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Rubens Eduardo Garcia MACHADO

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dirigée par:

Evangelie ATHANASSOULA

&

Ronaldo Eustáquio DE SOUZA

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Orientadores:

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&

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Résumé

Des simulations cosmologiques à N -corps indiquent que les halos de matière noire des galaxies devraient être généralement triaxiaux. Pourtant, on croit que la présence d'un disque baryonique modifie la forme des halos. L'objectif de cette thèse est d'étudier comment la formation des barres est affectée par la triaxialité des halos et comment, à son tour, la présence de la barre influence la forme du halo. Nous avons exploré ce thème avec l'aide de simulations numériques, tant non-collisionnelles qu'hydrodynamiques.

Nous réalisons un ensemble de simulations non-collisionnelle à N -corps de galaxies à disque avec des halos triaxiaux de matière noire, utilisant des disques elliptiques comme conditions initiales. Ces disques sont beaucoup plus proches de l'équilibre avec leurs halos que les disques circulaires, et l'ellipticité du disque initial dépend de l'ellipticité du potentiel gravitationnel du halo. À titre de comparaison, nous considérons également des modèles avec des disques initialement circulaires, et nous constatons que les différences sont très importantes. La masse du disque est introduite de façon quasi-adiabatique dans les halos, mais l'échelle de temps de croissance n'est pas très importante. Nous étudions des modèles avec différentes triaxialités et, pour étudier le comportement de la forme du halo en l'absence de formation de barre, nous analysons des modèles avec des différentes masses du disque, concentrations du halo, dispersions de vitesse du disque et aussi des modèles où la forme du disque est maintenue artificiellement axisymétrique. Nous constatons que l'introduction d'un disque massif, même si il n'est pas circulaire, provoque une diminution partielle de la triaxialité du halo. Une fois que le disque est complètement introduit, une forte barre stellaire se développe dans le halo, qui est encore non-axisymétrique, lui faisant perdre la non-axisymétrie qui lui restait. Si les paramètres du halo sont tels qu'une barre ne se forme pas, le halo est capable de rester triaxial et la circularisation de sa forme sur le plan du disque est limitée à la période de croissance du disque. Nous concluons qu'une partie de la circularisation du halo est due à la croissance du disque, mais une partie doit être attribuée à la formation d'une barre. Des barres dans la composante du halo, qui ont déjà été trouvées dans les halos sphériques, se trouvent également dans les halos triaxiaux. Nous constatons que les disques initialement circulaires réagissent excessivement au potentiel triaxial et deviennent très allongées. Ils perdent aussi plus de moment cinétique que les disques initialement elliptiques et forment ainsi des barres plus fortes. De ce fait, la circularisation que leurs barres provoque sur leurs halos est aussi plus rapide. Nous analysons les formes verticales des halos et observons que leurs aplatissements verticaux restent considérables, ce qui signifie que les halos deviennent approximativement oblate à la fin de la simulation. Finalement, nous analysons aussi la cinématique d'un sous-ensemble de particules du halo qui tournent comme le disque. Ces particules occupent une couche autour du plan du disque et leur rotation est plus importante dans le halo sphérique que dans les triaxiaux. Nous constatons également que, même si la forme finale du halo est à peu près indépendante de la forme initiale, les halos initialement triaxiaux sont capables de retenir

l'anisotropie de leurs dispersions de vitesse.

Nous effectuons également une série de simulations hydrodynamiques, dans lesquelles une partie de la masse du disque est sous forme de gaz. Dans ces simulations, la formation stellaire est incluse, de sorte que la fraction de gaz diminue avec le temps. Nous explorons des modèles avec des différentes triaxialités du halo et des différentes fractions initiales de gaz, ce qui nous permet d'évaluer comment chacun affecte la formation de la barre. Les modèles de toutes les formes de halo et de toutes les fractions initiales de gaz ont environ la même teneur en gaz à la fin de la simulation. Néanmoins, nous constatons que la présence de gaz dans les premières phases a des effets importants sur l'évolution ultérieure. Des modèles avec une fraction initiale de gaz élevée développent des barres qui sont sensiblement plus faibles. Nous trouvons les barres les plus fortes quand le halo est sphérique et quand il n'y a pas de gaz. Les barres sont généralement plus faibles si le contenu initial en gaz est plus grand et si la triaxialité du halo est plus grande. La présence du gaz, par contre, est un facteur plus efficace que la triaxialité du halo pour inhiber la formation d'une barre forte. La formation de barre rend les halos plus axisymétriques et nous observons la formation de la barre du halo en cas de fortes barres stellaires. Par contre, si la barre est faible, la partie centrale du halo devient circularisée. La rotation des particules du halo qui se comportent comme ceux du disque est également présente dans les simulations avec gaz, étant plus important pour le halo sphérique. Il y a une corrélation claire entre la force de la barre et la vitesse tangentielle maximale de la rotation de ces particules du halo. Dans toutes nos simulations avec du gaz, les vitesses de rotation des barres diminuent toujours avec le temps. Nous constatons que les étoiles récemment formées tiennent proportionnellement moins de moment cinétique, et les barres sont plus fortes dans la population plus jeune. La morphologie du gaz dans le disque est influencée par la force de la barre. Nous observons la formation d'une concentration de masse centrale, qui est entourée par une région annulaire considérable, dont la densité du gaz est très faible. Ce vide de gaz est plus grand pour les disques fortement barrés.

mots-clés: astrophysique – dynamique galactique – galaxies: évolution – galaxies: halos – méthodes: simulations numériques

Abstract

Cosmological N -body simulations indicate that the dark matter haloes of galaxies should be generally triaxial. Yet, the presence of a baryonic disc is believed to modify the shape of the haloes. The goal of this thesis was to study how bar formation is affected by halo triaxiality and how, in turn, the presence of the bar influences the shape of the halo. We have explored this topic with the help of numerical simulations, both collisionless and hydrodynamical.

We perform a set of collisionless N -body simulations of disc galaxies with triaxial dark matter haloes, using elliptical discs as initial conditions. Such discs are much closer to equilibrium with their haloes than the circular ones, and the ellipticity of the initial disc depends on the ellipticity of the halo gravitational potential. For comparison, we also consider models with initially circular discs, and find that the differences are very important. The mass of the disc is grown quasi-adiabatically within the haloes, but the time-scale of growth is not very important. We study models of different halo triaxialities and, to investigate the behaviour of the halo shape in the absence of bar formation, we run models with different disc masses, halo concentrations, disc velocity dispersions and also models where the disc shape is kept artificially axisymmetric. We find that the introduction of a massive disc, even if this is not circular, causes the halo triaxiality to be partially diluted. Once the disc is fully grown, a strong stellar bar develops within the halo that is still non-axisymmetric, causing it to lose its remaining non-axisymmetry. In triaxial haloes in which the parameters of the initial conditions are such that a bar does not form, the halo is able to remain triaxial and the circularisation of its shape on the plane of the disc is limited to the period of disc growth. We conclude that part of the circularisation of the halo is due to disc growth, but part must be attributed to the formation of a bar. Bars in the halo component, which have been already found in axisymmetric haloes, are also found in triaxial ones. We find that initially circular discs respond excessively to the triaxial potential and become highly elongated. They also lose more angular momentum than the initially elliptical discs and thus form stronger bars. Because of that, the circularisation that their bars induce on their haloes is also more rapid. We also analyse halo vertical shapes and observe that their vertical flattenings remain considerable, meaning that the haloes become approximately oblate by the end of the simulations. Finally, we also analyse the kinematics of a subset of halo particles that rotate in disc-like manner. These particles occupy a layer around the plane of the disc and their rotation is more important in the spherical halo than in triaxial ones. We also find that, even though the final shape of the halo is roughly independent of the initial shape, the initially triaxial ones are able to retain the anisotropy of their velocity dispersions.

We also perform a series of hydrodynamical simulations, in which part of the disc mass is in the form of gas. These simulations include star formation, so the fraction of gas decreases with time. We explore models of different halo triaxiality and also of different

initial gas fractions, which allows us to evaluate how each affects the formations of the bar. Models of all halo shapes and of all initial gas fractions reach approximately the same gas content at the end of the simulation. Nevertheless, we find that the presence of gas in the early phases has important effects on the subsequent evolution. Models with high initial gas fraction develop bars that are substantially weaker. We find the strongest bar in the model where the halo is spherical and in which there is no gas. Bars are generally weaker for larger initial gas content and for larger halo triaxiality. The presence of gas, however, is a more efficient factor in inhibiting the formation of a strong bar than halo triaxiality is. Bar formation causes the halos to become more axisymmetric and we observe the formation of the halo bar in cases of strong stellar bars, but if the bar is weak the central part of the halo becomes circularised. The rotation of the disc-like halo particles is also present in the simulations with gas, being more important for the spherical halo. There is a clear correlation between bar strength and the peak tangential velocity of these rotating halo particles. In all our gas simulations, the bar pattern speeds always decrease with time. We also find that the recently formed stars hold proportionally less angular momentum, and bars are stronger in the younger population. The morphology of gas in the disc is influenced by the strength of the bar. We observe the formation of a central mass concentration, which is surrounded by a considerable annular region of very low gas density. This gas void is larger for strongly barred discs.

keywords: astrophysics – galactic dynamics – galaxies: evolution – galaxies: haloes – methods: numerical simulations

Resumo

As simulações cosmológicas de N -corpos indicam que os halos de matéria escura das galáxias devem ser em geral triaxiais. Contudo, acredita-se que a presença de um disco bariônico seja capaz de alterar a forma do halo. O objetivo desta tese é o de estudar como a formação de barras é afetada pela triaxialidade do halo e como, por sua vez, a presença da barra influencia a forma do halo. Nós exploramos este tópico através de simulações numéricas, tanto acolisionais quanto hidrodinâmicas.

Nós realizamos um conjunto de simulações acolisionais de N -corpos de galáxias disco com halos triaxiais de matéria escura, utilizando discos elípticos como condições iniciais. Tais discos encontram-se mais próximos do equilíbrio do que discos circulares e a elipticidade do disco inicial depende da elipticidade do potencial gravitacional do halo. Para comparação, também consideramos modelos com discos inicialmente circulares e notamos que as diferenças são muito importantes. A massa do disco é introduzida nos halos de forma quase adiabática, mas a escala de tempo de crescimento do disco não é muito relevante. Nós estudamos modelos de diferentes triaxialidades e, para investigar o comportamento da forma do halo na ausência de formação de barra, nós analisamos modelos com diferentes massas do disco, concentrações do halo, dispersão de velocidades do disco e também modelos nos quais o formato do disco é mantido artificialmente axissimétrico. Encontramos que a introdução de um disco massivo, mesmo que este não seja circular, causa uma diluição parcial da triaxialidade do halo. Uma vez que o disco esteja completamente introduzido, uma barra estelar forte se desenvolve na presença de um halo que ainda é não-axissimétrico, causando a perda da triaxialidade restante do halo. Em halos triaxiais cujos parâmetros das condições iniciais são tais que a barra não se forma, o halo é capaz de permanecer triaxial e a circularização de seu formato no plano do disco fica limitada ao período de crescimento do disco. Concluimos que parte da circularização do halo é devida ao crescimento do disco, mas parte precisa ser atribuída à formação da barra. Barras no componente do halo, que já haviam sido encontradas em halos axissimétricos, são também encontradas em halos triaxiais. Nós notamos que discos inicialmente circulares reagem excessivamente ao potencial triaxial e se tornam altamente alongados. Tais discos também perdem mais momento angular do que discos inicialmente elípticos e portanto formam barras mais fortes. Por esta razão, a circularização que suas barras induzem em seus halos é também mais rápida. Nós também analisamos o formato vertical dos halos e observamos que seus achatamentos verticais permanecem consideráveis, o que significa que os halos se tornam aproximadamente oblatos ao fim das simulações. Finalmente, nós analisamos a cinemática de um sub-conjunto de partículas do halo que apresentam rotação típica de discos. Estas partículas ocupam uma camada ao redor do plano do disco e sua rotação é mais importante no halo esférico do que nos halos triaxiais. Nós também observamos que, embora os formatos finais dos halos sejam aproximadamente independentes do formato inicial, os halos triaxiais são capazes de reter a anisotropia das suas dispersões de velocidade.

Uma série de simulações hidrodinâmicas foi realizada, nas quais parte da massa do disco está na forma de gás. Estas simulações incluem formação estelar, de modo que a fração de gás diminui com o tempo. Foram explorados modelos de diferentes triaxialidades do halo e também de diferentes frações iniciais de gás, o que nos permitiu avaliar como cada um afeta a formação da barra. Modelos de todas as triaxialidades do halo e de todas as frações iniciais de gás alcançam aproximadamente o mesmo conteúdo de gás ao fim da simulação. Entretanto, notamos que a presença de gás nas fases iniciais tem importantes efeitos na evolução subsequente. Modelos com alta fração inicial de gás desenvolvem barras substancialmente mais fracas. A barra é mais forte se o halo é esférico e se não há gás. Barras são em geral mais fracas nos modelos com maior conteúdo de gás e com maior triaxialidade do halo. A presença de gás, no entanto, é um fator mais eficiente do que a triaxialidade do halo, ao inibir a formação de uma barra forte. A formação de barra novamente faz com que os halos se tornem mais axissimétricos e observamos a formação da barra do halo em casos de barras estelares fortes, mas se a barra é fraca a parte central do halo se circulariza. A rotação das partículas do halo que se comportam como as do disco está presente também nas simulações com gás, e é mais importante para o halo esférico. Existe uma correlação clara entre a força da barra e a velocidade tangencial máxima destas partículas em rotação do halo. Em todas as nossas simulações com gás, a velocidade de rotação da barra decresce com o tempo. Nós também obtemos o resultado de que as estrelas formadas recentemente apresentam proporcionalmente menos momento angular, e as barras são mais fortes nas populações mais jovens. A morfologia do gás no disco é influenciada pela força da barra. Nós observamos a formação de uma concentração central de massa, que é cercada por uma considerável região anular na qual a densidade do gás é muito baixa. Esta ausência de gás é maior para discos fortemente barrados.

palavras-chave: astrofísica – dinâmica galáctica – galáxias: evolução – galáxias: halos – métodos: simulações numéricas

Introduction

Galaxies are thought to be surrounded by approximately spherical, centrally concentrated haloes of dark matter. The dynamics of disc galaxies in general, and of barred galaxies in particular, are importantly influenced by the haloes. The gravitational evolution of such systems has been extensively studied with the aid of numerical simulations. This Chapter introduces aspects of dark matter haloes (Sect. 1.1) and of barred galaxies (Sect. 1.2) which are relevant to the specific issue of bar evolution within triaxial haloes (itself reviewed in Sect. 1.4). The numerical codes employed in this work are described in Sect. 1.3.

1.1 Dark Matter Haloes

Within the current Λ CDM (cold dark matter, where λ represents the cosmological constant) paradigm, structure formation in the universe is believed to take place hierarchically, with smaller objects collapsing early and larger systems growing by merging or accretion. Computer simulations are a central tool in the study of structure formation, because the processes by which dark matter density perturbations build up into large scale structures are highly nonlinear. N -body cosmological simulations, starting from primordial density fluctuations, are able to follow the formation and merging history of dark matter haloes through time.

Early simulations (Barnes & Efstathiou 1987, Frenk et al. 1988, Dubinski & Carlberg 1991, Warren et al. 1992, Cole & Lacey 1996) had already consistently shown that the pure dark matter haloes are on average not spherical. Indeed, haloes have been reported as being substantially triaxial, with a slight preference towards prolate (rather than oblate) shapes, in which major-to-minor axis ratio in excess of 2 are not uncommon. Later simulations (Jing & Suto 2002, Allgood et al. 2006, Novak et al. 2006) – with the aid of increased computer power as well as improved numerical algorithms – have a sufficiently large number of particles to allow an adequate statistical analysis of the halo properties. Typically it is found that such haloes have isodensity axis ratios of $b/a \sim 0.8$ (intermediate-to-major) and $c/a \sim 0.6$ (minor-to-major), depending on the mass of the halo and on the details of the cosmological simulations. In particular, more massive haloes tend to be more triaxial (or rather, prolate). This is presumably due to the fact that massive haloes undergo a larger number of merging events, which take place non-isotropically, along preferred directions linked to the filaments of the global large scale structure. Furthermore, the axis ratios show some radial dependence and the triaxiality is usually found to increase towards the centre, as seen by Hayashi et al. (2007), who measured the shapes of the isopotential surfaces of cosmological haloes from the Millennium Simulation (Springel et al. 2005). Furthermore,

the angular momentum of haloes seems to be often well aligned with their minor axes (e.g. Bailin & Steinmetz 2004; 2005), suggesting that disc galaxy formation should take place on the plane containing the major and intermediate axes.

Simulations of large scale structure used to be restricted to dark matter, ignoring the baryonic component, due to the computational cost. More recently, cosmological simulations that include gas and the treatment of several physical processes (such as gas cooling, star formation, chemical enrichment and supernova feedback, besides gravity) have been able to form discs. And it is found that haloes tend to become axisymmetric due to the disc (Kazantzidis et al. 2004, Tissera et al. 2010, Pedrosa et al. 2009; 2010). Indeed, studies of how the presence of baryons could alter halo shapes (Katz & Gunn 1991, Evrard et al. 1994, Tissera & Dominguez-Tenreiro 1998, Debattista et al. 2008) had generally found that pure dark matter haloes tend to be more triaxial than haloes in the presence of a gaseous dissipative component or of a disc potential. Dubinski (1994), for instance, carried out numerical experiments in which an analytic disc potential was grown into a triaxial N -body halo, altering the orbits of the halo particles and causing the halo to go from a prolate-triaxial to an approximately oblate shape.

In order to compare halo shapes in pure dark matter and in gas simulations, Tissera et al. (2010) resimulated six galaxy-sized haloes from the Aquarius Project (Springel et al. 2008), with the inclusion of baryons. Such haloes are selected from a lower resolution version of the Millennium-II Simulation (Boylan-Kolchin et al. 2009), and had been resimulated with much higher resolutions (Vogelsberger et al. 2009, Navarro et al. 2010). The simulations of Tissera et al. (2010) include metal-dependent cooling, star formation and the treatments of supernova feedback and chemical enrichment as implemented in Scannapieco et al. (2005; 2006), resulting in haloes that become more oblate in the baryon dominated regions, compared to the triaxial shapes of their dissipationless counterparts.

Thus a halo which contains a baryonic disc is expected to be less triaxial than a pure dark matter halo. Indeed, observational constraints on halo shape find that present-day haloes are very mildly elongated on the plane of the disc, or even consistent with an axisymmetric potential (Trachternach et al. 2008). This is also reflected on the statistics of disc galaxy shapes (Lambas, Maddox, & Loveday 1992; Fasano et al. 1993; Rix & Zaritsky 1995; Ryden 2004; 2006), whose ellipticities are small compared to the ellipticities of haloes from cosmological simulations.

The shapes of discs are deformed by the aspheric halo potentials, such that in the equilibrium configuration the discs should be generally elliptical. At the same time, the presence of the disc acts to oppose the halo ellipticity on the plane (Jog 2000, Bailin et al. 2007), since the disc elongation is perpendicular to the halo elongation. The interplay of the baryonic disc with its halo should somehow reconcile the highly triaxial shapes of pure dark matter haloes from cosmological simulations with the near circularity of present-day observed galaxies.

1.2 Barred Galaxies

Bars, weak or strong, are found in the majority of present-day disc galaxies, with an often quoted fraction of roughly two thirds (Eskridge et al. 2000, Marinova & Jogee 2007, Menéndez-Delmestre et al. 2007). Bars can also be found at higher redshifts (Abraham et al. 1999, Sheth et al. 2003), although there they constitute a smaller fraction of the disc galaxies than at low redshifts (Sheth et al. 2008). The torques they exert drive dynamical evolution and, in particular, redistribute angular momentum within their parent galaxy (Kormendy 1979, Sellwood 1980).

One of the consequences of angular momentum transfer is the slowing down of the bar (Tremaine & Tremaine 1984, Weinberg 1985, Athanassoula 1996, Debattista & Sellwood 1998; 2000, Valenzuela & Klypin 2003, Athanassoula & Misiriotis 2002, Athanassoula 2003, O’Neill & Dubinski 2003). Another important relation is that between angular momentum redistribution and the evolution of bar strength (Athanassoula & Misiriotis 2002, Athanassoula 2003). As the disc loses angular momentum to the halo, the bar slows down and becomes stronger. Angular momentum is also transferred from the inner to the outer parts of the disc itself, and such transfers are more efficient when the disc is cold, i.e. when its velocity dispersion is low. Additionally, more massive haloes are, in general, more capable of receiving angular momentum. Due to the decrease of the bar pattern speed, the corotation radius moves further out. In galaxies where the corotation radius becomes very large, most of the stellar disc will be located within it. Angular momentum will then be mostly absorbed by the halo alone.

In simulations, discs of strongly barred galaxies show a peanut-shaped structure when viewed edge-on (Combes & Sanders 1981, Combes et al. 1990, Raha et al. 1991, Athanassoula 2005b; 2008, Martinez-Valpuesta et al. 2006). The class of observed boxy/peanut “bulges” are understood to be a part of the bar seen edge-on. When viewed along the minor axis of the bar, the peanut structure consists of two vertical protuberances almost at the edges of the bar. This structure forms some time after the formation of the bar itself and it grows stronger after the vertical buckling of the disc, which is a period during which the disc temporarily loses symmetry with respect to the equatorial plane. The vertical thickening of the peanut is generally accompanied by a momentary weakening of the bar.

Another feature which is tightly linked to bar strength is the formation of the so-called “halo bar” (Athanassoula 2005a; 2007), or “dark matter bar” (Colín, Valenzuela, & Klypin 2006). Seen in simulations with spherical haloes, this is a structure that forms in the innermost parts of the dark matter halo. It is much shorter and less elongated than the stellar bar, but rotates approximately in tandem with it. A further known effect that barred galaxies have on the spherical halo is on the kinematics of halo particles near the equatorial plane. Within a certain layer around the stellar disc, there are halo particles that rotate considerably, in the same sense of disc rotation.

1.3 Numerical Simulations

1.3.1 N-body

In collisionless stellar dynamics, particles representing the stellar mass (and particles representing dark matter) move under the influence of their mutual gravitational interactions. Due to the long range nature of gravity, their motion is dominated by the collective mean field, rather than by close encounters. Furthermore, the number of stars in a galaxy is sufficiently large, so that the timescale of two-body relaxation far exceeds the Hubble time. If collisions are ignored, the only forces are gravitational forces and the problem at hand is a set of coupled non-linear second order ordinary differential equations that relate the acceleration of each particle to the positions of all other particles in the system. Ultimately, the practical difficulty of solving the N -body problem numerically consists in determining the distances between each particle and every other particle. In numerical simulations, the so-called “softening length” is introduced to render the equations of motion nonsingular, i.e. gravitational forces become truncated at a certain small scale in order to prevent them from reaching arbitrarily high values, as distances between particles vanish. This prevents the formation of bound pairs and also ensures that two-body relaxation time is large.

Effectively computing all mutual distances between particles – direct summation – is exceedingly time-consuming and unfortunately impracticable for large numbers of particles. In order to circumvent this difficulty, the so-called “tree codes” (Barnes & Hut 1986) avoid taking every single particle into account separately. When computing the potential at a given particle position, the tree code approximates the potentials due to distant groups of particles by the centre of mass of that cell (rather, by its multipole expansions). An “opening angle” parameter specifies how distant a group of particles must be in order to have their potentials approximated in this way. In this fashion, the number of terms in the original summation is greatly reduced by replacing partial sums due to particles in a given far away cell with only one term. Small force errors in the long range forces are nevertheless introduced.

The N -body code `GYRFALCON` (Dehnen 2000; 2002) implements one such type of method, but with additional improvements that make it even more efficient. By taking advantage of the symmetry of mutual cell-cell interactions, it reduces the computational effort significantly and, as a side-effect, it satisfies Newton’s third law by construction, thus conserving momentum exactly. `GYRFALCON` was developed within the framework of the `NEMO` tools (Teuben 1995) and it is the code used in the simulations of Chapter 2.

“Particle-mesh” methods, on the other hand, are based on dividing space into a grid and estimating the density on each node of this grid. Such mean density has to be assigned taking into account the masses of the nearby particles according to some shape function. The Poisson equation is then solved (usually by Fast Fourier Transform, FFT) to obtain the potentials at the nodes, which can then be interpolated to the actual particle locations. In this way, each particle interacts with an average field of the other particles. Such methods are quite fast but they are not well suited for high resolution.

The collisionless part of the `GADGET2` code (Springel et al. 2001, Springel 2005) uses a hybrid TreePM method which employs the tree algorithm restricted to short-range scales,

while using a particle-mesh algorithm for the long-range gravitational forces. `GADGET2` is the code used in the simulations of Chapter 3.

1.3.2 Hydrodynamical

In `GADGET2`, the gas interactions are described by the SPH (Smoothed Particle Hydrodynamics) method. In this method, the fluid (the gas) is treated as particles. In this sense, SPH is a ‘Lagrangian’ method, in which mass is discretised (as opposed to ‘Eulerian’ approaches, in which space is discretised). The main advantage of representing the fluid by particles – rather than by volume elements – is that the spacial resolution is automatically adjustable. From a set of discrete points – the gas particles – kernel interpolation (with a certain smoothing length) allows continuous fluid properties to be described. Apart from its mass, each gas particle also carries internal energy (or entropy). Artificial viscosity needs to be introduced so that shocks are captured. The code uses an improved SPH method, described in Springel & Hernquist (2002), in which both energy and entropy are conserved.

The version of `GADGET2` used in the simulations of Chapter 3 uses sub-grid physics described in Springel & Hernquist (2003). In this approach, an SPH fluid element (i.e. each gas particle) represents a region of the interstellar medium containing both cold gas clouds and hot ambient gas, the two in pressure equilibrium. This sub-resolution model is a statistical formulation, in the sense that it uses spatially averaged quantities to describe the interstellar medium. The processes specifically modelled are: star formation, cloud evaporation (due to supernovae) and cloud growth (due to cooling).

Out of the gas in cold clouds, stars are formed, on a characteristic timescale. It is assumed that a certain fraction of these newly-formed stars are very short-lived and die instantly as supernovae. As they do, they eject material in the form of hot gas and also release energy, which heats the ambient hot phase. Another ingredient is that cold clouds are evaporated in the presence of the supernova explosion, the mass of evaporated clouds being proportional to the mass in supernovae. Finally, radiative energy loss by the hot gas is the process by which the cold clouds arise and grow, and during this growth temperature and total volume of the two phases are kept constant.

1.4 Simulations of barred galaxies

Barred galaxies have been extensively studied, theoretically, observationally, as well as with the help of simulations. The latter have been particularly helpful, since they provide information on the evolution of the disc galaxies, on how the halo properties influence this, and have allowed detailed comparison with observations. Simulational work has by necessity relied on a number of simplifying approximations. We will discuss here two such approximations, and in Chapters 2 and 3, we will consider bar formation and evolution in their absence, i.e. in more realistic cases than in previous studies.

One approximation used in the majority of studies so far is that initially the halo is spherically symmetric. A second, often-used approximation, consists in neglecting the gas component. Observations indicate that present-day discs (and haloes) are considerably

axisymmetric in the equatorial plane, while the haloes in cosmological simulations including baryons are less aspherical. For this reason, it is necessary to understand the effect of baryons (including the gaseous component) on the evolution of the barred disc galaxies.

The vast majority of works studying bar formation and evolution use idealised galaxy models with an exponential, or near-exponential, disc and a spherical halo. Yet, cosmological simulations show that dark matter haloes should be triaxial, as described above.

Galactic discs, forming within such haloes, may develop bars, whose dynamics must be influenced both by the non-circular disc and the non-spherical halo. In order to investigate the effect of a triaxial halo on bar formation, Gadotti & de Souza (2003) performed N -body simulations of a spheroid embedded in a rigid triaxial halo potential. In their simulations, the spheroid was distorted into a bar-like structure, due to the halo triaxiality.

The effects of a cosmological setting were studied by Curir, Mazzei, & Murante (2006), who embedded circular discs in the haloes of cosmological simulations. They argue that the large scale anisotropies of the mass distribution influence the bar strengths and details of bar evolution, due to the continuous matter infall and substructures in the halo.

With simulations of considerably higher mass resolution, Berentzen, Shlosman, & Jogee (2006) examined the effect of mildly triaxial haloes on bar evolution. In two of their three simulations with a live triaxial halo, they found that the bar quickly dissolved. The third case had a very massive and very mildly triaxial halo. In particular, its isopotential axis ratio in the equatorial plane is about 0.9. In this last case the bar does not dissolve.

Berentzen & Shlosman (2006) performed N -body simulations in which they grew circular seed discs in assembling triaxial haloes, within a quasi-cosmological setting. The final shape of the halo depends on the mass of the disc, but not on the timescale of its growth. They show that massive discs completely wash out the halo prolateness and then develop long-lived bars, whereas discs that contribute less to the rotation curve are less efficient in axisymmetrising their haloes. They claim that in less massive discs, the bar instability is damped by the halo triaxiality.

Heller, Shlosman, & Athanassoula (2007) investigated the formation of discs by following the collapse of an isolated cosmological density perturbation. They include star formation and stellar feedback in their simulations, so that a baryonic disc forms inside the assembling dark matter halo. The halo triaxiality is decreased in their models during disc growth. Bars that are formed early decay within a few Gyr, but such bars are driven by the prolateness of the halo and do not follow the usual bar instability evolution. Also, they find that the tumbling of the triaxial halo figure is insignificant.

Widrow (2008) used the adiabatic squeezing method of Holley-Bockelmann et al. (2001) to produce triaxial halo models, which he applied to a study of F568-3. This work was focused on a study of the rotation curve of this galaxy and did not discuss the reasons for the changes of the halo and disc shapes. Debattista et al. (2008) carried out control simulations in which a rigid disc was adiabatically first grown and then evaporated and found that the haloes were substantially rounder when the disc was near full mass, but that they returned to their initial shape after the disc was evaporated. The main goal of this paper and of its sequel (Valluri et al. 2010) was to search for the changes of orbital structure that could account for the changes of halo shape due to the baryonic component.

The gaseous component has a considerable effect on the evolution of disc galaxies. Its

mass may be a small fraction of the total at present, but it has been much more important in the past (Hammer et al. 2009). Furthermore, gas can react quite strongly to gravitational perturbations. Numerous works have explored the effects of gas on disc evolution, and in particular in the context of barred galaxies (e.g. Athanassoula 1992, Shlosman & Noguchi 1993, Friedli & Benz 1993, Berentzen et al. 1998; 2007, Curir et al. 2007; 2008, Villa-Vargas et al. 2010). Yet, relatively few studies have been made with a sufficient number of particles, and, of these, a fair fraction neglects the physics of the gas, i.e. neglects its multiphase nature, as well as star formation, feedback and cooling. In these simplified cases, the amount of gas stays constant during the simulation, so, if the simulation spans several Gyr, the adopted (average) gas fraction is too low during the first part of the simulation and too high during the last part.

1.5 Outline of the thesis

The purpose of this thesis was to study bar formation and evolution in the presence of triaxial dark matter haloes. To achieve this goal, we use the tools of numerical simulations and, continuing along the line of the above-mentioned works, we employ carefully devised initial conditions in order to understand the effects of halo shape – but also those of gas content – on the evolution of barred galaxies.

In Chapter 2, we present the results of our collisionless simulations, i.e. models that consist of a dark matter halo and a stellar disc, without gas. As initial conditions, we use live elliptical discs that are gradually introduced in the triaxial halo, and are thus closer to equilibrium than circular discs. We then compare results of simulations with elliptical and circular discs. We also study the effects that disc growth and bar formation have on halo shape, and we run a series of specially designed simulations in order to evaluate the contributions of the two separate effects. We also study vertical shapes and the kinematics of the haloes.

In Chapter 3, we improve on the previous results by analysing bar formation in simulations in which a gas component is also present in the disc. With these simulations, which include star formation, we are able to study the combined effects of different halo shapes and different initial gas fractions. We present results of the effects of these parameters on the vertical and on the face-on structure of the discs, analysing separately the gas component, and the stellar component of different ages. We also evaluate halo kinematics, the density and temperature structures of the gas and the star formation rates.

Finally, in Chapter 4, we present a summary and discussions of the results. Appendix A gives the detailed derivations of the relations between ellipticities that were used to set up the elliptical disc initial conditions, using an epicyclic approximation. Appendix B details the method of using Fourier coefficients to measure bar strength. And Appendix C displays snapshots of the halo and disc for several of the collisionless models.

Loss of halo triaxiality due to bar formation: Collisionless simulations

In this Chapter, we present the results of our collisionless simulations, composed of disc plus halo systems, with no gas. These results are also presented in Machado & Athanassoula (2010).

We investigate bar formation and evolution in triaxial haloes and the corresponding effects on the halo properties. Does the triaxiality of the halo inhibit bar formation, or change drastically the bar properties? Or, alternatively, are strong bars able to form inside triaxial haloes and then cause them to lose their remaining triaxiality? We investigate different models and different types of initial conditions. As initial conditions, we use elliptical discs which are designed to be in equilibrium with the elliptical potential of the halo and compare the results with those obtained with the more straightforward, but out of equilibrium, initially circular discs. We also focus on evaluating the separate effects of two factors that contribute to changing the halo shape: the growth of the disc mass, and the formation of a bar.

In Sect. 2.1 we present our initial conditions and in Sect. 2.2 we discuss bar growth and compare the results in initially axisymmetric and in initially elongated discs. In Sects. 2.3 and 2.4 we consider different halo core sizes and different time-scales for disc growth, respectively. In Sect. 2.5 we present a series of simulations, made in order to distinguish how much of the evolution of the halo shape is due to the introduction of the disc and how much to the growth of the bar. Vertical shapes are the subject of Sect. 2.6. In Sect. 2.7 we consider the effect of triaxiality on halo kinematics.

2.1 Initial conditions

2.1.1 Halo initial conditions

The triaxial halo initial conditions for our simulations were created using the iterative method of Rodionov, Athanassoula, & Sotnikova (2009), which creates equilibrium N -body systems with a given mass distribution and, if desired, given kinematical constraints. In our case we did not wish to impose specific kinematical constraints, so we first adopted the desired mass distribution, as described below, and then found the corresponding kinematics so that the model is in equilibrium. For this we made a large number of successive evolutionary steps of small duration, and at the end of each such step brought back the

Table 2.1: Initial shapes of the haloes as given by the intermediate-to-major (b/a) and minor-to-major (c/a) axis ratios.

halo	b/a	c/a
1	1.0	1.0
2	0.8	0.6
3–	0.7	0.5
3	0.6	0.4
3+	0.5	0.3

mass distribution to the adopted density profile and shape, as described in detail in Rodionov et al. (2009). At the end of this sequence we obtain initial conditions which are both in equilibrium and have the desired mass distribution.

The initially spherical haloes have the density profile described in Hernquist (1993). They are then made triaxial by scaling the particle positions in the y and z directions by factors of b/a and c/a , respectively ($c < b < a$). The iterative method is employed to obtain the velocities of an equilibrium configuration with such shape. The density profile of these triaxial haloes is described by:

$$\rho_h(r') = \frac{M_h}{2\pi^{3/2}} \frac{\alpha}{r_c} \frac{\exp(-r'^2/r_c^2)}{r'^2 + \gamma^2}, \quad (2.1)$$

where

$$r' = \sqrt{(x/a)^2 + (y/b)^2 + (z/c)^2}, \quad (2.2)$$

M_h is the mass of the halo, γ is a core radius and r_c is the cutoff radius. The normalisation constant α is defined by

$$\alpha = \{1 - \sqrt{\pi}q \exp(q^2)[1 - \text{erf}(q)]\}^{-1} \quad (2.3)$$

where $q = \gamma/r_c$ (Hernquist 1993).

We employ in our simulations five haloes of different shapes: one spherical and four triaxial ones, with the initial axis ratios given in Table 2.1. All haloes have the same mass $M_h = 5$ and cutoff radius $r_c = 10$. The core radius is $\gamma = 0.5$ for all models (except in Section 2.3, in which a different core size is explored). Haloes 1, 2 and 3 are used throughout the analysis, whereas the additional haloes 3– and 3+ are used mainly in Sect. 2.7.

The haloes (with no disc) were evolved for 800 time units (units in Section 2.1.3) to make sure that their shapes remain unchanged. Their axis ratios are independent of radius at the beginning of the simulations and remain so for 800 time units. The overall shape of the halo, taking all particles into account, remains constant with time for all models. We may also measure the shape using only particles in the inner region. In order to define this region, we proceed as follows. We choose a radius, which represents the size of the region we wish to study, in this particular case $r = 3$. We count the number n of particles inside

a sphere of radius r . We sort all particles by density and define the inner region as the region occupied by the first n particles of highest local density. In this way, we select an ellipsoidal shape (bounded by an isodensity surface) and avoid delimiting the inner region by radius, which would introduce a bias in the calculation of the shape (Athanasoula & Misiriotis 2002). In the case of these pure haloes, however, the inner shape does not differ from the overall shape, and both are constant in time. Linear fits to the $b/a(t)$ and $c/a(t)$ of the pure halo models show that the axis ratios typically change with a slope of the order of 10^{-6} to 10^{-5} . This means that in a Hubble time ($t \approx 1000$), the change in b/a or c/a is of the order of 0.1% to 1%. This holds for both the entire halo and its inner region.

In the case of halo 3, we observe a vertical instability of the $m = 4$ type. When viewed on the xz plane, halo 3 develops a transient X-like structure in the beginning of the simulation. This instability is more pronounced if there is a disc, but it is also measurable in the pure halo model. The relative intensity of the $m = 4$ Fourier component peaks at about $t = 100$ but after that it strongly subsides in both cases.

2.1.2 Disc initial conditions

2.1.2.1 Epicyclic approximation to create elliptical discs

If a circular disc were to be introduced in a triaxial halo, it would be out of equilibrium. An elliptical disc whose shape is determined by the shape of the halo potential should be initially closer to equilibrium and thus more suitable as initial condition for the simulations. In order to set up the position and velocity coordinates for such an elliptical disc, we use the epicyclic approximation (Binney & Tremaine 1987, Gerhard & Vietri 1986; Franx, van Gorkom, & de Zeeuw 1994). In the presence of a non-axisymmetric halo potential, the disc particles are expected to have non-circular orbits. This approximation tells us how elliptical each of these orbits ought to be. Ultimately, the departure from circularity of each orbit is determined by two quantities: the shape of the halo potential, and also its mass distribution (in the form of circular velocity, v_c) – both as a function of radius.

The first step is to create a circular disc with an exponential density profile:

$$\rho_d(R, z) = \frac{M_d}{4\pi z_0 R_d^2} \exp\left(-\frac{R}{R_d}\right) \text{sech}^2\left(\frac{z}{z_0}\right), \quad (2.4)$$

where M_d is the disc mass, $R_d = 1$ is the scale length of the disc and $z_0 = 0.2$ is the scale height.

The elliptical disc will have an ellipticity of the orbits ε_R and an ellipticity of the velocities ε_v (ellipticities are defined as $\varepsilon = 1 - b/a$), both of which have a radial dependence. Because the epicyclic approximation does not take height into account, the vertical coordinates will remain unchanged, and the R and φ coordinates of the disc will be altered independently of z . The position coordinates on the plane are reassigned as follows:

$$R = R_0 \left[1 - \frac{\varepsilon_R}{2} \cos(2\varphi_0) \right] \quad (2.5)$$

$$\varphi = \varphi_0 + \frac{\varepsilon_R + \varepsilon_v}{4} \sin(2\varphi_0), \quad (2.6)$$

Table 2.2: Parameters of the initial conditions for all simulations: (1) model name; (2) halo (see halo shapes in Table 2.1); (3) disc shape; (4) disc mass; (5) halo mass; (6) halo core size; (7) Toomre parameter; (8) relative orientation of the disc and halo major axes; (9) time-scale of disc growth and (10) whether disc particles are live or rigid.

(1) name	(2) halo	(3) disc shape	(4) M_d	(5) M_h	(6) γ	(7) Q	(8) major axes	(9) t_{grow}	(10) disc
1C	1	circular	1	5	0.5	~ 1	-	100	live
2C	2	circular	1	5	0.5	~ 1	-	100	live
3C	3	circular	1	5	0.5	~ 1	-	100	live
2E	2	elliptical	1	5	0.5	~ 1	perpendicular	100	live
3E	3	elliptical	1	5	0.5	~ 1	perpendicular	100	live
1'C	1	circular	1	5	5.0	~ 1	-	100	live
2'C	2	circular	1	5	5.0	~ 1	-	100	live
3'C	3	circular	1	5	5.0	~ 1	-	100	live
2'E	2	elliptical	1	5	5.0	~ 1	perpendicular	100	live
3'E	3	elliptical	1	5	5.0	~ 1	perpendicular	100	live
1C t10	1	circular	1	5	0.5	~ 1	-	10	live
2E t10	2	elliptical	1	5	0.5	~ 1	perpendicular	10	live
3E t10	3	elliptical	1	5	0.5	~ 1	perpendicular	10	live
1C t200	1	circular	1	5	0.5	~ 1	-	200	live
2E t200	2	elliptical	1	5	0.5	~ 1	perpendicular	200	live
3E t200	3	elliptical	1	5	0.5	~ 1	perpendicular	200	live
1Cm	1	circular	0.3	5	0.5	~ 1	-	100	live
2Cm	2	circular	0.3	5	0.5	~ 1	-	100	live
3Cm	3	circular	0.3	5	0.5	~ 1	-	100	live
2Em	2	elliptical	0.3	5	0.5	~ 1	perpendicular	100	live
3Em	3	elliptical	0.3	5	0.5	~ 1	perpendicular	100	live
1C azi	1	circular	1	5	0.5	~ 1	-	100	constrained
2C azi	2	circular	1	5	0.5	~ 1	-	100	constrained
1C hot	1	circular	1	5	0.5	2.4	-	100	live
2E hot	2	elliptical	1	5	0.5	2.4	perpendicular	100	live
3E hot	3	elliptical	1	5	0.5	2.4	perpendicular	100	live
1C rigid	1	circular	1	5	0.5	-	-	100	rigid
2C rigid	2	circular	1	5	0.5	-	-	100	rigid
3C rigid	3	circular	1	5	0.5	-	-	100	rigid
3E 90	3	elliptical	1	5	0.5	~ 1	parallel	100	live

where (R_0, φ_0) are the position coordinates of the particles of the circular disc and (R, φ) are the position coordinates of the particles of the new elliptical disc. Similarly, the velocity coordinates will be:

$$v_R = v_c \varepsilon_R \sin(2\varphi_0) \quad (2.7)$$

$$v_\varphi = v_c \left[1 + \frac{\varepsilon_v}{2} \cos(2\varphi_0) \right], \quad (2.8)$$

where v_c is the circular velocity, v_R is the radial velocity and v_φ the tangential velocity. The ellipticities of the velocities, of the positions and of the potential can be shown to have a simple dependence:

$$\varepsilon_v = \varepsilon_R + \varepsilon_{pot}. \quad (2.9)$$

Besides, it is possible to show that the ellipticity of the orbit, ε_R , is related to the ellipticity of the potential ε_{pot} through:

$$\varepsilon_R = \varepsilon_{pot} \left[\left(\frac{2v_c^2}{R} + \frac{dv_c^2}{dR} \right) \left(\frac{2v_c^2}{R} - \frac{dv_c^2}{dR} \right)^{-1} \right]_{R_0}, \quad (2.10)$$

which is a generalisation for any $v_c(R)$ of the particular case employed by Franx et al. (1994), where the circular velocity was a power law. (See Appendix A for the full derivations.)

So, for a given triaxial halo, we measure the ellipticity of the potential as a function of radius. Then, measuring $v_c(R)$ and estimating its derivative allows us to calculate the ellipticity of the orbits for each disc particle. The particles in the disc have orbits whose ellipticities are not constant with radius even if the ellipticity of the halo potential is. The epicyclic approximation, however, is not valid when v_c is approximately proportional to R , and for this reason it cannot be applied to the innermost part the disc. To prevent it from diverging, ε_R is set to a constant value in the innermost region.

The initial shapes of the halo potentials are quite independent of radius and correspond to $(b/a)_{pot}$ of approximately 1, 0.85 and 0.72 for haloes 1, 2 and 3, respectively. The axis ratios of the potential are expected to be larger than the axis ratios of the density because the isopotential contours (since they refer to an integrated quantity) are always smoother and more circular than the isodensity contours. In order to measure the axis ratios of the potential, the halo particles are sorted by potential and the components of the inertia tensor are calculated in consecutive intervals containing equal number of particles. The $(b/a)_{pot}$ is obtained for each interval, interpolated to the disc particle positions and used to calculate ε_R and ε_v for each disc particle. The resulting shapes of the discs can be seen in the upper row of Fig. 2.1. In equilibrium, the orbits of the disc particles will be elongated in a direction perpendicular to that of the halo major axis, and this comes out naturally from the epicycle approximation. Besides creating elliptical discs, for comparisons, we also create equivalent models using circular discs in each of the triaxial haloes (Table 2.2).

2.1.2.2 Growing the disc

Instead of introducing the disc abruptly, we slowly grow the mass of the disc while the system evolves, in order to ensure a smooth adaptation of the halo and avoid transients

Figure 2.1: Discs of models 1C (left), 2E (middle) and 3E (right) on the xy plane at $t = 0$, $t = 100$ and at $t = 800$. Disc rotation is counterclockwise. Each frame is 10 by 10 units of length. Colour represents projected density and the range is the same for all panels.

in the disc. This is done by gradually increasing the mass m_i of each disc particle from (almost) zero to M_d/N_d , according to a smooth curve:

$$m_i(t) = \frac{M_d}{N_d} \times \begin{cases} \frac{1}{2} \left(1 - \cos \frac{\pi t}{t_{grow}} \right) & , 0 \leq t \leq t_{grow} \\ 1 & , t > t_{grow} \end{cases} \quad (2.11)$$

where M_d is the final mass of the disc, N_d is the number of disc particles and t_{grow} is the growth time of the disc mass. This procedure is applied for an interval of 100 time units, during which time both the halo and the disc are live. In the models described in Sect. 2.4 other growth times are explored.

At first, the disc particles are very light and they feel the halo potential and respond almost as test particles, without much self gravity and without affecting the halo abruptly. At $t = 100$ the disc is at full mass and, in some respects, this is the instant that ought to be regarded as the actual beginning of the simulation, since before this time both the total mass of the system and its total angular momentum are in fact increasing.

When the mass of the disc has finished growing ($t = 100$) the contributions of the disc and halo to the total circular velocity are comparable in the inner region, for the spherical case (1C). In the more triaxial model (3E), however, the halo is stronger even in the inner region. One of the reasons for this is that the discs are not identical in the three models. The (azimuthally averaged) rotation curve of the more elliptical disc (model 3E) is slightly lower in the very centre, if compared to that of the circular disc in model 1C (Fig. 2.2). Another reason for the different rotation curves is the fact that the density of the triaxial haloes is slightly higher in the centre if compared to the spherical halo. As can be seen in Fig. 2.3, the density profiles at $t = 0$ (solid lines) are already slightly larger in the centre for the triaxial haloes, because they are made triaxial by shifting the y and z coordinates of the particles by factors smaller than one, which causes the concentration of mass in the centre to increase. Apart from this fact, the inner halo density suffers a further small increase (in all three models) due to the introduction of the disc, presumably because the mass of the disc drags halo matter towards the equatorial plane. This is why at $t = 100$ (dotted lines in Fig. 2.3) there has been a systematic increase of the density in the innermost regions accompanied by a very mild but systematic density decline for $r > 10$. At $t = 100$ the halo density profiles of the three models are roughly similar, meaning that the initial differences in their inner densities were compensated by the growth of the disc mass. So, as far as halo density profiles are concerned, the three haloes are more similar at $t = 100$ than they were at $t = 0$. For clarity, further times are not shown in Fig. 2.3, but from $t = 100$ to $t = 800$ the changes are insignificant.

Using the isothermal sheet approximation, the vertical component of the velocity dispersion, σ_z^2 , is determined by z_0 :

$$\sigma_z^2 = \pi G \Sigma(R) z_0, \quad (2.12)$$

where $\Sigma(R)$ is the surface density. The velocity dispersions in the other two coordinates are acquired spontaneously by the disc during its evolution, while its mass grows. Because

Figure 2.2: Azimuthally averaged rotation curves at $t = 100$: disc (dotted lines), halo (dashed lines) and total (solid lines). Models with $M_d = 1$ are shown in the upper row and models with $M_d = 0.3$ in lower row. In the upper right corner of each panel we give the name of the corresponding model.

Figure 2.3: Azimuthally averaged halo density profiles before ($t = 0$, solid lines) and after ($t = 100$, dotted lines) the introduction of the disc, for models 1C, 2E and 3E, respectively.

of this method of growing the disc, where the velocity dispersions are gradually acquired by the evolving disc, we do not set a particular Toomre parameter Q to begin with. By measuring the epicyclic frequency κ and the surface mass density Σ we find that the values of Q for models 1C, 2E and 3E, averaged over the radius, are in the range of 0.7 to 1.1, between $t = 100$ and $t = 140$. It should be noted that the radially averaged values are meant as a rough estimate, since the radial dependence of Q is considerable. At later times the bar grows and the potential becomes strongly non-axisymmetric, so that the standard definition of Q is not very meaningful.

2.1.3 Miscellanea

The units used in this Chapter are such that the Newtonian gravitational constant is $G = 1$ and the scale length of the disc is $R_d = 1$. Furthermore, for the standard sequence of models $M_d = 1$. In physical units, if we take the disc scale length to be 3.5 kpc and the mass of

disc $5 \times 10^{10} M_{\odot}$, for example, then the unit of time is 1.4×10^7 yr and the unit of velocity is 248 km/s.

The models here are evolved for 800 units of time, which, in the above example, corresponds to 11.2 Gyr. In these simulations, the halo has 10^6 particles and the disc 2×10^5 particles. The mass of the halo is always $M_h = 5$ and, for the standard models, the mass of the disc is $M_d = 1$. The evolution is calculated using the N -body code *GYRFALCON* (Dehnen 2000; 2002). In all cases, we used a softening of 0.05 units of length, an opening angle of 0.6 and a time step of $1/64$ units of time. This led to an energy conservation of the order of 0.1%.

We calculate the b/a and c/a axis ratios using the eigenvalues of the inertia tensor. If the shapes are measured in circular shells with equally spaced radial bins, a strong bias towards sphericity is introduced. Instead, we sort the halo particles as a function of local density and we measure the shape inside density bins containing equal number of particles, as already described in Sect. 2.1.1. These bins are not necessarily spherical and they are not equally spaced in radius.

In order to measure the strength of the bars, we use the Fourier components of the bidimensional mass distribution as a function of cylindrical radius, computed in the following way:

$$a_m(R) = \sum_{i=0}^{N_R} m_i \cos(m\theta), \quad m = 0, 1, 2, \dots \quad (2.13)$$

$$b_m(R) = \sum_{i=0}^{N_R} m_i \sin(m\theta), \quad m = 1, 2, \dots \quad (2.14)$$

where N_R is the number of particles inside a given ring and m_i is the mass of each particle. The relative amplitude A_m is defined as:

$$A_m = \frac{\int_0^{R_{max}} \sqrt{a_m^2 + b_m^2} R dR}{\int_0^{R_{max}} a_0 R dR}. \quad (2.15)$$

The ‘bar strength’ is the quantity A_m (for $m = 2$), and the integration is done until a maximum radius $R_{max} = 3$, which is typically where the amplitude of the $m = 2$ component reaches a minimum. (See Appendix B.)

2.2 Loss of halo triaxiality due to elliptical or circular discs

As described above we introduce the disc gradually over 100 time units and then continue the fully self-consistent simulation to follow the evolution. The most striking difference with respect to the pure halo models is that the haloes in which (massive, bar-forming) discs are grown lose at least some of their triaxiality (Fig. 2.4). The effects on the vertical flattening c/a are presented in Sect. 2.6.1. Except for that section, expressions such as ‘circularisation’ and ‘loss of triaxiality’ refer to the face-on shape of the halo, i.e. they both mean that the halo becomes circular on the disc equatorial plane (b/a approaching 1), regardless of c/a . The snapshots of the disc and of the halo density, seen on the xy plane at $t = 0$, 100 and $t = 800$, are presented in Appendix C.

Figure 2.4: Haloes of models 1C (left), 2E (middle) and 3E (right) on the xy plane at $t = 0$, $t = 100$ and at $t = 800$. Each frame is 10 by 10 units of length. The projected density range is the same for all panels, and the same as in Fig. 2.1.

Figure 2.5: The evolution of the b/a radial profile. The first row shows the evolution of models with haloes 1, 2 and 3 and with their respective equilibrium discs. The second row shows the evolution of the same haloes, but with initially circular discs.

2.2.1 Standard models and models with initially circular discs

In all models where (bar-forming) discs were grown, the haloes had their shapes altered. Even in the case of a spherical halo with circular disc (1C) the innermost region of the halo becomes rather prolate, due to the formation of the bar: this is the “halo bar” (Athanasoula 2005a; 2007), or “dark matter bar” (Colín, Valenzuela, & Klypin 2006), which rotates together with the disc bar, but is less elongated and less strong. The formation of such a structure causes the halo b/a to reach 0.8 in the inner region (Fig. 2.5) in all models where the disc forms a strong bar. Further out, the initially triaxial haloes (models 2 and 3) lose their triaxiality almost entirely after 800 time units. Roughly, almost half of the loss takes place during disc growth (from solid to dashed lines in Fig. 2.5) and the second half takes place due to bar formation and evolution (from dashed to dotted lines in Fig. 2.5). This is approximately valid also if circular discs are used instead of elliptical discs, but there are interesting differences which will be discussed.

The time evolution of the shapes of models 1C, 2E and 3E are shown in Fig. 2.6. These are the standard models, because each of the three haloes contains its equilibrium disc. The variations with circular discs (2C, 3C) will be used for comparisons. The halo b/a are measured within three different radii. First, they are measured taking all particles into account. To measure the shape within a given radius r , we employ the procedure already described in Sect. 2.1.1. This is done for $r = 3$ and $r = 1$. Measuring the shapes in this fashion highlights respectively: the overall shape (all particles), the shape at the region where most of the disc mass is located ($r < 3$) and the shape in the region of the bar ($r < 1$).

As can be seen in Fig. 2.6, the shapes of the haloes in models 1C, 2E and 3E all tend to be same at $t = 800$, the region of the disc ($r < 3$) approaching circularity faster than the outer halo. This is presumably due to the fact that the dynamical time is much longer in

Figure 2.6: The b/a axis ratios of the halo, measured taking into account all particles (top), the particles in $r < 3$ (middle) and the particles in $r < 1$ (bottom), as described in the text. The dotted, dashed and solid lines show models 1C, 2E and 3E respectively.

the outer parts than in the region with $r < 3$. In $r < 1$, all models develop the same halo bar with $b/a = 0.8$. The evolution of the vertical flattening (discussed in Sect. 2.6.1) shows that even in the case of the spherical halo c/a also drops to 0.8, which means that the halo bar is a prolate structure (1:0.8:0.8, the circular plane containing the shorter axes).

The comparisons of the central parts of models 1C, 2E and 3E (and also 2C and 3C) are shown in Fig. 2.7. On the first row the halo b/a for $r < 3$ is shown again for the three standard models and then separately for models 2 and 3 comparing their respective elliptical and circular discs. The more relevant difference is that for the case of the circular disc inside the more triaxial halo (3C): the halo gets circularised faster than in the case of the elliptical disc. During disc growth, the halo b/a already increases more for the circular disc and, from then on, it remains always larger than that of model 3E, until about $t = 500$. The second row of Fig. 2.7 shows the evolution of the bar strengths. Bar strengths, given by the quantity $A_2(R)$ (Eq. 2.15), are measured from $R = 0$ to $R = 3$, because that's approximately the first minimum of $A_2(R)$ for all models with strong bars (Fig. 2.8). For the standard models, we note that at first, our measure of A_2 actually decreases in the E cases. We point out that there is no actual bar at such times: the non-zero A_2 of models E is simply a measure of the non-circularity of the initial conditions, so that A_2 is in reality indicating the non-axisymmetry, or non-circularity of the disc. During disc growth this

non-circularity is momentarily reduced, such that by $t = 100$ all three models have roughly the same A_2 value. Subsequently, the actual bar instability sets in and a true bar grows in all three models, but with a delay in the case of 3E. If we then compare what happens in the models with circular discs and with elliptical discs, it turns out that, contrary to what one might have naively expected, the bar is actually stronger in the *initially* circular discs. It should be emphasised, however, that these discs cease being circular immediately after $t = 0$, because they are presumably driven by the halo into an excessively elliptical shape. At first, the A_2 values of the circular discs increases very sharply, to values considerably higher than even those of the initially elliptical discs. This behaviour is reminiscent of the transients and overshoots observed in bar response calculations when the bar is not inserted gradually. As a matter of fact, avoiding such unreasonable behaviour was precisely one of the motivations for setting up initial conditions for elliptical discs which should be in equilibrium. By $t = 100$ the A_2 of the circular discs has decreased somewhat, but it is still higher than that of the elliptical discs. So by the time the bar begins to really form, it does so in a disc which is actually more elliptical than in the E models. The result is that in the C models, the bar is stronger than in the E models, during most of the evolution.

The fact that discs with stronger bars circularise the halo more is a first indication of the importance of bar formation and evolution in the loss of halo triaxiality. In models that form stronger bars, the axisymmetrisation of the halo is accomplished sooner. Comparing models 2E and 3E in Fig. 2.7, we see that during most of the time ($t = 100$ to $t = 400$ or 500) the bar in 2E is stronger than in 3E. At the same time, b/a has larger values. Similarly, in model 3, the E and C cases show that the bar of 3C is stronger during $100 < t < 700$ and also the halo is clearly more circular. In fact, the halo b/a for 3E doesn't even increase very much in $100 < t < 400$. It only becomes steeper at $t = 400$, when the bar strength of 3E has finally caught up with values comparable to those of 3C.

Another very important quantity in the evolution of these systems is the angular momentum, whose total values (per unit mass) are shown in Fig. 2.9. Each disc acquires a certain amount of angular momentum while it is growing, and this amount is not the same for all models. Thus the total angular momentum increases somewhat during a time equal to t_{grow} , but stays constant after that. There is, however, considerable angular momentum exchange between the disc and the halo component, as in models with axisymmetric haloes (e.g. Sellwood 1980, Debattista & Sellwood 2000, Athanassoula 2003, O'Neill & Dubinski 2003; Martinez-Valpuesta, Shlosman, & Heller 2006; Villa-Vargas, Shlosman, & Heller 2009). The net amount of angular momentum lost by the disc is gained by the halo in all models. At $t = 100$ the halo has nearly zero angular momentum, so that roughly the total angular momentum of the system is with the disc. Later, as the bar begins to form, it gets redistributed. One important feature of these models is that the initially more elliptical discs acquire more angular momentum. This can be clearly seen if we compare models 1C, 2E and 3E, our standard sequence of models with equilibrium discs (thick lines in Fig. 2.9). And it can also be seen if we compare E and C discs for one given halo: the initially circular discs (thin lines) have less angular momentum than the equivalent initially elliptical disc in the same halo, or, equivalently, in E models, the disc has acquired more angular momentum by $t = 100$ than in C models.

We may draw another similar observation from the lower panel of Fig. 2.7, which

Figure 2.7: Time evolution of inner halo b/a ($r < 3$), (top). A_2 (middle). Fractional angular momentum of the disc, per unit mass (bottom). The first column compares the spherical case with the two triaxial haloes containing their respective elliptical discs. The dotted, dashed and solid lines show models 1C, 2E and 3E respectively. The second and third columns (for haloes 2 and 3 respectively) compare the elliptical disc (thick lines) and circular disc (thin lines) cases.

Figure 2.8: Evolution of the $m = 2$ Fourier component as a function of radius for different models. The curves are measured at intervals of $\Delta t = 50$ and plotted with a vertical displacement for clarity. The scale corresponds to the uppermost curve.

Figure 2.9: The disc angular momentum is shown for models 1C (dotted), 2E (dashed) and 3E (solid lines). Before $t = 100$ the mass of the disc is still growing, but the angular momentum per unit mass is roughly constant for most models. Also shown is the angular momentum of models with circular discs (thin lines).

Figure 2.10: A_2 and halo b/a for models using the very triaxial halo 3+, with circular disc (dashed lines) and elliptical disc (solid lines).

shows the fractional angular momentum of the discs. We see that aside from having more total angular momentum, the initially more elliptical disc will also hold a larger fraction of the total angular momentum of the system than the fraction that will be held by an equivalent circular disc, at any given time. This is valid for any one given halo. And it is still true if we compare the elliptical discs of the two triaxial haloes. The consequence of this is immediate: if a disc is holding on to its angular momentum, it means that it is not forming a bar so well. So the discs that don't lose their angular momentum so efficiently will have smaller bar strengths. This is in good agreement with the results found for axisymmetric haloes, where a tight correlation can be found between the bar strength and the angular momentum gained by the halo and lost from the disc (Athanasoula 2003).

So we might propose the following scenario: as they form, elliptical discs acquire a larger amount of angular momentum. Also, they don't lose their angular momentum so efficiently and this causes them to develop weaker bars, which take more time to circularise their triaxial haloes.

One particularly interesting comparison between circular and elliptical discs is provided by models using halo 3+, which is more triaxial than halo 3. Halo 3-, being intermediate in shape between haloes 2 and 3, shows an evolution which is merely intermediate between those two cases. In the case of the very triaxial halo 3+, on the other hand, the difference between using a circular disc or an elliptical disc is drastic. Figure 2.10 shows the evolutions of A_2 and of halo b/a for models 3C+ and 3E+. For the model with a circular disc (3C+), A_2 peaks strongly very early on ($t \sim 10$) and then decays gradually to small but non-zero values. For the model with an elliptical disc (3E+), A_2 decreases at first and then starts growing in a manner similar to the other E cases, albeit with a considerable delay. Model 3C+ is the only one of our simulations in which halo triaxiality could be said to have inhibited bar formation. That, however, appears to be due to the inadequacy of using a circular disc as the initial conditions. When, instead, we use an elliptical disc in the same halo, a strong bar does form.

The evolution of halo shapes in models 3C+ and 3E+ (Fig. 2.10) shows that disc introduction causes some halo circularisation in both cases. It is also clear that in the case where there is bar formation (3E+), the halo suffers further circularisation and the period of more intense loss of triaxiality coincides with the period of faster bar growth. In the

case where there is no significant bar formation (3C+), the halo is able to remain triaxial throughout. These results point to the contribution of the bar as one of the factors causing loss of halo triaxiality.

These results are also indications of very important differences that arise depending on whether one uses circular, and manifestly out of equilibrium discs, or elliptical, and near equilibrium ones, when simulating bar formation within triaxial haloes.

2.2.2 Elliptical disc parallel to the halo major axis

The equilibrium configuration for the system of an elliptical disc inside a triaxial halo is such that the major axis of the disc is perpendicular to the major axis of the halo. Therefore, in all our initial conditions the elliptical disc is oriented in this way. We, nevertheless, also explored one simulation in which the major axes of disc and halo are parallel, knowing that this would be well off equilibrium. We took model 3E and turned by 90° the elliptical disc in the initial conditions (let us call it model 3E90). The result is that the evolution of this model is very similar to the evolution of model 3C. That is to say, in model 3E90 the bar is stronger than in model 3E. The angular momentum transfer is more steep and the circularisation of the halo takes place faster. As a matter of fact, the bar in model 3E90 is even slightly stronger than in model 3C, and thus the angular momentum is lost by the disc even faster and the halo b/a consequently increases more rapidly. The disc in model 3E90 behaves much in the same manner as the disc of 3C in the beginning of the simulation: it becomes excessively distorted in the direction perpendicular to the halo major axis. This confirms the general trend that a stronger bar will cause greater halo circularisation. It also indicates how strong the effect of out-of-equilibrium initial conditions can be, thereby stressing the importance of starting the simulation in near-equilibrium.

2.2.3 Position angles

In order to study the relative orientations of the various elongated structures, we distinguish between two regions of the disc and two regions of the halo, namely the inner and outer parts of each. We therefore define the following four components: the *disc bar* ($0 < R < 3$), the *halo bar* ($0 < R < 1$), the *outer disc* ($3 < R < 10$) and the *outer halo* ($3 < R < 30$). Using models 1C, 2E and 3E we then measure the position angles of each of these components as a function of time, using the particles contained within those radii. The angles are obtained from the Fourier analysis, and correspond to the direction of the elongation of the $m = 2$ component. The above definitions of the disc bar region and of the halo bar region take into account the typical lengths of these structures, that extend namely to about $R = 3$ and $R = 1$. The definitions of the outer disc and outer halo are meant to give an estimate of the direction of the overall orientation of such structures, without being contaminated by the bars. The outermost parts of the halo retain some residual triaxiality which, although small, is sufficient to allow a reasonable determination of the direction of its major axis.

Once both are formed, the disc bar and the halo bar rotate together. The halo bar forms sooner in the spherical case, while in the triaxial cases, the formation of the halo

bar is somewhat delayed. Once they are sufficiently strong, their position angles roughly coincide throughout the rest of simulation.

The orientation of the outer halo is ill defined in the spherical case. In the triaxial cases, the major axis turns very slowly. We find that it takes about 700 or 800 time units to turn by 90° . Such slow tumbling was also obtained by Heller et al. (2007), after the period of collapse. This is also in good agreement with the results of Bailin & Steinmetz (2004), who measured the figure rotation of haloes from cosmological simulations and obtained pattern speeds with a log-normal distribution centred at approximately $\Omega_p = 0.148 h \text{ km s}^{-1} \text{ kpc}^{-1}$. This means that a typical halo would rotate roughly 90° during a Hubble time.

The outer shape of the disc also bears some interesting relations to the bar. In the beginning of the simulation the elliptical disc is perpendicular to the major axis of the halo. At first, the position angle of the outer disc remains in the same direction. Gradually, after both bars are formed, the overall shape of the disc acquires figure rotation, with a pattern speed comparable to that of the bar but out of phase with respect to it. The phase difference between the disc bar and the outer disc is of 90° at first, but then decreases.

2.3 Different halo core sizes

In spherical haloes it was found that the core radius is an essential feature in determining the evolution of the galaxy (Athanasoula & Misiriotis 2002, Athanasoula 2003). So far, we have only presented results from haloes with a small core ($\gamma = 0.5$), of the type called MH in Athanasoula & Misiriotis (2002). In this section we will discuss the change of halo shape in simulations with large cores ($\gamma = 5.0$, i.e. of the type called MD in the aforementioned reference). These models, named 1'C, 2'E, 3'E (and 2'C, 3'C), are the equivalent of the standard set 1C, 2E, 3E (and 2C, 3C) in the sense that they have the same shapes as the standard set. However, they have different density profiles, namely they are less concentrated. Evolved without discs, these haloes with $\gamma = 5.0$ are just as stable as the haloes with $\gamma = 0.5$, as far as their density profile and shape are concerned, i.e. they retain their shapes and profiles until the end of the simulation.

Haloes with a large core have a peculiarity which causes the epicycle approximation to give worse results. Because the core is larger, the velocity curve is, at the start of the simulation, approximately linear ($v_c \propto R$) across a larger region. The epicyclic approximation breaks down in this case, which means that ε_R needs to be truncated at an arbitrary value over a good portion of the disc. In the spherical case, the disc is appropriately circular, but because of these difficulties, the disc shape is not set quite properly in the triaxial models of larger core. As a consequence we have some transients and overshoots: a disc which is not in equilibrium with its triaxial halo will respond to the ellipticity of the potential it feels by becoming excessively elongated at first, thus causing A_2 to increase too much. This also means that with such haloes, the behaviour of the elliptical discs are not much better than the circular ones in the same triaxial haloes.

The evolutions of halo b/a , A_2 , and angular momentum for haloes with large cores are shown in Fig. 2.11. They show clearly that these haloes are more susceptible to circularisation. The disc growth alone is capable of driving the halo b/a to 1 by $t = 100$ in

Figure 2.11: Same as Fig. 2.7, but for haloes with larger cores ($\gamma = 5.0$).

all cases. These now spherical, large-cored haloes don't receive much angular momentum from their discs, and thus don't develop strong bars. The fact that the departures from axisymmetry in models 2' and 3' are higher than in 1' is mostly due to the fact that the discs in the initially triaxial haloes were driven to be excessively elliptical, even though some of the non-axisymmetry in 2' can be attributed to the formation of a weak bar, that can be discerned on the snapshots. Moreover, there is some degree of angular momentum exchange.

The main result of this section is that less concentrated haloes are unable to retain their triaxial shape. The growth of disc mass inside such large-cored haloes is already enough to make them totally spherical. Furthermore, models with such haloes do not develop strong stellar bars, but in any case, there wouldn't be any triaxiality left to be erased by the bar later on.

2.4 Different time-scales for disc growth

Berentzen & Shlosman (2006) experimented with different ways of growing a seed disc into a triaxial halo by adding the stellar particles gradually. They found that the halo shape is not very sensitive to whether the disc is introduced abruptly or quasi-adiabatically. Following their experiments, we also re-ran models 1C, 2E, 3E both with a shorter ($t_{grow} = 10$) and a longer ($t_{grow} = 200$) time-scale of disc growth. For the first two models (top and middle panels of Fig. 2.12), the result is that the overall evolution of A_2 and halo b/a is merely shifted to earlier or later times. We know that the bar growth is faster in cases where the halo to disc mass ratio is smaller (e.g. Athanassoula & Sellwood 1986). So the temporal shift witnessed in the two upper panels of Fig. 2.12 could simply mean that disc mass has to reach a certain limiting value for the bar to start growing sufficiently rapidly. In models with smaller t_{grow} , the bar is comparatively stronger at earlier times and the final value of A_2 is somewhat larger.

In the case of model 3E, on the other hand, the evolutions of the bar strength are rather similar in the cases where $t_{grow} = 100$ and $t_{grow} = 200$. The A_2 usually begins to grow rapidly immediately after t_{grow} . In the case of model 3E with the standard $t_{grow} = 100$, however, it stalls for about another 100 time units, and is eventually caught up with by model 3E with $t_{grow} = 200$. In model 3E ($t_{grow} = 100$) the halo b/a suffers a steep increase during disc growth (going from 0.6 to 0.8). After that, this halo momentarily undergoes a slight gain of triaxiality, during $100 < t < 200$. During this period, instead of bar formation setting in, we have a small elongation of the halo. By the time the halo has settled at $b/a \sim 0.75$ it is indistinguishable in shape from model 3E ($t_{grow} = 200$). It is only then that angular momentum transfer begins, for both models, and their bar strengths increase simultaneously.

Figure 2.12: A_2 and halo b/a for simulations with different time-scales of disc growth: $t_{\text{grow}} = 10$ (dotted lines), 100 (solid lines) and 200 (dashed lines). Note the different time scale in the right panels.

2.5 Relative contributions of the disc and the bar to the loss of halo triaxiality

The loss of halo triaxiality can be partly due to the introduction of the disc and partly due to the growth of the bar. In order to disentangle these two separate effects and to assess their relative contributions, we ran a number of specifically designed simulations. This includes simulations with less massive discs, simulations in which bar growth was artificially suppressed, simulations with hot discs and simulations with rigid discs.

2.5.1 Less massive discs

As already discussed in Athanassoula (2002), the relative halo mass influences the bar in two quite different ways. First the halo-to-disc mass ratio influences the growth time of the bar, in the sense that relatively more massive haloes (i.e. relatively less massive discs) lead to slower bar growths (e.g. Athanassoula & Sellwood 1986). Then during the bar evolution, the halo helps the bar grow by absorbing at its resonances the angular momentum emitted by the bar (Athanassoula 2002). In fact the strongest bars form when there is optimum balance between emitters and absorbers and this can determine the location of corotation (Athanassoula 2003).

Thus, by adopting very low mass discs, we should obtain weak bars. For this reason, we ran simulations with a disc mass $M_d = 0.3$, i.e. less than a third of the disc used in all previously discussed models, which have $M_d = 1$. This is one of the cases that can allow

Figure 2.13: Time evolution of the halo non-axisymmetry (b/a) for models with a disc less massive than in our standard case ($M_d = 0.3$, instead of 1). The upper panel corresponds to the whole halo, the middle one to the inner halo and the lower one to the innermost halo (see Section 2.2 for definitions). The dotted, dashed and solid lines show models 1Cm, 2Em and 3Em respectively.

us to investigate whether it is the bar or the presence of the disc itself that causes the halo to become axisymmetric; or rather, to quantify the contributions of these effects. It is to be expected, however, that non-massive discs should obviously have smaller effects on the halo. The mass of these discs contributes little to the total circular velocity curves (Fig. 2.2). Also, halo density profiles do not suffer much increase in the inner region due to disc growth if $M_d = 0.3$.

Figure 2.13 shows the evolution of the halo b/a for models containing less massive discs, for the whole halo, the inner halo and the innermost halo (see Section 2.2 for definitions) and Fig. 2.14 includes information on the disc non-axisymmetry and the ratio of disc-to-total angular momentum. From the latter and from viewing sequences of snapshots which show the evolution, we see that in all the models with low disc mass there is no true bar formation, even though the inner disc becomes elliptically distorted. Yet there is some, albeit little, change of the halo shape, limited to $t < 100$. Arguments from the time during which these changes occur, the fact that there is no true bar and practically no angular momentum exchange, lead to the conclusions that the whatever loss of halo triaxiality is witnessed is due to the introduction of the disc and not to any subsequent bar formation.

And still we note that, during the period of disc growth, the halo b/a in Fig. 2.14 does increase. In model 2Em it goes up to about 0.85 and in model 3Em to little more than 0.65 (and slightly more so if the disc is initially circular). This means that introducing a non-bar-forming disc caused very small loss of halo triaxiality and only in the period of disc growth.

Growing the more massive disc causes much greater loss of triaxiality. In Fig. 2.7, there is also an increase of halo b/a due to disc introduction, but it is much larger, specially in the case of the more triaxial halo, where it grows from 0.6 to 0.8. This means that introducing a (massive) bar-forming disc had caused a loss of triaxiality of about 0.2 in model 3E. This is to be compared with a corresponding increase by 0.05 in model 3Em, which has low mass disc. In the models with the standard disc mass there is further loss of halo triaxiality after the disc has reached its full mass. Thus, by $t = 800$ the halo is very near circular ($b/a > 0.95$), which means that the amount by which the halo shape changed during and after disc growth are comparable. Equivalently in the case of the less triaxial halo 2E, the b/a increases from 0.8 to 0.9 during disc growth and afterwards gets to about 0.95 as well. For the models with lower mass discs we do not witness any further loss of halo triaxiality after the disc has grown and the total change of triaxiality is rather small.

To summarise, we can say the models with low mass discs suffer only a small loss of halo triaxiality and that all of it is due to the introduction of the disc. Although these models isolate the effect of the disc introduction, they can not give us much information on the relative effects of the disc and bar for other cases. They are, nevertheless, applicable to galaxies with low surface brightness discs, and argue that such galaxies should not have suffered much loss of halo triaxiality, and that their halo shape should be near what it was from galaxy formation and as due to effects of interactions and mergings.

Before turning to other ways of assessing the relative role of the disc and bar to the loss of halo triaxiality, we will discuss an interesting property observed in one of our models, model 3Cm. Here we see that the A_2 , the strength of $m = 2$ non-axisymmetry, shows oscillations. Viewing the evolution of this model, we see that it did not develop a proper bar but has an elliptical distortion in the centre (with some spiral structure) whose shape oscillates periodically. This elliptical distortion rotates and its elongation is more pronounced when it is perpendicular to the halo elongation. The A_2 amplitude always peaks when the orientation of the elliptical elongation is perpendicular to the halo elongation. The oval rotates with a period of about 50 time units and even after 30 alignments, its mean strength has not decreased.

The behaviour of these discs is in some respects analogous to that of galaxies with double-bars (also known as nested bars, or nuclear bars), in which there is a primary (outer) bar and a secondary (inner) bar. In our simulations of low-mass discs inside triaxial haloes, the discs don't develop bars, but the oval distortion in the disc rotates in the presence of an elongated halo potential. Thus the disc 'oval' (to avoid calling it a bar) is analogous to a 'secondary bar' and the triaxial halo itself is analogous to a 'primary bar', with the difference that the triaxial halo does not rotate and its major axis remains aligned with the x -axis.

In their theoretical approach to orbits within double bars, Maciejewski & Sparke (2000), and Maciejewski & Athanassoula (2007) find that the loops supporting the inner

Figure 2.14: Same as Fig. 2.7, but for a less massive disc ($M_d = 0.3$, instead of 1).

bar are thicker when the two bars are parallel; and that if the inner bar is a self-consistent bar made of particles trapped around those loops, it should be thinner when the two bars are perpendicular. Debattista & Shen (2007) describe collisionless N -body simulations of discs in rigid haloes, which form double bars. In such simulations, the bar strengths oscillate; the secondary bar becomes stronger when the two bars are perpendicular and weaker when they are parallel. Furthermore, the secondary bar rotates faster than average when they are perpendicular; and slower than average when they are parallel. For our analogous situation (which consists essentially of an elliptically distorted disc rotating in an elongated halo potential) we obtain the same correlations, which are also in agreement with the theoretical predictions of loops by Maciejewski & Sparke (2000). This would argue that low surface brightness galaxies whose discs show important oval distortion could be living in haloes which are still substantially non-axisymmetric.

The A_2 amplitude is calculated with bidimensional quantities. In order to have some estimate of the vertical shape of the oval, we compute c/a for the disc particles using the inertia tensor, in the same way as we do for halo shapes. The vertical thickness of the disc (in cases 3Em and 3Cm) also oscillates periodically. The oval is vertically thinner when it is perpendicular to the halo elongation; and it is vertically thicker when it is parallel to the halo elongation. It means that the oval's elongation correlates with its vertical flattening: when the oval is more elongated, it is also more flattened. Apart from oscillating, the mean thickness increases with time.

The haloes in simulations with low-mass discs remain triaxial. In models 3Em and 3Cm, the shape of the overall halo remains quite constant at about $b/a \sim 0.7$, with no significant radial dependence. In the innermost part of the halo ($r < 1$), however, the halo b/a oscillates, but by no more than 2%. These oscillations are very small, but measurable and quite regular. Furthermore, they anti-correlate sharply with the shape of the oval: when the oval is more elongated, the inner halo is less elongated. But this is not reminiscent of a 'halo bar', because there is no halo rotation at any radius: these haloes don't rotate importantly (except for tumbling slowly), which means that the major axis remains in the same direction.

2.5.2 Suppressing bar formation by imposing disc axisymmetry

In order to separate the effects of the bar and of the introduction of the disc, we need to analyse simulations in which the disc is standard (i.e. not low mass), but where there is no bar. We try to achieve this artificially, by randomising the azimuthal coordinate of the disc particles in regular time intervals ($\Delta t = 1$) during the evolution of the system. Artificially forcing the disc to retain axisymmetry throughout the evolution prevents any non-axisymmetric structure from forming. We use the very same discs as in the standard models, which would otherwise form strong bars, and observe the behaviour of their respective haloes in the case where their bar formation is suppressed by axisymmetrisation.

This technique consists in reassigning the φ coordinate of the disc particles to a random value between 0 and 2π , while keeping their cylindrical radius and their distance from the equatorial plane unaltered at each intervention. This procedure is applied repeatedly during the evolution, from the moment when the disc mass is fully grown, until the end of the

Figure 2.15: A_2 and halo b/a for models where bar formation has been suppressed by imposing disc axisymmetry (dashed lines). The disc is axisymmetrised at intervals of $\Delta t = 1$. For comparison, the solid lines show the results of the corresponding unconstrained simulations where the bars do form.

simulation and is straightforward in the case of axisymmetric haloes. In our case, however, the potential of the halo is not axisymmetric, so that the particles can find themselves in a region of deeper or shallower potential than before the rotation. Because of that, we re-assign their velocities in such a way as to conserve total energy, while at the same time keeping the angular momentum of each particle unaltered. Tests showed that if the randomisations are discontinued, then bar formation promptly sets in again.

This procedure works well for the spherical halo case because the discs in such haloes are indeed meant to be circular (axisymmetric). In the case of the triaxial haloes, making the disc perfectly axisymmetric causes it to be out of equilibrium with the halo potential. In the case of halo model 2, bar formation was successfully suppressed. However, in the case of halo model 3 it was not possible to apply the randomisations without causing the disc to become severely unstable. Experimentation showed that applying the interventions at different intervals also caused the disc of model 3 to be disrupted by the end of the simulation. Some disc particles end up gaining too much velocity and escape. Since it was not possible to have a permanently circular and stable disc inside a very triaxial halo, we exclude halo 3 from the analysis of this section.

Our purpose here is to evaluate the changes of halo shape in the absence of bar formation. The evolutions of A_2 and b/a are shown in Fig. 2.15, for models 1C and 2E and for the corresponding simulations where the axisymmetrisations described here were applied. The two evolutions before $t = 100$ are identical, because no axisymmetrisation was applied while the disc grew, and, for the initially triaxial halo, we witness an increase of the halo b/a from 0.8 to 0.9.

After we start applying the axisymmetrisation, however, the evolutions become different. In the unconstrained simulation, the bar forms and the triaxial halo completely loses its remaining triaxiality and becomes quite axisymmetric by the end of the simulation. In the simulations where the disc is continuously axisymmetrised, however, the bar does not

Figure 2.16: Comparison between models with hot discs ($Q = 2.4$, dashed lines) and models with cooler discs ($Q = 1$, solid lines). The left panels compare the A_2 and the right ones the halo b/a .

form and there is no further loss of halo triaxiality. So in the absence of bars, the halo of model 2E retains its b/a of 0.9, which otherwise would have gone to 1.

The conclusion is that, in these models, a certain fraction (approximately half) of the loss of halo triaxiality can be attributed to bar formation. It should, however, be remembered that the axisymmetrisation process artificially keeps the model out of equilibrium.

2.5.3 Hot discs

In axisymmetric haloes, hot discs are known to form oval distortions rather than strong bars (Athanassoula 1983; 2003; 2005a). We can thus use such discs to investigate whether they are able to completely circularise their haloes, as bar-forming discs do. As initial conditions, we first create circular discs (with Toomre parameter of $Q = 2.4$), using the method of Rodionov et al. (2009). These discs are meant to be in equilibrium with a spherical halo potential as that of halo 1. They are then made elliptical using the epicyclic approximation, so that their shapes will be in equilibrium with triaxial haloes 2 or 3. In these cases, however, only the positions are altered, while the velocities remain those of a circular disc model, so as to retain the velocity dispersions of the discs. Although this is not strictly correct, such models are nevertheless closer to equilibrium than models of circular discs inside triaxial haloes. During the period of disc mass growth, the velocities gradually adapt to the elliptical potential, while not losing their higher dispersions, which is important to this analysis.

In agreement with what was found in axisymmetric haloes, the models with hot discs

Figure 2.17: Comparison of halo b/a between models with a live circular disc (solid lines) and the corresponding models with a rigid disc (dashed lines).

can reasonably be said not to have formed bars (Fig. 2.16), as A_2 remains always below 0.1 (except for model 1Chot where it begins to grow a little towards the end of the simulation). The non-zero albeit small values of A_2 are merely due to slight oval distortions in the disc. These hot discs lose practically no angular momentum to their haloes (not shown here). The consequence of the lack of bar formation is that again the halo is able to remain triaxial (right panel of Fig. 2.16). The b/a of halo 2 goes from 0.8 to 0.9 and that of halo 3 goes from 0.6 to 0.8 in the period between $t = 0$ and $t = 100$, for both the hot and the normal discs. After that, however, the hot disc loses no further triaxiality, contrary to the standard models discussed in Sect. 2.2.

And so, as was the case also with the haloes of Sect. 2.5.2, there is no further circularisation besides that which was caused by the disc growth. This is compelling evidence that indeed the bar plays an important role in altering the shape of the halo.

2.5.4 Rigid discs

Another way of evaluating the effects of bar-forming discs, as opposed to non-bar-forming ones, is to replace the disc particles by an analytic potential. In such a case, the simulation consists only of the usual halo particles, but the disc is represented by a fixed potential, which is rigid and permanently axisymmetric. The halo particles feel their own self gravity and they feel the disc potential that has the same mass and scale lengths as in the simulations with live discs before the disc instability sets in. As in previous cases, we grew the potential gradually into the halo, according to a smooth function of time, between $t = 0$ and $t = 100$.

The results in Fig. 2.17 compare the halo shape evolution of models using live discs

and of models with rigid discs. The latter suffer a small amount of circularisation, only in the period of disc growth. The circularisation induced by rigid discs is much smaller than that caused by live discs, and it is in fact smaller even than that caused by low-mass live discs (Sect. 2.5.1). Once this rigid, circular, disc potential is in place, there is no further change of halo shape. Since there is evidently no bar formation, this again hints in the direction that the presence of the bar has determining effects on the halo shape evolution. For comparison, the models with live discs shown on Fig. 2.17 are the ones with circular discs, so that the initial conditions are identical in the sense that the shapes and masses of the disc are the same. The rigid disc simulations show us what happens if such circular discs are forced to remain circular and not develop a bar. Simulations with rigid potentials are, however, not realistic because there is no exchange of angular momentum.

2.6 Vertical shapes

2.6.1 Halo vertical flattening

The minor-to-major axis ratio c/a of the haloes is also affected both by disc growth and by bar formation, but to a lesser degree than the intermediate to major axis ratio b/a . The c/a generally increases, indicating that the halo tends to a less flattened configuration and this, taken together with the its circularisation, shows that the halo tends to become more spherical. Models 1, 2 and 3 start out with axis ratios of 1:1:1, 1:0.8:0.6 and 1:0.6:0.4, respectively and end with oblate shapes of roughly 1:1:0.9, 1:1:0.7 and 1:1:0.6 (as measured in $r < 3$). The only relevant radial dependence that is introduced in c/a is due to the presence of the disc, which makes the innermost regions somewhat more flattened than the overall shape. In the case of the spherical model, the inner region becomes significantly flatter and this change takes place mostly before $t = 100$. Presumably, due to the growth of disc mass, halo matter is pulled towards the plane $z = 0$ and parts of the halo in the immediate surroundings of the disc become flatter than the overall shape. Figure 2.18 shows the evolution of c/a for the standard models 1C, 2E, 3E as measured within three different radii.

In the simulations using the less massive disc, c/a remains virtually unaffected (Fig. 2.19), showing not even a slight increase during disc growth. These haloes remain approximately as triaxial as at $t = 0$. The models with a less concentrated halo suffer an increase of c/a only until $t = 100$ (the time during which their b/a goes to unity). After that, the c/a does not change any more and these haloes are oblate by the end of the simulation (Fig. 2.19).

In the other models where there is no bar formation (hot disc and axisymmetrised disc (Fig. 2.19)), the c/a increases only slightly until $t = 100$. The comparison of the non-bar-forming models with the standard models shows that in the presence of bar formation the triaxial haloes would have become still less flattened. This indicates that the bar acts not only on the shape of the halo on the equatorial plane, but also affects its vertical flattening; it causes the halo to tend towards sphericity by making it rounder in both directions. In the models with no bar formation, the final shape of the haloes is truly triaxial, being roughly 1:0.9:0.6 for halo 2 and 1:0.8:0.5 for halo 3. Finally, in the spherical model, the halo c/a

Figure 2.18: Halo c/a evolution for models 1C (dotted lines), 2E (dashed lines), 3E (solid lines), measured using particles within $r < 1$ (bottom), $r < 3$ (middle) and all particles (top). Thin lines correspond to the respective models with initially circular discs.

Figure 2.19: Comparison of c/a evolution between the standard models (thick lines: 1C, dotted; 2E, dashed; 3E, solid) and other models (thin lines). Four sets of models are shown: less massive disc (upper left), less concentrated halo (lower left), hotter disc (upper right) and azimuthally randomised disc (lower right).

Figure 2.20: Comparison of halo c/a between models with a rigid disc (dashed lines) and the corresponding models with a live circular disc (solid lines).

Figure 2.21: Halo c/a as a function of b/a for models with halo 2 (left) and with halo 3 (right). Note the different scales. The open circles mark the shapes of the halo at $t = 0$. The other symbols show the halo shape of each model at $t = 800$: standard models (asterisk), larger halo core (filled circle), less massive disc (cross), axisymmetrised disc (square), hot disc (triangle) and rigid disc (diamond).

suffers a small decrease due to disc introduction, in both rigid disc and live disc cases. In the triaxial models, live discs cause a small increase of c/a , but with the rigid discs, there is hardly any change of c/a (Fig. 2.20).

The final value of c/a depends on the model, and the bar-forming models are the ones in which c/a changes the most. The range of variation of c/a is narrower than that of b/a , but clearly there is a correlation between the amounts of b/a and c/a increase (Fig. 2.21).

2.6.2 Formation of boxy/peanut bulges

In simulations with axisymmetric haloes, the discs of strongly barred galaxies show a peanut-shaped structure, when viewed edge-on along the minor axis of the bar. The peanut consists of two prominent humps that swell vertically from the plane of the disc, on both sides (Fig. 2.22). It begins to grow some time after the bar and it becomes significantly stronger after the buckling, when the disc momentarily loses its symmetry with respect to the $z = 0$ plane (Combes & Sanders 1981, Combes et al. 1990, Raha et al. 1991, Athanassoula 2005b; 2008, Martinez-Valpuesta et al. 2006).

Such structures were also observed in the simulations of Berentzen et al. (2006) and Berentzen & Shlosman (2006), as well as in ours, showing that halo triaxiality does not inhibit their formation. We here want to go one step further and check quantitatively the effect of triaxiality on the peanut strength. In order to measure the latter quantity, we follow one of the methods proposed by Martinez-Valpuesta & Athanassoula (2008). We first determine the orientation of the bar and then rotate the disc such that the bar major axis lies in the direction of the x -axis. Considering then the disc particles projected on the xz plane, we measure the dispersion of the z coordinates in successive slices of Δx . This dispersion – denoted h_z , to avoid the symbol normally used for velocity dispersions – is an indicator of the thickness of the peanut as a function of x . When the peanut forms, h_z reaches a maximum at a position $|x|$ which is near but within the end of the bar, while remaining small in the centre (Fig. 2.23). As the peanut becomes stronger, the maximum of h_z increases, and the position of the maximum moves further out.

The evolution of peanut strength as a function of time, in simulations with spherical haloes, has been described by Athanassoula (2008). Fig. 2.24 shows the $h_{z,max}$ as a function of time, for different models. We find that in the triaxial models, the formation of the peanut is delayed with respect to the spherical case. Furthermore, for any given halo triaxiality, the C model forms the peanut sooner than the corresponding E model. This is consistent with the fact that the peanut strength is related to the bar strength (Martinez-Valpuesta & Athanassoula 2008). Since the C models tend to develop stronger bars slightly earlier than the E models, it is expected that they would also grow a strong peanut earlier.

We also measure the skewness S_z of the distribution of vertical coordinates, with respect to $z = 0$, which is a measure of departures from vertical symmetry: high values of S_z correspond to a buckling of the disc. Each sharp increase of the peanut strength $h_{z,max}$ coincides with a buckling episode (not shown here). The time of the first buckling increases with the triaxiality of the model. And a C model buckles before the corresponding E model.

Figure 2.22: Time evolution of the discs of the standard models seen edge-on (and side-on).

Figure 2.23: Dispersions of z along the bar major axis.

Figure 2.24: Strength of the peanut as a function of time.

2.7 Kinematics of the disc-like halo particles

Athanassoula (2007) showed that in strongly barred galaxies, the inner parts of the halo display some mean rotation in the same sense as the disc rotation. This is more important for particles nearer to the equatorial plane and decreases with increasing distance from it, but is always much smaller than the disc rotation. Here we extend this analysis to triaxial haloes and point out some kinematic properties that depend on the initial triaxiality of the haloes.

If we select halo particles in a region around the equatorial plane ($|z| < 0.5$) and measure their tangential velocities v_φ at a given time ($t = 800$) we already notice that there is some rotation, with peak velocities of about $v_\varphi = 0.1$. This definition, however, may include particles which happened to be passing by the equatorial region at $t = 800$, but that are not permanently staying close to it. We, therefore, use two alternative definitions to select these disc-like halo particles: we select the particles that are within $|z| < 0.5$ at $t = 800$, but that have remained inside this cylinder during $800 < t < 900$ (definition 1) or during $600 < t < 1200$ (definition 2). The first definition already removes many of the particles that were not truly rotating, but definition 2 is even more strict.

The radial profiles of tangential velocities are shown in Fig. 2.25, for particles simply within $|z| < 0.5$ at $t = 800$ and also for the two other definitions. Note that the tangential velocities are significantly higher when the more strict requirements are applied to define the region of disc-like particles; they range from $v_\varphi = 0.2$ to 0.6 , depending on the model and the definition. The spherical halo shows more rotation than the triaxial ones and indeed the peak tangential velocities decrease with increasing triaxiality. More precisely, it should be stated that the tangential velocity depends on the *initial* triaxiality of the haloes, because by $t = 800$, these five haloes have approximately the same shapes. And yet their kinematics at that time still depend systematically on the initial shape.

The peak tangential velocities of the five models are shown in the upper panel of Fig. 2.26. In models whose halo was initially more triaxial, there is less rotation. The lower panel of Fig. 2.26 shows the radii at which the tangential velocities peak.

The velocity dispersions of the spherical halo are isotropic. The initially triaxial haloes retain an anisotropy, even if by $t = 800$ they also have become spherical. The left panel of Fig. 2.27 shows the radial profiles of the tangential, radial and vertical velocity dispersions for the $|z| < 0.5$ region of the five haloes. The more triaxial haloes have systematically larger radial dispersions and systematically smaller vertical dispersions (the centre excluded). Such features one would expect to be due to construction of the triaxial haloes, but they are still present at $t = 800$ when all haloes have become roughly spherical. Additionally, it can be noticed that the departures from isotropy increase with initial triaxiality of the model. Again we note that although such anisotropy in the velocity dispersions would be obvious in the initial conditions, it is not evident that it would be retained after the haloes have been circularised and have all reached roughly the same shapes. In the right panel of Fig. 2.27, the velocity dispersions are shown for the two definitions of the region with disc-like kinematics. In this case, as one would expect for particles that are rotating, the vertical velocities are much smaller and the tangential velocities are much higher than the isotropic velocity dispersions of the spherical case (and slightly more so with definition 2 than with

Figure 2.25: Radial profile of tangential velocities, measured at $t = 800$, for the permanently disc-like halo particles, using definitions 1 (solid lines) and 2 (dotted lines), as discussed in the text. The corresponding profiles for the $|z| < 0.5$ region are also shown (dashed lines).

Figure 2.26: Top: Peak tangential velocities, measured at $t = 800$, of the disc-like halo particles using definitions 1 (solid squares) and 2 (open squares). Bottom: Radii of the peak tangential velocities.

definition 1). With these two definitions, there is no systematic dependence of anisotropy on initial shape (except perhaps in the innermost region, where the spherical halo is more isotropic). But generally, using definitions 1 and 2, the disc-like halo particles show the same regime of rotation for all five models, with the anisotropy peaking at about $r = 3$. Note also that the difference between the results from definitions 1 and 2 increases with increasing initial triaxiality. If the entire halo is taken in account, the velocity dispersions are similar to those of the left panel of Fig. 2.27.

In order to quantify the anisotropy, we use an anisotropy parameter β defined as:

$$\beta = 1 - \frac{1}{2} \left(\frac{\sigma_{\varphi}}{\sigma_R} \right)^2. \quad (2.16)$$

In the case of isotropy, β would be 0.5. The value of β , calculated from the whole halo, increases with increasing initial triaxiality, which means that the radial motions are correspondingly more important, even at $t = 800$ (upper panel of Fig. 2.28). If only particles within $|z| < 0.5$ are taken into account, the anisotropy is similar (upper panel of Fig. 2.28). When using the entire halo or the $|z| < 0.5$ particles, β does not have much radial dependence and does not show important changes with time. The lower panel of Fig. 2.28 shows the anisotropy for the disc-like halo particles using definitions 1 (solid symbols) and 2 (open symbols). With these definitions and when the anisotropy is measured at $R = 1$ (circles), there is some isotropy in the very centre, since there β is close to 0.5. When β is measured at its peaks ($R = 3$), the anisotropy is larger and shows more important tangential motions (note that the two panels have very different scales). With the more strict definition 2, the anisotropy is even higher. But with either definition and at any radii, there is not a significant dependence of anisotropy (of the disc-like halo particles) with halo model.

Figure 2.27: Left: velocity dispersions σ_φ (solid lines), σ_R (dashed lines) and σ_z (dotted lines) for the $|z| < 0.5$ region. For comparison, the average velocity dispersion of the spherical case is shown in all panels (dot-dashed line). Right: velocity dispersions for the permanently disc-like halo particles, by definitions 1 (thick lines) and 2 (thin lines), as discussed in the text. The line types are as for the left panels. All panels, both left and right, correspond to $t = 800$.

Figure 2.28: Top: anisotropy parameter for the entire halo (filled squares) and for the $|z| < 0.5$ region (open squares). Bottom: anisotropy parameter with definitions 1 (solid symbols) and 2 (open symbols), measured at $R = 1$ (circles) and at $R = 3$ (triangles). Note that the two panels have very different scales.

Hydrodynamical simulations: the effects of gas

In the present Chapter, we follow the formation and evolution of bars in disc galaxies which contain gas and are located within triaxial dark matter haloes. These results are part of Athanassoula, Machado, & Rodionov (2010).

Here we use haloes of three different shapes and, for each shape, we explore five different gas fractions. This allows us to evaluate the influence of both these parameters in bar formation and in numerous other properties linked to it.

In Sect. 3.1 we give information on the technical aspects of the work. In particular, we describe how the equilibrium initial conditions were derived and what their relevant properties are (Sect. 3.1.1), we describe the code we use, with the corresponding gas physics (Sect. 3.1.2) and we give information on the simulation parameters (Sect. 3.1.3). Results are given in Sections 3.2 to 3.8. Sect. 3.2 gives the evolution of the gas fraction and of the general morphology with time. Sect. 3.3 discusses the time evolution of the bar-related quantities best seen in the disc equatorial plane, such as the axial ratio of the halo on that plane (b/a), the corresponding angular momentum component, and the bar strength, length and pattern speed. Sect. 3.5 gives similar information, but now for quantities obtained from edge-on views, such as the halo minor-to-major axis ratios and the formation of boxy/peanut features. The differences in stellar populations of different ages are presented in Sect. 3.4. The density and temperature profiles of the gas, as well as the region of very low gas density are described in Sect. 3.6. Halo kinematics, in particular that of the rotating disc-like particles are studied in Sect. 3.7. Finally, star formation rates are the subject of Sect. 3.8.

3.1 Simulations with gas

3.1.1 Initial conditions

The basic properties of models discussed in this Chapter are listed in Table 3.1. The first column presents the name of the run, the second one is a number indicating which halo is used and the third and fourth columns show the initial shape of the halo, indicated by the b/a and c/a axis ratios. The fifth column gives the initial gas fraction in the disc component. Finally, the sixth and seventh columns provide the disc and gas particle numbers as described below.

Table 3.1: Parameters of the initial conditions: (i) name of the run, (ii) name of the halo, (iii) halo b/a , (iv) halo c/a , (v) gas fraction, (vi) number of `DISK` particles, and (vii) number of gas particles.

(i) run	(ii) halo	(iii) b/a	(iv) c/a	(v) gas fraction	(vi) N_{DISK}	(vii) N_{gas}
gtr101	1	1.0	1.0	0.0	2×10^5	0
gtr102	2	0.8	0.6	0.0	2×10^5	0
gtr003b	3	0.6	0.4	0.0	2×10^5	0
gtr106	1	1.0	1.0	0.2	1.6×10^5	2×10^5
gtr109	2	0.8	0.6	0.2	1.6×10^5	2×10^5
gtr110	3	0.6	0.4	0.2	1.6×10^5	2×10^5
gtr111	1	1.0	1.0	0.5	1×10^5	5×10^5
gtr114	2	0.8	0.6	0.5	1×10^5	5×10^5
gtr115	3	0.6	0.4	0.5	1×10^5	5×10^5
gtr116	1	1.0	1.0	0.75	5×10^5	7.5×10^5
gtr117	2	0.8	0.6	0.75	5×10^5	7.5×10^5
gtr118	3	0.6	0.4	0.75	5×10^5	7.5×10^5
gtr119	1	1.0	1.0	1.0	0	1×10^6
gtr120	2	0.8	0.6	1.0	0	1×10^6
gtr121	3	0.6	0.4	1.0	0	1×10^6

The initial conditions have been built using the iterative method of Rodionov et al. (2009). Each component is built in the total galactic potential after a series of short constrained iterative steps. Within the triaxial model, the disc acquires an elliptical shape, and so does the gas. During the iterations for the creation of the initial conditions, there is no star formation. The resulting circular velocity curves of the initial conditions are shown in Fig. 3.1. The total rotation curves are essentially the same for all models. The only differences are in the relative contributions of the gas and disc components, as a function of the initial gas fraction.

It should be stressed that these simulations are not directly comparable to those of Chapter 2, even though the initial halo shapes are the same. The haloes were built by somewhat different procedures. In Chapter 2, we first prepared an equilibrium halo model with the desired shape. Then, a disc was gradually grown into that halo, which brings about a certain amount of axisymmetrisation. So the halo potential that the bar feels as it grows is considerably less triaxial than that of the initial halo model. For example, when starting with a halo model with axis ratios $b/a = 0.6$ and $c/a = 0.4$, we obtained, after disc growth, $b/a = 0.8$ and $c/a = 0.45$. In that sense, the bars in Chapter 3 models can be thought of as growing in more triaxial haloes than in Chapter 2. An additional difference between the halo models is that in Chapter 2 the equilibrium haloes were made triaxial by squeezing in both directions; that is, by shifting the particle positions in the y and in the z directions by factors smaller than one. For the Chapter 3 models, we obtained the desired shapes by squeezing the halo in the y and z directions while stretching it in the x direction. This means that the resulting haloes are effectively larger, even if they have the same shapes. This has the advantage of maintaining the central density closer to that of the spherical case.

The haloes in this Chapter have the same Hernquist profile as given by Eq. 2.1, with core radius $\gamma = 1.5$ kpc, cut-off radius $r_c = 30$ kpc and mass of $M_h = 2.5 \times 10^{11} M_\odot$. The discs have the azimuthally averaged exponential profile given by Eq. 2.4, where the scale length is $R_d = 3$ kpc, the scale height is $z_0 = 0.6$ kpc and the total disc mass is always $M_d = 5 \times 10^{10} M_\odot$. We do not impose specific Toomre parameters when creating the discs, but by measuring them from the initial conditions, we find that they typically acquire $Q \sim 1 - 2$.

3.1.2 The code

We use a version of GADGET2 (Springel 2005) that includes gas physics, as explained in Sect. 1.3.2. The dark matter and the stars are followed by N -body particles and gravity is calculated with a TreePM code. The gas is modeled with an improved SPH method, as described in Springel & Hernquist (2002). The code uses sub-grid physics described in Springel & Hernquist (2003); i.e. it employs physically motivated prescriptions for processes (star formation, cloud evaporation due to supernovae and cloud growth due to cooling) that would take place in scales below the spatial resolution of the simulation. In this approach, each SPH particle represents a region of the interstellar medium containing both cold gas clouds and hot ambient gas, the two in pressure equilibrium.

Figure 3.1: Circular velocity curves of the initial conditions ($t = 0$): total (solid lines), halo (dashed lines), disc (dotted lines) and gas (dot-dashed lines). The three columns correspond to the three halo shapes, namely halo 1, halo 2 and halo 3. Each row corresponds to a different gas fraction (calculated as the fraction of gas in the disc component, i.e. the ratio of the gas mass to the gas plus disc mass.)

3.1.3 Numerical miscelanea

In the simulations described in this Chapter, the unit of length is always 1 kpc, the unit of mass is $1 \times 10^{10} M_{\odot}$ and the unit of velocity is 1 km/s. We continued all simulations up to 10 Gyr.

We adopted a softening length of 50 pc for all components, resulting in an energy conservation smaller than 0.5% in the collisionless models. In the models with gas, energy was conserved to within 1%. The timescale of star formation was 2.1 Gyr and the fraction of newly-formed stars assumed to die instantly as supernovae is of 10% (for details on star formation prescriptions, see Springel & Hernquist 2003).

With the GADGET2 code, we used four types of particles: HALO, DISK, GAS and STARS. The DISK particles represent stars and their number remains constant throughout the simulation. But as the simulation evolves, due to star formation, the gas particles give rise to new stars, represented by particles labelled as STARS. This means that both the DISK particles and the STARS particles represent the stars of the galaxy. In a sense, the DISK component may be thought of as representing an older stellar population, whereas the STARS particles represent relatively younger stars.

The halo always has 10^6 particles, such that the mass of each halo particle is $m_{halo} = 2.5 \times 10^5 M_{\odot}$. The total disc mass is always one fifth of the halo mass, but the disc particles are distributed into two components: the GAS particles and the DISK particles. The numbers of each of these components are given in Table 3.1. Since each set of three models has a different fraction of the total disc mass in the form of gas, the numbers of DISK and GAS particles are different. This is built in such a way that the mass of each gas particles is always the same in all initial conditions: $m_{gas} = 5 \times 10^4 M_{\odot}$. Likewise, $m_{DISK} = 2.5 \times 10^5 M_{\odot}$ is the same in all simulations. The DISK and HALO mass resolutions are the same, but the GAS mass has better resolution. Unless otherwise stated, we shall henceforth refer to these components merely as: halo, disc, gas and stars.

The same measuring techniques for the pure N -body simulations adopted in Chapter 2 are employed here. To measure the halo shapes, we first sort out the halo particles with respect to local density and then calculate the axial ratios from the eigenvalues of the inertia tensor. In this way we avoid the bias which would have been introduced had we sorted the particles with respect to distance from the centre.

The bar strength, A_2 , is measured by the normalised $m = 2$ Fourier component, integrated over a certain radial extend, as described in Sect. 2.1.3. Bar lengths are measured by fitting generalised ellipses to the mass distributions of the disc as a function of semi-major axis. The ellipticities are larger within the region of the bar. The end of the bar is determined as the point of steepest decrease of ellipticity.

3.2 Global Evolution

3.2.1 Gas fraction

Figure 3.2 shows the evolution of the gas fraction, which decreases with time, as expected due to star formation. The decrease is quite steep during the first 3 Gyr, when the gas

Figure 3.2: Gas fraction as a function of time.

Table 3.2: Evolution of the fraction of gas in the disc.

time (Gyr)	gas fraction			
0	20%	50%	75%	100%
2	7%	13%	16%	19%
5	4%	6%	8%	9%
10	3%	5%	6%	7%

fraction is still high, and then flattens out, starting to level at roughly 10% or less. We can also observe that the shape of the halo hardly, if at all, influences the total amount of stars formed and, therefore, the total amount of gas left at any time is independent of halo shape.

Table 3.2 presents the fraction of gas in the disc at given times during the simulation for the four different initial gas fractions. These values were obtained as the mean values for all the runs which start at $t = 0$ with the specified value of the initial gas fraction. The initial values cover the range between 0% to 100%, whereas the final range is restricted to 3% to 7%. The initial fractions represent reasonable values for discs-like galaxies at high redshifts, or their progenitors (Hammer et al. 2009), while the final values are more representative of present-day early-type spirals (McGaugh & de Blok 1997).

3.2.2 Face-on and edge-on morphology

Figures 3.3, 3.4 and 3.5 display the face-on and edge-on views of our simulations. The first three columns correspond to the old disc population, the next three to the stars created from the gas during the simulation, and the last three to the gas. The three columns in each set show the simulations with: halo 1, halo 2 and halo 3, respectively. Each row corresponds to one value of the initial gas fraction. From top to bottom these are 0, 20, 50, 75 and 100%. Figure 3.3 gives the face-on view at $t = 6$ Gyr and Fig. 3.4 gives the face-on view at $t = 10$ Gyr. The edge-on views at both times are given in Fig. 3.5.

It should be remembered that these morphologies do not cover all possible shapes occurring in nature, since the basic halo and disc stellar profiles are the same in all models, and the only change is in the halo shape and in the initial gas fraction.

The basic morphology can be roughly described as follows: A bar forms in some cases, surrounded by a more or less clear ring structure, of the same extent as the bar, i.e. it is an inner ring. Judging by eye, the bar is easier to discern in the disc population than in the stars.

The gas has a somewhat different morphology, the most striking difference being the absence of a bar. The gas morphology is reminiscent of what was found in the simulations of Athanassoula (1992). In the central region there is a strong concentration of gas, which we will hereafter call the central mass concentration (CMC). It is surrounded by a large and very low density annulus, whose radius is a function of the simulation parameters and of time. Surrounding this gas void, there is the disc of gas, in which there are clear spiral segments, but no clear-cut two-armed global spirals. Looking carefully, one may, at certain times, discern in the very low density annulus two thin stripes of gas, linking the CMC to the gas disc surrounding the very low density annulus. Their location with respect to the bar, as well as their extent, links them to the gas concentrations in the shocks on the leading edges of the bar, found for example in the purely hydrodynamical simulations of Athanassoula (1992). They are of course much less symmetric and less well outlined than in those simulations, but this should be expected, since the older simulations were simply response simulations of an isothermal gas in a model bar galaxy, and did not include things like feedback. Nevertheless, it is clear that these are the same kinds of features.

In those cases where the CMC is sufficiently extended for us to have adequate information on its shape, we can see that it is elongated and oriented perpendicularly to the low density region. This argues that this low density region is linked to the bar, as was shown already in Athanassoula (1992) and that the CMC is located within the inner Lindblad resonance.

Looking at times $t = 6$ and $t = 10$ Gyr, there is a clear gradient in bar strength from top left to bottom right in both sets of plots. The strongest bars are found for the spherical halo and no gas, and the strength decreases as we go towards more triaxial haloes and larger gas contents. This will be quantified in Sect. 3.3, but it can already be qualitatively seen in Figs. 3.3 and 3.4. Moreover, the extent of the low density region follows a related trend along the same diagonal. Namely, it is largest for spherical haloes and no gas content and decreases as the halo triaxiality decreases and/or as the initial gas fraction increases. Furthermore, these extents are always larger at $t = 10$ than at $t = 6$ Gyr. A more detailed discussion of the features of the low gas density annulus will be presented in Sect. 3.6.

Seen edge-on and side-on (i.e. with the line of sight along the bar minor axis) both the stars and disc population show a clear boxy/peanut feature. At $t = 6$ Gyr two of the simulations happen to be caught in the act of buckling, while at $t = 10$ Gyr the feature is already well formed and is either boxy, or peanut, or 'X'-shaped. One notices that these features are larger and more prominently peanut-shaped in the models with low gas fraction and/or spherical haloes (top left frames, both for the disc and star components). For the models with more triaxial haloes and/or greater gas content, this feature is smaller and more boxy-shaped.

The gas seen edge-on is much thinner than the disc or the stars, and does not depart significantly from the equatorial plane in the form of a box or a peanut. In one of the 20% gas cases, however, there are some peculiar vertical protuberances of the gas. The model in which this happens is the gaseous case with the strongest bar and also with the longest and strongest peanut shape. These two gaseous protuberances do not coincide with the radius at which the stellar peanut has its maximum height: they are located slightly further out, closer to the end of the peanut structure. The length of the peanut feature is shorter than the length of the bar, and so these gaseous protuberances are located near the extremities of the bar, but still within it. The vertical extents of disc, stars and gas particle populations, as wells as other peanut properties, will be discussed quantitatively in Sect. 3.5.

Figure 3.3: Face-on view of disc, stars and gaseous components at $t = 6$ Gyr. The first three columns correspond to the disc component (old stars), the next three to the stars that were formed during the simulation and the last three to the gas. The three columns of each set correspond to simulations with halo 1, halo 2 and halo 3, respectively. Different rows show different initial gas fractions. Disc rotation is counterclockwise and colour represents project density, whose range is the same for panels displaying the same component, and the scales are given in $10^{10} M_{\odot}/kpc^3$ by colour bars in the bottom of the plot. The size of each square frame is 40 kpc.

Figure 3.4: Same as Fig. 3.3, but for $t = 10 \text{ Gyr}$.

Figure 3.5: Edge-on view of disc, stars and gaseous components at $t = 6$ Gyr (top) and $t = 10$ Gyr (bottom). Particles are seen side-on, i.e. along the direction of the bar minor axis.

3.3 Bars and their effect on the evolution I: face-on

Figure 3.6 shows the radial profile of the halo equatorial axial ratio (b/a) at three different times during the simulation, for each model. Simulations with spherical halo (halo 1), or mildly triaxial halo (halo 2) and with initial gas fraction up to 50% have a considerably triaxial innermost region. This triaxial shape forms during the evolution and can also be seen, but to a much lesser degree, in the simulation with the strongly triaxial halo (halo 3) and no initial gas. This feature – the ‘halo bar’ – had been studied in spherical halo simulations (Athanasoula 2005a; 2007, Colín et al. 2006). The other simulations do not have this feature and show the opposite trend of an increase of b/a at small radii. The gas fraction that limits the simulations with a halo bar from those with a rounder centre depends on the shape of the halo and is approximately between 50 and 75% for the less triaxial halo and between 0 and 20% for the strongly triaxial one. Therefore, unless there is a sufficiently strong stellar bar inducing the formation of the elongated halo bar structure, the innermost ($r < 5$ kpc) regions of the triaxial haloes tends to become circular even if the overall halo remains triaxial.

The existence of this feature is clearly linked to the existence of a strong bar in the disc component, since the simulations that do not have it also have no bar or only a very weak one, at best. The dependence of the halo bar development with gas fraction (and therefore with the stellar bar strength) is particularly discernible in the halo 1 case, where the halo is already circular in the beginning and remains so throughout. The only departures from $b/a = 1$ are in the inner region and are due to the halo bar. In the absence of gas, with the strongest stellar bar, the halo bar is most elongated, with b/a as low as 0.75 in the centre. The 100% gas case, whose stellar bar is very weak, has a weak halo bar of $b/a \sim 0.9$ in the centre.

Apart from this inner feature, globally the halo evolves towards axisymmetry. In all triaxial cases, the halo axial ratio b/a increases, i.e. the haloes become more round in the equatorial plane. In Chapter 2 we had shown that this axisymmetrisation has two phases. Part of the axisymmetrisation occurred when the disc was introduced and grown to reach its final mass. The second part of this axisymmetrisation was due to the bar. Due to the different method of producing the initial conditions for the gas simulations, we dispense here with the first part, focusing only on the second phase, so that all the loss of triaxiality should be attributed to the bar. The amount of change in the halo b/a depends on the model. To quantify this point and its relation to the bar strength, we measure the average b/a of each halo (estimated in a radial extent from $r = 10$ to $r = 30$ kpc). This interval excludes the outermost part of the halo, and also the region of the halo bar, thus providing a quantity that represents the overall shape of each halo. We measure the radially averaged b/a for each model at two instants: $t = 5$ and $t = 10$ Gyr. The amount by which this average differs from the initial shape of each halo, $\Delta(b/a) = \langle b/a \rangle - (b/a)_0$, is roughly correlated to the bar strength, as shown in Fig. 3.7. The bars generally grow stronger from $t = 5$ to $t = 10$ Gyr (but the very weak ones, only marginally) and the $\Delta(b/a)$ also increases in that interval. In almost all cases, the following correspondence is present: considering each model at $t = 5$ Gyr (open circles and open squares in Fig. 3.7), their counterparts at $t = 10$ Gyr (filled circles and filled squares respectively) are always upper and to the right, meaning

Figure 3.6: Radial profile of the halo equatorial axial ratio (b/a) for three times during the simulation ($t = 0, 5$ and 10 Gyr) and for all simulations.

Figure 3.7: The increase in the halo equatorial axial ratio b/a as a function of bar strength. Shown here are halo 2 (circles) and halo 3 (squares) models at times $t = 5$ Gyr (open symbols, dashed lines) and $t = 10$ Gyr (filled symbols, solid lines). In nearly all cases, larger A_2 is associated with larger circularisation, i.e. for each open symbol in the dotted lines, its counterpart in the solid lines are upper and more to the right.

that increases in A_2 led to an increase of $\Delta(b/a)$.

Figure 3.8 shows the evolution of several quantities which are derived from the face-on view of the disc, namely the strength, length and pattern speed of the bar, as well as the fraction of total angular momentum held by the disc particles, and the halo axial ratio in the equatorial plane. The data are displayed so as to emphasise the effect of the initial gas fraction. The different line types display the results of different initial gas fractions for each one of the halo shapes. It is immediately clear that this effect is considerable.

The bar strengths presented in the first row of Fig. 3.8 are measured as the relative intensity of the $m = 2$ Fourier component of the density, inside $R = 12$ kpc (see Sect. 3.1.3). The resulting bar strength at the end of the simulations are clearly dependent on the initial gas fraction. All the other parameters being equal, the presence of the gas acts in the sense of inhibiting the formation of strong bars. This same effect holds true for each halo shape. The strongest bars are those belonging to models without gas in each case, and the weakest bars are the ones whose initial disc was composed exclusively of gas. In the cases of haloes 2 and 3, the initial values of A_2 are not strictly zero, reflecting the shapes of their discs, which are already non-circular in the initial conditions. The strongest bars (0% and 20% gas) grow by a steep increase of A_2 , while the weak ones grow more gradually. The weaker bars (50%, 75% and 100% gas) do not show noticeable decreases in A_2 , which, when present, are associated with buckling episodes (e.g. Athanassoula 2008). One peculiarity of the 100% gas cases is that they all suffer a short transient phase in the very beginning of the simulations: their A_2 undergoes an immediate increase at $t = 0$ which subsides in 1 Gyr, then joining approximately the same A_2 values of the other models.

It should be emphasised that the important parameter determining such different evolutions was the *initial* gas fraction, since star formation consumes most of the gas quite early on. Already by $t = 3$ Gyr, and thenceforth, all models have quite low – and quite similar – gas fractions (Sect. 3.2.1). This argues that the importance of gas to the dynamics of

Figure 3.8: Time evolution of several quantities which are derived from the face-on view of the disc. From top to bottom, the rows of plots give the bar strength, the halo axial ratio in the equatorial plane, the fraction of total angular momentum that is in the disc stellar particles, the pattern speed of the bar, and the length of the bar. The three columns correspond to the three different halo models and the different initial gas fractions are shown with lines of different types. In this display one can see easily the effect of the initial gas fraction on the results.

barred galaxies would not be sufficiently represented by considering a fixed amount of gas throughout the evolution. These results indicate that long after most of the initial gas has been consumed, the galaxy will still feel the consequences of this early phase. And that otherwise identical models (same disc-to-halo mass ratio) are quite sensitive to the fraction of the initial disc mass in the form of gas. This suggests that stars that were formed from the gas, and stars that were already present in the beginning behave distinctly. As we shall see in this and later Sections, bar strength will ultimately be correlated to almost all relevant parameters, meaning that the initial gas, coupled with its conversion into stars, is of crucial importance.

One such correlation is the dependence of bar strength with halo circularisation, as discussed above. The second row of panels in Fig. 3.8 shows the halos shapes as a function of time, measured at $r = 12$ kpc with the method mentioned in Sect. 3.1.3. The shapes of haloes 2 and 3 generally become more circular, going from $b/a = 0.8$ to about 0.95 and from $b/a = 0.6$ to 0.75–0.85, respectively, which means that halo 2 became very nearly axisymmetric, while halo 3 was able to remain considerably triaxial. The dependence of b/a with gas fraction is not strictly monotonic, but we may observe that the steep increases of A_2 are always associated with intense periods of circularisation, whereas for the very weak bars, both A_2 and b/a grow slowly and smoothly, in general.

An important dynamical quantity related to bar strength is the angular momentum redistribution. Its different shares in the young and old stellar components, among other parameters will be discussed in Sect. 3.4. Here, in the third row of Fig. 3.8, we plot the fraction of angular momentum held by the entire disc (i.e. disc + stars + gas) as a function of time. In the beginning of the simulation, nearly all of the angular momentum is due to the disc, and the halo has nearly no contribution. All of the angular momentum lost by the disc is necessarily gained by the halo. The general behaviour then is that the disc loses angular momentum, that loss being greater in the models with stronger bars, as one would expect. The dependence of angular momentum loss with initial gas fraction is also present, albeit with some ambiguity in the cases of the weakest bars. But clearly the discs that transfer more than 30% of their angular momentum to the halo are able to develop the strongest bars.

In all our models, bars slow down as they grow stronger. Moreover, at any given time, models with strong bars rotate more slowly than models with weak bars, as is clear from the fourth row of panels in Fig. 3.8. This means that models with more initial gas develop weak slowly rotating bars, whereas models with little or no gas have strong rapidly rotating bars. In no case did the pattern speeds increase with time. In fact, if something, the pattern speed seems to decrease more strongly with time, in cases with a large initial gas fraction than in cases with little or no gas. On the other hand, the initial pattern speed values are much higher for simulations with higher initial gas fraction than for simulations with little or no gas. Therefore, in spite of the faster decrease, the final pattern speed value is still considerably higher. This behaviour is contrary to previous results (Villa-Vargas et al. 2010), where in the cases with strong gas content the pattern speed was found to increase with time. The difference could be due to the different ways of modelling the gas, since here we use sub-grid physics to describe the multiphase medium.

Bar lengths are shown as a function of time in the fifth row of Fig. 3.8. The lengths

of the bars are computed as the radius having the steepest drop in ellipticity. We define the bar length as the distance from the centre of the galaxy to the edge of the bar. Stronger bars tend to be generally longer, but these measurements are independent and it need not be always the case. They become longer with time and their lengths are again systematically correlated to initial gas fraction. The lengths of the strongest bars are of $L_{bar} \sim 15$ kpc (gasless models) and the weakest bars are as short as $L_{bar} \sim 5 - 7$ kpc (100% gas models).

Figure 3.9 gives the same information as Fig. 3.8, but the data is displayed so as to emphasise the effect of halo triaxiality, allowing comparison of the models of different halo shapes, for each gas fraction. The general trend of bar strength as a function of halo shape is that the bars within triaxial haloes tend to be weaker. For the 20%, 50% and 75% gas cases, more triaxial haloes lead to weaker bars. For the gasless case, however, haloes 2 and 3 become quite indistinguishable after a certain time and their bars are equally strong. Likewise, haloes 2 and 3 of the 100% gas case have almost identically weak bars.

We should stress that these results are not directly comparable to those of Chapter 2, because here the bars (even in the gasless cases) were formed within haloes that were still considerably triaxial, whereas the haloes of Chapter 2 had lost part of their triaxiality before the bar began to form. Furthermore, in the present triaxial models, even though the bars are very weak, they do form by actual bar instability (with transfer of angular momentum) and are able to induce circularisation in their halo.

The comparison of b/a increase for different halo shapes in Fig. 3.9 shows that by the end of the simulations halo 3 has become as triaxial as halo 2 was in the beginning. Angular momentum transfer, bar pattern speed and bar length seem to be more importantly influenced by gas fraction than by halo shape. Angular momentum flows from the disc to the halo by approximately the same amounts, independently of halo shape. Pattern speeds decrease at roughly the same rate for each given gas fraction, but independently of halo shape. For the high gas fractions (75% and 100%), however, the weakest bars (triaxial haloes) are always rotating more rapidly than the spherical model's bars. For bars as slowly rotating as those of the 50% gas case, and those slower, the pattern speed is independent of halo shape. Bar lengths L_{bar} follow the same behaviour in the sense that it is independent of halo shape for 50% gas and below. For the 75% and 100% gas cases, bars are marginally shorter in the triaxial haloes. The dependences of these quantities on halo shape are less pronounced and less systematic than the dependences on gas fraction. The best correlated quantity is the strength of the bar, that decreases with triaxiality.

Figure 3.9: This figure shows the same information as Fig. 3.8, but now the data are displayed so as to show best the effect of halo triaxiality.

The gradual importance of the bar along the sequence of models can also be illustrated by the radial profiles of the $m = 2$ Fourier component. In order to obtain A_2 , this quantity is integrated over a certain radius and the normalised by the $m = 0$ component (i.e. the mass) of each models. But the radial profiles of $m = 2$ also provides some visual information, shown in Fig. 3.10, where the different lines show, for all models, three different times during the simulation. By comparing the solid lines ($t = 10$ Gyr) from left to right, as well as from top to bottom, one notices how the area below it generally decreases. The vertical lines in Fig. 3.10 mark the length of the bars at the three different times, with the matching line styles corresponding to the respective $m = 2$ curves at those times. We should notice that, in each case, the agreement of the vertical lines with the first minima of the $m = 2$ lines is quite good, even though these quantities are computed in totally independent ways. The progression of the vertical lines towards the left, when we observe each set from top to bottom, again indicates that the more initial gas we have, the weaker the bars are. In some models, however, the bar is still too weak at $t = 3$ Gyr to allow meaningful measurements. The $m = 2$ in some cases increases in the outer regions of the disc, after the minimum that corresponds to the end of the bar. This is due to a mild overall non-axisymmetry of the disc in the low density outer parts, and occasionally to some spiral structure, which is also a form of non-axisymmetry that can contribute to the $m = 2$ mode. In the 100% gas case, the outer growth of $m = 2$ is particularly large at $t = 3$ Gyr but then decreases.

At the corotation radius (R_{CR}), the pattern speed of the bar, Ω , is equal to v_c/R . Since the circular velocities are similar between different models, the different values of corotation radius are mainly the result of different pattern speeds. The presence of the gas produces rapidly rotating bars, implying that for the same rotation curve we may expect smaller R_{CR} . On the other hand, high gas fractions also give rise to shorter bars. This means that the ratio of corotation radius to bar length, R_{CR}/L_{bar} , may conceivably be the same for all gas fractions, as the effect of gas acts in the same direction on both quantities. Furthermore, bar lengths vary by a factor of 3 (roughly from 5 to 15 kpc), while the range of pattern speeds varies by the same factor, covering essentially the interval of 10 and 30 rad/Gyr. Since both Ω and L_{bar} seem to be affected by similar factors and at roughly uniform intervals, one possible way in which the effect of gas could lead to some dependence in R_{CR}/L_{bar} would be if the pattern speeds were sufficiently high, such that R_{CR} would be quite small and changes in Ω would no longer result in noticeable changes in R_{CR} , since v_c/R is steep in the centre. Then, the ratio R_{CR}/L_{bar} would be smaller for high gas content. Is it however doubtful that such dependence is reliably noticeable in Fig. 3.11. If that were the case, it could be due to the rapidly rotating bars of the high gas fraction models. Nevertheless, we do find that the ratios are in reasonable agreement with (or perhaps slightly above) the prediction $R_{CR}/L_{bar} = 1.2 \pm 0.2$ from gas flow simulations of Athanassoula (1992), as well as hydrodynamical models (Lindblad et al. 1996, Zánmar Sánchez et al. 2008) and also the typical ratios found from observations (e.g. Elmegreen 1996, Aguerri et al. 1998).

Figure 3.10: The relative $m = 2$ Fourier component of the mass as a function of radius at times $t = 3, 6$ and 10 Gyr. The layout, and line styles are the same as in Fig. 3.6. The thin vertical lines give the length of the bar at the three times, as given by the respective line styles.

Figure 3.11: Ratio of the corotation radius to the bar length as a function of time for all models.

Figure 3.12: Radius containing 90% of the disc mass, for each component: old stars (solid lines), young stars (dashed lines), youngest stars (dotted lines), and gas (dot-dashed lines). The evolutions of these quantities are very similar for all models, so this plot displays only one of them (halo 1 and 50% initial gas fraction).

3.4 The bar in the young and the old stellar populations

In this section, we discuss the properties of the bar, and also of the disc, when considering stellar populations of different ages.

We will use three distinct populations, which we will call the old, young and youngest components. The “old stars” are represented by the `disk` component, i.e. the particles that were already present in the beginning of the simulation. The “young stars” are represented by stellar particles (labelled `STARS`) that were created from the gas, during the simulation, due to star formation. They are relatively younger than the disc component, but since most of the star formation takes place early-on, it is also useful to define another set of stellar particles which is even younger. The “youngest stars” are a sub-set of the previous component, but considering only the particles whose stellar ages are smaller than 0.1 Gyr at each given time. The old population is composed of a fixed set of particles that does not change throughout the simulation. The young component is made of newly created particles, the majority of which is already present by $t = 3$ Gyr. The youngest set is composed of very recently created stars. The particles that constitute this third set do not remain the same at each time step, being constantly replaced.

The size of the disc is an overall property that depends on which one of these populations we are looking at. It also depends on time. Essentially, the older components are more extended than the younger. Figure 3.12 show R_{90} (the radius encompassing 90% of each component’s mass) as a function of time, for the old (solid lines), young (dashed lines), and youngest stars (dotted lines). The differences between the models are very small, so we display only one representative example (halo 1 and 50% initial gas fraction). The R_{90} radius of these three components expand slightly over time, by no more than 2 kpc. Also shown is the extent of the gaseous disc (dot-dashed lines), which extends to more than 20 kpc, further out than any of the stellar components. The gas R_{90} radius also shows the largest expansion, while the three stellar R_{90} increase only mildly.

Another quantity that helps in quantifying the disc sizes is their scale length. We measure the azimuthally averaged surface density profiles – for each stellar component as a function of time, for all models – performing a fit of an exponential profile to obtain the scale lengths R_d . The discs are evidently not axisymmetric during the evolution, and we

ignore the inner region in order to obtain the R_d from the outer parts. As seen in Fig. 3.13, the scale lengths generally increase, roughly in accordance with the factor of ~ 1.5 cited by Valenzuela & Klypin (2003). In the old stellar component, R_d increases more importantly in the gasless and in the 20% gas cases. In the higher gas content cases, there is little or no growth of R_d for the old stars. This argues that bar formation is responsible for increasing the scale length, and that it also affects the old population more than the younger ones. The youngest stars (dotted lines in Fig. 3.13) have always the smallest scale lengths. The evolution of the young stars (dashed lines) is not too dissimilar between models, and it is mainly the different behaviour of the old stars (solid lines) between 20% and 50% gas models that causes the chief change of regime. Because the old stars are less affected in the 50% gas model and above, their R_d is overtaken by the young stellar R_d . The fact that strong bars affect the scale length of old stars more than the young ones is probably linked to the different bar strength of these components.

Bar lengths are roughly the same if we look at the populations of different ages, but not bar strengths. In Fig. 3.14 we show the bar strengths measured separately for the old, young and youngest stars. It is clear, even for the weak ones, that bars are always stronger in the young component. This can be understood in the sense that a great deal of star formation takes place within the bar, presumably because the gas is able to reach sufficiently high densities due to shocks. At later times in the simulations, the lack of gas in the bar region is evidence that the gas consumption was more intense within the bar. One expects, therefore, that the newly formed stars are often found in the bar. That is the reason why the bars are strongest when we look at only the youngest stars. Also noteworthy is the fact that the young and the old populations remain distinct throughout the simulation, even though their ages are effectively not so different. By the end of the simulation, the old stars are 10 Gyr old, while the vast majority of the stars in the young component is over 7 Gyr old. Yet, the difference in bar strength in these two populations is quite as important as the difference between the 7 Gyr old. In other words, different dynamics would be expected for the very youngest stars, due to their recent formation. One might have expected the behaviour of the 7 Gyr old stars to be indistinguishable from the oldest population, but the former do carry some consequences of their different origin, in the sense that stars formed from the gas are more likely to be found in the bar than stars that were already present in the initial conditions.

As expected, the angular momentum held by each of the three stellar populations generally decreases with time. The decrease is more pronounced for the strongly barred galaxies. Since the number of particles in the three different populations are quite different, the loss of angular momentum can only be comparable if we normalise them by the mass of each component. This is shown in Fig. 3.15, where we plot the specific angular momentum of each population as a function of time for all models. The general behaviour is that the slope of the loss of angular momentum depends on bar strength (on initial gas fraction), as it decreases from left to right and/or from top to bottom. We can observe that the specific amount of angular momentum held by the younger stars is systematically smaller than that held by older ones. As it forms, the angular momentum of the young stellar component (dashed lines in Fig. 3.15) actually increases momentarily in the early phase of the simulation, during $t < 3$ Gyr.

Figure 3.13: Scale lengths of the fitted exponential surface density profiles for all models as a function of time, for each component: old stars (solid lines), young stars (dashed lines), and youngest (dotted lines).

Figure 3.14: Bar strength calculated separately for three stellar populations of different ages: old stars (solid lines), young stars (dashed lines), and youngest (dotted lines).

Figure 3.15: Specific angular momentum calculated separately for three stellar populations of different ages: old stars (solid lines), young stars (dashed lines), and youngest (dotted lines).

Additional information is provided by looking at the specific angular momentum as a function of radius. Figure 3.16 shows such profiles for the three populations at two different times, $t = 6$ Gyr and $t = 10$ Gyr. Some features are common in both cases. The younger populations generally hold less angular momentum. There is a small amount of old stellar particles (solid lines) at quite large radii who, due to their distance from centre, hold a non-negligible amount of angular momentum. The young and youngest components do not have particles at such large radii contributing to the angular momentum. Notice the vertical lines in Fig. 3.16, that mark the position of the corotation radius measured at $t = 6$ Gyr. It decreases systematically with increasing gas fraction.

By comparing each panel at $t = 6$ Gyr with the respective one at $t = 10$ Gyr, we notice that angular momentum is lost from the inner regions. In the case of the gasless models, for example, the angular momentum that was present within roughly $R < 10$ kpc is considerably reduced, while there is an increase in the outer parts, albeit less noticeable. This is accompanied by (or causes a) shift of the peak to slightly larger radii. The reduction of angular momentum in the inner parts is best seen in Fig. 3.17, which compares the profiles at $t = 6$ Gyr (dashed lines) and $t = 10$ Gyr (solid lines). One notices that (from dashed to solid lines) there is always a decrease of angular momentum in the inner part, and a relatively smaller increase in the outer parts. This is less important in the weakly barred galaxies. It means that the disc loses some of its momentum to the halo, but there is also transfer of angular momentum from the inner to the outer disc. We find that this internal flow is observed in all three stellar populations, as seen in Fig. 3.17.

Figure 3.16: Radial profile of specific angular momentum calculated at $t = 6$ Gyr (left) and at $t = 10$ Gyr (right) separately for three stellar populations of different ages. The vertical lines mark the location of the corotation radius measured at $t = 6$ Gyr.

Figure 3.17: The same data as in Fig. 3.16, comparing $t = 6$ Gyr (dashed lines) and at $t = 10$ Gyr (solid lines), separately for three stellar populations of different ages: old (black lines), young (red lines) and youngest (blue lines) stars. The vertical lines mark the location of the corotation radius measured at $t = 6$ Gyr.

3.5 Bars and their effect on the evolution II: edge-on

Seen side-on (i.e. edge-on and along the direction of the bar minor axis), the discs in the simulations of strongly barred galaxies develop prominent peanut-shaped features. In order to quantify the extent of these features, we measure the dispersion of the vertical coordinates. Since the peanut structure is usually symmetric above and below the disc plane (except at the moments of buckling) and since it is also symmetric with respect to the yz , when we align the bar major axis with the x axis, we reflect all position coordinates to the first quadrant of the xz plane. In order to avoid confusion with the symbol normally reserved for velocity dispersions, we call the dispersion of z position coordinates h_z . Figure 3.18 show h_z as a function of distance from the centre at three times for all models, using all stellar particles (i.e. disc and stars).

One notices that the peanut structure is more prominent for spherical haloes and for low gas contents. The vertical lines in Fig. 3.18 mark the length of this feature. Peanut lengths are determined as the point at which the absolute value of the derivative of h_z with respect to radius reaches a minimum, provided it is after the peak. The different line types show these peanut lengths at the respective times $t = 6$ Gyr and $t = 10$ Gyr, but not at $t = 3$ Gyr, because the peanut is not yet sufficiently formed to be measurable at that time. By the end of the simulation, it is clear how the peanut length depends on the initial gas fraction. The most vertically thick are also the longest ones.

Performing analogous measurements for the gas component (Fig. 3.19), we observe that the gas remains very close to the plane of the disc and does not depart vertically in the form of peanut-shaped bulges. We are however ignoring the extremely hot gas particles that actually escape from the disc (see Sect. 3.6). Apart from that, the only regions of the gaseous disc that show some vertical thickening are the outermost parts, in which we observe some flaring. Even so, the heights reached by the gas are always much smaller than those of the stars; note different scales of Figs. 3.18 and 3.19. The only case of peanut-like structures in the gas is that of halo 1 with 20% gas, in which the gas displays protuberances that are similar to peanuts, whose peaks are located further out than the peaks of the stellar peanuts. Such gaseous protuberances are only seen in this gaseous model with the strongest bar.

In order to quantify the intensity of the peanut structures, both in gas and in stars, we determine its strength simply as the maximum value of h_z at each time. This peanut strength, $h_{z,max}$, is shown in Fig. 3.20. The first three rows of panels show the peanut strength separately for the old, young and youngest stellar populations, respectively (as defined in Sect. 3.4). Focusing first on the old stars (first row), we notice that there are essentially two behaviours: there are those models in which the peanut strength grows by a steep sudden increase, and those in which it grows smoothly. The first class includes all gasless models and the haloes 1 and 2 of the 20% gas case. For these, the steep increase in $h_{z,max}$ is always associated with an important drop of A_2 . This occurs more than once in some models. In the 20% gas case with halo 3, and in all models with higher gas content, A_2 does not show evident drops and the slope of $h_{z,max}$ is always small. The models of the first type are the ones in which the peanut feature is very pronounced and even ‘X’-shaped. As a matter of fact, in the weak bar models, what we have been calling peanut for simplicity

Figure 3.18: Dispersions of z -coordinates (for the stellar component) along the semi-major axis of the bar, at times $t = 3, 6$ and 10 Gyr. The vertical lines mark the respective lengths of the peanuts.

Figure 3.19: This is analogous to Fig. 3.18, but shows the dispersions of z -coordinates for the *gas* component.

is in fact more boxy-shaped. The general evolution of $h_{z,max}$ for the young and youngest stars does not differ greatly from that of the old stars. Except perhaps that, in the youngest stars of the 20% gas case with halo 3, there might be a marginally noticeable jump in $h_{z,max}$ correlated to the early moment of A_2 drop, that was not visible in the old stars.

The fourth row of Fig. 3.20 has the $h_{z,max}$ for the gas component. It shows the in the beginning ($t < 3$ Gyr), while the gas is consumed to form stars, the maximum heights effectively decrease, i.e. the gas disc becomes even thinner. As we shall see in Sect. 3.6, the gas also cools down with time. For the rest of the simulation, the $h_{z,max}$ of the gas remains essentially in the vicinity of 0.4. For the gas, $h_{z,max}$ was calculated ignoring the outer flaring of the disc, since that is a low density region far from the centre and would not represent the overall thickness of the bulk of the gas.

Peanut lengths are shown in Fig. 3.21, separated into the old, young and youngest stellar components. This quantity is sometimes not well defined, when the peanut feature is too weak. The peanut lengths are gradually shorter for increasing gas fractions, but they are not significantly different when we look only at the youngest stars.

In Fig. 3.22 we show the ratio of peanut length to bar length. Peanut lengths are always shorter than bar lengths (Athanasoula 2005b) and the ratio in our simulations is roughly in the range of $L_{peanut}/L_{bar} \sim 0.6 - 0.8$.

The other quantity that derives from the edge-on view of the simulations is the halo vertical flattening, c/a . The minor-to-major axis ratio is computed in the same way as the intermediate-to-major axis ratio was, and it is shown in the fifth row of Fig. 3.20. Vertical changes in the spherical haloes are insignificant. The triaxial haloes become less vertically flattened, but the increases in c/a are smaller than those of b/a . Both act in the direction of bringing the haloes towards sphericity. Halo 2 goes from $c/a = 0.6$ to about 0.7 and halo 3 from $c/a = 0.4$ to 0.5–0.6. Gasless models do suffer the largest c/a increases, but systematic dependence with the other gas fractions is inconclusive.

Figure 3.20: The peanut strength is shown for the stellar component (separated into old, young and youngest stars) and for the gas component. Also shown is the halo vertical axial ratios c/a .

Figure 3.21: Peanut lengths as a function of time, calculated using three stellar populations of different ages.

Figure 3.22: Ratio of peanut length to bar length.

3.6 Density and temperature of the gas

The amount of gas in the disc decreases greatly with time, due to star formation (as discussed in Sect. 3.2.1). Other gas features that change remarkably over time are its radial profiles of temperature and density.

All models start out with an initially exponential gas density profile. As discussed in Sect. 3.2.2, the gas evolves to a morphology quite different from that of the disc, specially in the inner regions. In the very central region there is a mass concentration whose exact size and mass depend on the particular model and evolve over time. Typically, the CMC is about 1 kpc across, vertically flattened and holds roughly $2 \times 10^8 M_{\odot}$ (which amounts to about half of a percent of the total disc mass). It is, nevertheless, the region of highest gas density. Around the CMC, there is an annulus of very low gas density (the ‘gas void’), a few kiloparsecs wide and surrounded by the gas disc, where most of the gas mass actually is.

Figure 3.23 shows the radial profiles of gas density for different times, from $t = 0$ (dot-dashed lines) to $t = 10$ Gyr (solid lines). One notices that the void in the gas soon appears, but at different times for each model. And that the CMC is present in all cases. The general lowering of all profiles is due to the fact that the total amount of gas effectively decreases over time. Most of this decrease takes place before $t = 3$ Gyr (dotted lines in Fig. 3.23) because that is the period of most intense star formation. After that, the density of the overall gas disc does not change much in its outer parts. From then onwards, the most relevant change is the deepening of the absence of gas in the void region. The gas void grows and it moves outwards.

As for the temperatures (Fig. 3.24), their profiles start out from the values that were spontaneously acquired during the iterations to create the initial conditions, to sustain equilibrium with the density profile in question. The overall trend in the temperature evolution is that the gas basically cools over time. There is an inner region, of the size of the CMC, where the temperature remains relatively high. In the region of the gas void, average temperatures are very low merely due to the lack of particles. And in the outer regions, the profiles flatten at temperatures of the order of 10^4 K.

A fraction of the gas particles, between 0.2% and 0.02% of the total disc mass, depending on the model, manages to acquire such high temperatures that they effectively escape from the disc, forming two large cone-shaped distributions of very hot and very low density gas around the disc, that extend as far as 30 kpc in height. We remove this hot escaping gas from the temperature and density profiles by not taking into account gas particles that go beyond a height of $z = 0.5$ kpc and also ignoring those with temperatures $T > 8 \times 10^5$ K.

In order to quantify the properties of the gas void, we measure its size by determining the inner radius and the outer radius (illustrative example in Fig. 3.25). These gas structures are certainly not axisymmetric, but we use azimuthally averaged quantities to obtain reasonable estimates of their sizes. The boundaries between what we call gas void and regions of relatively higher gas density are often ill defined. For example, at certain times, before the formation of the CMC, there is some flow of gas along the direction of the bar. Evidently, the radii of the CMC and of the gas void are only meaningful after these structures are sufficiently formed in order to be measurable. To obtain these estimates at each

Figure 3.23: Density profiles of the gas at $t = 0$ (dot-dashed lines), $t = 3$ (dotted lines), $t = 6$ (dashed lines) and $t = 10$ Gyr (solid lines).

Figure 3.24: Temperature profiles of the gas at $t = 0$ (dot-dashed lines), $t = 3$ (dotted lines), $t = 6$ (dashed lines) and $t = 10$ Gyr (solid lines).

Figure 3.25: Illustrative example of the measurement of the inner and outer radii of the annulus of very low gas density. Shown here are the gas particles on the xy plane (units in kpc) of the 20% and 50% gas models, at $t = 10$ Gyr.

time, we use the radial profile of gas density, from which we locate the radius of minimum density. Then, to the right and to the left of this position, we look for the radii at which the cumulative mass function has its steepest increases. The cumulative mass function is less noisy than the density profile and thus helps avoid local overdensities that might lead to incorrect estimates. The circles in Fig. 3.25 represent the inner and outer radii estimated in this way for two examples, and suggest that this procedure gives reasonable results at times when the void and the CMC are well formed and nearly circular. Another useful property to quantify the intensity of the lack of gas is the ‘gas deficiency’, defined as the difference between the areas below the density profile curves, calculated at at given time t and at $t = 0$. This gas deficiency is normalised to the initial gas fraction of each model and thus allow for comparisons between the different models.

Figure 3.26 shows several properties of the gas void, for all models, as a function of time. These properties only become measurable after about $t = 4$ Gyr and even then they are not all well defined. The inner radius of the void (i.e. the radius of the CMC) is of the order of 1 kpc for most cases (first row of panels in Fig. 3.26). The outer radius increases constantly for all models and has a very systematic dependence on initial gas fraction. Models with higher initial gas fraction have always smaller outer radii (second row). This is also noticeable in the third row of panels, that shows the difference between outer and inner radii: the width of the void is larger in models with less gas. By the end of the simulation,

the width of the annulus ranges from about 4 to 8 kpc. Comparing halo shapes, the spherical ones usually have wider annuli than the triaxial ones. This indicates that the bar strength is ultimately what determines the size of the void. The other quantity that suggests this correlation is the ratio of outer radius to bar length, which is very nearly constant with time (indicating that the two quantities grow together) and has the same value for almost all of the models, approximately $R_{out}/L_{bar} \sim 0.6 - 0.7$ (fourth row). The gas deficiency (fifth row) tends to increase with time, without any strongly systematic dependence on models. For the majority of models, the mass of the CMC (sixth row) seems to reach approximately the same values by the end of the simulation. Furthermore – as least in the periods during which it is detached from the outer gaseous disc – the CMC seems to not be increasing its mass, suggesting it is not being fed by inflows of gas. Evidently, the methods of measuring the radii of the empty annulus introduce the bias that there are only measurements after the radial flows of gas have ceased.

Figure 3.26: Properties of the very low density annulus (the ‘gas void’), from top to bottom: inner radius, outer radius, difference between the previous two radii, ratio of outer radius to bar length, gas deficiency (see text), and mass of the CMC.

3.7 Kinematics of the haloes

In this section, we perform a study of the halo kinematics and, in particular, the kinematics of the disc-like halo particles. This analysis is analogous to that of Sect. 2.7, but extended now to the simulations with gas.

In the models whose discs have a gaseous component, we also observe that there are halo particles, in the vicinity of the disc, that show some mean rotation, in the same sense of the disc rotation. The rotation of the disc-like halo particles decreases with height, and is always smaller than the rotation of the disc itself.

First we select, at $t = 10$ Gyr, the halo particles located within $|z| < 2$ kpc. Even with this criterion, a small amount of rotation is already noticeable. The profiles of tangential velocities (dashed lines in Fig. 3.27) show that these particles reach as much as $v_\phi = 20$ km/s in the cases where the rotation is more pronounced. One also notices from these profiles that the rotation in the disc-like halo particles is more important in the spherical model with no gas, which has the strongest bar. As with several other features, here again we observe the correlation of this quantity with bar strength. In Fig. 3.27, the peak tangential velocity tends to decrease with increasing halo triaxiality and it decreases with increasing initial gas fraction.

Not all the halo particles located within $|z| < 2$ kpc at $t = 10$ Gyr are necessarily participating in the rotation. Some particles might have quite different orbits, and be merely passing through that region at that given instant. The other criterion we employ is to select particles that remain within the $|z| < 2$ kpc cylinder during the time interval $7 < t < 10$ Gyr and do not leave it. These sets of particles have higher average tangential velocities (solid lines in Fig. 3.27), ranging from about $v_\phi = 40$ to $v_\phi = 80$ km/s. The overall direction of the decrease of rotation is again along the diagonal that goes from top left to bottom right. In order to better illustrate this correlation, we plot in Fig. 3.28 the peak tangential velocities versus the bar strengths of each model. It is clear that models with stronger bars have higher peak rotation. In this sense, if the gas fraction is capable of influencing the dynamics of the disc-like halo particles, it probably occurs through the strength of the bar.

The velocity dispersions of the disc-like halo particles also provide some information about the effects of halo shape and/or gas fraction on the kinematics of the halo. Figure 3.29 shows the velocity dispersions separated into the radial, tangential and vertical components, for the halo particles within $|z| < 2$ kpc at $t = 10$ Gyr. In the case of the spherical halo (halo 1) these three nearly coincide. In the triaxial haloes, the anisotropy increases, in agreement with the results of Sect. 2.7. But as a matter of fact, here the anisotropies of the triaxial haloes were expected, because halo 2 and halo 3 models are indeed triaxial by the end of the simulations, whereas the (initially triaxial) haloes of Sect. 2.7 had become axisymmetrised. Another result from Fig. 3.29 is that the gas fraction has very little, if any, influence on the anisotropy of the velocity dispersions.

When considering the tangential velocities, the particles obtained from the $|z| < 2$ kpc region at $t = 10$ Gyr already provide some measurable rotation, albeit small. When dealing with the velocity dispersions, however, this information is lost (as was also seen in Sect. 2.7), and thus the curves in Fig. 3.29 are in fact quite similar to those of the halo taken as a whole. One feature that indicates that this set of particles is representative of the

Figure 3.27: Tangential velocity profiles of the halo disc-like particles. Dashed lines show the halo particles located within $|z| < 2$ kpc at $t = 10$ Gyr. Solid lines show the halo particles that remain within that region during the interval $7 < t < 10$ Gyr.

Figure 3.28: Correlation between bar strengths and the peak tangential velocities of the halo disc-like particles. Symbols represent different halo shapes: halo 1 (circles), halo 2 (squares) and halo 3 (triangles).

overall halo is the fact that the radial component of the velocity dispersions is always the largest one in the triaxial haloes, due to the more elongated orbits.

In order to better understand the kinematics of the disc-like halo particles, as opposed to that of the entire halo, it is necessary to use the second criterion (of the particles in $|z| < 2$ kpc during $7 < t < 10$ Gyr), whose velocity dispersions are shown in Fig. 3.30. When only these particles are considered, the anisotropies are much more evident. Since we are dealing with a set of particles with important rotation, the vertical velocity dispersions are always small (dotted lines in Fig. 3.30), while the tangential component is always larger than the radial one (solid and dashed lines in Fig. 3.30, respectively).

For the overall halo (or the merely $|z| < 2$ kpc particles) there is no much radial dependence apart from the fact that all dispersions are higher in the centre. With the particles of the second definition, the radial dependence is more complicated, making it less clear to notice the differences by eye. We therefore show more quantitative comparisons of the anisotropies in Fig. 3.31, where we use an anisotropy parameter as defined by Eq. 2.16 ($\beta = 0.5$ means isotropy; $\beta < 0.5$ means a predominance of rotation; $\beta > 0.5$ means a predominance of radial motions). Summarized in Fig. 3.31 are the anisotropy parameters of the halo particles as a function of gas fraction, where the symbols represent the halo shapes: halo 1 (circles), halo 2 (squares) and halo 3 (triangles). There are three sets of halo particles considered. The first two are: the entire halo and the $|z| < 2$ kpc particles at $t = 10$ Gyr. One notices that these two cases are similar to one another and that there is no dependence on gas fraction. The spherical halo is always near isotropy and the triaxial haloes are progressively anisotropic towards the region where radial motions are predominant. This indicates that β for these two sets of particles is measuring the anisotropy that is due to the overall halo shape. The other set of particles provides more information on the rotation. For the particles in $|z| < 2$ kpc during $7 < t < 10$ Gyr, we plot β as measured at an outer radius $R = 12$ kpc (which is typically the size of the strongest bars). These disc-like particles of the spherical halo are further away from isotropy, in the region of rotation predominance, while those of the triaxial haloes are progressively closer to $\beta = 0.5$. De-

Figure 3.29: Radial, tangential and vertical components of the velocity dispersions of the halo disc-like particles located within $|z| < 2$ kpc at $t = 10$ Gyr.

Figure 3.30: Radial, tangential and vertical components of the velocity dispersions of the halo disc-like particles that remain within $|z| < 2$ kpc during the interval $7 < t < 10$ Gyr.

Figure 3.31: Anisotropy parameter as a function of the initial gas fraction of the models. From top to bottom: all halo particles, halo particles within $|z| < 2$ kpc at $t = 10$ Gyr, halo particles within $|z| < 2$ kpc during $7 < t < 10$ Gyr (measured at $R = 12$ kpc). Symbols represent different halo shapes: halo 1 (circles), halo 2 (squares) and halo 3 (triangles). Note the different scales.

pendence on gas fraction is not sufficiently systematic to allow for reliable interpretation. These anisotropy results, again, are to be understood in the context that rotation of the halo particles is more pronounced in the models with stronger bars. That is, initial gas fraction does not seem to play a particularly direct role in determining the rotation of the disc-like halo particles, or their anisotropy, except in the sense that high initial gas fractions tend to inhibit the formation of very strong bars, and that weak bars are associated with less important halo disc-like rotation.

3.8 Star formation rates

3.8.1 Schmidt-Kennicutt Law: globally

In order to check the Schmidt-Kennicutt law globally, we measure the time-averaged star formation rate (SFR) for each model and also its gas surface density (Σ_{gas}) averaged over time and over the entire disc (out to R_{90}). The result is shown in Fig. 3.32, where we plot SFR versus Σ_{gas} for each one of the gaseous models, and the slope is of approximately $n = 2.3$. Haloes 1, 2 and 3 are represented by the different symbols. The models are grouped in four sets that are evidently the four gas fractions (20, 50, 75 and 100%), since the time-averaged gas surface density is always higher for the larger initial gas fraction models. We note that for each set of gas fraction models, the triaxial haloes of that set seem to show progressively higher star formation rates.

3.8.2 Schmidt-Kennicutt Law: locally

Much information is lost by averaging the SFR over time, since it is quite high during the first few gigayears of the simulation and then very low for the remainder of the evolution. The same is true for gas surface density, as it decreases with distance from the centre. We therefore also evaluate the Schmidt-Kennicutt law locally, for each galaxy. In order to achieve this, we measure the azimuthally averaged Σ_{gas} in successive concentric rings. This is done for three different times: $t = 1$, $t = 6$ and $t = 10$ Gyr. At each given time, the SFR was estimated in the following way. We measure the mass of the youngest stars (age smaller than 0.1 Gyr) within a given ring. This quantity, divided by the area of the ring and by 0.1 Gyr provides a rough estimator of the instantaneous rate of star formation, in mass of formed stars per unit time per unit area, at the given time. Of course, the gas present at a given time is not the gas that actually gave rise to those particular stars, but since the radial distributions of gas and stars do not change dramatically between, say, 5.9 and 6 Gyr it is reasonable to use the gas present at $t = 6$ as a proxy for the gas that – 0.1 Gyr earlier – gave rise to the youngest stars at that radius. Within the region of the bar, however, this argument might not hold, because of the radial motions within the bar.

At $t = 1$ Gyr there are no important bars present in any models yet. Star formation is operating at that time and as one would expect: regions of high density are forming more stars and the relations between SFR and Σ_{gas} are very nearly the same for all models. This is shown in Fig. 3.33. The values of Σ_{gas} are larger than the range of Fig. 3.32 because at $t = 1$ Gyr all galaxies still have large gas content, whereas the time-averaged densities

are much smaller. The points of high gas density and high SFR correspond to the inner regions of the disc; the points lower and to the left correspond to the outskirts of the disc. The general relations are in rough agreement with other simulation results (Schaye & Dalla Vecchia 2008; e.g.), including the thresholds of star formation. The slopes of the Schmidt-Kennicutt relation are larger locally (Fig. 3.33) than for the global relation (Fig. 3.32). For the 20% gas cases they are in the vicinity of $n = 2.7$, while for the models with larger gas content it is of about $n = 3.1$. Figure 3.34 would suggest that larger initial gas fraction is related to a steeper Schmidt-Kennicutt slope (measured at $t = 1$ Gyr) and that the shape of the halo has no relevant effect on it.

The presence of the bar introduces complications at later times (Fig. 3.32). Within the region of the bar, gas density is very low, as described in Sect. 3.6. Furthermore, most of the youngest stars are to be found in the stellar bar. These two facts combined explain the blue and green points in Fig. 3.35 that appear to have been shifted to the left from the expected relation. Within the bar, SFR is very important, but (as a consequence) Σ_{gas} has become extremely low. Above a certain SFR, the points no longer follow the relation. Note however that the uppermost and rightmost blue and green points are back on the normal relation: they are the CMC, the innermost region in which gas density is sufficiently high that the relation can be reliably measured. Presumably this difficulty is due to the radial flows of gas and stars within the bar, associated with the measurement of density and SFR within concentric circular annuli. In the outer parts of the disc, the newly formed stars do not immediately migrate radially away from their place of birth. That is, in the outskirts, both the recent stars and their originating gas are orbiting at approximately the same distance from the centre of the galaxy. Therefore, the gas currently present in the location of the new stars is a reasonable tracer. Within the bar, however, gas flow is predominantly radial, and so is the motion of stars. This means that new stars would not necessarily be found at the same annuli as the gas from which they sprang. In other words, within the bar, the current gas at a given radius is not a reasonable tracer of the gas that effectively gave rise to the stars at that radius. One would need to follow the actual gas that gave rise to the current young stars, and such gas would have been elsewhere earlier on.

For clarity, we simply ignore the complicated region of the bar and measure these quantities only in the outer parts, where there is no void in the gas and the stellar component is roughly axisymmetric. This restricts us the low densities, where the relations remains linear for at least some ranges (Fig. 3.36).

Figure 3.32: Global Schmidt-Kennicutt law: star formation rate versus gas surface density. Both quantities are averaged over the disc (out to R_{90}) and time-averaged over the entire evolution. Symbols represent different halo shapes: halo 1 (circles), halo 2 (squares) and halo 3 (triangles).

Figure 3.33: Local Schmidt-Kennicutt law at $t = 1$ Gyr.

Figure 3.34: Slopes of the Schmidt-Kennicutt law at $t = 1$ Gyr, as a function of initial gas fraction. For each gas fraction, we plot the average of the different halo shape models, whose range of values is represented by the vertical lines.

Figure 3.35: Local Schmidt-Kennicutt law at $t = 1$ Gyr (red squares), $t = 6$ Gyr (green circles) and $t = 10$ Gyr (blue triangles).

Figure 3.36: Local Schmidt-Kennicutt law at $t = 6$ Gyr (green circles) and $t = 10$ Gyr (blue triangles).

Summary and Outlook

4.1 Summary and discussion of collisionless simulations

Cosmological N -body simulations have shown that dark matter haloes of galaxies should be triaxial, at least in cases where there are no baryons (Allgood et al. 2006). This can be the result of asymmetric mergings, or of a radial orbit instability (Bellovary et al. 2008), coupled to tidal effects from other galaxies or from groups and clusters. The resulting prolate halo has very little or no figure rotation.

We investigated how such a halo will influence bar formation and, more generally, how it will influence the secular evolution of disc galaxies. Our combined disc and halo initial conditions were built so as to be as near equilibrium as possible, with discs which are initially elliptical. We have, nevertheless, also considered initially circular discs, to test what the effect of such less realistic initial conditions would be.

The growth and evolution of such discs drive the haloes rapidly towards axisymmetry, except for the innermost parts, where the final shape of the halo is elongated. This latter effect is independent of the initial halo triaxiality and is found also in initially circular haloes (e.g. Athanassoula 2005a; 2007, Colín et al. 2006). It is linked to the angular momentum exchange within the galaxy and the formation of a ‘halo bar’, which is shorter and less elongated than the disc bar, but rotates with the same pattern speed.

This innermost prolate elongation put aside, the remaining halo tends towards axisymmetry even for models with considerable initial triaxiality, and this from the moment the disc starts growing. One can distinguish two different axisymmetrisation phases. Initially, while the disc grows, this trend towards halo axisymmetry is quite rapid. The second phase depends on whether or not a bar is formed: in the presence of a bar, the circularisation continues.

The disc shape also changes with time. Initially it is elongated, its ellipticity depending on that of the halo. Thus the A_2 is non-zero at $t = 0$, although there is no bar. It decreases with time and reaches a minimum around the time that the disc has reached its maximum mass. After that, the bar starts forming and induces a further increase of halo axisymmetry. In our models, bar formation takes place in the presence of haloes that are still considerably triaxial, even though the triaxiality is later erased. In the more triaxial cases, bar formation is somewhat delayed, but the subsequent evolution of the bar proceeds in a manner similar to the spherical halo case. We have also presented triaxial models that do not develop bars at all; but in such cases, the equivalent spherical ones do not either. The general agreement of our results with previous studies is in the sense that a truly triaxial halo cannot coexist with a strong bar for very long: one of these non-axisymmetries must give way. We have

presented simulations in which the bar prevails and the halo triaxiality yields. This argues that in situations where the parameters are such that a bar is known to form in the spherical case, it would also form in the (initially) triaxial cases, further erasing the triaxiality as it does.

Using circular discs, instead of very near-equilibrium elliptical ones, may give rise to quite different A_2 evolutions, particularly if the triaxiality of the halo is important. In the case of a very triaxial halo containing a circular disc, the A_2 initially increases very abruptly to reach a strong peak and then decays. That was the only simulation in which halo triaxiality damped bar formation, but we stress that this is only when a circular disc was used. In that very triaxial halo, a circular disc is even further from equilibrium than in the less triaxial ones. There are other situations in which A_2 grows very abruptly in the very beginning and then decays without forming a bar, and this only happens if we use circular discs as initial conditions. For instance, in the case of low mass discs, the initial A_2 increase is due to the circular disc becoming excessively distorted. After the peak, the remaining non-zero A_2 is due to a mild oval distortion in the centre of the disc (and also some vague spiral structure), but none of this amounts to actual bar formation (furthermore, there is no exchange of angular momentum between disc and halo).

Such behaviour of A_2 might be analogous to those of Berentzen et al. (2006), who also have initially circular discs and find that the bars in their live triaxial haloes dissolve after a few Gyr (except in the case of their more cuspy halo, which does not retain its triaxiality). Similarly, Berentzen & Shlosman (2006), also with initially circular discs, find that bar formation is damped by the triaxiality of the halo. In their models, strong bars form in discs that have erased the halo prolateness almost completely.

In all our models, as in previously run models with initially spherical haloes, bar formation is always necessarily accompanied by angular momentum exchange; more specifically, angular momentum is transferred from the disc to the halo. In the beginning of the simulations, practically all the angular momentum is in the disc. In all models where there is bar formation, the disc-to-total ratio of angular momentum decreases steadily. Further properties of the angular momentum exchange are similar to those found in spherical haloes; e.g. it is related to the bar strength; strong bars causing more transfer from disc to halo. Also, in haloes with large core, the bars are weaker and the angular momentum lost by the disc is correspondingly small. There are simulations where the A_2 is non-zero due to oval distortions of the disc which, however, do not qualify as real bars. In such cases, there is very little, if any, angular momentum exchange between disc and halo. We also find differences in angular momentum transfer depending on whether we use circular or elliptical discs as initial conditions. The initially circular discs become excessively elongated during the very short phase of disc growth and they thus develop strong bars faster than the equivalent model with an elliptical disc. These discs, that host stronger bars, lose angular momentum faster. The other quantity that is also closely related to angular momentum transfer and bar strength is the shape of the halo: a stronger bar causes the halo to become axisymmetric faster.

Our standard models have haloes with a small core, of the type called MH in Athanassoula & Misiriotis (2002). Since the core radius is an essential feature in determining the evolution of the galaxy (Athanassoula & Misiriotis 2002, Athanassoula 2003), we also

ran simulations with large cores, i.e. of the MD type. Here the effect of the disc is even stronger. In all models, by the time the disc was fully grown, the halo has already become nearly axisymmetric, so that any further evolution follows closely that of the MD models of Athanassoula & Misiriotis (2002), namely, compared to MH models, the bar is less strong and there is less angular momentum exchange between the disc and the halo.

We also presented models in which the growth time of the disc was either much smaller, or much bigger than that of the standard models. We found that this does not affect much the results, limiting itself mostly to a temporal shift of the evolution, backwards or forwards in time, respectively. This is in good agreement with similar tests made by Berentzen & Shlosman (2006), albeit with different models.

A crucial question in this context is how much of the loss of the halo triaxiality is due to the introduction of the disc and how much to the formation of the halo. In the case of MD discs, i.e. of discs which are heavy relative to the halo in the inner parts of the galaxy, we saw that by the time the disc has reached its full mass, the halo has become nearly spherical. It is thus reasonable to expect that low mass discs would have a smaller effect on the halo shape, as already found in the simulations of Berentzen & Shlosman (2006). We repeated such simulations here, but with initially elliptical discs. In these cases, our disc has sufficiently low mass so that very little angular momentum can be emitted from its inner regions. Thus although the halo was ready to receive angular momentum, not much exchange was possible for lack of sufficient emitters. As a result, the bar was exceedingly weak, as in the very low mass discs of Athanassoula (2003), and so brought no effect on the halo triaxiality. Thus in these cases the slight decrease of the halo ellipticity when the disc is introduced, is not followed by further decrease caused by the bar. We point out that our corresponding spherical halo model with low mass disc does not develop a bar either, showing that it was not the triaxiality of the other models that was responsible for inhibiting bar formation. In fact, in none of our sets of models with initially near-equilibrium discs, do we find cases where a bar develops in the spherical halo, while failing to do so in the equivalent triaxial ones. Such models, with discs of low mass relative to the halo, give us useful information on galaxies with low surface brightness discs, and argue that such galaxies should not have suffered much loss of halo triaxiality, and that their halo shape should be near what it was from galaxy formation and as due to effects of interactions and mergings.

We made further simulations to find what fraction of the triaxiality loss can be attributed to the disc introduction and what to the formation and evolution of the bar. To accomplish this point we ran models where the disc was kept artificially axisymmetric, models where the disc was very hot (so that bar grew very slowly and was more of an oval than a strong bar) and models where the disc was rigid. In these three cases (as well as in the low mass case), the simulations were designed so as not to form bars. In all cases where there is no bar, the halo is able to remain triaxial until the end of the simulation. Any circularisation that takes place is restricted to the period of disc growth. This circularisation is particularly small if the disc mass is small or if the disc is rigid. The rigid discs – where the potential of a (permanently circular) disc is represented by an analytic potential instead of live particles – are the ones that cause the smallest effects on their haloes, suggesting that it is not merely the presence of disc mass that alters the halo shape, but also the active response of the disc

orbits that oppose the halo elongation.

We find that the vertical shape of the triaxial haloes is also affected both by disc growth and by bar formation, in the sense that haloes become less flattened with time. These changes are, however, always small compared to the changes in the equatorial shape. Nevertheless, in cases where the b/a increase is more pronounced, that of c/a is important also. The general evolution of vertical shapes is thus analogous to that of the halo shape on the plane of the disc: disc introduction causes some degree of change and the subsequent formation of a bar determines whether or not there will be further loss of flattening. But even in models where a strong bar forms (and the halo is thus completely axisymmetrised), the vertical flattening remains considerable, meaning that the haloes have become approximately oblate by the end of the simulation.

As the bar forms, the vertical structure of the disc is also affected. As with previous simulations of axisymmetric galaxies, the discs in the triaxial models also undergo buckling episodes and peanut formation. In our initially triaxial models that form strong bars, the first buckling of the disc occurs later than in the spherical case. Consequently, peanut formation is delayed in the triaxial models. The models with initially circular discs develop stronger peanuts before the corresponding models with elliptical discs. All this is in agreement with the fact that peanut strength is related to bar strength (Martinez-Valpuesta & Athanassoula 2008), and the fact that bars grow faster in initially circular discs.

The spherical halo particles in a layer near the equatorial plane acquire during the evolution some rotation in the same sense as the disc rotation, as in Athanassoula (2007). We extended that analysis to triaxial haloes. We select halo particles that are permanently in the vicinity of the $z = 0$ plane; by analysing their radial profiles of tangential velocities, we find that these ‘disc-like’ halo particles show considerable rotation, with velocities of the order of half of that of the disc particles. Such rotation is present as well in the triaxial models and the peak tangential velocities depend systematically on the initial triaxiality of the model. There is more rotation in the spherical halo than in the triaxial ones, even though their shapes are roughly the same by the end of the simulation. Apart from the rotation, there is another kinematic feature which is dependent on the initial triaxiality: the anisotropy of the velocity dispersions. If we consider the entire haloes, we see that the in spherical one the velocities are isotropic, but the initially triaxial models retain their anisotropy even after their shapes have become approximately spherical. We find that the velocity anisotropy at the end of the simulation depends systematically on the *initial* triaxiality.

To summarise, we have presented simulations of bar formation in triaxial haloes and showed that the haloes become more axisymmetric due to two separate factors: disc growth and bar formation. Typically half the circularisation can be attributed to the introduction of the disc and the other half to the formation of a strong bar. Halo vertical flattening is also affected, but to a much lesser degree, meaning that haloes become roughly oblate by the end of the simulation. This is the first study of bar formation and disc bar evolution in which live elliptical discs within live triaxial haloes were employed as the initial conditions for N -body simulations. Elliptical discs have the advantage of being closer to equilibrium and of not responding to the presence of the aspherical halo potential by becoming excessively and unphysically distorted, as circular discs tend to do. We have also analysed the kinematics

of the halo and pointed out that even after the haloes lose their triaxiality, they are able to retain the anisotropy of their velocity dispersions. We also note that the disc-like particles of the halo rotate less in the haloes that were initially triaxial.

4.2 Summary and discussion of hydrodynamical simulations

Numerical simulations of barred galaxies have often been studied in the absence of the gas component. Even if the fraction of disc mass in the form of gas is relatively small in present-day galaxies, larger gas content was available in the past. Furthermore, the dynamics of the collisional gas component is qualitatively different from that of the stars. Gas can respond more strongly to gravitational perturbation. It can also cool down and form stars. In order to improve on the results of our collisionless simulations of bars in triaxial haloes, we also performed hydrodynamical simulations that include star formation. This allowed us to explore the joint effects of these two parameters: initial halo shape and initial gas fraction in the disc.

We studied models in which the total disc-to-halo mass ratio is always the same, but with different initial fractions of disc mass in the form of gas and different halo shapes. We find that regardless of the initial amount, most of the gas is consumed relatively fast, leaving all models with roughly 5% of gas in the disc after 10 Gyr. In spite of the fact that all discs have the same total mass and approximately the same gas content in the end of the simulation, their evolutions are quite different, as evidenced by the several properties we measured over time. This suggests that the dynamics of the relatively younger stars (formed from the gas) differs from that of the old stars. It also indicates that the amount of gas present in the early phases of galaxy evolution plays a very important role in determining the future properties of the disc.

The most remarkable difference among the models, and perhaps the one that ultimately causes all others, is the strength of the bars. The strongest bar was found in the model with spherical halo and no gas; the weakest one in the very triaxial halo with 100% initial gas. We found that bar strength depends on both quantities: larger halo triaxiality caused weaker bars, and larger initial gas fraction also caused weak bars. The latter effect is more relevant than the former. Even though strong bars are usually also long bars, this need not be always the case. We found that bar lengths in our models are generally quite sensitive to gas fraction, but barely vary with halo shape. For all models, the corotation radius is located at approximately 1.4 times the length of the bar.

Another relevant property – which is more dependent on gas fraction than on halo shape – is the pattern speed. In our simulations, a strong bar always rotates more slowly than a weak bar. Looking at one given simulation as a function of time, the bars always slow down as they grow stronger. Comparing different simulations, if a strong bar forms, its rotation starts at a low pattern speed; if a weak bar forms, the rotation begins at high pattern speed. In either case, rotation subsequently decreases. In none of our simulations did we witness bar pattern speeds increasing.

An important way in which the dynamics of the younger stellar population differs from that of the old is that young stars hold proportionally less angular momentum. Both the

young and old stellar components lose angular momentum to the halo; it also flows from the inner to the outer disc. But the bar is stronger if we look only at the recently formed stars than if we consider all stars. This means that the young stars are mostly formed in or near the bar. The old stellar population also extends to further radii than the young stars, but not as far as the gaseous component.

When viewed edge-on, the discs of strongly barred galaxies present a distinct peanut-shaped feature. The peanut is strongest and longest in the case of the spherical halo with no gas. In models of higher gas fraction and/or higher halo triaxiality, this feature is much less pronounced: it is shorter, vertically thinner, and its shape is more boxy. The bar is longer than the peanut and we find that for most of our models, the ratio of peanut to bar length is approximately 0.7 or slightly higher for gasless models. In the strongly barred models, the formation of the peanut takes place by a steep sudden growth of the vertical protuberances. In the weakly barred ones (high gas fraction), the vertical features grow more slowly.

The distribution of gas in the disc is influenced by the strength of the bar. In the centre of the discs a very small high density central mass concentration of gas develops. In models with low initial gas fraction and/or low halo triaxiality, a considerable annulus of very low gas density exists around this CMC. This gas void is more important for strongly barred models. Its outer radius is almost always at $0.7 L_{bar}$, approximately.

In the hydrodynamical simulations, we did not study the halo circularisation during the period of disc growth, but we found that the formation of the bar did cause the triaxial haloes to lose part of their triaxiality. The mildly triaxial haloes became virtually axisymmetric on the equatorial plane and the most triaxial haloes suffered a certain amount of circularisation, but were still considerably triaxial by the end of the simulation. Moreover, the bars that developed within the triaxial haloes were generally weaker than those of the spherical models. The inner part of the haloes developed the so-called halo-bar when the stellar bar is sufficiently strong. This means that the halo bar is absent if the initial gas fraction is high, depending on initial halo shape. In models where the halo bar does not form, the innermost part of the halo becomes effectively more spherical, even when the overall halo shape is still triaxial. The vertical shapes of haloes undergo a thickening that is smaller than the one that operates on the equatorial plane, but both act to bring the halo towards sphericity.

The rotation of the disc-like halo particles is also present in the simulations with gas. The dependence of this rotation on bar strength is evidenced by the clear correlation of the peak tangential velocities of each model with A_2 . The rotation is more important in the spherical halo than in the triaxial ones. The velocity dispersions of the triaxial haloes remain anisotropic by the end of the simulations. This anisotropy is a function of halo shape but it is quite independent of gas fraction.

4.3 Outlook and perspectives

Interesting issues remain to be explored, since simulations such as the ones we performed, in which all components are live, can be used to study in detail the orbital structure, not

only during the period of disc growth, but also during the period of bar formation. It would be interesting to try to determine whether the axisymmetrisation of the halo during the phase of bar evolution is in some way related to the onset of chaos.

We have studied in detail the relevant influences of essentially two parameters: halo shape and gas content. Numerous other combinations of parameters remain to be explored, the most immediate of which would be to extend the present sets of simulations with new ones covering a range of disc-to-halo mass ratios and disc velocity dispersions. Further experiments could include a range of different halo core sizes and possibly simulations varying the details of the gas physics as well.

Moreover, there are current cosmological simulations that have had success in forming realistic discs. The study of the relation between the halo properties of such galaxies and the possible bar-like structures might contribute to the understanding of how the two affect one another.

One feature of particular interest is related to the gas particles that escape from the disc. In our simulations, these particles gave rise to two considerable cone-shaped structures above and below the disc, in which we find diffuse gas of very high temperatures. After characterising the properties of this feature, investigation is currently under way, in order to try to understand the detailed processes that lead to its formation.

Epicyclic Approximation

In this treatment, we assume that a non-circular disk is embedded in a non-spherical halo. The disk lies on the $z = 0$ plane and the ellipticity of the total potential in the plane of the disk is ε_{pot} . The ellipticity of orbits (that is, the ellipticity of the disk particles positions) is ε_R and the ellipticity of the velocities is ε_v . Using an epicyclic approximation, we derive relations between these ellipticities. If ε_{pot} is known, as well as the rotation curve, we are able to calculate the position and velocity coordinates for the disk particles, for general $v_c(R)$.

Following Binney & Tremaine (1987), in a frame of reference that rotates with pattern speed Ω_b , the equations of motion are:

$$\begin{aligned} \ddot{R} - R\dot{\varphi}^2 &= -\frac{\partial\Phi}{\partial R} + 2R\dot{\varphi}\Omega_b + \Omega_b^2 R \\ R\ddot{\varphi} + 2\dot{R}\dot{\varphi} &= -\frac{1}{R}\frac{\partial\Phi}{\partial\varphi} - 2\Omega_b\dot{R} \end{aligned} \quad (\text{A.1})$$

The total potential is an unperturbed potential, plus a small perturbation $\Phi_1(R, \varphi)$ such that $\left|\frac{\Phi_1}{\Phi_0} \ll 1\right|$

$$\Phi(R, \varphi) = \Phi_0(R) + \Phi_1(R, \varphi) \quad (\text{A.2})$$

and the position coordinates may be written as zero and first-order terms:

$$\begin{aligned} R(t) &= R_0 + R_1(t) \\ \varphi(t) &= \varphi_0 + \varphi_1(t) \end{aligned} \quad (\text{A.3})$$

that can be substituted separately into equation the first equation of motion

$$\ddot{R}_0 + \ddot{R}_1 - (R_0 + R_1)(\dot{\varphi}_0 + \dot{\varphi}_1)^2 = -\frac{\partial\Phi_0}{\partial R} - \frac{\partial\Phi_1}{\partial R} + 2(R_0 + R_1)(\dot{\varphi}_0 + \dot{\varphi}_1)\Omega_b + \Omega_b^2(R_0 + R_1) \quad (\text{A.4})$$

$$R_0\dot{\varphi}_0^2 = \left(\frac{\partial\Phi}{\partial R}\right)_{R_0} - 2R_0\dot{\varphi}_0\Omega_b - \Omega_b^2 R_0 \quad (\text{A.5})$$

to yield

$$R_0(\dot{\varphi}_0 + \Omega_b)^2 = \left(\frac{d\Phi_0}{dR}\right)_{R_0} \quad (\text{A.6})$$

If

$$\Omega_0 = \Omega(R_0) \quad (\text{A.7})$$

$$\Omega^2 \equiv \frac{1}{R} \frac{d\Phi_0}{dR} \quad (\text{A.8})$$

Ω is the circular frequency at R in the potential Φ_0 . The angular speed of the guiding centre $\dot{\varphi}_0$ may be obtained from equation (A.6):

$$\begin{aligned} (\dot{\varphi}_0 + \Omega_b)^2 &= \frac{1}{R_0} \left(\frac{d\Phi_0}{dR} \right)_{R_0} \\ (\dot{\varphi}_0 + \Omega_b)^2 &= \Omega_0^2 \\ \dot{\varphi}_0 &= \Omega_0 - \Omega_b \end{aligned} \quad (\text{A.9})$$

$$\varphi_0 = (\Omega_0 - \Omega_b)t \quad (\text{A.10})$$

The first order terms in equations (A.1) yield:

$$\begin{aligned} \ddot{R}_1 + \left(\frac{d^2\Phi_0}{dR^2} - \Omega^2 \right)_{R_0} R_1 - 2R_0\Omega_0\dot{\varphi}_1 &= - \left. \frac{\partial\Phi_1}{\partial R} \right|_{R_0} \\ \ddot{\varphi}_1 + 2\Omega_0 \frac{\dot{R}_1}{R_0} &= - \frac{1}{R_0^2} \left. \frac{\partial\Phi_1}{\partial\varphi} \right|_{R_0} \end{aligned} \quad (\text{A.11})$$

Now we assume a given form of Φ_1 :

$$\Phi_1(R, \varphi) = \Phi_b(R) \cos(m\varphi) \quad (\text{A.12})$$

We have assumed that φ_1 is small. If we also assume that φ itself is small, than we may replace φ by φ_0 in equation (A.11):

$$\ddot{R}_1 + \left(\frac{d^2\Phi_0}{dR^2} - \Omega^2 \right)_{R_0} R_1 - 2R_0\Omega_0\dot{\varphi}_1 = - \left(\frac{d\Phi_b}{dR} \right)_{R_0} \cos[m(\Omega_0 - \Omega_b)t] \quad (\text{A.13})$$

$$\ddot{\varphi}_1 + 2\Omega_0 \frac{\dot{R}_1}{R_0} = \frac{m\Phi_b(R_0)}{R_0^2} \sin[m(\Omega_0 - \Omega_b)t] \quad (\text{A.14})$$

Integrating equation (A.14):

$$\begin{aligned} \varphi_1 &= \int -\frac{2\Omega_0}{R_0} \frac{dR_1}{dt} dt + \frac{m\Phi_b(R_0)}{R_0^2} \int \sin[m(\Omega_0 - \Omega_b)t] dt \\ \dot{\varphi}_1 &= -2\Omega_0 \frac{R_1}{R_0} - \frac{\Phi_b(R_0)}{R_0^2(\Omega_0 - \Omega_b)t} \cos[m(\Omega_0 - \Omega_b)t] + cte \end{aligned} \quad (\text{A.15})$$

and replacing it in equation (A.13):

$$\begin{aligned} \ddot{R}_1 + \left(\frac{d^2\Phi_0}{dR^2} - \Omega^2 \right)_{R_0} R_1 + 4\Omega^2 R_1 + \frac{2\Phi_b(R_0)\Omega_0}{R_0(\Omega_0 - \Omega_b)} \cos [m(\Omega_0 - \Omega_b)t] &= - \frac{\partial\Phi_1}{\partial R} \Big|_{R_0} \\ \ddot{R}_1 + \left(\frac{d^2\Phi_0}{dR^2} + 3\Omega^2 \right)_{R_0} R_1 &= - \left[\frac{d\Phi_b}{dR} + \frac{2\Phi_b\Omega}{R(\Omega - \Omega_b)} \right]_{R_0} \cos [m(\Omega_0 - \Omega_b)t] \end{aligned} \quad (\text{A.16})$$

So we have

$$\ddot{R}_1 + \kappa_0^2 R_1 = - \left[\frac{d\Phi_b}{dR} + \frac{2\Phi_b\Omega}{R(\Omega - \Omega_b)} \right]_{R_0} \cos [m(\Omega_0 - \Omega_b)t] \quad (\text{A.17})$$

where

$$\kappa_0^2 = \left(\frac{d^2\Phi_0}{dR^2} + 3\Omega^2 \right)_{R_0} = \left(R \frac{d\Omega^2}{dR} + 4\Omega^2 \right)_{R_0} \quad (\text{A.18})$$

is the usual epicycle frequency. Equation (A.17) is the equation of motion of a harmonic oscillator of natural frequency κ_0 that is driven at frequency $m(\Omega_0 - \Omega_b)$. The general solution is:

$$R_1(t) = C_1 \cos(\kappa_0 t + \psi) - \left[\frac{d\Phi_b}{dR} + \frac{2\Phi_b\Omega}{R(\Omega - \Omega_b)} \right]_{R_0} \frac{\cos [m(\Omega_0 - \Omega_b)t]}{\kappa_0^2 - m^2(\Omega_0 - \Omega_b)^2} \quad (\text{A.19})$$

If $C_1 = 0$, $R_1(\varphi_0)$ is periodic with period $2\pi/m$ and we have a closed loop orbit.

Using equation (A.10) in (A.19), we have:

$$R_1(\varphi_0) = \frac{-1}{\kappa_0^2 - m^2(\Omega_0 - \Omega_b)^2} \left[\frac{d\Phi_b}{dR} + \frac{2\Phi_b\Omega}{R(\Omega - \Omega_b)} \right]_{R_0} \cos(m\varphi_0) \quad (\text{A.20})$$

Let us define

$$\mathbb{D}^2 \equiv \kappa_0^2 - m^2(\Omega_0 - \Omega_b)^2 \quad (\text{A.21})$$

Then

$$R_1(\varphi_0) = -\frac{1}{\mathbb{D}^2} \left[\frac{d\Phi_b}{dR} + \frac{2\Phi_b\Omega}{R(\Omega - \Omega_b)} \right]_{R_0} \cos(m\varphi_0) \quad (\text{A.22})$$

Now, taking $m = 2$ and $(\Omega - \Omega_b) = \Omega$

$$\mathbb{D}^2 \equiv \kappa_0^2 - 4\Omega_0^2 \quad (\text{A.23})$$

$$R_1(\varphi_0) = -\frac{1}{\mathbb{D}^2} \left[\frac{d\Phi_b}{dR} + \frac{2\Phi_b}{R} \right]_{R_0} \cos(2\varphi_0) \quad (\text{A.24})$$

where

$$\varphi_0 = \Omega_0 t \quad (\text{A.25})$$

But R_1 is the first-order component (perturbation) of $R(t)$ which is given by:

$$\begin{aligned}
 R(t) &= R_0 + R_1 \\
 R(\varphi_0) &= R_0 + R_1(\varphi_0) \\
 R(\varphi_0) &= R_0 + \left(-\frac{1}{\mathbb{D}^2} \right) \left[\frac{d\Phi_b}{dR} + \frac{2\Phi_b}{R} \right]_{R_0} \cos(2\varphi_0) \\
 R(\varphi_0) &= R_0 \left[1 + \left(-\frac{1}{R_0 \mathbb{D}^2} \right) \left[\frac{d\Phi_b}{dR} + \frac{2\Phi_b}{R} \right]_{R_0} \cos(2\varphi_0) \right]
 \end{aligned} \tag{A.26}$$

then

$$\begin{aligned}
 R(\varphi_0) &= R_0 \left[1 + \frac{\varepsilon_R}{2} \cos(2\varphi_0) \right] \\
 \text{or} \\
 R(t) &= R_0 \left[1 + \frac{\varepsilon_R}{2} \cos(2\Omega_0 t) \right]
 \end{aligned} \tag{A.27}$$

where the ellipticity of the orbits ε_R is:

$$\varepsilon_R = -\frac{2}{R_0} \frac{1}{\Delta^2} \left[\frac{d\Phi_b}{dR} + \frac{2\Phi_b}{R} \right]_{R_0} \tag{A.28}$$

but now

$$\Delta^2 \equiv 4\Omega_0^2 - \kappa_0^2 \tag{A.29}$$

and Δ^2 is always positive since $\Omega < \kappa < 2\Omega$. Alternatively, we could estimate ε_R noting that

$$\begin{aligned}
 R_{max} &= R_0 + |R_1|_{max} \\
 R_{min} &= R_0 - |R_1|_{min}
 \end{aligned} \tag{A.30}$$

$$\varepsilon_R = 1 - \frac{R_{min}}{R_{max}} = \frac{R_{max} - R_{min}}{R_{max}} = \frac{R_0 + |R_1| - R_0 + |R_1|}{R_0 + |R_1|} = \frac{2|R_1|}{R_0 + |R_1|} \tag{A.31}$$

but since $|R_0 + R_1| \simeq R_0$

$$\varepsilon_R \simeq \frac{2|R_1|_{max}}{R_0} \tag{A.32}$$

and because the maximum of $|R_1|$ occurs when $2\varphi_0 = 0$ in equation (A.24), we have again

$$\varepsilon_R \simeq \frac{2}{R_0} \frac{1}{\mathbb{D}^2} \left[\frac{d\Phi_b}{dR} + \frac{2\Phi_b}{R} \right]_{R_0} \tag{A.33}$$

We still need to compute $\varphi_1(t)$ by integrating equation (A.15). First we note that

$$\begin{aligned}
R_1(t) &= -\frac{1}{\mathbb{D}^2} \left[\frac{d\Phi_b}{dR} + \frac{2\Phi_b}{R} \right]_{R_0} \cos(2\Omega_0 t) \\
R_1(t) &= -\frac{R_0}{2} \varepsilon_R \cos(2\Omega_0 t)
\end{aligned} \tag{A.34}$$

So we may replace this R_1 in equation (A.15):

$$\begin{aligned}
\dot{\varphi}_1(t) &= 2 \frac{\Omega_0}{R_0} \frac{R_0}{2} \varepsilon_R \cos(2\Omega_0 t) - \frac{\Phi_b(R_0)}{\Omega_0 R_0^2} \cos(2\Omega_0 t) \\
\dot{\varphi}_1(t) &= \left[\Omega_0 \varepsilon_R - \frac{\Phi_b(R_0)}{\Omega_0 R_0^2} \right] \cos(2\Omega_0 t) \\
\varphi_1(t) &= \int_0^t \left[\Omega_0 \varepsilon_R - \frac{\Phi_b(R_0)}{\Omega_0 R_0^2} \right] \cos(2\Omega_0 t') dt' \\
\varphi_1(t) &= \left[\frac{\Omega_0 \varepsilon_R}{2\Omega_0} - \frac{\Phi_b(R_0)}{2\Omega_0^2 R_0^2} \right] \sin(2\Omega_0 t)
\end{aligned} \tag{A.35}$$

Let us define

$$A \equiv \left[\frac{\varepsilon_R}{2} - \frac{\Phi_b(R_0)}{2\Omega_0^2 R_0^2} \right] \tag{A.36}$$

so that $\varphi_1(t) = A \sin(2\Omega_0 t)$. Further, we write it as:

$$\begin{aligned}
A &= \frac{\varepsilon_R}{4} + \left[\frac{\varepsilon_R}{4} - \frac{\Phi_b(R_0)}{2\Omega_0^2 R_0^2} \right] \\
A &= \frac{\varepsilon_R}{4} + B
\end{aligned} \tag{A.37}$$

and replacing ε_R by equation (A.33):

$$\begin{aligned}
B &= \left[\frac{\varepsilon_R}{4} - \frac{\Phi_b(R_0)}{2\Omega_0^2 R_0^2} \right] \\
&= \frac{2}{4} \frac{1}{R_0} \frac{1}{\mathbb{D}^2} \frac{d\Phi_b}{dR} + \frac{2}{4} \frac{1}{R_0} \frac{1}{\mathbb{D}^2} \frac{2\Phi_b}{R_0} - \frac{\Phi_b}{2\Omega_0^2 R_0^2} \\
&= \frac{1}{4} \frac{2}{R_0 \mathbb{D}^2} \left[\frac{d\Phi_b}{dR} + \frac{2\Phi_b}{R_0} - \frac{\Phi_b \mathbb{D}^2}{\Omega_0^2 R_0^2} \right] \\
&= \frac{1}{4} \frac{2}{R_0 \mathbb{D}^2} \left[\frac{d\Phi_b}{dR} + \frac{6\Omega_0^2 - \kappa_0^2}{\Omega_0^2} \frac{\Phi_b(R_0)}{R_0} \right]
\end{aligned} \tag{A.38}$$

If the ellipticity of the velocities is given by:

$$\varepsilon_v \equiv \frac{2}{R_0 \mathbb{D}^2} \left[\frac{d\Phi_b}{dR} + \frac{6\Omega_0^2 - \kappa_0^2}{\Omega_0^2} \frac{\Phi_b(R_0)}{R_0} \right] \quad (\text{A.39})$$

then the perturbation to the angular position coordinate is simply:

$$\varphi_1(t) = \left(\frac{\varepsilon_R + \varepsilon_v}{4} \right) \sin(2\Omega_0 t) \quad (\text{A.40})$$

Finally,

$$\begin{aligned} \varphi(t) &= \varphi_0(t) + \varphi_1(t) \\ \varphi(t) &= \Omega_0 t + \frac{1}{4} (\varepsilon_R + \varepsilon_v) \sin(2\Omega_0 t) \end{aligned} \quad (\text{A.41})$$

And we also note that

$$\varepsilon_v = \varepsilon_R - \frac{2\Phi_b(R_0)}{R_0^2 \Omega_0^2} \quad (\text{A.42})$$

Now we have to compute the velocities:

$$\begin{aligned} \vec{v} &= \dot{R} \hat{\mathbf{e}}_R + R \dot{\varphi} \hat{\mathbf{e}}_\varphi + \dot{z} \hat{\mathbf{e}}_z \\ \vec{v} &= v_R \hat{\mathbf{e}}_R + v_\varphi \hat{\mathbf{e}}_\varphi \end{aligned} \quad (\text{A.43})$$

The radial component is:

$$\begin{aligned} v_R &= \dot{R} \\ v_R &= \frac{d}{dt} \left(R_0 - \frac{R_0 \varepsilon_R}{2} \cos(2\Omega_0 t) \right) \\ v_R &= R_0 \Omega_0 \varepsilon_R \sin(2\Omega_0 t) \\ v_R &= v_c \varepsilon_R \sin(2\Omega_0 t) \end{aligned} \quad (\text{A.44})$$

and the circular velocity at R_0 is simply $v_c = R_0 \Omega_0$. The tangential component of the velocity is:

$$\begin{aligned} \dot{\varphi} &= \frac{d}{dt} \left(\Omega_0 t + \frac{(\varepsilon_R + \varepsilon_v)}{4} \sin(2\Omega_0 t) \right) \\ \dot{\varphi} &= \Omega_0 + \frac{(\varepsilon_R + \varepsilon_v)}{4} 2\Omega_0 \cos(2\Omega_0 t) \end{aligned} \quad (\text{A.45})$$

$$\begin{aligned} v_\varphi &= R \dot{\varphi} \\ v_\varphi &= \left(R_0 - \frac{R_0 \varepsilon_R}{2} \cos(2\Omega_0 t) \right) \left(\Omega_0 + \frac{\varepsilon_R}{2} \Omega_0 \cos(2\Omega_0 t) + \frac{\varepsilon_v}{2} \Omega_0 \cos(2\Omega_0 t) \right) \\ v_\varphi &= R_0 \Omega_0 + \frac{\varepsilon_v}{2} R_0 \Omega_0 \cos(2\Omega_0 t) - \frac{\varepsilon_R^2}{4} R_0 \Omega_0 \cos^2(2\Omega_0 t) - \frac{\varepsilon_R \varepsilon_v}{4} R_0 \Omega_0 \cos^2(2\Omega_0 t) \end{aligned} \quad (\text{A.46})$$

If the ellipticities are small, we may keep only the first-order terms:

$$\begin{aligned} v_\varphi &= R_0 \Omega_0 + R_0 \Omega_0 \frac{\varepsilon_v}{2} \cos(2\Omega_0 t) + \mathcal{O}(\varepsilon^2) \\ v_\varphi &= v_c \left(1 + \frac{\varepsilon_v}{2} \cos(2\Omega_0 t) \right) \end{aligned} \quad (\text{A.47})$$

To summarize, the positions are given by:

$$\begin{cases} R = R_0 \left[1 - \frac{\varepsilon_R}{2} \cos(2\Omega_0 t) \right] \\ \varphi = \Omega_0 t + \frac{\varepsilon_R + \varepsilon_v}{4} \sin(2\Omega_0 t) \end{cases} \quad (\text{A.48})$$

and the velocities are given by:

$$\begin{cases} v_R = v_c \varepsilon_R \sin(2\Omega_0 t) \\ v_\varphi = v_c \left[1 + \frac{\varepsilon_v}{2} \cos(2\Omega_0 t) \right] \end{cases} \quad (\text{A.49})$$

Now we need to relate the ellipticities ε_R , ε_v with ε_{pot} . If the ratio between the maximum and the minimum values of the potential can be written analogously to equation (A.31), we have:

$$\varepsilon_{pot} \approx - \left| \frac{2\Phi_b}{\Phi_{max}} \right| \quad (\text{A.50})$$

and the minus sign comes from the fact that the potentials are all negative. Again, if the perturbation is small, $\Phi_{max} = \Phi_0 + \Phi_b \approx \Phi_0$ and then the maximum value of the potential is $|\Phi_0| = v_c^2 = \Omega_0^2 R_0^2$. So the ellipticity of the potential is:

$$\varepsilon_{pot} = \left(-\frac{2\Phi_b}{v_c^2} \right)_{R_0} = -\frac{2\Phi_b}{\left(R \frac{d\Phi}{dR} \right)_{R_0}} = -\frac{2\Phi_b}{\Omega_0^2 R_0^2} \quad (\text{A.51})$$

But because of equation (A.42) (which results from the definitions of ε_R and ε_v and is valid generally), the relation between the ellipticities is simply:

$$\varepsilon_v = \varepsilon_R + \varepsilon_{pot} \quad (\text{A.52})$$

Let us consider the general case of a velocity curve $v_c = v_c(R)$. If we replace equation (A.18) into (A.29), we may write:

$$\begin{aligned} \Delta^2 &= 4\Omega_0^2 - \kappa_0^2 = -R \frac{d\Omega^2}{dR} = -R \frac{d}{dR} \left(\frac{v^2}{R^2} \right) \\ \Delta^2 &= \frac{1}{R} \left[-\frac{dv^2}{dR} + \frac{2v^2}{R} \right]_{R_0} \end{aligned} \quad (\text{A.53})$$

Besides, from equation (A.51), we have that

$$\frac{2\Phi_b}{R} = \frac{2}{R} \left(-\frac{\varepsilon_{pot}}{2} \right) v_c^2 = -\frac{\varepsilon_{pot}}{R_0} v_c^2 \quad (\text{A.54})$$

from which we also get

$$\left(\frac{d\Phi_b}{dR} \right)_{R_0} = -\frac{\varepsilon_{pot}}{2} \frac{dv_c^2}{dR} \quad (\text{A.55})$$

Now, replacing these in equation (A.28):

$$\begin{aligned} \varepsilon_R &= -\frac{2}{R_0} \frac{1}{\Delta^2} \left[\frac{d\Phi_b}{dR} + \frac{2\Phi_b}{R} \right]_{R_0} \\ \varepsilon_R &= -\frac{2}{R_0} \frac{1}{\Delta^2} \left[-\frac{\varepsilon_{pot}}{2} \frac{dv_c^2}{dR} - \frac{\varepsilon_{pot}}{R_0} v_c^2 \right]_{R_0} \\ \varepsilon_R &= \frac{2}{R_0} \frac{1}{\Delta^2} \left[\frac{dv_c^2}{dR} + \frac{2v_c^2}{R} \right] \varepsilon_{pot} \end{aligned} \quad (\text{A.56})$$

and finally, using equation (A.53) we obtain the relation between the ellipticities ε_R and ε_{pot} :

$$\varepsilon_R = \left[\frac{\frac{2v_c^2}{R} + \frac{dv_c^2}{dR}}{\frac{2v_c^2}{R} - \frac{dv_c^2}{dR}} \right]_{R_0} \varepsilon_{pot} \quad (\text{A.57})$$

This relation is valid for general v_c . We may now consider the particular case of a power-law velocity curve:

$$v_c = v_{c0} \left(\frac{R}{R_0} \right)^\alpha \quad (\text{A.58})$$

Then we may compute these two terms:

$$\begin{aligned} \frac{2v_c^2}{R} &= 2v_{c0}^2 \frac{R^{2\alpha-1}}{R_0^{2\alpha}} \Big|_{R_0} = \frac{2v_{c0}^2}{R_0} \\ \frac{dv_c^2}{dR} &= 2\alpha v_{c0}^2 \frac{R^{2\alpha-1}}{R_0^{2\alpha}} \Big|_{R_0} = \frac{2\alpha v_{c0}^2}{R_0} \end{aligned} \quad (\text{A.59})$$

And we recover the relation between the ellipticities for the power-law v_c as in Franx et al. (1994):

$$\begin{aligned} \varepsilon_R &= \left[\frac{\frac{2v_{c0}^2}{R_0} + \alpha \frac{2v_{c0}^2}{R_0}}{\frac{2v_{c0}^2}{R_0} - \alpha \frac{2v_{c0}^2}{R_0}} \right] \varepsilon_{pot} \\ \varepsilon_R &= \left(\frac{1 + \alpha}{1 - \alpha} \right) \varepsilon_{pot} \end{aligned} \quad (\text{A.60})$$

Quantifying bar strength with Fourier coefficients

Let $M(\theta)$ be the mass (per unit surface area) as a function of azimuth. This function is defined in the interval $-\pi \leq \theta \leq \pi$ and it may be expressed as a Fourier series:

$$M(\theta) = \frac{a_0}{2} + \sum_{m=1}^{\infty} a_m \cos(m\theta) + \sum_{m=1}^{\infty} b_m \sin(m\theta) \quad (\text{B.1})$$

where the coefficients are defined as:

$$a_m = \frac{1}{\pi} \int_{-\pi}^{\pi} M(\theta) \cos(m\theta) d\theta, m = 0, 1, 2, \dots \quad (\text{B.2})$$

$$b_m = \frac{1}{\pi} \int_{-\pi}^{\pi} M(\theta) \sin(m\theta) d\theta, m = 1, 2, 3, \dots \quad (\text{B.3})$$

B.1 case 1: uniform distribution

Let us consider first the axisymmetric case, in which $M(\theta)$ does not depend on the azimuth and is equal to a constant M_o :

$$M(\theta) = M_o \quad (\text{B.4})$$

Then we may calculate the coefficient a_0 :

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} M(\theta) d\theta \quad (\text{B.5})$$

$$a_0 = \frac{M_o}{\pi} \int_{-\pi}^{\pi} d\theta \quad (\text{B.6})$$

$$a_0 = 2M_o \quad (\text{B.7})$$

All the other coefficients are zero:

$$a_m = \frac{M_o}{\pi} \int_{-\pi}^{\pi} \cos(m\theta) d\theta = 0 \quad (\text{B.8})$$

$$b_m = \frac{M_o}{\pi} \int_{-\pi}^{\pi} \sin(m\theta) d\theta = 0 \quad (\text{B.9})$$

And the series for $M(\theta)$ is simply:

$$M(\theta) = \frac{a_0}{2} \quad (\text{B.10})$$

$$M(\theta) = M_o \quad (\text{B.11})$$

B.2 case 2: two-peaked distribution

The disc of a strongly barred galaxy, seen on the xy plane will have an azimuthal distribution of mass that has two peaks. That is the reason why the relative amplitude of the $m = 2$ coefficients is a convenient way of quantifying the intensity of the bar.

Let us consider a very simple function $M(\theta)$ that has two peaks separated by an angle of π , each of width $2w$:

$$M(\theta) = \begin{cases} \frac{\pi M_o}{2w} & \text{if } (\theta_b - w) \leq \theta \leq (\theta_b + w) \\ \frac{\pi M_o}{2w} & \text{if } (\theta_b - \pi - w) \leq \theta \leq (\theta_b - \pi + w) \\ 0 & \text{elsewhere} \end{cases} \quad (\text{B.12})$$

One of the peaks occurs at θ_b and the other at $\theta_b - \pi$. The height of the peaks is chosen such that the integral of $M(\theta)$ will be the same as in case 1, i.e. the total mass is the same, but here it is concentrated inside two circular sectors opposed by the vertex. Figure B.1 shows three possible orientations, and Fig. B.2 show different widths.

B.2.1 Fourier coefficients for case 2

Let us calculate the coefficients a_m and b_m to obtain the Fourier series of the function given by equation B.12.

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} M(\theta) d\theta \quad (\text{B.13})$$

$$a_0 = \frac{1}{\pi} \frac{\pi M_o}{2w} \left[\int_{\theta_b - \pi - w}^{\theta_b - \pi + w} d\theta + \int_{\theta_b - w}^{\theta_b + w} d\theta \right] \quad (\text{B.14})$$

$$a_0 = \frac{1}{\pi} \frac{\pi M_o}{2w} [\theta_b - \pi + w - \theta_b - \pi + w + \theta_b + w - \theta_b + w] \quad (\text{B.15})$$

$$a_0 = 2M_o \quad (\text{B.16})$$

The odd coefficients $a_1, a_3, a_5, \dots$ and $b_1, b_3, b_5, \dots$ will be all zero:

$$a_m = \frac{1}{\pi} \int_{-\pi}^{\pi} M(\theta) \cos m\theta d\theta, \quad m = 1, 3, 5, \dots \quad (\text{B.17})$$

$$a_m = \frac{1}{\pi} \frac{\pi M_o}{2w} \left[\int_{\theta_b - \pi - w}^{\theta_b - \pi + w} \cos m\theta d\theta + \int_{\theta_b - w}^{\theta_b + w} \cos m\theta d\theta \right] \quad (\text{B.18})$$

$$a_m = \frac{M_o}{2w} \frac{1}{m} [\sin(m\theta_b - m\pi + mw) - \sin(m\theta_b - m\pi - mw) + \sin(m\theta_b + mw) - \sin(m\theta_b - mw)] \quad (\text{B.19})$$

but because

$$\sin(\alpha - m\pi) = -\sin(\alpha) \quad \text{if } m = 1, 3, 5, \dots \quad (\text{B.20})$$

we have that all the odd a_m will be:

$$a_m = \frac{M_o}{2w} \frac{1}{m} [-\sin(m\theta_b + mw) + \sin(m\theta_b - mw) + \sin(m\theta_b + mw) - \sin(m\theta_b - mw)] \quad (\text{B.21})$$

$$a_m = 0 \quad \text{if } m = 1, 3, 5, \dots \quad (\text{B.22})$$

Similarly for the odd b_m :

$$b_m = \frac{1}{\pi} \int_{-\pi}^{\pi} M(\theta) \sin m\theta d\theta \quad , m = 1, 3, 5, \dots \quad (\text{B.23})$$

$$b_m = \frac{1}{\pi} \frac{\pi M_o}{2w} \left[\int_{\theta_b - \pi - w}^{\theta_b - \pi + w} \sin m\theta d\theta + \int_{\theta_b - w}^{\theta_b + w} \sin m\theta d\theta \right] \quad (\text{B.24})$$

$$b_m = -\frac{M_o}{2w} \frac{1}{m} [\cos(m\theta_b - m\pi + mw) - \cos(m\theta_b - m\pi - mw) + \cos(m\theta_b + mw) - \cos(m\theta_b - mw)] \quad (\text{B.25})$$

$$+ \cos(m\theta_b + mw) - \cos(m\theta_b - mw)] \quad (\text{B.26})$$

and also

$$\cos(\alpha - m\pi) = -\cos(\alpha) \quad \text{if } m = 1, 3, 5, \dots \quad (\text{B.27})$$

we have that all the odd b_m will be:

$$b_m = -\frac{M_o}{2w} \frac{1}{m} [-\cos(m\theta_b + mw) + \cos(m\theta_b - mw) + \cos(m\theta_b + mw) - \cos(m\theta_b - mw)] \quad (\text{B.28})$$

$$b_m = 0 \quad \text{if } m = 1, 3, 5, \dots \quad (\text{B.29})$$

The even coefficients $a_2, a_4, a_6, \dots$ and $b_2, b_4, b_6, \dots$ will be non-zero:

$$a_m = \frac{1}{\pi} \int_{-\pi}^{\pi} M(\theta) \cos m\theta d\theta \quad , m = 2, 4, 6, \dots \quad (\text{B.30})$$

$$a_m = \frac{1}{\pi} \frac{\pi M_o}{2w} \left[\int_{\theta_b - \pi - w}^{\theta_b - \pi + w} \cos m\theta d\theta + \int_{\theta_b - w}^{\theta_b + w} \cos m\theta d\theta \right] \quad (\text{B.31})$$

$$a_m = \frac{M_o}{2w} \frac{1}{m} [\sin(m\theta_b - m\pi + mw) - \sin(m\theta_b - m\pi - mw) + \sin(m\theta_b + mw) - \sin(m\theta_b - mw)] \quad (\text{B.32})$$

but now

$$\sin(\alpha - m\pi) = \sin(\alpha) \quad \text{if } m = 2, 4, 6, \dots \quad (\text{B.33})$$

so we have the following for the even a_m :

$$a_m = \frac{M_o}{2w} \frac{1}{m} [2 \sin(m\theta_b + mw) - 2 \sin(m\theta_b - mw)] \quad (\text{B.34})$$

and because

$$\sin(\alpha + \beta) - \sin(\alpha - \beta) = 2 \cos(\alpha) \sin(\beta) \quad (\text{B.35})$$

we have

$$a_m = \frac{M_o}{w} \frac{1}{m} [2 \cos(m\theta) \sin(mw)] \quad (\text{B.36})$$

$$a_m = \frac{2M_o}{mw} \cos(m\theta) \sin(mw) \quad \text{if } m = 2, 4, 6, \dots \quad (\text{B.37})$$

And similarly for the even b_m :

$$b_m = \frac{1}{\pi} \int_{-\pi}^{\pi} M(\theta) \sin m\theta d\theta, \quad m = 2, 4, 6, \dots \quad (\text{B.38})$$

$$b_m = \frac{1}{\pi} \frac{\pi M_o}{2w} \left[\int_{\theta_b - \pi - w}^{\theta_b - \pi + w} \sin m\theta d\theta + \int_{\theta_b - w}^{\theta_b + w} \sin m\theta d\theta \right] \quad (\text{B.39})$$

$$b_m = -\frac{M_o}{2w} \frac{1}{m} [\cos(m\theta_b - m\pi + mw) - \cos(m\theta_b - m\pi - mw) + \cos(m\theta_b + mw) - \cos(m\theta_b - mw)] \quad (\text{B.40})$$

using this time

$$\cos(\alpha - m\pi) = \cos(\alpha) \quad \text{if } m = 2, 4, 6, \dots \quad (\text{B.41})$$

and

$$\cos(\alpha + \beta) - \cos(\alpha - \beta) = -2 \sin(\alpha) \sin(\beta) \quad (\text{B.42})$$

$$b_m = -\frac{M_o}{2w} \frac{1}{m} [2 \cos(m\theta_b + mw) - 2 \cos(m\theta_b - mw)] \quad (\text{B.43})$$

$$b_m = -\frac{M_o}{mw} [\cos(m\theta_b + mw) - \cos(m\theta_b - mw)] \quad (\text{B.44})$$

$$b_m = -\frac{M_o}{mw} [-2 \sin(m\theta_b) \sin(mw)] \quad (\text{B.45})$$

$$b_m = \frac{2M_o}{mw} \sin(m\theta_b) \sin(mw) \quad (\text{B.46})$$

$$(\text{B.47})$$

So, to summarise, the odd coefficients are all zero and the even coefficients are given by:

$$a_m = \frac{2M_o}{mw} \cos(m\theta_b) \sin(mw), \quad m = 2, 4, 6, \dots \quad (\text{B.48})$$

$$b_m = \frac{2M_o}{mw} \sin(m\theta_b) \sin(mw), \quad m = 2, 4, 6, \dots \quad (\text{B.49})$$

B.2.2 $m = 2$ coefficients

In particular, the $m = 2$ coefficients will be:

$$a_2 = \frac{M_o}{w} \cos(2\theta_b) \sin(2w) \quad (\text{B.50})$$

$$b_2 = \frac{M_o}{w} \sin(2\theta_b) \sin(2w) \quad (\text{B.51})$$

Incidentally, the orientation of the bar θ_b may be obtained from the ratio of these coefficients, as:

$$\theta_b = \frac{1}{2} \arctan\left(\frac{b_2}{a_2}\right) \quad (\text{B.52})$$

The function $M(\theta)$ is then approximated by:

$$M(\theta) \simeq \frac{a_0}{2} + a_2 \cos(2\theta) + b_2 \sin(2\theta) + \dots \quad (\text{B.53})$$

Figure B.3 shows the partial sums S_m of the series for S_0 until S_{14} .

We may define the relative amplitude of the $m = 2$ component as:

$$I_2 = \frac{\sqrt{a_2^2 + b_2^2}}{a_0} \quad (\text{B.54})$$

which can also be written as:

$$I_2 = \frac{M_o}{w} \frac{\sqrt{\cos^2(2\theta_b) \sin^2(2w) + \sin^2(2\theta_b) \sin^2(2w)}}{2M_o} \quad (\text{B.55})$$

$$I_2 = \frac{M_o \sin(2w)}{w 2M_o} \quad (\text{B.56})$$

$$I_2 = \frac{\sin(2w)}{2w} \quad (\text{B.57})$$

So in this simplified model, the relative amplitude is a function of the ‘width’ of the bar, $I_2(w)$ (see Fig. B.4) and evidently does not depend on the orientation θ_b . If the half-width is the maximum possible, $w = \pi/2$, then we have the case of a uniform distribution of mass, and the bar strength is $I_2(\pi/2) = 0$. If, on the other extreme, the width tends to zero, we have the case of the entire mass lying along one line, which is the maximum possible bar strength $I_2(0) = 1$.

B.2.3 Radial dependence

So far, we did not consider radial dependence, because for this simplified model, all radii are equivalent, so we may regard the previous results as applying for a certain annulus ΔR , or for the galaxy as a whole.

We may consider a ‘bar’ which is described not by two circular sectors, but by a rectangle on the xy plane, having dimensions $2R_b$ by $2p$. In this case, the angle $2w$, subtended by the width of the bar, is no longer constant, but depends on the distance from the center $R = \sqrt{x^2 + y^2}$ as:

$$w(R) = \arctan\left(\frac{p}{R}\right) \quad (\text{B.58})$$

and thus the I_2 will depend on radius as:

$$I_2(R) = \begin{cases} 0 & \text{if } R < p \\ \frac{\sin\left[2 \arctan\left(\frac{p}{R}\right)\right]}{2 \arctan\left(\frac{p}{R}\right)} & \text{if } p \leq R \leq R_b \\ 0 & \text{if } R > R_b \end{cases} \quad (\text{B.59})$$

This $I_2(R)$ is shown in Fig. B.5 for $R_b = 3$ and $p = 0.5$. Note that the $I_2(R)$ must drop to zero after the end of the bar; and it must also be zero for $R < p$ because this inner region (a circumference whose diameter is the width of the bar) is entirely contained inside the bar, and thus it is axisymmetric. The sharp discontinuities at $R = p$ and $R = R_b$ are due to the fact that in this model, the bar is exactly rectangular and the density is uniform inside the rectangle.

I created an N -body snapshot which consists of 200 000 particles uniformly distributed inside a rectangular region of dimensions 6 by 1 on the xy plane, in order to measure its $I_2(R)$ with the same software that has been used throughout the analyses. The results of this radial profile are shown as the blue symbols in Fig. B.5. In a bar from an actual N -body simulation, the edges of the bar would be smooth and the density would not be necessarily uniform. Furthermore, $I_2(R)$ does not reach 1 in actual bars, because there is mass in the rest of the disc. If the mock rectangular bar is added on top of an axisymmetric disc, the effect is that the $I_2(R)$ curve is lowered.

Figure B.1: The 'bars' and the functions $M(\theta)$ of three different orientations θ_b (for a given half-width $w = \pi/10$).

Figure B.2: The ‘bars’ and the functions $M(\theta)$ of three different half-widths w (for a given orientations $\theta_b = 0$).

Figure B.3: Partial sums of the Fourier series.

Figure B.4: $m = 2$ relative amplitude as a function of bar half-width $I_2(w)$.

Figure B.5: Left: N -body mock bar with 200 000 particles. Right: $m = 2$ relative amplitude as a function of radius for a rectangular bar model of half-length $R_b = 3$ and half-width $p = 0.5$. Red line is equation B.59 with $R_b=3$ and $p = 0.5$; blue symbols are the measurement of the N -body mock bar.

Snapshots of the collisionless models

The snapshots shown here are from the simulations of Chapter 2. Haloes and discs are seen on the xy plane at $t = 0$, $t = 100$ and at $t = 800$. Disc rotation is counterclockwise. Each frame is 10 by 10 units of length. Colour represents projected density and the range is the same for all panels. The first three columns show the halo and the last three columns show the disc. In each set, the columns are for models with halo 1, halo 2 and halo 3.

Figure C.1: The standard set.

Figure C.2: Models with initially circular discs.

Figure C.3: Models with less massive (initially elliptical) discs.

Figure C.4: Models with less massive (initially circular) discs.

Figure C.5: Models with larger-cored halo (initially elliptical discs).

Figure C.6: Models with larger-cored halo (initially circular discs).

Figure C.7: Models with hot discs.

Figure C.8: Models whose discs are axisymmetrised.

Figure C.9: Models with rigid discs. The discs are represented by an analytic potential.

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Dynamics of barred galaxies in triaxial dark matter haloes

Abstract: Cosmological N -body simulations indicate that the dark matter haloes of galaxies should be generally triaxial. Yet, the presence of a baryonic disc is believed to modify the shape of the haloes. The goal of this thesis is to study how bar formation is affected by halo triaxiality and how, in turn, the presence of the bar influences the shape of the halo. We performed a series of collisionless and hydrodynamical numerical simulations, using elliptical discs as initial conditions. Triaxial halos tend to become more spherical and we show that part of the circularisation of the halo is due to disc growth, but part must be attributed to the formation of a bar. We find that the presence of gas in the disc is a more efficient factor than halo triaxiality in inhibiting the formation of a strong bar.

keywords: astrophysics – galactic dynamics – galaxies: evolution – galaxies: haloes – methods: numerical simulations

Dinâmica de galáxias barradas em halos triaxiais de matéria escura

Resumo: As simulações cosmológicas de N -corpos indicam que os halos de matéria escura das galáxias devem ser em geral triaxiais. Contudo, acredita-se que a presença de um disco bariônico seja capaz de alterar a forma do halo. O objetivo desta tese é o de estudar como a formação de barras é afetada pela triaxialidade do halo e como, por sua vez, a presença da barra influencia a forma do halo. Nós realizamos uma série de simulações numéricas acolisionais e hidrodinâmicas, utilizando discos elípticos como condições iniciais. Os halos triaxiais tendem a se tornar mais esféricos e nós mostramos que parte da circularização do halo é devida ao crescimento do disco, mas parte precisa ser atribuída à formação da barra. Notamos que a presença de gás no disco é um fator mais eficiente do que a triaxialidade do halo em inibir a formação de uma barra forte.

palavras-chave: astrofísica – dinâmica galáctica – galáxias: evolução – galáxias: halos – métodos: simulações numéricas

Dynamique des galaxies barrées dans des halos triaxiaux de matière noire

Résumé: Des simulations cosmologiques à N -corps indiquent que les halos de matière noire des galaxies devraient être généralement triaxiaux. Pourtant, on croit que la présence d'un disque baryonique modifie la forme des halos. L'objectif de cette thèse est d'étudier comment la formation des barres est affectée par la triaxialité des halos et comment, à son tour, la présence de la barre influence la forme du halo. Nous réalisons un ensemble de simulations numériques non-collisionnelles et hydrodynamiques, utilisant des disques elliptiques comme conditions initiales. Les halos triaxiaux ont tendance à devenir plus sphérique et nous montrons qu'une partie de la circularisation du halo est due à la croissance du disque, mais une partie doit être attribuée à la formation d'une barre. Nous trouvons que la présence du gaz dans le disque est un facteur plus efficace que la triaxialité du halo pour inhiber la formation d'une barre forte.

mots-clés: astrophysique – dynamique galactique – galaxies: évolution – galaxies: halos – méthodes: simulations numériques