**UNIVERSITY OF SÃO PAULO
ENGINEERING SCHOOL OF SÃO CARLOS**

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**Increasing the Energy Efficiency of Parallel Manipulators
by means of Kinematic Redundancy and Model
Predictive Control**

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**Aumentando a Eficiência Energética dos Manipuladores
Paralelos por meio da Redundância Cinemática e do
Modelo de Controle Preditivo**

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This work is dedicated to my beloved colleague Carlos Perez, who had the ability to teach me how to study science.

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"Cada pessoa que passa na nossa vida,
passa sozinha, porque cada pessoa é
única e nenhuma substitui a outra.
Cada pessoa que passa pela nossa vida
passa sozinha, não nos deixa só,
porque deixa um pouco de si e leva um
pouquinho de nós. Essa é a mais bela
responsabilidade da vida e a prova de
que as pessoas não se encontram por
acaso."

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“It is change, continuing change, inevitable change, that is the dominant factor in society today. No sensible decision can be made any longer without taking into account not only the world as it is, but the world as it will be.”

Isaac Asimov

ABSTRACT

RUIZ, A. G. **Increasing the Energy Efficiency of Parallel Manipulators by means of Kinematic Redundancy and Model Predictive Control**. 2017. 125p. Thesis(PhD) - Engineering School of São Carlos, University of São Paulo, São Carlos, 2017.

The use of robotic manipulators in industrial applications is continuously growing. Therefore, the proposal of novel kinematic architectures for robotic manipulators can be a strategy for coping with the required performance of specific tasks. On this matter, the parallel manipulators represent an alternative to fulfill this gap. The objective of this manuscript is to prove that the energy efficiency of parallel manipulators can be increased by the use of kinematic redundancy. Due to the presence of kinematic redundancy, the number of solutions to the inverse kinematics problem become infinite. Hence, a redundancy resolution scheme is required to select a suitable one among the infinite solutions. In this work, a model predictive control (MPC) based method is proposed as redundancy resolution scheme. This proposal is evaluated numerically and experimentally by comparing the energy consumption of non-redundant and kinematically redundant manipulators during the execution of pre-defined tasks. The non-redundant manipulator under study is the planar parallel $3RRR$ manipulator. This manipulator consists of three identical kinematic chains containing one active revolute joint and two passive revolute joints. Kinematic redundancies were added to the manipulator by including one active prismatic joint in each kinematic chain. In this way, the kinematically redundant manipulator under study is the planar parallel $3PRRR$ manipulator. By activating or locking the prismatic joints, up to three levels of kinematic redundancy can be evaluated. Numerical kinematic and dynamic models of the manipulators under study were derived not only for their numerical evaluation but also for the derivation of the model-based redundancy resolution scheme. Experimental data was acquired using the prototype built at the Laboratory of Dynamics at São Carlos School of Engineering at University of São Paulo. This experimental data was exploited for assessing the usability of the MPC for deriving a redundancy resolution scheme and for evaluating the impact of several levels of kinematic redundancy on the manipulator's energy consumption. Based on this data, one can conclude that MPC can be a suitable alternative for solve redundancy resolution problems and that the redundant parallel manipulators presented a lower energy consumption than the non-redundant one to execute the pre-defined tasks. The rate of reduction on the energy consumption achieved by the redundant manipulators varied between 6% and 60% depending on the task. Nevertheless, the numerical and experimental data presented differences in some particular cases.

Keywords: Parallel Kinematic Manipulators, Kinematic Redundancy, Energy Efficiency, Model Predictive Control, Redundancy Resolution Scheme.

RESUMO

RUIZ, A. G. **Aumentando a Eficiência Energética dos Manipuladores Paralelos por meio da Redundância Cinemática e do Modelo de Controle Preditivo**. 2017. 125p. Tese (Doutorado) - Escola de Engenharia de São Carlos, Universidade de São Paulo, São Carlos, 2017.

O número de aplicações realizadas pelos manipuladores robóticos cresce continuamente. Assim, o desenvolvimento de novas arquiteturas para os manipuladores robóticos mais adaptadas a aplicações concretas é necessário. Destarte, os manipuladores paralelos constituem uma alternativa a ser considerada. O objetivo deste texto é provar que a eficiência energética dos manipuladores paralelos pode ser incrementada por meio da redundância cinemática. A presença de redundância cinemática implica um número infinito de soluções no problema da cinemática inversa. Logo, é preciso um esquema de resolução de redundância para escolher uma das soluções. No presente texto, um método baseado no modelo de controle preditivo (MPC), é proposto como esquema de resolução de redundância. Esta proposta é avaliada tanto numérica como experimentalmente comparando o consumo energético dos manipuladores não redundante e redundantes durante a execução de umas trajetórias predefinidas. O manipulador paralelo não redundante estudado é o $3RRR$. Este manipulador é composto por três cadeias cinemáticas idênticas que incluem uma junta rotativa ativa e duas juntas rotativas passivas. Redundâncias cinemáticas foram adicionadas ao manipulador incluindo uma junta prismática ativa em cada uma das três cadeias cinemáticas, obtendo assim, o manipulador redundante $3PRRR$. Ativando ou bloqueando as juntas prismáticas podem ser avaliados até três níveis de redundância cinemática. Modelos matemáticos dos manipuladores foram propostos tanto para a estabelecer uma avaliação numérica como para a dedução do esquema de resolução de redundância. Um protótipo do manipulador $3PRRR$ construído na Escola da Engenharia de São Carlos foi usado para realizar os experimentos. Os dados experimentais foram utilizados para comprovar a utilidade do MPC como esquema de resolução de redundância, e para avaliar os efeitos da redundância cinemática no consumo energético. Com fundamento nos resultados é possível concluir que o MPC pode ser uma alternativa adequada para resolver problemas de resolução de redundância e que os manipuladores paralelos redundantes apresentaram um menor consumo energético para realizar a mesma tarefa quando comparados aos não redundante. A taxa de redução da energia em favor dos manipuladores redundantes varia entre 6% e 60% dependendo da tarefa. Por outro lado, a análise numérica mostrou discrepâncias com a análise experimental em certas circunstâncias.

Palavras-chave: Manipuladores Robóticos Paralelos, Redundância Cinemática, Eficiência Energética, Modelo de Controle Preditivo, Esquema de Resolução de Redundância.

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LIST OF SYMBOLS

a_i	Distance between coordinate system $O - xy$ and the point A_i
$\mathbf{A}_{eq}$	Matrix of equality constraints for the redundancy resolution scheme
α	End-effector's orientation angle
$\mathbf{b}_{eq}$	Vector of equality constraints for the redundancy resolution scheme
β_i	Rotation angle of passive joint located in point C_i
$\dot{\mathbf{d}}_{ij}$	Partial velocity vector of body j for the chain i
δ_i	Positions of the active prismatic joints
E_{total}	Energy consumed by the manipulator
$\mathbf{F}_e$	End-effector's force
$\mathbf{F}_{ij}$	Force of body j for the chain i
$\mathbf{F}_{x_{ij}}$	Horizontal force of body j for the chain i
$\mathbf{F}_{y_{ij}}$	Vertical force of body j for the chain i
$\mathbf{F}_{x_e}$	Horizontal force of the end-effector
$\mathbf{F}_{y_e}$	Vertical force of the end-effector
F_{δ_i}	Force of the active prismatic joint i
g	Acceleration of gravity
$\mathbf{G}_e$	Angular part of Jacobian matrix
$\mathbf{G}_{ij}$	Partial angular velocity matrix of body j for the chain i
γ_i	Prismatic joints orientation angles
$\mathbf{H}$	Hessian matrix for the redundancy resolution scheme
I_a	Armature current of the DC motor
I_j	Moment of inertia of body j
I_N	End-effector's moment of inertia
i	Chain number sub-index

$\mathbf{H}_e$	Linear part of Jacobian matrix
$\mathbf{H}_{ij}$	Partial linear velocity matrix of body j for the chain i
$\mathbf{J}$	Jacobian matrix
j	Body number sub-index
K_b	Back electromotive constant of DC motor
K_m	Torque constant of DC motor
$\mathbf{K}_{ij}$	Partial velocity matrix of body j for the chain i
$\mathbf{l}_b$	Lower bounds for the variables of the redundancy resolution scheme
L	Armature impedance of DC motor
l	Lead of the ball screw
l_1	Length of link 1
l_2	Length of link 2
λ_i	End-effector's internal angles
$\mathbf{M}_e$	End-effector's moment
$\mathbf{M}_{ij}$	Moment of body j for the chain i
m_j	Mass of body j
m_N	End-effector's mass
μ	Friction coefficient of the ball screw
N	Prediction horizon of the model predictive control
P	Power of DC motor
$\dot{\phi}_{ij}$	Angular velocity of body j for the chain i
$\mathbf{p}_{ij}$	Vector of partial forces and moments of body j of the chain i
$\mathbf{p}_e$	Vector of partial forces and moments of the end-effector
q	Trajectory position profile
R	Armature resistance of DC motor
$\mathbf{r}_{C_i}$	Position vector of point C_i

$\mathbf{r}_{D_i}$	Position vector of point D_i
$\dot{r}_{x_{ij}}$	Horizontal velocity of body j for the chain i
$\dot{r}_{y_{ij}}$	Vertical velocity of body j for the chain i
s_j	Distance from center of mass of body j to its pivotal point
t_0	Initial time instant
t_f	Final time instant
t	Time instant
T	Time interval
$\boldsymbol{\vartheta}$	Vector of variables of the redundancy resolution scheme
$\tau_{g,i}$	Torque at the input of the gear heads
P_{total}	Total power consumption
$\tau_{p,i}$	Torque of the actuator located in the prismatic joint
τ_{θ_i}	Torque of the active rotary joint i
$\boldsymbol{\tau}$	Driving torques vector
θ_i	Rotation angle of the active rotary joints
$\boldsymbol{\Theta}$	Actuated joints position vector
$\dot{\boldsymbol{\Theta}}$	Actuated joints velocities vector
$\mathbf{u}_b$	Upper bounds for the variables of the redundancy resolution scheme
V	Voltage of DC motor
V_b	Back electromotive force of DC motor
W	Mass of the load on the ball screw
x_e	End-effector's horizontal coordinate
y_e	End-effector's vertical coordinate
$\mathbf{X}$	End-effector's positions vector
$\dot{\mathbf{X}}$	End-effector's velocities vector

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1 INTRODUCTION

The development of energy efficient machines and systems represents a matter of interest for companies, governments, and communities due to the need for environmental suitability and the requirement for reducing the costs associated with the manufacturing processes. The robotic manipulators are present in a large number of industrial areas performing specific tasks as pick-and-place, assembling or milling, among others. From the point of view of their kinematic architecture, robotic manipulators can be divided into two groups: the serial and the parallel manipulators. On the one hand, the end-effector of a serial kinematic manipulator is connected by a single kinematic chain to the manipulator's base. On the other hand, the end-effector of a parallel kinematic manipulator is connected by two or more kinematic chains to the manipulator's base. The parallel manipulators, also known as parallel kinematic machines (PKMs), present advantages compared to serial manipulators such as improved accuracy, increased stiffness, higher load carrying capacity and the ability to achieve higher velocities. However, because of their simplicity and larger workspace, serial manipulators are more often found in practical applications than PKMs. The number of applications suited for robotic manipulators is continuously growing, hence, the study of innovative manipulator's kinematic architectures may promote insights on new design alternatives that can cope with the latest requirements.

Due to their kinematic architecture, PKMs are composed of lighter moving parts. As a result, this architecture is a promising alternative for designing not only robotic manipulators with improved dynamic performance (SILVA et al., 2010) but also energy efficient robotic systems (LI; BONE, 2001). A comparison between serial and parallel kinematic architectures for a three degrees-of-freedom (DOFs) spatial manipulator has been done by Li and Bone (2001). It could be concluded that for similar workspace and actuators, the energy consumption of the PKM was up to 26% lower than the consumption of the serial manipulator. The main reason for this reduction is the fact that the mass of the moving parts of the parallel manipulator, is indeed, significant smaller.

Despite this interesting attribute, PKMs suffer from the presence of singularities in their workspace (GOSSELIN; ANGELES, 1990). As a consequence, the workspace of PKMs is rather limited imposing an important drawback. To overcome this, some researchers have explored the use of redundancy for reducing the singular configurations (KOTLARSKI; HEIMANN; ORTMAIER, 2012), (CHA; LASKY; VELINSKY, 2007), (GOSSELIN; LALIBERTÉ; VEILLETTE, 2015). Besides the enlargement of the workspace due to the reduction of the singularity *loci*, the addition of redundancy might also provide higher accuracy (KOTLARSKI; HEIMANN; ORTMAIER, 2011), better motion/force transmissibility (XIE; LIU; WANG, 2011) and improved dynamic performance (ZHAO;

GAO, 2009), (FONTES; SILVA, 2016). A recent review of redundant PKMs has discussed the impact of the use of redundancies on the workspace enlargement, singularity elimination/avoidance, and improved joint-torque distribution (LUCES; MILLS; BENHABIB, 2017). Two main kinds of redundancy can be used: the actuation and kinematic redundancies.

A robotic manipulator is called redundant if the number of active actuators that control the manipulators' motion is higher than the number of degrees-of-freedom (DOFs) of the end-effector. The actuation redundancy is presented when the number of kinematic chains is higher than the number of end-effector's DOFs. In this case, the inverse kinematics problem presents a unique solution, *i.e.* any end-effector pose can be achieved by only one configuration of the actuators. Conversely, the kinematic redundancy is achieved when additional active actuators are introduced into the kinematic chains. In this case the number of solutions of the inverse kinematics problem is infinite, *i.e.* a single end-effector pose can be achieved by infinite combinations of the actuator's positions. Thus, a scheme for selecting a suitable configuration among the possible ones is mandatory. This scheme is referred as redundancy resolution along this manuscript.

In this work, the assessment of the energy consumption of kinematically redundant manipulators is presented. The objective was to evaluate the impact of kinematic redundancy on the energy consumption of a PKM. The outcome of this evaluation can give the engineer insights into the design of energy efficient robotic manipulators by the exploitation of novel kinematic architectures. Due to the presence of kinematic redundancies, a redundancy resolution scheme was required. In order to do so, a novel redundancy resolution scheme based on the Model Predictive Control (MPC) technique is proposed in this work. This proposal was evaluated numerically and experimentally by comparing the energy consumption of non-redundant and kinematically redundant manipulators during pre-defined tasks. Numerical kinematic and dynamic models of the manipulators under study were derived not only for their numerical evaluation but also for the derivation of the model-based redundancy resolution scheme. Experimental data were acquired using the prototype built at the Laboratory of Dynamics at São Carlos School of Engineering at University of São Paulo.

1.1 Energy Efficiency

The energy efficiency can be defined as a metric that evaluates what part of the total energy consumed by an equipment or machine is wasted as losses and what part is used as effective work. However, in the current days, the term energy efficiency is referred

as the set of strategies that allows to reduce the energy costs, to reduce the pollution and to provide energy access to a wider number of human beings. As a consequence the energy efficiency has become a core issue in the energy policies worldwide.

There is a growing concern from part of governments and society about the need for reducing the ecological footprint of the activities produced by humans. In addition, the industrial community is continuously looking for strategies to reduce the costs of their processes. Moreover, the emerging economies are aware of the importance that the energy efficiency has on their development.

Despite of the previous statements, the world is far from an complete energy efficient panorama. According to the International Energy Agency (iea), the 70% of the total world's energy use is outside of any energy efficiency policy (INTERNATIONAL ENERGY AGENCY (IEA), 2016). Hence, the development of energy efficient strategies is a valuable field that needs to be explored. On this matter, researchers from different areas have been working out on this demand during the last decades. Some popular examples of energy efficient devices and methodologies are summarized hereafter.

Perhaps the most notable example of an energy efficient device is the electrical vehicle. Electric vehicles are able to convert from 59% to 62% of the electrical energy to power at the wheels. The conventional petrol vehicle only converts from 17% to 21% of the available energy in the gasoline to power at the wheels (MILLER et al., 2011). Figure 1.1 illustrates the BMW i8 concept, an hybrid car that uses an electric engine to power the front wheels and a gasoline engine to power the rear wheels.

The advances in the development of energy efficient systems are being made by the improvement on the measuring devices. For instance, in Krones and Müller (2014), the authors presented an approach to reduce the energy consumption of an industrial process by supplying energy related measurements of the different stages. This approach allows to rearrange and update the different units included in the system in order to increase the energy efficiency of the entire factory. Recent researches have extrapolated this idea to complete urban areas creating the concept of *smart city* (HASHEM et al., 2016).

Another important energy efficiency related area is the *energy harvesting*. In this technique, the energy originated by external sources is collected by means of devices such as piezoelectric transducers or solar cells and stored in batteries. An interesting example of an energy harvesting system can be found in the city of Rio de Janeiro in Brazil. Located at *Morro da Mineira favela*, the company *PAVEGEN* has built a football pitch equipped with a power-generating system on the pavement. This system is composed by 200 piezo-

Figure 1.1: Hybrid car BMW i8 concept.

Source: Photographer David Villarreal Fernández.

electric based kinetic tails. As footballers run around the pitch, the energy is stored in batteries and used to provide light to the pitch during the night hours (PAVEGEN, 2014). In Fig.1.2, the operators are installing the power-generating pavement on the football pitch.

Figure 1.2: Operators placing the kinetic tails in the pitch.

Source: PAVENGEN.

Figure 1.3 shows the worldwide energy consumption by sectors during the year 2011. The total amount of energy consumed in the world during the year 2011 was 524 quadrillion of British Thermal Units (BTU) or $524 \cdot 10^{18}$ Joules (J). Almost the half of that amount was used in the industrial sector, where the vast majority of robotic manipulators are present. According to the International Federation of Robotics (IFR), there exist around 1.2 million of robotic manipulators. This number is expected to increase to around 2.6 million units by 2019. In view of this perspective, the investigation of techniques to increase the energy efficiency of robotic manipulators is a valuable area of research.

Figure 1.3: Worldwide energy consumption by sector during the year 2011.

Source: Energy Information Administration.

1.2 Robotic Manipulators

A robotic manipulator is a computer controlled mechanism designed to move materials, tools or specific devices with programmable motions in order to perform different kind of tasks.

The first applications of robotic manipulators involved to transfer objects from a point A to a point B. This motion is usually denoted as a pick-and-place task. Due to their capacity to perform repetitive tasks with high accuracy and speed, robotic manipulators have attracted the interest of the industry. As a result, nowadays robotic manipulators are present in numerous applications in the manufacturing industry such as welding, assembling and painting, picking and placing components in circuit boards or machining metal pieces among others. As an example, Figure 1.4 illustrates a factory in which several robotic manipulators are assembling car parts.

Due to the technological advances of the last years, the robotic manipulators have drastically improved their abilities. It is a fact that, nowadays, it is possible to find them in miscellaneous areas such as aerospace, military or medical applications. Figure 1.5 illustrates the robot *Da Vinci*, a robotic manipulator designed to perform minimal invasive surgery.

Figure 1.4: Robotic manipulators assembling car parts in a factory.

Source: Licensed by Creative Commons Attribution-Share Alike 3.0 Unported.

Robotic manipulators are composed of a sequence of links and joints. The links are rigid members that are connected to each other by means of joints. The joints allow movement between adjacent links. Depending on the nature of the allowed movement, the joints can belong to two types: (i) Linear or Prismatic joints, which allow translation and (ii) Rotational or Revolute joints which allow rotation. There are alternative types of joints that are composed of a combination of the aforementioned ones. A detailed review of joints and linkages can be found in Hartenberg and Denavit (1964). Moreover, the joints can be passive or active. The passive joints allow the motion without actuating on it, *i.e.* there is no actuator embedded in it. In this way, the manipulator's motion depends on the actuators embedded in the active joints of the manipulator. The actuators are able to provide torque and force to the joints by means of energy consumption. Different types of mechanisms such as electrical motors or pneumatic pumps, among others, can be used as actuators. In addition, the actuators may be connected to the computers allowing the iteration between human and machine. Hence, the actuators are the ultimately responsible for executing the tasks and the control schemes defined by the user (LEWIS; DAWSON; ABDALLAH, 2004).

The robotic manipulators present two main parts: (i) the base which is the support of the manipulator and is fixed to the floor or a proper surface and (ii) the end-effector, also known as gripper, which is the part that interacts with the environment. The end-effector is the element of the manipulator where the tools are attached. Depending of the specific purpose, the tools can be drillers, welders or force-enclosures, among others.

Figure 1.5: Surgery robot *Da Vinci*.

Source: ©[2017] Intuitive Surgical, Inc.

1.3 Serial and Parallel Manipulators

Robotic manipulators can be classified by several criteria as geometry, application, power source or type of joints. Moreover, the robotic manipulators can be classified depending on its kinematic architecture. On this matter, a distinction between serial and parallel kinematic architectures for robotic manipulator can be established.

- **Serial manipulators:** They are composed by a single open kinematic chain that connects the base to the end-effector. In this architecture, each link is connected to the previous one. An example of serial manipulator is illustrated in Fig.1.6. At the same time, serial manipulators are divided into five categories depending on their geometry: articulated, spherical, SCARA, cylindrical or Cartesian as discussed in Spong, Hutchinson and Vidyasagar (2006).

Figure 1.6: Serial robot ABB.

Source: Photo shot by the author.

- **Parallel manipulators:** They are also known as Parallel Kinematic Machines (PKMs). In this architecture, the set of links form a closed kinematic chain. Specifically, the end-effector is connected to the base by two or more independent kinematic chains. An example of parallel manipulator is the *Stewart Platform* illustrated in Fig.1.7.

Figure 1.7: Stewart Platform.

Source: MRMOCO.

Both types of manipulators present advantages and disadvantages. The serial manipulators are built with heavy and bulky parts as the joints and links must support not only the load but also the weight of the subsequent parts. This characteristic makes the serial manipulator to present a low mass/load ratio. In addition, in serial manipulators, the position inaccuracies presented in one of the joint are transmitted to the following joints. This leads to problems of precision in the positioning of the end-effector. However, they present a large workspace and their mathematical models are relatively simple making their control design and implementation easier. Due to the previous two reasons, the serial manipulators are currently the most common manipulators in industry.

On the other hand, the end-effector of a PKM is supported by several kinematic chains connected to the base at different points providing a higher carrying capacity. In addition, they present higher position accuracy as their kinematic chains are shorter inducing a lower transmission of the position errors. Nevertheless, their workspace is composed by the shared individual workspaces of each kinematic chain leading to a smaller total workspace. In addition, due to their kinematic structure, their mathematical models requires more complex formulations.

1.4 Introduction to Kinematic Redundancy

It is a fact that PKMs represent an alternative to the design of more efficient robotic manipulators. Nevertheless, this kinematic architecture presents some disadvantages when compared to the serial one. One of this disadvantages is the limited workspace not only due to its nature (discussed in Section 1.3), but also due to the presence of singularities. An alternative to reduce or to eliminate the presence of singularities is the use of kinematic redundancy.

The concept of kinematic redundancy is illustrated, hereafter, by adding an actuator to a kinematic chain of the planar parallel manipulator $3\underline{R}RR$. The nomenclature is explained as follows: The number represents the number of kinematic chains. The rotative joints are depicted by the letter R. The prismatic joints are depicted by the letter P. The underlined letters denotes that the joint is active, if the letter is not underlined the joint is passive. In this way, the end-effector of the manipulator $3\underline{R}RR$ is connected to the base via three identical kinematic chains. Each kinematic chain presents three rotative joints (one active ($\underline{R}$), and two passive (RR)). This manipulator is illustrated in Fig.1.8(a). The end-effector has three degrees of freedom as it is characterized by two Cartesian coordinates (x, y) , and one orientation angle α . In this way, the manipulator $3\underline{R}RR$ is called non-redundant as three active rotative joints control the three degrees of freedom of the end-effector.

Figure 1.8 (b) illustrates the manipulator $\underline{P}RRR+2\underline{R}RR$. This manipulator is characterized by the addition of an active prismatic joint ($\underline{P}$) to one of the kinematic chains of the $3\underline{R}RR$. Hence, the manipulator $\underline{P}RRR+2\underline{R}RR$ is redundant because the three degrees of freedom of the end-effector must be controlled by four active joints. Moreover, this type of redundancy can be classified as *kinematic redundancy*.

In this work, the kinematic redundancy is exploited as an alternative to increase the energy efficiency of parallel manipulators. In order to do so, numerical and experimental approaches are used for comparing the energy consumption of the non-redundant parallel manipulator $3\underline{R}RR$ with its kinematically redundant versions $\underline{P}RRR+2\underline{R}RR$, $2\underline{P}RRR+\underline{R}RR$ and $3\underline{P}RRR$. These versions present up to three levels of kinematic redundancies.

The presence of kinematic redundancies introduces an extra challenge to the control and planning of the manipulators' motion. For the non-redundant manipulator the number of degrees of freedom of the end-effector is three (two Cartesian coordinates and one

Figure 1.8: Parallel manipulators, (a) $3RRR$, (b) $PRRR+2RRR$

Source: Elaborated by the author.

orientation angle). The three degrees of freedom must be controlled by three actuators in the non-redundant manipulator $3RRR$ in Fig.1.8(a). As a consequence, the inverse kinematics problem has a unique solution, *i.e.* there is only a single configuration of the actuators to a specific end-effector pose. For the kinematically redundant manipulator $PRRR+2RRR$, illustrated in Fig.1.8(b), the inverse kinematic problem presents an infinite number of solutions. In other words, any given end-effector pose can be achieved by infinite configurations of the actuators. This situation is illustrated Fig.1.9, as one can note that different configurations of the actuators can lead to the same end-effector pose.

Due to the presence of kinematic redundancy, the number of solutions to the inverse kinematics problem become infinite. Hence, a redundancy resolution scheme is required to select a suitable one among the infinite solutions. A suitable solution can be found taking into account important prerequisites such as maximum velocity, minimum energy or maximum end-effector accuracy, among others. In this work, the Model Predictive Control methodology (MPC), is used for the derivation of a redundancy resolution scheme. This novel approach is an important contribution of this work.

Figure 1.9: Three possible configurations for the same end-effector pose.

Source: Elaborated by the author.

1.5 Objectives

The main objectives of this work are: (i) to prove based on experimental and numerical results that the kinematic redundancy is a good alternative to increase the energy efficiency of planar parallel manipulators and (ii) to propose a novel redundancy resolution scheme based on the model predictive control (MPC) approach. In order to achieve the main objectives of this work, the following tasks were accomplished:

- the proposal and presentation of an approach to evaluate the energy consumption of the parallel manipulators under study experimentally.
- the derivation of kinematic and dynamical models of the parallel manipulators under study.
- the exploitation of the MPC methodology and the kinematic models for deriving a novel redundancy resolution scheme.
- the study of the economical viability of the proposal (the use of kinematic redundancy for improving energy efficiency) considering a real scenario.

1.6 Roadmap

In this section, a brief description of the subsequent chapters of this thesis is enumerated.

1. **Literature review:** The articles and books that served as a background for the accomplishment of this thesis are presented here. This chapter is divided in three parts: In the first part general information about the parallel kinematic machines is provided, containing historical remarks, classification and applications. The second part is dedicated to the kinematic redundancy and pretends to answer this question: What is the kinematic redundancy good for? The third section is dedicated to the redundancy resolution schemes. It provides examples of the different approaches used to treat this problem. The fourth section presents a brief introduction of the model predictive control (MPC): its applications and associated literature.
2. **Methodology:** This chapter describes the mathematical formulation used to obtain the kinematic and dynamic models of the parallel manipulators under study. In addition, a model of the actuators for obtain a mathematical expression of its electrical power consumption is provided. The approach used for generate the trajectories to be executed by the end-effector is also presented.
3. **Redundancy Resolution:** This chapter provides the methodology associated with the MPC based redundancy resolution scheme used in this work.
4. **Prototype, Data Acquisition and Processing:** A description of the 3PRRR prototype together with the procedure to collect the data are presented in this chapter.
5. **Results:** In this chapter, the numerical and experimental results are presented for sake of comparing the manipulators under study regarding their energy consumption. In addition, an economical study to assess the viability of the proposal is presented and discussed.
6. **Conclusions:** The last chapter discusses the obtained results. Positive outcomes and issues regarding the proposal are evaluated providing insights for future works.

2 LITERATURE REVIEW

This chapter summarizes the theoretical background and related studies of the topics treated in this manuscript. This chapter is divided in four sections. The first section presents a basic overview of parallel kinematic machines, including definitions, examples of applications and references to the articles and books that define the state of the art of PKMs. The second section is dedicated to the effects of kinematic redundancy in the performance of parallel manipulators. As previously discussed, the inverse kinematics problem of a kinematic redundant PKM admits a infinite number of solutions. In order to select a proper configuration, the formulation of a redundancy resolution scheme is required. During the past years, different authors have developed different redundancy resolution schemes based in diverse mathematical formulations. The third section outlines some different schemes employed by the authors to solve the inverse kinematics problem when kinematic redundancy is present. The redundancy resolution scheme used in this research is based on the Model Predictive Control (MPC) technique. The fourth section briefly reviews the articles that defines the state of the art of this method.

2.1 Parallel Kinematic Machines

Robotic manipulators are computer controlled mechanical systems. The manipulator's end-effector can move with respect to the fixed base. The end-effector's DOFs can be defined as the minimal set of generalized coordinates that can described its motion in an independent manner. It might present translational and rotational DOFs.

The most common robotic manipulators present a serial architecture. A serial manipulator is composed by a series of links, each of them being tied to its precedent and subsequent by joints. During the last years, enormous advances in the robotic manipulators have been developed. However, in most of the cases their architectures have remained unchanged, meaning that the vast majority of robotic manipulators in the industry are serial manipulators.

The serial architecture presents important drawbacks regarded to the load/mass ratio and accuracy. The load/mass ratio is defined as the relationship between the total mass of the manipulator and the mass of the maximum load that the manipulator can handle without damage its structure. The accuracy is defined as the distance between the desired and real end-effector position. Figure 2.1 illustrates a serial manipulator KUKA KR5 sixx. This

manipulator weights 29 kg and can handle a maximum load of 5 Kg yielding a mass/load ratio of 0.17. In general, the mass/load ratios of most serial manipulators are between the values of 0.03 and 0.2 (MERLET, 2010).

Figure 2.1: Serial manipulator KUKA KR5 sixx.

Source: Photo shot by the author.

The low mass/load ratio is an intrinsic characteristic of the serial architecture. As a consequence of the serial distribution of links and joints, each element must support the weight of the following links, joints and load. Therefore, links and joints must be built with heavy and bulky materials for assure the required stiffness. In this scenario, deformations are unavoidable affecting the system accuracy. In addition, the positioning errors of serial manipulators tend to be greater since the error in a link is transmitted to the next one and so on. At the end, the positioning error in the end-effector is the sum of the errors of all the previous links. This effect is even more appreciable at higher velocities as the heavy components are under higher inertial forces which can lead to additional deformation. In conclusion, the serial manipulators are not suited for tasks were high accuracy and handling of heavy loads are required (MERLET, 2010).

Due to the aforementioned drawbacks, several authors have been investigating the performance and applicability of parallel manipulators. A PKM can be defined as a closed-chain robotic manipulator. Its end-effector is connected to the fixed base by several independent kinematic chains. Due to its architecture, parallel manipulators present some advantages: (i) most of their motors remains fixed yielding lighter mechanism, (ii) the end-effector is linked to base by several chains each one supporting a fraction of the carrying load, (iii) the gear drives are not required yielding lighter and more precise mechanisms (GOSSELIN, 1985). These characteristics are relevant for designing a high dynamic performance manipulator capable of carrying high load capacity and reaching high speed.

The most famous parallel manipulators are the Stewart Platform (GOUGH, 1956), and the Delta robot (CLAVEL, 1991). The Stewart platform is also known as Gough Platform or hexapod. This machine is used not only for designing flight simulator, but also machine tools, surgical robots and mechanical bulls, among others. An example of the use of a Stewart platform as flight simulator for the airline Lufthansa is illustrated in Fig.2.2.

Figure 2.2: Stewart Platform supporting a simulated aircraft cabin.

Source: Photo by Ethan Arnold. Copyrighted by Creative Commons BY-SA 3.0 License.

The Delta robot consists of an end-effector connected to three kinematic chains. These chains are connected to the base by universal joints. The Delta robot was designed to pick and place objects at high speed. In this way, one common application is to pick and assemble components in circuit boards. Recently, the Delta robot has been adapted to work as a subsystem of the 3D printers. Figure 2.3 illustrates a Delta robot performing a task of pick and place eggs in a distribution center. PKMs present good performance in terms of accuracy, load carrying capacity and high speed of operation. It has been also

Figure 2.3: A chain of Delta robots picking eggs and placing them on plates.

Source: Image licensed under the Creative Commons Attribution-Share Alike 3.0 Unported.

proved that they might be more energy efficient when compared to serial ones during some tasks' execution (LI; BONE, 2001). These characteristics have made PKMs to become an important alternative in real applications as technology evolves and applications become more specific. For instance, on the aeronautical industry researchers from the aircraft manufacturer Boeign and the robot builder ABB have been working together to develop the five-axis horizontal Tau PKM (CROTHERS et al., 2009). This machine presents great capabilities for assembling and handling aircraft components when compared to classical serial robots. Another real application are the exploitation of PKMs in machining process. The *Exechon* is an example of PKM suited for this kind of application. The Exechon PKM has the capability of provide large machining forces with a light moving mass, leading to energy savings when compared to the classical machine tools (BI; WANG, 2012). An Exechon PKM is illustrated in Fig.2.4.

In this work, the use of kinematic redundancies is exploited. In order to do so, planar PKMs with three DOFs are investigated. The non-redundant PKM under study is the planar 3RRR manipulator, illustrated in Fig.2.5. This parallel manipulator has three active rotative joints located in the points A_1 , A_2 and A_3 . The rotative joints located in B_1 , B_2 , B_3 , C , D , and E are passive. The end-effector is defined by the equilateral triangle CDE . This robot has three degrees of freedom as the position of the end-effector is defined by two Cartesian coordinates of its barycenter (x, y) and its orientation angle α . The nomenclature associated to this kind of manipulator is further explained:

- The number represents the quantity of kinematic chains. The 3RRR has three kinematic chains A_1B_1C , A_2B_2D and A_3B_3E .

Figure 2.4: An *Exechon* parallel kinematic machine.

Source: Unison ltd.

- The letter represents the kind of the joint. R is a revolute joint. P is a prismatic joint. The $3\underline{\text{R}}\text{RR}$ has three rotative joints per kinematic chain.
- The underline in $\underline{\text{R}}$ means that this joint is active. The $3\underline{\text{R}}\text{RR}$ has one active rotative joint in each kinematic chain.

Figure 2.5: $3\underline{\text{R}}\text{RR}$ planar parallel manipulator.

Source: Elaborated by the author.

The actuators of the $3R\bar{R}R$ are fixed to the base. This implies in an inertia reduction of the manipulator during the execution of a task. This construction characteristic promotes the classical advantages of parallel manipulators such as high load capacity and increased velocity and accuracy performance. A detailed study of this manipulator can be found in Gosselin (1985). The manipulator $3R\bar{R}R$ has been studied by numerous researchers. In Wu, Wang and You (2011), a dynamic formulation of the $3R\bar{R}R$ is derived and its dynamic performance is compared with other similar parallel manipulators. In Kucuk (2013), an optimization problem is proposed for minimizing the inertia of the links and the end-effector of the $3R\bar{R}R$ aiming the design of an energy efficient system. In Furtado et al. (2014), a $3R\bar{R}R$ mechanism is used as an alternative to provide rehabilitation to patients with injured hands.

Even if the studies related to the planar parallel manipulator $3R\bar{R}R$ are about a fair number of topics, the vast majority of the literature is related to the problem of singularities. In general, the common presence of singularities in parallel manipulators is their major drawback. A robotic manipulator is said to be in a singular configuration or singularity when the Jacobian matrices become rank deficient. Jacobian matrices correlates the actuators' velocities with the end-effector's velocities. In these situations, the actuators are not able to control the motion of the end-effector leading to undesirable motions that can deteriorate the manipulator's accuracy and can cause damages to the mechanism and its surroundings.

Figure 2.6 illustrates a $3R\bar{R}R$ mechanism in a singular configuration. This configuration is called first kind singularity and occurs when one of the kinematic chains is totally extended. In this situation, a force applied to the end-effector in the direction of the extended leg has no effect in the actuators, making the manipulator uncontrollable. The first work that addressed the presence of singularities of the $3R\bar{R}R$ was wrote by Gosselin and Angeles (1990). Since then, numerous researches have proposed different approaches to solve this problem. Some of them have proposed the use of different algorithms to optimize and find suitable trajectories that avoid singular configurations. This strategy is denoted as motion path planning (PARSA; DANIALI; GHADERI, 2010),(ABO-SHANAB, 2014). The previous approach provides effective results against singularities but reduces the manipulator workspace. As the PKMs' workspace is rather limited due to their own architecture, some authors tried to solve the problem of singularities by using different approaches. One of these approaches is denoted as kinematic redundancy. This is the strategy under investigation in this work.

Figure 2.6: A $3\underline{R}$ RR mechanism in a singular configuration.

Source: Elaborated by the author.

2.2 Kinematic Redundancy

In this work, kinematic redundancies are added to the planar parallel manipulator $3\underline{R}$ RR by the inclusion of active prismatic joints. If a single prismatic actuator is included to one kinematic chain, the result will be the parallel manipulator with one level of kinematic redundancy $\underline{P}$ RRR+ $2\underline{R}$ RR illustrated in Fig.2.7. The nomenclature is further explained:

- $\underline{P}$ RRR: One kinematic chain with an active prismatic joint ($\underline{P}$), an active rotative joint ($\underline{R}$) and two rotative joints (RR).
- $2\underline{R}$ RR: Two kinematic chains, each chain has an active rotative joint ($\underline{R}$) and two passive rotative joints (RR).

In this respect, the manipulator $3\underline{R}$ RR is denoted as non-redundant as it has three actuators that should control the three degrees of freedom of the end-effector. For the case of the $\underline{P}$ RRR+ $2\underline{R}$ RR manipulator, the end-effector keeps three degrees of freedom but, as consequence of the introduction of the kinematic redundancy, the three degrees of freedom are controlled by four actuators. In fact, redundancy is the term used when there are more actuators than DOFs to be controlled. In a nutshell, there are two kind of redundancies: the kinematic and the actuation redundancies. Actuation redundancy can be introduced by the inclusion of an additional kinematic chain to non-redundant manipulator. An example of this is the parallel manipulator $4\underline{R}$ RR, illustrated in Fig.2.8. In this case, the manipulator has an extra identical kinematic chain when compared to the $3\underline{R}$ RR manipulator. The active revolute joints are located in the gray circles while the black circles represent revolute passive joints. The end-effector is a diamond shaped parallelogram with three degrees of freedom. The dynamic modeling of the $4\underline{R}$ RR can be found in Wu, Wang and You (2011). The terminology related to the kinematic and actuation redundancy in parallel manipulators can be confusing as there is not an unique

Figure 2.7: Parallel manipulator with kinematic redundancy $\underline{\text{P}}\underline{\text{R}}\underline{\text{R}}\underline{\text{R}}+2\underline{\text{R}}\underline{\text{R}}\underline{\text{R}}$.

Source: Elaborated by the author.

Figure 2.8: Parallel manipulator with actuation redundancy $4\underline{\text{R}}\underline{\text{R}}\underline{\text{R}}$.

Source: Elaborated by the author.

nomenclature accepted by the different authors (MULLER, 2013). In this work, the term kinematic redundancy is used when additional actuators are included to the kinematic chains of a $3\underline{\text{R}}\underline{\text{R}}\underline{\text{R}}$ manipulator, as depicted in Fig.2.7. On the other hand, the term actuation redundancy is used in this manuscript to define a parallel manipulator with more kinematic chains than the number of DOFs of its end-effector, as illustrated in Fig.2.8.

Several authors have demonstrated the advantages relative to the use of actuation redundancy. The benefits of actuation redundancy regarding to energy savings on robotic manipulators were studied in Lee et al. (2015). In Cheng et al. (2001), it is claimed that

Figure 2.9: Manipulator 3RRR in a singular configuration (a). Singularity avoided by means of kinematic redundancy (b).

Source: Elaborated by the author.

actuation redundancy in parallel manipulators can reduce the presence of singularities, can provide the homogenization of the torques and forces, and may increase the stiffness of the manipulator. Moreover, the actuation redundancy may reduce the required electrical power to perform a task (RUIZ; FONTES, 2015). In this work, the actuation redundancy is not considered, since there is a lack of studies regarding the impact of the inclusion of kinematic redundancy on the dynamic performance and energy consumption of PKM.

As have been stated in the previous section, the common presence of singularities is an important drawback of PKMs. The capability of kinematic redundancy to avoid singular configurations resides in its capacity of reconfiguration. An example of this is illustrated in Fig.2.9 (a). The manipulator 3RRR is in a singular configuration. The singular configuration can be showed by the intersection of the three lines of action of the links attached to the end-effector. In this configuration, no torque disturbances on the end-effector can be handled by the actuators demonstrating the lack of rigidity of the manipulator. Due to this fact, higher inputs are demanded from the actuators which can cause an increase in the energy consumption and can damage the mechanical and electrical devices.

As explained in Gosselin and Angeles (1990), some end-effector's poses of the non-redundant manipulator 3RRR can lead to singular configurations. The kinematic redundancy allows the manipulator to reach those end-effector's poses without falling in a singular configuration. As illustrated in Fig. 2.9 (b), the singular configuration can be avoided by the use of an additional prismatic actuator. Therefore, the use of kinematic redundancy allows the enlargement of the usable workspace of the manipulator 3RRR by reducing the presence of singularities (KOTLARSKI et al., 2009a).

Due to the benefits related to singularity avoidance achieved by the use of kinematic redundancy in PKMs, several authors have studied how the use of kinematic redundancy could be beneficial to other features. For instance, in Xie, Liu and Wang (2011), the performance of different PKMs was evaluated. The article concluded that the kinematic redundancy could greatly contribute to the singularity avoidance and motion/force transmission in PKMs. In Kotlarski, Abdellatif and Heimann (2008), the effects of kinematic redundancy in the pose accuracy of a manipulator $\underline{PR}+\underline{RRR}$ were studied, showing that kinematic redundancy could yield considerable reductions in the end-effector's positioning errors. The same authors experimentally proved the aforementioned benefits of kinematic redundancy by exploiting experimentally a prototype (KOTLARSKI; HEIMANN; ORTMAIER, 2011). A further example can be found in the work of Fontes and Silva (2016). In this article, the authors explained how the dynamic performance of planar parallel manipulators could be enhanced by the use of kinematic redundancy.

The kinematic redundancy provides numerous benefits to PKMs. However the presence of kinematic redundancies leads to an inverse kinematics problem with infinite number of solutions. This means that an end-effector's pose can be achieved by an infinite number of manipulator's configurations. Due to this issue, a method for solving the inverse kinematics problem by selecting an optimal solution is mandatory. This solution is obtained by means of a redundancy resolution scheme.

2.3 Redundancy Resolution Review

A strategy for overcoming the presence of singularities in the workspace of PKMs is the inclusion of an extra active joint in an active kinematic chain. As aforementioned, this strategy is known as kinematic redundancy. The kinematically redundant parallel manipulators present higher mobility than required for the desired task allowing the avoidance of singularities and obstacles. This capability is illustrated in Fig.2.10 for the case of the redundant parallel manipulator $\underline{PRRR}+2\underline{RRR}$.

Due to the presence of the kinematic redundancy, a strategy for selecting a suitable configuration among the infinite solutions of the inverse kinematics problem is required. The mathematical approach to solve the inverse kinematics problem of a kinematically redundant PKM is known as redundancy resolution and is treated in this section. There are several approaches for mathematically describing a redundancy resolution scheme. The earliest redundancy resolution schemes used the pseudoinverse of the Jacobian matrix (WHITNEY, 1969). This method minimizes the norm of the joint velocities in order to find the desired end-effector velocities. Some other methods have been proposed using

Figure 2.10: Three possible configurations for the same end-effector pose.

Source: Elaborated by the author.

different objective functions such as to minimize the required joint-torques to perform an end-effector motion (HOLLERBACH; SUH, 1987), to maximize the accuracy of the end-effector (LIEGEOIS, 1977), to avoid obstacles (COLBAUGH; SERAJI; GLASS, 1989), or to prevent motion near singular configurations (CHIAVERINI, 1997).

An example of a redundancy resolution scheme used in PKMs with the aim of avoiding motion near singular configurations can be found in Niemann et al. (2013). In this work, the strategy requires the selection of configurations that prevents from a loss of rank of the Jacobian matrix. Further redundancy resolution schemes included the use of the Jacobian matrix to find the configurations that increase the position accuracy (KOTLARSKI; ABDELLATIF; HEIMANN, 2008), or the use of dynamical indexes related with driving torques to increment the overall performance of the manipulator (FONTES; SILVA, 2016).

For the case of the PKM with kinematic redundancy $\underline{P}RRR+2\underline{R}RR$ depicted in Fig.2.7, a redundancy resolution scheme can be posed as an optimization problem in which the variables to be optimized are the positions of the prismatic joint δ . Using this approach, two redundancy resolution schemes for the parallel manipulators under study were proposed by Kotlarski, Abdellatif and Heimann (2008), and further exploited by Fontes and Silva (2016). These schemes are:

- **Pre-positioning approach:** The positions δ are kept constant during the task's execution. These fixed positions of the prismatic actuators δ are found by solving an optimization problem. This approach is the easiest one to implement in terms of computing efforts as the redundant actuators do not need to be controlled during the end-effector movement. This approach can lead to satisfactory results for the case of simple trajectories and repetitive tasks.
- **On-going approach:** The positions δ may vary during the task's execution. The

positions of the prismatic actuators δ are also found by solving an optimization problem that takes into account a mathematical description on how these positions can vary. This optimization problem has a larger amount of variables which might impose some challenges for its solution.

The previous two approaches are the basic methods for several redundancy resolution schemes (KOTLARSKI et al., 2010). For instance, in Kotlarski, Abdellatif and Heimann (2008), the redundancy resolution scheme consists in a combination of the pre-positioning and on-going approaches. The end-effector trajectory is divided in three segments and the pre-positioning method is used to find the positions of the redundant actuators at the beginning of the execution of each segment.

The on-going approach can be posed by imposing a mathematical description for the motion of the redundant actuators δ . For instance, the trajectory of δ can be described by a five degree polynomial. If the initial and final are known, the only variables to be found are the initial and final positions of the prismatic actuators δ (RUIZ; FONTES; SILVA, 2015),(FONTES; SILVA, 2016). By adopting this strategy, the complexity of the optimization problem can be reduced.

In previous numerical analyses, the on-going approach has shown a better performance than the pre-positioning approach (FONTES; SANTOS; SILVA, 2014). Nevertheless, the choice of one approach or another for practical matters depends on the application, the task and the limitations imposed by the mathematical formulation of the optimization problem (MERLET, 2007).

In the present work, the selected redundancy resolution scheme exploits the pre-positioning approach along with the MPC method (model predictive control). This methodology is presented and explained in chapter 4.

2.4 Introduction to MPC

The MPC refers to a family of control algorithms that employ a process model to predict the response of a plant during a future time interval. The MPC optimizes the future response of a plant by computing a sequence of future input samples. Then, the first input sample is transferred to the plant, and the process is repeated consecutively for the following time interval.

Back in the late seventies and eighties, the MPC based algorithms were first used in

the process control of petrol refineries and power plants (RICHALET et al., 1978). At that time, the MPC technologies raised their popularity due to their capacity to control multi-variable systems and to achieve the best economical operating point of their units. Since then, the number of industrial application areas where MPC was applied included chemical processes, air and gas providers, milling and metallurgy, food processing, polymers, aerospace and automotive industry, among others (QIN; BADGWELL, 2003),(LEE, 2011).

Various authors have addressed the theoretical questions regarding to MPC during the late eighties and the beginning of nineties, such as García, Prett and Morari (1989) or Muske and Rawlings (1993). Literature regarding the late issues of MPC theory can be found in Rawlings and Mayne (2009) or Mayne et al. (2000).

Due to its industrial acceptance, the MPC technologies have been further investigated by researchers in order to modernize, to improve the methods and to increase the spectrum of industrial applications suited for MPC (FORBES et al., 2015). The most common applications of MPC included linear models applied to large scale systems such as crude oil distillation units (RICHALET, 1993), (BADGWELL; QIN, 2013). In the academia, recent researchers include novel theories such as the non-linear MPC (CAMACHO; BORDONS, 2007), robust MPC (MAYNE; SERON; RAKOVIĆ, 2005), or EMPC (Economical Model Predictive Control) (ELLIS; DURAND; CHRISTOFIDES, 2014).

In this thesis, the MPC theory is exploited to the mathematical description of the optimization problem using the pre-positioning approach. This novel redundancy resolution scheme is an important contribution of this work. The impact of the use of kinematic redundancy to improve the energy efficiency of parallel manipulators is assessed in this work by the use of this novel MPC-based redundancy resolution scheme.

3 PLANAR PARALLEL MANIPULATORS MODELING

In this chapter, the kinematic and dynamic models of the planar parallel manipulators under study are derived. The methodology is applied to the particular case of the parallel manipulator with three levels of kinematic redundancy, the 3PRRR. The formulation of this manipulator is easily applicable to the remaining parallel manipulators with two, one or none levels of redundancy, respectively. The equations derived hereafter are essential for the control design, trajectory planning and motion planning of the PKMs under investigation. The methodology described in this chapter is based on the work of Fontes and Silva (2016).

The chapter is divided as follows. Firstly, the inverse kinematic model of the 3PRRR is derived in Section 3.1. In this problem, the joint parameters (joint coordinates) are calculated for a given end-effector's pose (Cartesian coordinates and angular positions of the end-effector). Secondly, the Jacobian Matrix is obtained in Section 3.2. The Jacobian matrix relates the vector of joint velocities with the vector of end-effector's pose velocities. The Principle of Virtual Work requires the derivation of the velocities and accelerations of each rigid body of the system under study. Jacobian matrices are used for this purpose in Section 3.3. By the exploitation of the Principle of Virtual Work, the dynamic model of the 3PRRR is derived in the Section 3.4. The inverse dynamical model allows the calculation of the required torques for executing a task (a given end-effector's motion). The electromechanical model of the actuators (DC motors) is detailed in Section 3.5. Moreover, the methodology for assessing the energy consumption during a task execution is also clarified. In the Section 3.6, the mathematical formulation for generating the end-effector's trajectories is presented.

3.1 Inverse Kinematics

In robotics, the inverse kinematics refers to the process to determine the joint coordinates that provide a given end-effector's pose. This calculation is useful since some applications of robotic manipulators requires the end-effector to perform a desired motion in the workspace. In this section, the inverse kinematics of the manipulator 3PRRR, depicted in Fig.3.1, are derived. This manipulator has 3 identical kinematic chains, each one of them with one active prismatic joints (P), one active revolute joints (R) and two passive revolute joints (RR).

Figure 3.1: Parallel manipulator 3PRRR

Source: Elaborated by the author.

Figure 3.2: Modeling scheme: (a) a single kinematic chain and (b) the end-effector of the 3PRRR manipulator.

Source: Own composition.

Figure 3.2 illustrates a single kinematic chain i (where $i = 1, 2, 3$) and the end-effector of the manipulator 3PRRR. The end-effector is an equilateral triangle where h represents the distance from the barycenter to one of the vertex. The barycenter of the triangle is defined by the end-effector's coordinates (x_e, y_e) and its angular position denoted as α . The active revolute joints are located in points B_i . The passive revolute joints are located in points C_i and D_i . The angles θ_i denote the joint coordinates of the active revolute joints. The angles β_i denote the joint coordinates of the passive revolute joints situated in C_i . The points A_i and the angles γ_i define the locations of the prismatic joints. The joint coordinates of the prismatic joints are denoted by δ_i . The lengths of the links, $\|B_i C_i\| = l_1$ and $\|C_i D_i\| = l_2$. The angles γ_i and λ_i are constant quantities that impose the kinematic constraints to derive the inverse kinematics model hereafter. Thus,

with respect to the coordinate system $O-xy$, the position vector $\mathbf{r}_{D_i}$ can be written as

$$\mathbf{r}_{D_i} = \mathbf{r}_{C_i} + l_2 \begin{bmatrix} \cos \beta_i \\ \sin \beta_i \end{bmatrix} = \begin{bmatrix} x_e \\ y_e \end{bmatrix} + h \begin{bmatrix} \cos(\alpha + \lambda_i) \\ \sin(\alpha + \lambda_i) \end{bmatrix}, \quad (3.1)$$

while the position vector $\mathbf{r}_{C_i}$ can be expressed as

$$\mathbf{r}_{C_i} = \mathbf{r}_{B_i} + l_1 \begin{bmatrix} \cos \theta_i \\ \sin \theta_i \end{bmatrix}, \quad (3.2)$$

and the position vector $\mathbf{r}_{B_i}$ can be written as

$$\mathbf{r}_{B_i} = a_i \begin{bmatrix} \cos \lambda_i \\ \sin \lambda_i \end{bmatrix} + \delta_i \begin{bmatrix} \cos \gamma_i \\ \sin \gamma_i \end{bmatrix}, \quad (3.3)$$

where the quantity a_i denotes the distance between the coordinate system $O-xy$ and the point A_i . The substitution of Eqs. 3.2 and 3.3 in Eq. 3.1 yields the following kinematic constraint

$$\begin{bmatrix} x_e \\ y_e \end{bmatrix} + h \begin{bmatrix} \cos(\alpha + \lambda_i) \\ \sin(\alpha + \lambda_i) \end{bmatrix} - a_i \begin{bmatrix} \cos \lambda_i \\ \sin \lambda_i \end{bmatrix} - \delta_i \begin{bmatrix} \cos \gamma_i \\ \sin \gamma_i \end{bmatrix} - l_1 \begin{bmatrix} \cos \theta_i \\ \sin \theta_i \end{bmatrix} = l_2 \begin{bmatrix} \cos \beta_i \\ \sin \beta_i \end{bmatrix}. \quad (3.4)$$

By using the definition of the vector norm and simplifying, the Eq.3.4 can be rewritten as:

$$\left\| \begin{bmatrix} \mu_i \\ \rho_i \end{bmatrix} - l_1 \begin{bmatrix} \cos \theta_i \\ \sin \theta_i \end{bmatrix} \right\| = l_2, \quad (3.5)$$

where

$$\begin{bmatrix} \mu_i \\ \rho_i \end{bmatrix} = \begin{bmatrix} x_e \\ y_e \end{bmatrix} + h \begin{bmatrix} \cos(\alpha + \lambda_i) \\ \sin(\alpha + \lambda_i) \end{bmatrix} - a_i \begin{bmatrix} \cos \lambda_i \\ \sin \lambda_i \end{bmatrix} - \delta_i \begin{bmatrix} \cos \gamma_i \\ \sin \gamma_i \end{bmatrix}. \quad (3.6)$$

The relation defined by Eq.3.5 can be simplified as

$$-2l_1\rho_i \sin \theta_i - 2l_1\mu_i \cos \theta_i + \mu^2 + \rho_i^2 + l_1^2 - l_2^2 = 0. \quad (3.7)$$

By using the substitutions found in Euler (1827), the angles θ_i can be related to a set of variables v_i as

$$\cos \theta_i = \frac{1 - v_i^2}{1 + v_i^2}, \quad (3.8)$$

and

$$\sin \theta_i = \frac{2v_i}{1 + v_i^2}, \quad (3.9)$$

where

$$v_i = \tan \frac{\theta_i}{2}. \quad (3.10)$$

Thus, using the Eqs.3.8 and 3.9, Eq.3.7 can be rewritten as

$$E_{1i} \frac{2v_i}{1+v_i^2} + E_{2i} \frac{1-v_i^2}{1+v_i^2} + E_{3i} = 0, \quad (3.11)$$

where

$$E_{1i} = -2l_1\rho_i, \quad (3.12)$$

$$E_{2i} = -2l_1\mu_i, \quad (3.13)$$

$$E_{3i} = \mu_i^2 + \rho_i^2 + l_1^2 - l_2^2. \quad (3.14)$$

Equation 3.11 can be rearranged in quadratic form as

$$(E_{3i} - E_{2i})v_i^2 + 2E_{1i}v_i + E_{3i} = 0. \quad (3.15)$$

Solving Eq.3.15 results in

$$v_i = \frac{-E_{1i} \pm \sqrt{E_{1i}^2 + E_{2i}^2 - E_{3i}^2}}{E_{3i} - E_{2i}}. \quad (3.16)$$

Replacing Eq.3.10 into Eq.3.16, the joint coordinates of the active revolute joints θ_i , are written as

$$\theta_i = 2 \tan^{-1} \frac{-E_{1i} \pm \sqrt{E_{1i}^2 + E_{2i}^2 - E_{3i}^2}}{E_{3i} - E_{2i}}, \quad (3.17)$$

The choice of the sign + or - in Eq.3.17 defines the working mode of the manipulator. This choice yields to eight possible working modes depicted in Fig. 3.3. In this manuscript, the plus sign was the choice for all the kinematic chains, this choice is illustrated by the working mode W1 in Fig. 3.3. Once the angles θ_i are known, the passive joint angles β_i can be calculated as

$$\beta_i = \tan^{-1} \frac{\rho - l_1 \sin \theta_i}{\mu - l_1 \cos \theta_i}. \quad (3.18)$$

The previous methodology can be easily adapted to any of the manipulators with 0, 1 or 2 levels of kinematic redundancy depicted in Fig. 3.4. For instance, the inverse kinematics problem of the non-redundant manipulator M1 can be obtained by making the values of δ_1 , δ_2 and δ_3 equal to zero. For obtaining the inverse kinematics problem of the remaining manipulators, the corresponding values of δ_i must be set to zero or set to a value accordingly.

Figure 3.3: The eight possible working modes for the manipulator $3\underline{P}RRR$.

Source: Own composition.

Figure 3.4: M1: $3RRR$ manipulator, M2-M4: $\underline{P}RRR+2\underline{R}RR$ manipulators, M5-M7: $2\underline{P}RRR+\underline{R}RR$ manipulators and M8: $3\underline{P}RRR$ manipulator.

Source: Elaborated by the author.

The quantities h , a_i , γ_i , λ_i , l_1 , l_2 are constant values referred as the geometrical parameters of the manipulator. Table 1 shows the values of the geometrical parameters for the particular case of the prototype under investigation in this work.

Table 1: Prototype's Geometrical Parameters

i	$h(mm)$	$a_i(mm)$	$\gamma_i(deg)$	$\lambda_i(deg)$	$l_1(m)$	$l_2(m)$
1	59.7	259.8	180	90	0.191	0.232
2	59.7	259.8	60	-30	0.191	0.232
3	59.7	259.8	300	210	0.191	0.232

Source: Composed by the author.

3.2 Jacobian Matrix

In robotics, the Jacobian matrix relates the vector of end-effector's velocities with the vector of the joint's velocities. For the particular case of the parallel manipulator 3P $\overline{R}$ RR, the Jacobian matrix correlates the vector of joint velocities, $\dot{\Theta} = [\dot{\theta}_1, \dot{\theta}_2, \dot{\theta}_3, \dot{\delta}_1, \dot{\delta}_2, \dot{\delta}_3]^T$, with the vector of end-effector's velocities, $\dot{X} = [\dot{x}_e, \dot{y}_e, \dot{\alpha}]^T$. The Jacobian matrix is present in several important aspects of robotics as the planning of smooth trajectories, the determination of singularities or the derivation of the dynamic models, among others. In order to derive the Jacobian matrix of the 3P $\overline{R}$ RR manipulator, Eq.3.7 must be rewritten as

$$(\mu_i - l_1 \cos \theta_i)l_2 \cos \beta_i + (\rho_i - l_1 \sin \theta_i)l_2 \sin \beta_i = l_2^2. \quad (3.19)$$

By taking the first derivative of Eq.3.19, the following equation can be obtained:

$$\begin{aligned} (\dot{x}_e - h \sin(\alpha + \lambda_i) + l_2 \dot{\delta}_i \cos \gamma_i + l_1 l_2 \dot{\theta}_i \sin \theta_i)l_2 \cos \beta_i + \dots \\ (\dot{y}_e + h \cos(\alpha + \lambda_i) + l_2 \dot{\delta}_i \sin \gamma_i + l_1 l_2 \dot{\theta}_i \cos \theta_i)l_2 \sin \beta_i = 0 \end{aligned} \quad (3.20)$$

Using the trigonometric addition formulas

$$\begin{aligned} \sin(\Lambda + \Upsilon) &= \sin \Lambda \cos \Upsilon + \cos \Lambda \sin \Upsilon, \\ \cos(\Lambda + \Upsilon) &= \cos \Lambda \cos \Upsilon - \sin \Lambda \sin \Upsilon, \end{aligned} \quad (3.21)$$

Eq.3.20 can be written in a matrix form:

$$\begin{bmatrix} P_{11} & P_{12} & P_{13} \\ P_{21} & P_{22} & P_{23} \\ P_{31} & P_{32} & P_{33} \end{bmatrix} \begin{bmatrix} \dot{x}_e \\ \dot{y}_e \\ \dot{\alpha} \end{bmatrix} = \begin{bmatrix} Q_{11} & 0 & 0 & Q_{14} & 0 & 0 \\ 0 & Q_{22} & 0 & 0 & Q_{25} & 0 \\ 0 & 0 & Q_{33} & 0 & 0 & Q_{36} \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \\ \dot{\delta}_1 \\ \dot{\delta}_2 \\ \dot{\delta}_3 \end{bmatrix}, \quad (3.22)$$

where

$$P_{i1} = l_2 \cos \beta_i, \quad P_{i2} = l_2 \sin \beta_i, \quad P_{i3} = l_2 h \sin(\beta_i - \lambda_i - \alpha), \quad (3.23)$$

$$Q_{ii} = l_1 l_2 \sin(\beta_i - \theta_i), \quad Q_{ii+3} = l_2 \cos(\beta_i - \gamma_i), \quad (3.24)$$

being

$$\mathbf{P} = \begin{bmatrix} P_{11} & P_{12} & P_{13} \\ P_{21} & P_{22} & P_{23} \\ P_{31} & P_{32} & P_{33} \end{bmatrix}, \quad (3.25)$$

and

$$\mathbf{Q} = \begin{bmatrix} Q_{11} & 0 & 0 & Q_{14} & 0 & 0 \\ 0 & Q_{22} & 0 & 0 & Q_{25} & 0 \\ 0 & 0 & Q_{33} & 0 & 0 & Q_{36} \end{bmatrix}. \quad (3.26)$$

Using the Eqs.3.25 and 3.26, the Jacobian matrix can be expressed as

$$\mathbf{J} = \mathbf{P}^{-1}\mathbf{Q}. \quad (3.27)$$

Hence, it is possible to correlate the end-effector velocities with the joint velocities using the Jacobian matrix as

$$\dot{\mathbf{X}} = \mathbf{J}\dot{\mathbf{\Theta}}. \quad (3.28)$$

3.3 Partial Velocities and Accelerations

In order to obtain the dynamic model of the manipulator 3PRRR, the Virtual Work Principle was used. This principle belongs to a group of methods that employs variational techniques to obtain mechanical models (DYM; SHAMES, 1973). The first step to apply the Virtual Work Principle consists in identify the different bodies of the manipulator 3PRRR. Then, the partial velocity and acceleration matrices of each body must be obtained.

As the Jacobian matrix is for the end-effector, the partial velocity matrices $\mathbf{K}_{ij}$ correlates the vector of partial velocities $\dot{\mathbf{d}}_{ij}$ with the vector of the actuated joint velocities $\dot{\mathbf{\Theta}}$ as

$$\dot{\mathbf{d}}_{ij} = \mathbf{K}_{ij}\dot{\mathbf{\Theta}}. \quad (3.29)$$

The indexes i and j denote the chain and body numbers, respectively. Therefore, there is a partial velocity matrix for each chain and each body. The partial velocity matrices can be divided in two blocks:

$$\mathbf{K}_{ij} = \begin{bmatrix} \mathbf{H}_{ij} \\ \mathbf{G}_{ij} \end{bmatrix}. \quad (3.30)$$

The block $\mathbf{H}_{ij}$ is related to the partial Cartesian velocities. The block $\mathbf{G}_{ij}$ is related to the partial angular velocities. Equivalently, the vector of partial velocities is divided in a Cartesian part and an angular part:

$$\dot{\mathbf{d}}_{ij} = \begin{bmatrix} \dot{r}_{x_{ij}} & \dot{r}_{y_{ij}} & \dot{\phi}_{ij} \end{bmatrix}^T. \quad (3.31)$$

In this work, the manipulator's bodies are described as follows. The actuator situated in B_i is the first body. The second body is the link B_iC_i with pivotal point in B_i . The third

body is the link $C_i D_i$ with pivotal point in C_i .

The position vector of the point B_i is given by

$$\begin{bmatrix} r_{x_{i1}} \\ r_{y_{i1}} \end{bmatrix} = a_i \begin{bmatrix} \cos \lambda_i \\ \sin \lambda_i \end{bmatrix} + \delta_i \begin{bmatrix} \cos \gamma_i \\ \sin \gamma_i \end{bmatrix}. \quad (3.32)$$

The time derivation of Eq.3.32 results

$$\begin{bmatrix} \dot{r}_{x_{i1}} \\ \dot{r}_{y_{i1}} \end{bmatrix} = \dot{\delta}_i \begin{bmatrix} \cos \gamma_i \\ \sin \gamma_i \end{bmatrix}. \quad (3.33)$$

Hence, the linear velocity matrix of the actuator situated in B_i is

$$\mathbf{H}_{i1} = \begin{bmatrix} \cos \gamma_i \\ \sin \gamma_i \end{bmatrix} \mathbf{e}_{i+3}, \quad (3.34)$$

where

$$\begin{bmatrix} \mathbf{e}_1 \\ \mathbf{e}_2 \\ \mathbf{e}_3 \\ \mathbf{e}_4 \\ \mathbf{e}_5 \\ \mathbf{e}_6 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}. \quad (3.35)$$

The actuator in the point B_i is considered to be a punctual mass. So, no angular DOF is associated with the first body and its angular velocity matrix can be defined as

$$\mathbf{G}_{i1} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \quad (3.36)$$

The linear velocity of the pivotal point situated in B_i is the same as the linear velocity of the actuator in B_i , then

$$\mathbf{H}_{i2} = \mathbf{H}_{i1}. \quad (3.37)$$

The angular velocity of the link $B_i C_i$ is $\dot{\theta}_i$; therefore, its angular velocity matrix can be expressed as

$$\mathbf{G}_{i2} = \mathbf{e}_i. \quad (3.38)$$

The position vector of point C_i is given by

$$\begin{bmatrix} r_{x_{i3}} \\ r_{y_{i3}} \end{bmatrix} = l_1 \begin{bmatrix} \cos \theta_i \\ \sin \theta_i \end{bmatrix} + a_i \begin{bmatrix} \cos \lambda_i \\ \sin \lambda_i \end{bmatrix} + \delta_i \begin{bmatrix} \cos \gamma_i \\ \sin \gamma_i \end{bmatrix}. \quad (3.39)$$

The time derivative of Eq.3.39 results

$$\begin{bmatrix} \dot{r}_{x_{i3}} \\ \dot{r}_{y_{i3}} \end{bmatrix} = \dot{\theta}_i \begin{bmatrix} -l_1 \sin \theta_i \\ l_1 \cos \theta_i \end{bmatrix} + \dot{\delta}_i \begin{bmatrix} \cos \gamma_i \\ \sin \gamma_i \end{bmatrix}. \quad (3.40)$$

Hence, the linear velocity matrix of the actuator situated in C_i is

$$\mathbf{H}_{i3} = \begin{bmatrix} -l_1 \sin \theta_i \\ l_1 \cos \theta_i \end{bmatrix} \mathbf{e}_i + \begin{bmatrix} \cos \gamma_i \\ \sin \gamma_i \end{bmatrix} \mathbf{e}_{i+3}. \quad (3.41)$$

The velocity of the point D_i is obtained with the time derivative of Eq.3.1

$$\dot{\mathbf{r}}_{D_i} = \begin{bmatrix} \dot{x}_e \\ \dot{y}_e \end{bmatrix} + h \begin{bmatrix} -\sin(\alpha + \lambda_i) \\ \cos(\alpha + \lambda_i) \end{bmatrix} \dot{\alpha}. \quad (3.42)$$

In a similar way to the partial velocity matrices, the Jacobian matrix can be divided in a linear part and an angular part as

$$\mathbf{J} = \begin{bmatrix} \mathbf{H}_e \\ \mathbf{G}_e \end{bmatrix}. \quad (3.43)$$

Then,

$$\begin{bmatrix} \mathbf{H}_e \\ \mathbf{G}_e \end{bmatrix} \dot{\boldsymbol{\Theta}} = \begin{bmatrix} \dot{x}_e \\ \dot{y}_e \\ \dot{\alpha} \end{bmatrix}. \quad (3.44)$$

Using Eq.3.44, Eq.3.42 can be rewritten as

$$\dot{\mathbf{r}}_{D_i} = \left[\mathbf{H}_e + h \begin{bmatrix} -\sin(\alpha + \lambda_i) \\ \cos(\alpha + \lambda_i) \end{bmatrix} \mathbf{G}_e \right] \dot{\boldsymbol{\Theta}}. \quad (3.45)$$

On the other hand, the time derivative of the remaining part of Eq.3.1 results

$$\dot{\mathbf{r}}_{D_i} = \dot{\mathbf{r}}_{C_i} + l_2 \begin{bmatrix} -\sin \beta_i \\ \cos \beta_i \end{bmatrix} \dot{\beta}_i. \quad (3.46)$$

Equation 3.46 can be rewritten as

$$\dot{\mathbf{r}}_{D_i} = \left[\begin{bmatrix} -l_1 \sin \theta_i \\ l_1 \cos \theta_i \end{bmatrix} \mathbf{e}_i + \begin{bmatrix} \cos \gamma_i \\ \sin \gamma_i \end{bmatrix} \mathbf{e}_{i+3} \right] \dot{\boldsymbol{\Theta}} + l_2 \begin{bmatrix} -\sin \beta_i \\ \cos \beta_i \end{bmatrix} \dot{\beta}_i. \quad (3.47)$$

Replacing Eqs.3.45 and 3.41 into Eq.3.47 results

$$\left[\mathbf{H}_e + h \begin{bmatrix} -\sin(\alpha + \lambda_i) \\ \cos(\alpha + \lambda_i) \end{bmatrix} \mathbf{G}_e - \mathbf{H}_{i3} \right] \dot{\boldsymbol{\Theta}} = l_2 \begin{bmatrix} -\sin \beta_i \\ \cos \beta_i \end{bmatrix} \dot{\beta}_i. \quad (3.48)$$

The angular velocity of the the link $C_i D_i$ is $\dot{\beta}_i$, which can be isolated from Eq.3.48

$$\dot{\beta}_i = \frac{1}{l_2} \begin{bmatrix} -\sin \beta_i & \cos \beta_i \end{bmatrix} \left[\mathbf{H}_e + h \begin{bmatrix} -\sin(\alpha + \lambda_i) \\ \cos(\alpha + \lambda_i) \end{bmatrix} \mathbf{G}_e - \mathbf{H}_{i3} \right] \dot{\boldsymbol{\Theta}}. \quad (3.49)$$

Finally, the angular velocity matrix of the link $C_i D_i$ is obtained from Eq.3.49 as

$$\mathbf{G}_{i3} = \frac{1}{l_2} \begin{bmatrix} -\sin \beta_i & \cos \beta_i \end{bmatrix} \left[\mathbf{H}_e + h \begin{bmatrix} -\sin(\alpha + \lambda_i) \\ \cos(\alpha + \lambda_i) \end{bmatrix} \mathbf{G}_e - \mathbf{H}_{i3} \right]. \quad (3.50)$$

Using the partial velocity matrices, the partial accelerations are calculated using the time derivative of Eq.3.29 as

$$\ddot{\mathbf{d}}_{ij} = \dot{\mathbf{K}}_{ij} \dot{\boldsymbol{\Theta}} + \mathbf{K}_{ij} \ddot{\boldsymbol{\Theta}}. \quad (3.51)$$

3.4 Dynamic Model

By using the Jacobian matrix, the partial velocities and acceleration matrices, the dynamic model of the manipulator 3PRRRR can be derived. In order to do so, the forces and moments of each body should be calculated using the Newton-Euler formulation. The use of this formulation permits to describe the translational and rotational dynamics of a rigid body (HAHN, 2014).

In the following formulation the quantities m_j denote the masses of each body, the distance from the mass center of each body to each pivotal point is symbolized by s_j and I_j denotes the moment of inertia of each body. Thus, the position of the center of mass of the body j of the chain i can be defined as

$$\mathbf{r}_{is_j} = \begin{bmatrix} r_{x_{ij}} + s_j \cos \phi_{ij} \\ r_{y_{ij}} + s_j \sin \phi_{ij} \end{bmatrix}. \quad (3.52)$$

By virtue of the Newton's second law, the inertial forces of the body j and chain i are defined as

$$\mathbf{F}_{ij} = m_j \ddot{\mathbf{r}}_{is_j}. \quad (3.53)$$

Equation 3.53 can be rewritten as

$$\mathbf{F}_{ij} = \begin{bmatrix} m_j(\ddot{r}_{x_{ij}} - s_j \ddot{\phi}_{ij} \sin \phi_{ij} - s_j \dot{\phi}_{ij}^2 \cos \phi_{ij}) \\ m_j(\ddot{r}_{y_{ij}} + s_j \ddot{\phi}_{ij} \cos \phi_{ij} - s_j \dot{\phi}_{ij}^2 \sin \phi_{ij}) \end{bmatrix}. \quad (3.54)$$

The external moment acting on the body j of chain i about its center of mass can be defined according to the Euler's second law as

$$M_{ij} = \mathbf{s}_j \times \ddot{\mathbf{r}}_{x_{ij}} m_j + \mathbf{s}_j \times \ddot{\mathbf{r}}_{y_{ij}} m_j + I_j \ddot{\phi}_{ij}. \quad (3.55)$$

Figure 3.5 illustrates the direction of the vectors included in the cross products of Eq.3.55.

Figure 3.5: Linear accelerations frame.

Source: Elaborated by the author.

Using the right hand rule, it is possible to compute the external moment of the body j and chain i as

$$M_{ij} = m_j s_j (-\ddot{r}_{x_{ij}} \sin \phi_{ij} + \ddot{r}_{y_{ij}} \cos \phi_{ij}) + I_j \ddot{\phi}_{ij}, \quad (3.56)$$

where

$$\dot{\mathbf{d}}_{ij} = \begin{bmatrix} \dot{r}_{x_{ij}} & \dot{r}_{y_{ij}} & \dot{\phi}_{ij} \end{bmatrix}^T, \quad (3.57)$$

and

$$\ddot{\mathbf{d}}_{ij} = \begin{bmatrix} \ddot{r}_{x_{ij}} & \ddot{r}_{y_{ij}} & \ddot{\phi}_{ij} \end{bmatrix}^T. \quad (3.58)$$

The end-effector's force and moment can be written as

$$\mathbf{F}_e = \begin{bmatrix} m_e \ddot{x} \\ m_e \ddot{y} \end{bmatrix}, \quad (3.59)$$

$$M_e = I_e \ddot{\alpha}, \quad (3.60)$$

where m_e and I_e denote the end-effector's mass and moment of inertia, respectively. Using the Virtual Work Principle (WU; WANG; YOU, 2011), the relationship between the vector of actuated torques $\boldsymbol{\tau}$ and the forces and moments of the individual manipulator bodies can be expressed as

$$\boldsymbol{\tau} \cdot \partial \boldsymbol{\Theta} = \mathbf{p}_e \cdot \partial \mathbf{X} + \sum_{j=1}^3 \sum_{i=1}^3 \mathbf{p}_{ij} \cdot \partial \mathbf{d}_{ij}, \quad (3.61)$$

where

$$\boldsymbol{\tau} = \begin{bmatrix} \tau_{\theta_1} \tau_{\theta_2} \tau_{\theta_3} F_{\delta_1} F_{\delta_2} F_{\delta_3} \end{bmatrix}^T. \quad (3.62)$$

and

$$\mathbf{p}_{ij} = \begin{bmatrix} F_{x_{ij}} & F_{y_{ij}} & M_{ij} \end{bmatrix}, \quad (3.63)$$

and

$$\mathbf{p}_e = \begin{bmatrix} F_{x_e} & F_{y_e} & M_e \end{bmatrix}. \quad (3.64)$$

Using the Jacobian and the partial velocity matrices obtained in the previous sections, the Eq.3.61 can be rewritten as

$$\boldsymbol{\tau} \cdot \partial \boldsymbol{\Theta} = \mathbf{p}_e \cdot \mathbf{J} \cdot \partial \boldsymbol{\Theta} + \sum_{j=1}^3 \sum_{i=1}^3 \mathbf{p}_{ij} \cdot \mathbf{K}_{ij} \cdot \partial \boldsymbol{\Theta}. \quad (3.65)$$

Therefore, the vector of driving torques and forces $\boldsymbol{\tau}$ is provided by the dynamic model as

$$\boldsymbol{\tau} = \mathbf{p}_e \cdot \mathbf{J} + \sum_{j=1}^3 \sum_{i=1}^3 \mathbf{p}_{ij} \cdot \mathbf{K}_{ij}. \quad (3.66)$$

The methodology described in this section is suitable for the derivation of the dynamic model of the three times redundant manipulator 3PRRRR. It can be adapted to the case of a non-redundant manipulator 3RRRR, and to the case of the redundant manipulators illustrated in Fig.3.4. In those cases, the displacements of the linear guides δ_i , must be settled to zero accordingly. In order to solve the dynamic model suggested by Eq.3.66, the physical parameters of the manipulators must be known. Table 2 shows the values of the

parameters used for the manipulator's bodies and end-effector in this case. These parameters were obtained by measuring the lengths s_j . The masses m_j and inertial moments I_j were obtained empirically by adjusting the values until a good correlation between the numerical and experimental data was found.

At this point, two important aspects should be included in these models in order to obtain

Table 2: The end-effector's and manipulator's bodies parameters.

j	$s_j(m)$	$m_j(Kg)$	$I_j(kg \cdot m^2)$	$I_e(kg \cdot m^2)$	$m_e(Kg)$
1	0	1.6	0	$2.5 \cdot 10^{-6}$	1.2
2	0.095	2	0.0036		
3	0.116	3	0.0064		

Source: Composed by the author.

the proper dynamic equations. On the one hand, the connection between the links and the actuators in the prototype is provided by means of gear heads with reduction coefficient of 3.5. Hence, the torque at the input of the gear head, $\tau_{g,i}$, could be found by using the following relation:

$$\tau_{g,i} = \tau_{\theta_i}/3.5, \quad (3.67)$$

On the other hand, in the prototype, the linear displacements in the prismatic joints are achieved by the actuation of lead screw mechanisms. In these mechanisms, the motion produced by a rotative actuator is used to turn a screw and the screw is connected to a platform supported by a rail system. As the screw turns, the platform is moved linearly along the rail system (SPOTTIS, 1961). The relationship between the force of the moving platform F_{δ_i} and the torque of the actuator $\tau_{p,i}$ is given by

$$\tau_{p,i} = \frac{3.5}{2\pi} l (F_{\delta_i} + \mu W g), \quad (3.68)$$

where l is the ball screw lead, μ is the friction coefficient of the sliding surface, W is the mass of the load and g is the gravity acceleration. The values of l , μ and W are specific of the built prototype. The parameters of the lead screw used in the prototype are specified in Tab.3.

Table 3: Lead Screw Parameters.

$l(m)$	μ	$W(kg)$
0.01	0.02	1.6

Source: Composed by the author.

3.5 Power Consumption

In the previous section, the dynamic model of the 3P $\overline{R}$ RR was obtained. This permits to know the torques that the actuators must generate in order to perform a given task, *i.e.* the end-effector trajectory. In a robotic manipulator, the torques in the active joints are drove by means of actuators. The actuators are mechanical devices responsible for the motion and control of the manipulator. The aim of this section is to provide a model of the actuators in order to get a mathematical formulation that correlates the energy consumption required by the actuators to produce the desired torques.

Figure 3.6: A scheme of the electric circuit of the armature of a DC motor.

Source: Elaborated by the author.

The actuators used in the prototype are brushless DC motors. For the sake of assessing their electrical energy consumption, they were modeled as regular DC motors. Since brush-less DC motors have 3 inductances, resistances and electromotive forces (1 per phase), the mathematical description of this type of motor is usually more complicated than regular DC motors. Nevertheless, since brush-less motors are usually more energy efficient than regular DC motors, this simplification may yield conservative results. For sake of comparison, these results are arguably viable. A scheme of a DC motor is depicted in Fig. 3.6. A DC motor has a rotor that can turn inside a stator. The stator contains devices that are capable of generating a magnetic field (a permanent magnet or a coil of wire with a current running through it). The charged particles responsible for the armature current I_a in the rotor (see Fig. 3.6) are under the Lorentz force since they are moving through a magnetic field. Under the influence of these forces, the rotor turns with a torque τ and a velocity $\dot{\theta}$. The torque τ is related to the armature current I_a , according to the Lorentz's Force Law, as

$$\tau = K_m \cdot I_a, \quad (3.69)$$

where K_m is a quantity known as torque constant. The voltage V can be obtained by using the Kirchhoff voltage law as

$$V = V_b + L \frac{dI_a}{dt} + RI_a, \quad (3.70)$$

where V_b , R and L are the back electromotive force, the armature resistance and the armature inductance, respectively. The back electromotive force can be expressed, according to Faraday's law of induction, as

$$V_b = K_b \cdot \dot{\theta}, \quad (3.71)$$

where K_b is the back electromotive constant. The power of a DC motor can be computed as

$$P = V \cdot I_a. \quad (3.72)$$

The previous formulation is intended to be applied to continuous variables. However, as the numerical approach is performed using the Matlab environment, all the variables are actually expressed by means of discrete vectors. The same is applied when the data is extracted from the experiments in the prototype. Hence, diverse discrete variables, such as, the actuators' individual currents, $\mathbf{I}_k = [I_{1,k} \dots I_{6,k}]^T$, and the actual actuators' angular positions, $\underline{\Phi}_k = [\phi_{1,k} \dots \phi_{6,k}]^T$, are acquired in discrete form, where k denotes the instant time under investigation. The sub-index a has been eliminated for sake of simplicity. The required voltage, $V_{i,k}$, for driving each actuator at the time step k can be calculated in discrete form as

$$V_{i,k} = K_b \cdot \frac{\phi_{i,k} - \phi_{i,k-1}}{t(k) - t(k-1)} + L \cdot \frac{I_{i,k} - I_{i,k-1}}{t(k) - t(k-1)} + R \cdot I_{i,k}, \quad (3.73)$$

where K_b is the actuator's torque constant, L is the terminal inductance and R is the terminal resistance. The time derivatives have been approximated due to the discrete nature of the variables.

The energy consumption of each manipulator during the execution of any task, E_{total} can be calculated by taking the time integral of the electric power. Due to the discrete nature of the variables, the definite integral has been approximated using the trapezoidal rule

$$E_{total} = \sum_{i=1}^{3+M} \left[\frac{st}{2} \sum_{k=1}^K (V_{i,k+1} I_{i,k+1} + V_{i,k} I_{i,k}) \right], \quad (3.74)$$

where $st = t(k) - t(k-1)$, K is the total number of time steps and M is number of redundant actuators under investigation. Since SI units can be used, the actuators' individual currents and required voltage are given in ampere (A) and volts (V), respectively. In this way, the E_{total} is given in joule (J).

The quantities K_m , K_b , R and L are different for each DC motor. For the case of the brushless DC motors used to drive the prototype, the values of these quantities are listed in Tab.4.

Table 4: Actuator's parameters

$K_m(mN \cdot m/A)$	$K_b(rpm/V)$	$R(ohm)$	$L(mH)$
114	83.4	1.1	0.864

Source: Maxon EC 60 flat motor data sheet.

3.6 Trajectory Planning

In this research, the procedure to assess the impact of kinematic redundancy on the energy efficiency of planar PKMs consisted in comparing the energy consumption of the non-redundant and redundant PKMs under investigation during the execution of the same end-effector's trajectory (task). The approach for generating the end-effector's trajectories is further addressed in this section. The trajectories position profiles used in this manuscript are defined by fifth degree polynomials of the form

$$q(t) = a_0 + a_1t + a_2t^2 + a_3t^3 + a_4t^4 + a_5t^5. \quad (3.75)$$

It is possible to obtain the expressions for velocity and acceleration by computing the first and second derivatives of Eq.3.75, respectively:

$$\dot{q}(t) = a_1 + 2a_2t + 3a_3t^2 + 4a_4t^3 + 5a_5t^4, \quad (3.76)$$

$$\ddot{q}(t) = 2a_2 + 6a_3t + 12a_4t^2 + 20a_5t^3. \quad (3.77)$$

Equations 3.75, 3.76 and 3.77 can be rewritten in terms of the initial and final instants of time as

$$q(t_0) = a_0 + a_1t_0 + a_2t_0^2 + a_3t_0^3 + a_4t_0^4 + a_5t_0^5, \quad (3.78)$$

$$\dot{q}(t_0) = a_1 + 2a_2t_0 + 3a_3t_0^2 + 4a_4t_0^3 + 5a_5t_0^4, \quad (3.79)$$

$$\ddot{q}(t_0) = 2a_2 + 6a_3t_0 + 12a_4t_0^2 + 20a_5t_0^3, \quad (3.80)$$

$$q(t_f) = a_0 + a_1t_f + a_2t_f^2 + a_3t_f^3 + a_4t_f^4 + a_5t_f^5, \quad (3.81)$$

$$\dot{q}(t_f) = a_1 + 2a_2t_f + 3a_3t_f^2 + 4a_4t_f^3 + 5a_5t_f^4, \quad (3.82)$$

$$\ddot{q}(t_f) = 2a_2 + 6a_3t_f + 12a_4t_f^2 + 20a_5t_f^3. \quad (3.83)$$

Equation 3.78-3.83 can be rewritten in matrix form as

$$\begin{bmatrix} 1 & t_0 & t_0^2 & t_0^3 & t_0^4 & t_0^5 \\ 0 & 1 & 2t_0 & 3t_0^2 & 4t_0^3 & 5t_0^4 \\ 0 & 0 & 2 & 6t_0 & 12t_0^2 & 20t_0^3 \\ 1 & t_f & t_f^2 & t_f^3 & t_f^4 & t_f^5 \\ 0 & 1 & 2t_f & 3t_f^2 & 4t_f^3 & 5t_f^4 \\ 0 & 0 & 2 & 6t_f & 12t_f^2 & 20t_f^3 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{bmatrix} = \begin{bmatrix} q(t_0) \\ \dot{q}(t_0) \\ \ddot{q}(t_0) \\ q(t_f) \\ \dot{q}(t_f) \\ \ddot{q}(t_f) \end{bmatrix}. \quad (3.84)$$

If the initial and final values of the position, velocity, acceleration and time are known, the coefficients $a_0 - a_5$ can be computed by solving the previous system. By doing this, it is possible to obtain the position, velocity and acceleration profiles.

In the parallel manipulators under study, the end-effector position is defined by three parameters, two Cartesian coordinates (x_e, y_e) and one orientation angle α . Hence, the previous process must be applied three times for defining an end-effector path. The end-effector trajectories used in this thesis are illustrated in Figs. 3.7, 3.9 and 3.11. The initial and final values of velocity and acceleration are zero for the three trajectories. Moreover, the orientation angle α has been fixed to a constant value of zero degrees during the entire execution for the three cases.

Figure 3.7: Trajectory I

Source: Elaborated by the author.

The trajectory I depicted in Fig. 3.7 is a vertical straight path started from point $(0.05, -0.1)$ and finishing in the point $(0.05, 0.1)$. This trajectory is executed in 0.5 seconds, hence $t_0 = 0$ and $t_f = 0.5$. In this case, the horizontal Cartesian coordinate is constant along the entire path, so that its horizontal position can be expressed as

$$x_{eI}(t) = 0.05. \quad (3.85)$$

The vertical coordinate is defined as

$$y_{eI}(t) = -0.1 + 0t + 0t^2 + 16t^3 - 48t^4 + 38.4t^5. \quad (3.86)$$

The position, velocity and acceleration profiles of trajectory I are illustrated in Fig.3.8.

Figure 3.8: Position, velocity and acceleration profiles for the trajectory I.

Source: Elaborated by the author.

Figure 3.9: Trajectory II

Source: Elaborated by the author.

The trajectory II depicted in Fig.3.9 is an arc of 60 degrees and radius of 0.15 meters that

starts at point (0.1,-0.1) and finishes in the point (0.1,0.1). This trajectory is an arc which position profile is defined by the expression below

$$q_{II}(t) = 3.6652 + 0t + 0t^2 - 83.7758t^3 + 251.3274t^4 - 201.0619t^5. \quad (3.87)$$

The Cartesian coordinates of the trajectory II are found using the Eq.3.94 as follows

$$x_{eII} = 0.2 + 0.15 \cdot \cos(q_{II}(t)), \quad (3.88)$$

$$y_{eII} = 0.15 \cdot \sin(q_{II}(t)). \quad (3.89)$$

The position, velocity and acceleration profiles of trajectory II are illustrated in Fig.3.10.

Figure 3.10: Position, velocity and acceleration profiles for the trajectory II.

Source: Elaborated by the author.

The trajectory III depicted in Fig.3.11 is a straight path started from point (0,0) and finishing in the point (-0.1,0.05). This trajectory is executed in 0.5 seconds, hence $t_0 = 0$ and $t_f = 0.5$. By carrying out the method explained at the beginning of this section the horizontal coordinate of the trajectory III can be defined as

$$x_{eIII}(t) = 0 + 0t + 0t^2 - 8t^3 + 24t^4 - 19.2t^5. \quad (3.90)$$

The vertical coordinate position equation can be expressed as

$$y_{eIII}(t) = 0 + 0t + 0t^2 - 4t^3 + 12t^4 - 9.6t^5. \quad (3.91)$$

Figure 3.11: Trajectory III

Source: Elaborated by the author.

Figure 3.12: Position, velocity and acceleration profiles for the trajectory III.

The position, velocity and acceleration profiles of trajectory III are illustrated in Fig.3.12. The trajectory IV depicted in Fig.3.13 is a straight path started from point $(-0.1, -0.5)$ and finishing in the point $(0.1, 0.05)$. This trajectory is executed in 0.5 seconds, hence $t_0 = 0$ and $t_f = 0.5$. By carrying out the method explained at the beginning of this section the horizontal coordinate of the trajectory IV can be defined as

$$x_{eIV}(t) = -0.05 + 0t + 0t^2 + 8t^3 - 24t^4 + 19.2t^5. \quad (3.92)$$

The vertical coordinate position equation can be expressed as

$$y_{eIV}(t) = -0.1 + 0t + 0t^2 + 16t^3 - 48t^4 + 38.4t^5. \quad (3.93)$$

Figure 3.13: Trajectory IV

Source: Elaborated by the author.

The position, velocity and acceleration profiles of trajectory IV are illustrated in Fig.3.14.

Figure 3.14: Position, velocity and acceleration profiles for the trajectory IV.

The trajectory V depicted in Fig.3.15 is a circle centered in the point (0,0) with a radius of 0.1 meters. The trajectory is executed in clockwise direction starting from point (0.1, 0). This trajectory is executed in 2 seconds, hence $t_0 = 0$ and $t_f = 2$. By carrying out the method explained at the beginning of this section the position profile of the trajectory V can be defined as

$$q_V(t) = 0 + 0t + 0t^2 + 7.854t^3 - 5.891t^4 + 1.1781t^5. \quad (3.94)$$

Figure 3.15: Trajectory V

V

Source: Elaborated by the author.

The Cartesian coordinates of the trajectory V are found using the Eq.3.94 as follows

$$x_{eV} = 0.1 \cdot \cos(q_V(t)), \quad (3.95)$$

$$y_{eV} = 0.1 \cdot \sin(q_V(t)). \quad (3.96)$$

The position, velocity and acceleration profiles of trajectory V are illustrated in Fig.3.16.

Figure 3.16: Position, velocity and acceleration profiles for the trajectory V.

Source: Elaborated by the author.

4 REDUNDANCY RESOLUTION SCHEME

In the section 3.1, the inverse kinematics problem was modeled by means of three equations (one for each kinematic chain), that relates the end-effector's pose with the active joint's inputs. For the case of the non redundant parallel manipulator $3\underline{R}RR$, given an end-effector's pose $\mathbf{X} = [x_e, y_e, \alpha]$, the inverse kinematics problem presents an unique solution. In other words, as the prismatic joints $[\delta_1, \delta_2, \delta_3]$ are zero, the active joints coordinates $\Theta = [\theta_1, \theta_2, \theta_3]$ are the only unknowns of the problem. Thus, the inverse kinematics problem becomes a system of three equations and three unknowns with an unique solution.

The last statement is not valid for the case of parallel manipulators with kinematic redundancy under investigation in this work. In those cases, the prismatic joints become variables, leading to an inverse kinematics problem with three equations and four to six unknowns depending on the number of levels of kinematic redundancy. The effect of kinematic redundancy in the inverse kinematics problem is graphically illustrated in Fig. 4.1. In this figure, the end-effector's pose remains constant for several joint's inputs due to the presence of a redundant active prismatic joint.

Figure 4.1: Illustration of the inverse kinematic problem with infinite solutions for the redundant manipulator $\underline{P}RRR+2\underline{R}RR$.

The capabilities of kinematic redundancy are fully related to this characteristic. By having infinite possible configurations for a single end-effector's pose, a suitable configuration that better fits the needs of an specific application can be selected. The process to make this choice is referred as redundancy resolution.

The redundancy resolution schemes can be classified into two main techniques according to the motion of the redundant actuators: the pre-positioning technique and the ongoing

technique. In the pre-positioning technique, the optimal positions of the redundant actuators must be found before the execution of the task (the end-effector's motion), and then, they should remain in that position until the task execution is finished. On the other hand, the ongoing technique, the optimal trajectories for the redundant actuators are also considered as unknowns. In this approach, the redundant actuator's positions vary during the task execution. The pre-positioning and ongoing approaches are also referred in the literature as off-line and on-line approaches, respectively (KOTLARSKI et al., 2010).

It has been proved numerically that the better dynamic performance of the the redundant PKMs under study can be achieved by using the ongoing approach than by using the pre-positioning approach (FONTES; SANTOS; SILVA, 2014). Fontes, Santos and Silva (2014) proposed a redundancy resolution scheme based on an optimization problem. The aim of this optimization problem was to minimize the maximum required torque to execute a task.

Alternatively, a novel redundant resolution scheme is based on the Model Predictive Control (MPC) technique is proposed in this work. This proposal is explained hereafter. This scheme is proposed using the pre-positioning approach. The reason for this choice is following explained. The redundant prismatic joints are actuated by means of a lead screw with a lead of 1cm in the prototype. This means that for each complete turn of the actuator, the prismatic platform is moved by 1cm. This leads to high power consumption when fast and large displacements of the prismatic actuators are performed, making the application of the on-going approach unpractical considering the main objective of this work.

4.1 Implementation of MPC

The MPC employs a mathematical model of a system to predict the response of a plant during a future time interval. In discrete time, the MPC technique can be summarized in four steps:

1. The current state of the system should be determined.
2. The future behavior of the process should be predicted and the best sequence of control on a limited time window of N steps starting at the current state of the system should be find.
3. The first control action on the system should be implemented.
4. The prediction horizon should be moved by one step.

This process should be repeated until the total time of the task is reached. The MPC technique can be perceived as an iterative method that solves individual optimization

problems. These individual problems consider N future samples during each optimization process. This process is repeated for every time step moving the N future samples forward. The value N is called Prediction Horizon. Due to this characteristic, the MPC is also known as Receding Horizon Control. In this section, the methodology to model the MPC technique into a redundancy resolution scheme suited for the purpose of this research is further detailed.

4.1.1 Individual Optimization Problem

As stated previously, the MPC technique solves individual optimization problems, one after another during a number of iterations. Each individual optimization problem can be formulated as

$$\text{minimize } \frac{1}{2} \boldsymbol{\vartheta}^T \mathbf{H} \boldsymbol{\vartheta} \quad (4.1.1)$$

$$\text{subject to } \mathbf{A}_{eq} \cdot \boldsymbol{\vartheta} = \mathbf{b}_{eq} \quad (4.1.2)$$

$$\mathbf{l}_b \leq \boldsymbol{\vartheta} \leq \mathbf{u}_b, \quad (4.1.3)$$

The cost function in Eq.4.1.1 is described by a quadratic function and the constraints in Eq.4.1.2 by linear relations. The vector $\boldsymbol{\vartheta}$ describes the decision variables. The lower and upper bounds of the variables are described by the vectors $\mathbf{l}_b$ and $\mathbf{u}_b$ in Eq.4.1.3.

In order to apply the MPC technique as a redundancy resolution scheme for the PKMs under study, the number of decision variables for each optimization problem must be N times the number of actuated joints of the manipulator. Hence, the set of optimization variables are defined as

$$\boldsymbol{\vartheta} = [\boldsymbol{\Theta}_1, \boldsymbol{\Theta}_2, \dots, \boldsymbol{\Theta}_N]. \quad (4.2)$$

For the case of the planar parallel manipulator with three levels of redundancy 3PRRR , the vector of actuated joints is defined as

$$\boldsymbol{\Theta} = [\theta_1 \quad \theta_2 \quad \theta_3 \quad \delta_1 \quad \delta_2 \quad \delta_3]. \quad (4.3)$$

This vector is composed of six components related to the six actuated joints. As a consequence, the number of variables of the vector $\boldsymbol{\vartheta}$ is $6N$. The decision variables are bounded by the physical limitations of the prototype's actuators. These limitations are represented by the lower and upper bounds $\mathbf{l}_b$ and $\mathbf{u}_b$ in Eq.4.1.3. The active revolute joints are bounded to the interval $[-2\pi, 2\pi]$, in radians, and the active prismatic joints are bounded to the interval $[-0.2, 0.2]$, in meters. Then, the vector of lower bounds can be expressed as

$$\mathbf{l}_b = [l_{b1}, l_{b2}, \dots, l_{bN}], \quad (4.4)$$

where

$$l_b = [-2\pi \quad -2\pi \quad -2\pi \quad -0.2 \quad -0.2 \quad -0.2]. \quad (4.5)$$

In a similar manner, the vector of upper bounds can be written as

$$\mathbf{u}_b = [u_{b1}, u_{b2}, \dots, u_{bN}], \quad (4.6)$$

where

$$u_b = [2\pi \quad 2\pi \quad 2\pi \quad 0.2 \quad 0.2 \quad 0.2]. \quad (4.7)$$

4.1.2 Cost Function

In optimization problems, the decision variables should be found in such a way that a cost function is minimized or maximized. In this work, the objective of the optimization problem is to find the inputs of the actuators (decision variables) that guarantee a reduced energy consumption during the execution of a task. A easy way to accomplish this objective is to minimize the motion of the actuated joints as much as possible during the task execution. A cost function that achieves the previous condition can be expressed as

$$\frac{1}{2} \sum_{j=t+1}^{t+N} \left[\sum_{i=1}^3 (\theta_{i,j-1} - \theta_{i,j})^2 + \sum_{i=1}^3 (\delta_{i,j-1} - \delta_{i,j})^2 \right], \quad (4.8)$$

where the sub-index i indicates the kinematic chain, and the sub-index j depicts the sample time. The parameter t represents the current time step which is increased in one step each new iteration. It is a fact that the cost function in Eq.4.8 attempts to minimize the difference between the inputs of the actuators at the current time step and their positions at the next time step.

The cost function in Eq.4.8 must be written in the quadratic form in order to fit the MPC technique (Eq.4.1.1). To do this, the matrix $\mathbf{H}$ must be written accordingly. For sake of illustration and clarity, an example is exploited hereafter. Supposing that the prediction horizon is $N = 3$, the set of optimization variables is

$$\boldsymbol{\vartheta}^T = [\Theta_1 \quad \Theta_2 \quad \Theta_3], \quad (4.9)$$

where

$$\Theta_k^T = [\theta_{1,k} \quad \theta_{2,k} \quad \theta_{3,k} \quad \delta_{1,k} \quad \delta_{2,k} \quad \delta_{3,k}], \quad k = 1, 2, 3 \quad (4.10)$$

In order to write the cost function in the quadratic form of Eq.4.1.1 for the case $N = 3$, the matrix $\mathbf{H}$ must be wrote as

$$\mathbf{H} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad (4.11)$$

The matrix $\mathbf{H}$ in Eq.4.11 has 18×18 dimensions or is a $6N \times 6N$. This idea can be generalized to higher values of the prediction horizon in order to get the $\mathbf{H}$ that proper fits the cost function in Eq.4.8.

As can be implied from Section.3.5, the power consumption of a DC motor depends on two factors, its torque and its velocity. The Eq.4.8 just minimizes the velocity and does not deal with the torque. The addition of the actuators driving torques to the cost function increases the complexity of each optimization problem because additional optimization variables must be added as well as a model for the torques in the constrains. On this matter, when the formulation related to torques was introduced to the actual redundancy resolution scheme, problems of convergence to a feasible solution were found. Hence, the author decided not to include the torques in the actual formulation. To adopt the easiest path, allowed to drastically reduce the computational cost of the optimization problem, nevertheless, additional issues were found. These issues are specified and commented in the results and conclusions sections.

4.1.3 Constrains

The constrains are a set of equations that should not be violated by the optimal solution. In this study, the constrains describe the inverse kinematics problem treated in Section 3.1. The solution of this problem can be expressed as

$$\theta_i = 2 \tan^{-1} \frac{-E_{1i} + \sqrt{E_{1i}^2 + E_{2i}^2 - E_{3i}^2}}{E_{3i} - E_{2i}}, \quad (4.12)$$

where

$$E_{1i} = -2l_1\rho_i, \quad E_{2i} = -2l_1\mu_i, \quad E_{3i} = \mu_i^2 + \rho_i^2 + l_1^2 - l_2^2, \quad (4.13)$$

and

$$\begin{bmatrix} \mu_i \\ \rho_i \end{bmatrix} = \begin{bmatrix} x_e \\ y_e \end{bmatrix} + h \begin{bmatrix} \cos(\alpha + \lambda_i) \\ \sin(\alpha + \lambda_i) \end{bmatrix} - a_i \begin{bmatrix} \cos \lambda_i \\ \sin \lambda_i \end{bmatrix} - \delta_i \begin{bmatrix} \cos \gamma_i \\ \sin \gamma_i \end{bmatrix}. \quad (4.14)$$

Equation 4.12 imposes the kinematic relation between the inputs of the active joints (θ_i, δ_i) and the end-effector's pose (x_e, y_e, α) . The MPC technique requires the description of the constraints as a set of linear relations. Therefore, a linearization of Eq.4.12 is required. The linearization of the Eq.4.12 was performed by using the expansion in Taylor series. It is known that a function $f(z)$ (being z a random variable), infinitely differentiable at a value c can be expressed in Taylor series as

$$f(z) = f(c) + f'(c)(x - c) + \frac{f''(c)}{2!}(x - c)^2 + \frac{f'''(c)}{3!}(x - c)^3 + \dots \quad (4.15)$$

If the point c is close to x , the Taylor series can be simplified by its linear terms as

$$f(z) = f(c) + f'(c)(x - c). \quad (4.16)$$

On the other hand, the solution to the inverse kinematics in Eq.4.12 can be rewritten as

$$\theta_i = f(\delta_i). \quad (4.17)$$

Equation 4.17 can be evaluated at discrete times using the subindex k , hence

$$\theta_{i,k} = f(\delta_{i,k}). \quad (4.18)$$

A close value to the point $\delta_{i,k}$ can be its previous time sample point $\delta_{i,k-1}$. Therefore, Eq.4.18 can be expressed in Taylor series as

$$f(\delta_{i,k}) = f(\delta_{i,k-1}) + f'(\delta_{i,k-1})(\delta_{i,k} - \delta_{i,k-1}). \quad (4.19)$$

Equation 4.19 can be rewritten as

$$\theta_{i,k} - \theta'_{i,k-1}\delta_{i,k} = \theta_{i,k-1} - \theta'_{i,k-1}\delta_{i,k-1}. \quad (4.20)$$

By exploiting this strategy for the three kinematic chains of the manipulator, Eq.4.20 can be expressed in matrix form as

$$\begin{bmatrix} 1 & 0 & 0 & -\theta'_{1,k-1} & 0 & 0 \\ 0 & 1 & 0 & 0 & -\theta'_{2,k-1} & 0 \\ 0 & 0 & 1 & 0 & 0 & -\theta'_{3,k-1} \end{bmatrix} \Theta_k = \begin{bmatrix} \theta_{1,k-1} - \theta'_{1,k-1}\delta_{1,k-1} \\ \theta_{2,k-1} - \theta'_{2,k-1}\delta_{2,k-1} \\ \theta_{3,k-1} - \theta'_{3,k-1}\delta_{3,k-1} \end{bmatrix}, \quad (4.21)$$

where

$$\Theta_k^T = \begin{bmatrix} \theta_{1,k} & \theta_{2,k} & \theta_{3,k} & \delta_{1,k} & \delta_{2,k} & \delta_{3,k} \end{bmatrix}. \quad (4.22)$$

It can be confirmed that Eq.4.21 has the same form as Eq.4.1.2. For sake of illustration, this approach is exploited for the case in which $N = 3$, Eq.4.21 becomes

$$\begin{pmatrix} A_1 & O & O \\ O & A_2 & O \\ O & O & A_3 \end{pmatrix} \boldsymbol{\vartheta} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}, \quad (4.23)$$

where

$$A_1 = \begin{bmatrix} 1 & 0 & 0 & -\theta'_{1,0} & 0 & 0 \\ 0 & 1 & 0 & 0 & -\theta'_{2,0} & 0 \\ 0 & 0 & 1 & 0 & 0 & -\theta'_{3,0} \end{bmatrix} \quad b_1 = \begin{bmatrix} \theta_{1,0} - \theta'_{1,0}\delta_{1,0} \\ \theta_{2,0} - \theta'_{2,0}\delta_{2,0} \\ \theta_{3,0} - \theta'_{3,0}\delta_{3,0} \end{bmatrix}, \quad (4.24)$$

$$A_2 = \begin{bmatrix} 1 & 0 & 0 & -\theta'_{1,1} & 0 & 0 \\ 0 & 1 & 0 & 0 & -\theta'_{2,1} & 0 \\ 0 & 0 & 1 & 0 & 0 & -\theta'_{3,1} \end{bmatrix} \quad b_2 = \begin{bmatrix} \theta_{1,1} - \theta'_{1,1}\delta_{1,1} \\ \theta_{2,1} - \theta'_{2,1}\delta_{2,1} \\ \theta_{3,1} - \theta'_{3,1}\delta_{3,1} \end{bmatrix}, \quad (4.25)$$

$$A_3 = \begin{bmatrix} 1 & 0 & 0 & -\theta'_{1,2} & 0 & 0 \\ 0 & 1 & 0 & 0 & -\theta'_{2,2} & 0 \\ 0 & 0 & 1 & 0 & 0 & -\theta'_{3,2} \end{bmatrix} \quad b_3 = \begin{bmatrix} \theta_{1,2} - \theta'_{1,2}\delta_{1,2} \\ \theta_{2,2} - \theta'_{2,2}\delta_{2,2} \\ \theta_{3,2} - \theta'_{3,2}\delta_{3,2} \end{bmatrix}, \quad (4.26)$$

and O are zero matrices. By obtaining Eq.4.21, the inverse kinematics solution is expressed in the form of a set of the linear equality constrains of the proposed optimization problem described in Eq.4.1.2. For the particular case of $N = 3$:

$$\mathbf{A}_{eq} = \begin{pmatrix} A_1 & O & O \\ O & A_2 & O \\ O & O & A_3 \end{pmatrix}, \quad (4.27)$$

and

$$\mathbf{b}_{eq} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}. \quad (4.28)$$

Contrarily to the matrix $\mathbf{H}$, the block matrices $\mathbf{A}_{eq}$ and $\mathbf{b}_{eq}$ must be calculated at each iteration. In order to do this, the values of the variables obtained by means of the previous optimization are used to compute the new matrices. As an example, for this case, the next iteration corresponds to the samples $k = 2, 3, 4$. Therefore the new linear equality constrains become

$$\begin{pmatrix} A_2 & O & O \\ O & A_3 & O \\ O & O & A_4 \end{pmatrix} \begin{bmatrix} \Theta_2 \\ \Theta_3 \\ \Theta_4 \end{bmatrix} = \begin{pmatrix} b_2 \\ b_3 \\ b_4 \end{pmatrix}. \quad (4.29)$$

As it can be seen, in order to express the inverse kinematics problem in the form of Taylor series, the partial derivative of the Eq.4.12 should be calculated (represented by $\theta'_{i,k}$). The analytically calculation of these derivatives can be tricky as several trigonometric functions are involved. The strategy to overcome this problem was to approximate the derivatives by the central differences as

$$\theta'_{i,k} \approx \frac{\theta_{i,k+\epsilon} - \theta_{i,k-\epsilon}}{2\epsilon}, \quad (4.30)$$

where ϵ is a value close to zero.

4.1.4 Complete Scheme

The previous section provides the necessary mathematical formulation to accomplish a single optimization in the MPC based redundancy resolution scheme. At the end of each optimization problem a set of optimization variables $\boldsymbol{\vartheta}^{(p)}$ are obtained, where the super-index (p) represents the iteration number. The vector $\boldsymbol{\vartheta}^{(p)}$ is used to compute the matrices $\mathbf{A}_{eq}$ and $\mathbf{b}_{eq}$ for the next iteration. As previously established, the set of optimization variables is a vector composed by the position of the actuators. Hence, after the first iteration is finished, the obtained set of variables are

$$\boldsymbol{\vartheta}^{(1)} = [\boldsymbol{\Theta}_1^{(1)}, \boldsymbol{\Theta}_2^{(1)}, \dots, \boldsymbol{\Theta}_N^{(1)}]. \quad (4.31)$$

The component $\boldsymbol{\Theta}_1^{(1)}$ constitute the first time step optimized set of actuator's inputs. At this point the component $\boldsymbol{\Theta}_1^{(1)}$ is implemented in the system and the second iteration is performed. At the end of the second iteration, a new set of optimized variables are found

$$\boldsymbol{\vartheta}^{(2)} = [\boldsymbol{\Theta}_2^{(2)}, \boldsymbol{\Theta}_3^{(2)}, \dots, \boldsymbol{\Theta}_{N+1}^{(2)}]. \quad (4.32)$$

Then, the component $\boldsymbol{\Theta}_2^{(2)}$ is implemented in the system and the operation window is advanced by one time step. The process is repeated until the end of the time execution. The duration of the execution depends of the length of the trajectory in samples. For this particular case, the trajectories have a length of 1000 samples. Therefore, the scheme is complete after perform 1000 single optimization problems. At the end, the optimal joint trajectories for a given end-effector trajectory are obtained. A convergence test was performed in order to select this sample rate. For the samples rates of 1000, 2000, 3000 and 10000 samples the variations in the total power consumption were in the range of 10^{-6} Watts. This error range is acceptable for the study presented in this work, hence, the sample rate of 1000 samples was chosen.

In the previous process, a set of trajectories for the redundant prismatic actuators have been selected. However, due to the limitations imposed by the prototype, the pre-positioning approach was selected. In other words, the inputs of the redundant prismatic actuators should remain the same during the task execution. The adopted approach to do use the mean values of these inputs found by the proposed MPC based redundancy resolution scheme. By adopting these values, the inputs for the actuators can be calculated by solving the inverse kinematics problem.

The previous formulation provides the required theory to implement the MPC based redundancy resolution scheme. Optimization problems described using the MPC approach are quadratic problems, which are convex problems, that can be solved using several algorithms, among them the Interior Point Method. This numerical solver is implemented in the Matlab function `quadprog`. The used value for the prediction horizon N was 10

Figure 4.2: Redundancy Resolution Scheme Map.

Source: Elaborated by the author.

samples. Small variations of the value of the prediction horizon did not represented large changes in the behavior of the system. Nevertheless, it was observed that larger values of the prediction horizon led to very time-consuming calculations. A complete scheme of the redundancy resolution scheme can be seen in Fig.4.2. This methodology was applied to the general case of the three times redundant manipulator 3PRRR, depicted as M8 in Fig.3.4. In order to apply this methodology to any of the manipulators M1 to M8, the set of lower and upper bounds of the redundant prismatic actuators must be settled to zero accordingly. As an example, for the case of the one time redundant manipulator PRRR+2RRR, depicted as M2 in Fig.3.4, the vectors of lower and upper bounds become

$$l_b = [-2\pi \quad -2\pi \quad -2\pi \quad -0.2 \quad 0 \quad 0], \quad (4.33)$$

and

$$u_b = [2\pi \quad 2\pi \quad 2\pi \quad 0.2 \quad 0 \quad 0], \quad (4.34)$$

Figure 4.3: Redundancy Resolution Scheme Algorithm.

```
1:  Define Prototype Parameters
2:  Define End-Effector Trajectory  $(x_e, y_e, \alpha)$ 
3:  Built matrix H
4:  Define bounds lb and ub
5:    for t=1: length(trajjectory)
6:      for k=1: N(horizon)
7:        Calculate Aeq and beq
8:         $\vartheta(t, k) = \text{quadprog}(H, Aeq, beq, lb, ub)$ 
9:      end
10: end
```

respectively. Figure 4.3 illustrates the redundancy resolution scheme algorithm. A similar extended algorithm was used in the MATLAB environment for solve the inverse kinematics problem for the case of the parallel manipulators with kinematic redundancy.

5 PROTOTYPE AND DATA ACQUISITION

This work provides numerical and experimental analysis in order to assess whether or not the kinematic redundancy is a good alternative to increase the energy efficiency of planar parallel manipulators. The required background to carry out simulations and obtain numerical results was presented in the previous chapters. In this chapter, the framework used to accomplish the experimental analysis is provided. The chapter is divided in two parts. The first part is dedicated to present the manipulator prototype built at the São Carlos School of Engineering facilities. The second part describes the procedure used to acquire and treat the data from the prototype's communications interface.

5.1 Prototype

In order to experimentally prove the benefits of kinematic redundancy regarding the energy efficiency of parallel manipulators, the prototype depicted in Fig.5.1 was used. The prototype is a 3PRRR parallel manipulator. This robot presents three identical kinematic chains. Each chain is composed by one active prismatic joint (P), one active revolute joint (R), and two passive revolute joints (RR). The prototype was built at the São Carlos School of Engineering at University of São Paulo. The robot can be found at the laboratory of Dynamics of the aforementioned institution.

Figure 5.1: The 3PRRR prototype

Source: EESC Laboratory of Dynamics group.

This setup possess a high versatility as by properly locking the prismatic joints the

robot can be reconfigured into different parallel manipulators. By locking all the prismatic joints the prototype becomes the 3RRR . By locking two of the three prismatic joints, the manipulator $\text{PRRR}+2\text{RRR}$ is obtained. By locking just one of the prismatic joints results in the robot $2\text{PRRR}+\text{RRR}$. When the three prismatic joints are active the prototype becomes the 3PRRR manipulator. Hence, by actuating or locking the prismatic joints the eight configurations illustrated in Fig.5.2 can be experimentally investigated. In this figure, the revolute joints are illustrated as circles and the prismatic joints as rectangles. The rectangles filled in black illustrate the prismatic joints when are locked. The rectangles filled in white illustrate the prismatic joints when they are activated. In this way, the configuration R1 represents the non-redundant 3RRR manipulator, the configurations R2-R4 represent the possible redundant $\text{PRRR}+2\text{RRR}$ manipulators, the configurations R5-R7 represent the possible redundant $2\text{PRRR}+\text{RRR}$ manipulators and the configuration R8 represents the 3PRRR .

Figure 5.2: R1: the 3RRR manipulator, R2-R4: the $\text{PRRR}+2\text{RRR}$ manipulators, R5-R7: the $2\text{PRRR}+\text{RRR}$ manipulators and R8: the 3PRRR

Source: Elaborated by the author.

The prototype's communications scheme is illustrated in Fig.5.3. The communications scheme permits iteration between the human and the prototype. Thus, the human operator can induce actions in the prototype, as the execution of a predefined trajectory. Moreover, the communication scheme provides output data relative to the torques, currents velocities and positions of the actuator among other parameters.

The communication's scheme is composed by four main parts: (1) the actuators M1-M6, (2) the control boards C1-C6, (3) the power supplies PS1-PS6, and (4) a computer (PC). Details regarded to each part are following described:

- **Actuators:** The prototype is actuated by six brushless DC motors, depicted as

Figure 5.3: Prototype's Communications Scheme.

Source: Elaborated by the author.

M1-M6 in Fig.5.3. The actuators are Maxon EC60 flat motors (Fig.5.4), with nominal torque of 0.257N.m @ 3580rpm. The actuators are connected to the Maxon planetary gear heads GP52C, with a reduction rate of 3.5:1. Due to the reduction rate of the planetary gear head, the resulting available nominal torque of each set is 0.82N.m @ 1200rpm.

Figure 5.4: Maxon EC60 brushless DC motor.

Source: Photo shot by the author.

For the case of the active rotary joints, the motion is provided by the actuators and transmitted to the links directly by the gear head. For the case of the prismatic joints, the gear head is connected to a mechanism that transform the rotary motion into linear motion. This mechanism consists of a table system with ball screw HIWIN KK60-10-C-E-600-A-1-F0-S3 (Fig.5.5). In this case, the system possesses a lead of 0.01 meters, this means that the platform experiments a displacement of 0.01 meters for each complete turn of the actuator.

Figure 5.5: Lead screw system HIWIN KK60.

Source: Photo shot by the author.

- **Control Boards:** Depicted as C1-C6 in Fig.5.3, these devices are Maxon EPOS2 50/5 (Fig.5.6). The control boards provide communication between the actuators and the computer. In summary, the controller board performs four basic processes:
 - Receives the discrete input data from the computer containing the information regarding to the task to execute.
 - Coverts the discrete input data from the computer to analog signals to drive the actuators.
 - Controls the actuator position by establish communication with the type Hall sensors present in the actuators.
 - Sends the output information to the computer in discrete form.

Figure 5.6: Control Board Maxon EPOS2 50/5.

placa.jpg

Source: Photo shot by the author.

- **Power Supplies:** Each control board is powered by a power supply as can be seen in Fig.5.7. The power supplies receives 220 volts at 60 Hertz of alternating current at the input and outputs 48 Volts at 10 Amperes of direct current.

The communication between the control boards is based on the CAN Protocol. In the other hand, the communication between the prototype and the computer is done through the board C1 via USB. A human interface in the Matlab environment was built to permit the

Figure 5.7: Power supplies MCE 48-10.

Source: Photo shot by the author.

users to implement the desired control strategies in the control boards. The experimental campaign described hereafter exploited the *Interpolated Position Mode*. In this mode, the user provides the desired positions and velocities at diverse time steps. This data is interpolated through splines by the board and is used as a reference signal to the control procedure. In this mode, linear position feedforward and position feedback controllers are used to guarantee performance and robustness during the execution of the desired task. The control parameters, the feedforward and feedback gains, have been adjusted manually. Further details on the setup can be found in Santos et al. (2015).

5.2 Data Acquisition

In this section, the procedures to acquire and to post process the data obtained from the simulation and from the prototype are explained. The initial procedure to carry out the experiments in the prototype and the numerical simulation is the same for both approaches. Figure 5.8 illustrates the initial procedure in the form of an scheme divided into two stages.

The stage 1 is common for both numerical and experimental approaches. It is carried out in the Matlab environment. At the end of stage 1, the required information for the dynamic modeling of the manipulators and the actuators is available. In this way, the numerical approach can be accomplished at Stage 2.a using the models described in chapter 3.

Similarly, at the end of stage 1, the required input data for the experimental campaign is available. This data is composed by vectors that describe the time and the positions for the end-effector, active joints and passive joints. Hence, the experimental campaign can be carried out at Stage 2.b. The approach used to perform the experiments on the prototype can be summarized in three steps:

Figure 5.8: Initial procedure for data acquisition.

Source: Elaborated by the author.

1. **Introduce the input data on the prototype:** The end-effector and joint coordinates must be introduced in the form of a Matlab structure in the computer responsible for the human-machine interface.
2. **Task execution:** The control boards collect the input data from the Matlab and send the corresponding signals to the actuators.
3. **Output data acquisition:** Once the task is concluded, the setup provides the data related to the execution. This data is presented in the form of Matlab vectors and matrices useful for conversion to graphics and charts.

The data obtained after the execution of a single experiment presents issues related to the limitations in the speed of communications between the actuators, control boards and computer. In this respect each single execution of a trajectory is repeated 20 times. The data referent to the current flow, position, velocity and time of the actuators is stored for each repetition. Once the 20 experiences are finished, an average signal is computed for each required data. By doing this, it is possible to obtain a more accurate and noise free representation of the experiments. Thus, the vectors of current, velocity and time of all actuators are available at the end of the experiments. With this, it is possible to compute

the total power consumption by using the methodology described by Eqs.3.70-3.74 in section 3.5.

Due to the use of the average values of the several experiments to present the outcomes, an statistical study was used in order to provide a likelihood ratio for the presented results. This statistical study revealed that the results shown in this work posses a confident interval of 10% above or below of the average values. Details about the used procedure to reach this value can be found in Cox and Hinkley (1974).

6 RESULTS

The purpose of this thesis is to prove whether the presence of kinematic redundancy together with the MPC based redundancy resolution scheme may increase the energy efficiency of parallel manipulators under study. In order to do this, the data obtained from the numerical simulations and the experiments carried out in the prototype are presented in this chapter.

The procedure to verify the aforementioned objective consisted in comparing the energy consumption of the non-redundant parallel manipulator 3RRR and the redundant manipulators with different levels of kinematic redundancy illustrated as R2, R5 and R8 in Fig.6.1. The energy consumption of the manipulators under study have been acquired during the execution of five different tasks as depicted in Fig.6.2 and Fig.6.3. The methodology to generate the trajectories are detailed in Section 3.6.

Figure 6.1: R1: the 3RRR manipulator, R2: the $\text{PRRR}+2\text{RRR}$ manipulator, R5: the $2\text{PRRR}+\text{RRR}$ manipulator and R8: the 3PRRR .

Source: Elaborated by the author.

The execution of such trajectories by the redundant manipulators requires the use of a proper redundancy resolution scheme. The proposed scheme is based in the MPC technique depicted in Section 4. This chapter is divided into four sections. In the the first section, the total energy consumption required for execute each trajectory is compared with each other for the manipulators under study. The second section presents the data relative to the currents that are required by the actuators of the different manipulators. The current profiles provide insights to understand the data related to the energy consumption. The third section presents the inputs of the active revolute joints during the execution of the three different trajectories. The last section presents an economical study that evaluates the viability of the proposal considering the execution of a repetitive task.

Figure 6.2: Trajectories I, II and III.

Source: Elaborated by the author.

Figure 6.3: Trajectories IV and V.

Source: Elaborated by the author.

6.1 Power Consumption

In this section, the data referred to the energy consumption required by the manipulators to execute the three different tasks is arranged in chart form. Figures 6.4, 6.5 and 6.6 depicts the energy consumed by manipulators R1, R2, R5 and R8 to execute the trajectories I, II and III, respectively. The power consumption is displayed for the experimental and the numerical cases in these figures.

For the experimental case, the manipulators with kinematic redundancy (R2, R5 and R8) were able to reduce the amount of power consumption with respect to the non-redundant manipulator R1 for the three trajectories. However, the rate of reduction was different for each trajectory. The manipulator with three levels of kinematic redundancy R8 was able to reduce the power consumption in 54% and 62% and 45% for trajectories I, II and III, respectively. The rates of reduction in percentage for all manipulator are summarized in Tab.5.

The MPC based redundancy resolution scheme finds the fixed inputs (positions) of the active prismatic joints δ_i that minimizes the movements of the active revolute joints. In this respect, Tab.6 details the positions of the prismatic joints for each trajectory and manipulator. It is observed that for trajectories I and II, the power consumption reduction achieved by the manipulator R5 is approximately the same as the one for manipulator R8. In addition, for trajectories I and II the chosen positions of the third prismatic joint δ_3 in R8 are closer to the zero value than δ_1 and δ_2 . Hence, for trajectories I and II, the prismatic joints δ_1 and δ_2 contribute more to the reduction of the total power consumption than the third prismatic joint δ_3 . On the other hand, the opposite phenomena is observed for trajectory III. In this case the manipulator R8 achieves a 45% of reduction in the power consumption with respect to R1, while R2 and R5 achieve just a 6%. Table 6 shows that the third prismatic joint δ_3 , that is only active in the manipulator R8, presents a higher value from the zero position than δ_1 and δ_2 . Hence, the power consumption reduction is mostly achieved by the third prismatic joint δ_3 in the case of trajectory III.

The numerical results present an acceptable similarity with the experiments. On the one hand, the pattern of the bar charts in Figs. 6.4, 6.5 and 6.6 presents the same trend between experimental and numerical cases. On the other hand, the absolute values of total energy consumption are slightly higher for the numerical case. In addition, the rate at which the energy consumption is reduced for each manipulator, is lower for the numerical case, as can be seen in Tab.5.

Figures.6.7 and 6.8 are referred to the energy consumption required by the manipulators R1, R2, R5 and R8 to execute the trajectories IV and V respectively. Both trajectories are special cases that deserve special mention. On the one hand, the trajectory IV represents the case in which the manipulator crosses a singularity during the execution for manipulators R5 and R8. On view of the experimental part of Fig.6.7, the presence of singularities does not affect to the power consumption, which is reduced around an 80% for manipulators R5 and R8 with respect to the non-redundant R1. However, vibrations in the end-effector are observed during the execution of the trajectory on the prototype. Even if this effect is not measurable in the actual prototype setup, it is verifiable at sight that these vibrations compromise the accuracy in the end-effectors pose. For the case of the numerical evaluation in Fig.6.7, it is observable that no data is displayed for the R5 and R8 cases. The numerical evaluation predicts actuation torques that leads to infinite due to the presence of the singular configuration. Hence, its power consumption leads to infinite in the same fashion. On the other hand, the Fig.6.8 represents a trajectory in which the addition of kinematic redundancy just achieved a reduction of 10% in the power consumption. For the numerical case the redundant manipulator presented higher values

of the total power consumed when compared to the non-redundant manipulator R1.

Figure 6.4: Comparative of the total power consumption of manipulators R1, R2, R5 and R8 between the numerical simulations and the experiments in the prototype regarding to execution of trajectory I.

Source: Elaborated by the author.

Table 5: Power consumption reduction rate for manipulators R2, R5 and R8 when compared to R1 for the three trajectories.

	Experimental				Numerical			
	R1	R2	R5	R8	R1	R2	R5	R8
I	0%	-20%	-55%	-54%	0%	-23%	-49%	-42%
II	0%	-28%	-59%	-62%	0%	-17%	-42%	-46%
III	0%	-6%	-6%	-45%	0%	+1%	+1%	-23%

Source: Composed by the author.

Figure 6.5: Comparative of the total power consumption of manipulators R1, R2, R5 and R8 between the numerical simulations and the experiments in the prototype regarding to execution of trajectory II.

Source: Elaborated by the author.

Figure 6.6: Comparative of the total power consumption of manipulators R1, R2, R5 and R8 between the numerical simulations and the experiments in the prototype regarding to execution of trajectory III.

Source: Elaborated by the author.

Table 6: Positions of the prismatic joints δ_i for trajectories I, II and III.

$\delta_i(\text{m})$	R1	R2	R5	R8
$\delta_{1\text{I}}$	0	-0.171	-0.171	-0.171
$\delta_{2\text{I}}$	0	0	0.1285	0.1286
$\delta_{3\text{I}}$	0	0	0	0.0893
$\delta_{1\text{II}}$	0	-0.191	-0.191	-0.190
$\delta_{2\text{II}}$	0	0	0.1623	0.1613
$\delta_{3\text{II}}$	0	0	0	0.0576
$\delta_{1\text{III}}$	0	-0.054	-0.058	-0.057
$\delta_{2\text{III}}$	0	0	0.011	0.012
$\delta_{3\text{III}}$	0	0	0	-0.138

Source: Composed by the author.

Figure 6.7: Comparative of the total power consumption of manipulators R1, R2, R5 and R8 between the numerical simulations and the experiments in the prototype regarding to execution of trajectory IV.

Source: Elaborated by the author.

Figure 6.8: Comparative of the total power consumption of manipulators R1, R2, R5 and R8 between the numerical simulations and the experiments in the prototype regarding to execution of trajectory V.

Source: Elaborated by the author.

6.2 Current Profiles

The redundancy resolution scheme used in this manuscript belongs to the pre-positioning type, *i.e.* the prismatic joints are positioned prior to the execution of the trajectory, and then, they are fixed in the optimal position during the task execution. As a consequence, the power consumption of the prismatic joints during the execution is close to zero. Hence, the majority of the energy is consumed by the motors responsible for the active revolute joints depicted as M1, M2 and M3 in Fig. 5.3.

Figure 6.9 illustrates the current profiles for the experimental case of manipulators R1 and R8 during the execution of trajectory I. A reduction in the current profiles of motors M1 and M2 of manipulator R8 with respect to manipulator R1 can be observed. On the contrary, the current profile of motor M3 remains similar for the two manipulators.

Figure 6.10 illustrates the current profiles for the numerical case of the manipulators R1 and R8 during the execution of trajectory I. As for the experimental case, the current profiles have been reduced causing a reduction in the total power consumption. However, for the numerical case the motors M3 and M2 are the ones that present the higher reduction. As a result, differences in the shapes of the current profiles can be observed when comparing to the experimental case.

Figure 6.11 illustrates the current profiles for the experimental case of manipulators R1 and R8 during the execution of trajectory II. In a similar manner to the case of trajectory I, a large amount of the reduction in the current profiles is achieved by motors M1 and M2, while the motor M3 remains barely unchanged for both manipulators R1 and R8. Analogously, for the numerical case in Fig. 6.12, the motors M2 and M3 present a reduction in the current profile while the current profile of motor M1 remains unaltered. Again, differences can be perceived between the numerical and experimental cases.

Figure 6.13 illustrates the current profiles for the experimental case of manipulators R1 and R8 during the execution of trajectory III. In this case, the reduction in the current profiles is much more pronounced for motor M3 when comparing the manipulator R8 to the R1. The motors M1 and M2 remains almost unaltered for the two manipulators. The numerical case of the current profiles for trajectory III is illustrated in Fig. 6.14. For this case, one can observe that the data obtained from the numerical and experimental cases are similar.

Figure 6.15 illustrates the current profiles for the experimental case of manipulators R1 and R8 during the execution of trajectory IV. For this case, a pronounced reduction in the motors M1 and M2 is observable when comparing the manipulator R8 to the R1.

However, in the numerical case, the Fig.6.16 presents an outstanding value at time 0.13 seconds. At that point, the manipulator R8 is adopting a singular configuration, which leads to high values of the actuator currents. This effect is not noticeable in the current profiles of the experimental case as the actuators can not handle high values of the driving current. However, a deviation from the desired end-effector path is observable looking at the execution in the prototype.

Figures 6.17 and 6.18 illustrate the current profiles for the experimental and numerical cases of manipulators R1 and R8 during the execution of trajectory V respectively. For this trajectory, the current profiles present a similar form for both cases. This result is in concordance with the previous results obtained for the power consumption, that show no notable reduction for any of the manipulators with kinematic redundancy.

Figure 6.9: Experimental current profiles of rotative actuators M1, M2 and M3 during the execution of trajectory I. (a) R1 and (b) R8.

Source: Elaborated by the author.

Figure 6.10: Numerical current profiles of rotative actuators M1, M2 and M3 during the execution of trajectory I. (a) R1 and (b) R8.

Source: Elaborated by the author.

Figure 6.11: Experimental current profiles of rotative actuators M1, M2 and M3 during the execution of trajectory II. (a) R1 and (b) R8.

Source: Elaborated by the author.

Figure 6.12: Numerical current profiles of rotative actuators M1, M2 and M3 during the execution of trajectory II. (a) R1 and (b) R8.

Source: Elaborated by the author.

Figure 6.13: Experimental current profiles of rotative actuators M1, M2 and M3 during the execution of trajectory III. (a) R1 and (b) R8.

Source: Elaborated by the author.

Figure 6.14: Numerical current profiles of rotative actuators M1, M2 and M3 during the execution of trajectory III. (a) R1 and (b) R8.

Source: Elaborated by the author.

Figure 6.15: Experimental current profiles of rotative actuators M1, M2 and M3 during the execution of trajectory IV. (a) R1 and (b) R8.

Source: Elaborated by the author.

Figure 6.16: Numerical current profiles of rotative actuators M1, M2 and M3 during the execution of trajectory IV. (a) R1 and (b) R8.

Source: Elaborated by the author.

Figure 6.17: Experimental current profiles of rotative actuators M1, M2 and M3 during the execution of trajectory V. (a) R1 and (b) R8.

Source: Elaborated by the author.

Figure 6.18: Numerical current profiles of rotative actuators M1, M2 and M3 during the execution of trajectory V. (a) R1 and (b) R8.

Source: Elaborated by the author.

6.3 Position Profiles

Figure 6.19 illustrates the prototype's position profiles of the active revolute joints during the execution of trajectory I for the non-redundant manipulator R1 and the redundant manipulator R8. When comparing both cases, it can be seen that the profiles described by the actuators of the manipulator R8 are shorter than the ones of the manipulator R1. The shortening is more evident for the motors M1 and M2. Similar results are observed for the case of trajectory II in Fig. 6.20.

Figure 6.21 illustrates the prototype's position profiles of the active revolute joints during the execution of trajectory III for the non-redundant manipulator R1 and the redundant manipulator R8. In this case, it can be observed that the shortening of the path was clearly for motor M3 while the motors M1 and M2 remains with barely the same path length.

Figure 6.23 illustrates the prototype's position profiles of the active revolute joints during the execution of trajectory IV for the non-redundant manipulator R1 and the redundant manipulator R8. For this case, the path was shortened for the case of motors M1 and M2. However, from the current profiles analysis, it is known that the manipulator R8 adopts a singular configuration during the execution of the trajectory IV. This leads to the execution of a different trajectory and hence, the conditions for proving the reduction in the path profiles are not valid anymore.

Figure 6.23 illustrates the prototype's position profiles of the active revolute joints during the execution of trajectory V for the non-redundant manipulator R1 and the redundant manipulator R8. In the same fashion as the previous results, the position profiles do not experience a noticeable change in any of the motors for this case as the kinematic redundancy did not achieved a noticeable improvement.

Figure 6.19: Experimental positions profiles of rotative actuators M1, M2 and M3 during the execution of trajectory I. (a) R1 and (b) R8.

Source: Elaborated by the author.

Figure 6.20: Experimental positions profiles of rotative actuators M1, M2 and M3 during the execution of trajectory II. (a) R1 and (b) R8.

Source: Elaborated by the author.

Figure 6.21: Experimental positions profiles of rotative actuators M1, M2 and M3 during the execution of trajectory III. (a) R1 and (b) R8.

Source: Elaborated by the author.

Figure 6.22: Experimental positions profiles of rotative actuators M1, M2 and M3 during the execution of trajectory IV. (a) R1 and (b) R8.

Source: Elaborated by the author.

Figure 6.23: Experimental positions profiles of rotative actuators M1, M2 and M3 during the execution of trajectory V. (a) R1 and (b) R8.

Source: Elaborated by the author.

6.4 Economical Study

In the experimental results section, it was shown that the energy consumption required by the planar parallel manipulator 3RRR during the execution of selected tasks can be reduced by the use of additional actuators in the form of kinematic redundancy. The presence of kinematic redundancy demands the use of additional components as actuators, lead screws, processors and power supplies that imperatively increases the manipulator's costs. Hence, the question here is whether or not the increased expenses are lower than the incomes generated by the energy savings. In order to address this dilemma, it is presented an economical study that intends to emulate an scenario in which a machine executes a task repeatedly over a long period of time. This situation is commonly found in the manufacturing industry as a part of an entire process such as a car assembly chain.

For sake of illustration, a repetitive task is selected for this study. This task is the trajectory II depicted in Fig. 6.2. The manipulators to be compared in this case are the R1 and R8 which consume 68 mW and 26 mW during the execution of the trajectory II, respectively. It is supposed that the executions are repeated during 18 hours per day during one year. As the the execution takes 0.5 seconds, each manipulator can perform it 7200 times during one hour. The average price of the kWh varies according to the country. For instance, in Germany, the average price of the kWh is \$0.30, while in China is \$0.08. In this manuscript, the price of the kWh is considered to be \$0.25.

Table 7 enumerates the approximate costs of the manipulator components. With this information, it is possible to compute the cost of the manipulators under investigation. The manipulator R1 has three actuators, three control boards, three power sources and three gear heads, hence its cost results in 3840 \$. On the other hand, the manipulator R8 possesses six actuators, six control boards, six power sources, six gear heads and three linear guides, resulting in a total price of 9570 \$. Table 8 summarizes the results

Table 7: Costs of the Manipulator Components.

Actuator	Linear Guide	Control Board	Power Source	Gear Head
300 \$	630 \$	630 \$	100 \$	250 \$

Source: Elaborated by the author.

of the economical study. It can be seen that the redundant manipulator R8 is able to reduce the total cost in electrical energy by approximately 500 \$ a year. The cost of the manipulator R8 is 6351 \$ higher than the price of the manipulator R1. Therefore, the additional investment introduced by the manipulator R8 can be recovered in 12 and a half years, approximately.

Table 8: Results of the Economical Study

Manipulators	R1	R8
Consumption during one hour (kWh)	0.490	0.187
Consumption during one year (kWh)	9570	3219
Cost of Manipulator (\$)	3840	11460
Cost of electricity per year (\$)	804	307

Source: Elaborated by the author.

7 CONCLUSIONS AND DISCUSSIONS

The number of applications suited for robotic manipulators are steadily increasing. Innovative designs have been investigated for improving the system's reliability and versatility, for reducing the energy consumption, for improving the dynamic performance, among others. The reduction of energy consumption yields the reduction of the costs and the improvement of the environmental sustainability.

The main objective of this work was to evaluate the effects of the kinematic redundancy in the energy efficiency of parallel manipulators. A parallel manipulator prototype with three levels of kinematic redundancy 3PRRR, along with its dynamical model was used to assess experimental and numerical data for this evaluation. The presence of kinematic redundancy on parallel manipulators leads to a infinite number of solutions in the inverse kinematics problem. Hence, the use of a redundancy resolution scheme to determine the best solution according to a given criteria is required. On this matter, a novel redundancy resolution scheme based on the model predictive control technique was proposed. The energy consumption of non-redundant and redundant manipulators has been compared during the execution of different tasks. The redundant manipulators could present 1, 2 and 3 levels of kinematic redundancy. The kinematic redundancy was included by the actuation of the prismatic joints. The MPC based redundancy resolution scheme was used in such a way that inputs of the active joints are minimal for a predefined end-effector's trajectory.

In summary, this research attempted to contribute in the field of parallel manipulators by answering two main questions:

- Is the kinematic redundancy a good alternative to increase the energy efficiency of parallel manipulators?
- Can the model predictive control formulation be used effectively as a redundancy resolution scheme for parallel manipulators with kinematic redundancy?

This chapter is divided as follows. Firstly, the main findings are presented and discussed. Secondly, the previous findings are contrasted with the state-of-the-art. In the third section, the limitations inherent to this study are discussed and presented. The fourth section indicates interesting open research topics in this field. Finally, the contributions of this research are presented.

7.1 Main Findings

Several findings have been derived from this research. In this section, the main findings regarding the experimental and numerical analysis, the economical study and the proposed redundancy resolution scheme are presented.

7.1.1 Experimental Analysis Discussion

The most relevant result regarding the experimental analysis showed that the parallel manipulators with kinematic redundancy were able to reduce the power consumption during the execution of the same task when compared to the non-redundant one. It was also observed that the rate at which the power consumption is reduced depends on the predefined end-effector trajectory. According to the information summarized in Tab.5, in the trajectories I and II when one level of redundancy was added (manipulator R2), the reduction rate was 20% and 28%, respectively. When two levels of redundancy were added (manipulator R3), the rate becomes 55% and 59%, respectively. When three levels of redundancy were added (manipulator R4), the rate became 54% and 62%, respectively. Both manipulators R3 and R4 presented similar rates of reduction. So, it can be concluded that for trajectories I and II the reduction was mainly achieved by the first and second redundancies while the addition of the third redundancy did not contribute to the power consumption reduction in a considerable fashion. On the contrary, for the trajectory III, the major reduction in the energy consumption was achieved when the third redundancy was included. One can conclude that the contribution of the different levels of redundancy to the power consumption reduction depends on the specific trajectory executed by the end-effector. The previous conclusion can be also verified when the current and position profiles of the active revolute joints of the non-redundant manipulator R1 and the three times redundant manipulator R4 (see Sections 6.2 and 6.3) are compared. It can be seen that for trajectories I and II, the profiles of the motors 1 and 2 are reduced while the profile of the motor 3 remains practically unaltered. The contrary effect is noticeable for the trajectory III, where the profiles of the motors 1 and 2 did not change, while the current profile of motor 3 presented a significant rate of reduction.

The required energy consumption for execute the trajectories I, II and III was reduced by means of the kinematic redundancy and the proposed redundancy resolution scheme. On the other hand the results related to the trajectory V exhibit no remarkable reduction of the power consumption for any of the manipulators with kinematic redundancy. Hence, it is fair to conclude that the kinematic redundancy collectively with redundancy resolution scheme are able to reconfigure the redundant manipulator to increase their energy efficiency but not for all trajectories. Thus, it is important to carefully choose the end-effector trajectory in order to take fully advantage of the kinematic redundancy and the proposed

redundancy resolution scheme in terms of energy efficiency.

7.1.2 Numerical Comparative Discussion

The same conclusions drawn considering the experimental data can be observed for the numerical case. However when comparing both cases certain discrepancies were observed. The main discrepancies were noticed in the values of the total power consumption and in the current profiles. Two major reasons can be the responsible for the aforementioned discrepancies. On the one hand, some manipulator's parameters, such as the masses and inertial moments of the components, have been roughly estimated. In fact, some values have been attributed in an attempt to improve the numerical and experimental data correlation. On the other hand, the communications between the prototype and the computer interface presents limitations relative to the data acquisition speed.

7.1.3 Economical Study Discussion

An economical evaluation was carried out by investigating a scenario in which two planar parallel manipulators, a non-redundant (R1) and a redundant (R4) ones, must execute a repetitive task during a period of one year. At the end of the year, the total costs in terms of energy consumption were compared with the cost of the manipulator components in order to evaluate the viability of the proposal. It was concluded that the redundant manipulator was able to reduce the cost in electrical energy by approximately 500 dollars per year. Since, the cost of a redundant manipulator was estimated to be 6351 dollars higher than the non-redundant, the additional investment could be recovered in approximately 12 years. After that period, a decrease of 500 dollars in the total cost of the process due to energy saving would be achieved per year.

The assessment of the proposal viability is a rather complicated task since 12 years for recovering the initial investment can be a long or a short period depending on the company and the technology involved. The numbers obtained also can vary depending on the electricity and component costs. In an scenario in which the electricity costs are higher and the component costs are lower, the use of kinematic redundancy is much more beneficial as the initial investment can be recovered in a shorter period. This scenario can be found in more developed countries as Germany or the United States. On the other hand, if the electricity costs are lower and the component costs are higher, the initial investment will be recovered in a longer period making the viability of the proposal to be doubtful, this situation arises in countries under development as India.

7.1.4 Redundancy Resolution Scheme Discussion

In this work, a Redundancy Resolution scheme based in the Model Predictive Control (MPC) technique was proposed. The method was exploited to find the optimal

inputs of the active prismatic joints that minimizes the motion to be executed by the active revolute joints. The position profiles illustrated in Figs. 6.19, 6.20 and 6.21 verified that the MPC based redundancy resolution scheme was able to shorten the motion of the active revolute joints (the rotative actuators) for the redundant manipulator. Due this result, it can be concluded that the method can be exploited as a redundancy resolution scheme.

7.2 Relation with Previous Research

The benefits of kinematic redundancy in parallel manipulators have been studied previously by several researchers. With this respect, this work presents an additional benefit that can be achieved by the inclusion of kinematic redundancies since it can be an alternative for improving the energy efficiency of parallel manipulators. This benefit is strongly related to the enhancement of the dynamic performance achieved by the reduction of torques shown in Fontes and Silva (2016), by the improvement of the pose accuracy discussed in Kotlarski, Abdellatif and Heimann (2008), or by the better motion/force transmissibility proved by Xie, Liu and Wang (2011).

In previous works, the MPC has been used in the field of robotics for control the trajectory tracking and navigation of wheeled mobile robots (KÜHNE; LAGES; SILVA, 2004), and the motion of industrial robots (INCREMONA; FERRARA; MAGNI, 2017). This work updates the state of the art of this technique by presenting a redundancy resolution scheme.

The MPC based redundancy resolution scheme proposed in this text presents a formulation limited to the use of the inverse kinematics, which is compelling for the purpose of this research. However, it can not be used to avoid motion near singularities, which is a common issue in parallel manipulators. The avoidance of singularities requires the use of some sort of Jacobian related indexes such as proposed by Kotlarski et al. (2009b) and Niemann et al. (2013). Other redundancy resolution schemes exploit a more complex formulation that allows the use of the Jacobian, the partial velocities/accelerations and the dynamic model to minimize dynamical indexes related with driving torques as discussed by Fontes, Santos and Silva (2014) or Santos et al. (2015).

7.3 Limitations of the Research

The first important limitation is related with the nature of the cost function. As have been said, the number one objective is to reduce the required energy consumption of the manipulator by means of the kinematic redundancy. However the power consumption model is not represented entirely in the associated cost function of the redundancy resolution scheme. In stead, the cost function is only dependent of the actuators velocities

and does not deal with the driving torques. Hence, the cost function does not directly reduce the power consumption in all cases. This explains why in some cases, to increase the number of redundant actuators leads to slightly higher power consumption. For instance the manipulator R8 with three levels of redundancy consumes more than the manipulator R5 with just two levels as can be seen in Fig.6.4.

The second more important limitation is related to singularities. When robotic manipulators fall in singular configuration, the behavior of the manipulator becomes erratic. This can lead to bad performance or damages in the robot and its surroundings. The effect of singular configurations are even more outstanding for the case of parallel manipulators due to their limited workspace. The proposed redundancy resolution scheme does not introduce any feature for singularity avoidance. Due to the singularity related problems of parallel manipulator and, on view of the results related to the trajectory IV, it is required to include a dedicated singularity avoidance formulation in the current redundancy resolution scheme.

In addition, the findings of this research are restricted to the speed at which the data is transmitted from the prototype to the computer. In spite of the speed of the present communication scheme being acceptable for this research, a system with a faster data acquisition rate could help to obtain more accurate and noise free sets of data.

Moreover, the current implementation of the MPC as redundancy resolution scheme is limited to the inverse kinematics equations. For the current formulation of the technique, the inclusion of constraints based in more advanced models would result in an excessive computational cost. Nevertheless, the technique was suited for this case of study as the trajectories to be executed were uncomplicated.

It is also important to note that the numerical analysis was limited by the approach used to identify the parameters used in the prototype. This imposes a limitation to the accuracy of the system mathematical model.

Finally, this research is limited to the evaluation of the power consumption of the manipulator's actuators. There is no reference to the energy consumption of any additional components such as controllers or power sources.

7.4 Future Recommendations

As previously commented, the current implementation of the MPC as redundancy resolution scheme is limited to the inverse kinematics problem. Future implementations of this technique could include more advanced models for the constraints in order to fit the dynamic model and the fully energy consumption model in the cost function. Moreover, the redundancy resolution scheme used a pre-positioning approach, *i.e* the redundant actuators

must be firstly moved to the optimal positions, and then, they must be locked during the task execution. Further investigation must be done using the on-going positioning strategy as it has shown improved results when applied on numerical simulations in previous works. On this matter, the use of lead screws with higher leads for the redundant actuators can help to perform larger translational movements with shorter turns of the rotational actuators. Therefore, the consumption of the redundant actuators could be reduced.

Some differences between the numerical and experimental data could be reduced if a parameter identification procedure is exploited for improving the correlation between this data. It is important to note that this identification procedure should include redundancy resolution schemes since the system should be excited by relevant input sets.

One of the most important parameters for evaluating the performance of a robotic manipulator is the accuracy. In the field of robotics, the accuracy can be defined as the parameter that determines how close the end-effector trajectory is to the desired trajectory. No investigation about the influence of the proposal in the accuracy of the manipulators was carried out in this work. As for any practical application the accuracy is an essential parameter, future researches must include techniques to evaluate the accuracy of the used prototype and its relationship with the kinematic redundancy and the proposed redundancy resolution schemes.

7.5 Contributions

Generally, this work contributes to the state of the art of robotic manipulators by proposing the use of new manipulator's kinematic architectures that can be more efficient to specific applications than the classical architectures.

The main objective of this work was accomplished as it has been showed that the presence of kinematic redundancy in planar parallel manipulators can lead to a reduction of the total required power consumption to perform a task. Hence, it can be concluded that the use of kinematic redundancy can be an alternative for increasing the energy efficiency of PKMs.

The MPC technique is commonly found in the control of a wide variety of industrial processes. The technique was applied with success to the case under study. The proposed method supplies a novel use for the MPC: the redundancy resolution scheme. Moreover, as the MPC is already a common tool for control processes in the industry, its use as redundancy resolution scheme can be beneficial for the adaptation of parallel manipulators to the industrial sector.

At last but not least, the concepts and proposals presented in this work aimed to reduce

the ecological fingerprint imposed by human activities to the planet.

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