

**Universidade de São Paulo**  
**Faculdade de Economia, Administração e Contabilidade**  
**Departamento de Economia**  
**Programa de Pós-Graduação em Economia**

**Short Selling Recall Option Pricing:  
empirical and theoretical approaches**

Precificação da Opção de Recompra em  
Operações de Venda Descoberta:  
abordagens empírica e teórica

**Leonardo Viana de Almeida**

Dissertação apresentada ao Programa de Pós-Graduação em Economia do Departamento de Economia da Faculdade de Economia, Administração e Contabilidade da Universidade de São Paulo, como requisito parcial para a obtenção do título de Mestre em Ciências.

Orientador: Prof. Dr. Fernando Daniel Chague

Versão Corrigida

(versão original disponível na Faculdade de Economia, Administração e Contabilidade)

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## **ABSTRACT**

Short selling is important for price efficiency as it helps negative information to be incorporated into prices. As short selling requires borrowing stock in advance, the equity lending market plays a central role in price efficiency. For instance, when the costs of borrowing certain equities are high, these stocks are likely to be overpriced. Unfortunately, not much is known about the equity lending market, particularly the Brazilian market. Here, we have investigated a particular feature of the equity lending contract, namely, the lender recall option. Lending contracts either i) allow the lender to recall the stock at an earlier date than initially agreed, or ii) allow no early recall, that is, they are fixed term contracts. We have derived a simple model for recall option pricing and confirmed the model empirically.

**Keywords:** Stock lending; short selling; option pricing; theoretical model; regression analysis.

## **Precificação da Opção de Recompra em Operações de Venda Descoberta: abordagens empírica e teórica**

### **RESUMO**

A venda descoberta desempenha uma importante participação na eficiência da precificação de ativos, pois permite incorporar informações negativas aos seus preços. Como a venda descoberta requer que um ativo seja alugado previamente, o mercado de aluguel de ativos tem um papel central na formação eficiente de preços. Por exemplo, quando os custos de aluguel são altos, ativos estão provavelmente sobrevalorizados. Infelizmente pouco se conhece a fundo sobre o mercado de aluguel de ativos. Neste artigo, investigamos uma característica do aluguel de ações, propriamente dita, a opção de liquidação antecipada pelo doador. Contratos de aluguel, quanto a este aspecto, podem i) permitir que o doador requeira suas ações antes do prazo acordado ou ii) não permitir esta opção, possuindo prazo fixo. Derivamos um modelo simples de precificação desta opção e confirmamos o modelo empiricamente.

**Palavras-chave:** Aluguel de ações; venda descoberta; precificação de opções; modelo teórico; regressão analítica.

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## List of Symbols and Abbreviations

BM&F BOVESPA: Brazilian Securities, Commodities and Futures Exchange (*Bolsa de Valores, Mercadorias e Futuros de São Paulo*)

FE: Fixed Effect

HHI: Herfindahl-Hirschman Index

$N(\mu, \sigma^2)$ : Normal distribution with mean  $\mu$  and variance  $\sigma^2$

SD: standard deviation

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## 1. INTRODUCTION

Short selling is important for price efficiency as it helps negative information to be incorporated into prices. As short selling requires borrowing stock in advance, the equity lending market plays a central role in price efficiency. For instance, when the costs of borrowing certain equities are high, these stocks are likely to be overpriced. Unfortunately, not much is known about the equity lending market, particularly the Brazilian market. Here, we have investigated a particular feature of the equity lending contract, namely, the lender recall option. Lending contracts either i) allow the lender to recall the stock at an earlier date than initially agreed, or ii) allow no early recall, that is, they are fixed term contracts. We have derived a simple model for the recall option pricing and confirmed the model empirically.

From the lender's perspective, the recall option is desirable as it allows the lender to sell the re-called stock at any time. As such, the lender would agree with a lower loan fee to have the recall option available. In turn, from the borrower's perspective, the recall option is not desirable as an early recall by the lender means that the borrower will have to locate another owner willing to lend it or, as a last resource, buy it in the spot market. In this case, the borrower would also require a lower loan cost (fee) for recall option. Thus, from an economic perspective, loan contract with recall option available should have lower loan fees, that is, *ceteris paribus*.

In order to show that loan contracts with recall option actually present lower loan fees, we used a comprehensive data set containing all stock loan contracts closed in Brazil from January 2009 to July 2011. Interestingly, our panel regressions show that the recall option does involve lower loan fees, but only in some circumstances. For instance, the recall option is to be priced in deals closed only when the conditions of equity lending market are unfavorable to the borrowers. That is, the loan fee is lower for deals in which recall option is available only when borrowers are likely to have a hard time locating new lenders.

We established a proxy for the borrower's difficulty in locating a new lender from the concentrated equity lending market, which is in the hands of a few lenders. More precisely, we computed the Herfindahl-Hirschman index (HHI) of the lender's past market share. This index ranges from zero to one: when there are plenty of lenders

and brokers offering a large quantity of stock, the index approaches zero, whereas when there are fewer lenders and brokers the index approaches one. At the limit the index is set to one, corresponding to a case of monopoly. The market concentration index, in this case, is a measure of how hard it is for borrowers to find a particular stock available for loan, like a loan search cost proxy.

An interactive variable, combining the market concentration index with the lender recall option dummy, which assigns 1 to re-callable cases or zero otherwise, has been shown to be significant after being regressed by the loan fee as a dependent variable. The negative coefficient demonstrates that lenders accept a lower loan fee while borrowers demand a lower loan cost (fee) if loan is re-callable and market is scarce. We performed regressions by using different market concentration indexes for both lender and brokers as well as at monthly or weekly aggregated concentrations. Assuming that the market is highly concentrated, the recall option price varies from 0 to 3% a year.

To give some theoretical foundation for these empirical findings, we have proposed a theoretical pricing model which involves two different preference values before outputting the recall option price; one preference value obtained under the borrower's perspective and the other under the lender's perspective (the latter being based on a particular rationale of an American option pricing model). The borrower's preference value framework refers to his perspective and corroborates our empirical findings.

The borrower's preference value approach is a flexible framework developed to accommodate the market concentration index, since it interferes with the impact of recall option on the loan fee. This framework establishes a loan fee process resulting in a new loan fee in the case of recall. The new fee changes in terms of both level and volatility, making use of market concentration as an argument. Through this new fee, the borrower can re-loan the stock after it is re-called, thus changing its final pay-off. The expected pay-off difference in such scenarios sets the preference value.

The lender's perspective for his preference value can be compared to a traditional American stock option, in which the option owner has the right to recall (similar to buying for a fixed price) a certain amount of shares at any time. Therefore, this framework was adapted from a binary model projected to price the American

stock option. Both frameworks support our empirical results, as well as the variable impact dynamics.

A short sale position, simultaneously held with a long position, can be justified by a distinct expectation formed by investors regarding future returns. Short positions increase the spot market supply as its curve shifts towards right, hence lowering the asset prices. Therefore, short selling restrictions may lead to asset over-valuation (Miller, E., 1977).

The global financial crisis in 2007-2009 and its repercussions in the present day have frightened many market regulators around the world, who in response have restricted or even banned short selling positions. However, there are still major controversies among researchers about its impact on the market, which enhances the debate (Chague, F.; et al., 2014).

Duffie, Gârleanu & Pedersen (GDP model, 2002) proposed a model which includes the cost for finding assets available for renting (search cost) and therefore lenders may act as local monopolists, which results in positive loan fees, even in the presence of loan supply excess.

Kolansiski, Reed & Ringgenberg (KRR model, 2013) suggested proxies for search costs, such as firm's size, bid-ask spread and stock concentration among lenders, and found that these variables raise both level and dispersion of loan fee as they intuitively increase search costs.

In 2015, Prado empirically tested the GDP model and observed that financial institutions increase their long positions as the loan fees of shares are raised, incorporating loan income into stock prices. As a final result, asset owners take into account loaning as a possible dividend gain, which raises the asset values (Prado, M., 2015).

There is a related strand of literature investigating option pricing through different approaches (e.g. equilibrium models), suggesting them for the US Treasury bonds loan market, the so-called "repo market" (Krishnamurthy, A., 2002). Krishnamurthy observed that investor heterogeneity leads to the simultaneous presence of both demand for renting bonds and owners not willing to lend them. This suggests the presence of different investment strategies or different expectations regarding future bond returns, as demonstrated by Miller's first argument (1977). After analysing data from June 1995 and November 1999, Krishnamurthy found no

opportunity for significant gain obtained by short selling 30-year new treasury bonds combined with buying (long position) 30 year-old bonds.

The evidence about the Brazilian equity lending market is incipient. Chague et al. (2015) have shown that the stock lending market is a market-based relationship, where low-cost borrowers (borrowers who face lower loan fees) are well connected to brokers who, in turn, are well connected to lenders who hold a large share of the lending market. The Brazilian stock loan market is highly inefficient. Our data show that some loan fees are three times higher than others for the same stock in similar quantities and on the same day.

In the American market, Avellaneda (2009) has proposed a model based on hard-to-borrow stocks (those which face regulatory short-selling restrictions or insufficient float available for lending): a dynamic model correlating stock price and buy-in rate (i.e. when the clearing chamber issues a buy order to the seller who has failed to deliver a previously sold stock). The author showed empirical evidence that buy-ins are followed by price jumps (Avellaneda, M., 2009).

Also in the American market, Engelberg, Reed & Ringgenberg (2014) have proposed proxies to measure short selling risks, such as short interest (loan market supply and actually borrowed shares), fee risk (past loan fee level and volatility) and recall risk (frequency of past recall events). By using linear regressions, it was observed that these variables are related to (negative) future returns, representing a premium for short sellers, that is, the bigger the risk, the larger the return from short positions.

## 2. THE LOAN MARKET

### 2.1 American and Brazilian Loan Markets

The Securities and Exchange Commission defines *short sale* as selling an asset (e.g. equity, private or public bonds) which does not currently belong to the seller or is not intended for delivery. Hence, in order to perform a short sale, the seller must rent it (D'Avolio, G., 2002).

One of the first large data collections from the American equity lending market was conducted by D'Avolio between April 2000 and September 2002, who combined data from several institutions and brokers intermediating loan contracts. The main empirical results are summarized bellow (D'Avolio, G., 2002):

- The aggregate loan market is quite liquid for lending any stock, whose average cost is 0.25% *per year*, and about 7% of the supply is in fact loaned.
- Most liquid stocks in the spot market are available for lending. Those unavailable for lending represent less than 1% of the market in value and are traded for very low nominal prices *per share*.
- Stocks of low nominal values with low liquidity are almost never loaned despite being available, representing 10% of the loan market in value.
- Institutional investors who lend abundant stocks usually offer large companies stocks with high liquidity, indicating that they are large passive funds that follow market indexes.
- 91% of the loans cost less than 1% *per year*, whereas the remaining 9% cost in average 4.3% *per year*.
- Some stocks are excessively expensive for lending; in general, they are of small companies with low cash-flow, high managerial turnover and poorly specified accounting books.
- Recall option is rarely taken; the monthly average is about 2% of the stocks loaned. The average time for recall option is 23 business days before contract expiry date. Within such events, the market volume may double and the short seller's gains diminished, on average (D'Avolio, G., 2002).

The current data show that the Brazilian equity loan market has several advantages over the American one. All tradings are recorded daily in a centralized

database containing information such as offers available for loaning, volume, quantity, financial intermediates, deals closed, loan and brokerage fees, and investor type. This contrasts with most of the lending markets, which are decentralized with only partially available information.

Studies on these data are relatively recent, whereas empirical analyses of American data began decades ago.

The Brazilian equity loan market features a singular micro-structure, where all transactions are centralized through BM&F BOVESPA, the world's fourth biggest stock exchange for equities, public and private bonds, future currency contracts, commodities, indexes and all sorts of derivatives.

The loan market is regulated by the Brazilian Securities and Exchange Commission and by the National Monetary Council.

Any asset traded by the stock exchange is available for loan, whose parties (i.e. lender and borrower) are intermediated by BM&F BOVESPA authorized brokers. The loan market has grown significantly in Brazil over the last decade. In 2011, transactions involved 290 companies *per* month involving individuals (40%), foreign investors (35%) and funds (25%) as lenders as well as funds (70%), foreign investors (25%) and individuals (5%) as borrowers (Chague, F.; et al., 2014).

As advertised by the Brazilian BM&F BOVESPA, equity or asset-rental activity increases the market liquidity, thus enhancing price efficiency and flexibility, which benefits investors with both short- and long-term strategies. A short sale trade requires the asset to be borrowed, either in advance or after selling it on the spot market, by rewarding the lender with a loan fee freely settled between lender and borrower. The market requires mechanisms to balance the price settlement accordingly, with asset renting and short selling making a significant contribution.

Formally after lending the asset, the owner is no longer entitled to it and therefore does not receive any designated rights from the issuing company. The loan system is, however, in charge of reimbursing the lender on the same date with the same amount, just as it would be if the lender owned the asset. Similarly, the loan system also withdraws this amount from the borrower.

By using data from the Brazilian Clearing and Depository Corporation, the table below demonstrates the rapid growth of the Brazilian loan market.

**Table 1.** Brazilian loan market's annual aggregate figures

Year	Volume (mi R\$)	Volume (mi US\$)	Number of Deals	Number of Intermediates	Number of Traded Assets
2006	109,674.05	50,496.18	271,210	1,002	1,869
2007	272,473.06	142,106.28	568,592	998	2,639
2008	303,505.51	174,568.50	627,414	1,000	3,006
2009	258,912.60	137,483.77	711,987	990	2,887
2010	465,605.78	265,892.32	971,558	999	3,125
2011	732,750.51	436,302.09	1,417,787	1,046	3,327
2012	785,927.87	405,854.85	1,313,355	1,019	3,322
2013	1,006,836.85	468,628.79	1,693,151	965	3,500
2014	735,017.70	321,967.85	1,518,369	935	3,481
2015	665,732.25	203,569.14	1,519,445	870	3,358

Source: BM&F BOVESPA (home/services/asset loan/variable income)

A significant market increase can be observed over the past decade until 2013, whether in volume, deals or traded assets, when the loan market roughly matched the spot market in volume (R\$ 1.8 trillion). The reverse tendency occurred in the last two years due to the Brazilian economic recession in 2014 and 2015. The volume in US\$ decreased even more severely due to the depreciation of the Brazilian currency.

**Table 2.** Brazilian loan market participation in 2015 according to investor type

Year	Type	Individuals	Commercial Banks	Social Retirement Funds	Mutual Funds	Other Civilian Social Associations	Foreign Investors	Private Enterprises
2015	Lender	15.59%	0.20%	1.59%	44.21%	1.09%	35.60%	1.71%
	Borrower	2.61%	2.37%	0.00%	48.25%	0.17%	45.83%	0.76%

Source: BM&F BOVESPA (home/services/asset loan/variable income)

Table 2 shows that mutual funds are the main participant in the loan market, followed by foreign investors, both acting as lenders and borrowers, whereas individuals participate mostly as asset lenders.

The Brazilian market involves mainly over-the-counter transactions (90% of the deals), with only a few deals being made through electronic means, where offers are available.

By using the Brazilian data set, Chague et al. (2013 and 2014) have found evidence that short sale agents are well-informed, indicating that market asset price may not take into account all the related information available. However, studies

found no evidence that such transactions manipulate the market or make prices unstable. Finally, there are indeed signs that privileged information benefits short seller agents (Chague, F.; et al. 2013 and 2014). Chague et al. have also shown that the use of electronic means by the loan market reduces both loan fee and volatility, thus encouraging its use in a market shadowed by the over-the-counter practice (Chague, et al., 2015, Brazilian equity lending market).

## **2.2 Loan Market Data: Variables and Features**

The available data present all loan offers to both lenders and borrowers registered in Brazil from January 2009 to July 2011 on a daily basis (i.e. 3,359,354 offers). For every lender's or borrower's loan offer there is a unique offer code, including date, ticker code (i.e. firm's share code), quantity (number of shares offered), loan fee, brokerage fee, lender's, borrower's and broker's identification numbers (formed by a random number invariant over time which uniquely identifies investor or intermediate), offer status (closed, available, partially available, or canceled due to some specific reason), time length, investor type, lender recall option, and borrower return option. Lender recall option means that the lender is able, although not obliged, to request their shares prior to the loan expiration date. Borrower return option means that the borrower is able, although not obliged, to deliver their loaned shares prior to the loan expiration date.

In general, we can identify the following approximate participation according to investor type:

- Lenders: 41% of individuals, 33% of funds, 15% of foreign investors.
- Borrowers: 45% of funds, 30% of individuals, 16% of foreign investors.

As our dataset presents both lender's and borrower's offers, every closed deal (closed offer) is exhibited at least as a duplicate observation. Moreover, several different borrowers may accept one lender's offer, which explains the excess of total borrower offers in our data set (56%).

A borrower may also partially accept a lender's offer, which makes the remaining shares available (partially available status) for other borrowers to lend.

These features have imposed filters for data treatment before our empirical analysis, meaning that we were restricted to lender closed offers only.

Almost 100% of the borrower offers are settled, thus allowing the borrower to have a return option in advance (> 99.9%), which makes us unable to analyse their impact on the market.

Meanwhile, the lender recall option increased over time, that is, from 10% in January 2009 to 50% in July 2011.

As previously mentioned, in order to avoid duplicity and promote coherence, our data were restricted to lender closed offers only, resulting in 1,065,425 offers.

Due to tax arbitrage to be mentioned, a window formed by 4 days prior to and 4 days after the equity capital interest payment date was removed, including the day itself.

In Brazil, share owners entitled to capital interest payments, usually taxed by 15%, make an arbitrage gain by loaning shares to a mutual fund. Mutual funds pay taxes at another instance (liquidation day), therefore the capital interest payments are not taxed by 15%, even when the shares are borrowed (due to a flaw in the tax system). This means that both parties may collect extra gains without risk by lending each other the asset only during the interest payment day or around it, which increases transactions enormously in our data set, providing misleading information for our empirical analysis.

Liquidity filters were also applied. Among the 425 different tickers borrowed in the period (Jan/2009-Jul/2011), more than 200 of them involved less than 100 transactions (less than once a week, on average).

After a rough overview, we observed high intraday loan fee dispersion (same stock, same day and similar quantities) measured by both standard deviation and range (see Appendix 1):

- High loan fee dispersion in time (same stock).
- High loan fee dispersion across stocks.

For borrowers, the lender recall option represents a high risk, therefore such investors should request to pay a reduced loan fee. On the other hand, for lenders, the recall option reduces the risk, therefore such investors should accept a lower loan fee.

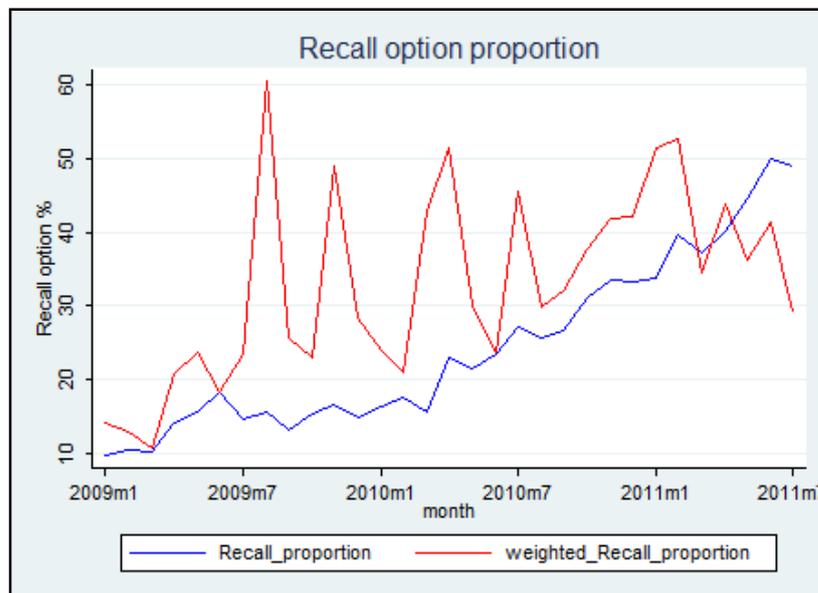
Average loan fees in time (for most liquid stocks) were, however, higher for offers with recall option (both adjusted or not by loan size) than without. Deal-by-deal regression using loan fee as dependent variable and recall option (dummy) and loan size as independent variables, as well as firm and time fixed effects, exhibited a positive coefficient, a finding contradicting our above-mentioned prediction.

These findings may be related to the fact that borrowers may have faced many liquidity problems for re-loaning, as well as to the remarkable opacity of the loan market.

### 2.3 The Variable Recall Option

Our dataset shows recall option as a variable of YES or NO string. We performed a proportional overlook without liquidity or capital gain filters in order to investigate its rough proportions. The graphic below shows its monthly proportion without and with weighted values regarding the quantities loaned. Only lender closed offers were considered due to previously explained reasons.

**Figure 1.** Monthly proportion of recall option



We can observe that rough proportion increases over time from 10% in January 2009 to 50% in July 2011. However, the proportion weighted by the amount of shares loaned is much higher and varies on a cyclical basis, with the bottom of the

peaks seeming to follow the rough proportion. In other words, lenders tend to use the recall option more often in large loan deals, and both types of deals demonstrate increased use of the recall option in the last few months of our data.

### 3. THEORETICAL MODEL

Our model framework begins by questioning what impacts of a recall event might be on a lender or on a borrower regarding their gains.

The recall option represents a security for lenders, allowing them to re-establish their stock prior to the loan expiration date. On the other hand, the recall possibility represents a great risk for borrowers, obligating them to re-loan the stock or buy it from the spot market in order to deliver it to its owner.

The theoretical model here proposed points out the different views on how to value an increase or reduction of that risk. After measuring these different values, we came up with a simple equilibrium model.

Although any option presents a unique price, a recall event involves different responses whether you are a lender or a borrower. If a lender recalls the stock, the borrower assumes all the risk of not being able to delivery it.

In order to support such a distinct view, we proposed a pay-off expectation exercise, where each investor looks first at one's preference value, which quantitatively expresses how the recall option is worth according to one's expectation. We named such values as *Investor (Lender/Borrower) Preference Value*.

$$\text{Lender Preference Value} = E_{t=0} \left[ \begin{array}{l} \textit{Payoff with} \\ \textit{recall option} \end{array} \right] - E_{t=0} \left[ \begin{array}{l} \textit{Payoff without} \\ \textit{recall option} \end{array} \right] \quad (\text{eq.1})$$

$$\text{Borrower Preference Value} = E_{t=0} \left[ \begin{array}{l} \textit{Payoff without} \\ \textit{recall option} \end{array} \right] - E_{t=0} \left[ \begin{array}{l} \textit{Payoff with} \\ \textit{recall option} \end{array} \right] \quad (\text{eq.2})$$

Once the recall option has been available for the lender, the expected pay-off will be equal or greater, meaning that the lender's preference value is positive. On the other hand, the recall option represents a risk for borrowers and thus their expected pay-off will be equal or lesser than otherwise. Therefore, both equations 1 and 2 were structured for a positive preference value.

Although they look quite the same, as we will see below, they were obtained by very different approaches.

### 3.1 Borrower's Preference Value

The borrower's preference value was developed based on the fact that, according to our empirical results, the recall option has positive values only in particular situations. Such situations are characterised by fewer owners or brokers offering shares for loaning. If loan availability is scarce in the market, borrowers are likely to be worried about finding new shares for loaning if these are to be recalled. When loans are in abundance, however, it is easy to re-loan stocks and recall situations are solved without cost. Therefore, the valuing framework incorporates the borrowers' concern about having their shares recalled or not, which may obligate them to re-loan the recalled shares by paying a new loan fee.

The valuation model was projected to accommodate the variables found to impact the recall price in our regression studies. The main variable is the market concentration index, which indicates whether the recall option is worth using or not, and even how valuable it may become.

The borrower valuation model establishes the following simple assumptions:

*i.* Once the borrower's shares have been recalled by the lender, the borrower may re-loan them from another owner by offering a new loan fee.

*ii.* Investors (borrowers) are risk-neutral, i.e., linear utility function,  $U(w) = w$ .

As a recall event takes place only once during a loan contract, a summarised three-period process is assumed to set the loan fee ( $b$ ).

The three periods are:

$t = 0$ :  $b_{t=0} = b_0$  Loan fee at the contract's beginning.

$t = 1$ :  $b_{t=1} = b_1 \geq 0$  Loan fee at a recall event, with probability  $p$ .

$t = 2 = T$ : Expiration date.

The loan fee ( $b$ ) process takes into account the loan market concentration. When the loan supply is more concentrated, the loan fee tends to increase and become more volatile (owners act as local monopolists), as supported by the literature. From our empirical results, we have discovered that the speed of market concentration actually raises the loan fee, also causing volatility.

The loan market concentration, measured by the Herfindahl-Hirschman index (HHI) and largely explained in Section 4, is assumed to act as an argument that generates the loan fee process by increasing (or reducing) constant,  $\mu$ . We will call this constant as loan fee process drift, since it raises (or reduces) the loan fee level in the next period ( $b_{t=1} = b_1$ ). Initially, we propose a linear function as simple as  $\mu \left( \frac{HHI_t - HHI_{t-1}}{HHI_{t-1}} \right) = c_{HHI} \mu$ , where  $c_{HHI}$  is an adjusting constant. The term  $\frac{HHI_t - HHI_{t-1}}{HHI_{t-1}}$  may be either positive or negative, causing the loan fee to increase or decrease, respectively; so that the drift  $\mu$  becomes coherently positive or negative ( $c_{HHI} \geq 0$ ).

We also suppose HHI to be an argument of the loan fee volatility function,  $\sigma \left( \frac{HHI_t - HHI_{t-1}}{HHI_{t-1}} \right) = c_{vol} \sigma$ , in a similar way.

The process variables act as follows:

$$b_{t=0} = b_0 \rightarrow b_1 = b_0(1 + \mu + \sigma\varepsilon)b_1 \geq 0 \quad (\text{eq. 3})$$

$\varepsilon \sim N(0,1), \text{independent}$

As the recall option is valued by the borrower's pay-off difference between contracts without and with recall option, the latter must be equal or less than the former in terms of expectation, so we have:

Pay-off( $\Pi$ ) at  $t = 2$  without recall option:

$$\Pi_2 = S_0 e^{2r} - S_0(e^{2b_0} - 1) - S_2 \quad (\text{eq. 4})$$

Pay-off( $\Pi$ ) at  $t = 2$  after recall at  $t = 1$ :

$$\Pi_{2'} = S_0 e^{2r} - S_0(e^{b_0} - 1) - S_1(e^{b_1} - 1) - S_2 \quad (\text{eq. 5})$$

where:

$S_{t=0}, S_{t=1}, S_{t=2}$  : stock prices at respective times.

$b_0, b_1$  : loan fees at respective times.

$r$  : risk-free interest rate

Loan fee and risk-free rate are both expressed as continuously compounded annual rates, but from equations 4 and 5, one may notice that we have a discrete

sum after loan fee shifted from  $b_0$  to  $b_1$  and thus the pay-off results will not be the same. Indeed, but the annual loan fee rates are usually 0 to 5%, on average, (see Appendix 1), and during a standard loan contract (34 days), such rates will be less than one tenth of that. It is easy to demonstrate that the difference between a discrete rate sum and a continuous rate calculation will be very small (0.1 to 1%) when very low rates are considered (0 to 1%).

Therefore, the valuation rationale will be the difference between (expected) pay-offs:

Pay-off difference:

$$\Pi_2 - \Pi_{2'} = S_0 e^{2r} - S_0(e^{2b_0} - 1) - S_2 - [S_0 e^{2r} - S_0(e^{b_0} - 1) - S_1(e^{b_1} - 1) - S_2]$$

From this point on, we establish an assumption that acts as an acceptable reasonable approximation,  $S_1 = S_0$ . The stock price is not actually supposed to be the same at a recall event; we largely agree that it does change. However, from the borrower's perspective, after having the shares recalled the borrower is not obligated to buy them from the spot market priced by  $S_1$ , instead the borrower can re-loan shares from another owner by offering a new loan fee,  $b_1$ , according to our model. In order to assume that  $S_1 = S_0$ , we propose an adjusted loan fee,  $b_{\bar{1}}$ , which aggregates the stock price movement in percentage,  $b_{\bar{1}} = b_1 \left( \frac{S_1 - S_0}{S_0} \right)$ . Thus, this adjusted loan fee incorporates the stock price change. We then continue calling the adjusted loan fee as  $b_1$  for simplification, assuming that the additional share is present within its drift,  $\mu$ .

After this proposal, the pay-off difference expression becomes simplified as follows:

$$\text{Pay-off difference} = S_0(e^{b_0} - e^{2b_0} + e^{b_1} - 1).$$

We now turn to the expected difference:

Expected pay-off difference  $= E_{t=0}[\Pi_2 - \Pi_{2'}] = E_{t=0}[S_0(e^{b_0} - e^{2b_0} + e^{b_1} - 1)] = S_0(e^{b_0} - e^{2b_0} + E_{t=0}[e^{b_1}] - 1)$ , since  $b_0$  and  $S_0$  are known at  $t = 0$ .

As  $b_1 = b_0(1 + \mu + \sigma\varepsilon)$  with probability  $p$  and  $b_1 = b_0$  with probability  $1 - p$ , the loan fee at  $t = 1$  is therefore  $pb_0(1 + \mu + \sigma\varepsilon) + (1 - p)b_0 = pb_0(\mu + \sigma\varepsilon) + b_0$ .

We have  $E_{t=0}[e^{b_1}] = E_{t=0}[e^{pb_0(\mu+\sigma\varepsilon)+b_0}]$  and  $\varepsilon \sim N(0,1)$ , with  $\mu$  and  $\sigma$  being the model-adjusting constants, so  $pb_0(\mu + \sigma\varepsilon) + b_0$  is also normally distributed.

According to normal distribution: if  $X \sim N(\mu, \sigma^2)$  then  $E[e^X] = e^{\mu + \frac{\sigma^2}{2}}$

Finally:  $E_{t=0}[e^{pb_0(\mu+\sigma\varepsilon)+b_0}] = e^{pb_0(\mu+\sigma)+b_0 + \frac{p^2b_0^2\sigma^2}{2}} = e^{pb_0\left(\mu+\sigma+\frac{pb_0\sigma^2}{2}\right)+b_0}$

Therefore, the expected pay-off difference may be expressed, without expectancy, as follows:

$$\text{Borrower Preference Value} = S_0 \left( e^{b_0} - e^{2b_0} + e^{pb_0\left(\mu+\sigma+\frac{pb_0\sigma^2}{2}\right)+b_0} - 1 \right) \quad (\text{eq. 6})$$

Model Parameters:

$$\mu(HHI, S_1) = c_{HHI} \left( \frac{S_1 - S_0}{S_0} \right) HHI = c_{\overline{HHI}} HHI = \mu_{numeric}: \text{loan fee drift.}$$

We call this as  $\mu_{numeric}$  because we simply test different values directly for it.

$$\sigma(HHI) = c_{vol} HHI = \sigma_{numeric}: \text{loan fee stochastic share.}$$

$p$ : recall event at  $t = 1$  probability.

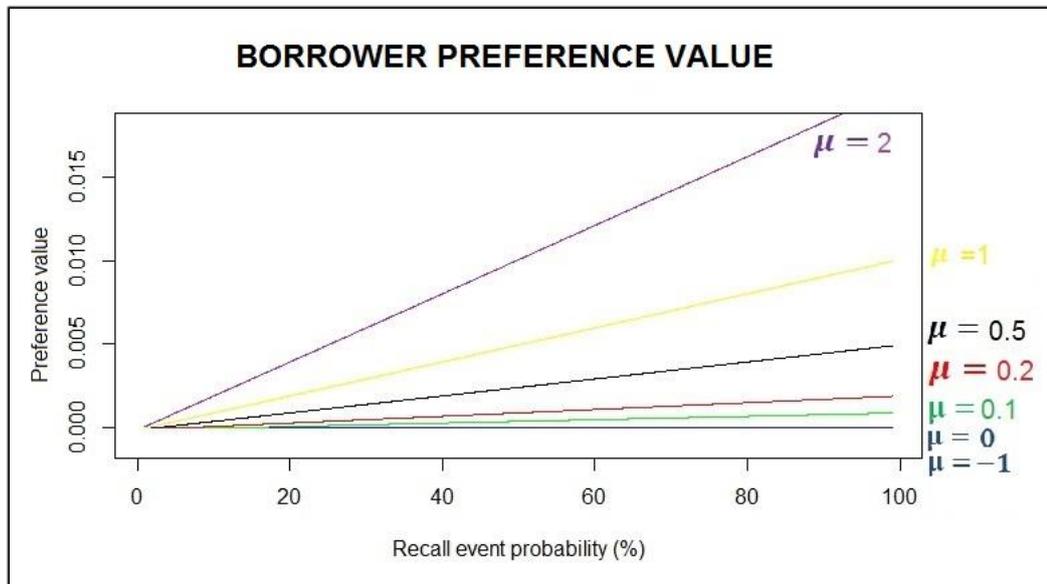
### 3.2 Borrower's Preference Value Simulation

We start by analysing the model parameter impacts ( $\mu$ ), loan fee drift ( $\sigma$ ), loan fee stochastic share; and recall event ( $p$ ) at  $t = 1$  probability.

The initial loan fee,  $b_0$ , was set to 1%, which is reasonable according to our empirical data (Appendix 1). Initial stock price was normalised as  $S_0 = 1$ .

In the first graph, we simulated the process without stochastic share,  $\sigma = 0$ , with recall probability varying from 1% to 99% among several loan fee drifts,  $\mu$  (-1; 0; 0.1; 0.2; 0.5; 1; and 2).

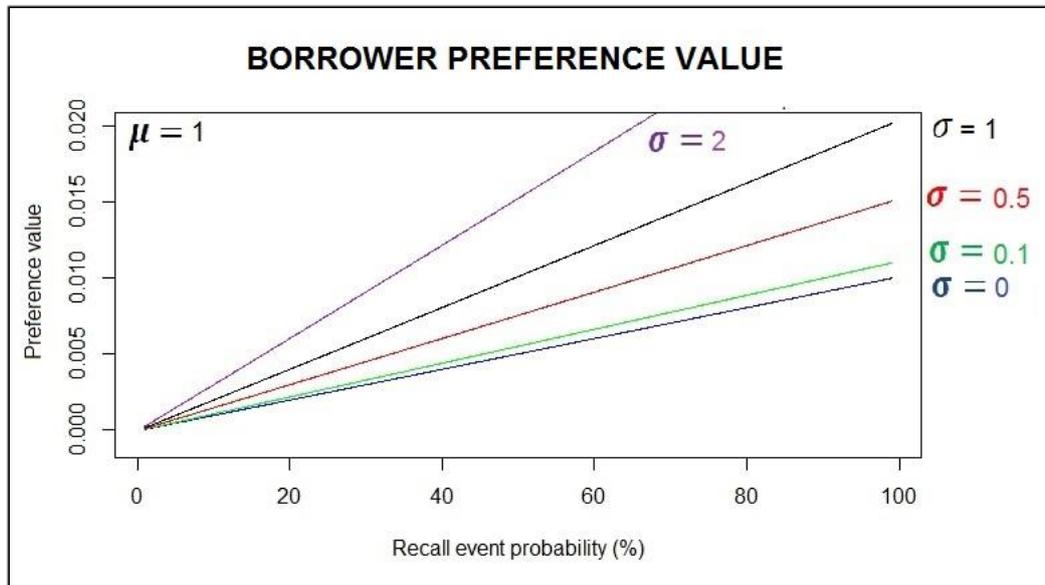
**Figure 2.** Borrower's preference value: recall probability variation among several loan fee drifts.



The preference value rises with recall probability and loan fee drift. Since zero or negative drifts reduce the loan fee, a recall event does not concern the borrower, rendering the recall option worthless.

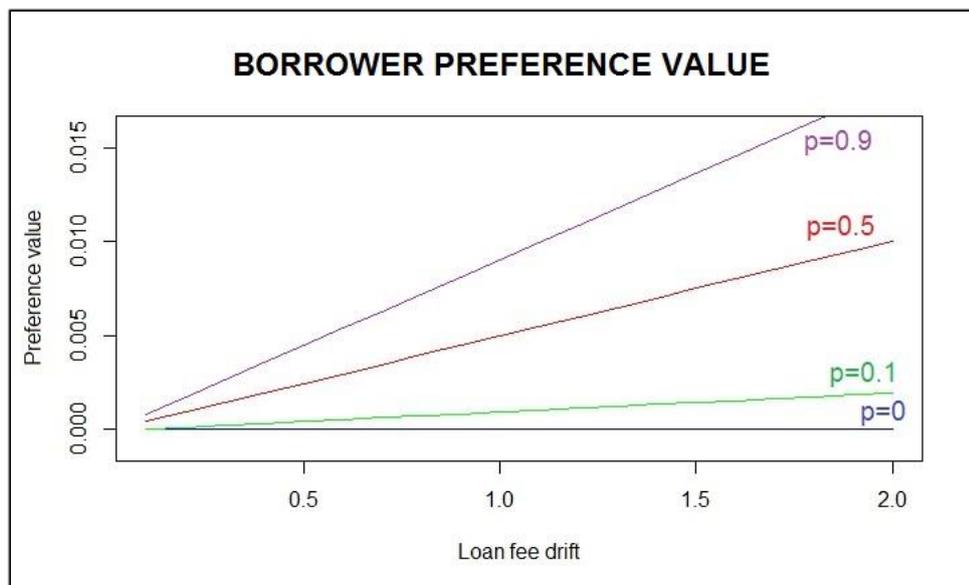
In the next graph, in order to measure the impact of loan fee stochastic share, we have a fixed loan fee drift,  $\mu = 1$ , with different stochastic shares,  $\sigma$  (0; 0.1; 0.5; 1 and 2), and recall probability varying from 1% to 99%.

**Figure 3.** Borrower's preference value: recall frequency variation, fixed loan fee drift, different loan fee shares.



Loan fee stochastic share also raises the preference value. The next one inverts the first graph, without stochastic share, by varying the loan fee drift among several recall probabilities ( $p = 0$ ;  $p = 0.1$ ;  $p = 0.5$  and  $p = 0.9$ ).

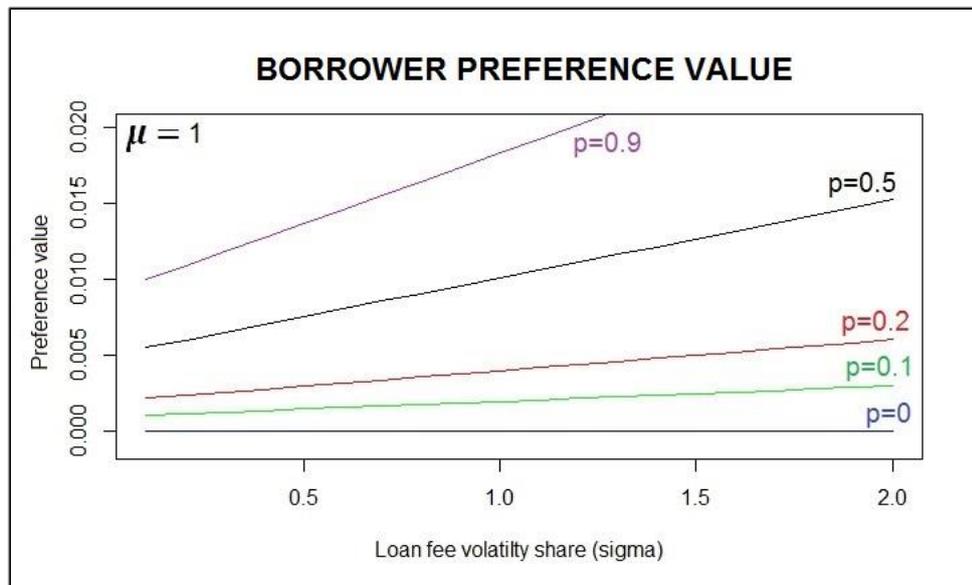
**Figure 4.** Borrower preference value: loan fee drift variation among different recall probabilities.



We can see the same parameter's impact as in the first figure.

Finally, we have set a fixed loan fee drift,  $\mu = 1$ , and varied the loan fee stochastic share among several recall event probabilities ( $p = 0$ ;  $p = 0.1$ ;  $p = 0.2$ ;  $p = 0.5$  and  $p = 0.9$ ).

**Figure 5.** Borrower preference value: stochastic share variation, fixed loan fee drift, different recall probabilities.



Unsurprisingly, we can see that the results are coherent with the previous figures.

### 3.3 Lender's Preference Value

A lender recall option may look like quite similar to a traditional American stock option, whose owner has the right to buy a certain amount of shares for a fixed price at any time until their expiry date. In the recall option, the lender is entitled to recall the loaned shares (at no cost) at any time until the loan expiry date. Therefore, we adapted a binary model, originally proposed for pricing American stock options, to obtain the lender preference value.

One of the simplest, most useful and a very popular technique for valuing an option involves constructing a binomial tree, especially if the option may be exercised at any time until the expiration date.

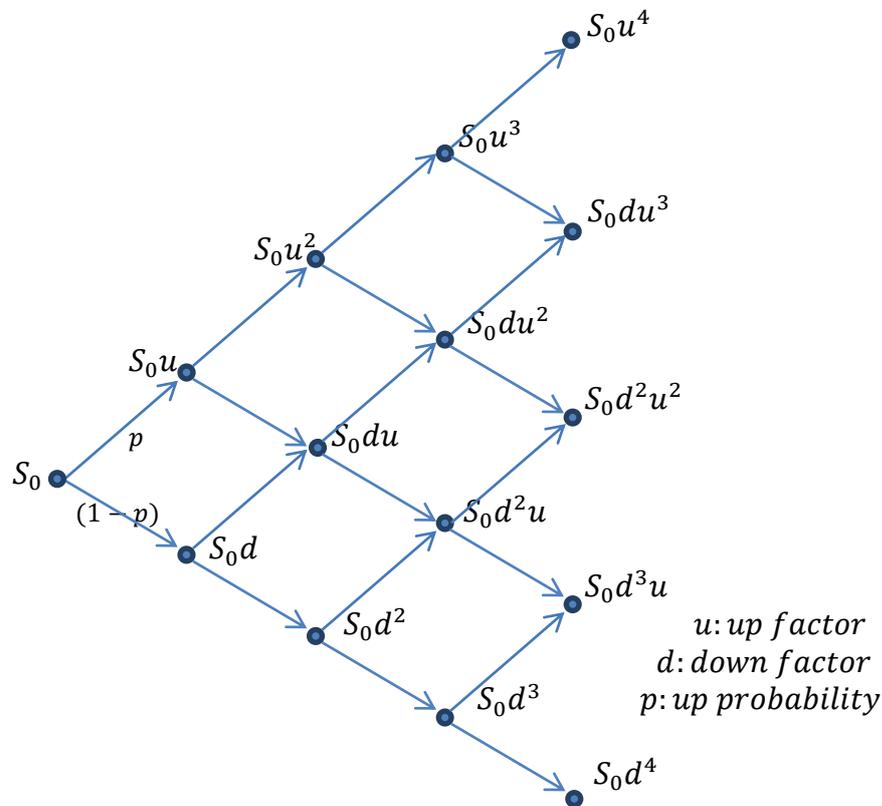
This American option pricing model proposes a binomial tree which represents different possible paths for stock price over the life of an option. Over each time step, the stock may move up with a certain probability by a certain percentage, or move down by a certain percentage with a certain probability (Hull, J., 2012).

As the time step becomes arbitrarily smaller, this model becomes time-continuous, like the Black-Scholes-Merton model. It can be shown that a European option price given by a binomial tree converges to the Black-Scholes price at the limit, and the only two assumptions required to price an option via a binomial tree are no-arbitrage and risk-neutral valuation (Cox, J.; et al., 1979).

As in the Cox model, we assume that the stock price, initially at  $S_0$ , follows a binomial process over discrete periods of time. From period  $t - 1$  to period  $t$ , the stock price may go up by a factor  $u$ , ( $u > 1$ ), with probability  $p$ , or go down by a factor  $d$ , ( $0 < d < 1$ ) with probability  $1 - p$ .

The following diagram illustrates a four steps binomial tree that represents the process followed by a stock price:

**Figure 6.** Four-step binomial tree stock price process



For solving the American option price, we proceed as follows: at the final node,  $t = 4$ , as illustrated above, American and European option prices match its prices; for a call option they value the final stock price minus the strike price, and for a put option they value the strike price minus the final stock price.

At the earlier node,  $t = 3$ , as in the example, one can measure the discounted expected future pay-off, after setting the upper node with probability  $p$  and pay-off as described above, and the lower node likewise. Moving from back-forwards we can set all expected pay-offs and compare them with the given value by immediately executing the option; the greater value establishes the option stop rule.

Our lender preference value framework establishes the following assumptions:

- i.* Stock price is driven by a binomial tree process.
- ii.* Investors (lenders) are risk-neutral.
- iii.* Lenders choose to recall a stock if they intend to sell it (otherwise they would wait longer or until the expiry date).

iv. Risk-free interest rate is known and fixed.

As mentioned earlier, the preference value is measured by the difference between lender pay-offs with and without a recall option. The latter must be equal or less than the former. The final pay-off node is set to zero, since the recall option is meaningless at the loan expiration date. Then, we move forwards and compare discounted expected pay-off with immediate pay-off, as in the American option price model. By contrast, in this framework, the lenders balance the earnings from loan fee,  $b$ , in comparison.

Pay-off without recall option:

$$\Pi_2 = S_0(e^{bT} - 1) + S_T \quad (\text{eq. 7})$$

Pay-off with recall option:

$$\Pi_{2'} = S_0(e^{b\tau} - 1) + S_\tau \quad \text{for some } 0 < \tau < T \quad (\text{eq. 8})$$

Difference between pay-offs with and without stock recalled at  $t = \tau$ :

$$\Pi_2 - \Pi_{2'} = S_0(e^{b\tau} - 1) + S_\tau - [S_0(e^{bT} - 1) + S_T] = S_0(e^{b\tau} - e^{bT}) + S_\tau - S_T$$

At each node, the lender compares today's with tomorrow's pay-offs:

$$1. \text{ Today: } \text{Payoff}_{f_{t=\tau}} = S_0(e^{b\tau} - e^{bT}) + S_\tau - e^{-r(T-\tau)} E_{t=\tau}[S_T] \quad (\text{eq. 9})$$

$$2. \text{ Tomorrow: } \text{Payoff}_{f_{t=\tau+1}} = e^{-r\Delta T} [p \cdot \text{payoff}_{f_p} + (1-p) \cdot \text{payoff}_{f_{1-p}}] \quad (\text{eq. 10})$$

For that node, we set the lender preference value as the maximum pay-off chosen from the greater value above, restricted to positive values, and move back until  $t = 0$ .

Model variables:

$S_0$ : Initial stock price.  $S_T$ : Final stock price.  $T$ : Loan number of periods.

$S_\tau$ : Stock price at  $t = \tau$ ,  $0 \leq \tau \leq T$ .

$b$ : Loan fee.  $r$ : risk-free interest rate. Both as continuously compounded.

$\text{payoff}_{f_p}$ : Future pay-off at upper node.

$\text{payoff}_{f_{1-p}}$ : Future pay-off at lower node.

$\Delta T$ : time taken at each step (measured in years, 1 business day =  $\frac{1}{252}$  year).

$E_{t=\tau}[S_T]$ : is measured according to the binomial tree probabilities and factors.

### 3.4 Lender's Preference Value Simulation

We have simulated the framework above by a numeric exercise after adopting special values for some parameters, and varying other parameters through a large range of values. Some parameters were fixed or normalised as follows:  $S_0 = 1$  (therefore, the preference value relative to the stock price is also normalised);  $T = 21 + 4 = 25$ , meaning the standard loan period. Although not recorded in our dataset, the loan period standardized by the Brazilian Stock Exchange is 34 days: one month (~21 business days) plus 4 days, which is the time length to re-loan the shares and keep the parties involved; and  $\Delta T = 1 \text{ business day} = \frac{1}{252} \text{ years}$ . Finally, the parameters  $p, u, d$  and  $r$  were allowed to vary in order to measure their impacts.

The preference value was set as the option value at time  $t = 0$ , after proceeding back-forwardly in a similar way as for the American option using the binomial tree model, as explained above.

Stock price volatility,  $\sigma$ , is a secondary parameter that enters the model in the following analysis.

The first set of parameters was chosen to obtain zero-expected net return, in other words, the asset returns, in probability, the same return as the risk-free interest rate.

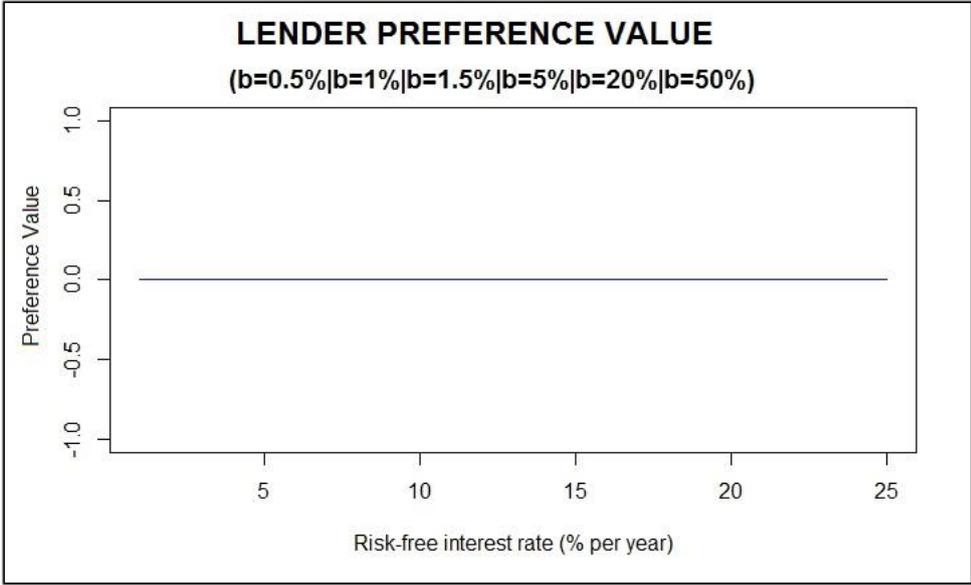
To match volatility, we set  $u = e^{\sigma\sqrt{\Delta T}}$  and  $d = e^{-\sigma\sqrt{\Delta T}}$ .

And to match the stock expected return and risk-free rate, we set  $p = \frac{e^{r\Delta T} - d}{u - d}$ , which comes from a risk-neutral expectation.

We then studied the impact of varying the variables risk-free interest tax, loan fee and asset volatility.

The next graph shows the risk-free interest tax variation between different loan fees, for zero expected net return.

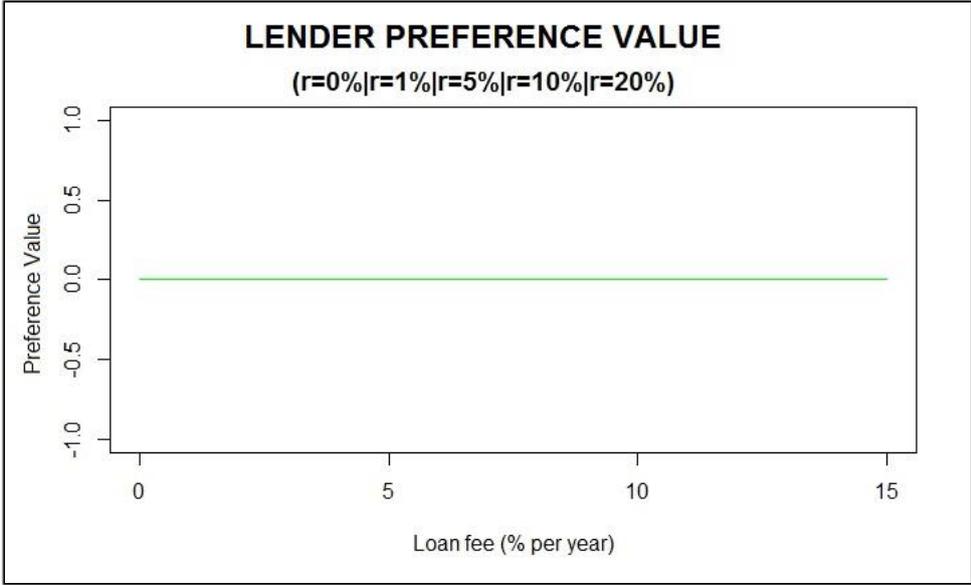
**Figure 7.** Lender's preference value: expected return, risk-free rate variation among several loan fees.



Both loan fee and risk-free rate, in this case, have no impacts on the preference value.

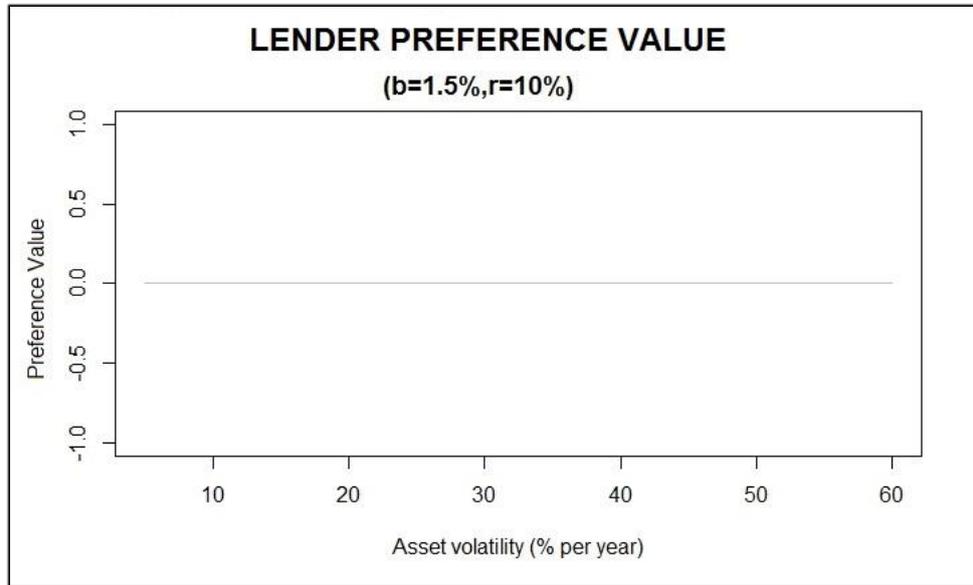
The next one varies the loan fee between different risk-free interest rates.

**Figure 8.** Lender's preference value: expected return, loan fee variation among several risk-free interest rates.



Again, we notice no parameter's impact. The last one varies stock price volatility within a set of risk-free interest rates and loan fees, but we obtained the same graph for any value of loan fee and risk-free interest rate combination.

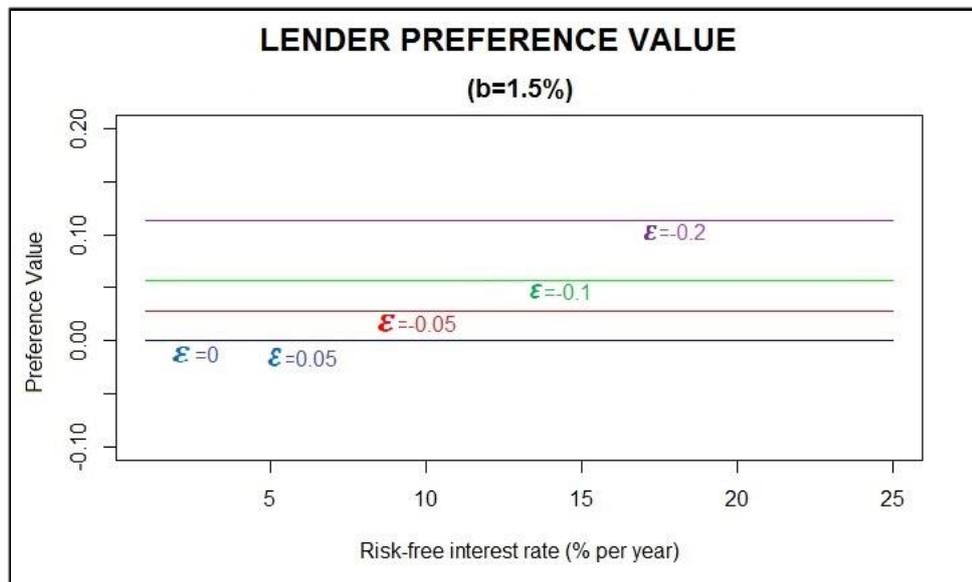
**Figure 9:** Lender's preference value: expected return, volatility variation



Since we have again found no response, we altered parameter  $p$ , leading the stock price process probability to move up or move down from zero-expected net return. We set  $p$  by adding a new parameter,  $\varepsilon$ , and varied the risk-free interest rate among different  $\varepsilon$  values (-0.2; -0.1; -0.05; 0 and +0.05), which increases or reduces the stock expected return as follows:

$$p = \frac{e^{r\Delta T} - d}{u - d} + \varepsilon \quad (\text{eq. 11})$$

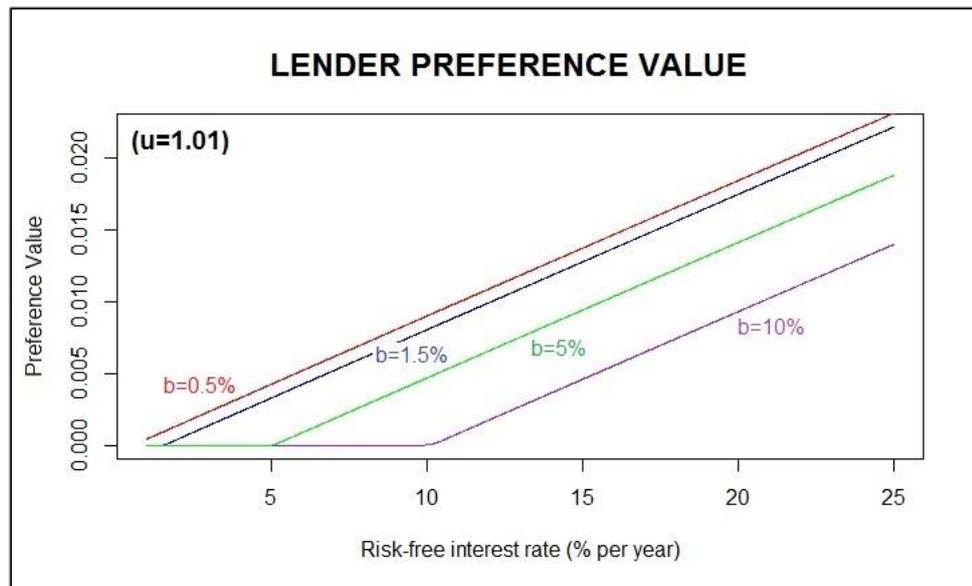
**Figure 10.** Lender's preference value: multiple expected returns, risk-free rate variation



In this case, the preference value responds to the stock expected returns, reaching positive values only for negative expected returns (as for the risk-free interest rate); the lower the expected return, the higher the preference value. Therefore, for every interest rate  $r$  we adjust the probability,  $p = \frac{e^{r\Delta T} - d}{u - d}$ , regardless of  $\varepsilon$  value, and the preference value does not change with risk-free interest rate variation, loan fee or stock volatility for similar reasons.

In the next set of simulations, we propose parameter combinations so that stock has zero-expected rough return relative to the risk-free interest rate. In the next graph we set the following combinations:  $u = 1.01$ ;  $d = 0.99$   $p = 0.5$ , with risk-free interest rate varying among different loan fees ( $b = 0.5\%$ ;  $b = 1.5\%$ ;  $b = 5\%$  and  $b = 10\%$ ).

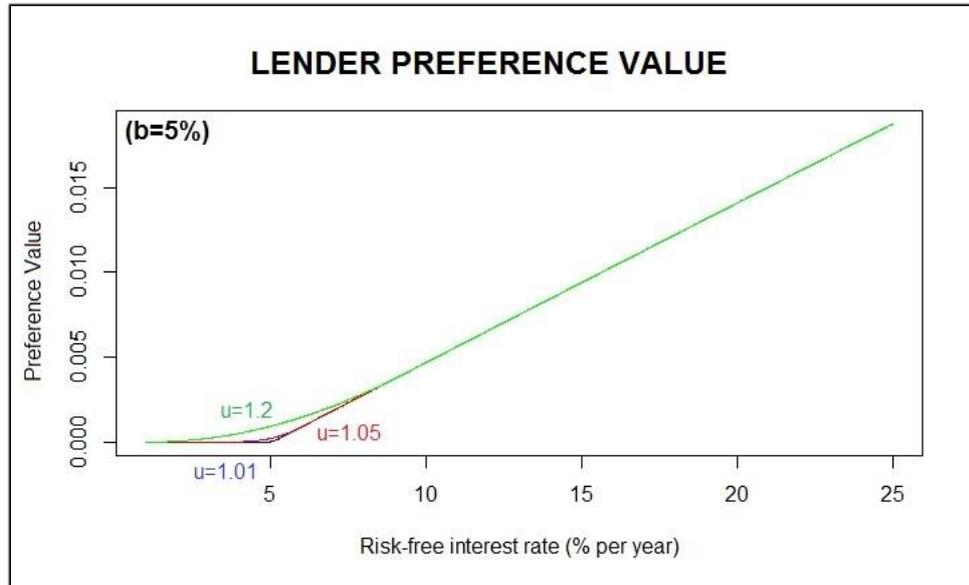
**Figure 11.** Lender's preference value: negative expected return, risk-free rate variation



We notice that, once the stock has a negative expected return, the preference value rises with the risk-free interest rate and lowers with the loan fee.

The following graph shows risk-free rate variation for a fixed loan fee ( $b = 5\%$ ) among different up-factors ( $u = 1.01$ ;  $u = 1.05$  and  $u = 1.2$ ), keeping a zero-expected rough return by setting  $u + d = 2(u > 0, d > 0)$  and  $p = 0.5$ .

**Figure 12.** Lender's preference value: negative expected return, risk-free rate variation



The up-factor increases the stock volatility and once the stock has a negative expected return for the same loan fee and risk-free rate, we have a slightly larger preference value as the volatility rises.

### 3.5 Equilibrium Model

Once we have obtained lender and borrower preference values – the reasonable value each investor believes it is worth the recall option (i.e. taking into account that the recall event either reduces or increases the risk), we propose an equilibrium model to price the recall option seen on our empirical results, that is, its market value.

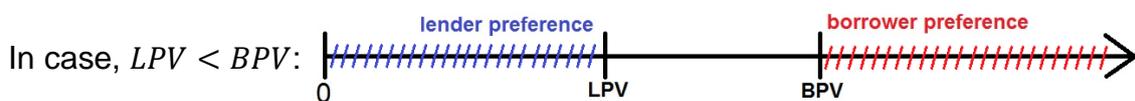
To begin with, let us think hypothetically that the recall option is set by a regulator authority. For example, the recall option is fixed at 30% of the loan fee freely agreed by lender and borrower for a loan contract without recall option. Therefore, for a 10% loan fee without recall option, the same contract would worth 7% with it.

The lender sees the above recall price (30%) as a discount to be given at the loan fee (reward) in order to settle a loan contract with recall option, thus the lender would wish the cheaper price as possible.

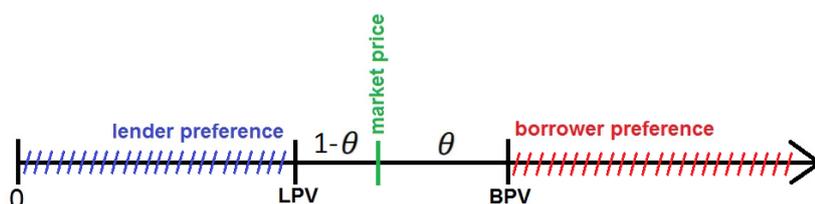
On the other hand, the borrower sees such a price as a bonus to be received, which lowers his loan cost, and thus the borrower would wish the highest price as possible.

Therefore, the preference value represents a preference range (a set): the lender accepts any value between zero and LPV to impose the recall option, whereas the borrower accepts any value between BPV and  $+\infty$ , which is bounded by the loan fee itself,  $[BPV, Loan\ Fee_{without\ recall}]$ .

We have then three possibilities:  $LPV < BPV$  or  $LPV > BPV$  or equality.

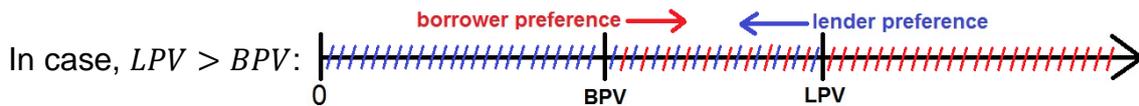


From this condition on, lenders and borrowers will establish an equilibrium point to set the market price. The recall price will be as close as possible to LPV or as close as possible to BPV, depending on each investor's market power.

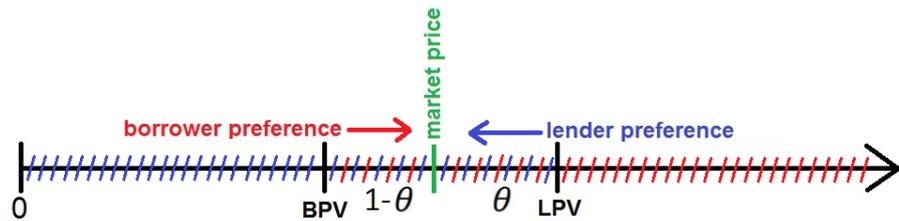


Here,  $\theta$  represents the lender's monopoly power,  $0 \leq \theta \leq 1$ , indicating that the closer to 1 the higher the monopoly market. We could propose  $\theta$  as a function from the lender market concentration index,  $\theta = \text{function}(HHI)$ . Therefore, the recall price would be:

$$\text{Recall Price} = LPV + (BPV - LPV)(1 - \theta) = \theta \cdot LPV + (1 - \theta) \cdot BPV \quad (\text{eq. 12})$$



In this case, the preference sets overlap each other as any value within the range  $[BPV, LPV]$  would be accepted by both investors. Knowing that, each investor will offer a more advantageous value which still would be within the other's preference range. This recursive exercise would repeat itself until an equilibrium is reached according to each investor's market power.



Where  $0 \leq \theta \leq 1$  is also the lender monopoly power. Therefore the recall price would be:

$$\text{Recall Price} = BPV + (LPV - BPV)(1 - \theta) = \theta \cdot BPV + (1 - \theta) \cdot LPV \quad (\text{eq. 13})$$

When  $\theta = 1$ , where the lender has total monopoly power, the recall price would be the minimum value between  $LPV$  and  $BPV$ . When the borrower has total monopoly, the price would be the highest one.

## 4. EMPIRICAL ANALYSIS

### 4.1 Initial Results

The first empirical analysis performed aimed to study the recall option impacts on the loan fee. We initiated by producing graphs to show the average loan fee in a given period (whether a day, week or month), separated by loan contracts with and without the recall option. Even after weighting the loan fee according to quantity or volume, the average fee for contracts with recall option tended to be higher than those without, which contradicts our prediction. We, therefore, restricted our graph to only one ticker, and after performing it for the 20 most liquid tickers, one by one, we found almost the same pattern, excluding the cross-sectional variance from our rough analysis.

Next, we performed several regression analyses using the recall option as a dummy variable, assuming values one and zero for deals with and without recall option, respectively. All the result coefficients were positive (meaning a higher loan fee for callable stock), thus reinforcing the conclusion drawn in the graphs. We then restricted the regressions to loan fee percentiles, obtaining either the same or even worthless results for high and low quartiles of loan fees.

By this point we were quite perplexed with the results, and we decided to ask a few routine questions to market dealers in order to see how the Brazilian loan market functions on the trade floor.

One mutual fund operator told us that, as a frequent short-term seller, he does not mind having to pay any loan fee as long as it seems reasonable or even if the loan contract is callable. However, the recall option becomes of concern when the loan supply shrinks, or in other words, when it is hard to find brokers with stock available for lending.

When liquidity is not sufficient for re-loans (after recall), the recall option may represent a great risk to borrowers, and therefore, these investors should demand a reduced loan fee when the loan contract offers the lender a recall option.

## 4.2 Market Concentration Index

In order to present a scarcity situation for borrowers in which a particular stock is hard to find at a reasonable loan price, we have proposed a market concentration index serving as a proxy for search cost, which has been widely discussed in the recent literature.

A common and simple accepted measure for market concentration is the Herfindahl-Hirschman Index.

This index is obtained by summing up the squares of individual market shares of each firm in the industry. The result is a value between zero and one. The closer a market is to being a monopoly, the higher the market's concentration (and the lower its competition). If, for example, there is only one firm in the industry, this firm would have 100% of the market share ( $s_j = 1$ ), and the HHI value would be 1, indicating a monopoly (it is also common to consider each market share as a percentage, with HHI values ranging from 0 to 10.000). If there are thousands of firms competing, each one would have nearly 0% of the market share, and the HHI would be close to zero, indicating almost perfect competition.

The U.S. Department of Justice considers a market with a result of less than 0.1 to be competitive, whereas that with 0.18 or greater to be a highly concentrated market. As a general rule, mergers and acquisitions increasing the HHI by more than 0.01 point in concentrated markets raise anti-trust concerns.

In order to measure the scarcity in the loan market, we have proposed the use of the Herfindahl-Hirschman Index, as follows:

$$HHI_{s,t} = \sum_{j=1}^J s_{j,s,t}^2$$

$HHI_{s,t}$ : Herfindahl-Hirschman index for stock (ticker)  $s$  at time  $t$  (month or week).

$s_{j,s,t}$ : market share of lender (or broker)  $j$  for stock (ticker)  $s$  at a given time  $t$  (month or week) in terms of quantity (proportion of the whole quantity).

$J$ : number of lenders (or brokers) offering (closed offers considered only) stock (ticker)  $s$  during a period of time  $t$  (month or week).

We have constructed H-H indexes for closed offers from brokers and lenders, both on a weekly and monthly basis, by using different frequencies filters, that is, the first 55, 100 and 150 most frequent tickers (included).

The next table summarises some basic statistics for the four different indexes constructed, considering three different liquidity cuts (55, 100, 150).

**Table 3.** Herfindahl-Hirschman index cross-sectional descriptive statistics in terms of the 55 most liquid tickers.

HHI	Lender	Broker	Lender	Broker
Percentile	Monthly	Monthly	Weekly	Weekly
10 <sup>th</sup>	0.041687	0.075485	0.064310	0.102847
25 <sup>th</sup>	0.058886	0.089463	0.090501	0.128861
median	0.084282	0.113261	0.135220	0.172815
75 <sup>th</sup>	0.121078	0.149118	0.209019	0.248695
90 <sup>th</sup>	0.180436	0.205300	0.317508	0.357245
Mean	0.102882	0.133522	0.172083	0.211753
S.D.	0.079094	0.082083	0.131264	0.134007

**Table 4.** Herfindahl-Hirschman index cross-sectional descriptive statistics in terms of the 100 most liquid tickers.

HHI	Lender	Broker	Lender	Broker
Percentile	Monthly	Monthly	Weekly	Weekly
10 <sup>th</sup>	0.048958	0.079199	0.074145	0.111299
25 <sup>th</sup>	0.067577	0.097466	0.106810	0.146053
median	0.099186	0.129564	0.168184	0.208466
75 <sup>th</sup>	0.153934	0.185676	0.279731	0.331203
90 <sup>th</sup>	0.248476	0.310038	0.463447	0.537489
Mean	0.135553	0.172499	0.231329	0.277809
S.D.	0.124850	0.137611	0.194928	0.202501

**Table 5.** Herfindahl-Hirschman index cross-sectional descriptive statistics in terms of the 150 most liquid tickers.

HHI	Lender	Broker	Lender	Broker
Percentile	Monthly	Monthly	Weekly	Weekly
10 <sup>th</sup>	0.054972	0.084934	0.083173	0.120673
25 <sup>th</sup>	0.080596	0.109910	0.127213	0.166270
median	0.131263	0.163832	0.223438	0.270169
75 <sup>th</sup>	0.247591	0.298822	0.434279	0.500370
90 <sup>th</sup>	0.443659	0.522599	0.792388	0.880664
Mean	0.202304	0.244205	0.324269	0.371937
S.D.	0.194574	0.206787	0.268743	0.271288

The statistics above reveal several features of the HHI distributions:

- All distributions seem to be asymmetric on the left (mean > median).
- Broker deals are more concentrated than the lender ones.
- Weekly aggregations are more concentrated than the monthly ones.
- Bottom (mostly illiquid) shares are more concentrated than the more liquid ones for borrowing.

### 4.3 Regression Analysis

Once we assembled the four HHI variables, we proposed a way of capturing the interaction between recall option and HHI variables and their loan fee impact. A simple and effective way to do so is to combine the variables by multiplying them. The recall option has value one when it is recallable, whereas HHI has value close to one when the market is highly concentrated. Therefore, the interaction within recallable loans increases as the market concentrates and the option becomes of concern, requiring intuitive direction.

As the HHI concentration indexes are measured for a given period of time (month or week), an observation at the beginning of that period would capture future events via contemporaneous H-H index. Despite this, investors are only able to study past market concentration, which affects the current search cost for a particular stock.

The liquidity filter separates the 55 most frequent tickers accounting for 744,922 observations for monthly aggregation and 761,958 for weekly aggregation on a deal-by-deal basis (i.e. 100 tickers: 904,750 observations for monthly aggregation, and 926,666 for weekly aggregation; 150 tickers: 945,769 and 968,922 observations, respectively). The following equation was performed by using the four HHI variables constructed (monthly and weekly, lender and broker).

The coefficients were estimated by using ordinary least square and its variance-covariance matrix by robust estimation in order to allow covariance between independent variables, as follows:

$$\widehat{V}_{rob} = \widehat{V} \left( \sum_{j=1}^N \hat{u}_j' \hat{u}_j \right) \widehat{V}$$

$\widehat{V}$ : estimated ordinary variance-covariance matrix.

$\hat{u}_j$ : estimated row  $j$  of regression deviation.

Equation 14

$$\begin{aligned}
 Fee_{i,s,t_{day},obs} = & \alpha + \beta_1.HHI_{s,t-1} + \beta_2.(HHI_{s,t-1} . Recall\_dummy_{i,s,t_{day},obs}) \\
 & + \beta_3.Recall\_dummy_{i,s,t_{day},obs} + \beta_4.Quantity_{i,s,t_{day},obs} + FE_{day} + FE_{firm} \\
 & + \varepsilon_{i,s,t_{day},obs}
 \end{aligned}$$

$Fee_{i,s,t_{day},obs}$ : loan fee asked by lender  $i$ , or through broker  $i$ , for lending stock (ticker)  $s$ , on business day  $t_{day}$ , within deal listed by observation  $obs$  (offer code), which identifies all deals in our data set.

$HHI_{s,t-1}$ : Herfindahl-Hirschman index calculated for stock (ticker)  $s$ , at the previous period,  $t - 1$  (month or week).

$Recall\_dummy_{i,s,t_{day},obs}$ : recall option dummy (= 1, if recallable, = 0, if not); for lender  $i$  offer, or through broker  $i$ , for stock (ticker)  $s$ , on business day  $t_{day}$ , within deal listed by observation  $obs$  (offer code).

$Quantity_{i,s,t_{day},obs}$ : Loan quantity (number of shares), for lender  $i$  offer, or through broker  $i$ , for stock (ticker)  $s$ , on business day  $t_{day}$ , within deal listed by observation  $obs$  (offer code).

$FE_{day}$ : Fixed effect for every business day on data set.

$FE_{firm}$ : Fixed effect for every stock (ticker) on data cut-off (55, 100 or 150).

$\varepsilon_{i,s,t_{day},obs}$ : regression error.

The following table shows coefficient estimation followed by its t-statistic for each different HHI, as well as the regression constant, with adjusted  $R^2$  at the bottom.

**Table 6.** Regression results for Equation 14 in terms of the 55 most liquid stocks.

	Lender Monthly		Broker Monthly		Lender Weekly		Broker Weekly	
Lagged HHI ( $\beta_1$ )	-3.469	-51.000	-1.588	-29.361	-0.658	-19.874	-0.717	-23.853
Lagged HHI*Recall ( $\beta_2$ )	-3.562	-26.185	-3.481	-28.566	-0.844	-10.677	-1.261	-16.683
Recall dummy ( $\beta_3$ )	0.488	33.825	0.580	34.485	0.297	22.530	0.400	25.787
Quantity ( $\beta_4$ )	8.25.10 <sup>-8</sup>	3.525	7.40.10 <sup>-8</sup>	3.293	6.99.10 <sup>-8</sup>	3.173	6.91.10 <sup>-8</sup>	3.148
Constant ( $\alpha$ )	2.184	40.136	2.004	37.229	1.947	36.076	1.958	36.128
Fixed Effect <sub>firm</sub>	YES		YES		YES		YES	
Fixed Effect <sub>day</sub>	YES		YES		YES		YES	
R <sup>2</sup> (adj.)	0.493988		0.491534		0.491187		0.491607	

Obs.: Coefficient estimations on the left and t-statistics on the right within each cell.

Every coefficient turned out to be significant. One may notice that only the recall option points to a coefficient with counter-intuitive direction (positive value), whereas the interaction points to a direction intuitively (negative).

Intuition would suggest that large deals (i.e. involving a large number of shares loaned) come from big investors with extensive market connections and great bargaining power. In the case of short-sellers, these investors may bargain for a reduced loan fee, but in the case of lenders, such investors may bargain for a higher loan fee. This thinking leads to a balance of power within the market. Although significant, the quantity coefficient shows that, on average, quantity barely changes loan fees for contracts lower than 1,000,000 shares, although it does indicate that lenders might have a greater bargaining power than borrowers due to the positive sign.

The lagged HHI alone seems to be counter-intuitive (due to their negative coefficients), indicating that past market concentration accounts for a loan fee reduction. Therefore, we proposed a loan fee dynamics aimed to capture the loan fee increasing. Rapid HHI growth may give investors the impression that the market will soon become scarce, generating concern around recall probability and increasing loan fees. Therefore, we constructed the HHI percentage growth as follows:

$$\Delta\% HHI = \frac{HHI_t - HHI_{t-1}}{HHI_{t-1}}$$

By substituting the lagged HHI with its percentage growth, we formulated the following equation:

Equation 15:

$$\begin{aligned}
Fee_{i,s,t_{day},obs} = & \alpha + \beta_1 \cdot \left( \frac{HHI_{s,t} - HHI_{s,t-1}}{HHI_{s,t-1}} \right) + \beta_2 \cdot (HHI_{s,t-1} \cdot Recall\_dummy_{i,s,t_{day},obs}) \\
& + \beta_3 \cdot Recall\_dummy_{i,s,t_{day},obs} + \beta_4 \cdot Quantity_{i,s,t_{day},obs} + FE_{day} + FE_{firm} \\
& + \varepsilon_{i,s,t_{day},obs}
\end{aligned}$$

We regressed again for the each four HHI variables (monthly and weekly, lender and broker) and obtained the coefficients and t-static estimations shown in the table below:

**Table 7.** Regression results for Equation 15 in terms of the 55 most liquid stocks.

	Lender Monthly		Broker Monthly		Lender Weekly		Broker Weekly	
$HHI_t - HHI_{t-1}/HHI_{t-1} (\beta_1)$	0.370	67.581	0.400	60.237	0.110	27.688	0.194	30.459
Lagged HHI*Recall ( $\beta_2$ )	-4.649	-36.755	-3.401	-29.648	-1.057	-14.173	-1.358	-18.793
Recall dummy ( $\beta_3$ )	0.578	42.181	0.572	35.711	0.325	25.729	0.417	28.075
Quantity ( $\beta_4$ )	4.02.10 <sup>-8</sup>	1.900	4.42.10 <sup>-8</sup>	2.149	4.85.10 <sup>-8</sup>	2.286	3.96.10 <sup>-8</sup>	1.937
Constant ( $\alpha$ )	1.902	35.297	1.795	33.515	1.843	34.384	1.819	33.648
Fixed Effect <sub>firm</sub>	YES		YES		YES		YES	
Fixed Effect <sub>day</sub>	YES		YES		YES		YES	
R <sup>2</sup> (adj.)	0.496795		0.493939		0.491737		0.492630	

Obs.: Coefficient estimations on the left and t-statistics on the right within each cell.

The interactive variable remains significant and intuitive (negative sign). In both equations, the absolute value of the interactive variable coefficient exceeds the coefficient of the recall dummy alone, indicating that recallable loans do indeed accept reduced loan fees. The overall sum demonstrates that the recall option costs from 0 to 4% a year, on average, assuming that the market is highly concentrated. This conclusion may be questioned if one multiplies the interaction variable coefficients by the mean (or median) HHI values. Nonetheless the recall event impacts the loan fee at higher HHI percentiles, probably above the 90<sup>th</sup> percentile, where the coefficient sums are still negative.

Equation 15 points out that such a growth is in line with our previous prediction, showing that as market concentration rises, so does the loan fee.

In order to confirm our results, we discuss several issues concerning robustness, as outlined in the next section.

## 5. Robustness

Our first concern about robustness refers to the frequency filter, and thus we applied the 100 and 150 most frequent tickers to Equations 14 and 15, respectively.

Equation 14:

$$\begin{aligned}
 Fee_{i,s,t_{day},obs} = & \alpha + \beta_1 \cdot HHI_{s,t-1} + \beta_2 \cdot (HHI_{s,t-1} \cdot Recall\_dummy_{i,s,t_{day},obs}) \\
 & + \beta_3 \cdot Recall\_dummy_{i,s,t_{day},obs} + \beta_4 \cdot Quantity_{i,s,t_{day},obs} + FE_{day} + FE_{firm} \\
 & + \varepsilon_{i,s,t_{day},obs}
 \end{aligned}$$

**Table 8.** Regression results for Equation 14 in terms of the 100 most liquid stocks.

	Lender Monthly		Broker Monthly		Lender Weekly		Broker Weekly	
Lagged HHI ( $\beta_1$ )	-2.361	-42.038	-0.763	-16.983	-0.513	-20.334	-0.516	-22.066
Lagged HHI*Recall ( $\beta_2$ )	-1.779	-17.004	-1.279	-13.208	-0.490	-9.346	-0.707	-14.135
Recall dummy ( $\beta_3$ )	0.347	27.819	0.335	23.167	0.253	23.304	0.313	25.188
Quantity ( $\beta_4$ )	1.16.10 <sup>-7</sup>	4.236	1.11.10 <sup>-7</sup>	4.123	1.14.10 <sup>-7</sup>	4.215	1.13.10 <sup>-7</sup>	4.189
Constant ( $\alpha$ )	2.924	41.510	2.709	38.684	2.713	38.904	2.731	39.154
Fixed Effect <sub>firm</sub>	YES		YES		YES		YES	
Fixed Effect <sub>day</sub>	YES		YES		YES		YES	
R <sup>2</sup> (adj.)	0.504458		0.502673		0.502948		0.503114	

Obs.: Coefficient estimations on the left and t-statistics on the right within each cell.

**Table 9.** Regression results for Equation 14, in terms of the 150 most liquid stocks.

	Lender Monthly		Broker Monthly		Lender Weekly		Broker Weekly	
Lagged HHI ( $\beta_1$ )	-1.935	-41.675	-0.880	-22.779	-0.580	-24.924	-0.595	-27.269
Lagged HHI*Recall ( $\beta_2$ )	-0.755	-9.741	-0.728	-10.145	-0.212	-4.800	-0.380	-9.053
Recall dummy ( $\beta_3$ )	0.259	24.102	0.277	22.747	0.218	21.286	0.259	22.659
Quantity ( $\beta_4$ )	1.25.10 <sup>-7</sup>	4.201	1.21.10 <sup>-7</sup>	4.117	1.22.10 <sup>-7</sup>	4.148	1.22.10 <sup>-7</sup>	4.145
Constant ( $\alpha$ )	2.867	41.445	2.733	39.700	2.726	39.738	2.751	40.106
Fixed Effect <sub>firm</sub>	YES		YES		YES		YES	
Fixed Effect <sub>day</sub>	YES		YES		YES		YES	
R <sup>2</sup> (adj.)	0.509169		0.508082		0.508447		0.508588	

Obs.: Coefficient estimations on the left and t-statistics on the right within each cell.

We observe the same results for Equation 14 applied within the 55 most frequent tickers. The interactive variable remains significant and has a negative sign. We may notice, however, that its absolute value has indeed decreased and together with the recall dummy coefficient, the overall sum remains negative or slightly negative.

Equation 15:

$$\begin{aligned}
Fee_{i,s,t_{day},obs} = & \alpha + \beta_1 \cdot \left( \frac{HHI_{s,t} - HHI_{s,t-1}}{HHI_{s,t-1}} \right) + \beta_2 \cdot (HHI_{s,t-1} \cdot Recall\_dummy_{i,s,t_{day},obs}) \\
& + \beta_3 \cdot Recall\_dummy_{i,s,t_{day},obs} + \beta_4 \cdot Quantity_{i,s,t_{day},obs} + FE_{day} + FE_{firm} \\
& + \varepsilon_{i,s,t_{day},obs}
\end{aligned}$$

**Table 10.** Regression results for Equation 15 in terms of the 100 most liquid stocks.

	Lender Monthly		Broker Monthly		Lender Weekly		Broker Weekly	
$HHI_t - HHI_{t-1}/HHI_{t-1} (\beta_1)$	0.343	72.304	0.386	66.196	0.101	28.379	0.182	33.110
Lagged HHI*Recall ( $\beta_2$ )	-2.335	-24.448	-0.912	-10.021	-0.645	-13.035	-0.731	-15.215
Recall dummy ( $\beta_3$ )	0.394	33.678	0.290	21.115	0.276	26.412	0.318	26.619
Quantity ( $\beta_4$ )	7.43.10 <sup>-8</sup>	2.945	8.29.10 <sup>-8</sup>	3.309	9.50.10 <sup>-8</sup>	3.630	8.57.10 <sup>-8</sup>	3.394
Constant ( $\alpha$ )	2.585	36.652	2.524	36.055	2.576	37.072	2.576	36.965
Fixed Effect <sub>firm</sub>	YES		YES		YES		YES	
Fixed Effect <sub>day</sub>	YES		YES		YES		YES	
R <sup>2</sup> (adj.)	0.507524		0.505258		0.503386		0.50405	

Obs.: Coefficient estimations on the left and t-statistics on the right within each cell.

**Table 11.** Regression results for Equation 15 in terms of the 150 most liquid stocks.

	Lender Monthly		Broker Monthly		Lender Weekly		Broker Weekly	
$HHI_t - HHI_{t-1}/HHI_{t-1} (\beta_1)$	0.322	71.762	0.364	65.867	0.098	28.375	0.179	33.535
Lagged HHI*Recall ( $\beta_2$ )	-1.145	-16.094	-0.585	-8.617	-0.421	-9.935	-0.480	-11.841
Recall dummy ( $\beta_3$ )	0.295	28.962	0.259	22.273	0.252	25.405	0.282	25.453
Quantity ( $\beta_4$ )	8.41.10 <sup>-8</sup>	2.998	9.27.10 <sup>-8</sup>	3.344	1.03.10 <sup>-7</sup>	3.597	9.45.10 <sup>-8</sup>	3.388
Constant ( $\alpha$ )	2.591	37.343	2.538	36.806	2.575	37.603	2.576	37.509
Fixed Effect <sub>firm</sub>	YES		YES		YES		YES	
Fixed Effect <sub>day</sub>	YES		YES		YES		YES	
R <sup>2</sup> (adj.)	0.511915		0.510311		0.508737		0.509347	

Obs.: Coefficient estimations on the left and t-statistics on the right within each cell.

Again, we can see the same conclusion as in Equation 14, after comparing the results for Equation 15 applied within the 55 most frequent tickers. The interactive variable remains significant and negative. Its absolute value has also decreased, but it surpasses the recall dummy coefficient.

Our second concern about robustness refers to the loan amount (number of shares), that may impact on our main results. Therefore, we omitted the quantity in order to check its impact on the results, with the other variables remaining exactly the same:

Equation 16:

$$Fee_{i,s,t_{day},obs} = \alpha + \beta_1 \cdot HHI_{s,t-1} + \beta_2 \cdot (HHI_{s,t-1} \cdot Recall\_dummy_{i,s,t_{day},obs}) + \beta_3 \cdot Recall\_dummy_{i,s,t_{day},obs} + FE_{day} + FE_{firm} + \varepsilon_{i,s,t_{day},obs}$$

The results are shown in the following three tables:

**Table 12.** Regression results for Equation 16 in terms of the 55 most liquid stocks.

	Lender Monthly		Broker Monthly		Lender Weekly		Broker Weekly	
Lagged HHI ( $\beta_1$ )	-3.467	-50.985	-1.584	-29.311	-0.657	-19.859	-0.716	-23.833
Lagged HHI*Recall ( $\beta_2$ )	-3.559	-26.165	-3.483	-28.590	-0.842	-10.657	-1.261	-16.678
Recall dummy ( $\beta_3$ )	0.490	34.018	0.582	34.681	0.298	22.684	0.402	25.928
Constant ( $\alpha$ )	2.183	40.122	2.004	37.212	1.947	36.065	1.957	36.117
Fixed Effect <sub>firm</sub>	YES		YES		YES		YES	
Fixed Effect <sub>day</sub>	YES		YES		YES		YES	
R <sup>2</sup> (adj.)	0.493975		0.491525		0.491178		0.491599	

Obs.: Coefficient estimations on the left and t-statistics on the right within each cell.

**Table 13.** Regression results for Equation 16 in terms of the 100 most liquid stocks.

	Lender Monthly		Broker Monthly		Lender Weekly		Broker Weekly	
Lagged HHI ( $\beta_1$ )	-2.359	-42.004	-0.760	-16.909	-0.512	-20.289	-0.515	-22.016
Lagged HHI*Recall ( $\beta_2$ )	-1.780	-17.014	-1.284	-13.259	-0.489	-9.345	-0.708	-14.157
Recall dummy ( $\beta_3$ )	0.350	28.120	0.339	23.446	0.256	23.616	0.316	25.480
Constant ( $\alpha$ )	2.924	41.505	2.709	38.675	2.712	38.899	2.731	39.148
Fixed Effect <sub>firm</sub>	YES		YES		YES		YES	
Fixed Effect <sub>day</sub>	YES		YES		YES		YES	
R <sup>2</sup> (adj.)	0.504435		0.502653		0.502927		0.503093	

Obs.: Coefficient estimations on the left and t-statistics on the right within each cell.

**Table 14.** Regression results for Equation 16 in terms of the 150 most liquid stocks.

	Lender Monthly		Broker Monthly		Lender Weekly		Broker Weekly	
Lagged HHI ( $\beta_1$ )	-1.932	-41.631	-0.876	-22.702	-0.578	-24.875	-0.593	-27.214
Lagged HHI*Recall ( $\beta_2$ )	-0.757	-9.764	-0.732	-10.210	-0.213	-4.818	-0.381	-9.088
Recall dummy ( $\beta_3$ )	0.262	24.478	0.280	23.108	0.221	21.652	0.262	23.002
Constant ( $\alpha$ )	2.867	41.440	2.732	39.691	2.725	39.733	2.750	40.099
Fixed Effect <sub>firm</sub>	YES		YES		YES		YES	
Fixed Effect <sub>day</sub>	YES		YES		YES		YES	
R <sup>2</sup> (adj.)	0.509143		0.508058		0.508422		0.508564	

Obs.: Coefficient estimations on the left and t-statistics on the right within each cell.

The coefficient results are practically the same, indicating that although quantity does affect the loan fee (demonstrated by its coefficient), it does not interfere with the other impacts.

Finally, as each ticker may play a key role due to robustness, we omitted the ticker fixed effect and adjusted the dependent variable (i.e. loan fee) by using its standardised value:

$$Standard\ loan\ fee_s = \left( \frac{Loan\ Fee - mean_s}{standard\ deviation_s} \right) \text{ for each ticker } s$$

Both loan fee mean and standard deviation were measured considering all the deals by that ticker (entire data period). This regression approach aimed to reduce the individual ticker effect on the impacts on other variables, being an alternative to the ticker fixed effect. The next equation summarises this attempt:

Equation 17:

$$\left( \frac{Fee_{i,s,t_{day},obs} - Mean_{fee_s}}{StandardDeviation_{fee_s}} \right) = \alpha + \beta_1 \cdot HHI_{s,t-1} + \beta_2 \cdot (HHI_{s,t-1} \cdot Recall_{dummy_{i,s,t_{day},obs}}) + \beta_3 \cdot dummy_{i,s,t_{day},obs} + \beta_4 \cdot Quantity_{i,s,t_{day},obs} + FE_{day} + \varepsilon_{i,s,t_{day},obs}$$

The next three tables show the estimation results.

**Table 15.** Regression results for Equation 17 in terms of the 55 most liquid stocks.

	Lender Monthly		Broker Monthly		Lender Weekly		Broker Weekly	
Lagged HHI ( $\beta_1$ )	-0.967	-29.151	-0.390	-14.091	-0.203	-12.720	-0.174	-11.407
Lagged HHI*Recall ( $\beta_2$ )	-1.372	-20.119	-1.072	-17.243	-0.297	-8.226	-0.350	-9.901
Recall dummy ( $\beta_3$ )	0.187	28.236	0.184	23.839	0.105	18.889	0.124	18.602
Quantity ( $\beta_4$ )	3.10.10 <sup>-8</sup>	2.609	3.02.10 <sup>-8</sup>	2.567	2.95.10 <sup>-8</sup>	2.529	2.90.10 <sup>-8</sup>	2.491
Constant ( $\alpha$ )	-0.282	-17.812	-0.308	-19.394	-0.313	-19.791	-0.312	-19.619
Fixed Effect <sub>firm</sub>	NO		NO		NO		NO	
Fixed Effect <sub>day</sub>	YES		YES		YES		YES	
R <sup>2</sup> (adj.)	0.067911		0.064125		0.062244		0.062260	

Obs.: Coefficient estimations on the left and t-statistics on the right within each cell.

**Table 16.** Regression results for Equation 17 in terms of the 100 most liquid stocks.

	Lender Monthly		Broker Monthly		Lender Weekly		Broker Weekly	
Lagged HHI ( $\beta_1$ )	-0.514	-20.401	-0.067	-3.296	-0.083	-7.188	-0.046	-4.253
Lagged HHI*Recall ( $\beta_2$ )	-0.659	-13.394	-0.317	-7.136	-0.129	-5.382	-0.130	-5.606
Recall dummy ( $\beta_3$ )	0.127	23.853	0.098	16.248	0.081	18.302	0.085	16.757
Quantity ( $\beta_4$ )	$3.68 \cdot 10^{-8}$	3.196	$3.87 \cdot 10^{-8}$	3.321	$3.93 \cdot 10^{-8}$	3.377	$3.93 \cdot 10^{-8}$	3.376
Constant ( $\alpha$ )	-0.245	-16.156	-0.270	-17.734	-0.265	-17.463	-0.267	-17.610
Fixed Effect <sub>firm</sub>	NO		NO		NO		NO	
Fixed Effect <sub>day</sub>	YES		YES		YES		YES	
R <sup>2</sup> (adj.)	0.056023		0.053970		0.053215		0.053127	

Obs.: Coefficient estimations on the left and t-statistics on the right within each cell.

**Table 17.** Regression results for Equation 17 in terms of the 150 most liquid stocks.

	Lender Monthly		Broker Monthly		Lender Weekly		Broker Weekly	
Lagged HHI ( $\beta_1$ )	-0.338	-18.112	-0.080	-5.071	-0.091	-9.587	-0.060	-6.529
Lagged HHI*Recall ( $\beta_2$ )	-0.145	-4.154	-0.029	-0.916	0.047	2.508	0.030	1.626
Recall dummy ( $\beta_3$ )	0.082	18.798	0.067	13.718	0.058	14.500	0.059	13.061
Quantity ( $\beta_4$ )	$3.79 \cdot 10^{-8}$	3.306	$3.98 \cdot 10^{-8}$	3.428	$3.93 \cdot 10^{-8}$	3.414	$3.96 \cdot 10^{-8}$	3.436
Constant ( $\alpha$ )	-0.254	-16.972	-0.270	-18.024	-0.267	-17.844	-0.268	-17.931
Fixed Effect <sub>firm</sub>	NO		NO		NO		NO	
Fixed Effect <sub>day</sub>	YES		YES		YES		YES	
R <sup>2</sup> (adj.)	0.049880		0.048989		0.048365		0.048297	

Obs.: Coefficient estimations on the left and t-statistics on the right within each cell.

All coefficients remained significant, pointing to the same directions as within the three previous equations and supporting our main result and thesis. However, the general adjusted R<sup>2</sup> result indicates that this regression approach strongly lowers our independent variable explanation power, from proximally 0.5 to 0.05. It suggests that ticker individuality is one of the main participants in explaining the loan fee. Despite this, a regression R<sup>2</sup> comparison is not of our concern, as our purpose was not to propose a predictive model with variables best explaining the loan fee. Instead, we have focused on capturing variable impacts in order to price the recall option in terms of loan fee reduction.

## 6. CONCLUDING REMARKS

As found in the literature on the Brazilian loan market, we have come across a great discrepancy in loan fees, even for the same stock, same date and similar quantities. Loan fee showed a high dispersion in both range and variance.

At first, the lender recall option does not seem to affect the loan fee, even after taking into account special cases and restrictions.

From a borrower's perspective, the flexible model proposed a loan fee process to render positive values depending on the parameter combination, particularly the recall event probability. Along with the recall rate, loan fee drift and volatility raise the recall option cost, demonstrating the connection between market concentration and recall option.

The lender recall option price model (Appendix 2) also assumes zero value for many parameter combinations. The recall price exhibits highly expected stock return. From a lender's perspective, the binomial model achieves positive values only when the expected asset net return is negative. Once the recall option is valuable, the impact of the variables involved has been found to follow an intuitive direction, that is, risk-free interest rate, asset volatility and negative expected return increase the option price, whereas the loan fee reduces it.

Our regression analysis points out the following conclusions:

- Recall option is worthless unless the loan market is concentrated.
- From our estimated coefficients (recall dummy and interactive variable) and their variances and assuming that the market is highly concentrated with HHI values close to one in a rough overview among all H-H indexes, recall option tends to value 0 to 3% a year.
- Loan quantity, although significant, barely affects loan fee, on average.
- Past market concentration does not raise the loan fee, however its growth rate does.
- Stock individuality accounts for roughly half of the loan fee variation.

After having investigated an intriguing feature of the Brazilian loan market and identified its contribution to the market opacity, we encourage future investigations and support the use of stock exchange chamber for open loan market, which would reduce the discrepancies and increase the pricing efficiency.

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## APPENDIX 1

**Table 18.** Main statistics for the 150 most liquid tickers: loan fee statistics (% per year) and recall option proportion.

Ticker	frequency	mean	median	min	max	SD	recall
VALE5	44054	0.278	0.250	0.000	30.000	0.230	0.138
GGBR4	34510	1.973	1.550	0.000	29.890	2.145	0.191
PETR4	33870	0.745	0.250	0.000	60.800	1.961	0.141
BBDC4	27630	0.297	0.250	0.000	7.000	0.256	0.145
RDCD3	22774	6.993	6.000	0.000	40.000	5.240	0.465
USIM5	21332	0.651	0.650	0.000	6.900	0.215	0.151
LAME4	19928	2.614	3.000	0.000	14.000	1.390	0.205
TNLP4	19354	3.484	2.900	0.000	20.000	2.271	0.299
ITUB4	19153	0.280	0.250	0.000	3.250	0.179	0.162
CMIG4	18412	1.523	1.250	0.000	12.000	0.758	0.206
NATU3	17013	4.862	5.000	0.000	29.890	2.164	0.269
LREN3	16118	5.851	6.300	0.000	20.000	2.670	0.348
MRVE3	16101	6.075	6.650	0.000	80.000	2.623	0.374
CYRE3	16068	5.033	3.500	0.000	27.000	3.855	0.375
CIEL3	16055	5.634	3.250	0.100	23.000	4.161	0.430
OGXP3	15891	1.550	1.300	0.003	38.000	1.113	0.436
CSNA3	15634	0.628	0.650	0.000	11.500	0.332	0.154
CPFE3	15503	3.356	3.250	0.000	25.000	1.451	0.179
TCSL4	15284	5.541	5.200	0.000	30.000	3.527	0.297
PETR3	15269	0.541	0.250	0.000	49.800	1.528	0.164
PDGR3	14654	1.652	0.600	0.000	36.000	1.976	0.419
GFSA3	14609	5.352	6.000	0.000	30.000	3.746	0.343
BVMF3	14507	0.554	0.360	0.000	12.000	0.469	0.228
AMBV4	14309	0.357	0.300	0.000	17.300	0.517	0.194
JBSS3	14073	7.419	7.000	0.000	30.000	4.270	0.374
BBAS3	13823	0.813	0.300	0.000	35.000	2.213	0.191
VALE3	13076	0.291	0.250	0.000	4.500	0.184	0.140
RSID3	12943	4.338	3.700	0.000	30.000	3.371	0.439
ELET6	12183	3.020	2.750	0.000	29.730	1.726	0.148
BRKM5	11468	1.171	1.000	0.000	7.870	0.586	0.276
VIVO4	10472	2.890	3.000	0.000	29.890	1.261	0.241
BTOW3	10095	4.688	4.500	0.000	29.890	2.517	0.409
SANB11	10058	4.172	4.000	0.000	50.000	2.005	0.447
MMXM3	9818	7.977	5.000	0.000	44.000	6.817	0.369

GOLL4	9818	4.051	3.500	0.000	35.000	3.689	0.271
CSAN3	9773	1.704	1.400	0.000	15.000	1.279	0.322
FIBR3	9662	1.979	1.550	0.000	13.100	1.118	0.208
KLBN4	9597	1.909	1.500	0.000	99.000	1.421	0.441
ITSA4	9478	0.343	0.250	0.000	35.000	0.549	0.238
TLPP4	8631	5.794	5.250	0.000	26.500	2.674	0.180
TAMM4	8423	3.323	2.700	0.000	20.000	2.963	0.332
CCRO3	8395	1.947	1.620	0.000	10.100	1.424	0.213
CRUZ3	8162	1.512	1.200	0.000	10.000	0.856	0.142
GETI4	8072	2.785	2.800	0.000	9.180	0.850	0.178
PCAR5	7982	1.338	0.850	0.001	12.500	0.868	0.299
LIGT3	7827	3.100	1.950	0.000	34.000	4.105	0.286
EMBR3	7572	2.128	2.000	0.000	17.600	1.458	0.211
CPLE6	7262	1.207	1.200	0.000	14.000	0.523	0.279
ELPL6	7124	1.764	1.800	0.000	21.000	0.723	0.125
UGPA4	6910	0.649	0.600	0.000	90.000	1.114	0.369
BRFS3	6809	0.711	0.550	0.000	11.000	0.612	0.242
USIM3	6569	5.121	2.100	0.000	29.000	5.550	0.274
BRTO4	6433	2.303	1.850	0.000	98.410	3.099	0.357
ALLL11	6268	1.310	1.000	0.000	15.000	0.809	0.182
NETC4	6034	1.296	1.200	0.000	15.500	1.118	0.189
SBSP3	6002	2.183	1.700	0.000	29.890	1.472	0.232
TBLE3	5823	1.545	1.100	0.100	7.000	0.903	0.174
RENT3	5648	3.421	3.500	0.010	6.500	1.055	0.227
MRFG3	5458	12.697	11.700	0.100	35.000	7.490	0.480
BOVA11	5287	3.278	3.300	0.010	12.500	0.679	0.364
HYPE3	5189	3.471	3.100	0.010	14.000	2.438	0.464
LLXL3	5178	2.939	2.250	0.000	30.000	2.470	0.408
TMAR5	5093	4.324	3.400	0.000	35.000	3.318	0.304
ELET3	5036	1.537	0.900	0.000	36.420	1.660	0.289
BRAP4	5023	0.577	0.500	0.000	10.950	0.530	0.182
CESP6	4946	1.076	1.000	0.000	10.890	0.545	0.303
ECOD3	4881	7.540	6.500	0.000	50.000	4.733	0.457
BRML3	4822	4.386	3.600	0.000	20.000	2.712	0.387
BISA3	4592	1.637	0.800	0.000	16.000	2.086	0.559
TRPL4	4470	1.785	1.600	0.000	17.000	1.167	0.178
GOAU4	4307	0.465	0.300	0.000	63.000	1.087	0.268
MPXE3	4263	2.425	2.100	0.000	50.000	1.525	0.646
PSSA3	4067	1.430	1.250	0.100	8.500	0.653	0.319
DTEX3	4057	0.923	0.700	0.100	7.870	0.869	0.314

ELPL4	4053	2.008	1.700	0.000	15.000	1.645	0.244
SUZB5	3895	2.371	2.200	0.100	14.000	1.134	0.344
PRGA3	3826	2.762	2.450	0.000	15.000	1.710	0.133
ALLL3	3716	0.555	0.500	0.010	8.000	0.365	0.359
GVTT3	3584	4.517	3.400	0.010	15.000	2.960	0.177
MULT3	3469	4.746	4.900	0.100	10.500	0.963	0.419
ENBR3	3419	4.481	4.000	0.000	28.000	2.664	0.261
ARCZ6	3335	7.413	7.000	0.000	35.000	3.497	0.164
TNLP3	3317	1.452	1.500	0.000	8.750	0.734	0.274
PCAR4	3309	2.871	2.000	0.000	40.000	3.589	0.358
ITAU4	3015	0.227	0.200	0.000	8.500	0.193	0.055
LUPA3	2844	7.484	7.500	1.500	30.890	1.583	0.190
WEGE3	2825	3.232	2.200	0.000	15.000	2.416	0.153
TCSL3	2775	1.148	1.050	0.000	9.500	0.815	0.419
VCPA4	2484	7.322	7.000	0.000	29.950	3.347	0.208
DASA3	2479	1.487	1.200	0.010	15.000	1.038	0.424
AMIL3	2138	6.547	6.000	0.100	13.000	1.351	0.548
ODPV3	2074	3.087	3.200	0.020	7.000	0.970	0.549
HGTX3	2026	2.924	2.700	0.100	9.000	1.295	0.563
DURA4	1996	1.726	1.800	0.000	6.000	0.520	0.112
FFTL4	1889	1.914	1.400	0.010	25.000	1.572	0.221
BRSR6	1839	3.684	2.000	0.100	20.000	3.223	0.495
SDIA4	1836	1.046	1.000	0.000	3.500	0.347	0.164
TGMA3	1739	3.597	3.500	0.000	8.000	0.749	0.600
CTIP3	1570	2.440	1.500	0.000	10.000	2.027	0.669
BRTP4	1483	2.921	1.950	0.000	9.200	1.985	0.304
GGBR3	1395	1.216	0.800	0.000	7.500	1.217	0.252
MAGG3	1382	3.423	3.500	0.010	25.600	1.643	0.418
EVEN3	1245	4.680	4.900	0.000	12.500	1.691	0.587
CSMG3	1226	3.357	2.700	0.010	78.000	3.080	0.312
VNET3	1183	4.940	5.000	0.050	15.000	1.553	0.342
TOTS3	1170	4.219	3.500	0.050	15.000	1.856	0.368
AEDU3	1157	4.626	4.150	0.010	24.000	1.862	0.661
IGTA3	1152	7.672	6.250	0.000	25.000	4.232	0.523
SULA11	1130	1.572	1.400	0.010	5.000	0.808	0.508
AMBV3	1105	0.727	0.400	0.075	16.330	1.339	0.107
CGAS5	1072	1.874	1.950	0.000	9.250	0.542	0.161
TCSA3	1016	4.001	3.930	0.000	8.000	1.186	0.527
OHLB3	995	5.867	6.000	0.050	11.300	1.924	0.259
POMO4	993	1.764	1.500	0.100	5.000	0.836	0.331

SLCE3	991	6.890	7.500	0.300	10.800	2.168	0.505
FLRY3	974	8.705	7.810	0.000	15.800	2.855	0.748
BICB4	958	1.921	1.800	0.010	17.470	1.275	0.426
STBP11	938	3.335	2.700	0.100	12.600	1.524	0.386
RAIA3	924	4.660	4.500	0.000	8.000	1.206	0.944
CLSC6	919	1.401	1.050	0.000	10.700	1.150	0.131
PRTX3	918	1.102	0.550	0.000	15.000	1.359	0.548
LPSB3	902	9.022	9.300	0.002	13.000	1.257	0.662
RAPT4	900	1.154	0.750	0.150	18.830	1.588	0.183
BBDC3	893	0.534	0.300	0.100	8.700	0.615	0.167
AMAR3	861	9.207	10.000	0.300	30.000	4.701	0.640
ITUB3	843	1.262	0.800	0.010	17.000	1.549	0.380
CNFB4	831	2.934	0.850	0.100	27.350	4.574	0.146
MYPK3	800	4.581	4.650	0.400	20.000	1.105	0.293
DROG3	713	5.034	5.500	0.010	20.000	2.037	0.477
H RTP3	699	13.932	13.000	0.100	60.000	9.731	0.828
ALSC3	697	1.967	1.500	0.500	10.120	1.882	0.818
ECOR3	683	3.331	4.000	0.000	7.100	1.836	0.565
BBRK3	679	2.690	0.800	0.000	20.000	2.998	0.552
POSI3	677	8.346	8.000	0.100	30.000	3.222	0.377
BNCA3	666	3.267	3.210	1.000	10.000	1.200	0.272
VCPA3	660	5.862	6.000	0.350	7.800	1.140	0.129
AEDU11	660	2.542	2.400	0.100	15.000	1.366	0.498
BRTO3	653	4.034	3.300	0.100	95.210	4.253	0.257
SMTO3	643	6.433	6.900	0.100	15.000	2.381	0.162
LOGN3	638	2.655	3.000	0.360	6.500	1.283	0.047
EZTC3	595	1.944	1.800	0.100	5.100	0.800	0.531
BEEF3	591	8.275	8.500	0.000	18.000	2.057	0.305
UBBR11	586	0.650	0.650	0.000	3.000	0.219	0.065
INEP4	584	3.604	3.450	0.500	12.000	1.460	0.264
AGEI3	584	4.171	4.200	0.000	20.000	1.651	0.354
MILS3	566	5.701	5.800	0.000	23.350	1.355	0.788
KROT11	558	6.282	6.500	0.001	12.000	2.433	0.609
AGIN3	546	7.304	7.000	1.000	15.000	2.079	0.520
CTAX4	542	1.860	2.000	0.100	3.200	0.664	0.275
COCE5	531	0.328	0.250	0.130	1.300	0.184	0.431

Obs.: Filters applied before statistics: 1. Lender closed offers only.

2. Capital interest payment window removed.