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AND WEAKLY NON LINEAR SEAKEEPING PROBLEMS**

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Research Area:

Naval Architecture and Ocean Engineering

Supervisor: Prof. Alexandre Nicolaos
Simos

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2016

Este exemplar foi revisado e corrigido em relação à versão original, sob responsabilidade única do autor e com a anuência de seu orientador.

São Paulo, _____ de _____ de _____

Assinatura do autor: _____

Assinatura do orientador: _____

Catálogo-na-publicação

Ruggeri, Felipe

A Higher Order Time Domain Panel Method for Linear and Weakly non linear seakeeping problems / F. Ruggeri – versão corr. – São Paulo, 2016. 228 p.

Tese (Doutorado) - Escola Politécnica da Universidade de São Paulo. Departamento de Engenharia Naval e Oceânica.

1.Comportamento em ondas 2.Método de elementos de contorno no domínio do tempo 3.Efeitos de segunda ordem em sistemas flutuantes 4.Interação onda-corrente I.Universidade de São Paulo. Escola Politécnica. Departamento de Engenharia Naval e Oceânica II.t.

I dedicate this thesis to my parents Mario and Lucia, and my sister, Marina.

I also dedicate this thesis to my lovely Mirreille, the woman of my life, for her love, inspiration and patience.

ACKNOWLEDGMENTS

I would like to acknowledge to my friend prof. Alexandre Simos, who supervised my activities in the last 6 years, including my previous Master Thesis. I really appreciate the support, discussions and dedication during these years.

I also would like to express my gratitude to Prof. Mardel Bongiovanni de Conti for the orientation and support when I started working with BEM, still in a bidimensional code. Probable without this support, including the several programming lessons, I would have never developed the 2D code that was the initial step to the method presented in this thesis.

I gratefully acknowledge Prof. Kazuo Nishimoto, Prof. Eduardo Aoun Tannuri and Prof. Claudio Mueller Sampaio, from the Numerical Offshore Tank (TPN), for the several opportunities regarding the research projects in the last years, including the experimental facilities for the experimental test performed.

A special thank to my colleagues Rafael Watai, Guilherme Feitosa, Rodrigo Lavieri, Raul Dotta, Thiago Peternella, Pedro Mello and others for the many discussions and experiences in the last years.

I am very grateful to Mireille for the complete support and motivation during the several hours, including some holidays and weekends, required to write this text. I would like to include my parents, Mario and Lucia, in this acknowledgment for the several hours dedicated to guarantee the conditions required to conclude this work, as for the education received.

I also acknowledge the financial support of FAPESP¹ PhD grant (2012/06681-7).

¹Fundação de Amparo à Pesquisa do Estado de São Paulo

RESUMO

Essa tese aborda o desenvolvimento de um método de Rankine de ordem alta no domínio do tempo (TDRPM) para o estudo de problemas lineares e fracamente não lineares, incluindo o efeito de corrente, envolvendo sistemas flutuantes. O método de ordem alta desenvolvido considera a geometria do corpo como descrita pelo padrão *Non-uniform Rational Basis Spline* (NURBS), que está disponível em diversos *softwares* de *Computed Aided Design* (CAD) disponíveis, sendo as diversas funções (potencial de velocidades, elevação da superfície-livre e outros) descritos usando B-splines de grau arbitrário. O problema é formulado considerando interações onda-corrente-estrutura para efeitos de até segunda ordem, os de ordem superior sendo calculados considerando as interações somente dos termos de ordem inferior. Para garantir a estabilidade numérica, o problema de contorno com valor inicial é formulado com relação ao potencial de velocidades e da parcela local do potencial de acelerações, este para garantir cálculos precisos da pressão dinâmica. O problema de ordem zero é resolvido usando a linearização de corpo-duplo ao invés da linearização de Neumann-Kelvin para permitir a análise de corpos rombudos, o que requer o cálculo de termos- m de grande complexidade. O método adota fontes de Rankine como funções de Green, que são integradas através de quadratura de Gauss-Legendre no domínio todo, exceto com relação aos termos de auto-influência que adotam um procedimento de dessingularização. O método numérico é inicialmente verificado considerando corpos de geometria simplificada (esfera e cilindro), considerando efeitos de primeira e segunda ordens, com e sem corrente. As derivadas do potencial de velocidades são verificadas comparando os termos- m obtidos numericamente com soluções analíticas disponíveis para a esfera em fluido infinito. As forças de deriva média e dupla-frequência são calculadas para estruturas fixas e flutuantes, sendo as funções calculadas (elevação da superfície-livre, campo de velocidades) comparadas com resultados disponíveis na literatura, incluindo o movimento da esfera flutuante sob a ação de corrente e ondas. São também estudados dois casos de aplicação prática, a resposta de segunda ordem de uma plataforma semi-submersível e o efeito de *wave-drift damping* para o ângulo de equilíbrio de uma plataforma FPSO ancorada através de sistema *turret*. No caso da semi-submersível, os ensaios foram projetados e realizados em tanque de provas. **PALAVRAS-CHAVE:** Comportamento em ondas, Método de Rankine no domínio do tempo, Efeitos de onda de segunda ordem, Interação onda-corrente.

ABSTRACT

This thesis addresses the development of a weakly non-linear Higher Order Time Domain Rankine Panel Method (TDRPM) for the linear and weakly non-linear seakeeping analysis of floating offshore structures, including wave-current interaction effects. A higher order boundary elements method is adopted based on the body geometry description using Non-uniform Rational B-splines (NURBS) formulation, which can be generated by many standard Computed Aided Design (CAD) softwares widely available, and the several computed quantities (velocity potential, free surface elevation and others) are described using a B-spline formulation of arbitrary degree. The problem is formulated considering wave-current-body interactions up to second order effects, these ones considering the terms obtained by interaction of zero/first order quantities. In order to provide numerical stability, the Initial Boundary Value Problem (IBVP) is formulated in terms of the velocity potential and the local acceleration potential, the later used to predict the hydrodynamic pressure accurately. The zeroth order problem is solved using the double-body linearization instead of the Neumann-Kelvin one in order to allow bluff bodies simulation, leading to very complex expressions regarding the m-terms computation. The method adopts the Rankine sources as Green's function, which are integrated using Gauss quadrature in the entire domain, but for the self-influence terms that are integrated using a desingularized procedure. The numerical method is verified initially considering simplified geometries (sphere and circular cylinder) for both, first and second-order computations, with and without current effects. The derivatives of the velocity potential are verified by comparing the numerical m-terms to the analytical solutions for a hemisphere under uniform flow. The mean and double frequency drift forces are computed for fixed and floating structures and the quantities involved in these computations (wave runup, velocity field) are also compared to literature results, including the free floating response of a sphere under current effects. Two practical cases are also studied, namely the wave-induced second order responses of a semi-submersible platform and the wave-drift-damping effect evaluated through the equilibrium angle of a turret moored FPSO. For the former, some specific model tests were designed and conducted in a wave-basin.

KEYWORDS: Seakeeping, Time Domain Rankine Panel Method, Second-order wave forces, Wave-current interaction.

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List of symbols

$[I_{ni}]$	inertia matrix computed in the non-inertial frame
$[K^{(0)}]$	stiffness matrix
$[K_H^{(0)}]$	hydrostatic restoration matrix
$[K_{ext}^{(0)}]$	external stiffness matrix
$[T]_{ni \rightarrow i}$	transformation matrix from non-inertial frame to inertial one
α_x	roll angle
α_y	pitch angle
α_z	yaw angle
$\bar{S}_{fb}$	mean wetted floating body surface
$\bar{S}_{fixed}$	mean wetted fixed surface
$\bar{S}_{fs}$	mean free surface boundary
$\bar{S}_{pm}$	mean wetted prescribed motion surface
β	incidence wave angle
δ	wave phase
$\frac{1}{r_{PQ}}$	Rankine sources
$\dot{\vec{\omega}}$	angular acceleration
$\epsilon = kA$	perturbation parameter in Stokes expansion
η	local ordinate in the desingularized domain during the self-influence terms computation

$\eta(t)$	free surface elevation
$\eta(x, y, t)$	free surface elevation
$\Omega(t)$	fluid region
ω_e	encounter frequency
$\phi^{(0)}$	current disturbance potential
$\phi_I^{(1)}$	first order incident wave potential
$\phi_{dist}^{(1)}$	first order disturbance potential due to the body
$\phi_{i,j}^p$	the b-spline potential coefficient corresponding to the i^{th} basis function in direction u, j^{th} basis function in direction v corresponding of the p^{th} patch of the domain.
Ψ	acceleration potential
ρ	specific mass
φ	velocity potential
φ_{curr}	incident current velocity potential
$\vec{\alpha}$	body rotational motions vector
$\vec{\omega}$	rotational velocity
$\vec{\omega}'$	angular velocity vector in the non-inertial frame
$\vec{a}_G$	center of gravity translational acceleration
$\vec{F}_{ext}^{(2)}$	second order external forces
$\vec{F}^{ext}$	external forces vector
$\vec{L}_0$	angular moment considering pole O
$\vec{M}_0^{ext}$	external torque considering pole O
$\vec{n}_s$	surface normal vector
$\vec{r}_i$	an arbitrary vector in system OXYZ

$\vec{r}_{ni}$	an arbitrary vector in system oxyz
$\vec{v}$	flow velocity field
$\vec{v}_G$	translational center of gravity velocity
$\vec{v}_P$	velocity vector of point P in the inertial frame
$\vec{v}_p$	flow velocity at point P
$\vec{v}_s$	surface velocity vector
$\vec{X}$	body translation motions vector
$\vec{x}_G$	vector of the center of gravity G in relation to the origin O
$\vec{x}_P$	vector of an arbitrary point P in relation to the origin O
ξ	local coordinate in the desingularized domain during the self-influence terms computation
A	wave amplitude
$f(u, v)$	arbitrary continuous function in (u,v) domain
Fn	Froude number
g	gravity acceleration
G_{PQ}	Green's function considering points P(field) and Q(source)
h	water depth
I_{Dk}	k-th component of the dipole self-influence integral term
I_{Sk}	k-th component of the source self-influence integral term
$J(u, v)$	Jacobian
k	wavenumber
KC	Keulegan-Carpenter number
m	body mass
$N_i^{k_u, p}(u)$	i^{th} basis function in direction u with degree k_u in the p^{th} path

$p(x, y, z, t)$ flow pressure

$S_{fb}(x, y, z, t)$ floating bodies boundaries

$S_{fixed}(x, y, z, t)$ fixed structures boundaries

$S_{fs}(x, y, z, t)$ free surface boundaries

$S_{pm}(x, y, z, t)$ prescribed motion boundaries

U ship (current) velocity

u_c collocation point u coordinate

v_c collocation point v coordinate

w_m Gaussian weight

OXYZ inertial coordinate system

oxyz non-inertial coordinate system

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Chapter 1

Introduction and motivation

The seakeeping analysis of floating structures is an important theme in the context of naval architecture and ocean engineering because there are many situations concerning both offshore platforms and ships that require the computation of the vessel motions in a particular wave condition. The evaluation of the dynamic behavior in waves is a critical test both for the design of a floating structure and for the analysis of the operations it shall perform during its lifetime. Regarding the latter, an accurate seakeeping analysis is essential for establishing the limit sea conditions and therefore the probability associated to the interruption at that particular operation (the so-called downtime).

There are several examples of these operations, for instance, the maximum acceleration levels required by a FPSO production plant to operate, the crew comfort of offshore platform or ship based on maximum accelerations/roll motions, the maximum allowed velocities/motions in the helideck of a platform for the landing/taking off of helicopters activities, the maximum relative motions for cargo (container) transfer between a FPSO and a Platform Supply Vessels (PSVs), the maximum loads induced in the mooring cables or hawser during a tandem/STS¹ cargo transfer operation, the maximum accelerations allowed in the connection joint of a SCR riser, the definition of the optimum position for a large turret structure to be installed in a FPSO platform, the UKC² computation of a ship in the approaching channel of a harbour etc. All these operations/phenomena have been studied in the University of São Paulo research projects in recent years.

Furthermore, there are important new topics that are becoming increasingly important as, for instance, the calculation of wave runup about cylindrical or conical shaped foundations

¹Ship-to-ship operation

²Underkeel clearance

of offshore wind turbines, which changes appreciably under current effects, as discussed by Buchmann et al. [2000]. The added resistance of ships has also regained attention nowadays due to the new regulations concerning CO_2 , SO_X and NO_X emissions levels required by the EEDI³, SEEMP⁴ and EEOI⁵ introduced by IMO⁶ in 2012, which are motivating the ship owners to improve ship design in order to reduce added resistance and fuel consumption aware that added resistance can increase ship resistance up to 35% (MAN [2011]). In the past, this problem was addressed following simplified methodologies, see for instance Havelock [1942] and Salvensen [1978], and nowadays there are several methods available for ship routing optimization, including empirical methods, Rankine Panel Methods and CFD⁷, as summarized by Lu et al. [2015].

Computational solution of diffraction-radiation wave effects on floating bodies through boundary element methods (BEM) has now been performed for more than forty years, during which many academic and commercial codes have been presented. Some of these codes are now widespread and frequently used by the offshore industry in the design and analysis of fixed and floating units. Linear frequency-domain codes appeared first in the 70's, mainly due to the limitations imposed by the computing power available at that time. Some examples, among many others, are the well-known codes AQWA (Ansys [2013]) and WAMIT (Lee and Newman [2005]). More recently, several new codes have been developed based on a time-domain approach, both with academic and commercial purposes, see for instance, SeaFEM (Serván-Camas and García-Espinosa [2013]). Nonetheless, these codes are not open source, rendering the integration with other numerical codes/simulators unfeasible.

At the University of São Paulo, the dynamic simulator DYNASIM (Nishimoto et al. [2002]) nowadays works as a time integrator of the environmental forces considering the hydrodynamic coefficients pre-computed by a frequency domain commercial code WAMIT, which does not take into account current effects in the computations. These interactions are only considered following a semi-analytical correction, Aranha [1994, 1996]. Moreover the coefficients are obtained by a frequency domain code, the hypothesis of linearity in the wave-structure interaction phenomena being automatically assumed in the transposition to time domain following Cummins [1962] approach. However, this procedure leads to some limitations, specially because the hydrodynamic problem in frequency domain can only be solved considering the first order and

³Energy Efficiency Design Index

⁴Ship Energy Efficiency Management Plan

⁵Energy Efficiency Operational Index

⁶International Maritime Organization

⁷Computational Fluid Dynamics

higher order solutions using Stokes series (see Stoker [1957]), leading to some difficulties during the evaluation of second order forces in time-domain (mainly the relative phases) acting on the vessel in the sum/difference frequencies.

In practice some important wave-body interactions should be evaluated using a non linear theory, for example, the rolling of a FPSO platform and the interaction with mooring lines (solved nowadays in the simulator by a finite elements method), which cannot be properly included in the frequency domain approach.

The linear approximation imposes yet another limitation concerning the fact that the boundary conditions are applied directly on the mean wetted surface and the hydrostatic properties are kept constant during the entire simulation, not taking into account the effects of the variation of wetted surface in time. As a result, for some structures with small drafts, as high speed vessels (catamaran, trimaran etc.) and monobuoys, the forces and motions are not predicted properly. A similar problem is also verified when multi-bodies are simulated and present large relative displacements, motivating some alternative strategies trying to overcome this limitation, like re-run the frequency domain code if the displacements exceed a specified value, as proposed by Tannuri et al. [2004] and Queiroz Filho and Tannuri [2009]. Nonetheless, these approaches are simplifications of the real physical problem and the evaluation of the involved modelling errors is difficult. Moreover, the "update" of hydrodynamic coefficients during the simulation provides some numerical problems in the convolution integral, requiring additional filtering and smoothing techniques to avoid numerical instabilities.

Those limitations were the main reasons that motivated the development of a Rankine Panel Method code, starting from zero in 2009 and initially considering only a 2D approach under a low order discretization (Ruggeri [2012]). This first work was very important for the comprehension of the the challenges concerning both formulation and numerical implementation. The method then evolved in 2013 to a 3D formulation, still considering a low order approach, as presented by Watai [2014]. This code included a re-meshing technique in order to study multi-body interactions with large relative displacements, which allowed comparing the new method with the simplified approach based on re-running the frequency domain code described earlier.

The present thesis addresses the evolution of the numerical method to a higher order approach, for both domain geometry and computed quantities, in order to study both wave-current and second order effects under a weakly non-linear approach, by which the wave effects are segregated in several orders of magnitude and solved successively in a sequence of linear boundary

value problems. This formulation is applied instead of the fully non-linear one, where all non-linear quantities are directly evaluated in time-domain, because it is a more stable numerical method and requires less computational resources. The higher order approach is implemented in order to guarantee the potential (free surface and pressure) continuity in the entire domain, thus allowing computation of the tangential derivatives that are required for inclusion of second order and current effects. The geometry is also described using a higher order approach (NURBS⁸ formulation) to provide a continuous representation of the normal vector and simplify the mesh generation by means of conventional CAD⁹ softwares.

The current effects change the system steady state, which is included in the wave first order solution considering the m-terms in the body boundary conditions, the body pressure quadratic term and free surface conditions. In the present method, the zeroth order potential is decomposed in two functions, the first one regarding the incident current, and the second one as a disturbance potential due to the presence of the body. The first order problem is also solved following a similar approach, where one potential represents the incident wave field, while the second one the disturbance due to body presence. This single potential takes into account radiation and diffraction effects together, instead of the usual decomposition in seven different potential widely considered for the frequency domain approach.

The current effects are also considered in the computation of second order forces acting in the body. However, the second order wave forces are evaluated considering only the quadratic interactions of zeroth/first order quantities, thus the second order potential is not solved in the present method. These quadratic terms are computed in time domain and applied in the second order motion equations in order to compute the wave induced second order motions.

In the pursuit of a time-domain HOBEM¹⁰ code, many different alternatives are in principle available for the solution of the various computations that must be performed. The upsides and disadvantages of each alternative, however, are often disputable and frequently important details regarding the algorithms required to perform such computations are absent in the specialized literature. For the code developer, this situation certainly makes his task substantially harder. Having this observation in mind, a detailed description of some important algorithms that are critical for the performance of the code will be presented along this thesis.

The text is structured as follows:

⁸Non-uniform b-splines

⁹Computer aided design

¹⁰Higher Order Boundary Elements Method

Chapter 2 presents the objectives of this thesis, including the new features of the method compared to the previous ones developed in Ruggeri [2012] and Watai [2014].

Chapter 3 presents a literature review considering the hypothesis under the potential flow approximation, mathematical formulation, the several methods available in the literature to solve Laplace's equation and the challenges described in the literature regarding the higher order approach.

Chapter 4 presents the mathematical formulation that sets the basis for the method, including the first and second order equations under the Stokes series approach, and including the simplifications (re-arrangement) of the boundary condition terms applied to reduce the computational effort. The acceleration potential is also introduced in the formulation to provide accurate predictions of the hydrodynamic pressure, important to achieve stability in the time integration. The several possibilities concerning the wave-current interaction linearization are presented and the choice for the double-body model linearization is discussed, including the problems regarding computation of the m-terms.

Chapter 5 presents the integral equations to be solved by the Boundary Elements Method formulation, obtained by the use of Green's second identity. The choice for the Rankine sources is also discussed, and also its implications. The most obvious one is the fact that free surface discretization cannot be avoided, because the free surface equations are not automatically satisfied.

Chapter 6 presents the Higher Order formulation considering the geometry description using NURBS¹¹, and the physical quantities (velocity/acceleration potentials, wave elevation, flow velocity etc) representation by a B-spline formulation of arbitrary degree. The desingularization procedure adopted during the integration of the source/dipole terms is also presented and the convergence of the integral is discussed, as well as selection of the collocation points. The formulation proposed adopts more than one collocation point per panel and therefore provides more equations than variables, requiring a least square method to solve the over-determined linear system.

Chapter 7 presents the numerical results computed to verify/validate the developed method. The simulations consider simplified geometries (hemisphere and circular cylinder) in order to compare the achieved results with the ones available in the literature. Furthermore, two practical cases are also evaluated: the first and second order motions of a floating semi-submersible

¹¹Non Uniform Rational Basis Spline

platform and the yaw equilibrium angle of a turret FPSO.

Chapter 8 presents the conclusions and suggested future developments that may extend the application of the numerical method in order to tackle additional phenomena that require more complex simulations.

Chapter 2

Objectives

The main objective of this thesis is to improve and extend the Time Domain Rankine Panel Method code after the initial developments of Ruggeri [2012] and Watai [2014].

The main contributions are:

- The mathematical formulation of the hydrodynamic problem including current effects is derived in the context of the existing numerical code in order to make progress on the numerical routines already programmed in the 2D/3D low order methods; The mixed geometry/time dependent terms are split for improving the numerical performance. (for instance, the influence matrices and boundary condition terms);
- Under the assumptions of the weakly non-linear approach, the mathematical formulation of the hydrodynamic problem for second order loads is presented and the algorithms to compute the second-order quantities are developed and tested. In this version, only the quadratic interactions are considered, neglecting the second order potential. The second order motions are computed by solving the second order motion equation coupled to the second order loads.
- Inclusion of the geometry description based on standard iges CAD format, important for the higher order formulation due to the "exact" geometrical description of the body. The geometrical description is generic to allow an arbitrary number/orientation of patches, which can be performed by several CAD softwares (e.g. Rhinoceros, AutoCAD, Solid-Works, Multisurf etc);
- Inclusion of the quantities description (potential, pressure, free surface elevation etc.) using B-spline formulation of arbitrary degree. A new algorithm is developed based on a "hidden

layer approach” together with a multiple collocation points per panel and a Least Square Method (LSQ) to solve the over-determined linear system.

- Inclusion of the velocity/acceleration field computation using the developed higher order formulation important, which is an feature for the solution of the m-terms required for wave-current interaction. A tracking algorithm for surface/surface intersection problem is developed for the computation of the parametric coordinates at the free surface patches, which is important for wave runup evaluation. The wave runup and velocity field are applied in the second order effects evaluation. The algorithms are tested and verified based on literature results.

It is important to clarify that the objective of this thesis is the development of the numerical method itself, not yet taking into consideration the many techniques associated to high-performance computing (HPC). The code is programmed and executed in Matlab® environment, and therefore is subjected to the several constraints regarding memory utilization and performance. The improvement of the computational performance (programming language , memory allocation, parallel processing etc.) is not in the scope of this thesis.

Chapter 3

Literature review

This chapter presents the literature review regarding the numerical methods to solve wave-current-body interactions, including the difficulties of these methods and the strategies applied to overcome the several limitations. These works were applied in this thesis to comprehend the hydrodynamic problems involved, numerical techniques developed and results that were used as paradigms in the comparisons.

The wave-current-body interaction phenomena have been widely studied in the literature in the last century due to the interest in predicting the behavior of floating structures (ships, oil platforms, floating wind turbines etc). Most of the methods applied to solve the body motions equations under wave action are based on the potential flow approach since the solution of the complete Navier-Stokes equation for a generic case still a challenge. The solutions for real flows are only available for simple cases, several of them summarized in Batchelor [2009] or Milne-Thomson [1968].

The challenge of solving Navier-Stokes equations without (or only considering small) simplifications led to the development of several numerical methods (CFD¹) in the last years, many of them summarized in Ferziger and Peric [2002], Tannehill et al. [1997]. However, even with the continuous improvements in computational capacity, the accuracy and time required to solve the fluid-structure interaction problem are still a challenge, turning unfeasible the application of this methodology to time domain simulations of 3D wave flows with complex structures. The additional hypotheses are traditionally to assume the fluid as ideal (homogeneous/inviscid) and the flow as irrotational and incompressible, therefore the velocity field can be described using a potential function (velocity potential).

¹Computational Fluid Dynamics

The assumption of potential flow is appropriate when the viscous effects on the flow dynamics can be neglected, which may occur when the Reynolds number is large, since the inertial forces becomes large compared to viscous ones. For oscillating bodies, such as oil platforms/ships on ocean waves, the ratio of inertial by viscous forces is given by the KC^2 number (3.1), see for instance Newman [1977], Sarpkaya and Isaacson [1981], Chakrabarti [1987] and Faltinsen [1990]. The KC number is evaluated assuming V as the velocity amplitude, T is the period of oscillation (ω is the frequency of oscillation) and L is a characteristic length. For harmonic oscillations with amplitude A , it corresponds to the ratio of the amplitude over a characteristic length. If this ratio is small the inertial forces dominate. In the seakeeping context of stationary floating bodies, this condition is satisfied under the hypothesis of small wave amplitudes and body motions³. Furthermore an additional hypothesis required in order to guarantee accurate results is the small ratio wave amplitude per body draft, since the problem is solved considering the wetted mean surface (wetted surface at rest), not taking into account the instantaneous wetted surface variations.

$$KC = \frac{VT}{L} = \frac{A\omega \frac{2\pi}{\omega}}{L} = \frac{2\pi A}{L} \quad (3.1)$$

The second non-dimensional number involved is the Froude number, that in the context of deep waters is given by (3.2), where U is the forward speed (or current), L a characteristic length and g the gravity acceleration. This number represents the ratio of inertial forces over gravitational ones and is well-known by naval architects, because it defines, for example, the importance of wave resistance in the total ship resistance (kelvin waves). If this non-dimensional number is small the gravitational effects are dominant so the influence of wave resistance is small (for example a tanker has less than 10% of the total resistance due to radiated waves). There are several formulations available in the literature according to the Froude number and number of terms (linear/non-linear) considered in the free surface/body boundary conditions. Some possible approximations, such as the Uniform flow, Newman-Kelvin, Double-body linearizations or the Non-linear body boundary condition etc. These approximations are discussed, for example, by Nakos et al. [1993], Kring [1994], Prins [1995], Raven [1996], Huang [1997], Bunnik [1999]. Newman [1970] also summarizes many possibilities under the slender-body hypothesis, similar to thin wing theory.

²Keulegan Carpenter

³In particular situations, however, the dynamic conditions may not meet this requirements. This is the case, for example, of resonant motions when small wave amplitudes may lead to large displacements, especially when the motion induces low potential damping.

$$F_n = \frac{U}{\sqrt{Lg}} \quad (3.2)$$

The potential flow hypothesis was also applied for several other correlated studies, for instance, propagation of waves (i.e Stoker [1957], Wehausen and Laitone [1960], Dean et al. [1959]), wave ship-resistance (i.e Havelock [1909]), wave-body interaction without current (i.e Havelock [1942], John [1949], John [1950], Mei et al. [2005]), and wave-current-body interaction (i.e John [1950], Timman and Newman [1962]) and many others, which verified the fine adherence and robustness of this hypothesis for the applications presented in this thesis.

The three major subdivisions in boundary elements method applied for naval and ocean applications are: advancing ships (problem with forward speed), platforms (floating stationary structures) and numerical wave tanks (NWT), the last one focusing on two main themes, non-linear free surface phenomena and fluid-structure interaction. Among the numerical codes based on Boundary Elements Methods available nowadays for seakeeping analysis, the main commercial softwares are WAMIT®(WAM [2015]), AQWA®(Ansys [2013]) and WADAM®(WAD [2010]), all adopting a frequency domain solution. Among the time domain softwares, which were mostly applied and developed for academic purposes, one may find TIMIT®(Korsmeyer [1988], Bingham [1994], Korsmeyer et al. [1999]), SWAN® (Kring [1994], Huang [1997], Sclavounos et al. [1997], Mantzaris [1998]) and SeaFEM (Serván-Camas and García-Espinosa [2013]) applying the time domain formulation.

Most studies assume the convenient perturbation theory (Stokes [1847], Nayfeh [1973]) coupled to Taylor series to segregate the complete non-linear problem into a collection of several linear ones, which is known as weakly non-linear approach, instead of solving the non-linear free-surface and body boundary conditions applied to time-variant boundaries, which is known as "Fully Non-Linear" approach.

These linear problems obtained under the weakly non-linear formation are solved successively assuming the low order solutions as forcing conditions into the higher order equations and the total solution is computed as the sum of the several order solutions, which is the simpler method to solve the non-linear problem. The completely non-linear boundary value problem (BVP) for a floating body is very difficult to solve because the boundary conditions are non-linear and applied to time variant boundaries, which are unknown a priori. These methods usually assume a "Mixed Eulerian-Lagrangian" (MEL) formulation for tracking the free surface evolution in

time (i.e Xue et al. [2001], Wu [2004] and Yan [2010]), which is not performed in the weakly non-linear approach because the free surface remains in the undisturbed position ($z=0$ outside the body).

In the present work, the numerical technique chosen to solve the BVP is the Boundary Elements Method (BEM), essentially based on the fact that, for potential flow, the velocities can be defined in terms of the values at the boundaries, thus not requiring the discretization of the entire fluid domain, as would be the case in a Finite Elements Method (FEM), Finite Volumes Method (FVM) or Finite Differences Method (FDM). The BEM method is also the most conventional method for these problems. The BEM approach requires the choice of a Green function that satisfies Laplace's equation and several possibilities are available, such as Kelvin sources and transient Green function, summarized for instance in Kuznetsov et al. [2004] and Hermans [2011]. Since these functions already satisfies the free surface/bottom boundary conditions, these boundaries do not need to be included in the numerical method. Nonetheless, a major inconvenient of these Green's functions are the evaluation of improper integrals of hard convergence, as discussed by Wehausen and Laitone [1960], Noblesse [1981], Newman [1984], Newman [1985], Newman [1987], Ohkusu and Iwashita [1989] and others. Some of these works also presented numerical algorithms to be applied in numerical codes.

However, it should also be reminded that these functions satisfy only the linearized free surface conditions and thus they require the free surface discretization for non-linear problems, losing one of the major advantages of these functions in the linear problem. An alternative is to consider Rankine sources as Green function, which requires a significantly smaller effort to be computed if compared to the transient ones, and allowing the direct application to second order problems. On the other hand, this function does not satisfy any boundary condition therefore all boundaries must be taken into account in the numerical method.

In the past decades some authors developed numerical codes that aim to compute strong non-linear interactions, which must be solved in time domain. For instance van Daalen [1993] formulated a fully non-linear time domain BEM for the evaluation of the 2D wave-maker problem, forced oscillation tests and decay tests. Greco [2001] followed a similar approach for the investigation of green water phenomenon, but added some additional effects like hydroelasticity. Tanizawa et al. [1999, 2000], Koo [2003], Koo and Kim [2004] and Kim and Koo [2005] applied the bidimensional fully non-linear approach for the evaluation of response to waves of floating bodies in a numerical wave tank simulation (NWT). Tanizawa and Naito [1997] used the NWT

for the study of parametric roll of a "bell" shaped body still in a 2D approach. Contento [2000] also studied the fully non-linear problem validating his results initially with Vugts [1968] experiments for forced oscillation and then performing decay tests and simulations of motion response to waves. This last study considered large initial displacements (40% of the draft) for heave decay tests and showed significant non-linear influence, although the same had not been observed by van Daalen [1993] concerning circular and rectangular sections. Tanizawa and Naito [1998] tried to reproduce chaos in roll motions still in a bidimensional approach using MEL approach.

Shao [2010] states that strong non-linear effects are also important in the study of slamming, green water, capsizing of ships and violent sloshing, motivating the development of a non-linear numerical code. This methodology was also adopted by some authors to study non-linear waves, for instance, Grilli et al. [2001] in the study of overturning waves and Grilli et al. [2002] for tsunamis. Xue et al. [2001] also studied 3D non-linear steep waves applying the fully non-linear approach, which was extended to wave-body interactions in Liu et al. [2001]. It should be noticed that almost all the fully non-linear methods adopt a smooth technique to reduce the wave-breaking instabilities in the free-surface, because this effect cannot be modelled in the scope of potential theory. However, even when applying these filtering procedures, the simulation can only be performed for a reduced amount of time before it is compromised by the cumulative numerical errors. Moreover, in the fully non-linear approach a remeshing scheme for both free surface and body submerged surface has to be performed, a very difficult task from the computational point of view due to the "so-called" surface-surface intersection problem and the time required for a robust evaluation, mainly for complex geometries (i.e. semi-submersible).

However several non-linear problems studied in the seakeeping context do not require this level of complexity in the formulation, including the ones evaluated in this thesis, namely the second order horizontal motions/forces and wave-drift-damping effects. Therefore the weakly non-linear (perturbation) formulation will be applied.

The weakly non-linear formulation set the basis for the development of the main academic and commercial codes, originally in frequency domain, such as WAMIT and AQWA, due to hardware limitations. The solution in frequency domain is based on separation of variables considering the motions as periodic, not allowing the analysis of transient effects. Furthermore, in frequency domain the fluid/structure interaction is not solved completely, since the body motion equation is not evaluated coupled to the hydrodynamic BVP, meaning that the dynamics is post-processed after the diffraction and radiation forces are computed. This procedure was

largely applied in the past for the computation of (quasi)analytical solutions, see for instance the Ursell [1949] study of a circular cylinder, Black et al. [1971] solution for 2D rectangular sections, Hulme [1982] solution for a hemisphere and the 3D circular cylinder solution of McCamy and Fuchs [1954] among others, who also widely applied the technique in validation procedures.

The time-domain simulations were originally performed using the hydrodynamic coefficients in frequency domain, under the convolution integral proposed by Cummins [1962]. This approach is still widely used in several time-domain simulators, although some problems are verified in the relative phases, mainly close to resonance or in second order problems. This methodology allows the inclusion of nonlinear effects for several applications, see for instance the mooring analysis coupled to the body motions (Nishimoto et al. [2002]) or non-linear roll motions Journee [1993]. However, the hydrodynamic coefficients (forces) computed in frequency domain do not change with body motions, which is a huge simplification compared to the "coupled" time domain solution, considering the influence of body motions into the flow and vice-versa. This direct time domain approach requires the body dynamics to be solved coupled with the fluid BVP, simplifying the inclusion/evaluation of non-linearities, either in the hydrodynamic problem or motion equation. The solution of the weakly non linear formulation directly in time domain usually requires more computational effort than the frequency domain one (or Cummins equation one), mainly due to the free surface discretization procedure (justifying the frequency domain approach in the first developed methods).

Some important "weakly" non-linear problems are the slow drifts (well discussed by Pinkster [1980], Pesce [1984] and Aranha and Pesce [1986]) and mean drift forces (see for instance Maruo [1960]) of moored offshore structures. The computation of slow drift forces requires the full QTF⁴ evaluation, which takes a long time to run even in frequency domain since the free surface must be included in the computations and the second order problem is solved for the several pairs of wave frequency. Based on this fact, in the past several methodologies were developed to reduced the computational effort, some of these to choose a reduce, but appropriated, number of frequency pairs according to the expected resonance frequency and wave spectrum. Moreover, several approximations (i.e Newman, Pinkster, Aranha etc.) were developed to overcome the computational limitation, each one with a different order of errors in the difference frequency ($\Delta\omega^n$) and wave spectra, as compared by Matsumoto [2014] for roll motions of FPSOs and applied in the study of surge slow motions of a wind turbine by Lopez-Pavon et al. [2015]. As

⁴Quadratic Transfer Function

discussed before the long time computation required for the second-order problem is mainly due to the free surface discretization, since the Green function adopted in frequency domain does not naturally satisfy the second order inhomogeneous free surface condition⁵. In the time-domain computations using Rankine sources, the free surface must be discretized for the application of the first order free surface conditions, therefore the additional effort required to compute the second order ones is smaller if compared to the frequency domain approach. The approach also allows the study of second order effects due to a group of waves, obtaining the forces and motions during one single computation, see for instance Isaacson and Cheung [1990], Isaacson and Cheung [1992], Kim et al. [1997], Wang and Wu [2007].

The weakly non linear approach was also adopted for the study of third order problems in the presence or absence of current, as presented in Malenica and Molin [1995] (without current) and Shao [2010] (the later also studied second order problems). Zhu [1997] formulated the third order diffraction problem, comparing a BEM solution to the long-wave approximation theory for the ringing⁶ effects for single and a multi-columns fixed structures. Stassen et al. [1998] described a BEM applied to the problem concerning the third-order free surface waves and argued that an additional condition must be imposed in order to correct the secular terms (see Nayfeh [1973]) that produced instabilities⁷.

The order of approximation of the geometry and the potential function leads to two kinds of numerical methods: the low order, that retains only the first term in the approximations, therefore the panels are plane and the quantities (i.e: potential) constant inside each panel, as introduced by Hess and Smith [1964], and the higher order methods, which assume other shape functions that retain more terms, providing continuity of the potential function and normal vector between neighbor panels. One example of low order method is the singularized one developed by Yee-Tak Ng [1993] to study second order effects on floating structures. However, the use of higher order numerical methods are justified due to the reduction of the computational effort, specially for solving second order problems, with or without forward speed, that require an accurate evaluation of the tangential derivatives of the velocity potential.

In fact, it is known that a low order approach is not the most appropriate for the computation of mean, slow and sum drift forces and also wave-current interaction loads, because they all

⁵This problem can be mitigated when additional simplifications are possible (i.e: neglect the forcing terms in the free surface second order condition or the second order potential completely etc).

⁶High frequency transient type response

⁷as introduced by Benjamin and Feir [1967] for waves propagating without any kind of dissipation (or at least negligible dissipation in this order of approximation)

require accurate computation of the derivatives of the velocity potential. Moreover, for the development of the code reported in this thesis, which aims at progressively including such non-linear effects, a higher order approach was adopted for both body geometry and velocity potential. Previous comparisons regarding computational time and accuracy for the low and higher order approaches were provided by Liu et al. [1991] that applied quadratic quadrilateral elements (8 nodes) and by Maniar [1995b], who applied a B-spline description, both works dealing only with frequency domain solutions for many simulation conditions. Maniar [1995b] also adopted a Galerkin procedure to obtain a determined linear system since, usually, the number of variables are larger than equations when a single collocation point per panel is assumed. Qiu [2001] and Qiu et al. [2006] presented the so-called panel free method (desingularized) for wave body interaction, with and without current, where the geometry is generically described by the coefficients of a single NURBS, as largely available in CAD packages. Gao and Zou [2008] presented a desingularized higher order method based on NURBS for the geometry description and B-spline for the potential function to study problems involving forward speed. Liu et al. [1993] computed the mean-drift forces and wave runup on the ISSC TLP using the quadratic elements in combination with the wave source Green function (Wehausen and Laitone [1960]) in order to avoid the need for free surface representation. In the same work Liu et al. [1993] presented results regarding the body velocity on the sphere surface in uniform flow to verify the accuracy of the quadratic elements derivatives formulation, achieving a good agreement with the analytical solution. Prins [1995] also studied first and second order (mean drift) loads, with and without current, assuming a low order approach, and computing the velocity field by means of a finite difference scheme. Liu et al. [1995] computed the mean and double frequency second order forces in frequency domain for the same ISSC TLP, which required the free surface discretization to solve the second order free surface condition, a procedure similar to the one that is adopted by WAMIT. Kim and Kim [1997] extended the method for wave-current interaction using the 9-node quadratic elements for both potential function and body geometry. Rankine source was employed as Green function, thus also the free surface was discretized. Shao [2010] also applied a similar approach using a 12-node cubic polynomial for both potential function and geometry in the study of first and second order quantities, both in the presence and absence of a small current.

Chapter 4

Mathematical problem

The governing flow equations concerned are defined and simplified based on the assumptions of potential flow and small Froude numbers in order to compute the hydrodynamic forces and body motions.

The mathematical problem is formulated for an arbitrarily floating body under the action of regular gravity waves and small current. The Stokes series approach is adopted to tackle this difficult non-linear problem. The difficulties involved for solving the non-linear problem are presented to justify the application of a Stokes series to simplify the mathematical problem. After proper boundary conditions are defined and the Boundary Value Problem (BVP) is posed, the Initial Value Problem (IVP) is treated by defining the initial conditions and procedures for time integration.

4.1 Governing equations

With the assumption of potential flow, the mass conservation is described by Laplace's equation (4.1), where $\varphi = \varphi(x, y, z, t)$ denotes velocity potential function ($\vec{v} = \nabla\varphi$), which is position and time dependent, applied to a fluid region $\Omega(t)$.

$$\nabla^2\varphi = 0 \text{ in } \Omega(t) \quad (4.1)$$

The conservation of linear momentum is simplified into Bernoulli's equation (4.2), where ρ is the specific mass, p the pressure and g the gravity acceleration.

$$\frac{\partial\varphi}{\partial t} + \frac{1}{2} \|\nabla\varphi\|^2 + \frac{p}{\rho} + gz = 0 \quad (4.2)$$

The boundary conditions required to particularize the solution of equation 4.1 are grouped in $S_{fixed}(x, y, z, t)$, $S_{pm}(x, y, z, t)$, $S_{fb}(x, y, z, t)$ and $S_{fs}(x, y, z, t)$, denoting the fixed (stationary), prescribed motions, floating body and free surface boundaries, respectively. Apart from the free surface boundary, which is always part of the fluid boundaries, this classification concerns the motions of the solid boundaries involved in the seakeeping problem.

Fixed boundaries may represent the sea bottom, a fixed oil platform (like gravity or jacket ones), the walls of a wave basin or a breakwater etc. The prescribed motion boundaries are the wetted surface of bodies with imposed motion, such as wave-makers or, as another example, bodies at forced oscillation test¹. Finally, the surfaces $S_{fb}(x, y, z, t)$ represent the wetted surface of floating bodies, which are those whose motions are not known a priori, but only as part of the IBVP solution.

The conditions at all boundaries but the free surface are simply the no-flux condition, which avoid the fluid particles to penetrate the structure, as given by (4.3) and (4.4), the last one already considering the velocity at the boundary described in terms of the velocities at the center of gravity and rotational velocity of the body using Poisson's formula for the rigid body. In the last equation $\vec{v}_G$ is the translational center of gravity velocity, $\vec{\omega}$ is the rotational velocity and $\vec{G}(t)$ is the center of gravity position. Since the problem treated is time dependent there is a boundary value problem coupled to an initial value one, both providing non-linearities in the mathematical treatment.

The Neumann conditions that describe the no flux are, a priori, non-linear at all boundaries since the quantities change in time (normal vector, velocity vector, center of gravity position etc) and quite complicated to be computed since the regions of evaluation are also time dependent.

$$\frac{\partial \varphi_Q}{\partial n_Q} = 0, \quad for \quad Q \in S_{fixed}(x, y, z, t) \quad (4.3)$$

$$\frac{\partial \varphi_Q}{\partial n_Q} = \vec{n}_Q(t) \cdot [\vec{v}_G(t) + \vec{\omega}(t) \wedge (Q - G(t))], \quad for \quad Q \in [S_{fb}(x, y, z, t), S_{pm}(x, y, z, t)] \quad (4.4)$$

In the context of seakeeping analysis concerning potential flows, the free surface is usually understood as a "membrane"² that segregates water from air at all time. This construction denies the possibility of breaking waves, since the membrane is assumed to be simply connected, an important hypothesis because the boundary region is simplified to a geometrical surface.

¹It should be noticed that even the fixed boundaries can change its wetted surface in time if waves or current are acting, so the notation "fixed" only concerns the motions of the solid boundary.

²Strictly speaking is not a "membrane" because there is no elasticity

The free surface elevation is measured from the undisturbed surface using a variable η , the z coordinate of the air-water interface, as shown in Figure (4.1).

The main goal is to find out a mathematical surface that describes this "membrane", assuming that no fluid particles can cross the "membrane", therefore any motion of the particles in the surface normal direction deforms it in order to keep the membrane always segregating the two phases.

Figure 4.1: Free surface

Suppose a generic surface given by (4.5). It can be expanded using Taylor series as shown in (4.6), where $\vec{v}_s$ is the surface velocity with components (v_x, v_y, v_z) .

$$S(x, y, z, t) = 0, \forall t \quad (4.5)$$

$$S(x + v_x \Delta t, y + v_y \Delta t, z + v_z \Delta t, t) = S(x, y, z, t) + \left(\frac{\partial S}{\partial t} + \vec{v}_s \cdot \nabla S \right) \Delta t + O(\Delta t^2) + O(\Delta t^3) + \dots \quad (4.6)$$

If we divide the expansion by Δt and take the limit case when Δt goes to zero, assuming all surface derivatives as finite, the equation (4.8) is obtained, since (4.5) is true at all times.

$$\lim_{\Delta t \rightarrow 0} \frac{S(x + v_x \Delta t, y + v_y \Delta t, z + v_z \Delta t, t) - S(x, y, z, t)}{\Delta t} = \lim_{\Delta t \rightarrow 0} \left[\frac{S(x, y, z, t) - S(x, y, z, t)}{\Delta t} + \frac{\left(\frac{\partial S}{\partial t} + \vec{v}_s \cdot \nabla S \right) \Delta t + O(\Delta t^2) + \dots}{\Delta t} \right] \quad (4.7)$$

$$\frac{\partial S}{\partial t} + \vec{v}_s \cdot \nabla S = 0 \quad (4.8)$$

Since the fluid has no viscosity, the essential relation of a generic fluid particle at the boundary

is that the projection of the particle velocity vector on the free surface normal direction should be the same as the projection of the surface velocity vector at the surface normal direction, as given by (4.9). Here $\vec{v}_p$ is the flow velocity at a point P adjacent to the surface S and $\vec{n}_s$ is the surface normal vector, which is equal to the surface gradient.

$$\vec{v}_p \cdot \vec{n}_s = \vec{v}_s \cdot \vec{n}_s \Rightarrow \vec{v}_p \cdot \nabla S = \vec{v}_s \cdot \nabla S, \quad P \in S(x, y, z, t) \quad (4.9)$$

Applying (4.8) at the right side of (4.9) and changing the flow velocity by the gradient of the velocity potential function leads to (4.10).

$$\nabla \varphi_p \cdot \nabla S_p = -\frac{\partial S}{\partial t} \Big|_p, \quad \forall t, \quad P \in S(x, y, z, t) \quad (4.10)$$

The free surface can be written as (4.11) and replacing it in equation (4.10) leads to condition (4.12).

$$S(x, y, z, t) = z - \eta(x, y, t) = 0 \quad (4.11)$$

$$\frac{\partial \eta_p}{\partial t} = \nabla \varphi_p \cdot \nabla S_p, P \in S(x, y, z, t) \Rightarrow \frac{\partial \eta}{\partial t} + \frac{\partial \varphi}{\partial x} \frac{\partial \eta}{\partial x} + \frac{\partial \varphi}{\partial y} \frac{\partial \eta}{\partial y} - \frac{\partial \varphi}{\partial z} = 0, \quad z = \eta(x, y, t) \quad (4.12)$$

This is known as the free surface kinematic condition and states that particles at the free surface will be always at the free surface, because the velocity of the fluid particles adjacent to the membrane on the surface normal direction should be the same of the surface normal velocity itself, which is assumed by construction. Therefore, the particles cannot "drop" from the surface.

The description of the free surface as (4.11) denies the possibility of overturning waves (waves that are almost breaking, see Figure (4.2)) because an elevation function η is assumed and it is single valued in space.

Figure 4.2: Sketch of an overturning wave

The first expression in (4.12) states the Neumann condition for the free surface, but since $\frac{\partial \eta}{\partial t}$ is unknown, the kinematic condition is not enough for defining the "membrane" motions in time. An additional equation is derived by applying Bernoulli's equation (4.2) in order to study the membrane dynamics, which is done by assuming the pressure at the free surface as atmospheric, thus equation (4.13) is obtained. This statement is known as the dynamic free surface condition and describes the equilibrium of forces at the air-water interface.

$$\frac{\partial \varphi}{\partial t} + \frac{1}{2}(\nabla \varphi \cdot \nabla \varphi) + g\eta = 0 \quad \text{at} \quad z = \eta(x, y, t) \quad (4.13)$$

In the same way as already discussed for the prescribed motion and floating body boundaries, the free surface conditions are non-linear, being difficult to find an easy solution procedure. The floating body wetted surface, at all times, can only be defined by solving Newton's second law applied to an inertial frame, equations (4.14) and (4.15), assuming the body mass (m) time independent and the external loads as the gravity, flow pressure and external damping/restoration³. Here $\vec{L}_0$ is the angular momentum considering a pole O and $\vec{M}_0^{ext}$ is the external torque considering the same pole.

$$\frac{d(m\vec{v}_G)}{dt} = m \frac{d\vec{v}_G}{dt} = \sum \vec{F}^{ext} \quad (4.14)$$

$$\frac{d\vec{L}_0}{dt} = \sum \vec{M}_O^{ext} \quad (4.15)$$

The body motion equations are formulated in the inertial frame, assuming a coordinate system located (OXYZ) at the CoG⁴ mean position with a time-independent basis vector. An

³These additional terms could be applied, for instance, to emulate simplified linear damping effects due to viscosity or restoration forces due to springs or mooring lines in the vessel.

⁴Center of gravity

additional non-inertial coordinate frame (oxyz) is positioned in the instantaneous CoG (fixed to the body) with a time-dependent basis vector, as shown in Figure 4.3. These coordinate systems are coincident at $t=0$ s with $\vec{O}o$ as the translation motions (surge, sway and heave) and the angle between the vectors $\vec{O}\vec{Y}$ and $\vec{o}\vec{y}$ (or $\vec{O}\vec{Z}$ and $\vec{o}\vec{z}$) is the roll angle, while the angle between $\vec{O}\vec{X}$ and $\vec{o}\vec{x}$ (or $\vec{O}\vec{Z}$ and $\vec{o}\vec{z}$) is assumed as pitch (the angle is measured after the roll is computed) and the angle between $\vec{O}\vec{X}$ and $\vec{o}\vec{x}$ (or $\vec{O}\vec{Y}$ and $\vec{o}\vec{y}$) the yaw angle (after both roll and pitch are computed).

Figure 4.3: Coordinates systems.

The transformation matrix from oxyz to OXYZ is given by equation (4.16), where $\vec{r}_{ni}$ is the vector coordinates in system oxyz, $\vec{r}_i$ the coordinates in system OXYZ and $[T]_{ni \rightarrow i}$ the transformation matrix expressed in (4.17), where α_x , α_y and α_z are the roll, pitch and yaw angles, respectively.

$$\vec{r}_i = \vec{O}o + [T]_{ni \rightarrow i} \vec{r}_{ni} \quad (4.16)$$

$$[T]_{ni \rightarrow i} = \begin{bmatrix} \cos \alpha_z & -\sin \alpha_z & 0 \\ \sin \alpha_z & \cos \alpha_z & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos \alpha_y & 0 & -\sin \alpha_y \\ 0 & 1 & 0 \\ -\sin \alpha_y & 0 & \cos \alpha_y \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \alpha_x & -\sin \alpha_x \\ 0 & -\sin \alpha_x & \cos \alpha_x \end{bmatrix} =$$

$$\begin{bmatrix} \cos \alpha_z \cdot \cos \alpha_y & \cos \alpha_z \cdot \sin \alpha_y \cdot \sin \alpha_x - \sin \alpha_z \cdot \cos \alpha_x & \cos \alpha_z \cdot \sin \alpha_y \cdot \cos \alpha_x + \sin \alpha_z \cdot \sin \alpha_x \\ \sin \alpha_z \cdot \cos \alpha_y & \sin \alpha_z \cdot \sin \alpha_y \cdot \sin \alpha_x + \cos \alpha_z \cdot \cos \alpha_x & \sin \alpha_z \cdot \sin \alpha_y \cdot \cos \alpha_x - \cos \alpha_z \cdot \sin \alpha_x \\ -\sin \alpha_y & \cos \alpha_y \cdot \sin \alpha_x & \cos \alpha_y \cdot \cos \alpha_x \end{bmatrix} \quad (4.17)$$

The translational motions are calculated based on equation (4.14) while the rotational ones are calculated in the inertial frame OXYZ using equation (4.15), which is not convenient because the body motions would lead to a time dependent inertia matrix, which may be difficult to re-compute in time domain. In order to overcome this problem, the transformation matrix described earlier is used to transfer the angular momentum from the inertial frame (OXYZ) to the non-inertial one (oxyz), as shown in (4.18) where the quantities with prime are computed in the non-inertial frame, since the inertia matrix is time independent in this coordinate system. Here $\vec{x}_P$ and $\vec{x}_G$ are the vectors of an arbitrary point P and the center of gravity G in relation to the origin O in the inertial frame, $\vec{v}_P$ is the velocity vector of point P also in the inertial frame.

$$\begin{aligned} \frac{d\vec{L}_G}{dt} &= \frac{d}{dt} \left[\iiint (\vec{x}_P - \vec{x}_G) \wedge \vec{v}_P dm_p \right] = \frac{d}{dt} \left[[T]_{ni \rightarrow i} \iiint (\vec{x}'_P - \vec{x}'_G) \wedge \vec{v}'_P dm_p \right] = \\ \frac{d[T]_{ni \rightarrow i}}{dt} \left[\iiint (\vec{x}'_P - \vec{x}'_G) \wedge \vec{v}'_P dm_p \right] &+ [T]_{ni \rightarrow i} \frac{d}{dt} \left[\iiint (\vec{x}'_P - \vec{x}'_G) \wedge \vec{v}'_P dm_p \right] = \sum \vec{M}_G^{ext} \quad (4.18) \end{aligned}$$

The second term is changed by the product of the moment of inertia by the angular acceleration in the non-inertial frame times the transformation matrix, shown in (4.19), where $[I_{ni}]$ is the inertia matrix given by equation (4.20) evaluated in the non-inertial coordinate system, with $I_{xx}, I_{yy}, I_{zz}, I_{xy}, I_{xz}, I_{yz}$ evaluated according to (4.21)⁵.

$$[T]_{ni \rightarrow i} \frac{d}{dt} \left[\iiint (\vec{x}'_P - \vec{x}'_G) \wedge \vec{v}'_P dm_p \right] = [T]_{ni \rightarrow i} [I_{ni}] \{\dot{\vec{\omega}}'\} \quad (4.19)$$

$$[I_{ni}] = \begin{bmatrix} I_{xx} & -I_{xy} & I_{xz} \\ -I_{xy} & I_{yy} & -I_{yz} \\ -I_{xz} & I_{yz} & I_{zz} \end{bmatrix} \quad (4.20)$$

$$\begin{aligned} I_{xx} &= \iiint [(y_p - y_G)^2 + (z_p - z_G)^2] dm_p & I_{xy} &= \iiint [(x_p - x_G)(y_p - y_G)] dm_p \\ I_{yy} &= \iiint [(x_p - x_G)^2 + (z_p - z_G)^2] dm_p & I_{xz} &= \iiint [(x_p - x_G)(z_p - z_G)] dm_p \\ I_{zz} &= \iiint [(x_p - x_G)^2 + (y_p - y_G)^2] dm_p & I_{yz} &= \iiint [(y_p - y_G)(z_p - z_G)] dm_p \end{aligned} \quad (4.21)$$

The first term in equation (4.18) is changed by the cross product given in equation (4.22), where the angular velocity vector $\vec{\omega}'$ is represented in the non-inertial frame combined to (4.19) and

⁵Some approximations for the main diagonal terms of inertia matrix are available in the literature for practical problems, see for instance Journee and Massie [2001] regarding merchant ships.

multiplied by $[T]_{ni \rightarrow i}^{-1} = [T]_{i \rightarrow ni}$ to provide equation (4.23).

$$\frac{d[T]_{ni \rightarrow i}}{dt} \left[\iiint (\vec{x}'_P - \vec{x}'_G) \wedge \vec{v}'_P dm \right] = [T]_{ni \rightarrow i} \{\vec{\omega}'\} \wedge [I_{ni}] \{\vec{\omega}'\} \quad (4.22)$$

$$[I_{ni}] \{\dot{\vec{\omega}}'\} + \{\vec{\omega}'\} \wedge [I_{ni}] \{\vec{\omega}'\} = [T]_{i \rightarrow ni} \sum \vec{M}_G^{ext} \quad (4.23)$$

The detailed deduction of the body dynamic equations can be found, for instance, in Ogilvie [1983], Kim and Yue [1990], Faltinsen [2005] and Shao [2010].

The boundary value problem (BVP) can be summarized in solving Laplace's equation (4.1) under the boundary conditions given by (4.3), (4.4) and (4.12). The dynamic free surface condition (4.13) and body motions equations (4.14)/(4.23) are applied for the definition of the time dependent surface boundaries.

Since the problem containing non-linear boundary conditions applied at unknown boundaries is hard to solve directly, a perturbation technique is usually adopted for turning the original non-linear BVP into a sequential series of linear problems. The application of this technique and the main assumptions involved are discussed next.

4.2 Stokes expansion

The Stokes expansion procedure is performed considering that all quantities (variables) concerning the problem (velocity potential, free surface elevation, normal vectors, moving bodies position vectors etc.) can be expanded using an unidimensional Stokes series, that is essentially a multi-scales expansion using a perturbation factor ϵ (wave-steepness). In this expansion the zeroth order terms refer to the steady state of the system⁶. It could be argued that, since there are two non-dimensional numbers governing the problem dynamics, namely the Keulegan-Carpenter and Froude number, a two dimensional Stokes series should be assumed. However, we are only interested in low-velocity effects, which means that only the effects of the current in the waves due to the steady pattern of the flow will be considered, which is the most relevant one in the asymptotic expansion.

This perturbation factor ϵ is actually kA , the wave steepness, where k is the wave number and A the wave amplitude, a parameter that should be small in order to guarantee wave stability (see for instance Stoker [1957]).

⁶In the context of the seakeeping model, they will be not null only when current effects are taken into consideration.

The quantities are expanded in Stokes series as described in equations (4.24) to (4.28), with the zeroth order term taking into account the steady state effect, which is the system equilibrium condition, so time independent. The $\vec{X}$ function in equation (4.27) and $\vec{\alpha}$ function in equation (4.28) denote the body linear and angular displacements and the normal vector can be expanded in Stokes series and computed as the gradient of the several order surfaces. The method proposes the problem to be solved by splitting the original problem into a collection of linear problems (one for each order), solving them successively by imposing the solutions of the lower orders problem into the higher order ones.

$$\varphi(x, y, z, t) = \varphi^{(0)}(x, y, z) + \sum_{i=1}^{\infty} \varphi^{(i)}(x, y, z, t) \cdot \epsilon^i \quad (4.24)$$

$$\eta(x, y, t) = \eta^{(0)}(x, y) + \sum_{i=1}^{\infty} \eta^{(i)}(x, y, t) \cdot \epsilon^i \quad (4.25)$$

$$\vec{n}(x, y, z, t) = \vec{n}^{(0)}(x, y, z) + \sum_{i=1}^{\infty} \vec{n}^{(i)}(x, y, z, t) \cdot \epsilon^i \quad (4.26)$$

$$\vec{X}(t) = \vec{X}^{(0)} + \sum_{i=1}^{\infty} \vec{X}^{(i)}(t) \cdot \epsilon^i \quad (4.27)$$

$$\vec{\alpha}(t) = \vec{\alpha}^{(0)} + \sum_{i=1}^{\infty} \vec{\alpha}^{(i)}(t) \cdot \epsilon^i \quad (4.28)$$

The required condition is that the function should be analytic, so the function can be described as series with terms that asymptotically converge to zero and the entire sum converges to a single value. Therefore, all individual potentials φ_i must be finite, meaning they are bounded by a positive real M ($|\varphi_i(x, y, z, t)| \leq M, M \in \mathcal{R}^+, i = 1, 2, 3, \dots, \infty$) and the perturbation factor modulus to be less than 1 since this series is bounded by a geometrical series, as shown in (4.29). This convergence condition is obviously extended to the remaining expansions. In practice, there are several offshore problems where this condition does not hold, for example, the slow motions of floating platforms that are much larger than the first order ones.

$$\sum_{i=0}^{\infty} \varphi_i(x, y, z, t) \epsilon^i \leq \sum_{i=0}^{\infty} M \epsilon^i = M \sum_{i=0}^{\infty} \epsilon^i \stackrel{|\epsilon| \leq 1}{=} \frac{M}{1 - \epsilon} \quad (4.29)$$

The rigid body velocity vectors can be written by time derivation of (4.27) and (4.28),

providing (4.30) and (4.31). The mean velocities are zero because the zeroth order terms are time independent.

$$\vec{V}(t) = \sum_{i=1}^{\infty} \vec{V}^{(i)}(t) \cdot \epsilon^i \quad (4.30)$$

$$\vec{\omega}(t) = \sum_{i=1}^{\infty} \vec{\omega}^{(i)}(t) \cdot \epsilon^i \quad (4.31)$$

4.2.1 The Zeroth order problem

As discussed before, the boundary conditions are defined at the instantaneous boundary, which is not convenient for the solution of the problem. In order to overcome this inconvenient, the quantities are expanded in Taylor series around the mean surfaces. In this procedure, the free surface mean position is the zeroth order elevation, which is unknown, a priori. Nonetheless, if we expand the velocity potential around the unknown zeroth order elevation (4.32) is obtained and the only zero order that remain are those related to $\varphi^{(0)}(x, y, z)$.

$$\varphi|_{z=\eta} = \varphi|_{z=\eta^{(0)}} + \sum_{j=1}^{\infty} \frac{\partial^j \varphi}{\partial z^j} \Big|_{z=\eta^{(0)}} \frac{(\eta - \eta^{(0)})^j}{j!} = \sum_{i=0}^{\infty} \epsilon^i \varphi^{(i)}|_{z=\eta^{(0)}} + \sum_{j=1}^{\infty} \sum_{i=0}^{\infty} \epsilon^i \frac{\partial^j \varphi^{(i)}}{\partial z^j} \Big|_{z=\eta^{(0)}} \frac{(\sum_{k=1}^{\infty} \epsilon^k \eta^{(k)})^j}{j!} \quad (4.32)$$

Replacing the zero order quantities in the dynamic free surface condition (4.13) considering a point in the infinite, where there is no disturbance due to the floating body (only due to the current of intensity U), provides equation (4.33). Since the zero order potential is time independent, the zero order free surface elevation is given by (4.34), which is, in a first analysis, a non-linear condition that should be solved iteratively. This is the procedure usually assumed in non-linear ship wave resistance computations, when the Froude number is high. However, in this work only small Froude numbers will be investigated and since the steady free surface is $O(Fn^2)$ and assuming that the disturbance of the body is small (even for a bluff one), the steady free surface will be neglected ($\eta^{(0)} = 0$).

$$\frac{\partial \varphi^{(0)}}{\partial t} + \frac{1}{2}(\nabla \varphi^{(0)} \cdot \nabla \varphi^{(0)}) + g\eta^{(0)} = \frac{1}{2}U^2 \quad \text{at } z = \eta^{(0)}(x, y) \quad (4.33)$$

$$\eta^{(0)} = \frac{1}{2g}(U^2 - |\nabla \varphi^{(0)}|^2) \quad \text{at } z = \eta^{(0)}(x, y) \quad (4.34)$$

One of the main difficulties that arise in the computation of ship wave resistance for low Froude

number is that the wave resistance is a second order quantity, so the accuracy is strongly dependent on the discretization method (i.e: low order vs higher order methods).

The kinematic free surface condition can be obtained by the zero order quantities in (4.12), obtaining (4.35), which is a symmetry (mirror) condition on the plane $z=0$.

$$\frac{\partial \eta^{(0)}}{\partial t} + \frac{\partial \varphi^{(0)}}{\partial x} \frac{\partial \bar{\eta}}{\partial x} + \frac{\partial \varphi^{(0)}}{\partial y} \frac{\partial \bar{\eta}}{\partial y} - \frac{\partial \varphi^{(0)}}{\partial z} = 0 \Rightarrow \frac{\partial \varphi^{(0)}}{\partial z} = 0, \quad z = 0 \quad (4.35)$$

The zeroth order body boundary conditions is also obtained replacing the series into (4.3) and (4.4) after expanding the quantities around the mean wetted surface using Taylor series. As an example, the expansion of the velocity vector is shown in (4.36) and taking the limit when $\epsilon \rightarrow 0$ provides the zero order condition (4.37) since there is no zero order velocity term. In (4.37) the bar indicates that the mean wetted surface is considered. The values of $\Delta x, \Delta y$ and Δz are obtained from (4.27) without the zero order value. Therefore all powers of the delta terms on Taylor expansion have at least order ϵ , but the zero power one, which is exactly the value at rest. The other velocity and normal vector components could be expanded in an analogous manner.

$$\begin{aligned} V_x(x_0 + \Delta x, y_0 + \Delta y, z_0 + \Delta z, t) = & V_x(x_0, y_0, z_0, t) + \frac{\partial V_x}{\partial x} \Big|_{(x_0, y_0, z_0)} \Delta x + \frac{\partial V_x}{\partial y} \Big|_{(x_0, y_0, z_0)} \Delta y + \\ & \frac{\partial V_x}{\partial z} \Big|_{(x_0, y_0, z_0)} \Delta z + \dots = \sum_{j=0}^{\infty} \sum_{k=0}^{\infty} \sum_{u=0}^{\infty} \frac{\Delta x^j \Delta y^k \Delta z^u}{j!k!u!} \frac{\partial^{j+k+u} V_x}{\partial x^j \partial y^k \partial z^u} \Big|_{(x_0, y_0, z_0)} \end{aligned} \quad (4.36)$$

$$\nabla \varphi_Q^{(0)} \cdot \vec{n}_Q^{(0)} = 0, \quad \text{for } Q \in \{\bar{S}_{fixed} \cup \bar{S}_{pm} \cup \bar{S}_{fb}\} \quad (4.37)$$

The zero order problem is linear, thus we can consider the flow as a superposition of the basic undisturbed current of intensity U propagating in the x direction and the disturbance due to the body ($\phi^{(0)}$), as presented in (4.38).

$$\varphi^{(0)} = \varphi_{curr} + \phi^{(0)} = Ux + \phi^{(0)} \quad (4.38)$$

The zero order BVP for the disturbance potential is now complete and consists of solving Laplace's equation (4.39) under the boundary conditions given by (4.40) and (4.41).

$$\nabla^2 \varphi^{(0)} = 0 \Rightarrow \nabla^2 \phi^{(0)} = 0, \quad \text{in } \bar{\Omega} \quad (4.39)$$

$$\frac{\partial \phi_Q^{(0)}}{\partial n_Q^{(0)}} = 0, \text{ for } Q \in \bar{S}_{fs} \quad (4.40)$$

$$\frac{\partial \phi_Q^{(0)}}{\partial n_Q^{(0)}} = -Un_x^{(0)}, \text{ for } Q \in \{\bar{S}_{fixed} \cup \bar{S}_{pm} \cup \bar{S}_{fb}\} \quad (4.41)$$

The problem stated has null zero order force acting on a single body simulation in the x, y directions, known as D'Alembert Paradox since the problem solved is analogous to a submerged body in infinite fluid under a potential flow⁷. However this fact is only valid because we are neglecting the higher order effects concerning the current, that would produce a steady wave pattern, which means that energy would be transferred from the body to the flow (irradiated waves) therefore a wave resistance force is present. There are several codes for ship resistance that solve the flow assuming the symmetry condition and then compute the wave resistance from the integration of the second order wave elevation, which can be computed from (4.34). However, in order to keep the formulation consistent (in the context of multiscale effects) this force will not be taken into account in the context of small currents velocities.

The zero order force can be computed from the integration of the body surface pressure, which is also expanded around the mean wetted surface ($\bar{S}_{fb}$), as shown in (4.42). The pressure function is also expanded in a Stokes series (4.43) providing (4.44) and (4.45). The integration of $\nabla \varphi^{(0)} \cdot \nabla \varphi^{(0)}$ in x and y directions is null due to D'Alembert Paradox for a single body and the component in z direction would generate static sinkage and trim, which is usually of interest in ship analysis. However, as discussed earlier this term is proportional to U^2 (or $Fn^2 = U^2/\sqrt{gL}^2$) and since we are interested in small current effects it will be neglected.

$$p|_{s_{fb}(t)} = p_{\bar{S}_{fb}} + \nabla p|_{\bar{S}_{fb}} \cdot \vec{x} + O(|\vec{x}|^2) \quad (4.42)$$

$$p(x, y, z, t) = \sum_{i=0}^{\infty} \epsilon^i p^{(i)}(x, y, z, t) \quad (4.43)$$

$$\vec{F}^{(0)} = \iint_{\bar{S}_{fb}} p_Q^{(0)} \vec{n}_Q^{(0)} dS_Q = -\rho \iint_{\bar{S}_{fb}} \left(\frac{1}{2} \nabla \varphi_Q^{(0)} \cdot \nabla \varphi_Q^{(0)} + gz \right) \vec{n}_Q^{(0)} dS_Q \quad (4.44)$$

$$\vec{M}^{(0)} = \iint_{\bar{S}_{fb}} p_Q^{(0)} [(Q - G) \wedge \vec{n}_Q^{(0)}] dS_Q = -\rho \iint_{\bar{S}_{fb}} \left(\frac{1}{2} \nabla \varphi_Q^{(0)} \cdot \nabla \varphi_Q^{(0)} + gz \right) [(Q - G) \wedge \vec{n}_Q^{(0)}] dS_Q \quad (4.45)$$

The integration of the hydrostatic pressure would provide the buoyancy force and hydrostatic

⁷This is not true in the case of a body close to a wall.

restoring moments in x and y directions. Assuming that the body is in initial equilibrium, then

$$\vec{F}^{(0)} = \rho g V \vec{k} = mg \vec{k}, \quad \vec{M}^{(0)} = \vec{0} \quad (4.46)$$

Now that the steady current effect is considered, the wave-body interaction will be described assuming additional terms of Stokes expansion.

4.2.2 First order problem for the velocity and acceleration potentials

The first order problem will evaluate the wave-current-structure interaction considering only first order quantities related to wave-steepness. Since we are only interested in first order quantities only the terms up to $O(\epsilon)$ need to be evaluated since the product of these quantities would be at least of first order.

One of the biggest challenges in time domain simulations is to achieve the necessary accuracy in the computation of the time derivative of the velocity potential, which is required to guarantee numerical stability. Simple techniques, such as the use of backward numerical differential scheme lead to poor predictions and unstable numerical schemes. Several alternatives have been proposed to overcome this limitation but in this work we follow the "acceleration potential" approach, which consists in the construction of a boundary value problem for the time derivative of the velocity potential, providing a mathematical problem that is analogous to the one for the velocity potential. For a better comprehension, both the velocity and acceleration potential boundary conditions are presented together to stress the similarity of both problems. The essence of the fluid-structure interaction for a free floating body is quite different from the one when the motion is prescribed, because in the latter the position does not depend on the hydrodynamic pressure, which is proportional to the time derivative of the potential function. For a free floating body, on the other hand, this dependence exists and any inaccurate evaluation of the pressure may turn the solution unstable, meaning that, after some time, the simulation will diverge.

In the context of potential flow the velocity field is assumed irrotational, which allows the determination of a potential function for the flow velocity, representing a conservative field, being the velocity completely defined by the values of this function at the boundaries. The same procedure may be proposed for the flow acceleration:

$$\vec{a} = \nabla \Psi \quad (4.47)$$

It is possible to establish an equation between the acceleration potential and the velocity potential, as shown in (4.48).

$$\vec{a} = \frac{\partial \vec{v}}{\partial t} + \nabla \vec{v} \cdot \vec{v} = \frac{\partial(\nabla \varphi)}{\partial t} + \nabla(\nabla \varphi) \cdot \nabla \varphi = \nabla \left(\frac{\partial \varphi}{\partial t} + \frac{1}{2} \nabla \varphi \cdot \nabla \varphi \right) = \nabla \Psi \quad (4.48)$$

The equation (4.49) is non-linear with a difficult convective term, which is also expanded using Stokes series, providing equation (4.50). The local (eulerian) velocity term satisfies Laplace's equations since the function is assumed to be continuous with continuous derivative.

$$\Psi = \frac{\partial \varphi}{\partial t} + \frac{1}{2} \nabla \varphi \cdot \nabla \varphi \quad (4.49)$$

$$\Psi^{(1)} = \frac{\partial \varphi^{(1)}}{\partial t} + \nabla \varphi^{(0)} \cdot \nabla \varphi^{(1)} \quad (4.50)$$

$$\nabla^2 \left(\frac{\partial \varphi^{(1)}}{\partial t} \right) = 0, \text{ in } \Omega(t) \quad (4.51)$$

The free surface conditions can be evaluated considering the zero and first order potentials expanded around the undisturbed free surface ($z=0$) as shown in (4.52), which are replaced in the kinematic and dynamic conditions, as shown in (4.53) and (4.54).

$$\varphi|_{z=\eta} = \varphi^{(0)}|_{z=0} + \epsilon \left[\varphi^{(1)}|_{z=0} + \frac{\partial \varphi^{(0)}}{\partial z} \Big|_{z=0} \eta^{(1)} \right] + O(\epsilon^2) \quad (4.52)$$

$$\begin{aligned} & \frac{\partial(\eta^{(0)} + \epsilon \eta^{(1)})}{\partial t} + \frac{\partial(\varphi^{(0)} + \epsilon \frac{\partial \varphi^{(0)}}{\partial z} \eta^{(1)} + \epsilon \varphi^{(1)})}{\partial x} \frac{\partial(\eta^{(0)} + \epsilon \eta^{(1)})}{\partial x} + \\ & \frac{\partial(\varphi^{(0)} + \epsilon \frac{\partial \varphi^{(0)}}{\partial z} \eta^{(1)} + \epsilon \varphi^{(1)})}{\partial y} \frac{\partial(\eta^{(0)} + \epsilon \eta^{(1)})}{\partial y} - \frac{\partial(\varphi^{(0)} + \epsilon \frac{\partial \varphi^{(0)}}{\partial z} \eta^{(1)} + \epsilon \varphi^{(1)})}{\partial z} = 0, \\ & z = 0 \end{aligned} \quad (4.53)$$

$$\begin{aligned} & \frac{\partial(\varphi^{(0)} + \epsilon \varphi^{(1)})}{\partial t} + g(\eta^{(0)} + \epsilon \eta^{(1)}) + \frac{1}{2} \nabla(\varphi^{(0)} + \epsilon \frac{\partial \varphi^{(0)}}{\partial z} \eta^{(1)} + \epsilon \varphi^{(1)}) \cdot \nabla(\varphi^{(0)} + \epsilon \frac{\partial \varphi^{(0)}}{\partial z} \eta^{(1)} + \epsilon \varphi^{(1)}) \\ & = \frac{1}{2} U^2, \text{ at } z = 0 \end{aligned} \quad (4.54)$$

Since $\frac{\partial \varphi^{(0)}}{\partial z} = \frac{\partial \varphi^{(0)}}{\partial t} = \eta^{(0)} = \frac{\partial \eta^{(0)}}{\partial t} = 0$, we divide both equations by ϵ and take the limit when $\epsilon \rightarrow 0$ leading to the kinematic and dynamic free surface first order conditions (4.55) and (4.56), respectively. It should be noticed that the effect of the steady flow in the waves is considered due to the zero order potential derivatives in space.

$$\frac{\partial \eta^{(1)}}{\partial t} + \frac{\partial \varphi^{(0)}}{\partial x} \frac{\partial \eta^{(1)}}{\partial x} + \frac{\partial \varphi^{(0)}}{\partial y} \frac{\partial \eta^{(1)}}{\partial y} - \frac{\partial \varphi^{(1)}}{\partial z} - \eta^{(1)} \frac{\partial^2 \varphi^{(0)}}{\partial z^2} = 0, \text{ at } z = 0 \quad (4.55)$$

$$\frac{\partial \varphi^{(1)}}{\partial t} + g\eta^{(1)} + \nabla \varphi^{(0)} \cdot \nabla \varphi^{(1)} = 0, \text{ at } z = 0 \quad (4.56)$$

The boundary conditions are obtained in relation to the mean wetted surface by expanding the quantities around this mean surface. The zero and first order potentials can be expanded around the mean wetted surface using Taylor series as shown in (4.57), where $\vec{x}^{(1)}$ is the first order motion of a specific point in the surface, that can be evaluated from the rigid body translations and rotations.

$$\varphi|_S = \varphi^{(0)}|_{\bar{S}} + \epsilon[\varphi^{(1)}|_{\bar{S}} + \nabla(\varphi^{(0)})|_{\bar{S}} \cdot \vec{x}^{(1)}] + O(\epsilon^2) \quad (4.57)$$

Replacing these quantities in the fixed body boundary condition dividing by ϵ and taking the limit for $\epsilon \rightarrow 0$ leads to (4.58), which is simplified into (4.59) since the body is fixed therefore $\vec{n}^{(1)} = \vec{x}^{(1)} = \vec{0}$.

$$\nabla \varphi_Q^{(1)} \cdot \vec{n}_Q^{(0)} + \nabla \varphi_Q^{(0)} \cdot \vec{n}_Q^{(1)} + \nabla[(\vec{x}_Q^{(1)} \cdot \nabla) \varphi_Q^{(0)}] \cdot \vec{n}_Q^{(0)} = 0, \quad Q \in \bar{S}_{fixed} \quad (4.58)$$

$$\nabla \varphi_Q^{(1)} \cdot \vec{n}_Q^{(0)} = 0, \quad Q \in \bar{S}_{fixed} \quad (4.59)$$

The boundary condition for the local acceleration potential problem can be obtained by an analogous process

$$\nabla \left(\frac{\partial \varphi_Q^{(1)}}{\partial t} \right) \cdot \vec{n}_Q^{(0)} = 0, \quad Q \in \bar{S}_{fixed} \quad (4.60)$$

The prescribed motion and floating body boundary condition for the velocity potential problem is expressed by (4.61), which can be simplified into (4.63) assuming the normal vector expansion in (4.62). This alternative formulation for the first order normal vector allows the

computation of the same boundary condition demonstrated by Timman and Newman [1962].

$$\nabla\varphi_Q^{(1)}\cdot\vec{n}_Q^{(0)}+\nabla\varphi_Q^{(0)}\cdot\vec{n}_Q^{(1)}+[(\vec{x}_Q^{(1)}\cdot\nabla)\nabla\varphi_Q^{(0)}]\cdot\vec{n}_Q^{(0)}=[\vec{v}_G^{(1)}+\vec{\omega}^{(1)}\wedge(Q^{(0)}-G^{(0)})]\cdot\vec{n}^{(0)}, \quad Q\in\{\bar{S}_{pm}\cup\bar{S}_{fb}\} \quad (4.61)$$

$$\vec{n}=\nabla S=\nabla\bar{S}+\nabla(\nabla\bar{S})\cdot\vec{x}+O(|\vec{x}|^2)\Rightarrow\vec{n}=\vec{n}^{(0)}+\epsilon\nabla(\vec{n}^{(0)})\cdot\vec{x}^{(1)}+O(\epsilon^2)\Rightarrow\vec{n}^{(1)}=(\vec{x}^{(1)}\cdot\nabla)\vec{n}^{(0)} \quad (4.62)$$

$$\nabla\varphi_Q^{(1)}\cdot\vec{n}_Q^{(0)}=[\vec{v}_G^{(1)}+\vec{\omega}^{(1)}\wedge(Q^{(0)}-G^{(0)})]\cdot\vec{n}^{(0)}-[(\nabla\varphi_Q^{(0)}\cdot\nabla)\vec{x}_Q^{(1)}-(\vec{x}_Q^{(1)}\cdot\nabla)\nabla\varphi_Q^{(0)}]\cdot\vec{n}_Q^{(0)}, \quad Q\in\{\bar{S}_{pm}\cup\bar{S}_{fb}\} \quad (4.63)$$

The first term in the right-hand side of (4.63) brings the so-called n-terms, usually defined in the radiation problems of the wave-body interaction without current, while those derived from the second term are called m-terms, as introduced by Ogilvie and Tuck [1969]. The m-terms couple the basis flow (current) to the first order motions due to waves. The identity (4.64), which can be verified using some vectorial algebra allows the decomposition of the m-terms into (4.67), which is a simpler condition to be evaluated.

$$[(\nabla\varphi_Q^{(0)}\cdot\nabla)\vec{x}_Q^{(1)}-(\vec{x}_Q^{(1)}\cdot\nabla)\nabla\varphi_Q^{(0)}]=\nabla\wedge(\vec{x}_Q^{(1)}\wedge\nabla\varphi_Q^{(0)})-\vec{x}_Q^{(1)}\nabla\cdot\nabla\varphi_Q^{(0)}+\nabla\varphi_Q^{(0)}\nabla\cdot\vec{x}_Q^{(1)} \quad (4.64)$$

$$\vec{x}^{(1)}\nabla^2\varphi^{(0)}=0 \quad (4.65)$$

$$\nabla\varphi^{(0)}\nabla\cdot\vec{x}^{(1)}=0 \quad (4.66)$$

$$\nabla\varphi_Q^{(1)}\cdot\vec{n}_Q^{(0)}=[\vec{v}_G^{(1)}+\vec{\omega}^{(1)}\wedge(Q^{(0)}-G^{(0)})]\cdot\vec{n}^{(0)}+[\nabla\wedge(\vec{x}^{(1)}\wedge\nabla\varphi_Q^{(0)})]\cdot\vec{n}_Q^{(0)}, \quad Q\in\{\bar{S}_{pm}\cup\bar{S}_{fb}\} \quad (4.67)$$

The first order position vector $\vec{x}_Q^{(1)}$ is given by (4.68), that replaced in the m-terms leads to (4.69).

$$\vec{x}_Q^{(1)}=\vec{X}^{(1)}+\vec{\alpha}^{(1)}\wedge(Q^{(0)}-G^{(0)}) \quad (4.68)$$

$$[\nabla\wedge(\vec{x}^{(1)}\wedge\nabla\varphi_Q^{(0)})]\cdot\vec{n}_Q^{(0)}=[\nabla\wedge(\vec{X}_1\wedge\nabla\varphi_Q^{(0)})]\cdot\vec{n}_Q^{(0)}+\{\nabla\wedge[(\vec{\alpha}_1\wedge(Q-G))\wedge\nabla\varphi_Q^{(0)}]\}\cdot\vec{n}_Q^{(0)} \quad (4.69)$$

After an exhaustive algebraic process, the m-terms can be rewritten as (4.70) and (4.71), which can be reformulated as (4.72), a convenient form from the numerical point of view, as will be discussed later.

$$\{p_1, p_2, p_3\}^T = \begin{cases} -x_G^{(1)} \left(\frac{\partial^2 \varphi^{(0)}}{\partial x^2} n_x^{(0)} + \frac{\partial^2 \varphi^{(0)}}{\partial x \partial y} n_y^{(0)} + \frac{\partial^2 \varphi^{(0)}}{\partial x \partial z} n_z^{(0)} \right) \\ -y_G^{(1)} \left(\frac{\partial^2 \varphi^{(0)}}{\partial y \partial x} n_x^{(0)} + \frac{\partial^2 \varphi^{(0)}}{\partial y^2} n_y^{(0)} + \frac{\partial^2 \varphi^{(0)}}{\partial y \partial z} n_z^{(0)} \right) \\ -z_G^{(1)} \left(\frac{\partial^2 \varphi^{(0)}}{\partial z \partial x} n_x^{(0)} + \frac{\partial^2 \varphi^{(0)}}{\partial z \partial y} n_y^{(0)} + \frac{\partial^2 \varphi^{(0)}}{\partial z^2} n_z^{(0)} \right) \end{cases} = [\{m_1^{(0)}, m_2^{(0)}, m_3^{(0)}\} \cdot \{x_G^{(1)}, y_G^{(1)}, z_G^{(1)}\}]^T \quad (4.70)$$

$$\{p_4, p_5, p_6\}^T = \begin{cases} \alpha_x^{(1)} \left[(y_Q^{(0)} - y_G^{(0)}) p_3^{(0)} - (z_Q^{(0)} - z_G^{(0)}) p_2^{(0)} + n_z^{(0)} \frac{\partial \varphi^{(0)}}{\partial y} - n_y^{(0)} \frac{\partial \varphi^{(0)}}{\partial z} \right] \\ \alpha_y^{(1)} \left[(z_Q^{(0)} - z_G^{(0)}) p_1^{(0)} - (x_Q^{(0)} - x_G^{(0)}) p_3^{(0)} + n_x^{(0)} \frac{\partial \varphi^{(0)}}{\partial z} - n_z^{(0)} \frac{\partial \varphi^{(0)}}{\partial x} \right] \\ \alpha_z^{(1)} \left[(x_Q^{(0)} - x_G^{(0)}) p_2^{(0)} - (y_Q^{(0)} - y_G^{(0)}) p_1^{(0)} + n_y^{(0)} \frac{\partial \varphi^{(0)}}{\partial x} - n_x^{(0)} \frac{\partial \varphi^{(0)}}{\partial y} \right] \end{cases} \quad (4.71)$$

$$[\nabla \wedge (\vec{x}^{(1)} \wedge \nabla \varphi^{(0)})] \cdot \vec{n}_Q^{(0)} = \{m_1^{(0)}, m_2^{(0)}, m_3^{(0)}, m_4^{(0)}, m_5^{(0)}, m_6^{(0)}\} \{x_G^{(1)}, y_G^{(1)}, z_G^{(1)}, \alpha_x^{(1)}, \alpha_y^{(1)}, \alpha_z^{(1)}\}^T \quad (4.72)$$

The boundary condition for the local acceleration potential can be obtained by deriving the velocity condition in time, taking into account that, the time derivative of zero order quantities are null and considering the m-terms vector $\vec{m}_Q^{(0)}$. The resulting condition is expressed in equation (4.73).

$$\nabla \left(\frac{\partial \varphi_Q^{(1)}}{\partial t} \right) \cdot \vec{n}_Q^{(0)} = [\vec{a}_G^{(1)} + \dot{\vec{\omega}}^{(1)} \wedge (Q^{(0)} - G^{(0)})] \cdot \vec{n}_Q^{(0)} + \vec{m}_Q^{(0)} \cdot \{\vec{v}_G^{(1)}, \vec{\omega}^{(1)}\}, \quad Q \in \{\bar{S}_{pm} \cup \bar{S}_{fb}\} \quad (4.73)$$

The main problems concerning the evaluation of the m-terms arise from the fact that the boundary elements methods usually applied to solve the mathematical problem are based on a lower order approach, with the quantities (i.e: potential, normal vector etc.) assumed constant inside each element. This leads to a poor estimative of the tangential derivatives that are essential for the evaluation of the "complete" boundary condition. In order to overcome this limitation, an alternative procedure based on a higher order approach will be presented in the following chapters.

At this point, a note on a different linearization procedure regarding the effects of the steady flow is convenient. The consideration of m-terms as presented above is known in the literature as double-body linearization, since the flow is symmetrical with respect to the $z=0$ plane. An alternative approach usually applied in the seakeeping analysis of ships is to take advantage of the fact that the ship is usually a slender structure, inspired in the thin foil theory from aeronautics. Following this assumption, the steady potential is given by (4.74), where $\delta = B/L$

(B=ship breadth and L = ship length) is a measure of the ship slenderness and $\phi^{(0,1)}$ is at zero order in relation to the Froude number and at first order with respect to the slenderness factor, providing that in the limit when $\delta \rightarrow 0$ the undisturbed flow is recovered.

$$\varphi^{(0)} = Ux + \phi^{(0)} \Rightarrow \phi^{(0)} = \delta\phi^{(0,1)} + O(\delta^2) \quad (4.74)$$

The derivatives of the y and z velocity components in relation to x in the hull surface are proportional to the slenderness of the ship, which leads to the fact that the disturbances in the flow in x direction are an order of magnitude smaller than in y or z, as shown in (4.75).

$$\frac{\Delta y}{\Delta x} \propto \delta; \quad \frac{\Delta z}{\Delta x} \propto \delta \Rightarrow \frac{\frac{\Delta\phi^{(0,1)}}{\Delta x}}{\frac{\Delta\phi^{(0,1)}}{\Delta y}} \propto \frac{\frac{\partial\phi^{(0,1)}}{\partial x}}{\frac{\partial\phi^{(0,1)}}{\partial y}} \propto \delta \Rightarrow \frac{\partial\phi^{(0,1)}}{\partial x} \propto \delta \frac{\partial\phi^{(0,1)}}{\partial y} \quad (4.75)$$

Replacing the expansions in the m-terms expressions and taking the limit when $\delta \rightarrow 0$ leads to (4.76), which is known as the Neumann-Kelvin linearization, which takes advantage of the ship slenderness. This linearization is obviously simpler than the double-body one since it essentially neglects the entire contribution of the disturbance potential into the zero order potential.

$$\{m_1, m_2, m_3, m_4, m_5, m_6\} = \{0, 0, 0, 0, -Un_z^{(0)}, Un_y^{(0)}\} \quad (4.76)$$

In the development of the zero order free surface boundary condition the assumption taken was that the current velocity is small that lead to a no flux condition on $z=0$. On the other hand, by following the Neumann-Kelvin approach, the combination of the dynamic and kinematic free surface conditions can be performed as shown in (4.77), where $\nabla^* = (\partial/\partial x, \partial/\partial y, 0)$. The development of the free surface boundary condition leads to (4.78) and dividing this expression by δ and performing the limit when $\delta \rightarrow 0$ leads to expression (4.79), the so called Neumann-Kelvin free surface condition. The ratio U^2/g is analogous to the square of the Froude number ($Fn = U/\sqrt{Lg}$) but for the ship characteristic length L.

It is important to notice the differences between the linearizations since the double-body one is based on the fact that the current (or forward speed) is small, thus the disturbance in the free surface due to the presence of the body is also small, while the Neumann-Kelvin one prescind from this limitation of small forward speed, but considers that the disturbance in the flow is

small because the body slenderness is high. The linear $O(\delta)$ free surface elevation is then given by (4.80).

$$\eta^{(0)} = \frac{1}{2g}(U^2 - \nabla\varphi^{(0)} \cdot \nabla\varphi^{(0)}) \Rightarrow \frac{1}{2g}\nabla^*\varphi^{(0)} \cdot \nabla^*(\nabla\varphi^{(0)} \cdot \nabla\varphi^{(0)}) + \frac{\partial\varphi^{(0)}}{\partial z} = 0, \text{ in } z = 0 \quad (4.77)$$

$$\begin{aligned} \frac{1}{2g} \left[\left(U + \frac{\partial\phi^{(0)}}{\partial x} \right) \vec{i} + \left(\frac{\partial\phi^{(0)}}{\partial y} \right) \vec{j} \right] \cdot \left[\left(2U \frac{\partial^2\phi^{(0)}}{\partial x^2} + 2 \frac{\partial\phi^{(0)}}{\partial x} \frac{\partial^2\phi^{(0)}}{\partial x^2} + 2 \frac{\partial\phi^{(0)}}{\partial y} \frac{\partial^2\phi^{(0)}}{\partial x \partial y} + 2 \frac{\partial\phi^{(0)}}{\partial z} \frac{\partial^2\phi^{(0)}}{\partial x \partial z} \right) \vec{i} + \right. \\ \left. \left(2U \frac{\partial^2\phi^{(0)}}{\partial y \partial x} + 2 \frac{\partial\phi^{(0)}}{\partial y} \frac{\partial^2\phi^{(0)}}{\partial y \partial x} + 2 \frac{\partial\phi^{(0)}}{\partial y} \frac{\partial^2\phi^{(0)}}{\partial y^2} + 2 \frac{\partial\phi^{(0)}}{\partial z} \frac{\partial^2\phi^{(0)}}{\partial y \partial z} \right) \vec{j} \right] + \frac{\partial\phi^{(0)}}{\partial z} = 0, \text{ in } z = 0 \quad (4.78) \end{aligned}$$

$$\frac{U^2}{g} \frac{\partial^2\phi^{(0,1)}}{\partial x^2} + \frac{\partial\phi^{(0,1)}}{\partial z} = 0, \text{ in } z = 0 \quad (4.79)$$

$$\eta^{(0,1)} = -\frac{U}{g} \frac{\partial\phi^{(0,1)}}{\partial x}, \text{ in } z = 0 \quad (4.80)$$

Unfortunately, this simplification cannot be adopted in our problem without imposing significant limitations regarding its applicability. Indeed, although there are floating units which are ship-shaped such as the FPSO⁸ platform, there are many other types that cannot be considered slender, for example, spars. Even a FPSO would only be considered slender for flow incidences from bow or stern, which is not always the case. For this reason, the double body linearization will be assumed. It will require the computation of the potential tangential derivatives on the panels, motivating the use of a higher order approach concerning the interpolating functions.

Resuming the formulation of the first order problem, the body forces and moments should also be computed from the mean wetted surface by integrating the flow pressure (4.82) on the mean wetted surface, considering the first order pressure terms (4.81). The integral over the waterline is performed neglecting the quadratic term that is proportional to U^2 ($F n^2$) and must be considered carefully, at first sight it may appear to be a first order quantity. However, replacing the hydrostatic term and performing the integration process leads to terms $O(\epsilon^2)$ which should be neglected in the first order problem (see(4.83)).

$$p^{(1)}|_S = p^{(1)}|_{\bar{S}} + \epsilon \nabla(p^{(0)})|_{\bar{S}} \cdot \vec{x}^{(1)} \quad (4.81)$$

$$\vec{F}^{(1)} = \iint_{\bar{S}} (p^{(1)} \vec{n}^{(0)} + p^{(0)} \vec{n}^{(1)}) dS + \oint_{WL} \int_0^{(\eta^{(1)} - z^{(1)})} p^{(0)} \vec{n}^{(0)} dz \quad (4.82)$$

⁸Floating Production and Storage Offloading Unit

$$\begin{aligned}
\oint_{WL} \int_0^{(\eta^{(1)}-z^{(1)})} p^{(0)} \vec{n}^{(0)} dS &= -\rho g \oint_{WL} \int_0^{(\eta^{(1)}-z^{(1)})} z(\vec{n}^{(0)}|_{z=0} + \frac{\partial \vec{n}^{(0)}}{\partial z} \Big|_{z=0} z) dz dl = \\
&- \rho g \oint_{WL} [\vec{n}^{(0)}|_{z=0} \int_0^{(\eta^{(1)}-z^{(1)})} z dz + \frac{\partial \vec{n}^{(0)}}{\partial z} \Big|_{z=0} \int_0^{(\eta^{(1)}-z^{(1)})} z^2 dz] dl = \\
&- \rho g [\vec{n}^{(0)}|_{z=0} \frac{z^2}{2} \Big|_0^{(\eta^{(1)}-z^{(1)})} + \frac{\partial \vec{n}^{(0)}}{\partial z} \Big|_{z=0} \frac{z^3}{3} \Big|_0^{(\eta^{(1)}-z^{(1)})}] dl_{WL} \quad (4.83)
\end{aligned}$$

The integration of the remaining terms is performed considering two components (1) and (2) as shown in (4.84) only to simplify the development, keeping only first order quantities (4.85), (4.86) and (4.87).

$$\vec{F}^{(1)} = \vec{F}_1^{(1)} + \vec{F}_2^{(1)} \quad (4.84)$$

It can be verified from Gauss theorem that most of the hydrostatic terms (integration of $\rho g(z_G^{(1)} + \alpha_y^{(1)}(x_Q^{(0)} - x_G^{(0)}) - \alpha_x^{(1)}(y_Q^{(0)} - y_G^{(0)}))$) are null but for the components shown in (4.86). However, the components in x and y direction are cancelled by the terms provided by the integration of $\vec{F}_2^{(1)}$ assuming now the first order normal vector given by $\vec{n}^{(1)} = \vec{\alpha}^{(1)} \wedge \vec{n}^{(0)}$.

$$\begin{aligned}
\vec{F}_1^{(1)} &= \iint_{\bar{S}} p^{(1)} \vec{n}^{(0)} dS = \\
&- \rho \iint_{\bar{S}} \left(\frac{\partial \varphi_Q^{(1)}}{\partial t} + \nabla \varphi_Q^{(0)} \cdot \nabla \varphi_Q^{(1)} + g[z_G^{(1)} + \alpha_y^{(1)}(x_Q^{(0)} - x_G^{(0)}) - \alpha_x^{(1)}(y_Q^{(0)} - y_G^{(0)})] \right) \vec{n}_Q^{(0)} dS_Q \\
\vec{F}_1^{(1)} &= -\rho \iint_{\bar{S}} \left(\frac{\partial \varphi_Q^{(1)}}{\partial t} + \nabla \varphi_Q^{(0)} \cdot \nabla \varphi_Q^{(1)} \right) \vec{n}_Q^{(0)} dS_Q + \vec{F}_H^{(1)} \quad (4.85)
\end{aligned}$$

with:

$$\vec{F}_H^{(1)} = -\rho g \left\{ [A_{WL} z_G^{(1)} + \alpha_y^{(1)} \iint_{\bar{S}} (x_Q^{(0)} - x_G^{(0)}) n_z^{(0)} dS - \alpha_x^{(1)} \iint_{\bar{S}} (y_Q^{(0)} - y_G^{(0)}) n_z^{(0)} dS] \vec{k} + \alpha_y^{(1)} V \vec{i} - \alpha_x^{(1)} V \vec{j} \right\} \quad (4.86)$$

$$\vec{F}_2^{(1)} = \iint_{\bar{S}} p^{(0)} \vec{n}^{(1)} dS = \vec{\alpha}^{(1)} \wedge \iint_{\bar{S}} p^{(0)} \vec{n}^{(0)} dS = \vec{\alpha}^{(1)} \wedge \rho g V \vec{k} = \rho g (\alpha_y^{(1)} V \vec{i} - \alpha_x^{(1)} V \vec{j}) \quad (4.87)$$

Therefore the first order forces can be summarized in (4.88), where $S_{35} = \iint_{\bar{S}} (x_Q^{(0)} - x_G^{(0)}) n_z^{(0)} dS$ and $S_{34} = \iint_{\bar{S}} (y_Q^{(0)} - y_G^{(0)}) n_z^{(0)} dS$. Following a similar procedure for the moments would lead

to (4.89), with $S_{44} = \iint_{\bar{S}} (y_Q^{(0)} - y_G^{(0)})^2 n_z^{(0)} dS$, $S_{45} = \iint_{\bar{S}} (x_Q^{(0)} - x_G^{(0)})(y_Q^{(0)} - y_G^{(0)}) n_z^{(0)} dS$ and $S_{55} = \iint_{\bar{S}} (x_Q^{(0)} - x_G^{(0)})^2 n_z^{(0)} dS$.

$$\vec{F}^{(1)} = -\rho \iint_{\bar{S}} \left(\frac{\partial \varphi_Q^{(1)}}{\partial t} + \nabla \varphi_Q^{(0)} \cdot \nabla \varphi_Q^{(1)} \right) \vec{n}_Q^{(0)} dS_Q - \rho g [A_{WL} z_G^{(1)} + \alpha_y^{(1)} S_{34} - \alpha_x^{(1)} S_{35}] \vec{k} \quad (4.88)$$

$$\begin{aligned} \vec{M}^{(1)} = & -\rho \iint_{\bar{S}} \left(\frac{\partial \varphi_Q^{(1)}}{\partial t} + \nabla \varphi_Q^{(0)} \cdot \nabla \varphi_Q^{(1)} \right) (\vec{r}_{PQ} \wedge \vec{n}_Q^{(0)}) dS_Q - \{[(\rho g S_{44} + \rho g V z_B^{(0)} - m g z_G^{(1)}) \alpha_x^{(1)} - \rho g S_{45} \alpha_y^{(1)} - \\ & (\rho g V x_B^{(0)} + m g x_G^{(0)}) \alpha_z^{(1)}] \vec{i} + [(\rho g S_{55} + \rho g V z_B^{(0)} - m g z_G^{(1)}) \alpha_y^{(1)} + (-\rho g V y_B^{(0)} + m g y_G^{(0)}) \alpha_z^{(1)}] \vec{j}\} \end{aligned} \quad (4.89)$$

The first order motion equation in the 6 DoF is evaluated following equation (4.90), where $[M]$ is the mass matrix, $[C_{ext}^{(0)}]$ is the external damping matrix, $[K_{ext}^{(0)}]$ is the external stiffness matrix. The latter may be a non-zero matrix if one needs to include linear external stiffness effects, as the ones that represent the moorings of a ship or floating platform. A non-zero linear damping matrix may be used, for example, in order to approximate viscous effects (linearized), a common procedure in the seakeeping analysis performed in frequency domain.

$$\begin{aligned} [M] \{ \ddot{\vec{X}}^{(1)} \} + [C_{ext}^{(0)}] \{ \dot{\vec{X}}^{(1)} \} + [K_{ext}^{(0)}] \{ \vec{X}^{(1)} \} = & \begin{Bmatrix} \vec{F}^{(1)} \\ \vec{M}^{(1)} \end{Bmatrix} = \\ & \begin{Bmatrix} -\rho \iint_{\bar{S}} \left(\frac{\partial \varphi_Q^{(1)}}{\partial t} + \nabla \varphi_Q^{(0)} \cdot \nabla \varphi_Q^{(1)} \right) \vec{n}_Q^{(0)} dS_Q - \rho g [A_{WL} z_G^{(1)} + \alpha_y^{(1)} S_{34} - \alpha_x^{(1)} S_{35}] \vec{k}; \\ -\rho \iint_{\bar{S}} \left(\frac{\partial \varphi_Q^{(1)}}{\partial t} + \nabla \varphi_Q^{(0)} \cdot \nabla \varphi_Q^{(1)} \right) (\vec{r}_{PQ} \wedge \vec{n}_Q^{(0)}) dS_Q - \{[(\rho g S_{44} + \rho g V z_B^{(0)} - m g z_G^{(1)}) \alpha_x^{(1)} - \\ - \rho g S_{45} \alpha_y^{(1)} - (\rho g V x_B^{(0)} + m g x_G^{(0)}) \alpha_z^{(1)}] \vec{i} + [(\rho g S_{55} + \rho g V z_B^{(0)} - m g z_G^{(1)}) \alpha_y^{(1)} + \\ (-\rho g V y_B^{(0)} + m g y_G^{(0)}) \alpha_z^{(1)}] \vec{j}\} \end{Bmatrix} \end{aligned} \quad (4.90)$$

The angular motion equation is evaluated in the non-inertial frame in order to avoid the computation of a time-dependent inertia matrix, as presented in (4.23), now generalized considering external damping and stiffness matrices:

$$[I_{ni}] \{ \dot{\vec{\omega}}' \} + \{ \vec{\omega}' \} \wedge [I_{ni}] \{ \vec{\omega}' \} + [C_{ext}] \{ \vec{\omega}' \} + [K_{ext}] \{ \vec{\alpha}' \} = [T]_{i \rightarrow ni} \vec{M}_G^{ext} = \vec{M}' \quad (4.91)$$

The conservation equation is expanded considering only first order quantities, as shown in (4.92), assuming that $\vec{\omega}^{(0)'} = \vec{0}$.

$$[I_{ni}]\{\dot{\vec{\omega}}^{(1)'}\} + [C_{ext}]\{\vec{\omega}^{(1)'}\} + [K_{ext}]\{\vec{\alpha}^{(1)'}\} = \{[T]_{i \rightarrow ni} \vec{M}_G^{ext}\}^{(1)} \quad (4.92)$$

The transformation matrix $[T]_{i \rightarrow ni}$ is evaluated assuming a Taylor series combined to a Stokes one, see for instance the first term expansion in equation (4.93), where a zero and a second order term are expanded. A similar procedure is applied for the several terms in the transformation matrix leading to the zero, first and second order transformation matrices shown in equation (4.94), (4.95) and (4.96).

$$\begin{aligned} \cos(\alpha_z) \cos(\alpha_y) &= \left[\left(\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} \alpha_z^{2n} \right) \left(\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} \alpha_y^{2n} \right) \right] = \\ &= \left[\left(\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} \left(\sum_{j=1}^{\infty} \epsilon^j \alpha_z^{(j)} \right)^{2n} \right) \left(\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} \left(\sum_{j=1}^{\infty} \epsilon^j \alpha_y^{(j)} \right)^{2n} \right) \right] \approx \\ &= \left\{ \left[1 - \frac{(\epsilon \alpha_z^{(1)} + \epsilon^2 \alpha_z^{(2)} + \dots)^2}{2!} \right] \left[1 - \frac{(\epsilon \alpha_y^{(1)} + \epsilon^2 \alpha_y^{(2)} + \dots)^2}{2!} \right] \right\} = 1 - \frac{\epsilon^2}{2} ((\alpha_y^{(1)})^2 + (\alpha_z^{(1)})^2) + \dots \end{aligned} \quad (4.93)$$

$$[T]_{i \rightarrow ni}^{(0)} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (4.94)$$

$$[T]_{i \rightarrow ni}^{(1)} = \begin{bmatrix} 0 & -\alpha_z^{(1)} & \alpha_y^{(1)} \\ \alpha_z^{(1)} & 0 & -\alpha_x^{(1)} \\ -\alpha_y^{(1)} & \alpha_x^{(1)} & 0 \end{bmatrix} \quad (4.95)$$

$$[T]_{i \rightarrow ni}^{(2)} = \begin{bmatrix} 0 & -\alpha_z^{(2)} & \alpha_y^{(2)} \\ \alpha_z^{(2)} & 0 & -\alpha_x^{(2)} \\ -\alpha_y^{(2)} & \alpha_x^{(2)} & 0 \end{bmatrix} - \frac{1}{2} \begin{bmatrix} [(\alpha_y^{(1)})^2 + (\alpha_z^{(1)})^2] & -2\alpha_x^{(1)} \alpha_y^{(1)} & -2\alpha_x^{(1)} \alpha_z^{(1)} \\ 0 & [(\alpha_x^{(1)})^2 + (\alpha_z^{(1)})^2] & -2\alpha_y^{(1)} \alpha_z^{(1)} \\ 0 & 0 & [(\alpha_x^{(1)})^2 + (\alpha_y^{(1)})^2] \end{bmatrix} \quad (4.96)$$

The first order moment in equation (4.90) is computed based on equation (4.97), keeping in

mind that the zero order moment is null, as already shown in (4.46).

$$\vec{M}^{(1)'} = [T]_{i \rightarrow ni}^{(0)} \vec{M}^{(1)} + [T]_{i \rightarrow ni}^{(1)} \vec{M}^{(0)} = [T]_{i \rightarrow ni}^{(0)} \vec{M}^{(1)} \quad (4.97)$$

The angular momentum conservation equation is simplified into equation (4.98). It should be notice that in first order the angular motions in the non-inertial frame are equal to the inertial frame ones since $\{\vec{\omega}^{(1)}\} = [T]_{i \rightarrow ni}^{(0)-1} \{\vec{\omega}^{(1)'}\} = [1]^{-1} \{\vec{\omega}^{(1)'}\} = \{\vec{\omega}^{(1)'}\}$.

$$[I_{ni}] \{\dot{\vec{\omega}}^{(1)'}\} + [C_{ext}] \{\vec{\omega}^{(1)'}\} + [K_{ext}] \{\vec{\alpha}^{(1)'}\} = [T]_{i \rightarrow ni}^{(0)} \vec{M}^{(1)} = \vec{M}^{(1)'} \quad (4.98)$$

The hydrostatic terms proportional to the first order motions are collected together with the external stiffness matrix, providing equation (4.99) regarding first order motions.

$$[M] \{\ddot{\vec{X}}^{(1)}\} + [C_{ext}^{(0)}] \{\dot{\vec{X}}^{(1)}\} + [K^{(0)}] \{\vec{X}^{(1)}\} = \left\{ \begin{array}{l} -\rho \iint_{\bar{S}} \left(\frac{\partial \varphi_Q^{(1)}}{\partial t} + \nabla \varphi_Q^{(0)} \cdot \nabla \varphi_Q^{(1)} \right) \vec{n}_Q^{(0)} dS_Q \\ -\rho \iint_{\bar{S}} \left(\frac{\partial \varphi_Q^{(1)}}{\partial t} + \nabla \varphi_Q^{(0)} \cdot \nabla \varphi_Q^{(1)} \right) (\vec{r}_{PQ} \wedge \vec{n}_Q^{(0)}) dS_Q \end{array} \right\} \quad (4.99)$$

where $[K^{(0)}] = [K_H^{(0)}] + [K_{ext}^{(0)}]$, the first one is the hydrostatic matrix and the second one the external stiffness matrix.

Since the problem is now completely linearized, the first order potential can be decomposed in two (or more) components (4.100), the first one regarding the disturbance in the flow due to the presence of the body ($\phi_{dist}^{(1)}$) and the other one that takes into account the incident undisturbed wave fields $\phi_I^{(1)}$, defined as (4.101) for deep water. Here A_i is the amplitude of the i-th wave component, ω_i is the wave frequency, ω_{ei} is the encounter frequency (4.102), k_i is the wave-number, β_i is the incidence angle and δ_i is the phase. It should that the disturbance potential ($\phi_{dist}^{(1)}$) takes into account all diffraction and radiation effects of the conventional "frequency" domain decomposition.

$$\varphi^{(1)} = \phi_{dist}^{(1)} + \phi_I^{(1)} \quad (4.100)$$

$$\phi_I^{(1)} = \sum_{i=1}^N \frac{A_i g}{\omega_i} e^{k_i z} \cos(k_i x \cos \beta_i + k_i y \sin \beta_i - \omega_{ei} t + \delta_i) \quad (4.101)$$

$$\omega_{ei} = |U k_i \cos(\beta_i) + \omega_i| \quad (4.102)$$

The shallow water simulations for a constant depth are performed considering the undis-

turbed incidence wave field defined in equation (4.103), where h is the water depth.

$$\phi_I^{(1)} = \sum_{i=1}^N \frac{A_i g}{\omega_i} \frac{\cosh[k_i(z+h)]}{\cosh(k_i h)} \cos(k_i x \cos \beta_i + k_i y \sin \beta_i - \omega_i t + \delta_i) \quad (4.103)$$

The first order problem concerning both the velocity and local acceleration potentials coupled to motion equations can be summarized from equation (4.104) to (4.110), already replacing the first/zero order potential decomposition.

$$\nabla^2 \phi_{dist}^{(1)} = 0, \quad \nabla^2 \left(\frac{\partial \phi_{dist}^{(1)}}{\partial t} \right) = 0, \quad \text{in } \bar{\Omega} \quad (4.104)$$

$$\begin{aligned} \frac{\partial \phi_{dist}^{(1)}}{\partial t} + g \eta_{dist}^{(1)} + \left(U + \frac{\partial \phi^{(0)}}{\partial x} \right) \left(\frac{\partial \phi_{dist}^{(1)}}{\partial x} + \frac{\partial \phi_I^{(1)}}{\partial x} \right) + \frac{\partial \phi^{(0)}}{\partial y} \left(\frac{\partial \phi_{dist}^{(1)}}{\partial y} + \frac{\partial \phi_I^{(1)}}{\partial y} \right) + \frac{\partial \phi^{(0)}}{\partial z} \left(\frac{\partial \phi_{dist}^{(1)}}{\partial z} + \frac{\partial \phi_I^{(1)}}{\partial z} \right) + \\ \left(\frac{\partial \phi_I^{(1)}}{\partial t} + g \eta_I^{(1)} \right) = 0, \quad z = 0 \end{aligned} \quad (4.105)$$

$$\begin{aligned} \frac{\partial \eta_{dist}^{(1)}}{\partial t} + \left(U + \frac{\partial \phi^{(0)}}{\partial x} \right) \left(\frac{\partial \eta_{dist}^{(1)}}{\partial x} + \frac{\partial \eta_I^{(1)}}{\partial x} \right) + \frac{\partial \phi^{(0)}}{\partial y} \left(\frac{\partial \eta_{dist}^{(1)}}{\partial y} + \frac{\partial \eta_I^{(1)}}{\partial y} \right) - \frac{\partial \phi_{dist}^{(1)}}{\partial z} - (\eta_{dist}^{(1)} + \eta_I^{(1)}) \frac{\partial^2 \phi^{(0)}}{\partial z^2} - \\ \left(\frac{\partial \phi_I^{(1)}}{\partial z} - \frac{\partial \eta_I^{(1)}}{\partial t} \right) = 0, \quad z = 0 \end{aligned} \quad (4.106)$$

$$\frac{\partial \phi_{distQ}^{(1)}}{\partial n_Q^{(0)}} = - \frac{\partial \phi_{IQ}^{(1)}}{\partial n_Q^{(0)}}, \quad \frac{\partial^2 \phi_{distQ}^{(1)}}{\partial n_Q^{(0)} \partial t} = - \frac{\partial^2 \phi_{IQ}^{(1)}}{\partial n_Q^{(0)} \partial t}, \quad Q \in \bar{S}_{fixed} \quad (4.107)$$

$$\frac{\partial \phi_{distQ}^{(1)}}{\partial n_Q^{(0)}} = [\vec{n}_Q^{(0)}, (Q^{(0)} - G^{(0)}) \wedge \vec{n}_Q^{(0)}] \cdot \{\vec{v}_G^{(1)}, \vec{\omega}^{(1)}\} + \vec{m}_Q^{(0)} \cdot \{\vec{x}_G^{(1)}, \vec{\alpha}^{(1)}\} - \frac{\partial \phi_{IQ}^{(1)}}{\partial n_Q^{(0)}}, \quad Q \in \{\bar{S}_{pm} \cup \bar{S}_{fb}\} \quad (4.108)$$

$$\frac{\partial^2 \phi_{distQ}^{(1)}}{\partial n_Q^{(0)} \partial t} = [\vec{n}_Q^{(0)}, (Q^{(0)} - G^{(0)}) \wedge \vec{n}_Q^{(0)}] \cdot \{\vec{a}_G^{(1)}, \dot{\vec{\omega}}^{(1)}\} + \vec{m}_Q^{(0)} \cdot \{\vec{v}_G^{(1)}, \vec{\omega}^{(1)}\} - \frac{\partial^2 \phi_{IQ}^{(1)}}{\partial n_Q^{(0)} \partial t}, \quad Q \in \{\bar{S}_{pm} \cup \bar{S}_{fb}\} \quad (4.109)$$

$$\begin{aligned} [M] \{ \ddot{X}^{(1)} \} + [C_{ext}^{(0)}] \{ \dot{X}^{(1)} \} + [K^{(0)}] \{ X^{(1)} \} = \\ - \rho \iint_{\bar{S}_{fb}} \left[\frac{\partial \phi_{distQ}^{(1)}}{\partial t} + \frac{\partial \phi_{IQ}^{(1)}}{\partial t} + \left(U + \frac{\partial \phi_Q^{(0)}}{\partial x} \right) \left(\frac{\partial \phi_{distQ}^{(1)}}{\partial x} + \frac{\partial \phi_{IQ}^{(1)}}{\partial x} \right) + \frac{\partial \phi_Q^{(0)}}{\partial y} \left(\frac{\partial \phi_{distQ}^{(1)}}{\partial y} + \frac{\partial \phi_{IQ}^{(1)}}{\partial y} \right) + \right. \\ \left. \frac{\partial \phi_Q^{(0)}}{\partial z} \left(\frac{\partial \phi_{distQ}^{(1)}}{\partial z} + \frac{\partial \phi_{IQ}^{(1)}}{\partial z} \right) \right] \left\{ \begin{matrix} \vec{n}_Q^{(0)} \\ (Q^{(0)} - G^{(0)}) \wedge \vec{n}_Q^{(0)} \end{matrix} \right\} dS_Q \end{aligned} \quad (4.110)$$

4.3 The second order problem

In the context of seakeeping analysis, several non-linear effects are important not only for a question of accuracy, but also to represent the floating body dynamics from a qualitative point of view. One example is the computation of mean and slow drift forces. In the absence of these effects, the first order theory predicts that the body will remain in the same mean position at all times, neglecting an important part of the body's dynamics.

The second order effects can be partially computed from the first order solution, considering the interaction between two arbitrary first order quantities. In ocean engineering, two important effects are the slow and mean drift forces, mentioned above, the first one due to resonant effects in the horizontal motions of moored offshore platforms and the second a steady force that contributes to the mean horizontal offset of the system. The second order problem is presented, including the important approximations assumed in this thesis and some additional works required to perform comparisons with frequency domain computations that will be presented ahead in Chapter 7.

The second order linear momentum conservation is computed from equation (4.111), which is similar to the first order motion equation, where $\ddot{\vec{X}}^{(2)}$ are the second order translational accelerations and $\vec{F}_{ext}^{(2)}$ the external second order forces in surge, sway and heave degrees of freedom. These force are decomposed in external damping/stiffness and hydrostatic/hydrodynamic loads, the first one presented in equation (4.112), where $\vec{F}_{HH}^{(2)}$ are the hydrostatic/hydrodynamic forces, which will be described next.

$$m\ddot{\vec{X}}^{(2)} = \vec{F}_{ext}^{(2)} \quad (4.111)$$

$$[M]\{\ddot{\vec{X}}^{(2)}\} + [C_{ext}]\{\dot{\vec{X}}^{(2)}\} + [K_{ext}]\{\vec{X}^{(2)}\} = \vec{F}_{HH}^{(2)} \quad (4.112)$$

The second order angular momentum conservation is computed based on equation (4.113), assuming the transformation matrices of order 0 and 1 described before and $\vec{M}^{(0)} = \vec{0}$. The angular velocity computed is defined in the non-inertial frame and can be transformed to the inertial one by computing $\{\vec{\omega}^{(2)}\} = [T]_{i \rightarrow ni}^{(0)-1}\{\vec{\omega}^{(2)'}\} + [T]_{i \rightarrow ni}^{(1)-1}\{\vec{\omega}^{(1)'}\}$.

$$\begin{aligned}
[I_{ni}]\{\vec{\omega}^{(2)'}\} + \{\vec{\omega}^{(1)'}\} \wedge [I_{ni}]\{\vec{\omega}^{(1)'}\} + [C_{ext}]\{\vec{\omega}^{(2)'}\} + [K_{ext}]\{\vec{\alpha}^{(2)'}\} = \\
[T]_{i \rightarrow ni}^{(0)} \vec{M}^{(2)} + [T]_{i \rightarrow ni}^{(1)} \vec{M}^{(1)} + [T]_{i \rightarrow ni}^{(2)} \vec{M}^{(0)} = \vec{M}^{(2)} + [T]_{i \rightarrow ni}^{(1)} \vec{M}^{(1)} \quad (4.113)
\end{aligned}$$

The second order forces can be computed for the general force expression (4.114). By following the weakly non-linear hypothesis⁹, the integral over the wetted surface can be split into two components: a mean wetted surface and a periodic component that oscillates along the body waterline considering all quantities expanded using Taylor series around these domains, as show in equations (4.115) and (4.116).

$$\vec{F} = -\rho \iint_{S_B(t)} \left(\frac{\partial \varphi_Q}{\partial t} + \frac{1}{2} \nabla \varphi_Q \cdot \nabla \varphi_Q + g z_Q \right) \vec{n}_Q dS \quad (4.114)$$

$$\varphi|_z = \varphi|_{z=0} + \frac{\partial \varphi}{\partial z} \Big|_{z=0} z + \dots \quad (4.115)$$

$$\varphi|_{S_W} = \varphi|_{\bar{S}_B} + \nabla \varphi \Big|_{\bar{S}_B} \cdot [\vec{x}_Q(t) - \vec{\bar{x}}_Q] + \dots \quad (4.116)$$

Assuming the fundamental hypothesis of low current velocity, the zero order free surface elevation can be neglected as discussed previously for the double flow linearization approximation, therefore the quantities above the mean water level can be collected only up to order one and only the first terms in Taylor series are maintained because the integration of variable z over the water columns would provide terms of order 2.

$$\vec{F}^{(2)} = \iint_{\bar{S}_B} \left(p_Q^{(0)} \vec{n}_Q^{(2)} + p_Q^{(1)} \vec{n}_Q^{(1)} + p_Q^{(2)} \vec{n}_Q^{(0)} \right) dS + \oint_{WL} \int_0^{(\eta_Q - z_Q)} \left(p_Q^{(0)} \vec{n}_Q^{(0)} + p_Q^{(1)} \vec{n}_Q^{(0)} + p_Q^{(1)} \vec{n}_Q^{(0)} \right) dz dl \quad (4.117)$$

The expansions in Taylor and Stokes series of the several quantities, assuming the normal vector $\vec{n}^{(i)} = \alpha^{(i)} \wedge \vec{n}^{(0)}$ is performed for the first term, as shown in (4.118) using Gauss theorem, where $\forall$ is the submerged volume and neglecting the term $\nabla \varphi_Q^{(0)} \cdot \nabla \varphi_Q^{(0)}$ which is proportional

⁹There forces that are usually defined using the stokes series approach instead of the fully non-linear one.

to U^2 .

$$\begin{aligned} \iint_{\bar{S}_B} p^{(0)} \vec{n}_Q^{(2)} dS &= -\rho \vec{\alpha}^{(2)} \wedge \iint_{\bar{S}_B} \left[\frac{1}{2} \nabla \varphi_Q^{(0)} \cdot \nabla \varphi_Q^{(0)} + gz \right] \vec{n}_Q^{(0)} dS \approx \\ &= -\rho g \forall \vec{\alpha}^{(2)} \wedge \vec{k} = \rho g [\forall \alpha_y^{(2)} \vec{i} - \forall \alpha_x^{(2)} \vec{j}] \quad (4.118) \end{aligned}$$

The second term is integrated similarly as show in (4.119), where the term $\vec{x}_Q^{(1)} \cdot \nabla(\nabla \varphi^{(0)}) \cdot \nabla \varphi^{(0)}$ is neglect since it is a term of order U^2 , where $\vec{F}^{(1)} = \iint_{\bar{S}_B} \left(\frac{\partial \varphi_Q^{(1)}}{\partial t} + \nabla \varphi_Q^{(0)} \cdot \nabla \varphi_Q^{(1)} \right) dS$.

$$\begin{aligned} \iint_{\bar{S}_B} p^{(1)} \vec{n}_Q^{(1)} dS &= -\rho \iint_{\bar{S}_B} \left(\frac{\partial \varphi_Q^{(1)}}{\partial t} + \nabla \varphi_Q^{(0)} \cdot \nabla \varphi_Q^{(1)} + \vec{x}_Q^{(1)} \cdot \nabla(\nabla \varphi^{(0)}) \cdot \nabla \varphi^{(0)} + gz_Q \right) \vec{\alpha}^{(1)} \wedge \vec{n}_Q^{(0)} dS \approx \\ &= -\rho \vec{\alpha}^{(1)} \wedge \iint_{\bar{S}_B} \left[\frac{\partial \varphi_Q^{(1)}}{\partial t} + \nabla \varphi_Q^{(0)} \cdot \nabla \varphi_Q^{(1)} + g(z_G^{(1)} + \alpha_y^{(1)}(x_Q^{(0)} - x_G^{(0)}) - \alpha_x^{(1)}(y_Q^{(0)} - y_G^{(0)})) \right] \vec{n}_Q^{(0)} dS \\ &= \vec{\alpha}^{(1)} \wedge \vec{F}^{(1)} - \rho g \alpha^{(1)} \wedge \iint_{\bar{S}_B} \left[z_G^{(1)} + \alpha_y^{(1)}(x_Q^{(0)} - x_G^{(0)}) - \alpha_x^{(1)}(y_Q^{(0)} - y_G^{(0)}) \right] \vec{n}_Q^{(0)} dS \\ &= \vec{\alpha}^{(1)} \wedge \vec{F}^{(1)} - \rho g \alpha^{(1)} \wedge [\alpha_y^{(1)} \forall \vec{i} + \alpha_x^{(1)} \forall \vec{j} + (A_{WL} z_G^{(1)} + \alpha_y^{(1)} S_{34} - \alpha_x^{(1)} S_{35}) \vec{k}] \quad (4.119) \end{aligned}$$

The third term is integrated based on equation (4.120), where the term $\vec{x}_Q^{(2)} \cdot \nabla(\nabla \varphi_Q^{(0)} \cdot \nabla \varphi_Q^{(0)})$ is neglected too and $\vec{F}^{(2)} = -\rho \iint_{\bar{S}_B} \left[\frac{\partial \varphi_Q^{(2)}}{\partial t} + \frac{1}{2} \nabla \varphi_Q^{(1)} \cdot \nabla \varphi_Q^{(1)} + \nabla \varphi_Q^{(0)} \cdot \nabla \varphi_Q^{(2)} + \vec{x}_Q^{(1)} \cdot \nabla \left(\frac{\partial \varphi_Q^{(1)}}{\partial t} \right) + \vec{x}_Q^{(1)} \cdot \nabla(\nabla \varphi_Q^{(0)} \cdot \nabla \varphi_Q^{(1)}) \right] \vec{n}_Q^{(0)} dS$.

$$\begin{aligned} \iint_{\bar{S}_B} p^{(2)} \vec{n}_Q^{(0)} dS &= -\rho \iint_{\bar{S}_B} \left[\frac{\partial \varphi_Q^{(2)}}{\partial t} + \frac{1}{2} \nabla \varphi_Q^{(1)} \cdot \nabla \varphi_Q^{(1)} + \nabla \varphi_Q^{(0)} \cdot \nabla \varphi_Q^{(2)} + \vec{x}_Q^{(1)} \cdot \nabla \left(\frac{\partial \varphi_Q^{(1)}}{\partial t} \right) + \vec{x}_Q^{(1)} \cdot \nabla(\nabla \varphi_Q^{(0)} \cdot \nabla \varphi_Q^{(1)}) + \right. \\ &\quad \left. \vec{x}_Q^{(2)} \cdot \nabla(\nabla \varphi_Q^{(0)} \cdot \nabla \varphi_Q^{(0)}) + gz_Q^{(2)} \right] \vec{n}_Q^{(0)} dS \\ &\approx -\rho \iint_{\bar{S}_B} \left[\frac{\partial \varphi_Q^{(2)}}{\partial t} + \frac{1}{2} \nabla \varphi_Q^{(1)} \cdot \nabla \varphi_Q^{(1)} + \nabla \varphi_Q^{(0)} \cdot \nabla \varphi_Q^{(2)} + \vec{x}_Q^{(1)} \cdot \nabla \left(\frac{\partial \varphi_Q^{(1)}}{\partial t} \right) + \vec{x}_Q^{(1)} \cdot \nabla(\nabla \varphi_Q^{(0)} \cdot \nabla \varphi_Q^{(1)}) \right] \vec{n}_Q^{(0)} dS - \\ &\quad \rho g \iint_{\bar{S}_B} [g(z_G^{(2)} + \alpha_y^{(2)}(x_Q^{(0)} - x_G^{(0)}) - \alpha_x^{(2)}(y_Q^{(0)} - y_G^{(0)}))] \vec{n}_Q^{(0)} dS \\ &= \vec{F}^{(2)} - \rho g [-\alpha_y^{(2)} \forall \vec{i} + \alpha_x^{(2)} \forall \vec{j} + (A_{WL} z_G^{(2)} + \alpha_y^{(2)} S_{34} - \alpha_x^{(2)} S_{35}) \vec{k}] \quad (4.120) \end{aligned}$$

The integral over the waterline is performed as shown in equation (4.121), neglecting the

term $\frac{1}{2}\nabla\varphi_Q^{(0)} \cdot \varphi_Q^{(0)}$ which is proportional to U^2 (Fn^2).

$$\begin{aligned}
\oint_{WL} \int_0^{(\eta_Q - z_Q)} p_Q^{(0)} \vec{n}_Q^{(0)} dz dl &= -\rho \oint_{WL} \int_0^{(\eta_Q - z_Q)} \left(\frac{1}{2} \nabla \varphi_Q^{(0)} \cdot \nabla \varphi_Q^{(0)} + gz \right) dz dl \approx -\rho g \oint_{WL} \int_0^{(\eta_Q - z_Q)} z dz dl = \\
&= -\frac{1}{2} \rho g \oint_{WL} [z^2]_0^{(\eta_Q - z_Q)} dl = -\frac{1}{2} \rho g \oint_{WL} (\eta_Q - z_Q)^2 dl \approx \\
&= -\frac{1}{2} \rho g \oint_{WL} (\eta_Q^{(1)} - [z_G^{(1)} + \alpha_y^{(2)}(x_Q^{(0)} - x_G^{(0)}) - \alpha_x^{(2)}(y_Q^{(0)} - y_G^{(0)})])^2 dl \quad (4.121)
\end{aligned}$$

It should be noticed that the second order forces contain the contribution of the second order potential and the product between first order quantities. The second order potential could be decomposed in a disturbance and incident potential (4.122), following a similar procedure to the first order potential.

$$\varphi^{(2)} = \phi_{dist}^{(2)} + \phi_I^{(2)} \quad (4.122)$$

The incident wave field is calculated following equation (4.123) for deep waters. However the calculation of the disturbed second order potential is quite complex since the second order boundary value problem must be solved using an integral equation coupled to the second order local acceleration potential. Moreover, due to the time domain approach the first order motions change the no-flux boundary conditions, leading to a quite difficult numerical problem because any unbalance in the motion equation can lead to second order instability. Furthermore, the second order incident potential requires a larger domain compared to the first order one since the waves are appreciably larger, which requires more computational capacity then available in Matlab® programming language. Therefore the complete second order problem is not solved in this thesis.

$$\phi_I^{(2)} = \sum_{i=1}^N \left\{ -\frac{3A_i^2 \omega_i}{8} e^{2k_i z} \cos[2(k_i x \cos \beta_i + k_i y \sin \beta_i - \omega_{ei} t + \delta_i)] \right\} \quad (4.123)$$

In the past, when there were even smaller computational resources, several possible approximations were developed (Newman, Pinkster, Aranha and others) to avoid the computation of the complete second order solution, namely the QTF¹⁰, some of these described by Pinkster

¹⁰Quadratic Transfer Function

[1980] and Kim and Yue [1990]. Actually $\phi_{dist}^{(2)}$ will only be important for submerged bodies, see for instance Aranha and Pesce [1986]. In this thesis the second order potential is neglected in the second order loads, simplifying equation (4.118) into (4.124).

$$\begin{aligned} \vec{F}^{(2)} = & -\rho \iint_{\vec{S}_B} \left\{ \frac{1}{2} \nabla \varphi_Q^{(1)} \cdot \varphi_Q^{(1)} + \nabla \frac{\partial \varphi_Q^{(1)}}{\partial t} \cdot [\vec{x}_G^{(1)} + \vec{\alpha}^{(1)} \wedge (Q^{(0)} - G^{(0)})] \right\} \vec{n}_Q^{(0)} dS + \\ & \alpha^{(1)} \wedge \vec{F}^{(1)} - \rho g A_{WL} \{ [z_G^{(2)} + y_{cf} \alpha_x^{(2)} - x_{cf} \alpha_y^{(2)}] + [\alpha_z^{(1)} (x_{cf} \alpha_x^{(1)} + y_{cf} \alpha_y^{(1)})] \} \vec{k} + \\ & \frac{1}{2} \rho g \oint_{WL} \{ \eta^{(1)} - [z_G^{(1)} + \alpha_x^{(1)} (y - y_G) - \alpha_y^{(1)} (x - x_G)] \}^2 \vec{n}^{(0)} dl \quad (4.124) \end{aligned}$$

In the presence of current the mean drift forces can change appreciably, the so called wave-drift damping effect, as described in several references, for instance Wichers [1988], Grue and Biberg [1993] and Aranha [1994]. In the present numerical method the computed forces already take into account the current effects in the mean drift force directly. However, several results in the literature are available in terms of the so-called wave drift damping coefficient, therefore it is computed in this thesis by numerical differential of the mean drift forces close to $U=0$, as shown in equation (4.125), in order to allow some comparisons.

$$\vec{F}^{(2)}(\omega, U) = \vec{F}^{(2)}(\omega, 0) + \left. \frac{\vec{F}^{(2)}(\omega, U)}{\partial U} \right|_{U=0} U + O(U^2) \Rightarrow Bxx = \left. \frac{\vec{F}^{(2)}(\omega, U)}{\partial U} \right|_{U=0} \quad (4.125)$$

The equations and boundary conditions presented so far are enough to guarantee an unique solution to the boundary value problem but the initial value problem is still not solved. The initial conditions for the initial value problem closure are discussed ahead, in section 4.4.

4.4 Initial conditions

The initial conditions for the first order problem must be defined in order to guarantee the solution uniqueness. The conditions concerning the floating body are basically that the initial velocity and body displacements to be null, since the body initially is at rest. The undisturbed free surface is flat $\eta(x, y, 0) = 0$ and the velocities/accelerations are taken as zero.

At the floating bodies, prescribed motion and fixed surfaces the variable to be evaluated in the BVP is the potential function, because the Neumann conditions are known (for the fixed and prescribed motion surfaces) or can be evaluated (for the floating bodies using the equations of motion) at all time. However, for the free surface neither the potential nor the flux are known,

which means that both could be chosen as variables to be evaluated in the BVP. In this work, the choice was for the imposition of a Dirichlet condition (the potential function) at the free surface and the evaluation of the flux, in order to turn the free surface partial differential equations into ordinary ones.

The flux through the free surface is zero and deriving the dynamic free surface condition leads to (4.126), that can be combined with the kinematic condition, providing (4.127). Since the first order elevation is initially null, the flux through the free surface can be computed from (4.128). Replacing this expression for the first order flux condition in the zero flux condition and since the integration domain is fixed in time, the condition (4.129) can be derived.

$$\frac{\partial \varphi^{(1)}}{\partial t} + g\eta^{(1)} + \nabla \varphi^{(0)} \cdot \nabla \varphi^{(1)} = 0 \Rightarrow \frac{\partial^2 \varphi^{(1)}}{\partial t^2} + g \frac{\partial \eta^{(1)}}{\partial t} + \nabla \varphi^{(0)} \cdot \nabla \left(\frac{\partial \varphi^{(1)}}{\partial t} \right) = 0 \quad (4.126)$$

$$\frac{\partial^2 \varphi^{(1)}}{\partial t^2} + g \nabla^* \varphi^{(0)} \cdot \nabla^* \eta^{(1)} + g \frac{\partial \varphi^{(1)}}{\partial z} + \nabla \varphi^{(0)} \cdot \nabla \frac{\partial \varphi^{(1)}}{\partial t} = 0 \quad (4.127)$$

$$\frac{\partial \varphi^{(1)}}{\partial z} = -\frac{1}{g} \left(\frac{\partial^2 \varphi^{(1)}}{\partial t^2} + \nabla \varphi^{(0)} \cdot \nabla \frac{\partial \varphi^{(1)}}{\partial t} \right), \quad t = 0 \quad (4.128)$$

$$\begin{aligned} \iint_{S_{fs}} \left(\frac{\partial \varphi^{(1)}}{\partial n} \right)_{t=0} dS &= \iint_{S_{fs}} \left(\frac{\partial \varphi^{(1)}}{\partial z} \right)_{t=0} dS = -\frac{1}{g} \iint_{S_{fs}} \left(\frac{\partial^2 \varphi^{(1)}}{\partial t^2} + \nabla \varphi^{(0)} \cdot \nabla \frac{\partial \varphi^{(1)}}{\partial t} \right)_{t=0} dS = \\ &= -\frac{1}{g} \left(\frac{\partial^2}{\partial t^2} \iint_{S_{fs}} \varphi^{(1)}|_{fs} dS + \frac{\partial}{\partial t} \iint_{S_{fs}} \nabla \varphi^{(0)} \cdot \nabla \varphi^{(1)}|_{fs} dS \right) = 0 \end{aligned} \quad (4.129)$$

The simplest function that satisfies this condition regardless of the zero order potential is the null function and so it was assumed for the whole free surface ($\varphi^{(1)}|_{fs}=0$). The BVP and IVP are now complete and a numerical method must be chosen to solve the mathematical problem. In this thesis, the choice is for a boundary elements method, which will be detailed next, in Chapters 5 and 6.

Chapter 5

Integral Equations for the Boundary Elements Method (BEM)

In this chapter, Green's second identity is applied for derivation of the integral equations that will be solved by the boundary elements method for the computation of the velocity and local acceleration potentials at the fluid boundaries. The Rankine source function is presented and adopted as Green's function. Together with proper boundary conditions presented in Chapter 4, a convenient set of integral equations is obtained for the problems in different order of magnitude (zero and first ones), which will be tackled by the numerical scheme in Chapter 6.

5.1 Green's second identity

As it was discussed in the previous chapters, the assumption of potential flow allows the velocity field to be completely described by the boundaries values of a potential function. Under this assumption the solution of the flow dynamics is based on BVPs governed by Laplace's equation.

However, this equation must be satisfied in the entire fluid region, and imposing it directly may not be convenient in terms of a numerical solution because then the whole volume domain should be discretized. Since the velocity field does not depend on the interior domain values directly, the essential idea of a boundary elements method is to relate the interior values of the potential function with its boundary values and this can be done by making use of Green's second identity (5.1).

Figure 5.1: Representation of Ω , $\partial\Omega$, $\partial\Omega_\epsilon$ and ϵ

$$\iiint_{\Omega} (G\nabla^2\varphi - \varphi\nabla^2G)d\Omega = \iint_{\partial\Omega} (G\frac{\partial\varphi}{\partial n} - \varphi\frac{\partial G}{\partial n})d\partial\Omega \quad (5.1)$$

The region Ω is assumed to be singly connected and bounded by piecewise smooth surfaces $\partial\Omega$, as shown in Figure (5.1), with the normal vector pointing outwards for an observer inside the fluid domain. The potential function φ and the Green's function G are assumed harmonic and C^2 -continuous, which means that both satisfy Laplace's equation ($\nabla^2\varphi = \nabla^2G = 0$) in Ω , but for a finite number of points where the functions may not be defined.

It is possible to isolate the singular points (being $\vec{x}$ the vector pointing from the origin of the coordinate system to those points) by using a small sphere of radius ϵ and surface $\partial\Omega_\epsilon$ centered at $\vec{x}$, if those points are placed in the interior of Ω . If those points are at the boundary $\partial\Omega$, half a sphere is used, taking advantage of the smooth piecewise boundary. For a more generic surface that is not smooth, such as a sharp corner, a spherical cap defined by the solid angle, it is, the interior space angle, should be taken.

Now that all precautions were taken and there are two surfaces bounding the points where the function is not defined, Green's identity can be applied to the region between those surfaces if the point is inside Ω or at the region $\Omega \setminus \Omega_\epsilon$ if the point is at the boundaries, leading to (5.2). Since both φ and G are harmonic, the left side of this equation is null, leading to (5.3).

$$\iiint_{\Omega} (G\nabla^2\varphi - \varphi\nabla^2G)d\Omega = \iint_{\partial\Omega + \partial\Omega_\epsilon} (G\frac{\partial\varphi}{\partial n} - \varphi\frac{\partial G}{\partial n})d\partial\Omega \quad (5.2)$$

$$\iint_{\partial\Omega} (G \frac{\partial\varphi}{\partial n} - \varphi \frac{\partial G}{\partial n}) d\partial\Omega = - \iint_{\partial\Omega_\epsilon} (G \frac{\partial\varphi}{\partial n} - \varphi \frac{\partial G}{\partial n}) d\partial\Omega \quad (5.3)$$

The next step is to evaluate the right side of (5.3). Since the radius ϵ is arbitrary, so is the surface $\partial\Omega_\epsilon$, one may take the limit $\epsilon \rightarrow 0$ (5.4). Before proceeding, however, the function G needs to be defined.

$$\lim_{\epsilon \rightarrow 0} \iint_{\partial\Omega_\epsilon} (G \frac{\partial\varphi}{\partial n} - \varphi \frac{\partial G}{\partial n}) d\partial\Omega \quad (5.4)$$

5.2 Rankine sources

The choice of the most appropriate Green's function is particular to each boundary problem, the convenience to the numerical method and approach adopted. In the context of seakeeping problems, there are some renowned ones, such as the transient Green function, the Kelvin sources and the Rankine sources, this last one being adopted in this work.

The transient Green function for deep water is given by (5.5), where r and r' are given by (5.6) and (5.7), K is the wave number, J_0 is a Bessel function, Q is the point where the singularity is placed and P the evaluation point. This singularity is usually adopted in the frequency domain approach, for it automatically satisfies the linear free surface and radiation conditions, which means that the free surface does not need to be discretized. However, this Green's function has the inconvenient of involving a improper integral containing a Bessel function in the integrand, which requires some computational effort to guarantee its numerical convergence. Furthermore, it only satisfies the linearized free surface condition and so non-linearities cannot be included easily.

$$G(P, Q) = \frac{1}{r} + \frac{1}{r'} + \frac{2K}{\pi} \int_0^\infty \frac{e^{k(z_P+z_Q)}}{k-K} J_0(kR) dk \quad (5.5)$$

$$r = \sqrt{(x_P - x_Q)^2 + (y_P - y_Q)^2 + (z_P - z_Q)^2} \quad (5.6)$$

$$r' = \sqrt{(x_P - x_Q)^2 + (y_P - y_Q)^2 + (z_P + z_Q)^2} \quad (5.7)$$

Despite the Rankine sources (5.8) do not satisfy any boundary condition automatically (like the free surface or a plane bottom), it is very simple to evaluate and convenient for future

developments that may include other non-linear effects¹, justifying it's use in this work. In (5.8), c_0 is an arbitrary constant (the source strength).

$$G(P, Q) = \frac{c_0}{r_{PQ}} \quad (5.8)$$

$$r_{PQ} = \sqrt{(x_P - x_Q)^2 + (y_P - y_Q)^2 + (z_P - z_Q)^2} \quad (5.9)$$

This expression should be replaced into Green's identity and, as discussed before, the precautions should be taken because the Rankine source is singular at $r_{PQ} = 0$, leading to an indetermination problem for interior or boundary points.

After choosing the Green function (5.8) the limiting process (5.4) can be continued but, for convenience, a spherical coordinate system will be used, where $(\theta : 0 \leq \theta \leq 2\pi)$ is the azimuth angle, $(\psi : 0 \leq \psi \leq \pi)$ is the polar angle and r is the radial distance. The jacobian for a spherical coordinate system is $r^2 \sin(\psi)$ and assuming the function φ_Q as finite, the first part of the limit can be evaluated as (5.10), which is independent of the limit polar angle in the integration interval ψ^* .

$$\begin{aligned} \lim_{\epsilon \rightarrow 0} \iint_{\partial\Omega_\epsilon} G \frac{\partial \varphi}{\partial n} d\partial\Omega &= \lim_{\epsilon \rightarrow 0} \int_0^{\psi^*} \int_0^{2\pi} \left[\frac{c_0}{r} \frac{\partial \varphi_Q}{\partial n_Q} r^2 \sin(\psi) \right]_{r=\epsilon} d\theta d\psi = \\ &= \lim_{\epsilon \rightarrow 0} \int_0^{\psi^*} \int_0^{2\pi} \left[c_0 r \frac{\partial \varphi_Q}{\partial n_Q} \sin(\psi) \right]_{r=\epsilon} d\theta d\psi = \lim_{\epsilon \rightarrow 0} 2\pi [-\cos(\psi)]_0^{\psi^*} \epsilon = 0 \end{aligned} \quad (5.10)$$

For the evaluation of the second part of the limit, the gradient operator in spherical coordinates is given by (5.11) and the term $\frac{\partial G}{\partial n}$ becomes (5.12), with the normal vector pointing outward the fluid region.

The limiting process is taken in (5.13).

$$\nabla \cdot = \left(\frac{\partial}{\partial r} \vec{e}_r + \frac{1}{r} \frac{\partial}{\partial \theta} \vec{e}_\theta + \frac{1}{r \sin(\theta)} \frac{\partial}{\partial \psi} \vec{e}_\psi \right). \quad (5.11)$$

$$\frac{\partial G}{\partial n} = \nabla G \cdot \vec{n} = \frac{\partial}{\partial r} \left(\frac{c_0}{r} \right) \vec{e}_r \cdot (-\vec{e}_r) = \frac{c_0}{r^2} \quad (5.12)$$

¹meaning other than those of the weakly non-linear approach adopted in this work.

$$\begin{aligned} \lim_{\epsilon \rightarrow 0} \iint_{\partial\Omega_\epsilon} \varphi \frac{\partial G}{\partial n} d\partial\Omega &= \lim_{\epsilon \rightarrow 0} \int_0^{\psi^*} \int_0^{2\pi} \left[\varphi_Q \frac{c_0}{r^2} r^2 \sin(\psi) \right]_{r=\epsilon} d\theta d\psi = \\ &= \lim_{\epsilon \rightarrow 0} \int_0^{\psi^*} \int_0^{2\pi} \varphi_Q c_0 \sin(\psi) d\theta d\psi = 2\pi \varphi_Q c_0 [-\cos(\psi)]_0^{\psi^*} \end{aligned} \quad (5.13)$$

As discussed before, if the point is interior to Ω then $\psi^* = \pi$. Otherwise, if the point is at the boundary $\partial\Omega$ and the surface is smooth, $\psi^* = \pi/2$.

This leads to the results in (5.14), where the integration region does not contain the point P, since the residues were already evaluated and moved to the right side of the expression. (For the boundary values it was considered only smooth surfaces). The constant c_0 is assumed as unit since it is arbitrary anyway.

$$\iint_{\partial\Omega-P} \left(\varphi_Q \frac{\partial G_{PQ}}{\partial n_Q} - G_{PQ} \frac{\partial \varphi_Q}{\partial n_Q} \right) d\partial\Omega_Q = \begin{cases} -4\pi\varphi_P, & \text{if } P \text{ is inside } \Omega \\ -2\pi\varphi_P, & \text{if } P \text{ is at } \partial\Omega \\ 0, & \text{if } P \text{ is outside } \Omega \end{cases} \quad (5.14)$$

5.3 Numerical scheme

The integral equation derived is the basis for the numerical scheme developed to solve the mathematical problem considering both the velocity and acceleration potential BVPs in zero and first orders. Due to the linearization procedure described in Chapter 4, the zero order problem can be solved regardless the first order solution. On the other hand, the first order solution can only be evaluated after the solution of the zero order one, and the second order solution requires both, the solutions of the zeroth and first order problems.

5.3.1 Integral equations for the zeroth order problem

The equation for the zero order problem can be formulated using (5.14) BVP presented earlier, by imposing the no-flux condition on each boundary according to (5.15).

$$\begin{aligned} 2\pi\phi_P^{(0)} + \iint_{\bar{S}_{fixed}} \phi_Q^{(0)} \frac{\partial}{\partial n_Q^{(0)}} \left(\frac{1}{r_{PQ}} \right) dS_Q + \iint_{\bar{S}_{pm}} \phi_Q^{(0)} \frac{\partial}{\partial n_Q^{(0)}} \left(\frac{1}{r_{PQ}} \right) dS_Q + \iint_{\bar{S}_{fb}} \phi_Q^{(0)} \frac{\partial}{\partial n_Q^{(0)}} \left(\frac{1}{r_{PQ}} \right) dS_Q + \\ \iint_{\bar{S}_{fs}} \phi_Q^{(0)} \frac{\partial}{\partial n_Q^{(0)}} \left(\frac{1}{r_{PQ}} \right) dS_Q = -U \quad \iint_{\bar{S}_{fixed} \cup \bar{S}_{pm} \cup \bar{S}_{fb}} n_{xQ} \left(\frac{1}{r_{PQ}} \right) dS_Q \end{aligned} \quad (5.15)$$

The numerical solution of this problem provides the zero order potential in each boundary.

The solution can be computed by considering the discretization of the rigid free surface or the double body model, in which the bodies are reflected by taking the plane $z=0$, and the flow solved considering the body completely submerged.

Both approaches have some advantages and drawbacks. The first one allows the direct computation of the zero order potential (and derivatives) in the free surface but has the disadvantage that the discretized free surface must be "large enough" to guarantee that the perturbation in the flow due to the body is negligible on the edges of the free surface. This approach usually provides a bigger linear system if compared to the second approach due to the discretization of the free surface.

The double-body model, in its turns, has a great advantage from the point of view of accuracy because the flow is solved considering an ideally infinite "fluid". However, since the flow is solved considering only the solid boundaries, the approximation of the potential in the free surface needs to be post-processed. This must be done considering that the points in the free surface are inside the flow region, therefore the solid angle is 4π and the source and dipole terms must be computed for each point in the free surface.

The double-body model is adopted in this work, but to simplify the computations only the submerged part of the body is modeled by the software. It would be straight forward to generate an "artificial" body by reflecting the submerged body over the $z=0$ plane. However, instead of doing this, the mirror source term over the $z=0$ plane is added to the Green's function, as in (5.16). With this procedure, the size of the influence matrix is reduced because only half of the hull (the submerged part) needs to be discretized.

$$G_{PQ} = \frac{1}{\sqrt{(x_P - x_Q)^2 + (y_P - y_Q)^2 + (z_P - z_Q)^2}} + \frac{1}{\sqrt{(x_P - x_Q)^2 + (y_P - y_Q)^2 + (z_P + z_Q)^2}} \quad (5.16)$$

Therefore the integral equation (5.15) is replaced by (5.17), neglecting the free-surface boundary.

$$2\pi\phi_P^{(0)} + \iint_{\bar{S}_{fixed}} \phi_Q^{(0)} \frac{\partial}{\partial n_Q^{(0)}} \left(\frac{1}{r_{PQ}} + \frac{1}{r'_{PQ}} \right) dS_Q + \iint_{\bar{S}_{pm}} \phi_Q^{(0)} \frac{\partial}{\partial n_Q^{(0)}} \left(\frac{1}{r_{PQ}} + \frac{1}{r'_{PQ}} \right) dS_Q + \iint_{\bar{S}_{fb}} \phi_Q^{(0)} \frac{\partial}{\partial n_Q^{(0)}} \left(\frac{1}{r_{PQ}} + \frac{1}{r'_{PQ}} \right) dS_Q = -U \iint_{\bar{S}_{fixed} \cup \bar{S}_{pm} \cup \bar{S}_{fb}} n_{x_Q} \left(\frac{1}{r_{PQ}} \right) dS_Q \quad (5.17)$$

5.3.2 Integral equations for the first order problem

The ordinary differential equations concerning the first order problem are summarized from (5.18) to (5.21), the last two concerning the motion equation for floating bodies, already written conveniently for numerical integration. The original partial differential equations may be changed by these ordinary ones since the values in the right hand side are known. The incident wave field $\phi_I^{(1)}$ was already defined in (4.101) and (4.102), and the terms $\phi^{(0)}$ are obtained from the zeroth order BVP.

$$\frac{\partial}{\partial t}(\phi_{dist}^{(1)}) = -g\eta_{dist}^{(1)} - \left(U + \frac{\partial\phi^{(0)}}{\partial x}\right)\left(\frac{\partial\phi_{dist}^{(1)}}{\partial x} + \frac{\partial\phi_I^{(1)}}{\partial x}\right) - \frac{\partial\phi^{(0)}}{\partial y}\left(\frac{\partial\phi_{dist}^{(1)}}{\partial y} + \frac{\partial\phi_I^{(1)}}{\partial y}\right) - \left(\frac{\partial\phi_I^{(1)}}{\partial t} + g\eta_I^{(1)}\right), \quad z = 0 \quad (5.18)$$

$$\begin{aligned} \frac{\partial}{\partial t}(\eta_{dist}^{(1)}) = & -\left(U + \frac{\partial\phi^{(0)}}{\partial x}\right)\left(\frac{\partial\eta_{dist}^{(1)}}{\partial x} + \frac{\partial\eta_I^{(1)}}{\partial x}\right) - \frac{\partial\phi^{(0)}}{\partial y}\left(\frac{\partial\eta_{dist}^{(1)}}{\partial y} + \frac{\partial\eta_I^{(1)}}{\partial y}\right) + \frac{\partial\phi_{dist}^{(1)}}{\partial z} + (\eta_{dist}^{(1)} + \eta_I^{(1)})\frac{\partial^2\phi^{(0)}}{\partial z^2} \\ & + \left(\frac{\partial\phi_I^{(1)}}{\partial z} - \frac{\partial\eta_I^{(1)}}{\partial t}\right), \quad z = 0 \quad (5.19) \end{aligned}$$

$$\frac{d}{dt}\{X^{(1)}\} = \dot{X}^{(1)} \quad (5.20)$$

$$\begin{aligned} \frac{d}{dt}\{\dot{X}^{(1)}\} = & [M]^{-1}\left\{-\rho \iint_{\bar{S}} \frac{\partial\phi_{distQ}^{(1)}}{\partial t}\{\vec{n}_Q^{*(0)}\}dS_Q - \rho \iint_{\bar{S}} \frac{\partial\phi_{IQ}^{(1)}}{\partial t}\{\vec{n}_Q^{*(0)}\}dS_Q - \right. \\ & \left. \rho \iint_{\bar{S}} \left(U + \frac{\partial\phi_Q^{(0)}}{\partial x}\right)\left(\frac{\partial\phi_{distQ}^{(1)}}{\partial x} + \frac{\partial\phi_{IQ}^{(1)}}{\partial x}\right) + \frac{\partial\phi_Q^{(0)}}{\partial y}\left(\frac{\partial\phi_{distQ}^{(1)}}{\partial y} + \frac{\partial\phi_{IQ}^{(1)}}{\partial y}\right)\{\vec{n}_Q^{*(0)}\}dS_Q - [C^{(0)}]\{\dot{X}^{(1)}\} - [K^{(0)}]\{X^{(1)}\}\right\} \quad (5.21) \end{aligned}$$

The quantities with the subscript "dist" are evaluated solving the integral equations for the first order velocity (5.22) and local acceleration (5.23) potential equations. In each boundary of the domain the potential or flux must be imposed to guarantee an unique solution. The flux condition is imposed in the body boundaries, while the potential function is provided in the free surface, therefore the values of $\phi_{dist}^{(1)}$ in the body boundaries and $\frac{\partial\phi_{dist}^{(1)}}{\partial z}$ in free surface are computed solving the velocity BVP coupled to the free surface equations. The acceleration BVP provides the derivative in time of these quantities.

$$\begin{aligned}
& 2\pi\phi_{dist_P}^{(1)} + \iint_{\bar{S}_{fixed}} \phi_{dist_Q}^{(1)} \frac{\partial}{\partial n_Q^{(0)}} \left(\frac{1}{r_{PQ}} \right) dS_Q + \iint_{\bar{S}_{pm}} \phi_{dist_Q}^{(1)} \frac{\partial}{\partial n_Q^{(0)}} \left(\frac{1}{r_{PQ}} \right) dS_Q + \iint_{\bar{S}_{fb}} \phi_{dist_Q}^{(1)} \frac{\partial}{\partial n_Q^{(0)}} \left(\frac{1}{r_{PQ}} \right) dS_Q - \\
& \iint_{\bar{S}_{fs}} \frac{\partial \phi_{dist_Q}^{(1)}}{\partial z} \left(\frac{1}{r_{PQ}} \right) dS_Q = - \iint_{\bar{S}_{fixed} \cup \bar{S}_{pm} \cup \bar{S}_{fb}} \frac{\partial \phi_{I_Q}^{(1)}}{\partial n_Q^{(0)}} \left(\frac{1}{r_{PQ}} \right) dS_Q + \{\dot{X}^{(1)}\}_{pm} \cdot \left[\iint_{\bar{S}_{pm}} \left(\frac{1}{r_{PQ}} \right) \{\vec{n}_Q^{*(0)}\} dS_Q \right] + \\
& \{X^{(1)}\}_{pm} \cdot \left[\iint_{\bar{S}_{pm}} \left(\frac{1}{r_{PQ}} \right) \{\vec{m}_Q^{(0)}\} dS_Q \right] + \{\dot{X}^{(1)}\}_{fb} \cdot \left[\iint_{\bar{S}_{fb}} \left(\frac{1}{r_{PQ}} \right) \{\vec{n}_Q^{*(0)}\} dS_Q \right] + \{X^{(1)}\}_{fb} \cdot \left[\iint_{\bar{S}_{fb}} \left(\frac{1}{r_{PQ}} \right) \{\vec{m}_Q^{(0)}\} dS_Q \right] \\
& - \iint_{\bar{S}_{fs}} \phi_{dist_Q}^{(1)} \frac{\partial}{\partial n_Q^{(0)}} \left(\frac{1}{r_{PQ}} \right) dS_Q
\end{aligned} \tag{5.22}$$

$$\begin{aligned}
& 2\pi \frac{\partial \phi_{dist_P}^{(1)}}{\partial t} + \iint_{\bar{S}_{fixed}} \frac{\partial \phi_{dist_Q}^{(1)}}{\partial t} \frac{\partial}{\partial n_Q^{(0)}} \left(\frac{1}{r_{PQ}} \right) dS_Q + \iint_{\bar{S}_{pm}} \frac{\partial \phi_{dist_Q}^{(1)}}{\partial t} \frac{\partial}{\partial n_Q^{(0)}} \left(\frac{1}{r_{PQ}} \right) dS_Q + \iint_{\bar{S}_{fb}} \frac{\partial \phi_{dist_Q}^{(1)}}{\partial t} \frac{\partial}{\partial n_Q^{(0)}} \left(\frac{1}{r_{PQ}} \right) dS_Q - \\
& \iint_{\bar{S}_{fs}} \frac{\partial^2 \phi_{dist_Q}^{(1)}}{\partial t \partial z} \left(\frac{1}{r_{PQ}} \right) dS_Q = - \iint_{\bar{S}_{fixed} \cup \bar{S}_{pm} \cup \bar{S}_{fb}} \frac{\partial^2 \phi_{dist_Q}^{(1)}}{\partial t \partial n_Q^{(0)}} \left(\frac{1}{r_{PQ}} \right) dS_Q + \{\ddot{X}^{(1)}\}_{pm} \cdot \left[\iint_{\bar{S}_{pm}} \left(\frac{1}{r_{PQ}} \right) \{\vec{n}_Q^{*(0)}\} dS_Q \right] + \\
& \{\dot{X}^{(1)}\}_{pm} \cdot \left[\iint_{\bar{S}_{pm}} \left(\frac{1}{r_{PQ}} \right) \{\vec{m}_Q^{(0)}\} dS_Q \right] + \{\ddot{X}^{(1)}\}_{fb} \cdot \left[\iint_{\bar{S}_{fb}} \left(\frac{1}{r_{PQ}} \right) \{\vec{n}_Q^{*(0)}\} dS_Q \right] + \{\dot{X}^{(1)}\}_{fb} \cdot \left[\iint_{\bar{S}_{fb}} \left(\frac{1}{r_{PQ}} \right) \{\vec{m}_Q^{(0)}\} dS_Q \right] \\
& - \iint_{\bar{S}_{fs}} \left[-g_{dist}^{(1)} - \left(U + \frac{\partial \phi^{(0)}}{\partial x} \right) \left(\frac{\partial \phi_{dist}^{(1)}}{\partial x} + \frac{\partial \phi_I^{(1)}}{\partial x} \right) - \frac{\partial \phi^{(0)}}{\partial y} \left(\frac{\partial \phi_{dist}^{(1)}}{\partial y} + \frac{\partial \phi_I^{(1)}}{\partial y} \right) \right] \frac{\partial}{\partial n_Q^{(0)}} \left(\frac{1}{r_{PQ}} \right) dS_Q
\end{aligned} \tag{5.23}$$

The acceleration-flux in the free floating bodies boundary condition in the acceleration BVP can be eliminated by replacing the motion equations (5.21), considering the algebraic work in (5.24). This term is dependent of the time derivative of the potential function, therefore it can

be collected together to the dipole term in the linear system left side.

$$\begin{aligned}
\{\ddot{X}^{(1)}\}_{fb} \left[\iint_{\bar{S}_{fb}} \left(\frac{1}{r_{PQ}} \right) \{\vec{n}_Q^{*(0)}\} dS_Q \right] &= [M]^{-1} \left\{ -\rho \iint_{\bar{S}_{fb}} \frac{\partial \phi_{distQ}^{(1)}}{\partial t} \{\vec{n}_Q^{*(0)}\} dS_Q - \rho \iint_{\bar{S}} \frac{\partial \phi_{IQ}^{(1)}}{\partial t} \{\vec{n}_Q^{*(0)}\} dS_Q - \right. \\
\rho \iint_{\bar{S}} \left(U + \frac{\partial \phi_Q^{(0)}}{\partial x} \right) \left(\frac{\partial \phi_{distQ}^{(1)}}{\partial x} + \frac{\partial \phi_{IQ}^{(1)}}{\partial x} \right) &+ \frac{\partial \phi_Q^{(0)}}{\partial y} \left(\frac{\partial \phi_{distQ}^{(1)}}{\partial y} + \frac{\partial \phi_{IQ}^{(1)}}{\partial y} \right) \{\vec{n}_Q^{*(0)}\} dS_Q - [C^{(0)}] \{\dot{X}^{(1)}\} - [K^{(0)}] \{X^{(1)}\} \Big\}. \\
\left[\iint_{\bar{S}_{fb}} \left(\frac{1}{r_{PQ}} \right) \{\vec{n}_Q^{*(0)}\} dS_Q \right] &= -\rho [M]^{-1} \left(\iint_{\bar{S}_{fb}} \frac{\partial \phi_{distQ}^{(1)}}{\partial t} \{\vec{n}_Q^{*(0)}\} dS_Q \right) \left(\iint_{\bar{S}_{fb}} \left(\frac{1}{r_{PQ}} \right) \{\vec{n}_Q^{*(0)}\} dS_Q \right) - \rho [M]^{-1} \{\vec{F}'\}
\end{aligned} \tag{5.24}$$

This procedure leads to be stable in time evolution since it couples the flow acceleration to the body motions. The algorithms adopted to discretize the integral equations, as the body geometry will be presented in Chapter 6. The importance of the higher order approach in the computation of the m-terms and spatial derivatives of the potential function will be also discussed.

5.3.3 Second order effects

When dealing with moored vessels, which are the main focus of the present development, several weakly non-linear wave effects play relevant roles in the seakeeping analysis. The most evident of those are the mean and low frequency components that are responsible for the drifts of the moored system in the horizontal plane. However, even when limiting the hydrodynamic model to the second-order, mean and slow drifts are not the only effects of interest. For example, resonant motions induced by second-order wave effects on the roll and pitch motions of a platform may have an impact on the definition of its minimum air gap and a second-order correction of the wave profile may provide better estimates of the wave run-up upon its columns (see, for example, Matsumoto et al. [2013]).

The second order effects can be partially computed from the first order solution by considering the interaction of two arbitrary first order quantities. The complete second-order solution, however, involves also the effects of the second-order potential. In the present stage of the development, the contribution from the second-order potential is neglected, as discussed previously. Moreover, although it cannot be considered a general statement, the approximation considering only quadratic effects has also proven to be reasonable in some situations, as for example, in

some analysis involving semi-submersibles Lopez-Pavon et al. [2015].

Differently from an approach in frequency domain, in time domain computations the second order forces already consider both the difference and sum-frequency components that arise from the action of a regular wave and, therefore, there is no need to solve the second order forces in a different way for the difference and sum-frequency problems. In the particular case of a single regular wave, the second order forces obtained in time domain contain a zero order force (mean drift, $\delta\omega = 0$) and the double frequency component ($\delta\omega = 2\omega$). The second order hydrodynamic motions are computed in this thesis by solving the motion equations defined in (4.112) and (4.113) considering the wave loads computed using (4.124) following a standard 4th order Runge-Kutta scheme (RK4).

5.3.4 Numerical beach

In order to truncate the numerical domain and avoid scattered reflected waves to influence the numerical solution a numerical beach zone (sponge layer) is applied, as proposed by Israeli and Orszag [1981], where a damping function $\nu(x, y)$ that holds a parabolic shape close to the domain edge (truncation region) is assumed. The free surface conditions are modified to equations (5.25) and (5.26).

The function $\nu(x, y)$ is defined in equation (5.27), where a and b are constants, λ is the largest expected wave length, ω the respective wave frequency and d_{beach} is the length of the damping zone. More details concerning the sponge layer approach assumed can be seen in Ruggeri [2012] and Watai [2014], where many tests regarding the sponge layer effectiveness are presented, as the theoretical background regarding the attenuation capability.

$$\begin{aligned} \frac{\partial \phi_{dist}^{(1)}}{\partial t} + g\eta_{dist}^{(1)} + \left(U + \frac{\partial \phi^{(0)}}{\partial x} \right) \left(\frac{\partial \phi_{dist}^{(1)}}{\partial x} + \frac{\partial \phi_I^{(1)}}{\partial x} \right) + \frac{\partial \phi^{(0)}}{\partial y} \left(\frac{\partial \phi_{dist}^{(1)}}{\partial y} + \frac{\partial \phi_I^{(1)}}{\partial y} \right) + \\ \frac{\partial \phi^{(0)}}{\partial z} \left(\frac{\partial \phi_{dist}^{(1)}}{\partial z} + \frac{\partial \phi_I^{(1)}}{\partial z} \right) - \left(\frac{\partial \phi_I^{(1)}}{\partial t} + g\eta_I^{(1)} \right) + \nu(x, y)\eta_{dist}^{(1)} = 0, \quad z = 0 \end{aligned} \quad (5.25)$$

$$\begin{aligned} \frac{\partial \eta_{dist}^{(1)}}{\partial t} + \left(U + \frac{\partial \phi^{(0)}}{\partial x} \right) \left(\frac{\partial \eta_{dist}^{(1)}}{\partial x} + \frac{\partial \eta_I^{(1)}}{\partial x} \right) + \frac{\partial \phi^{(0)}}{\partial y} \left(\frac{\partial \eta_{dist}^{(1)}}{\partial y} + \frac{\partial \eta_I^{(1)}}{\partial y} \right) - \\ \frac{\partial \phi_{dist}^{(1)}}{\partial z} + \left(\frac{\partial \phi_I^{(1)}}{\partial z} - \frac{\partial \eta_I^{(1)}}{\partial t} \right) - (\eta_{dist}^{(1)} + \eta_I^{(1)}) \frac{\partial^2 \phi^{(0)}}{\partial z^2} + \nu(x, y)\phi_{dist}^{(1)} = 0, \quad z = 0 \end{aligned} \quad (5.26)$$

$$\nu(x, y) = a\omega \left(\frac{[x^2 + y^2]^{1/2} - d_{beach}}{b\lambda} \right)^2 \quad (5.27)$$

5.3.5 Time integration

The integration in time of the ordinary differential equations is performed using a standard 4th order Runge-Kutta method (RK4), following equation 5.32.

$$k_0 = f(t_i, y_i) \quad (5.28)$$

$$k_1 = f(t_i + 0.5\Delta t, y_i + 0.5\Delta t k_0) \quad (5.29)$$

$$k_2 = f(t_i + 0.5\Delta t, y_i + 0.5\Delta t k_1) \quad (5.30)$$

$$k_3 = f(t_i + \Delta t, y_i + \Delta t k_2) \quad (5.31)$$

$$y_{i+1} = y_i + \frac{\Delta t}{6} (k_0 + 2k_1 + 2k_2 + k_3) \quad (5.32)$$

At each step of the RK4 the following calculations are performed:

1. Impose the free surface potential, body velocities and prescribed velocities in S_{fs} , S_{fb} and S_{pm} , as forcing term in integral equation regarding the velocity potential to compute at control points of the potential (φ_{dist}) in S_{fb} , S_{fixed} , S_{pm} and the control points of $\frac{\partial \varphi_{dist}}{\partial n}$ in S_{fs} .
2. Impose the time derivative of the free surface potential and prescribed accelerations in S_{fs} and S_{pm} , as forcing term in integral equation regarding the acceleration potential to compute the control points of the time derivative of the potential ($\frac{\partial \varphi_{dist}}{\partial t}$) in S_{fb} , S_{fixed} , S_{pm} and control points of $\frac{\partial^2 \varphi_{dist}}{\partial t \partial n}$ in S_{fs} .
3. Compute the control points of $\frac{\partial \eta_{dist}}{\partial t}$ from the kinematic free surface condition using $\frac{\partial \varphi_{dist}}{\partial n}$, computed in the velocity BVP.
4. Compute the control points of $\frac{\partial \varphi_{dist}}{\partial t}$ from the dynamic free surface condition using the control points of η_{dist} .
5. Compute the body forces and moments using $\frac{\partial \varphi_{dist}}{\partial t}$ using the control points calculated from the acceleration BVP.
6. Update the body velocity and position using RK4 scheme.

7. Update the free surface potential (ϕ_{dist}) and elevation (η_{dist}) control points using RK4 scheme.
8. Go back to step 1.

The procedure can be summarized in Figure 5.2 and the other steps are performed analogously.

Figure 5.2: Illustration of the first step in the Runge-Kutta scheme for solving wave-body interaction (The other steps are evaluated analogously).

Chapter 6

A higher order panel method

In this chapter the higher order panel method considering both geometry and velocity/acceleration potential is presented with the problem geometry described by NURBS and the unknown functions represented by splines surfaces. A validation procedure is performed together with a discussion about the choice of the collocation points chosen for the evaluation of the integral equations. The procedure adopted for numerical integration of the source/dipoles is also presented, with an special emphasis on how the difficulties associated to the singularity of the Rankine sources are dealt with.

The reader will apprehend that many of the numerical procedures are discussed in their full particularities. This is indeed intentional, for one of the aims of this thesis is to provide details on how to avoid computational problems that may become an obstacle to the numerical development. These are details that are often absent in the literature, making the task of implementing such a complex method even more cumbersome for those working.

6.1 Higher order formulation considering potential functions and geometry description

The works available in the literature usually segregate the panel methods in two different categories: the so-called low order and higher order methods. The first one was originally proposed by Hess and Smith [1964]. It considers the panel as quadrilaterals (or triangular) planes bounded by the vertexes¹ and the velocity potential (and the other physical quantities) is assumed to be constant over the entire panel. The higher order methods, on the other hand, consider the exact

¹actually, it could need some additional steps to adjust the vertexes for quadrangular panels since 4 points are not essentially coplanar

geometry (for engineering purposes) and describe it by splines (e.g. Maniar [1995b]), curvilinear panels (bi-quadratic) elements (e.g. Willis [2006]), NURBS (e.g. Qiu [2001]) or a generic parametric geometry (e.g. Lee et al. [1998]).

Regardless the approximation for the potential function, the use of a higher order approach for the geometric description has advantages, such as an easier convergence analysis and a more accurate representation of the boundary surfaces. Moreover, meshes generated using the surfaces directly can maintain the mesh topology easily, which is not the case if a non-structured mesh generator is adopted. Nowadays, a usual procedure for using a panel method is to generate the geometry using a CAD software first, and then mesh it using a specific software, but this second step can be avoided if the numerical method is able to handle the CAD geometry directly, which is the purpose of the method developed herein.

Previous knowledge of the mesh topology is essential if the velocity potential representation include requirements such as continuity of the potential function (and derivatives) across the panels of each patch, as also is the case for the method developed in this thesis.

6.1.1 Geometry representation using NURBS

One of the standard formats used by the CAD industry is the "igs" file, which in the context of a panel method contains essentially the surface representation using NURBS. This description is based on the superposition of spline functions properly weighted, which are generically given by expression (6.1), where p is the basis function degree, u is the point on the curve domain and u_i are the knots in the interior region. It should be noticed that the spline of degree p (considering $p > 0$) is defined by a recurrence equation, where the spline of degree 0 is given by equation (6.2), which is piecewise continuous.

$$N_{i,p}(u) = \frac{u - u_i}{u_{i+p} - u_i} N_{i,p-1}(u) + \frac{u_{i+p+1} - u}{u_{i+p+1} - u_{i+1}} N_{i+1,p-1}(u), \quad p > 0 \quad (6.1)$$

$$N_{i,0}(u) = \begin{cases} 1, & \text{if } u_i \leq u < u_{i+1} \\ 0, & \text{if } u < u_i \text{ or } u \geq u_{i+1} \end{cases} \quad (6.2)$$

An example of some basis functions of several degrees can be seen in Figure 6.1, where the black points denote the knots vector $\{0, 0.25, 0.5, 0.75, 1\}$ adopted for spline construction and each color corresponds to a basis function.

Figure 6.1: Example of shape functions using b-spline description.

Since a surface domain is bi-dimensional, it can be described using two orthogonal spline polynomials combined in generic directions u and v to provide the surface domain. Although the surface domain can be generic because the u knots and v knots could be arbitrary, it is convenient to "scale" it into the square of dimensions $[0,1] \times [0,1]$ in order to simplify the algebraic manipulations. The i^{th} basis function of degree k are null in the whole domain but in the range $[u_i, u_{i+k+1}]$, so the basis function have a local influence region, which allows almost any smooth continuous arbitrary function to be represented using NURBS without an excessive computational effort. This characteristic is very useful specially for complex surfaces, such as the stern or the bulb of a ship, since moving the surface control points will not change the entire surface, just a small region surrounding those particular control points. An example of the combined b-spline curves of degree 2 can be seen in Figure 6.2, where it is shown the combined $N_2(u)N_2(v)$ function for knot vectors $\{0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1\}$ and it is clear that the function has only a localized influence.

Figure 6.2: Example of shape function of degree 2.

The NURBS surface is created considering expression (6.3), where $\vec{C}_{ij}$ denotes the control points of the surface and w_{ij} are the respective weights.

$$\begin{Bmatrix} x(u, v) \\ y(u, v) \\ z(u, v) \end{Bmatrix} = S(u, v) = \frac{\sum_{i=1}^{N_u} \sum_{j=1}^{N_v} w_{ij} \vec{C}_{ij} N_{i,p}(u) N_{j,p}(v)}{\sum_{i=1}^{N_u} \sum_{j=1}^{N_v} w_{ij} N_{i,p}(u) N_{j,p}(v)}, \quad 0 \leq u \leq 1, \quad 0 \leq v \leq 1 \quad (6.3)$$

The parameterization functions $x(u, v)$, $y(u, v)$ and $z(u, v)$ map the surface domain into the the patch region, as illustrated in Figure 6.3, which means that all evaluations that should be performed in space can actually be performed in the parametric domain UV. This is an important characteristic because the UV domain is always a regular unitary region, and by using the Jacobian operator one can always rebat the operations that should be performed in physical space to those in the parametric region.

Figure 6.3: Mapping functions $x(u, v)$, $y(u, v)$ and $z(u, v)$.

For example, The integral over the surface is changed to the integral over the parametric domain considering the Jacobian of the transformation (6.4), which is also used for the normal vector (6.5).

$$|J(u, v)| = \left\| \frac{\partial S}{\partial u} \wedge \frac{\partial S}{\partial v} \right\| \quad (6.4)$$

$$\vec{n}(u, v) = \frac{\frac{\partial S}{\partial u} \wedge \frac{\partial S}{\partial v}}{\left\| \frac{\partial S}{\partial u} \wedge \frac{\partial S}{\partial v} \right\|} \quad (6.5)$$

The integrals of a generic function $\psi(x, y, z)$ over any patch of the domain can be performed considering its parameterization, as shown in (6.6), where $\partial\Omega_i$ is the patch considered. Since the integral equations are built in the parametric space, the functions are evaluated in these coordinates, which means that they should be transferred to spatial coordinates every time that it is required.

$$\iint_{\partial\Omega_i} \psi(x, y, z) d\partial\Omega_i = \int_0^1 \int_0^1 \psi(x(u, v), y(u, v), z(u, v)) |J(u, v)| dv du \quad (6.6)$$

There are some functions, such as the velocity potential, that are obtained directly from the solution of the integral equations. However, there are others that need to be evaluated after the potential function is known, such as the tangential velocities in the patches. A simple way to perform this operation is to consider the identity (chamber rule) presented generically for the u direction in (6.7), which can be applied analogously for the v direction.

$$\frac{\partial\varphi}{\partial u} \Big|_{(u,v)} = \frac{\partial\varphi}{\partial x} \Big|_{(x(u,v), y(u,v), z(u,v))} \frac{\partial x}{\partial u} \Big|_{(u,v)} + \frac{\partial\varphi}{\partial y} \Big|_{(x(u,v), y(u,v), z(u,v))} \frac{\partial y}{\partial u} \Big|_{(u,v)} + \frac{\partial\varphi}{\partial z} \Big|_{(x(u,v), y(u,v), z(u,v))} \frac{\partial z}{\partial u} \Big|_{(u,v)} \quad (6.7)$$

The main goal is the evaluation of $\partial\varphi/\partial x$, $\partial\varphi/\partial y$ and $\partial\varphi/\partial z$, and if we assume that $\partial\varphi/\partial u$ and $\partial\varphi/\partial v$ are known², there is a lack of equations because there are three variables and only two equations. However, an additional equation can be obtained considering the flux through the surface, as shown in (6.8). This allows the evaluation of the velocity components analytically (6.9), it means, without any additional numerical approximation (as would be the case if a finite difference method was used) since the potential and spatial coordinates can be differentiated analytically once the shape functions are polynomials.

$$\frac{\partial\varphi}{\partial n} \Big|_{(u,v)} = \left(\frac{\partial\varphi}{\partial x}, \frac{\partial\varphi}{\partial y}, \frac{\partial\varphi}{\partial z} \right) \Big|_{(x(u,v), y(u,v), z(u,v))} \cdot \vec{n}(u, v) \quad (6.8)$$

²Considering that the potential is represented using a higher order representation.

$$\begin{pmatrix} \frac{\partial \varphi}{\partial x} \\ \frac{\partial \varphi}{\partial y} \\ \frac{\partial \varphi}{\partial z} \end{pmatrix}_{(x(u,v),y(u,v),z(u,v))} = \begin{bmatrix} \frac{\partial x}{\partial u} & \frac{\partial y}{\partial u} & \frac{\partial z}{\partial u} \\ \frac{\partial x}{\partial v} & \frac{\partial y}{\partial v} & \frac{\partial z}{\partial v} \\ n_x & n_y & n_z \end{bmatrix}_{(u,v)}^{-1} \begin{pmatrix} \frac{\partial \varphi}{\partial u} \\ \frac{\partial \varphi}{\partial v} \\ \frac{\partial \varphi}{\partial n} \end{pmatrix}_{(u,v)} \quad (6.9)$$

The procedure discussed above enables the computation of the velocity potential spatial derivatives using the derivatives evaluated in the parametric domain. These derivatives are required, for instance, in the free surface conditions. The evaluation of the derivatives of the potential function in the parametric space can only be performed by following this approach because the potential is also described as higher order, while a finite difference scheme would be applied if a low order description was employed.

Furthermore, the computation of the complete m-terms assuming double body basis flow requires the computation of the second derivatives, which is performed using the second derivatives of the potential function, as shown in (6.10) concerning the u direction. A similar procedure can be followed for the computation of the v second derivative and the mixed (uv) one, providing three equations.

However, since there are 6 unknowns in these equations, three additional equations are required and they are provided by the derivatives of the no-flux boundary condition in u and v directions, as shown, for example for the u direction in (6.11). The additional equation to be included is the Laplace's equation itself, providing the closure to the problem, as shown in (6.12).

$$\begin{aligned} \frac{\partial^2 \varphi}{\partial u^2} = & \frac{\partial^2 \varphi}{\partial x^2} \left(\frac{\partial x}{\partial u} \right)^2 + \frac{\partial^2 \varphi}{\partial y^2} \left(\frac{\partial y}{\partial u} \right)^2 + \frac{\partial^2 \varphi}{\partial z^2} \left(\frac{\partial z}{\partial u} \right)^2 + 2 \frac{\partial^2 \varphi}{\partial x \partial y} \left(\frac{\partial x}{\partial u} \right) \left(\frac{\partial y}{\partial u} \right) + 2 \frac{\partial^2 \varphi}{\partial x \partial z} \left(\frac{\partial x}{\partial u} \right) \left(\frac{\partial z}{\partial u} \right) + \\ & 2 \frac{\partial^2 \varphi}{\partial y \partial z} \left(\frac{\partial y}{\partial u} \right) \left(\frac{\partial z}{\partial u} \right) + \frac{\partial \varphi}{\partial x} \left(\frac{\partial^2 x}{\partial u^2} \right) + \frac{\partial \varphi}{\partial y} \left(\frac{\partial^2 y}{\partial u^2} \right) + \frac{\partial \varphi}{\partial z} \left(\frac{\partial^2 z}{\partial u^2} \right) \end{aligned} \quad (6.10)$$

$$\begin{aligned} -U \frac{\partial n_x}{\partial u} = & \frac{\partial^2 \varphi}{\partial u^2} = \frac{\partial^2 \varphi}{\partial x^2} \left(\frac{\partial x}{\partial u} \right) n_x + \frac{\partial^2 \varphi}{\partial y^2} \left(\frac{\partial y}{\partial u} \right) n_y + \frac{\partial^2 \varphi}{\partial z^2} \left(\frac{\partial z}{\partial u} \right) n_z + \frac{\partial^2 \varphi}{\partial x \partial y} \left[\left(\frac{\partial x}{\partial u} \right) n_y + \left(\frac{\partial y}{\partial u} \right) n_x \right] + \\ & \frac{\partial^2 \varphi}{\partial x \partial z} \left[\left(\frac{\partial x}{\partial u} \right) n_z + \left(\frac{\partial z}{\partial u} \right) n_x \right] + \frac{\partial^2 \varphi}{\partial y \partial z} \left[\left(\frac{\partial y}{\partial u} \right) n_z + \left(\frac{\partial z}{\partial u} \right) n_y \right] + \frac{\partial \varphi}{\partial x} \frac{\partial n_x}{\partial u} + \frac{\partial \varphi}{\partial y} \frac{\partial n_y}{\partial u} + \frac{\partial \varphi}{\partial z} \frac{\partial n_z}{\partial u} \end{aligned} \quad (6.11)$$

$$\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} + \frac{\partial^2 \varphi}{\partial z^2} = 0 \quad (6.12)$$

The second order derivatives therefore can be computed solving the linear system shown in

(6.14), which is used for the computation of the double-body basis flow m-terms, presented in section 4.2.2.

$$M = \begin{bmatrix} \left(\frac{\partial x}{\partial u}\right)^2 & \left(\frac{\partial y}{\partial u}\right)^2 & \left(\frac{\partial z}{\partial u}\right)^2 & 2\left(\frac{\partial x}{\partial u}\right)\left(\frac{\partial y}{\partial u}\right) & 2\left(\frac{\partial x}{\partial u}\right)\left(\frac{\partial z}{\partial u}\right) & 2\left(\frac{\partial y}{\partial u}\right)\left(\frac{\partial z}{\partial u}\right) \\ \left(\frac{\partial x}{\partial v}\right)^2 & \left(\frac{\partial y}{\partial v}\right)^2 & \left(\frac{\partial z}{\partial v}\right)^2 & 2\left(\frac{\partial x}{\partial v}\right)\left(\frac{\partial y}{\partial v}\right) & 2\left(\frac{\partial x}{\partial v}\right)\left(\frac{\partial z}{\partial v}\right) & 2\left(\frac{\partial y}{\partial v}\right)\left(\frac{\partial z}{\partial v}\right) \\ \left(\frac{\partial x}{\partial u}\right)\left(\frac{\partial x}{\partial v}\right) & \left(\frac{\partial y}{\partial u}\right)\left(\frac{\partial y}{\partial v}\right) & \left(\frac{\partial z}{\partial u}\right)\left(\frac{\partial z}{\partial v}\right) & x_u y_v + x_v y_u & x_u z_v + x_v z_u & y_u z_v + y_v z_u \\ 1 & 1 & 1 & 0 & 0 & 0 \\ \frac{\partial x}{\partial u} n_x & \frac{\partial y}{\partial u} n_y & \frac{\partial z}{\partial u} n_z & \left(\frac{\partial x}{\partial u} n_y + \frac{\partial y}{\partial u} n_x\right) & \left(\frac{\partial x}{\partial u} n_z + \frac{\partial z}{\partial u} n_x\right) & \left(\frac{\partial y}{\partial u} n_z + \frac{\partial z}{\partial u} n_y\right) \\ \frac{\partial x}{\partial v} n_x & \frac{\partial y}{\partial v} n_y & \frac{\partial z}{\partial v} n_z & \left(\frac{\partial x}{\partial v} n_y + \frac{\partial y}{\partial v} n_x\right) & \left(\frac{\partial x}{\partial v} n_z + \frac{\partial z}{\partial v} n_x\right) & \left(\frac{\partial y}{\partial v} n_z + \frac{\partial z}{\partial v} n_y\right) \end{bmatrix} \quad (6.13)$$

$$\begin{pmatrix} \varphi_{xx} \\ \varphi_{yy} \\ \varphi_{zz} \\ \varphi_{xy} \\ \varphi_{xz} \\ \varphi_{yz} \end{pmatrix} = M^{-1} \begin{pmatrix} -\varphi_x \frac{\partial^2 x}{\partial y^2} - \varphi_y \frac{\partial^2 y}{\partial y^2} - \varphi_z \frac{\partial^2 z}{\partial y^2} + \frac{\partial^2 \varphi}{\partial y^2} \\ -\varphi_x \frac{\partial^2 x}{\partial v^2} - \varphi_y \frac{\partial^2 y}{\partial v^2} - \varphi_z \frac{\partial^2 z}{\partial v^2} + \frac{\partial^2 \varphi}{\partial v^2} \\ -\varphi_x \frac{\partial^2 x}{\partial u \partial v} - \varphi_y \frac{\partial^2 y}{\partial u \partial v} - \varphi_z \frac{\partial^2 z}{\partial u \partial v} + \frac{\partial^2 \varphi}{\partial u \partial v} \\ 0 \\ -\varphi_x \frac{\partial n_x}{\partial u} - \varphi_y \frac{\partial n_y}{\partial u} - \varphi_z \frac{\partial n_z}{\partial u} - U \frac{\partial n_x}{\partial u} \\ -\varphi_x \frac{\partial n_x}{\partial v} - \varphi_y \frac{\partial n_y}{\partial v} - \varphi_z \frac{\partial n_z}{\partial v} - U \frac{\partial n_x}{\partial v} \end{pmatrix} \quad (6.14)$$

6.1.2 Solution representation using B-Splines

In the numerical method implemented herein, the velocity and acceleration potentials are described by b-splines, which means that the solution is continuous inside each patch and that by increasing the order of the spline representation, more derivatives can be kept continuous inside the patches.

One could also require the solution to be continuous through patches, however this approach could be very complicated, specially for an arbitrary body geometry. First, it would require an additional step concerning the determination of relative positions among patches in order to determine the borders and a re-meshing scheme in order to guarantee that each panel is always surrounded by eight other ones. Moreover, this requirement may not be convenient for particular geometries (for example, when sharp corners exist and coincide with patch borders). An example of this is presented in Figure 6.4, that illustrates the problem concerning several patches with different panel distributions, but the problem can be much more complicated for typical geometries adopted for the hull of offshore system, such as those of semisubmersible

platforms (an example will be given in Chapter 7). So, in order to keep the method robust, the continuity between patches is not imposed in the development of the BEM code.

Figure 6.4: Example of body containing several discontinuous patches.

Since no hypothesis regarding the value of the potential function in the edges of a patch are made, an additional "hidden layer" of knots is required to avoid the potential function to be null at all edges. The hidden layer is created by repeating the first and last knots k times (the polynomial degree). A comparison between the "common spline" defined before and the multiple knots approach for a spline of degree 2 with 5 knots is shown in Figure 6.5.

Figure 6.5: The effect of a hidden layer (multiple knots) in the edges of the u vector. The b-spline in the upper figure does not contain multiple knots while the one in lower one does.

Since there is no previous imposition of the function continuity between patches, the numerical convergence of the solution can sometimes be improved by the user converting some contiguous patches into a single one, which is feasible if the geometry is continuous and assuming some expertise from the user.

The velocity and acceleration potentials and also the free surface elevation are all represented in the generic form of equation (6.15), where $\phi_{i,j}^p$ are the coefficients corresponding to the i^{th} basis function in direction u and j^{th} basis function in direction of the p^{th} patch of the domain.

The basis functions have degree k_u , k_v in the u and v directions, respectively³.

$$\varphi(u, v) = \sum_{p=0}^{N_p} \sum_{i=0}^{N_u} \sum_{j=0}^{N_v} \phi_{i,j}^p N_i^{k_u,p}(u) N_j^{k_v,p}(v) \quad (6.15)$$

Although the solution is higher order, the benefits concerning this approach can only be verified after analyzing the integration process assumed by the method, specially near the singular points of the integral equation, because different from the low order approach, there is significant contributions due to curvature effects in the self-influence terms.

In expression 6.9 the potential function first derivatives should be evaluated, which can be performed assuming the definition 6.15 concerning the potential function, which provides expression 6.16 using the analytic derivative of the shape function.

$$\frac{\partial \varphi}{\partial u}(u, v) = \sum_{p=0}^{N_p} \sum_{i=0}^{N_u} \sum_{j=0}^{N_v} \phi_{i,j}^p \frac{dN_i^{k_u,p}(u)}{du} N_j^{k_v,p}(v) \quad (6.16)$$

$$\frac{\partial \varphi}{\partial v}(u, v) = \sum_{p=0}^{N_p} \sum_{i=0}^{N_u} \sum_{j=0}^{N_v} \phi_{i,j}^p N_i^{k_u,p}(u) \frac{dN_j^{k_v,p}(v)}{dv}$$

The shape function derivative regarding the u component may be evaluated according to (6.17). The evaluation of the v component is analogous.

$$\begin{aligned} \frac{dN_{i,p}(u)}{du} &= \frac{1}{u_{i+p} - u_i} N_{i,p-1}(u) + \frac{u - u_i}{u_{i+p} - u_i} \frac{dN_{i,p-1}(u)}{du} - \\ &\quad \frac{1}{u_{i+p+1} - u_{i+1}} N_{i+1,p-1}(u) + \frac{u_{i+p+1} - u}{u_{i+p+1} - u_{i+1}} \frac{dN_{i+1,p-1}(u)}{du}, \quad p > 0 \end{aligned} \quad (6.17)$$

The second derivatives of the potential function will also require the shape function second derivatives, which can be evaluated following expression (6.18) and by comparison with (6.17) the recurrence relation shown in 6.19 becomes clear.

$$\begin{aligned} \frac{d^2 N_{i,p}(u)}{du^2} &= \frac{2}{u_{i+p} - u_i} \frac{dN_{i,p-1}(u)}{du} + \frac{u - u_i}{u_{i+p} - u_i} \frac{d^2 N_{i,p-1}(u)}{du^2} - \\ &\quad \frac{2}{u_{i+p+1} - u_{i+1}} \frac{dN_{i+1,p-1}(u)}{du} + \frac{u_{i+p+1} - u}{u_{i+p+1} - u_{i+1}} \frac{d^2 N_{i+1,p-1}(u)}{du^2}, \quad p > 0 \end{aligned} \quad (6.18)$$

³In the present work, the same degree was always used in both directions.

$$\begin{aligned} \frac{d^n N_{i,p}(u)}{du^n} = & \frac{n}{u_{i+p} - u_i} \frac{d^{n-1} N_{i,p-1}(u)}{du^{n-1}} + \frac{u - u_i}{u_{i+p} - u_i} \frac{d^n N_{i,p-1}(u)}{du^n} - \\ & \frac{n}{u_{i+p+1} - u_{i+1}} \frac{d^{n-1} N_{i+1,p-1}(u)}{du^{n-1}} + \frac{u_{i+p+1} - u}{u_{i+p+1} - u_{i+1}} \frac{d^n N_{i+1,p-1}(u)}{du^n}, \quad p > 0 \quad (6.19) \end{aligned}$$

Although this last expression looks like a simple derivative procedure, it should be noticed that the evaluation of the n th derivative requires the previous calculation of the lower degrees derivatives and, due to the "natural" recurrence behavior of the b-spline description, the evaluation of the i th shape function of degree p requires also the calculation of the $(i-1)$ th and i th shape functions of degree $p-1$.

Following the recurrence relation presented above, the shape function derivatives concerning a B-spline of degree 2 using the knot list $\{0,0,0,0.1,0.2,0.3,0.4,0.5,0.6,0.7,0.8,0.9,1.0,1.0,1.0\}$ can be evaluated, providing the results shown in Figure 6.6, considering both the function and its derivatives.

Figure 6.6: Shape functions of degree 2 (first picture, up to down), first derivatives (second picture), second derivatives (third picture) and the third derivatives (forth picture).

As expected the third derivative is null in the whole domain since the shape function considers only polynomials of degree 2.

6.2 Source/dipole integration procedure

The integral equations to be solved were presented in Chapter 5. The Green's identity presented before is reproduced in (6.20), considering the 3 possible positions of the field point. If the method is built considering only field points placed in the boundaries of the domain and in regular regions, the solid angle can always be defined as 2π . This is coherent with the geometry described by the NURBS surface (always continuous) if we do not choose exactly the edges of the patches, since in those regions the continuity of the solution cannot be guaranteed, as discussed before.

$$\iint_{\partial\Omega_{-P}} \left(\varphi_Q \frac{\partial G_{PQ}}{\partial n_Q} - G_{PQ} \frac{\partial \varphi_Q}{\partial n_Q} \right) d\partial\Omega_Q = \begin{cases} -4\pi\varphi_P, & \text{if } P \text{ is inside } \Omega \\ -2\pi\varphi_P, & \text{if } P \text{ is at } \partial\Omega \\ 0, & \text{if } P \text{ is outside } \Omega \end{cases} \quad (6.20)$$

The integrands will always have two main functions, the sources and dipoles, both containing singularities when the field and source points are coincident. The integral terms are given by (6.21) and (6.22), with the normal vector and the Jacobian operator to be obtained from geometric description using NURBS.

$$\iint_{\partial\Omega_{-P}} \varphi_Q \frac{\partial}{\partial n_Q} \left(\frac{1}{r_{PQ}} \right) d\partial\Omega_Q = PV \int_0^1 \int_0^1 \varphi(u, v) \frac{\partial}{\partial n(u, v)} \left(\frac{1}{r_{PQ}(u, v)} \right) |J(u, v)| dv du \quad (6.21)$$

$$\iint_{\partial\Omega_{-P}} \frac{\partial \varphi_Q}{\partial n_Q} \frac{1}{r_{PQ}} d\partial\Omega_Q = PV \int_0^1 \int_0^1 \frac{\partial \varphi}{\partial n}(u, v) \left(\frac{1}{r_{PQ}(u, v)} \right) |J(u, v)| dv du \quad (6.22)$$

Equations (6.21) and (6.22) can be discretized using the potential function approximation by B-spline, leading to (6.23) and (6.24), already taking advantage from the fact that the basis functions are null outside their influence regions in order to reduce the integration interval. This fact is important because it avoids the waste of computational resources, for unnecessary numerical integration over regions where the basis function is null by construction.

$$\int_0^1 \int_0^1 \varphi(u, v) \frac{\partial}{\partial n(u, v)} \left(\frac{1}{r_{PQ}(u, v)} \right) |J(u, v)| dv du = \sum_{p=0}^{N_p} \sum_{i=0}^{N_u} \sum_{j=0}^{N_v} \phi_{i,j}^p \int_{u_i}^{u_{i+k_u+1}} \int_{v_j}^{v_{j+k_v+1}} N_i^{k_u,p}(u) N_j^{k_v,p}(v) \frac{\partial}{\partial n(u, v)} \left(\frac{1}{r_{PQ}(u, v)} \right) |J(u, v)| dv du \quad (6.23)$$

$$\int_0^1 \int_0^1 \frac{\partial \varphi(u, v)}{\partial n} \left(\frac{1}{r_{PQ}(u, v)} \right) |J(u, v)| dv du = \sum_{p=0}^{N_p} \sum_{i=0}^{N_u} \sum_{j=0}^{N_v} \frac{\partial \phi_{i,j}^p}{\partial n} \int_{u_i}^{u_{i+k_u+1}} \int_{v_j}^{v_{j+k_v+1}} N_i^{k_u,p}(u) N_j^{k_v,p}(v) \left(\frac{1}{r_{PQ}(u, v)} \right) |J(u, v)| dv du \quad (6.24)$$

In order to improve the numerical integration procedure, there are different considerations to be done depending on the position of the collocation point (that may be in the same panel over which integration is being made, outside of this panel in the nearfield region or in the farfield).

In order to illustrate the difficulties associated to the integration process, the case of a sphere in uniform flow is used as an example for the evaluation of the influence matrix. The integrand in (6.23) is particularly difficult to be evaluated in the nearfield and for the self-influence due to the abrupt variations in the function values near the singular point. The computations are performed considering simple Gauss-Legendre quadrature, which consists basically in (6.25), where Ng_u and Ng_v are the number of gaussian points in u and v direction, w_m and w_n are the weights and u_m and v_n are the gaussian points.

$$\int_{u_i}^{u_{i+k_u+1}} \int_{v_j}^{v_{j+k_v+1}} f(u, v) dv du \approx \sum_{m=1}^{Ng_u} \sum_{n=1}^{Ng_v} w_m w_n f(u_m, v_n) \quad (6.25)$$

The influence matrix is composed of the dipoles and source elements, denoted by the A and B matrices, respectively. In order to simplify the analysis, in this example the potential is described by a zero order polynomial, meaning that the potential function has a constant value inside each panel, and the elements are evaluated considering 2, 4, 6, 8, 10, 20, 40 and 80 gaussian points. The sphere surface is discretized considering panels with areas of less than 0.2 m² for a sphere of unitary radius, generating a total number of 64 panels.

When the low order approach is used for describing the geometry, the principal value of the dipole self-influence terms is always null due to the orthogonal relation between the normal vector and the distance vector (which defines the panel plane), as shown in (6.26), so no contribution would be expected from this evaluation. However, in the higher order approach, there is a significant contribution due to the panel curvature effects, as discussed next.

$$PV \iint_{\Omega_1} \frac{(x_P - x_Q)n_{xQ} + (y_P - y_Q)n_{yQ} + (z_P - z_Q)n_{zQ}}{r_{PQ}^3} d\Omega_Q = PV \iint_{\Omega_1} \frac{\vec{r}_{PQ} \cdot \vec{n}_Q}{r_{PQ}^3} d\Omega_Q = 0 \quad (6.26)$$

The variation of the influence matrices considering 80 and 40 gaussian points is illustrated in Figure 6.7, where the red elements mean that the differences between the evaluations in their positions are greater than 10^{-6} , chosen as an arbitrary precision. Results therefore show that even considering this large number of gaussian points the integrals have not yet converged for the main diagonals, while all other elements have already achieved a difference of less than 10^{-10} in matrix A and 10^{-11} for matrix B, revealing the difficulty concerning the evaluation of the self-influence terms. This behavior indicates that an alternative method should be adopted in order to provide more accurate results for the evaluation of the self-influence coefficients.

Figure 6.7: Comparison of the influence matrices A(left) and B(right) for 80 and 40 gaussian points in the numerical integrations for a mesh of 64 panels.

A similar study is performed considering the numerical integrations using 6 and 4 gaussian points, which are more reasonable for the developed method, providing the results shown in Figure 6.8. More elements in which the integral variation is greater than 10^{-6} is verified, as expected. However, the number of matrix elements that are above this 10^{-6} criteria is 464 for the matrix B and 528 for matrix A, less than 13% of the total number of elements, indicating that a localized integration procedure could be used for these specific terms.

Figure 6.8: Comparison of the influence matrices A(left) and B(right) for 6 and 4 gaussian points in the numerical integrations for a mesh of 64 panels.

The time consumption for building the influence matrix when the number of gaussian points is changed can be seen in Table 6.1 and in Figure 6.9⁴.

Table 6.1: Computation time for influence matrices evaluation for a mesh of 64 panels

Number of gaussian points	Time elapsed (s)
2	0.13
4	0.21
6	0.35
8	0.55
10	0.80
20	3.05
40	13.99
80	85.06

Figure 6.9: Comparison of the influence matrices evaluation time by the number of gaussian points used for the numerical integrations considering a mesh of 64 panels.

⁴considering a Matlab programming language.

A convergence analysis considering both self-influence terms in matrices A and B can be seen in Figures 6.10 and 6.11. Also it is shown that the convergence is quite slow when using Gauss-Legendre quadrature for the numerical integration of those terms, since even for a large number of points (more than 40) the tolerance of less than 10^{-4} could not be achieved.

Figure 6.10: Convergence analysis of the A matrix the self-influence integral changing the number of gaussian points

Figure 6.11: Convergence analysis of the B matrix the self-influence integral changing the number of gaussian points

In order to improve this integration process, an analytic dessingularization method was adopted, considering the bi-quadratic transformation method, as introduced by Maniar [1995b] for a higher order approach for the geometry description. However, the method originally proposed was applied to a B-spline geometry representation, so an extension of the method is proposed here in order to take into account the NURBS geometry. Moreover, the approach in Maniar [1995b] requires the approximation of the integrand by a series expansion and an analytic

integration of each individual term, considering both the source/dipole terms and the Jacobian transformation, resulting in a complex procedure. An approximation of the basis function (polynomial) and the source/dipole term was also presented by Danmeier [1998], however his work provides no further details about the Jacobian treatment and whether the integral is performed by simple quadrature after the bi-quadratic transformation is applied or the terms are expanded and then integrated.

In this work, the procedure proposed by Kim et al. [2007] was followed, applying the bi-quadratic transformation for the analytic dessingularization and then Gauss-Legendre quadrature in the "new" non-singular integrand, already considering the NURBS geometry description. The basic idea consists of splitting the domain of integration in 4 different zones (triangles) keeping the singular point only at the vertex of each triangle, as illustrated in Figure 6.12. Here, u_c and v_c are the parametric coordinates of the collocation points and the right side picture is the integration domain after applying the bi-quadratic transformation, as described in (6.27) for the region 2 (the transformations of the other regions are obtained by analogy, as shown in (6.28), (6.29) and (6.30) for regions 1, 3 and 4, respectively).

Figure 6.12: Domain splitting into 4 triangles and integration domain considering the region (2) using bi-quadratic transformation.

$$u - u_c = \xi \tag{6.27}$$

$$v - v_c = \xi \eta$$

$$u - u_c = \xi \eta \tag{6.28}$$

$$v - v_c = \xi$$

$$\begin{aligned} u - u_c &= -\xi\eta \\ v - v_c &= -\xi \end{aligned} \tag{6.29}$$

$$\begin{aligned} u - u_c &= -\xi \\ v - v_c &= -\xi\eta \end{aligned} \tag{6.30}$$

Following this approach, the self-influence terms can be written as (6.31) and (6.32), where the terms I_{Dk} and I_{Sk} are the integrals over each individual triangle (region 2 is shown as example) and summing the contributions due to the 4 regions the integral over the entire element can be evaluated properly. These last expressions are not singular and can be evaluated numerically using Gauss-Legendre quadrature directly.

$$\begin{aligned} \int_{u_1}^{u_2} \int_{v_1}^{v_2} N_i(u) N_j(v) \frac{\partial}{\partial n(u, v)} \left(\frac{1}{r_{PQ}(u, v)} \right) |J(u, v)| dv du &= \sum_{k=1}^4 I_{Dk} \\ I_{D2} &= \int_0^{u_2-u_c} \int_{\frac{v_1-v_c}{u_2-u_c}}^{\frac{v_2-v_c}{u_2-u_c}} N_i(\xi+u_c) N_j(\xi\eta+v_c) \frac{\vec{r}_{PQ}(\xi+u_c, \xi\eta+v_c) \cdot \vec{N}(\xi+u_c, \xi\eta+v_c)}{r_{PQ}^3(\xi+u_c, \xi\eta+v_c)} \left| \frac{\partial \vec{r}_{PQ}}{\partial \xi} \wedge \frac{\partial \vec{r}_{PQ}}{\partial \eta} \right| d\eta d\xi \end{aligned} \tag{6.31}$$

$$\begin{aligned} \int_{u_1}^{u_2} \int_{v_1}^{v_2} N_i(u) N_j(v) \left(\frac{1}{r_{PQ}(u, v)} \right) |J(u, v)| dv du &= \sum_{k=1}^4 I_{Sk} \\ I_{S2} &= \int_0^{u_2-u_c} \int_{\frac{v_1-v_c}{u_2-u_c}}^{\frac{v_2-v_c}{u_2-u_c}} N_i(\xi+u_c) N_j(\xi\eta+v_c) \frac{1}{r_{PQ}(\xi+u_c, \xi\eta+v_c)} \left| \frac{\partial \vec{r}_{PQ}}{\partial \xi} \wedge \frac{\partial \vec{r}_{PQ}}{\partial \eta} \right| d\eta d\xi \end{aligned} \tag{6.32}$$

The numerical integration using 4 Gaussian points per region (16 in total concerning the entire panel) for the same sphere studied earlier provided an accuracy of less than 10^{-6} , which could not be achieved even using 80 Gaussian points in the singular formulation directly. It should be noticed, however, that this methodology cannot be applied directly in case the collocation points lie exactly in the panel borders, something that may happen in a generic procedure, for choosing the collocation points. This condition is dealt with by repeating the integration procedure for the two panels that share the same collocation point in their edges, but for each panel, only three regions are used, as shown in Figure 6.13. In the more remote possibility that the collocation point lies exactly in the vertex of a panel (4 panels sharing the same collocation point), this point is arbitrarily moved by a small amount (typically 1/100 of the panel diagonal length) to the previous panel (lower i and j index). The reason for this choice is that these pan-

els are closer to the patch edges, where it is probable that the flow variations are more intense, specially because no continuity is imposed between patches.

Figure 6.13: Two panels "sharing" a collocation point in the edge.

The integration concerning the other terms near the main diagonal of the linear system (but not the self-influence) can be performed using an adaptative method, increasing the number of Gaussian points during the integration process until the desired accuracy is achieved, a procedure that usually requires less than 9 points.

6.3 The importance of a wise choice of collocation points for setting up the linear system

Setting up the linear system is not a trivial task as in the low order method, since due to the additional (hidden) layers concerning the patch boundaries, usually there are more variables (coefficients to be determined) than equations even if we consider only one collocation point inside each panel. There are several ways to overcome this problem and since a "best" choice is not clear, three different methodologies were evaluated. Two of them define a number of collocation points equal to the number of b-spline unknown coefficients, thus obtaining a determined linear system, while the third approach assumes more than a single collocation point per panel, leading to an overdetermined linear system, that is solved by means of an specific procedure. The three approaches are discussed next, again taking as an example the case of a sphere under uniform flow in infinite water depth. A degree 2 b-spline basis was chosen for the comparisons, presented next.

In the method discussed in section 6.3.3, when there are more collocation points than un-

knowns, an overdetermined linear system is obtained. There are two main techniques usually employed to overcome this inconvenient, the first one based on a least square approach (Kim et al. [2007]) and the second one based on a Galerkin approach (Maniar [1995b]). The first one is adopted in this work.

6.3.1 The "extended" panel approach

The first method that was evaluated is the "extended" panel approach, where the panels are not considered as the interval between successive knots, but as the interval where a shape function (spline) extends without null values (u_i, u_{i+p+1}). The collocation points are chosen as the midpoint of the not null interval (6.33), providing one collocation point per shape function. The collocation points positions are illustrated in Figure 6.14 concerning the same case of a sphere under uniform flow, where it can be verified that there is a concentration of points near the patch edges mainly due to the hidden layer splines and a lack of points in some intermediate regions.

Figure 6.14: Example of the center of the "extended" panels as collocation points.

$$(u_{ci}, v_{ci}) = \left(\frac{u_i + u_{i+p+1}}{2}, \frac{v_i + v_{i+p+1}}{2} \right) \quad (6.33)$$

6.3.2 Gauss Legendre Roots

The second approach assumed was the distribution of collocation points using Legendre polynomial roots in the parametric space, which provided the distribution shown in Figure 6.15. One may realize that there is a concentration of points near the patch edges in this second approach, the opposite in the first approach, whereas the third procedure provides a more uniform distribution of collocation points.

Figure 6.15: Example of Legendre polynomials roots as collocation points.

6.3.3 Multiple collocation points per panel

The third approach assumed was the allocation of multiple collocation points inside each panel (interval of not coincident knots) in order to obtain an overdetermined linear system. The number of collocation points inside each panel is completely arbitrary but in this work 4 points were assumed, as illustrated in Figure 6.16.

The system of equations to be solved are generically given in (6.34), where m and n are the number of lines and columns, respectively ($m \geq n$). The least squares approach proposes the multiplication of both sides by the transpose of matrix A and then the inverse procedure of matrix $A^T A$ can be performed since this product is a square matrix, obtaining only the coefficients c , as shown in (6.35).

$$[A]_{m,n} \{c\}_{n,1} = \{B\}_{m,1} \quad (6.34)$$

$$([A]_{n,m}^T[A]_{m,n})\{c\}_{n,1} = [A]_{n,m}^T\{B\}_{m,1} \Rightarrow \{c\}_{n,1} = ([A]_{n,m}^T[A]_{m,n})^{-1}[A]_{n,m}^T\{B\}_{m,1} \quad (6.35)$$

Figure 6.16: Example of multiple collocation points approach.

6.4 Comparisons concerning the collocation points

The performance of the alternative approaches is compared for the case of a submerged sphere in uniform flow, with the analytic solution in (6.36), the right also considering the undisturbed incident flow. The numerical solutions regarding the disturbance potential are compared in Figure 6.17, considering a 64 panels mesh.

$$\phi^{(0)} = \frac{1}{2} \frac{xR^3}{(x^2 + y^2 + z^2)^{3/2}} \Rightarrow \varphi^{(0)} = Ux + \frac{1}{2} \frac{xR^3}{(x^2 + y^2 + z^2)^{3/2}} \quad (6.36)$$

The velocity potential distribution was evaluated in a grid of 60x60 in the parametric space, considering an uniform distribution, thus the potential is not evaluated in the collocation points itself, unless by coincidence.

It can be seen that only the multiple collocation points per panel had converged in the entire domain. The method using the Legendre roots showed a good agreement in the edges of the sphere but not in the central region, and this is probably due to the lower number of collocation points in the central region when following this specific approach. The extended panel method provides a good agreement near the center of the sphere but not in the edges and there are large

”fluctuations” in the values along the azimuth angle (for a constant x value) mainly in the edges region, which was not expected since the flow is symmetric about the incident flow axis (x axis). These fluctuations are much reduced in the Legendre roots approach and negligible in the case of multiple collocation points. This kind of result, with a fast convergence even for a reduced 64 panels mesh, motivated the adoption of the procedure with multiple collocation points per panel.

Figure 6.17: Comparison among several collocation points choice considering the disturbance potential due to an incident unitary flow.

Also, it should be noticed that, due to the lack of continuity of the velocity potential across the patches, the solution may change significantly depending on the user expertise concerning mesh generation. In general, it is always better to keep larger patches instead of splitting it in several small ones due to the continuity of the solution inside each individual patch, defined by construction. In order to investigate this behavior, the same sphere that was described considering a single patch in the previous analyses was split into two patches, as shown in Figure 6.18, where the differences in mesh topology can be observed.

Figure 6.18: Comparison of mesh topology concerning a sphere described by a single patch (right) and by two patches (left).

A similar comparison regarding the computation of the velocity potential function is shown in Figure 6.19, where it is verified that although there are significant differences in the results, the approach with the multiple collocation points per panel is once again less sensitive, with a general good agreement with the analytic solution. Again, this speaks in its favour as the preferable choice for building the linear system.

Figure 6.19: Comparison among several collocation points choice considering the disturbance potential due to an incident unitary flow, considering a sphere described by 2 patches.

6.5 Comparisons concerning the spline degree

In order to verify the influence of the spline degree on the distribution of the potential functions, the method of multiple collocation points per panel was used while varying the spline degree from 0 (constant inside each panel) to 3 (cubic).

The results regarding the distribution of the disturbance potential can be seen in Figure 6.20. As expected for dealing with a spheric body geometry, it can be seen that there is a gain in terms of accuracy of the distribution when changing from a low order (zero order) to a higher order approach and that, in the latter case, only small improvements arise when changing from a linear to a quadratic or cubic model.

Figure 6.20: Comparison among several degrees of spline approximation and analytic solution concerning the disturbance potential around a sphere under incident flow.

In order to evaluate the velocities inside the parametric space using the analytic derivatives of the shape functions, the velocity in the x direction was evaluated and the result compared to the analytic solution concerning the disturbance velocity, as shown in Figure 6.21. Once again, results show that all the higher order models provide good agreement with the analytical expression, although the linear distribution presented larger discrepancies, especially near the $\pm 0.8\text{m}$ position.

Figure 6.21: Comparison among several degrees of spline approximation and analytic solution concerning the disturbance velocity in x direction around a sphere under incident flow.

Number of panels

In order to investigate the convergence of the numerical scheme following the multiple collocation points per panel approach and B-spline solution of degree 2, a convergence analysis was performed changing the panel size, which changes the number of panels in u and v parametric dimensions. The results concerning potential distribution can be seen in Figure 6.22 and velocity in x direction in Figure 6.23.

It can be verified that the agreement concerning all meshes is fine, and in the case of 64 and 36 panels is remarkable the agreement in all points of the domain for both potential and velocity. Actually, in engineering terms even a mesh containing 16 panels can provide reliable results in the entire sphere, while the mesh containing 8 panels provides good results concerning the velocity in x direction but has some differences considering the potential distribution near the coordinate $x=1\text{m}$, where the edge of the sphere patch in the parametric coordinate system is present.

Figure 6.22: Convergence analysis of the velocity potential concerning a sphere under incident flow.

Figure 6.23: Convergence analysis of the velocity in x direction concerning a sphere under incident flow.

6.6 Computation of zero order potential and spatial derivatives in the free surface

As discussed in section 5.3.1 the zero order potential is computed assuming a double body approach, therefore the potential in the free surface and the spatial derivatives are unknown after the zero order solution computation, requiring an additional procedure to compute these contri-

butions that are important to the solution of the first order wave-current-interaction problem. Since the solution is described by a b-spline polynomial the coefficients must be computed in order to provide a complete description of the potential behavior on the free surface, that will be important in the integration and derivation procedures on this surface.

The potential in each point of the free surface can be numerically computed assuming Green's identity applied to the field points inside the fluid region, assuming a solid angle of 4π since a complete sphere is used to isolate the singularity, as discussed in Chapter 5. The integral equation 6.37 is used to evaluate the potential in every point P of the free surface after the splines coefficients are computed following equation (5.17). The procedure assumed in this work is to consider the collocation points located in each panel of the free surface and to solve an overdetermined linear system by a least square approach to compute the b-splines coefficients of the potential function on the free surface.

$$\phi_P^{(0)} = \frac{1}{4\pi} \left[\iint_{\partial\Omega} -U n_{xQ} \left(\frac{1}{r_{PQ}} \right) d\partial\Omega_Q - \iint_{\partial\Omega} \phi_Q^{(0)} \frac{\partial}{\partial n_Q} \left(\frac{1}{r_{PQ}} \right) d\partial\Omega_Q \right], Q \in \bar{S}_{fs} \quad (6.37)$$

A comparison between the numerical and analytical results for the disturbance potential in the free surface collocation points shown in Figure 6.24 can be seen in Figure 6.25, where the results are shown as function of the distance to the origin ($x=0, y=0$) for the several polar coordinates ($\theta = \text{atan}(y, x)$), considering the four quadrants. In general, a good agreement between the numerical results (TDRPM) with the analytical solution can be verified.

Figure 6.24: Collocation points assumed in computation of the free surface potential disturbance due a sphere (black) and patches subdivision lines (red).

Figure 6.25: Comparison of numerical and analytic prediction of the disturbance potential in the free surface collocation points provided by a sphere in infinite flow.

The same analysis is performed for the disturbance velocity in x and y directions, obtained from the derivation of the integral equation (6.37) in x_p direction, as shown in (6.38). The computation of the disturbance velocity in y direction is performed analogously. It can be verified that a good agreement between numerical and analytic prediction is achieved for both velocity components too, as can be seen in Figures 6.26 and 6.27.

$$\frac{\partial \phi_P^{(0)}}{\partial x} = \frac{1}{4\pi} \left[\iint_{\partial\Omega} -U n_{xQ} \left(\frac{x_Q - x_P}{r_{PQ}^3} \right) d\partial\Omega_Q - \iint_{\partial\Omega} \phi_Q^{(0)} \left(\frac{n_{xQ} r_{PQ}^2 - 3(x_P - x_Q) \vec{r}_{PQ} \cdot \vec{n}_Q}{r_{PQ}^5} \right) d\partial\Omega_Q \right], Q \in \bar{S}_{fs} \quad (6.38)$$

Figure 6.26: Comparison of numerical and analytic prediction of the disturbance velocity in x direction in the free surface collocation points provided by a sphere in infinite flow.

Figure 6.27: Comparison of numerical and analytic prediction of the disturbance velocity in y direction in the free surface collocation points provided by a sphere in infinite flow.

6.7 The wave-current-body interaction problem

For building the influence matrices, the procedure for the computation of source and dipole terms was already presented, which is enough for the zero order problem. However, the first order problem requires the computation of additional quantities such as the incident wave field, the free surface elevation, the product of the source terms by the normal vectors and m-terms, as can be seen in (6.39), (6.40) and (6.41). In this work we take advantage of the b-spline approach and compute the contributions of each patch by integrating the polynomial format. It should be noticed that since we perform the computations always in the mean boundaries surfaces, the source, dipoles, normal vectors, area and the regions of integration are all constant in time, so they only have to be computed once.

The integration of the terms containing the incident field (6.41) also does not require a new computation per time-step. Although $\phi_I^{(1)}$ is time dependent in (4.101) and (4.103), the cosine and sine functions may be expanded using the trigonometric relations shown in (6.42) and (6.43), therefore the time variant term can be removed from the integrand, simplifying the computations. Although there are now 4 terms to be integrated instead of a single one, these quantities are computed only once, not requiring any additional computation during the time evolution.

$$\iint_{\tilde{S}_{pm}} \left(\frac{1}{r_{PQ}} \right) \{ \vec{m}_Q^{(0)} \} dS_Q = \int_0^1 \int_0^1 \left(\frac{1}{r_{PQ}(u,v)} \right) \{ \vec{m}_Q^{(0)}(u,v) \} |J(u,v)| dv du \quad (6.39)$$

$$\iint_{\tilde{S}_{pm}} \left(\frac{1}{r_{PQ}} \right) \{ \vec{n}_Q^{*(0)} \} dS_Q = \int_0^1 \int_0^1 \left(\frac{1}{r_{PQ}(u,v)} \right) \{ \vec{n}_Q^{*(0)}(u,v) \} |J(u,v)| dv du \quad (6.40)$$

$$\begin{aligned} \iint_{\tilde{S}_{fixed} \cup \tilde{S}_{pm} \cup \tilde{S}_{fb}} \frac{\partial \phi_{IQ}^{(1)}}{\partial n_Q^{(0)}} \left(\frac{1}{r_{PQ}} \right) dS_Q &= \sum_{p=0}^{N_p} \sum_{i=0}^{N_u} \sum_{j=0}^{N_v} \sum_{w=0}^{N_w} \\ A_w \omega_w \int_0^1 \int_0^1 e^{k_w z(u,v)} \{ &[-\cos \beta_w n_x(u,v) - \sin \beta_w n_y(u,v)] \sin(k_w x(u,v) \cos \beta_w + k_w y(u,v) \sin \beta_w - \omega_w t + \delta_w) + \\ &n_z(u,v) \cos(k_w x(u,v) \cos \beta_w + k_w y(u,v) \sin \beta_w - \omega_w t + \delta_w) \} |J(u,v)| dv du \quad (6.41) \end{aligned}$$

$$\begin{aligned}
& \sin(k_w x(u, v) \cos \beta_w + k_w y(u, v) \sin \beta_w - \omega_w t + \delta_w) = \\
& \sin(k_w x(u, v) \cos \beta_w + k_w y(u, v) \sin \beta_w + \delta_w) \cos(\omega_w t) - \\
& \sin(\omega_w t) \cos(k_w x(u, v) \cos \beta_w + k_w y(u, v) \sin \beta_w + \delta_w) \quad (6.42)
\end{aligned}$$

$$\begin{aligned}
& \cos(k_w x(u, v) \cos \beta_w + k_w y(u, v) \sin \beta_w - \omega_w t + \delta_w) = \\
& \cos(k_w x(u, v) \cos \beta_w + k_w y(u, v) \sin \beta_w + \delta_w) \cos(\omega_w t) - \\
& \sin(\omega_w t) \sin(k_w x(u, v) \cos \beta_w + k_w y(u, v) \sin \beta_w + \delta_w) \quad (6.43)
\end{aligned}$$

6.7.1 Free surface condition

The ordinary linearized free surface conditions should also be solved following a least square approach including the damping effect of the numerical beach, presented in section 5.3.4, that introduces additional terms in the free surface conditions, as reproduced in (6.44) and (6.45).

$$\begin{aligned}
& \frac{\partial \eta_{dist}^{(1)}}{\partial t} + \left(U + \frac{\partial \phi^{(0)}}{\partial x} \right) \left(\frac{\partial \eta_{dist}^{(1)}}{\partial x} + \frac{\partial \eta_I^{(1)}}{\partial x} \right) + \frac{\partial \phi^{(0)}}{\partial y} \left(\frac{\partial \eta_{dist}^{(1)}}{\partial y} + \frac{\partial \eta_I^{(1)}}{\partial y} \right) - \frac{\partial \phi_{dist}^{(1)}}{\partial z} - \\
& (\eta_{dist}^{(1)} + \eta_I^{(1)}) \frac{\partial^2 \phi^{(0)}}{\partial z^2} + \nu \phi_{dist}^{(1)} = 0, \quad z = 0 \quad (6.44)
\end{aligned}$$

$$\begin{aligned}
& \frac{\partial \phi_{dist}^{(1)}}{\partial t} + g \eta_{dist}^{(1)} + \left(U + \frac{\partial \phi^{(0)}}{\partial x} \right) \left(\frac{\partial \phi_{dist}^{(1)}}{\partial x} + \frac{\partial \phi_I^{(1)}}{\partial x} \right) + \frac{\partial \phi^{(0)}}{\partial y} \left(\frac{\partial \phi_{dist}^{(1)}}{\partial y} + \frac{\partial \phi_I^{(1)}}{\partial y} \right) + \\
& \frac{\partial \phi^{(0)}}{\partial z} \left(\frac{\partial \phi_{dist}^{(1)}}{\partial z} + \frac{\partial \phi_I^{(1)}}{\partial z} \right) + \nu \eta_{dist}^{(1)} = 0, \quad z = 0 \quad (6.45)
\end{aligned}$$

The beach effect will be taken into account by applying the free surface condition in the collocation points and solving the coefficients $\frac{\partial \eta_{dist_{ij}}^{(1)}}{\partial t}$ and $\frac{\partial \phi_{dist_{ij}}^{(1)}}{\partial t}$ by a least square approach. For instance, the time derivative of the free surface applied in the collection of collocation points k can be written as (6.46) already considering 4 collocation points per panel. By considering (6.44) and (6.45) all the terms in these conditions can be written under a matrix format, as presented in (6.47), where the vectors A_1 , B_1 , A_2 and B_2 are the spatial derivatives of the incident wave elevation function expanded to isolate the time variation terms and $\otimes$ is used to denote that the product operation is done element by element.

$$\begin{aligned}
& \sum_{i=1}^{N_u} \sum_{j=1}^{N_v} \frac{\partial \eta_{dist_{ij}}^{(1)}}{\partial t} N_i(u_{P_k}) N_j(v_{P_k}) = \\
& \begin{bmatrix} N_1(u_{P1})N_1(v_{P1}) & N_1(u_{P1})N_2(v_{P1}) & \dots & N_{N_u}(u_{P1})N_{N_v}(v_{P1}) \\ N_1(u_{P2})N_1(v_{P2}) & N_1(u_{P2})N_2(v_{P2}) & \dots & N_{N_u}(u_{P2})N_{N_v}(v_{P2}) \\ \vdots & \vdots & \vdots & \vdots \\ N_1(u_{P4N_P})N_1(v_{P4N_P}) & N_1(u_{P4N_P})N_2(v_{P4N_P}) & \dots & N_{N_u}(u_{P4N_P})N_{N_v}(v_{P4N_P}) \end{bmatrix} \begin{pmatrix} \frac{\partial \eta_{dist_{11}}^{(1)}}{\partial t} \\ \frac{\partial \eta_{dist_{12}}^{(1)}}{\partial t} \\ \vdots \\ \frac{\partial \eta_{dist_{N_u N_v}}^{(1)}}{\partial t} \end{pmatrix} = \\
& \left[N_{ij}(u_P, v_P) \right]_{4.N_P X N_u . N_v} \cdot \left\{ \frac{\partial \eta_{dist_{ij}}^{(1)}}{\partial t} \right\}_{N_u . N_v X 1} = [N] \{ \eta_t^{(1)} \} \quad (6.46)
\end{aligned}$$

$$\begin{aligned}
[N] \{ \eta_t^{(1)} \} &= [N] \{ \phi_z^{(1)} \}_{dist} + [N] \{ \phi_{z2}^{(0)} \}_{dist} \otimes [N] \{ \eta_{dist}^{(1)} \} + [N] \{ \phi_{z2}^{(0)} \}_{dist} \otimes \{ A \} \cos(\omega t) \\
&- (U \{ 1 \} + [N] \{ \phi_x^{(0)} \}) \otimes ([N] \{ \eta_x^{(1)} \}_{dist} + \{ A_1 \} \cos(\omega t) + \{ B_1 \} \sin(\omega t)) - \\
&[N] \{ \phi_y^{(0)} \} \otimes ([N] \{ \eta_y^{(1)} \}_{dist} + \{ A_2 \} \cos(\omega t) + \{ B_2 \} \sin(\omega t)) - \{ \nu \} \{ \phi_{dist}^{(1)} \} = 0 \quad (6.47)
\end{aligned}$$

Since the zero order potential is already known from the solution of the zero order problem, these quantities are constant in the first order problem, simplifying the matrix terms. The kinematic first order condition can be solved according to (6.48), which appears to be a complex expression but is very convenient from a numerical point of view because all matrices can be pre-computed and only the vector containing the first order quantities needs to be updated on each time-step. The solution of the dynamic condition can be performed analogously.

$$\begin{aligned}
\{ \eta_t^{(1)} \} &= ([N]^T [N])^{-1} ([N]^T [N]) \{ \phi_z^{(1)} \}_{dist} + ([N]^T [N])^{-1} ([N] \{ \phi_{z2}^{(0)} \}_{dist} \otimes [N]) \{ \eta_{dist}^{(1)} \} \\
&+ ([N]^T [N])^{-1} [N] \{ \phi_{z2}^{(0)} \}_{dist} \otimes \{ A \} \cos(\omega t) - ([N]^T [N])^{-1} [N]^T U \{ 1 \} \otimes [N] \{ \eta_x^{(1)} \}_{dist} \\
&- ([N]^T [N])^{-1} [N]^T U \{ 1 \} \otimes \{ A_1 \} \cos(\omega t) - ([N]^T [N])^{-1} [N]^T U \{ 1 \} \otimes \{ B_1 \} \sin(\omega t) - \\
&([N]^T [N])^{-1} [N]^T [N] \{ \phi_x^{(0)} \} \otimes [N] \{ \eta_x^{(1)} \}_{dist} - ([N]^T [N])^{-1} [N]^T [N] \{ \phi_x^{(0)} \} \otimes \{ A_1 \} \cos(\omega t) - \\
&([N]^T [N])^{-1} [N]^T [N] \{ \phi_x^{(0)} \} \otimes \{ B_1 \} \sin(\omega t) - ([N]^T [N])^{-1} [N]^T [N] \{ \phi_y^{(0)} \} \otimes [N] \{ \eta_y^{(1)} \}_{dist} - \\
&([N]^T [N])^{-1} [N]^T [N] \{ \phi_y^{(0)} \} \otimes \{ A_2 \} \cos(\omega t) - ([N]^T [N])^{-1} [N]^T [N] \{ \phi_y^{(0)} \} \otimes \{ B_2 \} \sin(\omega t) - \\
&([N]^T [N])^{-1} [N]^T \{ \nu \} \{ \phi_{dist}^{(1)} \} = 0 \quad (6.48)
\end{aligned}$$

An alternative approach that will be tested in the future is to replace the beach damping function $\nu(x, y)$ by an equivalent spline approximation as the incident potential function in order to avoid the least square approach and update the coefficients of the differential equations directly using the coefficients of the splines. This procedure would be equivalent to applying the free surface condition continuously in the entire surface instead of only imposing it on the collocation points and should require less computational effort.

The computation of the second order forces is performed considering terms on the body mean wetted surface and body waterline. The only term that has not been discussed yet is the gradient of the time derivative of the potential function, which can be computed analogously to the potential function itself, as presented earlier. This gradient can be obtained based on equation (6.49), where the transformation matrix requires only the spline description concerning body geometry. The quantities $\frac{\partial^2 \phi_{dist}}{\partial u \partial t}$ and $\frac{\partial^2 \phi_{dist}}{\partial v \partial t}$ may be computed directly from the solution of the first order problem, while $\frac{\partial^2 \phi_{dist}}{\partial n \partial t}$ is obtained from the body boundary condition in the acceleration potential equation.

$$\begin{pmatrix} \frac{\partial^2 \phi_{dist}}{\partial x \partial t} \\ \frac{\partial^2 \phi_{dist}}{\partial y \partial t} \\ \frac{\partial^2 \phi_{dist}}{\partial z \partial t} \end{pmatrix}_{(x(u,v), y(u,v), z(u,v))} = \begin{bmatrix} \frac{\partial x}{\partial u} & \frac{\partial y}{\partial u} & \frac{\partial z}{\partial u} \\ \frac{\partial x}{\partial v} & \frac{\partial y}{\partial v} & \frac{\partial z}{\partial v} \\ n_x & n_y & n_z \end{bmatrix}_{(u,v)}^{-1} \begin{pmatrix} \frac{\partial^2 \phi_{dist}}{\partial u \partial t} \\ \frac{\partial^2 \phi_{dist}}{\partial v \partial t} \\ \frac{\partial^2 \phi_{dist}}{\partial n \partial t} \end{pmatrix}_{(u,v)} \quad (6.49)$$

Nonetheless, there is still a numerical problem concerning the definition of the waterline intersection for a generic body, which will be done by following the method presented next.

6.7.2 The computational definition of the waterline intersection

The computation of mean drift forces requires the integration of the free surface elevation along the body's waterline, which in its turn demands the solution of a particularly difficult mathematical problem, the surface-surface intersection (similar to the so-called "trim problem"), a difficult task when the mathematica surface is defined by multiple patches.

Many algorithms are presented in the literature for dealing with this problem, most of them requiring an iterative method or the solution of a system of partial differential equations, see for instance, Teixeira and Creus [2008], Tang and Medioni [1998], Hass et al. [2007]. Regarding the present numerical method, the intersection problem occurs for two different situations: the first one when only the mean wetted surface is provided and then there is no surface above the

undisturbed waterline; the second when the full body geometry is provided and, therefore, the surface bellow the instantaneous waterline must be defined.

In general, the integration of the body waterline (WL) is performed in the several curves defined by the intersection of the body and the undisturbed free surface elevation (WL_i), as shown in (6.50).

$$\oint_{WL} F(l)dl = \sum_{i=1}^N \oint_{WL_i} F(l)dl_i \quad (6.50)$$

It is performed assuming a parametric curve $\gamma_i(s) = (x_i(s), y_i(s), z_i(s))$ that maps the physical curve in the parametric space from 0 to L_i , where $F_i(s)$ is a generic curve, as illustrated in Figure 6.28, applying (6.51).

Figure 6.28: Mapping from the parametric to the physical space concerning the waterline.

$$\oint_{WL_i} F(l)dl_i = \int_0^{L_i} F_i(s) \|\gamma'_i(s)\| ds \quad (6.51)$$

The vector $\gamma'_i(s)$ is tangent to the intersection of both surfaces and since the undisturbed free surface is defined as $z=0$ the vector is defined as the cross product of both normal vectors.

$$\gamma'_i(s) = \left(\frac{\partial S}{\partial x}, \frac{\partial S}{\partial y}, \frac{\partial S}{\partial z} \right) \Big|_{(x(u(s),v(s)),y(u(s),v(s)),0)} \wedge (0,0,1) = \left(\frac{\partial S}{\partial y}, -\frac{\partial S}{\partial x}, 0 \right) \Big|_{(x(u(s),v(s)),y(u(s),v(s)),0)} \quad (6.52)$$

A simple technique to define the surface intersection is to define an initial point in the intersection and then solve the system of coupled ordinary differential equations (6.53), which is performed using a Runge-Kutta-Fehlberg technique with an adaptive step size. Nonetheless,

the solution requires an initial point, which is found by application of Newton-Raphson method in the NURBS space concerning the body patches, as shown in equation (6.54).

Since the initial point computed is not always located in the corner of a patch, the ordinary differential equation is solved considering both tangent vector orientations.

$$\begin{cases} \frac{dx}{ds} = \frac{\partial S}{\partial y} \Big|_{(x(u(s),v(s)),y(u(s),v(s)),0)} \\ \frac{dy}{ds} = -\frac{\partial S}{\partial x} \Big|_{(x(u(s),v(s)),y(u(s),v(s)),0)} \end{cases} \quad (6.53)$$

$$z(u(s=0), v(s=0)) = 0 \Rightarrow x(u(s=0), v(s=0)), y(u(s=0), v(s=0)) \quad (6.54)$$

The solution of this mathematical problem provides a set of point lists $p(j)$, each one regarding the waterline intersection. Therefore the integral can be approximated by equation (6.55) using a trapezoidal rule for non-equally spaced data, where α_i is the weights regarding 1 to the first and last points and 2 for the remaining.

$$\int_0^{L_i} F_i(s) \|\gamma'_i(s)\| ds \approx \sum_{j=0}^{N_c} \sum_{i=0}^{Np(j)-1} \alpha_i F_j(s_i) \sqrt{[x_j(s_{i+1}) - x_j(s_i)]^2 + [y_j(s_{i+1}) - y_j(s_i)]^2} \quad (6.55)$$

The contribution of body motions in the relative elevation of the waterline can be computed using the points obtained in terms of the parametric space. However, the free surface elevation requires an additional step since the elevation is computed in terms of the parametric coordinates (u,v) of the floating body patches, which are not the same of the free surface patches. The coordinates of the parametric space are defined by solving the problem (6.56) in terms of the parametric space of the free surface patch.

$$\begin{cases} x(u, v) = x_j(s_i) \\ y(u, v) = y_j(s_i) \end{cases}, i = 1, 2, \dots, N_p(j), j = 0, \dots, N_c \quad (6.56)$$

From the numerical point of view, the solution of the previous problem can be difficult due to the tolerance concerning the match of body and free surface patches. The points in the body patches are "projected" in the free surface assuming a small tolerance ε , using equation (6.57), where $\vec{n}^*$ is the unitary normal vector in the body waterline intersection pointing outward as

shown in Figure 6.29, neglecting the z-component for non "wall" sided bodies.

$$(x(u, v), y(u, v), 0)_{fs} = (x_j(s_i), y_j(s_i), 0)_{body} + \varepsilon \vec{n}^*|_{(x_j(s_i), y_j(s_i), 0)} \quad (6.57)$$

Figure 6.29: Projection of the points in the body surface intersection to the free surface.

Chapter 7

Verification of the numerical method: selected case-studies

A computational code was implemented based on the formation presented in Chapter 4 and 5 and making use the numerical scheme discussed in Chapter 6. From now on this code will be denoted by TDRPM. This chapter contains a collection of different case-studies selected for the verification of this code. They involved both simple (sphere, circular cylinder) and complex body geometries (semi-submersible, VLCC) and address the computation of first and second-order hydrodynamic forces and motions, fluid velocities and free surface elevation. The influence of current effects on the wave-induced forces is also evaluated considering different geometries. When analysing the correctness of the algorithms implementation and the accuracy of their results, many times comparisons are based on the corresponding results obtained with WAMIT®, a widely used BEM code. It should be noticed, however, that since WAMIT's solution is performed in frequency-domain, the comparisons require some sort of post-processing of data. This is done by calculating the amplitudes (and phases) of the different hydrodynamic quantities (forces, wave elevation) and body motion parameters once a steady-state response is obtained in the time-domain simulations. Whenever WAMIT results are used as the paradigm for the evaluation, a previous convergence analysis regarding the WAMIT mesh has been previously performed. Although the higher-order method was used for WAMIT computations, a direct comparison regarding the meshes is not appropriated, since the numerical schemes are different.

Some case-studies, on the other hand, involve complex practical geometries and the evaluation of demanding dynamic conditions (the slow resonant motions of a semi-submersible platform, the wave-drift damping effects on the heading of a turret FPSO). In this case, although

sometimes WAMIT results are also considered, the verification is fundamentally based on experimental results obtained in wave-basins or towing tanks. Again, many times the comparison cannot be made directly with respect to the time-history of the quantities, and some treatment of the results is required.

The performance of the numerical method in terms of computational processing will not be addressed for two reasons. The first one is that a similar BEM time-domain code on which this comparison could be made was not available. One must remind that although WAMIT is a BEM code, its approach and algorithms are essentially different from the ones employed in the time domain solution. The second is the fact that the present version of the time domain code is implemented in Matlab® environment, which is obviously not the best choice if the objective is to improve the computational performance. With this considerations in mind, the numerical results for the different case-studies will be presented in the following sequence:

1. The initial simulations were performed in order to verify the higher order methodology regarding the NURBS geometry description and potential (velocity etc) b-spline formulation for simplified geometries (sphere and circular cylinder). The first order solution was verified considering the computed RAOs¹ compared to numerical solutions provided by the frequency domain commercial software WAMIT in 6 DoF².
2. The verification of the first and second order potential derivatives ($\frac{\partial \phi}{\partial x}, \frac{\partial \phi}{\partial y}, \frac{\partial \phi}{\partial z}, \frac{\partial^2 \phi}{\partial x^2}, \frac{\partial^2 \phi}{\partial y^2}, \frac{\partial^2 \phi}{\partial z^2}, \frac{\partial^2 \phi}{\partial x \partial y}, \frac{\partial^2 \phi}{\partial x \partial z}, \frac{\partial^2 \phi}{\partial y \partial z}$) in the body surface was performed by computing the so-called m-terms for a sphere. The results were compared with the analytical solution for the zero order problem (current effect) considering the entire set of m-terms. Moreover, for practical purposes, they are also compared to the Neumann-Kelvin approximation, commonly applied for slender ships, that requires only the first order derivatives computation.
3. The wave runup along a fixed sphere is evaluated using TDRPM and compared to WAMIT predictions in order to validate the numerical implementation. The runup is an important contribution to the second order drift forces. The fluid velocity field on the body surface of the fixed sphere is also evaluated and compared to WAMIT's frequency domain results, since the accuracy of the velocity field computation is critical for proper evaluation of the

¹Response Amplitude Operator

²Degrees of Freedom

second-order quadratic pressure. These quantities were integrated over the body surface for the computation of the second order loads due to several regular waves to achieve the mean and double frequency forces in surge and heave, which are also compared to WAMIT's results. The simulations were performed for a fixed sphere in order to validate these loads considering a simplified case where the body motions are absent. It should be noticed that the time domain simulation computed by the TDRPM provides the mean and double frequency results combined, so a Fourier procedure is performed to segregate these components.

4. The second order loads are computed for a floating sphere in order to include the effects of the body motions and acceleration potential. The wave runup and velocity field are again computed and compared to frequency domain results.
5. The second order loads on the fixed sphere are computed considering a combination of several regular waves in order to evaluate the second order difference and sum frequencies loads. It should be reminded that only the quadratic terms regarding first order quantities are considered in the computation, therefore the influence of the second order potential is neglected.
6. The second order motions were evaluated in the case of a semi-submersible platform, which was tested experimentally in the Hydrodynamic Calibrator of the University of São Paulo (CH-TPN). The waves were defined in order to amplify the second order motions in surge and pitch degrees of freedom. The numerical results are computed by TDRPM and compared to the experimental data and WAMIT predictions in order to verify the numerical method capability.
7. The wave-current effects are investigated for a fixed hemisphere and a fixed circular cylinder considering the first order excitation force under negative, zero and positive Froude numbers. The results are compared to WAMIT predictions for the zero Froude condition, to Kim and Kim [2007] results for the positive and negative Froude numbers in the case of the sphere and to Shao [2010] results for the fixed cylinder. It should be noticed that both Kim and Kim [2007] and Shao [2010] are BEM higher order methods. However both methods adopt quadratic or cubic isoparametric shape functions for both body geometry and velocity potential, instead of NURBS geometrical description combined to the B-spline formulation for the velocity potential adopted in the developed method.

8. The wave-current effects are investigated in terms of second order quantities considering the mean drift forces for positive and negative Froude number for the same geometries described on the previous item. The evaluation of the wave drift damping coefficient in surge direction is compared to the numerical predictions provided by Kim and Kim [2007].
9. The first order RAOs are computed considering several Froude numbers for a floating hemisphere, which are compared to Huijsmans [1996] results in order to validate the body motion equations in the presence of current.
10. In order to verify the numerical predictions of the wave-current interaction for a more practical application, an analysis of the wave-drift-damping effect on the mean heading of a FPSO unit is performed. The results achieved by the TDRPM regarding the equilibrium yaw angle are compared to the results in Aranha et al. [2001], which studied the problem with the help of model tests.

7.1 First order response amplitude operator (RAO) computation without current

7.1.1 Hemisphere (Infinite water depth)

The linear response amplitude operators (RAOs) for a floating sphere were computed assuming a hemisphere of radius 1m built with 2 patches and a circular free-surface of radius 20m built considering a single patch, as shown in Figure 7.1.

In order to verify the numerical convergence of the method, three different meshes were used in the computations, each one considering 3 different degrees for the spline solution approximation (0, 1 and 2). The solutions were compared to the frequency domain WAMIT code to verify the correctness of the predictions. It should be noticed that although the zero order solution approximation is usually called "low order" in the literature, this is only so if the panels are also plane with constant normal vector, which is not the case in the developed method. In the present code the geometry will always be described following a NURBS pattern, providing a continuous representation of the geometry and normal vector.

Figure 7.1: Sphere and free surface geometries.

The numerical meshes adopted in the computations had the discretization shown in Table 7.1 and the subdivisions in the physical domain are illustrated in Figure 7.2.

Table 7.1: (U x V) number of subdivisions in the parametric space for the sphere

Discretization	Body patch 1	Body patch 2	Free surface
Fine	6X6	6X6	30X30
Medium	4X4	4X4	24X24
Coarse	2X2	2X2	16X16

Figure 7.2: Meshes adopted for the computation of the sphere: fine (left), medium (center) and coarse (right).

Since the simulations are performed in time domain, the responses achieved are time series of the motions in the several degrees of freedom, for example, the series shown in Figure 7.3 for the surge and heave motions. The wave heading chosen was 0 degrees (waves along the x-axis) therefore no sway, roll or yaw motions are expected due to the problem symmetry. Also, there is no pitch motions because the center of gravity is taken coincident to the sphere geometric center.

Figure 7.3: Example of motion time series of a sphere. (ξ_1 : surge, ξ_3 : heave)

The time series are post-processed to obtain the amplitude of the response in steady oscillation, thus neglecting the first cycles with ramp and other transient effects. The amplitudes are computed from the mean of the several maximum and minimums considering all cycles. The surge and heave transfer functions can be seen in Figures 7.4 and 7.5, showing that a good agreement is achieved for the fine mesh regardless of the spline degree, while the coarser meshes present larger discrepancies for the zero and first order splines, which is more evident in the heave amplitude operator.

Figure 7.4: Surge response amplitude operator for the sphere.

Figure 7.5: Heave response amplitude operator for the sphere.

The effect of the spline degree effect on the surge and heave responses computed with the fine meshes can be assessed in Tables 7.2 and 7.3, which present the mean difference between the Rankine panel method and WAMIT results. One may realize that the differences are in general lower degree 2 if compared to the first and zero order ones. However, it should also be noticed that all results are in good agreement since the mean differences are always less than 2.5% and may be considered negligible for practical applications. The largest differences are related to low wave periods (high frequencies) for the wavelengths are smaller and therefore the effect of free surface discretization in the numerical solution is more pronounced.

Some views of the wave field computed during the simulations of the free floating sphere can be seen in Figure 7.6 and Figure 7.7 for wave frequencies $\omega = 3.706$ rad/s ($kR=1.4$) and $\omega = 2.426$ rad/s ($kR=0.60$), respectively. It can be seen that for the higher wave period (Figure 7.7) almost no runup is observed and that the body vertical motion essentially follows the wave elevation. On the hand, for the lower wave period (Figure 7.6), there is a significant runup on the sphere and the body motions are appreciably smaller. The wave diffraction effects are clearly more pronounced in this case.

Table 7.2: Surge response amplitudes (m/m) for the fine mesh and differences with respect to WAMIT results for the sphere.

SURGE							
$\omega(rad/s)$	WAMIT	Degree			Difference		
		0	1	2	0	1	2
2.426	0.694	0.694	0.691	0.692	0.07%	0.47%	0.30%
2.621	0.645	0.645	0.642	0.643	0.08%	0.55%	0.39%
2.801	0.597	0.596	0.593	0.594	0.19%	0.76%	0.61%
2.971	0.552	0.554	0.550	0.550	0.28%	0.43%	0.31%
3.132	0.509	0.512	0.508	0.508	0.61%	0.25%	0.20%
3.285	0.468	0.473	0.468	0.468	1.02%	0.15%	0.16%
3.431	0.431	0.438	0.431	0.431	1.71%	0.19%	0.03%
3.571	0.396	0.406	0.397	0.396	2.58%	0.48%	0.22%
3.706	0.363	0.375	0.366	0.365	3.23%	0.78%	0.32%
3.836	0.334	0.346	0.336	0.334	3.72%	0.80%	0.11%
3.962	0.307	0.321	0.308	0.306	4.61%	0.44%	0.11%
4.084	0.282	0.295	0.281	0.279	4.71%	0.27%	1.16%
4.202	0.260	0.273	0.258	0.255	5.32%	0.73%	1.90%
MEAN					2.16%	0.49%	0.45%

Table 7.3: Heave response amplitudes (m/m) for the fine mesh and differences with respect to WAMIT results for the sphere.

HEAVE							
$\omega(rad/s)$	WAMIT	Degree			Difference		
		0	1	2	0	1	2
2.426	1.185	1.181	1.188	1.184	0.35%	0.60%	0.32%
2.621	1.304	1.295	1.307	1.301	0.67%	0.93%	0.46%
2.801	1.479	1.458	1.476	1.466	1.47%	1.25%	0.65%
2.971	1.712	1.688	1.714	1.698	1.39%	1.51%	0.94%
3.132	1.887	1.870	1.887	1.864	0.87%	0.92%	1.23%
3.285	1.726	1.787	1.749	1.739	3.56%	2.14%	0.60%
3.431	1.301	1.390	1.337	1.340	6.84%	3.79%	0.22%
3.571	0.927	0.960	0.929	0.936	3.49%	3.19%	0.75%
3.706	0.673	0.679	0.653	0.669	0.88%	3.73%	2.45%
3.836	0.504	0.493	0.479	0.493	2.25%	2.83%	2.83%
3.962	0.390	0.373	0.360	0.374	4.36%	3.43%	3.89%
4.084	0.309	0.292	0.290	0.301	5.56%	0.60%	4.01%
4.202	0.250	0.233	0.242	0.249	6.43%	3.68%	2.83%
MEAN					2.93%	2.20%	1.63%

Figure 7.6: Examples of the wave pattern and sphere displacements during the simulation for $\omega = 3.706$ rad/s, from $t_{ini}=10.0s$ to $t_{end}=12.4s$.

Figure 7.7: Examples of the wave pattern and sphere displacements during the simulation for $\omega = 2.426$ rad/s, from $t_{ini}=10.0s$ to $t_{end}=12.4s$.

7.1.2 Vertical Circular Cylinder (Infinite water depth)

The same analysis performed for the sphere was repeated for a circular cylinder with vertical axis in order to evaluate also a rotational motion (pitch). The main objective was to confirm the correct implementation of the method and the time integration of the analysis of the angular motion equations for this simple geometry, which was generated considering only two patches, one for the circular hull (patch 1) and another one for the bottom (patch 2). The geometry of the circular cylinder can be seen in Figure 7.8 and the meshes adopted for the computations are illustrated in Figure 7.9. Additionally this geometry was studied due to the presence of sharp corners, which could generate problems to the numerical method, since separation effects may occur in these regions, which is not modeled in the scope of the potential formulation.

Table 7.4: (U x V) number of subdivisions in the parametric space for the circular cylinder

Discretization	Body patch 1	Body patch 2	Free surface
Fine	8X8	8X8	30X30
Medium	6X6	6X6	24X24
Coarse	4X4	4X4	16X16

Figure 7.8: Free surface and circular cylinder geometry.

Figure 7.9: Meshes adopted for the computation of the circular cylinder: fine (left), medium (center) and coarse (right).

The first-order RAOs obtained with the different meshes are presented in Figure 7.10, 7.11 and 7.12 for the surge, heave and pitch motions, respectively. Again, it can be seen that there is a good agreement for the fine mesh, regardless of the spline degree. Mean differences with respect to the WAMIT results are provided in Tables 7.5, 7.6 and 7.7. As in the case of the sphere, the differences, in general are smaller when a degree 2 spline is used for representing the velocity and acceleration potentials.

Figure 7.10: Surge response amplitude operator for the vertical circular cylinder.

Figure 7.11: Heave response amplitude operator for the vertical circular cylinder.

Figure 7.12: Pitch response amplitude operator for the vertical circular cylinder.

Table 7.5: Surge response amplitudes (m/m) for the fine mesh and differences with respect to WAMIT results for the circular vertical cylinder.

SURGE (m/m)							
$\omega(rad/s)$	WAMIT	Degree			Difference		
		0	1	2	0	1	2
1.700	0.883	0.872	0.874	0.876	1.26%	1.00%	0.86%
1.756	0.880	0.865	0.868	0.869	1.74%	1.40%	1.23%
1.811	0.878	0.860	0.864	0.865	2.01%	1.64%	1.44%
1.867	0.877	0.855	0.858	0.860	2.49%	2.10%	1.85%
1.922	0.873	0.841	0.845	0.848	3.64%	3.25%	2.89%
1.978	0.855	0.816	0.817	0.822	4.51%	4.41%	3.87%
2.000	0.837	0.799	0.798	0.803	4.56%	4.66%	4.05%
2.033	0.797	0.767	0.764	0.769	3.78%	4.21%	3.55%
2.089	0.715	0.708	0.703	0.705	1.06%	1.75%	1.43%
2.144	0.668	0.674	0.670	0.669	0.95%	0.36%	0.15%
2.200	0.652	0.665	0.662	0.659	1.91%	1.41%	0.97%
2.333	0.641	0.648	0.646	0.643	1.10%	0.78%	0.43%
2.667	0.586	0.593	0.591	0.590	1.20%	0.88%	0.78%
3.000	0.509	0.517	0.514	0.514	1.58%	1.06%	1.10%
3.333	0.426	0.430	0.426	0.426	0.86%	0.14%	0.06%
3.667	0.345	0.357	0.352	0.353	3.45%	1.77%	2.06%
4.000	0.272	0.270	0.269	0.258	0.65%	1.18%	5.33%
4.333	0.210	0.211	0.212	0.201	0.67%	1.18%	4.34%
4.667	0.159	0.151	0.164	0.161	5.20%	2.86%	1.08%
5.000	0.120	0.132	0.124	0.121	9.88%	3.22%	0.72%
MEAN					2.62%	1.96%	1.91%

Table 7.6: Heave response amplitudes (m/m) for the fine mesh and differences with respect to WAMIT results for the circular vertical cylinder

HEAVE (m/m)							
$\omega(rad/s)$	WAMIT	Degree			Difference		
		0	1	2	0	1	2
1.700	1.126	1.135	1.133	1.132	0.80%	0.69%	0.60%
1.756	1.151	1.160	1.159	1.158	0.83%	0.70%	0.60%
1.811	1.181	1.194	1.192	1.190	1.09%	0.93%	0.80%
1.867	1.217	1.231	1.229	1.227	1.11%	0.91%	0.77%
1.922	1.262	1.277	1.274	1.271	1.16%	0.91%	0.73%
1.978	1.317	1.336	1.332	1.330	1.45%	1.14%	0.93%
2.000	1.343	1.364	1.359	1.356	1.53%	1.19%	0.95%
2.033	1.387	1.411	1.405	1.401	1.76%	1.36%	1.08%
2.089	1.474	1.511	1.504	1.498	2.51%	1.99%	1.62%
2.144	1.588	1.639	1.629	1.621	3.20%	2.55%	2.08%
2.200	1.740	1.821	1.807	1.796	4.68%	3.86%	3.24%
2.333	2.399	2.614	2.584	2.554	8.98%	7.72%	6.47%
2.667	2.229	2.104	2.175	2.215	5.63%	2.41%	0.64%
3.000	0.515	0.438	0.475	0.492	14.83%	7.70%	4.39%
3.333	0.201	0.187	0.194	0.197	6.97%	3.49%	2.00%
3.667	0.092	0.076	0.075	0.080	17.22%	18.91%	12.90%
4.000	0.045	0.042	0.042	0.048	7.25%	7.35%	4.90%
4.333	0.023	0.025	0.021	0.022	9.23%	8.47%	4.05%
4.667	0.012	0.012	0.012	0.012	1.93%	3.86%	0.86%
5.000	0.006	0.008	0.006	0.006	21.97%	1.75%	2.24%
MEAN					5.71%	3.89%	2.59%

Table 7.7: Pitch response amplitudes (rad/m) for the fine mesh and differences with respect to WAMIT results for the circular vertical cylinder

$\omega(rad/s)$	PITCH (rad/m)						
	WAMIT	Degree			Difference		
		0	1	2	0	1	2
1.700	0.690	0.691	0.699	0.694	0.13%	1.23%	0.47%
1.756	0.831	0.833	0.844	0.836	0.29%	1.60%	0.64%
1.811	1.019	1.029	1.045	1.033	0.97%	2.54%	1.35%
1.867	1.280	1.298	1.321	1.303	1.41%	3.21%	1.78%
1.922	1.641	1.655	1.686	1.661	0.83%	2.69%	1.17%
1.978	2.096	2.060	2.088	2.064	1.72%	0.35%	1.49%
2.000	2.270	2.196	2.200	2.219	3.25%	3.09%	2.25%
2.033	2.449	2.324	2.330	2.375	5.13%	4.88%	3.04%
2.089	2.372	2.262	2.238	2.263	4.66%	5.64%	4.63%
2.144	1.990	1.948	1.912	1.949	2.10%	3.89%	2.06%
2.200	1.611	1.591	1.559	1.593	1.28%	3.23%	1.17%
2.333	1.046	1.017	1.001	1.020	2.78%	4.31%	2.51%
2.667	0.541	0.533	0.527	0.534	1.59%	2.65%	1.27%
3.000	0.357	0.355	0.351	0.356	0.63%	1.82%	0.50%
3.333	0.257	0.254	0.249	0.253	1.24%	3.05%	1.65%
3.667	0.191	0.195	0.190	0.193	1.99%	0.99%	0.60%
4.000	0.145	0.144	0.140	0.144	0.67%	3.29%	0.51%
4.333	0.110	0.113	0.110	0.104	2.27%	0.06%	5.62%
4.667	0.085	0.089	0.088	0.084	4.44%	3.49%	1.08%
5.000	0.066	0.070	0.069	0.068	6.63%	4.03%	3.24%
MEAN					2.20%	2.80%	1.85%

7.2 Computation of m-terms over a sphere

The m-terms were computed for a sphere in order to validate the algorithms included in this computation, since for this geometry an analytical solution exists and is obtained from the dipole singularity potential, providing a very useful benchmark. In order to quantify the differences between the Neumann-Kelvin and the Double-body linearization procedures both computations are considered for this specific geometry, although the former is clearly not the best choice from the physical point of view for dealing with bluff bodies. The analytical solution of the zero order potential was already presented in section 6.4 to justify the choice for the method that adopts multiples collocation points per panel. The sphere is centered in the origin of the coordinate system with the flow aligned in the x-direction.

The computation of the m-terms requires the evaluation of the second derivatives of the potential function with respect to the spatial coordinates (x,y,z), which usually cannot be computed easily. However, by following the higher order approach these computations can be performed according to the linear system described in the expression (6.14).

The points where the comparison of the m-terms are performed correspond to 16 gaussian points per higher order panel in the parametric space, as illustrated in Figure 7.13, where the red lines indicate the limits of each higher order panel and the black dots indicate the gaussian points. The comparison between the computed m-terms and those derived analytically can be seen in Figure 7.14 and 7.15 for the x, y, z, yy and zz components, where the comparisons are presented as function of the the point ID, which is not directly related to x, y or z coordinate. The point ID is an identification to the point used during the integration process. A fine agreement is obtained with the double-body approximation, whose results are plotted in red circles.

The major differences are located near the limits (borders of patch) of the sphere, where there is a singularity in the Jacobian operator during the mapping transformation, which was also reported by Maniar [1995a] in his computation.

Figure 7.13: Gaussian points assumed in the m-terms computation.

Just for the sake of illustrating the differences between the approaches available for linearizing the free surface, the Neumann-Kelvin approximation was also used, although only the rotational terms over y and z directions are available (Figure 7.15), revealing a poor agreement. As it was mentioned before, this should already be expected since the sphere is not a slender body, a fundamental assumption of the Neumann-Kelvin approximation.

The normalized mean error achieved in the computation of the m-terms was evaluated following expression (7.1), where X_i^{num} is the numeric computation and X_i^{teo} the analytic value of an specific m-term. N is total number of points evaluated for calculating the mean differences shown in Table 7.8. The rotational term in x direction was descended since it can be shown that its contribution is null, in this case.

$$E = \frac{1}{N[\max(X) - \min(X)]} \sum_{i=1}^N (X_i^{num} - X_i^{teo}) \quad (7.1)$$

Table 7.8: Mean errors in the computation of the m-terms over a sphere considering the numerical results and analytical solution.

m-term	Normalized Mean error
m_{0x}	1.09%
m_{0y}	0.96%
m_{0z}	0.82%
m_{0yy}	0.56%
m_{0zz}	0.74%

The results attest the correct implementation of the velocity field computation, which will be also required for a proper evaluation of the second-order pressure terms.

Figure 7.14: Comparison of m-terms in the degrees of freedom x, y and z (translations).

Figure 7.15: Comparison of m-terms in the degrees of freedom yy and zz (rotations).

In order to allow a better visualization of the m-terms variation, the numerical values are plotted considering the sphere polar coordinates in Figures 7.16 and 7.17. Here the x-axis refers to the polar angle θ and y-axis is the azimuthal angle ψ according to the spherical coordinates in (7.2). The gaussian points can be seen in black dots and the color map represents the m-terms intensity.

$$\begin{aligned}
 x &= R \cos(\theta) \sin(\psi) \\
 y &= R \sin(\theta) \sin(\psi) \\
 z &= R \cos(\psi) \\
 0 \leq \theta \leq 2\pi, \quad \pi/2 \leq \psi \leq \pi
 \end{aligned}
 \tag{7.2}$$

(a) x

(b) y

(c) z

Figure 7.16: m-terms color map in x, y and z coordinates (translations) for a sphere.

(a) xx

(b) yy

(c) zz

Figure 7.17: m-terms color map in xx, yy and zz coordinates (rotations) for a sphere.

7.3 Mean drift and sum frequency forces on a fixed hemisphere in regular waves

The mean and double frequency second order forces were computed for a fixed hemisphere in order to verify the numerical implementation. It should be noticed that in time-domain simulations it is not possible to segregate the mean and double frequency effects prior to the simulations. Therefore, the results were post-processed for comparison with frequency domain results. In the case of a fixed body, the second order loads are computed based on equation (7.3), in the absence of current and body motions, thus they are only dependent of the wave runup and velocity field. For a more detailed analysis, the accuracy obtained in the computation of these quantities is presented first.

$$\vec{F}^{(2)} = -\frac{1}{2}\rho g \oint_{WL} [\eta^{(1)}]^2 \vec{n}^{(0)} dl + \frac{1}{2}\rho \iint_{\tilde{S}_B} \nabla \varphi^{(1)} \cdot \nabla \varphi^{(1)} \vec{n}^{(0)} dS \quad (7.3)$$

7.3.1 Wave runup

The wave runup along a half-submerged sphere at unitary radius was computed considering wave frequencies varying from $\omega = 2rad/s$ to $\omega = 4rad/s$ with increments of $0.2rad/s$, with unitary amplitude ($A = 1m$). The runup was compared for the entire waterline, assuming the grid and orientation defined in Figure 7.18. A collection of 200 points located in the waterline was used, with the parametric coordinates defined following the numerical algorithm presented in section 6.7.2, that projects the points of the body on the waterline into the free surface patches. It should be noticed that there is a numerical border effect close to edge of the free surface patch, therefore these points were defined for $R = 1.01m$ for the unitary radius sphere in order to avoid numerical problems. This procedure should not influence the results appreciably since the wavelengths are much larger than this fictitious distance. The b-spline degree assumed in the basis form of the solution was 2 for both body and free surface.

Figure 7.18: Grid points assumed in the computation of runup along the cylinder.

The results computed using the time domain code are compared to WAMIT1s frequency domain ones³. In order to obtain the wave runup at each point, the runup factor is computed using equation (7.4). An example of free surface elevation time series can be seen in Figure 7.19 for the point (1.01,0), located upstream of the flow and for $\omega=4$ rad/s. Here η_I is the incident wave elevation, η_{dist} is the disturbed free surface due to the body interaction and η_{total} the sum of both quantities.

$$|\eta_i| = \sqrt{2} \left[\frac{1}{t_{sim}} \int_0^{t_{sim}} \eta_i^2 dt \right]^{1/2}, \quad i = 1, 2, \dots, N_p \quad (7.4)$$

Figure 7.19: Example of free surface elevation for the point (1.01,0,0) close to the sphere for $\omega=4$ rad/s.

³The same points with a $R=1.01$ m distance from the origin were evaluated.

Figure 7.20⁴ brings a comparison of the wave runup along the waterline for the several wave frequencies. One may observe that for long waves (low frequencies) the diffraction effects are not pronounced since the waves are appreciably longer than the sphere diameter, while for shorter waves the diffractions effects are very important and the amplification factor reaches up to 60% of the incident wave amplitude. The agreement between WAMIT and Rankine results is good, the differences being always less than 8% considering all the points and frequencies analyzed. The major differences occur for short waves around $\theta = 0^\circ$. In fact this indicates that the free surface mesh should be more refined in this region since this is an "end patch" zone, more affected by local numerical effects due to the lack of continuity in the geometry.

⁴In this picture the results for $\omega = 4.0\text{rad/s}$ are suppressed for a better visualization of the other wave frequencies.

Figure 7.20: Wave Runup for the fixed sphere for wave frequencies from $\omega=2.0\text{rad/s}$ until $\omega=3.8\text{rad/s}$ considering increments of 0.2rad/s .

Table 7.9 presents a comparison of the differences obtained in the runup predictions according to the TDRPM and WAMIT. Mean differences represent the mean value for the 200 points.

Table 7.9: Comparison of TDRPM and WAMIT results regarding the wave runup over a fixed sphere.

ω (rad/s)	Maximum			Minimum			Mean difference (m)
	TDRPM	WAMIT	Difference (%)	TDRPM	WAMIT	Difference (%)	
2.0	1.08	1.07	1.49	0.85	0.86	0.80	0.02
2.2	1.13	1.15	1.69	0.83	0.83	0.17	0.01
2.4	1.22	1.26	3.15	0.80	0.80	0.23	0.01
2.6	1.30	1.36	3.90	0.77	0.76	1.19	0.02
2.8	1.39	1.44	3.40	0.73	0.73	0.12	0.02
3.0	1.44	1.49	3.70	0.69	0.69	0.16	0.02
3.2	1.48	1.53	3.58	0.65	0.66	1.01	0.02
3.4	1.50	1.58	4.59	0.60	0.59	2.12	0.04
3.6	1.52	1.59	4.73	0.55	0.55	0.05	0.04
3.8	1.54	1.64	6.40	0.52	0.50	4.21	0.05
4.0	1.56	1.68	6.62	0.50	0.47	7.12	0.06

7.3.2 Velocity field

As mentioned before, a good accuracy in the computation of the velocity field is very important because the quadratic velocity term plays an important role in the second order pressure, meaning that inaccurate predictions may also affect the mean drift forces appreciably.

The comparisons are performed over the entire body surface assumed for the velocity integration process during mean drift forces computation. The points are generated in the parametric space assuming 16 gaussian points for each "higher order" panel, following the same pattern employed in section 7.2 and illustrated in Figure 7.13.

Figure 7.21 presents an example of time series computed by TDRPM considering the three velocity components. In this example, velocities were evaluated on point $(x,y,z)=(0,1,0)$ and for a wave frequency up to $\omega = 4 \text{ rad/s}$. Figure 7.22 presents a comparison of the velocity amplitudes computed by both, TDRPM and WAMIT, for all the points of the grid in the range between 2.0 to 3.8 rad/s. In these figures, the x-axis represents the point ID, which is related to integration process, as already discussed in 7.2.

Figure 7.21: Example of velocity time series for a point in the body surface.

Once again, a good agreement is observed in general, except around the patch singularity close to the sphere "edge", where excessively high velocities are obtained in both numerical methods. However, it was verified that the spurious values occur only for a few singular points, and the contribution of these values for the total integral performed for the evaluation of the mean drift forces is not significant.

Table 7.10 presents a quantitative comparison of differences between both predictions considering mean and standard deviation values derived according to (7.5).

$$\begin{aligned}
 MeandiffV_x &= \frac{1}{N_p} \frac{\sum_{i=1}^{N_p} |V_{xiTDRPM} - V_{xiWAMIT}|}{A\omega} \\
 StddiffV_x &= \frac{1}{A\omega} \sqrt{\frac{\sum_{i=1}^{N_p} (V_{xiTDRPM} - V_{xiWAMIT})^2}{N_p - 1}}
 \end{aligned} \tag{7.5}$$

Figure 7.22: Velocity field for the fixed sphere for wave frequencies from $\omega=2.0\text{rad/s}$ until $\omega=3.8\text{rad/s}$ considering increments of 0.2rad/s

Table 7.10: Mean and standard deviation of the difference of the velocity field regarding WAMIT and TDRPM simulations for a fixed sphere.

ω (rad/s)	Mean diff Vx	Std diff Vx	Mean diff Vy	Std diff Vy	Mean diff Vz	Std diff Vz
2.0	0.02	0.02	0.00	0.08	0.02	0.07
2.2	0.01	0.02	0.00	0.09	0.03	0.07
2.4	0.00	0.02	0.00	0.09	0.03	0.08
2.6	0.00	0.02	0.00	0.09	0.02	0.07
2.8	0.00	0.02	0.00	0.08	0.02	0.07
3.0	0.00	0.02	0.00	0.08	0.02	0.07
3.2	0.00	0.02	0.00	0.08	0.02	0.07
3.4	0.01	0.02	0.00	0.08	0.01	0.07
3.6	0.01	0.02	0.00	0.08	0.01	0.07
3.8	0.01	0.02	0.00	0.08	0.01	0.08
4.0	0.01	0.02	0.00	0.08	0.01	0.08

An example of the evolution in time of the velocity field module (absolute values⁵) can be seen in Figure 7.23 for the wave frequency $\omega = 2.0$ rad/s and considering the time frames between 5.0s to 7.1s.

A visual comparison of the variation of the velocity amplitude over the wetted surface predicted by both methods can be done in Figure 7.24, Figure 7.25, Figure 7.26 and Figure 7.27, following the same orientation defined in section 7.2 regarding the polar coordinates.

⁵ $|\vec{v}^{(1)}| = \sqrt{(v_x^{(1)})^2 + (v_y^{(1)})^2 + (v_z^{(1)})^2}$

Figure 7.23: Example of the evolution of the velocity absolute values over the sphere for $\omega = 2.0\text{rad/s}$ from $t_{ini}=5.0\text{s}$ until $t_{end}=7.1\text{s}$

Figure 7.24: Velocity amplitudes over the fixed sphere for wave frequencies $\omega=2.0\text{rad/s}$ to $\omega=2.4\text{rad/s}$ considering increments of 0.2rad/s .

(a) 2.6 rad/s

(b) 2.8 rad/s

(c) 3.0 rad/s

Figure 7.25: Velocity amplitudes over the fixed sphere for wave frequencies $\omega=2.6\text{rad/s}$ to $\omega=3.0\text{rad/s}$ considering increments of 0.2rad/s .

(a) 3.2 rad/s

(b) 3.4 rad/s

(c) 3.6 rad/s

Figure 7.26: Velocity amplitudes over the fixed sphere for wave frequencies $\omega=3.2\text{rad/s}$ to $\omega=3.6\text{rad/s}$ considering increments of 0.2rad/s .

Figure 7.27: Velocity amplitudes over the fixed sphere for wave frequencies $\omega=3.8\text{rad/s}$ to $\omega=4.0\text{rad/s}$ considering increments of 0.2rad/s .

7.3.3 Mean drift and double frequency sum forces

Once the computations of wave elevation and velocity field have been verified, the next step is to evaluate the prediction of the second order forces in x and z directions. Since the simulations involved only regular (monochromatic) waves, these forces will encompass a mean (zero frequency $\Delta\omega = 0\text{rad/s}$) component and a component that oscillates with twice the wave frequency ($\Delta\omega = 2\omega$)⁶.

The same mesh parameters and b-spline degrees are adopted in the comparisons and an example of the time series of the second order forces can be seen in Figure 7.28. One should

⁶One should remind that only quadratic components are computed by the method for the 2nd order problem.

realize that the mean period of oscillation in this series is $T=0.78s$, corresponding to $\omega = 8.0rad/s$, which is twice the incident wave frequency, as expected. The existence of a non-null mean drift force in the x-direction is perceived as the signal does not present zero mean. The value of the mean drift component ($\Delta\omega = 0$) is extracted computing the mean value of the time series, neglecting the first 6s of transient effects.

$$\bar{F}_{x,z} = \frac{1}{\Delta t} \int_{t_i}^{t_i+\Delta t} F_{x,z}^{(2)}(t) dt \quad (7.6)$$

Figure 7.28: Example of the surge quadratic second order forces for a monochromatic wave for a fixed sphere $\omega=4$ rad/s.

The z-component of the mean forces has no influence of the waterline wave elevation since there is no z-component in the body normal vector, thus only the velocity field contributes to the heave mean force. The results are compared to WAMIT predictions in Figures 7.29 and 7.30. For the surge force the contributions arising from wave elevation and from the quadratic velocity term are also presented separately. One may realize that the behavior of each component is different since the waterline integral is positive while the velocity pressure term is negative, so the later term means a drop in pressure while the first one an increase.

A good agreement between the methods is observed both in trends and accuracy, since the differences are less than 5% and 10% for the heave and surge forces, respectively.

Figure 7.29: WAMIT and TDRPM mean drift forces in surge direction for a fixed sphere. The components obtained in TDRPM simulations are also presented separately.

Figure 7.30: WAMIT and TDRPM mean drift forces in heave direction for a fixed sphere.

The TDRPM sum-frequency forces can also be computed from the time series assuming a sinusoidal signal using equation (7.7). The results are compared to the ones obtained from WAMIT quadratic second order forces in Figure 7.31 and 7.32. The agreement is good for both directions and the values in surge are appreciably higher than those in the heave direction mainly due to the free surface integral contribution, which is not present in the heave force.

The comparison regarding the zero and double frequency second order loads for the fixed sphere considering Rankine and WAMIT results can be verified in Tables 7.11 and 7.12.

$$F_{x,z}^{(2\omega)} = \sqrt{2} \sqrt{\frac{1}{\Delta t} \int_{t_i}^{t_i+\Delta t} [F_{x,z}^{(2)}(t) - \bar{F}_{x,z}^{(2)}]^2 dt} \quad (7.7)$$

Figure 7.31: WAMIT and TDRPM sum drift forces in surge direction for a fixed sphere. The components obtained in TDRPM simulations are also presented separately.

Figure 7.32: WAMIT and TDRPM sum drift forces in heave direction for a fixed sphere.

Table 7.11: Comparison between WAMIT and TDRPM results regarding the zero frequency for a fixed sphere.

ω (rad/s)	$\bar{F}_x^{(2)}/(\rho g A^2)$			$\bar{F}_z^{(2)}/(\rho g A^2)$		
	WAMIT	TDRPM	Difference (%)	WAMIT	TDRPM	Difference (%)
2.00	0.09	0.13	31.3	0.47	0.45	4.0
2.20	0.14	0.16	10.3	0.51	0.49	3.9
2.40	0.22	0.25	10.7	0.54	0.53	3.6
2.60	0.29	0.32	11.7	0.56	0.55	2.6
2.80	0.36	0.41	11.4	0.56	0.55	2.8
3.00	0.43	0.48	10.8	0.55	0.53	3.7
3.20	0.47	0.53	11.2	0.53	0.51	3.3
3.40	0.50	0.56	11.1	0.50	0.48	3.2
3.60	0.51	0.58	11.4	0.48	0.46	4.4
3.80	0.51	0.57	10.0	0.46	0.43	5.6
4.00	0.52	0.55	5.1	0.44	0.41	6.8

Table 7.12: Comparison between WAMIT and TDRPM results regarding the double frequency for a fixed sphere.

ω (rad/s)	$\bar{F}_x^{(2\omega)}/(\rho g A^2)$			$\bar{F}_z^{(2\omega)}/(\rho g A^2)$		
	WAMIT	TDRPM	Difference (%)	WAMIT	TDRPM	Difference (%)
2.00	1.03	0.98	5.2	0.23	0.26	12.5
2.20	1.14	1.10	3.7	0.27	0.30	9.0
2.40	1.23	1.20	2.8	0.32	0.34	4.7
2.60	1.30	1.25	4.5	0.34	0.35	4.2
2.80	1.32	1.27	3.9	0.34	0.35	3.5
3.00	1.31	1.27	2.8	0.32	0.33	4.2
3.20	1.32	1.28	3.1	0.28	0.29	3.2
3.40	1.35	1.32	2.4	0.24	0.24	1.2
3.60	1.40	1.40	0.3	0.18	0.17	1.4
3.80	1.48	1.49	1.3	0.12	0.11	4.1
4.00	1.49	1.55	3.7	0.08	0.08	3.9

7.4 Mean drift and sum frequency forces on a floating hemisphere in regular waves

An analysis similar to the one in section 7.3 is presented herein, but now considering that the sphere is free to move. Nonetheless, one should remind that in this case the body velocities also contribute to the 2nd order forces, as it can be seen in expression (7.8).

$$\begin{aligned} \vec{F}^{(2)} = & -\frac{1}{2}\rho g \oint_{WL} [\eta^{(1)} - (z_G^{(1)} + (y_Q - y_G^{(0)})\alpha_x^{(1)} - (x_Q - x_G^{(0)})\alpha_y^{(1)})]^2 \vec{n}^{(0)} dl + \frac{1}{2}\rho \iint_{\bar{S}_B} \nabla\varphi^{(1)} \cdot \nabla\varphi^{(1)} \vec{n}^{(0)} dS + \\ & \rho \iint_{\bar{S}_B} \vec{x}_Q^{(1)} \cdot \nabla \left(\frac{\partial\varphi_Q^{(1)}}{\partial t} \right) \vec{n}^{(0)} dS + \vec{\alpha}^{(1)} \wedge \vec{F}^{(1)} \quad (7.8) \end{aligned}$$

7.4.1 Wave runup

The runup around a floating body can be significantly different from the situation when it is fixed, especially close to the dynamic resonances. Not only the body velocities change the pressure field, but also the motions will radiate waves, changing the free surface elevation pattern compared to the fixed condition. In order to validate the numerical implementation related to the computation of the runup over the floating sphere, simulations with the same wave conditions given in section 7.3 were performed, providing the results shown in Figure 7.33.

First, by comparing with similar results presented in the previous section (Figure 7.22), one may realize that the results for the floating condition are very different from the ones with the fixed setup. It can also be verified that the runup pattern also changes appreciably as an effect of the interaction of diffracted and radiated waves. The overall agreement with WAMIT predictions is fair, with small differences for low period waves (high frequency waves), which would require a finer free surface mesh⁷. Furthermore, for periods close to 3.2 rad/s the body is close to its heave resonance and since no additional external damping is included, the motion amplifications in this frequency range are large, as can be seen from the heave RAO depicted in Figure 7.5. The dynamic response therefore amplifies small differences regarding the computation of forces and wave elevations and this justifies the somewhat larger discrepancies observed around $\omega = 3.2\text{rad/s}$. It can be verified that the local effect caused by the sphere geometric topology still plays a role in the discontinuity of the wave runup curve close to 0° , 90° , 180° and 270° , similar to the results presented for the fixed sphere.

⁷A similar problem occurred for the fixed sphere and was discussed in section 7.3.1.

Figure 7.33: Wave Runup for the free floating sphere for wave frequencies from $\omega=2.0\text{rad/s}$ until $\omega=3.8\text{rad/s}$ considering increments of 0.2rad/s .

The comparisons regarding the maximum and minimum runup factors are summarized in Table 7.13 and some frames taken from the numerical simulations for wave frequencies $\omega = 2.4$ rad/s and $\omega = 3.8$ rad/s can be seen in Figure 7.34 and Figure 7.35. Here, the large amplification is evident for the shorter wave (Figure 7.34), while for the longer one (Figure 7.35), almost no runup is observed.

Table 7.13: Comparison of TDRPM and WAMIT results regarding the wave runup for a floating sphere

ω (rad/s)	Maximum			Minimum			Mean
	TDRPM	WAMIT	Difference (%)	TDRPM	WAMIT	Difference (%)	difference(m)
2.0	1.01	1.01	1.49	0.95	0.96	0.80	0.00
2.2	1.01	1.01	1.69	0.92	0.94	0.17	0.01
2.4	1.02	1.02	3.15	0.87	0.89	0.23	0.01
2.6	1.02	1.03	3.90	0.81	0.83	1.19	0.01
2.8	1.02	1.02	3.40	0.68	0.69	0.12	0.01
3.0	0.99	1.00	3.70	0.53	0.52	0.16	0.01
3.2	0.92	0.89	3.58	0.49	0.46	1.01	0.07
3.4	1.23	1.35	4.59	0.47	0.38	2.12	0.09
3.6	1.45	1.54	4.73	0.35	0.29	0.05	0.07
3.8	1.55	1.65	6.40	0.29	0.24	4.21	0.07
4.0	1.61	1.73	6.62	0.29	0.22	7.12	0.07

Figure 7.34: Example of the simulation for $\omega = 2.426$ rad/s from $t_{ini}=10.0s$ until $t_{end}=12.4s$. Both body motions and wave elevations are depicted.

Figure 7.35: Example of the simulation for $\omega = 3.706$ rad/s from $t_{ini}=10.0$ s until $t_{end}=12.4$ s. Both body motions and wave elevations are depicted.

7.4.2 Velocity field

Again it can be seen that the agreement for the velocity field is fine but for the points close to the patch borders. Figures 7.36 and 7.37 present the comparison for the three velocity components (V_x, V_y, V_z) over the sphere surface. Now, however, the discrepancies are a bit larger if compared to the fixed case, especially in the frequency range for which the heave motion amplification is more pronounced.

Figure 7.36: Velocity field for the fixed sphere for wave frequencies from $\omega=2.0\text{rad/s}$ until $\omega=3.0\text{rad/s}$ considering increments of 0.2rad/s .

Figure 7.37: Velocity field for the fixed sphere for wave frequencies from $\omega=3.2\text{rad/s}$ until $\omega=4.0\text{rad/s}$ considering increments of 0.2rad/s .

The comparison of the normalized mean differences and standard deviation (see eq. (7.5)) for the floating sphere configuration can be seen in Table 7.14, assuming WAMIT results as benchmark. By comparing the results of those in Table 7.10 (fixed sphere) the increase in the differences becomes clear.

Table 7.14: Mean and standard deviation of the difference of the velocity field regarding WAMIT and TDRPM simulations for a floating sphere.

ω (rad/s)	Mean diff Vx	Std diff Vx	Mean diff Vy	Std diff Vy	Mean diff Vz	Std diff Vz
2.0	0.02	0.01	0.02	0.07	0.02	0.06
2.2	0.02	0.01	0.02	0.06	0.01	0.06
2.4	0.02	0.01	0.01	0.06	0.00	0.05
2.6	0.03	0.01	0.01	0.05	0.00	0.05
2.8	0.04	0.01	0.01	0.06	0.00	0.05
3.0	0.05	0.01	0.02	0.06	0.01	0.06
3.2	0.03	0.02	0.04	0.07	0.03	0.06
3.4	0.01	0.01	0.03	0.07	0.02	0.06
3.6	0.01	0.01	0.01	0.06	0.01	0.06
3.8	0.01	0.01	0.01	0.06	0.01	0.06
4.0	0.00	0.01	0.01	0.06	0.02	0.06

The visual comparison regarding the variation of the velocity amplitude over the surface $|\vec{v}^{(1)}| = \sqrt{(v_x^{(1)})^2 + (v_y^{(1)})^2 + (v_z^{(1)})^2}$ for the free floating configuration can be seen in Figures 7.38, 7.39, 7.40 and 7.41. The more pronounced differences in the color maps for frequencies around $\omega = 3.2$ rad/s once more indicate the amplification of the discrepancies close to the heave resonance.

(a) 2.0 rad/s

(b) 2.2 rad/s

(c) 2.4 rad/s

Figure 7.38: Velocity amplitudes over the free floating sphere for wave frequencies $\omega=2.0\text{rad/s}$ to $\omega=2.4\text{rad/s}$ considering increments of 0.2rad/s .

(a) 2.6 rad/s

(b) 2.8 rad/s

(c) 3.0 rad/s

Figure 7.39: Velocity amplitudes over the free floating sphere for wave frequencies $\omega=2.6\text{rad/s}$ to $\omega=3.0\text{rad/s}$ considering increments of 0.2rad/s .

(a) 3.2 rad/s

(b) 3.4 rad/s

(c) 3.6 rad/s

Figure 7.40: Velocity amplitudes over the free floating sphere for wave frequencies $\omega=3.2\text{rad/s}$ to $\omega=3.6\text{rad/s}$ considering increments of 0.2rad/s .

Figure 7.41: Velocity amplitudes over the free floating sphere for wave frequencies $\omega=3.8\text{rad/s}$ to $\omega=4.0\text{rad/s}$ considering increments of 0.2rad/s .

7.4.3 Mean drift forces for a floating sphere

The total second order forces expressed in (7.8) may be decomposed in the three components (7.9), (7.10) and (7.11), which respectively include the effects of the wave elevation along the body waterline, the (quadratic) fluid velocity and acceleration.

$$\vec{F}_1^{(2)} = -\frac{1}{2}\rho g \oint_{WL} [\eta^{(1)} - (z_G^{(1)} + (y_Q - y_G^{(0)})\alpha_x^{(1)} - (x_Q - x_G^{(0)})\alpha_y^{(1)})]^2 \vec{n}^{(0)} dl \Rightarrow \text{Free surface} \quad (7.9)$$

$$\vec{F}_2^{(2)} = \frac{1}{2}\rho \iint_{\bar{S}_B} \nabla \varphi^{(1)} \cdot \nabla \varphi^{(1)} \vec{n}^{(0)} dS \Rightarrow \text{Fluid velocity} \quad (7.10)$$

$$\vec{F}_3^{(2)} = \rho \iint_{\vec{S}_B} \vec{x}_Q^{(1)} \cdot \nabla \left(\frac{\partial \varphi_Q^{(1)}}{\partial t} \right) \vec{n}^{(0)} dS \Rightarrow \text{Fluid acceleration} \quad (7.11)$$

Figures 7.42 Figure 7.43 present the comparison of mean drift predictions in surge and heave directions. The values of the different components obtained in the TDRPM simulations are also shown.

Agreement is good both qualitatively and quantitatively for the surge and heave mean drift forces. One may realize that the three force components are important for the surge forces, while for the heave force the effect of the first component is null. It is also interesting to observe that the phase shift associated to the heave resonance induces a change of sign of the vertical mean force.

Figure 7.42: WAMIT and Rankine mean drift forces comparison in surge direction for a floating sphere.

Figure 7.43: WAMIT and Rankine mean drift forces comparison in heave direction for a floating sphere.

7.5 Quadratic Second Order Forces on a fixed sphere

When a simulation is performed for a combination of two or more regular waves with different frequencies, several second-order force terms that oscillate in sum and difference frequencies are obtained. Once again taking the fixed sphere of unit radius as an example, Figure 7.46 presents an example of the time series of surge force under the action of a trichromatic wave that propagates along x-direction. Besides the total surge force, the figure also presents separately the first and second order components. The latter includes mean, difference and sum frequency terms.

Figure 7.44: Example of surge force time series for two regular wave components. (Total, second order and first order - up to down)

The Fourier analysis of the second order force signal several components that are mixed in the force time series, which can be seen in Figure 7.45. In the case of two regular waves there are 5 force components: the double frequencies of the first and second incident waves ($2\omega_1$ and $2\omega_2$), the sum of both frequencies ($\omega_1 + \omega_2$), the difference of both frequencies ($\omega_1 - \omega_2$) and the mean forces.

Figure 7.45: Force spectrum considering two regular waves

In case three regular waves were combined, there would be 10 different second order components, as illustrated in Figures 7.46 and 7.47. It is interesting to observe that this procedure may be extended to the concept of an incident wave composed of infinite frequency components, as for example one obtained from a white-noise wave spectrum. From such a simulation, all the force components in sum and difference frequencies could be obtained (both amplitudes and phases) from the analysis of the total force signal. This procedure is indeed much more expedite than performing separate simulations for the different frequency pairs. The only "drawback" is that the mean forces for each frequency cannot be derived from the same analysis. However, they can be computed with much less computational effort from simulations of regular waves.

Figure 7.46: Example of surge force time series for three regular wave components. (Total, second order and first order - up to down)

Figure 7.47: Force spectrum considering three regular waves.

The matrix that contains the information (amplitude and phase) of the second-order diffraction forces for the different frequency-pairs is known as QTF (Quadratic Transfer Function, see for instance, Faltinsen [1990]).

Next, some QTF values obtained from simulations of bichromatic waves with the fixed sphere are compared to those obtained from WAMIT in frequency domain⁸. Tables 7.15 and 7.16 present the results for eight different pairs of frequencies regarding the QTFs in surge and heave directions, respectively. The same results are represented graphically in Figures 7.48 and 7.49. From these results it can be inferred that the agreement with WAMIT' results is generally good, both for the difference and sum frequency forces. This fact once again attest not only the correctness of implementation of the algorithms involved in the computations of the second order forces, but also the accuracy of the predictions of the fluid velocity field on the body surface and wave elevation along the waterline.

⁸One could remind that only the "quadratic" part of the QTF is computed by the TDRPM code. Therefore, the WAMIT results used for comparison exclude the influence of the second-order potential.

Table 7.15: Comparison of quadratic second order forces computed from both TDRPM and WAMIT numerical methods in surge direction.

Second order surge drift forces										
ID	ω_1	ω_2	$\frac{F_1^{(2)}(\omega_1 - \omega_2)}{\rho g A_1 A_2}$		$\frac{F_1^{(2)}(\omega_1 + \omega_2)}{\rho g A_1 A_2}$		$\frac{F_1^{(2)}(2\omega_1)}{\rho g A_1^2}$		$\frac{F_1^{(2)}(2\omega_2)}{\rho g A_2^2}$	
			TDRPM	WAMIT	TDRPM	WAMIT	TDRPM	WAMIT	TDRPM	WAMIT
1	2.00	2.20	0.13	0.13	1.05	1.02	0.94	0.96	1.09	1.07
2	2.00	2.61	0.24	0.24	1.02	1.11	0.94	0.96	1.27	1.22
3	2.20	2.80	0.28	0.28	1.22	1.17	1.09	1.07	1.25	1.24
4	2.61	2.80	0.33	0.33	1.35	1.23	1.34	1.22	1.33	1.24
5	2.00	3.00	0.33	0.33	1.03	1.12	0.95	0.96	1.21	1.25
6	2.00	2.80	0.29	0.29	1.20	1.13	0.94	0.96	1.25	1.24
7	2.00	3.20	0.36	0.36	1.02	1.09	0.96	0.96	1.18	1.26
8	2.20	2.61	0.14	0.14	1.18	1.16	1.08	1.07	1.24	1.22

Figure 7.48: Comparison between WAMIT and TDRPM results for surge quadratic transfer function in both difference and sum frequencies.

As a final remark on the results of this section, it should be reminded that a similar comparison involving the QTF forces for bodies that are free to oscillate cannot be performed directly, since the forces computed from pressure integration on the body surface in time domain already include both, diffraction and radiation effects (something that was already discussed in section 7.4).

However, a comparison of the 2nd order results obtained from both computational methods in the analysis of a floating body will be presented in the next section regarding the slow resonant motions of a semi submersible model. Results will be also evaluated in the light of experimental data with the small-scale model in bichromatic wave tests performed in a wave basin. The scenario is actually much complex and rich because the paradigm for the numerical predictions are resonant motions induced by forces of small magnitude and also influenced by mooring and

viscous damping effects. This analysis is presented next.

Table 7.16: Comparison of quadratic second order forces computed from both TDRPM and WAMIT numerical methods in heave direction.

Second order heave drift forces										
ID	ω_1	ω_2	$\frac{F_3^{(2)}(\omega_1 - \omega_2)}{A_1 A_2}$		$\frac{F_3^{(2)}(\omega_1 + \omega_2)}{A_1 A_2}$		$\frac{F_3^{(2)}(2\omega_1)}{A_1^2}$		$\frac{F_3^{(2)}(2\omega_2)}{A_2^2}$	
			TDRPM	WAMIT	TDRPM	WAMIT	TDRPM	WAMIT	TDRPM	WAMIT
1	2.00	2.20	0.52	0.49	0.27	0.28	0.24	0.26	0.31	0.30
2	2.00	2.61	0.49	0.50	0.29	0.30	0.24	0.26	0.33	0.35
3	2.20	2.80	0.54	0.52	0.33	0.32	0.28	0.30	0.35	0.35
4	2.61	2.80	0.61	0.56	0.32	0.35	0.34	0.35	0.35	0.35
5	2.00	3.00	0.50	0.47	0.29	0.29	0.26	0.26	0.35	0.33
6	2.00	2.80	0.52	0.49	0.27	0.30	0.26	0.26	0.34	0.35
7	2.00	3.20	0.47	0.44	0.28	0.28	0.26	0.26	0.31	0.29
8	2.20	2.61	0.56	0.53	0.31	0.32	0.28	0.30	0.34	0.35

Figure 7.49: Comparison between WAMIT and TDRPM results for heave quadratic transfer function in both difference and sum frequencies.

7.6 Evaluation of second order loads on a semi-submersible platform

Aiming at the analysis of a practical case involving a more complex geometry, some experimental tests with the model (1:120) of a semi-submersible platform were performed. These tests were specifically designed for providing a basis of comparison regarding second-order loads and motions. The model presents features that are favorable for this purpose, especially the very high natural periods in roll and pitch, which guarantee some more intense resonant motions, due to second-order wave effects.

The experimental campaign was conducted in the Hydrodynamic Calibrator of the University of São Paulo (CH-TPN), shown in Figure 7.50, a squared 14mx14mx4m (length, width, depth) wave basin, equipped with 148 active-absorber wave generators. This equipment can generate the incident wave field and absorb those diffracted/radiated by the model, following specific control algorithms, as discussed by Mello et al. [2010].

Figure 7.50: Hydrodynamic Calibrator of the University of São Paulo.

From previous experiences, it is well-known fact that direct measurements of second order forces is not easy because, by definition, these forces are very small if compared to the first order ones. Since large first order loads are always present, specific load cells are required, which should be able to support these forces while measuring the second order quantities accurately in the same time, without excessive noise. Moreover, this experiment requires a captive model test with additional difficulties to keep the model restrained without introducing additional natural

frequencies from the model support⁹. Instead of measuring second order forces directly, the experimental test was planned to measure second order motions considering pair of waves with frequency difference close to the resonant frequencies. In these conditions, even small forces lead to large motions because the damping of the system is usually small. The motions were measured using Qualisys optical tracking system, shown in Figure 7.51, with an uncertainty smaller than 1.0 mm.

Figure 7.51: Qualisys Optical Tracking System.

The semi-submersible model studied built in a 1:120 scale can be seen in Figure 7.52, with the main properties shown in Table 7.17. This scale factor was selected following the generation/absorption recommendations in order achieve the optimum wave absorption performance for the selected wave periods.

Figure 7.52: Semi-submersible model.

⁹A recent example of captive model tests for measuring slow-drift forces was presented by Lopez-Pavon et al. [2015], when dealing with the analysis of the floater of a floating wind turbine.

Table 7.17: Semi-submersible properties (real scale)

Dimension	Value	Units
LOA	102.4	m
Beam	96.0	m
Depth	45.0	m
Draft	25.0	m
Displacement	80,560	ton
Pitch Moment of Inertia	$5.7 \cdot 10^7$	$\text{ton} \cdot \text{m}^2$
VCG	31.9	m
Pitch natural period	84.0	s
Heave natural period	24.1	s
Surge natural period	140.0	s

The tests were performed considering only headsea condition, therefore only the surge slow motions were studied in the horizontal plane, with the natural period defined based on the horizontal mooring stiffness. This stiffness is provided by 4 springs connected to the model and the basin corners, as illustrated in Figure 7.53, where the mooring arrangement is seen in (a) and the load cells attached to the springs are shown in (b). Each spring has a linear stiffness of 117.65 kN/m (8.17 N/m in model scale), providing a theoretical linear equivalent stiffness of 235.30 kN/m (16.34 N/m in model scale). Pull-out tests indicated a value of 241.06 kN/m (16.74 N/m in model scale) corresponding to a surge natural period of 140s (12.93s in model scale).

Figure 7.53: Model mooring arrangement.

Vertical slow motions (induced by heave, roll and pitch) have also been investigated in the literature, for instance, Matos et al. [2011, 2013] studied the roll and pitch slow motions of semi-submersible platforms following experimental and numerical approaches, with the results also compared to real scale measurements. Since the tested platform has a pitch natural period of $84.0s^{10}$ (7.7s in model scale), the slow pitch motions were also investigated. It should be noticed this platform has a reduced hydrostatic restoration, thus even the small influence of the mooring lines changed the pitch natural period to 66.0s (6.04s in model scale).

The experimental test was performed considering 114 bichromatic wave conditions defined based on the surge and pitch natural periods. In the context of the weakly non-linear theory, the second order forces/motions are proportional to the product $(A_1 A_2)$, where A_i is the amplitude of each individual wave frequency. However, in practice, it is well known that under large motions the damping may have a non-linear behavior due to, among other reasons, the large separation effects (high KC numbers). In order to investigate this behavior, at least 3 different pairs of wave amplitudes were tested for each pair of wave frequencies. The dynamic model implemented in the TDRPM code, however, considers only a single linearized external damping coefficient in order to maintain the coherence with the weakly non-linear formulation. A total of 20 pairs of bichromatic waves were generated with $\Delta\omega$ equal to the surge natural frequency (wave ID from 1 to 20 - $\Delta\omega_1 = 0.045rad/s$) comprising 4 pairs of wave heights (1.5m, 2.5m, 3.5m and 4.5m). It should be noticed that for the shorter waves the 4.5m wave height was not tested due to lack of stability in wave propagation (excessive wave steepness). Since the pitch natural period changed from 84.0s, in the decay tests without mooring lines, to 66.0s in the moored condition, 5 frequency-differences were selected ($\Delta\omega_2 = 0.1030rad/s$, $\Delta\omega_3 = 0.0952rad/s$, $\Delta\omega_4 = 0.0997rad/s$, $\Delta\omega_5 = 0.0898rad/s$ and $\Delta\omega_6 = 0.0748rad/s$) corresponding to difference periods 61.0, 63.0, 66.0, 70.0 and 84.0s, which are identified by wave IDs from 21 until 114. The complete list of tested conditions is summarized in Table 7.18.

Some examples of the experimental surge time series can be seen in Figure 7.54, for wave periods $T_1 = 9.00s$ and $T_2 = 8.45s$ ($\delta T = 139.5s$) under wave heights 1.5m and 2.5m. The time series presents the wave frequency motions, the double frequency ones, a mean value and the slow drift motion, the last ones the most important in the scope of this thesis. It should be noticed that for wave height 1.5m, the effect of the first order motions are more appreciable if compared to the first order motions. On the other hand, for wave height 2.5m the first order

¹⁰verified by the decay tests without mooring lines.

motions are almost negligible if compared to the second order motions, since the second-order forces are quadratic in amplitude.

(a) $H_1 = H_2 = 1.5\text{m}$

(b) $H_1 = H_2 = 2.5\text{m}$

Figure 7.54: Example of experimental surge motion time series for $T_1=9.00\text{s}$ and $T_2=8.45\text{s}$ ($\delta T = 139.5\text{s}$).

Table 7.18: Bichromatic wave conditions assumed in the experimental tests performed.

ID	$T_1(s)$	$T_2(s)$	$H_1(m)$	$H_2(m)$	$\delta T(s)$	ID	$T_1(s)$	$T_2(s)$	$H_1(m)$	$H_2(m)$	$\delta T(s)$
1	9.00	8.45	1.5	1.5	139.5	58	14.00	11.55	3.5	3.5	66.0
2	10.00	9.33	1.5	1.5	139.5	59	11.00	9.43	4.5	4.5	66.0
3	11.00	10.20	1.5	1.5	139.5	60	12.00	10.15	4.5	4.5	66.0
4	12.00	11.05	1.5	1.5	139.5	61	9.00	7.88	1.5	1.5	63.0
5	13.00	11.89	1.5	1.5	139.5	62	10.00	8.63	1.5	1.5	63.0
6	14.00	12.72	1.5	1.5	139.5	63	11.00	9.36	1.5	1.5	63.0
7	9.00	8.45	2.5	2.5	139.5	64	12.00	10.08	1.5	1.5	63.0
8	10.00	9.33	2.5	2.5	139.5	65	13.00	10.78	1.5	1.5	63.0
9	11.00	10.20	2.5	2.5	139.5	66	14.00	11.45	1.5	1.5	63.0
10	12.00	11.05	2.5	2.5	139.5	67	9.00	7.88	2.5	2.5	63.0
11	13.00	11.89	2.5	2.5	139.5	68	10.00	8.63	2.5	2.5	63.0
12	14.00	12.72	2.5	2.5	139.5	69	11.00	9.36	2.5	2.5	63.0
13	9.00	8.45	3.5	3.5	139.5	70	12.00	10.08	2.5	2.5	63.0
14	10.00	9.33	3.5	3.5	139.5	71	13.00	10.78	2.5	2.5	63.0
15	11.00	10.20	3.5	3.5	139.5	72	14.00	11.45	2.5	2.5	63.0
16	12.00	11.05	3.5	3.5	139.5	73	9.00	7.88	3.5	3.5	63.0
17	13.00	11.89	3.5	3.5	139.5	74	10.00	8.63	3.5	3.5	63.0
18	14.00	12.72	3.5	3.5	139.5	75	11.00	9.36	3.5	3.5	63.0
19	11.00	10.20	4.5	4.5	139.5	76	12.00	10.08	3.5	3.5	63.0
20	12.00	11.05	4.5	4.5	139.5	77	13.00	10.78	3.5	3.5	63.0
21	9.00	7.84	1.5	1.5	61.0	78	14.00	11.45	3.5	3.5	63.0
22	10.00	8.59	1.5	1.5	61.0	79	9.00	7.97	1.5	1.5	70.0
23	11.00	9.32	1.5	1.5	61.0	80	10.00	8.75	1.5	1.5	70.0
24	12.00	10.03	1.5	1.5	61.0	81	11.00	9.51	1.5	1.5	70.0
25	13.00	10.72	1.5	1.5	61.0	82	12.00	10.24	1.5	1.5	70.0
26	14.00	11.39	1.5	1.5	61.0	83	13.00	10.96	1.5	1.5	70.0
27	9.00	7.84	2.5	2.5	61.0	84	14.00	11.67	1.5	1.5	70.0
28	10.00	8.59	2.5	2.5	61.0	85	9.00	7.97	2.5	2.5	70.0
29	11.00	9.32	2.5	2.5	61.0	86	10.00	8.75	2.5	2.5	70.0
30	12.00	10.03	2.5	2.5	61.0	87	11.00	9.51	2.5	2.5	70.0
31	13.00	10.72	2.5	2.5	61.0	88	12.00	10.24	2.5	2.5	70.0
32	14.00	11.39	2.5	2.5	61.0	89	13.00	10.96	2.5	2.5	70.0
33	9.00	7.84	3.5	3.5	61.0	90	14.00	11.67	2.5	2.5	70.0
34	10.00	8.59	3.5	3.5	61.0	91	9.00	7.97	3.5	3.5	70.0
35	11.00	9.32	3.5	3.5	61.0	92	10.00	8.75	3.5	3.5	70.0
36	12.00	10.03	3.5	3.5	61.0	93	11.00	9.51	3.5	3.5	70.0
37	13.00	10.72	3.5	3.5	61.0	94	12.00	10.24	3.5	3.5	70.0
38	14.00	11.39	3.5	3.5	61.0	95	13.00	10.96	3.5	3.5	70.0
39	11.00	9.32	4.5	4.5	61.0	96	14.00	11.67	3.5	3.5	70.0
40	12.00	10.03	4.5	4.5	61.0	97	9.00	8.13	1.5	1.5	84.0
41	9.00	7.92	1.5	1.5	66.0	98	10.00	8.94	1.5	1.5	84.0
42	10.00	8.68	1.5	1.5	66.0	99	11.00	9.73	1.5	1.5	84.0
43	11.00	9.43	1.5	1.5	66.0	100	12.00	10.50	1.5	1.5	84.0
44	12.00	10.15	1.5	1.5	66.0	101	13.00	11.26	1.5	1.5	84.0
45	13.00	10.86	1.5	1.5	66.0	102	14.00	12.00	1.5	1.5	84.0
46	14.00	11.55	1.5	1.5	66.0	103	9.00	8.13	2.5	2.5	84.0
47	9.00	7.92	2.5	2.5	66.0	104	10.00	8.94	2.5	2.5	84.0
48	10.00	8.68	2.5	2.5	66.0	105	11.00	9.73	2.5	2.5	84.0
49	11.00	9.43	2.5	2.5	66.0	106	12.00	10.50	2.5	2.5	84.0
50	12.00	10.15	2.5	2.5	66.0	107	13.00	11.26	2.5	2.5	84.0
51	13.00	10.86	2.5	2.5	66.0	108	14.00	12.00	2.5	2.5	84.0
52	14.00	11.55	2.5	2.5	66.0	109	9.00	8.13	3.5	3.5	84.0
53	9.00	7.92	3.5	3.5	66.0	110	10.00	8.94	3.5	3.5	84.0
54	10.00	8.68	3.5	3.5	66.0	111	11.00	9.73	3.5	3.5	84.0
55	11.00	9.43	3.5	3.5	66.0	112	12.00	10.50	3.5	3.5	84.0
56	12.00	10.15	3.5	3.5	66.0	113	13.00	11.26	3.5	3.5	84.0
57	13.00	10.86	3.5	3.5	66.0	114	14.00	12.00	3.5	3.5	84.0

An example of surge motion response spectrum obtained from the analysis of the experimental data can be seen in Figure 7.55, where the energy associated to the 7 frequency components (namely ω_1 , ω_2 , $2\omega_1$, $2\omega_2$, $\omega_1 + \omega_2$, $\omega_1 - \omega_2$ and 0-mean drift) may be visualized.

Figure 7.55: Example of experimental motion spectrum.

The several components are separated using a FFT¹¹ filter. In this procedure we select and maintain only one wave frequency, setting the remaining ones as zero. This "new" spectrum is then transformed to time domain by applying the inverse Fourier transformation, where a single wave frequency is present. The amplitude value is then computed for this specific wave frequency by taking the standard deviation multiplied by $\sqrt{2}$. This procedure is adopted for the 6 wave frequencies presented before, but for the null frequency, which can be computed as the motion average value in time. The intervals assumed to segregate the wave frequencies are $0.5(\omega_2 - \omega_1)$, $0.5(\omega_2 - \omega_1 + \omega_2)$, $0.5(\omega_1 + \omega_2)$, $0.5(2\omega_1 + \omega_2)$, $0.5(2\omega_1 + \omega_2 + \omega_1)$ and $0.5(2\omega_2 + \omega_2 + \omega_1)$, exactly the midpoint between consecutive frequencies. For some cases, some overlaps between close frequencies were verified.

7.6.1 First order motions

Although we are more interested in second order motions, the first comparisons were performed for the first order motions in terms of response amplitude operators. This comparison is important to verify once again the developed method. Moreover, the first order solution is applied in the computation of second order forces; therefore the inaccuracies in these quantities could lead

¹¹Fast Fourier Transformation

(a) General overview of the free surface patches

(b) Details close to the body free surface intersection

Figure 7.57: Semi-submersible and free surface meshes considered in the simulations.

Two examples of the surge motions predicted by the TDRPM simulations are presented in Figure 7.58, for the same wave conditions of Figure 7.54. Here, the numerical simulation time series is shifted in time to synchronize with the experimental data, thus allowing a better comparison. The results are split in first order motions (top), second order ones (middle) and the combined motions (bottom), and the last one is compared to the experimental results.

The motions computed by the TDRPM code already include the first and second-order effects. Since the simulations are performed with a fixed mesh, small shifts in the phase at the first order motions due to the drift of the platform cannot be captured by the numerical model. They would actually be a higher order effect. However, as the natural frequency is low and the drift amplitudes are small compared to the body representative dimensions, it can be seen these effects are negligible in this case.

(a) $H_1 = H_2 = 1.5\text{m}$

(b) $H_1 = H_2 = 2.5\text{m}$

Figure 7.58: Comparison between numerical simulation using TDRPM method and experimental data for surge motion under $T_1=9.00\text{s}$ and $T_2=8.45\text{s}$ ($\delta T = 139.5\text{s}$).

The comparisons are performed for all wave conditions considering the experimental data, TDRPM and WAMIT frequency domain results for the surge, heave and pitch motions. The RAO estimations are presented in Figure 7.59. Here, the blue circles correspond to the first wave frequency (ω_1) and red ones to the second wave frequencies (ω_2) in the experimental test (the time domain results computed by TDRPM are cyan and magenta). Since for the same pair of wave frequencies there are several wave heights, the results are grouped in the vertical lines

in order to verify the variation of the RAO due to non linear effects¹². The RAOs computed by TDRPM do not change according to the wave amplitude, except for some small fluctuations in the FFT filtering procedure.

It can be verified that there are some appreciable variations in the surge RAO in the range between 7s and 9s regarding the experimental data, as well as some variations in the pitch ones from 10.0 to 12.5s. No appreciable discrepancies are observed in the heave RAO. The agreement between Rankine and WAMIT results is good overall.

The discrepancies in the surge motion for short waves may be due to an insufficient mesh refinement. The ones in pitch are more probably related to (nonlinear) variations in the mooring stiffness as the model drift in the basin (one should remind that a simplified linearized mooring is considered and that the pitch response is very sensitive to changes in the external stiffness).

¹²No additional treatments for this non-linear effects will be performed in the scope of this thesis

(a) Surge

(b) Heave

(c) Pitch

Figure 7.59: Comparison of surge, heave and pitch first order RAOs between experimental data (red and blue circles), Rankine method (magenta and cyan squared markers) and WAMIT predictions (black continuous line).

The second order motions are now evaluated considering the sum-frequency, mean and difference frequency motions, the last two the focus of the present thesis. However, since the sum frequency results are also available after the post-processing procedure described earlier, the comparisons are also performed for them. Some particularities arise in the mean drifts comparisons, since it is not possible to segregate the contribution of each specific wave frequency, therefore the comparisons are performed in terms of the total motion.

The comparisons are performed for the 3 degrees of freedom of the experimental test (surge, heave and pitch), although for several conditions a particular degree of freedom may have almost no response. One additional source of comparison are the WAMIT frequency domain results, computed using the second order module. In order to turn the numerical comparisons fair, these results only take into account the quadratic terms, neglecting the second order potential.

In the analysis of a similar semi-submersible platform, Simos et al. [2008] have previously verified that the most relevant components for the slow drift motions are the quadratic terms due to first order quantities interactions, indicating that the effects of the second order potential can be neglected. This is indeed a favorable aspect for the present analysis, for the TDRPM code computes only the quadratic terms.

7.6.2 Second order motions - sum frequency

The results regarding the sum frequency motion components can be verified in Figure 7.60, where the results are collected according to the difference period, which will be more convenient later in the slow motions comparisons. Here the continuous line represents the mean value computed from the experimental data considering the several different pairs of wave heights, which are represented by the bars around the mean value. The black circles indicate the WAMIT predictions while the red ones denote the Rankine computations. The motions are divided by the product of the amplitudes of the incident waves and presented as function of the first wave period (T_1), already presented in Table 7.18. It shall be noticed that since the pairs of frequencies were selected in order to amplify the slow motions, the sum frequency motions are very small. Moreover, although a sophisticated optical tracking system is used for monitoring the motions in the experiments, in some cases the motion amplitudes are only of some millimeters, reducing the reliability for these measurements. However, quite surprisingly, the agreement is not so poor, keeping at least the same orders of magnitude, although large discrepancies are verified in almost all comparisons performed (Rankine x WAMIT x experimental data).

(a) Surge

(b) Heave

(c) Pitch

Figure 7.60: Comparison among experimental data, WAMIT and Rankine numerical results regarding the second order motions due to the sum frequency forces.

7.6.3 Second order motions - mean drift

The mean drifts are also compared following the methodology described earlier. However, since WAMIT results are available only in terms of mean forces, the motions are obtained using the surge, heave and pitch stiffness coefficients according to equations (7.12), where A_{WL} , GM , $K_{x,l}$, $\bar{F}_1$, $\bar{F}_3$ and $\bar{M}_5$ denote respectively the waterline area, metacentric height, horizontal stiffness and the moment lever (vertical difference between the vessel CG and the spring vertical coordinate at the fairlead), surge, heave and pitch mean forces. The experimental data and Rankine predictions are taken directly from the mean level of the motion time series.

$$\bar{X}_1 = \frac{\bar{F}_1}{K_x}, \quad \bar{X}_3 = \frac{\bar{F}_3}{\rho g A_{WL}}, \quad \bar{X}_5 = \frac{\bar{M}_5}{GM \Delta g + K_x l^2} \quad (7.12)$$

It should be noticed that the wave heights were defined conveniently equal for the first and second wave frequencies (see Table 7.18) in order to guarantee that for each pair of frequencies a single value would be obtained in the numerical predictions. This fact is illustrated in equation (7.13). Here $\bar{F}_{11}$ and $\bar{F}_{12}$ are the mean drift forces in surge direction due to the first and second wave frequencies, which can be written as function of the mean drift coefficients ($C(\omega_i)$ ($i=1,2$)) and the square of the incident amplitude (A_i^2 ($i=1,2$)). Since the wave amplitudes are equal, the sum can be defined in terms only of the combined coefficient and the square of the wave amplitude.

$$\bar{F}_1 = \bar{F}_{11} + \bar{F}_{12} = \rho g [A_1^2 C(\omega_1) + A_2^2 C(\omega_2)] = \rho g A^2 [C(\omega_1) + C(\omega_2)] \quad (7.13)$$

The comparisons can be seen in Figure 7.61. A general good agreement can be verified for trends and accuracy for the surge and heave directions. As expected the mean forces tends to increase with the decrease of the wave period for the same pair of wave amplitudes. The agreement of experimental data and numerical results for pitch motions is poorer if compared to the surge and heave ones, and apparently the trends are captured differently for the lower values of ΔT . Nonetheless, one should realize that the mean angles involved are very small and therefore difficult to measure accurately, not to mention the sensitivity to the mooring lines, as discussed before.

(a) Surge

(b) Heave

(c) Pitch

Figure 7.61: Comparison among experimental data, WAMIT and Rankine numerical results regarding the second order motions due to the mean drift forces.

7.6.4 Second order motions - difference frequency

The comparisons regarding the slow motions are presented in Figure 7.62. It can be verified that the slow motions in surge are small for the difference periods selected in order to study the pitch slow motions. On the other hand, the amplification is large for the difference period close to surge resonance period ($\Delta T = 139.5s$), as it was expected. In this difference period, it can be realized that the variations associated to the wave height in the experimental results are much larger, which is probably due to the nonlinear viscous damping or changes in the mooring for large displacements, both of which are not considered in the numerical methods. The adherence of WAMIT and Rankine results is better for the longer periods, while for the shorter ones, when the amplification is higher, the discrepancies are more pronounced. However, both results are always inside the range of experimental data.

The comparison for the heave motions revealed the largest discrepancies are many times the correct trends are missed by the numerical predictions. This is more evident for the smaller difference periods ($\Delta T = 63 - 70s$) and for the largest one ($\Delta T = 139.5s$). In the latter even the numerical methods seem to predict different trends, with WAMIT results closer to the experimental data.

The comparisons regarding the pitch degree of freedom showed a general good agreement, with large amplifications in the range between 66.0s to 84.0s, where once again there is an apparently non-linear behavior, since the experimental data changes appreciably for different wave heights. Regarding the pitch slow motions, WAMIT results seem to underpredict the amplitudes, especially for the lower values of T_1 .

(a) Surge

(b) Heave

(c) Pitch

Figure 7.62: Comparison among experimental data, WAMIT and Rankine numerical results regarding the second order motions due to the slow drift forces.

7.7 Effects of wave-current interaction of First-order forces

At first verification of the numerical method developed for computing wave-current interaction effects was performed for a fixed sphere and a bottom mounted cylinder in order to simplify the analysis. The results are compared in terms of first order excitation forces for the sphere and cylinder. For the cylinder case, the modification of the wave runup due to current interaction, is also investigated. Results obtained with the TDRPM are confronted to those of other computational codes previously available in the literature.

7.7.1 Fixed sphere

The results presented by Kim and Kim [2007] will be used in this section for comparing the first order excitation forces, and also in section 7.8 for the analysis of mean drift-forces and wave drift damping coefficients. The numerical method developed by Kim and Kim [2007] is a BEM method. The wave-current interaction is investigated for a fixed sphere of unit radius and the analysis is made for three Froude numbers (-0.1, 0.0 and +0.1), where the value is positive when wave and current are in the same direction and negative when they are in opposite directions. The computational domain and discretization assumed is the same of the second order loads computation.

The results were computed in the range of ka (a =sphere radius) from 0.4 to 1.4, and also compared to WAMIT predictions for the zero current setup. Some examples of time series of surge first order force for $ka=0.4$ and $ka=1.4$ can be seen in Figure 7.63 and Figure 7.64, respectively. In each figure, results obtained in simulations with the three different Froude numbers are presented. Table 7.19 summarizes all the wave conditions that were tested. The time span is 40s regardless the wave period, and the force magnitude is computed from the last 20s of simulation to avoid transient effects. The differences in the force frequency that can be perceived for the different Froude numbers in Figures 7.63 and 7.64 are due to the encounter frequency effect, which also change the duration of the transient effects, mainly for the higher wave frequencies.

Table 7.19: Wave conditions assumed in the fixed condition simulations for excitation forces computation considering current effects.

$\omega(\text{rad/s})$	T(s)	ka (m/m)	$\lambda(m)$
1.98	3.17	0.40	15.7
2.21	2.84	0.50	12.6
2.43	2.59	0.60	10.5
2.62	2.40	0.70	9.0
2.80	2.24	0.80	7.9
2.97	2.12	0.90	7.0
3.13	2.00	1.00	6.3
3.28	1.91	1.10	5.7
3.43	1.83	1.20	5.2
3.57	1.76	1.30	4.8
3.71	1.70	1.40	4.5

Figure 7.63: First order excitation forces for a fixed sphere under several Froude numbers for ka=0.4.

Figure 7.64: First order excitation forces for a fixed sphere under several Froude numbers for ka=1.4.

The results concerning surge first order excitation forces are summarized in Figure 7.65, and a quite reasonable agreement can be verified between Rankine and Kim and Kim [2007] predictions regardless the Froude number.

Figure 7.65: Comparison of surge first order excitation forces for a fixed sphere under several Froude numbers.

7.7.2 Bottom mounted circular cylinder

Another verification regarding the effects of wave-current interaction was performed based on the results presented by Shao [2010] for a bottom mounted cylinder geometry. The circular cylinder taken into consideration has a radius of 1m and the water depth is 1m. The numerical method employed in Shao [2010] is a BEM method. Comparisons are made for both, the first order forces in surge direction and the wave runup along the cylinder waterline.

The simulations for the analysis of results regarding wave run-up were performed for the three Froude numbers ($Fn=-0.1, 0.0, +0.1$) considering a wave period of 2.0s ($kR=1.0$). Results are presented in Figure 7.66, where $\theta = 0$ means a point located in the coordinates ($X=-a, Y=0$), downstream of the flow. It should be noticed that the wave run-up pattern changes appreciably due to the current effects. The comparison for zero Froude number is made with WAMIT's results. A good agreement is observed in general, although some fluctuations were observed in TDRPM elevation time series, which could be due to the runup points proximity of the body.

Figure 7.66: Comparison of wave run-up on a cylinder for $ka=1$ and $F_n=-0.1$, 0.0 and $+0.1$ considering TDRPM, WAMIT and Shao [2010] results.

The analysis of first order surge forces involved wave periods in the range of $ka=0.8$ to 2.0 (increments of 0.1). Results for the nondimensional forces are presented in Figure 7.67. As expected, the effect of current is not as pronounced as for the wave runup, and once again the predictions obtained with the different methods are in good agreement.

Figure 7.67: Comparison of surge first order excitation forces on a cylinder for $ka=1$ and $F_n=-0.1$, 0.0 and $+0.1$ considering the TDRPM, WAMIT and Shao [2010] results.

Another aspect must be highlighted in the analysis of the results above. The simulations comprised waves in the kh range from 0.7 to 2.0 . Therefore, for the longer waves (typically for $kh < \pi$), the results are also influenced by bottom effects. Although a systematic analysis

regarding water depth effects is not performed in this thesis, the results above are a good indication that the numerical method is able to deal with them correctly. The same may be said with respect to second-order computations based on the results in section 7.8.2, where the same bottom mounted cylinder will be considered for the analysis of mean drift forces.

7.8 Effects of wave-current interaction on second-order mean forces

The second order loads in the presence of current are evaluated considering the mean drift forces and the so-called wave drift damping for the fixed sphere (Kim and Kim [2007]). Mean drift forces on the bottom mounted cylinder are also compared to predictions provided by Shao [2010].

7.8.1 Fixed sphere

The mean drift forces are computed from the second order time series following the same procedure described in item 7.3.3 and the results are shown in Figure 7.68. The results obtained from the different numerical methods present good concordance, especially concerning the variations induced by the current velocity in different directions.

Figure 7.68: Comparison of surge mean drift forces for a fixed sphere under several Froude numbers.

The wave drift-damping is computed based on equation (7.14), where $\bar{F}_x$ denotes the mean drift forces computed in the absence of current ($U=0$) and the second term is computed based on a central 3-points difference scheme. This numerical approximation of the derivative is performed using Froude numbers of ± 0.04 , as proposed by Kim and Kim [2007].

$$\bar{F}_x = \bar{F}_x + U \frac{\partial \bar{F}_x}{\partial U} \Big|_{U=0} + O(U^2) \Rightarrow B_1 = \frac{\partial \bar{F}_x}{\partial U} \Big|_{U=0} \quad (7.14)$$

The comparison of the wave-drift damping coefficients (B_1) can be seen in Figure 7.69.

Results also obtained by Grue and Biberg [1993] are also included in the figure, as presented in Kim and Kim [2007].

Figure 7.69: Comparison of surge wave-drift damping coefficients for a fixed sphere under several Froude numbers.

7.8.2 Bottom-mounted Circular cylinder

Mean drift forces were also compared with Shao [2010] results for the circular cylinder under $F_n=-0.1$, and results are presented in Figure 7.70. The agreement is fair for the whole range of wave frequencies.

Figure 7.70: Comparison of TDRPM and Shao [2010] results for surge mean drift forces on a cylinder for $ka=1$ and $F_n=-0.1$.

7.9 First order response amplitude operator (RAO) computation with current

Huijsmans [1996] presents the development of a BEM method in frequency domain and in this context, provides numerical predictions concerning the 1st order motions of a floating sphere (unit radius) including current effects. The results are considered in this section for a first verification of the motions computed with TDRPM in simulations involving non-zero current velocities. The same mesh adopted for the simulations of the fixed sphere in section 7.7.1 was adopted. The current velocities considered in the simulations are -0.125, +0.125 and 0.0m/s, corresponding to Froude numbers of -0.04, +0.04 and 0.0.

The comparisons regarding the response amplitude in surge and heave operators can be seen in Figure 7.71 and Figure 7.72, respectively. A good agreement is observed for the Froude number 0 for both degrees of freedom in the entire frequency range, corroborating the results already presented in section 7.1.

In general the surge motions are less sensitive to the variation of Froude number (in the range analyzed) than the heave motions, mainly for high wave numbers, although some appreciably variations are verified for low wave numbers. The motion amplitudes computed by the present method are also listed in Table 7.20. Both for surge and heave motions it can be said that the agreement with the results presented by Huijsmans [1996] is reasonable, although for the shorter waves some larger discrepancies are observed in surge and, in heave, a more pronounced disagreement is observed around $ka=1$. The latter is certainly linked to the heave resonance that occurs for $\omega \approx 3.20\text{rad/s}$. Despite these discrepancies, however, it should be noticed that the trends imposed by the current effects are very similar for both methods.

Table 7.20: Numerical results for surge and heave RAOs computed using the TDRPM.

ka	ω (rad/s)	Surge RAO			Heave RAO		
		Fn=-0.04	Fn=0.00	Fn=0.04	Fn=-0.04	Fn=0.00	Fn=0.04
0.40	1.98	0.86	0.85	0.81	1.00	1.02	1.06
0.50	2.21	0.77	0.78	0.74	1.07	1.07	1.08
0.60	2.43	0.72	0.69	0.68	1.20	1.18	1.13
0.70	2.62	0.65	0.64	0.64	1.38	1.30	1.22
0.80	2.80	0.60	0.59	0.60	1.70	1.47	1.39
0.90	2.97	0.56	0.55	0.56	1.94	1.70	1.58
1.00	3.13	0.51	0.51	0.52	1.74	1.86	1.98
1.10	3.28	0.45	0.47	0.48	1.24	1.74	2.31
1.20	3.43	0.43	0.43	0.45	0.78	1.34	1.95
1.30	3.57	0.39	0.40	0.41	0.50	0.94	1.24
1.40	3.71	0.36	0.37	0.38	0.35	0.67	0.76

Figure 7.71: Surge RAO for Froude numbers -0.04, 0.00 and +0.04 computed by TDRPM and Huijsmans [1996].

Figure 7.72: Heave RAO for Froude numbers -0.04, 0.00 and +0.04 computed by TDRPM and Huijsmans [1996].

7.10 An analysis of the wave current interaction effects on the heading of a turret moored FPSO

As a final test regarding the computation of the wave-current interaction effects, this section addresses the analysis of experimental results presented by Aranha et al. [2001]. The experiments were conducted for an FPSO model with a pivoting point that simulated the effects of a turret. The model was towed in the IPT towing tank (for emulating current velocities) and tested for several regular waves with different frequencies. In this configuration, the model is restricted in x and y directions, so the only degree of freedom concerning the motions in the horizontal plane is the yaw angle. The reason for including this case-study is that the wave-drift damping effects induce significant variations in the equilibrium yaw (heading) angle of the FPSO, providing a good experimental basis for the investigation of such effects. Not surprisingly, therefore, these results were employed in Aranha et al. [2001] in order to check the accuracy of the analytical Wave-drift damping formula previously proposed by Aranha [1994] (nowadays usually referred to as "Aranha's formula").

For the comparisons there is, however, a small drawback, which is related to the fact that besides the influence on the potential (mean second-order) yaw moment, the current effects induce viscous effects that also contribute to this moment considerably.

Therefore, for a proper comparison, the numerical results presented herein will also consider an "external" (viscous) yaw moment ($N_c(U, \psi)$) that is computed using exactly the same analytical formulation adopted by Aranha et al. [2001]. The moment is estimated according to eq. (7.15), where U is the current speed, T the vessel draft, L the vessel length, a the distance between the turret position from amidship and $\{C_{2C}(\psi), C_{6C}(\psi)\}$ are the current coefficients, computed using equation (7.16). Table 7.21 presents the values of the geometrical parameters of the VLCC model as well as the drag coefficient C_Y necessary for the estimation of the viscous moment. Figure 7.73 presents an sketch of the test configuration.

Table 7.21: Main characteristics of the VLCC tanker

Dimension	Value	Unit
C_B	0.832	-
Length	320.0	m
Beam	54.5	m
Draft	21.6	m
C_Y	0.84	-
l/L	0.059	-

Figure 7.73: Turret equilibrium experiment performed by Aranha et al. [2001]

$$N_c(U, \psi) = \frac{1}{2} \rho U^2 T L^2 [C_{6C}(\psi) - a C_{2C}(\psi)] \quad (7.15)$$

$$\begin{aligned} C_{2C}(\psi) &\approx \left(C_Y - \frac{\pi T}{2L} \right) \sin \psi |\sin \psi| + \frac{\pi T}{2L} \sin^3 \psi + \frac{\pi T}{L} \left(1 + 0.4 \frac{C_{BB}}{T} \right) \sin \psi |\cos \psi| \\ C_{6C}(\psi) &\approx -\frac{l}{L} \left(C_Y - \frac{\pi T}{2L} \right) \sin \psi |\sin \psi| + \frac{\pi T}{L} \sin \psi \cos \psi - \left(\frac{1 + |\cos \psi|}{2} \right)^2 \frac{\pi T}{L} \left(\frac{1}{2} - 2.4 \frac{T}{l} \right) \sin \psi |\cos \psi| \end{aligned} \quad (7.16)$$

At this point, one should remind that the experiments cannot be reproduced directly in the present version of TDRPM, since it is not able to reproduce the effects of mean heading variation during one simulation¹³. The following methodology was thus adopted: the yaw moment was evaluated for various ψ angles by adding up the mean second order moment (computed with TDRPM) and the viscous yaw angle (obtained with (7.15)). The equilibrium yaw angle (ψ_E) was then obtained from the stable equilibrium condition (7.18), where $N_w(U, \psi, \omega)$ is the yaw moment due to waves under current effects. These effects were computed by the TDRPM assuming the same methodology described earlier for the second order drift forces computation for a floating sphere (section 7.4). Now, however, the contributions of the double frequency effects are neglected, so only the mean loads are considered.

The mesh adopted in the simulations can be seen in Figure 7.74. The free surface mesh is

¹³Since the numerical method developed adopts a fixed mesh, the yaw equilibrium equation cannot be solved directly in time domain by imposing the current forces as external loads because the effects of variation of the mean yaw angle cannot be reproduced. The solution of the equilibrium equation would require a remeshing algorithm, similar to the method proposed by Watai [2014].

done for a 1000m radius with 30x32 higher order panels (the red lines are the limits of each panel). The VLCC geometry is described by 3 patches with 10x12, 10x12 and 6x8 subdivisions, which can be seen in details in Figure 7.75. The B-spline degree is set as 2 for both u and v directions.

Figure 7.74: Higher order mesh assumed in the VLCC simulations (the red lines are the limits of each panel).

Figure 7.75: Details of the VLCC mesh (the red lines are the limits of each panel).

The simulations are performed considering wave headings from $\psi = 0^\circ$ to $\psi = 30^\circ$ with increments of 5° for $kT = \omega^2 T/g$ from 0.2 to 1.0, providing a total of 35 combinations, as shown in Table 7.22. Only the current intensity of 1.90m/s is considered for comparison with the paper results and the smallest kT adopted was 0.2 due to physical domain limitations since the waves are almost the same length of the domain (see Table 7.22). The damping layer used in the computations has a length of 1λ (wavelength).

Table 7.22: Wave conditions assumed in the simulations.

kT	$\omega(rad/s)$	$T(s)$	$\lambda(m)$
0.2	0.30	20.8	678.6
0.4	0.43	14.7	339.3
0.6	0.52	12.0	226.2
0.8	0.60	10.4	169.6
1.0	0.67	9.3	135.7

The results computed by the TDRPM code are compared to the data obtained in the experimental test and the two formulations also presented in Aranha et al. [2001], shown in (7.17). In this equations, $c = g/\omega$ is the wave celerity, A is the wave amplitude, ω_e is the encounter frequency, $N_Z(\omega_e, \psi)$ the second order mean yaw moment computed assuming the center of gravity as pole and $D_y(\omega_e, \psi)$ is the second order mean sway force, the last two computed considering the encounter frequency. The first expression takes the wave drift damping effect into account according to the Aranha's formula, while the second one neglects this effect.

Neither the original WAMIT output data file containing the second order order loads, nor the geometrical surface used in Aranha et al. [2001] computations were available, therefore a similar

VLCC model was used in the computations presented in this thesis. In order to quantify the influence of these geometrical "differences" in the results¹⁴, Aranha's formula was implemented using the output data from the new VLCC geometry, which is called in the results as "Aranha's formula - present". This methodology was applied to provide a more fair comparison between Aranha's formula and TDRPM results considering the present VLCC geometry, since it would be unfeasible to perform a new experimental test with this new geometry.

$$N_{\omega}^W(U, \psi, \omega) = \frac{1}{2} \rho g A^2 L^2 \left(1 - 4 \frac{U}{c}\right) [N_Z(\omega_e, \psi) - a D_y(\omega_e, \psi)] \quad (7.17)$$

$$N_{\omega}^{WO}(U, \psi, \omega) = \frac{1}{2} \rho g A^2 L^2 [N_Z(\omega, \psi) - a D_y(\omega, \psi)]$$

The equilibrium configuration is computed from (7.18) on a bisection method assuming a linear interpolation algorithm, see for instance Figure 7.76 regarding $kT=0.6$ and $a = 0.3L$.

$$N_R(U, \psi_E, \omega) = N_c(U, \psi_E) + N_w(U, \psi_E, \omega) = 0 \quad (7.18)$$

$$\left(\frac{\partial N_R(U, \psi, \omega)}{\partial \psi} \right) \Big|_{\psi=\psi_E} < 0$$

Figure 7.76: Example of bisection method result for $kT=0.6$ and $a = 0.3L$ considering TDRPM results

The equilibrium angles (ψ_E) are evaluated for the several configurations tested and the results are summarized in Figures 7.77 and 7.78 for the turret position $a = 0.3L$ and $a = 0.2L$, respectively. It should be noticed that there are some small discrepancies between the original

¹⁴although no large differences were expected.

numerical result and the re-implementation performed for Aranha' formula (probably due to the geometrical differences in VLCC geometry, as explained earlier), which are more evident in the numerical results without the correction for wave current interaction. This may be due to the geometrical differences between the original hull and the present one, mainly for $kT > 0.5$. The yaw equilibrium angles obtained from the Rankine predictions, experimental data and the simplified formulation can be seen in Tables 7.23 and 7.24.

In the analysis of the results, since other sources of error are certainly involved (starting from the hull geometry), more than looking at the accuracy regarding the agreement of the predictions, the effects associated to the wave-drift damping should be observed. In fact, as mentioned earlier in this section, the variations introduced by the current effects on the mean yaw moment is significant and, judging from the results in Figures 7.77 and 7.78, they were well captured by the TDRPM simulations.

Figure 7.77: Comparison of the numerical results achieved with TDRPM, present Aranha's formula (re-implemented) and original results from Aranha et al. [2001] (experimental and numerical) for $a=0.3L$.

Table 7.23: Comparison of equilibrium angle considering experimental data and numerical results from the simplified formulation and Rankine simulations for $a=0.3L$.

kT	Equilibrium angle (°)		
	Experimental (Aranha 2001)	Aranha's formula w/WDD	TDRPM
0.2	5.7	4.1	5.7
0.4	6.8	9.3	12.8
0.6	15.6	18.8	23.6
0.8	19.3	20.1	26.5
1.0	20.1	20.1	27.8

Figure 7.78: Comparison of the numerical results achieved with TDRPM, present Aranha's formula (re-implemented) and original results from Aranha et al. [2001] (experimental and numerical) for $a=0.2L$.

Table 7.24: Comparison of equilibrium angle considering experimental data and numerical results from the simplified formulation and Rankine simulations for $a=0.2L$.

Equilibrium angle (°)			
kT	Experimental (Aranha 2001)	Aranha's formula w/WDD	TDRPM
0,2	23,8	19,0	19,0
0,4	26,0	24,2	26,3
0,6	31,1	35,4	38,2
0,8	32,6	39,0	43,4
1,0	37,9	39,4	45,4

Chapter 8

Conclusions and suggested future works

This thesis presented the mathematical formulation concerning the wave-current-body-interaction problem in the scope of a weakly non-linear approach, requiring the computation of the tangential derivatives of the potential function and other quantities over the numerical domain. The theoretical formulation was also extended to include the current effects in the local acceleration potential BVP, which is used to compute accurate values of the potential derivative, important for pressure evaluation. This formulation is required to provide stability to the numerical method, avoiding the use of iterative solvers for the motion equations and free surface conditions.

A higher order formulation based on B-splines of arbitrary degree description was developed in the BEM method, including the geometry description using NURBS formulation. The numerical domain is read by the developed method generically, therefore no additional hypothesis is imposed regarding the continuity between patches. This fact leads to more variables than equations if a single collocation point per panel (widely adopted in low order methods) is assumed together with the hidden layer b-spline approach, applied in this work. Some possibilities regarding the definition of the additional collocation points needed were evaluated and the multiple collocation points per panel (4) was chosen as the best option, leading to overdetermined linear systems. These systems are solved applying a Least-square-method (LSM), as well as the free surface differential equations, which are also solved considering the application in the collocation points. In the higher order formulation, the solution of the equations provides the spline coefficients, not the potential function (free surface elevation and others) directly, as in the case of low order methods.

In the higher order panel method, the contribution of the panel curvature in the source and dipole terms in the self-integration terms, which occurs when the field point and source point are located in the same panel, requires an specific procedure for proper evaluation. It was solved following a specific dessingularization procedure, providing accurate results that were compared to the simple Gauss quadrature. The other non-singular terms in the influence matrices are evaluated using Gauss quadrature directly.

The interaction of floating bodies with the current leads to the so-called m-terms, that represents a real challenge to be computed accurately due to the second derivatives that are present in these terms. These terms are computed from the solution of the zero order problem under the hypothesis of double-flow linearization. A comparison of the m-terms in the case of a sphere, (for which the zero order potential can be evaluated analytically) with the numerical predictions of the Rankine panel method attested the good performance of the algorithm. The first order response operators computed for a sphere and circular cylinder, two simplified geometries chosen for validation purposes, also have shown a good agreement with WAMIT frequency domain predictions.

One of the main goals of this thesis was the computation of second order quantities, namely slow and mean drift forces, that are very important in the context of moored offshore structures. The comparison of mean and double frequency forces was performed for a fixed sphere, initially comparing the wave run-up and velocity field with WAMIT frequency domain predictions, achieving a general fair agreement, which was also observed in the mean and double frequency forces.

In order to verify the motions effects in the mean drift forces, the same sphere was simulated in free floating condition. The results were again compared to literature results, achieving a good agreement, although some differences were observed mainly close to the heave resonance.

In a case-study involving a practical offshore system, the second order motions of a semi-submersible platform were also investigated and compared to experimental results conducted specifically with this purpose in the Hydrodynamic Calibrator of the Numerical Offshore Tank at University of São Paulo (CH-TPN). A reasonable agreement was verified one more time, even though several uncertainties are present, mainly because the forces involved are really small and non-linear viscous and mooring effects play a role in the experimental measurements.

The wave-current-structure interactions were evaluated initially for a fixed sphere under the action of waves under positive and negative Froude numbers considering the first and second

order excitation forces, obtaining good concordance with the first order forces presented by Kim et al. [2007]. On the other hand, the second order forces showed a worse agreement, mainly for the positive Froude number condition. The wave current interaction was also evaluated considering the wave runup and the diffraction forces on a fixed cylinder, showing a good agreement with Shao [2010] results for both conditions, although there were some differences in the symmetry plane upstream and downstream of the cylinder. A similar trend was verified in the fixed sphere for some singular periods. Larger variations are verified in the presence of current, regardless the negative or positive Froude numbers values compared to the zero current condition.

The wave current effects were also evaluated considering the responses of a free floating sphere, the results compared to Huijsmans [1996] predictions using a frequency domain method. The general trends, mainly the resonance frequency shift, are recovered comparing the present results with the literature reference.

The last comparisons were performed considering a turret moored FPSO tested in IPT Towing Tank, which was presented by Aranha et al. [2001] for the analysis of the wave-drift-damping effect on the yaw equilibrium angle. The VLCC geometry was rebuilt using a similar vessel and the simplified expressions proposed in the paper implemented to be compared to the experimental data, since the "exact" vessel geometry was not available. The viscous current loads were externally introduced following the formulations proposed in the paper for the equilibrium computation. Since the mesh is fixed in the numerical method, many different angles of incidence were evaluated, and the equilibrium was obtained searching for the angle of zero moment. The results revealed that the developed method was able to capture the trends properly, indicating that the wave-drift-damping effects are indeed well reproduced.

The future developments proposed in order to improve and extend the numerical method are:

- The implementation considering a more appropriate programming language and HPC techniques to provide a better memory management and overall computational performance;
- The inclusion of a remeshing algorithm during the free floating simulations, which could be applied, for instance, for dealing with conditions that involve changes in the mean wave angle (such as the case of the turret FPSO studied in Chapter 7.10) and multi-body system with large relative displacements.
- The introduction of a non-linear damping model to deal with large displacements, where

a linear coefficient is not enough to reproduce the physics of damping.

- The inclusion of second order approximations (Pinkster and others), as the implementation of the complete second order solution to extend the numerical code for other applications;
- The integration with other numerical codes can be envisaged, as for example, in order to provide a coupled analysis with moorings and risers dynamics in time domain;

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