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# Toward a richer history of the representative agent: the contributions of Tjalling Koopmans and Paul Samuelson before Robert Lucas

## Por uma história mais rica do agente representativo: as contribuições de Tjalling Koopmans e Paul Samuelson antes de Robert Lucas

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*To Louise Chu*

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# Resumo

O Agente representativo é atualmente uma ferramenta metodológica onipresente em economia. No entanto sua história ainda não está totalmente desenvolvida. Essa tese contribui com esse estudo histórico através da análise de três contextos separados, mas, sobrepostos.

O primeiro capítulo examina a ascensão do consumidor representativo nas décadas de 1950 a 1970 nas contribuições à economia intertemporal de Tjalling Koopmans. Na primeira década, de 1950, Koopmans foi um nome importante na incursão da Comissão Cowles na teoria da decisão e, como econometrista, também participou ativamente do debate sobre problemas de agregação na economia. Na década de 1960, Koopmans escreveu a maior parte de suas contribuições para o subcampo da economia do horizonte infinito (incluindo seu modelo de crescimento ótimo) e é nessa década que ele articulou de forma completa suas visões sobre o agente representativo. Finalmente, na década de 1970, Koopmans continuou contribuindo para a teoria das decisões individuais baseada nas preferências, levando-o à elaboração das funções de utilidade intertemporalmente separáveis. Ao longo dessas três décadas, Koopmans passou de um posicionamento ambíguo em relação ao consumidor representativo para outro mais inclusivo. Curiosamente, seu artigo de crescimento de 1965, que ajudou a disseminar o agente representativo em macroeconomia, pode ser visto como um ponto de virada no entendimento de Koopmans. Parte dessa mudança se deve ao uso cada vez maior do dispositivo na macroeconomia, um movimento que ele não iniciou mas ajudou a intensificar.

O segundo capítulo pergunta se o agente representativo pode ter emergido como o resultado de transformações que ocorreram na microeconomia ao longo das décadas de 1930 e 1940, especialmente no subcampo da teoria da demanda. Para contar essa história, começo com uma interpretação histórica particular desse subcampo, proposta por Wade Hands e Philip Mirowski na década de 1990, centrada nas formulações teóricas e nos testes econométricos subsequentes do sistema de funções de demanda que envolviam o matemático Harold Hotelling e o economista Henry Schultz, conhecido como o impasse de Hotelling-Schultz. Embora esse impasse tenha sido abandonado por Schultz e Hotelling ao final da década de

1930, o mesmo continuou na profissão, inclusive na Comissão Cowles, então dirigida por Koopmans. Ele desempenhou um papel importante no surgimento do agente representativo na microeconomia derivado dos problemas de agregação. O significado da introdução de preferências homotéticas de Paul Samuelson na teoria do Equilíbrio Geral e sua conexão com os escritos de Koopmans durante a década de 1950 também é examinado.

O terceiro capítulo identifica o surgimento do agente representativo no desenvolvimento da literatura de crescimento ótimo. Embora Paul Samuelson tenha usado consumidores representativos de vida infinita para lançar luz sobre tópicos macroeconômicos em seus trabalhos de meados da década de 1930 até o início da década de 1950, essa ferramenta só ganhou mais adeptos depois de ter sido "acordada" no início dos anos 60. É mostrado que o principal centro de pesquisa em economia de crescimento na época, o Instituto de Tecnologia de Massachusetts (MIT), reunindo membros do corpo docente e estudantes de pós-graduação que trabalhavam com a regra de ouro do crescimento, bem como a teoria do turnpike, ajudou a sancionar o agente representativo como ferramenta legítima para investigações macroeconômicas. Além disso, em comunidades além do MIT, economistas como Koopmans e Lionel McKenzie também poderiam ter desempenhado um papel na divulgação do dispositivo metodológico dado uma possível influência que Samuelson teve sobre eles.

**Palavra-Chaves**: Agente Representativo; Tjalling Koopmans; Paul Samuelson

# Abstract

The representative agent is nowadays a ubiquitous methodological tool used in modern economics. However, its history is not fully developed. This thesis begins to fill that gap by analyzing three separate, yet overlapping, contexts.

The first chapter examines the rise of the representative consumer from the 1950s to the 1970s in the contributions to intertemporal economics by Tjalling Koopmans. In the first decade, the 1950s, Koopmans was an important figure in the Cowles Commission's incursion into decision theory, and, as an econometrician, an active participant in the debate on aggregation problems in economics. In the 1960s, Koopmans wrote the bulk of his contributions to the subfield of infinite horizon economies (including his optimal growth model) and it is in this decade that he fully articulated his views on the representative agent. Finally, in the 1970s, Koopmans continued contributing to the preference-based approach to individual decision-making leading to his intertemporally separable utility functions. Over these three decades, Koopmans went from an ambiguous stance toward the representative consumer to a more supportive one. Interestingly, his 1965 growth paper, that helped spread the use of the representative agent in macroeconomics, can be seen as a turning point. Part of this change is due to the ever-increasing use of the device in macroeconomics, a movement that he did not initiate but helped intensify.

The second chapter asks whether the representative agent might have emerged as the outcome of transformations that occurred in microeconomics from the 1930s throughout the 1940s, especially in the subfield of demand theory. To tell this story, I begin with a particular historical interpretation of this subfield, propounded by Wade Hands and Philip Mirowski in the 1990s, centered on the theoretical formulations and the ensuing econometric testing of the system of demand functions that involved the mathematician Harold Hotelling and the economist Henry Schultz, known as the Hotelling-Schultz impasse. Although this impasse was abandoned by both authors by the end of the 1930s, it continued in such places as the Cowles Commission, then directed by Koopmans. He played an important role in the emergence of the representative agent in the microeconomics of aggregation problems. The significance of Paul Samuelson's introduction of homothetic preferences into general equilibrium theory and its connection to Koopmans's writings during the 1950s is also scrutinized.

The third chapter identifies the emergence of the representative agent in the development of the optimal growth literature. Although Paul Samuelson used infinitely-lived representative consumers to shed light on macroeconomic topics in his works from the mid-1930s to the early 1950s, this tool only gained more adepts after it was "agreed upon" at the beginning of the 1960s. It is shown that the main center of research in growth economics at the time, the Massachusetts Institute of Technology (MIT), by congregating faculty members and graduate students working with the golden-rule of growth as well as the turnpike theory, helped sanction the representative agent as a legitimate tool for macroeconomic investigations. Furthermore, in communities beyond MIT, economists such as Koopmans and Lionel McKenzie could have also played a role in spreading the methodological device, given the possible sway Samuelson had on them.

**Keywords**: Representative Agent; Tjalling Koopmans; Paul Samuelson

# Contents

# Introduction

*Fallacy of Composition: a fallacy in which what is true of a part is, on that account alone, alleged to be also true on the whole.*

- Paul Samuelson, *1955*

This dissertation is concerned with changes in economics that began to take shape only recently. By "recent" I mean the period after World War II, the era of the Cold War and, to to some extent, an age still feeling the effects of the great depression. It was also a time that witnessed the dawn of a new phase in the profession, which moved from a certain pluralism to the dominance of neoclassical economics. Historians, sociologists, and historians of economics have been studying time and again (not without reasons) these transformations. Although no "litmus test" can be provided as to what exactly neoclassicism consists of, it is broadly agreed that it has been responsible for the stabilization of a great number of economic ideas that are still with us today.[1,2]

The representative agent is one such idea. As I will try to show, this is not a story that took place exclusively within the field of modern macroeconomics, an approach made distinguishable by such eminent scholars as Ragnar Frisch, Jan Tinbergen and John Maynard Keynes.[3] It is above all a story focused on the least visible side, namely, the development of *microeconomics*, whose contributions came with no less illustrious names such as Tjalling Koopmans, William "Terence" Gorman and Paul Samuelson. Given the subject treated in this dissertation, related in no small part to the problems of aggregation in economics, one should also add the development of econometrics, a

[1] In this dissertation, I follow E. Roy Weintraub, for whom "(...) scientific work is knowledge creation in a context and that such knowledge is shared knowledge within a particular community" (Weintraub 1991, p. 4). Stabilization is, thus, a process in which a diverse group of people, belonging to different research communities, working in various places, and over a specific period of time, interacted and extended their networks and which ultimately resulted in the narrowing of their disagreements.

[2] Different strands within neoclassical economics fought for the hegemony of their ideas in the stabilization process of the postwar era. Diverse accounts of these episodes, focusing on different theoretical and empirical aspects, can be found, e.g., in Weintraub (1991, 2002), Morgan and Rutherford (Eds., 1998), Mirowski (2002, 2006, 2012), Hands (2010, 2016), Hands and Mirowski (1998), Duarte (2016), and Mirowski and Hands (1998).

[3] For an overview of the prehistory of macroeconomics (before Keynes's 1936 *General Theory*), see Dimand (2015).

development that adds to the complexity of this narrative. As Kevin Hoover stated, “the history of modern macroeconomics is importantly a history of its relationship to microeconomics and econometrics” (Hoover 2015, p. 400).

In writing this dissertation, I would like to point out how I intend to contribute to the history of economics literature. With the possible exceptions of Hoover (2012) and Hands (2017), it is puzzling to observe that the representative agent has been the subject of few historical studies.[4] Important publications, such as Kirman (1992), Janssen (1993), Hartley (1996, 1997), Hahn and Solow (1995), and King (2012), have paid attention to the theoretical and, above all, methodological shortcomings of the assumption of the representative agent, leaving unanswered the questions regarding the contexts by which it rose to its modern use.

In one of the first critical articles on macroeconomic models based on representative agents, Kirman (1992) said that the problem is their inability to deal with coordination problems (characteristic of capitalist economies) and, thus, “In such a world there would be no meaningful stock market, distributional considerations could not enter government policy and the very idea of asymmetric information would make no sense” (Kirman 1992, p. 118). In the book, *Microfoundations - a critical inquiry* (1993), Maarten Janssen argued that since both economic theories in general and those based upon microfoundations in particular are based on *individualism*, the representative agent could be a topic treated within the field of methodology; it is via this reasoning that the writings of the New Classical School, mostly of Robert Lucas, are seen throughout Janssen’s book.

In *The Representative Agent in Macroeconomics* (1997), James Hartley dedicated one chapter to the historical analysis of the representative agent, tracing its origin to Alfred Marhsall’s notion of a “representative firm.” The bulk of his analysis is, then, focused on such methodological discussions as the use of the representative agent by the New Classical School, first as an independent enterprise and later considered within the Walrasian tradition as well. In two other critical sources on the representative agent, Hahn

[4] A number of history articles have dealt with the representative agent, some in connection to the problems of aggregation (e.g., Boianovsky 2013, 2016a, 2016b; Hands 2016), one in connection to time discounting in growth models (Duarte 2016), and one that analyzes the representative agent within the context of the microfoundations movement of the late 1990s (Duarte 2012).

and Solow (1995) and King (2012) have also been (mostly) dedicated to the inadequacy of the device from a methodological perspective.

In John King's (2012) view, the use of RARE (a representative agent with rational expectations) is not adequate when applied to individual economic agents under a capitalist economic system, following the critical line advanced in Kirman (1992); still, according to King (2012), the first published versions of business cycle models based on such RARE microfoundations (derived from the Ramsey model) were Roberts Lucas's 1975 and 1977 articles. Finally, following a similar movement, on why they decided to write *A Critical Essay on Modern Macroeconomic Theory*, Hahn and Solow (1995, p. vi) stated: "We decided on this joint venture when we found that we share the same unease with the 'New Classical Macroeconomics' that was then just becoming dominant."

As the examples above show, there is an almost automatic (and correct) way of associating the representative agent with the research initiated by the New Classical School or with the methodological implications of its adoption for economics. In this vein, Hoover (2012) conducted a survey of microfoundations programs found throughout history and concluded that the one led by Robert Lucas was the "representative agent microfoundations program" (the only one that implied the "elimination of macroeconomics"). Hands (2017) has provided the literature with an account focused on the rise of the representative agent through complicated interactions in the field of microeconomics and macroeconomics, that is, on the role the representative agent has played in research in those two fields as well as in related subfields.

This thesis, thus, contributes to the history of the subject in three ways. First, it brings further details into Hands's (2017) examination, complementing, for example, his interpretation of what he called an "implicit" representative agent in microeconomics. Second, by demonstrating that the representative agent was already an inescapable issue in the early works of Koopmans, I believe I am doing "historical justice" to the concept, for in opposition to the studies I have cited, its initial use and later dominance in macroeconomics might not have initiated with the writings of Robert Lucas at the end of the 1960s and beginning of the 1970s. Third, by focusing on the canonical Ramsey-Cass-Koopmans model but with a particular view to the rise of the representative agent, this work will pick up from where Duarte (2010) left off, thereby analyzing how one of its proponents—Tjalling Koopmans—viewed and treated the artifice.

In the remaining pages of this introduction, I will explain what I mean by a representative agent and how it relates to aggregation problems found in economics, for example, in the subfield of international trade theory. This connection is intended to enhance the historical importance of the concepts, for an important economist in this dissertation, Paul Samuelson, made seminal contributions to both subjects.

As made clear by Wade Hands (2017), there have been several forms of representative units in economics: Alfred Marshall's "representative firm," each "representative class" in the classical economists' schemes (landlord, capitalists and workers) and Stanley Jevons's "trading bodies," to which I might add Tjalling Koopmans's notion of "subsistence farmer" (see chapter one of this dissertation). Since my intention is to study the representative agent as adopted in modern macroeconomics, the definition pursued here is the utility-maximizing representative agent, "(...) the 'rational economic agent' of mainstream microeconomics (...) as well as the rational individual agents in 'decision theory' and 'rational choice theory' in related fields" (ibid., p. 1686).

Following Acemoglu (2009), there are two ways to admit a representative agent in a model and in both cases the demand of a single person is taken to be equivalent to the demand of the entire economy (aggregate demand). In the first way, the *trivial way*, all households in an economy are infinitely-lived and identical and have the same discount factor $\beta^t$, the same sequence of effective labour endowment $\{e(t)\}_{t=0}^{\infty}$, and the same instantaneous utility function

$$u(c),$$

where $u : \mathbb{R}_+ \to \mathbb{R}$ is increasing and concave, and $c(t)$ is the consumption of household $h$. In this case, the representative agent can be used "(...) not only for positive analysis (the level of savings, e.g.), but also for normative analysis, such as evaluating the optimality of equilibria" (Acemoglu 2009, p. 150).

As Acemoglu argues, although the assumption of an economy populated with identical inhabitants is unrealistic, changing it to an economy with heterogeneous consumers and, therefore, modeling its aggregate demand has not been promising either; in fact, heterogeneous agent models developed so far have not led to the same level of usefulness as representative agent models; while the latter ones have been successful in generating insights into the process of growth due to the homogeneity of demand, heterogeneous agent models have failed to provide the same level of generality.[5]

To this nagging challenge the mainstream literature responded by trying to find out "(...) when an economy with heterogeneity can be modeled as if aggregate consumption level were generated by the optimization decision of a representative household" (ibid.). Then, to reach a solution, one generally begins with a general equilibrium model, a pure exchange economy with 2 consumers, and shows that its aggregate consumption, known as the aggregate excess demand function, is equivalent to the solution of a maximization problem for a single household.

However, not even an aggregation of this type is possible due to the independent findings by Hugo Sonnenschein (1973), Rolf Mantel (1974) and Gerard Debreu (1974): although the individual (excess) demand function satisfies the weak axiom of revealed preference (WARP) and have Slutsky matrices that are symmetric and negative semidefinite, the aggregate (excess) demand function does not satisfy them.[6,7] Thus, while individual demand curves are negatively sloped, aggregate demand curves can take any shape. Those results implied that if aggregate demand cannot be proved to exist (at least not the way economists wanted), every macro policy that is based on it would not make sense.

To enable the desired aggregation, an important restriction has to be imposed: the consumer must have a special type of preferences, denominated *homothetic preferences*, which leads to indirect utility of the Gorman form; with, and only with the imposition of

[5] Macroeconomic models with heterogeneous households (agents) have been substantially improved over the last ten years; see, for example, Baqaee and Farhi (2018) and the suggested references therein. The point argued here is that modeling with representative agents still pays off due to the homogeneity of demand it allows.

[6] For a historical analysis of the Sonnenschein-Mantel-Debreu theorems, see, e.g., Rizvi (2006) and Hands (2012b).

[7] In the appendix to chapter two, I discuss the properties of the individual demand function.

such a restriction, one reaches a second form to admit a representative agent in economic models, called the *technical way*.[8] This dissertation is, therefore, an attempt to tell the history of how this technicality came to be and, above all, the people involved in its creation and diffusion.

Aggregation has been a vexing topic in economics at least since Stanley Jevons's 1871 *Theory of Political Economy*, in which he resorted to a "fictitious mean," or an "average economical law", to grasp the complexity of an economy composed of heterogeneous agents and, therefore, as a way to replace the ordinary laws of supply and demand.[9] He justified his reasoning in the following manner: "(...) economical laws representing the conduct of large aggregates of individuals will never represent exactly the conduct of any one individual" since, he continued, "(...) a community is composed of persons differing widely in their powers, wants, habits, and possessions" (Jevons 1871, p. 90).

In the twentieth-century context, to obtain insights into the gains from trade between nations, trade economists deepened the search for a measure of society's overall welfare in which they brought out the concept of the "community indifference curve."[10] Contributing one of the earliest papers on the subject, Scitovsky (1942) admitted to a certain "irreversibility" in the profession on its pursuit of the analogy between persons and countries. Along this line, Paul Samuelson endorsed trade between countries as trade between individuals, for "in this way, the problem of weighting and combining different individuals' advantages within each country is avoided" (Samuelson 1938b, p. 262).

Two articles in the literature inaugurated a "modern era" in aggregation studies, both of which have proved (or supposedly so) the existence of a community indifference curve under very restrictive assumptions. The first one came out in 1953 in the journal *Econometrica* and was written by William "Terence" Gorman. According to Muellbauer (1976), Gorman's main results are sustainable as long as each consumer is assumed to have sufficient income and, in addition, their marginal propensity to consume for any good is the same. In other words, the gist of Gorman's argument is that income redistribution

[8] In the appendix to chapter two, I discuss very briefly the implications to economics of the adoption of homothetic preferences.
[9] Jevons (1871, chapter four) on the "Definition of Trading Bodies."
[10] A theme concomitantly researched in the field of welfare economics.

could not affect people's choices.

The second important paper was written by Paul Samuelson only three years later, in 1956, and was published in *The Quarterly Journal of Economics*. In this article, among other technical requirements, Samuelson built his proof of a well-behaved social indifference curve, denominated *social welfare contours*, by assuming a certain optimal reallocation of income within society capable of keeping "(...) the ethical worth of each person's marginal dollar equal" (Samuelson 1956, p. 21). Once again, resembling the argument also pushed forward in Gorman (1953), the redistribution of income across members of the society does not affect their subsequent choices.

The discussion of community indifference curves matters for two reasons, both of which serve as entrance to my historical examination into the representative consumer. First, as acknowledged by various authors such as Samuelson (1956), Chipman (1965a, b) and Muellbauer (1976), among others, both concepts are closely related. Second, specifically in the case of Samuelson (1956), the representative consumer was an offshoot of the proof of the social indifference curve and, as such, it became a common modeling device in other fields of study in economics such as general equilibrium theory (see chapter two) and optimal growth theory (see chapter three).

Nowhere in economics is the representative agent more explicit than in the subfield of economic growth.[11] As one would readily observe by either attending a current macro graduate course or skimming through the profession's most prestigious journals, the so-called Ramsey-Cass-Koopmans model became a predominant workhorse framework in the field, applied to topics as wide as fiscal and monetary policies, technology shocks, finance, human capital, and development issues, among others.[12]

In this thesis, my goal is to provide a historical account of how such a ubiquitous device emerged over the years and winded up taking on its present critical form. To do so, in the first chapter, I start by focusing on the rise of the representative agent in neoclassical

[11] See Hands (2017), for whom the assumption of the representative agent is explicit in macroeconomics, but implicit in microeconomics.

[12] The overlapping generations (OLG) model, which gained greater notoriety after Lucas's 1972 article, is its main rival framework in macroeconomics. See, for example, Geanakoplos (2008) and Assous and Duarte (2017).

choice theory as developed by Tjalling Koopmans in the roughly twenty-five-year period that began in the 1950s and continued until he won the economics Nobel prize in 1975, when he then decided to redirect his attention to more practical concerns.[13]

In the second chapter, I ask whether the representative agent might have been an offshoot of transformations that occurred in demand theory in the 1930s and 1940s. To tell this story, I begin with a particular interpretation of the field proposed in two separate articles by Wade Hands and Philip Mirowski in the 1990s, an episode they have dubbed the Hotelling-Schultz impasse. I argue that the representative agent was the result of the Cowles Commission's (failed) incursion into the field of applied microeconomics, an incursion initiated by the commission's research director Jacob Marschak, but later overturned by Koopmans when he took over the research directorship.

Finally, in the third chapter, I take heed of the emergence of optimal growth models in the 1960s as the inflection point in this investigation, a turning point for the representative agent in macroeconomics. I discuss how the emergence of the aforementioned subfield, sometimes under the turnpike theory variant, helped usher in the widespread use of the representative agent. Paramount in this process was the engagement of theorists from other research fields, especially general equilibrium and trade theories. I also discuss a possible role of the Massachussetts Institute of Technology's (MIT) economics department, specifically a graduate course co-taught in the 1960s by Edmund Phelps and Robert Solow.

I shall provide a justification for this narrative path. Modern macroeconomics, which has coalesced around dynamic stochastic general equilibrium (DSGE) models, emerged as a fusion of the Real Business Cycle (RBC) models and the New Keynesian literature. Edward Prescott, Finn Kydland, and their associates, in turn, used the optimal growth literature of the 1960s as their baseline models, whose major proponent was Tjalling Koopmans.[14] I then argue that the representative agent found in the early optimal growth models can also be traced back to the representative agent found in Koopmans's writings

[13] According to Scarf (1995, p. 286): "By the early 1970s Koopmans may have felt that the mathematical revolution led by him had been too successful - that elaborate mathematical arguments were being advanced throughout the profession to the neglect of more immediate practical concerns. He began to apply the techniques of growth theory to the study of exhaustible resources and, in particular, those resources used in the provision of energy."

[14] For an account of this episode see, for example, De Vroey (2016) and Duarte (2012).

in the field of consumer theory in the 1950s.

Although it seems a straightforward story, I shall consider some historical facts taking place at the same time and which, in my view, enhance the complexity of this historical narrative. First, initiated around the 1940s, a different economic theorizing was just under way, one characterized by the heavy use of axiomatic methods (Weintraub 1991, 2002). Second, "the time was ripe" (Düppe and Weintraub 2014) for the intertwined development of activity analysis (a particular form of linear programming) and general equilibrium theory; indeed, Koopmans used the studies in the first field to formulate the theory of production that other economists would use in the studies of the latter field (Mirowski 2002, chapters 5 and 6; Düppe and Weintraub 2014). As I will attempt to unravel in the first chapter, Koopmans's stances concerning the representative consumer were the result of, or perhaps the motivation for, his incursions into those two subfields. Understanding how those scientific communities overlapped, a point overlooked even by historians, may help in the construction of a clearer history of this episode. Third, even though new studies have proved the highly restrictive nature of aggregation problems, theoretical and applied research that relies upon the representative agent has continued to gain ground since the 1990s (see chapter two); casting light on this latter fact probably requires an assessment beyond economics and, hence, into the social studies of science.

To conclude this introduction, I would like to consider briefly why history matters and, by extension, why I chose this field for my economics doctoral dissertation. As Craufurd D. Goodwin asserted in his reflection on the state of the discipline in a not far away epoch, since "[History of economic thought] was thought of as simply an historical extension of theory, and the practitioner as simply a special kind of theorist with a long time horizon" (Goodwin 2008, p. 2), it is also my belief that "doing history of economic thought" is ultimately "doing economics." Thus, following Goodwin's reasoning, a historical investigation of optimal growth models and demand theory as a whole, and centered on the representative agent in particular, is fundamentally an investigation of modern macroeconomics. Along this line, I quote a passage by the econometrician Roy J. Epstein on what led him to write a PhD dissertation on the history of econometrics:

> (...) I discovered a long history of substantative debates over methodology that complements, and even extends, the critiques put forth recently by some of the most respected modern practitioners. It is my hope that readers will find this history as relevant and enlightening as I did in understanding the current state of macroeconomics (Epstein 1987, preface).

I hope this research, on the rise of the representative agent, will also help in understanding the current state of macroeconomics.

# 1 Preference, Utility, and Koopmans's Stance on the Representative Consumer

## 1.1 Introduction

A pivotal step in the development of modern macroeconomics consisted in the axiomatization of time preference and the ensuing specification of a utility function in discounted form.[1] In a series of papers, notably Koopmans (1950, 1953, 1960, 1964, 1972a, 1972b) and Koopmans, Diamond, and Williamson (1964), Tjalling Charles Koopmans made foundational contributions to the topic of intertemporal choices. With varying degrees of technical as well as historical importance, in common, those articles extended the ordinary formulation of preferences and the utility functions over time. The following passage, from one of the aforementioned articles, exemplifies an important advancement in this research effort:

> In a previous article one of the authors [8] studied some implications of a set of postulates concerning a preference ordering of consumption programs for an infinite future. The preference ordering was assumed to be representable by a numerical utility function defined on the *space* of consumption programs, and the postulates were formulated as properties of that function. While these postulates themselves appeared to be concerned only with properties more immediate and elementary than any questions of timing preference, it was found that the postulates implied, at least in certain parts of the program space, a preference for advancing the timing of future satisfaction. This conclusion was expressed by the concept of *impatience* (Koopmans et al. 1964, p. 82, italics added).[2]

[1] See Duarte (2016) on how mathematicians and economists strove to narrow their understanding of time discounting in the 1920s-1960s period, going from a nagging ethical subject to a pure technical requirement.

[2] The coinage of the term "impatience" owes first to the Austrian economist Eugen von Böhm-Bawerk, who introduced it into the theory of rate of interest (Christ and Hurwicz 2008) and, later, to Fisher (1930), for whom "(...) individuals facing the alternatives of having a given reward today or in the future opt for current reward" (Duarte 2016, p. 292).

The article “[8]” mentioned above referred to Koopmans’s 1960 “Stationary Ordinal Utility and Impatience.” Notwithstanding that impatience used to be regarded as an irrational behaviour, an economic disharmony according to Arthur Cecil Pigou, from Koopmans’s 1960 article onward it became a logical outcome out of a set of rational postulates underlying a utility function.[3]

But, as claimed by Christ and Hurwicz (2008), an important facet in Koopmans’s contributions to intertemporal economics was the seemingly trivial change in his research agenda in the 1970s: from the postulates underlying utility functions, Koopmans (1972a, b) turned the focus to the postulates underlying preference relations instead. Such a re-orientation in his research affected the way the author approached the representative consumer, going from a less enthusiastic view to a more favourable one as his career progressed.

Founded on, but not limited to, the assumptions of continuity, stationarity, and total independence over time in preference relations, he then proved that a utility function over an *infinite program* had now a certain desirable *additive property*. In other words, not only did the existence problem of the continuous utility function appear to be solved, Koopmans now wished to expand its scope to focus on the intertemporal additive properties of preferences.[4]

As I will try to substantiate in this chapter, such an episode seems to have led to an important historical feature: an ever-increasing explicitness of the representative agent in macroeconomics in the 1970s and 1980s.[5] As Kevin Hoover gauged using the JSTOR database, shown in table 1 below, the number of mentions of the terms “representative agent,” “representative consumer,” and “representative household,” for the periods “through 1969” and “1970 and after,” went from 0, 1, and 0 to 157, 107, and 69, respectively.

[3] For an examination of Arthur Pigou’s and other Cambridge economists’ thoughts on welfare analysis, particularly on their stances against discounting future utilities, see Collard (1996).

[4] Debreu (1959), section 4.6, was among the first to prove the existence of a continuous utility function.

[5] It might not be a coincidence that the main results contradicting the possibility of consumer aggregation, e.g., the Sonneschein-Mantel-Debreu theorem, were also established in the 1970s. See Rizvi (2006), Hands (2012b), and their suggested references for a historical overview of that theorem.

Table 1 – The use of expressions "representative agent," "representative consumer" and "representative household"

| | Through 1969 | 1970 and after |
|---|---|---|
| Representative Agent | 0 | 157 |
| Representative Consumer | 1 | 107 |
| Representative Household | 0 | 69 |

Source: Hoover (2012)

A possible reason is that the additive properties Koopmans helped build might have now become an important complement to the first established, but sometimes neglected, proof of aggregation across individuals provided in Gorman (1953) and, as such, strengthened the overall technical justification of that methodological device;[6] coupled with Samuelson's contributions regarding homothetic preferences (Hands 2016), it could have freed the representative agent to be used in many other subfields of macroeconomics as well, such as the stochastic optimal growth literature found in Brock and Mirman (1972).

The rest of the chapter is organized as follows. Section 1.2 presents Koopmans's initial works in the subfield of intertemporal economics in the 1950s, a contribution marked by aggregation concerns and outshone by his incursions into activity analysis. Section 1.3, focusing on Koopmans's writings in the 1960s, discusses consumer choices with an infinite horizon in a context of optimal growth problems. Section 1.4 describes Koopmans's research reorientation in the 1970s, when he shifted his attention to investigate postulates of preference relations. The possible outcome of this new research to modern macroeconomics is delineated. Section 2.5 offers some concluding remarks.[7]

6 Many authors consider Gorman (1953) to be the first proof of the representative consumer, for instance, Muellbauer (1976), Deaton and Muellbauer (1980), Lewbel (1994), Blackborby et al. (2008), and Acemoglu (2009).

7 With a view to giving greater clarity to Koopmans's writings, I chose to divide his articles into those published in the 1950s, 1960s and 1970s. A different way to discern a segmentation in his writings is through his optimal growth paper (Koopmans 1963). Even though the final versions of the articles published in the 1970s came out as early as the mid-1960s they were, nonetheless, written after his growth article was finalized. In other words, the aforesaid *change* in the emphasis of his papers took shape only after his foray into the realm of normative growth theory (Christ and Hurwicz 2008). I expect to demonstrate why in the subsequent sections.

## 1.2 Koopmans's intertemporal economics in the 1950s: overshadowed by Activity Analysis?

Koopmans's early incursion into the field of infinite horizon economies took place as early as the 1950s with two important short publications. As explained by a long-time colleague, in those writings he sought to study in earnest "(...) the desirability of postponing unnecessary future decisions" (Chipman 2006b, p. 531).[8]

Initially presented at the 1949 Boulder (Colorado) meeting of the Econometric Society, and published in 1950 as a report of the organization's journal *Econometrica*, "Utility Analysis of Decisions Affecting Future Well Being" laid down the initial reasoning and little of the technical ground that would later serve the author's long-term engagement with utility analysis. With the utility function based originally upon a complete and static ordering of objects, Koopmans contended that its sought-after dynamic generalization oftentimes ignored the desire for postponement of decisions in cases of unpredictable events. To deal with it, he understood that:

> Assets can then be entered in the utility function as representative of the *sets of consumption sequences* they give access to, through direct enjoyment, resale and purchase of other assets or consumption flows, alternatively or in succession, subject to later decision (Koopmans 1950, p. 175, italics added).

Koopmans continued by drawing special attention to the role played by borrowing, or consuming financing, to the well-being of consumers, since "(...) it gives access to the flow of services associated with the possession of durable consumer's goods at an earlier time than would otherwise be possible at the same rate of saving" (ibid.).

In the 1953 edition of *Cahiers du Séminaire d'Économétrie*, a special volume

[8] John Chipman completed his PhD at Johns Hopkins University in 1951 and during the 1950-51 academic year was a post-doctoral fellow at the University of Chicago's department of economics and a guest fellow at the Cowles Commission; his close contact with Koopmans took place while the latter served as a professor at the University of Chicago and had an appointment as research director at Cowles around the same period. Chipman has important publications in international trade theory, econometric theory and microeconomics, especially in demand theory. He has also written history of economics papers. Chipman taught at the University of Minnesota (Economics) from 1955 until his retirement in 2007.

dedicated to choice theory, Koopmans contributed a small article entitled "La Notion d'Utilité dans Le Cas de Décisions Concernant le Bien-Être Futur," in which he extended the central idea pursued in his previous article.[9] By contrasting the concept of "preferences" to that of "possibilities" he enabled the introduction of a set $Q$, or ordered sequences, into the utility function; and via the replacement of points by sets he then permitted uncertainty to play an explicit role in choice theory.[10] In the discrete case, as drawn from his original article and reproduced in figure 1, it is possible to represent the sets in *tree form* whose ramifications portray every possible choice which varies according to different contingencies.[11]

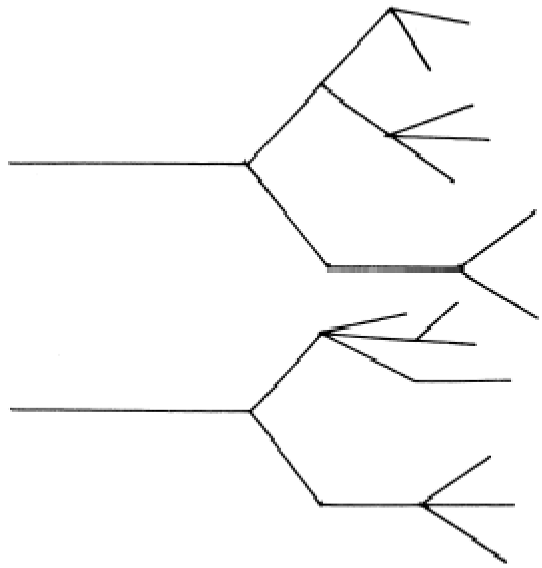

Figure 1 – Ramification of Choices

In light of Koopmans's early treatment of preferences over time in the 1950s, it is possible to suggest that those articles led him to rewrite the utility function as a *time*

[9] It is curious to observe that the 1950 and 1953 articles had similar titles. The French journal *Cahiers du Séminaire d'Économétrie* is published nowadays under the title *Annals of Economics and Statistics.*

[10] *Decision-making under uncertainty* became part of a broader research "package" in *decision theory* after Koopmans joined the Cowles Commission in 1943 and "reengineered" it under the influence of von Neumman. According to Mirowski (2002, chapter five), other packages included in this new wave were *linear programming*, *activity analysis*, and *Arrow-Debreu general equilibrium.*

[11] As I will show in section 1.4, the notion of choices representable in tree forms, also called *separable utility functions*, would be formalized (axiomatized) by Koopmans only in the articles published in the 1970s. Interestingly, even though it took Koopmans almost twenty years to re-engage with the formal properties of such utility functions, they continued to be studied by Robert Strotz (1957, 1959) and William Gorman (1959a, b). How Koopmans's contributions can be understood considering Strotz's and Gorman's will then be discussed in the same section.

*aggregator,* whose parts were composed of an *one-period utility* and a *prospective utility*, as seen in the articles “Stationary Ordinal Utility and Impatience” and “Stationary Utility and Time Perspective,” published in the 1960s. In the next section, I will discuss some of the technical aspects that enabled Koopmans to attain this outcome.

While Koopmans’s writings on the topic of preferences over time in the 1960s featured more advanced mathematical methods, which he would often use to analyze issues related to the representative consumer, his 1950s endeavors in the field were characterized by a dearth of such technicalities. It does not follow from this fact that the aggregation problem was not for him a nagging concern. In a series of letters exchanged between Koopmans and Marc Nerlove in January of 1957, the issue of aggregation emerged not only as a difficulty limited to a field, but a more general concern within economics.[12]

In a letter dated January 10, 1957, Koopmans asked Nerlove about his recent research activities at the Agricultural Marketing Service of the United States Department of Agriculture (USDA).[13] Nerlove replied with a rather detailed description of his inquiries into, *inter alia*, the statistical properties of distributed lags models, their applications to the estimation of supply functions for agricultural commodities, their intertwined relationship with demand functions (in which Milton Friedman’s permanent income hypothesis drew most of his interest) and, essentially, some problematic instances in the case such as the inapplicability of Friedman’s consumption function “to cover individual items of consumption.”[14,15]

12 In the summer of 1953, as a graduate student, Marc Nerlove worked as a research assistant for Tjalling Koopmans and Jacob Marschak at the Cowles Commission. In 1956, under the supervision of Carl Christ, he earned a PhD in Economics from Johns Hopkins University with a dissertation measuring the dynamics and elasticity of the U.S. agricultural supply. In 1956-1957, the period of the correspondence, Nerlove worked as an analytical statistician at the United States Department of Agriculture (USDA).

13 Letter from Koopmans to Nerlove, January 10, 1957, TKP, box 16, file folder 304.

14 Letter from Nerlove to Koopmans, January 27, 1957, TKP, box 16, file folder 304.

15 Milton Friedman’s intuitive notion of the *permanent income hypothesis* is derived from chapter three of his famous book *A Theory of the Consumption Function,* published in 1957. The gist of his argument is that agents’ consumption behaviours are better predicted with a non-directly observable “mean of the expected level of income in the very near-term” (Carroll 2001, p. 24), labeled “permanent income,” instead of the usual “recorded income” as postulated by Keynes (De Vroey 2016). What should be clear to better comprehend Nerlove’s interpretation of the Friedman’s consumption function, and by extension, the story presented in this thesis, is that both Friedman’s model and the neoclassical theory of consumer’ choice were initially theorized at the individual levels. For an empirical economist like Nerlove, whose works came down to estimating aggregate relationships, the incongruence between both approaches overshadowed his analyses. It is precisely this frustration that prompted Henri Theil (see footnote 17 for a brief sketch of his biography) to write the opening paragraph of his 1957 book and

Referring specifically to those, as well as to other concerns raised by Nerlove within econometrics, Koopmans then responded with the following overall intuition:[16]

> I have not given much thought to the problem of distributed lags in the last few years. However, I have developed a general feeling that the clue to a number of different problems in econometrics lies in further exploration of aggregation problems, that ties our notions of individual decision making with somewhat more aggregated relationships. I doubt that this reflection is of much value to you in regard to distributed lags, but it does seem to me that this is one of the problems that may stand to gain if progress should be made on aggregation. Theil's book is excellent as far as it goes. My main reservation about it is that it studies primarily the implications of current econometric procedures, rather than trying to answer the deeper question what procedures we should be using in view of aggregation problems.[17]

Even though the exchange of views on the aggregation problems between the authors arose in the field of applied econometrics, there was also a clear comprehension of its theoretical nature as captured in Koopmans's indication of the need to handle such a more difficult "deeper question" and in Theil's (1954, p. 1) own explanation of the problem:

> A serious gap exists between the greater part of rigorous economic theory and the pragmatic way in which economic systems are empirically analysed. Axiomatically founded theories refer mostly to individuals, for instance the consumer or the entrepreneur. Empirical descriptions of economic actions in

which I chose to quote at full length below.

[16] Letter from Koopmans to Nerlove, January 31, 1957, TKP, box 16, file folder 304.

[17] The book mentioned by Koopmans in the correspondence was Theil's *Linear Aggregation of Economic Relations*, published in 1954. Henri Theil was born in 1924 in Amsterdam and passed away in the United States in 2000. After earning his doctorate in Economics (with distinction) from the University of Amsterdam in 1951, Theil began his career as a macroeconomic analyst and forecaster for the Central Planning Bureau (CPB) of the government of the Netherlands under the directorship of Jan Tibergen. Between 1953 and 1966 he taught econometrics at Erasmus University Rotterdam (formerly the Netherlands School of Economics in Rotterdam), where he also founded and directed The Econometric Institute. From 1966 to 1981 he was a member of Chicago's department of economics. From 1981 until his passing, Theil remained affiliated with the University of Florida. Over a period of five decades he published more than 250 articles and 17 books, of which the one mentioned by Koopmans was his very first (Bewley 2000).

> large communities, on the other hand, are nearly always extremely global: they are confined to the behaviour of groups of individuals. The necessity of such a procedure can scarcely be questioned: How could one handle the tremendous mass of relations governing the actions of millions of individuals? But the introduction of relations pretending to describe the reactions of groups of individuals instead of single individuals raises questions of fundamental importance, which are not very well understood: What are the connections between the functional relationships postulated by the economic theories of individual households and the relationship for groups of individuals postulated by the empirical research worker? If the analysis shows that these relations are not of a "desirable" type, is it possible to indicate rules for the construction of group relationships that must be obeyed in order that certain optimum criteria are fulfilled? And how must these criteria been chosen?
>
> These are questions that must be answered by the theory of aggregation.[18]

In what follows, I shall make a few remarks concerning Koopmans's publications in the 1950s vis-à-vis his overall contributions to the subfield. As far as I am aware, among all appreciations written so far about Koopmans's contributions to economics or about his professional life, such as Koopmans's own Nobel Memorial Lecture written in 1975, Werin and Jungenfelt (1976), Niehans (1990), Scarf (1995), Chipman (2006b), and Christ and Hurwicz (2008), only Chipman (2006b) cited the two articles I mentioned above in the body of their texts as an indication of their importance to economics (although Werin and Jungenfelt (1976) did at least list the 1953 piece in their references). In the two-volume *Scientific Papers of Tjalling C. Koopmans*, published in 1970 and 1985, none of the 1950s papers regarding intertemporal economics were included, even though in the preface to the first volume Beckmann, Christ, and Nerlove (1970, p. v) wrote: "In making our selection

[18] Following Nerlove's research experiences, it is interesting to observe that many theoretical problems seem to surface more frequently in applied works than in theoretical ones proper. For example, aggregation issues were more of a problem in the "modeling Resource Group" (MRG) than in any other academic communities Koopmans belonged to. The MRG was an association of economists and other experts created to provide the Committee on Nuclear and Alternative Energy System (CONAES) with economic information obtained through mathematical models. In addition to Tjalling Koopmans, William Nordhaus, Hendrik Houthakker and Kenneth Arrow took part in the discussions of the group.

of his papers to be included in this volume, we have attempted to include significant contributions in each of the major areas of Koopmans' work."[19]

As to why the profession might have overlooked his contributions in intertemporal economics at the time, a possible answer is that the subject could have been upstaged by the parallel development of activity analysis, whose 1949 conference, "Activity Analysis of Production and Allocation," drew most of Koopmans's attention for the remaining years of the 1950s.[20] This can be observed in the ensuing publications of the conference volume (Koopmans 1951) and, especially, of his celebrated *Three Essays on the State of Economic Science* (1957). To get a more reliable answer to the question, though, bibliometric research would be worth conducting, helping us to shed light on such related questions as how have the citations of Koopmans's writings evolved since he published those papers in the 1950s.

By bringing further details into the works of the Dutch economist in the 1950s and, more important, by later tightening them up with his writings in subsequent sections, I intend to contribute with a narrative of Koopmans's works in the subfield of intertemporal economics. Such a narrative shall therefore aid in the understanding of his thoughts regarding the representative consumer.

## 1.3 Koopmans's Intertemporal Economics in the 1960s: articulation of the representative agent

The bulk of Koopmans's contributions to infinite horizon economies appeared in a series of papers concentrated in the first half of the 1960s. The intertemporal utility function with time discounting that appeared in Koopmans's 1960 "Stationary Ordinal Utility and Impatience" had the form[21]

$$U = \sum_{t=1}^{\infty} \alpha^{t-1} u(x_t), \tag{1.1}$$

[19] The authors considered such areas to be econometrics, activity analysis, and infinite horizon economies (intertemporal economics).

[20] The importance of the 1949 conference to the development of economic science in the second half of the twentieth century was only recently made more clear by Düppe and Weintraub (2014).

[21] The first version of this work was the Cowles Foundation Discussion Paper No. 81, dated November, 1959.

where $\alpha \in (0,1)$ denoted a (constant) exponential discount factor, $x_t$ a vector of goods, and $u(\cdot)$ an instantaneous utility function.[22] The expression, interpreted as representing a preference ordering of infinite program, had the following comment by the author (Koopmans 1960, p. 288):

> Flexibility of interpretation remains as to whether this ordering may serve as a first approximation to the preferences of an individual consumer, or may perhaps be an "impersonal" result of the aggregation of somewhat similar individual preferences (interpreting "consumption" as "consumption per head" in the case of a growing population), or finally may guide choices in a centrally planned economy. In each of these interpretations further modifications and refinements may be called for.

Jointly written with Peter Diamond and Richard Williamson, and published in *Econometrica* in 1964, "Stationary Utility and Time Perspective" consisted primarily of direct generalizations of the notion of *impatience* found in Koopmans (1960) by way of a discovery of a deeper property of preference ordering, called *time perspective* (hence the titles of both articles), applicable to a larger part of the commodity space.[23,24] Among other goals, the article demonstrated that in comparing two consumption programs of the same period, for example, programs $A$ and $B$, where the first is strictly preferred to the second, the more the consumer postponed his or her consumption, the smaller would be the difference assigned by the utility function; put another way, time mattered in the authors' account of preference and utility.

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[22] The instantaneous utility function is an essential concept in both macroeconomics and growth theory, and a description of the function is thus worth quoting at length from a major textbook on growth theory: "The instantaneous utility function captures the utility that an individual derives from consumption at time $t$. It is therefore *not* the same as a utility function specifying a complete preference ordering over all commodities - here consumption levels in all dates. For this reason, the instantaneous utility function is sometimes referred to as the 'felicity function'" (Acemoglu 2009, p. 180, italics added).

[23] Following Mas-Colell et al. (1995, p. 19), the consumption set is the set of all nonnegative bundles of commodities $X = \mathbb{R}^L_+ = \{x \in \mathbb{R}^L : x_l \geq 0 \text{ for } l = 1, ..., L\}$, where $\mathbb{R}^L$ is the commodity space.

[24] The first version of this paper was a Cowles Foundation Discussion Paper No. 142, dated August, 1962. In his 2010 Nobel memoir Peter Diamond recounted, with inescapable gratitude, how his mathematical ability helped reorient Koopmans's research while working as his research assistant in the summer of 1960 (when Diamond had not even yet begun graduate studies at MIT), a fact that stirred Koopmans to promote him as a co-author of the 1964 paper.

An example shall illustrate this point. Suppose an individual prefers apples to oranges today. A utility function then would assign a higher value for the first fruit. In Koopmans, Diamond, and Williamson's interpretation, since consumers are impatient, the preference for apples over oranges would be diluted when he/she tried to compare both fruits in, say, twenty months from today, leading to an ever-lower difference assigned by the utility function as time passes. Yet, to achieve this outcome, a real-valued utility function had to be capable of representing an ordering of sequences over time of commodity bundles in the first place.

If ${}_1x = (x_1, x_2, ..., x_t)$ represents an infinite-dimensional sequence (over time) of a particular commodity bundle, it can alternatively be written ${}_1x = (x_1, {}_2x)$, where ${}_2x = (x_2, x_3, ..., x_t)$ is a sequence of commodity bundles starting at $t = 2$. Thus, the initial notation can be rewritten as ${}_1x \equiv (x_1, {}_2x)$ or, considering only two periods, ${}_1x \equiv (x_1, x_2)$. In this definition $x_1$ and $x_2$ constitute the same commodity vector (consumption bundle) in the consumption set $X$ but for periods 1 and 2, respectively.[25] The authors' intention was to justify the formulation of $U({}_1x)$ as a function of the instantaneous utility function $u_n(x_n)$ and, therefore, attain an intertemporal utility function such as $U({}_1x) = (u_1(x_1), u_2(x_2), ..., u_n(x_n))$. By accepting the postulates P1 (existence and continuity), P2 (sensitivity), P3 (limited complementarity), P4 (stationarity), and P5 (extreme programs), and undertaking various proofs, $U({}_1x)$ could then be written in the form

$$U({}_1x) = V(u(x_1), U({}_2x)), \tag{1.2}$$

where $V(u, U)$ is continuous and increasing in its two arguments $u$ and $U$. As $u(x_1)$ and $U({}_2x)$ can be defined, respectively, as an *one-period* (or an *immediate*) *utility* and a *prospective utility*, $U({}_1x)$ can be redefined as an *aggregate utility*, where the function $V(\cdot)$ is the *time aggregator* (over all future time periods). Related to this function, Koopmans (1960, p. 292) stated that it "(...) indicates how any given pair of utility levels, immediate ($u_1$) and prospective ($U_2$) stacks up against any other pair in making choices for the entire

[25] Throughout this chapter, I follow the notations adopted by Koopmans in his examination of the subject. To clarify, a consumption vector $x_t$ can also be understood as $x_t = (x_{t1}, x_{t2}, ..., x_{tl})$, that is, assuming a $l-$dimensional commodity space.

future."[26] By iteration, (1.2) can be generalized to

$$U({}_1x) = V(u(x_1), u(x_2), u(x_3), ..., u(x_\tau), U({}_{\tau+1}x)) \tag{1.3}$$

for all $\tau$. Bearing on a result attributed to Debreu (1960), in which postulate $3'$ (independence) is added to the set of postulates, a monotonic transformation can be performed on (1.3) such that (using $t = 3$)

$$U({}_1x) = u_1(x_1) + u_2(x_2) + U_3({}_3x). \tag{1.4}$$

Finally, as Koopmans (1960) and Phlips (1983) argued, equation (1.4) is usually written in the literature as

$$U({}_1x) = \sum_{t=1}^{\infty} \kappa^{t-1} u(x_t), \quad \kappa = \frac{1}{1+\gamma}, \quad 0 < \kappa < 1 \tag{1.5}$$

or

$$U({}_1x) = \int_0^{\infty} e^{-\rho t} u(x_t) dt \tag{1.6}$$

for discrete and continuous time, respectively.[27] The significance of the additively separable preference assumption (postulates 3 and $3'$) cannot be underestimated. It is indispensable in an intertemporal problem solved by either a household or a central planner: in a workhorse Ramsey-Cass-Koopmans model, each reaches an Euler equation where the marginal rate of substitution in consumption between periods $t$ and $t + 1$ is independent of future periods.[28]

Koopmans's ingenious way of working out an intertemporal utility function depended unavoidably on how he aggregated the time-varying instantaneous utility function. Although he never explicitly identified such a function with the (aggregate) expression

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[26] The passage refers indeed to an expression obtained prior to equation (1.2), namely, $U({}_1x) = V(u_1(x_1), U_2({}_2x))$, where the difference consists in the appearance of time subscripts in both arguments of the function $V$. Nevertheless, since my concern is to focus on the aggregative feature of $U({}_1x)$, both expressions play the same role. See Phlips (1983, chapter XI) for further clarifications on this point.

[27] Koopmans stated that equation (1.5), where "aggregate utility is a discounted sum of all future one-period utilities, with a constant discount factor $\kappa$" (1960, p. 308), had been used "extensively" before, such as in Ramsey (1928), Samuelson and Solow (1956), and Strotz (1957), but with $\kappa = 1$ imposed in the first two cases.

[28] In such problems, an intertemporal utility function of the form (1.5) or (1.6) is optimized subject to a feasibility or technology constraint (in the centralized case) or subject to a wealth constraint (in the decentralized one).

"representative consumer," one could argue that the author always had the latter construct in mind: as is the case with intertemporal preference and intertemporal utility, the representative agent also enabled a correspondence between consumer and central planner.

In a 1964 sole-authored paper entitled "On Flexibility of Future Preferences," an examination of the effects of uncertainties on agents' preferences in coming, yet distant periods, Koopmans adopted an open stance regarding to whom should such a (future) preference belong by stating from the outset: "The economist's traditional model of choice - *whether consumer's choice or planner's choice* - is based on an analytical separation of preferences and opportunity" (Koopmans 1964, p. 469, italics added).[29] In another passage, Koopmans alluded to the usefulness (and therefore ability) of a central planner acting as some sort of aggregator of preferences. According to him:

> (...) taste evolves with experience. A model that freezes preferences by the adoption at an initial point in time of an ordering of programs for a future period of indefinite duration is likely to become an unacceptable straight jacket as time proceeds. At what age would the individual consumer be supposed to embrace the ordering that is to guide all his consumption choices for the remainder of his life? *And the economic planner, who presumably attempts to aggregate the preferences of the population - perhaps with some admixture of his own values - wouldn't he wish to retain flexibility so that he could respond in the future to newly perceived currents of taste and desire?* In fact, would it be morally defensible for one generation to presume, and act on, a very definite specification of the next generation's preferences? (ibid., p. 472, italics added).

As pointed out by Duarte (2016), some controversial points involved in the analysis of the individual's intertemporal allocation problem compared to the planner's were already raised by Amartya Sen and Stephen Marglin in the 1950s and 1960s; Koopmans's words quoted above could be a reaction to this discussion and, in this sense, his treatment of individual vis-à-vis centralizer figures, such as "the subsistence farmer" or the "the

[29] This short paper appeared first as a Cowles Foundation Discussion Paper dated No. 150, December, 1962.

economic planner" (Koopmans 1964, p. 470) might be taken as an example of the care Koopmans granted to the issue as well.

In a particular study centered on the context of axiomatizaton of time preference and the subsequent incorporation of time discounting in optimal growth models, also in the 1960s, Duarte (2016, p. 294) claimed: "While Cass advocates an infinite horizon, discounted optimization problem, he does not make an argument for basing social welfare on the representative agent's utility function." In contrast, "(...) Koopmans blurred the lines between individual and social intertemporal choices" and, as such, made "(...) more room for discounting to become a rather technical requirement" (ibid.).

In his famous optimal growth article written in 1963, "On the Concept of Optimal Economic Growth," Koopmans introduced a slightly modified version of (1.1), but now applied to examine normative features of economic growth:[30]

$$U = \sum_{t=1}^{\infty} \kappa^{t-1} u(x_t), \tag{1.7}$$

where $\kappa \in (0, 1)$ is the constant discount factor. Looking for mathematical simplicity, his analysis into the existence and characteristics of optimal growth paths resorted to the finite-horizon utility function $U = \int_0^T u(x_t)dt$, where the simple integration of the instantaneous utility flow $u(x_t)$ "(...) implies *noncomplementarity* between consumption in any two or more parts of the future" (Koopmans 1963, p. 20). Applying the discount factor $e^{\rho t}$ and pushing time horizon to infinity, the latter expression could then be rewritten as

$$V(\rho) = \int_0^{\infty} e^{-\rho t} u(x_t)dt, \tag{1.8}$$

where $\rho$ is a positive instantaneous discount rate strictly greater than 0, whilst other variables remain with the same definitions as in (1.1). Equation (1.8) is therefore almost identical to (1.6). Initially presented with a finite horizon, where "(...) the choice of the terminal capital stock is as much a part of the problem to be solved as the choice of the path" (ibid., p. 5), the optimal growth problem was re-framed as an infinite horizon one since in Koopmans's view it was "(...) perhaps a more natural specification in many formulations of the growth problem" (ibid., p. 6).

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[30] Cowles Foundation Discussion Paper No. 163, dated December, 1963.

Similar to all of the papers that dealt with the topic of infinite horizon economies seen above, the 1963 growth article also left open the interpretation by which equations (1.5) and (1.6) could be the utility function of some generic preference, that is, "(...) no discussion of whether these preferences were of individuals, a representative agent, or a social planner" (Duarte 2016, p. 292-3).

Koopmans's famous optimal growth paper was then published in definitive form in a 1965 volume that wrapped up the proceedings of a conference held in Vatican City.[31] But as can be implied from this paper, Koopmans's concern regarding the technical appropriateness of (1.7) as an optimality criterion did not encompass an explanation of whether it purported the preferences of an individual, a social planner, or a representative agent either; he carried on his investigation solely on the grounds that the most basic (and suitable) mathematical formulation of an optimality criterion "(...) is that of a preference ordering of growth paths" (Koopmans 1963, p. 2).

Not specifying whose preference is (1.7) could be a practice inherited from his previous work "Stationary Ordinal Utility and Impatience" (Koopmans 1960). In this article, following the derivation of equation (1.1), an expression almost identical to equation (1.7), Koopmans asserted that it had already been used extensively in the literature. But a perusal of the articles mentioned therein suggests that they did not include time discounting.[32]

We can derive two implications from this fact. First, as examined in Duarte (2016), it substantiates the claim in which Koopmans pioneered the axiomatization of time preference, for a clear difference can be observed between his 1960 article and other contemporaneous works analyzed by Duarte (2016). Second, due to Koopmans's early treatment of a discounted intertemporal utility function, it would be reasonable to expect a timid stance towards the wording of the expression "representative consumer" as well.

[31] Under the theme "Study Week on the Econometric Approach to Development Planning," the conference was organized by the Pontifical Academy of Sciences in 1963. In addition to Koopmans, who discussed his 1963 version of the paper, other important economists at the time also presented articles and participated in the overall discussion during the occasion, such as Richard Stone, Walter Isard, Ragnar Frisch, Wassily Leontief, Luigi Pasinetti, and Michio Morishima, among others. The changes that occurred between the 1963 and 1965 versions and the role played by the French economist Edmond Malinvaud are discussed in detail in Spear and Young (2014).

[32] The articles mentioned were Ramsey (1928), Samuelson and Solow (1956), and Strotz (1956).

This posture, nevertheless, did not go unscathed as seen in the quotation below:

> The effort economists such as Koopmans made to construct a behavioral basis of an ordinal intertemporal discounted utility function that characterized either the behavior of an individual or of a social planner did not prevent serious criticisms either to employing a social discount factor or to treating individual and social decisions in the same way (Duarte 2016, p. 296).

Irrespective of the technical reason behind Koopmans's treatment of individual and social decisions, there was still a noticeable vagueness in the way he viewed the aggregation of preferences and of utility functions in the 1950s and 1960s. Although I have tried to show such ambiguity through a close reading of Koopmans's writings and the perusal of the letters he exchanged with Marc Nerlove, a further investigation may answer *why* that was the case.

## 1.4 Koopmans's intertemporal economics in the 1970s: expansion of the representative agent appeal in macroeconomics

Initially part of a single 1966 Cowles Foundation discussion paper entitled "Structure of Preference Over Time," the articles "Representation of Preference Orderings with Independent Components of Consumption" (1972a) and "Representation of Preference Orderings Over Time" (1972b) appeared as separate pieces in a single volume in honor of Koopmans's long-time friend and mentor Jacob Marschak.[33] In common, and in opposition to his writings from the 1950s and 1960s, the 1970s articles signified a (re)orientation of his research interest toward the postulates of preference relations (which he called *ordering)* under which he sought the possibility of a more complete representation in terms of utility functions.[34]

[33] On Koopmans's meandering career before joining the Cowles Commission in July 1944, Mirowski (2012, p. 152) said: "Were it not for Marschak, one can easily imagine that the dissatisfied chap bouncing from one brief job to another, never really fitting in anywhere, might have left no mark whatsoever on the history of economics."

[34] A subtle, yet difference worth noting between the two articles was that rather than studying the postulates underlying preference orderings *on a prospect space*, in the second paper he focused on the

If in the 1960 article Koopmans proved the existence of a (continuous) utility function at the very outset, with the reformulated postulates set out in the 1970s, the author endeavored "(...) to construct a representation of $\succsim$ on the entire program space ${}_1\mathcal{X}$, or on as large a subspace of it as we can" (Koopmans 1972b, p. 108). Stated even more clearly, and in connection to the point I want to emphasize, the author made this complementary point in another passage:

> The postulates are modeled after those used in two earlier studies by KOOPMANS (1960) and by KOOPMANS, DIAMOND AND WILLIAMSON (1964). The main difference is that the former studies presupposed the existence of a continuous representation. In the present study, the postulates refer to a continuous ordering, and the proximate aim of the study is to derive the existence of a continuous representation. Further differences will be noted in connection with the *third* and fifth *postulates* (Koopmans 1972b, p. 105, italics added).

Aligned with the main results predicated on the assumption of *separable preferences* obtained in the 1972a article, the re-examination of the independence (third), monotonicity (fifth) and, especially, stationarity (fourth) postulates in the 1972b paper led to the continuous representation of the utility function with certain desirable additive properties.[35] The ultimate goal of such re-appraisals was therefore to find a theoretical device that would enable a better comparison of growth paths for an indefinite future or, in other words, a technical device that once applied to growth theories could broaden the understanding of infinite-time growth processes. Such was the direct importance of such an elaboration that it can be found, for example, in a prominent recent textbook *Introduction to Modern Economic Growth*, where the author states:

> The Solow growth model is predicated on a constant savings rate. Instead, it would be much more satisfactory to specify the *preference orderings* of

postulates underlying preference orderings *over time*, that is, a space ${}_1\mathcal{X}$ of programs, of infinite-time sequences.

[35] Koopmans used the term *independence of different components of consumption* instead of *separable preference*.

> individuals, as in standard general equilibrium theory, and derive their decisions from these preferences. This will enable us both to have a better understanding of the factors that affect savings decisions and also to discuss the "optimality of equilibria" - in other words, to pose and answer questions related to whether the (competitive) equilibria of growth models can be "improved upon." The notion of improvement here will be based on the standard concept of Pareto optimality, which asks whether some household can be made better off without others being worse off. Naturally, we can only talk of individuals or households being "better off" if we have some information about well-defined preference orderings (Acemoglu 2009, p. 215, italics added).

The conceptual and mathematical refinements Koopmans sought within intertemporal choice theory, especially with regard to its implications for the additivity properties of utility functions, turned out to have important sway in the profession's effort to integrate optimal growth theory into general equilibrium theory. In the next two subsections, in addition to providing a relatively thick outline of Koopmans's writings, I show how he dealt with aggregation problems and, by extension, with the representative agent.

### 1.4.1 Representation of preference ordering with independent components of consumption

Koopmans strove to fulfill two aims set out for his 1972a paper: first, summarized under *proposition 1*, provide proofs of the existence of continuous utility functions and, second, under *proposition 2*, provide proofs of an (additively) separable utility function from the assumption of (additively) separable preferences. As a starting point, in section 2, Koopmans defined and described the mathematical properties of a complete preference ordering on a prospect space. In section 3, drawing on the results found in Debreu (1959), Koopmans announced proposition 1 for continuous as well as non-continuous preference relations.[36]

[36] The actual announcement of proposition 1 was based on the assumption of continuity of $\succsim$.

In section 4, building on the previous works by Leontief (1947a, b), Samuelson (1947, chapter seven) and, especially, Debreu ([1959] 1960), Koopmans proved preliminarily the existence of a separable utility function under the assumption of two independent components of consumption.[37,38] After the definitions of some particular mathematical properties related to preference relations, the author attained what he called *Result A*:[39]

$$U(x) = F(u(x_P), v(x_Q)). \tag{4.4}$$

According to Koopmans (1972a, p. 88), "A function of this form has been called a *utility tree* by STROTZ (1956, 1959), and a *separable utility function* by GORMAN (1959a, b)."[40] The idea of separability in the utility function, therefore, implies that "(...) instead of one function $U$ of $n_P + n_Q$ variables (...)," there is "(...) a triple of functions, one ($F$) of two variables, one ($u$) of $n_P$, and one ($v$) of $n_Q$" (ibid.).

In section 5, mathematically the most challenging one, he began tackling the subject of an *additively* separable utility function under a Euclidean ($n$-dimensional) commodity space, partitioned into three independent subsets ($n = 3$). Unlike proposition 1, where Koopmans resorted to a definition found in Debreu (1959), the arguments used to attain proposition 2 were drawn from Debreu ([1959] 1960) instead related now to the definition of a cardinal utility function applied to "independent factors of the action set."[41] Koopmans then reached the following additive separable utility function (proposition 2):

$$U^*(x) = u^*(x_P) + v^*(x_Q) + w^*(x_R), \tag{5.4}$$

which is said to be unique up to a linear transformation.

In section 6, Koopmans extended the existence proof of an additively separable utility functions for the case of additively separable preferences defined in a commodity space with more than three variables (that is, the case of more than three independent

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[37] It is interesting to observe that Debreu himself referred to Samuelson's *Foundations of Economic Analysis* on the subject.

[38] "Preliminarily" because Koopmans will return to this case in section 7 of his article.

[39] To facilitate comparisons, I retain Koopmans's original equation numbers from here.

[40] There is an error in Koopmans's original text: Robert Strotz's papers that dealt with the concept of *utility tree* were his 1957 and 1959 articles, not his 1956 paper.

[41] Besides this case, Debreu ([1959] 1960) also studied the concept of cardinal utility in two other situations: "stochastic objects of choice" and "stochastic acts of choice."

components of consumption, $n \geq 3$). His important conclusion, called *result C*, presents the following continuous utility function,

$$U(x) = u_1(x_1) + u_2(x_2) + ... + u_k(x_k), \tag{6.3}$$

unique up to an increasing linear transformation.

In section 7, the author basically discussed why proving additively separable preferences in the case with two commodities ($n = 2$) is harder than in the case where there exists three or more commodities ($n \geq 3$). In Koopmans's (1972a, p. 101) own words, the latter case "(...) leads to a more special class of representation than the case $k = 2$." The whole point is tightened up to the question of why not every function of separable form, such as that obtained in *result A*, $U(x) = F(u(x_P), v(x_Q))$, can be expressed as an *additively* function of separable form

$$U^*(x) = u^*(x_P) + v^*(x_Q), \tag{7.1}$$

This discussion matters because this development would turn out to be important in macroeconomics.

### 1.4.2 Representation of preference ordering over time

Whilst the appraisals of preference orderings moved from an emphasis on prospect space (Koopmans 1972a) to an emphasis over time (Koopmans 1972b), the *propositions* obtained in the first article were used to attain the goal set out in the second one. I briefly discussed both propositions above, which concerned, first, the existence of a continuous utility function and, second, the existence of an additively separable utility function (first, for the $n = 3$ case, and later extended to $n \geq 3$).

If the first article fulfilled an essential role in introducing all necessary ingredients for a rigorous discussion of preferences on a prospect space, namely, the propositions and their corresponding proofs, the greatest merit of the second article consisted in its *de facto* re-assessment of the *postulates* or axioms (of preference relations) in light of such novel developments (the propositions). It is worth noting that Koopmans had already discussed painstakingly the very same postulates in a series of papers in the 1960s, including, of course, his paper on optimal growth (Koopmans 1963). By rescuing such discussions in a time setting in the 1970s, he intended to provide proof of existence of an additively separable utility function. Furthermore, in the author's own words, "(...) both propositions are applied in discussing the choice of a criterion for the evaluation of growth paths, starting from postulates about a preference ordering of such paths" (Koopmans 1972a, p. 81-2).

In section 2, titled "Postulates Concerning a Preference Ordering over Time," Koopmans sought to re-analyze all five postulates previously studied in Koopmans (1960) and Koopmans et al. (1964), this time enumerating them in the following arrangement: continuity (P1), sensitivity (P2), complete independence (P3) (consists in a stronger case, in which both the "limited" independence, P3′, and the "extended" independence, P3″, postulates hold), stationarity (P4), and monotonicity (P5) (which includes a stronger case denominated "extreme programs"); Koopmans aimed at re-framing them as axioms of choices in a time setting and no longer taking their algebraic representations as given.[42] In the article an important emphasis is then placed on the postulate of independence by encompassing P3′ and P3″, for they "(...) facilitate explorations of the implications of the fourth postulate, the real objective of this study (...)" (Koopmans 1972b, p. 107).

Specifically, instead of focusing on P3′, as he did in the aforementioned articles, the focus now rested on the implications of P3. The modification in the weight given to a crucial axiom was meant to "(...) preclude all complementarity between the consumption of different periods" (ibid.), that is, to ensure a separable utility function. Put in another way, if stationarity disregards whatever past values a consumption vector ${}_1x_{t-1}$ assumed before a certain time (in any consumption program), and disregards the exact time those changes began, then it can be attained through the validity of postulate P3 (P3′ and P3″ hold). With all the postulates propounded, Koopmans sought to build a representation of preference relations on the entire program space ${}_1\mathcal{X}$ "(...) or on as large a subspace of it as we can" (ibid.).

[42] According to Koopmans, the postulate P3 is attributable to Gorman (1968).

In section 3, the two propositions discussed in Koopmans (1972a) will then finally aid in the construction of such representations. The first outcome, called *result D*, churned out the following utility function for all time $T$:

$$U_T({}_1x_T) = u(x_1) + \alpha u(x_2) + ... + \alpha^{T-1}u(x_T), \qquad 0 < \alpha < 1, \tag{3.2}$$

where $u(x)$ is a continuous utility function defined on $\mathcal{X}$ and independent of $T$, while $\alpha$ is also independent of $T$.

In section 5, Koopmans reached probably one of his most (and last) important results in the field of infinite horizon economies, the attainment of a representation of preference ordering (a utility function) on the space of programs bounded in utility.[43,44] Formally, any consumption program ${}_1x = (x_1, x_2, ..., x_t, ...)$ is bounded in utility if there exist vectors $\overline{x}$ and $\overline{\overline{x}}$ in $\mathcal{X}$ such that $\overline{\overline{x}} \succsim x_t \succsim \overline{x}$, for all $t$.

Hence, on the large subspace of the program space bounded in preference (made possible with the application of propositions 1 and 2), for example, ${}_1\mathcal{X}^*$, the ordering $\succsim$ can therefore be represented by the continuous utility function

$$U({}_1x) \equiv \sum_{t=1}^{\infty} \alpha^{t-1}u(x_t), \qquad 0 < \alpha < 1, \tag{5.1}$$

which is a result that consists in his *proposition 3*.

As acknowledged by Koopmans in section 6, the utility function (5.1) surprisingly exhibited strong implications of the axioms used to derive it. Moreover, he emphasized the roles played by two mathematical concepts in the studies of program changes in future periods and, therefore, of direct concern to the evaluation of growth paths: while the function $u(x)$ allowed the comparison of utility differences in the same period, the discount factor $\alpha$ enabled the comparison of utility differences in different periods. Koopmans then went on to discuss cases with and without the discount factor (see Duarte (2016) for an overall analysis).

In section 7, Koopmans (1972b, p. 117) synthesized the results he had found thus far:

[43] I will skip directly to the principal arguments of section 5 since is more general and, furthermore, comprises the main result found in section 4.

[44] As pointed out by Christ and Hurwicz (2008), the term "bounded in preference" is preferable to the term "bounded in utility," used by Koopmans, because no numerical utility is involved in his definition.

"If instead of complete independence (P3) we postulate only limited independence (P3′), Proposition 2 is not available, and we must fall back on Result A." That is, without the assumption of separable preferences, guaranteed by the axiom of complete independence (P3), an *additively* separable utility function (proposition 2) would have been impossible and, accordingly, the best offshoot would be a separable utility function as obtained under *Result A* (equation 4.4). He nevertheless pointed to an interesting result obtained by Diamond (1965) (Existence Theorem) in which an *aggregate utility function* identical to expression (1.2) above was obtained through a different set of postulates, all applied to a preference ordering as well.[45]

Finally, as I anticipated, in the concluding part of the article the author highlighted the special role played by the stationary assumption (P4) in producing:

> (...) interesting special forms for the utility function $U({}_1x)$ in terms of simpler functions $u(x)$ and possibly $V(u, U)$, that facilitate the use of $U({}_1x)$ in models of *optimal economic growth*, and may perhaps suggest further parametrization or other specialization for econometric studies of individual consumption plans over time (Koopmans 1972b, p. 121-2, italics added).

In other words, the passage referred then to the equation obtained in proposition 3, that is, the expression (5.1), or the expression (1.2), which were previously obtained in Koopmans (1960) and Koopmans et al. (1964). Such expressions are also known to be in *recursive stationary* form, where the preference ordering would remain the same even if the timing of all periods were moved one period into the future.

I have discussed in this section two important changes in Koopmans's works in the 1970s. The first one was how he changed the emphasis from the axioms/postulates concerning utility functions to the those concerning preference orderings/relations. The second change was the extension of such an analysis over time. An important similarity between the 1970s articles and his earlier writings consisted, again, in the author's preoccupation with aggregation issues and, as a result, the representative consumer.

[45] The postulates were P1′, P2, P3′, P4, and P5′.

In essence, by asking "(...) [if] one may wish to examine whether, or under what conditions, an aggregate preference ordering over time can be imputed, on an 'as if' basis, to a society of individual decision-makers each guided by his own preference ordering over time" (Koopmans 1972b, p. 104), and by considering it "(...) the ultimate goal of a theory of preference over time for an economy with private wealth (...)" (ibid.), Koopmans indeed tried to tackle the same Post-World War II theoretical challenges raised by Kenneth Arrow, James Buchanan, Gordon Tullock, and Richard Musgrave in such different fields as Social Choice Theory, Public Choice Theory, and Public Finance.[46]

Interestingly, due to the key role of economic planning in the postwar context, and given the debates over whether the difference between social choice and public choice theories could be framed as positive or normative interpretations, Koopmans might have felt compelled to address the same issue.[47] In his words, "In regard to preference over time, the simplest interpretation of the orderings that have been studied most thus far is a normative one" (ibid.). In contrast, he also stated that: "Another possible interpretation is that one wishes to study descriptively the preference ordering of an individual with regard to his life-time consumption program, assuming that such an ordering is implicit in his decisions" (ibid.).

## 1.5 Concluding Remarks

The Dutch physicist and economist Tjalling Koopmans wrote seminal articles in three fields of research in economics: activity analysis, econometric theory and intertemporal economics (infinite horizon economies). For his contributions to activity analysis, or the optimal allocation of resources, he was awarded the 1975 Nobel Prize in Economics (shared with the Russian mathematician Leonid Kantorovich).

This chapter investigated Koopmans's contributions to the optimal growth literature,

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[46] The literature on Social Choice Theory began with Arrow ([1951] 1963); the field of Public Choice Theory is largely due to the publication of Buchanan and Tullock (1962); finally, Musgrave (1959) is considered to have laid the foundations of Public Finance.

[47] As explained by Judy Klein (2007), both World War II and the Cold War forced the US government into planning (or "programming"), even though such conflicts represented, on the one side, capitalism and the free-market and, on the other side, state corporatism and central planning.

a subfield also considered to be within the realm of the optimal allocation of resources, in an attempt to uncover the factors that might have influenced his stance toward a methodological device used in such models: the representative consumer. To do so, I focused specifically on his contributions to intertemporal economics in the 1950s, 1960s and 1970s.

By separating his contributions into those subperiods, I was able to observe in more detail the evolution of his thinking about the representative agent. From a greater concern with aggregation problems in the 1950s, he shifted to a more practical emphasis in the 1960s. In the 1970s, Koopmans's works became more interrelated with the instrumental needs of macroeconomists, understood as the use of additive separable utility functions. In the works of all those periods, a certain vagueness is noticeable in the use of the term "representative consumer."

The analysis in this chapter can be enhanced in at least one direction. Koopmans's construction of additive separable utility functions in the 1970s seems to have followed the overall change in economics that took place after the adoption of homothetic preferences in the 1950s (in an effort that took Paul Samuelson almost thirty years). These facts seem to have favoured a higher use of the representative agent in macroeconomics in the 1980s with the advent of the Real Business Cycle School and, later, in the 1990s with the emergence of the so-called New Neoclassical Synthesis.

# 2 The Representative Agent in Microeconomics: A Samuelson-Koopmans Thread?

*Samuelson is omnipresent in American and even world economics; (...) he appears at every turn of history and in every disguise. The unwilling college student (...) finds him there, expounding economic wisdom in eight hundred well chosen pages (1967). The graduate student is disciplined to our trade by study of the Foundations (1947) and two dozen assorted papers of Samuelson. In the pages of every journal and many a collective volume he appears, read equally by esoteric specialists in optimal growth or integrability conditions and by the most policy-oriented in practical central banking, impatient with theoretical niceties.*

- Kenneth Arrow, *1967*

*(...) microeconomic theory begins with choices. Indeed, the theory not only begins with choices; it remains focused on choices for a very long time.*

- David Kreps, *2013*

## 2.1 Introduction

It has become popular among economists that a major result in microeconomic theory is consumer rationality. The path to this outcome begins with axioms of preferences, proceeds with the existence proof of a utility function and continues with the solving of a (constrained) *utility maximization* problem that leads to Marshallian demand functions. Such functions are said to satisfy some desirable properties. It is possible to attain an equivalent result in a (constrained) *expenditure minimization* problem where, this time, Hicksian demand functions are obtained.

The connection between Hicksian and Marshallian demand functions is attributable to an expression proposed by Eugen Slutsky, a fundamental equation that breaks the quantity demanded of a good—due to variation in its price or in another good's price—into substitution and income effects. The symmetry of the Slutsky matrix, in conjunction with the Integrability Theorem, guarantees that Marshallian demand functions reverse to the axioms of preferences, thus connecting the outcome of the utility maximization problem to its initial definitions.

Yet, the development of microeconomic theory overcame more resistance than the history presented above (and elsewhere) suggests. Through a more careful look into the works and careers of particular economists one can observe how, operating in networks, they ignored, created and extended theoretical and/or empirical research whose final outcome will one day stand as the canonical history of the subject. The purpose in this chapter is to tell one of these stories.

Much like Kenneth Arrow's aim in his review of Paul Samuelson's *Collected Scientific Papers* in 1967, my intention in this essay also takes into account the influence Paul Samuelson exerted on the profession; specifically, I show that a significant part of Tjalling Koopmans's research in microeconomics was informed by Paul Samuelson's research in the field and this was especially so concerning the representative agent. To substantiate this claim I begin this narrative, in section 2.2, with an important yet overlooked episode in the development of microeconomics in the 1930s, the "Hotelling-Schultz dialogue," a research effort that consisted in, *inter alia*, testing the symmetry conditions of Hotelling demand functions (obtained through the solution of Harold Hotelling's consumption optimization problem).

With the advent of contributions from Eugen Slutsky, the Cowles Commission conducted more than the testing of the new symmetry conditions: under the influence of the Keynesian revolution the well-known Slutsky income effect, defined at the microeconomic level, was used to shed light on facts defined at the macroeconomic one, an objective of the Slutsky-Walras program that had engaged Koopmans since the 1940s. In section 2.3, I explain why this program was centered on the problem of aggregation and, above all, how from this effort a few considerations about the representative agent can be inferred. To bring further details into this analysis, in section 2.4, I connect Koopmans's research efforts, as discussed in the previous section, to Samuelson's contributions in microeconomics which also helped give rise to the representative agent. In section 2.5, I consider the meaning of dynamics in economics, a topic originating concomitantly in a important paper analyzed in this essay (Samuelson 1943), with interesting implications for the interpretation of the representative agent. In section 2.6 I provide some concluding remarks.

## 2.2 The Empirically-motivated Hotelling-Schultz Dialogue

### 2.2.1 The Context

Although the amount of empirical research in economics began to display an increasing trend as early as the 1970s, it took hold in the profession only in the 1980s for several reasons. Among these were the ever-increasing availability of data, the development of new statistical and econometric methods and the reduced cost of computer power, to which one might add a higher demand for works of "practical relevance."[1]

Even profiting from a whole new set of scientific and technological advances, however, certain fields have yet to see the expected improvement: in demand analysis, economists continue to face hurdles of a theoretical and empirical nature and, as the story involving Harold Hotelling and Henry Schultz will show, such a state goes back at least to the years before World War II.

In their quest to trace the rise of orthodox demand theory in the United States in the first half of the twentieth century, Wade Hands and Philip Mirowski have identified a point of origin, a sort of theoretical *tabula rasa*, where two "relatively minor figures" initiated an inquiry that ultimately determined the direction of a few, yet influential, economic research centers in the United States.[2] In the authors' interpretation, everything began

[1] Few papers have recently analyzed the *empirical turn* that occurred in economics over the last forty years. Combing through nearly 33 journal titles cited in selected years of the *American Economic Review* and using a *machine learning* algorithm, Angrist et al. (2017) concluded, among other things, that the profession's turn toward empirical research has been an evolution within instead of across fields. The volume edited by Backhouse and Cherrier (2017) provides various views on the rise of applied research in historical perspective.

[2] The two papers written by Wade Hands and Philip Mirowski were "Harold Hotelling and the Neoclassical Dream" and "A Paradox of Budgets: the Postwar Stabilization of American Neoclassical Demand Theory," both published in 1998. Though the authors changed the order of co-authorship, I will simply refer to "Hands and Mirowski" when referring to this story (in line with the satisfaction of the alphabetical protocol) and the correct order when a specific passage is used as a reference or, obviously,

when Henry Schultz and Harold Hotelling set out in the 1930s to find, *inter alia*, a theory capable of underpinning the empirical demand functions they had been (painstakingly) trying to estimate. Hands and Mirowski have dubbed this episode the "Hotelling-Schultz dialogue" or, interchangeably, the "Hotelling-Schultz impasse."

The first person in this story, Henry Schultz, was born in 1893 in Russian-occupied Poland.[3] After attending college at the City University of New York, Schultz began graduate work in economics at Columbia University under the early econometrician Henry Ludwell Moore, writing a dissertation on the statistical estimation of demand functions.[4] With the conclusion of the doctoral degree, Schultz was appointed professor of economics at the University of Chicago, where he carried out the major work of his career by founding a statistical laboratory to deepen the estimation of demand functions, "(...) a large program of research of a kind definitely projected by earlier studies" (Hotelling 1939, p. 98).[5]

In need of finding a reasonable theory behind the estimated equations, Schultz began to look into the general equilibrium approach pioneered by Leon Walras and Vilfredo Pareto. In his mind, such functions not only had pervasive interdependence at the market level, but more to the point, were the result of agents' utility maximizing behaviour (exactly *à la Walras*). As in the natural sciences, there had to be "laws" behind empirical regularities in microeconomics as well. What Schultz did not realize at the beginning, nonetheless, was how difficult his applied task would turn out to be, let alone contribute to the theory he so dearly praised.

A second contributor in those early studies of empirical demand analysis and utility theory was Harold Hotelling, raised in Seattle, Washington, but born in Minnesota in 1895.

quoted.

[3] His family emigrated to New York City when he was still a child. See Hotelling (1939) for more information on the life of Henry Schultz.

[4] In a Festschrift to Schultz published after his death, Harold Hotelling (1939, p. 98) wrote: "[Schultz's] inspiration was in the work of Henry L. Moore, whoose piooner attempt to derive demand curves from time series stirred his enthusiam." Henry Moore (1869-1958), considered one of the founding fathers of econometrics, earned a PhD in Economics from Johns Hopkins University. Spearheading a generation of Americans who crossed the Atlantic to learn from Europeans, Moore took classes in mathematical statistics from Karl Pearson at the University of London and also corresponded with luminaires of that time such as Alfred Marshall, Francis Ysidro Edgeworth, Vilfredo Pareto and Léon Walras. On this and additional details of Moore's career, see Mirowski (1990).

[5] Schultz taught at the University of Chicago from the fall of 1926 until his untimely death in 1938. His name is often associated with the early formation of the Chicago School of Economics.

Although graduating in journalism in 1919, he went on to find a more fulfilling career in mathematics, earning a master's degree in 1921. Three years later, Hotelling completed a doctorate also in mathematics at Princeton University with a dissertation on *analysis situs*, known nowadays as topology, a field where his adviser—Oswald Veblen—contributed with the first ever modern treatment of the topic (Darnell 1988; Mac Lane 1964).[6] As he would later recall of this graduate experience, after being rejected by the graduate program in economics at Columbia University, a fact that postponed his immediate desire to apply the science of mathematics to uncover "new truth" in economics, studying subjects as wide as mathematical physics, differential geometry and astrophysics gave him an even better grasp of the tools, which turned out to be helpful in his subsequent works (Darnell 1988).

Leaving university in 1924 when jobs for pure mathematicians were in short supply, Hotelling began his professional life at the recently created Stanford Food Research Institute (SRI) as a mathematical and statistical consultant (Hands and Mirowski 1998). It was at the institute, collaborating with his colleague Holbrook Working, and mirroring Schultz's experience with widespread economic crisis and uncertainty (especially in the food sector), that he developed his own interests in demand theory and estimation methods.[7] Indeed, as Arrow and Lehmann (2012, p. 903) put it, "It was during his Stanford period that he began to focus on the two fields - economics and statistics - in which he would do his life's work."

As was the case with some academicians in the United States, who used to travel abroad to learn from their counterparts, Hotelling spent six months in England in 1929

6 Oswald Veblen (1880-1960), nephew of the Institutionalist Thorstein Veblen, earned a PhD in mathematics at the University of Chicago in 1903. A famous geometer of his time, Oswald taught at Princeton University during his whole career. Also at Princeton he helped found the Institute of Advanced Study's School of Mathematics, a place famous for its faculty who included such names as Albert Einstein, John von Neumann and Hermann Weyl (a person we will meet again later in chapter three of this thesis).

7 Holbrook Working earned a PhD in Agricultural Economics from the University of Wisconsin-Madison in 1921 and joined SRI in 1925. Having made substantial contributions in the statistical analysis of commodity prices (JBES 1986), his influence on Hotelling's 1932 article can be gauged by the several times he was mentioned. Holbrook and his younger sibling, Elmer Working, were also two of the first economists who underscored the importance of *endogeneity problems* in the estimation of demand functions. Elmer's article, "What do Statistical 'Demand Curves' Show," published in 1927, remains even today a reading source in applied microeconomics courses such as Industrial Organization. Deaton and Muellbauer's *Almost Ideal Demand System* is built, among other models, on Holbrook Working's "Statistical Law of Family Expenditure," published in 1943.

working with a leading statistician, making him "(...) one of the few Americans who in the 1920s realized the revolution that R. A. Fisher had brought about in statistics (...)" (ibid.). So long-lasting was this influence that even following the relocation to Columbia University in 1931, "(...) most of his energy during the 15 years there was spent developing the first program in the modern (Fisherian) theory of statistics" (ibid.), a fact that certainly helped him hone his publications in microeconomics. The bulk of his involvement with Schultz, therefore, took place during this Columbia period precisely when, as editor of the *Journal of Political Economy* in 1932, Schultz received an article written by Harold Hotelling.[8]

Among other enquiries, Henry Schultz had been seeking answers to one particular question connected to the measurement of demand: could the Walrasian model fulfill the theoretical void he saw in the estimated curves? Schultz found a perfect interlocutor in Hotelling, for Hotelling's main article, "Edgeworth's Taxation Paradox and the Nature of Demand and Supply Functions," published in the influential *Journal of Political Economy* in 1932, tackled the objective above.[9]

Even though Hotelling searched for a more theoretical approach while Schultz was "(...) the champion of an empirical operationalism" (Hands and Mirowski 1998, p. 363), they shared the following features:

1. A concern with practical matters, i.e., the reality of the American countryside in the years leading to the great depression of 1929: thus, the estimation of the demand functions were considered paramount to better comprehend hidden events crippling the sector.[10]

2. A belief that economic theory was important only insofar as it shed light on tariffs, subsidies, production quotas, and related matters.

3. Particularly on the issue of pure theory, a belief that the neoclassical price version had

[8] As Hands and Mirowski (1998) have reminded us, Hotelling and Schultz had already exchanged correspondence as early as the 1920s, but on other matters.

[9] The University of Chicago's economics department, where Henry Schultz was a Professor, had been publishing the *Journal of Political Economy* since 1893. See the special issue edited by John List and Harald Uhlig (2017) on the combined influence on our profession of "Chicago economics" and its flagship journal.

[10] As the Stanford University historian David Kennedy once wrote: "Herbert Hoover needed no comprehensive study to know that the farm issue was urgent. Virtually his first act as president, even before he commissioned his wide-ranging examination of recent social trends, was to convene a special congressional session to resolve the farm crisis" (Kennedy 1999, p. 17-18). Then, he continued, "(...) as the agricultural depression of the 1920s merged with the great depression of the 1930s, (...) the misery of rural America knew no relief" (ibid., p. 19).

to be:

3.1 Grounded in mathematics.

3.2 Empirical: data should corroborate the theory.[11]

3.3 Somehow derived from Physics: "a true scientific method."

3.4 The basis of everything else in economics.

3.5 Conducive to the obtainment of interdependent demand functions.

A central takeaway of this section is that, ontologically, both authors believed that demand functions "existed out there in the world" in an interdependent form, "(...) functionally linked together by rational adjustments to other prices, incomes, as well as various accidents" (Hands and Mirowski 1998, p. 345), unlike the Marshallian approach in which a simple price-quantity plot would be acceptable. An important part of the following development in this literature then revolved around a key test called the *symmetry condition*. But as much as it was the economist' job to correctly find interdependency empirically, and better, to base it upon the writings of Léon Walras, Vilfredo Pareto and Francis Edgeworth, things did not work out well, neither in the past, nor in the present.

### 2.2.2 Everything Came Down to Symmetry Conditions: past and present

In section 5 of the "Edgeworth's Taxation" paper, Hotelling demonstrated that a certain *symmetry condition* should hold when one took the cross-partial derivative of demand functions with respect to prices, a feature he alternatively dubbed *integrability conditions*. After reading the article:

> (...) Schultz immediately set his laboratory to work calculating the demand functions and estimating the partial derivatives to test for the symmetry condition of the Hotelling Economy, as well as the quasi-correlation coefficient to measure complementarity (Hands and Mirowski 1998, p. 355).

[11] Both Henry Schultz and Harold Hotelling pioneered the applicability of linear regression analysis as well as Fisherian statistical hypothesis testing to demand theory. See, in particular, Schultz (1938) and Darnell (1988), this latter on Hotelling's contributions as well as his interaction with Schultz in statistics.

Such an attempt to test for the restriction was probably one of the first carried out in microeconomics, and following its transformation "to the status of core empirical content," the authors continued (ibid.):

> (...) the symmetry or integrability conditions had frequently failed to hold, and worse, sometimes the sign differed, so the complementarity coefficient would give conflicting results when the order of calculation was inverted.

When Schultz's tests went awry, several attempts to save the Hotelling economy were made and these included, but were not limited to, changes in the functional forms of the demand equations, modifications or extensions of the model to take account of possible aggregation problems and even the whole disposal of the Hotelling framework.[12]

What happened next is that a significant part of the dialogue lost its importance with the unexpected death of Henry Schultz in 1938, the definitive retreat of Harold Hotelling from demand theory and, indisputably the most important factor, the rediscovery of Eugen Slutsky's "Sulla Teoria del Bilancio del Consumatore," published as far back as 1915, but now considered the text that provided one vital step in the proof to recover a consumer's preferences from her/his demand behaviour.[13] When the Slutsky equation finally made its way into orthodox microeconomics (in the postwar period), becoming the "single most important result" in demand theory (Hands 2004) or its "fundamental equation" (Jehle and Reny 2011), another struggle began: this time to test the Slutsky symmetry conditions.

Then, roughly forty years later, and with a similar motivation, the leading empirical microeconomist Angus Deaton (1986, p. 1796) wrote:

> All the techniques of demand analysis so far discussed share a common approach of attempting to fit demand functions to the observed data and then enquiring as to the compatibility of these fitted functions with utility theory.

[12] Drawing on the correspondence between Hotelling and Schultz, Hands and Mirowski (1998, p. 356-7) list ten ways the protagonists attempted to overcome the failed empirical results, with Schultz always leaning toward the empirical whereas Hotelling toward the theoretical.

[13] See Chipman and Lenfant (2002) for how Slutsky's 1915 and 1927 contributions grew in importance in economics. For both historical and intellectual contexts involved in Slutsky's writings, see Barnett (2004, 2007).

As I show in appendix 2.A.2, for the above-mentioned compatibility to hold, the integrability condition should be satisfied, which is a different way of saying that the symmetry conditions should be satisfied. In another passage, after pointing out the considerable body of empirical literature that has carried out the tests, including his (and John Muellbauer's) *almost ideal demand system* of 1980, Deaton (ibid., p. 1791) granted:

> Although there is some variation in results through different data sets, different approximating functions, different estimation and testing strategies, and different commodity disaggregations, there is a good deal of accumulated evidence rejecting the restrictions.

Even taking into account some of the latest developments in the estimation of consumer behaviour, e.g., the nonparametric approach developed since the 1980s (Stoker 1989; Härdle et al. 1991; Haag et al. 2009) or the use of nonseparable models in the 2000s (Hoderlein 2011; Imbens and Newey 2009; Altonji and Matzkin 2005), contemporary economists still fall short of attaining what Harold Hotelling and Henry Schultz wanted to attain more than eighty years ago.

## 2.3 Koopmans and the Cowles Approach to Aggregation Problems

A central claim in Wade Hands and Philip Mirowski's recent project of re-evaluating American demand theory contends that had economists followed more closely Harold Hotelling's demand functions, as explicated in his 1932 "Edgeworth Taxation Paradox" paper, the *neoclassical dream* would have been fulfilled. Such a dream consisted in successfully addressing six theoretical concerns within the Arrow-Debreu-Mckenzie general equilibrium model: *(i)* existence, *(ii)* uniqueness, *(iii)* stability of the equilibrium price vector, *(iv)* comparative statics via the employment of the "correspondence principle," *(v)* welfare economics and, finally, *(vi)* the satisfaction of the physics heuristics.[14]

But partly because of its disappointing symmetry tests and partly due to the

[14] On this counterfactual exercise, Leonid Hurwicz (1998, p. 399) asserted: "Hotelling's unlimited budget model was not even under consideration for consumer choice theory, and hence there was no opportunity for rejection... Since there was no rejection, no one is guilty of it."

recently discovered contributions from Eugen Slutsky, the profession walked away from Hotelling's unlimited budget approach; while Slutsky's approach turned into an essential building block of neoclassical demand analysis, Hotelling's was relegated to the so-called "Hotelling Lemma."

What followed in the literature is that although there appeared "(...) three major reactions to the breakdown of the Hotelling and Schultz programme, each characterized by their treatment of the 'integrability' or [Slutsky] symmetry conditions (...)" (Hands and Mirowski 1998, p. 373), the representative institutions also differed in how the Slutsky-based demand functions could be used to cast light on two relevant, intertwined topics of the day: first, the Keynesian revolution, and, second, the compatibility of macroeconomics and Walras's microeconomics (brought out by the Keynesian revolution). These two concerns continued to engage Cowles and, especially, Tjalling Koopmans over the following decades.

On the first topic, there was a general perception in the United States of the revolutionary ideas presented in John Maynard Keynes's 1936 *General Theory*. Since the interwar period had witnessed a severe economic downturn, it was feared that it could happen in the aftermath of World War II as well. Keynes's theory of aggregate income became not only an important reference to better comprehend business cycles, but was seen as providing remedies to counterbalance them. It is with this discernment that Carl Christ recalled: "The applied econometric work of the Cowles Commission, inspired by Marschak and directed at the *improvement of macroeconomic policy*, had a definite Keynesian flavor" (Christ 1994, p. 35, italics added).

Yet, Keynes's theory of aggregate income, coupled with Slutsky's contributions, enabled another much-wanted interpretation in economics: the relationship of macroeconomics to microeconomics; by breaking it out as a separate magnitude, the Slutsky income effect considered at the agent level provided a theoretical basis to interpret income changes at the aggregate one, as though it provided some sort of microfoundations.[15,16]

15 For a more extended discussion on how Keynesian macroeconomics might have influenced the profession's adoption of the Slutsky equation through the interpretation of the income effect, see Hands (2012a).

16 As explained in Hoover (2012), the relationship of macroeconomics to microeconomics was already an important topic in the "prehistory" of microfoundational programs, a period which included research by such economists as Ragnar Frisch and John Maynard Keynes himself. Although "microfoundations" as a *systematic program of inquiry* did not exist prior to John Hicks's first general equilibrium program

Veering towards Slutsky and Keynes seemed a promising research strategy at Cowles: it made possible the pursuit of Alfred Cowles's and Jacob Marschak's dreams of predicting stock market returns and taming business cycles, respectively (Christ 1994). In fact, as is plainly documented in the historiography, under the research directorship of Marshack (1942-1948) the Cowles Commission spent time and resources in structural estimation econometrics, precisely a macroeconometric project whose outcome went awry (see, for example, Morgan 1990, Louçã 2007, and Mirowski (2002)).

What is, perhaps, slightly lesser known is that the Commission—possibly in pursuing Cowles's desire—also engaged in applied works of a microeconomic nature, a Walrasian-Slutsky program. This objective, like Schultz's efforts almost a decade before, failed as well. It did so because the estimations repeatedly rejected neoclassical price theory in the very crucial respect it rejected the Hotelling economy: the symmetry conditions.[17]

In hindsight, given that most empirical studies have dealt, and continue to deal, with aggregate demand, could this failure be associated with a previous warning by Hotelling and Schultz, for whom the Slutsky version did not have nice properties under aggregation and, therefore, were destined to failures?

The rejections of the symmetry hypothesis in both approaches to demand theory made aggregation problems an even larger concern in the decade of the 1940s.[18] Chipman (2006a) provided a historical overview on the substantial literature on the estimation of system of demand functions that have dealt with such a problem, underlining the pioneering studies of Lawrence Klein, Richard Stone, Herman Wold, and others. Around this period there also appeared other programs of inquiry dealing specifically with problems of aggregation on both theoretical and empirical grounds.

In the theory realm, as I mentioned before, there was John Hicks's 1939 *Value and Capital,* an attempt at constructing a Walrasian macro model, "(...) a general-equilibrium

by the end of the 1930s (Hicks [1939] 1946), and continued with Lawrence Klein's macroeconometric project of the mid-1940s and Robert Lucas's New Classical Macroeconomics in the 1970s, the term was first used by a mainstream author only in the early 1970s (Phelps 1970).

17 As claimed by Hands and Mirowski (1998, p. 375): "(...) actual quantitative restrictions implied by the neoclassical price rarely held in the empirical exercises undertaken at Cowles."

18 As I have mentioned in the "Introduction" to this dissertation, trade theorists have long striven with the concept of aggregation at least since William S. Jevons's concept of *trading bodies.*

microfoundational program, which is conceived as showing how Keynesian problems can arise directly from the interactions of individual agents" (Hoover 2012, p. 37).[19] In the empirical case, there was Lawrence Klein's 1946 *Econometrica* article, "Macroeconomics and the theory of rational behaviour," an attempt to build aggregate variables couched in the behaviours of both households and firms; this effort inaugurated, in methodological terms, a new tradition in macroeconometrics.[20]

After Klein's article, other works followed suit, for example, in determining aggregation in a one-industry model (May 1946), in building macroeconomic values in the so-called "general case" (Pu 1946), or in studying the impacts on macro values (aggregate production functions) of changes in micro values (sectoral production functions), "with which it is derived" (May 1947, p. 51). Following May (1947), Nataf (1948) tried to determine the conditions under which an aggregate production function corresponded exactly to the production functions of individual firms (rather than of isolated sectors). Analyzing instability features in systems of differential equations, Hawkins (1948) proposed aggregation methods that could curb them. Finally, also included in these articles was a report of papers presented at a session of the Chicago Meeting of the Econometric Society on the "Relationships Derived from Aggregate Data," with contributions from Kenneth Arrow (1948) and Francis Dresch (1948).[21]

To van Daal and Walker (1990), this spate of articles published in *Econometrica* between 1946 and 1948 consisted perhaps in the greatest testimony yet to the importance of aggregation problems in economics. More dramatically, still according to the authors, with the possible exception of a few remarks made in Cournot's famous 1838 book (*Recherches sur les Principes Mathématique de la Théorie du Richesse*), no economists had ever taken

[19] Weintraub's 1979 *Microfoundations - The Compatibility of Microeconomics and Macroeconomics*, contains a thorough discussion of Hicks's microfoundational program.

[20] Post-World War II macroeconometrics owes to Hicks, Modigliani and Keynes as much as it owes to Tinbergen, even though the latter two authors disagreed fiercely. Klein's innovations in the subfield came in the form of establishing a middle-ground between Keynes and Tinbergen (Hoover 2012). Pinzón-Fuchs (2017) details the history of Klein's macroeconometrics program.

[21] This annual meeting was held on December 27-30, 1947. As is customary, since the texts presented in one year are published in the following one as a joint report, the authors' contributions were dated 1948. A third paper listed in the session, by Ronald Shephard, was not presented and thus did not have its abstract included in the volume.

the matter seriously before.[22]

Tjalling Koopmans's research, on the one hand, was stirred by such an aggregation concern: a problem he decided to address by renouncing the empirical approach and adopting the theoretical one. This move coincided with the dismissal of Jacob Marschak as Cowles's research director in 1948; when Koopmans acceded to that position, rather than sanctioning research in applied works, he shifted the commission's focus to "(...) extract the 'abstract structure' of Walrasian system and subject it to the most rigorous axiomatic scrutiny" (Hands and Mirowski 1998, p. 375). As I discussed in chapter one of this dissertation, this research occupied most of his time from the 1950s to the mid-1970s, a research program that opened an important chapter in the history of axiomatization in economics.

An illustration of how aggregation and general equilibrium theory were interlinked subjects can be drawn from Leonid Hurwicz's appraisal of Hands and Mirowski's alleged objectives of the Arrow-Debreu research program:

> [Hands and Mirowski] impute to the neoclassicists (mainly Arrow, Debreu and Samuelson) a programme containing a set of five objectives. These are listed as existence, uniqueness and stability of competitive equilibria; comparative statics, and welfare economics (...). (*The problem of aggregation, discussed elsewhere, seems to have status akin to that of the five goals*) (Hurwicz 1998, p. 399, italics added).

But as much as Koopmans and other economists placed an enormous amount of effort into this aggregation project, and even though "(...) one can discern the skeleton of the Slutsky conditions buried within the [general equilibrium] system" (Hands and Mirowski 1998, p. 375), the authors maintained that:

> Unfortunately, axiomatic elaboration also eventually led to the conclusion that the system placed no effective restrictions upon excess demand functions, thus subverting the original impetus for the research program (ibid.).

[22] One can find a similar impression of Cournot's work, for example, in James Friedman's (2000) survey of the French economist's works and life.

This dim view was also shared by Hurwicz (1998, p. 401), for whom the framework was inevitably "(...) inadequate for aggregate prediction purposes."

Even though placed in an econometric setting, the same difficulty can be observed in a letter Koopmans wrote to Marc Nerlove after the Cowles Commission moved to Yale University in 1955:

> (...) I have developed a general feeling that the clue to a number of different problems in econometrics lies in further exploration of aggregation problems, that ties our notions of individual decision making with somewhat more aggregated relationships. I doubt that this reflection is of much value to you in regard to distributed lags, but it does seem to me that this is one of the problems that may stand to gain if progress should be made on aggregation. Theil's book is excellent as far as it goes. My main reservation about it is that it studies primarily the implications of current econometric procedures, rather than trying to answer the deeper question what procedures we should be using in view of aggregation problems.[23]

In this vein, and also highlighting the frequency with which aggregation concerns arise in theoretical as well as applied econometric works, Hurwicz wrote as follows:

> The issue of *aggregation* has, I believe, special status because of its relationship to econometric research. Since much econometric research (and this, I believe, includes the studies of Henry Schultz) uses data aggregated over certain populations or collections of firms, there arises the question to which properties known to hold for individual agents carry over to aggregates. One reason why this issue is important is that identifiability assumptions used (and needed) by econometricians are often derived from theoretical (or instropective) considerations applicable to *individual* economic agents (Hurwicz 1998, p. 401, italics added).

[23] Letter from Tjalling Koopmans to Marc Nerlove, January 31, 1957, TKP, box 16, file folder 304.

Koopmans's research, on the other hand, was also stirred by a quarrel with affiliates of the National Bureau of Economic Research (NBER), one of the world's oldest business cycle research institutes.[24] As will be shown below, this discussion encompassed the problem of aggregation and general equilibrium theory as well.

In reviewing the 1946 book *Measuring Business Cycles*, written by Arthur Burns and Wesley Mitchell, Koopmans accused their methods of investigation as being "(...) made with a minimum of assistance from theoretical conceptions and hypotheses regarding the nature of the economic processes by which the variables studied are generated" (Koopmans 1947, p. 161). Their theory, Koopmans reasoned, lacked an explicit formulation about the way an economic agent made choices, essential to explain "man's economic behaviour" and therefore the ultimate causes behind the cycles.[25]

In response to those charges, Rutledge Vining alluded to a certain vagueness in the alternative interpretation proposed by the Cowles economist, skeptical with what Koopmans thought stood behind people's choices:

> Koopmans doesn't give his hypotheses specific economic content (...); and suggests that the kind of content it should have in *general terms*, such as "the behaviour of groups of economic agents," "underlying human responses," "knowledge of man's behaviour and its motives" (Vining 1949, p. 79-80, italics added).

That Koopmans was ambiguous when he referred to individual vis-à-vis group choices, and that he conveyed his ideas through elaborate mathematical arguments specially after the 1960s, have already been stressed in my discussion of his writings (see chapter one of this dissertation). Perhaps it may come as a surprise that such characteristics were already pointed out by Vining as early as 1949, as the continuation of his text illustrates:

> But apparently *all he has to insist upon at present is the mathematical form*, and from his discussions it appears not unfair to regard the formal economic

[24] For an early history of the nber, see Morgan (1990, especially chapter two) and Fogel et al. 2013.

[25] This initial denunciation by Koopmans sparked animosity between members of the Cowles Commission and the NBER. It became known in the literature as the "measurement without theory" debate, named after the title of Koopmans's article.

> theory underlying his approach as being in the main available from works no later than those of Walras (ibid., italics added).

Another interesting feature that stands out in this passage relates to how closely connected, in Vining's discernment, Koopmans's approach appeared to be with Walras's. More comprehensively, in different excerpts, Vining spoke of Koopmans's attempt at providing aggregate formulations within the Walrasian framework. In one instance, Vining (ibid.: 81) stated:

> (...) some of us may feel that the unit of analysis and the entity the behaviour of which it is of interest to study is not the individual economizer in his conscious, problem solving state of mind. I believe that much of the statistical regularities that are to be observed in population phenomena and that are relevant for the discussion of economic problems involves the behaviour of social organisms that are distinctly more than simple algebraic aggregates of consciously economizing individuals. I think that in a positive sense the aggregate has an existence over and above the existence of Koopmans' individual units and behaviour characteristics that may not be deducible from the behaviour of these component parts.

In another passage, in criticizing Koopmans's econometric practice (inherited from Jan Tinbergen), he affirmed:

> In a sense, these are the only problems that have been attacked by this entire line of development - the problem of statistical estimation that would be presented by the empirical counterpart of the Walrasian conception. Add to Walras the simple notion of lagged effects (if it is not already there) and certain devices of the nature of the difference equation, and the problem is wholly statistical as contrasted with economic (ibid.: 80).

In closing this section, if the "measurement without theory" criticisms Tjalling Koopmans raised against the Bureau were to have some effects, he had to contribute with the Walrasian theory: building better theories of choice, perhaps with better aggregation properties, was Koopmans's task.

## 2.4 Samuelson on Homothetic Preferences, the Strong Axiom of Revealed Preference, and the Representative Agent

Among his various contributions to economics, Paul Samuelson made crucial inroads in the Walrasian general equilibrium theory. In a series of papers spanning almost thirty years, he introduced into the apparatus the assumption that all individuals in the economy had *identical* homothetic preferences and, by doing so, he helped establish certain desirable properties in the framework.

Beginning with the article "Social Indifference Curves," published in 1956, Samuelson adopted *homogeneous of degree one* utility functions as representative of homothetic preferences (see the appendix 2.A.3 for more discussion). An important aspect of this formulation is that demand functions resulting from such utility functions could be aggregated to form a *representative consumer* (or *representative agent*).[26] As stated in Hands (2016), in addition to the existence of a representative consumer, bearing on the assumption of homothetic preferences warranted three other properties worth noting: aggregation, market rationality, and welfare.[27]

Because Samuelson's intention in the 1950s had been to contribute to the international trade literature, i.e., the search for community indifference curves, making a case for homothetic preferences permitted also a simultaneous re-interpretation of aggregation problems. As Samuelson (1956, p. 21) summarized:

> Since most "individual" demand is really "family" demand, the argument can be made that such family demands have been shown to have none of the nice properties of modern consumption theory. However, if within the family

[26] See, e.g., Acemoglu (2009) and Blackborby et al. (2008) on the mathematics of the aggregation.

[27] All four implications following the assumption of homothetic preferences are unique in their importance to the general equilibrium model. However, to the question I am raising in this text, the most important one is the possibility of considering the market demand as if generated by a single, representative consumer.

> there can be assumed to take place an optimal reallocation of income so as to keep each member's dollar expenditure of equal ethical worth, then there can be derived for the whole family a set of well-behaved indifference contours relating the totals of what it consumes: the family can be said to *act as if* it maximizes such a group preference function. The same argument will apply to all of society if optimal reallocation of income can be assumed to keep the ethical worth of each person's marginal dollar equal.

It is, therefore, precisely due to this extension that the Walrasian general equilibrium model

> (...) did have sufficient structure at the *agent level* to be able to say very specific (and desirable) things about the *market-level* results generated by the competitive interaction of such agents. The model was the homothetic Santa Claus case of uniform homothetic tastes ... (Hands 2016, p. 427, italics added).

Along with this aggregative property, the existence of the representative agent meant that rather than a *sypply = demand type equilibrium* (for all goods in the economy), now it sufficed to arrive at the solution to a constrained optimization problem of that single agent. As a consequence, following all usual assumptions of demand theory, the rationality of a single individual is (as if) transferred to the whole system.[28]

Parallel to this event, in separate efforts, Samuelson (1950) and Houthakker (1950) developed the *strong axiom of revealed preference* (SARP) approach to consumer theory, one which emboldened the perception that a representative agent could in fact exist in the economy: "(...) if the SARP axiom holds on market (rather than individual) demand functions, then there always exists a rationalising representative agent: i.e. the so-called Wald case where the market reflects 'revealed group preference' (...)" (Hands 2016, p. 429).

Contrary to the reliance on the *weak axiom of revealed preference* (WARP), the fulfillment of SARP signified the satisfaction of integrability condition necessary for the

[28] The attainment of the representative agent means also the solution to two other specific problems within general equilibrium theory: uniqueness and stability. However, I will not pursue such topics here.

demand function to be treated as if it were generated by a budget-constrained utility maximizing agent. Hence, borrowing a last passage from Hands (ibid., p. 430)

> (...) if the SARP holds on market demand functions then the demand functions behave as if they were generated by a representative agent and the Walrasian general equilibrium of the whole economy reduces to the consumer's equilibrium of that agent.

An important point here is that more than enabling the construction of a particular breed of general equilibrium model, the elimination of the heterogeneity of agents and income effects—as a result of homothetic preferences—freed the representative agent to be used in other fields of economics as well, such as growth theory, international trade, and financial economics.[29] As a matter of fact, in the last chapter I investigate how, if at all, Samuelson's representative agent as used in macroeconomics made it to Koopmans's: it turns out that the channel of influence might have played out in the subfield of optimal growth theory. But first, I shall discuss a change in the meaning of dynamics as portrayed in Hands (2010).

## 2.5 The Postwar Stabilization of Consumer Choice Theory: redefining dynamics

It seems that with regard to the stabilization of consumer choice theory, the thesis of a "skein," or of an "interlocking competitive system," consisting of theories that culminated in the three American neoclassical hubs (the Cowles general equilibrium approach, the Chicago Marshallian variant and Samuelson's revealed preference doctrine) has been virtually uncontested.[30]

[29] "Financial Economics" studies the interactions of households, firms and financial intermediaries in both domestic and international settings. In the latter case, national economic policies might be even more important a force. Robert C. Merton defines it as the overlapping of finance, monetary economics and public finance.

[30] Besides Hurwicz (1998), Cartwright (1998), and other criticisms contained in Backhouse et al. (1998), Daniel Hammond also challenged the validity of the Hotelling-Schultz impasse; specifically, Hammond (2006) disagrees with chief aspects concerning the unfolding of events from the standpoint of the Chicago School.

In an article published in the *European Journal of the History of Economic Thought* in 2010, Wade Hands introduced into the former interpretative thread yet another simultaneous modifying feature: consumer choice theory ceased to be dynamic. This time, the source of amendment came from another series of contributions by Paul Samuelson, notably his PhD dissertation *Foundations of Economic Analysis*, published in 1947, a source that played a key role.[31] In his appreciation of Samuelson's contributions, Hands (2010, p. 332) wrote:

> By the 1950s "dynamic" meant "based explicitly on differential or difference equations involving time," and optimization problems - maximum or minimum - were *not* of this sort. Maximization was not a dynamic process; the Walrasian tâtonnement was.[32]

In another passage, Hands claimed that:

> The ultimate impact of the separation - or the impact of the profession generally accepting this separation - was that consumer choice theory, which was based on utility *maximization*, ceased to have anything to do with movements or dynamics. Of course no dynamics means no paths, no endowment effects, no reference dependence, no order of consumption, none of the other problems associated with integrability$_B$. The concept of economic dynamics is stabilized and in the process consumer choice theory is relieved of the responsibility for dealing with all of these potentially troublesome issues. Economic agents with well-ordered preferences defined over the entire choice space became the standard basis for consumer choice theory, and the non-integrable case and all the difficulties associated with it quietly left the stage. Stabilizing dynamics thus helped stabilize consumer choice (ibid.).[33]

[31] In Wade Hands's argument, since Samuelson's articles on stability were all incorporated into his 1947 volume, it suffices to refer only to this last work as his main contribution to the topic. Furthermore, on the simultaneity issue, such contributions by Samuelson coincide temporarily with the formation of the "skein," as defended by Hands and Mirowski (1998) and Mirowski and Hands (1998).

[32] So clear was the message that Samuelson wanted to convey in his book that he separated it into Part I, dedicated to optimization problems, and Part II, dealing with dynamics.

[33] For an explanation of integrability$_B$, see Hands (2006).

If one were to follow Hands's classification on what (additionally) might or might not belong to the stabilized body of consumer choice theory, Koopmans's articles in intertemporal economics would not pass the test. That is to say, Koopmans continued to carry out his studies in the 1950s and 1960s in the "old" way, searching for a dynamic nature within it even after a possible separation between optimization and dynamics became more visible in the literature.

Even though it is possible to insist that Koopmans did not undertake dynamic analysis proper since neither differential nor difference equations were used, Hands's exemplification quoted above, wherein "well-ordered preferences defined over the entire space became the standard basis for consumer choice theory" (ibid., p. 332-3), cannot be used to shed light on Koopmans's contributions either. As I have treated in some detail elsewhere, a crucial part of his analysis shored up on the notion of impatience and time perspective, thus making time a crucial matter.[34] Furthermore, as it has also been shown, Koopmans continued to pursue a distinctive preference-based demand theory in the 1970s by postulating preference orderings *over time* rather than *on a prospect space*. For several authors, such a distinction cannot be easily applied either.

In Louis Phlips's instructive *Applied Consumption Analysis* (1983), all chapters concerning Koopmans's contributions to consumer choice theory were allotted to a whole section titled "Dynamics." In appraising the case in which the consumer looks into the future, Phlips (1983, p. 263) stated:

> We want to explain the allocation of his budget among *n* commodities, when due attention is given to the fact that he is not maximizing an instantaneous "static" or an instantaneous "dynamic" utility function (in which the influence of the past behaviour is incorporated) as in previous chapters, but is maximizing an "intertemporal" utility function defined on sequences over time (from now to some future date) of commodity bundles.

Judy Klein (2007) made the case in which Richard Bellman used dynamic programming

[34] See chapter one of this dissertation.

intensively to carry out "economizing" efforts during World War II. Economists then began to apply the same functional equation formulation first in microeconomics and, later, in macroeconomics. A successful incursion of economic dynamics into the realm of microeconomics would surely have required its application in consumer choice theory. Lars Peter Hansen (2010) argued that the first use of recursive preferences can be traced to Koopmans (1960) and Koopmans et al. (1964). Stokey et al. (1989) added to this list the article by Beals and Koopmans (1969).

Following this reasoning, considering that for Shone (2002, p. 3) "By its very nature, dynamics involves time derivative, $dx/dt$, where $x$ is a continuous function of time, or difference equations, $x_t - x_{t-1}$ where time is considered discrete units," and that, in the context of a neoclassical growth model, Corbae et al. (2009, p. 92) asserted that "A dynamic optimization problem is one in which a decision must be made over time in which early decisions affect later options," the rupture as proposed by Wade Hands seems hardly straightforward.

Although providing a historical account that cannot be taken at face value, the graduate textbook *Recursive Methods in Economic Dynamics* (1989) has this to say regarding the inclusion of dynamic (and stochastic) features in economics:

> These theoretical developments are based on a wide variety of results in economics, mathematics, and statistics: the contingent-claim view of economic equilibria introduced by Arrow (1953) and Debreu (1959), the economic applications of the calculus of variation pioneered long ago by Ramsey (1928) and Hotelling (1931), the theory of dynamic programming of Bellman (1957) and Blackwell (1965) (Stokey et al. 1989, p. 3).

The passage shows that at least some of the publications that contributed seminally to dynamics, and which were later applied to consumption theory, came out at the approximate time of their alleged separation (stabilization).[35] Finally, it can be argued that dynamic optimization techniques solve problems posed, for example, in the Hamiltonian dynamic

[35] In the case of Ramsey (1928), there is even a clearer intersection between dynamics and choice theory.

system and they date back at least to Samuelson and Solow (1956) (Wulwick 1995).

In distinct ways, the excerpts above coincide with a key economic substance in Koopmans's research: the blurring of optimization (choice theory) and dynamics (intertemporal features). One can discern such a feature in the articles written in the 1950-1970 period, as well as in the only application he made of that theoretical framework: the 1963 normative growth model.

## 2.6 Concluding Remarks

Besides the historical value of providing a more accurate account of demand theory, building such an alternative version served another purpose: it allowed us to better understand the rise of the representative consumer in the 1950s and 1960s. Furthermore, considering that this period also coincided with the discussions concerning the compatibility between general equilibrium and demand theories, I have attempted to offer a history of the representative agent in an even more convoluted context.

Even though it was not my aim to emphasize the complex Hotelling and Schultz dialogue, it is worth quoting a passage from Mirowski and Hands (1998) for two reasons. In addition to conveying precisely the methodological and economic significance of Samuelson's contributions to the impasse, the passage *per se* also helps us observe the state of demand theory at that time, and hence is useful for comparing what the Cowles Commission was at the end of the 1930s with what it became after an important person in this essay, Tjalling Koopmans, took over the research directorship of the institution in 1948:

> The problem for Samuelson was to find a way of formulating the theory of demand that would be consistent with his positivist-operationalist methodology while simultaneously *avoiding the type of econometric testing* associated with Schultz. How could this possibly be done? (...) In essence his answer was to *change the place where the empiricism lived in the neoclassical theory of demand*. Instead of having empiricism enter at the back end - by testing the empirical implications deduced from theory - the revealed preference approach

> would place empiricism right up front at the beginning of the exercise. If the epistemologically dubious notion of subjective utility could be replaced with a strictly behaviorist - thus objective, observational, operational, and meaningful - concept of consumer action, demand theory could be reconstituted on what Samuelson considered legitimate scientific foundations (Mirowski and Hands 1998, p. 283, italics added).

In other words, in the 1930s, and for most of the 1940s, demand analysis was overwhelmingly an econometric enterprise.

An important remark in this chapter, therefore, is that if the Cowles Commission carried out applied econometric projects of microeconomic and macroeconomic natures before Koopmans, after his appointment as research director in 1948 the institution was transformed into a pole of pure theoretical microeconomics. More important, motivated by aggregation discussions taking place in economics, the new research program at Cowles was also directed to tackle such a problem.

I have also stressed in this chapter that a significant portion of this discussion, of the compatibility between micro and macroeconomics, can be traced to another historical fact: the widespread adoption of homothetic preferences, first introduced by Paul Samuelson in 1956. Then, I have showed that Koopmans's research purpose intersected with Samuelson's push for homothetic preferences.

Although consumer theory and dynamics stabilized as two separate branches within microeconomics in the 1950s and 1960s, Koopmans still tried to extend the first subfield through the addition of dynamic features, initially with respect to the structure of utility functions and, later, with respect to preferences. This observation, however, seems to be at odds with interpretations given, for example, in Hands (2010), and I provided some examples in the literature showing why it is hard to tell consumption theory and dynamics apart.

I hope the narrative in this chapter has identified a few junctures that made more transparent the relationship between the development of demand theory and the rise of the representative agent in microeconomics.

At least two important questions remain open for further investigations. The first one concerns Koopmans's own use of homothetic preferences and how much he interacted on this matter not only with Paul Samuelson, but with Gerard Debreu, an important mathematical economist who made essential contributions to the axiomatization of economics and who was also a member of the Cowles Commission. Following this query, a second question concerns a clear association between the Koopmans-Samuelson writings on homothetic preferences and the time-separability preferences used in modern macroeconomics, as discussed in Barro and King (1984).

# 2.A Appendix

## 2.A.1 From the Consumer's Problem to Testable Properties (restrictions) of Marshallian Demand Functions

### 2.A.1.1 The consumer's problem (CP)

*Preference relations* gives a consumer some rules of choice: by applying them on the *consumption set*, $X = \mathbb{R}^n_+$, he/she chooses the desired bundles.[36] Given some *consumer's circumstances*, though, she/he has to narrow it down to a subset called a *feasible set*. Then, applying preference relations on the feasible set, a *consumer behaviour* is delineated: the person always chooses the preferred bundles according to their feasibility. Formally, the consumer seeks

$$\boldsymbol{x}^* \in B \text{ such that } \boldsymbol{x}^* \succsim \boldsymbol{x} \text{ for all } \boldsymbol{x} \in B. \tag{2.1}$$

With the definition of both the *utility function* and the *budget set*, the CP can be recast in familiar terms using differential calculus:

$$\max_{\boldsymbol{x} \in \mathbb{R}^n_+} \mathrm{u}(\boldsymbol{x}) \qquad \text{s.t.} \qquad \boldsymbol{p} \cdot \boldsymbol{x} \leq \mathrm{y}. \tag{2.2}$$

If $u(\cdot)$ is continuous and $B$ is a compact set (nonempty, closed, and bounded), the Weierstrass Theorem guarantees a solution to problem (2.2). Such a solution, $\boldsymbol{x}^*$, consists in an ordinary demand function called the *Marshallian demand function*: a unique function from the set of prices and incomes to the set of quantities.

### 2.A.1.2 The indirect utility function

An *indirect utility function*, $v$, is simply a real-valued function that summarizes the elements contained in the budget-set, *prices and income*, and the *maximized value of utility*. Formally, $v : \mathbb{R}^n_+ \times \mathbb{R}_+ \to \mathbb{R}$ is defined as follows:

$$v(\boldsymbol{p}, y) \equiv \max_{\boldsymbol{x} \in \mathbb{R}^n_+} \mathrm{u}(\boldsymbol{x}) \qquad s.t. \qquad \boldsymbol{p}{\cdot}\boldsymbol{x} \leq \mathrm{y}, \tag{2.3}$$

[36] Unless otherwise stated, this appendix follows Jehle and Reny (2011).

There is a clear relationship between the indirect and the direct utility functions:

(i) if $u(\boldsymbol{x})$ is continuous, $v(\boldsymbol{p}, y)$ is well-defined for all $\boldsymbol{p} \gg 0$ and $y \geq 0$, since a solution to problem (2.3) is guaranteed to exist;

(ii) if $u(\boldsymbol{x})$ is strictly quasiconcave, the solution $\boldsymbol{x}(\boldsymbol{p}, y)$, is unique. Thus,

$$v(\boldsymbol{p}, y) = u(\boldsymbol{x}(\boldsymbol{p}, y)). \tag{2.4}$$

**Theorem 2A.1 Properties of the Indirect Utility Functions**

*If $u(\boldsymbol{x})$ is continuous and strictly increasing on $\mathbb{R}^n_+$, then $v(\boldsymbol{p}, y)$ defined in (2.3) is as follows:*

1. *Continuous on $\mathbb{R}^n_{++} \times \mathbb{R}_+$,*
2. *Homogeneous of degree zero in $(\boldsymbol{p}, y)$,*
3. *Strictly increasing in $y$,*
4. *Decreasing in $\boldsymbol{p}$,*
5. *Quasiconvex in $(\boldsymbol{p}, y)$.*

Moreover, it satisfies

6. *Roy's identity: If $v(\boldsymbol{p}, y)$ is differentiable at $(\boldsymbol{p}^0, y^0)$ and $\partial v(\boldsymbol{p}^0, y^0)/\partial y \neq 0$, then*

$$x_i(\boldsymbol{p}^0, y^0) = -\frac{\partial v(\boldsymbol{p}^0, y^0)/\partial p_i}{\partial v(\boldsymbol{p}^0, y^0)/\partial y}, \qquad i = 1, ..., n. \tag{2.5}$$

On Roy's identity, Nolan Miller (2006, p. 60) says: "(...) in many cases it will be easier to estimate an indirect utility function and derive the direct demand functions via Roy's identity than to derive the $x(p, w)$ directly."

#### 2.A.1.3 The expenditure function

As an extreme-value function, the expenditure function (EF) is obtained by solving the following problem:

$$e(\boldsymbol{p}, u) \equiv \min_{\boldsymbol{x} \in \mathbb{R}^n_+} \boldsymbol{p} \cdot \boldsymbol{x} \qquad \text{s.t} \qquad u(\boldsymbol{x}) \geq u \tag{2.6}$$

for all $\boldsymbol{p} \gg 0$ and all attainable utility levels $u$.

The solution to (2.6) leads to a system of *Hicksian demand functions*, which in practice, consists in an unobservable expression. This reasoning bears on the concept of *compensated demand*, which works in the following way: suppose that Maria faces prices $p_1$, $p_k$ with income $y$ and chooses the bundle $x(p_1, p_k, y)$. Suppose now that there occurred a price change, $\Delta p_k$, and, at the same time, someone required Maria to keep her utility level constant. How could this requirement be satisfied? The answer is simple: if the price fell, the only way to keep Maria's utility unchanged is through a *decrease* in income (otherwise her utility would have increased). On the other hand, if there was a price increase, only through an income increase would her utility be maintained constant. Thus, the demand function attained through a change in income level in the "backdrop" to offset the initial price change (keeping utility constant) is the Hicksian or compensated demand function.

Figure 2 illustrates the explanation in two parts. Panel (a) shows a decrease in the price of good 1 and a simultaneous "compensatory" decrease in income to keep utility constant, for the gyration of the isoexpenditure line along the indifference curve. Panel (b) summarizes the change in consumption from $x_1^h(p_1^0, p_2^0, u)$ to $x_1^h(p_1^1, p_2^0, u)$, giving rise to the *Hicksian demand curve*.

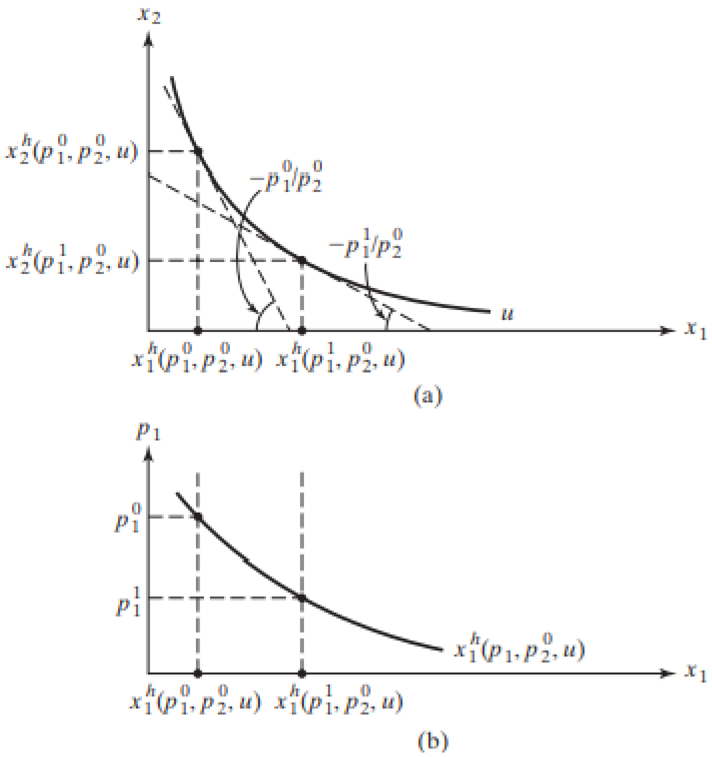


Figure 2 – Hicksian Demand Curve

Once the Hicksian demand functions have been obtained through (2.6), the expenditure function for a two-commodity case is

$$e(p_1, p_2, u) = p_1 \cdot x_1^h + p_2 \cdot x_2^h. \tag{2.7}$$

**Theorem 2A.2. Properties of the Expenditure Function**

*If $u$ is continuous and strictly increasing, then $e(\boldsymbol{p}, u)$ defined in 2.6 is as follows:*

1. *Zero when $u$ takes on the loIst level of the utility in $\mathcal{U}$,*
2. *Continuous on its domain $\mathbb{R}^n_{++} \times \mathcal{U}$,*
3. *For all $\boldsymbol{p} \gg \mathbf{0}$, strictly increasing and unbounded above in $u$,*
4. *Increasing in $\boldsymbol{p}$,*
5. *Homogeneous of degree 1 in $\boldsymbol{p}$,*
6. *Concave in $\boldsymbol{p}$.*

*If, in addition, $u(\cdot)$ is strictly quasiconcave, there is*

7. *Shephard's lemma: $e(\boldsymbol{p}, u)$ is differentiable in $\boldsymbol{p}$ at $(\boldsymbol{p}^0, u^0)$ with $\boldsymbol{p}^0 \gg 0$, and*

$$\frac{\partial e(\boldsymbol{p}^0, u^0)}{\partial p_i} = x_i^h(\boldsymbol{p}^0, u^0), \qquad i = 1, ..., n.$$

The Shephard's lemma allows applied economists to retrieve the unobservable Hicksian demand function by taking the partial derivative of $e(\boldsymbol{p}, u)$ with respect to the required price.

#### 2.A.1.4 Properties of demands

Theoretical properties achieved for Marshallian demand functions represent a capstone in the field of microeconomics. They guide research in empirical studies in the form of restrictions that estimated parameters should satisfy.

**(1) Budget Balancedness (Adding up or Walras's Law)**

This property implies two characteristics. First, only real variables, i.e., relative prices and real income, affect consumer's behaviour (no money illusion). Second, consumer spending will always exhaust income. They can be summarized in theorem 2A.3.

**Theorem 2A.3. Homogeneity and Budget Balancedness**

*If consumer's preference is complete, transitive, continuous, strictly monotonic and strictly quasi convex on* $\mathbb{R}^n_+$*, and additionally, it can be represented by a real-valued utility function, u, that is continuous, strictly increasing, and strictly quasiconcave on* $\mathbb{R}^n_+$*, then, the consumer demand function* $x_i(\boldsymbol{p}, y)$*,* $i = 1, ..., n$ *is homogeneous of degree zero in all prices and income, and it satisfies budget balancedness,* $\boldsymbol{p} \cdot \boldsymbol{x}(\boldsymbol{p}, y) = y$ *for all (*$\boldsymbol{p}$*,*$y$*).*

### (2) Symmetry and Negative Semidefiniteness of the Slutsky Matrix

To attain these other two independent properties of the system of demand functions, it is necessary to define *substitution* and *income* effects. These concepts are used to explain a very simple economic phenomenon: "How the quantity demanded of good $j$ changes due to a change in its price or a change in the price of a good $i$?" Eugen Slutsky provided the equation that linked both effects.

**Theorem 2A.4.** *The Slutsky Equation*

*Let* $\boldsymbol{x}(\boldsymbol{p}, y)$ *be the consumer's Marshallian demand system. Let* $u^*$ *be the level of utility the consumer achieves at prices* $\boldsymbol{p}$ *and income* $y$*. Then,*

$$\frac{\partial x_i(\boldsymbol{p}, y)}{\partial p_j} = \frac{\partial x_i^h(\boldsymbol{p}, u^*)}{\partial p_j} - x_j(\boldsymbol{p}, y) \cdot \frac{\partial x_i(\boldsymbol{p}, y)}{\partial y}, \qquad i, j = 1, ..., n.$$

Theorem 2A.4 shows that any total effect in quantity demanded caused by a variation in price can be decomposed into an unobservable *substitution* part and an observable *income* part: the left-hand side of the equation is the (Marshallian) total effect and the first right-hand term is the (Hicksian) substitution effect, while the second right-hand term is the income effect.

*There is also a special Slutsky equation: an own-price change*

$$\frac{\partial x_i(\boldsymbol{p}, y)}{\partial p_i} = \frac{\partial x_i^h(\boldsymbol{p}, u^*)}{\partial p_i} - x_i(\boldsymbol{p}, y) \cdot \frac{\partial x_i(\boldsymbol{p}, y)}{\partial y}.$$

The left-hand term gives the slope of the Marshallian demand curve for good $i$, while the first expression on the right-hand side is the slope of a Hicksian demand curve (the second expression on the right-hand side is again the income effect).

Since the ultimate intention consists in explaining the Marshallian demand curve, and because the Hicksian demand term is not directly observable, economists focus on the latter to find a way out.

**Theorem 2A.5. Negative Own-substitution Terms**

*Let $x_i^h(\boldsymbol{p}, u)$ be the Hicksian demand for good $i$. Then*

$$\frac{\partial x_i^h(\boldsymbol{p}, u)}{\partial p_i} \leq 0, \qquad i = 1, 2, ..., n.$$

**Proof:** Recall that the expenditure functions in the case of two commodities was $e(p_1, p_2, u) = p_1 x_1^h + p_2 x_2^h$. The seventh property of the expenditure function, Shephard's lemma, shows that $\dfrac{\partial e(\cdot)}{p_1} = x_1^h$. Taking a second-order partial derivative of the same expression results in the expression of this theorem: $\dfrac{\partial^2 e(\cdot)}{p_1^2} = \dfrac{\partial x_1^h}{\partial p_1}$. On its sign, the sixth property of the EF states that it is concave in $\boldsymbol{p}$ and, thus, according to a theorem on the sign of second-order partial derivatives for concave functions (see Jehle and Reny's (2011) Theorem A2.5), all of its second-order partial derivatives are non-positive.

**Theorem 2A.6. Symmetric Substitution Terms**

*Let $x^h(\boldsymbol{p}, u)$ be the consumer's system of Hicksian demands and suppose that $e(\cdot)$ is twice-continuously differentiable. Then,*

$$\frac{\partial x_i^h(\boldsymbol{p}, u)}{\partial p_j} = \frac{\partial x_j^h(\boldsymbol{p}, u)}{\partial p_i} \qquad i, j = 1, 2, ..., n. \tag{2.8}$$

To understand this theorem better, let us suppose again the two-commodity case where the EF is: $e(p_1, p_2, u) = p_1 x_1^h + p_2 x_2^h$. The following relations are straightforward:

$$\frac{\partial e(\boldsymbol{p}, u)}{\partial p_1} = x_1^h \tag{2.9}$$

$$\frac{\partial^2 e(\boldsymbol{p},u)}{\partial p_1^2} \equiv \frac{\partial x_1^h}{\partial p_1} \tag{2.10}$$

and

$$\frac{\partial^2 e(\boldsymbol{p},u)}{\partial p_2 \partial p_1} \equiv \frac{\partial x_1^h}{\partial p_2}, \tag{2.11}$$

where (2.10) and (2.11) are the "own-substitution" term and the "cross-substitution" terms, respectively. Likewise, for good 2

$$\frac{\partial e(\boldsymbol{p},u)}{\partial p_2} = x_2^h \tag{2.12}$$

$$\frac{\partial^2 e(\boldsymbol{p},u)}{\partial p_2^2} \equiv \frac{\partial x_2^h}{\partial p_2} \tag{2.13}$$

and

$$\frac{\partial^2 e(\boldsymbol{p},u)}{\partial p_1 \partial p_2} \equiv \frac{\partial x_2^h}{\partial p_1}, \tag{2.14}$$

where (2.13) and (2.14) are, again, the "own-substitution" and the "cross-substitution" terms, respectively.

It is possible to arrange the second-order partial derivatives of the EF (or the first-order partial derivatives of the Hicksian demand function, $x_i$) in a *substitution matrix* with the "own-substitution terms" on the principal diagonal and the "cross-substitution terms" *off* the principal diagonal. In the two-commodity case,

$$\sigma(p_1, p_2, u) = \begin{pmatrix} \dfrac{\partial x_1^h(\boldsymbol{p},u)}{\partial p_1} & \dfrac{\partial x_1^h(\boldsymbol{p},u)}{\partial p_2} \\ \\ \dfrac{\partial x_2^h(\boldsymbol{p},u)}{\partial p_1} & \dfrac{\partial x_2^h(\boldsymbol{p},u)}{\partial p_2} \end{pmatrix} \tag{2.15}$$

In the three-commodity case,

$$\sigma(p_1, p_2, p_3, u) = \begin{pmatrix} \dfrac{\partial x_1^h(\boldsymbol{p}, u)}{\partial p_1} & \dfrac{\partial x_1^h(\boldsymbol{p}, u)}{\partial p_2} & \dfrac{\partial x_1^h(\boldsymbol{p}, u)}{\partial p_3} \\ \\ \dfrac{\partial x_2^h(\boldsymbol{p}, u)}{\partial p_1} & \dfrac{\partial x_2^h(\boldsymbol{p}, u)}{\partial p_2} & \dfrac{\partial x_2^h(\boldsymbol{p}, u)}{\partial p_3} \\ \\ \dfrac{\partial x_3^h(\boldsymbol{p}, u)}{\partial p_1} & \dfrac{\partial x_3^h(\boldsymbol{p}, u)}{\partial p_2} & \dfrac{\partial x_3^h(\boldsymbol{p}, u)}{\partial p_3} \end{pmatrix} \tag{2.16}$$

By satisfying Young's Theorem, all cross-partial derivatives are equal. This fact makes the *substitution matrix symmetric.*

It is important to remember that the elements in the substitution matrix are not the *first-order* partial derivatives of the EF with respect to prices (as attained through Shephard's lemma), but the *second-order* partial derivatives:

$$\begin{pmatrix} \dfrac{\partial x_1^h(\boldsymbol{p}, u)}{\partial p_1} & \dfrac{\partial x_1^h(\boldsymbol{p}, u)}{\partial p_2} \\ \\ \dfrac{\partial x_2^h(\boldsymbol{p}, u)}{\partial p_1} & \dfrac{\partial x_2^h(\boldsymbol{p}, u)}{\partial p_2} \end{pmatrix} \equiv \begin{pmatrix} \dfrac{\partial^2 e(\boldsymbol{p}, u)}{\partial p_1^2} & \dfrac{\partial^2 e(\boldsymbol{p}, u)}{\partial p_2 \partial p_1} \\ \\ \dfrac{\partial^2 e(\boldsymbol{p}, u)}{\partial p_1 \partial p_2} & \dfrac{\partial^2 e(\boldsymbol{p}, u)}{\partial p_2^2} \end{pmatrix} \tag{2.17}$$

Since (2.17) is the Hessian matrix of the EF and being the EF concave with respect to $\mathbf{p}$ (property 6), the expression is negative semidefinite. This gives rise to the *negative semidefinite substitution matrix*, as stated in theorem 2A.7.

**Theorem 2A.7. Negative Semidefinite Substitution Matrix**

*Let $x^h(\boldsymbol{p}, u)$ be the consumer's system of Hicksian demands and let the matrix*

$$\sigma(\boldsymbol{p}, u) \equiv \begin{pmatrix} \dfrac{\partial x_1^h(\boldsymbol{p}, u)}{\partial p_1} & \cdots & \dfrac{\partial x_1^h(\boldsymbol{p}, u)}{\partial p_n} \\ \\ \vdots & \ddots & \vdots \\ \\ \dfrac{\partial x_n^h(\boldsymbol{p}, u)}{\partial p_1} & \cdots & \dfrac{\partial x_n^h(\boldsymbol{p}, u)}{\partial p_n} \end{pmatrix} \tag{2.18}$$

*called the substitution matrix contains all the Hicksian substitution terms. Then, the matrix* $\sigma(\boldsymbol{p}, u)$ *is negative semidefinite.*

Finally, to reach a *symmetric* and *negative semidefinite* Slutsky matrix, recall the $ij^{th}$ Slutsky equation (Theorem 2A.4):

$$\frac{\partial x_i(\boldsymbol{p}, y)}{\partial p_i} = \frac{\partial x_i^h(\boldsymbol{p}, u)}{\partial p_i} - x_j(\boldsymbol{p}, y)\frac{\partial x_i(\boldsymbol{p}, y)}{\partial y},$$

which can be rewritten as

$$\frac{\partial x_i^h(\boldsymbol{p}, u)}{\partial p_j} = \frac{\partial x_i(\boldsymbol{p}, y)}{\partial p_j} + x_j(\boldsymbol{p}, y)\frac{\partial x_i(\boldsymbol{p}, y)}{\partial y}. \tag{2.19}$$

Then, substituting (2.19) for elements of matrix (2.18), we obtain the Slutsky matrix:

$$S(\boldsymbol{p}, y) = \begin{pmatrix} \frac{\partial x_1(\boldsymbol{p}, y)}{\partial p_1} + x_1(\boldsymbol{p}, y)\frac{\partial x_1(\boldsymbol{p}, y)}{\partial y} & \cdots & \frac{\partial x_1(\boldsymbol{p}, y)}{\partial p_n} + x_n\frac{\partial x_1(\boldsymbol{p}, y)}{\partial y} \\ \vdots & \ddots & \vdots \\ \frac{\partial x_n(\boldsymbol{p}, y)}{\partial p_1} + x_1(\boldsymbol{p}, y)\frac{\partial x_n(\boldsymbol{p}, y)}{\partial y} & \cdots & \frac{\partial x_n(\boldsymbol{p}, y)}{\partial p_n} + x_n(\boldsymbol{p}, y)\frac{\partial x_n(\boldsymbol{p}, y)}{\partial y} \end{pmatrix} \tag{2.20}$$

which, like the substitution (Hicksian demand) version, is *symmetric and negative semidefinite.*

### 2.A.2 The Integrability Problem: connecting theoretical and applied microeconomics

Integrability is the process of recovering a consumer's utility from his/her demand function. It means the possibility of moving back and forth between the demand function and preferences, a result of utmost importance especially in applied works. This is the case because empirical economists generally specify (postulate) a demand function first and check whether it satisfies the theoretical properties later. To understand the integrability problem (in loose terms), we know that

$$\max_{\boldsymbol{x}\in\mathbb{R}^n_+} \mathrm{u}(\boldsymbol{x}) \qquad s.t. \qquad \boldsymbol{p}\cdot\boldsymbol{x} = \mathrm{y}$$

results in a system of demand functions. Then bringing into play diverse concepts such as indirect utility, the expenditure function, Hicksian demand and duality (see figure 3.G.3 in Mas-Colell et al. (1995, p.75)), the Slutsky matrix is shown to be symmetric and negative semidefinite.

Since every element of the Slutsky matrix is a *partial differential equation (PDE)*, which leads to a *system of PDE*, the *integrability conditions* state that the converse is also true: there will always be a system of Partial Differential Demand Equations whose solution (integral) is the utility function that generated the demand function.

**Theorem 2A.8. The Integrability Theorem**
*A continuously differentiable function* $\boldsymbol{x}\colon \mathbb{R}^{n+1}_{++} \to \mathbb{R}^n_+$ *is the demand function generated by some increasing, quasiconcave utility function if (and only if, when utility is continuous, strictly increasing, and strictly quasiconcave) it satisfies budget balancedness, symmetry, and negative semidefiniteness.*

A fuller discussion of this theorem is provided in, among other sources, Mas-Colell et al. (1995, section 3.H), Jehle and Reny (2011, section 2.2) and Kreps (2013, section 11.5), though a proof is provided in Hurwicz and Uzawa (1971), a classic article on the subject. A thorough survey can be found in Hurwicz (1971), while some historical questions are raised by Hands (2006, 2011) and Gardes and Garrouste (2006).

### 2.A.3 Homothetic Preferences and Implications

Preferences are homothetic if indifference curves are invariant to scaling up consumption bundles: doubling income doubles all the quantities. Indifference surfaces, in other words, are radial blowup of other indifference curves. As a result, their marginal rates of substitutions are constant along rays through the origin. Homothetic preferences are representable by the *homogeneous of degree one* utility functions: $u(\lambda \boldsymbol{x}) = \lambda u(\boldsymbol{x})$. Such a function carries an interesting feature: when maximized, it gives rise to a demand function whose ratio depends only on price, not on income. The concepts of *parallel* and *linear* Engel curves can help us grasp the implications of homothetic preferences.

In the case of parallel Engel curves, marginal changes in consumption caused by a redistribution of income is the same for all agents in the economy. For instance, take $1 away from Ana and give it to John. Ana's fall in consumption is exactly John's increase. Under this assumption, aggregate consumption as well as aggregate income remain constant. In the case of linear Engel curves, marginal changes in consumption caused by a redistribution of income is the same for all agents in the economy even if their incomes differ. This definition is more subtle. In the example of parallel curves, Ana and John had the same income level. In the case of *linearity*, we go a bit further and consider the same unchanged marginal effect in consumption even if Ana were poor and John rich. To sum up, here, taking $1 away from Ana and giving it to John will result in Ana's fall in consumption which is, exactly, John's increase: both consumers have an identical marginal propensity to consume.

# 3 The Representative Agent in Macroeconomics: The Samuelson-Koopmans Thread

*The tracing of influences is the most treacherous ground in the history of thought.*

- Friedrich Hayek, *1964*

*Samuelson is omnipresent in American and even world economics; (...) he appears at every turn of history and in every disguise. The unwilling college student (...) finds him there, expounding economic wisdom in eight hundred well chosen pages (1967). The graduate student is disciplined to our trade by study of the Foundations (1947) and two dozen assorted papers of Samuelson. In the pages of every journal and many a collective volume he appears, read equally by esoteric specialists in optimal growth or integrability conditions and by the most policy-oriented in practical central banking, impatient with theoretical niceties.*

- Kenneth Arrow, *1967*

## 3.1 Introduction

The representative agent gained prominence in the 1970s with the rise of the New Classical Macroeconomics.[1] In this tradition, the demand function obtained in the constrained utility maximization problem of a single consumer is taken to be the exact representation of an aggregate demand function. This, among other reasons, explains why the term "microfoundations" gained currency around the same period.[2] To most economists working in the 1970s (and after), a microeconomic phenomenon preceded every economic phenomenon, even those that were traditionally considered macroeconomic in nature such as output fluctuation, inflation, and unemployment.

[1] For a thorough historical analysis of the New Classical of Macroeconomics, see Hoover (1988).

[2] See Hoover (2012) for an analysis of microfoundational programs, especially the one associated with the New Classical Macroeconomics.

This chapter studies the history of the representative agent in the growing of optimal growth models, an important macroeconomics literature that flourished in the 1950s and 1960s. By re-centering the rise of that methodological device on a preceding period, I show that it was not exclusively a New Classical enterprise and, following this reasoning, rather than studying the contributions of Robert Lucas, the works of two other economists take center stage in this story: Tjalling Koopmans and Paul Samuelson. Since both authors made important contributions to the subdisciplines of choice as well as optimal growth theories, by focusing on their intertwined developments I identify a possible path by which the representative agent could have achieved its importance later in the literature.

Apart from this introduction, this chapter is organized as follows. Section 3.2 begins the study of what is, perhaps, the first article written by Paul Samuelson where a macroeconomic problem is solved using a consumer choice model. In the next section, 3.3, I show how the representative consumer evolved with the development of turnpike models, a story centralized in a seminal article written by Samuelson in 1965. In section 3.4, I examine a possible connection between the methodological device and the MIT graduate program in economics of the 1960s. Then, in section 3.5, I try to determine Samuelson's (1965) influence in a few yet important articles in the optimal growth literature written around the same time. From section 3.3 onward, I simultaneously investigate a connection, or a thread, between Samuelson and Koopmans, the latter also a fundamental contributor in the solution of macroeconomic problems using a consumer choice framework, as well as of the turnpike literature. Section 3.6 offers some concluding remarks.

## 3.2 The Infinitely-lived Representative Consumer in Macroeconomics

The earliest hint to what appeared to be an infinitely-lived consumer solving an utility maximization problem in the works of Paul Samuelson can be found in three articles. The 1937 "Note on Measurement of Utility," the 1943 "Dynamics, Statics, and the

Stationary State" and, finally, the 1951 "Theory of Optimal Taxation."[3] In the 1943 paper, motivated to examine the properties of dynamic models vis-à-vis static ones, Samuelson singled out a famous article by Lionel Robbins in which he criticized Joseph Schumpeter for what was the Harvard professor's misunderstanding of the implication of a zero interest rate in the steady state.[4] To illustrate that Schumpeter had not made the alleged mistake, Samuelson (1943, p. 62) wrote:

> Let us assume (...) a condition of perfect certainty and an economy consisting of one or more individuals. We further assume (...) that there is no intrinsic rate of time preference. We need not speculate as to whether or not this implies infinite life expectations for the individuals, for the family, etc., etc., since in any case we are not concerned here with the realism or the usefulness of the argument.

And, turning his attention to the utility (function) adopted by that individual, he concluded by saying:

> (...) it is the same function at each instant of time and the individual acts so as to maximize the sum of utilities thus defined over all future time (ibid.).

At least with regard to the issue of a zero interest rate discussed in the controversy, the 1943 article went unnoticed for almost thirty years until John K. Whitaker took on the matter in a paper that attempted to reinstate the accuracy in Robbins's arguments.[5] Thinking of a reason for this long neglect, Whitaker pointed to Samuelson's particular way of approaching the problem, one based upon the choices of identical individuals or of an aggregate entity that maximized a certain utility function with a long-run horizon:

[3] "Theory of Optimal Taxation" first appeared as a memorandum for the U.S. Treasury in 1951. It was later published in the *Journal of Public Economics* in 1986.

[4] This discussion began, in fact, between Joseph Schumpeter and his fellow Austrian economist Eugen Böhm von Bawerk and became known in the literature as "The Schumpeter-Böhm Bawerk controversy." For a review of this dispute, see Faber (1979, chapter seven). For a survey of other controversies in Austrian Capital Theory, see Garrison (1990).

[5] John K. Whitaker (1933-2016) was professor of Economics at the University of Virginia for more than thirty years. Past president of the History of Economics Society (HES), Whitaker was most known for his expertise on Alfred Marshall, having edited *The Early Economic Writings of Alfred Marshall (1867-1890)*, a two-volume collection published in 1975 based on the english economist's previously unknown works.

> Such a delay in reaction may seem surprising. But in fact it is only in the last few years that our understanding of the subtle problem of decision making with infinite horizons has advanced sufficiently to bring to light certain difficulties in Samuelson's attempt to confound Robbins' criticism of Schumpeter (Whitaker 1971, p. 389).

In mathematical form, Samuelson had resorted to the expression

$$U_t = \int_t^{\infty} u[c(v)]dv; \qquad u', -u'' > 0, \tag{3.1}$$

where $u[\cdot]$ is the utility function, $c$ is consumption and $v$ is time.[6] In Whitaker's words, (3.1) was a model of "immortal individuals, all maximizing at any date a Ramsey preference function (...) with zero time preference" (ibid.).

In a reply to Whitaker's (1971) charges, Samuelson (1971, p. 391) himself, as expected, resorted to a representative agent-like economy to substantiate his arguments. "[S]uppose we first deal with Robinson Crusoe. He lives forever and has no time discount," Samuelson wrote and went to propose the following model:

$$max \int_0^{\infty} u[c]dt, \tag{3.2}$$

where the budget constraint was a Ramsey-Solow type technology given by $C + \frac{dK}{dt} = F(K, L) = f(k)$, with $L$ constant and $f'(K) = 0$, a Schumpeterian golden-rule case.[7]

Two interesting historical questions can be raised in connection to Whitaker's justification concerning the profession's delay in addressing the problem posed by Samuelson in 1943. First, *how had the problem of decision making with infinite horizons evolved since Samuelson (1943) through the 1970s?* And, following the first question, *how did the problem of decision making with infinite horizons make its way to optimal growth models?*

In a reply to an inquiry by Whitaker on the Robbins-Schumpeter subject, Samuelson stated (italics added):[8]

6 Whitaker (1971) explained in footnote 7 how, by interpreting Samuelson (1943), he reached the utility function (3.1).

7 I will discuss the meaning of this formulation in more detail in section 3.3.

8 Letter from Samuelson to Whitaker, April 5, 1971, PSP, box 62, file folder "Review of Economics and Statistics, 1942-1969 and undated."

> I think that you have a point (that Robbins had a valid point). My cavalier rejection may well have to be rewritten *in line with today's progress in growth models*. I even have the impression that I have run across similar observations in the modern literature.

As this quotation seems to suggest, there is possibly a direct connection between the development of infinite-horizon decision making in microeconomics and its simultaneous accession to macroeconomics. In chapter two of this dissertation, I attempted to provide an answer to the first question, an account that associated the rise of the representative agent with the struggles within microeconomics, particularly in demand theory. In this chapter, I turn my attention to the second question, focusing on the rise of the representative agent in optimal growth models.

## 3.3 Optimal Growth Models, the Turnpike Theory, and the Initial Explicitness of the Representative Agent

The reconstruction efforts in the years following the end of World War II fostered new research currents in both development economics and growth theories in many parts of the world.[9] Of particular urgency were investigations into how through certain economic policies a nation-state could increase its rate of growth and, therefore, secure a higher standard of living for its citizens. In connection with this objective, a question economists (and policymakers alike) frequently asked was, What would be the optimal rate of capital accumulation in a given country? Normative growth theorists who pursued similar questions during peacetime were led inevitably to a pioneering article written by Frank Ramsey in 1928 entitled "A Mathematical Theory of Saving," published in the British

9 For example, in the United Kingdom, the Harrod (1939) model (during World War II); in Latin America the studies developed by the Comisión Económica para América Latina y el Caribe (CEPAL) (see Bielschowsky (ed., 2016)); in the United States the articles written by Evsey Domar (1946) and Robert Solow (1956, 1957). For a critical overview on the connection, or lack thereof, between growth theories and development economics after Solow (1956, 1957), see Toye (2009). Snowdon (2009) provides a comprehensive treatment of growth processes which include, but are not limited to, poverty traps, aids to poor countries, and the sometimes neglected notion of "big-push" in both the growth and development literatures.

*Economic Journal*, one of the profession's leading research venues.[10]

Since the publication of Ramsey (1928), the first influential papers that appeared in the United States in the mid-1950s to mid-1960s dealing with problems of the same nature were Tjalling Koopmans's 1963 "On the Concept of Optimal Economic Growth," discussed in chapter one of this dissertation, and David Cass's 1965 "Optimal Growth in an Aggregative Model of Capital Accumulation."[11] The three articles went down in history as having formed the rudimentary framework for the present-day dominant approach, acting as a sort of dorsal spine to macroeconomic models of neoclassical inspiration (Gaspard 2001).[12]

As I shall try to show below, by the time Koopmans's and Cass's articles were published, the representative agent had already started to gain wide acceptance in macroeconomics, first and foremost within a particular community a few years before, at the start of the 1960s. It began with a group of macroeconomists who turned out to be directly related to a graduate course in "Advanced Economic Theory" co-taught by Robert Solow and Edmund Phelps at MIT in the fall of 1962.

To substantiate this claim, I bring to the forefront of the representative agent history a particular variant of optimal growth models, a deterministic version studied under the turnpike literature. Shepherded into economics by, among others, Paul Samuelson as early as the 1940s, and refined in the ensuing decades by other economists, the turnpike was first "conjectured" in a research report Samuelson wrote for RAND in 1949, but made available to a wider research audience (much) later in the first volume of his *Collected*

[10] On the subject of progress in economic science, Koopmans (1967) contended that if "the great depression" or "the great crisis of capitalism" had not intervened immediately after Ramsey published his famous paper and thus "deflected economists from following Ramsey's powerful ideas," the optimal growth literature of the 1960s could have been written more than thirty years in advance. Koopmans (1967) was first published as Koopmans (1966b), a Cowles Foundation Discussion Paper number.

[11] Published in the *Review of Economic Studies*, Cass's renown article was a revised version of the first chapter of his PhD thesis written under the supervision of the Japanese economist Hirofumi Uzawa of Stanford University. For an interesting account on the relationship between Uzawa and Cass and other related issues, see the interview with Cass (Spear and Wright 1998).

[12] See Duarte (2009) on how Ramsey's growth model made it into the neoclassical growth literature and ended up becoming, as argued by the author, its sacred predecessor. Olivier Blanchard (2000) tells a story of the "rescue" of the Ramsey (1928) model by mainstream economists in the 1960s. When Michael Woodford presented a paper in 2008 in a session of the *American Economic Association* on the topic of "Convergence in Macroeconomics" and claimed "wide agreement" among macroeconomists regarding the use of "intertemporal general-equilibrium foundations," he meant also the model approach initiated by Ramsey (1928) and continued by Cass (1965) and Koopmans (1965).

*Papers* (Samuelson 1966).[13] As Samuelson would later recall in an *American Economic Review* article, in the 1949 research report the turnpike came "one clue" short of being formulated (and proved) in connection with the "(...) famous Ramsey problem of optimal saving" (Samuelson 1965, p. 486).[14] A formal, albeit incomplete, proof of the "turnpike theorem" appeared then in *Linear Programming and Economic Analysis* (1958), which Samuelson wrote (also for RAND) with his MIT colleague Robert Solow and Harvard economics professor Robert Dorfman.[15]

As I have treated in chapter two of this dissertation, in addition to being remarkable for several theoretical developments within economics, such as the stabilization of consumer choice and the existence proof in general equilibrium theories, the third quarter of the twentieth century also witnessed the rise of normative growth models. For Avinash Dixit, Samuelson's most significant contribution within this field was "(...) the development of nonsubstitution theorems and *turnpike theorems*" (Dixit 2012, p. 13, italics added).[16]

Meaning a sort of "ideal path" over which a balanced growth at a maximal rate could be achieved for such variables as a utility function, capital stock, output, or a combination of these or any other variables, the turnpike reasoning was rapidly incorporated into the optimal growth literature: it provided economists with a way of thinking about how a country could achieve such an ideal (maximal) growth pattern, if at all. In studying the turnpike theorems, McKenzie (1986, p. 1281) wrote: "We will be concerned with the long-term tendencies of paths of capital accumulation that maximize, in some sense, a utility sum for society over an unbounded time span." In other words, the turnpike theory consisted in formulating normative mechanisms, such as choosing a certain saving rate,

[13] After the end of World War II, in March 1946, the U.S. Air Force initiated a project, called project RAND, aimed at providing the military with research in decision problems. The contributions would come from the fields of applied mathematics, statistics, and economics. The project became independent in May 1948 and has been operating under the name the Rand Corporation ever since. It is still located in Santa Monica, California.

[14] Samuelson (1965) associated the turnpike theorem as used in economics with a counterpart concept found in mathematics called *catenary properties*. For him, the application of that mathematical property was first undertaken in economics by Ramsey (1928).

[15] Given this co-authorship, this version of the turnpike theorem became associated with the acronym "DOSSO" or "DSS." See, for example, McKenzie (2008) and Spear and Young (2015).

[16] In Dixit's (2012) view, Samuelson's writings on growth and capital theories can be put under the same label. Yet, it is interesting to note that although a "taking stock" of Samuelson's contributions to the latter field could be found in the early 1980s (see, e.g., Solow 1983), the same does not hold true for his contributions to growth theory, particularly, to turnpike theory.

that at least enabled an economy to approximate as much as possible the *vicinity* of such a maximal growth state.

When in 1949 Samuelson found himself close to writing down the turnpike theorem, he believed that consumption should in no way be included, since in that RAND report he was following von Neumann (1945), which was basically a production model lacking the consumer side. Samuelson's 1965 article, however, was an explicit attempt at formulating a *new* turnpike theorem in which the utility of consumption would play a major role or, in his words, would "[represent] the *desideratum* for the problem" (Samuelson 1965, p. 486).[17]

Indeed, in addition to the general desire to push for a utilitarian approach to economic growth in this period, consumption made its way into the turnpike analysis because Samuelson was now also addressing how to reach its optimal level, which, in the 1960s, became associated with a certain "golden rule": choosing the right saving rate that would maximize the consumption level for all generations, as first proposed in Edmund Phelps's 1961 "The Golden Rule of Accumulation: A Fable for Growthmen."[18]

In Samuelson's mind, it was yet another opportunity to tackle the famous Ramsey problem of optimal saving (or optimal growth), but now in connection with a most modern discussion: the turnpike and, especially, the golden rule for maximum consumption. The amalgamation of these topics at that time can be observed from the following examples. At the beginning of a note taken on a lecture delivered by Edmund Phepls in 1962 at MIT, MIT doctoral student Edwin Burmeister wrote: "Golden Rule," then followed it with "another way to look at the same problem to which Ramsey addressed himself: optimal saving ratio."[19] In another instance, while questioning Tjalling Koopmans about models

[17] In his American Economic Association "Ely Lecture," McKenzie (1998) recounted a very engaging story related to this episode, and knowing it may help clarify some aspects related to the history of turnpike theory. According to him, when von Neumann presented his famous 1945 article in a seminar in Cambridge, Massachusetts, the young Paul Samuelson objected to the lack of maximization of an objective function. The *idea* of the turnpike, which linked the production side in the activities model and the attainment of various types of maximum (for example, initial or terminal capital stock), was then born precisely in the 1949 RAND paper (although, as I stated before, proved only in 1958). The quarrel between Samuelson and von Neumann further illuminates why Samuelson might have wanted to concentrate only on the production side in his 1949 article.

[18] See Duarte (2016) and the references therein for a better treatment of the utilitarian approach to growth problems.

[19] Notes from classes, December 14, 1962, EBP, box 23, file folder "Notes and tests from classes taught by Solow and Modigliani."

with multiple turnpike equilibria, Samuelson wrote: "I would doubt this on 'golden-rule considerations'."[20]

It is important to consider from the very outset, though, that there was never a "golden rule" in Ramsey (1928). For him, the initial problem to be solved was

$$\max_{x_t, a_t} \int_0^\infty [U(x(t)) - V(a(t))]dt, \tag{3.3}$$

where $U(x)$ is total utility from consumption, $x$; $V(a)$ is total disutility from labour, $a$; and the difference $[U(x(t)) - V(a(t))]$ denotes collective "net enjoyment per unit of time," which is an increasing function *only* of capital stock.[21]

By not resorting to any form of time discounting, since it consisted in an "ethically indefensible practice" (Ramsey 1928, p. 543), no solution could be found to the maximization problem as it stood.[22] To achieve *convergence* in that improper integral Ramsey resorted to a satiation point, a "bliss" level which worked simply as an upper bound to the expression.

With a basic understanding of Ramsey (1928) at hand, I shall discuss how, in turn, Samuelson (1965) dealt with the problem posed by the Cambridge (UK) economist and, fundamentally, how he solved it. Samuelson rewrote Ramsey's problem in the following way:

$$\max_{[k(t)]} \int_0^\infty u[c(t)]dt = \int_0^\infty u[f(k(t)) - \dot{k}(t)]dt \qquad \text{s.t} \qquad k(0) = k^0, \tag{3.4}$$

where $u(c)$ is utility of consumption; $f(k)$ is annual output, which depends on the capital-labour ratio (initially labour grows at zero rate); and $\dot{k}$ is the net variation of capital over time (after depreciation) or, simply, the amount of saving, $f(k) - c$.

Searching for an upper bound for this infinite integral, Samuelson also took up

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20 Letter from Samuelson to Koopmans, March 21, 1972, PSP, box 45, file folder "Tjalling Koopmans, 1946-1985." In the letter, Samuelson reminded Koopmans of the model with multiple turnpike equilibria he had developed in Liviatan and Samuelson (1969), thus, incompatible with the notion of golden-rule.

21 This interpretation should be straightforward since such maximum net enjoyment "(...) subject to the condition that our expenditure $x$ is equal to what we can produce with labour $a$ and capital $c$" (Ramsey 1928, p. 544) is predicated on the assumption of a fixed population: hence, if the population growth rate is zero, in Ramsey's nomenclature, $\frac{\dot{a}}{a} = 0$, then net enjoyment is a function solely of capital.

22 Maybe also as a philosopher, Frank Ramsey did not see any reason why the utility of future generations should not be as highly esteemed as today's; besides, he also considered that such a practice represented a "(...) weakness of the imagination" (ibid.).

Ramsey's bliss saturation assumption by, in the first solution, pursuing a "Schumpeterian case," where the marginal product of capital reaches zero—in other words, "(...) where capital gets saturated at a finite level, producing a zero own-rate-of interest" (Samuelson 1965, p. 487). The bliss-convergence conditions require then

$$\begin{aligned} &f(k),\ \text{a concave, nonnegative function, } f'(k) \\ &f(k^*),\ \text{a unique maximum for } 0 < k^* < \infty \text{, where } f'(k) = 0 \\ &u(c),\ \text{an increasing concave function for } 0 \leq c \leq c^* = f(k^*) \end{aligned} \tag{3.5}$$

and, in order to avoid working with minimization of the deviation from the bliss point, he set $u(c^*) = 0$, which implied $u(c) < 0$.[23]

In Samuelson's interpretation there was yet another way to achieve the bliss point, this time via the saturation of $u(c)$. To solve it, and also as a way to bring the golden rule to the discussion, he proposed a "simple device" enabling the re-writing of the Ramsey problem in *per-capita terms*; it consisted in postulating a neoclassical production function *with* the assumption of a growing population:

$$C + \dot{K} = F(K, L) = LF\left(\frac{K}{L}, 1\right) \tag{3.6}$$

and

$$L(t) = L(0)exp(g \cdot t), \tag{3.7}$$

where the latter expression shows the exponential growth of the population at rate $g$. Next, by "following the crowd," it is possible to write

$$\max \int_0^T U\left(\frac{C}{L}\right) dt, \tag{3.8}$$

where $U\left(\frac{C}{L}\right)$ is the utility of the *representative man*.

As the author continued, "With minor changes in notation, we can now throw this problem into the Ramsey form, the only difference being that we intepret $k$, $c$, and $u$ as

23 Although mentioning it, at no moment did Samuelson write the minimization (of the integral) of the deviation of net enjoyment from the bliss state, as found in Ramsey's article.

per capita terms" (ibid., p. 495). Thus

$$k = \frac{K}{L}, \frac{\dot{K}}{K} - \frac{\dot{L}}{L} = \frac{\dot{K}}{K} - g = \frac{\dot{k}}{k}$$

$$\frac{C}{L} = c, U\left(\frac{C}{L}\right) = u(c). \tag{3.9}$$

Since $\frac{d}{dt}\left(\frac{K}{L}\right) = \frac{\dot{K}}{L} - k\frac{\dot{L}}{L}$, after dividing equation (3.6) by $L$ and subtracting $gk$, it becomes

$$c + \dot{k} = f(k) - gk \equiv \psi(k^*). \tag{3.10}$$

Finally, it has been shown that equations (3.4) and (3.10) are identical, except that the former is a special case of the latter (with $g = 0$).

It is from this point, then, that the Ramsey bliss condition, aided with the assumption of the population growing exponentially at rate $g$, becomes the Phelps-Swan-Robinson Golden-Rule state of *per capita* consumption, given by

$$\max_k \psi(k) = \max_k f(k) - gk = \psi(k^*), \tag{3.11}$$

where

$$f'(k^*) = \text{interest rate} = g, \text{the system's natural rate of growth.} \tag{3.12}$$

The gist of the argument is that since total utility is zero, $u(c^*) = 0$, the utility integral of the *representative man* will converge as $T$ approaches infinity (if it is measured as a negative divergence from the golden-rule state, $c^* = f(k^*)$). In Samuelson's understanding, what ensues is that all properties related to the per capita consumption turnpike theorem are straightforward from this point (for example, the catenaries and the saddlepoints, all in per capita terms).

On the whole, it is important to recapitulate the two facts I have singled out hitherto involving optimal growth theories, the turnpike and the golden rule. In the first, I argued that in the production model introduced by von Neumann (1945), Samuelson (1949) and Dorfman, Samuelson and Solow (1958) adapted the concept of turnpikes. The second fact is that in the utilitarian model of consumption pioneered by Ramsey (1928), the same notion of turnpike was adapted by Tjalling Koopmans (1965), David Cass (1966) and, again, Samuelson (1965). Along this line, on how these facts gave rise to optimal growth theories as we understand it today, McKenzie (1998, p. 2) stated:

> The subsequent history of models of optimal growth has featured an interplay of these two foundations. That is, the Ramsey objective of maximizing a utility sum over time has been introduced into the disaggregated model of von Neumann, and the von Neumann production sector featuring numerous activities has been introduced into the Ramsey model.

Another example of this development is that many scholars of the 1960s, who would later be recognized for their contributions to optimal growth and general equilibrium theories, were highly interested in the turnpike literature as well. In a letter written to Paul Samuelson, while holding a professorship at Osaka University (Japan) in 1960, Lawrence Klein reminded his former PhD adviser that[24]

> A paper has apparently been submitted to IER by Furuya and Inada on Balanced Growth and Intertemporal Efficiency in Capital Accumulation. There is some mix-up, and it might have gone to Econometrica. In any way, it deals with your work on balanced growth (Econometrica) and on turnpikes.[25]

Klein concluded by adding:[26]

> On our way here, I stopped at Berkeley where Frank Hahn and Roy Radner were considerably exercised over Turnpikes. When we arrived here, Hicks and Morishima were excited about the same problems.

In my view, an important turning point in the use of the representative agent in the optimal growth literature may be found in the research efforts that, in one way or the other,

[24] Letter from Klein to Samuelson, June 27, 1960, PSP, box 132, file folder “Lawrence Klein, 1944-2006.”

[25] “IER” stands for *International Economic Review*, a journal published by the University of Pennsylvania economics department.

[26] Letter from Klein to Samuelson, June 27, 1960, PSP, box 132, file folder “Lawrence Klein, 1944-2006.”

had Samuelson (1965) as a passage point: either to provide solutions to the golden-rule problem or to aid in the development of turnpike approaches, many subsequent articles took the advice of that "simple device" into consideration (as I will show below).

Moreover, even if the aforementioned article by Samuelson was not the first to pose explicitly the use of a representative agent (or "representative man" in his term), it might have served as *the* modeling example that other authors could have followed. In other words, given Samuelson's standing in the profession, his ideas and methods carried certain rhetorical force. But, what specific historical questions are raised with the above examination of Samuelson's (1965) use of the representative agent? As I see it, there are at least two problems worth investigating.

First, at its most fundamental level, *how did Samuelson "follow the crowd," as he suggested in his 1965 paper, in the way he modeled the representative agent in the first half of the 1960s?* Could this be the case of the stabilization of a concept in a sense put forward by Roy Weintraub (1991)?

Second, at a more practical level, *how influential really was Samuelson (1965) for the subsequent literature?* In the following sections, I shall try to answer both questions in the order posed.

## 3.4 The Massachusetts Institute of Technology (MIT) Economics Graduate Program in the 1960s

To understand how Samuelson's (1965) modeling strategy made an immediate splash, I will focus on a few seemingly minor events which took place prior to the publication of that article in the *American Economic Review*. As it turned out, they played a fundamental role in establishing how economists began to view Ramsey-type models to handle optimal growth (saving), turnpike and golden-rule problems.

As most early histories of the MIT economics department revolve around Paul Samuelson (see the articles in the volume edited by Weintraub (2014)), in my story he claimed that the representative agent apparatus was "accorded" a few years before in an

"Advanced Economic Theory" course co-taught by Robert Solow and Edmund Phelps in the fall of 1962.[27,28] In a footnote, by referring to the aforementioned device, Samuelson recalled:

> The year 1962-63 was a golden year for Golden-Rules at M.I.T. In the seminar of Robert Solow and Edmund Phelps (visiting from Yale), Christian von Weizsäcker (of Basel, Hamburg, and Berlin), Christopher Bliss (of Cambridge), and others proved all kinds of theorems. Professor Phelps reminds me that Weizsäker and T. Koopmans had independently developed this device, as did S. Chakravarty (of Delhi) during his 1963-64 stay at M.I.T. (Samuelson 1965, p. 487).

Samuelson favoured the use of the representative agent since it enabled the writing of the Ramsey analysis in per capita form, an important way to handle the golden-rule problem.

An interesting fact regarding Samuelson's memory of this event is that although he stated "in the seminar of Robert Solow and Edmund Phelps," as if those two taught it, it was indeed Samuelson who taught the seminar.

One student who attended the seminar, Edwin Burmeister (2009, p. 39), recalled: "In the Spring semester of 1963 Paul Samuelson taught a seminar - I do not remember its exact title - and most of us from 14.123 were enrolled." Since "14.123" was the number (code) of the "advanced theory course" I mentioned above (that started in September 1962 and ended in January 1963), it is possible that such a course took place *before* Samuelson's seminar. If so, it is likely that Samuelson then courteously gave the credit for the seminar he taught to Phelps and Solow because most attendees were also in their advanced theory course.

[27] This point is also made in Assaf (2016).

[28] Robert Solow earned an Economics PhD from Harvard University and currently holds the appointment of Institute Professor, Emeritus, at MIT. Known for his contributions to the theory of economic growth, he was awarded the Nobel Prize in Economics in 1987. Edmund Phelps, a Nobel Prize laureate in 2006, studied, among other themes, labour markets and macroeconomics (economic growth and the role of expectations). Of historical interest, he edited *Microeconomic Foundations of Employment and Inflation Theory*, the first (mainstream) publication to ever use the term "microfoundations" (Hoover 2012). He obtained his PhD in Economics from Yale University and currently directs the Center on Capitalism and Society at Columbia University.

What I have shown above were two important episodes that took place at the Massachusetts Institute of Technology: a course taught by two young professors and a seminar led by Paul Samuelson, who was not only one of the first to join the Industrial Economics PhD program, but a leading figure to all those involved in the institution.[29] Although no explicit references to "representative agent" or "representative man" can be found in the notes taken by Burmeister, the use of this methodological device can be illuminated by another MIT student of the 1960s: Robert C. Merton.[30]

As a protégé of Paul Samuelson, Merton attended the graduate program in economics in Cambridge (MA) between 1967 and 1970. In one of the first articles of his career, titled "A Golden Golden-Rule for Welfare Maximization in an Economy with a Varying Population Growth Rate," published in 1969, Merton solved a utility maximization problem of a *representative man*: regarding this formulation, he stated in the paper, "Because this derivation parallels Samuelson's derivation of the Per Capita Consumption Turnpike Theorem [6], I have tried to keep the notation" (Merton 1969, footnote 1). The work "6" Merton referred to was precisely Samuelson's "A Catenary Turnpike Theorem Involving Consumption and the Golden Rule," which came out only two years before and also used the representative man to shed light on growth problems.

As to why Merton decided to use the device, I underscore a passage taken from a letter Samuelson wrote to Wassily Leontief, chairman of the Society of Fellows of Harvard University, recommending his disciple for a Junior Fellowship:[31]

> In an advanced theory course I put a question on the final exam that Merton had solved in a paper to be published in the WESTERN ECONOMIC

[29] This statement should be read with a caveat: if we count the number of PhD students advised as a measure of influence of a faculty member, Robert Solow became more important in the 1960s when he advised almost 16% of all students; during the same decade, Samuelson advised only 2%. See Duarte (2014) for more discussion on this subject.

[30] Robert C. Merton is the son of the famous Columbia University Sociology of Science Professor Robert K. Merton. He attended the doctoral program at MIT under the supervision of Paul Samuelson, a friend of his father. Merton contributed to economics mainly in finance theory (life cycle finance and the pricing of securities) and over the course of his career received several awards, including the 1997 Nobel Prize in Economics (shared with Myron Scholes). In his Nobel biography Merton (1997) wrote: "I applied to half a dozen good departments, but only one, M.I.T., accepted me, and it gave me a full fellowship."

[31] Letter from Samuelson to Leontief, October 21, 1969, PSP, box 52, file folder "Bob C. Merton, 1969-2009."

> JOURNAL. His written answer was not of A$^+$ caliber even though his knowledge of this subject had been demonstrated to be greater than mine!

Put another way, the theoretical framework used by generations of students could be directly influenced by the form the instructors thought relevant, viz., in accordance with the problems they wished to tackle. In this case, Samuelson could have thought that, at the end of the 1960s, the appropriate way to handle turnpike (optimal growth) problems was using the representative agent, a device jointly formulated at the beginning of that decade.

Many other students who attended the course as well as the seminar would end up holding important faculty positions and, hence, introducing the device to their students. Of particular importance was Peter Diamond, who not only became professor at the same powerhouse economics department he graduated from (MIT), but contributed as a co-author with Tjalling Koopmans (and Richard Williamson) to a seminal paper on the axiomatization of time preference (see chapter one of this dissertation), a central component in the widespread adoption of the representative agent in macroeconomics.[32]

Although no "litmus test" can be provided as to how exactly the representative agent spread into macroeconomic theory, not to mention why, there are reasons to believe that it made its way more intensely in the literature through an ever higher interaction between the communities of optimal growth and turnpike theorists, all concerned, among other subjects, with the achievement of maximal growth rates at the highest consumption levels.

## 3.5 Samuelson and the Optimal Growth Literature

I shall now analyze the influence of Samuelson (1965) in the subsequent development of the optimal growth literature. To do so, I review some published sources of the protagonists in this story as well as profit from a narrative first presented by Stephen Spear and Warren Young (2015) on a related subject. In addition, to provide what I see as

[32] The list of students in Phelps and Solow's course can be found on www.irwincollier.com, under "M.I.T." artifacts, and then "Robert Solow's Advanced Economic Theory Course, 1962."

evidence of Samuelson's influence in the profession, I rely on archival sources, specifically some letters he exchanged with some figures in this narrative.

In "Two-Sector Growth, Optimal Growth, and the Turnpike: Amalgamation and Metamorphosis," published in *Macroeconomic Dynamics* in 2015, Spear and Young explore how the transition from positive one-sector growth models to normative two-sector (and multi-sector) growth models in the postwar period took place and, more important, how, by evolving so, such theoretical developments gave rise to a new understanding of the turnpike concept.[33] By taking the story in this section as a complement to the discussion carried out in the previous one, it may be argued that the representative agent could have strengthened its presence in macroeconomics during this process.

According to Spear and Young (2015), with the publication of a new interpretation of the turnpike theory by David Cass in 1966, Tjalling Koopmans and Lionel McKenzie endeavored—in independent efforts—to reconcile this new version of the theory with the Ramsey-Cass-Malinvaud-Koopmans optimal growth framework (which Koopmans called the *amalgam*).[34,35]

Irrespective of how this specific episode played out, what matters in this narrative is that both Koopmans and McKenzie carried out such re-interpretations by rescuing Samuelson's (1965) notion of catenary properties. And since the change in the notion of turnpike was undertaken by highly influential economists, I believe that more than just economic substance might have been passed down to incoming generations: in essence, by also placing a significant weight on Samuelson (1965) in their reworking of optimality vis-à-vis turnpike concepts, Koopmans and McKenzie could have pushed downstream a different economic methodology, inadvertently strengthening the use of the representative agent.

[33] Spear and Young (2015) propose a more comprehensive narrative, in which the transitions from various types of growth models have a common historical motivation: a Uzawa-Cass connection.

[34] In a 2014 article, the same Stephen Spear and Warren Young proposed renaming the "Ramsey-Cass-Koopmans" model the "Ramsey-Cass-Malinvaud-Koopmans" model. In their interpretation, the French economist Edmond Malinvaud also played an important role in the construction of the theoretical framework, a process that took place via the "cross-fertilization" of ideas among the authors. Since I base part of my narrative on Spear and Young (2015), I shall follow their interpretation.

[35] "Independent" in the sense that they were never co-authors; however, McKenzie seemed to have followed Koopmans in several points regarding the turnpike.

In section 9 of their paper, Spear and Young (2015) traced the change in the way Koopmans himself understood how the Ramsey-Cass-Koopmans model should be conceived. If Koopmans initially conceived such a framework to be of a "Ramsey-Cass-Malinvaud-Koopmans-type" in a Cowles Foundation discussion paper of 1966, in the version published in *Econometrica* in January 1967 Koopmans corrected it to be of a "Ramsey-Cass-Malinvaud-Koopmans-Samuelson-type." As Spear and Young (2015, p. 413) stated:

> In our view, the addition of the reference to Samuelson (1965), with its "catenary turnpike theorem," brought about the conflation of optimality and turnpike in the framework of, as Koopmans puts it, the "amalgam" of his model with those of Cass's 1965 and 1966 papers, Malinvaud, and Samuelson.

In this case of a late inclusion undertaken by Koopmans, how much of a sway could Samuelson have held over him? Although no historical document associated specifically with this episode can be found either in the Koopmans or the Samuelson archives, some indirect connections can be drawn from other materials found in such sources.

After a widely known conference on "Activity Analysis of Production and Allocation," organized by Koopmans in the summer of 1949 in Chicago, somewhere around 1950 he sent the "Introduction" to the conference volume to Samuelson, hoping to receive some comments and suggestions. He received a reply with the following remarks:[36]

> I have just returned from a trip to Washington and so have been delayed in sending on my comments on your excellent draft of the introduction to the activity monograph. It is a very good summary and my suggestions are very minor ones. (1) page 1 in connection with Schlesinger and Wald do you want to mention the names of Neisser and von Stackelberg who around the same time wrote articles on over-determinism and underdeterminism of Cassel-Walras system. (2) Bottom of page 2 do you want to mention Bergson's original 1938

[36] Letter from Samuelson to Koopmans, May 18, 1950, PSP, box 45, file folder "Tjalling Koopmans, 1946-1985."

> invention of the "social welfare function." (3) Top of page 3 you might add a sentence speaking of the "new welfare economics" and list after the name of Pareto, Barone, Lerner, Hotelling, Bergson, Kaldor, Hicks and others. Rather than give references you could refer to my references, chapter VIII. (4) Top of page 9 leave out name von Mises ????? (5) Page 13 and elsewhere there is a misspelling of the name Weyl. (6) Page 14 last paragraph if you wish to you might refer to the existence of some unpublished RAND memos of mine. Since my RAND monograph is proceeding so slowly, it would probably be best not to mention it.
>
> I think this is a very excellent introduction.

Although admitting its limited nature, the extent of Samuelson's influence on Koopmans can also be measured by considering how many of his six suggestions were effectively implemented. Interestingly, Koopmans carried out all changes proposed in the exact manner described in Samuelson's letter:

1. Neisser and von Stackelberg were cited.

2. Bergson's original 1938 work on the social welfare function was properly acknowledged.

3. The term "new welfare economics" as well as the list of other contributors to the subject were added in the exact way suggested by Samuelson, including the reference to his 1948 article, chapter VIII.

4. Since von Mises's name appeared with the expression "leave out" along with question marks, it is hard to know whether they carried a surprising connotation due either to an addition or to an elision (when Samuelson read the draft version). Given the fact that Samuelson was a "compulsive citer," the second case is more likely.[37] And, in

[37] The term "compulsive citer" was used by Robert Solow in a letter he wrote to E. Roy Weintraub (1991, p. 56). There are many illustrative instances of Samuelson's preoccupation in not neglecting to credit those who, he thought, first came up with an idea. In a letter to Christian Weizsäcker dated March 13, 1972, discussing the refereeing process of a paper on the Le Chatelier Principle, Samuelson complained that the author had not referred to one of his earlier articles in which he extended the theorem from symmetric matrixes of extrema problems to diagonally-dominant matrixes of the "Frobenius-Leontief-Metzler-Mosak-Hicks-Morishima type." In another example, the same Weizsäcker recollected an episode where Samuelson, upon being briefed on an idea Weizsäcker developed for an article, pointed out to him the weaknesses of the manuscript. After giving up writing the paper, another author published an

fact, “von Mises” appeared various times.

5. The name of Weyl was correctly spelled.

6. Finally, although allowed to be quoted, Koopmans did not make reference to Samuelson’s unpublished RAND memos, and no mention of his RAND monograph was made either.

Samuelson’s standing in the profession as a whole can also be appraised through his influence on other contemporary authors. Heavily connected to the literature of turnpike and modern growth theories was the University of Rochester Professor Lionel McKenzie. In “Turnpike Theory,” a survey article published in *Econometrica* in 1976, McKenzie stated that the theory was developed sequentially by Ramsey (1928), Koopmans (1965), Malinvaud (1965), and Cass (1966). Three years later, in a draft to his 1986 *Handbook of Mathematical Economics* chapter, which circulated in 1979 as the California Institute of Technology working paper 267 and titled “Optimal Economic Growth and Turnpike Theorems,” McKenzie did not mention any of the authors who contributed with the turnpike theory cited in the 1976 *Econometrica* article, but rescued two theorems first put forward in Koopmans (1967). In McKenzie’s actual chapter in the *Handbook*, however, he not only corrected his previous omission, but now added Samuelson (1965), pretty much the same way that Koopmans did in 1967.[38]

In sum, I have tried to highlight two facts from the story above. First, if Koopmans’s *new* amalgamation approach in the 1967 *Econometrica* paper led to the conflation of optimality and turnpike theorems in optimal growth and, as claimed by Spear and Young (2015), if this was the case after the inclusion of Samuelson (1965), then it possibly underlines a certain leadership role Samuelson played in the profession, while also stressing his sway over Koopmans. Second, if such a new amalgamation approach was also taken up by Lionel McKenzie in the ensuing decades, it could be argued that it was done out of respect for Koopmans’s and Samuelson’s technical expertise as well as professional

article whose central ideas were similar to what Weizsäcker had been trying to develop; as a way to make up for it, Samuelson later wrote an article in which he acknowledged a certain theory along the “Weizsäcker-Kennedy lines.” For more interesing examples, see Duarte (2010).

38 Interestingly, nevertheless, Malinvaud (1965) was removed from the final version of McKenzie (1986).

prestige.

This fact can be observed in a letter McKenzie wrote to Samuelson asking for advice regarding his job at Duke University around 1954 (interestingly, Koopmans was also mentioned in the passage):[39]

> I want to ask your advice on a very confidential basis. I hope you won't regard it as an intrusion, but the truth is, outside Duke, you, Hicks, and Koopmans are about the only seniors of the profession on whom I feel it possible for me to call, and Hicks, of course, is in England and, therefore, not very useful here.

As the story in the correspondence went, McKenzie asked for assistance to move to some northern or eastern university. In the Samuelson Papers, several letters of recommendation can then be found of Samuelson making sincere efforts in helping McKenzie find an appointment in some universities across the United States. In one instance, replying to a request to assess some candidates in connection with an open position at the University of Pennsylvania, Samuelson recommended McKenzie this way:[40]

> Some scholars are overrated in the bourse of trade; some carry quotations that are too low and represent surplus value. I believe McKenzie belongs in the latter class and that time is gradually improving his terms of trade to their proper high level.

By undertaking the discussion above, I share the view in which intellectual and professional connections may sometimes evolve into a more personal one, as can be observed in the correspondence between Samuelson and McKenzie. Further, considering that in a series of other papers McKenzie wrote the confluence of optimal growth and turnpike theories

[39] Letter from McKenzie to Samuelson, September 25, 1954, PSP, box 52, file folder "Lionel McKenzie, 1947-2002." I should add that even though the year "1954" did not appear on the letter I reproduced above, it could be inferred from another letter in which the content was a follow-up of that conversation. Other accounts also put McKenzie as a professor at Duke University in the first half of the 1950s (see, e.g., Weintraub (2011)).

[40] Letter from Samuelson to Irving B. Kravis, March 23, 1964, PSP, box 52, file folder "Lionel McKenzie, 1947-2002."

became even stronger, the same link between both theories and the representative agent could be made as well.

## 3.6 Concluding Remarks

This chapter began with two observations: first, Paul Samuelson and Tjalling Koopmans were important contributors to consumer choice and optimal growth theories (not to say of activity analysis and general equilibrium) and, second, in contrast with the standard narrative, the representative agent was also a commonly used tool in optimal growth models of the 1960s. Motivated by these facts and making use of published and unpublished sources, I developed a historical narrative about the rise of the representative agent centered on that period having as a backdrop the interaction between Koopmans and Samuelson.

Samuelson had been working with intertemporal choice models at least since the 1930s when, in "A Note on the Measurement of Utility," he wrote down problems such as the maximization of sums of future utilities with time discounting in mathematical form. This was not the case, however, when he used a similar framework to deal with a macroeconomic question in the 1943 article. This chapter showed that the next mathematical formulation with an infinitely-lived consumer took place precisely in Samuelson's studies of turnpike models by the middle of the 1960s. In the author's mind, as he admitted in correspondence, a newer application of the tool could only happen with the advancement of macroeconomics, particularly of optimal growth models.

I then showed that part of this delay could be attributed to a certain lack of "compromise," a state only achieved by a group of economists directly or indirectly related to the economics graduate program at MIT. On this "indirect" part, Tjalling Koopmans pushed for the use of the infinitely-lived representative consumer, and this chapter attempted to show how the connection between him and Samuelson took place.

In this chapter, the connections and influences among authors were undertaken through traditional methods in the history of thought, namely by means of a careful perusal of important published works and archival sources (correspondence). In recognizing that economists operate in broad networks, the analysis in this chapter can certainly be enhanced with other bibliometric research methods such as prosopography (collective biography) and social network analysis (SNA).

# Conclusion

Until the beginning of the 1930s economics was a pluralistic science, in a sense influenced by Institutionalism, but with "(...) no method to defend and no one economic theory to peddle," to use Morgan and Rutherford's (1998, p. 3) words. By the end of that decade things began to change and, after World War II, the profession had already taken an irreversible path, characterized by the dominance of neoclassicism. Although the representative agent emerged as one relevant outcome of this transformation, few historical studies have interpreted this particular case of the mathematization of economics.

In the preceding chapters I explored the rise of the representative agent in three different, yet overlapping contexts. In delving into Tjalling Koopmans's formulation of intertemporal choice theory, perusing the articles that dealt with the topic in the roughly three decades that covered the 1950s-1970s period, I showed that Koopmans maintained a hesitant posture regarding the use of the representative agent until 1963, when he then published the first version of his optimal growth article. A possible reason for such an ambiguity was the problem of aggregation and, as I showed in letters exchanged with Marc Nerlove, Koopmans was highly concerned with the subject within economics.

The subject of aggregation also explained, in part, the rise of the representative agent in microeconomics. Following a similar interpretation put forth in Hands (2012a) and, to a lesser extent, Hands and Mirowski (1998) and Mirowski and Hands (1998), I contended that the aggregate view of the economy brought about by the Keynesian revolution ultimately caused a change in the Walrasian-Slutsky program. Another circumstance where aggregation issues played a role in microeconomics concerned the so-called "measurement without theory" debate which involved the Cowles Commission and the NBER. The last case where the aggregation problem also became important began with the introduction of homothetic preferences by Paul Samuelson in the 1950s. I showed in this thesis that, taken these episodes together, the representative agent received an important thrust in microeconomics.

There is possibly a connection between the representative agent in microeconomics and its rise in macroeconomics. Paul Samuelson worked in the subfields of demand theory as well as economic growth and, as such, he used the intertemporal model of consumption to shed light on macroeconomic problems. The spread of the representative agent in the optimal growth literature is an interesting case study to understand Roy Weintraub's concept of *stabilization*; I discussed how different economists (Paul Samuelson, Tjalling Koopmans and Lionel McKenzie), with similar research interests (general equilibrium theory, trade theory and economic growth) and working in different communities (the institutions and research fields to which they belonged), narrowed down their disagreements over the years. I also emphasized a fact that might have enhanced this outcome: the influence Samuelson had in the profession, especially on Koopmans and McKenzie.

As I have emphasized in the introduction to this dissertation, the representative agent has always been strongly associated with the research program initiated by Robert Lucas and the ensuing writings of his associates who, together, founded the New Classical School of macroeconomics in the 1970s. In this thesis, I have tried to tell a different story by going further back in time and focusing on the rise of the tool as the result of transformations to economics that took place from the 1930s until the end of the 1960s.

If with the term "New Classical" one automatically associates the term "microfoundations," would it be more logical to find the representative agent in microeconomics or in macroeconomics? As it turned out, such a methodological device was used as early as the 1940s by Samuelson in macroeconomics and continued to be perfected in the subsequent decades mostly based on Koopmans's research in microeconomics. My thesis showed that the relationship between the two fields and, by extension, the resulting rise of the representative agent is sometimes blurred. Both authors made contributions in demand analysis and growth theories and both authors helped build and, above all, used the representative agent.

An interesting implication of this research is that although Lucas used to claim that economists should start using "economics" instead of the terms "macroeconomics" or "microeconomics," since every economic phenomenon (he claims) is microeconomic in nature, my research showed that both Koopmans and, especially, Samuelson did use the representative agent to shed light on macroeconomic issues, such as the optimal growth problem and the turnpike theory, and none of them ever intended to bring an end to macroeconomic analysis.

To conclude, in this dissertation I was more interested in identifying relationships in the historiography than in making evaluations of the scientific merit of the theories involved. In addition, I tried to construct a narrative in the spirit of Bruno Latour's *Science in Action* (1987), focusing on the *science in the making* side of the representative agent in economics, a side generally untold to both the neophyte and the practitioner of economics and which involves uncertainty, decisions, and people at work.

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