.49	90.33	81.60	94.12	1,028.45	47.02	57.36	47.02	59.82	86.99	62.35	86.99	59.79	59.01	59.61	24.77	52.04	12.46	87.40	34.73	
	1.10	76.30	72.74	76.03	76.19	73.43	78.21	188.92	57.05	60.34	57.05	61.88	73.58	62.97	73.58	61.27	63.28	63.66	42.87	59.36	28.49	76.30	145.56	
	1.00	65.08	62.95	61.89	61.77	63.83	63.30	44.10	68.54	63.03	68.54	63.69	58.45	63.08	58.45	62.68	63.51	63.43	83.50	66.01	123.42	65.06	994.36	
	0.90	55.81	56.40	51.83	51.40	56.07	53.93	16.06	89.83	71.10	89.83	69.87	49.68	66.95	49.68	67.45	67.54	66.67	211.06	78.06	1,075.91	55.81	1,898.70	
	0.80	48.40	50.58	43.26	42.69	49.53	43.83	8.42	125.58	72.58	125.58	69.13	41.85	64.45	41.85	66.13	72.57	70.85	671.98	93.33	1,965.27	48.40	1,996.13	
1.10	1.20	38.13	34.66	37.41	37.71	35.52	40.19	177.93	24.44	28.26	24.44	29.49	35.02	30.15	35.02	29.02	27.84	28.13	16.82	25.96	11.14	38.11	130.69	
	1.10	35.26	33.06	33.77	33.81	33.81	35.62	48.75	29.67	29.96	29.67	30.84	31.88	30.95	31.88	30.08	30.42	30.53	26.95	29.95	24.42	35.26	834.47	
	1.00	32.40	31.13	30.02	29.97	31.63	32.62	18.96	35.94	32.74	35.94	33.78	28.51	33.16	28.51	32.15	32.17	32.05	47.99	33.93	93.15	32.40	1,862.25	
	0.90	29.68	29.26	26.87	26.72	29.59	28.49	9.89	47.04	34.99	47.04	34.96	25.67	33.66	25.67	33.36	35.18	34.78	109.66	40.74	841.91	29.68	1,991.69	
	0.80	26.57	26.94	23.74	23.52	26.77	24.90	6.37	63.28	39.36	63.28	38.65	22.62	35.86	22.62	36.21	37.03	36.26	315.53	47.32	1,944.63	26.55	1,999.60	
1.20	1.20	14.19	13.26	13.59	13.61	13.50	14.18	21.15	11.57	11.57	11.57	12.11	12.91	12.19	12.91	11.72	11.95	11.99	10.37	11.73	9.07	14.19	1,785.92	
	1.10	13.71	12.98	12.91	12.90	13.22	13.36	11.87	13.33	12.28	13.33	12.75	12.18	12.70	12.18	12.26	12.64	12.66	14.43	12.94	17.92	13.71	1,985.53	
	1.00	13.41	12.85	12.45	12.42	13.09	13.18	7.70	15.86	13.56	15.86	13.85	11.80	13.63	11.80	13.30	13.75	13.70	22.56	14.71	62.10	13.41	1,999.15	
	0.90	12.84	12.55	11.69	11.63	12.68	12.73	5.54	19.70	15.24	19.70	15.29	11.11	14.83	11.11	14.61	14.63	14.53	41.49	16.86	472.34	12.84	2,000.00	
	0.80	12.40	12.28	11.08	11.00	12.38	11.73	4.39	25.43	16.61	25.43	16.49	10.58	15.52	10.58	15.48	15.87	15.61	101.39	19.74	1,853.03	12.39	2,000.00	
1.30	1.20	7.96	7.57	7.61	7.61	7.69	7.91	8.25	7.37	7.06	7.37	7.30	7.28	7.32	7.28	7.05	7.26	7.26	7.36	7.30	8.01	7.96	1,998.26	
	1.10	7.93	7.62	7.53	7.51	7.74	7.94	6.24	8.37	7.68	8.37	7.88	7.17	7.83	7.17	7.62	7.74	7.73	9.57	8.05	14.91	7.93	2,000.00	
	1.00	7.72	7.46	7.22	7.20	7.61	7.73	4.86	9.73	8.33	9.73	8.49	6.95	8.33	6.95	8.16	8.15	8.12	13.75	8.89	42.25	7.72	2,000.00	
	0.90	7.74	7.56	7.15	7.12	7.67	7.64	4.07	11.75	9.05	11.75	9.16	6.83	8.91	6.83	8.77	8.82	8.77	22.33	10.09	278.49	7.74	2,000.00	
	0.80	7.79	7.75	7.02	6.99	7.78	7.37	3.47	14.94	9.92	14.94	9.95	6.78	9.48	6.78	9.41	9.81	9.69	45.20	11.87	1,724.65	7.79	2,000.00	
1.40	1.20	5.62	5.43	5.42	5.42	5.50	5.57	5.25	5.62	5.29	5.62	5.43	5.21	5.42	5.21	5.26	5.37	5.37	5.88	5.49	7.16	5.62	2,000.00	
	1.10	5.50	5.36	5.26	5.25	5.44	5.49	4.34	6.25	5.57	6.25	5.69	5.09	5.65	5.09	5.52	5.58	5.58	7.32	5.87	12.64	5.50	2,000.00	
	1.00	5.55	5.40	5.24	5.22	5.51	5.46	3.75	7.24	6.00	7.24	6.10	5.06	6.01	5.06	5.89	6.04	6.02	9.93	6.57	32.21	5.55	2,000.00	
	0.90	5.57	5.48	5.21	5.19	5.55	5.37	3.31	8.51	6.48	8.51	6.52	5.02	6.35	5.02	6.28	6.41	6.38	14.86	7.29	179.76	5.57	2,000.00	
	0.80	5.57	5.54	5.14	5.11	5.57	5.40	2.96	10.35	7.19	10.35	7.21	4.98	6.92	4.98	6.86	7.00	6.93	26.00	8.35	1,502.91	5.57	2,000.00	
1.50	1.20	4.28	4.18	4.15	4.15	4.23	4.30	3.88	4.55	4.29	4.55	4.34	4.05	4.34	4.05	4.25	4.26	4.26	4.86	4.41	6.49	4.28	2,000.00	
	1.10	4.33	4.25	4.17	4.17	4.30	4.31	3.48	5.14	4.57	5.14	4.62	4.07	4.59	4.07	4.51	4.53	4.53	6.03	4.82	10.98	4.33	2,000.00	
	1.00	4.27	4.22	4.09	4.08	4.27	4.28	3.13	5.74	4.85	5.74	4.87	3.99	4.81	3.99	4.75	4.72	4.70	7.66	5.16	26.11	4.27	2,000.00	
	0.90	4.39	4.34	4.15	4.14	4.40	4.28	2.88	6.77	5.23	6.77	5.23	4.05	5.13	4.05	5.08	5.14	5.12	10.92	5.80	121.78	4.39	2,000.00	
	0.80	4.37	4.34	4.08	4.07	4.40	4.25	2.64	8.11	5.67	8.11	5.65	3.99	5.47	3.99	5.42	5.50	5.46	17.58	6.53	1,277.15	4.37	2,000.00	
	UCL	0.50	0.70	0.71	0.71	0.73	0.81	-18.37	0.77	0.66	0.77	0.75	0.79	0.87	0.79	0.79	0.72	0.77	3.52	0.75	4.45	0.79	-0.08	
	ARL0	0.09	170.01	170.05	170.02	170.03	170.02	170.03	170.00	170.01	170.00	170.09	170.06	170.08	170.06	170.02	170.01	170.00	170.00	170.09	170.04	170.05	170.04	170.01

decreases of ARL_1 are very slowly seeming that they are insensible for δ_X size. For the statistics of type $Z_{(\cdot)}$, we have the class of insensible as Z_1, Z_2, Z_3, Z_4 ; the statistic Z_5 follows the behavior of the class $\rho_{(\cdot)}$; the statistics Z_4^{**} and Z_5^* behave like W_1 and W_2 .

As expected EWMA charts provide better results than Shewhart chart. The values of in-control ratio influence strongly the speed for detection as cases with $\rho_0 = 0.4$ have worse performance than $\rho_0 = 0.7$ mainly in Shewhart charts and $\tau < 1.20$. Such influence is weaker in EWMA charts and $\tau \geq 1.20$. Similarly the in-control Poisson rates also affect the speed for detection. The condition $(\lambda_{10}, \lambda_{20}) = (28, 40)$ yield better performance than $(\lambda_{10}, \lambda_{20}) = (7, 10)$ in both charts (coherently the condition $(\lambda_{10}, \lambda_{20}) = (16, 40)$ outperforms than $(\lambda_{10}, \lambda_{20}) = (4, 10)$ for $\rho_0 = 0.4$).

All the simulated cases have as in-control ratios $\rho_0 = 0.4, 0.7$. The in-control average Poisson rates as also the in-control ratio influence strongly the speed of detection in both charts mainly for small shifts in the ratio (that is, low values of τ) as lower in-control average rates provide larger ARL_1 . Such effect becomes weaker for larger shifts in the ratio.

Tables 10-11 indicate the statistics which yield the best performance for every combination of shifts δ_x and τ condition as also the minimum value of ARL_1 from the simulation study respectively for $\rho_0 = 0.4$ and $\rho_0 = 0.7$.

Some interesting results from Table 10. For example the statistic Z_4^{**} also outperforms for $\delta_x \geq 1.10$ and $\tau \leq 1.10$ in EWMA control chart. For other combinations the statistic W_0 is the best option. These outcomes are also watched for Shewhart chart with in-control Poisson rates $\lambda_{10} = 16$ and $\lambda_{20} = 40$. Concerning on Shewhart chart for in-control Poisson rates $\lambda_{10} = 4$ and $\lambda_{20} = 10$, the statistics $\hat{\rho}_{as}$ and $\hat{\rho}_{ft}$ exhibit good performance for $\delta_x \leq 0.90$. For other values of δ_x , the statistics like Z_1 and Z_2 for $\tau > 1.20$ otherwise Z_3 may be chosen.

From Tables 10-11, we regard that for both types of control charts in case of small shifts on the ratio ($\tau \leq 1.20$), the shift size on the average rate ($\delta_x = (1+d)\lambda_{10}$) influences the speed of detection as minimum of ARL_1 decreases as $|d|$ increases. But for larger shifts ($\tau > 1.20$), this influence is weaker as minimum of ARL_1 are similar whatever are the values of δ_x mainly for EWMA charts. Additionally when the difference between λ_{10} and λ_{20} is large like $\lambda_{10} = 28$ and $\lambda_{20} = 40$ or $\lambda_{10} = 16$ and $\lambda_{20} = 40$, the best performance relies on the statistics Z_4^{**} and W_0 .

Table 10 – List of the statistics with best performance- case $\rho_0 = 0.4$

τ	δ_X	Shewhart				EWMA			
		$\lambda_{10} = 4$ and $\lambda_{20} = 10$		$\lambda_{10} = 16$ and $\lambda_{20} = 40$		$\lambda_{10} = 4$ and $\lambda_{20} = 10$		$\lambda_{10} = 16$ and $\lambda_{20} = 40$	
		Statistics	min ARL_1	Statistics	min ARL_1	Statistics	min ARL_1	Statistics	min ARL_1
1.02	1.20	Z_3	109.86	Z_4^{**}	34.92	Z_4^{**}	35.30	Z_4^{**}	13.35
	1.10	Z_3	129.15	Z_4^{**}	70.73	Z_4^{**}	65.42	Z_4^{**}	32.56
	1.00	Z_2	155.04	W_0	133.67	W_0	127.16	W_0	91.73
	0.90	$\hat{\rho}_{rs}$	112.47	W_0	74.64	W_0	62.09	W_0	24.84
	0.80	$\hat{\rho}_{as} \hat{\rho}_{ft}$	78.29	W_0	44.11	W_0	33.22	W_0	10.86
1.05	1.20	Z_3	96.89	Z_4^{**}	31.97	Z_4^{**}	32.33	Z_4^{**}	12.46
	1.10	Z_3	118.38	Z_4^{**}	64.86	Z_4^{**}	59.53	Z_4^{**}	28.49
	1.00	$\hat{\rho}_{rs}$	130.26	W_0	93.00	W_0	83.13	W_0	44.10
	0.90	$\hat{\rho}_{rs}$	95.84	W_0	55.18	W_0	43.73	W_0	16.06
	0.80	$\hat{\rho}_{as} \hat{\rho}_{ft}$	66.76	W_0	32.95	W_0	26.22	W_0	8.42
1.10	1.20	Z_3	81.89	Z_4^{**}	28.99	Z_4^{**}	29.22	Z_4^{**}	11.14
	1.10	Z_3	101.42	Z_4^{**}	56.00	Z_4^{**}	53.48	Z_4^{**}	24.42
	1.00	$\hat{\rho}_{rs}$	97.94	W_0	54.31	W_0	47.63	W_0	18.96
	0.90	$\hat{\rho}_{as} \hat{\rho}_{ft}$	73.64	W_0	34.93	W_0	29.04	W_0	9.89
	0.80	$\hat{\rho}_{as} \hat{\rho}_{ft}$	52.62	W_0	22.27	W_0	18.46	W_0	6.37
1.20	1.20	Z_3	62.31	Z_4^{**}	23.03	Z_4^{**}	23.93	Z_4^{**}	9.07
	1.10	$Z_1 Z_2$	69.24	$\hat{\rho}_{rs}^*$	31.55	W_0	31.39	W_0	11.87
	1.00	$\hat{\rho}_{rs}$	60.22	W_0	22.53	W_0	21.12	W_0	7.70
	0.90	$\hat{\rho}_{as} \hat{\rho}_{ft}$	46.55	W_0	15.91	W_0	15.97	W_0	5.54
	0.80	$\hat{\rho}_{as} \hat{\rho}_{ft}$	34.40	W_0	11.08	W_0	12.24	W_0	4.39
1.30	1.20	$Z_1 Z_2$	47.27	$W_1^{**} W_2^{**} W_3^{**} W_4^{**}$	16.86	Z_4^{**}	18.58	$W_1^{**} W_4^{**}$	7.05
	1.10	$\hat{\rho}_{rs}$	47.99	W_0	15.41	W_0	16.87	W_0	6.24
	1.00	$\hat{\rho}_{rs}$	40.42	W_0	11.69	W_0	13.33	W_0	4.86
	0.90	$\hat{\rho}_{as} \hat{\rho}_{ft}$	32.19	W_0	8.76	W_0	10.69	W_0	4.07
	0.80	$\hat{\rho}_{as} \hat{\rho}_{ft}$	23.83	W_0	6.59	W_0	9.00	W_0	3.47
1.40	1.20	$Z_1 Z_2$	35.45	$\hat{\rho}_{sd} \hat{\rho}_{sq} \hat{\rho}_{as} \hat{\rho}_{ft} \hat{\rho}_{rs} \hat{\rho}_{rs}^* \hat{\rho}_{rs} W_0 W_3^{**} W_4^{**} Z_1 Z_2 Z_5$	10.49	W_0	13.36	$W_3^{**} W_4^{**}$	5.21
	1.10	$\hat{\rho}_{rs}$	33.57	W_0	8.67	W_0	11.46	W_0	4.34
	1.00	$\hat{\rho}_{rs}$	28.84	W_0	6.88	W_0	9.68	W_0	3.75
	0.90	$\hat{\rho}_{as} \hat{\rho}_{ft}$	22.63	W_0	5.58	W_0	8.38	W_0	3.31
	0.80	$\hat{\rho}_{as} \hat{\rho}_{ft}$	18.37	W_0	4.50	W_0	7.27	W_0	2.96
1.50	1.20	$Z_1 Z_2$	27.76	W_0	6.58	W_0	9.73	W_0	3.88
	1.10	$\hat{\rho}_{rs}$	23.98	W_0	5.57	W_0	8.62	W_0	3.48
	1.00	$\hat{\rho}_{rs}^*$	20.98	W_0	4.66	W_0	7.74	W_0	3.13
	0.90	$\hat{\rho}_{as} \hat{\rho}_{ft}$	17.37	W_0	3.99	W_0	6.84	W_0	2.88
	0.80	$\hat{\rho}_{as} \hat{\rho}_{ft}$	14.06	W_0	3.29	W_0	6.21	W_0	2.64

Table 11 – List of the statistics with best performance - case $\rho_0 = 0.7$

		Shewhart				EWMA			
τ	δx	$\lambda_{10} = 7$ and $\lambda_{20} = 10$		$\lambda_{10} = 28$ and $\lambda_{20} = 40$		$\lambda_{10} = 7$ and $\lambda_{20} = 10$		$\lambda_{10} = 28$ and $\lambda_{20} = 40$	
		Statistics	min ARL_1	Statistics	min ARL_1	Statistics	min ARL_1	Statistics	min ARL_1
1.02	1.20	Z_4^{**}	63.05	Z_4^{**}	27.48	Z_4^{**}	23.03	Z_4^{**}	10.34
	1.10	Z_4^{**}	99.95	Z_4^{**}	60.04	Z_4^{**}	57.49	Z_4^{**}	25.48
	1.00	$\hat{W}_2^* W_4^{**}$	150.84	$\hat{\rho}_{as} \hat{\rho}_{it}$	112.26	W_1^*	112.26	W_1^*	85.56
	0.90	$\hat{\rho}_{rs} \hat{\rho}_{it}$	105.97	$\hat{\rho}_{rs}$	95.40	W_4^*	94.37	W_0^*	54.70
	0.80	$\hat{\rho}_{as} \hat{\rho}_{it} \hat{\rho}_{rs}$	75.32	$\hat{\rho}_{as} \hat{\rho}_{it} \hat{\rho}_{rs}$	68.42	$\hat{\rho}_{rs}$	75.32	W_0^*	33.12
1.05	1.20	Z_4^{**}	59.98	Z_4^{**}	24.08	Z_4^{**}	26.00	Z_4^{**}	9.25
	1.10	Z_4^{**}	92.64	Z_4^{**}	52.24	Z_4^{**}	49.53	Z_4^{**}	21.76
	1.00	$\hat{\rho}_{as} \hat{\rho}_{it} \hat{\rho}_{rs}$	124.77	$\hat{\rho}_{rs}$	90.82	W_4^*	77.75	$W_0^* W_1^*$	48.28
	0.90	$\hat{\rho}_{as} \hat{\rho}_{it} \hat{\rho}_{rs}$	88.48	$\hat{\rho}_{as} \hat{\rho}_{it} \hat{\rho}_{rs}$	68.58	W_4^*	65.98	W_0^*	31.24
	0.80	$\hat{\rho}_{as} \hat{\rho}_{it} \hat{\rho}_{rs}$	64.16	$\hat{\rho}_{as} \hat{\rho}_{it} \hat{\rho}_{rs}$	50.82	W_4^*	55.76	W_0^*	21.48
1.10	1.20	Z_4^{**}	52.99	Z_4^{**}	19.77	Z_4^{**}	22.24	Z_4^{**}	8.10
	1.10	Z_4^{**}	81.32	Z_4^{**}	43.48	Z_4^{**}	41.16	Z_4^{**}	16.96
	1.00	$\hat{\rho}_{rs}$	94.16	$\hat{\rho}_{rs}$	51.59	W_4^*	48.84	W_0^*	21.71
	0.90	$\hat{\rho}_{as} \hat{\rho}_{it}$	67.69	$\hat{\rho}_{as} \hat{\rho}_{it} W_3^* W_4^*$	41.62	W_4^*	42.68	W_0^*	17.07
	0.80	$\hat{\rho}_{as} \hat{\rho}_{it} \hat{\rho}_{rs}$	49.29	$\hat{\rho}_{as} \hat{\rho}_{it} W_3^* W_4^*$	32.47	W_4^*	36.01	W_0^*	13.46
1.20	1.20	Z_4^{**}	42.78	Z_4^{**}	14.67	Z_4^{**}	17.09	Z_4^{**}	6.61
	1.10	$W_2^* W_4^{**}$	63.49	$W_1^* W_2^* W_3^* W_4^{**} Z_1 Z_2 Z_3 Z_4$	25.98	W_4^*	25.74	W_1^*	9.19
	1.00	$\hat{\rho}_{rs}$	54.25	$\hat{\rho}_{as} \hat{\rho}_{it} \hat{\rho}_{rs} W_3^* W_4^*$	22.17	W_4^*	23.99	W_0^*	8.89
	0.90	$\hat{\rho}_{as} \hat{\rho}_{it} \hat{\rho}_{rs}$	41.75	$\hat{\rho}_{as} \hat{\rho}_{it} \hat{\rho}_{rs} W_3^* W_4^*$	18.66	W_4^*	21.83	W_0^*	8.01
	0.80	$\hat{\rho}_{as} \hat{\rho}_{it} \hat{\rho}_{rs}$	32.75	$\hat{\rho}_{as} \hat{\rho}_{it} \hat{\rho}_{rs} W_3^* W_4^*$	15.92	W_4^*	20.81	W_0^*	7.36
1.30	1.20	Z_4^{**}	38.16	Z_4^{**}	11.67	$W_1^* Z_4^{**}$	14.53	$W_1^* W_2^* W_3^* W_4^{**} Z_3 Z_4 Z_4^{**}$	5.27
	1.10	$W_2^* W_4^{**}$	42.86	$\hat{\rho}_{sd} \hat{\rho}_{sq} \hat{\rho}_{as} \hat{\rho}_{it} \hat{\rho}_{rs} W_3^* W_4^*$	12.90	W_4^*	15.44	any except $\{Z_4^{**} Z_5^{**}\}$	5.86
	1.00	$\hat{\rho}_{as} \hat{\rho}_{it}$	35.36	$\hat{\rho}_{sd} \hat{\rho}_{sq} \hat{\rho}_{as} \hat{\rho}_{it} \hat{\rho}_{rs} W_3^* W_4^* Z_5$	11.96	W_4^*	14.84	any except $\{Z_4^{**} Z_5^{**}\}$	5.65
	0.90	$\hat{\rho}_{as} \hat{\rho}_{it} \hat{\rho}_{rs}$	28.94	$\hat{\rho}_{as} \hat{\rho}_{it} \hat{\rho}_{rs} W_3^* W_4^*$	10.29	W_4^*	14.02	$\hat{\rho}_{as} \hat{\rho}_{it} W_0^* W_4^*$	5.47
	0.80	$\hat{\rho}_{as} \hat{\rho}_{it} \hat{\rho}_{rs}$	22.97	$\hat{\rho}_{as} \hat{\rho}_{it} \hat{\rho}_{rs} W_3^* W_4^*$	9.05	W_4^*	13.55	$W_0^* W_4^*$	5.27
1.40	1.20	$W_2^* W_4^{**}$	29.61	$W_1^* W_1^* W_2^* W_3^* W_4^{**} Z_1 Z_2 Z_3 Z_4$	7.68	$W_0^* W_1^* W_2^* W_3^* W_4^{**} W_4^* W_4^{**} Z_1 Z_3 Z_4$	10.79	any except $\{Z_3^{**} Z_5^{**}\}$	4.27
	1.10	$\hat{\rho}_{as} \hat{\rho}_{it} \hat{\rho}_{rs}$	30.99	$\hat{\rho}_{sd} \hat{\rho}_{sq} \hat{\rho}_{as} \hat{\rho}_{it} \hat{\rho}_{rs} W_3^* W_4^* Z_5$	7.39	W_4^*	10.61	any except $\{Z_3^{**} Z_5^{**}\}$	4.30
	1.00	$\hat{\rho}_{rs}$	24.94	$\hat{\rho}_{sd} \hat{\rho}_{sq} \hat{\rho}_{as} \hat{\rho}_{it} \hat{\rho}_{rs} W_3^* W_4^* Z_5$	6.96	W_4^*	10.38	$\hat{\rho}_{sd} \hat{\rho}_{sq} \hat{\rho}_{as} \hat{\rho}_{it} \hat{\rho}_{rs} W_0^* W_3^* W_4^* Z_5$	4.29
	0.90	$\hat{\rho}_{as} \hat{\rho}_{it} \hat{\rho}_{rs}$	20.65	$\hat{\rho}_{as} \hat{\rho}_{it} \hat{\rho}_{rs} W_3^* W_4^*$	6.27	W_4^*	10.37	$\hat{\rho}_{sd} \hat{\rho}_{sq} \hat{\rho}_{as} \hat{\rho}_{it} \hat{\rho}_{rs} W_0^* W_3^* W_4^* Z_5$	4.25
	0.80	$\hat{\rho}_{as} \hat{\rho}_{it} \hat{\rho}_{rs}$	16.74	$\hat{\rho}_{sd} \hat{\rho}_{sq} \hat{\rho}_{as} \hat{\rho}_{it} \hat{\rho}_{rs} W_3^* W_4^* Z_5$	5.89	W_4^*	10.13	$\hat{\rho}_{sd} \hat{\rho}_{sq} \hat{\rho}_{as} \hat{\rho}_{it} \hat{\rho}_{rs} W_0^* W_3^* W_4^*$	4.17
1.50	1.20	$W_2^* W_4^{**} Z_1 Z_2$	22.71	any except $\{W_0^* W_1^* W_2^* Z_4^* Z_5^*\}$	5.04	$W_0^* W_1^* W_2^* W_3^* W_4^{**} W_4^* W_4^{**} Z_1$	8.32	any except $\{Z_3^{**} Z_5^*\}$	3.40
	1.10	$\hat{\rho}_{as} \hat{\rho}_{it} \hat{\rho}_{rs}$	22.28	$\hat{\rho}_{sd} \hat{\rho}_{sq} \hat{\rho}_{as} \hat{\rho}_{it} \hat{\rho}_{rs} W_3^* W_4^*$	4.90	$W_3^* W_4^*$	8.30	any except $\{Z_4^{**} Z_5^*\}$	3.51
	1.00	$\hat{\rho}_{as} \hat{\rho}_{it} \hat{\rho}_{rs}$	18.69	$\hat{\rho}_{sd} \hat{\rho}_{sq} \hat{\rho}_{as} \hat{\rho}_{it} \hat{\rho}_{rs} W_3^* W_4^*$	4.69	$W_3^* W_4^*$	8.18	any except $\{W_1^* W_2^* Z_2 Z_3 Z_4 Z_4^{**} Z_5^*\}$	3.51
	0.90	$\hat{\rho}_{as} \hat{\rho}_{it} \hat{\rho}_{rs}$	15.50	$\hat{\rho}_{sd} \hat{\rho}_{sq} \hat{\rho}_{as} \hat{\rho}_{it} \hat{\rho}_{rs} W_3^* W_4^* Z_5$	4.57	$W_3^* W_4^*$	8.08	$\hat{\rho}_{sd} \hat{\rho}_{sq} \hat{\rho}_{as} \hat{\rho}_{it} \hat{\rho}_{rs} W_0^* W_3^* W_4^* Z_5$	3.54
	0.80	$\hat{\rho}_{as} \hat{\rho}_{it} \hat{\rho}_{rs}$	13.11	$\hat{\rho}_{sd} \hat{\rho}_{sq} \hat{\rho}_{as} \hat{\rho}_{it} \hat{\rho}_{rs} W_3^* W_4^* Z_5$	4.28	$W_3^* W_4^*$	7.95	$\hat{\rho}_{sd} \hat{\rho}_{sq} \hat{\rho}_{as} \hat{\rho}_{it} \hat{\rho}_{rs} W_0^* W_3^* W_4^* Z_5$	3.53

5 Numerical Example

In this chapter, a numerical example is presented to illustrate a real application.

Printing machine sales companies have rental contracts of printing machine for the customers. The rental contract usually consists of a fixed monthly fee and a cost per page fee.

In this case, the maintenance service with spare parts and consumption of consumables are included in the contract fees. So, the printing machine sales company should send spare parts and consumables to the customers by their costs. The control of spare parts and consumables is crucial as it impacts on the profitability of each contract. For example, the profitability will be worse if the company continues sending consumables items more than necessary.

In this numerical example, we focus on the delivery control of toner. There are 4 types of toner: Black (K toner), Yellow (Y toner), Magenta (M toner) and Cyan (C toner). The theoretical proportions of consumption of 4 types of toner as the print coverage of each color are defined in contract. Usually Y, M and C toners are more expensive than K toner. If the proportions of Y, M and C toners' consumption against K toner's one increase, the profitability will be worse. So the ratio of K toner delivery and the other toners delivery is worth to be monitored in order to control it.

In this numerical example, we use a specific rental contract customer case of printing machine company and monitor the ratio of C and K delivery quantities (C per K) by EWMA-type control chart with the statistic $Z4^{**}$. The customer contracted six rental machines for printing activity in November 2017 and the printing machine sales company has provided toners until June 2019. The C and K toners' standard monthly delivery quantities can be estimated by the defined coverage in contract and print volume which was researched and supposed for quotation. These standard quantities can be used as averages of delivery of each toner. Let λ_{10} and λ_{20} be in-control Conditions shown in Table 12. Then the index ρ_0 which is ratio of λ_{10} and λ_{20} is obtained. Our interest is to detect the increase of the ρ_0 . EWMA constant is selected as 0.4 and UCL is set with 3,04993 by Monte Carlo simulation considering the in-control rates.

Table 12 shows the historical delivery of C and K toners, the statistic $Z4^{**}$ of each sample and the statistic EWMA built with $Z4^{**}$. Figure 4 demonstrates EWMA-type control chart with UCL (red line). As a result, EWMA on 14th month surpassed UCL and assignable case is recognized.

However, as the control chart was not implemented at the printing machine sales

Table 12 – Example conditions and results

Conditions		Results			
λ_{10} (C Toner)	6.406	Month	C toner	K toner	Z4**
λ_{20} (K Toner)	17.733	1	13	13	2.87
ρ_0 (Index)	0.361	2	3	10	-0.36
EWMA Constant	0.400	3	11	10	2.76
UCL	3.04993	4	5	0	2.74
		5	8	25	-0.40
		6	5	0	2.74
		7	3	3	1.23
		8	8	12	1.53
		9	5	16	-0.37
		10	5	2	2.30
		11	24	25	3.92
		12	21	22	3.64
		13	16	25	2.14
		14	47	60	4.81
		15	21	32	2.57
		16	23	35	2.70
		17	19	32	2.10
		18	16	25	2.14

company, it only perceived abnormality lately at 18th month when the company required to renew the contract. The company searched and recognized that the toner delivery amounts were extremely high since 11th month however in customer's stock there are no excess toners (which means they were disappeared/missed or even stolen). The company could consider a shift in the ratio of C and K quantities from 11th month because the toners might be disappeared with different proportion. The control chart detected the change in 14th month which leads a run length of 4 months. Although this is not an ideal run length, but, it is much better than the discovery of abnormality only in 18th month as the company could notice the problem four months earlier as also could minimize the loss related to the toners if it had implemented a control chart. For this extraordinary problem, one may question why not a control chart to monitor the individual toner delivery quantity (for example, only C toner delivery quantity) as the quantities increased significantly. The main reason is that it may not be really informative if this increase may come from a print volume hike (which results a better profit as it increases the revenue by cost per click fee). In this example, the control chart for the ratio of two toner delivery quantities could give a signal related to the change of proportion of consumption earlier than the company's recognition.

Figure 4 – EWMA-type Control Chart with $Z4^{**}$ for numerical example

Final Remarks

In this study the performances of the control charts Shewhart and EWMA for monitoring the ratio of two Poisson rates are evaluated by an extensive simulation study. Several combination of statistics and normal transformations are evaluated.

EWMA control chart really improves the performance as expected. So the recommendation is for its use. The choice of the statistics depends on the three inputs: the in-control Poisson rates, the shifts on the ratio and in-control ratio. In summary, a practical recommendation: when the difference between the two in-control Poisson rates is large use the statistic Z_4^{**} for small shifts on the ratio and small increases on the average rate otherwise use the statistic W_0 . If the difference between the two in-control Poisson rates is small, the statistic W_4^* could be used instead of W_0 .

In case Shewhart control chart should be used, the statistics as $\hat{\rho}_{as}$ and $\hat{\rho}_{ft}$ can be chosen for decreases on the average Poisson rate. For other cases, the best is to run a simulation study to choose the best set of statistics.

Certainly the values of ARL_1 are still high (lowest values are around 3) and more research is needed to improve the performance. Result is not ideal as only individual observations are available to build control chart. For future study, one possibility is to consider the development of CUSUM-type chart.

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Appendix

APPENDIX A – Source code of stage I

<Data set of random variables X and Y>

```

lamb <-7
mu <- 10
rho0 <- lamb/mu

tau <- 1.00
rho1 <- tau*rho0

deltax <- 1.00

lamb1 <- deltax*lamb
mu1 <- lamb1/rho1

random.lamb <- matrix(NA,nrow=2000,ncol=5000)
random.mu <- matrix(NA,nrow=2000,ncol=5000)

for (m in 1:5000){
  random.lamb[,m] <- rpois(2000,lamb1)
  random.mu[,m] <- rpois(2000,mu1)
}

```

<Data set of the statistic $\hat{\rho}_{sd}^*$ >

```

ds.poisson <- matrix(NA,nrow=2000,ncol=5000)

for (j in 1:5000){
  for (i in 1:2000){
    ds.poisson[i,j] <- random.lamb[i,j]/(random.mu[i,j]+1)
  }
}

```

<Data set of the statistic $\hat{\rho}_{sq}^*$ >

```

ds.poisson <- matrix(NA,nrow=2000,ncol=5000)

for (j in 1:5000){
for (i in 1:2000){
ds.poisson[i,j] <- sqrt(random.lamb[i,j])/sqrt(random.mu[i,j]
]+1)
}
}

```

<Data set of the statistic $\hat{\rho}_{as}$ >

```

ds.poisson <- matrix(NA,nrow=2000,ncol=5000)

for (j in 1:5000){
for (i in 1:2000){
ds.poisson[i,j] <- (2*sqrt(random.lamb[i,j]+3/8))/(2*sqrt(
random.mu[i,j]+3/8))
}
}

```

<Data set of the statistic $\hat{\rho}_{ft}$ >

```

ds.poisson <- matrix(NA,nrow=2000,ncol=5000)

for (j in 1:5000){
for (i in 1:2000){
ds.poisson[i,j] <- (sqrt(random.lamb[i,j]+1)+sqrt(random.lamb
[i,j]))/(sqrt(random.mu[i,j]+1)+sqrt(random.mu[i,j]))
}
}

```

<Data set of the statistic $\hat{\rho}_{rs}^*$ >

```

ds.poisson <- matrix(NA,nrow=2000,ncol=5000)

for (j in 1:5000){
for (i in 1:2000){
ds.poisson[i,j] <- ((random.lamb[i,j]+2*sqrt(random.lamb[i,j]
* lamb))*sqrt(1/rho0))/(random.mu[i,j]+2*sqrt(random.mu[i,j]
]*mu)+1)
}
}

```

<Data set of the statistic $\hat{\rho}_{rs}^{**}$ >

```
ds.poisson <- matrix(NA,nrow=2000,ncol=5000)

for (j in 1:5000){
  for (i in 1:2000){
    ds.poisson[i,j] <- ((3*random.lamb[i,j]*sqrt(1/rho0)))/(3*
      random.mu[i,j]+1)
  }
}
```

<Data set of the statistic W_0 >

```
ds.poisson <- matrix(NA,nrow=2000,ncol=5000)

for (j in 1:5000){
  for (i in 1:2000){
    ds.poisson[i,j] <- random.lamb[i,j]-random.mu[i,j]
  }
}
```

<Data set of the statistic W_1 >

```
ds.poisson <- matrix(NA,nrow=2000,ncol=5000)

for (j in 1:5000){
  for (i in 1:2000){
    ds.poisson[i,j] <- (random.lamb[i,j]-random.mu[i,j]*rho0)
      /sqrt(lamb+mu*(rho0^2))
  }
}
```

<Data set of the statistic W_1^{**} >

```
ds.poisson <- matrix(NA,nrow=2000,ncol=5000)

for (j in 1:5000){
  for (i in 1:2000){
    ds.poisson[i,j] <- (random.lamb[i,j]-random.mu[i,j]*rho0)
      /sqrt(random.lamb[i,j]+random.mu[i,j]*(rho0^2)+1)
  }
}
```

<Data set of the statistic W_2 >

```

ds.poisson <- matrix(NA,nrow=2000,ncol=5000)

for (j in 1:5000){
  for (i in 1:2000){
    ds.poisson[i,j] <- (random.lamb[i,j]-random.mu[i,j]*rho0)
      /sqrt((lamb+mu)*rho0)
  }
}

```

<Data set of the statistic W_2^{**} >

```

ds.poisson <- matrix(NA,nrow=2000,ncol=5000)

for (j in 1:5000){
  for (i in 1:2000){
    ds.poisson[i,j] <- (random.lamb[i,j]-random.mu[i,j]*rho0)
      /sqrt((random.lamb[i,j]+random.mu[i,j])*rho0+1)
  }
}

```

<Data set of the statistic W_3^* >

```

ds.poisson <- matrix(NA,nrow=2000,ncol=5000)

for (j in 1:5000){
  for (i in 1:2000){
    ds.poisson[i,j] <- (log((random.lamb[i,j]+1)/(random.mu[i,
      j]+1))-log(rho0))/sqrt(1/(lamb)+1/(mu))
  }
}

```

<Data set of the statistic W_3^{**} >

```

ds.poisson <- matrix(NA,nrow=2000,ncol=5000)

for (j in 1:5000){
  for (i in 1:2000){
    ds.poisson[i,j] <- (log((random.lamb[i,j]+1)/(random.mu[i,
      j]+1))-log(rho0))/sqrt(1/(random.lamb[i,j]+1)+1/(
      random.mu[i,j]+1))
  }
}

```

<Data set of the statistic W_4^* >

```
ds.poisson <- matrix(NA,nrow=2000,ncol=5000)

for (j in 1:5000){
  for (i in 1:2000){
    ds.poisson[i,j] <- (log((random.lamb[i,j]+1)/(random.mu[i,
      j]+1))-log(rho0))/sqrt((2+1/rho0+rho0)/(lamb+mu))
  }
}
```

<Data set of the statistic W_4^{**} >

```
ds.poisson <- matrix(NA,nrow=2000,ncol=5000)

for (j in 1:5000){
  for (i in 1:2000){
    ds.poisson[i,j] <- (log((random.lamb[i,j]+1)/(random.mu[i,
      j]+1))-log(rho0))/sqrt((2+1/rho0+rho0)/(random.lamb[i,
      j]+random.mu[i,j]+1))
  }
}
```

<Data set of the statistic Z_1 >

```
ds.poisson <- matrix(NA,nrow=2000,ncol=5000)

for (j in 1:5000){
  for (i in 1:2000){
    ds.poisson[i,j] <- 2*(sqrt(random.lamb[i,j])-sqrt(rho0*
      random.mu[i,j]))/sqrt(1+rho0)
  }
}
```

<Data set of the statistic Z_2 >

```
ds.poisson <- matrix(NA,nrow=2000,ncol=5000)

for (j in 1:5000){
  for (i in 1:2000){
    ds.poisson[i,j] <- 2*(sqrt(random.lamb[i,j]+3/8)-sqrt(ro0
      *(random.mi[i,j]+3/8)))/sqrt(1+ro0)
  }
}
```

<Data set of the statistic Z_3 >

```
ds.poisson <- matrix(NA,nrow=2000,ncol=5000)

for (j in 1:5000){
  for (i in 1:2000){
    ds.poisson[i,j] <- (sqrt(random.lamb[i,j]+1)+sqrt(random.
      lamb[i,j]))-ro0*(sqrt(random.mi[i,j]+1)+sqrt(random.mi[
        i,j])))/sqrt(1+ro0^2)
  }
}
```

<Data set of the statistic Z_4 >

```
ds.poisson <- matrix(NA,nrow=2000,ncol=5000)

for (j in 1:5000){
  for (i in 1:2000){
    ds.poisson[i,j] <- (random.lamb[i,j]+2*sqrt(random.lamb[i
      ,j]*lamb))-rho0*(random.mu[i,j]+2*sqrt(random.mu[i,j]*
        mu))/sqrt(4*lamb+4*(rho0^2)*mu))
  }
}
```

<Data set of the statistic Z_4^{**} >

```
ds.poisson <- matrix(NA,nrow=2000,ncol=5000)

for (j in 1:5000){
  for (i in 1:2000){
    ds.poisson[i,j] <- (3*random.lamb[i,j]-3*rho0*sqrt(random
      .mu[i,j]))/(2*sqrt(random.lamb[i,j]+(rho0^2)*random.mu
        [i,j])+1)
  }
}
```

<Data set of the statistic Z_5 >

```
ds.poisson <- matrix(NA,nrow=2000,ncol=5000)

for (j in 1:5000){
  for (i in 1:2000){
```

```

ds.poisson[i,j] <- ((random.lamb[i,j]/(random.mu[i,j]+1))-(
  lamb/mu+lamb/(1+3*mu+mu^2)))/sqrt((lamb/mu)^2*(1/(1+lamb)
  +1/(1+mu)))
}
}

```

<Data set of the statistic Z_5^* >

```

ds.poisson <- matrix(NA,nrow=2000,ncol=5000)

for (j in 1:5000){
for (i in 1:2000){
ds.poisson[i,j] <- -((1+4*random.mu[i,j]+random.mu[i,j]^2)*
  sqrt(1+random.lamb[i,j]))/((1+3*random.mu[i,j]+random.mu[i,
  j]^2)*sqrt((1+random.mu[i,j])*(random.lamb[i,j]+random.mu
  [i,j]+2)))
}
}

```

<Data set of EWMA with the statistic $\hat{\rho}_{sd}^*$ >

```

const <- 0.20

ds.ewma.zi <- matrix(NA,nrow=2000,ncol=5000)

for (j in 1:5000){

  for (i in 2:2000){
    ds.ewma.zi[1,j] <- const*ds.poisson[1,j]+(1-const)*(lamb/
      (mu+1))
    ds.ewma.zi[i,j] <- const*ds.poisson[i,j]+(1-const)*ds.
      ewma.zi[i-1,j]
  }

}

```

<Data set of EWMA with the statistic $\hat{\rho}_{sq}^*$ >

```

const <- 0.20

ds.ewma.zi <- matrix(NA,nrow=2000,ncol=5000)

for (j in 1:5000){

```

```

for (i in 2:2000){
  ds.ewma.zi[1,j] <- const*ds.poisson[1,j]+(1-const)*(sqrt(
    lamb)/sqrt(mu+1))
  ds.ewma.zi[i,j] <- const*ds.poisson[i,j]+(1-const)*ds.
    ewma.zi[i-1,j]
}
}

```

<Data set of EWMA with the statistic $\hat{\rho}_{as}$ >

```

const <- 0.20

ds.ewma.zi <- matrix(NA,nrow=2000,ncol=5000)

for (j in 1:5000){

  for (i in 2:2000){
    ds.ewma.zi[1,j] <- const*ds.poisson[1,j]+(1-const)*((2*
      sqrt(lamb+3/8))/(2*sqrt(mu+3/8)))
    ds.ewma.zi[i,j] <- const*ds.poisson[i,j]+(1-const)*ds.
      ewma.zi[i-1,j]
  }
}

```

<Data set of EWMA with the statistic $\hat{\rho}_{ft}$ >

```

const <- 0.20

ds.ewma.zi <- matrix(NA,nrow=2000,ncol=5000)

for (j in 1:5000){

  for (i in 2:2000){
    ds.ewma.zi[1,j] <- const*ds.poisson[1,j]+(1-const)*((sqrt
      (lamb+1)+sqrt(lamb))/(sqrt(mu+1)+sqrt(mu)))
    ds.ewma.zi[i,j] <- const*ds.poisson[i,j]+(1-const)*ds.
      ewma.zi[i-1,j]
  }
}

```



```
}
```

<Data set of EWMA with the statistic $\hat{\rho}_{rs}^*$ >

```
const <- 0.20
```

```
ds.ewma.zi <- matrix(NA,nrow=2000,ncol=5000)
```

```
for (j in 1:5000){
```

```
  for (i in 2:2000){
```

```
    ds.ewma.zi[1,j] <- const*ds.poisson[1,j]+(1-const)*((lamb  
      +2*sqrt(lamb*lamb))*sqrt(1/rho0))/(mu+2*sqrt(mu*mu)+1)
```

```
    ds.ewma.zi[i,j] <- const*ds.poisson[i,j]+(1-const)*ds.  
      ewma.zi[i-1,j]
```

```
  }
```

```
}
```

<Data set of EWMA with the statistic $\hat{\rho}_{rs}^{**}$ >

```
const <- 0.20
```

```
ds.ewma.zi <- matrix(NA,nrow=2000,ncol=5000)
```

```
for (j in 1:5000){
```

```
  for (i in 2:2000){
```

```
    ds.ewma.zi[1,j] <- const*ds.poisson[1,j]+(1-const)*(3*  
      lamb*sqrt(1/rho0))/(3*mu+1)
```

```
    ds.ewma.zi[i,j] <- const*ds.poisson[i,j]+(1-const)*ds.  
      ewma.zi[i-1,j]
```

```
  }
```

```
}
```

<Data set of EWMA with the statistic W_0 >

```
const <- 0.2
```

```
ds.ewma.zi <- matrix(NA,nrow=2000,ncol=5000)
```

```
for (j in 1:5000){
```

```

for (i in 2:2000){
  ds.ewma.zi[1,j] <- const*ds.poisson[1,j]+(1-const)*(lamb-
    mu)
  ds.ewma.zi[i,j] <- const*ds.poisson[i,j]+(1-const)*ds.
    ewma.zi[i-1,j]
}
}

```

<Data set of EWMA with the statistic W_1 >

```

const <- 0.20

ds.ewma.zi <- matrix(NA,nrow=2000,ncol=5000)

for (j in 1:5000){

  for (i in 2:2000){
    ds.ewma.zi[1,j] <- const*ds.poisson[1,j]+(1-const)*((lamb
      -mu*rho0)/sqrt(lamb+mu*(rho0^2)))
    ds.ewma.zi[i,j] <- const*ds.poisson[i,j]+(1-const)*ds.
      ewma.zi[i-1,j]
  }
}

```

<Data set of EWMA with the statistic W_1^{**} >

```

const <- 0.20

ds.ewma.zi <- matrix(NA,nrow=2000,ncol=5000)

for (j in 1:5000){

  for (i in 2:2000){
    ds.ewma.zi[1,j] <- const*ds.poisson[1,j]+(1-const)*((lamb
      -mu*rho0)/sqrt(lamb+mu*(rho0^2)+1))
    ds.ewma.zi[i,j] <- const*ds.poisson[i,j]+(1-const)*ds.
      ewma.zi[i-1,j]
  }
}

```

```
}
```

```
<Data set of EWMA with the statistic  $W_2$ >
```

```
<Data set of EWMA with the statistic  $W_2^{**}$ >
```

```
const <- 0.20
```

```
ds.ewma.zi <- matrix(NA,nrow=2000,ncol=5000)
```

```
for (j in 1:5000){
```

```
  for (i in 2:2000){
```

```
    ds.ewma.zi[1,j] <- const*ds.poisson[1,j]+(1-const)*((lamb
      -mu*rho0)/sqrt((lamb+mu)*rho0+1))
```

```
    ds.ewma.zi[i,j] <- const*ds.poisson[i,j]+(1-const)*ds.
      ewma.zi[i-1,j]
```

```
  }
```

```
}
```

```
<Data set of EWMA with the statistic  $W_3^*$ >
```

```
const <- 0.20
```

```
ds.ewma.zi <- matrix(NA,nrow=2000,ncol=5000)
```

```
for (j in 1:5000){
```

```
  for (i in 2:2000){
```

```
    ds.ewma.zi[1,j] <- const*ds.poisson[1,j]+(1-const)*((log
      ((lamb+1)/(mu+1))-log(rho0))/sqrt(1/(lamb)+1/(mu)))
```

```
    ds.ewma.zi[i,j] <- const*ds.poisson[i,j]+(1-const)*ds.
      ewma.zi[i-1,j]
```

```
  }
```

```
}
```

```
<Data set of EWMA with the statistic  $W_3^{**}$ >
```

```
const <- 0.20
```

```
ds.ewma.zi <- matrix(NA,nrow=2000,ncol=5000)
```

```

for (j in 1:5000){

  for (i in 2:2000){
    ds.ewma.zi[1,j] <- const*ds.poisson[1,j]+(1-const)*(log((
      lamb+1)/(mu+1))-log(rho0))/sqrt(1/(lamb+1)+1/(mu+1))
    ds.ewma.zi[i,j] <- const*ds.poisson[i,j]+(1-const)*ds.
      ewma.zi[i-1,j]
  }
}

```

<Data set of EWMA with the statistic W_4^* >

```

const <- 0.20

ds.ewma.zi <- matrix(NA,nrow=2000,ncol=5000)

for (j in 1:5000){

  for (i in 2:2000){
    ds.ewma.zi[1,j] <- const*ds.poisson[1,j]+(1-const)*((log
      ((lamb+1)/(mu+1))-log(rho0))/sqrt((2+1/rho0+rho0)/(
      lamb+mu))))
    ds.ewma.zi[i,j] <- const*ds.poisson[i,j]+(1-const)*ds.
      ewma.zi[i-1,j]
  }
}

```

<Data set of EWMA with the statistic W_4^{**} >

```

const <- 0.20

ds.ewma.zi <- matrix(NA,nrow=2000,ncol=5000)

for (j in 1:5000){

  for (i in 2:2000){
    ds.ewma.zi[1,j] <- const*ds.poisson[1,j]+(1-const)*((log
      ((lamb+1)/(mu+1))-log(rho0))/sqrt((2+1/rho0+rho0)/(
      lamb+mu+1))))
  }
}

```

```

    ds.ewma.zi[i,j] <- const*ds.poisson[i,j]+(1-const)*ds.
      ewma.zi[i-1,j]
  }
}

```

<Data set of EWMA with the statistic Z_1 >

```

const <- 0.20

ds.ewma.zi <- matrix(NA,nrow=2000,ncol=5000)

for (j in 1:5000){

  for (i in 2:2000){
    ds.ewma.zi[1,j] <- const*ds.poisson[1,j]+(1-const)*(2*(
      sqrt(lamb)-sqrt(ro0*mi))/sqrt(1+ro0))
    ds.ewma.zi[i,j] <- const*ds.poisson[i,j]+(1-const)*ds.
      ewma.zi[i-1,j]
  }
}

```

<Data set of EWMA with the statistic Z_2 >

```

const <- 0.20

ds.ewma.zi <- matrix(NA,nrow=2000,ncol=5000)

for (j in 1:5000){

  for (i in 2:2000){
    ds.ewma.zi[1,j] <- const*ds.poisson[1,j]+(1-const)*(2*(
      sqrt(lamb+3/8)-sqrt(rho0*(mu+3/8)))/sqrt(1+rho0))
    ds.ewma.zi[i,j] <- const*ds.poisson[i,j]+(1-const)*ds.
      ewma.zi[i-1,j]
  }
}

```

<Data set of EWMA with the statistic Z_3 >

```

const <- 0.20

```

```

ds.ewma.zi <- matrix(NA,nrow=2000,ncol=5000)

for (j in 1:5000){

  for (i in 2:2000){
    ds.ewma.zi[1,j] <- const*ds.poisson[1,j]+(1-const)*(sqrt(
      lamb+1)+sqrt(lamb)-ro0*(sqrt(mi+1)+sqrt(mi)))/sqrt(1+
      ro0^2)
    ds.ewma.zi[i,j] <- const*ds.poisson[i,j]+(1-const)*ds.
      ewma.zi[i-1,j]
  }

}

```

<Data set of EWMA with the statistic Z_4 >

```

const <- 0.20

ds.ewma.zi <- matrix(NA,nrow=2000,ncol=5000)

for (j in 1:5000){

  for (i in 2:2000){
    ds.ewma.zi[1,j] <- const*ds.poisson[1,j]+(1-const)*((lamb
      +2*sqrt(lamb*lamb)-rho0*(mu+2*sqrt(mu*mu)))/(sqrt(4*
      lamb+4*(rho0^2)*mu)))
    ds.ewma.zi[i,j] <- const*ds.poisson[i,j]+(1-const)*ds.
      ewma.zi[i-1,j]
  }

}

```

<Data set of EWMA with the statistic Z_4^{**} >

```

const <- 0.40

ds.ewma.zi <- matrix(NA,nrow=2000,ncol=5000)

for (j in 1:5000){

  for (i in 2:2000){

```

```

    ds.ewma.zi[1,j] <- const*ds.poisson[1,j]+(1-const)*(3*
      lamb-3*rho0*sqrt(mu))/(2*sqrt(lamb+(rho0^2)*mu)+1)
    ds.ewma.zi[i,j] <- const*ds.poisson[i,j]+(1-const)*ds.
      ewma.zi[i-1,j]
  }
}

<Data set of EWMA with the statistic  $Z_5$ >
const <- 0.20

ds.ewma.zi <- matrix(NA,nrow=2000,ncol=5000)

for (j in 1:5000){

  for (i in 2:2000){
    ds.ewma.zi[1,j] <- const*ds.poisson[1,j]+(1-const)*((lamb
      /(mu+1))-(lamb/mu+lamb/(1+3*mu+mu^2)))/sqrt((lamb/mu)
      ^2*(1/(1+lamb)+1/(1+mu)))
    ds.ewma.zi[i,j] <- const*ds.poisson[i,j]+(1-const)*ds.
      ewma.zi[i-1,j]
  }
}

<Data set of EWMA with the statistic  $Z_5^*$ >
const <- 0.20

ds.ewma.zi <- matrix(NA,nrow=2000,ncol=5000)

for (j in 1:5000){

  for (i in 2:2000){
    ds.ewma.zi[1,j] <- const*ds.poisson[1,j]+(1-const)*-((1+4
      *mu+mu^2)*sqrt(1+lamb))/((1+3*mu+mu^2)*sqrt((1+mu)*(
      lamb+mu+2)))
    ds.ewma.zi[i,j] <- const*ds.poisson[i,j]+(1-const)*ds.
      ewma.zi[i-1,j]
  }
}

```

```
}

    <ARL0 Estimation>
UCL <- 0.9

repeat{

    UCL <- UCL+0.00001

    RL.TI <- NULL

    for (j in 1:5000){

        TI1 <- NULL

        for (i in 1:2000){
            TI1[i] <- ds.poisson[i,j]
            if(TI1[i]>UCL){break}
        }

        RL.TI[j] <- length(TI1)

    }

    ARL <- mean(RL.TI)

    if (ARL>170){break}
}
```


APPENDIX B – Source code of stage II

<ARL1 Estimation for Shewhart control chart in case λ_{10} and λ_{20} are known>

```

Parameter.tau <- c(1.02,1.05,1.1,1.2,1.3,1.4,1.5)
Parameter.deltax <- c(1.2,1.1,1,0.9,0.8)

lamb <- 16
mu <- 40
rho0 <- lamb/mu

UCLset <- c(0.77778, 0.88192, 0.89932, 0.89973, 1.00336,
            3.19343, -5.99999, 2.61998, 2.61998, 2.39568, 2.39568,
            2.44959, 2.47765, 5.19609, 2.54406)

ARL1.sd <- matrix(NA,nrow=5,ncol=7)
ARL1.ans <- matrix(NA,nrow=5,ncol=7)
ARL1.dif <- matrix(NA,nrow=5,ncol=7)
ARL1.FT <- matrix(NA,nrow=5,ncol=7)
ARL1.Rossi <- matrix(NA,nrow=5,ncol=7)
ARL1.SQR <- matrix(NA,nrow=5,ncol=7)
ARL1.VAR <- matrix(NA,nrow=5,ncol=7)
ARL1.W <- matrix(NA,nrow=5,ncol=7)
ARL1.W.log <- matrix(NA,nrow=5,ncol=7)
ARL1.W2 <- matrix(NA,nrow=5,ncol=7)
ARL1.W2.log <- matrix(NA,nrow=5,ncol=7)
ARL1.Z1 <- matrix(NA,nrow=5,ncol=7)
ARL1.Z2 <- matrix(NA,nrow=5,ncol=7)
ARL1.Z3 <- matrix(NA,nrow=5,ncol=7)
ARL1.Z4 <- matrix(NA,nrow=5,ncol=7)

UCL.sd <- UCLset[1]
UCL.SQR <- UCLset[2]
UCL.ans <- UCLset[3]
UCL.FT <- UCLset[4]

```

```

UCL.Rossi <- UCLset[5]
UCL.VAR <- UCLset[6]
UCL.dif <- UCLset[7]
UCL.W <- UCLset[8]
UCL.W2 <- UCLset[9]
UCL.W.log <- UCLset[10]
UCL.W2.log <- UCLset[11]
UCL.Z1 <- UCLset[12]
UCL.Z2 <- UCLset[13]
UCL.Z3 <- UCLset[14]
UCL.Z4 <- UCLset[15]

for (i in 1:7) {

  tau <- Parameter.tau[i]

  for (j in 1:5){

    deltax <- Parameter.deltax[j]

    rho1 <- tau*rho0

    lamb1 <- deltax*lamb
    mu1 <- lamb1/rho1

    random.lamb <- matrix(NA,nrow=2000,ncol=5000)
    random.mu <- matrix(NA,nrow=2000,ncol=5000)

    for (m in 1:5000){
      random.lamb[,m] <- rpois(2000,lamb1)
      random.mu[,m] <- rpois(2000,mu1)
    }

    ds.poisson <- matrix(NA,nrow=2000,ncol=5000)

    for (n in 1:5000){
      for (m in 1:2000){
        ds.poisson[m,n] <- random.lamb[m,n]/(random.mu[m,n
          ]+1)
      }
    }
  }
}

```

```

    }
  }

  ds.poisson.ans <- matrix(NA,nrow=2000,ncol=5000)

  for (n in 1:5000){
    for (m in 1:2000){
      ds.poisson.ans[m,n] <- (2*sqrt(random.lamb[m,n]+3/8))
        /(2*sqrt(random.mu[m,n]+3/8))
    }
  }

  ds.poisson.FT <- matrix(NA,nrow=2000,ncol=5000)

  for (n in 1:5000){
    for (m in 1:2000){
      ds.poisson.FT[m,n] <- (sqrt(random.lamb[m,n]+1)+sqrt(
        random.lamb[m,n]))/(sqrt(random.mu[m,n]+1)+sqrt(
        random.mu[m,n]))
    }
  }

  ds.poisson.Rossi <- matrix(NA,nrow=2000,ncol=5000)

  for (p in 1:5000){
    for (o in 1:2000){
      ds.poisson.Rossi[o,p] <- ((random.lamb[o,p]+2*sqrt(
        random.lamb[o,p]*lamb))*sqrt(1/rho0))/(random.mu[o
        ,p]+2*sqrt(random.mu[o,p]*mu)+1)
    }
  }

  ds.poisson.SQR <- matrix(NA,nrow=2000,ncol=5000)

  for (r in 1:5000){
    for (q in 1:2000){
      ds.poisson.SQR[q,r] <- sqrt(random.lamb[q,r])/sqrt(
        random.mu[q,r]+1)
    }
  }

```

```

}

ds.poisson.VAR <- matrix(NA,nrow=2000,ncol=5000)

for (v in 1:5000){
  for (u in 1:2000){
    ds.poisson.VAR[u,v] <- ((random.lamb[u,v]/(random.mu[
      u,v]+1))-(lamb/mu+lamb/(1+3*mu+mu^2)))/sqrt((lamb/
      mu)^2*(1/(1+lamb)+1/(1+mu)))
  }
}

ds.poisson.dif <- matrix(NA,nrow=2000,ncol=5000)

for (n in 1:5000){
  for (m in 1:2000){
    ds.poisson.dif[m,n] <- random.lamb[m,n]-random.mu[m,n]
  }
}

W <- matrix(NA,nrow=2000,ncol=5000)

for (n in 1:5000){
  for (m in 1:2000){
    W[m,n] <- (random.lamb[m,n]-random.mu[m,n]*rho0)/sqrt
      (lamb+mu*(rho0^2))
  }
}

W2 <- matrix(NA,nrow=2000,ncol=5000)

for (n in 1:5000){
  for (m in 1:2000){
    W2[m,n] <- (random.lamb[m,n]-random.mu[m,n]*rho0)/
      sqrt((lamb+mu)*rho0)
  }
}

```

```

W.log <- matrix(NA,nrow=2000,ncol=5000)

for (n in 1:5000){
  for (m in 1:2000){
    W.log[m,n] <- (log((random.lamb[m,n]+1)/(random.mu[m,
      n]+1))-log(rho0))/sqrt(1/(lamb)+1/(mu))
  }
}

W2.log <- matrix(NA,nrow=2000,ncol=5000)

for (n in 1:5000){
  for (m in 1:2000){
    W2.log[m,n] <- (log((random.lamb[m,n]+1)/(random.mu[m,
      n]+1))-log(rho0))/sqrt((2+1/rho0+rho0)/(lamb+mu))
  }
}

Z1 <- matrix(NA,nrow=2000,ncol=5000)

for (n in 1:5000){
  for (m in 1:2000){
    Z1[m,n] <- 2*(sqrt(random.lamb[m,n])-sqrt(rho0*random
      .mu[m,n]))/sqrt(1+rho0)
  }
}

Z2 <- matrix(NA,nrow=2000,ncol=5000)

for (n in 1:5000){
  for (m in 1:2000){
    Z2[m,n] <- 2*(sqrt(random.lamb[m,n]+3/8)-sqrt(rho0*(
      random.mu[m,n]+3/8)))/sqrt(1+rho0)
  }
}

Z3 <- matrix(NA,nrow=2000,ncol=5000)

for (n in 1:5000){

```

```

    for (m in 1:2000){
      Z3[m,n] <- (sqrt(random.lamb[m,n]+1)+sqrt(random.lamb
        [m,n]))-rho0*(sqrt(random.mu[m,n]+1)+sqrt(random.mu
        [m,n])))/sqrt(1+rho0^2)
    }
  }

Z4 <- matrix(NA,nrow=2000,ncol=5000)

for (n in 1:5000){
  for (m in 1:2000){
    Z4[m,n] <- (random.lamb[m,n]+2*sqrt(random.lamb[m,n]*
      lamb)-rho0*(random.mu[m,n]+2*sqrt(random.mu[m,n]*
      mu)))/(sqrt(4*lamb+4*(rho0^2)*mu))
  }
}

RL.TI <- NULL

for (n in 1:5000){

  TI <- NULL

  for (m in 1:2000){
    TI[m] <- ds.poisson[m,n]
    if(TI[m]>UCL.sd){break}
  }

  RL.TI[n] <- length(TI)

}

ARL1.sd[j,i] <- mean(RL.TI)

RL.TI <- NULL

for (n in 1:5000){

```

```
TI <- NULL

for (m in 1:2000){
  TI[m] <- ds.poisson.ans[m,n]
  if(TI[m]>UCL.ans){break}
}

RL.TI[n] <- length(TI)

}

ARL1.ans[j,i] <- mean(RL.TI)

RL.TI <- NULL

for (n in 1:5000){

  TI <- NULL

  for (m in 1:2000){
    TI[m] <- ds.poisson.dif[m,n]
    if(TI[m]>UCL.dif){break}
  }

  RL.TI[n] <- length(TI)

}

ARL1.dif[j,i] <- mean(RL.TI)

RL.TI <- NULL

for (n in 1:5000){

  TI <- NULL

  for (m in 1:2000){
    TI[m] <- ds.poisson.FT[m,n]
    if(TI[m]>UCL.FT){break}
```

```
}

RL.TI[n] <- length(TI)

}

ARL1.FT[j,i] <- mean(RL.TI)

RL.TI <- NULL

for (n in 1:5000){

  TI <- NULL

  for (m in 1:2000){
    TI[m] <- ds.poisson.Rossi[m,n]
    if(TI[m]>UCL.Rossi){break}
  }

  RL.TI[n] <- length(TI)

}

ARL1.Rossi[j,i] <- mean(RL.TI)

RL.TI <- NULL

for (n in 1:5000){

  TI <- NULL

  for (m in 1:2000){
    TI[m] <- ds.poisson.SQR[m,n]
    if(TI[m]>UCL.SQR){break}
  }

  RL.TI[n] <- length(TI)

}
```

```
ARL1.SQR[j,i] <- mean(RL.TI)

RL.TI <- NULL

for (n in 1:5000){

  TI <- NULL

  for (m in 1:2000){
    TI[m] <- ds.poisson.VAR[m,n]
    if(TI[m]>UCL.VAR){break}
  }

  RL.TI[n] <- length(TI)

}

ARL1.VAR[j,i] <- mean(RL.TI)

RL.TI <- NULL

for (n in 1:5000){

  TI <- NULL

  for (m in 1:2000){
    TI[m] <- W[m,n]
    if(TI[m]>UCL.W){break}
  }

  RL.TI[n] <- length(TI)

}

ARL1.W[j,i] <- mean(RL.TI)

RL.TI <- NULL
```

```

for (n in 1:5000){

  TI <- NULL

  for (m in 1:2000){
    TI[m] <- W2[m,n]
    if(TI[m]>UCL.W2){break}
  }

  RL.TI[n] <- length(TI)

}

ARL1.W2[j,i] <- mean(RL.TI)

RL.TI <- NULL

for (n in 1:5000){

  TI <- NULL

  for (m in 1:2000){
    TI[m] <- W.log[m,n]
    if(TI[m]>UCL.W.log){break}
  }

  RL.TI[n] <- length(TI)

}

ARL1.W.log[j,i] <- mean(RL.TI)

RL.TI <- NULL

for (n in 1:5000){

  TI <- NULL

  for (m in 1:2000){

```

```

        TI[m] <- W2.log[m,n]
        if(TI[m]>UCL.W2.log){break}
    }

    RL.TI[n] <- length(TI)

}

ARL1.W2.log[j,i] <- mean(RL.TI)

RL.TI <- NULL

for (n in 1:5000){

    TI <- NULL

    for (m in 1:2000){
        TI[m] <- Z1[m,n]
        if(TI[m]>UCL.Z1){break}
    }

    RL.TI[n] <- length(TI)

}

ARL1.Z1[j,i] <- mean(RL.TI)

RL.TI <- NULL

for (n in 1:5000){

    TI <- NULL

    for (m in 1:2000){
        TI[m] <- Z2[m,n]
        if(TI[m]>UCL.Z2){break}
    }

    RL.TI[n] <- length(TI)

```

```

}

ARL1.Z2[j,i] <- mean(RL.TI)

RL.TI <- NULL

for (n in 1:5000){

  TI <- NULL

  for (m in 1:2000){
    TI[m] <- Z3[m,n]
    if(TI[m]>UCL.Z3){break}
  }

  RL.TI[n] <- length(TI)

}

ARL1.Z3[j,i] <- mean(RL.TI)

RL.TI <- NULL

for (n in 1:5000){

  TI <- NULL

  for (m in 1:2000){
    TI[m] <- Z4[m,n]
    if(TI[m]>UCL.Z4){break}
  }

  RL.TI[n] <- length(TI)

}

ARL1.Z4[j,i] <- mean(RL.TI)

```

```

    }
}

ARL1.sd.ajt <- c(ARL1.sd[,1],ARL1.sd[,2],ARL1.sd[,3],ARL1.sd
[,4],ARL1.sd[,5],ARL1.sd[,6],ARL1.sd[,7])
ARL1.ans.ajt <- c(ARL1.ans[,1],ARL1.ans[,2],ARL1.ans[,3],ARL1
.ans[,4],ARL1.ans[,5],ARL1.ans[,6],ARL1.ans[,7])
ARL1.dif.ajt <- c(ARL1.dif[,1],ARL1.dif[,2],ARL1.dif[,3],ARL1
.dif[,4],ARL1.dif[,5],ARL1.dif[,6],ARL1.dif[,7])
ARL1.FT.ajt <- c(ARL1.FT[,1],ARL1.FT[,2],ARL1.FT[,3],ARL1.FT
[,4],ARL1.FT[,5],ARL1.FT[,6],ARL1.FT[,7])
ARL1.Rossi.ajt <- c(ARL1.Rossi[,1],ARL1.Rossi[,2],ARL1.Rossi
[,3],ARL1.Rossi[,4],ARL1.Rossi[,5],ARL1.Rossi[,6],ARL1.
Rossi[,7])
ARL1.SQR.ajt <- c(ARL1.SQR[,1],ARL1.SQR[,2],ARL1.SQR[,3],ARL1
.SQR[,4],ARL1.SQR[,5],ARL1.SQR[,6],ARL1.SQR[,7])
ARL1.VAR.ajt <- c(ARL1.VAR[,1],ARL1.VAR[,2],ARL1.VAR[,3],ARL1
.VAR[,4],ARL1.VAR[,5],ARL1.VAR[,6],ARL1.VAR[,7])
ARL1.W.ajt <- c(ARL1.W[,1],ARL1.W[,2],ARL1.W[,3],ARL1.W[,4],
ARL1.W[,5],ARL1.W[,6],ARL1.W[,7])
ARL1.W2.ajt <- c(ARL1.W2[,1],ARL1.W2[,2],ARL1.W2[,3],ARL1.W2
[,4],ARL1.W2[,5],ARL1.W2[,6],ARL1.W2[,7])
ARL1.W.log.ajt <- c(ARL1.W.log[,1],ARL1.W.log[,2],ARL1.W.log
[,3],ARL1.W.log[,4],ARL1.W.log[,5],ARL1.W.log[,6],ARL1.W.
log[,7])
ARL1.W2.log.ajt <- c(ARL1.W2.log[,1],ARL1.W2.log[,2],ARL1.W2.
log[,3],ARL1.W2.log[,4],ARL1.W2.log[,5],ARL1.W2.log[,6],
ARL1.W2.log[,7])
ARL1.Z1.ajt <- c(ARL1.Z1[,1],ARL1.Z1[,2],ARL1.Z1[,3],ARL1.Z1
[,4],ARL1.Z1[,5],ARL1.Z1[,6],ARL1.Z1[,7])
ARL1.Z2.ajt <- c(ARL1.Z2[,1],ARL1.Z2[,2],ARL1.Z2[,3],ARL1.Z2
[,4],ARL1.Z2[,5],ARL1.Z2[,6],ARL1.Z2[,7])
ARL1.Z3.ajt <- c(ARL1.Z3[,1],ARL1.Z3[,2],ARL1.Z3[,3],ARL1.Z3
[,4],ARL1.Z3[,5],ARL1.Z3[,6],ARL1.Z3[,7])
ARL1.Z4.ajt <- c(ARL1.Z4[,1],ARL1.Z4[,2],ARL1.Z4[,3],ARL1.Z4
[,4],ARL1.Z4[,5],ARL1.Z4[,6],ARL1.Z4[,7])

Result <- matrix(c(ARL1.sd.ajt,ARL1.SQR.ajt,ARL1.ans.ajt,ARL1
.FT.ajt,ARL1.Rossi.ajt,ARL1.VAR.ajt,ARL1.dif.ajt,ARL1.W.

```

```
ajt,ARL1.W2.ajt,ARL1.W.log.ajt,ARL1.W2.log.ajt,ARL1.Z1.ajt
,ARL1.Z2.ajt,ARL1.Z3.ajt,ARL1.Z4.ajt),nrow=35,ncol=15,
byrow=FALSE)
```

<ARL1 Estimation for Shewhart control chart in case λ_{10} and λ_{20} are unknown>

```
Parameter.tau <- c(1.02,1.05,1.1,1.2,1.3,1.4,1.5)
Parameter.deltax <- c(1.2,1.1,1,0.9,0.8)

lamb <- 16
mi <- 40
ro0 <- lamb/mi

UCLset <- c(1.25926, 1.12217, 1.13995, 1.13927, 1.26091,
-0.05629, 9.00001, 2.22584, 2.5508, 2.6347, 2.37087,
2.52739, 2.52552, 3.91676, 5.86602)

ARL1.sd <- matrix(NA,nrow=5,ncol=7)
ARL1.ans <- matrix(NA,nrow=5,ncol=7)
ARL1.dif <- matrix(NA,nrow=5,ncol=7)
ARL1.FT <- matrix(NA,nrow=5,ncol=7)
ARL1.Rossi <- matrix(NA,nrow=5,ncol=7)
ARL1.SQR <- matrix(NA,nrow=5,ncol=7)
ARL1.VAR <- matrix(NA,nrow=5,ncol=7)
ARL1.W <- matrix(NA,nrow=5,ncol=7)
ARL1.W.log <- matrix(NA,nrow=5,ncol=7)
ARL1.W2 <- matrix(NA,nrow=5,ncol=7)
ARL1.W2.log <- matrix(NA,nrow=5,ncol=7)
ARL1.Z1 <- matrix(NA,nrow=5,ncol=7)
ARL1.Z2 <- matrix(NA,nrow=5,ncol=7)
ARL1.Z3 <- matrix(NA,nrow=5,ncol=7)
ARL1.Z4 <- matrix(NA,nrow=5,ncol=7)

UCL.sd <- UCLset[1]
UCL.SQR <- UCLset[2]
UCL.ans <- UCLset[3]
UCL.FT <- UCLset[4]
UCL.Rossi <- UCLset[5]
UCL.VAR <- UCLset[6]
```

```

UCL.dif <- UCLset[7]
UCL.W <- UCLset[8]
UCL.W2 <- UCLset[9]
UCL.W.log <- UCLset[10]
UCL.W2.log <- UCLset[11]
UCL.Z1 <- UCLset[12]
UCL.Z2 <- UCLset[13]
UCL.Z3 <- UCLset[14]
UCL.Z4 <- UCLset[15]

for (i in 1:7) {

  tau <- Parameter.tau[i]

  for (j in 1:5){

    deltax <- Parameter.deltax[j]

    ro1 <- tau*ro0

    lamb1 <- deltax*lamb
    mi1 <- lamb1/ro1

    random.lamb <- matrix(NA,nrow=2000,ncol=5000)
    random.mi <- matrix(NA,nrow=2000,ncol=5000)

    for (m in 1:5000){
      random.lamb[,m] <- rpois(2000,lamb1)
      random.mi[,m] <- rpois(2000,mi1)
    }

    ds.poisson <- matrix(NA,nrow=2000,ncol=5000)

    for (n in 1:5000){
      for (m in 1:2000){
        ds.poisson[m,n] <- random.lamb[m,n]/(random.mi[m,n]
          ]+1)
      }
    }
  }
}

```

```

ds.poisson.ans <- matrix(NA,nrow=2000,ncol=5000)

for (n in 1:5000){
  for (m in 1:2000){
    ds.poisson.ans[m,n] <- (2*sqrt(random.lamb[m,n]+3/8))
      /(2*sqrt(random.mi[m,n]+3/8))
  }
}

ds.poisson.FT <- matrix(NA,nrow=2000,ncol=5000)

for (n in 1:5000){
  for (m in 1:2000){
    ds.poisson.FT[m,n] <- (sqrt(random.lamb[m,n]+1)+sqrt(
      random.lamb[m,n]))/(sqrt(random.mi[m,n]+1)+sqrt(
      random.mi[m,n]))
  }
}

ds.poisson.Rossi <- matrix(NA,nrow=2000,ncol=5000)

for (p in 1:5000){
  for (o in 1:2000){
    ds.poisson.Rossi[o,p] <- ((3*random.lamb[o,p]*sqrt(1/
      ro0)))/(3*random.mi[o,p]+1)
  }
}

ds.poisson.SQR <- matrix(NA,nrow=2000,ncol=5000)

for (r in 1:5000){
  for (q in 1:2000){
    ds.poisson.SQR[q,r] <- sqrt(random.lamb[q,r])/sqrt(
      random.mi[q,r]+1)
  }
}

ds.poisson.VAR <- matrix(NA,nrow=2000,ncol=5000)

```

```

for (v in 1:5000){
  for (u in 1:2000){
    ds.poisson.VAR[u,v] <- -(((1+4*random.mi[u,v]+random.
      mi[u,v]^2)*sqrt(1+random.lamb[u,v]))/((1+3*random.
      mi[u,v]+random.mi[u,v]^2)*sqrt((1+random.mi[u,v])*
      (random.lamb[u,v]+random.mi[u,v]+2))))
  }
}

ds.poisson.dif <- matrix(NA,nrow=2000,ncol=5000)

for (n in 1:5000){
  for (m in 1:2000){
    ds.poisson.dif[m,n] <- random.lamb[m,n]-random.mi[m,n]
  }
}

W <- matrix(NA,nrow=2000,ncol=5000)

for (n in 1:5000){
  for (m in 1:2000){
    W[m,n] <- (random.lamb[m,n]-random.mi[m,n]*ro0)/sqrt(
      random.lamb[m,n]+random.mi[m,n]*(ro0^2)+1)
  }
}

W2 <- matrix(NA,nrow=2000,ncol=5000)

for (n in 1:5000){
  for (m in 1:2000){
    W2[m,n] <- (random.lamb[m,n]-random.mi[m,n]*ro0)/sqrt
      ((random.lamb[m,n]+random.mi[m,n])*ro0+1)
  }
}

W.log <- matrix(NA,nrow=2000,ncol=5000)

```

```

for (n in 1:5000){
  for (m in 1:2000){
    W.log[m,n] <- (log((random.lamb[m,n]+1)/(random.mi[m,
      n]+1))-log(ro0))/sqrt(1/(random.lamb[m,n]+1)+1/(
        random.mi[m,n]+1))
  }
}

W2.log <- matrix(NA,nrow=2000,ncol=5000)

for (n in 1:5000){
  for (m in 1:2000){
    W2.log[m,n] <- (log((random.lamb[m,n]+1)/(random.mi[m,
      n]+1))-log(ro0))/sqrt((2+1/ro0+ro0)/(random.lamb[
        m,n]+random.mi[m,n]+1))
  }
}

Z1 <- matrix(NA,nrow=2000,ncol=5000)

for (n in 1:5000){
  for (m in 1:2000){
    Z1[m,n] <- 2*(sqrt(random.lamb[m,n])-sqrt(ro0*random.
      mi[m,n]))/sqrt(1+ro0)
  }
}

Z2 <- matrix(NA,nrow=2000,ncol=5000)

for (n in 1:5000){
  for (m in 1:2000){
    Z2[m,n] <- 2*(sqrt(random.lamb[m,n]+3/8)-sqrt(ro0*(
      random.mi[m,n]+3/8)))/sqrt(1+ro0)
  }
}

Z3 <- matrix(NA,nrow=2000,ncol=5000)

for (n in 1:5000){

```

```

    for (m in 1:2000){
      Z3[m,n] <- (sqrt(random.lamb[m,n]+1)+sqrt(random.lamb
        [m,n])-ro0*(sqrt(random.mi[m,n]+1)+sqrt(random.mi[
          m,n])))/sqrt(1+ro0^2)
    }
  }

Z4 <- matrix(NA,nrow=2000,ncol=5000)

for (n in 1:5000){
  for (m in 1:2000){
    Z4[m,n] <- (3*random.lamb[m,n]-3*ro0*sqrt(random.mi[m,
      n]))/(2*sqrt(random.lamb[m,n]+(ro0^2)*random.mi[m,
        n])+1)
  }
}

RL.TI <- NULL

for (n in 1:5000){

  TI <- NULL

  for (m in 1:2000){
    TI[m] <- ds.poisson[m,n]
    if(TI[m]>UCL.sd){break}
  }

  RL.TI[n] <- length(TI)

}

ARL1.sd[j,i] <- mean(RL.TI)

RL.TI <- NULL

for (n in 1:5000){

  TI <- NULL

```

```

    for (m in 1:2000){
      TI[m] <- ds.poisson.ans[m,n]
      if(TI[m]>UCL.ans){break}
    }

    RL.TI[n] <- length(TI)

  }

  ARL1.ans[j,i] <- mean(RL.TI)

  RL.TI <- NULL

  for (n in 1:5000){

    TI <- NULL

    for (m in 1:2000){
      TI[m] <- ds.poisson.dif[m,n]
      if(TI[m]>UCL.dif){break}
    }

    RL.TI[n] <- length(TI)

  }

  ARL1.dif[j,i] <- mean(RL.TI)

  RL.TI <- NULL

  for (n in 1:5000){

    TI <- NULL

    for (m in 1:2000){
      TI[m] <- ds.poisson.FT[m,n]
      if(TI[m]>UCL.FT){break}
    }
  }

```

```
    RL.TI[n] <- length(TI)

}

ARL1.FT[j,i] <- mean(RL.TI)

RL.TI <- NULL

for (n in 1:5000){

    TI <- NULL

    for (m in 1:2000){
        TI[m] <- ds.poisson.Rossi[m,n]
        if(TI[m]>UCL.Rossi){break}
    }

    RL.TI[n] <- length(TI)

}

ARL1.Rossi[j,i] <- mean(RL.TI)

RL.TI <- NULL

for (n in 1:5000){

    TI <- NULL

    for (m in 1:2000){
        TI[m] <- ds.poisson.SQR[m,n]
        if(TI[m]>UCL.SQR){break}
    }

    RL.TI[n] <- length(TI)

}
```

```
ARL1.SQR[j,i] <- mean(RL.TI)

RL.TI <- NULL

for (n in 1:5000){

  TI <- NULL

  for (m in 1:2000){
    TI[m] <- ds.poisson.VAR[m,n]
    if(TI[m]>UCL.VAR){break}
  }

  RL.TI[n] <- length(TI)

}

ARL1.VAR[j,i] <- mean(RL.TI)

RL.TI <- NULL

for (n in 1:5000){

  TI <- NULL

  for (m in 1:2000){
    TI[m] <- W[m,n]
    if(TI[m]>UCL.W){break}
  }

  RL.TI[n] <- length(TI)

}

ARL1.W[j,i] <- mean(RL.TI)

RL.TI <- NULL

for (n in 1:5000){
```

```
TI <- NULL

for (m in 1:2000){
  TI[m] <- W2[m,n]
  if(TI[m]>UCL.W2){break}
}

RL.TI[n] <- length(TI)

}

ARL1.W2[j,i] <- mean(RL.TI)

RL.TI <- NULL

for (n in 1:5000){

  TI <- NULL

  for (m in 1:2000){
    TI[m] <- W.log[m,n]
    if(TI[m]>UCL.W.log){break}
  }

  RL.TI[n] <- length(TI)

}

ARL1.W.log[j,i] <- mean(RL.TI)

RL.TI <- NULL

for (n in 1:5000){

  TI <- NULL

  for (m in 1:2000){
    TI[m] <- W2.log[m,n]
```

```

        if(TI[m]>UCL.W2.log){break}
    }

    RL.TI[n] <- length(TI)

}

ARL1.W2.log[j,i] <- mean(RL.TI)

RL.TI <- NULL

for (n in 1:5000){

    TI <- NULL

    for (m in 1:2000){
        TI[m] <- Z1[m,n]
        if(TI[m]>UCL.Z1){break}
    }

    RL.TI[n] <- length(TI)

}

ARL1.Z1[j,i] <- mean(RL.TI)

RL.TI <- NULL

for (n in 1:5000){

    TI <- NULL

    for (m in 1:2000){
        TI[m] <- Z2[m,n]
        if(TI[m]>UCL.Z2){break}
    }

    RL.TI[n] <- length(TI)

```

```
}

ARL1.Z2[j,i] <- mean(RL.TI)

RL.TI <- NULL

for (n in 1:5000){

  TI <- NULL

  for (m in 1:2000){
    TI[m] <- Z3[m,n]
    if(TI[m]>UCL.Z3){break}
  }

  RL.TI[n] <- length(TI)

}

ARL1.Z3[j,i] <- mean(RL.TI)

RL.TI <- NULL

for (n in 1:5000){

  TI <- NULL

  for (m in 1:2000){
    TI[m] <- Z4[m,n]
    if(TI[m]>UCL.Z4){break}
  }

  RL.TI[n] <- length(TI)

}

ARL1.Z4[j,i] <- mean(RL.TI)

}
```

```
}

```

```

ARL1.sd.ajt <- c(ARL1.sd[,1], ARL1.sd[,2], ARL1.sd[,3], ARL1.sd
[,4], ARL1.sd[,5], ARL1.sd[,6], ARL1.sd[,7])
ARL1.ans.ajt <- c(ARL1.ans[,1], ARL1.ans[,2], ARL1.ans[,3], ARL1
.ans[,4], ARL1.ans[,5], ARL1.ans[,6], ARL1.ans[,7])
ARL1.dif.ajt <- c(ARL1.dif[,1], ARL1.dif[,2], ARL1.dif[,3], ARL1
.dif[,4], ARL1.dif[,5], ARL1.dif[,6], ARL1.dif[,7])
ARL1.FT.ajt <- c(ARL1.FT[,1], ARL1.FT[,2], ARL1.FT[,3], ARL1.FT
[,4], ARL1.FT[,5], ARL1.FT[,6], ARL1.FT[,7])
ARL1.Rossi.ajt <- c(ARL1.Rossi[,1], ARL1.Rossi[,2], ARL1.Rossi
[,3], ARL1.Rossi[,4], ARL1.Rossi[,5], ARL1.Rossi[,6], ARL1.
Rossi[,7])
ARL1.SQR.ajt <- c(ARL1.SQR[,1], ARL1.SQR[,2], ARL1.SQR[,3], ARL1
.SQR[,4], ARL1.SQR[,5], ARL1.SQR[,6], ARL1.SQR[,7])
ARL1.VAR.ajt <- c(ARL1.VAR[,1], ARL1.VAR[,2], ARL1.VAR[,3], ARL1
.VAR[,4], ARL1.VAR[,5], ARL1.VAR[,6], ARL1.VAR[,7])
ARL1.W.ajt <- c(ARL1.W[,1], ARL1.W[,2], ARL1.W[,3], ARL1.W[,4],
ARL1.W[,5], ARL1.W[,6], ARL1.W[,7])
ARL1.W2.ajt <- c(ARL1.W2[,1], ARL1.W2[,2], ARL1.W2[,3], ARL1.W2
[,4], ARL1.W2[,5], ARL1.W2[,6], ARL1.W2[,7])
ARL1.W.log.ajt <- c(ARL1.W.log[,1], ARL1.W.log[,2], ARL1.W.log
[,3], ARL1.W.log[,4], ARL1.W.log[,5], ARL1.W.log[,6], ARL1.W.
log[,7])
ARL1.W2.log.ajt <- c(ARL1.W2.log[,1], ARL1.W2.log[,2], ARL1.W2.
log[,3], ARL1.W2.log[,4], ARL1.W2.log[,5], ARL1.W2.log[,6],
ARL1.W2.log[,7])
ARL1.Z1.ajt <- c(ARL1.Z1[,1], ARL1.Z1[,2], ARL1.Z1[,3], ARL1.Z1
[,4], ARL1.Z1[,5], ARL1.Z1[,6], ARL1.Z1[,7])
ARL1.Z2.ajt <- c(ARL1.Z2[,1], ARL1.Z2[,2], ARL1.Z2[,3], ARL1.Z2
[,4], ARL1.Z2[,5], ARL1.Z2[,6], ARL1.Z2[,7])
ARL1.Z3.ajt <- c(ARL1.Z3[,1], ARL1.Z3[,2], ARL1.Z3[,3], ARL1.Z3
[,4], ARL1.Z3[,5], ARL1.Z3[,6], ARL1.Z3[,7])
ARL1.Z4.ajt <- c(ARL1.Z4[,1], ARL1.Z4[,2], ARL1.Z4[,3], ARL1.Z4
[,4], ARL1.Z4[,5], ARL1.Z4[,6], ARL1.Z4[,7])

Result <- matrix(c(ARL1.sd.ajt, ARL1.SQR.ajt, ARL1.ans.ajt, ARL1
.FT.ajt, ARL1.Rossi.ajt, ARL1.VAR.ajt, ARL1.dif.ajt, ARL1.W.
ajt, ARL1.W2.ajt, ARL1.W.log.ajt, ARL1.W2.log.ajt, ARL1.Z1.ajt

```

```
,ARL1.Z2.ajt,ARL1.Z3.ajt,ARL1.Z4.ajt),nrow=35,ncol=15,
byrow=FALSE)
```

<ARL1 Estimation for EWMA-type control chart in case λ_{10} and λ_{20} are known>

```
Parameter.tau <- c(1.02,1.05,1.1,1.2,1.3,1.4,1.5)
Parameter.deltax <- c(1.2,1.1,1,0.9,0.8)

lamb <- 16
mu <- 40
rho0 <- lamb/mu

const <- 0.20

UCLset.EW <- c(0.5001, 0.69907, 0.71246, 0.71358, 0.73416,
              0.78689, -18.37338, 0.77053, 0.77053, 0.78903, 0.78903,
              0.72448, 0.76644, 3.51851, 0.74677)

ARL1.sd.EW <- matrix(NA,nrow=5,ncol=7)
ARL1.ans.EW <- matrix(NA,nrow=5,ncol=7)
ARL1.dif.EW <- matrix(NA,nrow=5,ncol=7)
ARL1.FT.EW <- matrix(NA,nrow=5,ncol=7)
ARL1.Rossi.EW <- matrix(NA,nrow=5,ncol=7)
ARL1.SQR.EW <- matrix(NA,nrow=5,ncol=7)
ARL1.VAR.EW <- matrix(NA,nrow=5,ncol=7)
ARL1.W.EW <- matrix(NA,nrow=5,ncol=7)
ARL1.W.log.EW <- matrix(NA,nrow=5,ncol=7)
ARL1.W2.EW <- matrix(NA,nrow=5,ncol=7)
ARL1.W2.log.EW <- matrix(NA,nrow=5,ncol=7)
ARL1.Z1.EW <- matrix(NA,nrow=5,ncol=7)
ARL1.Z2.EW <- matrix(NA,nrow=5,ncol=7)
ARL1.Z3.EW <- matrix(NA,nrow=5,ncol=7)
ARL1.Z4.EW <- matrix(NA,nrow=5,ncol=7)

UCL.sd.EW <- UCLset.EW[1]
UCL.SQR.EW <- UCLset.EW[2]
UCL.ans.EW <- UCLset.EW[3]
UCL.FT.EW <- UCLset.EW[4]
UCL.Rossi.EW <- UCLset.EW[5]
```

```

UCL.VAR.EW <- UCLset.EW[6]
UCL.dif.EW <- UCLset.EW[7]
UCL.W.EW <- UCLset.EW[8]
UCL.W2.EW <- UCLset.EW[9]
UCL.W.log.EW <- UCLset.EW[10]
UCL.W2.log.EW <- UCLset.EW[11]
UCL.Z1.EW <- UCLset.EW[12]
UCL.Z2.EW <- UCLset.EW[13]
UCL.Z3.EW <- UCLset.EW[14]
UCL.Z4.EW <- UCLset.EW[15]

for (i in 1:7) {

  tau <- Parameter.tau[i]

  for (j in 1:5){

    deltax <- Parameter.deltax[j]

    rho1 <- tau*rho0

    lamb1 <- deltax*lamb
    mu1 <- lamb1/rho1

    random.lamb <- matrix(NA,nrow=2000,ncol=5000)
    random.mu <- matrix(NA,nrow=2000,ncol=5000)

    for (m in 1:5000){
      random.lamb[,m] <- rpois(2000,lamb1)
      random.mu[,m] <- rpois(2000,mu1)
    }

    ds.poisson <- matrix(NA,nrow=2000,ncol=5000)

    for (n in 1:5000){
      for (m in 1:2000){
        ds.poisson[m,n] <- random.lamb[m,n]/(random.mu[m,n]
          +1)
      }
    }
  }
}

```

```

}

ds.poisson.ans <- matrix(NA,nrow=2000,ncol=5000)

for (n in 1:5000){
  for (m in 1:2000){
    ds.poisson.ans[m,n] <- (2*sqrt(random.lamb[m,n]+3/8))
      /(2*sqrt(random.mu[m,n]+3/8))
  }
}

ds.poisson.FT <- matrix(NA,nrow=2000,ncol=5000)

for (n in 1:5000){
  for (m in 1:2000){
    ds.poisson.FT[m,n] <- (sqrt(random.lamb[m,n]+1)+sqrt(
      random.lamb[m,n]))/(sqrt(random.mu[m,n]+1)+sqrt(
      random.mu[m,n]))
  }
}

ds.poisson.Rossi <- matrix(NA,nrow=2000,ncol=5000)

for (p in 1:5000){
  for (o in 1:2000){
    ds.poisson.Rossi[o,p] <- ((random.lamb[o,p]+2*sqrt(
      random.lamb[o,p]*lamb))*sqrt(1/rho0))/(random.mu[o
      ,p]+2*sqrt(random.mu[o,p]*mu)+1)
  }
}

ds.poisson.SQR <- matrix(NA,nrow=2000,ncol=5000)

for (r in 1:5000){
  for (q in 1:2000){
    ds.poisson.SQR[q,r] <- sqrt(random.lamb[q,r])/sqrt(
      random.mu[q,r]+1)
  }
}

```

```

ds.poisson.VAR <- matrix(NA,nrow=2000,ncol=5000)

for (v in 1:5000){
  for (u in 1:2000){
    ds.poisson.VAR[u,v] <- ((random.lamb[u,v]/(random.mu[
      u,v]+1))-(lamb/mu+lamb/(1+3*mu+mu^2)))/sqrt((lamb/
      mu)^2*(1/(1+lamb)+1/(1+mu)))
  }
}

ds.poisson.dif <- matrix(NA,nrow=2000,ncol=5000)

for (n in 1:5000){
  for (m in 1:2000){
    ds.poisson.dif[m,n] <- random.lamb[m,n]-random.mu[m,n
      ]
  }
}

W <- matrix(NA,nrow=2000,ncol=5000)

for (n in 1:5000){
  for (m in 1:2000){
    W[m,n] <- (random.lamb[m,n]-random.mu[m,n]*rho0)/sqrt
      (lamb+mu*(rho0^2))
  }
}

W2 <- matrix(NA,nrow=2000,ncol=5000)

for (n in 1:5000){
  for (m in 1:2000){
    W2[m,n] <- (random.lamb[m,n]-random.mu[m,n]*rho0)/
      sqrt((lamb+mu)*rho0)
  }
}

W.log <- matrix(NA,nrow=2000,ncol=5000)

```

```

for (n in 1:5000){
  for (m in 1:2000){
    W.log[m,n] <- (log((random.lamb[m,n]+1)/(random.mu[m,
      n]+1))-log(rho0))/sqrt(1/(lamb)+1/(mu))
  }
}

W2.log <- matrix(NA,nrow=2000,ncol=5000)

for (n in 1:5000){
  for (m in 1:2000){
    W2.log[m,n] <- (log((random.lamb[m,n]+1)/(random.mu[m,
      n]+1))-log(rho0))/sqrt((2+1/rho0+rho0)/(lamb+mu))
  }
}

Z1 <- matrix(NA,nrow=2000,ncol=5000)

for (n in 1:5000){
  for (m in 1:2000){
    Z1[m,n] <- 2*(sqrt(random.lamb[m,n])-sqrt(rho0*random
      .mu[m,n]))/sqrt(1+rho0)
  }
}

Z2 <- matrix(NA,nrow=2000,ncol=5000)

for (n in 1:5000){
  for (m in 1:2000){
    Z2[m,n] <- 2*(sqrt(random.lamb[m,n]+3/8)-sqrt(rho0*(
      random.mu[m,n]+3/8)))/sqrt(1+rho0)
  }
}

Z3 <- matrix(NA,nrow=2000,ncol=5000)

for (n in 1:5000){
  for (m in 1:2000){

```

```

      Z3[m,n] <- (sqrt(random.lamb[m,n]+1)+sqrt(random.lamb
        [m,n])-rho0*(sqrt(random.mu[m,n]+1)+sqrt(random.mu
          [m,n])))/sqrt(1+rho0^2)
    }
  }

Z4 <- matrix(NA,nrow=2000,ncol=5000)

for (n in 1:5000){
  for (m in 1:2000){
    Z4[m,n] <- (random.lamb[m,n]+2*sqrt(random.lamb[m,n]*
      lamb)-rho0*(random.mu[m,n]+2*sqrt(random.mu[m,n]*
        mu)))/(sqrt(4*lamb+4*(rho0^2)*mu))
  }
}

RL.TI <- NULL

for (n in 1:5000){

  TI <- NULL

  for (m in 2:2000){
    TI[1] <- const*ds.poisson[1,n]+(1-const)*(lamb/(mu+1)
      )
    TI[m] <- const*ds.poisson[m,n]+(1-const)*TI[m-1]
    if(TI[m]>UCL.sd.EW){break}
  }

  RL.TI[n] <- length(TI)

}

ARL1.sd.EW[j,i] <- mean(RL.TI)

RL.TI <- NULL

for (n in 1:5000){

```

```

TI <- NULL

for (m in 2:2000){
  TI[1] <- const*ds.poisson.ans[1,n]+(1-const)*((2*sqrt
    (lamb+3/8))/(2*sqrt(mu+3/8)))
  TI[m] <- const*ds.poisson.ans[m,n]+(1-const)*TI[m-1]
  if(TI[m]>UCL.ans.EW){break}
}

RL.TI[n] <- length(TI)

}

ARL1.ans.EW[j,i] <- mean(RL.TI)

RL.TI <- NULL

for (n in 1:5000){

  TI <- NULL

  for (m in 2:2000){
    TI[1] <- const*ds.poisson.dif[1,n]+(1-const)*(lamb-mu
      )
    TI[m] <- const*ds.poisson.dif[m,n]+(1-const)*TI[m-1]
    if(TI[m]>UCL.dif.EW){break}
  }

  RL.TI[n] <- length(TI)

}

ARL1.dif.EW[j,i] <- mean(RL.TI)

RL.TI <- NULL

for (n in 1:5000){

  TI <- NULL

```

```

for (m in 2:2000){
  TI[1] <- const*ds.poisson.FT[1,n]+(1-const)*((sqrt(
    lamb+1)+sqrt(lamb))/(sqrt(mu+1)+sqrt(mu)))
  TI[m] <- const*ds.poisson.FT[m,n]+(1-const)*TI[m-1]
  if(TI[m]>UCL.FT.EW){break}
}

RL.TI[n] <- length(TI)

}

ARL1.FT.EW[j,i] <- mean(RL.TI)

RL.TI <- NULL

for (n in 1:5000){

  TI <- NULL

  for (m in 2:2000){
    TI[m] <- ds.poisson.Rossi[m,n]
    TI[1] <- const*ds.poisson.Rossi[1,n]+(1-const)*((lamb
      +2*sqrt(lamb*lamb))*sqrt(1/rho0))/(mu+2*sqrt(mu*mu
      )+1)
    TI[m] <- const*ds.poisson.Rossi[m,n]+(1-const)*TI[m
      -1]
    if(TI[m]>UCL.Rossi.EW){break}
  }

  RL.TI[n] <- length(TI)

}

ARL1.Rossi.EW[j,i] <- mean(RL.TI)

RL.TI <- NULL

for (n in 1:5000){

```

```

TI <- NULL

for (m in 2:2000){
  TI[1] <- const*ds.poisson.SQR[1,n]+(1-const)*(sqrt(
    lamb)/sqrt(mu+1))
  TI[m] <- const*ds.poisson.SQR[m,n]+(1-const)*TI[m-1]
  if(TI[m]>UCL.SQR.EW){break}
}

RL.TI[n] <- length(TI)

}

ARL1.SQR.EW[j,i] <- mean(RL.TI)

RL.TI <- NULL

for (n in 1:5000){

  TI <- NULL

  for (m in 2:2000){
    TI[1] <- const*ds.poisson.VAR[1,n]+(1-const)*((lamb/(
      mu+1))-(lamb/mu+lamb/(1+3*mu+mu^2)))/sqrt((lamb/mu
      )^2*(1/(1+lamb)+1/(1+mu)))
    TI[m] <- const*ds.poisson.VAR[m,n]+(1-const)*TI[m-1]
    if(TI[m]>UCL.VAR.EW){break}
  }

  RL.TI[n] <- length(TI)

}

ARL1.VAR.EW[j,i] <- mean(RL.TI)

RL.TI <- NULL

for (n in 1:5000){

```

```

TI <- NULL

for (m in 2:2000){
  TI[1] <- const*W[1,n]+(1-const)*((lamb-mu*rho0)/sqrt(
    lamb+mu*(rho0^2)))
  TI[m] <- const*W[m,n]+(1-const)*TI[m-1]
  if(TI[m]>UCL.W.EW){break}
}

RL.TI[n] <- length(TI)

}

ARL1.W.EW[j,i] <- mean(RL.TI)

RL.TI <- NULL

for (n in 1:5000){

  TI <- NULL

  for (m in 2:2000){
    TI[1] <- const*W2[1,n]+(1-const)*((lamb-mu*rho0)/sqrt(
      ((lamb+mu)*rho0))
    TI[m] <- const*W2[m,n]+(1-const)*TI[m-1]
    if(TI[m]>UCL.W2.EW){break}
  }

  RL.TI[n] <- length(TI)

}

ARL1.W2.EW[j,i] <- mean(RL.TI)

RL.TI <- NULL

for (n in 1:5000){

```

```

TI <- NULL

for (m in 2:2000){
  TI[1] <- const*W.log[1,n]+(1-const)*((log((lamb+1)/(
    mu+1))-log(rho0))/sqrt(1/(lamb)+1/(mu)))
  TI[m] <- const*W.log[m,n]+(1-const)*TI[m-1]
  if(TI[m]>UCL.W.log.EW){break}
}

RL.TI[n] <- length(TI)

}

ARL1.W.log.EW[j,i] <- mean(RL.TI)

RL.TI <- NULL

for (n in 1:5000){

  TI <- NULL

  for (m in 2:2000){
    TI[1] <- const*W2.log[1,n]+(1-const)*((log((lamb+1)/(
      mu+1))-log(rho0))/sqrt((2+1/rho0+rho0)/(lamb+mu)))
    TI[m] <- const*W2.log[m,n]+(1-const)*TI[m-1]
    if(TI[m]>UCL.W2.log.EW){break}
  }

  RL.TI[n] <- length(TI)

}

ARL1.W2.log.EW[j,i] <- mean(RL.TI)

RL.TI <- NULL

for (n in 1:5000){

  TI <- NULL

```

```

    for (m in 2:2000){
      TI[1] <- const*Z1[1,n]+(1-const)*(2*(sqrt(lamb)-sqrt(
        rho0*mu))/sqrt(1+rho0))
      TI[m] <- const*Z1[m,n]+(1-const)*TI[m-1]
      if(TI[m]>UCL.Z1.EW){break}
    }

    RL.TI[n] <- length(TI)

  }

  ARL1.Z1.EW[j,i] <- mean(RL.TI)

  RL.TI <- NULL

  for (n in 1:5000){

    TI <- NULL

    for (m in 2:2000){
      TI[1] <- const*Z2[1,n]+(1-const)*(2*(sqrt(lamb+3/8)-
        sqrt(rho0*(mu+3/8)))/sqrt(1+rho0))
      TI[m] <- const*Z2[m,n]+(1-const)*TI[m-1]
      if(TI[m]>UCL.Z2.EW){break}
    }

    RL.TI[n] <- length(TI)

  }

  ARL1.Z2.EW[j,i] <- mean(RL.TI)

  RL.TI <- NULL

  for (n in 1:5000){

    TI <- NULL

```

```

    for (m in 2:2000){
      TI[1] <- const*Z3[1,n]+(1-const)*(sqrt(lamb+1)+sqrt(
        lamb)-rho0*(sqrt(mu+1)+sqrt(mu)))/sqrt(1+rho0^2)
      TI[m] <- const*Z3[m,n]+(1-const)*TI[m-1]
      if(TI[m]>UCL.Z3.EW){break}
    }

    RL.TI[n] <- length(TI)

  }

  ARL1.Z3.EW[j,i] <- mean(RL.TI)

  RL.TI <- NULL

  for (n in 1:5000){

    TI <- NULL

    for (m in 2:2000){
      TI[1] <- const*Z4[1,n]+(1-const)*((lamb+2*sqrt(lamb*
        lamb)-rho0*(mu+2*sqrt(mu*mu)))/(sqrt(4*lamb+4*(
        rho0^2)*mu)))
      TI[m] <- const*Z4[m,n]+(1-const)*TI[m-1]
      if(TI[m]>UCL.Z4.EW){break}
    }

    RL.TI[n] <- length(TI)

  }

  ARL1.Z4.EW[j,i] <- mean(RL.TI)

}

}

ARL1.sd.ajt.EW <- c(ARL1.sd.EW[,1],ARL1.sd.EW[,2],ARL1.sd.EW
[,3],ARL1.sd.EW[,4],ARL1.sd.EW[,5],ARL1.sd.EW[,6],ARL1.sd.
EW[,7])

```

```

ARL1.ans.ajt.EW <- c(ARL1.ans.EW[,1], ARL1.ans.EW[,2], ARL1.ans
  .EW[,3], ARL1.ans.EW[,4], ARL1.ans.EW[,5], ARL1.ans.EW[,6],
  ARL1.ans.EW[,7])
ARL1.dif.ajt.EW <- c(ARL1.dif.EW[,1], ARL1.dif.EW[,2], ARL1.dif
  .EW[,3], ARL1.dif.EW[,4], ARL1.dif.EW[,5], ARL1.dif.EW[,6],
  ARL1.dif.EW[,7])
ARL1.FT.ajt.EW <- c(ARL1.FT.EW[,1], ARL1.FT.EW[,2], ARL1.FT.EW
  [,3], ARL1.FT.EW[,4], ARL1.FT.EW[,5], ARL1.FT.EW[,6], ARL1.FT.
  EW[,7])
ARL1.Rossi.ajt.EW <- c(ARL1.Rossi.EW[,1], ARL1.Rossi.EW[,2],
  ARL1.Rossi.EW[,3], ARL1.Rossi.EW[,4], ARL1.Rossi.EW[,5], ARL1
  .Rossi.EW[,6], ARL1.Rossi.EW[,7])
ARL1.SQR.ajt.EW <- c(ARL1.SQR.EW[,1], ARL1.SQR.EW[,2], ARL1.SQR
  .EW[,3], ARL1.SQR.EW[,4], ARL1.SQR.EW[,5], ARL1.SQR.EW[,6],
  ARL1.SQR.EW[,7])
ARL1.VAR.ajt.EW <- c(ARL1.VAR.EW[,1], ARL1.VAR.EW[,2], ARL1.VAR
  .EW[,3], ARL1.VAR.EW[,4], ARL1.VAR.EW[,5], ARL1.VAR.EW[,6],
  ARL1.VAR.EW[,7])
ARL1.W.ajt.EW <- c(ARL1.W.EW[,1], ARL1.W.EW[,2], ARL1.W.EW[,3],
  ARL1.W.EW[,4], ARL1.W.EW[,5], ARL1.W.EW[,6], ARL1.W.EW[,7])
ARL1.W2.ajt.EW <- c(ARL1.W2.EW[,1], ARL1.W2.EW[,2], ARL1.W2.EW
  [,3], ARL1.W2.EW[,4], ARL1.W2.EW[,5], ARL1.W2.EW[,6], ARL1.W2.
  EW[,7])
ARL1.W.log.ajt.EW <- c(ARL1.W.log.EW[,1], ARL1.W.log.EW[,2],
  ARL1.W.log.EW[,3], ARL1.W.log.EW[,4], ARL1.W.log.EW[,5], ARL1
  .W.log.EW[,6], ARL1.W.log.EW[,7])
ARL1.W2.log.ajt.EW <- c(ARL1.W2.log.EW[,1], ARL1.W2.log.EW
  [,2], ARL1.W2.log.EW[,3], ARL1.W2.log.EW[,4], ARL1.W2.log.EW
  [,5], ARL1.W2.log.EW[,6], ARL1.W2.log.EW[,7])
ARL1.Z1.ajt.EW <- c(ARL1.Z1.EW[,1], ARL1.Z1.EW[,2], ARL1.Z1.EW
  [,3], ARL1.Z1.EW[,4], ARL1.Z1.EW[,5], ARL1.Z1.EW[,6], ARL1.Z1.
  EW[,7])
ARL1.Z2.ajt.EW <- c(ARL1.Z2.EW[,1], ARL1.Z2.EW[,2], ARL1.Z2.EW
  [,3], ARL1.Z2.EW[,4], ARL1.Z2.EW[,5], ARL1.Z2.EW[,6], ARL1.Z2.
  EW[,7])
ARL1.Z3.ajt.EW <- c(ARL1.Z3.EW[,1], ARL1.Z3.EW[,2], ARL1.Z3.EW
  [,3], ARL1.Z3.EW[,4], ARL1.Z3.EW[,5], ARL1.Z3.EW[,6], ARL1.Z3.
  EW[,7])
ARL1.Z4.ajt.EW <- c(ARL1.Z4.EW[,1], ARL1.Z4.EW[,2], ARL1.Z4.EW

```



```
[,3], ARL1.Z4.EW[,4], ARL1.Z4.EW[,5], ARL1.Z4.EW[,6], ARL1.Z4.EW[,7])
```

```
Result <- matrix(c(ARL1.sd.ajt.EW, ARL1.SQR.ajt.EW, ARL1.ans.ajt.EW, ARL1.FT.ajt.EW, ARL1.Rossi.ajt.EW, ARL1.VAR.ajt.EW, ARL1.dif.ajt.EW, ARL1.W.ajt.EW, ARL1.W2.ajt.EW, ARL1.W.log.ajt.EW, ARL1.W2.log.ajt.EW, ARL1.Z1.ajt.EW, ARL1.Z2.ajt.EW, ARL1.Z3.ajt.EW, ARL1.Z4.ajt.EW), nrow=35, ncol=15, byrow=FALSE)
```

<ARL1 Estimation for EWMA-type control chart in case λ_{10} and λ_{20} are unknown>

```
Parameter.tau <- c(1.02, 1.05, 1.1, 1.2, 1.3, 1.4, 1.5)
Parameter.deltax <- c(1.2, 1.1, 1, 0.9, 0.8)
```

```
lamb <- 7
mu <- 10
rho0 <- lamb/mu
```

```
const <- 0.20
```

```
UCLset.EW <- c(1.04798, 0.98399, 1.05389, 1.0523, 1.11371, -0.17911, 0.10567, 0.6745, 0.73371, 0.78192, 0.76359, 0.75087, 0.76307, 1.48459, 2.42239)
```

```
ARL1.sd.EW <- matrix(NA, nrow=5, ncol=7)
ARL1.ans.EW <- matrix(NA, nrow=5, ncol=7)
ARL1.dif.EW <- matrix(NA, nrow=5, ncol=7)
ARL1.FT.EW <- matrix(NA, nrow=5, ncol=7)
ARL1.Rossi.EW <- matrix(NA, nrow=5, ncol=7)
ARL1.SQR.EW <- matrix(NA, nrow=5, ncol=7)
ARL1.VAR.EW <- matrix(NA, nrow=5, ncol=7)
ARL1.W.EW <- matrix(NA, nrow=5, ncol=7)
ARL1.W.log.EW <- matrix(NA, nrow=5, ncol=7)
ARL1.W2.EW <- matrix(NA, nrow=5, ncol=7)
ARL1.W2.log.EW <- matrix(NA, nrow=5, ncol=7)
ARL1.Z1.EW <- matrix(NA, nrow=5, ncol=7)
ARL1.Z2.EW <- matrix(NA, nrow=5, ncol=7)
```

```

ARL1.Z3.EW <- matrix(NA,nrow=5,ncol=7)
ARL1.Z4.EW <- matrix(NA,nrow=5,ncol=7)

UCL.sd.EW <- UCLset.EW[1]
UCL.SQR.EW <- UCLset.EW[2]
UCL.ans.EW <- UCLset.EW[3]
UCL.FT.EW <- UCLset.EW[4]
UCL.Rossi.EW <- UCLset.EW[5]
UCL.VAR.EW <- UCLset.EW[6]
UCL.dif.EW <- UCLset.EW[7]
UCL.W.EW <- UCLset.EW[8]
UCL.W2.EW <- UCLset.EW[9]
UCL.W.log.EW <- UCLset.EW[10]
UCL.W2.log.EW <- UCLset.EW[11]
UCL.Z1.EW <- UCLset.EW[12]
UCL.Z2.EW <- UCLset.EW[13]
UCL.Z3.EW <- UCLset.EW[14]
UCL.Z4.EW <- UCLset.EW[15]

for (i in 1:7) {

  tau <- Parameter.tau[i]

  for (j in 1:5){

    deltax <- Parameter.deltax[j]

    rho1 <- tau*rho0

    lamb1 <- deltax*lamb
    mu1 <- lamb1/rho1

    random.lamb <- matrix(NA,nrow=2000,ncol=5000)
    random.mu <- matrix(NA,nrow=2000,ncol=5000)

    for (m in 1:5000){
      random.lamb[,m] <- rpois(2000,lamb1)
      random.mu[,m] <- rpois(2000,mu1)
    }
  }
}

```

```

ds.poisson <- matrix(NA,nrow=2000,ncol=5000)

for (n in 1:5000){
  for (m in 1:2000){
    ds.poisson[m,n] <- random.lamb[m,n]/(random.mu[m,n]
      ]+1)
  }
}

ds.poisson.ans <- matrix(NA,nrow=2000,ncol=5000)

for (n in 1:5000){
  for (m in 1:2000){
    ds.poisson.ans[m,n] <- (2*sqrt(random.lamb[m,n]+3/8))
      /(2*sqrt(random.mu[m,n]+3/8))
  }
}

ds.poisson.FT <- matrix(NA,nrow=2000,ncol=5000)

for (n in 1:5000){
  for (m in 1:2000){
    ds.poisson.FT[m,n] <- (sqrt(random.lamb[m,n]+1)+sqrt(
      random.lamb[m,n]))/(sqrt(random.mu[m,n]+1)+sqrt(
      random.mu[m,n]))
  }
}

ds.poisson.Rossi <- matrix(NA,nrow=2000,ncol=5000)

for (p in 1:5000){
  for (o in 1:2000){
    ds.poisson.Rossi[o,p] <- ((random.lamb[o,p]+2*sqrt(
      random.lamb[o,p]*lamb))*sqrt(1/rho0))/(random.mu[o
      ,p]+2*sqrt(random.mu[o,p]*mu)+1)
  }
}

```

```

ds.poisson.SQR <- matrix(NA,nrow=2000,ncol=5000)

for (r in 1:5000){
  for (q in 1:2000){
    ds.poisson.SQR[q,r] <- sqrt(random.lamb[q,r])/sqrt(
      random.mu[q,r]+1)
  }
}

ds.poisson.VAR <- matrix(NA,nrow=2000,ncol=5000)

for (v in 1:5000){
  for (u in 1:2000){
    ds.poisson.VAR[u,v] <- -((1+4*random.mu[u,v]+random.
      mu[u,v]^2)*sqrt(1+random.lamb[u,v]))/((1+3*random.
      mu[u,v]+random.mu[u,v]^2)*sqrt((1+random.mu[u,v])*
      (random.lamb[u,v]+random.mu[u,v]+2)))
  }
}

ds.poisson.dif <- matrix(NA,nrow=2000,ncol=5000)

for (n in 1:5000){
  for (m in 1:2000){
    ds.poisson.dif[m,n] <- random.lamb[m,n]-random.mu[m,n]
  }
}

W <- matrix(NA,nrow=2000,ncol=5000)

for (n in 1:5000){
  for (m in 1:2000){
    W[m,n] <- (random.lamb[m,n]-random.mu[m,n]*rho0)/sqrt
      (random.lamb[m,n]+random.mu[m,n]*(rho0^2)+1)
  }
}

W2 <- matrix(NA,nrow=2000,ncol=5000)

```

```

for (n in 1:5000){
  for (m in 1:2000){
    W2[m,n] <- (random.lamb[m,n]-random.mu[m,n]*rho0)/
      sqrt((random.lamb[m,n]+random.mu[m,n])*rho0+1)
  }
}

W.log <- matrix(NA,nrow=2000,ncol=5000)

for (n in 1:5000){
  for (m in 1:2000){
    W.log[m,n] <- (log((random.lamb[m,n]+1)/(random.mu[m,
      n]+1))-log(rho0))/sqrt(1/(random.lamb[m,n]+1)+1/(
      random.mu[m,n]+1))
  }
}

W2.log <- matrix(NA,nrow=2000,ncol=5000)

for (n in 1:5000){
  for (m in 1:2000){
    W2.log[m,n] <- (log((random.lamb[m,n]+1)/(random.mu[m,
      n]+1))-log(rho0))/sqrt((2+1/rho0+rho0)/(random.
      lamb[m,n]+random.mu[m,n]+1))
  }
}

Z1 <- matrix(NA,nrow=2000,ncol=5000)

for (n in 1:5000){
  for (m in 1:2000){
    Z1[m,n] <- 2*(sqrt(random.lamb[m,n])-sqrt(rho0*random
      .mu[m,n]))/sqrt(1+rho0)
  }
}

Z2 <- matrix(NA,nrow=2000,ncol=5000)

```

```

for (n in 1:5000){
  for (m in 1:2000){
    Z2[m,n] <- 2*(sqrt(random.lamb[m,n]+3/8)-sqrt(rho0*(
      random.mu[m,n]+3/8)))/sqrt(1+rho0)
  }
}

Z3 <- matrix(NA,nrow=2000,ncol=5000)

for (n in 1:5000){
  for (m in 1:2000){
    Z3[m,n] <- (sqrt(random.lamb[m,n]+1)+sqrt(random.lamb
      [m,n]))-rho0*(sqrt(random.mu[m,n]+1)+sqrt(random.mu
      [m,n])))/sqrt(1+rho0^2)
  }
}

Z4 <- matrix(NA,nrow=2000,ncol=5000)

for (n in 1:5000){
  for (m in 1:2000){
    Z4[m,n] <- (3*random.lamb[m,n]-3*rho0*sqrt(random.mu[
      m,n]))/(2*sqrt(random.lamb[m,n]+(rho0^2)*random.mu
      [m,n])+1)
  }
}

RL.TI <- NULL

for (n in 1:5000){

  TI <- NULL

  for (m in 2:2000){
    TI[1] <- const*ds.poisson[1,n]+(1-const)*(lamb/(mu+1)
      )
    TI[m] <- const*ds.poisson[m,n]+(1-const)*TI[m-1]
    if(TI[m]>UCL.sd.EW){break}
  }
}

```

```

    RL.TI[n] <- length(TI)

}

ARL1.sd.EW[j,i] <- mean(RL.TI)

RL.TI <- NULL

for (n in 1:5000){

    TI <- NULL

    for (m in 2:2000){
        TI[1] <- const*ds.poisson.ans[1,n]+(1-const)*((2*sqrt
            (lamb+3/8))/(2*sqrt(mu+3/8)))
        TI[m] <- const*ds.poisson.ans[m,n]+(1-const)*TI[m-1]
        if(TI[m]>UCL.ans.EW){break}
    }

    RL.TI[n] <- length(TI)

}

ARL1.ans.EW[j,i] <- mean(RL.TI)

RL.TI <- NULL

for (n in 1:5000){

    TI <- NULL

    for (m in 2:2000){
        TI[1] <- const*ds.poisson.dif[1,n]+(1-const)*(lamb-mu
            )
        TI[m] <- const*ds.poisson.dif[m,n]+(1-const)*TI[m-1]
        if(TI[m]>UCL.dif.EW){break}
    }
}

```

```

    RL.TI[n] <- length(TI)

}

ARL1.dif.EW[j,i] <- mean(RL.TI)

RL.TI <- NULL

for (n in 1:5000){

    TI <- NULL

    for (m in 2:2000){
        TI[1] <- const*ds.poisson.FT[1,n]+(1-const)*((sqrt(
            lamb+1)+sqrt(lamb))/(sqrt(mu+1)+sqrt(mu)))
        TI[m] <- const*ds.poisson.FT[m,n]+(1-const)*TI[m-1]
        if(TI[m]>UCL.FT.EW){break}
    }

    RL.TI[n] <- length(TI)

}

ARL1.FT.EW[j,i] <- mean(RL.TI)

RL.TI <- NULL

for (n in 1:5000){

    TI <- NULL

    for (m in 2:2000){
        TI[m] <- ds.poisson.Rossi[m,n]
        TI[1] <- const*ds.poisson.Rossi[1,n]+(1-const)*((lamb
            +2*sqrt(lamb*lamb))*sqrt(1/rho0))/(mu+2*sqrt(mu*mu
            )+1)
        TI[m] <- const*ds.poisson.Rossi[m,n]+(1-const)*TI[m
            -1]
    }
}

```

```

        if(TI[m]>UCL.Rossi.EW){break}
    }

    RL.TI[n] <- length(TI)

}

ARL1.Rossi.EW[j,i] <- mean(RL.TI)

RL.TI <- NULL

for (n in 1:5000){

    TI <- NULL

    for (m in 2:2000){
        TI[1] <- const*ds.poisson.SQR[1,n]+(1-const)*(sqrt(
            lamb)/sqrt(mu+1))
        TI[m] <- const*ds.poisson.SQR[m,n]+(1-const)*TI[m-1]
        if(TI[m]>UCL.SQR.EW){break}
    }

    RL.TI[n] <- length(TI)

}

ARL1.SQR.EW[j,i] <- mean(RL.TI)

RL.TI <- NULL

for (n in 1:5000){

    TI <- NULL

    for (m in 2:2000){
        TI[1] <- const*ds.poisson.VAR[1,n]+(1-const)*-((1+4*
            mu+mu^2)*sqrt(1+lamb))/((1+3*mu+mu^2)*sqrt((1+mu)*
            (lamb+mu+2)))
        TI[m] <- const*ds.poisson.VAR[m,n]+(1-const)*TI[m-1]
    }

```

```

        if (TI[m]>UCL.VAR.EW){break}
    }

    RL.TI[n] <- length(TI)

}

ARL1.VAR.EW[j,i] <- mean(RL.TI)

RL.TI <- NULL

for (n in 1:5000){

    TI <- NULL

    for (m in 2:2000){
        TI[1] <- const*W[1,n]+(1-const)*((lamb-mu*rho0)/sqrt(
            lamb+mu*(rho0^2)+1))
        TI[m] <- const*W[m,n]+(1-const)*TI[m-1]
        if (TI[m]>UCL.W.EW){break}
    }

    RL.TI[n] <- length(TI)

}

ARL1.W.EW[j,i] <- mean(RL.TI)

RL.TI <- NULL

for (n in 1:5000){

    TI <- NULL

    for (m in 2:2000){
        TI[1] <- const*W2[1,n]+(1-const)*((lamb-mu*rho0)/sqrt
            ((lamb+mu)*rho0+1))
        TI[m] <- const*W2[m,n]+(1-const)*TI[m-1]
        if (TI[m]>UCL.W2.EW){break}
    }

```

```

    }

    RL.TI[n] <- length(TI)

}

ARL1.W2.EW[j,i] <- mean(RL.TI)

RL.TI <- NULL

for (n in 1:5000){

    TI <- NULL

    for (m in 2:2000){
        TI[1] <- const*W.log[1,n]+(1-const)*((log((lamb+1)/(
            mu+1))-log(rho0))/sqrt(1/(lamb+1)+1/(mu+1)))
        TI[m] <- const*W.log[m,n]+(1-const)*TI[m-1]
        if(TI[m]>UCL.W.log.EW){break}
    }

    RL.TI[n] <- length(TI)

}

ARL1.W.log.EW[j,i] <- mean(RL.TI)

RL.TI <- NULL

for (n in 1:5000){

    TI <- NULL

    for (m in 2:2000){
        TI[1] <- const*W2.log[1,n]+(1-const)*((log((lamb+1)/(
            mu+1))-log(rho0))/sqrt((2+1/rho0+rho0)/(lamb+mu+1)
            ))
        TI[m] <- const*W2.log[m,n]+(1-const)*TI[m-1]
        if(TI[m]>UCL.W2.log.EW){break}
    }

```

```

    }

    RL.TI[n] <- length(TI)

}

ARL1.W2.log.EW[j,i] <- mean(RL.TI)

RL.TI <- NULL

for (n in 1:5000){

    TI <- NULL

    for (m in 2:2000){
        TI[1] <- const*Z1[1,n]+(1-const)*(2*(sqrt(lamb)-sqrt(
            rho0*mu))/sqrt(1+rho0))
        TI[m] <- const*Z1[m,n]+(1-const)*TI[m-1]
        if(TI[m]>UCL.Z1.EW){break}
    }

    RL.TI[n] <- length(TI)

}

ARL1.Z1.EW[j,i] <- mean(RL.TI)

RL.TI <- NULL

for (n in 1:5000){

    TI <- NULL

    for (m in 2:2000){
        TI[1] <- const*Z2[1,n]+(1-const)*(2*(sqrt(lamb+3/8)-
            sqrt(rho0*(mu+3/8)))/sqrt(1+rho0))
        TI[m] <- const*Z2[m,n]+(1-const)*TI[m-1]
        if(TI[m]>UCL.Z2.EW){break}
    }
}

```

```

    RL.TI[n] <- length(TI)

}

ARL1.Z2.EW[j,i] <- mean(RL.TI)

RL.TI <- NULL

for (n in 1:5000){

    TI <- NULL

    for (m in 2:2000){
        TI[1] <- const*Z3[1,n]+(1-const)*(sqrt(lamb+1)+sqrt(
            lamb)-rho0*(sqrt(mu+1)+sqrt(mu)))/sqrt(1+rho0^2)
        TI[m] <- const*Z3[m,n]+(1-const)*TI[m-1]
        if(TI[m]>UCL.Z3.EW){break}
    }

    RL.TI[n] <- length(TI)

}

ARL1.Z3.EW[j,i] <- mean(RL.TI)

RL.TI <- NULL

for (n in 1:5000){

    TI <- NULL

    for (m in 2:2000){
        TI[1] <- const*Z4[1,n]+(1-const)*(3*lamb-3*rho0*sqrt(
            mu))/(2*sqrt(lamb+(rho0^2)*mu)+1)
        TI[m] <- const*Z4[m,n]+(1-const)*TI[m-1]
        if(TI[m]>UCL.Z4.EW){break}
    }
}

```

```

    RL.TI[n] <- length(TI)

  }

  ARL1.Z4.EW[j,i] <- mean(RL.TI)

}

}

ARL1.sd.ajt.EW <- c(ARL1.sd.EW[,1], ARL1.sd.EW[,2], ARL1.sd.EW
[,3], ARL1.sd.EW[,4], ARL1.sd.EW[,5], ARL1.sd.EW[,6], ARL1.sd.
EW[,7])
ARL1.ans.ajt.EW <- c(ARL1.ans.EW[,1], ARL1.ans.EW[,2], ARL1.ans
.EW[,3], ARL1.ans.EW[,4], ARL1.ans.EW[,5], ARL1.ans.EW[,6],
ARL1.ans.EW[,7])
ARL1.dif.ajt.EW <- c(ARL1.dif.EW[,1], ARL1.dif.EW[,2], ARL1.dif
.EW[,3], ARL1.dif.EW[,4], ARL1.dif.EW[,5], ARL1.dif.EW[,6],
ARL1.dif.EW[,7])
ARL1.FT.ajt.EW <- c(ARL1.FT.EW[,1], ARL1.FT.EW[,2], ARL1.FT.EW
[,3], ARL1.FT.EW[,4], ARL1.FT.EW[,5], ARL1.FT.EW[,6], ARL1.FT.
EW[,7])
ARL1.Rossi.ajt.EW <- c(ARL1.Rossi.EW[,1], ARL1.Rossi.EW[,2],
ARL1.Rossi.EW[,3], ARL1.Rossi.EW[,4], ARL1.Rossi.EW[,5], ARL1
.Rossi.EW[,6], ARL1.Rossi.EW[,7])
ARL1.SQR.ajt.EW <- c(ARL1.SQR.EW[,1], ARL1.SQR.EW[,2], ARL1.SQR
.EW[,3], ARL1.SQR.EW[,4], ARL1.SQR.EW[,5], ARL1.SQR.EW[,6],
ARL1.SQR.EW[,7])
ARL1.VAR.ajt.EW <- c(ARL1.VAR.EW[,1], ARL1.VAR.EW[,2], ARL1.VAR
.EW[,3], ARL1.VAR.EW[,4], ARL1.VAR.EW[,5], ARL1.VAR.EW[,6],
ARL1.VAR.EW[,7])
ARL1.W.ajt.EW <- c(ARL1.W.EW[,1], ARL1.W.EW[,2], ARL1.W.EW[,3],
ARL1.W.EW[,4], ARL1.W.EW[,5], ARL1.W.EW[,6], ARL1.W.EW[,7])
ARL1.W2.ajt.EW <- c(ARL1.W2.EW[,1], ARL1.W2.EW[,2], ARL1.W2.EW
[,3], ARL1.W2.EW[,4], ARL1.W2.EW[,5], ARL1.W2.EW[,6], ARL1.W2.
EW[,7])
ARL1.W.log.ajt.EW <- c(ARL1.W.log.EW[,1], ARL1.W.log.EW[,2],
ARL1.W.log.EW[,3], ARL1.W.log.EW[,4], ARL1.W.log.EW[,5], ARL1
.W.log.EW[,6], ARL1.W.log.EW[,7])

```

```

ARL1.W2.log.ajt.EW <- c(ARL1.W2.log.EW[,1], ARL1.W2.log.EW
  [,2], ARL1.W2.log.EW[,3], ARL1.W2.log.EW[,4], ARL1.W2.log.EW
  [,5], ARL1.W2.log.EW[,6], ARL1.W2.log.EW[,7])
ARL1.Z1.ajt.EW <- c(ARL1.Z1.EW[,1], ARL1.Z1.EW[,2], ARL1.Z1.EW
  [,3], ARL1.Z1.EW[,4], ARL1.Z1.EW[,5], ARL1.Z1.EW[,6], ARL1.Z1.
  EW[,7])
ARL1.Z2.ajt.EW <- c(ARL1.Z2.EW[,1], ARL1.Z2.EW[,2], ARL1.Z2.EW
  [,3], ARL1.Z2.EW[,4], ARL1.Z2.EW[,5], ARL1.Z2.EW[,6], ARL1.Z2.
  EW[,7])
ARL1.Z3.ajt.EW <- c(ARL1.Z3.EW[,1], ARL1.Z3.EW[,2], ARL1.Z3.EW
  [,3], ARL1.Z3.EW[,4], ARL1.Z3.EW[,5], ARL1.Z3.EW[,6], ARL1.Z3.
  EW[,7])
ARL1.Z4.ajt.EW <- c(ARL1.Z4.EW[,1], ARL1.Z4.EW[,2], ARL1.Z4.EW
  [,3], ARL1.Z4.EW[,4], ARL1.Z4.EW[,5], ARL1.Z4.EW[,6], ARL1.Z4.
  EW[,7])

Result <- matrix(c(ARL1.sd.ajt.EW, ARL1.SQR.ajt.EW, ARL1.ans.
  ajt.EW, ARL1.FT.ajt.EW, ARL1.Rossi.ajt.EW, ARL1.VAR.ajt.EW,
  ARL1.dif.ajt.EW, ARL1.W.ajt.EW, ARL1.W2.ajt.EW, ARL1.W.log.
  ajt.EW, ARL1.W2.log.ajt.EW, ARL1.Z1.ajt.EW, ARL1.Z2.ajt.EW,
  ARL1.Z3.ajt.EW, ARL1.Z4.ajt.EW), nrow=35, ncol=15, byrow=FALSE
  )

```