Cesar Barbedo Rocha

**Energetics, baroclinic instability and models of
vertical structure in the Brazil Current region
(22°S – 28°S)**

(Corrected version)

A thesis submitted in partial fulfillment of the requirements for the degree of Master of Sciences in Oceanography, with emphasis in Physical Oceanography, *Instituto Oceanográfico, Universidade de São Paulo*.

Advisor: Prof. Dr. Ilson Carlos Almeida da Silveira.

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**UNIVERSIDADE DE SÃO PAULO
INSTITUTO OCEANOGRÁFICO**

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Evaluated in ____ / ____ / ____

Prof. Dr.

Grade

Prof. Dr.

Grade

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Grade

“When working with observations, ill-posedness is the norm, not the exception.”

Carl Wunsch

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[†]“*The verification of errors in the revised manuscript is the ultimate proof for the existence of demons.*” (Frequently attributed to Goethe, but I could not find a reliable source.)

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The main computational programs written for the calculations performed in this thesis and figures in high resolution can be obtained at ftp://ftp.io.usp.br/lado/rocha_papers or directly from the author (cesarbarbedorocha@gmail.com).

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Abstract

We use *four* current meter mooring records and quasi-synoptic hydrographic observations in conjunction with a one-dimensional quasi-geostrophic (QG) linear stability model to investigate the downstream changes in the Brazil Current (BC) System off the southeast Brazil (22°S – 28°S) as well as its implications. The dataset depicts the downstream thickening of the BC: Its vertical extension increases from 350 m at 22.7°S to 800 m at 27.9°S . Most of this deepening occurs between 25.5°S and 27.9°S , and it is likely linked to the Santos bifurcation. To the south of that bifurcation, the BC transport is increased by at least 5 Sv. Moreover, the analysis of the water column average kinetic energy (IKE) and its barotropic/baroclinic partition show that the Santos bifurcation is associated with a substantial increase in the barotropic component of the BC System: The IKE is, on average, 70 % baroclinic to the north and becomes 63 % barotropic to the south of that bifurcation. The water column average eddy kinetic energy (IEKE) and its ratio to the IKE quantitatively reveal the conspicuous mesoscale activity associated to the BC off the southeast Brazil; accordingly, the IEKE accounts for (30–60) % of the IKE. The linear stability model predicts southwestward-propagating fastest-growing waves [$\sim(180\text{--}190)$ km] within 25.5°S – 27.9°S and quasi-standing most-unstable waves (~ 230 km) at 22.7°S , roughly consistent with observations and previous work.

We also assess the ability of the QG modes and surface QG (SQG) solutions to represent the vertical structure of the sub-inertial time-varying flow in the southwestern Atlantic. At two moorings, which present a sharp near-surface decay in the vertical structure of the 1st empirical orthogonal function (EOF) of current meter time series, the SQG solutions are consistent with the data, accounting for up to 85 % of the 1st EOF variance. The SQG solutions are nonetheless indistinguishable from a *four* QG mode representation. In contrast, at a third mooring that do *not* present such sharp-decay, the vertical structure of the 1st EOF is fairly well-captured by the traditional barotropic/1st baroclinic mode combination, which accounts for 91 % of its variance. We argue that such vertical structures may be associated with the type of instability experimented by the mean flow in each region. “Charney-like” or surface-intensified “Phillips-like” instabilities may rationalize the observed SQG-like vertical structures depicted at two moorings. Mid-depth-intensified “Phillips-like” instabilities are consistent with a *two* QG mode representation at a third mooring.

Key words: Southwestern Atlantic, Brazil Current, energetics, mesoscale phenomena, baroclinic instability, QG modes, SQG solutions.

Resumo

Séries temporais correntográficas, observações hidrográficas quase-sinóticas e um modelo linear quase-geostrófico (QG) são combinados com o propósito de investigar as transformações no Sistema Corrente do Brasil (CB) ao largo da costa sudeste (22°S - 28°S) e suas implicações. O conjunto de dados revela o espessamento vertical da CB, que ocupa os 350 m superiores da coluna de água em $22,7^{\circ}\text{S}$ e atinge 800 m em $27,9^{\circ}\text{S}$. Parte significativa deste espessamento ocorre entre $25,5^{\circ}\text{S}$ e $27,9^{\circ}\text{S}$, provavelmente relacionado à Bifurcação de Santos. Ao sul desta bifurcação, o transporte da CB é pelo menos 5 Sv superior. Ademais, a análise da energia cinética média na coluna de água (ECM) e sua partição entre componentes barotrópica e baroclínica revela que a Bifurcação de Santos está associada ao aumento significativo da componente barotrópica do Sistema CB. A ECM é, em média, 70 % baroclínica ao norte da bifurcação, tornando-se 63 % barotrópica ao sul desta. A análise da energia cinética turbulenta média na coluna de água (ECTM) corrobora o importante papel da atividade de mesoescala do Sistema CB ao largo do sudeste do Brasil: A ECTM é responsável por (30–60) % da ECM. O modelo de estabilidade linear prevê ondas com maiores taxas de crescimento [$\sim(180\text{--}190)$ km] que se propagam para sudoeste entre $25,5^{\circ}\text{S}$ – $27,9^{\circ}\text{S}$. Em $22,7^{\circ}\text{S}$, as ondas mais instáveis (~ 230 km) crescem essencialmente sem propagação, consistente com as observações e também com informações presentes na literatura.

A habilidade dos modos QG e das soluções QG superficiais (QGS) em representar a variabilidade subinercial no Atlântico Sudoeste também é investigada. Em dois fundeios, a estrutura vertical da 1a função empírica ortogonal (FOE) apresenta um decaimento agudo. Este decaimento é consistente com soluções QGS, que contêm até 85 % da variância da 1a FOE. No entanto, estas soluções convergem para uma representação por *quatro* modos QG. Por outro lado, a estrutura vertical da 1a FOE em um terceiro fundeio *não* apresenta tal decaimento marcante. Consequentemente, a 1a FOE é bem representada pela tradicional combinação dos modos barotrópico/1o baroclínico. Argumentamos que estas estruturas podem estar associadas ao tipo de instabilidade experimentada pelo escoamento médio em cada região. Instabilidades tipo “Charney” ou “Phillips” (intensificadas em superfície) são consistentes com estruturas verticais tipo QGS presentes em dois fundeios. Instabilidades tipo “Phillips” (intensificadas em meia água) são consistentes com a representação por *dois* modos QG em um terceiro fundeio.

Descritores: Atlântico Sudoeste, Corrente do Brasil, energética, fenômenos de mesoescala, instabilidade baroclínica, modos QG, soluções SQG.

Chapter 1

Introduction to the thesis

1.1 On the thesis structure

This thesis is written in a manuscript style and it is composed of two major self-contained chapters (Chapters 2 and 3) in which we combine observations with simple results from the quasi-geostrophic (QG) theory to study the mean patterns and the mesoscale variability in the Brazil Current region (22°S – 28°S). In this chapter, we briefly describe the regional oceanography of the study region (Section 1.2), elaborating on past work and open questions; we also introduce the foundations of the theoretical ideas used throughout this thesis (Section 1.3). In addition, the thesis hypotheses and objectives are presented (Section 1.4). Chapter 4 provides a synthesis of the thesis results by summarizing the main findings from Chapters 2 and 3 and posing open questions to guide future work.

1.2 Regional oceanography of the southwestern Atlantic

1.2.1 The South Atlantic Subtropical Gyre and the Brazil Current

The most prominent large scale feature of the upper (~ 1000 m) subtropical South Atlantic ocean is an anticyclonic¹ gyre [*e.g.*, Talley *et al.*, 2011, Figure 1.1]. Analogous to other features in subtropical ocean basins, the South Atlantic Subtropical Gyre (SASG)

¹Anti-clockwise in the Southern Hemisphere.

is wind-driven and it is asymmetric owing to the combined effects of Earth's rotation and its sphericity in combination with the presence of continents. This asymmetry sets strong western boundary currents (WBC) as opposed to relatively weaker eastern boundary currents and nearly zonal flows connecting them.

The Brazil Current (BC) is the WBC of the SASG. In a broad sense, the BC could be thought as the Sverdrup's return transport, and therefore it is analogous to, *e.g.*, the Gulf Stream in the North Atlantic and the Kuroshio in the North Pacific. However, observations consistently depict the BC as a weak current as compared to its counterparts in the Northern Hemisphere [*e.g.*, Tomczak and Godfrey, 2001]. The uniqueness of this region may be explained in part by the interaction of the buoyancy-driven (in the upper ocean) and wind-driven circulations. Indeed, when comparing the transport of the western North Atlantic ("section 7" of his observations) with that of the South Atlantic ("section 15") on the basis of few hydrographic observations, Stommel noted that " (...) the difference in appearance between section 7 and section 15 is so remarkable as to call for further description and elaboration" [Stommel, 1965]. He gives his account:

(...) The marked difference between the function for the Gulf Stream and that for the Brazil Current must therefore be due to the different way in which the thermohaline western boundary currents enter. In the Gulf Stream the thermohaline circulation reinforces the poleward surface motion; in the Brazil Current the situation is quite contrary [Stommel, 1965]^{2,3}.

Stommel's idea to rationalize the small transport of the BC⁴ relative to the Gulf Stream is portrayed in Figure 1.2. As it will be reviewed in the next paragraphs, this tentative explanation proved to be roughly consistent with subsequent observations, at least to the

²Estimates for the wind-driven transport are fairly the same for both hemispheres, roughly 20 Sv at about 20°N/S [Stommel, 1965].

³Stommel uses the term "thermohaline circulation" in reference to that part of the circulation mainly driven by differences in surface heat and fresh-water fluxes that leads to the sinking of water in high latitudes (here termed "buoyancy-driven circulation"). Unfortunately, the term "thermohaline circulation" has been used vaguely in the recent literature [Wunsch, 2002]. In this context, we believe that "buoyancy-driven", more frequently used in modern textbooks [*e.g.*, Vallis, 2006], is a less ambiguous term.

⁴The term "Brazil Current" is regarded to the southwestern Atlantic southward flowing current in the upper ocean.

north of 25°S – there were *not* hydrographic sections in the western limb of the South Atlantic to the south of 23°S available at the time (see *Stommel* [1965]’s Figure 79).

More recent observations have shown that the subtropical western South Atlantic is indeed very different than the western North Atlantic. From a surface/near-surface perspective, the BC is formed at $\sim 15^\circ\text{S}$ as the southern branch of the South Equatorial Current (sSEC) impinges on the Brazilian continental margin and bifurcates [*e.g.*, *Stramma and England*, 1999]. In its site of origin ($\sim 15^\circ\text{S}$ - 20°S), the BC is depicted as a weak and shallow [$\sim (100\text{-}200)$ m] flow, strongly constrained by the local topography and possibly eddy-dominated [*Soutelino et al.*, 2011]. Within this latitude range, the BC only carries Tropical Water (TW) [*Stramma and England*, 1999, Figure 1.3]. Few BC transport estimates exist for this region; the geostrophic transport estimates range from ~ 2 Sv to ~ 5 Sv⁵ [*da Silveira et al.*, 2000b], likely highly inaccurate owing to the arbitrariness of the reference level choice.

This surface/near-surface perspective frequently depicted in textbooks hides an important feature of the SASG: The sSEC bifurcation is a complex-stratified (depth-dependent) feature⁶ [*Stramma and England*, 1999; *Rodrigues et al.*, 2007]. Indeed, at pycnocline levels [$\sim (200\text{-}500)$ m], this bifurcation is thought to occur at $\sim 20^\circ\text{S}$ (Figure 1.3), feeding the BC with South Atlantic Central Water (SACW), and thereby increasing its vertical extension to ~ 400 m [*e.g.*, *da Silveira et al.*, 2004]. Over the 20°S - 25°S latitude range, the geostrophic transport estimates varies from ~ 5 Sv to ~ 9 Sv [*da Silveira*, 2006].

Indeed, the region between 20°S and 25°S seems to be unique among WBCs in terms of vertical structure [*da Silveira et al.*, 2008]. Within this region, the São Paulo Plateau imposes gentler topographic gradients on the continental elevation. Consequently, the buoyancy-driven Deep Western Boundary Current (DWBC), which transports essentially North Atlantic Deep Water (NADW) to the south, is displaced to offshore portions of the continental margin [*da Silveira*, 2006]. Therefore, the DWBC⁷ and the BC seem to be

⁵Sv: volume transport unit often used in oceanography [after Harald Ulrik Sverdrup (1888 - 1957)]; $1 \text{ Sv} \equiv 10^6 \text{ m}^3 \text{ s}^{-1}$. This is the only non-SI unit used throughout this thesis.

⁶From a basin-wide perspective, there is “poleward shrinkage” [*Talley et al.*, 2011] of the SASG.

⁷There are just speculations regarding pathways of the DWBC off the southeast Brazil. Farther north, at $\sim 8^\circ\text{S}$, the DWBC breaks up into eddies [*Dengler et al.*, 2004; *Schott et al.*, 2005]. Between $\sim 19^\circ\text{S}$ and $\sim 27^\circ\text{S}$, a substantial portion of the DWBC transport seems to flow eastwards owing to potential vorticity

Figure 1.1: Schema of surface circulation of the South Atlantic. The main ocean currents are depicted in black; arrows indicate their sense. The major topographic features are depicted in light-gray. The South Atlantic Subtropical Gyre (SASG) is depicted between $\sim 10^{\circ}\text{S}$ and $\sim 40^{\circ}\text{S}$. The SASG is delimited to the north (south) by the South Equatorial Current (South Atlantic Current). The western limb of the SASG is occupied by the Brazil Current. Source: Talley *et al.* [2011] (<http://booksite.academicpress.com/DPO>).

Figure 1.2: Schematic representation for Stommel's possible explanation for the differences in transport observed in the western North and South Atlantic. Colored rectangles represent water masses flows. Adapted from *Stommel* [1965].

dynamically decoupled [*da Silveira et al.*, 2000b]. Furthermore, to the north of 25°S , the Antarctic Intermediate Water (AAIW) flows to the north/northeast as an Intermediate Western Boundary Current [IWBC, *Evans and Signorini*, 1985; *Böebel et al.*, 1999; *da Silveira et al.*, 2004]. From $\sim 20^{\circ}\text{S}$ to $\sim 25^{\circ}\text{S}$, the WBC system that flows over the slope is therefore composed by the BC, transporting TW and SACW to the south/southwest, and the underneath IWCB, transporting AAIW to the north/northeast. The BC-IWBC flow is often termed BC System [*e.g.*, *da Silveira et al.*, 2008].

At intermediate levels ($\sim 500\text{--}1000\text{ m}$), the sSEC bifurcation is thought to have its major axis at $\sim 25^{\circ}\text{S}$ [*Böebel et al.*, 1999]. Thus, within this region, the BC is fed with Antarctic Intermediate Water (AAIW) [Santos bifurcation⁸, *Böebel et al.*, 1999, Figure 1.3], and the entire upper $\sim 1000\text{ m}$ water column over the slope flows to the south/southwest [*Müller et al.*, 1998]. Just to the south of this region, geostrophic transport estimates vary substantially from $\sim 10\text{ Sv}$ to $\sim 20\text{ Sv}$ [*da Silveira et al.*, 2000b].

constraints in the presence of the Vitória-Trindade Ridge and the São Paulo Plateau [*Zazenberg and Siedler*, 1998]. In the southern limb of the southeast Brazil, the DWBC is apparently a continuous flow to the south of $\sim 28^{\circ}\text{S}$. However, velocity-derived transport estimates [*Müller et al.*, 1998] proved much smaller than those revealed by geostrophic calculations [*Zazenberg and Siedler*, 1998]. From an experimental point of view this still an open question.

⁸This bifurcation was termed the “Santos bifurcation” on the basis of float trajectory analysis: The major axis of this bifurcation is located roughly at the latitude of Santos city, Brazil ($\sim 25^{\circ}\text{S}$.)

Figure 1.3: Schema of large scale oceanic long-term mean circulation off east/southeast Brazil as inferred from observations by *Stramma and England* [1999] and *Böebel et al.* [1999]. The following water masses are depicted: Tropical Water (TW), South Atlantic Central Water (SACW), Antarctic Intermediate Water (AAIW) and North Atlantic Deep Water (NADW). In addition, the following currents are represented: Brazil Current (BC), Intermediate Western Boundary Current (IWBC), Deep Western Boundary Current (DWBC), North Brazil Under Current (NBUC) and North Brazil Current (NBC). Courtesy: R. Soutelino (IEAPM).

Nevertheless, these estimates indicate a BC transport fairly weaker than the mean transport for the Florida Current at the same latitude range [~ 30 Sv for the upper 1000 m, *Leaman et al.*, 1987]. To the south of 35°S , transport estimates for the BC reach even larger values (> 80 Sv) if one considers the BC as the total southward flow on the western boundary, that is, the wind-driven BC and buoyancy-driven DWBC [*Zemba*, 1991]; this representation of the BC is *not* frequently adopted however. While the BC transport is less than the Florida Current/Gulf Stream system, their downstream transport growth rate are similar [~ 5 percent per 100 km, *Gordon and Greengrove*, 1986]. For the BC, this transport growth is thought to be due to the combined effects of its thickening and a recirculation [*Gordon and Greengrove*, 1986; *Zemba*, 1991, Figure 1.1]. The BC flows to the south down to 36°S – 40°S where it separates from the coast upon its collision with the northward-flowing Malvinas Current [*e.g.*, *Gordon and Greengrove*, 1986; *Piola and*

Matano, 2001].

1.2.2 The mesoscale activity along the Brazil Current path

The BC develops strong mesoscale activity along its southward path, from its formation region [*Soutelino et al.*, 2011] down to the Brazil–Malvinas Confluence [*Gordon and Greengrove*, 1986], which is one of the most energetic regions of the world ocean [*Piola and Matano*, 2001]. Figure 1.4 displays the root mean square of the sea surface height based on 18 years (1993–2011) of multi-satellite altimeter data, thus representing a measure of variability of this property^{9,10}. The variability is substantial along the southeastern Brazilian continental margin. In particular, significant part of that variance seems to be accounted for by phenomena of the time scales ranging from 2 weeks¹¹ to 3 months, thus reflecting the prominence of the BC mesoscale activity.

Indeed, hydrographic observations indicate that large frontal meanders are ubiquitous features of the BC off the southeast Brazil [*e.g.*, *Mascarenhas et al.*, 1971; *Campos et al.*, 1995; *da Silveira et al.*, 2008]. These meanders may either grow in a quasi-standing fashion [*Garfield*, 1990; *da Silveira et al.*, 2008, Figure 1.5] or propagate downstream [*Garfield*, 1990; *Campos et al.*, 1995, 2000; *Fernandes et al.*, 2009]. Very few cloud-free sea surface temperature (SST) maps suggest that some of those meanders detach from the current; the eddies either propagate oceanwards or are reabsorbed by the BC main flow, eventually triggering a meandering downstream [*Garfield*, 1990] as depicted in one of the most classical SST images for the BC (Figure 1.6).

Studies addressing the BC dynamics are rare and localized. While *da Silveira et al.* [2008] and *Mano et al.* [2009] suggest that baroclinic instability is a key process in the meander growth at $\sim 23^\circ\text{S}$, *Oliveira et al.* [2009] point out that barotropic instabilities may be important along all the BC path. In fact, these processes do *not* preclude each other, and it is likely that both are important in generating the BC mesoscale variability.

⁹Although the thesis author carried out these calculations for the present work, this idea was based on *Ceccopieri* [2011].

¹⁰This variance is only consequence of phenomena with marked sea surface height expression, thus not necessarily depicting all the subinertial variability within this region.

¹¹This is the period associated with the Nyquist frequency for this multi-satellite altimeter product, which provides one map of SSH every 7 days.

Figure 1.4: Root mean square (rms) of sea surface height based on 18 years (1993–2011) of weekly multi-satellite altimeter data compiled by Collecte Localisation/Satellites Space Oceanography Division (CLS/SDO) and distributed by *Archiving, Validation and Interpretation of Satellite Oceanographic data* (AVISO; <http://www.aviso.oceanobs.com>). (a) Total signal (TS) and (b) Mesoscale (TS 3-month high-pass) signal. Based on *Ceccopieri* [2011].

Figure 1.5: Sequence of *Advanced High Resolution Radiometer* sea surface temperature (SST) images spanning 13 days and depicting a quasi-standing meander growing off Cape Frio (indicated by the black square). As the meander grows, it advects colder shelf waters oceanwards, thus becoming clearly observable to the radiometer. Offshore cold temperatures ($< 25^{\circ}\text{C}$) are likely low clouds. Courtesy: J. A. Lorenzzetti (INPE). Originally published by *da Silveira* [2006].

Figure 1.6: *Advanced High Resolution Radiometer* sea surface temperature (SST) image for July 20, 1993. The Brazil Current meandering off the southeast Brazil is clearly depicted as a sequence of crests (warm) and troughs (cold). Adapted from *Campos et al.* [1996].

1.2.3 An overlooked region

Despite its critical role in the general circulation of the ocean and global climate, the South Atlantic still one of the least studied regions of the world ocean [Garzoli and Matano, 2011; Garzoli *et al.*, 2012]. In particular, the southwestern Atlantic is badly undersampled [Campos *et al.*, 1996]. The unique vertical structure of the BC off the southeast Brazil was being described for the very first time on the basis of a single realization¹² of direct velocity measurements using the PEGASUS profiler [Evans and Signorini, 1985], while a contemporary study presented fairly reliable statistics for the Florida Current transport based on 16 sections (spanning ~ 2 years) of data from the same instrument [Leaman *et al.*, 1987]. Still, most of the current three-dimensional depiction of the southwestern Atlantic circulation is based on a rather sparse quasi-synoptic (mostly hydrographic) dataset. Consequently, primitive-equation numerical models (*e.g.*, global ocean state estimates, “operational” regional ocean models, etc) are poorly constrained in that region [Garzoli and Matano, 2011].

Although some recent progress on the understanding of the BC dynamics has been achieved on the basis of short-term velocity records [da Silveira *et al.*, 2008], drifters [Oliveira *et al.*, 2009] and process-oriented primitive-equation numerical simulations [Mano *et al.*, 2009; Calado *et al.*, 2008], many basic questions are still to be answered. For instance: (i) *How does the BC transform off the southeast Brazil?* and (ii) *What is the ability of simplified QG models of vertical structure to account for the variability in the BC region?* These questions are addressed in the present thesis. A systematic answer to (i) can provide a better description of the BC off the southeast Brazil and robust statistics to assess the performance of primitive-equation numerical models, whereas an answer to (ii) can shed light on the current debate of the use of those QG models to interpret and project surface information to depth, which has potential applications particularly for this undersampled region [*e.g.*, assimilation of surface fields into numerical models based on dynamical arguments, LaCasce and Mahadevan, 2006].

¹²This section was occupied three times over a 10-day period [Evans and Signorini, 1985]. However, from the standpoint of mesoscale phenomena, which may dominate the variability in this region, these three sections are *not* statistically independent. Therefore, it seems reasonable to assume that this dataset represent a single realization.

1.3 Theoretical background

1.3.1 Quasi-geostrophic approximation

The QG system is a well-known simplified set of equations used to model small Rossby number motions in the ocean and atmosphere [*e.g.*, Charney and Flierl, 1981]. Although it is perhaps the most common simplified set of equations used in theoretical studies [Vallis, 2006], it is also used in applications. For instance, the QG system was employed by Jule Charney, Ragnar Fjortoft and John von Neumann (among others) to produce the first consistent numerical weather prediction in 1950's [Lindzen *et al.*, 1990]. QG models have also been used to rationalize and synthesize observations in the ocean [*e.g.*, Szuts *et al.*, 2012].

The derivation of the QG system invokes four major simplifications [*e.g.*, Pedlosky, 1987; Vallis, 2006], and therefore assumes four scaling hypotheses, namely:

1. *The β -plane approximation*, which requires that the study domain is *not* very large, so that the variations of the Coriolis parameter are small. Specifically, this requires that $|\beta\mathcal{L}| \ll |f_0|$, where f_0 is the mean Coriolis parameter¹³ and β its meridional gradient; $\mathcal{L}$ ¹⁴ is length scale of the motion;¹⁵
2. *The time scale is advective*. Specifically, $\mathcal{T} = \mathcal{L}/\mathcal{U}$, where $\mathcal{T}$ and $\mathcal{U}$ stand for the characteristic time and velocity scales of the motion, respectively;
3. *The Rossby number is small*, so that the flow is close to the geostrophic balance. Specifically, this requires that $Ro \equiv \mathcal{U}/(|f_0|\mathcal{L}) \ll 1$; and
4. *The motion has a length scale not significantly larger than the deformation scale* ($L_d \equiv \mathcal{N}\mathcal{H}/\mathcal{L}$, where $\mathcal{N}$ is the characteristic stratification frequency, and $\mathcal{H}$ is the vertical (length) scale of the motion). Specifically, this requires that $Ro(\mathcal{L}/L_d)^2 = \mathcal{O}(Ro)$. For a continuously stratified fluid, this is equivalent to require that the

¹³The Coriolis parameter is defined as $f = f_0 + \beta y$

¹⁴Here, capital script letters stand for characteristic scales of the motion.

¹⁵For convenience one generally requires $|\beta\mathcal{L}|/|f_0| = \mathcal{O}(Ro)$.

variations of the stratification are $O(Ro)$ compared to the background stratification [Vallis, 2006]. Also, some texts present this assumption as $Bu \equiv (Ld/\mathcal{L})^2 = \mathcal{O}(1)$.

Using these scaling hypotheses in the primitive equations, and further combining the $\mathcal{O}(1)$ (geostrophic balance) and $\mathcal{O}(Ro)$ terms¹⁶, we obtain a scalar conservation statement [e.g., Vallis, 2006]

$$\frac{D_g}{Dt}q = 0, \quad (1.1)$$

where $D_g/Dt = \partial/\partial t + J(\psi, \cdot)$ is the derivative following the geostrophic flow $(u_g, v_g) = (-\partial\psi/\partial y, \partial\psi/\partial x)$, with J the horizontal Jacobian and ψ the geostrophic stream-function; q is the quasi-geostrophic potential vorticity

$$q = \beta y + \nabla^2 \psi + \frac{\partial}{\partial z} \left(\frac{f_0^2}{N^2(z)} \frac{\partial \psi}{\partial z} \right), \quad (1.2)$$

composed by the variations of the planetary vorticity (βy), relative vorticity ($\nabla^2 \psi$) and vortex-stretching $\left[\frac{\partial}{\partial z} \left(\frac{f_0^2}{N^2(z)} \frac{\partial \psi}{\partial z} \right) \right]$ terms. In the QG framework, $\nabla \equiv \frac{\partial}{\partial x} \hat{x} + \frac{\partial}{\partial y} \hat{y}$ stands for the horizontal gradient operator, where $(\hat{x}, \hat{y})$ are the unit vectors in the horizontal (x,y) directions, namely, zonal and meridional directions in a cartesian β -plane¹⁷. In addition, the evolution of q in the interior (Eqs. 1.1 and 1.2) is generally coupled¹⁸ with that of the buoyancy at the boundaries. By applying the same four scaling arguments to the thermodynamics equation [Vallis, 2006], we obtain

$$\frac{D_g}{Dt}b = 0, \quad \text{at } z = -H, 0, \quad (1.3)$$

which is a conservation statement for the buoyancy ($b \equiv -g\rho/\rho_0$, where ρ stands for density variations and ρ_0 is a reference density).

The elegance of the QG system comes from the fact that it is composed by only these two scalar conservation statements (Eqs. 1.1 and 1.3). Indeed, this represents a tremen-

¹⁶For a judicious mathematical treatment see Pedlosky [1987].

¹⁷In the early literature [e.g., Bretherton, 1966], q is also called pseudo potential vorticity because it represents a linearization of the Ertel potential vorticity using the scaling arguments presented above.

¹⁸In fact, this gives the boundary conditions to inverting 1.2 for ψ . Therefore one can also pose the problem as the evolution of q subject to boundary conditions given by the buoyancy equation.

dous simplification of the primitive equations. By assuming those four scaling hypotheses, relatively fast evolving high Ro motions (*e.g.*, internal gravity waves) are filtered out. Moreover, in the QG system, all other dynamical variables can be ultimately recovered from q [Vallis, 2006], namely:

- The stream-function by inverting Eq.1.2; that is, given q at time t , we invert the Elliptic operator in Eq.1.2 to obtain ψ ;
- The velocity ($\vec{u}_g$) by using the geostrophic relation $\vec{u}_g = \hat{z} \times \nabla\psi$, where $\hat{z}$ is the unit vector in the vertical direction;
- The pressure anomalies from the stream-function definition $p = \rho_0 f_0 \psi$; and
- The buoyancy anomalies from the hydrostatic equation $b = f_0 \partial\psi / \partial z$.

In the ocean, the QG equations seem roughly suitable for motions ranging from 10 km to 500 km [Charney and Flierl, 1981], thus spanning the scales of the mesoscale phenomena.

1.3.2 Projection onto vertical QG modes

In order to invert q for obtaining ψ (Eq. 1.2), it is common to project the solutions onto vertical QG modes. The QG modes arise from the separation of variables of the governing equations [*e.g.*, Pedlosky, 1987]. Consider the linearized form of Eqs. 1.1 and 1.2 for a mean motionless ocean

$$\beta \frac{\partial \psi}{\partial x} + \frac{\partial}{\partial t} \left[\nabla^2 \psi + \frac{\partial}{\partial z} \left(\frac{f_0^2}{N^2(z)} \frac{\partial \psi}{\partial z} \right) \right] = 0, \quad \text{at } -H < z < 0. \quad (1.4)$$

Assuming separable solutions in the form

$$\psi(x, y, z, t) = \Psi(x, y, t) \phi(z), \quad (1.5)$$

where $\Psi(x, y)$ ¹⁹ and $\phi(z)$ are the horizontal and vertical functions, and using Eq. 1.5, we

¹⁹One generally expands $\Psi(x, y, t)$ in Fourier modes [*e.g.*, Ferrari and Wunsch, 2010]; however this function can be much more general. As this thesis focus solely on the vertical structure (F), we keep Ψ as a generic function in space and time.

can reduce the partial differential Eq. 1.4 to one ordinary differential equation and one partial differential equation:

$$-\frac{1}{(\partial\Psi/\partial t)}\left(\beta\frac{\partial\Psi}{\partial x} + \frac{\partial\nabla^2\Psi}{\partial t}\right) = \frac{1}{\phi}\frac{d}{dz}\left(\frac{f_0^2}{N^2(z)}\frac{d\phi}{dz}\right) = -\lambda^2, \quad (1.6)$$

where $-\lambda^2$ is the separation constant. The vertical structure equation follows directly:

$$\frac{\partial}{\partial z}\left(\frac{f_0^2}{N^2(z)}\frac{\partial\phi}{\partial z}\right) + \lambda^2\phi = 0. \quad (1.7)$$

For the boundary conditions, one generally assumes rigid lids at the surface and the bottom (no vertical motion) [*e.g.*, *Pedlosky, 1987*]. This boundary condition is easily written in terms of ϕ by inserting Eq. 1.5²⁰ in Eq. 1.3 (and using the fact that in the QG system $b = f_0\partial\psi/\partial z$):

$$\frac{\partial\phi}{\partial z} = 0, \quad \text{at } z = 0, -H. \quad (1.8)$$

Eqs. 1.7 and 1.8 together form an eigenvalue problem of the Sturm-Liouville type. From the Sturm-Liouville theory, the solutions are an infinite set of eigenfunctions ϕ_j , with $j = \{0, 1, 2, \dots\}$, associated to its respective eigenvalues λ_j^2 [*Samelson, 2011*]. Multiplying Eq. 1.7 by ϕ and integrating (by parts) over the domain $[-H, 0]$, we obtain [*Samelson, 2011*]

$$\lambda^2 = \frac{\int_{-H}^0 \phi^2 dz}{\int_{-H}^0 \frac{f_0^2}{N^2(z)} \left(\frac{d\phi}{dz}\right)^2 dz}. \quad (1.9)$$

The integrands on the right hand side (rhs) are both positive. The corollary is that the eigenvalues λ_j^2 are all positive. Another neat result is obtained by writing Eq. 1.7 for an arbitrary j and multiplying it by a different eigenfunction (hereafter vertical QG modes), namely ϕ_i ($i \neq j$). After integrating twice (by parts), we obtain

$$(\lambda_i^{-2} - \lambda_j^{-2}) \int_{-H}^0 \phi_i \phi_j dz = 0. \quad (1.10)$$

²⁰Linearized to be consistent with this derivation.

As $\lambda_i^2 \neq \lambda_j^2$, it follows that the QG modes are orthogonal to each other.

Finally, it is convenient to normalize the QG modes, so that

$$\int_{-H}^0 \phi_i^2 dz = 1. \quad (1.11)$$

In summary, the QG modes represent eigenfunctions of the vortex-stretching operator²¹, and therefore are functionals of the stratification [Flierl, 1978]. Furthermore, the QG modes are infinite and orthogonal, spanning the subspace for solutions of ϕ . In other words, they constitute the best basis for the subspace of solutions of ϕ . Indeed, the orthogonality of the QG modes is a general result, and even solutions for the nonlinear Eqs. 1.1 and 1.2 are frequently projected onto these eigenmodes as a first approximation [e.g., Flierl, 1978; Ferrari and Wunsch, 2010]. These eigenmodes have also been used to consistently analyze and synthesize observations [e.g., Kundu et al., 1975; Wunsch, 1997; da Silveira et al., 2000a, 2008; Szuts et al., 2012].

1.3.3 Baroclinic instability

Baroclinic instability is the major mechanism of energy conversion from the mean flow to the perturbations in the ocean and atmosphere [e.g., Pedlosky, 1987; Vallis, 2006]. It gives rise to the ocean eddies (e.g., *Gulf Stream* rings), and its analogous in the atmosphere which are responsible for the weather [Charney and Flierl, 1981]. Therefore, baroclinic instability may be the geophysical phenomena that affect us most [Vallis, 2006].

Through baroclinic instability, perturbations grow in the expenses of the available potential energy (APE) associated to the mean horizontal buoyancy (related to the vertical shear through the thermal wind) of the mean currents [e.g., McWilliams, 2006]. Therefore, this mechanism tends to bring the potential density surfaces (or potential temperature surfaces in the atmosphere) back to the minimum state of potential energy, namely that of plane surfaces. The complete development of the instabilities is nonlinear, and therefore analytically untreatable [Pedlosky, 1987]. To overcome this difficulty, one frequently

²¹Similar modes are obtained outside of the QG theory, e.g., for the long-wave limit of the internal waves [Gill, 1982]

study the stability of the mean currents (under the QG approximation) to small-amplitude perturbations, and thereby linearizes the problem. Indeed, this assumption has proved successful to reveal the basic physics of this process and provides consistent predictions, at least for the initial stages of the perturbation growth [Pedlosky, 1987]. Classical examples are the early analytical models by *Eady* [1949] and *Charney* [1947].

More recently, linear QG baroclinic instability models have proved to provide consistent predictions for the length and time scales of the mesoscale variability in the ocean [e.g., *Smith*, 2007; *Tulloch et al.*, 2011]. On a regional scale, *Johns* [1988] showed that a 1-dimensional linear QG model successfully predicts the basic characteristics of the meandering in the Gulf Stream. *da Silveira et al.* [2008] further applied similar ideas to the BC meanders at 22.7°S, and showed that these meanders grow likely through baroclinic instability.

1.3.4 Simplified models of vertical structure

The traditional rigid lid boundary condition used in deriving the QG modes (Eq. 1.8) implicitly ignores buoyancy anomalies at the boundaries. This can be easily shown by combining Eqs. 1.6 and 1.8:

$$\frac{d\phi}{dz} = \frac{b}{\Psi f_0}. \quad (1.12)$$

In contrast, other QG paradigm can be posed to match the surface buoyancy (and to typically neglect the interior q). This simplified model gives rise to the so-called surface quasi-geostrophic solutions [SQG, *Held et al.*, 1995], which are exponential-like surface intensified modes (details in Chapter 3). Recently, the importance of the SQG solutions as opposed to the QG modes, and their ability to represent the actual oceanic flow have been intensely debated [*Lapeyre and Klein*, 2006; *LaCasce and Mahadevan*, 2006; *Le Traon et al.*, 2008; *Lapeyre*, 2009; *Ferrari and Wunsch*, 2010; *Xu and Fu*, 2011; *LaCasce*, 2012; *Xu and Fu*, 2012; *Scott and Furnival*, 2012, among others].

The appeal of the SQG solutions as a simplified model is that its vertical and horizontal structures are intrinsically related. In this context, it has been argued that the SQG frame-

work could be used in applications such as dynamically assimilating remotely-sensed fields into numerical models [*e.g.*, *LaCasce and Mahadevan*, 2006]. However, QG modes and SQG solutions are *not* orthogonal to each other [*e.g.*, *Ferrari and Wunsch*, 2010], and its likely that one contains the contribution of the other.

1.4 Thesis hypotheses and objectives

In Chapter 2, we test the hypothesis that *the vertical thickening of the Brazil Current observed off the southeast Brazil is accompanied by changes in the barotropic/baroclinic kinetic energy partition and on the properties of linear baroclinic instability*. In particular, Chapter 2's objectives are:

- To produce a first-order statistics of the BC System velocity field;
- To examine the BC System mean and synoptic vertical structures, in particular (i) the BC vertical extension (southward flow), (ii) the BC transport and (iii) the BC System vertical shear;
- To obtain estimations for the BC System water column average kinetic energy and eddy kinetic energy, as well as their partition between barotropic and baroclinic components; and
- To evaluate the BC System linear stability properties, in particular those associated with geophysical instabilities of vertical shear.

Current meter records, hydrographic (synoptic and climatological) observations in conjunction with a QG linear stability model are analyzed to achieve these objectives.

In Chapter 3, we investigate further the mesoscale variability in the Brazil Current region, particularly focusing on QG models of vertical structure. Specifically, we test the hypothesis that *the surface quasi-geostrophic solutions can account for the vertical structure of the time-dependent flow, but are nonetheless indistinguishable from a higher order QG mode combination*. Chapter 3's objectives are:

- To produce a concise representation of the vertical structure of the mesoscale variability in the BC region;
- To evaluate the ability of the QG modes and SQG solutions to account for the vertical structure of the mesoscale variability in the BC region; and
- To assess the similarities and differences between SQG solutions and QG modes vertical structures.

To fulfill these objectives, we test theoretical ideas against mooring observations.

Chapter 2

On the Brazil Current System at 22°S-28°S: Vertical structure, energetics and baroclinic instability

Abstract

We use *four* current meter mooring records and quasi-synoptic hydrographic observations in conjunction with a linear quasi-geostrophic (QG) stability model to investigate the changes in the vertical structure of the Brazil Current (BC) System off the southeast Brazil (22°S-28°S) as well as its implications. The mooring data depicts the downstream thickening of the BC; accordingly, the BC is 350 m deep at the MARLIM mooring (22.7°S), 400 m deep at the DFBS mooring (24.15°S), 550 m deep at the C3 mooring (25.5°S), and 800 m deep at the W333 mooring (27.9°S). Hydrographic observations suggest that the maximum increase in transport (5 Sv) occurs between the C3 and the W333 moorings. On average, the water column average kinetic energy (IKE) is highly baroclinic ($\sim 70\%$) at the MARLIM and C3 moorings, and it is substantially barotropic ($\sim 63\%$) at the W333 mooring, thus corroborating that the Santos bifurcation is associated with a strong increasing in the BC System barotropic component. At the DFBS mooring a virtual equipartition of the IKE is observed, likely owing to the dominance of the eddy field at that location. Relatively high levels of water column average eddy kinetic energy (IEKE) are observed; the IEKE/IKE ratio is 0.50, 0.60, 0.33, 0.42 at the MARLIM, DFBS, C3 and W333 moorings, respectively. These estimates confirm (and rather quantify) the substantial role of the BC mesoscale activity off the southeast Brazil. At the *four* moorings, the kinetic energy level and its partition between barotropic and baroclinic components in a given day departs significantly from the time-mean; the total flow can change from $\sim 90\%$ baroclinic to $\sim 90\%$ barotropic in few days. Typically, bursts of energy in the barotropic mode are associated with eddy events. While long-lived eddy events at the MARLIM and DFBS moorings are likely linked to quasi-standing meander growth, relatively short-lived eddy events at the C3 and W333 moorings may be related to meander propagation. A one-dimensional QG linear stability model proved successful in predicting some of

the observed characteristics. In particular, the model predicts quasi-standing waves at the MARLIM mooring and southwestward-propagating waves at the C3 and W333 moorings. The present estimates could be a test bed for numerical models.

Key words: Brazil Current, energetics, Santos bifurcation, baroclinic instability.

2.1 Introduction

The Brazil Current (BC) is the subtropical western boundary current of the South Atlantic. It is relatively poorly studied when compared to its North Atlantic counterpart, the *Gulf Stream*. From the experimental point of view, most of our understanding of the BC rely on quasi-synoptic (mostly hydrographic) observations [*e.g.*, Campos *et al.*, 1995; da Silveira *et al.*, 2004]. Studies based on analysis of current meter time series [Müller *et al.*, 1998; da Silveira *et al.*, 2008] and drifters [Oliveira *et al.*, 2009] are relatively rare, and this has hampered the quantitative description of the BC and the understanding of its dynamics.

The BC presents a singular vertical structure as it flows over the slope off the south-east Brazil from $\sim 20^{\circ}\text{S}$ to $\sim 25^{\circ}\text{S}$. In this region, the BC is depicted as a shallow current (~ 400 m), transporting Tropical Water (TW) at surface/near-surface levels and South Atlantic Central Water (SACW) at pycnocline levels [Campos *et al.*, 1995; da Silveira *et al.*, 2000b]. Underneath the BC [$\sim (500\text{--}1200)$ m] there is an opposing flow, the Intermediate Western Boundary Current (IWBC), transporting mainly Antarctic Intermediate Water (AAIW) to the north/northeast [*e.g.*, Evans and Signorini, 1985; da Silveira *et al.*, 2004]. This unique subtropical western boundary current system is linked to the complex-stratified subtropical gyre bifurcation system over the Brazilian continental margin [*e.g.*, Stramma and England, 1999]. At intermediate levels, the so-called Santos bifurcation [Böebel *et al.*, 1999], is thought to occur somewhere between 25°S and 27°S [Böebel *et al.*, 1999; Legeais *et al.*, 2013]. To the south of this region, at $\sim 28^{\circ}\text{S}$, all the water column over the slope flows to the south/southwest [Müller *et al.*, 1998], and therefore the BC transports TW, SACW and AAIW.

Few studies have addressed the BC dynamics. In a strenuous effort to extract dynamic information from a rather sparse dataset, da Silveira *et al.* [2008] showed that

the mean BC System¹ at 22.7°S is essentially a 1st baroclinic mode flow. Furthermore, these authors pointed out that the vertical shear associated with the BC/IWBC opposing flows likely prone the BC System to baroclinic instability, and that a 1-dimensional quasi-geostrophic (QG) model was successful in predicting the time and length scales of the mesoscale variability in this region. In particular, the QG linear model predicted unstable modes with very small propagation speeds, in agreement with few observed events in sea surface temperature (SST) imagery [Garfield, 1990]. Based on primitive-equation numerical simulations of the BC System in the same region, *Mano et al.* [2009] described the energy flux during a cyclonic meandering process off Cape Frio (~23°S). While the basic phenomenology depicted by these authors seem to be consistent with *da Silveira et al.* [2008]'s QG baroclinic instability arguments, the full meandering development seems to be much more complicated. Moreover, *Oliveira et al.* [2009] studied the surface energetics of the BC from surface drifter buoys. These authors showed that the (surface) eddy kinetic energy is significant along all the BC path, and that part of the BC variability may be accounted for by barotropic instabilities.

It is relatively well accepted that the BC experiments changes on its downstream path off the southeast Brazil [Müller *et al.*, 1998; *da Silveira et al.*, 2000b]. Nonetheless, no systematic quantifications of these transformations is available to date. In addition, the BC is generally associated with a rich mesoscale activity within this region [*e.g.*, *Campos et al.*, 1995, 2000; *da Silveira et al.*, 2008], but estimations of its relative importance are limited to the surface [*Oliveira et al.*, 2009].

Here we investigate the BC downstream transformations as it flows off the southeast Brazil (22°S-28°S)². The question to be addressed is therefore *How does the BC transform off the southeast Brazil?* The specific questions are: (1a) *How much does the BC thickens?* and (1b) *How much does the BC transport increase within this region?*; (2a) *What is the*

¹The BC System is defined as the western boundary current system over the slope off the southeast Brazil. From ~22°S to ~25°S it is composed by the BC and the IWCB. To the south of this region, it is composed solely by the BC. While this denomination was first introduced in the English-written scientific literature by *da Silveira et al.* [2008], it has been used in a couple of texts in Portuguese before [Godoi, 2005; *da Silveira*, 2006].

²This region encompasses the southern portion of the so-called *Abrolhos-Campos Region* and the whole South Brazil Bight [Castro and Miranda, 1998], spanning the Campos Basin (~21°S–~23°S) through the Santos Basin (~24°S–~28°S).

partition between barotropic and baroclinic components in the BC System? and (2b) *What are the downstream changes on this partition?*; (3) *How much of the BC System energy is in the eddy field?*; (4) *What are the downstream changes on the BC System linear instability properties?* To address these questions we analyze *four* current meter mooring records, *not* necessarily spanning the same time period, but nonetheless covering the study region in an unprecedented along-coast resolution. These moorings, when analyzed under the same framework, allow us to have a comprehensive description of the BC off the southeast Brazil as well as to have a glance on the impacts of the BC transformations on its dynamics.

2.2 Mooring observations

2.2.1 The dataset

The *four* current meter moorings are located in the BC domain (Figure 2.2.1) spanning the southeast Brazil (22°S-28°S), namely the MARLIM (22.7°S, 1250 m) mooring, the DEPROAS FBS (24.15°, 1200 m) mooring (hereafter DFBS), the COROAS 3 (25.5°, 1000 m) mooring (hereafter C3) and the WOCE ACM12 333 (27.9°S, 1200 m) mooring (hereafter W333). These records have lengths ranging from 1 to 2 years, thus the BC mesoscale variability is expected to be well resolved. Table 2.1 displays the geographic position, levels of instruments and time range for each mooring. The original reference (in case they were already used or quoted in the literature) is also listed.

Figure 2.1: The dataset spanning the southeast Brazil continental slope. Black dots indicate positions for MARLIM (22.7°S, 40.2°W, 1250 m), C3 (25.5°S, 44.9°W, 1000 m) and W333 (27.9°S, 46.7°W, 1200 m) moorings. Gray triangles represent the hydrographic stations near each mooring.

Table 2.1: Main characteristics for MARLIM, DFBS, C3 and W333 moorings. θ is BC thermal front angle estimated from *da Silveira et al.* [2008]’s and *Garfield* [1990]’s mean front positions. Positive angles ($\theta > 0$) are measured clockwise from the geographical north. $\partial\eta_b/\partial x$ represents the cross-front topographic gradient estimated from ETOPO2 (<http://www.ngdc.noaa.gov/mgg/global/etopo2.html>). Note that *da Silveira et al.* [2008] only used the second half of the MARLIM record, and *Campos et al.* [1996] only quoted the C3 mooring results. (We acknowledge that fractions of the C3 record were analyzed and presented in a couple texts in portuguese [*Moreira*, 1998; *Souza*, 2000].)

	Isobath (m)	θ	$\partial\eta_b/\partial x$	Instruments (m)	Record (days)	Reference
MARLIM (22.7°S, 40.2°W)	1250	37°	-1.8×10^{-2}	50; 100; 250; 350; 450; 650; 750; 950; 1050	308 Feb. 1992 - Dec. 1992	<i>da Silveira et al.</i> [2008]
DFBS (24.15°S, 42.4°W)	1200	75°	-1.1×10^{-2}	30; 50; 70; 500; 800; 1000;	482 Jan. 2003 - May 2004	—
C3 (25.5°S, 44.9°W)	1000	30°	-1.2×10^{-2}	29; 91; 293; 698	455 Dec. 1992 - Mar. 1994	<i>Campos et al.</i> [1996]
W333 (27.9°S, 46.7°W)	1200	13°	-0.8×10^{-2}	60; 77; 95; 112; 138; 155; 173; 230; 475; 680; 885	693 Jan. 1991 - Nov. 1992	<i>Müller et al.</i> [1998]

The velocity record at each current meter was low-pass filtered using a *Lanczos-cosine* filter [e.g., *Emery and Thomson*, 2001]. The cut-off period was set 40 h, which is 30 % greater than the inertial period at the northernmost mooring (MARLIM), and therefore near- and super-inertial motions were filtered out. Individual current meter gaps were filled using the empirical orthogonal functions (EOF) of the velocity anomaly time series [*Beckers and Rixen*, 2003]. This method was chosen because it does not change the series statistics (see Appendix A for details). A similar method was employed by *Dengler et al.* [2004] and *Schott et al.* [2005] with the purpose of filling gaps in current meter moorings off the northeast Brazil.

The C3 mooring presented a gap in all four instruments (04 May 1993 – 18 June 1993), which was filled in two steps. The uppermost instrument velocity series was filled using the EOF method (horizontally) with the uppermost instrument of the adjacent mooring (C2) which has a coherent spectrum to the C3 mooring uppermost record. The remaining three instruments gaps were then filled using the EOFs (vertically). Such twofold filling process was *not* possible for the 30-day gap (27 May 1992 – 23 June 1992) at the MARLIM mooring since *no* concurrent adjacent mooring were available.

The filtered velocity time series for the MARLIM mooring is presented in Figure 2.2a. The BC flow is clearly depicted in the upper 300 m, although strongly current inversions are observed at these levels. As near- and super-inertial motions were filtered out, it is likely that these inversions are due to strong BC meandering events [*da Silveira et al.*, 2008]. In contrast, the IWBC is consistently depicted from (650-1050) m flowing to the northeast. No current inversions are observed in the IWBC domain, suggesting that the meandering is mainly confined to the upper levels. Indeed, this pattern is evident in the velocity anomaly time series (Figure 2.2b).

The time series for the DFBS mooring depicts a much more convoluted scenario (Figure 2.3a), particularly in the BC domain. The three uppermost instruments (30, 50 and 70 m) present strong current inversions throughout the record. At these depths, the time series for the velocity anomalies (Figure 2.3b) are not (visually) very different from that of the total flow, suggesting vigorous BC eddy activity at this location, likely associated

to the BC adjustment as it overshoots the continental margin off Cape Frio ($\sim 23^\circ\text{S}$) as hypothesized by *Campos et al.* [1995]. At intermediate levels (800 m and 1000 m instruments) this mooring depicts the IWBC flowing to the northeast. However, it should be noted that the IWBC seems more variable at this location than at the MARLIM mooring. The flow at 500 m presents reversions throughout the record, suggesting the zero-crossing associated with the BC-IWBC inversion occurs near this depth.

At the C3 mooring (Figure 2.4a), the BC is depicted in the three upper instruments (29, 51, and 293 m) as a persistent southwest flow. Although the BC is also highly variable at this location (Figure 2.4b), just one strong current inversion is observed (Mar. 1993). The C3 mooring has just one instrument at intermediate depths (693 m). The time series for this instrument depicts a northeastward flow, suggesting that the IWBC is already well-formed at this location as quoted by *Campos et al.* [1996]. However, it is important to note that, contrary to what is observed at the MARLIM mooring, the northeast flow at the C3 mooring is clearly highly variable.

Farther downstream, at the W333 mooring (Figure 2.5a) the BC is depicted flowing southward/southwestward from the surface down to the instrument at 670 m depth. At this location, just two prominent velocity vector inversions are observed in the BC (Jan. 1991 and Oct. 1992). In addition, the flow at the deepermost instrument (885 m) is highly variable and presents no preferential direction. Thus, there is no evidence of the IWBC, suggesting that this mooring is located to the south of the Santos bifurcation, consistent with recent results on the basis of float observations [*Legeais et al.*, 2013]. In addition, the vertical shear seems relatively small in the upper 250 m at this location when compared to that at the MARLIM and C3 moorings.

Figure 2.2: MARLIM mooring velocity time series: (a) total flow and (b) anomalies. For clarity, only one vector per day is plotted. For reference, the north points upwards.

Figure 2.3: DFBS mooring velocity time series: (a) total flow and (b) anomalies. For clarity, the velocity record at 50 m is omitted and only one vector per day is plotted. For reference, the north points upwards.

Figure 2.4: C3 mooring velocity time series: (a) total flow and (b) anomalies. For clarity, only one vector per day is plotted. For reference, the north points upwards.

Figure 2.5: W333 mooring velocity time series: (a) total flow and (b) anomalies. For clarity, the velocity records at 77 m, 112 m and 155 m are omitted from this plot and only one vector per day is plotted. For reference, the north points upwards.

2.2.2 Velocity basic statistics

We anticipate that the time-mean velocity vector is approximately parallel to the mean BC thermal front, which denotes the BC inshore border [da Silveira *et al.*, 2008; Garfield, 1990] at each mooring (Figure 2.7). Hence, it is convenient to rotate the velocity vector for a local along-front/cross-front co-ordinate system³. This facilitates the inter-comparison amongst the moorings.

The along-front direction was computed based on the angle between the geographic north and BC mean thermal front (Table 2.1), which was computed using Garfield [1990]’s and da Silveira *et al.* [2008]’s estimates for the mean BC front position; at the DFBS mooring, we used the mean thermal front from the latter author. It should be noted that as the rotation is a linear operation, the total variance is preserved [Emery and Thomson, 2001]. Hereafter, u and v will be used to refer to the local cross- and along-front (low-pass filtered) velocity components, respectively.

For each mooring, basic statistics (mean and standard deviation) for the velocity components were estimated. In order to assess the robustness of these estimations, the standard errors were calculated following a Student’s test [*e.g.*, Emery and Thomson, 2001]. Accordingly, for a 95 % confidence interval we have the time-mean estimate

$$[\bar{u} \pm 2E_{\bar{u}}, \bar{v} \pm 2E_{\bar{v}}] = \left[\bar{u} \pm \frac{2\sqrt{\overline{u'^2}}}{\sqrt{N_{eu}}}, \bar{v} \pm \frac{2\sqrt{\overline{v'^2}}}{\sqrt{N_{ev}}} \right], \quad (2.1)$$

and the standard deviation estimate

$$[std_u \pm 2E_{std_u}, std_v \pm 2E_{std_v}] = \left[std_u \pm \frac{2\sqrt{\overline{u'^2}}}{\sqrt{2N_{eu}}}, std_v \pm \frac{2\sqrt{\overline{v'^2}}}{\sqrt{2N_{ev}}} \right], \quad (2.2)$$

where the overbar indicates averages and prime indicates the anomaly about the time-mean; N_{eu} and N_{ev} are the number of independent samples, here taken as the effective number of degrees of freedom (eDOF), defined as [Glover *et al.*, 2011]

$$[N_{eu}, N_{ev}] = \left[\frac{N\Delta t}{Ti_u}, \frac{N\Delta t}{Ti_v} \right], \quad (2.3)$$

³This represents a “quasi-natural” coordinate system.

where N is the total number of samples, Δt is the sample interval, and Ti_u and Ti_v are the integral time scales in the cross- and along-front direction, respectively. The integral time scales were estimated as [Stammer, 1997]

$$[Ti_u, Ti_v] = \left[\frac{1}{C_u^2(0)} \sum_{i=1}^N C_u^2(i) \Delta t, \frac{1}{C_v^2(0)} \sum_{i=1}^N C_v^2(i) \Delta t \right], \quad (2.4)$$

where C_u and C_v are the autocorrelation functions for the cross- and along-front velocity components, respectively, which are computed individually for each current meter at each mooring. Figure 2.2.2 depicts a typical autocorrelation functions for the along- (a) and cross-front (b) velocity components. The time scales are rather anisotropic, which leads to significant differences in the eDOF in each direction. The along-front time scales varies from 6 days to 9 days, whereas the cross-front time scale varies from 4 days to 5 days. Moreover, eDOF varies substantially (~ 30 -170) from mooring to mooring due to differences in the length of their records. While the highest eDOF is found for the cross-front velocity component at the W333 mooring (~ 170), the smallest eDOF are found for the along-front velocity component at the DFBS mooring (~ 30) and at the MARLIM mooring (~ 40).

Mean and standard deviation along with their uncertainties are presented in Tables 2.2, 2.3, 2.4 and 2.5, and the mean velocity vectors for the instruments closest to 50 m and 700 m are displayed in Figure 2.7. As anticipated, the mean flow essentially follows the mean BC thermal front, which is parallel to the local isobath at all moorings but at the DFBS mooring (Figure 2.7). Near this mooring, the mean BC front veers eastward accompanying the 200 m isobath; the deeper isobaths veering is more gentle, so that the IWBC flow at this location follows the local isobath, which is not in the same direction of the BC front. (Basic statistics for the non-rotated velocity series are tabulated in Appendix B, with the hope that these estimates could serve as a reference for future work.)

Near-surface BC along-front mean velocity is highest at the C3 mooring ($-0.51 \pm 0.05 \text{ m s}^{-1}$ at 29 m), as compared to the W333 mooring ($-0.34 \pm 0.04 \text{ m s}^{-1}$ at 50 m) and MARLIM mooring ($-0.31 \pm 0.12 \text{ m s}^{-1}$ at 50 m). This suggests that the C3 mooring is closer to the BC core than the other moorings. Furthermore, at the DFBS mooring, the

Figure 2.6: Autocorrelation function for (a) along-front and (b) cross-front velocity components for the 138 m (nominal depth) at the W333 mooring.

mean along-front BC velocity is much weaker ($-0.17 \pm 0.14 \text{ m s}^{-1}$ at 50 m). High near-surface along-front standard deviation is observed at the DFBS mooring ($0.39 \pm 0.10 \text{ m s}^{-1}$ at 50 m) and at the MARLIM mooring ($0.37 \pm 0.03 \text{ m s}^{-1}$ at 50 m), exceeding the mean values at the respective positions, thus confirming the high variability at these locations. Relatively high near-surface along-front standard deviation values are also observed at the C3 mooring ($0.24 \pm 0.04 \text{ m s}^{-1}$ at 29m) and at the W333 mooring ($0.23 \pm 0.03 \text{ m s}^{-1}$ at 60 m). Note that the high uncertainties associated with the mean estimates at the near-surface instruments at the DFBS and MARLIM moorings are consequence of the small eDOF and the high standard deviation at those locations. In particular, the mean values for the upper 3 instruments at the DFBS mooring have very low statistical significance. There are too few eDOF to estimate a reliable along-front mean. Physically, the BC experiments high variability in this region, as if continuously adjusting to conserve its potential vorticity after overshooting the continental margin off Cape Frio ($\sim 23^\circ\text{S}$) [Campos *et al.*, 1995]. Therefore, it is very difficult to establish a “stationary” flow at this location.

The BC cross-front mean velocity component is very weak at all moorings/instruments, with values smaller than 0.07 m s^{-1} (see Tables 2.2, 2.3, 2.4 and 2.5). On the other hand, relatively high cross-front standard deviation values are observed at the MARLIM mooring ($0.25 \pm 0.06 \text{ m s}^{-1}$ at 50 m), at the DFBS mooring ($0.17 \pm 0.03 \text{ m s}^{-1}$ at 50 m), at the C3 mooring ($0.12 \pm 0.02 \text{ m s}^{-1}$ at 29 m) and at the W333 mooring ($0.17 \pm 0.02 \text{ m s}^{-1}$ at 60 m). These standard deviation values are significant, although slightest smaller than those for the along-front component, suggesting that the time-varying flow is less anisotropic than the mean flow.

At intermediate depths, the MARLIM and DFBS moorings consistently depict a mean IWBC flowing northward/eastward. Maximum IWBC along-front velocity is observed in the north of the domain, at MARLIM mooring ($0.22 \pm 0.03 \text{ m s}^{-1}$ at 950 m). At the DFBS mooring, maximum along-front IWBC velocity is observed at 800 m ($0.11 \pm 0.02 \text{ m s}^{-1}$). At the C3 mooring, the single instrument at intermediate depths depicts a relatively weak (but significant) northward/eastward flow ($0.09 \pm 0.03 \text{ m s}^{-1}$ at 698 m), suggesting that the IWBC is relatively well formed at this location as quoted by *Campos et al.* [1996]. The along-front IWBC velocity component seems to present relatively small (as compared to the mean) standard deviation values at the MARLIM mooring ($0.08 \pm 0.02 \text{ m s}^{-1}$ at 950 m), but slightly greater at the DFBS mooring ($0.08 \pm 0.02 \text{ m s}^{-1}$ at 800 m). At the C3 mooring, the along-front IWBC standard deviation ($0.13 \pm 0.02 \text{ m s}^{-1}$ at 698 m) exceeds the mean value.

In contrast, a mean southward/westward along-front flow is observed at intermediate depths at the W333 mooring ($0.06 \pm 0.02 \text{ m s}^{-1}$ at 680 m), in agreement with *Müller et al.* [1998] for the same mooring. No mean northward/eastward flow is observed at this location, suggesting that the Santos bifurcation [*Böebel et al.*, 1999] occurs somewhere between 25.5°S and 27.9°S , in agreement with recent float observations [*Legeais et al.*, 2013]. Relatively high along-front standard deviation values are observed at intermediate depths at this mooring, exceeding the mean values ($0.10 \pm 0.01 \text{ m s}^{-1}$ at 680 m).

In summary, the basic statistics depict a highly variable BC off the southeast Brazil, thus corroborating the importance of its mesoscale activity within this region. The mean

along-front velocity measurements also depict the thickening of the BC, namely its downstream vertical extension growth. In order to better describe this thickening process, we now turn to the projection of the mean along-front velocity profile onto the dynamical modes and its synthesis.

Figure 2.7: Mean velocity vector for instruments closest to 50 m (red) and 700 m (blue) at each mooring. The mean thermal front, which denotes the inner edge of the Brazil Current, estimated by *Garfield* [1990] (diamonds) and *da Silveira et al.* [2008] (circles) is also shown.

Table 2.2: Velocity statistics for MARLIM mooring. The 95 % confidence interval was estimated following a Student's test. Stars indicate those estimates that are not statistically significant.

Instrument [m]	Along-front		Cross-front	
	Mean [m s ⁻¹]	Std [m s ⁻¹]	Mean [m s ⁻¹]	Std [m s ⁻¹]
50	-0.31 ± 0.12	0.37 ± 0.08	0.07 ± 0.08*	0.25 ± 0.06
100	-0.26 ± 0.11	0.36 ± 0.08	0.08 ± 0.07	0.23 ± 0.05
250	-0.08 ± 0.06	0.20 ± 0.04	0.08 ± 0.04	0.13 ± 0.03
350	0.02 ± 0.05*	0.16 ± 0.03	0.05 ± 0.03	0.09 ± 0.02
450	0.08 ± 0.03	0.10 ± 0.02	0.07 ± 0.03	0.09 ± 0.02
650	0.19 ± 0.02	0.07 ± 0.02	0.00 ± 0.01*	0.05 ± 0.01
750	0.19 ± 0.02	0.07 ± 0.01	0.03 ± 0.02	0.06 ± 0.01
950	0.22 ± 0.03	0.08 ± 0.02	0.03 ± 0.01	0.04 ± 0.01
1050	0.21 ± 0.03	0.10 ± 0.02	0.02 ± 0.01	0.03 ± 0.01

Table 2.3: Velocity statistics for DFBS mooring. The 95 % confidence interval was estimated following a Student's test. Stars indicate those estimates that are not statistically significant.

Instrument [m]	Along-front		Cross-front	
	Mean [m s ⁻¹]	Std [m s ⁻¹]	Mean [m s ⁻¹]	Std [m s ⁻¹]
30	-0.17 ± 0.14	0.38 ± 0.10	0.02 ± 0.04*	0.17 ± 0.03
50	-0.17 ± 0.14	0.39 ± 0.10	0.02 ± 0.04*	0.18 ± 0.03
70	-0.16 ± 0.14	0.40 ± 0.10	0.02 ± 0.04*	0.19 ± 0.03
500	0.04 ± 0.04*	0.12 ± 0.03	0.04 ± 0.01	0.06 ± 0.01
800	0.11 ± 0.03	0.08 ± 0.02	0.07 ± 0.01	0.04 ± 0.01
1000	0.09 ± 0.03	0.09 ± 0.02	0.05 ± 0.01	0.04 ± 0.01

Table 2.4: Velocity statistics for C3 mooring. The 95 % confidence interval was estimated following a Student's test. Stars indicate those estimates that are not statistically significant.

Instrument [m]	Along-front		Cross-front	
	Mean [m s ⁻¹]	Std [m s ⁻¹]	Mean [m s ⁻¹]	Std [m s ⁻¹]
29	-0.51 ± 0.05	0.24 ± 0.04	-0.01 ± 0.03*	0.12 ± 0.02
91	-0.49 ± 0.06	0.24 ± 0.04	-0.05 ± 0.03	0.12 ± 0.02
293	-0.23 ± 0.04	0.17 ± 0.03	-0.03 ± 0.01	0.06 ± 0.01
698	0.09 ± 0.03	0.13 ± 0.02	0.00 ± 0.01*	0.03 ± 0.01

Table 2.5: Velocity statistics for W333 mooring. The 95 % confidence interval was estimated following a Student's test. Stars indicate those estimates that are not statistically significant.

Instrument [m]	Along-front		Cross-front	
	Mean [m s ⁻¹]	Std [m s ⁻¹]	Mean [m s ⁻¹]	Std [m s ⁻¹]
60	-0.34 ± 0.04	0.23 ± 0.03	-0.07 ± 0.03	0.17 ± 0.02
77	-0.34 ± 0.05	0.23 ± 0.03	-0.06 ± 0.04	0.17 ± 0.03
95	-0.34 ± 0.05	0.23 ± 0.03	-0.07 ± 0.04	0.17 ± 0.03
112	-0.34 ± 0.05	0.23 ± 0.03	-0.07 ± 0.03	0.17 ± 0.02
138	-0.33 ± 0.05	0.23 ± 0.03	-0.06 ± 0.03	0.16 ± 0.02
155	-0.32 ± 0.05	0.22 ± 0.03	-0.06 ± 0.03	0.15 ± 0.02
173	-0.31 ± 0.05	0.21 ± 0.03	-0.06 ± 0.03	0.14 ± 0.02
230	-0.29 ± 0.05	0.20 ± 0.03	-0.06 ± 0.03	0.13 ± 0.02
475	-0.15 ± 0.03	0.12 ± 0.02	-0.04 ± 0.01	0.07 ± 0.01
680	-0.06 ± 0.02	0.10 ± 0.01	-0.02 ± 0.01	0.06 ± 0.01
885	0.00 ± 0.02*	0.11 ± 0.01	-0.02 ± 0.01	0.05 ± 0.01

2.2.3 The thickening of the Brazil Current

Linear dynamical modes

The linear flat-bottom dynamical modes are relatively well-known [*e.g.*, *Pedlosky*, 1987]. They are an infinite discrete set of solutions to the governing vertical structure equation. The modes were computed by solving numerically the eigenvalue problem (see Appendix D for details)

$$\frac{d}{dz} \left(\frac{f_0^2}{N^2} \frac{d\phi_j}{dz} \right) + \lambda_j^2 \phi_j = 0, \quad (2.5)$$

$$\frac{d\phi_j}{dz} = 0, \text{ at } z = 0, \quad z = -H, \quad (2.6)$$

where ϕ_j are the dynamical modes and λ_j^2 their respective eigenvalues (inverse of the deformation radii squared); and f_0^2 and $N^2(z)$ stand for the inertial and stratification frequencies squared, respectively. $N^2(z)$ profiles were computed using climatological World Ocean Atlas 2009 temperature [*Locarnini et al.*, 2010] and salinity [*Antonov et al.*, 2010] profiles at each mooring region; these profiles were interpolated to a regular vertical grid (10 m) and smoothed following *da Silveira et al.* [2000a]. The resulting $N^2(z)$ profile and its associated dynamical modes for the DFBS mooring are displayed in Figure 2.2.3.

Figure 2.8: (a) Climatological stratification (a) and five gravest dynamical modes (b) for DFBS mooring. The barotropic mode is depth-invariant and the number of zero-crossings is equivalent to the order of the respective baroclinic mode.

As the climatological stratification is very similar at all moorings, the QG modes for the other locations (not shown) are also very similar.

As the dynamical modes are orthogonal, they constitute the best basis for the subspace of solutions. That is, the solutions expanded in terms of these modes generally converge rapidly. Indeed, these modes have been successfully used to synthesize observations [*e.g.*, Kundu *et al.*, 1975; da Silveira *et al.*, 2008; Szuts *et al.*, 2012]. The basic idea is to expand the observations in terms of the dynamical modes [*e.g.*, Szuts *et al.*, 2012]. We write, in matrix notation,

$$[\mathbf{U}, \mathbf{V}] = \mathbf{F}[a_U, a_V] + [R_U, R_V], \quad (2.7)$$

where $[\mathbf{U}, \mathbf{V}]$ are the current meter time-mean velocity at L discrete depths; $\mathbf{F}$ is a $L \times M$ matrix, representing a finite number (M) of dynamical modes at L discrete depths; $[a_U, a_V]$ are the M mode amplitudes ($N \times 1$ vector); and $[R_U, R_V]$ are residuals at L depths. We solve for $[a_U, a_V]$ given the velocity observations. In particular, we compute the ampli-

tudes through a Gauss-Markov estimate⁴ [e.g., Szuts et al., 2012],

$$[\hat{a}_U, \hat{a}_V] = (E_0 \mathbf{F}^T)(\mathbf{F} E_0 \mathbf{F}^T + \sigma^2 \mathbf{I})^{-1}[\mathbf{U}, \mathbf{V}], \quad (2.8)$$

where $E_0 = \text{diag}(E_f / \sum E_f)$ is the *a priori* energy partition (diagonal) $M \times M$ matrix, where E_f is the *a priori* energy fraction in the 5 gravest modes. σ is the $L \times L$ uncertainty of $[\mathbf{U}, \mathbf{V}]$ at each depth. We chose to use 5 gravest modes (i.e, $N = 5$, the barotropic mode and 4 gravest baroclinic modes). Following Szuts et al. [2012], we set $E_f = \{1 \ 1 \ 1 \ 1 \ 1\}^T$.

Mean flow dynamical fit

The estimated mode amplitudes $[\hat{a}_U, \hat{a}_V]$ are then used to obtain a velocity profile at the same (vertical) grid of $N^2(z)$

$$[\mathbf{U}_i, \mathbf{V}_i] = \mathbf{F}_N [\hat{a}_U, \hat{a}_V], \quad (2.9)$$

where $\mathbf{F}_L$ is a $N \times 5$ matrix, representing the 5 gravest dynamical modes at N discrete depths. As the dynamical modes and their derivatives are smooth [da Silveira et al., 2008], we argue that this is a consistent method for obtaining interpolated velocity profiles on a regular grid. These profiles are further used along with the $N^2(z)$ profile as inputs for the linear stability model (Section 2.5.1).

The dynamically interpolated along-front velocity profiles together clearly shows the thickening of the BC (Figure 2.2.3). Accordingly, the BC is 350 m deep at the MARLIM mooring, 550 m deep at the C3 mooring and 850 m deep at the W333 mooring. Although the mean along-front velocity presents low statistical significance at the DFBS mooring, the BC vertical extension seems to be consistent at this location; the BC is 400 m deep at this location.

Another interesting feature that is clear in the dynamically synthesized velocity profiles concerns the downstream changes in mean flow vertical shear. Accordingly, the BC System presents a much more prominent shear at the MARLIM, DFBS and C3 moorings

⁴This is different from what is done in Chapter 3 because here the linear system (Eq. 2.7) is undetermined at the C3 mooring, that is $L < M$.

than at the W333 mooring, thus suggesting that the Santos bifurcation is associated with a strong increasing in the BC System barotropic component.

2.3 Quasi-synoptic patterns

In order to present a two-dimensional characterization of the downstream changes in the BC, we analyze the available hydrographic sections contemporary to the mooring records (Table 2.6). Note that the W333 does not present velocity measurements exactly concurrent to the A10 cruise. While this cruise recovered the W333 mooring, the last month (December 1992) of the mooring measurements were disregarded during the quality control process. Note that the mooring velocity compared to the geostrophic velocity estimates at this location represent those measurements made one month earlier (30 November 1992), and hence this comparison should be interpreted with caution.

The cross-section geostrophic velocities are computed via dynamic method. Two types of referencing for those computations are employed. For the MARLIM, DFBS and C3 moorings, we consider the level of no-motion associated to the BC-IWBC; this depth is estimated from the mooring velocity profile synthesized using the dynamical modes in a similar fashion as the mean flow. As the IWBC is *not* present in the W333 mooring, we use the velocity of the deepermost instrument as a reference. In both cases, the average for the time of the cruise occupation is taken as representative of the mooring observations. In order to estimate the geostrophic velocity at depths shallower than the reference level (outer shelf and slope), we employed the *Reid and Mantyla* [1978]'s method, which consists in extrapolating the geopotential anomaly according to its gradient. Basically, the isopycnals slopes are extrapolated shoreward based on the adjacent isopycnals slopes multiplied by a constant factor, here taken as 0.5; in other words, a given extrapolated isopycnal is considered to have a slope half as steep it has oceanwards. For MARLIM P2, COROAS HM2 and WOCE A10, no extrapolation was performed for depths shallower than 200 m (roughly the shelf-break); and for DEPROAS V, no extrapolation was carried out at all since the BC was fairly off-shore during that cruise. The BC geostrophic transport was estimated considering this current as the southward flow delimited by the 0.05

Figure 2.9: Time (ensemble) mean of the along-front velocity [m s⁻¹] for three moorings: MARLIM (22.7°S; red); DFBS (24.15°S; magenta); C3 (25.5°S; green) and W333 (27.9°S; blue). Filled circles indicate the current meter mean values; continuous lines represent a "dynamical fit" to the discrete values.

Table 2.6: Hydrographic cruises contemporary to mooring records. The geostrophic BC transport was estimated considering the flow delimited by the -0.05 m s^{-1} isotach.

Cruise	Mooring	Occupation date	Reference level	BC transport
MARLIM P2	MARLIM	17-20 Nov. 1992	330 dbar*	5.4 Sv
DEPROAS V	DFBS	27-29 Sep. 2003	380 dbar*	5.2 Sv
COROAS HM2	C3	23-25 Jul. 1993	410 dbar*	6.8 Sv
WOCE A10	W333	30-31 Dec. 1992	885 m [†]	11.8 Sv

*Level of no-motion based on the (cruise average) synthesized mooring velocity; [†] Based on the 885 m (nominal depth) cross-section mooring velocity.

m s^{-1} isotach, which was the weakest contour that well defined the BC in all sections. Therefore, these transport values represent conservative estimates.

The MARLIM P2 cruise depicts a shallow BC ($\sim 330 \text{ m}$), with maximum (surface) velocity of $\sim 0.5 \text{ m s}^{-1}$ (Figure 2.10a). The BC is slightly skewed shoreward, although it seems to be flowing essentially over the slope. Underneath the BC, a northward flow was not entirely sampled because the CTD⁵ casts stopped at 1000 m in this cruise. Nonetheless, the IWBC seems to be a consistent northward flow, with maximum geostrophic velocity of 0.2 m s^{-1} at its core ($\sim 800 \text{ m}$). The estimated geostrophic transport is 5.4 Sv, greater than that computed by *da Silveira et al.* [2008] on the basis of an application of a sectional version of the Princeton Ocean Model for the same cruise (4.24 Sv). The estimated geostrophic velocity is (at best) satisfactory (Figure 2.10b). The geostrophic IWBC seems fairly consistent with that measured at the MARLIM mooring. However, the geostrophic velocity overestimates the near-surface moored velocity magnitude and vertical shear. The depth-integrated RMS difference is 0.77.

The geostrophic velocity section for the DEPROAS V cruise depicts a relatively shallow BC ($\sim 400 \text{ m}$), with maximum surface velocity of $\sim 0.7 \text{ m s}^{-1}$ (Figure 2.11a). Underneath the BC, the IWCB is depicted extending deeper than $\sim 1200 \text{ m}$, with maximum velocity of $\sim 0.3 \text{ m s}^{-1}$ ($\sim 1000 \text{ m}$). During this cruise, a frontal cyclonic meander is observed, which apparently is confined to the BC domain. The BC geostrophic transport is estimated as 5.2 Sv, consistent with previous estimates close to this location [*da Silveira et al.*, 2004]. Moreover, the estimated geostrophic velocity seems fairly consistent

⁵Conductive/Temperature/Depth profiler.

Figure 2.10: (a) Geostrophic velocity for MARLIM P2 and (b) comparison of geostrophic velocity and mooring velocity (time-mean for the period of the cruise). The horizontal black line indicates the level of no-motion, which was matched to the synthesized mooring velocity. Black triangles represent the hydrographic stations. White triangles indicate the positions used in the geostrophic velocity calculation. The green triangle represents the point (along the section) closest to the MARLIM mooring. Gray dots indicate the current meter positions along the water column.

with the (cruise average) mooring velocity (Figure 2.11b). The RMS difference between the synthesized mooring velocity and the geostrophic velocity is 0.33. The major differences consist of an overestimation of the surface velocities and an underestimation of the velocities close to the IWBC core (~ 1000 m).

The HM2 cruise depicts a 410 m deep BC flowing southwards, with maximum (surface) geostrophic velocity of $\sim 0.8 \text{ m s}^{-1}$ (Figure 2.12a). The IWBC is depicted as a northward flow with maximum velocity of $\sim 0.17 \text{ m s}^{-1}$ at its core (~ 1000 m), thus weaker than that at the DEPROAS V cruise. At this location, contrary to what is observed upstream, the BC core does not seem aligned with the IWBC core, with the latter displaced offshore. (The C3 mooring is located very close to the BC core. This is consistent with the fact the C3 mooring presented highest velocity as compared to the other moorings *only* if this quasi-synoptic depiction is representative of the mean BC at that location.) For this cruise, the BC geostrophic transport is 6.8 Sv, *not* inconsistent with the estimates by Campos *et al.* [1995], although those authors considered a much deeper level of no-motion (7.3 Sv relative to 750 dbar). However, we acknowledge that this may be a lower bound estimate since part of the BC seems to be flowing onto the outer shelf.

Figure 2.11: (a) Geostrophic velocity for DEPROAS V and (b) comparison of geostrophic velocity and mooring velocity (time-mean for the period of the cruise). The horizontal black line indicates the level of no-motion, which was matched to the synthesized mooring velocity. Black triangles represent the hydrographic stations. White triangles indicate the positions used in the geostrophic velocity calculation. The green triangle represents the point (along the section) closest to the DFBS mooring. Gray dots indicate the current meter positions along the water column.

The agreement between the geostrophic velocity and that measured by the C3 mooring is satisfactory, particularly in representing the vertical shear (Figure 2.12b). However, the geostrophic velocity significantly overestimates the moored velocity near the surface and underestimates it close to the IWBC core. The RMS difference between those velocity profiles is 0.54.

Farther downstream, the WOCE A10 cruise depicts a significantly different scenario, namely a ~ 900 m deep BC, with maximum (surface) velocity of $\sim 0.55 \text{ m s}^{-1}$ (Figure 2.13a). The BC is also significantly skewed shoreward at this location. Consistent to what is observed by the W333 mooring, no significant northward flow is observed underneath the BC. At this location, the BC geostrophic transport is estimated as 11.8 Sv. As for the COROAS HM2 cruise, this is likely a lower bound on the BC transport estimate since substantial flow seems to occur onto the outer shelf. The few estimates of transport within this region range from 11.4 Sv to 27 Sv [*da Silveira et al.*, 2000b], depending on the reference level adopted (typically much deeper than that considered here). Direct velocity measurements point to a BC transporting 18 Sv just to the south of this region [$\sim 31^\circ\text{S}$; *Garfield*, 1990]. The geostrophic velocity is fairly representative of the current meter velocity, with the former slightly underestimating the latter near the surface (Figure

Figure 2.12: (a) Geostrophic velocity for COROAS HM2 and (b) comparison of geostrophic velocity and mooring velocity (time-mean for the period of the cruise). The horizontal black line indicates the level of no-motion, which was matched to the synthesized mooring velocity. Black triangles represent the hydrographic stations. White triangles indicate the positions used in the geostrophic velocity calculation. The green triangle represents the point (along the section) closest to the C3 mooring. Gray dots indicate the current meter positions along the water column.

2.13b). The RMS difference between those two velocity profiles is 0.25.

The description of the time-mean along-front flow (Section 2.2.3) and the quasi-synoptic patterns (Section 2.3) suggests that the BC presents a marked downstream thickening. In order to systematically quantify the BC System downstream changes, we now turn to the analysis of the water column average kinetic energy and its partition between barotropic and baroclinic components. We also use the energetics analysis to estimate the importance of the mesoscale activity in the BC System.

Figure 2.13: (a) Geostrophic velocity for A10 and (b) comparison of geostrophic velocity and mooring velocity (time-mean for the period of the cruise). The horizontal black line indicates the level of “known-motion”, which consists in the deepermost mooring instrument depth. Black triangles represent the hydrographic stations. White triangles indicate the positions used in the geostrophic velocity calculation. The green triangle represents the point (along the section) closest to the W333 mooring. Gray dots indicate the current meter positions along the water column.

2.4 Energetics

2.4.1 Kinetic energy estimate

Water column average kinetic energy

An estimate of the water column average kinetic energy per unit mass⁶ is given by [*e.g.* Wunsch, 1997]

$$KE(t) = \frac{1}{2H} \int_{-H}^0 [u^2(t, z) + v^2(t, z)] dz \approx \frac{1}{2H} \int_{-H}^0 ([\hat{a}_{ui}(t)\phi_i(z)]^2 + [\hat{a}_{vi}(t)\phi_i(z)]^2) dz, \quad (2.10)$$

where ϕ_i is the i th dynamical mode, and $[\hat{a}_{ui}, \hat{a}_{vi}]$ its associated amplitude in the $[u, v]$. Similarly to what was done for the mean along-front flow (Section 2.2.3), these mode amplitudes are estimated by projecting the instantaneous (discrete) velocity profiles onto the dynamical modes, thus producing a series of amplitudes. Note that the summation convention is assumed on the right hand side (rhs) of Eq. 2.10. Expanding the rhs, and

⁶Although we will not continue to mention it, kinetic energy in this text *always* should be read as “kinetic energy per unit mass”.

using the orthonormality condition,

$$\int_{-H}^0 \phi_i \phi_j dz = \delta_{ij}, \quad (2.11)$$

where δ_{ij} is the Dirac delta, we obtain

$$KE = \frac{1}{2H} [\hat{a}_{ui}^2(t) + \hat{a}_{vi}^2(t)]. \quad (2.12)$$

Eq. 2.12 allows us to estimate the partition of KE amongst the dynamical modes. In particular, we are interested in the barotropic/baroclinic⁷ partition; we define

$$\%KE_{bt} = \frac{KE_0}{KE} \times 100\%, \quad (2.13)$$

and

$$\%KE_{bc} = \frac{\sum_{j=1}^4 KE_j}{KE} \times 100\%. \quad (2.14)$$

Similar calculations are performed for the velocity anomalies $([u', v'])$.

Surface kinetic energy

The surface kinetic energy is estimated by computing Eq. 2.10 at the surface

$$KE_{surf} \approx \frac{1}{2H} ([\hat{a}_{ui}(t)Fi(0)]^2 + [\hat{a}_{vi}(t)Fi(0)]^2) dz, \quad (2.15)$$

which is an extrapolation of the KE to the surface.

In this case, the KE is only separable (amongst the dynamical modes) if the modes are not correlated on time [Wunsch, 1997]. This seems not be the case here. Nonetheless, we keep the total KE_{surf} in order to compare against those estimates from drifters buoys by Oliveira *et al.* [2009].

⁷Here, the term baroclinic energy stands for the sum of the energy associated with the four gravest baroclinic modes.

2.4.2 KE time average

First, we explore the time-mean values of depth mean total kinetic energy (IKE) and depth mean eddy kinetic energy (IEKE) and their partition between barotropic and baroclinic components. On average, the C3 mooring presents the highest IKE level ($0.045 \text{ m}^2 \text{ s}^{-2}$)⁸, followed by the MARLIM mooring ($0.038 \text{ m}^2 \text{ s}^{-2}$), the DFBS mooring ($0.037 \text{ m}^2 \text{ s}^{-2}$) and the W333 mooring ($0.032 \text{ m}^2 \text{ s}^{-2}$). However, as stated in Section 2.2.2, this may be influenced by the relative distance of the moorings to the BC core; caution should be taken when comparing these results in absolute values.

An interesting result is the magnitude of the IEKE and its ratio to the IKE. On average, at the MARLIM mooring, the IEKE is estimated as $0.019 \text{ m}^2 \text{ s}^{-2}$, half as big as the IKE. High levels of IEKE are observed at the DFBS mooring ($0.022 \text{ m}^2 \text{ s}^{-2}$), and its ratio to the IKE is ~ 0.6 . The C3 mooring presents a relatively weaker (but substantial) eddy field ($0.014 \text{ m}^2 \text{ s}^{-2}$; its ratio to the IKE is ~ 0.33). Farther south, at the W333 mooring, similar level of IEKE is found ($0.014 \text{ m}^2 \text{ s}^{-2}$), though its ratio to the IKE is 0.44. Although these IEKE levels are generally less than those observed in the Gulf Stream and the Kuroshio regions [*e.g.*, Wunsch, 1997], their relative importance to the IKE is a remarkable feature of the BC off the southeast Brazil. Thus, these results confirm (and rather quantify) the importance of the eddy field in the BC path off southeast Brazil, consistent with previous studies based on quasi-synoptic observations [*e.g.*, Campos *et al.*, 1995], mooring data [da Silveira *et al.*, 2008], surface drifters [Oliveira *et al.*, 2009] and SST imagery analysis [Lorenzetti *et al.*, 2009]. In particular, these estimates point to the dominance of the eddy field at the DFBS mooring.

As the only available kinetic energy estimates for the BC derive from drifters [Oliveira *et al.*, 2009], it is interesting to present estimates for the mean surface kinetic energy (SKE) and the mean surface eddy kinetic energy (SEKE). As expected, the highest SKE at the C3 mooring, where the mean SKE is $0.163 \text{ m}^2 \text{ s}^{-2}$; the SEKE is $0.034 \text{ m}^2 \text{ s}^{-2}$; the SEKE/SKE ratio is ~ 0.20 . At the MARLIM mooring, the mean SKE is 0.1500 m^2

⁸Sadly, the recent literature has been keeping the kinetic energy units in the old-fashioned CGS system (*i.e.*, $\text{cm}^2 \text{ s}^{-2}$; in order to easily compare with past work, they claim). We believe it is time to change, and present our estimates in the SI system ($\text{m}^2 \text{ s}^{-2}$) – the transformation of units is trivial anyway.

Figure 2.14: Time series of water-column average kinetic energy at the MARLIM mooring (22.7°S). (a) Total kinetic energy (KE). (b) Eddy kinetic energy (EKE). The fractions of energy (KE and EKE) in barotropic (bt) and baroclinic (bc) components are presented in magenta and yellow, respectively.

Figure 2.15: Time series of water-column average kinetic energy at the DFBS mooring (24.15°S). (a) Total kinetic energy (KE). (b) Eddy kinetic energy (EKE). The fractions of energy (KE and EKE) in barotropic (bt) and baroclinic (bc) components are presented in magenta and yellow, respectively.

Figure 2.16: Time series of water-column average kinetic energy at the C3 mooring (25.5°S). (a) Total kinetic energy (KE). (b) Eddy kinetic energy (EKE). The fractions of energy (KE and EKE) in barotropic (bt) and baroclinic (bc) components are presented in magenta and yellow, respectively.

Figure 2.17: Time series of water-column average kinetic energy at the W333 mooring (27.9°S). (a) Total kinetic energy (KE). (b) Eddy kinetic energy (EKE). The fractions of energy (KE and EKE) in barotropic (bt) and baroclinic (bc) components are presented in magenta and yellow, respectively.

s^{-2} ; in this case, the SEKE is substantial ($0.0960 \text{ m}^2 \text{ s}^{-2}$); its ratio to the SKE is ~ 0.64 . At the DFBS mooring, high levels of SKE are observed ($0.113 \text{ m}^2 \text{ s}^{-2}$), which is essentially in the eddy field ($0.111 \text{ m}^2 \text{ s}^{-2}$), thus producing a SEKE/SKE ratio of 0.98. The weakest level of SKE is found at the W333 mooring ($0.100 \text{ m}^2 \text{ s}^{-2}$); the SEKE is $0.040 \text{ m}^2 \text{ s}^{-2}$; the SEKE/SKE ratio is ~ 0.40 . Interestingly, at the DFBS and, to some extent, at the MARLIM mooring, the eddy kinetic energy represents a greater fraction of the total kinetic energy at the surface, consistent with the fact that the anomalies are very surface-intensified at these locations [*da Silveira et al.*, 2008, this work].

It is difficult to compare absolute values of these estimates against those of *Oliveira et al.* [2009]; while we estimate the SKE and SEKE in a single position, those authors have taken averages of drifters-derived velocity over $0.5^\circ \times 0.5^\circ$ bins. Nonetheless, our results are in general agreement with those of the cited authors, specifically in presenting high levels of SEKE at $\sim 22^\circ\text{-}23^\circ\text{S}$ (their transects IV and V, close to DFBS and MARLIM moorings, respectively) and relatively less SEKE levels at $\sim 25^\circ\text{S}$ and $\sim 28^\circ\text{S}$ (their transects VI and VII, close to C3 and W333 moorings, respectively).

2.4.3 Barotropic/baroclinic partition

At the MARLIM mooring, on average, the IKE is 70 % baroclinic (mainly 1st mode); the IEKE is slightly more barotropic (40 %). Based on synoptic direct velocity measurements, *da Silveira et al.* [2004] estimate that the BC is (75-80) % baroclinic, while *da Silveira et al.* [2008], using 152 days of the MARLIM series, indicate that the along-front BC is 98% baroclinic. We emphasize that our barotropic/baroclinic estimates are based on the IKE; as the cross-front component is mainly due to the eddy field, which is more barotropic, the partition will be more barotropic than the estimates based only on the along-isobath velocity component. Furthermore, the discrepancies with *da Silveira et al.* [2008] estimates are probably because we appended the early 150 days of the MARLIM record; as will be discussed in Section 2.4.4, the beginning of MARLIM series is marked by a strong mesoscale activity. At the DFBS mooring, on average, a virtual equipartition between barotropic and baroclinic energy for the total flow is observed, likely owing to

the dominance of the eddy field. Indeed, the kinetic energy partition is almost the same ($\sim 56\%$ barotropic) for the velocity anomaly field.

At C3 mooring, on average, the total flow presents a partition similar to that for the MARLIM mooring ($\sim 69\%$ baroclinic). However, its eddy field seems much more barotropic ($\sim 62\%$). In contrast, at the W333, a dramatic change in the IKE partition is observed; the total flow becomes $\sim 54\%$ barotropic, confirming our qualitatively estimate based on the mean along-front velocity profiles (Section 2.2.3). In this case, on average, the eddy field is just slightly more barotropic ($\sim 64\%$). While the relatively high barotropicity of the total flow at the DFBS mooring is likely due to the dominance of the eddy field over the mean flow [Oliveira *et al.*, 2009, this work], this phenomenon seems to be a manifestation of the high barotropicity of the mean flow itself at the W333 mooring (Figure 2.2.3).

2.4.4 Duration of eddy events

The results presented in Sections 2.4.2 and 2.4.3 are rather statistical estimates. However, the kinetic energy level in a given day departs significantly from the time-mean. At the MARLIM mooring, strong energetic events that occur at the beginning of the series (April-May 1992) do *not* repeat at the end. These events are associated with inversions in the surface velocity vector in the BC domain (see Figures 2.2). At the DFBS, C3 and W333 moorings (Figures 2.15, 2.16 and 2.17), strong energetic events seem to occur throughout the series; in that sense, the series seems more statistically more homogeneous at these moorings. Generally, bursts of energy in the barotropic mode (at all moorings) are associated with energetic eddy events.

At the MARLIM mooring, one such event lasts for ~ 37 days (March - April, 1992). Two similar long-lived barotropic energy events are observed at the DFBS mooring, lasting ~ 28 days (April - May, 2003) and ~ 32 days (August - September 2003). Indeed these events, are marked by strong surface current vector inversions (Figures 2.14 and 2.15) and they are likely associated with a quasi-standing meander growth events [da Silveira *et al.*, 2008]. In particular, the event depicted at the MARLIM mooring contrasts with the late

120 days of the record, when the total flow is essentially baroclinic and low levels of IEKE are observed. Hence, at the MARLIM mooring, there seems to be two regimes: (1) a “mean regime” (*not* perturbed), characterized by relatively low IEKE levels, and mainly baroclinic total flow; and (2) a “meandering regime” (perturbed), marked by high IEKE levels, and therefore strongly barotropic total flow. In (1), the BC is well defined and prone to baroclinic instability [da Silveira *et al.*, 2008; Mano *et al.*, 2009]; (2) may be linked to the cyclonic meander growth processes [da Silveira *et al.*, 2008; Mano *et al.*, 2009]. The transition from (2) to (1) may be related to the BC (re)stabilization [Mano *et al.*, 2009].

Although a similar “meandering regime” is observed at the DFBS mooring, a “mean regime” is *not* observed at this location, thus reflecting the vigorous BC eddy activity just to the of Cape Frio ($\sim 23^\circ\text{S}$) [*e.g.*, Campos *et al.*, 1995]. Indeed this fact was depicted in the drifter analysis by Oliveira *et al.* [2009], which presented a *not* statistically significant mean along-shore current at this location. It is also consistent with the BC thermal front analysis by Lorenzetti *et al.* [2009], which showed that the BC front presents a spread-out frontal density⁹ just to the south of Cape Frio (their Figures 1, 2 and 3), thus reflecting the high variability of the BC at this location.

At the C3 and W333 moorings these events are relatively short-lived, lasting for 20 days at maximum (typically ~ 10 days). These events may be related to the meander propagation between $\sim 24^\circ\text{S}$ and $\sim 28^\circ\text{S}$ as suggested by primitive-equation numerical simulations [Campos *et al.*, 2000; Fernandes *et al.*, 2009].

In summary, the energetic analysis points to the intense eddy activity of the BC along its path off the southeast Brazil. While the total BC flow is significantly baroclinic at the MARLIM mooring, it becomes substantially barotropic at the W333 mooring, with an intermediate scenario observed at the C3 mooring. At the DFBS mooring, the dominance of the eddy field leads to an equipartition of the kinetic energy between barotropic and baroclinic components. In order to investigate possible dynamical effects of the downstream changes in the BC, we now turn to the analysis of linear stability of the mean along-front

⁹As explained by the authors, high frontal density location indicates high probability of finding the front there.

flow.

2.5 Analysis of baroclinic instability

2.5.1 Model formulation and simplification

A one-dimensional QG model is employed to study the stability of the mean flow to small perturbations on an along-front f -plane channel. The model is similar to that used by *da Silveira et al.* [2008], who built up on *Johns* [1988]. Tests were made with a more generalized version of the model, which considers both meridional and zonal velocity components on a β -plane. The instabilities in this generalized model presented maximum growth curves approximately parallel to the mean BC thermal front [*da Silveira et al.*, 2008; *Garfield*, 1990], and the linear instability properties were very close to those obtained with the 1-dimensional channel model. Therefore, we keep the use of the 1-dimensional version on a f -plane for the sake of simplicity.

The formulation of the model is standard [*e.g.*, *Vallis*, 2006]. We linearize the QG potential vorticity equation about local slowly varying stratification and velocity profiles [*e.g.*, *Smith*, 2007; *da Silveira et al.*, 2008]

$$\left[\frac{\partial}{\partial t} + V \frac{\partial}{\partial y} \right] q' - \frac{\partial \psi}{\partial y} \frac{\partial Q}{\partial x} = 0, \quad -H < z < 0, \quad (2.16)$$

where

$$\frac{\partial Q}{\partial x} = \frac{\partial}{\partial z} \left(\frac{f_0^2}{N^2} \frac{\partial V}{\partial z} \right) \quad (2.17)$$

is the mean QG potential vorticity gradient, and

$$q' = \nabla^2 \psi + \frac{\partial}{\partial z} \left(\frac{f_0^2}{N^2} \frac{\partial \psi}{\partial z} \right) \quad (2.18)$$

is the eddy QG potential vorticity. V is the mean along-front velocity and ψ the eddy stream-function. Note that we have neglected the relative vorticity of the mean flow under the local approximation. Nevertheless, the neglect of this term completely isolates baro-

clinic instability phenomena, which is ultimately desirable for a first-order phenomenological understanding [da Silveira et al., 2008]. Barotropic and mixed instabilities are likely to take place along all the BC path [Oliveira et al., 2009], but they are not considered here as consequence of the limitation of our dataset (one mooring at each location).

The boundary conditions at the upper and lower surfaces are [da Silveira et al., 2008]

$$\left[\frac{\partial}{\partial t} + V \frac{\partial}{\partial y} \right] \frac{\partial \psi}{\partial z} - \frac{\partial \psi}{\partial y} \frac{\partial V}{\partial z} = 0 \quad \text{at } z = 0, \quad (2.19)$$

and

$$\left[\frac{\partial}{\partial t} + V \frac{\partial}{\partial y} \right] \frac{\partial \psi}{\partial z} - \frac{\partial \psi}{\partial y} \left[\frac{\partial V}{\partial z} + \frac{N^2}{f_0} \frac{\partial \eta_b}{\partial x} \right] = 0 \quad \text{at } z = -H, \quad (2.20)$$

where $\partial \eta_b / \partial x$ is the cross-front topographic gradient. For consistency with the QG scaling, we scale the topographic gradients by the Rossby number (Ro) as done by da Silveira et al. [2008]. Based on these authors' estimates, we assume $\text{Ro} = 10^{-1}$ for all moorings.

In addition, no normal-flow is imposed at the lateral boundaries, that is

$$\frac{\partial \psi}{\partial y} = 0 \quad \text{at } x = \pm L, \quad (2.21)$$

where L is the half width of the channel [da Silveira et al., 2008].

Assuming wave-like solutions confined in a channel for the eddy stream-function [e.g., Pedlosky, 1987]

$$\psi(x, y, z) = \text{Re}\{\varphi(z) \cos\left(\frac{\pi x}{2L}\right) e^{i(ly - \omega t)}\}, \quad (2.22)$$

and upon substituting Eq. 2.22 in Eqs. 2.16, 2.19 and 2.20, we obtain

$$\left\{ (Vl - \omega) \left[\frac{\partial}{\partial z} \left(\frac{f_0^2}{N^2} \frac{\partial}{\partial z} \right) - \left(K^2 + \frac{\pi^2}{4L^2} \right) \right] + l \frac{\partial Q}{\partial x} \right\} \varphi = 0, \quad -H < z < 0, \quad (2.23)$$

and

$$(Vl - \omega) \frac{\partial \varphi}{\partial z} - l \frac{\partial V}{\partial z} \varphi = 0 \quad \text{at } z = 0, \quad (2.24)$$

and

$$(Vl - \omega) \frac{\partial \varphi}{\partial z} - l \left[\frac{\partial V}{\partial z} + \frac{N^2}{f_0} \frac{\partial \eta_b}{\partial x} \right] \varphi = 0 \quad \text{at } z = -H, \quad (2.25)$$

which is an generalized eigenvalue problem for φ . If an eigenvalue is complex (*i.e.*, $\omega = \omega_r + i\omega_i$, $\omega_i \neq 0$) and positive¹⁰, the instability occurs. From Eq. 2.22, we have

$$\psi(x, y, z) = \text{Re}\{\varphi(z) e^{\omega_i t} \cos\left(\frac{\pi x}{2L}\right) e^{i(ly - \omega_r t)}\}, \quad (2.26)$$

thus, the unstable modes grow exponentially with growth rate ω_i . For the unstable modes, the vertical structure $[\varphi(z)]$ is also complex and can be written as

$$\varphi(z) = |\varphi(z)| e^{i\theta(z)}, \quad (2.27)$$

where $|\varphi(z)|$ is its amplitude and $\theta(z)$ its phase. Under a finite-amplitude assumption $e^{\omega_i t}$ ought to equilibrate at some time [Pedlosky, 1987]. In the real ocean, these instabilities are either damped by friction or experiment nonlinear cascades [Vallis, 2006]. The full development of the instabilities is completely nonlinear [Pedlosky, 1987]. Therefore, the predictions presented here should be considered as a first order approximation for the initial stages of the baroclinic instability process in the BC. Our motivation to use the linear QG instability approach comes from the fact that it proved to produce consistent predictions for time and length scales of mesoscale variability in the global ocean [*e.g.*, Tulloch *et al.*, 2011], in the Gulf Stream off Cape Hatteras [Johns, 1988] and in the BC System within the Campos Basin [da Silveira *et al.*, 2008].

For each mooring, Eqs. 2.23, 2.24 and 2.25 are solved numerically (see Appendix E for details) given the local $V(z)$ and $N^2(z)$ profiles. Similar to da Silveira *et al.* [2008], we

¹⁰Note that the complex eigenvalue problem is symmetric, so that there is a decaying mode for each growing mode.

use the mean along-front velocity profile as synthesized by the dynamical modes (Section 2.2.3; Figure 2.2.3). As the mean along-front at the DFBS mooring presents low statistical significance, the stability analysis at this location would be very inaccurate. Therefore, we perform the stability analysis only for the MARLIM, C3 and W333 moorings. The mean stratification is taken as the climatological profile (Figure 2.5.3) also used for computing the dynamical modes.

We set $L = 100$ km, which represents the BC width [da Silveira *et al.*, 2008]. Tests for different L were performed. Similar to what was obtained by Johns [1988], the results here proved insensitive to values greater than 100 km.

2.5.2 Necessary conditions for baroclinic instability

Before we show the results for the linear stability analysis, it is interesting to evaluate whether the necessary (but not sufficient) conditions for baroclinic instability are satisfied (see Appendix F for details). Generically, one can say that if the mean potential vorticity gradient ($\partial Q/\partial x$) inverts its sign within the (vertical) domain, all necessary conditions for baroclinic instability are fulfilled [*e.g.*, da Silveira *et al.*, 2008; Johns, 1988]. At the three moorings, $\partial Q/\partial x$ inverts the sign within the water column (Figure 2.19b). Therefore, the BC is potentially baroclinically unstable at those locations. Nonetheless, the moorings locations present different $\partial Q/\partial x$ magnitude and zero-crossing depths, thus differences in the baroclinic unstable modes are expected. The $\partial Q/\partial x$ zero-crossing depth occurs at 321 m, 366 m and 538 m at the MARLIM, C3 and W333 mooring, respectively.

In fact, the differences in the $\partial Q/\partial x$ profiles are linked to differences in the velocity shear ($\partial V/\partial z$) profile (Figure 2.19a), which is a consequence of the downstream vertical thickening of the BC. The $\partial V/\partial z$ is negative over the entire water column at the three moorings. However, it is near-surface intensified at the MARLIM and C3 mooring, and mid-depth intensified at the W333 mooring. The $\partial V/\partial z$ magnitude is greater at the MARLIM and C3 moorings than at the W333 mooring. The maximum $\partial V/\partial z$ magnitude is observed at the C3 mooring (slightly greater than that of the MARLIM mooring).

2.5.3 Linear instability properties

The linear instability properties for the three moorings are presented in Figure 2.20. The BC is indeed unstable at all moorings. In particular, at the MARLIM mooring QG waves from 125 km to 525 km are unstable. (Technically, longer waves (> 525 km) are unstable since $\omega_i \neq 0$. However, their growth rates are insignificant $< 0.002 \text{ day}^{-1}$, and it is likely that they are not physically meaningful.) At this mooring, the most-unstable mode¹¹ (230 km) has growth rate of 0.035 day^{-1} . The BC at the C3 mooring is unstable for waves from 100 km to 560 km, with maximum growth rate (0.056 day^{-1}) at 190 km. The growth rates are much greater for this mooring because it has a greater mean along-front velocity magnitude, likely owing to its relative position to the BC core. Farther downstream, at the W333 mooring, the BC is unstable to QG waves from 100 km to 400 km. The most-unstable mode (180 km) has growth rate of 0.030 day^{-1} .

The unstable modes at the MARLIM mooring (Figure 2.20b) have very small phase speeds, *e.g.* 0.03 m s^{-1} for the most-unstable mode (230 km), similar to those obtained by *da Silveira et al.* [2008]. Conversely, the unstable modes present significant phase speeds at the C3 and W333 moorings, in which the most-unstable modes propagate phase with -0.13 m s^{-1} and -0.15 m s^{-1} , respectively. The steering levels¹² for the most-unstable modes are 365 m, 380 m and 486 m for the MARLIM, C3 and W333 moorings, respectively. Thus, consistent with the predictions by *da Silveira et al.* [2008], the fastest-growing modes at the MARLIM mooring couple with and drain energy from the IWBC. This is also consistent with primitive-equation numerical results by *Mano et al.* [2009], who showed that the BC System meandering starts in the IWBC domain, although the steering levels in their simulations are deeper than those observed here. Conversely, at the C3 and W333 mooring, the most-unstable waves couple with and drain energy from the BC.

The vertical structure for the fastest-growing mode at each of the three moorings are displayed in Figure 2.21. The amplitude of these modes at the MARLIM and C3 moor-

¹¹Defined as the fastest growing mode [$\max(\omega_i)$].

¹²The steering level is defined as the depth at which $C_r = V$ [Pedlosky, 1987]. At this depth, the perturbations couple with the mean flow.

Figure 2.18: Climatological stratification $[N^2(z)]$ for MARLIM, C3 and W333 mooring.

ings is surface-intensified and present a sharp decay within the upper 250 m and 350 m, respectively. On the other hand, at the W333 mooring, the most-unstable mode vertical structure magnitude decreases from surface to mid-depth (500 m) and then increases towards deeper levels, characteristic of mid-depth “Phillips-like” instabilities (according *Tulloch et al.* [2011] terminology), in which solely $\partial Q/\partial x$ plays a role on its development.

Another interesting observation can be made upon inspecting the most-unstable mode phase (Figure 2.21b). The maximum phase shift is shallower at the MARLIM mooring than at the W333 mooring. An intermediate maximum phase shift is observed at the C3 mooring. This maximum phase shift is associated with the depth of maximum baroclinic conversion [*e.g.*, *Smith*, 2007], and therefore the maximum conversion is deeper at the W333 mooring, which is consistent with the fact that the steering level is much deeper at that location.

Figure 2.19: (a) Mean along-isobath velocity vertical shear and (b) cross-isobath mean potential vorticity gradient for MARLIM, C3 and W333 moorings.

Figure 2.20: Linear baroclinic instability properties for MARLIM, C3 and W333 moorings: (a) Growth rate and (b) phase speeds.

Figure 2.21: Most unstable-mode vertical structure for MARLIM, C3 and W333 moorings: (a) amplitude ($|\varphi|$) and (b) phase (θ).

2.6 Summary and discussion

The goal of the present work is to investigate the downstream transformations in the BC as it flows off the southeast Brazil (22°S - 28°S). We analyze *four* current meter mooring records spanning this region, namely MARLIM (22.7°S), DFBS (24.15°S), C3 (25.5°S) and W333 (27.9°S) moorings. These moorings, analyzed under the same framework, allow us to give a comprehensive description of the BC off the southeast Brazil.

The mooring data depicts the BC downstream vertical extension growth. The 350 m deep flow at the MARLIM mooring becomes a 800 m deep flow at W333 mooring; and intermediate scenario is observed at the C3 mooring, namely a 550 m deep BC. At the DFBS mooring the mean BC extends down to 400 m, although mean results at this mooring have low statistical significance owing to the high variability experimented by the BC at this location. At the MARLIM mooring, the BC is also highly variable, presenting a standard deviation that exceeds the mean in the uppermost instrument. Conversely, the underneath northward/eastward flowing IWBC proved much less variable at the MARLIM mooring. At the C3 mooring, a current meter at intermediate depths (~ 700 m) depicts a mean flow to the north/northeast, thus indicating the presence of the IWBC; however, at this loca-

tion, the standard deviation at this instrument exceeds the mean. Farther downstream, the entire water column over the slope seems to flow to the south/southeast. Therefore, the Santos bifurcation seems to occur somewhere between 25.5° and 27.9° S, consistent with recent descriptions on the basis of float trajectory analysis [Legeais *et al.*, 2013].

Quasi-synoptic observations contemporary to the mooring records are generally consistent with the scenario that emerge from the mean moored velocity analysis. Geostrophic velocity estimates fitted to the mooring data depict a shallow BC at the MARLIM (~ 320 m), DFBS (~ 380 m) and C3 (~ 410 m) moorings and relatively deeper at the W333 mooring (~ 800 m). A lower-bound estimate for the BC transport reveals that the major transport increase occurs from the C3 mooring (6.8 Sv) to the W333 mooring (11.8 Sv). Part of this 5 Sv increment is certainly associated with the Santos bifurcation. However, the presence of a recirculation [Peterson and Stramma, 1991] is *not* ruled out. Furthermore, the BC transport variability at any time scale between the two hydrographic sections occupations can contribute to these differences. Moreover, at those two locations (C3 and W333), there seems to be substantial flow onto the outer shelf.

The IWBC is synoptically well-developed at the MARLIM and DFBS moorings with maximum velocity reaching $\sim 0.25 \text{ m s}^{-1}$, in agreement with previous work [da Silveira *et al.*, 2004, 2008] and consistent with the mean moored velocity patterns. At the C3 mooring the IWCB is also clearly depicted, but it is relatively weaker, with maximum velocity of about $\sim 0.15 \text{ m s}^{-1}$.

The water column average kinetic energy (IKE) and its barotropic/baroclinic partition estimates reveal that the changes in the BC System vertical structure are accompanied by strong changes in the vertical partition of the IKE. Accordingly, the BC is, on average, highly baroclinic ($\sim 70\%$) at the MARLIM mooring, in agreement with previous estimates [da Silveira *et al.*, 2004] that point out to the high baroclinicity of the BC System close to this latitude; similar results are obtained at the C3 mooring. A dramatic change in the partition of the energy is observed to the south of the C3 mooring. Accordingly, the total flow becomes 63% barotropic at the W333 mooring. At the DFBS mooring an equipartition of the total flow between barotropic and baroclinic components was ob-

served, a manifestation of the high variability of the BC at this location [*e.g.*, *Oliveira et al.*, 2009; *Lorenzetti et al.*, 2009] likely due to intense BC meandering as it overshoots the continental margin off Cape Frio ($\sim 23^\circ\text{S}$) [*e.g.*, *Campos et al.*, 1995].

The water column average eddy kinetic energy (IEKE) and its ratio to the IKE confirms the importance of the BC mesoscale activity as it flows off the southeast: the IEKE accounts for from 33 % (W333 mooring) up to 60 % (DFBS mooring) of the IKE. This fact is indeed in agreement with quasi-synoptic (mostly hydrographic) observations [*e.g.*, *Campos et al.*, 1995, 2000; *da Silveira et al.*, 2004], drifter statistics [*Oliveira et al.*, 2009] and thermal front analysis [*Lorenzetti et al.*, 2009]. Moreover, this points to the dominance of the eddy field at the DFBS mooring, also consistent with [*Oliveira et al.*, 2009]. Furthermore, the IEKE is, on average, more barotropic than IKE. The exception is the W333 mooring, where the IEKE has approximate the same energy partition as the total flow (~ 64 % barotropic).

Indeed, the analysis of the IKE and IEKE time series reveals that these eddy events are associated with bursts of energy in the barotropic mode, thus supporting the idea that the BC meanders are more barotropic than the mean flow [*da Silveira et al.*, 2008]. At the MARLIM mooring, the persistence of the energy in the barotropic mode for ~ 37 days may be an indication of a quasi-standing meander growth in the region [*da Silveira et al.*, 2008; *Mano et al.*, 2009]. Similar long-lived eddy events are observed at the DFBS mooring. Conversely, at the C3 mooring, these bursts of energy in the barotropic mode last 20 days at maximum (typically much less), likely owing to meander propagation [*Campos et al.*, 2000; *Fernandes et al.*, 2009].

Motivated by the fact that the 1-dimensional QG baroclinic instability analysis provided consistent predictions for the length and time scales of the BC meandering [*da Silveira et al.*, 2008], we also evaluated the stability of the mean along-front BC to small-amplitude perturbations at the three moorings. The instability calculations were not performed at the DFBS mooring because the estimated time-mean BC flow at that location has low statistic significance. As far as the baroclinic instability is concerned, the BC downstream vertical extension growth has two major effects on the linear instability prop-

erties. The first remarkable fact is that the most-unstable waves are consistently shorter to the south. The estimated fastest growing modes were associated with wavelengths of 230 km, 190 km and 180 km, at the MARLIM, C3 and W333 moorings, respectively. Secondly, the unstable waves at the MARLIM mooring couple with the IWBC and present very small phase speeds ($\sim 0.03 \text{ m s}^{-1}$), while those at the C3 and W333 moorings couple with the BC and present significant phase speeds (-0.13 and -0.15 m s^{-1} , respectively).

The theoretical predictions of the linear stability model seem to roughly match the sparse information in the literature, as well as the observations analyzed here. At the MARLIM mooring, the fraction of the IKE in the barotropic mode is significant during marked eddy events, likely owing to the quasi-standing character of the BC meanders in this region [*da Silveira et al.*, 2008]. The vertical shear associated with the BC-IWBC system prone the flow to baroclinic instability and the growth of large frontal meanders [*e.g.*, *Garfield*, 1990]. A quasi-stationary mesoscale feature, the Vitória Eddy, was also observed just to the north of the MARLIM mooring region [*Schmid et al.*, 1995].

In contrast, within the Santos Basin, modeling studies have predicted the meander propagation to the south/southwest [*e.g.*, *Campos et al.*, 2000; *Fernandes et al.*, 2009]. Under the linear baroclinic instability approach, the explanation for this fact is that the BC is relatively deeper, and the maximum velocity shear is at deeper levels. Consequently, the steering levels are located in the BC domain. The theoretical fact that the vorticity waves are shorter to the south still needs to be proved from observations.

Fernandes et al. [2009] two-layer modeling results point out that 450 km vorticity waves propagate to the south/southeast with phase speeds of about 0.20 m s^{-1} . These authors point out that this fact apparently contradicts *da Silveira et al.* [2008]’s baroclinic instability predictions at 22.7°S , which showed that 450 km vorticity waves tend to propagate slowly to the north/northeast, similar to what was obtained here. The present estimates suggest that *Fernandes et al.* [2009]’s model, which considered a constant 500 m upper layer thickness from 20°S - 28°S , may be a better approximation for the BC within the Santos Basin (24°S - 27°S). As the BC is relatively thicker ($\sim 500 \text{ m}$ at the C3 mooring) at these locations than within the Campos Basin (20°S - 23°S), as far as the baroclinic

instability is concerned, the vorticity waves tend to propagate in the BC direction. Conversely, the BC is slightly shallower within the Campos Basin (350-400 m) [*da Silveira et al.*, 2008, this work]. Consequently, a unique background in which the waves grow in a quasi-standing fashion is set up [*da Silveira et al.*, 2008]. We therefore argue that the vertical thickening of the BC has to be taken into account in order to correctly represent the distinct features of the BC mesoscale variability off the southeast Brazil.

The region just to the south/southwest of Cape Frio ($\sim 23^\circ\text{S}$), where the DFBS mooring is located, is unique as to call for further discussion. Although synoptic observations frequently depict a well-organized BC flowing to the south/southeast near this location [*e.g.*, *da Silveira et al.*, 2004], our mooring record is not successful in producing reliable estimates for a mean BC owing to relatively small number of degrees of freedom and the high variability. A longer record could produce a statistically significant time-mean BC. However, it is possible that no persistent current exists at this location as it seems to be the case to the north and to the south of this region. Indeed, this is the only region where *da Silveira et al.* [2008]’s mean BC thermal front diverge from that estimated by *Garfield* [1990]. In addition, *Lorenzetti et al.* [2009]’s frontal density estimates spread out in this region, thus indicating the high variability of the BC thermal frontal. This high variability is somehow to be expected since the BC overshoots the continental margin at Cape Frio and this may trigger the meandering process in order to conserve its potential vorticity [*Campos et al.*, 1995]. Baroclinic instability may account for the meander growth in this region, perhaps in an intermediate scenario between those at the MARLIM and at the C3 moorings.

2.7 Conclusions

We conclude by answering the questions posed in the introduction: (1a) *How much does the BC thickens?* and (1b) *How much does the BC transport increase within this region?* The BC thickens from a 350 m deep flow at 22.7°S to a 800 m deep flow at 27.9°S ; Hydrographic observations suggest that the BC transport is increased by at least 5 Sv from 25.5°S to 27.9°S . (2a) *What is the partition between barotropic and baroclinic compo-*

nents in the BC? and (2b) *What are the downstream changes on this partition?* The total flow is mainly baroclinic ($\sim 70\%$) at 22.7°S and 25.5°S , and becomes mainly barotropic at ($\sim 63\%$) at 27.9°S ; However, it should be noted that an equipartition is observed for the total flow at 24.15°S , a manifestation of the dominance of the eddy field at this location. (3) How much of the BC energy is in the eddy field? In a water column integral sense, the eddy field accounts for (30-60) % of the total kinetic energy; (4) *What are the downstream changes on the BC linear instability properties?* The changes are twofold: (i) The fastest growing modes are shorter to the south (at 25.5°S and 27.9°S); (ii) At 25.5°S and 27.9°S , these unstable waves tend propagate to the south, as opposed to quasi-standing waves at 22.7°S .

It was *not* our intention at all to fully explain the observed mesoscale activity of the BC through the linear baroclinic stability analysis. Barotropic instabilities seem to be important along all the BC path [Oliveira *et al.*, 2009]. One important open question concerns the baroclinic and barotropic instabilities relative importance and how it changes as the BC thickens and becomes more barotropic. The full development of the meandering process is *not* captured by the linear models. Therefore, this question should be assessed by combining synoptic and mooring observations [*e.g.*, da Silveira *et al.*, 2008] and primitive-equation numerical models [*e.g.*, Mano *et al.*, 2009]. However, numerical models should prove to be consistent with the available statistics for the BC [*e.g.* Oliveira *et al.*, 2009]. We believe that the present estimates could also be a test bed for those models.

Chapter 3

Surface Quasi-geostrophic solutions in the Southwestern Atlantic*

Abstract

We investigate whether the Quasi-geostrophic (QG) modes and the Surface Quasi-geostrophic (SQG) solutions are consistent with the vertical structure of the subinertial variability off southeast Brazil. The 1st Empirical Orthogonal Function (EOF) of current meter time series is reconstructed using different QG mode combinations; the 1st EOF is compared against SQG solutions. At two out of three moorings, the traditional flat-bottom barotropic (BT) and 1st baroclinic (BC1) mode combination fails to represent the observed sharp near-surface decay, although this combination contains up to 78 % of the depth-integrated variance. A mesoscale broad-band combination of flat-bottom SQG solutions is consistent with the near-surface sharp decay, accounting for up to 85 % of the 1st EOF variance. A higher order QG mode combination is also consistent with the data. Similar results are obtained for a rough topography scenario, in which the velocity vanishes at the bottom. The projection of the SQG solutions onto the QG modes confirms that these two models are mutually dependent. Consequently, as far as the observed near-surface vertical structure is concerned, SQG solutions and four QG mode combination are indistinguishable. Tentative explanations for such vertical structures are given in terms of necessary conditions for baroclinic instability. “Charney-like” instabilities, or, surface-intensified “Phillips-like” instabilities may explain the SQG-like solutions at two moorings; traditional “Phillips-like” instabilities may rationalize the BT/BC1 mode representation at the third mooring. These results point out to the presence of a richer subinertial near-surface dynamics in some regions, which should be considered for the interpretation and projection of remotely-sensed surface fields to depth.

Key words: QG modes, SQG solutions, Southwestern Atlantic, Brazil Current.

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3.1 Introduction

An interesting conundrum has arisen in the scientific literature on the correct dynamics to represent the global remotely-sensed sea surface height (SSH) and sea surface temperature (SST) at mesoscales and their relationship to the vertical structure of the oceanic flows [*e.g.*, *Lapeyre, 2009; Ferrari and Wunsch, 2010*]. Two dynamical ideas have been invoked to extend the SSH and SST (and soon sea surface density, as surface salinity measurements from *Aquarius* [*Lagerloef et al., 2008*] become available) to sub-surface: traditional quasi-geostrophic (QG) modes and surface quasi-geostrophic (SQG) solutions.

The first idea stems from the work by *Wunsch [1997]*, who observationally investigated the vertical partition of the horizontal kinetic energy. Based on 107 moorings (mainly in the North Atlantic and Pacific oceans), this study concluded that, in general, the daily-averaged surface kinetic energy (KE) is mainly due to 1st baroclinic (BC1) motions. *Wunsch [1997]* argued that this is because the BC1 mode is surface intensified; consequently, altimeters may reflect this motion. This seems consistent with theoretical predictions [*Fu and Flierl, 1980*] and idealized numerical QG turbulence experiments [*e.g.*, *Scott and Arbic, 2007*]. *Scott and Furnival [2012]* investigated further the use of a phase-locked linear combination of the barotropic (BT) and BC1 modes to extrapolate the surface geostrophic velocity. The authors pointed out that this linear combination loses predictive skills below 400 m.

The second idea is based on the SQG approximation. In SQG models [*e.g.*, *Held et al., 1995*], the flow is driven by surface buoyancy anomalies, with constant (generally zero) interior Potential Vorticity (PV). Indeed, the surface buoyancy anomalies can be interpreted as a delta-function PV anomaly [*Bretherton, 1966*], producing surface-intensified solutions. The appeal of the SQG framework is that the vertical solutions depend on the horizontal structure. Hence, the subsurface dynamics can be recovered solely from surface information and the mean stratification profile [*e.g.*, *Lapeyre and Klein, 2006*]. Although the constant interior PV assumption seems to be too strong [*Lacasse, 2012*] (hereafter L12), it is remarkable that the SQG correctly predicts the flow in some regions. Comparisons against primitive-equation simulations [*Lapeyre and Klein,*

2006; *Isern-Fontanet et al.*, 2008] and observations [*LaCasce and Mahadevan*, 2006] have shown that the flow resembles the SQG recovered fields, although it underestimates the velocity at depth. This problem can be partially solved by applying an adjustable constant stratification [*Lapeyre and Klein*, 2006; *Isern-Fontanet et al.*, 2008] (the “effective” parameter in their terminology) or by seeking an empirical correlation between surface buoyancy and interior PV anomalies [*LaCasce and Mahadevan*, 2006].

It has also been argued that the SQG may be a better framework for the upper ocean balanced dynamics since the slope of (along-track) altimeter-derived SSH wavenumber spectra at mesoscales seem to be more consistent with the slope predicted by SQG turbulence theory [*e.g.*, *Le Traon et al.*, 2008] than with the slope predicted by the classical QG turbulence theory and previously used to rationalize SSH observations [*e.g.*, *Stammer*, 1997]. However, altimeter-derived spectra slopes do not seem to match those few estimates from *in-situ* observations [*Wang et al.*, 2010], likely owing to noise contamination in altimeter measurements even at mesoscales [*e.g.*, *Xu and Fu*, 2011, 2012].

These two views of the upper ocean balanced dynamics do not exclude each other. *Ferrari and Wunsch* [2010] showed that a phase-locked linear combination between BT and BC1 modes is consistent with the SQG solution, because it can produce a surface intensification depending on the phase. L12’s elegant analytical solutions show that long SQG waves project primarily onto the BT and BC1 modes, thus resembling a combination of these modes. In addition, the similarity between the BC1 mode and SQG solution is even more striking for an ocean with rough topography; the major difference is that the SQG vertical decay is slightly sharper than that of the BC1 mode.

The current meter time series Empirical Orthogonal Functions (EOFs) are frequently evaluated in terms of the combinations of linear QG dynamical modes [*e.g.*, *Kundu et al.*, 1975; *da Silveira et al.*, 2008]. Although L12 clarifies the connections and distinctions between QG modes and SQG solutions, these ideas have not yet been tested against observations. Here we investigate whether the SQG solutions are consistent with the vertical structure of current meter moorings statistics off southeast Brazil, and explore their similarities to the traditional QG modes. This is done by assessing the errors obtained by re-

constructing the EOFs with different QG mode combination and comparing against SQG solutions. We believe that this can shed light on the interpretation of surface fields (at mesoscale) as well as their projections to depth in this region, where in-situ observations are limited.

This paper is organized as follows. In Section 3.2, a brief theoretical review on the traditional QG modes and SQG solutions is presented, primarily focusing on the vertical structure. The data sets and methods used in this work are reported in Section 3.3. Section 3.4 describes the main results of projections of the EOFs onto QG modes as well as its comparisons against the SQG solutions. Sections 3.5 and 3.6 present discussion and concluding remarks, respectively.

3.2 Traditional QG modes and SQG solutions

The traditional linear QG modes and the SQG solutions arise from separation of variables of the governing equations [*e.g.*, Pedlosky, 1987]. For the QG modes, the vertical structure is governed by

$$\frac{d}{dz} \left(\frac{f_0^2}{N^2} \frac{d\phi}{dz} \right) + \lambda^2 \phi = 0, \quad (3.1)$$

where ϕ is the vertical structure, f_0 and $N(z)$ are the inertial and stratification frequencies, respectively, and $-\lambda^2$ is the separation constant. At the boundaries, one generally requires [*e.g.*, Pedlosky, 1987]

$$\frac{d\phi}{dz} = 0, \text{ at } z = 0, \quad z = -H. \quad (3.2)$$

Eq. 3.1 is derived assuming a mean motionless and linear ocean. Also, the boundary condition at $z = -H$ requires a flat bottom. These assumptions are, to some extent, violated everywhere in the ocean. In particular, in our study region, the presence of the Brazil Current and the sloping topography may affect this decomposition. Nevertheless, we treat this as a local and linear problem. Boundary conditions for rough topography are discussed in Section 3.4.4.

Eq. 3.1 along with the boundary conditions (Eq. 3.2) constitute a particular case of the classical Sturm-Liouville eigenvalue problem. The eigensolutions (ϕ_j) are the traditional QG modes. The eigenvalues λ_j^2 are (by definition) the inverse of the deformation radii squared.

An implication of the traditional QG modes is that buoyancy anomalies are not allowed at the boundaries [e.g., L12]. Conversely, the SQG problem is posed to allow density anomalies at the surfaces [e.g., *Held et al.*, 1995; *Lapeyre and Klein*, 2006; *LaCasce and Mahadevan*, 2006; *Lapeyre*, 2009]. In this case, the vertical structure is governed by

$$\frac{d}{dz} \left(\frac{f_0^2}{N^2} \frac{d\chi}{dz} \right) - K^2 \chi = 0, \quad (3.3)$$

where we follow L12's notation and change the variable for the vertical structure function to keep it different from the traditional QG modes. Also, for this case $\lambda^2 = -K^2$, where $K = \sqrt{k_x^2 + k_y^2}$; (k_x, k_y) is the wavenumber in the (x, y) direction. This follows directly from the constant interior PV assumption [L12]. The boundary conditions (assuming no buoyancy anomalies at the bottom for simplicity) are

$$\frac{d\chi}{dz} = 1, \text{ at } z = 0, \quad (3.4)$$

and

$$\frac{d\chi}{dz} = 0, \text{ at } z = -H. \quad (3.5)$$

Unlike the traditional QG modes, the SQG vertical structure [$\chi(z)$] is intrinsically dependent on the horizontal scale (K). Therefore, Eq. 3.3 along with the boundary conditions (Eqs. 3.4 and 3.5) do not form an eigenvalue problem¹; consequently, there is a continuum of SQG solutions in case K is continuous (infinite unbounded domain).

¹Eq. 3.3 is *not* associated to an eigenvalue problem since K is given.

Figure 3.1: Mooring locations for MARLIM, W333 and W335. Solid black lines indicate the bathymetric depths. The limits of the grids used to compute the SST spectra are shown in gray.

3.3 Data and methods

3.3.1 Current meter moorings

Three moorings are used to estimate the vertical structure of the time-dependent flow. The MARLIM mooring (22.7° S; 40.2° W; Figure 3.1) is in the Brazil Current domain at the 1250 m isobath off-shore the coast of Rio de Janeiro, Brazil. It had 9 electromagnetic Marsh-McBirney sensors [50; 100; 250; 350; 450; 650; 750; 950; 1050 m]. From February 1992 to December 1992 (with an approximately 30-day-long gap in May 1992), this mooring had 308 days of hourly current measurements. The second half of this series was analyzed by *da Silveira et al.* [2008], who showed that the mooring has adequate vertical resolution to describe the mean patterns and mesoscale variability of the Brazil Current.

The other two moorings analyzed in the present work come from the WOCE Experiment, as part of the German component (IFM-Kiel) of the “Deep Basin Southwestern Boundary” array. Both moorings span January 1991 through November 1992, providing

about 650 days of current measurements every 2 hours. The WOCE 333 mooring (27.9° S; 46.7° W; Figure 3.1; hereafter W333) is in the Brazil Current domain, farther south than the MARLIM mooring, at the 1200 m isobath. It had 4 Aanderaa RCM8 current meters [230; 475; 680; 885 m] and one upward-looking 150 kHz ADCP at about 200 m. The ADCP provides current measurements at 8 m resolution. We use 7 ADCP levels [51; 77; 95; 112; 138; 155; 173 m]. The W335 mooring (28.3° S; 45.3° W; Figure 3.1; hereafter W335) is located off the Brazil Current at the base of the continental slope (approximately 3300 m isobath) in the domain a recirculation flow. It had 6 Aanderaa RCM8 current meters [275; 515; 915; 1415; 2510; 3215 m] and one upward-looking 8 m-bin 150 kHz ADCP at about 250 m. We use 7 ADCP levels [55; 72; 98; 115; 150; 192; 237 m]. While measurements within the mixed layer are available, we have not used them here, as they include a substantial component of ageostrophic motion. A detailed description of both WOCE moorings configurations and basic statistics is provided by Müller *et al.* [1998].

The current meter time series are low-pass filtered using a *Lanczos-cosine* filter. The cutoff frequency is set $1/40 \text{ h}^{-1}$ in order to retain only the sub-inertial energy. Additionally, the temporal mean of the filtered series is removed.

3.3.2 Climatological stratification

The mean stratification is needed to compute the QG modes and SQG solutions. For each mooring, the stratification is computed using annual mean temperature and salinity profiles from the *World Ocean Atlas 2009* climatology [Locarnini *et al.*, 2010; Antonov *et al.*, 2010]. The 8 closest profiles of the 0.25 degree resolution climatology are averaged (shelf profiles are excluded in the MARLIM and W333 cases). The stratification frequency is then computed, gridded in the vertical for each 10 m and smoothed following da Silveira *et al.* [2000a]. The resulting profiles are shown in Figure 3.2.

Figure 3.2: Climatological $N^2(z)$ profiles for MARLIM (black), W333 (dashed black) and W335 (gray) moorings region.

3.3.3 SST imagery

SST images are used for estimating the horizontal scales necessary to compute the SQG solutions. We selected all cloud-free 7-day composite of 4 km resolution AVHRR images from Pathfinder (version 5.1, NOAA) spanning the period of the moorings. A square grid (4 km resolution) of approximately 600×600 km is selected for two regions: (i) The MARLIM region; and (ii) The WOCE regions. Due to the geometrical constraints of the continental margin, the MARLIM mooring region square grid is rotated clockwise by 30° such that the zonal component is approximately a cross-isobath component. The SST from the original Pathfinder grid is linearly interpolated onto this new grid. The current meter velocity vector is rotated in a similar fashion, and hereafter the $v - (u-)$ component refer to the along- (cross-)isobath. Additionally, in this case, the mooring is located close to the middle of the inshore edge of the grid (Figure 3.1). The WOCE grid is north-south/east-west oriented. The grid is centered at the positions of the W335 mooring. The W333 mooring is located close to the eastern edge of the grid.

In order to examine the existence of dominant horizontal scales, we compute the zonal

and meridional mean wavenumber spectra. As we are interested only in the mesoscale eddies passing through the moorings, the SST images are high-pass filtered using a *Butterworth* filter. The cutoff wavenumber is set as $1/300 \text{ km}^{-1}$; half of the domain size. The SST anomalies are de-trended and multiplied by a Hanning window (5 points). The fast Fourier transform is computed for a mirror symmetric domain [*e.g.*, *Isern-Fontanet et al.*, 2006, 2008]; the mean wavenumber spectra is then computed assuming isotropy of scales.

The number of high quality cloud-free images (4 [10] for the MARLIM [WOCE] region/period) is too small to provide a statistically significant result. Hence, this spectra should be considered as a first attempt to characterize of the horizontal scales of the variability within these regions during the mooring periods. Microwave imagery is not available for the time of the moorings.

3.3.4 Statistical mode computation

EOFs are used to characterize the coherent spatial (vertical) pattern of the subinertial variability as measured by the moorings. The statistical or empirical modes are indeed an orthogonal basis for the data covariance matrix [*e.g.* *Emery and Thomson*, 2001]. We compute the covariance matrices using the *Mathworks, Inc.* MATLAB[®] “nancov” script, that allows computing the covariance for gappy data (MARLIM mooring). The EOFs are computed by finding the eigenvalues (fraction of variance) and eigenvectors (EOF vertical structure, in this case) of the covariance matrix numerically.

In order to evaluate which EOFs are statistically meaningful, we use a Monte Carlo [*Preisendorfer*, 1988] process, which consists of computing the EOFs for 100 surrogate random matrix of the same size as the data matrix; the random series are also filtered the same way as the velocity series. The amount of variance contained in each statistical mode (*i.e.* the relative contribution to the total energy) is then averaged; two standard deviations interval represent its 95 % significance limits. Only the statistical modes above the significance limits are considered statistically meaningful.

Pressure sensors records reveals significant vertical displacements during highly en-

ergetic events, particularly at the W335 mooring. For instance, the 275 m instrument (nominal depth) of the W335 mooring reached depths as deep as 550 m during one of such events. This could bias the QG mode projection specifically during such events, as discussed by Wunsch [1997] and Ferrari and Wunsch [2010]. However, as these events are short-lived, they do not significantly affect the 1st EOF vertical structure. Therefore, the vertical displacement of the instruments do not impact the overall results of the present study.

3.3.5 Dynamical mode fit

In this work we explain the spatial (vertical) structure of variability with the traditional QG modes and SQG solutions using EOFs as a measure of variability. Although the EOFs are only a statistical measure, we assume that the subinertial variability in this region is dominated by 1st order geostrophic motion. We therefore try to attribute physical meaning to the statistical modes by projecting them onto the QG modes. EOFs are also compared against SQG solutions.

The fundamental decomposition used here is similar to that of the projection of the velocity profiles onto dynamical modes [*e.g.*, Wunsch, 1997]. We write, in matrix notation,

$$[EOF_u, EOF_v] = F[A_u, A_v] + [R_u, R_v], \quad (3.6)$$

where $EOF_{uM \times 1}$ ($EOF_{vM \times 1}$) is the dominant EOF matrix for the $u(v)$ velocity component; $A_{uN \times 1}$ ($A_{vN \times 1}$) is the modal amplitude matrix; $F_{M \times N}$ is QG mode matrix (column 1 stands for the BT mode; columns N stands for the N-1th baroclinic mode); $R_{uM \times 1}$ ($R_{vM \times 1}$) is the residual and should be considered as the sum of errors from: (i) instrument uncertainties (propagated to the EOF computation); (ii) numerical errors from the eigenvalue/eigenvector numerical algorithm; and (iii) breakdown of the assumption that the variability in this frequency band is due to QG motion.

We assume that the energy is concentrated in the 5 gravest modes (i.e $N = 5$ in Eq. 3.6). These QG modes are computed by solving the Sturm-Liouville problem (Eqs. 3.1

and 3.2) numerically given the climatological $N^2(z)$ profile². The modal amplitudes are obtained by projecting the EOFs onto the QG modes. As the system is overdetermined for all moorings (*i.e.* number of vertical levels greater than the number of dynamical modes), the projection is done by solving the normal equations:

$$[A_u, A_v] = (F^T F)^{-1} F^T [EOF_u, EOF_v]. \quad (3.7)$$

The EOFs are reconstructed using 1 to 5 modes. The ability of the dynamical modes to represent the EOFs is evaluated by comparing the reconstructed (synthesized) profile against the statistical modes. A statistical measure of this comparison is the normalized RMS difference:

$$[RMS_{udiff}, RMS_{vdiff}] = \frac{\left\| [EOF_u, EOF_v] - F^j [A_u^j, A_v^j] \right\|}{\left\| [EOF_u, EOF_v] \right\|}, \quad (3.8)$$

where the vertical bars denote the length and j is the number of modes used for a particular reconstruction (synthesis). For example, we reconstruct the EOF using a BT/BC1 mode combination by setting $j = 2$ ($A_u^2 = A_{u2 \times 1}$ and $F^2 = F_{M \times 2}$). The RMS_{diff} using 5 modes ($j = 5$) should be a measure of the residual (R). Furthermore, the complement of Eq. 3.8 (*i.e.*, $1 - RMS_{diff}$) is the fraction of the EOF variance accounted for by the QG mode synthesis.

3.3.6 SQG vs. 1st EOF

As the SQG vertical solution depends on the horizontal scales, we estimate the dominant wavelengths by computing the mean wavenumber spectra using the set of AVHRR images described above. The mean SST wavenumber spectrum for the W333 and W335 region is shown in Figure 3.3. The spectrum is broad-banded at mesoscales (from ≈ 50 km up to the cut-off wavelength). It has high uncertainty owing to the small number of cloud-free images, making these results a rough estimate. Consequently, no dominant

²Details of the numerical resolution are presented in Appendix D.

wavenumber could be identified. (The spectrum for the MARLIM mooring (not shown) region is similar.) Thus, the SQG vertical structure is computed as a combination of all the waves resolved in the spectrum. That is, in the physical space we write

$$\bar{\chi}(z) = \sqrt{\sum_{i=1}^N |\hat{V}|^2 \chi^2(K_i, z)}, \quad (3.9)$$

where the hatted quantity stands for the Fourier transform of the respective variable. In deriving Eq. 3.9, we used the Parseval's theorem. Note that, in the SQG framework, the velocity (kinetic energy) spectrum is related to the SST spectrum (assuming SST anomalies are representative of density variations): $|\hat{V}|^2 = |\hat{u}|^2 + |\hat{v}|^2 \propto \chi^2(K_i, z=0) K_i^2 |\hat{T}|^2$. Therefore, under this approach, either surface velocity or temperature measurements could be used to estimate $\bar{\chi}(z)$. Since our mooring data are prior to the multi-altimeter-derived geostrophic velocity era, we use SST data.

For each wavenumber, we compute the SQG vertical structure [*i.e.*, $\chi(K_i, z)$] by solving numerically the SQG vertical problem (Eq. 3.3 subject to Eqs. 3.4 and 3.5) given the climatological $N^2(z)$ profile³; the combination of SQG vertical structures of all resolved wavenumbers (Eq. 3.9) provides the SQG solution $[\bar{\chi}(z)]$ that is ultimately compared against the 1st EOF vertical structure. As it accounts for a range of wavelengths, we think this combination is a better way of representing the SQG solution for comparing it with the EOFs vertical structure.

Another method for combining the SQG waves was also tested, namely simply combining the SQG solutions weighted by the energy fraction in each wavenumber resolved in the SST spectrum. This linear combination produces SQG solutions (not presented) very similar to those obtained with Eq. 3.9. Another (more arbitrary) possibility is to look for which wavenumber produces the SQG solution that best fits the 1st EOF vertical structure. The results (not presented) point that wavelengths of 250-300 km best fit the data; these vertical structures do not significantly differ from the combination of SQG waves. This suggests that the summation in Eq. 3.9 is dominated by the largest wavelength, which has the highest energy fraction, although it does not represent a statistically

³Details of the numerical resolution are presented in Appendix D.

Figure 3.3: SSTA spectrum (black line) for W333 and W335 moorings region. The gray shadow represents the 95% confidence interval. For reference, the gray vertical line indicates the 1st baroclinic deformation radius (33 km) for W335 mooring region.

Table 3.1: Percent of variance accounted for by the the first empirical mode. EOF_v (EOF_u) stands for the y - (x -direction) velocity component EOF.

	1st EOF _v	1st EOF _u
MARLIM	83.4 % $\pm$ 11.8 %	81.8 % $\pm$ 11.8 %
WOCE 333	92.2 % $\pm$ 10.6 %	96.2 % $\pm$ 10.6 %
WOCE 335	89.6 % $\pm$ 8.9 %	94.4 % $\pm$ 8.9 %

significant peak in the spectrum.

In order to be compared against the unitless EOF vertical structure, the SQG solution $[\bar{\chi}(z)]$ and the 1st EOF are normalized by their respective value in the depth associated to the uppermost current meter position (*i.e.*, closest to the surface). The measure of the variance of the vertical structure of the EOF accounted for by the SQG solution is calculated using a similar criterion as in the QG mode synthesis (Eq. 3.8).

3.4 Results

Table 3.1 presents the results from EOF computations. For all moorings and both velocity components, only the 1st EOF is statistically significant. Therefore, hereafter we just comment on the dominant statistical mode (1st EOF) for the three moorings. In general the 1st EOF accounts for about 85-95 % of the depth-integrated variance. As there are small differences between the two components, we arbitrarily choose to focus the description of the results on the v -component, although, the results for both components are tabulated (Tables 3.2, 3.3) and the u -component results for the rough topography scenario are illustrated in Figures 3.7, 3.8 and 3.9. Significant differences are mentioned and discussed below.

3.4.1 EOFs projection onto QG modes

The results from QG mode synthesis to the 1st EOF are presented in Table 3.2. We do not present the results for the 5 mode combination (*i.e.*, including the 4th baroclinic mode), as it accounts for less than 2 – 3% of the 1st EOF depth-integrated variance at all moorings. For the MARLIM mooring, a linear combination of the BT and BC1 modes accounts for 77.8 % of the depth-integrated variance. This mode combination particularly fails

Table 3.2: Percent of the 1st EOF_v (EOF_u) depth-integrated variance accounted for by 1 QG mode (BT), 2 QG modes (BT+BC₁) and 4 QG modes (BT+BC₁+BC₂+BC₃), and SQG solution. The traditional flat bottom boundary condition is applied in this case.

	QG mode combination			SQG
	1 mode	2 modes	4 modes	
MARLIM	29.1 (27.2) %	77.8 (85.8) %	91.2 (88.6) %	82.5 (73.3) %
WOCE 333	42.8 (32.3) %	91.0 (89.5) %	94.8 (94.9) %	73.9 (77.5) %
WOCE 335	25.4 (22.8) %	69.1 (79.7) %	98.5 (97.6) %	85.0 (77.6) %

Figure 3.4: First EOF (filled circles) of the velocity anomalies for MARLIM mooring. Comparison against 2 QG modes (BT+BC₁) and 4 QG modes (BT+BC₁+BC₂+BC₃), and SQG solution for a flat-bottom scenario.

to capture the sharp vertical decay in the upper 300-400 m (Figure 3.4). Consequently, higher order baroclinic modes are necessary to account for this near-surface variance. The BC2 and BC3 modes together contain 13.4 % of the variance. These results are quite similar to those obtained by *da Silveira et al.* [2008] for the same mooring, though the series analyzed in the present work is 120 days longer. Indeed, if the time series is split in three-month pieces and the EOFs are computed, we notice that the vertical structure of the 1st EOF does not change significantly. This supports the assumption that the 1st EOF is dominated by mesoscale variability.

The projections are different for the u-component (cross-isobath in this case) for the

Figure 3.5: First EOF (filled circles) of the velocity anomalies for W333 mooring. Comparison against 2 QG modes (BT+BC₁) and 4 QG modes (BT+BC₁+BC₂+BC₃), and SQG solution for a flat-bottom scenario.

MARLIM mooring. The BT and BC1 (see Table 3.2) accounts for 85.8 % of the variance. This suggests some anisotropy of the variability, possibly constrained by the sloping topography.

The BT/BC1 mode combination seems to better represent the 1st EOF vertical structure at the W333 mooring (Figure 3.5; see also Table 3.2). The linear combination of these modes contains 91.0 % of the variance. Note that in this case the 1st EOF does not show a sharp decay in the upper 300-400 m. Thus the BC1 mode describes the near-surface structure fairly well. The BC2 and BC3 modes together account for 3.8 % of the variance.

Offshore of the Brazil Current domain, at the W335 mooring, BT and BC1 modes together account for 69.1 % of the 1st EOF variance (Figure 3.6; Table 3.2). In particular, it is clear that the BC1 mode fails to represent the sharp vertical decay in the upper 500 m. Higher modes are needed to account for this variance; BC2 and BC3 modes together contain 29.6 % of the variance.

Figure 3.6: First EOF (filled circles) of the velocity anomalies for W335 mooring. Comparison against 2 QG modes (BT+BC₁) and 4 QG modes (BT+BC₁+BC₂+BC₃), and SQG solution for a flat-bottom scenario.

The results from the MARLIM and W335 moorings are not consistent with the overall results from Wunsch [1997], who has shown that the bulk of the near-surface energy is accounted for by the BC1 mode. In particular, the near-surface energy of these moorings contain considerable amount of energy in the BC2 and BC3 modes. These outcomes motivated us to test whether SQG solutions would be a better model for the vertical structure of the variability in these regions. As pointed out by L12, the fact that a dominant EOF in a particular region decays more sharply than the BC1 mode “could be an indication of SQG motion.”

3.4.2 EOFs vs. SQG solutions

The results from comparisons of 1st EOF against the resulting SQG solution $[\bar{\chi}(z)]$ for each mooring are presented in Table 3.2. For the Marlim mooring (Figure 3.4), the SQG solution accounts for 82.5 % of the 1st EOF variance. In particular, the SQG solution seems consistent with the sharp vertical decay in the upper 300-400m. The u-component

Table 3.3: Percent of the 1st EOF_v (EOF_u) depth-integrated variance accounted for by 1 QG mode (BC₁), 2 QG modes (BC₁+BC₂+BC₃) and 3 QG modes (BC₁+BC₂+BC₃), and SQG solution. The rough topography boundary condition is applied in this case. The BT mode vanishes.

	QG mode combination			SQG
	1 mode	2 modes	3 modes	
MARLIM	67.9 (73.6) %	88.0 (90.0) %	92.8 (90.4) %	84.8 (77.1) %
WOCE 333	89.5 (82.3) %	93.3 (95.4) %	96.4 (99.2) %	68.2 (74.9) %
WOCE 335	60.2 (64.1) %	75.4 (86.6) %	95.9 (97.3) %	84.9 (84.9) %

(cross-isobath in this case) is significantly different; the SQG accounts for 73.3 % of the 1st EOF variance in this case.

For the W333 mooring (Figure 3.5; see also Table 3.2), the SQG solution contains 73.9 % of the variance. However, the SQG solution vertical decay is much sharper, and it diverges dramatically from the 1st EOF in the upper 300-400 m. This is to be expected since the 1st EOF vertical structure is well represented by the BT/BC1 mode combination.

For the W335 mooring (Figure 3.6), the SQG solution accounts for 85.0 % of the 1st EOF variance. Unlike the BT/BC1 linear combination, the SQG solution does account for the sharp decay in the upper 500 m. The u-component contains 77.6 % of the 1st EOF variance. In particular, the 1st EOF vertical structure goes almost to zero below 2500m, which may be an indication of effects of topography, or, bottom friction (see discussion in Section 3.4.4).

3.4.3 SQG projection onto QG modes

As the traditional linear QG modes are solutions of the Sturm-Liouville problem (Eqs. 3.1 and 3.2), they constitute an orthogonal basis for the subspace of solutions. Therefore, the SQG solutions project onto them [Ferrari and Wunsch, 2010, L12]. Following L12, we evaluate how the SQG solutions project onto the QG modes. (Other authors treat the QG traditional mode decomposition as incomplete. They argue that they are degenerate with respect to the upper boundary condition; a good discussion is provided by Lapeyre [2009].)

The projection results for all moorings are presented in Table 3.4. For the MARLIM mooring, the SQG solution projects primarily onto the BT and BC1 modes (43.6 % and 33.2 %, respectively), followed by the BC2 mode (14.8 %) and the BC3 mode (6.0 %).

Table 3.4: Projection of SQG solution onto QG modes using the flat bottom (rough topography) boundary condition.

	Mode			
	BT	BC ₁	BC ₂	BC ₃
MARLIM	43.6 %	33.2 (56.9) %	14.8 (25.9) %	6.0 (10.7) %
WOCE 333	56.5 %	28.8 (65.7) %	11.7 (24.7) %	2.9 (8.8) %
WOCE 335	30.5 %	34.75 (50.7) %	17.5 (23.6) %	11.1 (12.9) %

Figure 3.7: First EOF (filled circles) of the velocity anomalies for MARLIM mooring. Comparison against 1 QG mode (BC₁) and 3 QG modes (BC₁+BC₂+BC₃), and SQG solution for a rough topography scenario. The BT mode vanishes in this case.

For the W333 mooring, the SQG solution projects primarily onto the BT mode (56.5 %), followed by the BC1 mode (28.8 %). This suggests that the SQG solution contains a considerable amount of the variance of the 1st EOF in this region, owing to the variability in this region being significantly barotropic (Figure 3.5). The projection of the SQG onto the higher modes (11.7 % for the BC2) is consistent with the fact that the SQG solution is not consistent with the 1st EOF. For the W335 mooring, the SQG solution projects primarily onto the BC1 mode (34.75 %), followed by the BT mode (30.5 %). Considerable amount of energy is also found in the BC2 and BC3 modes (17.5 % and 11.1 %, respectively).

Figure 3.8: First EOF (filled circles) of the velocity anomalies for W333 mooring. Comparison against 1 QG mode (BC_1) and 3 QG modes ($BC_1+BC_2+BC_3$), and SQG solution for a rough topography scenario. The BT mode vanishes in this case.

In general these projection results are in agreement with the theoretical cases studied by L12, which showed that long SQG waves project primarily onto the BT and BC_1 modes, with the dominance of one or the other depending upon the decay of the stratification profile. Indeed, the WOCE 355 case is similar to the theoretical “longwave/shallow thermocline” case discussed by L12, projecting primarily onto the BC_1 and BT modes, although a significant fraction of energy is found in the BC_2 and BC_3 modes.

3.4.4 The rough topography scenario

So far we have considered the traditional linear QG modes and the SQG solutions in a classical flat-bottom fashion, which implies no vertical velocity at the bottom, and, consequently, no buoyancy variations are allowed. (Mathematically, this is expressed by a homogeneous Neumann bottom boundary condition for the vertical structure, that is, $d\phi/dz = 0$ at $z = -H$.) In a more realistic scenario, with topography and bottom friction, the vertical velocity is not zero at the bottom [e.g., Vallis, 2006; Ferrari and

Figure 3.9: First EOF (filled circles) of the velocity anomalies for W335 mooring. Comparison against 1 QG mode (BC_1) and 3 QG modes ($BC_1+BC_2+BC_3$), and SQG solution for a rough topography scenario. The BT mode vanishes in this case.

Wunsch, 2010]. Here we investigate how topography affects the vertical structure of the QG modes and SQG solution and its comparison against the 1st EOF. For simplicity, we neglect the bottom friction. This would be the case when low stratification at the bottom and relatively small viscosity combine to produce a negligible friction forcing term [*Ferrari and Wunsch, 2010*]. The linearized bottom boundary condition in terms of the vertical structure becomes

$$\frac{c_y f_0}{N^2} \frac{d\phi}{dz} - \frac{d\eta_b}{dx} \phi = 0, \quad \text{at } z = -H, \quad (3.10)$$

where ϕ is the vertical structure (although we are using the QG modes notation, the same boundary condition will be used for the SQG solution, substituting ϕ for χ); for simplicity, we considered that the topography only varies in the x -direction (*i.e.* the departure from the mean depth is $\eta_b = \eta_b(x)$; x being approximately the cross isobath direction); c_y is the phase speed of the QG/SQG waves in the y -direction. Eq. 3.10 is a mixed boundary

(Dirichlet/Neumann) condition and as it depends on the wavenumber ($c_y = c_y(l)$, where l is the y-direction wavenumber), the Sturm-Liouville problem does not hold; the QG modes are not orthogonal [*e.g.*, Szuts *et al.*, 2012]. L12 computed the QG modes and SQG solutions for two extreme cases in which the Sturm-Liouville problem holds: the classical flat bottom ($d\phi/dz = 0$) and rough topography ($\phi = 0$). It is interesting to evaluate the limits in which the homogeneous Dirichlet boundary condition ($\phi = 0$) could be applied. Assuming the allowable error to be 10% we could apply the Dirichlet condition provided

$$c_y \leq 0.1 \frac{N^2|_{-H} H \left| \frac{d\eta_b}{dx} \right|}{|f_0|}. \quad (3.11)$$

We estimate the right hand side of Eq. 3.11 for the three moorings. The topographic gradient was estimated from the ETOPO2 data base and H is taken to be the mooring local depth. Generally for the three mooring regions, the Dirichlet boundary condition holds for waves with phase speeds smaller than 0.2-0.3 m s⁻¹ (Table 3.5). Estimates for phase speeds within these regions are rare. *da Silveira et al.* [2008]’s analysis of baroclinic instability for the MARLIM mooring region suggests phase speeds of approximately 0.05 m s⁻¹ for the most unstable Brazil Current baroclinic waves. Therefore, in this case, the Dirichlet boundary condition should be applied. Indeed our scaling arguments point out that the classical flat-bottom QG modes would hold only for much gentler slopes (reciprocal of Eq. 3.11). Hence, in these three cases, the Dirichlet boundary condition (rough topography) may be a better approximation than the classical flat bottom boundary condition.

A high bottom friction regime would also lead to a homogeneous Dirichlet bottom boundary condition ($\phi = 0$). This does not seem to be the case since the 1st EOF of the cross-isobath component is also affected above the bottom; for instance, the cross-isobath eigenstructure for the W335 mooring almost vanishes at 2500 m depth (Figure 3.9), suggesting that the effect is not restricted to a boundary layer. Near-bottom horizontal velocity intensification occurs, *e.g.*, in regions with closed f/h (where $h = H + \eta_b$ is the total depth) contours [*e.g.*, Dewar, 1998], but the topography is almost meridionally oriented off Brazil. Furthermore, as this near-bottom variability is likely not coherent to

Table 3.5: The estimated limit for along-isobath phase speed c_y for the Dirichlet boundary condition (rough topography) to hold.

	Dirichlet ($c_y \leq [\text{m s}^{-1}]$)
MARLIM	0.33
WOCE 333	0.22
WOCE 335	0.26

that of the surface-intensified mesoscale eddies in this region, this effect would not be present in the 1st EOF.

We recompute the QG modes as well as the SQG solutions using this new bottom boundary condition (Table 3.3; Figures 3.7, 3.8 and 3.9). The rough topography seems to make the interpretation of the 1st EOF in terms of baroclinic modes worse. (Note that the BT mode vanishes in this case [L12].) In general, the representation close to the bottom, particularly for the x -component (Figure 3.9) is improved. For the MARLIM mooring, the BC1 mode just contains 67.9 % of the 1st EOF variance (compared to 77.8 % of the BT/BC1 linear combination for the flat bottom case). Similarly, for the W335 mooring the BC1 mode is inadequate, accounting for only 60.2 % of the 1st EOF variance (compared to 69.1 % of the BT/BC1 flat bottom case). For the W333 case the BC1 mode contains 89.5 % of the 1st EOF variance (compared to 91.0 % of the BT/BC1 flat bottom case).

On the other hand, the SQG solution still seems to be a fair model to represent the 1st EOF at the MARLIM and W335 moorings. The SQG solution is modified only close to the bottom and remains consistent with the data, accounting for 77.1 % (84.9 %) of the (x -component) 1st EOF variance at the MARLIM (W335) mooring. In particular, the near-surface portion of the SQG solution is not significantly affected by the rough topography. Therefore the SQG solutions remain a better model (as compared to the BT/BC1 combination) to represent the sharp decay observed in the MARLIM and W335 moorings (Figures 3.7 and 3.9).

The SQG solutions project primarily onto the BC1 for the rough topography scenario. At the W333 mooring, the BC1 accounts for 65.7 % of the variance; the BC2 and BC3 modes contain 24.7 % and 8.8 % of it, respectively. For the W335 (MARLIM) mooring, the BC1 mode contains 50.7 % (56.9 %) of the variance. A significant amount of energy is found in the BC2 and BC3 modes, which account for 23.6 % (25.9 %) and 12.9 % (10.7

Figure 3.10: Flat-bottom SQG solution and its synthesis in terms of 4 QG modes ($BT+BC_1+BC_2+BC_3$) for MARLIM mooring. The synthesis accounts for 95.0 % of the depth-integrated SQG solution variance.

%) of the 1st EOF variance, respectively.

3.5 Summary and discussion

We test two dynamical models of vertical structure (traditional QG modes and SQG solutions) against 1st EOF from two moorings in the Brazil Current domain (MARLIM and W333) and one farther offshore (W335). Traditionally, these EOFs are interpreted as a phase-locked BT and BC1 mode linear combination. However, this linear combination poorly captures the sharp decay observed in the upper 500 m (300 m) at the W335 (MARLIM) mooring, albeit it contains 69.6 % (77.8 %) of the depth-integrated 1st EOF variance. Conversely, at the W333 mooring, which does not exhibit a near-surface sharp decay, the BT/BC1 linear combination is a better representation and contains 91.0 % of the 1st EOF variance.

The second model of the vertical structure is a combination of SQG waves. The vertical decay of this SQG solution is consistent with the statistics of the variability at the

W335 mooring. For this mooring, it accounts for 85.0 % of the depth-integrated variance and, in particular, captures the exponential decay in the upper 500 m. To some extent, similar results were obtained for the MARLIM region. In this case, the SQG solution accounts for 82.5 % of the 1st EOF vertical structure. However, the SQG solution presents a sharp decay which is inconsistent with the observed variability at the W333 mooring.

Ferrari and Wunsch [2010] argue and present some mooring-based evidence that a phase relationship between BT and BC1 modes is consistent with the SQG solution. This does not seem to be true for the W335 and the MARLIM moorings. The phase-locked combination between BT and BC1 mode produces just part of the surface intensification; it does not account for the sharp vertical decay observed in this region.

Lapeyre [2009], analyzed numerical model results from the *Parallel Ocean Program* model in the North Atlantic, and concluded that the SQG solution dominates the upper 600 m in the Gulf Stream region. The results for the W335 mooring and, to some extent, MARLIM mooring are consistent with this. However, we recognize that the SQG solutions project onto the QG modes, and another explanation for the sharp decay is simply a richer baroclinic composition. Indeed, this is not inconsistent with the SQG solution model if one considers that the traditional QG modes span the subspace of solutions. The projection of the SQG solutions onto the QG modes is in agreement with this: Considerable energy in the higher baroclinic modes (BC2 and BC3) to account for the sharp decay in the upper 500 m. For instance, Figure 3.10 depicts the flat-bottom SQG solution for the MARLIM mooring and its synthesis using 4 QG modes. This synthesis accounts for 95.0 % of the depth-integrated SQG solution variance. Results for the WOCE 335 mooring are very similar, where the 4 QG mode synthesis accounts for 85.0 % of the depth-integrated SQG solution variance, particularly representing the sharp near-surface decay. This is consistent with the fact that the inclusion of the BC2 and BC3 allows for the QG modes to account for up 98.5 % of the 1st EOF variance. Consequently, as far as near-surface decay is concerned, SQG solutions and four QG mode combination are indistinguishable. In other words, the SQG solution converges to a 4 QG mode representation and both are consistent with the data.

The calculations are repeated for a rough topography scenario, in which the velocity vanishes at the bottom. In general, the rough topography does not improve the results when trying to interpret the 1st EOF on the basis of the traditional QG mode. On the other hand, the SQG solution only changes close to the bottom, and still accounts for the sharp vertical decay. In particular, the SQG solutions (and higher order QG mode combination) for rough topography better matches the 1st EOF for the x-component. Additionally, in line with L12, the results show that for a rough topography, the similarity between the BC1 mode and SQG is higher, although higher order QG modes are necessary to account for the near-surface sharp decay.

The consistency between the SQG solutions and the 1st EOF vertical structure implies that SQG-based methods for reconstructing the subsurface dynamics [*e.g.*, *Lapeyre and Klein, 2006; LaCasce and Mahadevan, 2006; Isern-Fontanet et al., 2008*] are likely to work, provided that the surface kinetic energy is matched. A natural question is whether these methods would represent the correct physics, that is, if the SQG dynamics is dominant in these regions. To answer this question, we evaluate whether the SQG solutions can account for the amplitude of the observed eddy field. We estimate the surface velocity for the SST snapshots following the SQG methodology, assuming that the surface density is dominated by SST gradients [*e.g.*, *LaCasce and Mahadevan, 2006*]. The lateral SST gradient predicts eddy amplitudes (here defined as the spatial root mean square (RMS) of the velocity field) of about 0.07 and 0.02 m s⁻¹ at the MARLIM and W335 regions, respectively. This represents only about 20 % of the RMS of the uppermost velocity measurements at these moorings. Although the spatial RMS of the snapshot may not be directly comparable to the temporal RMS in a single location, these estimates are (at least) suggestive of a more complex (surface/interior) eddy dynamics. Indeed, the surface-intensified stratification affects the penetration of SQG waves, reducing its magnitude even at the surface (L12). In addition, the constant interior PV assumption is not strictly valid in these regions. The presence of the vertical shear of the Brazil Current (MARLIM and W333) and of a recirculation flow (W335) are associated with interior PV gradients. Therefore, it is likely that these SQG-like vertical structures are a combination

of the surface buoyancy gradients and the surface-intensified interior PV. Nonetheless, it is remarkable that the SQG solutions correctly represent the sharp vertical decay at the MARLIM and W335 moorings.

How can one explain such SQG-like vertical structures? An explanation could be sought on the basis of necessary conditions for baroclinic instability. Specifically, the relative importance of surface and interior contributions depends on the velocity vertical shear near the surface [Lapeyre, 2009], and this may determine the type of instability that is taking place [*e.g.*, Pedlosky, 1987]. Although Lapeyre [2009] argues that local linear baroclinic instability does not fully explain the differences in interior/surface mode decomposition in the North Atlantic, arguments based on the Charney-Stern-Pedlosky criterion for linear baroclinic instability [*e.g.*, Vallis, 2006] seem to provide a consistent explanation for the present results. In particular, the vertical shear of the long-term mean flow is intensified close to the surface at the MARLIM and W335 moorings (Figures 3.11 and 3.13), producing a long-term mean PV gradient that presents a relatively shallow zero-crossing. (Under the local approximation, we neglect the contribution of the relative vorticity in the PV. This seems a consistent approximation for the study of mesoscale phenomena [Tulloch *et al.*, 2011].) The interaction of the surface shear with the PV gradient in the interior could lead to the development of Charney-like instabilities [Tulloch *et al.*, 2011], producing a SQG-like vertical structure⁴. (Here, as background flow is meridional, the condition for the Charney-like instability is that the surface vertical shear has the same sign of the zonal PV gradient somewhere in the interior [*e.g.*, Isachsen, 2011].) The conditions for “shallow” Phillips-like instabilities [Tulloch *et al.*, 2011] are also satisfied. This would be the case in which the SQG-like vertical structure is solely generated by the surface-intensified PV (without surface buoyancy variations at the surface).

In contrast, the shear at the W333 mooring is intensified at mid-depth (Figure 3.12), producing a deeper (as compared to the MARLIM and W335 moorings) zero-crossing in the PV gradient profile. In this case, it is likely that Phillips-like instabilities [Tulloch

⁴Details on the necessary conditions for baroclinic instability are given in Appendix F.

Figure 3.11: Mean y-component velocity vertical shear (dashed line) and mean potential vorticity x-component gradient (continuous line) for MARLIM mooring. The shear and PV gradient are normalized by their maximum magnitudes $1.3 \cdot 10^{-3} \text{ [s}^{-1}\text{]}$ and $4.9 \cdot 10^{-10} \text{ [m}^{-1} \text{ s}^{-1}\text{]}$, respectively.

Figure 3.12: Mean y-component velocity vertical shear (dashed line) and mean potential vorticity x-component gradient (continuous line) for W333 mooring. The shear and PV gradient are normalized by their maximum magnitudes $6.0 \cdot 10^{-4} [\text{s}^{-1}]$ and $4.4 \cdot 10^{-10} [\text{m}^{-1} \text{s}^{-1}]$, respectively.

et al., 2011] take place, consistent with the fact that the vertical structure of the 1st EOF is captured by a linear combination of two QG modes. These explanations for the observed structure are tentative, as it is difficult to accurately estimate the shear at the surface owing to the lack of instruments. In addition, it is likely that both Charney-like and Phillips-like instabilities are important. It is also well-known that the local linear baroclinic instability analysis “ignores many other dynamical possibilities” [Tulloch *et al.*, 2011]. Furthermore, forced solutions could also be important; indeed, the negative equivalent depth modes [*e.g.*, Philander, 1978] have SQG-like vertical structures. Therefore, the vertical structure may be a response to much more complicated surface/near-surface processes. Notwithstanding these caveats, the local linear QG baroclinic instability arguments seem to plausibly rationalize the results obtained here.

Figure 3.13: Mean y-component velocity vertical shear (dashed line) and mean potential vorticity x-component gradient (continuous line) for W335 mooring. The shear and PV gradient are normalized by their maximum magnitudes $6.5 \cdot 10^{-5} [\text{s}^{-1}]$ and $3.8 \cdot 10^{-11} [\text{m}^{-1} \text{s}^{-1}]$, respectively.

3.6 Concluding remarks

The present work shows that the SQG solution is consistent with the vertical structure of the 1st EOF at two (one in the Brazil Current domain and one offshore) out of the three moorings analyzed. In particular, the SQG solution can account for the observed sharp near-surface decay. However, the only conclusion we can reach is that the SQG is a better model than the traditional BT/BC1 linear combination in representing the vertical structure of the statistics of mesoscale variability in these regions, specifically its near-surface sharp decay. Nevertheless, our results point out to the presence of a more complicated near-surface structure. On a regional scale, this implies that care should be taken in interpreting altimeter data only as due to 1st BC mode motions. Although an SQG interpretation or simply a richer baroclinic mode composition are both consistent with the data, the observed vertical structures may be the response to much a more complicated surface/near-surface processes. SQG-based models to recover subsurface fields

are likely to correctly represent the flow in these regions, although the interior PV is clearly not constant. Regions where the main PV gradients are confined to the surface tend to present a SQG-like vertical structure. In fact, this is simply a generalization of the PV sheet argument [*Bretherton*, 1966] to a surface-trapped PV. Future work should combine global mooring dataset, SSH and SST observations, and realistic numerical simulations to further investigate these issues, and consider local linear baroclinic instability analysis to rationalize the results.

Chapter 4

Final remarks

4.1 Thesis summary

We studied the variability and dynamics in the Brazil Current (BC) region by applying simple results from the quasi-geostrophic (QG) theory to rationalize observations. In Chapter 2, we used *four* current meter mooring records and quasi-synoptic hydrographic observations in conjunction with a linear QG model to study the changes in BC System off the southeast Brazil (22° - 28° S) as well as its implications. We estimated that the BC thickens substantially; it is a 350 m deep flow at the MARLIM mooring (22.7° S) and becomes a 800 m deep flow at the W333 mooring (27.9° S). This deepening is accompanied by changes in the barotropic/baroclinic partition of kinetic energy. In particular, the total flow is mainly baroclinic at the MARLIM mooring ($\sim 70\%$ baroclinic) and becomes mainly barotropic at the W333 mooring ($\sim 63\%$ barotropic), thus corroborating that the Santos bifurcation is associated with a strong increasing in the barotropic component of the BC System. The eddy kinetic energy was estimated to be important along all the BC path, accounting from 33% (at the C3 mooring, 25.5° S) up to 60% (at the DFBS mooring, 24.15° S) of the total kinetic energy. Hydrographic observations reveal that the BC transport is increased by at least 5 Sv from 25.5° S to 27.9° S. Moreover, a 1-dimensional QG linear stability model proved successful in predicting some of the observed and previously reported characteristics of the BC meandering. In particular, the model predicts quasi-standing waves at the MARLIM mooring and southwestward-propagating waves

within the Santos Basin (at the C3 and W333 moorings).

In Chapter 3, we have investigated further the mesoscale variability in the Brazil Current region, particularly focusing on QG models of vertical structure. Specifically, we tested two simplified QG models of vertical structure, namely traditional QG modes and surface QG (SQG) solutions, against the vertical structure of the 1st empirical orthogonal function (EOF) from current meter observations (two in the BC domain and one offshore it). We showed that while in two scenarios (flat-bottom and rough-topography) at two moorings (MARLIM and W335) the SQG solutions seem to better match the data as compared to the traditional barotropic/1st baroclinic (BT/BC1) mode combination, the SQG solutions are indistinguishable from a *four* QG mode combination. At a third mooring (W333), the BT/BC1 mode combination successfully represented the vertical structure of the 1st EOF in both scenarios. Based on necessary condition for baroclinic instability, we argued that the SQG-like or BT/BC1 mode combination vertical structures may be explained by the type of instability that take place in each region. Indeed, this is consistent with the results from Chapter 2, which predicted a surface-intensified most-unstable mode at the MARLIM mooring (which has an SQG-like 1st EOF) and a mid-depth “Phillips-like” most-unstable mode at the W333 mooring (which is well captured by the BT/BC1 mode combination).

4.2 Future work

The picture that emerges is that of a BC significantly deepening off southeast Brazil (22°S – 28°S) and presenting high eddy kinetic energy levels (Chapter 2) and possibly different dynamical regimes (Chapter 3). Further investigation of these issues is instrumental to better understanding and modeling the ocean within this region. As discussed Chapters 2 and 3, there are numerous open questions that should be addressed in future work, for instance:

1. *Is there a stationary Brazil Current just to the south of Cape Frio ($\sim 23^{\circ}\text{S}$)?*
2. *Is the Brazil Current potentially baroclinically unstable to the south of $\sim 28^{\circ}\text{S}$?*

3. *What are the relative importance of barotropic and baroclinic instabilities in the Brazil Current?*
4. *How are the barotropically unstable waves within the Santos Basin?*
5. *What is the role of forced solutions in the BC? Can these forced solutions account for the observed SQG-like vertical structure of the subinertial variability?*

It is my personal view that these questions should be tackled using observations (in-situ and remote) to better describe this yet poorly studied region, and judiciously combining theory and process-oriented primitive-equation numerical simulations to test hypotheses and rationalize the experimental facts.

Appendix A

Filling gaps using EOFs

Current meter gaps were filled using the empirical orthogonal functions (EOFs) following the algorithm proposed by *Beckers and Rixen* [2003]. The method consists in five major steps:

1. *Substitute zeros in the gappy positions.* (Here it is applied to the velocity anomaly field. Note that the zeros represent an unbiased initial guess, assuming that the removed mean was unbiased.)
2. *Estimate the EOFs.*
3. *Reconstruct the series using the statically significant modes.* (We used a method based on 100 Monte Carlo simulations to determine which statistical modes are meaningful [*Preisendorfer*, 1988]. Other criteria are discussed in *Beckers and Rixen* [2003].)
4. *Substitute the reconstructed series in the gappy positions.*
5. *Run steps 2, 3 and 4 iteratively until its convergence.* (In order to determine the convergence, compare the reconstructed series at time t with that at time $t-1$. We considered that the method converged when this difference (in the gappy positions) was lower than 10^{-5} .)

The method does not work for instantaneous gaps in all positions (in all current meters) since the data matrix becomes singular [*Beckers and Rixen*, 2003].

Figure A.1: Time series for the along-isobath velocity anomalies for the 95 m instrument at the W333 mooring. Black line represents the series with false gaps, the blue line the reconstructed series, and the red dots represents the real (observed) values in the gappy positions. The RMS difference between the reconstructed series and observed values is 0.01.

A test was performed using the W333 series, which did not present gaps. False gaps were randomly created at different instuments and with different lengths. The RMS difference between the real (observed) along-isobath velocity anomalies and those reconstructed (“filled”) are typically less than 0.05. For instance, Figure A.1 presents the time series of the along-isobath velocity anomalies for the 95 m instrument at the W333 mooring. The “filling” process yielded a series (in the gappy positions) that are 1% different that the observed one.

Computational programs written for this algorithm can be obtained at
ftp://ftp.io.usp.br/lado/rocha_papers
 or directly from the author (cesarbarbedorocha@gmail.com).

The gappy and filled series for MARLIM, DFBS and C3 moorings are presented in Figures A.2, A.3 and A.4, respectively.

Figure A.2: Velocity vector time series for MARLIM mooring: (a) gappy series and (b) filled series.

Figure A.3: Velocity vector time series for DFBS mooring: (a) gappy series and (b) filled series.

Figure A.4: Velocity vector time series for C3 mooring: (a) gappy series and (b) filled series.

Appendix B

Statistics for zonal and meridional velocity components

Tables B.1, B.2, B.3 and B.4 presents the statistics for the non-rotated velocity records at the MARLIM, DFBS, C3 and W333 moorings, respectively. Technically, all estimates are statistically significant. However, estimates for the mean velocity components presents low statistical significance at the DFBS mooring.

Table B.1: Velocity statistics for MARLIM mooring. The 95% confidence interval was estimated following a Student's test.

Instrument [m]	u (zonal)		v (meridional)	
	Mean [m s ⁻¹]	Std [m s ⁻¹]	Mean [m s ⁻¹]	std [m s ⁻¹]
50	-0.13± 0.09	0.30± 0.07	-0.29± 0.11	0.34 ± 0.08
100	-0.10± 0.09	0.29± 0.07	-0.25± 0.10	0.31 ± 0.07
250	0.02± 0.05	0.17± 0.04	-0.12± 0.05	0.16 ± 0.04
350	0.05± 0.04	0.12± 0.03	-0.01± 0.04	0.13 ± 0.03
450	0.10± 0.03	0.10± 0.02	0.02± 0.03	0.09 ± 0.02
650	0.11± 0.02	0.05± 0.01	0.15± 0.02	0.07 ± 0.01
750	0.14± 0.02	0.07± 0.01	0.13± 0.02	0.06 ± 0.01
950	0.16± 0.02	0.06± 0.01	0.15± 0.02	0.07 ± 0.02
1050	0.14± 0.02	0.07± 0.02	0.16± 0.03	0.08 ± 0.02

Table B.2: Velocity statistics for FBS mooring. The 95% confidence interval was estimated following a Student's test.

Instrument [m]	u (zonal)		v (meridional)	
	Mean [m s ⁻¹]	Std [m s ⁻¹]	Mean [m s ⁻¹]	std [m s ⁻¹]
30	-0.16 ± 0.13	0.38 ± 0.09	0.07 ± 0.04	0.17 ± 0.03
50	-0.16 ± 0.14	0.39 ± 0.10	0.07 ± 0.04	0.18 ± 0.03
70	-0.15 ± 0.14	0.40 ± 0.10	0.06 ± 0.04	0.19 ± 0.03
500	0.05 ± 0.04	0.12 ± 0.03	0.03 ± 0.01	0.06 ± 0.01
800	0.12 ± 0.03	0.08 ± 0.02	0.04 ± 0.01	0.04 ± 0.01
1000	0.10 ± 0.03	0.09 ± 0.02	0.03 ± 0.01	0.04 ± 0.01

Table B.3: Velocity statistics for C3 mooring. The 95% confidence interval was estimated following a Student's test.

Instrument [m]	u (zonal)		v (meridional)	
	Mean [m s ⁻¹]	Std [m s ⁻¹]	Mean [m s ⁻¹]	std [m s ⁻¹]
29	-0.26 ± 0.04	0.16 ± 0.03	-0.43 ± 0.05	0.21 ± 0.03
91	-0.29 ± 0.04	0.17 ± 0.03	-0.40 ± 0.05	0.21 ± 0.03
293	-0.14 ± 0.03	0.12 ± 0.02	-0.18 ± 0.03	0.14 ± 0.02
698	0.04 ± 0.02	0.08 ± 0.01	0.07 ± 0.02	0.11 ± 0.02

Table B.4: Velocity statistics for W333 mooring. The 95% confidence interval was estimated following a Student's test.

Instrument [m]	Along-front		Cross-front	
	Mean [m s ⁻¹]	Std [m s ⁻¹]	Mean [m s ⁻¹]	std [m s ⁻¹]
60	-0.16 ± 0.03	0.17 ± 0.02	-0.30 ± 0.04	0.23 ± 0.03
77	-0.16 ± 0.04	0.18 ± 0.03	-0.31 ± 0.05	0.23 ± 0.03
95	-0.16 ± 0.03	0.17 ± 0.02	-0.31 ± 0.05	0.23 ± 0.03
112	-0.16 ± 0.03	0.17 ± 0.02	-0.30 ± 0.05	0.23 ± 0.03
138	-0.16 ± 0.03	0.16 ± 0.02	-0.29 ± 0.05	0.22 ± 0.03
155	-0.15 ± 0.03	0.15 ± 0.02	-0.29 ± 0.05	0.22 ± 0.03
173	-0.15 ± 0.03	0.15 ± 0.02	-0.28 ± 0.04	0.21 ± 0.03
230	-0.14 ± 0.03	0.13 ± 0.02	-0.26 ± 0.04	0.19 ± 0.03
475	-0.09 ± 0.01	0.07 ± 0.01	-0.13 ± 0.03	0.12 ± 0.02
680	-0.03 ± 0.01	0.05 ± 0.01	-0.05 ± 0.02	0.10 ± 0.01
885	-0.02 ± 0.01	0.03 ± 0.00	0.01 ± 0.02	0.11 ± 0.01

Appendix C

Dynamical fit: Residuals

To evaluate the residuals (Eq. 2.7) of the dynamical fit to the instantaneous velocity and velocity anomalies we compute the dynamically synthesized velocity at each time. The computation is similar to that for the mean flow. That is, at each time we estimated the mode amplitudes (Eq. 2.8) and then recover the velocity profiles (2.9). We then obtain a series of dynamically synthesized velocity for each record. Similar syntheses are done for the velocity anomalies series. The residuals is the difference between the observed and the synthesized series. For instance, Figures C.1 and C.2 present the velocity and velocity anomalies series for the 750 m instrument at the MARLIM mooring. The normalized RMS difference between the observed zonal component and the recovered velocity anomalies is ~ 0.25 . However, it is important to recognize that at each instant of time the depth-integrated is the best in the least-squares sense.

Tables C.1, C.2, C.3 and C.4 display the normalized RMS difference at all depths for MARLIM, DFBS, C3 and W333 moorings, respectively. Generally, the normalized RMS difference is small; the synthesized series account for more than 90-95% of the variance, with few exceptions (as the MARLIM mooring at 750 m).

Figure C.1: Time series of velocity at 750 m for MARLIM mooring: (a) meridional and (b) zonal. Blue, magenta and black curves represent the measured, synthesized by the dynamical modes and the residual, respectively.

Figure C.2: Time series of velocity anomalies at 750 m for MARLIM mooring: (a) meridional and (b) zonal. Blue, magenta and black curves represent the measured, synthesized by the dynamical modes and the residual, respectively.

Table C.1: Normalized RMS difference between synthesized velocity (u, v) and velocity anomaly (ua, va) series for MARLIM mooring.

Depth (m)	u	v	ua	va
50	0.00073	0.00077	0.04372	0.06272
100	0.00166	0.00185	0.05516	0.07396
250	0.01150	0.01409	0.08888	0.08784
350	0.04061	0.05737	0.15246	0.18103
450	0.04374	0.10111	0.15256	0.14200
650	0.09918	0.10723	0.13839	0.20663
750	0.10859	0.16358	0.24900	0.23811
950	0.10322	0.14918	0.22415	0.29021
1050	0.06488	0.08441	0.15316	0.20749

Table C.2: Normalized RMS difference between synthesized velocity (u, v) and velocity anomaly (ua, va) series for DFBS mooring.

Depth (m)	u	v	ua	va
30	0.03134	0.05936	0.06830	0.06214
50	0.05518	0.10707	0.06671	0.11587
70	0.02523	0.04909	0.11535	0.09522
500	0.00153	0.00226	0.02358	0.01118
800	0.00229	0.00465	0.03615	0.01698
1000	0.00148	0.00401	0.03339	0.01008

Table C.3: Normalized RMS difference between synthesized velocity (u, v) and velocity anomaly (ua, va) series for C3 mooring.

Depth (m)	u	v	ua	va
29	0.0026	0.0018	0.0113	0.0179
91	0.0073	0.0059	0.0264	0.0390
293	0.0082	0.0085	0.0161	0.0242
698	0.0136	0.0134	0.0092	0.0168

Table C.4: Normalized RMS difference between synthesized velocity (u , v) and velocity anomaly (ua , va) series for C3 mooring.

Depth (m)	u	v	ua	va
60	0.07796	0.06343	0.11435	0.12091
77	0.09450	0.07519	0.13392	0.12464
95	0.05929	0.04226	0.09214	0.07709
112	0.04711	0.02999	0.06929	0.05154
138	0.05001	0.03438	0.06516	0.05417
155	0.05356	0.03642	0.07327	0.05972
173	0.06531	0.05349	0.09722	0.08805
230	0.07282	0.05908	0.08017	0.08413
475	0.02361	0.01946	0.00951	0.01762
680	0.02331	0.01630	0.00862	0.02153
885	0.01337	0.00567	0.00345	0.02550

Appendix D

QG modes and SQG solutions:

Numerical resolution

There are two approaches to numerically solve for the vertical structure of the traditional quasi-geostrophic (QG) modes and surface quasi-geostrophic (SQG) solutions. The first (and most typical) approach is simplify Eqs. 3.1 and 3.3 to a discrete grid [*Kundu et al.*, 1975]. The second approach considers the problem as a layered hydrostatic model. This approach is appealing given the fact that observations are always specified at finite number of points [*Flierl*, 1978]. While the first approach is a mathematical simplification, the second one is a simplified physical system. Nonetheless, both lead to the same set of linear equations. Therefore, there are no operational differences between these two techniques. Indeed, *Pedlosky* [1987] discusses the equivalence of layered models to the “level” models (finite-difference approximation). In what follows, we give details on the eigenvalue problem for the layered hydrostatic model approach and the numerics of the mode numerical calculation.

Computational programs written for the QG modes and SQG solutions computations can be obtained at

`ftp://ftp.io.usp.br/lado/cesar/paper_sqg/tools`

or directly from the author (`cesarbarbedorocha@gmail.com`).

D.1 Linear quasi-geostrophic vertical modes

The derivation of the N-layer hydrostatic equations is standard and can be found in numerous textbooks. Specifically, the QG potential vorticity in each layer is defined by [Pedlosky, 1987; Vallis, 2006; McWilliams, 2006]

$$q_1 = \nabla^2 \psi_1 + \beta y - \frac{f_0^2}{g'_{1.5} H_1} (\psi_1 - \psi_2), \quad (\text{D.1a})$$

$$q_n = \nabla^2 \psi_n + \beta y - \frac{f_0^2}{g'_{n+0.5} H_n} (\psi_n - \psi_{n+1}) + \frac{f_0^2}{g'_{n-0.5} H_n} (\psi_{n-1} - \psi_n), \quad 1 < n < N, \quad (\text{D.1b})$$

$$q_N = \nabla^2 \psi_N + \beta y + \frac{f_0^2}{g'_{N-0.5} H_1} (\psi_{N-1} - \psi_N), \quad (\text{D.1c})$$

where, H_n is the thickness of the n_{th} layer, $g'_{n-0.5} = g \frac{\rho_n - \rho_{n-1}}{\rho_0}$, and $n - 0.5$ indicates the interface between the $(n - 1)_{th}$ and n_{th} layers. A scheme of the vertical grid used for the numerical solution is presented in Figure D.1. In deriving Eq. D.1a, we assumed no buoyancy anomalies at the surface. This is further generalized in the SQG problem numerical scheme (D.2).

Expanding the solutions in orthonormal eigenmodes [McWilliams, 2006]

$$\Psi = \mathbf{G} \hat{\Psi}, \quad (\text{D.2})$$

where $\psi = \{\psi_1, \psi_2, \dots, \psi_N\}^T$ is the $N \times 1$ stream function matrix, $\mathbf{G}_{N \times M}$ is the vertical structure of the m th ($m = 0, 1, 2, \dots, N - 1$) mode at N discrete layers, and $\hat{\psi}_{M \times 1}$ is the mode amplitude matrix. A similar decomposition is assumed for q_{QG} and $I\beta y$. Given the mode orthogonality, an inverse relation exists

$$\hat{\Psi} = \mathbf{G}^{-1} \Psi. \quad (\text{D.3})$$

Eqs. D.1a, D.1b, and D.1c, written in matrix notation, become

$$\mathbf{q}_{QG} = P\Psi + I\beta y, \quad (\text{D.4})$$

where I is the identity matrix, and P a matrix $N \times N$ representing the linear operator that accounts for the horizontal and vertical second-order derivatives related to the relative vorticity and vortex-stretching terms, namely,

$$\mathbf{P} = \mathbf{I}\nabla^2 - \mathbf{S}. \quad (\text{D.5})$$

Inserting Eqs. D.2 in Eq D.4, with Eq. D.5, we obtain [McWilliams, 2006]

$$\hat{q}_{QG} = \mathbf{G}^{-1}\mathbf{P}\mathbf{G}\hat{\psi} + \hat{\mathbf{I}}\beta y = [\mathbf{I}\nabla^2 - \mathbf{G}^{-1}\mathbf{S}\mathbf{G}]\hat{\psi} + \hat{\mathbf{I}}\beta y. \quad (\text{D.6})$$

As the modes are orthonormal, there is no cross-mode coupling. Therefore, $\mathbf{G}^{-1}\mathbf{S}\mathbf{G}$ ought to be a diagonal matrix such that

$$\mathbf{G}^{-1}\mathbf{S}\mathbf{G} = -\mathbf{\Lambda}^2, \quad (\text{D.7})$$

where $-\mathbf{\Lambda}^2$ is a diagonal matrix. Eq. D.7 leads to an eigenvalue problem,

$$\mathbf{S}\mathbf{G} = -\mathbf{\Lambda}^2\mathbf{G}, \quad (\text{D.8})$$

that is the linear normal QG modes are eigenvectors of the vortex-stretching matrix. Note that (by definition) $-\mathbf{\Lambda}^2 = \mathbf{R}^{-2} = \{R_1^{-2}, R_2^{-2}, \dots, R_m^{-2}\}$, where R_m is the m_{th} deformation

radius [McWilliams, 2006]. From Eqs. D.1a, D.1b, and D.1c, we can parse S elements:

$$\begin{aligned}
 S_{11} &= -\frac{1}{H_1^2} \frac{f_0^2}{N_{1.5}^2}, \quad S_{12} = \frac{1}{H_1^2} \frac{f_0^2}{N_{1.5}^2}, \quad S_{1,n} = 0, \quad n > 2, \\
 S_{21} &= -\frac{1}{H_2^2} \frac{f_0^2}{N_{1.5}^2}, \quad S_{22} = \frac{1}{H_2^2} \left(\frac{f_0^2}{N_{1.5}^2} + \frac{f_0^2}{N_{2.5}^2} \right), \quad S_{23} = -\frac{1}{H_2^2} \frac{f_0^2}{N_{2.5}^2}, \quad S_{2,n} = 0, \quad n > 3, \\
 &\dots \\
 S_{N1} &= 0, \quad n < N - 1, \quad S_{NN-1} = \frac{1}{H_N^2} \frac{f_0^2}{N_{N-0.5}^2}, \quad S_{NN} = -\frac{1}{H_N^2} \frac{f_0^2}{N_{N-0.5}^2},
 \end{aligned} \tag{D.9}$$

where $\bar{N}_{n-0.5}^2 \equiv -\frac{g'_{n-0.5}}{H_N}$ is the stratification frequency at $N - 1$ interfaces, that is $\bar{N}^2$ is given in $N - 1$ points. For observations vertically equispaced¹, we have $H_1 = H_2 = \dots = H_N \equiv \Delta z$. Thus, Eq. D.9 becomes

$$\mathbf{S}_{\mathbf{QG}} = \frac{f_0^2}{(\Delta z)^2} \begin{bmatrix} -\xi_1 & \xi_1 & 0 & 0 & \dots & 0 & 0 & 0 & 0 \\ \xi_1 & -(\xi_1 + \xi_2) & \xi_2 & 0 & \dots & 0 & 0 & 0 & 0 \\ 0 & \xi_2 & -(\xi_2 + \xi_3) & \xi_3 & \dots & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \dots & 0 & \xi_{N-1} & -(\xi_{N-1} + \xi_N) & \xi_N \\ 0 & 0 & 0 & 0 & \dots & 0 & 0 & \xi_N & -\xi_N \end{bmatrix}, \tag{D.10}$$

where $\xi_n = \bar{N}^{-2}(n)$, $n = 1, 2, 3 \dots N - 1$. This equation was derived for the classic flat-bottom (homogeneous Neumann boundary condition for velocity). For the rough topography case, we write $S_{NN-1} = 0$. The eigenvalue problem (Eq. D.8) is solved numerically using the Matlab Mathworks *Inc.* “eig.m” algorithm. (Similar open source algorithms are available for Python.)

In order to assess the accuracy numerical solution, we use an analytical² solution for

¹This is assumed for convenience.

²More accurately, this solution should be called semi-analytic because it ultimately leads to an eigenvalue problem that is solved numerically.

Ld (km)	1st BC	2nd BC	3rd BC
Analytical	30.5394	14.2045	9.2602
Numerical	30.6475	14.2551	9.2935

Table D.1: Three gravest baroclinic modes deformation radius (λ^{-2} in the eigenvalue problem for the baroclinic modes) for analytical and numerical solutions computed for an exponential stratification (see text for details).

an exponential stratification [LaCasce, 2012]

$$N^2(z) = N_0^2 e^{\alpha z}, \quad (\text{D.11})$$

where N_0 is constant and α^{-1} is an e-folding scale. Introducing Eq. D.11 in Eq. (QG modes), we obtain (see LaCasce [2012] for details)

$$J_0(2\gamma)Y_0(2\gamma e^{-\alpha H/2}) - Y_0(2\gamma)J_0(2\gamma e^{-\alpha H/2}) = 0, \quad (\text{D.12})$$

where J_i is the i_{th} order bessel function of the first kind, $\gamma = (N_0 \lambda^2)/\alpha f_0$. Eq. D.12 is an eigenvalue problem for γ . We solve this using a Newton-Rapshon algorithm [Strang, 2007]. The vertical solution for each mode is then [LaCasce, 2012]

$$\phi_j(z) = A e^{\alpha z/2} [Y_0(2\gamma)J_1(2\gamma e^{-\alpha H/2}) - J_0(2\gamma)Y_0(2\gamma e^{-\alpha H/2})] = 0, \quad (\text{D.13})$$

where ϕ_j is the j_{th} eigenfunction (QG vertical mode); Y_0 is the zeroth order bessel function of the first kind.

We compute the QG modes using Eqs. D.12 and D.13, and compare to those obtained numerically by using the exponential stratification (Eq. D.11; see Figure D.2) in Eqs. D.17 and D.8. The parameter of the exponential stratification (α and N_0^2) were obtained through a recursive least-squares fit [Wunsch, 2006] to the climatological stratification at the W335 mooring. The 3 gravest baroclinic deformation radius for analytical and numerical solutions are presented in Table D.1; the differences are very small (30-100 m). The RMS difference between the vertical modes computed analytically and those obtained numerically indicates that the modes are less than 1% different (Table D.1).

Figure D.1: Grid used for numerical solution of the QG mode and SQG solutions problems. Shown are arbitrary points $n-1$, n , $n+1$, $n+2$.

	1st BC	2nd BC	3rd BC
RMS_{diff}	0.0024	0.0048	0.0071

Table D.2: RMS difference between numerical and analytical solutions for the three gravest baroclinic modes computed for an exponential stratification (see text for details).

Figure D.2: (a) Exponential stratification (fit to the climatological stratification at W335 mooring). (b) 1st, 2nd and 3rd baroclinic linear QG modes. (c) SQG solution ($L = 280$ km). Analytical (blue) and numerical (magenta) solutions are visually indistinguishable.

D.2 Surface quasi-geostrophic solutions

As discussed in the text, the surface quasi-geostrophic (SQG) is not an eigenvalue problem. In particular, the SQG problem can be written, under the N-layer approximation (or simplify in finite differences), as

$$(\mathbf{S}_{\mathbf{QG}} - \mathbf{I}K^2)\chi = 0, \quad 1 < n \leq N, \quad (\text{D.14})$$

where K is the wavenumber, and $\mathbf{S}_{\mathbf{QG}}$ defined in Eq. D.17. At the upper boundary (Eq, upper boundary SQG), we have

$$\chi_1 - \chi_2 = \Delta z, \quad n = 1. \quad (\text{D.15})$$

Combining Eqs. D.14 and D.15, we obtain a linear system of N equations, easily recasted in matrix notation as

$$(\mathbf{S}_{\mathbf{SQG}} - \mathbf{I}K^2)\chi = \mathbf{b}, \quad 1 \leq n \leq N, \quad (\text{D.16})$$

where

$$\mathbf{S}_{\mathbf{SQG}} = \frac{f_0^2}{(\Delta z)^2} \begin{bmatrix} \Delta z & -\Delta z & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\ \xi_1 & -(\xi_1 + \xi_2) & \xi_2 & 0 & \cdots & 0 & 0 & 0 & 0 \\ 0 & \xi_2 & -(\xi_2 + \xi_3) & \xi_3 & \cdots & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 0 & \xi_{N-1} & -(\xi_{N-1} + \xi_N) & \xi_N \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 & \xi_N & -\xi_N \end{bmatrix}, \quad (\text{D.17})$$

and $\mathbf{b} = \{1, 0, \dots, 0\}^T$, and $\chi = \{\chi_1, \chi_2, \chi_3, \dots, \chi_N\}^T$ is the vertical structure at N depths. The SQG solution is then obtained by inverting the matrix $(\mathbf{S}_{\mathbf{SQG}} - \mathbf{I}K^2)$, or, more accurately, as the inverse may not exist, by projecting $\mathbf{b}$ onto the column space of $(\mathbf{S}_{\mathbf{SQG}} - \mathbf{I}K^2)$, namely

$$\chi = [(\mathbf{S}_{\mathbf{SQG}} - \mathbf{I}K^2)^T(\mathbf{S}_{\mathbf{SQG}} - \mathbf{I}K^2)]^{-1}(\mathbf{S}_{\mathbf{SQG}} - \mathbf{I}K^2)^T \mathbf{b}. \quad (\text{D.18})$$

Similar to the QG modes case, we test, for an exponential stratification (Eq. D.11), the numerical solution of the SQG vertical structure against that of an analytical solution [LaCasce, 2012]

$$\chi(z) = \frac{e^{\alpha z/2}}{\eta\alpha} \times \frac{K_0(2\eta e^{-\alpha z/2})I_1(2\eta e^{\alpha z/2}) + I_0(2\eta e^{-\alpha H/2})K_1(2\eta e^{\alpha z/2})}{I_0(2\eta)K_0(2\eta e^{-\alpha H/2}) - I_0(2\eta e^{-\alpha H/2})K_0(2\eta)}, \quad (\text{D.19})$$

where $\eta = N_0 K / (\alpha f_0)$, I_i (not to be confused with the identity matrix $\mathbf{I}$) and K_i (not to be confused with the wavenumber K) are the i_{th} order modified bessel functions of the first and second kind, respectively.

The numerical scheme correctly resolves the SQG vertical structure. For instance, for a wavelength of 280 km (Figure D.2), the numerical solution is less than 1% different than that obtained analytically; the RMS_{diff} between these solutions is 0.0064.

Appendix E

Instability model: Numerical resolution

The discretization of the eigenvalue problem for the local linear baroclinic instability is similar to that for the QG modes (Appendix D) as the vortex-stretching operator appears in several terms. Here for simplicity of notation we define

$$\Gamma = \frac{\partial}{\partial z} \left(\frac{f_0^2}{N^2} \frac{\partial}{\partial z} \right). \quad (\text{E.1})$$

It is very useful to recognize that the topographic term in Eq. 2.25 introduces a contribution to the mean QG potential vorticity (Q) at the bottom¹, that is [Smith, 2007]

$$Q = \left(\Gamma V + \frac{f_0}{\Delta z} \eta_b \right) \hat{x} \quad \text{at } z = -H, \quad (\text{E.2})$$

where $\hat{x}_{rot}$ is the unit vector in the cross-isobath direction. The other terms in Eqs. 2.25 and 2.24 are contained in the discretized² form of Γ [Smith, 2007]. Thus, we can easily recognize the generalized eigenproblem for ω , in matrix notation,

$$\mathbf{A}\varphi = \omega \mathbf{B}\varphi, \quad (\text{E.3})$$

¹This is similar to the homogeneous QG models [e.g., McWilliams, 2006]

²Or the layered model as discussed in D.

where $\mathbf{A}^3$ and $\mathbf{B}$ are $N \times N$ matrices. From Eqs. 2.23, 2.24 and 2.25, we have

$$\mathbf{B} = \mathbf{S}_{\mathbf{QG}} - \mathbf{I} \left(K^2 + \frac{\pi^2}{4L_c^2} \right), \quad (\text{E.4})$$

and

$$\mathbf{A} = -l(\mathbf{S}_{\mathbf{QG}}\mathbf{V} + \mathbf{Q}\mathbf{x}_N) + l\mathbf{V}\mathbf{B}, \quad (\text{E.5})$$

where L_c is the channel width, $\mathbf{I}_{N \times N}$ is the identity matrix, $\mathbf{V} = \{v_1, v_2, \dots, v_{N-1}, v_N\}^T$, $\mathbf{Q}\mathbf{x}_N = \{0, 0, \dots, 0, \frac{f_0}{\Delta z} \left| \frac{\partial \eta_b}{\partial x} \right| \}^T$, and $\mathbf{S}_{\mathbf{QG}}$ is defined in Eq. D.17. The generalized eigenvalue problem (Eq. E.3) is solved using the “eig.m” Matlab *Mathworks Inc.* function.

A loop then is performed in the wavenumber (100 wavenumbers corresponding to wavelengths of 10-1000 km). For each wavenumber (l), the eigenvalue problem is solved; the unstable mode at each wavenumber (l) is take as $\omega_i = \max[\text{Im}(\omega)]$, and the vertical structure its eigenvector.

Note that the resolution of this numerical scheme is $O[(\Delta z)^2]$ and $O(\Delta z)$ close to the boundaries [Smith, 2007]. After several tests, we found that the solution does not change $\Delta z \leq 10$ m. Therefore, with use a vertical resolution of 10 m, which gives about 100 vertical levels.

In order to test this numerical solution, it is useful to compute the instability properties for theoretical cases which have analytical solutions. One such case is the classical *Eady* [1949] model. This model assumes constant stratification and constant velocity vertical shear. We can write a mean velocity as

$$V(z) = V_0 + \Lambda z, \quad (\text{E.6})$$

where Λ is the vertical shear, and we have assumed that a strictly zonal mean flow. Using E.6 we construct a velocity profile (100 points, $H = 1000$ m) using $\Lambda z = 10^{-4}$; we also set $N^2 = 10^{-2} \text{ s}^{-2}$ and $f_0 = 10^{-4} \text{ s}^{-1}$. The results are shown in Figure E. The linear instability properties are very similar to the analytical solution of the Eady problem found

³Bold capital letter denote matrices.

in textbooks [e.g., *Pedlosky, 1987; Vallis, 2006*]. In particular, the numerical solution predicts the fastest growing mode at $lRd = 0.245$, which corresponds to a wavelength of ~ 409 km, consistent with the $L_{max} \approx 400$ km predicted by theory for the scales used here [*Vallis, 2006*]. The maximum growth rate is 0.25 day^{-1} , which is also consistent with that predicted by theory (0.26 day^{-1}) [*Vallis, 2006*].

Several other tests were performed, e.g. including the β -term, varying the vertical shear [*Gill et al., 1974*], etc.

Computational programs written to solve this local linear QG baroclinic instability problem can be obtained at

ftp://ftp.io.usp.br/lado/cesar/linear_instability

or directly from the author.

Figure E.1: Instability properties for the numerical solutions to the Eady problem: (a) growth rate, (b) amplitude of the fastest growing mode, and (c) Phase of the fastest growing mode. The wavenumber in (a) is scaled by the deformation radius ($Rd \equiv (NH)/f_0 = 100$ km).

Appendix F

Conditions for baroclinic instability

The Charney-Stern-Pedlosky criterion for baroclinic instability for zonal flows is derived in major textbooks for mean zonal flows [Pedlosky, 1987; Vallis, 2006]. This condition can be easily generalized for the geometry assumed in Chapter 2, which is equivalent to the conditions for a solely meridional flow in the f -plane, namely

$$\omega_i \left\{ \int_{-H}^0 \frac{-l \partial Q / \partial x}{|lV - \omega|^2} |\varphi|^2 dz + \left[\frac{f_0^2}{N^2} \frac{l(\partial V / \partial z + N^2 / f_0 \partial \eta_b / \partial x)}{|lV - \omega|^2} |\varphi|^2 \right]_{-H}^0 \right\} = 0, \quad (\text{F.1})$$

where ω is the frequency, and l is the associated wavenumber in the y -direction (along-channel); Q is the mean potential vorticity and V is the mean velocity; $\eta_b(x)$ represents the departures from the mean depth in the x -direction (cross-channel); φ is the vertical structure.

For the occurrence of instability, non-trivial imaginary frequency must exist (i.e., $\omega_i \neq 0$). Therefore, the inner curly brackets terms ought to vanish. When the integral term dominates the balance, “interior” instabilities¹ may occur (generically named “Phillips-like”² instabilities following [Tulloch *et al.*, 2011] classification); the necessary condition for this kind of instability is that the (i) *PV gradient change sign within the interior*. In addition, there are many possibilities for instabilities involving the interaction

¹As discussed in Chapter 2 this classification differs from that adopted by Johns [1988]

²The original Phillips model only considers two layers with no buoyancy at the boundaries, thus representing the simplest type of instability driven by PV gradient inversion.

of the boundary terms and the interior PV gradients (here generically called “Charney-like” instabilities following *Tulloch et al.* [2011] classification: (ii) *the interior PV, at some depth, has the same sign of the vertical shear at the surface* or (iii) *the interior PV, at some depth, has the opposite sign to the quantity $\partial V/\partial z + N^2/f_0\partial\eta_b/\partial x$*). Generically, one could state that all these conditions are satisfied if the PV gradient changes its sign within the interior [*Johns*, 1988; *da Silveira et al.*, 2008]. Finally, it is important to acknowledge that these conditions are necessary (but not sufficient) for the instability occurrence [e.g., *Vallis*, 2006].

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