**UNIVERSITY OF SÃO PAULO
SÃO CARLOS SCHOOL OF ENGINEERING**

Filipe Marques Barbosa

**Robust recursive path-following control for autonomous
heavy-duty vehicles**

São Carlos

2018

Filipe Marques Barbosa

**Robust recursive path-following control for autonomous
heavy-duty vehicles**

Master Dissertation submitted to the São Carlos School of Engineering of the University of São Paulo in fulfillment of the requirements for the degree of Master of Science - Graduate Program in Electrical Engineering

Research area: Dynamic Systems

Supervisor: Prof. Dr. Valdir Grassi Junior

São Carlos

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To Glauce, Márcio, Ana Clara and Deborah Barbosa

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“I just want my phone call.”

The Joker

ABSTRACT

BARBOSA, Filipe Marques **Robust recursive path-following control for autonomous heavy-duty vehicles**. 2018. 96p. Dissertation (Masters in Science) - São Carlos School of Engineering, University of São Paulo, 2018.

Path following and lateral stability are crucial issues for autonomous vehicles. Moreover, these problems increase in complexity when handling heavy-duty vehicles due to their poor manoeuvrability, large sizes and mass variation. In addition, uncertainties on mass may have the potential to significantly decrease the performance of the system, even to the point of destabilising it. These parametric variations must be taken into account during the design of the controller. However, robust control techniques usually require offline adjustment of auxiliary tuning parameters, which is not practical and leads to sub-optimal operation. Hence, this work presents an approach to path-following and lateral control for autonomous heavy-duty vehicles subject to parametric uncertainties by using a robust recursive regulator. The main advantage of the proposed controller is that it does not depend on the offline adjustment of tuning parameters. Parametric uncertainties were assumed to be on the payload, and an $\mathcal{H}_\infty$ controller was used for performance comparison in simulations. The performance of both controllers is evaluated in a double lane-change manoeuvre. Simulation results showed that the proposed method had better performance in terms of robustness, lateral stability, driving smoothness and safety, which demonstrates that it is a very promising control technique for practical applications. Ultimately, experiment tests in a rigid heavy-duty truck validate what was found in simulation results.

Keywords: Path following; lateral control; robust control; heavy-duty vehicles.

RESUMO

BARBOSA, Filipe Marques **Controle robusto recursivo para seguimento de caminho aplicado a veículo autônomo de carga**. 2018. 96p. Dissertação (Mestrado em Ciências) - Escola de Engenharia de São Carlos, Universidade de São Paulo, São Carlos, 2018.

O seguimento de caminho e a estabilidade lateral são questões cruciais para veículos autônomos. Além disso, devido à baixa capacidade de manobra, tamanho e grande variação de massa, estes problemas se tornam mais complexos quando se trata de veículos pesados. Adicionalmente, as incertezas na massa têm o potencial de diminuir significativamente o desempenho do sistema, chegando ao ponto de desestabilizá-lo, assim, essas variações paramétricas devem ser consideradas durante o projeto do controlador. No entanto, as técnicas de controle robusto geralmente exigem o ajuste *offline* de parâmetros auxiliares do controlador, o que não é prático e leva a uma operação sub-ótima. Assim, este trabalho apresenta uma abordagem de controle de seguimento de caminho e controle lateral para veículos pesados autônomos sujeitos a incertezas paramétricas usando um regulador robusto recursivo. A principal vantagem deste controlador é que ele não depende do ajuste *offline* de parâmetros. Assumiu-se que as incertezas paramétricas estavam na carga do veículo, e um controlador $\mathcal{H}_\infty$ foi usado para comparar o desempenho em simulação. O desempenho de ambos os controladores é avaliado em uma manobra de mudança de faixa. Os resultados de simulação mostraram que o método proposto apresentou melhor desempenho em termos de robustez, estabilidade lateral, suavidade na condução e segurança, o que o demonstra como uma técnica de controle bastante promissora para aplicações práticas. Por fim, testes experimentais em um caminhão rígido reforçam os resultados obtidos em simulação.

Palavras-chave: Seguimento de caminho; controle lateral; controle robusto; veículos pesados.

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LIST OF ABBREVIATIONS AND ACRONYMS

AGV	Autonomous Ground Vehicle
AHV	Articulated Heavy Vehicle
PID	Proportional Integral Derivative
MPC	Model Predictive Control
LQR	Linear Quadratic Regulator
CG	Centre of Gravity
RLQR	Robust Linear Quadratic Regulator
GPS	Global Position System

LIST OF SYMBOLS

ψ_{des}	Desired yaw angle
α	Steering angle
ρ	Lateral displacement
θ	Orientation error
a_m	Distance from the front axle to the CG
b_m	Distance from the rear axle to the CG
l_m	Wheelbase
d_m	Distance between the rear axle and the coupling point
h_1	Distance between coupling point and the first unit CG
l_m^*	Distance between the front axle and the coupling point
v	Forward velocity
$\dot{y}_1$	Lateral Velocity
m_m	Mass of the unit
J_m	Unit moment of inertia
ψ_m	Yaw angle
α	Steering angle
ϕ_1	First articulation angle
ϕ_2	Second articulation angle
x_1	State vector (non-articulated vehicle)
x_2	State vector (one-articulated vehicle)
x_3	State vector (two-articulated vehicle)
c_j	Cornering stiffness of the axle
F_{z_j}	Vertical load of the axle
f_j	Normalised cornering stiffness

e	State variable error
M	Inertia matrix
A	State matrix in continuous time
B	Input matrix in continuous time
F	State matrix in discrete time
G	Input matrix in discrete time
δF	State uncertainty matrix
δG	Input uncertainty matrix
Δ	Contraction matrix
u	Control input
$\bar{J}^\mu$	Cost function
μ	Penalty parameter
K	Feedback gain
Q	State weight matrix
R	Input weight matrix
γ	Robustness parameter of the $\mathcal{H}_\infty$ controller

Subscript "m" references vehicle units

$m = 1$	First unit (towing unit)
$m = 2$	Second unit (trailing unit)
$m = 3$	Third unit (trailing unit)

Subscript "j" references vehicle axles

$j = 1$	Front axle of the towing unit
$j = 2$	Rear axle of the towing unit
$j = 3$	Axle of the second unit
$j = 4$	Axle of the third unit

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1 INTRODUCTION

In the last few decades, a significant research effort in the motion control area has been made aiming to improve the driving experience. Technology advances in the automobile industry have provided features such as parking assist and adaptive cruise control. These features allow vehicles to steer and park themselves providing some level of autonomy to them. Furthermore, some companies aim to develop autonomous ground vehicles (AGVs) able to drive itself in different types of roadways and road conditions.

The advantages of autonomous ground vehicles are well-established in the academic literature. For example, reducing the number of accidents; easing the transportation of elderly and disabled people (WU et al., 2015); offering more profitable means of transportation to industry and more efficient transportation methods to the military (FU et al., 2015; SHIN; HUH; PARK, 2015); improving ride comfort and safety for passengers (FILHO; TERRA; WOLF, 2017; HU et al., 2016); increasing road utilisation (DIAS; PEREIRA; PALHARES, 2014), etc. These advantages may be decisive in the transformation of the current transportation system.

Nowadays, heavy load vehicles are responsible for much of cargo transportation. The use of non-articulated and articulated heavy-duty vehicles has been increasing due to their economic advantages (KATI; KÖROĞLU; FREDRIKSSON, 2016), efficiency in freight transportation (JUJNOVICH; CEBON, 2013) and the growing demand for high capacity transport vehicles (ISLAM; LAINE; JACOBSON, 2015). Furthermore, the same technologies used for autonomous cars can also be addressed to heavy-duty vehicles (FAGNANT; KOCKELMAN, 2015), additionally increasing productivity and reducing the cost of cargo transportation (NOORVAND; KARNATI; UNDERWOOD, 2017).

On the other hand, their large sizes, complex configurations, poor manoeuvrability and the lack of knowledge about model parameters resulting in modelling errors are critical issues for lateral stability and safety (JUJNOVICH; CEBON, 2013; ISLAM; LAINE; JACOBSON, 2015). Accidents caused by lateral instability are often responsible for life and economic losses. In addition, the autonomous control of heavy-duty vehicles remains an issue. As payload may be much greater than vehicle weight itself (KATI; KÖROĞLU; FREDRIKSSON, 2016), mass is a critical parameter in vehicle dynamics and those vehicles are especially affected by mass variations. Hence, a control technique that overcomes the parametric uncertainties in the vehicle model and ensures system stability and performance objectives for a range of parameter values is necessary (KATI; KÖROĞLU; FREDRIKSSON, 2016). With this mind, robust controllers may be designed and used to withstand mass variations.

Kati, Koroğlu & Fredriksson (2016) proposed an $\mathcal{H}_\infty$ controller to deal with uncertainties on the payload of the vehicle. However, as the $\mathcal{H}_\infty$ controller depends on the offline adjustment of the auxiliary parameter γ , this results in sub-optimal controller operation due to the mass variations. The $\mathcal{H}_\infty$ controller is furthermore robust, but it cannot ensure smoothness for steering control applications. In fact, the lower the γ value, the more optimality condition the controller reaches. On the other hand, it cannot guarantee driving smoothness as there is no parameter to deal with this. Consequently, a mixed $\mathcal{H}_2/\mathcal{H}_\infty$ controller is used in the literature, where smoothness, and robustness and optimisation are respectively handled (SCHERER, 1995). In order to address the sub-optimality problem, the contribution of this paper is a novel approach for the lateral control of an autonomous articulated heavy-duty vehicle, based on a Robust Linear Quadratic Regulator (RLQR) presented in (TERRA; CERRI; ISHIHARA, 2014) and (CERRI; TERRA; ISHIHARA, 2009). The main advantage of the proposed controller is that it does not require any auxiliary tuning parameters, since both smoothness and robustness are already foreseen through a certain penalty parameter μ , which vanishes in the limit when it tends to infinity. This feature maintains the optimality for the full range of parametric uncertainties. This is additionally useful for online applications. A continuous-time model for the articulated vehicle in state-space form is presented. Then, the model is discretised in order to apply discrete RLQ control for solving a path-following problem.

Since $\mathcal{H}_\infty$ control is widely used for path-tracking problems (HU et al., 2016) and for robustifying the control strategy in automotive applications (LI et al., 2018; ZHAO; ZHANG; LI, 2018), a standard $\mathcal{H}_\infty$ controller is also applied to the plants of the heavy-duty vehicles for the sake of comparison in numerical simulations. Uncertainties on vehicle mass are introduced, then the performance of both controllers are compared in different cases. Simulation tests evaluate robustness, steering behaviour, vehicle displacement error and orientation error.

The RLQR ensures stability for a range of possible payloads. On the other hand, the $\mathcal{H}_\infty$ controller is dependent on the auxiliary parameter γ . Therefore, it cannot maintain good performance (or even stability) for a wide range of payloads, unless γ is adjusted offline (TERRA; CERRI; ISHIHARA, 2014).

Finally, the robust recursive regulator is experimentally tested to perform the path following of a rigid heavy-duty vehicle in a mining environment. Experiments also evaluated robustness, steering behaviour, lateral displacement and orientation error. Ultimately, the experimental results confirm the findings in numerical simulations.

2 PATH FOLLOWING AND LATERAL CONTROL

This chapter brings a brief presentation of the path following concept as well as some important published works that are related to the problem of path-following and lateral control for both non-articulated and articulated ground vehicles.

2.1 Path control of non-articulated vehicles

Path following is a crucial issue for most types of autonomous vehicles, therefore authors have addressed it for different applications. For instance, Nelson et al. (2007) developed a method for path-following control based on vector field where aerial vehicles performed straight-line paths, circular arcs and orbits in the presence of wind disturbances. Aguiar & Hespanha (2007) solved the path-following problem for a hovercraft (planar movement) and underwater (3-D movement) vehicles considering parametric modelling uncertainty by applying an adaptive supervisory control algorithm with combined logic-based switching and iterative Lyapunov-based techniques. The result is the vehicle converging to a tube centred around the desired path with a given velocity profile.

The path-following control for autonomous ground vehicles consists of manoeuvring the vehicle automatically through the steering system. This aims to make the vehicle follows the desired path defined by the planning system (XU et al., 2016), (TAGNE; TALJ; CHARARA, 2016). Figure 2.1 illustrates the path-following problem, where $\dot{y}_1$ is the lateral velocity, v the forward velocity, α the steering angle and ψ_1 the vehicle yaw angle. It depends mainly on the feedback information of the position and pose of the vehicle to follow the desired path under different conditions. The input information for the path following includes lateral error from the desired path, and its output is the feedback gain to reduce lateral displacement via steering actuators (XU et al., 2016).

Initially, path-following control was treated as a simply steering control to keep the vehicle following its path (XIA et al., 2016). However, lateral stability control has become increasingly important nowadays especially when dealing with heavy-duty vehicles. Thus, the next subsection brings important papers concerning path following and lateral control for non-articulated vehicles.

2.1.1 Lateral control of non-articulated vehicles

Concerning non-articulated autonomous ground vehicles, many different approaches have been proposed to solve the path-following problem. Yoon et al. (2009) presented a model-predictive control for active steering systems in unmanned ground vehicles. This work considers online obstacle avoidance and navigation towards the destination. It is

Figure 2.1: Path following problem.

Source: Author.

possible thanks to the prediction of a future path by the model-predictive controller, solving an optimisation problem to plan collision-free trajectories. Simulation results showed that applications of the distance-based method may require too much computation effort, however, the application based on local path generator has better computational efficiency.

Marino, Scalzi & Netto (2011) designed a nested PID steering control for an AGV to perform path following on roads with an uncertain curvature. The active steering control is based on the yaw rate error, as well as the lane keeping control is based on the lateral offset calculated by a gyroscope and the vision system. The used nested control structure has two independent loops that allow designing PID controllers in a multivariable context (two outputs, one input), and it is additionally robust since it does not need system parameters. The proposed controller was compared through simulations in the CarSim environment with the MPC used in the CarSim. Finally, an experiment performed by a prototype vehicle validated the controller.

Wang et al. (2015) presented a sensing system model and a descriptive model of a reference path that adapt to different driving directions by using fuzzy inference engines. The driving speed is directly used in each inference of the fuzzy logic to achieve lateral control for both complex tasks with sharp corners and simple tasks for high-speed driving. Thus, different fuzzy rule bases were designed for lateral offset and heading angle errors. The steering controller can, therefore, adapt to different driving speeds, including forward and backward directions. Simulation and experiments were made showing the effectiveness of the presented method since the vehicle performed very well under different driving

tasks.

Xia et al. (2016) presented a tracking method for the control of displacement and yaw angle of an AGV. The used method takes advantage of differential flatness theory and active disturbance rejection control schemes, adopted to ensure both control accuracy and robustness. A dynamic single-track model was linearised by making a small angle approximation. The authors proved that the model is flat and calculate a flat output. Then, an equivalent model is obtained based on the flat output and its derivatives. Finally, the problem of lateral tracking with one input (steering angle) and two outputs (lateral displacement and yaw angle) was investigated in simulations.

Hu et al. (2016) proposed a robust $\mathcal{H}_\infty$ output-feedback control strategy based on the mixed genetic algorithms and linear matrix inequality to perform the path following of autonomous ground vehicles. The designed controller simultaneously considers the external disturbances and parameter uncertainties, and make the vehicle track the path with the desired lateral and longitudinal velocities. Simulations based on CarSim-Simulink environment verified the validity of the presented control approach.

Lastly, Xu et al. (2016) did not present a novel approach but described the design of the driving control system implemented in the Kuafu-II autonomous vehicle that combines both the lateral and longitudinal controller. The vehicle lateral behaviour was described as a single-track model, and its lateral controller consists of nonlinear feedback control and sliding mode control to improve the tracking accuracy at different speeds. Thus, the nonlinear feedback control is used at low speeds and the sliding mode control is used at high speeds. The integrated system was evaluated both in simulation and road experiments.

2.2 Articulated vehicles

Articulated heavy vehicles, shown in Figure 2.2, are vehicles composed of a towing unit (tractor or truck) and one or multiple towed units (trailers). The units are linked by a mechanical coupling at an articulation point (dolly, fifth wheel, pintle hitch, *etc*) (ZHU; HE, 2015). Generally used to freight transportation, AHVs are capable to carry large loads in many road conditions, showing their flexibility and transport efficiency. The longer the vehicle is, poorer their manoeuvrability and directional control are. Therefore, AHVs rely on mechanical couplings that add yaw degrees of freedom, allowing the vehicle to articulate and improve its manoeuvrability and directional control (JUNOVICH; CEBON, 2013).

However, the articulated vehicle can experience lateral instabilities such as jack-knifing and rearward amplification as a result of the additional articulation joint (ISLAM et al., 2014), (JUNOVICH; CEBON, 2013). Consequently, articulated heavy vehicles are often involved in severe accidents, causing life and economic losses as result of these lateral stability problems (ISLAM et al., 2014). In addition to these problems, the parametric un-

Figure 2.2: Example of the configurations of used articulated vehicles.

(a) Tractor semi-trailer.

(b) Truck-full trailer.

Source: Adapted from (MANSVELDERS, 2006).

certainties in the vehicle model must be taken into account for applications in autonomous articulated heavy vehicles (AHVs). Thus, the next subsection brings important works concerning path following and lateral control of articulated vehicles.

2.2.1 Lateral control of articulated vehicles

In the literature, different approaches for the lateral control of articulated vehicles have been exploited. Some authors have proposed the use of an active trailer steering system to improve the path following and attitude control of articulated vehicles (JUNOVICH; CEBON, 2013), (KIM et al., 2016), (GUAN; KIM; WANG, 2017). For instance, Guan, Kim & Wang (2017) considered different vehicle conditions in the control strategy of articulated vehicles for varying conditions. In order to improve lateral stability, a model predictive controller was designed to make the vehicle follow the predicted trailer path and yaw angle, as well as minimise its side-slip angle. Simulations showed that the proposed controller improved the lateral stability and off-tracking of the articulated vehicle as well as presented good adaptability to different vehicle conditions.

Regarding autonomous articulated vehicles, some control strategies have been used in the literature. Kim et al. (2016) designed an active steering controller based on the LQR theory for tractor-trailer combination with the aim to improve lateral stability at high speeds and manoeuvrability at low speeds. The active steering control was designed to make the vehicle follow the desired yaw rate and minimise the side slip angle which is crucial for the vehicle stability. Therefore, a three-degree-of-freedom linear model was used to describe the behaviour of the tractor-trailer with steered trailer axles. Additionally, the simulated annealing particle swarm optimisation algorithm was applied to identify the key

parameters of the model. Simulations based on the TruckSim-Simulink environment were performed at both low and high speeds. The results showed an improvement in vehicle manoeuvrability and lateral stability at low and high speed, respectively.

Yuan & Zhu (2016) proposed a lateral-longitudinal control scheme using automatic steering strategies to avoid jackknifing, considering input limitations. Therefore, a combined longitudinal and lateral control scheme is proposed to deal with the reverse path tracking problem in articulated vehicles. Both anti-jackknifing condition and input saturation are considered a priori and the control design can ensure that the constraints are respected. The control has an anti-windup mechanism to prevent the articulation angle to exceed specified critical values and avoid jackknifing. The proposed controller demonstrated its effectiveness when tested in a Matlab and CarSim joint simulation platform.

With respect to the path-following problem for articulated vehicles, Jujnovich & Cebon (2013) presented control strategies for this issue. These aim to achieve path following for both tractor and trailer in different paths, vehicle speeds and in the presence of external disturbances. Nonlinear controllers based on kinematics are required for low speeds due to large articulated and steer angles. Although, the dynamic stability of the vehicle is compromised if the kinematic-based controllers remain active at high speeds since the side-slip characteristics of the trailer exhibit sign-change with increasing speeds. Therefore, low and high-speed controllers are combined using a speed-dependent gain in the intermediate speed range. The performance of steering controllers was compared to conventional vehicles and vehicles with command steer controllers on their trailer axles. Simulations showed improvement at low speeds resulting in more manoeuvrability and more stability and safety at high speeds.

Goodarzi & Ghajar (2014) introduced a novel lane-keeping controller to deal with the lateral and angular position of an articulated vehicle in the lane as well as keep its stability in critical situations. A human driver is necessary for this work, and the system only actuates when the vehicle deviates from its desired path. If this happens, the system controls the steering and brake systems aiming to lead the vehicle to the desired path and alerts the driver to keep the vehicle in the lane. Therefore, an optimal control technique was used and a fuzzy supervisory system was used to adapt it to the different driving behaviours from one driver to another. Finally, the performance of the proposed controller was investigated with simulation results for different vehicle manoeuvres.

Michalek (2014) proposed a controller approach to solve the path-following problem for articulated robotic vehicles equipped with an arbitrary number of off-axle hitched trailers. It is based on a highly scalable nonlinear cascade-like control that does not require to set the shortest distance to a reference path. By introducing the so-called segment-platooning reference paths, the proposed controller law ensures asymptotic following of both constant-curvature and varying-curvature reference paths. Experimental results

illustrated the practical effectiveness of the control law showing its small sensitivity to parametric uncertainties.

A novel sliding mode control scheme to perform the trajectory tracking for articulated vehicles was designed by Nayl et al. (2018). In this paper, the presented controller aims to achieve a better tracking performance and fast minimisation of its tracking error as well as reduce the chattering phenomenon. The derivation of the sliding mode controller depends on an entirely nonlinear kinematic model and the overall stability is proven based on the Lyapunov's stability condition. The controller performance was experimentally evaluated using a small scale and realistic articulated vehicle in multiple path tracking scenarios.

Thus, a controller capable of dealing with disturbances and additional model uncertainties is necessary in order to improve the lateral stability and achieve the path following for heavy-duty vehicles.

Most of the authors adopt constant values of mass in vehicle model, furthermore, when dealing with heavy-duty vehicles Jujnovich & Cebon (2013) and Goodarzi & Ghajar (2014) consider the only the fully loaded vehicle. Moreover, Xu et al. (2016) and Marino, Scalzi & Netto (2011) deal with uncertainties on mass in their approaches, however the issue of sub-optimal operation, mentioned in chapter 1, remains.

Nevertheless, as mentioned before, mass is a critical parameter in the vehicle dynamics and heavy-duty vehicles are especially affected by mass variations. Hence, a control technique that is robust to such parametric uncertainties in the model is necessary in order to achieve the autonomous control of heavy-duty vehicles. In other words, the controller must guarantee the lateral stability of the vehicle and perform the path following, reducing its lateral displacement and orientation errors for a range of parameter values.

To this end, a description of the lateral behaviour for non-articulated and articulated vehicles is necessary in order to achieve the lateral control of the vehicle. Therefore, the following chapter presents the lateral models of the vehicles for design and discussion.

3 SYSTEM MODELLING

With the aim to make the non-articulated and the articulated heavy-duty vehicle follow the desired path, it is necessary to minimise the lateral offset and heading error as well as ensure the vehicle stability. Therefore, the system modelling must take into account the path following and dynamic variables.

This chapter introduces the path-following and lateral dynamic models for non-articulated vehicles, vehicles with one articulation and vehicles with two articulations. Every towing units are assumed to have two axles and all trailing units are assumed to have one axle.

3.1 Path-following model

In order to solve the path-following problem, the lateral control aims to reduce lateral displacement and orientation angle errors of the towing vehicle. Therefore, the path-following model adopted here is based on the equations presented by Skjetne & Fossen (2001). The lateral displacement of the vehicle to a given reference path is the distance ρ from first unit centre of gravity to the closest point D on the desired path and its orientation error is defined as $\theta = \psi_1 - \psi_{des}$, where ψ_1 is the yaw angle of the towing unit and ψ_{des} its desired yaw angle. To this end, only the path following of the towing unit is considered for all vehicle configurations. Figure 3.1 shows the schematic diagram for the path following of a non-articulated and an articulated vehicle.

Source: Author.

Based on Serret-Frenet equations (SKJETNE; FOSSEN, 2001), the path-following model of the autonomous ground vehicle is expressed as

$$\begin{aligned}\dot{\rho} &= v \sin \theta + \dot{y}_1 \cos \theta \\ \dot{\theta} &= \dot{\psi}.\end{aligned}\tag{3.1}$$

By assuming a small orientation error θ , the displacement error ρ can be rewritten in the linear form as follows

$$\dot{\rho} = v\theta + \dot{y}_1.\tag{3.2}$$

3.2 Single-track dynamic models

Single-track models are widely used in literature (ALCALA et al., 2018; JI et al., 2018; JUJNOVICH; CEBON, 2013; KATI; KÖROĞLU; FREDRIKSSON, 2016) and provide a mathematical description of the vehicle lateral behaviour without much modelling and parametrisation effort (SCHRAMM; HILLER; BARDINI, 2018). These models assume that the vehicle can be described by only one equivalent track in each axle, linked by the vehicle body. Consequently, it only takes into account the planar movement of the vehicle, disregarding roll and pitch effects. The nonholonomic linear model adopted here is based on the bicycle model presented by Molengraft-Luijten et al. (2012).

For all used models, the following assumptions are adopted:

- Differences between left and right track are ignored;
- Vehicle velocity parameter is constant;
- The mass of each unit is assumed to be concentrated at the centre of gravity;
- Lateral tyre forces are proportional to the tyre slip angles;
- There is no load transfer.

In order to improve the path following, the lateral displacement ρ and orientation error θ must be as small as possible. In addition, vehicle stability ought to be guaranteed in vehicle modelling. Hence, the lateral velocity $\dot{y}_1$, yaw angle rate $\dot{\psi}_1$, and when dealing with articulated vehicles, the articulation angle ϕ and articulation rate $\dot{\phi}$, must be well controlled. The following subsections present the lateral modelling for non-articulated vehicles as well as the lateral modelling for two configurations of articulated vehicles, tractor-semitrailer and truck-full trailer. Table 3.1 details the parameters used in every vehicle models. Note that the first unit in all represented vehicles are the towing one, and therefore the second and third are towed units. The motion equation of the vehicles can be expressed as:

$$M\dot{x} = Ax + B\alpha,\tag{3.3}$$

where M is the inertia matrix, A is the state matrix, B is the input matrix, x is the state vector and α is the control input.

Table 3.1: Description of vehicle parameters

Parameter	Meaning	Unit
a_1	Distance from the front axle to the CG on first unit	m
a_2	Distance from the coupling point to the CG on second unit	m
a_3	Distance from the coupling point to the CG on third unit	m
b_1	Distance from the rear axle to the CG on first unit	m
b_2	Distance from the rear axle to the CG on second unit	m
b_3	Distance from the rear axle to the CG on third unit	m
l_1	First unit wheelbase	m
l_2	Second unit wheelbase	m
l_3	Third unit wheelbase	m
d_1	Distance between the rear axle and the coupling point on first unit	m
d_2	Distance between the rear axle and the coupling point on second unit	m
h_1	Distance between coupling point and the first unit CG	m
l_1^*	Distance between the front axle and the coupling point on first unit	m
l_2^*	Distance between the coupling points on second unit	m
v	Forward velocity	m/s
$\dot{y}_1$	Lateral Velocity	m/s
m_1	First unit mass	kg
m_2	Second unit mass	kg
m_3	Third unit mass	kg
J_1	First unit moment of inertia	$kg\ m^2$
J_2	Second unit moment of inertia	$kg\ m^2$
J_3	Third unit moment of inertia	$kg\ m^2$
ψ_1	First unit yaw angle	rad
ψ_2	Second unit yaw angle	rad
ψ_3	Third unit yaw angle	rad
α	Steering angle	rad
ϕ_1	First articulation angle	rad
ϕ_2	Second articulation angle	rad

3.2.1 Lateral modelling for non-articulated vehicles

A general free body diagram of a non-articulated vehicle is shown in Figure 3.2. The state vector in this vehicle configuration is $x_1 = [\dot{y}_1, \dot{\psi}_1, \rho, \theta]^T$ and the state-space description of the lateral control model for the heavy-duty vehicle is written as

$$\begin{bmatrix} m_1 & 0 & 0 & 0 \\ 0 & J_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \dot{x}_1 = \begin{bmatrix} \frac{-c_1 - c_2}{v} & \frac{b_1 c_1 - a_1 c_1 - m_1 v^2}{v} & 0 & 0 \\ \frac{b_1 c_2 - a_1 c_1}{v} & \frac{-a_1^2 c_1 - b_1^2 c_1}{v} & 0 & 0 \\ 1 & 0 & 0 & v \\ 0 & 1 & 0 & 0 \end{bmatrix} x_1 + \begin{bmatrix} c_1 \\ a_1 c_1 \\ 0 \\ 0 \end{bmatrix} \alpha, \quad (3.4)$$

where c_1, c_2 are the cornering stiffness of the vehicle front and rear axles, respectively.

Figure 3.2: Single-track model for a non-articulated vehicle.

Source: Author.

See Appendix A for the complete derivation of the models presented in this section.

Many studies consider constant cornering stiffness. However, this hypothesis is not considered in this work since these coefficients may vary according to several vehicle parameters. In fact, Fancher (1989) demonstrated, as cited in (LUIJTEN, 2010), that the relation between the vertical load forces and the tyre cornering stiffness are approximately linear for truck tyres. The coefficient of proportionality is given by a normalised cornering stiffness f_j , and the cornering stiffness c_j escalates linearly with the vertical load force of the axle F_{z_j} . Therefore, the cornering stiffness parameter is calculated as:

$$c_j = f_j F_{z_j} \text{ with } j = 1, \dots, p, \quad (3.5)$$

where p is the number of axles in the vehicle, $j = 1$ corresponds to the front axle and $j = 2$ to the rear axle.

The vertical force in each axle can be calculated as

$$\begin{aligned} F_{z_1} &= m_1 g \frac{b_1}{l_1} \\ F_{z_2} &= m_1 g \frac{a_1}{l_1}, \end{aligned} \quad (3.6)$$

where g is the gravitational acceleration. Moreover, Houben (2008), as cited in (MOLENGRAFT-LUIJTEN et al., 2012), observed that the normalised cornering stiffness of drive, steer and towed unit tyres (in articulated vehicles) are approximately the same. Therefore, for non-articulated vehicles it is assumed $f_1 \approx f_2$.

3.2.2 Lateral modelling for articulated vehicles

This subsection presents the single track lateral models for articulated vehicles, defined in section 2.2. There are several types of AHV combinations, however, this study only investigates the path-following control for tractor-semitrailer (one articulation) and truck-full trailer (two articulations) configurations, see Figure 2.2.

As the centre of gravity is assumed to be on the ground in the linear single-track model, the roll moment of coupling between the units is not modelled. Therefore, the equations shown here can be applied to any articulated vehicle. Note that for the vehicles represented in Figure 3.3 and 3.4 the hitch point may be positioned behind the rear axle of the towing unit (e.g. truck-full trailers where $h_1 > b_1$ and $d_1 > 0$) or in front of it (e.g. tractor-semitrailers where $h_1 < b_1$ and $d_1 < 0$).

Vehicles with one articulation

Figure 3.3 shows the free body diagram of a vehicle with one articulation where ϕ_1 is the articulation angle between the towing and towed units. The yaw angle of the towed unit is given by ψ_2 with $\psi_2 = \psi_1 + \phi_1$. The state vector in this vehicle configuration is defined as $x_2 = [\dot{y}_1, \dot{\psi}_1, \dot{\phi}_1, \phi_1, \rho, \theta]^T$ and the state-space description of the lateral control model for the heavy-duty vehicle with one articulation is written as

$$\begin{bmatrix} m_1 + m_2 & -m_2(h_1 + a_2) & -m_2a_2 & 0 & 0 & 0 \\ -m_2h_1 & J_1 + m_2h_1(h_1 + a_2) & m_2h_1a_2 & 0 & 0 & 0 \\ -m_2a_2 & J_2 + m_2a_2(h_1 + a_2) & J_2 + m_2a_2^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \dot{x}_2 = \begin{bmatrix} \frac{-c_1 - c_2 - c_3}{v} & \frac{c_3(h_1 + l_2) - a_1c_1 + b_1c_2 - (m_1 + m_2)v^2}{v} & \frac{c_3l_2}{v} & c_3 & 0 & 0 \\ \frac{c_3h_1 - a_1c_1 + b_1c_2}{v} & \frac{m_2h_1v^2 - a_1^2c_1 - b_1^2c_2 - c_3h_1(h_1 + l_2)}{v} & \frac{-c_3h_1l_2}{v} & -c_3h_1 & 0 & 0 \\ \frac{c_3l_2}{v} & \frac{m_2a_2v^2 - c_3l_2(h_1 + l_2)}{v} & \frac{-c_3l_2^2}{v} & -c_3l_2 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & v \\ 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} x_2 + \begin{bmatrix} c_1 \\ a_1c_1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \alpha, \quad (3.7)$$

where c_1 , c_2 and c_3 are the cornering stiffness of the towing unit front axle, towing unit rear axle and towed axle, respectively.

As mentioned in subsection 3.2.1, the relation between the normalised cornering stiffness f_i and the vertical load forces is also adopted for articulated vehicles. Therefore,

the vertical force in each axle can be calculated as

$$\begin{aligned} F_{z_1} &= m_1 g \frac{b_1}{l_1} - m_2 g \frac{b_2 d_1}{l_2 l_1} \\ F_{z_2} &= m_1 g \frac{a_1}{l_1} + m_2 g \frac{b_2 l_1^*}{l_2 l_1} \\ F_{z_3} &= m_2 g \frac{a_2}{l_2}, \end{aligned} \quad (3.8)$$

and it is assumed $f_1 \approx f_2 \approx f_3$.

Figure 3.3: Single-track model for a vehicle with one articulation.

Source: Author.

Vehicles with two articulations

The truck full-trailer combination, shown in Figure 2.2b, is the vehicle with two articulations used in this study. In this combination, the steering dolly makes an angle with respect to the truck, and the trailer makes an additional angle with respect to the steering dolly. Simplifications can be made for it by assuming that the dolly mass and inertia are small in relation to trailer mass ($m_2 \approx 0$, $J_2 \approx 0$, $a_2 \approx l_2$, $b_2 \approx 0$) and that there is no mechanical trail at the front axle ($d_2 \approx 0$, $l_2^* \approx l_2$). Owing to the massless adopted steering dolly in the truck-full trailer combination, the nature of the coupling is no longer dynamic but kinematic. The trailer does not influence the towing vehicle dynamics, nevertheless, the steering angle of the steering dolly is a function of the trailer

position and orientation with respect to the towing unit. Furthermore, since there is no load transfer between the units due to the kinematic coupling, the truck-full trailer can be modelled as two non-articulated vehicles kinematic coupled.

Figure 3.4 shows the free body diagram of a vehicle with two articulations where ϕ_1 is the articulation angle between the towing and first towed units, and ϕ_2 is the articulation angle between the first towed and second towed units. The yaw angle of the second towed unit is given by ψ_3 with $\psi_3 = \psi_2 + \phi_2 = \psi_1 + \phi_1 + \phi_2$. Thus, the state vector in this vehicle configuration is defined as $x_3 = [\dot{y}_1, \dot{\psi}, \dot{\phi}_1, \dot{\phi}_2, \phi_1, \phi_2, \rho, \phi]^T$ and the state-space description of the lateral control model for the truck-full trailer combination is written as

$$\begin{bmatrix} m_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & J_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ m_3 & -m_3(h_1 + l_2 + a_3) & -m_3(l_2 + a_3) & -m_3a_3 & 0 & 0 & 0 & 0 \\ 0 & J_3 & J_3 & J_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \dot{x}_3 = \begin{bmatrix} \frac{-c_1 - c_2}{v} & \frac{b_1c_2 - a_1c_1 - m_1v^2}{v} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{b_1c_2 - a_1c_1}{v} & \frac{-a_1^2c_1 - b_1^2}{v} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{c_3 + c_4}{v} & \frac{(c_3 + c_4)(h_1 + l_2) + c_4l_3 - m_3v^2}{v} & \frac{(c_3 + c_4)l_2 + c_4l_3}{v} & \frac{c_4l_3}{v} & c_3 + c_4 & c_4 & 0 & 0 \\ \frac{c_4b_3 - c_3a_3}{v} & \frac{(c_3a_3 - c_4b_3)(h_1 + l_2) - c_4l_3b_3}{v} & \frac{(c_3a_3 - c_4b_3)l_2 - c_4l_3b_3}{v} & \frac{-c_4l_3b_3}{v} & (c_3a_3 - c_4b_3) & -c_4b_3 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & v \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} x_3 + \begin{bmatrix} c_1 \\ a_1c_1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \alpha, \quad (3.9)$$

where c_1 , c_2 , c_3 and c_4 are the cornering stiffness of the towing unit front axle, towing unit rear axle, dolly axle and trailer axle, respectively.

The relation between the normalised cornering stiffness f_i and the vertical load

Figure 3.4: Single-track model for a vehicle with two articulations.

Source: Author.

forces mentioned in subsection 3.2.1 is also adopted once more time for the truck-full trailer combination. Therefore, the vertical force in each axle can be calculated as

$$\begin{aligned} F_{z_1} &= m_1 g \frac{b_1}{l_1}, & F_{z_2} &= m_1 g \frac{a_1}{l_1}, \\ F_{z_3} &= m_3 g \frac{b_3}{l_3}, & F_{z_4} &= m_3 g \frac{a_3}{l_3}, \end{aligned} \quad (3.10)$$

and it is assumed $f_1 \approx f_2 \approx f_3 \approx f_4$.

As mentioned in the chapter 2, the main objective of this work is to perform the path-following as well as guarantee the lateral stability of non-articulated and articulated heavy-duty vehicles due to their large mass variations. Inasmuch as mass variations may affect their manoeuvrability, lateral stability and safety. The outlined lateral models in

this chapter describe the lateral behaviour of the heavy-duty vehicles and can be used to achieve this control issue.

However, in order to perform the lateral control of the vehicle, the state-space representation must be rewritten as (3.11):

$$\dot{x} = M^{-1}(Ax + B\alpha). \quad (3.11)$$

With this mind, the next chapter brings the robust recursive regulator presented by Terra, Cerri & Ishihara (2014).

4 ROBUST RECURSIVE REGULATOR

The goal of the Robust Linear Quadratic Regulator (RLQR) is to minimise a given cost function subject to the maximum influence of parametric uncertainties. It is made by implementing an optimal feedback law in the form $u_i = K_i x_i$, where K_i is the feedback gain. This section describes the robust recursive regulator presented by Terra, Cerri & Ishihara (2014) and Cerri, Terra & Ishihara (2009).

4.1 Problem formulation

Consider the following discrete-time linear system subject to parametric uncertainties

$$x_{i+1} = (F_i + \delta F_i)x_i + (G_i + \delta G_i)u_i, \quad (4.1)$$

where $i = 0, \dots, N$, $x_i \in \mathbb{R}^n$ is the state vector, $u_i \in \mathbb{R}^m$ is the control input, and $F_i \in \mathbb{R}^{n \times n}$ and $G_i \in \mathbb{R}^{n \times m}$ are known nominal model matrices. Uncertainty matrices δF_i and δG_i represent parametric uncertainties modelled as

$$\begin{bmatrix} \delta F_i & \delta G_i \end{bmatrix} = H_i \Delta_i \begin{bmatrix} E_{F_i} & E_{G_i} \end{bmatrix}, \quad (4.2)$$

where $i = 0, \dots, N$; $H_i \in \mathbb{R}^{n \times p}$; $E_{F_i} \in \mathbb{R}^{l \times n}$ and $E_{G_i} \in \mathbb{R}^{l \times m}$ are known matrices; and $\Delta_i \in \mathbb{R}^{p \times l}$ is an arbitrary matrix such that $\|\Delta_i\| \leq 1$.

In order to obtain the Robust Linear Quadratic Regulator, the following optimisation problem must be solved (TERRA; CERRI; ISHIHARA, 2014):

$$\min_{x_{i+1}, u_i} \max_{\delta F_i, \delta G_i} \bar{J}_i^\mu(x_{i+1}, u_i, \delta F_i, \delta G_i), \quad (4.3)$$

where $\bar{J}_i^\mu$ is the cost function

$$\begin{aligned} \bar{J}_i^\mu(x_{i+1}, u_i, \delta F_i, \delta G_i) = \\ \begin{bmatrix} x_{i+1} \\ u_i \end{bmatrix}^T \begin{bmatrix} P_{i+1}^r & 0 \\ 0 & R_i \end{bmatrix} \begin{bmatrix} x_{i+1} \\ u_i \end{bmatrix} + \Phi^T \begin{bmatrix} Q_i & 0 \\ 0 & \mu I \end{bmatrix} \Phi, \end{aligned} \quad (4.4)$$

with fixed penalty parameter $\mu > 0$, weight matrices $Q_i \succ 0$, $R_i \succ 0$, $P_{i+1} \succ 0$ and

$$\Phi = \left\{ \begin{bmatrix} 0 & 0 \\ I & -G_i - \delta G_i \end{bmatrix} \begin{bmatrix} x_{i+1} \\ u_i \end{bmatrix} - \begin{bmatrix} -I \\ F_i + \delta F_i \end{bmatrix} x_i \right\}.$$

Details on penalty function can be seen in (CERRI; TERRA; ISHIHARA, 2009).

Remark. The optimisation problem (4.3)-(4.4) is a particular case of the robust least-squares problem and will be treated below.

4.2 Regularised least squares

Consider the least-square minimisation problem defined by

$$\min_{x \in \mathbb{R}^m} \{J(x)\}, \quad (4.5)$$

where $J(x)$ is a regularised quadratic functional

$$\begin{aligned} J(x) &= \|x\|_Q^2 + \|Ax - b\|_W^2 \\ &= x^T Q x + (Ax - b)^T W (Ax - b), \end{aligned} \quad (4.6)$$

with $Q \in \mathbb{R}^{m \times m}$ (regularisation matrix) and $W \in \mathbb{R}^{n \times n}$ symmetric positive definite, $A \in \mathbb{R}^{n \times m}$ and $b \in \mathbb{R}^n$ known, and $x \in \mathbb{R}^m$ the unknown vector.

Lemma 4.2.1. *The optimal solution for the problem (4.5)-(4.6) is*

$$x^* = (Q + A^T W A)^{-1} A^T W b.$$

Proof. See (KAILATH; SAYED; HASSIBI, 2000). □

4.3 Robust regularised least-squares problem

In the regularised least-squares problem established in (4.5)-(4.6), now suppose that the matrix A and the vector b are under influence of uncertainties δA and δb , respectively. Consider the min-max optimisation problem defined in (SAYED; NASCIMENTO, 1999) in the form:

$$\min_x \max_{\delta A, \delta b} \{J(x, \delta A, \delta b)\}, \quad (4.7)$$

with $J(x, \delta A, \delta b)$ given by

$$J(x, \delta A, \delta b) = \|x\|_Q^2 + \|(A + \delta A)x - (b + \delta b)\|_W^2, \quad (4.8)$$

and the uncertainties δA and δb modelled as

$$\begin{bmatrix} \delta A & \delta b \end{bmatrix} = H \Delta \begin{bmatrix} E_A & E_b \end{bmatrix}, \quad (4.9)$$

with A , b , H , E_A , E_b , Q and W known matrices, Δ a contraction arbitrary matrix ($\|\Delta\| \leq 1$) and x an unknown vector. The optimal solution for the problem (4.7)-(4.9) is given below. See demonstration details in (SAYED; NASCIMENTO, 1999), where a general result is proposed.

Lemma 4.3.1. *The optimisation problem (4.7)-(4.9) has a unique solution*

$$x^* = (\hat{Q} + A^T \hat{W} A)^{-1} (A^T \hat{W} b + \hat{\lambda} E_A^T E_b),$$

with $\hat{Q}$ and $\hat{W}$ defined as

$$\begin{aligned}\hat{Q} &:= Q + \hat{\lambda} E_A^T E_A, \\ \hat{W} &:= W + WH(\hat{\lambda} I - H^T W H)^\dagger H^T W.\end{aligned}$$

The non-negative scalar parameter obtained from the minimisation problem

$$\hat{\lambda} = \arg \min_{\lambda \geq \|H^T W H\|} \{\Gamma(\lambda)\},$$

where $\Gamma(\lambda) := \|x(\lambda)\|_Q^2 + \lambda \|E_A x(\lambda) - E_b\|^2 + \|Ax(\lambda) - b\|_{W(\lambda)}^2$ with

$$\begin{aligned}Q(\lambda) &:= Q + \lambda E_A^T E_A, \\ \tilde{Q}(\lambda) &:= Q(\lambda) + A^T W(\lambda) A, \\ W(\lambda) &:= W + WH(\lambda I - H^T W H)^\dagger H^T W, \\ x(\lambda) &:= \tilde{Q}(\lambda)^{-1} (A^T W(\lambda) b + \lambda E_A^T E_b).\end{aligned}$$

Proof. See (SAYED; NASCIMENTO, 1999) □

For this type of problem, it is appropriate to redefine Lemma 4.3.1 in terms of an array of matrices. The following lemma shows an optimal solution for the problem (4.7)-(4.9) in an alternative structure to this fundamental theorem.

Lemma 4.3.2. *Suppose $Q \succ 0$ and $W \succ 0$. The solution x^* for the problem (4.7)-(4.9) can be rewritten as*

$$\begin{bmatrix} x^* \\ J(x^*) \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & b \\ 0 & E_b \\ I & 0 \end{bmatrix}^T \begin{bmatrix} Q^{-1} & 0 & 0 & I \\ 0 & \hat{W}^{-1} & 0 & A \\ 0 & 0 & \hat{\lambda}^{-1} I & E_A \\ I & A^T & E_A^T & 0 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ b \\ E_b \\ 0 \end{bmatrix},$$

with $\hat{W}$ and $\hat{\lambda}$ as in Lemma 4.3.1.

Proof. See (CERRI; TERRA; ISHIHARA, 2009). □

4.4 Robust Linear Quadratic Regulator

The optimisation problem (4.3)-(4.4) is solved based on the solution of a general robust regularised least-squares problem (TERRA; CERRI; ISHIHARA, 2014). Back to the solution presented in Lemma 4.3.1, with $\mu > 0$, the RLQR has an optimal operation point for each step k of the algorithm. When suitable identifications of (4.3)-(4.4) with (4.7)-(4.9) are carried out, the regularisation of the robust regulator is reached thanks to minimisation over both $x_{i+1}(\mu)$ and $u_i(\mu)$ (TERRA; CERRI; ISHIHARA, 2014):

$$Q \leftarrow \begin{bmatrix} P_{i+1} & 0 \\ 0 & R_i \end{bmatrix}, \quad x \leftarrow \begin{bmatrix} x_{i+1}(\mu) \\ u_k(\mu) \end{bmatrix}, \quad W \leftarrow \begin{bmatrix} Q_i & 0 \\ 0 & \mu I \end{bmatrix},$$

$$\begin{aligned}
A &\leftarrow \begin{bmatrix} 0 & 0 \\ I & -G_i \end{bmatrix}, \quad \delta A \leftarrow \begin{bmatrix} 0 & 0 \\ 0 & -\delta G_i \end{bmatrix}, \quad \Delta \leftarrow \Delta_i \\
b &\leftarrow \begin{bmatrix} -I \\ F_i \end{bmatrix} x_i, \quad \delta b \leftarrow \begin{bmatrix} 0 \\ \delta F_i \end{bmatrix} x_i, \\
H &\leftarrow \begin{bmatrix} 0 \\ H_i \end{bmatrix}, \quad E_A \leftarrow [0 - E_{G_i}], \quad E_b \leftarrow E_{F_i} x_i,
\end{aligned} \tag{4.10}$$

The following theorem shows a framework given in terms of an array of matrices with the purpose of calculating the optimal cost function, control input and state trajectory.

Theorem 4.4.1. *For each $\mu > 0$ in the optimisation problem (4.3)-(4.4), the optimal solution is given by*

$$\begin{bmatrix} x_{i+1}^*(\mu) \\ u_i^*(\mu) \\ \tilde{J}_i^\mu(x_{i+1}^*(\mu), u_i^*(\mu)) \end{bmatrix} = \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & x_i(\mu)^T \end{bmatrix}^T \begin{bmatrix} L_{i,\mu} \\ K_{i,\mu} \\ P_{i,\mu} \end{bmatrix} x_i, \tag{4.11}$$

where the closed-loop system matrix L_i and the feedback gain K_i result from the recursion

$$\begin{bmatrix} L_i \\ K_i \\ P_i \end{bmatrix} = \begin{bmatrix} 0 & 0 & -I & \mathcal{F}_i & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & I \\ 0 & 0 & 0 & 0 & I & 0 \end{bmatrix} \Xi^{-1} \begin{bmatrix} 0 \\ 0 \\ -I \\ \mathcal{F}_i \\ 0 \\ 0 \end{bmatrix}, \tag{4.12}$$

with

$$\Xi = \begin{bmatrix} P_{i+1}^{-1} & 0 & 0 & 0 & I & 0 \\ 0 & R_i^{-1} & 0 & 0 & 0 & I \\ 0 & 0 & Q_i^{-1} & 0 & 0 & 0 \\ 0 & 0 & 0 & \Sigma_i(\mu, \hat{\lambda}_i) & \mathcal{I} & -\mathcal{G}_i \\ I & 0 & 0 & \mathcal{I}^T & 0 & 0 \\ 0 & I & 0 & -\mathcal{G}^T & 0 & 0 \end{bmatrix},$$

$$\begin{aligned}
\Sigma_i &= \begin{bmatrix} \mu^{-1}I - \hat{\lambda}_i^{-1}H_iH_i^T & 0 \\ 0 & \hat{\lambda}_i^{-1}I \end{bmatrix}, \\
\mathcal{I} &= \begin{bmatrix} I \\ 0 \end{bmatrix}, \quad \mathcal{G}_i = \begin{bmatrix} G_i \\ E_{G_i} \end{bmatrix}, \quad \mathcal{F}_i = \begin{bmatrix} F_i \\ E_{F_i} \end{bmatrix},
\end{aligned}$$

where P_{i+1} is the solution of the associated Riccati Equation and $\lambda_i > \|\mu H_i^T H_i\|$ (SAYED, 2001). Furthermore, alternatively one has

$$\begin{aligned} P_{i,\mu} = & L_{i,\mu}^T P_{i+1} L_{i,\mu} + K_{i,\mu} R_i K_{i,\mu} + Q_i + \\ & (\mathcal{I} L_{i,\mu} - \mathcal{G}_i K_{i,\mu} - \mathcal{F}_i)^T \Sigma_{i,\mu}^{-1} (\mathcal{I} L_{i,\mu} - \mathcal{G}_i K_{i,\mu} - \mathcal{F}_i) \succ 0. \end{aligned} \quad (4.13)$$

Proof. It follows from Lemma 4.3.2, identifications performed in (4.10) and results shown in (CERRI; TERRA; ISHIHARA, 2009). $\square$

Algorithm 1 shows the Robust Linear Quadratic Regulator obtained with Lemma 4.3.1. The parameter μ is associated with system robustness. It is responsible for ensuring the RLQR regularisation and validity of the equality (4.1). For maximum robustness, $\mu \rightarrow \infty$ and consequently $\Sigma_i \rightarrow 0$.

Algorithm 1 The Robust Linear Quadratic Regulator

Uncertain model: Consider the model (4.1)-(4.2) and criterion (4.3)-(4.4) with known F_i , G_i , E_{F_i} , E_{G_i} , $Q_i \succ 0$, and $R_i \succ 0$ for all i .

Initial conditions: Define x_0 and $P_{i,N} \succeq 0$.

Step 1: (*Backward*) For all $i = N - 1, \dots, 0$, compute

$$\begin{bmatrix} L_i \\ K_i \\ P_i \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -I \\ 0 & 0 & F_i \\ 0 & 0 & E_{F_i} \\ I & 0 & 0 \\ 0 & I & 0 \end{bmatrix}^T \begin{bmatrix} P_{i+1}^{-1} & 0 & 0 & 0 & 0 & I & 0 \\ 0 & R_i^{-1} & 0 & 0 & 0 & 0 & I \\ 0 & 0 & Q_i^{-1} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & I & -G_i \\ 0 & 0 & 0 & 0 & 0 & 0 & -E_{G_i} \\ I & 0 & 0 & I & 0 & 0 & 0 \\ 0 & I & 0 & -G_i^T & -E_{G_i}^T & 0 & 0 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \\ -I \\ F_i \\ E_{F_i} \\ 0 \\ 0 \end{bmatrix}.$$

Step 2: (*Forward*) For each $i = 0, \dots, N - 1$, obtain

$$\begin{bmatrix} x_{i+1}^* \\ u_i^* \end{bmatrix} = \begin{bmatrix} L_i \\ K_i \end{bmatrix} x_i^*,$$

with the total cost given by $J_r^* = x_0^T P_0 x_0$.

For each iteration of (4.13), the matrix $P_{i,\mu}$ is finite and $\mathcal{I} L_{i,\mu} - \mathcal{G}_i K_{i,\mu} - \mathcal{F}_i \rightarrow 0$, as shown in (TERRA; CERRI; ISHIHARA, 2014). Therefore,

$$\begin{aligned} L_{i,\infty} &= F_i + G_i K_{i,\infty} \\ E_{F_i} + E_{G_i} K_{i,\infty} &= 0, \end{aligned} \quad (4.14)$$

and a sufficient condition that satisfy (4.14) is

$$\text{rank} \left(\begin{bmatrix} E_{F_i} & E_{G_i} \end{bmatrix} \right) = \text{rank} \left(E_{G_i} \right). \quad (4.15)$$

Convergence and stability analyses of the RLQR are made through direct identifications with the standard optimal regulator problem for systems not subject to uncertainties. It

resembles the standard LQR where the stability is directly related with the positiveness of the Riccati equation solution (TERRA; CERRI; ISHIHARA, 2014). More details on convergence and stability analysis can be found in (TERRA; CERRI; ISHIHARA, 2014).

5 RESULTS

Simulation and experimental results were obtained for the performance evaluation of the designed lateral control. Simulation results were obtained for the three heavy-duty vehicle configurations and the experimental results were obtained only for the non-articulated vehicle. This chapter shows the obtained simulation and experimental results.

5.1 Numerical results

For the controller validation, the RLQR was performed and compared with the $\mathcal{H}_\infty$ controller in various operational conditions. The Matlab simulation software was used for this purpose. Simulations consist of minimising the lateral displacement and orientation errors. A double lane-change manoeuvre was performed during 30 seconds with the sampling period being 0.01 seconds and nominal payload subject to uncertainties. Figure 5.1 shows an example of a double lane-change scenario for the tractor-trailer combination, where ϵ is the tractor width. This scenario was adopted for all of the studied cases and for all vehicle combinations. Furthermore, for all cases, the penalty parameter $\mu = 10^9$, and the initial values of lateral and orientation errors are $\rho_0 = 0.3m$ and $\theta_0 = -0.1rad$, respectively.

Figure 5.1: Double lane-change scenario.

Source: Author.

The normalised cornering stiffness was applied in all cases studied here as it is a satisfactory representation for most heavy-duty applications and conditions (MOLENGRAFT-LUIJTEN et al., 2012). Thus, it was calculated as a function of the vertical load by assuming $f = f_1 = f_2 = f_3 = f_4 = 5.73 rad^{-1}$. In addition, the maximum steering angle ($0.44 rad$) was taken into account in numerical results.

The linear systems must be rewritten in order to compare the $\mathcal{H}_\infty$ controller and the robust recursive regulator presented in this work. Hence, the robust control design considering the $\mathcal{H}_\infty$ method discussed by Hassibi, Sayed & Kailath (1999) was used. Its equations, identifications and formulation are given in Appendix B.

Figure 5.2 gives the block diagrams for both control techniques, where e_i is the error between the reference and output, and x_{ref} is the reference state vector. Both e_i and x_{ref} are obtained when a reference control signal is applied to the lateral model of the vehicle.

Figure 5.2: Block diagrams for Robust Linear Quadratic Regulator and $\mathcal{H}_\infty$ control systems.

Source: Author

For each vehicle combination, the robustness parameter γ was adjusted to the lowest possible value that ensures both $\mathcal{H}_\infty$ controller existence and driving smoothness. Furthermore, the nominal payload was applied to each vehicle combination, and the weight matrices Q , R , R^c and Q^c initially adjusted so that the maximum steering rate of both controllers were $\max\|\dot{u}\|_{RLQR} \approx \max\|\dot{u}\|_{\mathcal{H}_\infty}$ at the nominal payload. Subsequently, the corresponding matrices Q and R^c , and Q^c and R were adjusted to the same value. This procedure was afterwards used for every numerical simulations and vehicle combinations considering mass variation. The methodology used to calculate the uncertainty matrices E_F , E_G and H is described in Appendix C.

The heavy-duty vehicles behaviour was evaluated with numerical results by taking a given reference path and performing the path following. For this purpose, the lateral velocity, yaw rate, articulation angles rate, articulation angles, lateral displacement and orientation error of the vehicle were observed. Moreover, controller evaluation was done through graphic analysis, and by adopting maximum steering rate and $\mathcal{L}_2$ norm of the error as performance criteria.

5.1.1 System response for the non-articulated vehicle

Table 5.1 shows the vehicle parameters and the necessary information to calculate it, obtained from websites for commercial vehicles¹ and towing implement² manufacturers. For all simulated cases with non-articulated vehicle, the initial conditions are the same for both controllers, those being $x_0 = [0, 0, 0.3, -0.1]^T$, and

$$H = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, E_F = \begin{bmatrix} 6.8572 \times 10^{-5} \\ -8.6201 \times 10^{-5} \\ 0 \\ -666.66667 \times 10^{-5} \end{bmatrix}^T \text{ and } E_G = \begin{bmatrix} -666.66667 \times 10^{-5} \\ -666.66667 \times 10^{-5} \end{bmatrix}^T.$$

The robustness parameter γ , and weight matrices Q , R^c , R and Q^c used for the non-articulated vehicle are

$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 49230 & 0 \\ 0 & 0 & 0 & 100 \end{bmatrix}, R = \begin{bmatrix} 4870 & 0 \\ 0 & 4870 \end{bmatrix},$$

$$\gamma = 24590, R^c = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 500 & 0 \\ 0 & 0 & 0 & 10 \end{bmatrix} \text{ and } Q^c = \begin{bmatrix} 20660 & 0 \\ 0 & 20660 \end{bmatrix}.$$

Table 5.1: Description of the non-articulated vehicle parameters

Parameter	Meaning	Value
a_1	Distance from the front axle to the CG	3.19 <i>m</i>
b_1	Distance from the rear axle to the CG	1.62 <i>m</i>
l_1	Vehicle wheelbase	4.81 <i>m</i>
ϵ	Vehicle width	2.6 <i>m</i>
v	Forward velocity	16.67 <i>m/s</i>
m_1	Vehicle mass	16030 <i>kg</i>
Payload	The amount of payload carried by the vehicle	12550 <i>kg</i>
J_1	Vehicle moment of inertia	215717 <i>kg m²</i>
c_1	Front axle cornering stiffness	540419 <i>N/rad</i>
c_2	Rear axle cornering stiffness	1064462 <i>N/rad</i>

Table 5.2 and Table 5.3 show maximum steering rate, payload variations, and $\mathcal{L}_2$ norm of lateral displacement and orientation errors for both performed controllers,

¹ <https://www.scania.com>

² <http://www.librelato.com.br>

respectively. Payload values for every evaluated case were chosen for the best illustration of the influence of mass variation.

Table 5.2: Evaluated cases for both controllers for the non-articulated vehicle.

Case	Payload	$\max \ \dot{\alpha}_{RLQR}\ $	$\max \ \dot{\alpha}_{\mathcal{H}_\infty}\ $
1	100%	0.3444 rad/s	0.3444 rad/s
2	300%	0.4107 rad/s	0.3051 rad/s
3	0%	0.2748 rad/s	0.4358 rad/s
4	200%	0.2209 rad/s	10.5236 rad/s

Table 5.3: $\mathcal{L}_2$ norm of the lateral displacement and orientation errors for the non-articulated vehicle.

Case	$\ \rho\ _{\mathcal{L}_2}$		$\ \theta\ _{\mathcal{L}_2}$	
	RLQR	$\mathcal{H}_\infty$	RLQR	$\mathcal{H}_\infty$
1	0.6385	0.5513	0.1423	0.1106
2	1.1790	1.0614	0.2201	0.1971
3	0.3719	0.3267	0.1120	0.0938
4	1.3843	0.3301	0.1812	0.1181

The description of each evaluated case and the graphics of numerical results follows.

Case 1 Considering nominal payload, Figure 5.3 and Figure 5.4 show the system state variables, the global position of vehicle centre of mass and steering angle performed by both controllers.

Case 2 Considering 200% of overload over the payload nominal value, Figure 5.5 and Figure 5.6 show the system state variables, the global position of tractor centre of mass and steering angle performed by both controllers.

Case 3 Considering the vehicle without payload, Figure 5.7 and Figure 5.8 show the system state variables, the global position of tractor centre of mass and steering angle performed by both controllers.

Case 4 Lastly, considering 100% of overload and the weight matrices $Q = R^c$ and $R = Q^c$ as follows

$$Q = R^c = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 25000 & 0 \\ 0 & 0 & 0 & 100 \end{bmatrix}, R = Q^c = \begin{bmatrix} 67070 & 0 \\ 0 & 67070 \end{bmatrix},$$

the values were adjusted to ensure both $\mathcal{H}_\infty$ existence and the RLQR smoothness. Figure 5.9 shows the very beginning of the input signal for this situation.

Figure 5.3: System state variables of the non-articulated vehicle for case 1.

Figure 5.4: Global position of the non-articulated vehicle centre of mass and steering angle for case 1.

Figure 5.5: System state variables of the non-articulated vehicle for case 2.

Figure 5.6: Global position of the non-articulated vehicle centre of mass and steering angle for case 2.

Figure 5.7: System state variables of the non-articulated vehicle for case 3.

Figure 5.8: Global position of the non-articulated vehicle centre of mass and steering angle for case 3.

Figure 5.9: Input signal of both controllers at the very beginning for $Q = R^c$ and $R = Q^c$ for case 4.

5.1.2 System response for the vehicle with one articulation

Table 5.4 shows the parameters of the vehicle with one articulation and the necessary information to calculate it, obtained from websites for commercial vehicles³ and towing

³ <https://www.scania.com>

implement⁴ manufacturers. For all simulated cases with one-articulated vehicle, the initial conditions are the same for both controllers, those being $x_0 = [0, 0, 0, 0, 0.3, -0.1]^T$, and

$$H = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, E_F = \begin{bmatrix} 6.8572 \times 10^{-5} \\ -8.6201 \times 10^{-5} \\ -1.1401 \times 10^{-5} \\ -8.0034 \times 10^{-5} \\ 0 \\ -666.66667 \times 10^{-5} \end{bmatrix}^T \text{ and } E_G = \begin{bmatrix} -666.66667 \times 10^{-5} \\ -666.66667 \times 10^{-5} \end{bmatrix}^T.$$

The robustness parameter γ , and weight matrices Q , R^c , R and Q^c used for the vehicle with one articulation are

$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 15054 & 0 \\ 0 & 0 & 0 & 0 & 0 & 100 \end{bmatrix}, R = \begin{bmatrix} 4998 & 0 \\ 0 & 4998 \end{bmatrix},$$

$$\gamma = 14350, R^c = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1268 & 0 \\ 0 & 0 & 0 & 0 & 0 & 10 \end{bmatrix} \text{ and } Q^c = \begin{bmatrix} 25660 & 0 \\ 0 & 25660 \end{bmatrix}.$$

⁴ <http://www.librelato.com.br>

Table 5.4: Description of the parameters for the vehicle with one articulation

Parameter	Meaning	Value
a_1	Distance from the front axle to the CG on first unit	1.73 <i>m</i>
a_2	Distance from the coupling point to the CG on second unit	4.80 <i>m</i>
b_1	Distance from the rear axle to the CG on first unit	2.42 <i>m</i>
b_2	Distance from the rear axle to the CG on second unit	3.20 <i>m</i>
l_1	First unit wheelbase	4.15 <i>m</i>
l_2	Second unit wheelbase	8.00 <i>m</i>
d_1	Distance between the rear axle and the coupling point on first unit	0.29 <i>m</i>
h_1	Distance between coupling point and the first unit CG	2.13 <i>m</i>
l_1^*	Distance between the front axle and the coupling point on first unit	3.86 <i>m</i>
v	Forward velocity	16.67 <i>m/s</i>
m_1	First unit mass	8909 <i>kg</i>
m_2	Second unit mass	9370 <i>kg</i>
Payload	The amount of payload carried by the vehicle	25000 <i>kg</i>
J_1	First unit moment of inertia	41710 <i>kg m²</i>
J_2	Second unit moment of inertia	40377 <i>kg m²</i>
c_1	Front axle cornering stiffness of the first unit	345160 <i>N/rad</i>
c_2	Rear axle cornering stiffness of the first unit	927120 <i>N/rad</i>
c_3	Axle cornering stiffness of the second unit	1158000 <i>N/rad</i>

Table 5.5 and Table 5.6 show maximum steering rate, payload variations, and $\mathcal{L}_2$ norm of lateral displacement and orientation errors for both performed controllers, respectively. Payload values for every evaluated case were chosen for the best illustration of the influence of mass variation.

Table 5.5: Evaluated cases for both controllers for the vehicle with one articulation.

Case	Payload	$\max\ \dot{\alpha}_{RLQR}\ $	$\max\ \dot{\alpha}_{\mathcal{H}_\infty}\ $
1	100%	0.3444 rad/s	0.3444 rad/s
2	200%	0.4721 rad/s	0.4187 rad/s
3	0%	0.3194 rad/s	0.3260 rad/s
4	234%	0.1815 rad/s	8.8180 rad/s

Table 5.6: $\mathcal{L}_2$ norm of the lateral displacement and orientation errors for the vehicle with one articulation.

Case	$\ \rho\ _{\mathcal{L}_2}$		$\ \theta\ _{\mathcal{L}_2}$	
	RLQR	$\mathcal{H}_\infty$	RLQR	$\mathcal{H}_\infty$
1	0.2483	0.2504	0.1439	0.0896
2	0.3192	0.2778	0.1443	0.0897
3	0.2064	0.2165	0.1279	0.0925
4	0.4757	0.3762	0.1493	0.2414

The description of each evaluated case and the graphics of numerical results for the vehicle with one articulation follow.

Case 1 Considering nominal payload, Figure 5.10 and Figure 5.11 show the system state variables, the global position of vehicle centre of mass and steering angle performed by both controllers.

Case 2 Considering 100% of overload over the payload nominal value, Figure 5.12 and Figure 5.13 show the system state variables, the global position of tractor centre of mass and steering angle performed by both controllers.

Case 3 Considering the vehicle without payload, Figure 5.14 and Figure 5.15 show the system state variables, the global position of tractor centre of mass and steering angle performed by both controllers.

Case 4 Lastly, considering 134% of overload and the weight matrices $Q = R^c$ and $R = Q^c$ as follows

$$Q = R^c = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 25000 & 0 \\ 0 & 0 & 0 & 0 & 0 & 100 \end{bmatrix}, R = Q^c = \begin{bmatrix} 67070 & 0 \\ 0 & 67070 \end{bmatrix},$$

the values were adjusted to ensure both $\mathcal{H}_\infty$ existence and the RLQR smoothness. Figure 5.16 and Figure 5.17 show the system state variables, the global position of tractor centre of mass and steering angle performed by both controllers.

Figure 5.10: System state variables of the vehicle with one articulation for case 1.

Figure 5.11: Global position of the centre of mass and steering angle of the vehicle with one articulation for case 1.

Figure 5.12: System state variables of the vehicle with one articulation for case 2.

Figure 5.13: Global position of the centre of mass and steering angle of the vehicle with one articulation for case 2.

Figure 5.14: System state variables of the vehicle with one articulation for case 3.

Figure 5.15: Global position of the centre of mass and steering angle of the vehicle with one articulation for case 3.

Figure 5.16: System state variables of the vehicle with one articulation for the case 4.

Figure 5.17: Global position of centre of mass and steering angle of the vehicle with one articulation for case 4.

5.1.3 System response for the vehicle with two articulations

Table 5.7 shows the parameters of the vehicle with tow articulations and the necessary information to calculate it, obtained from websites for commercial vehicles⁵ and towing implement⁶ manufacturers. For all simulated cases with two-articulated vehicle, the initial conditions are the same for both controllers, those being $x_0 = [0, 0, 0, 0, 0, 0, 0.3, -0.1]^T$, and

$$H = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, E_F = \begin{bmatrix} 6.8572 \times 10^{-5} \\ -8.6201 \times 10^{-5} \\ -1.1401 \times 10^{-5} \\ -8.0034 \times 10^{-5} \\ -1.1401 \times 10^{-5} \\ -8.0034 \times 10^{-5} \\ 0 \\ -666.66667 \times 10^{-5} \end{bmatrix}^T \text{ and } E_G = \begin{bmatrix} -666.66667 \times 10^{-5} \\ -666.66667 \times 10^{-5} \end{bmatrix}^T.$$

The robustness parameter γ , and weight matrices Q , R^c , R and Q^c used for the vehicle with one articulation are

$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 159800 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 10 \end{bmatrix}, R = \begin{bmatrix} 470 & 0 \\ 0 & 470 \end{bmatrix},$$

$$\gamma = 27190, R^c = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 169 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 10 \end{bmatrix} \text{ and } Q^c = \begin{bmatrix} 51610 & 0 \\ 0 & 51610 \end{bmatrix}.$$

Table 5.8 and Table 5.9 show maximum steering rate, payload variations, and $\mathcal{L}_2$ norm of lateral displacement and orientation errors for both performed controllers,

⁵ <https://www.scania.com>

⁶ <http://www.librelato.com.br>

Table 5.7: Description of the parameters for the vehicle with two articulation

Parameter	Meaning	Value
a_1	Distance from the front axle to the CG on first unit	3.18 <i>m</i>
a_2	Distance from the coupling point to the CG on second unit	4.41 <i>m</i>
a_3	Distance from the coupling point to the CG on third unit	2.72 <i>m</i>
b_1	Distance from the rear axle to the CG on first unit	1.62 <i>m</i>
b_2	Distance from the rear axle to the CG on second unit	0 <i>m</i>
b_3	Distance from the rear axle to the CG on third unit	4.08 <i>m</i>
l_1	First unit wheelbase	4.81 <i>m</i>
l_2	Second unit wheelbase	4.41 <i>m</i>
l_3	Third unit wheelbase	6.80 <i>m</i>
d_1	Distance between the rear axle and the coupling point on first unit	1.93 <i>m</i>
d_2	Distance between the rear axle and the coupling point on second unit	0 <i>m</i>
h_1	Distance between coupling point and the first unit CG	3.54 <i>m</i>
l_1^*	Distance between the front axle and the coupling point on first unit	6.73 <i>m</i>
l_2^*	Distance between the coupling points on second unit	4.41 <i>m</i>
v	Forward velocity	16.67 <i>m/s</i>
m_1	First unit mass	16030 <i>kg</i>
m_2	Second unit mass	0 <i>kg</i>
m_3	Third unit mass	8010 <i>kg</i>
Payload 1	The amount of payload carried by the vehicle	12550 <i>kg</i>
Payload 1	The amount of payload carried by the vehicle	12550 <i>kg</i>
J_1	First unit moment of inertia	120992 <i>kg m²</i>
J_2	Second unit moment of inertia	0 <i>kg m²</i>
J_3	Third unit moment of inertia	171132 <i>kg m²</i>
c_1	Front axle cornering stiffness of the towing unit	540419 <i>N/rad</i>
c_2	Rear axle cornering stiffness of the first unit	1064462 <i>N/rad</i>
c_3	Front axle cornering stiffness of the first unit	692716 <i>N/rad</i>
c_4	Rear axle cornering stiffness of the second unit	461810 <i>N/rad</i>

respectively. Payload values for every evaluated case were chosen for the best illustration of the influence of mass variation.

Table 5.8: Evaluated cases for both controllers for the vehicle with two articulation.

Case	Payload 1	Payload 2	$\max\ \dot{\alpha}_{RLQR}\ $	$\max\ \dot{\alpha}_{\mathcal{H}_\infty}\ $
1	100%	100%	0.2010 rad/s	0.2010 rad/s
2	200%	100%	0.1984 rad/s	0.1521 rad/s
3	200%	200%	0.1984 rad/s	0.1644 rad/s
4	100%	200%	0.2010 rad/s	0.1543 rad/s
5	200%	200%	0.1163 rad/s	19.8258 rad/s

Table 5.9: $\mathcal{L}_2$ norm of the lateral displacement and orientation errors for the vehicle with two articulation.

Case	$\ \rho\ _{\mathcal{L}_2}$		$\ \theta\ _{\mathcal{L}_2}$	
	RLQR	$\mathcal{H}_\infty$	RLQR	$\mathcal{H}_\infty$
1	1.2021	1.1610	0.1291	0.1242
2	1.3450	1.5581	0.1302	0.1714
3	1.3450	1.6442	0.1302	0.1740
4	1.2021	1.2485	0.1291	0.1221
5	1.2901	0.4871	0.1456	0.1406

The description of each evaluated case and the graphics of numerical results for the vehicle with one articulation follows.

Case 1 Considering nominal payload in both first and third units, Figure 5.18 and Figure 5.19 show the system state variables, the global position of vehicle centre of mass and steering angle performed by both controllers.

Case 2 Considering 100% of overload over the payload nominal value of the first unit, Figure 5.20 and Figure 5.21 show the system state variables, the global position of tractor centre of mass and steering angle performed by both controllers.

Case 3 Considering 100% of overload over the payload nominal value of both first and third units, Figure 5.22 and Figure 5.23 show the system state variables, the global position of tractor centre of mass and steering angle performed by both controllers.

Case 4 Considering 100% of overload over the payload nominal value of the third unit, Figure 5.24 and Figure 5.25 show the system state variables, the global position of tractor centre of mass and steering angle performed by both controllers.

Case 5 Lastly, Considering 100% of overload over the payload nominal value of both first and third units, $\gamma = 31190$ and the weight matrices $Q = R^c$ and $R = Q^c$ as follows

$$Q = R^c = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 25000 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 100 \end{bmatrix}, R = Q^c = \begin{bmatrix} 67070 & 0 \\ 0 & 67070 \end{bmatrix}.$$

These values were adjusted to ensure both $\mathcal{H}_\infty$ existence and the RLQR smoothness. Figure 5.26 shows the very beginning of the input signal for this situation and Figure 5.27 shows the system state variables.

Figure 5.18: System state variables of the vehicle with two articulation for case 1.

Figure 5.19: Global position of the centre of mass and steering angle of the vehicle with two articulation for case 1.

Figure 5.20: System state variables of the vehicle with two articulation for case 2.

Figure 5.21: Global position of the centre of mass and steering angle of the vehicle with two articulation for case 2.

Figure 5.22: System state variables of the vehicle with two articulation for case 3.

Figure 5.23: Global position of the centre of mass and steering angle of the vehicle with two articulation for case 3.

Figure 5.24: System state variables of the vehicle with two articulation for case 4.

Figure 5.25: Global position of the centre of mass and steering angle of the vehicle with two articulation for case 4.

Figure 5.26: Input signal of the vehicle with two articulation for the case 5.

Figure 5.27: System state variables of the vehicle with two articulations for the case 5.

5.2 Experimental results

Road tests were carried out in a mining environment with the purpose of demonstrating the Robust Linear Quadratic Regulator performance in experiments. This section describes the procedure and shows the results of experimental tests.

5.2.1 Hardware description

Many sensors and actuators are necessary for the autonomous operation, however only the ones that are pertinent to the lateral control will be described here. Thus, Figure 5.28 shows the Scania P-420 CB8x4ESZ, the vehicle used in road experiments. Furthermore, Table 5.10 shows the calculate vehicle parameters, based on the technical specifications of the heavy-duty vehicle and tipper implement. The normalised cornering stiffness was also applied in these calculations by assuming $f_1 = f_2 = 5.73 \text{ rad}^{-1}$.

The vehicle used for experiments was equipped with a industrial personal computer, a Dell Embedded Box PC 5000⁷ (RAM memory: 16Gb and processor: Intel^R CoreTM i7-

⁷ i.dell.com/sites/doccontent/shared-content/data-sheets/en/Documents/specsheet-dell-embedded-box-PC-3000-5000.pdf

Figure 5.28: The heavy-duty vehicle used for experimental control validation.

Table 5.10: Parameters of the Scania P-420 CB8x4ESZ

Parameter	Meaning	Value
a_1	Distance from the front axle to the CG	3.19 m
b_1	Distance from the rear axle to the CG	1.62 m
l_1	Vehicle wheelbase	4.81 m
m_1	Vehicle mass	16030 kg
Payload	The amount of payload carried by the vehicle	12550 kg
J_1	Vehicle moment of inertia	215717 kg m ²
c_1	Front axle cornering stiffness	540419 N/rad
c_2	Rear axle cornering stiffness	1064462 N/rad

6820EQ CPU @ 2.80GHz), where the algorithm of the self-driving system was implemented by using Python programming language.

The autonomous steering system have three main components, a DC motor for actuation, a controller for it and a encoder for feedback control. The used actuator is the DC motor with transmission Bosch EFP ⁸ (24V 78,5W), the used DC motor controller is

⁸ [bosch-ibusiness.com/downloads/I-Buisness_E-Motoren_2017_Catalogue_en_75091.pdf#page=13](https://www.bosch-ibusiness.com/downloads/I-Buisness_E-Motoren_2017_Catalogue_en_75091.pdf#page=13)

the Roboteq MDC2460⁹ and the used encoder for feedback is the YUMO E6B2-CWZ3E¹⁰. The sampling period was 0.01 seconds due to the GPS frequency.

5.2.2 The experiment

The experiments tests were conducted in a mining environment on a road at Carajás Mountains, Brazil, shown in Figure 5.29. Furthermore, Figure 5.30 shows the both first and second performed paths respectively from A to B and from B to A, as shown in Figure 5.29.

Figure 5.29: Satellite view of the experiments location.

Source: Adapted from Google Earth.

The controller parameters are

$$\mu = 10^9,$$

$$H = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \quad E_F = \begin{bmatrix} -0.000405618009134 \\ 0.004949413574869 \\ 0.000000034165611 \\ 0.000024747067874 \end{bmatrix}^T \quad \text{and} \quad E_G = \begin{bmatrix} -0.00114326083475285 \\ -0.00114326083475285 \end{bmatrix}^T,$$

⁹ roboteq.com/index.php/docman/motor-controllers-documents-and-files/documentation/datasheets/mdc2xxx-datasheet-1/2-datasheet-mdc2xxx/file

¹⁰ mouser.com/ds/2/307/e6b2-c_ds_csm491-25665.pdf

Figure 5.30: Performed paths in experiments.

and the adjusted weight matrices are

$$Q = \begin{bmatrix} 0.1 & 0 & 0 & 0 \\ 0 & 0.1 & 0 & 0 \\ 0 & 0 & 100 & 0 \\ 0 & 0 & 0 & 15 \end{bmatrix} \text{ and } R = \begin{bmatrix} 10000 & 0 \\ 0 & 10000 \end{bmatrix}.$$

The usual value for the payload parameter in mining operations is in the order of 35 tons. However, in order to investigate the effects of parametric uncertainties on system performance, the payload parameter was set to 12550 *kg*, which is a suitable value for regional and local distribution, construction and some off-road applications, see Table 5.10. Consequently, the control law calculates its gain based on a payload value that is much lower than the actual value used for the heavy-duty operation.

It was not possible to get the yaw rate and lateral velocity data, therefore the control gain is calculated taking into account every state variables $x = [\dot{y}_1, \dot{\psi}_1, \rho, \theta]^T$ but only the lateral displacement ρ and orientation error θ can be evaluated. Hence, the block diagram for the experimental test is given by Figure 5.31, where $e_i^* = [\rho, \theta]^T$.

5.2.3 System response

The heavy-duty vehicle performed a path from A to B in the first test, see Figure 5.29. The maximum steering angle was delimited at 0.3491 *rad* due to safety recommendations, because of this the human driver was required to assume the control of the vehicle steering wheel near the point B. Figure 5.32 shows the vehicle lateral displacement and orientation

Figure 5.31: Block diagram for experiments.

Source: Author.

error performed by the Robust Linear Quadratic Regulator and indicates when the human driver took the control of its steering system.

Figure 5.32: Performed lateral displacement and orientation error from A to B.

(a) Lateral displacement performed by both the RLQR and driver.

(b) Orientation error performed by both the RLQR and driver.

Furthermore, Figure 5.33a shows the velocity performer by the vehicle during the first test, and Figure 5.33b shows the steering angle calculated by the RLQR and performed by the autonomous steering system and human driver. Moreover a $\max\|\dot{\alpha}\| = 0.2643 \text{ rad/s}$ was observed in the first test.

In the second test, the heavy-duty vehicle performed a path from point B to A. Similarly to the first test, Figure 5.34 shows the vehicle displacement and orientation error performed by Robust Linear Quadratic Regulator and indicates when the human driver

Figure 5.33: Performed velocity and steering angle from A to B.

(a) Velocity performed by both the RLQR and driver.

(b) Steering angle performed by both the RLQR and driver.

took the control of its steering system. The same considerations about the maximum steering angle and alignment problems of the first test are once more time observed in the second test.

Finally, Figure 5.35 shows the velocity, the steering angle calculated by the RLQR and the steering angle performed by the vehicle during the second test. Moreover, a $\max\|\dot{\alpha}\| = 0.5057 \text{ rad/s}$ was observed in the second test.

Figure 5.34: Performed lateral displacement and orientation error from B to A.

(a) Lateral displacement performed by both the RLQR and driver.

(b) Orientation error performed by both the RLQR and driver.

Figure 5.35: Performed velocity and steering angle from B to A.

(a) Velocity performed by both the RLQR and driver.

(b) Steering angle performed by both the RLQR and driver.

6 DISCUSSION

This chapter discusses the findings in chapter 5. The numerical results for the three vehicle combinations studied here and the experimental results are discussed in this chapter.

6.1 Discussion on numerical results

The performance of the Robust Linear Quadratic Regulator and the $\mathcal{H}_\infty$ controller was numerically compared in simulations. In order to make a fair comparison, the procedure to obtain the results has three steps. Firstly, the weight matrices Q , R , R^c and Q^c of both controllers were adjusted so that the RLQR and $\mathcal{H}_\infty$ controller have the same value of $\max\|\dot{u}\|$ when the nominal payload is applied. Secondly, the vehicle was subject to payload mass variation to express the uncertainty on the mass parameter, the numerical results were obtained with nominal payload, 100% of overload and without payload. Finally, the weight matrices of both controllers were adjusted to $Q = R^c$ and $R = Q^c$ and once again the numerical results were obtained.

The main goal of these evaluated cases was to show how the Robust Recursive Regulator deals with uncertainties in heavy-duty vehicles. Numerical results demonstrated that the RLQR performance is less affected by payload mass variation than $\mathcal{H}_\infty$ controller. For the adjustment of weight matrices as described in the first step, the robust regulator presented worst performance before the settling time for every vehicle combinations during mass variation. However, it is verified through graphics and $\mathcal{L}_2$ norm analysis that the lateral displacement and orientation error of the robust regulator are less affected by mass uncertainties than $\mathcal{H}_\infty$ controller.

Moreover, at third step, in the presence of uncertainties, the $\max\|\dot{u}_{\mathcal{H}_\infty}\|$ (maximum steering rate of the $\mathcal{H}_\infty$ controller) is severely influenced by parametric variations in every vehicle combination while $\max\|\dot{u}_{RLQR}\|$ (maximum steering rate of the RLQR) is much less affected. It can be verified in Figure 5.9, 5.16, 5.17, 5.26 and 5.27 that $\max\|\dot{u}_{\mathcal{H}_\infty}\|$ assumes soaring values and affect the vehicle performance. This is significant, since high steering angle rates mean abrupt driving, which is not be possible for the mechanical system of the vehicle, representing a safety limitation to the $\mathcal{H}_\infty$ controller. An additional safety limitation is shown in Figure 5.27, where ϕ_1 presents a value of -1.88 rad which could result in trailer swing or jackknifing.

Furthermore, in the vehicle with two articulations, it was not verified a big influence of the payloads ratio in first and third units. This apparent lack of correlation can be attributed to the type of manoeuvre performed and the assumption of a constant forward

velocity.

The performed results for lateral velocity $\dot{y}_1$, yaw rate $\dot{\psi}_1$, articulation angle rate $\dot{\phi}$ and articulation angle ϕ demonstrate that the RLQR deals better with vehicle lateral dynamic behaviour since its performance is much less affected by mass variations. Furthermore, the obtained results confirm that the robust recursive regulator is still more stable, smoother and safer than $\mathcal{H}_\infty$ in the presence of payload variations. For better performance of the $\mathcal{H}_\infty$ controller, the parameter γ needs an offline adjusted for each particular payload. This is very inefficient for practical applications, given the wide mass variations in heavy-duty vehicles. The advantage of the RLQR is that does not require offline adjustment of any auxiliary parameter, maintaining good performance for each evaluated case.

6.2 Discussion on experimental results

The Robust Linear Quadratic Regulator performance was experimentally tested in the path following of a heavy-duty truck operating in an actual mining environment. Once again the robust recursive regulator presented satisfactory robustness, driving smoothness and safety due to the large range of parameter uncertainties in mining operations. Moreover, the limitations on feedback data did not significantly affect the vehicle performance in its path following task. The driving smoothness during experiments can be verified by the obtained $\max\|\dot{\alpha}\| = 0.2643 \text{ rad/s}$ for the route A to B and $\max\|\dot{\alpha}\| = 0.5057 \text{ rad/s}$ for the route B to A.

The experimental results for lateral displacement ρ and orientation error θ demonstrate that the Robust Linear Quadratic Regulator has satisfactory performance in heavy-duty applications. The obtained results confirm the findings in numerical simulations since the vehicle presented stability, smoothness, robustness and safety along the performed path in the presence of mass uncertainties.

It is important to note that γ assumed different values for different vehicle conditions during simulations while the penalty parameter remains $\mu = 10^9$ for all numerical simulations and even experimental tests. Returning to the hypothesis posed at the beginning of this study, it is now possible to state that the Robust Linear Quadratic Regulator does not require the offline adjustment of any auxiliary tuning parameters.

7 CONCLUSION

The Robust Linear Quadratic Regulator has been applied to perform the lateral control of autonomous heavy-duty vehicles subject to parametric uncertainties. Considering uncertainty in the vehicle mass, the RLQR performance was better in terms of robustness, lateral stability, driving smoothness and safety when compared to numerical simulations of $\mathcal{H}_\infty$ robust control technique. The RLQ controller performs well for a wide range of payloads, while the performance of the H_∞ controller is significantly affected by higher payloads, given a constant γ . The findings of RLQR performance in numerical simulations was afterwards demonstrated in experimental tests when a heavy-duty vehicle executed the path following operating in an actual mining environment. Hence, the robust recursive regulator was demonstrated as a profitable control technique to deal with parametric uncertainties in the path-following context for heavy-duty vehicles. Nevertheless, vertical and roll stability cannot be guaranteed because it uses a model-based control design that only considers the planar motion.

The robust recursive regulator could be exploited in aerial and underwater vehicles in order to perform the path-following control. Future work will involve the used of Markov jump model to minimise the effects of linearisation in the vehicle lateral model.

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Appendix

APPENDIX A – EQUATIONS OF MOTION

The equations of motion derived in this appendix, can be applied on all vehicle combinations used in this study. The used equation of motion are derived for a vehicle with two articulations, the maximum adopted here. If vehicles with less articulations are used, the corresponding masses, inertias and cornering stiffness must be set to zero. Thus, the following assumptions are used for the equations of motion:

- The articulation and tyre slip angles are small;
- The left and right tyres in each axle can be united in one equivalent tyre;
- As the centres of gravity are assumed to be on the ground, there is no body roll;
- There are no pneumatic or mechanical trail at the centre point steering;
- The forward velocity is constant;
- Aerodynamics forces on the vehicle are ignored.

Figure A.1 shows the free body diagram of the single-track model of a vehicle with two articulations.

The tyre forces are expressed in local coordinates as:

$$\begin{aligned}
 F_{y_1} &= -\frac{1}{u}c_1(\dot{y}_1 + a_1\dot{\psi}_1) + c_1\alpha \\
 F_{y_2} &= -\frac{1}{u}c_2(\dot{y}_1 - b_1\dot{\psi}_1) \\
 F_{y_3} &= -\frac{1}{u}c_3(\dot{y}_1 - h_1\dot{\psi}_1 - l^2 - l_2\dot{\psi}_2) + c_3\phi_1 \\
 F_{y_4} &= -\frac{1}{u}c_4(\dot{y}_1 - h_1\dot{\psi}_1 - l_2^*\dot{\psi}_2 - l_3\dot{\psi}_3) + c_4(\phi_1 + \phi_2).
 \end{aligned} \tag{A.1}$$

The state vector is $x = [\dot{y}_1, \dot{\psi}_1, \dot{\phi}_1, \dot{\phi}_2, \phi_1, \phi_2]^T$ and the equation describing the longitudinal dynamics is not considered as the forward velocity v is constant. Thus the equations of motion are:

$$\begin{aligned}
 &(m_1 + m_2 + m_3)(\ddot{y}_1 + v\dot{\psi}_1) - (m_2(h_1 + a_2) + m^3(h_1l_2^* + a_3))\ddot{\psi}_1 \\
 &- (m_2a_2 + m_3(l_2^* + a_3))\ddot{\phi}_1 - m_3a_3\ddot{\phi}_2 = \\
 &- \frac{1}{u}((c_1 + c_2 + c_3 + c_4)\dot{y}_1 + (a_1c_1 - b_1c_2) - c_3(h_1 + l_2 - c_4(h_1 + l_2^* + l_3))\dot{\psi}_1 \\
 &- (c_3l_2 + c_4(l_2^* + l_3))\dot{\phi}_1 - c_4l_3\dot{\phi}_2 - (c_3 + c_4)v\phi_1 - c_4v\phi_2) + c_1\alpha
 \end{aligned}$$

$$\begin{aligned}
& -h_1(m_2 + m_3)(\ddot{y}_1 + v\dot{\psi}_1) + (J_1 + m_2h_1(h_1 + a_2) + m_3h_1(h_1 + l_2^* + a_3))\ddot{\psi}_1 \\
& + (m_2h_1a_2 + m_3h_1(l_2^* + a_3))\ddot{\phi}_1 + m_3h_1a_3\ddot{\phi}_2 = \\
& -\frac{1}{u}((a_1c_1 - b_1c_2 - (c_3 + c_4)h_1\dot{y}_1 + (a_1^2 + b_1c_2 + c_3h_1)(h_1 + l_2^* + l_3))\dot{\psi}_1 \\
& + (c_3h_1l_2 + c_4h_1(l_2^* + l_3))\dot{\phi}_1 + c_4h_1l_3\dot{\phi}_2 + (c_3 + c_4)h_1v\phi_1 + c_4h_1v\phi_2) + c_1a_1\alpha \\
& -(m_2a_2 + m_3l_2^*)(\ddot{y}_1 + v\dot{\psi}_1) + (J_2 + m_2a_2(h_1 + a_2) + m_3l_2^*(h_1 + l_2^* + a_3))\ddot{\psi}_1 \\
& + (J_2 + m_2a_2^2 + m_3l_2^* + m_3l_2^*(l_2^* + a_3))\ddot{\phi}_1 + m_3a_3l_2^*\ddot{\phi}_2 = \tag{A.2} \\
& -\frac{1}{u}(-(c_3l_2^2 + c_4l_2^*)\dot{y}_1 + c_3l_2(h_1 + l_2) + c_4l_2^*(h_1 + l_2^*(h_1 + l_2^* + l_3)))\dot{\psi}_1 \\
& + (c_3l_2^2 + c_4l_2^*(l_2^* + l_3))\dot{\phi}_1 + c_4l_3l_2^*\dot{\phi}_2 + (c_3l_2 + c_4l_2^*)v\phi_1 + c_4l_2^*v\phi_2) \\
& -m_3a_3(\ddot{y}_1 + v\dot{\psi}_1) + (J_3 + (m_3a_3))\ddot{\psi}_1 + (J_3 + m_3a_3(l_2^* + a_3))\ddot{\phi}_1 \\
& + (J_3 + m_3a_2^2)\ddot{\phi}_2 = -\frac{1}{u}(-c_4l_3\dot{y}_1 + c_4l_3(h_1 + l_2^* + l_3))\dot{y}_1 \\
& + c_4l_3(l_2^* + l_3)\dot{\phi}_1 + c_4l_3^2\dot{\phi}_2 + c_4l_3v\phi_1 + c_4l_3v\phi_2
\end{aligned}$$

It is worth noting that for a vehicle with one articulation $c_4 = 0$, $m_3 = 0$, $J_3 = 0$ and the state vector is $\mathbf{x} = [y_1, \dot{\psi}_1, \dot{\phi}_1, \phi_1]^T$. Furthermore, for a non-articulated vehicle $c_3 = c_4 = 0$, $m_2 = m_3 = 0$, $J_2 = J_3 = 0$ and $\mathbf{x} = [y_1, \dot{\psi}_1]^T$.

Figure A.1: Free body diagram of a vehicle with two articulations.

Source: Author.

APPENDIX B – $\mathcal{H}_\infty$ CONTROL

The robust control design considering $\mathcal{H}_\infty$ method mentioned in (HASSIBI; SAYED; KAILATH, 1999) is used for the following linear system

$$x_{i+1} = F_i x_i + G_{1,i} w_i + G_{2,i} u_i, \quad i = 0, \dots, N, \quad (\text{B.1})$$

where x_i is the state vector, u_i is the control input and w_i is the disturbance. In its sub-optimal formulation, this technique is based on finding a control strategy where for every x_0 and $\{w_i\}_{i=0}^N$,

$$\frac{x_{N+1}^{*T} P_{N+1}^c x_{N+1}^* + \sum_{i=0}^N (u_i^{*T} Q_i^c u_i^* + x_i^{*T} R_i^c x_i^*)}{x_0^{*T} \Pi_0^{-1} x_0^* + \sum_{i=0}^N (w_i^{*T} Q_i^w w_i^*)} < \gamma^2, \quad (\text{B.2})$$

for a suitable $\gamma > 0$, where P_{N+1}^c , Q_i^c , R_i^c , Π_0 and Q_i^w are non-negative definite weighing matrices. Such matrices are associated with the final state, control input, state, initial state and disturbance, respectively. The recursive solution of this problem is formulated in terms of backwards Riccati equation and the verification of some existence conditions is necessary.

To perform the control using this technique, uncertain system (4.1)-(4.2) must be rewritten as the system (B.1). Hence, the following immediate identifications are considered

$$F_i \leftarrow F_i, \quad G_{2,i} \leftarrow G_i \quad x_i \leftarrow x_i, \quad u_i \leftarrow u_i, \quad G_{1,i} \leftarrow H_i, \quad w_i \leftarrow \Delta_i \begin{bmatrix} E_{F_i} & E_{G_i} \end{bmatrix} \begin{bmatrix} x_i \\ u_i \end{bmatrix}, \quad (\text{B.3})$$

$$P_{N+1}^c \leftarrow P_{N+1}, \quad Q_i^c \leftarrow R_i, \quad R_i^c \leftarrow Q_i, \quad Q_i^w \leftarrow I, \quad \Pi_0 \leftarrow I.$$

In order to use the system (B.1) as the uncertain system (4.1)-(4.2), some algebraic manipulations are necessary. Considering $R_{G,i}^{c-1} = Q_i^c + G_{2,i}^T P_{N+1}^c G_{2,i}$, the control signal is obtained from (HASSIBI; SAYED; KAILATH, 1999) as

$$\begin{aligned} u_i &= -R_{G,i}^{c-1} G_{2,i}^T P_{N+1}^c F_i x_i - R_{G,i}^{c-1} G_{2,i}^T P_{N+1}^c G_{1,i} w_i \\ &= (-R_{G,i}^{c-1} G_{2,i}^T P_{N+1}^c) (F_i x_i + G_{1,i} w_i). \end{aligned} \quad (\text{B.4})$$

From identifications made in (B.3) substitutions can be made in $G_{1,i}$ and w_i

$$\begin{aligned} u_i &= (-R_{G,i}^{c-1} G_{2,i}^T P_{N+1}^c) ((F_i + H_i \Delta E_{F_i}) x_i + H_i \Delta E_{G_i} u_{i-1}) \\ &= -R_{G,i}^{c-1} G_{2,i}^T P_{N+1}^c (F_i + \delta F_i) x_i - R_{G,i}^{c-1} G_{2,i}^T P_{N+1}^c H_i \Delta E_{G_i} u_{i-1}, \end{aligned}$$

and adding G_2 in both sides of the equation the (B.4) becomes

$$\begin{aligned} u_i &= -R_{G,i}^{c-1} G_{2,i}^T P_{N+1}^c (F_i + \delta F_i) x_i - R_{G,i}^{c-1} G_{2,i}^T P_{N+1}^c (G_{2,i} + H_i \Delta E_{G_i}) u_{i-1} + \\ &\quad + R_{G,i}^{c-1} G_{2,i}^T P_{N+1}^c G_{2,i} u_{i-1} \\ &= -R_{G,i}^{c-1} G_{2,i}^T P_{N+1}^c (F_i + \delta F_i) x_i - R_{G,i}^{c-1} G_{2,i}^T P_{N+1}^c (G_{2,i} + \delta G_{2,i}) u_{i-1} + \\ &\quad + R_{G,i}^{c-1} G_{2,i}^T P_{N+1}^c G_{2,i} u_{i-1}. \end{aligned} \quad (\text{B.5})$$

Considering the sampling period sufficiently small so that $x_i \approx x_{i-1}$, the (B.5) can be rewritten as

$$u_i := -R_{G,i}^{c^{-1}} G_{2,i}^T P_{N+1}^c ((F_i + \delta F_i)x_{i-1} + (G_{2,i} + \delta G_{2,i})u_{i-1}) + R_G^{c^{-1}} G_2^T P_{N+1}^c G_{2,i} u_{i-1}.$$

Thus, as $u_{i-1} = z_i$, the system (B.1) can be rewritten as system (4.1)-(4.2)

$$\begin{aligned} x_{i+1} &= (F_i + \delta F_i)x_i + (G_{2,i} + \delta G_{2,i})u_i \\ \begin{bmatrix} \delta F_i & \delta G_i \end{bmatrix} &= H_i \Delta_i \begin{bmatrix} E_{F_i} & E_{G_i} \end{bmatrix}, \end{aligned}$$

where

$$u_i = -R_{G,i}^{c^{-1}} G_{2,i}^T P_{N+1}^c x_i + R_{G,i}^{c^{-1}} G_{2,i}^T P_{N+1}^c G_{2,i} z_i. \quad (\text{B.6})$$

See details in (HASSIBI; SAYED; KAILATH, 1999).

APPENDIX C – UNCERTAINTIES MATRICES

In order to estimate the uncertainties matrices E_F , E_G and H , we considered the inertia uncertainties for deriving a robust control strategy by adopting maximum and minimum values of payload, which results in $m_{p_{max}}$ and $m_{p_{min}}$, respectively. Setting these values of mass, the maximum variations of the matrices F and G are calculated as:

$$\Gamma_F = F_{m_{p_{min}}} - F_{m_{p_{max}}} \quad (C.1)$$

where $F_{m_{p_{min}}}$ and $F_{m_{p_{max}}}$ are the discretised state-space matrices when $m_{p_{max}}$ and $m_{p_{min}}$ are applied to the state-space system (3.7). Thus, we afterwards selected the row in Γ_F that is most affected by mass variations, here the fifth row.

In this work, $m_{p_{min}}$ and $m_{p_{max}}$ values correspond to the unloaded and 100% of overload vehicle operation. Consequently, this range of uncertainty may imply in large E_F and E_G values, denoted as $E_{F_{100\%}}$ and $E_{G_{100\%}}$. The selection of these values during the control design guarantees the robustness stability for any condition, satisfying the mass variability. However, it jeopardises the performance of the nominal case. Therefore, lower E_F and E_G values were considered to overcome this problem. This choice is capable of enhancing the robustness of the proposal without jeopardising the system performance.

Thus, the matrices E_{F_i} and E_{G_i} are obtained as follows:

$$E_{F_i} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 0.1 \end{bmatrix}^T \begin{bmatrix} \arg(|\Gamma_{F_{5,1}}|) & 0 & 0 & 0 & 0 & 0 \\ 0 & \arg(|\Gamma_{F_{5,2}}|) & 0 & 0 & 0 & 0 \\ 0 & 0 & \arg(|\Gamma_{F_{5,3}}|) & 0 & 0 & 0 \\ 0 & 0 & 0 & \arg(|\Gamma_{F_{5,4}}|) & 0 & 0 \\ 0 & 0 & 0 & 0 & \arg(|\Gamma_{F_{5,5}}|) & 0 \\ 0 & 0 & 0 & 0 & 0 & \arg(|\Gamma_{F_{5,6}}|) \end{bmatrix} \quad (C.2)$$

$$E_{G_i} = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}^T \begin{bmatrix} \arg\{\max(|\Gamma_{F_{5,j}}|)\} & 0 \\ 0 & \arg\{\max(|\Gamma_{F_{5,j}}|)\} \end{bmatrix} \quad (C.3)$$

and $H_i = [1, 1, 1, 1, 1, 1]^T$. This way, the uncertainties matrices are obtained through Equation 4.2

$$\begin{bmatrix} \delta F_i & \delta G_i \end{bmatrix} = H_i \Delta_i \begin{bmatrix} E_{F_i} & E_{G_i} \end{bmatrix},$$

where Δ_i is a scalar represented by the mass variation.