NICOLAU OYHENARD DOS SANTOS

**Study of the influence of offshore drilling rigs heave motion in drillstrings
dynamic behavior**

São Paulo

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dynamic behavior**

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the University of São Paulo to fulfill partial
requirements for obtaining the title of Master
of Sciences.

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To my family.

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“The science of today is the technology of tomorrow.”

(Edward Teller)

ABSTRACT

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The price of crude oil is a major concern for oil companies nowadays, maintaining costs in deepwater drilling activities is of utmost importance. The contribution of the heave movement from floating platforms to the vibrations of drillstrings is still a subject not fully appreciated in the literature. This work deals with the dynamic behavior of the drillstring through finite element analysis (FEA) when an oscillating axial displacement at the top is applied. Numerous drillstring models were generated using two-dimensional beam elements that have degrees of freedom for the axial, shear, bending, and torsion types of displacement. The models incorporated an axial displacement at the top of a vertical drillstring set at the bottom. Natural frequencies were obtained for all degrees of freedom considering different heave scenarios. The variation in the normal forces distribution due to the inertia effect on the entire structure caused a change in the stiffness for shear and bending degrees of freedom. The variation of natural frequencies related to shear were obtained. In addition, the displacement of the neutral point in the drill column was obtained. The results presented here should contribute to future research on complex dynamic behavior of drillstrings.

Keywords: Drillstring, Vibration, Finite element, Heave, Dynamic.

RESUMO

SANTOS, Nicolau Oyhenard. **Estudo da influência do movimento de heave de sondas de perfuração offshore sobre o comportamento dinâmico de drillstrings**. 2018. 78 p. Dissertação (Mestre em Ciências) – Escola Politécnica, Universidade de São Paulo, Santos, 2018.

O preço do petróleo bruto é uma grande preocupação para as empresas de petróleo hoje em dia, a manutenção de custos em atividades de perfuração em águas profundas é de extrema importância. A contribuição do movimento de heave das plataformas flutuantes para as vibrações das colunas de perfuração ainda é um assunto não totalmente apreciado na literatura. Este trabalho aborda o comportamento dinâmico da coluna de perfuração através da análise de elementos finitos (FEA) quando um deslocamento axial oscilante no topo é aplicado. Numerosos modelos de colunas de perfuração foram gerados usando elementos de vigas bidimensionais que possuem graus de liberdade axiais, de cisalhamento, flexão e torção. Os modelos incorporaram um deslocamento axial no topo de uma coluna de perfuração vertical engastada na parte inferior. Frequências naturais foram obtidas para todos os graus de liberdade considerando diferentes cenários de heave. A variação na distribuição de forças normais devido ao efeito de inércia em toda a estrutura causou uma mudança na rigidez para graus de liberdade de cisalhamento e flexão. As variações das frequências naturais relacionadas ao cisalhamento foram obtidas. Além disso, o deslocamento do ponto neutro na coluna de perfuração foi obtido. Os resultados aqui apresentados deverão contribuir para futuras pesquisas em comportamento dinâmico complexo de drillstrings.

Palavras-chaves: Coluna de Perfuração, Vibração, Elementos Finitos, Heave, Dinâmico.

LIST OF FIGURES

Figure 1 – Lucas Spindletop well in 1901	15
Figure 2 – A drillstring and the necessary basic elements for drilling.	16
Figure 3 – Usual organization on a drilling rig.	18
Figure 4 – A mobile land rig with a portable mast in the vertical position.	19
Figure 5 – Simplified example of a fixed rig.	20
Figure 6 – A jackup rig self elevating.	21
Figure 7 – An ultradeepwater semisubmersible platform from Transocean Inc.	22
Figure 8 – A modern drillship.	23
Figure 9 – Types of displacement that an offshore rig can perform.	25
Figure 10 – A basic schematic of a vibration isolator such as a passive heave com- pensator.	26
Figure 11 – Basic components of a drillstring.	28
Figure 12 – Primary vibration modes of a drillstring.	29
Figure 13 – Drill collar whirling.	31
Figure 14 – Usual path to a finite element analysis.	35
Figure 15 – Beam element with eight degrees of freedom.	36
Figure 16 – Illustration of cantilever beam subjected to different types of loading.	38
Figure 17 – Displacement by frequency for the axial degree of freedom.	42
Figure 18 – Displacement by frequency for the shear degree of freedom.	43
Figure 19 – Displacement by frequency for the torsional degree of freedom.	44
Figure 20 – Comparison of results with data from Example 9.4 in (BATHE, 2006).	46
Figure 21 – Last node axial displacement for a 1000 meters bar	47
Figure 22 – Configuration used for static example.	49
Figure 23 – Comparison of analytical and numerical results.	50
Figure 24 – First vibration mode of the beam.	51
Figure 25 – Second vibration mode of the beam.	51
Figure 26 – Third vibration mode of the beam.	52
Figure 27 – Fourth vibration mode of the beam.	52
Figure 28 – The 4 types of displacement used as input for the model.	54

Figure 29 – Normal forces distribution considering only the weight and the hoisting force.	55
Figure 30 – Simplified diagram of dynamic model of drillstring.	58
Figure 31 – Vertical displacement and axial force variation on the top node.	59
Figure 32 – Simplified diagram of dynamic model of drillstring.	60
Figure 33 – Numerical results for an axial degree of freedom in the middle of the structure on Scenario 1.	61
Figure 34 – Neutral point displacement for the first heave scenario.	62
Figure 35 – Variation of dimensionless natural frequency for the first heave scenario.	63
Figure 36 – Numerical results for an axial degree of freedom in the middle of the structure on Scenario 2.	64
Figure 37 – Neutral point displacement for the second heave scenario.	65
Figure 38 – Variation of dimensionless natural frequency for the second heave scenario.	66
Figure 39 – Numerical results for an axial degree of freedom in the middle of the structure on Scenario 3.	67
Figure 40 – Neutral point displacement for the third heave scenario.	68
Figure 41 – Variation of dimensionless natural frequency for the third heave scenario.	69
Figure 42 – Numerical results for an axial degree of freedom in the middle of the structure on Scenario 4.	70
Figure 43 – Neutral point displacement for the fourth heave scenario.	71
Figure 44 – Variation of dimensionless natural frequency for the fourth heave scenario.	72

LIST OF TABLES

Table 1 – Values of physical parameters for the static example.	39
Table 2 – Comparison between analytic and numerical calculations for a 1 m. beam.	39
Table 3 – Natural frequencies obtained through analytical and finite element calculations for the axial degree of freedom.	43
Table 4 – Natural frequencies obtained through analytical and finite element calculations for the shear degree of freedom.	43
Table 5 – Natural frequencies obtained through analytical and finite element calculations for the torsional degree of freedom.	44
Table 6 – Steel bar properties.	47
Table 7 – Material and geometric properties of the pipe.	49
Table 8 – Comparison of adimensional natural frequencies calculated numerically and analytically.	50
Table 9 – Forces present on each element of the system	54
Table 10 – Data respective to all different sections in the structure.	57

CONTENTS

1	Introduction	13
1.1	<i>Drilling</i>	13
1.1.1	History	14
1.1.2	Description	15
1.1.3	Drilling rigs	18
1.1.4	Drilling Rig Systems	23
1.1.5	Drilling rigs displacement	24
1.1.6	Heave compensation systems	25
1.2	<i>Drillstring</i>	27
1.2.1	Mechanics	28
1.3	<i>State-of-the-art</i>	31
1.4	<i>Problem description</i>	33
1.5	<i>Research objective</i>	33
2	The Finite Element Method	34
2.1	<i>Concept</i>	34
2.2	<i>Static analysis</i>	36
2.3	<i>Dynamic Analysis</i>	39
2.3.1	Frequency Domain	41
2.3.2	Time Domain	44
3	Axial Tension, Natural Frequencies and Vibration Modes . . .	48
3.1	<i>Analytic Model</i>	48
3.2	<i>Geometric Stiffness Matrix</i>	48
3.3	<i>Static model validation</i>	49
3.4	<i>Natural Frequencies and Vibration Modes</i>	50
4	Parametrical Analysis	53
4.1	<i>Heave Displacement</i>	53
4.2	<i>Normal Forces Distribution</i>	54
4.3	<i>Dynamic Model</i>	56

4.3.1	Hanging Structure	57
4.3.2	Fixed Structure	59
5	Conclusions	74
	Bibliography	75

1 INTRODUCTION

Drilling operations in ultra-deep waters demand high initial investments and, consequently, a great concern with economical viability of offshore projects. During periods of low barrel price, most companies prioritize production over drilling new wells in high water depths, particularly in the Brazilian Pre-salt (Moreira Matoso Ribeiro Gomes *et al.*, 2017).

To face these financial challenges drilling operations need to be the most secure, efficient and economical, they can be, and needs to avoid any additional expenditures. Regarding this issue, one of the most contributing factors of problems occurred during offshore drilling is equipment failure. Most offshore operations happen by renting Mobile Offshore Drilling Units (MODUs), which requires a substantial investment from the operator company. Each time equipment fails, a trip is necessary, that is the removal of the entire drillstring and posterior placing of the same. Sometimes, it takes a full day for such an operation.

Vibrations are the dominant cause of drillstring fatigue. An absolute path to avoiding equipment fatigue is the previous simulation of drillstring behavior. Finite Element Analysis (FEA) consists of an excellent tool for behavior prediction and simulation of different settings in complex systems. Besides, costs are diminished substantially by not applying the trial and error strategy on expensive drilling projects (KHULIEF; AL-NASER, 2005).

Earlier works have studied drillstring dynamic behavior concerning several aspects. However, the contribution of offshore platforms heave, vertical oscillatory movement, in drillstring dynamic behavior for deep or ultra-deep wells wasn't extensively studied. This work proposes bridging that gap by using finite element analysis.

1.1 Drilling

Primarily, this section covers basic concepts about drilling and differences between onshore and offshore operations. A description of a typical drillstring used in deep wells is given. Offshore drilling platforms are discussed with special attention to ultradeepwater units. The chapter finishes with a literature review of state-of-art research regarding drillstring dynamic behavior.

1.1.1 History

Petroleum is the energy necessary for almost every venture in the world, from transportation of people and commodities to the production of some types of clothing and medicine. This energy stays buried underground beneath miles and miles of rocks and, sometimes, many miles of water as in offshore fields. All the activities to reach and retrieve the oil from such a difficult place without disturbing the environment safely and effectively are part of drilling engineering.

There are reports of oil well drilling activities from A.D. 300 in China and discussion still exists on when and where was the first oil well drilled and who the pioneer was at that time, but the industry in a general way recognizes some critical milestones. The most famous landmark was the well drilled by Edwin L. Drake in 1859, 21.18 meters deep, considered the first well originated from "modern drilling," taking into account the previous methods, with the purpose of finding oil. The well produced initially 8 - 10 barrels/day of oil and the mechanism used is called today "cable tool drilling" (MITCHELL; MISKA, 2011).

Cable tool drilling consists on reaching the subsurface by repeatedly dropping a heavy iron bit attached to a cable, an engine at the surface is responsible for pushing back the tool so that it can fall once again. This method, also called "percussion drilling," was adopted until the 1930s when rotary drilling became the standard in the drilling industry.

With the advent of rotary drilling, the industry changed. At the of the nineteenth century and beginning of the twentieth century, Patillo Higgins and Captain Anthony Francis Lucas managed to reach a reservoir located 305 meters deep under a salt dome in Texas. At the time they used the more advanced rotary drilling method and, after facing some challenges with unconsolidated sand, they reached the target depth producing a burst and uncontrolled flow of 100 barrel per day of oil, as shown in Figure 1. That well, named Lucas Spindletop, is considered one of the most significant milestones for oil industry proving the superior capacity of the rotary drilling method and the presence of substantial accumulations of oil in the region (MITCHELL; MISKA, 2011).

Figure 1 – Lucas Spindletop well in 1901

Source: (John Edward Brantly, 1971)

The use of floating units was introduced in well drilling in the 1950s. Exploration of offshore fields added new challenges to the industry and innovative technologies had to be developed, and still are. Offshore drilling moved eventually to ultra-deep waters and now, is a source of investment for many international oil companies.

Drilling in deep and ultra-deep waters demands high initial investments and a low barrel price creates a doubt when it comes to economic viability. The industry must adapt itself and every time the crude oil prices decrease, technology improvements are required to balance the financial equation, especially in relation to offshore exploration.

1.1.2 Description

A well is a hole that connects an underground hydrocarbons reservoir to the surface. Generally, the objective is to reach a depth stipulated in a pre-defined diameter within the time and cost programmed, respecting a predetermined operational window and guaranteeing the safety of the workers involved.

A tool at the surface imposes rotation to a set of interconnected pipes or the drillstring, that rotation is eventually passed down to a drilling bit, placed at the bottom

and responsible for actually drilling the well by crushing or scraping the rocks. The weight of drillstring generates an axial downward force that helps to deepen the well. As the well increases, the drillstring keeps getting longer by connecting new tubes at the top.

To maintain a clean well, drilling fluid is circulated inside the drillstring from the surface to bottom of the well, reaching the annular between the drillstring and the borehole through orifices located in the drill bit. The function of this "mud" is to remove the cuttings and lift them to the surface. When the mud reaches the surface, it is treated so that it can once again circulate inside the well. Figure 2 shows the usual process of rotary drilling.

Figure 2 – A drillstring and the necessary basic elements for drilling.

Source: (J.J. Azar; G. Robello Samuel, 2007)

Wells can have many purposes, and the quantity of personnel, equipment and the types of services depend on the objective of the well. Exploratory wells, or wildcat wells, are the first wells drilled to discover a reservoir still unexplored. After finding a reservoir,

additional wells can be drilled to gather more information on the extension and possible productivity of the field; these wells are called appraisal wells (or extension wells). If the reservoir is considered economically viable, new wells with the purpose of producing oil and gas are called development wells. Additionally, wells can serve different purposes such as injection, stratigraphic tests, and blowout reliefs.

Before every drilling activity, a company needs to own the rights to operate on the field. In some cases, the financial risks and profits of exploring a field are divided by a group of companies or a consortium. In the latter case, one of the companies takes the lead and is called the operator, and the other companies can participate or not on operational decisions depending on the consortium contract. Assuming an operator company exists and a location to drill an exploratory well is pinpointed by the geological team, a drilling contractor is hired to actually drill the well by first planning the entire operation, considering possible risks and following industry's standards on Health, Safety and Environment (HSE), consulting the operator company and executing the task.

Possible actions may be necessary before moving equipment and personnel such as studying seabed capability of installing equipment or preparing a site if it's in a remote area. The operator may have a representative drilling engineer or a group of specialists present at the moment of drilling the well depending on the importance of the operation. The professional representing the operator company may change the well plan in case of new circumstances present during the operation, but normally there is a great level of communication between the rig and the onshore headquarters regarding operational decisions. The person responsible for rig personnel and operation as a whole is called the tool pusher, usually a qualified professional with many years of experience working on drilling rigs. Also, drilling service companies can participate by executing diverse tasks during drilling such as handling drilling fluid, performing well logging or monitoring the well. Figure 3 shows the typical drilling rig organization.

Figure 4 – A mobile land rig with a portable mast in the vertical position.

Source: (MITCHELL; MISKA, 2011)

The first drilling activities in water began inland above structures such as artificial islands and wharves where the drilling equipment would be installed to drill wells in order to reach reservoirs below lakes or rivers. Marine rigs came later even though there was evidence of the existence of underwater hydrocarbon reservoirs, mainly because of difficult logistics at the time. The first marine rigs were submersible drilling barges with capacity of operating in depths between 3 and 6 meters (MITCHELL; MISKA, 2011).

Fixed platforms appeared in the mid-1940s to be used in shallow waters where the maximum water depth are up to 1500 ft (460 m) (MITCHELL; MISKA, 2011). Fixed platforms are bottom supported structures that are transported and mounted on location. The primary type of fixed platforms uses a jacket structure between the seafloor and the surface. This type of rig usually remains in the area for post-drilling production and pre-processing. Many wells can be tied back to the same platform so that production is centralized and logistics are easier to handle. The drilling rig can stay or be removed from the fixed platform depending on the company's strategy. Figure 5 displays an example of a fixed rig.

Figure 5 – Simplified example of a fixed rig.

Source: (MITCHELL; MISKA, 2011)

Soon after, investing in mobile offshore drilling started. Fixed platforms weren't suitable to drill wells in multiple locations due to the necessity of high initial investments and the impossibility of moving the rig after drilling. As a transition from fixed rigs, the most used bottom supported, but mobile, are the jackup rigs. This type of vessel has from 3 to 5 retractable legs that, when on site, are lowered to the sea floor and the platform elevated to the air as shown in Figure 6. When operations end the platform is lowered to water level, and its legs are "jacked up".

Figure 6 – A jackup rig self elevating.

Source: (J.J. Azar; G. Robello Samuel, 2007)

Jackup rigs could move from place to place to drill new well, but the fact that they are bottom-supported gives a great limitation to its use in the sea. Maximum water depth for these rigs are approximately 106 meters (350 ft.) (MITCHELL; MISKA, 2011).

Floating platforms such as semisubmersibles and drillships became the preferred options when exploring in deep waters. The semisubmersible (Figure 7) is a floating platform with two hulls for buoyancy; a compartment is filled with water and, together with an anchoring system there is reasonable stability for drilling operations. The platform is usually toed to arrive at the well location, and after the activities are complete, the platform is guided to drill in another area. This type of platform also serves as a production platform with pre-processing capabilities in deepwater reservoirs. Modern semisubmersible rigs have dynamic position capabilities and sensors installed on the wellhead to provide greater stability and to assure that the rig remains above the well at all times (RITTO, 2010).

Figure 7 – An ultra-deepwater semisubmersible platform from Transocean Inc.

Source: (LAKE, 2006)

The drillship is a ship-like drilling platform. It is capable of moving into locations by itself. It can operate in the most profound water depths, up to 3000 meters. Modern versions capable of drilling in ultra-deep waters usually have dynamic positioning systems to control the ship's motion. Platforms like the one shown in Figure 8 is called Floating Production Storage and Offloading vessels because of its multifunctionality and high level of autonomy.

Figure 8 – A modern drillship.

Source: (MITCHELL; MISKA, 2011)

1.1.4 Drilling Rig Systems

Drilling rigs in an offshore area usually have systems responsible for particular functions while drilling a well. Those systems are:

- power system;
- hoisting system;
- circulating system;
- rotary system;
- well control system;
- well monitoring system;

The power system generates the energy required for the rig to operate. Diesel engines are the primary source of energy in the platform. That energy is transmitted all the other systems that require electricity.

The hoisting system is responsible for lifting or lowering equipment in the rig. It's mainly composed of the drawworks, the drilling line, the block and tackle, the derrick, the substructure and ancillary equipment.

The circulating system is responsible for circulating drilling fluid. The route is from the platform into the drillstring, through the drill bit orifices, into the annular and back to the platform while carrying the cuttings so that it can be cleaned and reused.

The rotary system provides rotation to the drill bit. In older rigs, the primary source of rotation is the rotary table, a device that has a geometrical orifice on its center so that a shaft with a usually square or hexagonal cross-section transmits the rotation to the drillstring. While "making a connection", i.e., connecting a new pipe to the drillstring as the well deepens, a new tube is attached below the swivel, a device that combines the hoisting system and the circulating system supplying fluid to the interior of the drillstring. Modern drilling rigs use top drives, a more advanced alternative capable of giving rotation and fluid flow without the need of a rotary table or kelly.

The well control system has a vital role during drilling. It is responsible for assuring that no undesired flow of formation fluids occur during drilling and, if that happens, it has to make sure that the situation doesn't evolve into a blowout when an uncontrolled flow of hydrocarbons reach the surface. Equipment such as Blowout Preventers (BOPs), pit-volume indicators and flow indicators are part of the set of this system.

The well monitoring system is composed of all devices and equipment that have a part in the control of essential parameters in the rig such as well depth, hook load, and pit level. In modern platforms, a centralized control unit is present at the engineer's office so that monitoring occurs during all times during drilling.

1.1.5 Drilling rigs displacement

In marine environments, an important aspect when drilling is the rig displacement. Sometimes environmental conditions can post a challenge for a rig to stay above the well location. Vertical, horizontal and angular movements occur, especially in rough sea conditions. To minimize horizontal displacement anchoring and dynamic positioning system are good options. However, in water depths greater than 700 meters (MITCHELL; MISKA, 2011), anchoring a floating rig is not recommended because of the weight of the chains and the difficulty of installing without support vessels. As a solution for this problem, dynamic positioning (DP) is a system composed of a set of thrusters that counter the forces from the marine currents measured by sensors. A control room inside the rig monitors the whole system.

The vertical movement, called "heave", due to ocean waves is also a problem during drilling operations. For the drilling to be efficient, the Weight On Bit (WOB) has to

remain constant. To compensate the vertical displacement a heave compensator is used and a pneumatic tensioning device on the traveling block maintains a constant hookload (MITCHELL; MISKA, 2011).

Figure 9 – Types of displacement that an offshore rig can perform.

Source: (PERERA; SANDARUWAN, 2009)

1.1.6 Heave compensation systems

The main objective of a heave compensation system is to separate the vessel motion from the load motion, reducing the tension variation on the suspended structure. Woodacre *et al.* (2015) makes a comprehensive review of the different mechanisms of vertical motion heave compensation systems. Two prevailing categories are passive and active heave compensation systems.

A passive heave compensation system is basically an equipment that isolates vibration. In order to attenuate the vessel motion and maintain a controlled displacement of the drillstring, a series of spring-damper sets are put between the rig and the suspended drillstring. Figure 10 gives a simplistic schematic of a passive heave compensation system. A small vessel, a subject of heave displacement due to ocean currents, suspending a concentrated load at the end of a line. The spring-damper symbolizes a passive heave compensation system that attenuates the vibration from the vessel and transmits a lower amplitude vibration to the load it carries. Passive heave compensators don't require energy

and have proved to serve as an important asset during drilling activities as suggested early in the seventies by Butler (1973).

Figure 10 – A basic schematic of a vibration isolator such as a passive heave compensator.

Source: (WOODACRE; BAUER; IRANI, 2015)

The active heave compensation systems use a control system that acts directly on column weight variations, enabling operations under more severe environmental conditions (DO; PAN, 2008). Passive systems act when subjected to a variation of traction due to the change of load suffered, acting as a kind of "spring". In semi-active systems, there is a combination of active and passive systems where an additional hydraulic force is applied to the latter to minimize friction losses and air pressure losses. The semi-active system consumes less energy than the active system (MELLO; VAZ, 2004). The absence of a heave compensation system may jeopardize other activities not directly related to drilling, such as the control and operation of Remotely Operated Vehicles (ROV) or the exact location of equipment that has to be moved below the surface of the sea, besides causing great risks during drilling.

Due to the high costs of drilling operations on large water depths, many studies have focused on the development of heave compensation systems in order to mitigate possible damages that vertical riser oscillation from the platform could cause (CUELLAR, 2014; DO; PAN, 2008).

In addition to the design of heave compensation systems, much attention is paid by the industry to the understanding of the riser's response to the vertical movement of the platform. A major initiative of the industry was to develop systems of prediction of movement of the riser. However, the major obstacle is to understand how the riser deforms with the static and dynamic forces acting simultaneously on it (Marcio Yamamoto, 2011).

1.2 Drillstring

The drillstring consists on a string of steel tubes that are connected by threads. When the complete set is assembled, it forms a long and flexible structure. This tubing has at its lower end a shorter and heavier segment composed of thicker ducts, called drill collars, with a cutting interface at its free end, the drill bit (TUCKER; WANG, 1999a). The function of the heavier pipes is to apply weight to the bit so that the formation is invaded until the purpose of the drilling project is reached, usually an oil and / or gas reservoir.

The functions of the drillstring are (DAREING, 2012):

- to transmit mechanical rotary power to the drill bit;
- to circulate fluids;
- to affect hole direction;
- to apply force to drill bits.

The drillstring is mostly formed by drill pipes. There are drill collars at the lower portion and tools can be present, such as stabilizers, Measuring While Drilling (MWD), drill bit, Positive Displacement Motor (PDM) and turbines.

Drill pipes are designed to work while tensile is applied. In contrast, drill collars have higher resistance to compression. The difference in physical parameters between drill pipes and drill collars can be attenuated by placing heavy weight drill pipes between the two portions of pipe.

The pipes are connected by shouldered threaded couplings, called tool joints. These tools are used mainly to protect the coupling and to give an outside diameter offset so that they are more easily lifted and suspended.

Figure 11 – Basic components of a drillstring.

Source: (MITCHELL; MISKA, 2011; API, 2007).

1.2.1 Mechanics

Drillstring vibrations are a complex research subject. Many factors such as bit/formation interaction, drillstring/wellbore interaction and hydraulics can influence drillstring dynamic behavior. Associated with these types of vibration are phenomena that can substantially decrease drilling performance like premature wear and damage of drilling equipment, and consequent decrease in the rate of penetration (ROP) (AADNOY *et al.*, 2009). The BHA is the location in the drillstring where most of the excitations happen, and the pertaining drill collars and adjacent heavyweight drill pipes have a higher probability of suffering damage

due to vibrations. Although vibrations can generate adverse effects such as equipment failure and well misdirection, there is research on benefits of axial vibration to drilling (XIAO; HURICH; BUTT, 2018a; XIAO; HURICH; BUTT, 2018b). Three primary modes of vibration exist during drilling: axial, torsional and lateral vibrations, as shown in Figure 12.

Figure 12 – Primary vibration modes of a drillstring.

Source: (J.J. Azar; G. Robello Samuel, 2007).

The axial vibration consists of the oscillation along the longitudinal axis of the drillstring. The source can be static or dynamic. The leading dynamic cause of axial vibrations is bit/formation interaction, especially in roller-cone bits (SKAUGEN, 1987). Symptoms such as kelly bounce and whipping of the drawworks cables at the rig can indicate axial vibrations (DAREING, 1984). The severity of the vibration can induce a fluctuation in WOB decreasing the ROP as a consequence (AADNOY *et al.*, 2009).

Axial vibrations may cause bit-bounce, a phenomenon where the bit temporarily lifts from the bottom ceasing to interact with the formation to, later, impact the bottom of the well. Bit-bounce may cause bit wear, excessive vibration, a decrease in ROP and worse drilling performances.

Torsional vibration relates to angular motion along the drillstring longitudinal axis. A constant ROP on the surface does not transmit correctly to the bit because of the torsional flexibility of the drillstring. This type of vibration can be transient, where the oscillation occurs at specific points in the drillstring, and steady-state, that happens for a longer time interval. It relates to a common drilling problem called "stick-slip". This motion consists of a very low angular velocity, or complete standstill, of the bit while the tubes accumulate energy and a sudden release of energy making the bit rotate as fast as ten times the original angular velocity. This phenomenon can cause extensive bit wear, fatigue and even failure of drillstring equipment (AADNOY *et al.*, 2009).

Lateral vibrations also called bending, transverse and flexural vibrations, are the leading cause of drillstring failure. Bit/formation interaction, drillstring/borehole interaction and an initially bent BHA can cause this type of vibration. A consequence of lateral vibration is drillstring whirling, a centrifugal induced bowing of the drillstring during bit rotation (VANDIVER; NICHOLSON; SHYU, 1990), this effect can be forward, backward or chaotic. Figure 13 shows the nature of this motion where the center of mass moves from the drillstring longitudinal axis. This effect may lead to a range of phenomena that result in equipment failures such as abrasion of drill collars external walls, connections fatigue and a decrease in ROP. Since the BHA is always subject to compression, this section has a higher tendency of suffering from buckling and whirling (AADNOY *et al.*, 2009).

Figure 13 – Drill collar whirling.

Source: (VANDIVER; NICHOLSON; SHYU, 1990).

In practice, the study of drillstring mechanics involve the three main types of vibration separately together with a range of effects. These effects can be linear and parametrical coupling between axial, lateral and torsional vibrations (GHASEMLOONIA; Geoff Rideout; BUTT, 2014a), bit-bounce, backward and forward whirl and stick-slip. Coupling mechanisms occur and increase the complexity of the drillstring dynamic behavior substantially.

1.3 State-of-the-art

Previous research has focused on multiple aspects of the drillstring dynamic behavior as mentioned in literature reviews addressing vibrations, modeling, and failure (ZHU; TANG; YANG, 2014; GHASEMLOONIA; Geoff Rideout; BUTT, 2015; ALBDIRY; ALMENSORY, 2016). As a general rule, the BHA is the location that suffers the most with vibration (DAREING, 1984).

GHASEMLOONIA *et al.* (2015) makes a review on earlier works related to drillstring vibrations modeling and shows the most significant research on drillstring behavior

predictions and vibrations mitigation. The literature review until the end of the research period will follow the works mentioned in the publication.

KHULIEF (2005) developed a dynamic model of the drillstring including drill pipes and drill collars. The shaft elements used had 12 degrees of freedom each. The gravitational force field, the coupling between torsional and bending inertia and the gyroscopic effect were considered. The author used a consistent mass matrix and the explicit method to study the structure of dynamic behavior. Later, KHULIEF (2007) performed the analysis of the stick-slip effect by coupling axial and bending degrees of freedom while adding stick-slip self-excited oscillations.

TUCKER (1999b) developed a numerical model that covers all modes of vibration in an integrated system which was composed of Bottom Hole Assembly (BHA) and drill pipes. In the analysis, the element used had 6 degrees of freedom. The research addressed the oscillating friction between bit and formation (bit bouncing) and helical buckling of the drillstring.

Experiments suggest that the drillstring reacts in diverse ways when changing operating parameters such as torque, drag force, temperature and weight on bit. CHENG (2011) assembled an experiment to measure the forces acting on the bit and BHA through a dynamic force sensor that measures axial, lateral and rotational forces.

KAPITANIAK (2015) assembled an experimental rig with commercial drill bits and rock-samples. Several types of phenomena could be studied, such as stick-slip, whirling, bit-bounce and helical buckling. A finite element model was developed using shaft elements. The results showed convergence between the experiment and the model, especially during stick-slip oscillations.

Niedzwecki & Thamphi (1988) studied the dynamic behavior of long drillstrings subjected to the heave movement of the platform, compensated by a passive system installed in the platform-column drilling interface. The sensitivity of the string to different environmental loading scenarios was evaluated through the dynamic amplitude, the phase angle and the dimensionless voltage ratio. Through this study, it became clear the need for a heave compensation system to minimize vertical oscillation during drilling operations.

El-Hawary & Mbamalu (1996) developed robust estimation techniques for underwater motion compensation. Cuellar & Fortaleza (2014), developed a hydropneumatic heave compensation system together with a semi-active control system for 6 km depth drillings. The objective was to maintain the performance of the drilling with the constant change of

the mass of the drillstring due to the action of the environmental agents on the platform. Through the said system, more robust, they obtained a good result in the compensation of the heave.

There is a recent interest in the application of axial vibrations to increase ROP and consequently drill wells more efficiently (GHASEMLOONIA; Geoff Rideout; BUTT, 2014b).

Studies were carried out on the dynamics of the drillstring, the heave movement of the platform and the heave compensation systems currently used. However, few studies have been carried out on the effect of the heave movement of floating platforms with compensation system installed in the sum of vibrations of the drillstring.

1.4 Problem description

The fatigue caused by vibrations in drillstrings is a primary motive for equipment failure during drilling operations. Previous works addressed various aspects of drillstring vibrations, but the study of heave contribution was not particularly made. The input of an oscillatory motion to the top of the drillstring may affect axial tension in the structure in different ways, and changes in the modal behavior should occur.

1.5 Research objective

The objective of this work is to study the contribution of heave motion from environmental loads in offshore platforms to drillstring dynamic behavior using finite element analysis. Natural frequencies and vibration modes will be calculated as a function of parameters such as wave amplitude and frequency.

2 THE FINITE ELEMENT METHOD

2.1 *Concept*

Finite element method (FEM) is a mathematical tool used in many fields such as heat transfer, electromagnetism, fluid mechanics and structural analysis. The primary focus of this research is on structural analysis. Given the existence of a physical problem, the use of the method creates the possibility of predicting complex structure behavior considering different geometries and loads in a static or dynamic environment.

Every physical problem can be simplified by a mathematical model. A mathematical model is a set of governing differential equations that determine the system behavior that assumes parameters such as geometry, kinematics and physical laws that apply to the problem. The process of solving these equations for every point in the domain is called analytical solution.

Real problems, however, tend to have complex mathematical models that make it solving analytically almost impossible. The finite element method comes as a tool (one of many) that bypasses this problem. By breaking down the structure in "finite elements," or discretizing the structure, each element has a set of simpler algebraic equations that determine its governing behavior. Between elements, there are "nodes." Each node has types of movement it can perform. Those types of displacement, linear or angular, are called degrees of freedom.

The mesh or grid is the discretized structure; its refinement determines the accuracy of the approximated solution. The mesh generates a significant number of linear equations that are solvable by a matrix approach. After creating the mesh, it is necessary to set displacement restrictions or boundary conditions. Boundary conditions enable eliminating some rows and columns of the solution matrix, making the system solvable. Usually, in structural analysis, the nodes displacement are the solution, and different parameters are obtainable from the displacement values such as stress and strain on the nodes degrees of freedom for the entire mesh. Figure 14 displays the usual path of a simulation.

Figure 14 – Usual path to a finite element analysis.

Source: Modified from (BATHE, 2006).

The model generates approximated numerical solutions for every degree of freedom of the system. The results may have satisfying accuracy or not; it's the responsibility of the engineer to determine the degree of error for the model. If it does not satisfy, it's

possible to refine the model and to generate a new mesh with more elements or, even, to change the mathematical model for the physical problem.

The most valuable characteristics of the finite element method are efficiency and reliability, especially after the advent of digital computers. Although FEM is a great tool, the process depends on the mathematical model, if the numerical model doesn't represent to a certain extent the system behavior, the solution using FEM won't be able to deliver reliable results (BATHE, 2006).

2.2 Static analysis

The negative values relate to the displacement subtraction of The finite element method is an excellent approach to both static and dynamic problems. For the static part, the use of the technique is much more straightforward. Equation 1 gives the general approach to static problems in structural analysis.

$$Kx = F \quad (1)$$

In Equation 1, K represents the stiffness matrix, x represents the displacement vector and F the load vector. It's clear the relationship of the static problem with the general linear spring equation. The stiffness parameter will have a different formula for each degree of freedom.

For all analysis during the research period, the two-dimensional beam is the chosen finite element. The beam element has eight degrees of freedom: axial, shear, flexural and torsional for each node. Figure 15 shows the beam element for the example.

Figure 15 – Beam element with eight degrees of freedom.

Source: Nicolau Oyhenard dos Santos (2018)

Equations 2, 3 and 4 set the uncoupled behavior of the degrees of freedom corresponding to axial, shear, bend and torsion, respectively. Equation 3 represents the coupled behavior of a translational and a rotational degrees of freedom, shear and bend.

$$\frac{d}{dx} \left(\frac{AEdu(x)}{dx} \right) = -p(x) \quad (2)$$

$$\frac{d^2}{dx^2} \left(\frac{EI d^2 v(x)}{dx^2} \right) = q(x) \quad (3)$$

$$\frac{d}{dx} \left(\frac{JG d\phi(x)}{dx} \right) = -t(x) \quad (4)$$

In Equations 3 and 4, I stands for the area moment of inertia, G represent the shear modulus, and J is the polar moment of inertia.

Equation 5 is the matrix form of Equation 1 for the beam element (Marcio Yamamoto, 2011). Matrix rows and columns 2 and 5 concern the degree of freedom due to shear. In the stiffness matrix, rows and columns 3 and 6 represent bending and rows and columns 4 and 8 represent the angular degree of freedom corresponding to torsion. Thus, the elastic stiffness matrix for the 2D beam is 8×8 .

$$\begin{bmatrix} \frac{EA}{L} & 0 & 0 & 0 & \frac{-EA}{L} & 0 & 0 & 0 \\ 0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & 0 & \frac{-12EI}{L^3} & \frac{6EI}{L^2} & 0 \\ 0 & \frac{6EI}{L^2} & \frac{4EI}{L} & 0 & 0 & \frac{-6EI}{L^2} & \frac{2EI}{L} & 0 \\ 0 & 0 & 0 & \frac{GJ}{L} & 0 & 0 & 0 & \frac{-GJ}{L} \\ \frac{-EA}{L} & 0 & 0 & 0 & \frac{EA}{L} & 0 & 0 & 0 \\ 0 & \frac{-12EI}{L^3} & \frac{-6EI}{L^2} & 0 & 0 & \frac{12EI}{L^3} & \frac{-6EI}{L^2} & 0 \\ 0 & \frac{6EI}{L^2} & \frac{2EI}{L} & 0 & 0 & \frac{-6EI}{L^2} & \frac{4EI}{L} & 0 \\ 0 & 0 & 0 & \frac{-GJ}{L} & 0 & 0 & 0 & \frac{GJ}{L} \end{bmatrix} \begin{bmatrix} u_1 \\ v_1 \\ \theta_1 \\ \phi_1 \\ u_2 \\ v_2 \\ \theta_2 \\ \phi_2 \end{bmatrix} = \begin{bmatrix} F_{X_1} \\ F_{Y_1} \\ M_{Z_1} \\ M_{X_1} \\ F_{X_2} \\ F_{Y_2} \\ M_{Z_2} \\ M_{X_2} \end{bmatrix} \quad (5)$$

Primarily, the learning path for the mathematical technique focused on static analysis. To show an example of the method used, Figure 16 displays the beam with loads on its free end.

Figure 16 – Illustration of cantilever beam subjected to different types of loading.

Source: Nicolau Oyhenard dos Santos (2018)

Equations 6, 7, 8 and 9 are the usual formula for analytical calculations of a cantilever beam subjected to axial, shear, flexural and torsional loading respectively (HIBBELER, 2010).

$$\Delta l = \frac{F_x L}{AE} \quad (6)$$

$$\Delta v = \frac{F_y L^3}{3EI} \quad (7)$$

$$\Delta \theta = \frac{F_y L^2}{2EI} \quad (8)$$

$$\Delta \phi = \frac{M_x L}{JG} \quad (9)$$

Table 1 presents the values for the physical parameters used for the analytic and numerical calculations of a cantilever beam with applied loads on all four degrees of freedom of the last node, separately. These values correspond to a $6\frac{5}{8}$ OD drillpipe (LYONS; PLISGA, 2005).

Young's Modulus (E)	210.10^9 Pa
Cross Area (A)	$4.6.10^{-3} \text{ m}^2$
Specific Mass (ρ)	$8.96.10^3 \text{ kg/m}^3$
Area Moment of Inertia (I)	$1.46.10^{-5} \text{ m}^4$
Shear Modulus (G)	80.10^9 Pa
Polar Moment of Inertia (J)	$2.92.10^{-5} \text{ m}^4$
Beam Length (L)	1 m
Applied Load (F_X, F_Y)	10^4 N (1 ton)
Applied Momentum (M_X)	10^4 Nm

Table 1 – Values of physical parameters for the static example.

Table 2 shows a comparison between analytical and numerical solutions for the beam. The values show that the computational results are acceptable when compared to analytical calculations. They also show results for three different types of discretization: 5, 20 and 100 elements.

Displacement		Axial [mm]	Shear [mm]	Bend [deg]	Torsion [deg]
Analytical		0.0104	1.0872	0.0934	0.2453
Numerical	5 elements	0.0104	1.0872	0.0934	0.2453
	20 elements	0.0104	1.0872	0.0934	0.2453
	100 elements	0.0104	1.0872	0.0934	0.2453

Table 2 – Comparison between analytic and numerical calculations for a 1 m. beam.

2.3 Dynamic Analysis

Most finite element simulations involve dynamic problems. There are two types of dynamic analysis: on the frequency domain and on the time domain. Equation 10 shows the general equation for dynamic analysis in time domain.

$$M\ddot{x} + C\dot{x} + Kx = F(t) \quad (10)$$

Equation 10 has a few additional parameters, M stands for the mass matrix, C is the damping matrix, $\dot{x}$ and $\ddot{x}$ are the first and second derivatives of the displacement, i.e., velocity and acceleration. The variation with time demands the input of the inertia parameter, the mass matrix, and the damping parameter.

The linear dynamic response of a system depends on three types of forces: inertia forces, damping forces and elastic forces. All of them are time-dependent. The same equation can be used for static analysis as long as inertia and damping forces are neglected.

It's the job of the engineer to provide justification for the choice of which type of analysis to perform. The analyst suffers from the risk of performing useless work. Particularly to nonlinear problems, this is critical.

Equation 10 represents, mathematically, a system of linear differential equations of second order that can be solved by conventional methods using constant coefficients. However, this approach is not applicable when very large matrices are involved.

The mass matrix for the beam element is of the same size as the stiffness matrix, four rows and four columns for each node. The matrix can be lumped or consistent. Equation 11 shows the lumped matrix for the beam element. Equation 12 represent the consistent mass matrix for the beam element (Marcio Yamamoto, 2011).

$$M_L = \begin{bmatrix} \frac{\bar{m}L}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{\bar{m}L}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\rho JL}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\bar{m}L}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\bar{m}L}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\rho JL}{2} \end{bmatrix} \quad (11)$$

$$M_C = \frac{\bar{m}L}{420} \begin{bmatrix} 140 & 0 & 0 & 0 & 70 & 0 & 0 & 0 \\ 0 & 156 & 22L & 0 & 0 & 54 & -13L & 0 \\ 0 & 22L & 4L^2 & 0 & 0 & 13L & -3L^2 & 0 \\ 0 & 0 & 0 & \frac{140\rho J}{\bar{m}} & 0 & 0 & 0 & \frac{70\rho J}{\bar{m}} \\ 70 & 0 & 0 & 0 & 140 & 0 & 0 & 0 \\ 0 & 54 & 13L & 0 & 0 & 156 & -22L & 0 \\ 0 & -13L & -3L^2 & 0 & 0 & -22L & 4L^2 & 0 \\ 0 & 0 & 0 & \frac{70\rho J}{\bar{m}} & 0 & 0 & 0 & \frac{140\rho J}{\bar{m}} \end{bmatrix} \quad (12)$$

In the equations above, ρ stands for specific mass in $[kg/m^3]$, $\bar{m}$ is the distributed mass per unit of length in $[kg/m]$.

The damping matrix is assembled by a combination of the stiffness and mass matrices using Rayleigh's coefficients (CLOUGH; PENZIEN, 2013). Equation 13 shows the usual method for the calculation of the damping matrix.

$$B = \alpha K + \beta M \quad (13)$$

Damping was not considered for the validation of the dynamic analysis in the time and frequency domains. However, for the dynamic models of the drillstring, better results were obtained when $\alpha = 0.05$ and $\beta = 0$, and these results are shown in the last chapter.

2.3.1 Frequency Domain

Another code was created for dynamic analysis of beams. The examples used for code validation also had a cantilever beam to check natural frequencies convergence between analytical and FEA calculations. The analytical calculations were done based on Equations 14, 15 and 16 (GRAFF, 1975).

$$\omega_n^{axial} = \frac{\pi(2n-1)}{2} \sqrt{\frac{AE}{mL^2}} \quad n = 1, 2, 3, 4, 5 \quad (14)$$

$$\omega_1^{shear} = (1.875)^2 \sqrt{\frac{EI}{mL^4}} \quad (15a)$$

$$\omega_2^{shear} = (4.694)^2 \sqrt{\frac{EI}{mL^4}} \quad (15b)$$

$$\omega_3^{shear} = (7.855)^2 \sqrt{\frac{EI}{mL^4}} \quad (15c)$$

$$\omega_n^{torsion} = \frac{\pi(2n-1)}{2L} \sqrt{\frac{G}{\rho}} \quad n = 1, 2, 3, 4, 5 \quad (16)$$

Table 3 shows the first five natural frequencies for the degree of freedom corresponding to axial displacement from analytical and computational calculations. Results show sufficient convergence. Figure 17 presents the displacement modulus variation by frequency.

Figure 17 – Displacement by frequency for the axial degree of freedom.

Source: Nicolau Oyhenard dos Santos (2018)

Axial	Natural Frequencies				
	1	2	3	4	5
Analytical	7604.6	22814.0	38023.0	53232.0	68441.0
Numerical	7600.0	22810.0	38010.0	53210.0	68380.0

Table 3 – Natural frequencies obtained through analytical and finite element calculations for the axial degree of freedom.

Table 4 presents natural frequencies obtained through the same methods. Results are satisfying and Figure 18 shows a plot of lateral displacement by frequency. Table 5 shows the same results for the last torsional degree of freedom for the beam. Figure 19 presents last node torsional displacement by frequency.

Transversal	Natural Frequencies		
	1	2	3
Analytical	958.9	6009.5	16829.0
Numerical	960.0	6010.0	16820.0

Table 4 – Natural frequencies obtained through analytical and finite element calculations for the shear degree of freedom.

Figure 18 – Displacement by frequency for the shear degree of freedom.

Source: Nicolau Oyhenard dos Santos (2018)

Torsional	Natural Frequencies				
	1	2	3	4	5
Analytical	4694.0	14081.0	23468.0	32856.0	42243.0
Numerical	4690.0	14080.0	23460.0	32840.0	42210.0

Table 5 – Natural frequencies obtained through analytical and finite element calculations for the torsional degree of freedom.

Figure 19 – Displacement by frequency for the torsional degree of freedom.

Source: Nicolau Oyhenard dos Santos (2018)

2.3.2 Time Domain

The learning path of the method also covered dynamic analysis of structures formed by beam elements in the time domain. What an usual time analysis does is to impose equilibrium between inertia, damping and elastic forces to the system subjected to external forces (Equation 10) for each time step regarding a total analysis time and a particular time discretization.

There are numerical methods that can do this type of integration, two of these are: direct integration and mode superposition. Although a little different, these two methods are closely related.

In the case of direct integration methods, the word "direct" corresponds to the fact that no transformation is performed on the equations prior to the step-by-step numerical procedure. One of the important aspect of these methods is that system equilibrium is only sought for specific points in time rather than for the entire time interval. Another important aspect is that a variation of displacements, velocities and accelerations is assumed and this contributes to the solution accuracy and stability.

The solution of the equilibrium equations for dynamic analysis performed by direct integration can happen by applying explicit or implicit methods. The explicit method calculates the solution for a particular discrete time point $t + \Delta t$ based on the equilibrium equations of the previous time point t . The implicit method, however, obtains the solution for $t + \Delta t$ based on the equilibrium equations for time $t + \Delta t$ as shown in Equation 17. Another difference between the implicit and explicit method is the dependency of the latter on the size of the time step used in the analysis.

$${}^{t+\Delta t}KU + {}^{t+\Delta t}C\dot{U} + {}^{t+\Delta t}M\ddot{U} = {}^{t+\Delta t}R \quad (17)$$

This research focused on a particular implicit method called Newmark Method. It is a widely used numerical integration method for structure analysis. Being one the most used methods in finite element analysis, it solves differential equations that describe the dynamic behavior of solids and structures, as well as other systems. Equations 18 and 19 represent the formulas used to obtain velocities and displacements in the Newmark method. By isolating ${}^{t+\Delta t}\ddot{U}$, accelerations are obtained (NEWMARK, 1959).

$${}^{t+\Delta t}\dot{U} = {}^t\dot{U} + [(1 - \delta) {}^t\ddot{U} + \delta {}^{t+\Delta t}\ddot{U}]\Delta t \quad (18)$$

$${}^{t+\Delta t}U = {}^tU + {}^t\dot{U}\Delta t + \left[\left(\frac{1}{2} - \alpha \right) {}^t\ddot{U} + \alpha {}^{t+\Delta t}\ddot{U} \right] \Delta t^2 \quad (19)$$

The parameters α and δ can vary according to the necessary accuracy and stability for the analysis. This work adopted the constant-average-acceleration method, where $\alpha = \frac{1}{2}$ and $\delta = \frac{1}{4}$, known for being unconditionally stable (NEWMARK, 1959).

To validate the application of the finite element method to dynamic analysis in the time domain, an example from (BATHE, 2006) was used. Equation 20 shows the governing equilibrium equations for the example.

$$\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \ddot{U}_1 \\ \ddot{U}_2 \end{bmatrix} + \begin{bmatrix} 6 & -2 \\ -2 & 4 \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 10 \end{bmatrix} \quad (20)$$

The solution of the system with two degrees of freedom was obtained for a sequence of 12 time steps. The time step for the solution was 0.28 seconds. Figure 20 shows the solution obtained for the first and second degrees of freedom as well as the numerical solution obtained by the use of the Newmark Method.

Figure 20 – Comparison of results with data from Example 9.4 in (BATHE, 2006).

Source: Nicolau Oyhenard dos Santos (2018)

Apart from the example in the literature, a more realistic model was assembled in order to show and validate the algorithm for solving dynamic systems. A steel bar with properties shown in Table 6 has a punctual axial load of 10000 N at each node.

Table 6 – Steel bar properties.

Length	1000 <i>m</i>
Discretization	20 <i>elements</i>
Young's modulus	$210 \cdot 10^9 \text{ Pa}$
Area	$1 \cdot 10^{-3} \text{ m}^2$
Specific mass	7800 kg/m^3

Source: Nicolau Oyhenard dos Santos, 2018

A dynamic analysis using 0.01 seconds as a time step and a time analysis of 2.5 seconds was performed. Results are shown in Figure 21. Using Equation 14, the fundamental natural frequency obtained was 8.1505 Hz and corresponding wave period was 0.77 s . Numerical results show good convergence.

Figure 21 – Last node axial displacement for a 1000 meters bar

Source: Nicolau Oyhenard dos Santos (2018)

3 AXIAL TENSION, NATURAL FREQUENCIES AND VIBRATION MODES

3.1 Analytic Model

Models with increasingly complexity were necessary until an ideal model could be used for the obtention of natural frequencies and vibration modes. The static behavior of the drillstring under axial tension can be modeled by Equation 21 (PATIL; TEODORIU, 2013). This is a basic model that doesn't take into account hydrostatic and hydrodynamic effects.

$$\frac{d^2}{dz^2} \left(EI \frac{d^2 x_R}{dz^2} \right) - T(z) \frac{d^2 x_R}{dz^2} - w \frac{dx_R}{dz} = f \quad (21)$$

where x_R refers to the drillstring deflection in the direction perpendicular to the longitudinal axis, z is the vertical coordinate, T_z is the axial tension along the beam, w corresponds to the weight per unit of length (N/m^2) and f is resultant force on the structure (Marcio Yamamoto, 2011).

3.2 Geometric Stiffness Matrix

The effect of the axial stress on the drillstring due to pipe weight, buoyancy, hook load and WOB cause a change on the buckling and shear stiffness that, if not addressed, create a inaccurate response of the system. Equation 22 represents the geometric portion of the stiffness for a beam element (Marcio Yamamoto, 2011).

$$K_{geo} = \frac{T_{ef}}{30L} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 36 & 3L & 0 & 0 & -36 & 3L & 0 \\ 0 & 3L & 4L^2 & 0 & 0 & -3L & -L^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -36 & -3L & 0 & 0 & 36 & -3L & 0 \\ 0 & 3L & -L^2 & 0 & 0 & -3L & 4L^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (22)$$

3.3 Static model validation

Previous to dynamic analysis, a static example from the literature (Marcio Yamamoto, 2011) was used as a base for validation of the static model. Figure 22 displays a pipe with a weight attached to its bottom end and a distributed lateral force of 1 N/m. Table 7 shows the material properties for the static simulation.

Figure 22 – Configuration used for static example.

Source: Nicolau Oyhenard dos Santos (2018)

Modulus of Elasticity E	4.5 MPa
Area Moment of Inertia I	295.35E12 m^4
Pipe's Length L	0.6 m
Outside Diameter OD	9.35 mm
Internal Diameter ID	6.35 mm

Table 7 – Material and geometric properties of the pipe.

The lateral displacements were calculated analytically and numerically. Figure 23 shows convergence between analytical and numerical results for the static example with lateral load and axial tension resulting from a concentrated weight at the bottom.

Figure 23 – Comparison of analytical and numerical results.

Source: Nicolau Oyhenard dos Santos (2018)

3.4 Natural Frequencies and Vibration Modes

		Mode 1	Mode 2	Mode 3	Mode 4
Analytical		37.9150	115.6421	198.8759	290.7694
Numerical	10 elements	37.9370	115.7125	199.0223	291.0960
	50 elements	37.9151	115.6423	198.8763	290.7701
	100 elements	37.9150	115.6421	198.8759	290.7695

Table 8 – Comparison of adimensional natural frequencies calculated numerically and analytically.

Figures 24, 25, 26 and 27 display the first four eigenvectors for the shear degree of freedom. The results converged with (Marcio Yamamoto, 2011) for the static example displayed in Figure 22.

Figure 24 – First vibration mode of the beam.

Source: Nicolau Oyhenard dos Santos (2018)

Figure 25 – Second vibration mode of the beam.

Source: Nicolau Oyhenard dos Santos (2018)

Figure 26 – Third vibration mode of the beam.

Source: Nicolau Oyhenard dos Santos (2018)

Figure 27 – Fourth vibration mode of the beam.

Source: Nicolau Oyhenard dos Santos (2018)

The figures in this chapter prove that all algorithms and implementations are working properly, allowing to incorporate the heave influence on the drillstring behavior.

4 PARAMETRICAL ANALYSIS

In order to fulfill the objective of studying the influence of the heave motion of MODUs to dynamic behavior of drillstrings, the research resulted into a parametrical analysis. By changing the environmental parameters, the drillstring suffers different influences causing a different physical behavior and consequently variation of natural frequencies and vibration modes.

After certificating that the model shows good results according to the wave data, the following step is performing a modal analysis. Acquiring natural frequencies and vibration modes can be performed computationally through LAPACK (Linear Algebra PACKage) subroutines.

4.1 *Heave Displacement*

Heave is the vertical displacement of a rig placed offshore. To impose a displacement to the drillstring, wave data (PIANCA; MAZZINI; SIEGLE, 2010), was used directly as motion amplitude and period on the top of the drillstring. The research could have used only a portion of the wave amplitude representing a passive heave compensation system, but since the average values were low, it was decided to not consider a motion compensation. This work considered average wave amplitudes and periods for all seasons of the year in Santos Basin, Brazil. Figure 28 demonstrates the varying wave behavior across the year taking an average of the wave amplitudes and periods.

Figure 28 – The 4 types of displacement used as input for the model.

Source: Nicolau Oyhenard dos Santos (2018)

4.2 Normal Forces Distribution

Normal forces in a drillstring is maximum at its top and minimum at the bottom, where it turns negative because of the change from a tensioned system to a compressed system. The amount of compressive force above the bit is also called Weight on Bit (WOB) and is a paramount parameter for every drilling operation.

Equation 23 displays all forces considered in this model. Table 9 explains the parameters in Equation 23.

$$F = F_g + F_p + T \quad (23)$$

Force	Explanation
F_g	Hoisting system
F_p	Distributed weight
T	Axial Tension
F	Total force

Table 9 – Forces present on each element of the system

Taking into account Equation 23 and replacing F in Equation 24, Equation 25 can be used to calculate the tension on the drillstring. After calculating $M\ddot{U}$, axial tension is calculated.

$$M\ddot{U} + C\dot{U} + KU = F(t) \quad (24)$$

$$T = F_p - F_g + M\ddot{U} \quad (25)$$

Figure 29 shows the variation of normal forces considering an static case. When considering only the weight of the drillstring and the force due to the hoisting system of the platform, most of the drillstring suffers tension while at the bottom the string is on a compressive state.

Normal forces on the drillstring vary from the static example because of the inertia effect. Neutral point is the location where axial stress is zero. If you cut the line at this point, no difference in weight will be felt during suspension of the string.

Figure 29 – Normal forces distribution considering only the weight and the hoisting force.

Source: Nicolau Oyhenard dos Santos (2018)

4.3 *Dynamic Model*

After performing and validating static and dynamic analysis, the research focused on a more realistic model of a drillstring. Two models were considered: a hanging structure and a structure fixed at the bottom.

The program structure to perform the dynamic analysis consisted of:

1. Input data such as drillstring length, time of analysis, time step and number of finite elements;
2. Assembling of organizing matrices based upon boundary conditions and input data (Thomas J. R. Hughes, 1987);
3. Mass matrix assembling;
4. FOR loop [0: time interval: time of analysis]:
 - a) IF beginning of the loop:
 - i. Assembling of axial forces in the drillstring without acceleration.
 - ii. Assembling of stiffness and damping matrices.
 - iii. Obtention of dimensionless natural frequencies corresponding to the axial and torsional degrees of freedom (non-time dependent).
 - iv. Load array assembling (time dependent - Dirichlet boundary conditions).
 - v. Newmark method (NEWMARK, 1959).
 - vi. Obtention of dimensionless natural frequencies corresponding to the degree of freedom of shear (time dependent).
 - b) ELSE:
 - i. Assembling of axial forces in the drillstring WITH acceleration for the particular time step.
 - ii. Assembling of stiffness and damping matrices (time dependent).
 - iii. Load array assembling (time dependent - Dirichlet boundary conditions).
 - iv. Newmark method.
 - v. Obtention of dimensionless natural frequencies corresponding to the degree of freedom of shear (time dependent).
5. Output data

Taking into consideration the dynamic simulation of the drillstrings, the axial force due to the drawworks was 0.02 the weight of the whole structure subtracted by the buoyancy on seawater to maintain a tension-compression relationship in the drillstring necessary for all drilling operations.

The model contemplated 4 scenarios of platform heave motion as seen in Figure 28 (PIANCA; MAZZINI; SIEGLE, 2010). The first scenario had a heave response amplitude of 1.5 meters and wave excitation of 11 seconds, the second scenario had a heave response amplitude of 2 meters and wave excitation of 11 seconds, the third scenario had a heave response amplitude of 2.5 m and wave excitation of 11 s and the fourth scenario considered a heave response amplitude of 1.5 m and wave excitation of 7 s. The drillstring model considered for the analysis was a 1000 meters long composed only of drillpipes, heavyweight drillpipes and drillcollars. Table 4.3 presents the parameters used for the different sections in the drillstring (LYONS; PLISGA, 2005).

Tubing type	Drillpipe	HW drillpipe	Drillcollar
Section length (L)	800 m	100 m	100 m
Young's modulus (E)	210.10 ⁹ Pa	210.10 ⁹ Pa	210.10 ⁹ Pa
Outside diameter (OD)	6 $\frac{5}{8}$ in	4 $\frac{1}{2}$ in	5 in
Inside diameter (ID)	5.901 in	2.25 in	2.25 in
Specific mass (ρ)	7860 kg/m^3	7860 kg/m^3	7860 kg/m^3
Shear modulus (G)	80.10 ⁹ Pa	80.10 ⁹ Pa	80.10 ⁹ Pa

Table 10 – Data respective to all different sections in the structure.

All analysis had a total time of 50 seconds and a time step size of 0.05 seconds. To show the convergence of numerical results for the displacements, velocities and accelerations using the Newmark method, the first excitation period was removed because of transient behavior of data. For better visualization numerical results for the displacements, velocities and acceleration were shown for the first 30 seconds of analysis. The wave periods are highlighted by different background colors on the graphs.

4.3.1 Hanging Structure

Figure 30 displays a simplified diagram of the drillstring for the first dynamic model. Only one boundary condition is present, a prescribed displacement at the top node representing the heave displacement of the rig. The lower end of the drillstring is free to

move. This model represents a drillstring being lowered or raised with its whole length inside the riser.

Figure 30 – Simplified diagram of dynamic model of drillstring.

Source: Nicolau Oyhenard dos Santos (2018)

Vertical displacement and axial force variation on the top node of the drillstring, shown in Figure 31, prove coherent physical behavior of the drillstring. The axial force variation with time shows an alternation between states of tension and compression at the top of the structure. Although small considering the much greater weight of the drillstring, the axial force variation states that when the string is being lowered due to the displacement at the top the structure enters in a compressive state showing a negative axial force. When the vertical displacement is positive the drillstring enters in a state of tension. This behavior appears even when the drillstring has no physical link at its lower end evidencing the elastic behavior of the long structure made of fortified steel.

Since the structure is hanging with its lower end free to move, the displacement relative to time for the entire drillstring was equivalent. The amplitude and period of

excitation at the top have been displayed in all nodes of the discretized model with a slight difference in amplitude due to the elastic behavior of the slender structure.

Figure 31 – Vertical displacement and axial force variation on the top node.

Source: Nicolau Oyhenard dos Santos (2018)

Although until this point the validation has accompanied all the models, to validate this model data from active rigs such as hoisting force in the draw works system, drillstring weight and compensated rig displacement would be necessary for validation. This research proposes the obtention of rig data from the industry and comparison with results obtained by the finite element analysis to validate this model.

4.3.2 Fixed Structure

Posterior to the analysis of the drillstring in a hanging situation, a new analysis was made to the same structure fixed at the bottom. This new problem can be classified as a boundary-value problem since no vertical displacement occurs at the lower end, but an imposed displacement is passed at the top end of the string.

The direct integration through the Newmark method allows the obtention of displacements, velocities, and acceleration for every node of the model. These values are

necessary for calculating the axial forces for each time step in every element. As explained before, the geometric stiffness matrix is dependent on the axial force on every element.

The analysis considered all four scenarios of heave displacement obtaining the dimensionless natural frequencies corresponding to the degrees of freedom of axial, shear and torsion. The axial and torsional behavior is not dependent on the tensioned or compressed state of the structure so they were obtained only at the initial time step of the analysis.

The variation of the dimensionless natural frequencies corresponding to the degree of freedom of shear is shown for all scenarios.

Figure 32 – Simplified diagram of dynamic model of drillstring.

Source: Nicolau Oyhenard dos Santos (2018)

Scenario 1: Heave response amplitude 1.5 m, wave excitation 11 s

The first scenario considered a harmonic vertical displacement at the top node of 1.5 meters of amplitude and 11 seconds of the natural period. Figure 4.3.2 shows 4

subplots, the first one representing the imposed displacement at the top, the second (to the right) showing the displacement numerically obtained for a node at the middle of the string and the last two graphs show the velocity and acceleration variation. The time scale used, 11 to 30 seconds, in the graphs is a smaller portion of a longer analysis to show the correct behavior of displacements, velocities, and accelerations and to validate the dynamic analysis one more time.

Figure 33 – Numerical results for an axial degree of freedom in the middle of the structure on Scenario 1.

Source: Nicolau Oyhenard dos Santos (2018)

The graphs show results starting from the second period due to the transient behavior of the initial seconds of analysis. Figure 4.3.2 shows the displacement of the neutral point in the drillstring. At this point, axial forces equal zero, above this point the structure is tensioned and below this point, the structure is compressed.

Figure 34 – Neutral point displacement for the first heave scenario.

Source: Nicolau Oyhenard dos Santos (2018)

There is clear periodicity on the displacement of the neutral point and range between 912.5 and 937.5 meters demonstrate that the neutral point varies inside the drill collar section. Figure 4.3.2 shows the variation of the dimensionless natural frequencies with time. The graph displays natural frequencies from the second to the fifth vibration mode, showing periodicity one more time and equivalent behavior for the transversal degree of freedom.

Figure 35 – Variation of dimensionless natural frequency for the first heave scenario.

Source: Nicolau Oyhenard dos Santos (2018)

The same analysis was made for Scenarios 2, 3 and 4 showing equivalent and coherent results.

Scenario 2: Heave response amplitude 2 m, wave excitation 11 s

Figure 36 – Numerical results for an axial degree of freedom in the middle of the structure on Scenario 2.

Source: Nicolau Oyhenard dos Santos (2018)

Figure 37 – Neutral point displacement for the second heave scenario.

Source: Nicolau Oyhenard dos Santos (2018)

Figure 38 – Variation of dimensionless natural frequency for the second heave scenario.

Source: Nicolau Oyhenard dos Santos (2018)

Scenario 3: Heave response amplitude 2.5 m, wave excitation 11 s

Figure 39 – Numerical results for an axial degree of freedom in the middle of the structure on Scenario 3.

Source: Nicolau Oyhenard dos Santos (2018)

Figure 40 – Neutral point displacement for the third heave scenario.

Source: Nicolau Oyhenard dos Santos (2018)

Figure 41 – Variation of dimensionless natural frequency for the third heave scenario.

Source: Nicolau Oyhenard dos Santos (2018)

Scenario 4: Heave response amplitude 1.5 m, wave excitation 7 s

Figure 42 – Numerical results for an axial degree of freedom in the middle of the structure on Scenario 4.

Source: Nicolau Oyhenard dos Santos (2018)

Figure 43 – Neutral point displacement for the fourth heave scenario.

Source: Nicolau Oyhenard dos Santos (2018)

Figure 44 – Variation of dimensionless natural frequency for the fourth heave scenario.

Source: Nicolau Oyhenard dos Santos (2018)

As seen in the four different scenarios of heave motion that an offshore rig may be subjected to, the model shows good convergence when comparing axial displacements, velocities, and accelerations of a node in the middle of the structure, acquired through the use of the Newmark method and the Dirichlet boundary conditions technique.

Also, the neutral point displacements are periodic and in accordance with heave data. Results also incorporate the variation of the normal forces along the drillstring with time.

Dimensionless natural frequencies for the degree of freedom corresponding to shear were obtained. Results show periodicity when comparing every wave excitation. The behavior of the natural frequencies can be associated with a nonlinear behavior of slender structures when subjected to periodic loading such as marine drift.

The complete nonlinear behavior of the drillstring even now is studied by companies and academia. This dynamic model could only be validated if the actual behavior of the drillstring could be summarized with the present considerations. The research intends on

continuing with the goal of approaching a closer dynamic model of the drillstring that could be used by companies in order to predict and prevent excessive vibrations when drilling offshore.

5 CONCLUSIONS

An increasing complexity approach was adopted to apply accurately the finite element method for drilling operations. A static example of a slender structure with a weight applied to the bottom and a distributed transverse force was simulated and compared to existing literature. Natural frequencies and vibration modes were acquired and validated.

Dynamic analysis in both time and frequency domain showed good convergence and accurate results when compared with analytic calculations. A varying distribution of normal forces with time due to the inertia effect was adopted and the behavior of a drillstring along a time interval was studied.

A finite element model of a drillstring considering an oscillatory movement on the top node simulating the heave displacement of an offshore rig was produced. Natural frequencies variation were obtained considering different types of heave environments. Additionally, neutral point displacement was obtained.

The validation of the final model could be realized through a comparison of data when the drillstring is being lowered or raised. The validation of the model when the structure is fixed is impossible with data from the industry due to the lack of important parameters such as rotation, ocean force, friction, impacts to the well, heave compensation mechanisms, hydrostatic and hydrodynamic effects.

A higher discretization of the structure would allow a better visualization of neutral point position across time. High performance computing is an important step in continuing to study the drillstring, specially when facing nonlinear factor such as friction and impacts to the well bore.

The author intends to increase the complexity of this final model by adding hydrostatic and hydrodynamic effects to the drillstring, angular velocity, marine loading and a limit to drillstring displacement when simulating a drillstring inside a well.

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