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**Income redistribution policies in a Schumpeterian
economy**

Políticas de redistribuição de renda em uma economia Schumpeteriana

Guilherme Strifezzi Leal

Orientador: Prof. Dr. David Daniel Turchick Rubin

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Prof. Dr. Marco Antonio Zago

Reitor da Universidade de São Paulo

Prof. Dr. Adalberto Américo Fischmann

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Prof. Dr. Hélio Nogueira da Cruz

Chefe do Departamento de Economia

Prof. Dr. Márcio Issao Nakane

Coordenador do Programa de Pós-Graduação em Economia

GUILHERME STRIFEZZI LEAL

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Orientador: Prof. Dr. David Daniel Turchick Rubin

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Resumo

O presente estudo analisa os efeitos de redistribuição de renda sobre crescimento, desigualdade e bem-estar em uma economia Schumpeteriana e compara os resultados de diferentes políticas de redistribuição. O modelo contempla uma estrutura de inovação vertical baseada no conceito de destruição criadora e contém agentes avessos ao risco e elasticidade na oferta de trabalho. Ao contrário de estudos anteriores, mostra-se que taxa excessiva desencoraja a atividade de pesquisa, mas redistribuição de renda moderada pode incentivá-la. Analisam-se os resultados de equilíbrio para três diferentes políticas: redistribuição igual entre os agentes, subsídio à pesquisa e bônus por inovação. Enquanto a primeira é a mais eficiente para reduzir a desigualdade de renda, as outras duas são mais eficientes para promover crescimento econômico. Mostra-se que a política ótima em termos de geração de bem-estar varia dependendo da massa de indivíduos qualificados na economia, e que os resultados obtidos não sustentam a hipótese de convergência.

Palavras chave: Redistribuição de renda, pesquisa, crescimento econômico, oferta de trabalho elástica.

Abstract

We study the effects of redistributive taxation on growth, inequality and welfare in a Schumpeterian economy and compare the outcomes of different policies. The model features a structure of vertical innovations based on the concept of creative destruction and contains risk-averse agents and elastic labor supply. Contrary to previous findings, we show that excessive tax rates discourage research effort, but moderate redistribution can stimulate it. We analyze the results for three different policies: equal redistribution to all agents, research subsidy and innovation bonus. While the first is better in terms of reducing inequality, the others are more efficient to promote research effort and economic growth. We show that the best policy in terms of welfare generation changes as we increase the amount of skilled individuals in the economy, and that the obtained outcomes do not support the hypothesis of convergence.

Keywords: Income redistribution, research, economic growth, elastic labor supply.

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1 Introduction

Aghion and Howitt's (1992) contribution was the first to integrate Schumpeterian growth to a general equilibrium model. Their work features a structure of vertical innovations based on the concept of creative destruction and adds to the literature of endogenous growth by focusing on a channel of knowledge accumulation that, until then, had received little attention: quality-improving innovations. Technological progress, in the model, stems from industrial innovations which make previous technologies obsolete, and the amount of research determines the expected growth rate of the economy. They argue that the prospect of future research discourages current research, since it threatens to destroy the rents created by current entrepreneurship. One implication of this relationship between current and future research is the existence of a "no-growth trap", an equilibrium in which there is no innovations and no growth for an infinite period.

An intrinsic factor to the Aghion and Howitt (1992) economy is the high risk to which the researchers are exposed. Empirical evidences support the common sense that entrepreneurs in fact face much larger risks than paid employees. Hamilton (2000) investigate earnings differentials between self-employment and paid employment and verify that the cross-sectional standard deviation of self-employment is substantially higher, while Dunne (1988) studies the US manufacturing industry and observes that 61.5% of all firms exit in the five years following their first census, and 79.5% of all firms exit within ten years.

According to Mayshar (1977), in most countries private insurance is insufficient to cover these risks due to incompleteness of the capital market structure, and, therefore, a government intervention can benefit the economy. Sinn (1996) defends the importance of redistributive taxation as social insurance, not only on the account that it creates safety but also that it stimulates income-generating risky activities.

García-Peñalosa and Wen (2008), then, add risk-averse agents to Aghion and Howitt (1992) and analyze the equilibrium effects of redistributive taxation. They conclude that redistribution provides insurance to entrepreneurs, diluting their risk, and encourages agents to engage in research, therefore stimulating growth.

They also study the effects of redistribution on inequality and welfare. They show that low tax rates increase inequality relative to *laissez-faire*, while high tax rates can simultaneously promote growth and reduce inequality. In terms of welfare, García-Peñalosa and Wen (2008) suggests that high tax rates (99.8% in their numerical exercise) not only maximize but generate close to first-best welfare.

García-Peñalosa and Wen’s study abstract from leisure utility, hence their model presents inelastic labor supply, which ends up generating unrealistically large optimal taxes (99.8% optimal tax rate as pointed above, as well as a 100% growth-maximizing tax rate). Meghir and Phillips (2010) examine the relationship between labor supply and taxation and emphasize the importance, when doing policy analysis, of considering the incentive effects of taxation, which is essentially how hours worked respond to changes in taxes and transfers. The present study, therefore, incorporates elastic labor supply to García-Peñalosa and Wen’s work, by adding positive leisure elasticity to agents’ utility functions. We, then, reanalyze the effects of redistributive taxation on growth, inequality and welfare, and compare the equilibrium outcomes for different redistribution policies.

Three types of redistributive taxation policies are presented in our model: equal redistribution between all agents, research subsidy, in which only researchers receive transfers, and an innovation bonus, in which total tax revenue goes to successful innovators.^a We contrast the policies in terms of welfare generation, and show that the government’s optimal strategy is actually conditioned to the economy’s level of development. On the one hand, equally redistributing total tax revenues between agents is the best policy for economies with a low mass of skilled individuals, since, as taxation increases, the welfare gains of reducing inequality and preserving initial output are greater than those of promoting growth. On the other hand, for economies with a large mass of skilled individuals, subsidizing research and rewarding innovations are more attractive policies due to the greater impact of growth on welfare.

The second section of this paper presents the derivation of the model, considering a

^aWe thank Mauro Rodrigues Junior for this suggestion

redistribution structure in which all agents receive the same transfer. In the third section, we define and study the equilibrium allocation, including a new version of the "no-growth trap", and show that research effort is not strictly increasing in redistribution as argued by García-Peñalosa and Wen (2008). The fourth section presents the social planner problem and discuss first-best allocation. The fifth section illustrates some dynamics regarding the model's variables under equal redistribution policy and the sixth section presents the two remaining policies: research subsidy and bonus payment for innovation. The seventh section shows how different values for the mass of skilled individuals in the economy affects the optimal policy choice and also that the results do not support the hypothesis of convergence among economies. Finally, the eighth section concludes the paper.

2 The model

2.1 Households

The economy is populated by a continuum of size N with individuals living infinite periods. Households are split into a mass L of unskilled agents, who can only be employed for the production of final good Y , and H of skilled agents, who choose between working as researchers or as manufacturers producing intermediate good x .

Researchers, or entrepreneurs, engage in R&D with the intention of making an innovation, and, thus, obtaining a patent and licensing it to the intermediate goods sector. At period t , there exist a mass R_t of entrepreneurs, $M_t = H - R_t$ of manufacturers and L of unskilled workers. Manufacturers and unskilled workers earn, respectively, w_t and v_t for one unit of labor.

All individuals have identical utility functions, that depend on consumption C and leisure l , $0 \leq l \leq 1$. Utility at period t , as in García-Peñalosa and Turnovsky (2006), is given by the following function:

$$U(C_t, l_t) = \frac{(C_t l_t^\eta)^{1-\sigma}}{1-\sigma}; \eta \geq 0, \sigma \geq 0, \eta(1-\sigma) \leq \min\{1, \sigma\} \quad (1)$$

where σ is the coefficient of relative risk aversion, and η is the elasticity of leisure. If we take $\eta = 0$, we obtain the framework in García-Peñalosa and Wen (2008) and if we take $\eta = 0$ and $\sigma = 0$, we get Aghion and Howitt (1992).

2.2 Firms

The economy produces a single homogeneous final good Y , to be treated as numeraire, and a single intermediate good x , with price p . The final good is produced in a competitive sector according to the following function:

$$Y_t = A_t x_t^\theta O_{u,t}^{1-\theta} \quad (2)$$

where O_u is the work effort of unskilled individuals, which in equilibrium equals $(1 - l_u^a) L$, where l_u^a is the aggregate leisure of unskilled individuals. Also, A_t is the technological index at time t and $0 < \theta < 1$.

Each innovation increases the value of the technological index A_t by a factor $\gamma > 1$, so that $A_{t+1} = \gamma A_t$ when an innovation occurs at t and $A_{t+1} = A_t$ when there is no innovation.

There is a large number of risk-neutral firms willing to produce intermediate goods. When an innovation occurs, one of these firms buys the exclusive right to produce the intermediate good according to the new blueprint, becoming a temporary monopolist, until a new innovation takes place. The production function for the intermediate good depends solely on skilled labor according to the following linear technology:

$$x_t = O_{m,t} \tag{3}$$

where, analogously, O_m is the work effort of manufacturers, which in equilibrium equals $(1 - l_m^a) M$, and l_m^a is the aggregate leisure of manufacturers.

2.3 Research

At period t , R_t individuals engage in research with the intent of developing the next innovation, which will define the new production design for intermediate goods, and receiving its corresponding dividends. Each entrepreneur has a probability $\lambda(1 - l_{r,t})$ of making the discovery, where $l_{r,t}$ is the leisure of the researcher at time t , and λ is the parameter of a Poisson distribution. Assuming independence, the aggregate probability of at least one innovation occurring at period t is $\lambda(1 - l_{r,t}^a) R_t$,^b where $l_{r,t}^a$ stands for aggregate researchers' leisure at t . Also, for further reference, let ξ be a random variable describing the aggregate number of innovations according to the mentioned Poisson distribution, and let $\chi_{t,(\xi_t > 0)}$ be its corresponding indicator function, that is, $\chi_{t,(\xi_t > 0)} = 1$ when

^b Approximation valid for small λ .

there is at least one innovation and $\chi_{t,(\xi_t>0)} = 0$ when there isn't. Additionally, we assume that when a discovery is made, the patent is assigned to a unit measure set of researchers. As a consequence, we have that $A_t = A_0 \gamma^{\sum_{i=0}^{t-1} \chi_{i, [\xi_i>0]}}$.

We define research effort at t as $(1 - l_{r,t}^a) R_t$.

The new invention will be licensed to an intermediary firm, that will start to produce the new intermediate good and receiving its profits at $t+1$. In this manner, the researcher who made the discovery receives at t the expected value of the innovation that will be employed in the manufacturing process in the next period, V_{t+1} .

2.4 Government

The government promotes redistribution between the agents with the goal of mitigating the dead weight stemming from the monopoly and the entrepreneurs' uncovered risks. We shall first consider a linear income tax system identical to the one in García-Peñalosa and Wen (2008). Individuals pay a tax rate τ , $0 \leq \tau \leq 1$, on their income and receive the same transfer B . Total collection at t equals τY_t , which, in equilibrium, corresponds to the individual transfer B_t times the number of agents in the economy. The government's budget constraint in equilibrium at time t is, then:

$$\tau Y_t = B_t N \quad (4)$$

Different systems of income redistribution shall be discussed later in the numerical analysis section.

2.5 Firms' maximization problem

Since there is perfect competition in the final-good sector, x and O_u are paid their marginal products. Differentiating (2) to obtain the inverse demand function for interme-

diate goods, we can express the intermediate firm's profit maximization problem as:

$$\max_{x_t} p_t x_t - w_t x_t \quad (5)$$

$$s.t \ p_t = A_t \theta x_t^{\theta-1} O_{u,t}^{1-\theta} \quad (6)$$

The profit maximizing production of x_t is then, $x_t = \left(\frac{\theta^2 A_t}{w_t} \right)^{\frac{1}{1-\theta}} O_{u,t}$, which leads to $p_t = \frac{w_t}{\theta}$ and to the following expressions for wages and profit:

$$w_t = \frac{\theta^2 Y_t}{(1 - l_{m,t}^a) (H - R_t)} \quad (7)$$

$$v_t = \frac{(1 - \theta) Y_t}{(1 - l_{u,t}^a) L} \quad (8)$$

$$\pi_t = (1 - \theta) \theta Y_t \quad (9)$$

where π_t is the intermediate firm's profit at t .

Note that wages w_t and v_t are negative functions of total work effort in their respective sectors, that is, $(1 - l_{m,t}^a) (H - R_t)$ and $(1 - l_{u,t}^a) L$.

The value of the patent V_{t+1} is determined by the asset condition:

$$V_{t+1} = \frac{\pi_{t+1}}{r + \lambda (1 - l_{r,t+1}^a) R_{t+1}} \quad (10)$$

in which the value of the innovation to the risk-neutral firm is equal to the stream of profits discounted by the exogenous interest rate r and the risk of losing its monopoly power, that is, the probability of being replaced if a new discovery is made.

2.6 Households' maximization problem

Unskilled individuals and manufacturers choose, at each period, C_t and l_t to maximize (1) subject to the following budget constraints:

$$C_{u,t} \leq (1 - l_{u,t}) v_t (1 - \tau) + B_t \quad (11)$$

$$C_{m,t} \leq (1 - l_{m,t}) w_t (1 - \tau) + B_t \quad (12)$$

Researchers, however, do not know whether or not they will make their discovery, thus they choose $C_{1,t}$, $C_{0,t}$ and $l_{r,t}$ to maximize the expected utility:

$$\lambda (1 - l_{r,t}) U(C_{1,t}, l_{r,t}) + (1 - \lambda (1 - l_{r,t})) U(C_{0,t}, l_{r,t}) \quad (13)$$

where $C_{1,t} \leq V_{t+1} (1 - \tau) + B_t$ and $C_{0,t} \leq B_t$

Skilled individuals choose between working as manufacturers or as entrepreneurs by comparing their respective utility levels. One could also be concerned that the skilled individuals would want to work in the final sector, if it granted them a higher utility level. However, as proved in the Appendix, as long as we meet García-Peñalosa and Wen's condition ensuring skilled wages exceed unskilled wages in their model, given by $\frac{L}{H} > \frac{(1-\theta)}{\theta^2}$, skilled utilities will always exceed unskilled utilities for $\eta > 0$ as well (except for the case $\tau = 1$, which shall be discussed in section 3.3).

Unskilled individuals' and manufacturers' utility maximizing choices of leisure, in equilibrium, will always come from interior solutions, respectively:

$$l_{u,t}^* = \frac{\eta}{\eta + 1} \left(1 + \frac{B_t}{v_t (1 - \tau)} \right) \quad (14)$$

$$l_{m,t}^* = \frac{\eta}{\eta + 1} \left(1 + \frac{B_t}{w_t (1 - \tau)} \right) \quad (15)$$

The agents' optimal leisure choices, therefore, are negative functions of their post-tax wages, $v_t (1 - \tau)$ and $w_t (1 - \tau)$, and positive functions of their transfers, B_t . Also, if an

individual receives no transfer (i.e., $B_t = 0$), his leisure choice will be invariant to taxation and equal to $\frac{\eta}{\eta+1}$. This stems from the income effect of τ exactly offsetting its substitution effect, given the Cobb-Douglas utility function in (1).

For the researcher, choosing $l_{r,t} = 0$ will never be an optimal solution, however, as shown in the Appendix, if $\lambda \frac{(B_t^{1-\sigma} - C_{1,t}^{1-\sigma})}{1-\sigma} + \eta B_t^{1-\sigma} \geq 0$ when $l_{r,t} = 1$, then it will be optimal to choose $l_{r,t}^* = 1$. If $\lambda \frac{(B_t^{1-\sigma} - C_{1,t}^{1-\sigma})}{1-\sigma} + \eta B_t^{1-\sigma} < 0$ when $l_{r,t} = 1$ the optimal choice will be:

$$l_{r,t}^* = \frac{\eta(1-\sigma)}{\eta(1-\sigma)+1} \left(1 + \frac{1}{\lambda} \frac{B_t^{1-\sigma}}{C_{1,t}^{1-\sigma} - B_t^{1-\sigma}} \right) \quad (16)$$

In this case, the researchers' leisure choice is a negative function of the successful entrepreneur's income $C_{1,t}$ and a positive function of the transfer B_t . Again, leisure is invariant to taxation when there is no transfer B_t to be received, and in this case equals $\frac{\eta(1-\sigma)}{\eta(1-\sigma)+1}$.

3 Equilibrium

An allocation is defined as a symmetric equilibrium if all agents have the same choice of leisure, the firms are maximizing profits, the agents are maximizing their utilities, the skilled workers do not want to change their occupational choice, and the market clearing condition and government constraint are met. Formally:

$(p, v, w, V, C_u, C_m, C_0, C_1, l_u, l_m, l_r, l_u^a, l_m^a, l_r^a, R, \xi, \tau, B)$ is an equilibrium allocation, if:

- (i) $((1 - l_{u,t})L, x_t) \in \arg \max_{O_{ut} \geq 0, x_t \geq 0} A_t x_t^\theta O_{ut}^{1-\theta} - p_t x_t - v_t O_{ut}$.
- (ii) $(1 - l_{m,t})(H - R_t)$ solves problem (5) s.t. (6).
- (iii) $(C_{u,t}, l_{u,t})$ maximizes (1) s.t. (11), with value function $Z_u = Z_u(v_t, \tau, B_t)$.
- (iv) $(C_{m,t}, l_{m,t})$ maximizes (1) s.t. (12), with value function $Z_m = Z_m(w_t, \tau, B_t)$.
- (v) $(C_{0t}, C_{1t}, l_{r,t})$ maximizes (13) s.t. $\tilde{C}_{0,t} \leq B_t$ and $\tilde{C}_{1,t} \leq V_{t+1}(1 - \tau) + B_t$, with value function $Z_r = Z_r(V_{t+1}, \tau, B_t)$.

(vi) $Z_m(w_t, \tau, B_t) > Z_r(V_{t+1}, \tau, B_t) \Rightarrow R_t = 0$.

(vii) $Z_m(w_t, \tau, B_t) < Z_r(V_{t+1}, \tau, B_t) \Rightarrow R_t = H$.

(viii) V_{t+1} satisfies (10).

(ix) Government constraint (4) is met.

(x) $LC_{u,t} + (H - R_t)C_{m,t} + R_tC_{0,t} + \chi_{[\xi_t > 0]}(C_{1,t} - C_{0,t}) =$

$$A_t((1 - l_{m,t})(H - R_t))^\theta((1 - l_{u,t})L)^{1-\theta}.$$

(xi) $O_{u,t} = (1 - l_{u,t})L$ and $O_{m,t} = (1 - l_{m,t})(H - R_t)$.

(xii) $l_{r,t}^a = l_{r,t}; l_{m,t}^a = l_{m,t}; l_{u,t}^a = l_{u,t}$

The last term in the left-hand side of condition (x) accounts for the uncertainty of innovations occurrence. If there is an innovation, $\chi_{[\xi_t > 0]}$ equals one and a mass $R - 1$ of researchers will have income C_0 , while a unit measure set of entrepreneurs will have income C_1 . If there is no innovation, on the other hand, a mass R of researchers will have income C_0 and no individual will have income C_1 .

Conditions (vii) refers to a scenario in which manufacturing ceases to exist, which, in section 3.3 will be proved to be an impossible equilibrium. Condition (vi) refers to a scenario in which research ceases to exist, which is a possible situation and will also be explored in section 3.3.

The reader may note that it is possible to derive laissez-faire equilibrium as a specific case of the one presented above by putting $\tau = 0$.

Finally, stationary equilibrium is given by $R^* = R_t = R_{t+1}$ (hence, $M^* = M_t = M_{t+1}$) and $l_t^* = l$ for all agents. Such an equilibrium is, modulo null sets (which do not affect the market clearing conditions), unique.

3.1 Expected stationary growth rate

Change in output between periods t and $t + 1$ is given by:

$$\ln \left(\frac{Y_{t+1}}{Y_t} \right) = \ln \left(\frac{A_{t+1}}{A_t} \right) \quad (17)$$

Growth in this economy follows a stochastic process, since it depends only on technological shocks. Let g_t be the expected growth rate of productivity at t , $g_t = E (\ln A_{t+1} - \ln A_t)$. In stationary equilibrium, the probability of innovation is constant and equal to $\lambda (1 - l_r^*) R^*$, therefore the expected stationary growth rate is:

$$g = \lambda (1 - l_r^*) R^* \ln \gamma \quad (18)$$

The stationary probability of innovation, and thus the expected growth rate, is increasing in the number of researchers and decreasing in their leisure.

3.2 Income distribution

In order to measure inequality, we use the Gini coefficient, which is given by:

$$Gini = \frac{N}{2Y} \sum_{i=1}^G \sum_{j=1}^G |Y_i - Y_j| n_i n_j \quad (19)$$

where G is the number of different groups in the population, Y is the aggregate income, Y_i and Y_j are the incomes of individuals of groups i and j , and n_i and n_j are the shares of the population of each group. When there is an innovation, there are four income groups and the Gini coefficient is given by:

$$Gini(in) = \frac{1 - \tau}{2N} \times \left[\theta^2 L - (1 - \theta) (H - R) + \theta (1 - \theta) \left(\frac{\gamma (N - 1)}{r + \lambda (1 - l_r) R} - (R - 2) \right) + R - 2 \right] \quad (20)$$

When there is no innovation, however, there are three different groups and the coeffi-

cient is:

$$Gini(no) = \frac{1-\tau}{N} [\theta^2 (L+R) - (1-\theta)(H-2R)] \quad (21)$$

The probability-weighted Gini coefficient is, then:

$$Gini = \lambda R (1-l_r) Gini(in) + (1-\lambda R (1-l_r)) Gini(no) \quad (22)$$

Note that since the Gini coefficient measures relative inequality and since the incomes of all individuals in this economy are linear functions of Y , then Gini will be constant in steady-state equilibrium.

The direct effect of taxation is to reduce inequality in both scenarios as we can see from the first term of expressions (20) and (21). However, taxation also affects both l_r and R , therefore its overall effect on inequality depends on how redistributive taxation affects the number of entrepreneurs and researchers' choice of leisure, which will be explored in section 3.4.

3.3 Possible stationary equilibria

In stationary equilibrium, manufacturers and unskilled agents' choices of leisure are given by, respectively:

$$l_m^* = \frac{\eta \left[(1-\tau) \theta^2 + \frac{(H-R^*)\tau}{N} \right]}{(\eta+1) (1-\tau) \theta^2 + \frac{\eta(H-R^*)\tau}{N}} \quad (23)$$

$$l_u^* = \frac{\eta \left[(1-\tau) (1-\theta) + \frac{L\tau}{N} \right]}{(\eta+1) (1-\tau) (1-\theta) + \frac{\eta L\tau}{N}} \quad (24)$$

Note that their leisure choices do not depend on Y , and are constant in steady-state equilibrium. Also, note that if $\tau = 1$, both manufacturers and unskilled agents will choose $l^* = 1$ and there will be no production in the economy.

Individuals' utility values, in equilibrium, are given by:

$$U(C_{u,t}^*, l_u^*) = \frac{\left(\left(\frac{(1-\tau)(1-\theta)Y_t}{L} + \frac{\tau Y_t}{N} \right) \left(\frac{\eta[(1-\tau)(1-\theta) + \frac{L\tau}{N}]}{(\eta+1)(1-\tau)(1-\theta) + \frac{\eta L\tau}{N}} \right)^\eta \right)^{1-\sigma}}{1-\sigma} \quad (25)$$

$$U(C_{m,t}^*, l_m^*) = \frac{\left(\left(\frac{(1-\tau)\theta^2 Y_t}{H-R^*} + \frac{\tau Y_t}{N} \right) \left(\frac{\eta[(1-\tau)\theta^2 + \frac{(H-R^*)\tau}{N}]}{(\eta+1)(1-\tau)\theta^2 + \frac{\eta(H-R^*)\tau}{N}} \right)^\eta \right)^{1-\sigma}}{1-\sigma} \quad (26)$$

$$E(U(C_{r,t}^*, l_r^*)) = \lambda(1-l_r^*) \frac{\left(\left(\frac{(1-\tau)\theta(1-\theta)\gamma Y_t}{r+\lambda(1-l_r^*)R^*} + \frac{\tau Y_t}{N} \right) l_r^{*\eta} \right)^{1-\sigma}}{1-\sigma} + (1-\lambda(1-l_r^*)) \frac{\left(\frac{\tau Y_t}{N} l_r^{*\eta} \right)^{1-\sigma}}{1-\sigma} \quad (27)$$

Note that $U(C_{u,t}^*, l_u^*)$, $U(C_{m,t}^*, l_m^*)$ and $E(U(C_{r,t}^*, l_r^*))$ correspond to the value functions $Z_u(v_t, \tau, B_t)$, $Z_m(w_t, \tau, B_t)$ and $Z_r(V_{t+1}, \tau, B_t)$ using the equilibrium values for wages and profits.

Lemma 1 *In a stationary equilibrium, $R^* < H$.*

Proof in Appendix.

In fact, $R^* = H$ would imply $M^* = 0$, in contradiction with one of the Inada properties of the manufacturing production function, since a finite wage cannot be equated to an infinite productivity of labor.^c

By definition, there are always workers in the final sector, and by Lemma 1 there are always workers in the intermediate sector in a stationary equilibrium. Also, equations (23) and (24) show that, except for $\tau = 1$, $l_u^* < 1$ and $l_m^* < 1$, which means that work effort in the productive sector will always be positive. As we shall see, for the research sector the same does not apply.

^cWe thank Eduardo Correia de Souza for this observation

Let us now define as type 1 an equilibrium in which $0 < R^* < H$ and $0 < l_r^* < 1$, and as type 2 an equilibrium in which research effort is zero, that is, in which either $R^* = 0$ or $l_r^* = 1$. In type 2 equilibrium, expected growth rate is zero, therefore we call it the no-growth trap.

Also, let us take the conditions below:

$$\frac{\lambda \left(\tau^{1-\sigma} - \left(\frac{N}{r} (1-\theta) \theta \gamma (1-\tau) + \tau \right)^{1-\sigma} \right)}{1-\sigma} + \eta \tau^{1-\sigma} \geq 0 \quad (28)$$

$$\left(\frac{(1-\tau) \theta^2}{H} + \frac{\tau}{N} \right) \left(1 - \frac{(1-\tau) \theta^2}{(\eta+1)(1-\tau) \theta^2 + \eta H \frac{\tau}{N}} \right)^\eta > \frac{\tau}{N} \left(\frac{\lambda(P-1)+1}{1+\eta(1-\sigma)} \right) \left(\frac{\eta(1-\sigma)}{\lambda(P-1)} \right)^\eta \quad (29)$$

where $P = \left(1 + \frac{1}{r\tau} N \theta \gamma (1-\theta) (1-\tau) \right)^{1-\sigma}$

Proposition 2 *In a stationary equilibrium,*

(i) *If condition (28) holds, $l_r^* = 1$ and the economy is in the no-growth trap. If condition (28) doesn't hold, $0 < l_r^* < 1$.*

(ii) *If condition (29) holds, $R^* = 0$ and the economy is in the no-growth trap.*

Proof in Appendix.

3.4 Redistribution and research effort

García-Peñalosa and Wen (2008) discussed two different effects of increasing τ on research: The incentive effect, that discourages skilled agents to engage in research due to the tax charged on the value of the innovation, and the insurance effect, which partially protects researchers from failures and thus encourages entrepreneurship. They show that the insurance effect is dominant for all levels of taxation as long as λ is small enough and they state that an equilibrium with $\tau \rightarrow 1$ generates greater research effort and expected growth rate than laissez-faire.^d Now, however, we have a third effect, which is the impact

^dIf we consider an economy in which skilled agents can allocate a part of their times in research and another part in manufacturing, García-Peñalosa and Wen (2008) results would still hold.

of taxation on entrepreneurs' choice of leisure.

Proposition 3 *In a stationary equilibrium,*

- (i) *there is $\bar{\tau} < 1$ such that, if $\tau \geq \bar{\tau}$, then $l_r^* = 1$ (and research effort is zero).*
- (ii) *if $R^* > 0$ and condition (28) does not hold, entrepreneurs' leisure l_r^* is strictly increasing in τ .*

Proof in Appendix.

We shall refer to the effect expressed in item (ii) above as leisure effect of taxation.

Proposition 3 item (i) contradicts García-Peñalosa and Wen's proposition 1.3, which means that, in our model, as long as the economy converges to a type 1 stationary equilibrium for $\tau = 0$, research effort is greater in laissez-faire than it is when $\tau = 1$.

The net effect of taxation on the number of researchers is equal to

$$\frac{dR}{d\tau} = R_\tau + R_{l_r^*} \frac{d(l_r^*)}{d\tau}$$

where R_τ and $R_{l_r^*}$ correspond to partial derivatives. As shown in the Appendix, the sign of $dR/d\tau$ is ambiguous and depends on the combination of the parameters, given that the first term of the expression is negative, while the second is positive. The net effect of taxation on research effort ($R^*(1 - l_r^*)$) turns out to be ambiguous as well. The numerical analysis section shows us how different redistribution policies and changes in parameters affect research effort.

4 Social planner

We consider a social planner who chooses the number of researchers in the economy, and the leisure and consumption of the agents to maximize an utilitarian welfare function given by:

$$W = \sum_{t=0}^{\infty} \delta^t W_t \quad (30)$$

where δ is the social discount factor and

$$W_t = L U_{u,t} + (H - R_t) U_{m,t} + R_t U_{r,t} \quad (31)$$

Defining $\bar{C} = \frac{C_{i,t}}{Y_t}$ and assuming the social planner gives always the same share of consumption to each agent and chooses the same amount of leisure and same number of researchers across time, we can rewrite the expression above as:

$$W_t = \frac{\left(A_t ((1 - l_m) (H - R))^\theta ((1 - l_u) L)^{1-\theta} \right)^{1-\sigma}}{1 - \sigma} \times \quad (32)$$

$$\frac{1}{\left(L (l_u^\eta \bar{C}_u)^{1-\sigma} + (H - R_t) (l_m^\eta \bar{C}_m)^{1-\sigma} + R (l_r^\eta \bar{C}_r)^{1-\sigma} \right)}$$

Except for the term A_t , which represents the evolution of productivity in this economy and grows at a rate γ when there is innovation, the rest of the terms on the expression above are time-invariant. The present value of the expected welfare, then, can be written as:

$$W = \sum_{t=0}^{\infty} \delta^t \sum_{s=0}^t \left(\frac{t!}{s! (s-t)!} \right) (\lambda (1 - l_r) R)^s (1 - \lambda (1 - l_r) R)^{t-s} (\gamma^{1-\sigma})^s W_0 \quad (33)$$

Using the binomial theorem to rewrite this expression:

$$W = \frac{\left(A_0 ((1 - l_m) (H - R))^\theta ((1 - l_u) L)^{1-\theta} \right)^{1-\sigma}}{1 - \delta [1 + (\gamma^{1-\sigma} - 1) \lambda (1 - l_r) R]} \times \quad (34)$$

$$\frac{1}{\left(L (l_u^\eta \bar{C}_u)^{1-\sigma} + (H - R) (l_m^\eta \bar{C}_m)^{1-\sigma} + R (l_r^\eta \bar{C}_r)^{1-\sigma} \right)}$$

The expression above gives us some intuitions on what factors have impact on welfare. The numerator of the first term is the initial product, while the denominator can be understood as a combination of the social discount factor and the expected rate of innovation (which determines the expected growth rate). The second term captures risk exposure and redistributive effects. As in García-Peñalosa and Wen (2008), research effort has a negative impact on welfare once it reduces initial product, and a positive effect since it increases the expected rate of innovation, and thus the expected growth rate. Besides that, the number of researchers now impacts welfare through another channel, the leisure of manufacturers. The greater the number of researchers, the higher is the manufacturers' wage and their willingness to work (this can be inferred by differentiating expression (23)), which, on one hand has a positive impact on welfare due to it increasing initial product, and, on the other, a negative effect due to decreasing manufacturers' utility (the balance between researchers' and manufacturers' utility levels still holds due to the lower expected value of an innovation that results from a larger number of researchers).

4.1 First-best

The social planner chooses the number of researchers (R^{FB}), the leisure of the agents ($l_m^{FB}, l_u^{FB}, l_r^{FB}$) and their consumption ($\bar{C}_m^{FB}, \bar{C}_u^{FB}, \bar{C}_r^{FB}$) to maximize (34) subject to the resource constraint $L\bar{C}_u + (H - R^{FB})\bar{C}_m + R^{FB}\bar{C}_r = 1$, so that:

$$W^{FB} = \frac{\left(A_0 \left((1 - l_m^{FB}) (H - R^{FB}) \right)^\theta \left((1 - l_u^{FB}) L \right)^{1-\theta} \right)^{1-\sigma}}{1 - \delta [1 + (\gamma^{1-\sigma} - 1) \lambda (1 - l_r^{FB}) R^{FB}]} \times \frac{\left(L (l_u^{FB\eta} \bar{C}_u^{FB})^{1-\sigma} + (H - R^{FB}) (l_m^{FB\eta} \bar{C}_m^{FB})^{1-\sigma} + R (l_r^{FB\eta} \bar{C}_r^{FB})^{1-\sigma} \right)}{1 - \sigma} \quad (35)$$

$\theta=0.25$	$\gamma=1.8$	$r=1.5$
$\sigma=0.1$	$\eta=0.05$	$\delta=0.4$
$\lambda=0.0005$	$H=1500$	$N=20000$

Figure 1: Baseline parameters

5 Numerical analysis

5.1 Parameters

We illustrate some of the dynamics regarding the economy's variables with numerical examples. We use the same parameter values as in García-Peñalosa and Wen (2008) to facilitate comparisons, except for the leisure elasticity, which was introduced on this model. We use a low value for leisure elasticity ($\eta = 0.05$) and show that, even with a small change in parameters, the changes in results are significant. The social and private discount rate are identical and one period of time corresponds to ten years. Figure 1 presents the values of these parameters:

5.2 Results for equal redistribution policy

Figure 2 presents the variations in the steady-state equilibrium, under the policy of equal redistribution, for different tax rates. The horizontal axis of all graphs correspond to the tax rate, in percentage points. Growth rate and leisure are given in percentage points as well, while welfare and initial output are defined as deviation rates from their laissez-faire values. Inequality corresponds to the Gini coefficient as explained in section 3.2. The purple curves correspond to García-Peñalosa and Wen's model (with $\eta = 0$), while the blue curves correspond to the model with $\eta = 0.05$ (Leisure and Welfare graphs correspond only to the model with $\eta = 0.05$). García-Peñalosa and Wen (2008) argue that research effort is always increasing on redistribution, due to the insurance effect, that encourages skilled individuals to engage in research, (as we can see in the figure, for $\eta = 0$ the number of researchers and the growth rate are in fact strictly increasing functions of the tax rate). This effect still exists in our model with positive η , but it's no

longer dominant for all levels of taxation. The graph shows that the expected stationary growth rate, and therefore research effort, increases with taxation up to a certain point, then starts decreasing. At first, the number of researchers increases stimulating growth, however it rapidly collapses and for high levels of the tax rate, the number of researchers tends to zero while their leisure tends to one. As a result, as we increase the tax rate, we shift from a type 1 steady-state equilibrium to a type 2, where there is no research sector and no growth. Similar to García-Pañalosa and Wen (2008), for low tax rate values, income inequality increases in relation to laissez-faire, but for higher tax rate values it decreases. Initial output is always decreasing in taxation, for low tax rate values due to skilled individuals shifting to the research sector, and for high tax rate values because of agents' increasing leisure. The same goes for the model with $\eta = 0$, however the inelastic labor supply smooths the fall of the initial output. The welfare maximizing tax rate isn't 99.8% as in García-Pañalosa and Wen (2008), but 58%. A more detailed analysis of how leisure elasticity affects welfare is presented in the next section.

5.3 Optimal tax rate and first-best welfare

We define the government's optimal tax rate as the welfare maximizing tax rate under the policy described so far. The third graph of Figure 2 shows the difference between the optimal tax (the highest point of the blue curve) and the first-best welfare (the red line). García-Pañalosa and Wen (2008), suggest that this difference is almost imperceptible (first-best welfare is 22.67% greater than in laissez-faire, while their optimal tax rate welfare is 22.63%), however their model abstracted from leisure in individuals' utility functions. We found that as leisure elasticity is increased, the greater is the difference between optimal tax rate welfare and first-best welfare. Figure 3 shows what happens to this difference as we increase leisure elasticity (η).

Since welfare levels comparison is ordinal instead of cardinal, the reader should take Figure 3 with a pinch of salt. The graph's purpose is merely to reinforce how sensitive García-Pañalosa and Wen's conclusions are to the parameter η .

Figure 2: Equilibrium results for different values of the tax rate under equal redistribution policy

Figure 3: Difference between optimal tax welfare and first-best welfare for different values of leisure elasticity

5.4 No-growth trap

As discussed in section 3.6, this economy can converge to two different types of steady-state equilibria: One with all sectors coexisting and positive growth, and one with no research and hence no growth, which we call the no-growth trap. High levels of r and low levels of λ , γ , and H cause (29) to be satisfied, thus the economy can converge to the no-growth trap for all levels of tax rate. This is also true for Aghion and Howitt (1992) and García-Peñalosa and Wen (2008), however the addition of leisure elasticity makes it easier for an economy to be caught on the trap. For instance, using the parameters presented earlier, but with $\eta = 0$ (which is the case for both Aghion and Howitt 1992 and García-Peñalosa and Wen 2008), the minimum H for an economy to present positive research effort (for at least one level of taxation) is 556, while if $\eta = 0.05$, this value goes up to 1220. Alternative policies can, however, shift the economy to a steady-state equilibrium with positive growth. Next, we study two different policies to contrast with the one presented so far.

6 Growth-enhancing policies

6.1 Research subsidy

The first growth-enhancing policy we present is research subsidy. Under this structure, all agents still pay a tax rate τ on their income, but total tax revenue goes only to entrepreneurs and, in this sense, each researcher receives at t a transfer of $\frac{\tau Y_t}{R^*}$. Numerical analysis suggests that research subsidy is more efficient than equal redistribution to promote research effort and helps economies to escape from the no-growth trap, although is still insufficient when r is too high or λ , H or γ are too low. Also, high levels of taxation under this policy can put the economy back on the trap, once encourages entrepreneurs to increase their leisure. Figure 4 illustrate steady-state equilibrium under different tax rate values for research subsidy using our baseline parameter values (with $\eta = 0.05$). Since all transfers go to researchers, this policy stimulates skilled agents to choose entrepreneurship over manufacturing, thus, the number of researchers is an increasing function of the tax rate. However, for high levels of the tax rate the economy enters the no-growth trap as the leisure of researchers becomes one. In this scenario, therefore, most skilled agents choose to become an entrepreneur only to receive the transfer, but without really doing research. Welfare, then, falls as taxation increases once unskilled agents, which are the majority in this economy, pay for researchers not to work. As a consequence, inequality promptly grows and initial output falls as skilled workers shift from the productive to the research sector and agents' leisure increases.

6.2 Bonus for innovation

We now analyze an extreme innovation bonus payment policy. Under this framework, all agents pay a tax rate τ on their income and total tax revenue (τY_t at t) goes to the unit measure set of successful researchers. We have decided to compare three isolated policies in our study, however convex combinations of the mentioned policies could also be considered. Figure 5 summarizes the stationary equilibrium results for different tax

Figure 4: Equilibrium results for different values of the tax rate under research subsidy policy

rate values (again, with $\eta = 0.05$). Innovation bonus is even more efficient than research subsidy to promote research effort and is effective to take an economy out of the no-growth trap for any value of the parameters. Also, high tax rate levels do not jeopardize growth once the leisure of all agents under this policy is invariant to taxation. Innovation bonus is, then, the policy that promotes the greatest expected growth rate but is not the best policy to promote welfare gains (under baseline parameters). Since all of the government's transfers go to a unit measure set of individuals, welfare falls drastically as we increase the tax rate under research bonus policy. Inequality, then, increases with taxation and initial output decreases as skilled workers migrate from the productive sector to research, just like in research subsidy. Although for high levels of taxation innovation bonus promotes greater growth rates, initial outputs and less inequality than research subsidy policy, the later generates slightly less welfare losses. This happens because individuals are risk-averse, and so researchers' expected utility is greater on research subsidy policy, since it dilutes the risk between entrepreneurs.

Figure 5: Equilibrium results for different values of the tax rate under innovation bonus policy

	Opt. tax	R*	Leisure	Initial output	Growth	Gini	Welfare
H=500							
Equal redistribution	63%	0	100%	(7.9%)	0%	0.008	10%
Research subsidy	0%	0	100%	0,0%	0%	0.021	0%
Innovation bonus	0%	0	100%	0,0%	0%	0.021	0%
H=1500							
Equal redistribution	58%	208	10.6%	(9.2%)	0.5%	0.008	9%
Research subsidy	2%	663	7.0%	(12.2%)	1.7%	0.045	5%
Innovation bonus	0%	94	4.3%	0,0%	0.3%	0.288	0%
H=2500							
Equal redistribution	51%	939	10.0%	(7.7%)	2.2%	0.028	8%
Research subsidy	7%	1724	10.0%	(17.9%)	3.8%	0.067	13%
Innovation bonus	13%	1484	4.3%	(12.2%)	3.6%	0.127	3%
H=3500							
Equal redistribution	43%	1663	9.3%	(6.3%)	3.7%	0.056	7%
Research subsidy	13%	2776	12.4%	(22.6%)	5.5%	0.086	29%
Innovation bonus	39%	3092	4.3%	(32.9%)	6.5%	0.389	36%

Figure 6: Welfare-maximizing tax rate results for all policies under different values for the mass of skilled agents

7 Increasing the mass of skilled agents

The results presented so far suggest that equal redistribution is preferable to the other two policies in terms of welfare maximization, but this is not always true. The mass of skilled agents in a population (H) depends, among other factors, on the level of development and the size of the economy and, therefore, the preferred policy may not always be the same. Figure 6 summarizes the outcomes for each policy as we increase the mass of skilled workers. For each value of H , we present the welfare maximizing tax rate for each policy and its results. Once gain, welfare and initial output are defined as deviation rates from their laissez-faire values, leisure refers to entrepreneurs and we approximate the mass of researchers.

For low values of H , the economy enters the no-growth trap. Although the government could subsidize research or reward innovations in order to promote research effort and escape from the trap, this would only lead to welfare losses (which explains why welfare-maximizing tax rates are zero in lines 2 and 3). The optimal policy, in this case, would be to remain with zero growth, but adopting a policy of equal redistribution to

reduce income inequality. However, when the number of skilled individuals rises, things change. The impact of growth on welfare increases and promoting research effort becomes more attractive. For $H = 2500$, the optimal policy is to subsidize research, which is a more growth-oriented policy than equal redistribution. A bonus payment scheme is not the optimal policy here, because the greater growth wouldn't compensate for the negative effects of risk exposure (as discussed in section 6.2). For $H = 3500$, growth has an even greater impact on welfare so that innovation bonus turns out to be more efficient on generating welfare than the other two policies. The best policy to promote welfare gains, therefore, is contingent on the economy's mass of skilled agents.

Note that these results show that there is divergence. Underdeveloped economies, with a low mass of skilled individuals, would choose to remain trapped and redistribute income to reduce inequality and therefore would never catch up with developed economies, that would choose to subsidize research or reward innovations and, then, would present positive growth.

8 Concluding Remarks

We have studied the impacts of redistributive taxation in a Schumpeterian growth model with risk-averse agents and elastic labor supply, and compared the welfare effects of three different types of redistributive policies: equal redistribution, research subsidy and innovation bonus. With an equal redistribution policy, increasing taxation in relation to laissez-faire provides insurance to entrepreneurs who engage in risky income-generating activities, promoting a welfare rise. For high levels of the tax rate, however, the negative effects of taxation on research (namely the incentive effect and the leisure effect, both discouraging skilled agents to engage in research) are dominant, and excessive tax rates can generate zero research effort.

It is possible for an economy to have zero research effort and zero growth for an infinite period, which is a no-growth trap. Growth-enhancing policies such as research subsidy or a bonus payment for innovation can be used to help economies stimulate research activity and escape the no-growth trap.

However, for economies with a low mass of skilled individuals, equal redistribution is more efficient in terms of welfare generation than the other two policies, even for economies in the no-growth trap. That means that the government's optimal move, for a trapped economy, is to remain with no growth but redistributing income to reduce inequality and preserve initial output. For economies with a greater mass of skilled agents, research subsidy and innovation bonus are preferable, since growth has a greater impact on welfare. If we were to consider a structure with the possibility of human capital accumulation, economies in the no-growth trap seeking welfare maximization would have to first invest in the qualification of its agents, and once these economies increased their mass of skilled individuals, then subsidizing research or rewarding innovations could be positive policies.

If we consider economies with a low mass of skilled individuals to be underdeveloped while economies with a high mass of skilled individuals to be developed, the results show divergence. Underdeveloped economies would invest in redistribution policies that reduce inequality but would not generate growth and, therefore, would never catch up with

developed economies, which would invest in research-generating and growth-enhancing policies and present positive growth.

We have abstracted from relevant factors in our model, that could be the object of future studies. The presented model does not control for scale effects, which could alter some of the findings. Here, research subsidy and innovation bonus are welfare-maximizing policies for economies with a large mass of skilled individuals, and not for economies with a high proportion of skilled individuals in the population. Aghion and Howitt (1998) present a method to control for this effect. Another possible addition to the model would be to incorporate temporary patent protection. This can be done by adding a term to the denominator of the value of an innovation expression that captures the expiration time of a patent. Finally, the model also abstracts from capital accumulation. One possible path for future studies could be to incorporate it in the model, since the exogeneity of " r " becomes an issue for García-Peñalosa and Wen (2008) model with risk aversion.

9 References

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10 Appendix

Before proceeding with the proofs of the propositions in the main text, for the reader's convenience, we collect below the assumptions on the parameters of the model:

- $A_0 > 0$;
- $\frac{L}{H} > \frac{1-\theta}{\theta^2}$;
- $\eta(1-\sigma) \leq 1$;
- $\sigma \leq 1$.

Let us recall that, in equilibrium, it must be the case that $Y > 0$ (for instance, from equation (8)). This implies that there will be no equilibrium with $\tau = 1$ – in such a case, as explained in Section 3.3, both manufacturers and unskilled agents would choose not to work at all, hence (2) would yield $Y = 0$.

Yet another preliminary task I ought to complete prior to tackling the propositions is to establish the validity of the following claim from Section 2.6: condition $L/H > (1-\theta)/\theta^2$ ensures that skilled individuals reach a greater utility level than unskilled ones.

Proof. First, define the function $f(G) = ((1-\tau)G + \frac{\tau}{N}) \left(\frac{\eta((1-\tau)G + \frac{\tau}{N})}{(\eta+1)(1-\tau)G + \eta\frac{\tau}{N}} \right)^\eta$, with $\frac{df}{dG} = \left(\frac{\eta(1-\tau)G + \eta\frac{\tau}{N}}{(\eta+1)(1-\tau)G + \eta\frac{\tau}{N}} \right)^{\eta-1} \left(\frac{\eta(1-\tau)^2G + \eta\frac{\tau}{N}(1-\tau)}{(\eta+1)(1-\tau)G + \eta\frac{\tau}{N}} \left(1 - \frac{\eta\frac{\tau}{N}}{(\eta+1)(1-\tau)G + \eta\frac{\tau}{N}} \right) \right)$, which is positive for $\eta > 0$ and $G \neq 0$, hence $f(G)$ is strictly increasing.

$U(C_{m,t}^*, l_m^*) > U(C_{u,t}^*, l_u^*)$ guarantees that skilled individuals have greater utility levels than unskilled ones, and, using (26) and (25), can be expressed as

$$\left(\frac{(1-\tau)\theta^2}{H} + \frac{\tau}{N} \right) \left(\frac{\eta((1-\tau)\theta^2 + \frac{H\tau}{N})}{(\eta+1)(1-\tau)\theta^2 + \eta H \frac{\tau}{N}} \right)^\eta > \left(\frac{(1-\tau)(1-\theta)}{L} + \frac{\tau}{N} \right) \left(\frac{\eta((1-\tau)(1-\theta) + \frac{L\tau}{N})}{(\eta+1)(1-\tau)(1-\theta) + \eta L \frac{\tau}{N}} \right)^\eta.$$

The left-hand side of the inequality can be rewritten as $\left(\frac{(1-\tau)\theta^2}{H} + \frac{\tau}{N} \right) \left(\frac{\eta((1-\tau)\frac{\theta^2}{H} + \frac{\tau}{N})}{(\eta+1)(1-\tau)\frac{\theta^2}{H} + \eta\frac{\tau}{N}} \right)^\eta$, which equals $f\left(\frac{\theta^2}{H}\right)$, and the right-hand side as $\left(\frac{(1-\tau)(1-\theta)}{L} + \frac{\tau}{N} \right) \left(\frac{\eta((1-\tau)\frac{(1-\theta)}{L} + \frac{\tau}{N})}{(\eta+1)(1-\tau)\frac{(1-\theta)}{L} + \eta\frac{\tau}{N}} \right)^\eta$, which equals $f\left(\frac{(1-\theta)}{L}\right)$. Therefore, $\frac{\theta^2}{H} > \frac{(1-\theta)}{L}$, which is equivalent to $\frac{L}{H} > \frac{(1-\theta)}{\theta^2}$, ensures that $U(C_{m,t}^*, l_m^*) > U(C_{u,t}^*, l_u^*)$. ■

Finally, for the propositions.

Lemma 1. *In a stationary equilibrium, $R^* < H$.*

Proof. Since $Y_t > 0$, by expression (2), $x_t > 0$, and by equilibrium condition (xi), $R^* < H$. ■

Proposition 2. *In a stationary equilibrium,*

(i) *if condition (28) holds, $l_r^* = 1$ and the economy is in the no-growth trap. If condition (28) does not hold, $0 \leq l_r^* < 1$.*

(ii) *if condition (29) holds, $R^* = 0$ and the economy is in the no-growth trap.*

Proof. (i) Let us first elaborate on the researchers' utility maximization problem described in section 2.6:

Researchers choose, at t , $l_{r,t}$ in order to maximize their expected utility:

$$E(U_{r,t}) = \lambda(1 - l_{r,t}) \frac{(C_{1,t} l_{r,t}^\eta)^{1-\sigma}}{1-\sigma} + (1 - \lambda(1 - l_{r,t})) \frac{(B_t l_{r,t}^\eta)^{1-\sigma}}{1-\sigma}$$

given $C_{1,t}$ and B_t , and $C_{1,t} > B_t$.

Differentiating the expression above with respect to $l_{r,t}$, we have

$$\frac{\partial E(U_{r,t})}{\partial l_{r,t}} = \lambda \frac{(B_t^{1-\sigma} - C_{1,t}^{1-\sigma}) l_{r,t}^{\eta(1-\sigma)}}{1-\sigma} + \eta \left((1 - \lambda(1 - l_{r,t})) B_t^{1-\sigma} + \lambda(1 - l_{r,t}) C_{1,t}^{1-\sigma} \right) l_{r,t}^{\eta(1-\sigma)-1}$$

which, in turn, has the same sign as

$$\lambda \frac{(B_t^{1-\sigma} - C_{1,t}^{1-\sigma}) l_{r,t}}{1-\sigma} + \eta \left((1 - \lambda(1 - l_{r,t})) B_t^{1-\sigma} + \lambda(1 - l_{r,t}) C_{1,t}^{1-\sigma} \right)$$

Note that the function above is linear in $l_{r,t}$, and let us call it $G(l_{r,t})$.

When $l_{r,t} = 0$, $G(0) = \eta(B_t^{1-\sigma} + \lambda(C_{1,t} - B_t)^{1-\sigma})$, which is positive for $\eta > 0$. Therefore, the derivative at this point is positive and $l_r^* = 0$ is never an optimal solution.

When $l_{r,t} = 1$, $G(1) = \lambda \frac{(B_t^{1-\sigma} - C_{1,t}^{1-\sigma})}{1-\sigma} + \eta B_t^{1-\sigma}$. Therefore, if $G(1) \geq 0$, then the researcher will choose $l_{r,t}^* = 1$. However, if $G(1) < 0$, the researcher will choose $l_{r,t}^* \in (0, 1)$

so that $G(l_{r,t}^*) = 0$, that is:

$$\lambda \frac{(B_t^{1-\sigma} - C_{1,t}^{1-\sigma}) l_{r,t}}{1 - \sigma} + \eta ((1 - \lambda(1 - l_{r,t})) B_t^{1-\sigma} + \lambda(1 - l_{r,t}) C_{1,t}^{1-\sigma}) = 0$$

and hence,

$$l_{r,t}^* = \frac{\eta(1 - \sigma)}{\eta(1 - \sigma) + 1} \left(1 + \frac{1}{\lambda} \frac{B_t^{1-\sigma}}{C_{1,t}^{1-\sigma} - B_t^{1-\sigma}} \right) \quad (\text{A.0})$$

Now, from (4), (13) and (10), we have that in a stationary equilibrium, $C_{1,t} = \left(\frac{(1-\theta)\theta\gamma Y_t}{r + \lambda(1-l_r^*)R^*} (1 - \tau) + \frac{\tau Y_t}{N} \right)$ and $B_t = \frac{\tau Y_t}{N}$. Substituting these expressions in function G we have that, in a stationary equilibrium:

$$G(1) = \frac{(\lambda + \eta(1 - \sigma)) \left(\frac{\tau Y_t}{N} \right)^{1-\sigma} - \lambda \left(\frac{(1-\theta)\theta\gamma Y_t}{r} (1 - \tau) + \frac{\tau Y_t}{N} \right)^{1-\sigma}}{1 - \sigma}$$

Since the left-hand side of (28) is simply $G(1)$ multiplied by $\left(\frac{Y_t}{N} \right)$, which is positive, one may conclude that if (28) holds, then $G(1) \geq 0$ and $l_r^* = 1$.

Similarly, if (28) does not hold, $G(1) < 0$ and $0 \leq l_r^* < 1$.

(ii) Suppose $R^* > 0$. By equilibrium condition (vi), $Z_r(V_{t+1}, \tau, B_t) \geq Z_m(w_t, \tau, B_t)$, which means that in a stationary equilibrium, $E(U(C_{r,t}^*, l_r^*)) \geq U(C_{m,t}^*, l_m^*)$.

First, let us check that if $E(U(C_{r,t}^*, l_r^*)) \geq U(C_{m,t}^*, l_m^*)$, for some $R^* > 0$, then $E(U(C_{r,t}^*, l_r^*))|_{R^*=0} > U(C_{m,t}^*, l_m^*)|_{R^*=0}$.

Define

$$\omega(U(C_t, l_t)) = U(C_t, l_t) \times \frac{1 - \sigma}{Y_t^{1-\sigma}} \quad (\text{A.1})$$

To simplify calculations, we will prove that if $\omega(E(U(C_{r,t}^*, l_r^*))) \geq \omega(U(C_{m,t}^*, l_m^*))$ for some $R^* > 0$, then $\omega(E(U(C_{r,t}^*, l_r^*)))|_{R^*=0} > \omega(U(C_{m,t}^*, l_m^*))|_{R^*=0}$, which is equivalent to the thesis above.

We will start by proving that $\omega(E(U(C_{r,t}^*, l_r^*)))|_{R^*=0} > \omega(E(U(C_{r,t}^*, l_r^*)))|_{R^*>0}$ for interior l_r^* and $\omega(E(U(C_{r,t}^*, l_r^*)))|_{R^*=0} = \omega(E(U(C_{r,t}^*, l_r^*)))|_{R^*>0}$ for $l_r^* = 1$, and then we will show that $\omega(U(C_{m,t}^*, l_m^*))|_{R^*>0} > \omega(U(C_{m,t}^*, l_m^*))|_{R^*=0}$.

It should be noted that, using (A.0), (4), (13) and (10), in a stationary equilibrium, an interior l_r^* is given by:

$$l_r^* = \frac{\eta(1-\sigma)}{\eta(1-\sigma)+1} \left(1 + \frac{1}{\lambda} \frac{1}{\left(N \frac{(1-\theta)\theta\gamma}{r+\lambda(1-l_r^*)R^*} \frac{1-\tau}{\tau} + 1 \right)^{1-\sigma} - 1} \right) \quad (\text{A.2})$$

Also, for future reference, we can take expression (A.2), and isolate R^* to obtain

$$R^*(l_r^*, \tau) = \frac{N\theta(1-\theta)\gamma\left(\frac{1}{\tau}-1\right)}{\lambda(1-l_r^*)\left(\lambda^{-1}D+1\right)^{\frac{1}{1-\sigma}} - \lambda(1-l_r^*)} - \frac{r}{\lambda(1-l_r^*)} \quad (\text{A.3})$$

where $D = \left(\left(1 + \frac{1}{\eta(1-\sigma)} \right) l_r^* - 1 \right)^{-1}$.

Still from (A.2), note that the following is true about an expression that, as shown in (27), is relevant to determine the researchers' utility in equilibrium:

$$\left(\frac{N\gamma\theta(1-\theta)}{r+\lambda(1-l_r^*)R^*} \frac{1-\tau}{\tau} + 1 \right)^{1-\sigma} = \frac{1}{\lambda \left(\frac{\eta(1-\sigma)+1}{\eta(1-\sigma)} l_r^* - 1 \right)} + 1 \quad (\text{A.4})$$

Now, from (A.1) and (27) we obtain

$$\begin{aligned} \omega(E(U(C_{r,t}^*, l_r^*))) &= \lambda(1-l_r^*) \left(\frac{\tau}{N} \right)^{1-\sigma} \left(\left(\frac{N\gamma\theta(1-\theta)}{r+\lambda(1-l_r^*)R^*} \frac{1-\tau}{\tau} + 1 \right) l_r^{*\eta} \right)^{1-\sigma} \\ &\quad + (1-\lambda(1-l_r^*)) \left(\frac{\tau}{N} l_r^{*\eta} \right)^{1-\sigma} \end{aligned}$$

which, using (A.4) and after some manipulations, can be rewritten as

$$\omega(E(U(C_{r,t}^*, l_r^*))) = \left(\frac{\tau}{N} l_r^{*\eta} \left(1 + \eta(1-\sigma) \left(1 - \frac{1}{l_r^*} \right) \right)^{\frac{1}{\sigma-1}} \right)^{1-\sigma} \quad (\text{A.5})$$

which for $\tau \neq 0$ has the same sign as

$$\Omega(E(U(C_{r,t}^*, l_r^*))) = l_r^{*\eta} \left(1 + \eta(1-\sigma) \left(1 - \frac{1}{l_r^*} \right) \right)^{\frac{1}{\sigma-1}}$$

Differentiating the expression above with respect to l_r^* gives us:

$$\begin{aligned}
\frac{\partial \Omega (E (U (C_{r,t}^*, l_r^*)))}{\partial l_r^*} &= \eta (l_r^*)^{\eta-1} \left(1 + \eta (1 - \sigma) \left(1 - \frac{1}{l_r^*} \right) \right)^{\frac{1}{\sigma-1}} + (l_r^*)^\eta \frac{1}{\sigma-1} \times \\
&\quad \left(1 + \eta (1 - \sigma) \left(1 - \frac{1}{l_r^*} \right) \right)^{\frac{1}{\sigma-1}-1} \frac{\eta (1 - \sigma)}{(l_r^*)^2} \\
&= (l_r^*)^{\eta-2} \left(1 + \eta (1 - \sigma) \left(1 - \frac{1}{l_r^*} \right) \right)^{\frac{1}{\sigma-1}-1} \times \\
&\quad \left[\eta l_r^* \left(1 + \eta (1 - \sigma) \left(1 - \frac{1}{l_r^*} \right) \right) - \eta \right] \\
&= (l_r^*)^{\eta-2} \left(1 + \eta (1 - \sigma) \left(1 - \frac{1}{l_r^*} \right) \right)^{\frac{1}{\sigma-1}-1} \eta [l_r^* + \eta (1 - \sigma) (l_r^* - 1) - 1] \\
&= (l_r^*)^{\eta-2} \left(1 + \eta (1 - \sigma) \left(1 - \frac{1}{l_r^*} \right) \right)^{\frac{1}{\sigma-1}-1} \eta (\eta (1 - \sigma) + 1) (l_r^* - 1)
\end{aligned}$$

that is,

$$\partial \Omega (E (U (C_{r,t}^*, l_r^*))) / \partial l_r^* < 0 \quad (\text{A.6})$$

From (A.2), it is straightforward that for an interior $l_r^*, l_r^*|_{R^*>0}$ is greater than $l_r^*|_{R^*=0}$, therefore from (A.6), $\omega (E (U (C_{r,t}^*, l_r^*)))|_{R^*=0} > \omega (E (U (C_{r,t}^*, l_r^*)))|_{R^*>0}$. Note that we ruled out $\tau = 0$ from this proof, however, from (A.0) and (4), one can see that for an interior solution $l_r^*|_{\tau=0} = \frac{\eta(1-\sigma)}{\eta(1-\sigma)+1}$ and, hence, researchers' leisure does not depend on R^* , therefore, from (27) and (A.1) $\omega (E (U (C_{r,t}^*, l_r^*)))|_{R^*=0} > \omega (E (U (C_{r,t}^*, l_r^*)))|_{R^*>0}$ holds.

For $l_r^* = 1$ it is straightforward from (27) and (A.1) that $\omega (E (U (C_{r,t}^*, l_r^*)))|_{R^*=0} = \omega (E (U (C_{r,t}^*, l_r^*)))|_{R^*>0}$.

Now, let us prove that $\omega (U (C_{m,t}^*, l_m^*))|_{R^*>0} > \omega (U (C_{m,t}^*, l_m^*))|_{R^*=0}$. Using (26), we can see that expression $\omega (U (C_{m,t}^*, l_m^*))$ in turn, behaves with respect to R^* the same as

$$\log \left(\frac{(1-\tau) \theta^2 Y_t}{H - R^*} + \frac{\tau Y_t}{N} \right) + \eta \log \left(1 - \frac{(1-\tau) \theta^2}{(\eta+1) (1-\tau) \theta^2 + \eta (H - R^*) \frac{\tau}{N}} \right)$$

Differentiating the expression above with respect to R^* gives us

$$\begin{aligned}
& \frac{1}{\frac{(1-\tau)\theta^2 Y_t}{H-R^*} + \frac{\tau Y_t}{N}} \frac{(1-\tau)\theta^2 Y_t}{(H-R^*)^2} - \eta \frac{(\eta+1)(1-\tau)\theta^2 + \eta(H-R^*)\frac{\tau}{N}}{\eta(1-\tau)\theta^2 + \eta(H-R^*)\frac{\tau}{N}} \times \\
& \frac{(1-\tau)\theta^2}{\left((\eta+1)(1-\tau)\theta^2 + \eta(H-R^*)\frac{\tau}{N}\right)^2} \eta \frac{\tau}{N} \\
= & \frac{1}{(1-\tau)\theta^2 + (H-R^*)\frac{\tau}{N}} \frac{(1-\tau)\theta^2}{H-R^*} - \frac{1}{(1-\tau)\theta^2 + (H-R^*)\frac{\tau}{N}} \times \\
& \frac{(1-\tau)\theta^2}{(\eta+1)(1-\tau)\theta^2 + \eta(H-R^*)\frac{\tau}{N}} \eta \frac{\tau}{N} \\
= & \frac{(1-\tau)\theta^2}{(1-\tau)\theta^2 + (H-R^*)\frac{\tau}{N}} \left(\frac{1}{H-R^*} - \frac{\eta\frac{\tau}{N}}{(\eta+1)(1-\tau)\theta^2 + \eta(H-R^*)\frac{\tau}{N}} \right) \\
= & \frac{(1-\tau)\theta^2}{(1-\tau)\theta^2 + (H-R^*)\frac{\tau}{N}} \left(\frac{(\eta+1)(1-\tau)\theta^2 + \eta(H-R^*)\frac{\tau}{N} - \eta(H-R^*)\frac{\tau}{N}}{(H-R^*)((\eta+1)(1-\tau)\theta^2 + \eta(H-R^*)\frac{\tau}{N})} \right) \\
= & \frac{(1-\tau)\theta^2}{(1-\tau)\theta^2 + (H-R^*)\frac{\tau}{N}} \frac{(\eta+1)(1-\tau)\theta^2}{(H-R^*)((\eta+1)(1-\tau)\theta^2 + \eta(H-R^*)\frac{\tau}{N})} \\
= & \frac{N^2\theta^4(1-\tau)^2(\eta+1)}{(H-R^*)((H-R^*)\tau + N\theta^2(1-\tau))((H-R^*)\tau\eta + N\theta^2(1-\tau)(1+\eta))},
\end{aligned}$$

which, making use of Lemma 1, is greater than zero. Therefore, $\omega(U(C_{m,t}^*, l_m^*))$ is strictly increasing in R^* and it is then proved that $E(U(C_{r,t}^*, l_r^*)) \geq U(C_{m,t}^*, l_m^*)$, for some $R^* > 0$ implies $E(U(C_{r,t}^*, l_r^*))|_{R^*=0} > U(C_{m,t}^*, l_m^*)|_{R^*=0}$.

Next, we prove that if $E(U(C_{r,t}^*, l_r^*))|_{R^*=0} > U(C_{m,t}^*, l_m^*)|_{R^*=0}$, then (29) does not hold, thus ending the proof of the proposition. We start by checking that l_r^* must be interior in this case.

Let us define $\chi := \frac{\tau H}{(1-\tau)N}$. Using the alternative Bernoulli inequality, we have that

$$\left(1 - \frac{\theta^2}{(\eta+1)\theta^2 + \eta\chi}\right)^{\eta+1} \geq 1 - \frac{(\eta+1)\theta^2}{(\eta+1)\theta^2 + \eta\chi}$$

always holds (note that $0 \leq \frac{\theta^2}{(\eta+1)\theta^2 + \eta\chi} \leq 1$). Since

$$1 - \frac{(\eta+1)\theta^2}{(\eta+1)\theta^2 + \eta\chi} = \left(1 - \frac{\theta^2}{\theta^2 + \chi}\right) \left(1 - \frac{\theta^2}{(\eta+1)\theta^2 + \eta\chi}\right)$$

then

$$\left(1 - \frac{\theta^2}{(\eta+1)\theta^2 + \eta\chi}\right)^{\eta+1} \geq \left(1 - \frac{\theta^2}{\theta^2 + \chi}\right) \left(1 - \frac{\theta^2}{(\eta+1)\theta^2 + \eta\chi}\right)$$

which is equivalent to

$$(\theta^2 + \chi) \left(1 - \frac{\theta^2}{(\eta+1)\theta^2 + \eta\chi}\right)^\eta - \chi \geq 0$$

and substituting χ and multiplying by $\frac{(1-\tau)}{H}$,

$$\left(\frac{(1-\tau)\theta^2}{H} + \frac{\tau}{N}\right) \left(1 - \frac{(1-\tau)\theta^2}{(\eta+1)(1-\tau)\theta^2 + \eta H \frac{\tau}{N}}\right)^\eta - \frac{\tau}{N} \geq 0$$

which, in turn, by expressions (26) and (27) is equivalent to $U(C_{m,t}^*, l_m^*)|_{R^*=0} \geq E(U(C_{r,t}^*, l_r^*))|_{R^*=0}$ for $l_r^* = 1$. Having established the interiority of l_r^* , let us now prove that if $E(U(C_{r,t}^*, l_r^*))|_{R^*=0} > U(C_{m,t}^*, l_m^*)|_{R^*=0}$, then (29) does not hold.

l_r^* comes from an interior solution, hence, from (A.2) :

$$l_r^*|_{R^*=0} = \frac{\eta(1-\sigma)}{\eta(1-\sigma) + 1} \left(1 + \frac{1}{\lambda} \frac{1}{\left(N^{\frac{(1-\theta)\theta\gamma}{r} \frac{1-\tau}{\tau}} + 1\right)^{1-\sigma}} - 1\right) \equiv l_r^0 \quad (\text{A.7})$$

Since $E(U(C_{r,t}^*, l_r^*))|_{R^*=0} > U(C_{m,t}^*, l_m^*)|_{R^*=0}$, then by expressions (26) and (27) :

$$\begin{aligned} & \lambda(1 - l_r^0) \left(\left(\frac{(1-\tau)\theta(1-\theta)\gamma Y_t}{r} + \frac{\tau Y_t}{N} \right) l_r^{0\eta} \right)^{1-\sigma} + (1 - \lambda(1 - l_r^0)) \left(\frac{\tau Y_t}{N} l_r^0 \right)^{1-\sigma} \\ & > \left(\left(\frac{(1-\tau)\theta^2 Y_t}{H} + \frac{\tau Y_t}{N} \right) \left(\frac{\eta[(1-\tau)\theta^2 + \frac{H\tau}{N}]}{(\eta+1)(1-\tau)\theta^2 + \frac{\eta H \tau}{N}} \right)^\eta \right)^{1-\sigma} \end{aligned}$$

which is equivalent to

$$\frac{\tau}{N} l_r^{0\eta} \left(1 + \eta(1-\sigma) \left(1 - \frac{1}{l_r^0}\right)\right)^{\frac{1}{\sigma-1}} > \left(\frac{(1-\tau)\theta^2}{H} + \frac{\tau}{N}\right) \left(1 - \frac{(1-\tau)\theta^2}{(\eta+1)(1-\tau)\theta^2 + \eta H \frac{\tau}{N}}\right)^\eta$$

and using expression (A.7), we get

$$\frac{\tau}{N} \left(\frac{\lambda(P-1)+1}{1+\eta(1-\sigma)} \right) \left(\frac{\eta(1-\sigma)}{\lambda(P-1)} \right)^\eta > \left(\frac{(1-\tau)\theta^2}{H} + \frac{\tau}{N} \right) \left(1 - \frac{(1-\tau)\theta^2}{(\eta+1)(1-\tau)\theta^2 + \eta H \frac{\tau}{N}} \right)^\eta$$

where $P = (1 + \frac{1}{r\tau} N\theta\gamma(1-\theta)(1-\tau))^{1-\sigma}$. That is, (29) does not hold. ■

Proposition 3. *In a stationary equilibrium,*

(i) *there is $\bar{\tau} < 1$ such that, if $\tau \geq \bar{\tau}$, then $l_r^* = 1$ (and research effort is zero).*

(ii) *if $R^* > 0$ and condition (28) does not hold, entrepreneurs' leisure l_r^* is strictly increasing in τ .*

Proof. (i) First note that the expression in the left-hand side of (28) evaluated at $\tau = 1$ gives η , which is nonnegative. Thus, if we are able to show that that expression is increasing in τ , a simple appeal to Proposition 2(i) would settle the case.

By looking at the expression, we see that it is enough to show that, for all $\tau \in (0, 1)$,

$$\frac{\partial}{\partial \tau} \left(\frac{\tau^{1-\sigma} - \left(\frac{N}{r} (1-\theta)\theta\gamma(1-\tau) + \tau \right)^{1-\sigma}}{1-\sigma} \right) > 0,$$

that is,

$$\tau^{-\sigma} - \left(\frac{N}{r} (1-\theta)\theta\gamma(1-\tau) + \tau \right)^{-\sigma} \left(-\frac{N}{r} (1-\theta)\theta\gamma + 1 \right) > 0,$$

or still,

$$I(\tau) := 1 + \frac{\frac{N}{r} (1-\theta)\theta\gamma - 1}{\left(1 + \frac{N}{r} (1-\theta)\theta\gamma \frac{1-\tau}{\tau} \right)^\sigma} > 0.$$

If it happens to be the (unlikely) case that the numerator $N/r(1-\theta)\theta\gamma - 1$ is negative, since the denominator is positive and strictly decreasing in τ , $I(\tau)$ will be strictly decreasing in τ , whence $I(\tau) > I(1) = N/r(1-\theta)\theta\gamma \geq 0$.

However, if the numerator appearing in the definition of I is nonnegative, then an application of Bernoulli's inequality to the denominator (whose exponent σ , one should

note, is less than or equal to 1) would lead to

$$I(\tau) \geq 1 + \frac{\frac{N}{r}(1-\theta)\theta\gamma - 1}{1 + \sigma\frac{N}{r}(1-\theta)\theta\gamma\frac{1-\tau}{\tau}} = \frac{N}{r}(1-\theta)\theta\gamma\left(\sigma\frac{1-\tau}{\tau} + 1\right) > 0.$$

Thus, in either case, we indeed have $I(\tau) > 0, \forall \tau \in (0, 1)$.

(ii) In a stationary equilibrium with $R^* > 0$, by Lemma 1 and equilibrium conditions (vi) and (vii), $E(U(C_{r,t}^*, l_r^*)) = U(C_{m,t}^*, l_m^*)$ must hold.

Use $R^*(l_r^*, \tau)$ from (A.3) and expressions (A.5) and (26) to rewrite $E(U(C_{r,t}^*, l_r^*)) = U(C_{m,t}^*, l_m^*)$ as:

$$l_r^{*\eta} \left(1 + \eta(1-\sigma) \left(1 - \frac{1}{l_r^*}\right)\right)^{\frac{1}{\sigma-1}} = (K(l_r^*, \tau) + 1) \left(1 - \frac{1}{\eta \left(\frac{1}{K(l_r^*, \tau)} + 1\right) + 1}\right)^\eta \quad (\text{A.8})$$

where $K(l_r^*, \tau) = \frac{N\theta^2(\frac{1}{\tau}-1)}{H-R^*(l_r^*, \tau)}$.

In order to show that $\frac{dl_r^*}{d\tau} > 0$, let us differentiate both sides of (A.8) with respect to τ . First, differentiate the left-hand side of the equation to get $\frac{dLH}{d\tau} = \frac{\partial LH}{\partial \tau} + \frac{\partial LH}{\partial l_r^*} \frac{dl_r^*}{d\tau} = \frac{\partial LH}{\partial l_r^*} \frac{dl_r^*}{d\tau}$. From (A.6), we know that $\frac{\partial LH}{\partial l_r^*} < 0$.

Differentiating the right-hand side of (A.8), we get:

$$\begin{aligned} \frac{dRH}{d\tau} &= \frac{dK}{d\tau} \left(1 - \frac{1}{\eta \left(\frac{1}{K} + 1\right) + 1}\right)^\eta - (K+1) \eta \left(1 - \frac{1}{\eta \left(\frac{1}{K} + 1\right) + 1}\right)^{\eta-1} \times \\ &\quad \frac{1}{\left(\eta \left(\frac{1}{K} + 1\right) + 1\right)^2} \eta \frac{1}{K^2} \frac{dK}{d\tau} \\ &= \left(1 - \frac{1}{\frac{(\eta+1)K+\eta}{K}} - \eta^2 (K+1) \frac{1}{\left(\frac{(\eta+1)K+\eta}{K}\right)^2} \frac{1}{K^2}\right) \left(1 - \frac{1}{\frac{(\eta+1)K+\eta}{K}}\right)^{\eta-1} \frac{dK}{d\tau} \\ &= \left(\frac{\eta(K+1)}{(\eta+1)K+\eta} - \frac{\eta^2(K+1)}{((\eta+1)K+\eta)^2}\right) \left(\frac{\eta(K+1)}{(\eta+1)K+\eta}\right)^{\eta-1} \frac{dK}{d\tau} \\ &= \left(1 - \frac{\eta}{(\eta+1)K+\eta}\right) \left(\frac{\eta(K+1)}{(\eta+1)K+\eta}\right)^\eta \frac{dK}{d\tau} \\ &= \frac{(\eta+1)K}{(\eta+1)K+\eta} \left(\frac{\eta(K+1)}{(\eta+1)K+\eta}\right)^\eta \frac{dK}{d\tau}. \end{aligned}$$

Now, differentiating (A.3) with respect to τ and l_r^* , we have

$$R_\tau = -\frac{r + \lambda(1 - l_r^*) R}{\lambda(1 - l_r^*) \tau(1 - \tau)} \quad (\text{A.9})$$

$$R_{l_r^*} = \frac{(r + \lambda(1 - l) R)}{(1 - \sigma) \lambda(1 - l)} \frac{F + 1}{l_r^* (\lambda F^2 + F)} \left(1 + \frac{(r + \lambda(1 - l) R)}{N\theta(1 - \theta) \gamma \left(\frac{1}{\tau} - 1 \right)} \right) + \frac{R}{(1 - l)} \quad (\text{A.10})$$

where $F = \left(1 + \frac{1}{\eta(1 - \sigma)} \right) l_r^* - 1$, and, therefore, $R_\tau < 0$ and $R_{l_r^*} > 0$.

Writing K_τ and K_l for the partial derivatives of K with respect to τ and l_r^* respectively, we have $dK/d\tau = K_\tau + K_l dl_r^*/d\tau$, where, given expressions (A.9) and (A.10)

$$K_\tau = \frac{-N\theta^2}{(H - R^*) \tau^2} \left(\frac{r + \lambda(1 - l_r^*) R^*}{\lambda(1 - l_r^*) (H - R^*)} + 1 \right)$$

which is negative, and

$$K_l = \frac{N\theta^2 \left(\frac{1}{\tau} - 1 \right)}{(H - R^*)^2} \left(\frac{(r + \lambda(1 - l_r^*) R^*)}{(1 - \sigma) \lambda(1 - l_r^*)} \frac{\left(1 + \frac{1}{\eta(1 - \sigma)} \right)}{\lambda F^2 + F} \left(1 + \frac{(r + \lambda(1 - l_r^*) R^*)}{N\theta(1 - \theta) \gamma \left(\frac{1}{\tau} - 1 \right)} \right) + \frac{R^*}{(1 - l_r^*)} \right)$$

which is positive.

Now, use $\frac{dLH}{d\tau} = \frac{dRH}{d\tau}$ to obtain

$$\frac{dl_r^*}{d\tau} = \frac{K_\tau \eta K (\eta + 1) (K + 1) \left(\frac{\eta(K+1)}{\eta + K(\eta+1)} \right)^{\eta-1}}{\left(\frac{\partial LH}{\partial l_r^*} - \frac{K_l \eta K (\eta+1) (K+1)}{(\eta + K(\eta+1))^2} \left(\frac{\eta(K+1)}{\eta + K(\eta+1)} \right)^{\eta-1} \right) (\eta + K(\eta + 1))^2}$$

which is positive. ■