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Estudo de juntas em material compósito
soldadas por resistência elétrica:
da fabricação ao comportamento mecânico

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Study of resistance welded composite joints:
from the manufacturing process to the mechanical behaviour

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I dedicate this thesis to my family.

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Abstract

Angélico, R. A. (2013). *Study of resistance welded composite joints: from the manufacturing process to the mechanical behaviour*. São Carlos (2013). Thesis (Doctorate). São Carlos School of Engineering, University of São Paulo & Ecole Normale Supérieure de Cachan.

This study is dedicated to thermoplastic composite joints obtained by an electrical resistance welding procedure. This welding process consists in joining two substrates with an electrical resistor which acts as a heating element melting the polymer substrates. The substrates considered herein are 2mm thick 7-layer hybrid composites, with the following stacking sequence $([0^\circ/90^\circ]G, [0^\circ/90^\circ]C, [45^\circ]C, [0^\circ/90^\circ]C, [45^\circ]C, [0^\circ/90^\circ]C, [0^\circ/90^\circ]G)$, where G and C denote plies with PPS matrix reinforced by continuous glass or carbon fibres, respectively. The heating element is a stainless metallic grid surrounded by two PPS amorphous films. For a better understanding of the time evolution of the temperature field in the welded zone, a heat transfer model was developed in finite element code Abaqus[®]. The prediction capabilities of the numerical tool were validated by comparing the numerical results with thermocouple measurements. The thermal properties required by the finite element model, viz. the specific heat and the thermal conductivities, were identified from DSC tests and from an inverse identification procedure, respectively. The inverse identification procedure is based on a Levenberg-Marquart algorithm applied to the analysis of specific experiments instrumented with thermocouples and an infra-red camera. Thermal or/and mechanical analyses of anisotropic composite laminates can lead to high computational costs even for linear analyses. Proper Generalized Decomposition constitutes a promising tool to reduce computational costs for multi-dimensional problems such as multi-parametric problems typical of manufacturing process simulations and/or problems with different length scales typical of composite laminates. To demonstrate its capabilities and its efficiency –including in terms of computation costs for small size problems– PGD technique is applied to the solution of an axisymmetric heat transfer problem. Specimens were manufactured (with a laboratory welding machine designed and built during this study) with different processing parameters –heating element geometry, intensity of the electrical current, time evolution of the pressure. DCB specimens were tested to characterize the mechanical toughness under mode I. The analysis with the compliance method of the tests results exhibits two non-negligible energy dissipation mechanisms, related to crack creation and localized plastic deformation, respectively. An original model developed within the internal variable thermodynamics framework is proposed and used to describe the R-curves representative of the ductile behaviour of the DCB specimens. A first sensitivity analysis of the processing parameters on the joint fracture toughness exhibits the key role of the pressure applied onto the joint during the cooling phase of the welding process.

Keywords: composite joints, resistance welding, thermoplastic composites.

Resumo

Angélico, R. A. (2013). *Estudo de juntas em material compósito soldadas por resistência elétrica: da fabricação ao comportamento mecânico*. São Carlos (2013). Tese (Doutorado). Escola de Engenharia de São Carlos, Universidade de São Paulo & Ecole Normale Supérieure de Cachan.

Este estudo é dedicado a juntas de compósitos termoplásticos soldadas pelo processo de soldagem por resistência elétrica. Este processo consiste em unir dois substratos com um resistor elétrico que atua como um elemento de aquecimento que funde o polímero dos substratos. Os substratos considerados neste trabalho são laminados compósitos híbridos, constituídos de 7 camadas que totalizam 2 mm de espessura, com a seguinte sequência de empilhamento $([0^\circ/90^\circ]G, [0^\circ/90^\circ]C, [45^\circ]C, [0^\circ/90^\circ]C, [45^\circ]C, [0^\circ/90^\circ]C, [0^\circ/90^\circ]G)$, onde G e C denotam camadas de PPS reforçadas com fibra de vidro ou carbono, respectivamente. O elemento de aquecimento utilizado é uma malha metálica de aço inoxidável entre dois filmes de PPS (amorfo). Para um melhor entendimento do histórico do campo de temperatura na região soldada, um modelo de transferência de calor foi desenvolvido no pacote de elementos finitos Abaqus®. As capacidades de previsão de temperatura do modelo computacional foram validadas a partir da comparação com resultados experimentais de termopares. As propriedades térmicas do modelo em elementos finitos, viz. o calor específico e as condutividades térmicas, foram identificadas a partir de ensaios DSC e de um procedimento de identificação inverso, respectivamente. O procedimento de identificação inversa foi baseado no algoritmo de Levenberg-Marquart aplicado na análise de experimentos específicos instrumentados com termopares e com uma câmera infra-vermelha. A análise térmica ou/e mecânica de laminados compósitos anisotrópicos podem apresentar elevados custos computacionais, mesmo para análises lineares. A técnica PGD (*Proper Generalized Decomposition*) é uma ferramenta promissora na redução de custos computacionais de problemas multidimensionais, típicos de simulação do processo de manufatura, e/ou problemas multi-escalas, típico de laminados compósitos. Para demonstrar sua capacidade e sua eficiência, a técnica PGD é aplicada na solução de um problema axissimétrico de transferência de calor. Corpos-de-prova foram fabricados (com a máquina de soldagem laboratorial desenvolvida e construída durante este estudo) com diferentes parâmetros de processamento -geometria do elemento de aquecimento, intensidade da corrente elétrica, histórico de pressão. Corpos-de-prova DCB foram testados para caracterizar a resistência mecânica à propagação de trinca em modo I. A análise com o método da flexibilidade dos resultados mostram dois mecanismos predominantes de dissipação de energia, correlatos com a criação da trinca e a localização de deformação plástica, respectivamente. Um modelo original desenvolvido baseado nas variáveis internas termodinâmicas é proposto e usado para descrição das curvas-R representativas do comportamento dúctil dos corpos-de-prova DCB. Uma primeira análise de sensibilidade da resistência à fratura ao variar os parâmetros de processamento mostra que a pressão aplicada na junta durante a etapa de resfriamento desempenha papel fundamental na resistência final da junta.

Palavras-chave: juntas compósitas, soldagem por resistência, compósitos termoplásticos.

Resumé

Angélico, R. A. (2013). *Étude des joint composite soudé par résistance électrique: du procédé de fabrication au comportement mécanique*. São Carlos (2013). Thèse (Doctorat). Ecole d'Ingenierie de São Carlos, Université de São Paulo & Ecole Normale Supérieure de Cachan.

Cette étude est consacrée à l'assemblage par joints de plaques composites thermoplastiques obtenus par le procédé de soudage par résistance électrique. Ce procédé consiste à assembler deux plaques par l'intermédiaire de deux films polymères qui fondent au contact d'une résistance électrique chauffante. Les plaques étudiées sont des plaques composites de 2 mm d'épaisseur. Elles sont constituées de 7 couches hybrides, avec la séquence d'empilage $[0^\circ/90^\circ]G$, $[0^\circ/90^\circ]C$, $[45^\circ]C$, $[0^\circ/90^\circ]C$, $[45^\circ]C$, $[0^\circ/90^\circ]C$, $[0^\circ/90^\circ]G$, où G et C représentent la matrice de PPS renforcé soit par des fibres de verre continues, soit par des fibres de carbone. L'élément chauffant est une grille métallique en acier inoxydable placée entre deux films amorphes PPS. Une machine de soudage de laboratoire instrumentée a été développée pour fabriquer des éprouvettes et étudier l'influence de certains paramètres du procédé de soudage sur les propriétés finales de l'assemblage. Pour acquérir un meilleur contrôle de l'évolution du champ de température dans la zone soudée lors de l'opération de soudage, un modèle numérique de transfert de chaleur a été développé dans le code d'éléments finis Abaqus[®]. La capacité de prévision de l'outil numérique a été validée en comparant les résultats numériques avec les mesures de thermocouples placés à diverses positions du joint de soudage. La chaleur spécifique a été identifiée à partir d'essais de calorimétrie différentielle à balayage (DSC). Les conductivités thermiques de la plaque supposée isotrope transverse ont été identifiées à partir d'essais de diffusion de la chaleur dans une plaque instrumentée avec des thermocouples et une caméra infra-rouge soumise dans une zone centrale à une source thermique. La procédure d'identification inverse des conductivités thermiques est basée sur un algorithme de Levenberg-Marquart. Les analyses thermiques et/ou mécaniques des stratifiés composites anisotropes peuvent conduire à des coûts de calcul élevés, même pour les analyses linéaires. La technique Proper Generalized Decomposition (PGD) est un moyen pour réduire les coûts de calcul des problèmes multidimensionnels, tels que des problèmes multi-paramétriques typiques de processus de fabrication, et/ou des problèmes impliquant différentes échelles, observés typiquement dans les stratifiés composites. Cette technique a été utilisée pour résoudre le problème de transfert de chaleur dans la plaque et son potentiel et son efficacité prouvés. Les éprouvettes servant à caractériser le comportement mécanique du joint ont été fabriquées en faisant varier certains paramètres (géométrie de la grille métallique, intensité du courant électrique, évolutions temporelles du chargement thermique et de la pression appliquée durant l'opération d'assemblage). Des éprouvettes de type Double Cantilever Beam (DCB) ont été testées pour caractériser la résistance mécanique à la rupture en mode I. L'analyse des résultats expérimentaux par la méthode de la compliance a permis de mettre en évidence deux mécanismes de dissipation d'énergie non négligeables, liés d'une part à la création de fissure et d'autre part à la déformation plastique localisée. Un modèle original a été développé dans le cadre de la thermodynamique des processus irréversibles à variables internes et a été utilisé pour décrire la courbe R représentative du comportement «ductile» des éprouvettes DCB. Une première analyse de la sensibilité des paramètres de processus sur la ténacité à la rupture a montré que la pression appliquée sur le joint lors de la phase de refroidissement du processus de soudage joue un rôle clé sur la tenue du joint.

mots clés: joint composite, soudage par résistance, composite thermoplastique.

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Notation

a_1, a_2, b_1, b_2 : constants
 A : crack surface
 A_i, B_i, C_i : constants
 A_p : plastic internal variable
 c : specific heat
 d : mesh wire diameter
 $\mathcal{D}$: intrinsic dissipation
 E : Young modulus
 E_1, E_2 : Young modulus at direction 1 and 2, respectively
 $\bar{E}$: total energy density
 $f, g, \mathcal{F}, \mathcal{H}$: functions
 G : energy strain release rate
 G_{12} : Shear modulus
 h : convection coefficient
 h_{cr} : equivalent radiative-convective coefficient
 h_b, h_{down} : convection at bottom surface
 h_t, h_{up} : convection at top surface
 k_1, k_2, k_3 : thermal conductivities at directions 1, 2 and 3, respectively
 $\mathbf{K}$: thermal conductivity tensor
 I : inertia
 $\mathcal{I}$: time domain
 i : electrical current, index
 $\mathbf{J}$: Jacobian matrix
 l, w : dimensions
 p : mesh step
 $\mathbf{p}$: set of parameters
 P : electrical power
 $\bar{P}$: mean electrical power
 $\mathcal{R}$: radial direction function
 Ra : Rayleigh number

R_{eq} : electrical resistance

S : mesh open surface

q : heat flux

Q : external load

t : time

$\mathcal{T}$: time function

T : temperature

T_g : glass transition temperature

T_m : melting temperature

v : displacement

W_{ext} : external work

$\mathcal{Z}$: z-direction function

$\varepsilon_1, \varepsilon_2, \varepsilon_{12}$: strain components

γ_{12} : shear strain

ν_{12}, ν_{21} : Poisson ratio

Ω : space domain

ϕ : finite element shape functions

Ψ : free energy

Ψ_e : elastic energy contribution

Ψ_p : plastic energy contribution

ρ : density

σ : standard deviation

$\sigma_1, \sigma_2, \sigma_{12}, \sigma_x$: stress components

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CHAPTER 1

Introduction

Contents

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1.1 Composite materials and their joints

Composites are materials combining two or more materials (of same or different nature: ceramic, polymeric and metallic) in order to obtain a new material with improved properties when compared to those of the isolated phases (Daniel and Ishai, 1994; Callister and Rethwisch, 2012; Chawla, 2012). Suitable combinations of several phases can provide materials with high specific properties, i.e. high ratio between mechanical properties such as strength or stiffness and density. This kind of material is of great interest for high performance structures, to be used for instance in the aeronautical industry. For aeronautical components, weight saving results in several benefits such as long range,

1 Introduction

smaller fuel consumption, lower emissions, etc. This study is devoted to the assembly of laminated composites constituted of a polymeric matrix (polyphenylene sulphide, PPS) reinforced by continuous carbon or glass fibres.

Polymeric matrices can be classified into thermosetting or thermoplastic. The usage of continuous fibre embedded in a thermosetting polymeric matrix in aeronautical structures dates from the beginning of the 70's (Niu, 1992). It started with ACEE (Aircraft Energy Efficiency) program coordinated by NASA (National Aeronautical and Space Administration). In that project, three primary and secondary structures were manufactured using composite materials with thermosetting matrix and were compared to the original monolithic structures. It is recalled that primary structures are those which would endanger the aircraft upon failure, while secondary structures are those which do not induce imminent risk to aircraft even in case of failure. These new materials should support and transfer the load to adjacent structures without affecting the aircraft airworthiness. It is worth noticing that no mention of the in-service lifetime was made during that project. These studies resulted in a weight reduction of 30% (Niu, 1992) compared to the original metallic structures, showing the potentialities of these materials in the replacement of some metallic components.

Reinforced thermoplastic composites have been used since the beginning of 90's. Advances in thermoplastic matrices technologies have enabled their use in high performance composite structures, increasing their use in aeronautical and other (automotive, civil, energy) industries. Nowadays, thermoplastic matrices exhibit interesting properties for these industries, such as recyclability, good mechanical and chemical performances, low volatile organic emissions during manufacturing and shorter processing time (Schell et al., 2009). Thanks to these properties, thermoplastic composite became an option to substitute metallic or thermosetting composite components for some products (Dubé et al., 2008). It exists a large literature on this subject, for example, van Hattum and van Breugel (2001) who presented and discussed some advantages and potentialities of long-fibre reinforced thermoplastic composites.

Owing to the manufacturing process, thermoset and thermoplastic composites cannot be used to obtain large complex geometries contrary to machined monolithic metallic structures. Thus, it is necessary to use joining techniques to assemble several components and produce large and complex structures. In general, thermoset composites components are joined by mechanical fastening or adhesive bonding (Collings and Kelly, 1994). Both techniques can also be applied to thermoplastic composites. However, these composites can also be welded thanks to the polymeric ability to melt and recrystallize (Ageorges and Ye, 2001). Considering this scenario shown above, this study is

dedicated to the application of a welding technique to PPS matrix composites. As it is known, joining techniques may introduce undesirable stress concentrations and/or material degradation so that it is very important to study the joining process and investigate how it can influence the mechanical resistance of the structure assembly.

Mechanical fastening is a flexible process which implementation into the industrial environment is easy in comparison to other joining techniques. An abundant literature is dedicated to this assembly process. Stress analysis of mechanical fastened joints was reviewed in details by Camanho and Matthews (1997). A review of mechanical fastened composite joints was also presented by Thoppul et al. (2009). However, besides extensive work and excessive time requirements, mechanical fasteners have the disadvantage of introducing stress concentrations, delamination during drilling, water intrusion, different thermal conductivities, weight increases and galvanic corrosion (Ageorges and Ye, 2001).

Adhesive bonding presents a more favourable stress distribution because of the major contact surface when compared to the mechanical fastening. In this assembly technique, the surface preparation is a key point which has great influence on the mechanical behaviour of the joint (Wingfield, 1993; Kanerva and Saarela, 2013; Katnam et al., 2013). Surface treatments generally employ chemical reagents to remove contaminants and to increase the surface energy, consequently, the adhesion. However, implementing these techniques into the industrial environment can be a challenge because of the accurate quality control this type of joints require (Stavrov and Bersee, 2005).

Inherent melting and consolidation properties of thermoplastic matrix enable the use of fusion bonding process to join thermoplastic composite parts. Fusion bonding process is interesting because it requires a minor surface preparation, compared to adhesive bonding techniques, and a short assembly processing time (Ageorges and Ye, 2001). This technique is detailed in next chapter.

1.2 Industrial applications

The manufacturing of thermoplastic composite aeronautical structures has been studied by several authors. For example, Mahieux (2001) analysed the cost of manufacturing a carbon-fibre flywheel (AS4/PEEK). Whereas, compression moulding and co-consolidation were studied by Hou et al. (1997) for the manufacturing of aileron ribs and hinges. Some polymers are commonly used as matrix such as PEEK (polyether ether ketone), PEI (polyethylenimine) and PPS (polyphenylene sulphide) because of their mechanical properties and ability to work at high temperatures (above 150 °C) when compared to others. PEEK based composites are not cost-effective to replace thermoset composites because of the high processing temperature required (Diaz and

1 Introduction

Rubio, 2003), while PPS and PEI based composites are cost-effective to replace thermoset matrix composite and metallic parts in some applications (Diaz and Rubio, 2003).

Until now, thermoplastic composites have been applied mainly to secondary structures. Some aeronautical applications of thermoplastic composites viz. Gulfstream G400 and G500 (roof panels), Gulfstream G550 and G650 (elevator and rudder), Airbus A340-600 and A380 (leading edge) and Embraer Phenom (landing gear doors) are presented in Figure 1.1. The use of thermoplastic composite materials for primary structures requires further developments and improvements in manufacturing and joining techniques.

Figure 1.1: Thermoplastic composites in aircraft structures.

Considering brazilian market, EMBRAER (Empresa Brasileira de Aeronáutica) has shown interests in the development of resistance welding and infra-red (IR) lamp welding techniques (Pereira et al., 2012) as joining solutions for thermoplastic composites assemblies. Studies of these welding processes have been carried out at UNESP Guaratinguetá (São Paulo State University) and EESC/USP (São Carlos School of Engineering from University of São Paulo) (Pereira et al., 2012).

1.3 Motivation and objectives of this study

The present work is dedicated to welding joining technique for thermoplastic composites and focuses on the studying of the assembly process by resistance welding and on the evaluation of composite joints toughness obtained by this process. This goal brings industrial and scientific challenges.

Industrial challenges concerning the assembly of thermoplastic composite parts by welding are associated to the applicability of these technologies for assembling large

1.3 Motivation and objectives of this study

composite parts while assuring reproducibility, quality, traceability and airworthiness to guarantee manufacturing process and product certification. The certification process guarantees the product capacity to resist to mechanical and thermal in-service loading. Other industrial challenges are related to the optimization the assembly process while looking for its application to large components, and the reduction of the manufacturing costs.

Scientific challenges with respect to the joining process and the in-service component can be also highlighted. Welding processes and in-service conditions involve multi-physical and multi-scale phenomena. Multi-physics is intrinsic to polymer melting and consolidation, as well as to electromagnetic, thermal and mechanical loadings. Multi-scale analyses must be performed to embrace auto adhesion polymer mechanisms on the interface ($\sim 1 \mu\text{m}$ associated to the surface roughness and $\sim 1 \text{ nm}$ associated to the polymer macromolecules) and various physical mechanisms related to the behaviour of the assembly up to the component scale ($\sim 1 \text{ m}$). Furthermore, the prediction during the component design stage of components strength and lifetime under in-service conditions requires a good knowledge of the mechanical behaviour of joint when it is submitted to various thermo-mechanical loading. A constitutive model for welded joints should account for both deformation and failure mechanisms. Besides, a model which parameters would relate the potential failure to the manufacturing process parameters could be used to understand better and to optimize the welding process as well as to take account of the possible variability of the process when applied to large complex structures.

Motivated by the industrial and scientific challenges support, the goal of this study which consists in studying thermoplastic hybrid composite joints welded by electrical resistance and evaluating their mechanical resistance at room temperature. Hybrid composites refers to a laminate composed of PPS matrix reinforced with continuous glass fibres (G/PPS) or carbon fibres (C/PPS), as it will be detailed on the next chapter. To achieve this objective, the work was split in different steps:

- A resistance welding machine was designed and built at laboratory scale to manufacture specimens of different geometries by controlling the electrical current and pressure applied. Thanks to this machine, specimens were manufactured using different controlled processing conditions.
- The temperature profiles within the specimens during the welding process was measured and analysed via a finite element heat transfer model to investigate the influence of manufacturing parameters. Prior to these analysis, the thermal properties of G/PPS and C/PPS plates were identified from the results of tests which were designed and instrumented for this purpose.

- The resistance for crack propagation of welded joints manufactured using different values of the manufacturing parameters was investigated via DCB (Double Cantilever Beam) tests and an original energetic model was developed to describe the “ductile” crack propagation observed.

1.4 Outline of this manuscript

In that first chapter, “Introduction”, the applicability of composites materials to aeronautical components and the necessity to assemble them is presented. The advances of thermoplastic composites technology allowed their use in aircraft components. Limitations associated to the manufacturing of these components prevent the production of monolithic complex large parts. Industrial and scientific challenges motivated the goal of this work, viz. a study of the resistance welding process and of the mechanical resistance of welded joints.

In the second chapter, “Electrical resistance welding of thermoplastic hybrid laminates: manufacturing of joints and heat transfer analysis of the process”, an overview of fusion bonding techniques is given. Focus is put on the resistance welding and the process parameters which may have an influence on the mechanical resistance of the joint. The C/PPS and G/PPS composite laminates of interest in this study are presented together with their mechanical and thermal properties. An instrumented welding machine which was designed and built at laboratory scale during this study is presented in details. For the better understanding of the welding process, finite element transient heat transfer analyses were performed. The results of these analyses - validated by comparison with experimental thermocouple measurements - are also presented in details. Finally, details are given about the manufacturing process of specimens at different conditions.

The third chapter, “Identification of mechanical and thermal properties of G/PPS and C/PPS composite laminates”, is dedicated to the approach chosen to identify the elastic and thermal properties (specific heat and thermal conductivities) of the C/PPS and G/PPS laminates considered herein. The mechanical properties were identified from the results of uniaxial tension tests performed on specimens cut along different directions and instrumented with full displacement field measurements by using Digital Image Correlation. Specific heat was identified from Differential Scanning Calorimetry tests. The in-plane and out-of-plane thermal conductivities were identified from tests designed and performed during this study together with an inverse analysis combining a finite element heat transfer analysis and experimental measurements from an IR camera. The full identification and prediction procedure were validated by comparing the results

of the numerical predictions to the experimental evidence derived from tests on the hybrid laminates.

The fourth chapter, “Characterization and modelling of welded joints toughness”, is dedicated to the identification of the mechanical resistance under mode I loading of the welded joints manufactured as explained in the second chapter. The results of DCB tests are discussed in terms of R-curve interpretation. Experimental tests show that the specimens fail by propagation of a crack along the welded joint and that the plastic deformation of the welded joint constitutes an important contribution to the energy dissipated during the crack propagation. An energetic model is proposed to model the ductile failure of these welded joints. A first analysis of the sensitivity considering their manufacturing parameters of the resistance of these welded joints to mode I failure was also performed.

The fifth chapter, “An introduction to PGD solution of multidimensional problems”, is devoted to a brief introduction to Proper Generalized Decomposition (PGD) solution procedure used as a powerful tool for the reduction of problem dimensional complexity. This technique is applied to a case study corresponding to an axisymmetric heat transfer problem. Even for simple cases, it is possible to elucidate the benefits of this technique on the reduction of computational cost. Last, potentialities of PGD solution procedure on parametrized problems and on the separation of length scales are presented.

Conclusions and Perspectives are presented at the beginning of each chapter - which is almost self-contained. However, all these texts are gathered in the general conclusions chapter closing this manuscript.

1 *Introduction*

Electrical resistance welding of thermoplastic hybrid laminates: manufacturing of joints and heat transfer analysis of the process

The first part of this chapter is dedicated to the presentation of the materials and assembly process studied herein. The materials – provided by Tencate® - are 2 mm thick hybrid thermoplastic laminates with the following stacking sequence: $[(0^\circ/90^\circ)_G, (0^\circ/90^\circ)_C, (\pm 45^\circ)_C, (0^\circ/90^\circ)_C, (\pm 45^\circ)_C, (0^\circ/90^\circ)_C, (0^\circ/90^\circ)_G]$, where subscripts G and C respectively refer to glass and carbon continuous fibres embedded in a PPS matrix. In the manuscript, these plies are referred to as G/PPS or C/PPS. The assembly process is a welding resistance process where the heating source is a metallic grid inserted between the laminates to be assembled and heated by Joule effect while pressure loading is applied on the welded joint in its normal direction. The joint is constituted of a stainless steel plain weave canvas acting as heating element, and PPS films inserted in between the metallic grid and each of the two composite laminates to be assembled. During the welding process, these films as well as the first layers of the composite laminates are heated, melt and re-consolidate during cooling.

The core of this chapter is dedicated to the description of the welding machine which was designed and built during this study in order to be able to manufacture 140 mm vs. 40 mm welded joints by controlling a certain number of manufacturing parameters –including the geometry of the metallic grid, the intensity of the heating source –via the intensity of the electrical current applied to the grid, from 20 to 200 A-, the pressure –up to 4 MPa-, and the duration of the dwell time during which the pressure is applied while the joint cools down. This machine was used to manufacture specimens corresponding to four different set of manufacturing parameters. The fracture toughness tests were performed on Double Cantilever Beam coupons which are described and analysed in Chapter 4. Future works on this machine should consist on its instrumentation to control the pressure applied to the joint, to monitor the temperature field in the vicinity of the joint, and, possibly, to control some conductivity heat exchanges.

2 Electrical resistance welding of thermoplastic hybrid laminates

The last part of this chapter is dedicated to a first 3D heat transfer analysis –performed using the finite element code Abaqus®– of the electrical resistance welding process. During these simulations the metallic heating grid is modelled as a heat flux, and the seven-layer hybrid composite laminates are modelled as three-layer plates with the following stacking sequence [G/PPS, C/PPS, G/PPS], and each layer –with the appropriate thickness– exhibits homogenized orthotropic properties. The identification of the different material properties used for this analysis was performed during this study and is described in details in Chapter 3. The finite element predictions are compared to thermocouple measurements made at different points within the assembled composite laminates during the welding process. These measurements were made during a first experiment while applying minimal pressure to the joint during the welding process. First, the numerical simulations show that the value of the Nussel number –i.e. the ratio of the convective heat flux to the conductive heat flux– is low enough for conductivities and heat sources playing the key role in the process. Besides, the results of the numerical simulation are very encouraging, in particular in terms of the prediction of the temperature field –including the maximal value– during the heating phase. However, it appears that the simulation of the cooling phase requires further developments. These developments should be made in parallel with the instrumentation of the welding machine.

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2.1 Fusion bonding techniques

Fusion bonding is a process where two or more parts are joined by melting and consolidating the material located at the interfaces between the parts. This process can be performed by applying or not a pressure on the joint. In the context of the thermoplastic polymers and composites with a thermoplastic matrix, fusion bonding processes take benefit from the flow property of the thermoplastic materials above their glass transition temperature, T_g , for amorphous polymers, and above their crystalline melting point, T_m , for semi-crystalline polymers (Stavrov and Bersee, 2005).

The principle of fusion bonding techniques is illustrated by Figure 2.1 (Prager, 1981; Voyustskii, 1963). When two surfaces with the same polymer basis are maintained in contact and heated (stage “a”), after enough energy is provided to decrease the polymer viscosity, an intimate contact is created between them, and the discontinuities due to the irregularities from surface roughness disappear (stage “b”). Then, the entangled macromolecules are free to move through the “virtual” surface (stage “c”) creating links which join the two substrates after cooling the parts. The “c” phenomena is called interdiffusion.

Figure 2.1: Fusion bonding principle: (a) two distinct interfaces; (b) achievement of intimate contact; (c) collapse of the interface by interdiffusion (based on Prager (1981); Voyustskii (1963)).

Benatar and Gutowski (1986) described fusion bonding process in five steps: surface preparation, heating, pressure, diffusion and cooling. The first step consists on removing release agents (oil, chemical contaminants) from the surfaces to be welded. Fusion bonding techniques are more tolerant to contaminants than adhesive bonding techniques (Grimm, 1990), since the mechanical performance results from the macromolecules that have crossed the interface. Heating step enables the substrates to reach T_g or T_m in the interface region. According to Ageorges and Ye (2002), the pressure limits the detrimental consequences of the fusion of the thermoplastic resin on warpage of fibres. Healing process and intimate contact have been studied by Kim and Wool (1983). Finally, cooling step solidifies the material. In semi-crystalline polymers, cooling rate is directly

associated with the degree of crystallinity and crystal size, and consequently influences the final mechanical properties of the joint (Costa and Rezende, 2006).

According to Ageorges et al. (2001), fusion bonding techniques can be classified with respect to the heating source type and, consequently, separated into four groups: bulk heating, frictional heating, electromagnetic heating, two-stage techniques (Figure 2.2). The fusion bonding technique applied depends on the industry facilities and the geometry of the components to be welded (Todd, S. M., 1990; Davies et al., 1991). The most commonly used fusion bonding techniques are the resistance welding, induction welding and ultrasonic welding (Pereira et al., 2012). Table 2.1 compares these techniques with mechanical fastening and adhesive bonding, considering several criteria associated with the industrial applicability. It can be observed that mechanical fastening and adhesive bonding present advantages compared to fusion bonding techniques, viz. possibility of assembling large components, repairability and flexibility. However, they have an inferior performance in terms of inspection and preparation. Mechanical fastening inspection in general consists in visual inspection that it is limited to the superficial level, while adhesive fastening requires more time for surface preparation.

Because of its low surface energy, the self adhesion or the adhesion of PPS with others materials requires special surface preparation (Leahy et al., 2001), which makes difficult the usage of adhesive bonding techniques in standard industrial environment. Other polymeric components with low surface energy, such as polyethylene (PE) and polypropylene (PP), require complex surface activation treatments (Grimm, 1995). Fusion bonding techniques are particularly adapted to these materials.

Table 2.1: Comparison between joining techniques (extracted from Silverman and Griesse (1989)).

	Mechanical fastening	Adhesive bonding	Resistance welding	Ultrasonic welding	Induction welding
Performance	4	10	10	10	10
Reproducibility	10	5	8	9	9
Durability	5	5	8	9	8
Processing-time	2	0	8	10	8
Flexibility	10	10	5	5	7
Application to repair	10	10	8	7	7
Large-scale joint	10	10	8	5	8
Inspection	5	10	10	10	10
Preparation	2	2	10	10	10

(0: worst - 10: best)

This work focuses on the resistance welding of hybrid composite plates with a PPS

Figure 2.2: Classification of the fusion bonding techniques by the heating type: bulk heating, frictional heating, electromagnetic heating and two-stage heating. (Ageorges et al., 2001)

matrix. The following sections present in details the resistance welding process, the joint of interest, the laboratory scale instrumented machine which was built and a finite element heat transfer model allowing to estimate the temperature field in the area of the joint during the welding process.

2.2 Resistance welding of thermoplastic laminates

In the case of resistance welding, two thermoplastic composite parts are joined via an electrical resistor acting as a heating element (Figure 2.3). When current flows through the resistor, the temperature increases, especially in the interface region. The temperature exceeds the glass temperature (if the matrix amorphous), T_g , or the melting temperature (if the matrix is semi-crystalline), T_m , so that, a progressive intimate contact and autoadhesion between the two substrates is created (Figure 2.1). After the dwell-time, the electrical current is switched off, consequently the cooling step. The cooling, under a suitable pressure, consolidates the thermoplastic matrix, joining the two parts.

The heating element is an electrical resistor of equivalent electrical resistance R_{eq} , so

Figure 2.3: Principle of resistance welding process. (Hou, 1999)

that, the power heat dissipated, P , can be written as

$$P = R_{eq} i^2 \quad (2.1)$$

where i denotes the electrical current intensity. The total dissipated heat obviously depends on the geometry of the parts involved. Conversely the heat power density dissipated in the joint during the welding, $\bar{P}$, constitutes an intrinsic welding process parameter (Eveno et al., 1988). It is defined as

$$\bar{P} = \frac{P}{lw} = \frac{R_{eq} i^2}{lw} \quad (2.2)$$

where l and w denotes heating element length and width, respectively. Another important parameter (Stavrov and Bersee, 2005) is the total energy density, $\bar{E}$, reached during the welding process, defined as

$$\bar{E} = \int_{\Delta t} \bar{P} dt \quad (2.3)$$

where Δt is the time interval during the application of electrical current.

According to (2.3), the same energy density can be obtained by using low power density during a long time, or higher power density levels during a shorter time (Stavrov and Bersee, 2005). Heat power density level, according to its intensity, can be classified in three groups: low ($10\text{-}75 \text{ kW/m}^2$), intermediate ($75\text{-}130 \text{ kW/m}^2$) and high (130 kW/m^2 and more) (Hou and Friedrich, 1992). Hou et al. (1999) studied LSS (Lap Shear Strength) specimens made of PEI reinforced by carbon fibres where the joints were obtained with four values of the power density ($80, 118, 140, 160 \text{ kW/m}^2$). The results shown that a value about $30\text{-}35 \text{ MPa}$ LSS strength can be reached for all four values of the power density, but lower values require higher welding energy due to higher heat losses. The value of the applied pressure, the time evolution of pressure and electrical current are other key parameters of the welding process.

The welding resistance process has already been used in other industries besides aeronautics one (Ageorges and Ye, 2002), such as to join plastic pipes (Hunt, 1990), containers and medical devices (Grimm, 1995), to patch and repair structures (Sherrick and Rosenthal, 1985; Davies et al., 1991; Xiao et al., 1994), to facilitate the melting and re-consolidation process of thick composites (Ing, 2002). A great advantage of resistance welding considering the manufacturing processes is the possibility of non-destructive inspection. After welding, the heat element remains in the joint so that it can be used to reprocess the component and/or submit it to non-destructive inspection by IR systems after heating it at low temperature (Nino et al., 2009).

2.3 Composite joint of interest

The joint of interest in the study is obtained by the resistance welding of two hybrid composites plates, formed of G/PPS and C/PPS layers manufactured by Tencate[®], using a stainless steel metallic mesh as heating element (Figure 2.4). The materials are described in following sub-sections.

Figure 2.4: Thermoplastic composite joint by resistance welding. A stainless steel grid and PPS amorphous films are used to join the two hybrid composite plates.

2.3.1 Composite plates

The hybrid plates to be welded consists in a 2-mm thick composite laminate composed of seven layers. The top and bottom layers are constituted of glass continuous fibres

with a PPS matrix (subscript G), and the five other layers are constituted of carbon continuous fibres with PPS matrix (subscript C). The stacking sequence of the plates is the following $[(0^\circ/90^\circ)_G, (0^\circ/90^\circ)_C, (\pm 45^\circ)_C, (0^\circ/90^\circ)_C, (\pm 45^\circ)_C, (0^\circ/90^\circ)_C, (0^\circ/90^\circ)_G]$ (Figure 2.4). The notation $(0^\circ/90^\circ)$ denotes a bidirectional layer where 0° and 90° refers to the angle between main fabric directions (warp and weft) and a reference direction based on the structure coordinate system. The square plates, ~ 500 mm in side dimension, were manufactured by Tencate®.

Other stacking sequences configurations, also supplied by Tencate®, were tested to identify independently the mechanical and thermal properties of a single ply of PPS matrix reinforced with glass or carbon continuous fibres. These tests and the identification procedure are presented in details in next chapters. The main results are gathered in Table 2.2

Table 2.2: Mass density, mechanical and thermal properties of a single ply constituted of a PPS matrix reinforced by continuous glass or carbon fibres.

Property	Unit	G/PPS	C/PPS
Density	kg m^{-3}	1920	1550
Young's modulus in direction 1 = 0° (E_1)	GPa	23.9	52.7
Young's modulus in direction 2 = 90° (E_2)	GPa	23.2	51.5
Poisson ratio (ν_{12})	-	0.12	0.06
In-planar shear modulus (G_{12})	GPa	3.9	4.0
In-plane thermal conductivity ($k_{1,2}$)	$\text{W m}^{-1} \text{K}^{-1}$	0.25	2.7
Out-of-plane thermal conductivity (k_3)	$\text{W m}^{-1} \text{K}^{-1}$	0.24	0.38
Specific heat at 23°C (c)	$\text{J kg}^{-1} \text{K}^{-1}$	1050	726
Melting enthalpies	kJ kg^{-1}	14.72	19.33

2.3.2 Heating element

Heating elements most commonly used for resistance welding of polymer matrix composites are made of carbon fibres (unidirectional and fabrics) or stainless steel metallic grids (Ageorges and Ye, 2002).

Carbon fibres mesh has the advantage of being compatible with laminate constituents. Ageorges et al. (1998) performed a study comparing unidirectional and fabric carbon fibre arrangements in terms of mechanical performance. In that study, the use of fabric increases the lap shear strength up to 69 % and the interlaminar fracture toughness up to 179 % compared to the results obtained with an unidirectional arrangement. Although they introduce an additional material from those present in the composite

plates, stainless steel metallic grid have been chosen for recent researches and industry applications (Figure 2.5).

Figure 2.5: Stainless steel heating element.

Stainless steel mesh has the advantage of having a larger processing window compared to carbon fibres fabric as heating element (Stavrov and Bersee, 2005; Hou et al., 1999), which makes easier the tuning of process parameters. However, it introduces in-service stress concentrations and environmental degradation (Beevers, 1991). In this study, two different geometries of a 316L stainless steel grids were used as heating element (Figure 2.5). These metallic meshes were bought from Giusti & Cia company. Stainless steel 316 is a stable austenitic steel with large ductility. Table 2.3 presents typical physical properties of 316L stainless steel.

Table 2.3: Typical properties of stainless steel 316L.

Property	Value
Density	7990 kg/m^3
Electrical resistivity	$7.40 \cdot 10^{-7} \Omega m$
Heat capacity	500 J/kgK
Thermal conductivity	16.2 W/mK at $100^\circ C$ 21.4 W/mK at $500^\circ C$

The metallic mesh geometry is defined in Figure 2.6. M200 mesh corresponds to a $45 \mu m$ wire diameter (d), and a $127 \mu m$ pitch (p); while M270 mesh corresponds to a $40 \mu m$ wire diameter and a $94 \mu m$ pitch (Figure 2.6). To open surface, S , of the mesh is defined as

$$S = \frac{(p - d)^2}{p^2} \quad (2.4)$$

The open surface are 42% and 33% for M200 and M270 meshes, respectively.

Figure 2.6: Heating element geometry.

2.3.3 Polyphenylene sulphide (PPS)

Because of their thermal and mechanical properties, PEEK (polyetheretherketone), PEI (polyetherimide) and PPS (polyphenylene sulphide) are the most common thermoplastic polymers used in aeronautical industry. PPS is a semi-crystalline organic polymer constituted by an aromatic ring linked to sulfides (Figure 2.7). Because of its non-flammability and high hardening and stiffness, it is used to manufacture components submitted to high mechanical and/or thermal loads (Anagreh et al., 2008). Besides, PPS exhibits good chemical resistance, low water absorption, good insulation behaviour and good dimensional stability (Ticona, 2013). These good properties explain why is also often used in other industries like automotive and electronics.

Figure 2.7: PPS elementary cell (Costa and Rezende, 2006)

Table 2.4 gives typical mechanical and thermal properties for PPS. In the joint of interest, PPS is present as the matrix phase of the composite plates, and also, in the polymeric films surrounding the heating element. The amorphous films on each side of the heating element (Figure 2.4) are $73 \mu m$ thick, which is large enough to fill the voids of the heating element and to avoid the electrical current to reach the composite substrates fibres. This electrical insulation phenomenon is also ensured by the presence

of the first composite plate layer of PPS reinforced with continuous glass fibre.

Table 2.4: Typical PPS mechanical and thermal properties. Data from Tencate[®] datasheet.

Property	Unit	Value
Density	kg m^{-3}	1350
Specific heat*	$\text{J kg}^{-1} \text{K}^{-1}$	1830
Thermal conductivity	$\text{W K}^{-1} \text{m}^{-1}$	0.19
Young's modulus (in tension)	GPa	3.80
Tension strength	MPa	90.3
Young's modulus (in compression)	GPa	2.96
Compression strength	MPa	148

* data extracted from Celanese Fortron 214 PPS unfilled

2.4 Laboratory scale welding machine

According to Stavrov and Bersee (2005), a typical welding setup for this process comprises: heating element, clamps, insulator components, a pressure device, a power supply, a voltmeter and an ammeter (Figure 2.8). Based on this typical setup, a laboratory scale welding machine was designed and built in São Carlos School of Engineering (EESC/USP). This machine was used to manufacture specimens by controlling the welding parameters in order to investigate the sensitivity to these parameters of the joint mechanical resistance.

Figure 2.8: Typical resistance welding setup, based on Stavrov and Bersee (2005).

The frame of the welding machine is divided in a mechanical device to apply the pressure and an electrical system for the welding itself. A sketch of the machine is shown in Figure 2.9. The directions of the coordinate system in given in Figure 2.9 will be used along the present chapter for the description and investigation of the welding system.

2 Electrical resistance welding of thermoplastic hybrid laminates

Figure 2.9: Welding machine design.

Four linear guidelines maintain the parallelism between the upper and lower metallic (AISI 1020 steel) platens constituting the mechanical devices. The platens were installed on a manual uniaxial hydraulic press which was available in the Aeronautical Engineering Department at EESC/USP (Figure 2.10). These platens could be installed in a universal testing machine in order to obtain a better control the mechanical loading applied to the specimen during the welding process. The parallelism being guaranteed by the platens, internal elements were designed to enable the resistance welding process.

(a) Manual hydraulic press frame

(b) Welding machine in detail.

Figure 2.10: Laboratory scale welding machine for manufacturing of thermoplastic composites welded joints.

The internal element consisting in insulating material and electrical copper connectors are shown on Figure 2.11. The connectors equipped with two screws clamp the heating element. The surfaces in contact with the heating element metallic mesh were machined to assure a good electrical coupling. Two 15 mm thick PTFE blocks located in between

the platens and the composite laminates contribute to a good uniformity of the pressure applied to the specimens, as well as, a thermal (an electrical) insulation. Particular case was taken to avoid any electrical contact between the heating element and metallic parts. High electrical power could induce the heating element rupture (by metal melting), and subsequently a short circuit with the machine frame welding. To avoid this, all elements surrounding the heating element, excluding the electrical connectors, were covered with a polyimide film. To insulate the electrical connectors, they were glued on a Bakelite plate.

Figure 2.11: Internal machine components.

The machine has a useful welding area of ($y = 45$ mm) vs. ($x = 140$ mm) and welding can be performed positioning the heating element along x - or y -direction, as shown by Figure 2.9. The machine has been designed to be submitted to a maximum load of 25 kN. The applied load is measured by a compression load cell (HBM C9B - 50 kN, accuracy 0.5%) located below lower platens (Figure 2.10).

The electrical current is supplied by an arc-welding power supply. This device enables to reach alternate currents up to 200 A. Tests showed that an electrical current of 60 A is enough to induce on a joint of 20 mm x 120 mm with a M200 heating element a short circuit due to melting. The power supply device controls electronically the electrical current, keeping it almost constant during the welding. The electrical current was monitored using a ammeter clamp (resolution ± 0.1 A). No oscillation of the electrical current was detected according to the instrument resolution.

The welding procedure used during this study corresponds to the time evolution given in Figure 2.12. An initial pressure, p_0 , is applied to guarantee a good contact between the parts and to help the polymer diffusion through the heating element. At time t_0 , the electrical current is turned on at an intensity i maintained constant up to t_h . The pressure remains applied up to instant t_p . To the authors' knowledge the influence of t_p on the welded joint resistance has not yet been investigated.

During the welding process, small variations of the pressure were observed (Figure

Figure 2.12: Welding procedure.

2.12). The first decrease of the pressure coincides with the melting of PPS films on each side of the heating element (point A in Figure 2.12). Once the polymer has filled the heating element voids, the pressure increases and exceeds the initial pressure, p_0 , as a consequence of the thermal expansion of the specimen and volume expansion due to melting. The value of the pressure is maximum at point B (Figure 2.12) where the electrical current is stopped. Then the pressure decreases slowly till the initial pressure, p_0 , is reached. These variations could be limited if the platens were mounted on a servo-hydraulic load controlled system - which was not the case during this study.

Figure 2.13 shows how the polymer fills progressively the voids between the wires of stainless steel heating element. This picture was taken from a joint obtained by stopping the electrical current before applying enough input energy to create the link between the substrates. Once polymer melts, it occupies the parallelepipeds between the wires. This interrupted test also shows that the heat is higher at the borders - region B - than at the centre - region A - of the specimen, see Figure 2.13. The welding was performed with the heating element positioned along x-direction.

Figure 2.13: Polymer moving through the heating element.

2.5 Specimens manufacturing

To investigate the mechanical behaviour of welded joints by crack propagation tests in mode I loading, samples 120 mm in length and 20 mm in width were manufactured with a 60 vs. 20 mm² welded region (Figure 2.14). The geometry and the dimensions were chosen to evaluate the mechanical strength under mode I and mode II loading conditions through DCB and ENF testing configuration. However, the present work was focused on mode I tests.

Figure 2.14: Welded specimen geometry (units: mm).

The specimens were welded with the heating element aligned with x-direction. The non-welded region corresponding to the pre-crack of a DCB specimen was obtained by substituting the PPS films on each side of the heating element by polyimide films (see Figure 2.15). Polyimide exhibits a second order transition which is considered the glass transition temperature between 360 °C and 410 °C and no melting temperature is defined (Dupont, 2011). The welding direction, x , was chosen to be similar to real industrial conditions, where the crack propagates in the direction in which the electrical current is imposed during the welding process.

Figure 2.15: Specimen stacking: composite plates, pps films, polyimide films and heating element.

The preparation of the joint welding was conducted according to the following procedure:

1. specimens were cut from composite plates using a diamond disc cooled by air;

2 Electrical resistance welding of thermoplastic hybrid laminates

2. cut edges were polished to remove about 1.3 mm and avoid any deterioration of the composite due to cutting (Figure 2.16a);
3. stainless steel heating element and polymer films were cut by a scalpel;
4. the area of welding was delimited by using polyimide films put on the composite surface (Figure 2.16b);
5. staking and positioning of the whole sequence: composite - polymer - heating element - polymer - composite (Figure 2.16c);
6. a 6 mm-width polyimide tape was put near to the specimen border to avoid misalignments due to handling

Step 5 is the most important because it defines the position of the initial crack tip by the overlap of the linear transition between the PPS and polyimide films in both sides of the heating element.

Specimens were manufactured using M200 and M270 metallic meshes and different values of the manufacturing parameters as summarized on Table 2.5.

Table 2.5: Specimens manufacturing parameters.

Identification	Mesh	$\bar{i}$ (A)	t_h (s)	t_p (s)	p_0 (MPa)
A	M200	30	17	77	1.4
B	M270	30	17	77	1.4
C	M200	25	17	197	1.4
D	M270	25	17	197	1.4

2.5.1 Manufacturing defects

Defects may be observed in the welded region specimen shown in Figure 2.17 was welded with an electrical current of 30 A, during 17 s, without applying additional pressure beyond the platens weight. The size of the voids observed close to the heating element is of the order of 400 μm . These defects may be created by the air entrapped in the welded zone.

The temperatures are higher on the boundaries than in the centre of the specimen. This can lead to local loss of polymer (PPS), misalignments of composite fabric and, specimen height decrease (Figure 2.18). These defects are detrimental for the structural integrity of the component because of the reduction of material stiffness, strength, as well as assembly thickness (which leads to reduction of bending stiffness).

2.5 Specimens manufacturing

(a)

(b)

(c)

Figure 2.16: Specimen preparation steps before welding.

Figure 2.17: Defects on the welded zone (heating element aligned on vertical direction)

Figure 2.18: Boundary effect

2.6 Heat transfer analysis of the welding process

2.6.1 Experimental investigation

The following experiments were performed at UFSCar/DEMA Materials Engineering Department and consisted in measuring via thermocouples the temperature history during the welding process at some points inside the composite plates.

Thermocouples were disposed according to Figure 2.19. They were introduced in holes drilled in the composite surface opposite to the surface in contact with the heating element. Their exact position is reported in Table 2.6. The coordinates of the centre of thermocouple head are calculated considering the head like a sphere of 0.8 mm diameter.

Figure 2.19: Thermocouples positions schema.

Table 2.6: Thermocouples position according to the coordinate system in Figure 2.19.

TMP	x (mm)	y (mm)	z (mm)
1	10.0	5.0	-0.9
2	0.0	0.0	-1.4
3	20.0	0.0	-1.9
4	15.0	0.0	-0.9
5	5.0	-7.0	-0.9

precision: $\pm 0.1\text{mm}$

Thermocouples signals were recorded in a frequency of 100 Hz by a Lynx ADS 2000 IP data acquisition system. A M200 specimen was welded applying an electrical current

of 30 A during 17 s, which provides to the system a power density of $\sim 100 \text{ kW/m}^2$. The pressure applied during the welding process, was limited to the weight of the upper platen i.e. to ($\sim 25 \text{ kPa}$).

The temperature history is shown in Figures 2.20(a) and (b). Although the electrical current was cut off at time $t_h = 17 \text{ s}$, the temperature increased up to approximately 21 s. The difference measured by temperature of thermocouples 1, 4 and 5 (see Figure 2.20a), positioned at same z-coordinate, confirms that a temperature gradient exists along the y-direction from the specimen's centre to its boundary. The difference of temperature measured by thermocouples 4, 3 and 2 (see Figure 2.20b), all positioned at $y = 0$ but the height z differing by 0.5 mm, shows that a large temperature gradient exists along the z-direction. During the cooling step, it can be noted that thermocouple 4 has a higher cooling rate. During the cooling phase, the heat flux is larger in x-direction than in z-direction due to the high thermal conductivity of the stainless steel mesh ($\sim 16 \text{ Wm}^{-1}\text{K}^{-1}$ at 100°C) compared to the thermal conductivities of the composite plies and surrounding insulator (PTFE).

2.6.2 Finite element analysis

Resistance welding process is sensitive to various parameters, viz. heating element geometry and material, distance between the components, environmental conditions, thermal properties of adjacent components, electrical current intensity, time, etc. Mapping experimentally all these variables would be time and cost consuming, so that numerical analysis should be a useful tool to better understand and optimize this assembly process.

A finite element approach was chosen to solve the transient heat transfer problem and to simulate the thermal behaviour of the assembly during the welding process. Considering a solid Ω surrounded by a surface Γ , the problem consists in finding the temperature field $T(\mathbf{x}, t)$ for each material point $\mathbf{x} \in \Omega$ and for every $t \in [0, \tau]$ which is solution of the heat transfer equation:

$$\rho c \dot{T} - \text{div}(\mathbf{K} \nabla T) + r = 0 \quad (2.5)$$

verifying initial and boundary conditions, where the dot denotes the time derivative. Variable r denotes the latent heat associated to the melting during heating and crystallization during cooling, and constants ρ , c and $\mathbf{K}$ are the density, specific heat and conductivity tensor, respectively, of a material point of coordinates $\mathbf{x}$. Assuming symmetry conditions, the geometry modeled (solid Ω) consists in one eight of the welding

(a) Temperature history for $z = -0.9$ mm

(b) Temperature history $y = 0$

Figure 2.20: Experimental temperature history.

machine limited to upper and lower platens, as shown in Figure 2.21.

Figure 2.21: Region simulated by finite element analysis

Initial conditions are given by the room temperature

$$T(\mathbf{x}, 0) = T_\infty \quad (2.6)$$

The heat generated by the heating element is modelled by a heat flux boundary conditions are imposed (Neumann boundary condition) on $\Gamma_q \equiv \{\mathbf{x} \in \Omega \mid \mathbf{x} = (a, b, 0), \forall a, b\}$, whereas adiabatic or symmetry condition are assumed on $\Gamma_0 \equiv \{\mathbf{x} \in \Omega \mid \mathbf{x} = (a, 0, b) \vee \mathbf{x} = (0, a, b), \forall a, b\}$, and convective-radiative exchange (mixed conditions) on $\Gamma_{cr} = \Gamma - (\Gamma_0 \cup \Gamma_q)$, so that

$$q(\mathbf{x}, t) = \frac{Ri^2}{2lw} = \frac{\bar{P}}{2}, \quad \mathbf{x} \in \Gamma_q \quad (2.7)$$

$$q(\mathbf{x}, t) = 0, \quad \mathbf{x} \in \Gamma_0 \quad (2.8)$$

$$q(\mathbf{x}, t) = h_{cr}(T - T_\infty), \quad \mathbf{x} \in \Gamma_{cr} \quad (2.9)$$

Because of the uncertainty on the value of thermal convection coefficient, heat transfer analysis was performed with three different values of h_{cr} , viz. 5, 10 and 20 W/m².

Figure 2.22: FEM mesh used and boundary conditions (heat flux and symmetry).

The composite laminate was modeled as three layers of transverse isotropic materials and the properties are given in Table 2.2. The properties of 15-mm thick PTFE insulator are given in Table 2.7.

Finite element software Abaqus[®] was used to solve the problem. The geometry was meshed using 20-node quadratic parallelepiped (DC3D20) and 10-node tetrahedron (DC3D10) elements (Figure 2.22). The problem was solved using an implicit time integration algorithm with an initial time increment of 0.01 s, and subsequent increments

Table 2.7: Isotropic thermal properties of PTFE.

	PTFE
Density (kg/m ³)	2150
Specific heat (J/(kg °C))	1050
Thermal conductivity (W/(m °C))	0.4

controlled by Abaqus[®] solution control. The maximum variation permitted for nodal temperature between two increments was set to 1.0 °C.

The temperature field for $h_{cr} = 10 \text{ W}/(\text{m}^2 \text{ °C})$ for three instants during heating and cooling steps is shown in Figure 2.23. During the heating step, the higher temperatures remains concentrated in the specimen (Figure 2.23a,b,c). During the cooling step, the heat diffuses through the insulator (PTFE), which reaches up to 45°C (Figure 2.23e). Temperature profiles along y and z-direction are given in Figure 2.24 and Figure 2.25, respectively. It can be noticed that the heat flux through the PTFE insulator vanishes before it reaches the interface with the metallic platens.

Figure 2.23: Temperature fields snapshots during heating and cooling.

(a) heating

(b) cooling

Figure 2.24: Temperature profiles along y-direction ((x,z) = (0,-1.0))

(a) heating

(b) cooling

Figure 2.25: Temperature profiles along z -direction ($(x,y) = (0,0)$).

2.6.3 Experimental vs. Numerical Analyses

As mentioned before, finite element simulations were performed for three values of the radiative-convective coefficient, viz. $h_{cr} = 5, 10$ and $20 \text{ W}/(\text{m}^2\text{°C})$. Figure 2.26 shows that this parameter has negligible influence on the temperature profile. Hence all results presented in the sequel correspond to $h_{cr} = 10 \text{ W}/(\text{m}^2 \text{°C})$.

Figure 2.26: Temperature sensitivity at relation h_{cr}

Finite element model predictions and experimental measurements are compared in Figure 2.27 (left column). The difference between the predictions and measurements is plotted for each thermocouple position $\mathbf{x}_j$ so that:

$$e_j = T_{fem}(\mathbf{x}_j, t) - T_{exp}(\mathbf{x}_j, t) \quad (2.10)$$

where T_{exp} and T_{fem} are the experimental and finite element model temperature fields, respectively, can be seen in Figure 2.27 (right column).

During heating, the melting of the polymer films on each side of the heating element changes the geometry, and consequently, the local temperature field which is not taken into account by the finite element model. In fact, finite element model does not at all take into account the thin layer of heating element and PPS films surrounding it at the centre of the joint. Neglecting the energy stored ($\rho c \dot{T}$) and the energy dissipated during the solidification of these layers is probably the main cause of difference between predictions and measurements especially during cooling.

Besides, the uncertainty associated to the thermocouple positioning is ± 0.1 (Table 2.6) which due to the high thermal gradients can leads to variations of $\pm 7.6 \text{ °C}$, as illustrated by Figure 2.28.

Finally, it can be concluded that the FE model developed makes it possible to estimate the temperature field during the heating phase of the welding process but that further

Figure 2.27: Comparison between numerically predicted and experimentally measured temperature history in the welded region.

Figure 2.28: Uncertainty associated to thermocouple positioning along z direction using the finite element model.

developments are required to simulate the cooling phase. A first improvement will consist in adding to the problem solved the complex zone corresponding to the PPS films surrounding the heating element.

Identification of mechanical and thermal properties of G/PPS and C/PPS composite laminates

This chapter is dedicated to the identification of the mechanical and thermal properties of G/PPS or C/PPS composite laminates. These laminates –provided by Tencate®- are assumed to be representative of the G/PPS or C/PPS plies within the hybrid composite laminates studied herein. The thermal properties were used for the numerical simulations –presented in Chapter 2- of the welding process of the hybrid composite laminates. The mechanical properties will be used for future detailed analyses of the crack propagation tests discussed in Chapter 4. In-plane mechanical properties of the G/PPS or C/PPS composite laminates were identified from the results of uniaxial tension tests performed on specimens cut at 0° , 90° and 45° with respect to warp/weft orientations. Since the tests were instrumented with Digital Image Correlation, it was verified that the strain field is uniform within the central part of the specimens. Both materials exhibit orthotropic in-plane elastic properties –but with similar values of Young’s modulus in 0° and 90° directions. The values of this Young’s modulus and shear modulus are in agreement with the values given in the specification sheets from the manufacturer. The specific heat, melting –during heating- and recrystallization –during cooling- enthalpies were identified for both materials via DSC tests.

The main part of this chapter is dedicated to experiments that were designed, performed and analysed during this study, to identify the anisotropic thermal conductivities of the G/PPS or C/PPS composite laminates. The experiment consists in two steps: first, the central part of the studied horizontal square plate is heated with an electrical bulb and, then, the plate naturally cools down. The experiment is instrumented with thermocouples on both faces of the plate, and an Infra-Red (IR) camera facing the plate opposite to the heating source.

3 Identification of mechanical and thermal properties

Since the temperature field measured by the IR camera appears to be axisymmetric the thermal conductivities of both laminates are assumed to be isotropic transverse. The experimental results exploited for the identification procedure are the time for temperature to diffuse through the specimen thickness during the heating phase just in front of the heat source –as detected by two thermocouples–, and the temperature history along a radius initiating from the centre of the plate –as measured by the IR camera. The parameters to be identified are the in-plane and out-of plane thermal conductivities, the convection-radiation coefficients on top and bottom faces of the plate, and several parameters to describe the intensity and the geometry of the heat source. Finally the value of eleven parameters is derived from an inverse identification procedure based on Levenberg-Marquart algorithm applied simultaneously to the results of the experiments performed on the G/PPS and C/PPS plates. This global identification procedure proved to be possible thanks to previous separate approximate evaluations of the out-of-plane conductivities and of the convection-radiation coefficients. The in-plane thermal conductivity exhibited by C/PPS plate is about seven times larger than its out-of-plane conductivity, whereas the thermal conductivity identified for the G/PPS plate is almost isotropic.

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3.1 In-plane mechanical properties

3.1.1 Composite laminates

If the local coordinate system with axes 1, 2 and 3, attached to a composite ply is positioned such that the direction 3 is aligned with the out-of-plane normal, whereas directions 1 and 2 are aligned with warp and weft directions, the constitutive equation of the ply for plane stress state is written as

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_{12} \end{Bmatrix} = \begin{bmatrix} \frac{E_1}{1 - \nu_{21}\nu_{12}} & \frac{\nu_{12}E_2}{1 - \nu_{21}\nu_{12}} & 0 \\ \frac{\nu_{21}E_1}{1 - \nu_{21}\nu_{12}} & \frac{E_2}{1 - \nu_{21}\nu_{12}} & 0 \\ 0 & 0 & G_{12} \end{bmatrix} \begin{Bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_{12} \end{Bmatrix} \quad (3.1)$$

where E_1 , E_2 are the Young's moduli in directions 1 and 2, respectively, ν_{12} and ν_{21} the Poisson's ratios and G_{12} the shear modulus. The compliance matrix $[C]^{-1}$ appears when (3.1) is inverted, so that:

$$\begin{Bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_{12} \end{Bmatrix} = \begin{bmatrix} \frac{1}{E_1} & -\frac{\nu_{12}}{E_1} & 0 \\ -\frac{\nu_{21}}{E_2} & \frac{1}{E_2} & 0 \\ 0 & 0 & \frac{1}{G_{12}} \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_{12} \end{Bmatrix} \quad (3.2)$$

By the principle of Maxwell-Betty, the stiffness and compliance matrices are symmetric, so that:

$$\frac{\nu_{12}}{E_1} = \frac{\nu_{21}}{E_2} \quad (3.3)$$

Considering the above equations, the in-planar mechanical properties can be identified by uniaxial tensile tests performed along directions. Whereas E_1 and ν_{12} can be identified from a uniaxial test in direction $x \equiv 1 \equiv 0^\circ$ - such that $\sigma_x = \sigma_1$ and $\sigma_2 = \sigma_{12} = 0$ - , E_2 can be identified from a test along direction $2 \equiv 90^\circ$. Shear modulus G_{12} may be identified from a uniaxial test along $x \equiv 45^\circ$ direction so that

$$\frac{\sigma_x}{2} = G_{12} \frac{\epsilon_x - \epsilon_y}{2} = G_{12} \gamma_{12} \quad (3.4)$$

where y refers to the direction perpendicular to loading direction x .

3.1.2 Experiments

Uniaxial mechanical tests were performed at EESC/USP Department of Aeronautic Engineering by using an Instron 5985 testing machine with a load capacity of 250 kN and a 0.5% accuracy for loads lower than 250N. Full field displacement measurements were derived from the Digital Image Correlation measurements. Specimens of 200 vs. 25 mm^2 were cut from $[(0/90)]_7$ G/PPS and C/PPS plates along directions 0° , 90° and 45° using a diamond disc cooled by air. After cutting the side faces of the specimens were polished using three grain sizes (P120, P200, P320).

Digital Image Correlation (DIC) technique consists in measuring the displacement field by correlating two images, viz. a reference image, $f(\mathbf{x})$, and a deformed one, $g(\mathbf{x})$, where $\mathbf{x}$ is the position vector. Hild and Roux (2006) presents the principles and in-depth the equations used. According to the conservation of the optical flux, the bidimensional signal at deformed configuration $g(\mathbf{x})$ can be written as

$$g(\mathbf{x}) = f(\mathbf{x} - \mathbf{u}(\mathbf{x})) + b(\mathbf{x}) \quad (3.5)$$

where $\mathbf{u}(\mathbf{x}) = \{u_x(\mathbf{x}) \ u_y(\mathbf{x})\}^T$ is the unknown displacement field and $b(\mathbf{x})$ is a general noise which comes from different sources.

The problem consists in determining the displacement field $\mathbf{u}$ based on measured signals f and g . For that, the following function is minimized.

$$\mathcal{F}(\mathbf{v}) = \int_{\Omega} [g(\mathbf{x}) - f(\mathbf{x} - \mathbf{v}(\mathbf{x}))]^2 d\mathbf{x} \quad (3.6)$$

where $v(x)$ is the displacement field to be identified. Function $\mathcal{F}$ tends to zero when $\mathbf{u} = \mathbf{v}$. Into Correli-Q4 software developed at LMT-Cachan and used during this study, $\mathcal{F}$ is minimized for a domain which is meshed in four-node quadrilateral 2D finite elements (Hild and Roux, 2006).

A texture or random pattern is painted on the specimen surface to give an “identity” to each element of the mesh. A photographic system takes pictures from the specimen at different times. The minimization of function $\mathcal{F}$ is performed for each photo of interest where the reference image can be chosen among all previous images including the first one. The experimental setup, including photographic camera, lighting system, specimen and testing machine can be seen in Figure 3.1. Photos were taken using a Canon EOS 50D with a 180 mm focal-length objective, aperture 6.3, exposure 1/20 s.

The exploitation of a uniaxial test performed in a C/PPS specimen cut along 0° is first presented. The Region of Interest (ROI) was discretised in 16 vs. 16 pixels elements as shows in Figure 3.2(a) for the initial configuration and in (b) for the deformed

Figure 3.1: Uniaxial test with Correli setup.

configuration at 120 MPa. The minimization of function $\mathcal{F}$ between images (a) and (b) makes it possible to identify the displacement $\mathbf{u}$ and in particular, the vertical component $u_x(\mathbf{x})$ shown in pixels in Figure 3.2(c). Once the displacement field $\mathbf{u}$ is identified, the strain field $\epsilon = \{\epsilon_1 \ \epsilon_2 \ \epsilon_{12}\}^T$ can be computed. The minimization procedure to identify the displacement field leaves a residue of the order of 10^{-3} pixels for a displacement of the order of 10 pixels.

Figure 3.2: Uniaxial tests in direction 0° on a C/PPS laminate instrumented by Correli Q4: (a) initial configuration; (b) deformed configuration at $\sigma_x = 120$ MPa; (c) u_x displacement field in pixels at 120 MPa.

It is worth noticing that DIC makes it possible to verify the homogeneity of the strain field within the specimen before the computing the average strain over a cross-section.

The stress vs. average strain for C/PPS loaded in direction 0° and 90° is plotted in Figure 3.3(a) and (b), respectively. The strain component ϵ_1 (aligned with loading direction) vs. ϵ_2 (orthogonal do the loading direction) for C/PPS is plotted in Figure 3.4. Stress component σ_{12} vs. average strain γ_{12} measured during a test at 45° is plotted in Figure 3.5.

3 Identification of mechanical and thermal properties

(a)

(b)

Figure 3.3: Stress vs. strain curves for C/PPS laminates loaded along direction 0° (a) and 90° (b).

Figure 3.4: Strain ϵ_2 vs. strain ϵ_1 for C/PPS plates loaded along direction 0° .

Figure 3.5: Stress vs. strain curve for C/PPS loaded along direction 45°.

3.1.3 Results

The elastic properties were identified from the tests briefly discussed in previous subsections. Table 3.1 compiles the in-plane mechanical properties of C/PPS and G/PPS composites plates. The values were identified on the first unloading of stress vs. strain curves. These values are close to the values taken from the specification sheets provided by Tencate.

Table 3.1: In-planar elastic properties for C/PPS and G/PPS composites.

Property	Plate	Unit	Tencate*	Measured
Young's modulus, E_1	G/PPS	GPa	26.0	23.9
	C/PPS	GPa	51.0	52.7
Young's modulus, E_2	G/PPS	GPa	24.0	23.2
	C/PPS	GPa	51.0	51.5
Poisson's coefficient, ν_{12}	G/PPS	-	-	0.12
	C/PPS	-	-	0.06
Shear modulus, G_{12}	G/PPS	GPa	3.7	3.9
	C/PPS	GPa	4.0	4.0

* data from Tencate datasheet at 23°C / 50% humidity

3.2 Identification of specific heat and melting enthalpies

Specific heat and melting enthalpy for G/PPS and C/PPS were identified from the results of DSC tests performed at UFSCar/DEMA Materials Engineering Department. Small portions of the composite plates (~ 10 mg) were cut and heated from 0 °C to 340 °C and then cooled down to room temperature under a rate of 5 °C/min. The corresponding DSC curves for G/PPS and C/PPS are given in Figure 3.6.

Figure 3.6: DSC plots for G/PPS and C/PPS.

The peak temperatures - often called melting temperatures - and the melting enthalpies identified from these curves are 280.3 and 282.2 °C, 14.72 and 19.33 kJ/kg for G/PPS and C/PPS, respectively. Considering the mass fraction of PPS, viz. 37% and 43% in G/PPS and C/PPS laminates (Tencate data), respectively, and the melting enthalpy of PPS equal to 43.1 kJ/kg, according to Costa and Rezende (2006), the value of melting enthalpy estimated for G/PPS and C/PPS is 15.95 and 18.53 kJ/(kg °C), respectively. The uncertainty on the PPS or reinforcement fraction in the small specimens analyzed by DSC is enough to explain the difference between these figures and the result of DSC analyses.

The identified specific heat is plotted in Figure 3.7 for both laminates. A linear interpolation fit the data from 20°C up to 220°C:

$$c^c = 681.2 + 1.92T, \quad [J/(kg^{\circ}C)] \quad (3.7)$$

and

$$c^g = 1003.4 + 1.88T, \quad [J/(kg^\circ C)] \quad (3.8)$$

where superscripts c and g refers to C/PPS and G/PPS composites, respectively.

Figure 3.7: Specific heat for C/PPS and G/PPS laminates.

3.3 Identification of thermal conductivities

Different authors worked on the identification of thermal properties of laminated composites (Bailleul et al., 1996; Monteau, 2008; Villière et al., 2013). An alternative to the identification of properties consists in using homogenization techniques (Ning, Qiong-Gong; Chou, 1995; Carson et al., 2005). The following subsections describes how thermal conductivities of G/PPS and C/PPS plates were identified by using an inverse procedure which combines experimental measurements and heat transfer finite element simulations.

The measurements were made with standard thermocouples and an IR camera - as already used by different authors (Speka et al., 2008; Boué and Holé, 2012; Chudzik, 2012; Rodríguez and de Paulo Nicolau, 2012)

3.3.1 Experimental setup

An original experimental setup (Figure 3.8) was built at LMT-Cachan to heat up the central part of the upper face considering an horizontal square composite plate for a

3 Identification of mechanical and thermal properties

short period of time by using an electrical bulb. The aim is to obtain the thermal conductivities from the transient temperature distribution on the surface and through the thickness of the composite plate. The composite plate was instrumented with thermocouples placed on top and bottom surfaces, while the full-field temperature on bottom surface $z = 0$ was measured using an infrared camera. The heating phase corresponds to a constant heat flux q applied during a time interval t_h . During the cooling stage, the heat diffuses in the plate by conduction mechanisms or dissipates by convection and radiation. Infrared camera and thermocouples acquisition was triggered by the heating source.

Figure 3.8: Experimental setup for identification of thermal conductivities of G/PPS and C/PPS (500x500)mm² plates.

Infrared images were acquired using LMT-Cachan infrared camera CEDIP JADE III

3.3 Identification of thermal conductivities

MWIR (InSb), with 3.6 to 5.1 micrometers range, 320 vs. 240 px resolution, and acquisition frequency up to 150 Hz, and a NETD (Noise Equivalent Temperature Difference) equal to 25 mK. Before the tests, the camera was calibrated using a black body with reference temperature varying from 15 to 80°C with 5°C increment. Figure 3.9 shows the calibration procedure via infrared camera and black body. To provide for G/PPS and C/PPS plates the same surface, both plates were covered with a thin layer of black paint (Figure 3.10).

Figure 3.9: Infrared camera calibration with black body.

Figure 3.10: Black paint applied on G/PPS and C/PPS composite plates.

Thermocouples were placed at different position on upper and lower surfaces as shown in Figure 3.8. Thermocouples were maintained onto the surface using a thermal compound with a high thermal conductivity equal to $4.6 \text{ W}/(\text{m}^\circ\text{C})$. Caution was also taken to the initial conditions of the problem, starting the experiment when the composite plate was at the room temperature.

The temperature gradients along r-direction and z-direction are given in Figure 3.11 and Figure 3.12, respectively.

Starting of heating at $r = 0$ on the bottom surface is detected at the same time by the thermocouples and IR camera (see Figure 3.13), whereas the delay corresponds to the diffusion of temperature through the thickness (Figure 3.12). However, despite the attention paid to the way the thermocouples were “glued” onto the surface of the

Figure 3.11: Temperature gradient along r-direction measured by thermocouples at $r = 0, 10, 30$ and 60 mm on top surface.

Figure 3.12: Temperature gradient along z-direction at $r = 0$ measured by thermocouples.

plates, the difference between thermocouple indication and the IR measurements become rapidly non-negligible, so that the in-plane thermal conductivities were identified from the IR measurements.

3.3.2 IR camera measurements

IR camera information is saved in a Digital Level (DL) format as T_{DL} which must be converted to temperature in degrees Celsius, T_{DC} , according to:

$$T_{DC} = f(T_{DL}), \quad (3.9)$$

Figure 3.13: Temperature at $r = 0$ measured by IR camera and thermocouple on bottom surface.

where f results from the calibration procedure by using reference black body from 15°C up to 80°C with steps of 5°C . The value of the black body surface emissivity is assumed to be approximately equals to 1.0. A third degree polynomial is used to map T_{DL} to T_{DC} :

$$f(x, y, \xi) = \alpha_i \xi^i, \text{ for } i = 0 \dots 3 \quad (3.10)$$

as illustrated by Figure 3.14.

Figure 3.14: Conversion of IR camera indication from digital level (DL) to degrees Celsius.

An axisymmetric behaviour was observed all around the heat source axis direction (z -axis) (Figure 3.15). This hypothesis being verified, each image raw data can be reduced

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to a radial profile, written as

$$T_\mu(r, t) = \frac{2}{\pi r} \int_0^{\pi/2} T_{DL}(r, \theta, t) \delta\theta \quad (3.11)$$

where, $T_{DL}(r, \theta, t)$ denotes the temperature read by IR camera at position (r, θ) at time t . The standard deviation can be calculated as

$$\sigma(r, t) = \left\{ \int_0^{\pi/2} [T_{DL}(r, \theta, t) - T_\mu(r, t)]^2 \delta\theta \right\}^{1/2} \quad (3.12)$$

Temperature profiles derived from (3.11) are given in Figure 3.16 for G/PPS and C/PPS plates.

Figure 3.15: Axisymmetric temperature distribution.

3.3.3 Finite element model

The transient heat transfer problem was numerically simulated with finite element code Abaqus[®]. The room temperature was considered zero in the finite element model. The initial and boundary conditions of the problem are the following

$$T(r, z, t = 0) = 0 \quad (3.13a)$$

$$k_z \frac{\partial T}{\partial z} \Big|_{z=l} = q + h_t(T - T_\infty) \quad (3.13b)$$

$$k_z \frac{\partial T}{\partial z} \Big|_{z=0} = h_b(T - T_\infty) \quad (3.13c)$$

$$\frac{\partial T}{\partial r} \Big|_{r=r_{max}} = 0 \quad (3.13d)$$

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(a) G/PPS

(b) C/PPS

Figure 3.16: Temperature ($^{\circ}\text{C}$) history for G/PPS and C/PPS plates.

where h_t and h_b are the radiative-convective exchange coefficients at top and bottom surfaces, respectively, r_{max} the radius where no temperature change was experimentally observed ($r_{max} \approx 150\text{mm}$). The heat flux, q , delivered by the electrical bulb is defined as

$$q = \begin{cases} a_1 \exp(-r^2/b_1^2) + a_2 \exp(-r^2/b_2^2), & t \leq t_h \\ 0, & t > t_h \end{cases} \quad (3.14)$$

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The finite element model is schematized in Figure 3.17. Equation (3.13b) was implemented via a DFLUX user subroutine which makes it possible to prescribe a heat flux as a function of nodal coordinates and nodal temperature.

Figure 3.17: Finite element model for transient heat transfer analysis.

3.3.4 Inverse procedure

Unknowns

Experiments were performed on G/PPS and C/PPS plates. For each experiment the sets of unknowns are denoted by $\mathbf{p}^g$ and $\mathbf{p}^c$, respectively, and written as

$$\mathbf{p}^g = [k_r^g \ k_z^g \ a_1 \ a_2 \ b_1 \ b_2 \ h_b \ h_t]^T \quad (3.15)$$

$$\mathbf{p}^c = [k_r^c \ k_z^c \ \eta a_1 \ \eta a_2 \ b_1 \ b_2 \ h_b \ h_t]^T \quad (3.16)$$

where k^g and k^c denote the thermal conductivities.

The convective / radiative coefficient, h_b and h_t , are assumed to be the same for C/PPS and G/PPS plates. The heat source described by parameters a_1 , a_2 , b_1 and b_2 , is assumed to exhibit the same geometry but different magnitudes for G/PPS and

C/PPS plates. Coupling both problems, the following set of unknowns is introduced:

$$\mathbf{p} = [k_r^g \quad k_z^g \quad k_r^c \quad k_z^c \quad \eta \quad a_1 \quad a_2 \quad b_1 \quad b_2 \quad h_b \quad h_t]^T \quad (3.17)$$

Levenberg-Marquart framework

To identify $\mathbf{p}$ that minimizes the “distance” between the numerically predicted temperature $T(r, t, \mathbf{p})$ and the temperature experimentally measured $T_\mu(r, t)$, the following quadratic function is introduced

$$\mathcal{H}(\mathbf{p}) = \frac{1}{2} \iint_{\Omega_t \times \Omega_r} [T_\mu(r, t) - T(r, t, \mathbf{p})]^2 dr dt. \quad (3.18)$$

Using the Levenberg-Marquart iterative algorithm, at each iteration k ,

$$\mathbf{p}_{k+1} = \mathbf{p}_k + \delta \mathbf{p}_k \quad (3.19)$$

and

$$[\mathbf{J}^T \mathbf{J} + \lambda \text{diag}(\mathbf{J}^T \mathbf{J})] \delta \mathbf{p}_k = \mathbf{J}^T [T_\mu(r, t) - T(r, t, \mathbf{p}_k)] \quad (3.20)$$

where $\mathbf{J}$ is the Jacobian matrix such that its column $\mathbf{J}_i$ is defined as

$$\mathbf{J}_i = \frac{\partial T(r, t, \mathbf{p})}{\partial p_i}. \quad (3.21)$$

This derivative was approximated by

$$\mathbf{J}_i = \frac{\partial T(r, t, \mathbf{p})}{\partial p_i} \approx \frac{T(r, t, \mathbf{p}_i) - T(r, t, \mathbf{p})}{\Delta p_i} \quad (3.22)$$

with

$$\mathbf{p}_i = [p_1, \quad \dots \quad p_{i-1}, \quad p_i + \Delta p_i, \quad p_{i+1}, \quad \dots, p_m]^T \quad (3.23)$$

and $\Delta p = \xi p$, where ξ was chosen equal to $5 \cdot 10^{-3}$.

Then, for m unknowns, to complete an iteration, $m + 1$ finite element computations are needed. Since the temperature field $T(r, t, \mathbf{p})$ is computed for discrete values of $\mathbf{p}$. The double integral in eq. (3.18) is computed in a discrete way, considering a 1 mm element length in radial direction and a 10 s time step.

The Levenberg-Marquart algorithm was implemented coupling Abaqus[®] and Matlab[®] (Figure 3.18). Matlab[®] controls the flow of Levenberg-Marquart algorithm and Jacobian computation, (3.22), while Abaqus[®] computes the temperature for the set of parameters $\mathbf{p}$ and $\mathbf{p}_i$.

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Figure 3.18: Inverse procedure implemented by using Matlab[®] and Abaqus[®].

3.3.5 Identification strategy

The problem involves several unknowns. To try and identify all these unknowns at same time without a first good approximation can lead to values without physical sense. To workaround these problems, a first approximation of some parameters was derived from closed-form expressions, as well as, an identification procedure along z-direction.

Estimation of convection coefficients

According to Bergman et al. (2011), convection coefficients for a horizontal plate with a hot surface can be approximated by:

$$h = 0.27 k_{air} Ra^{(1/4)} / l_c \quad (3.24)$$

where k_{air} is the air thermal conductivity and $Ra = Ra(l_c)$ the Rayleigh number, whose evolution in function of temperature is given in Figure 3.19 up to 400 °C. For a characteristic length $l_c = 125mm$ and $T = 23\text{ °C}$, $k_{air} = 0.026\text{ W/(m°C)}$ and $Ra = 3.15 \cdot 10^7$ which results in the following first estimation of the convection coefficient: $h = 8.38\text{ W/(m°C)}$.

Estimation of out-of-plane thermal conductivities

Out-of-plane thermal conductivities can be estimated using an inverse procedure based on the temperature measured at $r = 0$ on top and bottom surfaces during the first seconds after heating. Assuming that during the first seconds the phenomenon is predominantly unidirectional, the problem can be treated as two parallel planes submitted to a heat flux q at $z = l$ and convection at $z = 0$ in an homogeneous media. The closed-form

3.3 Identification of thermal conductivities

(a)

(b)

Figure 3.19: Air thermal conductivity (a) and Rayleigh number (b) evolution in function of temperature.

solution to this problem is given by Carslaw and Jaeger (1986) as

$$T_{1D}(z, t) = \frac{lq}{KL} \left[1 + L \left(1 - \frac{z}{l} \right) - \sum_{n=1}^{\infty} \frac{2L(\alpha_n^2 + L^2) \cos(\alpha_n z/l)}{\alpha_n^2(L + L^2 + \alpha_n^2)} \exp(-\alpha_n^2 T) \right] \quad (3.25)$$

where α_n are the roots of:

$$\alpha_n \tan \alpha_n = L \quad (3.26)$$

and

$$L = lh \quad \text{and} \quad T = \frac{kt}{\rho c l^2} \quad (3.27)$$

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It is important to mention that the analytical equation is written for the heat flux applied on $z = 0$ and convection at $z = l$, which is the opposite of the coordinate system adopted here (Figure 3.8).

A first inverse problem can be built with unknown parameters $\mathbf{p} = [k_z^c, k_z^g, q]$, where k_z^c and k_z^g are the thermal conductivities of C/PPS and G/PPS, respectively, and q the applied heat flux, considered equal for both plates as first approximation. Then, the function to be minimized is

$$\mathcal{H} = \frac{1}{2} \int_0^t (T_{1D}(t, \mathbf{p}) - T_\mu(0, 0, t))^2 dt \quad (3.28)$$

where T_{1D} is the temperature history at $z = r = 0$ for a set of parameters $\mathbf{p}$. The temperature history measured during first 20 seconds were used as objective function.

Using the Levenberg-Marquart procedure described previously, a first estimation of $k_z^c = 0.363 \text{ W}/(\text{m}^\circ\text{C})$, $k_z^g = 0.276 \text{ W}/(\text{m}^\circ\text{C})$ and $q = 990 \text{ W}/\text{m}^2$ was identified. Figure 3.20 shows the numerically predicted and measured temperature histories.

Figure 3.20: First identification of out-of-plane thermal conductivities using an analytical 1D heat transfer model and experimental temperature history at $r=0$.

3.3.6 Results from inverse problems

Table 3.2 presents the parameters derived from the global identification procedure described in sub-section 3.3.4 using the first approximation given in sub-section 3.3.5. Figure 3.21 and 3.22 presents a comparison between the temperature profiles got derived from the finite element model and the temperature experimentally measured for G/PPS and C/PPS plates, respectively. Among all profiles used for the minimization process,

3.3 Identification of thermal conductivities

a maximum temperature difference of 2.5 °C and 2.0 °C was observed for C/PPS and G/PPS, respectively. For both plates the largest difference was observed at the beginning of cooling step. Probably, this difference is due to the radiation of the heat source that remains acting in the system some seconds after it is shut down.

Table 3.2: Parameters identified by the inverse procedure.

Plate	k_r	k_z	h_{down}	h_{up}	η	a_1	a_2	b_1	b_2
G/PPS	0.248	0.236	5.45	9.42	-	776.0	235.0	0.0137	0.0430
C/PPS	2.73	0.381			1.3

(a) Temperature profile at $t = 200$ s

(b) Temperature difference

Figure 3.21: Comparison between IR measurements and finite element predictions using the identified parameters for G/PPS plates.

(a) Temperature profile at $t = 100$ s

(b) Temperature difference

Figure 3.22: Comparison between IR measurements and finite element predictions using the identified parameters for C/PPS plates.

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CHAPTER 4

Characterization and modelling of welded joints toughness

This chapter is dedicated to the characterization and the modelling of the fracture toughness of the welded joints studied herein. The first part of the chapter is dedicated to mode I crack propagation tests, performed during this study on Double Cantilever Beam specimens cut from the joints manufactured with different processing conditions as explained in Chapter 2. Each type of specimen was loaded up to final failure either monotonically or with regular unloadings to evaluate the evolution of the specimen stiffness as the crack propagates. Whatever the manufacturing parameters chosen during this study, the observed failure mechanism corresponds to the delamination of the welded joint –and more precisely, within the fibreglass layer of one of the two assembled composite laminates. The tests were instrumented with Digital Image Correlation, and specific image processing using edge-detection Sobel algorithm was used to measure the beam kinematics all along crack propagation. The comparison between this experimentally measured beam deflection and the response of a model proposed by Kanninen (1973) is used as input of a Levenberg-Marquart algorithm to carry out the inverse identification of the crack length all along the loading sequence.

Besides, the compliance method was used to analyse the global load vs. displacement response of the specimens during the loading-unloading sequence imposed during some of the tests. This analysis shows that the energy dissipated during crack propagation can be split into the energy associated to the formation of the free surface and a non-negligible non-linear energy probably associated to plasticity mechanisms in the vicinity of the welded joint. So-called R-curve with a positive slope –corresponding to the plot of crack energy release rate vs. crack length– is representative of this type of ductile failure mode.

The last part of the chapter is dedicated to a model developed within internal variable thermodynamic framework. Assuming two dissipative mechanisms associated to crack formation and localized plasticity in the vicinity of the welded joint, respectively, this approach makes it possible to model the R-curve. This model is used to analyse the

4 Characterization and modelling of welded joints toughness

sensitivity of the welded joint fracture toughness to different manufacturing parameters. First results obtained herein show that the time during which the pressure is applied onto the joint during the cooling phase of the welding process has a key influence on the fracture toughness of the joint.

Despite the very encouraging results obtained, more investigations must be made to check the possible influence on the results of the DCB tests of the hinges used to load the specimens. First, the stiffness of these hinges may have an impact on the load vs. displacement response of the joints especially during loading-unloading tests, and, second, the friction in these hinges may induce a dissipated energy not to be mixed up with the energy associated to non-linear dissipative mechanisms in the vicinity of the joint. Besides, this model is rate independent, which may prove and be not valid for certain loading rates and environments.

However, it is worth noticing that, because of its energetic form, the model proposed herein should be applied without any difficulty to mode II tests –such as End Notched Flexure tests- and mixed mode (I+II) tests –on ASTM assembly or Dubé specimens.

Furthermore, these tests and their analysis constitute a first step in the analysis of the sensitivity of the mechanical behaviour of welded joints to the manufacturing parameters related to the mechanical and microstructural state induced by the welding process. Such a study may lead to numerous studies to optimize the welding process, to predict the mechanical and microstructural state of the joint and, finally its mechanical behaviour when submitted to various loading in various environments.

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4.1 Experimental investigation

The thermoplastic composite joints welded according to the different conditions presented in Chapter 2, were submitted to mode I crack propagation tests under monotonic loads. The tests were performed at LMT-Cachan using an electromechanical testing machine Instron 6225 (capacity 100 kN). The tests were instrumented with Digital Image Correlation in-house software Correli-Q4.

Stainless steel hinges were glued on the upper and bottom surfaces of the welded DCB specimens. Before crack propagation tests, the glue was tested to estimate its strength and checking that failure of the specimen would occur in the welded region and not in the adhesive layer between the hinges and the hybrid composite surface. The hinges were glued using a Loctite 406 adhesive after applying a first layer of Loctite Primer 770 on the PPS surface.

A reflex Canon 60D with a 60 mm objective macro was facing one side of the specimen while a Pixelfly camera with a 0.5 magnitude telecentric objective was facing opposite side. The first camera gives a global view of the specimen with the hinges, pre-crack and welded regions. The second camera was focused on the welded region. A photograph of the experimental setup is given in Figure 4.1.

Figure 4.1: DCB specimen test setup.

4.1.1 Typical load vs. displacement curve

A typical example of the global load vs. displacement response is plotted in Figure 4.2. This response can be divided in two parts, viz. the “envelope” which would be the response under monotonous loading and the elastic response during unloading. These elastic responses may be extrapolated down to zero load so that the measured

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displacement may be split into elastic and plastic parts such that

$$u = u_e + u_p \quad (4.1)$$

as illustrated in Figure 4.3.

Figure 4.2: Load vs. displacement curve for specimen D

Figure 4.3: Partition of displacement.

4.1.2 Identification of crack length

The position of the crack tip was identified solving an inverse problem combining the beam kinematics measured by image processing and Kanninen solution for DCB

specimen geometry. Using Digital Image Correlation to identify the crack length proved to be difficult because of the limited number of pixels along the specimen thickness. These few pixels along thickness direction were not enough for a proper analysis using standard Correli-Q4 in house DIC software. However, the images have been saved for future processing using triangular discretization and a regularization of the displacement field - which have already been used successfully for the identification of the displacement field in similar specimens (Mathieu et al., 2013).

Beam kinematics

The beam kinematics has been identified by image processing thanks to Sobel operator (Kittler, 1983). Sobel operator is a 3×3 kernel that operates on the intensity of the image, here called F . The intensity on grayscale images takes values from 0 to $2^{n_{bits}}$, where n_{bits} denotes the number of bits, here $n_{bits} = 16$. Sobel operator can be defined for horizontal, A_H , and vertical, A_V , directions such that

$$A_H = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix}, \quad (4.2)$$

and

$$A_V = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}. \quad (4.3)$$

Mathematically, both consists in a combination of a smoothing operator and central differential operators (Kittler, 1983). The final image G is obtained by applying Sobel operators A_H and A_V to original image F , so that

$$G = \sqrt{(A_H * F)^2 + (A_V * F)^2} \quad (4.4)$$

where $*$ denotes the 2D convolution¹ operation. Figure 4.4 illustrates the application of the convolution operation in the two dimensional context.

Figure 4.4: Image convolution.

An example of an image before and after convolution with Sobel operator is given in Figure 4.5. It can be seen that the specimen boundary is clearly delimited by a white line. Image computations were performed using Image Processing Toolbox from Matlab[®].

Figure 4.5: Original image and after convolution with Sobel operator.

The image after application of Sobel operator is a black (value 0) and white (value 1) image. Scanning its column x , makes it possible to capture a signal where the unitary limits gives the top and bottom surfaces deflections, $v_t(x)$ and $v_b(x)$, respectively (Figure 4.6). From these deflections, beam opening, Δ , in pixels can be computed as:

$$\Delta(x) = v_t(x) - v_b(x) - 2t \cos \alpha \quad (4.6)$$

¹Considering two square matrices signals A and B , not necessarily with the same size, the convolution $C = A * B$ is given by:

$$C[m, n] = \sum_{j=-\infty}^{\infty} \sum_{i=-\infty}^{\infty} A[i, j] \cdot B[m - i, n - j] \quad (4.5)$$

where i, j, m and n are matrix index.

where t denotes the laminate thickness, α the angle of the beam middle plane with respect to the horizontal, and v_t and v_b the deflection of upper and lower beam surfaces. Rotation α is clearly observed at the beginning of the test when specimens weight is non-negligible compared to the load applied by the hinges. The largest value of the rotation observed during the tests was equal to 7.3° .

(a) Column selected.

(b) Profile AB signal.

Figure 4.6: Signal captured from image column for the identification of top and bottom surface deflections (specimen D).

The opening in pixels was computed for every picture using relation (4.6) as illustrated by Figure 4.8. The conversion from pixels to millimeters was made by using a reference length, viz. the beam total thickness, $2t$.

4.1.3 DCB deflection

Kanninen (1973) studied the DCB specimen taking into account the region beyond the crack tip. The kinematics of a DCB specimen is described as the solution of an Euler-Bernoulli beam lying on an elastic foundation (Figure 4.9). The crack tip is

Figure 4.7: Top and bottom DCB specimen deflection identified for specimen D.

Figure 4.8: Beam opening in pixels after identification of the deflection for specimen D.

located at distance a from the loading axis while the specimen length is L and the elastic foundation stiffness is equal to k .

Figure 4.9: Kanninen model for DCB specimen

Considering two sub-domains, $\Omega_1 = [-a, 0]$ and $\Omega_2 = [0, c]$, the beam deflection v is the solution of the following two differential equations:

$$EI \frac{d^4 v}{dx^4} = 0, \quad x \in \Omega_1 \quad (4.7)$$

$$EI \frac{d^4 v}{dx^4} + kv = 0, \quad x \in \Omega_2 \quad (4.8)$$

where E denotes the Young's modulus in direction x , I the cross section bending modulus in plane yz , and k the stiffness of the beam foundation. Equations 4.7 and 4.8 can be combined and rewritten as

$$\frac{d^4 v}{dx^4} + 4\lambda^4 H(x)v = 0 \quad (4.9)$$

where $H(x)$ is Heaviside function such that

$$H(x) = \begin{cases} 0, & x \in \Omega_1 \\ 1, & x \in \Omega_2 \end{cases} \quad (4.10)$$

and where $\lambda^4 = k/(4EI)$.

The general solution for (4.9) can be written as

$$v(x) = \begin{cases} v_1(x) = c_1 \frac{1}{6}x^3 + c_2 \frac{1}{2}x^2 + c_3x + c_4, & x \in \Omega_1; \\ v_2(x) = d_1 e^{xc} \sin(xc) + d_2 e^{xc} \cos(xc) + \\ + d_3 e^{-xc} \sin(xc) + d_4 e^{-xc} \cos(xc), & x \in \Omega_2. \end{cases} \quad (4.11)$$

where c_i and d_i , $i = 1 \cdots 4$, are constants to be identified. The displacement, rotation, bending moment and internal shear force must be continuous between domains Ω_1 and Ω_2 , so that

$$v_1 = v_2, \quad v'_1 = v'_2, \quad v''_1 = v''_2, \quad v'''_1 = v'''_2, \quad \text{at } x = 0 \quad (4.12)$$

The problem must satisfy the following boundary conditions:

$$v''(x = -a) = 0 \quad (\text{no flexural loads}) \quad (4.13)$$

$$v'''(x = -a) = P/(EI) \quad (\text{prescribed shear load}) \quad (4.14)$$

$$v'''(x = c) = w''(x = c) = 0 \quad (\text{free edge condition}) \quad (4.15)$$

To introduce the above conditions in general solution (4.11), it is convenient to write

the system in the following matrix form

$$\begin{bmatrix} \varphi_1''(x=-a) & 0 \\ \varphi_1'''(x=-a) & 0 \\ 0 & \varphi_2''(x=c) \\ 0 & \varphi_2'''(x=c) \\ \varphi_1(x=0) & -\varphi_2(x=0) \\ \varphi_1'(x=0) & -\varphi_2'(x=0) \\ \varphi_1''(x=0) & -\varphi_2''(x=0) \\ \varphi_1'''(x=0) & -\varphi_2'''(x=0) \end{bmatrix} \begin{Bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ d_1 \\ d_2 \\ d_3 \\ d_4 \end{Bmatrix} = \begin{Bmatrix} 0 \\ Q/(EI) \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \quad (4.16)$$

where

$$\varphi_1 = [x^3/6 \ x^2/2 \ x \ 1] \quad (4.17)$$

$$\varphi_2 = [e^{xc} \sin(xc) \ e^{xc} \cos(xc) \ e^{-xc} \sin(xc) \ e^{-xc} \cos(xc)] \quad (4.18)$$

$$c = \{c_1 \ c_2 \ c_3 \ c_4\}^T \quad (4.19)$$

$$d = \{d_1 \ d_2 \ d_3 \ d_4\}^T \quad (4.20)$$

The general solution for the deflection according to Kanninen model is:

$$v(x) = \frac{6P}{Ebh^3\lambda^3} \begin{cases} \frac{\lambda^3 x^3}{3} + a\lambda^3 x^3 - A\lambda x + B \\ a\lambda \sin \lambda x \sinh \lambda x - \left(\frac{A-1}{2}\right) \sin \lambda x \cosh \lambda x + \\ B \cos \lambda x \cosh \lambda x - \left(\frac{A+1}{2}\right) \cos \lambda x \sinh \lambda x \end{cases} \quad (4.21)$$

where constants A and B have the following expressions

$$A = \left(\frac{\sinh^2 \lambda c + \sin^2 \lambda c}{\sinh^2 \lambda c - \sin^2 \lambda c} \right) + 2a\lambda \left(\frac{\sinh \lambda c \cosh \lambda c + \sin \lambda c \cos \lambda c}{\sinh^2 \lambda c - \sin^2 \lambda c} \right) \quad (4.22)$$

$$B = \left(\frac{\sinh \lambda c \cosh \lambda c - \sin \lambda c \cos \lambda c}{\sinh^2 \lambda c - \sin^2 \lambda c} \right) - a\lambda \left(\frac{\sinh^2 \lambda c + \sin^2 \lambda c}{\sinh^2 \lambda c - \sin^2 \lambda c} \right) \quad (4.23)$$

To illustrate the combination of the two solutions, Figure 4.10 presents the solutions v_1 and v_2 for $EI = 10^5$ MPa mm⁴, $k = 5 \cdot 10^3$ MPa mm⁴, $L = 50$ mm, $a = 25$ mm and $Q = 100$ N. The combination of these functions gives the deflection plotted in Figure 4.11. It can be noted an oscillatory behaviour of v_2 close to $x = 0$, which can lead to negative values, as can be seen on zoom given in Figure 4.11. As higher the value of k , as smaller is the difference between DCB solution and the solution of a cantilever beam, *i.e.*, $v(x = 0) = 0$.

Figure 4.10: Solutions v_1 and v_2 Figure 4.11: Kanninen solution. The selected region shows the oscillation of the function around $x = 0$.

Beam kinematics appears as being function of three parameters, viz. crack position a , factor λ and multiplier C . Given this set of parameters $\mathbf{p} = [a \ \lambda \ C]$, beam deflection,

$v(x, \mathbf{p})$, is written as:

$$v(x, \mathbf{p}) = \frac{C}{\lambda^3} \begin{cases} \frac{\lambda^3 x^3}{3} + a\lambda^3 x^3 - A\lambda x + B \\ a\lambda \sin \lambda x \sinh \lambda x - \left(\frac{A-1}{2}\right) \sin \lambda x \cosh \lambda x + \\ B \cos \lambda x \cosh \lambda x - \left(\frac{A+1}{2}\right) \cos \lambda x \sinh \lambda x \end{cases} \quad (4.24)$$

where A and B are defined by (4.22) and (4.23).

4.1.4 Inverse identification

The identification of parameters using an inverse procedure has been presented in previous chapter and the same approach is adopted. To identify set of parameters $\mathbf{p}$ that minimizes the function

$$\mathcal{F} = \frac{1}{2} \int_L [v(x, \mathbf{p}) - v_{obj}(x)]^2 dx \quad (4.25)$$

where L denotes the specimen length, $v(x, \mathbf{p})$ the deflection derived from Kanninen model and v_{obj} the objective function.

The oscillatory behaviour of Kanninen solution in the vicinity of $x = 0$ can lead the algorithm to try and identify infinite values for stiffness k , and consequently, lead to an error in the identification of the crack tip. To workaround this limitation of the closed-form solution, the convergence criterion is chosen such that $v_{obj} = v_{exp}$ if $v_{exp} > 0.05 \max(v_{exp})$.

Following the same type of iterative approach as in Chapter 2, at iteration $k + 1$, the parameters are calculated as

$$\mathbf{p}_{k+1} = \mathbf{p}_k + \delta \mathbf{p}_k \quad (4.26)$$

where $\delta \mathbf{p}_k$ is the solution of the following equation

$$[\mathbf{J}^T \mathbf{J} + \beta \text{diag}(\mathbf{J}^T \mathbf{J})] \delta \mathbf{p}_k = -\mathbf{J}^T [v(x, \mathbf{p}) - v_{obj}(x)] \quad (4.27)$$

and where $\mathbf{J}$ is the Jacobian matrix, such that each column J_i is computed as

$$\mathbf{J}_i = \frac{\partial v(x, \mathbf{p})}{\partial p_i} \quad (4.28)$$

For each picture $j + 1$, the set of initial values, $\mathbf{p}_0$, is chosen as the set of the values identified at the end of previous step, so that $\mathbf{p}_0^j = \mathbf{p}^{j-1}$. The crack position is defined

as the distance between the loading axis and the crack tip. A typical example of the evolution of the crack position as identified for each picture j of the envelope of load vs. displacement curve is plotted in Figure 4.12.

The identification of the crack position is stopped at after photo 700 since Kanninen solution is not valid for $c > 2h$ (Kanninen, 1973). To avoid the influence of high-order phenomena, it is considered in the following that, $\max(a - a_0) = 25$ mm. Figure 4.13 shows the crack tip position identified (dashed white line) for three different photographs.

Figure 4.12: Crack position identified for specimen D.

Figure 4.13: Visualization of the crack position identified for photos 100 (a), 150 (b) and 200 (c) for specimen D.

4.2 Identification of dissipated energies

Once the crack length is identified, the so-called compliance method may be used to derive the energy strain release rate, G , from the experimental results, so that

$$G = \frac{-1}{2} u_e^2 \frac{dR}{dA} = \frac{-1}{2b} u_e^2 \frac{dR}{da} \quad (4.29)$$

where u_e denotes the elastic part of the displacement, R the structural stiffness and b the specimen width such that cracked surface, A , is equal to $A = b \cdot a$. The energy introduced in the system via the external loads from the beginning of the test until time t is computed as

$$W_{ext} = \int_t Q \dot{u} dt \quad (4.30)$$

whereas the elastic energy stored by the material can be written as

$$\Psi_e = \frac{1}{2} R u_e^2|_t \quad (4.31)$$

and the energy consumed by crack propagation as

$$\Psi_{crack} = \int_t G \dot{A} dt \quad (4.32)$$

A typical evolution of these energies is plotted in Figure 4.14. It can be seen that the sum of the energy stored elastically and the energy dissipated by crack propagation differs from the work done by the external loads by a non-negligible amount that is equal to approximately 22% of the total energy at $t = 4000$ s. This large dissipated energy must be considered when modelling the test as explored in next section. As a first approximation, this energy will be associated to dissipative mechanisms localized at the crack tip.

Figure 4.14: Energy contributions identified experimentally for specimen D.

4.3 Ductile crack propagation model

4.3.1 Thermodynamical approach

This section is dedicated to describing the mechanical behaviour of the composite welded joint submitted to mode I loading. This model is written within a thermodynamical framework. Figure 4.15 presents a solid with a single crack submitted to a single load Q .

Figure 4.15: Solid with crack

The state variables and the associated driving forces chosen to describe the thermo-mechanical state of the solid Ω are listed in Table 4.1. It is convenient for the mathematical development that follows to define the subset of the state variables $\mathbf{V}^* = [A \ A_p]$ associated to the subset of driving forces $\mathbf{A}^* = [G \ -G_p]$.

Table 4.1: State variables and associated driving forces

State variables	Associated forces
Temperature (T)	Specific entropy ($-s$)
Displacement (u)	Load (Q)
Plastic displacement (u_p)	Load (Q_p)
Cracked surface (A)	Strain energy release rate (G)
Plastic internal variables (A_p)	($-G_p$)

Helmoltz's free energy of solid Ω is postulated in the following form

$$\Psi(u, u_p, A, A_p) = \Psi_e(u - u_p, A) + \Psi_p(A_p) \quad (4.33)$$

where Ψ_e and Ψ_p denote the elastic and plastic energy contributions, respectively. It is considered that the temperature of the entire solid Ω remains equal to room temperature, even during the loading process. It is also considered that plastic energy, Ψ_p , is modelled as a function of a single internal variable denoted by A_p . The elastic energy can be expressed as

$$\Psi_e = \frac{1}{2}R(A)(u - u_p)^2 \quad (4.34)$$

where R is the solid stiffness, which is a function of crack surface, A . The plastic energy

is postulated as:

$$\Psi_p = \Lambda_0 A_p + \frac{\Lambda_1}{n+1} A_p^{n+1} \quad (4.35)$$

where Λ_0 and Λ_1 are constants to be identified. From (4.33), the state laws can be written as

$$Q = \frac{\partial \Psi}{\partial u} = \frac{\partial \Psi_e}{\partial u} = R(A)(u - u_p) \quad (4.36)$$

$$Q_p = -\frac{\partial \Psi}{\partial u_p} = -\frac{\partial \Psi_e}{\partial u} = -R(A)(u - u_p) = -Q \quad (4.37)$$

$$G = -\frac{\partial \Psi}{\partial A} = -\frac{\partial \Psi_e}{\partial A} = -\frac{1}{2}(u - u_p)^2 \frac{\partial R}{\partial A} \quad (4.38)$$

$$-G_p = -\frac{\partial \Psi}{\partial A_p} = -\frac{\partial \Psi_p}{\partial A_p} = -\Lambda_0 - \Lambda_1 A_p^n \quad (4.39)$$

The intrinsic system dissipation can be computed as

$$\mathcal{D} = P_{ext} - (\dot{\Psi} + s\dot{T}) \quad (4.40)$$

where $P_{ext} = Q\dot{u}$ is the power exercised by the exterior loads since the process is supposed to be isothermal, term $s\dot{T}$ vanishes from (4.40), so that (4.40) can be rewritten as

$$\mathcal{D} = Q\dot{u} - (Q\dot{u} - Q_p\dot{u}_p - G\dot{A} + G_p\dot{A}_p) = Q_p\dot{u}_p + G\dot{A} - G_p\dot{A}_p = \mathcal{D}^{(I)} + \mathcal{D}^{(II)} \quad (4.41)$$

where $\mathcal{D}^{(I)} = Q_p\dot{u}_p$ and $\mathcal{D}^{(II)} = G\dot{A} - G_p\dot{A}_p$. First term, $Q\dot{u}_p$, correspond to the energy dissipated by the plastic deformation. Second term, $G\dot{A}$, corresponds to the energy associated to the extension of crack area, A . Last term, $G_p\dot{A}_p$, is assumed to be related to non-linear mechanisms localized in the crack propagation zone. It is recalled that total dissipation must be such that $\mathcal{D} \geq 0$ for the process to be thermodynamically consistent.

Crack propagation is assumed to derive from a criterion function f written as

$$f(G, -G_p) = G - G_p - G_c \leq 0 \quad (4.42)$$

so that $f < 0$ corresponds to no crack propagation and $f = 0$ to crack propagation.

The evolution law for internal variables $\mathbf{V}^*$ is assumed to maximize the dissipated energy $\mathcal{D}^{(II)}$ under the constraint $f(G, -G_p) = 0$, which is equivalent to write

$$\min_{\mathbf{A}} \left[-\mathcal{D}^{(II)} + \dot{\lambda} \frac{\partial f}{\partial \mathbf{A}^*} \right] \quad (4.43)$$

which leads to the following generalized normality rule

$$\dot{\mathbf{V}} = \dot{\lambda} \frac{\partial f}{\partial \mathbf{A}^*} \quad (4.44)$$

Expanding (4.44), it writes

$$\dot{A} = \dot{\lambda} \frac{\partial f}{\partial G} = \dot{\lambda} \quad (4.45)$$

$$\dot{A}_p = -\dot{\lambda} \frac{\partial f}{\partial (-G_p)} = \dot{\lambda} \quad (4.46)$$

where λ is a scalar positive multiplier. From (4.45) and (4.46) it can be concluded that

$$\dot{A} = \dot{A}_p = \dot{\lambda} \quad (4.47)$$

It is worth noticing that from (4.47), the dissipation can be rewritten as $\mathcal{D} = Q_p \dot{u}_p + (G - G_p) \dot{A}$. Hence $\dot{u}_p$ and $\dot{A}$ are positive and $G \geq G_p$, the second thermodynamical principle $\mathcal{D} \geq 0$ is verified.

Assuming as a first approximation that the crack propagation process is rate independent - for low propagation rates - the following consistency condition is verified

$$\dot{f} = 0 \quad \therefore \quad \dot{G} - \dot{G}_p = 0 \quad \text{when } f = 0 \quad (4.48)$$

It is possible to derive A_p from (4.39) and (4.48), so that

$$A_p = \left(\frac{G - G_c - \Lambda_0}{\Lambda_1} \right)^{1/n} \quad (4.49)$$

Consequently, the crack propagation law can be derived from (4.47), so that

$$A - A_0 = \left(\frac{G - G_c - \Lambda_0}{\Lambda_1} \right)^{1/n} \quad (4.50)$$

or

$$G = G_c + \Lambda_0 + \Lambda_1 (A - A_0)^n \quad (4.51)$$

According to this model the energy release rate is equal to $G_c + \Lambda_0$ at crack initiation and increases during crack propagation leading to so-called R-curve.

4.3.2 Identification of model parameters G_c , Λ_0 , Λ_1 and n

The identification of the parameters was performed while assuming that the crack front is linear, so that $A - A_0 = b(a - a_0)$ where b is the specimen width. The results

of this identification procedure is illustrated by Figures 4.16 and 4.17 corresponding to specimen D. Parameters Λ_0 is identified as the angle at $a - a_0 = 0$ in Ψ_p vs. $a - a_0$ curve, see Figure 4.17. Parameter G_c is identified from the threshold and parameters Λ_1 and n from the nonlinear part of R-curve, see Figure 4.16.

Figure 4.16: R-curve and model fitting.

Figure 4.17: Energy consumed by crack propagation and plastic mechanisms.

4.4 Influence of the manufacturing parameters

The model developed in previous sections was applied to the tests performed on the specimens manufactured with the different processing conditions given in Table 2.5

First, it is important to notice that for all processing conditions, the predominant failure mode is the same, viz. the delamination between the glass fibres and the PPS matrix in the G/PPS layer close to the welded joint (Figure 4.18). In other words, whatever the manufacturing parameters - within the range investigated herein - the failure mechanisms and hence, the values of the parameters given in Table 4.2 are related to the behaviour of the outer layer of the hybrid composite laminate that is melted during the welding process. The experimental R-curve and the corresponding response of the model are plotted in Figure 4.19 for all joint manufactured according to the parameters in Table 4.2.

Figure 4.18: Delamination between the glass fibres and PPS matrix.

Table 4.2: Identified model parameters for different processing conditions.

	Mesh	$\bar{i}$	t_h	t_p	p_0	G_c	Λ_0	Λ_1	n	$G_c + \Lambda_0$
	-	[A]	[s]	[s]	[MPa]	[kJ/m ²]	[kJ/m ²]	SI*	SI*	[kJ/m ²]
A	M200	30	17	77	1.4	252.7	20.7	119.6	0.46	273.4
B	M270	25	17	77	1.4	142.3	86.4	94.63	0.40	228.7
C	M200	30	17	197	1.4	333.8	196.2	102.7	0.36	530.0
D	M270	25	17	197	1.4	396.5	180.5	102.9	0.43	577.0

* International System units

The load vs. displacement response of all specimens tested are plotted in Figure 4.20(a) for monotonic tests and 4.20(b) for monotonic tests with unloadings. More tests

Figure 4.19: R-curve for different processing conditions.

should be performed to verify the repeatability of these results and, at this stage, it is impossible to conclude on the possible influence of unloading on the post-peak response of the cracked specimens.

Such influence could be explained by non-negligible stiffness - including the unloading phases - of the hinges used to load the specimens.

However, it seems that the time during which the pressure is maintained on the joint during the cooling phase of the welding process has a first order influence on the toughness, as confirmed by the difference of $(G_c + \Lambda_0)$ values for specimens A and B, or C and D.

The influence of the grid spacing - which is different for specimens A and C or B and D - does not look significant.

(a) Monotonic tests

(b) Tests with unloadings

Figure 4.20: Load vs. displacement response for difference manufacturing conditions, see Table 2.5.

4.5 Remarks on the model

The model presented herein does not take into account the blunting phenomenon that corresponds to the plastic deformation localized at crack tip before crack propagates. To introduce this phenomenon, a new state variable, u_b , could be introduced with the associated driving force, Q_b . Then, Helmholtz's free energy potential would take the following form:

$$\Psi(u, u_p, u_b, A, A_p) = \Psi_e(u - u_p - u_b, A) + \Psi_p(A_p) + \Psi_b(u_b) \quad (4.52)$$

where the “non-elastic” displacement is split in two parts, viz. u_p and u_b . The term Ψ_b should increase during the first loading and then remain constant when crack propagates, as schematically illustrated by Figure 4.21.

Figure 4.21: A possible approach to model the blunting phenomenon.

Besides, it is worth noticing that consistency condition (4.48) is probably not valid at high loading rates for which the behaviour of the joint may become viscous.

Last, it is recalled that the composite plates do not exhibit any remaining deformation after the welded joint is fully broken, which justifies the assumption made herein according which the dissipative mechanisms modeled by state variable A_p are localized in the crack propagation zone - and would be fully consistent with a nonlinear cohesive model. However, further tests should be made to verify that part of the energy associated to this variable is not related to friction in the hinges used to load the DCB specimens.

To summarize, more investigations should be made before the approach proposed herein is fully validated. However, it is worth noticing that the model proposed is written in terms of the strain energy release rate so that it should be possible to apply it to a more complex situations where the joints are submitted to mode II or mixed mode (I + II) loading.

CHAPTER 5

An introduction to PGD solution of multidimensional problems

Because of the number of parameters and the very different length scales along in-plane and through the thickness directions, full 3D numerical simulations of heat transfer or/and mechanical problems related to the composite laminates studied herein are very expensive –even for linear cases. Model reduction techniques must be developed for this kind of numerical simulations to be performed at the structural scale. This chapter is dedicated to a very promising model reduction technique, viz. Proper Generalized Decomposition (PGD). PGD solution procedure is applied to an axisymmetric heat transfer problem using a fully separated temperature field. The temperature is written as the product of three 1D functions. A finite element framework is applied to the solution of space functions and a Runge-Kutta algorithm to the solution of the time function. The verification of the boundary conditions –which is a key point of PGD approach– is explained in details. The program developed in the Matlab[®] environment is executed to three cases. The numerical results are favourably compared to close-form or numerical reference solutions. It is shown that the PGD solution procedure leads to computational costs which are lower by an order of magnitude than the cost of standard finite element simulations –even for the simple cases tested. The framework developed during this study constitutes a solid base for future developments and extensions to solve more complex heat transfer or mechanical problems related to the materials and joints studied herein. In particular, the separation of different space –in-plane and out-of-plane–, time, and for instance material variables should prove and be efficient to solve direct or inverse

problems.

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5.1 Introduction

The cost of numerical simulations of multidimensional and/or multi-parametric models may be very high even in the case of linear systems. A typical case of such model with a very large number of variables could be the FE modelling of a multi-layered composite structure. Taking account of the different constituents and interfaces in order to make it possible to model the different damage mechanisms within the matrix or the fibres in each ply and at the interfaces between the plies. One strategy to reduce the computational costs consists in solving a reduced-order system (Anthony and Nouy, 2010; Ammar, 2009). One of the frameworks often used is the so-called POD (Proper Orthogonal Decomposition) technique, which consists in using a function space extracted from a previous solution to approximate the solution for problems with similar set of parameters or analogous boundary conditions. However, this technique depends on the knowledge of an a priori solution for a case not so distant from the case of interest. PGD (Proper Generalized Decomposition) solution procedure comes as an alternative for these limitations since it does not depend on a priori solution and presents an strategy to enhance the function basis if needed. PGD method can be treated as a generalization of POD method (Anthony and Nouy, 2010).

PGD solution procedure consists in approximating a generic field $u(x_1, \dots, x_p)$ by

$$u(x_1, \dots, x_p) \approx \sum_{i=1}^n \mathcal{F}_i^1(x_1) \times \dots \times \mathcal{F}_i^p(x_p) = \sum_{i=1}^n [\Pi_{j=1}^p F_i^j] \quad (5.1)$$

where $\mathcal{F}_i^j$ are functions defined in sub-spaces of u , and x_k are any usual coordinate (scalar or vectorial) (Chinesta et al., 2010a,b, 2011), so that the solution is approximated as the sum of n enrichments, computed as the product of functions. This decomposition of the solution reduces the size of the global system and hence the computational costs.

This approach has been successfully applied to several problems. For instance, this solution procedure has been applied to models with different time scales so that different time dependent functions are introduced (Ammar et al., 2011). PGD method has also been applied to solve fluid mechanics problems (Dumon et al., 2011), to model and optimize pultrusion process (Ghnatios et al., 2011, 2012) or to solve multidimensional parametric models (Pruliere et al., 2010).

During this study, PGD solution procedure was developed for an axisymmetric heat transfer problem. This work can be the basis of PGD-based inverse analysis to identify separately in-plane and out-of-plane material parameters of composite laminates.

5.2 Problem description

The goal is to find the temperature field, $T(r, z, t)$, in an axisymmetric solid Ω enclosed by a surface Γ , where $r \in \Omega_r \equiv [0, a]$, $z \in \Omega_z \equiv [0, b]$ and $t \in \mathcal{I} \equiv [0, \bar{t}]$ (Figure 5.1). The solid mass density is denote by ρ , specific heat capacity by c and thermal conductivities by k_r and k_z in radial and z-directions, respectively. Temperature $T(r, z, t)$ is solution of the heat transfer equation ¹ written as

$$\rho c \dot{T} - \text{div}(\mathbf{K} \nabla T) = 0 \quad (5.2)$$

where

$$\mathbf{K} = \begin{bmatrix} k_r & 0 \\ 0 & k_z \end{bmatrix} \quad (5.3)$$

Figure 5.1: Axisymmetric heat transfer problem.

The weak form of equation (5.2) writes²

$$\int_{\mathcal{I}} \int_{\Omega} \delta T \left[\rho c \dot{T} - \text{div}(\mathbf{K} \nabla T) \right] d\Omega dt = 0 \quad (5.4)$$

where δT denotes a virtual temperature field, enough continuous and homogeneous on essential boundary conditions.

¹The divergent in an axisymmetric coordinate system for a vector $\mathbf{v} = \{v_r, v_z\}$ is given by

$$\text{div}(\mathbf{v}) = \frac{1}{r} \frac{d}{dr} (r v_r) + \frac{dv_z}{dz}$$

²The volume $d\Omega$ in axisymmetric coordinate system is given by $d\Omega = 2\pi r dr dz$.

5.3 PGD solution

Using a PGD type solution, temperature $T(r, z, t)$ can be written via the following space-time decomposition

$$T(t, r, z) = T_0 + \sum_{i=1}^N \mathcal{T}_i(t) \cdot \mathcal{X}_i(r, z) \quad (5.5)$$

or, as proposed herein, as the following complete decomposition

$$T(t, r, z) = T_0 + \sum_{i=1}^n \mathcal{R}_i(r) \mathcal{Z}_i(z) \mathcal{T}_i(t) + \mathcal{R}_{n+1}(r) \mathcal{Z}_{n+1}(z) \mathcal{T}_{n+1}(t) \quad (5.6)$$

where each product of functions $\mathcal{R}_i$, $\mathcal{Z}_i$ and $\mathcal{T}_i$ is called enrichment of the solution. Three unidimensional functions and the enrichment computed as their product are illustrated by Figure 5.2

Figure 5.2: PGD functions (a) and enrichment (b).

Term T_0 is an initial approximation of the temperature field which must verify the initial and boundary conditions. It can be written as the combination of d enrichments,

that

$$T_0 = \sum_{i=1}^d \mathcal{R}_i(r) \mathcal{Z}_i(z) \mathcal{T}_i(t) \quad (5.7)$$

For simplicity sake, in the sequel the functions are denoted by: $R_i = R_i(r)$, $Z_i = Z_i(z)$ and $T_i = T_i(t)$. The solution is approximated by the sum of $d + n$ enrichments, as illustrated by Figure 5.3.

Figure 5.3: Typical PGD solution scheme.

Expanding (5.4), the weak form can be rewritten as

$$\int_{\mathcal{I}} \int_{\Omega} \delta T \left[\rho c \frac{\partial T}{\partial t} - \left(k_r \frac{1}{r} \frac{\partial T}{\partial r} + k_r \frac{\partial^2 T}{\partial r^2} + k_z \frac{\partial^2 T}{\partial z^2} \right) \right] d\Omega dt = 0 \quad (5.8)$$

Assuming that all n functions were computed, the objective is to determine R_{n+1} , Z_{n+1} , and T_{n+1} and subsequently, one more enrichment to the field approximation. The virtual temperature field $\delta\theta$ can be written as

$$\delta T = \delta \mathcal{R}_{n+1} \mathcal{Z}_{n+1} \mathcal{T}_{n+1} + \mathcal{R}_{n+1} \delta \mathcal{Z}_{n+1} \mathcal{T}_{n+1} + \mathcal{R}_{n+1} \mathcal{Z}_{n+1} \delta \mathcal{T}_{n+1} \quad (5.9)$$

The introduction of (5.6) and (5.9) into (5.8) results in a non-linear problem that

5.4 Computation of functions $\mathcal{R}_{n+1}$, $\mathcal{Z}_{n+1}$ and $\mathcal{T}_{n+1}$

can be solved using an iterative scheme. Without losing generality, the virtual field (eq. 5.9) can be taken in a convenient way to evaluate separately each function $\mathcal{R}_{n+1}$, $\mathcal{Z}_{n+1}$, and $\mathcal{T}_{n+1}$: assuming $\delta\mathcal{T}_{n+1} = 0$ and $\delta\mathcal{Z}_{n+1} = 0$ makes it possible to compute $\mathcal{R}_{n+1}$; assuming $\delta\mathcal{R}_{n+1} = 0$ and $\delta\mathcal{T}_{n+1} = 0$ makes it possible to compute $\mathcal{Z}_{n+1}$; assuming $\delta\mathcal{R}_{n+1} = 0$ and $\delta\mathcal{Z}_{n+1} = 0$ makes it possible to compute $\mathcal{T}_{n+1}$. More details of this approach are given in following sub-sections.

5.4 Computation of functions $\mathcal{R}_{n+1}$, $\mathcal{Z}_{n+1}$ and $\mathcal{T}_{n+1}$

5.4.1 Function $\mathcal{R}_{n+1}$

Taking $\delta\mathcal{Z} = \delta\mathcal{T} = 0$ and $\delta\mathcal{R} \neq 0$ in (5.9), and introducing it and (5.6) into (5.8) results in

$$\begin{aligned} \int_{\mathcal{I}} \int_{\Omega} \delta\mathcal{R}_{n+1} \mathcal{Z}_{n+1} \mathcal{T}_{n+1} \left[\rho c \mathcal{R}_{n+1} \mathcal{Z}_{n+1} \frac{d\mathcal{T}_{n+1}}{dt} - k_r \frac{1}{r} \frac{d}{dr} \left(r \frac{d\mathcal{R}_{n+1}}{dr} \right) \mathcal{Z}_{n+1} \mathcal{T}_{n+1} \right. \\ \left. - k_z \mathcal{R}_{n+1} \frac{d^2 \mathcal{Z}_{n+1}}{dz^2} \mathcal{T}_{n+1} \right] d\Omega dt = - \int_{\mathcal{I}} \int_{\Omega} \delta\mathcal{R}_{n+1} \mathcal{Z}_{n+1} \mathcal{T}_{n+1} \left[\rho c \sum_{i=1}^n \mathcal{R}_i \mathcal{Z}_i \frac{d\mathcal{T}_i}{dt} \right. \\ \left. - k_r \sum_{i=1}^n \frac{1}{r} \frac{d}{dr} \left(r \frac{d\mathcal{R}_i}{dr} \right) \mathcal{Z}_i \mathcal{T}_i - k_z \sum_{i=1}^n \mathcal{R}_i \frac{d^2 \mathcal{Z}_i}{dz^2} \mathcal{T}_i \right] d\Omega dt \end{aligned} \quad (5.10)$$

which makes it possible to compute function $\mathcal{R}$. Above equation can be integrated on Ω_z and $\mathcal{I}$, (5.10) so that

$$\begin{aligned} A_1 \int_{\Omega_r} \delta\mathcal{R}_{n+1} \mathcal{R}_{n+1} r dr - A_2 \int_{\Omega_r} \delta\mathcal{R}_{n+1} \frac{d}{dr} \left(r \frac{d\mathcal{R}_{n+1}}{dr} \right) dr = \\ - \int_{\Omega_r} (A_3 + A_4) \delta\mathcal{R}_{n+1} r dr - \int_{\Omega_r} A_5 \delta\mathcal{R}_{n+1} dr \end{aligned} \quad (5.11)$$

where

$$A_1 = \rho c z_{00} t_{01} - k_z z_{02} t_{00} \quad (5.12)$$

$$A_2 = k_r z_{00} t_{00} \quad (5.13)$$

$$A_3 = \sum_{i=1}^n (\rho c z_{00i} t_{01i} - k_z z_{02i} t_{00i}) \mathcal{R}_i \quad (5.14)$$

$$A_4 = k_r \sum_{i=1}^n z_{00i} t_{00i} \frac{d\mathcal{R}_i}{dr} \quad (5.15)$$

$$A_5 = k_r \sum_{i=1}^n z_{00i} t_{00i} \frac{d^2 \mathcal{R}_i}{dr^2} \quad (5.16)$$

5 An introduction to PGD solution of multidimensional problems

Coupling constants, the expression of which is given (5.12) up to (5.16), correspond to integrals with respect to variables z and t of the product of a function with itself or its derivative. The constants appearing in (5.12) up to (5.16) are given by

$$z_{00} = \int_{\Omega_z} \mathcal{Z}_{n+1} \mathcal{Z}_{n+1} dz \quad (5.17)$$

$$z_{02} = \int_{\Omega_z} \mathcal{Z}_{n+1} \frac{d^2 \mathcal{Z}_{n+1}}{dz^2} dz \quad (5.18)$$

$$z_{00i} = \int_{\Omega_z} \mathcal{Z}_{n+1} \mathcal{Z}_i dz \quad (5.19)$$

$$z_{02i} = \int_{\Omega_z} \mathcal{Z}_{n+1} \frac{d^2 \mathcal{Z}_i}{dz^2} dz \quad (5.20)$$

$$t_{00} = \int_{\mathcal{I}} \mathcal{T}_{n+1} \mathcal{T}_{n+1} dt \quad (5.21)$$

$$t_{01} = \int_{\mathcal{I}} \mathcal{T}_{n+1} \frac{d\mathcal{T}_{n+1}}{dt} dt \quad (5.22)$$

$$t_{00i} = \int_{\mathcal{I}} \mathcal{T}_{n+1} \mathcal{T}_i dt \quad (5.23)$$

$$t_{01i} = \int_{\mathcal{I}} \mathcal{T}_{n+1} \frac{d\mathcal{T}_i}{dt} dt \quad (5.24)$$

The order of required derivative to compute function $\mathcal{R}$ can be reduced by integrating by parts the term in second-order derivative appearing in (5.11), so that

$$\int_{\Omega_r} \delta \mathcal{R}_{n+1} \frac{d}{dr} \left(r \frac{d\mathcal{R}_{n+1}}{dr} \right) dr = \delta \mathcal{R}_{n+1} r \frac{d\mathcal{R}_{n+1}}{dr} \Big|_{\partial \Omega_r} - \int_{\Omega_r} \frac{d\delta \mathcal{R}_{n+1}}{dr} \frac{d\mathcal{R}_{n+1}}{dr} r dr \quad (5.25)$$

It must be noted that the first term on the right-hand side is equal to zero because $\delta \mathcal{R}$ is defined homogeneous on the essential boundary conditions (Dirichlet's condition) or $d\mathcal{R}/dr$ is homogeneous on prescribed heat flux (Newmann's condition). Introducing (5.25) into (5.11), leads to

$$\begin{aligned} A_1 \int_{\Omega_r} \delta \mathcal{R}_{n+1} \mathcal{R}_{n+1} r dr + A_2 \int_{\Omega_r} \frac{d\delta \mathcal{R}_{n+1}}{dr} \frac{d\mathcal{R}_{n+1}}{dr} r dr = \\ - \int_{\Omega_r} (A_3 + A_4) \delta \mathcal{R}_{n+1} r dr - \int_{\Omega_r} A_5 \delta \mathcal{R}_{n+1} dr \end{aligned} \quad (5.26)$$

5.4 Computation of functions $\mathcal{R}_{n+1}$, $\mathcal{Z}_{n+1}$ and $\mathcal{T}_{n+1}$

A finite element framework can be used to solve (5.26), using a Galerkin approximation to describe virtual field $\delta\mathcal{R}$. Fields $\mathcal{R}_{n+1}$ and $\delta\mathcal{R}_{n+1}$ are then written as

$$\mathcal{R}_{n+1} = \alpha_{rk}\phi_k(r) \quad (5.27)$$

$$\delta\mathcal{R}_{n+1} = \beta_{rj}\phi_j(r) \quad (5.28)$$

where functions ϕ_i are shape functions here chosen as being linear by parts between nodes $k-1$ and k , and, k and $k+1$, respectively, so that

$$\phi_k(r) = \begin{cases} \frac{r - r_{k-1}}{r_k - r_{k-1}}, & r \in [r_{k-1}, r_k] \\ 1 - \frac{r - r_k}{r_{k+1} - r_k}, & r \in [r_k, r_{k+1}] \\ 0, & \text{otherwise} \end{cases} \quad (5.29)$$

The introduction of (5.27) and (5.28) into (5.26) for every β_{rj} non-trivial results in the following linear system in α_r

$$[A_1(K_1)_{ij} + A_2(K_2)_{ij}] \alpha_{rj} = -[(A_3 + A_4)(f_1)_i + A_5(f_2)_i] \quad (5.30)$$

where matrices K_1 , K_2 and vectors f_1 , f_2 can be obtained by assembly of element matrices K_1^e , K_2^e and vectors f_1^e , f_2^e , which are given by

$$\begin{aligned} (K_1^e)_{ij} &= \int_{r_k}^{r_{k+1}} \phi_i^e \phi_j^e r \, dr = \int_{r_k}^{r_{k+1}} \begin{bmatrix} \phi_1^e \phi_1^e & \phi_1^e \phi_2^e \\ \phi_2^e \phi_1^e & \phi_2^e \phi_2^e \end{bmatrix} r \, dr = \\ &= \frac{h_e}{12} \begin{bmatrix} h_e + 4r_k & h_e + 2r_k \\ h_e + 2r_k & 3h_e + 4r_k \end{bmatrix} \end{aligned} \quad (5.31)$$

$$(K_2^e)_{ij} = \int_{r_k}^{r_k+h} \phi_i^{e'} \phi_j^{e'} r \, dr = \int_{r_k}^{r_{k+1}} \begin{bmatrix} \phi_1^{e'} \phi_1^{e'} & \phi_1^{e'} \phi_2^{e'} \\ \phi_2^{e'} \phi_1^{e'} & \phi_2^{e'} \phi_2^{e'} \end{bmatrix} r \, dr =$$

$$= \begin{bmatrix} r_k/h_e + 1/2 & -r_k/h_e - 1/2 \\ -r_k/h_e - 1/2 & r_k/h_e + 1/2 \end{bmatrix} \quad (5.32)$$

$$(f_1^e)_j = \int_{r_k}^{r_k+h} \phi_j r \, dr = \frac{h_e}{6} \begin{Bmatrix} h_e + 3r_k \\ 2h_e + 3r_k \end{Bmatrix} \quad (5.33)$$

and

$$(f_2^e)_j = \int_{r_k}^{r_k+h} \phi_j \, dr = \frac{h_e}{2} \begin{Bmatrix} 1 \\ 1 \end{Bmatrix} \quad (5.34)$$

where

$$h_e = r_{k+1} - r_k \quad (5.35)$$

5.4.2 Function $\mathcal{Z}_{n+1}$

A finite element framework can also be used to compute function $\mathcal{Z}_{n+1}$. Analogous to function $\mathcal{R}_{n+1}$, function $\mathcal{Z}_{n+1}$ can be computed as

$$B_1 \int_{\Omega_z} \delta \mathcal{Z}_{n+1} \mathcal{Z}_{n+1} \, dz + B_2 \int_{\Omega_z} \frac{d\delta \mathcal{Z}_{n+1}}{dz} \frac{d\mathcal{Z}_{n+1}}{dz} r \, dz =$$

$$- \int_{\Omega_r} B_3 \delta \mathcal{Z}_{n+1} \, dz - \int_{\Omega_z} B_4 \delta \mathcal{Z}_{n+1} \, dz \quad (5.36)$$

where

$$B_1 = \rho c r_{00} t_{01} - k_r (r_{01} + r_{02}) t_{00} \quad (5.37)$$

$$B_2 = k_z r_{00} t_{00} \quad (5.38)$$

$$B_3 = \sum_{i=1}^n [\rho c r_{00i} t_{01i} - k_r (r_{01i} + r_{02i}) t_{00i}] \mathcal{Z}_i \quad (5.39)$$

$$B_4 = k_z \sum_{i=1}^n r_{00i} t_{00i} \frac{d^2 \mathcal{Z}_i}{dz^2} \quad (5.40)$$

5.4 Computation of functions $\mathcal{R}_{n+1}$, $\mathcal{Z}_{n+1}$ and $\mathcal{T}_{n+1}$

with to following constants,

$$r_{00} = \int_{\Omega_r} \mathcal{R}_{n+1} \mathcal{R}_{n+1} r \, dr \quad (5.41)$$

$$r_{01} = \int_{\Omega_r} \mathcal{R}_{n+1} \frac{d\mathcal{R}_{n+1}}{dr} \, dr \quad (5.42)$$

$$r_{02} = \int_{\Omega_r} \mathcal{R}_{n+1} \frac{d^2\mathcal{R}_{n+1}}{dr^2} r \, dr \quad (5.43)$$

$$r_{00i} = \int_{\Omega_r} \mathcal{R}_{n+1} \mathcal{R}_i r \, dr \quad (5.44)$$

$$r_{01i} = \int_{\Omega_r} \mathcal{R}_{n+1} \frac{d\mathcal{R}_i}{dr} \, dr \quad (5.45)$$

$$r_{02i} = \int_{\Omega_r} \mathcal{R}_{n+1} \frac{d^2\mathcal{R}_i}{dr^2} r \, dr \quad (5.46)$$

Function $\mathcal{Z}_{n+1}$ and its virtual associated field $\delta\mathcal{Z}$ can be defined as

$$\mathcal{Z}_{n+1} = \alpha_{zk} \psi_k(z) \quad (5.47)$$

$$\delta\mathcal{Z} = \beta_{zj} \psi_j(z) \quad (5.48)$$

which leads for every nontrivial β_{zj} to the following linear system

$$[B_1(K_1)_{ij} + B_2(K_2)_{ij}] \alpha_{z,j} = -(B_3 + B_4)(f_1)_i \quad (5.49)$$

As presented before, matrices K_1 , K_2 and vector f_1 are obtained by the assembly of the element matrices, considering again linear shape functions

$$\begin{aligned} (K_1^e)_{ij} &= \int_{z_k}^{z_k+h} \psi_i^e \psi_j^e \, dz = \int_{z_k}^{z_{k+1}} \begin{bmatrix} \psi_1^e \psi_1^e & \psi_1^e \psi_2^e \\ \psi_2^e \psi_1^e & \psi_2^e \psi_2^e \end{bmatrix} dz = \\ &= \frac{h}{6} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \end{aligned} \quad (5.50)$$

$$\begin{aligned}
 (K_2^e)_{ij} &= \int_{z_k}^{z_k+h} \psi_i^{e'} \psi_j^{e'} dz = \int_{z_k}^{z_{k+1}} \begin{bmatrix} \psi_1^{e'} \psi_1^{e'} & \psi_1^{e'} \psi_2^{e'} \\ \psi_2^{e'} \psi_1^{e'} & \psi_2^{e'} \psi_2^{e'} \end{bmatrix} dz = \\
 &= \frac{1}{h} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}
 \end{aligned} \tag{5.51}$$

and

$$(f_1^e)_j = \int_{z_k}^{z_k+h} \psi_j dz = \frac{h}{2} \begin{Bmatrix} 1 \\ 1 \end{Bmatrix} \tag{5.52}$$

5.4.3 Function $\mathcal{T}_{n+1}$

Function $\mathcal{T}_{n+1}$ can be computed as

$$\begin{aligned}
 C_1 \int_{\mathcal{I}} \delta \mathcal{T}_{n+1} \frac{d\mathcal{T}_{n+1}}{dt} dt + C_2 \int_{\mathcal{I}} \delta \mathcal{T}_{n+1} \mathcal{T}_{n+1} dt = \\
 - \int_{\mathcal{I}} C_3 \delta \mathcal{T}_{n+1} dt - \int_{\mathcal{I}} C_4 \delta \mathcal{T}_{n+1} dt
 \end{aligned} \tag{5.53}$$

where

$$C_1 = \rho c r_{00} z_{00} \tag{5.54}$$

$$C_2 = -k_r(r_{01} + r_{02})z_{00} - k_z r_{00} z_{02} \tag{5.55}$$

$$C_3 = \sum_{i=1}^n \rho c r_{00i} z_{00i} \frac{d\mathcal{T}_i}{dt} \tag{5.56}$$

$$C_4 = \sum_{i=1}^n [-k_r(r_{01i} + r_{02i})z_{00i} - k_z r_{00} z_{02}] \mathcal{T}_i \tag{5.57}$$

Going back to the strong form, (5.53) rewrites

$$C_1 \frac{d\mathcal{T}_{n+1}}{dt} + C_2 \mathcal{T}_{n+1} = -C_3 - C_4 \tag{5.58}$$

If strong form (5.58) is solved using a Runge-Kutta fourth-order algorithm, the function value at t_{p+1} can be computed from the previous value $\mathcal{T}(t_p)$ as follows

$$\mathcal{T}_{n+1}(t_{p+1}) = \mathcal{T}_{n+1}(t_p) + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) + \mathcal{O}(h^5) \tag{5.59}$$

5.5 Boundary conditions on $\mathcal{R}_{n+1}$, $\mathcal{Z}_{n+1}$ and $\mathcal{T}_{n+1}$

where

$$\begin{aligned}
k_1 &= h f(t_p, \mathcal{T}_{n+1}(t_p)) \\
k_2 &= h f(t_p + h/2, \mathcal{T}_{n+1}(t_p) + k_1/2) \\
k_3 &= h f(t_p + h/2, \mathcal{T}_{n+1}(t_p) + k_2/2) \\
k_4 &= h f(t_p + h, \mathcal{T}_{n+1}(t_p) + k_3)
\end{aligned} \tag{5.60}$$

and

$$f(t_p, \mathcal{T}_{n+1}(t_p)) = \frac{d\mathcal{T}_{n+1}}{dt}(t_p) = \frac{1}{C_1}(-C_3 - C_4 - C_2 \mathcal{T}_{n+1}(t_p)) \tag{5.61}$$

5.5 Boundary conditions on $\mathcal{R}_{n+1}$, $\mathcal{Z}_{n+1}$ and $\mathcal{T}_{n+1}$

Gonz et al. (2010) present the use of PGD for problems subjected to non-homogeneous essential boundary conditions and how to consider parts with geometrical complexity.

Initial and boundary conditions are applied through term T_0 , in (5.6). This term can also be split into d enrichments, so that

$$T_0 = \sum_{i=1}^d \mathcal{R}_i \mathcal{Z}_i \mathcal{T}_i \tag{5.62}$$

The boundary conditions consisted herein are imposed temperature or heat flux. Mixed boundary conditions, such as convection, have not been considered during this study. In a similar way, an initial temperature field can be applied. Table 5.1 presents some examples of first enrichments for some boundary conditions.

Once the boundary conditions have been verified by the first enrichments, homogeneous boundary conditions are applied when computing of the following enrichments.

Table 5.1: Some examples of first enrichments.

BC	$\mathcal{R}$	$\mathcal{Z}$	$\mathcal{T}$
$T_{,r}(r=0) = 0$	$\mathcal{R} = 1$	$\mathcal{Z} = \frac{\beta - \alpha}{2}z^2 + \alpha z$	$\mathcal{T} = t$
$T_{,r}(r=1) = 0$			
$T_{,z}(z=0) = \alpha t$			
$T_{,z}(z=1) = \beta t$			
$T_{,r}(r=0) = 0$	$\mathcal{R} = \frac{\alpha}{2}r^2$	$\mathcal{Z} = 1$	$\mathcal{T} = t$
$T_{,r}(r=1) = \alpha t$			
$T_{,z}(z=0) = 0$			
$T_{,z}(z=1) = 0$			
$T_{,r}(r=0) = 0$	$\mathcal{R} = 1$	$\mathcal{Z} = 1$	$\mathcal{T} = \alpha t$
$T(r=1) = \alpha t$			
$T_{,z}(z=0) = 0$			
$T(z=1) = \alpha t$			

5.6 Function derivatives

To evaluate functions $\mathcal{R}_{n+1}$, $\mathcal{Z}_{n+1}$ and $\mathcal{T}_{n+1}$, the first and second derivatives of previous functions are needed to compute the right hand side of (5.26), (5.36) and (5.58). These derivatives were implemented using a finite difference approximation, so that the first and second derivatives, e.g. for function $\mathcal{R}$ write

$$\frac{d\mathcal{R}}{dr}(r=r_k) = \frac{\mathcal{R}(r_{k+1}) - \mathcal{R}(r_{k-1})}{2h_e} + \mathcal{O}(h_r^2) \quad (5.63)$$

$$\frac{d^2\mathcal{R}}{dr^2}(r=r_k) = \frac{\mathcal{R}(r_{k+1}) - 2\mathcal{R}(r_k) + \mathcal{R}(r_{k-1}))}{h_e^2} + \mathcal{O}(h_r^2) \quad (5.64)$$

where the space discretization $h_e = r_{k+1} - r_k$ is constant. It can be noticed that error $\mathcal{O}(h^2)$ was chosen so that it is consistent with the approximation made when solving previous equations. In order to avoid to numerical instabilities which could appear if approximations of different orders were used. Last, it is worth to notice that it was not chosen to derive the finite element shape functions, since this approach would lead to

discontinuous derivatives at the boundaries between elements.

5.7 Convergence criteria and algorithm

As presented above, the PGD solution is composed of a sum of functions products and each function at $n + 1$ can be computed solving (5.26), (5.36) and (5.58). Thus, obtaining a PGD solution consists in two big steps: an iterative step to compute the new enrichments functions, $\mathcal{R}_{n+1}$, $\mathcal{Z}_{n+1}$ and $\mathcal{T}_{n+1}$, and another step to combine them and derive a new solution approximation. The convergence criterion used to stop the iterative step is written as ξ_e , where

$$\xi_e = |\mathcal{R}_{n+1}^k \mathcal{Z}_{n+1}^k \mathcal{T}_{n+1}^k - \mathcal{R}_{n+1}^{k-1} \mathcal{Z}_{n+1}^{k-1} \mathcal{T}_{n+1}^{k-1}| < \xi_{emax} \quad (5.65)$$

whereas the solution is assumed to be obtained when the following global criterion is satisfied

$$\xi_g = \left| \frac{T_{ref} - \sum_{i=1}^n \mathcal{R}_i \mathcal{Z}_i \mathcal{T}_i}{T_{ref}} \right| < \xi_{gmax} \quad (5.66)$$

An overview of the general PDG algorithm is given in Figure 5.4.

One important step is the normalization during each enrichment computation. During this step, the values of new enrichment functions may diverge. For example, function $\mathcal{R}_{n+1}$ may increase indefinitely in order to compensate asymmetric decrease of function $\mathcal{Z}_{n+1}$. Normalization of the functions should eliminate this issue. Since functions $\mathcal{R}$ and $\mathcal{Z}$ are normalized, the norm of the new enrichment is given by $\mathcal{T}$ norm.

```

1  read first  $d$  enrichments
2
3  while  $\xi_g < \xi_{gmax}$ 
4       $\mathcal{R}_{n+1} = \mathcal{R}_n$ 
5       $\mathcal{Z}_{n+1} = \mathcal{Z}_n$ 
6       $\mathcal{T}_{n+1} = \mathcal{T}_n$ 
7
8      k = 0
9      while  $\xi_e < \xi_{emax}$ 
10         compute  $\mathcal{R}$ 
11         normalize  $\mathcal{R}$ 
12         compute  $\mathcal{Z}$ 
13         normalize  $\mathcal{Z}$ 
14         compute  $\mathcal{T}$ 
15         compute  $\xi_e$ 
16          $k = k + 1$ 
17     end
18
19     compute  $\xi_g$ 
20      $n = n + 1$ 
21 end

```

Figure 5.4: General PGD program algorithm.

5.8 Examples

5.8.1 Case 1: 1D r-direction heat transfer problem

First example herein corresponds to a particular case of the axisymmetric problem, viz. the unidirectional problem in the radial direction. The solid depicted in Figure 5.1 is chosen such that $a, b = 1$, and material properties are $\rho c = 1$ and $k_r = 0.1$. The solid has an homogeneous initial temperature, $T(t = 0) = 0$, and is subjected to the following boundary conditions

$$\frac{\partial T}{\partial r}(r = 0) = \frac{\partial T}{\partial z}(z = 0) = \frac{\partial T}{\partial z}(z = 1) = 0 \quad (5.67)$$

$$T(r = 1) = \gamma t = 1t \quad (5.68)$$

The closed-form solution, according to Carslaw and Jaeger (1986), for this problem writes

$$T(r, z, t) = T_{ref}(r, t) = \gamma \left(t - \frac{a^2 - r^2}{4\gamma} \right) + \frac{2\gamma}{ak} \sum_{n=1}^{\infty} e^{-\alpha\beta_n^2 t} \frac{J_0(r\beta_n)}{\beta_n^3 J_1(a\beta_n)} \quad (5.69)$$

where $\alpha = k/(\rho c)$ is the thermal diffusivity, β_n the roots of $J_0(a\beta_n) = 0$, and J_0 , J_1 first kind Bessel function³.

To verify boundary conditions (5.67) and (5.68), the following functions were chosen as first enrichments

$$\mathcal{R}(r) = r^2, \quad \mathcal{Z}(z) = 1, \quad \mathcal{T}(t) = t \quad (5.70)$$

A favorable comparison between the closed-form and PGD temperature fields is shown in Figure 5.5.

Figure 5.5: Comparison between analytical and PGD solution for 1D radial problem.

Figure 5.6 shows the error between the PGD solution and the analytical solution as a function of the number of enrichments used to compute the approximated solution. The error has been computed as

$$error = \max(|T_{ref} - T_{pgd}|) \quad (5.71)$$

The first five enrichments computed, $n = 1, \dots, 5$, can be seen in Figure 5.7. The first enrichment computed exhibits the highest magnitude. Functions $\mathcal{R}$ and $\mathcal{Z}$ have null value at the points where the boundary conditions were specified

$$\frac{d\mathcal{R}}{dr}(r=0) = 0, \quad \mathcal{R}(r=1) = 0, \quad \mathcal{Z}(z) = cte \quad (5.72)$$

³First kind Bessel functions are defined as

$$J_\alpha(x) = \sum_{m=0}^{\infty} \frac{(-1)^m}{m! \Gamma(m + \alpha + 1)} \left(\frac{x}{2}\right)^{(2m+\alpha)}$$

where $\Gamma(z)$ is the Gamma function.

and $\mathcal{T}(0) = 0$, verifying the homogeneous initial temperature condition.

Figure 5.6: PGD solution convergence: radial direction case.

5.8.2 Case 2: 1D z-direction heat transfer problem

Analogously to the previous case, an unidirectional problem is treated in z-direction. The solid depicted in Figure 5.1 is chosen such that $a, b = 1$, and material properties $\rho c = 1$ and $k_z = 0.05$. The solid has an homogeneous initial temperature, $T(t = 0) = 0$, and is subjected to the following boundary conditions

$$\frac{\partial T}{\partial r}(r = 0) = \frac{\partial T}{\partial r}(r = 1) = \frac{\partial T}{\partial z}(z = 0) = 0 \quad (5.73)$$

$$T(z = 1) = \gamma t = 1t \quad (5.74)$$

According to Carslaw and Jaeger (1986), the closed-form solution for this problem is given by

$$T(r, z, t) = T_{ref}(z, t) = \gamma t + \frac{\gamma(z^2 - b^2)}{2k} + \frac{16\gamma b^2}{k_z \pi^3} \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)^3} e^{\frac{\alpha(2n+1)^2 t}{4b^2}} \cos \frac{(2n+1)\pi z}{2b} \quad (5.75)$$

To verify boundary conditions (5.73) and (5.74), the following functions were chosen as first enrichments:

$$\mathcal{R}(r) = cte, \quad \mathcal{Z}(z) = z^2, \quad \mathcal{T}(t) = t \quad (5.76)$$

A favorable comparison between the closed-form and PGD temperature fields is shown in Figure 5.8.

Figure 5.9 shows the error between the PGD solution and the reference solution, given

Figure 5.7: PGD enrichments: radial direction case.

Figure 5.8: Comparison between analytical and PGD solution for 1D z-direction problem.

by (5.71), as a function of the number of enrichments used to compute the approximated solution. The first five enrichments computed, $n = 1, \dots, 5$, can be seen in Figure 5.10.

Figure 5.9: PGD solution convergence: z-direction case.

5.8.3 Case 3: Axisymmetric heat transfer problem

The solid depicted in Figure 5.1 is chosen such that $a, b = 1$, and material properties $\rho c = 1$, $k_r = 0.1$ and $k_z = 0.01$. The solid has an homogeneous initial temperature, $T(t = 0) = 0$, and is subjected to the following boundary conditions

$$\frac{\partial T}{\partial r}(r = 0) = \frac{\partial T}{\partial z}(z = 0) = 0 \quad (5.77)$$

$$T(z = 1) = T(r = 1) = \gamma t = 1t \quad (5.78)$$

Figure 5.10: PGD enrichments; z-direction case.

The closed-form solution for time-dependent boundary conditions could be derived from the radial and z-direction solutions for unit temperature applied to surfaces $r = 1$ and $z = 1$ together with Duhamel's theorem⁴. However, the reference solution for this case was obtained using a finite element model.

To verify boundary conditions (5.73) and (5.74), the following functions were chosen as first two enrichments

$$d = 1 : \quad \mathcal{R}(r) = 1, \quad \mathcal{Z}(z) = 1, \quad \mathcal{T}(t) = t \quad (5.79)$$

$$d = 2 : \quad \mathcal{R}(r) = r^4 - 2r^3 + r^2, \quad \mathcal{Z}(z) = z^4 - 2z^3 + z^2, \quad \mathcal{T}(t) = 1 \quad (5.80)$$

First enrichment, (5.79), imposes the desired initial and boundary conditions. Second enrichment, (5.80) was chosen in such that its derivative is to furnish a first enrichments not homogeneous, so that it introduces a magnitude on the right-hand side of (5.26), (5.36) and (5.58). The first five enrichments computed, $n = 1, \dots, 5$, can be seen in Figure 5.11.

Table 5.2 presents a comparison between the PGD and finite element computation time. The computational time of the case $n_r = n_z = 10$ and $n_t = 100$ was chosen as reference, so that the values presented are the computation time divided by the computation time of the reference case. PGD algorithm appears as being particularly efficient compared to finite element solution when the size of the problem increases.

⁴Duhamel's theorem writes that the solution for a time-dependent boundary condition problem can be obtained from the solution of a fundamental problem, $T_f(x, t)$. The fundamental problems is a problem with homogeneous initial condition subjected to a single, constant, and unity magnitude boundary condition. If the time-dependent boundary condition is $b(t)$, the correspondent solution, $T(x, t)$, according to Duhamel's theorem, writes

$$T(x, t) = \int_{\tau=0}^t T_f(x, t - \tau) \frac{db(\tau)}{d\tau} d\tau + b(t = 0) T_f(x, t)$$

Figure 5.11: PGD enrichments: axisymmetric case. The first two enrichments were used to prescribe initial and boundary conditions.

Table 5.2: Time comparison between PGD and finite element computations. During the computations, no output was required to avoid the influence of the time used to write output files.

		n_r, n_z		
		10	50	100
n_t	100	1.00 ○	6.45 ○	15.7 ○
		1.00 ●	10.5 ●	40.5 ●
	500	1.41 ○	12.1 ○	41.3 ○
		4.40 ●	52.1 ●	203 ●
	1000	1.71 ○	20.8 ○	80.0 ○
		8.97 ●	104 ●	403 ●
		○ PGD	● FEM	

5.9 PGD potentialities

5.9.1 Identification of material properties

The PGD framework makes it possible to introduce new functions to describe the problem solution, which can prove and be convenient for optimization problems. For instance, considering the axisymmetric problem described above, the thermal conductivity tensor could be considered as an unknown and introduced as a new function, so that the temperature field is written as

$$T(r, z, t, \mathbf{K}) = T_0 + \sum_{i=1}^n \mathcal{R}_i \mathcal{Z}_i \mathcal{T}_i \mathcal{K}_i + \mathcal{R}_{n+1} \mathcal{Z}_{n+1} \mathcal{T}_{n+1} \mathcal{K}_{n+1} \quad (5.81)$$

where $\mathcal{K} = \mathcal{K}(\mathbf{K})$ is the function associated to the thermal conductivity tensor. The weak form can be developed considering the following virtual temperature field

$$\begin{aligned} \delta T = & \delta \mathcal{R}_{n+1} \mathcal{Z}_{n+1} \mathcal{T}_{n+1} \mathcal{K}_{n+1} + \mathcal{R}_{n+1} \delta \mathcal{Z}_{n+1} \mathcal{T}_{n+1} \mathcal{K}_{n+1} + \\ & + \mathcal{R}_{n+1} \mathcal{Z}_{n+1} \delta \mathcal{T}_{n+1} \mathcal{K}_{n+1} + \mathcal{R}_{n+1} \mathcal{Z}_{n+1} \mathcal{T}_{n+1} \delta \mathcal{K}_{n+1} \end{aligned} \quad (5.82)$$

A same strategy analogous to the strategy described in previous sub-sections could be applied to obtain the equations making it possible to compute the functions $\mathcal{R}$, $\mathcal{Z}$, $\mathcal{T}$ and $\mathcal{K}$. An algebraic expression could then be obtained to compute the $\mathcal{K}$ function at each iteration. The introduction of this new function and hence the construction

of the solution as a function of the thermal conductivity would make it possible its identification or optimization.

In practical this approach requires to account for mixed boundary condition. This difficult task could not be accomplished in this study, so that the identification of thermal conductivities as presented in Chapter 3, remains a prospect of this work.

5.9.2 Separation of length scales

PGD could also be applied to the analysis of problems with different length scales. For instance, it could be applied to the analysis of composite shells where the displacement field, $u(x, y, z)$, is approximated by the product of a function representing the membrane solution, $\mathcal{X}(x, y)$, and another function, $\mathcal{Z}(z)$, so that

$$u(x, y, z) = \sum_{i=1}^n \mathcal{X}(x, y) \mathcal{Z}(z) \quad (5.83)$$

while function $\mathcal{Z}$ enhances the representation of the heterogeneities along the thickness direction which are related to the stacking sequence of plies with different orientations (Bognet et al., 2012). This separation of space variables into two groups with different length scales reduces the size of the global system. Thus, it could save time during the solution process. The stress field can also be split in an analogous way of equation 5.83.

CHAPTER 6

Conclusions

The first part of Chapter 2 was dedicated to the presentation of the materials and assembly process studied herein. The materials -provided by Tencate[®]- are 2 mm thick hybrid thermoplastic laminates with the following stacking sequence: $[(0^\circ/90^\circ)_G, (0^\circ/90^\circ)_C, (\pm 45^\circ)_C, (0^\circ/90^\circ)_C, (\pm 45^\circ)_C, (0^\circ/90^\circ)_G]$, where subscripts G and C respectively refer to fibreglass and carbon continuous fibres embedded in a PPS matrix. The assembly process is a welding resistance process where the heating source is a metallic grid inserted between the laminates to be assembled and heated by Joule effect while pressure is applied normally to the welded joint. The joint is constituted of a stainless steel plain weave canvas acting as heating element, and PPS films inserted in between the metallic grid and each of the two composite laminates to be assembled. During the welding process, these films as well as the first layers of the composite laminates are heated, melt and reconsolidate during cooling.

The core of Chapter 2 was dedicated to the description of the welding machine which has been designed and built during this study in order to be able to manufacture 140 mm x 40 mm welded joints while controlling a certain number of manufacturing parameters –including the geometry of the metallic grid, the intensity of the heating source –via the

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intensity of the electrical current applied to the grid, from 20 to 200 A-, the pressure –up to ca. 4MPa-, and the duration of the dwell time during which the pressure is applied while the joint cools down. This machine was used to manufacture specimens corresponding to four different set of manufacturing parameters. The fracture toughness tests performed during this study on the Double Cantilever Beam coupons cut from these specimens are described and analysed in Chapter 4. Future work on this machine should consist in its instrumentation to control the pressure applied to the joint, to monitor the temperature field in the vicinity of the joint, and, possibly, to control some conductivity heat exchanges.

The last part of Chapter 2 was dedicated to a first 3D heat transfer analysis –performed with the industrial finite element code Abaqus®- of the electrical resistance welding process. During these simulations the metallic heating grid was modelled as a heat flux, and the seven-layer hybrid composite laminates were modelled as three-layer plates with the following stacking sequence [G/PPS, C/PPS, G/PPS], each layer –with the appropriate thickness- exhibiting homogenized orthotropic properties. The identification of the different material properties used for this analysis has been performed during this study and was described in details in Chapter 3. The finite element predictions were compared to thermocouple measurements made at different points within the assembled composite laminates during the welding process. These measurements were made during a first experiment while applying minimal pressure to the joint during the welding process. First, the numerical simulations showed that the value of the Nusselt number –i.e. the ratio of the convective heat flux to the conductive heat flux- is low enough for conductivities and heat sources to play the key role in the process. Besides, the results of the numerical simulation are very encouraging, in particular in terms of the prediction of the temperature field –including the maximal value- during the heating phase. However, it appears that the simulation of the cooling phase requires further developments. These developments should be made in parallel with the instrumentation of the welding machine.

The Chapter 3 was dedicated to the identification of the mechanical and thermal properties of G/PPS or C/PPS composite laminates. These laminates were assumed to be representative of the G/PPS or C/PPS plies within the hybrid composite laminates studied herein. The thermal properties were used for the numerical simulations –presented in Chapter 2- of the welding process of the hybrid composite laminates. The mechanical properties could be used for future detailed analyses of the crack propagation tests discussed in Chapter 4. In-plane mechanical properties of the G/PPS or C/PPS composite laminates were identified from the results of uniaxial tension tests performed

during this study on specimens cut at 0° , 90° and 45° with respect to warp/weft orientations. Since the tests were instrumented with Digital Image Correlation, it could be verified that the strain field was uniform within the central part of the specimens. Both materials exhibit orthotropic in-plane elastic properties –but with similar values of Young’s modulus in 0° and 90° directions. The values of this Young’s modulus and of the shear modulus are in agreement with the values given in the specification sheets provided with the plates. The specific heat, melting –during heating- and recrystallization –during cooling- enthalpies were identified for both materials from DSC tests performed during this study.

The main part of Chapter 3 was dedicated to experiments that has been designed, performed and analysed during this study, to identify the anisotropic thermal conductivities of the G/PPS or C/PPS composite laminates. The experiment consists in two steps: first, the central part of the studied horizontal square plate is heated with an electrical bulb and, then, the plate naturally cools down. The experiment is instrumented with thermocouples on both faces of the plate, and an Infra-Red (IR) camera facing the plate opposite to the heating source. Since the temperature field measured by the IR camera appears as being axisymmetric the thermal conductivities of both laminates are assumed to be isotropic transverse. The experimental results exploited for the identification procedure are the time for temperature to diffuse through the specimen thickness during the heating phase just in front of the heat source –as detected by two thermocouples-, and the time evolution of the temperature along a radius initiating from the centre of the plate –as measured by the IR camera. The parameters to be identified are the in-plane and out-of plane thermal conductivities, the convection-radiation coefficients on top and bottom faces of the plate, and several parameters to describe the intensity and the geometry of the heat source. Finally the value of eleven parameters is derived from an inverse identification procedure based on Levenberg-Marquart algorithm applied simultaneously to the results of the experiments performed on the G/PPS and C/PPS plates. This global identification procedure proved to be possible thanks to previous separate approximate evaluations of the out-of-plane conductivities and of the convection-radiation coefficients. The in-plane thermal conductivity exhibited by C/PPS plate is about seven times larger than its out-of-plane conductivity, whereas the thermal conductivity identified for the G/PPS plate is almost isotropic.

The Chapter 4 was dedicated to the characterization and the modelling of the fracture toughness of the welded joints studied herein. The first part of that chapter was dedicated to mode I crack propagation tests, performed during this study on Double Cantilever Beam specimens cut from the joints manufactured with different processing

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conditions as explained in Chapter 2. Each type of specimen was loaded up to final failure either monotonically or with regular unloadings to evaluate the evolution of the specimen stiffness as the crack propagates. Whatever the manufacturing parameters chosen during this study, the observed failure mechanism corresponds to the delamination of the welded joint –and more precisely, within the fibreglass layer of one of the two assembled composite laminates. The tests were instrumented with Digital Image Correlation, and specific image processing using edge-detection Sobel algorithm was used to measure the beam kinematics all along crack propagation. The comparison between this experimentally measured beam deflection and the response of a model proposed by Kanninen (1973) is used as input of a Levenberg-Marquart algorithm to make the inverse identification of the crack length all along the loading sequence.

Besides, the compliance method was used to analyse the global load vs. displacement response of the specimens during the loading-unloading sequence imposed during some of the tests. This analysis shows that the energy dissipated during crack propagation can be split into the energy associated to the formation of the free surface and a non-negligible non-linear energy probably associated to plasticity mechanisms in the vicinity of the welded joint. So-called R-curve with a positive slope –corresponding to the plot of crack energy release rate vs. crack length- is representative of this type of ductile failure mode.

The last part of Chapter 4 was dedicated to a model developed within internal variable thermodynamic framework. Assuming two dissipative mechanisms associated to crack formation and localized plasticity in the vicinity of the welded joint, respectively, this approach makes it possible to model the R-curve. This model was used to analyse the sensitivity of the welded joint fracture toughness to different manufacturing parameters. First results obtained herein show that the time during which the pressure is applied onto the joint during the cooling phase of the welding process has a key influence on the fracture toughness of the joint.

Despite the very encouraging results obtained, more investigations must be made to check the possible influence on the results of the DCB tests of the hinges used to load the specimens. First, the stiffness of these hinges may have an impact on the load vs. displacement response of the joints especially during loading-unloading tests, and, second, the friction in these hinges may induce a dissipated energy not to be mixed up with the energy associated to non-linear dissipative mechanisms in the vicinity of the joint. Besides, this model is rate independent, which may prove and be not valid for certain loading rates and environments.

However, it is worth noticing that, because of its energetic form, the model proposed

herein should be applied without any difficulty to mode II tests –such as End Notched Flexure tests- and mixed mode (I+II) tests –on ASTM assembly or Dubé specimens.

Furthermore, these tests and their analysis constitute a first step in the analysis of the sensitivity of the mechanical behaviour of welded joints to the manufacturing parameters related to the mechanical and microstructural state induced by the welding process. Such a study may lead to numerous studies to optimize the welding process, to predict the mechanical and microstructural state of the joint and, finally its mechanical behaviour when submitted to various loading in various environments.

Because of the number of parameters and the very different length scales along in-plane and through the thickness directions, full 3D numerical simulations of heat transfer or/and mechanical problems related to the composite laminates studied herein are very expensive –even for linear cases. Model reduction techniques must be developed for this kind of numerical simulations to be performed at the structural scale. The Chapter 5 was dedicated to a very promising model reduction technique, viz. Proper Generalized Decomposition (PGD). PGD solution procedure is applied to an axisymmetric heat transfer problem using a fully separated temperature field. The temperature is described as the product of three 1D functions. A finite element framework is applied to the solution of space functions and a Runge-Kutta algorithm to the solution of the time function. The verification of the boundary conditions –which is a key point of PGD approach- is explained in details. The program developed in the Matlab[®] environment during this study is applied to three cases. The numerical results were favourably compared to close-form or numerical reference solutions. It has been shown that the PGD solution procedure leads to computational costs that were lower by an order of magnitude than the cost of standard finite element simulations -even for the simple cases tested. The framework developed during this study constitutes a solid base for future developments and extensions to solve more complex heat transfer or mechanical problems related to the materials and joints studied herein. In particular, the separation of different space –in-plane and out-of-plane-, time, and for instance material variables should prove and be efficient to solve direct or inverse problems.

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