Universidade de São Paulo
Departamento de Economia, Administração e Contabilidade
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TWO ESSAYS ON ECONOMIC GROWTH AND THE RELATIVE PRICE OF CAPITAL

Dejanir Henrique Silva

Orientador: Prof. Dr. Mauro Rodrigues Júnior

Outubro de 2011

Versão Original

São Paulo

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Dissertação apresentada ao Departamento de Economia da Faculdade de Economia, Administração e Contabilidade da Universidade de São Paulo como requisito para a obtenção do título de Mestre em Economia.

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RESUMO

Esta dissertação consiste em uma análise sobre as interligações entre o preço relativo do capital e crescimento econômico. O trabalho está dividido em dois ensaios. No primeiro, é realizado uma análise do impacto dinâmico de restrições ao comércio internacional sobre crescimento econômico e o preço relativo do capital. Essa análise é particularmente relevante para entender o caso brasileiro, que observou uma forte elevação do preço relativo do capital justamente em um período de substituição de importação e restrições ao comércio de bens de capital. O segundo ensaio procura olhar para as diferenças entre os países de renda per capita e do preço relativo do capital. Em consonância com trabalhos empíricos recentes, a análise enfatiza como diferenças em produtividade podem ser geradas e como isso pode impactar a dispersão de renda per capita e do preço relativo do capital.

ABSTRACT

This dissertation consists in an analysis about the interconnection between the relative price of capital and economic growth. The work is divided in two essays. In the first one, an analysis of the dynamic impact of trade restrictions is performed in order to understand the impact over economic growth and relative price of capital. This analysis is particularly relevant to understand the Brazilian case, which presented a strong rise in the relative price of capital, exactly at the time the country was adopting import substitution policies and restrictions to the trade of capital goods. The second essay emphasizes differences between countries of income per capita and relative price of capital. Consistent with recent empirical work, this analysis emphasizes how differences in productivity could be generated and how this could impact the dispersion of income per capita and the relative price of capital.

Sumário

1	INTRODUCTION	1
2	DYNAMIC IMPACT OF TRADE RESTRICTIONS	3
2.1	Introduction	3
2.2	A Simple Two-Sector Neoclassical Model	5
2.3	The Complete Model	6
2.3.1	Production	7
2.3.2	Households	9
2.3.3	Closed-economy Equilibrium	10
2.4	Closed-economy steady state	11
2.5	Open-economy equilibrium	11
2.5.1	Open-economy steady state	13
2.6	International Trade Restrictions	15
2.6.1	Implications for TFP	17
2.7	Balanced Growth Path	17
2.8	External Debt	18
2.8.1	Equilibrium with Debt Markets	18
2.8.2	Steady state	21
2.9	Conclusion	22
3	ECONOMIC GROWTH AND RELATIVE PRICES	23
3.1	Introduction	23
3.2	The Model	25
3.2.1	Production	25
3.2.2	Households	29
3.2.3	World Economy Equilibrium	30
3.3	World economy steady state	31

3.4	Cross-country implications	33
3.5	Quantitative implications	35
3.6	Conclusions	37
4	REFERÊNCIAS BIBLIOGRÁFICAS	39
Apêndice A – CLOSED-ECONOMY STEADY STATE		43
1	Derivations	43
2	Steady-State	44
Apêndice B – PROOFS OF PROPOSITIONS IN CHAPTER 2		49
Apêndice C – WORLD ECONOMY STEADY STATE		55
Apêndice D – PROOFS OF PROPOSITIONS IN CHAPTER 3		61

1 INTRODUCTION

The focus of this work is to understand the connection between the relative price of capital and economic growth. The importance of capital accumulation has been emphasized since the beginning of the field modern macroeconomics, but only very recently the attention has turned into question related to the relative price of capital and how it can be connected of why some countries end up with much more capital and more income per capita.

One example of why this can be an important factor, consider a country that saves 20% of your income. If, for some reason, the relative price of capital rises 80%, the country will have to save now 36% to get exactly the same amount of machines that the country had before. On the opposite side, if the relative price of capital drops, lets say 80%, then with the same amount of savings the country can augment your stock of capital almost 5 times. This kind of calculations illustrate how this forces can be.

Note that these are not just random numbers: the relative price of capital raised almost 80% in a period of less than 20 years in Brazil. The opposite pattern was seen in the USA and in other developed countries. In a period of approximately 20 years, the relative price of capital *declined* 80% in the United States. Trying to understand these phenomena and possibles underlying causes is the main objective of this work.

But, what kind of force is behind these differences in relative prices? To gain some intuition about which kind of factor is relevant, we develop a simple two-sector general equilibrium model. It turns out that differences in technology in the consumption good sector and the investment goods are the main factor in the determination of relative prices in general equilibrium. Thus, one way to understand this problem is to look at technological differences.

However, if we will try to understand how these technological differences could emerge, we need a theory that generates this differences endogenously in equilibrium, not one that take this differences as an assumption.

In the first essay, we look at a particular cause of productivity differences and in relative prices: restrictions to international trade. This essay tries to understand what are the dynamic impacts of trade restrictions over income per capita and the relative price of capital. This is important specially to understand, for example, the case of Brazil. Brazil implemented a series of policies trying to stimulate the domestic production of capital goods instead of importing these goods from abroad, a process call Import Substitution. What this essay shows is that this kind of policy produces exactly the kind of increase in the relative price of capital observed in Brazil on this period.

The second essay tries to answer a question raised in the recent literature in economic growth. Empirically, it has been documented that investment rates are an important factor to explain differences in income per capita. However, this is only true if we consider investment rates at *international* prices, not domestic prices. The difference between the two comes from the relative price of capital and this fact implies that the relative price of capital is negatively related to income.

This observation has led some economists to propose the following explanation: some countries has some distortions in investment (like taxation, corruption, bureaucracy) and this raises the relative price of capital and reduces income per capital. But, recent empirical work has shown that this does not appear to be case, distortions does not play a big role in the explanation of the dispersion of income and the relative price of capital.

In the second essay, we propose an explanation to this empirical observation without invoking investment distortions. Productivity differences, endogenously generated by other factors like the level of human capital, may generated the pattern observed in the data, even without any kind of distortion.

2 DYNAMIC IMPACT OF TRADE RESTRICTIONS

2.1 Introduction

The recent financial crisis has caused not just a recession in several countries across the globe, but also had serious implications for international trade and the rise of protectionism. There was a major drop in global trade in the Great Recession of 2008-2009, a decrease around 30 percent relative to GDP, and a figure even more prominent to durable goods¹.

Recently, countries have raised a concern involving a possible “currency war”, where some governments supposedly try to devalue their currencies in order to boost production and profits of domestic firms. This kind of discussion makes the environment even more prone to policies that restricts trade.

Countries can be tempted to use trade policies to deal with these questions. However, it is important to understand what are the costs involved in this kind of strategy, not just in the short run, but also the impact of trade restrictions on long-run growth.

In this chapter, we try to understand the dynamic impact of trade restrictions, paying specific attention to channels that have not been so emphasized by the literature on international trade. Recent work has highlighted the importance of the relative price of capital goods to the economic performance of countries². We analyse how international

¹See Eaton et al. (2010) for more details.

²See Jones (1994), Restuccia e Urrutia (2001), Hsieh e Klenow (2007) and McGrattan et al. (1999). For an historical account of the importance of capital-good prices for development see Collins e Williamson (2001).

trade restrictions impacts the relative price of capital and eventually the level of prosperity of a country.

This paper investigates this question using a two-sector dynamic Heckscher-Ohlin model with economies of scale in the investment good sector. Our model extends the work of Rodrigues (2010) and it is also closely related to Atkeson e Kehoe (2000) and Baxter (1992). The model also presents characteristics in common with the literature that applies models of monopolistic competition to international trade and economic growth, as in Helpman e Krugman (1985) and Romer (1990).

Another important point, specially to developing countries, is the interaction between trade and international finance matters. This question is explicitly dealt with by introducing external debt in the model and considering the possibility that the accumulation of external debt may be an optimal response in this environment.

The plunge of global trade is so recent that does not allow us to study the long run implications of trade restrictions. Therefore, the predictions of the model are compared to the historical experience of a country that has adopted policies that hinder the import of capital goods in the past, the Brazilian economy. In the seventies, Brazil has adopted a series of import substitution policies, specially in the capital goods sector. This makes the Brazilian economy an ideal laboratory to test the implications of the theoretical model.

The Brazilian economy witnessed the relative price of investment goods soar in the period following the adoption of trade restrictions. At the same time, the economy accumulates capital and presents a drop in the total factor productivity, i.e., the increase in capital-labor ratio was higher than the increase in income. This period was also characterized by a significant increase in the level of external debt, when the external almost tripled in only six years.³

This chapter is organized as follows: section 2 analyses the results of a simple two-sector neoclassical model, with exogenous sectoral TFPs. Section 3 develops the envi-

³For more details, see Solomon e Sachs (1981).

ronment of the closed-economy version of our model with endogenous TFP and section 4 describes the steady state of the model. Section 5 presents the open-economy equilibrium. Section 6 analyses the impact of trade restrictions and section 7 introduces long-run growth in the model. Section 8 analyse the model with external debt and section 9 concludes.

2.2 A Simple Two-Sector Neoclassical Model

In order to gain intuition in what factors determine the relative price of investment goods, we will use the closest model to the traditional neoclassical model which allow us to make inferences about relative prices. In the basic one-sector growth model, the relative price of investment goods is equal to one by construction⁴, since it is possible to consume or invest the final product freely. Therefore, we will work with a two-sector model.

Consider a model with two factors, capital and labor, and two sectors, a consumption good and an investment good sector. Suppose that technology is described by:

$$Y_j = A_j K_j^\alpha L_j^{1-\alpha}, j = C, I \quad (2.1)$$

where sector C produces consumption goods and sector I produces investment goods.

From profit maximization in both sectors, we obtain the following conditions

$$p_C \alpha A_C \left(\frac{K_C}{L_C} \right)^{\alpha-1} = p_I \alpha A_I \left(\frac{K_I}{L_I} \right)^{\alpha-1} = r \quad (2.2)$$

$$p_C (1 - \alpha) A_C \left(\frac{K_C}{L_C} \right)^\alpha = p_I (1 - \alpha) A_I \left(\frac{K_I}{L_I} \right)^\alpha = w \quad (2.3)$$

Combining the two equations, we find an expression for the relative price of investment:

⁴Actually, the relative price of investment goods could be different from one even in a frictionless one-sector growth model, but only in a extreme case. If investment is irreversible (nonnegative), the relative price of capital can be different from one if the nonnegativity constraint is binding. See Barro e Martin (2004) .

$$\frac{p_I}{p_C} = \frac{A_I}{A_C} \quad (2.4)$$

where we use the fact that $K_C/L_C = K_I/L_I$.

If we introduce a tax in the investment sector (τ_I), the relative price of capital is altered to

$$\frac{p_I}{p_C} = (1 + \tau_I) \frac{A_C}{A_I} \quad (2.5)$$

Part of the literature abstract from differences between A_C and A_I and focus only on τ_I . Actually, data on relative prices was used as a way to identify the size of investment distortions. However, this contrast with the fact that investment is no more expensive in poor countries than in rich countries. Therefore, sectoral TFP must have a role in explaining the importance of relative prices to economic development.

Thus, to understand why differences in A_C and A_I may arise, we develop a richer model that allow us to answer this kind of question.

2.3 The Complete Model

The model is non-stochastic, time is discrete and there are two factors, capital and labor, and two sectors, a consumption good and an investment good sector. Initially, we will describe the closed-economy version of the model and then we will obtain some implication for the cross-section relation between, relative prices, investment rates and income.

In order to simplify notation, we will subsume the time subscript when there no risk of confusion.

2.3.1 Production

The economy has two sectors with different capital intensities. A consumption good sector, labor-intensive, and an investment good sector, capital-intensive.

Consumption Goods Sector

The consumption sector has a standard production structure, where capital and labor are combined using a Cobb-Douglas production function:

$$Y_C = K_C^{\theta_C} L_C^{1-\theta_C}, 0 < \theta_C < 1 \quad (2.6)$$

We assume perfect competition, what give us the following first-order conditions:

$$r = p_C \theta_C k_C^{\theta_C-1} \quad (2.7)$$

$$w = p_C (1 - \theta_C) k_C^{\theta_C} \quad (2.8)$$

where r is the rental-rate of capital, w is the wage rate and k_C is the capital-labor ratio of the consumption sector.

Investment Goods Sector

Production takes place in two stages in the investment good sector. Initially, a measure n of producers combine capital and labor in a monopolistic environment. After that, a competitive firm combine those varieties to assemble the investment good. Following Dixit e Stiglitz (1977), we use a CES aggregator:

$$Y_I = \left(\int_0^n x_i^\gamma di \right)^{1/\gamma}, 0 < \gamma < 1 \quad (2.9)$$

where x_i is the quantity of the variety i used in the production of the investment good.

The problem of the second-stage firm is to choose the quantity of the variety i taking as given the price of these varieties and the price of the investment good, i.e., the firm

solves

$$\max_{\{x_i\}} \left\{ \left(\int_0^n x_i^\gamma di \right)^{1/\gamma} - \int_0^n p_i x_i di \right\} \quad (2.10)$$

where we normalize the price of investment goods to 1⁵. The solution of this problem is the demand for each variety:

$$x_i = p_i^{\frac{1}{\gamma-1}} Y_I \quad (2.11)$$

where p_i is the price of variety i .

Integrating the formula above, we can obtain:

$$\begin{aligned} \int_0^n x_i^\gamma di &= \left\{ \int_0^n p_i^{\frac{\gamma}{\gamma-1}} di \right\} Y_I^\gamma \\ 1 &= \left\{ \int_0^n p_i^{\frac{\gamma}{\gamma-1}} di \right\}^{\frac{\gamma-1}{\gamma}} \end{aligned} \quad (2.12)$$

where we use (3.43) to eliminate Y_I from the first expression.

The first stage firm combines capital and labor using a Cobb-Douglas production function. The firm has also to pay a fixed cost, in terms of its own product, thus this technology is subject to economies of scale:

$$x_i = K_i^{\theta_I} L_i^{1-\theta_I} - f \quad (2.13)$$

where $\theta_I > \theta_C$, i.e., the investment good sector is more capital-intensive than the consumption good sector.

First stage firms behave monopolistically, i.e., firms maximize profits subject to their demand curve (given in (3.48)). Thus, the problem of the firm i is:

$$\begin{aligned} \max_{p_i, x_i} \{ p_i x_i - c(x_i) \} \\ s.t. \ x_i = p_i^{\frac{1}{\gamma-1}} Y_I \end{aligned} \quad (2.14)$$

where $c(\cdot)$ is the cost function of the variety i .

⁵This choice of *numeraire* is similar to the one made by Hsieh e Klenow (2007), where they choose as *numeraire* the pre-tax investment good prices.

It is possible to show that the cost function is given by

$$c(x_i) = \psi x_i + \psi f, \quad \psi = \frac{r^{\theta_I} w^{1-\theta_I}}{\theta_I^{\theta_I} (1 - \theta_I)^{1-\theta_I}}$$

where ψ represents the marginal cost.

The solution to the problem above is the traditional result from the monopoly problem, the constant mark-up rule:

$$p_i = \frac{1}{\gamma} \psi \quad (2.15)$$

In addition, we have a free entry condition, which implies that profits must be zero for each variety:

$$p_i x_i = \psi(x_i + f) \quad (2.16)$$

Then, from (3.50) and (3.51), we obtain the level of production of first stage firms:

$$x_i = x = \frac{\gamma}{1 - \gamma} f \quad (2.17)$$

which is constant across time and varieties. As the problem of each variety is identical, they demand the same quantity of capital and labor, then we can write $K_i = K_I$ and $L_i = L_I$ for every i . In addition, from the cost minimization problem, we obtain:

$$\frac{w}{r} = \frac{1 - \theta_I}{\theta_I} k_I \quad (2.18)$$

where k_I is the capital-labor ratio of each variety.

2.3.2 Households

There is a measure N of households with infinite horizon. They are endowed with k units of capital at instant t and one unit of time, which is supplied inelastically. The problem of the household is to choose a sequence of consumption $\{c_t\}_{t=0}^{\infty}$ and capital $\{k_{t+1}\}_{t=0}^{\infty}$ which solves the following problem:

$$\max \left\{ \sum_{t=0}^{\infty} \beta^t u(c_t) : p_{1t} c_t + k_{t+1} - (1 - \delta) k_t \leq w_t + r_t k_t, \forall t \geq 0 \right\} \quad (2.19)$$

where $0 < \beta < 1$ is the discount factor and $0 < \delta < 1$ is the depreciation rate, $u'(\cdot) > 0$ and $u''(\cdot) < 0$. Note that because this is a two-sector model we have to take into account the price of consumption goods. It is not possible directly to save in terms of consumption goods, as in traditional models. It is necessary to convert consumption goods into investment goods in order to move consumption to the future. The presence of the relative price of investment slightly changes the traditional Euler equation:

$$u'(c_t) = \beta u'(c_{t+1}) \frac{p_{It+1}}{p_{It}} \{r_{t+1} + (1 - \delta)\} \quad (2.20)$$

where we have defined the relative price of investment as $p_I \equiv 1/p_C$.

2.3.3 Closed-economy Equilibrium

In this section, we are considering the case in which countries cannot trade with each other, then all domestic markets have to clear:

$$Nc = Y_C = K_C^{\theta_C} L_C^{1-\theta_C} \quad (2.21)$$

$$N(k' - (1 - \delta)k) = Y_I = \left(\int_0^n x_i^\gamma di \right)^{1/\gamma} \quad (2.22)$$

$$Nk = K_C + nK_I \quad (2.23)$$

$$N = L_C + nL_I \quad (2.24)$$

where k' indicates next period capital-labor ratio and equations (3.56)-(2.24) refers to the market clearing conditions in the consumption good, investment good, capital and labor markets, respectively.

The closed-economy equilibrium is a sequence of prices and allocations given by $\{p_i, p_C, w, r, K_C, K_I, L_C, L_I, n, x, c, k'\}$ where markets clear and, given prices and k_0 , the appropriate quantities solve the problems of households and firms.

2.4 Closed-economy steady state

The steady state of the main variables in the model can be expressed in closed-form. The appendix describe how to find expressions for the variables of interest. The effect of an increase in N will typically depend on parameter values and the assumption below guarantees that c , k and y are increasing functions of N . The same condition guarantees that the relative price of investment goods is decreasing in N .

Assumption 1. $\gamma > \theta_2$

Thus, given assumption 1, we have our first proposition⁶:

Proposition 1. *Given Assumption 1, steady state per capita income, capital (in both sectors) and consumption are increasing functions of N in a closed-economy. The steady state relative price of investment goods is a decreasing function of N in a closed-economy.*

Basically, since there are economies of scale in the investment sector, countries with a large scale can support a great number of varieties, which increases the productivity of the investment sector. Therefore, the more productive country display a greater level of consumption, capital and income per capita.

2.5 Open-economy equilibrium

We now introduce trade in the model. There are two countries, home and foreign, with sizes N and N^* (we will denote foreign variables by a asterisk throughout this chapter), where $N + N^* = 1$. All remaining parameters are the same in both economies.

Countries can trade consumption goods and varieties freely, but investment goods are assembled internally by each country. Therefore, the world market clearing condition for the consumption good is given now by $Nc + N^*c^* = K_C^{\theta_C} L_C^{\theta_C} + K_C^{*\theta_C} L_C^{*\theta_C}$, where c and c^* denotes domestic and foreign consumption, respectively.

⁶All proofs are provided in the appendix.

Countries can use domestic and foreign varieties in the production of the investment good. We also assume there is no overlap in the production of varieties, i.e., each variety is produced in only one country⁷. Thus, the quantities of investment good produced are $Y_I = (n + n^*)^{1/\gamma} x^d$ and $Y_I^* = (n + n^*)^{1/\gamma} x_t^{*d}$, where we made use of the fact that x_i is constant across time and varieties. Note that n is the number of varieties produced and x^d is the demand for each variety in the home country (analogously for the foreign country). The market clearing condition for each variety is $x = x^d + x^{*d}$.

Introducing international trade in the model opens the possibility that a country specializes in the production of consumption goods or varieties. Therefore, we have that marginal revenue is less than or equal to the marginal cost in each sector. The following condition follows from the problem of a firm in the consumption sector:

$$p_C \leq \frac{r^{\theta_C} w^{1-\theta_C}}{\theta_C^{\theta_C} (1 - \theta_C)^{1-\theta_C}}, \quad \text{with equality if } K_C, L_C > 0 \quad (2.25)$$

where p_C is the price of the consumption good (as this sector is competitive, marginal revenue equals price) and the right hand side is the marginal cost. For the producer of variety i , we have that

$$p_i \leq \frac{1}{\gamma} \frac{r^{\theta_I} w^{1-\theta_I}}{\theta_I^{\theta_I} (1 - \theta_I)^{1-\theta_I}}, \quad \text{with equality if } n > 0 \quad (2.26)$$

where the marginal revenue is given by γp_i . Analogous conditions hold for the foreign country.

Depending of the capital-labor ratio in each country either one (or both) of the conditions above may be binding. If the capital-labor ratio is low enough, the country will specialize in the production of the consumption good and condition (2.26) will not be binding. If the capital-labor ratio is high enough, the country will specialize in the capital-intensive sector, and condition (2.25) will not be binding. For intermediate values of the capital-labor ratio the country will diversify the production and both goods will be sup-

⁷Models presenting the hypothesis of differentiated goods by country is known in the international trade literature as Armington models.

plied. These intermediate values defines what is called the cone of diversification in the international trade theory.

From conditions (2.25) and (2.26) (and its counterparts in the foreign country), if home and foreign countries produce both goods (which implies that both conditions are binding), then producers in the two countries will face the same factor prices, i.e., the Factor Price Equalization theorem will hold for these economies.

The characterization of the economy is completed by the market clearing conditions for factors and for the investment good, as well the Euler equations for each country.

2.5.1 Open-economy steady state

In steady state, the Euler equations in both countries imply that $r = r^* = 1/\beta - (1 - \delta)$. Thus, the rental price of capital is the same in both countries in steady state. With this result in hands, we can show that the capital-labor ratio is equalized across countries, and the wage rate is also the same for the two economies. Therefore, the Factor Price Equalization theorem holds in steady state. This result is expressed in our next proposition:

Proposition 2. *In a open-economy steady state, r, w, p_I, k_C, k_I, K_I and L_I are equalized across countries.*

One of the implications of the Proposition 2 is that both countries produce consumption goods and varieties in steady state. The next proposition states that factors price and allocations in a open-economy steady state is the same as in a closed-economy of size 1.

Proposition 3. *Open-economy steady state values for $r, w, p, p_I, k_C, k_I, K_I$ and L_I coincide with their respective steady state values in a closed-economy of size 1.*

Our previous result allows us to obtain the world quantities in steady state. Defining

by $z(N)$ the closed-economy steady state value of the variable z , Proposition 3 establish that the open-economy value of variable z is given by $z(1)$.

Although the previous result allows to determine the value of world economy variables in steady state, we cannot proceed in the same way for the country-level variables. In particular, we cannot pins down the distribution of capital in each country. In contrast with the neoclassical growth model, where factor prices uniquely determine factor intensities, because the Factor Price Equalization theorem holds in steady state, we are unable to do the same in our case. Therefore, there are several ways of allocate capital across countries.

We have established that the capital-labor ratio lies within the cone of diversification in steady state in both countries. Thus, $k, k^* \in [\underline{k}, \bar{k}]$, where $\underline{k}$ and $\bar{k}$ are the limits of the cone of diversification. Consider initially the case of a country with capital-labor ratio $\underline{k}$. From (3.58), (2.24) and using that $K_I = L_I = 0$ when $k = \underline{k}$, we have

$$\begin{cases} N\underline{k} &= K_C \\ N &= L_C \end{cases} \Rightarrow \underline{k} = \frac{K_C}{L_C} = k_C(1) \quad (2.27)$$

Analogously for the superior limit:

$$\begin{cases} N\bar{k} &= nK_I \\ N &= nL_I \end{cases} \Rightarrow \bar{k} = \frac{K_I}{L_I} = k_I(1) \quad (2.28)$$

The number of varieties produced depend on the distribution of capital across countries. From (3.58) and (2.24),

$$\begin{cases} Nk &= k_C(1)L_C + nK_I(1) \\ N &= L_C + nL_I(1) \end{cases} \Rightarrow n = \frac{N}{L_2(1)} \frac{k - k_C(1)}{k_I(1) - k_C(1)} \quad (2.29)$$

The expression above gives the relationship between the capital labor ratio and the number of varieties in steady state. If $k = k_C(1)$, then $n = 0$ and the country is specialized in the production of consumption goods. If $k = k_I(1)$, then $N = nL_I(1)$ and all the

resources are being devoted to the production of varieties.

Finally, the allocation of capital across countries also determine the level of consumption and income in steady state. From the budget constraint of the household, we have that $y = w(1) + r(1)k$ and $c = p_I(1)(w(1) + (r(1) - \delta)k)$, which implies that steady state consumption and income per capita are increasing functions of k ⁸.

2.6 International Trade Restrictions

In this section we study the impact of international trade restrictions. In particular, the consequences of the adoption of import substitution policies will be analysed. Import substitution is modelled as an unanticipated move to a completely closed-economy situation.

We will assume that the home country is initially specialized in the consumption good, i.e., $k_{open} = k_C(1)$ and initial income and consumption per capita is given by $y_{open} = w(1) + r(1)k_C(1)$ and $c_{open} = p_I(1)(w(1) + (r(1) - \delta)k_C(1))$.

We will assume that the restrictions on trade will hold long enough, so that the economy will reach its closed-economy steady-state. Thus, in the new steady state, capital, income and consumption per capita are given by $k_{close} = k(N)$, $y_{closed} = y(N)$ and $c_{closed} = c(N)$.

Our next proposition analyses how the impact of import substitution on capital, income and consumption per capita depends on the country size.

Proposition 4.

1. *There exists a unique $\bar{N}_k \in (0, 1)$ such that $k_{closed} \geq k_{open}$ if $N \geq \bar{N}_k$ and $k_{closed} < k_{open}$ if $N < \bar{N}_k$.*

⁸Note that $(r(1) - \delta) = 1/\beta - 1$ which is positive.

2. There exists a unique $\bar{N}_y \in (0, 1)$ such that $y_{closed} \geq y_{open}$ if $N \geq \bar{N}_y$ and $y_{closed} < y_{open}$ if $N < \bar{N}_y$.
3. There exists a unique $\bar{N}_c \in (0, 1)$ such that $y_{closed} \geq y_{open}$ if $N \geq \bar{N}_c$ and $y_{closed} < y_{open}$ if $N < \bar{N}_c$.
4. $\bar{N}_k < \bar{N}_y$ and $\bar{N}_k < \bar{N}_c$.

The impact of IS is different depending if the country size is larger or not than a certain threshold. In a closed-economy, the country needs to produce both goods, as varieties are capital-intensive, the capital tends to increase. But, as the scale of economy is smaller now (compared to the open economy), then capital tends to shrink. If N is sufficiently low, then the capital-labor ratio falls.

These results are very similar with the results found by Rodrigues (2010), but the last one is not the same what was found by Rodrigues (2010). The main difference is that we cannot completely order the thresholds. Depending on parameter values, $\bar{N}_c < \bar{N}_y$ or $\bar{N}_y < \bar{N}_c$. One can show that, if a country has scale $\bar{N}_y$, the value of consumption will drop, then, if the price of consumption were constant, this would require a higher scale to maintain consumption constant (basically, this is the result found by Rodrigues (2010)). However, since the price of consumption is dropping, it is possible that the consumption is raising at scale $\bar{N}_y$, what would require a lower threshold for the consumption. Thus, depending on parameter values, it is possible that the inequality holds in either direction.

The impact of the policy change is stressed in the following Corollary:

Corollary 1. *For any $N \in (0, 1)$, $k_{closed}/y_{closed} = k_{closed}^*/y_{closed}^*$ and $k_{open}/y_{open} < k_{closed}/y_{closed}$.*

Corollary 1 indicates that convergence will happen in terms of capital-output ratios. Proposition 1 have indicated that the same does not occur in terms of capital and income per capita. Therefore, the policy change will induce capital deepening, but not necessarily will generate long-run welfare gains.

2.6.1 Implications for TFP

We can compute the TFP as usual in the literature, using a Cobb-Douglas production function for the aggregate economy, where $Y_t = A_t K_t^\alpha N_t^{1-\alpha}$, with $\alpha \in (\theta_C, \theta_I)$ and A_t denoting the TFP. Thus, the measured TFP growth (also called Solow Residual) is then $\hat{A} = \hat{y} - \alpha \hat{k}$, where $\hat{y} = (1/T) \ln(y_{closed}/y_{open})$ and $\hat{k} = (1/t) \ln(k_{closed}/k_{open})$ are the annual average growth rate of income and capital per capita.

2.7 Balanced Growth Path

This section introduces long-run growth in the model. It is not necessary to assume an exogenous rate of technological progress, the presence of economies of scale generates endogenous growth when we introduce population growth. Then, we will characterize the balance growth path when country size increase at the constant rate $g(N)$ ⁹ in both countries, more precisely, $N_t = N_0[1 + g(N)]^t$ and $N_t^* = N_0^*[1 + g(N)]^t$ with $N_0 + N_0^* = 1$.

In the appendix, we establish that the economy presents long run growth and that the growth rate of income, capital and the *value* of consumption grows at the same rate. The relative price of investment decreases in a balanced growth path. These results are summarized in the next proposition:

Proposition 5. *Assume that preferences are CRRA. Thus, on a balanced growth path, per capita income, capital and consumption (in terms of investment goods) grow at the same rate, which is given by:*

$$g(y) = g(k) = g(p_1 c) = [1 + g(N)]^{\frac{1-\gamma}{\gamma-\theta_2}} - 1 \quad (2.30)$$

The growth rate of the relative price of investment is given by

$$g(p_I) = [1 + g(N)]^{\frac{-(1-\theta_C)(1-\gamma)}{\gamma-\theta_2}} - 1 \quad (2.31)$$

⁹We will denote by $g(z)$ the growth rate of variable z in a balanced growth path.

Note that the number of varieties grows over time because of the increasing scale, which is reflected in the measured TFP growth:

$$1 + g(A) = [1 + g(N)]^{\frac{(1-\alpha)(1-\gamma)}{\gamma-\theta_2}} \quad (2.32)$$

In the usual way, we can solve the model by detrending all variables by their long-run growth rate. Basically, this will produce equations analogous to the previously founded, but for detrended variables. Therefore, Propositions 1 to 4 holds in this case as well, but referring to detrended variables.

2.8 External Debt

Now we will analyse the impact of IS policies in a different context. Instead of assuming that the economy moves to a completely closed situation, we will allow the possibility of the country to accumulate assets (or debt) from the rest of the world. Basically, the economy cannot import varieties, but it is possible that the production of consumption good be different from consumption.

2.8.1 Equilibrium with Debt Markets

Now we introduce into the economy an international bond. We will adopt a small open economy approach, where we will assume that households can borrow or lend freely on an international debt market and the interest rate is taken as given by the household, i.e., we assume that the economy is unable to affect this rate.

It is important to specify in which good the interest rate is based. As the economy has two sectors, then it is not the same to have a interest rate expressed in consumption or investment goods. We will assume that the household has access to a consumption-based loan with a constant rate $\bar{r}$. As capital and bonds are assumed perfect substitutes, we

have the following no-arbitrage condition:

$$1 + \bar{r} = \frac{p_{It+1}}{p_{It}} (r_{t+1} + (1 - \delta)), \forall t \geq 0 \quad (2.33)$$

The flow budget constraint of the household is given by

$$p_{Ct}c_t + k_{t+1} - k_t(1 - \delta) \leq w_t + r_t k_t + p_{Ct}(d_{t+1} - (1 + \bar{r})d_t) \quad (2.34)$$

where d_t is the level of debt per capita (in terms of consumption goods).

From the flow budget constraint, we can derive the intertemporal budget constraint.

For $t \geq 1$, we have

$$\begin{aligned} p_{Ct}c_t + k_{t+1} - k_t(1 - \delta) &\leq w_t + r_t k_t + p_{Ct}(d_{t+1} - (1 + \bar{r})d_t) \\ c_t + \frac{k_{t+1}}{p_{Ct}} &\leq \frac{w_t}{p_{Ct}} + (r_t + 1 - \delta) \frac{p_{Ct-1}}{p_{1t}} \frac{k_t}{p_{Ct-1}} + (d_{t+1} - (1 + \bar{r})d_t) \\ c_t + \left(\frac{k_{t+1}}{p_{Ct}} - d_{t+1} \right) &\leq \frac{w_t}{p_{Ct}} + (1 + \bar{r}) \left(\frac{k_t}{p_{Ct-1}} - d_t \right) \\ c_t + \left(\frac{k_{t+1}}{p_{Ct}} - d_{t+1} \right) &\leq \frac{w_t}{p_{Ct}} + (1 + \bar{r}) \left(\frac{k_t}{p_{Ct-1}} - d_t \right) \\ c_t + a_{t+1} &\leq \frac{w_t}{p_{Ct}} + (1 + \bar{r})a_t \end{aligned} \quad (2.35)$$

where we have defined $a_t \equiv \left(\frac{k_t}{p_{Ct-1}} - d_t \right)$.

For $t = 0$, we have

$$\begin{aligned} c_0 + \left(\frac{k_1}{p_{C0}} - d_1 \right) &\leq \frac{w_0}{p_{C0}} + \left((r_0 + 1 - \delta) \frac{k_0}{p_{C0}} - (1 + \bar{r})d_0 \right) \\ c_0 + a_1 &\leq \frac{w_0}{p_{C0}} + (1 + \bar{r})a_0 \end{aligned}$$

where we have defined $a_0 \equiv \left((r_0 + 1 - \delta) \frac{k_0}{p_{C0}} - (1 + \bar{r})d_0 \right)$. Therefore, (2.35) holds for every period.

We can iterate the flow budget constraint (2.35), to obtain:

$$a_t = \frac{1}{(1 + \bar{r})^n} a_{t+n} + \sum_{j=0}^{n-1} \left(\frac{1}{1 + \bar{r}} \right)^{j+1} c_{t+j} - \sum_{j=0}^{n-1} \left(\frac{1}{1 + \bar{r}} \right)^{j+1} \frac{w_{t+j}}{p_{Ct+j}}$$

where we already use the fact that budget constraint holds with equality.

Imposing the condition that $\lim_{n \rightarrow \infty} \frac{1}{(1+\bar{r})^n} a_{t+n} = 0$ and evaluating at $t = 0$, we obtain

$$a_0 = \sum_{t=0}^{\infty} \left(\frac{1}{1+\bar{r}} \right)^{t+1} c_t - \sum_{t=0}^{\infty} \left(\frac{1}{1+\bar{r}} \right)^{t+1} \frac{w_t}{p_{Ct}} \quad (2.36)$$

Therefore, the problem of the household can be expressed as

$$\max_{\{c_t\}_{t=0}^{\infty}} \left\{ \sum_{t=0}^{\infty} \beta^t u(c_t) : \text{subject to (2.36)} \right\}$$

where the value of a_0 is taken as given.

The Euler equation derived from this problem is

$$u'(c_t) = \beta u'(c_{t+1})(1+\bar{r}) \quad (2.37)$$

Therefore, consumption must be constant every period, i.e., $c_t = \bar{c}, \forall t \geq 0$.

Substituting $c_t = \bar{c}, \forall t$ in the intertemporal budget constraint, we have

$$\bar{c} = \frac{\bar{r}}{1+\bar{r}} \left((1+\bar{r})a_0 + \sum_{t=0}^{\infty} \left(\frac{1}{1+\bar{r}} \right)^t \frac{w_t}{p_{Ct}} \right) \quad (2.38)$$

The evolution of debt per capital is given by:

$$d_{t+1} = (1+\bar{r})d_t + \left(c_t - k_{Ct}^{\theta_1} l_{Ct}^{1-\theta_C} \right)$$

The evolution of capital per capita is given by:

$$k_{t+1} = (1-\delta)k_t + \frac{n_t^{1/\gamma} x}{N} \quad (2.39)$$

which is essentially equation (3.57).

This formulation reflects the fact that capital goods are assumed not tradable, while we allow the possibility of trade in consumption goods.

2.8.2 Steady state

As the production structure of the economy is basically the same compared to the base model, the steady state value of most variables is exactly the same as it was in the closed-economy model. This is precisely the content of the next proposition.

Proposition 6. *The steady state values for $r, w, p, p_I, k, k_C, k_I, L_C$ and L_I in an economy with external debt coincide with their respective steady state values in a closed-economy.*

The main insight behind the previous result is that the only difference between the closed-economy case and the one with external debt¹⁰ is the market clearing condition for consumption. However, this condition is not necessary to obtain the steady state value of the remaining variables, which implies that the steady state values of these variables coincide with their value in a closed-economy.

The value of consumption is determined by the intertemporal budget constraint and depends on the initial conditions of the economy. Suppose for sake of simplicity that $d_0 = 0$. If $k_0 = k(N)$, the steady state value, then it is possible to show that $\bar{c} = c(N)$ and $\bar{d} = 0$. The economy has no reason to incur debt and the profile of consumption is flat in the closed-economy level.

Suppose now that $k_0 < k(N)$. Again we can determine the value of consumption using the intertemporal budget constraint. It is possible to show that a_0 drops if we reduce the value of k_0 from $k(N)$ to a smaller value. The second term in the budget constraint is the average value of real wages¹¹. As real wages are increasing in capital per capital, the average value drops, since the weight is greater for earlier values. Therefore, the value of consumption also drops.

From the law of movement to debt per capital, we have that in steady state $\bar{r}\bar{d} + \bar{c} = k_{Ct}^{\theta_C} l_{Ct}^{1-\theta_C}$. Since the steady state value of the production does not change with k_0 and $\bar{c}$ decreases, thus the economy has a positive level of debt in steady state.

¹⁰At least in this setting, where we suppose that the country cannot trade investment goods.

¹¹By real wages I mean wages expressed in terms of consumption goods.

Basically, the introduction of external debt in the model allows the economy to smooth consumption. When the capital stock of the economy is low, compared to steady state, production of consumption goods is low and the economy imports consumption goods from abroad, incurring in debt to pay for the imports, since the economy cannot export capital goods. When the economy accumulates capital, production increases and the level of debt stabilizes.

2.9 Conclusion

This paper tries to understand the dynamic impact of trade restrictions. We use a two-sector dynamic Heckscher-Ohlin model with economies of scale in the investment sector in order to address this question.

We show that a movement towards a closed-economy may cause a rise in the relative price of capital, a drop in the level of productivity and accumulation of capital. In addition, if the country has access to an international bond, than the economy may start to accumulate foreign debt. All these features is observed in the Brazilian experience after the adoption of trade restrictions in the seventies.

As steps for further research, we intend to analyse the quantitative implications of the model and verify if the model is able to account for the Brazilian experience in this dimension.

3 ECONOMIC GROWTH AND RELATIVE PRICES

3.1 Introduction

One of the most robust facts of the empirical research in economic development is the positive correlation between real investment rates and economic performance. There is an extensive literature documenting this relationship and trying to assess the importance of capital accumulation for economic growth¹².

Recent work has showed, however, that investment rates at *domestic* prices are not correlated with income. This indicates that most of the correlation between real investment and income is driven by differences in the relative price of capital¹³. In fact, the work of Jones (1994) has found a strong association between relative prices and economic performance.

This empirical regularity has motivated a new wave of research trying to relate investment distortions to relative prices and to the stage of economic development. Chari et al. (1997) use a one-sector growth model with investment distortions to match the cross-country differences in income per capita, where differences in relative prices are entirely driven by such distortions. A similar strategy was followed by Restuccia e Urrutia (2001) and Parente e Prescott (2002). Typically, investment distortions are interpreted very broadly, reflecting taxes, bribes or corruption.

However, Hsieh e Klenow (2007) have emphasized that differences in relative prices cannot be explained by higher investment prices in poor countries, but instead by lower consumption prices¹⁴. If investment distortions were to explain these differences, they would have to appear as higher investment prices, which has not been verified empirically. This fact lead Hsieh e Klenow (2007) to dismiss explanations related to investment

¹²For empirical evidence, see Barro (1991), Martin (1997) and Mankiw et al. (1992). Long e Summers (1991) find a strong association between equipment investment and economic growth. The theoretical channel has been emphasized by many, since the seminal work of Solow (1956).

¹³See Restuccia e Urrutia (2001) and McGrattan et al. (1999). For a historical account of the importance of capital-good prices for development see Collins e Williamson (2001).

¹⁴This fact have also been noticed previously by Eaton e Kortum (2001) .

distortions.

In Hsieh and Klenow's model, the price of consumption is determined by differences in total factor productivity (TFP) in the investment and the consumption sector. In other words, poor countries have lower consumption good prices because they are less efficient in producing investment goods in exchange for consumption goods. This pushes our question a little further: to account for the relationship between relative prices and economic development, we need to understand what determines the relative productivity of investment and consumption goods across nations.

In this work, we try to shed light on this question using a two-sector dynamic model, where economies of scale play a prominent role. Although the model features no investment distortions, it still able of account for the stylized facts described above. Particularly, differences in human capital endowments are the main source of heterogeneity across countries, and can therefore explain the cross-country dispersion of the relative price of physical capital and income per capita.¹⁵

Most of the literature trying to understand differences in income per capita across countries work with closed economy models, where each economy is treated as a different "island". We consider an open-economy setting in which countries are allowed to trade intermediate goods with each other.

Our model is closely related to that of Acemoglu e Ventura (2003), although our research question is quite different from theirs. Related work can be found in Ventura (1997), Eaton e Kortum (2001), Atkeson e Kehoe (2000) and Rodrigues (2010). The model has also features in common with the literature that applies models of monopolistic competition to international trade and economic growth, as in Helpman e Krugman (1985), Grossman e Helpman (1992) and Romer (1990).

This paper is organized as follows. Section 2 analyzes the results of a simple two-sector neoclassical framework, with exogenous sectoral TFPs. Section 3 develops the complete

¹⁵For a model of international trade which emphasizes the role of human capital and skilled work, see Findlay e Kierzkowski (1983).

model with endogenous TFP and international trade. Section 4 describes the steady state of the model. Section 5 analyzes the model's cross-country implications for income per capita, investment rates and relative prices. Section 6 presents a quantitative application of the model and section 7 concludes.

3.2 The Model

Time is discrete and there is no uncertainty. There are three factors – capital, unskilled labor and skilled labor – and two sectors – a consumption good and an investment good sector. Production in the consumption good sector is carried on through a standard constant returns to scale technology, which combines physical capital and unskilled labor. But firms in the investment sector behave as monopolist competitors, using capital and skilled labor to generate differentiated varieties. This structure gives rise to economies of scale in the production of the investment good.

There are J countries which can freely trade intermediate goods (varities), but do not trade consumption or investment goods. There is no international borrowing or lending, so that trade has to be always balanced.

In order to simplify notation, we will omit time indicators when there is no room for ambiguity.

3.2.1 Production

Consumption Goods Sector

The consumption sector has a standard production structure, where capital and unskilled labor are combined using a Cobb-Douglas production function:

$$Y_C^j = (K_C^j)^\theta (A^j L_u^j)^{1-\theta}, 0 < \theta < 1, j = 1, 2, \dots, J \quad (3.40)$$

where Y_C^j represents the production of the consumption good in the country j . L_u^j is the amount of unskilled labor in country j and A^j represents total factor productivity of country j . This level of productivity is the same across sectors of each country.

Based on the empirical evidence that the capital goods sector tend to be more human capital intensive than the consumption goods sector¹⁶, we make the simplifying assumption that the consumption goods sector do not use skilled labor, and the capital goods sector do not use unskilled labor¹⁷.

Since this sector is characterized by perfect competition, we have the following first-order conditions:

$$r^j = p_C^j \theta (k_C^j)^{\theta-1} \quad (3.41)$$

$$w_u^j = p_C^j (1 - \theta) A^j (k_C^j)^\theta \quad (3.42)$$

where r is the rental rate of capital, w_u is the wage rate of unskilled labor and k_C is the ratio between capital and effective labor of the consumption sector.

Investment Goods Sector

Production takes place in two stages in the investment good sector. In the first stage, a measure μ^j of producers in each country j combine capital and skilled labor to generate differentiated varieties in an environment characterized by monopolist competition. In the later stage, a competitive firm combines those varieties to assemble the investment good. Each variety is produced in only one country.

The production function of the investment good is given by:

$$X^j = \left(\int_0^n (x_i^j)^\gamma di \right)^{\frac{1}{\gamma}}, 0 < \gamma < 1 \quad (3.43)$$

¹⁶For empirical evidence on this topic, see Chari et al. (1997) and Eaton e Kortum (2001).

¹⁷Actually, the fact that the investment good sector is human capital intensive is related to what is known in the international trade literature as “Leontief’s Paradox”, or the fact that the USA is exporting labor intensive goods instead of capital intensive goods. One of explanations find by the literature is that if we include human capital in the calculations the paradox disappears for the USA. See, for instance, Kenen (1965) and Keesing (1966).

where x_i^j is the quantity of the variety i used in the production of the investment good in the country j . Note that n is not indexed by j , since there is free trade in varieties. Therefore, n represents the total measure of varieties produced in the world and can be expressed as $n = \sum_{j=1}^J \mu^j$.

The problem of the second-stage firm can be written as:

$$\max_{\{x_i^j\}} \left\{ p_X^j \left(\int_0^n (x_i^j)^\gamma di \right)^{1/\gamma} - \int_0^n p_i x_i^j di \right\} \quad (3.44)$$

where p_X^j is the price of the investment good in the country j . Since there is no trade in the investment good, we allow, at least in principle, that prices differ across countries.

The demand for variety i is therefore:

$$x_i^j = \left(\frac{p_i}{p_X^j} \right)^{\frac{1}{\gamma-1}} X^j$$

Integrating the expression above, we can find an equation for the price of the investment good in terms of the price of varieties:

$$p_X^j = \left\{ \int_0^n p_i^{\frac{\gamma}{\gamma-1}} di \right\}^{\frac{\gamma-1}{\gamma}}$$

A convenient choice of *numeraire* is to set the ideal price index of the composite good to 1:

$$\left\{ \int_0^n p_i^{\frac{\gamma}{\gamma-1}} di \right\}^{\frac{\gamma-1}{\gamma}} = 1 \quad (3.45)$$

Thus, the price of the investment good and the demand for variety i are given by:

$$p_X^j = 1 \quad (3.46)$$

$$x_i^j = p_i^{\frac{-1}{1-\gamma}} X^j \quad (3.47)$$

Consider now the problem of the first stage firm that produces variety i in country j . Such firm combines capital and skilled labor using a Cobb-Douglas production function, but has also to pay a fixed cost in terms of its own product. As a result, this technology

is subject to economies of scale:

$$z_i^j = (K_i^j)^\theta (A^j L_i^j)^{1-\theta} - f$$

where L_i^j is skilled labor used in the production of variety i .¹⁸

Note that z_i^j represents the production of variety i (in country j), while x_i^j represents the use of this variety by country j . Since there is free trade in varieties, these two quantities can be different.

The total demand for variety i is given by the sum of the demand across countries:

$$x_i \equiv \sum_{j=1}^J x_i^j = p_i^{\frac{1}{\gamma-1}} \left(\sum_{j=1}^J X^j \right) \equiv p_i^{\frac{1}{\gamma-1}} X \quad (3.48)$$

First stage firms interact in a monopolistic competition setting, i.e., they maximize profits subject to their demand curve (given in (3.48)). Consider now the the problem of variety i 's producer located in country j :

$$\begin{aligned} \max_{p_i, x_i} \{ & p_i x_i - c^j(x_i) \} \\ \text{s.t. } & x_i = p_i^{\frac{1}{\gamma-1}} X \end{aligned} \quad (3.49)$$

where $c^j(\cdot)$ is the cost function of variety i , which is produced in country j .

It is possible to show that the cost function is given by:

$$c^j(x) = \psi^j x + \psi^j f, \quad \psi^j = \frac{(r^j)^\theta (w_s^j/A^j)^{1-\theta}}{\theta^\theta (1-\theta)^{1-\theta}}$$

where ψ^j represents the marginal cost and w_s^j is the wage rate of skilled workers at country j . Note that every producer located in country j face the same cost function.

The solution of the problem above yields a constant mark-up over the marginal cost:

$$p_i = \frac{1}{\gamma} \psi^j \quad (3.50)$$

¹⁸Note that the level of labor augmenting technological level is the same across sectors, so the relative productivity of the sectors does not depend on A^j .

Note that the price of variety is the same for producers located in the same country. In addition, free entry implies that profits must be zero for each variety:

$$p_i x_i = \psi^j (x_i + f) \quad (3.51)$$

Then, from (3.50) and (3.51), we obtain the level of production of first stage firms:

$$x_i = x = \frac{\gamma}{1 - \gamma} f \quad (3.52)$$

which is constant across time and varieties. As the problem of each variety is identical, they demand the same quantities of capital and labor, that is, $K_i^j = K_I^j$ and $L_i^j = L_s^j$ for every producer i located in country j . Further, from equations (3.48) through (3.52), the price of varieties and the marginal cost are also constant across countries, i.e., $p_i = p$ and $\psi^j = \psi$.

Moreover, from the cost minimization problem, we obtain:

$$\frac{w_s^j}{r^j} = \frac{1 - \theta}{\theta} A^j k_I^j \quad (3.53)$$

where k_I^j is the capital-skilled labor ratio of each variety in country j .

3.2.2 Households

In each country j , there is a measure N^j of identical households with infinite horizon. At each instant t , they are endowed with $k^j(t)$ units of capital, $l_u^j(t)$ units of unskilled labor and $l_s^j(t)$ units of skilled labor, where $l_u^j + l_s^j = 1$. Labor is supplied inelastically. Given $k^j(0) > 0$, the problem of the representative household in country j is to choose a sequence of consumption and capital $\{c^j(t), k^j(t + 1)\}_{t=0}^{\infty}$ which solves the following problem:

$$\max \left\{ \sum_{t=0}^{\infty} \beta^t u(c^j(t)) : p_C^j(t) c^j(t) + k^j(t + 1) - (1 - \delta) k^j(t) \leq y^j(t), \forall t \geq 0 \right\} \quad (3.54)$$

where $y^j(t) \equiv w_u^j(t)l_u^j(t) + w_s^j(t)l_s^j(t) + r^j(t)k^j(t)$ is income per capita. As usual, $0 < \beta < 1$ is the discount factor, $0 < \delta < 1$ is the depreciation rate, $u'(\cdot) > 0$ and $u''(\cdot) < 0$.

Note that because this is a two-sector model, we have to take into account the price of investment goods. It is not possible directly to save in terms of consumption goods, as in traditional models. It is necessary to convert consumption goods into investment goods in order to move consumption to the future. The presence of the relative price of investment slightly changes the traditional Euler equation:

$$u'(c_t) = \beta u'(c_{t+1}) \frac{p_I^j(t+1)}{p_I^j(t)} (r^j(t+1) + (1 - \delta)) \quad (3.55)$$

where $p_I^j \equiv 1/p_C^j$.

3.2.3 World Economy Equilibrium

Since countries cannot trade final goods or factors, then these domestic markets have to clear domestically for each country j :

$$N^j c^j = Y_C^j = (K_j^c)^\theta (A^j L_u^j)^{1-\theta} \quad (3.56)$$

$$N^j [k^j(t+1) - (1 - \delta)k^j(t)] = Y_j^I = \left(\int_0^{n_t} x_i^j(t)^\gamma di \right)^{\frac{1}{\gamma}} \quad (3.57)$$

$$N^j k^j = K_C^j + \mu^j K_I^j \quad (3.58)$$

$$N^j l_u^j = L_u^j \quad (3.59)$$

$$N^j l_s^j = \mu^j L_s^j \quad (3.60)$$

for $j = 1, 2, \dots, J$. Equations (3.56)-(3.60) refer, respectively, to the market clearing conditions for the consumption good, investment good, capital and (unskilled and skilled) labor markets.

Furthermore, since varieties can be traded freely, the following market clearing condi-

tion holds for the world economy:

$$x_i = \sum_{j'=1}^J x_i^{j'} = (K_I^j)^\theta (A^j L_s^j)^{1-\theta} - f \quad (3.61)$$

where variety i is produced in country j .

Finally, given that there is no international borrowing or lending, trade has to be balanced:

$$\int_{i \in \Omega_j} p_i z_i^j di - \int_{i \in \Omega_j} p_i x_i^j di = \int_{i \notin \Omega_j} p_i x_i^j di, \quad j = 1, \dots, J$$

where Ω_j is the set of varieties produced in country j .

The left hand side represents country j exports (i.e., the production minus use of domestically produced varieties) and the right hand side represents imports (i.e., the total value of varieties produced abroad, but used domestically). We can also express the balanced trade condition as:

$$\int_{i \in \Omega_j} p_i z_i^j di = \int_0^n p_i x_i^j di \quad (3.62)$$

The world equilibrium is a sequence of prices $\{p, p_C^j(t), w_u^j(t), w_s^j(t), r^j(t)\}_{t=0}^\infty$ and allocations $\{K_C^j(t), K_I^j(t), L_j^u(t), L_s^j(t), \mu^j(t), n_t, x_i^j(t), c^j(t), k^j(t+1)\}_{t=0}^\infty$, for $j = 1, 2, \dots, J$, such that markets clear and, given prices and $k^j(0)$, the appropriate quantities solve the problems of households and firms.

3.3 World economy steady state

In what follows, variables with no time subscript denote their respective steady state values. Given the choice of functional forms, our main variables of interest can be expressed in closed form in steady state. The appendix describes how to find these expressions. Proposition 1 shows that, since countries interact through international trade, values for our main variables of interest in each country depend on the world scale. Given that the sector subject to economies of scale (the investment sector) is also skilled-labor intensive,

the relevant scale in our case is the amount of skilled-labor in terms of efficiency units.

The relevant measure of world scale is given by:

$$\mathcal{W} = \sum_{j=1}^J A^j l_s^j N^j \quad (3.63)$$

The effect of an increase in $\mathcal{W}$ will typically depend on parameter values. The assumption below guarantees that c^j , k^j and y^j are increasing functions of $\mathcal{W}$, and that the relative price of investment is decreasing in $\mathcal{W}$.

Assumption 1. $\gamma > \theta$

Thus, given Assumption 1, we have our first proposition¹⁹:

Proposition 1. *In steady state, per capita income, capital (in both sectors) and consumption in each country are increasing functions of $\mathcal{W}$, while the relative price of investment is a decreasing function of $\mathcal{W}$ in a open-economy.*

Basically, since there are economies of scale in the investment sector, and each country uses the whole set of varieties available, domestically or abroad, a larger world scale can support a greater number of varieties, which raises the productivity of the investment sector in every country. The higher productivity allows each country to also display a greater level of consumption, capital and income.

Note that, if the world population is growing, the model features endogenous growth. The scale effect is at the income level, not at the growth rate as in Romer (1990), a result similar in spirit to the semi-endogenous growth literature²⁰. As growth comes from an increasing world scale, all countries will share the same rate of economic growth, so that the world income distribution remains stable, in the same vein as Acemoglu e Ventura (2003).

In addition, the appendix shows that our key variables only depend on the size of the economy (N^j) through $\mathcal{W}$. Therefore, the size of country j 's population does not directly

¹⁹All proofs are provided in the appendix.

²⁰See Jones (1999).

impact its consumption, income or capital per capita. The next proposition characterizes the cross-country dispersion of relative incomes, consumption and capital in steady state.

Proposition 2. *In steady state, relative income, capital, consumption per capita and the ratio of the relative price of investment between any two countries j and j' are given by*

$$\frac{y^j}{y^{j'}} = \frac{k^j}{k^{j'}} = \frac{A^j l_s^j}{A^{j'} l_s^{j'}} \quad (3.64)$$

$$\frac{c^j}{c^{j'}} = \frac{A^j (l_s^j)^\theta (l_u^j)^{1-\theta}}{A^{j'} (l_s^{j'})^\theta (l_u^{j'})^{1-\theta}} \quad (3.65)$$

$$\frac{p_I^j}{p_I^{j'}} = \left(\frac{l_s^j / l_u^j}{l_s^{j'} / l_u^{j'}} \right)^{-(1-\theta)} \quad (3.66)$$

The proposition states that characteristics of the country will determine its income level in steady state. Countries with higher education levels (represented by higher human capital per capita), higher fraction of skilled workers, or better overall technological level will be richer. Thus, we have that characteristics of the country determine the level of its main variables (such as income, capital, consumption), but growth rates are determined by world scale.

3.4 Cross-country implications

In this section, we compare the cross-sectional implications of our model with the basic facts outlined in the Introduction. The empirical regularities emphasized by the literature are:

1. Investment rates at international prices are positively correlated with income.
2. Investment rates at domestic prices are not correlated with income.
3. The relative price of investment is negatively correlated with income.
4. The relative price of investment has a tendency to fall in most countries.
5. Investment goods are no more expensive in poor countries.

6. Consumption goods are cheaper in poor countries.

The proposition below states that the results of the model are consistent with the basic facts described above. As is traditional in the literature, we focus on cross-country dispersions in steady state.

Proposition 3. *In steady state, the investment rate at domestic prices is not correlated with income, the investment rate at international prices is positively correlated with income and the price of consumption goods is positively correlated with income. In addition, if the world population is increasing, the relative price of investment goods is decreasing over time.*

The result that the investment rate at domestic prices does not correlate with income comes from the fact that the capital-output ratio in domestic prices is equalized across countries (this is shown in the appendix, but can also be deducted from proposition 2). Note that the investment rate at domestic prices is given by:

$$i_j^{dom} = \frac{\delta k^j}{y^j} = \frac{\delta k^j}{p_C^j c^j + \delta k^j}$$

so that equalization of capital-output ratios implies equalization of the investment rates (at domestic prices).

But equalization of capital-output ratios in domestic prices does not imply equalization of capital-output ratios in international prices. The investment rate in international prices is:

$$i_j^{int} = \frac{p_I \delta k^j}{p_C c^j + p_I \delta k^j} = \frac{\delta}{(p_C/p_I) (c^j/k^j) + \delta}$$

where P^C and P^I are the international prices of consumption and investment price, respectively.²¹

The expression above shows that the investment rate at international prices is a decreasing function of c^j/k^j . From Proposition 2, the consumption-capital ratio is a decreasing

²¹In the Penn World Tables (PWT), for instance, P_C is given by a weighted average of consumption prices, where the weight is given by the level of consumption in each country. However, any set of common prices would be enough to allow cross-country comparisons.

function of two variables that are positively correlated with income: human capital and the share of skilled workers. Therefore, investment rates (at international prices) are positively correlated with income.

The lack of correlation between income and the price of investment follows from free trade in varieties (see equation (3.46)). As a result, the relative price of investment (which is the inverse of the price of consumption) is negatively correlated with income (again, this follows from Proposition 2).

Finally, the fact that the relative price of investment tends to decrease over time can be derived from Proposition 1. If the world population grows over time, so does world scale, which implies that the relative price of investment displays a decreasing trend.

In summary, the model is able to account for the main stylized facts without resorting to any investment distortion. The presence of free trade in intermediate goods and differences in endowments of human capital is sufficient to generate these results.

3.5 Quantitative implications

In this section, we analyze the quantitative implications of the model, comparing them with figures commonly used by the literature that relates economic performance to the relative price of investment.

The first question is whether the model is able to account for the magnitude of the cross-country correlation between income per capita and the relative price of investment. In order to calculate the model's counterpart of this correlation, we need estimates of the level of human capital, the share of skilled workers and the level of technology in each country.

The level of human capital is obtained in a way that is consistent with Mincerian regressions, following Hall e Jones (1999) and Bils e Klenow (2000). Specifically, human

capital in each country is given by:

$$h_j = \exp(\phi(S_j)) \quad (3.67)$$

where $\phi(S_j)$ is a piecewise linear function and S_j represents the average number of schooling years in country j .

Hall e Jones (1999) adopt this specification to account for decreasing returns in the education investments. Basically, countries are divided in three groups, where those with relatively lower level of schooling present a higher return to education.

We measure the share of skilled workers as the percentage of the population with high school diploma in each country, using Barro and Lee's (2010) dataset. Obviously, the measure is not perfect, but it appears to be a valid proxy for our variable of interest, at least as a first approximation. As usual in the literature, we estimate A^j as a residual.

Given estimates of A^j , h_j and l_s^j , and using the same set of countries as Hall e Jones (1999), we can compute the predicted values of income per capita and the relative price of investment. The correlation between these predicted variables is -0.49 . Jones (1994) estimates the correlation between income and relative prices as -0.58 , with a standard deviation of 0.10 . Therefore, we cannot reject the hypothesis that our estimated correlation is different from that in the data, at usual levels of significance.

Another implication of the model is that the relative price of capital tend to decrease in time. Using the results in the appendix, we can show that the growth rate of the relative price of investment is given by:

$$g_p = -\frac{(1-\gamma)(1-\theta)}{\gamma-\theta}g_w \quad (3.68)$$

where g_w is the growth rate of the world scale.

To calculate g_p , we need values for the parameters γ and θ . We obtain γ from the study of Broda e Weinstein (2006). These authors estimated thousands of elasticities of substitution at different levels of aggregation. Following Feenstra (2010), we take the

median elasticity within each 4-digit Standard International Trade Classification (SITC), and then average γ across product group. This yields $\gamma = 0.595$, which is in the lower range considered by Feenstra (2010) and corresponds to an elasticity of substitution of 2.47.²²

As usual, we choose $\theta = 1/3$ ²³. In a balanced growth path, taking the share of skilled workers and the level of human capital as constant, the growth rate of the world scale is given by the sum of the growth rate of population and the growth rate of efficiency units, which we assume to be the same for all countries. Values usually attributed to these rates are 0.01 and 0.02, respectively. Therefore, we set $g_W = 0.03$.

Using these parameter values, we can compute the growth rate of the relative price of investment. The value obtained is $g_p = -3.1\%$, which is very similar to the actual average annual rate of decline in the relative price of equipment in the U.S. over the 1954-1990 period, as reported by Greenwood et al. (1997).

3.6 Conclusions

The quest for understanding the link between investment in physical capital and economic performance has led the profession to consider the connection between the relative price of capital and economic growth. In the literature, the correlation between relative prices and income per capita has often led to the interpretation that investment distortions are key to explain differences in the standard of living across nations.

However, as shown by Hsieh e Klenow (2007), this is not consistent with empirical evidence. They emphasize that the dispersion of relative prices across countries is largely explained by differences in relative sectoral productivity, not investment distortions.

In this paper, we develop a model that accounts for the main empirical regularities

²²Note, however, that the expression above for g_p is very sensitive to the choice of γ . Our objective is only to show that using empirically reasonable values for the parameters, the model is consistent with the main empirical regularities.

²³See Gollin (2002).

involving income per capita and relative prices without resorting to any kind of investment distortion. Actually, differences in human capital or in the level of the technology are sufficient to generate these results. Put differently, the model can endogenously generate differences in relative TFP between the investment and the consumption sector.

A calibration exercise is also performed, with the model being able to generate a correlation between income per capita and the relative price of capital which is consistent with that in the data, as well to match very closely the rate of decline in the price of equipment observed in the U.S.

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APÊNDICE A – CLOSED-ECONOMY STEADY STATE

1 Derivations

We will now derive some expressions that will allow us to calculate the steady state of this economy.

From (2.12) and (3.50), we have

$$\begin{aligned}
 1 &= n^{(\gamma-1)/\gamma} \frac{1}{\gamma} \frac{r^{\theta_I} w^{1-\theta_I}}{\theta_I^{\theta_I} (1-\theta_I)^{1-\theta_I}} \\
 1 &= n^{(\gamma-1)/\gamma} \frac{1}{\gamma} \frac{[p_C \theta_C k_C^{\theta_C-1}]^{\theta_I} [p_C (1-\theta_C) k_C^{\theta_C}]^{1-\theta_I}}{\theta_I^{\theta_I} (1-\theta_I)^{1-\theta_I}} \\
 \frac{1}{p_C} &= n^{(\gamma-1)/\gamma} \frac{1}{\gamma} \left(\frac{\theta_C}{\theta_I} \right)^{\theta_I} \left(\frac{1-\theta_C}{1-\theta_I} \right)^{1-\theta_I} k_C^{\theta_C-\theta_I} \\
 \frac{1}{p_C} &\equiv p_I = n^{(\gamma-1)/\gamma} T_\theta k_C^{\theta_C-\theta_I} = T_\theta n^{-(1-\gamma)/\gamma} k_C^{-(\theta_I-\theta_C)}. \tag{1.1}
 \end{aligned}$$

where we use (3.41) and (3.42) in the second equality and $T_\theta \equiv \frac{1}{\gamma} \left(\frac{\theta_C}{\theta_I} \right)^{\theta_I} \left(\frac{1-\theta_C}{1-\theta_I} \right)^{1-\theta_I}$. We also have use the definition of the relative price of investment.

Combining (3.41), (3.42) and (3.53) we get that $k_C = \lambda k_I$, $\lambda \equiv [(1-\theta_I)/\theta_I]/[(1-\theta_C)/\theta_C]$.

From (3.58), we have that

$$\begin{aligned}
 Nk &= K_C + nK_I \\
 &= k_C L_C + nk_I L_I \tag{1.2}
 \end{aligned}$$

We can find expressions for L_C and L_I using (3.52) and (2.24):

$$L_I = (x + f)k_I^{-\theta_I} \quad (1.3)$$

$$L_C = N - nL_I = N - n(x + f)k_I^{-\theta_I} \quad (1.4)$$

From these equations, we get

$$\begin{aligned} Nk &= k_C[N - n(x + f)k_I^{-\theta_I}] + nk_I[(x + f)k_I^{-\theta_I}] \\ &= N\lambda k_I - \lambda n(x + f)k_I^{1-\theta_I} + n(x + f)k_I^{1-\theta_I} \\ &= N\lambda k_I + (1 - \lambda)n(x + f)k_I^{1-\theta_I} \end{aligned} \quad (1.5)$$

2 Steady-State

In steady state, $r = 1/\beta - (1 - \delta)$. Eliminating r , we get

$$\begin{aligned} p_C \theta_C k_C^{\theta_C - 1} &= \frac{1}{\beta} - (1 - \delta) \\ \frac{\theta_C k_C^{\theta_C - 1}}{n^{(\gamma-1)/\gamma} T_\theta k_C^{\theta_C - \theta_I}} &= \frac{1}{\beta} - (1 - \delta) \\ \frac{\theta_C k_C^{\theta_I - 1}}{n^{(\gamma-1)/\gamma} T_\theta} &= \frac{1}{\beta} - (1 - \delta) \\ k_C &= T_0 n^{\frac{1-\gamma}{\gamma(1-\theta_I)}} \end{aligned} \quad (2.6)$$

where $T_0 \equiv \left(\left(\frac{1}{\beta} - (1 - \delta) \right) \frac{T_\theta}{\theta_C} \right)^{\frac{-1}{1-\theta_I}}$.

We can find another equation relating k_C (or k_I) and n using the market clearing condition for the investment good sector:

$$\begin{aligned} N\delta k &= n^{1/\gamma} x \\ \delta[N\lambda k_I + (1 - \lambda)n(x + f)k_I^{1-\theta_I}] &= n^{1/\gamma} x \\ \delta[NT_0 n^{\frac{1-\gamma}{\gamma(1-\theta_I)}} + (1 - \lambda)n(x + f)\lambda^{-(1-\theta_I)} T_0^{1-\theta_I} n^{\frac{1-\gamma}{\gamma}}] &= n^{1/\gamma} x \\ \delta[NT_0 n^{\frac{1-\gamma}{\gamma(1-\theta_I)}} + (1 - \lambda)(x + f)\lambda^{-(1-\theta_I)} T_0^{1-\theta_I} n^{1/\gamma}] &= n^{1/\gamma} x \end{aligned}$$

where we use (1.5) and the relationship between k_C and k_I .

Manipulating the expression above, we get

$$\begin{aligned}
\delta NT_0 n^{\frac{1-\gamma}{\gamma(1-\theta_I)}} &= n^{1/\gamma} [x - \delta(1-\lambda)(x+f)\lambda^{-(1-\theta_I)} T_0^{1-\theta_I}] \\
n^{\frac{\theta_I-\gamma}{\gamma(1-\theta_I)}} &= \frac{1}{\delta NT_0} [x - \delta(1-\lambda)(x+f)\lambda^{-(1-\theta_I)} T_0^{1-\theta_I}] \\
n &= \left\{ \frac{1}{\delta NT_0} [x - \delta(1-\lambda)(x+f)\lambda^{-(1-\theta_I)} T_0^{1-\theta_I}] \right\}^{\frac{-\gamma(1-\theta_I)}{\gamma-\theta_I}} \quad (2.7)
\end{aligned}$$

This expression determines the steady-state value of n . We can simplify the expression even further:

$$\begin{aligned}
(1-\lambda)(x+f) \left(\frac{T_0}{\lambda} \right)^{1-\theta_I} &= \left(1 - \frac{1-\theta_I}{1-\theta_C} \frac{\theta_C}{\theta_I} \right) \frac{f}{1-\gamma} \left(\frac{1-\theta_I}{1-\theta_C} \frac{\theta_C}{\theta_I} \right)^{-(1-\theta_I)} \left([1/\beta - (1-\delta)] \frac{T_\theta}{\theta_C} \right)^{-1} \\
&= \frac{\theta_I - \theta_C}{1-\theta_C} \frac{\theta_C}{\theta_I} \frac{\gamma f}{1-\gamma} \left[\frac{1-\theta_I}{1-\theta_C} \frac{\theta_C}{\theta_I} \right]^{\theta_I-1} \left[\frac{1}{\beta} - (1-\delta) \right]^{-1} \left[\frac{\theta_I}{\theta_C} \right]^{\theta_I} \left[\frac{1-\theta_I}{1-\theta_C} \right]^{1-\theta_I} \\
&= \frac{\theta_I - \theta_C}{1-\theta_C} \frac{\gamma}{1-\gamma} f \left[\frac{1}{\beta} - (1-\delta) \right]^{-1} \quad (2.8)
\end{aligned}$$

where we have used the definition of T_θ in the second equality.

Then, we have that

$$\begin{aligned}
x - \delta(1-\lambda)(x+f)\lambda^{-(1-\theta_I)} T_0^{1-\theta_I} &= \frac{\gamma}{1-\gamma} f \left(1 - \delta \left(\frac{\theta_I - \theta_C}{1-\theta_C} \right) \left(\frac{1}{\beta} - (1-\delta) \right)^{-1} \right) \\
&= x \left(1 - \delta \left(\frac{\theta_I - \theta_C}{1-\theta_C} \right) \left(\frac{1}{\beta} - (1-\delta) \right)^{-1} \right) \\
&= x \left(\frac{1}{\beta} - (1-\delta) \right)^{-1} \left\{ \frac{1}{\beta} - (1-\delta) - \delta \left(\frac{\theta_I - \theta_C}{1-\theta_C} \right) \right\} \\
&= x \left(\frac{1}{\beta} - (1-\delta) \right)^{-1} \left\{ \frac{1}{\beta} - (1-\delta) - \delta \left(1 - \frac{1-\theta_I}{1-\theta_C} \right) \right\} \\
&= x \left(\frac{1}{\beta} - (1-\delta) \right)^{-1} \left\{ \frac{1}{\beta} - 1 + \delta \left(\frac{1-\theta_I}{1-\theta_C} \right) \right\}
\end{aligned}$$

Thus, the expression for n is:

$$n = T_n N^{\frac{\gamma(1-\theta_I)}{\gamma-\theta_I}} \quad (2.9)$$

where $T_n = \left\{ \frac{\delta T_0}{x} \left(\frac{1}{\beta} - (1 - \delta) \right) \left\{ \frac{1}{\beta} - 1 + \delta \left(\frac{1 - \theta_I}{1 - \theta_C} \right) \right\}^{-1} \right\}^{\frac{\gamma(1 - \theta_I)}{\gamma - \theta_I}} > 0$.

From (3.57), we can obtain the value of k in steady state:

$$\begin{aligned} k &= \frac{1}{\delta N} n^{1/\gamma} x = \frac{1}{\delta N} \left(T_n N^{\frac{\gamma(1 - \theta_I)}{\gamma - \theta_I}} \right)^{1/\gamma} x \\ k &= \frac{x}{\delta} T_n^{1/\gamma} N^{\frac{1 - \gamma}{\gamma - \theta_I}} \\ k &= T_k N^{\frac{1 - \gamma}{\gamma - \theta_I}} \end{aligned} \tag{2.10}$$

where $T_k \equiv \frac{x}{\delta} T_n^{1/\gamma}$.

Using (2.6) and the relation between k_C and k_I , we have that

$$\begin{aligned} k_C &= T_0 \left(T_n N^{\frac{\gamma(1 - \theta_I)}{\gamma - \theta_I}} \right)^{\frac{1 - \gamma}{\gamma(1 - \theta_I)}} \\ &= T_0 T_n^{\frac{1 - \gamma}{\gamma(1 - \theta_I)}} N^{\frac{1 - \gamma}{\gamma - \theta_I}} \end{aligned}$$

Defining $T_{k_C} \equiv T_0 T_n^{\frac{1 - \gamma}{\gamma(1 - \theta_I)}}$ and $T_{k_I} \equiv \lambda^{-1} T_{k_C}$, we have

$$k_C = T_{k_C} N^{\frac{1 - \gamma}{\gamma - \theta_I}} \tag{2.11}$$

$$k_I = T_{k_I} N^{\frac{1 - \gamma}{\gamma - \theta_I}} \tag{2.12}$$

Determined k_C and k_I , we can obtain L_C and L_I :

$$\begin{aligned} L_I &= (x + f) k_I^{-\theta_I} = \frac{f}{1 - \gamma} \left(T_{k_I} N^{\frac{1 - \gamma}{\gamma - \theta_I}} \right)^{-\theta_I} = T_{L_I} N^{\frac{-\theta_I(1 - \gamma)}{\gamma - \theta_I}} \\ L_C &= N - n L_I = N - T_n N^{\frac{\gamma(1 - \theta_I)}{\gamma - \theta_I}} T_{L_I} N^{\frac{-\theta_I(1 - \gamma)}{\gamma - \theta_I}} \\ L_C &= N (1 - T_n T_{L_I}) = T_{L_C} N \end{aligned} \tag{2.13}$$

where $T_{L_I} \equiv \frac{f}{1 - \gamma} T_{k_I}^{-\theta_I}$ and $T_{L_C} \equiv 1 - T_n T_{L_I}$ (we can show that T_{L_C} is positive if we make Assumption 1).

Similarly, we can determine the steady state value of consumption, income and the relative price of investment:

$$\begin{aligned}
c &= \frac{1}{N} k_C^{\theta_C} L_C = \frac{1}{N} T_{k_C}^{\theta_C} N^{\frac{\theta_C(1-\gamma)}{\gamma-\theta_I}} T_{L_C} N = T_{k_C}^{\theta_C} T_{L_C} N^{\frac{\theta_C(1-\gamma)}{\gamma-\theta_I}} \\
p_I &= n^{(\gamma-1)/\gamma} T_{\theta} k_C^{\theta_C-\theta_I} = \left(T_n N^{\frac{\gamma(1-\theta_I)}{\gamma-\theta_I}} \right)^{(\gamma-1)/\gamma} T_{\theta} \left(T_{k_C} N^{\frac{1-\gamma}{\gamma-\theta_I}} \right)^{\theta_C-\theta_I} \\
p_I &= T_n^{\frac{\gamma-1}{\gamma}} T_{\theta} T_{k_C}^{\theta_C-\theta_I} N^{\frac{-(1-\theta_C)(1-\gamma)}{\gamma-\theta_I}} \\
y &= w + rk = p_C(1-\theta_C)k_C^{\theta_C} + p_C\theta_C k_C^{\theta_C-1}k \\
y &= T_n^{\frac{1-\gamma}{\gamma}} T_{\theta}^{-1} T_{k_C}^{\theta_I-\theta_C} N^{\frac{(1-\theta_C)(1-\gamma)}{\gamma-\theta_I}} (1-\theta_C) T_{k_C}^{\theta_C} N^{\frac{\theta_C(1-\gamma)}{\gamma-\theta_I}} + \\
&\quad + T_n^{\frac{1-\gamma}{\gamma}} T_{\theta}^{-1} T_{k_C}^{\theta_I-\theta_C} N^{\frac{(1-\theta_C)(1-\gamma)}{\gamma-\theta_I}} \theta_C T_{k_C}^{\theta_C-1} N^{\frac{(\theta_C-1)(1-\gamma)}{\gamma-\theta_I}} T_k N^{\frac{1-\gamma}{\gamma-\theta_I}} \\
y &= T_n^{\frac{1-\gamma}{\gamma}} T_{\theta}^{-1} T_{k_C}^{\theta_I} \left((1-\theta_C) + \theta_C T_{k_C}^{-1} T_k \right) N^{\frac{1-\gamma}{\gamma-\theta_I}} \tag{2.14}
\end{aligned}$$

Then, the steady state value for c , y and p_I is given by

$$c = T_c N^{\frac{\theta_C(1-\gamma)}{\gamma-\theta_I}} \tag{2.15}$$

$$y = T_y N^{\frac{1-\gamma}{\gamma-\theta_I}} \tag{2.16}$$

$$p_I = T_{p_I} N^{\frac{-(1-\theta_C)(1-\gamma)}{\gamma-\theta_I}} \tag{2.17}$$

where $T_c = T_{k_C}^{\theta_C} T_{L_C}$, $T_y \equiv T_n^{\frac{1-\gamma}{\gamma}} T_{\theta}^{-1} T_{k_C}^{\theta_I} \left((1-\theta_C) + \theta_C T_{k_C}^{-1} T_k \right)$ and $T_{p_I} = T_n^{\frac{\gamma-1}{\gamma}} T_{\theta} T_{k_C}^{\theta_C-\theta_I}$.

APÊNDICE B – PROOFS OF PROPOSITIONS IN CHAPTER 2

Proof of Proposition 1.

Immediate from assumption 1 and (0.20), (0.14), (0.13), (0.7), (0.12) and (0.16). $\square$

Proof of Proposition 2.

The conditions (2.25) and (2.26) must be binding for at least one country, once the world production of consumption goods and varieties must be positive. Without loss of generality, suppose that (2.25) is binding for the home economy and (2.26) is binding for the foreign economy. Then:

$$\begin{aligned} p_C &= \frac{r^{\theta_C} w^{1-\theta_C}}{\theta_C^{\theta_C} (1-\theta_C)^{1-\theta_C}} \leq \frac{r^{\theta_C} w^{*1-\theta_C}}{\theta_C^{\theta_C} (1-\theta_C)^{1-\theta_C}} \iff w \leq w^* \\ p_i &= \frac{1}{\gamma} \frac{r^{\theta_I} w^{*1-\theta_I}}{\theta_I^{\theta_I} (1-\theta_I)^{1-\theta_I}} \leq \frac{1}{\gamma} \frac{r^{\theta_I} w^{1-\theta_I}}{\theta_I^{\theta_I} (1-\theta_I)^{1-\theta_I}} \iff w^* \leq w \end{aligned}$$

Thus, we have that $w = w^*$. This also implies that conditions (3.41) and (3.42) hold with equality for both countries (otherwise (2.25) would not hold with equality) and, then, $k_C = k_C^*$. Then, from (3.53), $k_I = k_I^* = \frac{w}{r} \frac{\theta_I}{1-\theta_I}$. From (3.52), $L_I = L_I^* = (x+f)k_I^{-\theta_I}$ and $K_2 = K_2^* = k_I L_I$. The equality of the relative price of investment comes from (3.41) or (3.42) and the fact that $k_C = k_C^*$. $\square$

Proof of Proposition 3.

We will show that the open economy steady state variables also satisfy the equations for the steady state of a closed-economy of size 1. Note that, from Proposition 2, all marginal conditions holds as equalities for both countries. Since prices and factor intensities are equalized across countries, all the conditions coming from the firm and household problems hold for the global economy. We just need to check if the variables also satisfy the market clearing conditions of a closed-economy.

From the market clearing condition for capital in both countries, we have that $Nk = K_C + nK_I$ and $N^*k^* = K_C + n^*K_I$, which implies that

$$Nk + N^*k^* = (K_C + K_C^*) + (n + n^*)K_I$$

Similarly for the labor market:

$$N + N^* = (L_C + L_C^*) + (n + n^*)L_I$$

The market clearing condition for the investment good in each country gives $N\delta k = (n + n^*)^{1/\gamma}x^d$ and $N^*\delta k^* = (n + n^*)^{1/\gamma}x^{*d}$. Summing the two conditions:

$$\delta(Nk + N^*k^*) = (n + n^*)^{1/\gamma}(x^d + x^{*d}) = (n + n^*)^{1/\gamma}x$$

The market clearing condition for the consumption good is given by $Nc + N^*c^* = k_C^{\theta_C} L_C + k_C^{*\theta_C} L_C^*$. We can express this condition as:

$$Nc + N^*c^* = \left(\frac{K_C}{L_C}\right)^{\theta_C} (L_C + L_C^*) = \left(\frac{K_C + K_C^*}{L_C + L_C^*}\right)^{\theta_C} (L_C + L_C^*) = (K_C + K_C^*)^{\theta_C} (L_C + L_C^*)^{1-\theta_C}$$

where we use that $\frac{K_C}{L_C} = \frac{K_C^*}{L_C^*} = \frac{K_C + K_C^*}{L_C + L_C^*}$. □

Proof of Proposition 4.

1. $k_{closed} = k(N)$ is strictly increasing in N (Proposition 1), but k_{open} does not depend on N . Moreover, $k(0) = 0 < k_1(1)$ and $k(1) > k_C(1)$. Therefore, there is only one $\bar{N}_k \in (0, 1)$ such that $k(\bar{N}_k) = k_C(1)$.

2. $y_{closed} = y(N)$ is strictly increasing in N (Proposition 1), but $y_{open} = w(1) + rk_C(1)$

does not depend on N . Moreover, $y(0) = 0 < w(1) + rk_C(1)$ and $y(1) = w(1) + rk(1) > w(1) + rk_C(1)$. Therefore, there is only one $\bar{N}_y \in (0, 1)$ such that $y(\bar{N}_y) = w(1) + rk_C(1)$.

3. $c_{closed} = c(N)$ is strictly increasing in N (Proposition 1), but $c_{open} = p_I(1)(w(1) + (r(1) - \delta)k_C(1))$ does not depend on N . Moreover, $c(0) = 0 < p_I(1)(w(1) + (r - \delta)k_C(1))$ and $c(1) = p_I(1)(w(1) + (r - \delta)k(1)) > p_I(1)(w(1) + (r - \delta)k_C(1))$. Therefore, there is only one $\bar{N}_c \in (0, 1)$ such that $c(\bar{N}_c) = p_I(1)(w(1) + (r - \delta)k_C(1))$.

4. Initially, we will show that the wage rate is increasing in N :

$$\begin{aligned} w(N) &= p_1(1 - \theta_C)k_C^{\theta_C} \\ &= T_n^{\frac{1-\gamma}{\gamma}} T_\theta^{-1} T_{k_C}^{\theta_I - \theta_C} N^{\frac{(1-\theta_C)(1-\gamma)}{\gamma - \theta_I}} T_{k_C}^{\theta_C} N^{\frac{\theta_C(1-\gamma)}{\gamma - \theta_I}} \\ &= T_n^{\frac{1-\gamma}{\gamma}} T_\theta^{-1} T_{k_C}^{\theta_I} N^{\frac{1-\gamma}{\gamma - \theta_I}} \end{aligned}$$

By definition, we have that $y(\bar{N}_k) = w(\bar{N}_k) + rk_C(1)$ and $y_{open} = y(\bar{N}_y) = w(1) + rk_C(1)$. As r does not depend on N ($r = 1/\beta - 1 + \delta$) and $w(\cdot)$ is increasing, then $y(\bar{N}_y) > y(\bar{N}_k)$. As $y(\cdot)$ is increasing, then $\bar{N}_y > \bar{N}_k$.

We can determine the other inequality by comparing the following expressions:

$$\begin{aligned} c(\bar{N}_k) &= p_I(\bar{N}_k) (w(\bar{N}_k) + (r - \delta)k_C(1)) \\ c(\bar{N}_c) &= p_I(1) (w(1) + (r - \delta)k_C(1)) \end{aligned}$$

Define $f(N) \equiv p_I(N) (w(N) + (r - \delta)k_C(1))$. Thus, $f(\bar{N}_k) = c(\bar{N}_k)$ and $f(1) =$

$c(\bar{N}_1)$. We will analyze the sign of the derivative of $f(N)$:

$$\begin{aligned}
f(N) &= p_I(N)p_1(N)(1 - \theta_C)T_{k_C}^{\theta_C}N^{\frac{\theta_C(1-\gamma)}{\gamma-\theta_I}} + p_I(N)p_1(1)\theta_C T_{k_C}^{\theta_C-1}T_{k_C} - \delta T_{k_C}T_{p_I}N^{\frac{-(1-\theta_C)(1-\gamma)}{\gamma-\theta_I}} \\
f(N) &= T_{k_C}^{\theta_C} \left((1 - \theta_C)N^{\frac{\theta_C(1-\gamma)}{\gamma-\theta_I}} + \theta_C N^{\frac{-(1-\theta_C)(1-\gamma)}{\gamma-\theta_I}} - \delta T_n^{\frac{\gamma-1}{\gamma}} T_{\theta} T_{k_C}^{1-\theta_I} N^{\frac{-(1-\theta_C)(1-\gamma)}{\gamma-\theta_I}} \right) \\
f(N) &= T_{k_C}^{\theta_C} \left((1 - \theta_C)N^{\frac{\theta_C(1-\gamma)}{\gamma-\theta_I}} + \theta_C N^{\frac{-(1-\theta_C)(1-\gamma)}{\gamma-\theta_I}} - \delta \theta_C \left(\frac{1}{\beta} - (1 - \delta) \right) N^{\frac{-(1-\theta_C)(1-\gamma)}{\gamma-\theta_I}} \right) \\
\frac{df(N)}{dN} &= T_{k_C}^{\theta_C} \left((1 - \theta_C) \frac{\theta_C(1-\gamma)}{\gamma - \theta_I} N^{\frac{\theta_C(1-\gamma)}{\gamma-\theta_I}-1} + \right. \\
&\quad \left. - \theta_C \frac{(1 - \theta_C)(1 - \gamma)}{\gamma - \theta_I} \left(1 - \delta \left(\frac{1}{\beta} - (1 - \delta) \right) \right) N^{\frac{-(1-\theta_C)(1-\gamma)}{\gamma-\theta_I}-1} \right)
\end{aligned}$$

Thus, the sign of the derivative is given by

$$sign \left(\frac{df(N)}{dN} \right) = sign \left(1 - \left(1 - \delta \left(\frac{1}{\beta} - (1 - \delta) \right) \right) N^{\frac{-(1-\gamma)}{\gamma-\theta_I}} \right)$$

This derivative is positive if $N^{\frac{1-\gamma}{\gamma-\theta_I}} > 1 - \left(1 - \delta \left(\frac{1}{\beta} - (1 - \delta) \right) \right) = \left(\frac{1}{\beta} - (1 - \delta) \right)^{-1} \left(\frac{1}{\beta} - 1 \right)$.

We can show that $\bar{N}_k^{\frac{1-\gamma}{\gamma-\theta_I}} > \left(\frac{1}{\beta} - (1 - \delta) \right)^{-1} \left(\frac{1}{\beta} - 1 \right)$ using the definition of $\bar{N}_k$.

Therefore, the derivative above is positive and $\bar{N}_c > \bar{N}_k$. $\square$

Proof of Corollary 1.

From (0.12) and (0.20), $k_{closed}/y_{closed} = T_k/T_y$ does not depend on N . Moreover, for all N , $k_{open}/y_{open} = k_C(1)/[w(1) + rk_C(1)]$, which is independent of N . But the capital-income ratio rises for $N = \bar{N}_y$ (Proposition 4), since k increases, but y does not change. This implies that $k_{open}/y_{open} < k_{closed}/y_{closed}$ for every N . $\square$

Proof of Proposition 5.

From the Euler equation, and assuming CRRA preferences, we have

$$r = \frac{p_{Ct+1}}{p_{Ct}} \frac{[1 + g(c)]^\sigma}{\beta} - (1 - \delta) \quad (0.1)$$

where σ is the risk aversion coefficient.

From (0.1), we conclude that the rental rate is constant in a balanced growth path. Using (3.41), we find an expression relating the growth of the consumption good price and the capital-labor ration in that sector:

$$\begin{aligned} 1 &= \frac{p_{Ct+1}}{p_{Ct}} \left(\frac{k_{Ct+1}}{k_{Ct}} \right)^{\theta_C - 1} \\ [1 + g(k_C)]^{1 - \theta_C} &= \frac{p_{Ct+1}}{p_{Ct}} \end{aligned} \quad (0.2)$$

From (1.1), we have

$$\begin{aligned} \frac{p_{Ct+1}}{p_{Ct}} &= \left(\frac{n_{t+1}}{n_t} \right)^{\frac{1-\gamma}{\gamma}} \left(\frac{k_{Ct+1}}{k_{Ct}} \right)^{\theta_I - \theta_C} \\ [1 + g(k_C)]^{1 - \theta_C} &= [1 + g(n)]^{\frac{1-\gamma}{\gamma}} [1 + g(k_C)]^{\theta_I - \theta_C} \\ [1 + g(k_C)]^{1 - \theta_I} &= [1 + g(n)]^{\frac{1-\gamma}{\gamma}} \end{aligned} \quad (0.3)$$

From (1.3) and the fact that $n_t L_{It} / N_t$ is constant in a balanced growth path, we obtain

$$\begin{aligned} \frac{L_{It+1}}{L_{It}} &= \left(\frac{k_{It+1}}{k_{It}} \right)^{-\theta_I} \\ \frac{1 + g(N)}{1 + g(n)} &= [1 + g(k_I)]^{-\theta_I} \\ [1 + g(N)]^{\frac{1-\gamma}{\gamma}} &= [1 + g(k_C)]^{1 - \theta_I} [1 + g(k_I)]^{\frac{-\theta_I(1-\gamma)}{\gamma}} \\ [1 + g(N)]^{\frac{1-\gamma}{\gamma}} &= [1 + g(k_C)]^{\frac{\gamma - \theta_I}{\gamma}} \\ 1 + g(k_C) &= [1 + g(N)]^{\frac{1-\gamma}{\gamma - \theta_I}} \end{aligned} \quad (0.4)$$

where we use that k_C and k_I grow at the same rate in a balanced growth path.

From (3.58), the aggregate capital-labor ratio increase at the same rate that k_C , i.e., $g(k) = g(k_C) = g(k_I) = [1 + g(N)]^{\frac{1-\gamma}{\gamma - \theta_I}} - 1$. The growth rate of wages is determined using the first order condition for the firm in the consumption sector:

$$\begin{aligned} \frac{w_{t+1}}{w_t} &= \frac{p_{Ct+1}}{p_{Ct}} \left(\frac{k_{Ct+1}}{k_{Ct}} \right)^{\theta_C} \\ \frac{w_{t+1}}{w_t} &= \frac{k_{Ct+1}}{k_{Ct}} = [1 + g(N)]^{\frac{1-\gamma}{\gamma - \theta_I}} \end{aligned} \quad (0.5)$$

Finally, we can determine the growth rate of income and consumption:

$$y_t = w_t + rk_t$$

$$\frac{y_t}{k_t} = \frac{w_t}{k_t} + r$$

Since the right hand side is constant, we conclude that $g(y) = g(k) = [1+g(N)]^{\frac{1-\gamma}{\gamma-\theta_I}} - 1$.

$$p_{Ct}c_t = y_t - [k_{t+1} - (1-\delta)k_t]$$

$$\frac{p_{Ct}c_t}{k_t} = \frac{y_t}{k_t} - \frac{[k_{t+1} - (1-\delta)k_t]}{k_t}$$

$$\frac{p_{Ct}c_t}{k_t} = \frac{y_t}{k_t} - \frac{k_{t+1}}{k_t} + (1-\delta)$$

As the right hand side is constant, then we have that p_{CC} grows at the same rate as the capital-labor ratio. Thus, the growth rate of consumption is

$$1 + g(c) = \frac{1 + g(k)}{1 + g(p_C)} = [1 + g(N)]^{\frac{\theta_C(1-\gamma)}{\gamma-\theta_I}} \quad (0.6)$$

Finally, the growth rate of the relative price of investment is

$$1 + g(p_I) = [1 + g(N)]^{\frac{-(1-\theta_C)(1-\gamma)}{\gamma-\theta_I}} \quad (0.7)$$

□

Proof of Proposition 6.

We have that the only difference between the economy with external debt and the closed-economy case is the market clearing for consumption goods. Therefore, it is enough to verify that the market clearing for consumption goods is not used to derive the steady state value of the remaining variables. Thus, their value coincide with the closed-economy value. □

APÊNDICE C – WORLD ECONOMY STEADY STATE

From the Euler equation, we have

$$\begin{aligned} r^j &= \frac{1}{\beta} - 1 + \delta \\ \theta p_C^j (k_C^j)^{\theta-1} &= \frac{1}{\beta} - 1 + \delta \\ p_I^j &= \theta \left(\frac{1}{\beta} - 1 + \delta \right)^{-1} (k_C^j)^{-(1-\theta)} \end{aligned} \tag{0.1}$$

where we use the fact that $p_I^j = 1/p_C^j$.

From the choice of *numeraire*, we have

$$\begin{aligned} \left\{ \int_0^n p_i^{\frac{\gamma}{\gamma-1}} di \right\}^{\frac{\gamma-1}{\gamma}} &= 1 \\ n_t^{\frac{\gamma-1}{\gamma}} \frac{\psi}{\gamma} &= 1 \end{aligned} \tag{0.2}$$

where we use the fact that the price of varieties is equalized across countries.

From (3.61), we have that

$$\begin{aligned} x + f &= (k_I^j)^\theta A^j L_s^j \\ \frac{f}{1-\gamma} &= (k_I^j)^\theta A^j L_s^j \\ \frac{f}{1-\gamma} \mu^j &= (k_I^j)^\theta A^j L_s^j \mu^j \\ \mu^j &= \frac{1-\gamma}{f} A^j l_j^s N^j (k_I^j)^\theta \end{aligned} \tag{0.3}$$

where we have used the market clearing condition to skilled labor in the last equality.

From the cost minimization conditions in each sector, we have

$$\frac{w_s^j}{w_u^j} = \frac{k_I^j}{k_C^j} \quad (0.4)$$

Combining the expression of the marginal cost in the production of varieties with the factor demand expressions and (0.4), we obtain

$$\psi_j = \frac{(r^j)^\theta (w_s^j/A^j)^{1-\theta}}{\theta^\theta (1-\theta)^{1-\theta}} = \frac{(\theta p_C^j)^\theta (k_C^j)^{\theta(\theta-1)} ((1-\theta) p_C^j k_I^j/k_C^j)^{1-\theta} (k_C^j)^{\theta(1-\theta)}}{\theta^\theta (1-\theta)^{1-\theta}} = p_C^j \left(\frac{k_I^j}{k_C^j} \right)^{1-\theta} \quad (0.5)$$

Replacing this expression in (0.2), we obtain

$$\begin{aligned} n^{\frac{\gamma-1}{\gamma}} \frac{1}{\gamma} p_C^j \left(\frac{k_I^j}{k_C^j} \right)^{1-\theta} &= 1 \\ n^{\frac{\gamma-1}{\gamma}} \frac{1}{\gamma} \left(\frac{k_I^j}{k_C^j} \right)^{1-\theta} &= p_I^j \\ n^{\frac{\gamma-1}{\gamma}} \frac{1}{\gamma} \left(\frac{k_I^j}{k_C^j} \right)^{1-\theta} &= \theta \left(\frac{1}{\beta} - 1 + \delta \right)^{-1} (k_C^j)^{-(1-\theta)} \\ (k_I^j)^{1-\theta} &= n^{\frac{1-\gamma}{\gamma}} \gamma \theta \left(\frac{1}{\beta} - 1 + \delta \right)^{-1} \\ k_I^j &= n^{\frac{1-\gamma}{\gamma(1-\theta)}} \left\{ \gamma \theta \left(\frac{1}{\beta} - 1 + \delta \right)^{-1} \right\}^{\frac{1}{1-\theta}} \end{aligned}$$

Note that k_I^j does not depend on j , so we can write $k_I^j = k_I$. Since k_I is invariant across countries, the value of μ^j is determined by the scale of the country, measure by the total amount of skilled labor (in effective units).

Replacing the expression for k_I in (0.3) and summing across countries, we obtain an expression for n

$$\begin{aligned}
n &= (k_I)^\theta \frac{1-\gamma}{f} \left(\sum_{j=1}^J A^j l_s^j N^j \right) \\
n &= n^{\frac{\theta(1-\gamma)}{\gamma(1-\theta)}} \left\{ \gamma \theta \left(\frac{1}{\beta} - 1 + \delta \right)^{-1} \right\}^{\frac{\theta}{1-\theta}} \frac{1-\gamma}{f} \left(\sum_{j=1}^J A^j l_s^j N^j \right) \\
n^{\frac{1-\theta/\gamma}{(1-\theta)}} &= \left\{ \gamma \theta \left(\frac{1}{\beta} - 1 + \delta \right)^{-1} \right\}^{\frac{\theta}{1-\theta}} \frac{1-\gamma}{f} \left(\sum_{j=1}^J A^j l_s^j N^j \right) \\
n &= \left\{ \gamma \theta \left(\frac{1}{\beta} - 1 + \delta \right)^{-1} \right\}^{\frac{\theta}{1-\theta/\gamma}} \left\{ \frac{1-\gamma}{f} \left(\sum_{j=1}^J A^j l_s^j N^j \right) \right\}^{\frac{1-\theta}{1-\theta/\gamma}} \\
n &= T_n \mathcal{W}^{\frac{\gamma(1-\theta)}{\gamma-\theta}}
\end{aligned} \tag{0.6}$$

where $T_n \equiv \left\{ \gamma \theta \left(\frac{1}{\beta} - 1 + \delta \right)^{-1} \right\}^{\frac{\gamma\theta}{\gamma-\theta}} \left(\frac{1-\gamma}{f} \right)^{\frac{\gamma(1-\theta)}{\gamma-\theta}}$ and $\mathcal{W} = \left(\sum_{j=1}^J A^j l_s^j N^j \right)$.

Therefore, the total measure of varieties produced in the world depends on the effective world scale, i.e., the sum of the amount of skilled labor (in effective units) in each country.

The expression for k_I is given by

$$k_I = T_{k_I} \mathcal{W}^{\frac{1-\gamma}{\gamma-\theta}} \tag{0.7}$$

where $T_{k_I} \equiv \left\{ \gamma \theta \left(\frac{1}{\beta} - 1 + \delta \right)^{-1} \right\}^{\frac{\gamma}{\gamma-\theta}} \left(\frac{1-\gamma}{f} \right)^{\frac{1-\gamma}{\gamma-\theta}}$.

Using this expression for k_I , we obtain the following expression for μ^j

$$\mu^j = T_\mu \left(A^j l_s^j N^j \right) \mathcal{W}^{\frac{\theta(1-\gamma)}{\gamma-\theta}} \tag{0.8}$$

where $T_\mu \equiv \frac{1-\gamma}{f} T_{k_I}^\theta = \left\{ \gamma \theta \left(\frac{1}{\beta} - 1 + \delta \right)^{-1} \right\}^{\frac{\gamma\theta}{\gamma-\theta}} \left(\frac{1-\gamma}{f} \right)^{\frac{\gamma(1-\theta)}{\gamma-\theta}}$.

From the trade balance condition for country j , we obtain

$$\begin{aligned}\int_{i \in \Omega_j} p_i z_i^j di &= \int_0^n p_i x_i^j di \\ \mu^j \frac{\psi}{\gamma} \frac{\gamma f}{1 - \gamma} &= \int_0^n p_i^{\frac{\gamma}{\gamma-1}} X^j di \\ X^j &= \mu^j \frac{f}{1 - \gamma} \psi\end{aligned}\tag{0.9}$$

where the last equality comes from the choice of *numeraire*.

From the market clearing condition for the investment good, we have

$$N^j \delta k^j = n^{1/\gamma} p^{\frac{1}{\gamma-1}} X^j = X^j = \mu^j \frac{f}{1 - \gamma} \psi\tag{0.10}$$

using (0.2) in the second equality.

From (0.3), we have

$$\mu^j \frac{f}{1 - \gamma} = A^j l_s^j N^j (k_I)^\theta$$

Replacing in the previous equation, we obtain

$$N^j \delta k^j = A^j l_s^j N^j (k_I)^\theta \psi\tag{0.11}$$

Using the expression for ψ , we can obtain

$$\begin{aligned}N^j \delta k^j &= A^j l_s^j N^j (k_I)^\theta p_C^j \left(\frac{k_I}{k_C^j} \right)^{1-\theta} \\ N^j \delta k^j &= \theta^{-1} \left(\frac{1}{\beta} - 1 + \delta \right) A^j l_s^j N^j k_I \\ k^j &= (\delta \theta)^{-1} \left(\frac{1}{\beta} - 1 + \delta \right) A^j l_s^j k_I \\ k^j &= T_k (A^j l_s^j) \mathcal{W}^{\frac{1-\gamma}{\gamma-\theta}}\end{aligned}\tag{0.12}$$

where $T_k = (\delta \theta)^{-1} \left(\frac{1}{\beta} - 1 + \delta \right) T_{k_I} = \frac{\gamma}{\delta} \left\{ \gamma \theta \left(\frac{1}{\beta} - 1 + \delta \right)^{-1} \right\}^{\frac{\theta}{\gamma-\theta}} \left(\frac{1-\gamma}{f} \right)^{\frac{1-\gamma}{\gamma-\theta}}$.

From the market clearing condition for capital and the definition of k_C^j and k_I , we

have that

$$\begin{aligned}
\frac{k^j}{A^j} &= l_j^u k_C^j + l_s^j k_I^j \\
l_j^u k_C^j &= T_k (l_s^j) \mathcal{W}^{\frac{1-\gamma}{\gamma-\theta}} - (l_s^j) T_{k_I} \mathcal{W}^{\frac{1-\gamma}{\gamma-\theta}} \\
l_j^u k_C^j &= [T_k - T_{k_I}] (l_s^j) \mathcal{W}^{\frac{1-\gamma}{\gamma-\theta}} \\
l_j^u k_C^j &= \left[(\delta\theta)^{-1} \left(\frac{1}{\beta} - 1 + \delta \right) - 1 \right] T_{k_I} (l_s^j) \mathcal{W}^{\frac{1-\gamma}{\gamma-\theta}} \\
l_j^u k_C^j &= (\delta\theta)^{-1} \left(\frac{1}{\beta} - 1 + \delta(1 - \theta) \right) T_{k_I} (l_s^j) \mathcal{W}^{\frac{1-\gamma}{\gamma-\theta}} \\
k_C^j &= T_{k_C} \left(\frac{l_s^j}{l_j^u} \right) \mathcal{W}^{\frac{1-\gamma}{\gamma-\theta}}
\end{aligned} \tag{0.13}$$

where $T_{k_C} \equiv (\delta\theta)^{-1} \left(\frac{1}{\beta} - 1 + \delta(1 - \theta) \right) T_{k_I}$.

From the market clearing condition for consumption goods, we obtain

$$\begin{aligned}
c_j &= (k_C^j)^\theta A^j l_j^u \\
c_j &= T_c \left(A^j (l_s^j)^\theta (l_j^u)^{1-\theta} \right) \mathcal{W}^{\frac{\theta(1-\gamma)}{\gamma-\theta}}
\end{aligned} \tag{0.14}$$

where $T_c \equiv T_{k_C}^\theta$.

From (0.1), the price of consumption goods is given by

$$\begin{aligned}
p_C^j &= \theta^{-1} \left(\frac{1}{\beta} - 1 + \delta \right) p^I (k_C^j)^{1-\theta} \\
p_C^j &= T_{p^c} \left(\frac{l_s^j}{l_j^u} \right)^{1-\theta} \mathcal{W}^{\frac{(1/\gamma-1)(1-\theta)}{1-\theta 1/\gamma}}
\end{aligned} \tag{0.15}$$

where $T_{p^c} \equiv \theta^{-1} \left(\frac{1}{\beta} - 1 + \delta \right) T_{p^I} (T_{k_C})^{1-\theta} = \gamma \left[(\delta\theta)^{-1} \left(\frac{1}{\beta} - 1 + \delta(1 - \theta) \right) \right]^{1-\theta} T_n^{1/\gamma-1}$.

Combining the expressions for the price of consumption and investment goods we obtain the result for the relative price of investment goods

$$\frac{p^I}{p_C^j} = T_{p^{Ic}} \left(\frac{l_s^j}{l_j^u} \right)^{-(1-\theta)} \mathcal{W}^{\frac{-(1/\gamma-1)(1-\theta)}{1-\theta 1/\gamma}} \tag{0.16}$$

where $T_{p^{Ic}} = \frac{T_{p^I}}{T_{p^c}}$.

From the demand for factors in the consumption goods sector, we obtain

$$r^j = T_r \mathcal{W}^{\frac{-(1/\gamma-1/\gamma)(1-\theta)}{1-\theta 1/\gamma}} \quad (0.17)$$

where $T_r \equiv \theta T_{p^c} T_{k_C}^{-(1-\theta)} = \gamma \theta \left\{ \gamma \theta \left(\frac{1}{\beta} - 1 + \delta \right)^{-1} \right\}^{\frac{\theta/\gamma-1}{1-\theta 1/\gamma}} \left(\frac{1-\gamma}{f} \right)^{\frac{-(1/\gamma-1/\gamma)(1-\theta)}{1-\theta 1/\gamma}}$.

$$w_u^j = T_{w^u} \left(\frac{A^j l_s^j}{l_j^u} \right) \mathcal{W}^{\frac{(1/\gamma(1-\theta)+1/\gamma\theta)-1}{1-\theta 1/\gamma}} \quad (0.18)$$

where $T_{w^u} \equiv (1-\theta) T_{p^c} T_{k_C}^\theta = (1-\theta) \frac{\gamma f}{1-\gamma} (\delta \theta)^{-1} \left(\frac{1}{\beta} - 1 + \delta(1-\theta) \right) T_n^{1/\gamma}$.

Using the fact that $w_s^j = w_u^j k_I / k_{j^c}$, we obtain

$$w_s^j = T_{w^s} A^j \mathcal{W}^{\frac{-(1/\gamma-1/\gamma)(1-\theta)}{1-\theta 1/\gamma}} \quad (0.19)$$

where $T_{w^s} \equiv T_{w^s} \frac{T_{k_I}}{T_{k_C}} = (1-\theta) \frac{\gamma f}{1-\gamma} T_n^{1/\gamma}$.

Income per capita can be determined using the identity $y = w_u^j l_j^u + w_s^j l_s^j + r k^j$:

$$y = \left\{ T_{w^u} \mathcal{W}^{\frac{(1/\gamma(1-\theta)+1/\gamma\theta)-1}{1-\theta 1/\gamma}} + T_{w^s} \mathcal{W}^{\frac{-(1/\gamma-1/\gamma)(1-\theta)}{1-\theta 1/\gamma}} + T_r T_k \mathcal{W}^{\frac{-(1/\gamma-1/\gamma)(1-\theta)+1/\gamma-1}{1-\theta 1/\gamma}} \right\} (A^j l_s^j) \quad (0.20)$$

APÊNDICE D – PROOFS OF PROPOSITIONS IN CHAPTER 3

Immediate from assumption 1 and (0.20), (0.14), (0.13), (0.7), (0.12) and (0.16). $\square$

The result comes from the ratio of (0.20), (0.12), (0.14) and (0.16) for countries j and j' . $\square$

Investment rates at domestic prices are given by

$$i^{dom} = \frac{\delta k^j}{y_j} \quad (0.1)$$

from (0.12) and (0.20), we see that investment rates are constant across countries, so it is not correlated with income.

From assumption 1 and (0.16), we obtain that the relative price of investment goods is decreasing in time.