UNIVERSIDADE DE SÃO PAULO
ESCOLA DE ENGENHARIA DE SÃO CARLOS

ANTONIO CARLOS DAUD FILHO

Estudo de Dinâmica de Voo e Controle de um VANT com Decolagem e Pouso Vertical

São Carlos
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Dissertação apresentada à Escola de Engenharia de São Carlos da Universidade de São Paulo, como requisito para a obtenção do Título de Mestre em Engenharia Mecânica.

Orientador: Prof. Dr. Eduardo Morgado Belo

ESTE EXEMPLAR TRATA-SE DA VERSÃO CORRIGIDA. A VERSÃO ORIGINAL ENCONTRA-SE DISPONÍVEL JUNTO AO DEPARTAMENTO DE ENGENHARIA MECANICA DA EESC-USP.

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Aos meus pais e meu irmão pelo encorajamento, apoio, compreensão e paciência.

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“O esforço natural de cada indivíduo para melhorar sua própria condição, quando se permite que ele atue com liberdade e segurança, constitui um princípio tão poderoso que, por si só, e sem qualquer outra ajuda, não somente é capaz de levar a sociedade à riqueza e à prosperidade, como também de superar uma centena de obstáculos impertinentes com os quais a insensatez das leis humanas com excessiva frequência obstrui seu exercício, embora não se possa negar que o efeito desses obstáculos seja sempre interferir, em grau maior ou menor, na sua liberdade ou diminuir sua segurança.”

Adam Smith, A Riqueza das Nações, Investigações Sobre sua Natureza e suas Causas (1776).

RESUMO

DAUD FILHO, A. C. **Estudo da Dinâmica de Voo e Controle de um VANT com Decolagem e Pouso Vertical**. 2018. Dissertação de Mestrado – Escola de Engenharia de São Carlos, Universidade de São Paulo, São Carlos, 2018.

Esta dissertação apresenta o desenvolvimento da teoria de dinâmica de voo e o conceito de controle a ser aplicado na modelagem e simulação de voo de um VANT com decolagem e pouso vertical proposto. Um conceito de aeronave de asa semi-tandem é projetado e os coeficientes aerodinâmicos, propriedades inerciais e parâmetros de controle são estimados, o que permitiu a implementação da teoria proposta. O modelo fez uso das equações de movimento multi-corpos onde a aeronave é dividida em partes de forma que a asa, o estabilizador horizontal e os rotores sejam entidades independentes. Além disso, o sucesso da fase de transição de voo pairado para cruzeiro e de cruzeiro para voo pairado pode ser verificado se houver a possibilidade da aeronave trimar ao longo do regime de velocidades de voo, em outras palavras, se houver uma combinação de estados de movimento que mantenha a aeronave estável do voo pairado para a condição de cruzeiro. Assim, as curvas de trimagem que expressam os estados são calculadas usando a minimização de uma função de custo envolvendo a soma dos quadrados de alguns dos estados de movimento, definidos pelas equações de movimento mencionadas anteriormente. Tal minimização é realizada usando o algoritmo Simplex Sequencial. Além disso, é apresentada uma estratégia de controle que estabiliza a aeronave durante a transição de voo pairado para configuração de cruzeiro, que é testada em simulação computacional de um voo longitudinal acelerado e desacelerado, ou seja, de voo pairado para cruzeiro e de cruzeiro para voo pairado. Finalmente, um protótipo da aeronave estudada é apresentado.

Palavras-chave: VANT, decolagem e pouso vertical, dinâmica de voo, controle, equações de movimento multi-corpos, voo de transição.

ABSTRACT

DAUD FILHO, A. C. **Flight Dynamics and Control Study of a VTOL UAV**. 2018. Master Thesis – São Carlos School of Engineering, University of São Paulo, São Carlos, 2018.

This thesis presents the development of the flight dynamics theory and control concept to be applied in the modeling and flight simulation of a proposed VTOL UAV. A semi-tandem wing aircraft concept is designed and the aerodynamic coefficients, inertial properties and controls parameters are estimated, which allowed the implementation of the proposed theory. The model made use of the multi-body equations of motion where the aircraft is divided in parts so that the wing, horizontal stabilizer and rotors are independent entities. Additionally, the success of the transition phase from hovering to cruise and from cruise to hovering can be verified if there is the possibility of the aircraft to trim along the flight speed regime, in other words, if there is a combination of states of motion that keep the aircraft stable from hover to cruise condition. So, the trim curves expressing the states are computed using the minimization of a cost function involving the sum of the squares of some of the states of motion, defined through the equations of motion previously mentioned. Such minimization is performed using the Sequential Simplex algorithm. Moreover, a control strategy that stabilizes the aircraft while it transitions from hovering to cruise configuration is presented, which is tested in computer simulation of an accelerated and decelerated longitudinal flight, that is, from hovering to cruise condition, and from cruise to hovering condition. Finally, a prototype of the aircraft studied is presented.

Keywords: VTOL UAV, flight dynamics, control, multi-body equations of motion, transition flight.

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LIST OF ABBREVIATIONS AND ACRONYMS

VANT	_	Veículo aéreo não tripulado (Unmanned aerial vehicle)
VTOL	_	Vertical take-off and landing
UAV	_	Unmanned aerial vehicle
NASA	_	National aeronautics and space administration
CAD	_	Computer aided design
RPA	_	Remotely piloted aircraft
RC	_	Radio-controlled
Datcom	_	Data compendium
RPM	_	Revolution per minute
MTOW	_	Maximum take-off weight
Naca	_	National advisory committee for aeronautics
C.G.	_	Center of gravity
ICAO	_	International civil aviation organization
LQR	_	Linear quadratic regulator

LIST OF SIMBOLS

$\vec{a}_A$ Acceleration vector in an arbitrary reference frame A

$\vec{a}_B$ Acceleration vector in an arbitrary reference frame B

$\vec{a}_{rel_{A/B}}$ Relative acceleration vector of point A in relation to point B

$\vec{a}^B_{rel_{R_j/B}}$ Relative acceleration vector of rotor origin in relation to body origin

$\vec{a}^B_{rel_{W_i/B}}$ Relative acceleration vector of aerodynamic surface origin in relation to body origin

A Equivalent inertia matrix; jacobian matrix

A_p Jacobian matrix of the equilibrium point

A_W Equivalent inertia matrix in wind axes ($SAS^T = A_W$)

AR_{HT_e} Exposed horizontal tail aspect ratio

AR_{W_e} Exposed wing aspect ratio

b Aircraft wing span; minor axis of equivalent elliptical cross section

b' Average propeller blade chord.

b_{HT} Horizontal tail span

b_W Wing span

B Angular motion equation coefficient B; jacobian matrix; number of propeller blades

$\boldsymbol{B}$ Simplex vertex with the best outcome

B^B_E Rotation matrix from Earth fixed inertial reference frame to body coordinate frame

B_p Jacobian matrix of the equilibrium point

B_W Angular motion equation coefficient B in wind axes ($SBS^T = B_W$)

$\bar{c}$ Aircraft mean aerodynamic chord

c_{l_α} Section lift curve slope of the basic airfoil

$\left(c_{l_\alpha}\right)_M$ Section lift curve slope for specific Mach number

$\frac{c_{l_\alpha}}{\left(c_{l_\delta}\right)_{theory}}$ Empirical correction factor that accounts for the development of the boundary layer towards the airfoil trailing edge

$\left(c_{l_\delta}\right)_{theory}$ Theoretical control lift effectiveness for a given airfoil thickness ratio and control-chord-to-airfoil-chord ratio

$\frac{c_{l_\delta}}{\left(c_{l_\delta}\right)_{theory}}$ Empirical correction factor for lift effectiveness of plain trailing edge controls

C Angular motion equation coefficient C

C_D Aerodynamic drag coefficient

C_{D_B} Body drag coefficient

$C_{D_{HT_e}}$ Exposed horizontal tail drag coefficient

$C_{D_{LG}}$ Landing gear drag coefficient

$C_{D_{q_{WBT}}}$ Wing-body-drag drag coefficient dynamic derivative with respect to Q_W

$C_{D_{W_e}}$ Exposed wing drag coefficient

$C_{D_{WBT}}$ Wing-body-tail static aircraft drag coefficient

$C_{D_{\alpha^2}{}_B}$ Body drag coefficient derivative with respective to α^2

$C_{D_{\alpha^3}{}_B}$ Body drag coefficient derivative with respective to α^3

$(C_{D0})_{WB_e}$ Exposed wing and body zero lift drag coefficient

$(C_{D0})_{HT_e}$ Exposed horizontal tail and body zero lift drag coefficient

C_f Friction coefficient

C_l Aerodynamic rolling moment coefficient

$C_{l_{LG}}$ Landing gear rolling moment coefficient

$C_{l_{WBT}}$ Wing-body-tail rolling moment coefficient

$C_{l_{p_{WBT}}}$ Aircraft rolling moment derivative with respect to P_W

$C_{l_{r_{WBT}}}$ Aircraft rolling moment derivative with respect to R_W

$C_{l_{\beta_{WBT}}}$ Aircraft rolling moment coefficient derivative with respect to β

$C_{l_{\dot{\beta}_{WBT}}}$ Aircraft rolling moment derivative with respect to $\dot{\beta}$

${C'}_{l_\delta}$ Rolling effectiveness of two full chord controls antisymmetrically deflected

$C_{l_{\delta_a}}$ Aircraft rolling moment coefficient derivative with respect to δ_{a_L} or δ_{a_R}

$C_{l_{\delta_r}}$ Aircraft rolling moment coefficient derivative with respect to δ_r

C_L Aerodynamic lift coefficient

C_{L_B} Body lift coefficient

$C_{L_{HT_e}}$ Exposed horizontal tail lift coefficient

$C_{L_{LG}}$ Landing gear lift coefficient

$C_{L_{q_{WBT}}}$ Aircraft lift derivative with respect to Q_W

$C_{L_{W_e}}$ Exposed wing lift coefficient

$C_{L_{WBT}}$ Aircraft static lift coefficient

C_{L_α} Lift curve slope of the aerodynamic surface with control retracted

$\left(C_{L_\alpha}\right)_B$ Body lift curve slope

$\left(C_{L_\alpha}\right)_{HT_e}$ Exposed horizontal tail lift curve slope

$\left(C_{L_\alpha}\right)_{W_e}$ Exposed wing lift curve slope

$C_{L_{\dot{\alpha}_{WBT}}}$ Aircraft lift coefficient derivative with respect to $\dot{\alpha}$

$C_{L_{\delta_e}}$ Horizontal tail lift coefficient derivative with respect to elevator deflection angle

C_m Aerodynamic pitching moment coefficient

C_{m_B} Body pitching moment coefficient

$C_{m_{HT_e}}$ Exposed horizontal tail pitching moment coefficient

$C_{m_{LG}}$ Landing gear moment coefficient

$C_{m_{q_{WBT}}}$ Aircraft pitching moment coefficient derivative with respect to Q_W

$C_{m_{W_e}}$ Exposed wing pitching moment coefficient

$C_{m_{WBT}}$ Aircraft pitching moment coefficient

$\left(C_{m_\alpha}\right)_{HT_e}$ Exposed horizontal tail pitching moment curve slope

$\left(C_{m_\alpha}\right)_{W_e}$ Exposed wing pitching moment curve slope

$C_{m_{\dot{\alpha}_{WBT}}}$ Aircraft pitching moment coefficient derivative with respect to $\dot{\alpha}$

$(C_{m0})_{HT_e}$ Exposed horizontal tail zero lift pitching moment coefficient

$(C_{m0})_{W_e}$ Exposed wing zero lift pitching moment coefficient

C_n Aerodynamic yawing moment coefficient

$C_{n_{LG}}$ Landing gear yawing moment coefficient

$C_{n_{WBT}}$ Wing-body-tail yawing moment coefficient

$C_{n_{p_{WBT}}}$ Aircraft yawing moment coefficient derivative with respect to P_W

$C_{n_{r_{WBT}}}$ Aircraft yawing moment coefficient derivative with respect to R_W

$C_{n_{\beta_{WBT}}}$ Aircraft yawing moment coefficient derivative in relation to β

$C_{n_{\dot{\beta}_{WBT}}}$ Aircraft yawing moment coefficient derivative with respect to $\dot{\beta}$

$C_{n_{\delta_a}}$ Aircraft yawing moment coefficient derivative in relation to ailerons deflection

$C_{n_{\delta_r}}$ Aircraft yawing moment coefficient derivative in relation to rudder deflection

C_{N_B} Body normal force coefficient

$C_{N_{HT_e}}$ Exposed horizontal tail normal force coefficient

$C_{N_{W_e}}$ Exposed wing normal force coefficient

C_p Jacobian matrix of the equilibrium point

C_P Propeller power coefficient

C_Q Propeller torque coefficient

$\boldsymbol{C_R}$ Contraction vertex closer to the current

C_T Propeller thrust coefficient

C_W Angular motion equation coefficient C in wind axes ($SCS^T = C_W$)

$\boldsymbol{C_W}$ Contraction vertex closer to the worst

C_{X_B} Body axial force coefficient

$C_{X_{HT_e}}$ Exposed horizontal tail axial force coefficient

$C_{X_{W_e}}$ Exposed wing axial force coefficient

C_Y Aerodynamic side force coefficient

$C_{Y_{p_{WBT}}}$ Aircraft side force derivative with respect to P_W

$C_{Y_{r_{WBT}}}$ Aircraft side force derivative with respect to R_W

$C_{Y_{WBT}}$ Aircraft static side force coefficient

$C_{Y_{\beta_{LG}}}$ Aircraft landing gear side force derivative with respect to β

$C_{Y_{\beta_{WBT}}}$ Aircraft side force derivative with respect to β

$C_{Y_{\dot{\beta}_{WBT}}}$ Aircraft side force derivative with respect to $\dot{\beta}$

$C_{Y_{\delta_r}}$ Aircraft side force derivative with respect to δ_r

d Leg diameter

d_B Fuselage maximum diameter

d_{BHT} Body diameter at the horizontal tail section

d_w Landing gear wheel diameter

d_{WB} Body diameter at the wing section

D Angular motion equation coefficient D; propeller diameter; Aerodynamic drag

D_p Jacobian matrix of the equilibrium point

D_{Q_W} Aerodynamic drag derivative with respective to Q_W

D_{V_T} Aerodynamic drag derivative with respective to V_T

D_W Angular motion equation coefficient D in wind axes ($SDS^T = D_W$)

D_α Aerodynamic drag derivative with respective to α

D_β Aerodynamic drag derivative with respective to β

$D_{\delta_{a_L}}$ Aerodynamic drag derivative with respective to δ_{a_L}

D_{δ_e} Aerodynamic drag derivative with respective to δ_e

D_{δ_f} Aerodynamic drag derivative with respective to δ_f

$D_{\delta_{a_R}}$ Aerodynamic drag derivative with respective to δ_{a_R}

$D_{\delta_{HT}}$ Aerodynamic drag derivative with respective to δ_{HT}

D_{δ_W} Aerodynamic drag derivative with respective to δ_W

$D_{\omega_j^2}$ Aerodynamic drag derivative with respective to ω_j^2

e_{HT_e} Exposed horizontal tail Oswald factor

e_{W_e} Exposed wing Oswald factor

E Torque due to tilt movement of rotating parts; jacobian matrix

$\boldsymbol{E}$ Expansion vertex

E_W Torque due to tilt movement of rotating parts in wind axes ($SE = E_W$)

$\vec{f}$ Vector of n scalar nonlinear functions f_i

F Portion of the acceleration resulting from movement of the concentrated masses around the aircraft body coordinate frame

$\vec{F}^B$ Net applied force vector in the body coordinate frame

$\vec{F}_B^W$ Net applied force vector in the wind axes

$\vec{F}^E$ Net applied force vector in the Earth fixed inertial reference frame

$\vec{F}_{legs}^W$ Landing gear legs total aerodynamic forces vector

$\vec{F}_{LG}^W$ Landing gear wind axis reference frame the force vector

$\vec{F}_{wheels}^W$ Landing gear wheels total aerodynamic forces vector

F_x^W x direction net applied force vector in the wind axes

F_y^W y direction net applied force vector in the wind axes

F_z^W z direction net applied force vector in the wind axes

$\left(\frac{F}{T}\right)_{camber}$ Thrust factor accounting for wing camber

$\left(\frac{F}{T}\right)_{deflection}$ Thrust factor accounting for wing camber plus control deflection

$\left(\frac{F}{T}\right)_{d_{Wa_{L,R}}}$ Thrust factor accounting for horizontal tail camber plus elevator deflection

$\vec{g}^E$ gravity vector in the Earth fixed inertial reference frame

$\vec{G}_B^E$ Body linear momentum in the Earth fixed inertial reference frame

G_{HT}	Horizontal tail transmission gear ratio
$\vec{G}_{R_j}^{E}$	Rotor linear momentum in the Earth fixed inertial reference frame
$\vec{G}_{total}^{E}$	Total linear momentum in the Earth fixed inertial reference frame
G_W	Wing transmission gear ratio
$\vec{G}_{W_i}^{E}$	Aerodynamic surface linear momentum in the Earth fixed inertial reference frame
h	Altitude
$\dot{h}$	Altitude derivative
h_I	Altitude integral
$\vec{H}_B^B$	Body angular momentum in the body coordinate frame
$\vec{H}_B^E$	Body angular momentum in the Earth fixed inertial reference frame
$\vec{H}_{R_j}^B$	Rotor angular momentum in the body coordinate frame
$\vec{H}_{R_j}^E$	Rotor angular momentum in the Earth fixed inertial reference frame
$\vec{H}_{total}^B$	Total angular momentum in the body coordinate frame
$\vec{H}_{total}^E$	Total angular momentum in the Earth fixed inertial reference frame
$\vec{H}_{W_i}^B$	Aerodynamic surface angular momentum in the body coordinate frame
$\vec{H}_{W_i}^E$	Aerodynamic surface angular momentum in the Earth fixed inertial reference frame
I	Identity matrix
$\tilde{I}_B^B$	Inertia matrix of the body in the body coordinate frame
$\tilde{I}_B^E$	Inertia matrix of the body in the Earth fixed inertial reference frame
$\tilde{I}_{R_j}^B$	Inertia matrix of the rotor in the body coordinate frame
$\tilde{I}_{R_j}^E$	Inertia matrix of the rotor in the Earth fixed inertial reference frame
$\tilde{I}_{W_i}^B$	Inertia matrix of the aerodynamic surface in the body coordinate frame
$\tilde{I}_{W_i}^E$	Inertia matrix of the aerodynamic surface in the Earth fixed inertial reference frame
J	Propeller advance ratio
J'	Modified advance ratio
J_{0_T}	Propeller advance ratio at zero thrust
$\mathcal{J}$	Cost function
k_Q	Propeller torque coefficient
k_T	Propeller thrust coefficient
$k_{p,h}$, $k_{d,h}$ and $k_{I,h}$	Altitude hold control coefficients

K Empirical factor depending on planform geometry

K' Empirical correction factor that corrects Δc_l for nonlinear effects at high control deflections

$\overline{K}$ Gain matrix

K_{a_i} Actuator gain

K_b Control span fator

K_{BHT} Body-horizontal tail interference factor

K_{HT} Number of propellers on the horizontal tail

K_p Rotor gain

K_W Number of propellers on the wing

$K_{W_{a_{L,R}}}$ Number of propellers that affect the left and right ailerons respectively

K_{WB} Wing-body tail interference factor

K_Λ Conversion factor for a partial span control on a sweptback wing

l Landing gear leg length

l_B Fuselage length

l_V Longitudinal distance between vertical tail aerodynamic center and moment reference point

L Aerodynamic lift

L_{Q_W} Aerodynamic lift derivative with respective to Q_W

L_{V_T} Aerodynamic lift derivative with respective to V_T

L_α Aerodynamic lift derivative with respective to α

$L_{\dot{\alpha}}$ Aerodynamic lift derivative with respective to $\dot{\alpha}$

$L_{\delta_{a_L}}$ Aerodynamic lift derivative with respective to δ_{a_L}

$L_{\delta_{a_R}}$ Aerodynamic lift derivative with respective to δ_{a_R}

L_{δ_e} Aerodynamic lift derivative with respective to δ_e

L_{δ_f} Aerodynamic lift derivative with respective to δ_f

$L_{\delta_{HT}}$ Aerodynamic lift derivative with respective to δ_{HT}

L_{δ_W} Aerodynamic lift derivative with respective to δ_W

$L_{\omega_j^2}$ Aerodynamic lift derivative with respective to ω_j^2

$\overline{L}$ Aerodynamic rolling moment

$\overline{L}_{P_W}$ Aerodynamic rolling moment derivative with respective to P_W

$\bar{L}_{R_W}$ Aerodynamic rolling moment derivative with respective to R_W

$\bar{L}_{V_T}$ Aerodynamic rolling moment derivative with respective to V_T

$\bar{L}_{\beta}$ Aerodynamic rolling moment derivative with respective to β

$\bar{L}_{\dot{\beta}}$ Aerodynamic rolling moment derivative with respective to $\dot{\beta}$

$\bar{L}_{\delta_{a_L}}$ Aerodynamic rolling moment derivative with respective to δ_{a_L}

$\bar{L}_{\delta_{a_R}}$ Aerodynamic rolling moment derivative with respective to $\delta_{\delta_{a_R}}$

$\bar{L}_{\delta_r}$ Aerodynamic rolling moment derivative with respective to δ_r

$\bar{L}_{\omega_j^2}$ Aerodynamic rolling moment derivative with respective to ω_j^2

m_B Body mass

m_{R_j} Rotor mass

m_{W_i} Aerodynamic surface mass

M Total aircraft mass; aerodynamic pitching moment; Mach number

M_a Actuator torque

$M_{a_{HT}}$ Horizontal tail actuator torque

M_{a_W} Wing actuator torque

M_P Sum of concentrated masses torque acting on the aircraft body C.G. due to their weights

M_{P_W} Sum of concentrated masses torque acting on the aircraft body C.G. due to their weights in wind axes $\left(SM_P = M_{P_W}\right)$

M_{Q_W} Aerodynamic pitching moment derivative with respective to Q_W

M_{V_T} Aerodynamic pitching moment derivative with respective to V_T

M_{α} Aerodynamic pitching moment derivative with respective to α

$M_{\dot{\alpha}}$ Aerodynamic pitching moment derivative with respective to $\dot{\alpha}$

M_{β} Aerodynamic pitching moment derivative with respective to β

M_{δ_e} Aerodynamic pitching moment derivative with respective to δ_e

M_{δ_f} Aerodynamic pitching moment derivative with respective to δ_f

$M_{\delta_{HT}}$ Aerodynamic pitching moment derivative with respective to δ_{HT}

M_{δ_W} Aerodynamic pitching moment derivative with respective to δ_W

$M_{\omega_j^2}$ Aerodynamic pitching moment derivative with respective to ω_j^2

n Propeller revolutions per second

N Aerodynamic yawing moment

$\boldsymbol{N}$ Simplex vertex with the next to worst outcome

NLe Number of landing gear legs

NWh Number of landing gear wheels

N_{P_W} Aerodynamic yawing moment derivative with respective to P_W

N_{R_W} Aerodynamic yawing moment derivative with respective to R_W

N_{V_T} Aerodynamic yawing moment derivative with respective to V_T

N_{β} Aerodynamic yawing moment derivative with respective to β

$N_{\dot{\beta}}$ Aerodynamic yawing moment derivative with respective to $\dot{\beta}$

$N_{\delta_{a_L}}$ Aerodynamic yawing moment derivative with respective to δ_{a_L}

$N_{\delta_{a_R}}$ Aerodynamic yawing moment derivative with respective to δ_{a_R}

N_{δ_r} Aerodynamic yawing moment derivative with respective to δ_r

$N_{\omega_j^2}$ Aerodynamic yawing moment derivative with respective to ω_j^2

O_B Origin of the aircraft body coordinate frame

O_E Origin of the Earth fixed inertial reference frame

O_{HTL} Origin of the left horizontal tail coordinate frame

O_{HTR} Origin of the right horizontal tail coordinate frame

O_{Ri} Origin of the rotor coordinate frame

O_{WL} Origin of the left wing coordinate frame

O_{WR} Origin of the right wing coordinate frame

p Pressure

$p_{fuselage}$ Fuselage maximum cross section perimeter

$p_{pistons}$ Fuselage maximum cross section perimeter covered by the exposed engine pistons

P Roll angular velocity, propeller power; solution of the Algebraic Riccati Equation

$\boldsymbol{\overline{P}}$ Average combination of variables except the worst

PA Pressure altitude

P_{HT} Horizontal tail pivot point

P_W Wing pivot point; Roll angular velocity in the wind axes

$\dot{P}_W$ Roll angular velocity derivative in the wind axes

P_0 Pressure at sea level

P_1 Pivot dynamics coefficient 1

P_2 Pivot dynamics coefficient 2

q'' Slipstream dynamic pressure

q_∞ Free stream dynamic pressure

Q Pitch angular velocity; propeller torque; LQR matrix

Q_j Propeller torque

Q_W Pitch angular velocity in the wind axes

$\dot{Q}_W$ Pitch angular velocity derivative in the wind axes

$\vec{r}_{A/B}$ Position vector of point A in relation to point B

$\vec{r}_i$ Position vector of landing gear component aerodynamic forces center

$\vec{r}_{pivot_i/B}$ Position vector of pivot point origin in relation to body origin

$\vec{r}_{ref}$ Position vector of aircraft moment reference point

$\vec{r}_{W_i/B}$ Position vector of aerodynamic surface origin in relation to body origin

$\vec{r}_{W_i/pivot_i}$ Position vector of aerodynamic surface origin in relation to pivot point origin

R Yaw angular velocity; specific gas constant for dry air; LQR matrix

$\boldsymbol{R}$ Reflection of the worst outcome with respect to $\overline{\boldsymbol{P}}$

Re Reynolds number

R_A^B Rotation matrix between reference frames A to B

$\dot{R}_A^B$ Rotation matrix derivative between reference frames A to B

$R_{R_j}^B$ Rotation matrix between rotor reference frame to body coordinate frame

$\dot{R}_{R_j}^B$ Rotation matrix derivative between rotor reference frame to body coordinate frame

$\ddot{R}_{R_j}^B$ Rotation matrix second derivative between rotor reference frame to body coordinate frame

$\tilde{R}_{R_j}^B$ Rotor inertia translation matrix

R_W Yaw angular velocity in the wind axes

$\dot{R}_W$ Yaw angular velocity derivative in the wind axes

$R_{W_i}^B$ Rotation matrix between aerodynamic surface reference frame to body coordinate frame

$\dot{R}_{W_i}^B$ Rotation matrix derivative between aerodynamic surface reference frame to body coordinate frame

$\ddot{R}_{W_i}^B$ Rotation matrix second derivative between aerodynamic surface reference frame to body coordinate frame

$\tilde{R}_{W_i}^{B}$ Aerodynamic surface inertia translation matrix

S Transformation matrix of body axes to wind axes

$\dot{S}$ Transformation matrix derivative of body axes to wind axes

S^T Transformation matrix of body axes to wind axes transpose

$\dot{S}^T$ Transformation matrix derivative of body axes to wind axes transpose

S_b Aircraft body base area (maximum cross section area)

S_{HT_e} Exposed horizontal tail planform area

S_p Aircraft body planform area; Propeller disk area

S_{VT_e} Exposed vertical tail planform area

S_W Aircraft wing reference area

S_{W_e} Exposed wing planform area

$S_{W_{wake}}$ Wing, or horizontal tail, surface planform area under effect of propellers wake

$S_{W_{wake,control}}$ Surface planform area under effect of propellers wake that also have aerodynamic control

S_α Transformation matrix of body axes to stability axes

S_β Transformation matrix of stability axes to wind axes

T Temperature; Thrust per propeller or total thrust when used in thrust recovery factor

$\vec{T}_B^B$ Net torque acting at the aircraft body coordinate frame

$\vec{T}_B^W$ Net torque acting at the wind axes

$T_c^{"}$ Propeller thrust coefficient based on slipstream velocity and propeller disk area

$T_{c\ HT}^{"}$ Propellers thrust coefficient for the horizontal tail

$T_{c\ W}^{"}$ Propellers thrust coefficient for the wing

T_j Propeller thrust

$[T]_{R_j}$ Rotor inertia rotation matrix

$[T]_{R_j}^T$ Rotor inertia rotation matrix transpose

$[\dot{T}]_{R_j}$ Rotor inertia rotation matrix derivative

$[\dot{T}]_{R_j}^T$ Rotor inertia rotation matrix derivative transpose

$[T]_{W_i}$ Aerodynamic surface inertia rotation matrix

$[T]_{W_i}^T$ Aerodynamic surface inertia rotation matrix transpose

$[\dot{T}]_{W_i}$ Aerodynamic surface inertia rotation matrix derivative

$[\dot{T}]_{W_i}^T$ Aerodynamic surface inertia rotation matrix derivative transpose

T_0 Temperature at sea level

$\vec{u}$ Perturbation vector of controls from the equilibrium point

U Aircraft body velocity in the x direction

$\vec{U}$ Control vector

$\vec{U}_e$ Control vector in the equilibrium point

V Aircraft body velocity in the y direction

$\vec{V}_A$ Velocity vector in an arbitrary reference frame A

$\dot{\vec{V}}_A$ Velocity derivative vector in an arbitrary reference frame A

V_B Aircraft body volume

$\vec{V}_B$ Velocity vector in an arbitrary reference frame B

$\dot{\vec{V}}_B$ Velocity derivative vector in an arbitrary reference frame B

$\vec{V}_B^B$ Body velocity in the body coordinate frame

$\dot{\vec{V}}_B^B$ Body velocity derivative in the body coordinate frame

$\vec{V}_B^E$ Body velocity in the Earth fixed inertial reference frame

$\dot{\vec{V}}_B^E$ Body acceleration in the Earth fixed inertial reference frame

$\vec{V}_B^W$ Aircraft body velocity vector in the wind axes

V_{lat} Lateral component of velocity vector

V_{long} Longitudinal component of velocity vector

$\vec{V}_{rel_{A/B}}$ Relative velocity vector of point A in relation to point B

$\vec{V}_{rel_{R_j/B}}^B$ Relative velocity vector of rotor origin in relation to body origin

$\vec{V}_{rel_{W_i/B}}^B$ Relative velocity vector of aerodynamic surface origin in relation to body origin

$\vec{V}_{R_j}^E$ Rotor velocity in the Earth fixed inertial reference frame

$\dot{\vec{V}}_{R_j}^E$ Rotor acceleration in the Earth fixed inertial reference frame

V_T Aircraft body flight speed

$\dot{V}_T$ Aircraft body flight speed derivative

$\vec{V}_{W_i}^E$ Aerodynamic surface velocity in the Earth fixed inertial reference frame

$\dot{\vec{V}}_{W_i}^E$ Aerodynamic surface acceleration in the Earth fixed inertial reference frame

w Landing gear wheel width

w_i Cost function weight

W Aircraft body velocity in the z direction

$\boldsymbol{W}$ Simplex vertex whose outcome is the worst

$\vec{x}$ Perturbation vector of states from the equilibrium point

$\dot{\vec{x}}$ Perturbation vector of states derivatives from the equilibrium point

$x_{AC_{HT}}$ Coordinate x of horizontal tail aerodynamic center

x_{AC_W} Coordinate x of wing aerodynamic center

$\vec{x}_I$ State integral vector

x_{ref} x coordinate reference point for pitching moment

x_{refB} Coordinate x of body pitching moment reference point

$x_{R_j/B}$ x position of rotor origin in relation to body origin

$\dot{x}_{R_j/B}$ x velocity of rotor origin in relation to body origin

$x_{W_i/B}$ x position of aerodynamic surface origin in relation to body origin

$\dot{x}_{W_i/B}$ x velocity of aerodynamic surface origin in relation to body origin

$\vec{X}$ State vector

$\dot{\vec{X}}$ State vector derivative

$\vec{X}_e$ Steady state vector or equilibrium point

$\vec{y}$ Output disturbance vector

$y_{MAC_{Si_L}}$ Lateral position of left aileron mean aerodynamic chord of the wing region immersed on propeller wake

$y_{MAC_{Si_R}}$ Lateral position of right aileron mean aerodynamic chord of the wing region immersed on propeller wake

y_{ref} y coordinate reference point for pitching moment

$y_{R_j/B}$ y position of rotor origin in relation to body origin

$\dot{y}_{R_j/B}$ y velocity of rotor origin in relation to body origin

$y_{W_i/B}$ y position of aerodynamic surface origin in relation to body origin

$\dot{y}_{W_i/B}$ y velocity of aerodynamic surface origin in relation to body origin

Y Aerodynamic side force

$\vec{Y}$ System output vector

Y_{P_W} Aerodynamic side force derivative with respective to P_W

Y_{R_W} Aerodynamic side force derivative with respective to R_W

Y_α Aerodynamic side force derivative with respective to α

Y_β Aerodynamic side force derivative with respective to β

$Y_{\dot{\beta}}$ Aerodynamic side force derivative with respective to $\dot{\beta}$

Y_{V_T} Aerodynamic side force derivative with respective to V_T

Y_{δ_r} Aerodynamic side force derivative with respective to δ_r

$z_{AC_{HT}}$ Coordinate z of horizontal tail aerodynamic center

z_{AC_W} Coordinate z of wing aerodynamic center

z_{ref} z coordinate reference point for pitching moment

z_{refB} Coordinate z of body pitching moment reference point

$z_{R_j/B}$ z position of rotor origin in relation to body origin

$\dot{z}_{R_j/B}$ z velocity of rotor origin in relation to body origin

z_v Vertical distance between vertical tail aerodynamic center and moment reference point

$z_{W_i/B}$ z position of aerodynamic surface origin in relation to body origin

$\dot{z}_{W_i/B}$ z velocity of aerodynamic surface origin in relation to body origin

α Aircraft body angle of attack

$\dot{\alpha}$ Aircraft body angle of attack derivative

α' Angle between the landing gear leg and longitudinal velocity vector

α'' Angle between the landing gear leg and lateral velocity vector

α_W Wing, or horizontal tail, angle of attack

α_δ Section lift effectiveness

$\alpha_{0_{HT}}$ Exposed horizontal tail angle of attack of zero lift

α_{0_W} Exposed wing angle of attack of zero lift

$\frac{(\alpha_\delta)_{C_L}}{(\alpha_\delta)_{c_l}}$ Ratio of the three-dimensional control effectiveness parameter to the two-dimensional control effectiveness parameter

β Aircraft body sideslip angle

$\dot{\beta}$ Aircraft body sideslip angle derivative

γ Flight path angle

δ_{a_L} Left aileron deflection angle

$\delta_{a_L}^C$ Left aileron control signal

$\dot{\delta}_{a_L}$ Left aileron deflection angle derivative

δ_{a_R} Right aileron deflection angle

$\delta_{a_R}^C$ Right aileron control signal

$\dot{\delta}_{a_R}$ Right aileron deflection angle derivative

δ_e Elevator deflection angle

δ_e^C Elevator control signal

$\dot{\delta}_e$ Elevator deflection angle derivative

δ_f Flap deflection angle

δ_f^C Flap control signal

$\dot{\delta}_f$ Flap deflection angle derivative

δ_{HT} Horizontal tail tilt angle

δ_{HT}^C Horizontal tail control signal

$\dot{\delta}_{HT}$ Horizontal tail tilt angle derivative

$\ddot{\delta}_{HT}$ Horizontal tail tilt angle second derivative

δ_r Rudder deflection angle

δ_r^C Rudder control signal

$\dot{\delta}_r$ Rudder deflection angle derivative

δ_W Wing tilt angle

δ_W^C Wing control signal

$\dot{\delta}_W$ Wing tilt angle derivative

$\ddot{\delta}_W$ Wing tilt angle second derivative

$\delta_{\perp HL}$ Control surface deflection measured normal to the control hinge line, to the control streamwise deflection δ

Δc_l Two-dimensional lift coefficient increment due to control deflection δ

$\Delta C_{D_{p,HT}}$ Increment in horizontal tail drag coefficient due to propellers wake

$\Delta C_{D_{p_L}}$ and $\Delta C_{D_{p_R}}$ Increment in wing drag coefficient due to propellers wake acting

$\Delta C_{D_{p,W}}$ Increment in wing drag coefficient due to propellers wake on the left and right ailerons

$(\Delta C_{D0})_{eng}$ Drag coefficient increment due to engine

$(\Delta C_{D0})_{ra}$ Drag coefficient increment due to radial engine drag

ΔC_{l_p} Rolling moment coefficient increment due to propellers wake

ΔC_L Three-dimensional lift coefficient increment of the aerodynamic surface of the respective aerodynamic control

$\Delta C_{L_{p,HT}}$ Increment in horizontal lift coefficient due to propellers wake

$\Delta C_{L_{p_L}}$ and $\Delta C_{L_{p_R}}$ Increment in wing lift coefficient due to propellers wake acting on the left and right ailerons

$\Delta C_{L_{p,W}}$ Increment in wing lift coefficient due to propellers wake

$\Delta C_{m_{\delta_e}}$ Increment in pitching moment coefficient due to elevator deflection

$\frac{\Delta C'_m}{\Delta C_L}$ Ratio of pitching moment increment to lift increment for a full span flap on a rectangular wing

ΔC_{n_p} Yawing moment coefficient increment due to propellers wake

ΔISA Real temperature shift at sea level

ΔV_T Increment in flight speed at the horizontal tail

$\Delta\theta$ Slipstream turning angle increment due to wing camber and incidence between the wing chord plane and the thrust axis

$\mathcal{E}_c$ Controls states disturbances cost function

$\mathcal{E}_d$ States derivative cost function

$\mathcal{E}_s$ States disturbance cost function

$\mathcal{E}_{th_c}$ Controls states disturbances cost function threshold

$\mathcal{E}_{th_d}$ States derivative cost function threshold

$\mathcal{E}_{th_s}$ States disturbance cost function threshold

ϵ Downwash angle at the horizontal tail

ϵ_h Altitude error

$\epsilon_{\dot{h}}$ Vertical velocity error

ϵ_{h_I} Altitude integral error

ϵ_{HT} Horizontal tail position error

ϵ_W Wing position error

η Ratio of the drag on a finite cylinder to the drag on an infinite cylinder

θ Pitch angle; slipstream turning angle

$\dot{\theta}$ Pitch angle derivatives

θ_f Slipstream turning angle under conditions of zero incidence and zero camber

θ_{max} Maximum turning angle

λ Landing gear leg longitudinal angle in relation to the vertical axis

λ' Landing gear leg lateral angle in relation to the vertical axis

λ_{a_i} Actuator time constant

λ_j Propeller rotation direction index

λ_p Rotor time constant

$\Lambda_{c/4}$ Sweep of the wing quarter chord

Λ_{HL} Control surface hinge line angle with respect to the lateral axis y

ν Kinematic air viscosity

ρ Air density; scalar weighting parameter

ρ_0 Air density at sea level

σ_e Effective propeller solidity

ϕ Roll angle; angle of bank of the body about its longitudinal axis

$\dot{\phi}$ Roll angle derivative

ψ Yaw angle

$\dot{\psi}$ Yaw angle derivatives

$\vec{\omega}_B$ Angular velocity vector of frame B

$\dot{\vec{\omega}}_B$ Angular velocity derivative vector of frame B

$\vec{\omega}_B^B$ Body angular velocity vector in the body coordinate frame

$\dot{\vec{\omega}}_B^B$ Body angular velocity derivative vector in the body coordinate frame

ω_j Propeller angular speed

$\omega_{\mathrm{j}}^{2^C}$ Propeller rotation squared control signal

ω_{R_j} Rotor angular velocity

$\dot{\omega}_{R_j}$ Rotor angular acceleration

$\vec{\omega}_{R_j}^{R_j}$ Rotor angular velocity vector in respect to its own reference frame

$\dot{\vec{\omega}}_{R_j}^{R_j}$ Rotor angular velocity derivative vector in respect to its own reference frame

$\vec{\omega}_{W_i}^{W_i}$ Aerodynamic surface angular velocity vector in respect to its own reference frame

Ω_B Body angular velocity vector in the body coordinate frame cross product in matrix form

$\dot{\Omega}_B$ Body angular velocity derivative vector in the body coordinate frame cross product in matrix form

Ω_R Transformation matrix defined as $S\dot{S}^T$

Ω_{HT} Horizontal tail stepper motor speed

Ω_{W} Body angular velocity vector in the wind axes cross product in matrix form; wing stepper motor speed

$\dot{\Omega}_{W}$ Body angular velocity derivative vector in the wind axes cross product in matrix form

∇ Row vector of first partial derivatives

SUMMARY

1 INTRODUCTION

This work focuses in the flight dynamics modeling and design of control system of a VTOL UAV concept, which has the main feature of tilting both wing and horizontal tail, along with the rotors. There is great interest in VTOL aircraft configuration due to its advantage of needing little or no support equipment at all to allow take-off and landing, such as runways, catapult, parachute or landing net, making the operation easier and cheaper.

Moreover, there is a wide possibility of civil applications in the UAV market such as: surveillance, mapping, 3D modeling issues (NEX; REMONDINO, 2014), agriculture monitoring and crop spraying, first responders, search and rescue, forest fire management, pipeline and power line monitoring, cinema and TV applications, environment and wildlife survey, topographical survey, offshore monitoring, border monitoring, search and rescue, counter narcotics, news services, crowd and traffic monitoring. Most of these applications require for the aircraft a stabilized gimbal with some sensors, such as high definition or thermal cameras, which can be changed according to the operation.

Besides, there are some possible configurations of VTOL aircraft, for instance: tail sitters, helicopters in several configurations, tilt rotors, tilt wing and quad or multirotor. Some more configurations were studied by Fredericks et al (2013) being that they came to the conclusion of the best configuration for a high efficiency cruise with vertical take-off and landing is a hybrid-electric propulsion semi tandem wing configuration, the name they called for halfway between a tandem wing and conventional wing configuration. This configuration is shown in Figure 1, depicting the NASA Greased Lightning 10 CAD model (ROTHHAAR et al., 2014). The hybrid-electric propulsion is a combination of combustion engines and electric motors, where engines inside the fuselage generate power by means of alternators to drive several electric motors positioned in the wing and horizontal stabilizer, which can tilt. The distributed propulsion enables the entire wing to be blow by propeller wash in order to avoid stall during transition. Therefore, in this work we will propose a design based on this VTOL configuration.

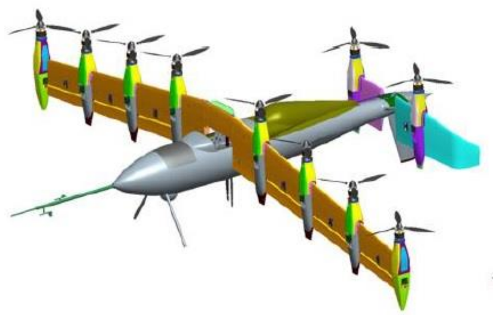


Figure 1: NASA Greased Lightning 10 (GL-10), semi tandem wing VTOL configuration (ROTHHAAR et al., 2014).

We will use some design goals in order to make the sizing of the aircraft for further analysis and design of control system and flight dynamics modeling. The maximum take-off weight must be less than 25 kg, which is the weight boundary for class 3 RPA (remotely piloted aircraft) of the brazilian regulation for UAVs (ANAC, 2017), which have less operation and project requirements by the authority and the construction and validation through experiments of a prototype should be cheaper. Additionally, most of the components should be commercial off the shelf, thus the combustion engine must use gasoline, because of high availability of this kind for RC aircraft. Besides that, we will not establish beforehand a range or endurance goal, since the first objective of this work is to model the flight dynamics and design the control system of the proposed VTOL UAV configuration, therefore there is no need for a long endurance design at this point.

Conceptually, the aircraft should be able to carry as payload a gyro stabilized gimbal with some sensors not defined in here, but possibly high definition and thermal cameras, or some other that makes possible the applications previously mentioned.

It is expected that the power required for take-off and landing should be much higher than that for cruise, thus the use of folding propellers is useful, so that during cruise some of the rotors can be turned off and the propeller drag is substantially reduced.

Furthermore, we will consider fixed tricycle landing gear, since there should be no room for retractable gear.

For the sizing, a search for possible commercial off the shelf components was made in order to have a database of weight and power of available components. Additionally, the structure should be made of composites, mostly carbon fiber with epoxy resin. However, no structural analysis was made, so the conceptual airframe for this study was done in a way that results in somewhat close to 40% airframe weight fraction, the same fraction used in the conceptual analysis of Fredericks et al (2013). The conceptual airframe layout is important so that we may have a reasonable mass distribution, which must be used to compute the inertia matrices.

Having the concept of the aircraft, we proceed with the flight dynamics modeling, where we present the equations of translation and angular motion. Yet, the traditional 6 degree-of-freedom rigid body equations would be an oversimplification of the system, since the aircraft configuration involves the tilt movement of the wing and horizontal stabilizer, both with spinning propellers, which results in shifting of the center of gravity and gyroscopic moments. In this way, a more complex formulation is required. It is appropriate to use the multi-body equations of motion, similar to those applied by Haixu et al (2010), where the aircraft is divided in parts so that the wing, horizontal stabilizer and rotors are independent entities. This allows the assessment of the linear and angular momentum in the Earth fixed inertial reference frame as well as in body coordinate frame for each part, which are subsequently derived to obtain the equations of motion. Additionally, dynamic equations of the attitude, altitude, actuators, rotors and the wing and horizontal tail complete the system dynamics.

For the control system design and analysis a state space formulation is used, therefore, the nonlinear equations of motion are algebraically linearized around equilibrium points. Thus, we define the state and control vectors, and then we proceed with the computation of every state space derivative with respect to the equations of motion and the equations describing the dynamics of the controls, attitude and altitude.

The required aerodynamics coefficients and derivatives were computed using the methods of Hoak (1965), a data compendium (Datcom) which presents various methods for the determination aerodynamics data. Also, for some coefficients, mainly drag coefficients, the methods of Hoerner (1965) were used.

The equilibrium points are the combination of states that trim the aircraft, which must be computed as a function of flight speed in order to have a successful transition between hovering to cruise condition, and from cruise to hovering. So a numerical algorithm called Sequential Simplex is used to compute the equilibrium points through minimization of a cost function, which involves the sum of the squares of some state derivatives. Some constrains are used for each flight condition, such as flight speed, flight path angle, rate of turn and altitude.

Then, the control system strategies, equations and architecture are presented, being that the state control employs a linear quadratic regulator in order to maintain the aircraft in the equilibrium point, additionally an altitude hold controller keeps the aircraft in the desired altitude, and the transition between desired flight speeds uses a strategy of successive changes and stabilization between target conditions so that results in a smooth and safe transition. Simulations were made applying the proposed control system to an accelerated and decelerated longitudinal flight, and the results are presented and discussed, showing the progress of the longitudinal states with respect to time.

So, the design of a prototype is presented and the next steps of the project are proposed and discussed, and this report ends with the conclusions.

Lastly, in order to facilitate the understanding of the equations and have a cleaner presentation, some terms and derivatives used in the demonstrations of the state equations are presented only in the Appendices. Besides, the parameters used in the simulations are all listed in the Appendices.

2 AIRCRAFT SIZING

The sizing begins by evaluating proper propulsion system components, that is, propellers, gasoline engine, an electric generator and electric motors. For the sizing we will use four propellers on the wing and two on the horizontal tail, giving a total of 6 propellers.

In order to obtain an initial value for the minimum required power for the propulsive system we assess the condition of hovering flight where the power required should be close to maximum. At this situation it is reasonable to consider static condition for the propellers, so the total static thrust would be the weight of the aircraft. We will be using a target total weight of 22 kg. In Figure 2, is shown the total static thrust as function of total required power, that is, the sum of the six propellers required power for the propeller diameters of 12, 13 and 14 inches, whose sizes have many suppliers. These curves were obtained by using the definitions of thrust and power coefficients, where such coefficients were taken from experimental results of Brandt (2005). We have used as mean values $C_T = 0{,}09$ and $C_P = 0{,}06$.

$$C_T = \frac{T}{\rho n^2 D^4} \qquad C_P = \frac{P}{\rho n^3 D^5}$$

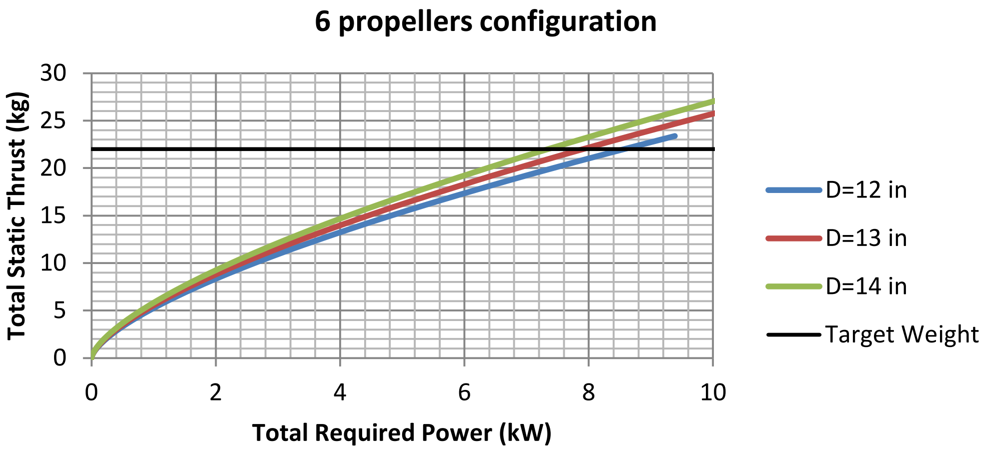


Figure 2: Total static thrust for 6 propellers configuration.

It is notable that for the larger the diameter of the propellers the less power is required, therefore we find the minimum required power for the rotors of 7.34 kW to have at least the target weight of 22 kg as total static thrust. Therefore, the minimum required power for the electric motors must be 1.22 kW.

Such power must be provided by the gasoline engine, electric generator and batteries, thus using the conversion efficiency of 95%, the same used in Fredericks et al. (2013), we find the minimum power required for the propulsive system of 7.72 kW.

With these minimum requirements it is possible to start the sizing by finding some commercial off-the-shelf components, which are listed in the Table 1. Note that the gasoline engine would not provide the full power required for hover at maximum weight, therefore the remaining power would be provided by batteries. It is worth mentioning that a full electric propulsive system, which means, central power provided only by batteries would be also possible, but the aircraft would have less autonomy, so it was decided to conceptually design the aircraft with the hybrid propulsive system.

Table 1: Propulsion system commercial off-the-shelf components.

Component	Specification	Quantity	Weight (kg)	Power (kW)
Gasoline Engine	DLE - 85 Gasoline Engine	1	2,550	6,34
Electric Generator	Turnigy RotoMax 80cc Size Brushless Outrunner Motor	1	1,916	6,60
Electric Motor	Turnigy Aerodrive SK3-5055-280kv Brushless Outrunner Motor	6	0,369	1,51
Propeller	Aeronaut Propellers CAM electric 14x9	6	0,047	-
Batteries	Multistar High Capacity 4S 8000 mAh Multirotor Lipo Pack Multistar High Capacity 6S 8000 mAh Multirotor Lipo Pack	1 1	0,643 0,956	1,38

The design objective is to use up to six propellers placed in the wing and horizontal tail, such that during take-off and landing all of them will be operating, and in cruise some may be turned off. Therefore, the use of folding propellers is advantageous to reduce drag when not operating. However, commercial off the shelf clockwise rotation folding propellers were not found, therefore the analyses will consider standard fixed blade propellers. Moreover, it would be advantageous to use propellers in the wing tip rotating opposite to the wing tip vortices, which may have benefits as an increase in propulsive efficiency or a reduction in induced drag (FREDERICKS et al., 2013).

Propeller performance data was obtained at Brandt (2005). The propeller chosen for the analysis was the Aeronaut CAM prop 14x9 from the manufacturer Aero-Naut Modellbau GmbH, which weights 47 g. However, since performance curves for this specific propeller is not available, we will use the performance curves of the APC Thin Electric 14x12, the closest available, shown in Figure 3 and Figure 4 for the static case. Additionally, the manufacturer informs that the maximum rotation speed is 10000 RPM.

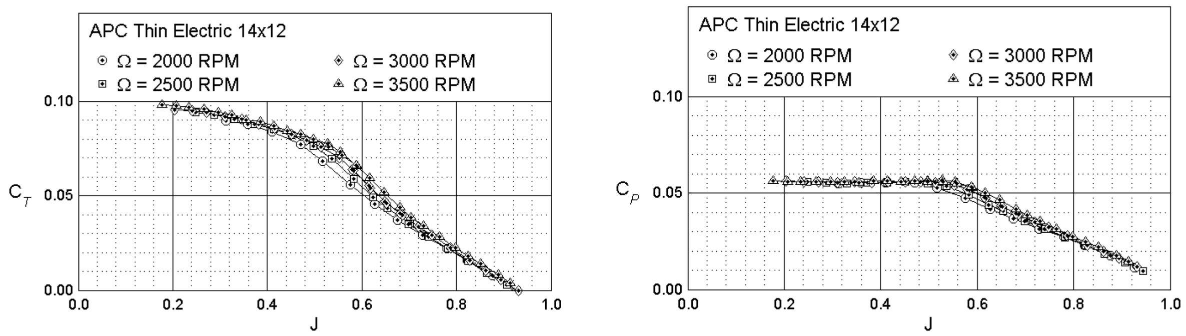


Figure 3: Propeller APC Thin Electric 14x12 thrust and power characteristics.

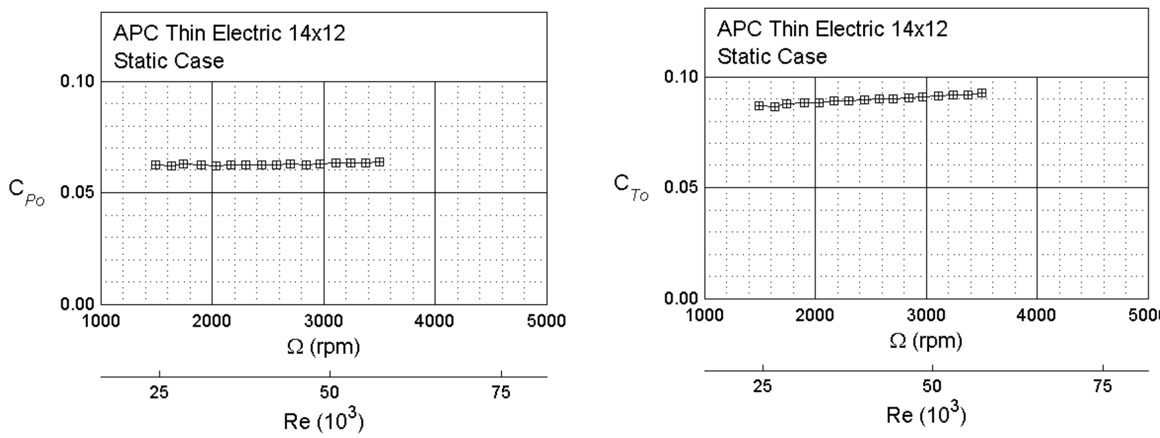


Figure 4: Propeller APC Thin Electric 14x12 static thrust and power characteristics.

The payload would be a gimbal with some sensors, which for now will not be defined, even to emphasize that the aircraft should be able to carry a variety of payloads. But, it should be reserved the weight of 0,8 kg.

For the fuel it was considered the density of 720 kg/m^3 for the aviation gasoline in a tank of 2,0 liters, which gives 1,44 kg of fuel.

The structures of the fuselage, wing, landing gear, horizontal and vertical tail should be made of carbon fiber with epoxy resin, with 50% fabric, which have density of 1600 kg/m^3. Since there will be no structural analysis in this work, the sizing of the structures were made so that the resulting airframe weight fraction would be close to 40% of the total weight, the same fraction used in the conceptual analysis of Fredericks et al. (2013).

Table 2: Aircraft parts weight.

	(kg)
MTOW	22
Fuel Weight	1,44
Gasoline Engine Weight	2,55
Electric Generator	1,916
Electric Motors	2,268
Payload Weight	0,800
Batteries	1,599
Miscellaneous	2,681
Structural Weight	8,8

The previous assumptions were used to determine the weights of the main parts of the aircraft, which are listed in Table 2. The components not yet defined, which would be servos, control board, antenna, wires and any other required equipment are grouped in the miscellaneous label. The sizing resulted in the airframe structural weight fraction of 40%

The sizing of the aerodynamic surfaces were made so that the aircraft trim would be possible for all flight speed range, that is, the trim algorithm, which will be explained later, would converge. The following Table 3 show some of the characteristics of the wing, horizontal and vertical tail, fuselage and propeller.

The wing airfoil is the S8036, designed for low speed aircraft (LYON et al., 1997). Whist for the horizontal and vertical tail the airfoil is the Naca 0012 (ABBOTT; DOENHOFF, 1959).

Finally, in Figure 5 and Figure 6 we have the four views of the resulting aircraft, with the cruise and hovering conditions respectively, where it is possible to note the wing and horizontal tail tilt in order to put the propellers in the proper position.

Table 3: Aircraft sizing results.

Wing Area (m^2)	0,36	Horizontal Tail Area (m^2)	0,16
Wing Aspect Ratio	7,11	Horizontal Tail Aspect Ratio	3,06
Wing Span (m)	1,6	Horizontal Tail Span (m)	0,7
Wing Taper Ratio	0,65	Horizontal Tail Taper Ratio	1
Wing Leading Edge Sweep (degrees)	1,71	Horizontal Tail Leading Edge Sweep (degrees)	0
Wing Dihedral Angle (degrees)	0	Horizontal Tail Dihedral Angle (degrees)	0
Wing Mean Aerodynamic Chord (m)	0,228	Horizontal Tail Mean Aerodynamic Chord (m)	0,228
Wing Mean Geometric Chord (m)	0,225	Horizontal Tail Mean Geometric Chord (m)	0,228
Wing Airfoil	S8036	Horizontal Tail Airfoil	Naca 0012
		Vertical Tail Area (m^2)	0,042
		Vertical Tail Aspect Ratio	0,84
		Vertical Tail Span (m)	0,188
		Vertical Tail Taper Ratio	0,65
Fuselage Length (m)	1,45	Vertical Tail Leading Edge Sweep (degrees)	13,7
Fuselage Cross Section Reference Area (m^2)	0,0303	Vertical Tail Mean Aerodynamic Chord (m)	0,226
Propeller Diameter (m)	0,355	Vertical Tail Mean Geometric Chord (m)	0,223
Wing Loading (kg/m^2)	61,1	Vertical Tail Airfoil	Naca 0012
Disc Loading (kg/m^2)	37	Vertical Tail Area (m^2)	0,042

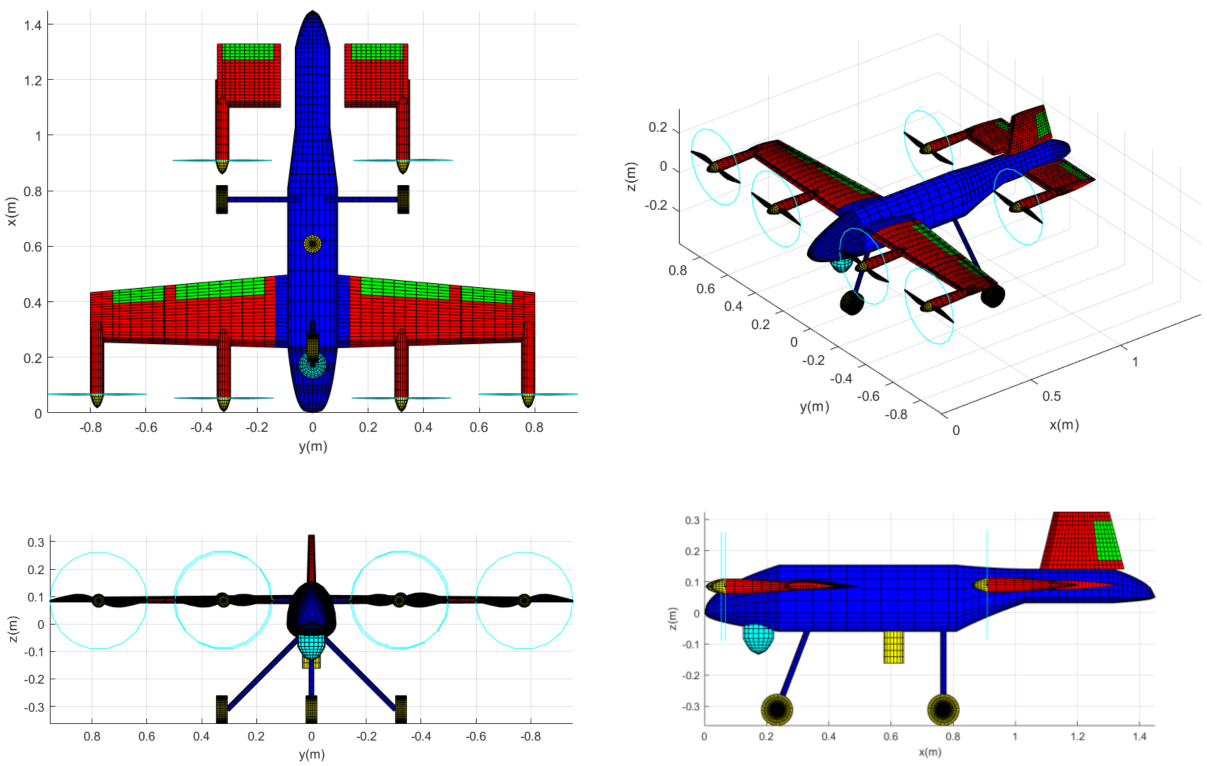


Figure 5: Aircraft cruise condition four views.

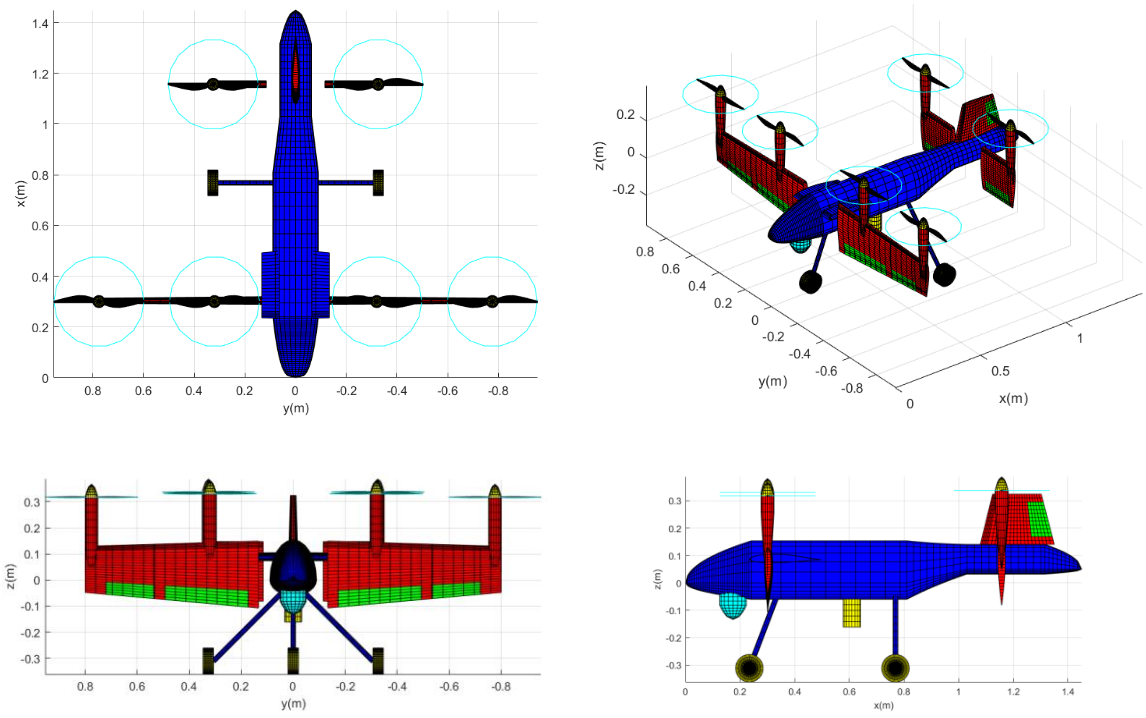


Figure 6: Aircraft hovering condition four views.

The structural concept and components layout are shown in Figure 7, where the wing and horizontal tail are tilted around the spar. The gasoline engine and generator are positioned in the mid of the fuselage, and the piston stays out of the fuselage so the cooling of such should be made by the airflow. The gimbal would be in the front of the fuselage facing down, where it would have the better angle of view. Lastly, the fuel tank would be right on top of the gasoline engine, in order to feed it.

Furthermore, in Figure 8 are shown the propellers rotation directions, where the wing and horizontal tail propellers rotates in opposition to the direction of the tip vortices. In this figure are also shown the center of gravity shift in the hovering condition with fuel tank full and empty. It is important that the C.G. be positioned somewhere between the wing and horizontal tail propellers in the hovering condition so that the control of the aircraft would be similar to a multirotor drone, where all propellers thrust points upwards, in other words, if the C.G. was in front of the wing propellers, the horizontal tail propellers would have thrust downwards in order to stabilize the aircraft in the hovering condition, which would be a waste of power and efficiency. In addition, since there are four front propellers and two rear propellers the best position for the C.G. would be a third of the distance between the front and rear propellers so that in the static condition the six would have the same thrust, minimizing the power required.

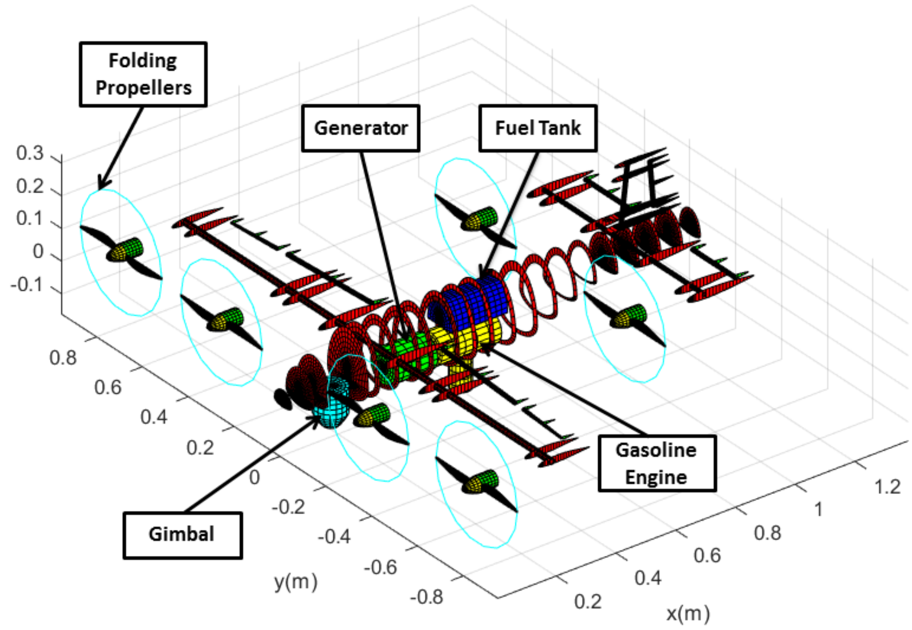


Figure 7: Aircraft components layout.

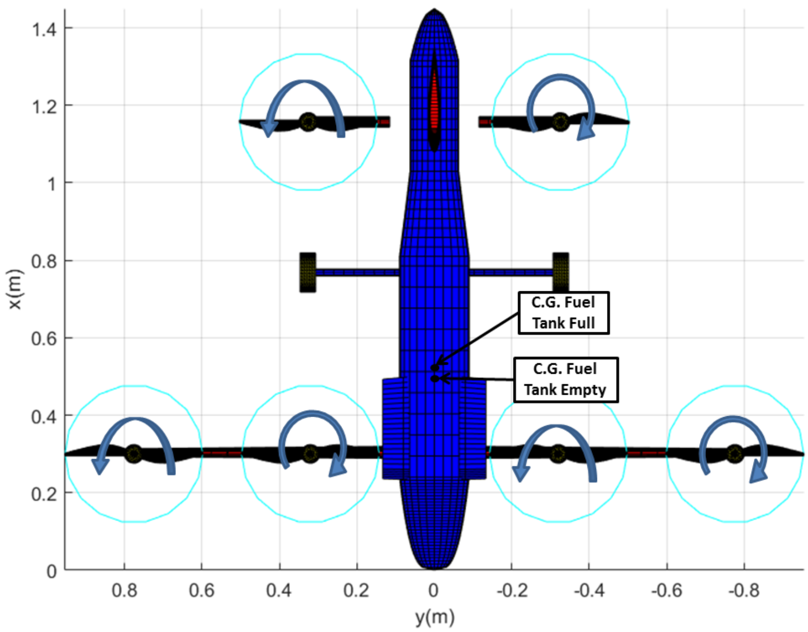


Figure 8: Propellers rotation directions and C.G. shift in hovering condition.

Besides, because of the wing and horizontal tail tilt, the aircraft center of gravity shifts simultaneously to such movement. In Figure 9 are shown the X and Z coordinates of the aircraft C.G. as function of the wing and respective horizontal tail tilt. The Y coordinate is always zero, because of the aircraft symmetry. Note that the C.G. is most aft at the hovering condition, and gradually moves forward while the surfaces tilt.

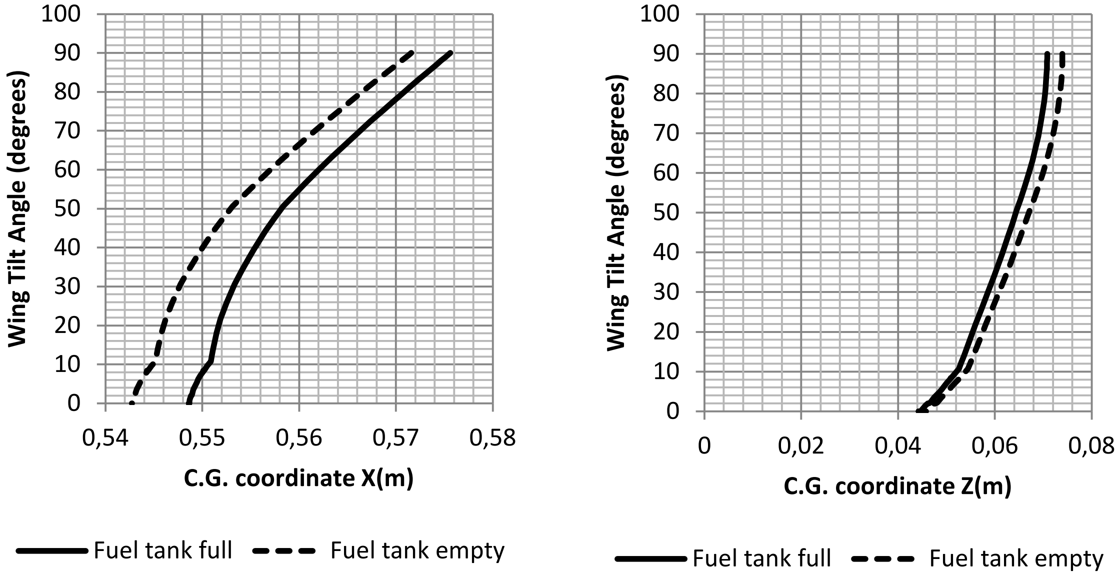


Figure 9: Aircraft C.G. shift as function of wing and respective horizontal tail tilt.

3 AIRCRAFT DYNAMIC SYSTEM MODEL

Most flight dynamic analysis uses the hypothesis that the aircraft behaves like a rigid body in the air, with the hypothesis that the mass of such is constant and there are no structural deformations. However, it would be an oversimplification of the system to apply the 6 degree-of-freedom rigid body equations of motion to the concept of aircraft of this work, since the wing and horizontal tail are supposed to tilt along with the spinning rotors, resulting in shifting of the center of gravity and gyroscopic moments.

Therefore, we will present a multi-body equations of motion that are a more appropriate approach, which are much similar to the equations presented in the work of Haixu et al. (2010). So, we will be dividing the aircraft in some parts and compute the inertial properties of each. Such parts are: the body, which involves the fuselage, landing gear, vertical tail and all its components; the right and left wing; right and left horizontal stabilizers; and each rotor a separate part. In this way, we also consider that each part has constant mass, even though fuel consumption reduces the body part mass over time, since the fuel tank is in the tail boom, the weight reduction is too slow to be considered in the dynamic analysis. And finally, no structure deformations are considered, that means that the parts dimensions are constant.

With the previous hypotheses we are able to define the aircraft dynamic system in Figure 10. In this figure we find the origin of the Earth fixed inertial reference frame O_E, and the origin of the aircraft body coordinate frame O_B, which is the position of the center of gravity of the aircraft body part, that can shift due to the quantity of fuel in the tank, but will not move because of the tilt of the wing or horizontal tail.

The wing and horizontal tail tilts with respect to the pivot points P_W and P_{HT}, which are fixed, and are positioned on the one quarter chord of the exposed root chords. The wing and horizontal tail are divided in right and left parts, each with its own concentrated mass, positioned in the respective center of gravity, with their own coordinate frame ($O_{WR}, O_{WL}, O_{HTR}, O_{HTL}$), in this manner, when the wing and horizontal tail tilts along the pivot points, their coordinate frames follows. Lastly, for every rotor in the wing and horizontal tail, there is also a coordinate frame ($O_{R1}, \dots, O_{R6}$), which are fixed with respect to their wing or horizontal stabilizer coordinate frame.

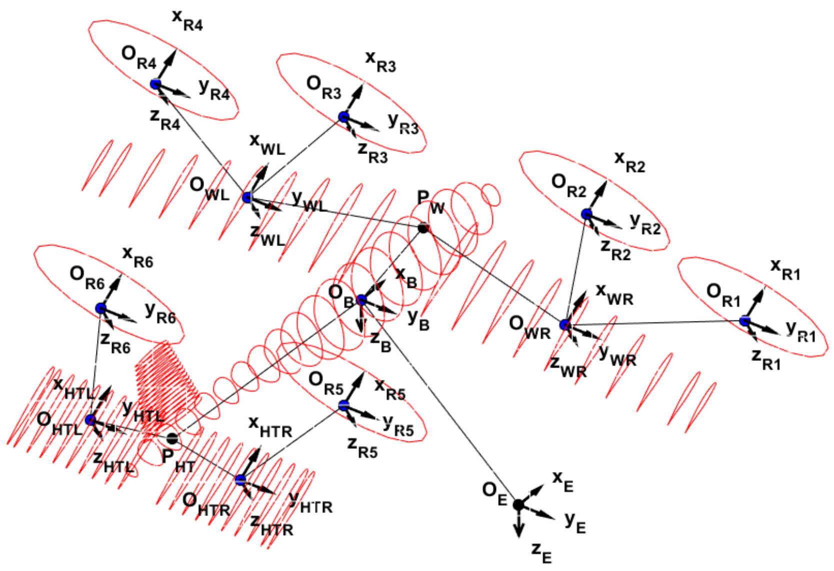


Figure 10: Reference frames and aircraft dynamic system.

Having the aircraft dynamic system model, we may proceed deriving the translational and angular equations of motion.

3.1 Translational motion

We begin by defining the total linear momentum in the Earth fixed inertial reference frame, as the sum of the linear momentum of each individual part. From now on, the subscript B will be referring to the aircraft body part, W_i to the right or left, wing or horizontal stabilizer part, R_j to the rotors, and the superscript will be referring to the reference frame of the vector, where in the following equation, E means Earth fixed inertial reference frame. So, the aircraft total linear momentum is,

$$\vec{G}_{total}^{E} = \vec{G}_B^E + \sum_{i=1}^{NW} \vec{G}_{W_i}^E + \sum_{j=1}^{NR} \vec{G}_{R_j}^E = m_B \vec{V}_B^E + \sum_{i=1}^{NW} m_{W_i} \vec{V}_{W_i}^E + \sum_{j=1}^{NR} m_{R_j} \vec{V}_{R_j}^E$$

Differentiation of the total linear momentum leads to the force equation in the Earth fixed inertial reference frame, where $\vec{F}^E$ is the net applied force vector.

$$\vec{F}^E = \frac{d}{dt}\left(\vec{G}_{total}^E\right) = \frac{d}{dt}\left(\vec{G}_B^E + \sum_{i=1}^{NW} \vec{G}_{W_i}^E + \sum_{j=1}^{NR} \vec{G}_{R_j}^E\right) = m_B \dot{\vec{V}}_B^E + \sum_{i=1}^{NW} m_{W_i} \dot{\vec{V}}_{W_i}^E + \sum_{j=1}^{NR} m_{R_j} \dot{\vec{V}}_{R_j}^E$$

Now, before we pass the equation to the body coordinate frame, it is necessary to introduce the theorem of Coriolis, which can be found at Stevens and Lewis (2016). Consider the transformation of a velocity vector in an arbitrary reference frame A to an arbitrary reference frame B, we have the rotation matrix R_A^B between reference frames A to B.

$$\vec{V}_B = R_A^B \vec{V}_A$$

The derivative of $\vec{V}_B$ with respect to the frame A is,

$$\frac{d}{dt_A}\left(\vec{V}_B\right) = R_A^B \dot{\vec{V}}_A$$

And with respect to the frame B,

$$\frac{d}{dt_B}\left(\vec{V}_B\right) = \dot{\vec{V}}_B = \dot{R}_A^B \vec{V}_A + R_A^B \dot{\vec{V}}_A$$

Combining we have,

$$\frac{d}{dt_A}\left(\vec{V}_B\right) = \dot{\vec{V}}_B - \dot{R}_A^B \vec{V}_A$$

Introducing the following relation, where $\vec{\omega}_B$ is the angular velocity vector of frame B relative to frame A.

$$\dot{R}_A^B = -\vec{\omega}_B \times R_A^B$$

Thus substituting we find,

$$\frac{d}{dt_A}\left(\vec{V}_B\right) = \dot{\vec{V}}_B + \vec{\omega}_B \times \vec{V}_B$$

Therefore, we use this equation to find the acceleration vector from Earth fixed inertial reference frame to the aircraft body coordinate frame,

$$\dot{\vec{V}}_B^E = \dot{\vec{V}}_B^B + \vec{\omega}_B^B \times \vec{V}_B^B$$

Being the aircraft body coordinate frame angular velocity vector,

$$\vec{\omega}_B^B = [P \quad Q \quad R]^T$$

The velocity vector for the concentrated masses of the right and left wing and horizontal tail in the Earth fixed inertial reference frame have additional term due to the relative movement with respect to the aircraft body concentrated mass. From Meriam (2012) we have the equation of relative acceleration of a moving point A with respect to a moving point B, wherein $\vec{r}_{A/B}$ is the position vector of point A in relation to point B, $\vec{V}_{rel_{A/B}}$ is the relative velocity vector of point A in relation to point B, and $\vec{a}_{rel_{A/B}}$ is the relative acceleration vector of point A in relation to point B.

$$\vec{a}_A = \vec{a}_B + \dot{\vec{\omega}}_B \times \vec{r}_{A/B} + \vec{\omega}_B \times \left(\vec{\omega}_B \times \vec{r}_{A/B}\right) + 2\vec{\omega}_B \times \vec{V}_{rel_{A/B}} + \vec{a}_{rel_{A/B}}$$

So, for the concentrated masses of the right and left wing and horizontal tail we have,

$$\dot{\vec{V}}^E_{W_i} = \dot{\vec{V}}^B_B + \vec{\omega}^B_B \times \vec{V}^B_B + \dot{\vec{\omega}}^B_B \times \vec{r}_{W_i/B} + \vec{\omega}^B_B \times \left(\vec{\omega}^B_B \times \vec{r}_{W_i/B}\right) + 2\vec{\omega}^B_B \times \vec{V}^B_{rel_{W_i/B}} + \vec{a}^B_{rel_{W_i/B}}$$

Similarly, for the concentrated masses of the rotors,

$$\dot{\vec{V}}^E_{R_j} = \dot{\vec{V}}^B_B + \vec{\omega}^B_B \times \vec{V}^B_B + \dot{\vec{\omega}}^B_B \times \vec{r}_{R_j/B} + \vec{\omega}^B_B \times \left(\vec{\omega}^B_B \times \vec{r}_{R_j/B}\right) + 2\vec{\omega}^B_B \times \vec{V}^B_{rel_{R_j/B}} + \vec{a}^B_{rel_{R_j/B}}$$

Thereby, we pass the force equation from the Earth fixed reference frame to the aircraft body coordinate frame.

$$\begin{aligned}\vec{F}^B + m_B B^B_E \vec{g}^E + \sum_{i=1}^{NW}\left\{m_{W_i} B^B_E \vec{g}^E\right\} + \sum_{j=1}^{NR}\left\{m_{R_j} B^B_E \vec{g}^E\right\} \\ = m_B\left(\dot{\vec{V}}^B_B + \vec{\omega}^B_B \times \vec{V}^B_B\right) \\ + \sum_{i=1}^{NW}\Big\{m_{W_i}\Big(\dot{\vec{V}}^B_B + \vec{\omega}^B_B \times \vec{V}^B_B + \dot{\vec{\omega}}^B_B \times \vec{r}_{W_i/B} + \vec{\omega}^B_B \times \left(\vec{\omega}^B_B \times \vec{r}_{W_i/B}\right) + 2\vec{\omega}^B_B \\ \times \vec{V}^B_{rel_{W_i/B}} + \vec{a}^B_{rel_{W_i/B}}\Big)\Big\} \\ + \sum_{j=1}^{NR}\Big\{m_{R_j}\Big(\dot{\vec{V}}^B_B + \vec{\omega}^B_B \times \vec{V}^B_B + \dot{\vec{\omega}}^B_B \times \vec{r}_{R_j/B} + \vec{\omega}^B_B \times \left(\vec{\omega}^B_B \times \vec{r}_{R_j/B}\right) + 2\vec{\omega}^B_B \\ \times \vec{V}^B_{rel_{R_j/B}} + \vec{a}^B_{rel_{R_j/B}}\Big)\Big\}\end{aligned}$$

Note that the terms added in the left side of the equation are vectors of weight of aircraft body, right and left wing or horizontal tail concentrated masses and rotors concentrated masses. Moreover, the vectors of weight use the rotation matrix from Earth fixed referential frame to body coordinate frame B_E^B, which is a function of the Euler angles: roll (ϕ), pitch (θ) and yaw (ψ). Positive yaw angle is a rotation about the z-axis, that is, nose right; positive pitch angle is a rotation about the new y-axis, that is, nose up; and positive roll is a rotation about the new x-axis, that is, right wing down, resulting in,

$$B_E^B = \begin{bmatrix} \cos\psi\cos\theta & \sin\psi\cos\theta & -\sin\theta \\ \cos\psi\sin\theta\sin\phi - \sin\psi\cos\phi & \sin\psi\sin\theta\sin\phi + \cos\psi\cos\phi & \cos\theta\sin\phi \\ \cos\psi\sin\theta\cos\phi + \sin\psi\sin\phi & \sin\psi\sin\theta\cos\phi - \cos\psi\sin\phi & \cos\theta\cos\phi \end{bmatrix}$$

We define now the relative velocity and acceleration vectors in the aircraft body coordinate frame. In the following equations $pivot_i$ and $pivot_j$ means the respective pivot point of the concentrated masses.

$$\vec{V}^B_{rel_{W_i/B}} = \frac{d}{dt}\left(\vec{r}_{W_i/B}\right) = \frac{d}{dt}\left(\vec{r}_{pivot_i/B} + R^B_{W_i}\vec{r}_{W_i/pivot_i}\right)$$
$$= \frac{d}{dt}\left(\vec{r}_{pivot_i/B}\right) + \dot{R}^B_{W_i}\vec{r}_{W_i/pivot_i} + R^B_{W_i}\frac{d}{dt}\left(\vec{r}_{W_i/pivot_i}\right) = \dot{R}^B_{W_i}\vec{r}_{W_i/pivot_i}$$

$$\vec{V}^B_{rel_{R_j/B}} = \frac{d}{dt}\left(\vec{r}_{R_j/B}\right) = \frac{d}{dt}\left(\vec{r}_{pivot_j/B} + R^B_{R_j}\vec{r}_{R_j/pivot_j}\right)$$
$$= \frac{d}{dt}\left(\vec{r}_{pivot_j/B}\right) + \dot{R}^B_{R_j}\vec{r}_{R_j/pivot_j} + R^B_{R_j}\frac{d}{dt}\left(\vec{r}_{R_j/pivot_j}\right) = \dot{R}^B_{R_j}\vec{r}_{R_j/pivot_j}$$

$$\vec{a}^B_{rel_{W_i/B}} = \frac{d}{dt}\left(\vec{V}^B_{rel_{W_i/B}}\right) = \frac{d}{dt}\left(\dot{R}^B_{W_i}\vec{r}_{W_i/pivot_i}\right) = \ddot{R}^B_{W_i}\vec{r}_{W_i/pivot_i} + \dot{R}^B_{W_i}\frac{d}{dt}\left(\vec{r}_{W_i/pivot_i}\right)$$
$$= \ddot{R}^B_{W_i}\vec{r}_{W_i/pivot_i}$$

$$\vec{a}^B_{rel_{R_j/B}} = \frac{d}{dt}\left(\vec{V}^B_{rel_{R_j/B}}\right) = \frac{d}{dt}\left(\dot{R}^B_{R_j}\vec{r}_{R_j/pivot_j}\right) = \ddot{R}^B_{R_j}\vec{r}_{R_j/pivot_j} + \dot{R}^B_{R_j}\frac{d}{dt}\left(\vec{r}_{R_j/pivot_j}\right)$$
$$= \ddot{R}^B_{R_j}\vec{r}_{R_j/pivot_j}$$

Wherein,

$\vec{r}_{pivot_i/B}$ or $\vec{r}_{pivot_j/B}$: Position vectors of the wing or horizontal tail pivot point to the origin of the aircraft body coordinate frame.

$\vec{r}^B_{W_i/pivot_i}$: Position vector of right or left wing or horizontal tail concentrated mass relative to respective pivot point.

$\vec{r}^B_{R_j/pivot_j}$: Position vector of rotor concentrated mass relative to respective pivot point.

Wing and horizontal tail tilt matrix with respect to wing or horizontal tail tilt angle (δ_W, δ_{HT}),

$$R^B_{W,HT} = \begin{bmatrix} \cos(\delta_{W,HT}) & 0 & \sin(\delta_{W,HT}) \\ 0 & 1 & 0 \\ -\sin(\delta_{W,HT}) & 0 & \cos(\delta_{W,HT}) \end{bmatrix}$$

Wing and horizontal tail tilt matrix derivative, considering wing or horizontal tail tilt velocity angle $(\dot{\delta}_W, \dot{\delta}_{HT})$.

$$\dot{R}^B_{W,HT} = \begin{bmatrix} -\dot{\delta}_{W,HT}\sin(\delta_{W,HT}) & 0 & \dot{\delta}_{W,HT}\cos(\delta_{W,HT}) \\ 0 & 0 & 0 \\ -\dot{\delta}_{W,HT}\cos(\delta_{W,HT}) & 0 & -\dot{\delta}_{W,HT}\sin(\delta_{W,HT}) \end{bmatrix}$$

Wing and horizontal tail second derivative tilt matrix, considering wing or horizontal tail tilt acceleration angle $(\ddot{\delta}_W, \ddot{\delta}_{HT})$.

$$\ddot{R}^B_{W,HT} = \begin{bmatrix} -(\ddot{\delta}_{W,HT}\sin(\delta_{W,HT}) + \dot{\delta}_{W,HT}\cos(\delta_{W,HT})) & 0 & (\ddot{\delta}_{W,HT}\cos(\delta_{W,HT}) - \dot{\delta}_{W,HT}\sin(\delta_{W,HT})) \\ 0 & 0 & 0 \\ -(\ddot{\delta}_{W,HT}\cos(\delta_{W,HT}) - \dot{\delta}_{W,HT}\sin(\delta_{W,HT})) & 0 & -(\ddot{\delta}_{W,HT}\sin(\delta_{W,HT}) + \dot{\delta}_{W,HT}\cos(\delta_{W,HT})) \end{bmatrix}$$

Rearranging the equation terms, and using the simplification ($\Omega_B = \vec{\omega}^B_B \times$),

$$\begin{aligned} \vec{F}^B + \left(m_B + \sum_{i=1}^{NW} m_{W_i} + \sum_{j=1}^{NR} m_{R_j}\right) B^B_E \vec{g}^E &= \left(m_B + \sum_{i=1}^{NW} m_{W_i} + \sum_{j=1}^{NR} m_{R_j}\right)\dot{\vec{V}}^B_B + \left(m_B + \sum_{i=1}^{NW} m_{W_i} + \sum_{j=1}^{NR} m_{R_j}\right)\Omega_B \vec{V}^B_B \\ &+ \sum_{i=1}^{NW}\left\{m_{W_i}\left[(\dot{\Omega}_B + \Omega_B\Omega_B)\vec{r}_{W_i/B} + \left(2\Omega_B\dot{R}^B_{W_i} + \ddot{R}^B_{W_i}\right)\vec{r}_{W_i/pivot_i}\right]\right\} \\ &+ \sum_{j=1}^{NR}\left\{m_{R_j}\left[(\dot{\Omega}_B + \Omega_B\Omega_B)\vec{r}_{R_j/B} + \left(2\Omega_B\dot{R}^B_{R_j} + \ddot{R}^B_{R_j}\right)\vec{r}_{R_j/pivot_j}\right]\right\} \end{aligned}$$

Being that,

$$\Omega_B = (\vec{\omega}_B \times \ ) = \begin{bmatrix} 0 & -R & Q \\ R & 0 & -P \\ -Q & P & 0 \end{bmatrix}$$

$$\dot{\Omega}_B = (\dot{\vec{\omega}}_B \times \ ) = \begin{bmatrix} 0 & -\dot{R} & \dot{Q} \\ \dot{R} & 0 & -\dot{P} \\ -\dot{Q} & \dot{P} & 0 \end{bmatrix}$$

Passing the aircraft body acceleration vector in the aircraft body coordinate frame to the left size we have,

$$\begin{aligned} \dot{\vec{V}}_B^B = -\Omega_B \vec{V}_B^B + \frac{\vec{F}_B^B}{M} + B_E^B \vec{g}^E \\ - \frac{\sum_{i=1}^{NW} \{ m_{W_i} [(\dot{\Omega}_B + \Omega_B \Omega_B) \vec{r}_{W_i/B} + (2\Omega_B \dot{R}_{W_i}^B + \ddot{R}_{W_i}^B) \vec{r}_{W_i/pivot_i}] \}}{M} \\ - \frac{\sum_{j=1}^{NR} \left\{ m_{R_j} \left[(\dot{\Omega}_B + \Omega_B \Omega_B) \vec{r}_{R_j/B} + \left(2\Omega_B \dot{R}_{R_j}^B + \ddot{R}_{R_j}^B\right) \vec{r}_{R_j/pivot_j} \right] \right\}}{M} \end{aligned}$$

In the previous equation we have used the following simplification for the total aircraft mass, being the sum of the concentrated masses of aircraft body, right and left wing and horizontal stabilizers, and rotors.

$$M = m_B + \sum_{i=1}^{NW} m_{W_i} + \sum_{j=1}^{NR} m_{R_j}$$

Additionally, the following terms are joined in a single term F. It represents the portion of the acceleration resulting from movement of the concentrated masses around the aircraft body coordinate frame.

$$\begin{aligned} F = \frac{\sum_{i=1}^{NW} \{ m_{W_i} [(\dot{\Omega}_B + \Omega_B \Omega_B) \vec{r}_{W_i/B} + (2\Omega_B \dot{R}_{W_i}^B + \ddot{R}_{W_i}^B) \vec{r}_{W_i/pivot_i}] \}}{M} \\ + \frac{\sum_{j=1}^{NR} \left\{ m_{R_j} \left[(\dot{\Omega}_B + \Omega_B \Omega_B) \vec{r}_{R_j/B} + \left(2\Omega_B \dot{R}_{R_j}^B + \ddot{R}_{R_j}^B\right) \vec{r}_{R_j/pivot_j} \right] \right\}}{M} \end{aligned}$$

Therefore, we have the aircraft translational motion equation in the body coordinate frame,

$$\dot{\vec{V}}_B^B = -\Omega_B \vec{V}_B^B + \frac{\vec{F}_B^B}{M} + B_E^B \vec{g}^E - F$$

3.2 Angular motion

For the aircraft angular motion equation we start by defining the total angular momentum in the Earth fixed inertial reference frame, again, being the sum of the portions of the aircraft body, right and left wing and horizontal stabilizers, and rotors. The terms $\tilde{I}$ are the inertia matrices of the concentrated masses, with the subscript indicating the part, and the superscript the reference frame. The terms $\vec{\omega}$ are angular velocity vector with subscript indicating the part and superscript the reference frame.

$$\vec{H}_{total}^{E} = \vec{H}_{B}^{E} + \sum_{i=1}^{NW} \vec{H}_{W_i}^{E} + \sum_{j=1}^{NR} \vec{H}_{R_j}^{E}$$

$$= \tilde{I}_{B}^{E}\vec{\omega}_{B}^{E} + \vec{r}_{B/E} \times \left(m_B \vec{V}_{B}^{E}\right) + \sum_{i=1}^{NW} \left\{\tilde{I}_{W_i}^{E}\vec{\omega}_{W_i}^{E} + \vec{r}_{W_i/E} \times \left(m_{W_i}\vec{V}_{W_i}^{E}\right)\right\}$$

$$+ \sum_{j=1}^{NR} \left\{\tilde{I}_{R_j}^{E}\vec{\omega}_{R_j}^{E} + \vec{r}_{R_j/E} \times \left(m_{R_j}\vec{V}_{R_j}^{E}\right)\right\}$$

Passing the angular motion equation in the Earth fixed inertial reference frame to the aircraft body coordinate frame,

$$\vec{H}_{total}^{B} = \vec{H}_{B}^{B} + \sum_{i=1}^{NW} \vec{H}_{W_i}^{B} + \sum_{j=1}^{NR} \vec{H}_{R_j}^{B}$$

$$= \tilde{I}_{B}^{B}\vec{\omega}_{B}^{B} + \sum_{i=1}^{NW} \left\{\tilde{I}_{W_i}^{B}\vec{\omega}_{W_i}^{B} + \vec{r}_{W_i/B} \times \left(m_{W_i}\vec{V}_{W_i}^{B}\right)\right\}$$

$$+ \sum_{j=1}^{NR} \left\{\tilde{I}_{R_j}^{B}\vec{\omega}_{R_j}^{B} + \vec{r}_{R_j/B} \times \left(m_{R_j}\vec{V}_{R_j}^{B}\right)\right\}$$

From Meriam (2012) we have the equation of relative velocity of a moving point A with respect to a moving point B.

$$\vec{V}_A = \vec{V}_B + \vec{\omega}_B \times \vec{r}_{A/B} + \vec{V}_{rel_{A/B}}$$

Therefore, we have the velocity of the concentrated masses with respect to the aircraft body coordinate frame,

$$\vec{V}_{W_i}^B = \vec{V}_B^B + \vec{\omega}_B^B \times \vec{r}_{W_i/B} + \vec{V}_{rel_{W_i/B}}^B = \vec{V}_B^B + \vec{\omega}_B^B \times \vec{r}_{W_i/B} + \dot{R}_{W_i}^B \vec{r}_{W_i/pivot_i}$$

$$\vec{V}_{R_j}^B = \vec{V}_B^B + \vec{\omega}_B^B \times \vec{r}_{R_j/B} + \vec{V}_{rel_{R_j/B}}^B = \vec{V}_B^B + \vec{\omega}_B^B \times \vec{r}_{R_j/B} + \dot{R}_{R_j}^B \vec{r}_{R_j/pivot_j}$$

The net torque $\vec{T}_B^B$ acting at the aircraft body coordinate frame comes from the rate of change of angular momentum. The added terms in the left side of the equation are the weights torques of the concentrated masses with respect to the body coordinate frame.

$$\vec{T}_B^B + \sum_{i=1}^{NW} \left\{ \vec{r}_{W_i/B} \times m_{W_i} B_E^B \vec{g}^E \right\} + \sum_{j=1}^{NR} \left\{ \vec{r}_{R_j/B} \times m_{R_j} B_E^B \vec{g}^E \right\} = \frac{d}{dt}\left(\vec{H}_{total}^B\right)$$

Expanding the derivative of the total angular momentum,

$$\frac{d}{dt}\left(\vec{H}_{total}^B\right) = \frac{d}{dt}\left(\vec{H}_B^B\right) + \sum_{i=1}^{NW} \frac{d}{dt}\left(\vec{H}_{W_i}^B\right) + \sum_{j=1}^{NR} \frac{d}{dt}\left(\vec{H}_{R_j}^B\right)$$

$$\begin{aligned}
\frac{d}{dt}\left(\vec{H}_{total}^B\right) = {} & \frac{d}{dt}\left(\tilde{I}_B^B \vec{\omega}_B^B\right) + \vec{\omega}_B^B \times \left(\tilde{I}_B^B \vec{\omega}_B^B\right) \\
& + \sum_{i=1}^{NW} \left\{ \frac{d}{dt}\left(\tilde{I}_{W_i}^B \vec{\omega}_{W_i}^B\right) + \vec{\omega}_B^B \times \left(\tilde{I}_{W_i}^B \vec{\omega}_{W_i}^B\right) \right. \\
& \left. + m_{W_i} \left[\frac{d\vec{r}_{W_i/B}}{dt} \times \vec{V}_{W_i}^B + \vec{r}_{W_i/B} \times \frac{d\vec{V}_{W_i}^B}{dt} \right] \right\} \\
& + \sum_{j=1}^{NR} \left\{ \frac{d}{dt}\left(\tilde{I}_{R_j}^B \vec{\omega}_{R_j}^B\right) + \vec{\omega}_B^B \times \left(\tilde{I}_{R_j}^B \vec{\omega}_{R_j}^B\right) + m_{R_j} \left[\frac{d\vec{r}_{R_j/B}}{dt} \times \vec{V}_{R_j}^B + \vec{r}_{R_j/B} \times \frac{d\vec{V}_{R_j}^B}{dt} \right] \right\}
\end{aligned}$$

Further expansion of the terms of the aircraft body, right and left wings and horizontal stabilizers, and rotors,

$$\frac{d}{dt}\left(\vec{H}_B^B\right) = \tilde{I}_B^B \dot{\vec{\omega}}_B^B + \vec{\omega}_B^B \times \left(\tilde{I}_B^B \vec{\omega}_B^B\right)$$

$$\sum_{i=1}^{NW}\frac{d}{dt}\left(\vec{H}_{W_i}^{B}\right)=\sum_{i=1}^{NW}\left\{\frac{d}{dt}\left(\tilde{I}_{W_i}^{B}\vec{\omega}_{W_i}^{B}\right)+\vec{\omega}_{B}^{B}\times\left(\tilde{I}_{W_i}^{B}\vec{\omega}_{W_i}^{B}\right)\right.$$
$$+m_{W_i}\left[\dot{R}_{W_i}^{B}\vec{r}_{W_i/pivot_i}\times\left(\vec{V}_{B}^{B}+\vec{\omega}_{B}^{B}\times\vec{r}_{W_i/B}+\dot{R}_{W_i}^{B}\vec{r}_{W_i/pivot_i}\right)+\vec{r}_{W_i/B}\right.$$
$$\times\left(\dot{\vec{V}}_{B}^{B}+\vec{\omega}_{B}^{B}\times\vec{V}_{B}^{B}+\dot{\vec{\omega}}_{B}^{B}\times\vec{r}_{W_i/B}+\vec{\omega}_{B}^{B}\times\left(\vec{\omega}_{B}^{B}\times\vec{r}_{W_i/B}\right)+2\vec{\omega}_{B}^{B}\times\dot{R}_{W_i}^{B}\vec{r}_{W_i/pivot_i}\right.$$
$$\left.\left.\left.+\ddot{R}_{W_i}^{B}\vec{r}_{W_i/pivot_i}\right)\right]\right\}$$

$$\sum_{j=1}^{NR}\frac{d}{dt}\left(\vec{H}_{R_j}^{B}\right)=\sum_{j=1}^{NR}\left\{\frac{d}{dt}\left(\tilde{I}_{R_j}^{B}\vec{\omega}_{R_j}^{B}\right)+\vec{\omega}_{B}^{B}\times\left(\tilde{I}_{R_j}^{B}\vec{\omega}_{R_j}^{B}\right)\right.$$
$$+m_{R_i}\left[\dot{R}_{R_j}^{B}\vec{r}_{R_j/pivot_j}\times\left(\vec{V}_{B}^{B}+\vec{\omega}_{B}^{B}\times\vec{r}_{R_j/B}+\dot{R}_{R_j}^{B}\vec{r}_{R_j/pivot_j}\right)+\vec{r}_{R_j/B}\right.$$
$$\times\left(\dot{\vec{V}}_{B}^{B}+\vec{\omega}_{B}^{B}\times\vec{V}_{B}^{B}+\dot{\vec{\omega}}_{B}^{B}\times\vec{r}_{R_j/B}+\vec{\omega}_{B}^{B}\times\left(\vec{\omega}_{B}^{B}\times\vec{r}_{R_j/B}\right)+2\vec{\omega}_{B}^{B}\times\dot{R}_{R_j}^{B}\vec{r}_{R_j/pivot_j}\right.$$
$$\left.\left.\left.+\ddot{R}_{R_j}^{B}\vec{r}_{R_j/pivot_j}\right)\right]\right\}$$

Because of wing and horizontal tail tilt, the inertia matrices of such surfaces and rotors respective to aircraft body coordinate frame are variables. The inertia matrices of the concentrated masses, with respect to the body coordinate frame, are obtained from the inertia matrices with respect to their own coordinate reference frames by translating and rotating the reference. This operation is demonstrated in the next equations for the concentrated masses of the panels and rotors respectively. There we have $[T]$ and $[T]^T$ the inertia rotation matrix and its transpose, and $\tilde{R}^B$ the translation matrix to the body coordinate frame. Moreover, the angular velocity vector in the body coordinate frame of the part is the sum of the angular velocity vector of the part with respect to its own reference frame, tilted to adjust the reference orientation, with the angular velocity vector of the body part with respect to its own reference frame.

$$\tilde{I}_{W_i}^{B}\vec{\omega}_{W_i}^{B}=[T]_{W_i}\left(\tilde{I}_{W_i}^{W_i}+m_{W_i}\tilde{R}_{W_i}^{B}\right)[T]_{W_i}^{T}\left(R_{W_i}^{B}\vec{\omega}_{W_i}^{W_i}+\vec{\omega}_{B}^{B}\right)$$

$$\tilde{I}_{R_j}^{B}\vec{\omega}_{R_j}^{B}=[T]_{R_j}\left(\tilde{I}_{R_j}^{R_j}+m_{R_j}\tilde{R}_{R_j}^{B}\right)[T]_{R_j}^{T}\left(R_{R_j}^{B}\vec{\omega}_{R_j}^{R_j}+R_{W_i}^{B}\vec{\omega}_{W_i}^{W_i}+\vec{\omega}_{B}^{B}\right)$$

Where,

$[T]_{W_i}$and $[T]_{R_j}$: Inertia rotation matrix: rotates the wing, horizontal tail or rotor inertia matrix to the aircraft body coordinate frame.

$\tilde{R}_{W_i}^{B}$ and $\tilde{R}_{R_j}^{B}$: Inertia translation matrix: transfers the wing, horizontal tail or rotor inertia matrix to the aircraft body coordinate frame.

$\vec{\omega}_{W_i}^{W_i}$: Right or left wing or horizontal tail concentrated mass angular velocity vector in respect to its own reference frame.

$\vec{\omega}_{R_j}^{R_j}$: Rotors concentrated mass angular velocity vector in respect to its own reference frame.

We can assume from axes alignment that the wing and horizontal tail angular velocity vector is fully aligned with the aircraft body y coordinate.

$$R_{W_i}^{B}\vec{\omega}_{W_i}^{W_i} = \begin{bmatrix} 0 & \dot{\delta}_{W_i} & 0 \end{bmatrix}^T$$

$$\dot{R}_{W_i}^{B}\vec{\omega}_{W_i}^{W_i} + R_{W_i}^{B}\dot{\vec{\omega}}_{W_i}^{W_i} = \begin{bmatrix} 0 & \ddot{\delta}_{W_i} & 0 \end{bmatrix}^T$$

Being that,

$$[T]_{W_i} = [T]_{W_i}^T = \begin{bmatrix} \cos\left(\delta_{W,HT_i}\right) & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \cos\left(\delta_{W,HT_i}\right) \end{bmatrix}$$

$$[T]_{R_j} = [T]_{R_j}^T = \begin{bmatrix} \cos\left(\delta_{W,HT_j}\right) & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \cos\left(\delta_{W,HT_j}\right) \end{bmatrix}$$

And their derivatives,

$$\left[\dot{T}\right]_{W_i} = \left[\dot{T}\right]_{W_i}^T = \begin{bmatrix} -\dot{\delta}_{W,HT_i}\sin\left(\delta_{W,HT_i}\right) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\dot{\delta}_{W,HT_i}\sin\left(\delta_{W,HT_i}\right) \end{bmatrix}$$

$$\left[\dot{T}\right]_{R_j} = \left[\dot{T}\right]_{R_j}^T = \begin{bmatrix} -\dot{\delta}_{W,HT_j}\sin\left(\delta_{W,HT_j}\right) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\dot{\delta}_{W,HT_j}\sin\left(\delta_{W,HT_j}\right) \end{bmatrix}$$

Also, considering the translation vector being $\vec{r}_{W_i/B} = [x_{W_i/B} \quad y_{W_i/B} \quad z_{W_i/B}]^T$, we have,

$$\tilde{R}_{W_i}^B = \begin{bmatrix} (y_{W_i/B})^2 + (z_{W_i/B})^2 & -x_{W_i/B}y_{W_i/B} & -x_{W_i/B}z_{W_i/B} \\ -y_{W_i/B}x_{W_i/B} & (x_{W_i/B})^2 + (z_{W_i/B})^2 & -y_{W_i/B}z_{W_i/B} \\ -z_{W_i/B}x_{W_i/B} & -z_{W_i/B}y_{W_i/B} & (x_{W_i/B})^2 + (y_{W_i/B})^2 \end{bmatrix}$$

Now, for the rotors the translation vector is $\vec{r}_{R_j/B} = [x_{R_j/B} \quad y_{R_j/B} \quad z_{R_j/B}]^T$, we also have,

$$\tilde{R}_{R_i}^B = \begin{bmatrix} \left(y_{R_j/B}\right)^2 + \left(z_{R_j/B}\right)^2 & -x_{R_j/B}y_{R_j/B} & -x_{R_j/B}z_{R_j/B} \\ -y_{R_j/B}x_{R_j/B} & \left(x_{R_j/B}\right)^2 + \left(z_{R_j/B}\right)^2 & -y_{R_j/B}z_{R_j/B} \\ -z_{R_j/B}x_{R_j/B} & -z_{R_j/B}y_{R_j/B} & \left(x_{R_j/B}\right)^2 + \left(y_{R_j/B}\right)^2 \end{bmatrix}$$

Rearranging the terms of the angular momentum derivatives,

$$\frac{d}{dt}\left(\vec{H}_B^B\right) = \tilde{I}_B^B \dot{\vec{\omega}}_B^B + \vec{\omega}_B^B \times (\tilde{I}_B^B \vec{\omega}_B^B)$$

$$\begin{aligned} \sum_{i=1}^{NW} \frac{d}{dt}\left(\vec{H}_{W_i}^B\right) = \sum_{i=1}^{NW} \Bigg\{ & \frac{d}{dt}\Big([T]_{W_i}\Big(\tilde{I}_{W_i}^{W_i} + m_{W_i}\tilde{R}_{W_i}^B\Big)[T]_{W_i}^T\Big(R_{W_i}^B \vec{\omega}_{W_i}^{W_i} + \vec{\omega}_B^B\Big)\Big) + \vec{\omega}_B^B \\ & \times \Big([T]_{W_i}\Big(\tilde{I}_{W_i}^{W_i} + m_{W_i}\tilde{R}_{W_i}^B\Big)[T]_{W_i}^T\Big(R_{W_i}^B \vec{\omega}_{W_i}^{W_i} + \vec{\omega}_B^B\Big)\Big) \\ & + m_{W_i}\Big[\dot{R}_{W_i}^B \vec{r}_{W_i/pivot_i} \times \Big(\vec{V}_B^B + \vec{\omega}_B^B \times \vec{r}_{W_i/B} + \dot{R}_{W_i}^B \vec{r}_{W_i/pivot_i}\Big) + \vec{r}_{W_i/B} \\ & \times \Big(\dot{\vec{V}}_B^B + \vec{\omega}_B^B \times \vec{V}_B^B + \dot{\vec{\omega}}_B^B \times \vec{r}_{W_i/B} + \vec{\omega}_B^B \times \left(\vec{\omega}_B^B \times \vec{r}_{W_i/B}\right) \\ & + \left(2\vec{\omega}_B^B \times \dot{R}_{W_i}^B + \ddot{R}_{W_i}^B\right)\vec{r}_{W_i/pivot_i}\Big)\Big]\Bigg\} \end{aligned}$$

$$\begin{aligned} \sum_{j=1}^{NR} \frac{d}{dt}\left(\vec{H}_{R_j}^B\right) = \sum_{j=1}^{NR} \Bigg\{ & \frac{d}{dt}\left([T]_{R_j}\left(\tilde{I}_{R_j}^{R_j} + m_{R_j}\tilde{R}_{R_j}^B\right)[T]_{R_j}^T\left(R_{R_j}^B \vec{\omega}_{R_j}^{R_j} + R_{W_i}^B \vec{\omega}_{W_i}^{W_i} + \vec{\omega}_B^B\right)\right) + \vec{\omega}_B^B \\ & \times \left([T]_{R_j}\left(\tilde{I}_{R_j}^{R_j} + m_{R_j}\tilde{R}_{R_j}^B\right)[T]_{R_j}^T\left(R_{R_j}^B \vec{\omega}_{R_j}^{R_j} + R_{W_i}^B \vec{\omega}_{W_i}^{W_i} + \vec{\omega}_B^B\right)\right) \\ & + m_{R_i}\left[\dot{R}_{R_j}^B \vec{r}_{R_j/pivot_j} \times \left(\vec{V}_B^B + \vec{\omega}_B^B \times \vec{r}_{R_j/B} + \dot{R}_{R_j}^B \vec{r}_{R_j/pivot_j}\right) + \vec{r}_{R_j/B}\right. \\ & \times \Big(\dot{\vec{V}}_B^B + \vec{\omega}_B^B \times \vec{V}_B^B + \dot{\vec{\omega}}_B^B \times \vec{r}_{R_j/B} + \vec{\omega}_B^B \times \left(\vec{\omega}_B^B \times \vec{r}_{R_j/B}\right) \\ & \left. + \left(2\vec{\omega}_B^B \times \dot{R}_{R_j}^B + \ddot{R}_{R_j}^B\right)\vec{r}_{R_j/pivot_j}\Big)\right]\Bigg\} \end{aligned}$$

Furthermore, expansion of the following derivatives,

$$\frac{d}{dt}\left(\tilde{I}^B_{W_i}\vec{\omega}^B_{W_i}\right) = \Big\{[\dot{T}]_{W_i}\left(\tilde{I}^{W_i}_{W_i} + m_{W_i}\tilde{R}^B_{W_i}\right)[T]^T_{W_i} + [T]_{W_i}m_{W_i}\frac{d}{dt}\left(\tilde{R}^B_{W_i}\right)[T]^T_{W_i}$$
$$+ [T]_{W_i}\left(\tilde{I}^{W_i}_{W_i} + m_{W_i}\tilde{R}^B_{W_i}\right)[\dot{T}]^T_{W_i}\Big\}\left(R^B_{W_i}\vec{\omega}^{W_i}_{W_i} + \vec{\omega}^B_B\right)$$
$$+ [T]_{W_i}\left(\tilde{I}^{W_i}_{W_i} + m_{W_i}\tilde{R}^B_{W_i}\right)[T]^T_{W_i}\left(\dot{R}^B_{W_i}\vec{\omega}^{W_i}_{W_i} + R^B_{W_i}\dot{\vec{\omega}}^{W_i}_{W_i} + \dot{\vec{\omega}}^B_B\right)$$

$$\frac{d}{dt}\left(\tilde{I}^B_{R_j}\vec{\omega}^B_{R_j}\right) = \Big\{[\dot{T}]_{R_j}\left(\tilde{I}^{R_j}_{R_j} + m_{R_j}\tilde{R}^B_{R_j}\right)[T]^T_{R_j} + [T]_{R_j}m_{R_j}\frac{d}{dt}\left(\tilde{R}^B_{R_j}\right)[T]^T_{R_j}$$
$$+ [T]_{R_j}\left(\tilde{I}^{R_j}_{R_j} + m_{R_j}\tilde{R}^B_{R_j}\right)[\dot{T}]^T_{R_j}\Big\}\left(R^B_{R_j}\vec{\omega}^{R_i}_{R_j} + R^B_{W_i}\vec{\omega}^{W_i}_{W_i} + \vec{\omega}^B_B\right)$$
$$+ [T]_{R_j}\left(\tilde{I}^{R_j}_{R_j} + m_{R_j}\tilde{R}^B_{R_j}\right)[T]^T_{R_j}\left(\dot{R}^B_{R_j}\vec{\omega}^{R_j}_{R_j} + R^B_{R_j}\dot{\vec{\omega}}^{R_j}_{R_j} + \dot{R}^B_{W_i}\vec{\omega}^{W_i}_{W_i} + R^B_{W_i}\dot{\vec{\omega}}^{W_i}_{W_i}\right.$$
$$\left. + \dot{\vec{\omega}}^B_B\right)$$

And the derivative of the inertia translation matrix,

$$\frac{d}{dt}\left(\tilde{R}^B_{W_i}\right) = \begin{bmatrix} 2\left(y_{W_i/B}\dot{y}_{W_i/B} + z_{W_i/B}\dot{z}_{W_i/B}\right) & -\left(\dot{x}_{W_i/B}y_{W_i/B} + x_{W_i/B}\dot{y}_{W_i/B}\right) & -\left(\dot{x}_{W_i/B}z_{W_i/B} + x_{W_i/B}\dot{z}_{W_i/B}\right) \\ -\left(\dot{y}_{W_i/B}x_{W_i/B} + y_{W_i/B}\dot{x}_{W_i/B}\right) & 2\left(x_{W_i/B}\dot{x}_{W_i/B} + z_{W_i/B}\dot{z}_{W_i/B}\right) & -\left(\dot{y}_{W_i/B}z_{W_i/B} + y_{W_i/B}\dot{z}_{W_i/B}\right) \\ -\left(\dot{z}_{W_i/B}x_{W_i/B} + z_{W_i/B}\dot{x}_{W_i/B}\right) & -\left(\dot{z}_{W_i/B}y_{W_i/B} + z_{W_i/B}\dot{y}_{W_i/B}\right) & 2\left(x_{W_i/B}\dot{x}_{W_i/B} + y_{W_i/B}\dot{y}_{W_i/B}\right) \end{bmatrix}$$

$$\frac{d}{dt}\left(\tilde{R}^B_{R_j}\right) = \begin{bmatrix} 2\left(y_{R_j/B}\dot{y}_{R_j/B} + z_{R_j/B}\dot{z}_{R_j/B}\right) & -\left(\dot{x}_{R_j/B}y_{R_j/B} + x_{R_j/B}\dot{y}_{R_j/B}\right) & -\left(\dot{x}_{R_j/B}z_{R_j/B} + x_{R_j/B}\dot{z}_{R_j/B}\right) \\ -\left(\dot{y}_{R_j/B}x_{R_j/B} + y_{R_j/B}\dot{x}_{R_j/B}\right) & 2\left(x_{R_j/B}\dot{x}_{R_j/B} + z_{R_j/B}\dot{z}_{R_j/B}\right) & -\left(\dot{y}_{R_j/B}z_{R_j/B} + y_{R_j/B}\dot{z}_{R_j/B}\right) \\ -\left(\dot{z}_{R_j/B}x_{R_j/B} + z_{R_j/B}\dot{x}_{R_j/B}\right) & -\left(\dot{z}_{R_j/B}y_{R_j/B} + z_{R_j/B}\dot{y}_{R_j/B}\right) & 2\left(x_{R_j/B}\dot{x}_{R_j/B} + y_{R_j/B}\dot{y}_{R_j/B}\right) \end{bmatrix}$$

Remembering that,

$$\frac{d}{dt}\left(\vec{r}_{W_i/B}\right) = [\dot{x}_{W_i/B} \quad \dot{y}_{W_i/B} \quad \dot{z}_{W_i/B}]^T = \dot{R}^B_{W_i}\vec{r}_{W_i/pivot_i}$$

$$\frac{d}{dt}\left(\vec{r}_{R_j/B}\right) = \left[\dot{x}_{R_j/B} \quad \dot{y}_{R_j/B} \quad \dot{z}_{R_j/B}\right]^T = \dot{R}^B_{R_j}\vec{r}_{R_j/pivot_j}$$

Therefore we have the full expansion of the angular momentum derivatives,

$$\frac{d}{dt}\left(\vec{H}^B_B\right) = \tilde{I}^B_B\dot{\vec{\omega}}^B_B + \vec{\omega}^B_B \times \left(\tilde{I}^B_B\vec{\omega}^B_B\right)$$

$$\sum_{i=1}^{NW}\frac{d}{dt}\left(\vec{H}_{W_i}^{B}\right)=\sum_{i=1}^{NW}\left\{\left\{[\dot{T}]_{W_i}\left(\tilde{I}_{W_i}^{W_i}+m_{W_i}\tilde{R}_{W_i}^{B}\right)[T]_{W_i}^{T}+[T]_{W_i}m_{W_i}\frac{d}{dt}\left(\tilde{R}_{W_i}^{B}\right)[T]_{W_i}^{T}\right.\right.$$
$$\left.+[T]_{W_i}\left(\tilde{I}_{W_i}^{W_i}+m_{W_i}\tilde{R}_{W_i}^{B}\right)[\dot{T}]_{W_i}^{T}\right\}\left(R_{W_i}^{B}\vec{\omega}_{W_i}^{W_i}+\vec{\omega}_{B}^{B}\right)$$
$$+[T]_{W_i}\left(\tilde{I}_{W_i}^{W_i}+m_{W_i}\tilde{R}_{W_i}^{B}\right)[T]_{W_i}^{T}\left(\dot{R}_{W_i}^{B}\vec{\omega}_{W_i}^{W_i}+R_{W_i}^{B}\dot{\vec{\omega}}_{W_i}^{W_i}+\dot{\vec{\omega}}_{B}^{B}\right)+\vec{\omega}_{B}^{B}$$
$$\times\left([T]_{W_i}\left(\tilde{I}_{W_i}^{W_i}+m_{W_i}\tilde{R}_{W_i}^{B}\right)[T]_{W_i}^{T}\left(R_{W_i}^{B}\vec{\omega}_{W_i}^{W_i}+\vec{\omega}_{B}^{B}\right)\right)$$
$$+m_{W_i}\left[\dot{R}_{W_i}^{B}\vec{r}_{W_i/pivot_i}\times\left(\vec{V}_{B}^{B}+\vec{\omega}_{B}^{B}\times\vec{r}_{W_i/B}+\dot{R}_{W_i}^{B}\vec{r}_{W_i/pivot_i}\right)+\vec{r}_{W_i/B}\right.$$
$$\times\left(\dot{\vec{V}}_{B}^{B}+\vec{\omega}_{B}^{B}\times\vec{V}_{B}^{B}+\dot{\vec{\omega}}_{B}^{B}\times\vec{r}_{W_i/B}+\vec{\omega}_{B}^{B}\times\left(\vec{\omega}_{B}^{B}\times\vec{r}_{W_i/B}\right)\right.$$
$$\left.\left.\left.+\left(2\vec{\omega}_{B}^{B}\times\dot{R}_{W_i}^{B}+\ddot{R}_{W_i}^{B}\right)\vec{r}_{W_i/pivot_i}\right)\right]\right\}$$

$$\sum_{j=1}^{NR}\frac{d}{dt}\left(\vec{H}_{R_j}^{B}\right)=\sum_{j=1}^{NR}\left\{\left\{[\dot{T}]_{R_j}\left(\tilde{I}_{R_j}^{R_j}+m_{R_j}\tilde{R}_{R_j}^{B}\right)[T]_{R_j}^{T}+[T]_{R_j}m_{R_j}\frac{d}{dt}\left(\tilde{R}_{R_j}^{B}\right)[T]_{R_j}^{T}\right.\right.$$
$$\left.+[T]_{R_j}\left(\tilde{I}_{R_j}^{R_j}+m_{R_j}\tilde{R}_{R_j}^{B}\right)[\dot{T}]_{R_j}^{T}\right\}\left(R_{R_j}^{B}\vec{\omega}_{R_j}^{R_i}+R_{W_i}^{B}\vec{\omega}_{W_i}^{W_i}+\vec{\omega}_{B}^{B}\right)$$
$$+[T]_{R_j}\left(\tilde{I}_{R_j}^{R_j}+m_{R_j}\tilde{R}_{R_j}^{B}\right)[T]_{R_j}^{T}\left(\dot{R}_{R_j}^{B}\vec{\omega}_{R_j}^{R_j}+R_{R_j}^{B}\dot{\vec{\omega}}_{R_j}^{R_j}+\dot{R}_{W_i}^{B}\vec{\omega}_{W_i}^{W_i}+R_{W_i}^{B}\dot{\vec{\omega}}_{W_i}^{W_i}\right.$$
$$\left.+\dot{\vec{\omega}}_{B}^{B}\right)+\vec{\omega}_{B}^{B}\times\left([T]_{R_j}\left(\tilde{I}_{R_j}^{R_j}+m_{R_j}\tilde{R}_{R_j}^{B}\right)[T]_{R_j}^{T}\left(R_{R_j}^{B}\vec{\omega}_{R_j}^{R_j}+R_{W_i}^{B}\vec{\omega}_{W_i}^{W_i}+\vec{\omega}_{B}^{B}\right)\right)$$
$$+m_{R_i}\left[\dot{R}_{R_j}^{B}\vec{r}_{R_j/pivot_j}\times\left(\vec{V}_{B}^{B}+\vec{\omega}_{B}^{B}\times\vec{r}_{R_j/B}+\dot{R}_{R_j}^{B}\vec{r}_{R_j/pivot_j}\right)+\vec{r}_{R_j/B}\right.$$
$$\times\left(\dot{\vec{V}}_{B}^{B}+\vec{\omega}_{B}^{B}\times\vec{V}_{B}^{B}+\dot{\vec{\omega}}_{B}^{B}\times\vec{r}_{R_j/B}+\vec{\omega}_{B}^{B}\times\left(\vec{\omega}_{B}^{B}\times\vec{r}_{R_j/B}\right)\right.$$
$$\left.\left.\left.+\left(2\vec{\omega}_{B}^{B}\times\dot{R}_{R_j}^{B}+\ddot{R}_{R_j}^{B}\right)\vec{r}_{R_j/pivot_j}\right)\right]\right\}$$

Rearranging terms it is possible to write in the simplified form,

$$\vec{T}_{B}^{B}+M_{P}=A\dot{\vec{\omega}}_{B}^{B}+B\vec{\omega}_{B}^{B}+C\dot{\vec{V}}_{B}^{B}+D\vec{V}_{B}^{B}+E$$

So that the terms are,

$$A = \tilde{I}_B^B + \sum_{i=1}^{NW} \left\{ [T]_{W_i} \left(\tilde{I}_{W_i}^{W_i} + m_{W_i} \tilde{R}_{W_i}^B \right) [T]_{W_i}^T - m_{W_i} \vec{r}_{W_i/B} \times \vec{r}_{W_i/B} \times \right\}$$

$$+ \sum_{j=1}^{NR} \left\{ [T]_{R_j} \left(\tilde{I}_{R_j}^{R_j} + m_{R_j} \tilde{R}_{R_j}^B \right) [T]_{R_j}^T - m_{R_j} \vec{r}_{R_j/B} \times \vec{r}_{R_j/B} \times \right\}$$

$$B = \Omega_B \tilde{I}_B^B + \sum_{i=1}^{NW} \left\{ [\dot{T}]_{W_i} \left(\tilde{I}_{W_i}^{W_i} + m_{W_i} \tilde{R}_{W_i}^B \right) [T]_{W_i}^T + [T]_{W_i} m_{W_i} \frac{d}{dt} \left(\tilde{R}_{W_i}^B \right) [T]_{W_i}^T \right.$$

$$+ [T]_{W_i} \left(\tilde{I}_{W_i}^{W_i} + m_{W_i} \tilde{R}_{W_i}^B \right) [\dot{T}]_{W_i}^T + \Omega_B [T]_{W_i} \left(\tilde{I}_{W_i}^{W_i} + m_{W_i} \tilde{R}_{W_i}^B \right) [T]_{W_i}^T$$

$$\left. - m_{W_i} \left[\left(\dot{R}_{W_i}^B \vec{r}_{W_i/pivot_i} \times \vec{r}_{W_i/B} \times \right) + \left(\vec{r}_{W_i/B} \times \Omega_B \vec{r}_{W_i/B} \times \right) \right] \right\}$$

$$+ \sum_{j=1}^{NR} \left\{ [\dot{T}]_{R_j} \left(\tilde{I}_{R_j}^{R_j} + m_{R_j} \tilde{R}_{R_j}^B \right) [T]_{R_j}^T + [T]_{R_j} m_{R_j} \frac{d}{dt} \left(\tilde{R}_{R_j}^B \right) [T]_{R_j}^T \right.$$

$$+ [T]_{R_j} \left(\tilde{I}_{R_j}^{R_j} + m_{R_j} \tilde{R}_{R_j}^B \right) [\dot{T}]_{R_j}^T + \Omega_B [T]_{R_j} \left(\tilde{I}_{R_j}^{R_j} + m_{R_j} \tilde{R}_{R_j}^B \right) [T]_{R_j}^T$$

$$\left. - m_{R_j} \left[\left(\dot{R}_{R_j}^B \vec{r}_{R_j/pivot_j} \times \vec{r}_{R_j/B} \times \right) + \left(\vec{r}_{R_j/B} \times \Omega_B \vec{r}_{R_j/B} \times \right) \right] \right\}$$

$$C = \sum_{i=1}^{NW} \left\{ m_{W_i} \vec{r}_{W_i/B} \times \right\} + \sum_{j=1}^{NR} \left\{ m_{R_j} \vec{r}_{R_j/B} \times \right\}$$

$$D = \sum_{i=1}^{NW} \left\{ m_{W_i} \left[\left(\dot{R}_{W_i}^B \vec{r}_{W_i/pivot_i} \times \right) + \vec{r}_{W_i/B} \times \Omega_B \right] \right\}$$

$$+ \sum_{j=1}^{NR} \left\{ m_{R_j} \left[\left(\dot{R}_{R_j}^B \vec{r}_{R_j/pivot_j} \times \right) + \vec{r}_{R_j/B} \times \Omega_B \right] \right\}$$

$$E = \sum_{i=1}^{NW}\Big\{\Big([\dot{T}]_{W_i}\big(\tilde{I}_{W_i}^{W_i} + m_{W_i}\tilde{R}_{W_i}^{B}\big)[T]_{W_i}^{T} + [T]_{W_i}m_{W_i}\frac{d}{dt}\big(\tilde{R}_{W_i}^{B}\big)[T]_{W_i}^{T}$$

$$+ [T]_{W_i}\big(\tilde{I}_{W_i}^{W_i} + m_{W_i}\tilde{R}_{W_i}^{B}\big)[\dot{T}]_{W_i}^{T}\Big) R_{W_i}^{B}\vec{\omega}_{W_i}^{W_i}$$

$$+ [T]_{W_i}\big(\tilde{I}_{W_i}^{W_i} + m_{W_i}\tilde{R}_{W_i}^{B}\big)[T]_{W_i}^{T}\big(\dot{R}_{W_i}^{B}\vec{\omega}_{W_i}^{W_i} + R_{W_i}^{B}\dot{\vec{\omega}}_{W_i}^{W_i}\big)$$

$$+ \Omega_B[T]_{W_i}\big(\tilde{I}_{W_i}^{W_i} + m_{W_i}\tilde{R}_{W_i}^{B}\big)[T]_{W_i}^{T}R_{W_i}^{B}\vec{\omega}_{W_i}^{W_i}$$

$$+ m_{W_i}\big[\dot{R}_{W_i}^{B}\vec{r}_{W_i/pivot_i} \times \dot{R}_{W_i}^{B} + \vec{r}_{W_i/B} \times \big(2\Omega_B\dot{R}_{W_i}^{B} + \ddot{R}_{W_i}^{B}\big)\big]\vec{r}_{W_i/pivot_i}\Big\}$$

$$+ \sum_{j=1}^{NR}\Big\{\Big([\dot{T}]_{R_j}\big(\tilde{I}_{R_j}^{R_j} + m_{R_j}\tilde{R}_{R_j}^{B}\big)[T]_{R_j}^{T} + [T]_{R_j}m_{R_j}\frac{d}{dt}\big(\tilde{R}_{R_j}^{B}\big)[T]_{R_j}^{T}$$

$$+ [T]_{R_j}\big(\tilde{I}_{R_j}^{R_j} + m_{R_j}\tilde{R}_{R_j}^{B}\big)[\dot{T}]_{R_j}^{T}\Big)\big(R_{R_j}^{B}\vec{\omega}_{R_j}^{R_j} + R_{W_j}^{B}\vec{\omega}_{W_j}^{W_j}\big)$$

$$+ [T]_{R_j}\big(\tilde{I}_{R_j}^{R_j} + m_{R_j}\tilde{R}_{R_j}^{B}\big)[T]_{R_j}^{T}\big(\dot{R}_{R_j}^{B}\vec{\omega}_{R_j}^{R_j} + R_{R_j}^{B}\dot{\vec{\omega}}_{R_j}^{R_j} + \dot{R}_{W_j}^{B}\vec{\omega}_{W_j}^{W_j} + R_{W_j}^{B}\dot{\vec{\omega}}_{W_j}^{W_j}\big)$$

$$+ \Omega_B[T]_{R_j}\big(\tilde{I}_{R_j}^{R_j} + m_{R_j}\tilde{R}_{R_j}^{B}\big)[T]_{R_j}^{T}\big(R_{R_j}^{B}\vec{\omega}_{R_j}^{R_j} + R_{W_j}^{B}\vec{\omega}_{W_j}^{W_j}\big)$$

$$+ m_{R_j}\Big[\dot{R}_{R_j}^{B}\vec{r}_{R_j/pivot_j} \times \dot{R}_{R_j}^{B} + \vec{r}_{R_j/B} \times \Big(2\Omega_B\dot{R}_{R_j}^{B} + \ddot{R}_{R_j}^{B}\Big)\Big]\vec{r}_{R_j/pivot_j}\Big\}$$

$$M_P = \sum_{i=1}^{NW}\big\{\vec{r}_{W_i/B} \times m_{W_i}B_E^B\vec{g}^E\big\} + \sum_{j=1}^{NR}\big\{\vec{r}_{R_j/B} \times m_{R_j}B_E^B\vec{g}^E\big\}$$

Each of the terms in the angular momentum equation accounts for a moment effect on the aircraft body center of gravity.

$\vec{T}_B^B$: Total aerodynamic and propulsive torques acting on the aircraft body C.G.

M_P: Sum of concentrated masses torque acting on the aircraft body C.G. due to their weights.

$A\dot{\vec{\omega}}_B^B$: Torque due to the aircraft angular inertia proportional to angular acceleration.

$B\vec{\omega}_B^B$: Torque due to the aircraft angular inertia proportional to angular velocity.

$C\dot{\vec{V}}_B^B$: Torque due to concentrated masses load factor with respect to aircraft body acceleration.

$D\vec{V}_B^B$: Torque due to concentrated masses load factor with respect to wing and horizontal tail tilt movement.

E: Torque due to tilt movement of rotating parts.

We can write the following terms in the form, considering the rotors rotation vector full aligned with the x coordinate of their own coordinate frame.

$$R_{R_j}^{B}\vec{\omega}_{R_j}^{R_j} = R_{R_j}^{B}\begin{Bmatrix}\omega_{R_j}\\ 0\\ 0\end{Bmatrix}$$

$$\dot{R}_{R_j}^{B}\vec{\omega}_{R_j}^{R_j} + R_{R_j}^{B}\dot{\vec{\omega}}_{R_j}^{R_j} = R_{R_j}^{B}\begin{Bmatrix}\dot{\omega}_{R_j}\\ 0\\ 0\end{Bmatrix}$$

Therefore, we have the aircraft angular motion equation in the body coordinate frame,

$$\dot{\vec{\omega}}_{B}^{B} = -A^{-1}B\vec{\omega}_{B}^{B} - A^{-1}D\vec{V}_{B}^{B} - A^{-1}C\dot{\vec{V}}_{B}^{B} + A^{-1}\left(\vec{T}_{B}^{B} + M_P - E\right)$$

Next, we expand the term of this equation not yet defined.

$$A^{-1} = (\tilde{I}_{B}^{B})^{-1} + \sum_{i=1}^{NW}\left\{\left[[T]_{W_i}\left(\tilde{I}_{W_i}^{W_i} + m_{W_i}\tilde{R}_{W_i}^{B}\right)[T]_{W_i}^{T}\right]^{-1} - \left(m_{W_i}\vec{r}_{W_i/B}\times\vec{r}_{W_i/B}\times\right)^{-1}\right\}$$
$$+ \sum_{j=1}^{NR}\left\{\left[[T]_{R_j}\left(\tilde{I}_{R_j}^{R_j} + m_{R_j}\tilde{R}_{R_j}^{B}\right)[T]_{R_j}^{T}\right]^{-1} - \left(m_{R_j}\vec{r}_{R_j/B}\times\vec{r}_{R_j/B}\times\right)^{-1}\right\}$$

Also,

$$\left[[T]_{W_i}\left(\tilde{I}_{W_i}^{W_i} + m_{W_i}\tilde{R}_{W_i}^{B}\right)[T]_{W_i}^{T}\right]^{-1} = \left([T]_{W_i}^{T}\right)^{-1}\left[\left(\tilde{I}_{W_i}^{W_i}\right)^{-1} + m_{W_i}\left(\tilde{R}_{W_i}^{B}\right)^{-1}\right]\left([T]_{W_i}\right)^{-1}$$

$$\left[[T]_{R_j}\left(\tilde{I}_{R_j}^{R_j} + m_{R_j}\tilde{R}_{R_j}^{B}\right)[T]_{R_j}^{T}\right]^{-1} = \left([T]_{R_j}^{T}\right)^{-1}\left[\left(\tilde{I}_{R_j}^{R_j}\right)^{-1} + m_{R_j}\left(\tilde{R}_{R_j}^{B}\right)^{-1}\right]\left([T]_{R_j}\right)^{-1}$$

It is also required the inverse of the inertia rotation matrix,

$$[T]_{W_i}^{-1} = \frac{1}{\cos\left(\delta_{W,HT_i}\right)}\begin{bmatrix}1 & 0 & 0\\ 0 & \cos\left(\delta_{W,HT_i}\right) & 0\\ 0 & 0 & 1\end{bmatrix}$$

The same for the rotors,

$$[T]_{R_j}^{-1} = \frac{1}{\cos\left(\delta_{W,HT_j}\right)}\begin{bmatrix}1 & 0 & 0\\ 0 & \cos\left(\delta_{W,HT_j}\right) & 0\\ 0 & 0 & 1\end{bmatrix}$$

The cross product of concentrated masses position with respect to the aircraft body coordinate frame can be written in the matrix form,

$$\vec{r}_{W_i/B}\times=\begin{bmatrix}0 & -z_{W_i/B} & y_{W_i/B}\\ z_{W_i/B} & 0 & -x_{W_i/B}\\ -y_{W_i/B} & x_{W_i/B} & 0\end{bmatrix}$$

Consequently, we obtain the matrices product,

$$\vec{r}_{W_i/B}\times\vec{r}_{W_i/B}\times=\begin{bmatrix}-\left(y_{W_i/B}\right)^2-\left(z_{W_i/B}\right)^2 & x_{W_i/B}y_{W_i/B} & x_{W_i/B}z_{W_i/B}\\ x_{W_i/B}y_{W_i/B} & -\left(z_{W_i/B}\right)^2-\left(x_{W_i/B}\right)^2 & y_{W_i/B}z_{W_i/B}\\ x_{W_i/B}z_{W_i/B} & y_{W_i/B}z_{W_i/B} & -\left(x_{W_i/B}\right)^2-\left(y_{W_i/B}\right)^2\end{bmatrix}$$

3.3 Transformation between reference axes

We have previously defined the equations of translational and angular motion relative to the body coordinate frame, or body axes, it is now necessary to define the stability in wind axes in order to make it easier the introduction of the aerodynamic forces and moments, which are defined with respect to these axes.

The aircraft velocity vector in the aircraft body coordinates frame, or body axes, and its components,

$$\vec{V}_B^B=[U\quad V\quad W]^T$$

And we define the aircraft body velocity vector in the wind axes, and its components,

$$\vec{V}_B^W=[V_T\quad 0\quad 0]^T$$

Wherein, V_T being the flight speed.

$$V_T=\sqrt{U^2+V^2+W^2}$$

The direction of the velocity vector comes from the angle of attack (α) and sideslip angle (β),

$$\alpha=\tan^{-1}\left(\frac{W}{U}\right)$$

$$\beta = \sin^{-1}\left(\frac{V}{V_T}\right)$$

Such that the transformation of body axes to stability axes, and wind axes are,

$$\begin{Bmatrix} x \\ y \\ z \end{Bmatrix}_{Stab} = \begin{bmatrix} \cos\alpha & 0 & \sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & 0 & \cos\alpha \end{bmatrix} \begin{Bmatrix} x \\ y \\ z \end{Bmatrix}_{Body} = S_\alpha \begin{Bmatrix} x \\ y \\ z \end{Bmatrix}_{Body}$$

$$\begin{Bmatrix} x \\ y \\ z \end{Bmatrix}_{Wind} = \begin{bmatrix} \cos\beta & \sin\beta & 0 \\ -\sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} x \\ y \\ z \end{Bmatrix}_{Stab} = S_\beta \begin{Bmatrix} x \\ y \\ z \end{Bmatrix}_{Stab}$$

Rearranging both transformations we find the transformation matrix S from body to wind axes,

$$\begin{Bmatrix} x \\ y \\ z \end{Bmatrix}_{Wind} = \begin{bmatrix} \cos\alpha\cos\beta & \sin\beta & \sin\alpha\cos\beta \\ -\cos\alpha\sin\beta & \cos\beta & -\sin\alpha\sin\beta \\ -\sin\alpha & 0 & \cos\alpha \end{bmatrix} \begin{Bmatrix} x \\ y \\ z \end{Bmatrix}_{Body} = S \begin{Bmatrix} x \\ y \\ z \end{Bmatrix}_{Body}$$

Therefore, we have the transformation between reference axes,

$$\vec{V}_B^W = S\vec{V}_B^B$$

It is valid for any vector in the body axes transformed to wind axes. Therefore, for the aircraft body angular velocity vector we have the transformation of the vector in the body axes to wind axes, defining both,

$$\vec{\omega}_B^B = [P \quad Q \quad R]^T$$

$$\vec{\omega}_B^W = [P_W \quad Q_W \quad R_W]^T$$

And the transformation,

$$\vec{\omega}_B^W = S\vec{\omega}_B^B$$

Then, we begin with the conversion of equation of translational motion in the body axes to wind axes. It is also described in Stevens and Lewis (2016) without the concentrated masses terms,

$$\dot{\vec{V}}_B^B = -\Omega_B \vec{V}_B^B + \frac{\vec{F}_B^B}{M} + B_E^B \vec{g}^E - F$$

$$S\frac{d}{dt}\left(S^T\vec{V}_B^W\right) = -S\Omega_B S\vec{V}_B^B + SB_E^B\vec{g}^E - SF + \frac{1}{M}S\vec{F}_B^B$$

$$S\dot{S}^T\vec{V}_B^W + SS^T\dot{\vec{V}}_B^W = -\Omega_W\vec{V}_B^W + SB_E^B\vec{g}^E - SF + \frac{1}{M}\vec{F}_B^W$$

Defining the term Ω_R,

$$\Omega_R = S\dot{S}^T = S_\beta S_\alpha\left(S_\beta\dot{S}_\alpha + \dot{S}_\beta S_\alpha\right)^T = S_\beta\left(S_\alpha\dot{S}_\alpha^T\right)S_\beta^T + S_\beta\dot{S}_\beta^T = \begin{bmatrix} 0 & -\dot{\beta} & -\dot{\alpha}\cos\beta \\ \dot{\beta} & 0 & \dot{\alpha}\sin\beta \\ \dot{\alpha}\cos\beta & -\dot{\alpha}\sin\beta & 0 \end{bmatrix}$$

It is required to derive the transpose of the transformation matrix from body to wind axes, so we have,

$$\dot{S}^T = \begin{bmatrix} -\dot{\alpha}\sin\alpha & -\left(\dot{\beta}\cos\alpha - \beta\dot{\alpha}\sin\alpha\right) & -\dot{\alpha}\cos\alpha \\ \dot{\beta} & 0 & 0 \\ \dot{\alpha}\cos\alpha & -\left(\dot{\beta}\sin\alpha + \beta\dot{\alpha}\cos\alpha\right) & -\dot{\alpha}\sin\alpha \end{bmatrix}$$

The transformation matrix S is orthogonal, thus $SS^T = I$. So, rearranging the equation,

$$\dot{\vec{V}}_B^W + \Omega_R\vec{V}_B^W = -\Omega_W\vec{V}_B^W + SB_E^B\vec{g}^E - SF + \frac{1}{M}\vec{F}_B^W$$

Where the left size is,

$$\dot{\vec{V}}_B^W + \Omega_R\vec{V}_B^W = \begin{Bmatrix} \dot{V}_T \\ 0 \\ 0 \end{Bmatrix} + \begin{bmatrix} 0 & -\dot{\beta} & -\dot{\alpha}\cos\beta \\ \dot{\beta} & 0 & \dot{\alpha}\sin\beta \\ \dot{\alpha}\cos\beta & -\dot{\alpha}\sin\beta & 0 \end{bmatrix}\begin{Bmatrix} V_T \\ 0 \\ 0 \end{Bmatrix} = \begin{Bmatrix} \dot{V}_T \\ \dot{\beta}V_T \\ \dot{\alpha}V_T\cos\beta \end{Bmatrix}$$

Furthermore, the concentrated masses term expands to,

$$SF = \frac{1}{M}\sum_{i=1}^{NW}\left\{m_{W_i}\left[\left(\dot{\Omega}_W + \Omega_W\Omega_W\right)S\vec{r}_{W_i/B} + \left(2\Omega_W S\dot{R}_{W_i}^B + S\ddot{R}_{W_i}^B\right)\vec{r}_{W_i/pivot_i}\right]\right\}$$
$$+\frac{1}{M}\sum_{j=1}^{NR}\left\{m_{R_j}\left[\left(\dot{\Omega}_W + \Omega_w\Omega_w\right)S\vec{r}_{R_j/B} + \left(2\Omega_W S\dot{R}_{R_j}^B + S\ddot{R}_{R_j}^B\right)\vec{r}_{R_j/pivot_j}\right]\right\}$$

Being that,

$$\Omega_W = S\Omega_B = \begin{bmatrix} 0 & -R_W & Q_W \\ R_W & 0 & -P_W \\ -Q_W & P_W & 0 \end{bmatrix}$$

$$\dot{\Omega}_W = \dot{S}\Omega_B + S\dot{\Omega}_B = \begin{bmatrix} 0 & -\dot{R}_W & \dot{Q}_W \\ \dot{R}_W & 0 & -\dot{P}_W \\ -\dot{Q}_W & \dot{P}_W & 0 \end{bmatrix}$$

$$\vec{\omega}_B^W = S\vec{\omega}_B^B = S\begin{Bmatrix} P \\ Q \\ R \end{Bmatrix} = \begin{Bmatrix} P_W \\ Q_W \\ R_W \end{Bmatrix}$$

$$\dot{\vec{\omega}}_B^W = \dot{S}\vec{\omega}_B^B + S\dot{\vec{\omega}}_B^B = \dot{S}\begin{Bmatrix} P \\ Q \\ R \end{Bmatrix} + S\begin{Bmatrix} \dot{P} \\ \dot{Q} \\ \dot{R} \end{Bmatrix} = \begin{Bmatrix} \dot{P}_W \\ \dot{Q}_W \\ \dot{R}_W \end{Bmatrix}$$

Moreover, the aerodynamic and propulsive forces in the wind axes are given by the following equation. The terms are: D, aerodynamic drag, Y, aerodynamic side force, L, aerodynamic lift, T_j, each propeller thrust.

$$\vec{F}_B^W = \begin{Bmatrix} F_x^W \\ F_y^W \\ F_z^W \end{Bmatrix} = \begin{Bmatrix} -D \\ Y \\ -L \end{Bmatrix} + S\sum_{j=1}^{NR}\left(R_{R_j}^B\begin{Bmatrix} T_j \\ 0 \\ 0 \end{Bmatrix}\right)$$

It is convenient to define the propeller thrust as follows, being that k_T, the propeller thrust coefficient and ω_j^2, propeller angular speed squared.

$$T_j = k_T\omega_j^2$$

Thus, we substitute it in the equation of translational motion in wind axes,

$$-M\begin{Bmatrix} \dot{V}_T \\ \dot{\beta}V_T \\ \dot{\alpha}V_T\cos\beta \end{Bmatrix} - M\Omega_W\begin{Bmatrix} V_T \\ 0 \\ 0 \end{Bmatrix} + MSB_E^B\vec{g}^E - MSF + \begin{Bmatrix} -D \\ Y \\ -L \end{Bmatrix} + S\sum_{j=1}^{NR}\left(R_{R_j}^B\begin{Bmatrix} k_T\omega_j^2 \\ 0 \\ 0 \end{Bmatrix}\right) = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}$$

The aerodynamic forces can be expressed as functions of aerodynamic coefficients, which will be useful later. So that C_D, the aerodynamic drag coefficient, C_Y, the aerodynamic side force coefficient, C_L, the aerodynamic lift coefficient, ρ, air density and S_W the aircraft wing reference area.

$$\begin{Bmatrix} -D \\ Y \\ -L \end{Bmatrix} = \frac{1}{2}\rho V_T^2 S_W\begin{Bmatrix} -C_D \\ C_Y \\ -C_L \end{Bmatrix}$$

Now we start the conversion of equation of angular motion in the body axes to wind axes.

$$A\dot{\vec{\omega}}_B^B = -B\vec{\omega}_B^B - D\vec{V}_B^B - C\dot{\vec{V}}_B^B + \vec{T}_B^B + M_P - E$$

$$SA\frac{d}{dt}(S^T\vec{\omega}_B^W) = -SBS^T\vec{\omega}_B^W - SDS^T\vec{V}_B^W - SC\frac{d}{dt}\left(S^T\vec{V}_B^W\right) + S\vec{T}_B^B + SM_P - SE$$

$$SAS^T\dot{\vec{\omega}}_B^W = -\left(SA\dot{S}^T + SBS^T\right)\vec{\omega}_B^W - \left(SDS^T + SC\dot{S}^T\right)\vec{V}_B^W - SCS^T\dot{\vec{V}}_B^W + \vec{T}_B^W + SM_P - SE$$

Making the following simplifications,

$$SAS^T = A_W \quad SBS^T = B_W \quad SCS^T = C_W \quad SDS^T = D_W \quad SE = E_W \quad SM_P = M_{P_W}$$

And substituting them in the angular motion equation,

$$A_W\dot{\vec{\omega}}_B^W = -\left(SA\dot{S}^T + B_W\right)\vec{\omega}_B^W - \left(D_W + SC\dot{S}^T\right)\vec{V}_B^W - C_W\dot{\vec{V}}_B^W + \vec{T}_B^W + M_{P_W} - E_W$$

Now we isolate the aircraft body angular acceleration in the wind axes to the left side,

$$\dot{\vec{\omega}}_B^W = -\left(S\dot{S}^T + A_W^{-1}B_W\right)\vec{\omega}_B^W - A_W^{-1}\left(D_W + SC\dot{S}^T\right)\vec{V}_B^W - A_W^{-1}C_W\dot{\vec{V}}_B^W + A_W^{-1}\vec{T}_B^W + A_W^{-1}\left(M_{P_W} - E_W\right)$$

Moreover the aerodynamic and propulsive moments expands. Being that: $\bar{L}$, aerodynamic rolling moment, M, aerodynamic pitching moment, N, aerodynamic yawing moment, λ_j, each propeller rotation direction index, Q_j, each propeller torque.

$$\vec{T}_B^W = \begin{Bmatrix} \bar{L} \\ M \\ N \end{Bmatrix} + S\sum_{j=1}^{NR}\left(R_{R_j}^B\begin{Bmatrix} \lambda_j Q_j \\ 0 \\ 0 \end{Bmatrix} + \vec{r}_{R_j/B} \times R_{R_j}^B\begin{Bmatrix} T_j \\ 0 \\ 0 \end{Bmatrix}\right)$$

For propellers rotating counterclockwise the propeller rotation index is +1, as for clockwise it is -1. Again, the propeller torque is also convenient to be expressed as a function of angular speed squared. Where, k_Q, propeller torque coefficient.

$$Q_j = k_Q\omega_j^2$$

Substituting we have the angular motion equations in wind axes,

$$-\begin{Bmatrix}\dot{P}_W\\ \dot{Q}_W\\ \dot{R}_W\end{Bmatrix}-\left(A_W^{-1}D_W+A_W^{-1}SC\dot{S}^T\right)\begin{Bmatrix}V_T\\ 0\\ 0\end{Bmatrix}-\left(\Omega_R+A_W^{-1}B_W\right)\begin{Bmatrix}P_W\\ Q_W\\ R_W\end{Bmatrix}+SA^{-1}S^T\begin{Bmatrix}\bar{L}\\ M\\ N\end{Bmatrix}$$

$$+SA^{-1}\sum_{j=1}^{NR}\left(R_{R_j}^B\begin{Bmatrix}\lambda_j k_Q\omega_j^2\\ 0\\ 0\end{Bmatrix}+\vec{r}_{R_j/B}\times R_{R_j}^B\begin{Bmatrix}k_T\omega_j^2\\ 0\\ 0\end{Bmatrix}\right)+A_W^{-1}\left(M_{P_W}-E_W\right)$$

$$-A_W^{-1}C_W\begin{Bmatrix}\dot{V}_T\\ 0\\ 0\end{Bmatrix}=\begin{Bmatrix}0\\ 0\\ 0\end{Bmatrix}$$

The aerodynamic moments can also be expressed as functions of aerodynamic coefficients, being C_l, the aerodynamic rolling moment coefficient, C_m, the aerodynamic pitching moment coefficient, C_n, the aerodynamic yawing moment coefficient, b, the aircraft wing span, $\bar{c}$, aircraft mean aerodynamic chord.

$$\begin{Bmatrix}\bar{L}\\ M\\ N\end{Bmatrix}=\frac{1}{2}\rho V_T^2 S_W\begin{Bmatrix}bC_l\\ \bar{c}C_m\\ bC_n\end{Bmatrix}$$

3.4 Attitude propagation equation

Another important equation is the attitude propagation equation (STEVENS; LEWIS, 2016), which relates Euler angles to aircraft angular velocity.

$$-\begin{Bmatrix}\dot{\phi}\\ \dot{\theta}\\ \dot{\psi}\end{Bmatrix}+\begin{bmatrix}1 & \tan\theta\sin\phi & \tan\theta\cos\phi\\ 0 & \cos\phi & -\sin\phi\\ 0 & \dfrac{\sin\phi}{\cos\theta} & \dfrac{\cos\phi}{\cos\theta}\end{bmatrix}S^T\begin{Bmatrix}P_W\\ Q_W\\ R_W\end{Bmatrix}=\begin{Bmatrix}0\\ 0\\ 0\end{Bmatrix}$$

3.5 Standard atmosphere model

Furthermore, it was used the ICAO Standard Atmosphere model (NACA, 1955), with the following parameters. Being P_0 the pressure at sea level, ρ_0 the air density at sea level and T_0 the temperature at sea level.

$$P_0=101325\ N/m^2=1013{,}25\,hPa$$

$$\rho_0 = 1{,}225\ kg/m^3$$

$$T_0 = 15°\text{C}$$

It is required to define the pressure altitude for the real temperature at sea level, where $\Delta ISA(°\text{C})$ is real temperature shift at sea level with respect to T_0.

$$T_{SL}(°\text{C}) = T_0(°\text{C}) + \Delta ISA(°\text{C})$$

$$PA(m) = -\frac{1000}{6{,}5}\Delta ISA(°\text{C})$$

Then, we find the equation for temperature $T(°\text{C})$, pressure p, air density ρ and specific gas constant for dry air R.

$$T(°\text{C}) = T_o(°\text{C}) - 6{,}5\frac{[h(m) + PA(m)]}{1000}$$

$$p = p_0\left(1 - 0{,}0065\frac{[h(m) + PA(m)]}{T_0(K)}\right)^{5{,}2561}$$

$$\rho = \frac{p}{RT}$$

$$R = 287{,}04\ \frac{Pa.m^3}{kg.K}$$

3.6 Wing and horizontal tail pivot dynamics

Next we derive the angular motion equation for the wing or horizontal tail pivot point in the reference of the pivot point. Considering the total angular momentum of the wing or horizontal tail, that is, the sum of the left and right aerodynamic surface angular momentum and the sum of the respectively rotors angular momentum. The index "P" means pivot point.

$$\vec{H}_{total}^{P} = \sum_{i=1}^{NW}\vec{H}_{W_i}^{P} + \sum_{j=1}^{NRW}\vec{H}_{R_j}^{P}$$

$$= \sum_{i=1}^{NW}\left\{\tilde{I}_{W_i}^{P}\vec{\omega}_{W_i}^{P} + \vec{r}_{W_i/P} \times \left(m_{W_i}\vec{V}_{W_i}^{P}\right)\right\} + \sum_{j=1}^{NRW}\left\{\tilde{I}_{R_j}^{P}\vec{\omega}_{R_j}^{P} + \vec{r}_{R_j/P} \times \left(m_{R_j}\vec{V}_{R_j}^{P}\right)\right\}$$

The net torque acting at the pivot point comes from the rate of change of angular momentum,

$$\vec{T}_P^P + \sum_{i=1}^{NW}\{\vec{r}_{W_i/P} \times m_{W_i} B_E^B \vec{g}^E\} + \sum_{j=1}^{NRW}\left\{\vec{r}_{R_j/P} \times m_{R_j} B_E^B \vec{g}^E\right\} = \frac{d}{dt}\left(\vec{H}_{total}^P\right)$$

Making the derivative of the total angular momentum,

$$\frac{d}{dt}\left(\vec{H}_{total}^P\right) = \sum_{i=1}^{NW}\frac{d}{dt}\left(\vec{H}_{W_i}^P\right) + \sum_{j=1}^{NRW}\frac{d}{dt}\left(\vec{H}_{R_j}^P\right)$$

$$\frac{d}{dt}\left(\vec{H}_{total}^P\right) = \sum_{k=1}^{NW}\left\{\frac{d}{dt}\left(\tilde{I}_{W_i}^P \vec{\omega}_{W_i}^P\right) + \vec{\omega}_P^P \times \left(\tilde{I}_{W_i}^P \vec{\omega}_{W_i}^P\right) + m_{W_i}\left[\frac{d\left[\vec{r}_{W_i/P}\right]}{dt} \times \vec{V}_{W_i}^P + \vec{r}_{W_i/P} \times \frac{d\vec{V}_{W_i}^P}{dt}\right]\right\}$$
$$+ \sum_{j=1}^{NRW}\left\{\frac{d}{dt}\left(\tilde{I}_{R_j}^P \vec{\omega}_{R_j}^P\right) + \vec{\omega}_P^P \times \left(\tilde{I}_{R_j}^P \vec{\omega}_{R_j}^P\right) + m_{R_j}\left[\frac{d\left[\vec{r}_{R_j/P}\right]}{dt} \times \vec{V}_{R_j}^P + \vec{r}_{R_j/P} \times \frac{d\vec{V}_{R_j}^P}{dt}\right]\right\}$$

The velocity vector of the pivot point, and its derivative, are the same as the velocity and acceleration vector of the aircraft body coordinate frame. So, we have,

$$\vec{V}_P^P = \vec{V}_B^B$$

$$\dot{\vec{V}}_P^P = \dot{\vec{V}}_B^B$$

The angular velocity of the pivot point can be written as follows.

$$\vec{\omega}_P^P = \vec{\omega}_{W_i}^P = \begin{bmatrix}0 & \dot{\delta}_{W_i} & 0\end{bmatrix}^T$$

$$\dot{\vec{\omega}}_P^P = \begin{bmatrix}0 & \ddot{\delta}_{W_i} & 0\end{bmatrix}^T$$

Continuing with the previous equation, we expand the derivatives,

$$\frac{d\left[\vec{r}_{W_i/P}\right]}{dt} = \frac{d}{dt}\left(R_{W_i}^B \vec{r}_{W_i/pivot_i}\right) = \dot{R}_{W_i}^B \vec{r}_{W_i/pivot_i} + R_{W_i}^B \frac{d}{dt}\left(\vec{r}_{W_i/pivot_i}\right) = \dot{R}_{W_i}^B \vec{r}_{W_i/pivot_i}$$

$$\frac{d\left[\vec{r}_{R_j/P}\right]}{dt} = \frac{d}{dt}\left(R_{R_j}^B \vec{r}_{R_j/pivot_j}\right) = \dot{R}_{R_j}^B \vec{r}_{R_j/pivot_j} + R_{R_j}^B \frac{d}{dt}\left(\vec{r}_{R_j/pivot_j}\right) = \dot{R}_{R_j}^B \vec{r}_{R_j/pivot_j}$$

And we define the velocity vectors,

$$\vec{V}_{W_i}^P = \vec{V}_P^P + \vec{\omega}_P^P \times \vec{r}_{W_i/P} + \vec{V}_{rel_{W_i/P}}^P = \vec{V}_B^B + \vec{\omega}_P^P \times \vec{r}_{W_i/P} + \dot{R}_{W_i}^B \vec{r}_{W_i/pivot_i}$$

$$\vec{V}_{R_j}^P = \vec{V}_P^P + \vec{\omega}_P^P \times \vec{r}_{R_j/P} + \vec{V}_{rel_{R_j/P}}^P = \vec{V}_B^B + \vec{\omega}_P^P \times \vec{r}_{R_j/P} + \dot{R}_{R_j}^B \vec{r}_{R_j/pivot_j}$$

And the acceleration vectors,

$$\frac{d\vec{V}_{W_i}^P}{dt} = \dot{\vec{V}}_P^P + \vec{\omega}_P^P \times \vec{V}_P^P + \dot{\vec{\omega}}_P^P \times \vec{r}_{W_i/P} + \vec{\omega}_P^P \times \left(\vec{\omega}_P^P \times \vec{r}_{W_i/P}\right) + 2\vec{\omega}_P^P \times \vec{V}_{rel_{W_i/P}}^P + \vec{a}_{rel_{W_i/P}}^P$$
$$= \dot{\vec{V}}_B^B + \vec{\omega}_P^P \times \vec{V}_B^B + \dot{\vec{\omega}}_P^P \times \vec{r}_{W_i/P} + \vec{\omega}_P^P \times \left(\vec{\omega}_P^P \times \vec{r}_{W_i/P}\right) + 2\vec{\omega}_P^P \times \dot{R}_{W_i}^B \vec{r}_{W_i/pivot_i}$$
$$+ \ddot{R}_{W_i}^B \vec{r}_{W_i/pivot_i}$$

$$\frac{d\vec{V}_{R_j}^P}{dt} = \dot{\vec{V}}_P^P + \vec{\omega}_P^P \times \vec{V}_P^P + \dot{\vec{\omega}}_P^P \times \vec{r}_{R_j/P} + \vec{\omega}_P^P \times \left(\vec{\omega}_P^P \times \vec{r}_{R_j/P}\right) + 2\vec{\omega}_P^P \times \vec{V}_{rel_{R_j/P}}^P + \vec{a}_{rel_{R_j/P}}^P$$
$$= \dot{\vec{V}}_B^B + \vec{\omega}_P^P \times \vec{V}_B^B + \dot{\vec{\omega}}_P^P \times \vec{r}_{R_j/P} + \vec{\omega}_P^P \times \left(\vec{\omega}_P^P \times \vec{r}_{R_j/P}\right) + 2\vec{\omega}_P^P \times \dot{R}_{R_j}^B \vec{r}_{R_j/pivot_j}$$
$$+ \ddot{R}_{R_j}^B \vec{r}_{R_j/pivot_j}$$

Substituting we find,

$$\sum_{i=1}^{NW} \frac{d}{dt}\left(\vec{H}_{W_i}^P\right) = \sum_{k=1}^{NW} \left\{ \frac{d}{dt}\left(\tilde{I}_{W_i}^P \vec{\omega}_{W_i}^P\right) + \vec{\omega}_P^P \times \left(\tilde{I}_{W_i}^P \vec{\omega}_{W_i}^P\right) \right.$$
$$+ m_{W_i} \left[\dot{R}_{W_i}^B \vec{r}_{W_i/pivot_i} \times \left(\vec{V}_B^B + \vec{\omega}_P^P \times \vec{r}_{W_i/P} + \dot{R}_{W_i}^B \vec{r}_{W_i/pivot_i}\right) + \vec{r}_{W_i/P} \right.$$
$$\times \left(\dot{\vec{V}}_B^B + \vec{\omega}_P^P \times \vec{V}_B^B + \dot{\vec{\omega}}_P^P \times \vec{r}_{W_i/P} + \vec{\omega}_P^P \times \left(\vec{\omega}_P^P \times \vec{r}_{W_i/P}\right) + 2\vec{\omega}_P^P \right.$$
$$\left.\left.\left. \times \dot{R}_{W_i}^B \vec{r}_{W_i/pivot_i} + \ddot{R}_{W_i}^B \vec{r}_{W_i/pivot_i}\right)\right]\right\}$$

$$\sum_{j=1}^{NRW} \frac{d}{dt}\left(\vec{H}_{R_j}^P\right) = \sum_{j=1}^{NRW} \left\{ \frac{d}{dt}\left(\tilde{I}_{R_j}^P \vec{\omega}_{R_j}^P\right) + \vec{\omega}_P^P \times \left(\tilde{I}_{R_j}^P \vec{\omega}_{R_j}^P\right) \right.$$
$$+ m_{R_j} \left[\dot{R}_{R_j}^B \vec{r}_{R_j/pivot_j} \times \left(\vec{V}_B^B + \vec{\omega}_P^P \times \vec{r}_{R_j/P} + \dot{R}_{R_j}^B \vec{r}_{R_j/pivot_j}\right) + \vec{r}_{R_j/P} \right.$$
$$\times \left(\dot{\vec{V}}_B^B + \vec{\omega}_P^P \times \vec{V}_B^B + \dot{\vec{\omega}}_P^P \times \vec{r}_{R_j/P} + \vec{\omega}_P^P \times \left(\vec{\omega}_P^P \times \vec{r}_{R_j/P}\right) + 2\vec{\omega}_P^P \times \dot{R}_{R_j}^B \vec{r}_{R_j/pivot_j} \right.$$
$$\left.\left.\left. + \ddot{R}_{R_j}^B \vec{r}_{R_j/pivot_j}\right)\right]\right\}$$

Additionally, we need to transform the inertia terms to the pivot reference point, thus we have,

$$\tilde{I}_{W_i}^P \vec{\omega}_{W_i}^P = [T]_{W_i} \left(\tilde{I}_{W_i}^{W_i} + m_{W_i} \tilde{R}_{W_i}^P \right) [T]_{W_i}^T \vec{\omega}_P^P$$

$$\tilde{I}_{R_j}^P \vec{\omega}_{R_j}^P = [T]_{R_j} \left(\tilde{I}_{R_j}^{R_j} + m_{R_j} \tilde{R}_{R_j}^P \right) [T]_{R_j}^T \left(R_{R_j}^P \vec{\omega}_{R_j}^{R_j} + \vec{\omega}_P^P \right)$$

Where,

$[T]_{W_i}$and $[T]_{R_j}$: Inertia tilt matrix: rotates the inertia matrix of the wing, horizontal tail or rotor reference frame to pivot point coordinate frame.

$\tilde{R}_{W_i}^P$ and $\tilde{R}_{R_j}^P$: Inertia translation matrix: transfers the inertia matrix of the wing, horizontal tail or rotor reference frame to the pivot point coordinate frame.

$\vec{\omega}_{R_j}^{R_j}$: Rotors concentrated mass angular velocity vector in respect to its own reference frame.

Computing the following derivatives,

$$\frac{d}{dt}\left(\tilde{I}_{W_i}^P \vec{\omega}_{W_i}^P\right) = \Big\{ [\dot{T}]_{W_i} \left(\tilde{I}_{W_i}^{W_i} + m_{W_i} \tilde{R}_{W_i}^P \right) [T]_{W_i}^T + [T]_{W_i} m_{W_i} \frac{d}{dt}\left(\tilde{R}_{W_i}^P\right) [T]_{W_i}^T$$
$$+ [T]_{W_i} \left(\tilde{I}_{W_i}^{W_i} + m_{W_i} \tilde{R}_{W_i}^P \right) [\dot{T}]_{W_i}^T \Big\} \vec{\omega}_P^P + [T]_{W_i} \left(\tilde{I}_{W_i}^{W_i} + m_{W_i} \tilde{R}_{W_i}^P \right) [T]_{W_i}^T \dot{\vec{\omega}}_P^P$$

$$\frac{d}{dt}\left(\tilde{I}_{R_j}^P \vec{\omega}_{R_j}^P\right) = \Big\{ [\dot{T}]_{R_j} \left(\tilde{I}_{R_j}^{R_j} + m_{R_j} \tilde{R}_{R_j}^P \right) [T]_{R_j}^T + [T]_{R_j} m_{R_j} \frac{d}{dt}\left(\tilde{R}_{R_j}^P\right) [T]_{R_j}^T$$
$$+ [T]_{R_j} \left(\tilde{I}_{R_j}^{R_j} + m_{R_j} \tilde{R}_{R_j}^P \right) [\dot{T}]_{R_j}^T \Big\} \left(R_{R_j}^P \vec{\omega}_{R_j}^{R_j} + \vec{\omega}_P^P \right)$$
$$+ [T]_{R_j} \left(\tilde{I}_{R_j}^{R_j} + m_{R_j} \tilde{R}_{R_j}^P \right) [T]_{R_j}^T \left(\dot{R}_{R_j}^B \vec{\omega}_{R_j}^{R_j} + R_{R_j}^B \dot{\vec{\omega}}_{R_j}^{R_j} + \dot{\vec{\omega}}_P^P \right)$$

Substituting we find,

$$\sum_{i=1}^{NW} \frac{d}{dt}\left(\vec{H}_{W_i}^P\right) = \sum_{k=1}^{NW} \Big\{ \Big\{ [\dot{T}]_{W_i} \left(\tilde{I}_{W_i}^{W_i} + m_{W_i} \tilde{R}_{W_i}^P \right) [T]_{W_i}^T + [T]_{W_i} m_{W_i} \frac{d}{dt}\left(\tilde{R}_{W_i}^P\right) [T]_{W_i}^T$$
$$+ [T]_{W_i} \left(\tilde{I}_{W_i}^{W_i} + m_{W_i} \tilde{R}_{W_i}^P \right) [\dot{T}]_{W_i}^T \Big\} \vec{\omega}_P^P + [T]_{W_i} \left(\tilde{I}_{W_i}^{W_i} + m_{W_i} \tilde{R}_{W_i}^P \right) [T]_{W_i}^T \dot{\vec{\omega}}_P^P + \vec{\omega}_P^P$$
$$\times \left([T]_{W_i} \left(\tilde{I}_{W_i}^{W_i} + m_{W_i} \tilde{R}_{W_i}^P \right) [T]_{W_i}^T \vec{\omega}_P^P \right)$$
$$+ m_{W_i} \Big[\dot{R}_{W_i}^B \vec{r}_{W_i/pivot_i} \times \left(\vec{V}_B^B + \vec{\omega}_P^P \times \vec{r}_{W_i/P} + \dot{R}_{W_i}^B \vec{r}_{W_i/pivot_i} \right) + \vec{r}_{W_i/P}$$
$$\times \Big(\dot{\vec{V}}_B^B + \vec{\omega}_P^P \times \vec{V}_B^B + \dot{\vec{\omega}}_P^P \times \vec{r}_{W_i/P} + \vec{\omega}_P^P \times \left(\vec{\omega}_P^P \times \vec{r}_{W_i/P} \right) + 2\vec{\omega}_P^P \times \dot{R}_{W_i}^B \vec{r}_{W_i/pivot_i}$$
$$+ \ddot{R}_{W_i}^B \vec{r}_{W_i/pivot_i} \Big) \Big] \Big\}$$

$$\sum_{j=1}^{NRW}\frac{d}{dt}\left(\vec{H}_{R_j}^{P}\right)=\sum_{j=1}^{NRW}\left\{\left\{[\dot{T}]_{R_j}\left(\tilde{I}_{R_j}^{R_j}+m_{R_j}\tilde{R}_{R_j}^{P}\right)[T]_{R_j}^{T}+[T]_{R_j}m_{R_j}\frac{d}{dt}\left(\tilde{R}_{R_j}^{P}\right)[T]_{R_j}^{T}\right.\right.$$
$$\left.+[T]_{R_j}\left(\tilde{I}_{R_j}^{R_j}+m_{R_j}\tilde{R}_{R_j}^{P}\right)[\dot{T}]_{R_j}^{T}\right\}\left(R_{R_j}^{P}\vec{\omega}_{R_j}^{R_j}+\vec{\omega}_{P}^{P}\right)$$
$$+[T]_{R_j}\left(\tilde{I}_{R_j}^{R_j}+m_{R_j}\tilde{R}_{R_j}^{P}\right)[T]_{R_j}^{T}\left(\dot{R}_{R_j}^{B}\vec{\omega}_{R_j}^{R_j}+R_{R_j}^{B}\dot{\vec{\omega}}_{R_j}^{R_j}+\dot{\vec{\omega}}_{P}^{P}\right)+\vec{\omega}_{P}^{P}$$
$$\times\left([T]_{R_j}\left(\tilde{I}_{R_j}^{R_j}+m_{R_j}\tilde{R}_{R_j}^{P}\right)[T]_{R_j}^{T}\left(R_{R_j}^{P}\vec{\omega}_{R_j}^{R_j}+\vec{\omega}_{P}^{P}\right)\right)$$
$$+m_{R_j}\left[\dot{R}_{R_j}^{B}\vec{r}_{R_j/pivot_j}\times\left(\vec{V}_{B}^{B}+\vec{\omega}_{P}^{P}\times\vec{r}_{R_j/P}+\dot{R}_{R_j}^{B}\vec{r}_{R_j/pivot_j}\right)+\vec{r}_{R_j/P}\right.$$
$$\times\left(\dot{\vec{V}}_{B}^{B}+\vec{\omega}_{P}^{P}\times\vec{V}_{B}^{B}+\dot{\vec{\omega}}_{P}^{P}\times\vec{r}_{R_j/P}+\vec{\omega}_{P}^{P}\times\left(\vec{\omega}_{P}^{P}\times\vec{r}_{R_j/P}\right)+2\vec{\omega}_{P}^{P}\times\dot{R}_{R_j}^{B}\vec{r}_{R_j/pivot_j}\right.$$
$$\left.\left.\left.+\ddot{R}_{R_j}^{B}\vec{r}_{R_j/pivot_j}\right)\right]\right\}$$

Rearranging terms it is possible to write in the simplified form,

$$\vec{T}_{P}^{P}+(M_P)_P=(A)_P\dot{\vec{\omega}}_{P}^{P}+(B)_P\vec{\omega}_{P}^{P}+(C)_P\dot{\vec{V}}_{B}^{B}+(D)_P\vec{V}_{B}^{B}+(E)_P$$

So that the terms are,

$$(A)_P=\sum_{i=1}^{NW}\left\{[T]_{W_i}\left(\tilde{I}_{W_i}^{W_i}+m_{W_i}\tilde{R}_{W_i}^{P}\right)[T]_{W_i}^{T}-m_{W_i}\vec{r}_{W_i/P}\times\vec{r}_{W_i/P}\times\right\}$$
$$+\sum_{j=1}^{NR}\left\{[T]_{R_j}\left(\tilde{I}_{R_j}^{R_j}+m_{R_j}\tilde{R}_{R_j}^{P}\right)[T]_{R_j}^{T}-m_{R_j}\vec{r}_{R_j/P}\times\vec{r}_{R_j/P}\times\right\}$$

$$(B)_P=\sum_{i=1}^{NW}\left\{[\dot{T}]_{W_i}\left(\tilde{I}_{W_i}^{W_i}+m_{W_i}\tilde{R}_{W_i}^{P}\right)[T]_{W_i}^{T}+[T]_{W_i}m_{W_i}\frac{d}{dt}\left(\tilde{R}_{W_i}^{P}\right)[T]_{W_i}^{T}\right.$$
$$+[T]_{W_i}\left(\tilde{I}_{W_i}^{W_i}+m_{W_i}\tilde{R}_{W_i}^{P}\right)[\dot{T}]_{W_i}^{T}+\Omega_P[T]_{W_i}\left(\tilde{I}_{W_i}^{W_i}+m_{W_i}\tilde{R}_{W_i}^{P}\right)[T]_{W_i}^{T}$$
$$\left.-m_{W_i}\left[\left(\dot{R}_{W_i}^{P}\vec{r}_{W_i/pivot_i}\times\vec{r}_{W_i/P}\times\right)+\left(\vec{r}_{W_i/P}\times\Omega_P\vec{r}_{W_i/P}\times\right)\right]\right\}$$
$$+\sum_{j=1}^{NR}\left\{[\dot{T}]_{R_j}\left(\tilde{I}_{R_j}^{R_j}+m_{R_j}\tilde{R}_{R_j}^{P}\right)[T]_{R_j}^{T}+[T]_{R_j}m_{R_j}\frac{d}{dt}\left(\tilde{R}_{R_j}^{P}\right)[T]_{R_j}^{T}\right.$$
$$+[T]_{R_j}\left(\tilde{I}_{R_j}^{R_j}+m_{R_j}\tilde{R}_{R_j}^{P}\right)[\dot{T}]_{R_j}^{T}+\Omega_P[T]_{R_j}\left(\tilde{I}_{R_j}^{R_j}+m_{R_j}\tilde{R}_{R_j}^{P}\right)[T]_{R_j}^{T}$$
$$\left.-m_{R_j}\left[\left(\dot{R}_{R_j}^{P}\vec{r}_{R_j/pivot_j}\times\vec{r}_{R_j/P}\times\right)+\left(\vec{r}_{R_j/P}\times\Omega_P\vec{r}_{R_j/P}\times\right)\right]\right\}$$

$$(C)_P=\sum_{i=1}^{NW}\left\{m_{W_i}\vec{r}_{W_i/P}\times\right\}+\sum_{j=1}^{NR}\left\{m_{R_j}\vec{r}_{R_j/P}\times\right\}$$

$$(D)_P = \sum_{i=1}^{NW}\{m_{W_i}[(\dot{R}_{W_i}^{P}\vec{r}_{W_i/pivot_i}\times) + \vec{r}_{W_i/P}\times\Omega_P]\} + \sum_{j=1}^{NR}\left\{m_{R_j}\left[\left(\dot{R}_{R_j}^{P}\vec{r}_{R_j/pivot_j}\times\right) + \vec{r}_{R_j/P}\times\Omega_P\right]\right\}$$

$$\begin{aligned}(E)_P = &\sum_{i=1}^{NW}\{m_{W_i}[\dot{R}_{W_i}^{P}\vec{r}_{W_i/pivot_i}\times\dot{R}_{W_i}^{P} + \vec{r}_{W_i/P}\times(2\Omega_P\dot{R}_{W_i}^{P} + \ddot{R}_{W_i}^{P})]\vec{r}_{W_i/pivot_i}\}\\
&+\sum_{j=1}^{NR}\left\{\left([\dot{T}]_{R_j}\left(\tilde{I}_{R_j}^{R_j} + m_{R_j}\tilde{R}_{R_j}^{P}\right)[T]_{R_j}^{T} + [T]_{R_j}m_{R_j}\frac{d}{dt}\left(\tilde{R}_{R_j}^{P}\right)[T]_{R_j}^{T}\right.\right.\\
&\left.+[T]_{R_j}\left(\tilde{I}_{R_j}^{R_j} + m_{R_j}\tilde{R}_{R_j}^{P}\right)[\dot{T}]_{R_j}^{T}\right)R_{R_j}^{P}\vec{\omega}_{R_j}^{R_j}\\
&+[T]_{R_j}\left(\tilde{I}_{R_j}^{R_j} + m_{R_j}\tilde{R}_{R_j}^{P}\right)[T]_{R_j}^{T}\left(\dot{R}_{R_j}^{B}\vec{\omega}_{R_j}^{R_j} + R_{R_j}^{B}\dot{\vec{\omega}}_{R_j}^{R_j}\right)\\
&+\Omega_P[T]_{R_j}\left(\tilde{I}_{R_j}^{R_j} + m_{R_j}\tilde{R}_{R_j}^{P}\right)[T]_{R_j}^{T}R_{R_j}^{P}\vec{\omega}_{R_j}^{R_j}\\
&\left.+m_{R_j}\left[\dot{R}_{R_j}^{P}\vec{r}_{R_j/pivot_j}\times\dot{R}_{R_j}^{P} + \vec{r}_{R_j/P}\times\left(2\Omega_P\dot{R}_{R_j}^{P} + \ddot{R}_{R_j}^{P}\right)\right]\vec{r}_{R_j/pivot_j}\right\}\end{aligned}$$

$$(M_P)_P = \sum_{i=1}^{NW}\{\vec{r}_{W_i/P}\times m_{W_i}B_E^B\vec{g}^E\} + \sum_{j=1}^{NR}\left\{\vec{r}_{R_j/P}\times m_{R_j}B_E^B\vec{g}^E\right\}$$

So that,

$$\Omega_P = (\vec{\omega}_P^P\times) = \begin{bmatrix}0 & 0 & \dot{\delta}_{W_i}\\ 0 & 0 & 0\\ -\dot{\delta}_{W_i} & 0 & 0\end{bmatrix}$$

It is necessary to transform the pivot dynamic equation to wind axes. Remembering that,

$$\vec{V}_B^B = S^T\vec{V}_B^W$$

Now substituting,

$$\vec{T}_P^W + S(M_P)_P = S(A)_P\dot{\vec{\omega}}_P^P + S(B)_P\vec{\omega}_P^P + S(C)_P\frac{d}{dt}\left(S^T\vec{V}_B^W\right) + S(D)_PS^T\vec{V}_B^W + S(E)_P$$

$$S(A)_P\dot{\vec{\omega}}_P^P = \vec{T}_P^W - S(B)_P\vec{\omega}_P^P - \left[S(C)_P\dot{S}^T + S(D)_PS^T\right]\vec{V}_B^W - S(C)_PS^T\dot{\vec{V}}_B^W + S[(M_P)_P - (E)_P]$$

Passing only $\dot{\vec{\omega}}_P^P$ to the left side,

$$\begin{aligned}\dot{\vec{\omega}}_P^P = &(A)_P^{-1}S^T\vec{T}_P^W - (A)_P^{-1}(B)_P\vec{\omega}_P^P - (A)_P^{-1}\left[(C)_P\dot{S}^T + (D)_PS^T\right]\vec{V}_B^W - (A)_P^{-1}(C)_PS^T\dot{\vec{V}}_B^W\\
&+(A)_P^{-1}[(M_P)_P - (E)_P]\end{aligned}$$

We now expand the total torque vector in aerodynamic moments of the wing or horizontal tail, the thrust moments with respect to pivot point and pivot actuator torque components $\left(\vec{T}_P^P\right)_a$,

$$\vec{T}_P^W = \begin{Bmatrix} \bar{L} \\ M \\ N \end{Bmatrix}_W + S \sum_{j=1}^{NRW} \left(R_{R_j}^B \begin{Bmatrix} \lambda_j Q_j \\ 0 \\ 0 \end{Bmatrix} + \vec{r}_{R_j/P} \times R_{R_j}^B \begin{Bmatrix} T_j \\ 0 \\ 0 \end{Bmatrix} \right) + S\left(\vec{T}_P^P\right)_a$$

We may write the actuator torque as,

$$\left(\vec{T}_P^P\right)_a = \begin{Bmatrix} 0 \\ M_a \\ 0 \end{Bmatrix}$$

Thus we find the pivot dynamics equation, from which we are only interested in the second line.

$$\begin{aligned}\begin{Bmatrix} 0 \\ \ddot{\delta}_{W_i} \\ 0 \end{Bmatrix} = (A)_P^{-1} \left[S^T \begin{Bmatrix} \bar{L} \\ M \\ N \end{Bmatrix}_W + \sum_{j=1}^{NRW} \left(R_{R_j}^B \begin{Bmatrix} \lambda_j k_Q \omega_j^2 \\ 0 \\ 0 \end{Bmatrix} + \vec{r}_{R_j/P} \times R_{R_j}^B \begin{Bmatrix} k_T \omega_j^2 \\ 0 \\ 0 \end{Bmatrix} \right) + \begin{Bmatrix} 0 \\ M_a \\ 0 \end{Bmatrix} \right] \\ - (A)_P^{-1}(B)_P \begin{Bmatrix} 0 \\ \dot{\delta}_{W_i} \\ 0 \end{Bmatrix} - (A)_P^{-1} \left[(C)_P \dot{S}^T + (D)_P S^T \right] \begin{Bmatrix} V_T \\ 0 \\ 0 \end{Bmatrix} - (A)_P^{-1}(C)_P S^T \begin{Bmatrix} \dot{V}_T \\ 0 \\ 0 \end{Bmatrix} \\ + (A)_P^{-1}[(M_P)_P - (E)_P]\end{aligned}$$

From where we extract the second line, using the subscript 2.

$$\begin{aligned}\ddot{\delta}_{W_i} = \left[(A)_P^{-1} S^T \begin{Bmatrix} \bar{L} \\ M \\ N \end{Bmatrix}_W \right]_2 + \sum_{j=1}^{NRW} \left(\left[(A)_P^{-1} R_{R_j}^B \begin{Bmatrix} \lambda_j k_Q \omega_j^2 \\ 0 \\ 0 \end{Bmatrix} + (A)_P^{-1} \vec{r}_{R_j/P} \times R_{R_j}^B \begin{Bmatrix} k_T \omega_j^2 \\ 0 \\ 0 \end{Bmatrix} \right]_2 \right) \\ + M_a - [(A)_P^{-1}(B)_P]_2 \dot{\delta}_{W_i} - \left[(A)_P^{-1} \left[(C)_P \dot{S}^T + (D)_P S^T \right] \begin{Bmatrix} V_T \\ 0 \\ 0 \end{Bmatrix} \right]_2 \\ - \left[(A)_P^{-1}(C)_P S^T \begin{Bmatrix} \dot{V}_T \\ 0 \\ 0 \end{Bmatrix} \right]_2 + \left[(A)_P^{-1}[(M_P)_P - (E)_P] \right]_2\end{aligned}$$

This is written as,

$$\ddot{\delta}_{W_i} = P_1 \dot{\delta}_{W_i} + P_2 + M_a$$

Being,

$$P_1 = -[(A)_P^{-1}(B)_P]_2$$

$$P_2 = \left[(A)_P^{-1}S^T \begin{Bmatrix} \bar{L} \\ M \\ N \end{Bmatrix}_W\right]_2 + \sum_{j=1}^{NRW}\left(\left[(A)_P^{-1}R_{R_j}^B \begin{Bmatrix} \lambda_j k_Q \omega_j^2 \\ 0 \\ 0 \end{Bmatrix} + (A)_P^{-1}\vec{r}_{R_j/P} \times R_{R_j}^B \begin{Bmatrix} k_T \omega_j^2 \\ 0 \\ 0 \end{Bmatrix}\right]_2\right)$$
$$- \left[\left[(A)_P^{-1}(C)_P\dot{S}^T + (A)_P^{-1}(D)_P S^T\right]\begin{Bmatrix} V_T \\ 0 \\ 0 \end{Bmatrix}\right]_2 - \left[(A)_P^{-1}(C)_P S^T \begin{Bmatrix} \dot{V}_T \\ 0 \\ 0 \end{Bmatrix}\right]_2$$
$$+ [(A)_P^{-1}(M_P)_P - (A)_P^{-1}(E)_P]_2$$

Now we use the state space representation of the pivot dynamic system. Using $x_1 = \delta_{W_i}$, $x_2 = \dot{\delta}_{W_i}$, we derive,

$$\begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{Bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & P_1 \end{bmatrix}\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} + \begin{Bmatrix} 0 \\ P_2 + M_a \end{Bmatrix}$$

And the output is as follows,

$$\begin{Bmatrix} \delta_{W_i} \\ \dot{\delta}_{W_i} \end{Bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix}$$

Therefore, we finish the description of the equations of motion in this chapter which allows the formulation of the state equations derived in the next chapter, which will then be used to derive the concept of the aircraft control system.

4 STATE EQUATIONS

We begin now with the state equation formulation, which comes from the small perturbation equations with respect to a steady-state flight condition, which are linear equations derived algebraically from nonlinear equations, where the nonlinear aerodynamic coefficients are replaced by terms involving the stability derivatives (STEVENS; LEWIS, 2016).

So we can assume that a series of nonlinear equation that describe the aircraft system dynamics com be expressed as follows, being that: $\vec{f}$, vector of n scalar nonlinear functions f_i, $\vec{X}$, the state vector, $\dot{\vec{X}}$, the state vector derivative, $\vec{U}$, the control vector.

$$\vec{f}\left(\dot{\vec{X}}, \vec{X}, \vec{U}\right) = 0$$

Which means that exist a combination of $\dot{\vec{X}}, \vec{X}, \vec{U}$ that nullifies all equations. Thus, the state vector that satisfies the condition of $\dot{\vec{X}} = 0$ and $\vec{U} = 0$ or constant, is called an equilibrium point, which we will call $\vec{X} = \vec{X}_e$, and the control vector in the equilibrium $\vec{U} = \vec{U}_e$. From this approach is possible to examine the behavior of the system near the equilibrium point by slightly perturbing some of the variables. That way, the steady-state aircraft flight can be defined as a condition in which all of the motion variables are constant or zero [13], such condition is represented by the vectors $\vec{X}_e$ and $\vec{U}_e$.

Now we begin with the linearization, considering small perturbations from the steady-state condition $\vec{X}_e$, $\vec{U}_e$, and derive a set of linear constant-coefficient state equations. Expanding the nonlinear state equations in a Taylor series about the equilibrium point, and keep only the first order terms, we find that the perturbations in the state, state derivative, and control vectors must satisfy the following equations [13],

$$\begin{gathered}
\nabla_{\dot{X}} f_1 \delta\dot{\vec{X}} + \nabla_X f_1 \delta\vec{X} + \nabla_U f_1 \delta\vec{U} = 0 \\
\nabla_{\dot{X}} f_2 \delta\dot{\vec{X}} + \nabla_X f_2 \delta\vec{X} + \nabla_U f_2 \delta\vec{U} = 0 \\
\vdots \\
\nabla_{\dot{X}} f_n \delta\dot{\vec{X}} + \nabla_X f_n \delta\vec{X} + \nabla_U f_n \delta\vec{U} = 0
\end{gathered}$$

The ‘delta’ operator represents (δ) small perturbations, and the ‘nabla’ operator (∇) represents a row vector of first partial derivatives, as in,

$$\nabla_X f_i = \begin{bmatrix} \frac{\partial f_i}{\partial X_1} & \frac{\partial f_i}{\partial X_2} & \cdots & \frac{\partial f_i}{\partial X_n} \end{bmatrix}$$

The system of equations can be written in implicit linear state-variable form as,

$$E\dot{\vec{x}} = A\vec{x} + B\vec{u}$$

Wherein, $\dot{\vec{x}}$, $\vec{x}$ and $\vec{u}$ are perturbation vectors from the equilibrium point, such that the state and control vectors are the sums of the equilibrium state and control and the perturbation state and control, respectively.

$$\vec{X} = \vec{X}_e + \vec{x}$$

$$\vec{U} = \vec{U}_e + \vec{u}$$

And the coefficient matrices or Jacobian matrices must be calculated at the equilibrium point. So, we define,

$$E = -\begin{bmatrix} \nabla_{\dot{X}} f_1 \\ \vdots \\ \nabla_{\dot{X}} f_n \end{bmatrix}_{\substack{\vec{X}=\vec{X}_e \\ \vec{U}=\vec{U}_e}} \quad A = \begin{bmatrix} \nabla_X f_1 \\ \vdots \\ \nabla_X f_n \end{bmatrix}_{\substack{\vec{X}=\vec{X}_e \\ \vec{U}=\vec{U}_e}} \quad B = \begin{bmatrix} \nabla_U f_1 \\ \vdots \\ \nabla_U f_n \end{bmatrix}_{\substack{\vec{X}=\vec{X}_e \\ \vec{U}=\vec{U}_e}}$$

At this point we must define the state vector in order to make the state space formulation, which enables assessment of flight characteristics and simulation. For this aircraft project will be considered the following state vector and control vector,

$$\vec{X}^T = \left(V_T, \beta, \alpha, P_W, Q_W, R_W, \phi, \theta, \psi, h, \delta_f, \delta_e, \delta_r, \delta_{a_L}, \delta_{a_R}, \delta_W, \dot{\delta}_W, \delta_{HT}, \dot{\delta}_{HT}, \omega_1^2, \omega_2^2, \omega_3^2, \omega_4^2, \omega_5^2, \omega_6^2\right)$$

The vector elements are: V_T, flight speed, β, sideslip angle, α, aircraft body angle of attack, P_W, wind axes rolling rate, Q_W, wind axes pitching rate, R_W, wind axes yawing rate, ϕ, roll angle, θ, pitch angle, ψ, yaw angle, h, aircraft altitude, δ_f, flap deflection angle, δ_e, elevator deflection angle, δ_r, rudder deflection angle, δ_{a_L}, left aileron deflection angle, δ_{a_R}, right aileron deflection angle, δ_W, wing tilt angle, $\dot{\delta}_W$, wilt tilt velocity, δ_{HT}, horizontal tail tilt angle, $\dot{\delta}_{HT}$, horizontal tail tilt velocity, ω_1^2 to ω_6^2, propeller 1 to 6 angular speed squared. And the control vector,

$$\vec{U}^T = \left(\delta_f^C, \delta_e^C, \delta_r^C, \delta_{a_L}^C, \delta_{a_R}^C, \delta_W^C, \delta_{HT}^C, {\omega_1^2}^C, {\omega_2^2}^C, {\omega_3^2}^C, {\omega_4^2}^C, {\omega_5^2}^C, {\omega_6^2}^C\right)$$

For the control vector we have the superscript C to represent the commanded signal.

We need therefore a total of 25 equations in order to have a complete state space representation of the system, which are: 3 equation of the translational motion, 3 equations of the angular motion, 3 equations from the attitude propagation. The 10th equation comes from the definition of the aircraft rate of climb, which is the vertical component of the velocity vector, being γ the flight path angle.

$$\dot{h} = V_T \sin\gamma$$

From Stevens and Lewis (2016) we have the expansion,

$$\sin\gamma = \cos\alpha\cos\beta\sin\theta - (\sin\phi\sin\beta + \cos\phi\sin\alpha\cos\beta)\cos\theta$$

Therefore, we have the rate of climb equation,

$$-\dot{h} + V_T[\cos\alpha\cos\beta\sin\theta - (\sin\phi\sin\beta + \cos\phi\sin\alpha\cos\beta)\cos\theta] = 0$$

The 11th to 15th equations come from the dynamics of the actuators. Actuator dynamics can be expressed as follows. The terms being: λ_{a_i}, actuator time constant and K_{a_i}, actuator gain. This is applied to flap, elevator, rudder and ailerons actuators.

$$-\dot{\delta}_i + \lambda_{a_i}\left(K_{a_i}\delta^c{}_i - \delta_i\right) = 0$$

Moreover, for the wing and horizontal tail dynamics we use the pivot dynamics equations, previously derived, which forms the 16th to 19th equations.

$$\begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{Bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & P_1 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} + \begin{Bmatrix} 0 \\ P_2 + M_a \end{Bmatrix}$$

$$\begin{Bmatrix} \delta_{W_i} \\ \dot{\delta}_{W_i} \end{Bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix}$$

Finally, the 20th to 25th equations are the rotors dynamics. It is assumed to be as follows, with λ_p, rotors time constant and K_p, rotors gain.

$$-\left(\dot{\omega_j^2}\right) + \lambda_p\left(K_p\left(\omega_j^2\right)^c - \omega_j^2\right) = 0$$

So, applying the actuator and rotors dynamic equation for all the remaining states we fulfill the 25 equations required.

Next step is the assembly of the Jacobian matrices terms. Starting with the first three rows of matrix E, we make the first partial derivatives of the translational motion equations with respect to the state derivative vector. When the aerodynamic derivative is significant it will appear in the form of the component with the variable subscribed.

Remembering the translational motion equations, they are written as follows,

$$\begin{pmatrix} f_1 \\ f_2 \\ f_3 \end{pmatrix} = -M \begin{Bmatrix} \dot{V}_T \\ \dot{\beta} V_T \\ \dot{\alpha} V_T \cos\beta \end{Bmatrix} - M\Omega_W \begin{Bmatrix} V_T \\ 0 \\ 0 \end{Bmatrix} + MSB_E^B \vec{g}^E - MSF + \begin{Bmatrix} -D \\ Y \\ -L \end{Bmatrix} + S \sum_{j=1}^{NR} \left(R_{R_j}^B \begin{Bmatrix} k_T \omega_j^2 \\ 0 \\ 0 \end{Bmatrix} \right)$$

And the state derivative vector is the vector assembling all the state derivatives,

$$\dot{\vec{X}}^T$$

$$= \left(\dot{V}_T, \dot{\beta}, \dot{\alpha}, \dot{P}_W, \dot{Q}_W, \dot{R}_W, \dot{\phi}, \dot{\theta}, \dot{\psi}, \dot{h}, \dot{\delta}_e, \dot{\delta}_r, \dot{\delta}_{a_L}, \dot{\delta}_{a_R}, \dot{\delta}_W, \ddot{\delta}_W, \dot{\delta}_{HT}, \ddot{\delta}_{HT}, \dot{\omega^2}_1 \dot{\omega^2}_2, \dot{\omega^2}_3, \dot{\omega^2}_4, \dot{\omega^2}_5, \dot{\omega^2}_6 \right)$$

In this way, follows the partial derivatives of the translational motion equations with respect to the state derivative vector,

$$\frac{\partial}{\partial \dot{V}_T} \begin{pmatrix} f_1 \\ f_2 \\ f_3 \end{pmatrix} = -M \begin{Bmatrix} 1 \\ 0 \\ 0 \end{Bmatrix}$$

$$\frac{\partial}{\partial \dot{\beta}} \begin{pmatrix} f_1 \\ f_2 \\ f_3 \end{pmatrix} = -M \begin{Bmatrix} 0 \\ V_T \\ 0 \end{Bmatrix} + \begin{Bmatrix} 0 \\ Y_{\dot{\beta}} \\ 0 \end{Bmatrix}$$

$$\frac{\partial}{\partial \dot{\alpha}} \begin{pmatrix} f_1 \\ f_2 \\ f_3 \end{pmatrix} = -M \begin{Bmatrix} 0 \\ 0 \\ V_T \end{Bmatrix} + \begin{Bmatrix} 0 \\ 0 \\ -L_{\dot{\alpha}} \end{Bmatrix}$$

$$\frac{\partial}{\partial \dot{P}_W} \begin{pmatrix} f_1 \\ f_2 \\ f_3 \end{pmatrix} = -\frac{\partial (MSF)}{\partial \dot{P}_W} = -\left(\sum_{i=1}^{NW} \left\{ m_{W_i} \frac{\partial \dot{\Omega}_W}{\partial \dot{P}_W} S\vec{r}_{W_i/B} \right\} + \sum_{j=1}^{NR} \left\{ m_{R_j} \frac{\partial \dot{\Omega}_W}{\partial \dot{P}_W} S\vec{r}_{R_j/B} \right\} \right)$$

$$\frac{\partial}{\partial \dot{Q}_W} \begin{pmatrix} f_1 \\ f_2 \\ f_3 \end{pmatrix} = -\frac{\partial (MSF)}{\partial \dot{Q}_W} = -\left(\sum_{i=1}^{NW} \left\{ m_{W_i} \frac{\partial \dot{\Omega}_W}{\partial \dot{Q}_W} S\vec{r}_{W_i/B} \right\} + \sum_{j=1}^{NR} \left\{ m_{R_j} \frac{\partial \dot{\Omega}_W}{\partial \dot{Q}_W} S\vec{r}_{R_j/B} \right\} \right)$$

$$\frac{\partial}{\partial \dot{R}_W} \begin{pmatrix} f_1 \\ f_2 \\ f_3 \end{pmatrix} = -\frac{\partial (MSF)}{\partial \dot{R}_W} = -\left(\sum_{i=1}^{NW} \left\{ m_{W_i} \frac{\partial \dot{\Omega}_W}{\partial \dot{R}_W} S\vec{r}_{W_i/B} \right\} + \sum_{j=1}^{NR} \left\{ m_{R_j} \frac{\partial \dot{\Omega}_W}{\partial \dot{R}_W} S\vec{r}_{R_j/B} \right\} \right)$$

$$\frac{\partial}{\partial \dot{\delta}_W}\begin{pmatrix} f_1 \\ f_2 \\ f_3 \end{pmatrix} = -\frac{\partial (MSF)}{\partial \dot{\delta}_W}$$

$$\frac{\partial}{\partial \dot{\delta}_{HT}}\begin{pmatrix} f_1 \\ f_2 \\ f_3 \end{pmatrix} = -\frac{\partial (MSF)}{\partial \dot{\delta}_{HT}}$$

The remaining state derivatives are all null vectors.

$$\frac{\partial}{\partial \dot{\phi}}\begin{pmatrix} f_1 \\ f_2 \\ f_3 \end{pmatrix} = \frac{\partial}{\partial \dot{\theta}}\begin{pmatrix} f_1 \\ f_2 \\ f_3 \end{pmatrix} = \frac{\partial}{\partial \dot{\psi}}\begin{pmatrix} f_1 \\ f_2 \\ f_3 \end{pmatrix} = \frac{\partial}{\partial \dot{h}}\begin{pmatrix} f_1 \\ f_2 \\ f_3 \end{pmatrix} = \frac{\partial}{\partial \dot{\delta}_i}\begin{pmatrix} f_1 \\ f_2 \\ f_3 \end{pmatrix} = \frac{\partial}{\partial \left(\dot{\omega_j^2}\right)}\begin{pmatrix} f_1 \\ f_2 \\ f_3 \end{pmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}$$

Wherein $\dot{\delta}_i$ represents derivatives with respective to elevator, rudder and ailerons actuators, and $\left(\dot{\omega_j^2}\right)$ represents derivatives of every rotor.

Next we make the same procedure to the 4th, 5th and 6th rows of the Jacobian matrix E, which are the first partial derivatives of the angular motion equations with respect to the state derivative vector.

Remembering, the angular motion equations can be written in the following way,

$$\begin{pmatrix} f_4 \\ f_5 \\ f_6 \end{pmatrix} = -\begin{Bmatrix} \dot{P}_W \\ \dot{Q}_W \\ \dot{R}_W \end{Bmatrix} - \left(A_W^{-1} D_W + A_W^{-1} SC\dot{S}^T\right)\begin{Bmatrix} V_T \\ 0 \\ 0 \end{Bmatrix} - \left(\Omega_R + A_W^{-1} B_W\right)\begin{Bmatrix} P_W \\ Q_W \\ R_W \end{Bmatrix} + SA^{-1}S^T\begin{Bmatrix} \bar{L} \\ M \\ N \end{Bmatrix}$$
$$+ SA^{-1}\sum_{j=1}^{NR}\left(R_{R_j}^B\begin{Bmatrix} \lambda_j k_Q \omega_j^2 \\ 0 \\ 0 \end{Bmatrix} + \vec{r}_{R_j}^B \times R_{R_j}^B\begin{Bmatrix} k_T \omega_j^2 \\ 0 \\ 0 \end{Bmatrix}\right) + A_W^{-1}\left(M_{P_W} - E_W\right)$$
$$- A_W^{-1} C_W\begin{Bmatrix} \dot{V}_T \\ 0 \\ 0 \end{Bmatrix}$$

Follows the derivatives with respect to the state derivative vector,

$$\frac{\partial}{\partial \dot{V}_T}\begin{pmatrix} f_4 \\ f_5 \\ f_6 \end{pmatrix} = -SA^{-1}CS^T\begin{Bmatrix} 1 \\ 0 \\ 0 \end{Bmatrix}$$

$$\frac{\partial}{\partial \dot{\beta}}\begin{pmatrix} f_4 \\ f_5 \\ f_6 \end{pmatrix} = -SA^{-1}C\frac{\partial \dot{S}^T}{\partial \dot{\beta}}\begin{Bmatrix} V_T \\ 0 \\ 0 \end{Bmatrix} - \frac{\partial \Omega_R}{\partial \dot{\beta}}\begin{Bmatrix} P_W \\ Q_W \\ R_W \end{Bmatrix} + SA^{-1}S^T\begin{Bmatrix} \bar{L}_{\dot{\beta}} \\ 0 \\ N_{\dot{\beta}} \end{Bmatrix}$$

$$\frac{\partial}{\partial\dot{\alpha}}\begin{pmatrix}f_4\\f_5\\f_6\end{pmatrix}=-SA^{-1}C\frac{\partial\dot{S}^T}{\partial\dot{\alpha}}\begin{Bmatrix}V_T\\0\\0\end{Bmatrix}-\frac{\partial\Omega_R}{\partial\dot{\alpha}}\begin{Bmatrix}P_W\\Q_W\\R_W\end{Bmatrix}+SA^{-1}S^T\begin{Bmatrix}0\\M_{\dot{\alpha}}\\0\end{Bmatrix}$$

$$\frac{\partial}{\partial\dot{P}_W}\begin{pmatrix}f_4\\f_5\\f_6\end{pmatrix}=-\begin{Bmatrix}1\\0\\0\end{Bmatrix}$$

$$\frac{\partial}{\partial\dot{Q}_W}\begin{pmatrix}f_4\\f_5\\f_6\end{pmatrix}=-\begin{Bmatrix}0\\1\\0\end{Bmatrix}$$

$$\frac{\partial}{\partial\dot{R}_W}\begin{pmatrix}f_4\\f_5\\f_6\end{pmatrix}=-\begin{Bmatrix}0\\0\\1\end{Bmatrix}$$

$$\frac{\partial}{\partial\dot{\delta}_W}\begin{pmatrix}f_4\\f_5\\f_6\end{pmatrix}=-SA^{-1}\frac{\partial D}{\partial\dot{\delta}_W}S^T\begin{Bmatrix}V_T\\0\\0\end{Bmatrix}-SA^{-1}\frac{\partial B}{\partial\dot{\delta}_W}S^T\begin{Bmatrix}P_W\\Q_W\\R_W\end{Bmatrix}-SA^{-1}\frac{\partial E}{\partial\dot{\delta}_W}$$

$$\frac{\partial}{\partial\ddot{\delta}_W}\begin{pmatrix}f_4\\f_5\\f_6\end{pmatrix}=-SA^{-1}\frac{\partial E}{\partial\ddot{\delta}_W}$$

$$\frac{\partial}{\partial\dot{\delta}_{HT}}\begin{pmatrix}f_4\\f_5\\f_6\end{pmatrix}=-SA^{-1}\frac{\partial D}{\partial\dot{\delta}_{HT}}S^T\begin{Bmatrix}V_T\\0\\0\end{Bmatrix}-SA^{-1}\frac{\partial B}{\partial\dot{\delta}_{HT}}S^T\begin{Bmatrix}P_W\\Q_W\\R_W\end{Bmatrix}-SA^{-1}\frac{\partial E}{\partial\dot{\delta}_{HT}}$$

$$\frac{\partial}{\partial\ddot{\delta}_{HT}}\begin{pmatrix}f_4\\f_5\\f_6\end{pmatrix}=-SA^{-1}\frac{\partial E}{\partial\ddot{\delta}_{HT}}$$

$$\frac{\partial}{\partial\left(\dot{\omega_J^2}\right)}\begin{pmatrix}f_4\\f_5\\f_6\end{pmatrix}=-SA^{-1}\frac{\partial E}{\partial\left(\dot{\omega_J^2}\right)}$$

The remaining state derivatives are all null vectors.

$$\frac{\partial}{\partial\dot{\phi}}\begin{pmatrix}f_4\\f_5\\f_6\end{pmatrix}=\frac{\partial}{\partial\dot{\theta}}\begin{pmatrix}f_4\\f_5\\f_6\end{pmatrix}=\frac{\partial}{\partial\dot{\psi}}\begin{pmatrix}f_4\\f_5\\f_6\end{pmatrix}=\frac{\partial}{\partial\dot{h}}\begin{pmatrix}f_4\\f_5\\f_6\end{pmatrix}=\frac{\partial}{\partial\dot{\delta}_i}\begin{pmatrix}f_4\\f_5\\f_6\end{pmatrix}=\begin{Bmatrix}0\\0\\0\end{Bmatrix}$$

It is worth noting that $\dot{\delta}_i$ represents derivatives in relation to elevator, rudder and ailerons actuators.

Now for the 7th, 8th and 9th rows of the Jacobian matrix E, we have the first partial derivatives of the attitude propagation equations with respect to the state derivative vector.

The attitude propagation equations can be written as follows,

$$\begin{pmatrix} f_7 \\ f_8 \\ f_9 \end{pmatrix} = -\begin{Bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{Bmatrix} + \begin{bmatrix} 1 & \tan\theta\sin\phi & \tan\theta\cos\phi \\ 0 & \cos\phi & -\sin\phi \\ 0 & \dfrac{\sin\phi}{\cos\theta} & \dfrac{\cos\phi}{\cos\theta} \end{bmatrix} S^T \begin{Bmatrix} P_W \\ Q_W \\ R_W \end{Bmatrix}$$

Follows the derivatives with respect to the state derivative vector,

$$\frac{\partial}{\partial\dot{\phi}}\begin{pmatrix} f_7 \\ f_8 \\ f_9 \end{pmatrix} = -\begin{Bmatrix} 1 \\ 0 \\ 0 \end{Bmatrix}$$

$$\frac{\partial}{\partial\dot{\theta}}\begin{pmatrix} f_7 \\ f_8 \\ f_9 \end{pmatrix} = -\begin{Bmatrix} 0 \\ 1 \\ 0 \end{Bmatrix}$$

$$\frac{\partial}{\partial\dot{\psi}}\begin{pmatrix} f_7 \\ f_8 \\ f_9 \end{pmatrix} = -\begin{Bmatrix} 0 \\ 0 \\ 1 \end{Bmatrix}$$

All the others derivatives are null vectors,

$$\frac{\partial}{\partial\dot{V}_T}\begin{pmatrix} f_7 \\ f_8 \\ f_9 \end{pmatrix} = \frac{\partial}{\partial\dot{\beta}}\begin{pmatrix} f_7 \\ f_8 \\ f_9 \end{pmatrix} = \frac{\partial}{\partial\dot{\alpha}}\begin{pmatrix} f_7 \\ f_8 \\ f_9 \end{pmatrix} = \frac{\partial}{\partial\dot{P}_W}\begin{pmatrix} f_7 \\ f_8 \\ f_9 \end{pmatrix} = \frac{\partial}{\partial\dot{Q}_W}\begin{pmatrix} f_7 \\ f_8 \\ f_9 \end{pmatrix} = \frac{\partial}{\partial\dot{R}_W}\begin{pmatrix} f_7 \\ f_8 \\ f_9 \end{pmatrix} = \frac{\partial}{\partial\dot{h}}\begin{pmatrix} f_7 \\ f_8 \\ f_9 \end{pmatrix}$$
$$= \frac{\partial}{\partial\dot{\delta}_i}\begin{pmatrix} f_7 \\ f_8 \\ f_9 \end{pmatrix} = \frac{\partial}{\partial\dot{\left(\omega_j^2\right)}}\begin{pmatrix} f_7 \\ f_8 \\ f_9 \end{pmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}$$

Where $\dot{\delta}_i$ represents derivatives in relation to all actuators, and $\dot{\left(\omega_j^2\right)}$ represents derivatives for every rotor.

Following, the first partial derivatives of the rate of climb equation with respect to the state derivative vector, or the 10^{th} row of the Jacobian matrix E.

Remembering the rate of climb equation, is written in the following way,

$$f_{10} = -\dot{h} + V_T[\cos\alpha\cos\beta\sin\theta - (\sin\phi\sin\beta + \cos\phi\sin\alpha\cos\beta)\cos\theta]$$

And the derivatives with respect to the state derivative vector,

$$\frac{\partial f_{10}}{\partial \dot{h}} = -1$$

All the others derivatives are null vectors.

$$\frac{\partial f_{10}}{\partial \dot{V}_T} = \frac{\partial f_{10}}{\partial \dot{\beta}} = \frac{\partial f_{10}}{\partial \dot{\alpha}} = \frac{\partial f_{10}}{\partial \dot{P}_W} = \frac{\partial f_{10}}{\partial \dot{Q}_W} = \frac{\partial f_{10}}{\partial \dot{R}_W} = \frac{\partial f_{10}}{\partial \dot{\phi}} = \frac{\partial f_{10}}{\partial \dot{\theta}} = \frac{\partial f_{10}}{\partial \dot{\psi}} = \frac{\partial f_{10}}{\partial \dot{\delta}_i} = \frac{\partial f_{10}}{\partial \dot{\left(\omega_j^2\right)}} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}$$

Remembering that $\dot{\delta}_i$ represents derivatives with respective to every actuator, and $\dot{\left(\omega_j^2\right)}$ represents derivatives for every rotor.

The 11th to the 15th rows of the jacobian matrix E refer to the actuators equations dynamics, respectively to the deflection angles of flap (δ_f), elevator (δ_e), rudder (δ_r), left aileron (δ_{a_L}), right aileron (δ_{a_R}), therefore we proceed with the derivatives. Remembering the actuators dynamic equations, written as,

$$f_{11-15} = -\dot{\delta}_i + \lambda_{a_i}\left(K_{a_i}{\delta^c}_i - \delta_i\right)$$

The only not null derivatives with respect to the state derivatives vector the following,

$$\frac{\partial f_{11}}{\partial \dot{\delta}_f} = \frac{\partial f_{12}}{\partial \dot{\delta}_e} = \frac{\partial f_{13}}{\partial \dot{\delta}_r} = \frac{\partial f_{14}}{\partial \dot{\delta}_{a_L}} = \frac{\partial f_{15}}{\partial \dot{\delta}_{a_R}} = -1$$

And the remaining null derivatives,

$$\frac{\partial f_{11-15}}{\partial \dot{V}_T} = \frac{\partial f_{11-15}}{\partial \dot{\beta}} = \frac{\partial f_{11-15}}{\partial \dot{\alpha}} = \frac{\partial f_{11-15}}{\partial \dot{P}_W} = \frac{\partial f_{11-15}}{\partial \dot{Q}_W} = \frac{\partial f_{11-15}}{\partial \dot{R}_W} = \frac{\partial f_{11-15}}{\partial \dot{\phi}} = \frac{\partial f_{11-15}}{\partial \dot{\theta}}$$
$$= \frac{\partial f_{11-15}}{\partial \dot{\psi}} = \frac{\partial f_{11-15}}{\partial \dot{h}} = \frac{\partial f_{11-15}}{\partial \dot{\left(\omega_j^2\right)}} = 0$$

The 16th to 19th rows refers to wing and horizontal tail dynamics, therefore we have,

$$\begin{pmatrix} f_{16} \\ f_{17} \end{pmatrix} = -\begin{Bmatrix} \dot{x}_{1_W} \\ \dot{x}_{2_W} \end{Bmatrix} + \begin{bmatrix} 0 & 1 \\ 0 & P_{1_W} \end{bmatrix} \begin{Bmatrix} x_{1_W} \\ x_{2_W} \end{Bmatrix} + \begin{Bmatrix} 0 \\ P_{2_W} + M_{a_W} \end{Bmatrix}$$

Being that, remembering that the subscript 2 means second line,

$$\begin{Bmatrix} \delta_W \\ \dot{\delta}_{\mathrm{W}} \end{Bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{Bmatrix} x_{1W} \\ x_{2W} \end{Bmatrix}$$

We find the not null derivatives,

$$\frac{\partial f_{16}}{\partial \dot{x}_{1W}} = \frac{\partial f_{16}}{\partial \dot{\delta}_{\mathrm{W}}} = -1$$

$$\frac{\partial f_{17}}{\partial \dot{V}_T} = \frac{\partial P_{2W}}{\partial \dot{V}_T} = -\left[(A)_P^{-1}{}_W (C)_{PW} S^T \begin{Bmatrix} 1 \\ 0 \\ 0 \end{Bmatrix} \right]_2$$

$$\frac{\partial f_{17}}{\partial \dot{\beta}} = \frac{\partial P_{2W}}{\partial \dot{\beta}} = \left[(A)_P^{-1}{}_W S^T \begin{Bmatrix} \bar{L}_{\dot{\beta}} \\ 0 \\ N_{\dot{\beta}} \end{Bmatrix}_W \right]_2$$

$$\frac{\partial f_{17}}{\partial \dot{\alpha}} = \frac{\partial P_{2W}}{\partial \dot{\alpha}} = \left[(A)_P^{-1}{}_W S^T \begin{Bmatrix} 0 \\ M_{\dot{\alpha}} \\ 0 \end{Bmatrix}_W \right]_2$$

$$\frac{\partial f_{17}}{\partial \dot{x}_{1W}} = \frac{\partial f_{17}}{\partial \dot{\delta}_{\mathrm{W}}} = P_{1W} + \frac{\partial P_{2W}}{\partial \dot{\delta}_{\mathrm{W}}}$$

$$\frac{\partial f_{17}}{\partial \dot{x}_{2W}} = \frac{\partial f_{17}}{\partial \ddot{\delta}_{\mathrm{W}}} = -1 + \frac{\partial P_{2W}}{\partial \ddot{\delta}_{\mathrm{W}}}$$

$$\frac{\partial f_{17}}{\partial \left(\dot{\omega}_j^2\right)} = \frac{\partial P_{2W}}{\partial \left(\dot{\omega}_j^2\right)} = -\left[(A)_P^{-1}{}_W \frac{\partial (E)_{PW}}{\partial \left(\dot{\omega}_j^2\right)} \right]_2 , for\ j = 1:4$$

And for the horizontal tail, the 18th and 19th equations,

$$\begin{pmatrix} f_{18} \\ f_{19} \end{pmatrix} = -\begin{Bmatrix} \dot{x}_{1HT} \\ \dot{x}_{2HT} \end{Bmatrix} + \begin{bmatrix} 0 & 1 \\ 0 & P_{1HT} \end{bmatrix} \begin{Bmatrix} x_{1HT} \\ x_{2HT} \end{Bmatrix} + \begin{Bmatrix} 0 \\ P_{2HT} + M_{aHT} \end{Bmatrix}$$

Being that,

$$\begin{Bmatrix} \delta_{HT} \\ \dot{\delta}_{\mathrm{HT}} \end{Bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{Bmatrix} x_{1HT} \\ x_{2HT} \end{Bmatrix}$$

We find the not null derivatives,

$$\frac{\partial f_{18}}{\partial \dot{x}_{1HT}} = \frac{\partial f_{18}}{\partial \dot{\delta}_{\mathrm{HT}}} = -1$$

$$\frac{\partial f_{18}}{\partial \dot{V}_T} = \frac{\partial P_{2_{HT}}}{\partial \dot{V}_T} = -\left[(A)_P^{-1}{}_{HT}(C)_{P_{HT}} S^T \begin{Bmatrix}1\\0\\0\end{Bmatrix}\right]_2$$

$$\frac{\partial f_{19}}{\partial \dot{\beta}} = \frac{\partial P_{2_{HT}}}{\partial \dot{\beta}} = \left[(A)_P^{-1}{}_{HT} S^T \begin{Bmatrix}\bar{L}_{\dot{\beta}}\\0\\N_{\dot{\beta}}\end{Bmatrix}_{HT}\right]_2$$

$$\frac{\partial f_{19}}{\partial \dot{\alpha}} = \frac{\partial P_{2_{HT}}}{\partial \dot{\alpha}} = \left[(A)_P^{-1}{}_{HT} S^T \begin{Bmatrix}0\\M_{\dot{\alpha}}\\0\end{Bmatrix}_{HT}\right]_2$$

$$\frac{\partial f_{19}}{\partial \dot{x}_{1_{HT}}} = \frac{\partial f_{18}}{\partial \dot{\delta}_{\mathrm{HT}}} = P_{1_{HT}} + \frac{\partial P_{2_W}}{\partial \dot{\delta}_{\mathrm{W}}}$$

$$\frac{\partial f_{19}}{\partial \dot{x}_{2_{HT}}} = \frac{\partial f_{18}}{\partial \ddot{\delta}_{\mathrm{HT}}} = -1 + \frac{\partial P_{2_{HT}}}{\partial \ddot{\delta}_{\mathrm{HT}}}$$

$$\frac{\partial f_{19}}{\partial \left(\dot{\omega_j^2}\right)} = \frac{\partial P_{2_{HT}}}{\partial \left(\dot{\omega_j^2}\right)} = -\left[(A)_P^{-1}{}_{HT} \frac{\partial (E)_{P_{HT}}}{\partial \left(\dot{\omega_j^2}\right)}\right]_2 , for\ j = 1{:}\,4$$

At last, the 20th to 25th rows refers to the rotors dynamics. Remembering the rotor dynamics equation,

$$f_{20-25} = -\left(\dot{\omega_j^2}\right) + \lambda_p\left(K_p\left(\omega_j^2\right)^c - \omega_j^2\right)$$

Again, the only not null derivatives are just a few,

$$\frac{\partial f_{20}}{\partial(\dot{\omega_1^2})} = \frac{\partial f_{21}}{\partial(\dot{\omega_2^2})} = \frac{\partial f_{22}}{\partial(\dot{\omega_3^2})} = \frac{\partial f_{23}}{\partial(\dot{\omega_4^2})} = \frac{\partial f_{24}}{\partial(\dot{\omega_5^2})} = \frac{\partial f_{25}}{\partial(\dot{\omega_6^2})} = -1$$

So we have completed the Jacobian E matrix.

Starting now with the assembly of the Jacobian matrix A, in which the terms are the first partial derivatives with respect to the state vector. The first three rows refer to the translational motion equations. Follows the derivatives with respect to the state vector,

$$\frac{\partial}{\partial V_T}\begin{pmatrix}f_1\\f_2\\f_3\end{pmatrix} = -M\begin{Bmatrix}0\\\dot{\beta}\\\dot{\alpha}\cos\beta\end{Bmatrix} - M\Omega_W\begin{Bmatrix}1\\0\\0\end{Bmatrix} + \begin{Bmatrix}-D_{V_T}\\Y_{V_T}\\-L_{V_T}\end{Bmatrix}$$

$$\frac{\partial}{\partial\beta}\begin{pmatrix}f_1\\f_2\\f_3\end{pmatrix} = -M\begin{Bmatrix}0\\0\\-\dot{\alpha}V_T\sin\beta\end{Bmatrix} + M\frac{\partial S}{\partial\beta}B_E^B\vec{g}^E - \frac{\partial(MSF)}{\partial\beta} + \begin{Bmatrix}-D_\beta\\Y_\beta\\0\end{Bmatrix} + \frac{\partial S}{\partial\beta}\sum_{j=1}^{NR}\left(R_{R_j}^B\begin{Bmatrix}k_T\omega_j^2\\0\\0\end{Bmatrix}\right)$$

$$\frac{\partial}{\partial\alpha}\begin{pmatrix}f_1\\f_2\\f_3\end{pmatrix} = M\frac{\partial S}{\partial\alpha}B_E^B\vec{g}^E - \frac{\partial(MSF)}{\partial\alpha} + \begin{Bmatrix}-D_\alpha\\Y_\alpha\\-L_\alpha\end{Bmatrix} + \frac{\partial S}{\partial\alpha}\sum_{j=1}^{NR}\left(R_{R_j}^B\begin{Bmatrix}k_T\omega_j^2\\0\\0\end{Bmatrix}\right)$$

$$\frac{\partial}{\partial P_W}\begin{pmatrix}f_1\\f_2\\f_3\end{pmatrix} = -M\frac{\partial\Omega_W}{\partial P_W}\begin{Bmatrix}V_T\\0\\0\end{Bmatrix} - \frac{\partial(MSF)}{\partial P_W} + \begin{Bmatrix}0\\Y_{P_W}\\0\end{Bmatrix}$$

$$\frac{\partial}{\partial Q_W}\begin{pmatrix}f_1\\f_2\\f_3\end{pmatrix} = -M\frac{\partial\Omega_W}{\partial Q_W}\begin{Bmatrix}V_T\\0\\0\end{Bmatrix} - \frac{\partial(MSF)}{\partial Q_W} + \begin{Bmatrix}-D_{Q_W}\\0\\-L_{Q_W}\end{Bmatrix}$$

$$\frac{\partial}{\partial R_W}\begin{pmatrix}f_1\\f_2\\f_3\end{pmatrix} = -M\frac{\partial\Omega_W}{\partial R_W}\begin{Bmatrix}V_T\\0\\0\end{Bmatrix} - \frac{\partial(MSF)}{\partial R_W} + \begin{Bmatrix}0\\Y_{R_W}\\0\end{Bmatrix}$$

$$\frac{\partial}{\partial\phi}\begin{pmatrix}f_1\\f_2\\f_3\end{pmatrix} = MS\frac{\partial B_E^B}{\partial\phi}\vec{g}^E$$

$$\frac{\partial}{\partial\theta}\begin{pmatrix}f_1\\f_2\\f_3\end{pmatrix} = MS\frac{\partial B_E^B}{\partial\theta}\vec{g}^E$$

$$\frac{\partial}{\partial\psi}\begin{pmatrix}f_1\\f_2\\f_3\end{pmatrix} = MS\frac{\partial B_E^B}{\partial\psi}\vec{g}^E$$

$$\frac{\partial}{\partial h}\begin{pmatrix}f_1\\f_2\\f_3\end{pmatrix} = \frac{\partial\rho}{\partial h}\frac{1}{2}V_T^2S_W\begin{Bmatrix}-C_D\\C_Y\\-C_L\end{Bmatrix} + S\sum_{j=1}^{NR}\left(R_{R_j}^B\begin{Bmatrix}\frac{\partial k_T}{\partial h}\omega_j^2\\0\\0\end{Bmatrix}\right)$$

$$\frac{\partial}{\partial\delta_f}\begin{pmatrix}f_1\\f_2\\f_3\end{pmatrix} = \begin{Bmatrix}-D_{\delta_f}\\0\\-L_{\delta_f}\end{Bmatrix}$$

$$\frac{\partial}{\partial\delta_e}\begin{pmatrix}f_1\\f_2\\f_3\end{pmatrix} = \begin{Bmatrix}-D_{\delta_e}\\0\\-L_{\delta_e}\end{Bmatrix}$$

$$\frac{\partial}{\partial \delta_r}\begin{pmatrix} f_1 \\ f_2 \\ f_3 \end{pmatrix} = \begin{Bmatrix} 0 \\ Y_{\delta_r} \\ 0 \end{Bmatrix}$$

$$\frac{\partial}{\partial \delta_{a_L}}\begin{pmatrix} f_1 \\ f_2 \\ f_3 \end{pmatrix} = \begin{Bmatrix} -D_{\delta_{a_L}} \\ 0 \\ -L_{\delta_{a_L}} \end{Bmatrix}$$

$$\frac{\partial}{\partial \delta_{a_R}}\begin{pmatrix} f_1 \\ f_2 \\ f_3 \end{pmatrix} = \begin{Bmatrix} -D_{\delta_{a_R}} \\ 0 \\ -L_{\delta_{a_R}} \end{Bmatrix}$$

$$\frac{\partial}{\partial \delta_W}\begin{pmatrix} f_1 \\ f_2 \\ f_3 \end{pmatrix} = -\frac{\partial (MSF)}{\partial \delta_W} + \begin{Bmatrix} -D_{\delta_W} \\ 0 \\ -L_{\delta_W} \end{Bmatrix} + S\sum_{j=1}^{NR}\left(\frac{\partial R_{R_j}^B}{\partial \delta_W}\begin{Bmatrix} k_T\omega_j^2 \\ 0 \\ 0 \end{Bmatrix}\right)$$

$$\frac{\partial}{\partial \dot{\delta}_W}\begin{pmatrix} f_1 \\ f_2 \\ f_3 \end{pmatrix} = -\frac{\partial (MSF)}{\partial \dot{\delta}_W}$$

$$\frac{\partial}{\partial \delta_{HT}}\begin{pmatrix} f_1 \\ f_2 \\ f_3 \end{pmatrix} = -\frac{\partial (MSF)}{\partial \delta_{HT}} + \begin{Bmatrix} -D_{\delta_{HT}} \\ 0 \\ -L_{\delta_{HT}} \end{Bmatrix} + S\sum_{j=1}^{NR}\left(\frac{\partial R_{R_j}^B}{\partial \delta_{HT}}\begin{Bmatrix} k_T\omega_j^2 \\ 0 \\ 0 \end{Bmatrix}\right)$$

$$\frac{\partial}{\partial \dot{\delta}_{HT}}\begin{pmatrix} f_1 \\ f_2 \\ f_3 \end{pmatrix} = -\frac{\partial (MSF)}{\partial \dot{\delta}_{HT}}$$

$$\frac{\partial}{\partial \omega_j^2}\begin{pmatrix} f_1 \\ f_2 \\ f_3 \end{pmatrix} = \begin{Bmatrix} -D_{\omega_j^2} \\ 0 \\ -L_{\omega_j^2} \end{Bmatrix} + SR_{R_j}^B k_T \begin{Bmatrix} 1 \\ 0 \\ 0 \end{Bmatrix}$$

Next we proceed with the 4th, 5th and 6th rows of the Jacobian matrix A, with derivation with respect to the angular motion equations.

$$\frac{\partial}{\partial V_T}\begin{pmatrix} f_4 \\ f_5 \\ f_6 \end{pmatrix} = -\left(SA^{-1}DS^T + SA^{-1}C\dot{S}^T\right)\begin{Bmatrix} 1 \\ 0 \\ 0 \end{Bmatrix} + SA^{-1}S^T\begin{Bmatrix} \bar{L}_{V_T} \\ M_{V_T} \\ N_{V_T} \end{Bmatrix}$$

$$\frac{\partial}{\partial\,\beta}\begin{pmatrix} f_4 \\ f_5 \\ f_6 \end{pmatrix} = -\left(\frac{\partial S}{\partial \beta}A^{-1}DS^T + SA^{-1}D\frac{\partial S^T}{\partial\,\beta} + \frac{\partial S}{\partial \beta}A^{-1}C\dot{S}^T + SA^{-1}C\frac{\partial \dot{S}^T}{\partial \beta}\right)\begin{Bmatrix} V_T \\ 0 \\ 0 \end{Bmatrix}$$

$$-\left(\frac{\partial \Omega_R}{\partial\,\beta} + \frac{\partial S}{\partial \beta}A^{-1}BS^T + SA^{-1}B\frac{\partial S^T}{\partial\,\beta}\right)\begin{Bmatrix} P_W \\ Q_W \\ R_W \end{Bmatrix} + \left(\frac{\partial S}{\partial \beta}A^{-1}S^T + SA^{-1}\frac{\partial S^T}{\partial\,\beta}\right)\begin{Bmatrix} \bar{L} \\ M \\ N \end{Bmatrix}$$

$$+ SA^{-1}S^T\begin{Bmatrix} \bar{L}_\beta \\ M_\beta \\ N_\beta \end{Bmatrix} + \frac{\partial S}{\partial\,\beta}A^{-1}\sum_{j=1}^{NR}\left(R_{R_j}^B k_Q\begin{Bmatrix} \lambda_j\omega_j^2 \\ 0 \\ 0 \end{Bmatrix} + \vec{r}_{R_j/B} \times R_{R_j}^B k_T\begin{Bmatrix} \omega_j^2 \\ 0 \\ 0 \end{Bmatrix}\right)$$

$$+ \frac{\partial S}{\partial\,\beta}A^{-1}(M_P - E) - \left(\frac{\partial S}{\partial\,\beta}A^{-1}CS^T + SA^{-1}C\frac{\partial S^T}{\partial\,\beta}\right)\begin{Bmatrix} \dot{V}_T \\ 0 \\ 0 \end{Bmatrix}$$

$$\frac{\partial}{\partial\,\alpha}\begin{pmatrix} f_4 \\ f_5 \\ f_6 \end{pmatrix} = -\left(\frac{\partial S}{\partial\,\alpha}A^{-1}DS^T + SA^{-1}D\frac{\partial S^T}{\partial\,\alpha} + \frac{\partial S}{\partial\,\alpha}A^{-1}C\dot{S}^T + SA^{-1}C\frac{\partial \dot{S}^T}{\partial\,\alpha}\right)\begin{Bmatrix} V_T \\ 0 \\ 0 \end{Bmatrix}$$

$$-\left(\frac{\partial S}{\partial\,\alpha}A^{-1}BS^T + SA^{-1}B\frac{\partial S^T}{\partial\,\alpha}\right)\begin{Bmatrix} P_W \\ Q_W \\ R_W \end{Bmatrix} + \left(\frac{\partial S}{\partial\,\alpha}A^{-1}S^T + SA^{-1}\frac{\partial S^T}{\partial\,\alpha}\right)\begin{Bmatrix} \bar{L} \\ M \\ N \end{Bmatrix}$$

$$+ SA^{-1}S^T\begin{Bmatrix} 0 \\ M_\alpha \\ 0 \end{Bmatrix} + \frac{\partial S}{\partial\,\alpha}A^{-1}\sum_{j=1}^{NR}\left(R_{R_j}^B\begin{Bmatrix} \lambda_j k_Q\omega_j^2 \\ 0 \\ 0 \end{Bmatrix} + \vec{r}_{R_j/B} \times R_{R_j}^B\begin{Bmatrix} k_T\omega_j^2 \\ 0 \\ 0 \end{Bmatrix}\right)$$

$$+ \frac{\partial S}{\partial\,\alpha}A^{-1}(M_P - E) - \left(\frac{\partial S}{\partial\,\alpha}A^{-1}CS^T + SA^{-1}C\frac{\partial S^T}{\partial\,\alpha}\right)\begin{Bmatrix} \dot{V}_T \\ 0 \\ 0 \end{Bmatrix}$$

$$\frac{\partial}{\partial P_W}\begin{pmatrix} f_4 \\ f_5 \\ f_6 \end{pmatrix} = -SA^{-1}\frac{\partial D}{\partial P_W}S^T\begin{Bmatrix} V_T \\ 0 \\ 0 \end{Bmatrix} - (\Omega_R + SA^{-1}BS^T)\begin{Bmatrix} 1 \\ 0 \\ 0 \end{Bmatrix} - SA^{-1}\frac{\partial B}{\partial\,P_W}S^T\begin{Bmatrix} P_W \\ Q_W \\ R_W \end{Bmatrix}$$

$$+ SA^{-1}S^T\begin{Bmatrix} \bar{L}_{P_W} \\ 0 \\ N_{P_W} \end{Bmatrix} + SA^{-1}\frac{\partial(M_P - E)}{\partial P_W}$$

$$\frac{\partial}{\partial Q_W}\begin{pmatrix} f_4 \\ f_5 \\ f_6 \end{pmatrix} = -SA^{-1}\frac{\partial D}{\partial Q_W}S^T\begin{Bmatrix} V_T \\ 0 \\ 0 \end{Bmatrix} - (\Omega_R + SA^{-1}BS^T)\begin{Bmatrix} 0 \\ 1 \\ 0 \end{Bmatrix} - SA^{-1}\frac{\partial B}{\partial\,Q_W}S^T\begin{Bmatrix} P_W \\ Q_W \\ R_W \end{Bmatrix}$$

$$+ SA^{-1}S^T\begin{Bmatrix} 0 \\ M_{Q_W} \\ 0 \end{Bmatrix} + SA^{-1}\frac{\partial(M_P - E)}{\partial Q_W}$$

$$\frac{\partial}{\partial R_W}\begin{pmatrix} f_4 \\ f_5 \\ f_6 \end{pmatrix} = -SA^{-1}\frac{\partial D}{\partial R_W}S^T\begin{Bmatrix} V_T \\ 0 \\ 0 \end{Bmatrix} - (\Omega_R + SA^{-1}BS^T)\begin{Bmatrix} 0 \\ 0 \\ 1 \end{Bmatrix} - SA^{-1}\frac{\partial B}{\partial R_W}S^T\begin{Bmatrix} P_W \\ Q_W \\ R_W \end{Bmatrix}$$

$$+ SA^{-1}S^T\begin{Bmatrix} \bar{L}_{R_W} \\ 0 \\ N_{R_W} \end{Bmatrix} + SA^{-1}\frac{\partial(M_P - E)}{\partial R_W}$$

$$\frac{\partial}{\partial \phi}\begin{pmatrix} f_4 \\ f_5 \\ f_6 \end{pmatrix} = SA^{-1}\frac{\partial(M_P - E)}{\partial \phi}$$

$$\frac{\partial}{\partial \theta}\begin{pmatrix} f_4 \\ f_5 \\ f_6 \end{pmatrix} = SA^{-1}\frac{\partial(M_P - E)}{\partial \theta}$$

$$\frac{\partial}{\partial \psi}\begin{pmatrix} f_4 \\ f_5 \\ f_6 \end{pmatrix} = SA^{-1}\frac{\partial(M_P - E)}{\partial \psi}$$

$$\frac{\partial}{\partial h}\begin{pmatrix} f_4 \\ f_5 \\ f_6 \end{pmatrix} = SA^{-1}S^T\frac{\partial \rho}{\partial h}\frac{1}{2}V_T^2 S_W\begin{Bmatrix} bC_l \\ \bar{c}C_m \\ bC_n \end{Bmatrix}$$

$$+ SA^{-1}\sum_{j=1}^{NR}\left(R_{R_j}^B\begin{Bmatrix} \lambda_j \frac{\partial k_Q}{\partial h}\omega_j^2 \\ 0 \\ 0 \end{Bmatrix} + \vec{r}_{R_j/B} \times R_{R_j}^B\begin{Bmatrix} \frac{\partial k_T}{\partial h}\omega_j^2 \\ 0 \\ 0 \end{Bmatrix}\right)$$

$$\frac{\partial}{\partial \delta_f}\begin{pmatrix} f_4 \\ f_5 \\ f_6 \end{pmatrix} = SA^{-1}S^T\begin{Bmatrix} 0 \\ M_{\delta_f} \\ 0 \end{Bmatrix}$$

$$\frac{\partial}{\partial \delta_e}\begin{pmatrix} f_4 \\ f_5 \\ f_6 \end{pmatrix} = SA^{-1}S^T\begin{Bmatrix} 0 \\ M_{\delta_e} \\ 0 \end{Bmatrix}$$

$$\frac{\partial}{\partial \delta_r}\begin{pmatrix} f_4 \\ f_5 \\ f_6 \end{pmatrix} = SA^{-1}S^T\begin{Bmatrix} \bar{L}_{\delta_r} \\ 0 \\ N_{\delta_r} \end{Bmatrix}$$

$$\frac{\partial}{\partial \delta_{a_L}}\begin{pmatrix} f_4 \\ f_5 \\ f_6 \end{pmatrix} = SA^{-1}S^T\begin{Bmatrix} \bar{L}_{\delta_{a_L}} \\ 0 \\ N_{\delta_{a_L}} \end{Bmatrix}$$

$$\frac{\partial}{\partial \delta_{a_R}}\begin{pmatrix} f_4 \\ f_5 \\ f_6 \end{pmatrix} = SA^{-1}S^T \begin{Bmatrix} \bar{L}_{\delta_{a_R}} \\ 0 \\ N_{\delta_{a_R}} \end{Bmatrix}$$

$$\begin{aligned}
\frac{\partial}{\partial \delta_W}\begin{pmatrix} f_4 \\ f_5 \\ f_6 \end{pmatrix} = &-\left(S\frac{\partial A^{-1}}{\partial\, \delta_W}DS^T + SA^{-1}\frac{\partial D}{\partial\, \delta_W}S^T + S\frac{\partial A^{-1}}{\partial\, \delta_W}C\dot{S}^T\right)\begin{Bmatrix} V_T \\ 0 \\ 0 \end{Bmatrix} \\
&-\left(S\frac{\partial A^{-1}}{\partial\, \delta_W}BS^T + SA^{-1}\frac{\partial B}{\partial\, \delta_W}S^T\right)\begin{Bmatrix} P_W \\ Q_W \\ R_W \end{Bmatrix} + S\frac{\partial A^{-1}}{\partial\, \delta_W}S^T\begin{Bmatrix} \bar{L} \\ M \\ N \end{Bmatrix} + SA^{-1}S^T\begin{Bmatrix} 0 \\ M_{\delta_W} \\ 0 \end{Bmatrix} \\
&+ S\frac{\partial A^{-1}}{\partial\, \delta_W}\sum_{j=1}^{NR}\left(R_{R_j}^B k_Q\begin{Bmatrix} \lambda_j\omega_j^2 \\ 0 \\ 0 \end{Bmatrix} + \vec{r}_{R_j/B} \times R_{R_j}^B k_T\begin{Bmatrix} \omega_j^2 \\ 0 \\ 0 \end{Bmatrix}\right) \\
&+ SA^{-1}\sum_{j=1}^{NR}\left(\frac{\partial R_{R_j}^B}{\partial\, \delta_W}k_Q\begin{Bmatrix} \lambda_j\omega_j^2 \\ 0 \\ 0 \end{Bmatrix} + \left[\frac{\partial\left(\vec{r}_{R_j/B}\times\right)}{\partial\, \delta_W}R_{R_j}^B + \vec{r}_{R_j/B} \times \frac{\partial R_{R_j}^B}{\partial\, \delta_W}\right]k_T\begin{Bmatrix} \omega_j^2 \\ 0 \\ 0 \end{Bmatrix}\right) \\
&+ S\frac{\partial A^{-1}}{\partial\, \delta_W}M_P + SA^{-1}\frac{\partial M_P}{\partial \delta_W} - S\frac{\partial A^{-1}}{\partial\, \delta_W}E - SA^{-1}\frac{\partial E}{\partial \delta_W} \\
&-\left(S\frac{\partial A^{-1}}{\partial \delta_W}CS^T + SA^{-1}\frac{\partial C}{\partial \delta_W}S^T\right)\begin{Bmatrix} \dot{V}_T \\ 0 \\ 0 \end{Bmatrix}
\end{aligned}$$

$$\frac{\partial}{\partial \dot{\delta}_W}\begin{pmatrix} f_4 \\ f_5 \\ f_6 \end{pmatrix} = -SA^{-1}\frac{\partial D}{\partial \dot{\delta}_W}S^T\begin{Bmatrix} V_T \\ 0 \\ 0 \end{Bmatrix} - SA^{-1}\frac{\partial B}{\partial \dot{\delta}_W}S^T\begin{Bmatrix} P_W \\ Q_W \\ R_W \end{Bmatrix} - SA^{-1}\frac{\partial E}{\partial \dot{\delta}_W}$$

$$\frac{\partial}{\partial \delta_{HT}}\begin{pmatrix} f_4 \\ f_5 \\ f_6 \end{pmatrix} = -\left(S\frac{\partial A^{-1}}{\partial\, \delta_{HT}} D S^T + S A^{-1} \frac{\partial D}{\partial\, \delta_{HT}} S^T + S\frac{\partial A^{-1}}{\partial\, \delta_{HT}} C \dot{S}^T \right)\begin{Bmatrix} V_T \\ 0 \\ 0 \end{Bmatrix}$$

$$- \left(S\frac{\partial A^{-1}}{\partial\, \delta_{HT}} B S^T + S A^{-1} \frac{\partial B}{\partial\, \delta_{HT}} S^T \right)\begin{Bmatrix} P_W \\ Q_W \\ R_W \end{Bmatrix} + S\frac{\partial A^{-1}}{\partial\, \delta_{HT}} S^T \begin{Bmatrix} \bar{L} \\ M \\ N \end{Bmatrix} + S A^{-1} S^T \begin{Bmatrix} 0 \\ M_{\delta_{HT}} \\ 0 \end{Bmatrix}$$

$$+ S\frac{\partial A^{-1}}{\partial\, \delta_{HT}} \sum_{j=1}^{NR} \left(R_{R_j}^B k_Q \begin{Bmatrix} \lambda_j \omega_j^2 \\ 0 \\ 0 \end{Bmatrix} + \vec{r}_{R_j/B} \times R_{R_j}^B k_T \begin{Bmatrix} \omega_j^2 \\ 0 \\ 0 \end{Bmatrix} \right)$$

$$+ S A^{-1} \sum_{j=1}^{NR} \left(\frac{\partial R_{R_j}^B}{\partial\, \delta_{HT}} k_Q \begin{Bmatrix} \lambda_j \omega_j^2 \\ 0 \\ 0 \end{Bmatrix} \right.$$

$$\left. + \left[\frac{\partial \left(\vec{r}_{R_j/B} \times \right)}{\partial\, \delta_{HT}} R_{R_j}^B + \vec{r}_{R_j/B} \times \frac{\partial R_{R_j}^B}{\partial\, \delta_{HT}} \right] k_T \begin{Bmatrix} \omega_j^2 \\ 0 \\ 0 \end{Bmatrix} \right) + S\frac{\partial A^{-1}}{\partial\, \delta_{HT}} M_P + S A^{-1} \frac{\partial M_P}{\partial \delta_{HT}}$$

$$- S\frac{\partial A^{-1}}{\partial\, \delta_{HT}} E - S A^{-1} \frac{\partial E}{\partial \delta_{HT}} - \left(S\frac{\partial A^{-1}}{\partial \delta_{HT}} C S^T + S A^{-1} \frac{\partial C}{\partial \delta_{HT}} S^T \right)\begin{Bmatrix} \dot{V}_T \\ 0 \\ 0 \end{Bmatrix}$$

$$\frac{\partial}{\partial \dot{\delta}_{HT}}\begin{pmatrix} f_4 \\ f_5 \\ f_6 \end{pmatrix} = -S A^{-1} \frac{\partial D}{\partial \dot{\delta}_{HT}} S^T \begin{Bmatrix} V_T \\ 0 \\ 0 \end{Bmatrix} - S A^{-1} \frac{\partial B}{\partial \dot{\delta}_{HT}} S^T \begin{Bmatrix} P_W \\ Q_W \\ R_W \end{Bmatrix} - S A^{-1} \frac{\partial E}{\partial \dot{\delta}_{HT}}$$

$$\frac{\partial}{\partial\, \omega_j^2}\begin{pmatrix} f_4 \\ f_5 \\ f_6 \end{pmatrix} = S A^{-1} S^T \begin{Bmatrix} \bar{L}_{\omega_j^2} \\ M_{\omega_j^2} \\ N_{\omega_j^2} \end{Bmatrix} + S A^{-1} \left(R_{R_j}^B k_Q \begin{Bmatrix} \lambda_j \\ 0 \\ 0 \end{Bmatrix} + \vec{r}_{R_j/B} \times R_{R_j}^B k_T \begin{Bmatrix} 1 \\ 0 \\ 0 \end{Bmatrix} \right) - S A^{-1} \frac{\partial \mathrm{E}}{\partial\, \omega_j^2}$$

And now the 7th, 8th and 9th rows terms of the Jacobian matrix A, associated with the attitude propagation equations.

$$\frac{\partial}{\partial \beta}\begin{pmatrix} f_7 \\ f_8 \\ f_9 \end{pmatrix} = \begin{bmatrix} 1 & \tan\theta \sin\phi & \tan\theta\cos\phi \\ 0 & \cos\phi & -\sin\phi \\ 0 & \dfrac{\sin\phi}{\cos\theta} & \dfrac{\cos\phi}{\cos\theta} \end{bmatrix} \frac{\partial S^T}{\partial \beta} \begin{Bmatrix} P_W \\ Q_W \\ R_W \end{Bmatrix}$$

$$\frac{\partial}{\partial \alpha}\begin{pmatrix} f_7 \\ f_8 \\ f_9 \end{pmatrix} = \begin{bmatrix} 1 & \tan\theta \sin\phi & \tan\theta\cos\phi \\ 0 & \cos\phi & -\sin\phi \\ 0 & \dfrac{\sin\phi}{\cos\theta} & \dfrac{\cos\phi}{\cos\theta} \end{bmatrix} \frac{\partial S^T}{\partial \alpha} \begin{Bmatrix} P_W \\ Q_W \\ R_W \end{Bmatrix}$$

$$\frac{\partial}{\partial P_W}\begin{pmatrix} f_7 \\ f_8 \\ f_9 \end{pmatrix} = \begin{bmatrix} 1 & \tan\theta\sin\phi & \tan\theta\cos\phi \\ 0 & \cos\phi & -\sin\phi \\ 0 & \dfrac{\sin\phi}{\cos\theta} & \dfrac{\cos\phi}{\cos\theta} \end{bmatrix} S^T \begin{Bmatrix} 1 \\ 0 \\ 0 \end{Bmatrix}$$

$$\frac{\partial}{\partial Q_W}\begin{pmatrix} f_7 \\ f_8 \\ f_9 \end{pmatrix} = \begin{bmatrix} 1 & \tan\theta\sin\phi & \tan\theta\cos\phi \\ 0 & \cos\phi & -\sin\phi \\ 0 & \dfrac{\sin\phi}{\cos\theta} & \dfrac{\cos\phi}{\cos\theta} \end{bmatrix} S^T \begin{Bmatrix} 0 \\ 1 \\ 0 \end{Bmatrix}$$

$$\frac{\partial}{\partial R_W}\begin{pmatrix} f_7 \\ f_8 \\ f_9 \end{pmatrix} = \begin{bmatrix} 1 & \tan\theta\sin\phi & \tan\theta\cos\phi \\ 0 & \cos\phi & -\sin\phi \\ 0 & \dfrac{\sin\phi}{\cos\theta} & \dfrac{\cos\phi}{\cos\theta} \end{bmatrix} S^T \begin{Bmatrix} 0 \\ 0 \\ 1 \end{Bmatrix}$$

$$\frac{\partial}{\partial \phi}\begin{pmatrix} f_7 \\ f_8 \\ f_9 \end{pmatrix} = \begin{bmatrix} 0 & \tan\theta\cos\phi & -\tan\theta\sin\phi \\ 0 & -\sin\phi & -\cos\phi \\ 0 & \dfrac{\cos\phi}{\cos\theta} & -\dfrac{\sin\phi}{\cos\theta} \end{bmatrix} S^T \begin{Bmatrix} P_W \\ Q_W \\ R_W \end{Bmatrix}$$

$$\frac{\partial}{\partial \theta}\begin{pmatrix} f_7 \\ f_8 \\ f_9 \end{pmatrix} = \begin{bmatrix} 0 & \dfrac{\sin\phi}{(\cos\theta)^2} & \dfrac{\cos\phi}{(\cos\theta)^2} \\ 0 & 0 & 0 \\ 0 & \sin\phi\dfrac{\sin\theta}{(\cos\theta)^2} & \cos\phi\dfrac{\sin\theta}{(\cos\theta)^2} \end{bmatrix} S^T \begin{Bmatrix} P_W \\ Q_W \\ R_W \end{Bmatrix}$$

The remaining derivatives are null vectors.

$$\frac{\partial}{\partial V_T}\begin{pmatrix} f_7 \\ f_8 \\ f_9 \end{pmatrix} = \frac{\partial}{\partial \psi}\begin{pmatrix} f_7 \\ f_8 \\ f_9 \end{pmatrix} = \frac{\partial}{\partial h}\begin{pmatrix} f_7 \\ f_8 \\ f_9 \end{pmatrix} = \frac{\partial}{\partial \delta_i}\begin{pmatrix} f_7 \\ f_8 \\ f_9 \end{pmatrix} = \frac{\partial}{\partial \omega_j^2}\begin{pmatrix} f_7 \\ f_8 \\ f_9 \end{pmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}$$

Again, δ_i represents derivatives in relation to every actuator, and ω_j^2 represents derivatives for every rotor.

Now in the 10th row of the Jacobian matrix A, we have partial derivatives of the rate of climb equation with respect to the state vector.

$$\frac{\partial f_{10}}{\partial V_T} = \cos\alpha\cos\beta\sin\theta - (\sin\phi\sin\beta + \cos\phi\sin\alpha\cos\beta)\cos\theta$$

$$\frac{\partial f_{10}}{\partial \beta} = -V_T[\cos\alpha\sin\beta\sin\theta + (\sin\phi\cos\beta - \cos\phi\sin\alpha\sin\beta)\cos\theta]$$

$$\frac{\partial f_{10}}{\partial \alpha} = -V_T(\sin\alpha\cos\beta\sin\theta + \cos\phi\cos\alpha\cos\beta\cos\theta)$$

$$\frac{\partial f_{10}}{\partial \phi} = -V_T(\cos\phi\sin\beta - \sin\phi\sin\alpha\cos\beta)\cos\theta$$

$$\frac{\partial f_{10}}{\partial \theta} = V_T(\cos\alpha\cos\beta\cos\theta + \cos\phi\sin\alpha\cos\beta\sin\theta)$$

The remaining derivatives are null.

$$\frac{\partial f_{10}}{\partial P_W} = \frac{\partial f_{10}}{\partial Q_W} = \frac{\partial f_{10}}{\partial R_W} = \frac{\partial f_{10}}{\partial \psi} = \frac{\partial f_{10}}{\partial \dot{\delta}_i} = \frac{\partial f_{10}}{\partial \omega_j^2} = 0$$

And the 11th to the 15th rows of the Jacobian matrix A referring to the actuator equations dynamics, respectively to the angles of flap (δ_f), elevator (δ_e), rudder (δ_r), left aileron (δ_{a_L}), right aileron (δ_{a_R}), therefore we proceed with the derivatives. We have the only not null derivatives with respect to the state derivative vector the following,

$$\frac{\partial f_{11}}{\partial \delta_f} = -\lambda_{\delta_f}$$

$$\frac{\partial f_{12}}{\partial \delta_e} = -\lambda_{\delta_e}$$

$$\frac{\partial f_{13}}{\partial \delta_r} = -\lambda_{\delta_r}$$

$$\frac{\partial f_{14}}{\partial \delta_{a_L}} = -\lambda_{\delta_{a_L}}$$

$$\frac{\partial f_{15}}{\partial \delta_{a_R}} = -\lambda_{\delta_{a_R}}$$

And the remaining null derivatives,

$$\frac{\partial f_{11-15}}{\partial V_T} = \frac{\partial f_{11-15}}{\partial \beta} = \frac{\partial f_{11-15}}{\partial \alpha} = \frac{\partial f_{11-15}}{\partial P_W} = \frac{\partial f_{11-15}}{\partial Q_W} = \frac{\partial f_{11-15}}{\partial R_W} = \frac{\partial f_{11-15}}{\partial \phi} = \frac{\partial f_{11-15}}{\partial \theta}$$
$$= \frac{\partial f_{11-15}}{\partial \psi} = \frac{\partial f_{11-15}}{\partial h} = \frac{\partial f_{11-15}}{\partial \omega_j^2} = 0$$

The 16th to 19th rows refers to wing and horizontal tail dynamics, therefore we have,

$$\frac{\partial f_{16}}{\partial x_{2_W}} = \frac{\partial f_{16}}{\partial \dot{\delta}_{\mathrm{W}}} = 1$$

$$\frac{\partial f_{17}}{\partial V_T} = \frac{\partial P_{2_W}}{\partial V_T}$$

$$\frac{\partial f_{17}}{\partial \beta} = \frac{\partial P_{2_W}}{\partial \beta}$$

$$\frac{\partial f_{17}}{\partial \alpha} = \frac{\partial P_{2_W}}{\partial \alpha}$$

$$\frac{\partial f_{17}}{\partial \phi} = \frac{\partial P_{2_W}}{\partial \phi}$$

$$\frac{\partial f_{17}}{\partial \theta} = \frac{\partial P_{2_W}}{\partial \theta}$$

$$\frac{\partial f_{17}}{\partial \psi} = \frac{\partial P_{2_W}}{\partial \psi}$$

$$\frac{\partial f_{17}}{\partial h} = \frac{\partial P_{2_W}}{\partial h}$$

$$\frac{\partial f_{17}}{\partial \delta_f} = \frac{\partial P_{2_W}}{\partial \delta_f}$$

$$\frac{\partial f_{17}}{\partial \delta_{a_L}} = \frac{\partial P_{2_W}}{\partial \delta_{a_L}}$$

$$\frac{\partial f_{17}}{\partial \delta_{a_R}} = \frac{\partial P_{2_W}}{\partial \delta_{a_R}}$$

$$\frac{\partial f_{17}}{\partial x_{1_W}} = \frac{\partial f_{17}}{\partial \delta_{\mathrm{W}}} = \frac{\partial P_{1_W}}{\partial \delta_{\mathrm{W}}} \dot{\delta}_{\mathrm{W}} + \frac{\partial P_{2_W}}{\partial \delta_{\mathrm{W}}}$$

$$\frac{\partial f_{17}}{\partial x_{2_W}} = \frac{\partial f_{17}}{\partial \dot{\delta}_{\mathrm{W}}} = P_{1_W} + \frac{\partial P_{2_W}}{\partial \dot{\delta}_{\mathrm{W}}}$$

$$\frac{\partial f_{17}}{\partial \omega_j^2} = \frac{\partial P_{2_W}}{\partial \omega_j^2}, for\ j = 1{:}\,4$$

Now for the horizontal tail, we find the 18th to 19th rows.

$$\frac{\partial f_{18}}{\partial x_{2_{HT}}} = \frac{\partial f_{18}}{\partial \dot{\delta}_{\mathrm{HT}}} = 1$$

$$\frac{\partial f_{19}}{\partial V_T} = \frac{\partial P_{2_{HT}}}{\partial V_T}$$

$$\frac{\partial f_{19}}{\partial \beta} = \frac{\partial P_{2_{HT}}}{\partial \beta}$$

$$\frac{\partial f_{19}}{\partial \alpha} = \frac{\partial P_{2_{HT}}}{\partial \alpha}$$

$$\frac{\partial f_{19}}{\partial \phi} = \frac{\partial P_{2_{HT}}}{\partial \phi}$$

$$\frac{\partial f_{19}}{\partial \theta} = \frac{\partial P_{2_{HT}}}{\partial \theta}$$

$$\frac{\partial f_{19}}{\partial \psi} = \frac{\partial P_{2_{HT}}}{\partial \psi}$$

$$\frac{\partial f_{19}}{\partial h} = \frac{\partial P_{2_{HT}}}{\partial h}$$

$$\frac{\partial f_{19}}{\partial \delta_e} = \frac{\partial P_{2_{HT}}}{\partial \delta_e}$$

$$\frac{\partial f_{19}}{\partial x_{1_{HT}}} = \frac{\partial f_{19}}{\partial \delta_{\mathrm{HT}}} = \frac{\partial P_{1_{HT}}}{\partial \delta_{\mathrm{HT}}} \dot{\delta}_{\mathrm{HT}} + \frac{\partial P_{2_{HT}}}{\partial \delta_{\mathrm{HT}}}$$

$$\frac{\partial f_{19}}{\partial x_{2_{HT}}} = \frac{\partial f_{19}}{\partial \dot{\delta}_{\mathrm{HT}}} = P_{1_{HT}} + \frac{\partial P_{2_{HT}}}{\partial \dot{\delta}_{\mathrm{HT}}}$$

$$\frac{\partial f_{19}}{\partial \omega_j^2} = \frac{\partial P_{2_{HT}}}{\partial \omega_j^2}, for\ j = 5{:}\,6$$

The last rows of the Jacobian matrix A, i.e., 20^{th} to 25th rows refer to the rotors dynamics. Again, the only not null derivatives are just a few,

$$\frac{\partial f_{20}}{\partial \omega_1^2} = \frac{\partial f_{21}}{\partial \omega_2^2} = \frac{\partial f_{22}}{\partial \omega_3^2} = \frac{\partial f_{23}}{\partial \omega_4^2} = \frac{\partial f_{24}}{\partial \omega_5^2} = \frac{\partial f_{25}}{\partial \omega_6^2} = -\lambda_p$$

So we have completed the Jacobian matrix A.

Lastly, we assemble the Jacobian matrix B. Its terms are first partial derivatives with respect to the control vector $\vec{U}$. From the first to the 10th row all the terms in the Jacobian matrix B are zeros, since there is no dependency in the translational, angular, attitude propagation and rate of climb equations regarding any of the variables in the control vector. It worth remembering that the control vector contains the control signals, not the control states.

For the 11th to 15th rows of the Jacobian matrix B, concerning the actuator dynamics equations, we have the only nonzero terms,

$$\frac{\partial f_{11}}{\partial \delta_f^C} = \lambda_{\delta_f} K_{\delta_f}$$

$$\frac{\partial f_{12}}{\partial \delta_e^C} = \lambda_{\delta_e} K_{\delta_e}$$

$$\frac{\partial f_{13}}{\partial \delta_r^C} = \lambda_{\delta_r} K_{\delta_r}$$

$$\frac{\partial f_{14}}{\partial \delta_{a_L}^C} = \lambda_{\delta_{a_L}} K_{\delta_{a_L}}$$

$$\frac{\partial f_{15}}{\partial \delta_{a_R}^C} = \lambda_{\delta_{a_R}} K_{\delta_{a_R}}$$

The last rows of the Jacobian matrix B, i.e., 20th to 25th rows regarding the rotors dynamic equations, have the only nonzero terms,

$$\frac{\partial f_{20}}{\partial \omega_1^{2^C}} = \frac{\partial f_{21}}{\partial \omega_2^{2^C}} = \frac{\partial f_{22}}{\partial \omega_3^{2^C}} = \frac{\partial f_{23}}{\partial \omega_4^{2^C}} = \frac{\partial f_{24}}{\partial \omega_5^{2^C}} = \frac{\partial f_{25}}{\partial \omega_6^{2^C}} = \lambda_p K_p$$

Finally we have accomplished the Jacobian matrix B.

It is notable that the derivation of the Jacobian matrices involved the partial derivatives of many terms, which are presented in the next chapters and in the Appendix.

5 AERODYNAMIC FORCES, MOMENTS AND DERIVATIVES

Previously we have used in the translational and angular motion equations the aerodynamic forces and moments, which were also written as functions of aerodynamic coefficients, and in the state space equations those terms were derived with respect to the state and state derivative vectors. Therefore, in this chapter we are going to define the aerodynamic coefficients and the relevant derivatives.

First, the aircraft drag coefficient,

$$C_D = C_{D_{WBT}} + C_{D_{q_{WBT}}} \frac{Q_W \bar{c}}{2V_T}$$

Wherein $C_{D_{WBT}}$ is the static aircraft drag coefficient and $C_{D_{q_{WBT}}}$ is the dynamic derivative with respect to Q_W. The subscript WBT means that the coefficient is referred to the aircraft wing-body-and-tail all together. We expand the static terms in body drag coefficient C_{D_B}, exposed wing drag coefficient $C_{D_{W_e}}$, exposed horizontal tail drag coefficient $C_{D_{HT_e}}$ and landing gear drag coefficient $C_{D_{LG}}$. Note that from now on the subscripts B will be referring to aircraft body, W_e to aircraft exposed wing, HT_e to aircraft exposed horizontal tail and LG to landing gear. So we find S_{W_e} the exposed wing planform area, S_{HT_e} the exposed horizontal tail planform area, ΔV_T the increment in flight speed at the horizontal tail.

$$C_{D_{WBT}} = C_{D_B} \frac{S_b}{S_W} + C_{D_{W_e}} \frac{S_{W_e}}{S_W} + C_{D_{HT_e}} \frac{(V_T + \Delta V_T)^2}{V_T^2} \frac{S_{HT_e}}{S_W} + C_{D_{LG}}$$

From Hoak (1965), we find the aircraft body drag coefficient,

$$C_{D_B} = C_{D_{\alpha^2{}_B}} \alpha^2 + C_{D_{\alpha^3{}_B}} \alpha^3$$

The derivatives are defined in the following equations. Being V_B the aircraft body volume, S_p the aircraft body planform area, S_b the aircraft body base area (maximum cross section area), η the ratio of the drag on a finite cylinder to the drag on an infinite cylinder, obtained as a function of the body fineness ratio (fuselage length to fuselage maximum diameter), (l_B/d_B) in Figure 11, and c_{dc} the experimental steady-state cross flow drag coefficient of a circular cylinder of infinite length, obtained as a function of the cross-flow Mach number at a given angle of attack in Figure 12.

$$C_{D_{\alpha^2 B}} = \frac{2}{V_B^{2/3}}$$

$$C_{D_{\alpha^3 B}} = \frac{\eta c_{dc}}{V_B^{2/3}} \frac{S_p}{S_b}$$

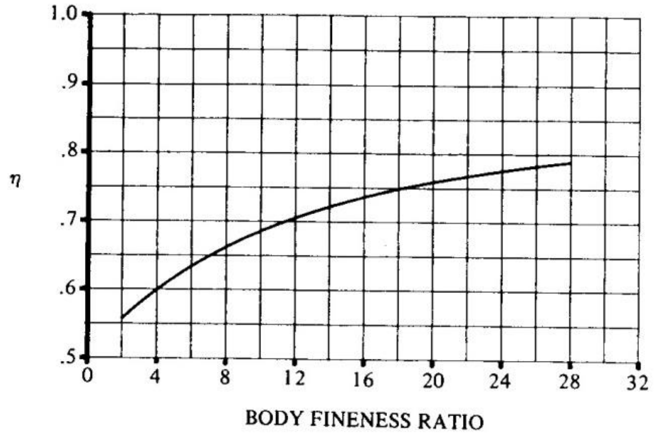


Figure 11: Figure 4.2.1.2-35a of Hoak (1965).

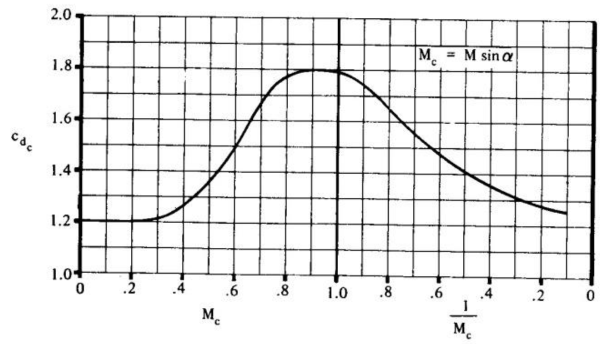


Figure 12: Figure 4.2.1.2-35b of Hoak (1965).

And the exposed wing and horizontal tail drag coefficients respectively. The terms are: $(C_{D0})_{WB_e}$ the exposed wing and body zero lift drag coefficient, $(C_{D0})_{HT_e}$ the exposed horizontal tail and body zero lift drag coefficient, $C_{L_{W_e}}$ the exposed wing lift coefficient, $C_{L_{HT_e}}$ the exposed horizontal tail lift coefficient, AR_{W_e} the exposed wing aspect ratio, AR_{HT_e} the exposed horizontal tail aspect ratio, e_{W_e} the exposed wing Oswald factor, e_{HT_e} the exposed horizontal tail Oswald factor, $\Delta C_{D_{p,W}}$ the increment in wing drag coefficient due to propellers wake, $\Delta C_{D_{p,HT}}$ the increment in horizontal tail drag coefficient due to propellers wake, $\Delta C_{D_{p_L}}$ and $\Delta C_{D_{p_R}}$ the increment in wing drag coefficient due to propellers wake acting on the left and right ailerons.

$$C_{D_{W_e}} = (C_{D0})_{WB_e} + \frac{C_{L_{W_e}}^2}{\pi AR_{W_e} e_{W_e}} + \Delta C_{D_{p,W}} + \Delta C_{D_{p_L}} + \Delta C_{D_{p_R}}$$

$$C_{D_{HT_e}} = (C_{D0})_{HT_e} + \frac{C_{L_{HT_e}}^2}{\pi AR_{HT_e} e_{HT_e}} + \Delta C_{D_{p,HT}}$$

Resulting in the aircraft total drag force,

$$D = \frac{\rho V_T^2}{2}\left(C_{D_B} S_b + C_{D_{W_e}} S_{W_e}\right) + \frac{\rho (V_T + \Delta V_T)^2}{2} C_{D_{HT_e}} S_{HT_e} + \frac{\rho V_T S_W \bar{c} Q_W}{4} C_{D_{q_{WBT}}} + D_{LG}$$

Thereafter, we have the aircraft total drag force derivatives with respect to the state vector $\vec{X}$,

$$D_{V_T} = \rho V_T \left(C_{D_B} S_b + C_{D_{W_e}} S_{W_e}\right) + \rho (V_T + \Delta V_T) C_{D_{HT_e}} S_{HT_e} + \frac{\rho \bar{c} Q_W}{4} C_{D_{q_{WBT}}} + \left(D_{V_T}\right)_{LG}$$

$$D_\beta = \left(D_\beta\right)_{LG}$$

$$D_\alpha = \frac{\rho V_T^2 S_{W_e}}{2\pi AR_{W_e} e_{W_e}}\left(\frac{\partial C_{L_{W_e}}^2}{\partial \alpha} + \frac{\partial \Delta C_{D_{p,W}}}{\partial \alpha} + \frac{\partial \Delta C_{D_{p_L}}}{\partial \alpha} + \frac{\partial \Delta C_{D_{p_R}}}{\partial \alpha}\right)$$
$$+ \frac{\rho V_T^2}{2}\left(C_{D_{\alpha^2 B}} 2\alpha + C_{D_{\alpha^3 B}} 3\alpha^2\right) S_b + \frac{\rho (V_T + \Delta V_T)^2 S_{HT_e}}{2\pi AR_{HT_e} e_{HT_e}}\left(\frac{\partial C_{L_{HT_e}}^2}{\partial \alpha} + \frac{\partial \Delta C_{D_{p,HT}}}{\partial \alpha}\right)$$
$$+ (D_\alpha)_{LG}$$

$$D_{Q_W} = \frac{\rho V_T S_W \bar{c}}{4} C_{D_{q_{WBT}}}$$

$$D_h = \frac{\partial \rho}{\partial h}\left[\frac{V_T^2}{2}\left(C_{D_B} S_b + C_{D_{W_e}} S_{W_e}\right) + \frac{(V_T + \Delta V_T)^2}{2} C_{D_{HT_e}} S_{HT_e} + \frac{V_T S_W \bar{c} Q_W}{4} C_{D_{q_{WBT}}}\right] + (D_h)_{LG}$$

$$D_{\delta_e} = \frac{\rho (V_T + \Delta V_T)^2 S_{HT_e}}{2}\left(\frac{1}{\pi AR_{HT_e} e_{HT_e}} \frac{\partial C_{L_{HT_e}}^2}{\partial \delta_e} + \frac{\partial \Delta C_{D_{p,HT}}}{\partial \delta_e}\right)$$

$$D_{\delta_W} = \frac{\rho V_T^2 S_{W_e}}{2}\left(\frac{1}{\pi AR_{W_e} e_{W_e}} \frac{\partial C_{L_{W_e}}^2}{\partial \delta_W} + \frac{\partial \Delta C_{D_{p,W}}}{\partial \delta_W} + \frac{\partial \Delta C_{D_{p_L}}}{\partial \delta_W} + \frac{\partial \Delta C_{D_{p_R}}}{\partial \delta_W}\right)$$

$$D_{\delta_{HT}} = \frac{\rho (V_T + \Delta V_T)^2 S_{HT_e}}{2}\left(\frac{1}{\pi AR_{HT_e} e_{HT_e}} \frac{\partial C_{L_{HT_e}}^2}{\partial \delta_{HT}} + \frac{\partial \Delta C_{D_{p,HT}}}{\partial \delta_{HT}}\right)$$

$$D_{\omega_j^2} = \begin{cases} \frac{\rho V_T^2 S_{W_e}}{2}\left(\frac{1}{\pi AR_{W_e} e_{W_e}} \frac{\partial C_{L_{W_e}}^2}{\partial \omega_j^2} + \frac{\partial \Delta C_{D_{p,W}}}{\partial \omega_j^2} + \frac{\partial \Delta C_{D_{p_L}}}{\partial \omega_j^2} + \frac{\partial \Delta C_{D_{p_R}}}{\partial \omega_j^2}\right), for\ j = 1{:}4 \\ \frac{\rho (V_T + \Delta V_T)^2 S_{HT_e}}{2}\left(\frac{1}{\pi AR_{HT_e} e_{HT_e}} \frac{\partial C_{L_{HT_e}}^2}{\partial \omega_j^2} + \frac{\partial \Delta C_{D_{p,HT}}}{\partial \omega_j^2}\right), for\ j = 5{:}6 \end{cases}$$

And for the remaining state vector terms, all zeros,

$$D_{P_W} = D_{R_W} = D_{R_W} = D_{\phi} = D_{\theta} = D_{\psi} = D_{\delta_r} = D_{\delta_{a_L}} = D_{\delta_{a_R}} = 0$$

All the derivatives with respect to the states derivative vector $\dot{\vec{X}}$ will be considered zeros.

Next the aircraft side force coefficient. Here we have $C_{Y_{WBT}}$the aircraft static side force coefficient, $C_{Y_{\beta_{WBT}}}$ the aircraft side force derivative with respect to β, $C_{Y_{\beta_{LG}}}$ the aircraft landing gear side force derivative with respect to β, $C_{Y_{p_{WBT}}}$the aircraft side force derivative with respect to P_W, $C_{Y_{r_{WBT}}}$the aircraft side force derivative with respect to R_W, $C_{Y_{\dot{\beta}_{WBT}}}$the aircraft side force derivative with respect to $\dot{\beta}$, $C_{Y_{\delta_r}}$the aircraft side force derivative with respect to δ_r.

$$C_Y = C_{Y_{WBT}} + C_{Y_{p_{WBT}}} \frac{P_W b}{2V_T} + C_{Y_{r_{WBT}}} \frac{R_W b}{2V_T} + C_{Y_{\dot{\beta}_{WBT}}} \frac{\dot{\beta} b}{2V_T}$$

Expanding the static term $C_{Y_{WBT}}$,

$$C_{Y_{WBT}} = C_{Y_{\beta_{WBT}}} \beta + C_{Y_{\delta_r}} \delta_r + C_{Y_{LG}}$$

Resulting in the aircraft total side force,

$$Y = \frac{\rho V_T^2 S_W}{2}\left(C_{Y_{\beta_{WBT}}} \beta + C_{Y_{\delta_r}} \delta_r\right) + \frac{\rho V_T S_W b}{4}\left(C_{Y_{p_{WBT}}} P_W + C_{Y_{r_{WBT}}} R_W + C_{Y_{\dot{\beta}_{WBT}}} \dot{\beta}\right) + Y_{LG}$$

And now we find the derivatives with respect to the state vector $\vec{X}$,

$$Y_{V_T} = \rho V_T S_W \left(C_{Y_{\beta_{WBT}}} \beta + C_{Y_{\delta_r}} \delta_r\right) + \frac{\rho S_W b}{4}\left(C_{Y_{p_{WBT}}} P_W + C_{Y_{r_{WBT}}} R_W + C_{Y_{\dot{\beta}_{WBT}}} \dot{\beta}\right) + \left(Y_{V_T}\right)_{LG}$$

$$Y_{\beta} = \frac{\rho V_T^2 S_W}{2} C_{Y_{\beta_{WBT}}} + \left(Y_{\beta}\right)_{LG}$$

$$Y_{\alpha} = (Y_{\alpha})_{LG}$$

$$Y_{P_W} = \frac{\rho V_T S_W b}{4} C_{Y_{p_{WBT}}}$$

$$Y_{R_W} = \frac{\rho V_T S_W b}{4} C_{Y_{r_{WBT}}}$$

$$Y_h = \frac{\partial \rho}{\partial h}\left[\frac{V_T^2 S_W}{2}\left(C_{Y_{\beta_{WBT}}}\beta + C_{Y_{\delta_r}}\delta_r\right) + \frac{V_T S_W b}{4}\left(C_{Y_{p_{WBT}}} P_W + C_{Y_{r_{WBT}}} R_W + C_{Y_{\dot{\beta}_{WBT}}}\dot{\beta}\right)\right] + (Y_h)_{LG}$$

$$Y_{\delta_r} = \frac{\rho V_T^2 S_W C_{Y_{\delta_r}}}{2}$$

And for the remaining state vector terms, all zeros,

$$Y_{Q_W} = Y_\phi = Y_\theta = Y_\psi = Y_{\delta_f} = Y_{\delta_e} = Y_{\delta_{a_L}} = Y_{\delta_{a_R}} = Y_{\delta_W} = Y_{\delta_{HT}} = Y_{\omega_j^2} = 0$$

Now for the derivatives with respect to the state derivatives $\dot{\vec{X}}$, we will consider only the one with respect to $\dot{\beta}$ to be nonzero.

$$Y_{\dot{\beta}} = \frac{\rho V_T S_W b}{4} C_{Y_{\dot{\beta}_{WBT}}}$$

Following, the aircraft lift coefficient. There is the aircraft static lift coefficient $C_{L_{WBT}}$, and the dynamic coefficients $C_{L_{q_{WBT}}}$ the aircraft lift derivative with respect to Q_W and $C_{L_{\dot{\alpha}_{WBT}}}$ the aircraft lift coefficient derivative with respect to $\dot{\alpha}$.

$$C_L = C_{L_{WBT}} + C_{L_{q_{WBT}}} \frac{Q_W \bar{c}}{2V_T} + C_{L_{\dot{\alpha}_{WBT}}} \frac{\dot{\alpha}\bar{c}}{2V_T}$$

We expand the aircraft wing-body-tail static lift coefficient in portions relative to body, exposed wing, exposed horizontal tail and landing gear. There we find: $C_{L_{W_e}}$ the exposed wing lift coefficient, C_{L_B} body lift coefficient, $C_{L_{HT_e}}$ the exposed horizontal tail lift coefficient and $C_{L_{LG}}$ the landing gear lift coefficient.

$$C_{L_{WBT}} = K_{WB}\frac{S_{W_e}}{S_W} C_{L_{W_e}} + K_{WB}\frac{S_b}{S_W} C_{L_B} + K_{BHT}\frac{S_{HT_e}}{S_W}\frac{(V_T + \Delta V_T)^2}{V_T^2} C_{L_{HT_e}} + C_{L_{LG}}$$

Wherein, K_{WB} and K_{BHT} are wing-body and body-horizontal tail interference factors respectively (HOAK, 1965), given by, where d_{WB} is the body diameter at the wing section,

b_W the wing span, d_{BHT} the body diameter at the horizontal tail section and b_{HT} the horizontal tail span.

$$K_{WB} = \left(\frac{d_{WB}}{b_W} + 1\right)^2$$

$$K_{BHT} = \left(\frac{d_{BHT}}{b_{HT}} + 1\right)^2$$

The parts lift coefficients have the following inner terms, where: $\left(C_{L_\alpha}\right)_{W_e}$ the exposed wing lift curve slope, α_{0_W} the exposed wing angle of attack of zero lift, $\Delta C_{L_{p,W}}$ the increment in wing lift coefficient due to propellers wake, $\Delta C_{L_{p_L}}$ and $\Delta C_{L_{p_R}}$ the increment in wing lift coefficient due to propellers wake acting on the left and right ailerons, $\left(C_{L_\alpha}\right)_{HT_e}$ the exposed horizontal tail lift curve slope, $\alpha_{0_{HT}}$ the exposed horizontal tail angle of attack of zero lift, ϵ the downwash angle at the horizontal tail, $C_{L_{\delta_e}}$ the horizontal tail lift coefficient derivative with respect to elevator deflection angle, $\Delta C_{L_{p,HT}}$ the increment in horizontal lift coefficient due to propellers wake, $\left(C_{L_\alpha}\right)_B$ the body lift curve slope.

$$C_{L_{W_e}} = \left(C_{L_\alpha}\right)_{W_e}\left(\alpha + \delta_W - \alpha_{0_W}\right) + \Delta C_{L_{p,W}} + \Delta C_{L_{p_L}} + \Delta C_{L_{p_R}}$$

$$C_{L_{HT_e}} = \left(C_{L_\alpha}\right)_{HT_e}\left(\alpha + \delta_{HT} - \alpha_{0_{HT}} - \epsilon\right) + C_{L_{\delta_e}}\delta_e + \Delta C_{L_{p,HT}}$$

$$C_{L_B} = \left(C_{L_\alpha}\right)_B \alpha$$

Resulting in the aircraft total lift,

$$L = \frac{\rho V_T^2}{2} K_{WB}\left(S_{W_e} C_{L_{W_e}} + S_b C_{L_B}\right) + \frac{\rho S_{HT_e}}{2} K_{BHT}(V_T + \Delta V_T)^2 C_{L_{HT_e}} + \frac{\rho V_T S_W \bar{c}}{4}\left(C_{L_{q_{WBT}}} Q_W + C_{L_{\dot{\alpha}_{WBT}}}\dot{\alpha}\right) + L_{LG}$$

And now we find the derivatives with respect to the state vector $\vec{X}$,

$$L_{V_T} = \rho V_T\left(K_{WB} S_{W_e} C_{L_{W_e}} + K_{WB} S_b C_{L_B}\right) + \rho(V_T + \Delta V_T) K_{BHT} S_{HT_e} C_{L_{HT_e}} + \frac{\rho S_W \bar{c}}{4}\left(C_{L_{q_{WBT}}} Q_W + C_{L_{\dot{\alpha}_{WBT}}}\dot{\alpha}\right) + \left(L_{V_T}\right)_{LG}$$

$$L_\beta = (L_\beta)_{LG}$$

$$L_\alpha = \frac{\rho V_T^2}{2} K_{WB} \left(S_{W_e} \left[(C_{L_\alpha})_{W_e} + \frac{\partial \Delta C_{L_{p,W}}}{\partial \alpha} + \frac{\partial \Delta C_{L_{p_L}}}{\partial \alpha} + \frac{\partial \Delta C_{L_{p_R}}}{\partial \alpha} \right] + S_b (C_{L_\alpha})_B \right) + \frac{\rho S_{HT_e}}{2} K_{BHT} (V_T + \Delta V_T)^2 \left((C_{L_\alpha})_{HT_e} + \frac{\partial \Delta C_{L_{p,HT}}}{\partial \alpha} \right) + (L_\alpha)_{LG}$$

$$L_{Q_W} = \frac{\rho V_T S_W \bar{c}}{4} C_{L_{q_{WBT}}}$$

$$L_h = \frac{\partial \rho}{\partial h} \left[\frac{V_T^2}{2} K_{WB} \left(S_{W_e} C_{L_{W_e}} + S_b C_{L_B} \right) + \frac{S_{HT_e}}{2} K_{BHT} (V_T + \Delta V_T)^2 C_{L_{HT_e}} + \frac{V_T S_W \bar{c}}{4} \left(C_{L_{q_{WBT}}} Q_W + C_{L_{\dot{\alpha}_{WBT}}} \dot{\alpha} \right) \right] + (L_h)_{LG}$$

$$L_{\delta_e} = \frac{\rho S_{HT_e}}{2} K_{BHT} (V_T + \Delta V_T)^2 \left(C_{L_{\delta_e}} + \frac{\partial \Delta C_{L_{p,HT}}}{\partial \delta_e} \right)$$

$$L_{\delta_{a_L}} = \frac{\partial \Delta C_{L_{p_L}}}{\partial \delta_{a_L}}$$

$$L_{\delta_{a_R}} = \frac{\partial \Delta C_{L_{p_R}}}{\partial \delta_{a_R}}$$

$$L_{\delta_W} = \frac{\rho V_T^2}{2} K_{WB} S_{W_e} \left((C_{L_\alpha})_{W_e} + \frac{\partial \Delta C_{L_{p,W}}}{\partial \delta_W} + \frac{\partial \Delta C_{L_{p_L}}}{\partial \delta_W} + \frac{\partial \Delta C_{L_{p_R}}}{\partial \delta_W} \right)$$

$$L_{\delta_{HT}} = \frac{\rho S_{HT_e}}{2} K_{BHT} (V_T + \Delta V_T)^2 \left((C_{L_\alpha})_{HT_e} + \frac{\partial \Delta C_{L_{p,HT}}}{\partial \delta_{HT}} \right)$$

$$L_{\omega_j^2} = \begin{cases} \frac{\rho V_T^2 K_{WB} S_{W_e}}{2} \left(\frac{\partial \Delta C_{L_{p,W}}}{\partial \omega_j^2} + \frac{\partial \Delta C_{L_{prop_L}}}{\partial \omega_j^2} + \frac{\partial \Delta C_{L_{prop_R}}}{\partial \omega_j^2} \right), for\ j = 1{:}\,4 \\ \frac{\rho S_{HT_e}}{2} K_{BHT} (V_T + \Delta V_T)^2 \frac{\partial \Delta C_{L_{p,HT}}}{\partial \omega_j^2}, for\ j = 5{:}\,6 \end{cases}$$

The remaining terms are null derivatives,

$$L_{P_W} = L_{R_W} = L_\phi = L_\theta = L_\psi = 0$$

As for the derivatives in relation to state derivatives vector, we will consider only $L_{\dot{\alpha}}$ to be nonzero.

$$L_{\dot{\alpha}} = \frac{\rho V_T S_W \bar{c}}{4} C_{L_{\dot{\alpha}_{WBT}}}$$

The first aerodynamic moment is the rolling moment coefficient, which has the aircraft static portion $C_{l_{WBT}}$ and the dynamic portions $C_{l_{p_{WBT}}}$ the aircraft rolling moment derivative with respect to P_W, $C_{l_{r_{WBT}}}$ the aircraft rolling moment derivative with respect to R_W and $C_{l_{\dot{\beta}_{WBT}}}$ the aircraft rolling moment derivative with respect to $\dot{\beta}$.

$$C_l = C_{l_{WBT}} + C_{l_{p_{WBT}}} \frac{b}{2V_T} P_W + C_{l_{r_{WBT}}} \frac{b}{2V_T} R_W + C_{l_{\dot{\beta}_{WBT}}} \frac{b}{2V_T} \dot{\beta}$$

Expanding the static rolling moment coefficient, we find $C_{l_{\beta_{WBT}}}$ the aircraft rolling moment coefficient derivative with respect to β, $C_{l_{\delta_a}}$ the aircraft rolling moment coefficient derivative with respect to δ_{a_L} or δ_{a_R}, $C_{l_{\delta_r}}$ the aircraft rolling moment coefficient derivative with respect to δ_r, ΔC_{l_p} the rolling moment coefficient increment due to propellers wake and $C_{l_{LG}}$ the landing gear rolling moment coefficient.

$$C_{l_{WBT}} = C_{l_{\beta_{WBT}}} \beta + C_{l_{\delta_a}} \frac{\left(\delta_{a_L} - \delta_{a_R}\right)}{2} + C_{l_{\delta_r}} \delta_r + \Delta C_{l_p} + C_{l_{LG}}$$

Resulting in the aircraft total rolling moment,

$$\bar{L} = \frac{\rho V_T^2 S_W b}{2} \left(C_{l_{\beta_{WBT}}} \beta + C_{l_{\delta_a}} \frac{\left(\delta_{a_L} - \delta_{a_R}\right)}{2} + C_{l_{\delta_r}} \delta_r + \Delta C_{l_p} \right) + \frac{\rho V_T S_W b^2}{4} \left(C_{l_{p_{WBT}}} P_W + C_{l_{r_{WBT}}} R_W + C_{l_{\dot{\beta}_{WBT}}} \dot{\beta} \right) + (\bar{L})_{LG}$$

And now we find the derivatives with respect to the state vector $\vec{X}$,

$$\bar{L}_{V_T} = \rho V_T S_W b \left(C_{l_{\beta_{WBT}}} \beta + C_{l_{\delta_a}} \frac{\left(\delta_{a_L} - \delta_{a_R}\right)}{2} + C_{l_{\delta_r}} \delta_r + \Delta C_{l_p} \right) + \frac{\rho S_W b^2}{4} \left(C_{l_{p_{WBT}}} P_W + C_{l_{r_{WBT}}} R_W + C_{l_{\dot{\beta}_{WBT}}} \dot{\beta} \right) + \left(\bar{L}_{V_T}\right)_{LG}$$

$$\bar{L}_\beta = \frac{\rho V_T^2 S_W b}{2} C_{l_{\beta_{WBT}}} + \left(\bar{L}_\beta\right)_{LG}$$

$$\bar{L}_\alpha = \frac{\rho V_T^2 S_W b}{2} \frac{\partial \Delta C_{l_p}}{\partial \alpha} + (\bar{L}_\alpha)_{LG}$$

$$\bar{L}_{P_W} = \frac{\rho V_T S_W b^2}{4} C_{l_{p_{WBT}}}$$

$$\bar{L}_{R_W} = \frac{\rho V_T S_W b^2}{4} C_{l_{r_{WBT}}}$$

$$\bar{L}_h = \frac{\partial \rho}{\partial h}\left[\frac{V_T^2 S_W b}{2}\left(C_{l_{\beta_{WBT}}}\beta + C_{l_{\delta_a}}\frac{\left(\delta_{a_L} - \delta_{a_R}\right)}{2} + C_{l_{\delta_r}}\delta_r + \Delta C_{l_p}\right) + \frac{V_T S_W b^2}{4}\left(C_{l_{p_{WBT}}}P_W + C_{l_{r_{WBT}}}R_W + C_{l_{\dot{\beta}_{WBT}}}\dot{\beta}\right)\right] + (\bar{L}_h)_{LG}$$

$$\bar{L}_{\delta_r} = \frac{\rho V_T^2 S_W b}{2} C_{l_{\delta_r}}$$

$$\bar{L}_{\delta_{a_L}} = \frac{\rho V_T^2 S_W b}{2}\left(\frac{C_{l_{\delta_a}}}{2} + \frac{\partial \Delta C_{l_p}}{\partial \delta_{a_L}}\right)$$

$$\bar{L}_{\delta_{a_R}} = \frac{\rho V_T^2 S_W b}{2}\left(-\frac{C_{l_{\delta_a}}}{2} + \frac{\partial \Delta C_{l_p}}{\partial \delta_{a_R}}\right)$$

$$\bar{L}_{\omega_j^2} = \frac{\rho V_T^2 S_W b}{2}\frac{\partial \Delta C_{l_p}}{\partial \omega_j^2}$$

The remaining terms are null derivatives,

$$\bar{L}_{Q_W} = \bar{L}_\phi = \bar{L}_\theta = \bar{L}_\psi = \bar{L}_{\delta_f} = \bar{L}_{\delta_e} = \bar{L}_{\delta_W} = \bar{L}_{\delta_{HT}} = 0$$

Regarding the derivatives in relation to state derivatives vector, we will consider only $\bar{L}_{\dot{\beta}}$ to be nonzero.

$$\bar{L}_{\dot{\beta}} = \frac{\rho V_T S_W b^2}{4} C_{l_{\dot{\beta}_{WBT}}}$$

And now we derive the aircraft pitching moment coefficient. Again, there is the static portion $C_{m_{WBT}}$ and the dynamic coefficients $C_{m_{q_{WBT}}}$ the aircraft pitching moment coefficient derivative with respect to Q_W and $C_{m_{\dot{\alpha}_{WBT}}}$ the aircraft pitching moment coefficient derivative with respect to $\dot{\alpha}$.

$$C_m = C_{m_{WBT}} + C_{m_{q_{WBT}}}\frac{\bar{c}}{2V_T}Q_W + C_{m_{\dot{\alpha}_{WBT}}}\frac{\bar{c}}{2V_T}\dot{\alpha}$$

Expanding the wing-body-tail static moment coefficient term with respect to a reference point $(x_{ref}, y_{ref}, z_{ref})$. Note that the reference point of our dynamic system is the origin of the aircraft body coordinate frame. There we find: C_{N_B} body normal force coefficient, C_{X_B} body axial force coefficient, C_{m_B} body pitching moment coefficient, $C_{N_{W_e}}$ exposed wing normal force coefficient, $C_{X_{W_e}}$ exposed wing axial force coefficient, $C_{m_{W_e}}$ exposed wing pitching moment coefficient, $C_{N_{HT_e}}$ exposed horizontal tail normal force coefficient, $C_{X_{HT_e}}$ exposed horizontal tail axial force coefficient, $C_{m_{HT_e}}$ exposed horizontal tail pitching moment coefficient, $C_{m_{LG}}$ landing gear moment coefficient, x_{refB} and z_{refB} coordinates of body pitching moment reference point, x_{AC_W} and z_{AC_W} coordinates of wing aerodynamic center, $x_{AC_{HT}}$ and $z_{AC_{HT}}$ coordinates of horizontal tail aerodynamic center.

$$
\begin{aligned}
C_{m_{WBT}} = {} & \frac{\left(x_{ref} - x_{refB}\right)}{\bar{c}_W}\frac{S_b}{S_W}K_{WB}C_{N_B} + \frac{\left(z_{ref} - z_{refB}\right)}{\bar{c}_W}\frac{S_b}{S_W}K_{WB}C_{X_B} + C_{m_B}\frac{S_b}{S_W} \\
& + \frac{\left(x_{ref} - x_{AC_W}\right)}{\bar{c}_W}\frac{S_{W_e}}{S_W}K_{WB}C_{N_{W_e}} + \frac{\left(z_{ref} - z_{AC_W}\right)}{\bar{c}_W}\frac{S_{W_e}}{S_W}K_{WB}C_{X_{W_e}} + C_{m_{W_e}}\frac{S_{W_e}}{S_W} \\
& + \frac{\left(x_{ref} - x_{AC_{HT}}\right)}{\bar{c}_{HT}}\frac{(V_T + \Delta V_T)^2}{V_T^2}\frac{S_{HT_e}\bar{c}_{HT}}{S_W\bar{c}_W}K_{BHT}C_{N_{HT_e}} \\
& + \frac{\left(z_{ref} - z_{AC_{HT}}\right)}{\bar{c}_{HT}}\frac{(V_T + \Delta V_T)^2}{V_T^2}\frac{S_{HT_e}\bar{c}_{HT}}{S_W\bar{c}_W}K_{BHT}C_{X_{HT_e}} + C_{m_{HT_e}}\frac{S_{HT_e}\bar{c}_{HT}}{S_W\bar{c}_W} + C_{m_{LG}}
\end{aligned}
$$

Making further expansions, for the body terms,

$$C_{N_B} = C_{L_B}\cos\alpha + C_{D_B}\sin\alpha$$

$$C_{X_B} = C_{L_B}\sin\alpha - C_{D_B}\cos\alpha$$

$$C_{m_B} = \left(C_{m_{\alpha 1}}\right)_B \sin(2\alpha)\cos\left(\frac{\alpha}{2}\right) + \left(C_{m_{\alpha 2}}\right)_B \sin^2\alpha$$

Wherein from Hoak (1965), where l_B is the body length, d_b the body maximum equivalent diameter (diameter of circular cross section with same area), V_B body volume, x_{Bc} distance from the nose to the centroid of the body planform area, S_p body planform area, η and c_{dc} the same used in computation of body drag.

$$\left(C_{m_{\alpha 1}}\right)_B = \left(\frac{C_N}{C_{N_{circ}}}\right)_{SB}\left[\frac{V_B - S_b\left(l_B - x_{refB}\right)}{S_b d_b}\right]$$

$$\left(C_{m_{\alpha 2}}\right)_B = -\left(\frac{C_N}{C_{N_{circ}}}\right)_{NT} \eta c_{dc} \frac{S_p}{S_b} \left[\frac{x_{refB} - x_{Bc}}{d_b}\right]$$

$$\left(\frac{C_N}{C_{N_{circ}}}\right)_{SB} = \frac{a}{b}\cos^2\phi + \frac{b}{a}\sin^2\phi$$

For the body major axis of equivalent body elliptical cross section perpendicular to the cross flow velocity.

$$\left(\frac{C_N}{C_{N_{circ}}}\right)_{NT} = \frac{3}{2}\sqrt{\frac{a}{b}}\left\{\frac{-b^2/a^2}{\left(1-\frac{b^2}{a^2}\right)^{3/2}}\ln\left[\frac{a}{b}\left(1+\sqrt{1-\frac{b^2}{a^2}}\right)\right] + \frac{1}{1-\frac{b^2}{a^2}}\right\}$$

Being: a the major axis of equivalent body elliptical cross section, b the minor axis of equivalent elliptical cross section, ϕ angle of bank of the body about its longitudinal axis with $\phi = 0$ with the major axis horizontal and $\phi = 90°$ with the minor axis horizontal.

Now referring to the wing,

$$C_{N_{W_e}} = C_{L_{W_e}}\cos\alpha + C_{D_{W_e}}\sin\alpha$$

$$C_{X_{W_e}} = C_{L_{W_e}}\sin\alpha - C_{D_{W_e}}\cos\alpha$$

$$C_{m_{W_e}} = (C_{m0})_{W_e} + \left(C_{m_\alpha}\right)_{W_e}\left(\alpha + \delta_W - \alpha_{0_W}\right)$$

We have: $(C_{m0})_{W_e}$ exposed wing zero lift pitching moment coefficient, $\left(C_{m_\alpha}\right)_{W_e}$ exposed wing pitching moment curve slope.

And horizontal tail terms,

$$C_{N_{HT_e}} = C_{L_{HT_e}}\cos(\alpha - \epsilon) + C_{D_{HT_e}}\sin(\alpha - \epsilon)$$

$$C_{X_{HT_e}} = C_{L_{HT_e}}\sin(\alpha - \epsilon) - C_{D_{HT_e}}\cos(\alpha - \epsilon)$$

$$C_{m_{HT_e}} = (C_{m0})_{HT_e} + \left(C_{m_\alpha}\right)_{HT_e}\left(\alpha + \delta_{HT} - \alpha_{0_{HT}} - \epsilon\right) + \Delta C_{m_{\delta_e}}$$

Similarly: $(C_{m0})_{HT_e}$ exposed horizontal tail zero lift pitching moment coefficient, $\left(C_{m_\alpha}\right)_{HT_e}$ exposed horizontal tail pitching moment curve slope, $\Delta C_{m_{\delta_e}}$ increment in pitching moment coefficient due to elevator deflection.

Resulting in the aircraft total pitching moment,

$$M = \frac{\rho V_T^2 S_W \bar{c}_W}{2} C_{m_{WBT}} + \frac{\rho V_T S_W \bar{c}_W^2}{4}\left(C_{m_{q_{WBT}}} Q_W + C_{m_{\dot{\alpha}_{WBT}}} \dot{\alpha}\right)$$

And now we find the derivatives with respect to the state vector $\vec{X}$,

$$\begin{aligned}
M_{V_T} = \rho V_T \Big[& (x_{ref} - x_{refB}) S_b K_{WB} (C_{L_B} \cos\alpha + C_{D_B} \sin\alpha) \\
& + (z_{ref} - z_{refB}) S_b K_{WB} (C_{L_B} \sin\alpha - C_{D_B} \cos\alpha) + C_{m_B} \bar{c}_W S_b \\
& + (x_{ref} - x_{AC_W}) S_{W_e} K_{WB} \left(C_{L_{W_e}} \cos\alpha + C_{D_{W_e}} \sin\alpha\right) \\
& + (z_{ref} - z_{AC_W}) S_{W_e} K_{WB} \left(C_{L_{W_e}} \sin\alpha - C_{D_{W_e}} \cos\alpha\right) + C_{m_{W_e}} \bar{c}_W S_{W_e} \Big] \\
& + \rho (V_T + \Delta V_T) S_{HT_e} \Big[(x_{ref} - x_{AC_{HT}}) K_{BHT} \left(C_{L_{HT_e}} \cos(\alpha - \epsilon) + C_{D_{HT_e}} \sin(\alpha - \epsilon)\right) \\
& + (z_{ref} - z_{AC_{HT}}) K_{BHT} \left(C_{L_{HT_e}} \sin(\alpha - \epsilon) - C_{D_{HT_e}} \cos(\alpha - \epsilon)\right) + C_{m_{HT_e}} \bar{c}_{HT} \Big] \\
& + \frac{\rho S_W \bar{c}_W^2}{4} \left(C_{m_{q_{WBT}}} Q_W + C_{m_{\dot{\alpha}_{WBT}}} \dot{\alpha}\right) + \left(M_{V_T}\right)_{LG}
\end{aligned}$$

$$M_\beta = \left(M_\beta\right)_{LG}$$

$$\begin{aligned}
M_\alpha = \frac{\rho V_T^2}{2} \Bigg[& (x_{ref} - x_{refB}) S_b K_{WB} \left(\frac{\partial C_{L_B}}{\partial\alpha} \cos\alpha - C_{L_B} \sin\alpha + \frac{\partial C_{D_B}}{\partial\alpha} \sin\alpha + C_{D_B} \cos\alpha\right) \\
& + (z_{ref} - z_{refB}) S_b K_{WB} \left(\frac{\partial C_{L_B}}{\partial\alpha} \sin\alpha + C_{L_B} \cos\alpha - \frac{\partial C_{D_B}}{\partial\alpha} \cos\alpha + C_{D_B} \sin\alpha\right) \\
& + \frac{\partial C_{m_B}}{\partial\alpha} \bar{c} S_b \\
& + (x_{ref} - x_{AC_W}) S_{W_e} K_{WB} \left(\frac{\partial C_{L_{W_e}}}{\partial\alpha} \cos\alpha - C_{L_{W_e}} \sin\alpha + \frac{\partial C_{D_{W_e}}}{\partial\alpha} \sin\alpha + C_{D_{W_e}} \cos\alpha\right) \\
& + (z_{ref} - z_{AC_W}) S_{W_e} K_{WB} \left(\frac{\partial C_{L_{W_e}}}{\partial\alpha} \sin\alpha + C_{L_{W_e}} \cos\alpha - \frac{\partial C_{D_{W_e}}}{\partial\alpha} \cos\alpha + C_{D_{W_e}} \sin\alpha\right) \\
& + \frac{\partial C_{m_{W_e}}}{\partial\alpha} \bar{c}_W S_{W_e} \Bigg] \\
& + \frac{\rho (V_T + \Delta V_T)^2 S_{HT_e}}{2} \Bigg[(x_{ref} - x_{AC_{HT}}) K_{BHT} \left(\frac{\partial C_{L_{HT_e}}}{\partial\alpha} \cos(\alpha - \epsilon) - C_{L_{HT_e}} \sin(\alpha - \epsilon)\right. \\
& \left. + \frac{\partial C_{D_{HT_e}}}{\partial\alpha} \sin(\alpha - \epsilon) + C_{D_{HT_e}} \cos(\alpha - \epsilon)\right) \\
& + (z_{ref} - z_{AC_{HT}}) K_{BHT} \left(\frac{\partial C_{L_{HT_e}}}{\partial\alpha} \sin(\alpha - \epsilon) + C_{L_{HT_e}} \cos(\alpha - \epsilon) - \frac{\partial C_{D_{HT_e}}}{\partial\alpha} \cos(\alpha - \epsilon)\right. \\
& \left. + C_{D_{HT_e}} \sin(\alpha - \epsilon)\right) + \frac{\partial C_{m_{HT_e}}}{\partial\alpha} \bar{c}_{HT} \Bigg] + (M_\alpha)_{LG}
\end{aligned}$$

$$M_{Q_W} = \frac{\rho V_T S_W \bar{c}_W^2}{4} C_{m_{q_{WBT}}}$$

$$M_h = \frac{\partial \rho}{\partial h}\left[\frac{V_T^2 S_W \bar{c}_W}{2} C_{m_{WBT}} + \frac{V_T S_W \bar{c}_W^2}{4}\left(C_{m_{q_{WBT}}} Q_W + C_{m_{\dot{\alpha}_{WBT}}} \dot{\alpha}\right)\right]$$

$$M_{\delta_e} = \frac{\rho (V_T + \Delta V_T)^2 \bar{c}_{HT} S_{HT_e}}{2} \frac{\partial C_{m_{HT_e}}}{\partial \delta_e} + \frac{\rho (V_T + \Delta V_T)^2 K_{BHT} S_{HT_e}}{2}\left[\left(x_{ref} - x_{AC_{HT}}\right)\left(\frac{\partial C_{L_{HT_e}}}{\partial \delta_e}\cos(\alpha - \epsilon) + \frac{\partial C_{D_{HT_e}}}{\partial \delta_e}\sin(\alpha - \epsilon)\right) + \left(z_{ref} - z_{AC_{HT}}\right)\left(\frac{\partial C_{L_{HT_e}}}{\partial \delta_e}\sin(\alpha - \epsilon) - \frac{\partial C_{D_{HT_e}}}{\partial \delta_e}\cos(\alpha - \epsilon)\right)\right]$$

$$M_{\delta_W} = \frac{\rho V_T^2}{2}\left[\left(x_{ref} - x_{AC_W}\right) S_{W_e} K_{WB}\left(\frac{\partial C_{L_{W_e}}}{\partial \delta_W}\cos\alpha + \frac{\partial C_{D_{W_e}}}{\partial \delta_W}\sin\alpha\right) + \left(z_{ref} - z_{AC_W}\right) S_{W_e} K_{WB}\left(\frac{\partial C_{L_{W_e}}}{\partial \delta_W}\sin\alpha - \frac{\partial C_{D_{W_e}}}{\partial \delta_W}\cos\alpha\right) + \frac{\partial C_{m_{W_e}}}{\partial \delta_W}\bar{c}_W S_{W_e}\right]$$

$$M_{\delta_{HT}} = \frac{\rho (V_T + \Delta V_T)^2 \bar{c}_{HT} S_{HT_e}}{2} \frac{\partial C_{m_{HT_e}}}{\partial \delta_{HT}} + \frac{\rho (V_T + \Delta V_T)^2 K_{BHT} S_{HT_e}}{2}\left[\left(x_{ref} - x_{AC_{HT}}\right)\left(\frac{\partial C_{L_{HT_e}}}{\partial \delta_{HT}}\cos(\alpha - \epsilon) + \frac{\partial C_{D_{HT_e}}}{\partial \delta_{HT}}\sin(\alpha - \epsilon)\right) + \left(z_{ref} - z_{AC_{HT}}\right)\left(\frac{\partial C_{L_{HT_e}}}{\partial \delta_{HT}}\sin(\alpha - \epsilon) - \frac{\partial C_{D_{HT_e}}}{\partial \delta_{HT}}\cos(\alpha - \epsilon)\right)\right]$$

$$\left(M_{\omega_j^2}\right)_W = \frac{\rho V_T^2 K_{WB} S_{W_e}}{2}\left[\left(x_{ref} - x_{AC_W}\right)\left(\frac{\partial C_{L_{W_e}}}{\partial \omega_j^2}\cos\alpha + \frac{\partial C_{D_{W_e}}}{\partial \omega_j^2}\sin\alpha\right) + \left(z_{ref} - z_{AC_W}\right)\left(\frac{\partial C_{L_{W_e}}}{\partial \omega_j^2}\sin\alpha - \frac{\partial C_{D_{W_e}}}{\partial \omega_j^2}\cos\alpha\right)\right]$$

$$\left(M_{\omega_j^2}\right)_{HT} = \frac{\rho (V_T + \Delta V_T)^2 K_{BHT} S_{HT_e}}{2}\left[\left(x_{ref} - x_{AC_{HT}}\right)\left(\frac{\partial C_{L_{HT_e}}}{\partial \omega_j^2}\cos(\alpha - \epsilon) + \frac{\partial C_{D_{HT_e}}}{\partial \omega_j^2}\sin(\alpha - \epsilon)\right) + \left(z_{ref} - z_{AC_{HT}}\right)\left(\frac{\partial C_{L_{HT_e}}}{\partial \omega_j^2}\sin(\alpha - \epsilon) - \frac{\partial C_{D_{HT_e}}}{\partial \omega_j^2}\cos(\alpha - \epsilon)\right)\right]$$

The remaining terms are null derivatives,

$$M_{P_W} = M_{R_W} = M_{\phi} = M_{\theta} = M_{\psi} = M_{\delta_r} = M_{\delta_{a_L}} = M_{\delta_{a_R}} = 0$$

Regarding the derivatives in relation to state derivatives vector, we will consider only $M_{\dot{\alpha}}$ to be nonzero.

$$M_{\dot{\alpha}} = \frac{\rho V_T S_W \bar{c}^2}{4} C_{m_{\dot{\alpha}_{WBT}}}$$

Finally we define the aircraft yawing moment coefficient. We have the static portion $C_{n_{WBT}}$ and in the dynamic portion $C_{n_{p_{WBT}}}$ the aircraft yawing moment coefficient derivative with respect to P_W, $C_{n_{r_{WBT}}}$ the aircraft yawing moment coefficient derivative with respect to R_W and $C_{n_{\dot{\beta}_{WBT}}}$ the aircraft yawing moment coefficient derivative with respect to $\dot{\beta}$.

$$C_n = C_{n_{WBT}} + C_{n_{p_{WBT}}} \frac{b}{2V_T} P_W + C_{n_{r_{WBT}}} \frac{b}{2V_T} R_W + C_{n_{\dot{\beta}_{WBT}}} \frac{b}{2V_T} \dot{\beta}$$

Expanding the wing-body-tail static yawing coefficient, we find $C_{n_{\beta_{WBT}}}$ the aircraft yawing moment coefficient derivative in relation to β, $C_{n_{\delta_a}}$ the aircraft yawing moment coefficient derivative in relation to ailerons deflection, $C_{n_{\delta_r}}$ the aircraft yawing moment coefficient derivative in relation to rudder deflection, ΔC_{n_p} the yawing moment coefficient increment due to propellers wake and $C_{n_{LG}}$ the landing gear yawing moment coefficient.

$$C_{n_{WBT}} = C_{n_{\beta_{WBT}}} \beta + C_{n_{\delta_a}} \frac{\left(\delta_{a_L} - \delta_{a_R}\right)}{2} + C_{n_{\delta_r}} \delta_r + \Delta C_{n_p} + C_{n_{LG}}$$

Resulting in the total yawing moment,

$$N = \frac{\rho V_T^2 S_W b}{2} \left(C_{n_{\beta_{WBT}}} \beta + C_{n_{\delta_a}} \frac{\left(\delta_{a_L} - \delta_{a_R}\right)}{2} + C_{n_{\delta_r}} \delta_r + \Delta C_{n_p} \right)$$
$$+ \frac{\rho V_T S_W b^2}{4} \left(C_{n_{p_{WBT}}} P_W + C_{n_{r_{WBT}}} R_W + C_{n_{\dot{\beta}_{WBT}}} \dot{\beta} \right) + (N)_{LG}$$

And now we find the derivatives with respect to the state vector $\vec{X}$,

$$N_{V_T} = \rho V_T S_W b \left(C_{n_{\beta_{WBT}}} \beta + C_{n_{\delta_a}} \frac{\left(\delta_{a_L} - \delta_{a_R}\right)}{2} + C_{n_{\delta_r}} \delta_r + \Delta C_{n_p} \right)$$
$$+ \frac{\rho S_W b^2}{4} \left(C_{n_{p_{WBT}}} P_W + C_{n_{r_{WBT}}} R_W + C_{n_{\dot{\beta}_{WBT}}} \dot{\beta} \right) + + \left(N_{V_T}\right)_{LG}$$

$$N_\beta = \frac{\rho V_T^2 S_W b}{2} C_{n_{\beta_{WBT}}} + \left(N_\beta\right)_{LG}$$

$$N_{\alpha} = \frac{\rho V_T^2 S_W b}{2} \frac{\partial \Delta C_{n_p}}{\partial \alpha} + (N_{\alpha})_{LG}$$

$$N_{P_W} = \frac{\rho V_T S_W b^2}{4} C_{n_{p_{WBT}}}$$

$$N_{R_W} = \frac{\rho V_T S_W b^2}{4} C_{n_{r_{WBT}}}$$

$$N_h = \frac{\partial \rho}{\partial h} \left[\frac{V_T^2 S_W b}{2} \left(C_{n_{\beta_{WBT}}} \beta + C_{n_{\delta_a}} \frac{\left(\delta_{a_L} - \delta_{a_R}\right)}{2} + C_{n_{\delta_r}} \delta_r + \Delta C_{n_p} \right) + \frac{V_T S_W b^2}{4} \left(C_{n_{p_{WBT}}} P_W + C_{n_{r_{WBT}}} R_W + C_{n_{\dot{\beta}_{WBT}}} \dot{\beta} \right) \right] + (N_h)_{LG}$$

$$N_{\delta_r} = \frac{\rho V_T^2 S_W b}{2} C_{n_{\delta_r}}$$

$$N_{\delta_{a_L}} = \frac{\rho V_T^2 S_W b}{2} \left(\frac{C_{n_{\delta_a}}}{2} + \frac{\partial \Delta C_{n_p}}{\partial \delta_{a_L}} \right)$$

$$N_{\delta_{a_R}} = \frac{\rho V_T^2 S_W b}{2} \left(-\frac{C_{n_{\delta_a}}}{2} + \frac{\partial \Delta C_{n_p}}{\partial \delta_{a_R}} \right)$$

$$N_{\omega_j^2} = \frac{\rho V_T^2 S_W b}{2} \frac{\partial \Delta C_{n_p}}{\partial \omega_j^2}$$

The remaining terms are null derivatives,

$$N_{Q_W} = N_{\phi} = N_{\theta} = N_{\psi} = N_{\delta_f} = N_{\delta_e} = N_{\delta_W} = N_{\delta_{HT}} = 0$$

Regarding the derivatives in relation to state derivatives vector, we will consider only $N_{\dot{\beta}}$ to be nonzero.

$$N_{\dot{\beta}} = \frac{\rho V_T S_W b^2 C_{n_{\dot{\beta}_{WBT}}}}{4}$$

6 LANDING GEAR AERODYNAMIC FORCES AND MOMENTS

For the computation of landing gear aerodynamic forces and moments we will divide it in legs and wheels, and calculate then for each part. Moreover, we start by defining the longitudinal and lateral components of velocity vector.

$$V_{long} = V_T \cos\beta$$

$$V_{lat} = V_T \sin\beta$$

Beginning with the forces on the legs, which we will consider cylinders, we have a representation of the aerodynamic forces coefficients in Figure 13. Where the parameters are: λ landing gear leg longitudinal angle in relation to the vertical axis, α' angle between the landing gear leg and longitudinal velocity vector, α aircraft angle of attack, l landing gear leg length, d leg diameter.

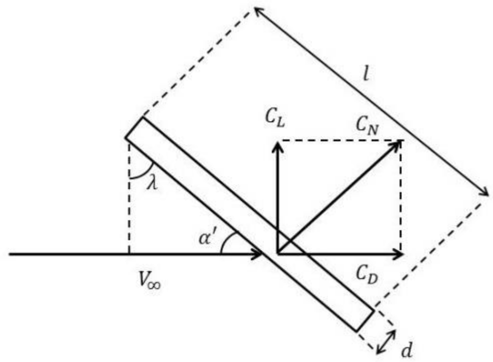


Figure 13: Aerodynamic forces coefficients on landing gear leg.

Defining α',

$$\alpha' = \alpha + \frac{\pi}{2} - \lambda$$

So we have to calculate the lift and drag coefficient of the landing gear leg. Therefore we start with the definition of friction coefficient from Hoerner (1965),

$$C_f = \frac{0.427}{(\log_{10} Re - 0.407)^{2.64}}$$

Being that Re is the Reynolds number acting on the landing gear leg on the longitudinal or lateral direction. Wherein, ν the kinematic air viscosity.

$$(Re)_{long} = \frac{V_{long} d}{\nu}$$

$$(Re)_{lat} = \frac{V_{lat} d}{\nu}$$

Additionally we need the drag coefficient of cylinders as a function of Reynolds number, which we find in Houghton and Carpenter (2003), shown in Figure 14.

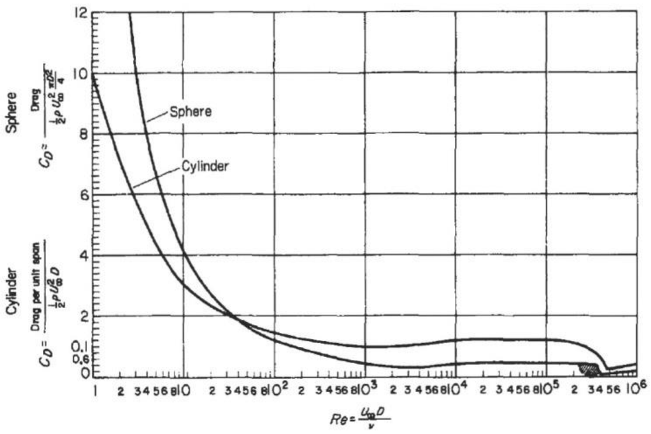


Figure 14: Drag coefficient as a function of Reynolds number for cylinder and sphere (HOUGHTON; CARPENTER, 2003).

In order to adjust the coefficient for the global reference of forces we make the following transformations,

$$\left(C_{D0_{long}}\right)_{global} = C_{D0_{long}} \frac{V_{long}^2}{V_T^2} = C_{D0_{long}} \cos^2 \beta$$

$$\left(C_{D0_{lat}}\right)_{global} = C_{D0_{lat}} \frac{V_{lat}^2}{V_T^2} = C_{D0_{lat}} \sin^2 \beta$$

Next, we have from Hoerner (1965) the equations for the landing gear leg longitudinal lift and drag coefficient,

$$(C_D)_{long} = \left(C_{D0_{long}}\right)_{global} \sin^3(\alpha') + (\Delta C_{D0})_{long}$$

$$(C_L)_{long} = \left(C_{D0_{long}}\right)_{global} \sin^2(\alpha') \cos(\alpha')$$

$$(\Delta C_{D0})_{long} = \pi \left(C_f\right)_{long}$$

Additionally, for the lateral aerodynamic forces we find,

$$(C_Y)_{lat} = \left(C_{D0_{lat}}\right)_{global} \sin^3(\alpha'') + (\Delta C_{D0})_{lat}$$

$$(C_L)_{lat} = \left(C_{D0_{lat}}\right)_{global} \sin^2(\alpha'') \cos(\alpha'')$$

Being: λ' the landing gear leg lateral angle in relation to the vertical axis, α'' angle between the landing gear leg and lateral velocity vector.

$$\alpha'' = \frac{\pi}{2} - \lambda'$$

Then summing the coefficients we come to,

$$C_D = (C_D)_{long}$$

$$C_Y = -(C_Y)_{lat}$$

$$C_L = (C_L)_{long} + (C_L)_{lat}$$

Such that, the aerodynamic forces on the landing gear legs, that is, drag, side force and lift are,

$$D = \frac{1}{2}\rho V_T^2 l d C_D$$

$$Y = \frac{1}{2}\rho V_T^2 l d C_Y$$

$$L = \frac{1}{2}\rho V_T^2 l d C_L$$

Moreover, we define now the computation for the aerodynamics forces on the landing gear wheels. From Hoerner (1965) we find the drag coefficients for some wheels forms in Figure 15.

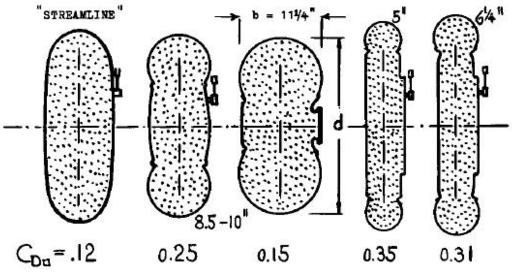


Figure 15: Drag coefficients for some aircraft wheels, (HOERNER, 1965).

And for the lateral direction we will use the definition from Hoerner (1965) for cylindrical bodies with blunt nose, in Figure 16.

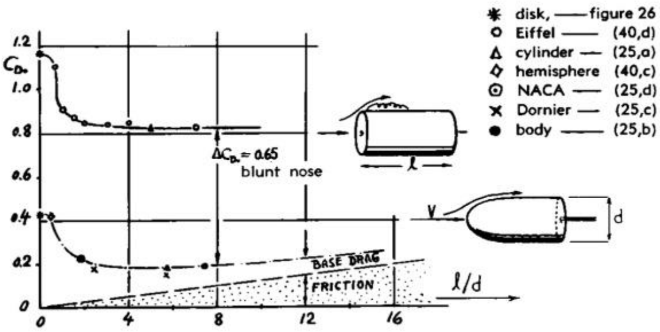


Figure 16: Drag coefficient of cylindrical bodies in axial flow, (HOERNER, 1965).

Once again, we adjust the coefficient for global reference with the following transformations,

$$\left(C_{D0_{long}}\right)_{global} = C_{D0_{long}} \frac{V_{long}^2}{V_T^2} = C_{D0_{long}} \cos^2 \beta$$

$$\left(C_{D0_{lat}}\right)_{global} = C_{D0_{lat}} \frac{V_{lat}^2}{V_T^2} = C_{D0_{lat}} \sin^2 \beta$$

Thus, we sum the terms and get the aerodynamic force coefficients for the landing gear wheels,

$$C_D = \left(C_{D0_{long}}\right)_{global}$$

$$C_Y = \left(C_{D0_{lat}}\right)_{global}$$

$$C_L = 0$$

And finally we find the aerodynamics forces on the wheels, where w is the landing gear wheel width and d_w the landing gear wheel diameter.

$$D = \frac{1}{2} \rho V_T^2 w d_w C_D$$

$$Y = \frac{1}{2} \rho V_T^2 w d_w C_Y$$

$$L = \frac{1}{2}\rho V_T^2 w d_w C_L$$

It remains now for us to sum the aerodynamic forces of every landing gear leg and wheel. Therefore, we find in the wind axis reference frame the force vector $\vec{F}_{LG}^{W}$ being the sum of the aerodynamic forces on the legs and on the wheels. In the equation we find $\vec{F}_{legs}^{W}$ landing gear legs total aerodynamic forces vector, $\vec{F}_{wheels}^{W}$ landing gear wheels total aerodynamic forces vector, NLe number of landing gear legs, NWh number of landing gear wheels.

$$\vec{F}_{LG}^{W} = \sum_{i=1}^{NLe} \vec{F}_{legs}^{W} + \sum_{j=1}^{NWh} \vec{F}_{wheels}^{W} = \sum_{i=1}^{NLe} \begin{Bmatrix} -D_i \\ Y_i \\ -L_i \end{Bmatrix} + \sum_{j=1}^{NWh} \begin{Bmatrix} -D_j \\ Y_j \\ -L_j \end{Bmatrix}$$

$$= \frac{1}{2}\rho V_T^2 \sum_{i=1}^{NLe} \begin{Bmatrix} -(ldC_D)_i \\ (ldC_Y)_i \\ -(ldC_L)_i \end{Bmatrix} + \frac{1}{2}\rho V_T^2 \sum_{j=1}^{NWh} \begin{Bmatrix} -(wd_r C_D)_j \\ (wd_r C_Y)_j \\ -(wd_r C_L)_j \end{Bmatrix}$$

Expanding the terms we find,

$$\vec{F}_{LG}^{W} = \frac{1}{2}\rho V_T^2 \left(\sum_{i=1}^{NLe} \begin{Bmatrix} -\left(ld\left[C_{D0_{long}} \cos^2\beta \sin^3\left(\alpha + \frac{\pi}{2} - \lambda\right) + \pi\left(C_f\right)_{long}\right]\right)_i \\ -\left(ld\left[C_{D0_{lat}} \sin^2\beta \sin^3\left(\frac{\pi}{2} - \lambda'\right) + \pi\left(C_f\right)_{lat}\right]\right)_i \\ -\left(ld\left[C_{D0_{long}} \cos^2\beta \sin^2\left(\alpha + \frac{\pi}{2} - \lambda\right)\cos\left(\alpha + \frac{\pi}{2} - \lambda\right) + C_{D0_{lat}} \sin^2\beta \sin^2\left(\frac{\pi}{2} - \lambda'\right)\cos\left(\frac{\pi}{2} - \lambda'\right)\right]\right)_i \end{Bmatrix} + \sum_{j=1}^{NWh} \begin{Bmatrix} -\left(wd_r C_{D0_{long}}\right)_j \cos^2\beta \\ \left(wd_r C_{D0_{lat}}\right)_j \sin^2\beta \\ 0 \end{Bmatrix} \right)$$

As for the aerodynamic moments acting on the aircraft due to the landing gear forces, we just have to define the distance of the aerodynamics forces on the landing gear components to the aircraft moment reference point and compute the moments, like in the following equation. We define: $\vec{r}_{ref}$ the position vector of aircraft moment reference point, $\vec{r}_i$ position vector of landing gear component aerodynamic forces center, $\vec{F}_{legs_i}^{W}$ landing gear leg aerodynamic forces vector, $\vec{F}_{wheels_i}^{W}$ landing gear wheel aerodynamic forces vector.

$$\vec{M}_{LG}^{W} = \sum_{i=1}^{NLe} \left[\left(\vec{r}_{ref} - \vec{r}_i\right) \times \vec{F}_{legs_i}^{W}\right] + \sum_{i=1}^{NWh} \left[\left(\vec{r}_{ref} - \vec{r}_i\right) \times \vec{F}_{wheels_i}^{W}\right]$$

Furthermore, for the aircraft flight dynamics modeling we will need the derivatives of the landing gear aerodynamic forces and moments with respect to the state vector $\vec{X}$ and state derivatives vector $\dot{\vec{X}}$, thus we will do it next.

Beginning with the landing gear aerodynamic forces derivatives with respect to V_T.

$$\frac{\partial \vec{F}_{LG}^{W}}{\partial V_T} = \begin{Bmatrix} -\left(D_{V_T}\right)_{LG} \\ \left(Y_{V_T}\right)_{LG} \\ -\left(L_{V_T}\right)_{LG} \end{Bmatrix}$$

Having the forces derivatives,

$$\begin{aligned}\left(D_{V_T}\right)_{LG} = \rho V_T \Bigg(& \sum_{i=1}^{NLe} \left(ld \left[C_{D0_{long}} \cos^2\beta \sin^3\left(\alpha + \frac{\pi}{2} - \lambda\right) + \pi \left(C_f\right)_{long} \right] \right)_i \\ & + \sum_{j=1}^{NWh} \left(wd_r C_{D0_{long}} \right)_j \cos^2\beta \Bigg) \\ & + \frac{1}{2}\rho V_T^2 \left(\sum_{i=1}^{NLe} \left(ld \left[\frac{\partial C_{D0_{long}}}{\partial V_T} \cos^2\beta \sin^3\left(\alpha + \frac{\pi}{2} - \lambda\right) + \pi \frac{\partial \left(C_f\right)_{long}}{\partial V_T} \right] \right)_i \right. \\ & \left. + \sum_{j=1}^{NWh} \left(wd_r \frac{\partial C_{D0_{long}}}{\partial V_T} \right)_j \cos^2\beta \right)\end{aligned}$$

$$\begin{aligned}\left(Y_{V_T}\right)_{LG} = \rho V_T \Bigg(& -\sum_{i=1}^{NLe} \left(ld \left[C_{D0_{lat}} \sin^2\beta \sin^3\left(\frac{\pi}{2} - \lambda'\right) + \pi \left(C_f\right)_{lat} \right] \right)_i \\ & + \sum_{j=1}^{NWh} \left(wd_r C_{D0_{lat}} \right)_j \sin^2\beta \Bigg) \\ & + \frac{1}{2}\rho V_T^2 \left(-\sum_{i=1}^{NLe} \left(ld \left[\frac{\partial C_{D0_{lat}}}{\partial V_T} \sin^2\beta \sin^3\left(\frac{\pi}{2} - \lambda'\right) + \pi \frac{\partial \left(C_f\right)_{lat}}{\partial V_T} \right] \right)_i \right. \\ & \left. + \sum_{j=1}^{NWh} \left(wd_r \frac{\partial C_{D0_{lat}}}{\partial V_T} \right)_j \sin^2\beta \right)\end{aligned}$$

$$\left(L_{V_T}\right)_{LG} = \rho V_T \left(\sum_{i=1}^{NLe} \left(ld \left[C_{D0_{long}} \cos^2\beta \sin^2\left(\alpha + \frac{\pi}{2} - \lambda\right) \cos\left(\alpha + \frac{\pi}{2} - \lambda\right) + C_{D0_{lat}} \sin^2\beta \sin^2\left(\frac{\pi}{2} - \lambda'\right) \cos\left(\frac{\pi}{2} - \lambda'\right)\right]\right)_i\right) + \frac{1}{2}\rho V_T^2 \left(\sum_{i=1}^{NLe} \left(ld \left[\frac{\partial C_{D0_{long}}}{\partial V_T} \cos^2\beta \sin^2\left(\alpha + \frac{\pi}{2} - \lambda\right) \cos\left(\alpha + \frac{\pi}{2} - \lambda\right) + \frac{\partial C_{D0_{lat}}}{\partial V_T} \sin^2\beta \sin^2\left(\frac{\pi}{2} - \lambda'\right) \cos\left(\frac{\pi}{2} - \lambda'\right)\right]\right)_i\right)$$

Wherein,

$$\frac{\partial C_{D0_{long}}}{\partial V_T} = \frac{\partial C_{D0_{long}}}{\partial (Re)_{long}} \frac{\partial (Re)_{long}}{\partial V_T} = \frac{\partial C_{D0_{long}}}{\partial (Re)_{long}} \frac{\cos\beta\, d}{\nu}$$

$$\frac{\partial C_{D0_{lat}}}{\partial V_T} = \frac{\partial C_{D0_{lat}}}{\partial (Re)_{lat}} \frac{\partial (Re)_{lat}}{\partial V_T} = \frac{\partial C_{D0_{lat}}}{\partial (Re)_{lat}} \frac{\sin\beta\, d}{\nu}$$

$$\frac{\partial \left(C_f\right)_{long}}{\partial V_T} = \frac{\partial \left(C_f\right)_{long}}{\partial (Re)_{long}} \frac{\partial (Re)_{long}}{\partial V_T} = \frac{\partial \left(C_f\right)_{long}}{\partial (Re)_{long}} \frac{\cos\beta\, d}{\nu}$$

$$\frac{\partial \left(C_f\right)_{lat}}{\partial V_T} = \frac{\partial \left(C_f\right)_{lat}}{\partial (Re)_{lat}} \frac{\partial (Re)_{lat}}{\partial V_T} = \frac{\partial \left(C_f\right)_{lat}}{\partial (Re)_{lat}} \frac{\sin\beta\, d}{\nu}$$

Additionally,

$$\frac{\partial \left(C_f\right)_{long}}{\partial (Re)_{long}} = -\frac{0.4896}{(Re)_{long}\left(\log_{10}(Re)_{long} - 0.407\right)^{4.28}}$$

$$\frac{\partial \left(C_f\right)_{lat}}{\partial (Re)_{lat}} = -\frac{0.4896}{(Re)_{lat}(\log_{10}(Re)_{lat} - 0.407)^{4.28}}$$

Derivative with respect to β:

$$\frac{\partial \vec{F}_{LG}^{W}}{\partial \beta} = \begin{Bmatrix} -\left(D_\beta\right)_{LG} \\ \left(Y_\beta\right)_{LG} \\ -\left(L_\beta\right)_{LG} \end{Bmatrix}$$

And the forces derivatives,

$$\left(D_\beta\right)_{LG} = \frac{1}{2}\rho V_T^2 \left(\sum_{i=1}^{NLe}\left\{\left(ld\left[\left(\frac{\partial C_{D0_{long}}}{\partial\beta}\cos^2\beta - C_{D0_{long}}\sin(2\beta)\right)\sin^3\left(\alpha+\frac{\pi}{2}-\lambda\right) + \pi\frac{\partial\left(C_f\right)_{long}}{\partial\beta}\right]\right)_i\right\} + \sum_{j=1}^{NWh}\left[wd_r\left(\frac{\partial C_{D0_{long}}}{\partial\beta}\cos^2\beta - C_{D0_{long}}\sin(2\beta)\right)\right]_j\right)$$

$$\left(Y_\beta\right)_{LG} = \frac{1}{2}\rho V_T^2 \left(-\sum_{i=1}^{NLe}\left(ld\left[\left(\frac{\partial C_{D0_{lat}}}{\partial\beta}\sin^2\beta + C_{D0_{lat}}\sin(2\beta)\right)\sin^3\left(\frac{\pi}{2}-\lambda'\right) + \pi\frac{\partial\left(C_f\right)_{lat}}{\partial\beta}\right]\right)_i + \sum_{j=1}^{NWh}\left[wd_r\left(\frac{\partial C_{D0_{lat}}}{\partial\beta}\sin^2\beta + C_{D0_{lat}}\sin(2\beta)\right)\right]_j\right)$$

$$\left(L_\beta\right)_{LG} = \frac{1}{2}\rho V_T^2 \sum_{i=1}^{NLe}\left\{\left(ld\left[\left(\frac{\partial C_{D0_{long}}}{\partial\beta}\cos^2\beta - C_{D0_{long}}\sin(2\beta)\right)\sin^2\left(\alpha+\frac{\pi}{2}-\lambda\right)\cos\left(\alpha+\frac{\pi}{2}-\lambda\right) + \left(\frac{\partial C_{D0_{lat}}}{\partial\beta}\sin^2\beta + C_{D0_{lat}}\sin(2\beta)\right)\sin^2\left(\frac{\pi}{2}-\lambda'\right)\cos\left(\frac{\pi}{2}-\lambda'\right)\right]\right)_i\right\}$$

Wherein,

$$\frac{\partial C_{D0_{long}}}{\partial\beta} = \frac{\partial C_{D0_{long}}}{\partial(Re)_{long}}\frac{\partial(Re)_{long}}{\partial\beta} = -\frac{\partial C_{D0_{long}}}{\partial(Re)_{long}}\frac{V_T d}{\nu}\sin\beta$$

$$\frac{\partial C_{D0_{lat}}}{\partial\beta} = \frac{\partial C_{D0_{lat}}}{\partial(Re)_{lat}}\frac{\partial(Re)_{lat}}{\partial\beta} = \frac{\partial C_{D0_{lat}}}{\partial(Re)_{lat}}\frac{V_T d}{\nu}\cos\beta$$

$$\frac{\partial\left(C_f\right)_{long}}{\partial\beta} = \frac{\partial\left(C_f\right)_{long}}{\partial(Re)_{long}}\frac{\partial(Re)_{long}}{\partial\beta} = -\frac{\partial\left(C_f\right)_{long}}{\partial(Re)_{long}}\frac{V_T d}{\nu}\sin\beta$$

$$\frac{\partial\left(C_f\right)_{lat}}{\partial\beta} = \frac{\partial\left(C_f\right)_{lat}}{\partial(Re)_{lat}}\frac{\partial(Re)_{lat}}{\partial\beta} = \frac{\partial\left(C_f\right)_{lat}}{\partial(Re)_{lat}}\frac{V_T d}{\nu}\cos\beta$$

Next the derivative with respect to α:

$$\frac{\partial \vec{F}_{LG}^{W}}{\partial \alpha} = \begin{Bmatrix} -(D_\alpha)_{LG} \\ (Y_\alpha)_{LG} \\ -(L_\alpha)_{LG} \end{Bmatrix}$$

$$= \frac{1}{2}\rho V_T^2 \left(\sum_{i=1}^{NPe} \begin{Bmatrix} -\left(ld \left[3C_{D0_{long}} \cos^2\beta \sin^2\left(\alpha + \frac{\pi}{2} - \lambda\right) \right] \right)_i \\ 0 \\ -\left(ldC_{D0_{long}} \cos^2\beta \left[\sin(2\alpha + \pi - 2\lambda) - \sin^3\left(\alpha + \frac{\pi}{2} - \lambda\right) \right] \right)_i \end{Bmatrix} \right)$$

Derivative with respect to h:

$$\frac{\partial \vec{F}_{LG}^{W}}{\partial h} = \begin{Bmatrix} -(D_h)_{LG} \\ (Y_h)_{LG} \\ -(L_h)_{LG} \end{Bmatrix}$$

And the forces derivatives,

$$(D_h)_{LG} = \frac{1}{2}\frac{\partial \rho}{\partial h} V_T^2 \left(\sum_{i=1}^{NLe} \left(ld \left[C_{D0_{long}} \cos^2\beta \sin^3\left(\alpha + \frac{\pi}{2} - \lambda\right) + \pi (C_f)_{long} \right] \right)_i + \sum_{j=1}^{NWh} \left(wd_r C_{D0_{long}} \right)_j \cos^2\beta \right)$$

$$(Y_h)_{LG} = \frac{1}{2}\frac{\partial \rho}{\partial h} V_T^2 \left(-\sum_{i=1}^{NLe} \left(ld \left[C_{D0_{lat}} \sin^2\beta \sin^3\left(\frac{\pi}{2} - \lambda'\right) + \pi (C_f)_{lat} \right] \right)_i + \sum_{j=1}^{NWh} \left(wd_r C_{D0_{lat}} \right)_j \sin^2\beta \right)$$

$$(L_h)_{LG} = \frac{1}{2}\frac{\partial \rho}{\partial h} V_T^2 \left(\sum_{i=1}^{NLe} \left(ld \left[C_{D0_{long}} \cos^2\beta \sin^2\left(\alpha + \frac{\pi}{2} - \lambda\right) \cos\left(\alpha + \frac{\pi}{2} - \lambda\right) + C_{D0_{lat}} \sin^2\beta \sin^2\left(\frac{\pi}{2} - \lambda'\right) \cos\left(\frac{\pi}{2} - \lambda'\right) \right] \right) \right)$$

All the remaining state vector derivatives are zeros,

$$\frac{\partial \vec{F}_{LG}^{W}}{\partial P_W} = \frac{\partial \vec{F}_{LG}^{W}}{\partial Q_W} = \frac{\partial \vec{F}_{LG}^{W}}{\partial R_W} = \frac{\partial \vec{F}_{LG}^{W}}{\partial \phi} = \frac{\partial \vec{F}_{LG}^{W}}{\partial \theta} = \frac{\partial \vec{F}_{LG}^{W}}{\partial \psi} = \frac{\partial \vec{F}_{LG}^{W}}{\partial \delta_i} = \frac{\partial \vec{F}_{LG}^{W}}{\partial \omega_j^2} = 0$$

Now for the aerodynamic moments, we find the derivative with respect to V_T:

$$\frac{\partial \vec{M}_{LG}^{W}}{\partial V_T} = \sum_{i=1}^{NP} \left[\left(\vec{r}_{ref} - \vec{r}_i \right) \times \frac{\partial \vec{F}_{leg_i}^{W}}{\partial V_T} \right] + \sum_{i=1}^{NRo} \left[\left(\vec{r}_{ref} - \vec{r}_i \right) \times \frac{\partial \vec{F}_{wheel_i}^{W}}{\partial V_T} \right]$$

Derivative with respect to β:

$$\frac{\partial \vec{M}_{LG}^{W}}{\partial \beta} = \sum_{i=1}^{NP} \left[\left(\vec{r}_{ref} - \vec{r}_i \right) \times \frac{\partial \vec{F}_{leg_i}^{W}}{\partial \beta} \right] + \sum_{i=1}^{NRo} \left[\left(\vec{r}_{ref} - \vec{r}_i \right) \times \frac{\partial \vec{F}_{wheel_i}^{W}}{\partial \beta} \right]$$

Derivative with respect to α:

$$\frac{\partial \vec{M}_{LG}^{W}}{\partial \alpha} = \sum_{i=1}^{NP} \left[\left(\vec{r}_{ref} - \vec{r}_i \right) \times \frac{\partial \vec{F}_{leg_i}^{W}}{\partial \alpha} \right] + \sum_{i=1}^{NRo} \left[\left(\vec{r}_{ref} - \vec{r}_i \right) \times \frac{\partial \vec{F}_{wheel_i}^{W}}{\partial \alpha} \right]$$

Derivative with respect to h:

$$\frac{\partial \vec{M}_{LG}^{W}}{\partial h} = \sum_{i=1}^{NP} \left[\left(\vec{r}_{ref} - \vec{r}_i \right) \times \frac{\partial \vec{F}_{leg_i}^{W}}{\partial h} \right] + \sum_{i=1}^{NRo} \left[\left(\vec{r}_{ref} - \vec{r}_i \right) \times \frac{\partial \vec{F}_{wheel_i}^{W}}{\partial h} \right]$$

Again, all the remaining state vector derivatives are zeros,

$$\frac{\partial \vec{M}_{LG}^{W}}{\partial P_W} = \frac{\partial \vec{M}_{LG}^{W}}{\partial Q_W} = \frac{\partial \vec{M}_{LG}^{W}}{\partial R_W} = \frac{\partial \vec{M}_{LG}^{W}}{\partial \phi} = \frac{\partial \vec{M}_{LG}^{W}}{\partial \theta} = \frac{\partial \vec{M}_{LG}^{W}}{\partial \psi} = \frac{\partial \vec{M}_{LG}^{W}}{\partial \delta_i} = \frac{\partial \vec{M}_{LG}^{W}}{\partial \omega_j^2} = 0$$

Moreover, all the derivatives with respect to the state derivatives vector are zeros.

7 ENGINE AND APPENDAGES DRAG

Hoerner (1965) summarizes results of experiments on radial engines drag (WEIK, 1928), which are represented on Figure 17.

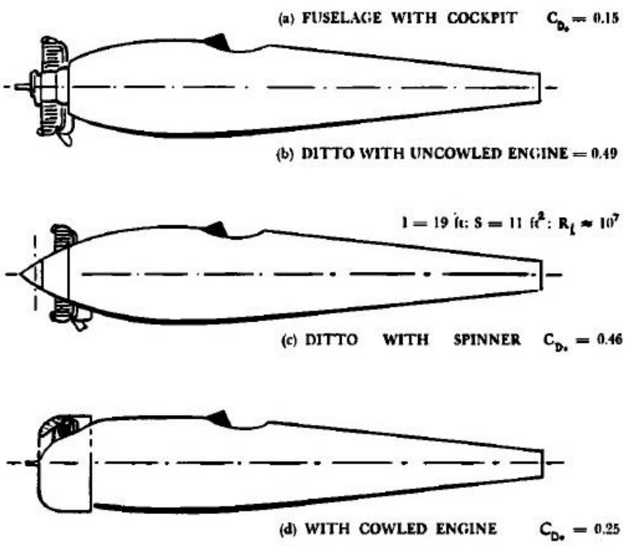


Figure 17: Radial engines drag.

Therefore, for a radial engine with spinner there is an increase on fuselage C_{D0} of 0,15 to 0,46.

$$(\Delta C_{D0})_{ra} = \frac{(0{,}46 - 0{,}15)}{0{,}15} = 2{,}067$$

However, the experiment considered a radial engine, which means that it covered the full perimeter of the fuselage cross section.

Thus, for a single or more pistons engine, is reasonable to assume that the effect of higher drag is proportional to the size of the pistons in relation to the fuselage maximum cross section perimeter.

$$(\Delta C_{D0})_{eng} = (\Delta C_{D0})_{ra} \frac{p_{pistons}}{p_{fuselage}}$$

Where, $p_{pistons}$ is the fuselage maximum cross section perimeter covered by the exposed engine pistons, and $p_{fuselage}$ is the fuselage maximum cross section perimeter.

Now, for appendage, which is the case of the gimbal, Hoerner (1965) summarizes results of experiments on drag of appendages designed to be used for armaments or other devices, which are represented on Figure 18. Drag coefficient based on added frontal area (S_{app}).

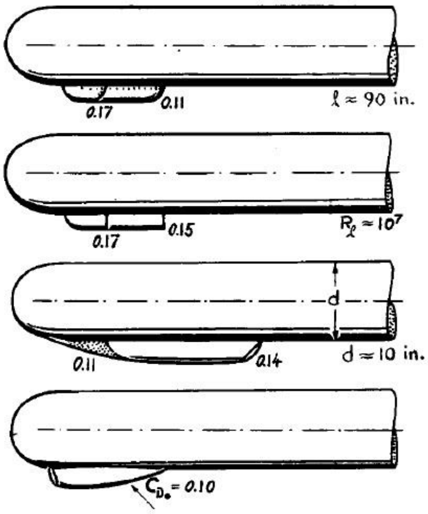


Figure 18: Appendages drag.

Camera gimbal drag can be assumed an appendage drag, closer to the form of added C_{D0} of 0,17 in the Figure 18.

Thus the added drag coefficient is given by:

$$(\Delta C_{D0})_{app} = (C_{D0})_{app} \frac{S_{app}}{S_W}$$

8 PROPELLER THRUST

Experiments on propellers are usually presented as thrust, power and torque coefficients as functions of propeller advance ratio J, which we will define now respectively C_T, C_P and C_Q, as functions of T propeller thrust, P propeller power, Q propeller torque, D propeller diameter and ρ the air density.

$$C_T = \frac{T}{\rho n^2 D^4} \quad C_P = \frac{P}{\rho n^3 D^5} \quad C_Q = \frac{Q}{\rho n^2 D^5}$$

$$J = \frac{V}{nD}$$

In the definition of advance ratio, the velocity term V is the velocity in the direction of thrust axis, and being the propeller revolution per second n relationship with propeller angular speed ω,

$$n = \frac{\omega}{2\pi}$$

Additionally we can relate the power and torque coefficients using the following,

$$C_Q = \frac{C_P}{2\pi}$$

However, for the flight dynamics modeling it is useful to describe propeller thrust as a linear function of the angular speed squared, or as in the following expression,

$$T = k_T \omega^2$$

Thus, we have the relation between propeller thrust coefficients, so that we obtain the experimental curve of C_T and make the transformation to k_T.

$$k_T = \frac{C_T \rho D^4}{(2\pi)^2}$$

And similarly to the torque coefficient,

$$Q = k_Q \omega^2$$

$$k_Q = \frac{C_Q \rho D^5}{(2\pi)^2} = \frac{C_P \rho D^5}{(2\pi)^3}$$

Furthermore, since we predict that the propeller will function at high angles of attack it is necessary to make a relationship between the propeller thrust coefficient and angle of attack. Thereby, we find in Hoak (1965) this relationship.

$$\frac{C_T(\alpha, J')}{C_T(0, J')} = 1 + \frac{3\left(\frac{J'}{J_{0_T}}\right)^2}{4\left(1 - \frac{J'}{J_{0_T}}\right)} \sin(\beta + 5) \left[\tan(\beta + 5) + \sigma_e \left(1 + \sqrt{1 + \frac{2}{\sigma_e}\tan(\beta + 5)}\right)(1 - \cos\alpha)\right] \tan^2\alpha$$

Wherein, J' the modified advance ratio, defined as below.

$$J' = J\cos\alpha$$

Also, $C_T(0, J')$ is the propeller thrust coefficient at zero angle of attack but with velocity equal to $V\cos\alpha$, β the propeller blade angle at 0,75 radius, in degrees, J_{0_T} propeller advance ratio at zero thrust, σ_e effective propeller solidity (propeller solidity based on average blade chord), defined as follows.

$$\sigma_e = \frac{4B\overline{b'}}{3\pi D}$$

Having, B number of propeller blades, b' the average propeller blade chord.

Concluding, we find the propeller thrust coefficient at specific angle of attack.

$$C_T(\alpha, J') = \frac{C_T(\alpha, J')}{C_T(0, J')} C_T(0, J')$$

9 AERODYNAMIC CONTROL DERIVATIVES

The aerodynamic control derivatives have been estimated using Hoak (1965) methods, which we will report hereafter.

9.1 Lift coefficient increment due to aerodynamic control surface deflection

The first analyzed is the lift coefficient increment due to aerodynamic control surface deflection, which is applied to elevator, rudder and ailerons deflections. For every control surface we shall use the following expressions to derive the increment in lift coefficient due to plain trailing edge control deflection with sealed gaps.

So, we have the two-dimensional lift coefficient increment Δc_l due to control deflection δ. The inner terms being $\left(c_{l_\delta}\right)_{theory}$ the theoretical control lift effectiveness for a given airfoil thickness ratio and control-chord-to-airfoil-chord ratio computed using Figure 19, $\frac{c_{l_\delta}}{\left(c_{l_\delta}\right)_{theory}}$ the empirical correction factor for lift effectiveness of plain trailing edge controls computed using Figure 20, which requires $\frac{c_{l_\alpha}}{\left(c_{l_\delta}\right)_{theory}}$ the empirical correction factor that accounts for the development of the boundary layer towards the airfoil trailing edge computed using Figure 21, and lastly K' also an empirical correction factor that corrects Δc_l for nonlinear effects at high control deflections, computed using Figure 22.

$$\Delta c_l = \delta \left(\frac{c_{l_\delta}}{\left(c_{l_\delta}\right)_{theory}} \right) \left(c_{l_\delta}\right)_{theory} K'$$

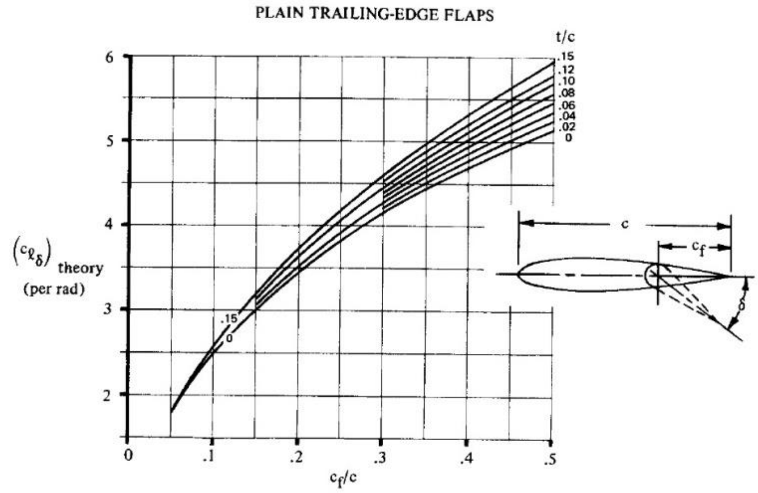


Figure 19: Theoretical control lift effectiveness for a given airfoil thickness ratio and control-chord-to-airfoil-chord ratio (HOAK, 1965).

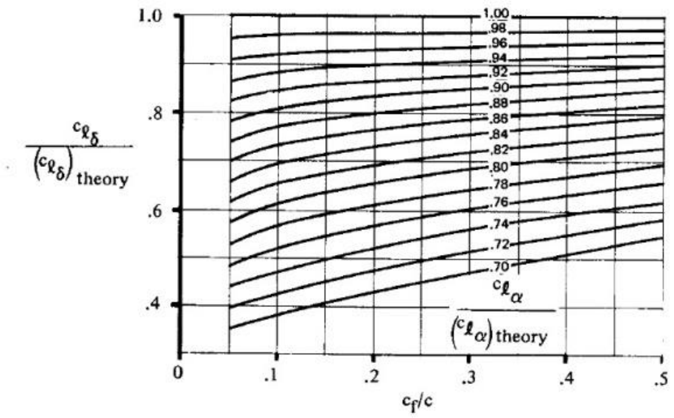


Figure 20: Empirical correction factor for lift effectiveness of plain trailing edge controls (HOAK, 1965).

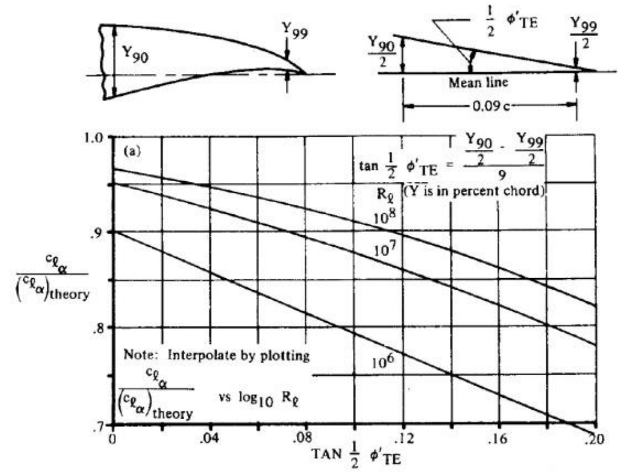


Figure 21: Empirical correction factor that accounts for the development of the boundary layer towards the airfoil trailing edge (HOAK, 1965).

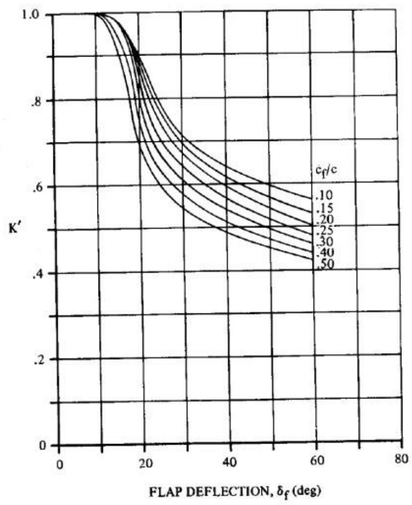


Figure 22: Empirical correction factor, corrects Δc_l for nonlinear effects at high control deflections (HOAK, 1965).

The two-dimensional lift increment due to control deflection is used to compute the three-dimensional lift coefficient increment ΔC_L of the aerodynamic surface of the respective aerodynamic control, that is, wing, horizontal or vertical tail, which is given by the following equation. At this equation we have the terms: C_{L_α} lift curve slope of the aerodynamic surface with control retracted, based on aerodynamic surface reference area, c_{l_α} section lift curve slope of the basic airfoil, including the effects of compressibility, $\frac{(\alpha_\delta)_{C_L}}{(\alpha_\delta)_{c_l}}$ ratio of the three-dimensional control effectiveness parameter to the two-dimensional control effectiveness parameter computed using Figure 23, K_b control span factor computed using Figure 24.

$$\Delta C_L = \Delta c_l \left(\frac{C_{L_\alpha}}{c_{l_\alpha}}\right)\left[\frac{(\alpha_\delta)_{C_L}}{(\alpha_\delta)_{c_l}}\right] K_b$$

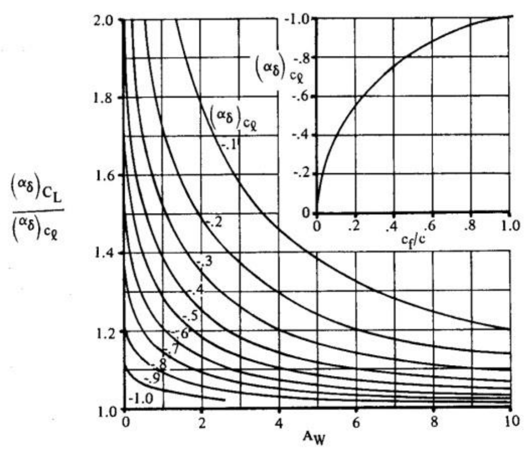


Figure 23: Control-chord factor (HOAK, 1965).

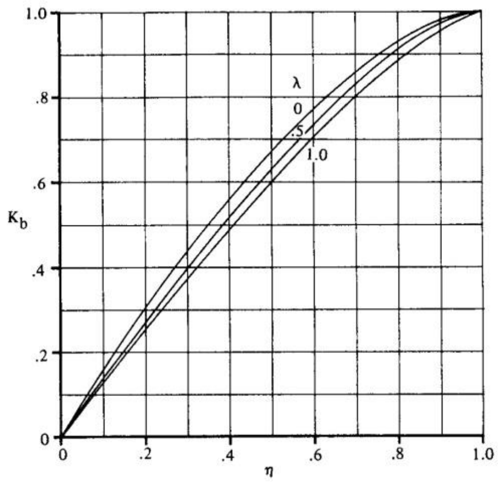


Figure 24: Control span factor (HOAK, 1965).

Additionally we need the following definition, in Figure 25, of control span in order to calculate K_b.

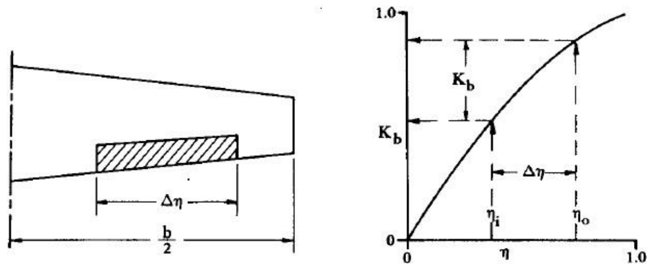


Figure 25: Definition for control span terms (HOAK, 1965).

Furthermore, the aerodynamic control surface configuration usually has an angle in relation to the direction of flight, therefore in order to correlate the control surface streamwise deflection to the deflection of the control surface hinge line, we shall use the following expression, which relates the Λ_{HL} the control surface hinge line angle with respect to the lateral axis y and $\delta_{\perp_{HL}}$ the control surface deflection measured normal to the control hinge line, to the control streamwise deflection δ.

$$\delta = \tan^{-1}\left(\cos \Lambda_{HL} \tan \delta_{\perp_{HL}}\right)$$

Rearranging the equations for two-dimensional and three dimensional lift increment we are able to compute the derivative of the lift coefficient with respect to the control surface streamwise deflection C_{L_δ}.

$$C_{L_\delta} = \left(\frac{c_{l_\delta}}{\left(c_{l_\delta}\right)_{theory}}\right)\left(c_{l_\delta}\right)_{theory} K' \left(\frac{C_{L_\alpha}}{c_{l_\alpha}}\right)\left[\frac{(\alpha_\delta)_{C_L}}{(\alpha_\delta)_{c_l}}\right] K_b$$

Thus, we have the final equation for the three dimensional lift coefficient increment.

$$\Delta C_L = C_{L_\delta}\delta$$

9.2 Lift coefficient increment due to propeller wake, wing, wing camber and aerodynamic control surface deflection combined

Next we will derive the increment in lift coefficient due to the effect of propeller wake, wing camber and control surface deflection combined that we call $(\Delta C_L)_{prop+wing+deflection}$. We will first divide the effects in lift coefficient increment due to propeller wake acting on the wing, or horizontal tail, alone $(\Delta C_L)_{camber}$ and the increment in lift coefficient due to propeller wake acting on the wing, or horizontal tail, plus control $(\Delta C_L)_{deflection}$, since there may be wing, or horizontal tail, regions affected by the propellers that might have aerodynamic controls and regions that don't. The wing, or horizontal tail, surface planform area under effect of propellers wake is $S_{W_{wake}}$, and the surface planform area under effect of propellers wake that also have aerodynamic control is $S_{W_{wake,control}}$.

$$(\Delta C_L)_{prop+wing+deflection} = (\Delta C_L)_{camber}\frac{S_{W_{wake}}}{S_{W_{wake}} + S_{W_{wake,control}}} + (\Delta C_L)_{deflection}\frac{S_{W_{wake,control}}}{S_{W_{wake}} + S_{W_{wake,control}}}$$

From Hoak (1965), we have the expressions for the increments in lift coefficient,

$$(\Delta C_L)_{camber} = 1{,}6\left(\frac{F}{T}\right)_{camber} T''_{c\,W}\sqrt{1 - T''_{c\,W}}\,\frac{K_W S_p}{S_W}\sin(\Delta\theta + \alpha_W)$$

$$(\Delta C_L)_{deflection} = 1{,}6\left(\frac{F}{T}\right)_{deflection} T''_{c\,W}\sqrt{1 - T''_{c\,W}}\,\frac{K_W S_p}{S_W}\sin\left(\theta_f + \Delta\theta + \alpha_W\right)$$

Where we have the inner terms,

$$\theta = \theta_f + \Delta\theta$$

$$\theta_f = \frac{\theta}{\delta}\delta$$

$$\Delta\theta = \frac{\theta}{\delta_e}\delta_e$$

We define: θ slipstream turning angle, θ_f slipstream turning angle under conditions of zero incidence and zero camber, $\Delta\theta$ slipstream turning angle increment due to wing camber and incidence between the wing chord plane and the thrust axis, K_W number of propellers on the wing, K_{HT} number of propellers on the horizontal tail, S_p propeller disk area, α_W wing, or horizontal tail, angle of attack, δ_e equivalent control deflection angle defined as the angle between the thrust axis and the mean camber line at the wing trailing edge, $\frac{\theta}{\delta}$ ratio obtained from Figure 26, $\frac{\theta}{\delta_e}$ ratio obtained from Figure 26, $\left(\frac{F}{T}\right)_{camber}$ thrust factor accounting for wing camber computed using Figure 27, $\left(\frac{F}{T}\right)_{deflection}$ thrust factor accounting for wing camber plus control deflection computed using Figure 27, $T_c^{"}$ propeller thrust coefficient based on slipstream velocity and propeller disk area, T thrust per propeller or total thrust when used in thrust recovery factor, $q^{"}$ slipstream dynamic pressure, q_∞ free stream dynamic pressure.

In addition we need the following equations that define $T_c^{"}$ and $q^{"}$.

$$T_c^{"} = \frac{T}{q^{"}S_p}$$

$$q^{"} = q_\infty + \frac{T}{S_p}$$

Moreover, we have the propellers thrust coefficient for the wing $T_{c\,W}^{"}$ and horizontal tail $T_{c\,HT}^{"}$,

$$T_{c\,W}^{"} = \frac{T_W}{q^{"}S_p} = \frac{T_W}{\left(q_\infty + \frac{T_W}{S_p}\right)S_p} = \frac{\sum_{j=1}^{K_W}\{\omega_j^2\}}{\sum_{j=1}^{K_W}\{\omega_j^2\} + \frac{\rho V_T^2 \pi D_p^2 K_W}{8k_T}}$$

$$T_{c\,HT}^{"} = \frac{T_{HT}}{q^{"}S_p} = \frac{T_{HT}}{\left(q_\infty + \frac{T_{HT}}{S_p}\right)S_p} = \frac{\sum_{j=1}^{K_{HT}}\{\omega_j^2\}}{\sum_{j=1}^{K_{HT}}\{\omega_j^2\} + \frac{\rho V_T^2 \pi D_p^2 K_{HT}}{8k_T}}$$

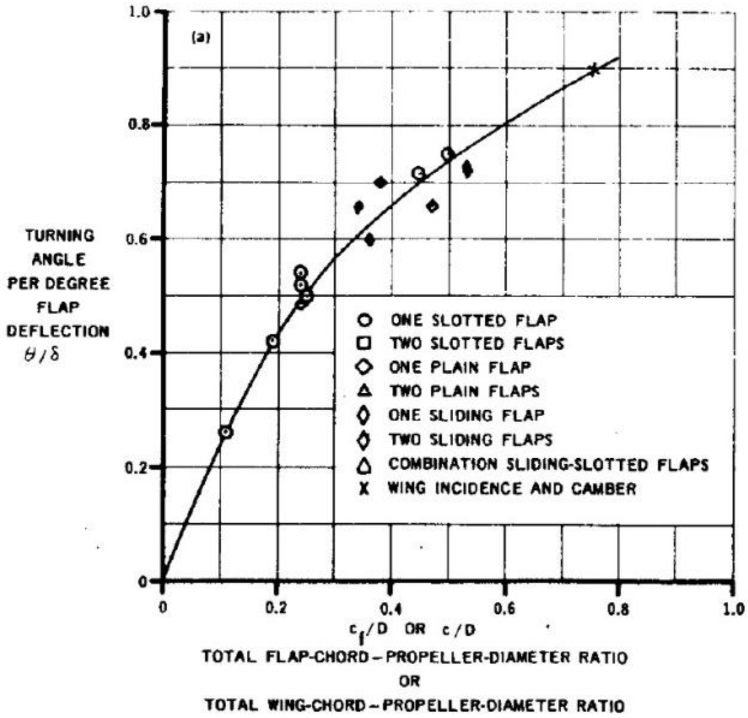


Figure 26: Variation of turning angle with control deflection (HOAK, 1965).

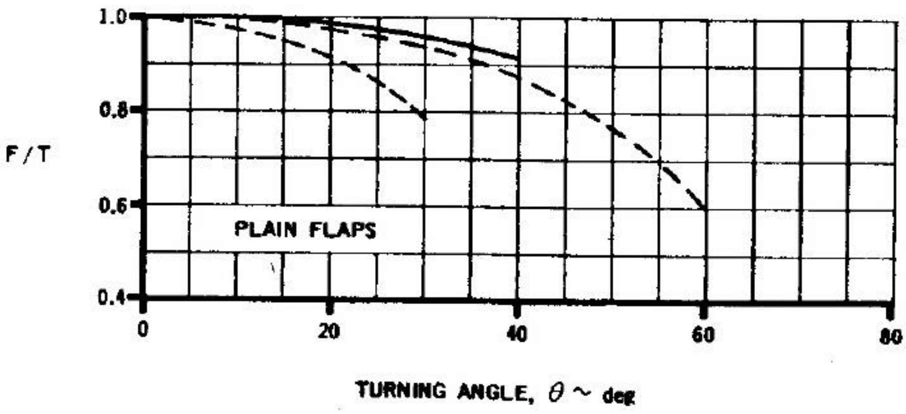


Figure 27: Thrust factor for plain trailing edge flaps (HOAK, 1965).

The procedure is valid if the turning angle obtained is over the linear part of the curve of variation of turning angle with control deflection. The maximum turning angle θ_{max} is defined in Figure 28, and must satisfy the following,

$$\frac{\theta_f}{\theta_{max}} \leq 0{,}95$$

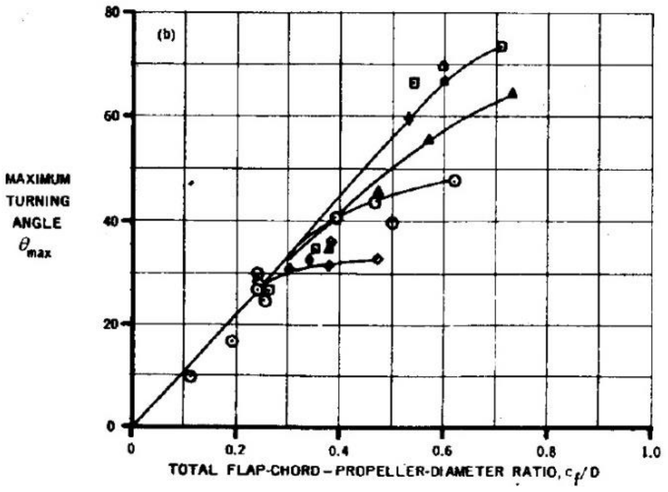


Figure 28: Maximum turning angle (HOAK, 1965).

Therefore, summing the effects on aircraft lift coefficient of all the aerodynamic control surfaces, we have,

$$\Delta C_{L_{WBT}} = (\Delta C_L)_{elevator}\frac{S_{HT_e}}{S_W} + (\Delta C_L)_{prop+wing+deflection} + (\Delta C_L)_{prop+htail+deflection}\frac{S_{HT_e}}{S_W}$$

Or yet, in the following form, using: $C_{L_{\delta_f}}$ aircraft lift coefficient derivative with respect to flap deflection, $C_{L_{\delta_e}}$ aircraft lift coefficient derivative with respect to elevator deflection, $\Delta C_{L_{p,W}}$ aircraft lift coefficient increment due to propeller-wing-flap interaction, $\Delta C_{L_{p,HT}}$ aircraft lift coefficient increment due to propeller-horizontal-elevator interaction.

$$\Delta C_{L_{WBT}} = C_{L_{\delta_f}}\delta_f + C_{L_{\delta_e}}\delta_e + \Delta C_{L_{p,W}} + \Delta C_{L_{p,HT}}$$

So that the terms are,

$$C_{L_{\delta_f}} = \left[\left(\frac{c_{l_\delta}}{\left(c_{l_\delta}\right)_{theory}}\right)\left(c_{l_\delta}\right)_{theory}K'\left(\frac{C_{L_\alpha}}{c_{l_\alpha}}\right)\left[\frac{(\alpha_\delta)_{C_L}}{(\alpha_\delta)_{c_l}}\right]K_b\right]_W\frac{S_{W_e}}{S_W}$$

$$C_{L_{\delta_e}} = \left[\left(\frac{c_{l_\delta}}{\left(c_{l_\delta}\right)_{theory}}\right)\left(c_{l_\delta}\right)_{theory}K'\left(\frac{C_{L_\alpha}}{c_{l_\alpha}}\right)\left[\frac{(\alpha_\delta)_{C_L}}{(\alpha_\delta)_{c_l}}\right]K_b\right]_{HT}\frac{S_{HT_e}}{S_W}$$

$$\Delta C_{L_{p,W}} = \frac{1{,}6T_{c\,W}^{"}\sqrt{1-T_{c\,W}^{"}}K_W S_p}{S_W\left(S_{W_{wake}}+S_{W_{wake,flap}}\right)}\left[\left(\frac{F}{T}\right)_{c_W}\sin\left(\frac{\theta}{\delta_{f_e}}\delta_{f_e}+\alpha+\delta_W\right)S_{W_{wake}} + \left(\frac{F}{T}\right)_{f_W}\sin\left(\frac{\theta}{\delta_f}\delta_f+\frac{\theta}{\delta_{f_e}}\delta_{f_e}+\alpha+\delta_W\right)S_{W_{wake,flap}}\right]$$

With, $S_{W_{wake,flap}}$ wing surface under effect of propellers wake that also have flap control surface, $\left(\frac{F}{T}\right)_{c_W}$ thrust factor accounting for wing camber, $\left(\frac{F}{T}\right)_{f_W}$ thrust factor accounting for wing camber plus flap deflection.

$$\Delta C_{L_{p,HT}} = \frac{1{,}6T_{c\,HT}^{"}\sqrt{1-T_{c\,HT}^{"}}K_{HT} S_p}{S_{HT}\left(S_{HT_{wake}}+S_{HT_{wake,elevator}}\right)}\left[\left(\frac{F}{T}\right)_{c_{HT}}\sin\left(\frac{\theta}{\delta_{e_e}}\delta_{e_e}+\alpha+\delta_{HT}-\epsilon\right)S_{HT_{wake}} + \left(\frac{F}{T}\right)_{e_{HT}}\sin\left(\frac{\theta}{\delta_e}\delta_e+\frac{\theta}{\delta_{e_e}}\delta_{e_e}+\alpha+\delta_{HT}-\epsilon\right)S_{HT_{wake,elevator}}\right]\frac{S_{HT_e}}{S_W}$$

Having, S_{HT} horizontal tail planform area, $S_{HT_{wake}}$ horizontal tail surface under effect of propellers wake, $S_{HT_{wake,elevator}}$ horizontal tail surface under effect of propellers wake that also have elevator control surface, $\left(\frac{F}{T}\right)_{c_{HT}}$ thrust factor accounting for horizontal tail camber, $\left(\frac{F}{T}\right)_{e_{HT}}$ thrust factor accounting for horizontal tail camber plus elevator deflection.

9.3 Drag coefficient increment due to propeller wake, wing, wing camber and aerodynamic control surface deflection combined

Similarly to the increment in lift coefficient due to propeller wake, wing camber and aerodynamic control surface deflection combined, we will divide the increment in drag coefficient corresponding to the increment due to propeller wake acting on the wing, or horizontal tail, alone $(\Delta C_D)_{camber}$ and the increment in drag coefficient due to propeller wake acting on the wing, or horizontal tail, plus control $(\Delta C_D)_{deflection}$.

$$(\Delta C_D)_{prop+wing+deflection} = (\Delta C_D)_{camber} \frac{S_{W_{wake}}}{S_{W_{wake}} + S_{W_{wake,control}}} + (\Delta C_D)_{deflection} \frac{S_{W_{wake,control}}}{S_{W_{wake}} + S_{W_{wake,conttrol}}}$$

We have from Hoak (1965), the expressions for the increments in drag coefficient,

$$(\Delta C_D)_{câmber} = 1{,}6\left(\frac{F}{T}\right)_{camber} T_c^{"}\sqrt{1 - T_c^{"}}\frac{K_W S_p}{S_W}[1 - \cos(\Delta\theta + \alpha_W)]$$

$$(\Delta C_D)_{deflexão} = 1{,}6\left(\frac{F}{T}\right)_{deflection} T_c^{"}\sqrt{1 - T_c^{"}}\frac{K_W S_p}{S_W}\left[1 - \cos\left(\theta_f + \Delta\theta + \alpha_W\right)\right]$$

And summing the increment in drag coefficient for the effects of propellers on wing and horizontal tail, we have,

$$\Delta C_{D_{WBT}} = \Delta C_{D_{p,W}} + \Delta C_{D_{p,HT}}$$

Remembering that there is and additional increment in induced drag because of the increment in lift coefficient, so when calculating the induced drag for the wing and horizontal tail, we must consider the increment in lift coefficient due to propellers effect.

So, applying the equations for the wing and horizontal tail we find,

$$\Delta C_{D_{p,W}} = \frac{1{,}6 T_{c\,W}^{"}\sqrt{1 - T_{c\,W}^{"}}K_W S_p}{S_W\left(S_{W_{wake}} + S_{W_{wake,flap}}\right)}\left\{\left(\frac{F}{T}\right)_{c_W}\left[1 - \cos\left(\frac{\theta}{\delta_{f_e}}\delta_{f_e} + \alpha + \delta_W\right)\right]S_{W_{wake}} + \left(\frac{F}{T}\right)_{f_W}\left[1 - \cos\left(\frac{\theta}{\delta_f}\delta_f + \frac{\theta}{\delta_{f_e}}\delta_{f_e} + \alpha + \delta_W\right)\right]S_{W_{wake,flap}}\right\}$$

$$\Delta C_{D_{p,HT}} = \frac{1{,}6 T_{c\,HT}^{"}\sqrt{1 - T_{c\,HT}^{"}}K_{HT} S_p}{S_{HT}\left(S_{HT_{wake}} + S_{HT_{wake,elevator}}\right)}\left\{\left(\frac{F}{T}\right)_{c_{Ht}}\left[1 - \cos\left(\frac{\theta}{\delta_{e_e}}\delta_{e_e} + \alpha + \delta_{Ht} - \epsilon\right)\right]S_{HT_{wake}} + \left(\frac{F}{T}\right)_{e_{HT}}\left[1 - \cos\left(\frac{\theta}{\delta_e}\delta_e + \frac{\theta}{\delta_{e_e}}\delta_{e_e} + \alpha + \delta_{Ht} - \epsilon\right)\right]S_{HT_{wake,elevator}}\right\}\frac{S_{HT_e}}{S_W}$$

9.4 Side force coefficient increment due to rudder deflection

We must use the equations for increment in lift coefficient due to aerodynamic control surface deflection previously described, to calculate the increment in side force coefficient due to rudder deflection. Therefore, we find the following equation applied to the vertical tail parameters and rudder geometry. Wherein, S_{VT_e} is the exposed vertical tail planform area.

$$(\Delta C_Y)_{rudder} = \left[\left(\frac{c_{l_\delta}}{\left(c_{l_\delta}\right)_{theory}}\right)\left(c_{l_\delta}\right)_{theory} K' \left(\frac{C_{L_\alpha}}{c_{l_\alpha}}\right)\left[\frac{(\alpha_\delta)_{C_L}}{(\alpha_\delta)_{c_l}}\right] K_b\right]_{Vtail} \delta_r \frac{S_{VT_e}}{S_W}$$

Thus we find the derivative of side force coefficient with respect to rudder deflection,

$$C_{Y_{\delta_r}} = \left[\left(\frac{c_{l_\delta}}{\left(c_{l_\delta}\right)_{theory}}\right)\left(c_{l_\delta}\right)_{theory} K' \left(\frac{C_{L_\alpha}}{c_{l_\alpha}}\right)\left[\frac{(\alpha_\delta)_{C_L}}{(\alpha_\delta)_{c_l}}\right] K_b\right]_{Vtail} \frac{S_{VT_e}}{S_W}$$

And the total effect on aircraft side force coefficient,

$$\Delta C_{Y_{WBT}} = C_{Y_{\delta_r}} \delta_r$$

9.5 Pitching moment coefficient increment due to control deflection

Once again, we have from Hoak (1965), the expressions for the increment in pitching moment coefficient due to control deflection,

$$\Delta C_{m_\delta} = \Delta C_m + K_\Lambda \left(\frac{A}{1{,}5}\right) \Delta C_{L_\delta} \tan \Lambda_{c/4}$$

Expanding the inner terms,

$$\Delta C_m = K_p \left\{\left(\frac{\Delta C'_m}{\Delta C_L}\right) \Delta C_{L_\delta} \left(\frac{c'}{c}\right)^2 - 0{,}25 C_L \left(\frac{c'}{c}\right)\left(\frac{c'}{c} - 1\right) + C_m \left[\left(\frac{c'}{c}\right)^2 - 1\right]\right\}$$

However, the term ΔC_{L_δ} must be for the particular case of full span control surface, wing aspect ratio of 6 and no sweep ($\Lambda_{c/2} = 0$). So we find,

$$\Delta C_{m_\delta} = K_p\left\{\left(\frac{\Delta C'_m}{\Delta C_L}\right)\left[C_{L_\delta}\right]_{sup\,fullspan,A=6,\Lambda_{c/2=0}}\delta\left(\frac{c'}{c}\right)^2 - 0{,}25C_L\left(\frac{c'}{c}\right)\left(\frac{c'}{c}-1\right) + C_m\left[\left(\frac{c'}{c}\right)^2 - 1\right]\right\} + K_\Lambda\left(\frac{AR_W}{1{,}5}\right)\left[C_{L_\delta}\right]_{sup\,fullspan,A=6,\Lambda_{c/2=0}}\delta\tan\Lambda_{c/4}$$

Being that, K_p control span factor as a function of taper ratio and control location computed using Figure 29, $\frac{\Delta C'_m}{\Delta C_L}$ ratio of pitching moment increment to lift increment for a full span flap on a rectangular wing computed using Figure 30, ΔC_{L_δ} lift coefficient increment due to control deflection, $\frac{c'}{c}$ ratio of extended wing chord to the retracted wing chord, C_L wing-body lift coefficient with control retracted, C_m wing-body pitching moment coefficient with control retracted, K_Λ conversion factor for a partial span control on a sweptback wing, $\Lambda_{c/4}$ sweep of the wing quarter chord computed using Figure 31.

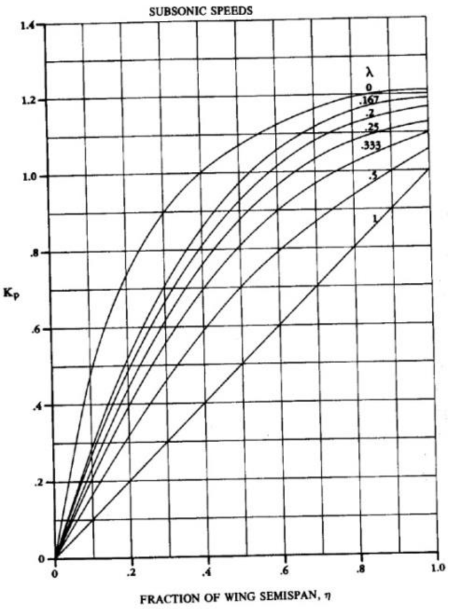


Figure 29: Conversion factor for partial span control (HOAK, 1965).

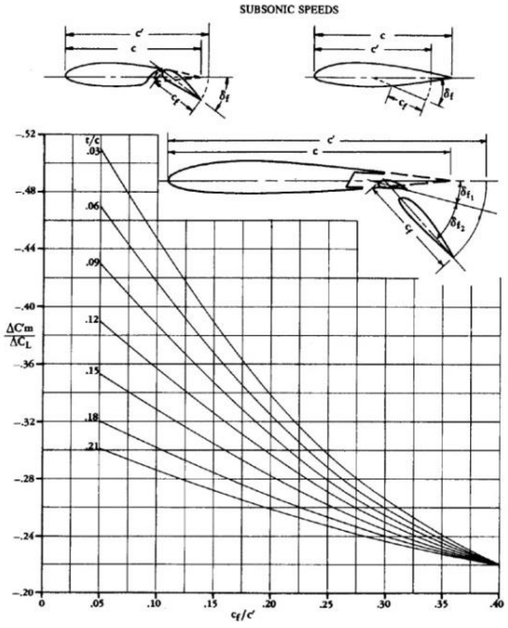
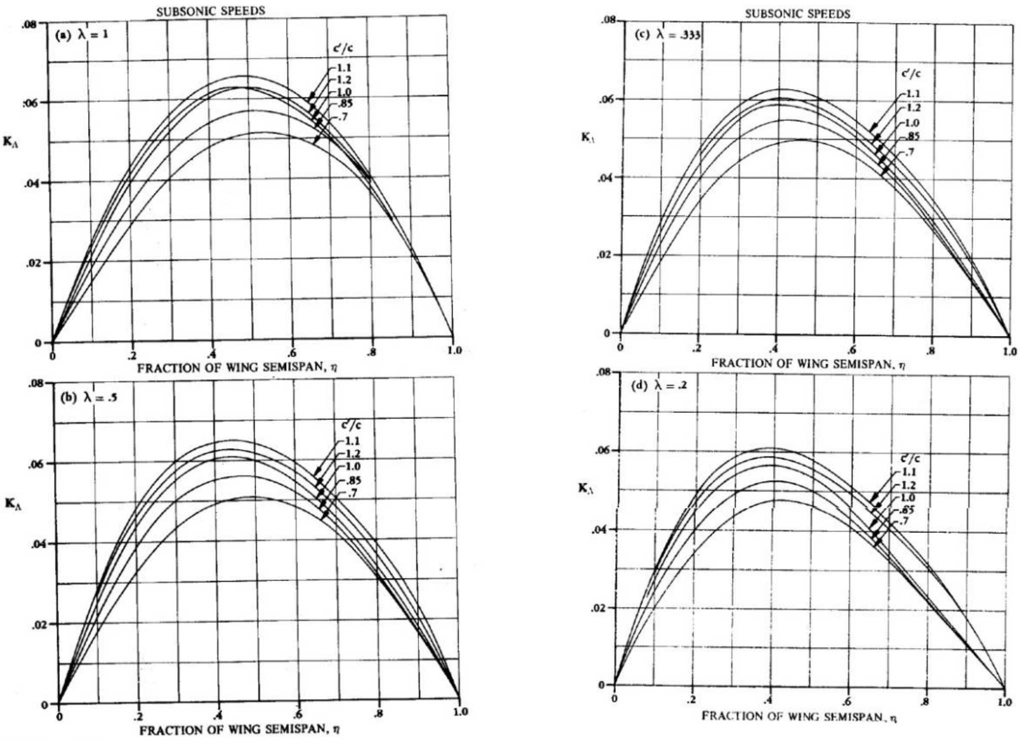


Figure 30: Ratio of pitching moment coefficient increment to lift coefficient increment due to control for unswept wings (HOAK, 1965).



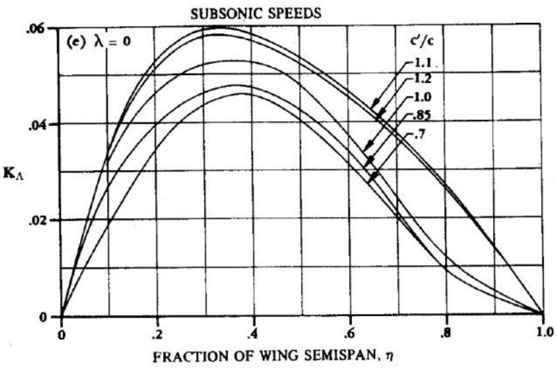


Figure 31: Span factor for swept wings (HOAK, 1965).

Thus, we apply the equations for pitching moment increment for the wing and horizontal tail,

$$\Delta C_{m_f} = \left\{ K_p \left(\frac{\Delta C'_m}{\Delta C_L}\right) \left[C_{L_{\delta_f}}\right]_{W\, fullspan, A=6, \Lambda_{c/2=0}} \left(\frac{c'}{c}\right)^2 + K_\Lambda \left(\frac{AR_W}{1,5}\right) \left[C_{L_{\delta_f}}\right]_{W\, fullspan, A=6, \Lambda_{c/2=0}} \tan \Lambda_{c/4} \right\} \frac{S_{W_e}}{S_W} \delta_f - K_p \left\{ 0,25 C_{L_{W_e}} \left(\frac{c'}{c}\right)\left(\frac{c'}{c} - 1\right) - C_{m_{W_e}} \left[\left(\frac{c'}{c}\right)^2 - 1\right] \right\} \frac{S_{W_e}}{S_W}$$

$$\Delta C_{m_e} = \left\{ K_p \left(\frac{\Delta C'_m}{\Delta C_L}\right) \left[C_{L_{\delta_e}}\right]_{HT\, fullspan, A=6, \Lambda_{c/2=0}} \left(\frac{c'}{c}\right)^2 + K_\Lambda \left(\frac{AR_{HT}}{1,5}\right) \left[C_{L_{\delta_e}}\right]_{HT\, fullspan, A=6, \Lambda_{c/2=0}} \tan \Lambda_{c/4} \right\} \frac{S_{HT_e}}{S_W} \delta_e - K_p \left\{ 0,25 C_{L_{HT_e}} \left(\frac{c'}{c}\right)\left(\frac{c'}{c} - 1\right) - C_{m_{HT_e}} \left[\left(\frac{c'}{c}\right)^2 - 1\right] \right\} \frac{S_{HT_e}}{S_W}$$

Now, summing the effects of increment in pitching moment coefficient for the wing and horizontal tail,

$$\Delta C_{m_{WBT}} = \Delta C_{m_f} + \Delta C_{m_e}$$

Additionally there is the effect on aircraft pitching moment due to the increment in lift and drag coefficient on the wing and horizontal tail, that is considered in the definition of aircraft pitching moment.

9.6 Rolling moment coefficient increment due to control deflection

From Hoak (1965), we find the expressions for the increment in rolling moment coefficient due to control deflection. There is the effect of aileron, rudder deflections and the propellers effect on ailerons. We begin with aileron effects. We first define rolling effectiveness of two full chord controls antisymmetrically deflected C'_{l_δ},

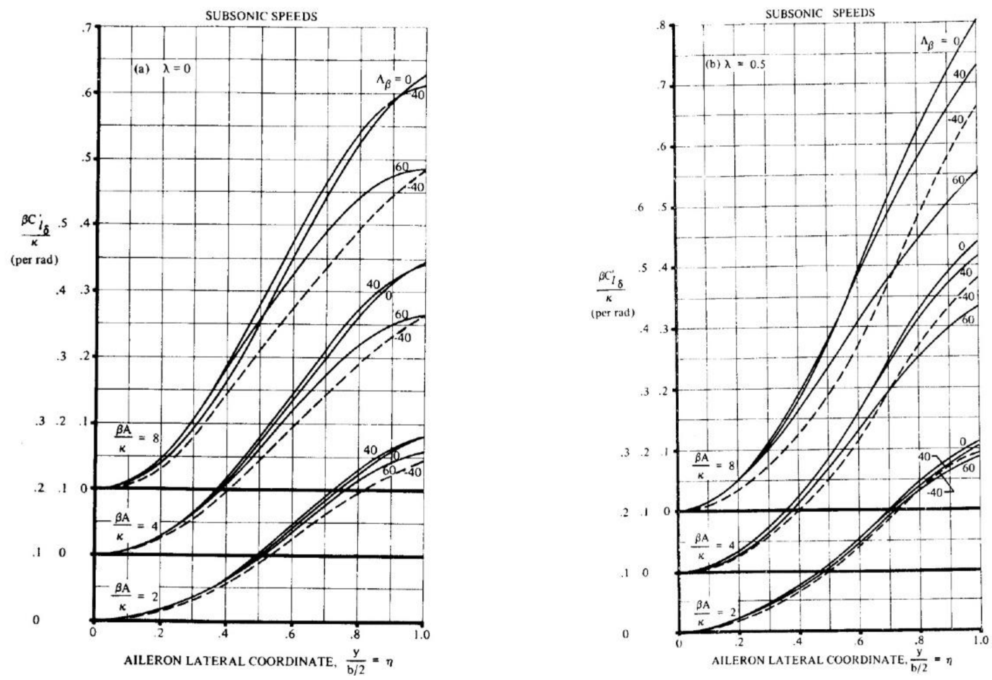
$$C'_{l_\delta} = \frac{\kappa}{\beta}\left(\frac{\beta C'_{l_\delta}}{\kappa}\right)$$

Being that,

$$\kappa = \frac{\left(c_{l_\alpha}\right)_M}{\left(\frac{2\pi}{\beta}\right)}$$

$$\beta = \sqrt{1 - M^2}$$

Where, $\left(c_{l_\alpha}\right)_M$ the section lift curve slope for specific Mach number, $\frac{\beta C'_{l_\delta}}{\kappa}$ ratio obtained from Figure 32, M the Mach number.



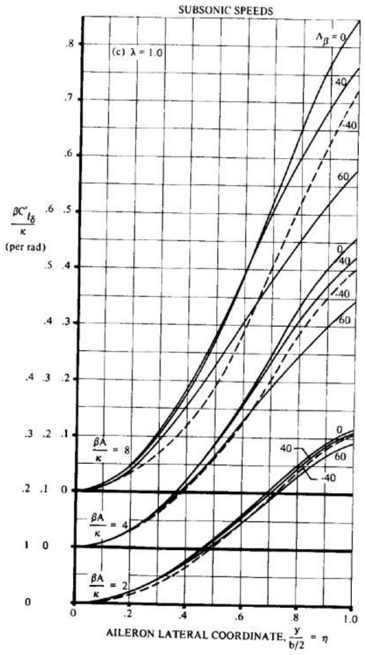


Figure 32: Subsonic aileron rolling moment parameter (HOAK, 1965).

In addition, we define the rolling moment coefficient derivative with respect to control deflection,

$$C_{l_\delta} = |\alpha_\delta| {C'}_{l_\delta}$$

That uses α_δ the section lift effectiveness,

$$\alpha_\delta = -\frac{(c_{l_\delta})_\alpha}{(c_{l_\alpha})_\delta} = -\frac{\Delta c_l}{c_{l_\alpha}\delta} = -\frac{\left(\frac{c_{l_\delta}}{(c_{l_\delta})_{theory}}\right)(c_{l_\delta})_{theory}K'}{c_{l_\alpha}}$$

Thus, summing the effect of left and right ailerons we find,

$$\Delta C_l = \left(\frac{C_{l_\delta}}{2}\right)_L \delta_{a_L} - \left(\frac{C_{l_\delta}}{2}\right)_R \delta_{a_R}$$

Remembering that we may have a hinge line angle, so the relationship between the streamwise derivative and the corresponding value defined normal to the hinge line is,

$$C_{l_\delta} = C_{l_{\delta_{\perp HL}}} \cos \Lambda_{HL}$$

Therefore, the increment in rolling moment coefficient due to aileron deflections is,

$$(\Delta C_l)_{aileron} = C_{l_{\delta_a}} \frac{\left(\delta_{a_L} - \delta_{a_R}\right)}{2}$$

And we can expand the derivative,

$$C_{l_{\delta_a}} = \left| \frac{\left(\frac{c_{l_\delta}}{\left(c_{l_\delta}\right)_{theory}}\right)\left(c_{l_\delta}\right)_{theory} K'}{c_{l_\alpha}} \right| \frac{\kappa}{\beta}\left(\frac{\beta C'_{l_\delta}}{\kappa}\right)\frac{S_{W_e}}{S_W}$$

Now, we derive the rudder deflection effect on the increment of rolling moment coefficient. Wherein, z_v is the vertical distance between vertical tail aerodynamic center and moment reference point and b_W the wing span.

$$(\Delta C_l)_{rudder} = (\Delta C_Y)_{rudder} \frac{z_v}{b_W}$$

So we get to,

$$(\Delta C_l)_{rudder} = C_{l_{\delta_r}} \delta_r$$

Where the rolling moment derivative with respect to rudder deflection $C_{l_{\delta_r}}$ is,

$$C_{l_{\delta_r}} = C_{Y_{\delta_r}} \frac{z_v}{b_W}$$

Next we derive the propellers contribution to the rolling moment coefficient. Such effect comes from the increment in normal force coefficient due to combination of propellers and ailerons deflections.

$$(\Delta C_l)_{prop+wing+aileron} = \Delta C_{l_p} = \frac{(\Delta C_N)_{prop_L} y_{MAC_{Si_L}} - (\Delta C_N)_{prop_R} y_{MAC_{Si_R}}}{b_W/2}$$

Having, $y_{MAC_{Si_L}}$ the lateral position of left aileron mean aerodynamic chord of the wing region immersed on propeller wake, $y_{MAC_{Si_R}}$ lateral position of right aileron mean aerodynamic chord of the wing region immersed on propeller wake.

Being the normal force coefficient increment for the left and right ailerons respectively,

$$(\Delta C_N)_{prop_L} = \Delta C_{L_{prop_L}} \cos\alpha + \Delta C_{D_{prop_L}} \sin\alpha$$

$$(\Delta C_N)_{prop_R} = \Delta C_{L_{prop_R}} \cos\alpha + \Delta C_{D_{prop_R}} \sin\alpha$$

And the inner terms, the increments in wing lift and drag coefficient due to the effect of propeller wake, wing camber and left and right ailerons deflection combined,

$$\Delta C_{L_{prop_{L,R}}} = \frac{1{,}6 T''_{c\,W a_{L,R}} \sqrt{1 - T''_{c\,W a_{L,R}}} K_{W_{a_{L,R}}} S_p}{S_W} \left(\frac{F}{T}\right)_{dWa_{L,R}} \sin\left(\frac{\theta}{\delta_{a_{L,R}}}\delta_{a_{L,R}} + \frac{\theta}{\delta_{a_{L,R_e}}}\delta_{a_{L,R_e}} + \alpha + \delta_W\right)$$

$$\Delta C_{D_{prop_{L,R}}} = \frac{1{,}6 T''_{c\,W a_{L,R}} \sqrt{1 - T''_{c\,W a_{L,R}}} K_{W_{a_{L,R}}} S_p}{S_W} \left(\frac{F}{T}\right)_{dWa_{L,R}} \left[1 - \cos\left(\frac{\theta}{\delta_{a_{L,R}}}\delta_{a_{L,R}} + \frac{\theta}{\delta_{a_{L,R_e}}}\delta_{a_{L,R_e}} + \alpha + \delta_W\right)\right]$$

This uses the following: $K_{W_{a_{L,R}}}$ number of propellers that affect the left and right ailerons respectively, $\left(\frac{F}{T}\right)_{dWa_{L,R}}$ thrust factor accounting for horizontal tail camber plus elevator deflection, $\delta_{a_{L,R}}$ left or right aileron streamwise deflection, $\delta_{a_{L,R_e}}$ equivalent left or right aileron deflection angle defined as the angle between the thrust axis and the mean camber line at the wing trailing edge.

And the propellers coefficients that affects the left and right ailerons,

$$T''_{c\,W_{a_L}} = \frac{\sum_{j=1}^{K_{W_{a_L}}}\{\omega_j^2\}}{\sum_{j=1}^{K_{W_{a_L}}}\{\omega_j^2\} + \frac{\rho V_T^2 \pi D_p^2 K_{W_{a_L}}}{8k_T}}$$

$$T''_{c\,W_{a_R}} = \frac{\sum_{j=1}^{K_{W_{a_R}}}\{\omega_j^2\}}{\sum_{j=1}^{K_{W_{a_R}}}\{\omega_j^2\} + \frac{\rho V_T^2 \pi D_p^2 K_{W_{a_R}}}{8k_T}}$$

Thus, summing the effects we have,

$$\Delta C_{l_{WBT}} = C_{l_{\delta_a}} \frac{(\delta_{a_L} - \delta_{a_R})}{2} + C_{l_{\delta_r}}\delta_r + \Delta C_{l_p}$$

9.7 Yawing moment coefficient increment due to control deflection

Similarly to the increment in rolling moment coefficient, we have the effects of ailerons, rudder and propellers on the yawing moment coefficient. We find from Hoak (1965), the ailerons effect,

$$(\Delta C_n)_{aileron} = K C_L C_{l_{\delta_a}} \frac{(\delta_{a_L} - \delta_{a_R})}{2}$$

Being that: K the empirical factor depending on planform geometry computed using Figure 33 and C_L the lift coefficient for zero aileron deflection.

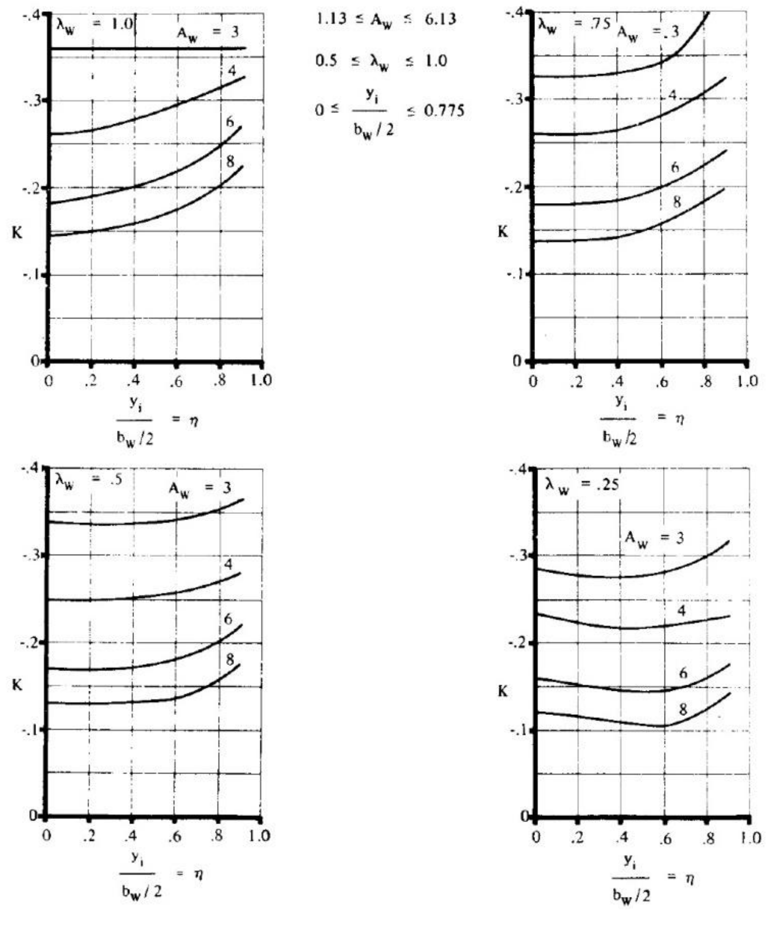


Figure 33: Correlation constant for determining yawing moment due to aileron deflection (HOAK, 1965).

We express the equation in the following form,

$$(\Delta C_n)_{aileron} = C_{n_{\delta_a}} \frac{(\delta_{a_L} - \delta_{a_R})}{2}$$

Wherein, the derivative of yawing moment coefficient with respect to aileron deflection is,

$$C_{n_{\delta_a}} = K C_L C_{l_{\delta_a}}$$

Additionally, we find the increment on yawing moment due to rudder deflection, as a function of l_V the longitudinal distance between vertical tail aerodynamic center and moment reference point.

$$(\Delta C_n)_{rudder} = -(\Delta C_Y)_{rudder} \frac{l_V}{b_W}$$

Expressing in the form,

$$(\Delta C_n)_{rudder} = C_{n_{\delta_r}} \delta_r$$

Where the yawing moment derivative coefficient with respect to rudder deflection is,

$$C_{n_{\delta_r}} = -C_{Y_{\delta_r}} \frac{l_V}{b_W}$$

Finally, for the effect of propellers we find,

$$(\Delta C_n)_{prop+wing+aileron} = \Delta C_{n_p} = \frac{(\Delta C_X)_{prop_L} y_{MAC_{Si_L}} - (\Delta C_X)_{prop_R} y_{MAC_{Si_R}}}{b_W/2}$$

Where the inner terms are,

$$(\Delta C_X)_{prop_L} = \Delta C_{L_{prop_L}} \sin\alpha - \Delta C_{D_{prop_L}} \cos\alpha$$

$$(\Delta C_X)_{prop_R} = \Delta C_{L_{prop_R}} \sin\alpha - \Delta C_{D_{prop_R}} \cos\alpha$$

Thus, summing the effects we have,

$$\Delta C_{n_{WBT}} = C_{n_{\delta_a}} \frac{(\delta_{a_L} - \delta_{a_R})}{2} + C_{n_{\delta_r}} \delta_r + \Delta C_{n_p}$$

10 FLIGHT CONDITIONS

Previously we have stated that for computation of the equilibrium points we must define the flight condition for which we must calculate the state and control variables that satisfies the trim condition. Therefore the subject of this chapter is the definition of such flight conditions.

The basic flight conditions are: hovering flight, steady-state longitudinal flight, and steady state turning flight.

The steady-state hovering flight is the condition where the aircraft has no desired flight velocity and operates similarly to a multirotor helicopter, with the propellers generating the thrust required to sustain the aircraft.

The steady-state longitudinal flight is the condition where the aircraft maintains its flight speed for a specific flight path angle and the wing and horizontal tail are kept in a fixed tilt angle.

In the steady state turning flight the aircraft makes a turn at a specific rate and flight path angle maintaining its flight speed, with the wing and horizontal tail tilt angle fixed.

In this way, we can summarize the flight condition like in the Table 4, where we have the input variables. The subscript "des" means desired input, "der" means derived input.

Table 4: Flight conditions input variables.

Flight Condition	V_T	$\dot{V}_T$	γ	P_W	Q_W	R_W	ϕ	ψ	$\dot{\psi}$
Steady-state hovering flight	0	0	0	0	0	0	0	$(\psi)_{des}$	0
Steady-state longitudinal flight	$(V_T)_{des}$	0	$(\gamma)_{des}$	0	0	0	0	$(\psi)_{des}$	0
Steady state turning flight	$(V_T)_{des}$	0	$(\gamma)_{des}$	$(P_W)_{der}$	$(Q_W)_{der}$	$(R_W)_{der}$	$(\phi)_{der}$	$(\psi)_{des}$	$(\dot{\psi})_{des}$

Wherein, the derived angular velocities come from the following equation, which is derived from the attitude propagation equation.

$$\begin{Bmatrix} P_W \\ Q_W \\ R_W \end{Bmatrix} = \begin{bmatrix} \cos\alpha\cos\beta & \sin\beta & \sin\alpha\cos\beta \\ -\cos\alpha\sin\beta & \cos\beta & -\sin\alpha\sin\beta \\ -\sin\alpha & 0 & \cos\alpha \end{bmatrix} \begin{bmatrix} 1 & 0 & -\sin\theta \\ 0 & \cos\phi & \cos\theta\sin\phi \\ 0 & -\sin\phi & \cos\theta\cos\phi \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ (\dot{\psi})_{des} \end{Bmatrix}$$

Furthermore, we have the constraint for rate of climb, which we find in Stevens and Lewis (2016). The coordinate transformation of the velocity vector from wind axes to Earth fixed inertial reference frame is as follows,

$$\vec{V}_E = \left(S_\beta S_\alpha B_E^B\right)^T \vec{V}_W = B_B^E S_\alpha^T S_\beta^T \vec{V}_E$$

Expanding we find,

$$\begin{Bmatrix} * \\ * \\ -V_T\sin\gamma \end{Bmatrix} = B_B^E \begin{bmatrix} \cos\alpha\cos\beta & -\cos\alpha\sin\beta & -\sin\alpha \\ \sin\beta & \cos\beta & 0 \\ \sin\alpha\cos\beta & -\sin\alpha\sin\beta & \cos\alpha \end{bmatrix} \begin{Bmatrix} V_T \\ 0 \\ 0 \end{Bmatrix}$$

The asterisks indicate “don’t care” components, and then arranging the third equation we find,

$$\sin\gamma = \sin\theta\cos\alpha\cos\beta - \cos\theta\sin\phi\sin\beta - \cos\theta\cos\phi\sin\alpha\cos\beta$$

And solving for the pitch angle θ, we find,

$$\theta = \tan^{-1}\left(\frac{ab + \sin\gamma\sqrt{a^2 - \sin^2\gamma + b^2}}{a^2 - \sin^2\gamma}\right), \theta \neq \frac{\pi}{2}$$

Where the equation terms are,

$$a = \cos\alpha\cos\beta$$

$$b = \sin\phi\sin\beta + \cos\phi\sin\alpha\cos\beta$$

Now for the turn coordination constraint, we have from the translational motion equations in the body axes reference frame,

$$\dot{\vec{V}}_B = -\Omega_B\vec{V}_B + B_E^B\vec{g}^E - F + \frac{\vec{F}_B^B}{M}$$

Expanding the terms,

$$\begin{Bmatrix}\dot{U}\\\dot{V}\\\dot{W}\end{Bmatrix} = -\begin{bmatrix}0 & -R & Q\\ R & 0 & -P\\ -Q & P & 0\end{bmatrix}\begin{Bmatrix}U\\V\\W\end{Bmatrix} + \begin{bmatrix}\cos\psi\cos\theta & \sin\psi\cos\theta & -\sin\theta\\ \cos\psi\sin\theta\sin\phi - \sin\psi\cos\phi & \sin\psi\sin\theta\sin\phi + \cos\psi\cos\phi & \cos\theta\sin\phi\\ \cos\psi\sin\theta\cos\phi + \sin\psi\sin\phi & \sin\psi\sin\theta\cos\phi - \cos\psi\sin\phi & \cos\theta\cos\phi\end{bmatrix}\begin{Bmatrix}0\\0\\g\end{Bmatrix} + \frac{1}{M}\begin{Bmatrix}F_{Bx}\\F_{By}\\F_{Bz}\end{Bmatrix} - F$$

For this condition we wish that $F_{By} = 0$, and we impose $\dot{V} = 0$. So we obtain for the second equation,

$$0 = -RU + PW + g\cos\theta\sin\phi - F(2)$$

Additionally, we impose $\dot{\phi} = \dot{\theta} = 0$, thus we find the angular velocities in body axes,

$$\begin{Bmatrix}P\\Q\\R\end{Bmatrix} = \begin{bmatrix}1 & 0 & -\sin\theta\\ 0 & \cos\phi & \cos\theta\sin\phi\\ 0 & -\sin\phi & \cos\theta\cos\phi\end{bmatrix}\begin{Bmatrix}\dot{\phi}\\\dot{\theta}\\\dot{\psi}\end{Bmatrix} = \begin{bmatrix}1 & 0 & -\sin\theta\\ 0 & \cos\phi & \cos\theta\sin\phi\\ 0 & -\sin\phi & \cos\theta\cos\phi\end{bmatrix}\begin{Bmatrix}0\\0\\\dot{\psi}\end{Bmatrix} = \begin{Bmatrix}-\dot{\psi}\sin\theta\\ \dot{\psi}\cos\theta\sin\phi\\ \dot{\psi}\cos\theta\cos\phi\end{Bmatrix}$$

We also make the conversion of the velocity vector in body axes to wind axes,

$$\vec{V}_B = \begin{Bmatrix}U\\V\\W\end{Bmatrix} = S^T\vec{V}_W = S^T\begin{Bmatrix}V_T\\0\\0\end{Bmatrix} = \begin{Bmatrix}V_T\cos\alpha\cos\beta\\ V_T\sin\beta\\ V_T\sin\alpha\cos\beta\end{Bmatrix}$$

Substituting R, U, P, W we find,

$$0 = -\dot{\psi}\cos\theta\cos\phi\, V_T\cos\alpha\cos\beta - \dot{\psi}\sin\theta\, V_T\sin\alpha\cos\beta + g\cos\theta\sin\phi - F(2)$$

Now we will solve for ϕ,

$$\sin\phi - \frac{\dot{\psi}V_T\cos\beta}{g}\cos\alpha\cos\phi = \frac{\dot{\psi}V_T\cos\beta}{g}\tan\theta\sin\alpha + \frac{F(2)}{g\cos\theta}$$

We get to an equation in the following form,

$$a\sin\phi + b\cos\phi = c$$

Where,

$$a = 1$$

$$b = -\frac{\dot{\psi} V_T \cos\beta}{g} \cos\alpha$$

$$c = \frac{\dot{\psi} V_T \cos\beta}{g} \tan\theta \sin\alpha + \frac{F(2)}{g\cos\theta}$$

Then we proceed with the algebraic manipulation,

$$a \sin\phi = c - b\cos\phi$$

$$a^2 \sin^2\phi = c^2 - 2bc\cos\phi + b^2\cos^2\phi$$

$$a^2 \sin^2\phi - b^2\cos^2\phi + 2bc\cos\phi - c^2 = 0$$

$$a^2 - a^2\cos^2\phi - b^2\cos^2\phi + 2bc\cos\phi - c^2 = 0$$

$$-(a^2 + b^2)\cos^2\phi + 2bc\cos\phi - c^2 + a^2 = 0$$

$$(a^2 + b^2)\cos^2\phi - 2bc\cos\phi + c^2 - a^2 = 0$$

$$\Delta = (2bc)^2 - 4(a^2 + b^2)(c^2 - a^2) = 4b^2c^2 - 4(a^2c^2 - a^4 + b^2c^2 - a^2b^2)$$
$$= 4b^2c^2 - 4a^2c^2 + 4a^4 - 4b^2c^2 + 4a^2b^2 = 4a^2(a^2 + b^2 - c^2)$$

Finally we come to,

$$\cos\phi = \frac{2bc \pm \sqrt{4a^2(a^2 + b^2 - c^2)}}{2(a^2 + b^2)} = \frac{bc \pm a\sqrt{a^2 + b^2 - c^2}}{a^2 + b^2}$$

Substituting the a, b, c terms we find the equation for ϕ,

$$\phi = \cos^{-1}\left(\frac{-\frac{\dot{\psi} V_T \cos\beta}{g}\cos\alpha\left(\frac{\dot{\psi} V_T \cos\beta}{g}\tan\theta\sin\alpha + \frac{F(2)}{g\cos\theta}\right) + \sqrt{1 + \left(\frac{\dot{\psi} V_T \cos\beta}{g}\cos\alpha\right)^2 - \left(\frac{\dot{\psi} V_T \cos\beta}{g}\tan\theta\sin\alpha + \frac{F(2)}{g\cos\theta}\right)^2}}{1 + \left(\frac{\dot{\psi} V_T \cos\beta}{g}\cos\alpha\right)^2} \right)$$

In order to maintain the consistency of the relation between $\dot{\psi}$ and ϕ the sign of the square root must be positive.

Moreover, a numerical approach is necessary to find ϕ and θ that satisfies both equations. An easy way to do this is to solve first the equation for θ with an initial value for ϕ (may be $\phi = 0$), and substitute the result of θ to find the new value for ϕ, that is used again to recalculate θ. After a few number of iterations a reasonable solution can be found.

Therefore, we have now the derived constraints for P_W, Q_W, R_W, θ and ϕ, which have to be inserted in the aircraft numerical trim algorithm in order to find the remaining state vector variables corresponding to the input flight condition.

11 AIRCRAFT TRIM

Aircraft simulation using the state space equations requires the equilibrium point, or steady-state condition, around which the flight behavior can be studied. Such point is represented by the state vector $\vec{X}_e$ and control vector $\vec{U}_e$ that make the state derivatives $\dot{V}_T, \dot{\beta}, \dot{\alpha}, \dot{P}_W, \dot{Q}_W, \dot{R}_W$ identically zero. Therefore for each specific flight condition we must be able to find the combination of state and control variables that meets with all null state derivatives.

We achieve this goal with a numerical algorithm in the following way. First we define a cost function from the sum of the squares of the state derivatives (STEVENS; LEWIS, 2016) previously mentioned, which is the following equation.

$$\mathcal{J} = \dot{V}_T^2 + \dot{\beta}^2 + \dot{\alpha}^2 + \dot{P}_W^2 + \dot{Q}_W^2 + \dot{R}_W^2$$

In the equilibrium point the cost function should be zero, because the state derivatives must be zero. Thus, if we successively compute the value of the cost function for some chosen state vector $\vec{X}$ and control vector $\vec{U}$ using the translational and angular motion equations to compute the state derivatives, in order to gradually approach the cost function to zero, it would be possible to find the state and control vector, for the specific flight condition, that nullifies the cost function.

An effective algorithm to solve this problem is the Sequential Simplex, described in Walters et al. (1991) and Nelder and Mead (1965), which is based on the search of optimum from experimentation and measurement of outcome from a combination of variables, which is explained from now on.

Given some arbitrary k variables whose combination leads to an outcome that can be measured and seeks to improve, it is made $k + 1$ outcome measurements from $k + 1$ combination of variables. The assembly of $k + 1$ outcome points, that we may call vertices, is a geometric figure that we call simplex whose coordinates are the value of each variable. The simplex vertex whose outcome is the worst we label $\boldsymbol{W}$, the one that has the best outcome we label $\boldsymbol{B}$ and the next to worst outcome we label $\boldsymbol{N}$. Next we make the average combination of variables except the worst ($\boldsymbol{W}$) and we label it $\overline{\boldsymbol{P}}$ ($\overline{\boldsymbol{P}} = \boldsymbol{\Sigma}/\boldsymbol{k}$). Then we define the reflection of the worst outcome with respect to $\boldsymbol{\overline{P}}$, that we label $\boldsymbol{R}$, such that $\boldsymbol{R} = \boldsymbol{\overline{P}} + (\boldsymbol{\overline{P}} - \boldsymbol{W})$. So we make a new outcome measurement given by the combination of variables of the vertex $\boldsymbol{R}$.

From now on we proceed with the following algorithm, understanding that " $>$ " means "better than", " $<$ " means "worse than", " $\geq$ " means "equal or better than" and " $\leq$ " means "equal or worse than".

Furthermore, there are restriction on the variables, thus if any of the variables of the vertex is out of boundaries, such vertex must be declared invalid. This examination is indicated by the factor f, so that if $f = 0$ all the variables in the vertex are within the boundaries, and if $f = 1$ at least one of the vertex variables is out of boundaries.

A. If $\boldsymbol{N} \leq \boldsymbol{R} \leq \boldsymbol{B}$ and $\boldsymbol{f_R = 0}$, proceed with simplex $\boldsymbol{BNR}$.
B. If $\boldsymbol{R} > \boldsymbol{B}$, calculate and measure $\boldsymbol{E}$.
   i. If $\boldsymbol{E} \geq \boldsymbol{B}$ and $\boldsymbol{f_E = 0}$, proceed with simplex $\boldsymbol{BNE}$.
   ii. If $\boldsymbol{E} < \boldsymbol{B}$ or $\boldsymbol{f_E = 1}$, proceed with simplex $\boldsymbol{BNR}$.
C. If $\boldsymbol{R} < \boldsymbol{N}$ or $\boldsymbol{f_R = 1}$:
   Calculate and measure $\boldsymbol{C_R}$
   i. If $\boldsymbol{R} \geq \boldsymbol{W}$ and $\boldsymbol{f_{C_R} = 0}$, proceed with simplex $\boldsymbol{BNC_R}$.
   ii. If $\boldsymbol{R} < \boldsymbol{W}$ or $\boldsymbol{f_{C_R} = 1}$, calculate and measure $\boldsymbol{C_W}$, proceed with simplex $\boldsymbol{BNC_W}$.

Being that $\boldsymbol{E}$ is an expansion that is calculated when the vertex $\boldsymbol{R}$ outcome is better than $\boldsymbol{B}$, so that a new vertex is created in order to evaluate if there is an even better outcome in the direction of $\boldsymbol{R}$, so $\boldsymbol{E} = \boldsymbol{B} + (\boldsymbol{\overline{P}} - \boldsymbol{W})$. The vertex $\boldsymbol{C_R}$ is a contraction that is calculated when the reflection vertex does not have a better outcome than the next to worst vertex, so the algorithm tries a new vertex closer to the current, $\boldsymbol{C_R} = \boldsymbol{\overline{P}} + \frac{1}{2}(\boldsymbol{\overline{P}} - \boldsymbol{W})$. If the reflection vertex is worse than the worst, the new vertex is a contraction closer to the worst, $\boldsymbol{C_W} = \boldsymbol{\overline{P}} - \frac{1}{2}(\boldsymbol{\overline{P}} - \boldsymbol{W})$.

In Figure 34 is shown, geometrically, an algorithm step where the function optimization involves the combination of only variable 1 and variable 2. Note the initial simplex with vertices $\boldsymbol{WNB}$, from where the algorithm tries to find the direction of function optimization, in other words, the next vertex that must substitute the worst vertex ($\boldsymbol{W}$) and form a new simplex. This new vertex can be $\boldsymbol{C_W}$, $\boldsymbol{C_R}$, $\boldsymbol{R}$ or $\boldsymbol{E}$, according to the algorithm steps.

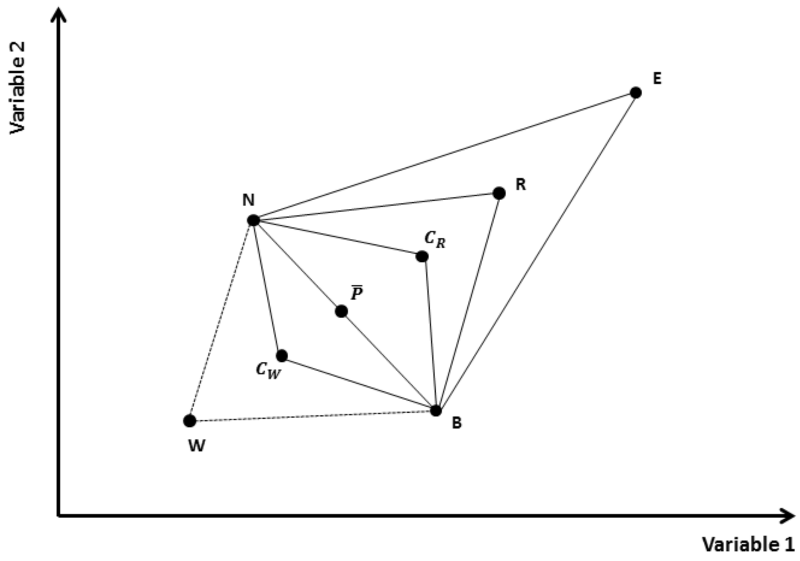


Figure 34: Variable size simplex definitions.

Therefore the vertex with the worst outcome $\boldsymbol{W}$ must be progressively removed and substituted by a new vertex that may be $\boldsymbol{R}$, $\boldsymbol{E}$, $\boldsymbol{C_R}$ or $\boldsymbol{C_W}$. The procedure continues until the best outcome $\boldsymbol{B}$ reaches some goal or stopping criterion. In Figure 35, from Walters et al. (1991), it is possible to observe the algorithm steps creating new simplexes that shows a movement towards the region where the combination of the variables X1 and X2 would provide successively better outcomes, given that the contour lines would represent points with the same value of the cost function. The dots not connected represent measurements taken during the algorithm procedure that were not included in the successive simplexes.

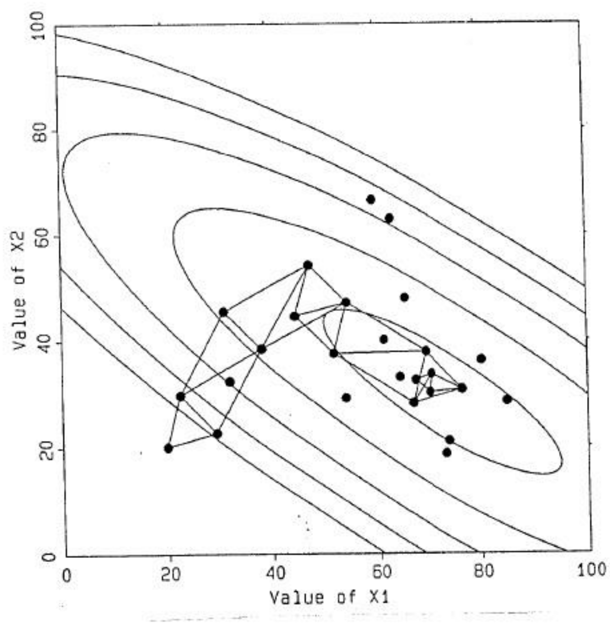


Figure 35: Variable size simplex algorithm sequence (WALTERS et al., 1991).

Applying the algorithm to the aircraft configuration we have the state vector with 25 variables to be defined,

$$\vec{X}^T = \left(V_T, \beta, \alpha, P_W, Q_W, R_W, \phi, \theta, \psi, h, \delta_f, \delta_e, \delta_r, \delta_{a_L}, \delta_{a_R}, \delta_W, \dot{\delta}_W, \delta_{HT}, \dot{\delta}_{HT}, \omega_1^2, \omega_2^2, \omega_3^2, \omega_4^2, \omega_5^2, \omega_6^2\right)$$

And the terms involved in the cost function are outcomes of the translational and angular motion equations,

$$\begin{Bmatrix} \dot{V}_T \\ \dot{\beta} \\ \dot{\alpha} \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \dfrac{1}{V_T} & 0 \\ 0 & 0 & \dfrac{1}{V_T \cos\beta} \end{bmatrix} \left[-\Omega_W \begin{Bmatrix} V_T \\ 0 \\ 0 \end{Bmatrix} + S B_E^B \vec{g}^E - SF + \frac{1}{M} \begin{Bmatrix} -D \\ Y \\ -L \end{Bmatrix} + \frac{1}{M} S \sum_{j=1}^{NR} \left(R_{R_j}^B \begin{Bmatrix} k_T \omega_j^2 \\ 0 \\ 0 \end{Bmatrix} \right) \right]$$

$$\begin{Bmatrix} \dot{P}_W \\ \dot{Q}_W \\ \dot{R}_W \end{Bmatrix} = -\left(SA^{-1}DS^T + SA^{-1}C\dot{S}^T\right) \begin{Bmatrix} V_T \\ 0 \\ 0 \end{Bmatrix} - \left(\Omega_R + SA^{-1}BS^T\right) \begin{Bmatrix} P_W \\ Q_W \\ R_W \end{Bmatrix} + SA^{-1}S^T \begin{Bmatrix} \bar{L} \\ M \\ N \end{Bmatrix} + SA^{-1} \sum_{j=1}^{NR} \left(R_{R_j}^B \begin{Bmatrix} \lambda_j k_Q \omega_j^2 \\ 0 \\ 0 \end{Bmatrix} + \vec{r}_{R_j/B} \times R_{R_j}^B \begin{Bmatrix} k_T \omega_j^2 \\ 0 \\ 0 \end{Bmatrix} \right) + SA^{-1}(M_P - E) - SA^{-1}CS^T \begin{Bmatrix} \dot{V}_T \\ 0 \\ 0 \end{Bmatrix}$$

For the hover condition we must use the aircraft body coordinate frame,

$$\begin{Bmatrix} \dot{U} \\ \dot{V} \\ \dot{W} \end{Bmatrix} = -\Omega_B \begin{Bmatrix} U \\ V \\ W \end{Bmatrix} + B_E^B \vec{g}^E - F + \frac{1}{M} \sum_{j=1}^{NR} \left(R_{R_j}^B \begin{Bmatrix} k_T \omega_j^2 \\ 0 \\ 0 \end{Bmatrix} \right)$$

$$\begin{Bmatrix} \dot{P} \\ \dot{Q} \\ \dot{R} \end{Bmatrix} = A^{-1} \left(-B \begin{Bmatrix} P \\ Q \\ R \end{Bmatrix} - D \begin{Bmatrix} U \\ V \\ W \end{Bmatrix} - C \begin{Bmatrix} \dot{U} \\ \dot{V} \\ \dot{W} \end{Bmatrix} + \sum_{j=1}^{NR} \left(R_{R_j}^B \begin{Bmatrix} \lambda_j k_Q \omega_j^2 \\ 0 \\ 0 \end{Bmatrix} + \vec{r}_{R_j/B} \times R_{R_j}^B \begin{Bmatrix} k_T \omega_j^2 \\ 0 \\ 0 \end{Bmatrix} \right) + M_P - E \right)$$

And the respective cost function,

$$\mathcal{J} = \dot{U}^2 + \dot{V}^2 + \dot{W}^2 + \dot{P}^2 + \dot{Q}^2 + \dot{R}^2$$

So being, it is necessary to define for each flight condition which variables are inputs and which must be calculated by the algorithm. From the flight conditions we have the input variables, or states. Once we define the variables, we must start the algorithm with an initial simplex, in other words, the first $k + 1$ combination of variables where the method begins. We will use the Corner Initial Method, described in Walters et al. (1991), which is best depicted in the following Table 5, where we have the vertex 1 with the basic variables S_i, that can be all zeros, and the other vertexes have incremented amounts s_i in one variable.

Table 5: Corner Initial Simplex.

Vertex	Variable 1	Variable 2	Variable 3	·	Variable k
1	S_1	S_2	S_3	$\cdots$	S_k
2	$S_1 + s_1$	S_2	S_3	$\cdots$	S_k
3	S_1	$S_2 + s_2$	S_3	$\cdots$	S_k
4	S_1	S_2	$S_3 + s_3$	$\cdots$	S_k
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$
$k + 1$	S_1	S_2	S_3	$\cdots$	$S_k + s_k$

Finally, the stopping criterion used is when the cost function reaches a value less than 1e-15.

12 TRIM RESULTS

Using the aircraft trim algorithm we have calculated the equilibrium points for hovering and steady-state longitudinal flight, which will be displayed as states or relevant properties, such as propeller power and rotations per minute, as functions of flight speed. Some states are all zeros in the equilibrium point in the steady-state longitudinal flight $(\beta, P_W, Q_W, R_W, \phi, \delta_r, \delta_{a_L}, \delta_{a_R})$. Furthermore, for the time being we will consider flap deflection δ_f not needed to trim the aircraft.

Additionally, it has been established that for every flight condition the wing and horizontal tail tilt angle must be as in the Figure 36, which was obtained using the trim algorithm and smoothing the curves for wing and horizontal tilt angle as functions of flight speed, which were used for ensuing recalculation of the other state and control states, for the condition of full load, flight path angle zero and altitude of 100 m. Therefore, for every other flight condition the wing and horizontal tail tilt angle was inputs to compute the other states.

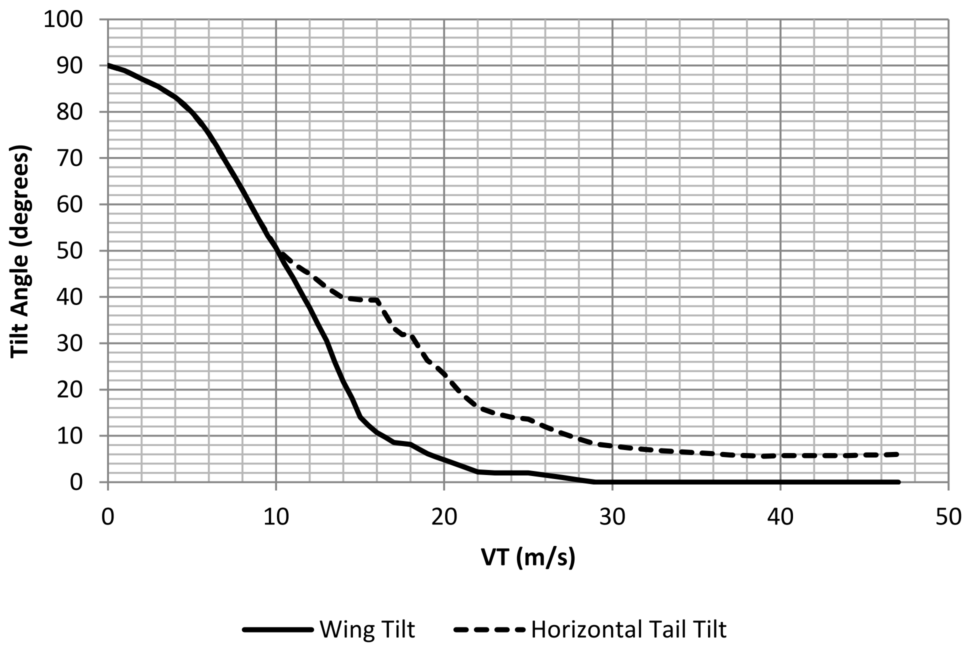


Figure 36: Wing and horizontal tail tilt angle x VT.

The curves express that from 0 to 10 m/s the wing and horizontal tail tilt angle are the same, starting in the hovering condition at 90° where the propellers are all pointing upwards. From 10 m/s forward, the curves differ, being that the wing tilts more until the wing tilt angle becomes zero at 29 m/s, does not become fully horizontal.

So we proceed with some relevant state of the trim curves for the maximum weight. Moreover it was computed for flight angle (γ) zero and altitude of 100 m.

First, in the Figure 37 we find the aircraft angle of attack which is equivalent to the pitch angle for flight angle zero, and elevator angle versus flight speed. The elevator is only used from 16 m/s forward, since for low flight speeds there is not much dynamic pressure in the aerodynamic controls and the wing and horizontal tail tilt together with propellers thrust is enough to trim the aircraft.

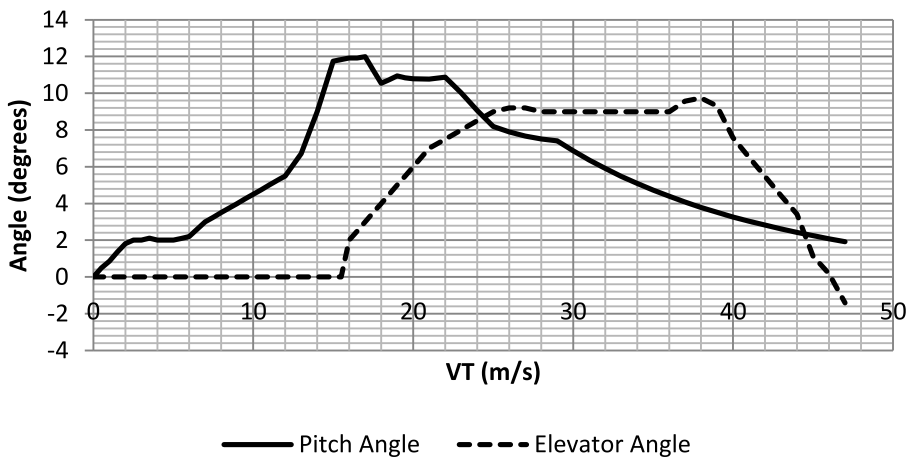


Figure 37: Aircraft angle of attack and elevator angle x VT, for flight path angle zero and altitude of 100 m.

For the longitudinal flight it has been determined that the four propellers in the wing will have the same command, thus the same steady power, being the same for the two propellers at the horizontal tail. From Figure 38 we can see that in the hover condition there is high power required from the wing and tail propellers, which is equivalent in the Figure 39 with the high values of propeller speed. As the flight speed increases the power and RPM are lowered, since some of the lift force is transferred to aerodynamic lift. Also, from 16 m/s on the tail propellers are turned off and the elevator assumes the role to trim the aircraft. It is necessary to do so because there is a limit on propeller RPM (10000 RPM), thus it is not possible to adjust all of the six propellers at the desired advance ratio at high speeds in order to trim the aircraft.

In high speeds the aircraft aerodynamic drag increases thus requiring more power from the propellers, therefore, there is a range of flight speed with minimum total power required, which happens between 14 and 20 m/s, for both maximum and minimum weight, seen in the Figure 40.

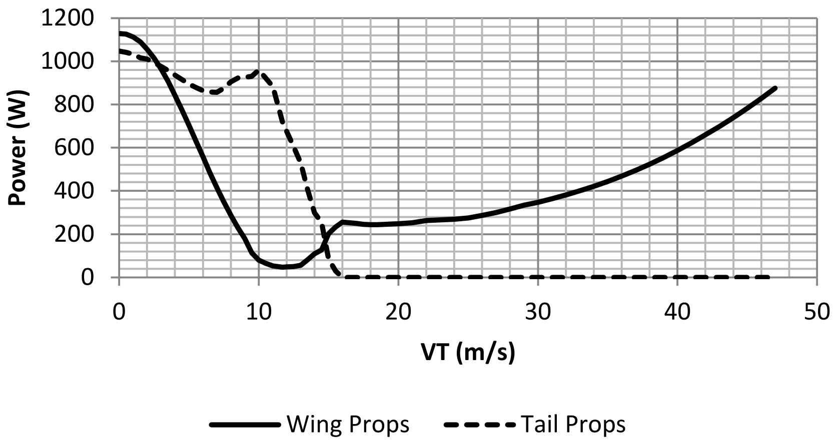


Figure 38: Each wing and tail propellers power x VT, for flight path angle zero and altitude of 100 m.

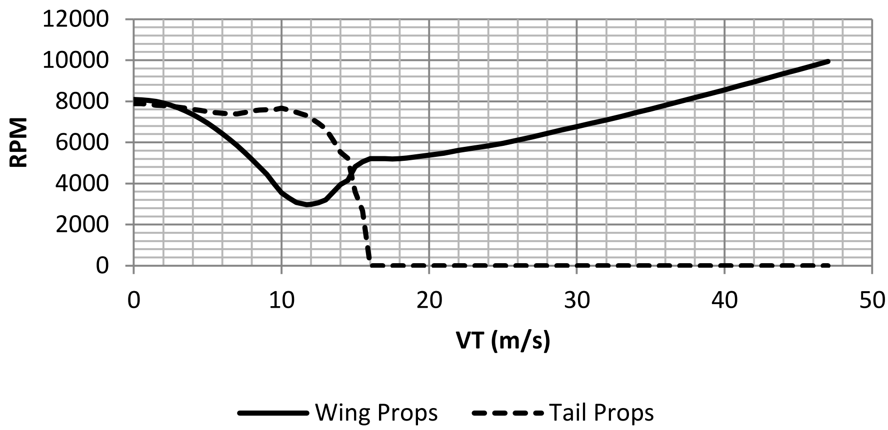


Figure 39: Each wing and tail propellers RPM x VT, for flight path angle zero and altitude of 100 m.

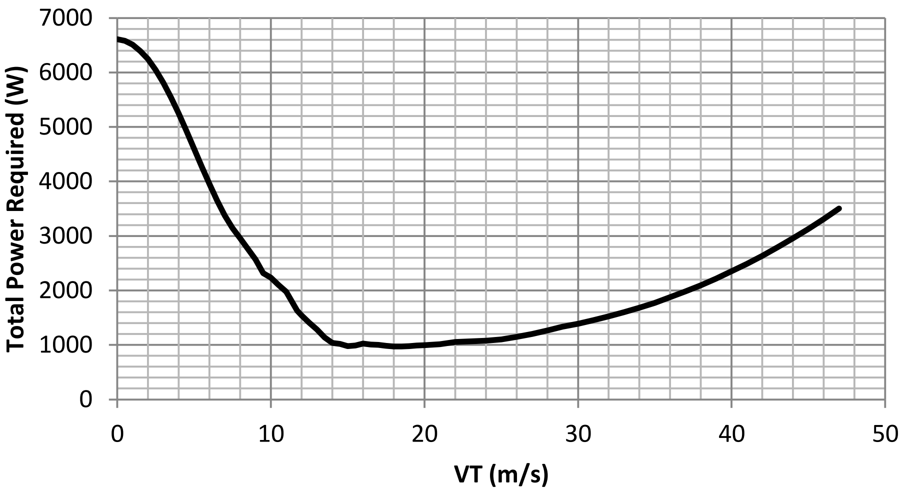


Figure 40: Propellers total power required x VT, for flight path angle zero and altitude of 100 m.

Moreover, since we don’t have yet experimental curves of wing and horizontal tail aerodynamic coefficients, the lift and drag coefficients can be only estimated. Therefore, from the linear model of the aerodynamic forces we obtain the theoretical lift coefficients for the wing and horizontal tail, which comes as results of the trim algorithm. Thereby, we find in Figure 41 the curves for theoretical aerodynamic lift coefficient. It is noteworthy that the lift coefficient for low speed is very high for both wing and horizontal tail, however the aerodynamic lift force is low since the dynamic pressure is also low, therefore the difference with respect to the real case should not be high. Anyway, if the aerodynamic lift coefficient is lower in the real case, the impact in the trim curves will be an increase in the power required for the propellers in the low flight speed range, which can only be accurately determined through experiments.

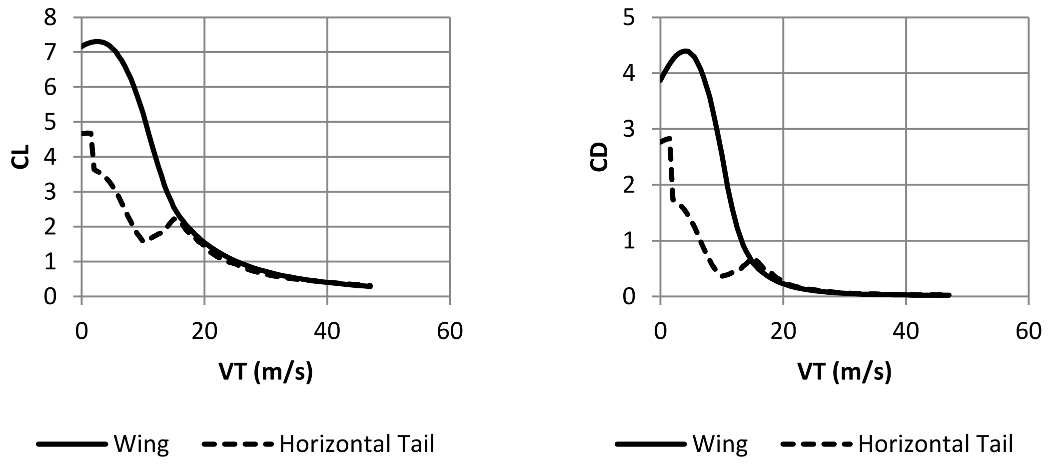


Figure 41: Theoretical wing and horizontal tail lift coefficient x VT, for flight path angle zero and altitude of 100 m.

With the aerodynamic and thrust forces we can assess the fraction of aerodynamic and thrust vertical forces with respect to the total weight, which means, how much of the total weight is sustained through aerodynamic forces and how much by the propellers thrust. In the Figure 42 it is observable that in the hovering condition the weight is fully sustained by the thrust forces and as the aircraft accelerates the aerodynamic forces gains more relevance. If in the real case the wing and horizontal tail lift coefficient don’t reach the values previously mentioned, the thrust fraction at low speeds should be higher.

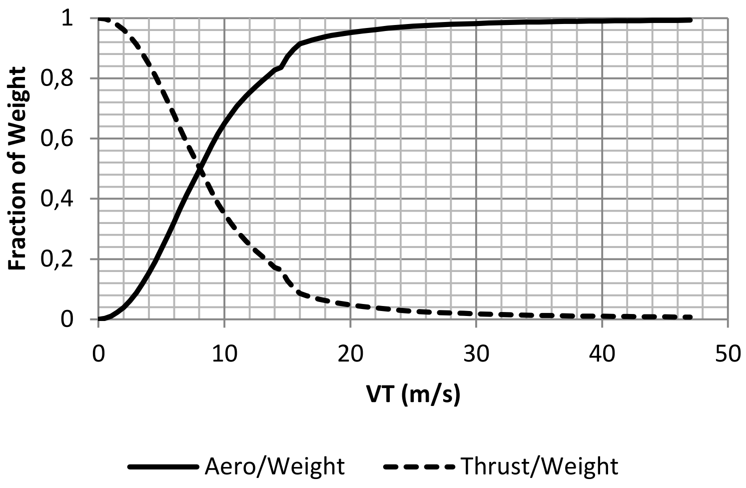


Figure 42: Aerodynamic and Thrust fraction of vertical forces with respect to weight, for flight path angle zero and altitude of 100 m.

13 FLIGHT CONTROL

The control of the aircraft for the longitudinal flight will be divided in four controllers, that is, the wing and horizontal tail position control (13.1), altitude hold controller (13.2), the aircraft state regulator (13.3) and flight speed control (13.4), which shall be described next.

Therefore, in order to increase or decrease the aircraft flight speed, the wing and horizontal tail controllers have to position the surfaces in the equilibrium positions given by the trim curves, while the state regulator and altitude hold controller keep the aircraft stable and in the proper altitude.

13.1 Wing and horizontal tail position control

The wing and horizontal tail should be positioned using stepper motors attached to gear mechanisms, so that, the motors position can be obtained by pulse counter method, which is, for each input pulse the stepper motor moves one step of some degrees given by the motor structure. Therefore, in order to reach the desired motor position it is only required to input the respective number of pulses.

So, for a given desired wing and horizontal tail position $\left(\delta_{W_e}, \delta_{HT_e}\right)$ we define the position error $(\epsilon_W, \epsilon_{HT})$ with respect to the current position (δ_W, δ_{HT}), which is indirectly obtained counting the input pulses,

$$\epsilon_W = \delta_{W_e} - \delta_W$$

$$\epsilon_{HT} = \delta_{HT_e} - \delta_{HT}$$

In order to simplify the model, we will consider the wing and horizontal tail stepper motors with constant speed (Ω_W, Ω_{HT}), given by a pulse counter method with constant pulse frequency. Additionally, the aerodynamic surfaces tilt speed $\left(\dot{\delta}_W, \dot{\delta}_{HT}\right)$ would be reduced by the wing and horizontal tail transmission gear ratio (G_W, G_{HT}), so we find,

$$\dot{\delta}_W = \frac{\Omega_W}{G_W}$$

$$\dot{\delta}_{HT} = \frac{\Omega_{HT}}{G_{HT}}$$

We may use the wing and horizontal tail control signals in the following way. If the position error is positive ($\epsilon_W > 0$) the stepper motor must turn counterclockwise ($\Omega_W > 0$) and the control signal is positive ($\delta_W^C = 1$). Else if the position error is negative ($\epsilon_W < 0$) the stepper motor must turn clockwise ($\Omega_W < 0$) and the control signal would be negative ($\delta_W^C = -1$). And if the error is zero it must be stopped ($\Omega_W = 0$), with ($\delta_W^C = 0$).

Therefore, in order to reduce the complexity of the dynamic model and adapt it to the control strategy and the actuators planned to be used in the wing and horizontal tail, which is, stepper motors, we will be using the previous equations to compute the dynamic response of both surfaces, and the wing and horizontal tail pivot dynamic equations will be used only to compute the respective torque required to their positioning.

13.2 Altitude hold controller

The altitude hold controller has the goal of keeping the aircraft flying in the defined altitude or even to change its altitude. Hence, for a given desired altitude (h_e) we define the altitude error (ϵ_h) with respect to the current altitude (h),

$$\epsilon_h = h - h_e$$

Its control method would be the proportional-integral-derivative (PID) where it is also required to define the vertical velocity error ($\epsilon_{\dot{h}}$) which is the same as the altitude derivative, and the altitude integral error $\left(\epsilon_{h_I}\right)$ which is the integral of the altitude error (h_I),

$$\epsilon_{\dot{h}} = \dot{h}$$

$$\epsilon_{h_I} = h_I = \int \epsilon_h dt$$

Next we compute a desired flight path angle (γ_{des}) which must be followed in order to correct the aircraft altitude, which is the outcome of the following equation, being $k_{p,h}$, $k_{d,h}$ and $k_{I,h}$ the altitude hold control coefficients,

$$\gamma_{des} = k_{p,h}\epsilon_h + k_{d,h}\epsilon_{\dot{h}} + k_{I,h}\epsilon_{h_I}$$

Also we compute the variation of the flight path angle desired, which is the difference to the current flight path angle defined as follows,

$$\Delta\gamma_{des} = \gamma_{des} - \gamma$$

Remembering the computation of the flight path angle,

$$\gamma = \sin^{-1}(\cos\alpha\cos\beta\sin\theta - (\sin\phi\sin\beta + \cos\phi\sin\alpha\cos\beta)\cos\theta)$$

Therefore, the variation on the desired flight path angle would imply in the correction of the pitch angle,

$$\theta_{des} = \theta_e + \Delta\gamma_{des}$$

Finally, the computed desired pitch angle replaces the equilibrium pitch in the state regulator in order to compute the necessary controls to adjust the aircraft trajectory. It is noteworthy that if the desired flight path angle is zero the respective desired pitch angle is equal to the equilibrium pitch required to trim the aircraft, computed using the trim algorithm. In Figure 43 is shown the block diagram for the altitude hold controller.

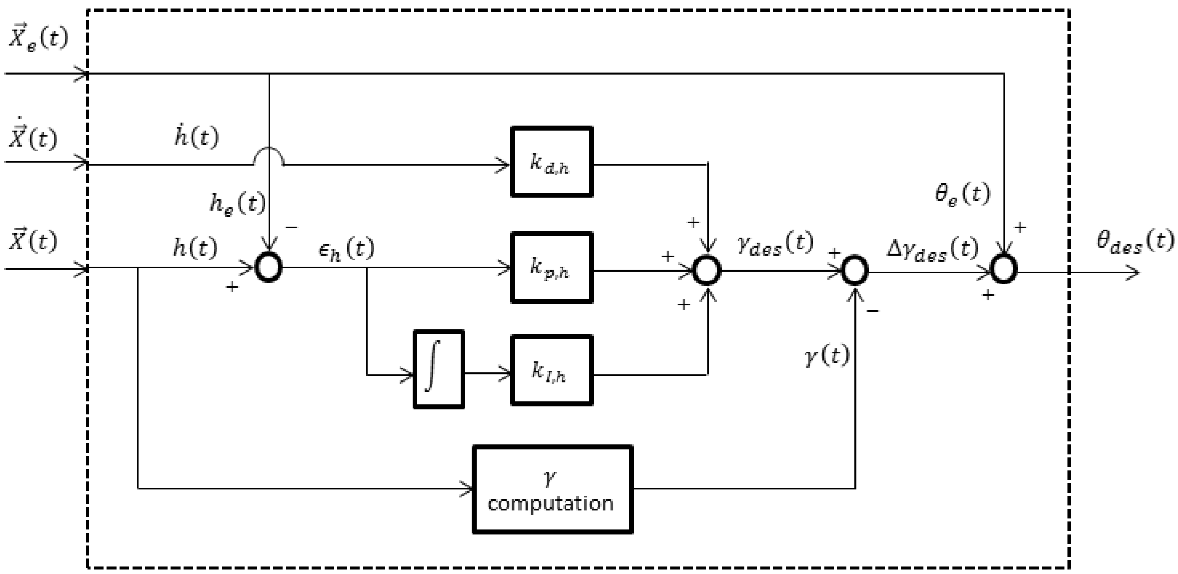


Figure 43: Altitude hold controller.

13.3 State regulator

Now, the state regulator has to stabilize the aircraft as it transitions from one equilibrium point to the next, that is, accelerates or decelerates, and stabilize the same if subject to state disturbances. So, given a specific flight condition, we must compute the equilibrium point and then the Jacobian matrices in order to assemble the aircraft dynamic systems equations. Additionally, we define $\vec{y}$ the output disturbance vector. Therefore, we get the system of linearized dynamic equations, remembering that $\vec{x}$, $\dot{\vec{x}}$ and $\vec{u}$ are disturbances of the state derivatives, state and control vectors respectively.

$$\dot{\vec{x}} = A_p\vec{x} + B_p\vec{u}$$

$$\vec{y} = C_p\vec{x} + D_p\vec{u}$$

Where the matrices A_p and B_p are given by the Jacobian matrices of the equilibrium point,

$$A_p = E^{-1}A$$

$$B_p = E^{-1}B$$

The matrix C_p form the combination of states disturbances that gives the output disturbance vector $\vec{y}$. There is no direct relation between the output and the control input, therefore D_p is a null matrix.

Also, for the control of the aircraft trajectory and ensure zero steady state error for some variables, it is useful to augment the system with integrators. Therefore, for some relevant system output $\vec{Y}$, add additional states $\vec{x}_I$. We may relate the system output $\vec{Y}$ to the output disturbance vector trough the matrix C which selects the suitable variable, and to have the full output we add the respective reference for the variable. So we get,

$$\vec{Y} = C\vec{y} + \vec{r}$$

Also, we define an error vector between the desired reference vector and the output of the desired variables, whose integral is the state integral vector $\vec{x}_I$,

$$\vec{e} = \vec{r} - \vec{Y}$$

$$\vec{x}_I = \int \vec{e}\, dt$$

Thus, its derivative is,

$$\dot{\vec{x}}_I = \vec{e} = \vec{r} - \vec{Y} = \vec{r} - (C\vec{y} + \vec{r}) = -C\left(C_p\vec{x} + D_p\vec{u}\right)$$

This is the state space internal model, or the integrators derivatives equation. Therefore, we have the augmented system consisting of the plant and internal model:

$$\begin{Bmatrix} \dot{\vec{x}} \\ \dot{\vec{x}}_I \end{Bmatrix} = \begin{bmatrix} A_p & 0 \\ -CC_p & 0 \end{bmatrix} \begin{Bmatrix} \vec{x} \\ \vec{x}_I \end{Bmatrix} + \begin{bmatrix} B_p \\ -CD_p \end{bmatrix} \vec{u}$$

Now we define $\vec{\bar{x}} = [\vec{x} \quad \vec{x}_I^T]^T$, so we can write the augmented system as:

$$\dot{\vec{\bar{x}}} = \bar{A}\vec{\bar{x}} + \bar{B}\vec{u}$$

The objective of the state regulator is to eliminate any disturbance in the states from the equilibrium point, which means that it must bring to zero the vector $\vec{\bar{x}}$, and consequently its derivative $\dot{\vec{\bar{x}}}$. In order to do so, we define now the state feedback control law for the augmented system in the form:

$$\vec{u} = -K\vec{x} - K_I\vec{x}_I = -[K \quad K_I] \begin{Bmatrix} \vec{x} \\ \vec{x}_I \end{Bmatrix} = -\bar{K}\vec{\bar{x}}$$

Such augmented system regulator architecture is represented in the Figure 44. Note that command saturation is added in order to safeguard the aircraft controls.

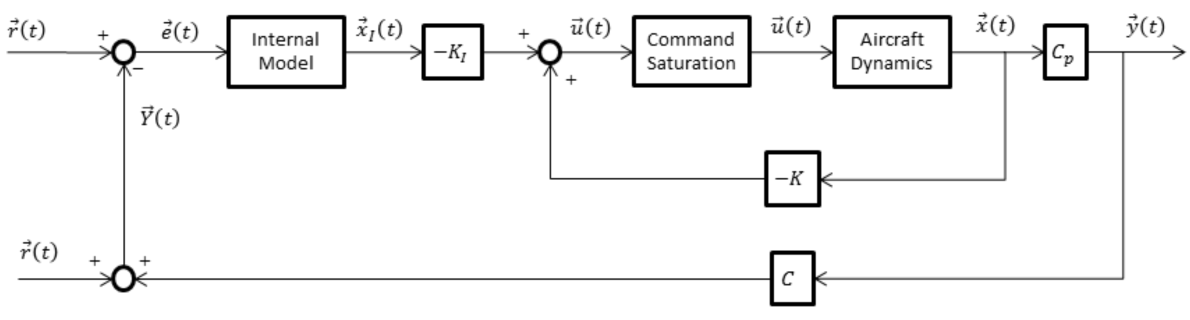


Figure 44: Augmented system regulator architecture.

It is required now to compute the gain matrix $\bar{K}$. In order to do so, we will use the linear quadratic regulator (LQR) to control the aircraft, thus we have the optimal feedback from the minimization of the quadratic cost function (BHATTACHARYYA et al., 2009),

$$J = \int_0^{\infty} \left(\vec{\tilde{x}}^T Q \vec{\tilde{x}} + \rho \vec{u}^T R \vec{u}\right) dt$$

With,

$$Q = Q^T \geq 0 \text{ and } R = R^T > 0$$

And ρ is a scalar weighting parameter.

The optimal control gain matrix for the corresponding LQR problem is,

$$\bar{K} = R^{-1} \bar{B}^T P$$

Provided P be a solution of the Algebraic Riccati Equation,

$$\bar{A}^T P + P\bar{A} - P\bar{B}R^{-1}\bar{B}^T P + Q = 0$$

Furthermore, it is useful to separate the aircraft state regulator in longitudinal control and lateral control. So that the longitudinal control involves the stabilization of the following states,

$$\vec{X}^T{}_{long} = \left(V_T, \alpha, Q_W, \theta, h, \delta_f, \delta_e, \omega_1^2, \omega_2^2, \omega_3^2, \omega_4^2, \omega_5^2, \omega_6^2\right)$$

Employing the longitudinal control vector,

$$\vec{U}^T{}_{long} = \left(\delta_f^C, \delta_e^C, {\omega_1^2}^C, {\omega_2^2}^C, {\omega_3^2}^C, {\omega_4^2}^C, {\omega_5^2}^C, {\omega_6^2}^C\right)$$

And the lateral control is the stabilization of the next,

$$\vec{X}^T{}_{lat} = \left(\beta, P_W, R_W, \phi, \psi, \delta_r, \delta_{a_L}, \delta_{a_R}, \omega_1^2, \omega_4^2\right)$$

Being the control vector,

$$\vec{U}^T{}_{lat} = \left(\delta_r^C, \delta_{a_L}^C, \delta_{a_R}^C, {\omega_1^2}^C, {\omega_4^2}^C\right)$$

So that, the total controls is sum of both longitudinal and lateral controls.

$$\vec{U}^T = \vec{U}^T{}_{long} + \vec{U}^T{}_{lat}$$

In this way, it is necessary to pick the longitudinal and lateral state corresponding rows and columns from the Jacobian matrices to compute both A_p and B_p for both cases. Then, if there is any integrator, compute $\bar{A}_{long}$, $\bar{A}_{lat}$, $\bar{B}_{long}$ and $\bar{B}_{lat}$, which are used to compute the longitudinal and lateral control gain matrices $(\bar{K}_{long}, \bar{K}_{lat})$. This procedure is illustrated in Figure 45, where the gain matrices are updated as a function of the input equilibrium states $\vec{X}_e$.

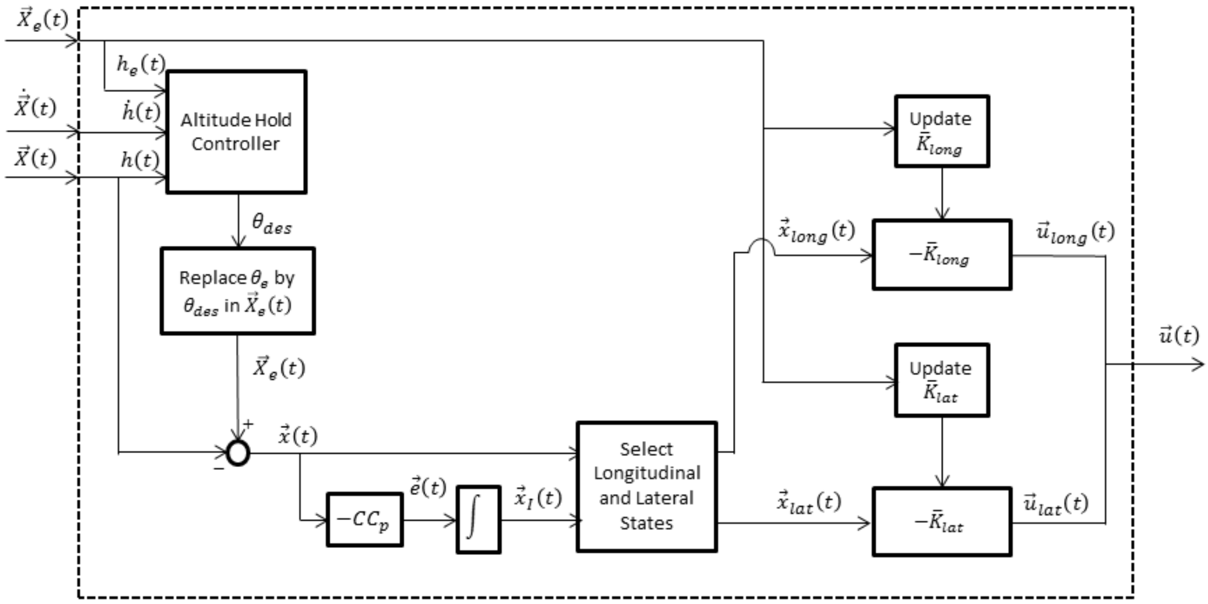


Figure 45: State regulator.

At this stage we recall that he altitude hold controller computes the desired pitch angle in order to adjust the aircraft path angle, which will then correct the aircraft altitude. It must be replaced with the equilibrium pitch in order to compute the angle disturbance (note this procedure in the block diagram of Figure 45), so remembering the definition of the state disturbance vector,

$$\vec{x} = \vec{X}_e - \vec{X}$$

We replace the equilibrium pitch angle with the desired angle computed in the altitude hold controller,

$$\theta_e = \theta_{des}$$

So the state regulator shall compute the controls required to adjust the aircraft trajectory concurrently with the stabilization of the aircraft. Additionally, with this method it is possible to tune how quickly the aircraft corrects its trajectory handling the altitude hold control coefficients.

13.4 Flight velocity control

So we have the aircraft control model for a fixed equilibrium point, which we have made as function of flight velocity, however we must be able to transition between desired flight speeds, which means, there must be a way to successfully and smoothly transition between equilibrium points.

Therefore, we introduce the control of aircraft velocity, which involves the tilt of the wing and horizontal tail to allow a successful transition from hovering to cruise flight, and from cruise to hover.

Previously we have described the trim algorithm that finds the combination of states and controls that keeps the aircraft on a steady-state, which we have made as functions of flight velocity, aircraft weight, flight path angle and turn speed. Therefore, it is possible to establish some target points in the range of flight velocities so that we control the change of flight velocity by moving from one target point to the next. Thus, in order to accelerate the aircraft from hover to maximum speed it would be required to gradually move between target points. To make easier to understand the concept of the target point we can see in the Figure 46 the curves of wing and horizontal tail tilt angles as functions of aircraft flight velocity, being that each markers corresponds to the tilt angle in the respective flight velocity target point, it is worth mentioning that this graph is the same as in Figure 36, but discretized. Therefore, for every target point we will have a combination of states, or equilibrium point $\vec{X}_e$.

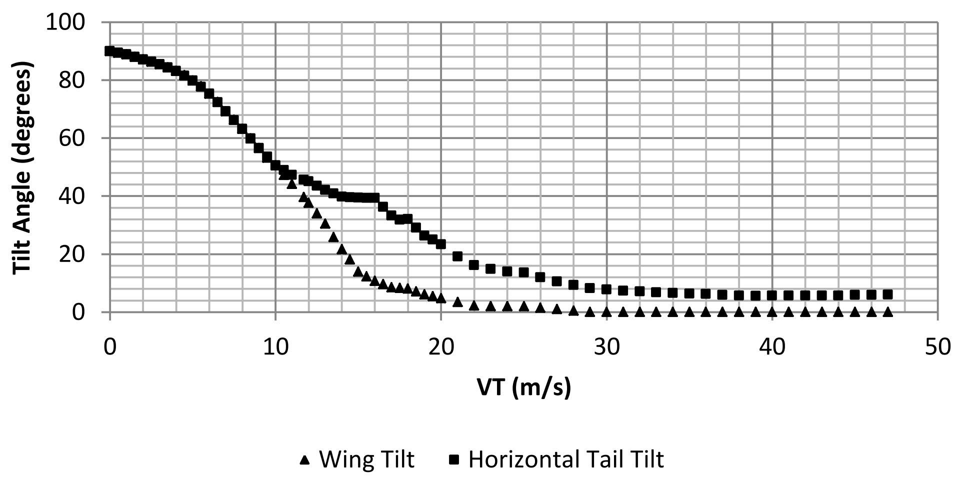


Figure 46: Wing and horizontal tail target points for accelerated flight.

So, in order to accelerate the aircraft we must assess the flight dynamics at each target point, stabilizing it so that the disturbance state vector would be the difference between the equilibrium point $(\vec{X}_e)$ in the target point and the current state vector $(\vec{X})$, therefore the control system goal is to successively control the aircraft, minimizing the disturbance state vector and only switch to the next target point if the aircraft has been stabilized in the current target point. We can write the disturbance state vector $(\vec{x})$ in the following way,

$$\vec{x} = \vec{X} - \vec{X}_e$$

We must now determine a criteria to switch from one target point to the next, we do that by defining the cost functions $\mathcal{E}_s$, $\mathcal{E}_c$ and $\mathcal{E}_d$, being the first the sum of ratios of the disturbance states $(V_T, \beta, \alpha, P_W, Q_W, R_W, \phi, \theta, \psi)$ to respective equilibrium states squared and multiplied by certain weight (w_i) so that some states may contribute more than others (NS is the number of states). But if the respective equilibrium state is zero that term is only the disturbance state squared multiplied by the weight. This cost function measures how far the aircraft current state is from the desired state target point.

$$\mathcal{E}_s = \sum_{i=1}^{NS} e_i$$

$$\begin{cases} e_i = w_i \sqrt{\left(\dfrac{x_i}{X_{e_i}}\right)^2}, & if\ X_{e_i} \neq 0 \\ e_i = w_i \sqrt{x_i{}^2}, & if\ X_{e_i} = 0 \end{cases}$$

Additionally we define $\mathcal{E}_c$ the cost function related to the current deviation of the controls states from the equilibrium position. The subtitle “c” refers to the controls states $\left(\delta_f, \delta_e, \delta_r, \delta_{a_L}, \delta_{a_R}, \omega_1^2, \omega_2^2, \omega_3^2, \omega_4^2, \omega_5^2, \omega_6^2\right)$, and NC is the number of controls.

$$\mathcal{E}_c = \sum_{i=1}^{NC} e_{i_c}$$

$$\begin{cases} e_{i_c} = w_{i_c} \sqrt{\left(\dfrac{x_{i_c}}{X_{e_{i_c}}}\right)^2}, & if\ X_{e_{i_c}} \neq 0 \\ e_{i_c} = w_{i_c} \sqrt{x_{i_c}{}^2}, & if\ X_{e_{i_c}} = 0 \end{cases}$$

Finally, we define the cost function $\mathcal{E}_d$ which measures how much the aircraft is oscillating, that is, how far the aircraft is from the steady condition measuring the states derivatives $\left(\dot{V}_T, \dot{\beta}, \dot{\alpha}, \dot{P}_W, \dot{Q}_W, \dot{R}_W, \dot{\phi}, \dot{\theta}, \dot{\psi}\right)$. The subtitle "d" means states derivatives.

$$\mathcal{E}_d = \sum_{i=1}^{NS} e_{i_d}$$

$$e_{i_d} = w_{i_d}\sqrt{{x_{i_d}}^2}$$

Consequently, we can define some threshold for the cost functions $\left(\mathcal{E}_{th_s}, \mathcal{E}_{th_c}, \mathcal{E}_{th_d}\right)$ by which is acceptable to switch from a target point to the next.

With this we are able to define the overall aircraft flight velocity control architecture that can be visualized in the Figure 47. Thereby, the control concept main strategy is to define an objective flight speed V_T, in order to gradually accelerates or decelerates towards it. The box written "Find Next Equilibrium Point" is a function that evaluates what is the next target point to be reached in order to eventually reach the objective flight speed, which provides the respective state $\vec{X}_e$ and control $\vec{U}_e$ equilibrium vectors. Remember that for each equilibrium point correspond different Jacobian matrices, which must be computed to feed the state regulator box in order to update the regulator gain matrices. Additionally, there is the wing and horizontal tail control box that provides tilt control whose purpose is to correctly position both wing and horizontal tail according to the desired position of the equilibrium point. Thus, while the regulator stabilizes the aircraft, the cost functions $(\mathcal{E}_s, \mathcal{E}_c, \mathcal{E}_d)$ must be constantly evaluated so that when they are simultaneously less than their thresholds $\left(\mathcal{E}_{th_s}, \mathcal{E}_{th_c}, \mathcal{E}_{th_d}\right)$, the system may switch to the next equilibrium point, which will set a new desired position for the wing and horizontal tail. So, the procedure continues until the objective flight speed is reached.

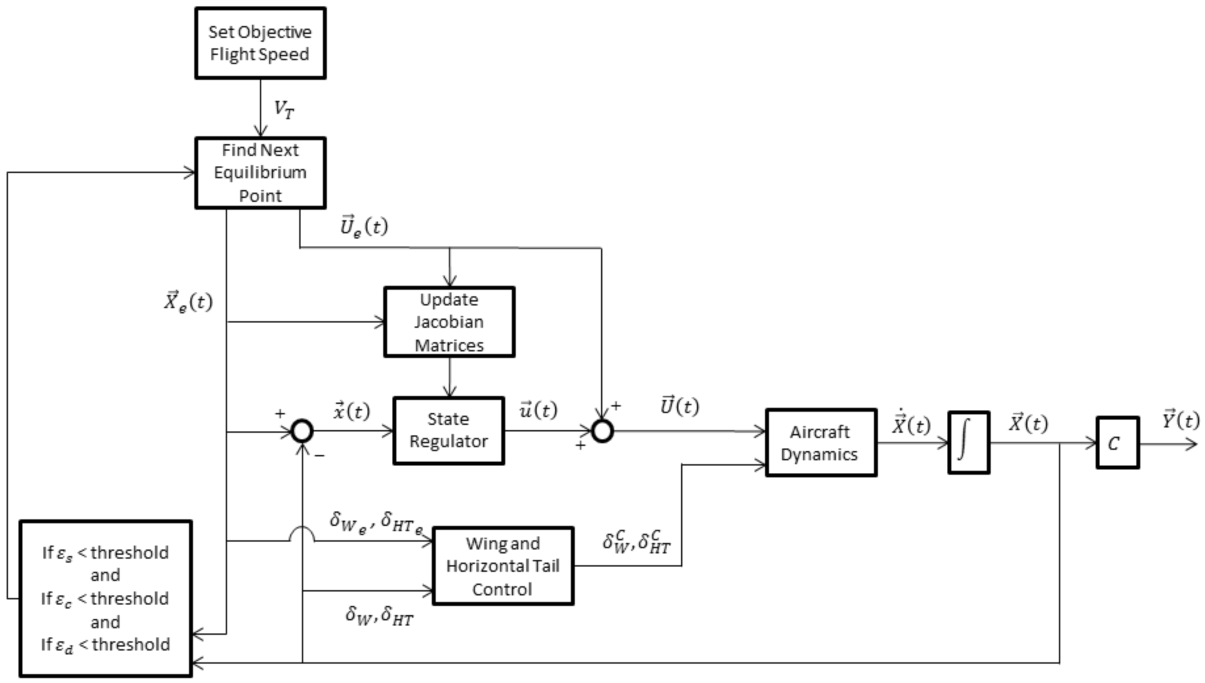


Figure 47: Flight velocity control architecture.

Doing this way we can have a successful transition between desired flight velocities, which is valid for both accelerated and decelerated flight.

13.5 Flight simulation

The aircraft dynamics box of Figure 47, for the purpose of flight simulation, is the grouping of all the dynamic equations that describes the aircraft dynamics. Those are the nonlinear translational motion equations, angular motion equations, attitude propagation equations, rate of climb equation, wing and horizontal tail tilting dynamics equations, the actuators dynamics equations, and the rotors dynamics equations. Thereby, in order to numerically solve the differential equations we`ve used the forth order Runge-Kutta numerical integration technique, described next.

Given a set of differential equations, or the initial-value problem, in the form of,

$$\dot{\vec{X}} = f\left(t, \vec{X}\right)$$

Knowing the vector at the beginning,

$$\vec{X}(t_0) = \vec{X}_0$$

The development of the function in time is computed using the following equations, for $n = 0,1,2,3, \dots$

$$\vec{X}_{n+1} = \vec{X}_n + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

Being h the time interval, so that we also have the evolution in time,

$$t_{n+1} = t_n + h$$

Additionally, the previous coefficients are computed as,

$$k_1 = f\left(t_n, \vec{X}_n\right)$$

$$k_2 = f\left(t_n + \frac{h}{2}, \vec{X}_n + \frac{h}{2}k_1\right)$$

$$k_3 = f\left(t_n + \frac{h}{2}, \vec{X}_n + \frac{h}{2}k_2\right)$$

$$k_4 = f\left(t_n + h, \vec{X}_n + hk_3\right)$$

14 SIMULATION RESULTS

This chapter presents the results of simulations in MatLab (2015) software environment of accelerated and decelerated longitudinal flights, thus we will not consider for now any hovering, turning, climbing or descent simulation. Therefore, we are interested here in the stabilization of the longitudinal state vector $\left(\vec{X}^T{}_{long}\right)$.

$$\vec{X}^T{}_{long} = \left(V_T, \alpha, Q_W, \theta, h, \delta_f, \delta_e, \omega_1^2, \omega_2^2, \omega_3^2, \omega_4^2, \omega_5^2, \omega_6^2\right)$$

Having the regulator longitudinal controls $\vec{U}^T{}_{long}$,

$$\vec{U}^T{}_{long} = \left(\delta_f^C, \delta_e^C, \omega_1^{2^C}, \omega_2^{2^C}, \omega_3^{2^C}, \omega_4^{2^C}, \omega_5^{2^C}, \omega_6^{2^C}\right)$$

And the state and controls of the wing and horizontal tail $(\delta_W, \delta_{HT}, \delta_W^C, \delta_{HT}^C)$.

The objective of the simulations was to verify if the theory and control model developed for this project is reasonable, and would allow a successful control of the aircraft while it accelerates and decelerates, in other words, permit the transition from hovering to cruise flight, and cruise to hovering flight.

Having all the aircraft dimensions, inertial parameters, aerodynamics static and dynamic coefficients, and propellers parameters, what we need now is the control parameters, that is, the definition of the LQR weighting scalar parameter ρ, the matrices Q and R, and the controls coefficients.

Such parameters where defined through trial and error, until a good response was reached, however for the Q and R matrices we have used a guideline presented in Stevens and Lewis (2016), that defines the matrices as diagonal,

$$Q = diag\{q_i\}$$

$$R = diag\{r_i\}$$

With, q_i and r_i as function of the maximum allowable deviations in the states x_{iM} and in the maximum allowable deviations controls u_{iM},

$$q_i = \frac{1}{x_{iM}^2}$$

$$r_i = \frac{1}{u_{iM}^2}$$

Furthermore, each equilibrium point requires its own control parameters, since each has different gain matrix. The final control parameters are shown in the Appendix.

We start with the results for the simulation of the accelerated flight from 3,0 m/s to 40 m/s, and decelerated flight from 40 m/s to 3,0 m/s. That is because there is still some work to be done adjust the controls for the hovering condition (close to 0 m/s). So is presented the changes over time for the longitudinal states and controls, and the cost functions.

In Figure 48 and Figure 49 are shown the results of flight speed over time, where the solid line represents the state, in this figure the flight speed, and the dotted line the equilibrium point reference flight speed. Moreover, the results for the accelerated flight are blue lines, and for the decelerated are red lines.

In the accelerated flight it takes 410 seconds to fully achieve the maximum speed of 40 m/s. Note that at the beginning of the simulation, where the wing and horizontal tail starts at 90 degrees tilt position, and slowly incline, the acceleration is the fastest and almost constant until it reaches 8,5 m/s, where the acceleration reduces. From 8,5 m/s to 16 m/s it has the slowest acceleration. From the flight speed of 16 m/s forth, the aerodynamic controls (flap and elevator) take action. The use of the flap to stabilize the aircraft is suitable, since the aircraft center of gravity is aft the wing, so the flap action is required to balance the elevator comands and the excess of wing lift. Moreover, from 16 m/s to 40 m/s it was decided to have the rear propellers turned off, due to RPM limits. Thus from 16 m/s to 22 m/s, we find a fast transition region and then up until 40 m/s it keeps an almost constant acceleration and smooth transition. Such smoothness is probably because of the smaller differences between wing and tilt equilibrium angle positions, so there is fewer and quicker actions in wing and horizontal tail positioning, and also higher aerodynamic controls effectiviness because of higher dynamic pressure.

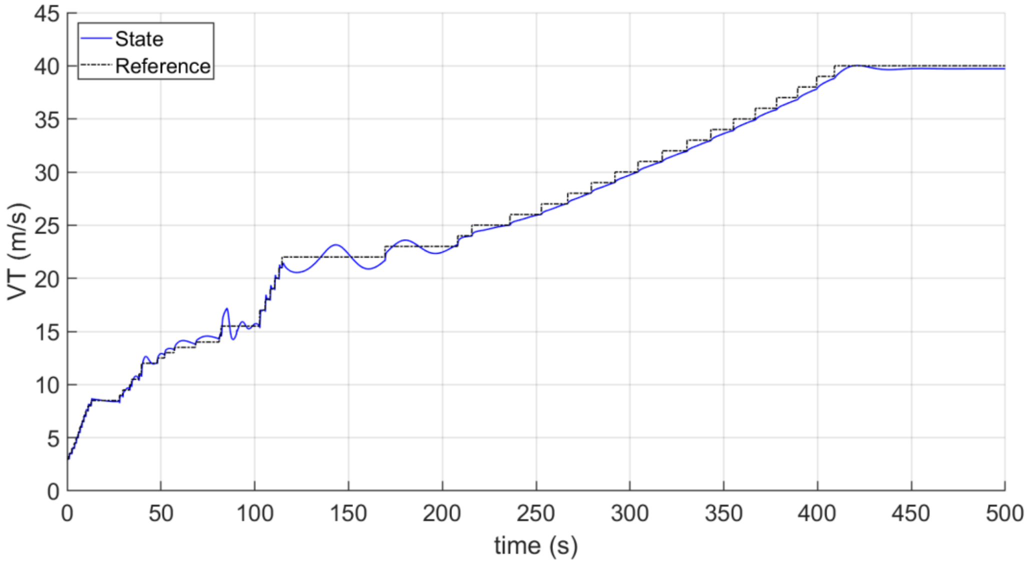


Figure 48: Accelerated flight simulation, results for V_T.

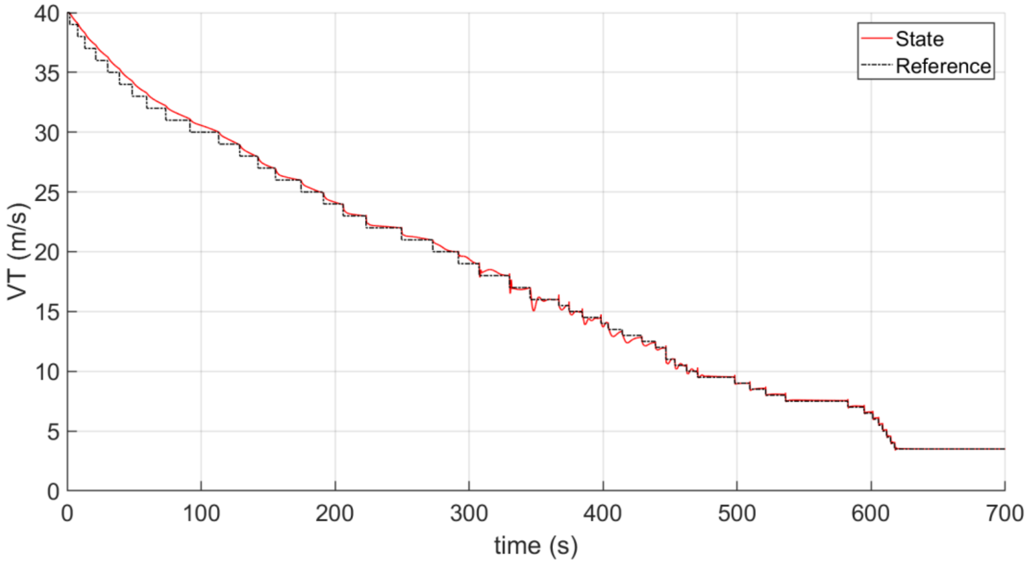


Figure 49: Decelerated flight simulation, results for V_T.

In the decelerated flight from cruise at 40 m/s to 3,0 m/s, the transition takes 618 seconds to reach the objective speed. At the beginning of the simulation, where the aircraft is at high speed, the deceleration is almost continuous, due to the easier and smoother control of the aircraft, where the aerodynamic controls have their maximum effectiveness. From 16 m/s to lower flight speeds there is a region where it is noted an oscilation to stabilize the velocity in the desired value, however the mean deceleration over time is kept almost constant, until the velocity of 7,5 m/s, which requires more time to stabilize. Then, an even faster mean deceleration takes place until the lowest flight speed of 3,5 m/s is reached.

Next we analyse the results of the attitude parameters, that is, angle of attack, the pitch angular velocity in the wind axes and the aircraft pitch angle.

In Figure 50 and Figure 51 we find the results for angle of attack for the accelerated flight and decelerated flight condition respectivelly. Again, the solid lines represent the state and the dotted lines the reference at the equilibrium point.

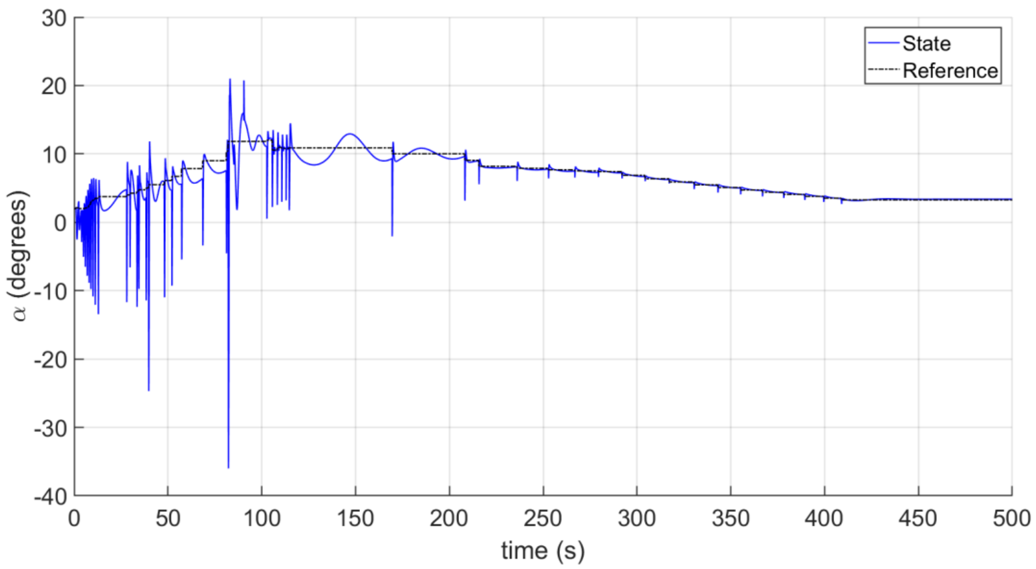


Figure 50: Accelerated flight simulation, results for α.

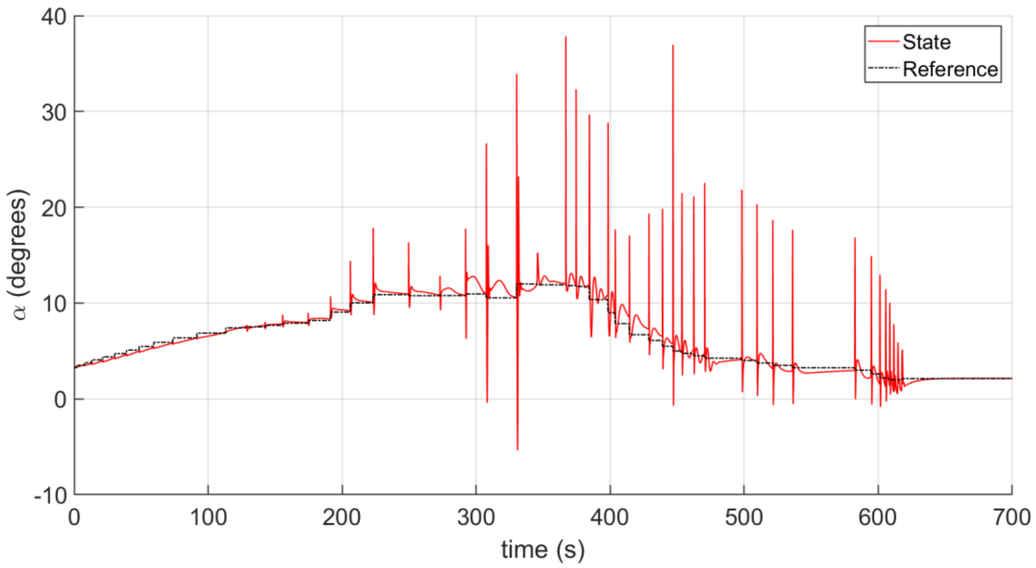


Figure 51: Decelerated flight simulation, results for α.

And the pitch angular velocity in the wind axes in Figure 52 and Figure 53.

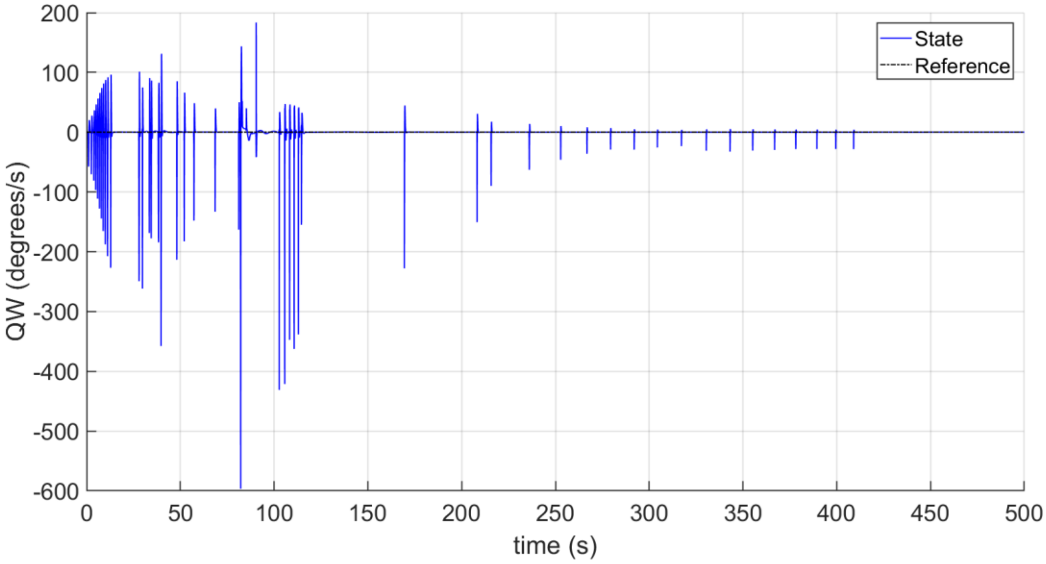


Figure 52: Accelerated flight simulation, results for Q_W.

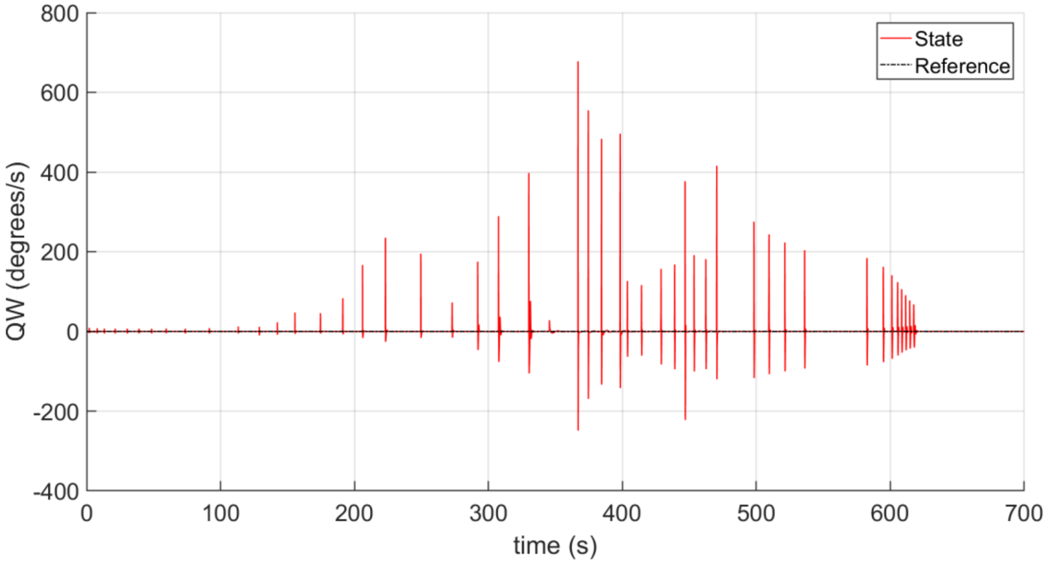


Figure 53: Decelerated flight simulation, results for Q_W.

Finally, the pitch angle results in Figure 54 and Figure 55.

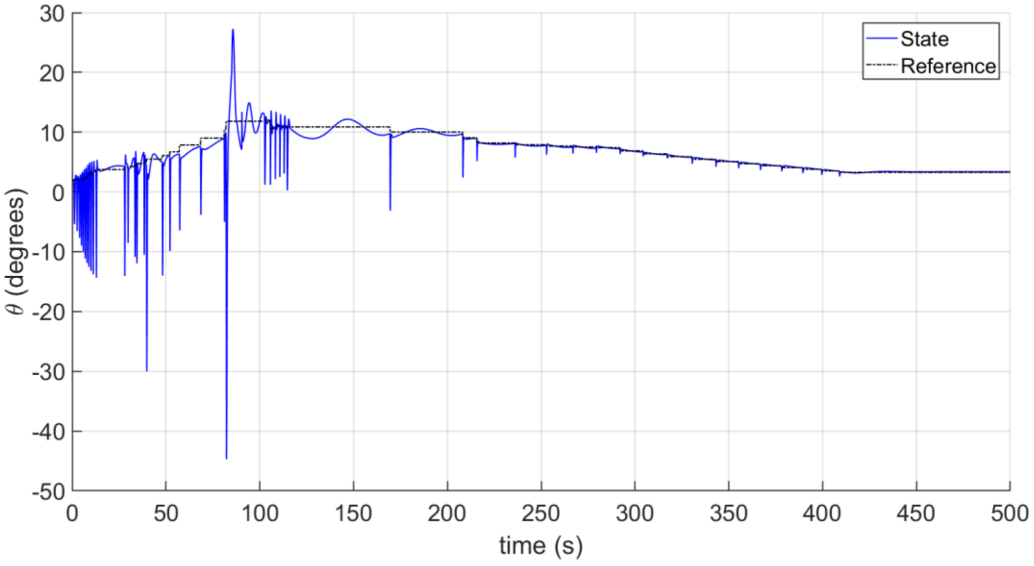


Figure 54: Accelerated flight simulation, results for $\boldsymbol{\theta}$.

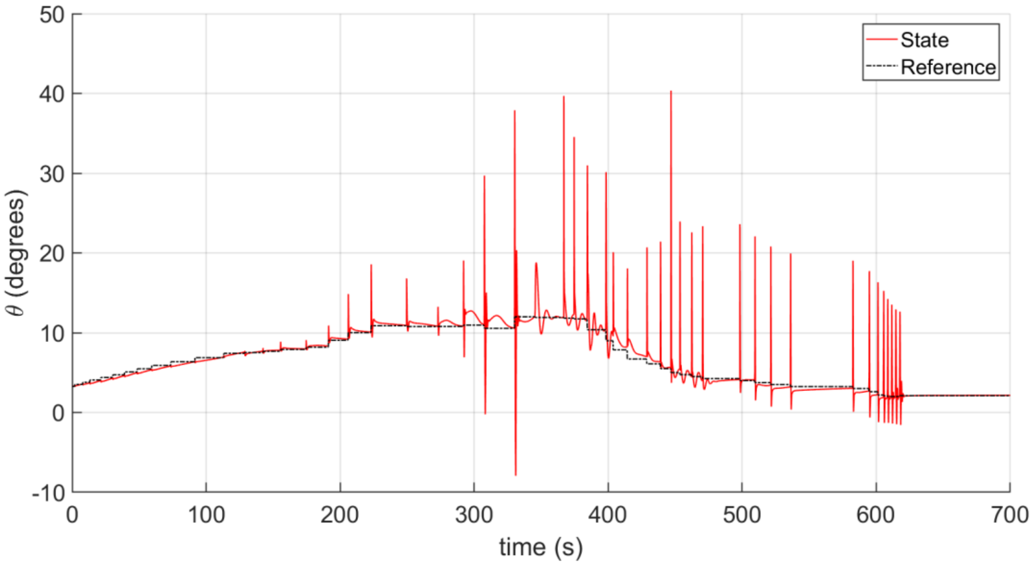


Figure 55: Decelerated flight simulation, results for $\boldsymbol{\theta}$.

Note that the higher attitude disturbances amplitudes with respect to the reference lines, for both accelerated and decelerated flight, occur at low flight speeds, where there is more changes between equilibrium positions for the wing and horizontal tail, so it is expected to have also higher changes in attitude. Possibly, the increase in the number of target points in this flight speed region would reduce such peaks. Moreover, the wing and horizontal tail position control is faster then the aircraft attitude control, so at the moment the control system changes it target point, there is a peak at aircraft attitude, which is gradually smoothed through the state regulator. The attitude angle amplitude is the highest at the transition from

15,5 m/s to 17 m/s, exactly the point where the rear controls switch between tail propellers and elevator. Again, in the high speed region, the attitude changes are the lowest, due to less changes in wing and horizontal tail position and higher aerodynamic controls effectiviness.

In Figure 56 and Figure 57 there are the results in the altitude of the aircraft for the accelerated and decelerated flight respectivelly. Remembering that the equilibrium were previoulsy computed for the altitude of 100 m, so that is the initial condition, and since it was also computed for no rate of climb, the simulation is supposed to keep the altitude of 100 m. It is observable that the control system succesfully maintain the aircraft altitude, having only few oscilations around the target altitude, being the largest displacement of less than 8,0 m, which happens when the control system switch from rear propeller to elevator control. Moreover, the oscilations occurs more frequently at low flight speeds, where the state controls comes from the propellers, which is also a consequence of fewer attitude control.

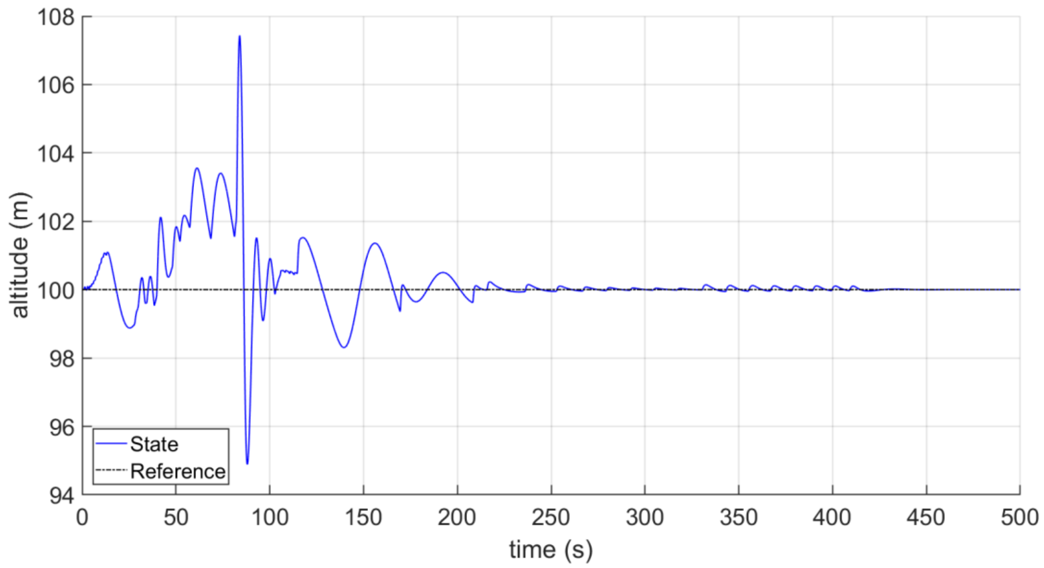


Figure 56: Accelerated flight simulation, results for altitude.

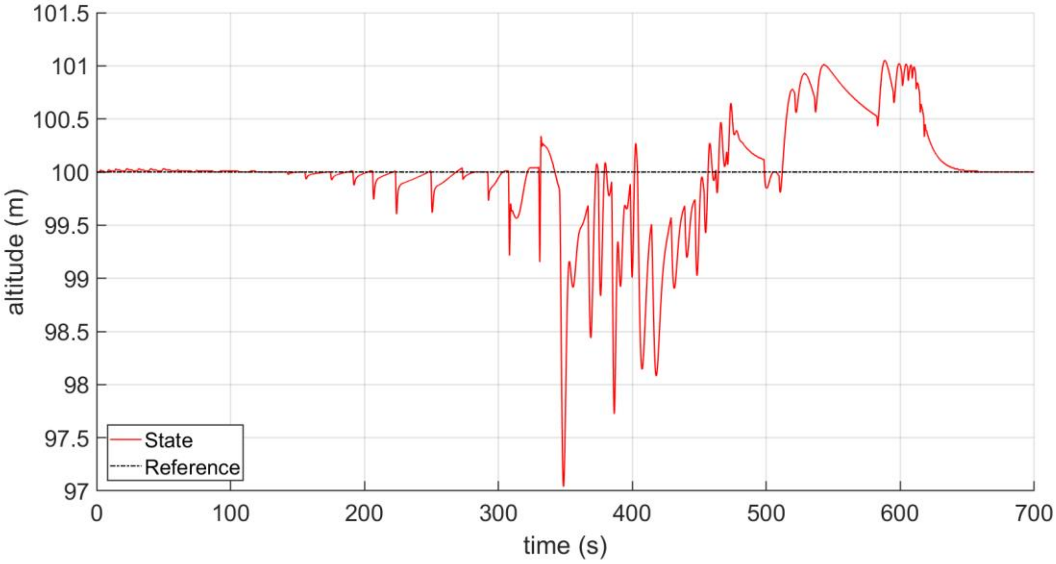


Figure 57: Decelerated flight simulation, results for altitude.

Follows now the controls results. In Figure 58 and Figure 59 we find respectivelly the flap deflections for the accelerated and decelerated flight conditions.

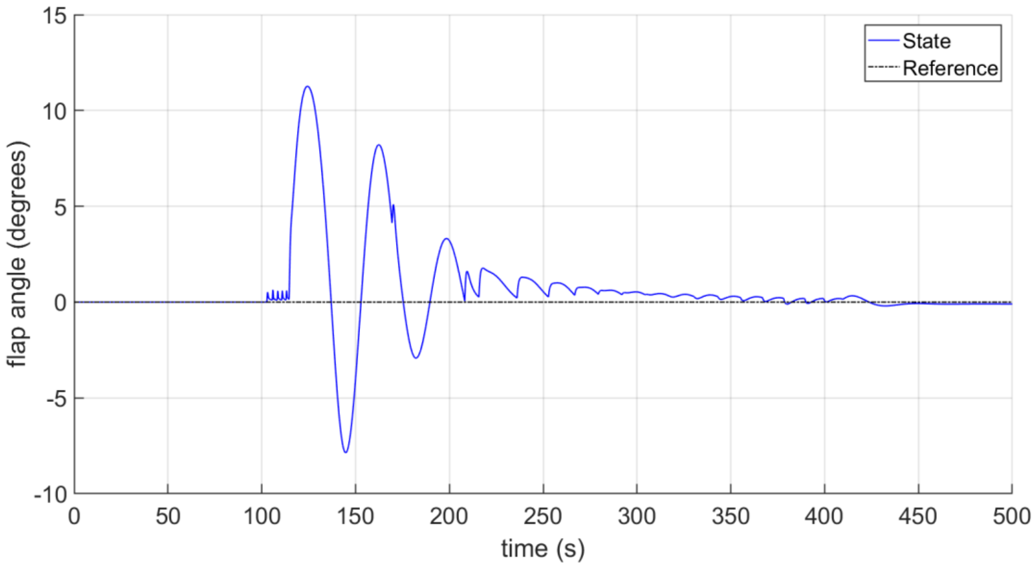


Figure 58: Accelerated flight simulation, results for δ_f.

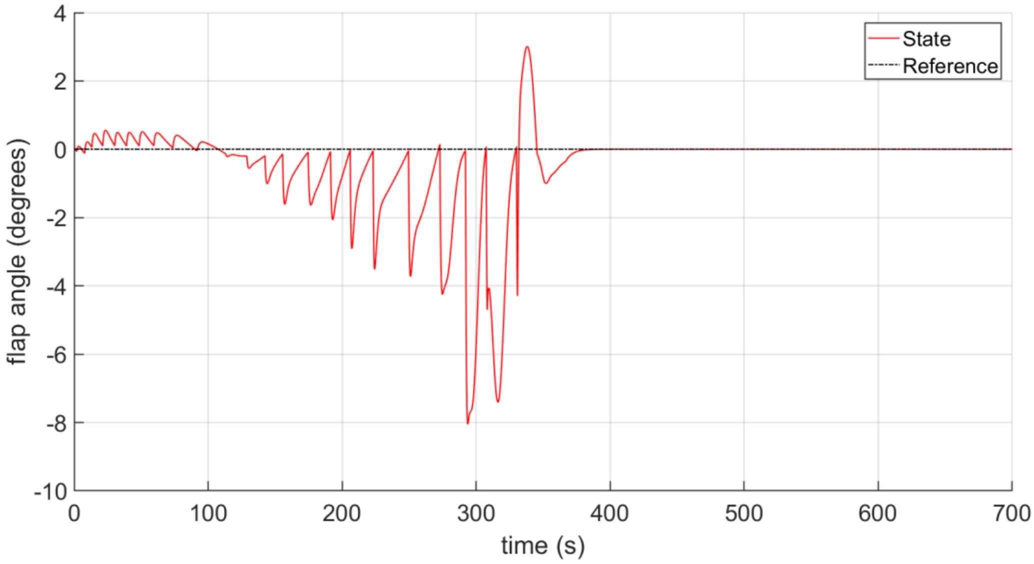


Figure 59: Decelerated flight simulation, results for δ_f.

In Figure 60 and Figure 61 we find respectivelly the elevator deflections for the accelerated and decelerated flight conditions.

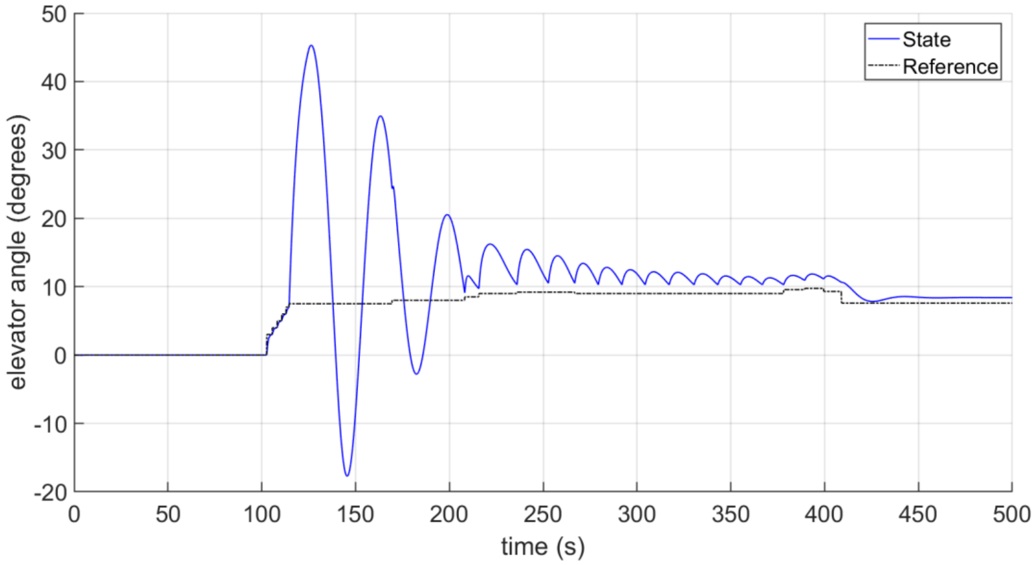


Figure 60: Accelerated flight simulation, results for δ_e.

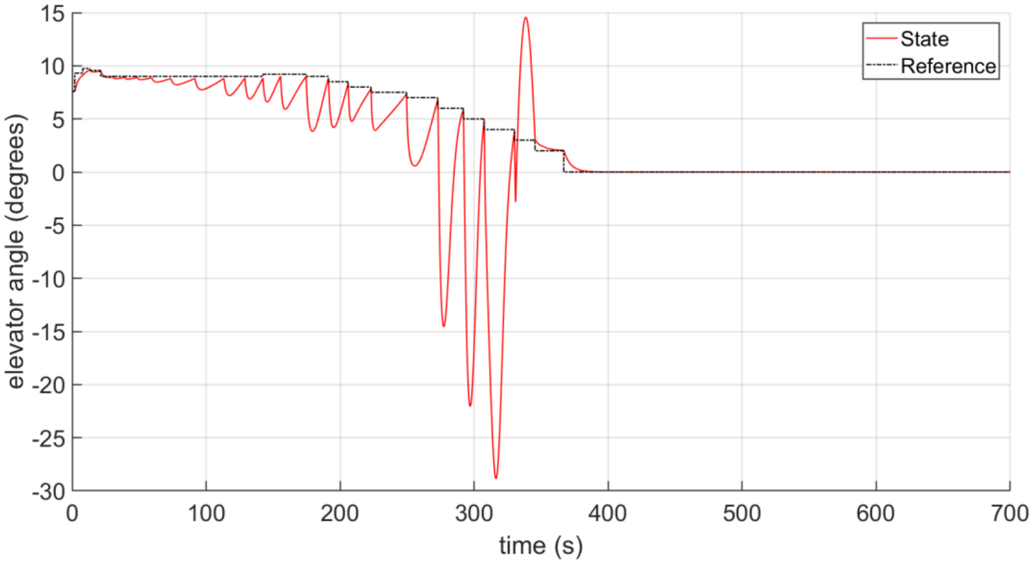


Figure 61: Decelerated flight simulation, results for δ_e.

At low speeds, until 16 m/s, there is no use of flap or elevator, since there is low dynamic pressure, the aerodynamic control would not be effective. Moreover, it was defined a range of allowable deflections of -50 degrees to +50 degrees, for both flap and elevator. Note that the region where both aerodynamic surfaces are most deflected is where it is imediately turned on (close to 16 m/s), which is reasonable, since the free stream dynamic pressure is lower than at higher speeds.

Now, the results for the wing and horizontal tail tilt angles in Figure 62 for the accelerated flight and in Figure 63 for the decelerated flight.

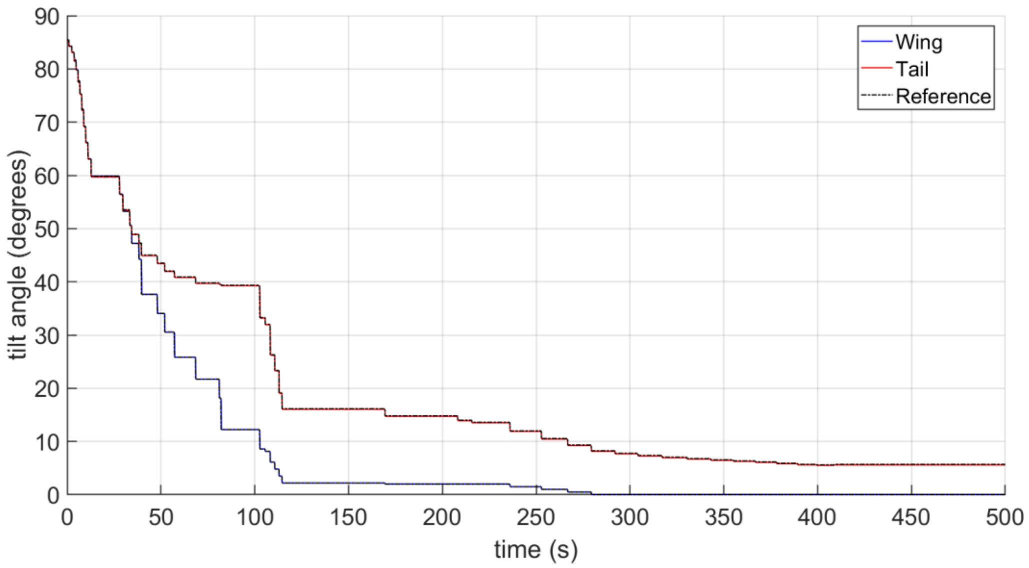


Figure 62: Accelerated flight simulation, results for δ_W and δ_{HT}.

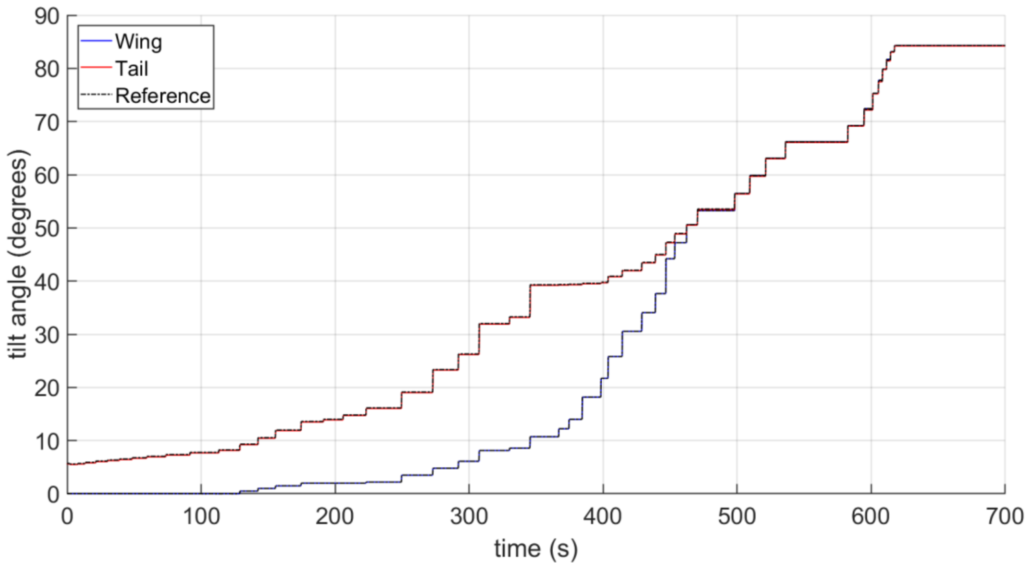


Figure 63: Decelerated flight simulation, results for δ_W and δ_{HT}.

The results shows the wing and horizontal tail tilt angle over time. Note that because of the quick action of the stepper motors to position the aerodynamic surfaces, their movement to reach the desired position is almost imperceptible, which is also desirable in the sense of quickly move the aircraft dynamic system to the target condition, so that the controls are more effective. It is important to remember that the state regulator computes the controls required to stabilize the aircraft for a specific situation where the dynamic system has fixed jacobian matrices, which means for this system, a fixed position of wing and horizontal tail. Therefore, the use of wing and horizontal tilt angle control to stabilize the aircraft while it transitions would not be recommended, since the aircraft dynamic system would be significantly altered while it constantly tilts the aerodynamic surfaces, in addition to consume more power from the wing and horizontal tail tilt actuators.

The propellers angular rotations squared are shown in Figure 64 for the accelerated flight and in Figure 65 for the decelerated flight.

It is observable that at high flight speeds the tail propellers have no angular rotation, since the power required is low and there is no need to trim the aircraft using all the six propellers. Also is visible the higher control amplitudes at low speeds where most of the control comes from the propellers. Furthermore, its is clear that at every target point transition there is a peak in propellers action to quickly stabilize the aircraft towards the equilibrium point.

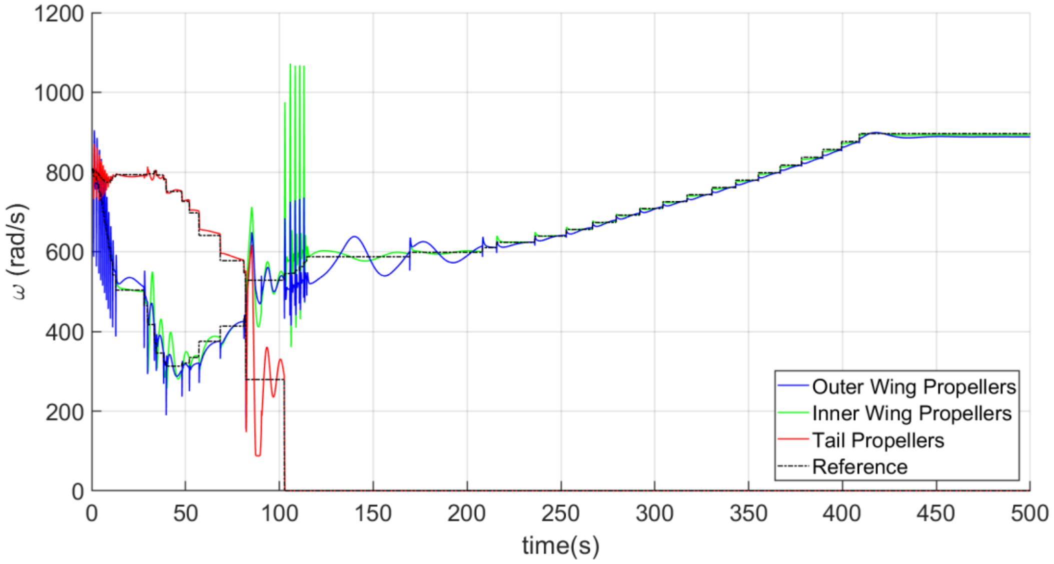


Figure 64: Accelerated flight simulation, results for ω_1 to ω_6.

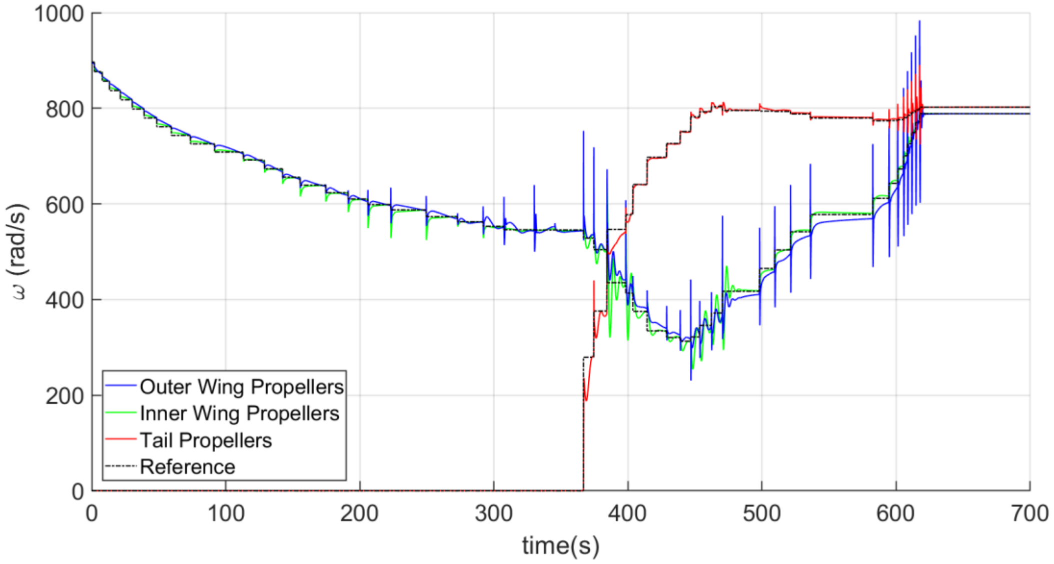


Figure 65: Decelerated flight simulation, results for ω_1 to ω_6.

In Figure 66 and Figure 67 are displayed the transition cost functions results, continuously computed through the simulations, responsible for the changes between target points. The cost function ε_s measures how disturbed is the aircraft state from the steady state condition, while the cost function ε_c measures how far are the controls from the position of steady state flight. This last one is particularly important because the aircraft should not transition to the next equilibrium point with its controls too far from the desired position, because if it does, there is a risk to start the stabilization of the next condition with the controls too far from the steady condition, which might destabilize the aircraft. Moreover, if it successively transitions between equilibrium points without the stabilization of the controls, there is the risk of eventually start the stabilization of the aircraft in the next equilibrium point with the controls too close to their limit, so there will be no room to effectivelly stabilize the transition. Lastly, the cost function ε_d measures how much the aircraft is oscilating, so it is important to move to the next equilibrium point only when the oscilation is minimum. Threfore, the transition to the next equilibrium point is only allowed if the three cost functions are bellow their thresholds. The thresholds used at each transition are listed in the Appendix.

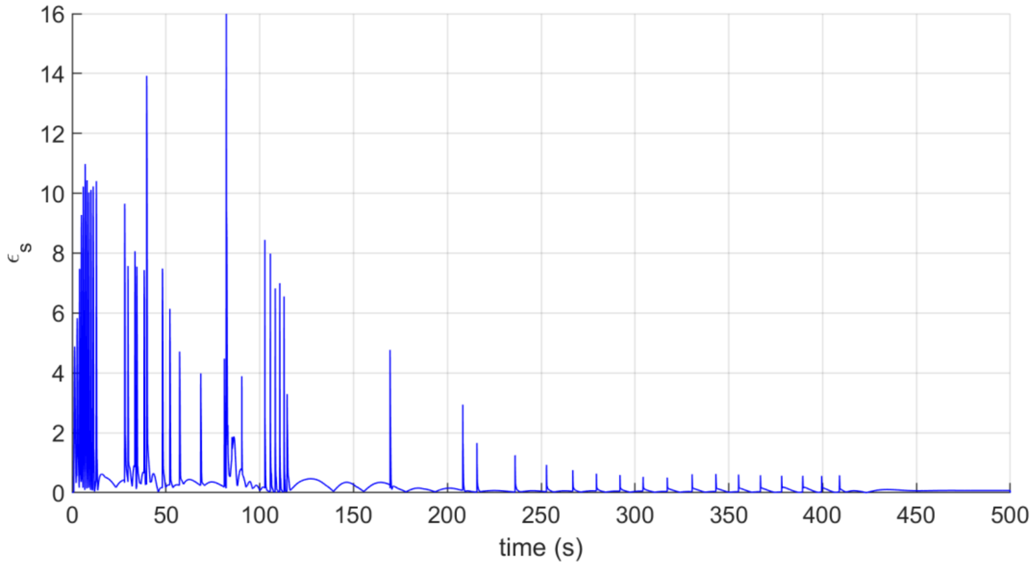


Figure 66: Accelerated flight simulation, results for transition cost function ε_s.

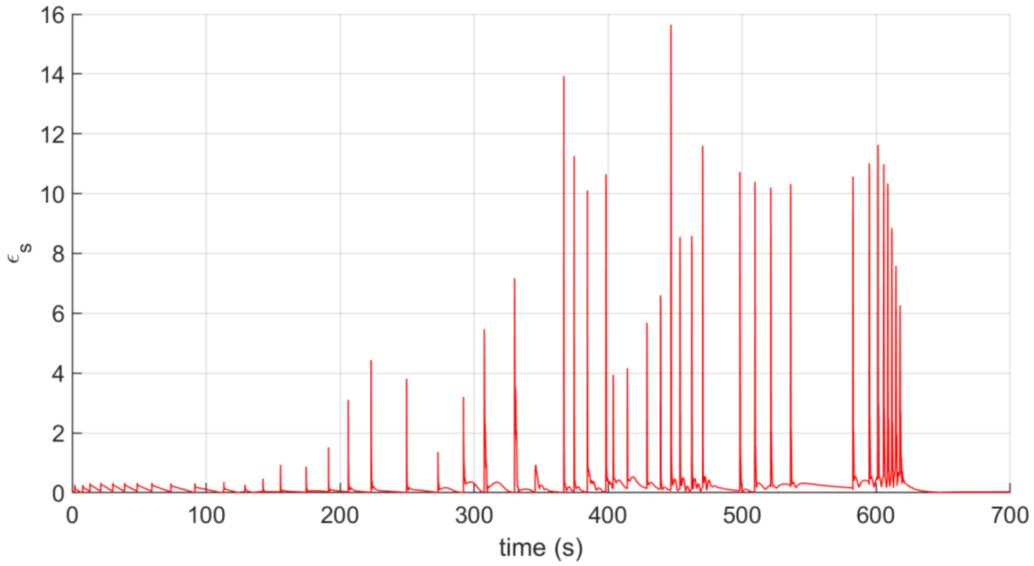


Figure 67: Decelerated flight simulation, results for transition cost function ε_s.

It is visible that the cost function ε_s reach higher peaks at the beginning of the simulation for the accelerated flight where it has the lower flight speeds, and also the higher disturbances in the equilibrium states, due to the higher transitions between equilibrium points. The same happens at the end of the simulation of the decelerated flght, where it passes through the lowest flight speeds. However, the state regulator quickly stabilizes the aircraft in order to move to the next target point, which can be seen as the successives quickly decay at the cost function.

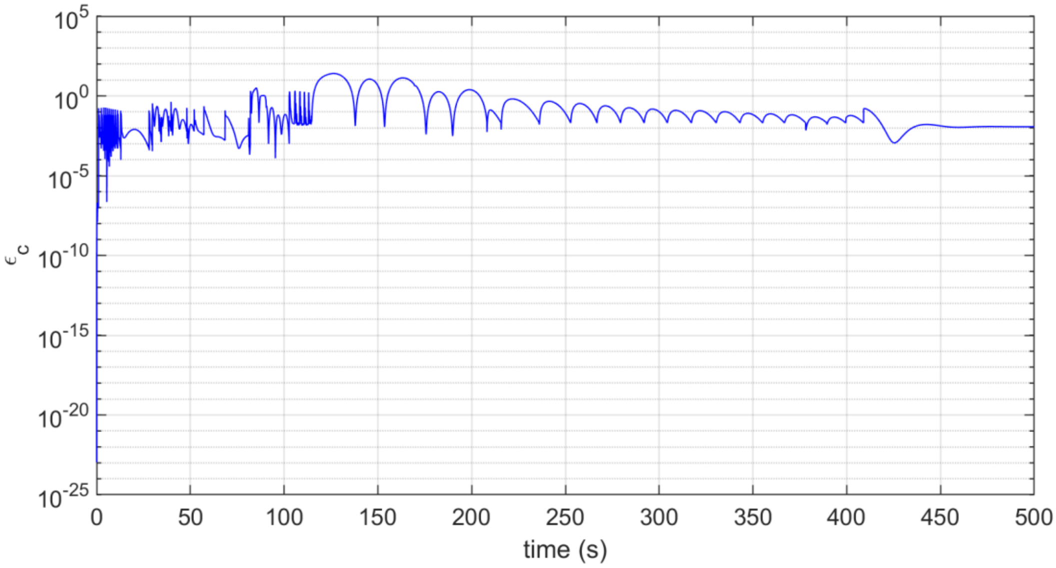


Figure 68: Accelerated flight simulation, results for transition cost function ε_c.

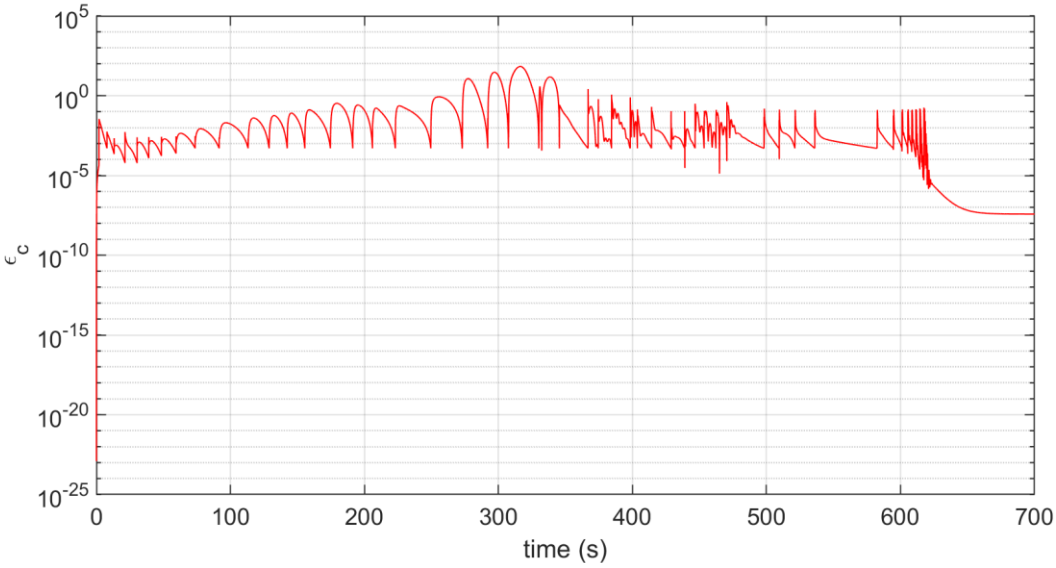


Figure 69: Decelerated flight simulation, results for transition cost function ε_c.

Now for the cost function with respect to controls deflections ε_c displayed in Figure 68 and Figure 69 the amplitudes are higher in the region where the control system switch between rear propellers and elevator, being that at high flight speeds it takes longer to stabilize due to slower action of the aerodynamic controls with respect to propellers used exclusively at low flight speeds. The same behavior is noted in Figure 70 and Figure 71 with results for the cost function of the state derivatives ε_d.

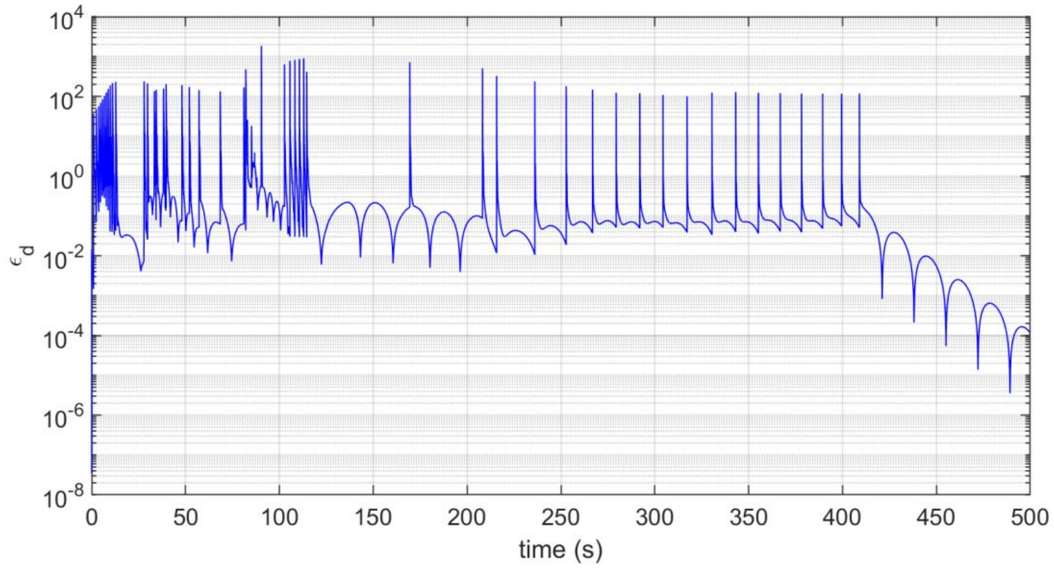


Figure 70: Accelerated flight simulation, results for transition cost function ε_d.

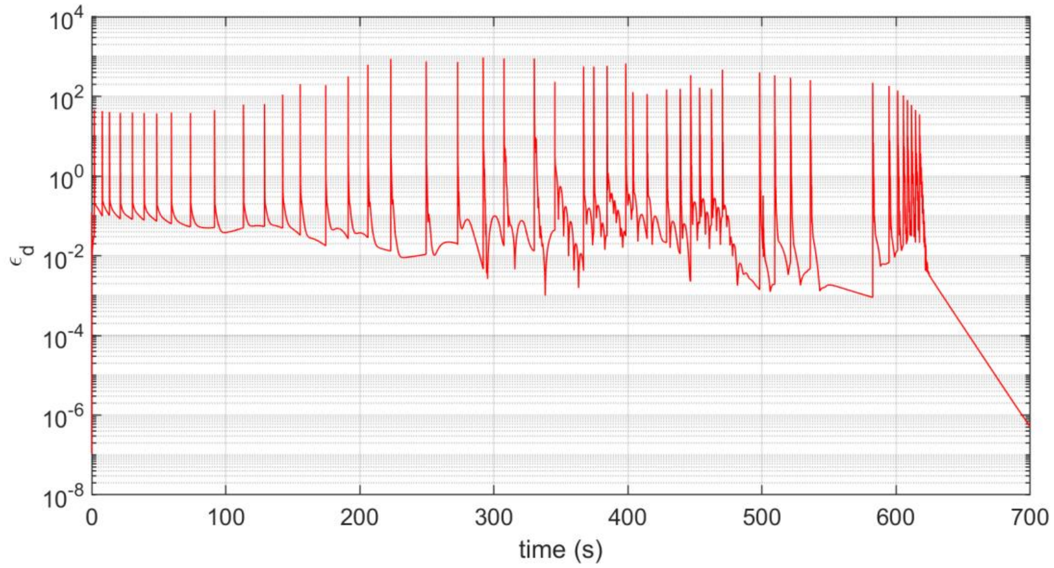


Figure 71: Decelerated flight simulation, results for transition cost function ε_d.

In Figure 72 and Figure 73 are shown the pitch angle desired which is computed by the altitude hold controller, and the steady condition pitch angle for each equilibrium point. Oscilation occurs most at the region between 9,0 m/s to 20 m/s, which is reasonable given that it is also the region with more oscilation in the other states of motion. Note that the desired value gradually aproaches the equilibrium value, which means that the altitude hold controller is succesfully working to stabilize the aircraft in the target altitude.

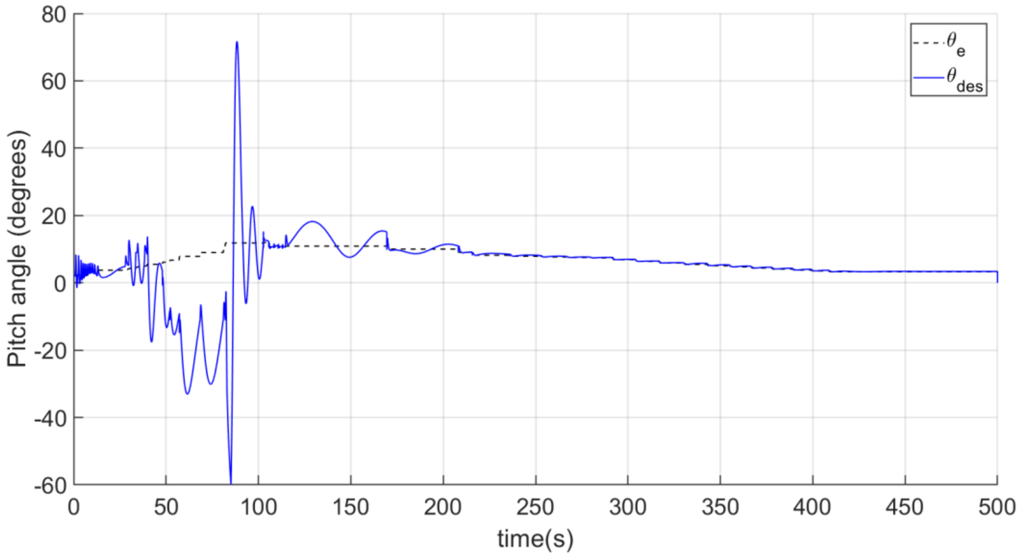


Figure 72: Accelerated flight simulation, results for pitch desired angle for the altitude hold controller.

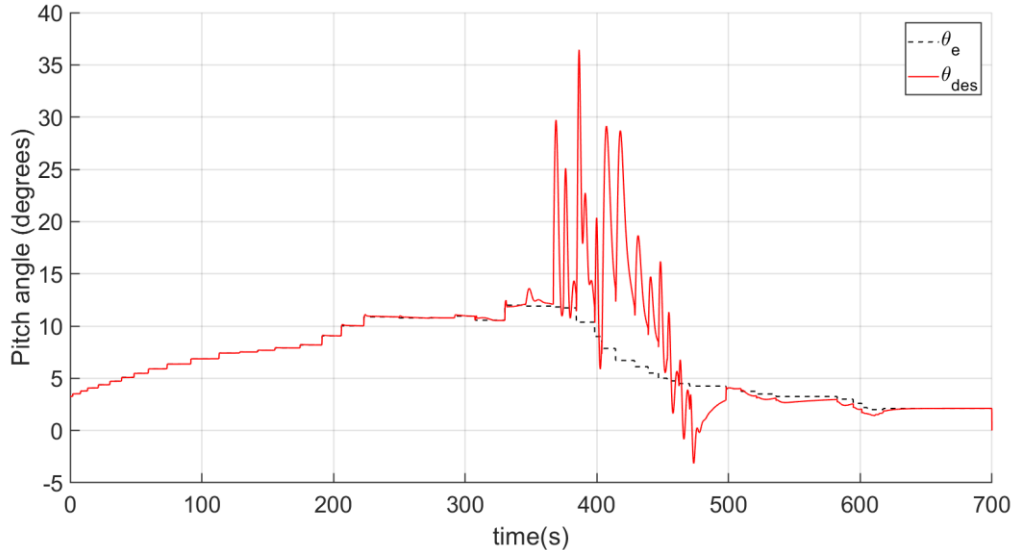


Figure 73: Decelerated flight simulation, results for pitch desired angle for the altitude hold controller.

Finally, in Figure 74 and Figure 75 are shown the horizontal distance travelled to complete the transition for the accelerated and decelerated longitudinal flight simulation. There we find the total distance of 9700 m (at 410 s) for the accelerated flight and 11990 m (at 618 s) for the decelerated flight.

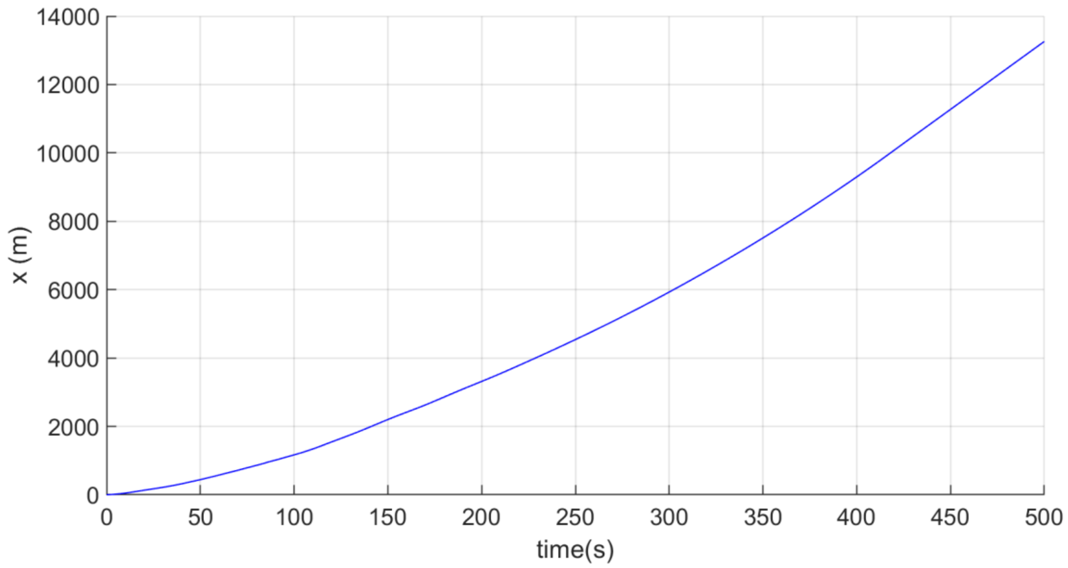


Figure 74: Accelerated flight simulation, results for the horizontal distance travelled while transitioning.

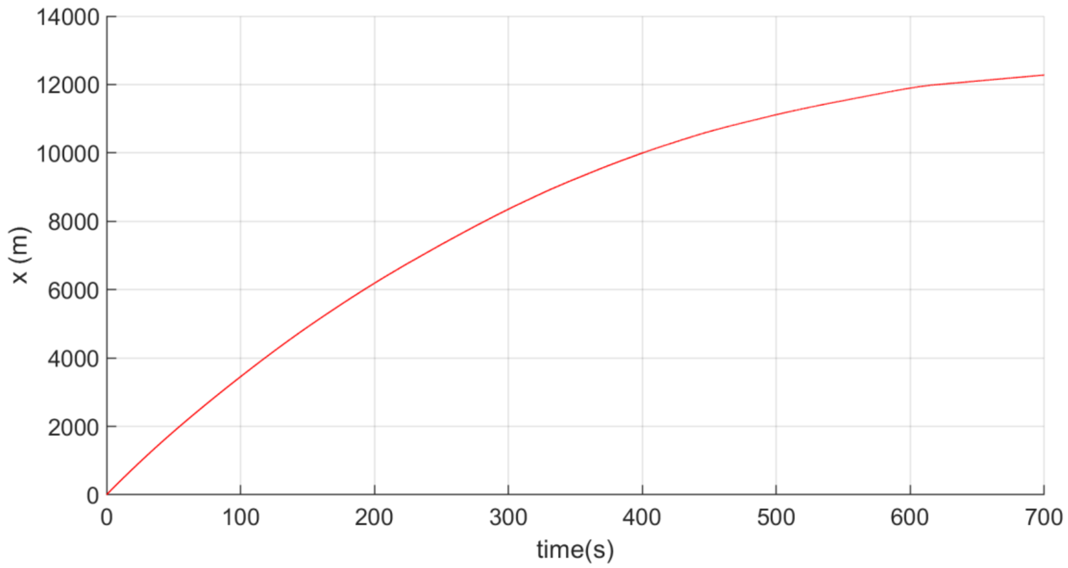


Figure 75: Decelerated flight simulation, results for the horizontal distance travelled while transitioning.

15 PROTOTYPE CONSTRUCTION

A prototype of the aircraft was designed and is currently being constructed so that the aerodynamic coefficients might be obtained through wind tunnel testing, and the inertial and control parameters be measured in order to be used in the embedded flight control software.

In the following Figure 76 and Figure 77 it is shown the aircraft prototype CAD design in Solidworks (2014).

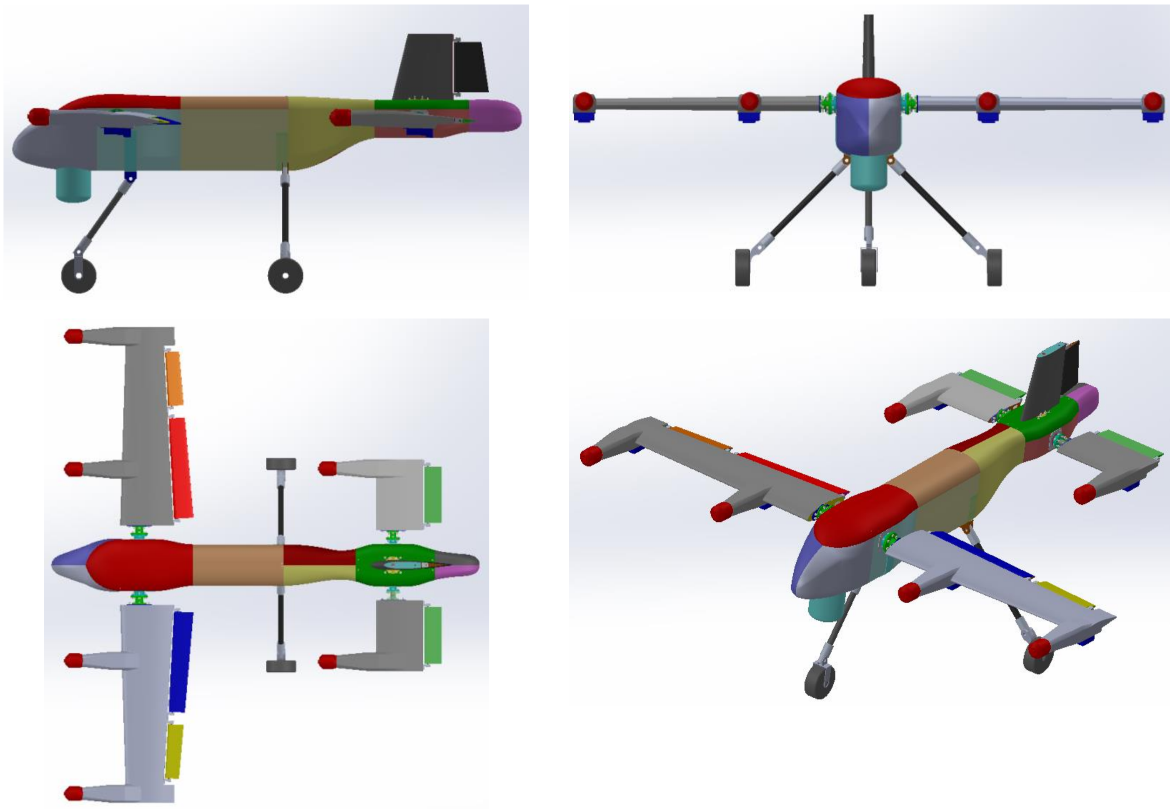
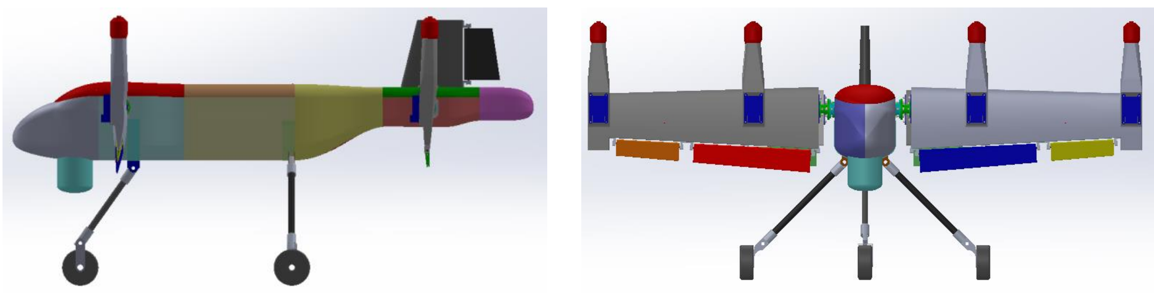


Figure 76: Aircraft prototype CAD model, cruise configuration.



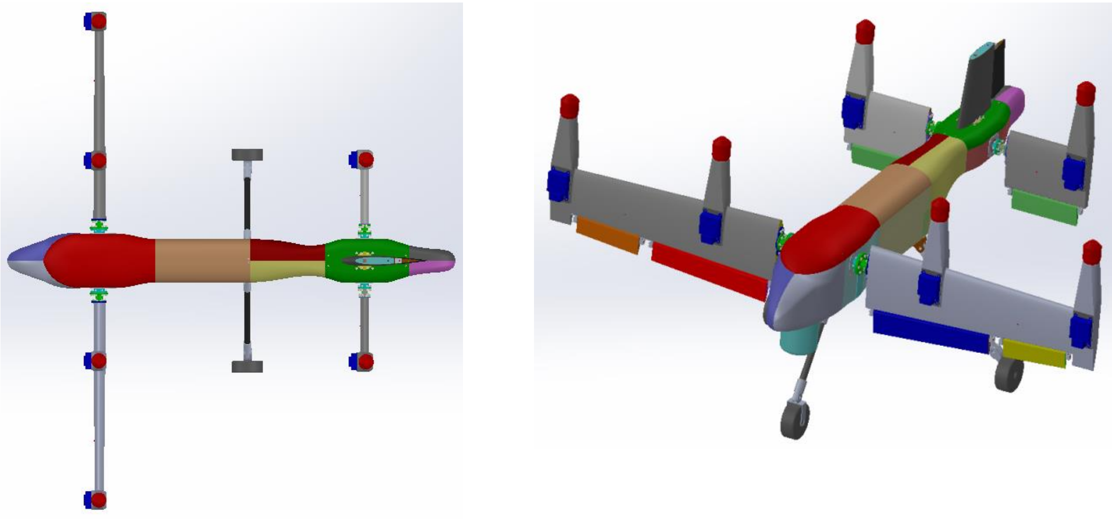


Figure 77: Aircraft prototype CAD model, hovering configuration.

Furthermore, the current status of the aircraft prototype is shown in Figure 78, where it can be seen the fuselage, left wing and left horizontal tail.

Figure 78: Aircraft prototype current status.

16 CONCLUSION

This thesis presents the theory developed to study the flight dynamics and control system of a proposed VTOL aircraft. It was used a promising configuration with wing and horizontal tail tilt through which an aircraft was designed and the system dynamics was modeled using multi-body equations of motion and the linearized state equations were derived. Also, was presented the methods used to estimate the aerodynamic coefficients and derivatives. Next, the Sequential Simplex algorithm was used to compute the aircraft trim conditions, defining the equilibrium vector of state and controls. The equilibrium points were the base for the proposed aircraft flight control system that uses successive state regulation procedure towards a desired flight speed, while holding its altitude. Lastly, the results for accelerated and decelerated longitudinal flight were presented, showing that the transition from hovering to cruise condition, and from cruise condition to hover speed would be possible for the proposed VTOL aircraft configuration using the control system presented.

In order to simulate the proposed control system it was required to estimate the aerodynamic coefficients and derivatives, which was done using references of high credibility, but in the proposed aircraft configuration it is expected for the wing and horizontal tail to operate in very high angles of attack, therefore in the post stall region, and yet under the effect of the propellers wake. However, the aerodynamic coefficients can only be properly obtained through experiments in wind tunnel so that all the complex aerodynamic interactions are measured. Additionally, it is necessary to measure the dynamics of the actuators and sensors which will allow the design of state observers. Thus, having the aerodynamic, control and sensors coefficients from experiments would allow the design of a more robust control system.

Finally, it is also challenging to develop a method to improve dynamic response during transition flight, which can be done by modifying the weighting scalar parameter ρ and the matrices Q and R of the Linear Quadratic Regulator, and adjusting the threshold for the cost functions of transition, in order the obtain a less oscillatory transition, with faster and shorter complete transition from hovering to cruise, and from cruise to hovering flight.

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18 APPENDIX A – PARTIAL DERIVATIVES

18.1 Partial derivatives of $\boldsymbol{S}$ and $\boldsymbol{S^T}$:

$$\frac{\partial S}{\partial \beta}=\frac{\partial}{\partial \beta}\begin{bmatrix}\cos\alpha\cos\beta & \sin\beta & \sin\alpha\cos\beta\\ -\cos\alpha\sin\beta & \cos\beta & -\sin\alpha\sin\beta\\ -\sin\alpha & 0 & \cos\alpha\end{bmatrix}=\begin{bmatrix}-\cos\alpha\sin\beta & \cos\beta & -\sin\alpha\sin\beta\\ -\cos\alpha\cos\beta & -\sin\beta & -\sin\alpha\cos\beta\\ 0 & 0 & 0\end{bmatrix}$$

$$\frac{\partial S}{\partial \alpha}=\frac{\partial}{\partial \alpha}\begin{bmatrix}\cos\alpha\cos\beta & \sin\beta & \sin\alpha\cos\beta\\ -\cos\alpha\sin\beta & \cos\beta & -\sin\alpha\sin\beta\\ -\sin\alpha & 0 & \cos\alpha\end{bmatrix}=\begin{bmatrix}-\sin\alpha\cos\beta & 0 & \cos\alpha\cos\beta\\ \sin\alpha\sin\beta & 0 & -\cos\alpha\sin\beta\\ -\cos\alpha & 0 & -\sin\alpha\end{bmatrix}$$

$$\frac{\partial S^T}{\partial \beta}=\frac{\partial}{\partial \beta}\begin{bmatrix}\cos\alpha\cos\beta & -\cos\alpha\sin\beta & -\sin\alpha\\ \sin\beta & \cos\beta & 0\\ \sin\alpha\cos\beta & -\sin\alpha\sin\beta & \cos\alpha\end{bmatrix}=\begin{bmatrix}-\cos\alpha\sin\beta & -\cos\alpha\cos\beta & 0\\ \cos\beta & -\sin\beta & 0\\ -\sin\alpha\sin\beta & -\sin\alpha\cos\beta & 0\end{bmatrix}$$

$$\frac{\partial S^T}{\partial \alpha}=\frac{\partial}{\partial \alpha}\begin{bmatrix}\cos\alpha\cos\beta & -\cos\alpha\sin\beta & -\sin\alpha\\ \sin\beta & \cos\beta & 0\\ \sin\alpha\cos\beta & -\sin\alpha\sin\beta & \cos\alpha\end{bmatrix}=\begin{bmatrix}-\sin\alpha\cos\beta & \sin\alpha\sin\beta & -\cos\alpha\\ 0 & 0 & 0\\ \cos\alpha\cos\beta & -\cos\alpha\sin\beta & -\sin\alpha\end{bmatrix}$$

18.2 Partial derivatives of $\boldsymbol{\dot{S}^T}$:

$$\frac{\partial \dot{S}^T}{\partial \dot{\beta}}=\frac{\partial}{\partial \dot{\beta}}\begin{bmatrix}-\dot{\alpha}\sin\alpha & -(\dot{\beta}\cos\alpha-\beta\dot{\alpha}\sin\alpha) & -\dot{\alpha}\cos\alpha\\ \dot{\beta} & 0 & 0\\ \dot{\alpha}\cos\alpha & -(\dot{\beta}\sin\alpha+\beta\dot{\alpha}\cos\alpha) & -\dot{\alpha}\sin\alpha\end{bmatrix}=\begin{bmatrix}0 & -\cos\alpha & 0\\ 1 & 0 & 0\\ 0 & -\sin\alpha & 0\end{bmatrix}$$

$$\frac{\partial \dot{S}^T}{\partial \dot{\alpha}}=\frac{\partial}{\partial \dot{\alpha}}\begin{bmatrix}-\dot{\alpha}\sin\alpha & -(\dot{\beta}\cos\alpha-\beta\dot{\alpha}\sin\alpha) & -\dot{\alpha}\cos\alpha\\ \dot{\beta} & 0 & 0\\ \dot{\alpha}\cos\alpha & -(\dot{\beta}\sin\alpha+\beta\dot{\alpha}\cos\alpha) & -\dot{\alpha}\sin\alpha\end{bmatrix}=\begin{bmatrix}-\sin\alpha & \beta\sin\alpha & -\cos\alpha\\ 0 & 0 & 0\\ \cos\alpha & -\beta\cos\alpha & -\sin\alpha\end{bmatrix}$$

$$\frac{\partial \dot{S}^T}{\partial \beta}=\frac{\partial}{\partial \beta}\begin{bmatrix}-\dot{\alpha}\sin\alpha & -(\dot{\beta}\cos\alpha-\beta\dot{\alpha}\sin\alpha) & -\dot{\alpha}\cos\alpha\\ \dot{\beta} & 0 & 0\\ \dot{\alpha}\cos\alpha & -(\dot{\beta}\sin\alpha+\beta\dot{\alpha}\cos\alpha) & -\dot{\alpha}\sin\alpha\end{bmatrix}=\begin{bmatrix}0 & \dot{\alpha}\sin\alpha & 0\\ 0 & 0 & 0\\ 0 & -\dot{\alpha}\cos\alpha & 0\end{bmatrix}$$

$$\frac{\partial \dot{S}^T}{\partial \alpha}=\frac{\partial}{\partial \alpha}\begin{bmatrix}-\dot{\alpha}\sin\alpha & -(\dot{\beta}\cos\alpha-\beta\dot{\alpha}\sin\alpha) & -\dot{\alpha}\cos\alpha\\ \dot{\beta} & 0 & 0\\ \dot{\alpha}\cos\alpha & -(\dot{\beta}\sin\alpha+\beta\dot{\alpha}\cos\alpha) & -\dot{\alpha}\sin\alpha\end{bmatrix}=\begin{bmatrix}-\dot{\alpha}\cos\alpha & \dot{\beta}\sin\alpha+\beta\dot{\alpha}\cos\alpha & \dot{\alpha}\sin\alpha\\ 0 & 0 & 0\\ -\dot{\alpha}\sin\alpha & -\dot{\beta}\cos\alpha+\beta\dot{\alpha}\sin\alpha & -\dot{\alpha}\cos\alpha\end{bmatrix}$$

18.3 Partial derivatives of $\boldsymbol{B_E^B}$:

$$\frac{\partial B_E^B}{\partial \phi}=\begin{bmatrix}0 & 0 & 0\\ \cos\psi\sin\theta\cos\phi+\sin\psi\sin\phi & \sin\psi\sin\theta\cos\phi-\cos\psi\sin\phi & \cos\theta\cos\phi\\ -\cos\psi\sin\theta\sin\phi+\sin\psi\cos\phi & -\sin\psi\sin\theta\sin\phi-\cos\psi\cos\phi & -\cos\theta\sin\phi\end{bmatrix}$$

$$\frac{\partial B_E^B}{\partial \theta}=\begin{bmatrix}-\cos\psi\sin\theta & -\sin\psi\sin\theta & -\cos\theta\\ \cos\psi\cos\theta\sin\phi & \sin\psi\cos\theta\sin\phi & -\sin\theta\sin\phi\\ \cos\psi\cos\theta\cos\phi & \sin\psi\cos\theta\cos\phi & -\sin\theta\cos\phi\end{bmatrix}$$

$$\frac{\partial B_E^B}{\partial \psi}=\begin{bmatrix}-\sin\psi\cos\theta & \cos\psi\cos\theta & 0\\ -\sin\psi\sin\theta\sin\phi-\cos\psi\cos\phi & \cos\psi\sin\theta\sin\phi-\sin\psi\cos\phi & 0\\ -\sin\psi\sin\theta\cos\phi+\cos\psi\sin\phi & \cos\psi\sin\theta\cos\phi+\sin\psi\sin\phi & 0\end{bmatrix}$$

18.4 Partial derivatives of $\boldsymbol{\Omega_W}$:

$$\frac{\partial \Omega_W}{\partial P_W} = \frac{\partial}{\partial P_W}\begin{bmatrix} 0 & -R_W & Q_W \\ R_W & 0 & -P_W \\ -Q_W & P_W & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$\frac{\partial \Omega_W}{\partial Q_W} = \frac{\partial}{\partial Q_W}\begin{bmatrix} 0 & -R_W & Q_W \\ R_W & 0 & -P_W \\ -Q_W & P_W & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix}$$

$$\frac{\partial \Omega_W}{\partial R_W} = \frac{\partial}{\partial R_W}\begin{bmatrix} 0 & -R_W & Q_W \\ R_W & 0 & -P_W \\ -Q_W & P_W & 0 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

18.5 Partial derivatives of $\boldsymbol{\Omega_W \Omega_W}$:

$$\frac{\partial \Omega_W \Omega_W}{\partial P_W} = \frac{\partial}{\partial P_W}\begin{bmatrix} -Q_W^2 - R_W^2 & P_W Q_W & P_W R_W \\ P_W Q_W & -P_W^2 - R_W^2 & Q_W R_W \\ P_W R_W & Q_W R_W & -P_W^2 - Q_W^2 \end{bmatrix} = \begin{bmatrix} 0 & Q_W & R_W \\ Q_W & -2P_W & 0 \\ R_W & 0 & -2P_W \end{bmatrix}$$

$$\frac{\partial \Omega_W \Omega_W}{\partial Q_W} = \frac{\partial}{\partial Q_W}\begin{bmatrix} -Q_W^2 - R_W^2 & P_W Q_W & P_W R_W \\ P_W Q_W & -P_W^2 - R_W^2 & Q_W R_W \\ P_W R_W & Q_W R_W & -P_W^2 - Q_W^2 \end{bmatrix} = \begin{bmatrix} -2Q_W & P_W & 0 \\ P_W & 0 & R_W \\ 0 & R_W & -2Q_W \end{bmatrix}$$

$$\frac{\partial \Omega_W \Omega_W}{\partial R_W} = \frac{\partial}{\partial R_W}\begin{bmatrix} -Q_W^2 - R_W^2 & P_W Q_W & P_W R_W \\ P_W Q_W & -P_W^2 - R_W^2 & Q_W R_W \\ P_W R_W & Q_W R_W & -P_W^2 - Q_W^2 \end{bmatrix} = \begin{bmatrix} -2R_W & 0 & P_W \\ 0 & -2R_W & Q_W \\ P_W & Q_W & 0 \end{bmatrix}$$

18.6 Partial derivatives of $\boldsymbol{\dot{\Omega}_W}$:

$$\frac{\partial \dot{\Omega}_W}{\partial \dot{P}_W} = \frac{\partial}{\partial \dot{P}_W}\begin{bmatrix} 0 & -\dot{R}_W & \dot{Q}_W \\ \dot{R}_W & 0 & -\dot{P}_W \\ -\dot{Q}_W & \dot{P}_W & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$\frac{\partial \dot{\Omega}_W}{\partial \dot{Q}_W} = \frac{\partial}{\partial \dot{Q}_W}\begin{bmatrix} 0 & -\dot{R}_W & \dot{Q}_W \\ \dot{R}_W & 0 & -\dot{P}_W \\ -\dot{Q}_W & \dot{P}_W & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix}$$

$$\frac{\partial \dot{\Omega}_W}{\partial \dot{R}_W} = \frac{\partial}{\partial \dot{R}_W}\begin{bmatrix} 0 & -\dot{R}_W & \dot{Q}_W \\ \dot{R}_W & 0 & -\dot{P}_W \\ -\dot{Q}_W & \dot{P}_W & 0 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

18.7 Partial derivatives of $\boldsymbol{\Omega_P}$:

$$\frac{\partial \Omega_{P_W}}{\partial \dot{\delta}_{\mathrm{W}}} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix}$$

$$\frac{\partial \Omega_{P_{HT}}}{\partial \dot{\delta}_{\mathrm{HT}}} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix}$$

18.8 Partial derivatives of $\boldsymbol{\Omega_R}$:

$$\frac{\partial \Omega_R}{\partial \dot{\beta}} = \frac{\partial}{\partial \dot{\beta}}\begin{bmatrix} 0 & -\dot{\beta} & -\dot{\alpha}\cos\beta \\ \dot{\beta} & 0 & \dot{\alpha}\sin\beta \\ \dot{\alpha}\cos\beta & -\dot{\alpha}\sin\beta & 0 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\frac{\partial\Omega_R}{\partial\dot{\alpha}} = \frac{\partial}{\partial\dot{\alpha}}\begin{bmatrix} 0 & -\dot{\beta} & -\dot{\alpha}\cos\beta \\ \dot{\beta} & 0 & \dot{\alpha}\sin\beta \\ \dot{\alpha}\cos\beta & -\dot{\alpha}\sin\beta & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -\cos\beta \\ 0 & 0 & \sin\beta \\ \cos\beta & -\sin\beta & 0 \end{bmatrix}$$

$$\frac{\partial\Omega_R}{\partial\,\beta} = \frac{\partial}{\partial\,\beta}\begin{bmatrix} 0 & -\dot{\beta} & -\dot{\alpha}\cos\beta \\ \dot{\beta} & 0 & \dot{\alpha}\sin\beta \\ \dot{\alpha}\cos\beta & -\dot{\alpha}\sin\beta & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & \dot{\alpha}\sin\beta \\ 0 & 0 & \dot{\alpha}\cos\beta \\ -\dot{\alpha}\sin\beta & -\dot{\alpha}\cos\beta & 0 \end{bmatrix}$$

18.9 Partial derivatives of $\boldsymbol{MSF}$:

$$\frac{\partial(MSF)}{\partial\beta} = \sum_{i=1}^{NW}\left\{m_{W_i}\left[\left(\dot{\Omega}_W + \Omega_W\Omega_W\right)\frac{\partial S}{\partial\beta}\vec{r}_{W_i/B} + \left(2\Omega_W\frac{\partial S}{\partial\beta}\dot{R}^B_{W_i} + \frac{\partial S}{\partial\beta}\ddot{R}^B_{W_i}\right)\vec{r}_{W_i/pivot_i}\right]\right\} + \sum_{j=1}^{NR}\left\{m_{R_j}\left[\left(\dot{\Omega}_W + \Omega_w\Omega_w\right)\frac{\partial S}{\partial\beta}\vec{r}_{R_j/B} + \left(2\Omega_W\frac{\partial S}{\partial\beta}\dot{R}^B_{R_j} + \frac{\partial S}{\partial\beta}\ddot{R}^B_{R_j}\right)\vec{r}_{R_j/pivot_j}\right]\right\}$$

$$\frac{\partial(MSF)}{\partial\alpha} = \sum_{i=1}^{NW}\left\{m_{W_i}\left[\left(\dot{\Omega}_W + \Omega_W\Omega_W\right)\frac{\partial S}{\partial\alpha}\vec{r}_{W_i/B} + \left(2\Omega_W\frac{\partial S}{\partial\alpha}\dot{R}^B_{W_i} + \frac{\partial S}{\partial\alpha}\ddot{R}^B_{W_i}\right)\vec{r}_{W_i/pivot_i}\right]\right\} + \sum_{j=1}^{NR}\left\{m_{R_j}\left[\left(\dot{\Omega}_W + \Omega_w\Omega_w\right)\frac{\partial S}{\partial\alpha}\vec{r}_{R_j/B} + \left(2\Omega_W\frac{\partial S}{\partial\alpha}\dot{R}^B_{R_j} + \frac{\partial S}{\partial\alpha}\ddot{R}^B_{R_j}\right)\vec{r}_{R_j/pivot_j}\right]\right\}$$

$$\frac{\partial(MSF)}{\partial P_W} = \sum_{i=1}^{NW}\left\{m_{W_i}\left[\frac{\partial\Omega_W\Omega_W}{\partial P_W}S\vec{r}_{W_i/B} + 2\frac{\partial\Omega_W}{\partial P_W}S\dot{R}^B_{W_i}\vec{r}_{W_i/pivot_i}\right]\right\} + \sum_{j=1}^{NR}\left\{m_{R_j}\left[\frac{\partial\Omega_W\Omega_W}{\partial P_W}S\vec{r}_{R_j/B} + 2\frac{\partial\Omega_W}{\partial P_W}S\dot{R}^B_{R_j}\vec{r}_{R_j/pivot_j}\right]\right\}$$

$$\frac{\partial(MSF)}{\partial Q_W} = \sum_{i=1}^{NW}\left\{m_{W_i}\left[\frac{\partial\Omega_W\Omega_W}{\partial Q_W}S\vec{r}_{W_i/B} + 2\frac{\partial\Omega_W}{\partial Q_W}S\dot{R}^B_{W_i}\vec{r}_{W_i/pivot_i}\right]\right\} + \sum_{j=1}^{NR}\left\{m_{R_j}\left[\frac{\partial\Omega_W\Omega_W}{\partial Q_W}S\vec{r}_{R_j/B} + 2\frac{\partial\Omega_W}{\partial Q_W}S\dot{R}^B_{R_j}\vec{r}_{R_j/pivot_j}\right]\right\}$$

$$\frac{\partial(MSF)}{\partial R_W} = \sum_{i=1}^{NW}\left\{m_{W_i}\left[\frac{\partial\Omega_W\Omega_W}{\partial R_W}S\vec{r}_{W_i/B} + 2\frac{\partial\Omega_W}{\partial R_W}S\dot{R}^B_{W_i}\vec{r}_{W_i/pivot_i}\right]\right\} + \sum_{j=1}^{NR}\left\{m_{R_j}\left[\frac{\partial\Omega_W\Omega_W}{\partial R_W}S\vec{r}_{R_j/B} + 2\frac{\partial\Omega_W}{\partial R_W}S\dot{R}^B_{R_j}\vec{r}_{R_j/pivot_j}\right]\right\}$$

$$\frac{\partial(MSF)}{\partial\delta_W} = \sum_{i=1}^{NW}\left\{m_{W_i}\left(\left(\dot{\Omega}_W + \Omega_W\Omega_W\right)S\frac{\partial R^B_{W_i}}{\partial\delta_W} + 2\Omega_W S\frac{\partial\dot{R}^B_{W_i}}{\partial\delta_W} + S\frac{\partial\ddot{R}^B_{W_i}}{\partial\delta_W}\right)\vec{r}_{W_i/pivot_i}\right\} + \sum_{j=1}^{NR}\left\{m_{R_j}\left(\left(\dot{\Omega}_W + \Omega_w\Omega_w\right)S\frac{\partial R^B_{R_j}}{\partial\delta_W} + 2\Omega_W S\frac{\partial\dot{R}^B_{R_j}}{\partial\delta_W} + S\frac{\partial\ddot{R}^B_{R_j}}{\partial\delta_W}\right)\vec{r}_{R_j/pivot_j}\right\}$$

$$\frac{\partial(MSF)}{\partial\delta_{HT}} = \sum_{i=1}^{NW}\left\{m_{W_i}\left(\left(\dot{\Omega}_W + \Omega_W\Omega_W\right)S\frac{\partial R^B_{W_i}}{\partial\delta_{HT}} + 2\Omega_W S\frac{\partial\dot{R}^B_{W_i}}{\partial\delta_{HT}} + S\frac{\partial\ddot{R}^B_{W_i}}{\partial\delta_{HT}}\right)\vec{r}_{W_i/pivot_i}\right\} + \sum_{j=1}^{NR}\left\{m_{R_j}\left(\left(\dot{\Omega}_W + \Omega_w\Omega_w\right)S\frac{\partial R^B_{R_j}}{\partial\delta_{HT}} + 2\Omega_W S\frac{\partial\dot{R}^B_{R_j}}{\partial\delta_{HT}} + S\frac{\partial\ddot{R}^B_{R_j}}{\partial\delta_{HT}}\right)\vec{r}_{R_j/pivot_j}\right\}$$

$$\frac{\partial(MSF)}{\partial\dot{\delta}_W} = \sum_{i=1}^{NW}\left\{m_{W_i}\left(2\Omega_W S\frac{\partial\dot{R}^B_{W_i}}{\partial\dot{\delta}_W} + S\frac{\partial\ddot{R}^B_{W_i}}{\partial\dot{\delta}_W}\right)\vec{r}_{W_i/pivot_i}\right\} + \sum_{j=1}^{NR}\left\{m_{R_j}\left(2\Omega_W S\frac{\partial\dot{R}^B_{R_j}}{\partial\dot{\delta}_W} + S\frac{\partial\ddot{R}^B_{R_j}}{\partial\dot{\delta}_W}\right)\vec{r}_{R_j/pivot_j}\right\}$$

$$\frac{\partial(MSF)}{\partial\dot{\delta}_{HT}} = \sum_{i=1}^{NW}\left\{m_{W_i}\left(2\Omega_W S\frac{\partial\dot{R}^B_{W_i}}{\partial\dot{\delta}_{HT}} + S\frac{\partial\ddot{R}^B_{W_i}}{\partial\dot{\delta}_{HT}}\right)\vec{r}_{W_i/pivot_i}\right\} + \sum_{j=1}^{NR}\left\{m_{R_j}\left(2\Omega_W S\frac{\partial\dot{R}^B_{R_j}}{\partial\dot{\delta}_{HT}} + S\frac{\partial\ddot{R}^B_{R_j}}{\partial\dot{\delta}_{HT}}\right)\vec{r}_{R_j/pivot_j}\right\}$$

18.10 Partial derivatives of $\boldsymbol{R_{W_i}^B}$:

$$\frac{\partial R_{W_i}^B}{\partial \delta_W} = \frac{\partial}{\partial \delta_W}\begin{bmatrix} \cos(\delta_{W,HT_i}) & 0 & \sin(\delta_{W,HT_i}) \\ 0 & 1 & 0 \\ -\sin(\delta_{W,HT_i}) & 0 & \cos(\delta_{W,HT_i}) \end{bmatrix} = \left\{\begin{bmatrix} -\sin\delta_W & 0 & \cos\delta_W \\ 0 & 0 & 0 \\ -\cos\delta_W & 0 & -\sin\delta_W \end{bmatrix} or \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\right\}$$

$$\frac{\partial R_{W_i}^B}{\partial \delta_{HT}} = \frac{\partial}{\partial \delta_{HT}}\begin{bmatrix} \cos(\delta_{W,HT_i}) & 0 & \sin(\delta_{W,HT_i}) \\ 0 & 1 & 0 \\ -\sin(\delta_{W,HT_i}) & 0 & \cos(\delta_{W,HT_i}) \end{bmatrix} = \left\{\begin{bmatrix} -\sin\delta_{HT} & 0 & \cos\delta_{HT} \\ 0 & 0 & 0 \\ -\cos\delta_{HT} & 0 & -\sin\delta_{HT} \end{bmatrix} or \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\right\}$$

18.11 Partial derivatives of $\boldsymbol{R_{R_j}^B}$:

$$\frac{\partial R_{R_j}^B}{\partial \delta_W} = \frac{\partial}{\partial \delta_W}\begin{bmatrix} \cos\left(\delta_{W,HT_j}\right) & 0 & \sin\left(\delta_{W,HT_j}\right) \\ 0 & 1 & 0 \\ -\sin\left(\delta_{W,HT_j}\right) & 0 & \cos\left(\delta_{W,HT_j}\right) \end{bmatrix} = \left\{\begin{bmatrix} -\sin\delta_W & 0 & \cos\delta_W \\ 0 & 0 & 0 \\ -\cos\delta_W & 0 & -\sin\delta_W \end{bmatrix} or \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\right\}$$

$$\frac{\partial R_{R_j}^B}{\partial \delta_{HT}} = \frac{\partial}{\partial \delta_{HT}}\begin{bmatrix} \cos\left(\delta_{W,HT_j}\right) & 0 & \sin\left(\delta_{W,HT_j}\right) \\ 0 & 1 & 0 \\ -\sin\left(\delta_{W,HT_j}\right) & 0 & \cos\left(\delta_{W,HT_j}\right) \end{bmatrix} = \left\{\begin{bmatrix} -\sin\delta_{HT} & 0 & \cos\delta_{HT} \\ 0 & 0 & 0 \\ -\cos\delta_{HT} & 0 & -\sin\delta_{HT} \end{bmatrix} or \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\right\}$$

18.12 Partial derivatives of $\boldsymbol{\dot{R}_{W_i}^B}$:

$$\frac{\partial \dot{R}_{W_i}^B}{\partial \delta_W} = \frac{\partial}{\partial \delta_W}\left(\dot{\delta}_{W,HT_i}\begin{bmatrix} -\sin(\delta_{W,HT_i}) & 0 & \cos(\delta_{W,HT_i}) \\ 0 & 0 & 0 \\ -\cos(\delta_{W,HT_i}) & 0 & -\sin(\delta_{W,HT_i}) \end{bmatrix}\right) = \left\{\dot{\delta}_W\begin{bmatrix} -\cos\delta_W & 0 & -\sin\delta_W \\ 0 & 0 & 0 \\ \sin\delta_W & 0 & -\cos\delta_W \end{bmatrix} or \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\right\}$$

$$\frac{\partial \dot{R}_{W_i}^B}{\partial \delta_{HT}} = \frac{\partial}{\partial \delta_{HT}}\left(\dot{\delta}_{W,HT_i}\begin{bmatrix} -\sin(\delta_{W,HT_i}) & 0 & \cos(\delta_{W,HT_i}) \\ 0 & 0 & 0 \\ -\cos(\delta_{W,HT_i}) & 0 & -\sin(\delta_{W,HT_i}) \end{bmatrix}\right) = \left\{\dot{\delta}_{HT}\begin{bmatrix} -\cos\delta_{HT} & 0 & -\sin\delta_{HT} \\ 0 & 0 & 0 \\ \sin\delta_{HT} & 0 & -\cos\delta_{HT} \end{bmatrix} or \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\right\}$$

$$\frac{\partial \dot{R}_{W_i}^B}{\partial \dot{\delta}_W} = \frac{\partial}{\partial \dot{\delta}_W}\left(\dot{\delta}_{W,HT_i}\begin{bmatrix} -\sin(\delta_{W,HT_i}) & 0 & \cos(\delta_{W,HT_i}) \\ 0 & 0 & 0 \\ -\cos(\delta_{W,HT_i}) & 0 & -\sin(\delta_{W,HT_i}) \end{bmatrix}\right) = \left\{\begin{bmatrix} -\sin\delta_W & 0 & \cos\delta_W \\ 0 & 0 & 0 \\ -\cos\delta_W & 0 & -\sin\delta_W \end{bmatrix} or \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\right\}$$

$$\frac{\partial \dot{R}_{W_i}^B}{\partial \dot{\delta}_{HT}} = \frac{\partial}{\partial \dot{\delta}_{HT}}\left(\dot{\delta}_{W,HT_i}\begin{bmatrix} -\sin(\delta_{W,HT_i}) & 0 & \cos(\delta_{W,HT_i}) \\ 0 & 0 & 0 \\ -\cos(\delta_{W,HT_i}) & 0 & -\sin(\delta_{W,HT_i}) \end{bmatrix}\right) = \left\{\begin{bmatrix} -\sin\delta_{HT} & 0 & \cos\delta_{HT} \\ 0 & 0 & 0 \\ -\cos\delta_{HT} & 0 & -\sin\delta_{HT} \end{bmatrix} or \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\right\}$$

18.13 Partial derivatives of $\boldsymbol{\dot{R}_{R_j}^B}$:

$$\frac{\partial \dot{R}_{R_j}^B}{\partial \delta_W} = \frac{\partial}{\partial \delta_W}\left(\dot{\delta}_{W,HT_j}\begin{bmatrix} -\sin\left(\delta_{W,HT_j}\right) & 0 & \cos\left(\delta_{W,HT_j}\right) \\ 0 & 0 & 0 \\ -\cos\left(\delta_{W,HT_j}\right) & 0 & -\sin\left(\delta_{W,HT_j}\right) \end{bmatrix}\right) = \left\{\dot{\delta}_W\begin{bmatrix} -\cos\delta_W & 0 & -\sin\delta_W \\ 0 & 0 & 0 \\ \sin\delta_W & 0 & -\cos\delta_W \end{bmatrix} or \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\right\}$$

$$\frac{\partial \dot{R}_{R_j}^B}{\partial \delta_{HT}} = \frac{\partial}{\partial \delta_{HT}}\left(\dot{\delta}_{W,HT_i}\begin{bmatrix} -\sin\left(\delta_{W,HT_j}\right) & 0 & \cos\left(\delta_{W,HT_j}\right) \\ 0 & 0 & 0 \\ -\cos\left(\delta_{W,HT_j}\right) & 0 & -\sin\left(\delta_{W,HT_j}\right) \end{bmatrix}\right) = \left\{\dot{\delta}_{HT}\begin{bmatrix} -\cos\delta_{HT} & 0 & -\sin\delta_{HT} \\ 0 & 0 & 0 \\ \sin\delta_{HT} & 0 & -\cos\delta_{HT} \end{bmatrix} or \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\right\}$$

$$\frac{\partial \dot{R}_{R_j}^B}{\partial \dot{\delta}_W} = \frac{\partial}{\partial \dot{\delta}_W}\left(\dot{\delta}_{W,HT_i}\begin{bmatrix} -\sin(\delta_{W,HT_i}) & 0 & \cos(\delta_{W,HT_i}) \\ 0 & 0 & 0 \\ -\cos(\delta_{W,HT_i}) & 0 & -\sin(\delta_{W,HT_i}) \end{bmatrix}\right) = \left\{\begin{bmatrix} -\sin\delta_W & 0 & \cos\delta_W \\ 0 & 0 & 0 \\ -\cos\delta_W & 0 & -\sin\delta_W \end{bmatrix} or \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\right\}$$

$$\frac{\partial \dot{R}_{R_j}^B}{\partial \dot{\delta}_{HT}} = \frac{\partial}{\partial \dot{\delta}_{HT}}\left(\dot{\delta}_{W,HT_i}\begin{bmatrix} -\sin(\delta_{W,HT_i}) & 0 & \cos(\delta_{W,HT_i}) \\ 0 & 0 & 0 \\ -\cos(\delta_{W,HT_i}) & 0 & -\sin(\delta_{W,HT_i}) \end{bmatrix}\right) = \left\{\begin{bmatrix} -\sin\delta_{HT} & 0 & \cos\delta_{HT} \\ 0 & 0 & 0 \\ -\cos\delta_{HT} & 0 & -\sin\delta_{HT} \end{bmatrix} or \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\right\}$$

18.14 Partial derivatives of $\ddot{\boldsymbol{R}}_{\boldsymbol{W_i}}^{\boldsymbol{B}}$:

$$\frac{\partial \ddot{R}_{W_i}^B}{\partial \delta_W} = \frac{\partial}{\partial \delta_W}\begin{bmatrix} -\left(\ddot{\delta}_{W,HT_i}\sin(\delta_{W,HT_i}) + \dot{\delta}_{W,HT_i}\cos(\delta_{W,HT_i})\right) & 0 & \left(\ddot{\delta}_{W,HT_i}\cos(\delta_{W,HT_i}) - \dot{\delta}_{W,HT_i}\sin(\delta_{W,HT_i})\right) \\ 0 & 0 & 0 \\ -\left(\ddot{\delta}_{W,HT_i}\cos(\delta_{W,HT_i}) - \dot{\delta}_{W,HT_i}\sin(\delta_{W,HT_i})\right) & 0 & -\left(\ddot{\delta}_{W,HT_i}\sin(\delta_{W,HT_i}) + \dot{\delta}_{W,HT_i}\cos(\delta_{W,HT_i})\right) \end{bmatrix}$$

$$= \left\{\begin{bmatrix} -\left(\ddot{\delta}_W\cos\delta_W - \dot{\delta}_W\sin\delta_W\right) & 0 & \left(-\ddot{\delta}_W\sin\delta_W - \dot{\delta}_W\cos\delta_W\right) \\ 0 & 0 & 0 \\ \left(\ddot{\delta}_W\sin\delta_W + \dot{\delta}_W\cos\delta_W\right) & 0 & -\left(\ddot{\delta}_W\cos\delta_W - \dot{\delta}_W\sin\delta_W\right) \end{bmatrix} or \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\right\}$$

$$\frac{\partial \ddot{R}_{W_i}^B}{\partial \delta_{HT}} = \frac{\partial}{\partial \delta_{HT}}\begin{bmatrix} -\left(\ddot{\delta}_{W,HT_i}\sin(\delta_{W,HT_i}) + \dot{\delta}_{W,HT_i}\cos(\delta_{W,HT_i})\right) & 0 & \left(\ddot{\delta}_{W,HT_i}\cos(\delta_{W,HT_i}) - \dot{\delta}_{W,HT_i}\sin(\delta_{W,HT_i})\right) \\ 0 & 0 & 0 \\ -\left(\ddot{\delta}_{W,HT_i}\cos(\delta_{W,HT_i}) - \dot{\delta}_{W,HT_i}\sin(\delta_{W,HT_i})\right) & 0 & -\left(\ddot{\delta}_{W,HT_i}\sin(\delta_{W,HT_i}) + \dot{\delta}_{W,HT_i}\cos(\delta_{W,HT_i})\right) \end{bmatrix}$$

$$= \left\{\begin{bmatrix} -\left(\ddot{\delta}_{HT}\cos\delta_{HT} - \dot{\delta}_{HT}\sin\delta_{HT}\right) & 0 & \left(-\ddot{\delta}_{HT}\sin\delta_{HT} - \dot{\delta}_{HT}\cos\delta_{HT}\right) \\ 0 & 0 & 0 \\ \left(\ddot{\delta}_{HT}\sin\delta_{HT} + \dot{\delta}_{HT}\cos\delta_{HT}\right) & 0 & -\left(\ddot{\delta}_{HT}\cos\delta_{HT} - \dot{\delta}_{HT}\sin\delta_{HT}\right) \end{bmatrix} or \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\right\}$$

$$\frac{\partial \ddot{R}_{W_i}^B}{\partial \dot{\delta}_W} = \frac{\partial}{\partial \dot{\delta}_W}\begin{bmatrix} -\left(\ddot{\delta}_{W,HT_i}\sin(\delta_{W,HT_i}) + \dot{\delta}_{W,HT_i}\cos(\delta_{W,HT_i})\right) & 0 & \left(\ddot{\delta}_{W,HT_i}\cos(\delta_{W,HT_i}) - \dot{\delta}_{W,HT_i}\sin(\delta_{W,HT_i})\right) \\ 0 & 0 & 0 \\ -\left(\ddot{\delta}_{W,HT_i}\cos(\delta_{W,HT_i}) - \dot{\delta}_{W,HT_i}\sin(\delta_{W,HT_i})\right) & 0 & -\left(\ddot{\delta}_{W,HT_i}\sin(\delta_{W,HT_i}) + \dot{\delta}_{W,HT_i}\cos(\delta_{W,HT_i})\right) \end{bmatrix}$$

$$= \left\{\begin{bmatrix} -\cos\delta_W & 0 & -\sin\delta_W \\ 0 & 0 & 0 \\ \sin\delta_W & 0 & -\cos\delta_W \end{bmatrix} or \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\right\}$$

$$\frac{\partial \ddot{R}_{W_i}^B}{\partial \dot{\delta}_{HT}} = \frac{\partial}{\partial \delta_{HT}}\begin{bmatrix} -\left(\ddot{\delta}_{W,HT_i}\sin(\delta_{W,HT_i}) + \dot{\delta}_{W,HT_i}\cos(\delta_{W,HT_i})\right) & 0 & \left(\ddot{\delta}_{W,HT_i}\cos(\delta_{W,HT_i}) - \dot{\delta}_{W,HT_i}\sin(\delta_{W,HT_i})\right) \\ 0 & 0 & 0 \\ -\left(\ddot{\delta}_{W,HT_i}\cos(\delta_{W,HT_i}) - \dot{\delta}_{W,HT_i}\sin(\delta_{W,HT_i})\right) & 0 & -\left(\ddot{\delta}_{W,HT_i}\sin(\delta_{W,HT_i}) + \dot{\delta}_{W,HT_i}\cos(\delta_{W,HT_i})\right) \end{bmatrix}$$

$$= \left\{\begin{bmatrix} -\cos\delta_{HT} & 0 & -\sin\delta_{HT} \\ 0 & 0 & 0 \\ \sin\delta_{HT} & 0 & -\cos\delta_{HT} \end{bmatrix} or \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\right\}$$

$$\frac{\partial \ddot{R}_{W_i}^B}{\partial \ddot{\delta}_W} = \frac{\partial}{\partial \ddot{\delta}_W}\begin{bmatrix} -\left(\ddot{\delta}_{W,HT_i}\sin(\delta_{W,HT_i}) + \dot{\delta}_{W,HT_i}\cos(\delta_{W,HT_i})\right) & 0 & \left(\ddot{\delta}_{W,HT_i}\cos(\delta_{W,HT_i}) - \dot{\delta}_{W,HT_i}\sin(\delta_{W,HT_i})\right) \\ 0 & 0 & 0 \\ -\left(\ddot{\delta}_{W,HT_i}\cos(\delta_{W,HT_i}) - \dot{\delta}_{W,HT_i}\sin(\delta_{W,HT_i})\right) & 0 & -\left(\ddot{\delta}_{W,HT_i}\sin(\delta_{W,HT_i}) + \dot{\delta}_{W,HT_i}\cos(\delta_{W,HT_i})\right) \end{bmatrix}$$

$$= \left\{\begin{bmatrix} -\sin\delta_W & 0 & \cos\delta_W \\ 0 & 0 & 0 \\ -\cos\delta_W & 0 & -\sin\delta_W \end{bmatrix} or \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\right\}$$

$$\frac{\partial \ddot{R}_{W_i}^B}{\partial \ddot{\delta}_{HT}} = \frac{\partial}{\partial \ddot{\delta}_{HT}}\begin{bmatrix} -\left(\ddot{\delta}_{W,HT_i}\sin(\delta_{W,HT_i}) + \dot{\delta}_{W,HT_i}\cos(\delta_{W,HT_i})\right) & 0 & \left(\ddot{\delta}_{W,HT_i}\cos(\delta_{W,HT_i}) - \dot{\delta}_{W,HT_i}\sin(\delta_{W,HT_i})\right) \\ 0 & 0 & 0 \\ -\left(\ddot{\delta}_{W,HT_i}\cos(\delta_{W,HT_i}) - \dot{\delta}_{W,HT_i}\sin(\delta_{W,HT_i})\right) & 0 & -\left(\ddot{\delta}_{W,HT_i}\sin(\delta_{W,HT_i}) + \dot{\delta}_{W,HT_i}\cos(\delta_{W,HT_i})\right) \end{bmatrix}$$

$$= \left\{\begin{bmatrix} -\sin\delta_{HT} & 0 & \cos\delta_{HT} \\ 0 & 0 & 0 \\ -\cos\delta_{HT} & 0 & -\sin\delta_{HT} \end{bmatrix} or \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\right\}$$

18.15 P Partial derivatives of $\ddot{\boldsymbol{R}}_{\boldsymbol{R_j}}^{\boldsymbol{B}}$:

$$\frac{\partial \ddot{R}_{R_j}^B}{\partial \delta_W} = \frac{\partial}{\partial \delta_W}\begin{bmatrix} -\left(\ddot{\delta}_{W,HT_j}\sin\left(\delta_{W,HT_j}\right)+\dot{\delta}_{W,HT_j}\cos\left(\delta_{W,HT_j}\right)\right) & 0 & \left(\ddot{\delta}_{W,HT_j}\cos\left(\delta_{W,HT_j}\right)-\dot{\delta}_{W,HT_j}\sin\left(\delta_{W,HT_j}\right)\right) \\ 0 & 0 & 0 \\ -\left(\ddot{\delta}_{W,HT_j}\cos\left(\delta_{W,HT_j}\right)-\dot{\delta}_{W,HT_j}\sin\left(\delta_{W,HT_j}\right)\right) & 0 & -\left(\ddot{\delta}_{W,HT_j}\sin\left(\delta_{W,HT_j}\right)+\dot{\delta}_{W,HT_j}\cos\left(\delta_{W,HT_j}\right)\right) \end{bmatrix}$$

$$= \left\{\begin{bmatrix} -(\ddot{\delta}_W\cos\delta_W - \dot{\delta}_W\sin\delta_W) & 0 & (-\ddot{\delta}_W\sin\delta_W - \dot{\delta}_W\cos\delta_W) \\ 0 & 0 & 0 \\ (\ddot{\delta}_W\sin\delta_W + \dot{\delta}_W\cos\delta_W) & 0 & -(\ddot{\delta}_W\cos\delta_W - \dot{\delta}_W\sin\delta_W) \end{bmatrix} \; or \; \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\right\}$$

$$\frac{\partial \ddot{R}_{R_j}^B}{\partial \delta_{HT}} = \frac{\partial}{\partial \delta_{HT}}\begin{bmatrix} -\left(\ddot{\delta}_{W,HT_j}\sin\left(\delta_{W,HT_j}\right)+\dot{\delta}_{W,HT_j}\cos\left(\delta_{W,HT_j}\right)\right) & 0 & \left(\ddot{\delta}_{W,HT_j}\cos\left(\delta_{W,HT_j}\right)-\dot{\delta}_{W,HT_j}\sin\left(\delta_{W,HT_j}\right)\right) \\ 0 & 0 & 0 \\ -\left(\ddot{\delta}_{W,HT_j}\cos\left(\delta_{W,HT_j}\right)-\dot{\delta}_{W,HT_j}\sin\left(\delta_{W,HT_j}\right)\right) & 0 & -\left(\ddot{\delta}_{W,HT_j}\sin\left(\delta_{W,HT_j}\right)+\dot{\delta}_{W,HT_j}\cos\left(\delta_{W,HT_j}\right)\right) \end{bmatrix}$$

$$= \left\{\begin{bmatrix} -(\ddot{\delta}_{HT}\cos\delta_{HT} - \dot{\delta}_{HT}\sin\delta_{HT}) & 0 & (-\ddot{\delta}_{HT}\sin\delta_{HT} - \dot{\delta}_{HT}\cos\delta_{HT}) \\ 0 & 0 & 0 \\ (\ddot{\delta}_{HT}\sin\delta_{HT} + \dot{\delta}_{HT}\cos\delta_{HT}) & 0 & -(\ddot{\delta}_{HT}\cos\delta_{HT} - \dot{\delta}_{HT}\sin\delta_{HT}) \end{bmatrix} \; or \; \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\right\}$$

$$\frac{\partial \ddot{R}_{R_j}^B}{\partial \dot{\delta}_W} = \frac{\partial}{\partial \dot{\delta}_W}\begin{bmatrix} -(\ddot{\delta}_{W,HT_i}\sin(\delta_{W,HT_i})+\dot{\delta}_{W,HT_i}\cos(\delta_{W,HT_i})) & 0 & (\ddot{\delta}_{W,HT_i}\cos(\delta_{W,HT_i})-\dot{\delta}_{W,HT_i}\sin(\delta_{W,HT_i})) \\ 0 & 0 & 0 \\ -(\ddot{\delta}_{W,HT_i}\cos(\delta_{W,HT_i})-\dot{\delta}_{W,HT_i}\sin(\delta_{W,HT_i})) & 0 & -(\ddot{\delta}_{W,HT_i}\sin(\delta_{W,HT_i})+\dot{\delta}_{W,HT_i}\cos(\delta_{W,HT_i})) \end{bmatrix}$$

$$= \left\{\begin{bmatrix} -\cos\delta_W & 0 & -\sin\delta_W \\ 0 & 0 & 0 \\ \sin\delta_W & 0 & -\cos\delta_W \end{bmatrix} \; or \; \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\right\}$$

$$\frac{\partial \ddot{R}_{R_j}^B}{\partial \dot{\delta}_{HT}} = \frac{\partial}{\partial \delta_{HT}}\begin{bmatrix} -(\ddot{\delta}_{W,HT_i}\sin(\delta_{W,HT_i})+\dot{\delta}_{W,HT_i}\cos(\delta_{W,HT_i})) & 0 & (\ddot{\delta}_{W,HT_i}\cos(\delta_{W,HT_i})-\dot{\delta}_{W,HT_i}\sin(\delta_{W,HT_i})) \\ 0 & 0 & 0 \\ -(\ddot{\delta}_{W,HT_i}\cos(\delta_{W,HT_i})-\dot{\delta}_{W,HT_i}\sin(\delta_{W,HT_i})) & 0 & -(\ddot{\delta}_{W,HT_i}\sin(\delta_{W,HT_i})+\dot{\delta}_{W,HT_i}\cos(\delta_{W,HT_i})) \end{bmatrix}$$

$$= \left\{\begin{bmatrix} -\cos\delta_{HT} & 0 & -\sin\delta_{HT} \\ 0 & 0 & 0 \\ \sin\delta_{HT} & 0 & -\cos\delta_{HT} \end{bmatrix} \; or \; \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\right\}$$

18.16 Partial derivatives of $\boldsymbol{B}$:

$$\frac{\partial B}{\partial P_W} = S^T\frac{\partial \Omega_W}{\partial P_W}\tilde{I}_B^B + \sum_{i=1}^{NW}\left\{S^T\frac{\partial \Omega_W}{\partial P_W}[T]_{W_i}(\tilde{I}_{W_i}^{W_i} + m_{W_i}\tilde{R}_{W_i}^B)[T]_{W_i}^T - m_{W_i}\vec{r}_{W_i/B}\times S^T\frac{\partial \Omega_W}{\partial P_W}\vec{r}_{W_i/B}\times\right\}$$
$$+\sum_{j=1}^{NR}\left\{S^T\frac{\partial \Omega_W}{\partial P_W}[T]_{R_j}\left(\tilde{I}_{R_j}^{R_j} + m_{R_j}\tilde{R}_{R_j}^B\right)[T]_{R_j}^T - m_{R_j}\vec{r}_{R_j/B}\times S^T\frac{\partial \Omega_W}{\partial P_W}\vec{r}_{R_j/B}\times\right\}$$

$$\frac{\partial B}{\partial Q_W} = S^T\frac{\partial \Omega_W}{\partial Q_W}\tilde{I}_B^B + \sum_{i=1}^{NW}\left\{S^T\frac{\partial \Omega_W}{\partial Q_W}[T]_{W_i}(\tilde{I}_{W_i}^{W_i} + m_{W_i}\tilde{R}_{W_i}^B)[T]_{W_i}^T - m_{W_i}\vec{r}_{W_i/B}\times S^T\frac{\partial \Omega_W}{\partial Q_W}\vec{r}_{W_i/B}\times\right\}$$
$$+\sum_{j=1}^{NR}\left\{S^T\frac{\partial \Omega_W}{\partial Q_W}[T]_{R_j}\left(\tilde{I}_{R_j}^{R_j} + m_{R_j}\tilde{R}_{R_j}^B\right)[T]_{R_j}^T - m_{R_j}\vec{r}_{R_j/B}\times S^T\frac{\partial \Omega_W}{\partial Q_W}\vec{r}_{R_j/B}\times\right\}$$

$$\frac{\partial B}{\partial R_W} = S^T\frac{\partial \Omega_W}{\partial R_W}\tilde{I}_B^B + \sum_{i=1}^{NW}\left\{S^T\frac{\partial \Omega_W}{\partial R_W}[T]_{W_i}(\tilde{I}_{W_i}^{W_i} + m_{W_i}\tilde{R}_{W_i}^B)[T]_{W_i}^T - m_{W_i}\vec{r}_{W_i/B}\times S^T\frac{\partial \Omega_W}{\partial R_W}\vec{r}_{W_i/B}\times\right\}$$
$$+\sum_{j=1}^{NR}\left\{S^T\frac{\partial \Omega_W}{\partial R_W}[T]_{R_j}\left(\tilde{I}_{R_j}^{R_j} + m_{R_j}\tilde{R}_{R_j}^B\right)[T]_{R_j}^T - m_{R_j}\vec{r}_{R_j/B}\times S^T\frac{\partial \Omega_W}{\partial R_W}\vec{r}_{R_j/B}\times\right\}$$

$$\frac{\partial B}{\partial \delta_W} = \sum_{i=1}^{NW} \left\{ \frac{\partial\left[[\dot{T}]_{W_i}\left(\tilde{I}_{W_i}^{W_i} + m_{W_i}\tilde{R}_{W_i}^{B}\right)[T]_{W_i}^{T}\right]}{\partial \delta_W} + \frac{\partial\left[[T]_{W_i} m_{W_i} \dot{\tilde{R}}_{W_i}^{B} [T]_{W_i}^{T}\right]}{\partial \delta_W} + \frac{\partial\left[[T]_{W_i}\left(\tilde{I}_{W_i}^{W_i} + m_{W_i}\tilde{R}_{W_i}^{B}\right)[\dot{T}]_{W_i}^{T}\right]}{\partial \delta_W} \right.$$

$$\left. + \Omega_B \frac{\partial\left[[T]_{W_i}\left(\tilde{I}_{W_i}^{W_i} + m_{W_i}\tilde{R}_{W_i}^{B}\right)[T]_{W_i}^{T}\right]}{\partial \delta_W} - m_{W_i} \frac{\partial\left[\left(\dot{R}_{W_i}^{B}\vec{r}_{W_i/pivot_i} \times \vec{r}_{W_i/B} \times\right) + \left(\vec{r}_{W_i/B} \times \Omega_B \vec{r}_{W_i/B} \times\right)\right]}{\partial \delta_W} \right\}$$

$$+ \sum_{j=1}^{NR} \left\{ \frac{\partial\left[[\dot{T}]_{R_j}\left(\tilde{I}_{R_j}^{R_j} + m_{R_j}\tilde{R}_{R_j}^{B}\right)[T]_{R_j}^{T}\right]}{\partial \delta_W} + \frac{\partial\left[[T]_{R_j} m_{R_j} \dot{\tilde{R}}_{R_j}^{B} [T]_{R_j}^{T}\right]}{\partial \delta_W} + \frac{\partial\left[[T]_{R_j}\left(\tilde{I}_{R_j}^{R_j} + m_{R_j}\tilde{R}_{R_j}^{B}\right)[\dot{T}]_{R_j}^{T}\right]}{\partial \delta_W} \right.$$

$$\left. + \Omega_B \frac{\partial\left[[T]_{R_j}\left(\tilde{I}_{R_j}^{R_j} + m_{R_j}\tilde{R}_{R_j}^{B}\right)[T]_{R_j}^{T}\right]}{\partial \delta_W} - m_{R_j} \frac{\partial\left[\left(\dot{R}_{R_j}^{B}\vec{r}_{R_j/pivot_j} \times \vec{r}_{R_j/B} \times\right) + \left(\vec{r}_{R_j/B} \times \Omega_B \vec{r}_{R_j/B} \times\right)\right]}{\partial \delta_W} \right\}$$

$$\frac{\partial B}{\partial \delta_{HT}} = \sum_{i=1}^{NW} \left\{ \frac{\partial\left[[\dot{T}]_{W_i}\left(\tilde{I}_{W_i}^{W_i} + m_{W_i}\tilde{R}_{W_i}^{B}\right)[T]_{W_i}^{T}\right]}{\partial \delta_{HT}} + \frac{\partial\left[[T]_{W_i} m_{W_i} \dot{\tilde{R}}_{W_i}^{B} [T]_{W_i}^{T}\right]}{\partial \delta_{HT}} + \frac{\partial\left[[T]_{W_i}\left(\tilde{I}_{W_i}^{W_i} + m_{W_i}\tilde{R}_{W_i}^{B}\right)[\dot{T}]_{W_i}^{T}\right]}{\partial \delta_{HT}} \right.$$

$$\left. + \Omega_B \frac{\partial\left[[T]_{W_i}\left(\tilde{I}_{W_i}^{W_i} + m_{W_i}\tilde{R}_{W_i}^{B}\right)[T]_{W_i}^{T}\right]}{\partial \delta_{HT}} - m_{W_i} \frac{\partial\left[\left(\dot{R}_{W_i}^{B}\vec{r}_{W_i/pivot_i} \times \vec{r}_{W_i/B} \times\right) + \left(\vec{r}_{W_i/B} \times \Omega_B \vec{r}_{W_i/B} \times\right)\right]}{\partial \delta_{HT}} \right\}$$

$$+ \sum_{j=1}^{NR} \left\{ \frac{\partial\left[[\dot{T}]_{R_j}\left(\tilde{I}_{R_j}^{R_j} + m_{R_j}\tilde{R}_{R_j}^{B}\right)[T]_{R_j}^{T}\right]}{\partial \delta_{HT}} + \frac{\partial\left[[T]_{R_j} m_{R_j} \dot{\tilde{R}}_{R_j}^{B} [T]_{R_j}^{T}\right]}{\partial \delta_{HT}} + \frac{\partial\left[[T]_{R_j}\left(\tilde{I}_{R_j}^{R_j} + m_{R_j}\tilde{R}_{R_j}^{B}\right)[\dot{T}]_{R_j}^{T}\right]}{\partial \delta_{HT}} \right.$$

$$\left. + \Omega_B \frac{\partial\left[[T]_{R_j}\left(\tilde{I}_{R_j}^{R_j} + m_{R_j}\tilde{R}_{R_j}^{B}\right)[T]_{R_j}^{T}\right]}{\partial \delta_{HT}} - m_{R_j} \frac{\partial\left[\left(\dot{R}_{R_j}^{B}\vec{r}_{R_j/pivot_j} \times \vec{r}_{R_j/B} \times\right) + \left(\vec{r}_{R_j/B} \times \Omega_B \vec{r}_{R_j/B} \times\right)\right]}{\partial \delta_{HT}} \right\}$$

$$\frac{\partial B}{\partial \dot{\delta}_W} = \sum_{i=1}^{NW} \left\{ \frac{\partial [\dot{T}]_{W_i}}{\partial \dot{\delta}_W}\left(\tilde{I}_{W_i}^{W_i} + m_{W_i}\tilde{R}_{W_i}^{B}\right)[T]_{W_i}^{T} + [T]_{W_i} m_{W_i} \frac{\partial \dot{\tilde{R}}_{W_i}^{B}}{\partial \dot{\delta}_W}[T]_{W_i}^{T} + [T]_{W_i}\left(\tilde{I}_{W_i}^{W_i} + m_{W_i}\tilde{R}_{W_i}^{B}\right)\frac{\partial [\dot{T}]_{W_i}^{T}}{\partial \dot{\delta}_W} \right.$$

$$\left. - m_{W_i}\left(\frac{\partial \dot{R}_{W_i}^{B}}{\partial \dot{\delta}_W}\vec{r}_{W_i/pivot_i} \times \vec{r}_{W_i/B} \times\right) \right\}$$

$$+ \sum_{j=1}^{NR} \left\{ \frac{\partial [\dot{T}]_{R_j}}{\partial \dot{\delta}_W}\left(\tilde{I}_{R_j}^{R_j} + m_{R_j}\tilde{R}_{R_j}^{B}\right)[T]_{R_j}^{T} + [T]_{R_j} m_{R_j} \frac{\partial \dot{\tilde{R}}_{R_j}^{B}}{\partial \dot{\delta}_W}[T]_{R_j}^{T} + [T]_{R_j}\left(\tilde{I}_{R_j}^{R_j} + m_{R_j}\tilde{R}_{R_j}^{B}\right)\frac{\partial [\dot{T}]_{R_j}^{T}}{\partial \dot{\delta}_W} \right.$$

$$\left. - m_{R_j}\left(\frac{\partial \dot{R}_{R_j}^{B}}{\partial \dot{\delta}_W}\vec{r}_{R_j/pivot_j} \times \vec{r}_{R_j/B} \times\right) \right\}$$

$$\frac{\partial B}{\partial \dot{\delta}_{HT}} = \sum_{i=1}^{NW} \left\{ \frac{\partial [\dot{T}]_{W_i}}{\partial \dot{\delta}_{HT}}\left(\tilde{I}_{W_i}^{W_i} + m_{W_i}\tilde{R}_{W_i}^{B}\right)[T]_{W_i}^{T} + [T]_{W_i} m_{W_i} \frac{\partial \dot{\tilde{R}}_{W_i}^{B}}{\partial \dot{\delta}_{HT}}[T]_{W_i}^{T} + [T]_{W_i}\left(\tilde{I}_{W_i}^{W_i} + m_{W_i}\tilde{R}_{W_i}^{B}\right)\frac{\partial [\dot{T}]_{W_i}^{T}}{\partial \dot{\delta}_{HT}} \right.$$

$$\left. - m_{W_i}\left(\frac{\partial \dot{R}_{W_i}^{B}}{\partial \dot{\delta}_{HT}}\vec{r}_{W_i/pivot_i} \times \vec{r}_{W_i/B} \times\right) \right\}$$

$$+ \sum_{j=1}^{NR} \left\{ \frac{\partial [\dot{T}]_{R_j}}{\partial \dot{\delta}_{HT}}\left(\tilde{I}_{R_j}^{R_j} + m_{R_j}\tilde{R}_{R_j}^{B}\right)[T]_{R_j}^{T} + [T]_{R_j} m_{R_j} \frac{\partial \dot{\tilde{R}}_{R_j}^{B}}{\partial \dot{\delta}_{HT}}[T]_{R_j}^{T} + [T]_{R_j}\left(\tilde{I}_{R_j}^{R_j} + m_{R_j}\tilde{R}_{R_j}^{B}\right)\frac{\partial [\dot{T}]_{R_j}^{T}}{\partial \dot{\delta}_{HT}} \right.$$

$$\left. - m_{R_j}\left(\frac{\partial \dot{R}_{R_j}^{B}}{\partial \dot{\delta}_{HT}}\vec{r}_{R_j/pivot_j} \times \vec{r}_{R_j/B} \times\right) \right\}$$

18.17 Partial derivatives of $\boldsymbol{C}$:

$$\frac{\partial C}{\partial \delta_W} = \sum_{i=1}^{NW}\left\{m_{W_i}\frac{\partial[\vec{r}_{W_i/B}\times]}{\partial \delta_W}\right\} + \sum_{j=1}^{NR}\left\{m_{R_j}\frac{\partial\left[\vec{r}_{R_j/B}\times\right]}{\partial \delta_W}\right\}$$

$$\frac{\partial C}{\partial \delta_{HT}} = \sum_{i=1}^{NW}\left\{m_{W_i}\frac{\partial[\vec{r}_{W_i/B}\times]}{\partial \delta_{HT}}\right\} + \sum_{j=1}^{NR}\left\{m_{R_j}\frac{\partial\left[\vec{r}_{R_j/B}\times\right]}{\partial \delta_{HT}}\right\}$$

18.18 Partial derivatives of $\boldsymbol{D}$:

$$\frac{\partial D}{\partial P_W} = \sum_{i=1}^{NW}\left\{m_{W_i}\vec{r}_{W_i/B}\times S^T\frac{\partial \Omega_W}{\partial P_W}\right\} + \sum_{j=1}^{NR}\left\{m_{R_j}\vec{r}_{R_j/B}\times S^T\frac{\partial \Omega_W}{\partial P_W}\right\}$$

$$\frac{\partial D}{\partial Q_W} = \sum_{i=1}^{NW}\left\{m_{W_i}\vec{r}_{W_i/B}\times S^T\frac{\partial \Omega_W}{\partial Q_W}\right\} + \sum_{j=1}^{NR}\left\{m_{R_j}\vec{r}_{R_j/B}\times S^T\frac{\partial \Omega_W}{\partial Q_W}\right\}$$

$$\frac{\partial D}{\partial R_W} = \sum_{i=1}^{NW}\left\{m_{W_i}\vec{r}_{W_i/B}\times S^T\frac{\partial \Omega_W}{\partial R_W}\right\} + \sum_{j=1}^{NR}\left\{m_{R_j}\vec{r}_{R_j/B}\times S^T\frac{\partial \Omega_W}{\partial R_W}\right\}$$

$$\frac{\partial D}{\partial \delta_W} = \sum_{i=1}^{NW}\left\{m_{W_i}\left(\frac{\partial \dot{R}^B_{W_i}}{\partial \delta_W}(\vec{r}_{W_i/pivot_i}\times) + \frac{\partial[\vec{r}_{W_i/B}\times]}{\partial \delta_W}S^T\Omega_W\right)\right\} + \sum_{j=1}^{NR}\left\{m_{R_j}\left(\frac{\partial \dot{R}^B_{R_j}}{\partial \delta_W}\left(\vec{r}_{R_j/pivot_j}\times\right) + \frac{\partial\left[\vec{r}_{R_j/B}\times\right]}{\partial \delta_W}S^T\Omega_W\right)\right\}$$

$$\frac{\partial D}{\partial \delta_{HT}} = \sum_{i=1}^{NW}\left\{m_{W_i}\left(\frac{\partial \dot{R}^B_{W_i}}{\partial \delta_{HT}}(\vec{r}_{W_i/pivot_i}\times) + \frac{\partial[\vec{r}_{W_i/B}\times]}{\partial \delta_{HT}}S^T\Omega_W\right)\right\} + \sum_{j=1}^{NR}\left\{m_{R_j}\left(\frac{\partial \dot{R}^B_{R_j}}{\partial \delta_{HT}}\left(\vec{r}_{R_j/pivot_j}\times\right) + \frac{\partial\left[\vec{r}_{R_j/B}\times\right]}{\partial \delta_{HT}}S^T\Omega_W\right)\right\}$$

$$\frac{\partial D}{\partial \dot{\delta}_W} = \sum_{i=1}^{NW}\left\{m_{W_i}\left(\frac{\partial \dot{R}^B_{W_i}}{\partial \dot{\delta}_W}\vec{r}_{W_i/pivot_i}\times\right)\right\} + \sum_{j=1}^{NR}\left\{m_{R_j}\left(\frac{\partial \dot{R}^B_{R_j}}{\partial \dot{\delta}_W}\vec{r}_{R_j/pivot_j}\times\right)\right\}$$

$$\frac{\partial D}{\partial \dot{\delta}_{HT}} = \sum_{i=1}^{NW}\left\{m_{W_i}\left(\frac{\partial \dot{R}^B_{W_i}}{\partial \dot{\delta}_{HT}}\vec{r}_{W_i/pivot_i}\times\right)\right\} + \sum_{j=1}^{NR}\left\{m_{R_j}\left(\frac{\partial \dot{R}^B_{R_j}}{\partial \dot{\delta}_{HT}}\vec{r}_{R_j/pivot_j}\times\right)\right\}$$

18.19 Partial derivatives of $(\boldsymbol{M_P} - \boldsymbol{E})$:

$$\frac{\partial(M_P - E)}{\partial P_W} = -\sum_{i=1}^{NW}\left\{S^T\frac{\partial \Omega_W}{\partial P_W}[T]_{W_i}(\tilde{I}^{W_i}_{W_i} + m_{W_i}\tilde{R}^B_{W_i})[T]^T_{W_i}R^B_{W_i}\vec{\omega}^{W_i}_{W_i} + m_{W_i}\vec{r}_{W_i/B}\times\left(2S^T\frac{\partial \Omega_W}{\partial P_W}\dot{R}^B_{W_i}\vec{r}_{W_i/pivot_i}\right)\right\}$$

$$-\sum_{j=1}^{NR}\left\{S^T\frac{\partial \Omega_W}{\partial P_W}[T]_{R_j}\left(\tilde{I}^{R_j}_{R_j} + m_{R_j}\tilde{R}^B_{R_j}\right)[T]^T_{R_j}\left(R^B_{R_j}\vec{\omega}^{R_j}_{R_j} + R^B_{W_j}\vec{\omega}^{W_j}_{W_j}\right) + m_{R_j}\vec{r}_{R_j/B}\right.$$

$$\left.\times\left(2S^T\frac{\partial \Omega_W}{\partial P_W}\dot{R}^B_{R_j}\vec{r}_{R_j/pivot_j}\right)\right\}$$

$$\frac{\partial(M_P - E)}{\partial Q_W} = -\sum_{i=1}^{NW}\left\{S^T \frac{\partial \Omega_W}{\partial Q_W}[T]_{W_i}\left(\tilde{I}_{W_i}^{W_i} + m_{W_i}\tilde{R}_{W_i}^{B}\right)[T]_{W_i}^{T} R_{W_i}^{B}\vec{\omega}_{W_i}^{W_i} + m_{W_i}\vec{r}_{W_i/B} \times \left(2S^T \frac{\partial \Omega_W}{\partial Q_W}\dot{R}_{W_i}^{B}\vec{r}_{W_i/pivot_i}\right)\right\}$$

$$-\sum_{j=1}^{NR}\left\{S^T \frac{\partial \Omega_W}{\partial Q_W}[T]_{R_j}\left(\tilde{I}_{R_j}^{R_j} + m_{R_j}\tilde{R}_{R_j}^{B}\right)[T]_{R_j}^{T}\left(R_{R_j}^{B}\vec{\omega}_{R_j}^{R_j} + R_{W_j}^{B}\vec{\omega}_{W_j}^{W_j}\right) + m_{R_j}\vec{r}_{R_j/B}\right.$$

$$\left.\times\left(2S^T \frac{\partial \Omega_W}{\partial Q_W}\dot{R}_{R_j}^{B}\vec{r}_{R_j/pivot_j}\right)\right\}$$

$$\frac{\partial(M_P - E)}{\partial R_W} = -\sum_{i=1}^{NW}\left\{S^T \frac{\partial \Omega_W}{\partial R_W}[T]_{W_i}\left(\tilde{I}_{W_i}^{W_i} + m_{W_i}\tilde{R}_{W_i}^{B}\right)[T]_{W_i}^{T} R_{W_i}^{B}\vec{\omega}_{W_i}^{W_i} + m_{W_i}\vec{r}_{W_i/B} \times \left(2S^T \frac{\partial \Omega_W}{\partial R_W}\dot{R}_{W_i}^{B}\vec{r}_{W_i/pivot_i}\right)\right\}$$

$$-\sum_{j=1}^{NR}\left\{S^T \frac{\partial \Omega_W}{\partial R_W}[T]_{R_j}\left(\tilde{I}_{R_j}^{R_j} + m_{R_j}\tilde{R}_{R_j}^{B}\right)[T]_{R_j}^{T}\left(R_{R_j}^{B}\vec{\omega}_{R_j}^{R_j} + R_{W_j}^{B}\vec{\omega}_{W_j}^{W_j}\right) + m_{R_j}\vec{r}_{R_j/B}\right.$$

$$\left.\times\left(2S^T \frac{\partial \Omega_W}{\partial R_W}\dot{R}_{R_j}^{B}\vec{r}_{R_j/pivot_j}\right)\right\}$$

$$\frac{\partial(M_P - E)}{\partial \phi} = \sum_{i=1}^{NW}\left\{\vec{r}_{W_i/B} \times m_{W_i}\frac{\partial B_E^B}{\partial \phi}\vec{g}^E\right\} + \sum_{j=1}^{NR}\left\{\vec{r}_{R_j/B} \times m_{R_j}\frac{\partial B_E^B}{\partial \phi}\vec{g}^E\right\}$$

$$\frac{\partial(M_P - E)}{\partial \theta} = \sum_{i=1}^{NW}\left\{\vec{r}_{W_i/B} \times m_{W_i}\frac{\partial B_E^B}{\partial \theta}\vec{g}^E\right\} + \sum_{j=1}^{NR}\left\{\vec{r}_{R_j/B} \times m_{R_j}\frac{\partial B_E^B}{\partial \theta}\vec{g}^E\right\}$$

$$\frac{\partial(M_P - E)}{\partial \psi} = \sum_{i=1}^{NW}\left\{\vec{r}_{W_i/B} \times m_{W_i}\frac{\partial B_E^B}{\partial \psi}\vec{g}^E\right\} + \sum_{j=1}^{NR}\left\{\vec{r}_{R_j/B} \times m_{R_j}\frac{\partial B_E^B}{\partial \psi}\vec{g}^E\right\}$$

18.20 Partial derivatives of $\boldsymbol{M_P}$:

$$\frac{\partial M_P}{\partial \delta_W} = \sum_{i=1}^{NW}\left\{\frac{\partial\left[\vec{r}_{W_i/B}\times\right]}{\partial \delta_W} m_{W_i} B_E^B \vec{g}^E\right\} + \sum_{j=1}^{NR}\left\{\frac{\partial\left[\vec{r}_{R_j/B}\times\right]}{\partial \delta_W} m_{R_j} B_E^B \vec{g}^E\right\}$$

$$\frac{\partial M_P}{\partial \delta_{HT}} = \sum_{i=1}^{NW}\left\{\frac{\partial\left[\vec{r}_{W_i/B}\times\right]}{\partial \delta_{HT}} m_{W_i} B_E^B \vec{g}^E\right\} + \sum_{j=1}^{NR}\left\{\frac{\partial\left[\vec{r}_{R_j/B}\times\right]}{\partial \delta_{HT}} m_{R_j} B_E^B \vec{g}^E\right\}$$

18.21 Partial derivatives of $\boldsymbol{E}$:

$$\frac{\partial E}{\partial \delta_W} = \frac{\partial E_W}{\partial \delta_W} + \frac{\partial E_R}{\partial \delta_W}$$

Expanding the terms,

$$\frac{\partial E_W}{\partial \delta_W} = \sum_{i=1}^{NW}\left\{\left[\frac{\partial\left([\dot{T}]_{W_i}\left(\tilde{I}_{W_i}^{W_i} + m_{W_i}\tilde{R}_{W_i}^{B}\right)[T]_{W_i}^{T}\right)}{\partial \delta_W} + \frac{\partial\left([T]_{W_i} m_{W_i}\dot{\tilde{R}}_{W_i}^{B}[T]_{W_i}^{T}\right)}{\partial \delta_W} + \frac{\partial\left([T]_{W_i}\left(\tilde{I}_{W_i}^{W_i} + m_{W_i}\tilde{R}_{W_i}^{B}\right)[\dot{T}]_{W_i}^{T}\right)}{\partial \delta_W}\right] R_{W_i}^{B}\vec{\omega}_{W_i}^{W_i}\right.$$

$$+ \frac{\partial\left[[T]_{W_i}\left(\tilde{I}_{W_i}^{W_i} + m_{W_i}\tilde{R}_{W_i}^{B}\right)[T]_{W_i}^{T}\right]}{\partial \delta_W}\left(\dot{R}_{W_i}^{B}\vec{\omega}_{W_i}^{W_i} + R_{W_i}^{B}\dot{\vec{\omega}}_{W_i}^{W_i}\right) + \Omega_B \frac{\partial\left[[T]_{W_i}\left(\tilde{I}_{W_i}^{W_i} + m_{W_i}\tilde{R}_{W_i}^{B}\right)[T]_{W_i}^{T}\right]}{\partial \delta_W} R_{W_i}^{B}\vec{\omega}_{W_i}^{W_i}$$

$$\left.+ m_{W_i}\frac{\partial\left[\dot{R}_{W_i}^{B}\vec{r}_{W_i/pivot_i} \times \left(\dot{R}_{W_i}^{B}\vec{r}_{W_i/pivot_i}\right) + \vec{r}_{W_i/B} \times \left(2\Omega_B \dot{R}_{W_i}^{B} + \ddot{R}_{W_i}^{B}\right)\vec{r}_{W_i/pivot_i}\right]}{\partial \delta_W}\right\}$$

$$\frac{\partial E_R}{\partial \delta_W} = \sum_{j=1}^{NR} \left\{ \left[\frac{\partial \left([\dot{T}]_{R_j} \left(\tilde{I}_{R_j}^{R_j} + m_{R_j} \tilde{R}_{R_j}^{B} \right) [T]_{R_j}^{T} \right)}{\partial \delta_W} + \frac{\partial \left([T]_{R_j} m_{R_j} \dot{\tilde{R}}_{R_j}^{B} [T]_{R_j}^{T} \right)}{\partial \delta_W} + \frac{\partial \left([T]_{R_j} \left(\tilde{I}_{R_j}^{R_j} + m_{R_j} \tilde{R}_{R_j}^{B} \right) [\dot{T}]_{R_j}^{T} \right)}{\partial \delta_W} \right] \left(R_{R_j}^{B} \vec{\omega}_{R_j}^{R_j} \right.\right.$$

$$\left. + R_{W_j}^{B} \vec{\omega}_{W_j}^{W_j} \right)$$

$$+ \left[[\dot{T}]_{R_j} \left(\tilde{I}_{R_j}^{R_j} + m_{R_j} \tilde{R}_{R_j}^{B} \right) [T]_{R_j}^{T} + [T]_{R_j} m_{R_j} \dot{\tilde{R}}_{R_j}^{B} [T]_{R_j}^{T} + [T]_{R_j} \left(\tilde{I}_{R_j}^{R_j} + m_{R_j} \tilde{R}_{R_j}^{B} \right) [\dot{T}]_{R_j}^{T} \right] \frac{\partial R_{R_j}^{B}}{\partial \delta_W} \vec{\omega}_{R_j}^{R_j}$$

$$+ \frac{\partial \left[[T]_{R_j} \left(\tilde{I}_{R_j}^{R_j} + m_{R_j} \tilde{R}_{R_j}^{B} \right) [T]_{R_j}^{T} \right]}{\partial \delta_W} \left(\frac{\partial \dot{R}_{R_j}^{B}}{\partial \delta_W} \vec{\omega}_{R_j}^{R_j} + \frac{\partial R_{R_j}^{B}}{\partial \delta_W} \dot{\vec{\omega}}_{R_j}^{R_j} \right)$$

$$+ [T]_{R_j} \left(\tilde{I}_{R_j}^{R_j} + m_{R_j} \tilde{R}_{R_j}^{B} \right) [T]_{R_j}^{T} \frac{\partial \left(\dot{R}_{R_j}^{B} \vec{\omega}_{R_j}^{R_j} + R_{R_j}^{B} \dot{\vec{\omega}}_{R_j}^{R_j} + \dot{R}_{W_j}^{B} \vec{\omega}_{W_j}^{W_j} + R_{W_j}^{B} \dot{\vec{\omega}}_{W_j}^{W_j} \right)}{\partial \delta_W}$$

$$+ \Omega_B \left[\frac{\partial \left[[T]_{R_j} \left(\tilde{I}_{R_j}^{R_j} + m_{R_j} \tilde{R}_{R_j}^{B} \right) [T]_{R_j}^{T} \right]}{\partial \delta_W} \left(R_{R_j}^{B} \vec{\omega}_{R_j}^{R_j} + R_{W_j}^{B} \vec{\omega}_{W_j}^{W_j} \right) + [T]_{R_j} \left(\tilde{I}_{R_j}^{R_j} + m_{R_j} \tilde{R}_{R_j}^{B} \right) [T]_{R_j}^{T} \frac{\partial R_{R_j}^{B}}{\partial \delta_W} \vec{\omega}_{R_j}^{R_j} \right]$$

$$\left. + m_{R_j} \frac{\partial \left[\dot{R}_{R_j}^{B} \vec{r}_{R_j/pivot_j} \times \left(\dot{R}_{R_j}^{B} \vec{r}_{R_j/pivot_j} \right) + \vec{r}_{R_j/B} \times \left(2\Omega_B \dot{R}_{R_j}^{B} + \ddot{R}_{R_j}^{B} \right) \vec{r}_{R_j/pivot_j} \right]}{\partial \delta_W} \right\}$$

$$\frac{\partial E}{\partial\ \delta_{HT}} = \frac{\partial E_W}{\partial \delta_{HT}} + \frac{\partial E_R}{\partial \delta_{HT}}$$

Expanding the terms,

$$\frac{\partial E_{HT}}{\partial \delta_{HT}} = \sum_{i=1}^{NW} \left\{ \left[\frac{\partial \left([\dot{T}]_{W_i} (\tilde{I}_{W_i}^{W_i} + m_{W_i} \tilde{R}_{W_i}^{B}) [T]_{W_i}^{T} \right)}{\partial \delta_{HT}} + \frac{\partial \left([T]_{W_i} m_{W_i} \dot{\tilde{R}}_{W_i}^{B} [T]_{W_i}^{T} \right)}{\partial \delta_{HT}} + \frac{\partial \left([T]_{W_i} (\tilde{I}_{W_i}^{W_i} + m_{W_i} \tilde{R}_{W_i}^{B}) [\dot{T}]_{W_i}^{T} \right)}{\partial \delta_{HT}} \right] R_{W_i}^{B} \vec{\omega}_{W_i}^{W_i} \right.$$

$$+ \frac{\partial \left[[T]_{W_i} (\tilde{I}_{W_i}^{W_i} + m_{W_i} \tilde{R}_{W_i}^{B}) [T]_{W_i}^{T} \right]}{\partial \delta_{HT}} \left(\dot{R}_{W_i}^{B} \vec{\omega}_{W_i}^{W_i} + R_{W_i}^{B} \dot{\vec{\omega}}_{W_i}^{W_i} \right) + \Omega_B \frac{\partial \left[[T]_{W_i} (\tilde{I}_{W_i}^{W_i} + m_{W_i} \tilde{R}_{W_i}^{B}) [T]_{W_i}^{T} \right]}{\partial \delta_{HT}} R_{W_i}^{B} \vec{\omega}_{W_i}^{W_i}$$

$$\left. + m_{W_i} \frac{\partial \left[\dot{R}_{W_i}^{B} \vec{r}_{W_i/pivot_i} \times (\dot{R}_{W_i}^{B} \vec{r}_{W_i/pivot_i}) + \vec{r}_{W_i/B} \times (2\Omega_B \dot{R}_{W_i}^{B} + \ddot{R}_{W_i}^{B}) \vec{r}_{W_i/pivot_i} \right]}{\partial \delta_{HT}} \right\}$$

$$\frac{\partial E_R}{\partial \delta_{HT}} = \sum_{j=1}^{NR} \left\{ \left[\frac{\partial \left([\dot{T}]_{R_j} \left(\tilde{I}_{R_j}^{R_j} + m_{R_j} \tilde{R}_{R_j}^{B} \right) [T]_{R_j}^{T} \right)}{\partial \delta_{HT}} + \frac{\partial \left([T]_{R_j} m_{R_j} \dot{\tilde{R}}_{R_j}^{B} [T]_{R_j}^{T} \right)}{\partial \delta_{HT}} + \frac{\partial \left([T]_{R_j} \left(\tilde{I}_{R_j}^{R_j} + m_{R_j} \tilde{R}_{R_j}^{B} \right) [\dot{T}]_{R_j}^{T} \right)}{\partial \delta_{HT}} \right] \left(R_{R_j}^{B} \vec{\omega}_{R_j}^{R_j} \right.\right.$$

$$\left. + R_{W_j}^{B} \vec{\omega}_{W_j}^{W_j} \right)$$

$$+ \left[[\dot{T}]_{R_j} \left(\tilde{I}_{R_j}^{R_j} + m_{R_j} \tilde{R}_{R_j}^{B} \right) [T]_{R_j}^{T} + [T]_{R_j} m_{R_j} \dot{\tilde{R}}_{R_j}^{B} [T]_{R_j}^{T} + [T]_{R_j} \left(\tilde{I}_{R_j}^{R_j} + m_{R_j} \tilde{R}_{R_j}^{B} \right) [\dot{T}]_{R_j}^{T} \right] \frac{\partial R_{R_j}^{B}}{\partial \delta_{HT}} \vec{\omega}_{R_j}^{R_j}$$

$$+ \frac{\partial \left[[T]_{R_j} \left(\tilde{I}_{R_j}^{R_j} + m_{R_j} \tilde{R}_{R_j}^{B} \right) [T]_{R_j}^{T} \right]}{\partial \delta_{HT}} \left(\dot{R}_{R_j}^{B} \vec{\omega}_{R_j}^{R_j} + R_{R_j}^{B} \dot{\vec{\omega}}_{R_j}^{R_j} + \dot{R}_{W_j}^{B} \vec{\omega}_{W_j}^{W_j} + R_{W_j}^{B} \dot{\vec{\omega}}_{W_j}^{W_j} \right)$$

$$+ [T]_{R_j} \left(\tilde{I}_{R_j}^{R_j} + m_{R_j} \tilde{R}_{R_j}^{B} \right) [T]_{R_j}^{T} \left(\frac{\partial \dot{R}_{R_j}^{B}}{\partial \delta_{HT}} \vec{\omega}_{R_j}^{R_j} + \frac{\partial R_{R_j}^{B}}{\partial \delta_{HT}} \dot{\vec{\omega}}_{R_j}^{R_j} \right)$$

$$+ \Omega_B \left[\frac{\partial \left[[T]_{R_j} \left(\tilde{I}_{R_j}^{R_j} + m_{R_j} \tilde{R}_{R_j}^{B} \right) [T]_{R_j}^{T} \right]}{\partial \delta_{HT}} \left(R_{R_j}^{B} \vec{\omega}_{R_j}^{R_j} + R_{W_j}^{B} \vec{\omega}_{W_j}^{W_j} \right) + [T]_{R_j} \left(\tilde{I}_{R_j}^{R_j} + m_{R_j} \tilde{R}_{R_j}^{B} \right) [T]_{R_j}^{T} \frac{\partial R_{R_j}^{B}}{\partial \delta_{HT}} \vec{\omega}_{R_j}^{R_j} \right]$$

$$\left. + m_{R_j} \frac{\partial \left[\dot{R}_{R_j}^{B} \vec{r}_{R_j/pivot_j} \times \left(\dot{R}_{R_j}^{B} \vec{r}_{R_j/pivot_j} \right) + \vec{r}_{R_j/B} \times \left(2\Omega_B \dot{R}_{R_j}^{B} + \ddot{R}_{R_j}^{B} \right) \vec{r}_{R_j/pivot_j} \right]}{\partial \delta_{HT}} \right\}$$

$$\frac{\partial E}{\partial \dot{\delta}_W} = \frac{\partial E_W}{\partial \dot{\delta}_W} + \frac{\partial E_R}{\partial \dot{\delta}_W}$$

Expanding the terms,

$$\frac{\partial E_W}{\partial \dot{\delta}_W} = \sum_{i=1}^{NW} \left\{ \left(\frac{\partial [\dot{T}]_{W_i}}{\partial \dot{\delta}_W} \left(\tilde{I}_{W_i}^{W_i} + m_{W_i} \tilde{R}_{W_i}^B \right) [T]_{W_i}^T + [T]_{W_i} m_{W_i} \frac{\partial \dot{\tilde{R}}_{W_i}^B}{\partial \dot{\delta}_W} [T]_{W_i}^T + [T]_{W_i} \left(\tilde{I}_{W_i}^{W_i} + m_{W_i} \tilde{R}_{W_i}^B \right) \frac{\partial [\dot{T}]_{W_i}}{\partial \dot{\delta}_W} \right) \begin{Bmatrix} 0 \\ \dot{\delta}_W \\ 0 \end{Bmatrix} \right.$$
$$+ \left([\dot{T}]_{W_i} \left(\tilde{I}_{W_i}^{W_i} + m_{W_i} \tilde{R}_{W_i}^B \right) [T]_{W_i}^T + [T]_{W_i} m_{W_i} \frac{d}{dt} \left(\tilde{R}_{W_i}^B \right) [T]_{W_i}^T + [T]_{W_i} \left(\tilde{I}_{W_i}^{W_i} + m_{W_i} \tilde{R}_{W_i}^B \right) [\dot{T}]_{W_i}^T \right) \begin{Bmatrix} 0 \\ 1 \\ 0 \end{Bmatrix}$$
$$+ \Omega_B [T]_{W_i} \left(\tilde{I}_{W_i}^{W_i} + m_{W_i} \tilde{R}_{W_i}^B \right) [T]_{W_i}^T \begin{Bmatrix} 0 \\ 1 \\ 0 \end{Bmatrix}$$
$$\left. + m_{W_i} \left[\frac{\partial \dot{R}_{W_i}^B}{\partial \dot{\delta}_W} \vec{r}_{W_i/pivot_i} \times \dot{R}_{W_i}^B + \dot{R}_{W_i}^B \vec{r}_{W_i/pivot_i} \times \frac{\partial \dot{R}_{W_i}^B}{\partial \dot{\delta}_W} + \vec{r}_{W_i/B} \times \left(2\Omega_B \frac{\partial \dot{R}_{W_i}^B}{\partial \dot{\delta}_W} + \frac{\partial \ddot{R}_{W_i}^B}{\partial \dot{\delta}_W} \right) \right] \vec{r}_{W_i/pivot_i} \right\}$$

$$\frac{\partial E_R}{\partial \dot{\delta}_W} = \sum_{j=1}^{NR} \left\{ \left(\frac{\partial [\dot{T}]_{R_j}}{\partial \dot{\delta}_W} \left(\tilde{I}_{R_j}^{R_j} + m_{R_j} \tilde{R}_{R_j}^B \right) [T]_{R_j}^T + [T]_{R_j} m_{R_j} \frac{\partial \dot{\tilde{R}}_{R_j}^B}{\partial \dot{\delta}_W} [T]_{R_j}^T + [T]_{R_j} \left(\tilde{I}_{R_j}^{R_j} + m_{R_j} \tilde{R}_{R_j}^B \right) \frac{\partial [\dot{T}]_{R_j}}{\partial \dot{\delta}_W} \right) \left(R_{R_j}^B \begin{Bmatrix} \omega_{R_j} \\ 0 \\ 0 \end{Bmatrix} + \begin{Bmatrix} 0 \\ \dot{\delta}_W \\ 0 \end{Bmatrix} \right) \right.$$
$$+ \left([\dot{T}]_{R_j} \left(\tilde{I}_{R_j}^{R_j} + m_{R_j} \tilde{R}_{R_j}^B \right) [T]_{R_j}^T + [T]_{R_j} m_{R_j} \frac{d}{dt} \left(\tilde{R}_{R_j}^B \right) [T]_{R_j}^T + [T]_{R_j} \left(\tilde{I}_{R_j}^{R_j} + m_{R_j} \tilde{R}_{R_j}^B \right) [\dot{T}]_{R_j}^T \right) \begin{Bmatrix} 0 \\ 1 \\ 0 \end{Bmatrix}$$
$$+ \Omega_B [T]_{R_j} \left(\tilde{I}_{R_j}^{R_j} + m_{R_j} \tilde{R}_{R_j}^B \right) [T]_{R_j}^T \begin{Bmatrix} 0 \\ 1 \\ 0 \end{Bmatrix}$$
$$\left. + m_{R_j} \left[\frac{\partial \dot{R}_{R_j}^B}{\partial \dot{\delta}_W} \vec{r}_{R_j/pivot_j} \times \dot{R}_{R_j}^B + \dot{R}_{R_j}^B \vec{r}_{R_j/pivot_j} \times \frac{\partial \dot{R}_{R_j}^B}{\partial \dot{\delta}_W} + \vec{r}_{R_j/B} \times \left(2\Omega_B \frac{\partial \dot{R}_{R_j}^B}{\partial \dot{\delta}_W} + \frac{\partial \ddot{R}_{R_j}^B}{\partial \dot{\delta}_W} \right) \right] \vec{r}_{R_j/pivot_j} \right\}$$

$$\frac{\partial E}{\partial \ddot{\delta}_W} = \frac{\partial E_W}{\partial \ddot{\delta}_W} + \frac{\partial E_R}{\partial \ddot{\delta}_W}$$

Expanding the terms,

$$\frac{\partial E_W}{\partial \ddot{\delta}_W} = \sum_{i=1}^{NW} \left\{ [T]_{W_i} \left(\tilde{I}_{W_i}^{W_i} + m_{W_i} \tilde{R}_{W_i}^B \right) [T]_{W_i}^T \begin{Bmatrix} 0 \\ 1 \\ 0 \end{Bmatrix} + m_{W_i} \vec{r}_{W_i/B} \times \frac{\partial \ddot{R}_{W_i}^B}{\partial \ddot{\delta}_W} \vec{r}_{W_i/pivot_i} \right\}$$

$$\frac{\partial E_R}{\partial \ddot{\delta}_W} = \sum_{j=1}^{NR} \left\{ [T]_{R_j} \left(\tilde{I}_{R_j}^{R_j} + m_{R_j} \tilde{R}_{R_j}^B \right) [T]_{R_j}^T \begin{Bmatrix} 0 \\ 1 \\ 0 \end{Bmatrix} + m_{R_j} \vec{r}_{R_j/B} \times \frac{\partial \ddot{R}_{R_j}^B}{\partial \ddot{\delta}_W} \vec{r}_{R_j/pivot_j} \right\}$$

$$\frac{\partial E}{\partial \dot{\delta}_{HT}} = \frac{\partial E_{HT}}{\partial \dot{\delta}_{HT}} + \frac{\partial E_R}{\partial \dot{\delta}_{HT}}$$

Expanding the terms,

$$\frac{\partial E_{HT}}{\partial \dot{\delta}_{HT}} = \sum_{i=1}^{NW} \Bigg\{ \left(\frac{\partial [\dot{T}]_{W_i}}{\partial \dot{\delta}_{HT}} \left(\tilde{I}_{W_i}^{W_i} + m_{W_i} \tilde{R}_{W_i}^{B} \right) [T]_{W_i}^{T} + [T]_{W_i} m_{W_i} \frac{\partial \dot{\tilde{R}}_{W_i}^{B}}{\partial \dot{\delta}_{HT}} [T]_{W_i}^{T} + [T]_{W_i} \left(\tilde{I}_{W_i}^{W_i} + m_{W_i} \tilde{R}_{W_i}^{B} \right) \frac{\partial [\dot{T}]_{W_i}}{\partial \dot{\delta}_{HT}} \right) \begin{Bmatrix} 0 \\ \dot{\delta}_{HT} \\ 0 \end{Bmatrix}$$

$$+ \left([\dot{T}]_{W_i} \left(\tilde{I}_{W_i}^{W_i} + m_{W_i} \tilde{R}_{W_i}^{B} \right) [T]_{W_i}^{T} + [T]_{W_i} m_{W_i} \frac{d}{dt} \left(\tilde{R}_{W_i}^{B} \right) [T]_{W_i}^{T} + [T]_{W_i} \left(\tilde{I}_{W_i}^{W_i} + m_{W_i} \tilde{R}_{W_i}^{B} \right) [\dot{T}]_{W_i}^{T} \right) \begin{Bmatrix} 0 \\ 1 \\ 0 \end{Bmatrix}$$

$$+ \Omega_B [T]_{W_i} \left(\tilde{I}_{W_i}^{W_i} + m_{W_i} \tilde{R}_{W_i}^{B} \right) [T]_{W_i}^{T} \begin{Bmatrix} 0 \\ 1 \\ 0 \end{Bmatrix}$$

$$+ m_{W_i} \left[\frac{\partial \dot{R}_{W_i}^{B}}{\partial \dot{\delta}_{HT}} \vec{r}_{W_i/pivot_i} \times \dot{R}_{W_i}^{B} + \dot{R}_{W_i}^{B} \vec{r}_{W_i/pivot_i} \times \frac{\partial \dot{R}_{W_i}^{B}}{\partial \dot{\delta}_{HT}} + \vec{r}_{W_i/B} \times \left(2\Omega_B \frac{\partial \dot{R}_{W_i}^{B}}{\partial \dot{\delta}_{HT}} + \frac{\partial \ddot{R}_{W_i}^{B}}{\partial \dot{\delta}_{HT}} \right) \right] \vec{r}_{W_i/pivot_i} \Bigg\}$$

$$\frac{\partial E_{R}}{\partial \dot{\delta}_{HT}} = \sum_{j=1}^{NR} \Bigg\{ \left(\frac{\partial [\dot{T}]_{R_j}}{\partial \dot{\delta}_{HT}} \left(\tilde{I}_{R_j}^{R_j} + m_{R_j} \tilde{R}_{R_j}^{B} \right) [T]_{R_j}^{T} + [T]_{R_j} m_{R_j} \frac{\partial \dot{\tilde{R}}_{R_j}^{B}}{\partial \dot{\delta}_{HT}} [T]_{R_j}^{T} + [T]_{R_j} \left(\tilde{I}_{R_j}^{R_j} + m_{R_j} \tilde{R}_{R_j}^{B} \right) \frac{\partial [\dot{T}]_{R_j}}{\partial \dot{\delta}_{HT}} \right) \left(R_{R_j}^{B} \begin{Bmatrix} \omega_{R_j} \\ 0 \\ 0 \end{Bmatrix} + \begin{Bmatrix} 0 \\ \dot{\delta}_{HT} \\ 0 \end{Bmatrix} \right)$$

$$+ \left([\dot{T}]_{R_j} \left(\tilde{I}_{R_j}^{R_j} + m_{R_j} \tilde{R}_{R_j}^{B} \right) [T]_{R_j}^{T} + [T]_{R_j} m_{R_j} \frac{d}{dt} \left(\tilde{R}_{R_j}^{B} \right) [T]_{R_j}^{T} + [T]_{R_j} \left(\tilde{I}_{R_j}^{R_j} + m_{R_j} \tilde{R}_{R_j}^{B} \right) [\dot{T}]_{R_j}^{T} \right) \begin{Bmatrix} 0 \\ 1 \\ 0 \end{Bmatrix}$$

$$+ \Omega_B [T]_{R_j} \left(\tilde{I}_{R_j}^{R_j} + m_{R_j} \tilde{R}_{R_j}^{B} \right) [T]_{R_j}^{T} \begin{Bmatrix} 0 \\ 1 \\ 0 \end{Bmatrix}$$

$$+ m_{R_j} \left[\frac{\partial \dot{R}_{R_j}^{B}}{\partial \dot{\delta}_{HT}} \vec{r}_{R_j/pivot_j} \times \dot{R}_{R_j}^{B} + \dot{R}_{R_j}^{B} \vec{r}_{R_j/pivot_j} \times \frac{\partial \dot{R}_{R_j}^{B}}{\partial \dot{\delta}_{HT}} + \vec{r}_{R_j/B} \times \left(2\Omega_B \frac{\partial \dot{R}_{R_j}^{B}}{\partial \dot{\delta}_{HT}} + \frac{\partial \ddot{R}_{R_j}^{B}}{\partial \dot{\delta}_{HT}} \right) \right] \vec{r}_{R_j/pivot_j} \Bigg\}$$

$$\frac{\partial E}{\partial \ddot{\delta}_{HT}} = \frac{\partial E_{HT}}{\partial \ddot{\delta}_{HT}} + \frac{\partial E_{R}}{\partial \ddot{\delta}_{HT}}$$

Expanding the terms,

$$\frac{\partial E_{HT}}{\partial \ddot{\delta}_{HT}} = \sum_{i=1}^{NW} \left\{ [T]_{W_i} \left(\tilde{I}_{W_i}^{W_i} + m_{W_i} \tilde{R}_{W_i}^{B} \right) [T]_{W_i}^{T} \begin{Bmatrix} 0 \\ 1 \\ 0 \end{Bmatrix} + m_{W_i} \vec{r}_{W_i/B} \times \frac{\partial \ddot{R}_{W_i}^{B}}{\partial \ddot{\delta}_{HT}} \vec{r}_{W_i/pivot_i} \right\}$$

$$\frac{\partial E_{R}}{\partial \ddot{\delta}_{HT}} = \sum_{j=1}^{NR} \left\{ +[T]_{R_j} \left(\tilde{I}_{R_j}^{R_j} + m_{R_j} \tilde{R}_{R_j}^{B} \right) [T]_{R_j}^{T} \begin{Bmatrix} 0 \\ 1 \\ 0 \end{Bmatrix} + m_{R_j} \vec{r}_{R_j/B} \times \frac{\partial \ddot{R}_{R_j}^{B}}{\partial \ddot{\delta}_{HT}} \vec{r}_{R_j/pivot_j} \right\}$$

$$\frac{\partial E}{\partial \omega_j^2} = \left([\dot{T}]_{R_j} \left(\tilde{I}_{R_j}^{R_j} + m_{R_j} \tilde{R}_{R_j}^{P} \right) [T]_{R_j}^{T} + [T]_{R_j} m_{R_j} \frac{d}{dt} \left(\tilde{R}_{R_j}^{P} \right) [T]_{R_j}^{T} + [T]_{R_j} \left(\tilde{I}_{R_j}^{R_j} + m_{R_j} \tilde{R}_{R_j}^{P} \right) [\dot{T}]_{R_j}^{T} \right) R_{R_j}^{P} \begin{Bmatrix} \frac{1}{2\sqrt{\omega_{R_j}^2}} \\ 0 \\ 0 \end{Bmatrix}$$

$$+ [T]_{R_j} \left(\tilde{I}_{R_j}^{R_j} + m_{R_j} \tilde{R}_{R_j}^{P} \right) [T]_{R_j}^{T} \dot{R}_{R_j}^{B} \begin{Bmatrix} \frac{1}{2\sqrt{\omega_{R_j}^2}} \\ 0 \\ 0 \end{Bmatrix} + \Omega_B [T]_{R_j} \left(\tilde{I}_{R_j}^{R_j} + m_{R_j} \tilde{R}_{R_j}^{P} \right) [T]_{R_j}^{T} R_{R_j}^{P} \begin{Bmatrix} \frac{1}{2\sqrt{\omega_{R_j}^2}} \\ 0 \\ 0 \end{Bmatrix}$$

$$\frac{\partial E}{\partial \left(\dot{\omega_j^2} \right)} = [T]_{R_j} \left(\tilde{I}_{R_j}^{R_j} + m_{R_j} \tilde{R}_{R_j}^{B} \right) [T]_{R_j}^{T} R_{R_j}^{B} \begin{Bmatrix} \frac{1}{2\sqrt{\dot{\omega}_{R_j}^2}} \\ 0 \\ 0 \end{Bmatrix}$$

18.22 Partial derivatives of $\boldsymbol{A^{-1}}$:

$$\frac{\partial A^{-1}}{\partial \delta_W} = \sum_{i=1}^{NW}\left\{\frac{\partial\left[[T]_{W_i}\left(\tilde{I}_{W_i}^{W_i} + m_{W_i}\tilde{R}_{W_i}^{B}\right)[T]_{W_i}^{T}\right]^{+}}{\partial \delta_W} - m_{W_i}\frac{\left(\vec{r}_{W_i/B}\times\vec{r}_{W_i/B}\times\right)^{-1}}{\partial \delta_W}\right\} + \sum_{j=1}^{NR}\left\{\frac{\partial\left[[T]_{R_j}\left(\tilde{I}_{R_j}^{R_j} + m_{R_j}\tilde{R}_{R_j}^{B}\right)[T]_{R_j}^{T}\right]^{+}}{\partial \delta_W} - m_{R_j}\frac{\left(\vec{r}_{R_j/B}\times\vec{r}_{R_j/B}\times\right)^{-1}}{\partial \delta_W}\right\}$$

$$\frac{\partial A^{-1}}{\partial \delta_{HT}} = \sum_{i=1}^{NW}\left\{\frac{\partial\left[[T]_{W_i}\left(\tilde{I}_{W_i}^{W_i} + m_{W_i}\tilde{R}_{W_i}^{B}\right)[T]_{W_i}^{T}\right]^{+}}{\partial \delta_{HT}} - m_{W_i}\frac{\left(\vec{r}_{W_i/B}\times\vec{r}_{W_i/B}\times\right)^{-1}}{\partial \delta_{HT}}\right\} + \sum_{j=1}^{NR}\left\{\frac{\partial\left[[T]_{R_j}\left(\tilde{I}_{R_j}^{R_j} + m_{R_j}\tilde{R}_{R_j}^{B}\right)[T]_{R_j}^{T}\right]^{+}}{\partial \delta_{HT}} - m_{R_j}\frac{\left(\vec{r}_{R_j/B}\times\vec{r}_{R_j/B}\times\right)^{-1}}{\partial \delta_{HT}}\right\}$$

Remembering that,

$$\frac{\partial U^{-1}}{\partial x} = -U^{-1}\frac{\partial U}{\partial x}U^{-1}$$

We have,

$$\begin{aligned}\frac{\partial A^{-1}}{\partial \delta_W} = \sum_{i=1}^{NW}\Bigg\{&-\left[[T]_{W_i}\left(\tilde{I}_{W_i}^{W_i} + m_{W_i}\tilde{R}_{W_i}^{B}\right)[T]_{W_i}^{T}\right]^{+}\frac{\partial\left[[T]_{W_i}\left(\tilde{I}_{W_i}^{W_i} + m_{W_i}\tilde{R}_{W_i}^{B}\right)[T]_{W_i}^{T}\right]}{\partial \delta_W}\left[[T]_{W_i}\left(\tilde{I}_{W_i}^{W_i} + m_{W_i}\tilde{R}_{W_i}^{B}\right)[T]_{W_i}^{T}\right]^{+} \\ &+ m_{W_i}\left(\vec{r}_{W_i/B}\times\vec{r}_{W_i/B}\times\right)^{-1}\frac{\partial\left(\vec{r}_{W_i/B}\times\vec{r}_{W_i/B}\times\right)}{\partial \delta_W}\left(\vec{r}_{W_i/B}\times\vec{r}_{W_i/B}\times\right)^{-1}\Bigg\} \\ + \sum_{j=1}^{NR}\Bigg\{&-\left[[T]_{R_j}\left(\tilde{I}_{R_j}^{R_j} + m_{R_j}\tilde{R}_{R_j}^{B}\right)[T]_{R_j}^{T}\right]^{+}\frac{\partial\left[[T]_{R_j}\left(\tilde{I}_{R_j}^{R_j} + m_{R_j}\tilde{R}_{R_j}^{B}\right)[T]_{R_j}^{T}\right]}{\partial \delta_W}\left[[T]_{R_j}\left(\tilde{I}_{R_j}^{R_j} + m_{R_j}\tilde{R}_{R_j}^{B}\right)[T]_{R_j}^{T}\right]^{+} \\ &+ m_{R_j}\left(\vec{r}_{R_j/B}\times\vec{r}_{R_j/B}\times\right)^{-1}\frac{\partial\left(\vec{r}_{R_j/B}\times\vec{r}_{R_j/B}\times\right)}{\partial \delta_W}\left(\vec{r}_{R_j/B}\times\vec{r}_{R_j/B}\times\right)^{-1}\Bigg\}\end{aligned}$$

$$\begin{aligned}\frac{\partial A^{-1}}{\partial \delta_{HT}} = \sum_{i=1}^{NW}\Bigg\{&-\left[[T]_{W_i}\left(\tilde{I}_{W_i}^{W_i} + m_{W_i}\tilde{R}_{W_i}^{B}\right)[T]_{W_i}^{T}\right]^{+}\frac{\partial\left[[T]_{W_i}\left(\tilde{I}_{W_i}^{W_i} + m_{W_i}\tilde{R}_{W_i}^{B}\right)[T]_{W_i}^{T}\right]}{\partial \delta_{HT}}\left[[T]_{W_i}\left(\tilde{I}_{W_i}^{W_i} + m_{W_i}\tilde{R}_{W_i}^{B}\right)[T]_{W_i}^{T}\right]^{+} \\ &+ m_{W_i}\left(\vec{r}_{W_i/B}\times\vec{r}_{W_i/B}\times\right)^{-1}\frac{\partial\left(\vec{r}_{W_i/B}\times\vec{r}_{W_i/B}\times\right)}{\partial \delta_{HT}}\left(\vec{r}_{W_i/B}\times\vec{r}_{W_i/B}\times\right)^{-1}\Bigg\} \\ + \sum_{j=1}^{NR}\Bigg\{&-\left[[T]_{R_j}\left(\tilde{I}_{R_j}^{R_j} + m_{R_j}\tilde{R}_{R_j}^{B}\right)[T]_{R_j}^{T}\right]^{+}\frac{\partial\left[[T]_{R_j}\left(\tilde{I}_{R_j}^{R_j} + m_{R_j}\tilde{R}_{R_j}^{B}\right)[T]_{R_j}^{T}\right]}{\partial \delta_{HT}}\left[[T]_{R_j}\left(\tilde{I}_{R_j}^{R_j} + m_{R_j}\tilde{R}_{R_j}^{B}\right)[T]_{R_j}^{T}\right]^{+} \\ &+ m_{R_j}\left(\vec{r}_{R_j/B}\times\vec{r}_{R_j/B}\times\right)^{-1}\frac{\partial\left(\vec{r}_{R_j/B}\times\vec{r}_{R_j/B}\times\right)}{\partial \delta_{HT}}\left(\vec{r}_{R_j/B}\times\vec{r}_{R_j/B}\times\right)^{-1}\Bigg\}\end{aligned}$$

Wherein,

$$\frac{\partial\left[[T]_{W_i}\left(\tilde{I}_{W_i}^{W_i} + m_{W_i}\tilde{R}_{W_i}^{B}\right)[T]_{W_i}^{T}\right]}{\partial \delta_W} = \frac{\partial [T]_{W_i}}{\partial \delta_W}\left(\tilde{I}_{W_i}^{W_i} + m_{W_i}\tilde{R}_{W_i}^{B}\right)[T]_{W_i}^{T} + m_{W_i}[T]_{W_i}\frac{\partial \tilde{R}_{W_i}^{B}}{\partial \delta_W}[T]_{W_i}^{T} + [T]_{W_i}\left(\tilde{I}_{W_i}^{W_i} + m_{W_i}\tilde{R}_{W_i}^{B}\right)\frac{\partial [T]_{W_i}^{T}}{\partial \delta_W}$$

$$\frac{\partial\left[[T]_{R_j}\left(\tilde{I}_{R_j}^{R_j}+m_{R_j}\tilde{R}_{R_j}^{B}\right)[T]_{R_j}^{T}\right]}{\partial\delta_W}=\frac{\partial[T]_{R_j}}{\partial\delta_W}\left(\tilde{I}_{R_j}^{R_j}+m_{R_j}\tilde{R}_{R_j}^{B}\right)[T]_{R_j}^{T}+m_{R_j}[T]_{R_j}\frac{\partial\tilde{R}_{R_j}^{B}}{\partial\delta_W}[T]_{R_j}^{T}+[T]_{R_j}\left(\tilde{I}_{R_j}^{R_j}+m_{R_j}\tilde{R}_{R_j}^{B}\right)\frac{\partial[T]_{R_j}^{T}}{\partial\delta_W}$$

$$\frac{\partial[[T]_{W_i}(\tilde{I}_{W_i}^{W_i}+m_{W_i}\tilde{R}_{W_i}^{B})[T]_{W_i}^{T}]}{\partial\delta_{HT}}=\frac{\partial[T]_{W_i}}{\partial\delta_{HT}}(\tilde{I}_{W_i}^{W_i}+m_{W_i}\tilde{R}_{W_i}^{B})[T]_{W_i}^{T}+m_{W_i}[T]_{W_i}\frac{\partial\tilde{R}_{W_i}^{B}}{\partial\delta_{HT}}[T]_{W_i}^{T}+[T]_{W_i}(\tilde{I}_{W_i}^{W_i}+m_{W_i}\tilde{R}_{W_i}^{B})\frac{\partial[T]_{W_i}^{T}}{\partial\delta_{HT}}$$

$$\frac{\partial\left[[T]_{R_j}\left(\tilde{I}_{R_j}^{R_j}+m_{R_j}\tilde{R}_{R_j}^{B}\right)[T]_{R_j}^{T}\right]}{\partial\delta_{HT}}=\frac{\partial[T]_{R_j}}{\partial\delta_{HT}}\left(\tilde{I}_{R_j}^{R_j}+m_{R_j}\tilde{R}_{R_j}^{B}\right)[T]_{R_j}^{T}+m_{R_j}[T]_{R_j}\frac{\partial\tilde{R}_{R_j}^{B}}{\partial\delta_{HT}}[T]_{R_j}^{T}+[T]_{R_j}\left(\tilde{I}_{R_j}^{R_j}+m_{R_j}\tilde{R}_{R_j}^{B}\right)\frac{\partial[T]_{R_j}^{T}}{\partial\delta_{HT}}$$

$$\frac{\partial(\vec{r}_{W_i/B}\times\vec{r}_{W_i/B}\times)}{\partial\delta_W}=\frac{\partial[\vec{r}_{W_i/B}\times]}{\partial\delta_W}(\vec{r}_{W_i/B}\times)+\vec{r}_{W_i/B}\times\frac{\partial[\vec{r}_{W_i/B}\times]}{\partial\delta_W}$$

$$\frac{\partial\left(\vec{r}_{R_j/B}\times\vec{r}_{R_j/B}\times\right)}{\partial\delta_W}=\frac{\partial\left[\vec{r}_{R_j/B}\times\right]}{\partial\delta_W}\left(\vec{r}_{R_j/B}\times\right)+\vec{r}_{R_j/B}\times\frac{\partial\left[\vec{r}_{R_j/B}\times\right]}{\partial\delta_W}$$

$$\frac{\partial(\vec{r}_{W_i/B}\times\vec{r}_{W_i/B}\times)}{\partial\delta_{HT}}=\frac{\partial[\vec{r}_{W_i/B}\times]}{\partial\delta_{HT}}(\vec{r}_{W_i/B}\times)+\vec{r}_{W_i/B}\times\frac{\partial[\vec{r}_{W_i/B}\times]}{\partial\delta_{HT}}$$

$$\frac{\partial\left(\vec{r}_{R_j/B}\times\vec{r}_{R_j/B}\times\right)}{\partial\delta_{HT}}=\frac{\partial\left[\vec{r}_{R_j/B}\times\right]}{\partial\delta_{HT}}\left(\vec{r}_{R_j/B}\times\right)+\vec{r}_{R_j/B}\times\frac{\partial\left[\vec{r}_{R_j/B}\times\right]}{\partial\delta_{HT}}$$

18.23 Partial derivatives of $[\boldsymbol{T}]_{\boldsymbol{W_i}}$ and $[\boldsymbol{T}]_{\boldsymbol{W_i}}^{\boldsymbol{T}}$:

$$\frac{\partial[T]_{W_i}}{\partial\delta_W}=\frac{\partial[T]_{W_i}^{T}}{\partial\delta_W}=\frac{\partial}{\partial\delta_W}\begin{bmatrix}\cos(\delta_{W,HT_i}) & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & \cos(\delta_{W,HT_i})\end{bmatrix}=\left\{\begin{bmatrix}-\sin\delta_W & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & -\sin\delta_W\end{bmatrix} or \begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}\right\}$$

$$\frac{\partial[T]_{W_i}}{\partial\delta_{HT}}=\frac{\partial[T]_{W_i}^{T}}{\partial\delta_{HT}}=\frac{\partial}{\partial\delta_{HT}}\begin{bmatrix}\cos(\delta_{W,HT_i}) & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & \cos(\delta_{W,HT_i})\end{bmatrix}=\left\{\begin{bmatrix}-\sin\delta_{HT} & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & -\sin\delta_{HT}\end{bmatrix} or \begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}\right\}$$

18.24 Partial derivatives of $[\boldsymbol{T}]_{\boldsymbol{R_j}}$ and $[\boldsymbol{T}]_{\boldsymbol{R_j}}^{\boldsymbol{T}}$:

$$\frac{\partial[T]_{R_j}}{\partial\delta_W}=\frac{\partial[T]_{R_j}^{T}}{\partial\delta_W}=\frac{\partial}{\partial\delta_W}\begin{bmatrix}\cos\left(\delta_{W,HT_j}\right) & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & \cos\left(\delta_{W,HT_j}\right)\end{bmatrix}=\left\{\begin{bmatrix}-\sin\delta_W & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & -\sin\delta_W\end{bmatrix} or \begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}\right\}$$

$$\frac{\partial[T]_{R_j}}{\partial\delta_{HT}}=\frac{\partial[T]_{R_j}^{T}}{\partial\delta_{HT}}=\frac{\partial}{\partial\delta_{HT}}\begin{bmatrix}\cos\left(\delta_{W,HT_j}\right) & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & \cos\left(\delta_{W,HT_j}\right)\end{bmatrix}=\left\{\begin{bmatrix}-\sin\delta_{HT} & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & -\sin\delta_{HT}\end{bmatrix} or \begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}\right\}$$

18.25 Partial derivatives of $[\dot{\boldsymbol{T}}]_{\boldsymbol{W_i}}$:

$$\frac{\partial[\dot{T}]_{W_i}}{\partial\delta_W}=\frac{\partial}{\partial\,\delta_W}\begin{bmatrix}-\dot{\delta}_{W,HT_i}\sin(\delta_{W,HT_i}) & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & -\dot{\delta}_{W,HT_i}\sin(\delta_{W,HT_i})\end{bmatrix}=\left\{\begin{bmatrix}-\dot{\delta}_W\cos\delta_W & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & -\dot{\delta}_W\cos\delta_W\end{bmatrix} or \begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}\right\}$$

$$\frac{\partial[\dot{T}]_{W_i}}{\partial\delta_{HT}}=\frac{\partial}{\partial\,\delta_{HT}}\begin{bmatrix}-\dot{\delta}_{W,HT_i}\sin(\delta_{W,HT_i}) & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & -\dot{\delta}_{W,HT_i}\sin(\delta_{W,HT_i})\end{bmatrix}=\left\{\begin{bmatrix}-\dot{\delta}_{HT}\cos\delta_{HT} & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & -\dot{\delta}_{HT}\cos\delta_{HT}\end{bmatrix} or \begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}\right\}$$

$$\frac{\partial[\dot{T}]_{W_i}}{\partial\dot{\delta}_W}=\frac{\partial}{\partial\,\dot{\delta}_W}\begin{bmatrix}-\dot{\delta}_{W,HT_i}\sin\left(\delta_{W,HT_i}\right) & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & -\dot{\delta}_{W,HT_i}\sin\left(\delta_{W,HT_i}\right)\end{bmatrix}=\left\{\begin{bmatrix}-\sin\delta_W & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & -\sin\delta_W\end{bmatrix} or \begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}\right\}$$

$$\frac{\partial[\dot{T}]_{W_i}}{\partial\dot{\delta}_{HT}}=\frac{\partial}{\partial\,\dot{\delta}_{HT}}\begin{bmatrix}-\dot{\delta}_{W,HT_i}\sin\left(\delta_{W,HT_i}\right) & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & -\dot{\delta}_{W,HT_i}\sin\left(\delta_{W,HT_i}\right)\end{bmatrix}=\left\{\begin{bmatrix}-\sin\delta_{HT} & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & -\sin\delta_{HT}\end{bmatrix} or \begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}\right\}$$

18.26 Partial derivatives of $[\dot{\boldsymbol{T}}]_{\boldsymbol{R_j}}$:

$$\frac{\partial[\dot{T}]_{R_j}}{\partial\delta_W}=\frac{\partial}{\partial\,\delta_W}\begin{bmatrix}-\dot{\delta}_{W,HT_j}\sin\left(\delta_{W,HT_j}\right) & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & -\dot{\delta}_{W,HT_j}\sin\left(\delta_{W,HT_j}\right)\end{bmatrix}=\left\{\begin{bmatrix}-\dot{\delta}_W\cos\delta_W & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & -\dot{\delta}_W\cos\delta_W\end{bmatrix} or \begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}\right\}$$

$$\frac{\partial[\dot{T}]_{R_j}}{\partial\delta_{HT}}=\frac{\partial}{\partial\,\delta_W}\begin{bmatrix}-\dot{\delta}_{W,HT_j}\sin\left(\delta_{W,HT_j}\right) & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & -\dot{\delta}_{W,HT_j}\sin\left(\delta_{W,HT_j}\right)\end{bmatrix}=\left\{\begin{bmatrix}-\dot{\delta}_{HT}\cos\delta_{HT} & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & -\dot{\delta}_{HT}\cos\delta_{HT}\end{bmatrix} or \begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}\right\}$$

$$\frac{\partial[\dot{T}]_{R_j}}{\partial\dot{\delta}_W}=\frac{\partial}{\partial\,\dot{\delta}_W}\begin{bmatrix}-\dot{\delta}_{W,HT_i}\sin\left(\delta_{W,HT_i}\right) & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & -\dot{\delta}_{W,HT_i}\sin\left(\delta_{W,HT_i}\right)\end{bmatrix}=\left\{\begin{bmatrix}-\sin\delta_W & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & -\sin\delta_W\end{bmatrix} or \begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}\right\}$$

$$\frac{\partial[\dot{T}]_{R_j}}{\partial\dot{\delta}_{HT}}=\frac{\partial}{\partial\,\dot{\delta}_{HT}}\begin{bmatrix}-\dot{\delta}_{W,HT_i}\sin\left(\delta_{W,HT_i}\right) & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & -\dot{\delta}_{W,HT_i}\sin\left(\delta_{W,HT_i}\right)\end{bmatrix}=\left\{\begin{bmatrix}-\sin\delta_{HT} & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & -\sin\delta_{HT}\end{bmatrix} or \begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}\right\}$$

18.27 Partial derivatives of $\vec{\boldsymbol{r}}_{\boldsymbol{W_i/B}}$:

$$\frac{\partial\left[\vec{r}_{W_i/B}\times\right]}{\partial\delta_W}=\left\{\begin{matrix}\begin{bmatrix}0 & \cos\delta_W\,x_{\frac{W_i}{pivot_i}}+\sin\delta_W\,z_{\frac{W_i}{pivot_i}} & 0\\ -\cos\delta_W\,x_{\frac{W_i}{pivot_i}}-\sin\delta_W\,z_{\frac{W_i}{pivot_i}} & 0 & \sin\delta_W\,x_{\frac{W_i}{pivot_i}}-\cos\delta_W\,z_{\frac{W_i}{pivot_i}}\\ 0 & -\sin\delta_W\,x_{\frac{W_i}{pivot_i}}+\cos\delta_W\,z_{\frac{W_i}{pivot_i}} & 0\end{bmatrix}\\ or \begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}\end{matrix}\right\}$$

$$\frac{\partial\left[\vec{r}_{W_i/B}\times\right]}{\partial\delta_{HT}}$$

$$=\left\{\begin{matrix}\begin{bmatrix}0 & \cos\delta_{HT}\,x_{\frac{W_i}{pivot_i}}+\sin\delta_{HT}\,z_{\frac{W_i}{pivot_i}} & 0\\ -\cos\delta_{HT}\,x_{\frac{W_i}{pivot_i}}-\sin\delta_{HT}\,z_{\frac{W_i}{pivot_i}} & 0 & \sin\delta_{HT}\,x_{\frac{W_i}{pivot_i}}-\cos\delta_{HT}\,z_{\frac{W_i}{pivot_i}}\\ 0 & -\sin\delta_{HT}\,x_{\frac{W_i}{pivot_i}}+\cos\delta_{HT}\,z_{\frac{W_i}{pivot_i}} & 0\end{bmatrix}\\ or \begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}\end{matrix}\right\}$$

18.28 Partial derivatives of $\vec{\boldsymbol{r}}_{\boldsymbol{R_j/B}}$:

$$\frac{\partial\left[\vec{r}_{R_j/B}\times\right]}{\partial\delta_W}=\left\{\begin{matrix}\begin{bmatrix}0 & \cos\delta_W\, x_{\frac{R_j}{pivot_j}}+\sin\delta_W\, z_{\frac{R_j}{pivot_j}} & 0\\ -\cos\delta_W\, x_{\frac{R_j}{pivot_j}}-\sin\delta_W\, z_{\frac{R_j}{pivot_j}} & 0 & \sin\delta_W\, x_{\frac{R_j}{pivot_j}}-\cos\delta_W\, z_{\frac{R_j}{pivot_j}}\\ 0 & -\sin\delta_W\, x_{\frac{R_j}{pivot_j}}+\cos\delta_W\, z_{\frac{R_j}{pivot_j}} & 0\end{bmatrix}\\ or\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}\end{matrix}\right\}$$

$$\frac{\partial\left[\vec{r}_{R_j/B}\times\right]}{\partial\delta_{HT}}$$
$$=\left\{\begin{matrix}\begin{bmatrix}0 & \cos\delta_{HT}\, x_{\frac{R_j}{pivot_j}}+\sin\delta_{HT}\, z_{\frac{R_j}{pivot_j}} & 0\\ -\cos\delta_{HT}\, x_{\frac{R_j}{pivot_j}}-\sin\delta_W\, z_{\frac{R_j}{pivot_j}} & 0 & \sin\delta_{HT}\, x_{\frac{R_j}{pivot_j}}-\cos\delta_{HT}\, z_{\frac{R_j}{pivot_j}}\\ 0 & -\sin\delta_{HT}\, x_{\frac{R_j}{pivot_j}}+\cos\delta_{HT}\, z_{\frac{R_j}{pivot_j}} & 0\end{bmatrix}\\ or\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}\end{matrix}\right\}$$

18.29 Partial derivatives of $\widetilde{\boldsymbol{R}}_{\boldsymbol{W_i}}^{\boldsymbol{B}}$:

$$\frac{\partial\tilde{R}_{W_i}^B}{\partial\delta_W}=\begin{bmatrix}\frac{\partial\left[\left(y_{W_i/B}\right)^2+\left(z_{W_i/B}\right)^2\right]}{\partial\delta_W} & \frac{\partial\left[x_{W_i/B}y_{W_i/B}\right]}{\partial\delta_W} & \frac{\partial\left[z_{W_i/B}x_{W_i/B}\right]}{\partial\delta_W}\\ \frac{\partial\left[x_{W_i/B}y_{W_i/B}\right]}{\partial\delta_W} & \frac{\partial\left[\left(z_{W_i/B}\right)^2+\left(x_{W_i/B}\right)^2\right]}{\partial\delta_W} & \frac{\partial\left[y_{W_i/B}z_{W_i/B}\right]}{\partial\delta_W}\\ \frac{\partial\left[z_{W_i/B}x_{W_i/B}\right]}{\partial\delta_W} & \frac{\partial\left[y_{W_i/B}z_{W_i/B}\right]}{\partial\delta_W} & \frac{\partial\left[\left(x_{W_i/B}\right)^2+\left(y_{W_i/B}\right)^2\right]}{\partial\delta_W}\end{bmatrix}$$

$$\frac{\partial\tilde{R}_{W_i}^B}{\partial\delta_{HT}}=\begin{bmatrix}\frac{\partial\left[\left(y_{W_i/B}\right)^2+\left(z_{W_i/B}\right)^2\right]}{\partial\delta_{HT}} & \frac{\partial\left[x_{W_i/B}y_{W_i/B}\right]}{\partial\delta_{HT}} & \frac{\partial\left[z_{W_i/B}x_{W_i/B}\right]}{\partial\delta_{HT}}\\ \frac{\partial\left[x_{W_i/B}y_{W_i/B}\right]}{\partial\delta_{HT}} & \frac{\partial\left[\left(z_{W_i/B}\right)^2+\left(x_{W_i/B}\right)^2\right]}{\partial\delta_{HT}} & \frac{\partial\left[y_{W_i/B}z_{W_i/B}\right]}{\partial\delta_{HT}}\\ \frac{\partial\left[z_{W_i/B}x_{W_i/B}\right]}{\partial\delta_{HT}} & \frac{\partial\left[y_{W_i/B}z_{W_i/B}\right]}{\partial\delta_{HT}} & \frac{\partial\left[\left(x_{W_i/B}\right)^2+\left(y_{W_i/B}\right)^2\right]}{\partial\delta_{HT}}\end{bmatrix}$$

18.30 Partial derivatives of $\widetilde{\boldsymbol{R}}_{\boldsymbol{R_j}}^{\boldsymbol{B}}$:

$$\frac{\partial\tilde{R}_{R_j}^B}{\partial\delta_W}=\begin{bmatrix}\frac{\partial\left[\left(y_{R_j/B}\right)^2+\left(z_{R_j/B}\right)^2\right]}{\partial\delta_W} & \frac{\partial\left[x_{R_j/B}y_{R_j/B}\right]}{\partial\delta_W} & \frac{\partial\left[z_{R_j/B}x_{R_j/B}\right]}{\partial\delta_W}\\ \frac{\partial\left[x_{R_j/B}y_{R_j/B}\right]}{\partial\delta_W} & \frac{\partial\left[\left(z_{R_j/B}\right)^2+\left(x_{R_j/B}\right)^2\right]}{\partial\delta_W} & \frac{\partial\left[y_{R_j/B}z_{R_j/B}\right]}{\partial\delta_W}\\ \frac{\partial\left[z_{R_j/B}x_{R_j/B}\right]}{\partial\delta_W} & \frac{\partial\left[y_{R_j/B}z_{R_j/B}\right]}{\partial\delta_W} & \frac{\partial\left[\left(x_{R_j/B}\right)^2+\left(y_{R_j/B}\right)^2\right]}{\partial\delta_W}\end{bmatrix}$$

$$\frac{\partial \tilde{R}_{R_j}^B}{\partial \delta_{HT}} = \begin{bmatrix} \frac{\partial\left[\left(y_{R_j/B}\right)^2 + \left(z_{R_j/B}\right)^2\right]}{\partial \delta_{HT}} & \frac{\partial\left[x_{R_j/B}y_{R_j/B}\right]}{\partial \delta_{HT}} & \frac{\partial\left[z_{R_j/B}x_{R_j/B}\right]}{\partial \delta_{HT}} \\ \frac{\partial\left[x_{R_j/B}y_{R_j/B}\right]}{\partial \delta_{HT}} & \frac{\partial\left[\left(z_{R_j/B}\right)^2 + \left(x_{R_j/B}\right)^2\right]}{\partial \delta_{HT}} & \frac{\partial\left[y_{R_j/B}z_{R_j/B}\right]}{\partial \delta_{HT}} \\ \frac{\partial\left[z_{R_j/B}x_{R_j/B}\right]}{\partial \delta_{HT}} & \frac{\partial\left[y_{R_j/B}z_{R_j/B}\right]}{\partial \delta_{HT}} & \frac{\partial\left[\left(x_{R_j/B}\right)^2 + \left(y_{R_j/B}\right)^2\right]}{\partial \delta_{HT}} \end{bmatrix}$$

1.1 Partial derivatives of terms in $\frac{\partial \tilde{R}_{W_i}^B}{\partial \delta_W}$ and $\frac{\partial \tilde{R}_{R_j}^B}{\partial \delta_W}$:

$$\frac{\partial\left[\left(y_{W_i/B}\right)^2 + \left(z_{W_i/B}\right)^2\right]}{\partial \delta_W} = 2y_{W_i/B}\frac{\partial y_{W_i/B}}{\partial \delta_W} + 2z_{W_i/B}\frac{\partial z_{W_i/B}}{\partial \delta_W} \quad and \quad \frac{\partial\left[\left(y_{R_j/B}\right)^2 + \left(z_{R_j/B}\right)^2\right]}{\partial \delta_W} = 2y_{R_j/B}\frac{\partial y_{R_j/B}}{\partial \delta_W} + 2z_{R_j/B}\frac{\partial z_{R_j/B}}{\partial \delta_W}$$

$$\frac{\partial\left[\left(z_{W_i/B}\right)^2 + \left(x_{W_i/B}\right)^2\right]}{\partial \delta_W} = 2z_{W_i/B}\frac{\partial z_{W_i/B}}{\partial \delta_W} + 2x_{W_i/B}\frac{\partial x_{W_i/B}}{\partial \delta_W} \quad and \quad \frac{\partial\left[\left(z_{R_j/B}\right)^2 + \left(x_{R_j/B}\right)^2\right]}{\partial \delta_W} = 2z_{R_j/B}\frac{\partial z_{R_j/B}}{\partial \delta_W} + 2x_{R_j/B}\frac{\partial x_{R_j/B}}{\partial \delta_W}$$

$$\frac{\partial\left[\left(x_{W_i/B}\right)^2 + \left(y_{W_i/B}\right)^2\right]}{\partial \delta_W} = 2x_{W_i/B}\frac{\partial x_{W_i/B}}{\partial \delta_W} + 2y_{W_i/B}\frac{\partial y_{W_i/B}}{\partial \delta_W} \quad and \quad \frac{\partial\left[\left(x_{R_j/B}\right)^2 + \left(y_{R_j/B}\right)^2\right]}{\partial \delta_W} = 2x_{R_j/B}\frac{\partial x_{R_j/B}}{\partial \delta_W} + 2y_{R_j/B}\frac{\partial y_{R_j/B}}{\partial \delta_W}$$

$$\frac{\partial\left[x_{W_i/B}y_{W_i/B}\right]}{\partial \delta_W} = \frac{\partial x_{W_i/B}}{\partial \delta_W}y_{W_i/B} + x_{W_i/B}\frac{\partial y_{W_i/B}}{\partial \delta_W} \quad and \quad \frac{\partial\left[x_{R_j/B}y_{R_j/B}\right]}{\partial \delta_W} = \frac{\partial x_{R_j/B}}{\partial \delta_W}y_{R_j/B} + x_{R_j/B}\frac{\partial y_{R_j/B}}{\partial \delta_W}$$

$$\frac{\partial\left[y_{W_i/B}z_{W_i/B}\right]}{\partial \delta_W} = \frac{\partial y_{W_i/B}}{\partial \delta_W}z_{W_i/B} + y_{W_i/B}\frac{\partial z_{W_i/B}}{\partial \delta_W} \quad and \quad \frac{\partial\left[y_{R_j/B}z_{R_j/B}\right]}{\partial \delta_W} = \frac{\partial y_{R_j/B}}{\partial \delta_W}z_{R_j/B} + y_{R_j/B}\frac{\partial z_{R_j/B}}{\partial \delta_W}$$

$$\frac{\partial\left[z_{W_i/B}x_{W_i/B}\right]}{\partial \delta_W} = \frac{\partial z_{W_i/B}}{\partial \delta_W}x_{W_i/B} + z_{W_i/B}\frac{\partial x_{W_i/B}}{\partial \delta_W} \quad and \quad \frac{\partial\left[z_{R_j/B}x_{R_j/B}\right]}{\partial \delta_W} = \frac{\partial z_{R_j/B}}{\partial \delta_W}x_{R_j/B} + z_{R_j/B}\frac{\partial x_{R_j/B}}{\partial \delta_W}$$

1.2 Partial derivatives of terms in $\frac{\partial \tilde{R}_{W_i}^B}{\partial \delta_{HT}}$ and $\frac{\partial \tilde{R}_{R_j}^B}{\partial \delta_{HT}}$:

$$\frac{\partial\left[\left(y_{W_i/B}\right)^2 + \left(z_{W_i/B}\right)^2\right]}{\partial \delta_{HT}} = 2y_{W_i/B}\frac{\partial y_{W_i/B}}{\partial \delta_{HT}} + 2z_{W_i/B}\frac{\partial z_{W_i/B}}{\partial \delta_{HT}} \quad and \quad \frac{\partial\left[\left(y_{R_j/B}\right)^2 + \left(z_{R_j/B}\right)^2\right]}{\partial \delta_{HT}} = 2y_{R_j/B}\frac{\partial y_{R_j/B}}{\partial \delta_{HT}} + 2z_{R_j/B}\frac{\partial z_{R_j/B}}{\partial \delta_{HT}}$$

$$\frac{\partial\left[\left(z_{W_i/B}\right)^2 + \left(x_{W_i/B}\right)^2\right]}{\partial \delta_{HT}} = 2z_{W_i/B}\frac{\partial z_{W_i/B}}{\partial \delta_{HT}} + 2x_{W_i/B}\frac{\partial x_{W_i/B}}{\partial \delta_{HT}} \quad and \quad \frac{\partial\left[\left(z_{R_j/B}\right)^2 + \left(x_{R_j/B}\right)^2\right]}{\partial \delta_{HT}} = 2z_{R_j/B}\frac{\partial z_{R_j/B}}{\partial \delta_{HT}} + 2x_{R_j/B}\frac{\partial x_{R_j/B}}{\partial \delta_{HT}}$$

$$\frac{\partial\left[\left(x_{W_i/B}\right)^2 + \left(y_{W_i/B}\right)^2\right]}{\partial \delta_{HT}} = 2x_{W_i/B}\frac{\partial x_{W_i/B}}{\partial \delta_{HT}} + 2y_{W_i/B}\frac{\partial y_{W_i/B}}{\partial \delta_{HT}} \quad and \quad \frac{\partial\left[\left(x_{R_j/B}\right)^2 + \left(y_{R_j/B}\right)^2\right]}{\partial \delta_{HT}} = 2x_{R_j/B}\frac{\partial x_{R_j/B}}{\partial \delta_{HT}} + 2y_{R_j/B}\frac{\partial y_{R_j/B}}{\partial \delta_{HT}}$$

$$\frac{\partial\left[x_{W_i/B}y_{W_i/B}\right]}{\partial \delta_{HT}} = \frac{\partial x_{W_i/B}}{\partial \delta_{HT}}y_{W_i/B} + x_{W_i/B}\frac{\partial y_{W_i/B}}{\partial \delta_{HT}} \quad and \quad \frac{\partial\left[x_{R_j/B}y_{R_j/B}\right]}{\partial \delta_{HT}} = \frac{\partial x_{R_j/B}}{\partial \delta_{HT}}y_{R_j/B} + x_{R_j/B}\frac{\partial y_{R_j/B}}{\partial \delta_{HT}}$$

$$\frac{\partial\left[y_{W_i/B}z_{W_i/B}\right]}{\partial \delta_{HT}} = \frac{\partial y_{W_i/B}}{\partial \delta_{HT}}z_{W_i/B} + y_{W_i/B}\frac{\partial z_{W_i/B}}{\partial \delta_{HT}} \quad and \quad \frac{\partial\left[y_{R_j/B}z_{R_j/B}\right]}{\partial \delta_{HT}} = \frac{\partial y_{R_j/B}}{\partial \delta_{HT}}z_{R_j/B} + y_{R_j/B}\frac{\partial z_{R_j/B}}{\partial \delta_{HT}}$$

$$\frac{\partial\left[z_{W_i/B}x_{W_i/B}\right]}{\partial\delta_{HT}}=\frac{\partial z_{W_i/B}}{\partial\delta_{HT}}x_{W_i/B}+z_{W_i/B}\frac{\partial x_{W_i/B}}{\partial\delta_{HT}}\quad and\quad \frac{\partial\left[z_{R_j/B}x_{R_j/B}\right]}{\partial\delta_{HT}}=\frac{\partial z_{R_j/B}}{\partial\delta_{HT}}x_{R_j/B}+z_{R_j/B}\frac{\partial x_{R_j/B}}{\partial\delta_{HT}}$$

18.31 Partial derivatives of $\boldsymbol{x_{W_i/B}}$, $\boldsymbol{y_{W_i/B}}$, $\boldsymbol{z_{W_i/B}}$, $\boldsymbol{x_{R_j/B}}$, $\boldsymbol{y_{R_j/B}}$, $\boldsymbol{z_{R_j/B}}$:

$$\frac{\partial x_{W_i/B}}{\partial\delta_W}=\left[-\sin\delta_W\,x_{W_i/pivot_i}+\cos\delta_W\,z_{W_i/pivot_i}\right]\,or\;0\quad and\quad \frac{\partial x_{R_j/B}}{\partial\delta_W}=\left[-\sin\delta_W\,x_{R_j/pivot_j}+\cos\delta_W\,z_{R_j/pivot_j}\right]\,or\;0$$

$$\frac{\partial y_{W_i/B}}{\partial\delta_W}=0\quad and\quad \frac{\partial y_{R_j/B}}{\partial\delta_W}=0$$

$$\frac{\partial z_{W_i/B}}{\partial\delta_W}=\left[-\cos\delta_W\,x_{W_i/pivot_i}-\sin\delta_W\,z_{W_i/pivot_i}\right]\,or\;0\quad and\quad \frac{\partial z_{R_j/B}}{\partial\delta_W}=\left[-\cos\delta_W\,x_{R_j/pivot_j}-\sin\delta_W\,z_{R_j/pivot_j}\right]\,or\;0$$

$$\frac{\partial x_{W_i/B}}{\partial\delta_{HT}}=\left[-\sin\delta_{HT}\,x_{W_i/pivot_i}+\cos\delta_{HT}\,z_{W_i/pivot_i}\right]\,or\;0\quad and\quad \frac{\partial x_{R_j/B}}{\partial\delta_{HT}}=\left[-\sin\delta_{HT}\,x_{R_j/pivot_j}+\cos\delta_{HT}\,z_{R_j/pivot_j}\right]\,or\;0$$

$$\frac{\partial y_{W_i/B}}{\partial\delta_{HT}}=0\quad and\quad \frac{\partial y_{R_j/B}}{\partial\delta_{HT}}=0$$

$$\frac{\partial z_{W_i/B}}{\partial\delta_{HT}}=\left[-\cos\delta_{HT}\,x_{W_i/pivot_i}-\sin\delta_{HT}\,z_{W_i/pivot_i}\right]\,or\;0\quad and\quad \frac{\partial z_{R_j/B}}{\partial\delta_{HT}}=\left[-\cos\delta_{HT}\,x_{R_j/pivot_j}-\sin\delta_{HT}\,z_{R_j/pivot_j}\right]\,or\;0$$

18.32 Partial derivatives of $\boldsymbol{\dot{\tilde{R}}^B_{W_i}}$:

$$\frac{\partial\dot{\tilde{R}}^B_{W_i}}{\partial\delta_W}=\begin{bmatrix}2\frac{\partial\left(y_{W_i/B}\dot{y}_{W_i/B}+z_{W_i/B}\dot{z}_{W_i/B}\right)}{\partial\delta_W} & \frac{\partial\left(\dot{x}_{W_i/B}y_{W_i/B}+x_{W_i/B}\dot{y}_{W_i/B}\right)}{\partial\delta_W} & \frac{\partial\left(\dot{z}_{W_i/B}x_{W_i/B}+z_{W_i/B}\dot{x}_{W_i/B}\right)}{\partial\delta_W}\\ \frac{\partial\left(\dot{x}_{W_i/B}y_{W_i/B}+x_{W_i/B}\dot{y}_{W_i/B}\right)}{\partial\delta_W} & 2\frac{\partial\left(x_{W_i/B}\dot{x}_{W_i/B}+z_{W_i/B}\dot{z}_{W_i/B}\right)}{\partial\delta_W} & \frac{\partial\left(\dot{y}_{W_i/B}z_{W_i/B}+y_{W_i/B}\dot{z}_{W_i/B}\right)}{\partial\delta_W}\\ \frac{\partial\left(\dot{z}_{W_i/B}x_{W_i/B}+z_{W_i/B}\dot{x}_{W_i/B}\right)}{\partial\delta_W} & \frac{\partial\left(\dot{y}_{W_i/B}z_{W_i/B}+y_{W_i/B}\dot{z}_{W_i/B}\right)}{\partial\delta_W} & 2\frac{\partial\left(x_{W_i/B}\dot{x}_{W_i/B}+y_{W_i/B}\dot{y}_{W_i/B}\right)}{\partial\delta_W}\end{bmatrix}$$

$$\frac{\partial\dot{\tilde{R}}^B_{W_i}}{\partial\delta_{HT}}=\begin{bmatrix}2\frac{\partial\left(y_{W_i/B}\dot{y}_{W_i/B}+z_{W_i/B}\dot{z}_{W_i/B}\right)}{\partial\delta_{HT}} & \frac{\partial\left(\dot{x}_{W_i/B}y_{W_i/B}+x_{W_i/B}\dot{y}_{W_i/B}\right)}{\partial\delta_{HT}} & \frac{\partial\left(\dot{z}_{W_i/B}x_{W_i/B}+z_{W_i/B}\dot{x}_{W_i/B}\right)}{\partial\delta_{HT}}\\ \frac{\partial\left(\dot{x}_{W_i/B}y_{W_i/B}+x_{W_i/B}\dot{y}_{W_i/B}\right)}{\partial\delta_{HT}} & 2\frac{\partial\left(x_{W_i/B}\dot{x}_{W_i/B}+z_{W_i/B}\dot{z}_{W_i/B}\right)}{\partial\delta_{HT}} & \frac{\partial\left(\dot{y}_{W_i/B}z_{W_i/B}+y_{W_i/B}\dot{z}_{W_i/B}\right)}{\partial\delta_{HT}}\\ \frac{\partial\left(\dot{z}_{W_i/B}x_{W_i/B}+z_{W_i/B}\dot{x}_{W_i/B}\right)}{\partial\delta_{HT}} & \frac{\partial\left(\dot{y}_{W_i/B}z_{W_i/B}+y_{W_i/B}\dot{z}_{W_i/B}\right)}{\partial\delta_{HT}} & 2\frac{\partial\left(x_{W_i/B}\dot{x}_{W_i/B}+y_{W_i/B}\dot{y}_{W_i/B}\right)}{\partial\delta_{HT}}\end{bmatrix}$$

$$\frac{\partial\dot{\tilde{R}}^B_{W_i}}{\partial\dot{\delta}_W}=\begin{bmatrix}2\frac{\partial\left(y_{W_i/B}\dot{y}_{W_i/B}+z_{W_i/B}\dot{z}_{W_i/B}\right)}{\partial\dot{\delta}_W} & \frac{\partial\left(\dot{x}_{W_i/B}y_{W_i/B}+x_{W_i/B}\dot{y}_{W_i/B}\right)}{\partial\dot{\delta}_W} & \frac{\partial\left(\dot{z}_{W_i/B}x_{W_i/B}+z_{W_i/B}\dot{x}_{W_i/B}\right)}{\partial\dot{\delta}_W}\\ \frac{\partial\left(\dot{x}_{W_i/B}y_{W_i/B}+x_{W_i/B}\dot{y}_{W_i/B}\right)}{\partial\dot{\delta}_W} & 2\frac{\partial\left(x_{W_i/B}\dot{x}_{W_i/B}+z_{W_i/B}\dot{z}_{W_i/B}\right)}{\partial\dot{\delta}_W} & \frac{\partial\left(\dot{y}_{W_i/B}z_{W_i/B}+y_{W_i/B}\dot{z}_{W_i/B}\right)}{\partial\dot{\delta}_W}\\ \frac{\partial\left(\dot{z}_{W_i/B}x_{W_i/B}+z_{W_i/B}\dot{x}_{W_i/B}\right)}{\partial\dot{\delta}_W} & \frac{\partial\left(\dot{y}_{W_i/B}z_{W_i/B}+y_{W_i/B}\dot{z}_{W_i/B}\right)}{\partial\dot{\delta}_W} & 2\frac{\partial\left(x_{W_i/B}\dot{x}_{W_i/B}+y_{W_i/B}\dot{y}_{W_i/B}\right)}{\partial\dot{\delta}_W}\end{bmatrix}$$

$$\frac{\partial\dot{\tilde{R}}^B_{W_i}}{\partial\dot{\delta}_{HT}}=\begin{bmatrix}2\frac{\partial\left(y_{W_i/B}\dot{y}_{W_i/B}+z_{W_i/B}\dot{z}_{W_i/B}\right)}{\partial\dot{\delta}_{HT}} & \frac{\partial\left(\dot{x}_{W_i/B}y_{W_i/B}+x_{W_i/B}\dot{y}_{W_i/B}\right)}{\partial\dot{\delta}_{HT}} & \frac{\partial\left(\dot{z}_{W_i/B}x_{W_i/B}+z_{W_i/B}\dot{x}_{W_i/B}\right)}{\partial\dot{\delta}_{HT}}\\ \frac{\partial\left(\dot{x}_{W_i/B}y_{W_i/B}+x_{W_i/B}\dot{y}_{W_i/B}\right)}{\partial\dot{\delta}_{HT}} & 2\frac{\partial\left(x_{W_i/B}\dot{x}_{W_i/B}+z_{W_i/B}\dot{z}_{W_i/B}\right)}{\partial\dot{\delta}_{HT}} & \frac{\partial\left(\dot{y}_{W_i/B}z_{W_i/B}+y_{W_i/B}\dot{z}_{W_i/B}\right)}{\partial\dot{\delta}_{HT}}\\ \frac{\partial\left(\dot{z}_{W_i/B}x_{W_i/B}+z_{W_i/B}\dot{x}_{W_i/B}\right)}{\partial\dot{\delta}_{HT}} & \frac{\partial\left(\dot{y}_{W_i/B}z_{W_i/B}+y_{W_i/B}\dot{z}_{W_i/B}\right)}{\partial\dot{\delta}_{HT}} & 2\frac{\partial\left(x_{W_i/B}\dot{x}_{W_i/B}+y_{W_i/B}\dot{y}_{W_i/B}\right)}{\partial\dot{\delta}_{HT}}\end{bmatrix}$$

18.33 Partial derivatives of $\dot{\tilde{\boldsymbol{R}}}_{\boldsymbol{R_j}}^{\boldsymbol{B}}$:

$$\frac{\partial \dot{\tilde{R}}_{R_j}^{B}}{\partial \delta_W} = \begin{bmatrix} 2\frac{\partial\left(y_{R_j/B}\dot{y}_{R_j/B}+z_{R_j/B}\dot{z}_{R_j/B}\right)}{\partial \delta_W} & \frac{\partial\left(\dot{x}_{R_j/B}y_{R_j/B}+x_{R_j/B}\dot{y}_{R_j/B}\right)}{\partial \delta_W} & \frac{\partial\left(\dot{x}_{R_j/B}z_{R_j/B}+x_{R_j/B}\dot{z}_{R_j/B}\right)}{\partial \delta_W} \\ \frac{\partial\left(\dot{x}_{R_j/B}y_{R_j/B}+x_{R_j/B}\dot{y}_{R_j/B}\right)}{\partial \delta_W} & 2\frac{\partial\left(x_{R_j/B}\dot{x}_{R_j/B}+z_{R_j/B}\dot{z}_{R_j/B}\right)}{\partial \delta_W} & \frac{\partial\left(\dot{y}_{R_j/B}z_{R_j/B}+y_{R_j/B}\dot{z}_{R_j/B}\right)}{\partial \delta_W} \\ \frac{\partial\left(\dot{x}_{R_j/B}z_{R_j/B}+x_{R_j/B}\dot{z}_{R_j/B}\right)}{\partial \delta_W} & \frac{\partial\left(\dot{y}_{R_j/B}z_{R_j/B}+y_{R_j/B}\dot{z}_{R_j/B}\right)}{\partial \delta_W} & 2\frac{\partial\left(x_{R_j/B}\dot{x}_{R_j/B}+y_{R_j/B}\dot{y}_{R_j/B}\right)}{\partial \delta_W} \end{bmatrix}$$

$$\frac{\partial \dot{\tilde{R}}_{R_j}^{B}}{\partial \delta_{HT}} = \begin{bmatrix} 2\frac{\partial\left(y_{R_j/B}\dot{y}_{R_j/B}+z_{R_j/B}\dot{z}_{R_j/B}\right)}{\partial \delta_{HT}} & \frac{\partial\left(\dot{x}_{R_j/B}y_{R_j/B}+x_{R_j/B}\dot{y}_{R_j/B}\right)}{\partial \delta_{HT}} & \frac{\partial\left(\dot{x}_{R_j/B}z_{R_j/B}+x_{R_j/B}\dot{z}_{R_j/B}\right)}{\partial \delta_{HT}} \\ \frac{\partial\left(\dot{x}_{R_j/B}y_{R_j/B}+x_{R_j/B}\dot{y}_{R_j/B}\right)}{\partial \delta_{HT}} & 2\frac{\partial\left(x_{R_j/B}\dot{x}_{R_j/B}+z_{R_j/B}\dot{z}_{R_j/B}\right)}{\partial \delta_{HT}} & \frac{\partial\left(\dot{y}_{R_j/B}z_{R_j/B}+y_{R_j/B}\dot{z}_{R_j/B}\right)}{\partial \delta_{HT}} \\ \frac{\partial\left(\dot{x}_{R_j/B}z_{R_j/B}+x_{R_j/B}\dot{z}_{R_j/B}\right)}{\partial \delta_{HT}} & \frac{\partial\left(\dot{y}_{R_j/B}z_{R_j/B}+y_{R_j/B}\dot{z}_{R_j/B}\right)}{\partial \delta_{HT}} & 2\frac{\partial\left(x_{R_j/B}\dot{x}_{R_j/B}+y_{R_j/B}\dot{y}_{R_j/B}\right)}{\partial \delta_{HT}} \end{bmatrix}$$

$$\frac{\partial \dot{\tilde{R}}_{R_j}^{B}}{\partial \dot{\delta}_W} = \begin{bmatrix} 2\frac{\partial\left(y_{R_j/B}\dot{y}_{R_j/B}+z_{R_j/B}\dot{z}_{R_j/B}\right)}{\partial \dot{\delta}_W} & \frac{\partial\left(\dot{x}_{R_j/B}y_{R_j/B}+x_{R_j/B}\dot{y}_{R_j/B}\right)}{\partial \dot{\delta}_W} & \frac{\partial\left(\dot{x}_{R_j/B}z_{R_j/B}+x_{R_j/B}\dot{z}_{R_j/B}\right)}{\partial \dot{\delta}_W} \\ \frac{\partial\left(\dot{x}_{R_j/B}y_{R_j/B}+x_{R_j/B}\dot{y}_{R_j/B}\right)}{\partial \dot{\delta}_W} & 2\frac{\partial\left(x_{R_j/B}\dot{x}_{R_j/B}+z_{R_j/B}\dot{z}_{R_j/B}\right)}{\partial \dot{\delta}_W} & \frac{\partial\left(\dot{y}_{R_j/B}z_{R_j/B}+y_{R_j/B}\dot{z}_{R_j/B}\right)}{\partial \dot{\delta}_W} \\ \frac{\partial\left(\dot{x}_{R_j/B}z_{R_j/B}+x_{R_j/B}\dot{z}_{R_j/B}\right)}{\partial \dot{\delta}_W} & \frac{\partial\left(\dot{y}_{R_j/B}z_{R_j/B}+y_{R_j/B}\dot{z}_{R_j/B}\right)}{\partial \dot{\delta}_W} & 2\frac{\partial\left(x_{R_j/B}\dot{x}_{R_j/B}+y_{R_j/B}\dot{y}_{R_j/B}\right)}{\partial \dot{\delta}_W} \end{bmatrix}$$

$$\frac{\partial \dot{\tilde{R}}_{R_j}^{B}}{\partial \dot{\delta}_{HT}} = \begin{bmatrix} 2\frac{\partial\left(y_{R_j/B}\dot{y}_{R_j/B}+z_{R_j/B}\dot{z}_{R_j/B}\right)}{\partial \dot{\delta}_{HT}} & \frac{\partial\left(\dot{x}_{R_j/B}y_{R_j/B}+x_{R_j/B}\dot{y}_{R_j/B}\right)}{\partial \dot{\delta}_{HT}} & \frac{\partial\left(\dot{x}_{R_j/B}z_{R_j/B}+x_{R_j/B}\dot{z}_{R_j/B}\right)}{\partial \dot{\delta}_{HT}} \\ \frac{\partial\left(\dot{x}_{R_j/B}y_{R_j/B}+x_{R_j/B}\dot{y}_{R_j/B}\right)}{\partial \dot{\delta}_{HT}} & 2\frac{\partial\left(x_{R_j/B}\dot{x}_{R_j/B}+z_{R_j/B}\dot{z}_{R_j/B}\right)}{\partial \dot{\delta}_{HT}} & \frac{\partial\left(\dot{y}_{R_j/B}z_{R_j/B}+y_{R_j/B}\dot{z}_{R_j/B}\right)}{\partial \dot{\delta}_{HT}} \\ \frac{\partial\left(\dot{x}_{R_j/B}z_{R_j/B}+x_{R_j/B}\dot{z}_{R_j/B}\right)}{\partial \dot{\delta}_{HT}} & \frac{\partial\left(\dot{y}_{R_j/B}z_{R_j/B}+y_{R_j/B}\dot{z}_{R_j/B}\right)}{\partial \dot{\delta}_{HT}} & 2\frac{\partial\left(x_{R_j/B}\dot{x}_{R_j/B}+y_{R_j/B}\dot{y}_{R_j/B}\right)}{\partial \dot{\delta}_{HT}} \end{bmatrix}$$

18.34 Partial derivatives of terms in $\frac{\boldsymbol{\partial \dot{\tilde{R}}_{W_i}^{B}}}{\boldsymbol{\partial\ \delta_W}}$ and $\frac{\boldsymbol{\partial \dot{\tilde{R}}_{R_j}^{B}}}{\boldsymbol{\partial\ \delta_W}}$:

$$2\frac{\partial\left(y_{W_i/B}\dot{y}_{W_i/B}+z_{W_i/B}\dot{z}_{W_i/B}\right)}{\partial \delta_W} = 2\left(\frac{\partial y_{W_i/B}}{\partial \delta_W}\dot{y}_{W_i/B} + y_{W_i/B}\frac{\partial \dot{y}_{W_i/B}}{\partial \delta_W} + \frac{\partial z_{W_i/B}}{\partial \delta_W}\dot{z}_{W_i/B} + z_{W_i/B}\frac{\partial \dot{z}_{W_i/B}}{\partial \delta_W}\right)$$

$$2\frac{\partial\left(x_{W_i/B}\dot{x}_{W_i/B}+z_{W_i/B}\dot{z}_{W_i/B}\right)}{\partial \delta_W} = 2\left(\frac{\partial x_{W_i/B}}{\partial \delta_W}\dot{x}_{W_i/B} + x_{W_i/B}\frac{\partial \dot{x}_{W_i/B}}{\partial \delta_W} + \frac{\partial z_{W_i/B}}{\partial \delta_W}\dot{z}_{W_i/B} + z_{W_i/B}\frac{\partial \dot{z}_{W_i/B}}{\partial \delta_W}\right)$$

$$2\frac{\partial\left(x_{W_i/B}\dot{x}_{W_i/B}+y_{W_i/B}\dot{y}_{W_i/B}\right)}{\partial \delta_W} = 2\left(\frac{\partial x_{W_i/B}}{\partial \delta_W}\dot{x}_{W_i/B} + x_{W_i/B}\frac{\partial \dot{x}_{W_i/B}}{\partial \delta_W} + \frac{\partial y_{W_i/B}}{\partial \delta_W}\dot{y}_{W_i/B} + y_{W_i/B}\frac{\partial \dot{y}_{W_i/B}}{\partial \delta_W}\right)$$

$$\frac{\partial\left(\dot{x}_{W_i/B}y_{W_i/B}+x_{W_i/B}\dot{y}_{W_i/B}\right)}{\partial \delta_W} = \frac{\partial \dot{x}_{W_i/B}}{\partial \delta_W}y_{W_i/B} + \dot{x}_{W_i/B}\frac{\partial y_{W_i/B}}{\partial \delta_W} + \frac{\partial x_{W_i/B}}{\partial \delta_W}\dot{y}_{W_i/B} + x_{W_i/B}\frac{\partial \dot{y}_{W_i/B}}{\partial \delta_W}$$

$$\frac{\partial\left(\dot{z}_{W_i/B}x_{W_i/B}+z_{W_i/B}\dot{x}_{W_i/B}\right)}{\partial \delta_W} = \frac{\partial \dot{z}_{W_i/B}}{\partial \delta_W}x_{W_i/B} + \dot{z}_{W_i/B}\frac{\partial x_{W_i/B}}{\partial \delta_W} + \frac{\partial z_{W_i/B}}{\partial \delta_W}\dot{x}_{W_i/B} + z_{W_i/B}\frac{\partial \dot{x}_{W_i/B}}{\partial \delta_W}$$

$$\frac{\partial\left(\dot{y}_{W_i/B}z_{W_i/B}+y_{W_i/B}\dot{z}_{W_i/B}\right)}{\partial\delta_W}=\frac{\partial\dot{y}_{W_i/B}}{\partial\delta_W}z_{W_i/B}+\dot{y}_{W_i/B}\frac{\partial z_{W_i/B}}{\partial\delta_W}+\frac{\partial y_{W_i/B}}{\partial\delta_W}\dot{z}_{W_i/B}+y_{W_i/B}\frac{\partial\dot{z}_{W_i/B}}{\partial\delta_W}$$

$$2\frac{\partial\left(y_{R_j/B}\dot{y}_{R_j/B}+z_{R_j/B}\dot{z}_{R_j/B}\right)}{\partial\delta_W}=2\left(\frac{\partial y_{R_j/B}}{\partial\delta_W}\dot{y}_{R_j/B}+y_{R_j/B}\frac{\partial\dot{y}_{R_j/B}}{\partial\delta_W}+\frac{\partial z_{R_j/B}}{\partial\delta_W}\dot{z}_{R_j/B}+z_{R_j/B}\frac{\partial\dot{z}_{R_j/B}}{\partial\delta_W}\right)$$

$$2\frac{\partial\left(x_{R_j/B}\dot{x}_{R_j/B}+z_{R_j/B}\dot{z}_{R_j/B}\right)}{\partial\delta_W}=2\left(\frac{\partial x_{R_j/B}}{\partial\delta_W}\dot{x}_{R_j/B}+x_{R_j/B}\frac{\partial\dot{x}_{R_j/B}}{\partial\delta_W}+\frac{\partial z_{R_j/B}}{\partial\delta_W}\dot{z}_{R_j/B}+z_{R_j/B}\frac{\partial\dot{z}_{R_j/B}}{\partial\delta_W}\right)$$

$$2\frac{\partial\left(x_{R_j/B}\dot{x}_{R_j/B}+y_{R_j/B}\dot{y}_{R_j/B}\right)}{\partial\delta_W}=2\left(\frac{\partial x_{R_j/B}}{\partial\delta_W}\dot{x}_{R_j/B}+x_{R_j/B}\frac{\partial\dot{x}_{R_j/B}}{\partial\delta_W}+\frac{\partial y_{R_j/B}}{\partial\delta_W}\dot{y}_{R_j/B}+y_{R_j/B}\frac{\partial\dot{y}_{R_j/B}}{\partial\delta_W}\right)$$

$$\frac{\partial\left(\dot{x}_{R_j/B}y_{R_j/B}+x_{R_j/B}\dot{y}_{R_j/B}\right)}{\partial\delta_W}=\frac{\partial\dot{x}_{R_j/B}}{\partial\delta_W}y_{R_j/B}+\dot{x}_{R_j/B}\frac{\partial y_{R_j/B}}{\partial\delta_W}+\frac{\partial x_{R_j/B}}{\partial\delta_W}\dot{y}_{R_j/B}+x_{R_j/B}\frac{\partial\dot{y}_{R_j/B}}{\partial\delta_W}$$

$$\frac{\partial\left(\dot{x}_{R_j/B}z_{R_j/B}+x_{R_j/B}\dot{z}_{R_j/B}\right)}{\partial\delta_W}=\frac{\partial\dot{x}_{R_j/B}}{\partial\delta_W}z_{R_j/B}+\dot{x}_{R_j/B}\frac{\partial z_{R_j/B}}{\partial\delta_W}+\frac{\partial x_{R_j/B}}{\partial\delta_W}\dot{z}_{R_j/B}+x_{R_j/B}\frac{\partial\dot{z}_{R_j/B}}{\partial\delta_W}$$

$$\frac{\partial\left(\dot{y}_{R_j/B}z_{R_j/B}+y_{R_j/B}\dot{z}_{R_j/B}\right)}{\partial\delta_W}=\frac{\partial\dot{y}_{R_j/B}}{\partial\delta_W}z_{R_j/B}+\dot{y}_{R_j/B}\frac{\partial z_{R_j/B}}{\partial\delta_W}+\frac{\partial y_{R_j/B}}{\partial\delta_W}\dot{z}_{R_j/B}+y_{R_j/B}\frac{\partial\dot{z}_{R_j/B}}{\partial\delta_W}$$

18.35 Partial derivatives of terms in $\frac{\partial\dot{\tilde{R}}_{W_i}^{B}}{\partial\,\delta_{HT}}$ and $\frac{\partial\dot{\tilde{R}}_{R_j}^{B}}{\partial\,\delta_{HT}}$:

$$2\frac{\partial\left(y_{W_i/B}\dot{y}_{W_i/B}+z_{W_i/B}\dot{z}_{W_i/B}\right)}{\partial\delta_{HT}}=2\left(\frac{\partial y_{W_i/B}}{\partial\delta_{HT}}\dot{y}_{W_i/B}+y_{W_i/B}\frac{\partial\dot{y}_{W_i/B}}{\partial\delta_{HT}}+\frac{\partial z_{W_i/B}}{\partial\delta_{HT}}\dot{z}_{W_i/B}+z_{W_i/B}\frac{\partial\dot{z}_{W_i/B}}{\partial\delta_{HT}}\right)$$

$$2\frac{\partial\left(x_{W_i/B}\dot{x}_{W_i/B}+z_{W_i/B}\dot{z}_{W_i/B}\right)}{\partial\delta_{HT}}=2\left(\frac{\partial x_{W_i/B}}{\partial\delta_{HT}}\dot{x}_{W_i/B}+x_{W_i/B}\frac{\partial\dot{x}_{W_i/B}}{\partial\delta_{HT}}+\frac{\partial z_{W_i/B}}{\partial\delta_{HT}}\dot{z}_{W_i/B}+z_{W_i/B}\frac{\partial\dot{z}_{W_i/B}}{\partial\delta_{HT}}\right)$$

$$2\frac{\partial\left(x_{W_i/B}\dot{x}_{W_i/B}+y_{W_i/B}\dot{y}_{W_i/B}\right)}{\partial\delta_{HT}}=2\left(\frac{\partial x_{W_i/B}}{\partial\delta_{HT}}\dot{x}_{W_i/B}+x_{W_i/B}\frac{\partial\dot{x}_{W_i/B}}{\partial\delta_{HT}}+\frac{\partial y_{W_i/B}}{\partial\delta_{HT}}\dot{y}_{W_i/B}+y_{W_i/B}\frac{\partial\dot{y}_{W_i/B}}{\partial\delta_{HT}}\right)$$

$$\frac{\partial\left(\dot{x}_{W_i/B}y_{W_i/B}+x_{W_i/B}\dot{y}_{W_i/B}\right)}{\partial\delta_{HT}}=\frac{\partial\dot{x}_{W_i/B}}{\partial\delta_{HT}}y_{W_i/B}+\dot{x}_{W_i/B}\frac{\partial y_{W_i/B}}{\partial\delta_{HT}}+\frac{\partial x_{W_i/B}}{\partial\delta_{HT}}\dot{y}_{W_i/B}+x_{W_i/B}\frac{\partial\dot{y}_{W_i/B}}{\partial\delta_{HT}}$$

$$\frac{\partial\left(\dot{z}_{W_i/B}x_{W_i/B}+z_{W_i/B}\dot{x}_{W_i/B}\right)}{\partial\delta_{HT}}=\frac{\partial\dot{z}_{W_i/B}}{\partial\delta_{HT}}x_{W_i/B}+\dot{z}_{W_i/B}\frac{\partial x_{W_i/B}}{\partial\delta_{HT}}+\frac{\partial z_{W_i/B}}{\partial\delta_{HT}}\dot{x}_{W_i/B}+z_{W_i/B}\frac{\partial\dot{x}_{W_i/B}}{\partial\delta_{HT}}$$

$$\frac{\partial\left(\dot{y}_{W_i/B}z_{W_i/B}+y_{W_i/B}\dot{z}_{W_i/B}\right)}{\partial\delta_{HT}}=\frac{\partial\dot{y}_{W_i/B}}{\partial\delta_{HT}}z_{W_i/B}+\dot{y}_{W_i/B}\frac{\partial z_{W_i/B}}{\partial\delta_{HT}}+\frac{\partial y_{W_i/B}}{\partial\delta_{HT}}\dot{z}_{W_i/B}+y_{W_i/B}\frac{\partial\dot{z}_{W_i/B}}{\partial\delta_{HT}}$$

$$2\frac{\partial\left(y_{R_j/B}\dot{y}_{R_j/B}+z_{R_j/B}\dot{z}_{R_j/B}\right)}{\partial\delta_{HT}}=2\left(\frac{\partial y_{R_j/B}}{\partial\delta_{HT}}\dot{y}_{R_j/B}+y_{R_j/B}\frac{\partial\dot{y}_{R_j/B}}{\partial\delta_{HT}}+\frac{\partial z_{R_j/B}}{\partial\delta_{HT}}\dot{z}_{R_j/B}+z_{R_j/B}\frac{\partial\dot{z}_{R_j/B}}{\partial\delta_{HT}}\right)$$

$$2\frac{\partial\left(x_{R_j/B}\dot{x}_{R_j/B}+z_{R_j/B}\dot{z}_{R_j/B}\right)}{\partial\delta_{HT}}=2\left(\frac{\partial x_{R_j/B}}{\partial\delta_{HT}}\dot{x}_{R_j/B}+x_{R_j/B}\frac{\partial\dot{x}_{R_j/B}}{\partial\delta_{HT}}+\frac{\partial z_{R_j/B}}{\partial\delta_{HT}}\dot{z}_{R_j/B}+z_{R_j/B}\frac{\partial\dot{z}_{R_j/B}}{\partial\delta_{HT}}\right)$$

$$2\frac{\partial\left(x_{R_j/B}\dot{x}_{R_j/B}+y_{R_j/B}\dot{y}_{R_j/B}\right)}{\partial\delta_{HT}}=2\left(\frac{\partial x_{R_j/B}}{\partial\delta_{HT}}\dot{x}_{R_j/B}+x_{R_j/B}\frac{\partial\dot{x}_{R_j/B}}{\partial\delta_{HT}}+\frac{\partial y_{R_j/B}}{\partial\delta_{HT}}\dot{y}_{R_j/B}+y_{R_j/B}\frac{\partial\dot{y}_{R_j/B}}{\partial\delta_{HT}}\right)$$

$$\frac{\partial\left(\dot{x}_{R_j/B}y_{R_j/B}+x_{R_j/B}\dot{y}_{R_j/B}\right)}{\partial\delta_{HT}}=\frac{\partial\dot{x}_{R_j/B}}{\partial\delta_{HT}}y_{R_j/B}+\dot{x}_{R_j/B}\frac{\partial y_{R_j/B}}{\partial\delta_{HT}}+\frac{\partial x_{R_j/B}}{\partial\delta_{HT}}\dot{y}_{R_j/B}+x_{R_j/B}\frac{\partial\dot{y}_{R_j/B}}{\partial\delta_{HT}}$$

$$\frac{\partial\left(\dot{x}_{R_j/B}z_{R_j/B}+x_{R_j/B}\dot{z}_{R_j/B}\right)}{\partial\delta_{HT}}=\frac{\partial\dot{x}_{R_j/B}}{\partial\delta_{HT}}z_{R_j/B}+\dot{x}_{R_j/B}\frac{\partial z_{R_j/B}}{\partial\delta_{HT}}+\frac{\partial x_{R_j/B}}{\partial\delta_{HT}}\dot{z}_{R_j/B}+x_{R_j/B}\frac{\partial\dot{z}_{R_j/B}}{\partial\delta_{HT}}$$

$$\frac{\partial\left(\dot{y}_{R_j/B}z_{R_j/B}+y_{R_j/B}\dot{z}_{R_j/B}\right)}{\partial\delta_{HT}}=\frac{\partial\dot{y}_{R_j/B}}{\partial\delta_{HT}}z_{R_j/B}+\dot{y}_{R_j/B}\frac{\partial z_{R_j/B}}{\partial\delta_{HT}}+\frac{\partial y_{R_j/B}}{\partial\delta_{HT}}\dot{z}_{R_j/B}+y_{R_j/B}\frac{\partial\dot{z}_{R_j/B}}{\partial\delta_{HT}}$$

18.36 Partial derivatives of terms in $\frac{\partial\dot{\vec{R}}_{W_i}^{B}}{\partial\,\dot{\delta}_W}$ and $\frac{\partial\dot{\vec{R}}_{R_j}^{B}}{\partial\,\dot{\delta}_W}$:

$$2\frac{\partial\left(y_{W_i/B}\dot{y}_{W_i/B}+z_{W_i/B}\dot{z}_{W_i/B}\right)}{\partial\dot{\delta}_W}=2\left(y_{W_i/B}\frac{\partial\dot{y}_{W_i/B}}{\partial\dot{\delta}_W}+z_{W_i/B}\frac{\partial\dot{z}_{W_i/B}}{\partial\dot{\delta}_W}\right)$$

$$2\frac{\partial\left(x_{W_i/B}\dot{x}_{W_i/B}+z_{W_i/B}\dot{z}_{W_i/B}\right)}{\partial\dot{\delta}_W}=2\left(x_{W_i/B}\frac{\partial\dot{x}_{W_i/B}}{\partial\dot{\delta}_W}+z_{W_i/B}\frac{\partial\dot{z}_{W_i/B}}{\partial\dot{\delta}_W}\right)$$

$$2\frac{\partial\left(x_{W_i/B}\dot{x}_{W_i/B}+y_{W_i/B}\dot{y}_{W_i/B}\right)}{\partial\dot{\delta}_W}=2\left(x_{W_i/B}\frac{\partial\dot{x}_{W_i/B}}{\partial\dot{\delta}_W}+y_{W_i/B}\frac{\partial\dot{y}_{W_i/B}}{\partial\dot{\delta}_W}\right)$$

$$\frac{\partial\left(\dot{x}_{W_i/B}y_{W_i/B}+x_{W_i/B}\dot{y}_{W_i/B}\right)}{\partial\dot{\delta}_W}=\frac{\partial\dot{x}_{W_i/B}}{\partial\dot{\delta}_W}y_{W_i/B}+x_{W_i/B}\frac{\partial\dot{y}_{W_i/B}}{\partial\dot{\delta}_W}$$

$$\frac{\partial\left(\dot{z}_{W_i/B}x_{W_i/B}+z_{W_i/B}\dot{x}_{W_i/B}\right)}{\partial\dot{\delta}_W}=\frac{\partial\dot{z}_{W_i/B}}{\partial\dot{\delta}_W}x_{W_i/B}+z_{W_i/B}\frac{\partial\dot{x}_{W_i/B}}{\partial\dot{\delta}_W}$$

$$\frac{\partial\left(\dot{y}_{W_i/B}z_{W_i/B}+y_{W_i/B}\dot{z}_{W_i/B}\right)}{\partial\dot{\delta}_W}=\frac{\partial\dot{y}_{W_i/B}}{\partial\dot{\delta}_W}z_{W_i/B}+y_{W_i/B}\frac{\partial\dot{z}_{W_i/B}}{\partial\dot{\delta}_W}$$

$$2\frac{\partial\left(y_{R_j/B}\dot{y}_{R_j/B}+z_{R_j/B}\dot{z}_{R_j/B}\right)}{\partial\dot{\delta}_W}=2\left(y_{R_j/B}\frac{\partial\dot{y}_{R_j/B}}{\partial\dot{\delta}_W}+z_{R_j/B}\frac{\partial\dot{z}_{R_j/B}}{\partial\dot{\delta}_W}\right)$$

$$2\frac{\partial\left(x_{R_j/B}\dot{x}_{R_j/B}+z_{R_j/B}\dot{z}_{R_j/B}\right)}{\partial\dot{\delta}_W}=2\left(x_{R_j/B}\frac{\partial\dot{x}_{R_j/B}}{\partial\dot{\delta}_W}+z_{R_j/B}\frac{\partial\dot{z}_{R_j/B}}{\partial\dot{\delta}_W}\right)$$

$$2\frac{\partial\left(x_{R_j/B}\dot{x}_{R_j/B}+y_{R_j/B}\dot{y}_{R_j/B}\right)}{\partial\dot{\delta}_W}=2\left(x_{R_j/B}\frac{\partial\dot{x}_{R_j/B}}{\partial\dot{\delta}_W}+y_{R_j/B}\frac{\partial\dot{y}_{R_j/B}}{\partial\dot{\delta}_W}\right)$$

$$\frac{\partial\left(\dot{x}_{R_j/B}y_{R_j/B}+x_{R_j/B}\dot{y}_{R_j/B}\right)}{\partial\dot{\delta}_W}=\frac{\partial\dot{x}_{R_j/B}}{\partial\dot{\delta}_W}y_{R_j/B}+x_{R_j/B}\frac{\partial\dot{y}_{R_j/B}}{\partial\dot{\delta}_W}$$

$$\frac{\partial\left(\dot{x}_{R_j/B}z_{R_j/B}+x_{R_j/B}\dot{z}_{R_j/B}\right)}{\partial\dot{\delta}_W}=\frac{\partial\dot{x}_{R_j/B}}{\partial\dot{\delta}_W}z_{R_j/B}+x_{R_j/B}\frac{\partial\dot{z}_{R_j/B}}{\partial\dot{\delta}_W}$$

$$\frac{\partial\left(\dot{y}_{R_j/B}z_{R_j/B}+y_{R_j/B}\dot{z}_{R_j/B}\right)}{\partial\dot{\delta}_W}=\frac{\partial\dot{y}_{R_j/B}}{\partial\dot{\delta}_W}z_{R_j/B}+y_{R_j/B}\frac{\partial\dot{z}_{R_j/B}}{\partial\dot{\delta}_W}$$

18.37 Partial derivatives of terms in $\frac{\partial \dot{\tilde{R}}^{B}_{W_i}}{\partial\, \dot{\delta}_{HT}}$ and $\frac{\partial \dot{\tilde{R}}^{B}_{R_j}}{\partial\, \dot{\delta}_{HT}}$:

$$2\frac{\partial\left(y_{W_i/B}\dot{y}_{W_i/B}+z_{W_i/B}\dot{z}_{W_i/B}\right)}{\partial\dot{\delta}_{HT}}=2\left(y_{W_i/B}\frac{\partial\dot{y}_{W_i/B}}{\partial\dot{\delta}_{HT}}+z_{W_i/B}\frac{\partial\dot{z}_{W_i/B}}{\partial\dot{\delta}_{HT}}\right)$$

$$2\frac{\partial\left(x_{W_i/B}\dot{x}_{W_i/B}+z_{W_i/B}\dot{z}_{W_i/B}\right)}{\partial\dot{\delta}_{HT}}=2\left(x_{W_i/B}\frac{\partial\dot{x}_{W_i/B}}{\partial\dot{\delta}_{HT}}+z_{W_i/B}\frac{\partial\dot{z}_{W_i/B}}{\partial\dot{\delta}_{HT}}\right)$$

$$2\frac{\partial\left(x_{W_i/B}\dot{x}_{W_i/B}+y_{W_i/B}\dot{y}_{W_i/B}\right)}{\partial\dot{\delta}_{HT}}=2\left(x_{W_i/B}\frac{\partial\dot{x}_{W_i/B}}{\partial\dot{\delta}_{HT}}+y_{W_i/B}\frac{\partial\dot{y}_{W_i/B}}{\partial\dot{\delta}_{HT}}\right)$$

$$\frac{\partial\left(\dot{x}_{W_i/B}y_{W_i/B}+x_{W_i/B}\dot{y}_{W_i/B}\right)}{\partial\dot{\delta}_{HT}}=\frac{\partial\dot{x}_{W_i/B}}{\partial\dot{\delta}_{HT}}y_{W_i/B}+x_{W_i/B}\frac{\partial\dot{y}_{W_i/B}}{\partial\dot{\delta}_{HT}}$$

$$\frac{\partial\left(\dot{z}_{W_i/B}x_{W_i/B}+z_{W_i/B}\dot{x}_{W_i/B}\right)}{\partial\dot{\delta}_{HT}}=\frac{\partial\dot{z}_{W_i/B}}{\partial\dot{\delta}_{HT}}x_{W_i/B}+z_{W_i/B}\frac{\partial\dot{x}_{W_i/B}}{\partial\dot{\delta}_{HT}}$$

$$\frac{\partial\left(\dot{y}_{W_i/B}z_{W_i/B}+y_{W_i/B}\dot{z}_{W_i/B}\right)}{\partial\dot{\delta}_{HT}}=\frac{\partial\dot{y}_{W_i/B}}{\partial\dot{\delta}_{HT}}z_{W_i/B}+y_{W_i/B}\frac{\partial\dot{z}_{W_i/B}}{\partial\dot{\delta}_{HT}}$$

$$2\frac{\partial\left(y_{R_j/B}\dot{y}_{R_j/B}+z_{R_j/B}\dot{z}_{R_j/B}\right)}{\partial\dot{\delta}_{HT}}=2\left(y_{R_j/B}\frac{\partial\dot{y}_{R_j/B}}{\partial\dot{\delta}_{HT}}+z_{R_j/B}\frac{\partial\dot{z}_{R_j/B}}{\partial\dot{\delta}_{HT}}\right)$$

$$2\frac{\partial\left(x_{R_j/B}\dot{x}_{R_j/B}+z_{R_j/B}\dot{z}_{R_j/B}\right)}{\partial\dot{\delta}_{HT}}=2\left(x_{R_j/B}\frac{\partial\dot{x}_{R_j/B}}{\partial\dot{\delta}_{HT}}+z_{R_j/B}\frac{\partial\dot{z}_{R_j/B}}{\partial\dot{\delta}_{HT}}\right)$$

$$2\frac{\partial\left(x_{R_j/B}\dot{x}_{R_j/B}+y_{R_j/B}\dot{y}_{R_j/B}\right)}{\partial\dot{\delta}_{HT}}=2\left(x_{R_j/B}\frac{\partial\dot{x}_{R_j/B}}{\partial\dot{\delta}_{HT}}+y_{R_j/B}\frac{\partial\dot{y}_{R_j/B}}{\partial\dot{\delta}_{HT}}\right)$$

$$\frac{\partial\left(\dot{x}_{R_j/B}y_{R_j/B}+x_{R_j/B}\dot{y}_{R_j/B}\right)}{\partial\dot{\delta}_{HT}}=\frac{\partial\dot{x}_{R_j/B}}{\partial\dot{\delta}_{HT}}y_{R_j/B}+x_{R_j/B}\frac{\partial\dot{y}_{R_j/B}}{\partial\dot{\delta}_{HT}}$$

$$\frac{\partial\left(\dot{x}_{R_j/B}z_{R_j/B}+x_{R_j/B}\dot{z}_{R_j/B}\right)}{\partial\dot{\delta}_{HT}}=\frac{\partial\dot{x}_{R_j/B}}{\partial\dot{\delta}_{HT}}z_{R_j/B}+x_{R_j/B}\frac{\partial\dot{z}_{R_j/B}}{\partial\dot{\delta}_{HT}}$$

$$\frac{\partial\left(\dot{y}_{R_j/B}z_{R_j/B}+y_{R_j/B}\dot{z}_{R_j/B}\right)}{\partial\dot{\delta}_{HT}}=\frac{\partial\dot{y}_{R_j/B}}{\partial\dot{\delta}_{HT}}z_{R_j/B}+y_{R_j/B}\frac{\partial\dot{z}_{R_j/B}}{\partial\dot{\delta}_{HT}}$$

18.38 Time derivatives of $\vec{r}_{W_i/B}$ and $\vec{r}_{R_j/B}$:

$$\frac{d}{dt}\left(\vec{r}_{W_i/B}\right)=\begin{Bmatrix}\dot{x}_{W_i/B}\\ \dot{y}_{W_i/B}\\ \dot{z}_{W_i/B}\end{Bmatrix}=\dot{R}^{B}_{W_i}\vec{r}_{W_i/pivot_i}=\begin{Bmatrix}-\dot{\delta}_{W,HT_i}\sin\left(\delta_{W,HT_i}\right)x_{W_i/pivot_i}+\dot{\delta}_{W,HT_i}\cos\left(\delta_{W,HT_i}\right)z_{W_i/pivot_i}\\ 0\\ -\dot{\delta}_{W,HT_i}\cos\left(\delta_{W,HT_i}\right)x_{W_i/pivot_i}-\dot{\delta}_{W,HT_i}\sin\left(\delta_{W,HT_i}\right)z_{W_i/pivot_i}\end{Bmatrix}$$

$$\frac{d}{dt}\left(\vec{r}_{R_j/B}\right)=\begin{Bmatrix}\dot{x}_{R_j/B}\\ \dot{y}_{R_j/B}\\ \dot{z}_{R_j/B}\end{Bmatrix}=\dot{R}^{B}_{R_j}\vec{r}_{R_j/pivot_j}=\begin{Bmatrix}-\dot{\delta}_{W,HT_j}\sin\left(\delta_{W,HT_j}\right)x_{R_j/pivot_j}+\dot{\delta}_{W,HT_j}\cos\left(\delta_{W,HT_j}\right)z_{R_j/pivot_j}\\ 0\\ -\dot{\delta}_{W,HT_j}\cos\left(\delta_{W,HT_j}\right)x_{R_j/pivot_j}-\dot{\delta}_{W,HT_j}\sin\left(\delta_{W,HT_j}\right)z_{R_j/pivot_j}\end{Bmatrix}$$

18.39 Partial derivatives of $\dot{x}_{W_i/B}$, $\dot{y}_{W_i/B}$, $\dot{z}_{W_i/B}$, $\dot{x}_{R_j/B}$, $\dot{y}_{R_j/B}$, $\dot{z}_{R_j/B}$:

$$\frac{\partial \dot{x}_{W_i/B}}{\partial \delta_W} = -\dot{\delta}_{W,HT_i}\left(\cos\delta_W\, x_{W_i/pivot_i} - \sin\delta_W\, z_{W_i/pivot_i}\right) \ or\ 0$$

$$\frac{\partial \dot{x}_{R_j/B}}{\partial \delta_W} = -\dot{\delta}_{W,HT_j}\left(\cos\delta_W\, x_{R_j/pivot_j} - \sin\delta_W\, z_{R_j/pivot_j}\right) \ or\ 0$$

$$\frac{\partial \dot{y}_{W_i/B}}{\partial \delta_W} = 0 \quad and \quad \frac{\partial \dot{y}_{R_j/B}}{\partial \delta_W} = 0$$

$$\frac{\partial \dot{z}_{W_i/B}}{\partial \delta_W} = \dot{\delta}_{W,HT_i}\left(\sin\delta_W\, x_{W_i/pivot_i} + \cos\delta_W\, z_{W_i/pivot_i}\right) \ or\ 0$$

$$\frac{\partial \dot{z}_{R_j/B}}{\partial \delta_W} = \dot{\delta}_{W,HT_j}\left(\sin\delta_W\, x_{R_j/pivot_j} + \cos\delta_W\, z_{R_j/pivot_j}\right) \ or\ 0$$

$$\frac{\partial \dot{x}_{W_i/B}}{\partial \delta_{HT}} = -\dot{\delta}_{W,HT_i}\left(\cos\delta_{HT}\, x_{W_i/pivot_i} - \sin\delta_{HT}\, z_{W_i/pivot_i}\right) \ or\ 0$$

$$\frac{\partial \dot{x}_{R_j/B}}{\partial \delta_{HT}} = -\dot{\delta}_{W,HT_j}\left(\cos\delta_{HT}\, x_{R_j/pivot_j} - \sin\delta_{HT}\, z_{R_j/pivot_j}\right) \ or\ 0$$

$$\frac{\partial \dot{y}_{W_i/B}}{\partial \delta_{HT}} = 0 \quad and \quad \frac{\partial \dot{y}_{R_j/B}}{\partial \delta_{HT}} = 0$$

$$\frac{\partial \dot{z}_{W_i/B}}{\partial \delta_{HT}} = \dot{\delta}_{W,HT_i}\left(\sin\delta_{HT}\, x_{W_i/pivot_i} + \cos\delta_{HT}\, z_{W_i/pivot_i}\right) \ or\ 0$$

$$\frac{\partial \dot{z}_{R_j/B}}{\partial \delta_{HT}} = \dot{\delta}_{W,HT_j}\left(\sin\delta_{HT}\, x_{R_j/pivot_j} + \cos\delta_{HT}\, z_{R_j/pivot_j}\right) \ or\ 0$$

$$\frac{\partial \dot{x}_{W_i/B}}{\partial \dot{\delta}_W} = -\sin\delta_W\, x_{W_i/pivot_i} + \cos\delta_W\, z_{W_i/pivot_i} \ or\ 0$$

$$\frac{\partial \dot{x}_{R_j/B}}{\partial \dot{\delta}_W} = -\sin\delta_W\, x_{R_j/pivot_j} + \cos\delta_W\, z_{R_j/pivot_j} \ or\ 0$$

$$\frac{\partial \dot{y}_{W_i/B}}{\partial \dot{\delta}_W} = 0 \quad and \quad \frac{\partial \dot{y}_{R_j/B}}{\partial \dot{\delta}_W} = 0$$

$$\frac{\partial \dot{z}_{W_i/B}}{\partial \dot{\delta}_W} = -\cos\delta_W\, x_{W_i/pivot_i} - \sin\delta_W\, z_{W_i/pivot_i} \ or\ 0$$

$$\frac{\partial \dot{z}_{R_j/B}}{\partial \dot{\delta}_W} = -\cos\delta_W\, x_{R_j/pivot_j} - \sin\delta_W\, z_{R_j/pivot_j} \ or\ 0$$

$$\frac{\partial \dot{x}_{W_i/B}}{\partial \dot{\delta}_{HT}} = -\sin\delta_{HT}\, x_{W_i/pivot_i} + \cos\delta_{HT}\, z_{W_i/pivot_i} \ or\ 0$$

$$\frac{\partial \dot{x}_{R_j/B}}{\partial \dot{\delta}_{HT}} = -\sin\delta_{HT}\, x_{R_j/pivot_j} + \cos\delta_{HT}\, z_{R_j/pivot_j} \ or\ 0$$

$$\frac{\partial \dot{y}_{W_i/B}}{\partial \dot{\delta}_{HT}} = 0 \quad and \quad \frac{\partial \dot{y}_{R_j/B}}{\partial \dot{\delta}_{HT}} = 0$$

$$\frac{\partial \dot{z}_{W_i/B}}{\partial \dot{\delta}_{HT}} = -\cos\delta_{HT}\, x_{W_i/pivot_i} - \sin\delta_{HT}\, z_{W_i/pivot_i}\ or\ 0$$

$$\frac{\partial \dot{z}_{R_j/B}}{\partial \dot{\delta}_{HT}} = -\cos\delta_{HT}\, x_{R_j/pivot_j} - \sin\delta_{HT}\, z_{R_j/pivot_j}\ or\ 0$$

18.40 Partial derivatives of $\left([\dot{\boldsymbol{T}}]_{\boldsymbol{W_i}}\left(\tilde{\boldsymbol{I}}_{\boldsymbol{W_i}}^{\boldsymbol{W_i}} + \boldsymbol{m}_{\boldsymbol{W_i}}\tilde{\boldsymbol{R}}_{\boldsymbol{W_i}}^{\boldsymbol{B}}\right)[\boldsymbol{T}]_{\boldsymbol{W_i}}^{\boldsymbol{T}}\right)$ and $\left([\dot{\boldsymbol{T}}]_{\boldsymbol{R_j}}\left(\tilde{\boldsymbol{I}}_{\boldsymbol{R_j}}^{\boldsymbol{R_j}} + \boldsymbol{m}_{\boldsymbol{R_j}}\tilde{\boldsymbol{R}}_{\boldsymbol{R_j}}^{\boldsymbol{B}}\right)[\boldsymbol{T}]_{\boldsymbol{R_j}}^{\boldsymbol{T}}\right)$:

$$\frac{\partial\left[[\dot{T}]_{W_i}\left(\tilde{I}_{W_i}^{W_i} + m_{W_i}\tilde{R}_{W_i}^B\right)[T]_{W_i}^T\right]}{\partial\delta_W} = \frac{\partial[\dot{T}]_{W_i}}{\partial\delta_W}\left(\tilde{I}_{W_i}^{W_i} + m_{W_i}\tilde{R}_{W_i}^B\right)[T]_{W_i}^T + m_{W_i}[\dot{T}]_{W_i}\frac{\partial\tilde{R}_{W_i}^B}{\partial\delta_W}[T]_{W_i}^T + [\dot{T}]_{W_i}\left(\tilde{I}_{W_i}^{W_i} + m_{W_i}\tilde{R}_{W_i}^B\right)\frac{\partial[T]_{W_i}^T}{\partial\delta_W}$$

$$\frac{\partial\left[[\dot{T}]_{R_j}\left(\tilde{I}_{R_j}^{R_j} + m_{R_j}\tilde{R}_{R_j}^B\right)[T]_{R_j}^T\right]}{\partial\delta_W} = \frac{\partial[\dot{T}]_{R_j}}{\partial\delta_W}\left(\tilde{I}_{R_j}^{R_j} + m_{R_j}\tilde{R}_{R_j}^B\right)[T]_{R_j}^T + m_{R_j}[\dot{T}]_{R_j}\frac{\partial\tilde{R}_{R_j}^B}{\partial\delta_W}[T]_{R_j}^T + [\dot{T}]_{R_j}\left(\tilde{I}_{R_j}^{R_j} + m_{R_j}\tilde{R}_{R_j}^B\right)\frac{\partial[T]_{R_j}^T}{\partial\delta_W}$$

$$\frac{\partial\left[[\dot{T}]_{W_i}\left(\tilde{I}_{W_i}^{W_i} + m_{W_i}\tilde{R}_{W_i}^B\right)[T]_{W_i}^T\right]}{\partial\delta_{HT}} = \frac{\partial[\dot{T}]_{W_i}}{\partial\delta_{HT}}\left(\tilde{I}_{W_i}^{W_i} + m_{W_i}\tilde{R}_{W_i}^B\right)[T]_{W_i}^T + m_{W_i}[\dot{T}]_{W_i}\frac{\partial\tilde{R}_{W_i}^B}{\partial\delta_{HT}}[T]_{W_i}^T + [\dot{T}]_{W_i}\left(\tilde{I}_{W_i}^{W_i} + m_{W_i}\tilde{R}_{W_i}^B\right)\frac{\partial[T]_{W_i}^T}{\partial\delta_{HT}}$$

$$\frac{\partial\left[[\dot{T}]_{R_j}\left(\tilde{I}_{R_j}^{R_j} + m_{R_j}\tilde{R}_{R_j}^B\right)[T]_{R_j}^T\right]}{\partial\delta_{HT}} = \frac{\partial[\dot{T}]_{R_j}}{\partial\delta_{HT}}\left(\tilde{I}_{R_j}^{R_j} + m_{R_j}\tilde{R}_{R_j}^B\right)[T]_{R_j}^T + m_{R_j}[\dot{T}]_{R_j}\frac{\partial\tilde{R}_{R_j}^B}{\partial\delta_{HT}}[T]_{R_j}^T + [\dot{T}]_{R_j}\left(\tilde{I}_{R_j}^{R_j} + m_{R_j}\tilde{R}_{R_j}^B\right)\frac{\partial[T]_{R_j}^T}{\partial\delta_{HT}}$$

18.41 Partial derivatives of $\left([\boldsymbol{T}]_{\boldsymbol{W_i}}\boldsymbol{m}_{\boldsymbol{W_i}}\dot{\tilde{\boldsymbol{R}}}_{\boldsymbol{W_i}}^{\boldsymbol{B}}[\boldsymbol{T}]_{\boldsymbol{W_i}}^{\boldsymbol{T}}\right)$ and $\left([\boldsymbol{T}]_{\boldsymbol{R_j}}\boldsymbol{m}_{\boldsymbol{R_j}}\dot{\tilde{\boldsymbol{R}}}_{\boldsymbol{R_j}}^{\boldsymbol{B}}[\boldsymbol{T}]_{\boldsymbol{R_j}}^{\boldsymbol{T}}\right)$:

$$\frac{\partial\left[[T]_{W_i}m_{W_i}\dot{\tilde{R}}_{W_i}^B[T]_{W_i}^T\right]}{\partial\delta_W} = m_{W_i}\left(\frac{\partial[T]_{W_i}}{\partial\delta_W}\dot{\tilde{R}}_{W_i}^B[T]_{W_i}^T + [T]_{W_i}\frac{\partial\dot{\tilde{R}}_{W_i}^B}{\partial\delta_W}[T]_{W_i}^T + [T]_{W_i}\dot{\tilde{R}}_{W_i}^B\frac{\partial[T]_{W_i}^T}{\partial\delta_W}\right)$$

$$\frac{\partial\left[[T]_{R_j}m_{R_j}\dot{\tilde{R}}_{R_j}^B[T]_{R_j}^T\right]}{\partial\delta_W} = m_{R_j}\left(\frac{\partial[T]_{R_j}}{\partial\delta_W}\dot{\tilde{R}}_{R_j}^B[T]_{R_j}^T + [T]_{R_j}\frac{\partial\dot{\tilde{R}}_{R_j}^B}{\partial\delta_W}[T]_{R_j}^T + [T]_{R_j}\dot{\tilde{R}}_{R_j}^B\frac{\partial[T]_{R_j}^T}{\partial\delta_W}\right)$$

$$\frac{\partial\left[[T]_{W_i}m_{W_i}\dot{\tilde{R}}_{W_i}^B[T]_{W_i}^T\right]}{\partial\delta_{HT}} = m_{W_i}\left(\frac{\partial[T]_{W_i}}{\partial\delta_{HT}}\dot{\tilde{R}}_{W_i}^B[T]_{W_i}^T + [T]_{W_i}\frac{\partial\dot{\tilde{R}}_{W_i}^B}{\partial\delta_{HT}}[T]_{W_i}^T + [T]_{W_i}\dot{\tilde{R}}_{W_i}^B\frac{\partial[T]_{W_i}^T}{\partial\delta_{HT}}\right)$$

$$\frac{\partial\left[[T]_{R_j}m_{R_j}\dot{\tilde{R}}_{R_j}^B[T]_{R_j}^T\right]}{\partial\delta_{HT}} = m_{R_j}\left(\frac{\partial[T]_{R_j}}{\partial\delta_{HT}}\dot{\tilde{R}}_{R_j}^B[T]_{R_j}^T + [T]_{R_j}\frac{\partial\dot{\tilde{R}}_{R_j}^B}{\partial\delta_{HT}}[T]_{R_j}^T + [T]_{R_j}\dot{\tilde{R}}_{R_j}^B\frac{\partial[T]_{R_j}^T}{\partial\delta_{HT}}\right)$$

18.42 Partial derivatives of $\left([\boldsymbol{T}]_{\boldsymbol{W_i}}\left(\tilde{\boldsymbol{I}}_{\boldsymbol{W_i}}^{\boldsymbol{W_i}} + \boldsymbol{m}_{\boldsymbol{W_i}}\tilde{\boldsymbol{R}}_{\boldsymbol{W_i}}^{\boldsymbol{B}}\right)[\dot{\boldsymbol{T}}]_{\boldsymbol{W_i}}^{\boldsymbol{T}}\right)$ and $\left([\boldsymbol{T}]_{\boldsymbol{R_j}}\left(\tilde{\boldsymbol{I}}_{\boldsymbol{R_j}}^{\boldsymbol{R_j}} + \boldsymbol{m}_{\boldsymbol{R_j}}\tilde{\boldsymbol{R}}_{\boldsymbol{R_j}}^{\boldsymbol{B}}\right)[\dot{\boldsymbol{T}}]_{\boldsymbol{R_j}}^{\boldsymbol{T}}\right)$:

$$\frac{\partial\left[[T]_{W_i}\left(\tilde{I}_{W_i}^{W_i} + m_{W_i}\tilde{R}_{W_i}^B\right)[\dot{T}]_{W_i}^T\right]}{\partial\delta_W} = \frac{\partial[T]_{W_i}}{\partial\delta_W}\left(\tilde{I}_{W_i}^{W_i} + m_{W_i}\tilde{R}_{W_i}^B\right)[\dot{T}]_{W_i}^T + m_{W_i}[T]_{W_i}\frac{\partial\tilde{R}_{W_i}^B}{\partial\delta_W}[\dot{T}]_{W_i}^T + [T]_{W_i}\left(\tilde{I}_{W_i}^{W_i} + m_{W_i}\tilde{R}_{W_i}^B\right)\frac{\partial[\dot{T}]_{W_i}^T}{\partial\delta_W}$$

$$\frac{\partial\left[[T]_{R_j}\left(\tilde{I}_{R_j}^{R_j} + m_{R_j}\tilde{R}_{R_j}^B\right)[\dot{T}]_{R_j}^T\right]}{\partial\delta_W} = \frac{\partial[T]_{R_j}}{\partial\delta_W}\left(\tilde{I}_{R_j}^{R_j} + m_{R_j}\tilde{R}_{R_j}^B\right)[\dot{T}]_{R_j}^T + m_{W_i}[T]_{R_j}\frac{\partial\tilde{R}_{W_i}^B}{\partial\delta_W}[\dot{T}]_{R_j}^T + [T]_{R_j}\left(\tilde{I}_{R_j}^{R_j} + m_{R_j}\tilde{R}_{R_j}^B\right)\frac{\partial[\dot{T}]_{R_j}^T}{\partial\delta_W}$$

$$\frac{\partial\left[[T]_{W_i}\left(\tilde{I}_{W_i}^{W_i}+m_{W_i}\tilde{R}_{W_i}^{B}\right)[\dot{T}]_{W_i}^{T}\right]}{\partial\delta_{HT}}=\frac{\partial[T]_{W_i}}{\partial\delta_{HT}}\left(\tilde{I}_{W_i}^{W_i}+m_{W_i}\tilde{R}_{W_i}^{B}\right)[\dot{T}]_{W_i}^{T}+m_{W_i}[T]_{W_i}\frac{\partial\tilde{R}_{W_i}^{B}}{\partial\delta_{HT}}[\dot{T}]_{W_i}^{T}+[T]_{W_i}\left(\tilde{I}_{W_i}^{W_i}+m_{W_i}\tilde{R}_{W_i}^{B}\right)\frac{\partial[\dot{T}]_{W_i}^{T}}{\partial\delta_{HT}}$$

$$\frac{\partial\left[[T]_{R_j}\left(\tilde{I}_{R_j}^{R_j}+m_{R_j}\tilde{R}_{R_j}^{B}\right)[\dot{T}]_{R_j}^{T}\right]}{\partial\delta_{HT}}=\frac{\partial[T]_{R_j}}{\partial\delta_{HT}}\left(\tilde{I}_{R_j}^{R_j}+m_{R_j}\tilde{R}_{R_j}^{B}\right)[\dot{T}]_{R_j}^{T}+m_{W_i}[T]_{R_j}\frac{\partial\tilde{R}_{W_i}^{B}}{\partial\delta_{HT}}[\dot{T}]_{R_j}^{T}+[T]_{R_j}\left(\tilde{I}_{R_j}^{R_j}+m_{R_j}\tilde{R}_{R_j}^{B}\right)\frac{\partial[\dot{T}]_{R_j}^{T}}{\partial\delta_{HT}}$$

18.43 Partial derivatives of $\left([\boldsymbol{T}]_{\boldsymbol{W_i}}\left(\tilde{\boldsymbol{I}}_{\boldsymbol{W_i}}^{\boldsymbol{W_i}}+\boldsymbol{m}_{\boldsymbol{W_i}}\tilde{\boldsymbol{R}}_{\boldsymbol{W_i}}^{\boldsymbol{B}}\right)[\boldsymbol{T}]_{\boldsymbol{W_i}}^{\boldsymbol{T}}\right)$ and $\left([\boldsymbol{T}]_{\boldsymbol{R_j}}\left(\tilde{\boldsymbol{I}}_{\boldsymbol{R_j}}^{\boldsymbol{R_j}}+\boldsymbol{m}_{\boldsymbol{R_j}}\tilde{\boldsymbol{R}}_{\boldsymbol{R_j}}^{\boldsymbol{B}}\right)[\boldsymbol{T}]_{\boldsymbol{R_j}}^{\boldsymbol{T}}\right)$:

$$\frac{\partial\left[[T]_{W_i}\left(\tilde{I}_{W_i}^{W_i}+m_{W_i}\tilde{R}_{W_i}^{B}\right)[T]_{W_i}^{T}\right]}{\partial\delta_{W}}=\frac{\partial[T]_{W_i}}{\partial\delta_{W}}\left(\tilde{I}_{W_i}^{W_i}+m_{W_i}\tilde{R}_{W_i}^{B}\right)[T]_{W_i}^{T}+m_{W_i}[T]_{W_i}\frac{\partial\tilde{R}_{W_i}^{B}}{\partial\delta_{W}}[T]_{W_i}^{T}+[T]_{W_i}\left(\tilde{I}_{W_i}^{W_i}+m_{W_i}\tilde{R}_{W_i}^{B}\right)\frac{\partial[T]_{W_i}^{T}}{\partial\delta_{W}}$$

$$\frac{\partial\left[[T]_{R_j}\left(\tilde{I}_{R_j}^{R_j}+m_{R_j}\tilde{R}_{R_j}^{B}\right)[T]_{R_j}^{T}\right]}{\partial\delta_{W}}=\frac{\partial[T]_{R_j}}{\partial\delta_{W}}\left(\tilde{I}_{R_j}^{R_j}+m_{R_j}\tilde{R}_{R_j}^{B}\right)[T]_{R_j}^{T}+m_{W_i}[T]_{R_j}\frac{\partial\tilde{R}_{W_i}^{B}}{\partial\delta_{W}}[T]_{R_j}^{T}+[T]_{R_j}\left(\tilde{I}_{R_j}^{R_j}+m_{R_j}\tilde{R}_{R_j}^{B}\right)\frac{\partial[T]_{R_j}^{T}}{\partial\delta_{W}}$$

$$\frac{\partial\left[[T]_{W_i}\left(\tilde{I}_{W_i}^{W_i}+m_{W_i}\tilde{R}_{W_i}^{B}\right)[T]_{W_i}^{T}\right]}{\partial\delta_{HT}}=\frac{\partial[T]_{W_i}}{\partial\delta_{HT}}\left(\tilde{I}_{W_i}^{W_i}+m_{W_i}\tilde{R}_{W_i}^{B}\right)[T]_{W_i}^{T}+m_{W_i}[T]_{W_i}\frac{\partial\tilde{R}_{W_i}^{B}}{\partial\delta_{HT}}[T]_{W_i}^{T}+[T]_{W_i}\left(\tilde{I}_{W_i}^{W_i}+m_{W_i}\tilde{R}_{W_i}^{B}\right)\frac{\partial[T]_{W_i}^{T}}{\partial\delta_{HT}}$$

$$\frac{\partial\left[[T]_{R_j}\left(\tilde{I}_{R_j}^{R_j}+m_{R_j}\tilde{R}_{R_j}^{B}\right)[T]_{R_j}^{T}\right]}{\partial\delta_{HT}}=\frac{\partial[T]_{R_j}}{\partial\delta_{HT}}\left(\tilde{I}_{R_j}^{R_j}+m_{R_j}\tilde{R}_{R_j}^{B}\right)[T]_{R_j}^{T}+m_{W_i}[T]_{R_j}\frac{\partial\tilde{R}_{W_i}^{B}}{\partial\delta_{HT}}[T]_{R_j}^{T}+[T]_{R_j}\left(\tilde{I}_{R_j}^{R_j}+m_{R_j}\tilde{R}_{R_j}^{B}\right)\frac{\partial[T]_{R_j}^{T}}{\partial\delta_{HT}}$$

18.44 Partial derivatives of $\left(\dot{\boldsymbol{R}}_{\boldsymbol{W_i}}^{\boldsymbol{B}}\vec{\boldsymbol{r}}_{\boldsymbol{W_i/pivot_i}}\times\vec{\boldsymbol{r}}_{\boldsymbol{W_i/B}}\times\right)$ and $\left(\dot{\boldsymbol{R}}_{\boldsymbol{R_j}}^{\boldsymbol{B}}\vec{\boldsymbol{r}}_{\boldsymbol{R_j/pivot_j}}\times\vec{\boldsymbol{r}}_{\boldsymbol{R_j/B}}\times\right)$:

$$\frac{\partial\left[\dot{R}_{W_i}^{B}\vec{r}_{W_i/pivot_i}\times\vec{r}_{W_i/B}\times\right]}{\partial\delta_{W}}=\frac{\partial\dot{R}_{W_i}^{B}}{\partial\delta_{W}}\left(\vec{r}_{W_i/pivot_i}\times\vec{r}_{W_i/B}\times\right)+\left(\dot{R}_{W_i}^{B}\vec{r}_{W,pivo_i}^{B}\times\right)\frac{\partial\left[\vec{r}_{W_i/B}\times\right]}{\partial\delta_{W}}$$

$$\frac{\partial\left[\dot{R}_{R_j}^{B}\vec{r}_{R_j/pivot_j}\times\vec{r}_{R_j/B}\times\right]}{\partial\delta_{W}}=\frac{\partial\dot{R}_{R_j}^{B}}{\partial\delta_{W}}\left(\vec{r}_{R_j/pivot_j}\times\vec{r}_{R_j/B}\times\right)+\left(\dot{R}_{R_j}^{B}\vec{r}_{R_j/pivot_j}\times\right)\frac{\partial\left[\vec{r}_{R_j/B}\times\right]}{\partial\delta_{W}}$$

$$\frac{\partial\left[\dot{R}_{W_i}^{B}\vec{r}_{W_i/pivot_i}\times\vec{r}_{W_i/B}\times\right]}{\partial\delta_{HT}}=\frac{\partial\dot{R}_{W_i}^{B}}{\partial\delta_{HT}}\left(\vec{r}_{W_i/pivot_i}\times\vec{r}_{W_i/B}\times\right)+\left(\dot{R}_{W_i}^{B}\vec{r}_{W_i/pivot_i}\times\right)\frac{\partial\left[\vec{r}_{W_i/B}\times\right]}{\partial\delta_{HT}}$$

$$\frac{\partial\left[\dot{R}_{R_j}^{B}\vec{r}_{R_j/pivot_j}\times\vec{r}_{R_j/B}\times\right]}{\partial\delta_{HT}}=\frac{\partial\dot{R}_{R_j}^{B}}{\partial\delta_{HT}}\left(\vec{r}_{R_j/pivot_j}\times\vec{r}_{R_j/B}\times\right)+\left(\dot{R}_{R_j}^{B}\vec{r}_{R_j/pivot_j}\times\right)\frac{\partial\left[\vec{r}_{R_j/B}\times\right]}{\partial\delta_{HT}}$$

18.45 Partial derivatives of $\left(\vec{\boldsymbol{r}}_{\boldsymbol{W_i/B}}\times\boldsymbol{\Omega_B}\vec{\boldsymbol{r}}_{\boldsymbol{W_i/B}}\times\right)$ and $\left(\vec{\boldsymbol{r}}_{\boldsymbol{R_j/B}}\times\boldsymbol{\Omega_B}\vec{\boldsymbol{r}}_{\boldsymbol{R_j/B}}\times\right)$:

$$\frac{\partial\left[\vec{r}_{W_i/B}\times\Omega_B\vec{r}_{W_i/B}\times\right]}{\partial\delta_{W}}=\frac{\partial\left[\vec{r}_{W_i/B}\times\right]}{\partial\delta_{W}}\left(\Omega_B\vec{r}_{W_i/B}\times\right)+\left(\vec{r}_{W_i/B}\times\Omega_B\right)\frac{\partial\left[\vec{r}_{W_i/B}\times\right]}{\partial\delta_{W}}$$

$$\frac{\partial\left[\vec{r}_{R_j/B}\times\Omega_B\vec{r}_{R_j/B}\times\right]}{\partial\delta_{W}}=\frac{\partial\left[\vec{r}_{R_j/B}\times\right]}{\partial\delta_{W}}\left(\Omega_B\vec{r}_{R_j/B}\times\right)+\left(\vec{r}_{R_j/B}\times\Omega_B\right)\frac{\partial\left[\vec{r}_{R_j/B}\times\right]}{\partial\delta_{W}}$$

$$\frac{\partial\left[\vec{r}_{W_i/B}\times\Omega_B\vec{r}_{W_i/B}\times\right]}{\partial\delta_{HT}}=\frac{\partial\left[\vec{r}_{W_i/B}\times\right]}{\partial\delta_{HT}}\left(\Omega_B\vec{r}_{W_i/B}\times\right)+\left(\vec{r}_{W_i/B}\times\Omega_B\right)\frac{\partial\left[\vec{r}_{W_i/B}\times\right]}{\partial\delta_{HT}}$$

$$\frac{\partial\left[\vec{r}_{R_j/B}\times\Omega_B\vec{r}_{R_j/B}\times\right]}{\partial\delta_{HT}}=\frac{\partial\left[\vec{r}_{R_j/B}\times\right]}{\partial\delta_{HT}}\left(\Omega_B\vec{r}_{R_j/B}\times\right)+\left(\vec{r}_{R_j/B}\times\Omega_B\right)\frac{\partial\left[\vec{r}_{R_j/B}\times\right]}{\partial\delta_{HT}}$$

18.46 Partial derivatives of $\left[\dot{R}^B_{W_i}\vec{r}_{W_i/pivot_i}\times\left(\dot{R}^B_{W_i}\vec{r}_{W_i/pivot_i}\right)\right]$ and $\left[\dot{R}^B_{R_j}\vec{r}_{R_j/pivot_j}\times\left(\dot{R}^B_{R_j}\vec{r}_{R_j/pivot_j}\right)\right]$:

$$\frac{\partial\left[\dot{R}^B_{W_i}\vec{r}_{W_i/pivot_i}\times\left(\dot{R}^B_{W_i}\vec{r}_{W_i/pivot_i}\right)\right]}{\partial\delta_W}=\left(\frac{\partial\dot{R}^B_{W_i}}{\partial\delta_W}\vec{r}_{W_i/pivot_i}\times\dot{R}^B_{W_i}+\dot{R}^B_{W_i}\vec{r}_{W_i/pivot_i}\times\frac{\partial\dot{R}^B_{W_i}}{\partial\delta_W}+\frac{\partial\left[\vec{r}_{W_i/B}\times\right]}{\partial\delta_W}2\Omega_B\dot{R}^B_{W_i}\right)\vec{r}_{W_i/pivot_i}$$

$$\frac{\partial\left[\dot{R}^B_{R_j}\vec{r}_{R_j/pivot_j}\times\left(\dot{R}^B_{R_j}\vec{r}_{R_j/pivot_j}\right)\right]}{\partial\delta_W}=\left(\frac{\partial\dot{R}^B_{R_j}}{\partial\delta_W}\vec{r}_{R_j/pivot_j}\times\dot{R}^B_{R_j}+\dot{R}^B_{R_j}\vec{r}_{R_j/pivot_j}\times\frac{\partial\dot{R}^B_{R_j}}{\partial\delta_W}+\frac{\partial\left[\vec{r}_{R_j/B}\times\right]}{\partial\delta_W}2\Omega_B\dot{R}^B_{R_j}\right)\vec{r}_{R_j/pivot_j}$$

$$\frac{\partial\left[\dot{R}^B_{W_i}\vec{r}_{W_i/pivot_i}\times\left(\dot{R}^B_{W_i}\vec{r}_{W_i/pivot_i}\right)\right]}{\partial\delta_{HT}}=\left(\frac{\partial\dot{R}^B_{W_i}}{\partial\delta_{HT}}\vec{r}_{W_i/pivot_i}\times\dot{R}^B_{W_i}+\dot{R}^B_{W_i}\vec{r}_{W_i/pivot_i}\times\frac{\partial\dot{R}^B_{W_i}}{\partial\delta_{HT}}+\frac{\partial\left[\vec{r}_{W_i/B}\times\right]}{\partial\delta_{HT}}2\Omega_B\dot{R}^B_{W_i}\right)\vec{r}_{W_i/pivot_i}$$

$$\frac{\partial\left[\dot{R}^B_{R_j}\vec{r}_{R_j/pivot_j}\times\left(\dot{R}^B_{R_j}\vec{r}_{R_j/pivot_j}\right)\right]}{\partial\delta_{HT}}=\left(\frac{\partial\dot{R}^B_{R_j}}{\partial\delta_{HT}}\vec{r}_{R_j/pivot_j}\times\dot{R}^B_{R_j}+\dot{R}^B_{R_j}\vec{r}_{R_j/pivot_j}\times\frac{\partial\dot{R}^B_{R_j}}{\partial\delta_{HT}}+\frac{\partial\left[\vec{r}_{R_j/B}\times\right]}{\partial\delta_{HT}}2\Omega_B\dot{R}^B_{R_j}\right)\vec{r}_{R_j/pivot_j}$$

18.47 Partial derivatives of $\left[\boldsymbol{\vec{r}_{W_i/B}}\times\left(\boldsymbol{2\Omega_B\dot{R}^B_{W_i}}+\boldsymbol{\ddot{R}^B_{W_i}}\right)\boldsymbol{\vec{r}_{W_i/pivot_i}}\right]$ and $\left[\boldsymbol{\vec{r}_{R_j/B}}\times\left(\boldsymbol{2\Omega_B\dot{R}^B_{R_j}}+\boldsymbol{\ddot{R}^B_{R_j}}\right)\boldsymbol{\vec{r}_{R_j/pivot_j}}\right]$:

$$\frac{\partial\left[\vec{r}_{W_i/B}\times\left(2\Omega_B\dot{R}^B_{W_i}+\ddot{R}^B_{W_i}\right)\vec{r}_{W_i/pivot_i}\right]}{\partial\delta_W}=\left(\vec{r}_{W_i/B}\times 2\Omega_B\frac{\partial\dot{R}^B_{W_i}}{\partial\delta_W}+\frac{\partial\left[\vec{r}_{W_i/B}\times\right]}{\partial\delta_W}\ddot{R}^B_{W_i}+\vec{r}_{W_i/B}\times\frac{\partial\ddot{R}^B_{W_i}}{\partial\delta_W}\right)\vec{r}_{W_i/pivot_i}$$

$$\frac{\partial\left[\vec{r}_{R_j/B}\times\left(2\Omega_B\dot{R}^B_{R_j}+\ddot{R}^B_{R_j}\right)\vec{r}_{R_j/pivot_j}\right]}{\partial\delta_W}=\left(\vec{r}_{R_j/B}\times 2\Omega_B\frac{\partial\dot{R}^B_{R_j}}{\partial\delta_W}+\frac{\partial\left[\vec{r}_{R_j/B}\times\right]}{\partial\delta_W}\ddot{R}^B_{R_j}+\vec{r}_{R_j/B}\times\frac{\partial\ddot{R}^B_{R_j}}{\partial\delta_W}\right)\vec{r}_{R_j/pivot_j}$$

$$\frac{\partial\left[\vec{r}_{W_i/B}\times\left(2\Omega_B\dot{R}^B_{W_i}+\ddot{R}^B_{W_i}\right)\vec{r}_{W_i/pivot_i}\right]}{\partial\delta_{HT}}=\left(\vec{r}_{W_i/B}\times 2\Omega_B\frac{\partial\dot{R}^B_{W_i}}{\partial\delta_{HT}}+\frac{\partial\left[\vec{r}_{W_i/B}\times\right]}{\partial\delta_{HT}}\ddot{R}^B_{W_i}+\vec{r}_{W_i/B}\times\frac{\partial\ddot{R}^B_{W_i}}{\partial\delta_{HT}}\right)\vec{r}_{W_i/pivot_i}$$

$$\frac{\partial\left[\vec{r}_{R_j/B}\times\left(2\Omega_B\dot{R}^B_{R_j}+\ddot{R}^B_{R_j}\right)\vec{r}_{R_j/pivot_j}\right]}{\partial\delta_{HT}}=\left(\vec{r}_{R_j/B}\times 2\Omega_B\frac{\partial\dot{R}^B_{R_j}}{\partial\delta_{HT}}+\frac{\partial\left[\vec{r}_{R_j/B}\times\right]}{\partial\delta_{HT}}\ddot{R}^B_{R_j}+\vec{r}_{R_j/B}\times\frac{\partial\ddot{R}^B_{R_j}}{\partial\delta_{HT}}\right)\vec{r}_{R_j/pivot_j}$$

18.48 Partial derivatives of $\boldsymbol{C^2_{L_{W_e}}}$:

$$\frac{\partial C^2_{L_{W_e}}}{\partial\alpha}=2\left[\left(C_{L_\alpha}\right)_{W_e}\left(\alpha+\delta_W-\alpha_{0_W}\right)+C_{L_{\delta_f}}\delta_f+\Delta C_{L_{p,W}}+\Delta C_{L_{p_L}}+\Delta C_{L_{p_R}}\right]\left(\left(C_{L_\alpha}\right)_{W_e}+\frac{\partial\Delta C_{L_{p,W}}}{\partial\alpha}\right)$$

$$\frac{\partial C^2_{L_{W_e}}}{\partial\delta_f}=2\left[\left(C_{L_\alpha}\right)_{W_e}\left(\alpha+\delta_W-\alpha_{0_W}\right)+C_{L_{\delta_f}}\delta_f+\Delta C_{L_{p,W}}+\Delta C_{L_{p_L}}+\Delta C_{L_{p_R}}\right]\left(C_{L_{\delta_f}}+\frac{\partial\Delta C_{L_{p,W}}}{\partial\delta_f}\right)$$

$$\frac{\partial C^2_{L_{W_e}}}{\partial\delta_W}=2\left[\left(C_{L_\alpha}\right)_{W_e}\left(\alpha+\delta_W-\alpha_{0_W}\right)+C_{L_{\delta_f}}\delta_f+\Delta C_{L_{p,W}}+\Delta C_{L_{p_L}}+\Delta C_{L_{p_R}}\right]\left(\left(C_{L_\alpha}\right)_{W_e}+\frac{\partial\Delta C_{L_{p,W}}}{\partial\delta_W}\right)$$

$$\frac{\partial C^2_{L_{W_e}}}{\partial\omega_j^2}=2\left[\left(C_{L_\alpha}\right)_{W_e}\left(\alpha+\delta_W-\alpha_{0_W}\right)+C_{L_{\delta_f}}\delta_f+\Delta C_{L_{p,W}}+\Delta C_{L_{p_L}}+\Delta C_{L_{p_R}}\right]\frac{\partial\Delta C_{L_{p,W}}}{\partial\omega_j^2}$$

18.49 Partial derivatives of $\boldsymbol{C^2_{L_{HT_e}}}$:

$$\frac{\partial C^2_{L_{HT_e}}}{\partial \alpha} = 2\left[\left(C_{L_\alpha}\right)_{HT_e}\left(\alpha + \delta_{HT} - \alpha_{0_{HT}} - \epsilon\right) + C_{L_{\delta_e}}\delta_e + \Delta C_{L_{p,HT}}\right]\left(\left(C_{L_\alpha}\right)_{HT_e} + \frac{\partial \Delta C_{L_{p,HT}}}{\partial \alpha}\right)$$

$$\frac{\partial C^2_{L_{HT_e}}}{\partial \delta_e} = 2\left[\left(C_{L_\alpha}\right)_{HT_e}\left(\alpha + \delta_{HT} - \alpha_{0_{HT}} - \epsilon\right) + C_{L_{\delta_e}}\delta_e + \Delta C_{L_{p,HT}}\right]\left(C_{L_{\delta_e}} + \frac{\partial \Delta C_{L_{p,HT}}}{\partial \delta_e}\right)$$

$$\frac{\partial C^2_{L_{HT_e}}}{\partial \delta_{HT}} = 2\left[\left(C_{L_\alpha}\right)_{HT_e}\left(\alpha + \delta_{HT} - \alpha_{0_{HT}} - \epsilon\right) + C_{L_{\delta_e}}\delta_e + \Delta C_{L_{p,HT}}\right]\left(\left(C_{L_\alpha}\right)_{HT_e} + \frac{\partial \Delta C_{L_{p,HT}}}{\partial \delta_{HT}}\right)$$

$$\frac{\partial C^2_{L_{HT_e}}}{\partial \omega_j^2} = 2\left[\left(C_{L_\alpha}\right)_{HT_e}\left(\alpha + \delta_{HT} - \alpha_{0_{HT}} - \epsilon\right) + C_{L_{\delta_e}}\delta_e + \Delta C_{L_{p,HT}}\right]\frac{\partial \Delta C_{L_{p,HT}}}{\partial \omega_j^2}$$

18.50 Partial derivatives of $\Delta\boldsymbol{C_{L_{p,W}}}$:

$$\frac{\partial \Delta C_{L_{p,W}}}{\partial \alpha} = \frac{1{,}6T''_{c\,W}\sqrt{1 - T''_{c\,W}}K_W S_p}{S_W\left(S_{W_{wake}} + S_{W_{wake,flap}}\right)}\left[\left(\frac{F}{T}\right)_{c_W}\cos\left(\frac{\theta}{\delta_{f_e}}\delta_{f_e} + \alpha + \delta_W\right)S_{W_{wake}} + \left(\frac{F}{T}\right)_{d_W}\cos\left(\frac{\theta}{\delta_f}\delta_f + \frac{\theta}{\delta_{f_e}}\delta_{f_e} + \alpha + \delta_W\right)S_{W_{wake,flap}}\right]$$

$$\frac{\partial \Delta C_{L_{p,W}}}{\partial \delta_f} = \frac{1{,}6T''_{c\,W}\sqrt{1 - T''_{c\,W}}K_W S_p}{S_W\left(S_{W_{wake}} + S_{W_{wake,flap}}\right)}\left(\frac{F}{T}\right)_{d_W}\frac{\theta}{\delta_f}\cos\left(\frac{\theta}{\delta_f}\delta_f + \frac{\theta}{\delta_{f_e}}\delta_{f_e} + \alpha + \delta_W\right)S_{W_{wake,flap}}$$

$$\frac{\partial \Delta C_{L_{p,W}}}{\partial \delta_W} = \frac{1{,}6T''_{c\,W}\sqrt{1 - T''_{c\,W}}K_W S_p}{S_W\left(S_{W_{wake}} + S_{W_{wake,flap}}\right)}\left[\left(\frac{F}{T}\right)_{c_W}\cos\left(\frac{\theta}{\delta_{f_e}}\delta_{f_e} + \alpha + \delta_W\right)S_{W_{wake}} + \left(\frac{F}{T}\right)_{d_W}\cos\left(\frac{\theta}{\delta_f}\delta_f + \frac{\theta}{\delta_{f_e}}\delta_{f_e} + \alpha + \delta_W\right)S_{W_{wake,flap}}\right]$$

$$\frac{\partial \Delta C_{L_{p,W}}}{\partial \omega_j^2} = \frac{1{,}6K_W S_p}{S_W\left(S_{W_{wake}} + S_{W_{wake,flap}}\right)}\frac{\partial T''_{c\,W}}{\partial \omega_j^2}\frac{2 - 3T''_{c\,W}}{2\sqrt{1 - T''_{c\,W}}}\left[\left(\frac{F}{T}\right)_{c_W}\sin\left(\frac{\theta}{\delta_{f_e}}\delta_{f_e} + \alpha + \delta_W\right)S_{W_{wake}} + \left(\frac{F}{T}\right)_{d_W}\sin\left(\frac{\theta}{\delta_f}\delta_f + \frac{\theta}{\delta_{f_e}}\delta_{f_e} + \alpha + \delta_W\right)S_{W_{wake,flap}}\right]$$

18.51 Partial derivatives of $\Delta\boldsymbol{C_{L_{p,HT}}}$:

$$\frac{\partial \Delta C_{L_{p,HT}}}{\partial \alpha} = \frac{1{,}6T''_{c\,Ht}\sqrt{1 - T''_{c\,Ht}}K_{HT} S_p}{S_{Ht}\left(S_{HT_{wake}} + S_{HT_{wake,elevator}}\right)}\left[\left(\frac{F}{T}\right)_{c_{Ht}}\cos\left(\frac{\theta}{\delta_{e_e}}\delta_{e_e} + \alpha + \delta_{Ht} - \epsilon\right)S_{HT_{wake}} + \left(\frac{F}{T}\right)_{d_{Ht}}\cos\left(\frac{\theta}{\delta_e}\delta_e + \frac{\theta}{\delta_{e_e}}\delta_{e_e} + \alpha + \delta_{Ht} - \epsilon\right)S_{HT_{wake,elevator}}\right]\frac{S_{HT_e}}{S_W}$$

$$\frac{\partial \Delta C_{L_{p,HT}}}{\partial \delta_e} = \frac{1{,}6T''_{c\,Ht}\sqrt{1 - T''_{c\,Ht}}K_{HT} S_p}{S_{Ht}\left(S_{HT_{wake}} + S_{HT_{wake,elevator}}\right)}\left(\frac{F}{T}\right)_{d_{Ht}}\frac{\theta}{\delta_e}\cos\left(\frac{\theta}{\delta_e}\delta_e + \frac{\theta}{\delta_{e_e}}\delta_{e_e} + \alpha + \delta_{Ht} - \epsilon\right)S_{HT_{wake,elevator}}\frac{S_{HT_e}}{S_W}$$

$$\frac{\partial \Delta C_{L_{p,HT}}}{\partial \delta_{HT}} = \frac{1{,}6T^{"}_{c\,Ht}\sqrt{1-T^{"}_{c\,Ht}}K_{HT}S_p}{S_{Ht}\left(S_{HT_{wake}}+S_{HT_{wake,elevator}}\right)}\left[\left(\frac{F}{T}\right)_{c_{Ht}}\cos\left(\frac{\theta}{\delta_{e_e}}\delta_{e_e}+\alpha+\delta_{Ht}-\epsilon\right)S_{HT_{wake}} + \left(\frac{F}{T}\right)_{d_{Ht}}\cos\left(\frac{\theta}{\delta_e}\delta_e+\frac{\theta}{\delta_{e_e}}\delta_{e_e}+\alpha+\delta_{Ht}-\epsilon\right)S_{HT_{wake,elevator}}\right]\frac{S_{HT_e}}{S_W}$$

$$\frac{\partial \Delta C_{L_{p,HT}}}{\partial \omega_j^2} = \frac{1{,}6K_{HT}S_p}{S_{Ht}\left(S_{HT_{wake}}+S_{HT_{wake,elevator}}\right)}\frac{\partial T^{"}_{c\,HT}}{\partial \omega_j^2}\frac{2-3T^{"}_{c\,HT}}{2\sqrt{1-T^{"}_{c\,HT}}}\left[\left(\frac{F}{T}\right)_{c_{Ht}}\sin\left(\frac{\theta}{\delta_{e_e}}\delta_{e_e}+\alpha+\delta_{Ht}-\epsilon\right)S_{HT_{wake}} + \left(\frac{F}{T}\right)_{d_{Ht}}\sin\left(\frac{\theta}{\delta_e}\delta_e+\frac{\theta}{\delta_{e_e}}\delta_{e_e}+\alpha+\delta_{Ht}-\epsilon\right)S_{HT_{wake,elevator}}\right]\frac{S_{HT_e}}{S_W}$$

18.52 Partial derivatives of $\Delta \boldsymbol{C_{D_{p,W}}}$:

$$\frac{\partial \Delta C_{D_{p,W}}}{\partial \alpha} = \frac{1{,}6T^{"}_{c\,W}\sqrt{1-T^{"}_{c\,W}}K_W S_p}{S_W\left(S_{W_{wake}}+S_{W_{wake,flap}}\right)}\left\{\left(\frac{F}{T}\right)_{c_W}\sin\left(\frac{\theta}{\delta_{f_e}}\delta_{f_e}+\alpha+\delta_W\right)S_{W_{wake}} + \left(\frac{F}{T}\right)_{d_W}\sin\left(\frac{\theta}{\delta_f}\delta_f+\frac{\theta}{\delta_{f_e}}\delta_{f_e}+\alpha+\delta_W\right)S_{W_{wake,flap}}\right\}$$

$$\frac{\partial \Delta C_{D_{p,W}}}{\partial \delta_f} = \frac{1{,}6T^{"}_{c\,W}\sqrt{1-T^{"}_{c\,W}}K_W S_p}{S_W\left(S_{W_{wake}}+S_{W_{wake,flap}}\right)}\left(\frac{F}{T}\right)_{d_W}\frac{\theta}{\delta_f}\sin\left(\frac{\theta}{\delta_f}\delta_f+\frac{\theta}{\delta_{f_e}}\delta_{f_e}+\alpha+\delta_W\right)S_{W_{wake,flap}}$$

$$\frac{\partial \Delta C_{D_{p,W}}}{\partial \delta_W} = \frac{1{,}6T^{"}_{c\,W}\sqrt{1-T^{"}_{c\,W}}K_W S_p}{S_W\left(S_{W_{wake}}+S_{W_{wake,flap}}\right)}\left\{\left(\frac{F}{T}\right)_{c_W}\sin\left(\frac{\theta}{\delta_{f_e}}\delta_{f_e}+\alpha+\delta_W\right)S_{W_{wake}} + \left(\frac{F}{T}\right)_{d_W}\sin\left(\frac{\theta}{\delta_f}\delta_f+\frac{\theta}{\delta_{f_e}}\delta_{f_e}+\alpha+\delta_W\right)S_{W_{wake,flap}}\right\}$$

$$\frac{\partial \Delta C_{D_{p,W}}}{\partial \omega_j^2} = \frac{1{,}6K_W S_p}{S_W\left(S_{W_{wake}}+S_{W_{wake,flap}}\right)}\frac{\partial T^{"}_{c\,W}}{\partial \omega_j^2}\frac{2-3T^{"}_{c\,W}}{2\sqrt{1-T^{"}_{c\,W}}}\left\{\left(\frac{F}{T}\right)_{c_W}\left[1-\cos\left(\frac{\theta}{\delta_{f_e}}\delta_{f_e}+\alpha+\delta_W\right)\right]S_{W_{wake}} + \left(\frac{F}{T}\right)_{d_W}\left[1-\cos\left(\frac{\theta}{\delta_f}\delta_f+\frac{\theta}{\delta_{f_e}}\delta_{f_e}+\alpha+\delta_W\right)\right]S_{W_{wake,flap}}\right\}$$

18.53 Partial derivatives of $\Delta \boldsymbol{C_{D_{p,HT}}}$:

$$\frac{\partial \Delta C_{D_{p,HT}}}{\partial \alpha} = \frac{1{,}6T^{"}_{c\,Ht}\sqrt{1-T^{"}_{c\,Ht}}K_{HT}S_p}{S_{Ht}\left(S_{HT_{wake}}+S_{HT_{wake,elevator}}\right)}\left\{\left(\frac{F}{T}\right)_{c_{Ht}}\sin\left(\frac{\theta}{\delta_{f_e}}\delta_{f_e}+\alpha+\delta_{Ht}-\epsilon\right)S_{HT_{wake}} + \left(\frac{F}{T}\right)_{d_{Ht}}\sin\left(\frac{\theta}{\delta_f}\delta_f+\frac{\theta}{\delta_{f_e}}\delta_{f_e}+\alpha+\delta_{Ht}-\epsilon\right)S_{HT_{wake,elevator}}\right\}\frac{S_{HT_e}}{S_W}$$

$$\frac{\partial \Delta C_{D_{p,HT}}}{\partial \delta_e} = \frac{1{,}6T^{"}_{c\,Ht}\sqrt{1-T^{"}_{c\,Ht}}K_{HT}S_p}{S_{Ht}\left(S_{HT_{wake}}+S_{HT_{wake,elevator}}\right)}\left(\frac{F}{T}\right)_{d_{Ht}}\frac{\theta}{\delta_e}\sin\left(\frac{\theta}{\delta_e}\delta_e+\frac{\theta}{\delta_{e_e}}\delta_{e_e}+\alpha+\delta_{Ht}-\epsilon\right)S_{HT_{wake,elevator}}\frac{S_{HT_e}}{S_W}$$

$$\frac{\partial \Delta C_{D_{p,HT}}}{\partial \delta_{HT}} = \frac{1{,}6 T''_{c_{Ht}} \sqrt{1 - T''_{c_{Ht}}} K_{HT} S_p}{S_{Ht}\left(S_{HT_{wake}} + S_{HT_{wake,elevator}}\right)} \left\{ \left(\frac{F}{T}\right)_{c_{Ht}} \sin\left(\frac{\theta}{\delta_{e_e}} \delta_{e_e} + \alpha + \delta_{Ht} - \epsilon\right) S_{HT_{wake}} + \left(\frac{F}{T}\right)_{d_{Ht}} \sin\left(\frac{\theta}{\delta_e} \delta_e + \frac{\theta}{\delta_{e_e}} \delta_{e_e} + \alpha + \delta_{Ht} - \epsilon\right) S_{HT_{wake,elevator}} \right\} \frac{S_{HT_e}}{S_W}$$

$$\frac{\partial \Delta C_{D_{p,HT}}}{\partial \omega_j^2} = \frac{1{,}6 K_{HT} S_p}{S_{Ht}\left(S_{HT_{wake}} + S_{HT_{wake,elevator}}\right)} \frac{\partial T''_{c_{HT}}}{\partial \omega_j^2} \frac{2 - 3T''_{c_{HT}}}{2\sqrt{1 - T''_{c_{HT}}}} \left\{ \left(\frac{F}{T}\right)_{c_{Ht}} \left[1 - \cos\left(\frac{\theta}{\delta_{e_e}} \delta_{e_e} + \alpha + \delta_{Ht} - \epsilon\right)\right] S_{HT_{wake}} + \left(\frac{F}{T}\right)_{d_{Ht}} \left[1 - \cos\left(\frac{\theta}{\delta_e} \delta_e + \frac{\theta}{\delta_{e_e}} \delta_{e_e} + \alpha + \delta_{Ht} - \epsilon\right)\right] S_{HT_{wake,elevator}} \right\} \frac{S_{HT_e}}{S_W}$$

18.54 Partial derivatives of $\boldsymbol{T''_{cW}}$:

$$\frac{\partial T''_{c_W}}{\partial \omega_j^2} = \frac{\frac{\rho V_T^2 \pi D_p^2}{8 k_{T_j}}}{\left(\omega_j^2 + \frac{\rho V_T^2 \pi D_p^2}{8 k_{T_j}}\right)^2}$$

18.55 Partial derivatives of $\boldsymbol{T''_{cHT}}$:

$$\frac{\partial T''_{c_{HT}}}{\partial \omega_j^2} = \frac{\frac{\rho V_T^2 \pi D_p^2}{8 k_{T_j}}}{\left(\omega_j^2 + \frac{\rho V_T^2 \pi D_p^2}{8 k_{T_j}}\right)^2}$$

18.56 Partial derivatives of $\Delta \boldsymbol{C_{l_p}}$:

$$\frac{\partial \Delta C_{l_p}}{\partial \alpha} = \frac{y_{MAC_{Si_L}}}{b_W/2} \left(\frac{\partial \Delta C_{L_{p_L}}}{\partial \alpha} \cos\alpha - \Delta C_{L_{p_L}} \sin\alpha + \frac{\partial \Delta C_{D_{p_L}}}{\partial \alpha} \sin\alpha + \Delta C_{D_{p_L}} \cos\alpha \right) - \frac{y_{MAC_{Si_R}}}{b_W/2} \left(\frac{\partial \Delta C_{L_{p_R}}}{\partial \alpha} \cos\alpha - \Delta C_{L_{p_R}} \sin\alpha + \frac{\partial \Delta C_{D_{p_R}}}{\partial \alpha} \sin\alpha + \Delta C_{D_{p_R}} \cos\alpha \right)$$

$$\frac{\partial \Delta C_{l_p}}{\partial \delta_{a_L}} = \frac{y_{MAC_{Si_L}}}{b_W/2} \left(\frac{\partial \Delta C_{L_{p_L}}}{\partial \delta_{a_L}} \cos\alpha + \frac{\partial \Delta C_{D_{p_L}}}{\partial \delta_{a_L}} \sin\alpha \right)$$

$$\frac{\partial \Delta C_{l_p}}{\partial \delta_{a_R}} = -\frac{y_{MAC_{Si_R}}}{b_W/2} \left(\frac{\partial \Delta C_{L_{p_R}}}{\partial \delta_{a_R}} \cos\alpha + \frac{\partial \Delta C_{D_{p_R}}}{\partial \delta_{a_R}} \sin\alpha \right)$$

$$\frac{\partial \Delta C_{l_p}}{\partial \omega_j^2} = \frac{\left(\frac{\partial \Delta C_{L_{prop_L}}}{\partial \omega_j^2} \cos\alpha + \frac{\partial \Delta C_{D_{prop_L}}}{\partial \omega_j^2} \sin\alpha \right) y_{MAC_{Si_L}} - \left(\frac{\partial \Delta C_{L_{prop_R}}}{\partial \omega_j^2} \cos\alpha + \frac{\partial \Delta C_{D_{prop_R}}}{\partial \omega_j^2} \sin\alpha \right) y_{MAC_{Si_R}}}{b_W/2}$$

18.57 Partial derivatives of $\Delta \boldsymbol{C_{L_{p_{L,R}}}}$:

$$\frac{\partial \Delta C_{L_{p_{L,R}}}}{\partial \alpha} = \frac{1{,}6 T''_{c_{W a_{L,R}}} \sqrt{1 - T''_{c_{W a_{L,R}}}} K_{W_{a_{L,R}}} S_p}{S_W} \left(\frac{F}{T}\right)_{d_{W a_{L,R}}} \cos\left(\frac{\theta}{\delta_{a_{L,R}}} \delta_{a_{L,R}} + \frac{\theta}{\delta_{a_{L,R_e}}} \delta_{a_{L,R_e}} + \alpha + \delta_W\right)$$

$$\frac{\partial \Delta C_{L_{p_L}}}{\partial \delta_{a_L}} = \frac{1{,}6 T''_{c_{W a_L}} \sqrt{1 - T''_{c_{W a_L}}} K_{W_{a_L}} S_p}{S_W} \left(\frac{F}{T}\right)_{d_{W a_L}} \frac{\theta}{\delta_{a_L}} \cos\left(\frac{\theta}{\delta_{a_L}} \delta_{a_L} + \frac{\theta}{\delta_{a_{L_e}}} \delta_{a_{L_e}} + \alpha + \delta_W\right)$$

$$\frac{\partial \Delta C_{L_{p_R}}}{\partial \delta_{a_R}} = \frac{1{,}6 T_{c\,Wa_R}^{"} \sqrt{1 - T_{c\,Wa_R}^{"}} K_{W_{a_R}} S_p}{S_W} \left(\frac{F}{T}\right)_{dWa_R} \frac{\theta}{\delta_{a_R}} \cos\left(\frac{\theta}{\delta_{a_R}} \delta_{a_R} + \frac{\theta}{\delta_{a_{R_e}}} \delta_{a_{R_e}} + \alpha + \delta_W\right)$$

$$\frac{\partial \Delta C_{L_{p_{L,R}}}}{\partial \delta_W} = \frac{1{,}6 T_{c\,Wa_{L,R}}^{"} \sqrt{1 - T_{c\,Wa_{L,R}}^{"}} K_{W_{a_{L,R}}} S_p}{S_W} \left(\frac{F}{T}\right)_{dWa_{L,R}} \cos\left(\frac{\theta}{\delta_{a_{L,R}}} \delta_{a_{L,R}} + \frac{\theta}{\delta_{a_{L,R_e}}} \delta_{a_{L,R_e}} + \alpha + \delta_W\right)$$

$$\frac{\partial \Delta C_{L_{prop_L}}}{\partial \omega_j^2} = \frac{1{,}6 K_{WL} S_p}{S_W} \frac{\partial T_{c\,Wa_L}^{"}}{\partial \omega_j^2} \frac{2 - 3T_{c\,Wa_{L,R}}^{"}}{2\sqrt{1 - T_{c\,Wa_{L,R}}^{"}}} \left(\frac{F}{T}\right)_{dWa_L} \sin\left(\frac{\theta}{\delta_{a_L}} \delta_{a_L} + \frac{\theta}{\delta_{a_{L_e}}} \delta_{a_{L_e}} + \alpha + \delta_W\right)$$

$$\frac{\partial \Delta C_{L_{prop_R}}}{\partial \omega_j^2} = \frac{1{,}6 K_{WR} S_p}{S_W} \frac{\partial T_{c\,Wa_R}^{"}}{\partial \omega_j^2} \frac{2 - 3T_{c\,Wa_R}^{"}}{2\sqrt{1 - T_{c\,Wa_R}^{"}}} \left(\frac{F}{T}\right)_{dWa_L} \sin\left(\frac{\theta}{\delta_{a_R}} \delta_{a_R} + \frac{\theta}{\delta_{a_{R_e}}} \delta_{a_{R_e}} + \alpha + \delta_W\right)$$

18.58 Partial derivatives of $\Delta \boldsymbol{C_{D_{p_{L,R}}}}$:

$$\frac{\partial \Delta C_{D_{p_{L,R}}}}{\partial \alpha} = \frac{1{,}6 T_{c\,Wa_{L,R}}^{"} \sqrt{1 - T_{c\,Wa_{L,R}}^{"}} K_{W_{a_{L,R}}} S_p}{S_W} \left(\frac{F}{T}\right)_{dWa_{L,R}} \sin\left(\frac{\theta}{\delta_{a_{L,R}}} \delta_{a_{L,R}} + \frac{\theta}{\delta_{a_{L,R_e}}} \delta_{a_{L,R_e}} + \alpha + \delta_W\right)$$

$$\frac{\partial \Delta C_{D_{p_L}}}{\partial \delta_{a_L}} = \frac{1{,}6 T_{c\,Wa_L}^{"} \sqrt{1 - T_{c\,Wa_L}^{"}} K_{W_{a_L}} S_p}{S_W} \left(\frac{F}{T}\right)_{dWa_L} \frac{\theta}{\delta_{a_L}} \sin\left(\frac{\theta}{\delta_{a_L}} \delta_{a_L} + \frac{\theta}{\delta_{a_{L_e}}} \delta_{a_{L_e}} + \alpha + \delta_W\right)$$

$$\frac{\partial \Delta C_{D_{p_R}}}{\partial \delta_{a_R}} = \frac{1{,}6 T_{c\,Wa_R}^{"} \sqrt{1 - T_{c\,Wa_R}^{"}} K_{W_{a_R}} S_p}{S_W \left(S_{Wa_{R_{wake}}} + S_{Wa_{R_{wake,aileron}}}\right)} \left(\frac{F}{T}\right)_{dWa_R} \frac{\theta}{\delta_{a_R}} \sin\left(\frac{\theta}{\delta_{a_R}} \delta_{a_R} + \frac{\theta}{\delta_{a_{R_e}}} \delta_{a_{R_e}} + \alpha + \delta_W\right)$$

$$\frac{\partial \Delta C_{D_{p_{L,R}}}}{\partial \delta_W} = \frac{1{,}6 T_{c\,Wa_{L,R}}^{"} \sqrt{1 - T_{c\,Wa_{L,R}}^{"}} K_{W_{a_{L,R}}} S_p}{S_W} \left(\frac{F}{T}\right)_{dWa_{L,R}} \sin\left(\frac{\theta}{\delta_{a_{L,R}}} \delta_{a_{L,R}} + \frac{\theta}{\delta_{a_{L,R_e}}} \delta_{a_{L,R_e}} + \alpha + \delta_W\right)$$

$$\frac{\partial \Delta C_{D_{prop_L}}}{\partial \omega_j^2} = \frac{1{,}6 K_{WL} S_p}{S_W} \frac{\partial T_{c\,Wa_L}^{"}}{\partial \omega_j^2} \frac{2 - 3T_{c\,Wa_L}^{"}}{2\sqrt{1 - T_{c\,Wa_L}^{"}}} \left(\frac{F}{T}\right)_{dWa_L} \left[1 - \cos\left(\frac{\theta}{\delta_{a_L}} \delta_{a_L} + \frac{\theta}{\delta_{a_{L_e}}} \delta_{a_{L_e}} + \alpha + \delta_W\right)\right]$$

$$\frac{\partial \Delta C_{D_{prop_R}}}{\partial \omega_j^2} = \frac{1{,}6 K_{WR} S_p}{S_W} \frac{\partial T_{c\,Wa_R}^{"}}{\partial \omega_j^2} \frac{2 - 3T_{c\,Wa_R}^{"}}{2\sqrt{1 - T_{c\,Wa_R}^{"}}} \left(\frac{F}{T}\right)_{dWa_R} \left[1 - \cos\left(\frac{\theta}{\delta_{a_R}} \delta_{a_R} + \frac{\theta}{\delta_{a_{R_e}}} \delta_{a_{R_e}} + \alpha + \delta_W\right)\right]$$

18.59 Partial derivatives of $\boldsymbol{C_{L_B}}$:

$$\frac{\partial C_{L_B}}{\partial \alpha} = \left(C_{L_\alpha}\right)_B$$

18.60 Partial derivatives of $\boldsymbol{C_{D_B}}$:

$$\frac{\partial C_{D_B}}{\partial \alpha} = C_{D_{\alpha^2 B}} 2\alpha + C_{D_{\alpha^3 B}} 3\alpha^2$$

18.61 Partial derivatives of $\boldsymbol{C_{m_B}}$:

$$\frac{\partial C_{m_B}}{\partial \alpha} = \left(C_{m_{\alpha 1}}\right)_B \left[2\cos(2\alpha)\cos\left(\frac{\alpha}{2}\right) - \frac{1}{2}\sin(2\alpha)\sin\left(\frac{\alpha}{2}\right)\right] + \left(C_{m_{\alpha 2}}\right)_B 2\sin\alpha\cos\alpha$$

18.62 Partial derivatives of $\boldsymbol{C_{L_{W_e}}}$:

$$\frac{\partial C_{L_{W_e}}}{\partial \alpha} = \left(C_{L_\alpha}\right)_{W_e} + \frac{\partial \Delta C_{L_{p,W}}}{\partial \alpha}$$

$$\frac{\partial C_{L_{W_e}}}{\partial \delta_f} = C_{L_{\delta_f}} + \frac{\partial \Delta C_{L_{p,W}}}{\partial \delta_f}$$

$$\frac{\partial C_{L_{W_e}}}{\partial \delta_W} = \left(C_{L_\alpha}\right)_{W_e} + \frac{\partial \Delta C_{L_{p,W}}}{\partial \delta_W}$$

$$\frac{\partial C_{L_{W_e}}}{\partial \omega_j^2} = \frac{\partial \Delta C_{L_{p,W}}}{\partial \omega_j^2}$$

18.63 Partial derivatives of $\boldsymbol{C_{D_{W_e}}}$:

$$\frac{\partial C_{D_{W_e}}}{\partial \alpha} = \frac{1}{\pi AR_{W_e} e_{W_e}} \frac{\partial C_{L_{W_e}}^2}{\partial \alpha} + \frac{\partial \Delta C_{D_{p,W}}}{\partial \alpha}$$

$$\frac{\partial C_{D_{W_e}}}{\partial \delta_f} = \frac{1}{\pi AR_{W_e} e_{W_e}} \frac{\partial C_{L_{W_e}}^2}{\partial \delta_f} + \frac{\partial \Delta C_{L_{p,W}}}{\partial \delta_f}$$

$$\frac{\partial C_{D_{W_e}}}{\partial \delta_W} = \frac{1}{\pi AR_{W_e} e_{W_e}} \frac{\partial C_{L_{W_e}}^2}{\partial \delta_W} + \frac{\partial \Delta C_{D_{p,W}}}{\partial \delta_W}$$

$$\frac{\partial C_{D_{W_e}}}{\partial \omega_j^2} = \frac{1}{\pi AR_{W_e} e_{W_e}} \frac{\partial C_{L_{W_e}}^2}{\partial \omega_j^2} + \frac{\partial \Delta C_{D_{p,W}}}{\partial \omega_j^2}$$

18.64 Partial derivatives of $\boldsymbol{C_{m_{W_e}}}$:

$$\frac{\partial C_{m_{W_e}}}{\partial \alpha} = \left(C_{m_\alpha}\right)_{W_e} + \frac{\partial \Delta C_{m_f}}{\partial \alpha}$$

$$\frac{\partial C_{m_{W_e}}}{\partial \delta_f} = \frac{\partial \Delta C_{m_f}}{\partial \delta_f} = \left\{K_p\left(\frac{\Delta C'_m}{\Delta C_L}\right)\left[C_{L_{\delta_f}}\right]_{W\,fullspan, A=6, \Lambda_{c/2=0}}\left(\frac{c'}{c}\right)^2 + K_\Lambda\left(\frac{A}{1{,}5}\right)\left[C_{L_{\delta_f}}\right]_{W\,fullspan, A=6, \Lambda_{c/2=0}}\tan\Lambda_{c/4}\right\}\frac{S_{W_e}}{S_W}$$

$$\frac{\partial C_{m_{W_e}}}{\partial \delta_W} = \left(C_{m_\alpha}\right)_{W_e}$$

18.65 Partial derivatives of $\boldsymbol{C_{L_{HT_e}}}$:

$$\frac{\partial C_{L_{HT_e}}}{\partial \alpha} = \left(C_{L_\alpha}\right)_{HT_e} + \frac{\partial \Delta C_{L_{p,HT}}}{\partial \alpha}$$

$$\frac{\partial C_{L_{HT_e}}}{\partial \delta_e} = C_{L_{\delta_e}} + \frac{\partial \Delta C_{L_{p,HT}}}{\partial \delta_e}$$

$$\frac{\partial C_{L_{HT_e}}}{\partial \delta_{HT}} = \left(C_{L_\alpha}\right)_{HT_e} + \frac{\partial \Delta C_{L_{p,HT}}}{\partial \delta_{HT}}$$

$$\frac{\partial C_{L_{HT_e}}}{\partial \omega_j^2} = \frac{\partial \Delta C_{L_{p,HT}}}{\partial \omega_j^2}$$

18.66 Partial derivatives of $\boldsymbol{C_{D_{HT_e}}}$:

$$\frac{\partial C_{D_{HT_e}}}{\partial \alpha} = \frac{1}{\pi AR_{HT_e} e_{HT_e}} \frac{\partial C_{L_{HT_e}}^2}{\partial \alpha} + \frac{\partial \Delta C_{D_{p,HT}}}{\partial \alpha}$$

$$\frac{\partial C_{D_{HT_e}}}{\partial \delta_e} = \frac{1}{\pi AR_{HT_e} e_{HT_e}} \frac{\partial C_{L_{HT_e}}^2}{\partial \delta_e} + \frac{\partial \Delta C_{D_{p,HT}}}{\partial \delta_e}$$

$$\frac{\partial C_{D_{HT_e}}}{\partial \delta_{HT}} = \frac{1}{\pi AR_{HT_e} e_{HT_e}} \frac{\partial C_{L_{HT_e}}^2}{\partial \delta_{HT}} + \frac{\partial \Delta C_{D_{p,HT}}}{\partial \delta_{HT}}$$

$$\frac{\partial C_{D_{HT_e}}}{\partial \omega_j^2} = \frac{1}{\pi AR_{HT_e} e_{HT_e}} \frac{\partial C_{L_{HT_e}}^2}{\partial \omega_j^2} + \frac{\partial \Delta C_{D_{p,HT}}}{\partial \omega_j^2}$$

18.67 Partial derivatives of $\boldsymbol{C_{m_{HT_e}}}$:

$$\frac{\partial C_{m_{HT_e}}}{\partial \alpha} = \left(C_{m_\alpha}\right)_{HT_e} + \frac{\partial \Delta C_{m_e}}{\partial \alpha}$$

$$\frac{\partial C_{m_{HT_e}}}{\partial \delta_e} = \frac{\partial \Delta C_{m_e}}{\partial \delta_e} = \left\{K_p \left(\frac{\Delta C'_m}{\Delta C_L}\right) \left[C_{L_{\delta_e}}\right]_{HT\ fullspan, A=6, \Lambda_{c/2=0}} \left(\frac{c'}{c}\right)^2 + K_\Lambda \left(\frac{A}{1{,}5}\right) \left[C_{L_{\delta_e}}\right]_{HT\ fullspan, A=6, \Lambda_{c/2=0}} \tan \Lambda_{c/4}\right\} \frac{S_{HT_e}}{S_W}$$

$$\frac{\partial C_{m_{HT_e}}}{\partial \delta_{HT}} = \left(C_{m_\alpha}\right)_{HT_e}$$

18.68 Partial derivatives of $\Delta\boldsymbol{C_{m_f}}$:

$$\frac{\partial \Delta C_{m_f}}{\partial \alpha} = -K_p \left\{0{,}25 \left(\frac{\partial C_{L_{W_e}}}{\partial \alpha}\right)_{no\ flap} \left(\frac{c'}{c}\right)\left(\frac{c'}{c} - 1\right) - \left(\frac{\partial C_{m_{W_e}}}{\partial \alpha}\right)_{no\ flap} \left[\left(\frac{c'}{c}\right)^2 - 1\right]\right\} \frac{S_{W_e}}{S_W}$$

18.69 Partial derivatives of $\Delta\boldsymbol{C_{m_e}}$:

$$\frac{\partial \Delta C_{m_e}}{\partial \alpha} = -K_p \left\{0{,}25 \left(\frac{\partial C_{L_{HT_e}}}{\partial \alpha}\right)_{no\ elevator} \left(\frac{c'}{c}\right)\left(\frac{c'}{c} - 1\right) - \left(\frac{\partial C_{m_{HT_e}}}{\partial \alpha}\right)_{no\ elevator} \left[\left(\frac{c'}{c}\right)^2 - 1\right]\right\} \frac{S_{HT_e}}{S_W}$$

18.70 Partial derivatives of $\Delta\boldsymbol{C_{n_p}}$:

$$\frac{\partial \Delta C_{n_p}}{\partial \alpha} = \frac{y_{MAC_{Si_L}}}{b_W/2} \left(\frac{\partial \Delta C_{L_{p_L}}}{\partial \alpha} \sin \alpha + \Delta C_{L_{p_L}} \cos \alpha - \frac{\partial \Delta C_{D_{p_L}}}{\partial \alpha} \cos \alpha + \Delta C_{D_{p_L}} \sin \alpha\right)$$
$$- \frac{y_{MAC_{Si_R}}}{b_W/2} \left(\frac{\partial \Delta C_{L_{p_R}}}{\partial \alpha} \sin \alpha + \Delta C_{L_{p_R}} \cos \alpha - \frac{\partial \Delta C_{D_{p_R}}}{\partial \alpha} \cos \alpha + \Delta C_{D_{p_R}} \sin \alpha\right)$$

$$\frac{\partial \Delta C_{n_p}}{\partial \delta_{a_L}} = \frac{y_{MAC_{Si_L}}}{b_W/2} \left(\frac{\partial \Delta C_{L_{p_L}}}{\partial \delta_{a_L}} \sin \alpha - \frac{\partial \Delta C_{D_{p_L}}}{\partial \delta_{a_L}} \cos \alpha\right)$$

$$\frac{\partial \Delta C_{n_p}}{\partial \delta_{a_R}} = -\frac{y_{MAC_{Si_R}}}{b_W/2}\left(\frac{\partial \Delta C_{L_{p_R}}}{\partial \delta_{a_R}}\sin\alpha - \frac{\partial \Delta C_{D_{p_R}}}{\partial \delta_{a_R}}\cos\alpha\right)$$

$$\frac{\partial \Delta C_{n_p}}{\partial \omega_j^2} = \frac{\left(\frac{\partial \Delta C_{L_{prop_L}}}{\partial \omega_j^2}\sin\alpha - \frac{\partial \Delta C_{D_{prop_L}}}{\partial \omega_j^2}\cos\alpha\right) y_{MAC_{Si_L}} - \left(\frac{\partial \Delta C_{L_{prop_R}}}{\partial \omega_j^2}\sin\alpha - \frac{\partial \Delta C_{D_{prop_R}}}{\partial \omega_j^2}\cos\alpha\right) y_{MAC_{Si_R}}}{b_W/2}$$

18.71 Partial derivatives of $(\boldsymbol{A})_{\boldsymbol{P}}^{-\boldsymbol{1}}$:

$$\frac{\partial (A)_P^{-1}{}_W}{\partial \delta_W} = \sum_{i=1}^{NW}\left\{\frac{\partial\left[[T]_{W_i}\left(\tilde{I}_{W_i}^{W_i} + m_{W_i}\tilde{R}_{W_i}^B\right)[T]_{W_i}^T\right]^+}{\partial \delta_W} - m_{W_i}\frac{\partial\left(\vec{r}_{W_i/P}\times\vec{r}_{W_i/P}\times\right)^{-1}}{\partial \delta_W}\right\}$$
$$+\sum_{j=1}^{NR}\left\{\frac{\partial\left[[T]_{R_j}\left(\tilde{I}_{R_j}^{R_j} + m_{R_j}\tilde{R}_{R_j}^B\right)[T]_{R_j}^T\right]^+}{\partial \delta_W} - m_{R_j}\frac{\partial\left(\vec{r}_{R_j/P}\times\vec{r}_{R_j/P}\times\right)^{-1}}{\partial \delta_W}\right\}$$

$$\frac{\partial (A)_P^{-1}{}_{HT}}{\partial \delta_{HT}} = \sum_{i=1}^{NW}\left\{\frac{\partial\left[[T]_{W_i}\left(\tilde{I}_{W_i}^{W_i} + m_{W_i}\tilde{R}_{W_i}^B\right)[T]_{W_i}^T\right]^+}{\partial \delta_{HT}} - m_{W_i}\frac{\partial\left(\vec{r}_{W_i/P}\times\vec{r}_{W_i/P}\times\right)^{-1}}{\partial \delta_{HT}}\right\}$$
$$+\sum_{j=1}^{NR}\left\{\frac{\partial\left[[T]_{R_j}\left(\tilde{I}_{R_j}^{R_j} + m_{R_j}\tilde{R}_{R_j}^B\right)[T]_{R_j}^T\right]^+}{\partial \delta_{HT}} - m_{R_j}\frac{\partial\left(\vec{r}_{R_j/P}\times\vec{r}_{R_j/P}\times\right)^{-1}}{\partial \delta_{HT}}\right\}$$

The next steps are much similar to the respective partial derivatives of A^{-1}, previously derived, but for the wing or horizontal tail alone with respect to their pivot point. Therefore, it will be omitted here.

18.72 Partial derivatives of $(\boldsymbol{B})_{\boldsymbol{P}}$:

$$\frac{\partial (B)_{P_W}}{\partial \delta_W} = \sum_{i=1}^{NW}\left\{\frac{\partial\left[[\dot{T}]_{W_i}\left(\tilde{I}_{W_i}^{W_i} + m_{W_i}\tilde{R}_{W_i}^B\right)[T]_{W_i}^T\right]}{\partial \delta_W} + \frac{\partial\left[[T]_{W_i} m_{W_i}\dot{\tilde{R}}_{W_i}^B[T]_{W_i}^T\right]}{\partial \delta_W} + \frac{\partial\left[[T]_{W_i}\left(\tilde{I}_{W_i}^{W_i} + m_{W_i}\tilde{R}_{W_i}^B\right)[\dot{T}]_{W_i}^T\right]}{\partial \delta_W}\right.$$
$$\left. + \Omega_{P_W}\frac{\partial\left[[T]_{W_i}\left(\tilde{I}_{W_i}^{W_i} + m_{W_i}\tilde{R}_{W_i}^B\right)[T]_{W_i}^T\right]}{\partial \delta_W} - m_{W_i}\frac{\partial\left[\left(\dot{R}_{W_i}^B\vec{r}_{W_i/pivot_i}\times\vec{r}_{W_i/P}\times\right) + \left(\vec{r}_{W_i/P}\times\Omega_{P_W}\vec{r}_{W_i/P}\times\right)\right]}{\partial \delta_W}\right\}$$
$$+\sum_{j=1}^{NR}\left\{\frac{\partial\left[[\dot{T}]_{R_j}\left(\tilde{I}_{R_j}^{R_j} + m_{R_j}\tilde{R}_{R_j}^B\right)[T]_{R_j}^T\right]}{\partial \delta_W} + \frac{\partial\left[[T]_{R_j} m_{R_j}\dot{\tilde{R}}_{R_j}^B[T]_{R_j}^T\right]}{\partial \delta_W} + \frac{\partial\left[[T]_{R_j}\left(\tilde{I}_{R_j}^{R_j} + m_{R_j}\tilde{R}_{R_j}^B\right)[\dot{T}]_{R_j}^T\right]}{\partial \delta_W}\right.$$
$$\left. + \Omega_{P_W}\frac{\partial\left[[T]_{R_j}\left(\tilde{I}_{R_j}^{R_j} + m_{R_j}\tilde{R}_{R_j}^B\right)[T]_{R_j}^T\right]}{\partial \delta_W} - m_{R_j}\frac{\partial\left[\left(\dot{R}_{R_j}^B\vec{r}_{R_j/pivot_j}\times\vec{r}_{R_j/P}\times\right) + \left(\vec{r}_{R_j/P}\times\Omega_{P_W}\vec{r}_{R_j/P}\times\right)\right]}{\partial \delta_W}\right\}$$

$$\frac{\partial (B)_{P_W}}{\partial \dot{\delta}_W} = \sum_{i=1}^{NW}\left\{\frac{\partial[\dot{T}]_{W_i}}{\partial \dot{\delta}_W}\left(\tilde{I}_{W_i}^{W_i} + m_{W_i}\tilde{R}_{W_i}^B\right)[T]_{W_i}^T + [T]_{W_i} m_{W_i}\frac{\partial \dot{\tilde{R}}_{W_i}^B}{\partial \dot{\delta}_W}[T]_{W_i}^T + [T]_{W_i}\left(\tilde{I}_{W_i}^{W_i} + m_{W_i}\tilde{R}_{W_i}^B\right)\frac{\partial[\dot{T}]_{W_i}^T}{\partial \dot{\delta}_W}\right.$$
$$\left. + \frac{\partial \Omega_{P_W}}{\partial \dot{\delta}_W}[T]_{W_i}\left(\tilde{I}_{W_i}^{W_i} + m_{W_i}\tilde{R}_{W_i}^P\right)[T]_{W_i}^T - m_{W_i}\left(\frac{\partial \dot{R}_{W_i}^B}{\partial \dot{\delta}_W}\vec{r}_{W_i/pivot_i}\times\vec{r}_{W_i/P}\times\right)\right\}$$
$$+\sum_{j=1}^{NR}\left\{\frac{\partial[\dot{T}]_{R_j}}{\partial \dot{\delta}_W}\left(\tilde{I}_{R_j}^{R_j} + m_{R_j}\tilde{R}_{R_j}^B\right)[T]_{R_j}^T + [T]_{R_j} m_{R_j}\frac{\partial \dot{\tilde{R}}_{R_j}^B}{\partial \dot{\delta}_W}[T]_{R_j}^T + [T]_{R_j}\left(\tilde{I}_{R_j}^{R_j} + m_{R_j}\tilde{R}_{R_j}^B\right)\frac{\partial[\dot{T}]_{R_j}^T}{\partial \dot{\delta}_W}\right.$$
$$\left. + \frac{\partial \Omega_{P_W}}{\partial \dot{\delta}_W}[T]_{R_j}\left(\tilde{I}_{R_j}^{R_j} + m_{R_j}\tilde{R}_{R_j}^P\right)[T]_{R_j}^T - m_{R_j}\left(\frac{\partial \dot{R}_{R_j}^B}{\partial \dot{\delta}_W}\vec{r}_{R_j/pivot_j}\times\vec{r}_{R_j/P}\times\right)\right\}$$

$$\frac{\partial(B)_{P_{HT}}}{\partial\delta_{\mathrm{HT}}}=\sum_{i=1}^{NW}\left\{\frac{\partial\left[[\dot{T}]_{W_i}\left(\tilde{I}_{W_i}^{W_i}+m_{W_i}\tilde{R}_{W_i}^{B}\right)[T]_{W_i}^{T}\right]}{\partial\delta_{HT}}+\frac{\partial\left[[T]_{W_i}m_{W_i}\dot{\tilde{R}}_{W_i}^{B}[T]_{W_i}^{T}\right]}{\partial\delta_{HT}}+\frac{\partial\left[[T]_{W_i}\left(\tilde{I}_{W_i}^{W_i}+m_{W_i}\tilde{R}_{W_i}^{B}\right)[\dot{T}]_{W_i}^{T}\right]}{\partial\delta_{HT}}\right.$$

$$+\Omega_{P_{HT}}\frac{\partial\left[[T]_{W_i}\left(\tilde{I}_{W_i}^{W_i}+m_{W_i}\tilde{R}_{W_i}^{B}\right)[T]_{W_i}^{T}\right]}{\partial\delta_{HT}}$$

$$\left.-m_{W_i}\frac{\partial\left[\left(\dot{R}_{W_i}^{B}\vec{r}_{W_i/pivot_i}\times\vec{r}_{W_i/P}\times\right)+\left(\vec{r}_{W_i/P}\times\Omega_{P_{HT}}\vec{r}_{W_i/P}\times\right)\right]}{\partial\delta_{HT}}\right\}$$

$$+\sum_{j=1}^{NR}\left\{\frac{\partial\left[[\dot{T}]_{R_j}\left(\tilde{I}_{R_j}^{R_j}+m_{R_j}\tilde{R}_{R_j}^{B}\right)[T]_{R_j}^{T}\right]}{\partial\delta_{HT}}+\frac{\partial\left[[T]_{R_j}m_{R_j}\dot{\tilde{R}}_{R_j}^{B}[T]_{R_j}^{T}\right]}{\partial\delta_{HT}}+\frac{\partial\left[[T]_{R_j}\left(\tilde{I}_{R_j}^{R_j}+m_{R_j}\tilde{R}_{R_j}^{B}\right)[\dot{T}]_{R_j}^{T}\right]}{\partial\delta_{HT}}\right.$$

$$\left.+\Omega_{P_{HT}}\frac{\partial\left[[T]_{R_j}\left(\tilde{I}_{R_j}^{R_j}+m_{R_j}\tilde{R}_{R_j}^{B}\right)[T]_{R_j}^{T}\right]}{\partial\delta_{HT}}-m_{R_j}\frac{\partial\left[\left(\dot{R}_{R_j}^{B}\vec{r}_{R_j/pivot_j}\times\vec{r}_{R_j/B}\times\right)+\left(\vec{r}_{R_j/B}\times\Omega_{P_{HT}}\vec{r}_{R_j/P}\times\right)\right]}{\partial\delta_{HT}}\right\}$$

$$\frac{\partial(B)_{P_{HT}}}{\partial\dot{\delta}_{\mathrm{HT}}}=\sum_{i=1}^{NW}\left\{\frac{\partial[\dot{T}]_{W_i}}{\partial\dot{\delta}_{HT}}\left(\tilde{I}_{W_i}^{W_i}+m_{W_i}\tilde{R}_{W_i}^{B}\right)[T]_{W_i}^{T}+[T]_{W_i}m_{W_i}\frac{\partial\dot{\tilde{R}}_{W_i}^{B}}{\partial\dot{\delta}_{HT}}[T]_{W_i}^{T}+[T]_{W_i}\left(\tilde{I}_{W_i}^{W_i}+m_{W_i}\tilde{R}_{W_i}^{B}\right)\frac{\partial[\dot{T}]_{W_i}^{T}}{\partial\dot{\delta}_{HT}}\right.$$

$$\left.+\frac{\partial\Omega_{P_{HT}}}{\partial\dot{\delta}_{HT}}[T]_{W_i}\left(\tilde{I}_{W_i}^{W_i}+m_{W_i}\tilde{R}_{W_i}^{P}\right)[T]_{W_i}^{T}-m_{W_i}\left(\frac{\partial\dot{R}_{W_i}^{B}}{\partial\dot{\delta}_{HT}}\vec{r}_{W_i/pivot_i}\times\vec{r}_{W_i/P}\times\right)\right\}$$

$$+\sum_{j=1}^{NR}\left\{\frac{\partial[\dot{T}]_{R_j}}{\partial\dot{\delta}_{HT}}\left(\tilde{I}_{R_j}^{R_j}+m_{R_j}\tilde{R}_{R_j}^{B}\right)[T]_{R_j}^{T}+[T]_{R_j}m_{R_j}\frac{\partial\dot{\tilde{R}}_{R_j}^{B}}{\partial\dot{\delta}_{HT}}[T]_{R_j}^{T}+[T]_{R_j}\left(\tilde{I}_{R_j}^{R_j}+m_{R_j}\tilde{R}_{R_j}^{B}\right)\frac{\partial[\dot{T}]_{R_j}^{T}}{\partial\dot{\delta}_{HT}}\right.$$

$$\left.+\frac{\partial\Omega_{P_{HT}}}{\partial\dot{\delta}_{HT}}[T]_{R_j}\left(\tilde{I}_{R_j}^{R_j}+m_{R_j}\tilde{R}_{R_j}^{P}\right)[T]_{R_j}^{T}-m_{R_j}\left(\frac{\partial\dot{R}_{R_j}^{B}}{\partial\dot{\delta}_{HT}}\vec{r}_{R_j/pivot_j}\times\vec{r}_{R_j/P}\times\right)\right\}$$

The next steps are much similar to the respective partial derivatives of B, previously derived, but for the wing or horizontal tail alone with respect to their pivot point. Therefore, it will be omitted here.

18.73 Partial derivatives of $(\boldsymbol{C})_{\boldsymbol{P}}$:

$$\frac{\partial(C)_{P_W}}{\partial\delta_{\mathrm{W}}}=\sum_{i=1}^{NW}\left\{m_{W_i}\frac{\partial\left[\vec{r}_{W_i/P}\times\right]}{\partial\delta_{\mathrm{W}}}\right\}+\sum_{j=1}^{NR}\left\{m_{R_j}\frac{\partial\left[\vec{r}_{R_j/P}\times\right]}{\partial\delta_{\mathrm{W}}}\right\}$$

$$\frac{\partial(C)_{P_{HT}}}{\partial\delta_{\mathrm{HT}}}=\sum_{i=1}^{NW}\left\{m_{W_i}\frac{\partial\left[\vec{r}_{W_i/P}\times\right]}{\partial\delta_{\mathrm{HT}}}\right\}+\sum_{j=1}^{NR}\left\{m_{R_j}\frac{\partial\left[\vec{r}_{R_j/P}\times\right]}{\partial\delta_{\mathrm{HT}}}\right\}$$

18.74 Partial derivatives of $(\boldsymbol{D})_{\boldsymbol{P}}$:

$$\frac{\partial(D)_{P_W}}{\partial\delta_{\mathrm{W}}}=\sum_{i=1}^{NW}\left\{m_{W_i}\left[\frac{\partial\dot{R}_{W_i}^{P}}{\partial\delta_{\mathrm{W}}}\left(\vec{r}_{W_i/pivot_i}\times\right)+\frac{\partial\left[\vec{r}_{W_i/P}\times\right]}{\partial\delta_{\mathrm{W}}}\Omega_{P_W}\right]\right\}+\sum_{j=1}^{NR}\left\{m_{R_j}\left[\frac{\partial\dot{R}_{R_j}^{P}}{\partial\delta_{\mathrm{W}}}\left(\vec{r}_{R_j/pivot_j}\times\right)+\frac{\partial\left[\vec{r}_{R_j/P}\times\right]}{\partial\delta_{\mathrm{W}}}\Omega_{P_W}\right]\right\}$$

$$\frac{\partial(D)_{P_W}}{\partial\dot{\delta}_{\mathrm{W}}}=\sum_{i=1}^{NW}\left\{m_{W_i}\left[\frac{\partial\dot{R}_{W_i}^{P}}{\partial\dot{\delta}_{\mathrm{W}}}\left(\vec{r}_{W_i/pivot_i}\times\right)+\vec{r}_{W_i/P}\times\frac{\partial\Omega_{P_W}}{\partial\dot{\delta}_{W}}\right]\right\}+\sum_{j=1}^{NR}\left\{m_{R_j}\left[\frac{\partial\dot{R}_{R_j}^{P}}{\partial\dot{\delta}_{\mathrm{W}}}\left(\vec{r}_{R_j/pivot_j}\times\right)+\vec{r}_{R_j/P}\times\frac{\partial\Omega_{P_W}}{\partial\dot{\delta}_{W}}\right]\right\}$$

$$\frac{\partial(D)_{P_{HT}}}{\partial\delta_{\mathrm{HT}}}=\sum_{i=1}^{NW}\left\{m_{W_i}\left[\frac{\partial\dot{R}_{W_i}^{P}}{\partial\delta_{\mathrm{HT}}}\left(\vec{r}_{W_i/pivot_i}\times\right)+\frac{\partial\left[\vec{r}_{W_i/P}\times\right]}{\partial\delta_{\mathrm{HT}}}\Omega_{P_{HT}}\right]\right\}+\sum_{j=1}^{NR}\left\{m_{R_j}\left[\frac{\partial\dot{R}_{R_j}^{P}}{\partial\delta_{\mathrm{HT}}}\left(\vec{r}_{R_j/pivot_j}\times\right)+\frac{\partial\left[\vec{r}_{R_j/P}\times\right]}{\partial\delta_{\mathrm{HT}}}\Omega_{P_{HT}}\right]\right\}$$

$$\frac{\partial(D)_{P_{HT}}}{\partial\dot{\delta}_{\mathrm{HT}}}=\sum_{i=1}^{NW}\left\{m_{W_i}\left[\frac{\partial\dot{R}_{W_i}^{P}}{\partial\dot{\delta}_{\mathrm{HT}}}\left(\vec{r}_{W_i/pivot_i}\times\right)+\vec{r}_{W_i/P}\times\frac{\partial\Omega_{P_{HT}}}{\partial\dot{\delta}_{HT}}\right]\right\}+\sum_{j=1}^{NR}\left\{m_{R_j}\left[\frac{\partial\dot{R}_{R_j}^{P}}{\partial\dot{\delta}_{\mathrm{HT}}}\left(\vec{r}_{R_j/pivot_j}\times\right)+\vec{r}_{R_j/P}\times\frac{\partial\Omega_{P_{HT}}}{\partial\dot{\delta}_{HT}}\right]\right\}$$

18.75 Partial derivatives of $(\boldsymbol{M_P})_{\boldsymbol{P}}$:

$$\frac{\partial(M_P)_{P_{W,HT}}}{\partial\phi}=\sum_{i=1}^{NW}\left\{\vec{r}_{W_i/P}\times m_{W_i}\frac{\partial B_E^B}{\partial\phi}\vec{g}^E\right\}+\sum_{j=1}^{NR}\left\{\vec{r}_{R_j/P}\times m_{R_j}\frac{\partial B_E^B}{\partial\phi}\vec{g}^E\right\}$$

$$\frac{\partial(M_P)_{P_{W,HT}}}{\partial\theta}=\sum_{i=1}^{NW}\left\{\vec{r}_{W_i/P}\times m_{W_i}\frac{\partial B_E^B}{\partial\theta}\vec{g}^E\right\}+\sum_{j=1}^{NR}\left\{\vec{r}_{R_j/P}\times m_{R_j}\frac{\partial B_E^B}{\partial\theta}\vec{g}^E\right\}$$

$$\frac{\partial(M_P)_{P_{W,HT}}}{\partial\psi}=\sum_{i=1}^{NW}\left\{\vec{r}_{W_i/P}\times m_{W_i}\frac{\partial B_E^B}{\partial\psi}\vec{g}^E\right\}+\sum_{j=1}^{NR}\left\{\vec{r}_{R_j/P}\times m_{R_j}\frac{\partial B_E^B}{\partial\psi}\vec{g}^E\right\}$$

$$\frac{\partial(M_P)_{P_W}}{\partial\delta_{\mathrm{W}}}=\sum_{i=1}^{NW}\left\{\frac{\partial\left[\vec{r}_{W_i/P}\times\right]}{\partial\delta_{\mathrm{W}}}m_{W_i}B_E^B\vec{g}^E\right\}+\sum_{j=1}^{NR}\left\{\frac{\partial\left[\vec{r}_{R_j/P}\times\right]}{\partial\delta_{\mathrm{W}}}m_{R_j}B_E^B\vec{g}^E\right\}$$

$$\frac{\partial(M_P)_{P_{HT}}}{\partial\delta_{\mathrm{HT}}}=\sum_{i=1}^{NW}\left\{\frac{\partial\left[\vec{r}_{W_i/P}\times\right]}{\partial\delta_{\mathrm{HT}}}m_{W_i}B_E^B\vec{g}^E\right\}+\sum_{j=1}^{NR}\left\{\frac{\partial\left[\vec{r}_{R_j/P}\times\right]}{\partial\delta_{\mathrm{HT}}}m_{R_j}B_E^B\vec{g}^E\right\}$$

18.76 Partial derivatives of $(\boldsymbol{E})_{\boldsymbol{P}}$:

$$\frac{\partial(E)_{P_W}}{\partial\delta_{\mathrm{W}}}=\frac{\partial\,(E)_{P_W}}{\partial\delta_{\mathrm{W}}}+\frac{\partial\,(E)_{P_R}}{\partial\delta_{\mathrm{W}}}$$

Expanding the terms,

$$\frac{\partial\,(E)_{P_W}}{\partial\delta_{\mathrm{W}}}=\sum_{i=1}^{NW}\left\{m_{W_i}\left[\frac{\partial\dot{R}_{W_i}^{P}}{\partial\delta_{\mathrm{W}}}\vec{r}_{W_i/pivot_i}\times\dot{R}_{W_i}^{P}+\dot{R}_{W_i}^{P}\vec{r}_{W_i/pivot_i}\times\frac{\partial\dot{R}_{W_i}^{P}}{\partial\delta_{\mathrm{W}}}+\vec{r}_{W_i/P}\times\left(2\Omega_{P_W}\frac{\partial\dot{R}_{W_i}^{P}}{\partial\delta_{\mathrm{W}}}+\frac{\partial\ddot{R}_{W_i}^{P}}{\partial\delta_{\mathrm{W}}}\right)\right]\vec{r}_{W_i/pivot_i}\right\}$$

$$\frac{\partial\,(E)_{P_R}}{\partial \delta_W} = \sum_{j=1}^{NR}\left\{\left(\frac{\partial\left[[\dot{T}]_{R_j}\left(\tilde{I}_{R_j}^{R_j} + m_{R_j}\tilde{R}_{R_j}^{P}\right)[T]_{R_j}^{T}\right]}{\partial \delta_W} + \frac{\partial\left[[T]_{R_j} m_{R_j}\frac{d}{dt}\left(\tilde{R}_{R_j}^{P}\right)[T]_{R_j}^{T}\right]}{\partial \delta_W}\right.\right.$$

$$\left. + \frac{\partial\left[[T]_{R_j}\left(\tilde{I}_{R_j}^{R_j} + m_{R_j}\tilde{R}_{R_j}^{P}\right)[\dot{T}]_{R_j}^{T}\right]}{\partial \delta_W}\right) R_{R_j}^{P}\begin{Bmatrix}\omega_{R_j}\\0\\0\end{Bmatrix}$$

$$+\left([\dot{T}]_{R_j}\left(\tilde{I}_{R_j}^{R_j} + m_{R_j}\tilde{R}_{R_j}^{P}\right)[T]_{R_j}^{T} + [T]_{R_j} m_{R_j}\frac{d}{dt}\left(\tilde{R}_{R_j}^{P}\right)[T]_{R_j}^{T} + [T]_{R_j}\left(\tilde{I}_{R_j}^{R_j} + m_{R_j}\tilde{R}_{R_j}^{P}\right)[\dot{T}]_{R_j}^{T}\right)\frac{\partial R_{R_j}^{P}}{\partial \delta_W}\begin{Bmatrix}\omega_{R_j}\\0\\0\end{Bmatrix}$$

$$+\frac{\partial\left[[T]_{R_j}\left(\tilde{I}_{R_j}^{R_j} + m_{R_j}\tilde{R}_{R_j}^{P}\right)[T]_{R_j}^{T}\right]}{\partial \delta_W}\left(\dot{R}_{R_j}^{P}\begin{Bmatrix}\omega_{R_j}\\0\\0\end{Bmatrix} + R_{R_j}^{P}\begin{Bmatrix}\dot{\omega}_{R_j}\\0\\0\end{Bmatrix}\right)$$

$$+[T]_{R_j}\left(\tilde{I}_{R_j}^{R_j} + m_{R_j}\tilde{R}_{R_j}^{P}\right)[T]_{R_j}^{T}\left(\frac{\partial \dot{R}_{R_j}^{P}}{\partial \delta_W}\begin{Bmatrix}\omega_{R_j}\\0\\0\end{Bmatrix} + \frac{\partial R_{R_j}^{P}}{\partial \delta_W}\begin{Bmatrix}\dot{\omega}_{R_j}\\0\\0\end{Bmatrix}\right)$$

$$+\Omega_{P_W}\left[\frac{\partial\left[[T]_{R_j}\left(\tilde{I}_{R_j}^{R_j} + m_{R_j}\tilde{R}_{R_j}^{P}\right)[T]_{R_j}^{T}\right]}{\partial \delta_W} R_{R_j}^{P}\begin{Bmatrix}\omega_{R_j}\\0\\0\end{Bmatrix} + [T]_{R_j}\left(\tilde{I}_{R_j}^{R_j} + m_{R_j}\tilde{R}_{R_j}^{P}\right)[T]_{R_j}^{T}\frac{\partial R_{R_j}^{P}}{\partial \delta_W}\begin{Bmatrix}\omega_{R_j}\\0\\0\end{Bmatrix}\right]$$

$$\left. + m_{R_j}\frac{\partial\left[\dot{R}_{R_j}^{P}\vec{r}_{R_j/pivot_j}\times\dot{R}_{R_j}^{P} + \vec{r}_{R_j/P}\times\left(2\Omega_{P_W}\dot{R}_{R_j}^{P} + \ddot{R}_{R_j}^{P}\right)\right]}{\partial \delta_W}\vec{r}_{R_j/pivot_j}\right\}$$

The next steps are much similar to the respective partial derivatives of E, previously derived, but for the wing or horizontal tail alone with respect to their pivot point. Therefore, it will be omitted here.

$$\frac{\partial\,(E)_P}{\partial \dot{\delta}_W} = \frac{\partial\,(E)_{P_W}}{\partial \dot{\delta}_W} + \frac{\partial\,(E)_{P_R}}{\partial \dot{\delta}_W}$$

Expanding the terms,

$$\frac{\partial\,(E)_{P_W}}{\partial \dot{\delta}_W} = m_{W_i}\left[\frac{\partial \dot{R}_{W_i}^{P}}{\partial \dot{\delta}_W}\vec{r}_{W_i/pivot_i}\times\dot{R}_{W_i}^{P} + \dot{R}_{W_i}^{P}\vec{r}_{W_i/pivot_i}\times\frac{\partial \dot{R}_{W_i}^{P}}{\partial \dot{\delta}_W} + \vec{r}_{W_i/P}\times\left(2\frac{\partial \Omega_{P_W}}{\partial \dot{\delta}_W}\dot{R}_{W_i}^{P} + 2\Omega_{P_W}\frac{\partial \dot{R}_{W_i}^{P}}{\partial \dot{\delta}_W} + \frac{\partial \ddot{R}_{W_i}^{P}}{\partial \dot{\delta}_W}\right)\right]\vec{r}_{W_i/pivot_i}$$

$$\frac{\partial (E)_{P_R}}{\partial \dot{\delta}_W} = \left(\frac{\partial [\dot{T}]_{R_j}}{\partial \dot{\delta}_W}\left(\tilde{I}_{R_j}^{R_j} + m_{R_j}\tilde{R}_{R_j}^{P}\right)[T]_{R_j}^{T} + [T]_{R_j} m_{R_j}\frac{\partial \dot{\tilde{R}}_{R_j}^{P}}{\partial \dot{\delta}_W}[T]_{R_j}^{T} + [T]_{R_j}\left(\tilde{I}_{R_j}^{R_j} + m_{R_j}\tilde{R}_{R_j}^{P}\right)\frac{\partial [\dot{T}]_{R_j}}{\partial \dot{\delta}_W}\right) R_{R_j}^{P}\begin{Bmatrix}\omega_{R_j}\\0\\0\end{Bmatrix}$$

$$+[T]_{R_j}\left(\tilde{I}_{R_j}^{R_j} + m_{R_j}\tilde{R}_{R_j}^{P}\right)[T]_{R_j}^{T}\frac{\partial \dot{R}_{R_j}^{P}}{\partial \dot{\delta}_W}\begin{Bmatrix}\omega_{R_j}\\0\\0\end{Bmatrix} + \frac{\partial \Omega_{P_W}}{\partial \dot{\delta}_W}[T]_{R_j}\left(\tilde{I}_{R_j}^{R_j} + m_{R_j}\tilde{R}_{R_j}^{P}\right)[T]_{R_j}^{T} R_{R_j}^{P}\begin{Bmatrix}\omega_{R_j}\\0\\0\end{Bmatrix}$$

$$+ m_{R_j}\left[\frac{\partial \dot{R}_{R_j}^{P}}{\partial \dot{\delta}_W}\vec{r}_{R_j/pivot_j}\times\dot{R}_{R_j}^{P} + \dot{R}_{R_j}^{P}\vec{r}_{R_j/pivot_j}\times\frac{\partial \dot{R}_{R_j}^{P}}{\partial \dot{\delta}_W} + \vec{r}_{R_j/P}\right.$$

$$\left.\times\left(2\frac{\partial \Omega_{P_W}}{\partial \dot{\delta}_W}\dot{R}_{R_j}^{P} + 2\Omega_{P_W}\frac{\partial \dot{R}_{R_j}^{P}}{\partial \dot{\delta}_W} + \frac{\partial \ddot{R}_{R_j}^{P}}{\partial \dot{\delta}_W}\right)\right]\vec{r}_{R_j/pivot_j}$$

$$\frac{\partial\,(E)_P}{\partial \ddot{\delta}_W} = \frac{\partial\,(E)_{P_W}}{\partial \ddot{\delta}_W} + \frac{\partial\,(E)_{P_R}}{\partial \ddot{\delta}_W}$$

Expanding the terms,

$$\frac{\partial\,(E)_{P_W}}{\partial \ddot{\delta}_W} = \sum_{i=1}^{NW}\left\{m_{W_i}\vec{r}_{W_i/P}\times\frac{\partial \ddot{R}_{W_i}^{P}}{\partial \ddot{\delta}_W}\vec{r}_{W_i/pivot_i}\right\}$$

$$\frac{\partial\,(E)_{P_R}}{\partial\ddot{\delta}_W}=\sum_{j=1}^{NR}\left\{m_{R_j}\vec{r}_{R_j/P}\times\frac{\partial\ddot{R}^P_{W_i}}{\partial\ddot{\delta}_{HT}}\vec{r}_{R_j/pivot_j}\right\}$$

$$\frac{\partial(E)_{P_{HT}}}{\partial\delta_{\mathrm{HT}}}=\frac{\partial\,(E)_{P_{HT}}}{\partial\delta_{\mathrm{HT}}}+\frac{\partial\,(E)_{P_R}}{\partial\delta_{\mathrm{HT}}}$$

Expanding the terms,

$$\frac{\partial\,(E)_{P_W}}{\partial\delta_{\mathrm{HT}}}=\sum_{i=1}^{NW}\left\{m_{W_i}\left[\frac{\partial\dot{R}^P_{W_i}}{\partial\delta_{\mathrm{HT}}}\vec{r}_{W_i/pivot_i}\times\dot{R}^P_{W_i}+\dot{R}^P_{W_i}\vec{r}_{W_i/pivot_i}\times\frac{\partial\dot{R}^P_{W_i}}{\partial\delta_{\mathrm{HT}}}+\vec{r}_{W_i/P}\times\left(2\Omega_{P_{HT}}\frac{\partial\dot{R}^P_{W_i}}{\partial\delta_{\mathrm{HT}}}+\frac{\partial\ddot{R}^P_{W_i}}{\partial\delta_{\mathrm{HT}}}\right)\right]\vec{r}_{W_i/pivot_i}\right\}$$

$$\begin{aligned}\frac{\partial\,(E)_{P_R}}{\partial\delta_{\mathrm{HT}}}=\sum_{j=1}^{NR}\Bigg\{&\Bigg(\Bigg(\frac{\partial\left[[\dot{T}]_{R_j}\left(\tilde{I}^{R_j}_{R_j}+m_{R_j}\tilde{R}^P_{R_j}\right)[T]^T_{R_j}\right]}{\partial\delta_{\mathrm{HT}}}+\frac{\partial\left[[T]_{R_j}m_{R_j}\frac{d}{dt}\left(\tilde{R}^P_{R_j}\right)[T]^T_{R_j}\right]}{\partial\delta_{\mathrm{HT}}}\\
&+\frac{\partial\left[[T]_{R_j}\left(\tilde{I}^{R_j}_{R_j}+m_{R_j}\tilde{R}^P_{R_j}\right)[\dot{T}]^T_{R_j}\right]}{\partial\delta_{\mathrm{HT}}}\Bigg)R^P_{R_j}\begin{Bmatrix}\omega_{R_j}\\0\\0\end{Bmatrix}\\
&+\left([\dot{T}]_{R_j}\left(\tilde{I}^{R_j}_{R_j}+m_{R_j}\tilde{R}^P_{R_j}\right)[T]^T_{R_j}+[T]_{R_j}m_{R_j}\frac{d}{dt}\left(\tilde{R}^P_{R_j}\right)[T]^T_{R_j}+[T]_{R_j}\left(\tilde{I}^{R_j}_{R_j}+m_{R_j}\tilde{R}^P_{R_j}\right)[\dot{T}]^T_{R_j}\right)\frac{\partial R^P_{R_j}}{\partial\delta_{\mathrm{HT}}}\begin{Bmatrix}\omega_{R_j}\\0\\0\end{Bmatrix}\\
&+\frac{\partial\left[[T]_{R_j}\left(\tilde{I}^{R_j}_{R_j}+m_{R_j}\tilde{R}^P_{R_j}\right)[T]^T_{R_j}\right]}{\partial\delta_{\mathrm{HT}}}\left(\dot{R}^P_{R_j}\begin{Bmatrix}\omega_{R_j}\\0\\0\end{Bmatrix}+R^P_{R_j}\begin{Bmatrix}\dot{\omega}_{R_j}\\0\\0\end{Bmatrix}\right)\\
&+[T]_{R_j}\left(\tilde{I}^{R_j}_{R_j}+m_{R_j}\tilde{R}^P_{R_j}\right)[T]^T_{R_j}\left(\frac{\partial\dot{R}^P_{R_j}}{\partial\delta_{\mathrm{HT}}}\begin{Bmatrix}\omega_{R_j}\\0\\0\end{Bmatrix}+\frac{\partial R^P_{R_j}}{\partial\delta_{\mathrm{HT}}}\begin{Bmatrix}\dot{\omega}_{R_j}\\0\\0\end{Bmatrix}\right)\\
&+\Omega_{P_{HT}}\left[\frac{\partial\left[[T]_{R_j}\left(\tilde{I}^{R_j}_{R_j}+m_{R_j}\tilde{R}^P_{R_j}\right)[T]^T_{R_j}\right]}{\partial\delta_{\mathrm{HT}}}R^P_{R_j}\begin{Bmatrix}\omega_{R_j}\\0\\0\end{Bmatrix}+[T]_{R_j}\left(\tilde{I}^{R_j}_{R_j}+m_{R_j}\tilde{R}^P_{R_j}\right)[T]^T_{R_j}\frac{\partial R^P_{R_j}}{\partial\delta_{\mathrm{HT}}}\begin{Bmatrix}\omega_{R_j}\\0\\0\end{Bmatrix}\right]\\
&+m_{R_j}\frac{\partial\left[\dot{R}^P_{R_j}\vec{r}_{R_j/pivot_j}\times\dot{R}^P_{R_j}+\vec{r}_{R_j/P}\times\left(2\Omega_{P_{HT}}\dot{R}^P_{R_j}+\ddot{R}^P_{R_j}\right)\right]}{\partial\delta_{\mathrm{HT}}}\vec{r}_{R_j/pivot_j}\Bigg\}\end{aligned}$$

$$\frac{\partial(E)_{P_{HT}}}{\partial\dot{\delta}_{\mathrm{HT}}}=\frac{\partial\,(E)_{P_{HT}}}{\partial\dot{\delta}_{\mathrm{HT}}}+\frac{\partial\,(E)_{P_R}}{\partial\dot{\delta}_{\mathrm{HT}}}$$

Expanding the terms,

$$\frac{\partial\,(E)_{P_W}}{\partial\dot{\delta}_{HT}}=m_{W_i}\left[\frac{\partial\dot{R}^P_{W_i}}{\partial\dot{\delta}_{HT}}\vec{r}_{W_i/pivot_i}\times\dot{R}^P_{W_i}+\dot{R}^P_{W_i}\vec{r}_{W_i/pivot_i}\times\frac{\partial\dot{R}^P_{W_i}}{\partial\dot{\delta}_{HT}}+\vec{r}_{W_i/P}\times\left(2\frac{\partial\Omega_{P_{HT}}}{\partial\dot{\delta}_{HT}}\dot{R}^P_{W_i}+2\Omega_{P_{HT}}\frac{\partial\dot{R}^P_{W_i}}{\partial\dot{\delta}_{HT}}+\frac{\partial\ddot{R}^P_{W_i}}{\partial\dot{\delta}_{HT}}\right)\right]\vec{r}_{W_i/pivot_i}$$

$$\begin{aligned}\frac{\partial(E)_{P_R}}{\partial\dot{\delta}_{HT}}=&\left(\frac{\partial[\dot{T}]_{R_j}}{\partial\dot{\delta}_{HT}}\left(\tilde{I}^{R_j}_{R_j}+m_{R_j}\tilde{R}^P_{R_j}\right)[T]^T_{R_j}+[T]_{R_j}m_{R_j}\frac{\partial\dot{\tilde{R}}^P_{R_j}}{\partial\dot{\delta}_{HT}}[T]^T_{R_j}+[T]_{R_j}\left(\tilde{I}^{R_j}_{R_j}+m_{R_j}\tilde{R}^P_{R_j}\right)\frac{\partial[\dot{T}]_{R_j}}{\partial\dot{\delta}_{HT}}\right)R^P_{R_j}\begin{Bmatrix}\omega_{R_j}\\0\\0\end{Bmatrix}\\
&+[T]_{R_j}\left(\tilde{I}^{R_j}_{R_j}+m_{R_j}\tilde{R}^P_{R_j}\right)[T]^T_{R_j}\frac{\partial\dot{R}^P_{R_j}}{\partial\dot{\delta}_{HT}}\begin{Bmatrix}\omega_{R_j}\\0\\0\end{Bmatrix}+\frac{\partial\Omega_{P_{HT}}}{\partial\dot{\delta}_{HT}}[T]_{R_j}\left(\tilde{I}^{R_j}_{R_j}+m_{R_j}\tilde{R}^P_{R_j}\right)[T]^T_{R_j}R^P_{R_j}\begin{Bmatrix}\omega_{R_j}\\0\\0\end{Bmatrix}\\
&+m_{R_j}\left[\frac{\partial\dot{R}^P_{R_j}}{\partial\dot{\delta}_{HT}}\vec{r}_{R_j/pivot_j}\times\dot{R}^P_{R_j}+\dot{R}^P_{R_j}\vec{r}_{R_j/pivot_j}\times\frac{\partial\dot{R}^P_{R_j}}{\partial\dot{\delta}_{HT}}+\vec{r}_{R_j/P}\right.\\
&\left.\times\left(2\frac{\partial\Omega_{P_{HT}}}{\partial\dot{\delta}_{HT}}\dot{R}^P_{R_j}+2\Omega_{P_{HT}}\frac{\partial\dot{R}^P_{R_j}}{\partial\dot{\delta}_{HT}}+\frac{\partial\ddot{R}^P_{R_j}}{\partial\dot{\delta}_{HT}}\right)\right]\vec{r}_{R_j/pivot_j}\end{aligned}$$

$$\frac{\partial\,(E)_P}{\partial\ddot{\delta}_{HT}} = \frac{\partial\,(E)_{P\,HT}}{\partial\ddot{\delta}_{HT}} + \frac{\partial\,(E)_{P\,R}}{\partial\ddot{\delta}_{HT}}$$

Expanding the terms,

$$\frac{\partial\,(E)_{P\,HT}}{\partial\ddot{\delta}_{HT}} = \sum_{i=1}^{NW}\left\{m_{W_i}\vec{r}_{W_i/P} \times \frac{\partial\ddot{R}^P_{W_i}}{\partial\ddot{\delta}_{HT}}\vec{r}_{W_i/pivot_i}\right\}$$

$$\frac{\partial\,(E)_{P\,R}}{\partial\ddot{\delta}_{HT}} = \sum_{j=1}^{NR}\left\{m_{R_j}\vec{r}_{R_j/P} \times \frac{\partial\ddot{R}^P_{W_i}}{\partial\ddot{\delta}_{HT}}\vec{r}_{R_j/pivot_j}\right\}$$

$$\frac{\partial(E)_{P\,W,HT}}{\partial\omega_j^2} = \left([\dot{T}]_{R_j}\left(\tilde{I}^{R_j}_{R_j} + m_{R_j}\tilde{R}^P_{R_j}\right)[T]^T_{R_j} + [T]_{R_j}m_{R_j}\frac{d}{dt}\left(\tilde{R}^P_{R_j}\right)[T]^T_{R_j} + [T]_{R_j}\left(\tilde{I}^{R_j}_{R_j} + m_{R_j}\tilde{R}^P_{R_j}\right)[\dot{T}]^T_{R_j}\right)R^P_{R_j}\begin{Bmatrix}\frac{1}{2\sqrt{\omega^2_{R_j}}}\\0\\0\end{Bmatrix}$$
$$+ [T]_{R_j}\left(\tilde{I}^{R_j}_{R_j} + m_{R_j}\tilde{R}^P_{R_j}\right)[T]^T_{R_j}\dot{R}^B_{R_j}\begin{Bmatrix}\frac{1}{2\sqrt{\omega^2_{R_j}}}\\0\\0\end{Bmatrix} + \Omega_{P\,W,HT}[T]_{R_j}\left(\tilde{I}^{R_j}_{R_j} + m_{R_j}\tilde{R}^P_{R_j}\right)[T]^T_{R_j}R^P_{R_j}\begin{Bmatrix}\frac{1}{2\sqrt{\omega^2_{R_j}}}\\0\\0\end{Bmatrix}$$

$$\frac{\partial(E)_{P\,W,HT}}{\partial\left(\dot{\omega_j^2}\right)} = [T]_{R_j}\left(\tilde{I}^{R_j}_{R_j} + m_{R_j}\tilde{R}^P_{R_j}\right)[T]^T_{R_j}R^B_{R_j}\begin{Bmatrix}\frac{1}{2\sqrt{\dot{\omega}^2_{R_j}}}\\0\\0\end{Bmatrix}$$

18.77 Partial derivatives of $\boldsymbol{P_{1W}}$:

$$\frac{\partial P_{1W}}{\partial\delta_W} = -\left[\frac{\partial(A)^{-1}_{P\ W}}{\partial\delta_W}(B)_{P\,W} + (A)^{-1}_{P\ W}\frac{\partial(B)_{P\,W}}{\partial\delta_W}\right]_2$$

$$\frac{\partial P_{1W}}{\partial\dot{\delta}_W} = -\left[(A)^{-1}_{P\ W}\frac{\partial(B)_{P\,W}}{\partial\dot{\delta}_W}\right]_2$$

18.78 Partial derivatives of $\boldsymbol{P_{2W}}$:

$$\frac{\partial P_{2W}}{\partial V_T} = \left[(A)^{-1}_{P\ W}S^T\begin{Bmatrix}\bar{L}_{V_T}\\M_{V_T}\\N_{V_T}\end{Bmatrix}_W\right]_2 - \left[\left[(A)^{-1}_{P\ W}(C)_{P\,W}\dot{S}^T + (A)^{-1}_{P\ W}(D)_{P\,W}S^T\right]\begin{Bmatrix}1\\0\\0\end{Bmatrix}\right]_2$$

$$\frac{\partial P_{2W}}{\partial\beta} = \left[(A)^{-1}_{P\ W}\frac{\partial S^T}{\partial\beta}\begin{Bmatrix}\bar{L}\\M\\N\end{Bmatrix}_W + (A)^{-1}_{P\ W}S^T\begin{Bmatrix}\bar{L}_\beta\\0\\N_\beta\end{Bmatrix}_W\right]_2 - \left[\left[(A)^{-1}_{P\ W}(C)_{P\,W}\frac{\partial\dot{S}^T}{\partial\beta} + (A)^{-1}_{P\ W}(D)_{P\,W}\frac{\partial S^T}{\partial\beta}\right]\begin{Bmatrix}V_T\\0\\0\end{Bmatrix}\right]_2$$
$$-\left[(A)^{-1}_{P\ W}(C)_{P\,W}\frac{\partial S^T}{\partial\beta}\begin{Bmatrix}\dot{V}_T\\0\\0\end{Bmatrix}\right]_2$$

$$\frac{\partial P_{2W}}{\partial \alpha} = \left[(A)_{P}^{-1}{}_{W} \frac{\partial S^T}{\partial \alpha} \begin{Bmatrix} \bar{L} \\ M \\ N \end{Bmatrix}_W + (A)_{P}^{-1}{}_{W} S^T \begin{Bmatrix} \bar{L}_\alpha \\ M_\alpha \\ N_\alpha \end{Bmatrix}_W \right]_2 - \left[\left[(A)_{P}^{-1}{}_{W} (C)_{PW} \frac{\partial \dot{S}^T}{\partial \alpha} + (A)_{P}^{-1}{}_{W} (D)_{PW} \frac{\partial S^T}{\partial \alpha} \right] \begin{Bmatrix} V_T \\ 0 \\ 0 \end{Bmatrix} \right]_2 - \left[(A)_{P}^{-1}{}_{W} (C)_{PW} \frac{\partial S^T}{\partial \alpha} \begin{Bmatrix} \dot{V}_T \\ 0 \\ 0 \end{Bmatrix} \right]_2$$

$$\frac{\partial P_{2W}}{\partial P_W} = \left[(A)_{P}^{-1}{}_{W} S^T \begin{Bmatrix} \bar{L}_{P_W} \\ 0 \\ N_{P_W} \end{Bmatrix}_W \right]_2$$

$$\frac{\partial P_{2W}}{\partial Q_W} = \left[(A)_{P}^{-1}{}_{W} S^T \begin{Bmatrix} 0 \\ M_{Q_W} \\ 0 \end{Bmatrix}_W \right]_2$$

$$\frac{\partial P_{2W}}{\partial R_W} = \left[(A)_{P}^{-1}{}_{W} S^T \begin{Bmatrix} \bar{L}_{R_W} \\ 0 \\ N_{R_W} \end{Bmatrix}_W \right]_2$$

$$\frac{\partial P_{2W}}{\partial \phi} = \left[(A)_{P}^{-1}{}_{W} \frac{\partial (M_P)_{PW}}{\partial \phi} \right]_2$$

$$\frac{\partial P_{2W}}{\partial \theta} = \left[(A)_{P}^{-1}{}_{W} \frac{\partial (M_P)_{PW}}{\partial \theta} \right]_2$$

$$\frac{\partial P_{2W}}{\partial \psi} = \left[(A)_{P}^{-1}{}_{W} \frac{\partial (M_P)_{PW}}{\partial \psi} \right]_2$$

$$\frac{\partial P_{2W}}{\partial h} = \left[(A)_{P}^{-1}{}_{W} S^T \frac{\partial \rho}{\partial h} \frac{1}{2} V_T^2 S_W \begin{Bmatrix} bC_l \\ \bar{c}C_m \\ bC_n \end{Bmatrix}_W \right]_2 + \sum_{j=1}^{NRW} \left(\left[(A)_{P}^{-1}{}_{W} R_{R_j}^B \begin{Bmatrix} \lambda_j \frac{\partial k_Q}{\partial h} \omega_j^2 \\ 0 \\ 0 \end{Bmatrix} + (A)_{P}^{-1}{}_{W} \vec{r}_{R_j/P} \times R_{R_j}^B \begin{Bmatrix} \frac{\partial k_T}{\partial h} \omega_j^2 \\ 0 \\ 0 \end{Bmatrix} \right]_2 \right)$$

$$\frac{\partial P_{2W}}{\partial \delta_f} = \left[(A)_{P}^{-1}{}_{W} S^T \begin{Bmatrix} 0 \\ M_{\delta_f} \\ 0 \end{Bmatrix}_W \right]_2$$

$$\frac{\partial P_{2W}}{\partial \delta_{a_L}} = \left[(A)_{P}^{-1}{}_{W} S^T \begin{Bmatrix} \bar{L}_{\delta_{a_L}} \\ 0 \\ N_{\delta_{a_L}} \end{Bmatrix}_W \right]_2$$

$$\frac{\partial P_{2W}}{\partial \delta_{a_R}} = \left[(A)_{P}^{-1}{}_{W} S^T \begin{Bmatrix} \bar{L}_{\delta_{a_R}} \\ 0 \\ N_{\delta_{a_R}} \end{Bmatrix}_W \right]_2$$

$$\frac{\partial P_{2W}}{\partial \delta_W} = \left[\frac{\partial (A)_P^{-1}{}_W}{\partial \delta_W} S^T \begin{Bmatrix} \bar{L} \\ M \\ N \end{Bmatrix}_W + (A)_P^{-1}{}_W S^T \begin{Bmatrix} 0 \\ M_{\delta_W} \\ 0 \end{Bmatrix}_W\right]_2$$
$$+ \sum_{j=1}^{NRW} \left(\left[\frac{\partial (A)_P^{-1}{}_W}{\partial \delta_W} R_{R_j}^B \begin{Bmatrix} \lambda_j k_Q \omega_j^2 \\ 0 \\ 0 \end{Bmatrix} + (A)_P^{-1}{}_W \frac{\partial R_{R_j}^B}{\partial \delta_W} \begin{Bmatrix} \lambda_j k_Q \omega_j^2 \\ 0 \\ 0 \end{Bmatrix} + \frac{\partial (A)_P^{-1}{}_W}{\partial \delta_W} \vec{r}_{R_j/P} \times R_{R_j}^B \begin{Bmatrix} k_T \omega_j^2 \\ 0 \\ 0 \end{Bmatrix}\right.\right.$$
$$\left.\left. + (A)_P^{-1}{}_W \left[\frac{\partial \left[\vec{r}_{R_j/P} \times\right]}{\partial \delta_W} R_{R_j}^B + \vec{r}_{R_j/P} \times \frac{\partial R_{R_j}^B}{\partial \delta_W}\right] \begin{Bmatrix} k_T \omega_j^2 \\ 0 \\ 0 \end{Bmatrix}\right]_2 \right)$$
$$- \left[\left[\frac{\partial (A)_P^{-1}{}_W}{\partial \delta_W} (C)_{P_W} \dot{S}^T + (A)_P^{-1}{}_W \frac{\partial (C)_{P_W}}{\partial \delta_W} \dot{S}^T + \frac{\partial (A)_P^{-1}{}_W}{\partial \delta_W} (D)_{P_W} S^T + (A)_P^{-1}{}_W \frac{\partial (D)_{P_W}}{\partial \delta_W} S^T\right] \begin{Bmatrix} V_T \\ 0 \\ 0 \end{Bmatrix}\right]_2$$
$$- \left[\left[\frac{\partial (A)_P^{-1}{}_W}{\partial \delta_W} (C)_{P_W} S^T + (A)_P^{-1}{}_W \frac{\partial (C)_{P_W}}{\partial \delta_W} S^T\right] \begin{Bmatrix} \dot{V}_T \\ 0 \\ 0 \end{Bmatrix}\right]_2$$
$$+ \left[\frac{\partial (A)_P^{-1}{}_W}{\partial \delta_W} (M_P)_{P_W} + (A)_P^{-1}{}_W \frac{\partial (M_P)_{P_W}}{\partial \delta_W} - \frac{\partial (A)_P^{-1}{}_W}{\partial \delta_W} (E)_{P_W} - (A)_P^{-1}{}_W \frac{\partial (E)_{P_W}}{\partial \delta_W}\right]_2$$

$$\frac{\partial P_{2W}}{\partial \dot{\delta}_W} = -\left[\left[(A)_P^{-1}{}_W \frac{\partial (D)_{P_W}}{\partial \dot{\delta}_W} S^T\right] \begin{Bmatrix} V_T \\ 0 \\ 0 \end{Bmatrix}\right]_2 - \left[(A)_P^{-1}{}_W \frac{\partial (E)_{P_W}}{\partial \dot{\delta}_W}\right]_2$$

$$\frac{\partial P_{2W}}{\partial \ddot{\delta}_W} = -\left[(A)_P^{-1}{}_W \frac{\partial (E)_{P_W}}{\partial \ddot{\delta}_W}\right]_2$$

$$\frac{\partial P_{2W}}{\partial \omega_j^2} = \left[(A)_P^{-1}{}_W S^T \begin{Bmatrix} \bar{L}_{\omega_j^2} \\ M_{\omega_j^2} \\ N_{\omega_j^2} \end{Bmatrix}_W\right]_2 + \sum_{j=1}^{NRW} \left(\left[(A)_P^{-1}{}_W R_{R_j}^B \begin{Bmatrix} \lambda_j k_Q \\ 0 \\ 0 \end{Bmatrix} + (A)_P^{-1}{}_W \vec{r}_{R_j/P} \times R_{R_j}^B \begin{Bmatrix} k_T \\ 0 \\ 0 \end{Bmatrix}\right]_2 \right) - \left[(A)_P^{-1}{}_W \frac{\partial (E)_{P_W}}{\partial \omega_j^2}\right]_2$$

$$\frac{\partial P_{2W}}{\partial \left(\dot{\omega_j^2}\right)} = -\left[(A)_P^{-1}{}_W \frac{\partial (E)_{P_W}}{\partial \left(\dot{\omega_j^2}\right)}\right]_2$$

18.79 Partial derivatives of $\boldsymbol{P_{1_{HT}}}$:

$$\frac{\partial P_{1_{HT}}}{\partial \delta_{HT}} = -\left[\frac{\partial (A)_P^{-1}{}_{HT}}{\partial \delta_{HT}} (B)_{P_{HT}} + (A)_P^{-1}{}_{HT} \frac{\partial (B)_{P_{HT}}}{\partial \delta_{HT}}\right]_2$$

$$\frac{\partial P_{1_{HT}}}{\partial \dot{\delta}_{HT}} = -\left[(A)_P^{-1}{}_{HT} \frac{\partial (B)_{P_{HT}}}{\partial \dot{\delta}_{HT}}\right]_2$$

18.80 Partial derivatives of $\boldsymbol{P_{2_{HT}}}$:

$$\frac{\partial P_{2_{HT}}}{\partial V_T} = \left[(A)_P^{-1}{}_{HT} S^T \begin{Bmatrix} \bar{L}_{V_T} \\ M_{V_T} \\ N_{V_T} \end{Bmatrix}_{HT}\right]_2 - \left[\left[(A)_P^{-1}{}_{HT} (C)_{P_{HT}} \dot{S}^T + (A)_P^{-1}{}_{HT} (D)_{P_{HT}} S^T\right] \begin{Bmatrix} 1 \\ 0 \\ 0 \end{Bmatrix}\right]_2$$

$$\frac{\partial P_{2_{HT}}}{\partial \beta} = \left[(A)_{P}^{-1}{}_{HT}\frac{\partial S^T}{\partial \beta}\begin{Bmatrix}\bar{L}\\M\\N\end{Bmatrix}_{HT} + (A)_{P}^{-1}{}_{HT}S^T\begin{Bmatrix}\bar{L}_\beta\\0\\N_\beta\end{Bmatrix}_{HT}\right]_2 - \left[\left[(A)_{P}^{-1}{}_{HT}(C)_{P_{HT}}\frac{\partial \dot{S}^T}{\partial \beta} + (A)_{P}^{-1}{}_{HT}(D)_{P_{HT}}\frac{\partial S^T}{\partial \beta}\right]\begin{Bmatrix}V_T\\0\\0\end{Bmatrix}\right]_2 - \left[(A)_{P}^{-1}{}_{HT}(C)_{P_{HT}}\frac{\partial S^T}{\partial \beta}\begin{Bmatrix}\dot{V}_T\\0\\0\end{Bmatrix}\right]_2$$

$$\frac{\partial P_{2_{HT}}}{\partial \alpha} = \left[(A)_{P}^{-1}{}_{HT}\frac{\partial S^T}{\partial \alpha}\begin{Bmatrix}\bar{L}\\M\\N\end{Bmatrix}_{HT} + (A)_{P}^{-1}{}_{HT}S^T\begin{Bmatrix}0\\M_\alpha\\0\end{Bmatrix}_{HT}\right]_2 - \left[\left[(A)_{P}^{-1}{}_{HT}(C)_{P_{HT}}\frac{\partial \dot{S}^T}{\partial \alpha} + (A)_{P}^{-1}{}_{HT}(D)_{P_{HT}}\frac{\partial S^T}{\partial \alpha}\right]\begin{Bmatrix}V_T\\0\\0\end{Bmatrix}\right]_2 - \left[(A)_{P}^{-1}{}_{HT}(C)_{P_{HT}}\frac{\partial S^T}{\partial \alpha}\begin{Bmatrix}\dot{V}_T\\0\\0\end{Bmatrix}\right]_2$$

$$\frac{\partial P_{2_{HT}}}{\partial P_W} = \left[(A)_{P}^{-1}{}_{HT}S^T\begin{Bmatrix}\bar{L}_{P_W}\\0\\N_{P_W}\end{Bmatrix}_{HT}\right]_2$$

$$\frac{\partial P_{2_{HT}}}{\partial Q_W} = \left[(A)_{P}^{-1}{}_{HT}S^T\begin{Bmatrix}0\\M_{Q_W}\\0\end{Bmatrix}_{HT}\right]_2$$

$$\frac{\partial P_{2_{HT}}}{\partial R_W} = \left[(A)_{P}^{-1}{}_{HT}S^T\begin{Bmatrix}\bar{L}_{R_W}\\0\\N_{R_W}\end{Bmatrix}_{HT}\right]_2$$

$$\frac{\partial P_{2_{HT}}}{\partial \phi} = \left[(A)_{P}^{-1}{}_{HT}\frac{\partial (M_P)_{P_{HT}}}{\partial \phi}\right]_2$$

$$\frac{\partial P_{2_{HT}}}{\partial \theta} = \left[(A)_{P}^{-1}{}_{HT}\frac{\partial (M_P)_{P_{HT}}}{\partial \theta}\right]_2$$

$$\frac{\partial P_{2_{HT}}}{\partial \psi} = \left[(A)_{P}^{-1}{}_{HT}\frac{\partial (M_P)_{P_{HT}}}{\partial \psi}\right]_2$$

$$\frac{\partial P_{2_{HT}}}{\partial h} = \left[(A)_{P}^{-1}{}_{HT}S^T\frac{\partial \rho}{\partial h}\frac{1}{2}V_T^2 S_W\begin{Bmatrix}bC_l\\\bar{c}C_m\\bC_n\end{Bmatrix}_{HT}\right]_2 + \sum_{j=1}^{NRW}\left(\left[(A)_{P}^{-1}{}_{HT}R_{R_j}^{B}\begin{Bmatrix}\lambda_j\frac{\partial k_Q}{\partial h}\omega_j^2\\0\\0\end{Bmatrix} + (A)_{P}^{-1}{}_{HT}\vec{r}_{R_j/P}\times R_{R_j}^{B}\begin{Bmatrix}\frac{\partial k_T}{\partial h}\omega_j^2\\0\\0\end{Bmatrix}\right]_2\right)$$

$$\frac{\partial P_{2_{HT}}}{\partial \delta_e} = \left[(A)_{P}^{-1}{}_{HT}S^T\begin{Bmatrix}0\\M_{\delta_e}\\0\end{Bmatrix}_{HT}\right]_2$$

$$\frac{\partial P_{2_{HT}}}{\partial \delta_{\mathrm{HT}}} = \left[\frac{\partial (A)_{P}^{-1}{}_{HT}}{\partial \delta_{\mathrm{HT}}} S^T \begin{Bmatrix} \bar{L} \\ M \\ N \end{Bmatrix}_{HT} + (A)_{P}^{-1}{}_{HT} S^T \begin{Bmatrix} 0 \\ M_{\delta_{HT}} \\ 0 \end{Bmatrix}_{HT}\right]_2$$

$$+ \sum_{j=1}^{NRW} \left(\left[\frac{\partial (A)_{P}^{-1}{}_{HT}}{\partial \delta_{\mathrm{HT}}} R_{R_j}^B \begin{Bmatrix} \lambda_j k_Q \omega_j^2 \\ 0 \\ 0 \end{Bmatrix} + (A)_{P}^{-1}{}_{HT} \frac{\partial R_{R_j}^B}{\partial \delta_{\mathrm{HT}}} \begin{Bmatrix} \lambda_j k_Q \omega_j^2 \\ 0 \\ 0 \end{Bmatrix} + \frac{\partial (A)_{P}^{-1}{}_{HT}}{\partial \delta_{\mathrm{HT}}} \vec{r}_{R_j/P} \times R_{R_j}^B \begin{Bmatrix} k_T \omega_j^2 \\ 0 \\ 0 \end{Bmatrix} \right.\right.$$

$$\left.\left. + (A)_{P}^{-1}{}_{HT} \left[\frac{\partial \left[\vec{r}_{R_j/P}\right]}{\partial \delta_{\mathrm{HT}}} \times R_{R_j}^B + \vec{r}_{R_j/P} \times \frac{\partial R_{R_j}^B}{\partial \delta_{\mathrm{HT}}} \right] \begin{Bmatrix} k_T \omega_j^2 \\ 0 \\ 0 \end{Bmatrix} \right]_2 \right)$$

$$- \left[\left[\frac{\partial (A)_{P}^{-1}{}_{HT}}{\partial \delta_{\mathrm{HT}}} (C)_{P_{HT}} \dot{S}^T + (A)_{P}^{-1}{}_{HT} \frac{\partial (C)_{P_{HT}}}{\partial \delta_{\mathrm{HT}}} \dot{S}^T + \frac{\partial (A)_{P}^{-1}{}_{HT}}{\partial \delta_{\mathrm{HT}}} (D)_{P_{HT}} S^T \right.\right.$$

$$\left.\left. + (A)_{P}^{-1}{}_{HT} \frac{\partial (D)_{P_{HT}}}{\partial \delta_{\mathrm{HT}}} S^T \right] \begin{Bmatrix} V_T \\ 0 \\ 0 \end{Bmatrix} \right]_2 - \left[\left[\frac{\partial (A)_{P}^{-1}{}_{HT}}{\partial \delta_{\mathrm{HT}}} (C)_{P_{HT}} S^T + (A)_{P}^{-1}{}_{HT} \frac{\partial (C)_{P_{HT}}}{\partial \delta_{\mathrm{HT}}} S^T \right] \begin{Bmatrix} \dot{V}_T \\ 0 \\ 0 \end{Bmatrix} \right]_2$$

$$+ \left[\frac{\partial (A)_{P}^{-1}{}_{HT}}{\partial \delta_{\mathrm{HT}}} (M_P)_{P_{HT}} + (A)_{P}^{-1}{}_{HT} \frac{\partial (M_P)_{P_{HT}}}{\partial \delta_{\mathrm{HT}}} - \frac{\partial (A)_{P}^{-1}{}_{HT}}{\partial \delta_{\mathrm{HT}}} (E)_{P_{HT}} - (A)_{P}^{-1}{}_{HT} \frac{\partial (E)_{P_{HT}}}{\partial \delta_{\mathrm{HT}}} \right]_2$$

$$\frac{\partial P_{2_{HT}}}{\partial \dot{\delta}_{\mathrm{HT}}} = - \left[\left[(A)_{P}^{-1}{}_{HT} \frac{\partial (D)_{P_{HT}}}{\partial \dot{\delta}_{\mathrm{HT}}} S^T \right] \begin{Bmatrix} V_T \\ 0 \\ 0 \end{Bmatrix} \right]_2 - \left[(A)_{P}^{-1}{}_{HT} \frac{\partial (E)_{P_{HT}}}{\partial \dot{\delta}_{\mathrm{HT}}} \right]_2$$

$$\frac{\partial P_{2_{HT}}}{\partial \omega_j^2} = \left[(A)_{P}^{-1}{}_{HT} S^T \begin{Bmatrix} \bar{L}_{\omega_j^2} \\ M_{\omega_j^2} \\ N_{\omega_j^2} \end{Bmatrix}_{HT} \right]_2 + \sum_{j=1}^{NRW} \left(\left[(A)_{P}^{-1}{}_{HT} R_{R_j}^B \begin{Bmatrix} \lambda_j k_Q \\ 0 \\ 0 \end{Bmatrix} + (A)_{P}^{-1}{}_{HT} \vec{r}_{R_j/P} \times R_{R_j}^B \begin{Bmatrix} k_T \\ 0 \\ 0 \end{Bmatrix} \right]_2 \right) - \left[(A)_{P}^{-1}{}_{HT} \frac{\partial (E)_{P_{HT}}}{\partial \omega_j^2} \right]_2$$

$$\frac{\partial P_{2_{HT}}}{\partial \left(\dot{\omega}_j^2\right)} = - \left[(A)_{P}^{-1}{}_{W} \frac{\partial (E)_{P_{HT}}}{\partial \left(\dot{\omega}_j^2\right)} \right]_2$$

18.81 Partial derivatives of $\boldsymbol{\rho}$:

$$\frac{\partial \rho}{\partial h} = \frac{1}{R} \left(\frac{\frac{\partial p}{\partial h} T - p \frac{\partial T}{\partial h}}{T^2} \right)$$

18.82 Partial derivatives of $\boldsymbol{p}$:

$$\frac{\partial p}{\partial h} = -0.0342 \frac{p_0}{T_0(K)} \left(1 - 0.0065 \frac{h + PA}{T_0(K)} \right)^{4.2561}$$

18.83 Partial derivatives of $\boldsymbol{T}$:

$$\frac{\partial T}{\partial h} = -0.0065$$

19 Appendix B – Control parameters

Follows tables of the control parameters used, for accelerated and decelerated flight.

Table 6: Accelerated flight control parameters, from 0 m/s to 6 m/s.

V_T (m/s)	0	0,5	1	1,5	2	2,5	3	3,5	4	4,5	5	5,5	6
ρ	1,0E+18	1,0E+18	1,0E+18	1,0E+18	1,0E+18	1,0E+18	1,0E+18	1,0E+18	1,0E+18	1,0E+18	1,0E+18	1,0E+18	1,0E+18
ε_{th_s}	2	2	2	2	2	2	2	2	2	2	2	2	2
ε_{th_c}	0,0005	0,0005	0,0005	0,0005	0,0005	0,0005	0,0005	0,0005	0,0005	0,0005	0,0005	0,0005	0,0005
ε_{th_d}	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5
$\left(V_{T_p}\right)_{max}$	1,0E-09	1,0E-09	1,0E-09	1,0E-09	1,0E-09	1,0E-09	1,0E-09	1,0E-09	1,0E-09	1,0E-09	1,0E-09	1,0E-09	1,0E-09
$(\alpha_p)_{max}$	0,000175	0,000175	0,000175	0,000175	0,000175	0,000175	0,000175	0,000175	0,000175	0,000175	0,000175	0,000175	0,000175
$\left(Q_{W_p}\right)_{max}$	1,745	1,745	1,745	1,745	1,745	1,745	0,175	1,745	1,745	1,745	1,745	1,745	1,745
$(\theta_p)_{max}$	0,000175	0,000175	0,000175	0,000175	0,000175	0,000175	0,000175	0,000175	0,000175	0,000175	0,000175	0,000175	0,000175
$(h_p)_{max}$	1,0E-09	1,0E-09	1,0E-09	1,0E-09	1,0E-09	1,0E-08	1,0E-08	1,0E-08	1,0E-08	1,0E-08	1,0E-08	1,0E-08	1,0E-08
$\left(\delta_{f_p}\right)_{max}$	1,047	1,047	1,047	1,047	1,047	1,047	1,047	1,047	1,047	1,047	1,047	1,047	1,047
$\left(\delta_{e_p}\right)_{max}$	1,047	1,047	1,047	1,047	1,047	1,047	1,047	1,047	1,047	1,047	1,047	1,047	1,047
$\left(\omega_{1_p}\right)_{max}$	500	500	500	500	500	500	500	500	500	500	500	500	500
$\left(\omega_{2_p}\right)_{max}$	500	500	500	500	500	500	500	500	500	500	500	500	500
$\left(\omega_{3_p}\right)_{max}$	500	500	500	500	500	500	500	500	500	500	500	500	500
$\left(\omega_{4_p}\right)_{max}$	500	500	500	500	500	500	500	500	500	500	500	500	500
$\left(\omega_{5_p}\right)_{max}$	500	500	500	500	500	500	500	500	500	500	500	500	500
$\left(\omega_{6_p}\right)_{max}$	500	500	500	500	500	500	500	500	500	500	500	500	500
$\left(\delta_{f\,p}^{C}\right)_{max}$	1	1	1	1	1	1	1	1	1	1	1	1	1
$\left(\delta_{e\,p}^{C}\right)_{max}$	1	1	1	1	1	1	1	1	1	1	1	1	1
$\left(\omega_{1_p}^{C}\right)_{max}$	37	37	37	37	37	37	37	37	37	37	37	37	37
$\left(\omega_{2_p}^{C}\right)_{max}$	37	37	37	37	37	37	37	37	37	37	37	37	37
$\left(\omega_{3_p}^{C}\right)_{max}$	37	37	37	37	37	37	37	37	37	37	37	37	37
$\left(\omega_{4_p}^{C}\right)_{max}$	37	37	37	37	37	37	37	37	37	37	37	37	37
$\left(\omega_{5_p}^{C}\right)_{max}$	37	37	37	37	37	37	37	37	37	37	37	37	37
$\left(\omega_{6_p}^{C}\right)_{max}$	37	37	37	37	37	37	37	37	37	37	37	37	37
$k_{p,h}$	0,01	0,01	0,01	0,01	0,01	0,01	0,01	0,01	0,01	0,01	0,01	0,01	0,01
$k_{d,h}$	0	0	0	0	0	0	0	0	0	0	0	0	0
$k_{I,h}$	-1,0E-05	-1,0E-05	-1,0E-05	-1,0E-05	-1,0E-05	-1,0E-05	-1,0E-05	-1,0E-05	-1,0E-05	-1,0E-05	-1,0E-05	-1,0E-05	-1,0E-05

Table 7: Accelerated flight control parameters, from 6,5 m/s to 13 m/s.

V_T (m/s)	6,5	7	7,5	8	8,5	9	9,5	10	10,5	11	12	12,5	13
ρ	1,0E+18	1,0E+18	1,0E+18	1,0E+18	1,0E+18	1,0E+18	1,0E+15	1,0E+15	1,0E+15	1,0E+15	1,0E+15	1,0E+15	1,0E+15
$\mathcal{E}_{th_s}$	2	2	2	2	2	2	1	1	1	1	1	1	1
$\mathcal{E}_{th_c}$	0,0005	0,0005	0,0005	0,0005	0,0005	0,0005	0,006	0,006	0,006	0,006	0,006	0,006	0,006
$\mathcal{E}_{th_d}$	0,5	0,5	0,5	0,5	0,5	0,5	0,01	0,01	0,01	0,01	0,01	0,01	0,01
$\left(V_{T_p}\right)_{max}$	1,0E-09	1,0E-09	1,0E-09	1,0E-09	1,0E-09	1,0E-09	1,0E-08	1,0E-07	1,0E-07	1,0E-07	8,0E-07	8,0E-07	8,0E-07
$(\alpha_p)_{max}$	0,000175	0,000175	0,00017	0,00017	0,00017	0,00017	0,000175	0,00017	0,00017	0,00017	0,00017	0,000175	0,00017
$\left(Q_{W_p}\right)_{max}$	1,745	1,745	1,745	1,745	1,745	1,745	1,745	1,745	1,745	1,745	1,745	1,745	1,745
$(\theta_p)_{max}$	0,000175	0,000175	0,00017	0,00017	0,00017	0,00017	0,000175	0,00017	0,00017	0,00017	0,00017	0,000175	0,00017
$(h_p)_{max}$	1,0E-08	1,0E-08	1,0E-08	1,0E-08	1,0E-08	1,0E-08	5,0E-06	1,0E-06	1,0E-06	1,0E-06	1,0E-06	1,0E-06	1,0E-06
$\left(\delta_{f_p}\right)_{max}$	1,047	1,047	1,047	1,047	1,047	1,047	1,047	1,047	1,047	1,047	1,047	1,047	1,047
$\left(\delta_{e_p}\right)_{max}$	1,047	1,047	1,047	1,047	1,047	1,047	1,047	1,047	1,047	1,047	1,047	1,047	1,047
$\left(\omega_{1_p}\right)_{max}$	500	500	500	500	500	500	500	500	500	500	500	500	500
$\left(\omega_{2_p}\right)_{max}$	500	500	500	500	500	500	500	500	500	500	500	500	500
$\left(\omega_{3_p}\right)_{max}$	500	500	500	500	500	500	500	500	500	500	500	500	500
$\left(\omega_{4_p}\right)_{max}$	500	500	500	500	500	500	500	500	500	500	500	500	500
$\left(\omega_{5_p}\right)_{max}$	500	500	500	500	500	500	500	500	500	500	500	500	500
$\left(\omega_{6_p}\right)_{max}$	500	500	500	500	500	500	500	500	500	500	500	500	500
$\left(\delta_{f\,p}^{C}\right)_{max}$	1	1	1	1	1	1	1	1	1	1	1	1	1
$\left(\delta_{e\,p}^{C}\right)_{max}$	1	1	1	1	1	1	1	1	1	1	1	1	1
$\left(\omega_{1_p}^{C}\right)_{max}$	37	37	37	37	37	37	37	37	37	37	37	37	37
$\left(\omega_{2_p}^{C}\right)_{max}$	37	37	37	37	37	37	37	37	37	37	37	37	37
$\left(\omega_{3_p}^{C}\right)_{max}$	37	37	37	37	37	37	37	37	37	37	37	37	37
$\left(\omega_{4_p}^{C}\right)_{max}$	37	37	37	37	37	37	37	37	37	37	37	37	37
$\left(\omega_{5_p}^{C}\right)_{max}$	37	37	37	37	37	37	37	37	37	37	37	37	37
$\left(\omega_{6_p}^{C}\right)_{max}$	37	37	37	37	37	37	37	37	37	37	37	37	37
$k_{p,h}$	0,01	0,01	0,01	0,01	0,01	0,01	0,2	0,2	0,2	0,2	0,2	0,2	0,2
$k_{d,h}$	0	0	0	0	0	0	0	0	0	0	0	0	0
$k_{I,h}$	-1,0E-05	-1,0E-05	-1,0E-05	-1,0E-05	-1,0E-05	-1,0E-05	0,01	0,01	0,01	0,01	0,01	0,01	0,01

Table 8: Accelerated flight control parameters, from 13,5 m/s to 23 m/s.

V_T (m/s)	13,5	14	14,5	15,5	17	18	19	20	21	22	23
ρ	1,0E+15	1,0E+15	1,0E+15	1,0E+15	1,0E+03	1,0E+03	1,0E+03	1,0E+03	1,0E+03	3,0E+05	3,0E+05
ε_{th_s}	0,2	0,2	0,2	0,2	0,05	0,05	0,05	0,05	0,05	0,5	0,5
ε_{th_c}	0,001	0,001	0,001	0,001	0,02	0,02	0,02	0,02	0,02	0,02	0,02
ε_{th_d}	0,5	0,5	0,5	0,5	0,03	0,03	0,03	0,03	0,03	0,1	0,1
$\left(V_{T_p}\right)_{max}$	8,0E-07	8,0E-07	8,0E-07	8,0E-07	8,0E-07	0,1	0,1	0,1	0,1	0,01	0,01
$(\alpha_p)_{max}$	0,00017	0,00017	0,00017	0,00017	0,0087	0,0087	0,0087	0,0087	0,0087	0,0017	0,0017
$\left(Q_{W_p}\right)_{max}$	1,745	1,745	1,745	1,745	1,745	1,745	1,745	1,745	1,745	1,745	1,745
$(\theta_p)_{max}$	0,00017	0,00017	0,00017	0,00017	0,0087	0,0087	0,0087	0,0087	0,0087	0,0174	0,0174
$(h_p)_{max}$	1,0E-06	1,0E-06	1,0E-06	1,0E-07	2,0E-01	2,0E-01	2,0E-01	2,0E-01	2,0E-01	5,0E-03	5,0E-03
$\left(\delta_{f_p}\right)_{max}$	1,047	1,047	1,047	1,047	0,003	0,003	0,003	0,003	0,003	0,009	0,009
$\left(\delta_{e_p}\right)_{max}$	1,047	1,047	1,047	1,047	0,003	0,003	0,003	0,003	0,003	0,009	0,009
$\left(\omega_{1_p}\right)_{max}$	500	500	500	500	500	500	500	500	500	0,5	0,5
$\left(\omega_{2_p}\right)_{max}$	500	500	500	500	500	500	500	500	500	0,5	0,5
$\left(\omega_{3_p}\right)_{max}$	500	500	500	500	500	500	500	500	500	0,5	0,5
$\left(\omega_{4_p}\right)_{max}$	500	500	500	500	500	500	500	500	500	0,5	0,5
$\left(\omega_{5_p}\right)_{max}$	500	500	500	500	500	500	500	500	500	0,5	0,5
$\left(\omega_{6_p}\right)_{max}$	500	500	500	500	500	500	500	500	500	0,5	0,5
$\left(\delta_{f\ p}^C\right)_{max}$	1	1	1	1	1	1	1	1	1	1	1
$\left(\delta_{e\ p}^C\right)_{max}$	1	1	1	1	1	1	1	1	1	1	1
$\left(\omega_{1_p}^C\right)_{max}$	37	37	37	37	37	37	37	37	37	37	37
$\left(\omega_{2_p}^C\right)_{max}$	37	37	37	37	37	37	37	37	37	37	37
$\left(\omega_{3_p}^C\right)_{max}$	37	37	37	37	37	37	37	37	37	37	37
$\left(\omega_{4_p}^C\right)_{max}$	37	37	37	37	37	37	37	37	37	37	37
$\left(\omega_{5_p}^C\right)_{max}$	37	37	37	37	37	37	37	37	37	37	37
$\left(\omega_{6_p}^C\right)_{max}$	37	37	37	37	37	37	37	37	37	37	37
$k_{p,h}$	0,2	0,2	0,2	0,2	0,01	0,01	0,01	0,01	0,01	0,01	0,01
$k_{d,h}$	0	0	0	0	0	0	0	0	0	0	0
$k_{I,h}$	-1,0E-05	-1,0E-05	-1,0E-05	1,0E-05	0,01	0,01	0,01	0,01	0,01	0,01	0,01

Table 9: Accelerated flight control parameters, from 24 m/s to 36 m/s.

V_T (m/s)	24	25	26	27	28	29	30	31	32	33	34	35	36
ρ	3,0E+05	3,0E+05	3,0E+05	3,0E+05	3,0E+05	3,0E+05	3,0E+05	3,0E+05	3,0E+05	3,0E+05	3,0E+05	3,0E+05	3,0E+05
ε_{th_s}	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5
ε_{th_c}	0,02	0,02	0,02	0,02	0,02	0,02	0,02	0,02	0,02	0,02	0,02	0,02	0,02
ε_{th_d}	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1
$\left(V_{T_p}\right)_{max}$	0,01	0,01	0,01	0,01	0,01	0,01	0,01	0,01	0,01	0,01	0,01	0,01	0,01
$(\alpha_p)_{max}$	0,0017	0,0017	0,0017	0,0017	0,0017	0,0017	0,0017	0,0017	0,0017	0,0017	0,0017	0,0017	0,0017
$\left(Q_{W_p}\right)_{max}$	1,745	1,745	1,745	1,745	1,745	1,745	1,745	1,745	1,745	1,745	1,745	1,745	1,745
$(\theta_p)_{max}$	0,0174	0,0174	0,0174	0,0174	0,0174	0,0174	0,0174	0,0174	0,0174	0,0174	0,0174	0,0174	0,0174
$(h_p)_{max}$	5,0E-03	5,0E-03	5,0E-03	5,0E-03	5,0E-03	5,0E-03	5,0E-03	5,0E-03	5,0E-03	1,0E-02	1,0E-02	1,0E-02	1,0E-02
$\left(\delta_{f_p}\right)_{max}$	0,009	0,009	0,009	0,009	0,009	0,009	0,009	0,009	0,009	0,009	0,009	0,009	0,009
$\left(\delta_{e_p}\right)_{max}$	0,009	0,009	0,009	0,009	0,009	0,009	0,009	0,009	0,009	0,009	0,009	0,009	0,009
$\left(\omega_{1_p}\right)_{max}$	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5
$\left(\omega_{2_p}\right)_{max}$	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5
$\left(\omega_{3_p}\right)_{max}$	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5
$\left(\omega_{4_p}\right)_{max}$	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5
$\left(\omega_{5_p}\right)_{max}$	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5
$\left(\omega_{6_p}\right)_{max}$	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5
$\left(\delta^C_{f_p}\right)_{max}$	1	1	1	1	1	1	1	1	1	1	1	1	1
$\left(\delta^C_{e_p}\right)_{max}$	1	1	1	1	1	1	1	1	1	1	1	1	1
$\left(\omega^C_{1_p}\right)_{max}$	37	37	37	37	37	37	37	37	37	37	37	37	37
$\left(\omega^C_{2_p}\right)_{max}$	37	37	37	37	37	37	37	37	37	37	37	37	37
$\left(\omega^C_{3_p}\right)_{max}$	37	37	37	37	37	37	37	37	37	37	37	37	37
$\left(\omega^C_{4_p}\right)_{max}$	37	37	37	37	37	37	37	37	37	37	37	37	37
$\left(\omega^C_{5_p}\right)_{max}$	37	37	37	37	37	37	37	37	37	37	37	37	37
$\left(\omega^C_{6_p}\right)_{max}$	37	37	37	37	37	37	37	37	37	37	37	37	37
$k_{p,h}$	0,01	0,01	0,01	0,01	0,01	0,01	0,01	0,01	0,01	0,01	0,01	0,01	0,01
$k_{d,h}$	0	0	0	0	0	0	0	0	0	0	0	0	0
$k_{I,h}$	0,01	0,01	0,01	0,01	0,01	0,01	0,01	0,01	0,01	0,01	0,01	0,01	0,01

Table 10: Accelerated flight control parameters, from 37 m/s to 47 m/s.

V_T (m/s)	37	38	39	40	41	42	43	44	45	46	47
ρ	3,0E+05	3,0E+05	3,0E+05	3,0E+05	3,0E+05	3,0E+05	3,0E+05	3,0E+05	3,0E+05	3,0E+05	3,0E+05
ε_{th_s}	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5
ε_{th_c}	0,02	0,02	0,02	0,02	0,02	0,02	0,02	0,02	0,02	0,02	0,02
ε_{th_d}	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1
$\left(V_{T_p}\right)_{max}$	0,01	0,01	0,01	0,01	0,01	0,01	0,01	0,01	0,01	0,01	0,01
$\left(\alpha_p\right)_{max}$	0,0017	0,0017	0,0017	0,0017	0,0017	0,0017	0,0017	0,0017	0,0017	0,0017	0,0017
$\left(Q_{W_p}\right)_{max}$	1,745	1,745	1,745	1,745	1,745	1,745	1,745	1,745	1,745	1,745	1,745
$\left(\theta_p\right)_{max}$	0,0174	0,0174	0,0174	0,0174	0,0174	0,0174	0,0174	0,0174	0,0174	0,0174	0,0174
$\left(h_p\right)_{max}$	1,0E-02	1,0E-02	1,0E-02	1,0E-02	1,0E-02	1,0E-02	1,0E-02	1,0E-02	1,0E-02	1,0E-02	1,0E-02
$\left(\delta_{f_p}\right)_{max}$	0,009	0,009	0,009	0,009	0,009	0,009	0,009	0,009	0,009	0,009	0,009
$\left(\delta_{e_p}\right)_{max}$	0,009	0,009	0,009	0,009	0,009	0,009	0,009	0,009	0,009	0,009	0,009
$\left(\omega_{1_p}\right)_{max}$	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5
$\left(\omega_{2_p}\right)_{max}$	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5
$\left(\omega_{3_p}\right)_{max}$	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5
$\left(\omega_{4_p}\right)_{max}$	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5
$\left(\omega_{5_p}\right)_{max}$	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5
$\left(\omega_{6_p}\right)_{max}$	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5
$\left(\delta^C_{f_p}\right)_{max}$	1	1	1	1	1	1	1	1	1	1	1
$\left(\delta^C_{e_p}\right)_{max}$	1	1	1	1	1	1	1	1	1	1	1
$\left(\omega^C_{1_p}\right)_{max}$	37	37	37	37	37	37	37	37	37	37	37
$\left(\omega^C_{2_p}\right)_{max}$	37	37	37	37	37	37	37	37	37	37	37
$\left(\omega^C_{3_p}\right)_{max}$	37	37	37	37	37	37	37	37	37	37	37
$\left(\omega^C_{4_p}\right)_{max}$	37	37	37	37	37	37	37	37	37	37	37
$\left(\omega^C_{5_p}\right)_{max}$	37	37	37	37	37	37	37	37	37	37	37
$\left(\omega^C_{6_p}\right)_{max}$	37	37	37	37	37	37	37	37	37	37	37
$k_{p,h}$	0,01	0,01	0,01	0,01	0,01	0,01	0,01	0,01	0,01	0,01	0,01
$k_{d,h}$	0	0	0	0	0	0	0	0	0	0	0
$k_{I,h}$	0,01	0,01	0,01	0,01	0,01	0,01	0,01	0,01	0,01	0,01	0,01

Table 11: Decelerated flight control parameters, from 0 m/s to 6 m/s.

V_T (m/s)	0	0,5	1	1,5	2	2,5	3	3,5	4	4,5	5	5,5	6
ρ	1,0E+18	1,0E+18	1,0E+18	1,0E+18	1,0E+18	1,0E+18	1,0E+18	1,0E+18	1,0E+18	1,0E+18	1,0E+18	1,0E+18	1,0E+18
ε_{th_s}	2	2	2	2	2	2	2	2	2	2	2	2	2
ε_{th_c}	5,0E-04	5,0E-04	5,0E-04	5,0E-04	5,0E-04	5,0E-04	5,0E-04	5,0E-04	5,0E-04	5,0E-04	5,0E-04	5,0E-04	5,0E-04
ε_{th_d}	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5
$\left(V_{T_p}\right)_{max}$	1,0E-09	1,0E-09	1,0E-09	1,0E-09	1,0E-09	1,0E-09	1,0E-09	1,0E-09	1,0E-09	1,0E-09	1,0E-09	1,0E-09	1,0E-09
$\left(\alpha_p\right)_{max}$	1,75E-04	1,75E-04	1,75E-04	1,75E-04	1,75E-04	1,75E-04	1,75E-04	1,75E-04	1,75E-04	1,75E-04	1,75E-04	1,75E-04	1,75E-04
$\left(Q_{W_p}\right)_{max}$	1,745	1,745	1,745	1,745	1,745	1,745	0,175	1,745	1,745	1,745	1,745	1,745	1,745
$\left(\theta_p\right)_{max}$	1,75E-04	1,75E-04	1,75E-04	1,75E-04	1,75E-04	1,75E-04	1,75E-04	1,75E-04	1,75E-04	1,75E-04	1,75E-04	1,75E-04	1,75E-04
$\left(h_p\right)_{max}$	1,0E-09	1,0E-09	1,0E-09	1,0E-09	1,0E-09	1,0E-08	1,0E-08	1,0E-08	1,0E-08	1,0E-08	1,0E-08	1,0E-08	1,0E-08
$\left(\delta_{f_p}\right)_{max}$	1,047	1,047	1,047	1,047	1,047	1,047	1,047	1,047	1,047	1,047	1,047	1,047	1,047
$\left(\delta_{e_p}\right)_{max}$	1,047	1,047	1,047	1,047	1,047	1,047	1,047	1,047	1,047	1,047	1,047	1,047	1,047
$\left(\omega_{1_p}\right)_{max}$	500	500	500	500	500	500	500	500	500	500	500	500	500
$\left(\omega_{2_p}\right)_{max}$	500	500	500	500	500	500	500	500	500	500	500	500	500
$\left(\omega_{3_p}\right)_{max}$	500	500	500	500	500	500	500	500	500	500	500	500	500
$\left(\omega_{4_p}\right)_{max}$	500	500	500	500	500	500	500	500	500	500	500	500	500
$\left(\omega_{5_p}\right)_{max}$	500	500	500	500	500	500	500	500	500	500	500	500	500
$\left(\omega_{6_p}\right)_{max}$	500	500	500	500	500	500	500	500	500	500	500	500	500
$\left(\delta_{f\ p}^{C}\right)_{max}$	1	1	1	1	1	1	1	1	1	1	1	1	1
$\left(\delta_{e\ p}^{C}\right)_{max}$	1	1	1	1	1	1	1	1	1	1	1	1	1
$\left(\omega_{1_p}^{C}\right)_{max}$	37	37	37	37	37	37	37	37	37	37	37	37	37
$\left(\omega_{2_p}^{C}\right)_{max}$	37	37	37	37	37	37	37	37	37	37	37	37	37
$\left(\omega_{3_p}^{C}\right)_{max}$	37	37	37	37	37	37	37	37	37	37	37	37	37
$\left(\omega_{4_p}^{C}\right)_{max}$	37	37	37	37	37	37	37	37	37	37	37	37	37
$\left(\omega_{5_p}^{C}\right)_{max}$	37	37	37	37	37	37	37	37	37	37	37	37	37
$\left(\omega_{6_p}^{C}\right)_{max}$	37	37	37	37	37	37	37	37	37	37	37	37	37
$k_{p,h}$	0,01	0,01	0,01	0,01	0,01	0,01	0,01	0,01	0,01	0,01	0,01	0,01	0,01
$k_{d,h}$	0	0	0	0	0	0	0	0	0	0	0	0	0
$k_{I,h}$	1,0E-05	1,0E-05	1,0E-05	1,0E-05	1,0E-05	1,0E-05	1,0E-05	1,0E-05	1,0E-05	1,0E-05	1,0E-05	1,0E-05	1,0E-05

Table 12: Decelerated flight control parameters, from 6,5 m/s to 13 m/s.

V_T (m/s)	6,5	7	7,5	8	8,5	9	9,5	10	10,5	11	12	12,5	13
ρ	1,0E+18	1,0E+18	1,0E+18	1,0E+18	1,0E+18	1,0E+18	1,0E+15	1,0E+15	1,0E+15	1,0E+15	1,0E+15	1,0E+15	1,0E+15
$\mathcal{E}_{th_s}$	2	2	2	2	2	2	1	1	1	1	1	1	1
$\mathcal{E}_{th_c}$	5,0E-04	5,0E-04	5,0E-04	5,0E-04	5,0E-04	5,0E-04	5,0E-04	5,0E-04	5,0E-04	5,0E-04	5,0E-04	5,0E-04	5,0E-04
$\mathcal{E}_{th_d}$	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5
$\left(V_{T_p}\right)_{max}$	1,0E-09	1,0E-09	1,0E-09	1,0E-09	1,0E-09	1,0E-09	1,0E-08	1,0E-07	1,0E-07	1,0E-07	1,0E-07	1,0E-07	1,0E-07
$(\alpha_p)_{max}$	1,75E-04	1,75E-04	1,75E-04	1,75E-04	1,75E-04	1,75E-04	1,75E-04	1,75E-04	1,75E-04	1,75E-04	1,75E-04	1,75E-04	1,75E-04
$\left(Q_{W_p}\right)_{max}$	1,745	1,745	1,745	1,745	1,745	1,745	1,745	1,745	1,745	1,745	1,745	1,745	1,745
$(\theta_p)_{max}$	1,75E-04	1,75E-04	1,75E-04	1,75E-04	1,75E-04	1,75E-04	1,75E-04	1,75E-04	1,75E-04	1,75E-04	1,75E-04	1,75E-04	1,75E-04
$(h_p)_{max}$	1,0E-08	1,0E-08	1,0E-08	1,0E-08	1,0E-08	1,0E-08	5,0E-06	1,0E-06	1,0E-06	1,0E-06	1,0E-06	1,0E-06	1,0E-06
$\left(\delta_{f_p}\right)_{max}$	1,047	1,047	1,047	1,047	1,047	1,047	1,047	1,047	1,047	1,047	1,047	1,047	1,047
$\left(\delta_{e_p}\right)_{max}$	1,047	1,047	1,047	1,047	1,047	1,047	1,047	1,047	1,047	1,047	1,047	1,047	1,047
$\left(\omega_{1_p}\right)_{max}$	500	500	500	500	500	500	500	500	500	500	500	500	500
$\left(\omega_{2_p}\right)_{max}$	500	500	500	500	500	500	500	500	500	500	500	500	500
$\left(\omega_{3_p}\right)_{max}$	500	500	500	500	500	500	500	500	500	500	500	500	500
$\left(\omega_{4_p}\right)_{max}$	500	500	500	500	500	500	500	500	500	500	500	500	500
$\left(\omega_{5_p}\right)_{max}$	500	500	500	500	500	500	500	500	500	500	500	500	500
$\left(\omega_{6_p}\right)_{max}$	500	500	500	500	500	500	500	500	500	500	500	500	500
$\left(\delta^c_{f\,p}\right)_{max}$	1	1	1	1	1	1	1	1	1	1	1	1	1
$\left(\delta^c_{e\,p}\right)_{max}$	1	1	1	1	1	1	1	1	1	1	1	1	1
$\left(\omega^c_{1_p}\right)_{max}$	37	37	37	37	37	37	37	37	37	37	37	37	37
$\left(\omega^c_{2_p}\right)_{max}$	37	37	37	37	37	37	37	37	37	37	37	37	37
$\left(\omega^c_{3_p}\right)_{max}$	37	37	37	37	37	37	37	37	37	37	37	37	37
$\left(\omega^c_{4_p}\right)_{max}$	37	37	37	37	37	37	37	37	37	37	37	37	37
$\left(\omega^c_{5_p}\right)_{max}$	37	37	37	37	37	37	37	37	37	37	37	37	37
$\left(\omega^c_{6_p}\right)_{max}$	37	37	37	37	37	37	37	37	37	37	37	37	37
$k_{p,h}$	0,01	0,01	0,01	0,01	0,01	0,01	0,2	0,2	0,2	0,2	0,2	0,2	0,2
$k_{d,h}$	0	0	0	0	0	0	0	0	0	0	0	0	0
$k_{I,h}$	1,0E-05	1,0E-05	1,0E-05	1,0E-05	1,0E-05	1,0E-05	-1,0E-05	-1,0E-05	-1,0E-05	-1,0E-05	-1,0E-05	-1,0E-05	-1,0E-05

Table 13: Decelerated flight control parameters, from 13,5 m/s to 23 m/s.

V_T (m/s)	13,5	14	14,5	15	15,5	16	17	18	19	20	21	22	23
ρ	1,0E+15	1,0E+15	1,0E+15	1,0E+15	1,0E+15	1,0E+05	1,0E+05	1,0E+05	1,0E+05	1,0E+05	1,0E+05	1,0E+05	1,0E+05
$\mathcal{E}_{th_s}$	1	1	1	1	0,5	0,05	0,05	0,05	0,05	0,05	0,05	0,05	0,05
$\mathcal{E}_{th_c}$	5,0E-04	5,0E-04	5,0E-04	5,0E-04	5,0E-04	5,0E-04	5,0E-04	5,0E-04	5,0E-04	5,0E-04	5,0E-04	5,0E-04	5,0E-04
$\mathcal{E}_{th_d}$	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5
$\left(V_{T_p}\right)_{max}$	1,0E-07	1,0E-08	1,0E-08	1,0E-08	1,0E-08	1,0E+01	1,0E+01	1,0E+01	1,0E+01	1,0E+00	1,0E+00	3,0E-02	3,0E-02
$\left(\alpha_p\right)_{max}$	1,75E-04	1,75E-04	1,75E-04	1,75E-04	1,75E-04	1,75E-02	1,75E-02	1,75E-02	1,75E-02	1,75E-03	1,75E-03	1,75E-03	1,75E-03
$\left(Q_{W_p}\right)_{max}$	1,745	1,745	1,745	1,745	1,745	1,745	1,745	1,745	1,745	1,745	1,745	1,745	1,745
$\left(\theta_p\right)_{max}$	1,75E-04	1,75E-04	1,75E-04	1,75E-04	1,75E-04	1,75E-02	1,75E-02	1,75E-02	1,75E-02	1,75E-03	1,75E-03	1,75E-02	1,75E-02
$\left(h_p\right)_{max}$	1,0E-06	1,0E-07	1,0E-07	1,0E-07	1,0E-07	1,0E-01	2,0E-03	2,0E-03	1,0E-03	1,0E-03	5,0E-03	5,0E-03	5,0E-03
$\left(\delta_{f_p}\right)_{max}$	1,047	1,047	1,047	1,047	1,047	0,017	0,017	0,017	0,002	0,175	0,009	0,002	0,002
$\left(\delta_{e_p}\right)_{max}$	1,047	1,047	1,047	1,047	1,047	0,017	0,017	0,017	0,002	0,175	0,009	0,002	0,002
$\left(\omega_{1_p}\right)_{max}$	500	500	500	500	500	0,25	0,25	0,25	0,25	0,25	1	1	1
$\left(\omega_{2_p}\right)_{max}$	500	500	500	500	500	0,25	0,25	0,25	0,25	0,25	1	1	1
$\left(\omega_{3_p}\right)_{max}$	500	500	500	500	500	0,25	0,25	0,25	0,25	0,25	1	1	1
$\left(\omega_{4_p}\right)_{max}$	500	500	500	500	500	0,25	0,25	0,25	0,25	0,25	1	1	1
$\left(\omega_{5_p}\right)_{max}$	500	500	500	500	500	0,25	0,25	0,25	0,25	0,25	1	1	1
$\left(\omega_{6_p}\right)_{max}$	500	500	500	500	500	0,25	0,25	0,25	0,25	0,25	1	1	1
$\left(\delta_{f\,p}^{C}\right)_{max}$	1	1	1	1	1	1	1	1	1	1	1	1	1
$\left(\delta_{e\,p}^{C}\right)_{max}$	1	1	1	1	1	1	1	1	1	1	1	1	1
$\left(\omega_{1_p}^{C}\right)_{max}$	37	37	37	37	37	37	37	37	37	37	37	37	37
$\left(\omega_{2_p}^{C}\right)_{max}$	37	37	37	37	37	37	37	37	37	37	37	37	37
$\left(\omega_{3_p}^{C}\right)_{max}$	37	37	37	37	37	37	37	37	37	37	37	37	37
$\left(\omega_{4_p}^{C}\right)_{max}$	37	37	37	37	37	37	37	37	37	37	37	37	37
$\left(\omega_{5_p}^{C}\right)_{max}$	37	37	37	37	37	37	37	37	37	37	37	37	37
$\left(\omega_{6_p}^{C}\right)_{max}$	37	37	37	37	37	37	37	37	37	37	37	37	37
$k_{p,h}$	0,2	0,2	0,2	0,2	0,2	0,01	0,01	0,01	0,01	0,01	0,01	0,01	0,01
$k_{d,h}$	0	0	0	0	0	0	0	0	0	0	0	0	0
$k_{I,h}$	-1,0E-05	-1,0E-05	-1,0E-05	-1,0E-05	-1,0E-05	1,0E-05	1,0E-05	1,0E-05	1,0E-05	1,0E-05	1,0E-05	1,0E-05	1,0E-05

Table 14: Decelerated flight control parameters, from 24 m/s to 36 m/s.

V_T (m/s)	24	25	26	27	28	29	30	31	32	33	34	35	36
ρ	1,0E+05	1,0E+05	1,0E+05	1,0E+05	1,0E+05	1,0E+05	1,0E+05	1,0E+05	1,0E+05	1,0E+05	1,0E+05	1,0E+05	1,0E+05
$\mathcal{E}_{th_s}$	0,05	0,05	0,05	0,05	0,05	0,05	0,05	0,05	0,05	0,05	0,05	0,05	0,05
$\mathcal{E}_{th_c}$	5,0E-04	5,0E-04	5,0E-04	5,0E-04	5,0E-04	5,0E-04	5,0E-04	5,0E-04	5,0E-04	5,0E-04	5,0E-04	5,0E-04	5,0E-04
$\mathcal{E}_{th_d}$	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5
$\left(V_{T_p}\right)_{max}$	3,0E-02	3,0E-02	3,0E-02	6,0E-02	6,0E-02	6,0E-02	6,0E-02	6,0E-02	6,0E-02	8,0E-02	8,0E-02	8,0E-02	8,0E-02
$\left(\alpha_p\right)_{max}$	1,75E-03	1,75E-03	1,75E-03	1,75E-03	1,75E-03	1,75E-03	1,75E-03	1,75E-03	1,75E-03	1,75E-03	1,75E-03	1,75E-03	1,75E-03
$\left(Q_{W_p}\right)_{max}$	1,745	1,745	1,745	1,745	1,745	1,745	1,745	1,745	1,745	1,745	1,745	1,745	1,745
$\left(\theta_p\right)_{max}$	1,75E-02	1,75E-02	1,75E-02	1,75E-02	1,75E-02	1,75E-02	1,75E-02	1,75E-02	1,75E-02	1,75E-02	1,75E-02	1,75E-02	1,75E-02
$\left(h_p\right)_{max}$	5,0E-03	5,0E-03	5,0E-03	5,0E-03	5,0E-03	5,0E-03	5,0E-03	5,0E-03	5,0E-03	1,0E-02	1,0E-02	1,0E-02	1,0E-02
$\left(\delta_{f_p}\right)_{max}$	0,003	0,005	0,002	0,002	0,002	0,002	0,002	0,002	0,002	0,002	0,002	0,002	0,002
$\left(\delta_{e_p}\right)_{max}$	0,003	0,005	0,002	0,002	0,002	0,002	0,002	0,002	0,002	0,002	0,002	0,002	0,002
$\left(\omega_{1_p}\right)_{max}$	1	1	1	1	1	1	1	1	1	1	1	1	1
$\left(\omega_{2_p}\right)_{max}$	1	1	1	1	1	1	1	1	1	1	1	1	1
$\left(\omega_{3_p}\right)_{max}$	1	1	1	1	1	1	1	1	1	1	1	1	1
$\left(\omega_{4_p}\right)_{max}$	1	1	1	1	1	1	1	1	1	1	1	1	1
$\left(\omega_{5_p}\right)_{max}$	1	1	1	1	1	1	1	1	1	1	1	1	1
$\left(\omega_{6_p}\right)_{max}$	1	1	1	1	1	1	1	1	1	1	1	1	1
$\left(\delta_{f\,p}^{C}\right)_{max}$	1	1	1	1	1	1	1	1	1	1	1	1	1
$\left(\delta_{e\,p}^{C}\right)_{max}$	1	1	1	1	1	1	1	1	1	1	1	1	1
$\left(\omega_{1_p}^{C}\right)_{max}$	37	37	37	37	37	37	37	37	37	37	37	37	37
$\left(\omega_{2_p}^{C}\right)_{max}$	37	37	37	37	37	37	37	37	37	37	37	37	37
$\left(\omega_{3_p}^{C}\right)_{max}$	37	37	37	37	37	37	37	37	37	37	37	37	37
$\left(\omega_{4_p}^{C}\right)_{max}$	37	37	37	37	37	37	37	37	37	37	37	37	37
$\left(\omega_{5_p}^{C}\right)_{max}$	37	37	37	37	37	37	37	37	37	37	37	37	37
$\left(\omega_{6_p}^{C}\right)_{max}$	37	37	37	37	37	37	37	37	37	37	37	37	37
$k_{p,h}$	0,01	0,01	0,01	0,01	0,01	0,01	0,01	0,01	0,01	0,01	0,01	0,01	0,01
$k_{d,h}$	0	0	0	0	0	0	0	0	0	0	0	0	0
$k_{I,h}$	1,0E-05	1,0E-05	1,0E-05	1,0E-05	1,0E-05	1,0E-05	1,0E-05	1,0E-05	1,0E-05	1,0E-05	1,0E-05	1,0E-05	1,0E-05

Table 15: Decelerated flight control parameters, from 37 m/s to 47 m/s.

V_T (m/s)	37	38	39	40	41	42	43	44	45	46	47
ρ	1,0E+05	1,0E+05	1,0E+05	1,0E+05	1,0E+05	1,0E+05	1,0E+05	1,0E+05	1,0E+05	1,0E+05	1,0E+05
$\mathcal{E}_{th_s}$	0,05	0,05	0,05	0,05	0,05	0,05	0,05	0,05	0,05	0,05	0,05
$\mathcal{E}_{th_c}$	5,0E-04	5,0E-04	5,0E-04	5,0E-04	5,0E-04	5,0E-04	5,0E-04	5,0E-04	5,0E-04	5,0E-04	5,0E-04
$\mathcal{E}_{th_d}$	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5
$\left(V_{T_p}\right)_{max}$	8,0E-02	8,0E-02	8,0E-02	8,0E-02	8,0E-02	8,0E-02	8,0E-02	8,0E-02	8,0E-02	8,0E-02	8,0E-02
$\left(\alpha_p\right)_{max}$	1,75E-03	1,75E-03	1,75E-03	1,75E-03	1,75E-03	1,75E-03	1,75E-03	1,75E-03	1,75E-03	1,75E-03	1,75E-03
$\left(Q_{W_p}\right)_{max}$	1,745	1,745	1,745	1,745	1,745	1,745	1,745	1,745	1,745	1,745	1,745
$\left(\theta_p\right)_{max}$	1,75E-02	1,75E-02	1,75E-02	1,75E-02	1,75E-02	1,75E-02	1,75E-02	1,75E-02	1,75E-02	1,75E-02	1,75E-02
$\left(h_p\right)_{max}$	1,0E-02	1,0E-02	1,0E-02	1,0E-02	1,0E-02	1,0E-02	1,0E-02	1,0E-02	1,0E-02	1,0E-02	1,0E-02
$\left(\delta_{f_p}\right)_{max}$	0,002	0,002	0,002	0,002	0,002	0,002	0,002	0,002	0,002	0,002	0,002
$\left(\delta_{e_p}\right)_{max}$	0,002	0,002	0,002	0,002	0,002	0,002	0,002	0,002	0,002	0,002	0,002
$\left(\omega_{1_p}\right)_{max}$	1	1	1	1	1	1	1	1	1	1	1
$\left(\omega_{2_p}\right)_{max}$	1	1	1	1	1	1	1	1	1	1	1
$\left(\omega_{3_p}\right)_{max}$	1	1	1	1	1	1	1	1	1	1	1
$\left(\omega_{4_p}\right)_{max}$	1	1	1	1	1	1	1	1	1	1	1
$\left(\omega_{5_p}\right)_{max}$	1	1	1	1	1	1	1	1	1	1	1
$\left(\omega_{6_p}\right)_{max}$	1	1	1	1	1	1	1	1	1	1	1
$\left(\delta_{f\ p}^{C}\right)_{max}$	1	1	1	1	1	1	1	1	1	1	1
$\left(\delta_{e\ p}^{C}\right)_{max}$	1	1	1	1	1	1	1	1	1	1	1
$\left(\omega_{1_p}^{C}\right)_{max}$	37	37	37	37	37	37	37	37	37	37	37
$\left(\omega_{2_p}^{C}\right)_{max}$	37	37	37	37	37	37	37	37	37	37	37
$\left(\omega_{3_p}^{C}\right)_{max}$	37	37	37	37	37	37	37	37	37	37	37
$\left(\omega_{4_p}^{C}\right)_{max}$	37	37	37	37	37	37	37	37	37	37	37
$\left(\omega_{5_p}^{C}\right)_{max}$	37	37	37	37	37	37	37	37	37	37	37
$\left(\omega_{6_p}^{C}\right)_{max}$	37	37	37	37	37	37	37	37	37	37	37
$k_{p,h}$	0,01	0,01	0,01	0,01	0,01	0,01	0,01	0,01	0,01	0,01	0,01
$k_{d,h}$	0	0	0	0	0	0	0	0	0	0	0
$k_{I,h}$	1,0E-05	1,0E-05	1,0E-05	1,0E-05	1,0E-05	1,0E-05	1,0E-05	1,0E-05	1,0E-05	1,0E-05	1,0E-05

20 Appendix C – Geometric Parameters

In Table 16 are listed the necessary geometric parameters to compute aerodynamic coefficients. The index “e” refers to exposed surface, “W” refers to wing, “Ht” to horizontal tail and “B” to body or fuselage.

Table 16: Geometric parameters.

AR_{W_e}	6,46	η	0,65
AR_{HT_e}	2,51	x_{AC_W} (m)	0,30
e_{W_e}	0,655	z_{AC_W} (m)	0,085
e_{HT_e}	0,1	$x_{AC_{HT}}$ (m)	1,15
S_W (m^2)	0,36	$z_{AC_{HT}}$ (m)	0,09
S_{W_e} (m^2)	0,31	x_{refB} (m)	0,57
S_{HT} (m^2)	0,16	z_{refB} (m)	0,028
S_{HT_e} (m^2)	0,13	l_B (m)	1,45
S_B (m^2)	0,03	d_b (m)	0,19
$\bar{c}_W$ (m)	0,228	S_p (m^2)	0,21
$\bar{c}_{HT}$ (m)	0,228	x_{Bc} (m)	0,65
b_W (m)	1,60	$\left(\frac{C_N}{C_{N_{circ}}}\right)_{NT}$	-0.45
b_{HT} (m)	0,7	$\left(\frac{C_N}{C_{N_{circ}}}\right)_{SB}$	0,84
V_B (m^3)	0,029	S_b (m^2)	0,03

21 Appendix D – Aerodynamic Longitudinal Parameters

Table 17: Aerodynamics longitudinal parameters.

$\left(C_{D0}\right)_{W_e}$	0,012	$\left(C_{m0}\right)_{W_e}$	-0,015
$\left(C_{D0}\right)_{HT_e}$	0,011	$\left(C_{m0}\right)_{HT_e}$	0
$\left(C_{L\alpha}\right)_{W_e}$	4,48	$\left(C_{m\alpha}\right)_{W_e}$	0
$\left(C_{L\alpha}\right)_{HT_e}$	2,96	$\left(C_{m\alpha}\right)_{HT_e}$	0
$\left(\alpha_{C_L=0}\right)_{W_e}$	-1,51	K_{WB}	1,23
$\left(\alpha_{C_L=0}\right)_{HT_e}$	0	K_{BHT}	1,39
$\left(C_{L\alpha}\right)_B$	0,268		

22 Appendix E – Aerodynamic Forces, Moments and Coefficients

Table 18: Equilibrium points aerodynamic forces, moments and coefficients, from 0 m/s to 6 m/s.

V_T (m/s)	0	0,5	1	1,5	2	2,5	3	3,5	4	4,5	5	5,5	6
D (N)	0	0,25	1,01	2,31	3,82	6,02	8,73	11,90	15,49	19,45	23,64	28,01	32,36
Y (N)	0	0	0	0	0	0	0	0	0	0	0	0	0
L (N)	0	0,55	2,22	5,00	8,45	13,17	18,90	25,56	33,11	41,42	50,36	59,81	69,56
$\bar{L}$ (Nm)	0	0	0	0	0	0	0	0	0	0	0	0	0
M (Nm)	0	0,06	0,23	0,53	1,42	2,25	3,28	4,51	5,93	7,56	9,35	11,34	13,42
N (Nm)	0	0	0	0	0	0	0	0	0	0	0	0	0
C_D	0	4,53	4,62	4,70	4,37	4,41	4,44	4,45	4,43	4,40	4,33	4,24	4,12
C_Y	0	0	0	0	0	0	0	0	0	0	0	0	0
C_L	0	10,10	10,14	10,17	9,67	9,65	9,62	9,56	9,47	9,37	9,22	9,05	8,85
C_l	0	0	0	0	0	0	0	0	0	0	0	0	0
C_m	0	4,57	4,63	4,69	7,14	7,23	7,31	7,39	7,44	7,49	7,50	7,51	7,48
C_n	0	0	0	0	0	0	0	0	0	0	0	0	0
D_W (N)	0	0,19	0,77	1,77	3,21	5,09	7,41	10,16	13,30	16,81	20,55	24,55	28,55
Y_W (N)	0	0	0	0	0	0	0	0	0	0	0	0	0
L_W (N)	0	0,42	1,70	3,83	6,84	10,69	15,38	20,87	27,12	34,09	41,62	49,72	58,14
$\bar{L}_W$ (Nm)	0	0	0	0	0	0	0	0	0	0	0	0	0
M_W (Nm)	0	0,000	-0,001	-0,001	-0,003	-0,004	-0,006	-0,008	-0,011	-0,013	-0,016	-0,020	-0,024
N_W (Nm)	0	0	0	0	0	0	0	0	0	0	0	0	0
C_{D_W}	0	3,45	3,53	3,61	3,68	3,73	3,77	3,80	3,81	3,80	3,76	3,72	3,63
C_{Y_W}	0	0	0	0	0	0	0	0	0	0	0	0	0
C_{L_W}	0	7,73	7,77	7,80	7,83	7,83	7,83	7,80	7,76	7,71	7,62	7,53	7,39
C_{l_W}	0	0	0	0	0	0	0	0	0	0	0	0	0
C_{m_W}	0	-0,013	-0,013	-0,013	-0,013	-0,013	-0,013	-0,013	-0,013	-0,013	-0,013	-0,013	-0,013
C_{n_W}	0	0	0	0	0	0	0	0	0	0	0	0	0
D_{HT} (N)	0	0,06	0,22	0,51	0,55	0,85	1,19	1,56	1,96	2,34	2,71	3,01	3,26
Y_{HT} (N)	0	0	0	0	0	0	0	0	0	0	0	0	0
L_{HT} (N)	0	0,13	0,52	1,17	1,61	2,49	3,53	4,71	6,01	7,36	8,77	10,13	11,47
$\bar{L}_{HT}$ (Nm)	0	0	0	0	0	0	0	0	0	0	0	0	0
M_{HT} (Nm)	0	0	0	0	0	0	0	0	0	0	0	0	0
N_{HT} (Nm)	0	0	0	0	0	0	0	0	0	0	0	0	0
$C_{D_{HT}}$	0	2,294	2,312	2,322	1,420	1,393	1,361	1,314	1,262	1,191	1,117	1,024	0,932
$C_{Y_{HT}}$	0	0	0	0	0	0	0	0	0	0	0	0	0
$C_{L_{HT}}$	0	5,337	5,342	5,338	4,153	4,101	4,041	3,958	3,867	3,745	3,616	3,451	3,281
$C_{l_{HT}}$	0	0	0	0	0	0	0	0	0	0	0	0	0
$C_{m_{HT}}$	0	0	0	0	0	0	0	0	0	0	0	0	0
$C_{n_{HT}}$	0	0	0	0	0	0	0	0	0	0	0	0	0

Table 19: Equilibrium points aerodynamic forces, moments and coefficients, from 6,5 m/s to 13 m/s.

V_T (m/s)	6,5	7	7,5	8	8,5	9	9,5	10	10,5	11	12	12,5	13
D (N)	36,61	40,53	44,08	47,16	49,72	51,47	52,32	52,63	51,54	50,10	46,60	45,11	43,89
Y (N)	0	0	0	0	0	0	0	0	0	0	0	0	0
L (N)	79,40	89,15	98,10	106,57	115,54	124,69	132,89	140,14	146,59	152,48	162,29	166,55	170,51
$\bar{L}$ (Nm)	0	0	0	0	0	0	0	0	0	0	0	0	0
M (Nm)	15,73	18,06	20,78	23,57	25,73	27,09	28,25	29,69	29,36	28,85	26,07	24,18	22,11
N (Nm)	0	0	0	0	0	0	0	0	0	0	0	0	0
C_D	3,97	3,79	3,59	3,37	3,15	2,91	2,65	2,41	2,14	1,90	1,48	1,32	1,19
C_Y	0	0	0	0	0	0	0	0	0	0	0	0	0
C_L	8,61	8,33	7,99	7,62	7,32	7,05	6,74	6,42	6,09	5,77	5,16	4,88	4,62
C_l	0	0	0	0	0	0	0	0	0	0	0	0	0
C_m	7,46	7,39	7,41	7,38	7,14	6,71	6,28	5,95	5,34	4,78	3,63	3,10	2,62
C_n	0	0	0	0	0	0	0	0	0	0	0	0	0
D_W (N)	32,60	36,38	40,02	43,25	45,75	47,23	47,83	48,04	46,28	44,18	38,87	36,30	33,93
Y_W (N)	0	0	0	0	0	0	0	0	0	0	0	0	0
L_W (N)	66,91	75,71	84,67	93,51	101,87	109,40	116,21	122,91	127,08	130,73	134,74	135,84	136,56
$\bar{L}_W$ (Nm)	0	0	0	0	0	0	0	0	0	0	0	0	0
M_W (Nm)	-0,028	-0,032	-0,037	-0,042	-0,048	-0,053	-0,060	-0,066	-0,073	-0,080	-0,095	-0,103	-0,112
N_W (Nm)	0	0	0	0	0	0	0	0	0	0	0	0	0
C_{D_W}	3,53	3,40	3,26	3,09	2,90	2,67	2,43	2,20	1,92	1,67	1,24	1,06	0,92
C_{Y_W}	0	0	0	0	0	0	0	0	0	0	0	0	0
C_{L_W}	7,25	7,07	6,89	6,69	6,46	6,18	5,90	5,63	5,28	4,95	4,28	3,98	3,70
C_{l_W}	0	0	0	0	0	0	0	0	0	0	0	0	0
C_{m_W}	-0,013	-0,013	-0,013	-0,013	-0,013	-0,013	-0,013	-0,013	-0,013	-0,013	-0,013	-0,013	-0,013
C_{n_W}	0	0	0	0	0	0	0	0	0	0	0	0	0
D_{HT} (N)	3,35	3,37	3,15	2,85	2,77	2,88	2,95	2,86	3,32	3,77	5,07	5,81	6,56
Y_{HT} (N)	0	0	0	0	0	0	0	0	0	0	0	0	0
L_{HT} (N)	12,55	13,51	13,52	13,15	13,77	15,41	16,83	17,39	19,69	21,95	27,79	30,98	34,25
$\bar{L}_{HT}$ (Nm)	0	0	0	0	0	0	0	0	0	0	0	0	0
M_{HT} (Nm)	0	0	0	0	0	0	0	0	0	0	0	0	0
N_{HT} (Nm)	0	0	0	0	0	0	0	0	0	0	0	0	0
$C_{D_{HT}}$	0,816	0,710	0,614	0,529	0,451	0,386	0,338	0,294	0,311	0,321	0,363	0,383	0,400
$C_{Y_{HT}}$	0	0	0	0	0	0	0	0	0	0	0	0	0
$C_{L_{HT}}$	3,061	2,845	2,637	2,441	2,243	2,066	1,927	1,791	1,840	1,869	1,988	2,043	2,088
$C_{l_{HT}}$	0	0	0	0	0	0	0	0	0	0	0	0	0
$C_{m_{HT}}$	0	0	0	0	0	0	0	0	0	0	0	0	0
$C_{n_{HT}}$	0	0	0	0	0	0	0	0	0	0	0	0	0

Table 20: Equilibrium points aerodynamic forces, moments and coefficients, from 13,5 m/s to 23 m/s.

V_T (m/s)	13,5	14	14,5	15,5	17	18	19	20	21	22	23
D (N)	43,42	43,34	43,49	44,76	42,08	39,59	38,20	37,08	36,03	35,68	34,38
Y (N)	0	0	0	0	0	0	0	0	0	0	0
L (N)	174,40	178,45	180,20	192,95	199,82	202,23	203,89	205,27	206,44	207,32	208,29
$\bar{L}$ (Nm)	0	0	0	0	0	0	0	0	0	0	0
M (Nm)	18,60	15,29	14,15	5,32	4,09	4,31	4,80	5,10	5,39	5,65	5,71
N (Nm)	0	0	0	0	0	0	0	0	0	0	0
C_D	1,09	1,01	0,95	0,85	0,67	0,56	0,48	0,42	0,37	0,34	0,30
C_Y	0	0	0	0	0	0	0	0	0	0	0
C_L	4,38	4,17	3,92	3,68	3,17	2,86	2,59	2,35	2,14	1,96	1,80
C_l	0	0	0	0	0	0	0	0	0	0	0
C_m	2,05	1,56	1,35	0,44	0,28	0,27	0,27	0,26	0,24	0,23	0,22
C_n	0	0	0	0	0	0	0	0	0	0	0
D_W (N)	31,52	29,44	27,81	24,70	21,86	20,02	18,53	17,12	15,88	14,79	13,76
Y_W (N)	0	0	0	0	0	0	0	0	0	0	0
L_W (N)	135,68	135,18	135,75	135,13	139,21	141,11	143,24	144,74	146,07	147,12	147,82
$\bar{L}_W$ (Nm)	0	0	0	0	0	0	0	0	0	0	0
M_W (Nm)	-0,120	-0,129	-0,139	-0,159	-0,191	-0,214	-0,238	-0,264	-0,291	-0,319	-0,349
N_W (Nm)	0	0	0	0	0	0	0	0	0	0	0
C_{D_W}	0,79	0,69	0,61	0,47	0,35	0,28	0,24	0,20	0,16	0,14	0,12
C_{Y_W}	0	0	0	0	0	0	0	0	0	0	0
C_{L_W}	3,41	3,16	2,96	2,58	2,21	1,99	1,82	1,66	1,52	1,39	1,28
C_{l_W}	0	0	0	0	0	0	0	0	0	0	0
C_{m_W}	-0,013	-0,013	-0,013	-0,013	-0,013	-0,013	-0,013	-0,013	-0,013	-0,013	-0,013
C_{n_W}	0	0	0	0	0	0	0	0	0	0	0
D_{HT} (N)	7,88	9,12	9,88	12,47	10,94	10,46	9,13	8,45	7,46	6,82	6,39
Y_{HT} (N)	0	0	0	0	0	0	0	0	0	0	0
L_{HT} (N)	39,06	43,66	44,88	58,34	61,24	61,80	61,41	61,38	61,31	61,24	61,59
$\bar{L}_{HT}$ (Nm)	0	0	0	0	0	0	0	0	0	0	0
M_{HT} (Nm)	0	0	0	0	-0,01302	-0,01855	-0,02646	-0,03449	-0,04609	-0,05452	-0,06356
N_{HT} (Nm)	0	0	0	0	0	0	0	0	0	0	0
$C_{D_{HT}}$	0,446	0,480	0,536	0,536	0,390	0,349	0,267	0,227	0,175	0,145	0,124
$C_{Y_{HT}}$	0	0	0	0	0	0	0	0	0	0	0
$C_{L_{HT}}$	2,208	2,297	2,435	2,509	2,183	2,062	1,795	1,652	1,441	1,304	1,199
$C_{l_{HT}}$	0	0	0	0	0	0	0	0	0	0	0
$C_{m_{HT}}$	0	0	0	0	-0,00203	-0,00271	-0,00338	-0,00406	-0,00474	-0,00508	-0,00542
$C_{n_{HT}}$	0	0	0	0	0	0	0	0	0	0	0

Table 21: Equilibrium points aerodynamic forces, moments and coefficients, from 24 m/s to 36 m/s.

V_T (m/s)	24	25	26	27	28	29	30	31	32	33	34	35	36
D (N)	33,15	32,22	31,93	31,88	32,06	32,46	32,31	32,30	32,43	32,68	33,04	33,51	34,13
Y (N)	0	0	0	0	0	0	0	0	0	0	0	0	0
L (N)	209,13	209,82	210,33	210,75	211,10	211,39	211,72	212,01	212,26	212,48	212,67	212,84	212,99
$\bar{L}$ (Nm)	0	0	0	0	0	0	0	0	0	0	0	0	0
M (Nm)	5,73	5,76	5,86	5,97	6,08	6,20	6,23	6,27	6,31	6,35	6,40	6,45	6,51
N (Nm)	0	0	0	0	0	0	0	0	0	0	0	0	0
C_D	0,26	0,24	0,22	0,20	0,19	0,18	0,16	0,15	0,15	0,14	0,13	0,13	0,12
C_Y	0	0	0	0	0	0	0	0	0	0	0	0	0
C_L	1,66	1,54	1,42	1,32	1,23	1,15	1,08	1,01	0,95	0,89	0,84	0,80	0,75
C_l	0	0	0	0	0	0	0	0	0	0	0	0	0
C_m	0,20	0,18	0,17	0,16	0,16	0,15	0,14	0,13	0,12	0,12	0,11	0,11	0,10
C_n	0	0	0	0	0	0	0	0	0	0	0	0	0
D_W (N)	12,84	12,04	11,38	10,80	10,30	9,87	9,48	9,14	8,86	8,61	8,41	8,24	8,11
Y_W (N)	0	0	0	0	0	0	0	0	0	0	0	0	0
L_W (N)	148,33	148,71	149,16	149,55	149,87	150,16	150,27	150,35	150,40	150,41	150,40	150,38	150,32
$\bar{L}_W$ (Nm)	0	0	0	0	0	0	0	0	0	0	0	0	0
M_W (Nm)	-0,380	-0,412	-0,446	-0,481	-0,517	-0,555	-0,594	-0,634	-0,676	-0,719	-0,763	-0,808	-0,855
N_W (Nm)	0	0	0	0	0	0	0	0	0	0	0	0	0
C_{D_W}	0,10	0,09	0,08	0,07	0,06	0,05	0,05	0,04	0,04	0,04	0,03	0,03	0,03
C_{Y_W}	0	0	0	0	0	0	0	0	0	0	0	0	0
C_{L_W}	1,18	1,09	1,01	0,94	0,88	0,82	0,76	0,72	0,67	0,63	0,60	0,56	0,53
C_{l_W}	0	0	0	0	0	0	0	0	0	0	0	0	0
C_{m_W}	-0,013	-0,013	-0,013	-0,013	-0,013	-0,013	-0,013	-0,013	-0,013	-0,013	-0,013	-0,013	-0,013
C_{n_W}	0	0	0	0	0	0	0	0	0	0	0	0	0
D_{HT} (N)	6,06	5,86	5,47	5,13	4,86	4,65	4,48	4,34	4,23	4,14	4,07	4,00	3,95
Y_{HT} (N)	0	0	0	0	0	0	0	0	0	0	0	0	0
L_{HT} (N)	61,98	62,34	62,48	62,60	62,71	62,81	63,09	63,37	63,64	63,91	64,18	64,44	64,73
$\bar{L}_{HT}$ (Nm)	0	0	0	0	0	0	0	0	0	0	0	0	0
M_{HT} (Nm)	-0,0729	-0,0815	-0,0915	-0,1000	-0,1059	-0,1136	-0,1216	-0,1297	-0,1374	-0,1449	-0,1522	-0,1594	-0,1667
N_{HT} (Nm)	0	0	0	0	0	0	0	0	0	0	0	0	0
$C_{D_{HT}}$	0,109	0,100	0,085	0,073	0,064	0,057	0,051	0,047	0,043	0,040	0,037	0,035	0,033
$C_{Y_{HT}}$	0	0	0	0	0	0	0	0	0	0	0	0	0
$C_{L_{HT}}$	1,117	1,064	0,972	0,891	0,824	0,769	0,722	0,680	0,645	0,614	0,587	0,563	0,541
$C_{l_{HT}}$	0	0	0	0	0	0	0	0	0	0	0	0	0
$C_{m_{HT}}$	-0,0057	-0,0060	-0,0062	-0,0062	-0,0060	-0,0060	-0,0060	-0,0060	-0,0060	-0,0060	-0,0060	-0,0060	-0,0060
$C_{n_{HT}}$	0	0	0	0	0	0	0	0	0	0	0	0	0

Table 22: Equilibrium points aerodynamic forces, moments and coefficients, from 37 m/s to 47 m/s.

V_T (m/s)	37	38	39	40	41	42	43	44	45	46	47
D (N)	34,84	35,64	36,52	37,48	38,52	39,62	40,80	42,04	43,35	44,72	46,16
Y (N)	0	0	0	0	0	0	0	0	0	0	0
L (N)	213,13	213,25	213,37	213,47	213,57	213,66	213,75	213,83	213,91	213,99	214,06
$\bar{L}$ (Nm)	0	0	0	0	0	0	0	0	0	0	0
M (Nm)	6,57	6,63	6,70	6,77	6,84	6,92	7,00	7,08	7,16	7,25	7,34
N (Nm)	0	0	0	0	0	0	0	0	0	0	0
C_D	0,12	0,11	0,11	0,11	0,10	0,10	0,10	0,10	0,10	0,10	0,10
C_Y	0	0	0	0	0	0	0	0	0	0	0
C_L	0,71	0,68	0,64	0,61	0,58	0,55	0,53	0,51	0,48	0,46	0,44
C_l	0	0	0	0	0	0	0	0	0	0	0
C_m	0,10	0,09	0,09	0,08	0,08	0,08	0,08	0,07	0,07	0,07	0,07
C_n	0	0	0	0	0	0	0	0	0	0	0
D_W (N)	8,01	7,93	7,88	7,84	7,83	7,84	7,87	7,91	7,97	8,04	8,13
Y_W (N)	0	0	0	0	0	0	0	0	0	0	0
L_W (N)	150,27	150,19	150,09	149,94	149,80	149,64	149,48	149,31	149,08	148,89	148,67
$\bar{L}_W$ (Nm)	0	0	0	0	0	0	0	0	0	0	0
M_W (Nm)	-0,904	-0,953	-1,004	-1,056	-1,109	-1,164	-1,220	-1,278	-1,336	-1,397	-1,458
N_W (Nm)	0	0	0	0	0	0	0	0	0	0	0
C_{D_W}	0,03	0,03	0,02	0,02	0,02	0,02	0,02	0,02	0,02	0,02	0,02
C_{Y_W}	0	0	0	0	0	0	0	0	0	0	0
C_{L_W}	0,50	0,48	0,45	0,43	0,41	0,39	0,37	0,35	0,34	0,32	0,31
C_{l_W}	0	0	0	0	0	0	0	0	0	0	0
C_{m_W}	-0,013	-0,013	-0,013	-0,013	-0,013	-0,013	-0,013	-0,013	-0,013	-0,013	-0,013
C_{n_W}	0	0	0	0	0	0	0	0	0	0	0
D_{HT} (N)	3,91	3,87	3,84	3,82	3,80	3,78	3,78	3,78	3,78	3,79	3,81
Y_{HT} (N)	0	0	0	0	0	0	0	0	0	0	0
L_{HT} (N)	65,00	65,28	65,58	65,92	66,25	66,58	66,92	67,27	67,67	68,03	68,43
$\bar{L}_{HT}$ (Nm)	0	0	0	0	0	0	0	0	0	0	0
M_{HT} (Nm)	-0,18499	-0,19682	-0,19612	-0,16697	-0,14983	-0,13115	-0,11093	-0,08922	-0,03109	-0,00499	0,041542
N_{HT} (Nm)	0	0	0	0	0	0	0	0	0	0	0
$C_{D_{HT}}$	0,031	0,030	0,028	0,027	0,026	0,024	0,023	0,022	0,021	0,021	0,020
$C_{Y_{HT}}$	0	0	0	0	0	0	0	0	0	0	0
$C_{L_{HT}}$	0,520	0,500	0,481	0,463	0,446	0,430	0,414	0,399	0,384	0,370	0,357
$C_{l_{HT}}$	0	0	0	0	0	0	0	0	0	0	0
$C_{m_{HT}}$	-0,00647	-0,00659	-0,00630	-0,00514	-0,00442	-0,00370	-0,00300	-0,00231	-0,00077	-0,00012	0,00095
$C_{n_{HT}}$	0	0	0	0	0	0	0	0	0	0	0

23 Appendix F – Propeller Coefficients

Table 23: Propeller rotation direction indexes.

λ_1	λ_2	λ_3	λ_4	λ_5	λ_6
1	-1	1	-1	1	-1

Table 24: Equilibrium points propeller coefficients, from 0 m/s to 6 m/s.

V_T (m/s)	0	0,5	1	1,5	2	2,5	3	3,5	4	4,5	5	5,5	6
k_{T_1}	5,10E-05	5,10E-05	5,10E-05	5,09E-05	5,08E-05	5,07E-05	5,05E-05	5,04E-05	5,02E-05	4,99E-05	4,97E-05	4,94E-05	4,91E-05
k_{T_2}	5,10E-05	5,10E-05	5,10E-05	5,09E-05	5,08E-05	5,07E-05	5,05E-05	5,04E-05	5,02E-05	4,99E-05	4,97E-05	4,94E-05	4,91E-05
k_{T_3}	5,10E-05	5,10E-05	5,10E-05	5,09E-05	5,08E-05	5,07E-05	5,05E-05	5,04E-05	5,02E-05	4,99E-05	4,97E-05	4,94E-05	4,91E-05
k_{T_4}	5,10E-05	5,10E-05	5,10E-05	5,09E-05	5,08E-05	5,07E-05	5,05E-05	5,04E-05	5,02E-05	4,99E-05	4,97E-05	4,94E-05	4,91E-05
k_{T_5}	5,08E-05	5,08E-05	5,07E-05	5,07E-05	5,07E-05	5,06E-05	5,06E-05	5,05E-05	5,05E-05	5,04E-05	5,04E-05	5,03E-05	5,03E-05
k_{T_6}	5,08E-05	5,08E-05	5,07E-05	5,07E-05	5,07E-05	5,06E-05	5,06E-05	5,05E-05	5,05E-05	5,04E-05	5,04E-05	5,03E-05	5,03E-05
k_{Q_1}	1,86E-06	1,86E-06	1,86E-06	1,86E-06	1,86E-06	1,85E-06	1,85E-06	1,85E-06	1,85E-06	1,84E-06	1,84E-06	1,83E-06	1,83E-06
k_{Q_2}	1,86E-06	1,86E-06	1,86E-06	1,86E-06	1,86E-06	1,85E-06	1,85E-06	1,85E-06	1,85E-06	1,84E-06	1,84E-06	1,83E-06	1,83E-06
k_{Q_3}	1,86E-06	1,86E-06	1,86E-06	1,86E-06	1,86E-06	1,85E-06	1,85E-06	1,85E-06	1,85E-06	1,84E-06	1,84E-06	1,83E-06	1,83E-06
k_{Q_4}	1,86E-06	1,86E-06	1,86E-06	1,86E-06	1,86E-06	1,85E-06	1,85E-06	1,85E-06	1,85E-06	1,84E-06	1,84E-06	1,83E-06	1,83E-06
k_{Q_5}	1,86E-06	1,85E-06	1,85E-06	1,85E-06	1,85E-06	1,85E-06	1,85E-06	1,85E-06	1,85E-06	1,85E-06	1,85E-06	1,85E-06	1,85E-06
k_{Q_6}	1,86E-06	1,85E-06	1,85E-06	1,85E-06	1,85E-06	1,85E-06	1,85E-06	1,85E-06	1,85E-06	1,85E-06	1,85E-06	1,85E-06	1,85E-06
J'_1	0	1,20E-05	9,92E-05	0,000347	0,000786	0,001606	0,002921	0,004899	0,007761	0,011646	0,017227	0,024369	0,034071
J'_2	0	1,20E-05	9,92E-05	0,000347	0,000786	0,001606	0,002921	0,004899	0,007761	0,011646	0,017227	0,024369	0,034071
J'_3	0	1,20E-05	9,92E-05	0,000347	0,000786	0,001606	0,002921	0,004899	0,007761	0,011646	0,017227	0,024369	0,034071
J'_4	0	1,20E-05	9,92E-05	0,000347	0,000786	0,001606	0,002921	0,004899	0,007761	0,011646	0,017227	0,024369	0,034071
J'_5	0	1,23E-05	0,000102	0,000355	0,000797	0,001617	0,002909	0,004816	0,007503	0,011371	0,015929	0,022297	0,029544
J'_6	0	1,23E-05	0,000102	0,000355	0,000797	0,001617	0,002909	0,004816	0,007503	0,011371	0,015929	0,022297	0,029544

Table 25: Equilibrium points propeller coefficients, from 6,5 m/s to 13 m/s.

V_T (m/s)	6,5	7	7,5	8	8,5	9	9,5	10	10,5	11	12	12,5	13
k_{T_1}	4,88E-05	4,85E-05	4,81E-05	4,78E-05	4,74E-05	4,70E-05	4,86E-05	4,80E-05	4,72E-05	4,56E-05	4,06E-05	3,82E-05	3,65E-05
k_{T_2}	4,88E-05	4,85E-05	4,81E-05	4,78E-05	4,74E-05	4,70E-05	4,86E-05	4,80E-05	4,72E-05	4,56E-05	4,06E-05	3,82E-05	3,65E-05
k_{T_3}	4,88E-05	4,85E-05	4,81E-05	4,78E-05	4,74E-05	4,70E-05	4,86E-05	4,80E-05	4,72E-05	4,56E-05	4,06E-05	3,82E-05	3,65E-05
k_{T_4}	4,88E-05	4,85E-05	4,81E-05	4,78E-05	4,74E-05	4,70E-05	4,86E-05	4,80E-05	4,72E-05	4,56E-05	4,06E-05	3,82E-05	3,65E-05
k_{T_5}	5,03E-05	5,03E-05	5,04E-05	5,05E-05	5,06E-05	5,06E-05	5,06E-05	5,07E-05	5,06E-05	5,05E-05	4,87E-05	4,84E-05	4,81E-05
k_{T_6}	5,03E-05	5,03E-05	5,04E-05	5,05E-05	5,06E-05	5,06E-05	5,06E-05	5,07E-05	5,06E-05	5,05E-05	4,87E-05	4,84E-05	4,81E-05
k_{Q_1}	1,82E-06	1,82E-06	1,81E-06	1,80E-06	1,80E-06	1,79E-06	1,56E-06	1,55E-06	1,57E-06	1,59E-06	1,56E-06	1,53E-06	1,50E-06
k_{Q_2}	1,82E-06	1,82E-06	1,81E-06	1,80E-06	1,80E-06	1,79E-06	1,56E-06	1,55E-06	1,57E-06	1,59E-06	1,56E-06	1,53E-06	1,50E-06
k_{Q_3}	1,82E-06	1,82E-06	1,81E-06	1,80E-06	1,80E-06	1,79E-06	1,56E-06	1,55E-06	1,57E-06	1,59E-06	1,56E-06	1,53E-06	1,50E-06
k_{Q_4}	1,82E-06	1,82E-06	1,81E-06	1,80E-06	1,80E-06	1,79E-06	1,56E-06	1,55E-06	1,57E-06	1,59E-06	1,56E-06	1,53E-06	1,50E-06
k_{Q_5}	1,85E-06	1,85E-06	1,85E-06	1,85E-06	1,85E-06	1,85E-06	1,85E-06	1,85E-06	1,85E-06	1,85E-06	1,59E-06	1,57E-06	1,56E-06
k_{Q_6}	1,85E-06	1,85E-06	1,85E-06	1,85E-06	1,85E-06	1,85E-06	1,85E-06	1,85E-06	1,85E-06	1,85E-06	1,59E-06	1,57E-06	1,56E-06
J'_1	0,046153	0,061725	0,080478	0,103568	0,132504	0,16847	0,216273	0,271738	0,330474	0,394594	0,494428	0,526543	0,546496
J'_2	0,046153	0,061725	0,080478	0,103568	0,132504	0,16847	0,216273	0,271738	0,330474	0,394594	0,494428	0,526543	0,546496
J'_3	0,046153	0,061725	0,080478	0,103568	0,132504	0,16847	0,216273	0,271738	0,330474	0,394594	0,494428	0,526543	0,546496
J'_4	0,046153	0,061725	0,080478	0,103568	0,132504	0,16847	0,216273	0,271738	0,330474	0,394594	0,494428	0,526543	0,546496
J'_5	0,038718	0,048785	0,05976	0,071198	0,084326	0,098667	0,112463	0,125826	0,13855	0,151996	0,179508	0,197003	0,217165
J'_6	0,038718	0,048785	0,05976	0,071198	0,084326	0,098667	0,112463	0,125826	0,13855	0,151996	0,179508	0,197003	0,217165

Table 26: Equilibrium points propeller coefficients, from 13,5 m/s to 23 m/s.

V_T (m/s)	13,5	14	14,5	15,5	17	18	19	20	21	22	23
k_{T_1}	3,74E-05	3,82E-05	3,80E-05	4,05E-05	3,78E-05	3,50E-05	3,26E-05	3,04E-05	2,83E-05	2,65E-05	2,45E-05
k_{T_2}	3,74E-05	3,82E-05	3,80E-05	4,05E-05	3,78E-05	3,50E-05	3,26E-05	3,04E-05	2,83E-05	2,65E-05	2,45E-05
k_{T_3}	3,74E-05	3,82E-05	3,80E-05	4,05E-05	3,78E-05	3,50E-05	3,26E-05	3,04E-05	2,83E-05	2,65E-05	2,45E-05
k_{T_4}	3,74E-05	3,82E-05	3,80E-05	4,05E-05	3,78E-05	3,50E-05	3,26E-05	3,04E-05	2,83E-05	2,65E-05	2,45E-05
k_{T_5}	4,78E-05	4,75E-05	4,73E-05	3,44E-05	4,09E-05	4,09E-05	4,09E-05	4,09E-05	4,09E-05	4,09E-05	4,09E-05
k_{T_6}	4,78E-05	4,75E-05	4,73E-05	3,44E-05	4,09E-05	4,09E-05	4,09E-05	4,09E-05	4,09E-05	4,09E-05	4,09E-05
k_{Q_1}	1,53E-06	1,54E-06	1,54E-06	1,58E-06	1,54E-06	1,50E-06	1,45E-06	1,40E-06	1,34E-06	1,30E-06	1,24E-06
k_{Q_2}	1,53E-06	1,54E-06	1,54E-06	1,58E-06	1,54E-06	1,50E-06	1,45E-06	1,40E-06	1,34E-06	1,30E-06	1,24E-06
k_{Q_3}	1,53E-06	1,54E-06	1,54E-06	1,58E-06	1,54E-06	1,50E-06	1,45E-06	1,40E-06	1,34E-06	1,30E-06	1,24E-06
k_{Q_4}	1,53E-06	1,54E-06	1,54E-06	1,58E-06	1,54E-06	1,50E-06	1,45E-06	1,40E-06	1,34E-06	1,30E-06	1,24E-06
k_{Q_5}	1,55E-06	1,55E-06	1,56E-06	1,37E-06	1,71E-06	1,71E-06	1,71E-06	1,71E-06	1,71E-06	1,71E-06	1,71E-06
k_{Q_6}	1,55E-06	1,55E-06	1,56E-06	1,37E-06	1,71E-06	1,71E-06	1,71E-06	1,71E-06	1,71E-06	1,71E-06	1,71E-06
J'_1	0,529091	0,514504	0,516907	0,47311	0,515667	0,551795	0,580174	0,604917	0,627102	0,64452	0,66413
J'_2	0,529091	0,514504	0,516907	0,47311	0,515667	0,551795	0,580174	0,604917	0,627102	0,64452	0,66413
J'_3	0,529091	0,514504	0,516907	0,47311	0,515667	0,551795	0,580174	0,604917	0,627102	0,64452	0,66413
J'_4	0,529091	0,514504	0,516907	0,47311	0,515667	0,551795	0,580174	0,604917	0,627102	0,64452	0,66413
J'_5	0,245415	0,282246	0,301511	0,614489	0	0	0	0	0	0	0
J'_6	0,245415	0,282246	0,301511	0,614489	0	0	0	0	0	0	0

Table 27: Equilibrium points propeller coefficients, from 24 m/s to 36 m/s.

V_T (m/s)	24	25	26	27	28	29	30	31	32	33	34	35	36
k_{T_1}	2,27E-05	2,11E-05	1,98E-05	1,87E-05	1,78E-05	1,71E-05	1,62E-05	1,54E-05	1,48E-05	1,42E-05	1,36E-05	1,32E-05	1,28E-05
k_{T_2}	2,27E-05	2,11E-05	1,98E-05	1,87E-05	1,78E-05	1,71E-05	1,62E-05	1,54E-05	1,48E-05	1,42E-05	1,36E-05	1,32E-05	1,28E-05
k_{T_3}	2,27E-05	2,11E-05	1,98E-05	1,87E-05	1,78E-05	1,71E-05	1,62E-05	1,54E-05	1,48E-05	1,42E-05	1,36E-05	1,32E-05	1,28E-05
k_{T_4}	2,27E-05	2,11E-05	1,98E-05	1,87E-05	1,78E-05	1,71E-05	1,62E-05	1,54E-05	1,48E-05	1,42E-05	1,36E-05	1,32E-05	1,28E-05
k_{T_5}	4,09E-05	4,09E-05	4,09E-05	4,09E-05	4,09E-05	4,09E-05	4,09E-05	4,09E-05	4,09E-05	4,09E-05	4,09E-05	4,09E-05	4,09E-05
k_{T_6}	4,09E-05	4,09E-05	4,09E-05	4,09E-05	4,09E-05	4,09E-05	4,09E-05	4,09E-05	4,09E-05	4,09E-05	4,09E-05	4,09E-05	4,09E-05
k_{Q_1}	1,19E-06	1,14E-06	1,10E-06	1,06E-06	1,03E-06	1,01E-06	9,79E-07	9,52E-07	9,28E-07	9,07E-07	8,88E-07	8,71E-07	8,57E-07
k_{Q_2}	1,19E-06	1,14E-06	1,10E-06	1,06E-06	1,03E-06	1,01E-06	9,79E-07	9,52E-07	9,28E-07	9,07E-07	8,88E-07	8,71E-07	8,57E-07
k_{Q_3}	1,19E-06	1,14E-06	1,10E-06	1,06E-06	1,03E-06	1,01E-06	9,79E-07	9,52E-07	9,28E-07	9,07E-07	8,88E-07	8,71E-07	8,57E-07
k_{Q_4}	1,19E-06	1,14E-06	1,10E-06	1,06E-06	1,03E-06	1,01E-06	9,79E-07	9,52E-07	9,28E-07	9,07E-07	8,88E-07	8,71E-07	8,57E-07
k_{Q_5}	1,71E-06	1,71E-06	1,71E-06	1,71E-06	1,71E-06	1,71E-06	1,71E-06	1,71E-06	1,71E-06	1,71E-06	1,71E-06	1,71E-06	1,71E-06
k_{Q_6}	1,71E-06	1,71E-06	1,71E-06	1,71E-06	1,71E-06	1,71E-06	1,71E-06	1,71E-06	1,71E-06	1,71E-06	1,71E-06	1,71E-06	1,71E-06
J'_1	0,681975	0,697298	0,709063	0,719052	0,727447	0,734477	0,742888	0,750285	0,756775	0,762487	0,767533	0,771988	0,775773
J'_2	0,681975	0,697298	0,709063	0,719052	0,727447	0,734477	0,742888	0,750285	0,756775	0,762487	0,767533	0,771988	0,775773
J'_3	0,681975	0,697298	0,709063	0,719052	0,727447	0,734477	0,742888	0,750285	0,756775	0,762487	0,767533	0,771988	0,775773
J'_4	0,681975	0,697298	0,709063	0,719052	0,727447	0,734477	0,742888	0,750285	0,756775	0,762487	0,767533	0,771988	0,775773
J'_5	0	0	0	0	0	0	0	0	0	0	0	0	0
J'_6	0	0	0	0	0	0	0	0	0	0	0	0	0

Table 28: Equilibrium points propeller coefficients, from 37 m/s to 47 m/s.

V_T (m/s)	37	38	39	40	41	42	43	44	45	46	47
k_{T_1}	1,25E-05	1,22E-05	1,19E-05	1,17E-05	1,15E-05	1,13E-05	1,11E-05	1,10E-05	1,09E-05	1,08E-05	1,07E-05
k_{T_2}	1,25E-05	1,22E-05	1,19E-05	1,17E-05	1,15E-05	1,13E-05	1,11E-05	1,10E-05	1,09E-05	1,08E-05	1,07E-05
k_{T_3}	1,25E-05	1,22E-05	1,19E-05	1,17E-05	1,15E-05	1,13E-05	1,11E-05	1,10E-05	1,09E-05	1,08E-05	1,07E-05
k_{T_4}	1,25E-05	1,22E-05	1,19E-05	1,17E-05	1,15E-05	1,13E-05	1,11E-05	1,10E-05	1,09E-05	1,08E-05	1,07E-05
k_{T_5}	4,09E-05	4,09E-05	4,09E-05	4,09E-05	4,09E-05	4,09E-05	4,09E-05	4,09E-05	4,09E-05	4,09E-05	4,09E-05
k_{T_6}	4,09E-05	4,09E-05	4,09E-05	4,09E-05	4,09E-05	4,09E-05	4,09E-05	4,09E-05	4,09E-05	4,09E-05	4,09E-05
k_{Q_1}	8,45E-07	8,34E-07	8,24E-07	8,15E-07	8,07E-07	8,00E-07	7,94E-07	7,89E-07	7,84E-07	7,80E-07	7,76E-07
k_{Q_2}	8,45E-07	8,34E-07	8,24E-07	8,15E-07	8,07E-07	8,00E-07	7,94E-07	7,89E-07	7,84E-07	7,80E-07	7,76E-07
k_{Q_3}	8,45E-07	8,34E-07	8,24E-07	8,15E-07	8,07E-07	8,00E-07	7,94E-07	7,89E-07	7,84E-07	7,80E-07	7,76E-07
k_{Q_4}	8,45E-07	8,34E-07	8,24E-07	8,15E-07	8,07E-07	8,00E-07	7,94E-07	7,89E-07	7,84E-07	7,80E-07	7,76E-07
k_{Q_5}	1,71E-06	1,71E-06	1,71E-06	1,71E-06	1,71E-06	1,71E-06	1,71E-06	1,71E-06	1,71E-06	1,71E-06	1,71E-06
k_{Q_6}	1,71E-06	1,71E-06	1,71E-06	1,71E-06	1,71E-06	1,71E-06	1,71E-06	1,71E-06	1,71E-06	1,71E-06	1,71E-06
J'_1	0,779115	0,782072	0,784691	0,787014	0,789075	0,790904	0,792528	0,793971	0,795254	0,796394	0,797408
J'_2	0,779115	0,782072	0,784691	0,787014	0,789075	0,790904	0,792528	0,793971	0,795254	0,796394	0,797408
J'_3	0,779115	0,782072	0,784691	0,787014	0,789075	0,790904	0,792528	0,793971	0,795254	0,796394	0,797408
J'_4	0,779115	0,782072	0,784691	0,787014	0,789075	0,790904	0,792528	0,793971	0,795254	0,796394	0,797408
J'_5	0	0	0	0	0	0	0	0	0	0	0
J'_6	0	0	0	0	0	0	0	0	0	0	0

24 Appendix G – Aerodynamic Controls Coefficients

Table 29: Equilibrium points aerodynamic controls coefficients, from 0 m/s to 6 m/s.

V_T (m/s)	0	0,5	1	1,5	2	2,5	3	3,5	4	4,5	5	5,5	6
$C_{L\delta_f}$	0,4519	0,4519	0,4519	0,4519	0,4519	0,4519	0,4519	0,4519	0,4519	0,4519	0,4519	0,4519	0,4519
$C_{L\delta_e}$	1,852	1,852	1,852	1,852	1,852	1,852	1,852	1,852	1,852	1,852	1,852	1,852	1,852
$C_{Y\delta_r}$	0,00592	0,00592	0,00592	0,00592	0,00592	0,00592	0,00592	0,00592	0,00592	0,00592	0,00592	0,00592	0,00592
$C_{l\delta_{a_L}}$	0,0943	0,0943	0,0943	0,0943	0,0943	0,0943	0,0943	0,0943	0,0943	0,0943	0,0943	0,0943	0,0943
$C_{l\delta_{a_R}}$	0,0943	0,0943	0,0943	0,0943	0,0943	0,0943	0,0943	0,0943	0,0943	0,0943	0,0943	0,0943	0,0943
$C_{l\delta_r}$	0,00066	0,00066	0,00066	0,00066	0,00066	0,00066	0,00066	0,00066	0,00066	0,00066	0,00066	0,00066	0,00066
$C_{n\delta_r}$	-0,0021	-0,0021	-0,0021	-0,0021	-0,0021	-0,0021	-0,0021	-0,0021	-0,0021	-0,0021	-0,0021	-0,0021	-0,0021
$C_{n\delta_{a_R}}$	-0,119	-0,119	-0,119	-0,120	-0,120	-0,119	-0,119	-0,118	-0,117	-0,116	-0,115	-0,113	-0,110
$C_{n\delta_{a_L}}$	-0,119	-0,119	-0,119	-0,120	-0,120	-0,119	-0,119	-0,118	-0,117	-0,116	-0,115	-0,113	-0,110

Table 30: Equilibrium points aerodynamic controls coefficients, from 6,5 m/s to 13 m/s.

V_T (m/s)	6,5	7	7,5	8	8,5	9	9,5	10	10,5	11	12	12,5	13
$C_{L\delta_f}$	0,4519	0,4519	0,4519	0,4519	0,4519	0,4519	0,4519	0,4519	0,4519	0,4519	0,4519	0,4519	0,4519
$C_{L\delta_e}$	1,852	1,852	1,852	1,852	1,852	1,852	1,852	1,852	1,852	1,852	1,852	1,852	1,852
$C_{Y\delta_r}$	0,00592	0,00592	0,00592	0,00592	0,00592	0,00592	0,00592	0,00592	0,00592	0,00592	0,00592	0,00592	0,00592
$C_{l\delta_{a_L}}$	0,0943	0,0943	0,0943	0,0943	0,0943	0,0943	0,0943	0,0943	0,0943	0,0943	0,0943	0,0943	0,0943
$C_{l\delta_{a_R}}$	0,0943	0,0943	0,0943	0,0943	0,0943	0,0943	0,0943	0,0943	0,0943	0,0943	0,0943	0,0943	0,0943
$C_{l\delta_r}$	0,00066	0,00066	0,00066	0,00066	0,00066	0,00066	0,00066	0,00066	0,00066	0,00066	0,00066	0,00066	0,00066
$C_{n\delta_r}$	-0,0021	-0,0021	-0,0021	-0,0021	-0,0021	-0,0021	-0,0021	-0,0021	-0,0021	-0,0021	-0,0021	-0,0021	-0,0021
$C_{n\delta_{a_R}}$	-0,108	-0,105	-0,102	-0,098	-0,095	-0,090	-0,086	-0,082	-0,077	-0,073	-0,063	-0,059	-0,055
$C_{n\delta_{a_L}}$	-0,108	-0,105	-0,102	-0,098	-0,095	-0,090	-0,086	-0,082	-0,077	-0,073	-0,063	-0,059	-0,055

Table 31: Equilibrium points aerodynamic controls coefficients, from 13,5 m/s to 23 m/s.

V_T (m/s)	13,5	14	14,5	15,5	17	18	19	20	21	22	23
$C_{L\delta_f}$	0,4519	0,4519	0,4519	0,4519	0,4519	0,4519	0,4519	0,4519	0,4519	0,4519	0,4519
$C_{L\delta_e}$	1,852	1,852	1,852	1,852	1,852	1,852	1,852	1,852	1,852	1,852	1,852
$C_{Y\delta_r}$	0,00592	0,00592	0,00592	0,00592	0,00592	0,00592	0,00592	0,00592	0,00592	0,00592	0,00592
$C_{l\delta_{a_L}}$	0,0943	0,0943	0,0943	0,0943	0,0943	0,0943	0,0943	0,0943	0,0943	0,0943	0,0943
$C_{l\delta_{a_R}}$	0,0943	0,0943	0,0943	0,0943	0,0943	0,0943	0,0943	0,0943	0,0943	0,0943	0,0943
$C_{l\delta_r}$	0,00066	0,00066	0,00066	0,00066	0,00066	0,00066	0,00066	0,00066	0,00066	0,00066	0,00066
$C_{n\delta_r}$	-0,0021	-0,0021	-0,0021	-0,0021	-0,0021	-0,0021	-0,0021	-0,0021	-0,0021	-0,0021	-0,0021
$C_{n\delta_{a_R}}$	-0,050	-0,046	-0,043	-0,038	-0,032	-0,029	-0,027	-0,024	-0,022	-0,021	-0,019
$C_{n\delta_{a_L}}$	-0,050	-0,046	-0,043	-0,038	-0,032	-0,029	-0,027	-0,024	-0,022	-0,021	-0,019

Table 32: Equilibrium points aerodynamic controls coefficients, from 24 m/s to 36 m/s.

V_T (m/s)	24	25	26	27	28	29	30	31	32	33	34	35	36
$C_{L_{\delta_f}}$	0,4519	0,4519	0,4519	0,4519	0,4519	0,4519	0,4519	0,4519	0,4519	0,4519	0,4519	0,4519	0,4519
$C_{L_{\delta_e}}$	1,852	1,852	1,852	1,852	1,852	1,852	1,852	1,852	1,852	1,852	1,852	1,852	1,852
$C_{Y_{\delta_r}}$	0,00592	0,00592	0,00592	0,00592	0,00592	0,00592	0,00592	0,00592	0,00592	0,00592	0,00592	0,00592	0,00592
$C_{l_{\delta_{a_L}}}$	0,0943	0,0943	0,0943	0,0943	0,0943	0,0943	0,0943	0,0943	0,0943	0,0943	0,0943	0,0943	0,0943
$C_{l_{\delta_{a_R}}}$	0,0943	0,0943	0,0943	0,0943	0,0943	0,0943	0,0943	0,0943	0,0943	0,0943	0,0943	0,0943	0,0943
$C_{l_{\delta_r}}$	0,00066	0,00066	0,00066	0,00066	0,00066	0,00066	0,00066	0,00066	0,00066	0,00066	0,00066	0,00066	0,00066
$C_{n_{\delta_r}}$	-0,0021	-0,0021	-0,0021	-0,0021	-0,0021	-0,0021	-0,0021	-0,0021	-0,0021	-0,0021	-0,0021	-0,0021	-0,0021
$C_{n_{\delta_{a_R}}}$	-0,018	-0,016	-0,015	-0,014	-0,013	-0,012	-0,011	-0,011	-0,010	-0,010	-0,009	-0,008	-0,008
$C_{n_{\delta_{a_L}}}$	-0,018	-0,016	-0,015	-0,014	-0,013	-0,012	-0,011	-0,011	-0,010	-0,010	-0,009	-0,008	-0,008

Table 33: Equilibrium points aerodynamic controls coefficients, from 37 m/s to 47 m/s.

V_T (m/s)	37	38	39	40	41	42	43	44	45	46	47
$C_{L_{\delta_f}}$	0,4519	0,4519	0,4519	0,4519	0,4519	0,4519	0,4519	0,4519	0,4519	0,4519	0,4519
$C_{L_{\delta_e}}$	1,852	1,852	1,852	1,852	1,852	1,852	1,852	1,852	1,852	1,852	1,852
$C_{Y_{\delta_r}}$	0,00592	0,00592	0,00592	0,00592	0,00592	0,00592	0,00592	0,00592	0,00592	0,00592	0,00592
$C_{l_{\delta_{a_L}}}$	0,0943	0,0943	0,0943	0,0943	0,0943	0,0943	0,0943	0,0943	0,0943	0,0943	0,0943
$C_{l_{\delta_{a_R}}}$	0,0943	0,0943	0,0943	0,0943	0,0943	0,0943	0,0943	0,0943	0,0943	0,0943	0,0943
$C_{l_{\delta_r}}$	0,00066	0,00066	0,00066	0,00066	0,00066	0,00066	0,00066	0,00066	0,00066	0,00066	0,00066
$C_{n_{\delta_r}}$	-0,0021	-0,0021	-0,0021	-0,0021	-0,0021	-0,0021	-0,0021	-0,0021	-0,0021	-0,0021	-0,0021
$C_{n_{\delta_{a_R}}}$	-0,008	-0,007	-0,007	-0,006	-0,006	-0,006	-0,006	-0,005	-0,005	-0,005	-0,005
$C_{n_{\delta_{a_L}}}$	-0,008	-0,007	-0,007	-0,006	-0,006	-0,006	-0,006	-0,005	-0,005	-0,005	-0,005

25 Appendix H – Wing Aerodynamic Derivatives

Table 34: Equilibrium points wing aerodynamic derivatives, from 0 m/s to 6 m/s.

V_T (m/s)	0	0,5	1	1,5	2	2,5	3	3,5	4	4,5	5	5,5	6
$(C_{L_{\dot{\alpha}}})_W$	0	0	0	0	0	0	0	0	0	0	0	0	0
$(C_{L_{Q_W}})_W$	2,22	2,21	2,21	2,19	2,18	2,14	2,08	1,99	1,85	1,67	1,40	1,08	0,67
$(C_{Y_{\dot{\beta}}})_W$	0	0	0	0	0	0	0	0	0	0	0	0	0
$(C_{Y_{R_W}})_W$	0	0	0	0	0	0	0	0	0	0	0	0	0
$(C_{Y_{P_W}})_W$	2,2E-11	-2,5E-05	-1,1E-04	-2,6E-04	-4,5E-04	-7,3E-04	-1,1E-03	-1,5E-03	-2,0E-03	-2,5E-03	-3,0E-03	-3,5E-03	-3,9E-03
$(C_{Y_{\beta}})_W$	8,5E-31	2,7E-17	4,5E-16	2,4E-15	6,8E-15	1,7E-14	3,8E-14	7,3E-14	1,3E-13	2,0E-13	3,0E-13	4,0E-13	5,0E-13
$(C_{D_{Q_W}})_W$	2,9125	2,910	2,903	2,892	2,878	2,857	2,830	2,796	2,755	2,708	2,649	2,585	2,508
$(C_{l_{P_W}})_W$	0,1149	0,114	0,114	0,113	0,112	0,110	0,107	0,1029	0,0957	0,0861	0,0720	0,0551	0,0333
$(C_{l_{R_W}})_W$	0,0000	0	0,0014	0,0033	0,0055	0,0088	0,0129	0,0179	0,0237	0,0297	0,0364	0,0422	0,0472
$(C_{l_{\dot{\beta}}})_W$	0	0	0	0	0	0	0	0	0	0	0	0	0
$(C_{l_{\beta}})_W$	6,5E-12	-3,6E-05	-1,5E-04	-3,5E-04	-5,8E-04	-9,4E-04	-1,3E-03	-1,9E-03	-2,5E-03	-3,1E-03	-3,9E-03	-4,5E-03	-5,0E-03
$(C_{m_{Q_W}})_W$	-8,44	-8,44	-8,44	-8,44	-8,44	-8,44	-8,44	-8,44	-8,44	-8,44	-8,44	-8,44	-8,44
$(C_{m_{\dot{\alpha}}})_W$	0	0	0	0	0	0	0	0	0	0	0	0	0
$(C_{n_{P_W}})_W$	0	-3,1E+01	-6,5E+00	-2,5E+00	-1,3E+00	-8,3E-01	-5,4E-01	-3,6E-01	-2,5E-01	-1,8E-01	-1,2E-01	-8,2E-02	-4,84E-02
$(C_{n_{R_W}})_W$	-0,125	-0,125	-0,125	-0,125	-0,125	-0,125	-0,125	-0,125	-0,124	-0,124	-0,1232	-0,1219	-0,1202
$(C_{n_{\dot{\beta}}})_W$	0	0	0	0	0	0	0	0	0	0	0	0	0
$(C_{n_{\beta}})_W$	1,1E-23	3,6E-10	6,1E-09	3,2E-08	9,1E-08	2,3E-07	5,1E-07	9,8E-07	1,7E-06	2,7E-06	4,0E-06	5,4E-06	6,8E-06

Table 35: Equilibrium points wing aerodynamic derivatives, from 6,5 m/s to 13 m/s.

V_T (m/s)	6,5	7	7,5	8	8,5	9	9,5	10	10,5	11	12	12,5	13
$(C_{L_{\dot{\alpha}}})_W$	0	0	0	0	0	0	0	0	0	0	0	0	0
$(C_{L_{Q_W}})_W$	0,22	-0,27	-0,72	-1,12	-1,44	-1,66	-1,75	-1,72	-1,56	-1,31	-0,48	0,02	0,53
$(C_{Y_{\dot{\beta}}})_W$	0	0	0	0	0	0	0	0	0	0	0	0	0
$(C_{Y_{R_W}})_W$	0	0	0	0	0	0	0	0	0	0	0	0	0
$(C_{Y_{P_W}})_W$	-4,2E-03	-4,2E-03	-3,9E-03	-3,4E-03	-2,7E-03	-1,7E-03	-6,6E-04	1,7E-04	1,2E-03	2,0E-03	3,2E-03	3,3E-03	3,2E-03
$(C_{Y_{\beta}})_W$	5,6E-13	5,6E-13	4,9E-13	3,7E-13	2,2E-13	8,9E-14	1,3E-14	9,4E-16	4,6E-14	1,3E-13	3,1E-13	3,4E-13	3,1E-13
$(C_{D_{Q_W}})_W$	2,4278	2,3373	2,2477	2,1557	2,0587	1,9573	1,8601	1,7834	1,6819	1,5922	1,3963	1,2995	1,2053
$(C_{l_{P_W}})_W$	0,0099	-0,0159	-0,0395	-0,0604	-0,0775	-0,0891	-0,0934	-0,0919	-0,0835	-0,0704	-0,0270	-0,0008	0,0262
$(C_{l_{R_W}})_W$	0,0498	0,0497	0,0465	0,0404	0,0313	0,0198	0,0077	-0,0020	-0,0143	-0,0237	-0,0370	-0,0387	-0,0369
$(C_{l_{\dot{\beta}}})_W$	0	0	0	0	0	0	0	0	0	0	0	0	0
$(C_{l_{\beta}})_W$	-5,34E-03	-5,33E-03	-4,99E-03	-4,33E-03	-3,36E-03	-2,12E-03	-8,23E-04	2,18E-04	1,53E-03	2,54E-03	3,96E-03	4,14E-03	3,95E-03
$(C_{m_{Q_W}})_W$	-8,44	-8,44	-8,44	-8,44	-8,44	-8,44	-8,44	-8,44	-8,44	-8,44	-8,44	-8,44	-8,44
$(C_{m_{\dot{\alpha}}})_W$	0	0	0	0	0	0	0	0	0	0	0	0	0
$(C_{n_{P_W}})_W$	-2,442E-02	-6,354E-03	5,954E-03	1,467E-02	2,076E-02	2,457E-02	2,628E-02	2,650E-02	2,561E-02	2,383E-02	1,745E-02	1,335E-02	8,906E-03
$(C_{n_{R_W}})_W$	-0,1181	-0,1154	-0,1125	-0,1092	-0,1054	-0,1011	-0,0966	-0,0928	-0,0876	-0,0828	-0,0719	-0,0664	-0,0611
$(C_{n_{\dot{\beta}}})_W$	0	0	0	0	0	0	0	0	0	0	0	0	0
$(C_{n_{\beta}})_W$	7,60E-06	7,56E-06	6,62E-06	4,99E-06	3,00E-06	1,20E-06	1,80E-07	1,27E-08	6,20E-07	1,71E-06	4,17E-06	4,56E-06	4,15E-06

Table 36: Equilibrium points wing aerodynamic derivatives, from 13,5 m/s to 23 m/s.

V_T (m/s)	13,5	14	14,5	15,5	17	18	19	20	21	22	23	13,5	14
$\left(C_{L_{\dot{\alpha}}}\right)_W$	0	0	0	0	0	0	0	0	0	0	0	0	0
$\left(C_{L_{Q_W}}\right)_W$	1,20	1,86	2,53	5,51	12,14	14,61	21,00	29,99	-2,64	-3,95	-5,01	1,20	1,86
$\left(C_{Y_{\dot{\beta}}}\right)_W$	0	0	0	0	0	0	0	0	0	0	0	0	0
$\left(C_{Y_{R_W}}\right)_W$	0	0	0	0	0	0	0	0	0	0	0	0	0
$\left(C_{Y_{P_W}}\right)_W$	2,6E-03	1,7E-03	7,5E-04	-3,0E-03	-9,4E-03	-1,4E-02	-2,1E-02	-2,6E-02	-2,6E-02	-2,6E-02	-2,4E-02	2,6E-03	1,7E-03
$\left(C_{Y_{\beta}}\right)_W$	2,0E-13	8,3E-14	1,5E-14	2,2E-13	2,2E-12	5,2E-12	1,1E-11	1,8E-11	1,8E-11	1,7E-11	1,6E-11	2,0E-13	8,3E-14
$\left(C_{D_{Q_W}}\right)_W$	1,0897	0,9935	0,9236	0,7784	0,6658	0,6045	0,5522	0,5043	0,4620	0,4232	0,3889	1,0897	0,9935
$\left(C_{l_{P_W}}\right)_W$	0,0614	0,0964	0,1316	0,2878	0,6358	0,7643	1,0984	1,5681	-0,1470	-0,2155	-0,2703	0,0614	0,0964
$\left(C_{l_{R_W}}\right)_W$	-0,0297	-0,0192	-0,0082	0,0311	0,0983	0,1519	0,2215	0,2824	0,2842	0,2751	0,2640	-0,0297	-0,0192
$\left(C_{l_{\dot{\beta}}}\right)_W$	0	0	0	0	0	0	0	0	0	0	0	0	0
$\left(C_{l_{\beta}}\right)_W$	3,18E-03	2,05E-03	8,75E-04	-3,33E-03	-1,05E-02	-1,63E-02	-2,37E-02	-3,03E-02	-3,05E-02	-2,95E-02	-2,83E-02	3,18E-03	2,05E-03
$\left(C_{m_{Q_W}}\right)_W$	-8,44	-8,44	-8,44	-8,44	-8,44	-8,44	-8,44	-8,44	-8,44	-8,44	-8,44	-8,44	-8,44
$\left(C_{m_{\dot{\alpha}}}\right)_W$	0	0	0	0	0	0	0	0	0	0	0	0	0
$\left(C_{n_{P_W}}\right)_W$	2,693E-03	-1,807E-03	-6,449E-03	-2,558E-02	-6,215E-02	-8,914E-02	-1,234E-01	-1,591E-01	-1,001E-01	-9,544E-02	-8,817E-02	2,693E-03	-1,807E-03
$\left(C_{n_{R_W}}\right)_W$	-0,0546	-0,0492	-0,0454	-0,0372	-0,0294	-0,0236	-0,0156	-0,0070	-0,0059	-0,0057	-0,0055	-0,0546	-0,0492
$\left(C_{n_{\dot{\beta}}}\right)_W$	0	0	0	0	0	0	0	0	0	0	0	0	0
$\left(C_{n_{\beta}}\right)_W$	2,70E-06	1,12E-06	2,03E-07	2,95E-06	2,95E-05	7,04E-05	1,50E-04	2,44E-04	2,47E-04	2,31E-04	2,13E-04	2,70E-06	1,12E-06

Table 37: Equilibrium points wing aerodynamic derivatives, from 24 m/s to 36 m/s.

V_T (m/s)	24	25	26	27	28	29	30	31	32	33	34	35	36
$\left(C_{L_{\dot{\alpha}}}\right)_W$	0	0	0	0	0	0	0	0	0	0	0	0	0
$\left(C_{L_{Q_W}}\right)_W$	-5,87	-6,57	-7,15	-7,61	-7,83	-9,31	-9,30	-9,29	-9,27	-9,25	-9,23	-9,20	-9,18
$\left(C_{Y_{\dot{\beta}}}\right)_W$	0	0	0	0	0	0	0	0	0	0	0	0	0
$\left(C_{Y_{R_W}}\right)_W$	0	0	0	0	0	0	0	0	0	0	0	0	0
$\left(C_{Y_{P_W}}\right)_W$	-2,3E-02	-2,1E-02	-2,0E-02	-1,9E-02	-1,8E-02	-1,6E-02	-1,5E-02	-1,4E-02	-1,3E-02	-1,3E-02	-1,2E-02	-1,1E-02	-1,1E-02
$\left(C_{Y_{\beta}}\right)_W$	1,4E-11	1,3E-11	1,2E-11	1,0E-11	9,2E-12	8,1E-12	7,1E-12	6,3E-12	5,5E-12	4,9E-12	4,3E-12	3,9E-12	3,4E-12
$\left(C_{D_{Q_W}}\right)_W$	0,3579	0,3297	0,3041	0,2807	0,2594	0,2399	0,2222	0,2060	0,1911	0,1773	0,1646	0,1529	0,1420
$\left(C_{l_{P_W}}\right)_W$	-0,3150	-0,3516	-0,3813	-0,4055	-0,4163	-0,4940	-0,4931	-0,4921	-0,4908	-0,4895	-0,4881	-0,4867	-0,4853
$\left(C_{l_{R_W}}\right)_W$	0,2519	0,2393	0,2266	0,2142	0,2021	0,1894	0,1774	0,1665	0,1564	0,1471	0,1386	0,1308	0,1235
$\left(C_{l_{\dot{\beta}}}\right)_W$	0	0	0	0	0	0	0	0	0	0	0	0	0
$\left(C_{l_{\beta}}\right)_W$	-2,7E-02	-2,5E-02	-2,4E-02	-2,2E-02	-2,1E-02	-2,0E-02	-1,9E-02	-1,7E-02	-1,6E-02	-1,5E-02	-1,4E-02	-1,4E-02	-1,3E-02
$\left(C_{m_{Q_W}}\right)_W$	-8,44	-8,44	-8,44	-8,44	-8,44	-8,44	-8,44	-8,44	-8,44	-8,44	-8,44	-8,44	-8,44
$\left(C_{m_{\dot{\alpha}}}\right)_W$	0	0	0	0	0	0	0	0	0	0	0	0	0
$\left(C_{n_{P_W}}\right)_W$	-8,0E-02	-7,4E-02	-6,6E-02	-6,2E-02	-5,8E-02	-5,2E-02	-4,9E-02	-4,6E-02	-4,3E-02	-4,0E-02	-3,8E-02	-3,6E-02	-3,4E-02
$\left(C_{n_{R_W}}\right)_W$	-0,005	-0,005	-0,0047	-0,0045	-0,0043	-0,0041	-0,0039	-0,0037	-0,0036	-0,0035	-0,0034	-0,0033	-0,0032
$\left(C_{n_{\dot{\beta}}}\right)_W$	0	0	0	0	0	0	0	0	0	0	0	0	0
$\left(C_{n_{\beta}}\right)_W$	1,9E-04	1,7E-04	1,5E-04	1,3E-04	1,2E-04	1,0E-04	9,5E-05	8,3E-05	7,4E-05	6,5E-05	5,8E-05	5,1E-05	4,5E-05

Table 38: Equilibrium points wing aerodynamic derivatives, from 37 m/s to 47 m/s.

V_T (m/s)	37	38	39	40	41	42	43	44	45	46	47
$(C_{L\dot{\alpha}})_W$	0	0	0	0	0	0	0	0	0	0	0
$(C_{L_{Q_W}})_W$	-9,16	-9,13	-9,11	-9,08	-9,06	-9,04	-9,01	-8,99	-8,97	-8,95	-8,93
$(C_{Y_{\dot{\beta}}})_W$	0	0	0	0	0	0	0	0	0	0	0
$(C_{Y_{R_W}})_W$	0	0	0	0	0	0	0	0	0	0	0
$(C_{Y_{P_W}})_W$	-9,9E-03	-9,4E-03	-8,9E-03	-8,4E-03	-7,9E-03	-7,5E-03	-7,1E-03	-6,8E-03	-6,4E-03	-6,1E-03	-5,8E-03
$(C_{Y_\beta})_W$	3,1E-12	2,8E-12	2,5E-12	2,2E-12	2,0E-12	1,8E-12	1,6E-12	1,5E-12	1,4E-12	1,2E-12	1,1E-12
$(C_{D_{Q_W}})_W$	0,1320	0,1226	0,1139	0,1058	0,0983	0,0912	0,0846	0,0785	0,0727	0,0673	0,0622
$(C_{l_{P_W}})_W$	-0,4839	-0,4825	-0,4811	-0,4798	-0,4785	-0,4772	-0,4760	-0,4748	-0,4736	-0,4725	-0,4715
$(C_{l_{R_W}})_W$	0,1168	0,1106	0,1048	0,0995	0,0945	0,0898	0,0855	0,0815	0,0777	0,0742	0,0708
$(C_{l_{\dot{\beta}}})_W$	0	0	0	0	0	0	0	0	0	0	0
$(C_{l_\beta})_W$	-1,25E-02	-1,18E-02	-1,12E-02	-1,06E-02	-1,01E-02	-9,56E-03	-9,10E-03	-8,67E-03	-8,26E-03	-7,88E-03	-7,52E-03
$(C_{m_{Q_W}})_W$	-8,44	-8,44	-8,44	-8,44	-8,44	-8,44	-8,44	-8,44	-8,44	-8,44	-8,44
$(C_{m_{\dot{\alpha}}})_W$	0	0	0	0	0	0	0	0	0	0	0
$(C_{n_{P_W}})_W$	-3,242E-02	-3,067E-02	-2,905E-02	-2,753E-02	-2,612E-02	-2,480E-02	-2,357E-02	-2,242E-02	-2,133E-02	-2,032E-02	-1,936E-02
$(C_{n_{R_W}})_W$	-0,0031	-0,0031	-0,0030	-0,0030	-0,0029	-0,0029	-0,0029	-0,0029	-0,0028	-0,0028	-0,0028
$(C_{n_{\dot{\beta}}})_W$	0	0	0	0	0	0	0	0	0	0	0
$(C_{n_\beta})_W$	4,11E-05	3,68E-05	3,30E-05	2,97E-05	2,67E-05	2,42E-05	2,19E-05	1,98E-05	1,80E-05	1,64E-05	1,49E-05

26 Appendix I – Horizontal Tail Aerodynamic Derivatives

Table 39: Equilibrium points horizontal tail aerodynamic derivatives, from 0 m/s to 6 m/s.

V_T (m/s)	0	0,5	1	1,5	2	2,5	3	3,5	4	4,5	5	5,5	6
$\left(C_{D_{Q_W}}\right)_{HT}$	0	0	0	0	0	0	0	0	0	0	0	0	0
$\left(C_{Y_\beta}\right)_{HT}$	-6,74	-6,76	-6,81	-6,89	-6,96	-6,75	-6,49	-6,14	-5,73	-5,17	-4,55	-3,73	-2,88
$\left(C_{Y_{Q_W}}\right)_{HT}$	0	0	0	0	0	0	0	0	0	0	0	0	0
$\left(C_{Y_{R_W}}\right)_{HT}$	0	0	0	0	0	0	0	0	0	0	0	0	0
$\left(C_{Y_{\dot\beta}}\right)_{HT}$	0,0000	0,0004	0,0018	0,0043	0,1604	0,1685	0,1771	0,1879	0,1987	0,2115	0,2235	0,2362	0,2466
$\left(C_{L_{Q_W}}\right)_{HT}$	0	0	0	0	0	0	0	0	0	0	0	0	0
$\left(C_{L_{\dot\alpha}}\right)_{HT}$	0,0000	0,0160	0,0275	0,0438	0,0579	0,0639	0,0639	0,0676	0,0639	0,0639	0,0639	0,0671	0,0703
$\left(C_{l_{P_W}}\right)_{HT}$	0,0946	0,0949	0,0956	0,0968	0,0945	0,0912	0,0872	0,0816	0,0753	0,0666	0,0571	0,0448	0,0320
$\left(C_{l_{R_W}}\right)_{HT}$	0,0000	0,0003	0,0014	0,0031	0,1171	0,1230	0,1292	0,1370	0,1448	0,1541	0,1627	0,1718	0,1793
$\left(C_{l_{\dot\beta}}\right)_{HT}$	0	0	0	0	0	0	0	0	0	0	0	0	0
$\left(C_{l_\beta}\right)_{HT}$	0,0000	-0,0003	-0,0012	-0,0027	-0,0999	-0,1049	-0,1103	-0,1170	-0,1237	-0,1317	-0,1391	-0,1471	-0,1535
$\left(C_{n_{P_W}}\right)_{HT}$	-36,93	-36,93	-36,93	-36,93	-36,93	-36,93	-36,93	-36,93	-36,93	-36,93	-36,93	-36,93	-36,93
$\left(C_{n_{R_W}}\right)_{HT}$	0	0	0	0	0	0	0	0	0	0	0	0	0
$\left(C_{n_{\dot\beta}}\right)_{HT}$	2,74E-23	8,63E-10	1,46E-08	7,83E-08	1,10E-04	1,21E-04	1,34E-04	1,51E-04	1,69E-04	1,91E-04	2,13E-04	2,38E-04	2,60E-04
$\left(C_{n_\beta}\right)_{HT}$	0	0	0	0	0	0	0	0	0	0	0	0	0
$\left(C_{m_{Q_W}}\right)_{HT}$	0,0000	0,0064	0,0012	-0,0002	-0,0352	-0,0371	-0,0391	-0,0416	-0,0440	-0,0469	-0,0495	-0,0524	-0,0547
$\left(C_{m_{\dot\alpha}}\right)_{HT}$	-0,2957	-0,2957	-0,2957	-0,2957	-0,2704	-0,2678	-0,2647	-0,2606	-0,2559	-0,2494	-0,2421	-0,2323	-0,2216

Table 40: Equilibrium points horizontal tail aerodynamic derivatives, from 6,5 m/s to 13 m/s.

V_T (m/s)	6,5	7	7,5	8	8,5	9	9,5	10	10,5	11	12	12,5	13
$\left(C_{D_{Q_W}}\right)_{HT}$	0	0	0	0	0	0	0	0	0	0	0	0	0
$\left(C_{Y_\beta}\right)_{HT}$	-1,77	-0,68	0,36	1,30	2,17	2,75	2,92	2,59	2,76	2,84	2,91	2,85	2,78
$\left(C_{Y_{Q_W}}\right)_{HT}$	0	0	0	0	0	0	0	0	0	0	0	0	0
$\left(C_{Y_{R_W}}\right)_{HT}$	0	0	0	0	0	0	0	0	0	0	0	0	0
$\left(C_{Y_{\dot\beta}}\right)_{HT}$	0,2561	0,2612	0,2622	0,2600	0,2547	0,2477	0,2411	0,2348	0,2367	0,2378	0,2430	0,2453	0,2472
$\left(C_{L_{Q_W}}\right)_{HT}$	0	0	0	0	0	0	0	0	0	0	0	0	0
$\left(C_{L_{\dot\alpha}}\right)_{HT}$	0,0831	0,0959	0,1039	0,1119	0,1199	0,1279	0,1359	0,1439	0,1519	0,1599	0,1759	0,1952	0,2145
$\left(C_{l_{P_W}}\right)_{HT}$	0,0156	-0,0003	-0,0152	-0,0285	-0,0405	-0,0484	-0,0504	-0,0454	-0,0478	-0,0490	-0,0503	-0,0497	-0,0487
$\left(C_{l_{R_W}}\right)_{HT}$	0,1861	0,1898	0,1906	0,1889	0,1851	0,1801	0,1754	0,1709	0,1723	0,1731	0,1768	0,1785	0,1798
$\left(C_{l_{\dot\beta}}\right)_{HT}$	0	0	0	0	0	0	0	0	0	0	0	0	0
$\left(C_{l_\beta}\right)_{HT}$	-0,1594	-0,1626	-0,1633	-0,1619	-0,1586	-0,1542	-0,1501	-0,1462	-0,1474	-0,1481	-0,1513	-0,1527	-0,1539
$\left(C_{n_{P_W}}\right)_{HT}$	-36,93	-36,93	-36,93	-36,93	-36,93	-36,93	-36,93	-36,93	-36,93	-36,93	-36,93	-36,93	-36,93
$\left(C_{n_{R_W}}\right)_{HT}$	0	0	0	0	0	0	0	0	0	0	0	0	0
$\left(C_{n_{\dot\beta}}\right)_{HT}$	2,80E-04	2,91E-04	2,94E-04	2,88E-04	2,77E-04	2,62E-04	2,48E-04	2,35E-04	2,39E-04	2,41E-04	2,52E-04	2,57E-04	2,61E-04
$\left(C_{n_\beta}\right)_{HT}$	0	0	0	0	0	0	0	0	0	0	0	0	0
$\left(C_{m_{Q_W}}\right)_{HT}$	-0,0568	-0,0579	-0,0582	-0,0577	-0,0565	-0,0549	-0,0535	-0,0521	-0,0525	-0,0527	-0,0539	-0,0544	-0,0548
$\left(C_{m_{\dot\alpha}}\right)_{HT}$	-0,2068	-0,1914	-0,1758	-0,1606	-0,1449	-0,1307	-0,1196	-0,1092	-0,1124	-0,1142	-0,1226	-0,1266	-0,1299

Table 41: Equilibrium points horizontal tail aerodynamic derivatives, from 13,5 m/s to 23 m/s.

V_T (m/s)	13,5	14	14,5	15,5	17	18	19	20	21	22	23
$\left(C_{D_{Q_W}}\right)_{HT}$	0	0	0	0	0	0	0	0	0	0	0
$\left(C_{Y_\beta}\right)_{HT}$	2,44	2,10	1,50	0,58	1,87	2,40	2,89	2,27	-0,65	-4,87	-2,84
$\left(C_{Y_{Q_W}}\right)_{HT}$	0	0	0	0	0	0	0	0	0	0	0
$\left(C_{Y_{R_W}}\right)_{HT}$	0	0	0	0	0	0	0	0	0	0	0
$\left(C_{Y_{\dot\beta}}\right)_{HT}$	0,2520	0,2552	0,2590	0,2619	0,2568	0,2524	0,2396	0,2324	0,2281	0,2310	0,2414
$\left(C_{L_{Q_W}}\right)_{HT}$	0	0	0	0	0	0	0	0	0	0	0
$\left(C_{L_{\dot\alpha}}\right)_{HT}$	0,2511	0,2878	0,3316	0,3781	0,3836	0,3374	0,3501	0,3448	0,3445	0,3478	0,3203
$\left(C_{l_{P_W}}\right)_{HT}$	-0,0442	-0,0395	-0,0312	-0,0183	-0,0364	-0,0437	-0,0499	-0,0406	0,0012	0,0613	0,0317
$\left(C_{l_{R_W}}\right)_{HT}$	0,1833	0,1856	0,1884	0,1905	0,2038	0,2063	0,2028	0,2033	0,2058	0,2108	0,2212
$\left(C_{l_{\dot\beta}}\right)_{HT}$	0	0	0	0	0	0	0	0	0	0	0
$\left(C_{l_\beta}\right)_{HT}$	-0,1569	-0,1589	-0,1613	-0,1631	-0,1600	-0,1572	-0,1493	-0,1448	-0,1421	-0,1439	-0,1504
$\left(C_{n_{P_W}}\right)_{HT}$	-36,93	-36,93	-36,93	-36,93	-36,93	-36,93	-36,93	-36,93	-36,93	-36,93	-36,93
$\left(C_{n_{R_W}}\right)_{HT}$	0	0	0	0	0	0	0	0	0	0	0
$\left(C_{n_{\dot\beta}}\right)_{HT}$	2,71E-04	2,78E-04	2,86E-04	2,93E-04	2,81E-04	2,71E-04	2,45E-04	2,30E-04	2,22E-04	2,27E-04	2,48E-04
$\left(C_{n_\beta}\right)_{HT}$	0	0	0	0	0	0	0	0	0	0	0
$\left(C_{m_{Q_W}}\right)_{HT}$	-0,0559	-0,0566	-0,0574	-0,0581	-0,0562	-0,0549	-0,0518	-0,0500	-0,0488	-0,0493	-0,0515
$\left(C_{m_{\dot\alpha}}\right)_{HT}$	-0,1391	-0,1463	-0,1573	-0,1724	-0,1507	-0,1400	-0,1173	-0,1052	-0,0895	-0,0809	-0,0762

Table 42: Equilibrium points horizontal tail aerodynamic derivatives, from 24 m/s to 36 m/s.

V_T (m/s)	24	25	26	27	28	29	30	31	32	33	34	35	36
$\left(C_{D_{Q_W}}\right)_{HT}$	0	0	0	0	0	0	0	0	0	0	0	0	0
$\left(C_{Y_\beta}\right)_{HT}$	7,24	6,90	7,81	9,99	11,56	12,76	13,69	14,44	15,00	15,46	16,05	16,59	16,92
$\left(C_{Y_{Q_W}}\right)_{HT}$	0	0	0	0	0	0	0	0	0	0	0	0	0
$\left(C_{Y_{R_W}}\right)_{HT}$	0	0	0	0	0	0	0	0	0	0	0	0	0
$\left(C_{Y_{\dot\beta}}\right)_{HT}$	0,2415	0,2344	0,2237	0,2124	0,2012	0,1905	0,1803	0,1706	0,1619	0,1543	0,1471	0,1407	0,1345
$\left(C_{L_{Q_W}}\right)_{HT}$	0	0	0	0	0	0	0	0	0	0	0	0	0
$\left(C_{L_{\dot\alpha}}\right)_{HT}$	0,2897	0,2618	0,2525	0,2454	0,2403	0,2370	0,2196	0,2035	0,1888	0,1752	0,1626	0,1511	0,1403
$\left(C_{l_{P_W}}\right)_{HT}$	-0,1120	-0,1067	-0,1191	-0,1496	-0,1714	-0,1879	-0,2008	-0,2110	-0,2186	-0,2249	-0,2331	-0,2405	-0,2449
$\left(C_{l_{R_W}}\right)_{HT}$	0,2241	0,2218	0,2153	0,2071	0,1980	0,1902	0,1829	0,1758	0,1695	0,1640	0,1589	0,1542	0,1497
$\left(C_{l_{\dot\beta}}\right)_{HT}$	0	0	0	0	0	0	0	0	0	0	0	0	0
$\left(C_{l_\beta}\right)_{HT}$	-0,1505	-0,1461	-0,1394	-0,1324	-0,1254	-0,1187	-0,1124	-0,1064	-0,1009	-0,0962	-0,0918	-0,0878	-0,0839
$\left(C_{n_{P_W}}\right)_{HT}$	-36,93	-36,93	-36,93	-36,93	-36,93	-36,93	-36,93	-36,93	-36,93	-36,93	-36,93	-36,93	-36,93
$\left(C_{n_{R_W}}\right)_{HT}$	0	0	0	0	0	0	0	0	0	0	0	0	0
$\left(C_{n_{\dot\beta}}\right)_{HT}$	2,48E-04	2,34E-04	2,13E-04	1,92E-04	1,72E-04	1,54E-04	1,38E-04	1,24E-04	1,11E-04	1,01E-04	9,18E-05	8,40E-05	7,67E-05
$\left(C_{n_\beta}\right)_{HT}$	0	0	0	0	0	0	0	0	0	0	0	0	0
$\left(C_{m_{Q_W}}\right)_{HT}$	-0,0513	-0,0496	-0,0472	-0,0447	-0,0423	-0,0399	-0,0376	-0,0355	-0,0336	-0,0319	-0,0303	-0,0289	-0,0275
$\left(C_{m_{\dot\alpha}}\right)_{HT}$	-0,0711	-0,0665	-0,0607	-0,0554	-0,0504	-0,0459	-0,0419	-0,0382	-0,0351	-0,0325	-0,0302	-0,0282	-0,0264

Table 43: Equilibrium points horizontal tail aerodynamic derivatives, from 37 m/s to 47 m/s.

V_T (m/s)	37	38	39	40	41	42	43	44	45	46	47
$\left(C_{D_{Q_W}}\right)_{HT}$	0	0	0	0	0	0	0	0	0	0	0
$\left(C_{Y_\beta}\right)_{HT}$	17,18	17,37	17,38	17,38	17,38	17,39	17,39	17,39	17,39	17,39	17,39
$\left(C_{Y_{Q_W}}\right)_{HT}$	0	0	0	0	0	0	0	0	0	0	0
$\left(C_{Y_{R_W}}\right)_{HT}$	0	0	0	0	0	0	0	0	0	0	0
$\left(C_{Y_{\dot\beta}}\right)_{HT}$	0,1271	0,1209	0,1165	0,1158	0,1136	0,1115	0,1095	0,1077	0,1095	0,1079	0,1082
$\left(C_{L_{Q_W}}\right)_{HT}$	0	0	0	0	0	0	0	0	0	0	0
$\left(C_{L_{\dot\alpha}}\right)_{HT}$	0,1304	0,1212	0,1126	0,1046	0,0971	0,0901	0,0836	0,0775	0,0718	0,0665	0,0614
$\left(C_{l_{P_W}}\right)_{HT}$	-0,2484	-0,2510	-0,2510	-0,2510	-0,2510	-0,2510	-0,2510	-0,2510	-0,2510	-0,2510	-0,2510
$\left(C_{l_{R_W}}\right)_{HT}$	0,1475	0,1440	0,1383	0,1282	0,1205	0,1131	0,1058	0,0988	0,0873	0,0807	0,0720
$\left(C_{l_{\dot\beta}}\right)_{HT}$	0	0	0	0	0	0	0	0	0	0	0
$\left(C_{l_\beta}\right)_{HT}$	-0,0793	-0,0754	-0,0727	-0,0723	-0,0709	-0,0696	-0,0684	-0,0672	-0,0684	-0,0674	-0,0676
$\left(C_{n_{P_W}}\right)_{HT}$	-36,93	-36,93	-36,93	-36,93	-36,93	-36,93	-36,93	-36,93	-36,93	-36,93	-36,93
$\left(C_{n_{R_W}}\right)_{HT}$	0	0	0	0	0	0	0	0	0	0	0
$\left(C_{n_{\dot\beta}}\right)_{HT}$	6,84E-05	6,19E-05	5,75E-05	5,68E-05	5,46E-05	5,25E-05	5,07E-05	4,90E-05	5,06E-05	4,91E-05	4,93E-05
$\left(C_{n_\beta}\right)_{HT}$	0	0	0	0	0	0	0	0	0	0	0
$\left(C_{m_{Q_W}}\right)_{HT}$	-0,0257	-0,0243	-0,0234	-0,0237	-0,0235	-0,0233	-0,0231	-0,0230	-0,0239	-0,0238	-0,0243
$\left(C_{m_{\dot\alpha}}\right)_{HT}$	-0,0243	-0,0226	-0,0215	-0,0214	-0,0208	-0,0203	-0,0198	-0,0194	-0,0199	-0,0195	-0,0195

27 Appendix J – Wing Body Tail Aerodynamic Derivatives

Table 44: Equilibrium points wing body tail aerodynamic derivatives, from 0 m/s to 6 m/s.

V_T (m/s)	0	0,5	1	1,5	2	2,5	3	3,5	4	4,5	5	5,5	6
$\left(C_{D_{Q_W}}\right)_{WBT}$	-7,7829	-7,7876	-7,7767	-7,7548	-2,9938	-2,8193	-2,6434	-2,4086	-2,1775	-1,8777	-1,5669	-1,1885	-0,8143
$\left(C_{Y_\beta}\right)_{WBT}$	0,6061	0,6061	0,6061	0,6061	2,3012	2,3566	2,4107	2,4855	2,5579	2,6484	2,7611	2,8865	3,0338
$\left(C_{Y_{Q_W}}\right)_{WBT}$	10,76	10,75	10,75	10,73	10,71	10,68	10,61	10,52	10,38	10,18	9,91	9,57	9,14
$\left(C_{Y_{R_W}}\right)_{WBT}$	-0,0163	-0,0174	-0,0183	-0,0195	-0,0207	-0,0212	-0,0212	-0,0215	-0,0212	-0,0212	-0,0212	-0,0215	-0,0217
$\left(C_{Y_{\dot\beta}}\right)_{WBT}$	0,0000	0,0008	0,0013	0,0020	0,0025	0,0025	0,0022	0,0019	0,0013	0,0008	0,0002	-0,0001	-0,0004
$\left(C_{L_{Q_W}}\right)_{WBT}$	0,0923	0,0925	0,0927	0,0929	0,0931	0,0932	0,0932	0,0933	0,0932	0,0932	0,0932	0,0933	0,0933
$\left(C_{L_{\dot\alpha}}\right)_{WBT}$	-0,0525	-0,0522	-0,0521	-0,0519	-0,0518	-0,0520	-0,0525	-0,0530	-0,0538	-0,0544	-0,0552	-0,0558	-0,0563
$\left(C_{l_{P_W}}\right)_{WBT}$	-0,00124	-0,00125	-0,00124	-0,00124	-0,00123	-0,00122	-0,00122	-0,00121	-0,00122	-0,00122	-0,00122	-0,00121	-0,00121
$\left(C_{l_{R_W}}\right)_{WBT}$	0,1190	0,1188	0,1184	0,1177	0,1165	0,1144	0,1111	0,1060	0,0986	0,0886	0,0741	0,0567	0,0344
$\left(C_{l_{\dot\beta}}\right)_{WBT}$	0,0103	0,0104	0,0112	0,0128	0,0148	0,0180	0,0221	0,0271	0,0329	0,0389	0,0456	0,0514	0,0563
$\left(C_{l_\beta}\right)_{WBT}$	-8,32	-8,04	-7,71	-7,24	-11,18	-11,15	-11,29	-11,38	-11,67	-11,90	-12,19	-12,42	-12,70
$\left(C_{n_{P_W}}\right)_{WBT}$	-55,26	-54,38	-53,32	-51,80	-50,48	-49,92	-49,92	-49,58	-49,92	-49,92	-49,92	-49,62	-49,32
$\left(C_{n_{R_W}}\right)_{WBT}$	-0,00497	-0,00497	-0,00497	-0,00497	-0,00497	-0,00497	-0,00497	-0,00497	-0,00497	-0,00497	-0,00497	-0,00497	-0,00497
$\left(C_{n_{\dot\beta}}\right)_{WBT}$	0,0086	0,0092	0,0097	0,0104	0,0110	0,0112	0,0112	0,0114	0,0112	0,0112	0,0112	0,0114	0,0115
$\left(C_{n_\beta}\right)_{WBT}$	########	-31,1822	-6,5909	-2,5166	-1,3955	-0,8345	-0,5465	-0,3694	-0,2603	-0,1847	-0,1253	-0,0833	-0,0498
$\left(C_{m_{Q_W}}\right)_{WBT}$	-0,1587	-0,1589	-0,1590	-0,1592	-0,1593	-0,1593	-0,1592	-0,1590	-0,1585	-0,1579	-0,1570	-0,1558	-0,1541
$\left(C_{m_{\dot\alpha}}\right)_{WBT}$	-0,1405	-0,1405	-0,1405	-0,1405	-0,1405	-0,1405	-0,1405	-0,1405	-0,1405	-0,1405	-0,1405	-0,1405	-0,1405

Table 45: Equilibrium points wing body tail aerodynamic derivatives, from 6,5 m/s to 13 m/s.

V_T (m/s)	6,5	7	7,5	8	8,5	9	9,5	10	10,5	11	12	12,5	13
$\left(C_{D_{Q_W}}\right)_{WBT}$	-0,3100	0,1563	0,5608	0,9111	1,2240	1,4513	1,5796	1,7064	1,2709	0,9222	0,0932	-0,3034	-0,6642
$\left(C_{Y_\beta}\right)_{WBT}$	3,2273	3,4296	3,4316	3,3710	3,5438	3,9318	4,2262	4,3225	4,2373	4,1635	4,0320	3,9641	3,9060
$\left(C_{Y_{Q_W}}\right)_{WBT}$	8,67	8,15	7,22	6,25	5,97	6,31	6,60	6,65	6,82	7,08	7,94	8,46	9,00
$\left(C_{Y_{R_W}}\right)_{WBT}$	-0,0229	-0,0235	-0,0236	-0,0237	-0,0237	-0,0238	-0,0239	-0,0239	-0,0240	-0,0241	-0,0242	-0,0244	-0,0245
$\left(C_{Y_{\dot\beta}}\right)_{WBT}$	0,0000	0,0007	0,0013	0,0023	0,0034	0,0048	0,0063	0,0075	0,0090	0,0102	0,0122	0,0133	0,0142
$\left(C_{L_{Q_W}}\right)_{WBT}$	0,0935	0,0936	0,0937	0,0938	0,0939	0,0940	0,0941	0,0942	0,0943	0,0944	0,0946	0,0948	0,0950
$\left(C_{L_{\dot\alpha}}\right)_{WBT}$	-0,0563	-0,0560	-0,0554	-0,0545	-0,0533	-0,0518	-0,0502	-0,0489	-0,0473	-0,0460	-0,0441	-0,0434	-0,0431
$\left(C_{l_{P_W}}\right)_{WBT}$	-0,00119	-0,00114	-0,00108	-0,00103	-0,00098	-0,00093	-0,00088	-0,00082	-0,00077	-0,00071	-0,00061	-0,00047	-0,00034
$\left(C_{l_{R_W}}\right)_{WBT}$	0,0102	-0,0164	-0,0406	-0,0621	-0,0798	-0,0917	-0,0961	-0,0944	-0,0861	-0,0731	-0,0299	-0,0036	0,0234
$\left(C_{l_{\dot\beta}}\right)_{WBT}$	0,0587	0,0584	0,0550	0,0488	0,0396	0,0279	0,0156	0,0058	-0,0066	-0,0162	-0,0297	-0,0318	-0,0304
$\left(C_{l_\beta}\right)_{WBT}$	-12,83	-12,97	-12,74	-12,35	-12,56	-13,32	-13,84	-13,85	-13,39	-12,96	-12,15	-11,40	-10,67
$\left(C_{n_{P_W}}\right)_{WBT}$	-48,11	-46,88	-44,93	-42,76	-42,16	-42,85	-43,04	-42,34	-41,57	-40,80	-39,27	-37,40	-35,53
$\left(C_{n_{R_W}}\right)_{WBT}$	-0,00497	-0,00497	-0,00497	-0,00497	-0,00497	-0,00497	-0,00497	-0,00497	-0,00512	-0,00561	-0,00652	-0,00694	-0,00735
$\left(C_{n_{\dot\beta}}\right)_{WBT}$	0,0122	0,0125	0,0126	0,0126	0,0126	0,0127	0,0127	0,0128	0,0128	0,0128	0,0129	0,0130	0,0131
$\left(C_{n_\beta}\right)_{WBT}$	-0,0260	-0,0081	0,0040	0,0126	0,0185	0,0222	0,0237	0,0238	0,0228	0,0208	0,0141	0,0097	0,0048
$\left(C_{m_{Q_W}}\right)_{WBT}$	-0,1521	-0,1496	-0,1467	-0,1435	-0,1398	-0,1355	-0,1311	-0,1274	-0,1223	-0,1175	-0,1067	-0,1014	-0,0962
$\left(C_{m_{\dot\alpha}}\right)_{WBT}$	-0,1405	-0,1405	-0,1405	-0,1405	-0,1405	-0,1405	-0,1405	-0,1405	-0,1405	-0,1405	-0,1405	-0,1405	-0,1405

Table 46: Equilibrium points wing body tail aerodynamic derivatives, from 13,5 m/s to 23 m/s.

V_T (m/s)	13,5	14	14,5	15,5	17	18	19	20	21	22	23
$\left(C_{D_{Q_W}}\right)_{WBT}$	-1,2263	-1,6722	-2,1383	-2,8901	-2,6188	-2,4883	-2,0610	-1,8664	-1,5254	-1,3362	-1,1950
$\left(C_{Y_{\beta}}\right)_{WBT}$	3,8319	3,7747	3,4240	3,6662	3,6893	3,6320	3,7108	3,6829	3,8206	3,8487	3,9050
$\left(C_{Y_{Q_W}}\right)_{WBT}$	9,69	10,38	10,34	14,17	21,12	23,33	30,18	39,41	5,64	4,32	3,22
$\left(C_{Y_{R_W}}\right)_{WBT}$	-0,0248	-0,0251	-0,0254	-0,0257	-0,0257	-0,0254	-0,0255	-0,0255	-0,0255	-0,0255	-0,0253
$\left(C_{Y_{\dot{\beta}}}\right)_{WBT}$	0,0155	0,0165	0,0178	0,0166	0,0104	0,0033	-0,0027	-0,0085	-0,0087	-0,0077	-0,0078
$\left(C_{L_{Q_W}}\right)_{WBT}$	0,0953	0,0956	0,0959	0,0962	0,0962	0,0959	0,0960	0,0960	0,0960	0,0960	0,0958
$\left(C_{L_{\dot{\alpha}}}\right)_{WBT}$	-0,0430	-0,0433	-0,0435	-0,0468	-0,0544	-0,0617	-0,0694	-0,0766	-0,0768	-0,0756	-0,0750
$\left(C_{l_{P_W}}\right)_{WBT}$	-0,00008	0,00019	0,00052	0,00087	0,00091	0,00056	0,00066	0,00062	0,00062	0,00064	0,00043
$\left(C_{l_{R_W}}\right)_{WBT}$	0,0587	0,0939	0,1295	0,2863	0,6336	0,7617	1,0956	1,5656	-0,1477	-0,2136	-0,2698
$\left(C_{l_{\dot{\beta}}}\right)_{WBT}$	-0,0239	-0,0141	-0,0039	0,0345	0,1016	0,1561	0,2254	0,2864	0,2882	0,2790	0,2686
$\left(C_{l_{\beta}}\right)_{WBT}$	-9,38	-8,13	-5,91	-5,12	-5,02	-6,27	-6,08	-6,17	-6,53	-6,51	-7,48
$\left(C_{n_{P_W}}\right)_{WBT}$	-31,97	-28,38	-22,21	-19,53	-19,06	-22,63	-21,85	-21,98	-22,75	-22,54	-25,23
$\left(C_{n_{R_W}}\right)_{WBT}$	-0,00774	-0,00812	-0,00849	-0,00919	-0,01015	-0,01075	-0,01131	-0,01184	-0,01235	-0,01284	-0,01330
$\left(C_{n_{\dot{\beta}}}\right)_{WBT}$	0,0132	0,0134	0,0136	0,0137	0,0137	0,0136	0,0136	0,0136	0,0136	0,0136	0,0135
$\left(C_{n_{\beta}}\right)_{WBT}$	-0,0021	-0,0073	-0,0128	-0,0329	-0,0696	-0,0956	-0,1301	-0,1657	-0,1068	-0,1021	-0,0943
$\left(C_{m_{Q_W}}\right)_{WBT}$	-0,0899	-0,0848	-0,0812	-0,0732	-0,0655	-0,0594	-0,0515	-0,0429	-0,0418	-0,0416	-0,0412
$\left(C_{m_{\dot{\alpha}}}\right)_{WBT}$	-0,1405	-0,1405	-0,1405	-0,1405	-0,1405	-0,1405	-0,1405	-0,1405	-0,1405	-0,1405	-0,1405

Table 47: Equilibrium points wing body tail aerodynamic derivatives, from 24 m/s to 36 m/s.

V_T (m/s)	24	25	26	27	28	29	30	31	32	33	34	35	36
$\left(C_{D_{Q_W}}\right)_{WBT}$	-1,0895	-1,0390	-0,9120	-0,8077	-0,7313	-0,6711	-0,6189	-0,5741	-0,5396	-0,5125	-0,4902	-0,4711	-0,4542
$\left(C_{Y_{\beta}}\right)_{WBT}$	3,9393	3,9003	3,9721	4,0354	4,0727	4,0832	4,1144	4,1380	4,1435	4,1353	4,1182	4,0961	4,0721
$\left(C_{Y_{Q_W}}\right)_{WBT}$	2,26	1,32	0,83	0,44	0,27	-1,28	-1,27	-1,26	-1,29	-1,33	-1,39	-1,45	-1,51
$\left(C_{Y_{R_W}}\right)_{WBT}$	-0,0251	-0,0249	-0,0248	-0,0248	-0,0247	-0,0247	-0,0246	-0,0244	-0,0243	-0,0242	-0,0241	-0,0240	-0,0239
$\left(C_{Y_{\dot{\beta}}}\right)_{WBT}$	-0,0079	-0,0079	-0,0067	-0,0060	-0,0052	-0,0042	-0,0041	-0,0039	-0,0038	-0,0037	-0,0036	-0,0035	-0,0034
$\left(C_{L_{Q_W}}\right)_{WBT}$	0,0956	0,0954	0,0953	0,0953	0,0952	0,0952	0,0950	0,0949	0,0947	0,0946	0,0944	0,0943	0,0942
$\left(C_{L_{\dot{\alpha}}}\right)_{WBT}$	-0,0744	-0,0737	-0,0724	-0,0712	-0,0699	-0,0685	-0,0676	-0,0667	-0,0659	-0,0652	-0,0645	-0,0639	-0,0633
$\left(C_{l_{P_W}}\right)_{WBT}$	0,00020	0,00000	-0,00007	-0,00012	-0,00015	-0,00018	-0,00030	-0,00041	-0,00052	-0,00061	-0,00070	-0,00077	-0,00085
$\left(C_{l_{R_W}}\right)_{WBT}$	-0,3206	-0,3569	-0,3872	-0,4126	-0,4244	-0,5028	-0,5024	-0,5018	-0,5008	-0,4998	-0,4987	-0,4976	-0,4963
$\left(C_{l_{\dot{\beta}}}\right)_{WBT}$	0,2570	0,2449	0,2324	0,2201	0,2082	0,1955	0,1838	0,1732	0,1634	0,1544	0,1461	0,1384	0,1314
$\left(C_{l_{\beta}}\right)_{WBT}$	-8,49	-9,24	-9,70	-10,08	-10,32	-10,45	-11,06	-11,60	-12,05	-12,44	-12,77	-13,05	-13,31
$\left(C_{n_{P_W}}\right)_{WBT}$	-28,07	-30,25	-31,44	-32,39	-33,02	-33,34	-35,04	-36,57	-37,88	-39,02	-40,02	-40,91	-41,72
$\left(C_{n_{R_W}}\right)_{WBT}$	-0,01375	-0,01417	-0,01458	-0,01498	-0,01535	-0,01572	-0,01607	-0,01642	-0,01675	-0,01707	-0,01738	-0,01768	-0,01798
$\left(C_{n_{\dot{\beta}}}\right)_{WBT}$	0,0134	0,0133	0,0133	0,0132	0,0132	0,0132	0,0131	0,0130	0,0130	0,0129	0,0129	0,0128	0,0127
$\left(C_{n_{\beta}}\right)_{WBT}$	-0,0865	-0,0793	-0,0711	-0,0668	-0,0630	-0,0575	-0,0537	-0,0503	-0,0471	-0,0443	-0,0416	-0,0392	-0,0369
$\left(C_{m_{Q_W}}\right)_{WBT}$	-0,0408	-0,0404	-0,0401	-0,0398	-0,0396	-0,0394	-0,0391	-0,0388	-0,0385	-0,0383	-0,0381	-0,0379	-0,0378
$\left(C_{m_{\dot{\alpha}}}\right)_{WBT}$	-0,1405	-0,1405	-0,1405	-0,1405	-0,1405	-0,1405	-0,1405	-0,1405	-0,1405	-0,1405	-0,1405	-0,1405	-0,1405

Table 48: Equilibrium points wing body tail aerodynamic derivatives, from 37 m/s to 47 m/s.

V_T (m/s)	37	38	39	40	41	42	43	44	45	46	47
$\left(C_{D_{Q_W}}\right)_{WBT}$	-0,4278	-0,4089	-0,4019	-0,4187	-0,4233	-0,4279	-0,4322	-0,4364	-0,4643	-0,4681	-0,4838
$\left(C_{Y_\beta}\right)_{WBT}$	4,0485	4,0268	4,0077	3,9918	3,9794	3,9703	3,9644	3,9613	3,9608	3,9625	3,9662
$\left(C_{Y_{Q_W}}\right)_{WBT}$	-1,57	-1,62	-1,66	-1,69	-1,71	-1,73	-1,73	-1,73	-1,73	-1,72	-1,70
$\left(C_{Y_{R_W}}\right)_{WBT}$	-0,0238	-0,0238	-0,0237	-0,0236	-0,0236	-0,0235	-0,0229	-0,0224	-0,0219	-0,0214	-0,0210
$\left(C_{Y_{\dot\beta}}\right)_{WBT}$	-0,0033	-0,0032	-0,0031	-0,0031	-0,0030	-0,0029	-0,0029	-0,0028	-0,0028	-0,0027	-0,0027
$\left(C_{L_{Q_W}}\right)_{WBT}$	0,0941	0,0940	0,0939	0,0938	0,0937	0,0936	0,0935	0,0934	0,0933	0,0932	0,0932
$\left(C_{L_{\dot\alpha}}\right)_{WBT}$	-0,0628	-0,0623	-0,0618	-0,0614	-0,0610	-0,0607	-0,0603	-0,0600	-0,0597	-0,0594	-0,0592
$\left(C_{l_{P_W}}\right)_{WBT}$	-0,00091	-0,00097	-0,00103	-0,00108	-0,00113	-0,00117	-0,00119	-0,00120	-0,00121	-0,00122	-0,00122
$\left(C_{l_{R_W}}\right)_{WBT}$	-0,4950	-0,4937	-0,4923	-0,4909	-0,4896	-0,4883	-0,4870	-0,4858	-0,4846	-0,4835	-0,4824
$\left(C_{l_{\dot\beta}}\right)_{WBT}$	0,1248	0,1188	0,1132	0,1080	0,1031	0,0986	0,0944	0,0904	0,0867	0,0833	0,0801
$\left(C_{l_\beta}\right)_{WBT}$	-13,54	-13,76	-13,97	-14,16	-14,35	-14,53	-14,70	-14,87	-15,04	-15,20	-15,35
$\left(C_{n_{P_W}}\right)_{WBT}$	-42,46	-43,15	-43,80	-44,42	-45,00	-45,56	-46,10	-46,62	-47,11	-47,59	-48,05
$\left(C_{n_{R_W}}\right)_{WBT}$	-0,01826	-0,01854	-0,01881	-0,01908	-0,01933	-0,01958	-0,01983	-0,02007	-0,02030	-0,02053	-0,02076
$\left(C_{n_{\dot\beta}}\right)_{WBT}$	0,0127	0,0127	0,0126	0,0126	0,0125	0,0125	0,0122	0,0119	0,0116	0,0114	0,0111
$\left(C_{n_\beta}\right)_{WBT}$	-0,0349	-0,0329	-0,0311	-0,0295	-0,0279	-0,0265	-0,0251	-0,0238	-0,0227	-0,0215	-0,0205
$\left(C_{m_{Q_W}}\right)_{WBT}$	-0,0376	-0,0375	-0,0373	-0,0372	-0,0371	-0,0370	-0,0369	-0,0368	-0,0367	-0,0367	-0,0366
$\left(C_{m_{\dot\alpha}}\right)_{WBT}$	-0,1405	-0,1405	-0,1405	-0,1405	-0,1405	-0,1405	-0,1405	-0,1405	-0,1405	-0,1405	-0,1405

28 Appendix K – Inertial Parameters

Table 49: Concentrated masses position with respect to aircraft nose for wing and horizontal tail tilt angle zero degrees.

	m(kg)	x(m)	y(m)	z(m)
Body	16,32	0,571	0	0,0286
Right Wing	0,977	0,295	0,468	0,084
Left Wing	0,977	0,295	-0,468	0,084
Right Horizontal Tail	0,507	1,152	0,242	0,086
Left Horizontal Tail	0,507	1,152	-0,242	0,086
Rotor 1	0,448	0,0918	0,776	0,085
Rotor 2	0,448	0,0782	0,320	0,085
Rotor 3	0,448	0,0782	-0,320	0,085
Rotor 4	0,448	0,0918	-0,776	0,085
Rotor 5	0,448	0,933	0,325	0,090
Rotor 6	0,448	0,933	-0,325	0,090

Table 50: Wing and horizontal tail pivot positions.

	x(m)	y(m)	z(m)
Wing Pivot	0,300	0	0,085
Horizontal Tail Pivot	1,157	0,000	0,090

Inertia matrices of the concentrated masses with respect to their owns reference axis. The indexes “r” means right and the indexes “l” means left.

$$\tilde{I}_B^B = \begin{bmatrix} 0.0274 & -0.0034 & -0.0385 \\ -0.0034 & 1.5603 & -5.18e-4 \\ -0.0385 & -5.18e-4 & 0.5376 \end{bmatrix}$$

$$\tilde{I}_{W_r}^{W_r} = \begin{bmatrix} 0.0153 & 6.10e-4 & -1.90e-5 \\ 6.10e-4 & 0.0046 & -8.08e-5 \\ -1.90e-5 & -8.08e-5 & 0.0169 \end{bmatrix}$$

$$\tilde{I}_{W_l}^{W_l} = \begin{bmatrix} 0.0153 & -6.10e-4 & -1.90e-5 \\ -6.10e-4 & 0.0046 & 8.08e-5 \\ -1.90e-5 & 8.08e-5 & 0.0169 \end{bmatrix}$$

$$\tilde{I}_{HT_r}^{HT_r} = \begin{bmatrix} 0.0012 & 2.49e-5 & -1.08e-5 \\ 2.49e-5 & 0.0021 & 2.64e-5 \\ -1.08e-5 & 2.64e-5 & 0.0019 \end{bmatrix}$$

$$\tilde{I}_{HT_l}^{HT_l} = \begin{bmatrix} 0.0012 & -2.49e-5 & -1.08e-5 \\ -2.49e-5 & 0.0021 & -2.64e-5 \\ -1.08e-5 & -2.64e-5 & 0.0019 \end{bmatrix}$$

$$\tilde{I}_{R}^{R} = \begin{bmatrix} 8.61e-4 & 0 & 0 \\ 0 & 5.59e-4 & 0 \\ 0 & 0 & 5.39e-4 \end{bmatrix}$$

29 Appendix L – Dynamics Parameters

Table 51: Dynamics parameters.

λ_p	20	$\lambda_{a_{\delta_r}}$	0.2
K_p	29.3	$K_{a_{\delta_r}}$	0.87
$\lambda_{a_{\delta_f}}$	0.2	$\lambda_{a_{\delta_{a_L}}}$	0.2
$K_{a_{\delta_f}}$	0.87	$K_{a_{\delta_{a_L}}}$	0.87
$\lambda_{a_{\delta_e}}$	0.2	$\lambda_{a_{\delta_{a_R}}}$	0.2
$K_{a_{\delta_e}}$	0.87	$K_{a_{\delta_{a_R}}}$	0.87